ON THE DETERMINATION OF MAGNETIC SUSCEPTIBILITIES OF ROCKS IN SITU

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INTRODUCTION

In geophysical prospecting by magnetic methods it is ordinarily advantageous and often essential to have a knowledge of the values of the magnetic susceptibilities of the rocks and formations present in the area under survey. To this end the usual procedure has been to take the geological samples in the field and to measure their volume susceptibilities in the laboratory, by means of one of the numerous methods available. In none of these methods is the measuring instrument entirely self contained or suited for use in the field.

In most cases the sample is pulverized; in some it is machined to shape to fit certain coils or instruments, and in one method, (that of C. A. Heiland), a cast of the sample is made and the susceptibility of the rock compared with that of a material of known susceptibility of the same size and shape, made from the cast of the original sample.

Some writers assert that the difference in susceptibility of a rock sample in the powdered and solid forms is negligible, and while this is doubtless true in many cases, there are other cases where the difference becomes appreciable. Again, the susceptibility obtained in the laboratory from a small specimen is liable to differ from that of the rock from which it was taken due to the

stresses involved by the hammering or other forces used to obtain the sample. In susceptibility measurements of a sample of granite, Wilson[1] found that an individual specimen had about the same susceptibility in the solid as in the pulverized form, but that tests on pieces from different parts of the rock varied as much as 750% due to the unequal distribution of magnetite in the rock.

It is obvious that the most accurate value would be that obtained from averaging a number of tests on the undisturbed rock instead of from one or two samples. No such procedures have been published to date. It is the purpose of this paper to describe a method by which the susceptibilities of rocks may be determined in place by means of a portable instrument.

CHOICE OF METHODS

It was decided to use a method employing alternating current. The use of alternating current has the advantage over direct current and magnetometric methods that it is free from extraneous magnetic disturbances.

The principle involved, as in other inductive methods, is the change in inductance of a coil due to the presence in its magnetic field of a paramagnetic or diamagnetic medium. In this case, however, the windings of the coil do not enclose any part of the rock under test. Instead, the coil is circular, having small cross-sectional dimensions as compared to its radius and consists of several hundred turns of fine wire. It is lain flat on the rock. Thus only part of its magnetic field penetrates the rock under test. The relation between the change in field strength or inductance and the susceptibility of the medium for this case will be developed later.

To measure the change in the inductance of the coil due to the presence of a magnetic material, three methods suggested themselves:

(1) An alternating current inductance bridge.

(2) The use of a beat frequency oscillator whose frequency is altered by the change in inductance of the coil.

(3) The change of the resonant point of an oscillating circuit with the change in inductance of the coil.

Since time was a factor, and only one method could be investigated, an alternating current bridge was chosen. It required the least amount of equipment, was not dependent on a fixed frequency source, and seemed therefore the most portable.

DESCRIPTION OF APPARATUS

Of the many alternating-current bridge methods available, (see Hague [2] for example), the one shown diagramatically in Figure 1 on page 6 was chosen on account of its simplicity and sensitivity, and the fact that a current source of fixed frequency was not required. This will be recognized as Owen's bridge [3]. Referring to the figure, LP_1 is the test coil previously mentioned, which is connected to the bridge by about eight feet of bifilar flexible wire. It is 9.4 cm mean diameter and contains 1000 turns of #31 enamelled wire with a resistance of 139.1 ohms. The cross-sectional dimensions are 1 cm by 0.5 cm. Accurate measurements gave its inductance as 0.17995 henrys. If the coil is symmetrically wound in regular layers its inductance may be closely calculated by several formulae, to which but two references will be given, Fleming [4] and Rosa [5]. A larger diameter coil than the experimental one is recommended for field work. The smaller one was used for the experimental work to cut down on the size of the samples needed.

P_2 and Q are standard variable resistances reading in ohms up to 5000 ohms. The dial type is preferable to the plug type on account of the greater ease of manipulation. Each has a vernier reading to at least one-tenth ohm.

