APPLICATION OF META-HEURISTIC OPTIMIZATION TO PORTFOLIO ANALYSIS IN THE OIL AND GAS INDUSTRY

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ABSTRACT

Technological advances have made enormous improvements in the exploration and development of petroleum prospects, but these improvements have not resulted in higher return on net assets. The exploration and production (E&P) industry has reported disappointing seven percent return on investment (ROI), which on average is less than the cost of capital (Brashear 2000). Improper capital rationing, project selection criteria, and lack of financial risk analysis are some of the reasons behind the insufficient attainment of shareholder goals.

It was seen first hand in this dissertation the difficulties faced by the oil companies in terms of additional amounts of analysis time required to use portfolio analysis. In the optimization stage using thirty investment projects, the analysis took more than six days of computer time.

The hypotheses in this dissertation are:

-Portfolio analysis gives improved financial results in terms of economic performance over traditional methods,

-Probabilistic economics are an essential part in risk analysis,

-The use of different risk measures helps in giving a more complete picture of the uncertainties,

-Meta-Heuristics methods allow for more detailed portfolio optimizations within a reasonable time frame.

This dissertation proposes a new model that incorporates portfolio management and financial risk analysis through the use of Meta-Heuristics methods. This new model improves project selections and capital budgeting through the combination of probabilistic modeling, portfolio analysis, and optimization methods. The value of a company's assets are maximized as a result of this recommended combination.

This research has shown that the use of Meta-Heuristics methods as a part of portfolio optimization is superior to traditional project selection. Through portfolio analysis, risk and uncertainty can be evaluated and managed. Companies should not rely on one risk definition, as this research shows. The use of different risk measures is essential to get different perspectives on the uncertainties in the project selection. Stochastic analysis is an essential part of the model with it risks and rewards can be defined as probabilities rather than static numbers.

The use of efficient frontier theory in portfolio optimization is shown to provide helpful insight in evaluating petroleum prospects. The new model was built using a spreadsheet platform to make it more adaptive to unconventional evaluation techniques, which are not offered by commercial packages. Twenty hypothetical, but realistic, investment projects were created as the basis of analysis for this new model. In testing the new model sixteen different portfolio scenarios were evaluated, with each portfolio scenario considering all twenty projects.

The use of this model is recommended for oil companies to improve their project evaluation process and financial performance.

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DEDICATION

I dedicate this thesis to the souls of my parents, may they rest in peace.

CHAPTER ONE

INTRODUCTION

In recent years technological advances have made enormous improvements in the exploration and development of petroleum prospects, but these improvements have not resulted in higher return on net assets. Improper capital rationing, project selection criteria, and lack of financial risk analysis are some of the reasons behind the insufficient attainment of shareholder goals.

For these reasons, the hypotheses of this dissertation are:

- Portfolio analysis gives improved financial results in terms of economic performance over traditional methods,
- Probabilistic economics are an essential part in risk analysis,
- The use of different risk measures helps in giving a more complete picture of the uncertainties,
- Meta-Heuristics methods allow for more detailed portfolio optimizations within a reasonable time frame.

One might think that investment management within the oil and gas industry is straightforward, where the goal is to find oil and gas and produce it efficiently and economically. In an ideal world a company would develop all the oil and gas it can find that passes an earnings hurdle rate, such as rate of return (ROR). Unfortunately, companies cannot invest in all available projects, so oil companies usually follow a procedure to choose the set of projects for investment. Such a procedure is usually based on capital budgeting constraints, where a company chooses among current projects until it runs out of capital to invest.

Project selections usually result from a ranking procedure, based on their net present value (NPV) or rate of return (ROR). This procedure allows projects to be ranked from best to worst. Capital is then spent on projects according to their ranks until the capital budget is exhausted. This will tend to favor large projects over small ones, since there might be a combination of small projects that has a higher NPV than a single large one, but this will not be discovered because each small project is evaluated on its own small NPV merits.

A variation of this method is the profit-to-investment ratio (PIR) as an economic yardstick for ranking projects. This method allocates capital efficiently, but it cannot handle multiple constraints. These constraints could include a contractual obligation for the supply of oil or gas, minimum cash flow for a given year, or a reserve addition goal. Also, constraints could come from conflicting aims of different departments inside the same organization. In summary, risk analysis is difficult to incorporate into portfolio optimization under current corporate practices.

The objective of this dissertation is to provide the oil and gas industry with a new model that will fill the voids left by current practices. In this new model probabilistic modeling, portfolio analysis, and optimization methods are combined and published for the first time in order to maximize the value of the company's assets. Also, capital rationing and other constraints are incorporated. Risk is quantified and the analysis performed through the portfolio optimization process. Correlations among variables in a single project, and between different projects, are also included in this study.

The new model utilizes the portfolio theory developed by the Nobel prize winner Harry Markowitz (1957). His theory states that increased return generally implies assuming increased risk. Also, the new model uses Markowitz's "efficient frontier" of stock portfolios, which describes the optimal investments for investors with differing aversions to risk.

Finding the best mix of projects for the company is not an easy task. If a company has only ten prospects to evaluate and decides to consider participating in 25-percent increments of the working interest, that will lead to more than one million possible outcomes (4^{10}), or combinations. With the addition of a few more projects, the task of evaluating all possible permutations becomes unfeasible.

One way to overcome this problem is through the use of Meta-Heuristics methods. Heuristics have been used for many years to provide approximate solutions to complex problems. Meta-Heuristics optimization methods have only recently been introduced to the oil and gas industry. Two of the best known Meta-Heuristics are Genetic Algorithms (GA) and Tabu Search. Combinations of Tabu Search and Scatter method are used in this model as an optimization algorithm. These terms are fully covered in the next chapter.

Monte Carlo simulation is used in this portfolio analysis to transform the model from deterministic to a stochastic one. Monte Carlo simulation is a statistics-based analysis tool that yields probability versus value relationships for key parameters, including oil and gas reserves, capital exposure, and various economic yardsticks, such as net present value (NPV) and return on investment (ROI). Hypothetical but realistic projects have been used herein to examine the effects of using different economic yardsticks through Meta-Heuristic optimization methods. Monte Carlo simulation is used to provide the probabilistic models.

The first of two stages in this study employ the model to generate a distribution of the net present value (NPV) of the portfolio. The second and final stage identifies the most efficient portfolio using the limited number of projects and considering the human resources to meet the objectives, while working within any constraints.

CHAPTER TWO

LITERATURE REVIEW

This chapter gives brief background information on each method mentioned in the Introduction, to aid the readers in familiarizing themselves with the topic discussed. First an overview of the stochastic models is presented, followed by a discussion of portfolio theory. Then optimization methodology is covered, followed by the various tools that are used in the research reported herein.

Stochastic Models

The terms stochastic and probabilistic modeling will be used interchangeably throughout this dissertation. Monte Carlo simulation is popular in stochastic modeling, and there are several other types of stochastic technique. For the purpose of this dissertation, the focus will be on Monte Carlo simulation as the only technique in stochastic modeling.

Monte Carlo simulation was named for Monte Carlo, Monaco, where the primary attractions are casinos containing games of chance. Games of chance, such as roulette wheels, dice, and slot machines, exhibit random behavior.

The random behavior in games of chance is similar to how Monte Carlo simulation selects variable values at random to simulate reality. When a die is rolled, the outcome is either a 1, 2, 3, 4, 5, or 6, but it is unknown for any particular roll. It is the same with any variables that have a known range of values, but an uncertain value for any particular time or event (e.g. interest rates, well costs, oil prices, production rates, etc).

Stochastic simulation is an analytical method meant to imitate a real-life system, especially when other analyses are too mathematically complex or too difficult to reproduce. Without the aid of simulation, a spreadsheet analysis will reveal only a single outcome, generally the most likely or average scenario.

Spreadsheet risk analysis uses both a spreadsheet model and simulation to automatically analyze the effects of varying inputs on outputs of the modeled system. Monte Carlo simulation is one type of spreadsheet simulation, which randomly generates an enormous number of values for uncertain variables in order to simulate reality.

Hertz (1964) has shown that the Monte Carlo technique is applicable to business decisions. His model was a production line for a manufacturing company. It was not until 1975 that two authors published two books detailing the use of Monte Carlo simulation in oil and gas economic evaluations. These authors were Newendorp (1975) and McCray (1975). Since then, several other authors have demonstrated the importance of recognizing dependencies among input variables, but the general approach remains similar to that presented in 1975.

In 1975 Newendorp considered a state-of-the-art risk analysis evaluation to be one which would account for all uncertainties relating to geologic and economic factors. Kelliher (2000) used Monte Carlo simulation as a tool to quantify the inherent uncertainties surrounding many of the estimates used to model long-term capital investment decisions.

Monte Carlo simulation is a statistics-based analysis tool that yields probability versus value relationships for key parameters, including oil and gas reserves, capital exposure, and various economic yardsticks, such as net present value (NPV) and return on investment (ROI). Murtha (1997) has published many papers on this subject. His paper "Monte Carlo Simulation: Its Status and Future" (1997) is a must read for the method users.

These probability relationships help the user answer such questions as, "What is the probability that the NPV of this prospect will exceed the target of $1,500,000?", or, "How likely is it that the reserves added from this year's exploration program will fall short of our planned production? " Monte Carlo simulation is a part of risk analysis and is sometimes performed in conjunction with or as an alternative to decision analysis.

Among the numerous words and phrases associated with risk analysis are decision analysis, risk assessment, risk management, portfolio management and optimization, and strategic planning. In some contexts, these words are used only in a qualitative sense, but the focus herein is quantitative.

Decision analysis, in its broadest form, includes problem identification, specification of objectives and constraints, modeling, uncertainty analysis, sensitivity analysis, and rules that lead to a decision. Generally speaking, risk analysis and assessment refer to the quantification of uncertainty, almost always in the context of possible investments. In the oil and gas business, although much of the analysis might pertain to reserve size, capital cost, production forecasting, and the like, the bottom line universally is monetary value.

Gali (1999) made a comparison between three methods for evaluating oil projects: Option Pricing, Decision Trees, and Monte Carlo simulations. His work concluded that although the methods appear to be different, they are part of the same general framework. Gali pointed out the flexibility of Monte Carlo simulation and that it is a natural extension of the deterministic NPV case.

Komlosi (2001) did thorough research on risk evaluation using the stochastic approach (Monte Carlo), and some of his findings are used herein in determining the risk involved with each project and the portfolio as a whole.

Portfolio Analysis

In recent years the practice of portfolio management and optimization has gained increasing importance in the oil and gas industry. With the increase in project opportunities, corporations are faced with an increasingly difficult task.

A portfolio is an aggregation of investment opportunities. Portfolio managers mix their prospects to reduce collective risk and enhance return on investment. Optimization is often taken as maximizing some measure of reward, such as NPV or profit-to-investment ratio, subject to constraints on risk. Strategic planning involves portfolio management, but may include more intangible aspects of investments, such as maintaining an annual minimum production of gas to meet a contractual obligation.

Decision and Portfolio Analysis in Exploration and Production

The pioneering work of Markowitz (1957) formalized the insight that increased economic return generally implies assuming increased risk. His efficient frontier of stock portfolios described the optimal investments for investors with differing aversions to risk. His 1957 book on portfolio selection remains an excellent introduction to this subject. Sharpe (1964) streamlined and expanded on Markowitz's work with his Capital Asset Pricing Model (CAPM). Both of these researchers received Nobel prizes in economics for their discoveries.

Decision analysis was first applied to Exploration and Production (E&P) on a project-by-project basis by Grayson (1960) and Kaufman (1963), and popularized by McCray(1975), Newendorp (1975), Megill (1977) and (1979), Cozzolino (1987), and others.

Ball (1983) proposed that petroleum E&P strategy be based on the Markowitz model, and he discussed examples based on mainframe computer computations. In a follow-up article Ball (1983) proposed the development of efficient frontiers for E&P projects using personal computers. According to Howell (2001), Portfolio Analysis can be used to help develop corporate strategy.

Brashear (1999) has done a study on the interdependencies among E&P projects and portfolio risk management. He concluded that explicit analysis of above ground uncertainties and interdependencies in evaluating the risk of individual E&P projects improves the quality of investment decision making.

Delfiner (2000) suggested a simple model to deal with below ground uncertainties by modeling dependencies between geologic risks in multiple target prospects. His model relies on aggregating geologic risk factors for each of the prospect targets. The input data for his model is updated with new information acquired during the drilling process. This updating can influence the geologic risk factors for the prospect's different targets. The risk factors in turn will affect the recovery factors estimated for the prospect.

Classical Optimization and Meta-Heuristics Methods

Optimization deals with finding the best (optimal) solution to problems. The problem in general can be expressed in the form of an objective function to be optimized and a set of constraints which restrict the values of the decision variables.

Linear Programming

The best known optimization tool is the linear programming model, which assumes that both the objective function and constraints can be expressed using linear functions. Linear programming techniques are able to find optimal solutions to problems without the need to evaluate all possible alternatives. Models with thousands, or even millions, of variables can be solved with reasonable amounts of computer time.

However, not all problems can be expressed by means of a linear objective and linear equalities or inequalities. Many complex systems may not even have a convenient mathematical representation, linear or non-linear. Techniques such as linear programming and its cousins, non-linear programming and integer programming, generally require a number of simplifying assumptions about the real system to be able to properly frame the problem.

Heuristics Methods

Heuristics have been used for many years to provide approximate solutions to complex problems. For example, in production scheduling, a production heuristic may be to give priority to jobs with the shortest estimated processing time. Depending on the context, this heuristic (or processing rule) may actually work fairly well. However, in some other situations the results may be disastrous, with dire consequences for equipment utilization, production lead times, and work-in-progress inventory.

Meta-Heuristics Methods

A Meta-Heuristic is an iterative master process that guides the operations of subordinate Heuristics. Two of the best-known Meta-Heuristics are Genetic Algorithms and Tabu Search. Genetic Algorithms (GA) procedures were developed by Holland in the early 1970's at the University of Michigan (Holland 1975). Parallel to the development of GA's, Fred Glover, at the University of Colorado, established the principles and operational rules for Tabu Search (TS) and a related methodology known as Scatter Search (Glover 1977).

Scatter Search has some interesting commonalties with GA ideas, although it has a number of quite distinct features. Several of these features have come to be incorporated into GA approaches after an intervening period of approximately a decade, while others remain largely unexplored in the GA context.

Scatter Search (Glover 1994) is designed to operate on a set of points, called reference points, that constitute good solutions obtained from previous solution efforts. The approach systematically generates linear combinations of the reference points to create new points, each of which is mapped into an associated feasible point. Tabu Search is then superimposed to control the composition of reference points at each stage.

Tabu Search (Glover and Laguna 1993) has its roots in the field of artificial intelligence as well as in the field of optimization. The heart of Tabu Search lies in its use of adaptive memory, which provides the ability to take advantage of the search history in order to guide the solution process.

In its simplest manifestations, adaptive memory is exploited to prohibit the search from reinvestigating solutions that have already been evaluated. However, the use of memory in this implementation is much more complex and calls upon memory functions that encourage search diversification and intensification. These memory components allow the search to escape from locally optimal solutions and in many cases find a globally optimal solution.

Similarities are immediately evident between Scatter Search and the original GA proposals. Both are instances of what are sometimes called "population based" approaches. Both incorporate the idea that a key aspect of producing new elements is to generate some form of combination of existing elements. On the other hand, several contrasts between these methods may be noted.

The early GA approaches were predicated on the idea of choosing parents randomly to produce offspring, and further on introducing randomization to determine which components of the parents should be combined. By contrast, the Scatter Search approach does not correspondingly make recourse to randomization, in the sense of being indifferent to choices among alternatives.

However, the Scatter Search approach is designed to incorporate strategic probabilistic biases, taking into account the evaluations and history. Scatter Search focuses on generating relevant outcomes without losing the ability to produce diverse solutions, because of the way the generation process is implemented. For example, the approach includes the generation of new points that are not direct combinations of the original points. The new points then may contain information that is not contained in the original reference points.

Scatter Search is an information driven approach, exploiting knowledge derived from the search space, high-quality solutions found within the space, and trajectories through the space over time. The combination of these factors creates a highly effective solution process. The incorporation of such designs is responsible for endowing this system with the ability to solve complex simulation-based problems with unprecedented efficiency (Glover 1994).

Merger of Optimization and Simulation

A long standing goal has been the ability to guide a series of simulations in the most effective way, instead of blindly itemizing scenarios with the hope that at least one of those itemized will be the one that is most desirable to implement. The integration of simulation and optimization is putting this goal within practical reach. Figure 2.1 shows this integration.

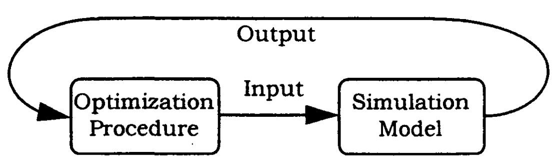

Figure 2.1 Coordination Between Optimization and Simulation

Early attempts to create methods for optimizing simulations relied on the user to run through a cumbersome "seat of the pants" analysis. Alternatively, they have been based on stochastic approximation designs whose main focus involves the analysis of convergence behavior in an infinite time frame. Not surprisingly, the results of such efforts have left a great deal to be desired from a practical standpoint.

As a step in the direction of greater rigor within a finite time horizon, a systematic catalog of all possible alternatives may be examined by complete enumeration algorithms. Although this approach guarantees optimal solutions, it has very limited application. As an example of an exceedingly simple setting where enumeration may be practicable, suppose that a simulation model depends on only two input factors, that is, a portfolio consisting of only two possible projects. If a feasible investment strategy would allow amounts from \$1,000 to \$10,000 to be invested in each of the two projects in multiples of \$1000, then 100 simulation runs are needed to enumerate all possibilities. If each simulation is relatively fast (e.g., 3 seconds), then the entire process could be done in only five minutes of computer time.

However, instead of only two investment projects, if a very modest increase up to five different investments is considered, then enumerating all alternatives to find an optimal one would require $10^5 = 100{,}000$ simulations, or approximately 3.5 days of computer time (Glover 1996). Of course, most simulation settings are not really so simple as this one described. It is easily possible for complete enumeration to take weeks or even months of computer time to carry out.

Analysis Tools

The model developed in this research uses commercial software available in the market place. For stochastic analysis the two major packages available are Crystal Ball and @Risk. Crystal Ball was chosen as the stochastic tool for this research reported herein. As for optimization, there are two Windows based programs, Genocop and OptQuest. Research by Laguna (1997a) has shown that OptQuest out performs Genocop when using stochastic analysis, and for that reason it is used herein for the model.

Crystal Ball

Crystal Ball is a risk analysis software (Decisioneering, Inc.) which automates the cumbersome "what-if" process using Monte Carlo simulation by applying a range of values, or a probability distribution, to each uncertain variable. The program generates random values from within the defined probability ranges, and then recalculates the model. Simulation is performed and observations are collected for forecast values of interest. Crystal Ball operates as an add-in function to Excel.

Genocop

Genocop is a Genetic Algorithm-based program for constrained and unconstrained optimization developed by Michalewicz (1994). Genocop allows decision variables to be either continuous or discrete (both general integers and Boolean). The system aims at finding a global optimum (minimum or maximum) of a function, while meeting constraints. Genocop can handle nonlinear constraints as well as nonlinear objective functions. Genocop is an experimental research code written in C and runs without changes on most UNIX systems. The system, however, can also be compiled in DOS and Windows machines. This software is the property of Zbigniew Michalewicz, but permission is granted to be copied and used for scientific, noncommercial purposes. The following figure shows the Genocop system configuration.

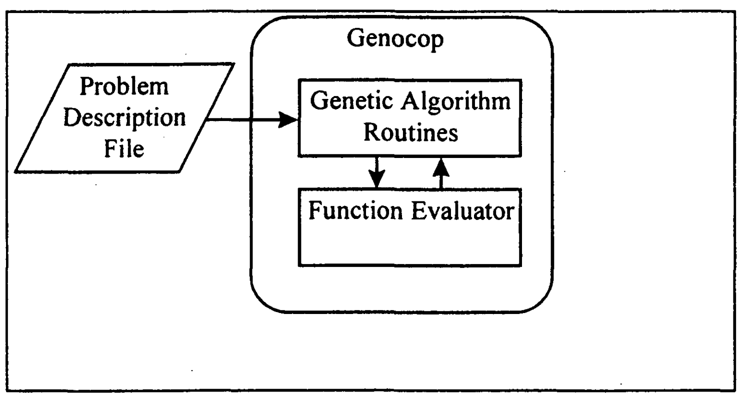

Figure. 2.2 Genocop Representation (after Laguna 1997a).

OptQuest

OptQuest is a general-purpose optimizer developed by Glover, Kelly and Laguna (1999) using the Scatter Search methodology. Scatter Search is a population-based approach that shares some common elements with Genetic Algorithms but uses a fundamentally different search philosophy. A detailed description of Scatter Search can be found in Glover and Laguna (1997b).

OptQuest is one of many functions integrated into Crystal Ball. The simulation model software returns the objective function value corresponding to a particular set of input factors (i.e., decision variables). OptQuest generates new input factors based on Scatter Search strategies and the process repeats. OptQuest then searches for optimal values of the decision variables.

OptQuest can handle more than one return value from the evaluator function, from which one of these values is associated with the objective function and the others are considered goals. The constraints set on the decision variables are defined inside the system and not within the function evaluator (see Figure 2.3).

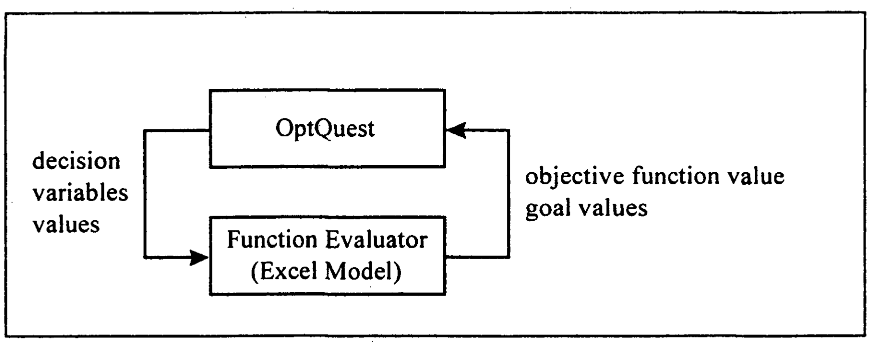

Figure. 2.3 OptQuest Representation (after Laguna 1997a).

In OptQuest, goals are values that can be known only after the function evaluation is performed. In risk analysis, for example, a user may wish to optimize the expected return of an investment portfolio while limiting the standard deviation to be less than a desired value. Genocop would handle this problem by constructing a nonlinear function that penalizes portfolios with standard deviations that are larger than the desired limit. OptQuest, on the other hand, handles this situation by defining an objective to be maximized (i.e., the expected return) and a goal to be satisfied (i.e., a bound on the standard deviation of returns).

CHAPTER THREE

METHODOLOGY

The Spreadsheet Model

The uniqueness of this research model lies in its integration of stochastic modeling with Meta-Heuristics methods for portfolio optimization in project evaluation. The research model consists of two major stages. The first stage is the stochastic stage. This stage is comprised of probability analysis of the stochastic outcome for each individual investment project. The net present value (NPV) of the project is calculated for each case. Monte Carlo simulation is the driving force. All the investment projects are evaluated in this stage.

The purpose of the second stage is to find the best projects that should go into the company's portfolio. Optimization algorithms are the driving force. The portfolio NPV is maximized in this stage while the constraints are enforced.

Stochastic Stage

In this stage, development of the hypothetical investment project is shown. Decline curve formulas were used to estimate the production rates of the oil and gas. Decline curve analyses are based on the following assumptions.

1. Sufficient past production performance is available in order to make a reasonable match of this performance and extrapolate its future performance.
2. The past production history is based on capacity (unrestricted) production with no changes in operational policy, such as artificial lift, stimulation, etc. It is assumed that the property will continue to be operated in the same manner in the future.

In this research, both exponential and hyperbolic decline curves were used. A simple example of the use of Monte Carlo simulation would be to use the hyperbolic decline equation to calculate remaining reserves. The equation is as follows:

$$ERR = \frac{q_i^{\,b}}{(1-b)D_i}\left(q_i^{\,1-b} - q_{el}^{\,1-b}\right)$$

This form of the equation is used here since it easier to convert to the exponential form just by making b=0.

For the purposes of illustration, the following deterministic or single-value estimates of the parameters:

$q_i = 100$ bpd, Initial production,

$q_{el} = 5$ bpd, End of life production,

$D_i = 0.6$, Initial nominal decline rate,

$b = 0.3$, Hyperbolic exponent,

ERR, Estimated Remaining Reserves.

With these values, the Estimated Remaining Reserves (ERR) are calculated to be 76,231 bbls. However, if all the above values are not known with certainty, Monte Carlo simulation is used to calculate the expected value for ERR as well as a range of values and their associated probabilities. Assume the uncertainty in all of the values can be represented by triangular distributions, and the mean of the triangular distributions will be the above values. The mean of a triangular distribution is the sum of the minimum, most likely, and maximum values divided by three. Also, assume that the variables are independent; for instance, q_{el} is not a function of q_i. A set of the example distributions is shown in Table 3.1. For more information on distributions see the later section Investment Projects.

Variable	Minimum	Most Likely	Maximum
qi bpd	90	100	110
qel bpd	2	4	9
Di	0.4	0.65	0.75
b	0	0.2	0.7

Table 3.1 Values used in the Monte Carlo Simulation Example.

This set of distributions was run through a Monte Carlo Simulator 10,000 times, and the results are shown in Table 3.2.

Variable Name	Minimum calculated or sampled value	Mean calculated or sampled value	Maximum calculated or sampled value	5% of values are less than	5% of values are greater than
ERR (bbl)	44,131	79,088	183,078	57,027	111,288
qi (bpd)	90.11	100.0	109.88	93.16	106.83
qel (bpd)	2.01	5.0	8.95	2.84	7.68
Di	0.402	0.6	0.748	0.466	0.708
b	0.00325	0.3	0.6974	0.0836	0.5676

Table 3.2 Results of Monte Carlo Simulation with 10,000 iterations.

There are several important results. Note first that the mean value of ERR from the Monte Carlo simulation (79,088 bbl) is not the same as the deterministic value (76,231 bbl), even though the means for the input to the Monte Carlo simulation were the deterministic values. Also, note that the value of ERR using the "Most Likely" values for each variable (68,847 bbl) is considerably different. These results occur because of the complex nature of the equation for calculating ERR, having uncertain variables raised to powers of other uncertain variables, and because the input distributions were not symmetrical about the most likely values.

The minimum and maximum values calculated for ERR are not particularly meaningful since they can vary considerably with the number of simulation passes. However, note that 90 percent of the values lie between 57,027 bbl and 111,288 bbl. This is a very wide range and gives an indication of the magnitude of the uncertainty.

Table 3.3 contains the complete distribution of ERR as calculated in this simulation. In this table two Monte Carlo simulation runs are shown. The reason for the second run is to observe the change in key variables, since it is, after all, controlled by a random number generator. Table 3.3 is useful when asking the question: what is the probability that the reserve will be below a certain number?

Cumulative Frequency	PASS1 Estimated Remaining Reserves, bbl	PASS2 Estimated Remaining Reserves, bbl
0%	44,131	44,071
5%	57,028	56,707
10%	59,903	59,874
15%	62,307	62,233
20%	64,444	64,451
25%	66,428	66,430
30%	68,387	68,280
35%	70,262	70,169
40%	71,963	72,064
45%	73,814	74,123
50%	75,978	76,110
55%	78,129	78,203
60%	80,441	80,407
65%	82,861	82,948
70%	85,671	85,802
75%	88,713	88,603
80%	92,162	92,066
85%	96,561	96,241
90%	102,550	102,143
95%	111,289	111,988
100%	183,078	184,589

Table 3.3 Distribution of Estimated Remaining Reserves for Two Simulation Passes.

Running the simulation for a second time without changing parameters might produce different results. Running a large number of trials in Monte Carlo simulation to reduce the error in the output functions minimizes the difference between multiple runs using identical input. In the second set of calculations (PASS2), the zero percent and 100 percent numbers could change considerably, but the numbers near the center of the distribution would not change significantly. A second simulation run was made as shown in Table 3.3. In the PASS2 column it can be seen that the median (50 percent) values vary by less than 0.2 percent, and in this case even the zero percent and 100 percent values do not change significantly from those in PASS1.

If the 10 percent value is chosen as the official value for remaining reserves, that value would be 59,903 bbl, which is 21 percent less than the value calculated deterministically. A graph of the data in the cumulative frequency Table 3.3 is shown in Figure 3.1.

