A LINEAR PROGRAMMING ALGORITHM TO MINIMIZE GLASS BATCH COSTS

by

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ABSTRACT

A model of the glass batch blending process at the Coors Glass Plant has been developed with minimizing cost as the objective. The algorithm is solved by iterative linear programming using the Simplex Method and the solution is found on an IBM compatible computer using Microsoft Excel. Output serves as an aid to the operations manager in making the daily glass batch composition decisions. Each percentage reduction in the cost of the raw materials associated with a glass batch would increase profit by approximately $400,000 per year.

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Chapter 1

INTRODUCTION

1.1 General Overview

Glass manufacturers are experiencing shrinking markets, cost pressures, competition, oversupply and decreasing demand. Strong market resistance to price increases continues. These variables are forcing glass manufacturers to study their operations for productivity gains, process and product improvements, and energy conservation. [1]

In order to cope with the problems being encountered in the glass industry, manufacturers are faced with the necessity of lowering total cost. It will be necessary for manufacturers to understand their own processes and to determine what elements contribute to cost relative to those processes. With no measurement system in place it will be difficult to achieve significant cost reductions because managers will be relying on 'gut feel' and intuition rather than facts. [1]

Minimizing glass batch costs is one area of glass manufacturing that can contribute to lowering total cost. Optimizing batch calculations using a blending algorithm can reduce the cost of the raw materials used in the glass manufacturing process. An algorithm used to optimize glass batch calculations can also reduce the time managers spend determining the optimal composition of each glass batch, and saving time is equivalent to saving money. Blending algorithms have been the subject of extensive study for many decades. [6], [8], [9], [11], [12], [13], [14]

Blending problems arise in a number of manufacturing activities and refer to situations in which multiple components are mixed together to yield one or more products. In these situations, the resources contain one or more essential ingredient that must be blended in such a manner that the final products will contain specific percentages of each ingredient. There are normally restrictions on the available quantities of the raw materials, restrictions on the quality of the products, and restrictions on the quantities of the products to be produced. Typically, there are many different ways in which raw materials can be blended together to form the final products that satisfy the various constraints. In most of these applications the operations manager must decide how much of each resource to purchase in order to satisfy product specifications and product demands while minimizing cost. [2], [3]

Many operations management decisions involve trying to make the most effective use of an organization's resources. The operations manager must decide which activities can be pursued without over using limited resources and while obtaining maximum reward. Linear programming is a widely used mathematical technique designed to help production and operations managers in planning and decision making while also keeping in mind the tradeoffs necessary to allocate resources. [4], [7]

There are many traditional applications of linear programming in the area of blending, some of which are product blending, product mixing, and the diet problem. There are a wide variety of problems in which certain basic raw materials are combined, or blended, to produce a product that satisfies certain specifications. The specifications of the blend along with the restrictions are given, and the task is to produce a blend that minimizes total cost while satisfying these restrictions. [5]

The product-mix problem is typical of linear programming problems in which there is competition for limited resources among several products. The problem is to allocate resources so as to maximize profit (or minimize cost). [5]

The diet problem is also a specific type of blending problem that arises in a number of manufacturing activities. The model behind the diet problem can be used to create menus that satisfy one's daily need for calories, vitamins, and minerals while limiting fat and cholesterol intake.

1.2 Overview of Existing System

The first step in solving a problem is to define the problem. Defining the problem includes specifying the organization's objectives and the parts of the organization (or system) that must be studied before the problem can be solved [15].

Daily, the ingredients that are used in glass manufacturing, such as lien sand, soda ash and limestone, are brought into the Coors Glass Plant via tanker trucks and these ingredients are stored in large silos. Each ingredient consists of a combination of the oxides important in glass manufacturing and the percentages of the oxides can vary slightly from day to day. All raw material arrivals at the Glass Plant coincide with an incoming fax informing the operations manager of the new raw material characteristics. Currently, the manager in charge of glass manufacturing manually calculates the quantities of each ingredient that are going to be blended together to create a glass batch.

There are three furnaces at the glass plant and three spreadsheets associated with each furnace. One spreadsheet contains information on the amounts, in pounds, of each of the different types of sand, or ingredients involved in glass manufacturing. Another spreadsheet performs calculations, and the third spreadsheet contains the percentages of the oxides present in the sands.

First, in the batch calculation, the percentages of the oxides for each ingredient are entered into the respective spreadsheet. Calculations are then performed based on the percentages of the oxides present in the raw materials. These calculations also take into account the idiosyncrasies and the requirements of the three furnaces, and include certain theoretical physical properties associated with the desired characteristics of the glass. Finally, the manager adjusts the amounts, in pounds, of the different ingredients until all the constraints are within their acceptable bounds.

The procedure currently used at the Coors Glass Plant for determining the composition of the daily glass batch relies on the knowledge of the operations manager. There are several potential problems with this procedure:

1. Without a system in place to calculate the optimal blend of the glass batch there is no assurance on the quality of the end product.
2. While the present manager has learned the technique of calculating a batch composition by hand, requiring only about 20 minutes per furnace per day, it is difficult for others to learn the same intuition and to apply that knowledge when the manager is out for the day.
3. This process does not take cost into consideration. With the problems being faced in the glass industry manufacturers are striving to lower total cost and ignoring cost does not lead to the low cost solution.

4. Currently, this system cannot evaluate the economic viability of purchasing raw materials from different sources.

1.3 Proposed System

Recently, the Coors Brewing Company - Glass Division - has been employing operations research methodology to make improvements in their glass manufacturing operations. One such topic of study is minimizing the total cost in glass batch production.

The objective of this thesis is to develop a linear programming algorithm, based on a version of the product-mix blending algorithm. This algorithm will determine the optimal amounts of the raw materials to be used in the composition of a glass batch, while minimizing cost. The Coors' Glass Manufacturing Plant in Wheat Ridge, Colorado will be the basis for this algorithm.

The decision variables will be the amounts, in pounds, of each of the different raw materials, or sands. The objective function will minimize cost and the solution will be constrained by the following:

- The furnace requirements and idiosyncrasies will be taken into account.

- The theoretical glass composition requirements, for example, the total percentage of SiO_2 found in the batch, will stay within a narrow acceptable range.
- The batch properties and requirements will be maintained.
- The theoretical physical properties requirements of the glass will also stay with in a narrow acceptable range.

A program will be developed to perform the algorithm. Flexibility will also be built into the program so that when the operations manager wants to change certain requirements, for example, wanting a slightly different color in the glass, those changes will be easily implemented.

Chapter 2

LITERATURE REVIEW

The following reference material assisted in the development of this thesis. The Colorado School of Mines Library, as well as at the Denver Public Library and the Dr. Maurer Library, provided the majority of the reference materials.

Wu, Nesa and Richard Coppins. 1981. *Linear Programming and Extensions.* New York: McGraw-Hill, Inc.

This text was used in the linear programming course offered at the Colorado School of Mines. Chapter 2 gives a basic overview of linear programming. The formulation and graphical interpretation of linear programming are discussed, along with examples of how this technique can be used to select the best course of action from a set of alternatives. This text provided the basic linear programming knowledge used in this thesis.

The three basic steps in the formulation of a linear programming model must include the following: (1) definition of the decision variables, (2) definition of the objective function, and (3) formulation of the constraints. These three steps lead to the single-objective linear programming model. The general form of the standard linear program in symbols is as follows [16]:

Optimize $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

subject to

$$a_{1,1}x_1 + a_{1,2}x_2 + \bullet\bullet\bullet + a_{1,n}x_n \{\leq, =, \geq\}\, b_1$$

$$a_{2,1}x_1 + a_{2,2}x_2 + \bullet\bullet\bullet + a_{2,n}x_n \{\leq, =, \geq\}\, b_2$$

● ●

$$a_{i,1}x_1 + a_{i,2}x_2 + \bullet\bullet\bullet + a_{i,n}x_n \{\leq, =, \geq\}\, b_i$$

● ●

$$a_{m,1}x_1 + a_{m,2}x_2 + \bullet\bullet\bullet + a_{m,n}x_n \{\leq, =, \geq\}\, b_m$$

$x_1, x_2, \bullet\bullet\bullet, x_n \geq 0$ (non-negativity)

where

$x_1, x_2, \bullet\bullet\bullet, x_n$ = decision variables

z = objective function

$c_1, c_2, \bullet\bullet\bullet, c_n$ = coefficients of the decision variables in the objective function

$a_{i,1}$, $a_{i,2}$, •••, $a_{i,n}$ = coefficients of the decision variables in the *i*th constraint

b_i = constant (or right-hand side) of the *i*th constraint

Nahmias, Steven. 1993. *Production and Operations Analysis*. Boston: Richard D. Irwin, Inc.

Steven Nahmias provides a supplemental chapter in his book, *Production and Operations Analysis*, on linear programming. Within this supplemental chapter lies an overview of the simplex method for solving linear programming problems. The simplex method is used in this thesis to solve the linear programming blending algorithm for minimizing glass batch costs. The simplex method is an algorithm that moves sequentially from one extreme point to another until it reaches the optimal solution. If the origin is a feasible solution, it will serve as the initial extreme point. At each iteration, the method considers all adjacent extreme points (those that can be reached by moving along an edge), and moves to the one that gives the best improvement in the objective function. The algorithm continues to move from one adjacent extreme point to another, finally terminating when the optimal solution is reached. [17]

User's Guide - Microsoft Excel - Version 5.0. Microsoft Office. Microsoft Corporation.

The production manager at Coors Brewing Company - Glass Division presently employs spreadsheets in Microsoft Excel to determine the daily glass batch compositions. These spreadsheets perform calculations in order to assist with the determination of the optimal batch size and composition to be used in the glass bottle production. In order to provide a program that would be utilized, familiar to the operations manager, and easily integrated into the existing system this model will solve the linear programming blending algorithm with Microsoft Excel. Excel has a tool called Solver that can be used to solve linear programming problems. Solver uses the Simplex Method for solving these problems. To use Microsoft Excel Solver with a spreadsheet model the problem must be defined by identifying (1) a target cell, or objective function, (2) the changing cells, or decision variables, and (3) the constraints to be used in the analysis. After the problem is defined and the solution process begins, Solver finds values that satisfy the constraints and optimize the objective function. Solver then displays the resulting values on the spreadsheet. The Excel User's Guide was very helpful in the implementation of the Coors glass blending algorithm. [18]

Winston, Wayne. 1991. *Operations Research: Applications and Algorithms*. Boston: PWS-Kent Publishing Company.

Chapter 1 of this text provided a good discussion on how to use operations research techniques to plan and solve problems. The seven step procedure outlined in this text was followed in establishing the framework for the study at the Coors Glass Plant. Wayne Winston believes when operations research is used to solve a problem, the following seven-step procedure should be followed (Table 1). [15]

Table 1

The Operations Research Methodology

Step 1.	Formulate the problem
Step 2.	Observe the system
Step 3.	Formulate a mathematical model of the problem
Step 4.	Verify the model and use the model for prediction
Step 5.	Select a suitable alternative
Step 6.	Present the results and conclusions of the study to the organization
Step 7.	Implement and evaluate recommendation

Anderson, David, Dennis Sweeney, and Thomas Williams. 1994. *An Introduction to Management Science: Quantitative Approaches to Decision Making*. New York: West Publishing Company.

This management science text touched upon the same problem solving steps discussed in the text by Wayne Winston [15]. Although each of these problem solving steps were used in this thesis, the final step, implementation, meet with some difficulty. The information regarding implementation found in this book was very useful. Since implementation frequently requires people to do things differently, it often meets with resistance. People tend to feel threatened by new procedures, believing their previous course of action is being called into question. This text promotes the view that the most effective way to ensure a successful implementation is to attain as much user involvement as possible throughout the problem solving process. If the users feel they have been involved in identifying the problem and developing the solutions, they are much more likely to enthusiastically implement the results. The likelihood for successful implementation of a project solution is much greater for those projects in which there has been extensive user involvement. [3]

Anderson, D. W. 1994. "Minimizing Glass Batch Costs Through Linear Programming." *Ceramic Engineering and Science Proceeding*, **15**(2): 19-26.

D. W. Anderson wrote, "Minimizing Glass Batch Costs Through Linear Programming", in which he discussed how the calculation of a glass batch may be viewed as the solution set to a set of simultaneous linear equations. In the glass business the manufacturer employing the most economical batch formula compatible with its melting process gains an advantage over its competitors. The practical problem of selecting the optimal low-cost solution requires analysis of countless possible batch combinations. [8]

This article discusses the algorithms that are available to examine the economy of a glass batch and determines the best technique available is linear programming. The glass batch problem adapts readily to analysis and solution using linear programming. It is possible to describe the glass batch problem as a set of simultaneous linear equations to be solved in such a manner that a linear cost function is optimized. [8]

Peng, Y. B., Xingye Lei and D. E. Day. 1991. "A Computer Programme for Optimising Batch Calculations." *Glass Technology*, **32**(4): 123-130.

In Y. B. Peng, Xingye Lei and D. E. Day's article they described how linear programming has been applied to batch calculation problems. This paper discusses a program called 'All-Purpose Batch Calculation' that was developed by the authors and outlines the general algorithm used in the program. Linear programming and the simplex method are used to minimize cost. The optimization is found through the creation of artificial free energies, which generally lead to batches with the smallest weight loss and a low melting temperature. This program takes the oxide batch information and converts it into a raw materials batch. [9], [10]

This program is fast and accurate and can find the best solution in terms of cost, chemistry, etc. But, even though this program is interesting, the 'All-Purpose Batch Calculation' is not appropriate for the Coors glass manufacturing situation. This algorithm does not take into account variations in the furnaces, the glass color requirements, and it only can handle 10 oxides. Coors uses raw materials that contain 13 oxides in their glass manufacturing operation. In order to encompass all of Coors's requirements, a program needed to be designed specifically for them.

Bailey, Janet. 1989. "How Glass Producer Needs Affect Raw Material Suppliers." *Glass Industry*, **70**(13): 8-10.

This article describes the problems the glass industry is facing with shrinking markets, cost pressures, competition, oversupply and under demand. The information contained in this article was very important as this author had very little knowledge of the problems occurring presently in the glass industry. The main point of this article is that both suppliers and manufacturers need to understand their processes. Measuring systems also must be in place in order to effect significant reductions in cost. Finally, managers relying on intuition rather than facts will not be in position to find the lowest total cost solutions necessary to compete in the glass industry today.

Chapter 3

FORMULATION OF THE BLENDING ALGORITHM

A general statement of the blending problem at the Coors Glass Manufacturing Plant would be to determine the optimal quantities of the raw materials which should be incorporated in the formation of a glass batch while minimizing cost. This problem statement is not adequate for modeling purposes though. Formulation requires a well-defined problem statement that contains a concise objective function and constraints that include relationships among the decision variables, relationships between the decision variables and external factors, and relationships between the decision variables and internal factors. It is the specific considerations that occur at the Coors Glass Plant, or internal constraints, which prohibit defining a mathematical model general enough to be applicable to all glass manufacturing facilities.

3.1 Problem Statement

The primary purpose of this blending algorithm is to create an optimal glass batch while minimizing cost. This algorithm will be used to determine the optimal quantities of the raw materials that are utilized by the Coors Glass plant during the glass manufacturing process. In addition to optimizing the blend of ingredients, this algorithm will allow the operations manager to determine the economic viability of utilizing raw materials from different sources.

This model must be able to make use of the initial inputs (the percentages of the different oxides present in each raw material) and turn those values into the optimal quantities, in pounds, for each specific raw material. These optimal quantities will then be blended together to create a glass batch. Included in the model must be the capability to account for requirements in glass color, furnace idiosyncrasies, and environmental restrictions, while maintaining glass properties and minimizing costs. After inputting the initial percentages of the oxides present in the raw materials the manager will then use this model to determine the daily glass batch compositions.

3.2 Decision Variables

The decision variables within a problem are those variables which one can control [5]. The objective of the daily glass batch blending decision identified in the problem statement may be condensed. The goal of this model is to determine the optimal quantities of the raw materials which will then be blended together to create a glass batch. The obvious choice of decision variables for this model is to let x_i be the optimal amount, in pounds, for each of the different raw materials. There are presently twelve different raw materials used by the Coors glass plant that will be considered as decision variables (See Table 2).

Table 2

Decision Variables

Variables	**Definition of Variables**		
L. P. Sand - Lien Sand	x_1	=	amount, in pounds of L. P. Sand
Oklahoma Sand - Sandy Flats	x_2	=	amount, in pounds of Oklahoma Sand
Limestone	x_3	=	amount, in pounds, of Limestone
Soda Ash	x_4	=	amount, in pounds, of Soda Ash
Nepheline Syenite	x_5	=	amount, in pounds, of Nepheline Syenite
B. H. Calumite	x_6	=	amount, in pounds, of B. H. Calumite
Melite	x_7	=	amount, in pounds, of Melite
Iron Pyrite	x_8	=	amount, in pounds, of Iron Pyrite
Salt Cake	x_9	=	amount, in pounds, of Salt Cake
Asbury Graphite	x_{10}	=	amount, in pounds, of Asbury Graphite
Mixed Cullet	x_{11}	=	amount, in pounds, of Mixed Cullet
Amber Cullet	x_{12}	=	amount, in pounds, of Amber Cullet

3.3 Objective Function

The objective of this glass batch blending algorithm is to minimize cost. Letting c_i be the cost, in dollars per pound, associated with buying the raw material i, the objective of minimizing cost may be expressed as

$$\text{minimize} \quad \sum c_i * x_i \qquad \text{for } i = 1 \text{ to } 12.$$

The objective function coefficients, or the costs of the raw materials, are readily available from the suppliers. Daily, the different glass inputs arrive at Coors and there is a price associated with each particular raw material. These prices are normally negotiated in advance and remain constant for a predetermined period of time.

3.4 External Constraints

For this algorithm to be implemented it must be written for an IBM compatible computer, using Microsoft Excel. In terms of problem formulation, this is the only external constraint that needs to be considered. The computers currently being used by the operations manager at Coors Glass Division are IBM compatible computers that, presently, have very little memory. This constraint places limitations on the problem formulation by eliminating more sophisticated solution algorithms that have sizable memory requirements.

All the pertinent spreadsheets now being used by the operations manager for calculating the daily glass batch compositions are on Microsoft Excel. Implementing and running this glass batch blending algorithm on Microsoft Excel could conceivably enhance the likelihood of this model being utilized. Also, it would be easier to update or change the algorithm, without a great deal of computer training, if the operations manager is familiar with the system and knowledgeable about the software.