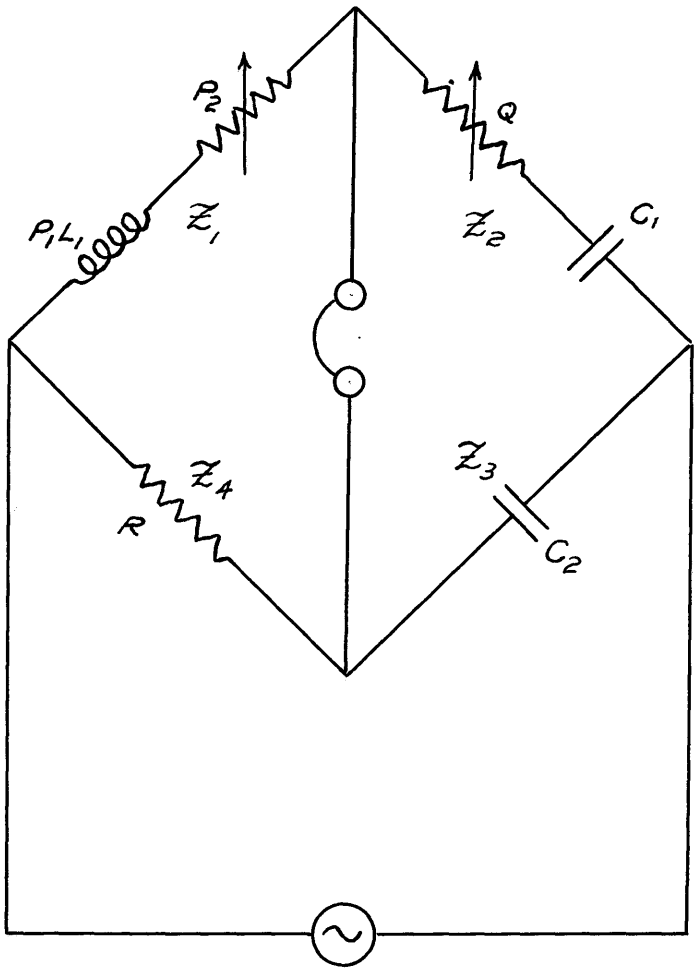

Figure 1. Diagram of Owen's Inductance Bridge.

C_1 and C_2 are fixed standard condensers of values 0.13 and 0.06 mfd respectively. As it was decided to use a frequency of 500 cycles, and branch impedances of 5000 ohms, (see the chapter on Experimental Work), these values of capacity were calculated to give 5000 ohms impedance. The nearest values avaliable were then used. R is a fixed standard resistance of 5000 ohms.

The source of current may be a buzzer, a maintained tuning fork, a motor-driven alternator, or a vacuum tube oscillator. The first two sources mentioned both emit a loud note and the device must be placed some distance away and in a sound proof box, so that the direct note will not be heard if telephones are to be used for detecting the point of balance. Also the buzzer has a rather impure wave form. Motor-driven alternators give a satisfactory wave form and are suitable for low frequencies but are usually too cumbersome for use in the field. For frequencies above say, 200 cycles, the vacuum tube oscillator is undoubtedly the most suited for precise tests. It gives an adequate output at a steady frequency and with a good wave form. A variable frequency vacuum tube oscillator was used in the experiments, so that the effects of different frequencies could be investigated.

Whatever the source chosen, it is advisable to connect

it to the bridge through a small transformer. By providing an earthed screen between the two windings, capacity effects between the source and bridge are minimized. It is also well to earth such parts as one side of the amplifier for example, to make the whole instrument more stable. It was found that wiring the parts in such a manner as to reduce the internal capacities between wires and units also helped in the performance of the bridge.

For detecting the point of balance, headphones were used, connected through a two stage transformer-coupled amplifier which happened to be available. (This amplifier is not shown in the circuit). Three stages of resistance coupling would probably be better, and a low pass filter is recommended to cut out harmonics and thus obtain a better null point. With a constant frequency source of not too high a value, a vibration galvonometer is satisfactory. Indeed, for out of doors work, where the noise of wind, traffic, etc. may cause the use of headphones to be impracticable, it may be found necessary to use a vibration galvonometer. These may be made in a very small size.

In using the bridge, balance is obtained by varying only the resistances P_2 and Q. This is easy, as the two adjustments for balance are independent of each other. In the circuit shown in Figure 1, the branch impedances are:

$z_1 = P + j\omega L$ where $P = P_1 + P_2$

$z_2 = Q - \frac{j}{\omega C_1}$

$z_3 = -\frac{j}{\omega C_2}$

$z_4 = R$

At balance,

$z_1 z_3 = z_2 z_4$

or

$$(P + j\omega L)(-\frac{j}{\omega C_2}) = (Q - \frac{j}{\omega C_1}) R \quad -----(1)$$

Separating reals and imaginaries

$$C_1 P = C_2 R \quad -----(2)$$

and

$$L = R Q C_2 \quad -----(3)$$

In practice, the inductance of the test coil in air is first measured, and then its inductance when lying flat on the face of the rock whose susceptibility is to be determined. The difference in the two values obtained is a measure of the susceptibility.

THEORY

The relation between the change in inductance or flux of the coil and the permeability or susceptibility of the medium on which it is laid is somewhat complex, and the writer knows of no article dealing directly with this subject. However, an outline of one method which may prove of use will be given. It is used in the calculation of magnetic fields in electrical machines and involves the determination of the permeance of field paths. This method is referred to by Karapetoff [6] but insufficient data are given by him.

The field due to a simple current or element of current at the interface of two media is shown approximately in Figure 2 on page 11. The lines of force will be closer together in the medium of higher permeability. The general principle applies that the lines of force and the equipotential surfaces assume such shapes and directions that the total permeance becomes a maximum, or the reluctance a minimum.

Since the permeance, $\mathcal{P}$, and the permeability, μ, are connected by the relation $\mathcal{P} = \frac{\mu A}{l}$, where A is the area and l the length of the path, the permeance is measured from the distribution of the field and the permeability then found. Due to mathematical difficulties, however, analytical calculation of permeances is feasible only in the simplest cases.

