15/05/008/001

Minutes of the Tenth Regular Meeting of the SHALE OIL TECHNICAL ADVISORY COMMITTEE held at Sinclair Oil & Gas Regional Office Denver, Colorado on January 17, 1966

Those present for the Committee were:

W. L. Jensen	Continental Oil Company
J. H. Smith	Continental Oil Company
W. H. Decker	Sinclair Research, Inc.
R. T. Ellington, Chairman	Sinclair Oil & Gas Company
D. C. Smith	Phillips Petroleum Company
R. Charles	Phillips Petroleum Company
N. P. Peet	Humble Oil & Refining Company
B. L. Schulman	Esso Research & Engineering Company
R. Mungen	Pan American Petroleum Corporation
K. L. Berry	Pan American Petroleum Corporation
S. L. Meisel	Socony Mobil Oil Company, Inc.
K. M. Elliott	Socony Mobil Oil Company, Inc.

Also attending were P. W. Snyder, J. E. Lawson and R. H. Cramer, Program Manager of the Anvil Points staff. The meeting was called to order at 8:40 a.m.

I. Old Business

The minutes of the Ninth TAC Meeting were approved in the form previously distributed.

Cramer reported that the next audit of the project will be carried out on February 14 by J. L. Pelej, Socony Mobil, and T. E. Davis, Humble Oil Company. The audit will cover the period from the last audit up to and including January 31, 1966.

At the last meeting question had been raised regarding the handling of patent cases in which there was valid co-inventorship and the individuals were from different Participating Parties. It was agreed that this situation could be handled relatively easy. Each inventor will file his concept with his own company in the normal manner with acknowledgment of the participation of his co-inventor. He will assign his share of the invention to his company in the normal manner. The actual filing and prosecution of the patent will then be carried out by one company with the agreement of the other. All of the contract provisions of the Project will apply so there will be no question as to the relative positions of the companies involved, or subsequent licensing of the patent.

II. New Business

Preliminary arrangements were made for the next TAC meeting with the consideration that they might be revised after report had been made on the experimental program and plans for the next phase of the experimental work were presented. The revised dates for the next Technical Observers' meeting are Tuesday, March 1, and Wednesday, March 2, 1966. The next TAC meeting will be held in Rifle on Wednesday, March 9, with the possibility of carry-over to Thursday, March 10, 1966.

Cramer opened his general remarks with the report that total project expenditures through December 31, 1965 amount to $2,185,624; they also have accrued costs amounting to $95,969 for a total cost of $2,281,593. This gives a variance with respect to the budget of minus $43,871. He reported that the main components of the overrun variance were $15,300 for mine road graveling and $33,000 for materials and supplies for retort modification. On this basis he estimated that the $2,720,000 defining Stage I of the Project will run out sometime during the third week in April, 1966.

In reporting personnel matters, Cramer reported that the CSM administrative manager had been replaced on November 26 and at the end of December, the staff consisted of 21 oil company people, with a total project staff of 106. Of the oil company personnel, 18 are technical people, including the three technical observers. Cramer next reported the presentations which had been made regarding an NLRB election of the CSM Research Foundation operating personnel and NLRB's ruling that it did have jurisdiction. The election was scheduled for January 21, 1966. The choice is between: the Oil Chemical and Atomic Workers; United Mine Workers, District 15; Operating Engineers International; or no union at all. The CSM Research Foundation has filed an appeal with NLRB regarding the election, but this has not been ruled upon. There was some discussion and conjecture regarding the factors, including safety, which might have caused the employees to seek the election. Cramer also reported that a copy of the Contracts had been put on file at the NLRB hearing to show the temporary nature of the Project. The question was then asked as to whether a union could force continuation of the Project beyond Stage I if the Parties voted to stop. Opinions will be sought.

To initiate the technical discussion, Cramer reviewed the objectives and conclusions of the experimental program for November and December and briefly discussed the major conclusions that can be made on the basis of the data obtained. The broad objective was to define and understand the Gas Combustion Retorting process in sufficient depth so that economic operation can be achieved in commercial retorts. The stated local objective was to obtain yields approaching 90% of Fischer Assay with 0.75 to 1.5-inch shale at a mass flow rate of 500 lb/hr-ft^2. The major hypothesis guiding the investigation was that kinetic effects governed by heat transfer control kerogen decomposition, coke and gas combustion, and carbonate decomposition. It is also hypothesized that the reaction sequence is complicated by dust and oil recirculation and that the oil recirculation is initiated and controlled by impaction of mist particles and condensation on the surface of downcoming dust and shale. He stated that they felt that the primary requirements for control of the kinetic effect are: (1) uniform heat supply, (2) minimization of dust accumulation in the retorting zone, and (3) sufficient retort zone residence time and temperature. When a run meets these conditions, yields close to 90% can be expected. It was acknowledged, however, that yields of the order of 105% of Fischer Assay might be obtainable if complicating phenomena were removed. The limiting factor appears to the staff to be mist impaction.

As a result of the test work and especially that conducted most recently, the major conclusions of the staff are as follows: the utilization of dilution gas injected with input air has little benefit. The majority of the generation of dust occurs in the retorting zone; consequently, dust removal from the combustion zone would appear to be of little benefit. Very low air injection velocities have no benefit. Substantial variation has been made in time-temperature profiles above 800°F without improving yield above about 90% F.A. The impingement of oil shale mist on the surface of dust and incoming shale appears to be the major cause for reducing the obtainable yield below a potential limit of 105% F.A.; the presence of dust in the retorting and condensing zone is definitely suspected as a contributing factor. High yields have been obtained with shale blends averaging 35 to 36 GPT; 38 to 40 GPT blends have also been run but with some difficulty. This does seem to permit consideration of "high-grading" operation in developing a commercial project.

Next, discussion of reordering experimental program items was taken up. The Committee recommended that the program items on shale richness and the sloping wall in Retort 2 should be carried on as scheduled. After this, the effect of oil refluxing would be determined in Retort 1 while Retort 2 was being prepared for the demonstration run. The run is to be started near February 1. Program Item 13, shale size and rate study, will follow the demonstration run and after this, other promising leads. Item 9, dust removal, will be removed from the program for the present and Item 11, external heating, will be displaced backward in timing. The Committee agreed to postpone further recommendations on experimental programming until the next meeting.

After considerable discussion of the objectives of Stage II, there seemed to be general agreement that Stage II should be so planned and executed as to yield experimental data and operating experience that the engineering department of any Participating Party or contractor could design completely a prototype plant. Such a plant would be of sufficient size that it could constitute a module that would be repeated as necessary to develop full plant capacity. The sense of considerable further discussion was that all possible care should be taken in planning Stage II and especially in designing Retort 3 to maximize the probability of attaining the stated objectives. Cramer was questioned regarding whether present staff would have time to carry out the planning studies in sufficient detail. He was asked to inform the TAC immediately if short-term assignment of other personnel might be necessary to carry out planning and analysis for Stage II with the necessary detail.

Cramer was asked to update the scoping study for Stage II and have it ready for discussion before or at the next meeting. It was agreed that this study should estimate the cost of all parts of the proposed program as accurately as possible. The desirability of having the proposal show the point at which the $3,000,000 originally projected in the Project Agreement would run out was discussed. New areas of research proposed for Stage II were discussed in the following order: investigations previously planned on Retorts 1, 2 and 3; investigation of retorting under pressure; extended mining research; and a farm-out study of crushing parameters. Voting on the items was tabled until the next meeting.

Alternative areas were discussed for mining because of concern for the condition of the roof in Able haulageway. This situation is to be given extended consideration before the next meeting.

In the executive session, the first matter of consideration was the new method of presenting technical material. Cramer, Lawson and Synder were complimented on the way this went off. The proposed amendment of the Research Agreement to permit use of technical personnel from all Participating Parties to expand the Anvil Points staff for Stage II was reviewed and tabled until the next meeting.

The meeting adjourned at 10:10 p.m.

R. T. Ellington/SLM

R. T. Ellington, Chairman

S L Meisel

S. L. Meisel, Secretary

TAC HANDOUT

Jan. 17, 1966 Meeting

file

POTENTIAL MECHANISMS WHICH MOST LIKELY AFFECT YIELD

1. OIL REFLUXING - Impingement and condensation of oil on shale which polymerizes and cracks on revaporization.

2. DUST CIRCULATION - Internally circulating dust adsorbs oil which polymerizes or cracks on revaporization.

3. INADEQUATE TIME-TEMPERATURE - Retorting at temperatures above 900° F causes cracking to coke and gas.

4. ERRATIC SHALE FLOW - Unretorted oil shale and oil laden dust drop into the combustion zone where the oil is burned or cracked.

PWSnyder
1/12/65

CALCULATED IMPACTION EFFICIENCY VERSUS MIST PARTICLE SIZE

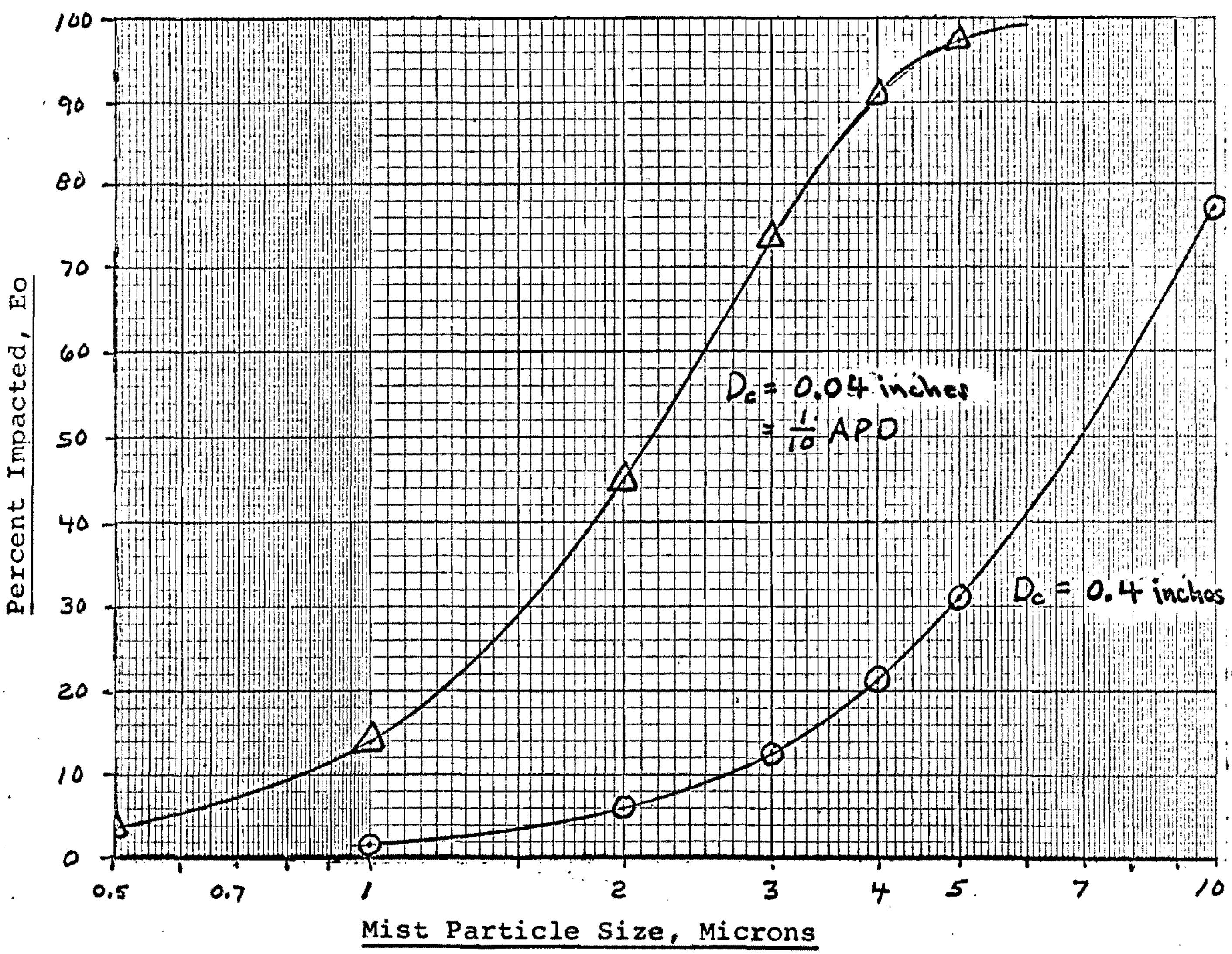
Operating Conditions:

Shale rate	= 500 lb/(hr)(ft^2)
Shale Size (APD)	= 0.4 inch
Air	= 4,300 SCF/T
Recycle	= 16,000 SCF/T
AT Top Temperature	= 130° F

~~Percent~~ FRACTION Impacted, $E_o = 1 - e^{-KUSdp^2/\epsilon d_c}$

Where: K = Small function of temperature ~2.6 X 10^{-4}
U = Offgas rate, MSCF/T
dp = Mist particle size, microns
ϵ = Void fraction
d_c = Mean path diameter (function of shale size), inche
S = Shale rate, lb/(hr)(ft^2)

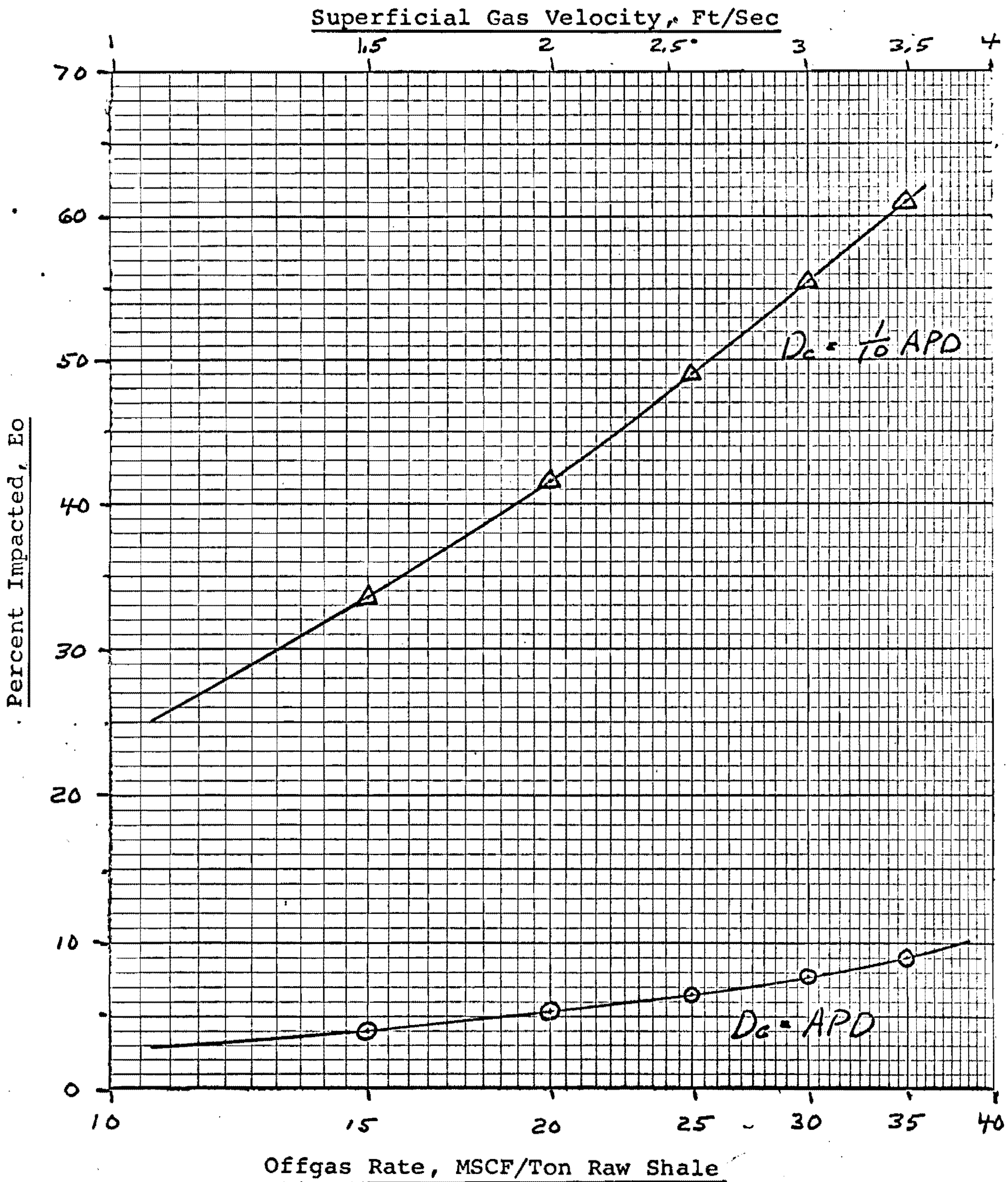
SURELY THE FRACTION IMPACTED ALSO DEPENDS ON THE DEPTH OF THE SHALE BED.