3.5 Constraints

There are four types of constraints found in this blending algorithm. These constraints can be grouped into variable, batch, theoretical glass composition, and theoretical physical property constraints. The variable constraints consist of non-negativity restrictions placed on all the decision variables. The batch, theoretical glass composition, and theoretical physical property constraints have a goal value, an allowable deviation, and both an upper and lower bound based on furnace requirements, environmental restrictions, and the desired final glass composition and physical properties [See Table 3]. The goal values and allowable deviations were determined from looking at past data and from discussions with the operations manager. In the instances where the goal values of the constraints are zero,

Table 3

An Example of the Constraints Relating to Furnace #1

Variable Constraints
Non-negativity Constraints Apply To All Variables

No.	Batch Constraints	Goal	Minimum	Maximum	Deviation
1	Batch Weight	8589.40	8580.81	8597.99	0.10%
2	Percentage of Fusion Loss	11.91%	11.89%	11.93%	0.20%
3	Glass Equivalency (in tons)	3.78	3.78	3.79	0.15%
4	Redox	-99.31	-98.81	-99.81	0.50%
5	Salt Cake per Ton of Sand	1.43	1.40	1.46	2.00%
6	Salt Cake per Ton of Glass	0.69	0.68	0.70	2.00%
7	Salt Cake per 5500 pounds of Batch	1.67	1.64	1.70	1.00%
8	Sulfur Input per Ton of Glass	1.42	1.41	1.43	1.00%
9	Percentage of Mixed Cullet	7.80%	7.79%	7.81%	0.10%
10	Percentage of Amber Cullet	18.23%	18.18%	18.28%	0.30%

No.	Theoretical Glass Composition Constraints	Goal	Minimum	Maximum	Deviation
11	Percentage $Si0_2$	71.87%	71.73%	72.01%	0.20%
12	Percentage Al_2O_3	2.00%	1.96%	2.04%	2.00%
13	Percentage Fe_2O_3	0.30%	0.27%	0.33%	10.00%
14	Percentage CaO	10.70%	10.67%	10.73%	0.30%
15	Percentage MgO	0.19%	0.18%	0.20%	3.00%
16	Percentage BaO	0.00%	0.00%	0.00%	1.00%
17	Percentage Na_2O	14.40%	14.34%	14.46%	0.45%
18	Percentage K_2O	0.42%	0.41%	0.43%	2.00%
19	Percentage SO_3	0.01%	0.01%	0.02%	10.00%
20	Percentage Sulfur	0.06%	0.06%	0.07%	2.00%
21	Percentage MnO	0.00%	0.00%	0.00%	1.00%

No.	Theoretical Physical Property Constraints	Goal	Minimum	Maximum	Deviation
22	Glass Density	2.5059	2.5046	2.5072	0.05%
23	Melt Viscosity	2631.50	2630.18	2632.82	0.05%
24	Gob Viscosity	2161.50	2160.42	2162.58	0.05%
25	Softening Point	1331.50	1331.23	1331.77	0.02%
26	Conveyor Viscosity	1167.20	1166.62	1167.78	0.05%
27	Annealing Point	1036.29	1035.77	1036.81	0.05%
28	Expansion Coefficient	90.95	90.76	91.13	0.20%
29	Cooling Time	89.60	89.51	89.69	0.10%
30	Liquid Temperature	1877.50	1876.56	1878.44	0.05%

which is currently the case for both the Percentage BaO and the Percentage MnO constraints, the minimum bounds are also assumed to be zero but the maximum bounds, in each case, are a small positive number.

The allowable deviations and the goal values, specifically determined for each constraint, were used to determine the minimum and maximum bounds allowable on the constraints. Generally, the minimum and maximum bounds on each constraint are determined using two steps. The first step is to multiply the goal value and the allowable deviation, and the second step consists of either adding or subtracting that figure to the goal value. These limits may be expressed as

bl_k for $k = 1$ to 30 and

bu_k for $k = 1$ to 30

where k counts the number of batch, theoretical glass composition, and theoretical physical property constraints. There are 30 constraints, excluding the variable constraints, included in this model. The same constraints exist for each of the three different furnaces but the goal values and the allowable deviations vary slightly due to furnace idiosyncrasies.

3.5.1 Batch Constraints

The batch constraints consist of ten individual constraints, as seen in Table 3, all relating to the desired final glass batch composition. In order to define the batch constraints, terms for both the redox factors and for the percentages of oxides present in the raw materials must be defined. The redox factor describes the oxidation-reduction potential of each raw material, so let R_i for i from 1 to 12 describe the redox factors corresponding to each ingredient. Also, there are thirteen oxides potentially present in each raw material, so let O_{ij} describe the oxides present in the raw materials with i from 1 to 12 denoting the 12 different raw materials, and j from 1 to 13 describing the 13 individual oxides present in each raw material [See Table 4].

Table 4

Factors Associated With Each Oxide

Oxides	Factors
SiO_2	O_{i1}
Al_2O_3	O_{i2}
Fe_2O_3	O_{i3}
Cr_2O_3	O_{i4}
CaO	O_{i5}
MgO	O_{i6}
BaO	O_{i7}
Na_2O	O_{i8}
K_2O	O_{i9}
SO_3	O_{i10}
Sulfur	O_{i11}
MnO	O_{i12}
C	O_{i13}

Presently, there is a spreadsheet updated daily which helps determine the composition of each glass batch. Upon receiving the raw material deliveries, the operations manager enters the percentages of the oxides present in the raw materials and updates the redox factors associated with each raw material [See Table 5]. This spreadsheet, which contains the percentages of the oxides present in the raw materials and the redox factors associated with those same raw materials, is then used to perform calculations. The calculations, which are performed on a separate spreadsheet, give value to the thirty previously defined constraints relating to each furnace. Finally, the quantities associated with each raw material are adjusted until all the constraints fall within their goal ranges. See Appendix A for examples of the spreadsheets associated with each of the three different furnaces.

Table 5

A Typical Example of the Percentage of the Oxides and Redox Factors Present in the Raw Materials

	SiO_2	Al_2O_3	Fe_2O_3	Cr_2O_3	CaO	MgO	BaO	Na_2O	K_2O	SO_3	S--	MnO	C	Cost ($/ Ton	Yield	Redox
Lien Sand	99.2%	0.5%	0.3%		0.0%	0.0%		0.0%	0.0%					$ 39.04	100.0%	
Sandy Flats	97.8%	1.2%	0.1%		0.3%	0.1%		0.1%	0.5%					$ 39.04	100.0%	
Soda Ash								58.4%						$ 92.84	58.4%	
Limestone					53.8%	0.7%								$ 26.42	54.5%	COD?
Nepheline Syenite	60.6%	23.0%	0.3%		0.5%	0.0%		10.1%	4.9%					$ 89.75	99.4%	
Melite 40 (Greenville)	48.7%	20.8%	21.3%		4.5%	0.9%		0.5%	1.5%	0.1%			0.3%	$ 92.08	98.6%	-0.10
Salt Cake								43.5%		56.2%				$ 162.20	99.8%	0.67
Ashland S. C. Mix								47.7%	0.8%	12.8%				$ 93.80	61.3%	
Asbury Graphite													91.0%	$ 261.00	91.0%	-6.10
Carbocite													72.0%	$ 136.89	72.0%	
Feldspar	67.9%	19.0%	0.1%		1.7%	0.1%		7.1%	4.1%					$ 59.00	99.9%	
B. Harb. Calumite	36.8%	10.9%	0.2%		39.3%	10.8%		0.3%	0.4%	0.2%	0.8%	0.5%	0.1%	$ 68.96	100.3%	-0.07
Billings Iron Pyrite	4.5%		61.2%								48.3%			$ 250.00	114.0%	-1.20
Coors Amber Cullet	71.1%	2.1%	0.2%		10.8%	0.3%		14.8%	0.5%	0.1%	0.1%			$ 55.00	99.9%	
Amber Cullet	72.3%	2.2%	0.2%		10.5%	0.3%		13.8%	0.6%	0.0%				$ 25.00	100.0%	
Flint Cullet	72.1%	1.8%	0.1%		10.9%	0.5%		14.2%	0.3%	0.2%		0.0%		$ 25.00	100.0%	
Green Cullet	72.1%	2.0%	0.2%	0.2%	11.4%	0.1%		12.8%	1.0%	0.2%				$ 25.00	100.0%	
Flat Glass Cullet	73.3%	0.2%	0.1%		8.8%	3.7%		13.5%	0.0%	0.2%					99.9%	
Mixed Cullet	72.1%	2.0%	0.2%	0.1%	11.0%	0.3%	0.0%	13.5%	0.7%	0.1%	0.0%	0.0%	0.0%	$ 25.00	100.0%	

The ten batch constraints are defined below [See Figure 1]. The batch constraints make up the first ten of the thirty total constraints assumed in this model. These ten constraints are bounded by previously defined upper and lower bounds. These limits may be expressed as

bl_k for $k = 1$ to 10 and

bu_k for $k = 1$ to 10.

These ten constraints exist for each of the three furnaces, but the goal values and the deviations vary slightly due to furnace idiosyncrasies.

Figure 1

Initial Batch Constraints

Batch Weight

$bl_1 \leq [\sum x_i] \leq bu_1$ for $i = 1$ to 12

Fusion Loss

$bl_2 \leq [\sum x_i - \sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] / \sum x_i \leq bu_2$

for $i = 1$ to 12 and for $j = 1$ to 13

Glass Equivalency

$bl_3 \leq [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] / 2000 \leq bu_3$

for $i = 1$ to 12 and for $j = 1$ to 13

Redox

$bl_4 \leq [2.725 * \%\ \text{moisture} / 3 + \sum R_i * x_i] * [x_1 + x_2] / 2000 \leq bu_4$

for $i = 1$ to 12

Salt Cake per Ton of Sand

$bl_5 \leq [x_9 / (x_1 + x_2) * 2000] \leq bu_5$

Salt Cake per Ton of Glass

$bl_6 \leq [x_9 / \sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10}) * 2000] \leq bu_6$

for $i = 1$ to 12 and for $j = 1$ to 13

Salt Cake per 5500 pounds of Batch

$bl_7 \leq [5500 / (\sum x_i) * x_9] \leq bu_7$ for $i = 1$ to 12

Sulfur Input per Ton of Glass

$bl_8 \leq [\sum (O_{i10} * x_i) * .2 + \sum (O_{i11} * x_i)] / [\sum (x_i * O_{ij} - O_{i13} * x_i - O_{i10} * x_i) / 2000] \leq bu_8$

for $i = 1$ to 12 and for $j = 1$ to 13

Percentage of Mixed Cullet

$bl_9 \leq [x_{11} / \sum x_i] \leq bu_9$ for $i = 1$ to 12

Percentage of Amber Cullet

$bl_{10} \leq [x_{12} / \sum x_i] \leq bu_{10}$ for $i = 1$ to 12

The ten batch constraints defined in Figure 1 were retrieved from the spreadsheets presently being used by the operations manager. The difficulty with these constraints is they are not all in the form of linear equations or inequalities. As optimization problems can be formulated as linear programs only when all the constraints are expressed as linear functions of the decision variables [17] some of these constraints need to be altered. Three of the original ten batch constraints are linear while seven are currently non-linear. The seven non-linear constraints are fusion loss, salt cake per ton of sand, salt cake per ton of glass, salt cake per 5500 pounds of batch, sulfur input per ton of glass, percentage of mixed cullet, and percentage of amber cullet.

In order to create a linear program it is necessary to change these non-linear constraints into linear inequalities. By looking at the seven non-linear constraints it is evident that the non-linear factors can be removed. The method employed, in reference to the batch constraints, to create linear restrictions consists of multiplying both the lower and upper bound on the constraint by the non-linear factor and then subtracting out the remaining linear piece of the equation. This method leads to a constraint that is either less than or greater than zero depending on whether the lower or upper bound is used.

The three denominators found in the seven non-linear constraints are

$[\sum x_i]$,

$[\sum (x_i * O_{ij} - O_{i13} * x_i - O_{i10} * x_i)]$,

and $[x_1 + x_2]$.

For example, looking at the equation for fusion loss in Figure 1, it is possible to take the denominator in the equation,

$\sum xi$,

and multiply it by both the upper and lower bound on the constraint, and then subtract out the linear portion of the constraint,

$[\sum x_i - \sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})]$.

This process reduces the original constraint to two linear equations which are either less than or greater than zero depending on whether the upper or lower bound on the constraint was used. The two linear constraints formed from the original constraint for fusion loss can be expressed as

$bl_2 * \sum x_i - [\sum x_i - (\sum x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10}) \leq 0$

for i = 1 to 12 and for j = 1 to 13 and

$bu_2 * \sum x_i - [\sum x_i - (\sum x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10}) \geq 0$

for i = 1 to 12 and for j = 1 to 13.

In order to make all the constraints consistently of the same form it is necessary to multiply the upper and lower bounds on the constraints by the denominators (those equations that are presently linear have denominators of 1). Then, the remaining portions of the original equations are subtracted from the multiplication factors times the constraint bounds to yield two linear constraints for each original constraint [See Figure 2].

Figure 2

Final Batch Constraints

Batch Weight

$bl_1 * 1 - [\sum x_i] \leq 0$ for i = 1 to 12

$bu_1 * 1 - [\sum x_i] \geq 0$ for i = 1 to 12

Fusion Loss

$bl_2 * [\sum x_i] - [\sum x_i - \sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_2 * [\sum x_i] - [\sum x_i - \sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] \geq 0$ for i = 1 to 12 and for j = 1 to 13

Glass Equivalency

$bl_3 * 1 - [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] / 2000 \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_3 * 1 - [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] / 2000 \geq 0$ for i = 1 to 12 and for j = 1 to 13

Redox

$bl_4 * 1 - [2.725 * \% \text{ moisture} / 3 + \sum R_i * x_i] * [x_1 + x_2] / 2000 \leq 0$

for i = 1 to 12

$bu_4 * 1 - [2.725 * \% \text{ moisture} / 3 + \sum R_i * x_i] * [x_1 + x_2] / 2000 \geq 0$

for i = 1 to 12

Salt Cake per Ton of Sand

$bl_5 * [x_1 + x_2] - [x_9 * 2000] \leq 0$

$bu_5 * [x_1 + x_2] - [x_9 * 2000] \geq 0$

Salt Cake per Ton of Glass

$bl_6 * [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10}] - [x_9 * 2000] \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_6 * [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10}] - [x_9 * 2000] \geq 0$ for i = 1 to 12 and for j = 1 to 13

Salt Cake per 5500 pounds of Batch

$bl_7 * [\sum x_i] - [5500 * x_9] \leq 0$ for i = 1 to 12

$bu_7 * [\sum x_i] - [5500 * x_9] \geq 0$ for i = 1 to 12

Sulfur Input per Ton of Glass

$bl_8 * [\sum (x_i * O_{ij} - O_{i13} * x_i - O_{i10} * x_i)] - [\sum (O_{i10} * x_i) * .2 + \sum (O_{i11} * x_i) * 2000] \leq 0$

for i = 1 to 12 and for j = 1 to 13

$bu_8 * [\sum (x_i * O_{ij} - O_{i13} * x_i - O_{i10} * x_i)] - [\sum (O_{i10} * x_i) * .2 + \sum (O_{i11} * x_i) * 2000] \geq 0$

for i = 1 to 12 and for j = 1 to 13

Percentage of Mixed Cullet

$bl_9 * [\sum x_i] - [x_{11}] \leq 0$ for i = 1 to 12

$bu_9 * [\sum x_i] - [x_{11}] \geq 0$ for i = 1 to 12

Percentage of Amber Cullet

$bl_{10} * [\sum x_i] - [x_{12}] \leq 0$ for i = 1 to 12

$bu_{10} * [\sum x_i] - [x_{12}] \geq 0$ for i = 1 to 12

3.5.2 Theoretical Glass Composition Constraints

The theoretical glass composition constraints consist of eleven individual constraints. These constraints calculate the total percentages of the oxides present in each of the different raw materials. These eleven constraints are bounded by previously defined upper and lower bounds. These limits may be expressed as

bl_k for k = 11 to 21 and

bu_k for k = 11 to 21.

The eleven theoretical glass composition constraints are defined below [See Figure 3].

Figure 3

Initial Theoretical Glass Composition Constraints

Percentage of SiO_2

$bl_{11} \le [\sum O_{i1} * x_i \ / \ \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{11}$ for i = 1 to 12 and for j = 1 to 13

Percentage of Al_2O_3

$bl_{12} \le [\sum O_{i2} * x_i \ / \ \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{12}$ for i = 1 to 12 and for j = 1 to 13

Percentage of Fe_2O_3

$bl_{13} \le [\sum O_{i3} * x_i \ / \ \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{13}$ for i = 1 to 12 and for j = 1 to 13

Percentage of CaO

$bl_{14} \le [\sum O_{i5} * x_i / \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{14}$ for i = 1 to 12 and for j = 1 to 13

Percentage of MgO

$bl_{15} \le [\sum O_{i6} * x_i / \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{15}$ for i = 1 to 12 and for j = 1 to 13

Percentage of BaO

$bl_{16} \le [\sum O_{i7} * x_i / \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{16}$ for i = 1 to 12 and for j = 1 to 13

Percentage of Na_2O

$bl_{17} \le [\sum O_{i8} * x_i / \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{17}$ for i = 1 to 12 and for j = 1 to 13

Percentage of K_2O

$bl_{18} \le [\sum O_{i9} * x_i / \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{18}$ for i = 1 to 12 and for j = 1 to 13

Percentage of SO_3

$bl_{19} \le [\sum O_{i10} * x_i / \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{19}$ for i = 1 to 12 and for j = 1 to 13

Percentage of Input Sulfur

$bl_{20} \le [\sum O_{i11} * x_i / \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{20}$ for i = 1 to 12 and for j = 1 to 13

Percentage of MnO

$bl_{21} \le [\sum O_{i12} * x_i / \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})] \le bu_{21}$ for i = 1 to 12 and for j = 1 to 13

All eleven of the theoretical glass composition constraints are non-linear in their present form. The method previously used with the batch constraints to create linear equations is also used with the glass composition constraints. The first step in creating the linear equation is to confine the constraint between its respective lower and upper bound. The second step consists of removing the non-linear portion of the equation by multiplying the entire equation by the non-linear factor. Finally, the third step entails subtracting the remaining portion of the constraint from the part of the equation that includes the constraint bound. This method leads to two linear constraints for each original constraint that are either greater or less than zero, depending on whether the upper or lower bound is used on the constraint.