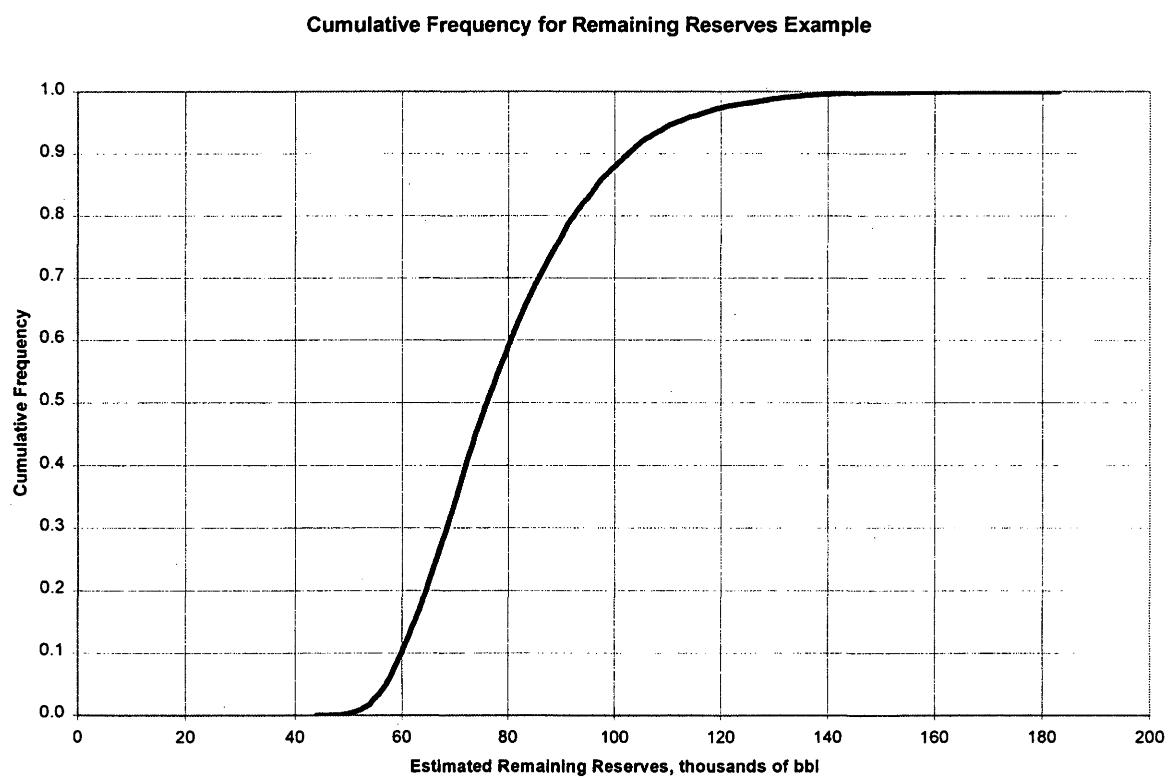

Figure 3.1 Cumulative Frequency of Estimated Remaining Reserves.

Figure 3.1 and its associated Table 3.3 are very common "outputs" from a Monte Carlo simulation. Another common output is a histogram in which the relative frequency of a particular range of outcomes is plotted, as shown in Figure 3.2. The values plotted on the x-axis are the midpoints of the bars. It can be seen that the most common result of the calculation is a value between 75,000 bbl and 85,000 bbl, which occurs approximately 26 percent of the time.

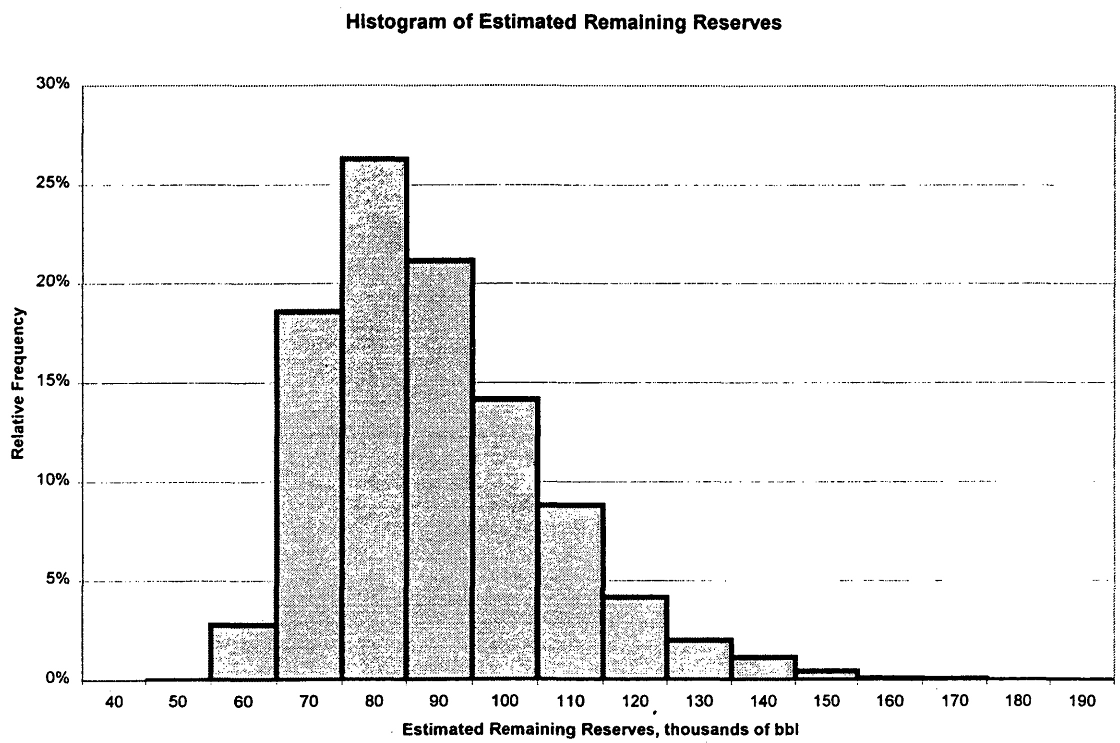

Figure 3.2 Histogram of Estimated Remaining Reserves.

The correlation coefficient between the various input variables and the outcome allows the user to determine which input variables have the greatest effect on the outcomes. This effect is, of course, based on the input distributions associated with each input variable and will change if the distribution parameters are changed. Table 3.4 shows which input variables have the greatest effect on the remaining reserves for this example simulation.

Variable	Correlation Coefficient "r"
b	0.730
Di	-0.607
qi (bpd)	0.206
qel (bpd)	-0.146

Table 3.4 Correlation Coefficient for Input Parameters to Calculate Estimated Remaining Reserves.

The correlation coefficient is the square root of the r^2 value from a linear fit of ERR to each of the input variables. It can be seen that "b" has a correlation coefficient of +0.73. The positive correlation coefficient means increases in "b" result in increases in ERR. The square of the correlation coefficient $(0.73)^2$ equals 0.53, which means that 53 percent of the variability of ERR can be explained by the variation in "b".

Figure 3.3 shows the correlation between ERR and "b". The least squares fit of the data has an r^2 value of 0.53. Similarly, D_i has a large effect on ERR, as shown in Table 3.4. However, since the correlation coefficient is negative, increases in D_i result in decreases in ERR. It can be seen that the estimate of q_{el} has little effect on remaining reserves for this example. Only 2 percent (-0.146^2) of the variability in ERR is explained by the variability in q_{el}. These same results are sometimes presented in a "Tornado" diagram, such as Figure 3.4.

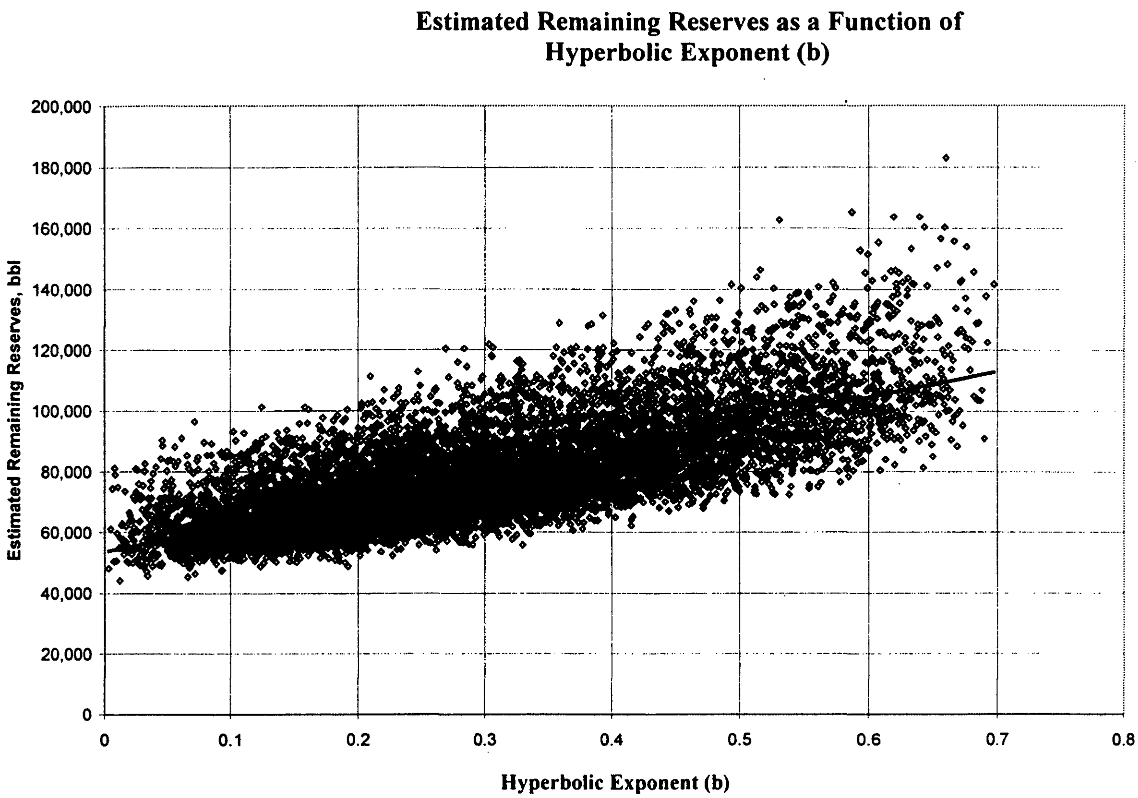

Figure 3.3 Correlation of Estimated Remaining Reserves with Hyperbolic Exponent.

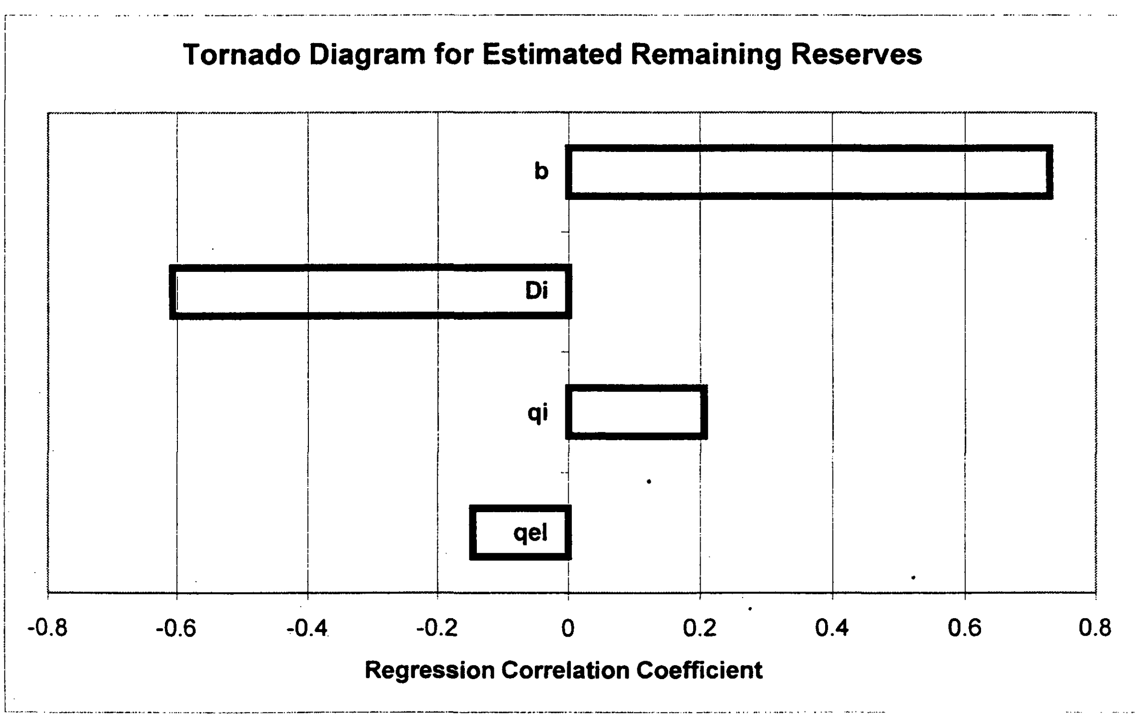

Figure 3.4 Tornado Diagram for Estimated Remaining Reserves.

Interpretation of Results

One of the more important numbers is the mean, or expected value, of distribution, and a decision can be made with this number alone. However, much more information is available, such as the probability that the project will lose money, assuming the Expected Monetary Value (EMV) was calculated in the simulation, and what the chances are that the project will make a large amount of money. Ideally, the entire cumulative frequency graph would be presented and compared with other projects. In this way the decision maker could see the full spectrum of anticipated possibilities.

Actual Production Time

In order to add realism and complexity to the hypothetical projects, a distribution of actual production time was used. When calculating total days on production, a uniform distribution was set with varying maximum and minimum values for each well. This is necessary to account for down time of the well, as in testing or work-overs. These data can be gathered in real life by reviewing the well's history and making a down time distribution. This distribution could be for the well itself, the sand group, or the whole reservoir. This procedure has not been reported in the literature. Figure 3.5 shows one of the well's actual production time. This figure was based on a normal distribution with a mean of 347 days and a standard deviation of 10 days.

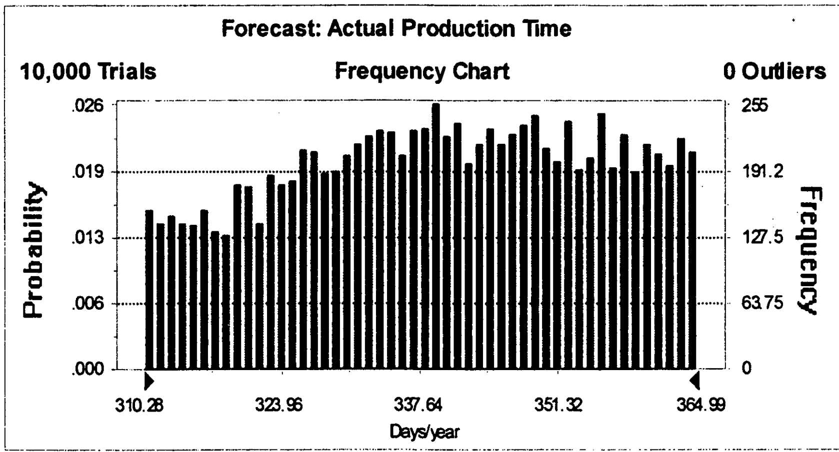

Figure 3.5 Actual Production Time.

Probability of Success

The probability of success for an investment project is the most significant variable. It denotes whether hydrocarbons are present in the formation, or whether the drilled well will be a success. The literature has numerous papers on calculating this value for an oil or gas project. Rose (2001) has a thorough explanation of probability of success (POS) calculations.

In this research model, a custom distribution was used to represent the POS of each of the investments. The distribution gives two outcomes, 1 and 2, and each has its own probability of occurrence, and it is different from one investment to the other. The outcome of the distribution acts as a switch for the project. One outcome denotes success and the other represents a dry-hole scenario. Each outcome has its own costs and profits. For example, if an investment has a dry-hole scenario, then abandonment cost and lease bonuses must be included in the analysis.

Dependency Among Variables

The interplay between projects is more complex, in that their economic outcomes are interrelated. This is known as statistical dependence. The simplest type of statistical dependence is correlation, which is a characteristic where change in one variable is systematically related to change in another variable (also called association). Correlation occurs in two variables:

- Positive correlation: when a given outcome for one project increases the chance of an outcome in the same direction for the other project. This type of correlation diminishes the effect of diversification.
- Negative correlation: when a given outcome for one project decreases the chance of an outcome in the same direction for the other project. This type enhances the effect of diversification.

Correlation arises in two ways: dependency and common influence. Dependency is when an event may partially depend on (is influenced by) one or more other events. Common influence is when two events may both be dependent upon another common driver.

Statistical dependence may be due to many sources. The four listed below are not meant to be exhaustive, but are widely encountered in E&P projects. These four are places, prices, profiles and politics.

Places

The economic outcomes of two E&P sites in very close proximity (for example in the same field) will be positively correlated through geological similarities, and would not constitute a very diversified portfolio. On the other hand, two sites in widely distant locations will display little or no geological correlation, and hence would be more diversified. "Places" can have corresponding implications for pricing (especially gas) and political issues, as well as for geologic ones.

Prices

Petroleum projects produce crude oil and natural gas in various proportions. Crude oil prices generally track each other very closely worldwide. Thus, the economic outputs of oil projects worldwide are positively correlated relative to fluctuations in crude price. However, this is not true for natural gas. Natural gas prices in many parts of the world do not track either world crude oil prices or each other very well. Thus, there would be a tendency for a portfolio consisting of a gas project and an oil project to be less positively correlated and therefore better diversified, relative to price, than a portfolio consisting of two oil projects. An example of this phenomenon is the economy of the city of Houston, which had suffered during the crude oil price drop of 1986, and had weathered a subsequent price drop successfully because it had diversified between oil and gas (Wall Street Journal 1993).

Profiles

A frequent concern is the timing of the flows of various elements of projects, which may extend for many years into the future. These flows might include such elements as cash flow, hydrocarbon production, reserve additions, and staff requirements. Often the more nearly constant these flows can be, the better. The correlation among these elements can be taken into consideration to minimize fluctuations in cash flow.

Politics

Petroleum investments have always been subject to political uncertainties, from the anti-trust decision against Standard Oil of 1911, environmental regulations, the Gulf War of 1991, the current strike in Venezuela, and beyond. Projects subject to disruption in the same direction due to the same political event will be positively correlated. Negative correlation of projects may also be induced through political uncertainty. For example, consider two politically distinct regions that supply natural gas through two different pipelines to a single market. The political disruption of production in either of the two regions could lead to market shortages, and hence to increased prices and/or demands for the non-disrupted region. A portfolio consisting of one project in each negatively correlated region would thus be protected against political risk in either region.

Ownership Interest Calculation

The computation of ownership interest is a complicated topic. Only the parts that pertain to this research (royalty, working interest, and the net revenue interest) are covered. Royalty is a share of the revenue free of all costs of development and production. The royalty is paid to the owner of the mineral interest under the land associated with the well. Typical royalty rates in the U.S. range from 12.5 percent to 25 percent of the production. Royalties in other countries can range from zero to more than 30 percent. Overriding royalty is the same as a royalty except that it does not come about because of ownership of the mineral interest. An "override" is a classic way for a lease broker or geologist to be compensated for buying leases or putting a deal together.

The royalty in the spreadsheet model is included in the term LRI (Less Royalty Interest). LRI is 100 percent minus the total royalty burden for the property. Since a company has little control on the royalty it pays, a fixed 20 percent royalty is used for all the projects in this research.

Working Interest

Working Interest (WI) is the company share of the costs. There are different types of working interest: Joint, Pooled, and Unitized. The total of all the working interests in a well must be equal to one. The working interest is one of the decision variables that was addressed in this model. Usually along with the share of the costs comes a reduced share of the revenue. It is quite common for a company to own less than 100 percent of the working interest in a well or project. Owning smaller interests in many projects, called diversification, is one of the ways to manage the risk involved in drilling for oil and gas. Working interest may also change over time as a result of "oil field deals."

Net Revenue Interest

Associated with each working interest is a net revenue interest (NRI), reduced proportionately by non-operating working interest. For an investor in an oil or gas deal, NRI is the key economic figure, which measures the percent of revenue earned by the investor. This NRI is earned by a specific working interest. In many cases, the working interest owners receive approximately 80 percent of the gross revenues, although sometimes it may be as high as 87.5 percent or as low as 70 percent (or less). If there is a 12.5 percent royalty, a 7.5 percent overriding royalty, and company "A" own a 50-percent working interest, then company A's net revenue interest is 40 percent ((100 percent less 12.5 percent less 7.5 percent) times 50 percent).

Gross Production

Throughout this dissertation production is always reported in standard conditions for gas and stock-tank barrels for oil. The term MCF is used for gas, which means thousand cubic feet. As for oil, the term is MBBL (thousand stock-tank barrels).

Barrels of Oil Equivalent

In the portfolio data sheet, Barrels of Oil Equivalent (BOE) is calculated. This is basically converting the gas production into equivalent oil production by using the BTU content of the gas and converting it to the BTU content of oil. BTU is British Thermal Unit. The BTU content of a gas is dependent on the composition of the gas. Since the model uses a portfolio of projects, 6.0 MCF per BBL was used to convert gas production to equivalent oil. On average one barrel of oil has a BTU content approximately equivalent to 6,000 cubic feet of gas. This BOE can be used as a constraint for the portfolio, such as a minimum BOE per year.

Revenue and Net Revenue

The revenue is the cash generated from the sale of the hydrocarbons. The product of the gross production times the price of oil or gas is the total revenue for the project. Net revenue is the result of multiplying total revenue by NRI.

Operating Cost

Operating cost, or operating expenditure (OPEX), is a periodic cost which is necessary for the day to day operation of the field. In the model this cost is split into two categories: fixed and variable costs. The fixed cost is a yearly amount for maintenance and workover of the field. The variable cost is a dollar amount per MCF or BBL produced from the field. The total operating cost varies between $500 and $4,000 per month per well.

Lease Bonus and Facility Costs

Upon acquiring mineral rights, the lessee pays a sign-up bonus to the lessor. The amount ranges from $1 per acre to $3,000 per acre, depending on the exploration activity within or nearby the leased property. Facility costs, or Capital Expenditure (CAPEX), with the lease bonus are front end costs. These costs are incurred at the beginning of the project. For this reason, both are combined in the model to simplify the procedure. Facility cost consists of all the equipment needed to put the well on production at the beginning of the field life. Other facilities needed may not be used until later, such as artificial lift. These are accounted for in the model according to the year they are incurred.

Capitalized Costs

Capitalized costs were handled separately from other costs in order to make it easier for depreciation and amortization calculations. This step makes the model compatible with any tax algorithm to be added later on. Lease and well equipment (L&WE) includes items such as the well head, flow line, separators, production tanks, and other equipment necessary to operate the property.

Intangible drilling costs (IDC) are any costs that have no apparent salvage values, such as fuel, labor, and power required to drill the well. Also, the cost of development dry holes is included in the capitalized cost with respect to economic calculations.

Expensed Costs

Geology and Geophysics (G&G) costs are pre-drilling exploration costs. These costs include topographical, geological, and geophysical studies. The geophysical costs include rights of access to properties as well as acquisition, processing, and interpretation of seismic surveys. These costs are capital in nature, but for tax calculation purposes they are expensed in the year they are incurred.

Hydrocarbon Prices in the Model

Crude oil prices were considered as a global variable in the model, meaning that for each iteration of the stochastic simulation, each project used consistent prices. Natural gas prices were treated in the model as if they belong to the same geographic location in order to achieve the proper correlation. The model can easily be modified to accept different natural gas price distributions.

When devising the price distribution, care was taken to include the seasonality of gas prices and how the time of the year affects the price. As for crude oil prices, they are affected by the API grade and the sulfur content. West Texas Intermediate (WTI) prices were the basis for the distribution shape. The lows of 1998 ($10/bbl) and the highs of the fall 2000 ($36/bbl) were used as truncation points of the distribution. The price is generated at the portfolio level and is fed back at the project level.

Optimization Stage

An optimization model is a model that seeks to maximize or minimize some quantity, such as profit or risk. The optimization stage has three major elements: decision variables, constraints, and an objective function. More information on the optimization algorithm is available in Chapter Two. Figures 3.6 and 3.7 show the flow diagram and the schematics of the optimization model, respectively.

Decision variables are quantities which can be controlled. The model used the working interest (WI) to determine the percentage of ownership in the project. Decision variables have an upper and a lower limit where they vary in between. The variation could be discrete or continuous. For the discrete case, the step size in possible WI increments can vary from zero percent to 100 percent. If the step size is 100 percent, then there are only two possible values for WI, zero percent or 100 percent. The step size can take any value; the smaller it is, the larger the number of outcomes to enumerate. When the WI number is allowed to fall anywhere between zero percent and 100 percent, including fractions, then it is a continuous variable.

Constraints describe relationships among decision variables that restrict the values of the decision variables. For example, a constraint might ensure that the number of projects undertaken in a portfolio does not exceed a fixed value. Constraints can be used to control mutually exclusive projects from being selected together in one portfolio.

The objective function gives a mathematical representation of the model's objective, such as maximizing NPV or minimizing costs, in terms of decision variables.

Objective functions are the forecasts from our stochastic model. The forecast will have the function statistical summary, such as mean, median, and standard deviation. The optimization can be run in two modes: deterministic or stochastic.

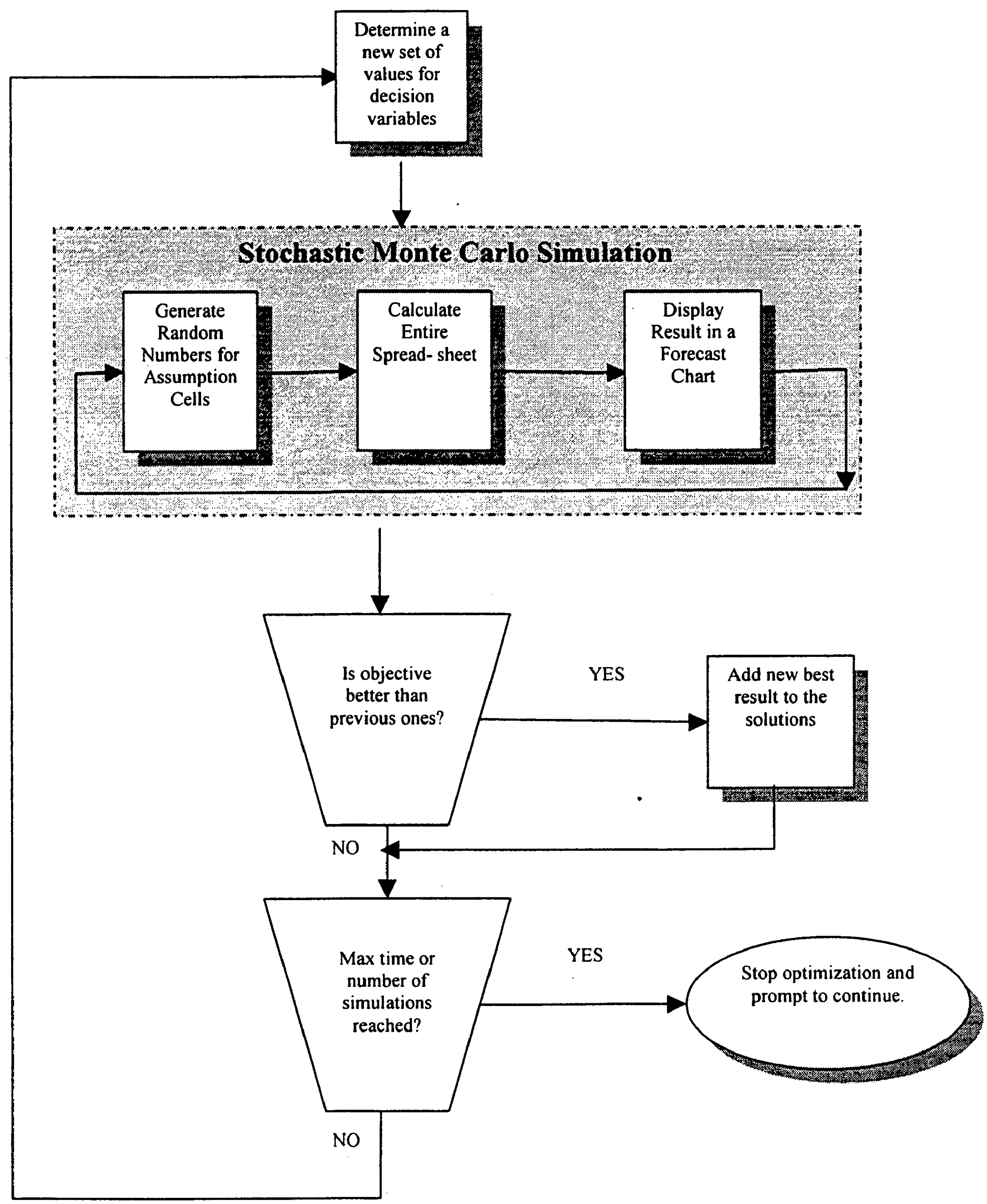

Figure 3.6 Portfolio Optimization Flow Chart

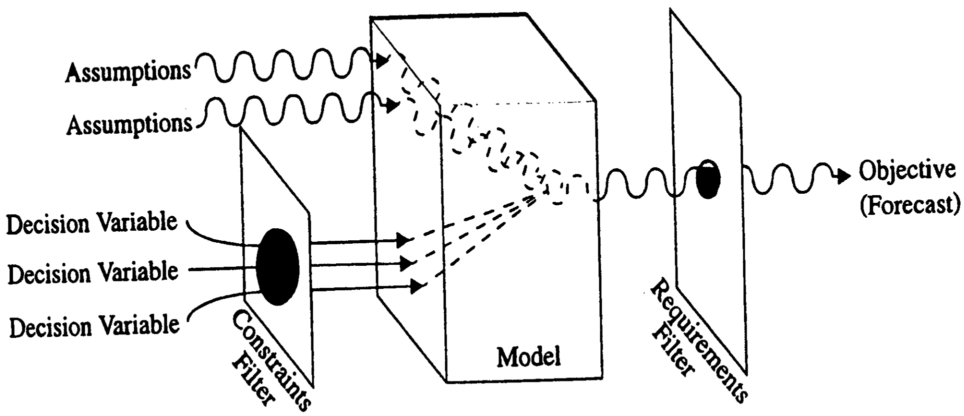

Figure 3.7 Optimization of Stochastic Model (after OptQuest 2000)

Neural Network

The optimization algorithm has a built-in neural network accelerator which is designed to improve performance by predicting the result of the simulation after approximately 10 percent of the trials have been run. The model contains more than thirty decision variables, and due to the large number of active files, the simulation is very slow. The neural network accelerator helps in reducing the time taken to evaluate inferior simulations. Inferior simulations are those where the objective function is lower than previous simulations. When the objective function is minimized, then a higher result than a previous simulation is considered an inferior simulation.

The algorithm collects information at the beginning of the simulation and, after a sufficient number of trials, it predicts the solution of the objective function. If the solution is inferior to previous trials, the simulation stops and a new set of decision variables are used. For our model, the neural network accelerator is used only after a preliminary run is performed, because the accelerator might rule out a superior solution, since it is only an approximation. A superior solution is when the model yields an optimal objective compared to the current solution.