PHGifford
1/11/66

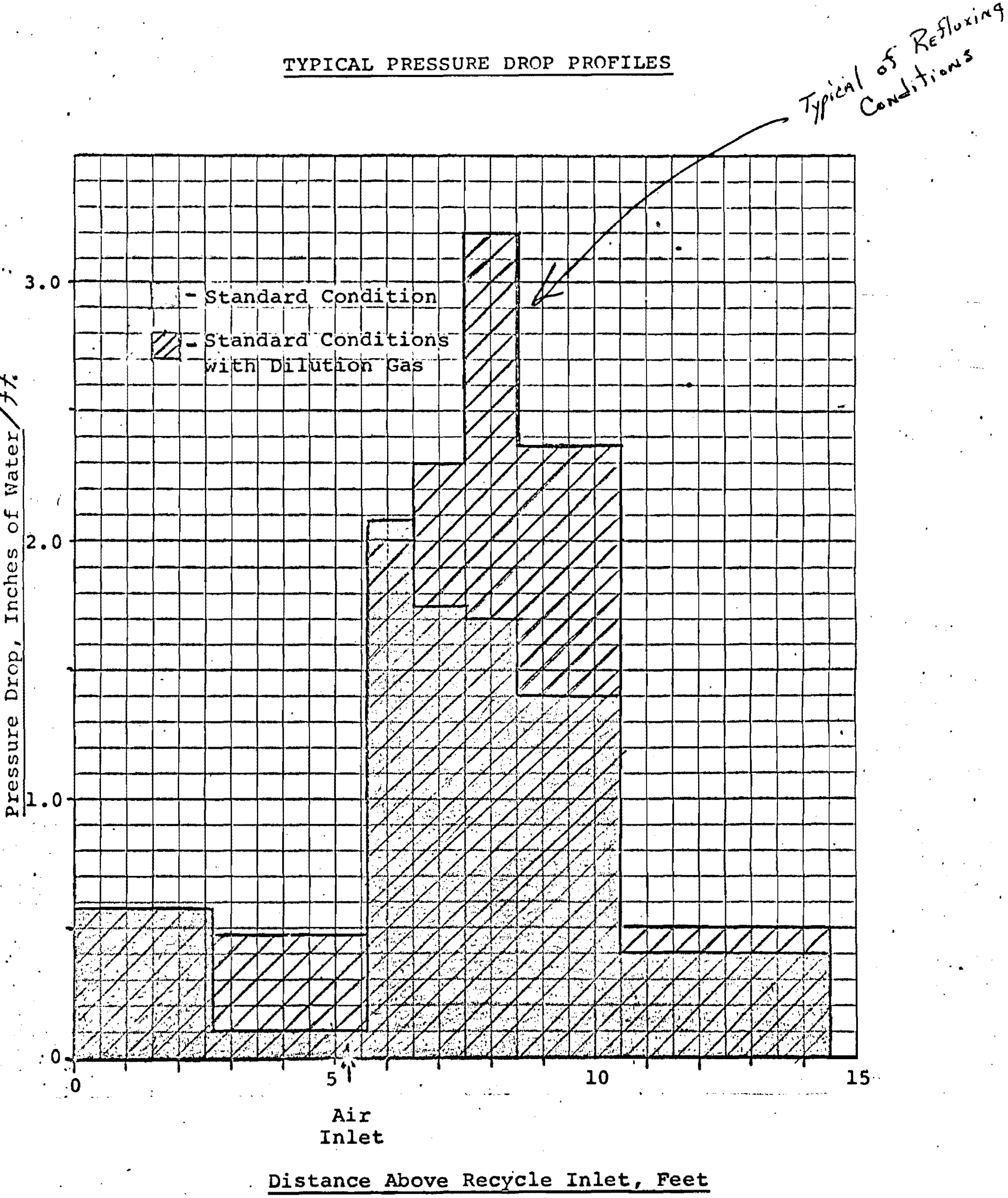
CALCULATED IMPACTION EFFICIENCY VERSUS OFFGAS RATE

Operating Conditions:
Shale rate = 500 lb/(hr)(ft^2)
Shale Size (APD) = 0.4 Inches
Oil Mist Size = 2 Microns



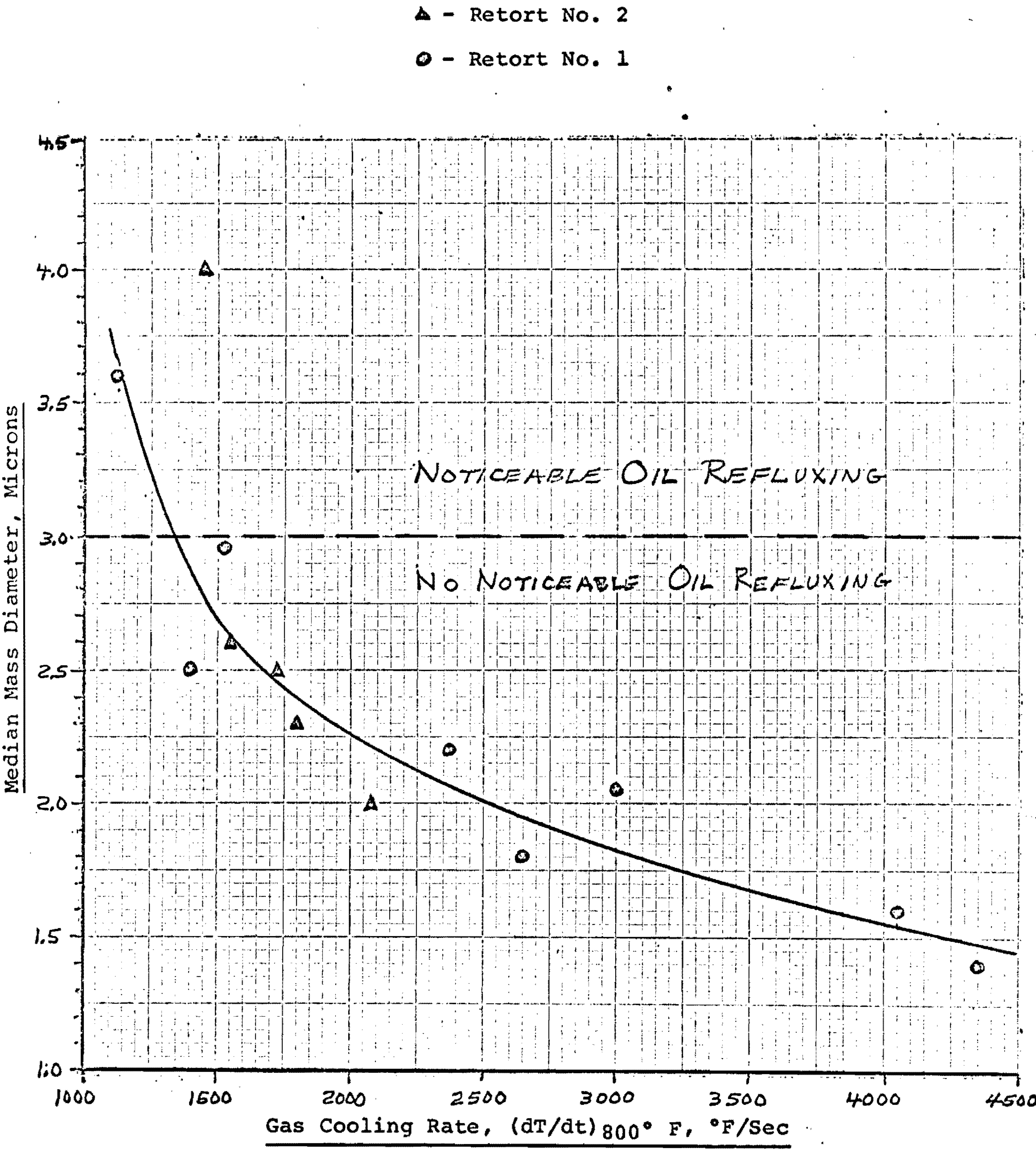
PHGifford
1/11/66

TYPICAL PRESSURE DROP PROFILES



JEBurchfield
1/13/66

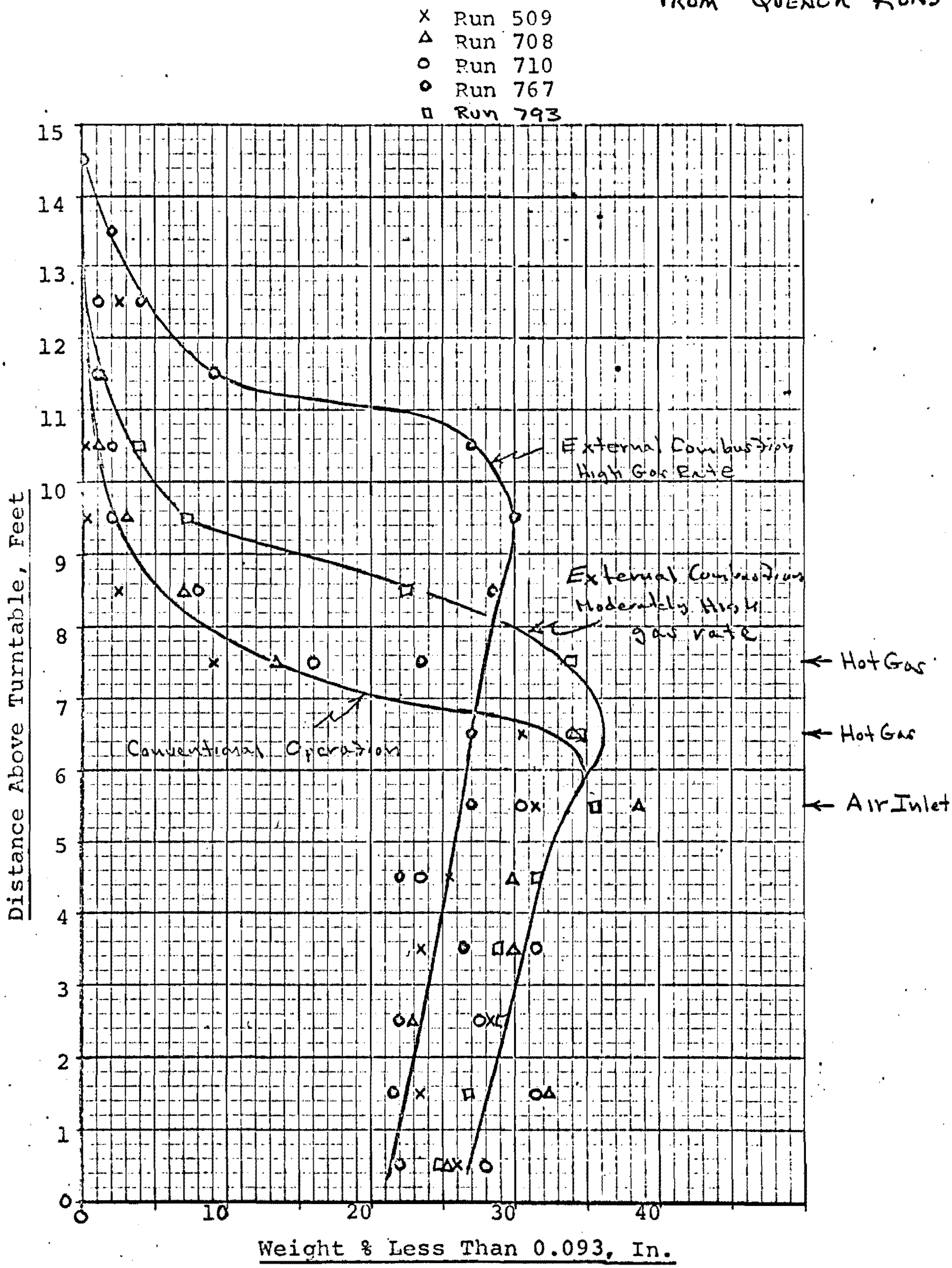
EFFECT OF GAS COOLING RATE ON
OIL MIST SIZE LEAVING RETORT



PHGifford
1/12/66

FINES CONCENTRATION VERSUS BED HEIGHT

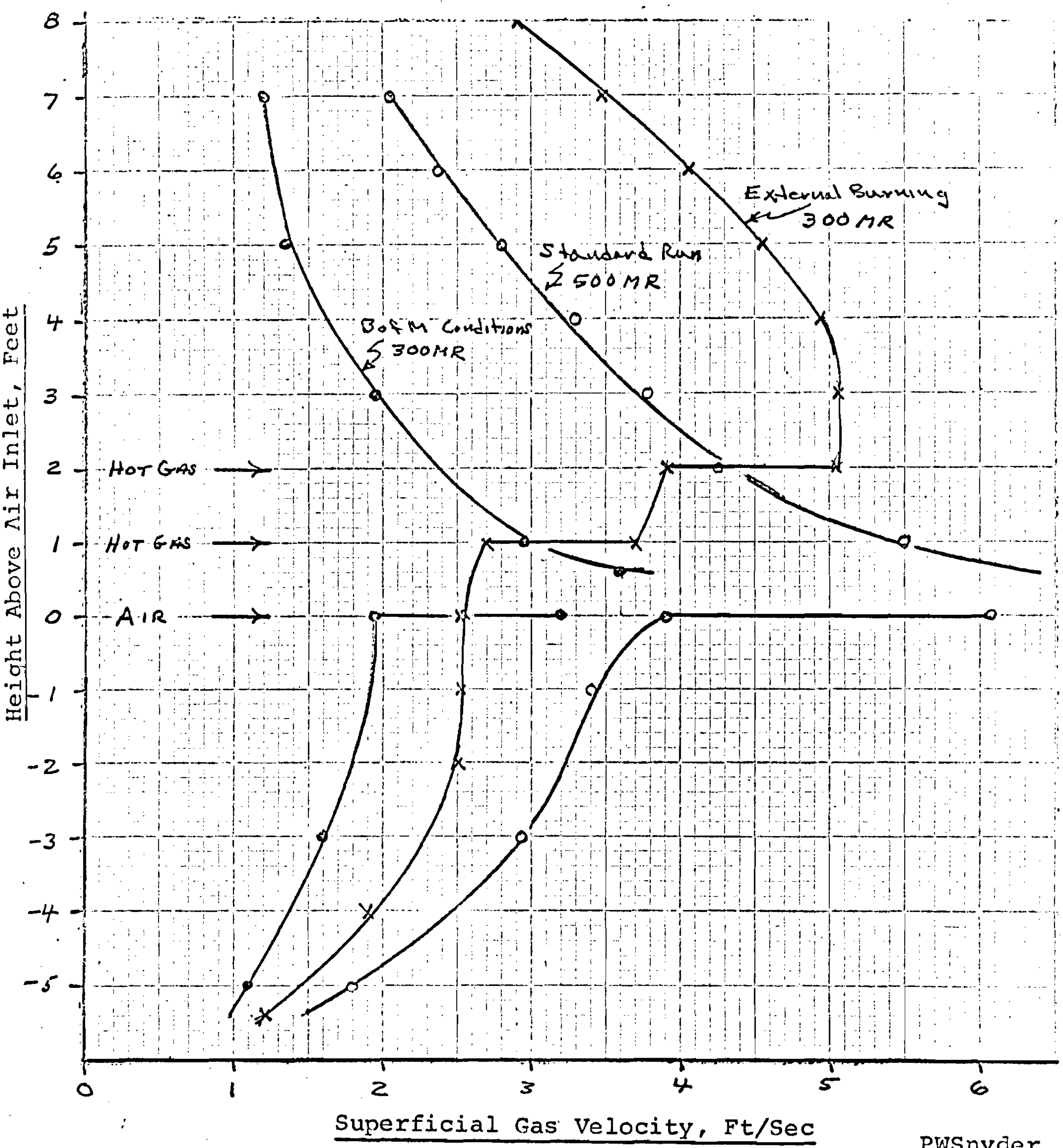
From Quench Runs



JEBurchfield
1/12/66

TYPICAL GAS VELOCITY PROFILES IN THE GAS-COMBUSTION RETORT

Measured how? If calculated, was oil vapor included?