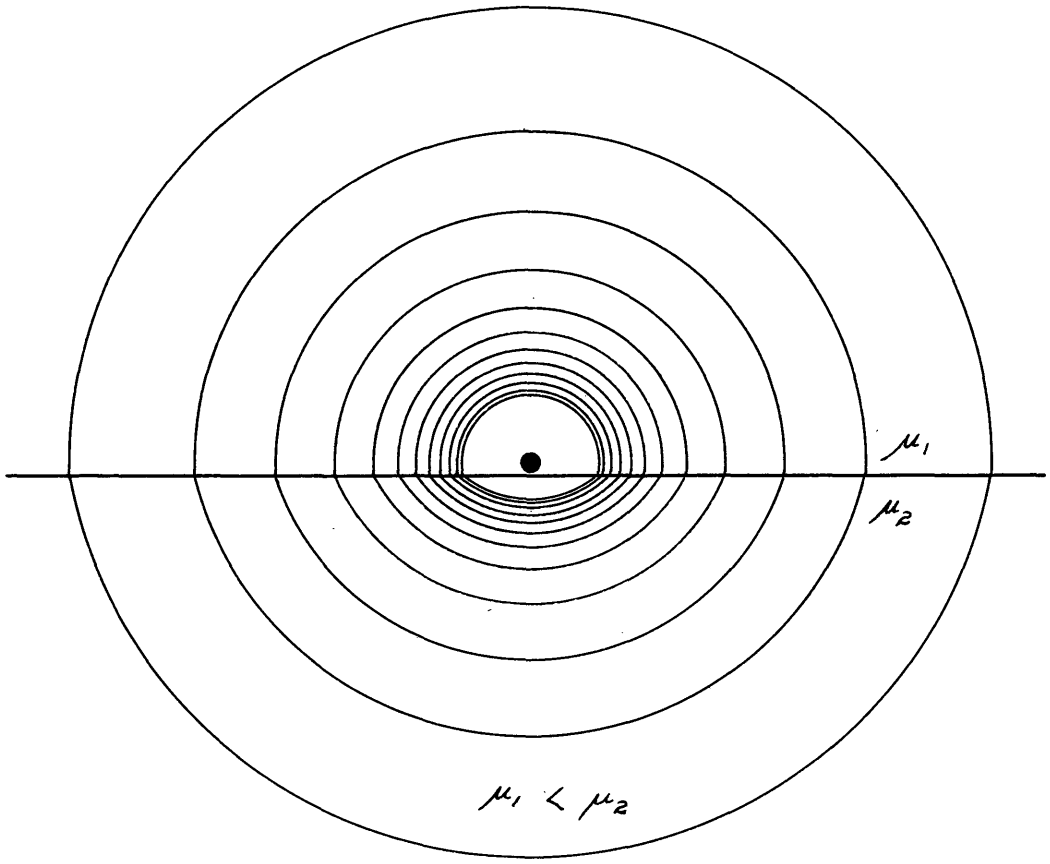

Figure 2. Magnetic Field due to a straight current at an interface.

In practical three-dimensional problems recourse must be had to a graphical method of trial and approximation in order to obtain the maximum permeance. Procedures have been worked out by Lord Rayleigh and Lehmann [7] for laying out a field. The general method is to map the field into small cells by means of lines of force and equipotential surfaces, using one's own judgment. The total permeance is then calculated by properly combining the permeances of the cells in series and parallel. Then, the assumed directions are modified a little and the permeance calculated again, until by successive trials, the positions of the lines of force are found for which the permeance becomes a maximum.

Consider a single loop of wire. The electromagnetic energy possessed by the loop at time t during the building up of flux is

$$dW = i_t e_t \, dt$$

$$dW = i_t \frac{d\phi_t}{d\phi} dt$$

$$dW = i_t \, d\phi_t \qquad \text{- - - - - - - - - - (4)}$$

where W is in joules, i_t and ϕ_t are the instantaneous values of the current in amperes and the flux in webers at the time t, and $e_t = \frac{d\phi_t}{dt}$ is the instantaneous e.m.f. The total energy supplied from the electrical source to build up the field to its final value W is

$$W = \int_0^i i_t \, d\phi_t \qquad \text{- - - - - - - - - - (5)}$$

In a medium of constant permeability, $\phi_t = \mathcal{P} i_t$, where $\mathcal{P}$ is the permeance of the magnetic circuit in henrys. We may obtain any one of the three following expressions for the electromagnetic energy stored in the loop:

$$W = \tfrac{1}{2} i \phi \qquad \text{- - - - - - - - - - - - (6)}$$

$$W = \tfrac{1}{2} i^2 \mathcal{P} \qquad \text{- - - - - - - - - - - - (7)}$$

$$W = \tfrac{1}{2} \phi^2 / \mathcal{P} \qquad \text{- - - - - - - - - - - - (8)}$$

Equation (6) expresses the fact that the magnetic energy stored in a loop is equal to one-half the product of the flux and the current. Equation (7) shows that the stored energy is proportional to the square of the current and to the permeance of the magnetic circuit.

Taking the more general case of a coil of n turns, we have partial as well as complete linkages. Partial linkages means that the flux which links with part of the turns is now of a magnitude comparable with that of the flux which links with all the turns of the coil. Considering first the complete linkages, or the energy due to the flux which links with all the turns of the coil, we find that, by repeating the reasoning given above for a single loop,

$$W_c = \tfrac{1}{2} n^2 i^2 \mathcal{P}_c \qquad \text{- - - - - - - - - - - - (9)}$$

The energy of the partial linkages is calculated in a

similar manner and may be shown to be: (8)

$$W_p = \tfrac{1}{2} i^2 \Sigma n_p^2 \Delta \mathcal{P}_p \qquad \text{- - - - - - - - - - (10)}$$

and the total energy of the coil is

$$W = \tfrac{1}{2} i^2 \left(n^2 \mathcal{P}_c + \Sigma n_p^2 \Delta \mathcal{P}_p \right) \qquad \text{- - - - - - - - - - (11)}$$