Investments Projects

Hypothetical but realistic projects are used in this dissertation. Appendix A has all the projects that have been used in this research. A summery of one of the investment projects (Oil#2) is shown in Appendix C. The rest of the investment projects are in the compact disk in Appendix D. Projects are prepared in a way that most of the inputs to the cash flow model are stochastic ones. Simple distribution functions (uniform, triangular, normal, Log-Normal, and custom distribution) have been used throughout the projects. However, if sufficient historical data are available, any type of distribution can be used. When using distributions for forecasting, an assumption is made that future distributions will be similar to past distributions.

Uniform Distribution

In the uniform distribution, all values between the minimum and maximum occur with equal likelihood. Three conditions apply when dealing with a uniform distribution:

-The minimum value is fixed,

-The maximum value is fixed,

-All the values between the minimum and the maximum have the same probability of occurring.

Figure 3.8 shows a uniform distribution for lease and well equipment.

Assumption: L&WE M$ / Well

Uniform distribution with parameters:

Minimum	$225.00
Maximum	$275.00

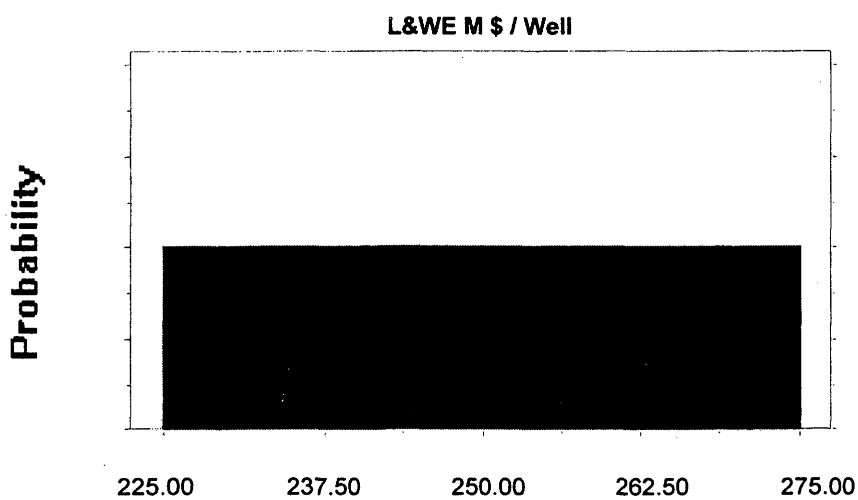

Figure 3.8 Uniform Distribution.

Triangular Distribution

The triangular distribution describe a situation where minimum, maximum, and most likely values are known. The three conditions underlying the triangular distribution are:

-The minimum number is fixed,

-The maximum number is fixed,

-The most likely value falls between the minimum and maximum values, forming a triangle-shape distribution, which shows that values near the minimum or maximum are less likely to occur than values near the most likely value.

Figure 3.9 shows a triangular distribution of intangible drilling costs, IDC's.

Assumption: IDC's M$/Well

Triangular distribution with parameters:

Minimum	$675.00
Likeliest	$700.00
Maximum	$825.00

Selected range is from $675.00 to $825.00

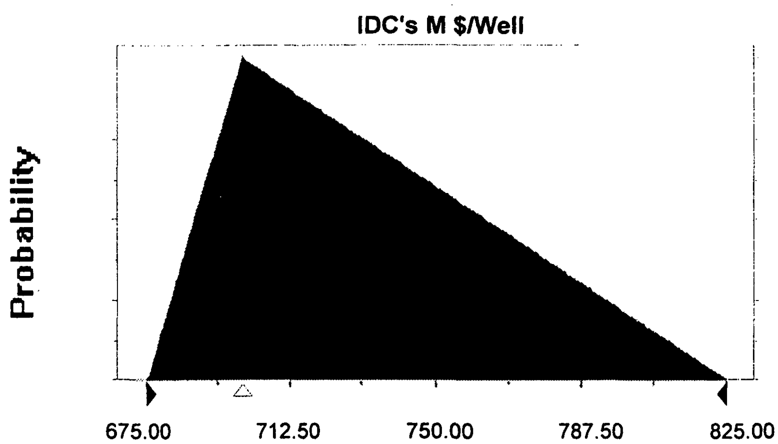

Figure 3.9 Triangular Distribution.

Normal Distribution

The normal distribution is the most important distribution in probability theory because it describes many natural phenomena. Two values are needed to use this distribution:

-The mean (most likely),

-The standard deviation.

Figure 3.10 shows a normal distribution.

Assumption: gas price $/MCF

Normal distribution with parameters:

Mean $2.50

Standard Dev. $1.00

Selected range is from $1.00 to $6.00

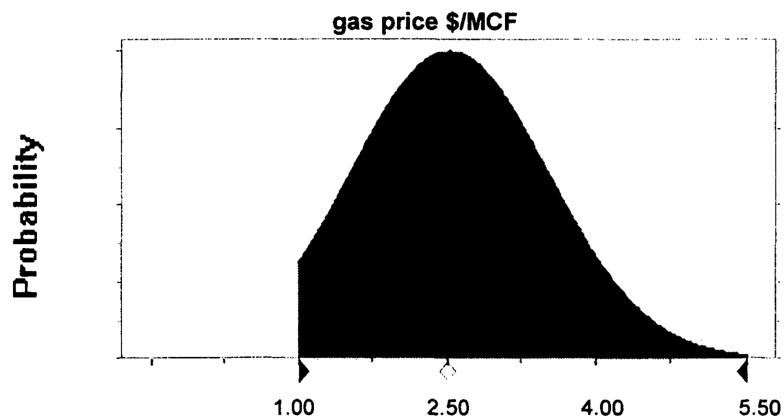

Figure 3.10 Normal Distribution.

Log-Normal Distribution

Log-Normal distribution is widely used in situations where values are positively skewed. The three conditions underlying the Log-Normal distribution are:

- The uncertain variable can increase without limits but cannot fall below zero,
- The uncertain variable is positively skewed with most of the values near the lower limits,
- The natural logarithm of the uncertain variable yields a normal distribution.

Figure 3.11 shows a Log-Normal distribution of reserves.

Assumption: Reserve MBBL

Log-Normal distribution with parameters:

Mean 500.00 MBBL

Standard Dev. 50.00 MBBL

Selected range is from 350.00 MBBL to 600.00 MBBL

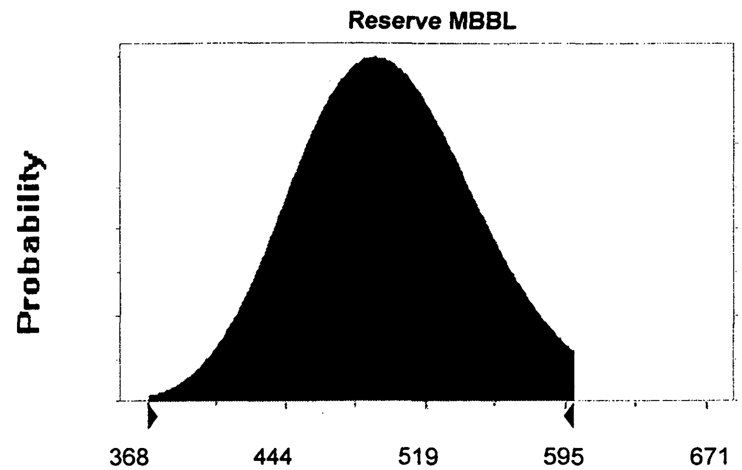

Figure 3.11 Log-Normal Distribution.

Custom Distribution

Custom distribution represents a unique situation that cannot be described by other distributions. Custom distribution is useful when using a bimodal variable and many other cases. Although probability of success can be modeled by a binomial distribution, a custom distribution was used for POS herein. Figure 3.12 shows the POS using a custom distribution. The custom distribution gives two possible outcomes. The first outcome has a value of 1.00, and the second outcome has a value of 2.00, where 1.00 stands for project success and 2.00 stands for project failure.

Assumption: POS

Custom distribution with parameters:		Relative Probability.
Outcome	1.00	0.300000
Outcome	2.00	0.700000
Total Relative Probability		1.000000

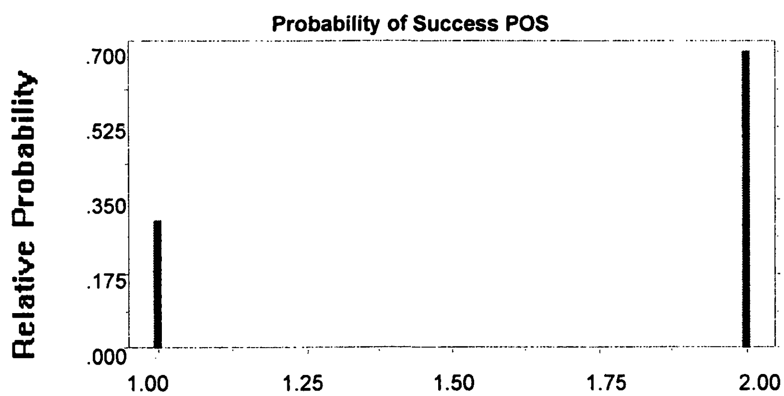

Figure 3.12 POS Custom Distribution.

Project Ranking

The traditional method of ranking projects is to sort them according to their NPV, or similar yardsticks, in a descending order. The project cost is then taken from the budget until the fund is exhausted. If the budget is not enough to take a whole project, then a percentage of the project is taken. Table 3.5 shows traditional projects ranking using a $200 million budget. The cumulative NPV in Table 3.5 is $508.277 million. Unlike portfolio optimization, traditional methods do not take risk into account or the uncertainties in the outcome. Some E&P executives claim that they prefer to "shoot the moon" (Ball 1999) aiming for the highest possible expected return, regardless of the risk. They believe they will arrive at the highest-return highest-risk point (efficient frontier Figure 3.34) by using traditional ranking methods. Ball (1999) states that this argument is false and will lead them to an arbitrary and unexamined tradeoff between risk and return. A comparison between traditional methods and portfolio analysis can be found in Chapter Four under the heading scenario A2.

Project	Mean NPV $M	Cost $M	Budget Unused $M $ 200,000	Cumulative NPV $M
Oil#8	$268,927	$ 45,773	$ 154,227	$ 268,927
Oil#3	$ 73,761	$ 29,050	$ 125,177	$ 342,688
Oil#9	$ 65,378	$ 39,173	$ 86,004	$ 408,066
Oil#1	$ 54,379	$ 7,838	$ 78,166	$ 462,445
Oil#11	$ 43,693	$ 63,172	$ 14,994	$ 506,138
Gas#6	$ 35,005	$ 15,970	$ 6% is taken	$ 508,277
Oil#2	$ 31,917	$ 9,774	Unavailable to invest in because of budgetary restraints	
Gas#7	$ 29,994	$ 31,241		
Oil#6	$ 23,056	$ 5,293		
Oil#5	$ 16,054	$ 7,799		
Gas#5	$ 15,227	$ 50,424		
Oil#10	$ 10,171	$ 35,367		
Gas#4	$ 9,378	$ 1,628		
Oil#4	$ 5,520	$ 3,484		
Gas#1	$ 5,058	$ 4,191		
Gas#2	$ 4,091	$ 1,965		
Gas#3	$ 2,939	$ 2,266		
Oil#7	$ 2,875	$ 2,935		
Gas#8	$ 1,411	$ 1,895		
Gas#9	$ 1,321	$ 4,414		

Table 3.5 Traditional Project Ranking by NPV.

Figures 3.13 to 3.28 show the investment projects NPV's. Complete investment projects, forecast cost, and estimated ultimate recovery EUR are shown in Appendix A. The NPV plots are shown after 1,000 iterations of Monte Carlo simulation. The NPV values are plotted as frequency diagrams.

Forecast: Gas#1 NPV

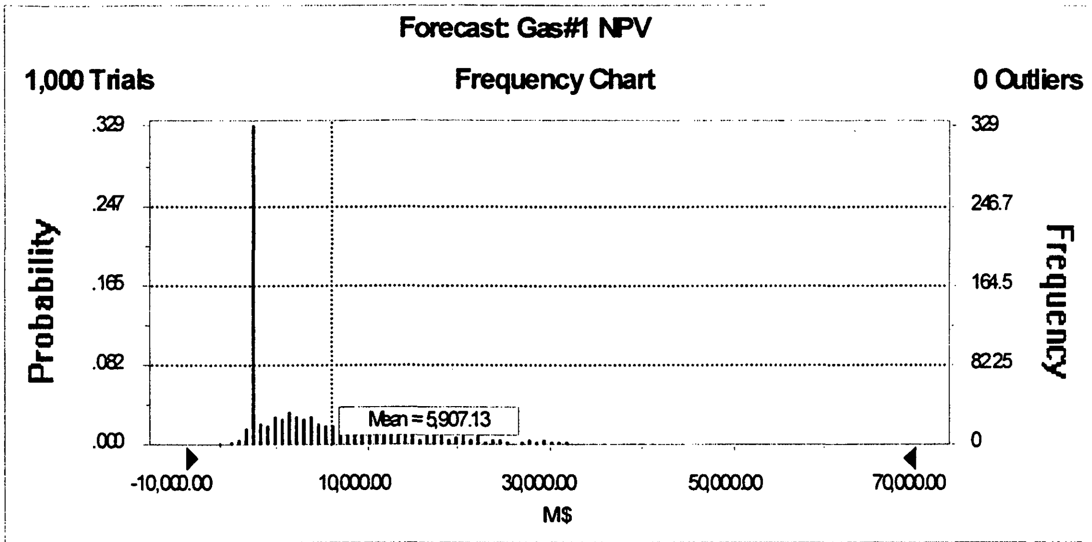

Figure 3.13 Gas#1 NPV.

Forecast: Gas#2 NPV

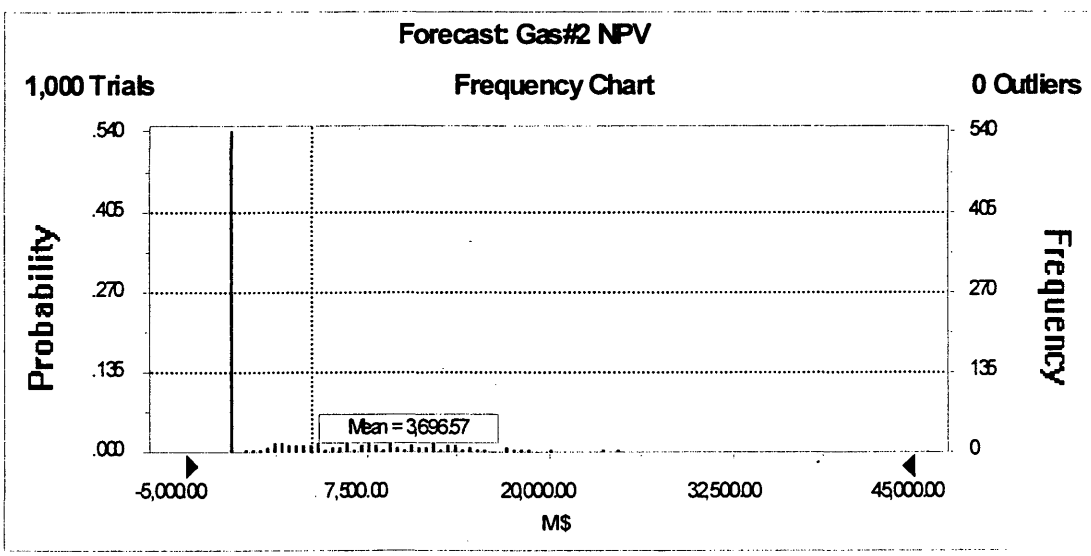

Figure 3.14 Gas#2 NPV.

Forecast: Gas#3 NPV

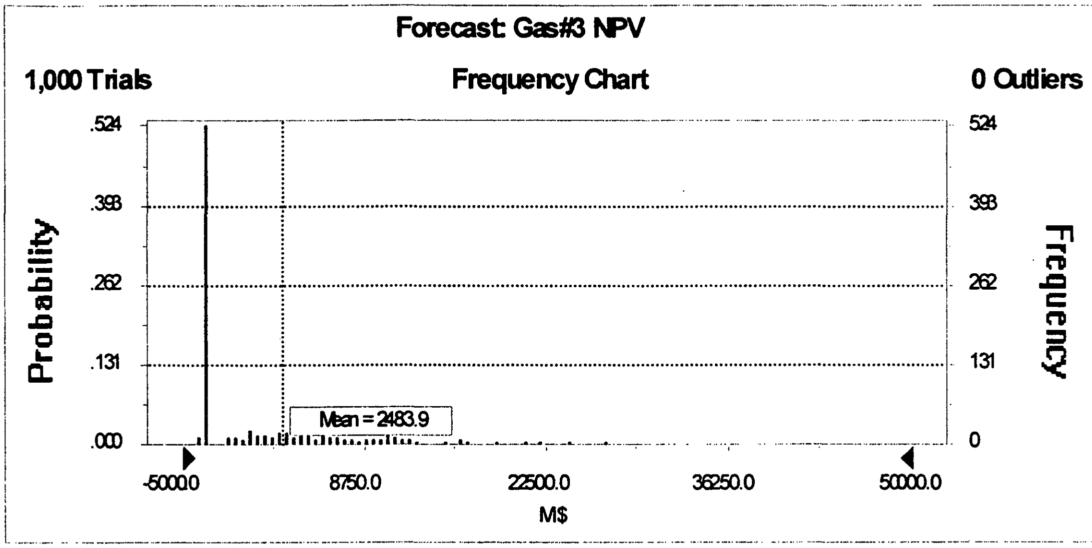

Figure 3.15 Gas#3 NPV.

Forecast: Gas#4 NPV

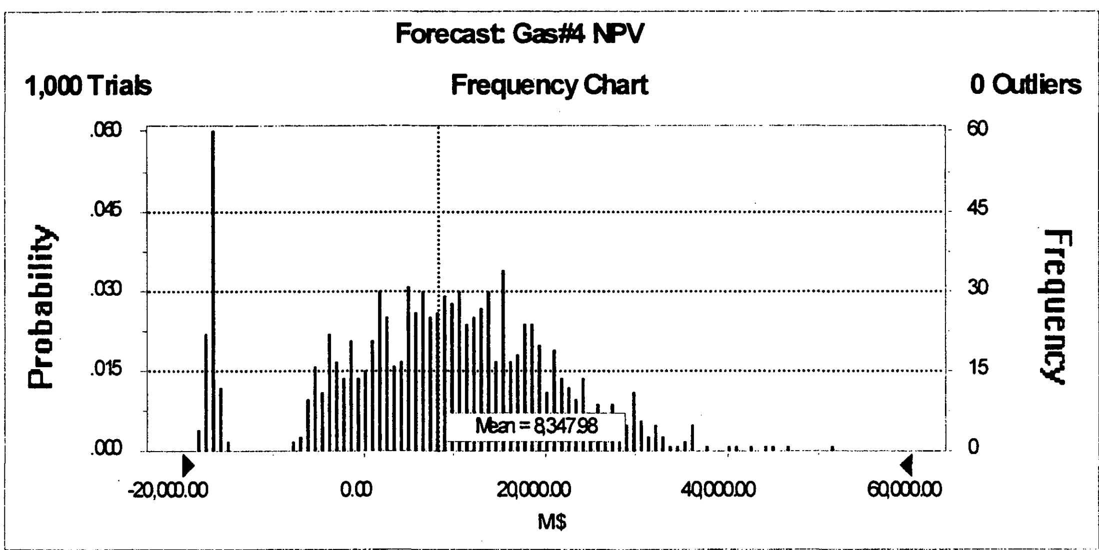

Figure 3.16 Gas#4 NPV.

Forecast: Gas#5 NPV

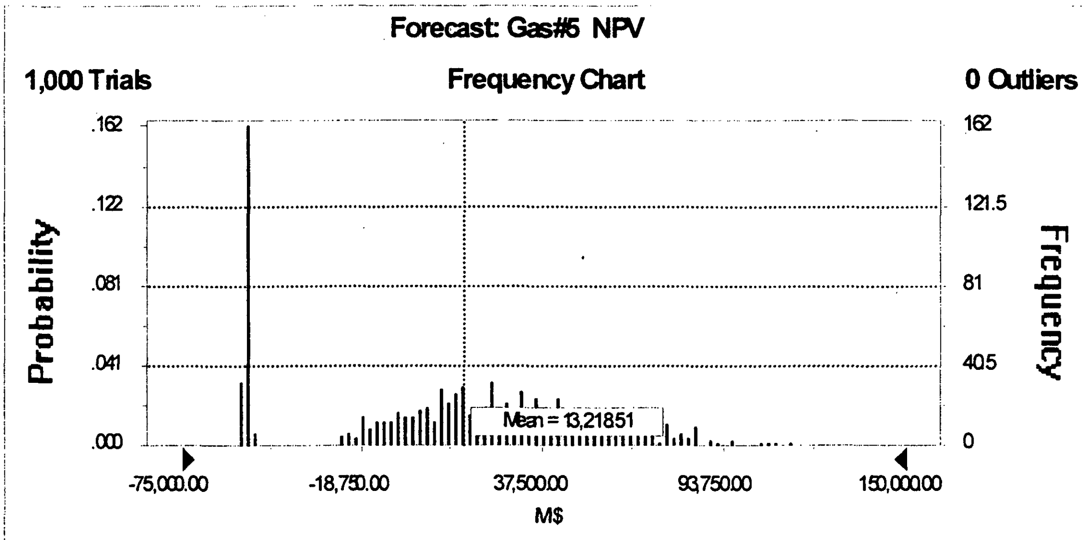

Figure 3.17 Gas#5 NPV.

Forecast: Gas#6 NPV

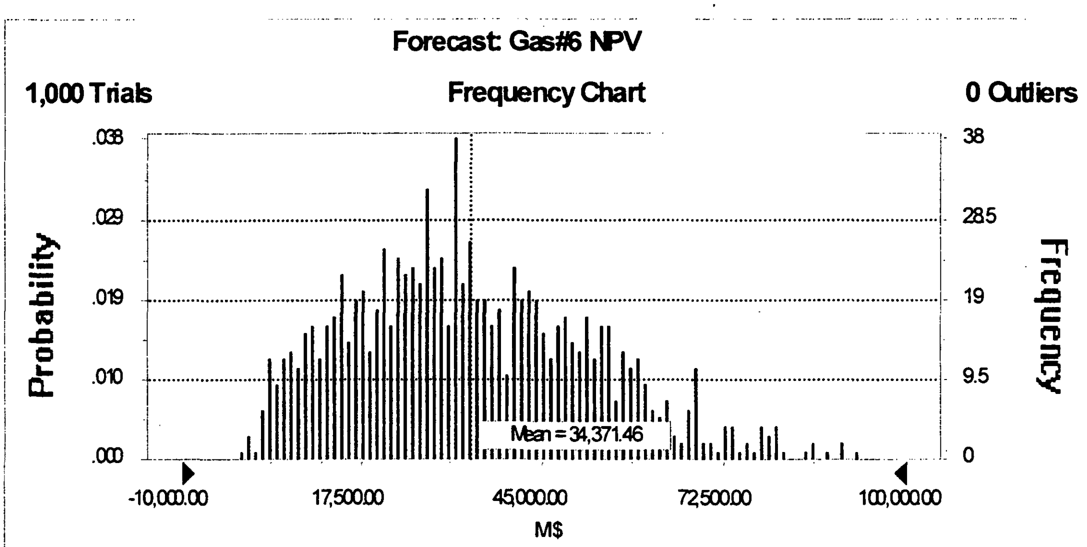

Figure 3.18 Gas#6 NPV.

Forecast: Gas#7 NPV

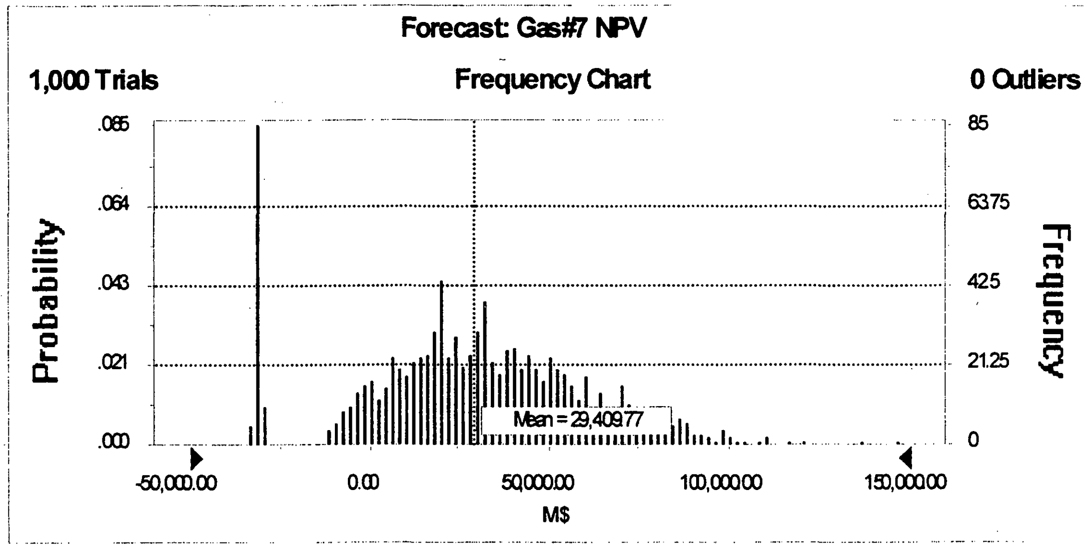

Figure 3.19 Gas#7 NPV.

Forecast: Gas#8 NPV

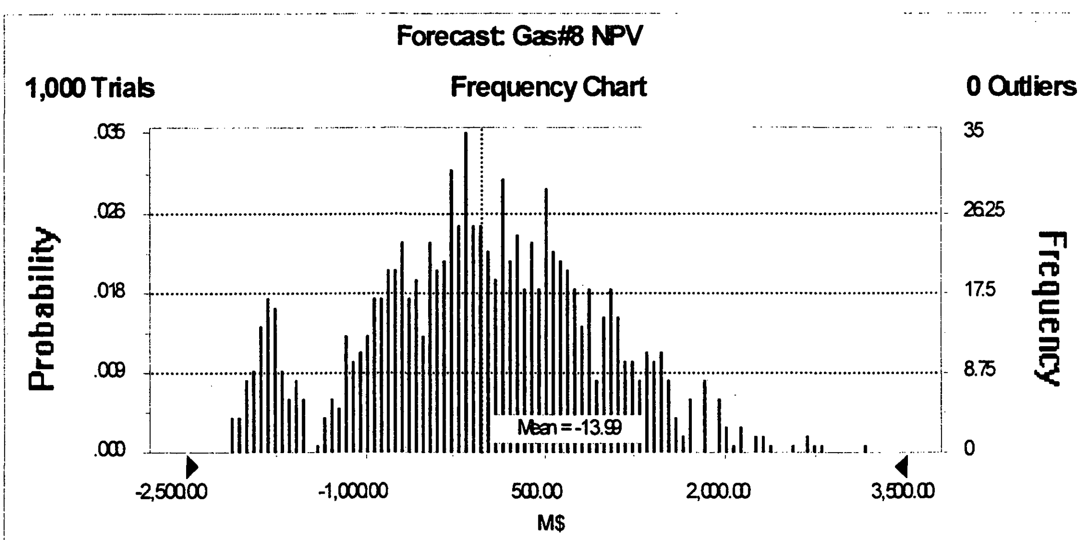

Figure 3.20 Gas#8 NPV.

Forecast: Gas#9 NPV

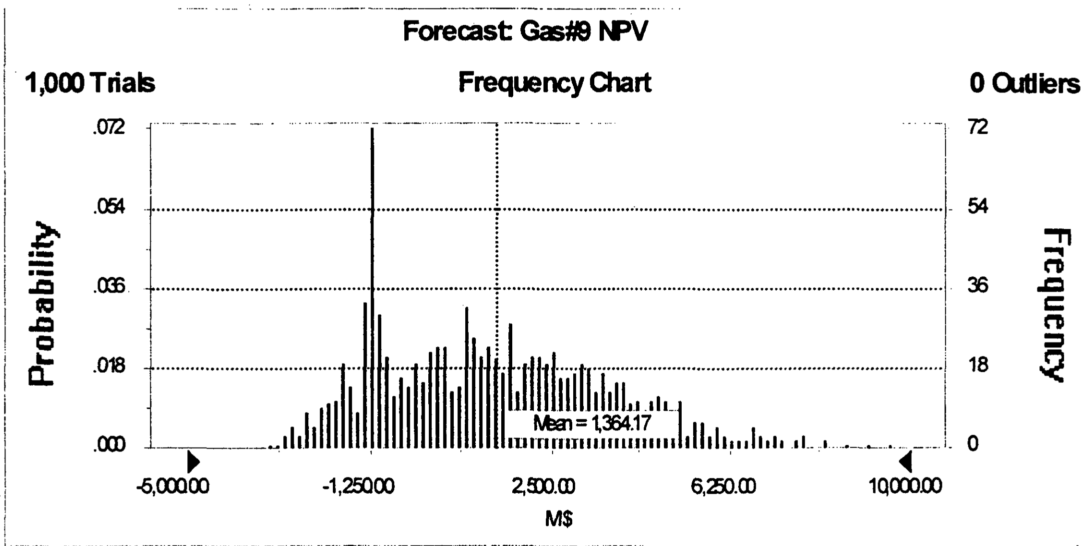

Figure 3.21 Gas#9 NPV.