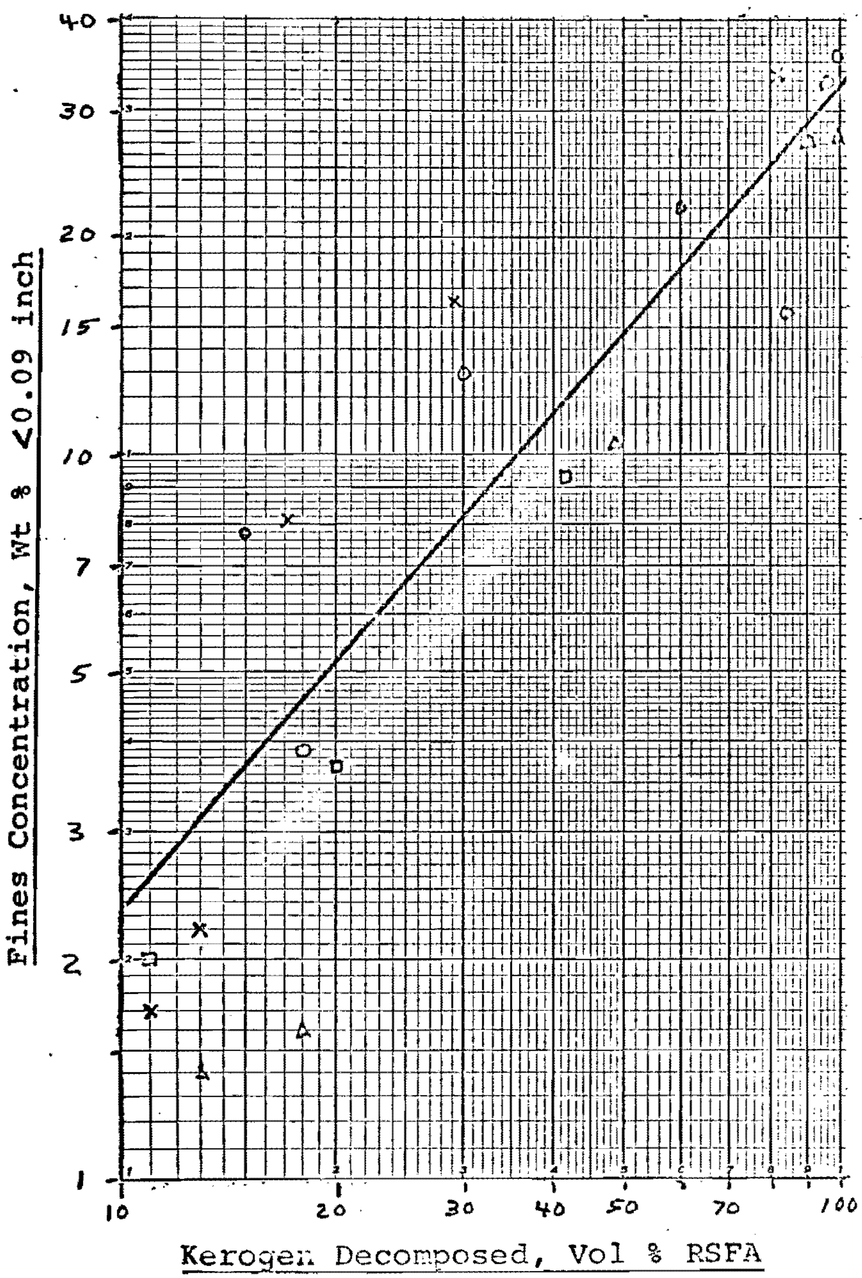


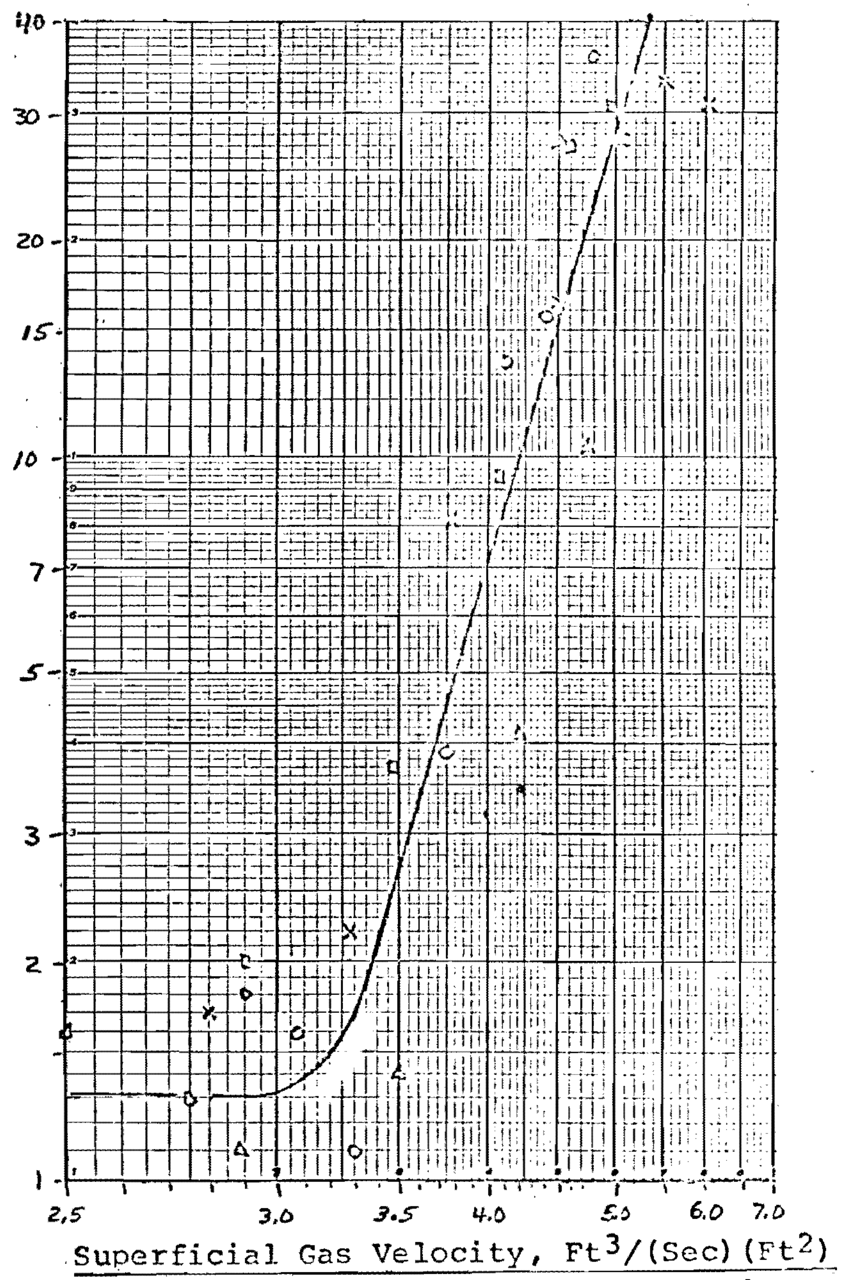
PWSnyder
1/12/65

Calc'd from Quench Runs

FINES CONCENTRATION IN THE GAS-COMBUSTION RETORT

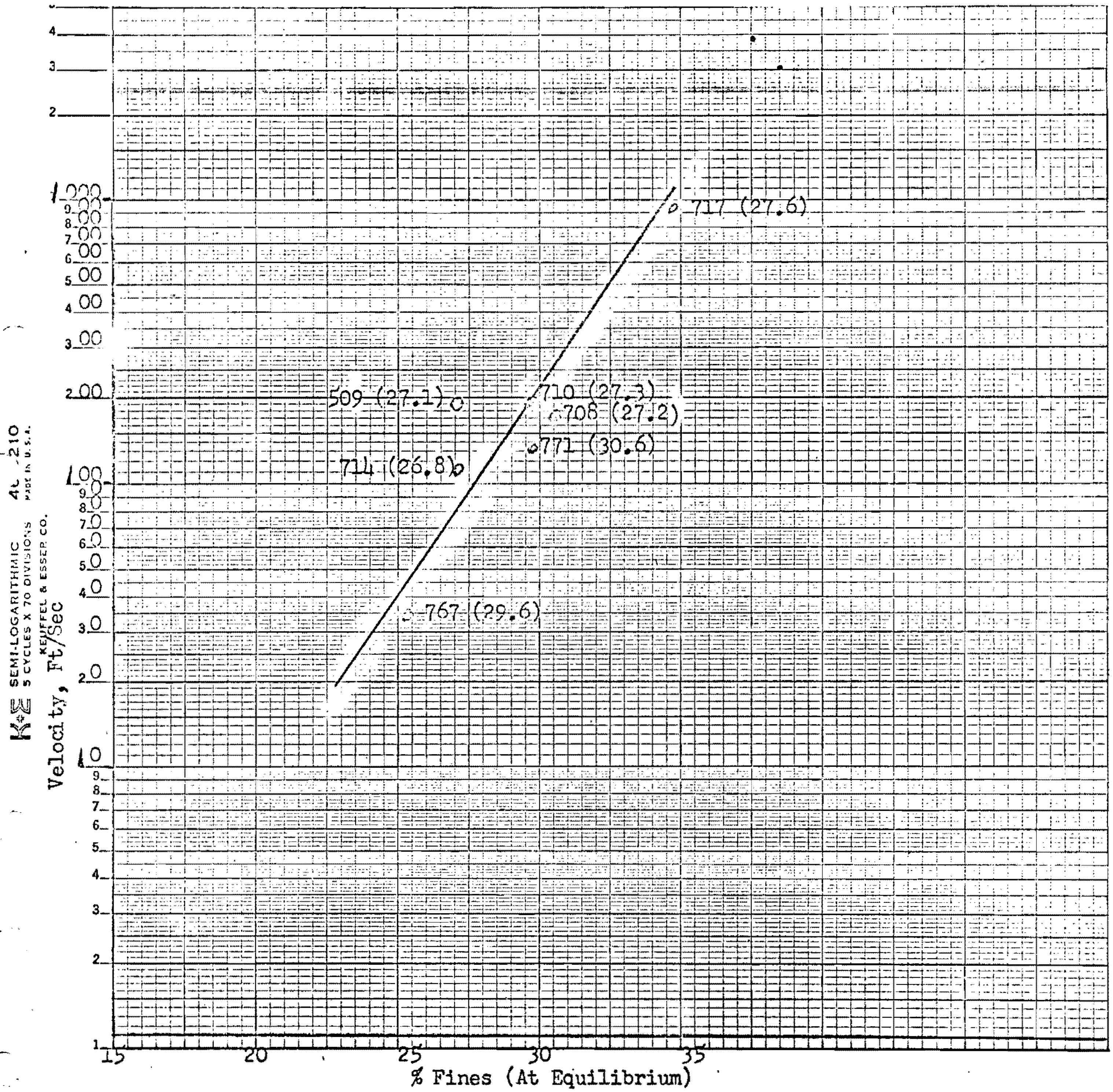
TYPE OF OPERATION:	Standard	High Cp Operation: Internal Burning	High Cp Operation: Moderate Gas Rate	High Cp Operation: High Gas Rate
SYMBOL:	X	Δ	O,O	□





EFFECT OF DISTRIBUTOR VELOCITY ON FINES GENERATION.
RETORT NO. 1

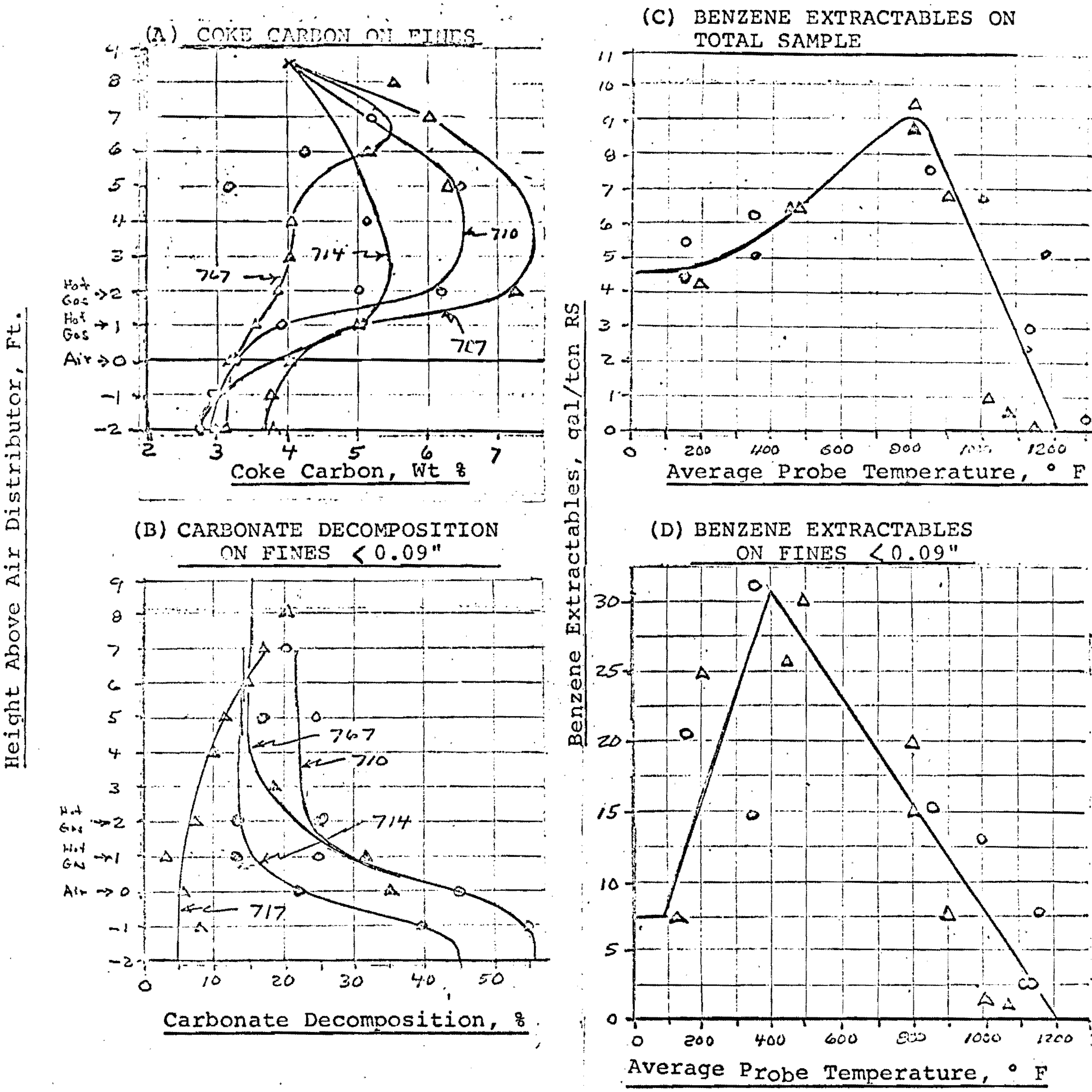
Legend
Run No. (Fischer Assay)



EETurner
12/15/65

SUMMARY OF RESULTS FROM RECENT QUENCH TESTS

Identification:	Conventional Operation	High Cp Internal Burning	High Cp-External Burning Moderate Gas Ratio	High Cp-External Burning Very High Gas Ratio
Symbol:	○	⊙	△	▲



