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# HEURISTIC NONLINEAR CONSTRAINED MAXIMIZATION OF ORE RESERVE TONNAGE BY BLENDING FOR MULTIPLE MINING AREAS USING CUTOFF GRADE AS THE DISCRIMINATOR

by

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# ABSTRACT

The typical objective of today's mining company is to maximize the net present value of deposits. Information about the available mineral reserves in a deposit is required to perform economic mine valuation. Determination of the maximum total amount of material available in a deposit is useful in the valuation process. Delineation techniques yield information about the spatial grade distribution from which a mineral inventory is built. The mineral inventory is used to evaluate mine economics and production logistics.

Cutoff grade analysis is used to graphically represent the average ore grade and the mineralized tonnage in reserve considering only material at, or above, the "cutoff grade". Economic and technical analysis is typically performed using global cutoff grade analysis. Cutoff grade analysis can also be performed selectively for individual mineralized areas.

Blending during production is used to smooth the quality and quantity of ore removed from different areas before it is fed to a mill. Blending decisions made during reserve analysis allow dissimilar material to be mixed to meet limiting economic constraints placed on the ore while maximizing total reserve tonnage.

A technique which uses blending to approach a heuristic constrained maximum total reserve from multiple areas represented by non-linear cutoff grade curves is developed in this thesis. A method to calculate cutoff grade for the individual areas to be blended is also presented.

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# INTRODUCTION

Mining is one of the most heavily capitalized industries in the world. To minimize financial risk and exposure, it is imperative for mining operations to adopt techniques which will optimize the efficient use of resources in targeting, delineating, extracting, beneficiating, and marketing mineral products. Mathematical techniques known as Operations Research have been developed which aid in efficient allocation and use of scarce resources.

The maximization of total available mineral resources for a mine or several mines of similar mineralization is one particular area in which Operations Research can be used. Difficulty in applying such mathematical techniques to specific mining problems may arise from the lack of information about an area. Mining problems are sometimes too complex to be modeled mathematically. Occasionally techniques can be developed which achieve a useful goal for a particular mine or for the industry in general.

A direct benefit of maximizing overall ore reserves is an increased value of the mining property. The efficient exploitation of mineral resources indirectly increases the mineral reserve base of a nation, lessens the environmental impact of procuring such resources, and reduces the total effort required to maintain a comparable rate of mineral production. However, the amount of ore which can be economically produced from one or several mining areas may be limited. The limiting constraints may relate to the market for the mineral resource being exploited. Other factors involved may be the mine and/or mill capacities, ore quality requirements for concentration, or the financial limits by which a mine operates.

Considering financial, production, and marketing concerns it is possible to define specific grades of ore and tonnages which can be mined and meet the given constraints. Cutoff grade is the term most often used to represent this specific grade. Cutoff grade (as used in this paper) is the grade of a mineral in rock, above which, one particular option will be exercised, below which, another option will be followed. This definition allows for several cutoff grades to be in effect (for any mining area) to meet the limiting constraints.

Any combination of factors may be taken into consideration to arrive at cutoff grades. Such a situation occurs when several ore products are produced from the same mining area (such as high grade ore, mill run ore, low grade ore, and waste). The definition of cutoff grade is specific since any mineralized or unmineralized rock will be considered either at, or above, a specific cutoff grade or below that cutoff grade. The selection of cutoff grade must take into account any and all factors which may effect the production over the period of exploitation. This includes all factors of finance, production, market, and timing. An infinite number of scenarios can occur considering all the possible factors; similarly, an infinite number of values for cutoff grade can be determined.

The grade of gangue minerals will occasionally prohibit operations at a particular cutoff grade. Processing concerns such as reagent consumption, a high or low variation in specific gravity between minerals, the surface chemistry of different minerals, or any other milling factor associated with a mineral's physical, chemical, or magnetic characteristics could be used as part of an analysis to determine a cutoff grade.

Commodity price and market effect cutoff grade significantly. Price, supply, demand, volume, competitors, substitutes, trends, and costs all influence cutoff grade in a mining area. Cutoff grade can also be influenced by both political and environmental climate, elevation, natural disaster, and environmental concerns. These issues are all of serious consequence considering the heavy capitalization required in the mining industry. The variability of these factors make long term decisions, based on the reserve analysis, uncertain over the life of a mine.

Blending ores from various locations in a mine or in a mining district can assist in optimizing the extraction, beneficiation, and marketing of ores. Blending over the short term is used to reduce wide fluctuations in ore quantity and quality. Minimizing variations provides a basis for consistent material flow resulting in higher efficiency in processing and lower final product variation. Blending may be required in a mining district due to substantial variation in ore grade, tonnage, hardness, or other inherent qualities characteristic of an ore.

Blending of ores from various mining areas is typically necessary to implement smooth development and production of operations on a day-to-day basis. The logistics of bench development, including drilling, blasting, mucking, and ore haulage, will often dictate the blending required in an open pit operation. The number and location of predrilled sites on various benches represents the currently available reserves. The placement of shovels or draglines in a pit represents where stripping or ore production may immediately commence if desired. Haulage system capacity and length of haul from a particular mining area to the mill influences the immediate production capacity. Underground operations must have a sufficient number of working faces in production so ore can be drawn from multiple areas to produce the proper blend. A sufficient number of bell raises must be producing in sublevel stoping, vertical crater retreat, or block caving operations to supply an efficient blend of ore to the mill. The design and development of underground mines must examine the ore blending requirements more thoroughly before implementing plans since ore is typically less accessible (in both development and production) in underground operations than in an open pit. The implementation of plans which allow the smooth and consistent production of ore must be based on overall production logistics.

Milling and concentration of the mineral contained within an ore is the primary phase of beneficiation which occurs after an ore has been extracted from the earth. A mill is typically built to process an ore from one particular mine or from several mines in a vicinity which have similar mineralization. The process used to extract the minerals is usually specific for a particular ore. To efficiently run a mineral processing plant it is necessary to identify the typical ore mineralization, develop system requirements which are economically best suited to extract the minerals from the ore, and operate the mill so that the economic system requirements are met. The economic system requirements for the milling process will change for any slight variation in ore grade, ore hardness, feed tonnage, or ratio of mineral content. These factors are expected to have small changes with time and are taken into account during the milling process. Large changes in these factors are intolerable for efficient mill operation.

In all milling processes some technique of concentration is used to separate the valuable minerals from the gangue. Blending is essential to concentration processes for efficient operation. The difference in specific gravity between valuable and gangue minerals is utilized in specific gravity separation processes. If large amounts of a valuable minerals (assumed to be the heavy mineral) enter the circuit at one time, the circuit is choked with heavy minerals and performance deteriorates. This is reflected either as a loss in recovery or as a drop in the concentration ratio (i.e., a lower grade concentrate is produced). Collectors, frothers, and modifiers are used in flotation circuits to collect the mineral being floated (by making it hydrophobic), create a stable froth, and alter the surface chemistry of the mineral. If a sudden large feed grade increase occurs in the mineral being floated, the quantity of collector being used in the mill circuit will not be adequate to recover as much mineral as possible. Similarly, if a sudden decrease in feed grade occurs, costly flotation reagents will be wasted. A similar situation occurs with depressants used in the flotation process which prevent gangue minerals from floating. If sudden increases or decreases occur in gangue mineral content, it is difficult to adjust the depressants quickly enough to maintain a smooth and economic operation. Leaching, roasting, pelletizing, cobbing, smelting, and all other concentration processes are highly inefficient when substantial uncontrolled changes in concentrator feed occur.

Blending, as a long term consideration, should be used during mine planning to evaluate total reserve tonnage. Ore reserve estimation is the evaluation of contained mineralization through sampling and interpolation. Such an analysis is used to quantify a mineral inventory representing the grade, tonnage, and location of mineralized rock. Blending should be considered in the reserve analysis so as to maximize the total tonnage of mineralized rock which can be considered ore. It is wise to consider the mineral inventory model made during the delineation phase in conjunction with mining capacity and blending opportunities to determine the ore reserve tonnage.

The consistency of mineral product quality and quantity is necessary to command respect in the market place. The credibility of the mining company in the market is achieved by operating mine and mill in a manner which produces consistent product. It is occasionally necessary to blend final concentrate if too much variance exists between different lots. The expense of blending final concentrate lots can be forgone if lot consistency occurs congruently with mine and mill operations.

Financially it is possible to have a climate favorable to producing a small amount of high grade ore. Similarly a large amount of lower grade ore may fulfill financial requirements. The corporate strength, financial assets, debts, liabilities, projections, and business direction all impact this decision. A smooth production and market operation promotes the overall economic health of a company. Consistent concentrate production leads to a corporate image of low risk and high predictability which is the character of a blue chip stock.

Blending can aid in the attempt to maximize the total economic tonnage of reserves when it is used in the evaluation of mine life and total contained tonnage for a mining area. This analysis is used to estimate reserves for the long term life of a mining area and serves a different purpose than the short term blending of ores.

Once the overall optimum cutoff grade or average grade for an entire mine or mining district is established, each reserve area under consideration has different total tonnages and mineralogy which could potentially contribute to the total reserve. The blend of grade and tonnage required from each mining area must be determined to maximize total reserves. A heuristic technique is presented in this thesis which finds the various optimal cutoff grades for each individual mining area given the economic cutoff grade constraints. This results in a heuristic maximum reserve tonnage blend which meets the required overall average grades for the entire mining district.

A literature survey was performed to gather background information concerning mineral deposit delineation, ore reserve analysis, cutoff grade theory, and ore blending. A significant amount of information was found in all areas studied except in the area of blending.

The section following the literature survey contains a theoretical discussion on blending and the various approaches which may be taken to maximize the reserve tonnage in a mining district. It considers the information obtained through the literature survey to arrive at the solution technique.

The section entitled "Solution Technique" describes a step by step heuristic process which can be used to solve blending problems of the type under consideration. It is iterative and allows the user flexibility to add, subtract, or change the variables or constraints.

An example problem is then presented to demonstrate the use of the solution technique and its application to problems involving several limiting constraints, multiple deposits, and multiple contained minerals. The example illustrates the proposed technique utilizing three constraints, eight deposits, and three minerals.

The final chapter summerizes the development of the solution technique, its applications in industry, other possible areas of application, and alternative methods which may be developed to solve similar problems.

# LITERATURE SURVEY

Literature was reviewed in the areas of resource estimation, cutoff grade theory, blending, and applied mathematical techniques to examine blending of different ores from different locations within a mining district. The objective was not to gather all of the information ever generated in these areas, rather to gain a more thorough insight into the work which had already been performed and the possible avenues by which this problem could be approached.

## Resource Estimation

Various delineation processes have been developed to estimate the extent of mineralization within a deposit. Using these processes a mineral inventory model can be generated which shows the grade, tonnage, and location of ore amenable to production. The overall objectives to be achieved in the delineation process are:

1. To give an accurate account of the sampling program and the results obtained during the delineation process.

2. To insure that sampling which has been performed in a mining area is sufficient for sound decisions to be made

about future efforts.

3. To have delineation results presented in a format that can easily be used in further evaluation.

The delineation process involves collecting geologic information within an area, typically by exposing the mineralization for examination through drilling. Geophysical and geochemical techniques can occasionally give quantitative results sufficient for reserve estimation although qualitative rather than quantitative results are obtained from these analysis.

## Geometric and Distance Weighting Methods

Geometric and distance weighting methods used to estimate reserves are discussed in detail by many authors. Work by Watermann and Hazen (1968) indicates that the more popular methods are: (1) triangular, (2) perpendicular bisectors (i.e., polygonal), (3) cross sectional, (4) rectangular blocks, and (5) inverse distance weighting.

The triangular method estimates the grade of a triangular block by averaging the grades from three adjacent drill hole samples. This average is assigned to the area circumscribed by the lines between the holes. It assumes a linear grade relationship between the drill holes.

The method of perpendicular bisectors is used to estimate the grade and tonnage of an area cicumscribed by the intersection of bisecting lines between drill holes. Angular bisectors is a similar technique using lines radiating from a drill hole obtained from bisecting the angles between adjacent drill holes to generate the cicumscribed area. These techniques give greater weight to isolated drill holes.

The results of these techniques are typically difficult to work with as the shapes, areas, and volumes generated in such analysis are different than the dimensions of the ore blocks which will be mined.

The method of cross sectional area is used to estimate grade and tonnage by assuming a linear grade relationship between projected cross sections within a deposit. This technique has the same problem as the triangular method with respect to grade linearity assumptions.

The rectangular block method uses drill hole results from either random location or uniform grid sampling. A rectangular grid is placed over the drill holes and the area circumscribed by the rectangle is assigned the grade of the sample in the rectangle.

Distance weighting techniques such as the inverse distance method, the inverse distance squared method, and the inverse distance to a power method are used to estimate the grade of an ore block by using the weighted average of samples surrounding an area to be estimated. Although this technique is considered to more accurately predict the grades of ore blocks than the previous methods, it has some drawbacks. The number of holes which should be included in the analysis, the power to be used on the distance between the holes, and the size of the block which should be established is up to the user. This implies that the results from such an analysis are somewhat subjective to the users state of mind at the time of analysis and that different results can be obtained by different analysts for the same deposit.

## Geostatistical Methods

Geostatistics (Matheron, 1963; David, 1975; Journal, 1978; Knudsen and Kim, 1978) uses the theory of regionalized variables developed by Matheron in France during the early 1960's. It is a non-bias estimating technique which reduces the variance in the error of the estimation. Geostatistics is the state of the art method to make reserve estimates. It assumes that samples may be spatially correlated and are not necessarily independent of one another. A spatial variance relationship is established to obtain the geostatistical correlation between sample grades and their spatial distribution. This relationship is displayed graphically as a variogram. From the variogram an estimation variance based on the spatial grade distribution can be determined.

Kriging (Krige, 1977) is the technique used to estimate the grade of blocks. The grade of a block is estimated by weighting the grade of sample assays such that the weighted estimation variance is minimized.

## Grade -Tonnage Curves

A mineral inventory can be constructed for a delineated deposit showing the estimated grades and tonnages of selective mining units (SMU's), (Stanley, 1979). A mineral inventory contains information on grade, tonnage, location, and production amenability such as ore hardness, mineral dissemination, gangue constituents, consistency, and any other characteristics which influence the production or quality of the final product. A mineral inventory is typically summarized with grade - tonnage curves.

Grade - Tonnage curves (David, 1975) graphically show the average mineral content and total tonnage contained in a deposit with respect to a specific grade, below which the mineralized rock is no longer considered ore. Grade -

Tonnage curves can be determined from either a statistical model using global data or from a block model using quantities of blocks and block grades in a deposit. The purpose of such curves is to represent the amount of mineral contained in a reserve if only the material of equal or greater value than the specified grade is considered. Geostatistical techniques have been shown to yield a reliable and unbiased estimate in the evaluation of reserves.

Erickson (1985) best summarizes mineral inventory estimation by stating that orebody delineation is the planning process which should yield "the contoured metal (mineral) values in both ore and waste (rock)."

## Cutoff Grade Theory

Early cutoff grade theory assumes that a fixed cutoff grade can be determined for the life of a mine which maximizes profit. Callaway (1958) developed an expression which relates a "break-even grade" (based on cost equals profit) to recovery grade, market price, and the cost of production at the recovery grade. The conclusions from this analysis indicate that maximum profit coincides with the production capacity. Net present value is not accounted for in this analysis.

Vickers (1961) considered marginal cost and marginal revenue in his analysis. He defines cutoff grade as the point that maximizes total profits of an orebody. His analysis assumes a constant production rate. Marginal cost and marginal revenue are evaluated at different cutoff grades, each with an associated average grade and tonnage. Again, net present value is not considered in the analysis.

The concept of "Profcost" (Erickson, 1968) was developed to yield cutoff grades which include the profit required from an operation (before taxes) to cover the investment risk. Profcost analysis concludes that there are two basic ways that cutoff grade can be calculated. One way to calculate cutoff grade is by "evaluating the average mine metal values with the average mining and processing costs either for the mine as a whole or for major portions of the mine". The other cutoff grade calculation involves "incrementally evaluating each block of material, taking into consideration metal values, type of ore material, and location of the ore in the mine". The first technique yields a single cutoff grade value for the mine or part of the mine under consideration. The second method yields dollar values or equivelent metal values for each block of material.

Douglass (1971) makes the assumption that "projects should be evaluated at the lowest acceptable rate of return." This relates the financial stature of a company to the deposit under consideration. A deposit which could deliver a favorable rate of return to a small mining company may often be unfavorable to other companies which can invest in other prospects which have much higher rates of return. Douglass makes the same fundamental assumptions about cutoff grade similar to those of the previous authors, however, he optimizes the cutoff grade to maximize net present value rather than maximizing profits.

Barry (1922) suggests that high grading a deposit in the early years of a mine is the optimal policy since such a policy returns the owners investments more rapidly than using a fixed cutoff grade for the mine life. His analysis is further justified when net present value is taken into account.

Halls et al., (1969) incorporates the concept of depreciation into the cutoff grade calculation. Cutoff grade in this analysis is the grade of ore which covers the costs of mining and processing, the cost of depreciation, and the "cost" of a predetermined profit. The optimal cutoff grade is the grade which produces the maximum net present value for the deposit. Mine life is fixed in the analysis and the chosen cutoff grade is to be used globally.

Henning (1963) observed that choosing the optimal cutoff grade is dependent upon whether or not the life of a mine is fixed. This can be related financially to the mine and mill capacities and the influence of these capacities on the rate at which ore can be processed.

John (1985) presents an analysis of cutoff grade based on maximizing net present value. His assumptions state that concentrator capacity is fixed, the mining rate is variable, and no market restrictions exist with respect to the ability to sell all concentrate produced. One of his main conclusions implies that the market price of the mineral under consideration has a significant effect on the cutoff grade analysis. That is to say, cutoff grade is extremely sensitive to commodity price.

Wells (1978) presents an optimization analysis which considers detailed effects on the cutoff grade. He points out that a large number of financial, production, and marketing parameters must be evaluated and considered in performing a cutoff grade analysis. This paper is intended to be used as an outline which provides guidelines for optimizing the outcome of a property evaluation and, inturn, optimizing the cutoff grade to be used during exploitation. He emphasizes the importance of scheduling ore extraction in a sequence which removes progressively lower grades of ore during the life of the mine.

Lillico (1973) considers a cutoff grade strategy which maximizes net present value. His work considers a dynamic cutoff grade theory rather than a static cutoff grade theory. This implies that cutoff grade is allowed to vary to achieve the maximum net present value rather than considering cutoff grade to be fixed. He shows by example that material which does not meet a global cutoff grade may be economic to mill if no other ore is available. If a block of material is not worth mining as ore but must be mined anyway to expose ore, then the mining cost for such a block is incurred whether the material is milled or wasted. The mining cost is sunk and inturn the calculation of cutoff grade should be performed without the mining cost. This lowers the marginal break-even cutoff grade for the block of ore under consideration.

Henning (1963) assumes that the distribution of grade within ore blocks follows an exponential law. In his example, mathematical optimum cutoff grades were determined for hypothetical economic objectives which maximized the spread between the annual operations profits and the investment costs. His paper shows that maximum profit occurs when a declining cutoff grade policy is adhered to.

Lane (1964) developed a technique which yields an optimum cutoff grade directly related to prices, costs, capacities and the grade distribution of the deposit. It was shown that cutoff grade effects determination of mine, mill, and market capacity. These different capacity constraints inturn effect the break-even cutoff grade requirements (i.e., the solution to the optimal cutoff grade will be iterative). He has also shown that maximum net present value is achieved with a declining cutoff grade policy. In 1979, Lane analyzed the effects of price and cost variations on the cutoff grade decision. He concluded that the optimal cutoff grade policy is to increase cutoff grade when prices are high and to decrease cutoff grade when prices are low. This is in agreement with the most basic laws of economics, namely the law of supply and demand. A shortage will occur and prices will rise if demand is high and supply is fixed. If capacities are fixed, the way to meet the increased demand is to produce more metal by raising the cutoff grade. The inverse relationship exists for oversupply, low prices, and sluggish demand. Lowering cutoff grade at a fixed capacity would reduce supply and stabilize the market.