All of the theoretical glass composition constraints have the same denominator. With simple algebra that denominator can be removed to yield linear equations. This factor can be expressed as

$$[\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})].$$

By looking at the equation for the Percentage of SiO_2 in Figure 3, and using the lower bound on the constraint, it is possible to take the denominator,

$$[\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})],$$

and multiply it through the entire equation, leaving

$$bl_{11} * [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] \leq \sum x_i * O_{ij} .$$

Now it is possible to subtract the right hand side of the equation from each side. This process reduces the constraint to two linear equations that are either less than or greater than zero depending on whether the upper or lower bound on the constraint is used. The two linear constraints associated with the Percentage of SiO_2 constraint can be expressed as

$$bl_{11} * [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] - \sum x_i * O_{ij} \leq 0$$

for i = 1 to 12 and j = 1 to 13 and

$$bu_{11} * [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] - \sum x_i * O_{ij} \geq 0$$

for i = 1 to 12 and j = 1 to 13.

This method for creating linear constraints from non-linear inequalities is used with all the theoretical glass composition constraints. These constraints are now all of the same linear form and are found below [See Figure 4].

Figure 4

Final Theoretical Glass Composition Constraints

Percentage of SiO_2

$bl_{11} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i1} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{11} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i1} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of Al_2O_3

$bl_{12} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i2} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{12} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i2} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of Fe_2O_3

$bl_{13} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i3} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{13} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i3} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of CaO

$bl_{14} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i5} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{14} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i5} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of MgO

$bl_{15} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i6} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{15} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i6} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of BaO

$bl_{16} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i7} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{16} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i7} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of Na_2O

$bl_{17} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i8} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{17} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i8} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of K_2O

$bl_{18} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i9} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{18} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i9} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of SO_3

$bl_{19} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i10} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{19} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i10} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of Input Sulfur

$bl_{20} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i11} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{20} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i11} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Percentage of MnO

$bl_{21} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i12} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{21} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i12} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

3.5.3 Theoretical Physical Property Constraints

The theoretical physical property constraints consist of nine individual constraints, as seen in Table 3, all relating to the desired physical properties associated with a glass batch. In order to characterize the physical property constraints ten factors associated with these constraints must be defined. The ten factors needed to describe these constraints are density, standard reference, melt viscosity, gob viscosity, softening point, conveyor viscosity, annealing, expansion, cooling time, and liquid temperature. Each of these ten factors is associated with the percentages of the oxides present in the raw materials [See Table 6].

Table 6

Additional Factors Associated With Each Oxide

Definitions		**Factors**
Density Factor	associated with each oxide	D_j
Standard Reference Factor	associated with each oxide	S_j
Melt Viscosity Factor	associated with each oxide	M_j
Gob Viscosity Factor	associated with each oxide	G_j
Softening Point Factor	associated with each oxide	F_j
Conveyor Viscosity Factor	associated with each oxide	N_j
Annealing Factor	associated with each oxide	A_j
Expansion Factor	associated with each oxide	E_j
Cooling Time Factor	associated with each oxide	T_j
Liquid Temperature Factor	associated with each oxide	L_j

The nine theoretical physical property constraints are defined below [See Figure 5]. The physical property constraints make up the last nine of the thirty total constraints assumed in this model. These nine constraints are bounded by previously defined upper and lower bounds, as seen in Table 3. These limits may be expressed as

bl_k for k = 22 to 30 and

bu_k for k = 22 to 30.

While the identical constraints exist for each of the three different furnaces, the goal values and the deviations do vary slightly between furnaces.

Figure 5

Initial Theoretical Physical Property Constraints

Glass Density

$$bl_{22} \le [\ 2.5043 + (\sum D_j * (\sum x_i * O_{ij} / (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})) - S_j)\ ] \le bu_{22}$$

for i = 1 to 12 and j = 1 to 13

Melt Viscosity

$$bl_{23} \le [\ \sum M_j * (\sum x_i * O_{ij} / (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})) - S_j\ ] \le bu_{23}$$

for i = 1 to 12 and j = 1 to 13

Gob Viscosity

$$bl_{24} \le [\ \sum G_j * (\sum x_i * O_{ij} / (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})) - S_j\ ] \le bu_{24}$$

for i = 1 to 12 and j = 1 to 13

Softening Point

$$bl_{25} \le [\ \sum F_j * (\sum x_i * O_{ij} / (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})) - S_j\ ] \le bu_{25}$$

for i = 1 to 12 and j = 1 to 13

Conveyor Viscosity

$$bl_{26} \le [\ \sum N_j * (\sum x_i * O_{ij} / (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})) - S_j\ ] \le bu_{26}$$

for i = 1 to 12 and j = 1 to 13

Annealing Point

$$bl_{27} \le [\ \sum A_j * (\sum x_i * O_{ij} / (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})) - S_j\ ] \le bu_{27}$$

for i = 1 to 12 and j = 1 to 13

Expansion Coefficient

$$bl_{28} \le [\ \sum E_j * (\sum x_i * O_{ij} / (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})) - S_j\ ] \le bu_{28}$$

for i = 1 to 12 and j = 1 to 13

Cooling Time

$$bl_{29} \le [\ \sum T_j * (\sum x_i * O_{ij} / (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})) - S_j\ ] \le bu_{29}$$

for i = 1 to 12 and j = 1 to 13

Liquid Temperature

$$bl_{30} \le [\ \sum L_j * (\sum x_i * O_{ij} / (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})) - S_j\ ] \le bu_{30}$$

for i = 1 to 12 and j = 1 to 13

The nine theoretical physical property constraints defined in Figure 5 are of a form that is incompatible with utilizing linear programming. None of these restrictions are in the form of linear equations. In order to proceed with the proposed algorithm these constraints need to be restructured into linear functions.

By looking at these non-linear constraints it is evident that the denominators can be removed from the equations. The method employed with the theoretical physical property constraints to create linear equations consists of setting each equation to either less than or equal to its lower bound or greater than or equal to its upper bound. The entire equation is then multiplied through by the denominator. Finally, all the linear terms must be subtracted from the segment of the equation that contains either the upper or lower bound multiplied by the denominator. This method leads to an equation that is either less than or greater than zero depending on whether the lower or upper bound on the constraint is used.

All of the non-linear constraints have the same denominator which can be expressed as

$$[\sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})].$$

For use in all the theoretical physical property constraints, *nl* will be the symbol for this denominator. So, looking at the equation for glass density in Figure 5, and using the lower bound, it is possible to take the denominator,

$$\sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}),$$

and multiply that factor through the entire equation, leaving

$$bl22 * nl \leq 2.5043 * nl + \sum (Dj * xi * Oij) - Sj * Dj * nl.$$

Now it is possible to subtract the right hand side of the equation from both sides. This process reduces the constraint to a linear equation that is either less than or greater than zero depending on whether the upper or lower bound on the constraint is used. The linear form of the glass density equation looks like

$$bl_{22} * nl - 2.5043 * nl + \sum (D_j * x_i * O_{ij}) - S_j * D_j * nl \leq 0$$

for i = 1 to 12 and for j = 1 to 13 and

$$bu_{22} * nl - 2.5043 * nl + \sum (D_j * x_i * O_{ij}) - S_j * D_j * nl \geq 0$$

for i = 1 to 12 and for j = 1 to 13.

This method for creating linear constraints from non-linear inequalities is used with all the theoretical physical property constraints. The theoretical physical property constraints are now of the same linear form and are found below [See Figure 6].

Figure 6

Final Theoretical Physical Property Constraints

Glass Density

$bl_{22} * \mathit{nl} - 2.5043 * \mathit{nl} - \sum (D_j * x_i * O_{ij}) + \sum (S_j * D_j *) * \mathit{nl} \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{22} * \mathit{nl} - 2.5043 * \mathit{nl} - \sum (D_j * x_i * O_{ij}) + \sum (S_j * D_j *) * \mathit{nl} \geq 0$ for i = 1 to 12 and j = 1 to 13

Melt Viscosity

$bl_{23} * \mathit{nl} - \sum (M_j * x_i * O_{ij}) + \sum (S_j * M_j) * \mathit{nl} \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{23} * \mathit{nl} - \sum (M_j * x_i * O_{ij}) + \sum (S_j * M_j) * \mathit{nl} \geq 0$ for i = 1 to 12 and j = 1 to 13

Gob Viscosity

$bl_{24} * \mathit{nl} - \sum (G_j * x_i * O_{ij}) + \sum (S_j * G_j) * \mathit{nl} \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{24} * \mathit{nl} - \sum (G_j * x_i * O_{ij}) + \sum (S_j * G_j) * \mathit{nl} \geq 0$ for i = 1 to 12 and j = 1 to 13

Softening Point

$bl_{25} * \mathit{nl} - \sum (F_j * x_i * O_{ij}) + \sum (S_j * F_j) * \mathit{nl} \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{25} * \mathit{nl} - \sum (F_j * x_i * O_{ij}) + \sum (S_j * F_j) * \mathit{nl} \geq 0$ for i = 1 to 12 and j = 1 to 13

Conveyor Viscosity

$bl_{26} * \mathit{nl} - \sum (N_j * x_i * O_{ij}) + \sum (S_j * N_j) * \mathit{nl} \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{26} * \mathit{nl} - \sum (N_j * x_i * O_{ij}) + \sum (S_j * N_j) * \mathit{nl} \geq 0$ for i = 1 to 12 and j = 1 to 13

Annealing Point

$bl_{27} * \mathit{nl} - \sum (A_j * x_i * O_{ij}) + \sum (S_j * A_j) * \mathit{nl} \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{27} * \mathit{nl} - \sum (A_j * x_i * O_{ij}) + \sum (S_j * A_j) * \mathit{nl} \geq 0$ for i = 1 to 12 and j = 1 to 13

Expansion Coefficient

$bl_{28} * \mathit{nl} - \sum (E_j * x_i * O_{ij}) + \sum (S_j * E_j) * \mathit{nl} \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{28} * \mathit{nl} - \sum (E_j * x_i * O_{ij}) + \sum (S_j * E_j) * \mathit{nl} \geq 0$ for i = 1 to 12 and j = 1 to 13

Cooling Time

$bl_{29} * \mathit{nl} - \sum (T_j * x_i * O_{ij}) + \sum (S_j * T_j) * \mathit{nl} \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{29} * \mathit{nl} - \sum (T_j * x_i * O_{ij}) + \sum (S_j * T_j) * \mathit{nl} \geq 0$ for i = 1 to 12 and j = 1 to 13

Liquid Temperature

$bl_{30} * \mathit{nl} - \sum (L_j * x_i * O_{ij}) + \sum (S_j * L_j) * \mathit{nl} \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{30} * \mathit{nl} - \sum (L_j * x_i * O_{ij}) + \sum (S_j * L_j) * \mathit{nl} \geq 0$ for i = 1 to 12 and j = 1 to 13

Note - *nl* refers to $[\sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})]$

3.6 Model Formulation

As constructed, this model is a valid representation of the glass batch blending problem at the Coors Glass Plant. The complete model formulation is shown below [See Figure 7]. The solution to this model accounts for glass color requirements, furnace idiosyncrasies, and environmental restrictions, while maintaining required final physical glass properties and minimizing costs. Derivation of a solution to the glass batch blending algorithm will now be discussed.

Figure 7

The Model Formulation

minimize

$\sum ci * xi$ for i = 1 to 12

subject to

Batch Constraints

$bl_1 * 1 - [\sum x_i] \leq 0$ for i = 1 to 12

$bu_1 * 1 - [\sum x_i] \geq 0$ for i = 1 to 12

$bl_2 * [\sum x_i] - [\sum x_i - \sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_2 * [\sum x_i] - [\sum x_i - \sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_3 * 1 - [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] / 2000 \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_3 * 1 - [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10})] / 2000 \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_4 * 1 - [2.725 * \%\ \text{moisture} / 3 + \sum R_i * x_i] * [x_1 + x_2] / 2000 \leq 0$ for i = 1 to 12

$bu_4 * 1 - [2.725 * \%\ \text{moisture} / 3 + \sum R_i * x_i] * [x_1 + x_2] / 2000 \geq 0$ for i = 1 to 12

$bl_5 * [x_1 + x_2] - [x_9 * 2000] \leq 0$

$bu_5 * [x_1 + x_2] - [x_9 * 2000] \geq 0$

$bl_6 * [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10}] - [x_9 * 2000] \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_6 * [\sum (x_i * O_{ij} - x_i * O_{i13} - x_i * O_{i10}] - [x_9 * 2000] \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_7 * [\sum x_i] - [5500 * x_9] \leq 0$ for i = 1 to 12

$bu_7 * [\sum x_i] - [5500 * x_9] \geq 0$ for i = 1 to 12

$bl_8 * [\sum (x_i * O_{ij} - O_{i13} * x_i - O_{i10} * x_i)] - [\sum (O_{i10} * x_i) * .2 + \sum (O_{i11} * x_i) * 2000] \leq 0$
for i = 1 to 12 and for j = 1 to 13

$bu_8 * [\sum (x_i * O_{ij} - O_{i13} * x_i - O_{i10} * x_i)] - [\sum (O_{i10} * x_i) * .2 + \sum (O_{i11} * x_i) * 2000] \geq 0$
for i = 1 to 12 and for j = 1 to 13

$bl_9 * [\sum x_i] - [x_{11}] \leq 0$ for i = 1 to 12

$bu_9 * [\sum x_i] - [x_{11}] \geq 0$ for i = 1 to 12

$bl_{10} * [\sum x_i] - [x_{12}] \leq 0$ for i = 1 to 12

$bu_{10} * [\sum x_i] - [x_{12}] \geq 0$ for i = 1 to 12

(Continued)

Figure 7 (Continued)

Theoretical Glass Composition Constraints

$bl_{11} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i1} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{11} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i1} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{12} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i2} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{12} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i2} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{13} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i3} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{13} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i3} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{14} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i5} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{14} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i5} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{15} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i6} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{15} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i6} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{16} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i7} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{16} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i7} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{17} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i8} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{17} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i8} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{18} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i9} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{18} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i9} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{19} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i10} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{19} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i10} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{20} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i11} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{20} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i11} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

$bl_{21} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i12} * x_i) \leq 0$ for i = 1 to 12 and for j = 1 to 13

$bu_{21} * \sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13}) - \sum (O_{i12} * x_i) \geq 0$ for i = 1 to 12 and for j = 1 to 13

Theoretical Physical Property Constraints

$bl_{22} * nl - 2.5043 * nl - \sum (D_j * x_i * O_{ij}) + \sum (S_j * D_j *) * nl \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{22} * nl - 2.5043 * nl - \sum (D_j * x_i * O_{ij}) + \sum (S_j * D_j *) * nl \geq 0$ for i = 1 to 12 and j = 1 to 13

$bl_{23} * nl - \sum (M_j * x_i * O_{ij}) + \sum (S_j * M_j) * nl \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{23} * nl - \sum (M_j * x_i * O_{ij}) + \sum (S_j * M_j) * nl \geq 0$ for i = 1 to 12 and j = 1 to 13

$bl_{24} * nl - \sum (G_j * x_i * O_{ij}) + \sum (S_j * G_j) * nl \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{24} * nl - \sum (G_j * x_i * O_{ij}) + \sum (S_j * G_j) * nl \geq 0$ for i = 1 to 12 and j = 1 to 13

$bl_{25} * nl - \sum (F_j * x_i * O_{ij}) + \sum (S_j * F_j) * nl \leq 0$ for i = 1 to 12 and j = 1 to 13

$bu_{25} * nl - \sum (F_j * x_i * O_{ij}) + \sum (S_j * F_j) * nl \geq 0$ for i = 1 to 12 and j = 1 to 13

(Continued)

Figure 7 (Continued)

Theoretical Physical Property Constraints (Continued)

$bl_{26} * nl - \sum (N_j * x_i * O_{ij}) + \sum (S_j * N_j) * nl \le 0$ for i = 1 to 12 and j = 1 to 13

$bu_{26} * nl - \sum (N_j * x_i * O_{ij}) + \sum (S_j * N_j) * nl \ge 0$ for i = 1 to 12 and j = 1 to 13

$bl_{27} * nl - \sum (A_j * x_i * O_{ij}) + \sum (S_j * A_j) * nl \le 0$ for i = 1 to 12 and j = 1 to 13

$bu_{27} * nl - \sum (A_j * x_i * O_{ij}) + \sum (S_j * A_j) * nl \ge 0$ for i = 1 to 12 and j = 1 to 13

$bl_{28} * nl - \sum (E_j * x_i * O_{ij}) + \sum (S_j * E_j) * nl \le 0$ for i = 1 to 12 and j = 1 to 13

$bu_{28} * nl - \sum (E_j * x_i * O_{ij}) + \sum (S_j * E_j) * nl \ge 0$ for i = 1 to 12 and j = 1 to 13

$bl_{29} * nl - \sum (T_j * x_i * O_{ij}) + \sum (S_j * T_j) * nl \le 0$ for i = 1 to 12 and j = 1 to 13

$bu_{29} * nl - \sum (T_j * x_i * O_{ij}) + \sum (S_j * T_j) * nl \ge 0$ for i = 1 to 12 and j = 1 to 13

$bl_{30} * nl - \sum (L_j * x_i * O_{ij}) + \sum (S_j * L_j) * nl \le 0$ for i = 1 to 12 and j = 1 to 13

$bu_{30} * nl - \sum (L_j * x_i * O_{ij}) + \sum (S_j * L_j) * nl \ge 0$ for i = 1 to 12 and j = 1 to 13

Variable Constraints

$x_i \ge 0$ for i = 1 to 12

Where

x_i - amount, in pounds, of each raw material
c_i - cost, in pounds, associated with buying each raw material i
bl_k - lower bound on constraint k
bu_k - upper bound on constraint k
R_i - redox factor associated with each raw material i
O_{ij} - percentages of oxides present in each raw material i
D_j - density factor associated with each oxide
S_j - standard reference factor associated with each oxide
M_j - melt viscosity factor associated with each oxide
G_j - gob viscosity factor associated with each oxide
F_j - softening point factor associated with each oxide
N_j - conveyor viscosity factor associated with each oxide
A_j - annealing factor associated with each factor
E_j - expansion factor associated with each oxide
T_j - cooling time factor associated with each oxide
L_j - liquid temperature factor associated with each oxide

Note - *nl* refers to $[\sum (x_i * O_{ij} - x_i * O_{i10} - x_i * O_{i13})]$

Chapter 4

THE PROGRAM AND IMPLEMENTATION

With the problem statement formulated and the mathematical model constructed, a solution technique must be chosen. Given the external constraint, which suggests solving the glass batch blending algorithm on an IBM compatible computer using Microsoft Excel, the obvious choice for a solution technique is linear programming, using the Solver Tool, in Microsoft Excel. As formulated, the proportionally, additivity, divisibility, and certainty assumptions of linear programming and the simplex method [16] are all satisfied.

