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OPTIMAL DRILLING PARAMETERS VIA GEOMETRIC PROGRAMMING

by

Jeffrey S. Daniels

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A Thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science in Mineral Economics.

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ABSTRACT

This thesis develops a general model for oil well drilling at minimum cost. The state-of-the-art in drilling operations is reviewed and the need for efficiency is discussed. A model is developed using the Galle and Woods equations, and the assumptions for the development are presented. Some approximations are introduced, verified, and the model is altered to reflect the approximations. Geometric Programming and the theory of condensation are briefly introduced and applied to the model. A computer program implementing Geometric Programming is discussed and the results of examples are interpretted with regard to accuracy and significance. The contribution this paper has made in the area of optimal drilling is presented and, finally, areas of future research are discussed.

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TABLE OF CONTENTS

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

A(f)	Formation abrasiveness parameter
B(x)	Fraction of bearing life expended
C(f)	Formation drillability parameter
D	Bit tooth dullness, fraction worn away
D(f)	Final bit tooth dullness
F	Distance drilled by bit, Feet
F(f)	Final distance drilled by bit
L	Tabulated function of $\bar{W}$
N	Rotary speed, R.P.M.
S	Drilling fluid parameter
T(f)	Final rotating time, Hours
W	Bit weight, thousands of pounds
$\bar{W}$	Equivalent bit weight on 7-7/8 inch bit
a	$0.928125D^2 + 6.0D + 1$, effect of tooth height on tooth dulling rate
i	$N + 4.348\times10^{-5}(N^3)$, effect of rotary speed on tooth dulling rate
m	$1359.1 - 714.19(\mathrm{LOG10}(\bar{W}))$, effect of bit weight on tooth dulling rate
k	Exponent on weight in drilling rate equation
p	Exponent on "a" in drilling rate equation

ACKNOWLEDGMENTS

The author wishes to express his gratitude to his thesis advisor, Dr. R. E. D. Woolsey, to his Department Chairman, Dr. Jean P. Mather, and to Dr. Billy J. Mitchell for serving on his committee and providing direction in research.

The author is grateful for the financial assistance provided during his studies by the Colorado School of Mines in the form of a Teaching Assistant position and an H. E. W. Fellowship.

Introduction

Modern oil well drilling involves outlays of capital at virtually each step in the operation. Sophisticated, off-shore deep-water drilling platforms can exceed $100,000 in cost per day, whether the rig is drilling or not. Drilling targets are rapidly increasing in depth, so that the cost contribution of the actual drilling is steadily rising. Efficiency has become a byword again rather than a secondary objective.

This paper is a study of optimal drilling technique via a mathematical model. The parameters that a drilling engineer controls are used to develop a computer program that calculates the optimum values of the parameters that yield the minimum cost per foot.

One purpose of this paper is to explore the application of Geometric Programming to a model originally devel- in 1960 by Galle and Woods of Hughes Tool Company. The resulting model will then be tested against problems from the literature and from the field. Further, a more field oriented program will be developed for application on operating drilling platforms.

This paper is intended for any reader interested in the latest mathematical technique in optimization and its application to a laboratory developed and field tested model of optimal drilling. The rigor employed to develop the model is sufficient to challenge the reader without becoming incomprehensible to the non-mathematician.

To introduce the reader to a general idea of the various operations involved in drilling, a brief description of drilling will be given within this Introduction. Then, the intuitive model described in the Introduction will be modified with the equations developed by Galle and Woods. Substitutions shall be examined and employed in the model. Following the above, the optimization technique, Geometric Programming, will be briefly introduced, as well as the condensation technique that is applied to the model. A computer program shall then be discussed, that employs the fully developed model, as to its speed, accuracy, and applicability to current drilling procedure. A short cost analysis feature of Geometric Programming will be included. Finally, the results and contributions of this paper will be considered, and areas of further research and development shall be presented.

To introduce the reader to drilling, it is assumed the problems of optimal rig placement(in the case of multiple well completions), geological and geophysical data interpretation, and the availability of personnel

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and equipment requirements have been dealt with. The drilling engineer is faced with the following problem. Suppose a drilling target is at a depth, call it D(t). What would be the minimum cost associated with drilling to the target depth? Cost is generated by segments of drilling activity followed by intervals of bit retrieving and changing operations - commonly called a "round trip". The engineer can minimize total cost per foot drilled by minimizing the cost of each bit run, which is composed of the cost of the bit itself, the cost of the round trip for that bit, and the cost of the drilling itself. A measure of efficiency would then be the cost of each bit run divided by the footage attained during the bit run. The above is an extremely simplified description of the problem.

Drilling operations are still an art in themselves. Every well drilled will be unique, even in the same geological area. Under the worst of circumstances, a model of the type to be developed in this paper will present the drilling engineer with a guideline that can be modified to suit the unexpected circumstances. Under the best of conditions, the model will give the engineer the best possible drilling procedure.

Chapter Two

The method that will be used to develop a drilling program using constant bit weight and rotary speed will given in this section. This will be followed by the assumptions used in the model, the development of the model and some techniques previously used to solve the constant bit weight and rotary speed problem. Then, the procedure used in this paper to solve the problem shall be discussed and the model will be further refined.

Thick, homogeneous geological formations are encountered frequently in oil well drilling. These formations require numerous drill bits to penetrate. The problem facing the drilling engineer on an oil rig is to determine the drilling parameters that he controls so as to minimize the cost per foot drilled per bit. The problem can be approached by taking information from the initial bit's drilling run in the formation. From this information, the optimum or best drilling weight, rotary speed, and final bit dullness may be computed for the next bit run via a model. This process can be repeated through the formation, thus minimizing total cost. It is seen that the current drill bit is providing new information for the next bit run, which is desirable since geological formations are rarely "homogeneous" in the strict sense of the word. The above synopsis defines in general the problem and the method that will be developed to provide the optimum parameters for minimum cost drilling.

Several assumptions must be made in order to develop the model that shall be utilized:

1. Proper attention has been given to bit selection, drilling fluid and the hydraulics of the cleaning action and lubrication of the bit.(1)
2. Maximum weight on the drill bit will not be dictated by the upper path of the drill hole.(2)
3. Rotary speed may be varied over a wide range without adversely affecting the hydraulics of the cleaning action and lubrication of the bit.(3)
4. A similar drilling fluid will be used for the next bit.
5. The previous bit run was operated at constant bit weight and rotary speed during the drilling.

The above assumptions represent the minimum criteria that must be met under the varied conditions that are encountered in drilling operations to use the following model.

The parameters that a drilling engineer can control are:

1. Weight on the bit, symbolized as W, in thousands of pounds.

2. Rotary speed of the bit, symbolized as N, in R.P.M.

3. Dullness of the bit, symbolized as D, which is dimensionless.

These three parameters shall be the independent variables of the model to follow. The model is a mathematical description that expresses total cost per foot drilled per bit run as a function of the independent variables for a given drilling fluid and formation.

The form of the model that will be developed is:

(1) PHI= (C(B) + C(T) C(R))/F, where

PHI= Operational variable (Dollars per foot per bit)

C(B)= Rental or purchase price of the bit (Dollars)

C(T)= Cost of a round trip of the bit (Dollars)

C(R)= Cost of rotating(drilling) the bit (Dollars)

F= Total footage drilled by the bit (Feet per bit)

The rental or purchase price of each bit is known explicitly. The cost of a round trip for the bit is calculated according to the formula:

(2) C(T)= (0.6)DEPTH(CRIG/24), where

(0.6)= Conversion factor (Hours per thousand feet)

DEPTH= Current depth of the hole (Thousands of feet)

CRIG= Daily operating cost of the rig (Dollars per day)

(CRIG/24)= Hourly cost of the rig (Dollars per hour)

Cost items such as labor, depreciation, fuel cost, drilling fluid cost and so on, are included in the CRIG term. The cost of rotating the bit is expressed as:

(3) $C(R) = (CRIG/24)T(f)$, where

$T(f) =$ Final or total rotating time of the bit (Hours)

The equations that allow expressions for T(f) and F to be derived shall be introduced next.