Assuming that there is no mutual inductance,

$$W = \tfrac{1}{2} L i^2 \qquad \text{by definition,}$$

where L is the inductance in henrys,

or

$$L = 2 W / i^2$$

$$= n^2 \mathcal{P}_c + \Sigma n_p^2 \Delta \mathcal{P}_p \qquad \text{- - - - - - - - - -(12)}$$

or replacing the summation by an integral,

$$L = n^2 \mathcal{P}_c + \int_o^n n_p^2 \, d\mathcal{P}_p \qquad \text{- - - - - - - - - - (13)}$$

If we replace the actual coil by a fictitious coil of equal inductance and the same number of turns, but without partial linkages, we may write

$$L = n^2 \mathcal{P} \qquad \text{- - - - - - - - - - (14)}$$

$$= n^2 \frac{\mu A}{l} \qquad \text{- - - - - - - - - - (15)}$$

The above theory assumes that the permeability is constant. In the case under consideration however, approximately

one-half of the field is in air and the other half in a medium of permeability μ . It is therefore necessary to calculate separately the permeance for the part of the field that is in air, and that for the part in the medium, to arrive at a correct value for the inductance L.

Permeability may be changed to susceptibility by the relation

$$\mu = 1 + 4\pi k$$

where k is the volume susceptibility in C.G.S. units.

EXPERIMENTAL WORK

The bridge as described was set up and connected to a variable frequency oscillator so that any stray linkages were a minimum.

Experimenting was first carried out to determine the influence of change in frequency, and of capacity and electrical conductivity of the samples. The effect of different branch impedances was also investigated, and the value which gave the most sensitive response to the bridge was found to be around 5000 ohms, which was the internal impedance of the oscillator output.

Frequency was shown to be immaterial to at least 2000 cycles by running tests up to that frequency with a small piece of iron in the coil. Brown (9) has compiled much data on this subject and has shown that, in the case of properly laminated iron, the permeability may remain constant, at the value of that in stationary fields, at frequencies up to one million. Since the frequency was not critical, 500 cycles was chosen chiefly because it gave a good note in the headphones used with the bridge.

To investigate the effects of any capacity in the sample, measurements of the inductances of the coil when in air and when suspended over the surface of a body of distilled water were made. No appreciable change in inductance was observed.

In a similar test with a saturated solution of sodium chloride of high electrical conductivity, the change in inductance was so small as to be within the working error of the instrument. Since the values of the dielectric constant and electrical conductivity in these two samples are well above any which would ordinarily be encountered in the field, they may be safely ignored. This is in accordance with experiments carried out by Barret.(10)

Tests were made on some ferric chloride solutions and on blocks of moulding clay and sand containing evenly distributed amounts of iron powder. The susceptibilities of these samples were known.

All the samples were made about the same size. With a thickness of 18 cm, the field strength at the far face of the specimen was about 2% of that of the center of the coil. As this small error was constant it was neglected entirely. See Figure 6 on page 28.

In the case of the ferric chloride solutions, Koenigsberger(11) gives the following relation between the percentage of Fe_2Cl_6 and the volume susceptibility of the solution,

$$K \cdot 10^6 = (88.78\ p - 0.78) / d \qquad \text{- - - - - - - - - (16)}$$

where p is the percentage of Fe_2Cl_6 and d is the density of the solution.

The Smithsonian Physical Tables, 8th Edition, 1933, p. 475 also gives a relation between the percentage of $FeCl_3$

and the mass susceptibility as follows,

$$K = (p/100)H + (1 - p/100)H_0 \quad - - - - - - - - - (17)$$

where H is the mass susceptibility of pure $FeCl_3$, given as $90 \cdot 10^{-6}$, and H_0 is the mass susceptibility of water, given as $-0.79 \cdot 10^{-6}$.

To obtain the volume susceptibility, the above must be divided by the density of the solution used, which may be found in most chemical handbooks.

Susceptibilities obtained by the above two formulae were found to agree within less than 2%.

The susceptibilities of the clay and sand samples were found by a method developed by C. A. Heiland, which follows.

Two identical solenoids, about 3.2 cm radius and 16.6 cm long were placed vertically 15 cm north and south of a vertical intensity magnetometer of known scale value, so that a line joining their centers passed through, and at right angles to, the center of the magnetic system. The solenoid windings were connected in opposition to a battery and rheostat so that, when the two coils were properly centered and current sent through them, there was no deflection of the balance. However when a paramagnetic sample is inserted in one of the solenoids, a stronger field is induced in it than is present in the one on the opposite side of the magnetometer, the effect being that of placing a magnet there

in the third position of Gauss, which will cause the magnet system to be deflected.