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12/15/65

SUMMARY OF AVAILABLE DATA ON THE EFFECT OF TIME-TEMPERATURE ON YIELD

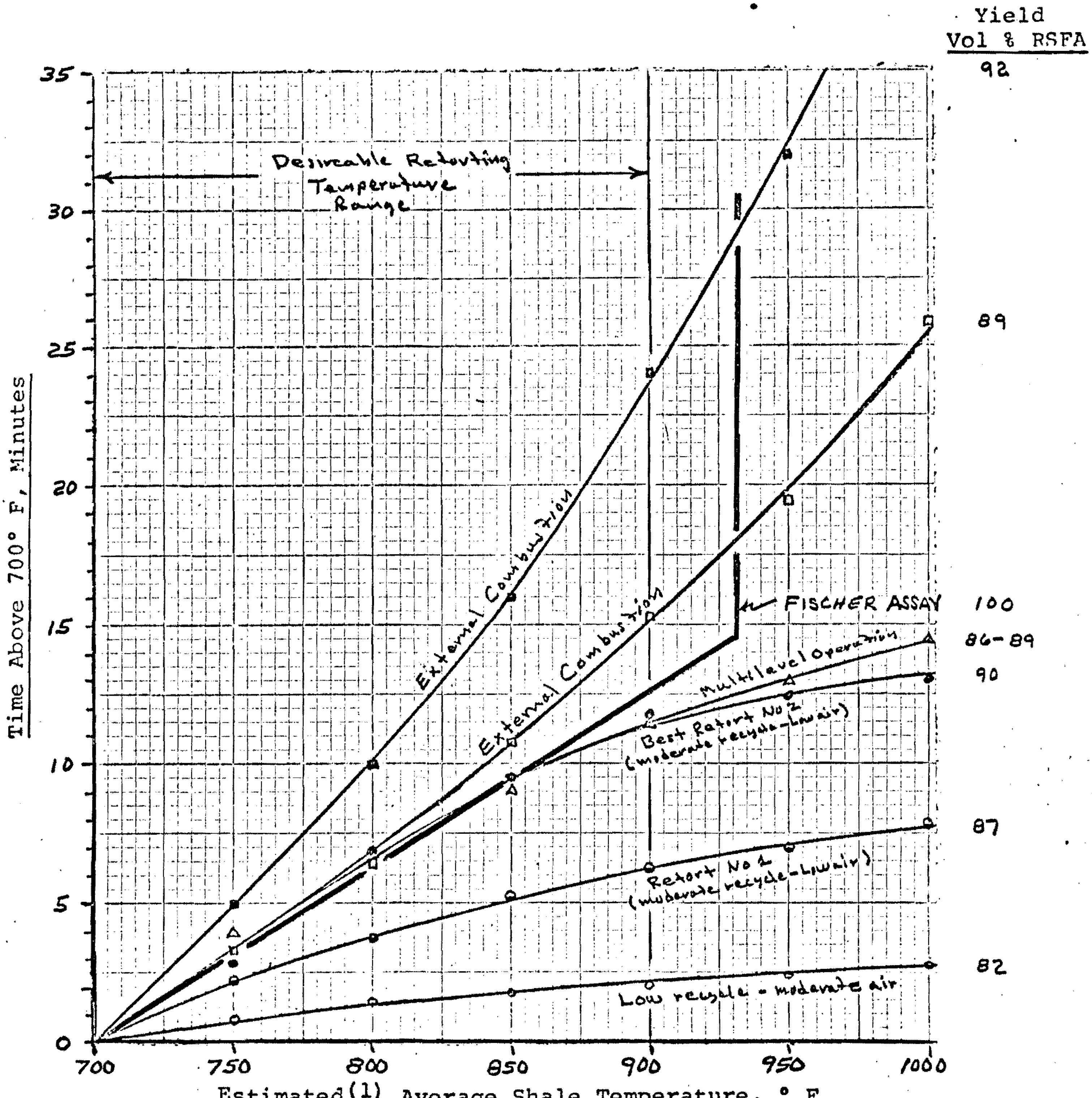
Bureau of Mines Information

Process	Maximum Shale Temp., °F	Shale Residence Time, Minutes	Oil Yield Vol % RSFA
Fischer Assay	1020	50	98
	930	40	100
	840	50	104
Royster	1150	100	94
	1090	120	99
	1030	130	101
	970	170	104
Gas Flow	1020	40	95
	980	50	96
	940	70	102

PWSnyder
1/12/66

RETORTING TIME-TEMPERATURE PROFILES FOR THE GAS-COMBUSTION RETORT

TYPE OF RUNS:	Retort No. 2 Low Recycle	Retort No. 2 Best Operation	Retort No. 1 Standard	Retort No. 1 External Combustion 764	Retort No. 1 External Combustion 767	Retort No. 1 Multi-Level
Shale Rate, lb/(hr)(ft^2)	500	500	500	300	300	500
Offgas, MSCF/T	19	22	22	27	33	22
Symbol	○	●	⊖	■	□	△



(1) Average probe temperature minus 100° F

PWS 1/10/66

DEFINITIVE EXPERIMENTS TO HELP RESOLVE THE EFFECTS OF OIL REFLUXING

A. OBJECTIVES:

1. Develop the effect of oil refluxing on yield.
2. Develop the cause and potential amount of refluxing.
3. Develop the effect of ~~retorting~~ operating conditions on mist size.

B. IN PROGRESS OR DEFINITELY PLANNED:

1. Evaluate the effect of adding a known amount of shale oil to the shale charged to a Fischer Assay.
2. Evaluate the effects of recycling oil product into the condensing zone of Retort No. 1 on yield, temperature profile, pressure profile, and oil properties.
3. Evaluate the effect of water injection in the oil condensing zone of the retort when operating at conditions which produce large mist size.
4. Develop the impingement characteristics of crushed oil shale (bench scale).
5. Continue to obtain mist size measurements from gas taken at various positions within the retort as well as from the offgas.

C. BEING CONSIDERED:

1. Develop the mist size - gas cooling rate relationship by controlled cooling of a simulated retort gas.
2. Evaluate injection of very fine oil mist in the condensing zone as a nucleating agent.

PWSnyder
1/13/66

DEFINITIVE EXPERIMENTS TO HELP RESOLVE THE EFFECTS AND SOURCE OF DUST IN THE RETORTING ZONE

A. OBJECTIVES:

1. Establish how dust affects yield when present in the retorting zone.

2. Establish the source of dust occurring in the retorting zone.

B. IN PROGRESS OR DEFINITELY PLANNED:

1. Investigate the extent of yield loss when evaporating shale oil from oil soaked spent shale fines.

2. Investigate the extent of yield loss when adding 10 to 30 per cent normal spent shale fines, retorted only fines, and raw shale fines to the normal Fischer Assay charge.

3. Run quench tests on Retort No. 1 after retorting lean and rich shales to establish the effect of richness on the fines concentration profile.

4. Tag about 100 pounds of spent shale fines with $MnCl_2$, inject the tagged fines into Retort No. 1 at the combustion zone, run a quench test, and establish the distribution of the tagged fines.

5. Modify Retort No. 2 to permit removing dust from the gas leaving the combustion zone.

C. BEING CONSIDERED:

1. Radioactively tag spent shale fines, inject the tagged fines continuously into the combustion zone of Retort No. 2 and measure the equilibrium distribution during operation of the retort with a scintillator.

PWSnyder
1/13/66

DEFINITIVE EXPERIMENTS TO HELP RESOLVE THE EFFECT OF TIME-TEMPERATURE

A. OBJECTIVE:

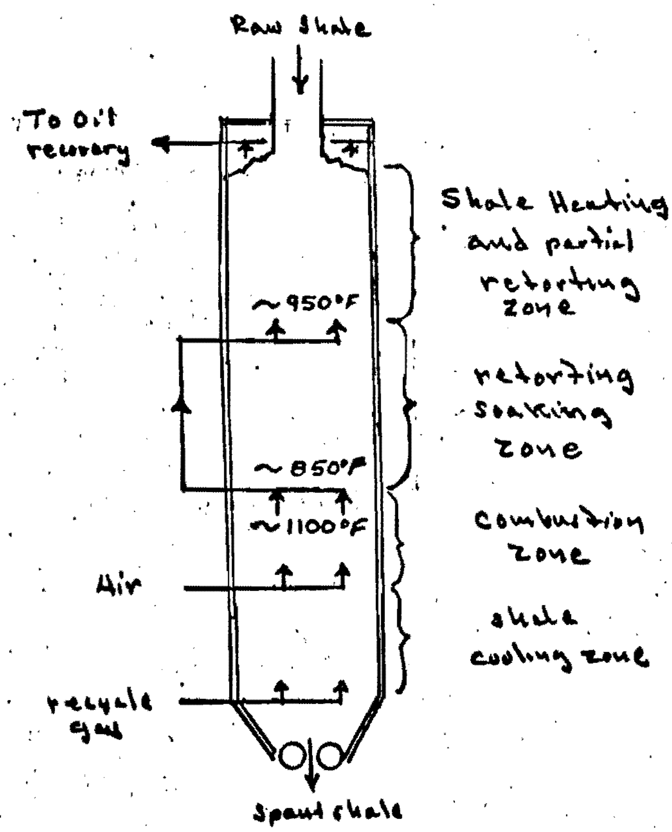
Establish the maximum yield possible from optimum retorting time at temperatures below 900° F.

B. IN PROGRESS:

Bench scale study of the effect on oil yield of varying the time-temperature profile of the Fischer Assay.

C. BEING CONSIDERED:

Modify Retort No. 2 to provide a retorting soaking zone after rapidly heating the shale to about 950° F. The concept of this modification is as follows:



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DEFINITIVE EXPERIMENTS TO HELP RESOLVE THE EFFECTS AND EXTENT OF ERRATIC SHALE FLOW

A. **OBJECTIVES:**

1. Minimize wall effect.
2. Establish the extent of erratic and/or bridging during retort operation.

B. **IN PROGRESS OR DEFINITELY PLANNED:**

Evaluate the effect of sloping walls on the performance of Retort No. 2 as measured by yield, temperature profile, pressure gradients, ring flow tests, and physical probing of the bed.

C. **BEING CONSIDERED:**

1. Use nuclear density gauging to measure bed density profiles during retort operation.
2. Run a drawdown test on Retort No. 2, primarily to establish the average flowing density.

PWSnyder
1/13/66

Jan. 17, 1966 Meeting file

PILOT RETO PERATION SUMMARY FOR NOVEMBER AND DECEMBER 1965

--	1		2		3	4	4		7	
	High Recycle Rate		Effect of Dilution Gas		Very Low		External Combustion		Multi-Level Air-Gas	
Best Prior Operation	High Recycle Low Air	Low Recycle High Air	BOM Condition	High Shale Rate	Air Injection Velocity	Retort No. 1 Performance	Best Condition	High Recycle	Hot	Cold
2	2	2	2	2	2	1	1	1	1	1
510 31	400 30	500 31	300 30	500 29	500 28	490 32	320 28	300 30	490-500 27-30	400 28
4,300 --	4,100 --	5,800 --	5,000 1,200	4,500 1,100	4,500 --	4,400 --	-- --	-- --	4,500 --	6,000 --
-- -- --	-- -- --	-- -- --	-- -- --	-- -- --	-- -- --	-- -- --	1,360 16,600 1,300	2,040 17,700 1300/1100	-- 6,000 900/1200	-- 7,500 120
15,900	19,200	11,400	14,300	15,800	16,000	16,400	12,000	15,500	12,000	14,000
9 --	9 --	9 --	7 --	9 --	9 --	9 --	9 8	9 8/7(3)	9 6-8	7 5
90 32	88 33	85 42	91 34	88 31	86 32	86 20	92 7	89 29	86-89 25-30	83 38
Very Good	Very Good	Very Good	Very Good	Very Good	Very Good	Very Good	Very Good	Good	Very Good	Very Good
30 15 0.9	20 7 0.7	30 4 0.5	20 5 0.3	30 6 1.2	10 6 1.1	150 7 0.6	50 22 0.7	30 14 0.7	30-150 8-12 0.6-0.8	200 5 0.5

eight per cent moisture on raw shale for Best Operation and Programs 1 - 3; improved raw shale rate moisture measurement used
Program No. 4.
00° F gas temperatures.
lly between two levels.

Program Number:

Program Descriptions:

Retort No.:

Operating Conditions:
Shale Rate, lb/(hr)(ft^2)
Fischer Assay, gal/ton

Gas Rates, SCF/T
Air Rate
Dilution Gas Rate

Oxygen
Hot Gas
Temperature, °F

Recycle Rate

Bed Height, Ft.
Above Air Distr.
Above Hot Gas Distr.

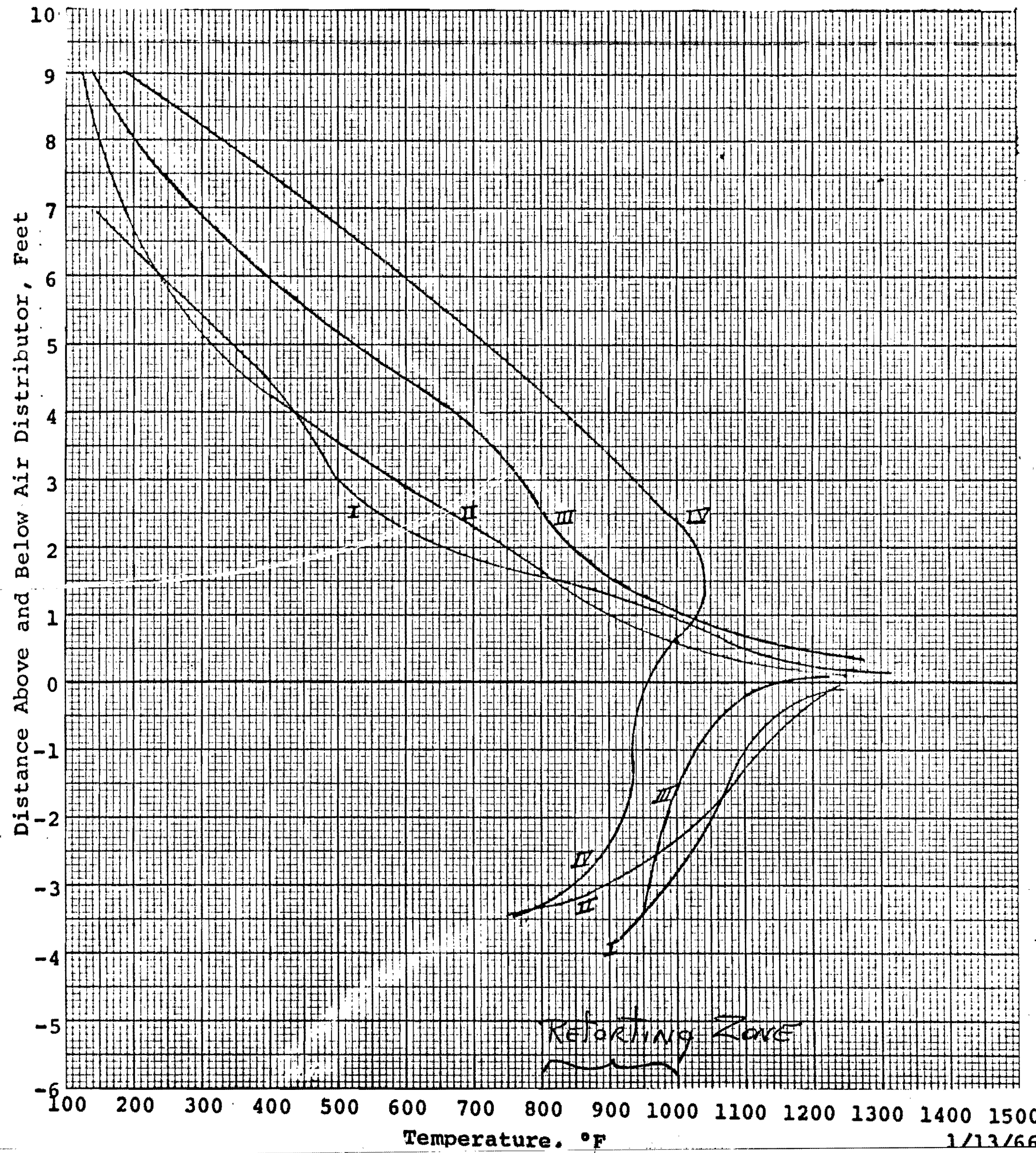
Test Results:
Oil Yield, Vol % RSFA(1)
Carbonate Decomposition, %

Operating Characteristics:
Performance
Air or Gas Injection
Velocity, ft/sec
Shale Residence, Mins.(2)
ΔP, inches H_2O/ft

(1)Yields adjusted to 1.3 we
beginning with runs for Pr
(2)Zone between 800 and 1,00
(3)Hot gas split about equal