Johnson (1969) examined static and dynamic cutoff grade theories. He concluded that for a given set of constraints an extraction sequence and schedule could be generated (using linear-integer programming techniques) which produces a maximum net present value for a deposit. A definite algorithm was not developed to solve for the optimal cutoff grade; rather a technique was developed which gives the optimum net present value decision as to mine, mill, or waste based on the specific and current situation of the operation. If an optimal sequence and schedule of decisions can be made considering the entire mineral inventory, the mine - mill logistics, and the current financial status of costs and revenues regarding the extraction and beneficiation or wasting of material, then that sequence will yield the maximum net present value without ever calculating a cutoff grade. Cutoff grade calculations are indeed made in the decision process which produces the sequence and schedule; however, the cutoff grade may be different for each block and may change many times in the evaluation considering its economic worth as it relates to other blocks and mining alternatives.

Roman (1973) did not concern himself with cutoff grades directly. He used a dynamic programming technique to determine an optimum production rate which results in the highest net present value over a mine's life. His analysis indicates that small operations with short lives and large profit margins show the greatest net present value increase when dynamic programming is used to determine the production schedule.

Blending

Barnes (1981) has described the use of linear programming as an economic planning tool to assist in optimizing operations by determining preferred blend quantities from different locations within a pit. Blending alternatives are optimized using linear programming techniques to insure that the average quality of ore to be mined will maintain a mill feed as close to the selected target as possible. Topuz (1979) descibes a linear programming technique known as "Goal Programming" which performs exactly this task. Goal Programming is a linear programming formulation method which allows for a progressive penalty to be paid as the blend of ores stray from the optimal cutoff grade. Typical linear programming constraints used in the blending problem are:

1. Entire blocks are assumed to be mined (i.e., integer values are used for blocks and no partial block removal is allowed).

2. A minimum tonnage will be produced for mill feed.

3. A maximum tonnage can be removed from any point in the mine subject to equipment constraints.

4. The mill has a maximum feed capacity.

5. Mill feed grade is maintained as close as possible to the optimal grade.

Other constraints can be formulated depending on the specific requirements of the mine.

Cutoff grade is an extremely detailed topic which takes into consideration many aspects of finance, production, and marketing involved in the development of a deposit. Cutoff grade theory has progressed from simple static and global analysis. Currently it may be possible to operate a mine using an optimal sequence and schedule arrived at by dynamic programming techniques (Dagdelen, 1985) without ever getting involved with global cutoff grade analysis.

## Summary

Work in the area of ore body delineation and mineral inventory analysis was reviewed to evaluate the basis of cutoff grade theory and blending. Literature concerning cutoff grade shows that many approaches have been used to determine the best cutoff grade for a mine. The literature indicates that determination of the "best" or "optimal" cutoff grade is directly dependent on financial, production, and market factors associated to the mining venture. The definition of cutoff grade used in this paper implies that a marker (measured in terms of mineral content) exists which differentiates blocks of ore from blocks of waste. This definition is consistent with the definition given by Taylor (1972); "Cutoff grade is any grade that for any specified reason is used to separate two courses of action, e.g., to mine or to leave, to mill or to dump."

Very little work has been done in the area of blending to determine the correct cutoff grades which achieve a desired blend. Mol and Gillies (1984) present a graphic solution technique for three deposits and three constraints. Their solution is heuristic and iterative in nature. It requires (1) retrieving average grades and tonnages from cutoff grade curves, (2) using marginal analysis to estimate the combined grade and tonnage, (3) determining which mines to draw more ore from and which mines to draw less ore from, and (4) evaluating the blend of ore which is obtained by performing the analysis. These steps are iterated until the results of blending the ores are sufficiently close to the tight constraint(s).

Blending ores has a significant impact on maintaining a smooth and consistent feed grade to the mill. To obtain the proper blend of ores it is necessary for the cutoff grade issue to be resolved. Limiting constraints can be formulated once an economic optimal cutoff grade is determined in the classical sense. The unresolved problem is to determine (analytically in an open form) the cutoff grade for each of several mining areas such that the blend of ores from several mines (or mining areas in the same mine) maximize tonnage yet meet the overall limiting constraints.

# THEORETICAL DISCUSSION

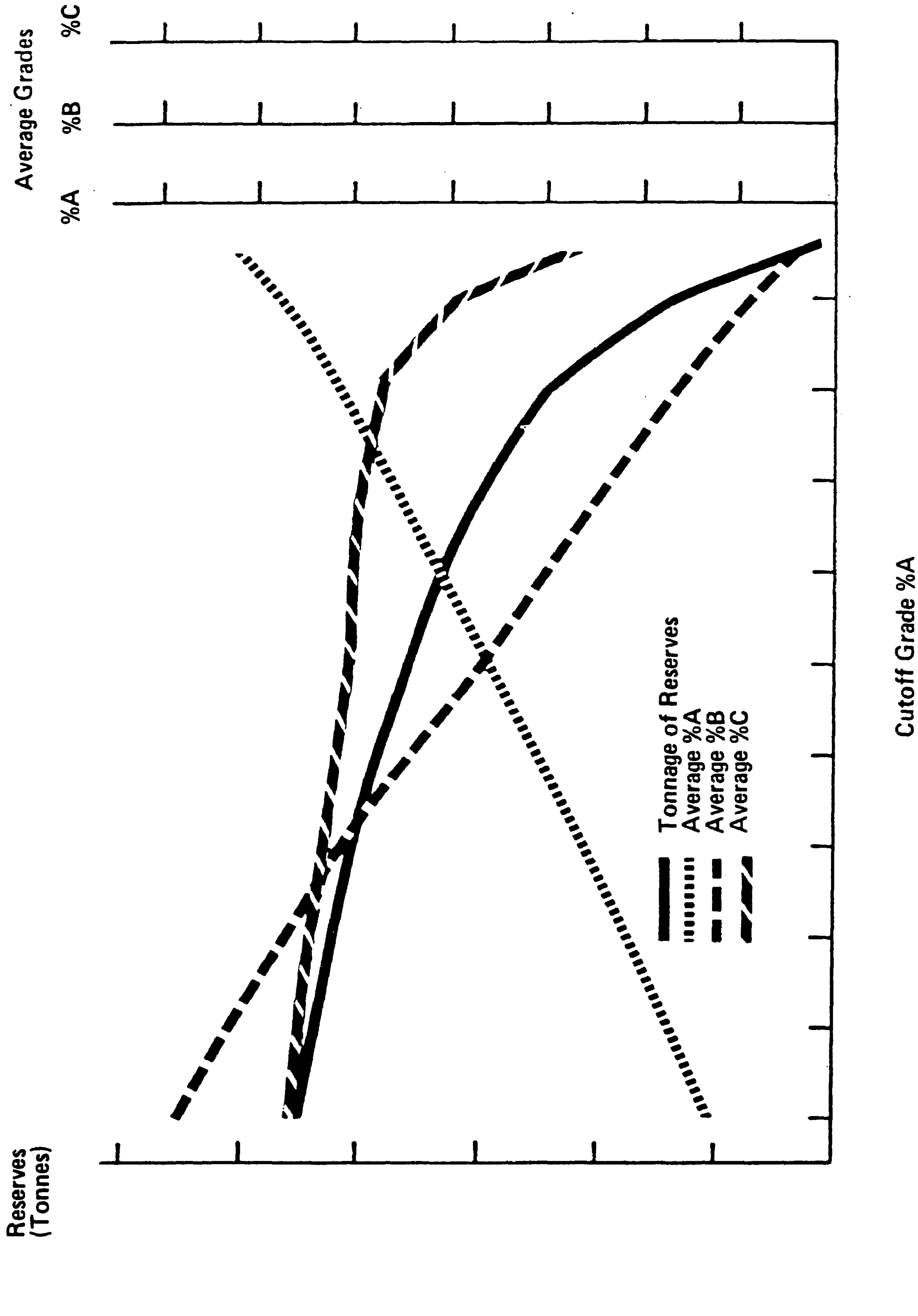
Various techniques can be employed to define and quantify ore reserve tonnage during the delineation process. Techniques used to make these estimates are the uniform spacing method, mining blocks, polygons (both perpendiculars at midpoints between holes and angular bisectors), triangles, random rectangular blocks, cross sections, and Geostatistical Kriging. From these models come estimates of tonnages and mineral grades for the entire deposit. To use these estimates, block models are built which show the location, tonnage, and grade of mineralized blocks in the deposit. The size of these blocks is decided upon by considering the quality of the estimates, the homogeneity of the mineralization, and the probable mining method which will be used to extract the ore. These blocks are typically called selective mining units (SMU'S). Once a mineral deposit has been targeted and delineated, a graphic technique can be used to describe the grade and tonnage of the reserves. Cutoff grade curves are generated from the information gathered during the delineation process.

To produce cutoff grade curves, the total number of SMU's at, or above, some specific grade (typically for the valuable mineral) are summed to find the total tonnes ( 1 metric tonne = 1.102 short tons) at, or above, the specified grade. The average valuable mineral content and the average gangue mineral content are independently summed for the same blocks found to be at, or above, the specified grade. The average grade of mineral above the specific grade is found by dividing the mineral content (in tonnes) by the total tonnes above the specified grade. At intervals over a selected range of cutoff grades, estimates are made of total tonnage above cutoff grade and average mineral content above cutoff grade. Cutoff grade curves are generated from this analysis. Such curves can be produced for an area within a mine, for an entire mine, or for an entire mining district. An example of a cutoff grade curve is given for a generic ore deposit Q, containing tonnes of valuable mineral A and gangue minerals B and C, (see Figure I).

Economic constraints are imposed on the grade of valuable or gangue minerals by the mining logistics, the beneficiation plant capacities, and the market requirements. Due to these limitations, mines which have an abundance of valuable mineral contained somewhere within the deposit may not produce an acceptable product because the grade of the gangue minerals exceed the acceptable limits. Similarly, these may be mines which are well within the acceptable gangue grade requirements but may be slightly under the limits in valuable mineral content to allow economic

FIGURE I

*GRADE - TONNAGE CURVE FOR HYPOTHETICAL OREBODY Q*



exploitation. If blending is used there is a possibility to meet all grade constraints by using proper proportions of material from the various ore locations. Provided that cutoff grade constraints are met, the objective of mine owners in this scenario is to maximize the reserve tonnage considering all mining areas combined.

Considering the generic cutoff grade curve situation described, assume there are N mines with similar mineralization, each mine with tonnage T to meet an overall minimum acceptable valuable mineral A grade of X, and maximum acceptable grades Y and Z of gangue minerals B and C respectively. Each mine has a minimum total tonnage of zero and a maximum total tonnage Tmax. Considering the objective of mine owners is to maximize the total reserves of all mining areas, this problem is mathematically stated:

$$\begin{aligned} \text{Maximize:} \quad & \sum Ti \\ \text{Subject to:} \quad & \sum Ai > \sum X\,(Ti) \\ & \sum Bi < \sum Y\,(Ti) \\ & \sum Ci < \sum Z\,(Ti) \end{aligned}$$

where: Ti is the total tonnage from the ith mining area
Ai is the average mineral content of A for the ith deposit expressed as a quadratic equation in Ti
Bi is the average mineral content of B for the ith deposit expressed as a quadratic equation in Ti
Ci is the average mineral content of C for the

ith deposit expressed as a quadratic equation in $T_i$
X is the total allowable percentage of mineral A in the total tonnage
Y is the total allowable percentage of mineral B in the total tonnage
Z is the total allowable percentage of mineral C in the total tonnage

Several theoretical guidelines to solution techniques which may apply to this problem have been developed (Woolsey, 1975; Hillier and Lieberman, 1967; Churchman et al., 1957). There is currently no known closed form solution technique which solves this problem. The different techniques have advantages and disadvantages which make each technique more or less favorable to use.

The graphic solution technique (Mol and Gillies, 1984) allows the user high visibility and understanding of the results obtained from the analysis. It is an iterative method which requires entering the graph, picking average grade and tonnage values, and estimating the total tonnes. A drawback to this technique is that the complexity of solution increases substantially as the number of mining areas under consideration increases.

Analysis using line segments to approximate curves (Jacoby et al., 1972; Hillier and Lieberman, 1967) is a technique used to determine optimum points by breaking the curves into linear segments and optimizing on these segments using linear programming techniques. This technique is also iterative by definition to achieve an acceptable quality of estimate. Two significant drawbacks with this technique are the large number of variables which must be handled to solve a relatively small problem using a linear programming format (Dantzig, 1963) and the lack of visibility given to the problem during the formal analysis. After entering variables and the respective coefficients into a linear programming problem solver and arriving at a solution, the results come back from the "black-box" with no easily interchangeable interface with the original curves. It is difficult to gain insight into what the algorithm is doing until the values are replotted and another iteration is performed. Much manipulation is required if both convex and concave curves are being considered simultaneously.

Use of Lagrange multipliers (Woolsey, 1975; Jacoby et al., 1972; Adby and Dempster, 1974) is a very powerful solution technique used to solve nonlinear problems. It has the distinct advantage of obtaining a closed form analytical solution which requires no iterative technique. The drawback of Lagrange multipliers is the large number of solution possibilities which arise in the formal analysis. If the problem is not positive-definite (i.e., if all terms in a polynomial to be evaluated do not have a positive coefficient) there is the possibility of not getting a solution at all. Over 500 equations were generated for the simple problem of blending ores from 3 mines and observing 3 constraints. Although many of the equations can be eliminated by inspection, the final solution step for such a problem requires the solution of multiple nonlinear equations with a similar number of unknowns. Another drawback to this analysis is the total lack of interface with the original cutoff grade curves which leaves the user totally dependent on a "black-box" analysis.

## SOLUTION TECHNIQUE

The following solution technique was developed to solve this constrained nonlinear optimization problem without the use of Lagrange Multipliers. It is labor intensive in that a substantial amount of data related to the various mining areas is generated during the process. It is easily adaptable to larger problems which consider a large number of mining areas. This solution technique keeps the user directly in touch with the data gathered from the cutoff grade curves and the reasoning used to solve the problem. Although it is not an iterative technique in the true sense, it does require original estimates to be made and updated estimates to be considered until the optimal solution is obtained.

Marginal value analysis (Koopmans and Reiter, 1951) and the Kuhn-Tucker sufficiency conditions (Hillier and Lieberman, 1967) are the foundation of this technique. The marginal value associated with the rate of change of average mineral content with respect to total tonnage is equal for all mining areas except when the marginal value cannot be met. If the valuable mineral abounds in a particular mining area the analysis will indicate that the entire tonnage should be extracted. Conversely, the analysis will indicate that no tonnage should be used from an area if the rate of change of average mineral content with respect to total tonnage is below an acceptable marginal value.

Step 1: Generation of average mineral content curves from cutoff grade curves.

Starting with cutoff grade curves as previously defined, new curves are generated which show the average mineral content at any cutoff grade. Conversion to content is necessary to implement this technique. Average mineral content curves are generated by multiplying the total tonnage at a specific cutoff grade by the average percent of mineral at that cutoff grade. This is done for each of the valuable and gangue minerals at several specific cutoff grades over a selected range of cutoff grades to insure an accurate representation of contained mineral.

Step 2: Observation of total tonnage and corresponding average mineral content.

Observations are taken and a list is made of total tonnage and the respective average mineral content for the valuable and gangue minerals for each mineral content curve and total tonnage curve at specific points within the cutoff grade range under consideration.

Step 3: Second order polynomial regression analysis to solve for average mineral content in terms of total tonnage.

A second order polynomial relationship is found between total tonnage in a specific mining area (total tonnes being the independent variable) and average mineral content in that mining area for all valuable and gangue minerals. This allows average mineral content to be expressed as a function of total tonnage.

Step 4: Use of a computerized spreadsheet to show average mineral content given total tonnage at multiple estimating points.

The equations of the curves relating total tonnage to average mineral content are entered into a spreadsheet program. These formula are used to estimate the average mineral content from the total tonnage for any mineral in any deposit. To display a range of total tonnages and related average mineral contents for a specific mining area, total tonnages at a desired interval over a range are entered into the first column of the spreadsheet; the dependent variables (i.e., the valuable and gangue average mineral tonnages) are generated in the next columns from the second order polynomial equations.

Step 5: Taking derivatives of the quadratic equations for all minerals and solving for the rate of change (marginal value) of the average mineral content with respect to total tonnage.

The derivatives of each polynomial are taken with respect to total tonnes and their calculated values are shown in the next column. This is done for the same intervals and ranges for all mining areas as described in Step 4.

Step 6: Taking rates of change (marginal value) between one average mineral content and another average mineral content at the same total tonnage for a specific mining area.

To gain visibility to the rate of change of one mineral content to another, the last columns show the ratios of the derivatives of all the minerals under consideration to the others. This gives the rate of change (marginal value) between one average mineral content and another average mineral content at the same total tonnage within a mining area. This is done for the same intervals and ranges for all mining areas as described in Step 4.

Step 7: Gathering tonnages from all mining areas at a specific marginal value to find minimum and maximum tonnages which meet the overall cutoff grade constraints.

Marginal analysis is applied in conjunction with the Kuhn-Tucker sufficiency conditions to determine the optimal blend of ores which maximize total tonnage yet meet the system constraints. Basically, the Kuhn-Tucker conditions state that the marginal values of a tight constraint (or of tight constraints) must be equal at optimality; a principle which holds true in both linear and nonlinear optimization. Deviation from this rule occurs if the marginal value of average mineral content with respect to tonnage (for one or several of the mining areas under consideration to be blended) is greater or less than the largest or smallest marginal value of all other mining areas. If a mining area has a marginal value greater than all other mining areas and the tight constraint requires some minimum value (i.e., the blend of mineral must be greater than or equal to the constraint), then this entire area should be mined since it can only assist in increasing the overall value of the blend above the constraining value. Similarly, if a mining area has a marginal value less than some maximum value (i.e., the blend of mineral must be less than the constraint), then this entire area should be mined since it can only assist in reducing the average mineral content of the overall blend below the constraining value. Diametrically opposed to these situations is the occurence of marginal values for a particular mining area (or mining areas) which are greater or less than the greatest or least acceptable marginal values at the optimal blend. If a mining area has a marginal value greater than the largest acceptable marginal value for the other deposits, and the tight constraint is a maximum (i.e., the blend of minerals must be less than the tight constraint), then material should not be removed from that deposit since it cannot contribute to meeting that constraint. Similarly, if a mining area has a marginal value smaller than the least acceptable marginal values for the other deposits, and the tight constraint is a minimum (i.e., the blend minerals must be greater than or equal to the tight constraint), then material should not be removed from that deposit since it cannot contribute to meeting that constraint.

The minimum possible total tonnage (which is the lower bound of the mathematical statement) is found by evaluating each mining area separately considering all constraints. The maximum possible tonnage (which is the upper bound of the mathematical statement) is found by evaluating each constraint separately and finding the smallest tonnage allowed by any one of the constraints for all mining areas combined. Taking one constraint at a time, the tonnage of average mineral content associated to that constraint and the total tonnage, which have the same marginal value for all reserves, are extracted from the tables. If the percentage of valuable or gangue mineral (depending on which constraint is under consideration), is greater or less than the required constrained valued, another marginal value is chosen and the analysis is redone. These steps are repeated until the marginal value is found at which point the average mineral content (and inturn the percentage of mineral content with respect to total tonnage) match the value of the constraint under consideration.