The following equations(4) were developed by Galle and Woods:

(4) Drilling rate equation: $dF/dt = (C(f)\bar{W}^{k} r)/a$

(5) Dulling rate equation: $dD/dt = (1/A(f))(i/am)$

(6) Bearing life equation: $dB(x)/dt = N/SL$

The interested reader is directed to the List of Symbols, which gives complete definitions for the notation used in Equations (4), (5) and (6). The above differential expressions were developed through laboratory work and modified to conform with field application and experience.(5)

An expression for final rotating time T(f) can be derived from the reciprocal of Equation (5):

(7) $dt/dD = A(f)(am/i)$

Multiply both sides of the equality by the differential dD.

(8) $dt = A(f)(am/i)dD$

Integrate the left hand side (LHS) to T(f) and integrate the right hand side (RHS) to final dullness.

(9) $dt = A(f)(m/i)\int_{0}^{D(f)} a \, dD$ or,

(10) $T(f) = A(f)(m/i)\int_0^{D(f)} a\, dD$

An expression for total footage F can be derived from the product of Equation (4) and the reciprocal of Equation (5):

(11) $(dF/dt)(dt/dD) = (A(f)C(f)\overline{W}^k rma^{1-p})/i$

Simplifying the LHS yields:

(12) $dF/dD = (A(f)C(f)\overline{W}^k rma^{1-p})/i$

Multiply both sides of Equation (12) by the differential dD. Then, integrate the LHS to final footage F and integrate the RHS to final dullness D(f):

(13) $F(f) = A(f)C(f)\overline{W}^k r(m/i)\int_0^{D(f)} a^{1-p}\, dD$

With the above expressions for T(f) and F, Equation (1) can be expressed as:

(14) $$PHI = \frac{C(B) + C(T) + (CRIG/24)A(f)(m/i)\int_0^{D(f)} a\, dD}{A(f)C(f)\overline{W}^k r(m/i)\int_0^{D(f)} a^{1-p}\, dD}$$

Equation (14) is now in terms of the three independent variables of weight, rotary speed and dullness of the bit.

One classical technique for finding the minimum of the cost function represented by Equation (14) would be to apply partial differentiation with respect to each of the independent variables. The resulting set of equations would be set equal to zero and then solved to determine the values of the independent variables at the minima.

Another technique to find the minima of Equation (14) would be to use the Calculus of Variations. Galle and Woods used this technique in their presentations of the variable weight and rotary speed model(6) and the constant bit weight and rotary speed model.(7) In both articles graphs were used to summarize several types of typical formation drilling situations. The reader matched the appropriate graph to the current drilling situation and read the optimal values of bit weight, rotary speed, final dullness and rotating time from the graph. The procedures to determine the optimal drilling program in both papers by Galle and Woods are complex and require a high order of sophistication by the user. In fact, one the reasons for this thesis is to provide a more intuitive method for field use.

A relatively recent and powerful development in mathematical programming allows a straight forward algorithm to be developed to determine the minima of Equation (14) and the associated values of the independent variables. The technique, Geometric Programming, will be discussed in general and applied to the results of the current chapter in Chapter Three. The remainder of this chapter will be devoted to the further refinement of Equation (14) so that an algorithm may be devised that will yield an optimal drilling program. Several key substitutions will be presented, the model will be modified accordingly and a general description of the algorithm will be given that will be developed fully in Chapter Three.

In normal drilling practice, the normalized weight $\overline{W}$ on a drill bit lies between twenty and eighty thousand pounds.(8) Further, the rotary speed N applied to a bit under normal circumstances lies between thirty and two hundred revolutions per minute(RPM).(9) Using these ranges of values as a basis, developed from years of experience in the field of drilling engineering(10), several terms of the Galle and Woods model will be examined to determine if simplification is possible and justifiable.

The term r is defined for hard formations as:

(15) $r = e^{-100/N^2} N^{.428} + 0.2N(1 - e^{-100/N^2})$

Equation (15) is equivalent to:

(16) $r = e^{-100/N^2}(N^{.428} - 0.2N) + 0.2N$

For soft formations, r is defined as:

(17) $r = e^{-100/N^2} N^{.75} + 0.5N(1 - e^{-100/N^2})$

Equation (17) is equivalent to:

(18) $r = e^{-100/N^2}(N^{.75} - 0.5N) + 0.5N$

Dr. Billy J. Mitchell, Petroleum Engineering Department, Colorado School of Mines, proposed the following simplifications for Equations (16) and (18). Let,

(19) $r \cong N^{.428}$ for hard formations, and

(20) $r \cong N^{.75}$ for soft formations.

The results of analyses run on the approximations represented by Equations (19) and (20) are shown as Appendices A and B, respectively. Further substitutions were investigated and the following approximations were arrived at:

(21) $r = N^{.43363}$ for hard formations.

(22) $r = N^{.7515}$ for soft formations.

The data and analyses are shown as Appendices C and D, respectively. In both cases reduction of the average and maximum error resulted for the approximations represented by Equations (21 and (22).

The term m is defined as:

(23) $m = 1359.1 - 714.19(\text{Log10}(\bar{W}))$.

A suitable approximation to Log10($\bar{W}$) was needed. Initially, a linear approximation was used. However, the error introduced with the linear approximation was considerable. Another substitution was investigated of the form αW^{β}. A direct search method was employed to determine the parameters. The values of α and β were determined to be:

$\alpha = 0.57879928$, and

$\beta = 0.2742$,

over the domain of values for $\bar{W}$ from twenty to eighty thousand pounds. Appendix E is a comparison of the approximation that shows the maximum error to be 1.14 % and

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the average error to be 0.539 %. Substituting for Log10($\overline{W}$),

(24) $m = 1359.1 - 714.19((0.57879928)\overline{W}^{.2742})$.

Equation (24) simplifies to:

(25) $m = 1359.1 - 413.3726578\overline{W}^{.2742}$.

One further substitution will be considered which involves the term L. Galle and Woods used a tabulation of $\overline{W}$ and L in their methodologies.(11) Appendix F shows the results of a multiple linear regression and the data on the terms Log10(L) and $\overline{W}$. The on campus computer's library statistics program was utilized to run the regression analysis. The correlation coefficient showed a high correlation of .9963. The regression indicated the following expression as an approximation to Log10(L):

(26) $\mathrm{Log10}(L) = 3.9422681 - 0.01782401\ \overline{W}$.