VERTICAL TEMPERATURE PROFILE - RETORT NO. 1

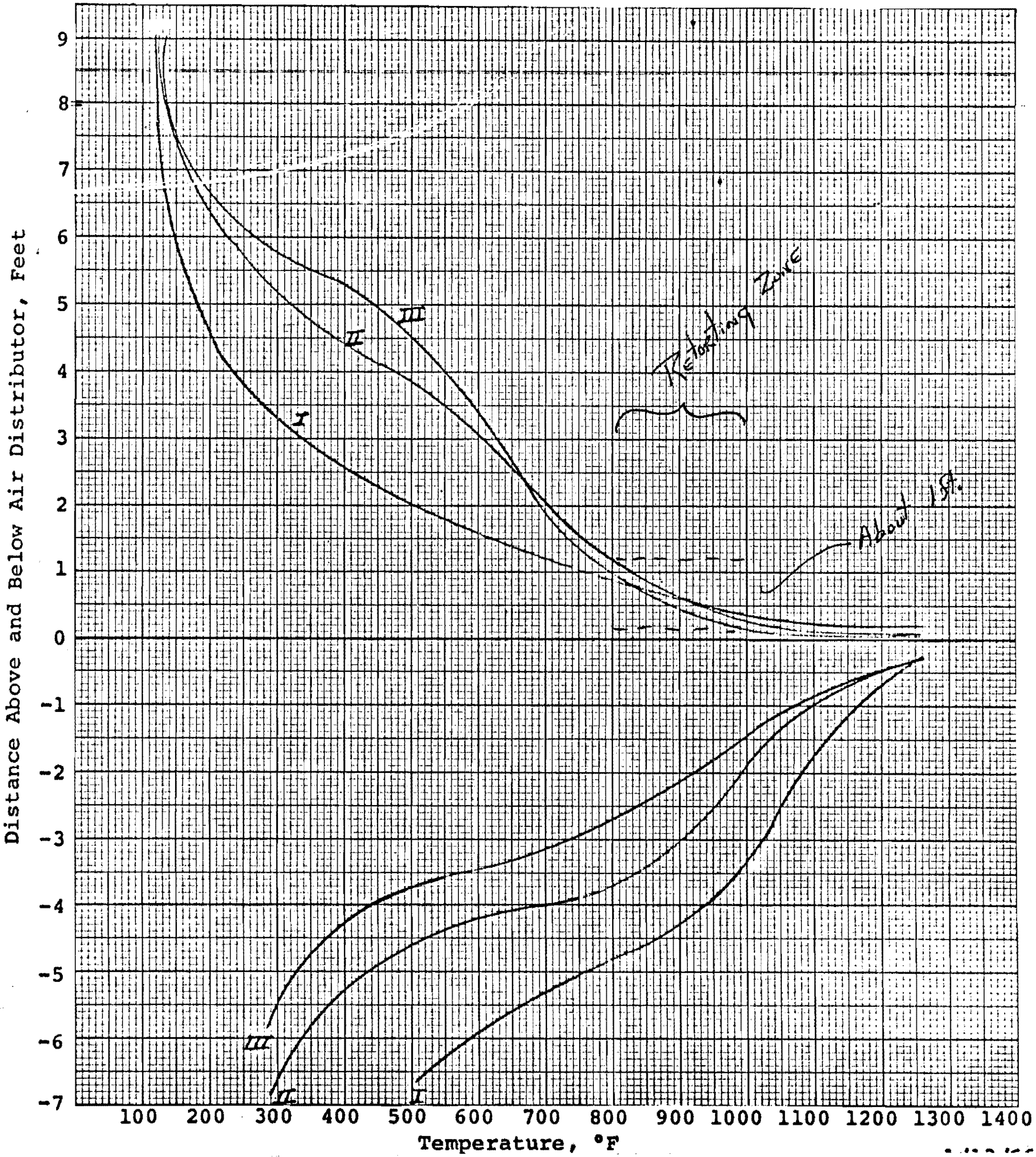
Curve	I	II	III	IV
Run Type	Cold Gas	Base	Hot Gas	High Cp
Total Gas, SCF/T	27,500	22,500	23,500	30,000
Mass Rate, lb/(hr)(ft^2)	400	500	500	300
Yield, Vol % FA	83	88	89	92
Retorting Time, Mins.	5	8	12	22
Mist Size, Microns	1.4	1.5	2.3	4.0



1/13/66

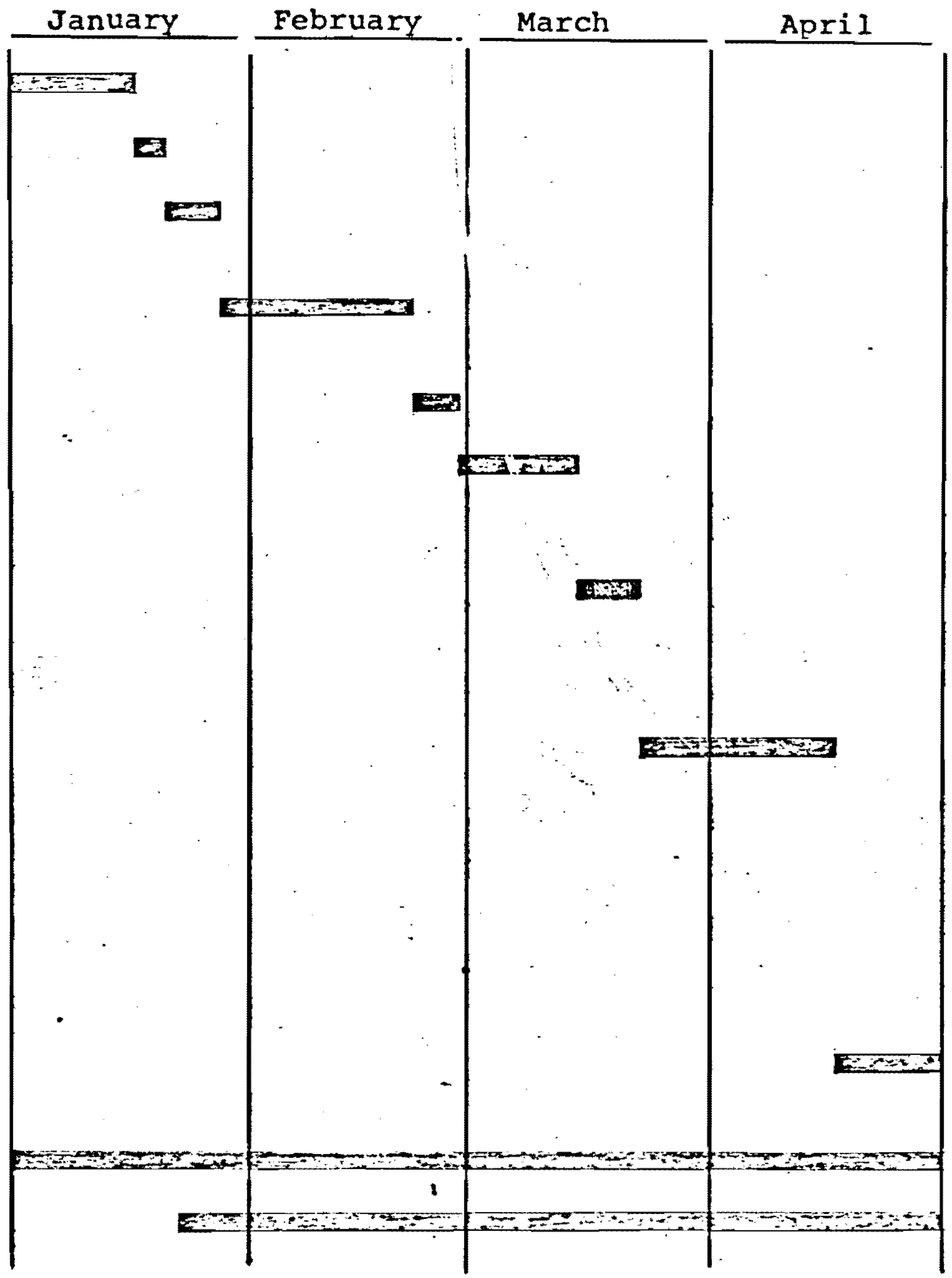
VERTICAL TEMPERATURE PROFILE - RETORT NO. 2

Curve	I	II	III
Run Type	High Air - Low Recycle	Low Air - High Recycle	Very High Recycle
Total Gas, SCF/T	19,300	22,500	26,000
Mass Rate, lb/(hr)(ft^2)	500	500	400
Yield, Vol % FA	83	88	88
Retorting Time, Mins.	5	7	9
Mist Size, Microns	1.9	2.5	3.6



LIGNMENT OF RETORTING STUDIES DURING MAINDER OF STAGE I

Retort No.	Est. No. of Runs	Est. Date Compl.
1	12	1/16
2	3	1/20
1	6	1/27
2	24	2/21
1	6	2/27
2	14	3/14
1	7	3/22
2	24	4/16
1	2	
2	6	
2	3	
1 or 2	3	4/30
2	--	4/30
--	--	4/30



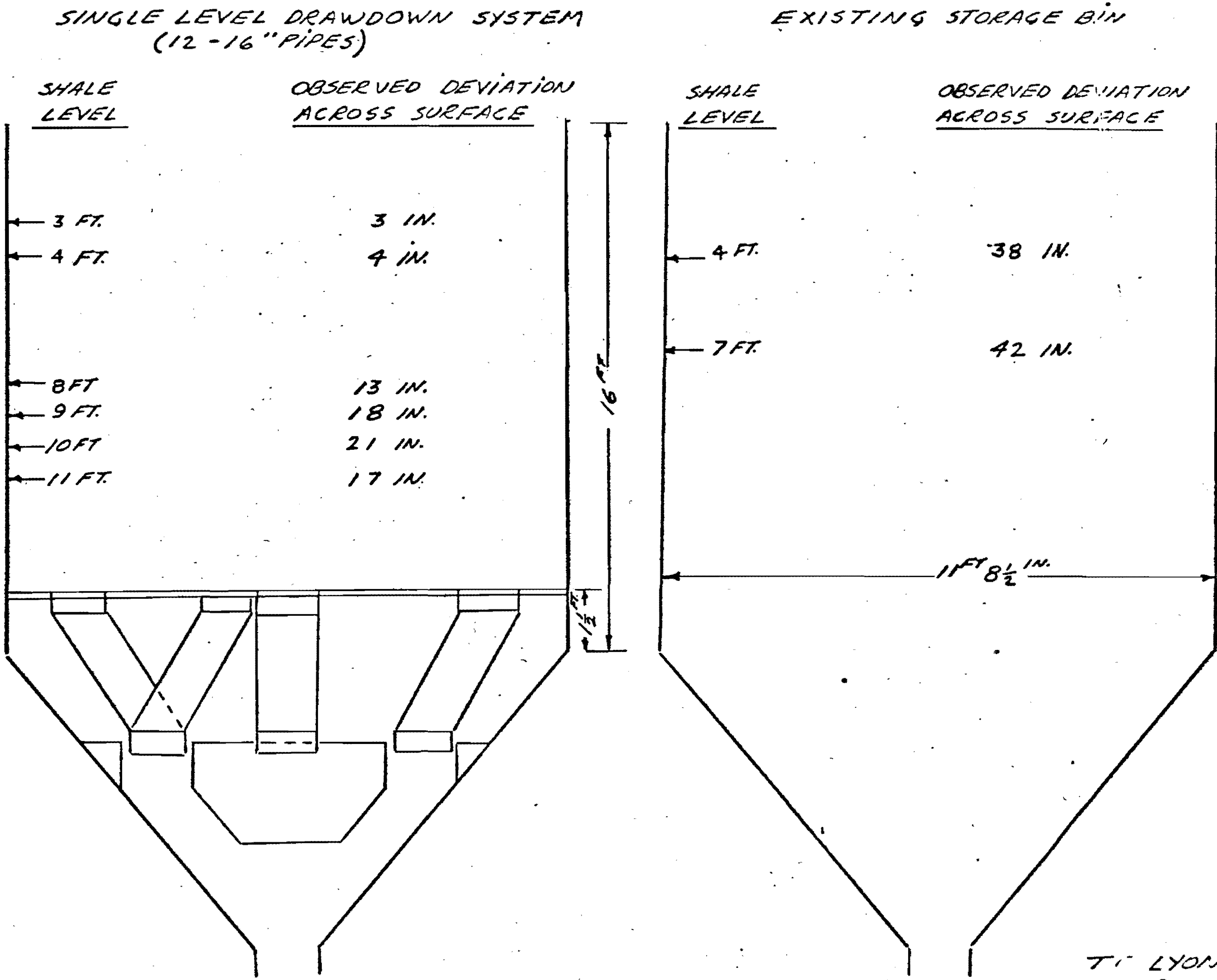
Members, 11/16/65.

TENTATIV RE

Program No.(1)	
8	Shale Richness.
--	Evaluate Sloping Walls
12	Effect of Oil Refluxing
13	Effect of Shale Size and Rate (Min. Study, 1/4-3, 1 1/2-3 In.)
? 11	External Heating of Gas (Confirmation of O_2 Study)- Hi C_p
--	14-Day Demonstration Run
15	Pursuit of Promising Leads: a. Oil or water injection in oil condensation zone b. Other Studies (Tagging Fines)
Out 9	Dust Removal From Gas Leaving Combustion Zone and Control of Time-Temperature
5, 14, 15	Slack Time for More Extensive Evaluation of Promising Programs a. Water mist injection in recycle gas b. Evaluation of horizontal distributor c. Confirmation of Multi-level-hot gas distributor d. Other studies
16	Evaluation of Mist Recovery Equipment
17	Bench Scale Retorting Studies