If the problem has only one tight constraint, that constraint will be the one which allows the minimum tonnage to be extracted from all mining areas as a blend. If more than one constraint is tight, it is necessary to optimize the system using the equal marginal values of the ratios of the various margins for the constraints which appear to be tight.

Step 8: Determination of optimum cutoff grades for all mining areas by inspection and correlation to the optimum total tonnages.

After all constraints have been evaluated and the maximum allowable tonnage is found for the tight constraint, the cutoff grades for the various mining areas are found by taking the total tonnage associated with the optimum blend for each mining area and extracting the cutoff grade associated to that tonnage for that mining area from the original cutoff grade curves.

# APPLICATION OF THE SOLUTION TECHNIQUE

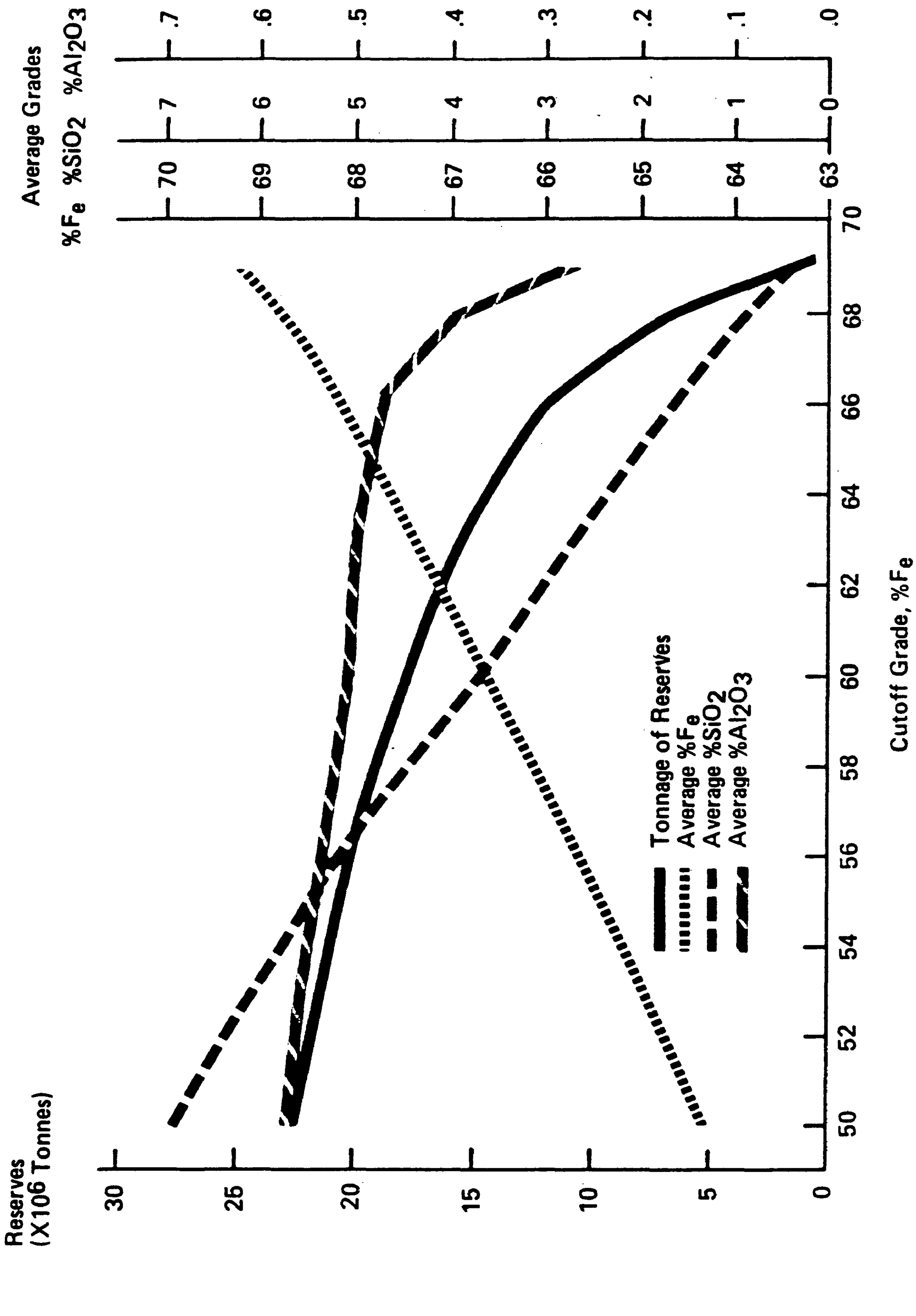
To evaluate the usefulness of the previously described technique, mining areas described in the paper by Mols and Gillies (1984) along with 5 other hypothetical orebodies are considered as prospective contributers to an optimal blend of ore. Figure II shows the cutoff grade curves for reserve #1 orebody. Cutoff grade curves for all reserves are shown in Appendix I. Depicted along the ordinate is cutoff grade as an average percent Fe. Total tonnes above cutoff grade are depicted along the left abscissa. Average percent Fe, SiO2, and Al2O3 are shown along the right abscissa of the graph. From these curves, it is possible to determine the total tonnage above cutoff grade and the average percent Fe, SiO2, and Al2O3 grades above cutoff grade.

Step 1: The average percent above cutoff grade is converted to average mineral content above cutoff grade so that the values can be mathematically manipulated. This is done by multiplying the total tonnes above cutoff grade times the average percent of mineral above cutoff grade to determine the average mineral content above cutoff grade.

Step 2: This process is applied to estimate the average mineral content of the Fe, SiO2, and Al2O3 at several points along the range of cutoff grades for all

FIGURE II

*GRADE-TONNAGE CURVES FOR RESERVE 1*



deposits. Results of this manipulation are shown for reserve #1 in Table I. Results for all reserves are shown in Appendix II.

Step 3: A quadratic equation is found for each average mineral content curve using total tonnes above cutoff grade as the independent variable and average mineral content above cutoff grade as the dependent variable. Equations resulting from the second order polynomial regression analysis of total tonnes versus average Fe, average SiO2, and average Al2O3 mineral content for reserve #1 are shown in Table II. Quadratic equations for the mineral suites in all the deposits are given in Appendix III. The regression analysis estimates are very good considering the correlation coefficient and goodness of fit (defined as the correlation coefficient squared) achieved in the regression of the curves. This is true for all deposits under consideration.

Step 4: Using the quadratic equations for all of the minerals being considered, a spreadsheet is developed using Lotus 123 to show the average mineral content as a function of total tonnes. Depicted is a range of total tonnes above cutoff grade and estimates of average mineral content above cutoff grade for all minerals and all deposits. This

Table I

Mineral Content Above Cutoff Grade
for Reserve #1

Reserve #1 (Tonnes x '10^6)

| Cutoff Grade | Total Tonnes | %Fe | Fe Tonnes | %SiO2 | SiO2 Content | % Al2O3 | Al2O3 Tonnes |
|---|---|---|---|---|---|---|---|
| 50 | 22.30 | 64.3% | 14.34 | 7.0% | 1.55 | 0.6% | 0.127 |
| 52 | 22.00 | 64.7% | 14.23 | 6.3% | 1.39 | 0.6% | 0.123 |
| 54 | 21.00 | 65.1% | 13.67 | 5.8% | 1.22 | 0.5% | 0.113 |
| 56 | 20.000 | 65.6% | 13.12 | 5.2% | 1.04 | 0.5% | 0.106 |
| 58 | 18.000 | 66.1% | 11.90 | 4.5% | 0.81 | 0.5% | 0.094 |
| 60 | 17.500 | 66.6% | 11.66 | 3.7% | 0.65 | 0.5% | 0.088 |
| 62 | 16.500 | 67.1% | 11.07 | 3.1% | 0.51 | 0.5% | 0.082 |
| 64 | 14.50 | 67.6% | 9.80 | 2.4% | 0.34 | 0.5% | 0.071 |
| 66 | 12.00 | 68.1% | 8.17 | 1.7% | 0.20 | 0.5% | 0.057 |
| 68 | 7.00 | 68.7% | 4.81 | 1.0% | 0.07 | 0.4% | 0.027 |

Table II

Quadratic Regression Analysis for Reserve #1

Reserve #1

| | |
|---|---|
| Fe: | $-398.3 + .779T - (0.523\ E{-}5)T^2$ |
| Goodness of Fit: | .99992 |
| Correlation Coefficient: | .99996 |
| $SiO_2$: | $577.3 - .122T + (0.733\ E{-}5)T^2$ |
| Goodness of Fit: | .99414 |
| Correlation Coefficient: | .99707 |
| $Al_2O_3$: | $-1.838 + (0.361\ E{-}2)T + (0.927\ E{-}7)T^2$ |
| Goodness of Fit: | .99748 |
| Correlation Coefficient: | .99874 |

spreadsheet is given for reserve #1 in Table III and for all deposits in Appendix IV. The first column shows the tonnage used in the quadratic equation to estimate the mineral content. The second, third, and fourth columns give the quadratic regression estimate of the average Fe, average $SiO_2$, and average $Al_2O_3$ mineral content respectively. Adding up these tonnages indicates that another constituent makes up the remainder of the tonnage not accounted for; however, it is not involved in the constraints and need not be considered in the analysis other than as part of the total tonnage.

Step 5: To perform marginal analysis it is necessary to determine the rate of change of average mineral content with respect to the tonnage for all minerals under consideration. The derivatives of average mineral content (i.e., the derivatives of the quadratic equations) with respect to total tonnes were taken to evaluate the rates of change. These derivative functions are given for reserve #1 in Table IV and for all deposits in Appendix V.

Steps 6: Another spreadsheet is constructed using Lotus 123 showing tonnage, $dFe/dT$, $dSiO_2/dT$, and $dAl_2O_3/dT$ for a range of tonnages for all deposits. This allows comparisons to be made between the rates of change of

Table III

Quadratic Equation Estimates of Mineral Content
for the Mineral Suite in Reserve #1

Reserve #1

| Tonnes | Fe Function | Percent Fe | Si Function | Percent Si | Al Function | Percent Al |
|---|---|---|---|---|---|---|
| 23000 | 14758 | 64.2% | 1637 | 7.1% | 130 | 0.6% |
| 21000 | 13660 | 65.0% | 1237 | 5.9% | 115 | 0.5% |
| 19000 | 12520 | 65.9% | 896 | 4.7% | 100 | 0.5% |
| 17000 | 11338 | 66.7% | 613 | 3.6% | 86 | 0.5% |
| 15000 | 10114 | 67.4% | 389 | 2.6% | 73 | 0.5% |
| 13000 | 8848 | 68.1% | 224 | 1.7% | 61 | 0.5% |
| 11000 | 7540 | 68.5% | 117 | 1.1% | 49 | 0.4% |
| 9000 | 6191 | 68.8% | 69 | 0.8% | 38 | 0.4% |
| 7000 | 4800 | 68.6% | 79 | 1.1% | 28 | 0.4% |
| 5000 | 3367 | 67.3% | 148 | 3.0% | 19 | 0.4% |
| 3000 | 1892 | 63.1% | 276 | 9.2% | 10 | 0.3% |

Table IV

Derivatives of the Quadratic Regression Analysis for the Mineral Suite in Reserve #1

Reserve #1

Fe: 0.779 - 2*(0.523 E-5)T

SiO2: - 0.122 + 2*(0.733 E-5)T

Al2O3: 0.361 E-2 + 2*(0.927 E-7)T

average mineral content with respect to total tonnage for all mineral constituents in all deposits. Table V shows this spreadsheet for reserve #1. The spreadsheets for all deposits are given in Appendix VI.

To show the rates of change between the various mineral constituents at various tonnages another spreadsheet is prepared using Lotus 123. This spreadsheet gives the tonnage in the first column followed by the ratios of dFe/dSiO2, dFe/dAl2O3, and dSiO2/dAl2O3 in the second, third, and fourth columns respectively. These ratios are required for the analysis if more than one constraint is tight. Table VI shows this spreadsheet for reserve #1. Appendix VII contains the spreadsheet for all the deposits.

Step 7: With all the aforementioned spreadsheets complete, it is possible to determine the maximum tonnage which can be considered in reserve given the Fe, SiO2, and Al2O3 constraints provided blending is allowed. The constraints placed on the reserves indicate that a minimum of 66% Fe be contained in the ore with a maximum of 3.4% SiO2 and 1.2% Al2O3.

The problem may be stated as follows:

Maximize: $\sum Ti$

Subject to: $\sum Fi > 0.660*( \sum Fi)$

Table V

Quadratic Equation Derivative Estimates of Mineral Content for the Mineral Suite in Reserve #1

Reserve #1

| Tonnage | Fe Tonnage | dFe/dT | Si Tonnage | dSi/dT | Al Tonnage | dAl/dT |
|---|---|---|---|---|---|---|
| 23000 | 14758 | 0.54 | 1637 | 0.215 | 130 | 0.0079 |
| 21000 | 13660 | 0.56 | 1237 | 0.185 | 115 | 0.0075 |
| 19000 | 12520 | 0.58 | 896 | 0.156 | 100 | 0.0071 |
| 17000 | 11338 | 0.60 | 613 | 0.127 | 86 | 0.0068 |
| 15000 | 10114 | 0.62 | 389 | 0.097 | 73 | 0.0064 |
| 13000 | 8848 | 0.64 | 224 | 0.068 | 61 | 0.0060 |
| 11000 | 7540 | 0.66 | 117 | 0.039 | 49 | 0.0056 |
| 9000 | 6191 | 0.69 | 69 | 0.009 | 38 | 0.0053 |
| 7000 | 4800 | 0.71 | 79 | -0.020 | 28 | 0.0049 |
| 5000 | 3367 | 0.73 | 148 | -0.049 | 19 | 0.0045 |
| 3000 | 1892 | 0.75 | 276 | -0.079 | 10 | 0.0042 |

Table VI

Marginal Values (Rates of Change) Between
Various Minerals in Reserve #1

Reserve #1

| Tonnage | dFe/dSi | dFe/dAl | dSi/dFe | dSi/dAl | dAl/dFe | dAl/dSi |
|---|---|---|---|---|---|---|
| 23000 | 2.51 | 68.45 | 0.40 | 27.27 | 0.015 | 0.037 |
| 21000 | 3.02 | 74.62 | 0.33 | 24.71 | 0.013 | 0.040 |
| 19000 | 3.72 | 81.43 | 0.27 | 21.88 | 0.012 | 0.046 |
| 17000 | 4.75 | 88.99 | 0.21 | 18.75 | 0.011 | 0.053 |
| 15000 | 6.39 | 97.42 | 0.16 | 15.24 | 0.010 | 0.066 |
| 13000 | 9.45 | 106.89 | 0.11 | 11.31 | 0.009 | 0.088 |
| 11000 | 17.14 | 117.60 | 0.06 | 6.86 | 0.009 | 0.146 |
| 9000 | 72.58 | 129.82 | 0.01 | 1.79 | 0.008 | 0.559 |
| 7000 | -35.52 | 143.88 | -0.03 | -4.05 | 0.007 | -0.247 |
| 5000 | -14.78 | 160.24 | -0.07 | -10.85 | 0.006 | -0.092 |
| 3000 | -9.52 | 179.52 | -0.10 | -18.85 | 0.006 | -0.053 |

$$\sum Si < 0.034*( \sum Si)$$

$$\sum Ai < 0.012*( \sum Ai)$$

where: Ti is the total tonnes for Reserve i.
Fi is the average Fe content in tonnes for Reserve i.
Si is the average SiO2 content in tonnes for Reserve i.
Ai is the average Al2O3 content in tonnes for Reserve i.

Each deposit is evaluated individually considering all constaints to obtain a lower bound. Using estimated values of average mineral content from Appendix IV, Table VII is constructed to find the lower bound. Notice that various constraints are tight for the different deposits. The results of this analysis indicates that a minimum total reserve tonnage of roughly 41 million tonnes could be obtained from the deposits individually. Each deposit is estimated for possible reserves considering only one constraint at a time to obtain an upper bound. Table VIII shows the maximum total tonnage which could be obtained by enforcing only one constraint at a time. Roughly 64 million tonnes, 94 million tonnes, and 103 million tonnes of ore could be considered as reserves by independently enforcing the Fe, SiO2, and Al2O3 constraints respectively. A reserve of 64 million tonnes of ore is the most tonnage which could be obtained considering one constraint at a time, the Fe

Table VII

Maximum Tonnage By Inspection Considering
Fe, SiO2, and Al2O3 - One Reserve at a Time

| Reserve | Total | Fe | % Fe | SiO2 | % SiO2 | Al2O3 | % Al2O3 | Tight Constraint |
|---|---|---|---|---|---|---|---|---|
| 1 | 16600 | 11100 | 66.9% | 568 | 3.4% | 83 | 0.5% | Si |
| 2 | 0 | NA | NA | NA | NA | NA | NA | Fe & Al2O3 |
| 3 | 620 | 410 | 66.1% | 21 | 3.4% | 6 | 1.1% | Si |
| 4 | 14420 | 9510 | 65.9% | 119 | 0.8% | 87 | 0.6% | Fe |
| 5 | 0 | NA | NA | NA | NA | NA | NA | Al2O3 |
| 6 | 4300 | 2890 | 67.2% | 146 | 3.4% | 28 | 0.6% | Si |
| 7 | 5000 | 3510 | 70.1% | 170 | 3.4% | 36 | 0.7% | Si |
| 8 | 0 | NA | NA | NA | NA | NA | NA | Al2O3 |

Table VIII

These are the Maximum Tonnages Available Considering One Constraint at a Time

Reserve Tonnages @ 66% Fe

| Reserve | Tonnage | Fe | % Fe |
|---|---|---|---|
| 1 | 18750 | 12360 | 65.9% |
| 2 | NA | NA | NA |
| 3 | 650 | 430 | 66.2% |
| 4 | 14420 | 9510 | 66.0% |
| 5 | 6400 | 4220 | 65.9% |
| 6 | 9300 | 6130 | 65.9% |
| 7 | 12670 | 8360 | 66.0% |
| 8 | 1740 | 1150 | 66.1% |
| | 63930 | 42160 | 65.9% |

Reserve Tonnages @ 3.4% SiO2

| Reserve | Tonnage | SiO2 | % SiO2 |
|---|---|---|---|
| 1 | 16600 | 570 | 3.4% |
| 2 | 8330 | 280 | NA |
| 3 | 620 | 20 | 3.2% |
| 4 | 30000 | 1030 | 3.4% |
| 5 | 5200 | 180 | 3.5% |
| 6 | 4300 | 150 | 3.5% |
| 7 | 5000 | 170 | 3.4% |
| 8 | 24200 | 820 | 3.4% |
| | 94250 | 3220 | 3.4% |

Reserve Tonnages @ 1.2% Al2O3

| Reserve | Tonnage | Al2O3 | % Al2O3 |
|---|---|---|---|
| 1 | 23000 | 130 | 0.6% |
| 2 | NA | NA | NA |
| 3 | 770 | 9 | 1.2% |
| 4 | 42000 | 472 | 1.1% |
| 5 | NA | NA | NA |
| 6 | 12800 | 150 | 1.2% |
| 7 | 24500 | 240 | 1.0% |
| 8 | NA | NA | NA |
| | 103070 | 1001 | 1.0% |

constraint being the tight constraint in this analysis. Hence the optimal solution is bounded by a lower limit of roughly 41 million tonnes and an upper limit of roughly 64 million tonnes.

At optimality the marginal rate of change of the average mineral content with respect to total tonnes for the tight constraint must be equal (except as previously described) for all mining areas. The marginal values of average mineral content for one mineral with respect to another must be equal if more than one constraint is tight. At least one constaint will be tight at optimality.