4.1 The Program

The thirty constraints and the twelve decision variables used in the glass batch blending model have been defined in the spreadsheets presently being used by the operations manager. Now to recapitulate, the manager receives information on the percentages of the oxides present in the raw materials and the redox factors associated with those raw materials daily, upon their arrival at the Glass Plant. Upon receipt of the raw material information the operations manager enters these values into a spreadsheet, as seen in Table 5. Then, going into a separate spreadsheet, the amounts, in pounds, of the raw materials are manipulated until all the constraints are bound within a narrow range. In order to create the linear programming algorithm to optimize the glass batch blending problem it was necessary to maneuver and condense each constraint into two summarizing linear equations.

There are three spreadsheets used for batch calculations associated with each furnace; the Calculation, Input, and Constraint and Final Batch Composition spreadsheets, for examples see Appendix A. The file 'glssbtch' in Microsoft Excel holds the spreadsheets used for this algorithm and the seven spreadsheets are entitled:

A: Constraint and Final Batch Composition Spreadsheet for Furnace #1
B: Constraint and Final Batch Composition Spreadsheet for Furnace #2
C: Constraint and Final Batch Composition Spreadsheet for Furnace #3
D: Raw Material Composition
E: Calculation Spreadsheet for Furnace #1
F: Calculation Spreadsheet for Furnace #2
G: Calculation Spreadsheet for Furnace #3.

On each of the three calculation spreadsheets a separate table was constructed to create linear factors from the non-linear equations for each of the theoretical physical property constraints and two of the batch constraints; fusion loss and redox [See Table 7].

Table 7

Calculation Spreadsheet for Furnace #1

Calculations	Batch lbs	SiO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	C	Yield	Redox	Redox-LP
																4.94	2.73
L P Sand	3625.0	3597.3	16.31	9.57	0.00	1.63	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3625.0	0.00	0.00
Oklahoma Sand	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Soda Ash	1283.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	749.40	0.00	0.00	0.00	0.00	0.00	749.40	0.00	0.00
Limestone	1055.0	0.00	0.00	0.00	0.00	567.59	7.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	575.19	0.00	0.00
Nephelite Syenite	352.0	213.31	80.96	1.13	0.00	1.62	0.15	0.00	35.55	17.25	0.00	0.00	0.00	0.00	349.97	0.00	0.00
B. H. Calumite	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Melite	14.0	6.82	2.91	2.98	0.00	0.63	0.13	0.00	0.07	0.21	0.02	0.00	0.00	0.04	13.80	-2.54	-1.40
Iron Pyrite	10.0	0.45	0.00	6.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.83	0.00	0.00	11.40	-21.75	-12.00
Salt Cake	2.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.13	0.00	1.46	0.00	0.00	0.00	2.59	3.16	1.74
Asbury Graphite	7.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.83	6.83	-82.88	-45.73
Mixed Cullet	670.0	483.30	13.61	1.14	0.48	73.58	1.91	0.00	90.44	4.49	0.90	0.00	0.04	0.00	669.90	0.00	0.00
Amber Cullet	1565.0	1131.5	34.43	3.60	0.00	164.33	4.70	0.00	215.97	9.86	0.31	0.00	0.00	0.00	1564.7	0.00	0.00
TOTAL	8584.1	5432.7	148.23	24.53	0.48	809.38	14.69	0.00	1092.6	31.81	1.35	4.83	0.04	6.87	7560.5	-99.07	-54.66

Resultant Oxides		SiO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MnO		Std.	Totals
%	100.0%	71.86%	1.96%	0.32%	0.01%	10.71%	0.19%	0.00%	14.45%	0.42%	0.01%	0.06%	0.00%			
Standard Reference		72.39%	1.83%	0.05%		11.13%	0.76%	0.00%	13.35%	0.29%						
Variance (%)		-0.535	0.131	0.275		-0.425	-0.566	0.000	1.101	0.131						
Density Factors		-0.002	0.002	0.011		0.011	0.007	0.017	0.005	0.003			0.013			
Density		0.0013	0.0002	0.0030		-0.005	-0.004	0.0000	0.0053	0.0004	0.0000	0.0000	0.000		2.5043	2.5059

Lakatos Factors										Deg F	Deg C
LOG 2 -Melt		1.53		17.29	14.65	0.00	-53.06	-3.48	2655	2632	1444
LOG 3 -Gob		1.27	1.38	10.92	6.39	0.00	-43.15	-2.65	2188	2162	1183
LOG 7.65 -Softening		0.94	1.40	0.93	1.02	0.00	-22.57	-1.35	1351	1332	722
LOG 10 - Conveyor		0.91		-0.98	1.30	0.00	-18.05	-1.11	1185	1167	631
LOG 13.37 -Annealing		0.91		-2.46	1.92	0.00	-14.31	-0.89	1051	1036	558
Expansion	-0.03	0.02		-0.69	-0.25	0.00	4.76	0.51	86.4	90.7	
Cooling Time	0.01	-0.09	0.09	0.74	0.40		1.89	0.15		89.6	
Liquid Temperature	0.27	2.42	5.86	-12.27	-5.37		-2.39	-1.32		1025.4	

(Continued)

Table 7 (Continued)

Linear Programming Factors	SiO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	C	Yield	Total
Density	9.7	1.8	22.7	0.0	-34.0	-31.6	0.0	39.9	3.4	0.0	0.0	0.1	0.0	18934	18946
Melt Viscosity		11545.8			130693	110766	0.0	-401156	-26294					20074010	19899566
Gob Viscosity		9572.2	10413.3		82526	48327	0.0	-326251	-20066					16540210	16344731
Softening Point		7105.1	10621.6		7065	7698	0.0	-170616	-10181					10215812	10067504
Conveyor Viscosity		6907.7			-7386	9836	0.0	-136493	-8402					8960005	8824469
Annealing Point		6907.7			-18625	14541	0.0	-108196	-6722					7946136	7834042
Expansion Coefficient	-202.2	167.8			-5234	-1925	0.0	35954	3855					654139	686755
Cooling Time	80.9	-651.3	645.6		5587	3036		14315	1107					653383	677504
Liquid Temperature	2021.5	18266.0	44319.1		-92804	-40586		-18089	-9984					7849663	14196989
Fusion Loss															1023.55
Redox															-54.66

Finally, on the Constraint and Final Batch Composition spreadsheet, a new table was inserted called the Optimization Sheet [See Table 8]. This table first lists the constraints with their goal values, allowable deviations, and acceptable minimum and maximum values. Then, below that section, another list displays the constraints along with four additional columns associated with each constraint. The first column lists the actual values associated with the original constraints, most of which are obtained from non-linear equations in the Calculation spreadsheet. The second column shows the adjusted linear values of the constraints, most of which are obtained from linear equations in the new table constructed in the Calculation spreadsheet. The two following columns calculate the values associated with the summarizing linear constraints that should be either greater than or less than zero depending on which bound is used.

Table 8

Optimization Sheet for Furnace #1

No.	Constraint	Goal	Minimum	Maximum	Deviation
1	Batch Weight	8589.40	8580.81	8597.99	0.10%
2	Percentage of Fusion Loss	11.91%	11.88%	11.93%	0.20%
3	Glass Equivalence (in tons)	3.7833	3.7776	3.7890	0.15%
4	Redox	-99.31	-98.81	-99.81	0.50%
5	Salt Cake per Ton of Sand	1.4300	1.4014	1.4586	2.00%
6	Salt Cake per Ton of Glass	0.6900	0.6762	0.7038	2.00%
7	Salt Cake per 5500 pounds of Batch	1.6700	1.6366	1.7034	2.00%
8	Sulfur Input per Ton of Glass	1.4200	1.4058	1.4342	1.00%
9	Percentage of Mixed Cullet	7.80%	7.79%	7.81%	0.10%
10	Percentage of Amber Cullet	18.23%	18.18%	18.28%	0.30%
11	Percentage $Si0_2$	71.87%	71.73%	72.01%	0.20%
12	Percentage Al_20_3	2.00%	1.96%	2.04%	2.00%
13	Percentage Fe_20_3	0.30%	0.27%	0.33%	10.00%
14	Percentage Ca0	10.70%	10.67%	10.73%	0.30%
15	Percentage MgO	0.190%	0.18%	0.20%	3.00%
16	Percentage BaO	0.00%	0.00%	0.00%	1.00%
17	Percentage Na_20	14.40%	14.34%	14.46%	0.45%
18	Percentage K_2O	0.42%	0.41%	0.43%	2.00%
19	Percentage SO_3	0.0140%	0.01260%	0.0154%	10.00%
20	Percentage Sulfur	0.064%	0.06%	0.07%	2.00%
21	Percentage MnO	0.00%	0.00%	0.00%	1.00%
22	Glass Density	2.5059	2.5046	2.5072	0.05%
23	Melt Viscosity	2631.50	2630.18	2632.82	0.05%
24	Gob Viscosity	2161.50	2160.42	2162.58	0.05%
25	Softening Point	1331.50	1331.23	1331.77	0.02%
26	Conveyor Viscosity	1167.20	1166.62	1167.78	0.05%
27	Annealing Point	1036.29	1035.77	1036.81	0.05%
28	Expansion Coefficient	90.95	90.76	91.13	0.20%
29	Cooling Time	89.60	89.51	89.69	0.10%
30	Liquid Temperature	1877.50	1876.56	1878.44	0.05%

(Continued)

Table 8 (Continued)

No.	Constraint	Actual Values	Adjusted Linear Values	Minimum Constraint [must be <=0]	Maximum Constraint [must be >=0]
1	Batch Weight	8580.8106	8580.8106	0.0000	17.1788
2	Percentage of Fusion Loss	0.1188	1019.7593	0.0000	4.0872
3	Glass Equivalence (in tons)	3.7805	3.7805	-0.0029	0.0084
4	Redox	-62.6379	-34.4833	-20.4620	-19.9153
5	Salt Cake per Ton of Sand	1.4586	5299.0162	-207.8046	0.0000
6	Salt Cake per Ton of Glass	0.7008	5299.0162	-186.2333	22.4517
7	Salt Cake per 5500 pounds of Batch	1.6982	14572.2946	-528.9399	44.2582
8	Sulfur Input per Ton of Glass	1.4342	10844.0597	-214.7339	0.0000
9	Percentage of Mixed Cullet	0.0779	668.6339	0.0000	1.3386
10	Percentage of Amber Cullet	0.1818	1559.5889	0.0000	9.3857
11	Percentage Si02	0.7190	5436.4006	-13.1413	8.5952
12	Percentage Al203	0.0196	148.1966	0.0000	6.0488
13	Percentage Fe203	0.0030	22.7388	-2.3239	2.2127
14	Percentage Ca0	0.1067	806.7695	-0.1641	4.6901
15	Percentage MgO	0.0019	14.5765	-0.6415	0.2205
16	Percentage BaO	0.0000	0.0000	0.0000	0.0764
17	Percentage Na20	0.1446	1093.5691	-9.6772	0.1219
18	Percentage K2O	0.0042	32.0453	-0.9240	0.3463
19	Percentage SO3	0.0001	1.0818	-0.1292	0.0826
20	Percentage Sulfur	0.0006	4.8811	-0.1388	0.0547
21	Percentage MnO	0.0000	0.0401	-0.0401	0.0362
22	Glass Density	2.5052	18941.8401	-4.0753	14.8719
23	Melt Viscosity	2632.8157	19906854.895	-19896.9064	0.0000
24	Gob Viscosity	2162.0959	16347717.894	-12677.1592	3666.0531
25	Softening Point	1331.2540	10065679.613	-153.3424	3873.6735
26	Conveyor Viscosity	1166.8630	8822710.9850	-1864.5610	6960.6980
27	Annealing Point	1035.7895	7831657.6528	-125.9859	7709.4635
28	Expansion Coefficient	90.7435	686872.6761	-608.1471	2142.4121
29	Cooling Time	89.6896	678147.6648	-1354.9404	0.0000
30	Liquid Temperature	1024.7563	14188775.841	0.0000	14195.8738

In order to use the optimization algorithm the following steps must be followed:

1. Open the file 'glssbtch' in Microsoft Excel.
2. Update the information on the Raw Material Composition spreadsheet.
3. Open the Constraint and Final Batch Composition spreadsheet.
4. Select the Solver Utility under the 'Tools' heading in Microsoft Excel. The objective function, the decision variables, and the batch, theoretical physical property, and theoretical glass composition constraints, along with the twelve non-negativity variable constraints have been defined within Solver.
5. Click on the 'Solve' button located in the upper right hand corner of the Solver window. This model takes about two minutes to run.
6. When Solver finds the solution a Summary Report is generated [See Table 9]. This self-explanatory report shows the optimal amounts, in pounds, for each of the raw materials, displays the cost associated with the objective function, gives the final values for each of the constraints, and also reveals the 'tight' constraints, constraints that have reached one of their bounds. The final solution is also displayed directly on the Constraint and Final Batch Composition spreadsheet.
7. Return to step 3 and go through the following steps for each of the other two furnaces.

Table 9

Summary Report For Furnace #1

Target Cell (Min.)

Cell	Name	Original Value	Final Value
E6	[Total Cost per Batch]	$ 214.83	$ 214.16

Adjustable Cells

Cell	Name	Original Value	Final Value
C12	L. P. SAND [Total Cost per Ton]	3625.0	3632.9
C13	OKLAHOMA SAND [Total Cost per Ton]	0.0	0.0
C14	LIMESTONE [Total Cost per Ton]	1056.0	1052.2
C15	SODA ASH [Total Cost per Ton]	1284.0	1284.9
C16	NEPHELINE SYENITE [Total Cost per Ton]	355.0	360.4
C17	B. H. CALUMITE [Total Cost per Ton]	0.0	0.0
C18	MELITE [Total Cost per Ton]	16.0	5.1
C19	IRON PYRITE [Total Cost per Ton]	10.0	10.1
C20	SALT CAKE [Total Cost per Ton]	2.6	2.6
C21	ASBURY GRAPHITE [Total Cost per Ton]	7.5	4.3
C22	MIXED CULLET [Total Cost per Ton]	670.0	668.6
C27	AMBER CULLET [Total Cost per Ton]	1565.0	1559.6

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
E109	Batch Weight [must be <=0]	0.0000	E109<=0	Binding	0.0000
F109	Batch Weight [must be >=0]	17.1788	F109>=0	Not Binding	17.1788
E110	Percentage of Fusion Loss [must be <=0]	0.0000	E110<=0	Binding	0.0000
F110	Percentage of Fusion Loss [must be >=0]	4.0872	F110>=0	Not Binding	4.0872
E111	Glass Equivalence (in tons) [must be <=0]	-0.0029	E111<=0	Not Binding	0.0029
F111	Glass Equivalence (in tons) [must be >=0]	0.0084	F111>=0	Not Binding	0.0084
E113	Salt Cake per Ton of Sand [must be <=0]	-207.8046	E113<=0	Not Binding	207.8046
F113	Salt Cake per Ton of Sand [must be >=0]	0.0000	F113>=0	Binding	0.0000
E114	Salt Cake per Ton of Glass [must be <=0]	-186.2333	E114<=0	Not Binding	186.2333
F114	Salt Cake per Ton of Glass [must be >=0]	22.4517	F114>=0	Not Binding	22.4517
E115	Salt Cake per 5500 pounds of Batch [must be <=0]	-528.9399	E115<=0	Not Binding	528.9399
F120	Percentage Al_20_3 [must be >=0]	6.0488	F120>=0	Not Binding	6.0488
E120	Percentage Al_20_3 [must be <=0]	0.0000	E120<=0	Binding	0.0000
F115	Salt Cake per 5500 pounds of Batch [must be >=0]	44.2582	F115>=0	Not Binding	44.2582
E116	Sulfur Input per Ton of Glass [must be <=0]	-214.7339	E116<=0	Not Binding	214.7339
F116	Sulfur Input per Ton of Glass [must be >=0]	0.0000	F116>=0	Binding	0.0000
F117	Percentage of Mixed Cullet [must be >=0]	1.3386	F117>=0	Not Binding	1.3386
E118	Percentage of Amber Cullet [must be <=0]	0.0000	E118<=0	Binding	0.0000
F118	Percentage of Amber Cullet [must be >=0]	9.3857	F118>=0	Not Binding	9.3857

(Continued)

Figure 8 (Continued)