(1) Program Numbers refer to ltr RHC/TAC

SINGLE LEVEL DRAWDOWN SYSTEM

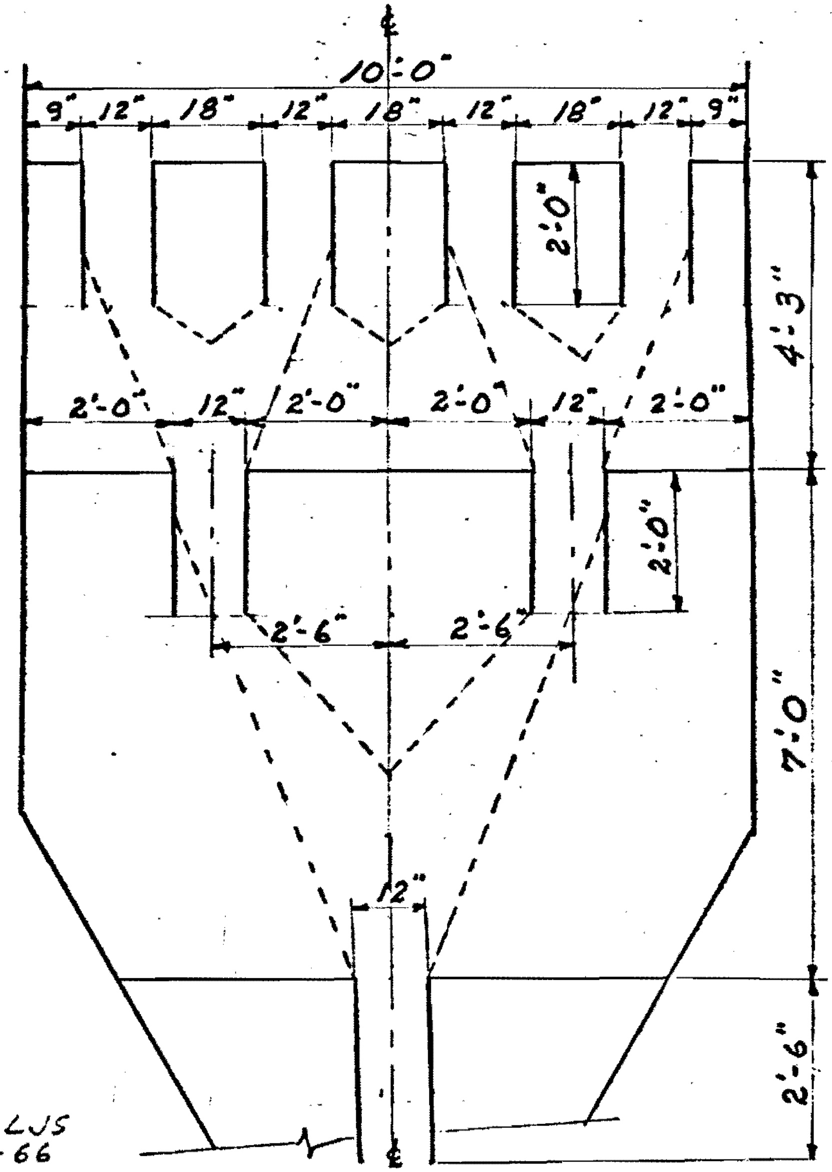


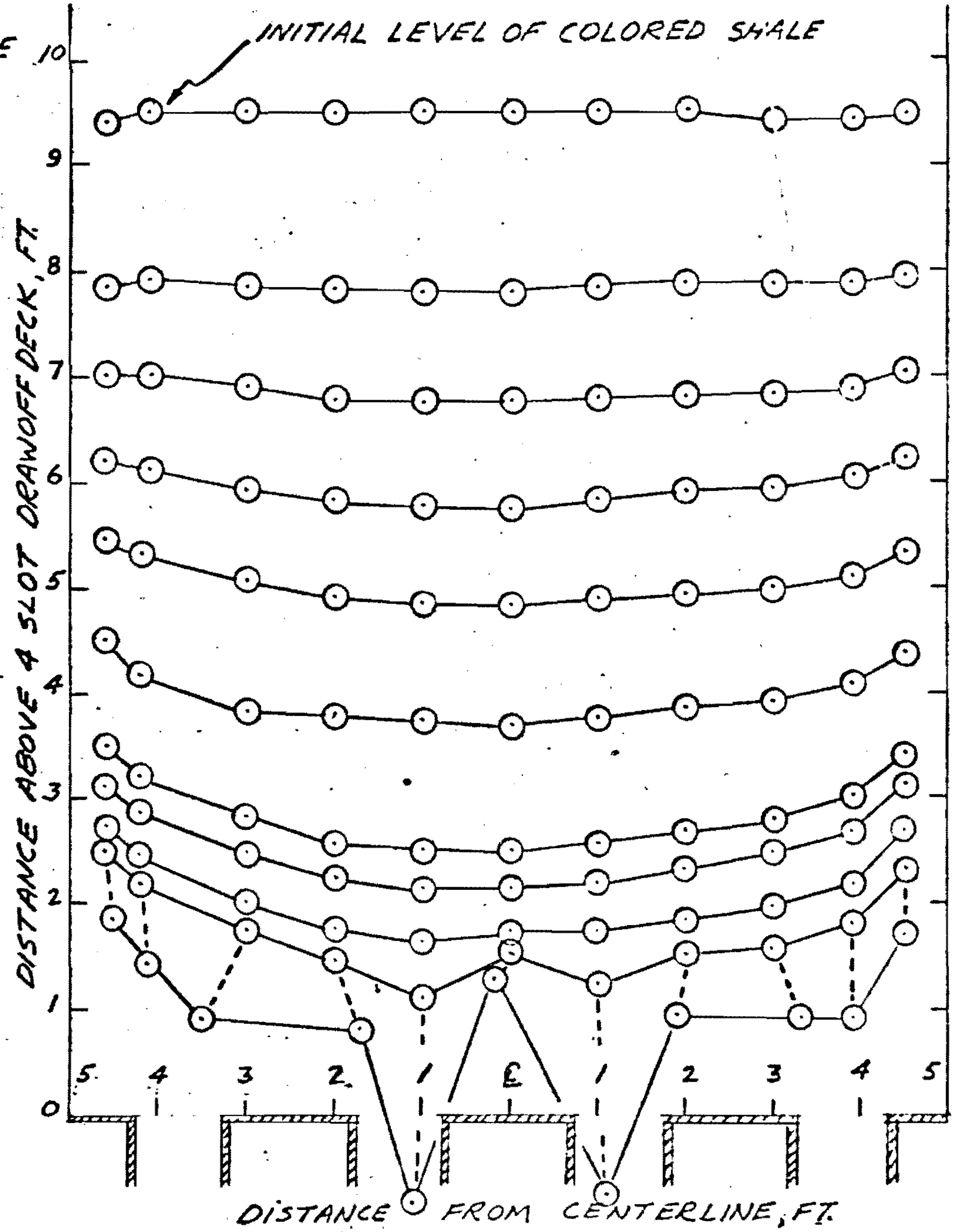
T. LYONS
12-10-65

HANDOUT 5 JEL

MULTI-LEVEL DRAWDOWN TECHNIQUE

4 SLOT SYSTEM 3/4-1 1/2 IN. SHALE



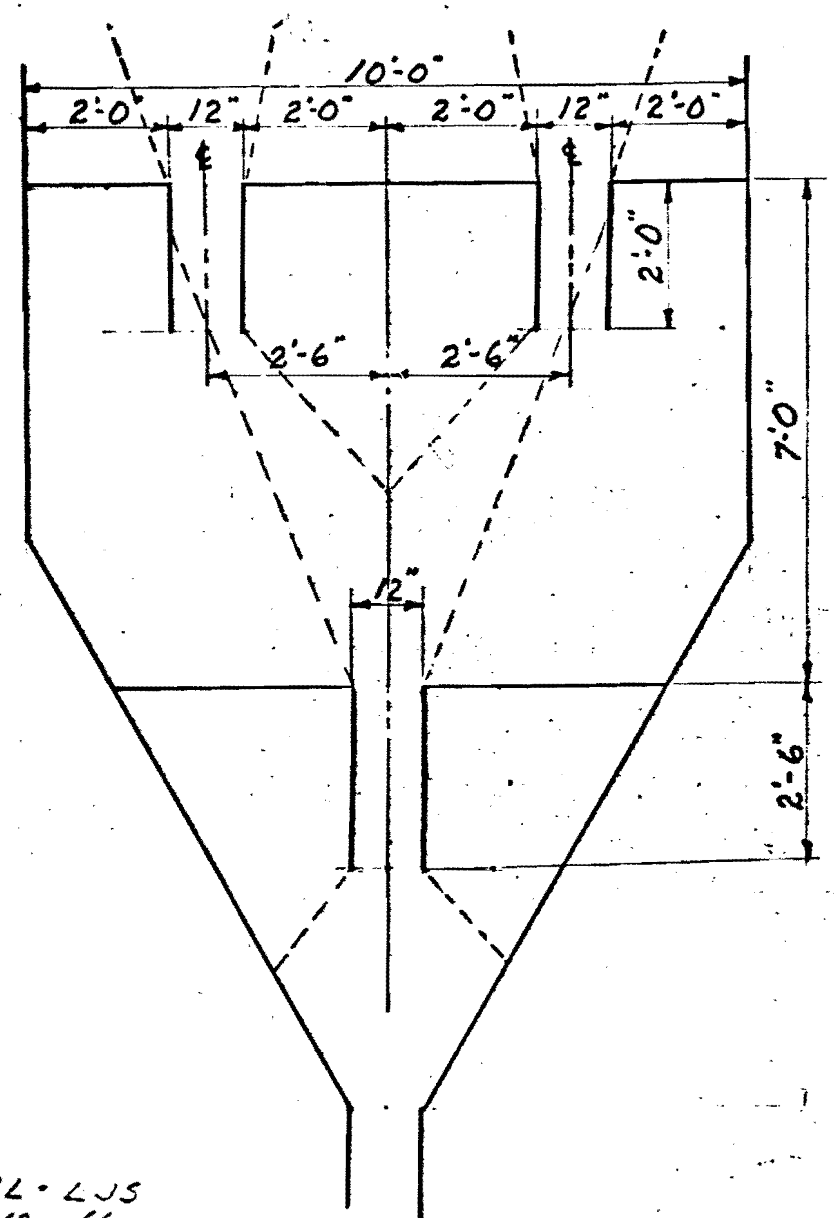


TCL-LJS
1-12-66

HANDOUT 6 JEL

MULTI-LEVEL DRAWDOWN TECHNIQUE

2 SLOT SYSTEM   3/4-1 1/2 IN. SHALE



TCL · LJS
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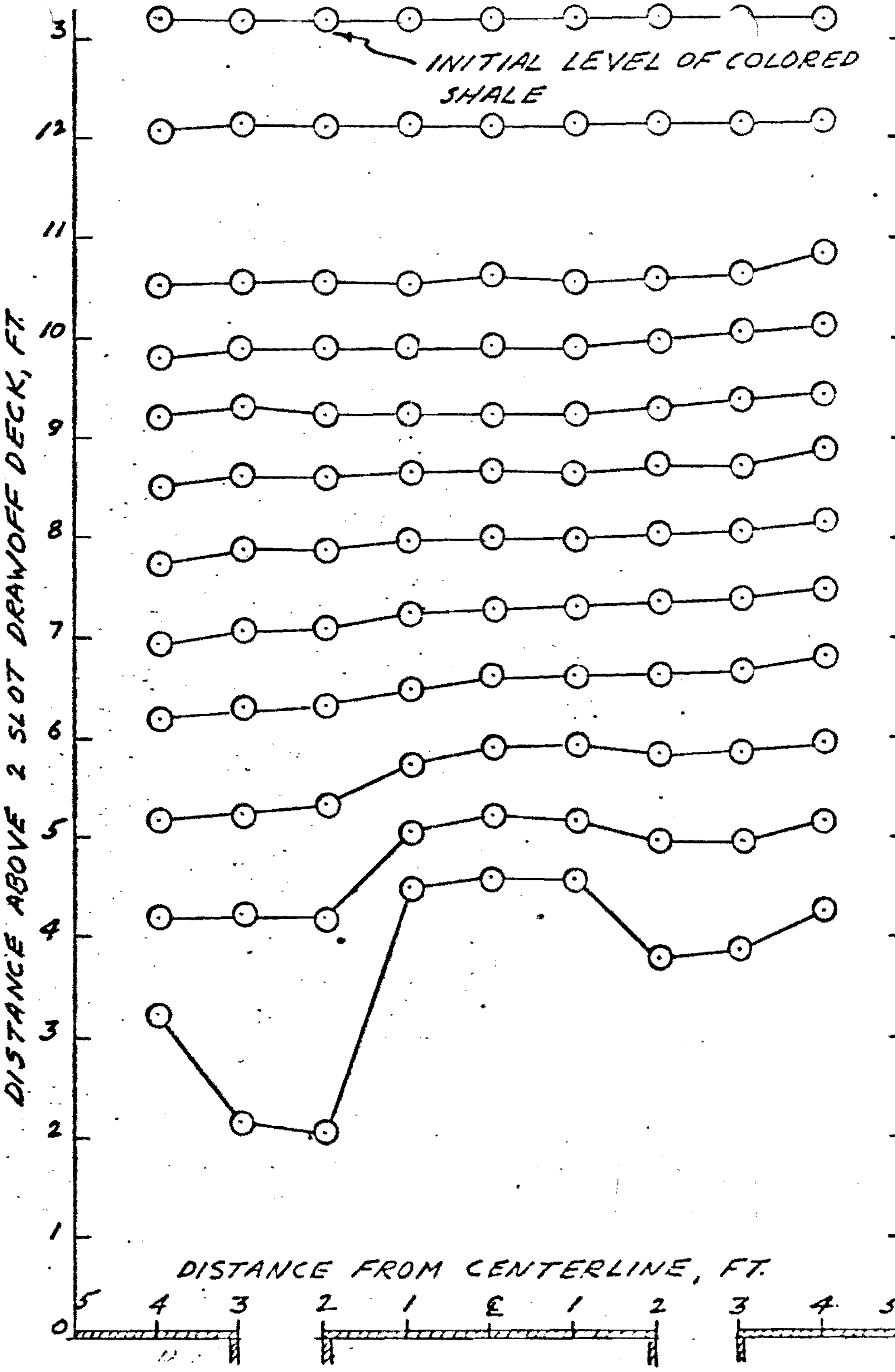


HANDOUT 7 JEL

SHALE FLOW PROFILES IN MULTI-LEVEL DRAWOFF SYSTEMS

EVALUATIONS BASED ON MARKED SHALE TECHNIQUE

	2 Slot Deck		Addition of 4 Slot Deck			
	Distances Measured Above 2 Slot Deck		Distances Measured Above 4 Slot Deck		Distances Corrected To 2 Slot Deck	
Shale Size	Initial Change, Ft.	Final Break, Ft.	Initial Change, Ft.	Final Break, Ft.	Initial Change, Ft.	Final Break, Ft.
1/4 - 1 in.	7(1)	4 1/2	7	1 1/2	11	5 1/2
3/4 - 1 1/2 in.	11	5 1/2	6 1/2	1 1/2	10 1/2	5 1/2
1 - 3 in.	11 1/2	6 1/2	>8	2	>12	6

(1) Evidence of change at 10 1/2 feet but pattern recovered.

TCLyons
1/12/66

HANDOUT 8 JEL

JWH

PROPOSED AGENDA
TENTH TECHNICAL ADVISORY COMMITTEE MEETING
DENVER, COLORADO
JANUARY 17 AND 18, 1966

I. Old Business
   A. Approval of Minutes
   B. Timing of Next Audit
   C. Problem of Joint Inventorship by Representatives of Different Companies
   D. Other

II. New Business
   A. Arrangements for Next Meeting
   B. Technical Presentation
      1. General Remarks
      2. Retort Program
         PWS a. Discussion of major reasons for discrepency between actual and theoretical yields
         JL b. Summary of program results, November to January
         PWS c. Definitive experiments to resolve yield discrepencies
         EWC d. Summary of major conclusions
         JL e. Future Plans
      JL 2. Mechanical Models
   C. Stage II Considerations
      1. May 1965 Scoping Study
      2. Stage II Additions
         a. Mining research proposed at October TAC meeting
         b. Crushing study
         c. Pressurized retorting
         d. Other
   D. Other

III. Executive Session
   A. Consider proposal to amend the Research Agreement to provide for possible use of technical personnel from all Participating Parties on the Anvil Points staff.

OUTLINE FOR FIRST PART OF RETORTING PRESENTATION TO JAN 17 TAC MEETING

a. DISCUSSION OF MAJOR REASONS FOR DISCREPANCY BETWEEN ACTUAL AND THEORETICAL YIELDS

Possible Fig

1. Ultimate yield is about 120 vol% of Fischer Assay; however practical yield is more like 105 vol%.

2. Potential mechanisms which are most likely reducing yield:

   a. Inadequate time-temperature during retorting causing excessive cracking to gas and coke.

   b. Excessive dust circulation in the retorting zone adsorbing the shale oil resulting in polymerization and cracking on revaporization.

   c. Excessive oil refluxing from impingement and/or condensation on shale resulting in polymerization and cracking on revaporization.

   d. Erratic shale flow resulting in unretorted oil shale and/or oil laden dust to fall into the combustion zone before the oil has had time to vaporize.

3. Available data indicating the presence of these mechanisms:

   a. Inadequate time-temperature:

      1) Bureau of Mines data on effect of time-temperature using Fischer Retort.

Fig

2 2) BofM data on the Royster and Gas Flow retorts.

3 3) Gas-Combustion Rate – Temperature profiles:

a) 12,000 vs 16,000 SCF/T recycle on Retort No 2.
b) 12,000 & 16,000 SCF/T recycle on Retort No 1.
c) External high Cp runs on Retort No 1.
d) Multi-level runs on Retort No 2.

b. <u>Excessive dust in retorting zone</u>:

4 1) Fines concentration versus bed height.
5 2) Fines concentration versus Kerogen decomposition
5 3) Fines concentration versus gas velocity.
6 4) Fines concentration versus injection velocity.
5) Large quantities of fines observed in thief samples.

7 6) Large concentration of oil on fines.

7 7) High concentration of coke on fines.

7 8) High carbonate decomposition on fines from some operations.

c. <u>Oil refluxing</u>:

8 1) Calculated impingement efficiency versus particle size with ~~gas velocity~~ packing size parameters. (gas velocity and mist)

9 2) Typical gas velocity profiling.

10 3) Oil mist size versus gas cooling rate.

11 4) Pressure gradient profiles.

Fig

12 5) Gas cooling rate versus gas rate — simplified mathematical derivation. (mist particle size is function of gas cooling rate.)

13 6) Math model simulation of temperature profiles when refluxing oil.

d. Erratic Oil Shale flows:

1) Ease of probing retort during operation.

14 2) Ring tests on Retort No 2.

14 3) Flowing density calculations on Retort No 1 from quench test data.