Examining the Fe constraint, tonnages estimated for equal values of dFe/dT are summed for the Fe content and the respective total tonnage. The average Fe content is divided by the total tonnage and an average percent Fe is determined. If the average percent Fe determined in the analysis is too low, the dFe/dT is raised and the process of evaluating the average percent Fe is reworked until 66% Fe is obtained. Table IX shows the respective tonnages required from each mine to obtain 66% Fe for the sum of the reserves. Table X and Table XI show the results of the same analysis performed for the SiO2 and the Al2O3 constraints respectively.

From these analysis, the constraint which produces the

Table IX

Maximum Obtainable Tonnages Considering the Fe Constraint

| Reserve | Total Tonnes | Fe Tonnes | % Fe | SiO2 Tonnes | % SiO2 | Al2O3 Tonnes | % Al2O3 |
|---|---|---|---|---|---|---|---|
| 1 | 11000 | 7540 | 68.5% | 117 | 1.1% | 49 | 0.4% |
| 2 | 500 | 328 | 65.6% | 10 | 2.0% | 12 | 2.4% |
| 3 | 350 | 238 | 67.9% | 5 | 1.4% | 3 | 0.9% |
| 4 | 18000 | 11566 | 64.3% | 236 | 1.3% | 117 | 0.7% |
| 5 | 6200 | 4106 | 66.2% | 229 | 3.7% | 116 | 1.9% |
| 6 | 8800 | 5815 | 66.1% | 347 | 3.9% | 76 | 0.9% |
| 7 | 12500 | 8263 | 66.1% | 594 | 4.8% | 103 | 0.8% |
| 8 | NA | NA | NA | NA | NA | NA | NA |
| | 57350 | 37856 | 66.0% | 1538 | 2.7% | 476 | 0.8% |

Table X

Maximum Obtainable Tonnages Considering the $SiO_2$ Constraint

| Reserve | Total Tonnes | Fe Tonnes | % Fe | $SiO_2$ Tonnes | % $SiO_2$ | $Al_2O_3$ Tonnes | % $Al_2O_3$ |
|---|---|---|---|---|---|---|---|
| 1 | 11000 | 7540 | 68.5% | 117 | 1.1% | 49 | 0.4% |
| 2 | 4000 | 2570 | 64.3% | 91 | 2.3% | 125 | 3.1% |
| 3 | 350 | 238 | 67.9% | 5 | 1.4% | 3 | 0.9% |
| 4 | 34000 | 18862 | 55.5% | 1434 | 4.2% | 326 | 1.0% |
| 5 | 4200 | 2871 | 68.4% | 129 | 3.1% | 72 | 1.7% |
| 6 | 800 | 552 | 69.0% | 5 | 0.6% | 15 | 1.9% |
| 7 | 7500 | 5168 | 68.9% | 284 | 3.8% | 57 | 0.8% |
| 8 | 25400 | 15228 | 60.0% | 871 | 3.4% | 820 | 3.2% |
| | 87250 | 53029 | 60.8% | 2936 | 3.4% | 1467 | 1.7% |

Table XI

Maximum Obtainable Tonnages Considering the Al2O3 Constraint

| Reserve | Total Tonnes | Fe Tonnes | % Fe | SiO2 Tonnes | % SiO2 | Al2O3 Tonnes | % Al2O3 |
|---|---|---|---|---|---|---|---|
| 1 | 23000 | 14758 | 64.2% | 1637 | 7.1% | 130 | 0.6% |
| 2 | NA | NA | NA | NA | NA | NA | NA |
| 3 | 770 | 502 | 65.2% | 36 | 4.7% | 9 | 1.2% |
| 4 | 42000 | 21358 | 50.9% | 2436 | 5.8% | 472 | 1.1% |
| 5 | 3200 | 2225 | 69.5% | 86 | 2.7% | 52 | 1.6% |
| 6 | 19800 | 12600 | 63.6% | 941 | 4.8% | 358 | 1.8% |
| 7 | 24500 | 14442 | 58.9% | 1774 | 7.2% | 239 | 1.0% |
| 8 | NA | NA | NA | NA | NA | NA | NA |
| | 113270 | 65885 | 58.2% | 6910 | 6.1% | 1260 | 1.1% |

lowest tonnage is the tight constaint. In this example the Fe constraint is the tight constraint limiting the total tonnage to roughly 58 million tonnes. The SiO2 and Al2O3 percentages are evaluated for the tonnages predicted by the tight constraint. If the SiO2 and Al2O3 percentages are within bounds, the solution is optimal. If either SiO2 or Al2O3 is above the allowed constrained values, it is necessary to use equivalent values of dFe/dSiO2, dFe/dAl2O3, or dSi/dAl2O3 (from Appendix VI) and perform the analysis based on more than one constraint being tight. The same rational of analysis used when one constraint is tight applies to multiple tight constraints in as much as the ratios of the rates of change must be equal at optimality.

Step 8: In this example only the Fe constraint is tight. The optimal solution is given in Table IX. To determine the optimum cutoff grade for each mining area, it is necessary to back estimate the cutoff grades at the tonnages found at optimality from the COG curves for each mining area. These cutoff grades are summarized in Table XII.

The previous application was related to the maximization of reserves within an entire mining district. The availability of ore within a mine or mining district is much lower during the mining phase than the 100%

Table XII

These are the Cutoff Grades for the Mining Areas at the Optimal Blend

| Reserve | Tonnage | COG %Fe | Ave %Fe |
|---|---|---|---|
| 1 | 11000 | 68.2% | 68.5% |
| 2 | 500 | 63.7% | 65.6% |
| 3 | 350 | 67.4% | 67.9% |
| 4 | 18000 | 62.6% | 64.3% |
| 5 | 6200 | 66.0% | 66.2% |
| 6 | 8800 | 66.0% | 66.1% |
| 7 | 12500 | 65.9% | 66.1% |
| 8 | NA | NA | |
| | 57350 | | 66.0% |

availability used in this example to encompass total contained reserves. It is necessary to use the grade - tonnage curves or the delineation information for the currently available reserves in order to find the optimal operating blend of material for the time frame under consideration.

# CONCLUSION

Cutoff grade curves are produced from the information gathered during the delineation phase of ore reserve analysis. They are designed to show the mineral content of the ore contained in a mining area. From the cutoff grade curves (or from the delineation information) it is possible to develop regression models which describe the mineral content in terms of the total tonnes of ore contained in the area.

When several mining areas (to be exploited) are located in the same locale it is desirable to maximize the total reserves contained within the deposits by blending. Several constraints are typically placed on ore grade for both the valuable and gangue mineral in the mill feed. These constraints make it difficult to estimate the maximum tonnage of ore which can be considered in reserve in each mining area and the cutoff grade which should be adhered to in mining the deposits so as to produce the optimal tonnage from each deposit and the maximum tonnage from the mining district.

This sort of analysis is required for both day to day mining operations (to achieve a blend of ore to feed the mill) and for long range reserve estimates. The short term optimize due to variations in the market price of the valuable commodity, equipment and labor availability, current location of operations, the geometry of the mining layout, and the production sequence being used to develop and produce from the mining area. This solution technique can be applied to daily production operations by using grade - tonnage curves for the available ore rather than for the entire mining area.

Cutoff grade curves which represent the ore grades and tonnages in the various mining areas are usually nonlinear. Typically these curves can be modeled by second order polynomial equations. This was done for the cutoff grade curves for hypothetical reserve #1 through reserve #8.

It is possible to determine the maximum ore reserves for a collection of mining areas under consideration if blending is allowed. The method of solution using line segments was considered; however, the number of iterations necessary for resolution, and the low visibility of the mathematics as it pertains to the original cutoff grade curves implies that this technique may not be the best solution method. Lagrange multipliers were considered as a solution technique but were found to be extremely complex for only three deposits with three constraints. Although a closed form solution may be obtained using Lagrange multipliers, the mathematics also give low visibility to the original problem and leaves a large number of nonlinear equations to be solved simultaneously even for the simplest problems.

A solution technique was developed which overcomes the problems presented by the other techniques. It requires a polynomial regression fit for each curve such that estimates can be made of the cutoff grade curves to generate the mineral content curves. Second order polynomials are typically sufficient to define the mineral content (being the dependent variable) in terms of total tonnage (being the independent variable). By taking the derivative of the quadratic equations, the marginal rates of change of the mineral content with respect to total tonnage for deposits can be determined at several intervals throughout a cutoff grade range. Considering that at optimality all marginal values must be the equal for the tight constraint, the values of total tonnage and inturn cutoff grade can be found for all deposits. If more than one constraint is tight, the ratios of marginal value between the tight constaints have to be equal. If this occurs the total tonnage and cutoff grade is determined at equal ratios of marginal value for the respective constraints. This solution process is labor intensive and requires an iterative approach to obtain the optimum solution. It does give high visibility to the problem throughout its solution.

Considering the economics of ore reserve estimation there are many benefits gained from maximizing the total reserve tonnage. The attainment of the largest possible ore reserve tonnage for the sum of the mining areas under consideration and the cutoff grades to be adhered to in each mining area is important. Processing costs are minimized as an understanding is gained of the best blend to ship to the mill from each mining area. The solution technique itself allows for virtually any addition and/or change in constraints since the marginal values presented in the analysis do not change; they are fixed as is the mineral content contained in the reserves. Sensitivity analysis can be performed by inspection. This allows many "what if" questions to be asked without having to redo the analysis. Changes in the information used to solve for the optimal blend can be made easily.

The results of any of these analysis are statistically as good as the information gathered during the delineation phase. No one optimization method is statistically better than another considering they all use the same raw data. The statistics concerning the sampling process and grade analysis gathered during the delineation phase typically have a much lower variance than the changes which will occur in commodity price to change the optimum operating conditions over the life of operations in the mineral district.

These analysis are extremely useful from a mine planning perspective. The capacities required to develop and produce ore from various locations can be predicted using this technique. This allows the estimation of proper equipment placement in the various mining areas, operation schedules, manpower requirements for each mining area, the time that may be required to develop reserves, and the longevity of each mining area.

The results of the example problem shows the cutoff grade and tonnage which should come from each mining area to optimize the sum of the reserves given the 3 limiting constraints. Other scenarios occur where this technique may be applicable. Different mining costs or profits could be placed in the objective function for the ores in each area however, the solution techniques would have to be modified to obtain the correct results from such an analysis. Manditory proportions imposed by the mill, stockholders of the company, or government could be added to the constraints. A multitude of problems related to the blending of ores can be solved by examining the marginal values of the various mineral constituents with respect to total tonnage.

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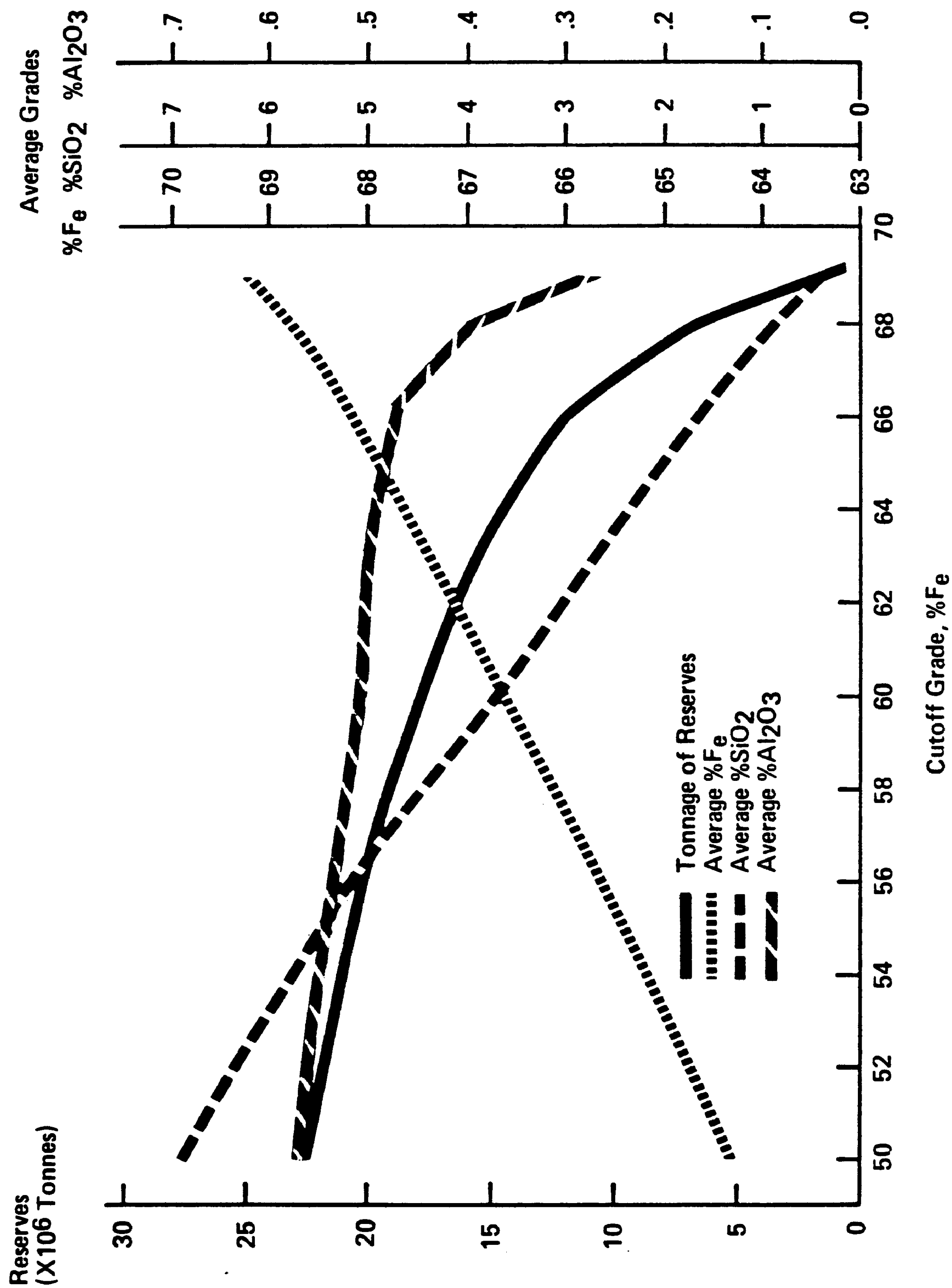
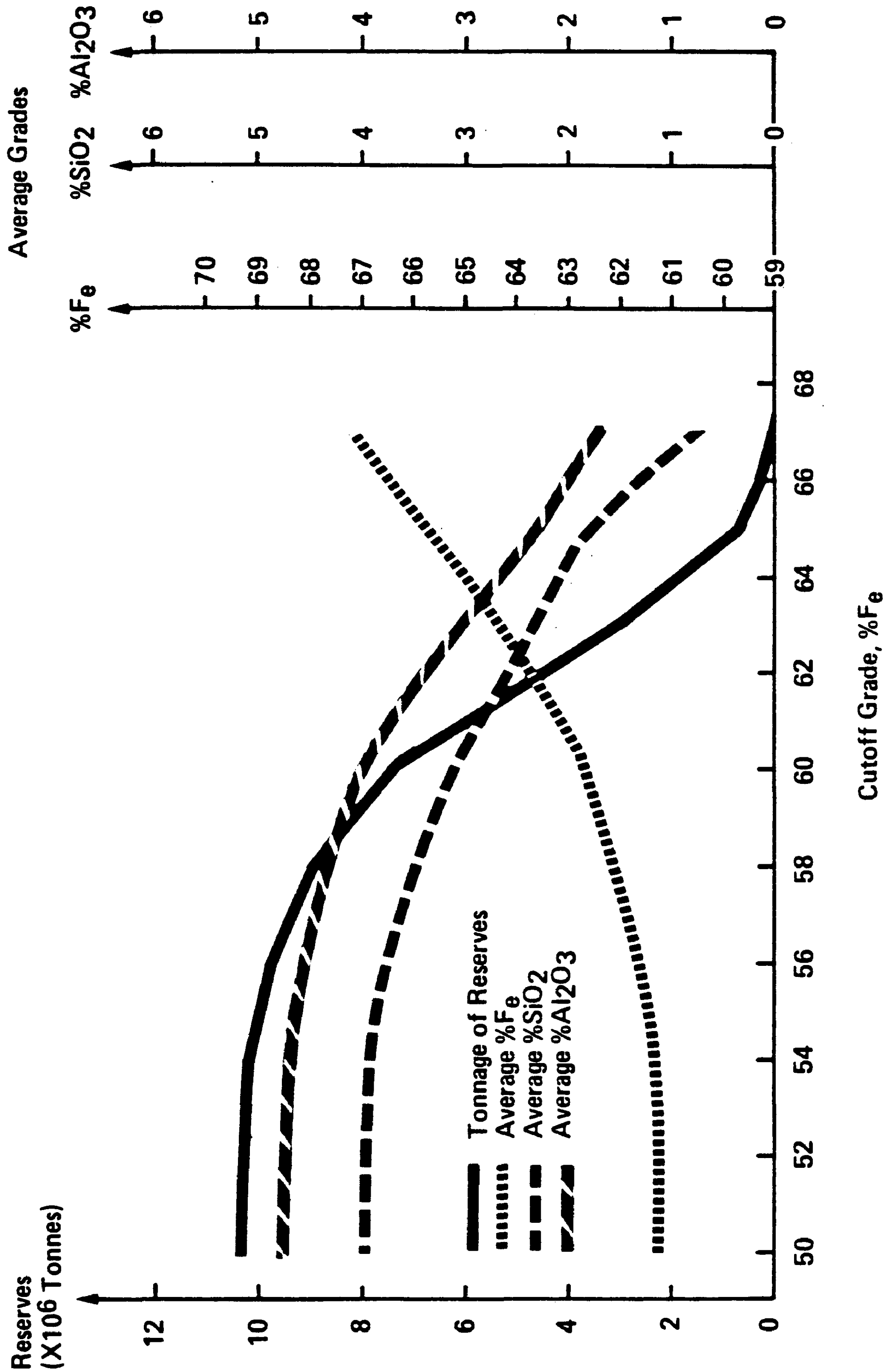
# APPENDIX I