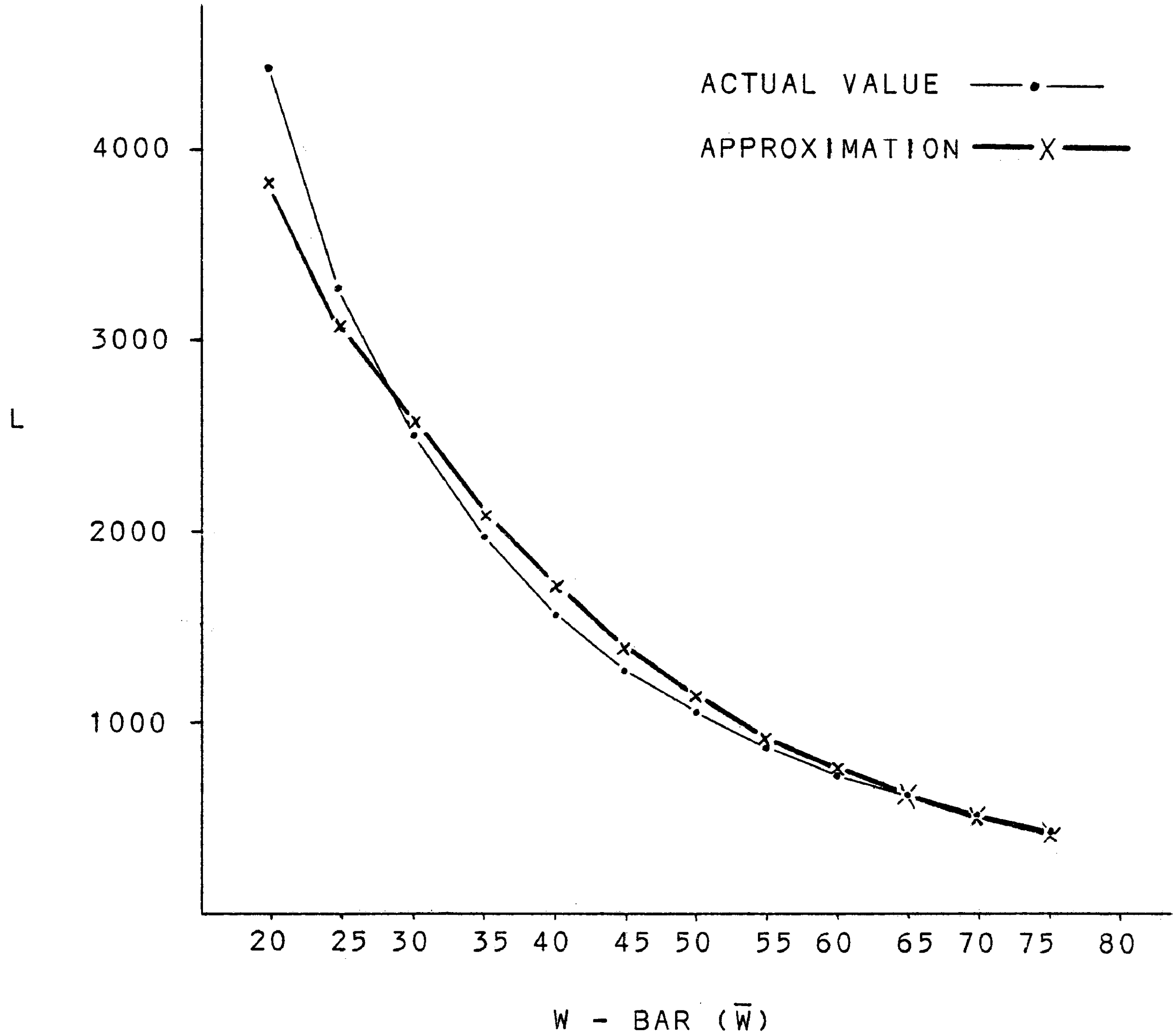
Taking the antilogarithm of Equation (23) yields:

(27) $L = 8755.240896(10^{-0.01782401\overline{W}})$.

A graph showing the original values of L and the associated values of $\overline{W}$ and the approximation represented by Equation (27) is included as Figure 1.

The three substitutions shown above as Equations (21), (22), (25) and (27) represent the necessary changes in the original Galle and Woods formulation to allow Geometric Programming to be applied to the model.

FIGURE 1



$L = 8755.240896(10^{-0.01782401\overline{W}})$

AVERAGE % ERROR: 5.345 %

MAXIMUM % ERROR: 14.080 %

Chapter Three

This chapter will be devoted to the application of Geometric Programming to the model developed in Chapter Two. Geometric Programming, or for brevity, GP, will be discussed briefly and then applied to the model so that a generalized algorithm can be developed. This will be followed by some examples taken from the literature and field data to verify the technique.

GP was formalized by Duffin, Petersen, and Zener(12). GP exploits the relationship between the arithmetic and geometric means:

(28) $\sum w_i U_i \geq \prod (U_i / w_i)^{w_i}$

if $\sum w_i = 1$, to provide a rapid method of designing for minimum cost.

Design problems involve finding the combination of variables that fulfills the constraints of the problem at minimum cost. Total cost, TC, can be expressed as the sum of component costs, U_i:

(29) $TC = \sum U_i$

Component cost is often expressed as a power function:

(30) $U_i = c_i \prod_j X_j^{a_{ij}}$

where the coefficient c_i is a positive constant, the exponents a_{ij} are arbitrary real constants, and the design variables X_j are positive. With these conditions, TC can be called posynomial, short for positive polynomial.(13) Equation (29) represents the primal function of the GP formulation as it is in terms of the design variables.

A dual function is developed from the generalized form of the geometric inequality:

(31) $$TC \geq \sum_i \prod_j (c_i / w_i)^{w_i} \quad X_j^{D_j}$$

Where D_j is defined by:

(32) $$D_j = \sum_i \sum_j w_i \; a_{ij}$$

Note that if a set of w_i can be found such that all $D_j = 0$, then Equation (31) is in terms of the constants c_i and the w_i associated with them. Duffin, et al, showed(14) that if a set of w_i could be found that satisfy the condition that all $D_j = 0$, then the primal function's value will be identical to the dual function's value. The GP technique resolves to finding a set of w_i such that:

(33) $$\sum w_i = 1.0 \text{ and,}$$

(34) $$\sum_i \sum_j w_i \; a_{ij} = 0.$$

Equation (33) is called the normality condition, and Equation (34) is called the orthogonality condition. It is seen that GP can solve certain types of highly non-linear optimization problems by solving a set of linear equations in a dual space.

Each term in the cost function and constraints gives rise to a w_i. The concept of degree of difficulty, DD, deals with the uniqueness of a solution set of w_i. DD is defined as:

(35) DD = No. of terms - No. of variables - 1.

Geometric Programming can best be used on problems with DD = 0. A problem of zero - DD implies that a unique set of w_i is associated with the minima. If the DD is positive, the value of DD represents the number of dimensions that must be searched to determine a feasible set of w_i, if such a set exists.

The method to solve the minimum cost drilling problem used in this paper consecutively iterates on final dullness, D(f). Then, the optimum values for bit weight and rotary speed are computed for each D(f) via GP. The set of drilling parameters associated with the global minimum of cost per foot is chosen as the best drilling program from the series. With the above understanding, the model will be further refined and GP will be applied to it. Equation (14) appears as:

$$(36) \qquad PHI = \frac{C(B) + C(T) + (CRIG/24)A(f)(m/i)\int_0^{D(f)} a \, dD}{A(f)C(f)\overline{W}^k r(m/i)\int_0^{D(f)} a^{1-P} \, dD}$$

In Equation (36) the only unknown values are weight, rotary speed and final dullness of the bit. The algorithm uses the values 0.1 to 1.0 for D(f) in increments of 0.1 to generate the optimal weight and rotary speed values for each D(f). The range and increments were suggested by Dr. B. J. Mitchell. Assume a value of D(f) has been chosen. Let the following constants be established:

(37) $K(1) = C(B) + C(T)$

(38) $K(2) = (CRIG/24)A(f)\int_0^{D(f)} a\ dD$

(39) $K(3) = A(f)C(f)\int_0^{D(f)} a^{1-P}\ dD$

Then, Equation (36) appears as:

(40) $$PHI = \frac{K(1) + K(2)(m/i)}{K(3)\overline{W}^{k}r(m/i)}$$

Let,

(41) $K(4) = K(1) / K(3)$, and

(42) $K(5) = K(2) / K(3)$.