Forecast: Gas$10 NPV

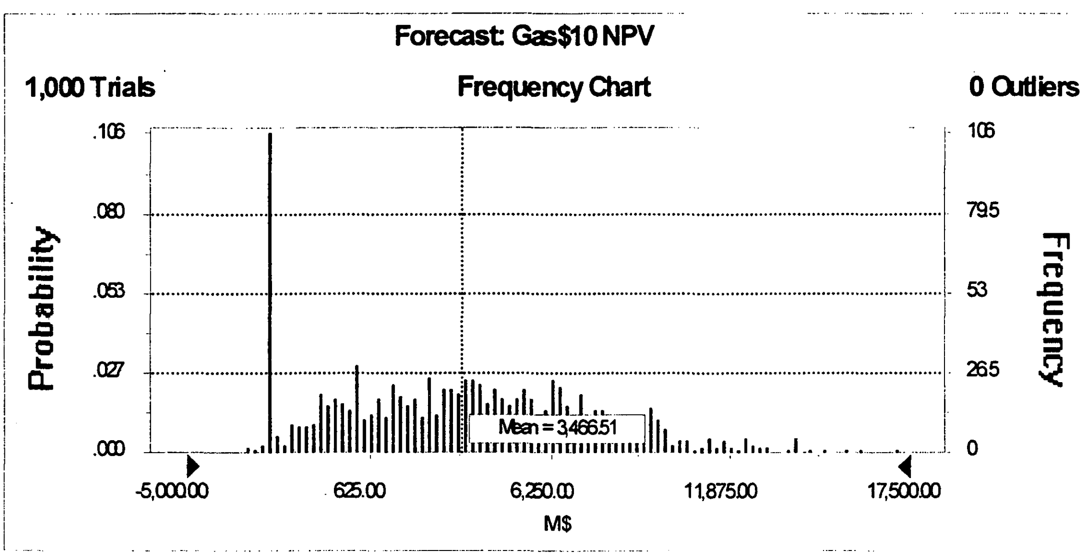

Figure 3.22 Gas#10 NPV.

Forecast: Gas#11 NPV

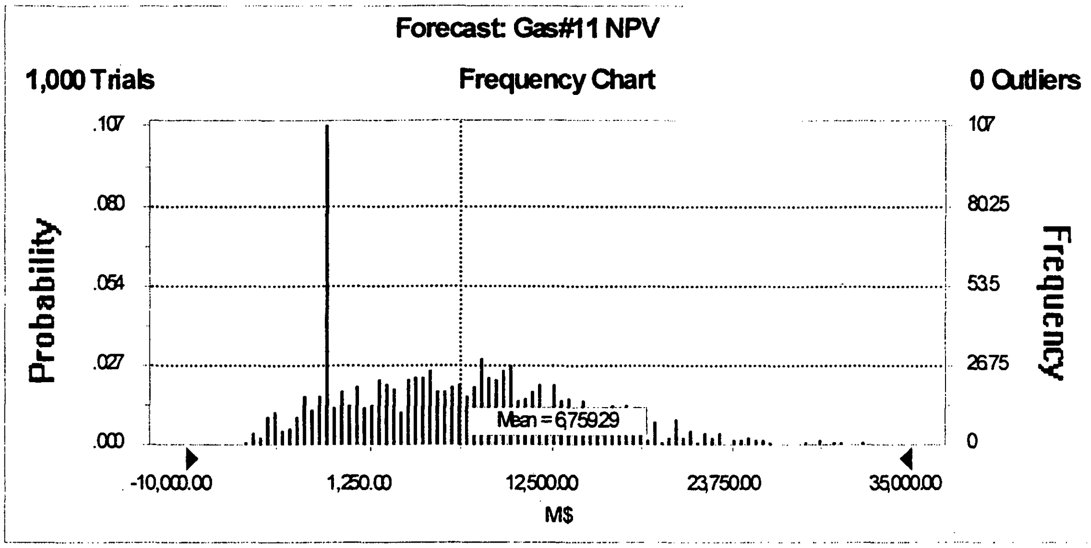

Figure 3.23 Gas#11 NPV.

Forecast: Gas#12 EUR

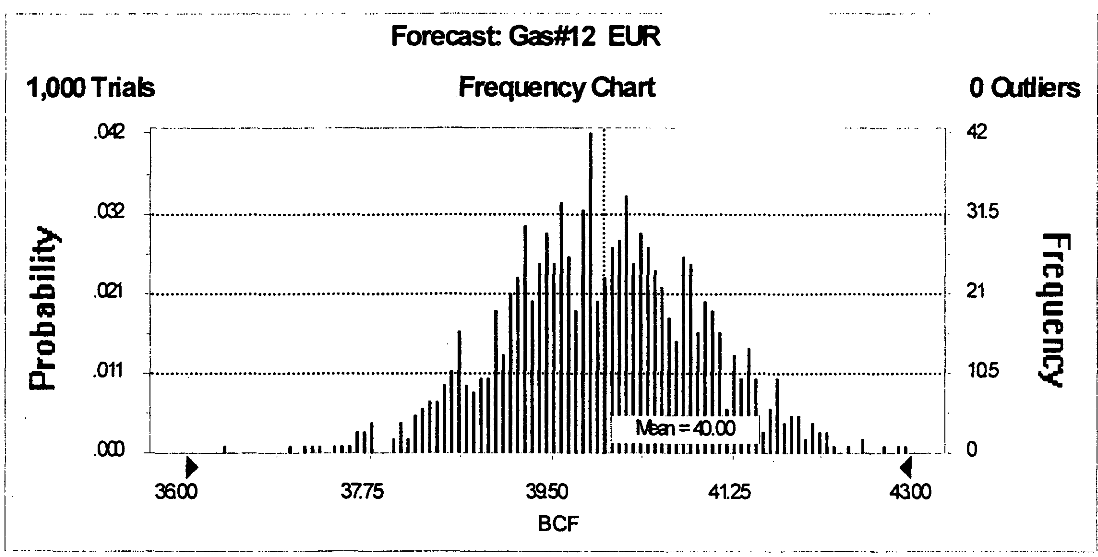

Figure 3.24 Gas#12 NPV.

Forecast: Gas#13 NPV

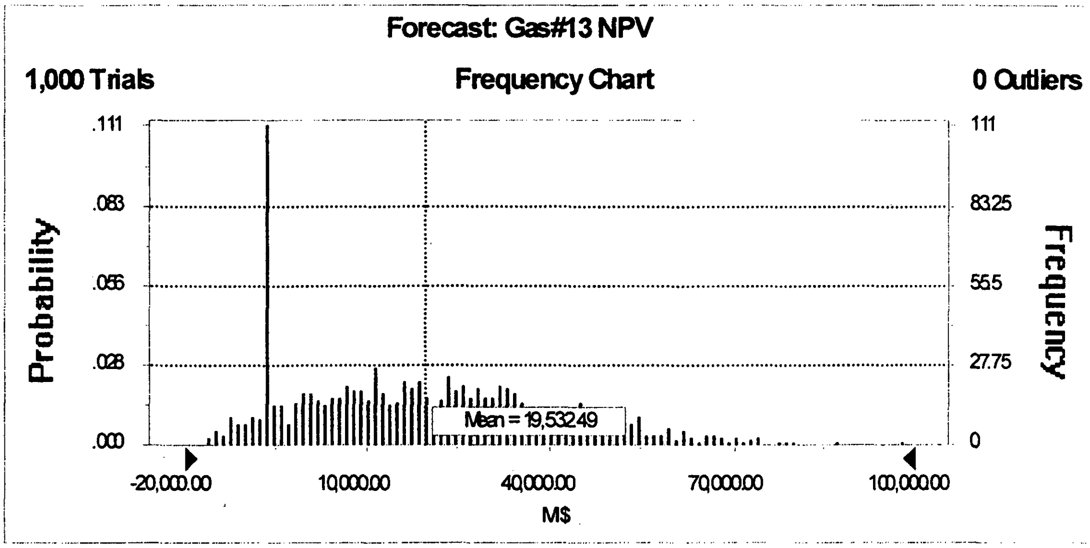

Figure 3.25 Gas#13 NPV.

Forecast: Gas#14 NPV

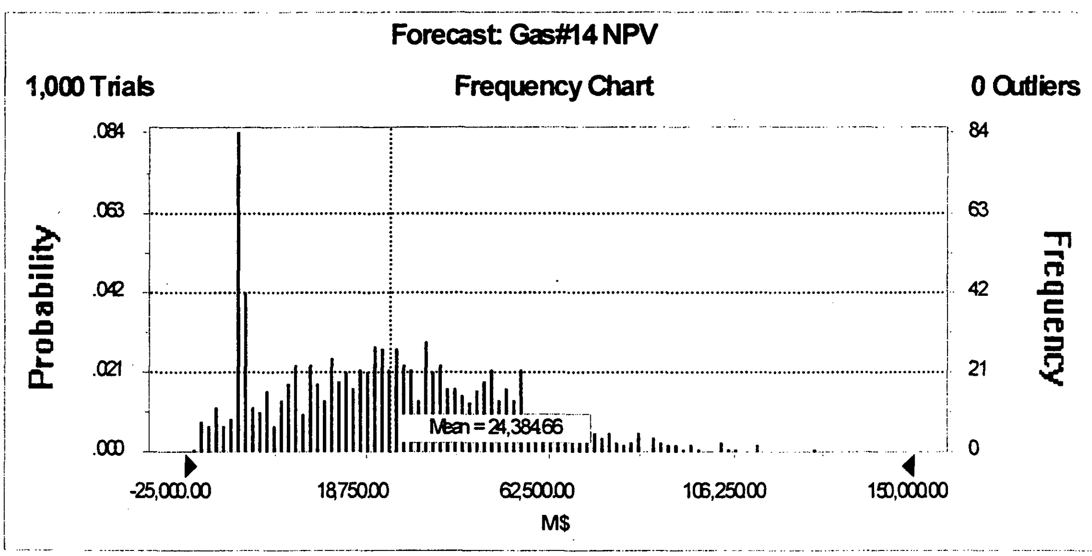

Figure 3.26 Gas#14 NPV.

Forecast: Oil#1 NPV

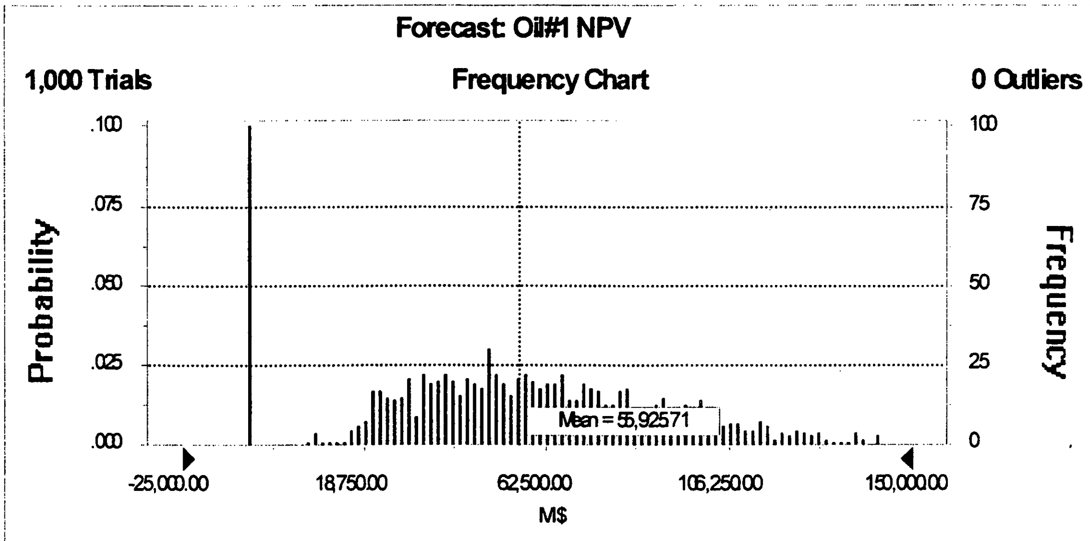

Figure 3.27 Oil#1 NPV.

Forecast: Oil#2 NPV

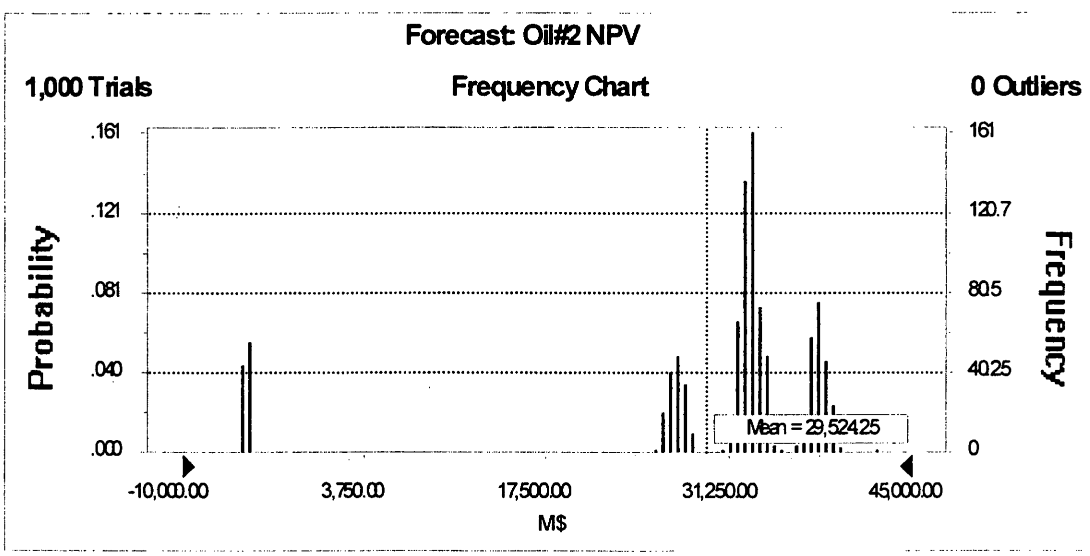

Figure 3.28 Oil#2 NPV.

Forecast: Oil#3 NPV

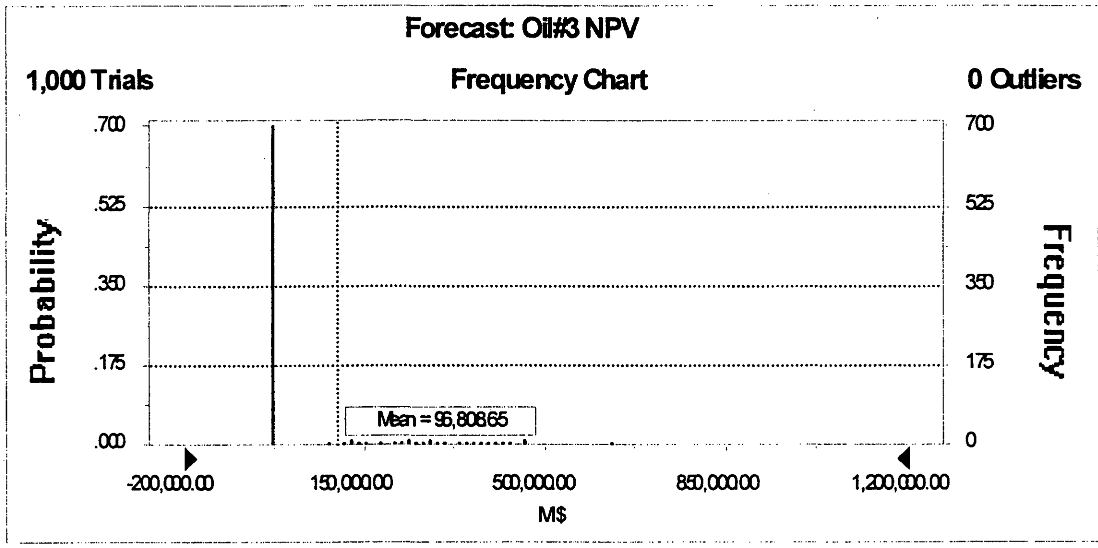

Figure 3.29 Oil#3 NPV.

Forecast: Oil#4 NPV

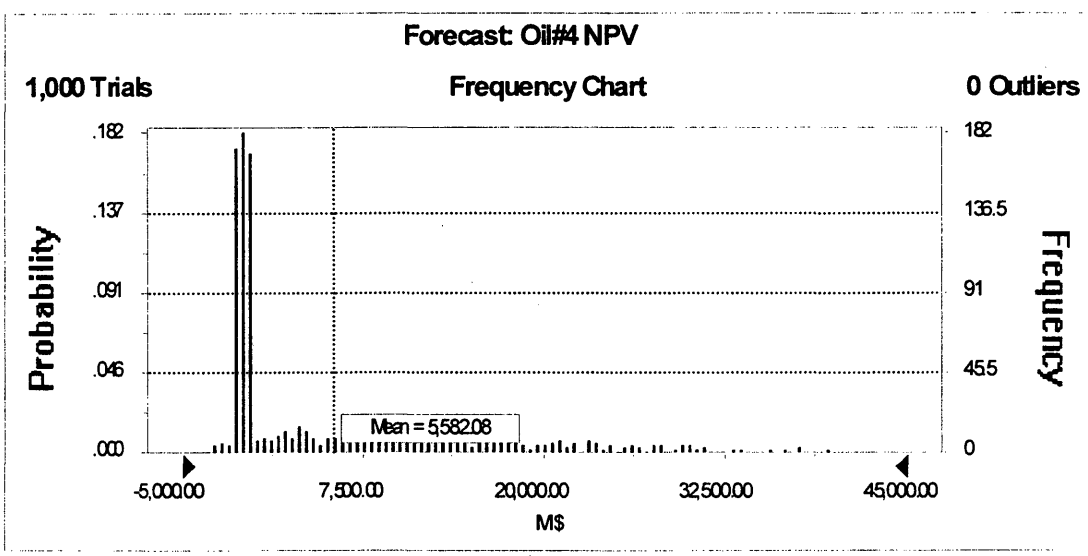

Figure 3.30 Oil#4 NPV.

Forecast: Oil#5 NPV

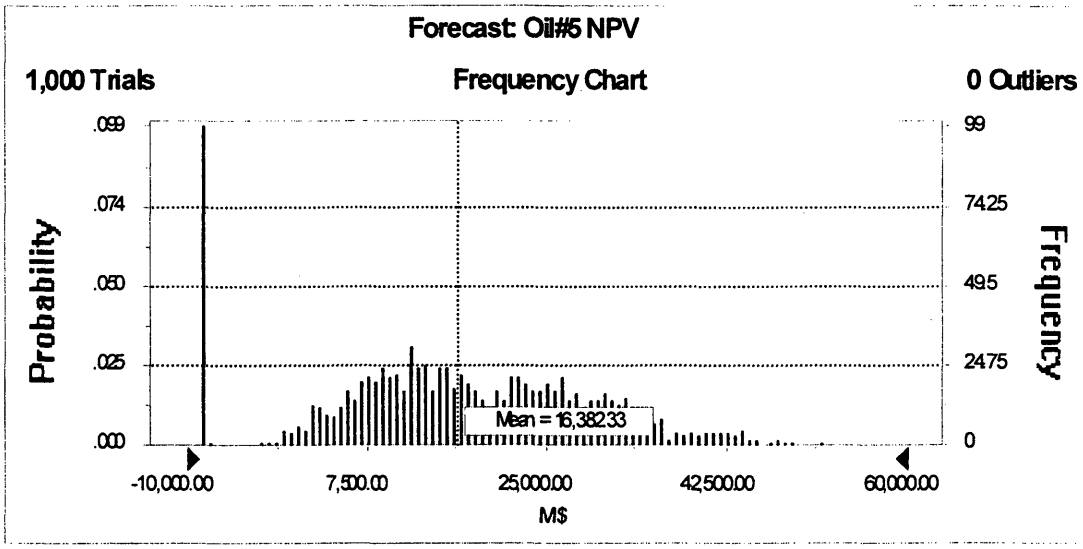

Figure 3.31 Oil#5 NPV.

Forecast: Oil#6 NPV

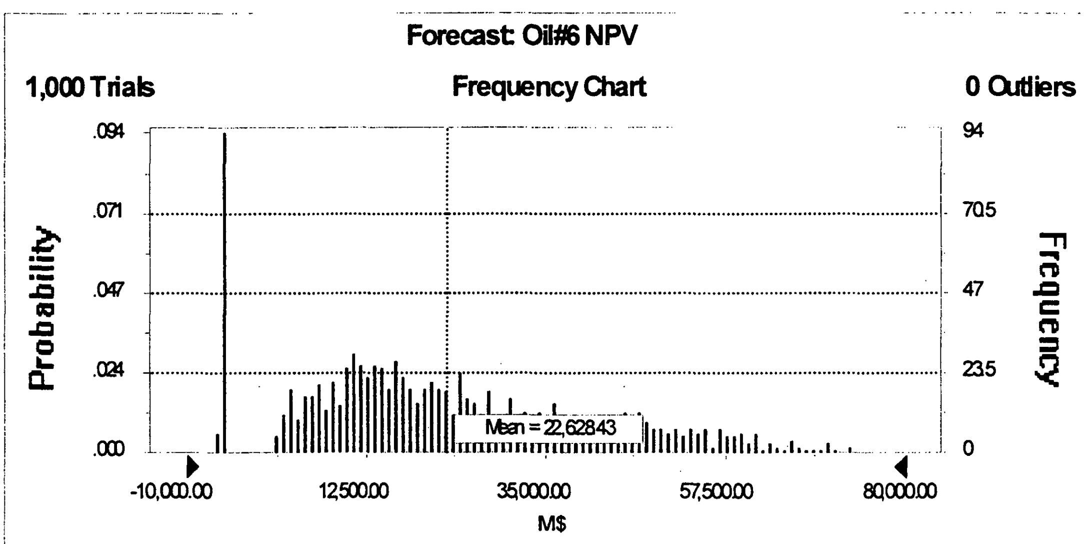

Figure 3.32 Oil#6 NPV.

Forecast: Oil#7 NPV

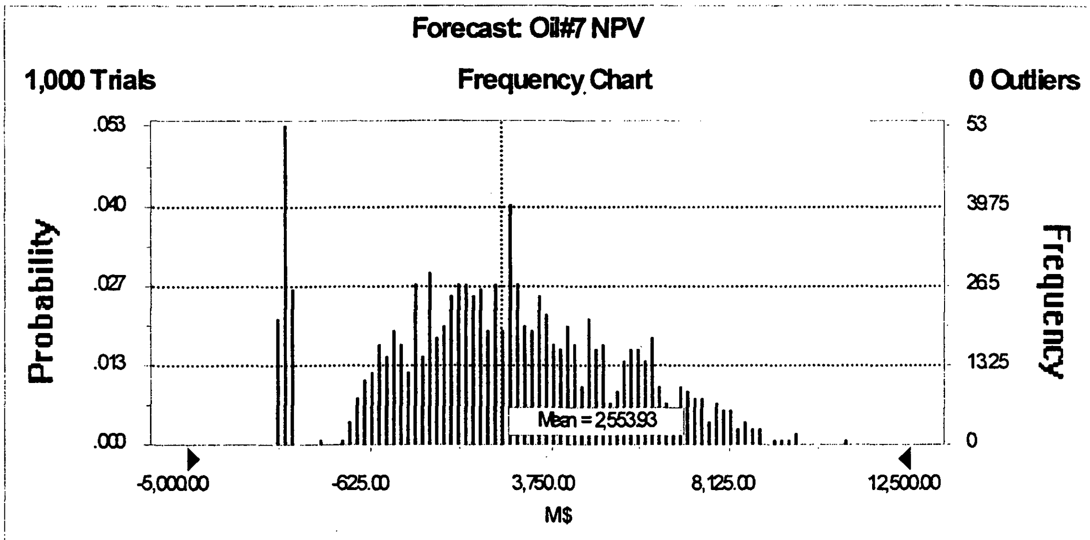

Figure 3.33 Oil#7 NPV.

Forecast: Oil#8 NPV

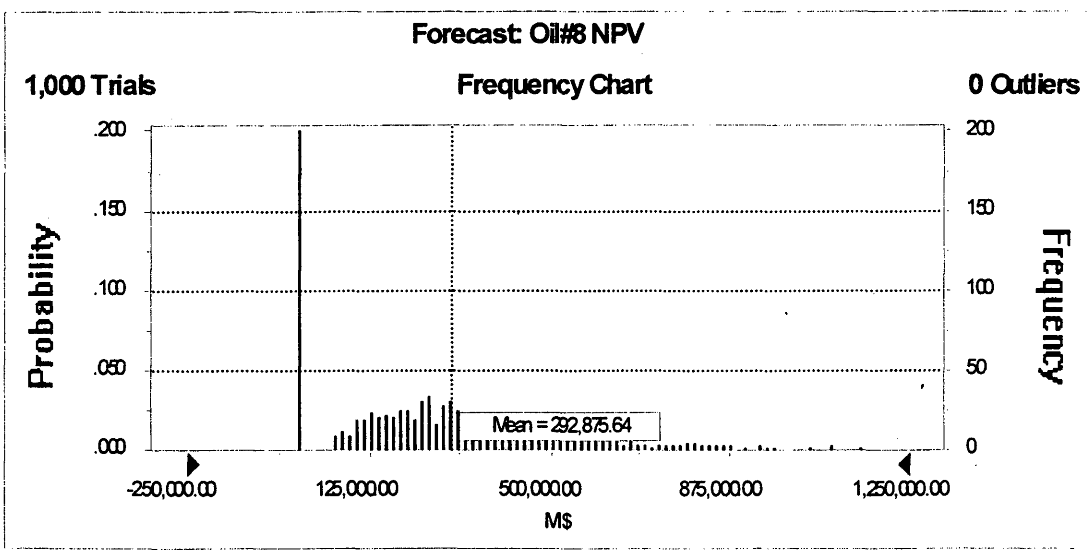

Figure 3.34 Oil#8 NPV.

Forecast: Oil#9 NPV

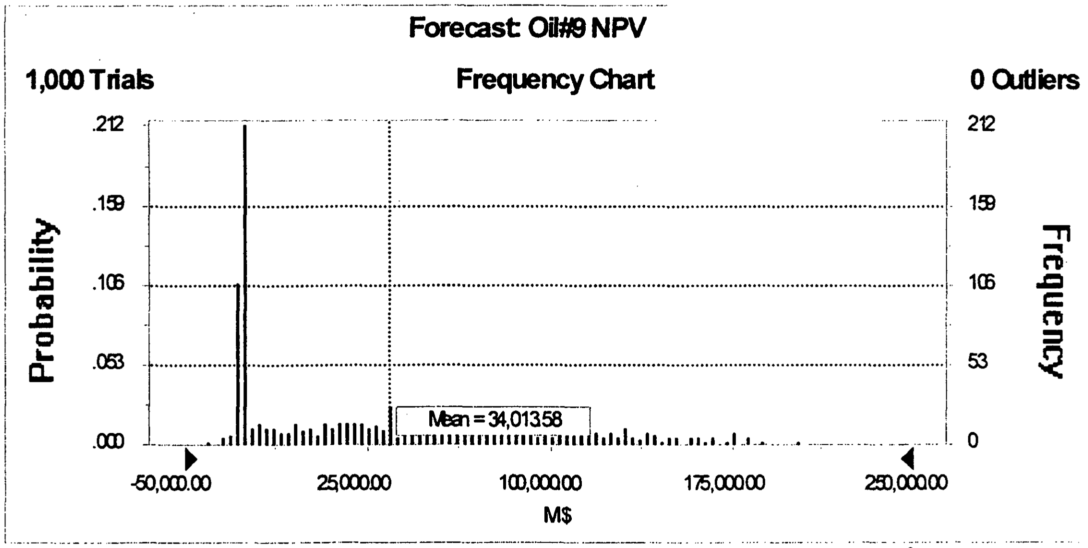

Figure 3.35 Oil#9 NPV.

Forecast: Oil#10 NPV

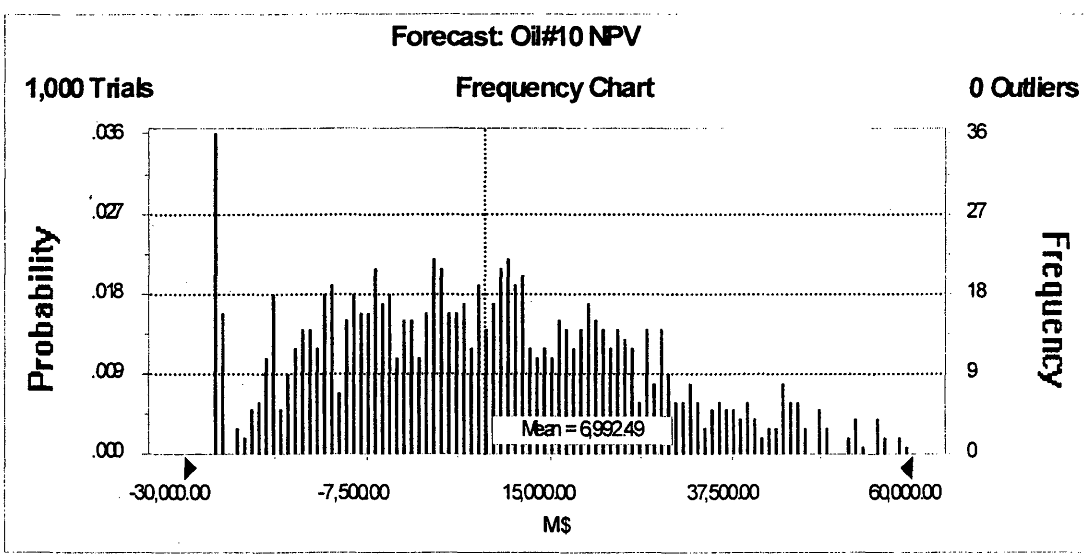

Figure 3.36 Oil#10 NPV.

Forecast: Oil#11 NPV

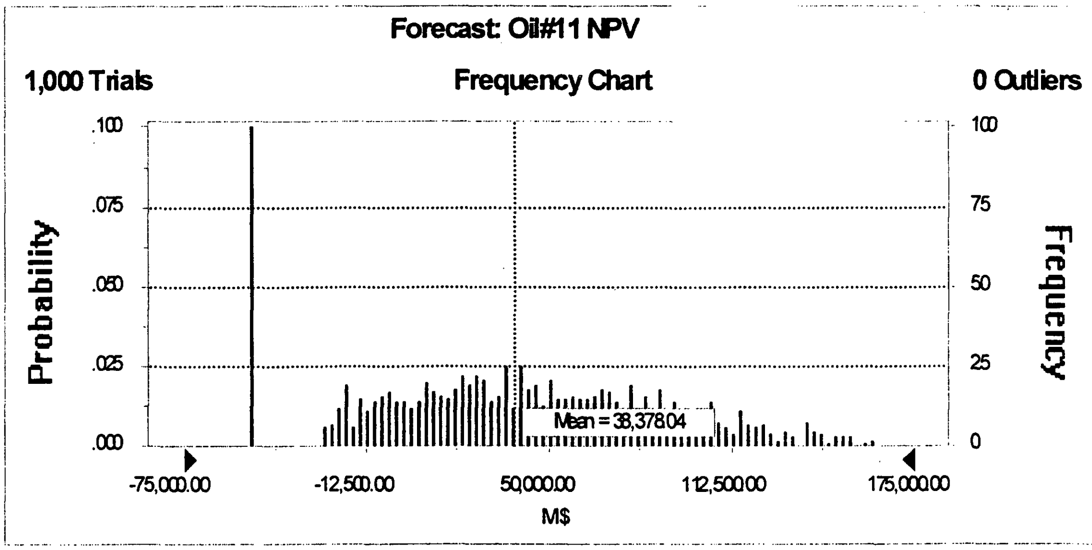

Figure 3.37 Oil#11 NPV.