## CUTOFF GRADE CURVES FOR ALL RESERVES

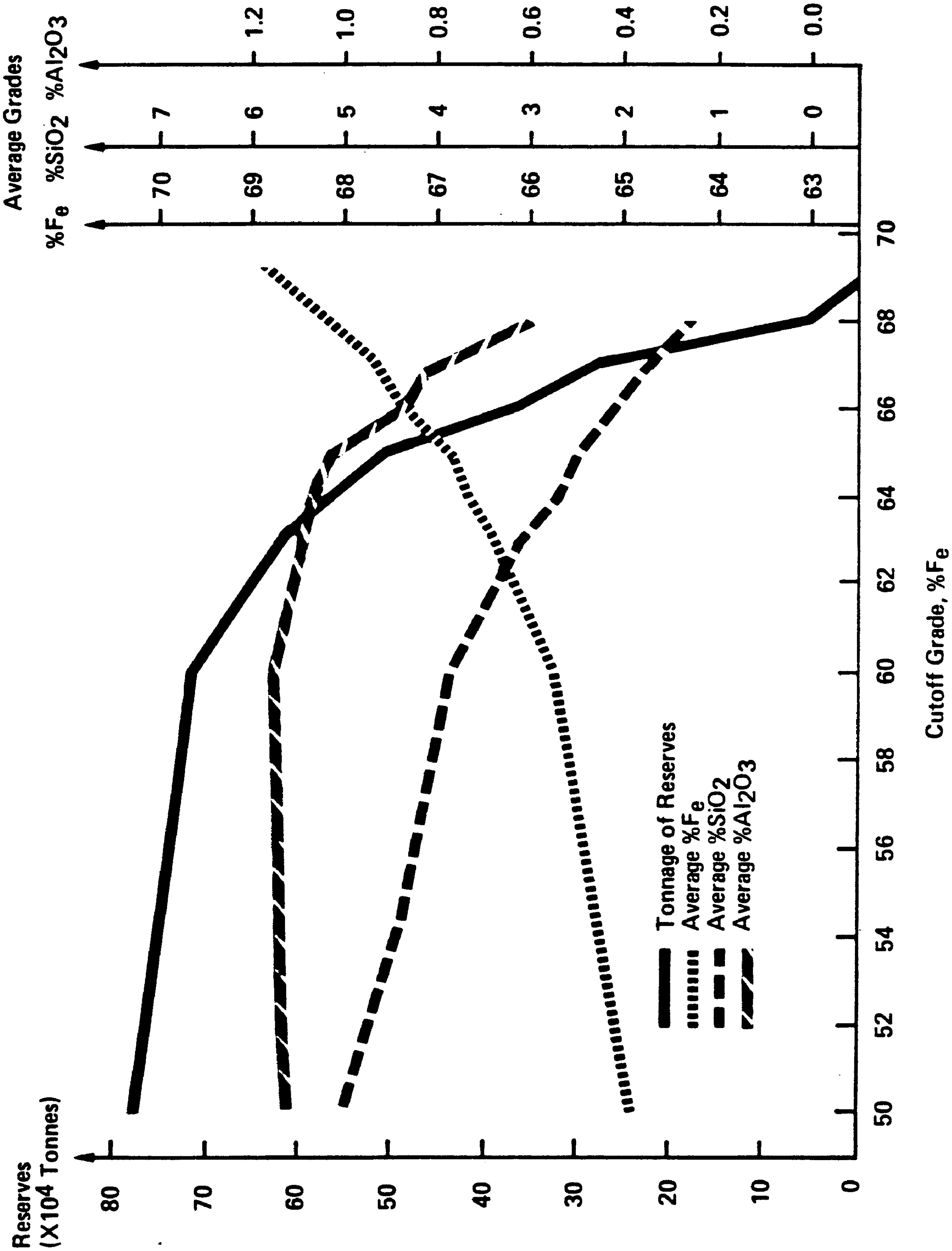
GRADE-TONNAGE CURVES FOR RESERVE 1



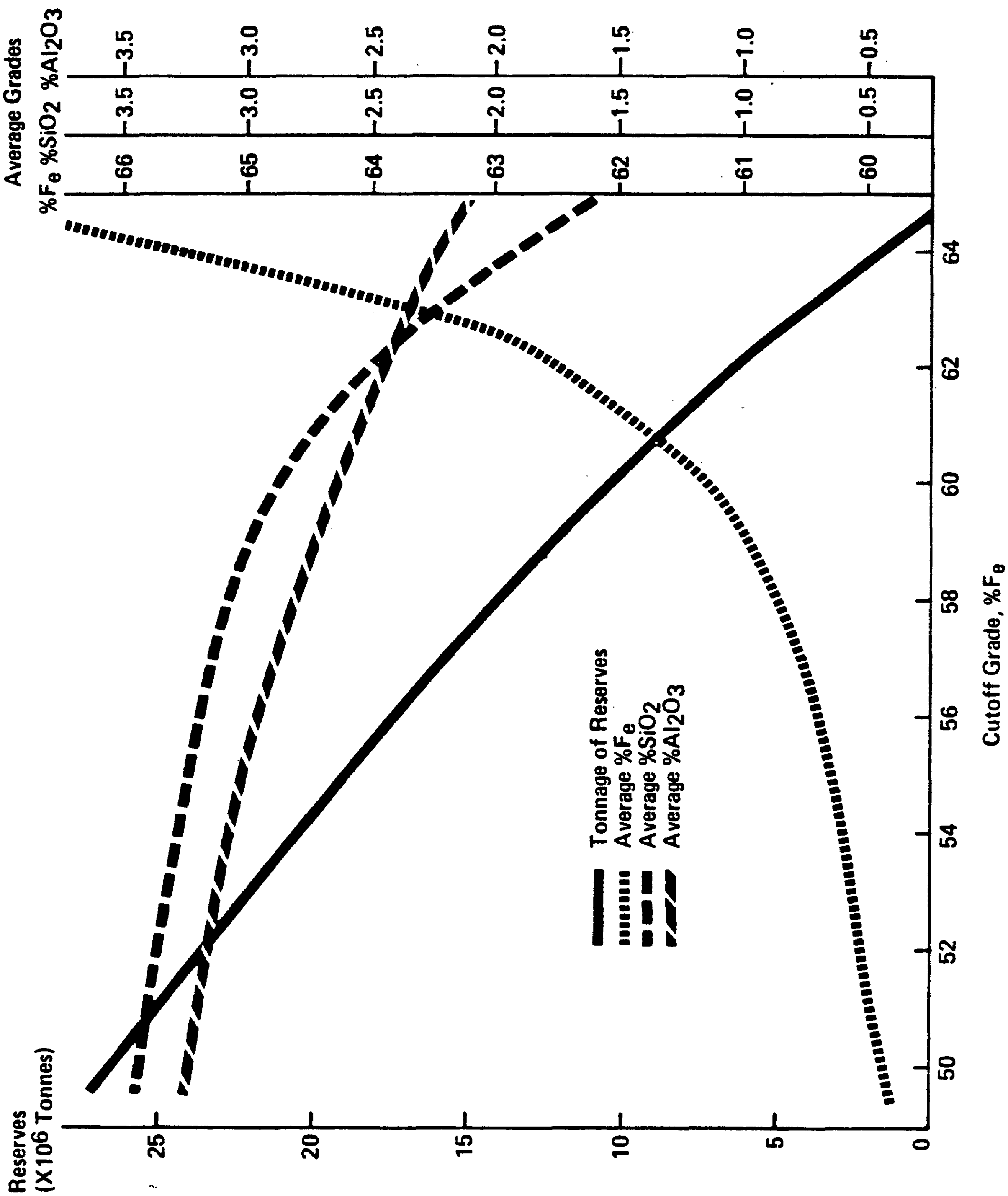
GRADE-TONNAGE CURVES FOR RESERVE 2



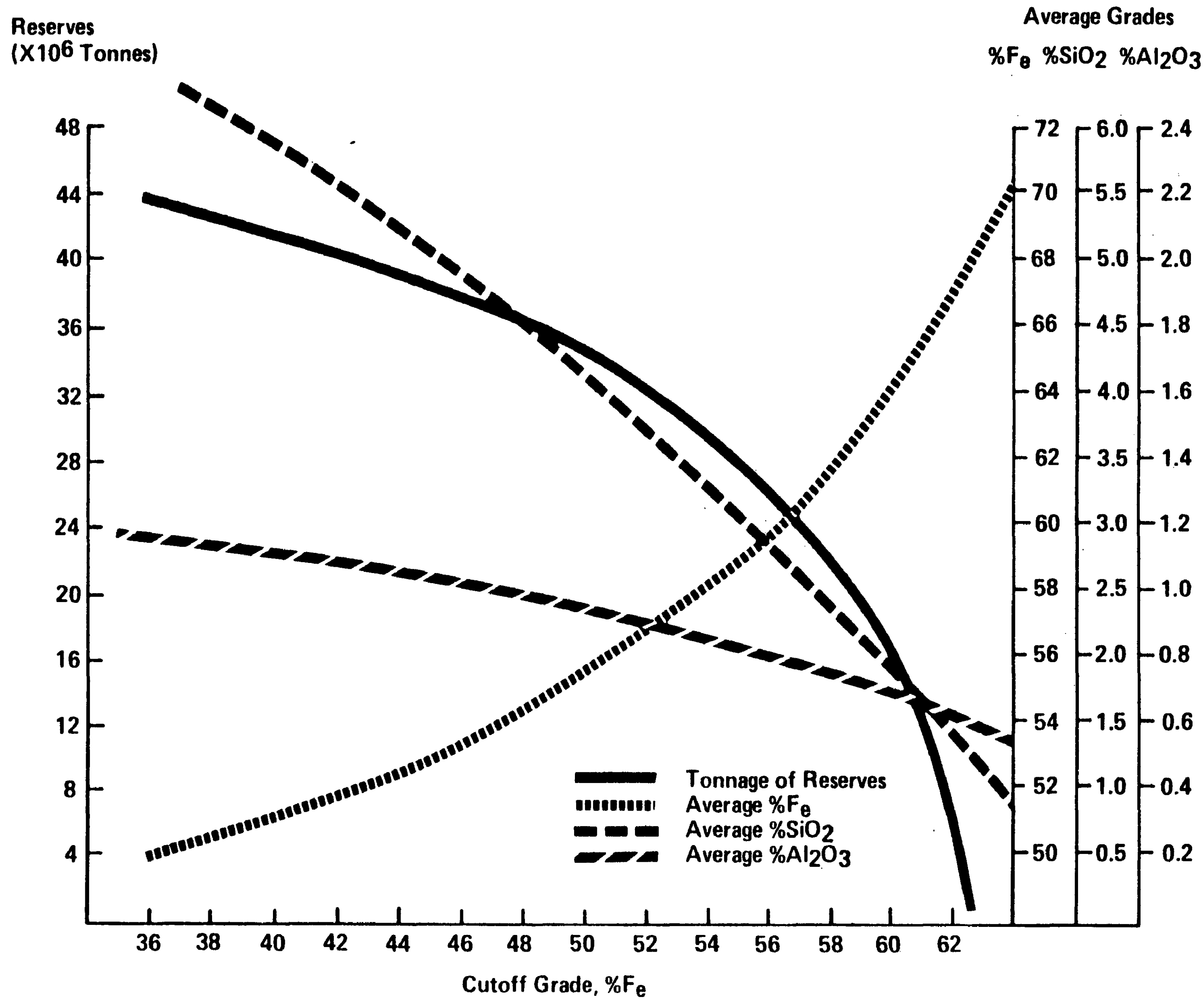
*GRADE-TONNAGE CURVES FOR RESERVE 3*



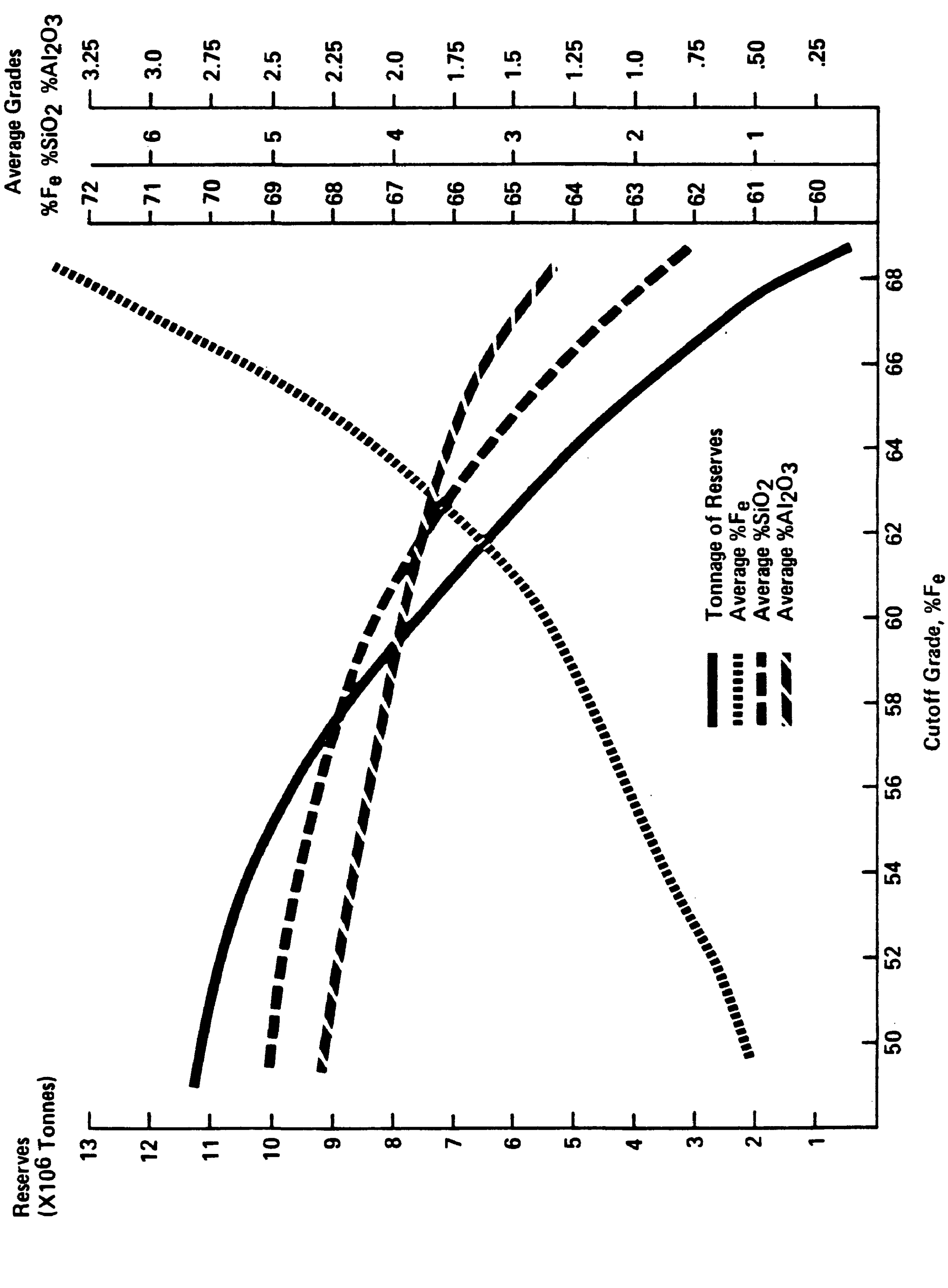
GRADE-TONNAGE CURVES FOR RESERVE 8



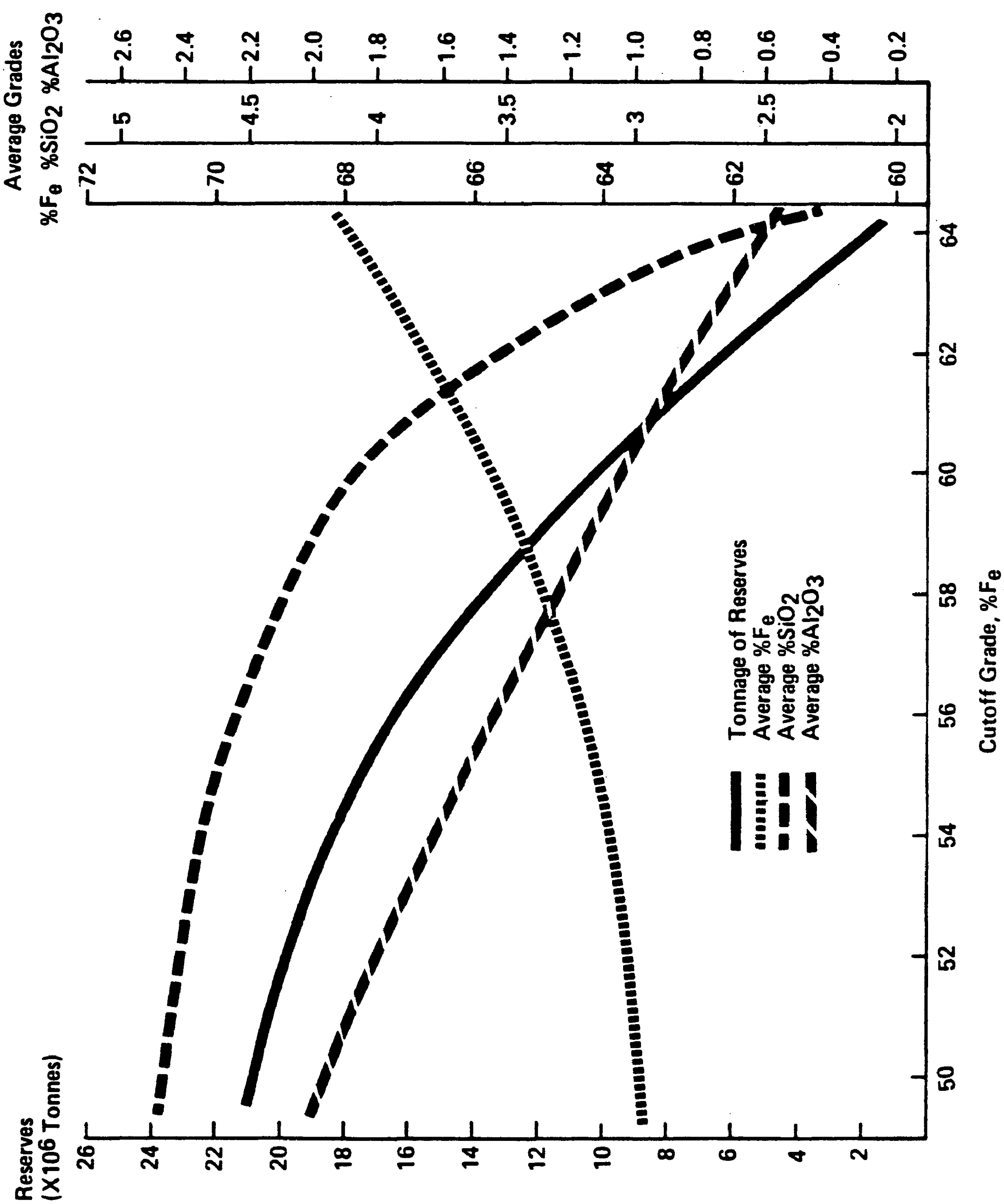
GRADE-TONNAGE CURVES FOR RESERVE 4



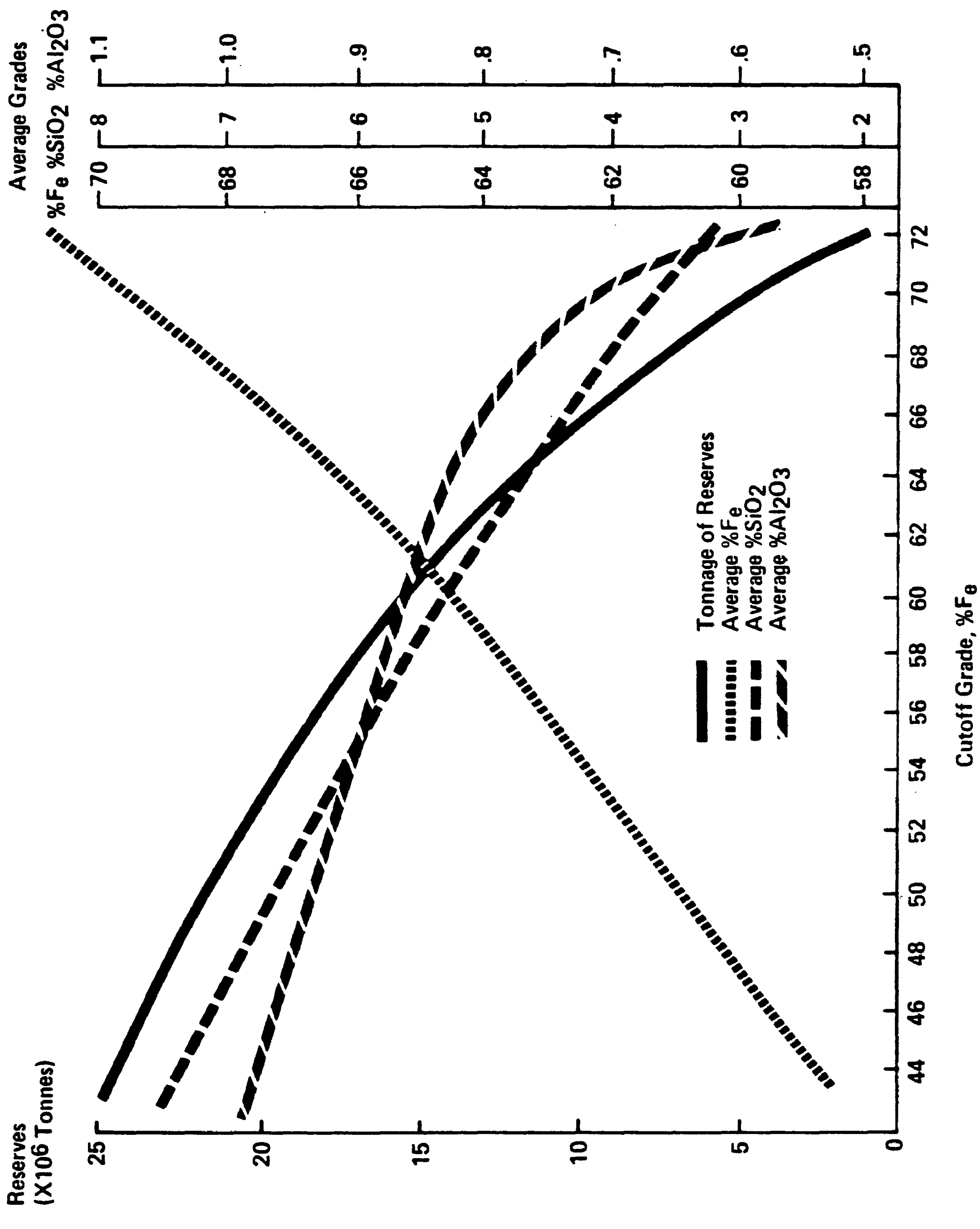
GRADE-TONNAGE CURVES FOR RESERVE 5



GRADE-TONNAGE CURVES FOR RESERVE 6



*GRADE-TONNAGE CURVES FOR RESERVE 7*



# APPENDIX II

# MINERAL CONTENT ABOVE CUTOFF GRADE FOR ALL RESERVES

Reserve #1 (Tonnes x '10^6)