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$E\$119	Percentage $Si0_2$ [must be <=0]	-13.1413	\$E\$119<=0	Not Binding	13.1413
\$F\$119	Percentage $Si0_2$ [must be >=0]	8.5952	\$F\$119>=0	Not Binding	8.5952
\$E\$121	Percentage Fe_20_3 [must be <=0]	-2.3239	\$E\$121<=0	Not Binding	2.3239
\$E\$122	Percentage Ca0 [must be <=0]	-0.1641	\$E\$122<=0	Not Binding	0.1641
\$E\$123	Percentage MgO [must be <=0]	-0.6415	\$E\$123<=0	Not Binding	0.6415
\$E\$124	Percentage BaO [must be <=0]	0.0000	\$E\$124<=0	Binding	0.0000
\$E\$125	Percentage Na_20 [must be <=0]	-9.6772	\$E\$125<=0	Not Binding	9.6772
\$E\$126	Percentage K_2O [must be <=0]	-0.9240	\$E\$126<=0	Not Binding	0.9240
\$E\$127	Percentage SO_3 [must be <=0]	-0.1292	\$E\$127<=0	Not Binding	0.1292
\$E\$128	Percentage Sulfur [must be <=0]	-0.1388	\$E\$128<=0	Not Binding	0.1388
\$E\$129	Percentage MnO [must be <=0]	-0.0401	\$E\$129<=0	Not Binding	0.0401
\$E\$130	Glass Density [must be <=0]	-4.0753	\$E\$130<=0	Not Binding	4.0753
\$E\$131	Melt Viscosity [must be <=0]	-19896.91	\$E\$131<=0	Not Binding	19896.91
\$E\$132	Gob Viscosity [must be <=0]	-12677.16	\$E\$132<=0	Not Binding	12677.16
\$E\$133	Softening Point [must be <=0]	-153.34	\$E\$133<=0	Not Binding	153.34
\$E\$134	Conveyor Viscosity [must be <=0]	-1864.56	\$E\$134<=0	Not Binding	1864.56
\$E\$135	Annealing Point [must be <=0]	-125.99	\$E\$135<=0	Not Binding	125.99
\$E\$136	Expansion Coefficient [must be <=0]	-608.15	\$E\$136<=0	Not Binding	608.15
\$E\$137	Cooling Time [must be <=0]	-1354.94	\$E\$137<=0	Not Binding	1354.94
\$E\$138	Liquid Temperature [must be <=0]	0.00	\$E\$138<=0	Binding	0.00
\$F\$138	Liquid Temperature [must be >=0]	14195.87	\$F\$138>=0	Not Binding	14195.87
\$F\$137	Cooling Time [must be >=0]	0.00	\$F\$137>=0	Binding	0.00
\$F\$136	Expansion Coefficient [must be >=0]	2142.41	\$F\$136>=0	Not Binding	2142.41
\$F\$135	Annealing Point [must be >=0]	7709.46	\$F\$135>=0	Not Binding	7709.46
\$F\$134	Conveyor Viscosity [must be >=0]	6960.70	\$F\$134>=0	Not Binding	6960.70
\$F\$133	Softening Point [must be >=0]	3873.67	\$F\$133>=0	Not Binding	3873.67
\$F\$132	Gob Viscosity [must be >=0]	3666.05	\$F\$132>=0	Not Binding	3666.05
\$F\$131	Melt Viscosity [must be >=0]	0.00	\$F\$131>=0	Binding	0.00
\$F\$130	Glass Density [must be >=0]	14.8719	\$F\$130>=0	Not Binding	14.8719
\$F\$129	Percentage MnO [must be >=0]	0.0362	\$F\$129>=0	Not Binding	0.0362
\$F\$128	Percentage Sulfur [must be >=0]	0.0547	\$F\$128>=0	Not Binding	0.0547
\$F\$127	Percentage SO_3 [must be >=0]	0.0826	\$F\$127>=0	Not Binding	0.0826
\$F\$126	Percentage K_2O [must be >=0]	0.3463	\$F\$126>=0	Not Binding	0.3463
\$F\$125	Percentage Na_20 [must be >=0]	0.1219	\$F\$125>=0	Not Binding	0.1219
\$F\$124	Percentage BaO [must be >=0]	0.0764	\$F\$124>=0	Not Binding	0.0764
\$F\$123	Percentage MgO [must be >=0]	0.2205	\$F\$123>=0	Not Binding	0.2205
\$F\$122	Percentage Ca0 [must be >=0]	4.6901	\$F\$122>=0	Not Binding	4.6901
\$F\$121	Percentage Fe_20_3 [must be >=0]	2.2127	\$F\$121>=0	Not Binding	2.2127
\$C\$12	L. P. SAND [Total Cost per Ton]	3632.9	\$C\$12>=0	Not Binding	3632.9
\$C\$13	OKLAHOMA SAND [Total Cost per Ton]	0.0	\$C\$13=0	Binding	0.0
\$C\$14	LIMESTONE [Total Cost per Ton]	1052.2	\$C\$14>=0	Not Binding	1052.2
\$C\$15	SODA ASH [Total Cost per Ton]	1284.9	\$C\$15>=0	Not Binding	1284.9
\$C\$16	NEPHELINE SYENITE [Total Cost per Ton]	360.4	\$C\$16>=0	Not Binding	360.4
\$C\$17	B. H. CALUMITE [Total Cost per Ton]	0.0	\$C\$17=0	Binding	0.0
\$C\$18	MELITE [Total Cost per Ton]	5.1	\$C\$18>=0	Not Binding	5.1
\$C\$19	IRON PYRITE [Total Cost per Ton]	10.1	\$C\$19>=0	Not Binding	10.1
\$C\$20	SALT CAKE [Total Cost per Ton]	2.6	\$C\$20>=0	Not Binding	2.6
\$C\$21	ASBURY GRAPHITE [Total Cost per Ton]	4.3	\$C\$21>=0	Not Binding	4.3
\$C\$22	MIXED CULLET [Total Cost per Ton]	668.6	\$C\$22>=0	Not Binding	668.6
\$C\$27	AMBER CULLET [Total Cost per Ton]	1559.6	\$C\$27>=0	Not Binding	1559.6

4.2 Implementation

The model has been loaded onto the operations manager's computer at Coors and there has been limited testing of this optimization algorithm by the relevant personnel at the Coors Glass Plant. The likelihood of this model's eventual acceptance is a function of the success of this testing phase and the support of management. Both management and the actual operator were involved in the initial formulation of this model and so accept the validity of the model. Unfortunately, there is much wariness with changing the status quo by the operator.

As long as the intuition of the current decision maker is believed to be adequate, the perceived risk of adopting this new procedure does not outweigh the potentially significant benefits. That this model may never gain complete acceptance may be attributed to the failure of this analyst to demonstrate successfully that this model is not a substitute for the current operator, but is just a tool that can be used for greater accuracy, speed, and cost effectiveness.

Chapter 5

CONCLUSIONS

The purpose of this study is to formulate a glass batch blending algorithm for the Coors Glass Plant, which will aid decision makers, manufacturing personnel, and management by determining the optimum quantities of the raw materials which should be used during the glass manufacturing process while minimizing cost. Currently, the cost associated with the raw materials is approximately $214.50 per batch, roughly 500 glass batches are run through the Coors Glass Plant each day, and the plant operates continuously 365 days per year. Every percentage reduction in total cost per glass batch could decrease cost and increase profits by approximately $400,000 per year.

The algorithm described in chapters 3 and 4 provides an effective and efficient process for determining the optimal composition of a glass batch. The method chosen is straightforward, easy to understand, and requires little training which makes it simple to implement and quite easy to use for management as well as for the manufacturing personnel. It is also capable of providing an optimal solution in a reasonable amount of time. The algorithm was designed with flexibility so that if changes occur in the constraint optimal values or in allowable deviations those changes would be very easy to update.

In addition to the financial benefits derived from using this optimization algorithm there is also the benefit that derives from the automation of a time consuming manual process. Without a site-specific model, the decision maker must rely on intuition, an inadequate tool for considering the all the factors that go into determining the final glass batch composition for each furnace. In view of these comments this approach has the potential to provide increased tangible and intangible saving for Coors glass plant.

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APPENDIX A

Constraint and Final Batch Composition Spreadsheet - Furnace #1

Amber Soda/Lime Glass

Object: To minimize cost while satisfying all the theoretical physical property requirements.

Objective Function: **[Total Cost per Batch] $ 214.16**
[Total Cost per Ton] $ 49.92

DATE	12/30/95			
CHANGE NO.	2208	2207	2206	
Variables	2/3/95	1/27/95	1/3/95	Average
L. P. SAND	3632.9	3625.0	3625.0	3627.6
OKLAHOMA SAND	0.0	0.0	0.0	0.0
LIMESTONE	1052.2	1055.0	1057.0	1054.7
SODA ASH	1284.9	1283.0	1285.0	1284.3
NEPHELINE SYENITE	360.4	352.0	360.0	357.5
B. H. CALUMITE	0.0	0.0	0.0	0.0
MELITE	5.1	14.0	18.0	12.4
IRON PYRITE	10.1	10.0	10.0	10.0
SALT CAKE	2.6	2.6	2.6	2.6
ASBURY GRAPHITE	4.3	7.5	7.5	6.4
MIXED CULLET	668.6	670.0	670.0	669.5
Flint - % Flint	30.0	30.0	30.0	30.0
Amber - % Amber	30.0	30.0	30.0	30.0
Green - % Green	40.0	40.0	40.0	40.0
Flat - % Flat	0.0	0.0	0.0	0.0
AMBER CULLET	1559.6	1565.0	1565.0	1563.2
RECYCLE - % MOISTURE	3.0	3.0	3.0	3.0

Theoretical Glass Composition Constraints

% SiO_2	71.90	71.85	71.86	71.87
% Al_2O_3	1.96	1.96	1.96	1.96
% Fe_2O_3	0.30	0.31	0.30	0.30
% CaO	10.67	10.73	10.72	10.71
% MgO	0.19	0.19	0.19	0.19
% BaO	0.00	0.00	0.00	0.00
% Na_2O	14.46	14.45	14.45	14.45
% K_2O	0.42	0.42	0.43	0.42
Estimated Total Sulfur at % SO_3	0.01	0.01	0.01	0.01
Input % S--	0.06	0.06	0.06	0.06
% MnO	0.00	0.00	0.00	0.00

Constraint and Final Batch Composition Spreadsheet - Furnace #1

Batch Constraints

DATE	12/30/95			
CHANGE NO.	2208	2207	2206	
	2/3/95	1/27/95	1/3/95	Average
BATCH WEIGHT	8580.81	8584.10	8600.10	8588.34
FUSION LOSS %	0.12	0.12	0.12	0.12
GLASS EQUIVALENT (Tons)	3.78	3.78	3.79	3.78
REDOX	-62.64	-99.07	-99.80	-87.17
EQUIV. SALT CAKE PER TON SAND	1.46	1.43	1.43	1.44
EQUIV. SALT CAKE PER TON GLASS	0.70	0.69	0.69	0.69
SLAG CONSUMPTION RATE	0.00	0.00	0.00	0.00
EQUIV. SALT CAKE PER 5500 # BATCH	1.70	1.67	1.66	1.68
SULFUR INPUT PER TON OF GLASS	1.43	1.42	1.42	1.42

Theoretical Glass Composition Constraints

GLASS DENSITY - (GM/CC)	2.51	2.51	2.51	2.51
MELT VISCOSITY - (LOG 2)	2632.82	2631.30	2631.30	2631.81
GOB VISCOSITY - (LOG 3)	2162.10	2161.40	2161.30	2161.60
SOFTENING POINT - (LOG 7.65)	1331.25	1331.60	1331.40	1331.42
CONVEYOR VISCOSITY - (LOG 10)	1166.86	1167.30	1167.20	1167.12
ANNEALING POINT - (LOG 13.37)	1035.79	1036.40	1036.30	1036.16
EXPANSION COEFFICIENT. - (X 10 -7)	90.84	91.00	91.00	90.95
WORKING RANGE INDEX - (S-A)	164.15	164.00	164.00	164.05
RELATIVE MACHINE SPEED - (%)	111.33	111.50	111.50	111.44
COOLING TIME - (Seconds)	89.69	89.60	89.60	89.63
LIQUID TEMP. - (degrees F)	1876.56	1877.30	1877.30	1877.05

Constraint and Final Batch Composition Spreadsheet - Furnace #1

Optimization Sheet

Constraint	Goal	Minimum	Maximum	Deviation
Batch Weight	8589.40	8580.81	8597.99	0.10%
Percentage of Fusion Loss	0.12	0.12	0.12	0.20%
Glass Equivalence (in tons)	3.78	3.78	3.79	0.15%
Redox	-99.31	-98.81	-99.81	0.50%
Salt Cake per Ton of Sand	1.43	1.40	1.46	2.00%
Salt Cake per Ton of Glass	0.69	0.68	0.70	2.00%
Salt Cake per 5500 pounds of Batch	1.67	1.64	1.70	2.00%
Sulfur Input per Ton of Glass	1.42	1.41	1.43	1.00%
Percentage of Mixed Cullet	7.80%	7.79%	7.81%	0.10%
Percentage of Amber Cullet	18.23%	18.18%	18.28%	0.30%
Percentage SiO_2	71.87%	71.73%	72.01%	0.20%
Percentage Al_2O_3	2.00%	1.96%	2.04%	2.00%
Percentage Fe_2O_3	0.30%	0.27%	0.33%	10.00%
Percentage CaO	10.70%	10.67%	10.73%	0.30%
Percentage MgO	0.19%	0.18%	0.20%	3.00%
Percentage BaO	0.00%	0.00%	0.00%	1.00%
Percentage Na_2O	14.40%	14.34%	14.46%	0.45%
Percentage K_2O	0.42%	0.41%	0.43%	2.00%
Percentage SO_3	0.01%	0.01%	0.02%	10.00%
Percentage Sulfur	0.06%	0.06%	0.07%	2.00%
Percentage MnO	0.00%	0.00%	0.00%	1.00%
Glass Density	2.51	2.50	2.51	0.05%
Melt Viscosity	2631.50	2630.18	2632.82	0.05%
Gob Viscosity	2161.50	2160.42	2162.58	0.05%
Softening Point	1331.50	1331.23	1331.77	0.02%
Conveyor Viscosity	1167.20	1166.62	1167.78	0.05%
Annealing Point	1036.29	1035.77	1036.81	0.05%
Expansion Coefficient	90.95	90.76	91.13	0.20%
Cooling Time	89.60	89.51	89.69	0.10%
Liquid Temperature	1877.50	1876.56	1878.44	0.05%

Constraint and Final Batch Composition Spreadsheet - Furnace #1

Optimization Sheet

Constraint	Actual Values	Adjusted Linear Values	Minimum Constraint [must be <=0]	Maximum Constraint [must be >=0]
Batch Weight	8580.81	8580.81	0.0000	17.1788
Percentage of Fusion Loss	0.12	1019.76	0.0000	4.0872
Glass Equivalence (in tons)	3.78	3.78	-0.0029	0.0084
Redox	-62.64	-34.48	-20.4620	-19.9153
Salt Cake per Ton of Sand	1.46	5299.02	-207.8046	0.0000
Salt Cake per Ton of Glass	0.70	5299.02	-186.2333	22.4517
Salt Cake per 5500 pounds of Batch	1.70	14572.29	-528.9399	44.2582
Sulfur Input per Ton of Glass	1.43	10844.06	-214.7339	0.0000
Percentage of Mixed Cullet	7.79%	668.63	0.0000	1.3386
Percentage of Amber Cullet	18.18%	1559.59	0.0000	9.3857
Percentage Si02	71.90%	5436.40	-13.1413	8.5952
Percentage Al203	1.96%	148.20	0.0000	6.0488
Percentage Fe203	0.30%	22.74	-2.3239	2.2127
Percentage Ca0	10.67%	806.77	-0.1641	4.6901
Percentage MgO	0.19%	14.58	-0.6415	0.2205
Percentage BaO	0.00%	0.00	0.0000	0.0764
Percentage Na20	14.46%	1093.57	-9.6772	0.1219
Percentage K2O	0.42%	32.05	-0.9240	0.3463
Percentage SO3	0.01%	1.08	-0.1292	0.0826
Percentage Sulfur	0.06%	4.88	-0.1388	0.0547
Percentage MnO	0.00%	0.04	-0.0401	0.0362
Glass Density	2.51	18941.84	-4.0753	14.8719
Melt Viscosity	2632.82	19906854.90	-19896.9064	0.0000
Gob Viscosity	2162.10	16347717.89	-12677.1592	3666.0531
Softening Point	1331.25	10065679.61	-153.3424	3873.6735
Conveyor Viscosity	1166.86	8822710.99	-1864.5610	6960.6980
Annealing Point	1035.79	7831657.65	-125.9859	7709.4635
Expansion Coefficient	90.74	686872.68	-608.1471	2142.4121
Cooling Time	89.69	678147.66	-1354.9404	0.0000
Liquid Temperature	1024.76	14188775.84	0.0000	14195.8738

Constraint and Final Batch Composition Spreadsheet - Furnace #2

Amber Soda/Lime Glass

Object: To minimize cost while satisfying all the theoretical physical property requirements.

Objective Function:	**[Total Cost per Batch]**	**$**	**214.49**
	[Total Cost per Ton]	**$**	**50.08**

DATE	1/6/96			
CHANGE NO.	3122	3121	3120	
Variables	2/3/95	1/27/95	1/25/95	Average
L. P. SAND	3625.0	3625.0	3625.0	3625.0
OKLAHOMA SAND	0.0	0.0	0.0	0.0
LIMESTONE	1047.0	1045.0	1045.0	1045.7
SODA ASH	1284.0	1286.0	1287.0	1285.7
NEPHELINE SYENITE	360.0	357.0	360.0	359.0
B. H. CALUMITE	0.0	0.0	0.0	0.0
MELITE	4.0	10.0	11.0	8.3
IRON PYRITE	10.0	10.0	10.0	10.0
SALT CAKE	9.0	9.0	9.0	9.0
ASBURY GRAPHITE	7.0	7.5	8.0	7.5
MIXED CULLET	666.0	670.0	670.0	668.7
Flint - % Flint	30.0	30.0	30.0	30.0
Amber - % Amber	30.0	30.0	30.0	30.0
Green - % Green	40.0	40.0	40.0	40.0
Flat - % Flat	0.0	0.0	0.0	0.0
AMBER CULLET	1554.0	1560.0	1560.0	1558.0
RECYCLE - % MOISTURE	2.5	2.5	2.5	2.5

Theoretical Glass Composition Constraints

% SiO_2	71.83	71.85	71.87	71.85
% Al_2O_3	1.97	1.96	1.96	1.96
% Fe_2O_3	0.31	0.30	0.29	0.30
% CaO	10.66	10.65	10.65	10.65
% MgO	0.19	0.19	0.19	0.19
% BaO	0.00	0.00	0.00	0.00
% Na_2O	14.50	14.51	14.50	14.50
% K_2O	0.43	0.42	0.04	0.30
Estimated Total Sulfur at % SO_3	0.03	0.03	0.03	0.03
Input % S--	0.06	0.06	0.06	0.06
% MnO	0.00	0.00	0.00	0.00

Constraint and Final Batch Composition Spreadsheet - Furnace #2

DATE	1/6/96			
CHANGE NO.	3122	3121	3120	
	2/3/95	1/27/95	1/25/95	Average
BATCH WEIGHT	8563.93	8579.50	8585.00	8576.14
FUSION LOSS %	0.12	0.12	0.12	0.12
GLASS EQUIVALENT (Tons)	3.77	3.78	3.78	3.78
REDOX	-8.24	-91.40	-97.10	-65.58
EQUIV. SALT CAKE PER TON SAND	4.97	4.97	4.97	4.97
EQUIV. SALT CAKE PER TON GLASS	2.38	2.38	2.38	2.38
SLAG CONSUMPTION RATE	0.00	0.00	0.00	0.00
EQUIV. SALT CAKE PER 5500 # Batch	5.78	5.77	5.77	5.77
SULFUR INPUT PER TON OF GLASS	1.61	1.61	1.61	1.61

Theoretical Glass Composition Constraints

GLASS DENSITY - (GM/CC)	2.51	2.51	2.51	2.51
MELT VISCOSITY - (LOG 2)	2631.53	2631.40	2631.60	2631.51
GOB VISCOSITY - (LOG 3)	2161.00	2160.80	2160.80	2160.87
SOFTENING POINT - (LOG 7.65)	1330.61	1330.40	1330.30	1330.44
CONVEYOR VISCOSITY - (LOG 10)	1166.29	1166.10	1166.10	1166.16
ANNEALING POINT - (LOG 13.37)	1035.32	1035.10	1035.10	1035.17
EXPANSION COEFFICIENT. - (X 10 -7)	91.00	91.00	91.00	91.00
WORKING RANGE INDEX - (S-A)	164.05	164.00	164.00	164.02
RELATIVE MACHINE SPEED - (%)	111.23	111.20	111.20	111.21
COOLING TIME - (Seconds)	89.77	89.80	89.80	89.79
LIQUID TEMP. - (degrees F)	1877.14	1876.70	1875.70	1876.51