Forecast: Oil#12 NPV

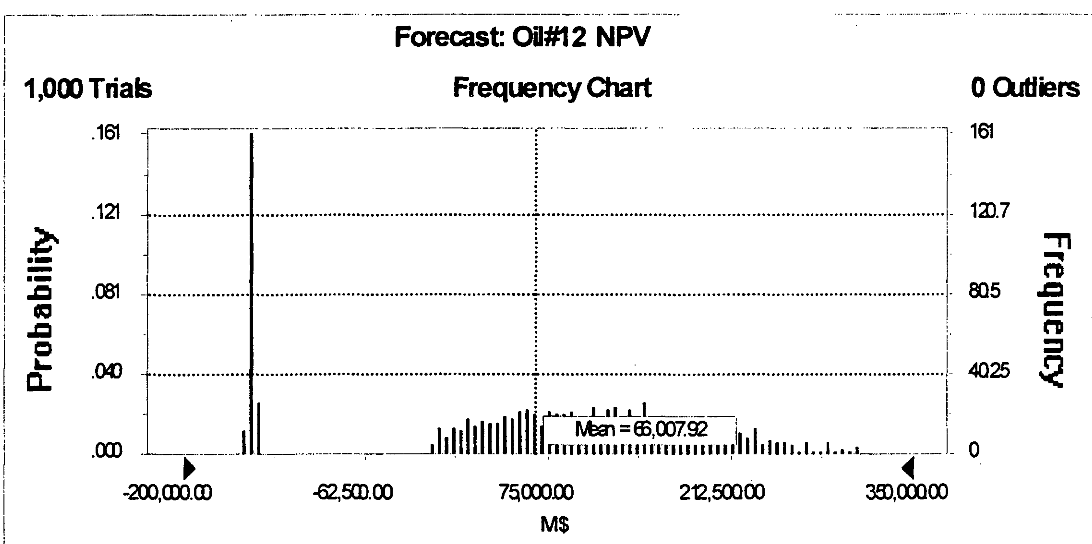

Figure 3.38 Oil#12 NPV.

Forecast: Oil#13 NPV

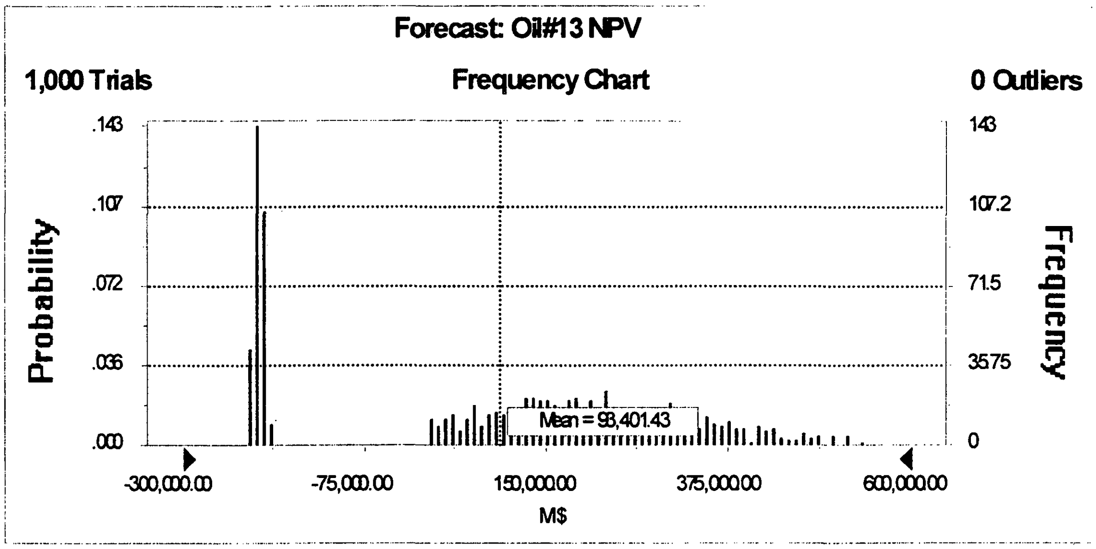

Figure 3.39 Oil#13 NPV.

Forecast: Oil#14 NPV

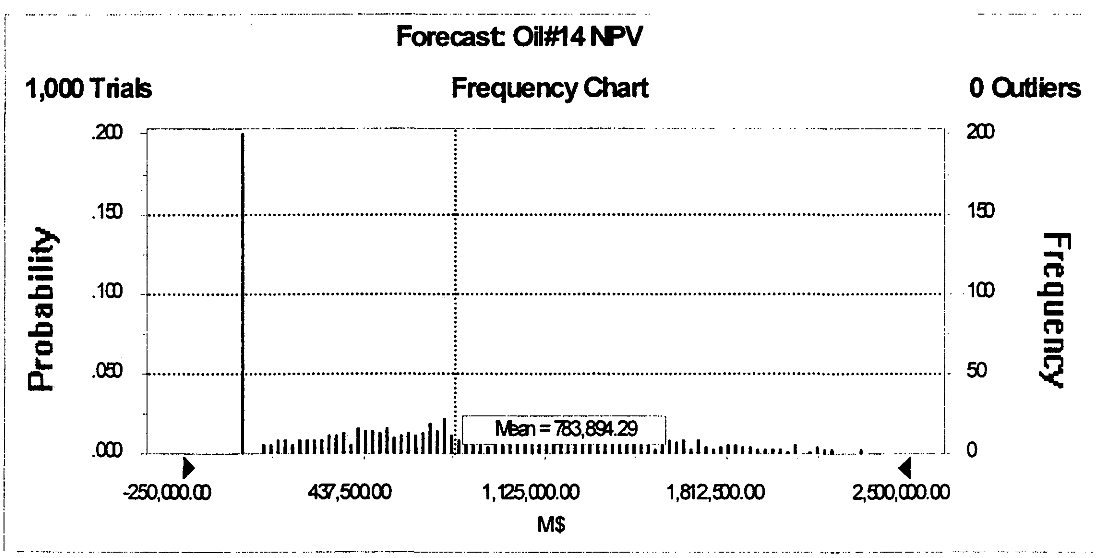

Figure 3.40 Oil#14 NPV.

Forecast: Oil#15 NPV

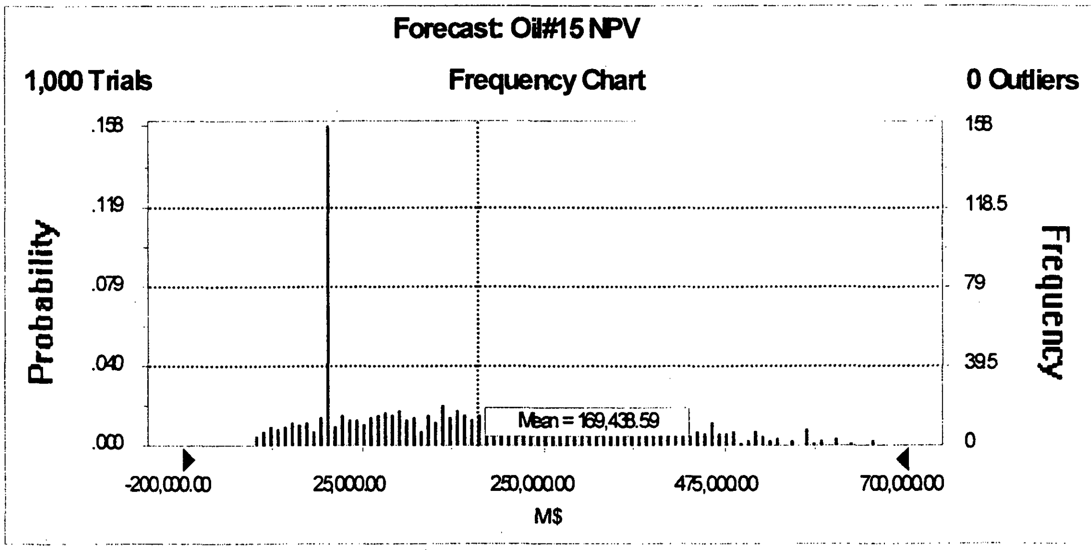

Figure 3.41 Oil#15 NPV.

Forecast: Oil#16 NPV

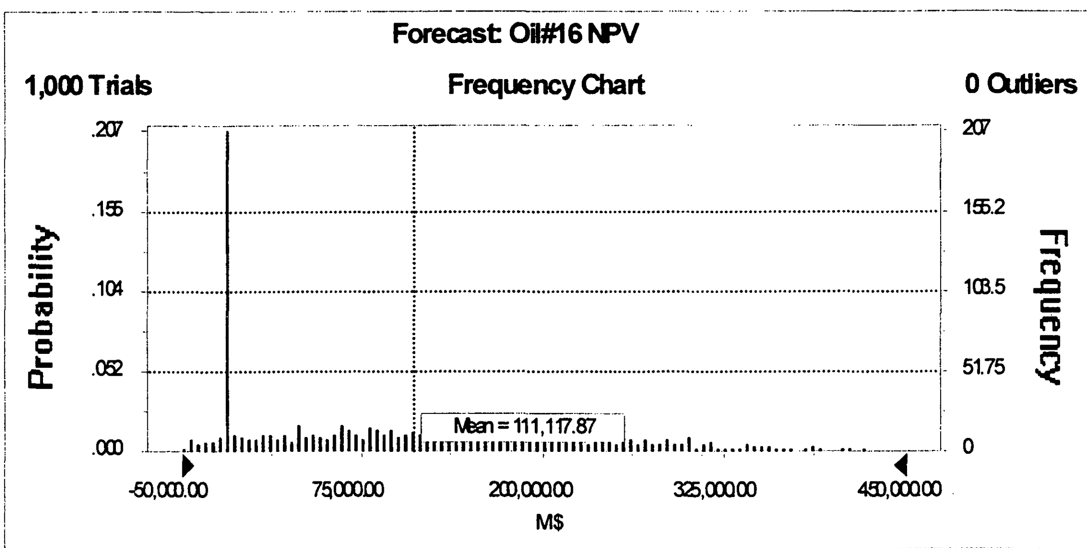

Figure 3.42 Oil#16 NPV.

Efficient Portfolios

If all possible combinations of investment projects were examined, each portfolio would have a specific mean NPV and a standard deviation of NPV associated with it. These are available only when doing a stochastic analysis of the projects. Standard deviation and mean NPV are used here as risk and reward, where the standard deviation represents the risk of not achieving the goal, and mean NPV as the reward of achieving that goal.

Plotting the means on one axis and the standard deviations on another axis generates a graph like Figure 3.43. Points on, or under, the curve represents possible combinations of investment projects. Points could lie above the curve when they represent infeasible solutions, for example, if they cost more than the allowed budget. These points are unobtainable combinations given the particular set of projects available and the budgetary restraints.

For any given mean NPV, there is one portfolio that has the smallest standard deviation possible. This portfolio lies on the curve at the point that intersects the mean NPV. Similarly, for any given standard deviation of NPV, there is one portfolio that has the highest mean NPV obtainable. This portfolio lies on the curve at the point that intersects the standard deviation of the NPV.

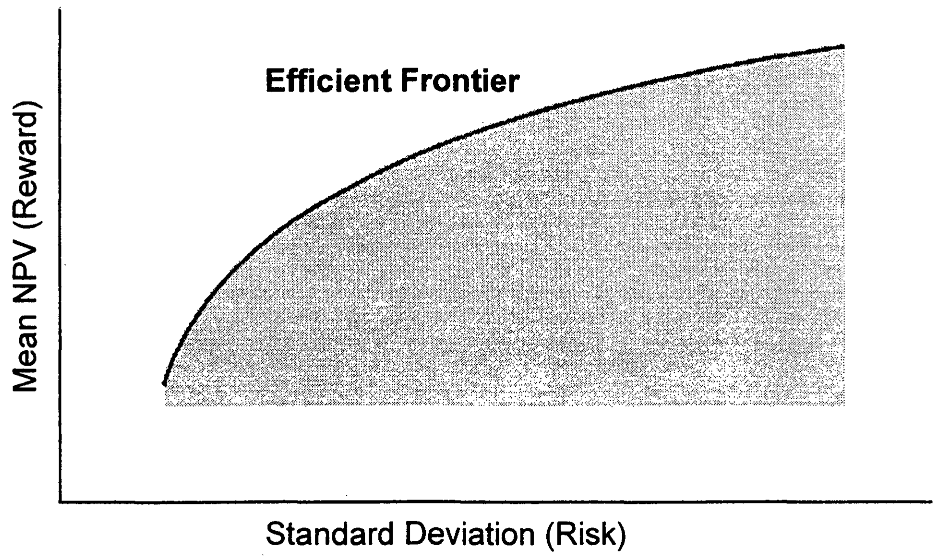

Figure 3.43 Efficient Frontier for Portfolios.

Portfolios that lie directly on the curve are called efficient (Markowitz 1957), since it is impossible to obtain a higher mean NPV without generating a higher standard deviation (risk) or a lower standard deviation (risk) without generating a lower mean NPV. The curve of efficient portfolios is often called the Efficient Frontier (EF).

Portfolios that lie below the curve are called inefficient, meaning better portfolios exist with either higher mean NPV (reward) or lower standard deviation (risk), or both. Other criteria can be used instead of mean NPV for selecting the portfolio, for example the tenth percentile (P10) of NPV. Also, maximizing the mean of the estimated ultimate recovery (EUR) can be used as an objective function. Portfolio cost is an example of minimizing the objective function.

Unfortunately, only one objective function can be used at one time to be maximized or minimized. Still, there is a way around this deficiency in the current optimizer, by applying a multi-objective function. This function can combine, through mathematical formulation, two objectives at the same time. One example used herein is to divide mean NPV by standard deviation, or some other risk measure, and maximize the resulting function. This way, mean NPV is increased while decreasing the standard deviation. This technique is called multi-objective (or multi-criteria) optimization. Conflicting objectives can be evaluated at the same time with this technique.

Correlation in the Model

Correlations among different projects and variables are included in part of the scenarios in order to observe the effects they have on the optimization process. Dependencies among variables are included only inside the project economic calculations. Unlike dependencies, correlations exist between different projects in the portfolio, and among the project variables themselves.

An example of dependencies is demonstrated in Table 3.6, where the success of certain layers is dependent on other layers. Success denotes that a layer is productive and hydrocarbons can be extracted economically. A random number generator, based on a custom distribution, determines which case is applied. If a layer is productive, then the amount it produces is uncertain, and another distribution is used to predict the outcome. In this example a Log-Normal distribution was used for each layer Estimated Ultimate Recovery (EUR), with varying limits to account for the different layer's capacity.

Layers Success	Layers Failure	Distribution Outcome	Probability of Outcome	Custom Distribution	EUR (BCF)
ABC	None	1	0.141	Range from 1-8	27.354
AB	C	2	0.049		0.000
AC	B	3	0.052		0.000
A	BC	4	0.046		0.000
BC	A	5	0.091		0.000
B	AC	6	0.032		0.000
C	AB	7	0.053		0.000
None	ABC	8	0.536		0.000
			1.000	Total EUR (BCF)	27.354

Table 3.6 Dependencies Among Gas#2 Layers.

Correlations among projects can arise from different sources. For instance, if two reservoirs are in the same sand formation and in the same locality, chances are if one is unproductive the other is too. Table 3.7 shows a correlation matrix of one of the scenarios of optimization. A correlation matrix is a table that contains correlation coefficients. There are two forms to introduce correlation coefficients into the portfolio. One is the correlation matrix and the other is by embedding the coefficient into the variables. The matrix form was chosen over variable-embedded correlation coefficients, because it can be switched on and off, and it is visible all the time. The correlation matrix in Table 3.7 has a series of stochastic variables, and this series is listed as the matrix first row and first column. Variables intersect at a single point inside the matrix, and that intersection point represents the correlation coefficient for both variables. The diagonal in the matrix represents variables intersecting with themselves.

Lower Triangle Correlation Matrix for Scenario A11	Oil#2 Chance of Dry Hole	Oil#2 Reserves Proved	Oil#3 Size of Discovery	Oil#3 Chance of Dry Hole	Oil#11 Chance of Dry Hole	Oil#11 Reserves MBBL	Oil#10 Chance of Dry Hole	Oil#10 Reserves MBBL	Oil#1 Chance of Dry Hole	Gas#9 Project Dry Hole	Gas#9 Reserves BCF	Gas#8 Project Dry Hole	Gas#8 Reserves BCF	Gas#7 Project Dry Hole	Oil#8 Size of Discovery	Oil#8 Chance of Dry Hole	Oil#5 Reserves MBBL	Oil#5 Chance of Dry Hole	Gas#2 Custom Distribution	Gas#3 Custom Distribution
Oil#2 Chance of Dry Hole	1.0																			
Oil#2 Reserves Proved		1.0																		
Oil#3 Size of Discovery			1.0																	
Oil#3 Chance of Dry Hole				1.0																
Oil#11 Chance of Dry Hole	1.0				1.0															
Oil#11 Reserves MBBL		1.0				1.0														
Oil#10 Chance of Dry Hole							1.0													
Oil#10 Reserves MBBL								1.0												
Oil#1 Chance of Dry Hole									1.0											
Gas#9 Project Dry Hole										1.0										
Gas#9 Reserves BCF											1.0									
Gas#8 Project Dry Hole										1.0		1.0								
Gas#8 Reserves BCF											1.0		1.0							
Gas#7 Project Dry Hole														1.0						
Oil#8 Size of Discovery			1.0												1.0					
Oil#8 Chance of Dry Hole				1.0												1.0				
Oil#5 Reserves MBBL								1.0									1.0			
Oil#5 Chance of Dry Hole							1.0											1.0		
Gas#2 Custom Distribution																			1.0	
Gas#3 Custom Distribution																			1.0	1.0

Table 3.7 Correlation Matrix (blanks represent zero value) .

Risk Analysis Measurements

Risk is a subjective matter, with no standard measure for it. Techniques for accounting for risk differ from one company to the other, making a universal measure for it impossible. For the model in hand, three risk measures have been used:

-Value at Risk (VaR),

-Standard deviation,

-Tenth percentile (P10).

Value at Risk (VaR) is a financial risk measure which is gaining attention in the business world. Many measures utilize it or its offspring, such as reward per risk (RPR). VaR is calculated in this model by subtracting the fifth percentile of the NPV distribution from the mean NPV (NPV mean-P5).

Standard deviation is a measure of dispersion about the mean NPV. It is the square root of the distribution variance. Standard deviation measures the volatility in the forecast, and it is a good indicator of risk. The tenth percentile (P10) means that 90 percent of the time the portfolio, or the forecast, will have values greater than P10. Plotting mean NPV versus the tenth percentile results in a comparison between expected values and actual value.

CHAPTER FOUR

DISCUSSION AND RESULTS

This chapter presents the results obtained from the model and discussion on optimization speed and risk measurements. Sixteen different scenarios have been evaluated. These scenarios represent a broad range of an oil company's needs from the portfolio optimization standpoint. Results from selected scenarios are discussed here, while the rest can be found in Appendix B. A plot of Efficient Frontier (Figure 3.43 from Chapter Three) is included with every case when applicable. Also, projects comprising the portfolios are included. Table 4.1 is a summary of the cases that have been evaluated.

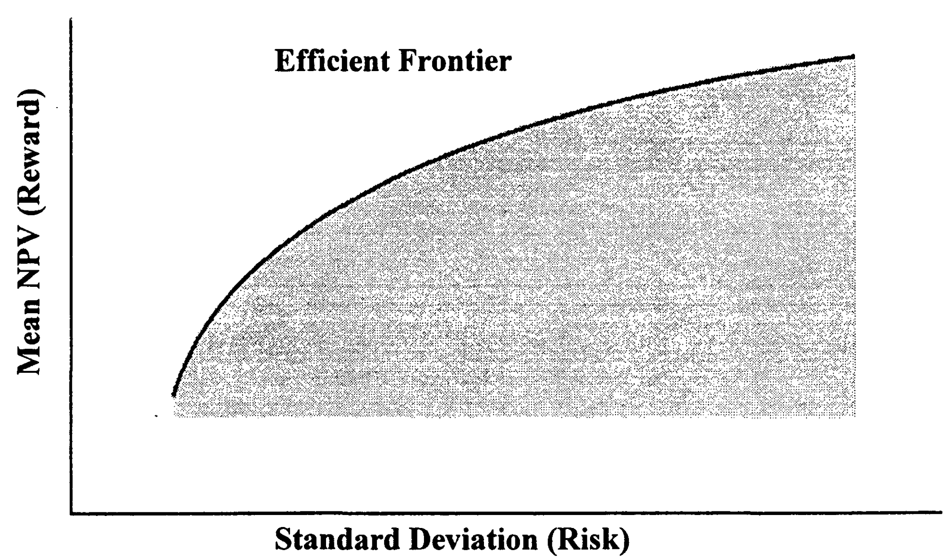

Figure 3.43 Efficient Frontier for Portfolios.

Scenario No.	Objective Function	Budget $million	Risk Measure $million	Correlations	Comments
A1	Maximize mean NPV	< 400	Value at Risk [10-1000]	OFF	
A2	Maximize mean NPV	< 200	Value at Risk [100-1000]	OFF	
A3	Maximize mean NPV	< 100	Value at Risk [1-1000]	OFF	
A4	Maximize mean NPV	< 400	Tenth percentile P10 [1-1000]	OFF	
A5	Maximize mean NPV	< 200	Tenth percentile P10 [1-1000]	OFF	
A6	Maximize mean NPV	< 400	Value at Risk [1-1000]	OFF	25% WI increments
A7	Maximize mean NPV	< 100	Tenth percentile P10 [1-1000]	OFF	
A8	Maximize mean NPV	< 200	Standard deviation [1-1000]	OFF	
A9	Maximize mean NPV	< 400	Standard deviation [1-1000]	OFF	25% WI increments
A10	Maximize mean Oil EUR	< 200	N/A	OFF	
A11	Maximize mean NPV	< 400	Value at Risk [1-1000]	ON	
A12	Maximize mean NPV	< 200	Value at Risk [1-1000]	ON	
A13	Maximize mean NPV	< 400	Tenth percentile P10 [1-1000]	ON	Positive Correlation
A14	Maximize mean NPV	< 400	Tenth percentile P10 [1-1000]	ON	Negative Correlation
A15	Maximize mean NPV	< 200	Standard deviation [1-1000]	ON	
A16	Maximize mean Oil EUR	Variable [10-400]	N/A	OFF	

Table 4.1 Summary of Optimization Scenarios Evaluated.

Optimization Speed

The first few optimizations consisting of thirty projects took approximately six days of simulation time on a Pentium-4, 2 GHz computer. A portfolio consisting of thirty investment projects has one billion possible combinations (2^{30}), and that's only considering an "all or nothing" approach to working interest WI. Also, each portfolio undergoes 500 iterations in the stochastic stage. Both of these reasons are the leading causes of slowing the optimization process.

The number of projects in the portfolio was reduced from thirty to twenty projects. This reduced the number of combinations from one billion to one million (2^{20}) possibilities. As for the iterations in the stochastic stage, testing was performed on the Latin Hypercube sampling method versus Monte Carlo sampling method, and the former method managed to achieve similar results as the latter with fewer iterations. The number of iterations was reduced from 500 to 250.

Latin Hypercube sampling is basically dividing the distribution of the variable into different non-overlapping segments, each having the same probability. These segments were then populated with data generated randomly. The advantage of this method is the whole distribution is sampled with less iteration than the Monte Carlo sampling method. These steps reduced the optimization time on average to less than forty-eight hours.

Scenario A1

In this scenario the mean NPV of the portfolio was maximized under the restriction of a $400 million budget. The efficient frontier graph is plotted using the value at risk (VaR) as a risk measure. Figure 4.2 shows the efficient frontier graph for scenario A1. The graph, as expected, is not as smooth as the ideal efficient frontier because Figure 3.43 is based on corporate stocks, not investment projects. Also, differences can be attributed to working interest (WI) percentage increments, the smaller WI increment, the smoother the graph. This difference has no effect on portfolio selection.

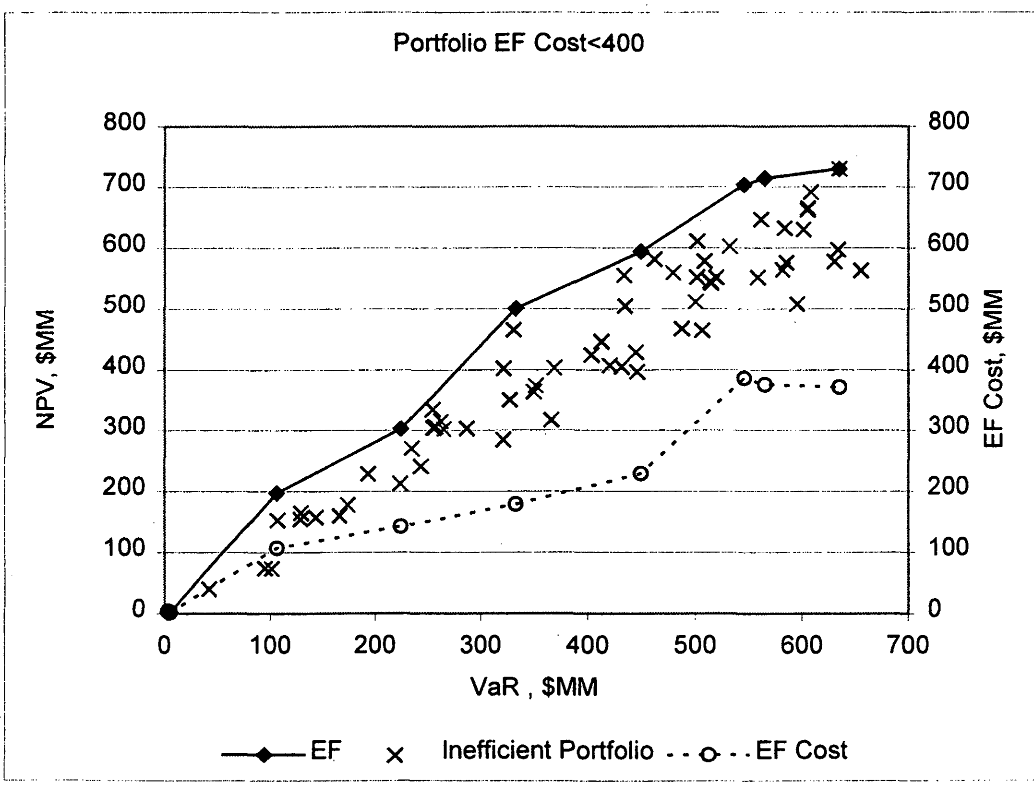

Figure 4.2 Scenario A1 Efficient Frontier.

Portfolios that lie on the solid line are said to be efficient, and under these conditions no other portfolio has a higher NPV with the same risk, or less risk. Portfolios marked with the asterisk are called inefficient. The optimization model actually generated a higher number of inefficient portfolios, but for clarity only a selected few are plotted.

Although the same budget is used for all portfolios, not all of them have the same cost. Efficient portfolio costs can be found by dropping vertically to the dashed cost line in Figure 4.2. From the graph it can be seen that the cost of the efficient portfolios are rising steadily with NPV for the lower risk portfolios. For higher NPV portfolios, an increase in risk resulted in a cost reduction. This last point is an example that although some projects are cheaper, because they contribute more risk to the portfolio, they are not chosen until higher risk portfolios are allowed. A clear example is project Oil#11, where the high uncertainty in its NPV is causing it to be chosen only in higher risk portfolios. Figure 4.3 and Table 4.2 show efficient portfolios compositions.

Figures and tables of portfolio compositions go hand in hand. In finding where a project is in terms of portfolios, it is easier to see it in the graph, such as Figure 4.3. However, if the compositions of a certain portfolio are sought, then the table is more appropriate. In the graph the twenty projects are in the X-axis, portfolio NPV's are in the Y-axis, and the project WI increments (zero or 1.0) are in the Z-axis.