Then,

$I = M/V$ where M is the magnetic moment

$M = IV$ I the intensity of magnetization

V the volume of the sample

$I = kH$ K the magnetic susceptibility

$M = kHV$ H the field strength in the sample

In the third position of Gauss, the field due to a magnet is

$$H = M/r^3 = \Delta Z = \epsilon \cdot \Delta s$$

$\Delta Z = kHV/r^3$ r is the distance from the center of the magnet

$\epsilon \cdot \Delta s = kHV/r^3$ ΔZ the change in field strength

ϵ the scale value of the instrument

Δs the change in scale reading

whence,

$$k = \epsilon \Delta s r^3/HV \qquad \text{- - - - - - - - - - - - - - - (18)}$$

and $$I = \epsilon \Delta s r^3/V \qquad \text{- - - - - - - - - - - - - - - (19)}$$

The fields of the solenoids were calculated from the formula

$$H' = 0.4\pi N i/\ell \qquad \text{- - - - - - - - - - - - - - - (20)}$$

where H' is in Gauss

N the total number of turns

i the current in amperes

ℓ the length in cm

This takes no account of the demagnetizing factor. The true field, H_0, is

$$H_0 = H' - DI \qquad \text{- - - - - - - - - - - - - - - - (21)}$$

where I is the intensity of magnetization and D the demagnetizing factor. From the dimensions of the coil, (ℓ/d = 5.38), the Smithsonian Physical Tables, 1933, p. 470 gives 0.64 as the value of D.

As, however, the susceptibilities of the samples used were fairly low, this correction became negligible. For example, a specimen of susceptibility $2060 \cdot 10^{-6}$, had an intensity of magnetization of 0.206 c.g.s. units at a maximum field strength of 100 Gauss.

Then,

$$H_0 = H' - DI$$

$$= 100 - 0.64 \times 0.206$$

$$= 100 - 0.13$$

$$= 99.87$$ a difference of only 1.3 in 1000 at 100 gauss. (This sample had the highest susceptibility of any used).

To determine how the calculated values of susceptibility

compared to those found by the deflection magnetometer. × investigations were made at various field strengths with solutions of 30% and 50% ferric chloride. $FeCl_3$ was used and the calculated values were obtained from equation (17). The observed and calculated values at a field strength of 100 gauss follow. Susceptibilities are in 10^{-6} c.g.s. units.

	k obs.	k calc.	Δk.
30% $FeCl_3$	15.6	16.1	+ 0.5
50% $FeCl_3$	29.6	28.9	- 0.7

The results are shown graphically in Figure 3 on page 22 using an exaggerated vertical scale.

Figure 4 on page 23 shows a series of I-H curves obtained with this instrument. Readings were taken at every 10 Gauss up to 100 Gauss.

The table below shows the computations for curve No. 3 in Figure 4, it being typical of all the others.

H in Gauss	Field at balance Gammas	Field at balance Gauss	I from Eq. (19)	$k \cdot 10^6$ Eq. (18)
10	32	0.00032	0.01000	1000
20	66	0.00066	0.02060	1030
30	106	0.00106	0.03310	1103
40	150	0.00150	0.04680	1170
50	193	0.00193	0.06020	1204
60	232	0.00232	0.07250	1208
70	276	0.00276	0.08610	1230
80	319	0.00319	0.09950	1243
90	362	0.00362	0.11310	1256
100	405	0.00405	0.12680	1263