Constraint and Final Batch Composition Spreadsheet - Furnace #2

Optimization Sheet

Constraint	Goal	Minimum	Maximum	Deviation
Batch Weight	8576.80	8563.93	8589.67	0.15%
Percentage of Fusion Loss	0.12	0.12	0.12	0.30%
Glass Equivalence (in tons)	3.78	3.77	3.79	0.20%
Redox	-92.00	-85.56	-98.44	7.00%
Salt Cake per Ton of Sand	4.97	4.97	4.97	0.10%
Salt Cake per Ton of Glass	2.38	2.38	2.39	0.20%
Salt Cake per 5500 pounds of Batch	5.77	5.76	5.78	0.20%
Sulfur Input per Ton of Glass	1.61	1.61	1.61	0.15%
Percentage of Mixed Cullet	7.80%	7.77%	7.82%	0.30%
Percentage of Amber Cullet	18.14%	18.09%	18.19%	0.30%
Percentage Si02	71.86%	71.79%	71.93%	0.10%
Percentage Al203	2.00%	1.95%	2.05%	2.50%
Percentage Fe203	0.30%	0.29%	0.31%	3.00%
Percentage Ca0	10.65%	10.64%	10.66%	0.10%
Percentage MgO	0.19%	0.19%	0.19%	1.00%
Percentage BaO	0.00%	0.00%	0.00%	1.00%
Percentage Na20	14.51%	14.49%	14.52%	0.10%
Percentage K2O	0.42%	0.41%	0.43%	2.00%
Percentage SO3	0.03%	0.03%	0.03%	7.00%
Percentage Sulfur	0.06%	0.06%	0.06%	1.00%
Percentage MnO	0.00%	0.00%	0.00%	1.00%
Glass Density	2.51	2.50	2.51	0.05%
Melt Viscosity	2631.56	2630.24	2632.88	0.05%
Gob Viscosity	2160.80	2159.72	2161.88	0.05%
Softening Point	1330.34	1330.08	1330.61	0.02%
Conveyor Viscosity	1166.08	1165.50	1166.66	0.05%
Annealing Point	1035.08	1034.56	1035.60	0.05%
Expansion Coefficient	91.00	90.91	91.09	0.10%
Cooling Time	89.81	89.72	89.90	0.10%
Liquid Temperature	1876.20	1875.26	1877.14	0.05%

Constraint and Final Batch Composition Spreadsheet - Furnace #2

Optimization Sheet

Constraint	Actual Values	Adjusted Linear Values	Minimum Constrain [must be <=0]	Maximum Constrain [must be >=0]
Batch Weight	8563.93	8563.93	0.0000	25.7304
Percentage of Fusion Loss	0.12	1016.08	-0.0266	6.0880
Glass Equivalence (in tons)	3.77	3.77	-0.0035	0.0116
Redox	-8.24	-4.55	-49.7980	-42.6872
Salt Cake per Ton of Sand	4.97	17986.74	0.0000	36.0095
Salt Cake per Ton of Glass	2.38	17986.74	-43.7001	28.2159
Salt Cake per 5500 pounds of Batch	5.78	49463.53	-148.4537	49.2020
Sulfur Input per Ton of Glass	1.61	12177.84	-36.4788	0.0000
Percentage of Mixed Cullet	7.82%	669.65	-4.0059	0.0000
Percentage of Amber Cullet	18.09%	1548.84	0.0000	9.3210
Percentage Si02	71.83%	5421.50	-2.8798	7.9683
Percentage Al203	1.97%	148.89	-1.7096	5.8383
Percentage Fe203	0.31%	23.32	-1.3586	0.0000
Percentage Ca0	10.66%	804.65	-1.6077	0.0000
Percentage MgO	0.19%	14.34	-0.1447	0.1421
Percentage BaO	0.00%	0.00	0.0000	0.0762
Percentage Na20	14.50%	1094.56	-0.7673	1.4225
Percentage K2O	0.43%	32.09	-1.0263	0.2417
Percentage SO3	0.03%	2.51	-0.2641	0.0741
Percentage Sulfur	0.06%	4.83	-0.0514	0.0452
Percentage MnO	0.00%	0.04	-0.0402	0.0361
Glass Density	2.51	18911.46	-13.5309	5.3765
Melt Viscosity	2631.53	19862396.76	-9688.8544	10173.7849
Gob Viscosity	2161.00	16310898.79	-9643.5454	6665.8645
Softening Point	1330.61	10043254.86	-4016.4986	0.0000
Conveyor Viscosity	1166.29	8803017.71	-6013.0279	2788.3774
Annealing Point	1035.32	7814472.83	-5743.0663	2069.5697
Expansion Coefficient	90.90	686832.90	-664.7427	708.9673
Cooling Time	89.77	677581.09	-385.9018	969.8443
Liquid Temperature	1025.08	14168370.42	-14161.2898	0.0000

Constraint and Final Batch Composition Spreadsheet - Furnace #3

Amber Soda/Lime Glass

Object: To minimize cost while satisfying all the theoretical physical property requirements.

Objective Function:	**[Total Cost per Batch]**	**$**	**217.37**
	[Total Cost per Ton]	**$**	**49.83**

DATE	**1/6/96**			
CHANGE NO.	**3953**	**3952**	**3951**	
Variables	**2/3/95**	**1/27/95**	**1/3/95**	**Average**
L. P. SAND	3625.6	3625.0	3625.0	3625.2
OKLAHOMA SAND	0.0	0.0	0.0	0.0
LIMESTONE	1031.6	1028.0	1030.0	1029.9
SODA ASH	1286.6	1288.0	1290.0	1288.2
NEPHELINE SYENITE	356.4	353.0	363.0	357.5
B. H. CALUMITE	0.0	0.0	0.0	0.0
MELITE	5.6	8.0	11.0	8.2
IRON PYRITE	0.0	0.0	0.0	0.0
SALT CAKE	16.0	16.0	16.0	16.0
ASBURY GRAPHITE	3.9	9.5	9.5	7.6
MIXED CULLET	719.5	723.0	723.0	721.8
Flint - % Flint	30.0	30.0	30.0	30.0
Amber - % Amber	30.0	30.0	30.0	30.0
Green - % Green	40.0	40.0	40.0	40.0
Flat - % Flat	0.0	0.0	0.0	0.0
AMBER CULLET	1679.7	1688.0	1688.0	1685.2
RECYCLE - % MOISTURE	0.0	0.0	0.0	0.0

Theoretical Glass Composition Constraints

% SiO2	72.00	72.00	72.01	72.00
% Al2O3	1.96	1.96	1.96	1.96
% Fe2O3	0.22	0.22	0.20	0.21
% CaO	10.56	10.56	10.56	10.56
% MgO	0.19	0.19	0.19	0.19
% BaO	0.00	0.00	0.00	0.00
% Na2O	14.57	14.58	14.58	14.58
% K2O	0.43	0.43	0.43	0.43
Estimated Total Sulfur at % SO3	0.05	0.05	0.05	0.05
Input % S--	0.00	0.00	0.00	0.00
% MnO	0.00	0.00	0.00	0.00

Constraint and Final Batch Composition Spreadsheet - Furnace #3

DATE	1/6/96			
CHANGE NO.	3953	3952	3951	
	2/3/95	1/27/95	1/3/95	Average
BATCH WEIGHT	8724.85	8738.50	8755.50	8739.62
FUSION LOSS %	0.12	0.12	0.12	0.12
GLASS EQUIVALENT (Tons)	3.85	3.86	3.87	3.86
REDOX	-24.68	-87.00	-87.55	-66.41
EQUIV. SALT CAKE PER TON SAND	8.82	8.83	8.83	8.83
EQUIV. SALT CAKE PER TON GLASS	4.15	4.15	4.14	4.15
SLAG CONSUMPTION RATE	0.00	0.00	0.00	0.00
EQUIV. SALT CAKE PER 5500 # Batch	10.08	10.07	10.05	10.07
SULFUR INPUT PER TON OF GLASS	0.53	0.53	0.53	0.53

Theoretical Glass Composition Constraints

GLASS DENSITY - (GM/CC)	2.50	2.50	2.50	2.50
MELT VISCOSITY - (LOG 2)	2632.02	2631.60	2631.60	2631.74
GOB VISCOSITY - (LOG 3)	2160.20	2159.90	2159.80	2159.97
SOFTENING POINT - (LOG 7.65)	1328.80	1328.60	1328.50	1328.63
CONVEYOR VISCOSITY - (LOG 10)	1164.77	1164.70	1164.60	1164.69
ANNEALING POINT - (LOG 13.37)	1033.69	1033.60	1033.50	1033.60
EXPANSION COEFFICIENT. - (X 10 -7)	91.15	91.20	91.20	91.18
WORKING RANGE INDEX - (S-A)	163.95	163.90	163.90	163.92
RELATIVE MACHINE SPEED - (%)	110.86	110.80	110.80	110.82
COOLING TIME - (Seconds)	90.05	90.10	90.10	90.08
LIQUID TEMP. - (degrees F)	1873.09	1872.90	1871.90	1872.63

Constraint and Final Batch Composition Spreadsheet - Furnace #3

Optimization Sheet

Constraint	Goal	Minimum	Maximum	Deviation
Batch Weight	8742.33	8724.85	8759.82	0.20%
Percentage of Fusion Loss	0.12	0.12	0.12	0.20%
Glass Equivalence (in tons)	3.86	3.85	3.87	0.20%
Redox	-85.10	-80.85	-89.36	5.00%
Salt Cake per Ton of Sand	8.83	8.82	8.84	0.10%
Salt Cake per Ton of Glass	4.15	4.14	4.15	0.10%
Salt Cake per 5500 pounds of Batch	10.07	10.05	10.08	0.15%
Sulfur Input per Ton of Glass	0.53	0.53	0.53	0.50%
Percentage of Mixed Cullet	8.26%	8.25%	8.28%	0.20%
Percentage of Amber Cullet	19.29%	19.25%	19.33%	0.20%
Percentage Si02	72.00%	71.99%	72.01%	0.01%
Percentage Al203	2.00%	1.95%	2.05%	2.50%
Percentage Fe203	0.21%	0.20%	0.22%	5.00%
Percentage Ca0	10.56%	10.53%	10.58%	0.20%
Percentage MgO	0.19%	0.19%	0.19%	1.00%
Percentage BaO	0.00%	0.00%	0.00%	1.00%
Percentage Na20	14.58%	14.57%	14.60%	0.10%
Percentage K2O	0.43%	0.43%	0.43%	0.70%
Percentage SO3	0.05%	0.05%	0.05%	5.00%
Percentage Sulfur	0.00%	0.00%	0.00%	5.00%
Percentage MnO	0.00%	0.00%	0.00%	1.00%
Glass Density	2.50	2.50	2.50	0.05%
Melt Viscosity	2631.64	2630.32	2632.95	0.05%
Gob Viscosity	2159.85	2158.77	2160.93	0.05%
Softening Point	1328.53	1328.27	1328.80	0.02%
Conveyor Viscosity	1164.60	1164.02	1165.18	0.05%
Annealing Point	1033.51	1032.99	1034.03	0.05%
Expansion Coefficient	91.20	91.15	91.25	0.05%
Cooling Time	90.10	90.05	90.14	0.05%
Liquid Temperature	1872.54	1871.60	1873.47	0.05%

Constraint and Final Batch Composition Spreadsheet - Furnace #3

Optimization Sheet

Constraint	Actual Values	Adjusted Linear Values	Minimum Constrain [must be <=0]	Maximum Constrain [must be >=0]
Batch Weight	8724.85	8724.85	0.0000	34.9693
Percentage of Fusion Loss	0.12	1016.15	0.0000	4.0728
Glass Equivalence (in tons)	3.85	3.85	-0.0011	0.0144
Redox	-24.68	-13.61	-35.6794	-30.9849
Salt Cake per Ton of Sand	8.82	31978.14	0.0000	64.0203
Salt Cake per Ton of Glass	4.15	31978.14	-49.8537	14.0668
Salt Cake per 5500 pounds of Batch	10.08	87939.89	-247.3051	16.1679
Sulfur Input per Ton of Glass	0.53	4121.53	-41.0103	0.0000
Percentage of Mixed Cullet	8.25%	719.49	0.0000	2.8837
Percentage of Amber Cullet	19.25%	1679.66	0.0000	6.7321
Percentage Si02	72.00%	5549.98	-0.2790	0.8310
Percentage Al203	1.96%	151.04	-0.7206	6.9881
Percentage Fe203	0.22%	17.00	-1.6188	0.0000
Percentage Ca0	10.56%	813.88	-1.7798	1.4751
Percentage MgO	0.19%	14.72	-0.2185	0.0744
Percentage BaO	0.00%	0.00	0.0000	0.0779
Percentage Na20	14.57%	1123.40	-0.4450	1.8031
Percentage K2O	0.43%	32.96	-0.1185	0.3445
Percentage SO3	0.05%	4.12	-0.3850	0.0082
Percentage Sulfur	0.00%	0.00	-0.0008	0.0801
Percentage MnO	0.00%	0.04	-0.0432	0.0347
Glass Density	2.50	19298.09	-12.8710	6.4239
Melt Viscosity	2632.02	20289418.40	-13082.5906	7203.8885
Gob Viscosity	2160.20	16652342.68	-11027.3792	5622.2610
Softening Point	1328.80	10243288.41	-4096.4961	0.0000
Conveyor Viscosity	1164.77	8978850.85	-5785.8257	3191.7281
Annealing Point	1033.69	7968368.50	-5331.0260	2635.9950
Expansion Coefficient	91.05	702681.46	0.0000	703.0330
Cooling Time	90.05	694190.73	0.0000	694.5380
Liquid Temperature	1022.83	14439046.80	-11447.7572	2987.0593

Percentage of the Oxides and Redox Factors Present in the Raw Materials

	SiO_2	Al_2O_3	Fe_2O_3	Cr_2O_3	CaO	MgO	BaO	Na_2O	K_2O	SO_3	S--	MnO	C	Cost ($/ Ton	Yield	Redox
Lien Sand	99.2%	0.5%	0.3%		0.0%	0.0%		0.0%	0.0%					$ 39.04	100.0%	
Sandy Flats	97.8%	1.2%	0.1%		0.3%	0.1%		0.1%	0.5%					$ 39.04	100.0%	
Soda Ash								58.4%						$ 92.84	58.4%	
Limestone					53.8%	0.7%								$ 26.42	54.5%	COD?
Nepheline Syenite	60.6%	23.0%	0.3%		0.5%	0.0%		10.1%	4.9%					$ 89.75	99.4%	
Melite 40 (Greenville)	48.7%	20.8%	21.3%		4.5%	0.9%		0.5%	1.5%	0.1%			0.3%	$ 92.08	98.6%	-0.10
Salt Cake								43.5%		56.2%				$ 162.20	99.8%	0.67
Ashland S. C. Mix								47.7%	0.8%	12.8%				$ 93.80	61.3%	
Asbury Graphite													91.0%	$ 261.00	91.0%	-6.10
Carbocite													72.0%	$ 136.89	72.0%	
Feldspar	67.9%	19.0%	0.1%		1.7%	0.1%		7.1%	4.1%					$ 59.00	99.9%	
B. Harb. Calumite	36.8%	10.9%	0.2%		39.3%	10.8%		0.3%	0.4%	0.2%	0.8%	0.5%	0.1%	$ 68.96	100.3%	-0.07
Billings Iron Pyrite	4.5%		61.2%								48.3%			$ 250.00	114.0%	-1.20
Coors Amber Cullet	71.1%	2.1%	0.2%		10.8%	0.3%		14.8%	0.5%	0.1%	0.1%			$ 55.00	99.9%	
Amber Cullet	72.3%	2.2%	0.2%		10.5%	0.3%		13.8%	0.6%	0.0%				$ 25.00	100.0%	
Flint Cullet	72.1%	1.8%	0.1%		10.9%	0.5%		14.2%	0.3%	0.2%		0.0%		$ 25.00	100.0%	
Green Cullet	72.1%	2.0%	0.2%	0.2%	11.4%	0.1%		12.8%	1.0%	0.2%				$ 25.00	100.0%	
Flat Glass Cullet	73.3%	0.2%	0.1%		8.8%	3.7%		13.5%	0.0%	0.2%					99.9%	
Mixed Cullet	72.1%	2.0%	0.2%	0.1%	11.0%	0.3%	0.0%	13.5%	0.7%	0.1%	0.0%	0.0%	0.0%	$ 25.00	100.0%	

Calculation Spreadsheet for Furnace #1

Calculations	Batch lbs	SiO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	C	Yield	Redox	Redox-LP
																4.95	2.73
L. P Sand	3632.9	3605.2	16.35	9.59	0.00	1.63	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3632.9	0.00	0.00
Oklahoma Sand	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Soda Ash	1284.9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	750.52	0.00	0.00	0.00	0.00	0.00	750.52	0.00	0.00
Limestone	1052.2	0.00	0.00	0.00	0.00	566.06	7.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	573.63	0.00	0.00
Nephelite Syenite	360.4	218.40	82.89	1.15	0.00	1.66	0.15	0.00	36.40	17.66	0.00	0.00	0.00	0.00	358.31	0.00	0.00
B. H. Calumite	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Melite	5.1	2.49	1.06	1.09	0.00	0.23	0.05	0.00	0.03	0.08	0.01	0.00	0.00	0.02	5.04	-0.93	-0.51
Iron Pyrite	10.1	0.46	0.00	6.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.88	0.00	0.00	11.52	-22.03	-12.13
Salt Cake	2.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.15	0.00	1.49	0.00	0.00	0.00	2.64	3.22	1.78
Asbury Graphite	4.3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.93	3.93	-47.86	-26.35
Mixed Cullet	668.6	482.32	13.58	1.14	0.48	73.43	1.91	0.00	90.25	4.48	0.90	0.00	0.04	0.00	668.53	0.00	0.00
Amber Cullet	1559.6	1127.6	34.31	3.59	0.00	163.76	4.68	0.00	215.22	9.83	0.31	0.00	0.00	0.00	1559.3	0.00	0.00
TOTAL	8580.8	5436.4	148.20	22.74	0.48	806.77	14.58	0.00	1093.6	32.05	1.35	4.88	0.04	3.95	7561.1	-62.64	-34.48