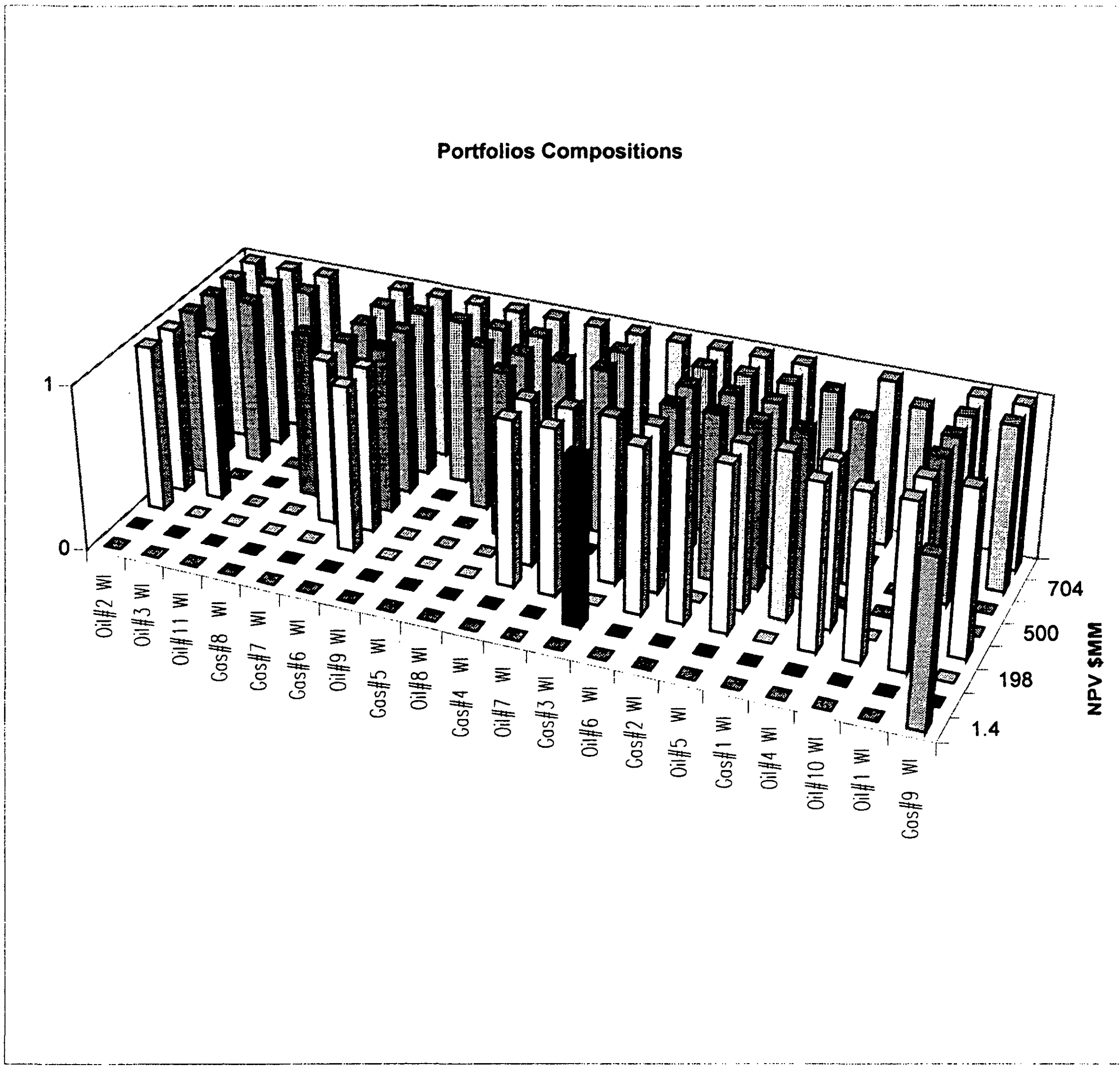

Figure 4.3 Efficient Portfolios Compositions.

NPV $MM	Cost $MM	Risk $MM	Oil#2 WI	Oil#3 WI	Oil#11 WI	Gas#8 WI	Gas#7 WI	Gas#6 WI	Oil#9 WI	Gas#5 WI	Oil#8 WI	Gas#4 WI	Oil#7 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Gas#1 WI	Oil#4 WI	Oil#10 WI	Oil#1 WI	Gas#9 WI
1.4	4.3	3.4																				1
2.8	2.3	5.8												1								
198	107	106	1					1				1	1		1	1	1		1	1	1	
303	143	224	1	1			1	1				1	1	1	1		1	1	1		1	1
****500***	***179***	***333***	**1**			**1**	**1**	**1**			**1**				**1**	**1**	**1**	**1**			**1**	
594	229	450	1	1			1	1		1	1	1	1		1	1	1		1		1	
704	386	545	1	1	1		1	1	1	1	1		1		1	1	1	1		1	1	1
715	376	565	1	1	1		1	1	1	1	1	1	1	1	1	1	1		1		1	1

Table 4.2 Efficient Portfolios Compositions.

The efficient portfolio with an NPV of 500 $MM and a cost of 179 $MM was chosen for further analysis. Extensive stochastic analysis was performed to refine the statistics of the portfolio. The choice of this portfolio is arbitrary, since every company has its own risk criteria. Some companies would prefer less risk exposure for their capital, even if it means less profit. Others are very aggressive in their risk taken. Figures 4.4, 4.5, 4.6, and 4.7 show NPV, Cost, Oil EUR, and Gas EUR, respectively, for this portfolio.

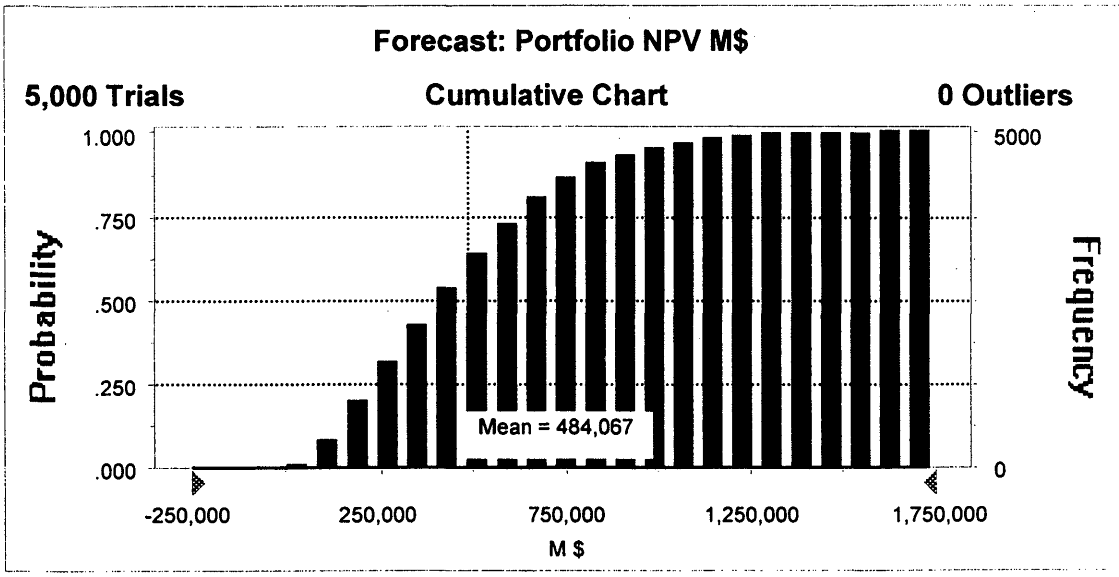

Figure 4.4 A1 Chosen Efficient Portfolio NPV Cumulative Frequencies.

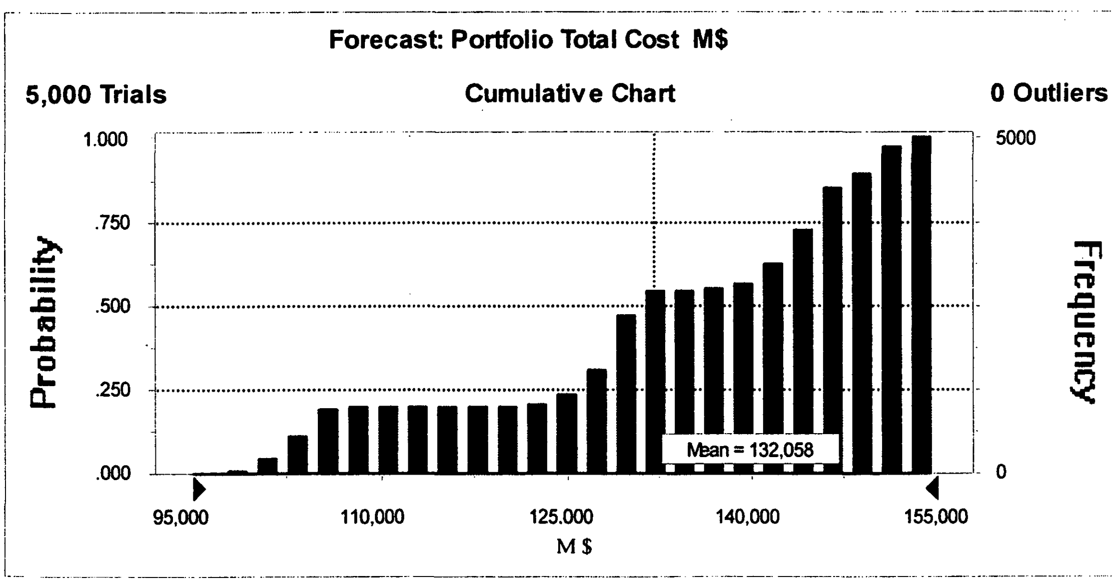

Figure 4.5 A1 Chosen Efficient Portfolio Cost Cumulative Frequencies.

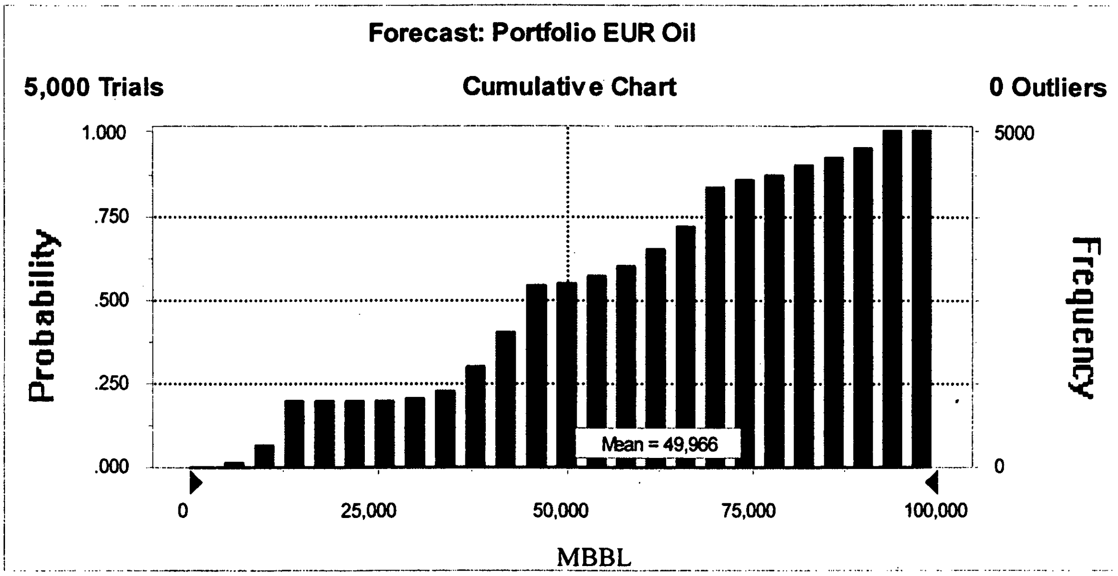

Figure 4.6 A1 Chosen Efficient Portfolio Oil EUR Cumulative Frequencies.

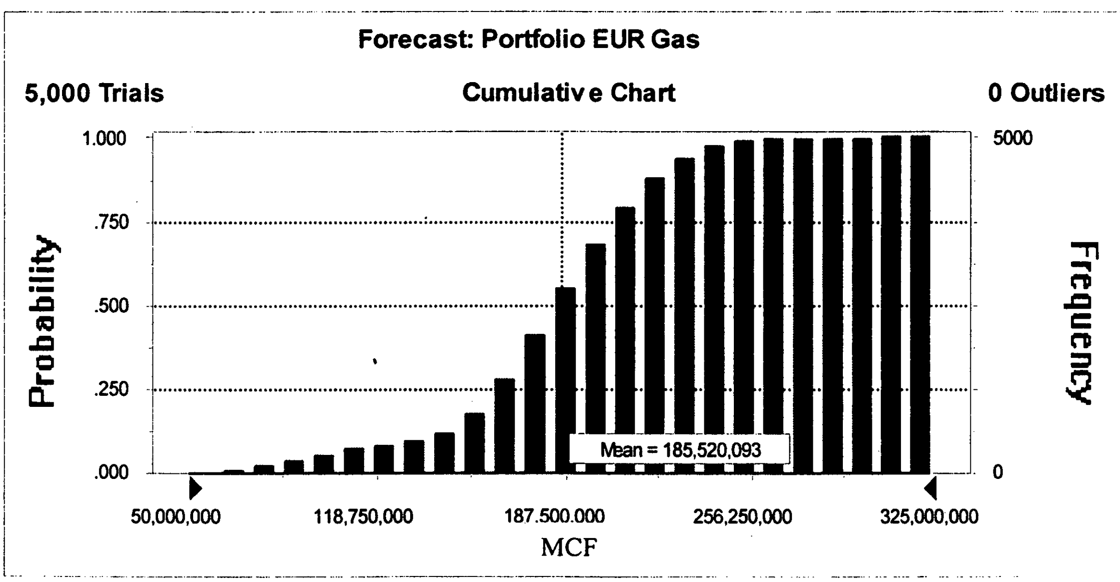

Figure 4.7 A1 Chosen Efficient Portfolio Gas EUR Cumulative Frequencies.

The first thing to notice is the difference in the NPV from the optimization model and the results in Figure 4.4. The optimization NPV is $500 million, and in Figure 4.4 the mean NPV is $484 million, a difference of $16 million. The difference is approximately three percent, which is considered small compared to the difference in the number of iterations. The extensive stochastic analysis used 5,000 iterations, compared to the 250 iterations during the optimization stage. This significant increase in iterations gave only a three percent change in the forecast. This is acceptable since we are dealing with a stochastic model.

Figures 4.6 and 4.7 are useful in determining the three reserve categories, Proven, Probable, and Possible. The word "Outliers", which can be seen in all the graphs, is just a range display; if the certainty is reduced "outliers" indicates the number of points outside the graph.

Scenario A2

The scenario A2 objective function is to maximize the mean NPV, with a budget restraint of less than $200 million. Half of the efficient portfolios costs in scenario A1 were less than $200 million, so in order to examine that part of the efficient portfolio a budget of $200 million was set for scenario A2. Value at Risk is the risk measure in this scenario. In order to observe the effect of a different budget, most of the variables were kept the same as in scenario A1. Unlike A1, the risk range was limited to portfolios with a risk greater than \$100 million. Figure 4.8 shows the efficient portfolios graph for scenario A2.

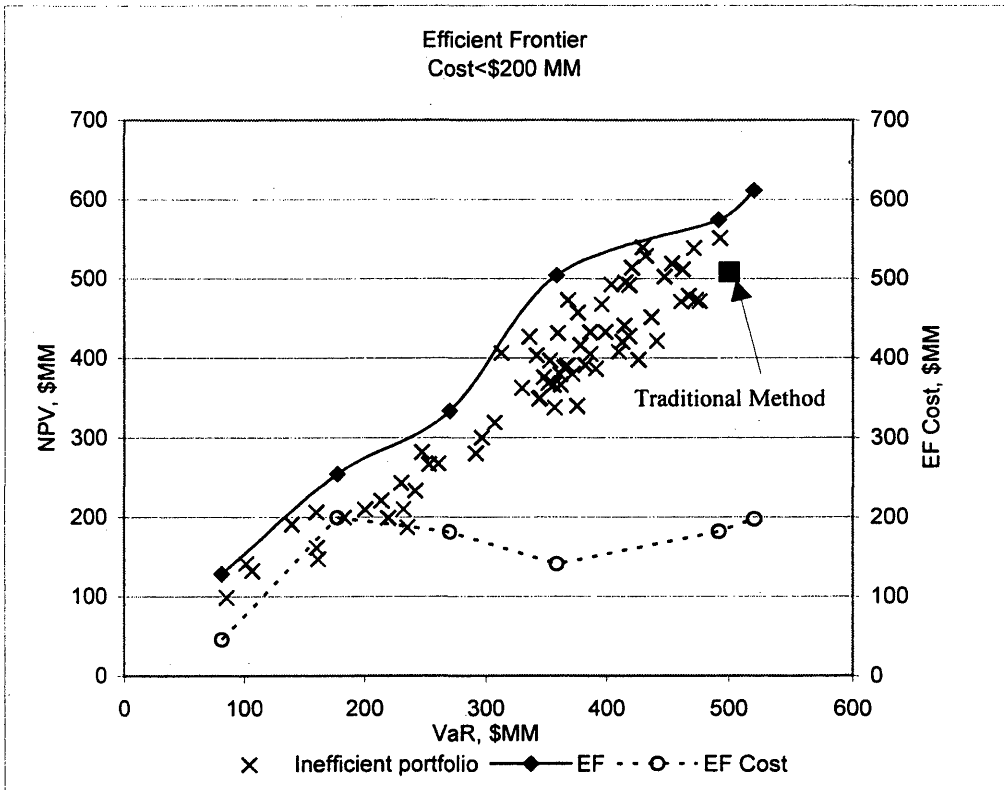

Figure 4.8 Scenario A2 Efficient Frontier.

As expected, the new efficient portfolios found in scenario A2 still lie on the same efficient frontier as in scenario A1, since they share the same set of conditions. Figure 4.9 and Table 4.3 show the efficient portfolios composition for scenario A2. As both cases A1 and A2 are similar, no extensive stochastic analysis was performed for this scenario A2.

In Figure 4.8 a point that represents the traditional method is plotted in the efficient frontier graph. Risk is infrequently addressed in traditional methods. However, for the example in Table 3.5 in Chapter Three, risk was calculated in order to compare the performance of traditional methods and the portfolio analysis. The VaR for the traditional method result was $501.012 MM and the mean NPV was $508.277 MM. The traditional method point lies below the efficient frontier line with the other inefficient portfolios. In the graph of Figure 4.8 there are other portfolios with a higher mean NPV and a lower risk. Therefore, if the traditional method portfolio is chosen, the company is undertaking unnecessary risk without any gain in NPV.

The discussion above demonstrates that the portfolio analysis methodology proposed herein gives improved results in terms of economic performance over the traditional methods, thus the first hypothesis has been addressed.

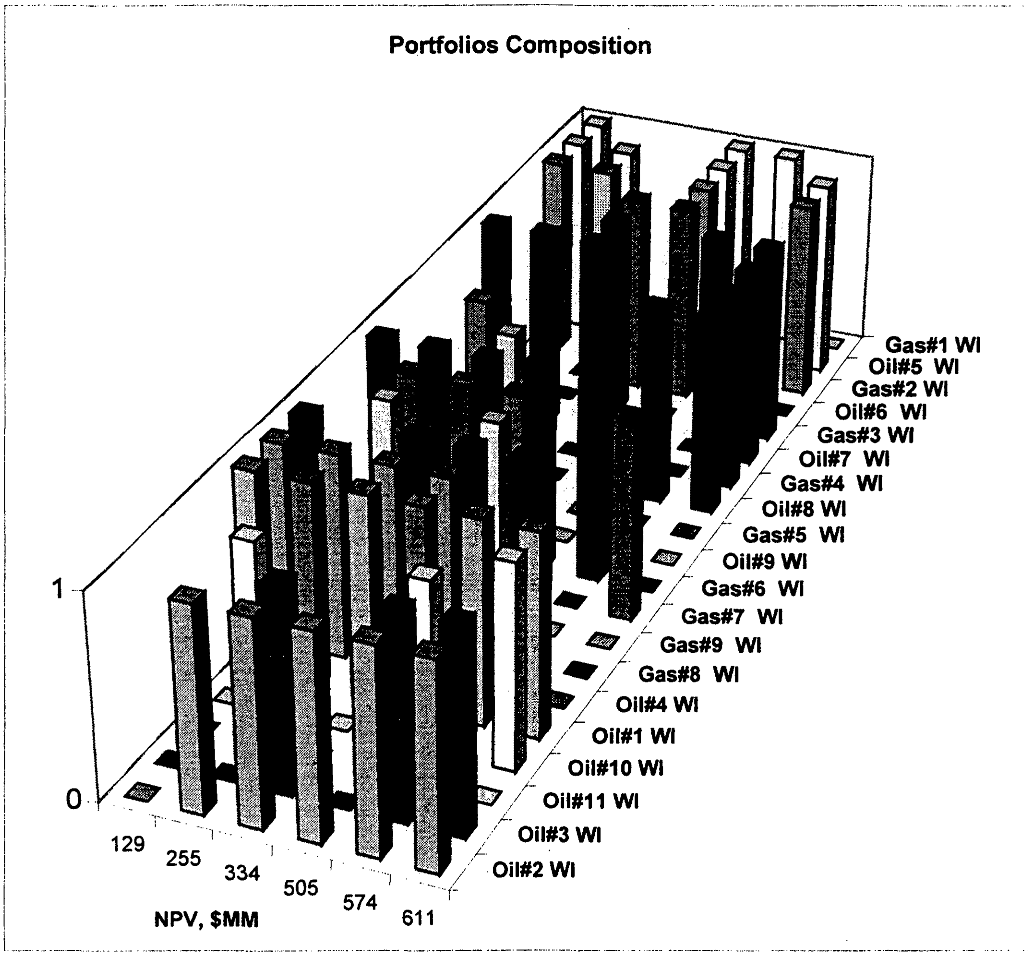

Figure 4.9 Scenario A2 Efficient Portfolios Compositions (WI equals 0 or 1).

NPV $MM	Cost $MM	Risk $MM	Oil#2 WI	Oil#3 WI	Oil#11 WI	Oil#10 WI	Oil#1 WI	Oil#4 WI	Gas#8 WI	Gas#9 WI	Gas#7 WI	Gas#6 WI	Oil#9 WI	Gas#5 WI	Oil#8 WI	Gas#4 WI	Oil#7 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Gas#1 WI
129	46	82					1	1	1			1					1			1	1	1
255	199	178	1		1		1	1		1	1	1		1			1			1	1	
334	181	270	1	1			1	1	1		1	1	1				1	1	1			
505	141	359	1				1	1	1	1	1	1			1				1	1	1	1
574	182	492	1	1	1		1		1			1			1			1				1
611	198	521	1	1		1	1				1				1	1	1	1		1	1	

Table 4.3 Scenario A2 Efficient Portfolios Compositions.

Scenario A4

Changing the risk measure in the portfolio provides a different perspective of the risk analysis. The risk measure is what differentiates this scenario from previous ones. The tenth percentile P10 is used here as the risk measure. The objective function is to maximize the mean NPV, while using a budget of $400 million or less. Figure 4.10 shows the efficient frontier graph for scenario A4 with risk defined as the tenth percentile of NPV. The shape of the graph is different because of the risk measure used here. Note it is desirable to be farther to the right on this graph, since that represents a higher estimate value of NPV tenth percentile. Using the tenth percentile as a risk measure yields different efficient portfolios as compared with value at risk (VaR). For instance, investment project Oil#5 is a dominant project in most of the portfolios in scenarios A1 and A2, but it is not included in the A4 efficient portfolios. Figure 4.10 and Table 4.4 shows scenario A4 efficient frontier and the portfolios compositions, respectively.

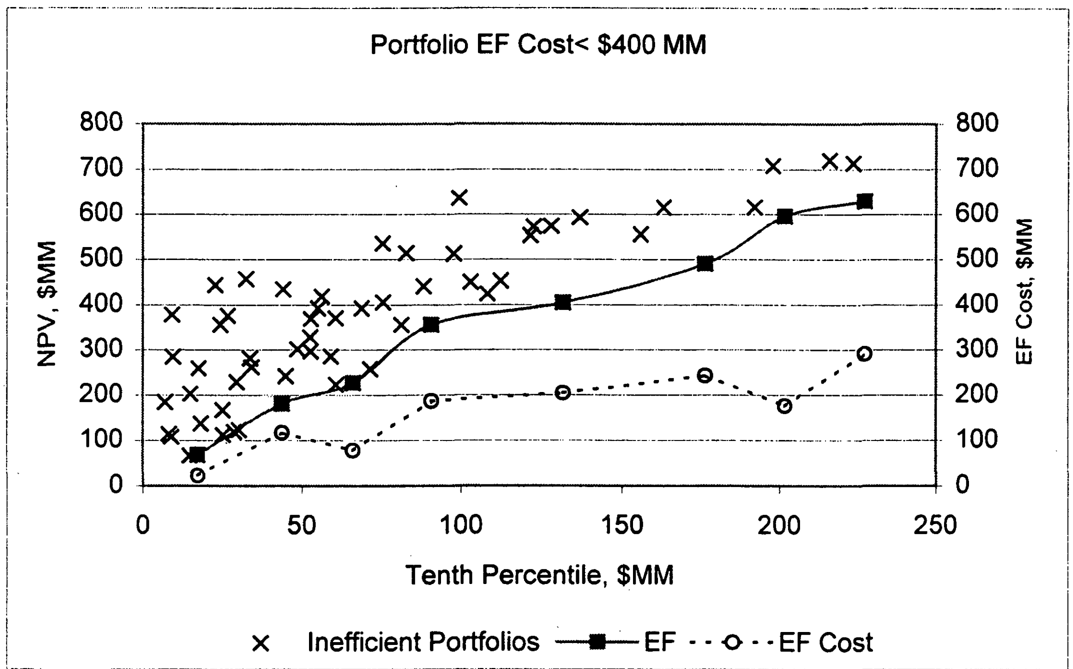

Figure 4.10 Scenario A4 Efficient Frontier.

NPV $MM	Cost $MM	Risk $MM	Oil#2 WI	Oil#3 WI	Oil#11 WI	Oil#10 WI	Oil#1 WI	Oil#4 WI	Gas#9 WI	Gas#8 WI	Gas#6 WI	Gas#7 WI	Oil#9 WI	Gas#5 WI	Oil#8 WI	Gas#4 WI	Oil#7 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Gas#1 WI
68	23	17					1	1	1								1	1		1		
182	118	44	1	1		1				1		1					1	1	1			
227	79	66	1	1			1	1			1						1	1	1	1		
356	187	91	1			1		1	1	1		1		1	1				1			
405	204	132	1			1		1	1	1		1		1	1				1			
491	243	177	1		1					1	1		1		1	1	1		1			1
596	177	202	1	1		1	1			1		1			1		1	1				1
630	288	228	1	1	1		1	1		1	1	1		1	1	1	1					1

Table 4.4 Scenario A4 Efficient Portfolios Compositions.

The efficient portfolio with an NPV of $596 MM and a cost of $177 MM is chosen for further analysis. The reason in this case is to be able to compare it to the scenario A1 portfolio, since both costs are almost the same. The idea behind this is to compare how much hydrocarbons can be produced with a $177 MM capital expenditure. Figures 4.11, 4.12, 4.13, and 4.14 show scenario A4's chosen portfolio NPV, Cost, Oil EUR, and Gas EUR, respectively.

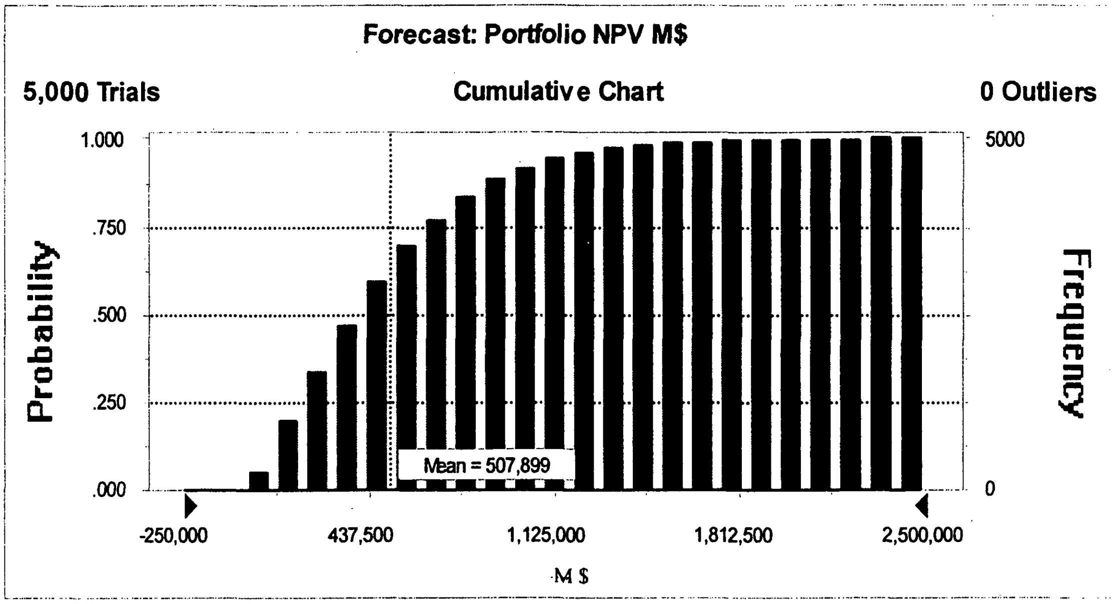

Figure 4.11 A4 Chosen Efficient Portfolio NPV Cumulative Frequencies.

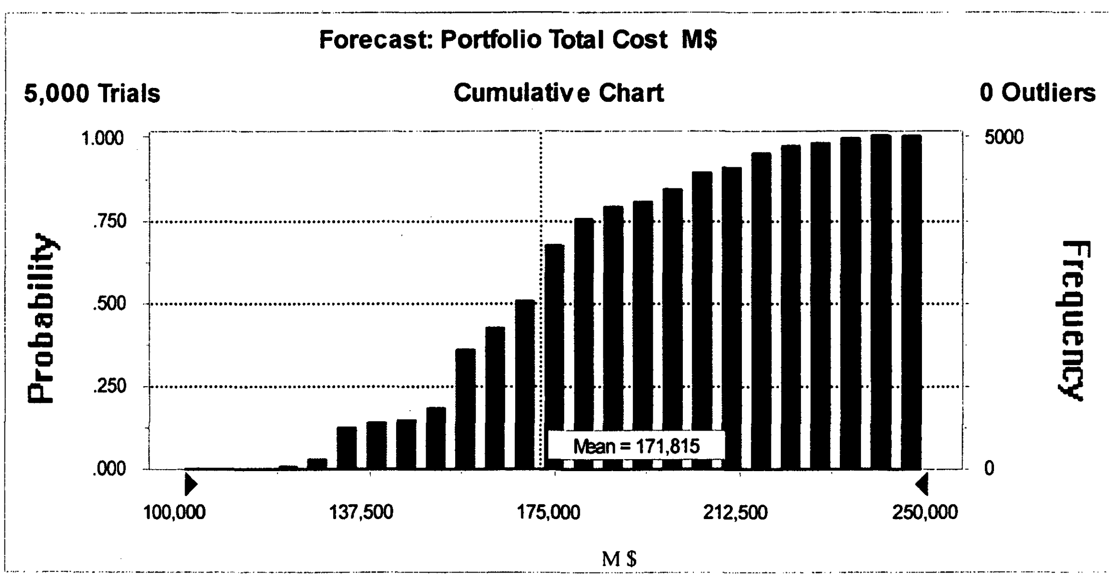

Figure 4.12 A4 Chosen Efficient Portfolio Cost Cumulative Frequencies.