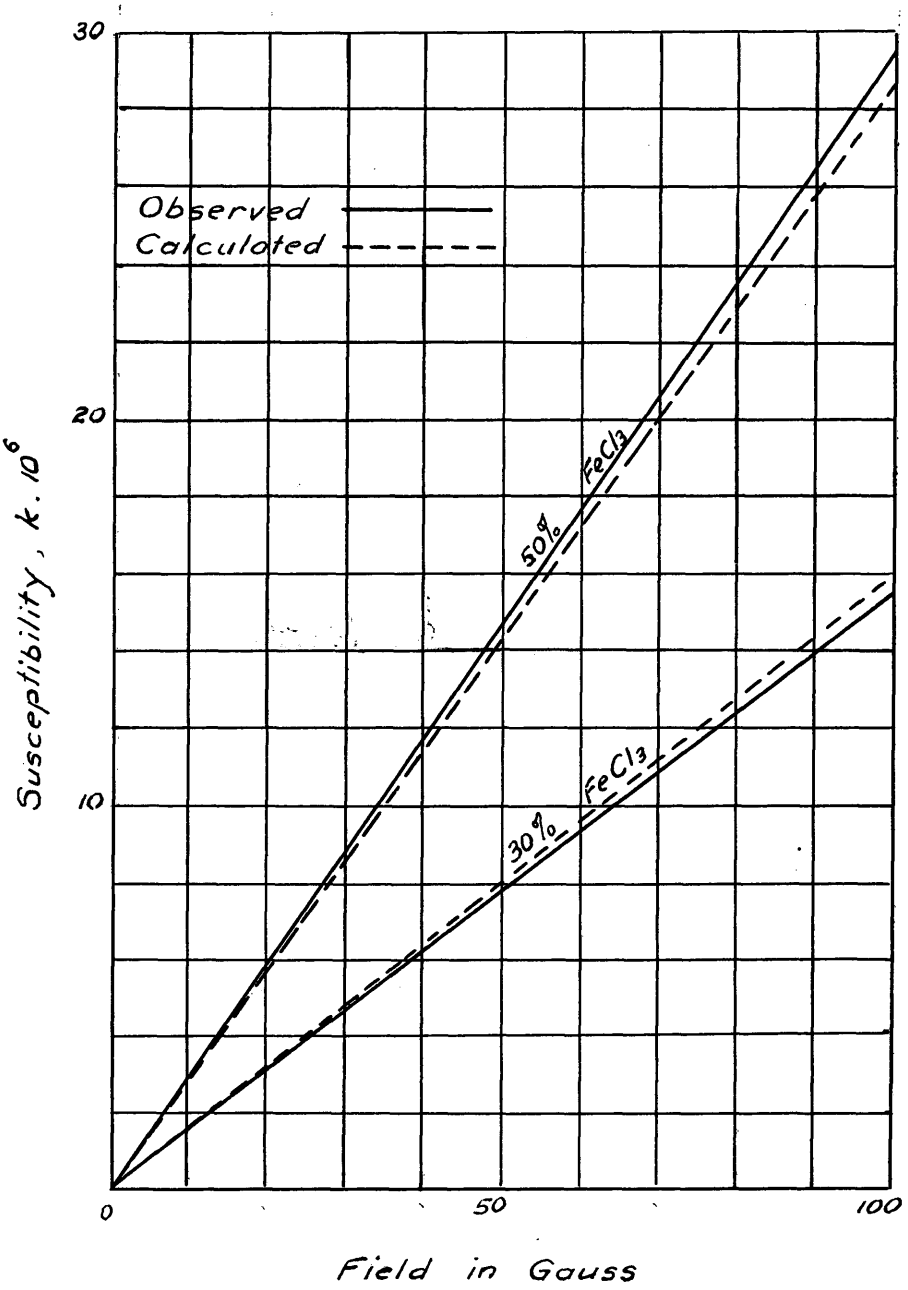

Figure 3. Calculated and observed values of k for Ferric Chloride solutions.

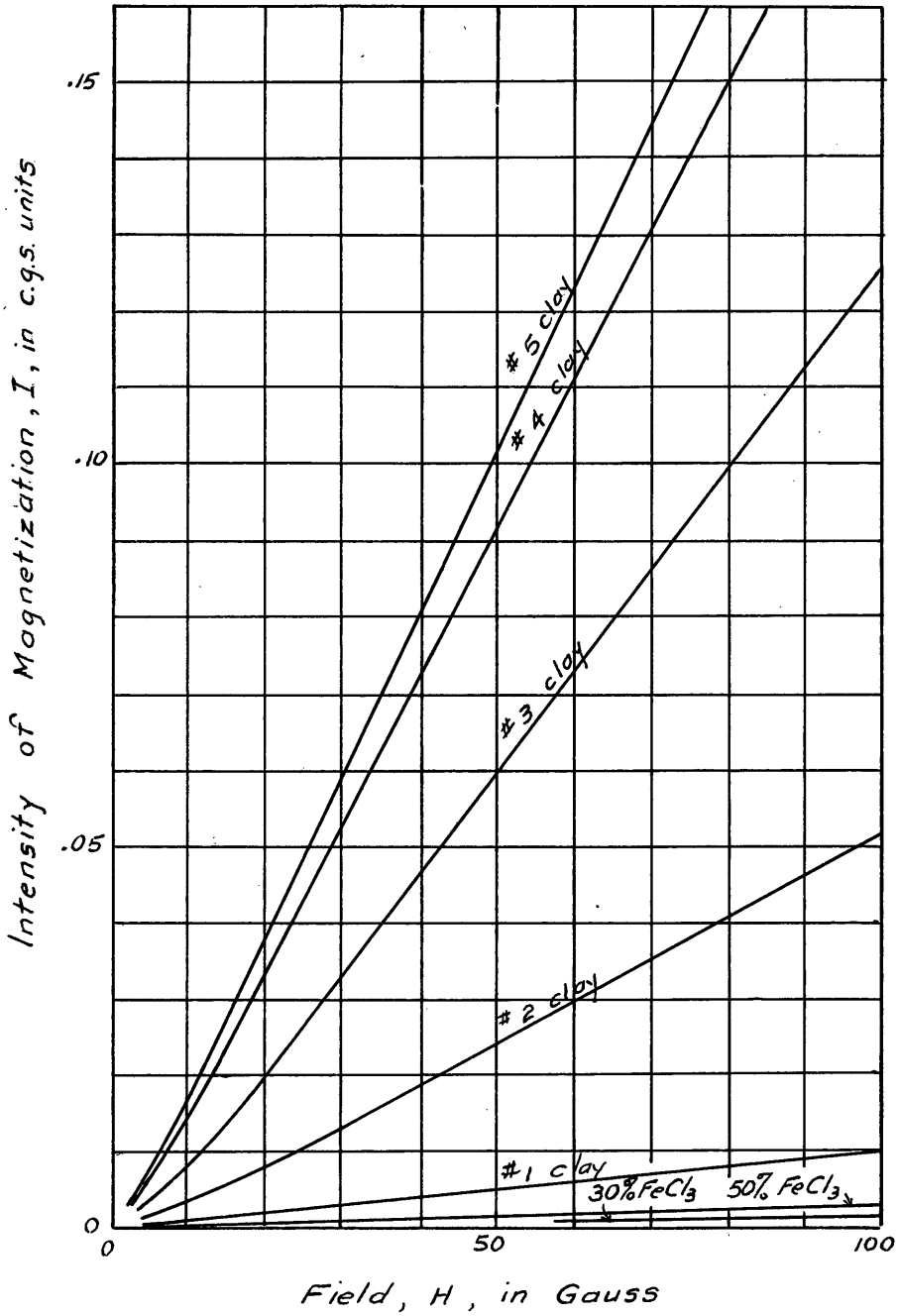

Figure 4. I-H Curves.