4) Visual observations in mechanical models.

5) Initial effect of stainless steel liner.

14 6) Observation of partially retorted oil shale nearly one foot below air distributor on some thief samples.

P. W. Snyder
1/9/66

Fig

C. DEFINITIVE EXPERIMENTS TO RESOLVE YIELD DISCREPENCIES

1. Effect of Retorting Time-Temperature:

a. Bench scale study of the effect of time-temperature on yield from retorting in Fischer Assay type of retort.

b. Modify Retort No 2 to provide a retorting soaking time after rapidly heating the shale to about 950°F.

2. Effect of Spent Shale Fines in Retorting Zone

a. Tag spent shale dust with a metallic salt, feed dust in at the combustion zone, run a quench test an examine shale in the retorting zone for tagged dust.

b. Investigate extent of yield loss from spent shale fines:

1) evaporation of oil from oil soaked spent shale fines.

2) Fischer Assay on raw shale containing 10-30% spent shale fines, retorted fines, and raw shale fines.

c. Measure the fines concentration profile on retort runs charging lean and rich shale. to establish the effect of shale richness on fines production.

d. Radioactively tag spent shale fines, introduce into the combustion zone, and measure the equilibrium distribution of these tagged fines during operation of the retort to develop residence time of fines in the retorting and combustion zones.

e. Modify Retort No 2 to permit removing dust from the gas leaving the combustion zone.

3. <u>Effect of Oil Refluxing</u>

a. Evaluate effect of adding shale oil to raw and spent shale on yields from Fischer Assay.

b. Evaluate effects of recycling shale oil during normal retort operations on yield, temperature profile, and pressure gradients.

c. Develop impingement characteristics of crushed oil shale.

d. Develop mist size - gas cooling rate relationships when cooling a simulated retort gas.

e. Evaluate effect of water injection in the oil condensing zone.

4. Effect of Erratic Shale Flow

a. Evaluate effect of slopping walls to eliminate erratic shale flow.

b. Use nuclear guaging to measure bed density profiles during retort operation in order to establish the extent of this problem.

c. Run a quench test on Retort No 2 to develop flowing density measurement

P W Snyder
1-10-66

PILOT RETORT OPERATION SUMMARY FOR NOVEMBER AND DECEMBER 1965

--	1		2		3	4	4		7	
	High Recycle Rate		Effect of Dilution Gas		Very Low		External Combustion		Multi-Level Air-Gas	
Best Prior Operation	High Recycle Low Air	Low Recycle High Air	BOM Condition	High Shale Rate	Air Injection Velocity	Retort No. 1 Performance	Best Condition	High Recycle	Hot	Cold
2	2	2	2	2	2	1	1	1	1	1
510	400	500	300	500	500	490	320	300	490-500	400
31	30	31	30	29	28	32	28	30	27-30	28
4,300	4,100	5,800	5,000	4,500	4,500	4,400	--	--	4,500	6,000
--	--	--	1,200	1,100	--	--	--	--	--	--
--	--	--	--	--	--	--	1,360	2,040	--	--
--	--	--	--	--	--	--	16,600	17,700	6,000	7,500
--	--	--	--	--	--	--	1,300	1300/1100	900/1200	120
15,900	19,200	11,400	14,300	15,800	16,000	16,400	12,000	15,500	12,000	14,000
9	9	9	7	9	9	9	9	9	9	7
--	--	--	--	--	--	--	8	8/7(3)	6-8	5
90	88	85	91	88	86	86	92	89	86-89	83
32	33	42	34	31	32	20	7	29	25-30	38
Very Good	Very Good	Very Good	Very Good	Very Good	Very Good	Very Good	Very Good	Good	Very Good	Very Good
30	20	30	20	30	10	150	50	30	30-150	200
15	7	4	5	6	6	7	22	14	8-12	5
0.9	0.7	0.5	0.3	1.2	1.1	0.6	0.7	0.7	0.6-0.8	0.5

eight per cent moisture on raw shale for Best Operation and Programs 1 - 3; improved raw shale rate moisture measurement used
Program No. 4.
00° F gas temperatures.
lly between two levels.

Program Number:

Program Descriptions:

Retort No.:

Operating Conditions:
Shale Rate, lb/(hr)(ft^2)
Fischer Assay, gal/ton

Gas Rates, SCF/T
Air Rate
Dilution Gas Rate

Oxygen
Hot Gas
Temperature, °F

Recycle Rate

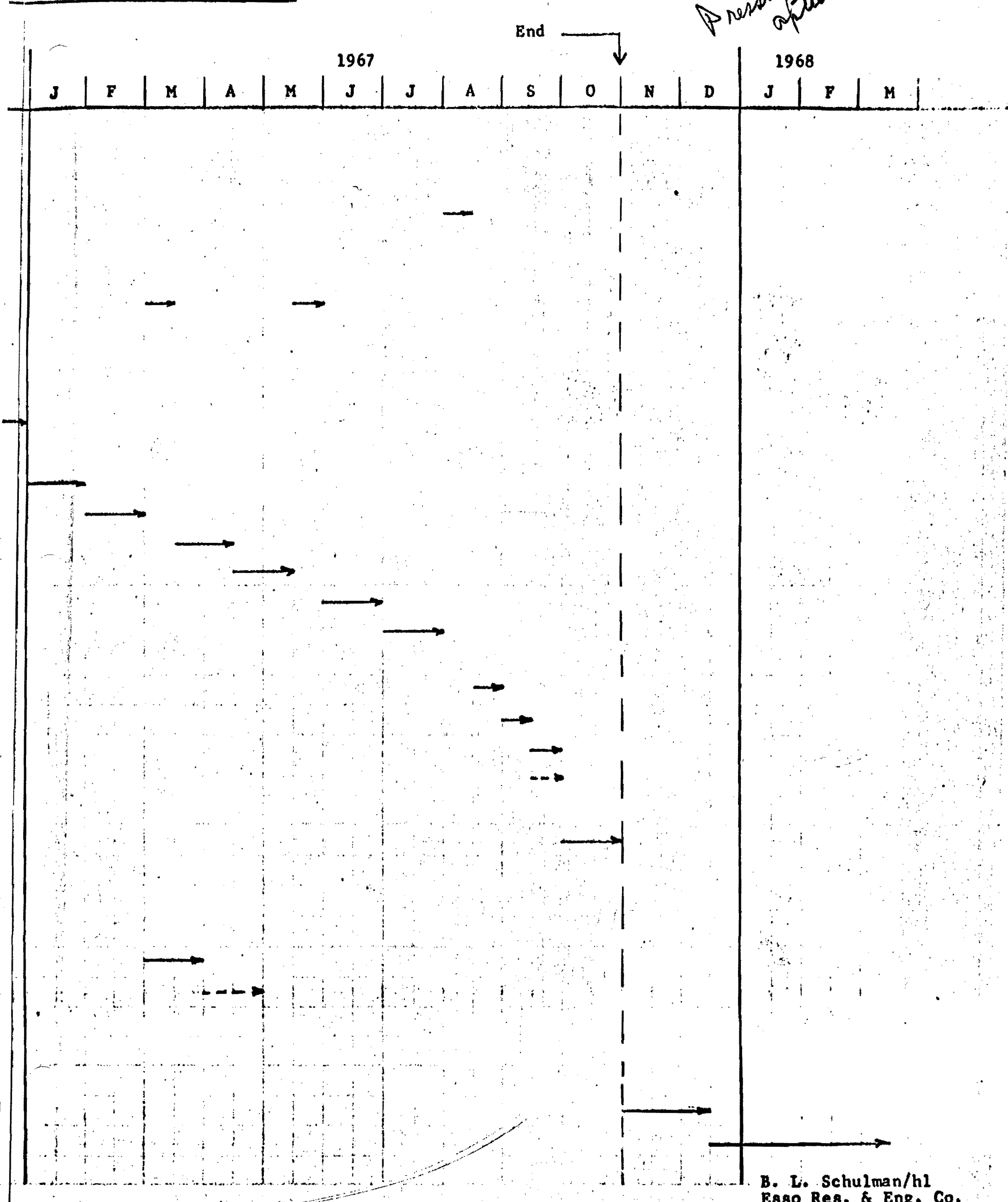
Bed Height, Ft.
Above Air Distr.
Above Hot Gas Distr.

Test Results:
Oil Yield, Vol % RSFA[(1)]
Carbonate Decomposition, %

Operating Characteristics:
Performance
Air or Gas Injection
Velocity, ft/sec
Shale Residence, Mins.[(2)]
Δ P, inches H_2O/ft

(1) Yields adjusted to 1.3 we
beginning with runs for P
(2) Zone between 800 and 1,00
(3) Hot gas split about equal

LUS OPTION ON PRESSURE STUDIES



B. L. Schulman/hl
Esso Res. & Eng. Co.
Dec. 1, 1965

POSSIBLE STAGE II TIMING PL[illegible]

ITEM	Retort 1	Retort 2	Retort 3	M (Begin, 1966)	J	J	A	S	O	N	D
•Proc. Var. Study: 1-1/2 - 3 inch		2		→							
" " " 1/4 - 3 inch		2			→						
•Mist Recovery Equip. Tests		2				→					
•Vacation							→				
•Gas Diseng'mnt + Dust Removal	1						→				
•Base Point Yields	1	2						→			
•Brainstorming, Other	1	2							→		→
•Shakedown #3			3							→	
•Bu. of Mines Check			3								
•Part. Size Studies			3								
- 1/4-3/4 inch											
- 3/4 - 1-1/2 inch											
- 1-1/2 - 3 inch											
- 1/4 - 3 inch											
•Mist Recovery Units			3								
•Multi-Level Air Injection, Other Distributors			3								
•Mechanical Internals			3								
•Shale Richness			3								
•Minimum Air			3								
•Water Injection			3							→	
•Put Plant in Standby											
Pressure Option											
•Shakedown #1	1						→	→			
•Test Runs	1								→		
•Evaluate Press Runs, Decide if worthwhile to try large unit											
•Shakedown Large Unit			3′								
•Demonstrate Opt. Press. on Best #3 Atmos. Cond.			3′								

HUMBLE OIL & REFINING COMPANY
HOUSTON, TEXAS 77001

SHALE OIL PROJECT

POST OFFICE BOX 2180

PROPOSED REEXAMINATION OF STAGE II

Objective

Provide detailed engineering and product data on mining, crushing, and retorting to permit direct scale-up to design of commercial plant.

Procedure

I. Select alternatives for evaluation

   A. Retorting
      1. Gas combustion
         a. Atmospheric pressure
         b. 0-60 psig
      2. Externally heated
      3. Multiple vessel
   B. Mining research
   C. Crushing research

II. Define scope of evaluation

   A. Initial cost of facilities (including modern instruments, additional analytical equipment, etc.)
      1. Mining
      2. Crushing
      3. Retorting
   B. Outline of research program
      1. Variables to be investigated
      2. Order in which to be investigated
      3. Approximate time schedule
   C. Schedule
      1. Preparation of facilities
         a. Mining
         b. Crushing
         c. Retorting
      2. Carrying out of experimental program
         a. Retorts 1 and 2
         b. Retort 3
   D. Staffing
      1. Technical
         a. Groups (operations, mechanical, analytical, etc.)
         b. Number in each group
      2. Non-technical
         a. Groups
         b. Number in each group
   E. Overall cost
      1. Initial cost of facilities
      2. Operations, including modifications to equipment

III. TAC to appoint special Task Force of Technical Observers to assist Anvil Points staff to
   A. Reexamine Stage II
   B. Help Program Manager keep Stage II plans updated, analyzed, and reported to TAC

Financing

I. Inform respective managements of estimated overall cost, schedule for Stage II

II. Ask for approval of Stage II as per present contract ($3MM or 18 mos., whichever reached first) with understanding that 1 or 2 extensions will be required and that overall cost, schedule will be approximately as shown in I.

NPPeet:me
12/10/65

ADVANTAGE FOR PRESSURE RETORTING

No credit for power recovery turbines

Basis:

An advantage of $13 MM in onsite investment cost (present day dollars) for a retort complex producing 120,000 B/CD of retorted oil from 30 gal./ton shale.

40-45 psig
1100 lb/hr ft²
shale rate

F. A. yield	90%
Income tax	50%
Investment credit	7%
Depreciation	6.6%
Offsites	Standoff
Operating costs	"
Interest rate	10% after taxes

4% yield loss will offset any benefit.
1100 lb/hr ft²

Assumptions:

Addition of 50,000 B/CD capacity each year for ten years beginning 1970. An investment savings of $5.41 MM is obtained for each 50,000 B/CD increment based on a linear proration of the 120,000 B/CD costs (actual savings would probably be somewhat higher for the smaller plants). Depreciation and investment credit started in year following commitment of investment corresponding to plant start-up. Depreciation calculated on double declining balance basis. (This gives the most conservative result, i.e., lowest advantage for pressure.) Note: The scheduling shown is for illustrative purposes only and should not be construed as the actual timetable for any company or group of companies.

Conclusions:

Cash flow table attached. Present worth of project including incremental research cost is $15.7 MM. This is $31.40/B/CD or 8.6¢/B/yr. Incremental research costs of $1.71 MM generate gross investment savings of $54.1 MM on the 500,000 B/CD total capacity. Profitability of incremental research (present worth of savings divided by present worth of incremental research cost) is 22.5.

offgas avail at 15 psi

No comm'l ESP avail for 60 psi

Paper 64-8 Air Polution Control Assn 6/21/64 BuMines

HT & HP ESP

H.P.Dengler/hl
Petroleum Staff
Dec. 9, 1965

CASH FLOW TABLE FOR PRESSURE RETORTING PROJECT

(All cost data in millions of dollars. Advantage for pressure shown as)
(positive values, debits have a minus sign proceding them.)

Year (Jan.)	Incremental Research Cost	Inv. Saving	Inv. Credit	Depreciation	Undepreciated Inv.	Cash Flow A.T.	P.W. Factor	P.W.
1966	-.04					-.02	1.0	-.02
1967	-.42					-.21	.91	-.19
1968	-1.25					-.63	.83	-.52
1969						-	.75	-
1970		5.41	~~.38~~	-	5.41	5.41	.68	3.68
1971		5.41	-.38	-.72	10.10	4.67	.62	2.90
1972		5.41	-.38	-1.33	14.18	4.37	.56	2.45
1973		5.41	-.38	-1.87	17.72	4.10	.51	2.09
1974		5.41	-.38	-2.34	20.79	3.86	.47	1.81
1975		5.41	-.38	-2.74	23.46	3.66	.42	1.54
1976		5.41	-.38	-3.10	25.77	3.48	.39	1.36
1977		5.41	-.38	-3.40	27.78	3.33	.35	1.13
1978		5.41	-.38	-3.66	29.53	3.20	.32	1.02
1979		5.41	-.38	-3.89	31.05	3.07	.29	.70
1980			-.38	-4.10	26.95	-2.43	.26	-.63
1981				-3.55	23.40	-1.77	.24	-.43
1982				-3.08	20.32	-1.54	.22	-.34
1983				-2.68	17.64	-1.34	.20	-.27
1984				-2.33	15.31	-1.16	.18	-.21
1985				-2.02	12.64*	-1.01	.16	-.16
1986				-1.67	10.32	-.83	.15	-.12
1987				-1.36	8.31	-.68	.14	-.10
1988				-1.10	6.56	-.55	.12	-.07
1989				-0.87	5.04	-.43	.11	-.05
1990				-0.66	3.73	-.33	.10	-.03
1991				-0.49	2.59	-.24	.09	-.02
1992				-0.34	1.60	-.17	.08	-.01
1993				-0.21	0.74	-.10	.08	-.01
1994				-0.10	0.64	-.05	.07	0
1995				0	0			
Total	-1.71	54.10	3.80	47.61	-	25.66		15.70

* Beginning in 1985 residual undepreciated investment of $0.65 MM is deducted each year since each plant is depreciated over a 15 year period (6.6%/yr. on a straight line basis),

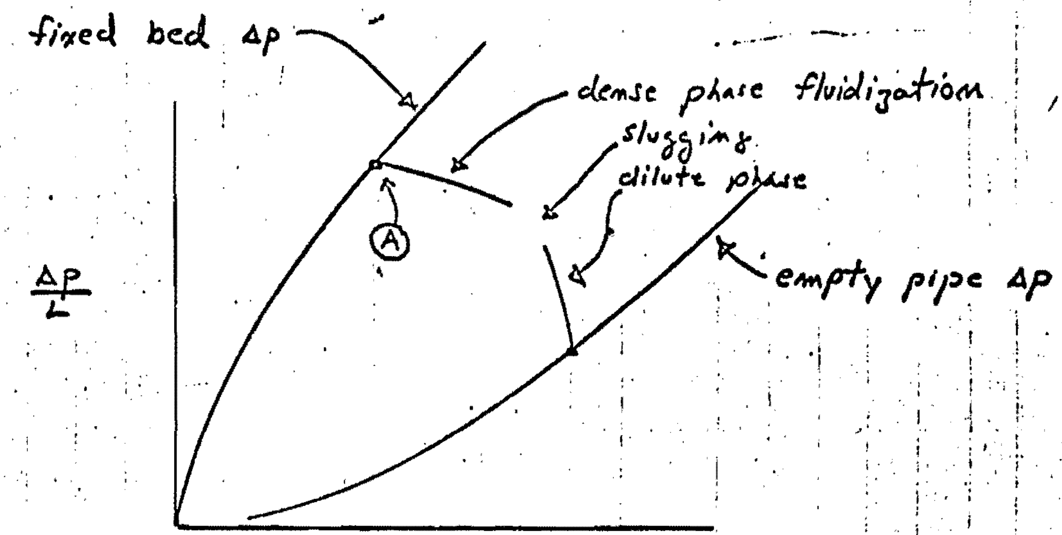
SUMMARY OF ADDITIONAL COSTS FOR INVESTIGATING PRESSURE

Alternative	Extra Cost ($M) for Operations	Investment	Total
A. Run Retort No. 1 in Stage II, backing out marginal work on this unit	0	40	40
B. Expand Stage II by addition of 3½ months for pressure study on Retort No. 1	420	40	460
C. Expand Stage II by:			
Addition of 3½ mos. for pressure study on Retort No. 1	420	40	460
Addition of 4½ mos. for study on large retort, if results from work on Retort No. 1 look good Scrubber	750	500	1250
Alternative C total	1170	540	1710

Incremental ?