Then, Equation (40) becomes:

(43) $PHI = K(4)\overline{W}^{-k}r^{-1}(i/m) + K(5)\overline{W}^{-k}r^{-1}$

Introduce another variable, V, for m and add a constraint to the total cost function represented by Equation (40). Since PHI is to be minimized, the constraint must be of the form:

(44) $V \leq m = 1359.1 - 413.3726578\ (\overline{W}^{.2742})$.

Then, Equation (44) becomes,

(45) $V + 413.3726578(\overline{W}^{.2742}) \leq 1359.1$

Divide both sides of Equation (45) by the constant on the right:

(46) $(0.00073578)V + (0.30415176)\overline{W}^{.2742} \leq 1.0$

Let,

(47) $K(10) = 0.00073578$ and

(48) $K(11) = 0.30415176$.

The model now appears as:

(49) $\text{MIN:} \quad PHI = K(4)\overline{W}^{-k} r^{-1}\, i V^{-1} + K(5)\overline{W}^{-k} r^{-1}$

(50) $\text{SUBJECT TO:} \quad K(10)V + K(11)\overline{W}^{.2742} \leq 1.0$

Let B represent the exponent on N in the substitution for r and express i in full form. Equation (49) becomes:

(51) $\text{MIN: } PHI = K(4)\overline{W}^{-k} N^{-B}(N+4.348\times10^{-5} N^{3})V^{-1} + K(5)\overline{W}^{-k} N^{-B}$

Let,

(52) $K(6) = 4.348\times10^{-5}$

(53) $\text{MIN:} \quad PHI = K(4)\overline{W}^{-k} V^{-1} N^{1-B} + K(4)K(6)\overline{W}^{-k} N^{3-B} V^{-1} + K(5)\overline{W}^{-k} N^{-B}$

(54) $\text{SUBJECT TO:} \quad K(10)V + K(11)\overline{W}^{.2742} \leq 1.0$

The model, represented by Equations (53) and (54) has the following degree of difficulty:

(55) $DD = 5-3-1 = 1$.

To reduce the degree of difficulty, condensation shall be used on terms 1 and 2 in the objective function, Equation (53). Condensation is a transformation that is theoretically based on the geometric inequality. Condensation appoximates a multiterm posynomial function with a monomial function. Beightler and Phillips(15) show that the feasible region for the original problem entirely contains the feasible region of the condensed problem. Then, any feasible solution to the condensed problem will be feasible, but not necessarily optimal, for the original problem. Optimality for both problems occurs when the original feasible vector around which the condensation takes place, is equivalent to the optimal vector for the condensed problem.

In simple form, the condensation algorithm is:

1. Choose a feasible original vector.
2. Determine the optimal condensed vector.
3. Compare the values from Steps 1 and 2, then:
   a. If they are equal, STOP.
   b. Otherwise, let the optimal consensed vector be the new feasible original vector. GO TO 2.

Rotary speed, N, is the primal variable used in the condensation. The condensation algorithm terminates when the feasible original value of N is within $\pm$ 0.001 of the optimal condensed value for N. The term,

$$K(4) + K(4)K(6)N^2$$

shall be condensed in the objective function, Equation (53). By the geometric inequality,

(56) $K(4) + K(6)N^2 \geq (K(4)/\lambda_1)^{\lambda_1} (K(4)K(6)/\lambda_2)^{\lambda_2}$

if and only if $\lambda_1 + \lambda_2 = 1$, and $\lambda_1, \lambda_2 \geq 0$.

Then, λ_2 can be described as:

(57) $\lambda_2 = 1 - \lambda_1$

Substituting into Equation (56),

(58) $K(4) + K(4)K(6)N^2 \geq (K(4)/\lambda_1)^{\lambda_1} (K(4)K(6)/1-\lambda_1)^{1-\lambda_1} N^{2(1-\lambda_1)}$

If a feasible value of λ_1 is chosen, the term,

$$(K(4)/\lambda_1)^{\lambda_1} (K(4)K(6)/1-\lambda_1)^{1-\lambda_1}$$

is a constant, call it K(7).

Then, Equation (54) becomes:

(59) MIN: $PHI = K(7)\overline{W}^{-k}V^{-1}N^{3-B-2\lambda_1} + K(5)\overline{W}^{-k}N^{-B}$

(60) SUBJECT TO: $K(10)V + K(11)\overline{W}^{.2742} \leq 1.0$

The problem now has a degree of difficulty equal to:

(61) DD= 4-3-1 =0.

If a feasible solution can be found in the condensed problem represented by Equations (59) and (60) that meets the optimality criteria given above, then the optimal condensed solution equals the optimal original solution. Let w_n represent the weight on the n^{th} term. The normality condition is:

(62) $w_1 + w_2 = 1.0$

The orthogonality condition for W is:

(63) $(-k)w_1 + (-k)w_2 + (.2742)w_4 = 0$; which implies

(64) $w_4 = (3.64697)k$.

The orthogonality condition for N is:

(65) $(3 - B - 2\lambda_1)w_1 + (-B)w_2 = 0$.

Equation (65) simplifies to:

(66) $(3 - 2\lambda_1)w_1 = B$.

The orthogonality condition for V is:

(67) $-w_1 + w_3 = 0$ or

(68) $w_1 = w_3$.

One of the properties of GP(16) allows the following expression:

(69) $$\frac{K(7)N^{3-B-2\lambda_1}\,\overline{W}^{-k}\,V^{-1}}{w_1} = \frac{K(5)\overline{W}^{-k}N^{-B}}{w_2}$$

Simplifying Equation (69) yields:

(70) $$\frac{K(7)N^{3-2\lambda_1}\;V^{-1}}{w_1} = \frac{K(5)}{w_2} = \frac{K(5)}{1-w_1}$$

(71) $$V = \frac{K(7)(1-w_1)N^{3-2\lambda_1}}{K(5)w_1}$$

Another property of GP(17) allows the following expression:

(72) $w_3 = K(10)V(w_3 + w_4)$.

Equation (72) is equivalent to:

(73) $$V = \frac{w_3}{K(10)(w_3 + w_4)}$$

Set Equations (71) and (73) equal. This yields:

(74) $$\frac{w_3}{K(10)(w_3 + w_4)} = \frac{K(7)(1-w_1)N^{3-2\lambda_1}}{K(5)w_1}$$

Equations (64) and (68) allow Equation (74) to be written:

$$(75) \quad \frac{w_1}{K(10)(w_1 + 3.64697k)} = \frac{K(7)(1-w_1)N^{3-2\lambda_1}}{K(5)w_1}$$

Solving for N yields:

$$(76) \quad N^{3-2\lambda_1} = \frac{K(5)w_1^2}{K(7)K(10)(1-w_1)(w_1 + 3.64697k)}$$

Equation (76) implies:

$$(77) \quad N = \left[\frac{K(5)w_1}{K(7)K(10)(1-w_1)(w_1 + 3.64697k)}\right]^{\frac{1}{3-2\lambda_1}}$$

Equation (66) expresses w_1 as a function of λ_1. Equation (78) expresses N in terms of λ_1.