| Cutoff Grade | Total Tonnes | %Fe | Fe Tonnes | %SiO2 | SiO2 Content | % Al2O3 | Al2O3 Tonnes |
|---|---|---|---|---|---|---|---|
| 50 | 22.30 | 64.3% | 14.34 | 7.0% | 1.55 | 0.6% | 0.127 |
| 52 | 22.00 | 64.7% | 14.23 | 6.3% | 1.39 | 0.6% | 0.123 |
| 54 | 21.00 | 65.1% | 13.67 | 5.8% | 1.22 | 0.5% | 0.113 |
| 56 | 20.000 | 65.6% | 13.12 | 5.2% | 1.04 | 0.5% | 0.106 |
| 58 | 18.000 | 66.1% | 11.90 | 4.5% | 0.81 | 0.5% | 0.094 |
| 60 | 17.500 | 66.6% | 11.66 | 3.7% | 0.65 | 0.5% | 0.088 |
| 62 | 16.500 | 67.1% | 11.07 | 3.1% | 0.51 | 0.5% | 0.082 |
| 64 | 14.50 | 67.6% | 9.80 | 2.4% | 0.34 | 0.5% | 0.071 |
| 66 | 12.00 | 68.1% | 8.17 | 1.7% | 0.20 | 0.5% | 0.057 |
| 68 | 7.00 | 68.7% | 4.81 | 1.0% | 0.07 | 0.4% | 0.027 |

Reserve #2 (Tonnes x '10^6)

| Cutoff Grade | Total Tonnes | %Fe | Fe Tonnes | %SiO2 | SiO2 Content | % Al2O3 | Al2O3 Tonnes |
|---|---|---|---|---|---|---|---|
| 50 | 10.30 | 61.3% | 6.31 | 3.95% | 0.41 | 4.70% | 0.484 |
| 52 | 10.20 | 61.3% | 6.25 | 3.93% | 0.40 | 4.67% | 0.476 |
| 54 | 10.10 | 61.5% | 6.21 | 3.90% | 0.39 | 4.63% | 0.468 |
| 56 | 9.60 | 61.7% | 5.92 | 3.70% | 0.36 | 4.45% | 0.427 |
| 58 | 8.40 | 62.2% | 5.22 | 3.40% | 0.29 | 4.20% | 0.353 |
| 60 | 7.40 | 62.8% | 4.65 | 3.03% | 0.22 | 3.95% | 0.292 |
| 62 | 4.40 | 63.9% | 2.81 | 2.50% | 0.11 | 3.30% | 0.145 |
| 64 | 1.90 | 65.1% | 1.24 | 2.05% | 0.04 | 2.65% | 0.050 |
| 66 | 0.10 | 66.6% | 0.07 | 1.20% | .00 | 2.00% | 0.002 |

Reserve #3 (Tonnes x '10^4)

| Cutoff Grade | Total Tonnes | %Fe | Fe Tonnes | %SiO2 | SiO2 Content | % Al2O3 | Al2O3 Tonnes |
|---|---|---|---|---|---|---|---|
| 50 | 77.00 | 65.00% | 50.05 | 5.00% | 3.85 | 1.12% | 0.862 |
| 52 | 76.00 | 65.10% | 49.48 | 4.75% | 3.61 | 1.13% | 0.859 |
| 54 | 75.00 | 65.30% | 48.98 | 4.50% | 3.38 | 1.14% | 0.855 |
| 56 | 74.00 | 65.45% | 48.43 | 4.30% | 3.18 | 1.15% | 0.851 |
| 58 | 72.90 | 65.60% | 47.82 | 4.10% | 2.99 | 1.16% | 0.846 |
| 60 | 71.50 | 65.80% | 47.05 | 3.90% | 2.79 | 1.17% | 0.837 |
| 62 | 65.00 | 66.20% | 43.03 | 3.40% | 2.21 | 1.10% | 0.715 |
| 64 | 55.00 | 66.70% | 36.69 | 2.70% | 1.49 | 1.06% | 0.583 |
| 66 | 36.00 | 67.35% | 24.25 | 2.10% | 0.76 | 0.86% | 0.310 |
| 68 | 5.00 | 68.20% | 3.41 | 1.30% | 0.07 | 0.60% | 0.030 |

Reserve #4 (Tonnes x '10^6)

| Cutoff Grade | Total Tonnes | %Fe | Fe Tonnes | %SiO2 | SiO2 Content | % Al2O3 | Al2O3 Tonnes |
|---|---|---|---|---|---|---|---|
| 40 | 42.00 | 51.00% | 21.42 | 5.90% | 2.48 | 1.13% | 0.475 |
| 42 | 41.00 | 51.70% | 21.20 | 5.70% | 2.34 | 1.10% | 0.451 |
| 46 | 38.00 | 56.13% | 21.33 | 4.90% | 1.86 | 1.05% | 0.399 |
| 50 | 35.00 | 55.60% | 19.46 | 4.20% | 1.47 | 0.95% | 0.333 |
| 54 | 30.00 | 58.20% | 17.46 | 3.30% | 0.99 | 0.87% | 0.261 |
| 56 | 22.00 | 61.70% | 13.57 | 2.40% | 0.53 | 0.75% | 0.165 |
| 62 | 4.00 | 67.25% | 2.69 | 1.30% | 0.05 | 0.63% | 0.025 |

Reserve #5 (Tonnes x '10^6)

| Cutoff Grade | Total Tonnes | %Fe | Fe Tonnes | %SiO2 | SiO2 Content | % Al2O3 | Al2O3 Tonnes |
|---|---|---|---|---|---|---|---|
| 50 | 11.20 | 61.00% | 6.83 | 5.00% | 0.56 | 2.25% | 0.252 |
| 52 | 10.80 | 61.70% | 6.66 | 4.90% | 0.53 | 2.20% | 0.238 |
| 54 | 10.30 | 62.50% | 6.44 | 4.75% | 0.49 | 2.15% | 0.222 |
| 56 | 9.60 | 63.20% | 6.07 | 4.65% | 0.45 | 2.10% | 0.202 |
| 58 | 8.80 | 63.80% | 5.61 | 4.40% | 0.39 | 2.05% | 0.180 |
| 60 | 7.60 | 64.50% | 4.90 | 4.14% | 0.32 | 1.98% | 0.151 |
| 62 | 6.40 | 65.70% | 4.20 | 3.75% | 0.24 | 1.87% | 0.120 |
| 64 | 5.00 | 67.50% | 3.38 | 3.25% | 0.16 | 1.77% | 0.089 |
| 66 | 3.40 | 69.59% | 2.37 | 2.65% | 0.09 | 1.67% | 0.057 |
| 68 | 1.50 | 72.30% | 1.08 | 1.80% | 0.03 | 1.33% | 0.020 |

Reserve #6 (Tonnes x '10^6)

| Cutoff Grade | Total Tonnes | %Fe | Fe Tonnes | %SiO2 | SiO2 Content | % Al2O3 | Al2O3 Tonnes |
|---|---|---|---|---|---|---|---|
| 50 | 20.80 | 63.40% | 13.19 | 4.80% | 1.00 | 1.95% | 0.41 |
| 52 | 19.70 | 63.70% | 12.55 | 4.75% | 0.94 | 1.80% | 0.35 |
| 54 | 18.50 | 63.90% | 11.82 | 4.68% | 0.87 | 1.63% | 0.30 |
| 56 | 16.40 | 64.40% | 10.56 | 4.53% | 0.74 | 1.45% | 0.24 |
| 58 | 13.60 | 65.00% | 8.84 | 4.35% | 0.59 | 1.24% | 0.17 |
| 60 | 10.20 | 65.70% | 6.70 | 4.10% | 0.42 | 1.03% | 0.11 |
| 62 | 6.20 | 66.80% | 4.14 | 3.50% | 0.22 | 0.83% | 0.05 |
| 64 | 1.70 | 68.00% | 1.16 | 2.63% | 0.04 | 0.60% | 0.01 |

Reserve #7 (Tonnes x '10^6)

| Cutoff Grade | Total Tonnes | %Fe | Fe Tonnes | %SiO2 | SiO2 Content | % Al2O3 | Al2O3 Tonnes |
|---|---|---|---|---|---|---|---|
| 52 | 24.50 | 58.80% | 14.41 | 7.30% | 1.79 | 0.98% | 0.24 |
| 56 | 20.50 | 61.60% | 12.63 | 6.30% | 1.29 | 0.92% | 0.19 |
| 60 | 15.50 | 64.50% | 10.00 | 5.27% | 0.82 | 0.86% | 0.13 |
| 64 | 11.80 | 66.30% | 7.82 | 4.70% | 0.55 | 0.82% | 0.10 |
| 68 | 7.40 | 68.50% | 5.07 | 4.00% | 0.30 | 0.76% | 0.06 |
| 72 | 1.00 | 70.80% | 0.71 | 3.20% | 0.03 | 0.60% | 0.01 |

Reserve #8 (Tonnes x '10^6)

| Cutoff Grade | Total Tonnes | %Fe | Fe Tonnes | %SiO2 | SiO2 Content | % Al2O3 | Al2O3 Tonnes |
|---|---|---|---|---|---|---|---|
| 50 | 26.60 | 59.90% | 15.93 | 3.45% | 0.92 | 3.25% | 0.86 |
| 54 | 20.50 | 60.25% | 12.35 | 3.30% | 0.68 | 3.10% | 0.64 |
| 58 | 14.00 | 60.75% | 8.51 | 3.10% | 0.43 | 2.80% | 0.39 |
| 60 | 10.40 | 61.30% | 6.38 | 2.85% | 0.30 | 2.65% | 0.28 |
| 62 | 6.30 | 62.30% | 3.92 | 2.52% | 0.16 | 2.48% | 0.16 |
| 64 | 1.50 | 65.50% | 0.98 | 1.90% | 0.03 | 2.25% | 0.03 |

# APPENDIX III

## QUADRATIC REGRESSION ANALYSIS FOR ALL RESERVES

Reserve #1

Fe: -398.3 + .779T - (0.523 E-5)T^2
Goodness of Fit: .99992
Correlation Coefficient: .99996

SiO2: 577.3 - .122T + (0.733 E-5)T^2
Goodness of Fit: .99414
Correlation Coefficient: .99707

Al2O3: -1.838 + (0.361 E-2)T + (0.927 E-7)T^2
Goodness of Fit: .99748
Correlation Coefficient: .99874

Reserve #2

Fe: -1.708 + .662T - (0.467 E-5)T^2
Goodness of Fit: .99999
Correlation Coefficient: .99999

SiO2: 4.517 + 0.0105T + (0.276 E-5)T^2
Goodness of Fit: .99906
Correlation Coefficient: .99953

Al2O3: .551 + (0.214 E-1)T + (0.245 E-5)T^2
Goodness of Fit: .99983
Correlation Coefficient: .99991

Reserve #3

Fe: $-1.532 + .707T - (0.689\ E{-4})T^2$
Goodness of Fit: .99994
Correlation Coefficient: .99997

SiO2: $2.251 - 0.0231T + (0.863\ E{-4})T^2$
Goodness of Fit: .98185
Correlation Coefficient: .99089

Al2O3: $-0.1756 + (0.798\ E{-2})T + (0.497\ E{-5})T^2$
Goodness of Fit: .99571
Correlation Coefficient: .99785

Reserve #4

Fe: $-313.7 + .768T - (0.592\ E{-5})T^2$
Goodness of Fit: .99980
Correlation Coefficient: .99990

SiO2: $174.6 - 0.0344T + (0.210\ E{-5})T^2$
Goodness of Fit: .99571
Correlation Coefficient: .99785

Al2O3: $15.82 + (0.172\ E{-2})T + (0.217\ E{-6})T^2$
Goodness of Fit: .99901
Correlation Coefficient: .99950

Reserve #5

Fe: $+34.87 + 0.714T - (0.928\ E\text{-}5)T^2$
Goodness of Fit: .99987
Correlation Coefficient: .99993

SiO2: $-20.04 + 0.0256T + (0.234\ E\text{-}5)T^2$
Goodness of Fit: .99970
Correlation Coefficient: .99985

Al2O3: $-2.758 + (0.148\ E\text{-}1)T + (0.692\ E\text{-}6)T^2$
Goodness of Fit: .99977
Correlation Coefficient: .99988

Reserve #6

Fe: $10.61 + .679T - (0.216\ E\text{-}5)T^2$
Goodness of Fit: .99999
Correlation Coefficient: .99999

SiO2: $-24.51 - 0.0370T + (0.593\ E\text{-}6)T^2$
Goodness of Fit: .99978
Correlation Coefficient: .99989

Al2O3: $15.77 - (0.144\ E\text{-}2)T + (0.945\ E\text{-}6)T^2$
Goodness of Fit: .99685
Correlation Coefficient: .99843

Reserve #7

Fe: $-47.44 + .741T - (0.612\ E{-5})T^2$
Goodness of Fit: .99995
Correlation Coefficient: .99997

SiO2: $20.32 + 0.0191T + (0.214\ E{-5})T^2$
Goodness of Fit: .99938
Correlation Coefficient: .99969

Al2O3: $-0.753 + (0.677\ E{-2})T + (0.123\ E{-6})T^2$
Goodness of Fit: .99996
Correlation Coefficient: .99998

Reserve #8

Fe: $81.94 + .610T - (0.544\ E{-6})T^2$
Goodness of Fit: .99999
Correlation Coefficient: .99999

SiO2: $-20.49 + 0.0282T + (0.273\ E{-6})T^2$
Goodness of Fit: .99957
Correlation Coefficient: .99978

Al2O3: $-5.439 + (0.238\ E{-1})T + (0.341\ E{-6})T^2$
Goodness of Fit: .99969
Correlation Coefficient: .99985

# APPENDIX IV

## QUADRATIC EQUATION ESTIMATES OF MINERAL CONTENT FOR THE MINERAL SUITES OF ALL RESERVES

Reserve #1

| Tonnes | Fe Function | Percent Fe | Si Function | Percent Si | Al Function | Percent Al |
|---|---|---|---|---|---|---|
| 23000 | 14758 | 64.2% | 1637 | 7.1% | 130 | 0.6% |
| 21000 | 13660 | 65.0% | 1237 | 5.9% | 115 | 0.5% |
| 19000 | 12520 | 65.9% | 896 | 4.7% | 100 | 0.5% |
| 17000 | 11338 | 66.7% | 613 | 3.6% | 86 | 0.5% |
| 15000 | 10114 | 67.4% | 389 | 2.6% | 73 | 0.5% |
| 13000 | 8848 | 68.1% | 224 | 1.7% | 61 | 0.5% |
| 11000 | 7540 | 68.5% | 117 | 1.1% | 49 | 0.4% |
| 9000 | 6191 | 68.8% | 69 | 0.8% | 38 | 0.4% |
| 7000 | 4800 | 68.6% | 79 | 1.1% | 28 | 0.4% |
| 5000 | 3367 | 67.3% | 148 | 3.0% | 19 | 0.4% |
| 3000 | 1892 | 63.1% | 276 | 9.2% | 10 | 0.3% |

Reserve #2

| Tonnes | Fe Function | Percent Fe | Si Function | Percent Si | Al Function | Percent Al |
|---|---|---|---|---|---|---|
| 10300 | 6317 | 61.3% | 405 | 3.9% | 481 | 4.7% |
| 10000 | 6147 | 61.5% | 385 | 3.8% | 460 | 4.6% |
| 9000 | 5574 | 61.9% | 322 | 3.6% | 392 | 4.4% |
| 8000 | 4992 | 62.4% | 265 | 3.3% | 329 | 4.1% |
| 7000 | 4400 | 62.9% | 213 | 3.0% | 271 | 3.9% |
| 6000 | 3800 | 63.3% | 167 | 2.8% | 217 | 3.6% |
| 5000 | 3189 | 63.8% | 126 | 2.5% | 169 | 3.4% |
| 4000 | 2570 | 64.2% | 91 | 2.3% | 125 | 3.1% |
| 3000 | 1941 | 64.7% | 61 | 2.0% | 87 | 2.9% |
| 2000 | 1303 | 65.1% | 36 | 1.8% | 53 | 2.7% |
| 1000 | 655 | 65.5% | 18 | 1.8% | 24 | 2.4% |
| 500 | 328 | 65.6% | 10 | 2.1% | 12 | 2.4% |

Reserve #3

| Tonnes | Fe Function | Percent Fe | Si Function | Percent Si | Al Function | Percent Al |
|---|---|---|---|---|---|---|
| 770 | 502 | 65.2% | 36 | 4.6% | 9 | 1.2% |
| 750 | 490 | 65.3% | 34 | 4.5% | 9 | 1.1% |
| 700 | 460 | 65.7% | 28 | 4.1% | 8 | 1.1% |
| 650 | 429 | 66.0% | 24 | 3.7% | 7 | 1.1% |
| 600 | 398 | 66.3% | 19 | 3.2% | 6 | 1.1% |
| 550 | 367 | 66.6% | 16 | 2.9% | 6 | 1.0% |
| 500 | 335 | 67.0% | 12 | 2.5% | 5 | 1.0% |
| 450 | 303 | 67.3% | 9 | 2.1% | 4 | 1.0% |
| 400 | 270 | 67.6% | 7 | 1.7% | 4 | 1.0% |
| 350 | 238 | 67.9% | 5 | 1.4% | 3 | 0.9% |
| 300 | 204 | 68.1% | 3 | 1.0% | 3 | 0.9% |
| 250 | 171 | 68.4% | 2 | 0.8% | 2 | 0.9% |
| 200 | 137 | 68.6% | 1 | 0.5% | 2 | 0.8% |
| 150 | 103 | 68.7% | 1 | 0.5% | 1 | 0.8% |
| 100 | 68 | 68.5% | 1 | 0.8% | 1 | 0.7% |
| 50 | 34 | 67.3% | 1 | 2.6% | 0 | 0.5% |