300 M Labor
150 M OH
50 Matl

Esso Research and Engineering Company
B. L. Schulman
12/3/65



pt. (A) represents the place where the frictional pressure drop on the bed is equal to the weight of the bed, and if the bed is not restrained, it will 'fluidize'.

The conventional Bureau of Mines atmospheric retort operates with some demonstrated gas velocity, v. The gas passing through the bed loses energy (ie.-pressure) to the rock through friction - the larger particles gain internal energy (heat) and some of the smaller particles are lifted or fluidized (potential and kenetic energy).

The basic criteria for designing the pressurized version of the Bureau of Mines retort is to keep the energy imparted to the shale per unit of bed length, or the $(\Delta p/L)$ constant. In this way, the solids lifting effect (or entrainment) should be the same for both retort versions.

If a relationship between $(\Delta p/L)$ and velocity, density, pressure, etc for the gas exists, then this relationship can be used to determine how to interchange the parameters to keep $(\Delta p/L)$ constant.

Assume the type of correlation developed by Leva, et. al. "Fluid Flow through Packed and Fluidized Systems," U.S. Bureau of Mines Bull. 504, 1951 holds for the retort.

$$\frac{\Delta P}{L} = \frac{2 \div G^2}{\rho} \times \frac{(1-\epsilon)^{3-m}}{\epsilon^3} \frac{2f}{g D_p \lambda^{3-m}}$$

G · mass velocity, gas
ρ · gas density
ϵ porosity
f "friction factor" as a function of particle N_{RE}
g gravity constant
D_p particle diameter
λ particle shape factor

If we wish to double G, what pressure π is necessary to keep $(\Delta P/L)$ constant?

ϵ, D_p, λ are constant for same shale bed
f assumed to be constant at high N_{RE} (this is conservative since f actually will decrease slightly)

then $$\left(\frac{\Delta P}{L}\right)_0 = \frac{\alpha G_0^2}{\rho_0} = \frac{\beta G_0^2}{\pi_0}$$

$$\therefore \quad G = \sqrt{\pi} \sqrt{\frac{\Delta P}{L\beta}}$$

Therefore if we double G, then we must raise the pressure by a factor of 4 for constant $(\Delta P/L)$. The new velocity is ½ as great as the original:

$$G_0 = \rho_0 v_0 \qquad G = 2G_0 = 2\rho_0 v_0 , \qquad \frac{G}{G_0} = 2 = \sqrt{\frac{\pi}{\pi_0}}$$

$$\therefore \quad \pi/\pi_0 = 4 = \rho/\rho_0$$

$$\rho v = 2\rho_0 v_0 = 4\rho_0 v , \quad \therefore \; v = \left(\frac{2\rho_0}{4\rho_0}\right) v_0 = \frac{1}{2} v_0$$

Conclusion: The gas mass velocity can be doubled for the same $(\Delta P/L)$ if the pressure is increased — 4 times.

The generalized correlation by Zenz defines a friction factor, which is a function of particle N_{Re} and voidage ϵ.

$$f = \frac{g D_p}{2 \rho_f v^2}\left(\frac{\Delta p}{L}\right), \text{ or } \left(\frac{\Delta p}{L}\right) = \frac{2 f G^2}{\rho g D_p}$$

To double G at constant $(\Delta p/L)$ $\quad 1 = \left(\frac{f_2 G_2^2}{\rho_2}\right)\left(\frac{\rho_1}{f_1 G_1^2}\right)$

Since $\rho \propto \pi$, $G_2 = 2G_1$ $\quad \pi_2 = 4 \cdot \frac{f_2}{f_1}$

Assume a base case $N_{Re} = 800$, then $f_2 = 8.3$, $f_1 = 9.2$

Therefore $\pi_2 = 4 \frac{8.3}{9.2} = 3.6 \pi_1$ or we might be somewhat conservative.

Several other equations appear in the litterature, such as proposed by Ergun which includes kenetic energy losses.

$$\frac{\Delta p}{L} = \frac{G}{\rho g D_p} \cdot \frac{1-\epsilon}{\epsilon^3}\left[\frac{150 (1-\epsilon)\mu}{D_p} + 1.75\ G\right]$$

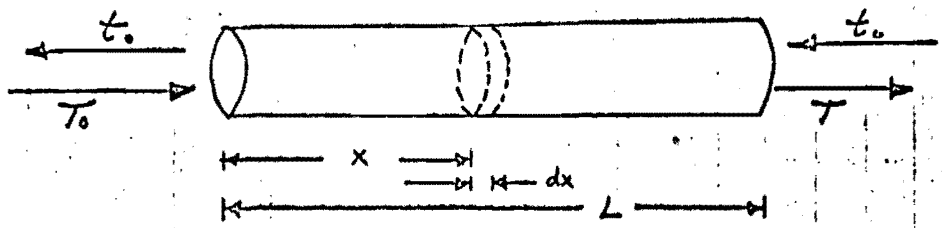
This is too complicated for simple analysis. Happel (Ind. Eng. Chem., 41, 1161, 1949) published an equation for moving fixed beds:

$$\frac{\Delta p}{L} = \frac{2 f G^2 (1-\epsilon)^3}{g \rho D_p}$$

where f decreases as $D_p G (1-\epsilon)/\mu$, a modified Reynolds number, increases. Note that this is the same form as the basic equation.

Conclusion is that by maintaining $\Delta p/L$ constant, the force exerted on the bed and entrainment will remain at about the same level as the atmospheric retort. Since $\Delta p/L$ is a function of both gas velocity and density, the gas mass rate can be double if the pressure is quadrupled and the $\Delta p/L$ will remain constant.

Heat Transfer - Pressurized Retort



a = heat transfer area ft^2/ft^3 or $1/ft$
A = cross sectional area ft^2
S, G Solid and Gas mass velocity $lb/ft^2 hr$
C_s, C_G specific heats of solid and gas BTU/lb °F
h heat transfer coefficient BTU/ ft^2 hr °F
t, T temperature of gas and solid respectively, °F
θ time, hr

Energy Balance to x

$$S C_s (T_x - T_0) = G C_G (t_x - t_0) \qquad (1)$$

Heat transfer in increment dx = $(a A dx) h (t_x - T_x)$

note assumes that $t_x - T_x \cong t_{x+dx} - T_{x+dx}$

The heat transfer change the temperature of the solid and gas by

$$(A S C_s)\,(T_{x+dx} - T_x) = (a A dx)(h)(t_x - T_x) \qquad (2)$$

$$(A G C_G)\,(t_{x+dx} - t_x) = (a A dx)(h)(t_x - T_x) \qquad (3)$$

The three equations can be solved for temperature t_x, T_x

$$t_x = \left[t_0 + \frac{G C_G t_0 - T_0 S C_s}{S C_s - G C_G} \right] e^{\left(\frac{ah}{G C_G} - \frac{ah}{S C_s}\right)x} - \frac{G C_G t_0 - T_0 S C_s}{S C_s - G C_G}$$

$$T_x = \left[T_0 + \frac{G C_G T_0 - T_0 S C_s}{S C_s - G C_G} \right] e^{\left(\frac{ah}{G C_G} - \frac{ah}{S C_s}\right)x} - \frac{G C_G t_0 - T_0 S C_s}{S C_s - G C_G}$$

These equations give the temperature profile when
(1) $SC_s \neq GC_G$
(2) t_0, T_0 are retort top temperatures.

Note the exponent on $e = \left(\frac{ah}{GC_G} - \frac{ah}{SC_s}\right)x$

let L be the calculated retort length for a given t_0, t_L, T_0, T_L

define $y = \frac{x}{L}$, $x = Ly$

That exponent becomes

$$\left(\frac{ah}{GC_G} - \frac{ah}{SC_s}\right)Ly = \left(\frac{ahL}{GC_G} - \frac{ahL}{SC_s}\right)y$$

This means if the ratio of $\frac{GC_G}{SC_s}$ is held constant between cases but the value of h changes, it is only necessary to change the retort length L to obtain an identical temperature profile.

It is now necessary to know how h varies with G for heat transfer between fluid and shale particles. One must be careful not to use correlations for wall coefficients. For fixed beds, Glaser developed

$$\frac{h}{C_p G}\left(\frac{C_p \mu}{k}\right)^{2/3} = \frac{.535}{(\phi Re)^{.3} - 1.6} \qquad 100 < \phi Re < 9200$$

k = thermal conductivity

$Re = \frac{\sqrt{A_p}\, G}{\mu(1-\epsilon)}$

A_p surface of particle

ϕ particle sphericity

ϵ void fraction

$\therefore \quad h \propto \frac{G}{(\phi Re)^{.3} - 1.6}$

since the denominator is fairly insensitive to (ϕRe)

assume that $(\phi Re)_2 = 2000$, $(\phi Re)_1 = 1000$. $(Re \propto G)$, $G_2 = 2G_1$

$$\frac{h_2 L_2}{G_2} = \frac{h_1 L_1}{G_1}$$

$$L_2 = \left(\frac{G_2}{G_1}\right) \frac{G_1}{(\phi Re)_1^{.3} - 1.6} \frac{(\phi Re)_2^{.3} - 1.6}{G_2} L_1 = \frac{19.8 - 1.6}{7.9 - 1.6} = \frac{18.2}{6.3} L_1 = 1.3 L_1 = (2)^{.38} L_1$$

For fluid beds, Glamson and Chu proposed

$$\frac{h}{C_p G} \left(\frac{C_p \mu}{k}\right)^{2/3} = 1.77 \left[\frac{D_p G}{\mu(1-\epsilon)}\right]^{-.44} \quad \frac{D_p G}{\mu(1-\epsilon)} > 30$$

$$h \propto \frac{G}{G^{.44}} = G^{.56}$$

$$\therefore \quad L_2 = \left(\frac{G_2}{G_1}\right)\left(\frac{G_1}{G_2}\right)^{.56} L_1 = \left(\frac{G_2}{G_1}\right)^{.44} L_1 = 1.36 L_1$$

Note both exponents agree quite well .38 versus .44, an exponent of .5 was chosen to be conservative, such as any change required for internal heat transfer in the shale. This resulted in a 41.5 % increase in height and a 29 % reduction in shale residence time.

Conclusions:

The temperature profile will remain constant if the ratio of GC_G / SC_S is held constant since the quantity

$$\frac{a}{C_G} \frac{hL}{G} - \frac{a}{C_S} \frac{hL}{S}$$ is held constant

from eq. ① $T = T_0 + \frac{GC_c}{SC_s}(t - t_0)$

from eq (3) $(t_{x+dx} - t_x) = \left(\frac{ah}{GC_G}\right)(t - T)\,dx$

eq(2) $(T_{x+dx} - T_x) = \frac{ah}{SC_s}(t - T)\,dx$

define $\sigma = \frac{ah}{SC_s}$ $\gamma = \frac{ah}{GC_G}$ units $= \frac{BTU \cdot ft^2 h \cdot lb\,°F}{ft \cdot ft^2 h\,°F \cdot lb \cdot BTU} = \frac{1}{ft}$

then $\frac{GC_c}{SC_s} = \frac{\sigma}{\gamma}$

by Taylor expansion $t_{x+dx} = t_x + \left(\frac{dt}{dx}\right)dx$

eq 1 $T = \frac{\sigma}{\gamma}t - \frac{\sigma}{\gamma}t_0 + T_0$

eq 1 + 3. $t_x + \left(\frac{dt}{dx}\right)dx - t_x = \gamma\left[t - \frac{\sigma}{\gamma}t + \frac{\sigma}{\gamma}t_0 - T_0\right]dx$

$$\frac{dt}{dx} = (\gamma - \sigma)t + \sigma t_0 - \gamma T_0 = (\gamma - \sigma)t + \beta$$ ④

$\beta = \sigma t_0 - \gamma T_0$

$\frac{dt}{dx} - (\gamma - \sigma)t = \beta$ 1st order linear

$$e^{-(\gamma+\sigma)x}dt - (\gamma - \sigma)e^{-(\gamma-\sigma)x}t\,dx = \beta e^{-(\gamma-\sigma)x}dx$$

$$\int_{0,t_0}^{x,t} d\left[t\,e^{-(\gamma-\sigma)x}\right] = \beta\int_0^x e^{-(\gamma-\sigma)x}dx$$

$$t e^{-(\gamma-\sigma)x} - t_0 = -\frac{\beta}{\gamma-\sigma}\left[e^{-(\gamma-\sigma)x} - 1\right]$$

or $t = t_0 e^{(\gamma-\sigma)x} - \frac{\beta}{\gamma-\sigma} + \frac{\beta}{\gamma-\sigma}e^{(\gamma-\sigma)x}$

$$\frac{\beta}{\gamma-\sigma} = \frac{\sigma t_0 - \gamma T_0}{\gamma - \sigma} = \frac{\frac{t_0}{SC_s} - \frac{T_0}{GC_G}}{\frac{1}{GC_G} - \frac{1}{SC_s}} = \frac{GC_G t_0 - T_0 SC_s}{SC_s - GC_G}$$

$$t = \left[t_0 + \frac{GC_G t_0 - T_0 SC_S}{SC_S - GC_G}\right] e^{\left(\frac{ah}{GC_G} - \frac{ah}{SC_S}\right)x} - \frac{GC_G t_0 - T_0 SC_S}{SC_S - GC_G}$$

repeat to find T

from eq (1) $$t = t_0 + \frac{SC_S}{GC_G}(T - T_0) = t_0 + \frac{\gamma}{\sigma}(T - T_0)$$

from eq (2)

$$\left(T_{x+1}\left(\frac{dT}{dx}\right)dx - T_x\right) = \sigma\left[\frac{\gamma}{\sigma}T - \frac{\gamma}{\sigma}T_0 + t_0 - T_0\right]dx$$

$$\frac{dT}{dx} = (\gamma - \sigma)T - \gamma T_0 + \sigma t_0 = (\gamma - \sigma)T + \beta \qquad (6)$$

note that equation 6 is identical to equation (5), therefore

$$T = \left[T_0 + \left(\frac{GC_G t_0 - T_0 SC_S}{SC_S - GC_G}\right)\right] e^{\left(\frac{ah}{GC_G} + \frac{ah}{SC_S}\right)x} - \frac{GC_G t_0 - T_0 SC_S}{SC_S - GC_G}$$

References

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