It will be noticed that there is a slight bend at the bottom of each curve, which otherwise is linear. Since the field strength of the search coil used in the experiments was only of the order of four Gauss, the susceptibilities of the samples were taken at this value when plotting the calibration curve, Figure 5, page 25. This will necessitate keeping the current in the bridge constant, and specifying at what field the susceptibilities were measured; in this case four Gauss.

In the tests made on the specimens with the bridge, the search coil was first laid on a wooden stool free from iron, and it was made sure that its inductance was the same as if it were suspended freely in air. The inductance, L_1, with the coil in air was determined from the equation of the bridge

$$L_1 = RQC_2 \qquad \text{- - - - - - - - - - - - - - - (3)}$$

Next, the sample was placed on the stool and the coil laid flat on its surface. Several readings were taken with the coil reversed on the block and the block also reversed, and the mean of these values used in the calculation of L_2 in the same way that L_1 was calculated. The difference, ΔL in the inductances of the coil, when laid flat on a magnetic

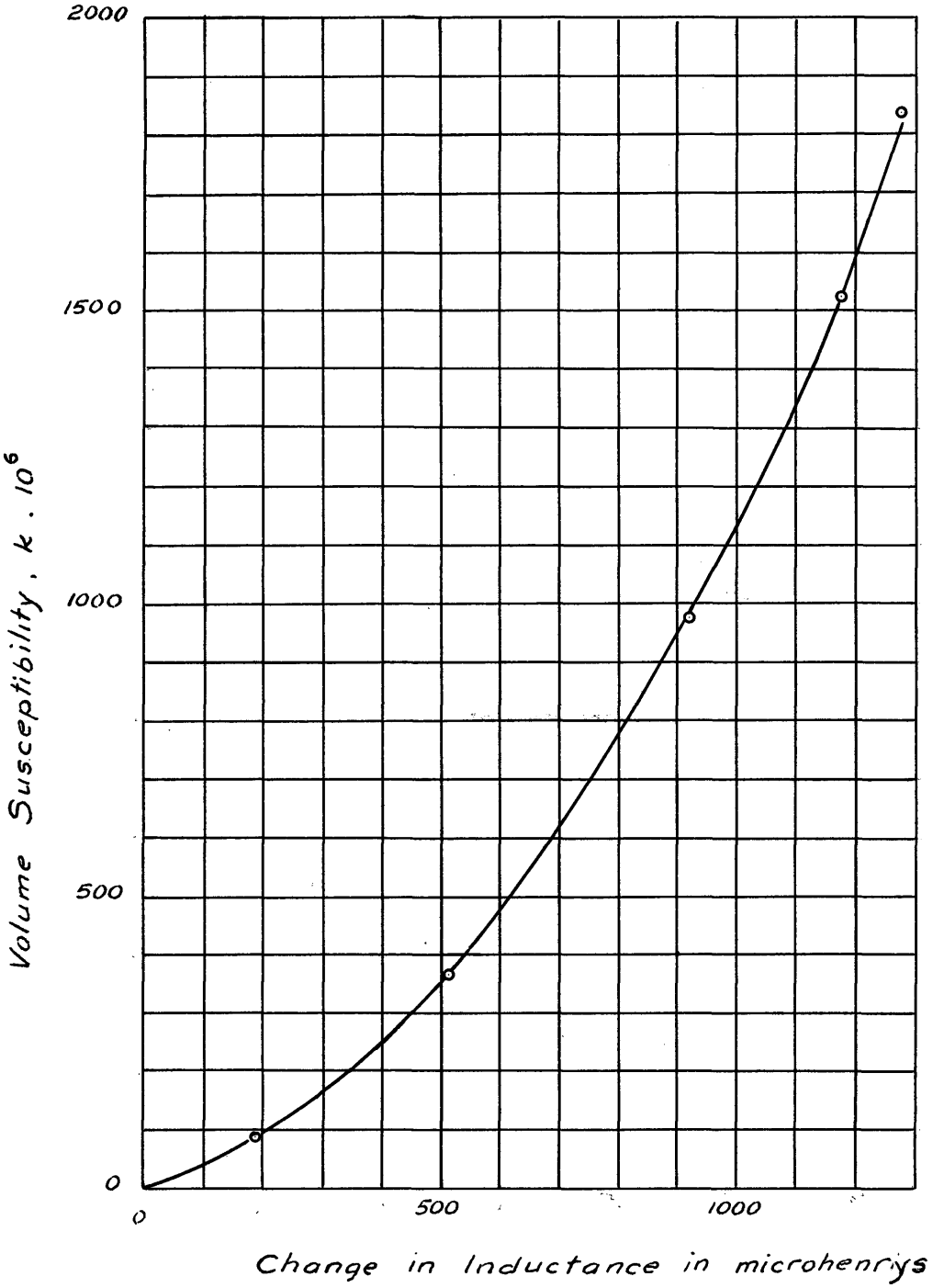

Figure 5. Calibration Curve

medium and when in air is thus found. Care was exercised to see that the inductance of the coil came back to the normal value for air after readings had been taken with a sample present. The inductance was found for each value of known susceptibility, from which the calibration curve, Figure 5, on page 25 was constructed.

Thus, to find the change in inductance caused by, say, sample No. 2, the bridge was balanced first in air. At the point of balance, the following values were obtained

$$R_1 = 5000 \text{ ohms}$$

$$Q_1 = 3599 \text{ ohms}$$

$$C_2 = 0.01 \text{ microfarads}$$

From equation (3), L_1 was found to be 0.179950 henrys.