Reserve #4

| Tonnes | Fe Function | Percent Fe | Si Function | Percent Si | Al Function | Percent Al |
|---|---|---|---|---|---|---|
| 42000 | 21358 | 50.9% | 2436 | 5.8% | 472 | 1.1% |
| 38000 | 20206 | 53.2% | 1902 | 5.0% | 395 | 1.0% |
| 34000 | 18862 | 55.5% | 1434 | 4.2% | 326 | 1.0% |
| 30000 | 17326 | 57.8% | 1034 | 3.4% | 263 | 0.9% |
| 26000 | 15598 | 60.0% | 701 | 2.7% | 208 | 0.8% |
| 22000 | 13678 | 62.2% | 435 | 2.0% | 159 | 0.7% |
| 18000 | 11566 | 64.3% | 236 | 1.3% | 117 | 0.7% |
| 14000 | 9262 | 66.2% | 105 | 0.7% | 83 | 0.6% |
| 10000 | 6766 | 67.7% | 41 | 0.4% | 55 | 0.5% |
| 6000 | 4078 | 68.0% | 44 | 0.7% | 34 | 0.6% |
| 2000 | 1198 | 59.9% | 114 | 5.7% | 20 | 1.0% |

Reserve #5

| Tonnes | Fe Function | Percent Fe | Si Function | Percent Si | Al Function | Percent Al |
|---|---|---|---|---|---|---|
| 11200 | 6870 | 61.3% | 561 | 5.0% | 250 | 2.2% |
| 10200 | 6354 | 62.3% | 485 | 4.8% | 220 | 2.2% |
| 9200 | 5820 | 63.3% | 414 | 4.5% | 192 | 2.1% |
| 8200 | 5267 | 64.2% | 348 | 4.2% | 165 | 2.0% |
| 7200 | 4696 | 65.2% | 286 | 4.0% | 140 | 1.9% |
| 6200 | 4106 | 66.2% | 229 | 3.7% | 116 | 1.9% |
| 5200 | 3498 | 67.3% | 177 | 3.4% | 93 | 1.8% |
| 4200 | 2871 | 68.4% | 129 | 3.1% | 72 | 1.7% |
| 3200 | 2225 | 69.5% | 86 | 2.7% | 52 | 1.6% |
| 2200 | 1561 | 71.0% | 48 | 2.2% | 33 | 1.5% |
| 1200 | 879 | 73.2% | 14 | 1.2% | 16 | 1.3% |

Reserve #6

| Tonnes | Fe Function | Percent Fe | Si Function | Percent Si | Al Function | Percent Al |
|---|---|---|---|---|---|---|
| 20800 | 13191 | 63.4% | 1002 | 4.8% | 395 | 1.9% |
| 19800 | 12600 | 63.6% | 941 | 4.8% | 358 | 1.8% |
| 18800 | 12005 | 63.9% | 881 | 4.7% | 323 | 1.7% |
| 17800 | 11405 | 64.1% | 823 | 4.6% | 290 | 1.6% |
| 16800 | 10801 | 64.3% | 765 | 4.6% | 258 | 1.5% |
| 15800 | 10193 | 64.5% | 709 | 4.5% | 229 | 1.4% |
| 14800 | 9581 | 64.7% | 653 | 4.4% | 201 | 1.4% |
| 13800 | 8964 | 65.0% | 599 | 4.3% | 176 | 1.3% |
| 12800 | 8343 | 65.2% | 547 | 4.3% | 152 | 1.2% |
| 11800 | 7717 | 65.4% | 495 | 4.2% | 130 | 1.1% |
| 10800 | 7088 | 65.6% | 445 | 4.1% | 110 | 1.0% |
| 9800 | 6453 | 65.9% | 395 | 4.0% | 92 | 0.9% |
| 8800 | 5815 | 66.1% | 347 | 3.9% | 76 | 0.9% |
| 7800 | 5172 | 66.3% | 300 | 3.9% | 62 | 0.8% |
| 6800 | 4525 | 66.5% | 255 | 3.7% | 50 | 0.7% |
| 5800 | 3874 | 66.8% | 210 | 3.6% | 39 | 0.7% |
| 4800 | 3218 | 67.0% | 167 | 3.5% | 31 | 0.6% |
| 3800 | 2558 | 67.3% | 125 | 3.3% | 24 | 0.6% |
| 2800 | 1894 | 67.6% | 84 | 3.0% | 19 | 0.7% |
| 1800 | 1225 | 68.1% | 44 | 2.4% | 16 | 0.9% |
| 800 | 552 | 69.0% | 5 | 0.7% | 15 | 1.9% |

Reserve #7

| Tonnes | Fe Function | Percent Fe | Si Function | Percent Si | Al Function | Percent Al |
|---|---|---|---|---|---|---|
| 24500 | 14442 | 58.9% | 1774 | 7.2% | 239 | 1.0% |
| 23500 | 13994 | 59.6% | 1652 | 7.0% | 226 | 1.0% |
| 22500 | 13535 | 60.2% | 1534 | 6.8% | 214 | 0.9% |
| 21500 | 13062 | 60.8% | 1421 | 6.6% | 201 | 0.9% |
| 20500 | 12578 | 61.4% | 1312 | 6.4% | 189 | 0.9% |
| 19500 | 12082 | 62.0% | 1207 | 6.2% | 178 | 0.9% |
| 18500 | 11573 | 62.6% | 1107 | 6.0% | 166 | 0.9% |
| 17500 | 11052 | 63.2% | 1010 | 5.8% | 155 | 0.9% |
| 16500 | 10518 | 63.7% | 919 | 5.6% | 144 | 0.9% |
| 15500 | 9973 | 64.3% | 831 | 5.4% | 134 | 0.9% |
| 14500 | 9415 | 64.9% | 748 | 5.2% | 123 | 0.8% |
| 13500 | 8845 | 65.5% | 669 | 5.0% | 113 | 0.8% |
| 12500 | 8263 | 66.1% | 594 | 4.8% | 103 | 0.8% |
| 11500 | 7668 | 66.7% | 523 | 4.6% | 93 | 0.8% |
| 10500 | 7062 | 67.3% | 457 | 4.4% | 84 | 0.8% |
| 9500 | 6443 | 67.8% | 395 | 4.2% | 75 | 0.8% |
| 8500 | 5812 | 68.4% | 338 | 4.0% | 66 | 0.8% |
| 7500 | 5168 | 68.9% | 284 | 3.8% | 57 | 0.8% |
| 6500 | 4513 | 69.4% | 235 | 3.6% | 48 | 0.7% |
| 5500 | 3845 | 69.9% | 190 | 3.5% | 40 | 0.7% |
| 4500 | 3165 | 70.3% | 150 | 3.3% | 32 | 0.7% |
| 3500 | 2472 | 70.6% | 113 | 3.2% | 24 | 0.7% |
| 2500 | 1768 | 70.7% | 82 | 3.3% | 17 | 0.7% |
| 1500 | 1051 | 70.0% | 54 | 3.6% | 10 | 0.6% |
| 500 | 322 | 64.3% | 30 | 6.1% | 3 | 0.5% |

Reserve #8

| Tonnes | Fe Function | Percent Fe | Si Function | Percent Si | Al Function | Percent Al |
|---|---|---|---|---|---|---|
| 26600 | 15926 | 59.9% | 922 | 3.5% | 869 | 3.3% |
| 25400 | 15228 | 60.0% | 871 | 3.4% | 820 | 3.2% |
| 24200 | 14528 | 60.0% | 821 | 3.4% | 771 | 3.2% |
| 23000 | 13827 | 60.1% | 772 | 3.4% | 723 | 3.1% |
| 21800 | 13124 | 60.2% | 723 | 3.3% | 676 | 3.1% |
| 20600 | 12420 | 60.3% | 675 | 3.3% | 630 | 3.1% |
| 19400 | 11714 | 60.4% | 629 | 3.2% | 585 | 3.0% |
| 18200 | 11006 | 60.5% | 582 | 3.2% | 541 | 3.0% |
| 17000 | 10297 | 60.6% | 537 | 3.2% | 498 | 2.9% |
| 15800 | 9586 | 60.7% | 493 | 3.1% | 456 | 2.9% |
| 14600 | 8874 | 60.8% | 449 | 3.1% | 415 | 2.8% |
| 13400 | 8160 | 60.9% | 406 | 3.0% | 375 | 2.8% |
| 12200 | 7444 | 61.0% | 364 | 3.0% | 336 | 2.8% |
| 11000 | 6727 | 61.2% | 322 | 2.9% | 298 | 2.7% |
| 9800 | 6009 | 61.3% | 282 | 2.9% | 261 | 2.7% |
| 8600 | 5289 | 61.5% | 242 | 2.8% | 225 | 2.6% |
| 7400 | 4567 | 61.7% | 203 | 2.7% | 190 | 2.6% |
| 6200 | 3844 | 62.0% | 165 | 2.7% | 155 | 2.5% |
| 5000 | 3119 | 62.4% | 127 | 2.5% | 122 | 2.4% |
| 3800 | 2393 | 63.0% | 90 | 2.4% | 90 | 2.4% |

# APPENDIX V

## DERIVATIVES OF THE QUADRATIC EQUATION ESTIMATES OF MINERAL CONTENT FOR THE MINERAL SUITE OF ALL RESERVES

Reserve #1

Fe: $0.779 - 2*(0.523\ E{-5})T$

SiO2: $-\ 0.122 + 2*(0.733\ E{-5})T$

Al2O3: $0.361\ E{-2} + 2*(0.927\ E{-7})T$

Reserve #2

Fe: $0.662 - 2*(0.467\ E{-5})T$

SiO2: $0.0105 + 2*(0.276\ E{-5})T$

Al2O3: $0.214\ E{-1} + 2*(0.245\ E{-5})T$

Reserve #3

Fe: 0.707 - 2*(0.689 E-4)T

SiO2: - 0.0231 + 2*(0.863 E-4)T

Al2O3: 0.798 E-2 + 2*(0.497 E-5)T

Reserve #4

Fe: 0.768 - 2*(0.592 E-5)T

SiO2: - 0.0344 + 2*(0.210 E-5)T

Al2O3: 0.172 E-2 + 2*(0.217 E-6)T

Reserve #5

Fe: $0.714 - 2*(0.928\ E{-5})T$

SiO2: $0.0256 + 2*(0.234\ E{-5})T$

Al2O3: $0.148\ E{-1} + 2*(0.692\ E{-6})T$

Reserve #6

Fe: $0.679 - 2*(0.216\ E{-5})T$

SiO2: $-\ 0.0370 + 2*(0.593\ E{-6})T$

Al2O3: $-0.144\ E{-2} + 2*(0.945\ E{-6})T$

Reserve #7

Fe: 0.741 - 2*(0.612 E-5)T

SiO2: 0.0191 + 2*(0.214 E-5)T

Al2O3: 0.677 E-2 + 2*(0.123 E-6)T

Reserve #8

Fe: 0.610 - 2*(0.544 E-6)T

SiO2: 0.0282 + 2*(0.273 E-6)T

Al2O3: 0.238 E-1 + 2*(0.341 E-6)T

# APPENDIX VI

## QUADRATIC EQUATION DERIVATIVE ESTIMATES OF THE MINERAL SUITES OF ALL RESERVES

Reserve #1

| Tonnage | Fe Tonnage | dFe/dT | Si Tonnage | dSi/dT | Al Tonnage | dAl/dT |
|---|---|---|---|---|---|---|
| 23000 | 14758 | 0.54 | 1637 | 0.215 | 130 | 0.0079 |
| 21000 | 13660 | 0.56 | 1237 | 0.185 | 115 | 0.0075 |
| 19000 | 12520 | 0.58 | 896 | 0.156 | 100 | 0.0071 |
| 17000 | 11338 | 0.60 | 613 | 0.127 | 86 | 0.0068 |
| 15000 | 10114 | 0.62 | 389 | 0.097 | 73 | 0.0064 |
| 13000 | 8848 | 0.64 | 224 | 0.068 | 61 | 0.0060 |
| 11000 | 7540 | 0.66 | 117 | 0.039 | 49 | 0.0056 |
| 9000 | 6191 | 0.69 | 69 | 0.009 | 38 | 0.0053 |
| 7000 | 4800 | 0.71 | 79 | -0.020 | 28 | 0.0049 |
| 5000 | 3367 | 0.73 | 148 | -0.049 | 19 | 0.0045 |
| 3000 | 1892 | 0.75 | 276 | -0.079 | 10 | 0.0042 |

Reserve #2

| Tonnage | Fe Tonnage | dFe/dT | Si Tonnage | dSi/dT | Al Tonnage | dAl/dT |
|---|---|---|---|---|---|---|
| 10300 | 6317 | 0.57 | 405 | 0.067 | 481 | 0.0720 |
| 10000 | 6147 | 0.57 | 385 | 0.066 | 460 | 0.0705 |
| 9000 | 5574 | 0.58 | 322 | 0.060 | 392 | 0.0656 |
| 8000 | 4992 | 0.59 | 265 | 0.055 | 329 | 0.0607 |
| 7000 | 4400 | 0.60 | 213 | 0.049 | 271 | 0.0558 |
| 6000 | 3800 | 0.61 | 167 | 0.044 | 217 | 0.0509 |
| 5000 | 3189 | 0.61 | 126 | 0.038 | 169 | 0.0460 |
| 4000 | 2570 | 0.62 | 91 | 0.033 | 125 | 0.0410 |
| 3000 | 1941 | 0.63 | 61 | 0.027 | 87 | 0.0361 |
| 2000 | 1303 | 0.64 | 36 | 0.021 | 53 | 0.0312 |
| 1000 | 655 | 0.65 | 18 | 0.016 | 24 | 0.0263 |
| 500 | 328 | 0.66 | 10 | 0.013 | 12 | 0.0239 |

Reserve #3

| Tonnage | Fe Tonnage | dFe/dT | Si Tonnage | dSi/dT | Al Tonnage | dAl/dT |
|---|---|---|---|---|---|---|
| 770 | 502 | 0.60 | 36 | 0.110 | 9 | 0.0156 |
| 750 | 490 | 0.60 | 34 | 0.106 | 9 | 0.0154 |
| 700 | 460 | 0.61 | 28 | 0.098 | 8 | 0.0149 |
| 650 | 429 | 0.62 | 24 | 0.089 | 7 | 0.0144 |
| 600 | 398 | 0.62 | 19 | 0.081 | 6 | 0.0139 |
| 550 | 367 | 0.63 | 16 | 0.072 | 6 | 0.0134 |
| 500 | 335 | 0.64 | 12 | 0.063 | 5 | 0.0129 |
| 450 | 303 | 0.65 | 9 | 0.055 | 4 | 0.0125 |
| 400 | 270 | 0.65 | 7 | 0.046 | 4 | 0.0120 |
| 350 | 238 | 0.66 | 5 | 0.037 | 3 | 0.0115 |
| 300 | 204 | 0.67 | 3 | 0.029 | 3 | 0.0110 |
| 250 | 171 | 0.67 | 2 | 0.020 | 2 | 0.0105 |
| 200 | 137 | 0.68 | 1 | 0.011 | 2 | 0.0100 |
| 150 | 103 | 0.69 | 1 | 0.003 | 1 | 0.0095 |
| 100 | 68 | 0.69 | 1 | -0.006 | 1 | 0.0090 |
| 50 | 34 | 0.70 | 1 | -0.014 | 0 | 0.0085 |

Reserve #4

| Tonnage | Fe Tonnage | dFe/dT | Si Tonnage | dSi/dT | Al Tonnage | dAl/dT |
|---|---|---|---|---|---|---|
| 42000 | 21358 | 0.52 | 2436 | 0.054 | 472 | 0.0109 |
| 38000 | 20206 | 0.54 | 1902 | 0.045 | 395 | 0.0100 |
| 34000 | 18862 | 0.56 | 1434 | 0.037 | 326 | 0.0091 |
| 30000 | 17326 | 0.59 | 1034 | 0.029 | 263 | 0.0082 |
| 26000 | 15598 | 0.61 | 701 | 0.020 | 208 | 0.0074 |
| 22000 | 13678 | 0.64 | 435 | 0.012 | 159 | 0.0065 |
| 18000 | 11566 | 0.66 | 236 | 0.003 | 117 | 0.0056 |
| 14000 | 9262 | 0.68 | 105 | -0.005 | 83 | 0.0048 |
| 10000 | 6766 | 0.71 | 41 | -0.013 | 55 | 0.0039 |
| 6000 | 4078 | 0.73 | 44 | -0.022 | 34 | 0.0030 |
| 2000 | 1198 | 0.76 | 114 | -0.030 | 20 | 0.0022 |

Reserve #5

| Tonnage | Fe Tonnage | dFe/dT | Si Tonnage | dSi/dT | Al Tonnage | dAl/dT |
|---|---|---|---|---|---|---|
| 11200 | 6870 | 0.61 | 561 | 0.052 | 250 | 0.0226 |
| 10200 | 6354 | 0.62 | 485 | 0.050 | 220 | 0.0219 |
| 9200 | 5820 | 0.63 | 414 | 0.047 | 192 | 0.0212 |
| 8200 | 5267 | 0.64 | 348 | 0.045 | 165 | 0.0205 |
| 7200 | 4696 | 0.65 | 286 | 0.042 | 140 | 0.0198 |
| 6200 | 4106 | 0.66 | 229 | 0.040 | 116 | 0.0191 |
| 5200 | 3498 | 0.67 | 177 | 0.038 | 93 | 0.0184 |
| 4200 | 2871 | 0.68 | 129 | 0.035 | 72 | 0.0177 |
| 3200 | 2225 | 0.68 | 86 | 0.033 | 52 | 0.0170 |
| 2200 | 1561 | 0.69 | 48 | 0.031 | 33 | 0.0163 |
| 1200 | 879 | 0.70 | 14 | 0.028 | 16 | 0.0156 |

Reserve #6

| Tonnage | Fe Tonnage | dFe/dT | Si Tonnage | dSi/dT | Al Tonnage | dAl/dT |
|---|---|---|---|---|---|---|
| 20800 | 13191 | 0.63 | 1002 | 0.049 | 395 | 0.0182 |
| 19800 | 12600 | 0.64 | 941 | 0.049 | 358 | 0.0173 |
| 18800 | 12005 | 0.64 | 881 | 0.048 | 323 | 0.0163 |
| 17800 | 11405 | 0.64 | 823 | 0.048 | 290 | 0.0154 |
| 16800 | 10801 | 0.64 | 765 | 0.047 | 258 | 0.0144 |
| 15800 | 10193 | 0.64 | 709 | 0.046 | 229 | 0.0135 |
| 14800 | 9581 | 0.65 | 653 | 0.046 | 201 | 0.0125 |
| 13800 | 8964 | 0.65 | 599 | 0.045 | 176 | 0.0116 |
| 12800 | 8343 | 0.65 | 547 | 0.045 | 152 | 0.0107 |
| 11800 | 7717 | 0.65 | 495 | 0.044 | 130 | 0.0097 |
| 10800 | 7088 | 0.66 | 445 | 0.043 | 110 | 0.0088 |
| 9800 | 6453 | 0.66 | 395 | 0.043 | 92 | 0.0078 |
| 8800 | 5815 | 0.66 | 347 | 0.042 | 76 | 0.0069 |
| 7800 | 5172 | 0.66 | 300 | 0.042 | 62 | 0.0059 |
| 6800 | 4525 | 0.66 | 255 | 0.041 | 50 | 0.0050 |
| 5800 | 3874 | 0.67 | 210 | 0.040 | 39 | 0.0040 |
| 4800 | 3218 | 0.67 | 167 | 0.040 | 31 | 0.0031 |
| 3800 | 2558 | 0.67 | 125 | 0.039 | 24 | 0.0021 |
| 2800 | 1894 | 0.67 | 84 | 0.039 | 19 | 0.0012 |
| 1800 | 1225 | 0.67 | 44 | 0.038 | 16 | 0.0003 |
| 800 | 552 | 0.68 | 5 | 0.038 | 15 | -0.0007 |