From Beightler and Phillips(18), define λ_1 as:

$$(79) \quad \lambda_1 = \frac{K(4)}{K(4) + K(4)K(6)N^2}$$

Equations (77) and (79) represent the iterative approximation of the condensation. A digital computer program written in FORTRAN is included as Appendix G. The code briefly introduces its use and limitations to the user and then asks for the data it requires for calculation. Due to the nature of the relation between the algebraic and geometric means, the condensed solution to the problem represents a lower bound on the optimal problem's solution. A computer code, developed by Dr. Billy J. Mitchell and the code developed in this paper, converge to different values of the optimal drilling program for a given final dullness and identical input parameters. This is probably due to to the different substitutions for r and Log10($\bar{W}$) used in the different codes.

A summary of the performance of the two codes in terms of cost is presented in Table 1. The column labeled "DUAL" represents the lower bound on the optimal cost due to the nature of the condensation technique. The expected performance, using the optimal primal parameters generated by the code developed in this paper, is summarized by the column labled "PRIMAL". The maximum difference between the primal and dual costs is 1.69 % occurring in Case II. Appendix H presents the input data for the four cases used as tests for the two codes. Appendix I presents the CPU time or the time needed to perform the calculations to optimality for both codes. Appendices J through M present the optimal parameters calculated by each code for Cases I through IV, respectively.

The performance of the code developed herein is promising, as it generates values close to the code developed by Dr. Mitchell, which has been used in field operations successfully.

TABLE 1

COMPUTED OPTIMAL COSTS

(Dollars per Foot Drilled)

	MITCHELL	PRIMAL	DUAL
Case I	4.950	5.072	4.997
Case II	19.006	18.865	18.546
Case III	24.668	24.028	23.915
Case IV	53.889	52.548	52.020

CONCLUSION

A fully operational optimal drilling program has been developed and verified in this thesis. The computational efficiency and theoretical performance of the thesis' program has been shown to be at least as good as a program previously developed, using the same theoretical basis, and field tested by an experienced consultant in drilling operations, Dr. Billy J. Mitchell.

The model and algorithm developed in the thesis need to be field tested for verification of the GP technique used to solve the constant bit weight and rotary speed problem. The accuracy of the code is a function of the condensed solution space and the validity of the original Galle and Woods equations. The approximations introduced above undoubtedly influence the solution, but the errors introduced by all of the substitutions would be on the order of 5 % or less, unless the extremum of the ranges for both bit weight and rotary speed were encountered simultaneously.

This thesis has basically established a lower bound for the constant bit weight and rotary speed problem. It has also demonstrated that GP can be applied to a field that is known for the highly non-linear characteristic equations that describe its functional relations.

Some suggested areas of further research would be:

1. Better substitutions for the terms r, L, and $\text{Log10}(\overline{W})$ used above.

2. Other convergence techniques applicable to Geometric Programming.

3. Other mathematical forms of the constant bit weight and rotary speed problem.

As a method, this thesis and its results can provide guidelines for the problem of minimum cost drilling. Further laboratory and field development should continue to refine the problem description and solution.

APPENDIX A

MITCHELL'S APPROXIMATION TO R (HARD)

ROT SPEED	R	APPROX	COL 3/COL 2
30.000000	4.467621	4.287537	.959691
40.000000	5.040219	4.849329	.962127
50.000000	5.518209	5.335304	.966854
60.000000	5.939032	5.768312	.971255
70.000000	6.320064	6.161720	.974946
80.000000	6.671038	6.524128	.977978
90.000000	6.998116	6.861448	.980471
100.000000	7.305524	7.177943	.982536
110.000000	7.596337	7.476805	.984265
120.000000	7.872881	7.760498	.985725
130.000000	8.136977	8.030965	.986972
140.000000	8.390082	8.289776	.988045
150.000000	8.633388	8.538214	.988976
160.000000	8.867885	8.777348	.989791
170.000000	9.094405	9.008078	.990508
180.000000	9.313660	9.231167	.991143
190.000000	9.526258	9.447274	.991709
200.000000	9.732732	9.656969	.992216

AVERAGE % ERROR = 1.85997 %

MAXIMUM % ERROR = 4.0309 %

APPENDIX B

MITCHELL'S APPROXIMATION TO R (SOFT)

ROT SPEED	R	APPROX	COL 3/COL 2
30.000000	13.048007	12.818610	.982419
40.000000	16.153492	15.905414	.984642
50.000000	19.046002	18.803015	.987242
60.000000	21.789513	21.558247	.989386
70.000000	24.418619	24.200454	.991066
80.000000	26.955039	26.749610	.992379
90.000000	29.413727	29.220111	.993418
100.000000	31.805632	31.622775	.994251
110.000000	34.139126	33.966008	.994929
120.000000	36.420819	36.256505	.995488
130.000000	38.656056	38.499713	.995956
140.000000	40.849259	40.700150	.996350
150.000000	43.004125	42.861605	.996686
160.000000	45.123805	44.987303	.996975
170.000000	47.211008	47.080024	.997226
180.000000	49.268084	49.142173	.997444
190.000000	51.297093	51.175865	.997637
200.000000	53.299853	53.182956	.997807

AVERAGE % ERROR = 0.65945 %

MAXIMUM % ERROR = 1.7581 %

APPENDIX C

DANIELS' APPROXIMATION TO R (HARD)

ROT SPEED	R	APPROX	COL 3/COL 2
30.000000	4.467621	4.370429	.978245
40.000000	5.040219	4.951095	.982317
50.000000	5.518209	5.454116	.988385
60.000000	5.939032	5.902822	.993903
70.000000	6.320064	6.310879	.998547
80.000000	6.671038	6.687086	1.002406
90.000000	6.998116	7.037496	1.005627
100.000000	7.305524	7.366480	1.008344
110.000000	7.596337	7.677310	1.010660
120.000000	7.872881	7.972515	1.012655
130.000000	8.136977	8.254091	1.014393
140.000000	8.390082	8.523648	1.015919
150.000000	8.633388	8.782506	1.017272
160.000000	8.867885	9.031763	1.018480
170.000000	9.094405	9.272344	1.019566
180.000000	9.313660	9.505037	1.020548
190.000000	9.526258	9.730517	1.021442
200.000000	9.732732	9.949371	1.022259

AVERAGE % ERROR = 1.37874 %

MAXIMUM % ERROR = 2.2259 %

APPENDIX D

DANIELS' APPROXIMATION TO R (SOFT)

ROT SPEED	R	APPROX	COL 3/COL 2
30.000000	13.048007	12.884175	.987444
40.000000	16.153492	15.993668	.990106
50.000000	19.046002	18.913676	.993052
60.000000	21.789513	21.691055	.995481
70.000000	24.418619	24.355170	.997402
80.000000	26.955039	26.926016	.998923
90.000000	29.413727	29.418006	1.000145
100.000000	31.805632	31.841973	1.001143
110.000000	34.139126	34.206339	1.001969
120.000000	36.420819	36.517808	1.002663
130.000000	38.656056	38.781841	1.003254
140.000000	40.849259	41.002961	1.003763
150.000000	43.004125	43.184965	1.004205
160.000000	45.123805	45.331087	1.004594
170.000000	47.211008	47.444115	1.004938
180.000000	49.268084	49.526459	1.005244
190.000000	51.297093	51.580237	1.005520
200.000000	53.299853	53.607312	1.005768

AVERAGE % ERROR = 0.44887 %

MAXIMUM % ERROR = 1.2556 %

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APPENDIX E

APPROXIMATION TO LOG10 W-BAR

W-BAR	LOG10(W-BAR)	APPROX	COL 3/COL 2
20.00	1.301030	1.316044	1.011540
25.00	1.397940	1.399082	1.000817
30.00	1.477121	1.470803	0.995723
35.00	1.544068	1.534304	0.993676
40.00	1.602060	1.591523	0.993423
45.00	1.653213	1.643762	0.994283
50.00	1.698970	1.691942	0.995864
55.00	1.740363	1.736742	0.997920
60.00	1.778151	1.778677	1.000296
65.00	1.812913	1.818146	1.002886
70.00	1.845098	1.855470	1.005621
75.00	1.875061	1.890905	1.008450
80.00	1.903090	1.924665	1.011337