Resultant Oxides		SiO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MnO	Std.	Totals
%	100.0%	71.90%	1.96%	0.30%	0.01%	10.67%	0.19%	0.00%	14.46%	0.42%	0.01%	0.06%	0.00%		
Standard Reference		72.39%	1.83%	0.05%		11.13%	0.76%	0.00%	13.35%	0.29%					
Variance (%)		-0.490	0.130	0.252		-0.460	-0.567	0.000	1.113	0.134					
Density Factors		-0.002	0.002	0.011		0.011	0.007	0.017	0.005	0.003			0.013		
Density		0.0012	0.0002	0.0027		-0.005	-0.004	0.0000	0.0053	0.0005	0.0000	0.0000	0.000	2.5043	2.5052

Lakatos Factors										Deg F	Deg C
LOG 2 -Melt		1.52		18.72	14.69	0.00	-53.66	-3.56	2655	2633	1445
LOG 3 -Gob		1.26	1.26	11.82	6.41	0.00	-43.64	-2.72	2188	2162	1183
LOG 7.65 -Softening		0.94	1.28	1.01	1.02	0.00	-22.82	-1.38	1351	1331	722
LOG 10 - Conveyor		0.91		-1.06	1.30	0.00	-18.26	-1.14	1185	1167	630
LOG 13.37 -Annealing		0.91		-2.67	1.93	0.00	-14.47	-0.91	1051	1036	558
Expansion	-0.02	0.02		-0.75	-0.26	0.00	4.81	0.52	86.4	90.7	
Cooling Time	0.01	-0.09	0.08	0.80	0.40		1.91	0.15		89.7	
Liquid Temperature	0.24	2.41	5.36	-12.31	-5.38		-2.45	-1.35		1024.8	

(Continued)

Calculation Spreadsheet for Furnace #1 (Continued)

Linear Programming Factors	SIO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	C	Yield	Total
Density	8.9	1.8	20.7	0.0	-36.9	-31.7	0.0	40.4	3.4	0.0	0.0	0.1	0.0	18935	18942
Melt Viscosity		11500.4			141536	111079	0.0	-405693	-26914					20075347	19906855
Gob Viscosity		9534.5	9516.9		89373	48463	0.0	-329941	-20540					16541312	16347718
Softening Point		7077.1	9707.3		7651	7720	0.0	-172546	-10422					10216492	10065680
Conveyor Viscosity		6880.6			-7998	9864	0.0	-138037	-8600					8960602	8822711
Annealing Point		6880.6			-20170	14582	0.0	-109419	-6880					7946665	7831658
Expansion Coefficient	-185.2	167.1			-5668	-1930	0.0	36361	3946					654182	686873
Cooling Time	74.1	-648.7	590.0		6051	3045		14477	1133					653426	678148
Liquid Temperature	1852.2	18194.2	40504.0		-93066	-40700		-18516	-10219					7850186	14188776
Fusion Loss															1019.76
Redox															-34.48

Calculation Spreadsheet for Furnace #2

Calculations	Batch lbs	SiO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	C	Yield	Redox	Redox-LP
																4.12	2.27
L.P. Sand	3625.0	3597.3	16.31	9.57	0.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3624.8	0.00	0.00
Oklahoma Sand	0.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Soda Ash	1284.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	749.98	0.00	0.00	0.00	0.00	0.00	749.98	0.00	0.00
Limestone	1047.0	0.0	0.00	0.00	0.00	563.29	7.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	570.82	0.00	0.00
Nephelite Syenite	360.0	218.2	82.80	1.15	0.00	1.66	0.15	0.00	36.36	17.64	0.00	0.00	0.00	0.00	357.92	0.00	0.00
B. H. Calumite	0.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Melite	4.0	1.9	0.83	0.85	0.00	0.18	0.04	0.00	0.02	0.06	0.01	0.00	0.00	0.01	3.94	-0.73	-0.40
Iron Pyrite	10.0	0.5	0.00	6.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.83	0.00	0.00	11.40	-21.75	-12.00
Salt Cake	9.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	3.92	0.00	5.06	0.00	0.00	0.00	8.98	10.93	6.03
Asbury Graphite	7.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.37	6.37	-77.36	-42.68
Mixed Cullet	666.0	480.4	13.53	1.13	0.48	73.14	1.90	0.00	89.90	4.47	0.89	0.00	0.04	0.00	665.90	0.00	0.00
Amber Cullet	1554.0	1123.5	34.19	3.57	0.00	163.17	4.66	0.00	214.45	9.79	0.31	0.00	0.00	0.00	1553.7	0.00	0.00
TOTAL	8566.0	5421.8	147.66	22.40	0.48	803.07	14.29	0.00	1094.6	31.96	3.14	4.83	0.04	6.38	7544.3	-84.79	-46.78

Resultant Oxides		SiO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	Std.	Totals
%	99.99%	71.87%	1.96%	0.30%	0.01%	10.64%	0.19%	0.00%	14.51%	0.42%	0.03%	0.06%	0.00%		
Standard Reference		72.39%	1.83%	0.05%		11.13%	0.76%	0.00%	13.35%	0.29%					
Variance (%)		-0.524	0.127	0.248		-0.485	-0.571	0.000	1.159	0.134					
Density Factors		-0.002	0.002	0.011		0.011	0.007	0.017	0.005	0.003			0.0125		
Density		0.0013	0.0002	0.0027		-0.005	-0.004	0.0000	0.0056	0.0005	0.0000	0.0000	0.0000	2.5043	2.5051

Lakatos Factors										Deg F	Deg C
LOG 2 - Melt		1.489		19.751	14.779	0.000	-55.884	-3.553	2655.1	2631.7	1444.3
LOG 3 -Gob		1.235	1.239	12.472	6.448	0.000	-45.449	-2.712	2187.7	2160.9	1182.7
LOG 7.65 -Softening		0.916	1.264	1.068	1.027	0.000	-23.768	-1.376	1351.2	1330.3	721.3
LOG 10 - Conveyor		0.891		-1.116	1.312	0.000	-19.014	-1.135	1185.1	1166.0	630.0
LOG 13.37 -Annealing		0.891		-2.815	1.940	0.000	-15.072	-0.908	1051.0	1035.0	557.2
Expansion	-0.026	0.022		-0.791	-0.257	0.000	5.009	0.521	86.4	90.9	
Cooling Time	0.010	-0.084	0.077	0.844	0.405		1.994	0.150		89.8	
Liquid Temperature	0.262	2.356	5.275	-12.383	-5.415		-2.445	-1.349		1024.5	

(Continued)

Calculation Spreadsheet for Furnace #2 (Continued)

Linear Programming Factors	SIO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	C	Yield	Total
Density	9.5	1.7	20.4	0.0	-38.8	-31.9	0.0	42.0	3.4	0.0	0.0	0.0	0.0	18893	18900
Melt Viscosity		11235.2			149010	111498	0.0	-421601	-26807					20030806	19854141
Gob Viscosity		9314.6	9351.1		94092	48646	0.0	-342879	-20458					16504611	16302678
Softening Point		6914.0	9538.1		8055	7749	0.0	-179312	-10380					10193825	10036389
Conveyor Viscosity		6721.9			-8421	9901	0.0	-143449	-8566					8940721	8796908
Annealing Point		6721.9			-21235	14637	0.0	-113710	-6853					7929033	7808594
Expansion Coefficient	-197.6	163.2			-5968	-1937	0.0	37787	3930					652731	686509
Cooling Time	79.0	-633.8	579.8		6370	3057		15045	1129					651976	677602
Liquid Temperature	1975.6	17774.6	39798.1		-93417	-40854		-18443	-10179					7832769	14154380
Fusion Loss															1021.72
Redox															-46.78

Calculation Spreadsheet for Furnace #3

Calculations	Batch lb.	SiO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	C	Yield	Redox	Redox-LP
																0.00	0.00
L. P. Sand	3625.0	3597.3	16.31	9.57	0.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3624.8	0.00	0.00
Oklahoma Sand	0.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Soda Ash	1288.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	752.32	0.00	0.00	0.00	0.00	0.00	752.32	0.00	0.00
Limestone	1030.0	0.0	0.00	0.00	0.00	554.14	7.42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	561.56	0.00	0.00
Nephelite Syenite	357.0	216.3	82.11	1.14	0.00	1.64	0.15	0.00	36.06	17.49	0.00	0.00	0.00	0.00	354.94	0.00	0.00
B. H. Calumite	0.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Melite	4.0	1.9	0.83	0.85	0.00	0.18	0.04	0.00	0.02	0.06	0.01	0.00	0.00	0.01	3.94	-0.73	-0.40
Iron Pyrite	0.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Salt Cake	16.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	6.96	0.00	9.00	0.00	0.00	0.00	15.96	19.43	10.72
Asbury Graphite	9.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.19	8.19	-99.46	-54.87
Mixed Cullet	721.0	520.1	14.65	1.23	0.52	79.18	2.06	0.00	97.32	4.84	0.97	0.00	0.04	0.00	720.89	0.00	0.00
Amber Cullet	1683.0	1216.8	37.03	3.87	0.00	176.72	5.05	0.00	232.25	10.60	0.34	0.00	0.00	0.00	1682.7	0.00	0.00
Total	8733.0	5552.5	150.93	16.66	0.52	813.49	14.71	0.00	1124.9	32.99	5.15	0.00	0.04	8.20	7711.9	-80.75	-44.55

Resultant Oxides		SiO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	Std.	Totals
%	100.0%	72.0%	2.0%	0.2%	0.0%	10.5%	0.2%	0.0%	14.6%	0.4%	0.1%	0.0%	0.0%		
Standard Reference		72.39%	1.83%	0.05%		11.13%	0.76%	0.00%	13.35%	0.29%					
Variance (%)		-0.391	0.127	0.167		-0.581	-0.569	0.000	1.237	0.138					
Density Factors		-0.002	0.002	0.011		0.011	0.007	0.017	0.005	0.003			0.0125		
Density		0.0009	0.0002	0.0018		-0.006	-0.004	0.0000	0.0059	0.0005	0.0000	0.0000	0.0000	2.5043	2.5033

Lakatos Factors															Deg F	Deg C
LOG 2 -Melt			1.487			23.67	14.745	0.000	-59.62	-3.665				2655.1	2631.7	1444.3
LOG 3 -Gob			1.233	0.835		14.94	6.433	0.000	-48.49	-2.797				2187.7	2159.9	1182.1
LOG 7.65 -Softening			0.915	0.852		1.28	1.025	0.000	-25.36	-1.419				1351.2	1328.5	720.3
LOG 10 - Conveyor			0.890			-1.34	1.309	0.000	-20.29	-1.171				1185.1	1164.5	629.2
LOG 13.37 -Annealing			0.890			-3.37	1.936	0.000	-16.08	-0.937				1051.0	1033.4	556.4
Expansion		-0.020	0.022			-0.95	-0.256	0.000	5.34	0.537				86.4	91.1	
Cooling Time		0.008	-0.084	0.052		1.01	0.404		2.13	0.154					90.1	
Liquid Temperature		0.196	2.352	3.554		-12.35	-5.403		-2.52	-1.392					1022.7	

(Continued)

Calculation Spreadsheet for Furnace #3 (Continued)

Linear Programming Factors	SIO2	Al2O3	Fe2O3	Cr2O3	CaO	MgO	BaO	Na2O	K2O	SO3	S--	MNO	C	Yield	Total
Density	7.2	1.8	14.0	0.0	-47.5	-32.5	0.0	45.8	3.6	0.0	0.0	0.1	0.0	19313	19305
Melt Viscosity		11466.9			182506	113709	0.0	-459820	-28266					20475838	20295434
Gob Viscosity		9506.7	6440.3		115243	49611	0.0	-373961	-21572					16871301	16656569
Softening Point		7056.5	6569.1		9865	7903	0.0	-195567	-10945					10420305	10245187
Conveyor Viscosity		6860.5			-10314	10098	0.0	-156453	-9032					9139360	8980519
Annealing Point		6860.5			-26008	14927	0.0	-124018	-7226					8105196	7969732
Expansion Coefficient	-150.9	166.6			-7309	-1976	0.0	41212	4144					667233	703320
Cooling Time	60.4	-646.8	399.3		7802	3117		16409	1190					666462	694793
Liquid Temperature	1508.8	18141.2	27409.9		-95270	-41664		-19446	-10733					8006792	14442911
Fusion Loss															1021.11
Redox															-44.55

Summary Report For Furnace #1

Target Cell (Min.)

Cell	Name	Original Value	Final Value
E6	[Total Cost per Batch]	$ 214.83	$ 214.16

Adjustable Cells

Cell	Name	Original Value	Final Value
C12	L. P. SAND [Total Cost per Ton]	3625.0	3632.9
C13	OKLAHOMA SAND [Total Cost per Ton]	0.0	0.0
C14	LIMESTONE [Total Cost per Ton]	1056.0	1052.2
C15	SODA ASH [Total Cost per Ton]	1284.0	1284.9
C16	NEPHELINE SYENITE [Total Cost per Ton]	355.0	360.4
C17	B. H. CALUMITE [Total Cost per Ton]	0.0	0.0
C18	MELITE [Total Cost per Ton]	16.0	5.1
C19	IRON PYRITE [Total Cost per Ton]	10.0	10.1
C20	SALT CAKE [Total Cost per Ton]	2.6	2.6
C21	ASBURY GRAPHITE [Total Cost per Ton]	7.5	4.3
C22	MIXED CULLET [Total Cost per Ton]	670.0	668.6
C27	AMBER CULLET [Total Cost per Ton]	1565.0	1559.6

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
E109	Batch Weight [must be <=0]	0.0000	E109<=0	Binding	0.0000
F109	Batch Weight [must be >=0]	17.1788	F109>=0	Not Binding	17.1788
E110	Percentage of Fusion Loss [must be <=0]	0.0000	E110<=0	Binding	0.0000
F110	Percentage of Fusion Loss [must be >=0]	4.0872	F110>=0	Not Binding	4.0872
E111	Glass Equivalence (in tons) [must be <=0]	-0.0029	E111<=0	Not Binding	0.0029
F111	Glass Equivalence (in tons) [must be >=0]	0.0084	F111>=0	Not Binding	0.0084
E113	Salt Cake per Ton of Sand [must be <=0]	-207.8046	E113<=0	Not Binding	207.8046
F113	Salt Cake per Ton of Sand [must be >=0]	0.0000	F113>=0	Binding	0.0000
E114	Salt Cake per Ton of Glass [must be <=0]	-186.2333	E114<=0	Not Binding	186.2333
F114	Salt Cake per Ton of Glass [must be >=0]	22.4517	F114>=0	Not Binding	22.4517
E115	Salt Cake per 5500 pounds of Batch [must be <=0]	-528.9399	E115<=0	Not Binding	528.9399
F120	Percentage Al203 [must be >=0]	6.0488	F120>=0	Not Binding	6.0488
E120	Percentage Al203 [must be <=0]	0.0000	E120<=0	Binding	0.0000
F115	Salt Cake per 5500 pounds of Batch [must be >=0]	44.2582	F115>=0	Not Binding	44.2582
E116	Sulfur Input per Ton of Glass [must be <=0]	-214.7339	E116<=0	Not Binding	214.7339
F116	Sulfur Input per Ton of Glass [must be >=0]	0.0000	F116>=0	Binding	0.0000
F117	Percentage of Mixed Cullet [must be >=0]	1.3386	F117>=0	Not Binding	1.3386
E118	Percentage of Amber Cullet [must be <=0]	0.0000	E118<=0	Binding	0.0000
F118	Percentage of Amber Cullet [must be >=0]	9.3857	F118>=0	Not Binding	9.3857
E119	Percentage Si02 [must be <=0]	-13.1413	E119<=0	Not Binding	13.1413
F119	Percentage Si02 [must be >=0]	8.5952	F119>=0	Not Binding	8.5952

Summary Report For Furnace #1

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
E121	Percentage Fe203 [must be <=0]	-2.3239	E121<=0	Not Binding	2.3239
E122	Percentage Ca0 [must be <=0]	-0.1641	E122<=0	Not Binding	0.1641
E123	Percentage MgO [must be <=0]	-0.6415	E123<=0	Not Binding	0.6415
E124	Percentage BaO [must be <=0]	0.0000	E124<=0	Binding	0.0000
E125	Percentage Na20 [must be <=0]	-9.6772	E125<=0	Not Binding	9.6772
E126	Percentage K2O [must be <=0]	-0.9240	E126<=0	Not Binding	0.9240
E127	Percentage SO3 [must be <=0]	-0.1292	E127<=0	Not Binding	0.1292
E128	Percentage Sulfur [must be <=0]	-0.1388	E128<=0	Not Binding	0.1388
E129	Percentage MnO [must be <=0]	-0.0401	E129<=0	Not Binding	0.0401
E130	Glass Density [must be <=0]	-4.0753	E130<=0	Not Binding	4.0753
E131	Melt Viscosity [must be <=0]	-19896.91	E131<=0	Not Binding	19896.91
E132	Gob Viscosity [must be <=0]	-12677.16	E132<=0	Not Binding	12677.16
E133	Softening Point [must be <=0]	-153.34	E133<=0	Not Binding	153.34
E134	Conveyor Viscosity [must be <=0]	-1864.56	E134<=0	Not Binding	1864.56
E135	Annealing Point [must be <=0]	-125.99	E135<=0	Not Binding	125.99
E136	Expansion Coefficient [must be <=0]	-608.15	E136<=0	Not Binding	608.15
E137	Cooling Time [must be <=0]	-1354.94	E137<=0	Not Binding	1354.94
E138	Liquid Temperature [must be <=0]	0.00	E138<=0	Binding	0.00
F138	Liquid Temperature [must be >=0]	14195.87	F138>=0	Not Binding	14195.87
F137	Cooling Time [must be >=0]	0.00	F137>=0	Binding	0.00
F136	Expansion Coefficient [must be >=0]	2142.41	F136>=0	Not Binding	2142.41
F135	Annealing Point [must be >=0]	7709.46	F135>=0	Not Binding	7709.46
F134	Conveyor Viscosity [must be >=0]	6960.70	F134>=0	Not Binding	6960.70
F133	Softening Point [must be >=0]	3873.67	F133>=0	Not Binding	3873.67
F132	Gob Viscosity [must be >=0]	3666.05	F132>=0	Not Binding	3666.05
F131	Melt Viscosity [must be >=0]	0.00	F131>=0	Binding	0.00
F130	Glass Density [must be >=0]	14.8719	F130>=0	Not Binding	14.8719
F129	Percentage MnO [must be >=0]	0.0362	F129>=0	Not Binding	0.0362
F128	Percentage Sulfur [must be >=0]	0.0547	F128>=0	Not Binding	0.0547
F127	Percentage SO3 [must be >=0]	0.0826	F127>=0	Not Binding	0.0826
F126	Percentage K2O [must be >=0]	0.3463	F126>=0	Not Binding	0.3463
F125	Percentage Na20 [must be >=0]	0.1219	F125>=0	Not Binding	0.1219
F124	Percentage BaO [must be >=0]	0.0764	F124>=0	Not Binding	0.0764
F123	Percentage MgO [must be >=0]	0.2205	F123>=0	Not Binding	0.2205
F122	Percentage Ca0 [must be >=0]	4.6901	F122>=0	Not Binding	4.6901
F121	Percentage Fe203 [must be >=0]	2.2127	F121>=0	Not Binding	2.2127
C12	L. P. SAND [Total Cost per Ton]	3632.9	C12>=0	Not Binding	3632.9
C13	OKLAHOMA SAND [Total Cost per Ton]	0.0	C13=0	Binding	0.0
C14	LIMESTONE [Total Cost per Ton]	1052.2	C14>=0	Not Binding	1052.2
C15	SODA ASH [Total Cost per Ton]	1284.9	C15>=0	Not Binding	1284.9
C16	NEPHELINE SYENITE [Total Cost per Ton]	360.4	C16>=0	Not Binding	360.4
C17	B. H. CALUMITE [Total Cost per Ton]	0.0	C17=0	Binding	0.0
C18	MELITE [Total Cost per Ton]	5.1	C18>=0	Not Binding	5.1
C19	IRON PYRITE [Total Cost per Ton]	10.1	C19>=0	Not Binding	10.1
C20	SALT CAKE [Total Cost per Ton]	2.6	C20>=0	Not Binding	2.6
C21	ASBURY GRAPHITE [Total Cost per Ton]	4.3	C21>=0	Not Binding	4.3
C22	MIXED CULLET [Total Cost per Ton]	668.6	C22>=0	Not Binding	668.6
C27	AMBER CULLET [Total Cost per Ton]	1559.6	C27>=0	Not Binding	1559.6

Summary Report For Furnace #2

Target Cell (Min.)