Then with the coil placed flat on the sample, Q had to be changed to 3609.3 ohms to again balance the bridge. The new inductance L_2 was calculated in the same manner and was found to be 0.180460. The difference, ΔL, between L_1 and L_2, was then 0.000510 henrys or 510 microhenrys.

In the calibration curve showing the susceptibility, k, reduced to a field of four Gauss, is plotted against the change in inductance, ΔL, of the search coil from its zero value. Since the inductance is directly proportional to the resistance Q however, the latter may, if wished be plotted against k and a certain amount of labor eliminated.

In the field work it is of course necessary to have the coil flat on the rock at all times. If this is not so, a part of the magnetic field will not pass through the rock, resulting in a smaller value of k than the correct one being recorded. The value of the field strength at the center of a circular coil of n turns and radius r cm is

$$H = 2\pi ni/r \qquad \text{- - - - - - - - - - - - (22)}$$

The field along the axis of the coil drops off rapidly with an increase in distance from the coil. At a distance x from its center

$$H = 2\pi nr^2 i/(x^2 + r^2)^{\frac{3}{2}} \qquad \text{- - - - - - - (23)}$$

This relation is shown graphically in Figure 6 on page 28 where the ratio of the distance x and the radius r is plotted against the percentage of H at the center of the coil. It is seen that at a distance from the coil equal to three times its radius, the magnetic field has dropped to about 3% of the maximum value. Thus, the larger the coil, the greater is the volume of rock under test. This may be important in the case of say a not too compact rock containing magnetite, where weathering may have altered the magnetite to hematite or limonite for a short distance from the surface. It is best therefore to use as large a coil as is practicable.

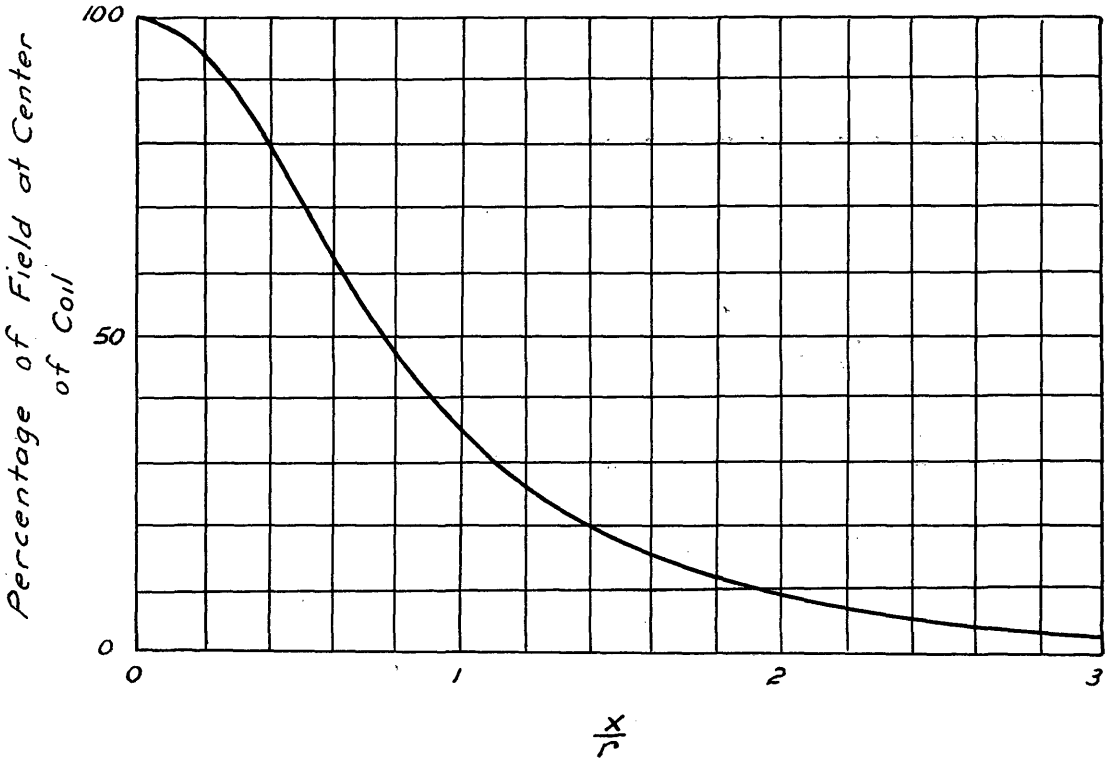

Figure 6. Variation of Field Strength with Distance from Center of Circular Coil.

CONCLUSIONS

It is believed that the best values of susceptibilities of rocks and formations are those determined in the field by readings taken on the undisturbed rock. A method for doing this by the use of an alternating current bridge has been devised and shown to be practicable. Integral parts of the instrument may, after experimentation has determined their best values, be replaced by standards of those values having small dimensions, so that the complete instrument will be small, compact, and readily portable.

It is to be regretted that time was not available to try out more than one method. Some modification of the present bridge, or the use of one of the other methods mentioned earlier in the paper, may result in a circuit which would be still more sensitive and yet have the same stability and simplicity of operation as the bridge used. It is suggested that there is room for further research on this point.

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