AVERAGE % ERROR = 0.53891 %

MAXIMUM % ERROR = 1.1540 %

APPENDIX F

APPROXIMATION TO L

WBAR	LOG10(L)	L	APPROX
20.0	3.647285	4439	3814.2
25.0	3.514548	3270	3115.3
30.0	3.397245	2496	2544.4
35.0	3.292920	1963	2078.2
40.0	3.198107	1578	1697.4
45.0	3.109916	1288	1386.4
50.0	3.026533	1063	1132.3
55.0	2.496452	884	924.8
60.0	2.868644	739	755.4
65.0	2.792392	620	617.0
70.0	2.716003	520	503.9
75.0	2.637490	434	411.6

MULTIPLE CORRELATION COEFFICIENT: 0.99634

ESTIMATED CONSTANT TERM: 3.9422681

REGRESSION COEFFICIENT: -0.01782401

AVERAGE % ERROR: 5.345 %

MAXIMUM % ERROR: 14.080 %

APPENDIX G

```
C
C     OPTIMAL DRILLING PARAMETERS PROGRAM
C
      COMMON/A1/A,B,CBIT,BSIZE,CRIG,FF,DHOLE,TF,WEIGHT,XN,
     1FORTYP,XPHI,DF,BXF,WBAR,AX,BX,CX,ZA,ZB,ZC,
     1ZL,ZDF,ZERO,CTRIP,ASUBF,CSUBF,DSUBF,VZINT,XK1,XK2,XK3,
     1XK4,XK5,XK6,XK7,XK8,XK1Ø,XK11,D1,XNSTAR,WSTAR,
     1TSUBF,XWSTAR
      DATA IN,IO/4,4/
1     FORMAT(/' DO YOU WANT INFORMATION ON THIS PROGRAM? '/
     1' ENTER "YES" OR "NO" (CARRIAGE RETURN) ',$)
      WRITE(4,1)
2     FORMAT(A3)
      READ(IN,2) ANS
      IF(ANS .EQ. 'YES') CALL INFO
3     FORMAT(/,' BIT COST =  ',$)
4     FORMAT(' BIT SIZE =  ',$)
5     FORMAT(' RIG COST =  ',$)
6     FORMAT(' FOOTAGE DRILLED BY BIT =  ',$)
7     FORMAT(' CURRENT DEPTH OF HOLE =  ',$)
8     FORMAT(' TIME ROTATING THE BIT =  ',$)
9     FORMAT(' WEIGHT ON BIT =  ',$)
1Ø    FORMAT(' ROTARY SPEED OF BIT =  ',$)
11    FORMAT(' FORMATION HARDNESS =  ',$)
12    FORMAT(' TOOTH WEAR TYPE =  ',$)
13    FORMAT(' DULLNESS OF BIT =  ',$)
14    FORMAT(' FINAL BEARING CONDITION =  ',$)
15    FORMAT(12G)
      WRITE(IO,3)
      READ(4,15) CBIT
      WRITE(IO,4)
      READ(4,15) BSIZE
      WRITE(IO,5)
      READ(4,15) CRIG
      WRITE(IO,6)
      READ(4,15) FF
      WRITE(IO,7)
      READ(4,15) DHOLE
      WRITE(IO,8)
      READ(4,15) TF
      WRITE(IO,9)
      READ(4,15) WEIGHT
      WRITE(IO,1Ø)
      READ(4,15) XN
      WRITE(IO,11)
      READ(4,15) FORTYP
      WRITE(IO,12)
      READ(4,15) XPHI
```

```
      WRITE(IO,13)
      READ(4,15) DF
      WRITE(IO,14)
      READ(4,15) BXF
C
C     WEIGHT NORMALIZATION
C
      WBAR= 7.88*WEIGHT/BSIZE
      CALL FORMAY
100   IF(FORTYP-2.Ø)1Ø1,1Ø2,1Ø3
1Ø1   A= Ø.6
      B= Ø.7515
      GO TO 1Ø5
1Ø2   A= Ø.85
      B= Ø.65
      GO TO 1Ø5
1Ø3   A= 1.Ø
      B= Ø.43363
1Ø5   CONTINUE
C
C     SUBROUTINE ZINT CONSTANTS
C
      AX= Ø.928125
      BX= 6.Ø
      CX= 1.Ø
C
C     CTRIP CALCULATION
C
      CTRIP= (Ø.6*DHOLE)*(CRIG/24.Ø)
      DF= Ø.1
4ØØ   CALL OPTICO
      CALL CONDEN
C
C     OPTIMAL WEIGHT CALCULATION
C
      XWSTAR= ((3.64697*A)/(XK11*(D1 + (3.64697*A))))**3.64697
      WSTAR= (XWSTAR*BSIZE)/7.88
C
C     TOTAL ROTATING TIME CALCULATION
C
      URN= 1359.1 - 714.19*(ALOG1Ø(XWSTAR))
      URF= (Ø.3Ø9375)*(DF**3.Ø) + 3.Ø*(DF**2.Ø) + DF
      URD= XNSTAR + (Ø.ØØØ04348*(XNSTAR**3.Ø))
      TSUBF= (ASUBF*URN*URF)/URD
C
C     BEARING LIFE FAILURE TIME CALCULATION
C
      XL= (971Ø.416944)*(2.718281828**(-Ø.Ø4286995*WBAR))
      S= (TF*XN)/(BXF*XL)
      XLSTAR=(971Ø.416944)*(2.718281828**(-Ø.Ø4286995*XWSTAR))
      TSUBX= (S*XLSTAR)/(XNSTAR)
```

```
      IF(DF .GT. Ø.95) GO TO 44Ø
      IF(TSUBX .GT. TSUBF) GO TO 42Ø
      CALL APPROX
42Ø   CALL OUTPUT
      DF= DF+ Ø.1
      GO TO 4ØØ
44Ø   CALL OUTPUT
      STOP
      END
```

```
      SUBROUTINE OPTICO
      COMMON/A1/A,B,CBIT,BSIZE,CRIG,FF,DHOLE,TF,WEIGHT,XN,
     1FORTYP,XPHI,DF,BXF,WBAR,AX,BX,CX,ZA,ZB,ZC,
     1ZL,ZDF,ZERO,CTRIP,ASUBF,CSUBF,DSUBF,VZINT,XK1,XK2,XK3,
     1XK4,XK5,XK6,XK7,XK8,XK10,XK11,D1,XNSTAR,WSTAR,
     1TSUBF,XWSTAR
400   XK1= CBIT + CTRIP
      XK2= (CRIG/24.0)*ASUBF*(0.309375*DF**3.0+3.0*DF**2.0*DF)
       IF(XPHI .EQ. 1.0) GO TO 410
      CALL ZINT
      XK3= ASUBF*CSUBF*VZINT
      GO TO 411
410   XK3= ASUBF*CSUBF*DF
411   CONTINUE
      XK4= XK1/XK3
      XK5= XK2/XK3
      XK6= 0.00004348
      XK10= 0.00073578
      XK11= 0.30415176
      RETURN
      END
```

```
      SUBROUTINE CONDEN