Cell	Name	Original Value	Final Value
E6	[Total Cost per Batch]	$ 214.49	$ 213.74

Adjustable Cells

Cell	Name	Original Value	Final Value
C12	L. P. SAND [Total Cost per Ton]	3625.0	3622.7
C13	OKLAHOMA SAND [Total Cost per Ton]	0.0	0.0
C14	LIMESTONE [Total Cost per Ton]	1047.0	1049.8
C15	SODA ASH [Total Cost per Ton]	1284.0	1284.0
C16	NEPHELINE SYENITE [Total Cost per Ton]	360.0	361.6
C17	B. H. CALUMITE [Total Cost per Ton]	0.0	0.0
C18	MELITE [Total Cost per Ton]	4.0	8.4
C19	IRON PYRITE [Total Cost per Ton]	10.0	10.0
C20	SALT CAKE [Total Cost per Ton]	9.0	9.0
C21	ASBURY GRAPHITE [Total Cost per Ton]	7.0	0.0
C22	MIXED CULLET [Total Cost per Ton]	666.0	669.6
C27	AMBER CULLET [Total Cost per Ton]	1554.0	1548.8

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
F109	Batch Weight [must be >=0]	25.7304	F109>=0	Not Binding	25.7304
E109	Batch Weight [must be <=0]	0.0000	E109<=0	Binding	0.0000
E110	Percentage of Fusion Loss [must be <=0]	-0.0266	E110<=0	Not Binding	0.0266
F110	Percentage of Fusion Loss [must be >=0]	6.0880	F110>=0	Not Binding	6.0880
E111	Glass Equivalence (in tons) [must be <=0]	-0.0035	E111<=0	Not Binding	0.0035
F111	Glass Equivalence (in tons) [must be >=0]	0.0116	F111>=0	Not Binding	0.0116
E113	Salt Cake per Ton of Sand [must be <=0]	0.0000	E113<=0	Binding	0.0000
F113	Salt Cake per Ton of Sand [must be >=0]	36.0095	F113>=0	Not Binding	36.0095
E114	Salt Cake per Ton of Glass [must be <=0]	-43.7001	E114<=0	Not Binding	43.7001
F114	Salt Cake per Ton of Glass [must be >=0]	28.2159	F114>=0	Not Binding	28.2159
E115	Salt Cake per 5500 pounds of Batch [must be <=0]	-148.4537	E115<=0	Not Binding	148.4537
F115	Salt Cake per 5500 pounds of Batch [must be >=0]	49.2020	F115>=0	Not Binding	49.2020
E116	Sulfur Input per Ton of Glass [must be <=0]	-36.4788	E116<=0	Not Binding	36.4788
F117	Percentage of Mixed Cullet [must be >=0]	0.0000	F117>=0	Binding	0.0000
E117	Percentage of Mixed Cullet [must be <=0]	-4.0059	E117<=0	Not Binding	4.0059
F118	Percentage of Amber Cullet [must be >=0]	9.3210	F118>=0	Not Binding	9.3210
E118	Percentage of Amber Cullet [must be <=0]	0.0000	E118<=0	Binding	0.0000
F119	Percentage Si02 [must be >=0]	7.9683	F119>=0	Not Binding	7.9683
E119	Percentage Si02 [must be <=0]	-2.8798	E119<=0	Not Binding	2.8798
F120	Percentage Al203 [must be >=0]	5.8383	F120>=0	Not Binding	5.8383
E120	Percentage Al203 [must be <=0]	-1.7096	E120<=0	Not Binding	1.7096
F121	Percentage Fe203 [must be >=0]	0.0000	F121>=0	Binding	0.0000
E121	Percentage Fe203 [must be <=0]	-1.3586	E121<=0	Not Binding	1.3586

Summary Report For Furnace #2

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
F122	Percentage Ca0 [must be >=0]	0.0000	F122>=0	Binding	0.0000
E122	Percentage Ca0 [must be <=0]	-1.6077	E122<=0	Not Binding	1.6077
E123	Percentage MgO [must be <=0]	-0.1447	E123<=0	Not Binding	0.1447
E124	Percentage BaO [must be <=0]	0.0000	E124<=0	Binding	0.0000
E125	Percentage Na20 [must be <=0]	-0.7673	E125<=0	Not Binding	0.7673
E126	Percentage K2O [must be <=0]	-1.0263	E126<=0	Not Binding	1.0263
E127	Percentage SO3 [must be <=0]	-0.2641	E127<=0	Not Binding	0.2641
E128	Percentage Sulfur [must be <=0]	-0.0514	E128<=0	Not Binding	0.0514
E129	Percentage MnO [must be <=0]	-0.0402	E129<=0	Not Binding	0.0402
E130	Glass Density [must be <=0]	-13.5309	E130<=0	Not Binding	13.5309
E131	Melt Viscosity [must be <=0]	-9688.85	E131<=0	Not Binding	9688.85
E132	Gob Viscosity [must be <=0]	-9643.55	E132<=0	Not Binding	9643.55
E133	Softening Point [must be <=0]	-4016.50	E133<=0	Not Binding	4016.50
E134	Conveyor Viscosity [must be <=0]	-6013.03	E134<=0	Not Binding	6013.03
E135	Annealing Point [must be <=0]	-5743.07	E135<=0	Not Binding	5743.07
E136	Expansion Coefficient [must be <=0]	-664.74	E136<=0	Not Binding	664.74
E137	Cooling Time [must be <=0]	-385.90	E137<=0	Not Binding	385.90
E138	Liquid Temperature [must be <=0]	-14161.29	E138<=0	Not Binding	14161.29
F123	Percentage MgO [must be >=0]	0.1421	F123>=0	Not Binding	0.1421
F124	Percentage BaO [must be >=0]	0.0762	F124>=0	Not Binding	0.0762
F125	Percentage Na20 [must be >=0]	1.4225	F125>=0	Not Binding	1.4225
F126	Percentage K2O [must be >=0]	0.2417	F126>=0	Not Binding	0.2417
F127	Percentage SO3 [must be >=0]	0.0741	F127>=0	Not Binding	0.0741
F128	Percentage Sulfur [must be >=0]	0.0452	F128>=0	Not Binding	0.0452
F129	Percentage MnO [must be >=0]	0.0361	F129>=0	Not Binding	0.0361
F130	Glass Density [must be >=0]	5.3765	F130>=0	Not Binding	5.3765
F131	Melt Viscosity [must be >=0]	10173.78	F131>=0	Not Binding	10173.78
F116	Sulfur Input per Ton of Glass [must be >=0]	0.0000	F116>=0	Binding	0.0000
F132	Gob Viscosity [must be >=0]	6665.86	F132>=0	Not Binding	6665.86
F133	Softening Point [must be >=0]	0.00	F133>=0	Binding	0.00
F134	Conveyor Viscosity [must be >=0]	2788.38	F134>=0	Not Binding	2788.38
F135	Annealing Point [must be >=0]	2069.57	F135>=0	Not Binding	2069.57
F136	Expansion Coefficient [must be >=0]	708.97	F136>=0	Not Binding	708.97
F137	Cooling Time [must be >=0]	969.84	F137>=0	Not Binding	969.84
F138	Liquid Temperature [must be >=0]	0.00	F138>=0	Binding	0.00
C12	L. P. SAND [Total Cost per Ton]	3622.7	C12>=0	Not Binding	3622.7
C13	OKLAHOMA SAND [Total Cost per Ton]	0.0	C13=0	Binding	0.0
C14	LIMESTONE [Total Cost per Ton]	1049.8	C14>=0	Not Binding	1049.8
C15	SODA ASH [Total Cost per Ton]	1284.0	C15>=0	Not Binding	1284.0
C16	NEPHELINE SYENITE [Total Cost per Ton]	361.6	C16>=0	Not Binding	361.6
C17	B. H. CALUMITE [Total Cost per Ton]	0.0	C17=0	Binding	0.0
C18	MELITE [Total Cost per Ton]	8.4	C18>=0	Not Binding	8.4
C19	IRON PYRITE [Total Cost per Ton]	10.0	C19>=0	Not Binding	10.0
C20	SALT CAKE [Total Cost per Ton]	9.0	C20>=0	Not Binding	9.0
C21	ASBURY GRAPHITE [Total Cost per Ton]	0.0	C21>=0	Binding	0.0
C22	MIXED CULLET [Total Cost per Ton]	669.6	C22>=0	Not Binding	669.6
C27	AMBER CULLET [Total Cost per Ton]	1548.8	C27>=0	Not Binding	1548.8

Summary Report For Furnace #3

Target Cell (Min.)

Cell	Name	Original Value	Final Value
E6	[Total Cost per Batch]	$ 218.13	$ 217.97

Adjustable Cells

Cell	Name	Original Value	Final Value
C12	L. P. SAND [Total Cost per Ton]	3625.0	3625.6
C13	OKLAHOMA SAND [Total Cost per Ton]	0.0	0.0
C14	LIMESTONE [Total Cost per Ton]	1030.0	1031.6
C15	SODA ASH [Total Cost per Ton]	1288.0	1286.6
C16	NEPHELINE SYENITE [Total Cost per Ton]	357.0	356.4
C17	B. H. CALUMITE [Total Cost per Ton]	0.0	0.0
C18	MELITE [Total Cost per Ton]	4.0	5.6
C19	IRON PYRITE [Total Cost per Ton]	0.0	0.0
C20	SALT CAKE [Total Cost per Ton]	16.0	16.0
C21	ASBURY GRAPHITE [Total Cost per Ton]	9.0	3.9
C22	MIXED CULLET [Total Cost per Ton]	721.0	719.5
C27	AMBER CULLET [Total Cost per Ton]	1683.0	1679.7

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
E110	Batch Weight [must be <=0]	0.0000	E110<=0	Binding	0.0000
F110	Batch Weight [must be >=0]	34.9693	F110>=0	Not Binding	34.9693
E111	Percentage of Fusion Loss [must be <=0]	0.0000	E111<=0	Binding	0.0000
F111	Percentage of Fusion Loss [must be >=0]	4.0728	F111>=0	Not Binding	4.0728
E112	Glass Equivalence (in tons) [must be <=0]	-0.0011	E112<=0	Not Binding	0.0011
F112	Glass Equivalence (in tons) [must be >=0]	0.0144	F112>=0	Not Binding	0.0144
E114	Salt Cake per Ton of Sand [must be <=0]	0.0000	E114<=0	Binding	0.0000
F114	Salt Cake per Ton of Sand [must be >=0]	64.0203	F114>=0	Not Binding	64.0203
E115	Salt Cake per Ton of Glass [must be <=0]	-49.8537	E115<=0	Not Binding	49.8537
F115	Salt Cake per Ton of Glass [must be >=0]	14.0668	F115>=0	Not Binding	14.0668
E116	Salt Cake per 5500 pounds of Batch [must be <=0]	-247.3051	E116<=0	Not Binding	247.3051
F116	Salt Cake per 5500 pounds of Batch [must be >=0]	16.1679	F116>=0	Not Binding	16.1679
E117	Sulfur Input per Ton of Glass [must be <=0]	-41.0103	E117<=0	Not Binding	41.0103
F117	Sulfur Input per Ton of Glass [must be >=0]	0.0000	F117>=0	Binding	0.0000
E118	Percentage of Mixed Cullet [must be <=0]	0.0000	E118<=0	Binding	0.0000
F118	Percentage of Mixed Cullet [must be >=0]	2.8837	F118>=0	Not Binding	2.8837
E119	Percentage of Amber Cullet [must be <=0]	0.0000	E119<=0	Binding	0.0000
F119	Percentage of Amber Cullet [must be >=0]	6.7321	F119>=0	Not Binding	6.7321
E120	Percentage Si02 [must be <=0]	-0.2790	E120<=0	Not Binding	0.2790
F120	Percentage Si02 [must be >=0]	0.8310	F120>=0	Not Binding	0.8310
E121	Percentage Al203 [must be <=0]	-0.7206	E121<=0	Not Binding	0.7206
F121	Percentage Al203 [must be >=0]	6.9881	F121>=0	Not Binding	6.9881

Summary Report For Furnace #3

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
E122	Percentage Fe203 [must be <=0]	-1.6188	E122<=0	Not Binding	1.6188
F122	Percentage Fe203 [must be >=0]	0.0000	F122>=0	Binding	0.0000
E123	Percentage Ca0 [must be <=0]	-1.7798	E123<=0	Not Binding	1.7798
E124	Percentage MgO [must be <=0]	-0.2185	E124<=0	Not Binding	0.2185
E125	Percentage BaO [must be <=0]	0.0000	E125<=0	Binding	0.0000
E126	Percentage Na20 [must be <=0]	-0.4450	E126<=0	Not Binding	0.4450
E127	Percentage K2O [must be <=0]	-0.1185	E127<=0	Not Binding	0.1185
E128	Percentage SO3 [must be <=0]	-0.3850	E128<=0	Not Binding	0.3850
E129	Percentage Sulfur [must be <=0]	-0.0008	E129<=0	Binding	0.0000
E130	Percentage MnO [must be <=0]	-0.0432	E130<=0	Not Binding	0.0432
E131	Glass Density [must be <=0]	-12.8710	E131<=0	Not Binding	12.8710
E132	Melt Viscosity [must be <=0]	-13082.59	E132<=0	Not Binding	13082.59
E133	Gob Viscosity [must be <=0]	-11027.38	E133<=0	Not Binding	11027.38
E134	Softening Point [must be <=0]	-4096.50	E134<=0	Not Binding	4096.50
E135	Conveyor Viscosity [must be <=0]	-5785.83	E135<=0	Not Binding	5785.83
E136	Annealing Point [must be <=0]	-5331.03	E136<=0	Not Binding	5331.03
E137	Expansion Coefficient [must be <=0]	0.00	E137<=0	Binding	0.00
E138	Cooling Time [must be <=0]	0.00	E138<=0	Binding	0.00
E139	Liquid Temperature [must be <=0]	-11447.76	E139<=0	Not Binding	11447.76
F123	Percentage Ca0 [must be >=0]	1.4751	F123>=0	Not Binding	1.4751
F124	Percentage MgO [must be >=0]	0.0744	F124>=0	Not Binding	0.0744
F125	Percentage BaO [must be >=0]	0.0779	F125>=0	Not Binding	0.0779
F126	Percentage Na20 [must be >=0]	1.8031	F126>=0	Not Binding	1.8031
F127	Percentage K2O [must be >=0]	0.3445	F127>=0	Not Binding	0.3445
F128	Percentage SO3 [must be >=0]	0.0082	F128>=0	Not Binding	0.0082
F129	Percentage Sulfur [must be >=0]	0.0801	F129>=0	Not Binding	0.0801
F130	Percentage MnO [must be >=0]	0.0347	F130>=0	Not Binding	0.0347
F131	Glass Density [must be >=0]	6.42	F131>=0	Not Binding	6.42
F132	Melt Viscosity [must be >=0]	7203.8885	F132>=0	Not Binding	7203.8885
F133	Gob Viscosity [must be >=0]	5622.26	F133>=0	Not Binding	5622.26
F134	Softening Point [must be >=0]	0.00	F134>=0	Binding	0.00
F135	Conveyor Viscosity [must be >=0]	3191.73	F135>=0	Not Binding	3191.73
F136	Annealing Point [must be >=0]	2635.99	F136>=0	Not Binding	2635.99
F137	Expansion Coefficient [must be >=0]	703.03	F137>=0	Not Binding	703.03
F138	Cooling Time [must be >=0]	694.54	F138>=0	Not Binding	694.54
F139	Liquid Temperature [must be >=0]	2987.06	F139>=0	Not Binding	2987.06
C12	L. P. SAND [Total Cost per Ton]	3625.6	C12>=0	Not Binding	3625.6
C13	OKLAHOMA SAND [Total Cost per Ton]	0.0	C13=0	Binding	0.0
C14	LIMESTONE [Total Cost per Ton]	1031.6	C14>=0	Not Binding	1031.6
C15	SODA ASH [Total Cost per Ton]	1286.6	C15>=0	Not Binding	1286.6
C16	NEPHELINE SYENITE [Total Cost per Ton]	356.4	C16>=0	Not Binding	356.4
C17	B. H. CALUMITE [Total Cost per Ton]	0.0	C17=0	Binding	0.0
C18	MELITE [Total Cost per Ton]	5.6	C18>=0	Not Binding	5.6
C19	IRON PYRITE [Total Cost per Ton]	0.0	C19>=0	Not Binding	0.0
C20	SALT CAKE [Total Cost per Ton]	16.0	C20>=0	Not Binding	16.0
C21	ASBURY GRAPHITE [Total Cost per Ton]	3.9	C21>=0	Not Binding	3.9
C22	MIXED CULLET [Total Cost per Ton]	719.5	C22>=0	Not Binding	719.5
C27	AMBER CULLET [Total Cost per Ton]	1679.7	C27>=0	Not Binding	1679.7