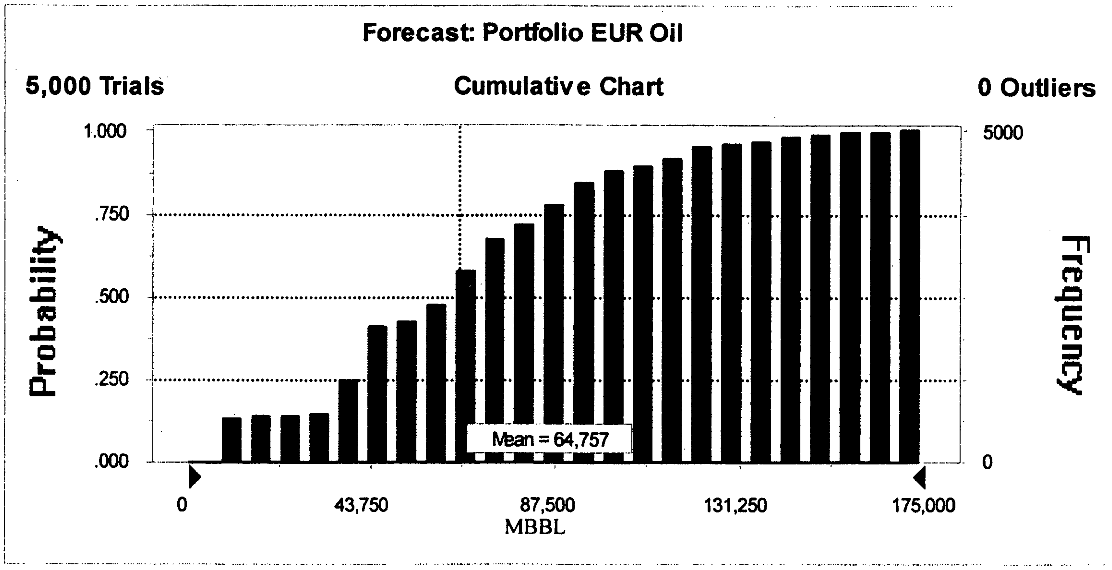

Figure 4.13 A4 Chosen Efficient Portfolio Oil EUR Cumulative Frequencies.

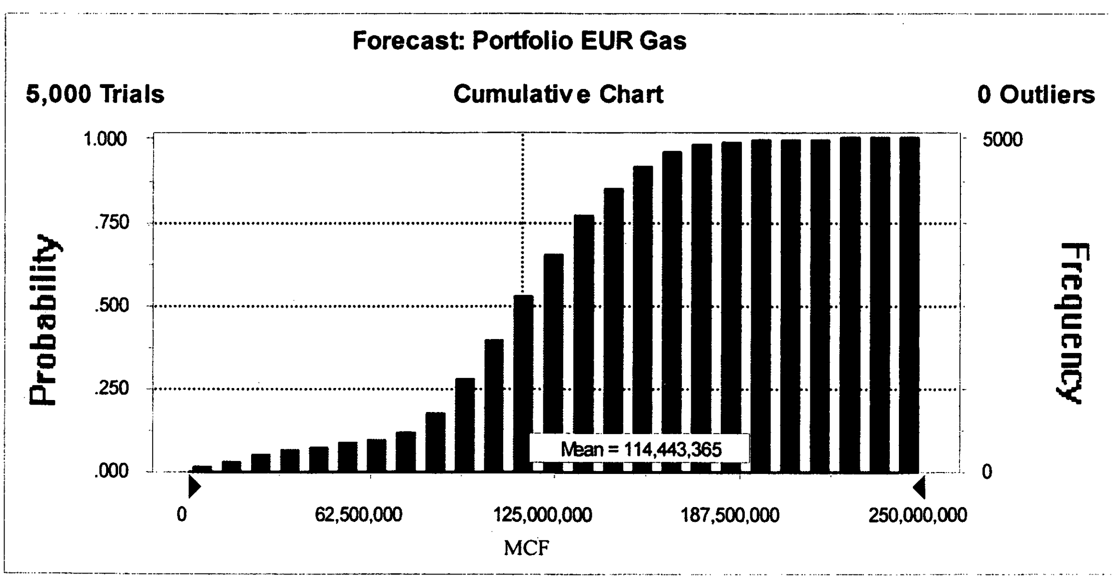

Figure 4.14 A4 Chosen Efficient Portfolio Gas EUR Cumulative Frequencies.

Although the scenarios A1 and A4 have similar costs, their compositions are different. Table 4.5 shows a comparison between the two scenarios A1 and A4 compositions.

	Oil EUR MMBBL	Gas EUR BCF	Oil#2 WI	Oil#3 WI	Oil#11 WI	Oil#10 WI	Oil#1 WI	Oil#4 WI	Gas#9 WI	Gas#8 WI	Gas#6 WI	Gas#7 WI	Oil#9 WI	Gas#5 WI	Oil#8 WI	Gas#4 WI	Oil#7 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Gas#1 WI
A1	50	186	1				1			1	1	1			1				1	1	1	1
A4	65	114	1	1		1	1			1		1			1		1	1				1

Table 4.5 Comparisons Between A1 and A4 Scenarios.

Scenario A4 has three oil investment projects that are not part of A1. They are Oil#3, Oil#7, and Oil#10, and this explains the increase in oil EUR from 50 million barrels to 65 million barrels. The increase in oil EUR came with the expense of a lower gas EUR for the A4 portfolio. In order to gain the above-mentioned oil projects, the A4 portfolio has to drop projects Gas#2 and Gas#6. Both portfolios are efficient, but the choice between them is dependent on the company's goals, whether it desires to increase oil or gas production.

Scenario A8

The objective function of this scenario is to maximize the mean NPV with a budget of $200 million. Risk is defined as the standard deviation from the mean NPV of the portfolio. The standard deviation is a way of quantifying the width of the distribution. In other words, it is a measure of uncertainty in the value of the portfolio. Figure 4.15 shows scenario A8's efficient frontier. The general shape of the A8 efficient frontier graph is similar to A2 efficient frontier graph Figure 4.8.

The lower side of the efficient frontier graph consists mainly of smaller projects with both low NPV and low standard deviation, namely Oil#10 and Oil#11, while the upper side sees the introduction of projects with high uncertainty in their NPV distributions, namely Oil#3 and Oil#8. Project Oil#9 is not part of any efficient portfolio in this scenario because it cannot compete with other projects in terms of higher NPV or lower standard deviation. The entire projects NPV's can be found in Chapter Three, and the portfolio compositions can be seen in Table 4.6

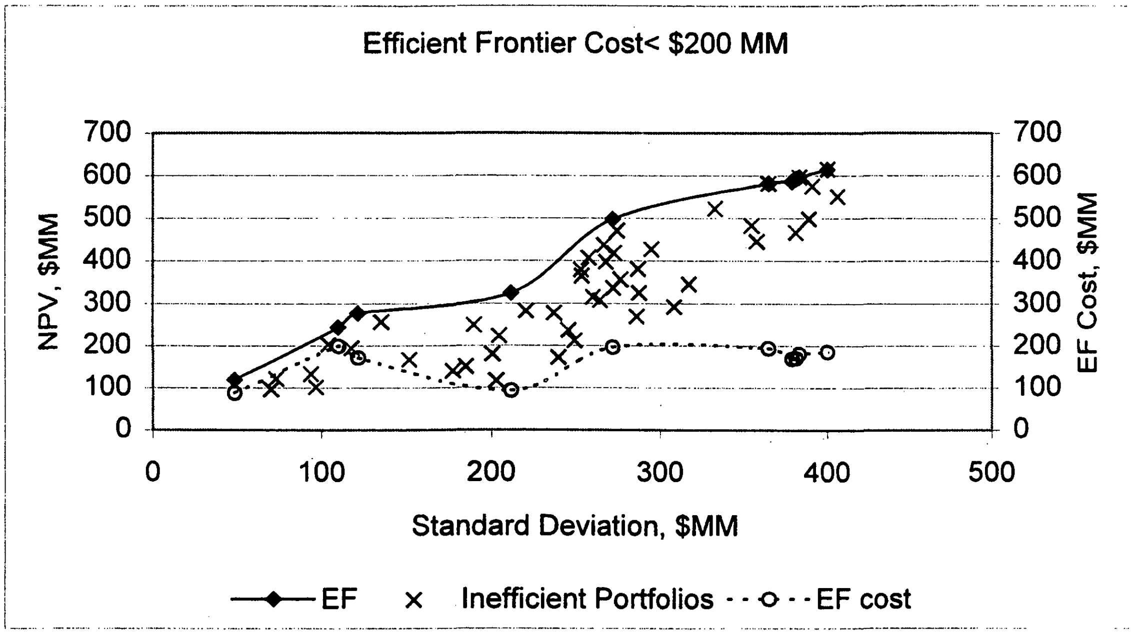

Figure 4.15 Scenario A8 Efficient Frontier.

NPV $MM	Cost $MM	Risk $MM	Oil#11 WI	Oil#2 WI	Oil#3 WI	Oil#10 WI	Oil#1 WI	Gas#9 WI	Gas#8 WI	Gas#7 WI	Gas#6 WI	Oil#9 WI	Gas#5 WI	Oil#8 WI	Gas#4 WI	Oil#7 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Gas#1 WI	Oil#4 WI
120	88	49		1		1		1			1					1	1		1	1	1	1
244	199	110	1	1			1	1	1	1	1		1			1				1		1
277	171	122	1	1			1	1	1	1	1				1			1		1	1	1
325	93	212						1			1			1	1		1		1		1	
499	196	272		1			1	1		1	1		1	1	1		1	1	1			1
586	169	379		1	1		1		1	1				1	1	1		1	1	1	1	
590	162	413		1	1		1	1		1	1			1			1	1		1		
597	180	383		1	1		1		1	1	1			1	1			1	1	1	1	
615	186	400		1	1		1	1	1	1	1			1	1			1	1	1	1	

Table 4.6 Scenario A8 Efficient Portfolios Compositions.

Scenario A11

Scenarios A11 and A1 have the same objective function and risk measure. They both maximize NPV while using VaR as a risk measure. The only difference is that correlations among projects are taken into account in scenario A11, whereas they were not in A1. Perfect positive correlations were used among some of the investment projects for A11, namely probability of success and reserves size. Table 4.8 shows the correlation matrix for scenario A11.

The efficient frontier plots for scenarios A1 and A11 show they both have the same general shape. Figures 4.2 and 4.16 show efficient frontiers for A1 and A11, respectively. The introduction of correlation into the stochastic model made the efficient frontier shape smoother for scenario A11. The reason for that is the positive correlation, which reduced the noise in the NPV distribution.

NPV $MM	Cost $MM	Risk $MM	Oil#3 WI	Oil#4 WI	Gas#1 WI	Oil#5 WI	Gas#2 WI	Oil#6 WI	Gas#3 WI	Oil#7 WI	Oil#8 WI	Gas#5 WI	Oil#9 WI	Gas#6 WI	Gas#7 WI	Gas#8 WI	Gas#9 WI	Oil#2 WI	Oil#1 WI	Oil#10 WI	Oil#11 WI	Gas#4 WI
177	91	108		1		1		1	1						1	1	1	1	1			1
284	212	218		1		1		1				1		1	1			1	1		1	1
474	162	334			1	1	1		1	1	1			1				1	1		1	
601	339	429		1	1	1		1	1	1	1	1	1	1	1	1		1	1		1	1
686	344	529	1		1	1		1	1	1	1	1	1	1	1		1	1	1	1		1
724	384	638	1	1	1			1	1	1	1	1	1	1	1		1	1	1	1	1	

Table 4.7 Scenario A11 Efficient Portfolios Compositions.

Lower Triangle Correlation Matrix for Scenario A11	Oil#2 Chance of Dry Hole	Oil#2 Reserves Proved	Oil#3 Size of Discovery	Oil#3 Chance of Dry Hole	Oil#11 Chance of Dry Hole	Oil#11 Reserves MBBL	Oil#10 Chance of Dry Hole	Oil#10 Reserves MBBL	Oil#1 Chance of Dry Hole	Gas#9 Project Dry Hole	Gas#9 Reserves BCF	Gas#8 Project Dry Hole	Gas#8 Reserves BCF	Gas#7 Project Dry Hole	Oil#8 Size of Discovery	Oil#8 Chance of Dry Hole	Oil#5 Reserves MBBL	Oil#5 Chance of Dry Hole	Gas#2 Custom Distribution	Gas#3 Custom Distribution
Oil#2 Chance of Dry Hole	1.0																			
Oil#2 Reserves Proved		1.0																		
Oil#3 Size of Discovery			1.0																	
Oil#3 Chance of Dry Hole				1.0																
Oil#11 Chance of Dry Hole	1.0				1.0															
Oil#11 Reserves MBBL		1.0				1.0														
Oil#10 Chance of Dry Hole							1.0													
Oil#10 Reserves MBBL								1.0												
Oil#1 Chance of Dry Hole									1.0											
Gas#9 Project Dry Hole										1.0										
Gas#9 Reserves BCF											1.0									
Gas#8 Project Dry Hole										1.0		1.0								
Gas#8 Reserves BCF											1.0		1.0							
Gas#7 Project Dry Hole														1.0						
Oil#8 Size of Discovery			1.0												1.0					
Oil#8 Chance of Dry Hole				1.0												1.0				
Oil#5 Reserves MBBL								1.0									1.0			
Oil#5 Chance of Dry Hole							1.0											1.0		
Gas#2 Custom Distribution																			1.0	
Gas#3 Custom Distribution																			1.0	1.0

Table 4.8 Scenario A11 Correlations Matrix.

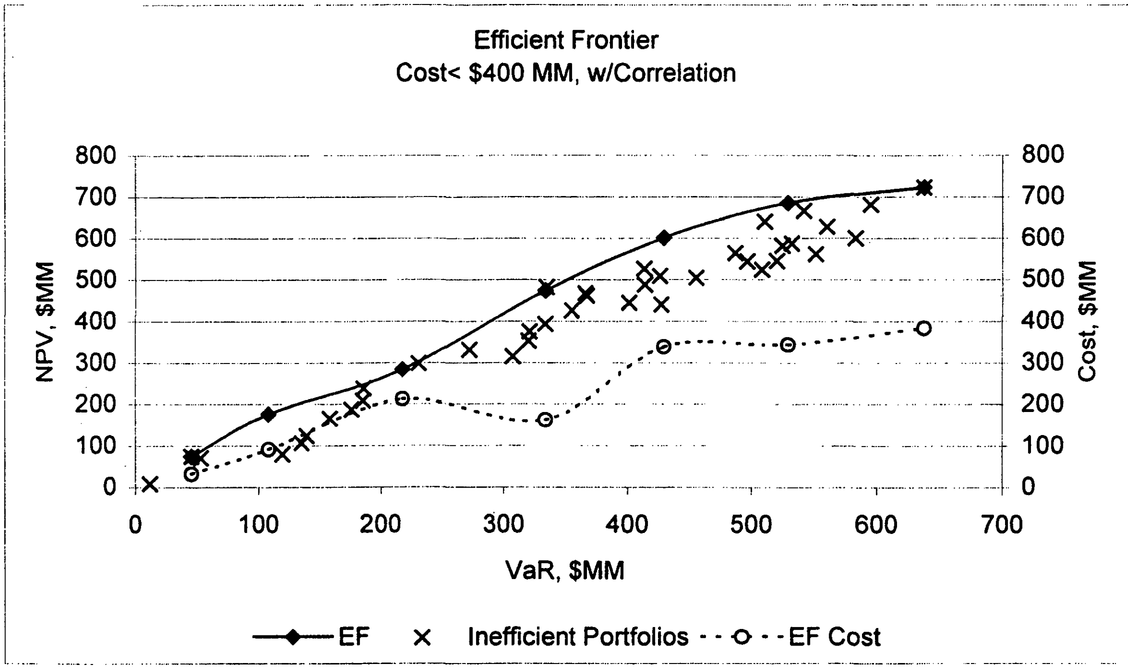

Figure 4.16 Scenario A11 Efficient Frontier

Scenario A15

The objective of scenario A15 is to maximize NPV, while using the standard deviation of NPV as the risk measure. The data used for the optimization is similar to scenario A8, with the exception of correlations. Different correlations among projects were introduced in this scenario. Correlation coefficient values have been changed from previous examples and they can be seen in Table 4.9. Correlation coefficients do not change with risk measure in reality, but for the sake of adding complexity to the model, they have been modified in this scenario.

Lower Triangle Correlation Matrix for Scenario A15	Oil#2 Chance of Dry Hole	Oil#2 Reserves Proved	Oil#3 Size of Discovery	Oil#3 Chance of Dry Hole	Oil#11 Chance of Dry Hole	Oil#11 Reserves MBBL	Oil#10 Chance of Dry Hole	Oil#10 Reserves MBBL	Oil#1 Chance of Dry Hole	Gas#9 Project Dry Hole	Gas#9 Reserves BCF	Gas#8 Project Dry Hole	Gas#8 Reserves BCF	Gas#7 Project Dry Hole	Oil#8 Size of Discovery	Oil#8 Chance of Dry Hole	Oil#5 Reserves MBBL	Oil#5 Chance of Dry Hole	Gas#2 Custom Distribution	Gas#3 Custom Distribution
Oil#2 Chance of Dry Hole	1.0																			
Oil#2 Reserves Proved		1.0																		
Oil#3 Size of Discovery			1.0																	
Oil#3 Chance of Dry Hole				1.0																
Oil#11 Chance of Dry Hole	0.5				1.0															
Oil#11 Reserves MBBL		0.5				1.0														
Oil#10 Chance of Dry Hole							1.0													
Oil#10 Reserves MBBL								1.0												
Oil#1 Chance of Dry Hole									1.0											
Gas#9 Project Dry Hole										1.0										
Gas#9 Reserves BCF											1.0									
Gas#8 Project Dry Hole										1.0		1.0								
Gas#8 Reserves BCF											1.0		1.0							
Gas#7 Project Dry Hole														1.0						
Oil#8 Size of Discovery			0.3												1.0					
Oil#8 Chance of Dry Hole				0.3												1.0				
Oil#5 Reserves MBBL								-0.8									1.0			
Oil#5 Chance of Dry Hole							-0.8											1.0		
Gas#2 Custom Distribution																			1.0	
Gas#3 Custom Distribution																			-0.5	1.0

Table 4.9 Correlation Matrix for A15.

Another significant change in this scenario is the choice of 25 percent working interest increments. This increased the number of possible combinations from one million (2^{20}) to 95 trillion (5^{20}). The optimization process took much longer computer time than other cases, but it was desired to examine the effect of WI increments on the portfolio. Scenario A15's efficient frontier and portfolio compositions are presented in Figure 4.17 and Table 4.10, respectively.

The efficient frontier plot shows that a smoother objective function is apparent, and the reason for that is the correlation among projects. Efficient portfolio costs hover closely around the \$200 million budget; the reason for that is the WI increments, where parts of projects can be added to the portfolio. Project Oil#9, which was inefficient in scenario A8, is now part of some of the portfolios.

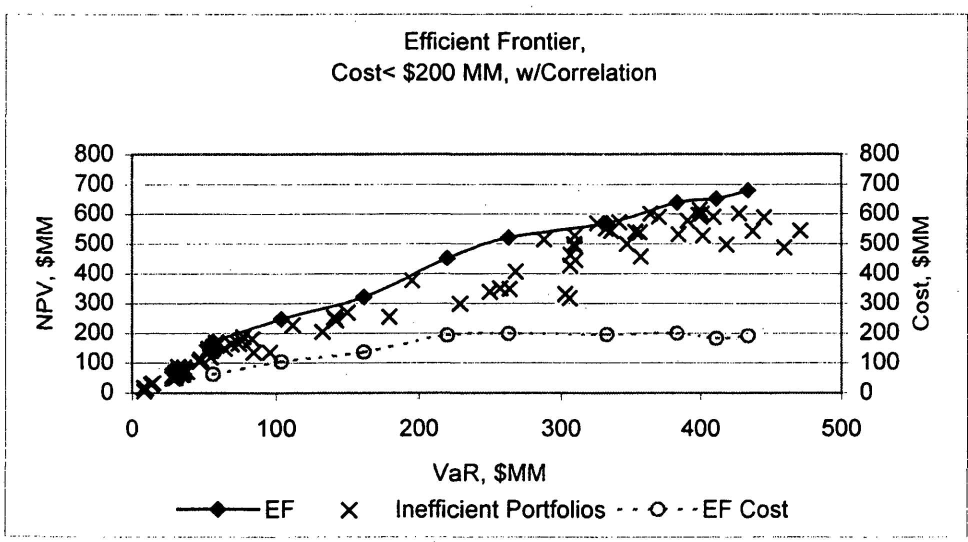

Figure 4.17 Scenario A15 Efficient Frontier

NPV $MM	172	247	322	452	522	570	639	652	679
Cost $MM	63	105	137	194	198	196	199	183	191
Risk $MM	56	104	161	220	263	332	383	411	434
Oil#3 WI					0.5	0.75	1	1	1
Oil#4 WI	0.5	0.75	0.5	0.75	0.75	0.75	0.25	0.75	0.5
Gas#1 WI	0.25	0.25	0.5	1	0.75	1	0.75	1	0.5
Oil#5 WI	0.75	0.75	0.5	1	1	1	1	1	1
Gas#2 WI	1	1		1	1	1	1	0.5	1
Oil#6 WI	1	0.75	0.5	1	1	1	1	0.75	1
Gas#3 WI			0.5		0.75	0.5	1		1
Oil#7 WI				0.25	0.5	0.5	0.5	0.75	1
Oil#8 WI		0.25	0.5	0.75	0.75	0.75	1	1	1
Gas#5 WI	0.25		0.25	1	0.75	0.5	0.25		0.25
Oil#9 WI		0.25							
Gas#6 WI	1	1	0.5	1	1	1	1	1	1
Gas#7 WI		0.5	1	1	1	1	1	1	0.75
Gas#8 WI	0.5	0.5		0.5	0.25	0.25	0.75		0.25
Gas#9 WI		0.25	1	0.75	0.75	1	0.75	0.75	1
Oil#11 WI			0.5						
Oil#1 WI	1	0.5	0.5	1	1	1	1	1	1
Oil#10 WI		0.25							
Oil#2 WI	1	1	1	1	1	1	1	1	1
Gas#4 WI		0.25		1	1	1	1	0.75	0.75

Table 4.10 Scenario A15 Efficient Portfolios Compositions.

Scenario A16

The objective function of this scenario is to maximize oil estimated ultimate recovery (EUR) for different budgets. This optimization is not a common one, since most oil companies use profit as the bottom line. However, this scenario is applicable when dealing with national oil companies, where production quotas must be met. As can be seen in Figure 4.18, a certain production quota can be achieved with different portfolios, and these portfolios are called efficient if they provide the same EUR with lowest possible cost. Table 4.14 shows the efficient portfolio compositions.

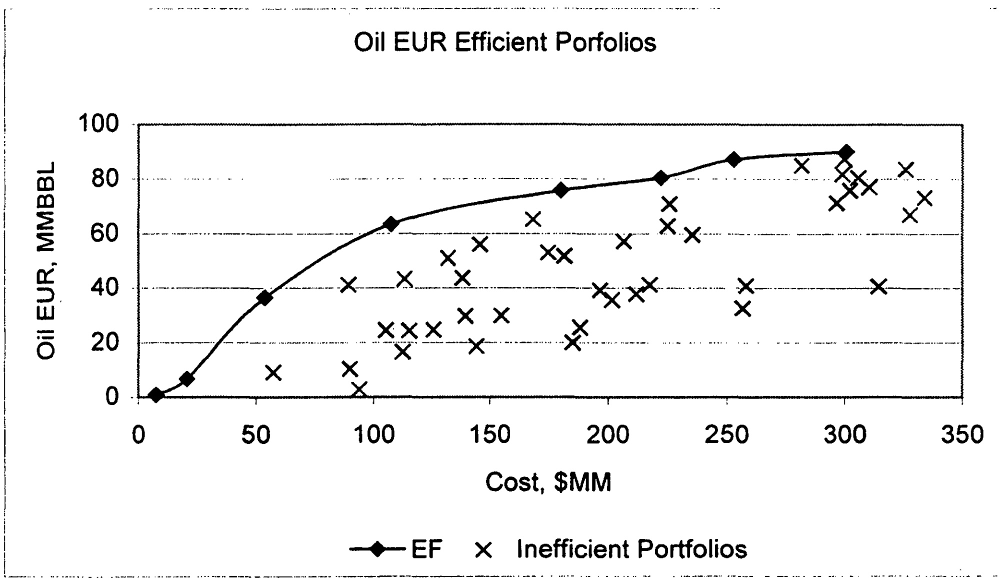

Figure 4.18 Scenario A16 Efficient Frontier.

Oil EUR MMBBL	Cost $MM	Oil#2 WI	Oil#3 WI	Oil#11 WI	Oil#10 WI	Oil#1 WI	Oil#4 WI	Gas#9 WI	Gas#8 WI	Gas#7 WI	Gas#6 WI	Oil#9 WI	Gas#5 WI	Oil#8 WI	Gas#4 WI	Oil#7 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Gas#1 WI
1	8						1		1								1				
7	21					1	1	1								1			1		
37	54	1												1							
63	107		1			1		1	1					1		1		1	1		1
76	180	1	1			1			1			1		1				1	1	1	
80	222	1	1		1	1	1		1			1		1		1	1	1	1		1
87	253	1	1	1		1	1					1		1				1	1	1	1
90	301	1	1	1	1	1			1			1		1	1	1	1	1	1	1	

Table 4.14 Scenario A16 Efficient Portfolios Compositions.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This dissertation has demonstrated the difficulties faced by the petroleum companies in terms of additional computer analysis time required to use portfolio analysis. In the optimization stage using thirty investment projects, the analysis took over six days of computer time.

In concluding this thesis it was demonstrated that the model developed herein can improve a company's financial performance. Providing accurate representation of the investment projects is essential for the success of the evaluations. The hypotheses set out at the beginning of the thesis were found to be true. The thesis conclusions are:

1. Meta-Heuristics methods can provide improvements over traditional project selection methods which handle the project individually and not part of a portfolio. The use of Tabu Search and Scatter method for portfolio optimization in the oil and gas industry is unique to this dissertation.
2. Portfolio optimization can handle budget rationing, maximizing cash flow, and maximizing the company's asset worth.

3. Portfolio optimization is greatly affected by the choice of risk measure. Furthermore, by using different risk measures valuable insight is achieved, resulting in better portfolio management.
4. Stochastic analysis assists in decision making, providing the geo-technical and cash flow model inputs are accurate representations.
5. Portfolio optimization using efficient frontier theory, combined with stochastic simulation, is a powerful technique to maximize a company's assets.
6. Building this model using a spreadsheet platform makes it easier to adapt to an unconventional project evaluation, unlike commercial evaluation packages.
7. Incorporating risk analysis in portfolio selection gives a robust and significant wealth of useful information. Risk is rarely used in traditional methods.
8. A company's goals can be incorporated into the project evaluation process through portfolio analysis.
9. Portfolio optimization allows the use of additional constraints, for example, production quotas, reserve replacements, and the exploration budget.

Recommendations and Further Work.

Recommendations

1. It is recommended that petroleum companies adapt the model presented herein to improve their project evaluation process and financial results.
2. The use of parallel computing or dual processor computers is recommended to speed the optimization process, resulting in improvements in the usability of the model.

Further work

1. The addition of tax calculating algorithms would be valuable and a possible next step in order to determine the optimum after-tax portfolio.
2. Inclusion of a complete sub-surface stochastic analysis would be another next step.
3. Meta-Heuristic methods may be applicable to reservoir simulation as an accelerator for history matching.