      COMMON/A1/A,B,CBIT,BSIZE,CRIG,FF,DHOLE,TF,WEIGHT,XN,
     1FORTYP,XPHI,DF,BXF,WBAR,AX,BX,CX,ZA,ZB,ZC,
     1ZL,ZDF,ZERO,CTRIP,ASUBF,CSUBF,DSUBF,VZINT,XK1,XK2,XK3,
     1XK4,XK5,XK6,XK7,XK8,XK10,XK11,D1,XNSTAR,WSTAR,
     1TSUBF,XWSTAR
      XNBAR= XN
607   XLAMB1= XK4/(XK4 + XK4*XK6*(XNBAR**2.0))
      D1= B/(3.0 - 2.0*XLAMB1)
      Z1= 1.0/(3.0 - 2.0*XLAMB1)
      Q1= 1.0 - XLAMB1
      XK7= ((XK4/XLAMB1)**XLAMB1)*((XK4*XK6/Q1)**Q1)
      XNUM= (XK5*(D1**2.0))**Z1
      XDEN= (XK10*XK7*(1.0-D1)*(D1+(3.64697*A)))**Z1
      XNSTAR= XNUM/XDEN
      IF(ABS(XNSTAR-XNBAR) .LT. 0.001) GO TO 630
      XNBAR=XNSTAR
      GO TO 607
630   RETURN
      END
```

```
      SUBROUTINE OUTPUT
      COMMON/A1/A,B,CBIT,BSIZE,CRIG,FF,DHOLE,TF,WEIGHT,XN,
     1FORTYP,XPHI,DF,BXF,WBAR,AX,BX,CX,ZA,ZB,ZC,
     1ZL,ZDF,ZERO,CTRIP,ASUBF,CSUBF,DSUBF,VZINT,XK1,XK2,XK3,
     1XK4,XK5,XK6,XK7,XK8,XK10,XK11,D1,XNSTAR,WSTAR,
     1TSUBF,XWSTAR
600   FORMAT(/' BEST WEIGHT= ',1X,F6.2,1X,' THOUSAND POUNDS ')
601   FORMAT(' BEST ROTARY SPEED= ',1X,F6.2,1X,' R.P.M. ')
602   FORMAT(' TOTAL ROTATING TIME= ',1X,F6.2,1X,' HOURS ')
603   FORMAT(' TOTAL FOOTAGE= ',1X,F7.2,1X,' FEET ')
604   FORMAT(' COST PER FOOT= ',1X,F8.4,1X,' $ PER FT ')
605   FORMAT(' FINAL BIT DULLNESS= ',1X,F5.3)
C
C     COST PER FOOT CALCULATION
C
C     TOTAL BIT FOOTAGE CALCULATION
C
      D2= 1.0 - D1
      URN= 1359.1-714.19*(ALOG10(XWSTAR))
      URD= XNSTAR + (0.00004348)*(XNSTAR**3.0)
      DIP= (ASUBF*CSUBF*URN*(XWSTAR**A)*(XNSTAR**B))/URD
      IF(XPHI .EQ. 1.0) GO TO 650
      CALL ZINT
      FSUBF= DIP*VZINT
      GO TO 611
650   FSUBF= DIP*DF
611   CONTINUE
      D4= 3.64697*A
      CPERF= ((XK7/D1)**D1)*((XK5/D2)**D2)*((XK10/D1)**D1)
     1*((XK11/D4)**D4)*((D1 + D4)**(D1 + D4))
      DSUBF= DF
      WRITE(4,600) WSTAR
      WRITE(4,601) XNSTAR
      WRITE(4,602) TSUBF
      WRITE(4,603) FSUBF
      WRITE(4,604) CPERF
      WRITE(4,605) DSUBF
690   PCPF= (CBIT + CTRIP + TSUBF*(CRIG/24.0))/FSUBF
632   FORMAT(' PRIMAL COST PER FOOT= ',F7.3)
      WRITE(4,632)PCPF
      RETURN
      END
```

```
      SUBROUTINE ZINT
      COMMON/A1/A,B,CBIT,BSIZE,CRIG,FF,DHOLE,TF,WEIGHT,XN,
     1FORTYP,XPHI,DF,BXF,WBAR,AX,BX,CX,ZA,ZB,ZC,ZL,ZDF,
     1ZERO,CTRIP,ASUBF,CSUBF,DSUBF,VZINT,XK1,XK2,XK3,
     1XK4,XK5,XK6,XK7,XK8,XK10,XK11,XNSTAR,WSTAR,
     1TSUBF,XWSTAR
      ZA= (2.0*AX*DF + BX)/(4.0*AX)
      ZB= (4.0*AX*CX - (BX**2.0))/(8.0*AX*(SQRT(AX)))
      ZC= SQRT(AX*(DF**2.0) + BX*DF + CX)
      ZL= ALOG(ZC + DF*(SQRT(AX)) + BX/(2*(SQRT(AX))))
      ZDF= ZA*ZC + ZB*ZL
      ZERO= (BX/(4*AX)) + ZB*(ALOG(1 + BX/(2*(SQRT(AX)))))
      VZINT= ZDF - ZERO
      RETURN
      END
```

```
      SUBROUTINE APPROX
      COMMON/A1/A,B,CBIT,BSIZE,CRIG,FF,DHOLE,TF,WEIGHT,XN,
     1FORTYP,XPHI,DF,BXF,WBAR,AX,BX,CX,ZA,ZB,ZC,
     1ZL,ZDF,ZERO,CTRIP,ASUBF,CSUBF,DSUBF,VZINT,XK1,XK2,XK3,
     1XK4,XK5,XK6,XK7,XK8,XK10,XK11,D1,XNSTAR,WSTAR,
     1TSUBF,XWSTAR
710   DF= 0.98*DF
      CALL OPTICO
      CALL CONDEN
C
C     OPTIMAL WEIGHT CALCULATION
C
630   XWSTAR= ((3.64698*A)/(XK11*(D1 + (3.64697*A))))**3.64697
      WSTAR= (XWSTAR*BSIZE)/7.88
C
C     TOTAL ROTATING TIME CALCULATION
C
      URN= 1359.1 - 714.19*(ALOG10(XWSTAR))
      URF= (0.309375)*(DF**3.0) + 3.0*(DF**2.0) + DF
      URD= XNSTAR + (0.00004348*(XNSTAR**3.0))
      TSUBF= (ASUBF*URN*URF)/URD
C
C     BEARING LIFE FAILURE TIME CALCULATION
C
      XL= (9710.416944)*(2.718281828**(-0.04286995*WBAR))
      S= (TF*XN)/(BXF*XL)
      XLSTAR=(9710.416944)*(2.718281828**(-0.04286995*XWSTAR))
      TSUBX= (S*XLSTAR)/(XNSTAR)
      IF(ABS(TSUBX-TSUBF) .LT. 0.1) GO TO 730
      GO TO 710
730   DSUBF= DF
      CALL OUTPUT
740   FORMAT(//' WARNING:  DO NOT EXCEED TOTAL ROTATING '/
     1' TIME AS BEARING FAILURE IS LIMITING THE '/
     1' DRILLING TIME.  THE LAST SET OF PARAMETERS IS '/
     1' USUALLY THE OPTIMAL DRILLING PROGRAM AT MINIMUM '/
     1' COST.  CHECK THE OTHER PROGRAMS FOR LOWER COST.  ')
      WRITE(4,740)
      STOP
      END
```