Reserve #7

| Tonnage | Fe Tonnage | dFe/dT | Si Tonnage | dSi/dT | Al Tonnage | dAl/dT |
|---|---|---|---|---|---|---|
| 24500 | 14442 | 0.59 | 1774 | 0.072 | 239 | 0.0098 |
| 23500 | 13994 | 0.60 | 1652 | 0.069 | 226 | 0.0096 |
| 22500 | 13535 | 0.60 | 1534 | 0.067 | 214 | 0.0095 |
| 21500 | 13062 | 0.61 | 1421 | 0.065 | 201 | 0.0094 |
| 20500 | 12578 | 0.62 | 1312 | 0.063 | 189 | 0.0093 |
| 19500 | 12082 | 0.62 | 1207 | 0.061 | 178 | 0.0092 |
| 18500 | 11573 | 0.63 | 1107 | 0.059 | 166 | 0.0090 |
| 17500 | 11052 | 0.63 | 1010 | 0.057 | 155 | 0.0089 |
| 16500 | 10518 | 0.64 | 919 | 0.054 | 144 | 0.0088 |
| 15500 | 9973 | 0.65 | 831 | 0.052 | 134 | 0.0087 |
| 14500 | 9415 | 0.65 | 748 | 0.050 | 123 | 0.0085 |
| 13500 | 8845 | 0.66 | 669 | 0.048 | 113 | 0.0084 |
| 12500 | 8263 | 0.66 | 594 | 0.046 | 103 | 0.0083 |
| 11500 | 7668 | 0.67 | 523 | 0.044 | 93 | 0.0082 |
| 10500 | 7062 | 0.68 | 457 | 0.042 | 84 | 0.0081 |
| 9500 | 6443 | 0.68 | 395 | 0.039 | 75 | 0.0079 |
| 8500 | 5812 | 0.69 | 338 | 0.037 | 66 | 0.0078 |
| 7500 | 5168 | 0.70 | 284 | 0.035 | 57 | 0.0077 |
| 6500 | 4513 | 0.70 | 235 | 0.033 | 48 | 0.0076 |
| 5500 | 3845 | 0.71 | 190 | 0.031 | 40 | 0.0074 |
| 4500 | 3165 | 0.71 | 150 | 0.029 | 32 | 0.0073 |
| 3500 | 2472 | 0.72 | 113 | 0.027 | 24 | 0.0072 |
| 2500 | 1768 | 0.73 | 82 | 0.024 | 17 | 0.0071 |
| 1500 | 1051 | 0.73 | 54 | 0.022 | 10 | 0.0069 |
| 500 | 322 | 0.74 | 30 | 0.020 | 3 | 0.0068 |

Reserve #8

| | Fe | | Si | | Al | |
|---|---|---|---|---|---|---|
| Tonnage | Tonnage | dFe/dT | Tonnage | dSi/dT | Tonnage | dAl/dT |
| 26600 | 15926 | 0.60 | 922 | 0.035 | 869 | 0.0329 |
| 25400 | 15228 | 0.60 | 871 | 0.035 | 820 | 0.0325 |
| 24200 | 14528 | 0.60 | 821 | 0.035 | 771 | 0.0321 |
| 23000 | 13827 | 0.60 | 772 | 0.034 | 723 | 0.0317 |
| 21800 | 13124 | 0.60 | 723 | 0.034 | 676 | 0.0313 |
| 20600 | 12420 | 0.60 | 675 | 0.034 | 630 | 0.0308 |
| 19400 | 11714 | 0.60 | 629 | 0.033 | 585 | 0.0304 |
| 18200 | 11006 | 0.60 | 582 | 0.033 | 541 | 0.0300 |
| 17000 | 10297 | 0.60 | 537 | 0.033 | 498 | 0.0296 |
| 15800 | 9586 | 0.60 | 493 | 0.032 | 456 | 0.0292 |
| 14600 | 8874 | 0.60 | 449 | 0.032 | 415 | 0.0288 |
| 13400 | 8160 | 0.60 | 406 | 0.032 | 375 | 0.0284 |
| 12200 | 7444 | 0.60 | 364 | 0.031 | 336 | 0.0280 |
| 11000 | 6727 | 0.60 | 322 | 0.031 | 298 | 0.0276 |
| 9800 | 6009 | 0.60 | 282 | 0.031 | 261 | 0.0272 |
| 8600 | 5289 | 0.61 | 242 | 0.031 | 225 | 0.0268 |
| 7400 | 4567 | 0.61 | 203 | 0.030 | 190 | 0.0263 |
| 6200 | 3844 | 0.61 | 165 | 0.030 | 155 | 0.0259 |
| 5000 | 3119 | 0.61 | 127 | 0.030 | 122 | 0.0255 |
| 3800 | 2393 | 0.61 | 90 | 0.029 | 90 | 0.0251 |

# APPENDIX VII

## MARGINAL VALUES (RATES OF CHANGE) BETWEEN VARIOUS MINERALS OF ALL RESERVES

Reserve #1

| Tonnage | dFe/dSi | dFe/dAl | dSi/dFe | dSi/dAl | dAl/dFe | dAl/dSi |
|---|---|---|---|---|---|---|
| 23000 | 2.51 | 68.45 | 0.40 | 27.27 | 0.015 | 0.037 |
| 21000 | 3.02 | 74.62 | 0.33 | 24.71 | 0.013 | 0.040 |
| 19000 | 3.72 | 81.43 | 0.27 | 21.88 | 0.012 | 0.046 |
| 17000 | 4.75 | 88.99 | 0.21 | 18.75 | 0.011 | 0.053 |
| 15000 | 6.39 | 97.42 | 0.16 | 15.24 | 0.010 | 0.066 |
| 13000 | 9.45 | 106.89 | 0.11 | 11.31 | 0.009 | 0.088 |
| 11000 | 17.14 | 117.60 | 0.06 | 6.86 | 0.009 | 0.146 |
| 9000 | 72.58 | 129.82 | 0.01 | 1.79 | 0.008 | 0.559 |
| 7000 | -35.52 | 143.88 | -0.03 | -4.05 | 0.007 | -0.247 |
| 5000 | -14.78 | 160.24 | -0.07 | -10.85 | 0.006 | -0.092 |
| 3000 | -9.52 | 179.52 | -0.10 | -18.85 | 0.006 | -0.053 |

Reserve #2

| Tonnage | dFe/dSi | dFe/dAl | dSi/dFe | dSi/dAl | dAl/dFe | dAl/dSi |
|---|---|---|---|---|---|---|
| 10300 | 8.41 | 7.85 | 0.12 | 0.93 | 0.127 | 1.070 |
| 10000 | 8.66 | 8.06 | 0.12 | 0.93 | 0.124 | 1.075 |
| 9000 | 9.61 | 8.80 | 0.10 | 0.92 | 0.114 | 1.092 |
| 8000 | 10.75 | 9.67 | 0.09 | 0.90 | 0.103 | 1.112 |
| 7000 | 12.15 | 10.69 | 0.08 | 0.88 | 0.094 | 1.137 |
| 6000 | 13.91 | 11.90 | 0.07 | 0.86 | 0.084 | 1.168 |
| 5000 | 16.17 | 13.38 | 0.06 | 0.83 | 0.075 | 1.208 |
| 4000 | 19.19 | 15.21 | 0.05 | 0.79 | 0.066 | 1.262 |
| 3000 | 23.46 | 17.53 | 0.04 | 0.75 | 0.057 | 1.338 |
| 2000 | 29.91 | 20.59 | 0.03 | 0.69 | 0.049 | 1.452 |
| 1000 | 40.80 | 24.79 | 0.02 | 0.61 | 0.040 | 1.646 |
| 500 | 49.66 | 27.53 | 0.02 | 0.55 | 0.036 | 1.803 |

Reserve #3

| Tonnage | dFe/dSi | dFe/dAl | dSi/dFe | dSi/dAl | dAl/dFe | dAl/dSi |
|---|---|---|---|---|---|---|
| 770 | 5.47 | 38.46 | 0.18 | 7.03 | 0.026 | 0.142 |
| 750 | 5.67 | 39.13 | 0.18 | 6.90 | 0.026 | 0.145 |
| 700 | 6.25 | 40.89 | 0.16 | 6.55 | 0.024 | 0.153 |
| 650 | 6.93 | 42.78 | 0.14 | 6.17 | 0.023 | 0.162 |
| 600 | 7.76 | 44.80 | 0.13 | 5.78 | 0.022 | 0.173 |
| 550 | 8.78 | 46.96 | 0.11 | 5.35 | 0.021 | 0.187 |
| 500 | 10.09 | 49.30 | 0.10 | 4.89 | 0.020 | 0.205 |
| 450 | 11.81 | 51.82 | 0.08 | 4.39 | 0.019 | 0.228 |
| 400 | 14.18 | 54.55 | 0.07 | 3.85 | 0.018 | 0.260 |
| 350 | 17.64 | 57.51 | 0.06 | 3.26 | 0.017 | 0.307 |
| 300 | 23.18 | 60.75 | 0.04 | 2.62 | 0.016 | 0.382 |
| 250 | 33.47 | 64.29 | 0.03 | 1.92 | 0.016 | 0.521 |
| 200 | 59.27 | 68.18 | 0.02 | 1.15 | 0.015 | 0.869 |
| 150 | 241.99 | 72.49 | .00 | 0.30 | 0.014 | 3.338 |
| 100 | -119.70 | 77.27 | -0.01 | -0.65 | 0.013 | -1.549 |
| 50 | -48.55 | 82.61 | -0.02 | -1.70 | 0.012 | -0.588 |

Reserve #4

| Tonnage | dFe/dSi | dFe/dAl | dSi/dFe | dSi/dAl | dAl/dFe | dAl/dSi |
|---|---|---|---|---|---|---|
| 42000 | 9.58 | 47.55 | 0.10 | 4.96 | 0.021 | 0.202 |
| 38000 | 11.88 | 54.10 | 0.08 | 4.55 | 0.018 | 0.220 |
| 34000 | 15.23 | 61.89 | 0.07 | 4.06 | 0.016 | 0.246 |
| 30000 | 20.53 | 71.32 | 0.05 | 3.47 | 0.014 | 0.288 |
| 26000 | 30.25 | 82.99 | 0.03 | 2.74 | 0.012 | 0.365 |
| 22000 | 53.78 | 97.76 | 0.02 | 1.82 | 0.010 | 0.550 |
| 18000 | 192.89 | 117.10 | 0.01 | 0.61 | 0.009 | 1.647 |
| 14000 | -137.26 | 143.48 | -0.01 | -1.05 | 0.007 | -0.957 |
| 10000 | -52.88 | 181.63 | -0.02 | -3.43 | 0.006 | -0.291 |
| 6000 | -33.59 | 241.68 | -0.03 | -7.20 | 0.004 | -0.139 |
| 2000 | -25.04 | 350.06 | -0.04 | -13.98 | 0.003 | -0.072 |

Reserve #5

| Tonnage | dFe/dSi | dFe/dAl | dSi/dFe | dSi/dAl | dAl/dFe | dAl/dSi |
|---|---|---|---|---|---|---|
| 11200 | 11.77 | 27.05 | 0.08 | 2.30 | 0.037 | 0.435 |
| 10200 | 12.51 | 28.33 | 0.08 | 2.26 | 0.035 | 0.442 |
| 9200 | 13.33 | 29.69 | 0.08 | 2.23 | 0.034 | 0.449 |
| 8200 | 14.23 | 31.15 | 0.07 | 2.19 | 0.032 | 0.457 |
| 7200 | 15.24 | 32.71 | 0.07 | 2.15 | 0.031 | 0.466 |
| 6200 | 16.36 | 34.38 | 0.06 | 2.10 | 0.029 | 0.476 |
| 5200 | 17.62 | 36.17 | 0.06 | 2.05 | 0.028 | 0.487 |
| 4200 | 19.04 | 38.11 | 0.05 | 2.00 | 0.026 | 0.500 |
| 3200 | 20.67 | 40.20 | 0.05 | 1.95 | 0.025 | 0.514 |
| 2200 | 22.54 | 42.48 | 0.04 | 1.88 | 0.024 | 0.531 |
| 1200 | 24.73 | 44.95 | 0.04 | 1.82 | 0.022 | 0.550 |

Reserve #6

| Tonnage | dFe/dSi | dFe/dAl | dSi/dFe | dSi/dAl | dAl/dFe | dAl/dSi |
|---|---|---|---|---|---|---|
| 20800 | 12.84 | 34.79 | 0.08 | 2.71 | 0.029 | 0.369 |
| 19800 | 13.04 | 36.82 | 0.08 | 2.82 | 0.027 | 0.354 |
| 18800 | 13.24 | 39.08 | 0.08 | 2.95 | 0.026 | 0.339 |
| 17800 | 13.45 | 41.63 | 0.07 | 3.09 | 0.024 | 0.323 |
| 16800 | 13.67 | 44.50 | 0.07 | 3.26 | 0.022 | 0.307 |
| 15800 | 13.89 | 47.78 | 0.07 | 3.44 | 0.021 | 0.291 |
| 14800 | 14.12 | 51.55 | 0.07 | 3.65 | 0.019 | 0.274 |
| 13800 | 14.35 | 55.94 | 0.07 | 3.90 | 0.018 | 0.257 |
| 12800 | 14.59 | 61.11 | 0.07 | 4.19 | 0.016 | 0.239 |
| 11800 | 14.83 | 67.28 | 0.07 | 4.54 | 0.015 | 0.220 |
| 10800 | 15.09 | 74.78 | 0.07 | 4.96 | 0.013 | 0.202 |
| 9800 | 15.35 | 84.10 | 0.07 | 5.48 | 0.012 | 0.182 |
| 8800 | 15.61 | 95.98 | 0.06 | 6.15 | 0.010 | 0.163 |
| 7800 | 15.89 | 111.65 | 0.06 | 7.03 | 0.009 | 0.142 |
| 6800 | 16.17 | 133.26 | 0.06 | 8.24 | 0.008 | 0.121 |
| 5800 | 16.46 | 164.99 | 0.06 | 10.02 | 0.006 | 0.100 |
| 4800 | 16.76 | 216.12 | 0.06 | 12.90 | 0.005 | 0.078 |
| 3800 | 17.07 | 312.27 | 0.06 | 18.30 | 0.003 | 0.055 |
| 2800 | 17.38 | 559.65 | 0.06 | 32.19 | 0.002 | 0.031 |
| 1800 | 17.71 | 2629.12 | 0.06 | 148.44 | .000 | 0.007 |
| 800 | 18.05 | -983.15 | 0.06 | -54.47 | -0.001 | -0.018 |

Reserve #7

| Tonnage | dFe/dSi | dFe/dAl | dSi/dFe | dSi/dAl | dAl/dFe | dAl/dSi |
|---|---|---|---|---|---|---|
| 24500 | 8.26 | 60.53 | 0.12 | 7.32 | 0.017 | 0.137 |
| 23500 | 8.61 | 61.93 | 0.12 | 7.20 | 0.016 | 0.139 |
| 22500 | 8.97 | 63.37 | 0.11 | 7.06 | 0.016 | 0.142 |
| 21500 | 9.36 | 64.85 | 0.11 | 6.93 | 0.015 | 0.144 |
| 20500 | 9.78 | 66.36 | 0.10 | 6.79 | 0.015 | 0.147 |
| 19500 | 10.22 | 67.92 | 0.10 | 6.65 | 0.015 | 0.150 |
| 18500 | 10.70 | 69.52 | 0.09 | 6.50 | 0.014 | 0.154 |
| 17500 | 11.21 | 71.16 | 0.09 | 6.35 | 0.014 | 0.158 |
| 16500 | 11.76 | 72.85 | 0.09 | 6.19 | 0.014 | 0.161 |
| 15500 | 12.36 | 74.59 | 0.08 | 6.03 | 0.013 | 0.166 |
| 14500 | 13.01 | 76.38 | 0.08 | 5.87 | 0.013 | 0.170 |
| 13500 | 13.72 | 78.22 | 0.07 | 5.70 | 0.013 | 0.175 |
| 12500 | 14.49 | 80.11 | 0.07 | 5.53 | 0.012 | 0.181 |
| 11500 | 15.34 | 82.06 | 0.07 | 5.35 | 0.012 | 0.187 |
| 10500 | 16.28 | 84.07 | 0.06 | 5.17 | 0.012 | 0.194 |
| 9500 | 17.32 | 86.14 | 0.06 | 4.98 | 0.012 | 0.201 |
| 8500 | 18.47 | 88.28 | 0.05 | 4.78 | 0.011 | 0.209 |
| 7500 | 19.77 | 90.49 | 0.05 | 4.58 | 0.011 | 0.218 |
| 6500 | 21.24 | 92.76 | 0.05 | 4.37 | 0.011 | 0.229 |
| 5500 | 22.91 | 95.12 | 0.04 | 4.15 | 0.011 | 0.241 |
| 4500 | 24.82 | 97.55 | 0.04 | 3.93 | 0.010 | 0.254 |
| 3500 | 27.05 | 100.06 | 0.04 | 3.70 | 0.010 | 0.270 |
| 2500 | 29.67 | 102.66 | 0.03 | 3.46 | 0.010 | 0.289 |
| 1500 | 32.78 | 105.35 | 0.03 | 3.21 | 0.009 | 0.311 |
| 500 | 36.56 | 108.14 | 0.03 | 2.96 | 0.009 | 0.338 |

Reserve #8

| Tonnage | dFe/dSi | dFe/dAl | dSi/dFe | dSi/dAl | dAl/dFe | dAl/dSi |
|---|---|---|---|---|---|---|
| 26600 | 16.82 | 18.11 | 0.06 | 1.08 | 0.055 | 0.928 |
| 25400 | 16.99 | 18.36 | 0.06 | 1.08 | 0.054 | 0.926 |
| 24200 | 17.17 | 18.61 | 0.06 | 1.08 | 0.054 | 0.922 |
| 23000 | 17.35 | 18.87 | 0.06 | 1.09 | 0.053 | 0.919 |
| 21800 | 17.54 | 19.14 | 0.06 | 1.09 | 0.052 | 0.916 |
| 20600 | 17.73 | 19.42 | 0.06 | 1.10 | 0.052 | 0.913 |
| 19400 | 17.92 | 19.70 | 0.06 | 1.10 | 0.051 | 0.910 |
| 18200 | 18.12 | 19.99 | 0.06 | 1.10 | 0.050 | 0.906 |
| 17000 | 18.32 | 20.29 | 0.05 | 1.11 | 0.049 | 0.903 |
| 15800 | 18.52 | 20.59 | 0.05 | 1.11 | 0.049 | 0.900 |
| 14600 | 18.73 | 20.91 | 0.05 | 1.12 | 0.048 | 0.896 |
| 13400 | 18.95 | 21.23 | 0.05 | 1.12 | 0.047 | 0.892 |
| 12200 | 19.16 | 21.57 | 0.05 | 1.13 | 0.046 | 0.889 |
| 11000 | 19.39 | 21.91 | 0.05 | 1.13 | 0.046 | 0.885 |
| 9800 | 19.61 | 22.26 | 0.05 | 1.13 | 0.045 | 0.881 |
| 8600 | 19.85 | 22.63 | 0.05 | 1.14 | 0.044 | 0.877 |
| 7400 | 20.08 | 23.00 | 0.05 | 1.15 | 0.043 | 0.873 |
| 6200 | 20.33 | 23.39 | 0.05 | 1.15 | 0.043 | 0.869 |
| 5000 | 20.57 | 23.79 | 0.05 | 1.16 | 0.042 | 0.865 |