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```
      SUBROUTINE FORMAY
      COMMON/A1/A,B,CBIT,BSIZE,CRIG,FF,DHOLE,TF,WEIGHT,XN,
     1FORTYP,XPHI,DF,BXF,WBAR,AX,BX,CX,ZA,ZB,ZC,
     1ZL,ZDF,ZERO,CTRIP,ASUBF,CSUBF,DSUBF,VZINT,XK1,XK2,XK3,
     1XK4,XK5,XK6,XK7,XK8,XK10,XK11,D1,XNSTAR,WSTAR,
     1TSUBF,XWSTAR
      AX= 0.928125
      BX= 6.0
      CX= 1.0
100   IF(FORTYP-2.0)101,102,103
101   A= 0.6
      B= 0.75
      GO TO 104
102   A= 0.85
      B= 0.65
      GO TO 104
103   A= 1.0
      B= 0.428
104   CONTINUE
C
C     ASUBF CALCULATION
C
200   XNUM= TF*(XN+ (0.00004348)*(XN**3.0))
      XIAP= 1359.1 - 714.19*ALOG10( WBAR )
      XINTAF= (0.309375)*(DF**3.0) + 3.0*(DF**2.0) + DF
      XDENOM= XIAP*XINTAF
      ASUBF=XNUM/XDENOM
210   FORMAT(/' A SUB F = ',F)
      WRITE(4,210)ASUBF
C
C     CSUBF CALCULATION
C
300   XI= XN + (0.00004348)*(XN**3.0)
      XM= 1359.1 - 714.19*(ALOG10(WBAR))
      IF(XPHI .EQ. 1.0) GO TO 309
      CALL ZINT
      CSUBF= (FF*XI)/(ASUBF*(WBAR**A)*(XN**B)*XM*VZINT)
      GO TO 311
309   CSUBF= (FF*XI)/(ASUBF*(WBAR**A)*(XN**B)*XM*DF)
310   FORMAT(/' C SUB F = ',F)
311   WRITE(4,310)CSUBF
      RETURN
      END
```

```
      SUBROUTINE INFO
701   FORMAT(/' THIS PROGRAM IS DESIGNED TO CALCULATE THE '/
     1' CONSTANT BIT WEIGHT AND ROTARY SPEED THAT MINIMIZES '/
     1' COST PER FOOT DRILLED FOR TOOTHED BIT RUNS.  THIS '/
     1' PROGRAM TAKES DATA FROM THE PREVIOUS BIT RUN AND '/
     1' GENERATES OPTIMAL DRILLING WEIGHT AND ROTARY SPEED '/
     1' FOR THE NEXT BIT. THE FOLLOWING IS ASSUMED ABOUT THE '/
     1' NEXT BIT: '//
 1'   1.  THE BITS ARE THE SAME SIZE AND COST. '/
 1'   2.  THE BITS ARE FROM THE SAME MANUFACTURER. '/
 1'   3.  THE SAME OR VERY SIMILAR DRILLING FLUID WILL '/
 1'        BE USED. '/
 1'   4.  THE FORMATION TYPE DOES NOT CHANGE DRAS- '/
 1'        TICALLY. '//
     1' IF THESE BASIC ASSUMPTIONS CAN NOT BE MET, THIS '/
     1' PROGRAM CANNOT GUARANTEE OPTIMAL DRILLING PROCEDURE. '//
     1' TO START A DRILLING PROGRAM USING THIS CODE YOU '/
     1' MUST HAVE THE FOLLOWING DATA IN THE INDICATED UNITS '/
     1' FROM THE PREVIOUS BIT RUN! '//
     1' BIT COST                     DOLLARS '/
     1' BIT SIZE                     INCHES '/
     1' RIG COST                     DOLLARS PER DAY '/
     1' FOOTAGE DRILLED BY BIT               FEET '/
     1' TIME ROTATING THE BIT                HOURS '/
     1' WEIGHT ON THE BIT                    THOUSANDS OF LBS '/
     1' ROTARY SPEED OF THE BIT              R.P.M. '/
     1' FORMATION HARDNESS                   1 = SOFT '/
 1'                                  2 = MEDIUM '/
 1'                                  3 = HARD '/
     1' TOOTH WEAR TYPE                      1.0=FLAT WEAR '/
 1'                                  0.5=CHIP WEAR '/
     1' DULLNESS OF BIT                      (SEE BELOW) '/
     1' FINAL BEARING CONDITION              (SEE BELOW) '//
     1' USE THE FOLLOWING TABLE FOR THE LAST ENTRIES: '//
     1' NO WEAR      0             5/8       0.625 '/
     1' 1/8          0.125         3/4       0.75 '/
     1' 1/4          0.25          7/8       0.875 '/
     1' 3/8          0.375         WORN OUT  1.0 '/
     1' 1/2          0.5 '//
     1' DO NOT INCLUDE DIMENSIONS ON THE DATA ENTERED - '/
     1' THE MACHINE IS PROGRAMMED TO ACCEPT NUMBERS ONLY. ')
      WRITE(4,701)
      RETURN
      END
```

APPENDIX H

TEST CASE DATA

PARAMETER	CASE I	CASE II	CASE III	CASE IV
BIT COST	200	400	1000	650
BIT SIZE	8.75	8.75	10.75	6.5
RIG COST	1200	4800	6000	12000
FINAL FOOTAGE	200	200	235	130
DEPTH	10	12	8	15
ROTATION TIME	12	10	17	7
WEIGHT	40	45	60	40
ROTARY SPEED	110	90	100	120
FORMATION	SOFT	SOFT	HARD	HARD
PHI	CHIP	FLAT	CHIP	FLAT
FINAL DULLNESS	0.375	0.75	0.9	0.5
FINAL BEARINGS	0.75	0.375	0.75	0.675

FOR A SOFT FORMATION: $K = 0.6$

$B = 0.75$

FOR A HARD FORMATION: $K = 1.0$

$B = 0.428$

PHI = 0.5 (FOR CHIP WEAR)

= 1.0 (FOR FLAT WEAR)

APPENDIX I

COMPUTATIONAL TIME

(CPU TIME, MIN:SEC)

	MITCHELL	DANIELS
CASE I	01:20.26	00:02.60
CASE II	00:43.23	00:03.81
CASE III	03:15.93	00:01.74
CASE IV	00:18.48	00:01:71

APPENDIX J

OPTIMAL VALUES FOR CASE I

	MITCHELL	DANIELS
COST PER FOOT	5.08	5.07
ROTATING TIME	13.38	12.75
WEIGHT	43.31	44.24
ROTARY SPEED	115.00	116.62
BIT DULLNESS	0.50	0.45
BEARING WEAR	1.0	1.0

APPENDIX K

OPTIMAL VALUES FOR CASE II

	MITCHELL	DANIELS
COST PER FOOT	19.02	18.88
ROTATING TIME	11.39	10.24
WEIGHT	41.08	42.19
ROTARY SPEED	110.00	101.04
BIT DULLNESS	0.90	0.90
BEARING WEAR	0.45	N/C

NOTE: N/C MEANS " NOT CALCULATED "

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APPENDIX L

OPTIMAL VALUES FOR CASE III

	MITCHELL	DANIELS
COST PER FOOT	24.67	24.03
ROTATING TIME	19.54	18.85
WEIGHT	77.76	77.41
ROTARY SPEED	70.00	73.05
BIT DULLNESS	1.00	1.00
BEARING WEAR	1.05	N/C

NOTE: N/C MEANS " NOT CALCULATED "

APPENDIX M

OPTIMAL VALUES FOR CASE IV

	MITCHELL	DANIELS
COST PER FOOT	53.71	52.55
ROTATING TIME	16.61	16.52
WEIGHT	45.37	44.20
ROTARY SPEED	40.00	42.82
BIT DULLNESS	0.40	0.40
BEARING WEAR	0.71	N/C

NOTE: N/C MEANS " NOT CALCULATED "

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