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NOMENCLATURE

API	American Petroleum Institute
B	Billion
b	Hyperbolic Exponent
BCF	Billion Cubic Feet
b/d	Barrels per Day
BOE	Barrels of Oil Equivalent
BTU	British Thermal Unit
CAPEX	Capital Expenditure
D_i	Initial Nominal Decline Rate
EF	Efficient Frontier
E&P	Exploration and Production
ERR	Estimated Remaining Reserves
EUR	Estimated Ultimate Recovery
GA	Genetic Algorithms
G&G	Geological and Geophysical
IDC	Intangible Drilling Cost
LRI	Less Royalty Interest
L&WE	Lease and Well Equipment
M	Thousand
MBBL	Thousand Barrels
MCF	Thousand Cubic Feet
MM	Million
MMBBL	Million Barrels
NPV	Net Present Value
NRI	Net Revenue Interest
P5	Fifth Percentile
P10	Tenth Percentile
PIR	Profit-to-Investment Ratio
POS	Probability of Success
q_{el}	End of Life Production Rate
q_i	Initial Production Rate
ROI	Return on Investment
ROR	Rate of Return
RPR	Reward per Risk
SC	Scatter Search
TS	Tabu Search
VaR	Value at Risk
WI	Working Interest
WTI	West Texas Intermediate

APPENDIX A

PROJECTS SUMMARY TABLE AND INVESTMENT PROJECT FORECASTS

Thirty investment projects are included in this appendix. Twenty were used for analysis and they are: Oil#1 through 11 and Gas#1 through 9. The discarded projects are: Oil#12 through 16 and Gas#10 through 14. Included for each of the 30 projects: NPV, Cost, and EUR. Projects can be found as follows:

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Oil#3	page 113
Oil#4	page 104
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Project Summary Table
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Project name	Type	Recoverable Reserves Oil MMBBL	Recoverable Reserves Gas BCF	No. of Wells	Probability of Success POS	NPV $M	Cost $M	Comments
Gas # 1	gas		34.25	2	70%	18,000	3,000	3 layers, each combination has POS
Gas # 2	gas		15.4	1	46%	7,100	2,018	3 layers, each combination has POS
Gas # 3	gas		11.2	1	46%	6,800	2,200	3 layers, each combination has POS
Gas # 4	gas		30.3	15	90%	25,600	22,999	15 well development, 5 wells every year
Oil # 1	Oil	0.5		1	90%	4,700	2,600	1 well horizontal 3 initial rates, high, low , med
Gas # 5	gas		90	20	80%	11,000	60,000	30 $MM Facilities cost, 5 wells every year
Oil # 2	Oil	3		3	90%	32,500	7,500	high well costs
Oil # 3	Oil	31-54-76		depends	30%	152,000	16,000	Wild cat 3 discovery sizes no. of wells depends on size
Oil # 4	Oil	150-400, 300-700		1--8	80%	14,500	6,000	Facility cost 2,500$M water-flood field
Gas # 6	gas		52	15	100%	60,000	35,000	Existing field, Facility cost 5,000$M
Gas # 7	gas		70	15	90%	29,000	36,000	
Oil # 5	Oil	2		1	90%	16,000	8,600	Facility cost & lease bonus 6000$M
Oil # 6	Oil	3		1	90%	22,600	5,700	Facility cost & lease bonus 3000$M
Oil # 7	Oil	0.5		1	90%	4,300	3,100	Facility cost & lease bonus 500$M
Oil # 8	Oil	37			80%	79,000	10,500	
Gas# 8	gas		2	2	90%	512	1,300	no Facility cost & lease bonus
Gas# 9	gas		6	3	90%	1,866	4,112	no Facility cost & lease bonus
Gas# 10	gas		10	5	90%	3,569	6,409	500 $M Facility cost & lease bonus

Project name	Type	Recoverable Reserves Oil MMBBL	Recoverable Reserves Gas BCF	No. of Wells	Probability of Success POS	NPV $M	Cost $M	Comments
Gas# 11	gas		20	10	90%	7,031	13,443	no Facility cost & lease bonus
Gas# 12	gas		40	14	90%	13,043	27,508	5000 $M Facility cost & lease bonus
Gas# 13	gas		60	21	90%	19,585	37,877	5000 $M Facility cost & lease bonus
Gas# 14	gas		80	27	90%	25,067	53,127	10,000 $M Facility cost & lease bonus
Oil # 9	Oil	16		16	70%	66,000	45,000	25,000 $M Facility cost & lease bonus
Oil # 10	Oil	4		8	95%	18,000	27,000	25,000 $M Facility cost & lease bonus
Oil # 11	Oil	10		12	90%	38,000	69,000	50,000 $M Facility cost & lease bonus
Oil # 12	Oil	21		12	80%	70,000	104,000	60,000 $M Facility cost & lease bonus
Oil # 13	Oil	35		20	70%	93	158,000	80,000 $M Facility cost & lease bonus
Oil # 14	Oil	160		30	80%	778,000	450,000	400,000 $M Facility cost & lease bonus
Oil # 15	Oil	57		20	85%	176,000	247,000	200,000 $M Facility cost & lease bonus
Oil # 16	Oil	34		15	80%	116,000	137,000	100,000 $M Facility cost & lease bonus

Project Summary Table Continued.

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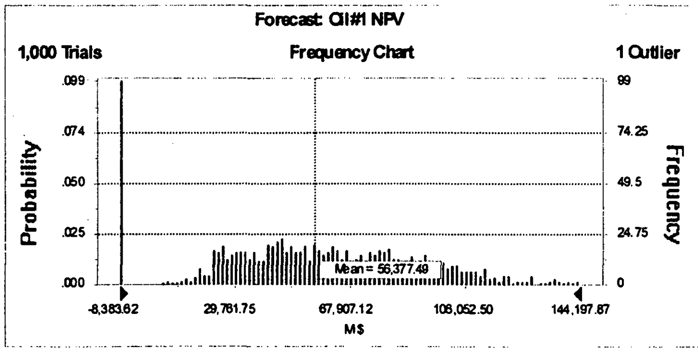

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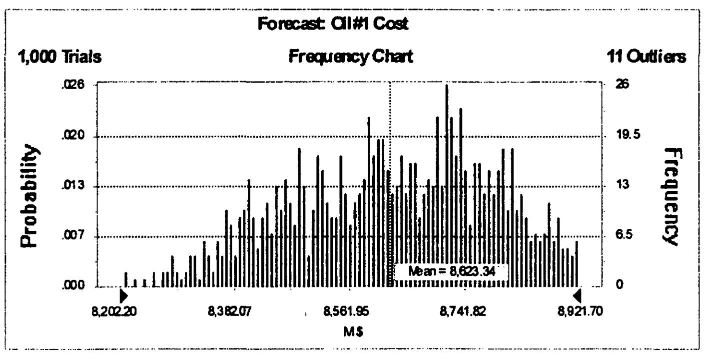
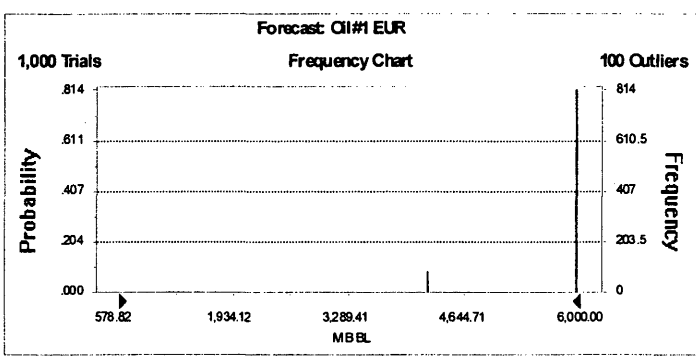


Forecast: Oil#1 EUR



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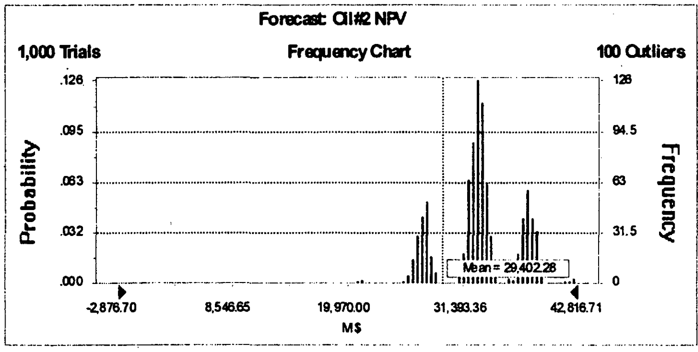
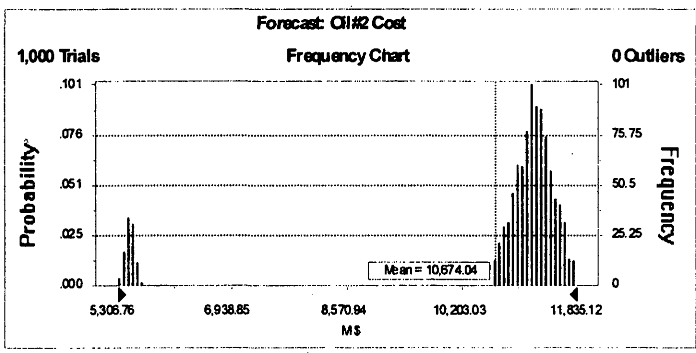
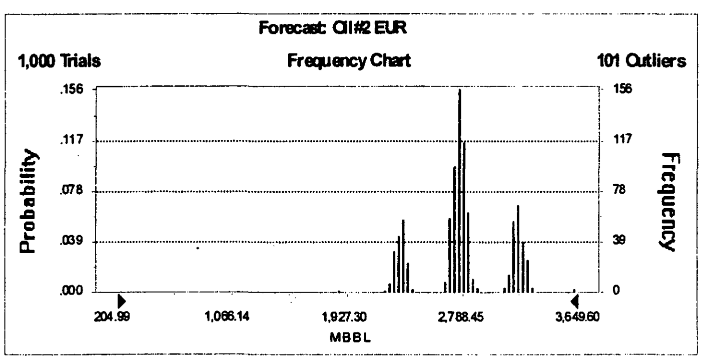


Forecast: Oil#2 Cost



Forecast: Oil#2 EUR



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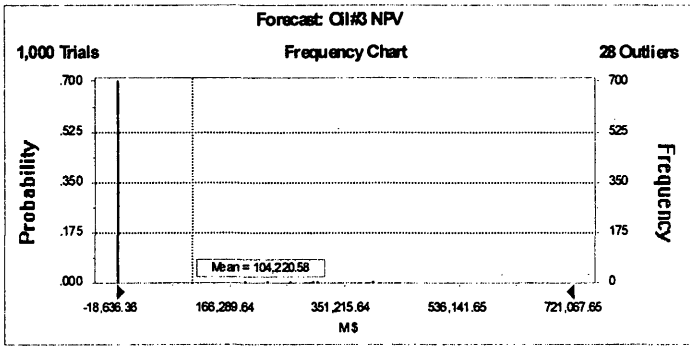
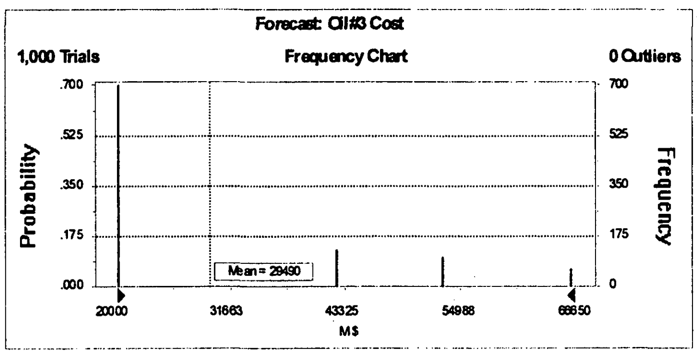
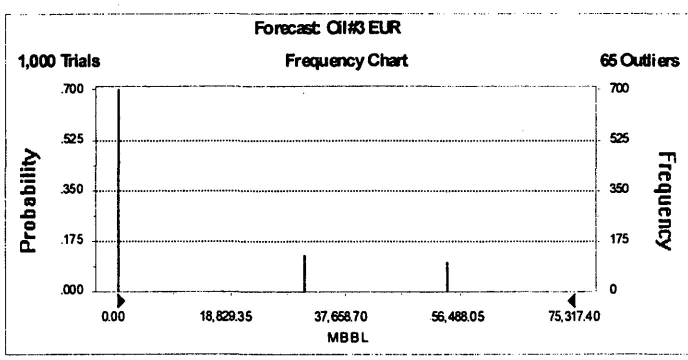


Forecast: Oil#3 Cost



Forecast: Oil#3 EUR



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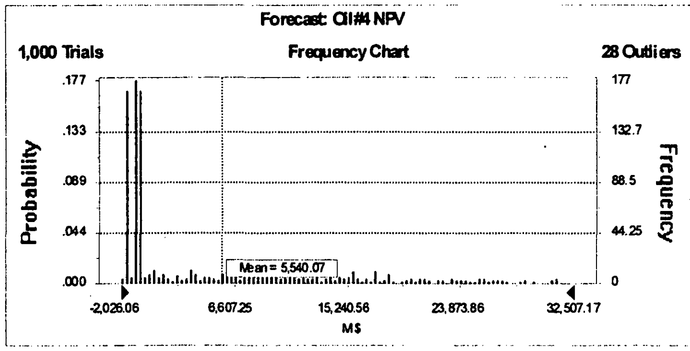

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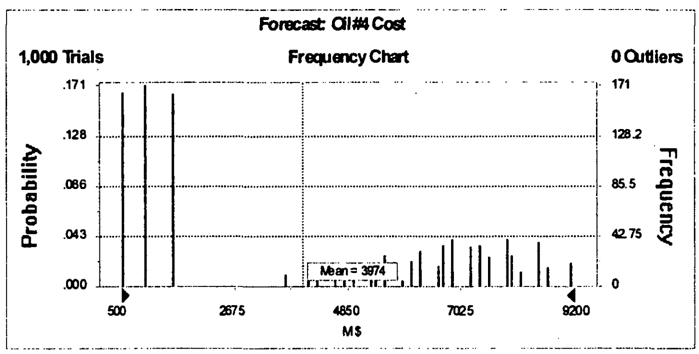
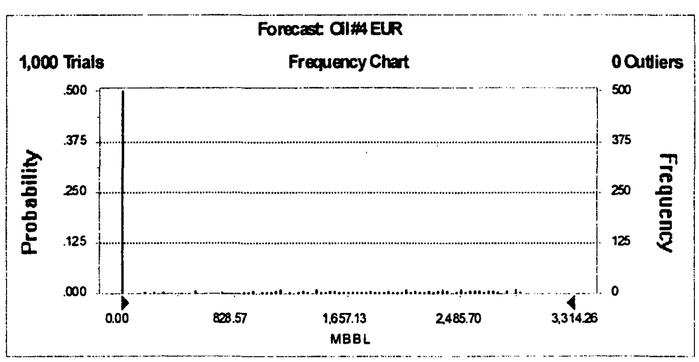


Forecast: Oil#4 EUR



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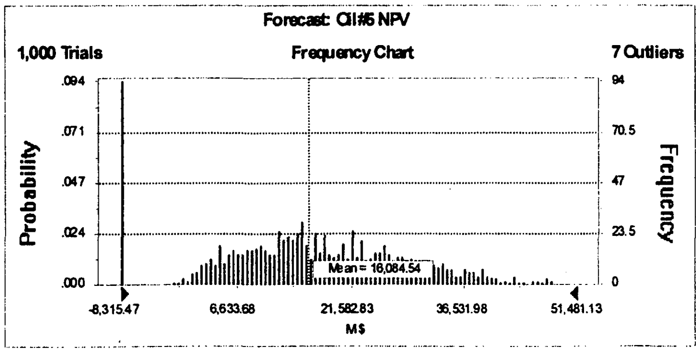
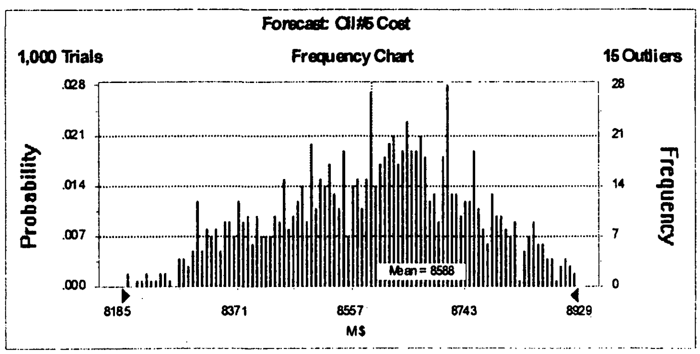
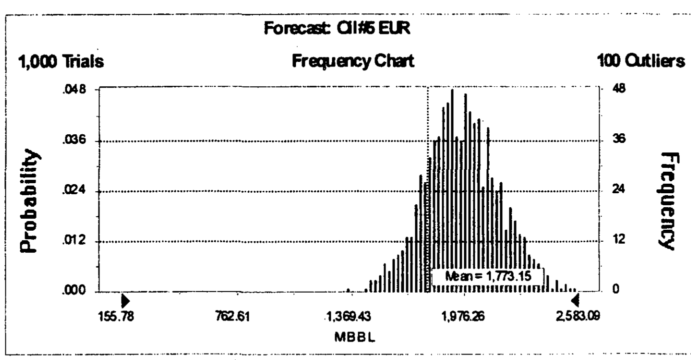


Forecast: Oil#5 Cost



Forecast: Oil#5 EUR



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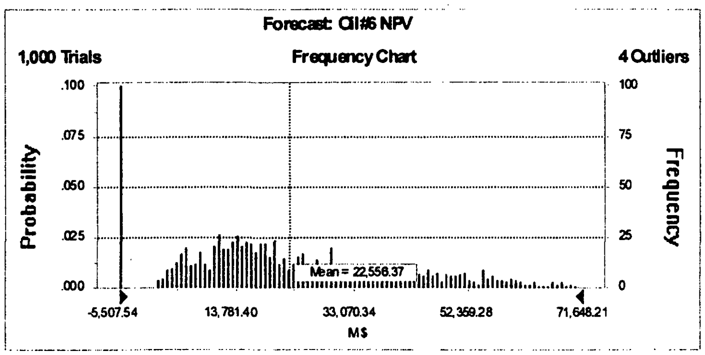

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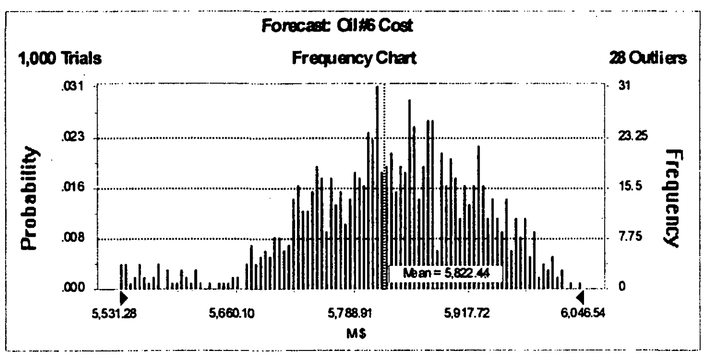

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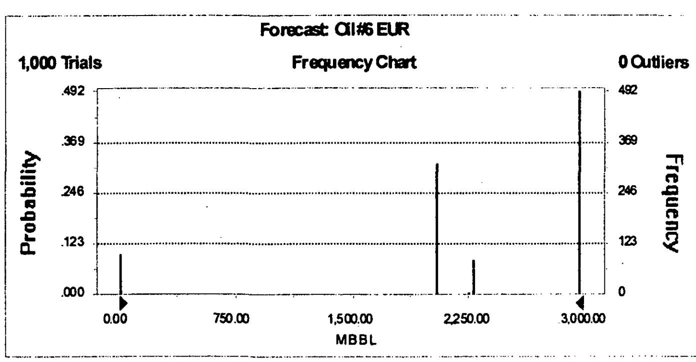

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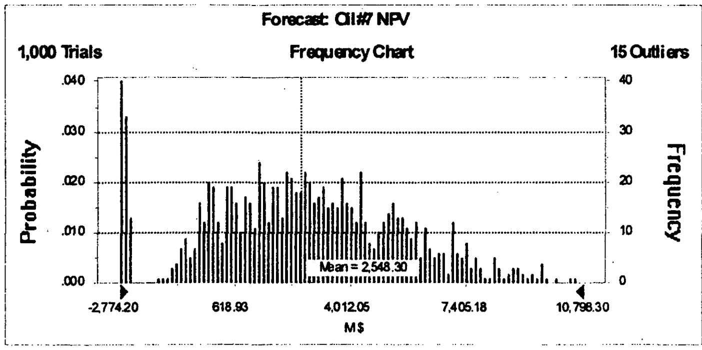

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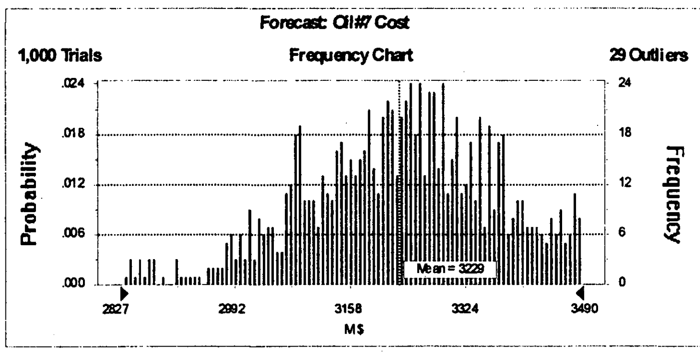

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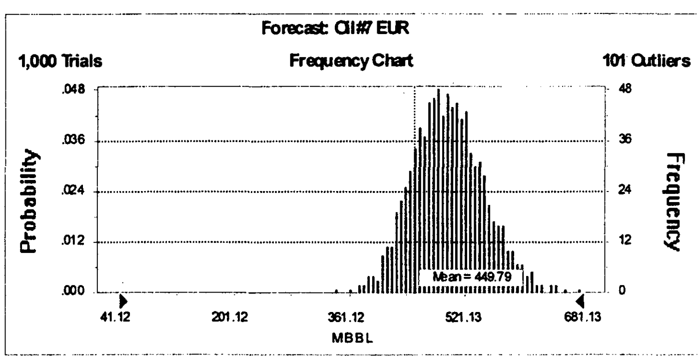

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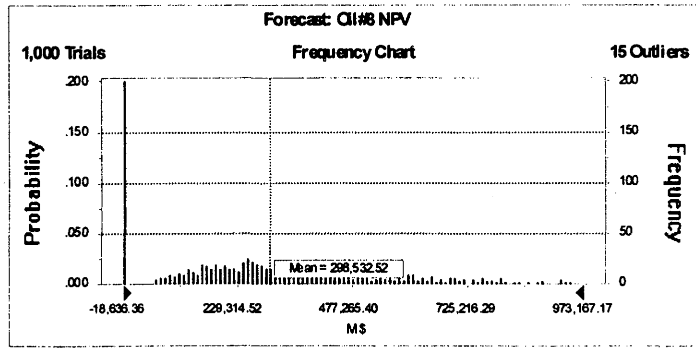

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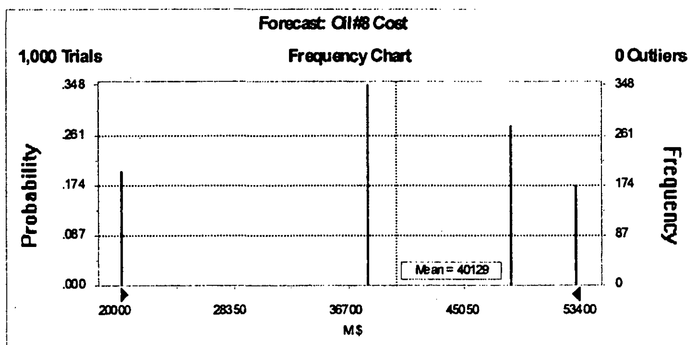

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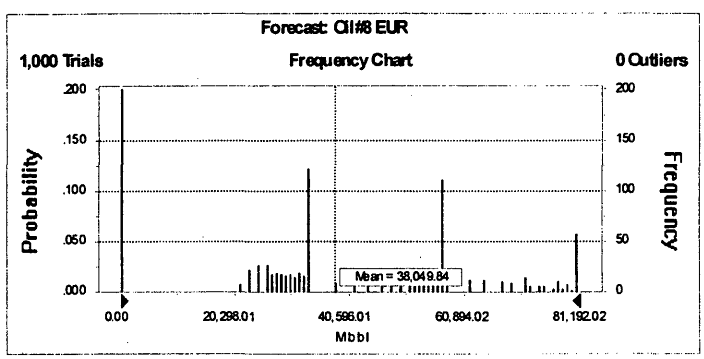

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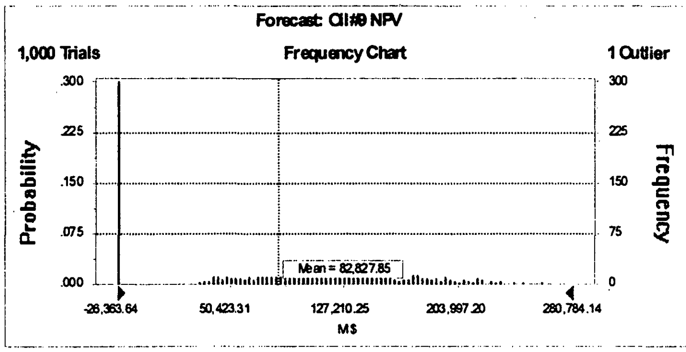
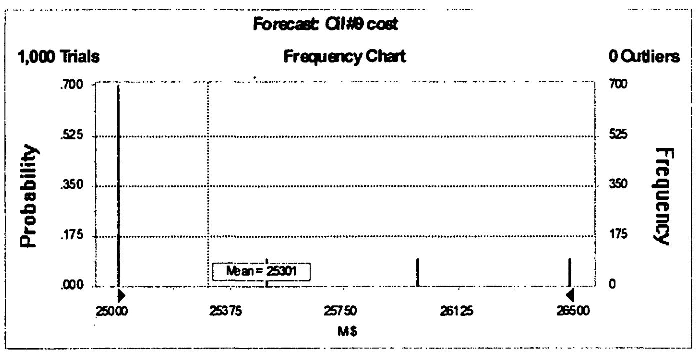


Forecast: Oil#9 cost



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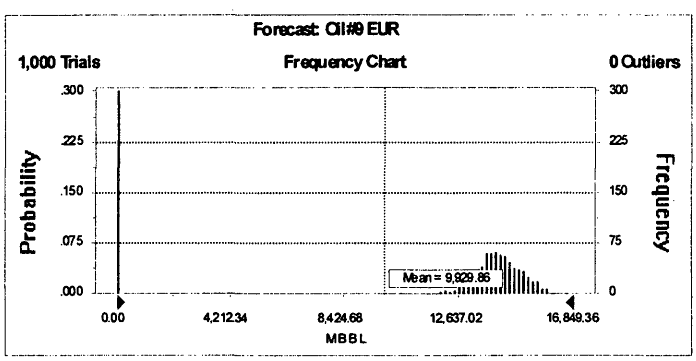

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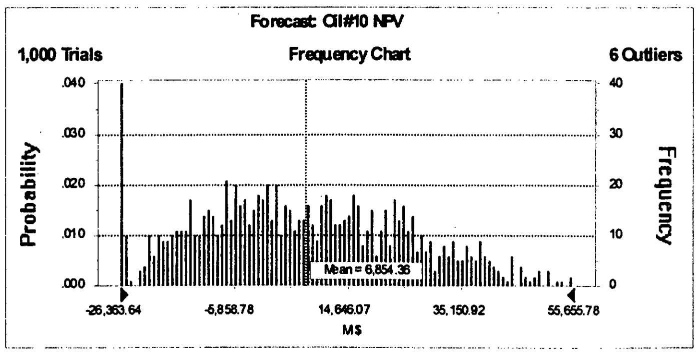

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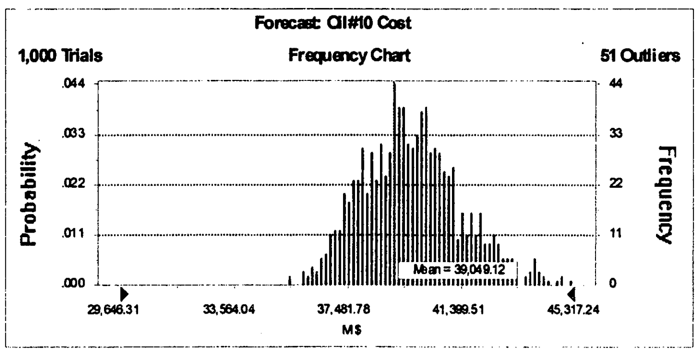

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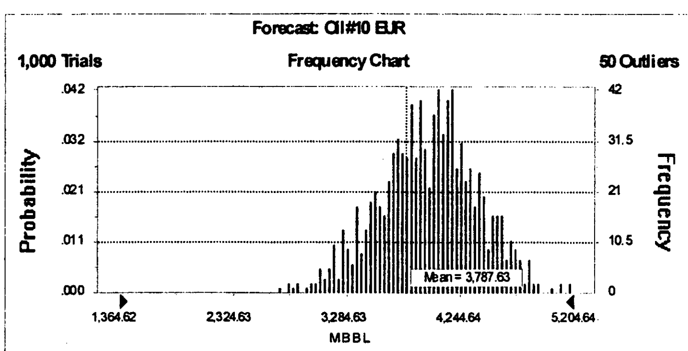

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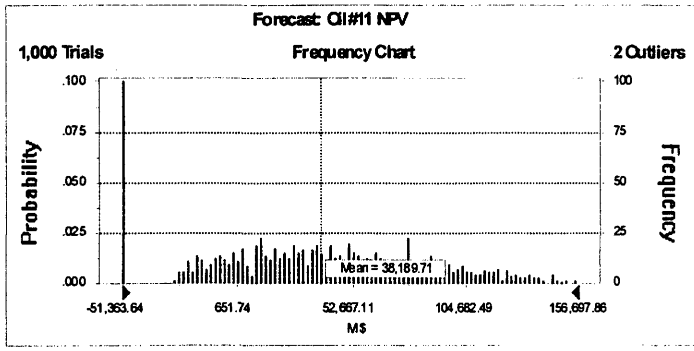

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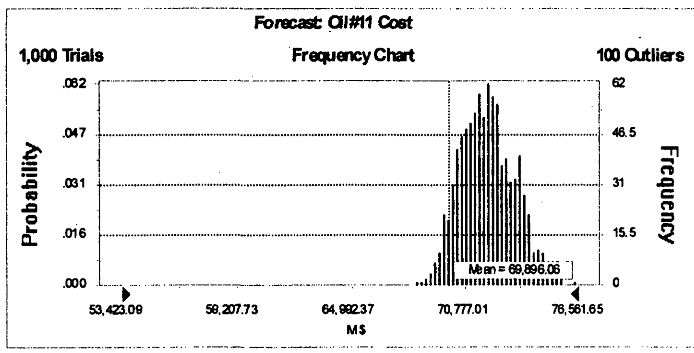
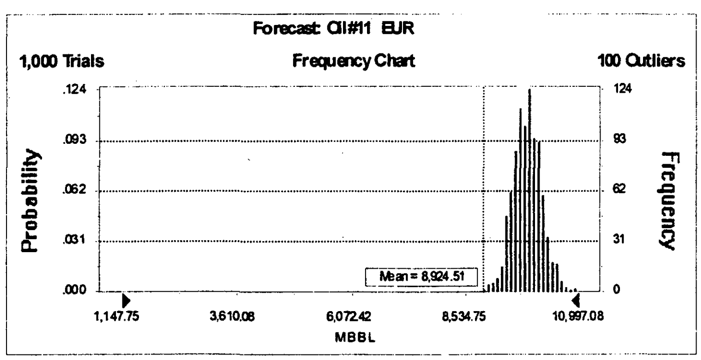


Forecast: Oil#11 EUR



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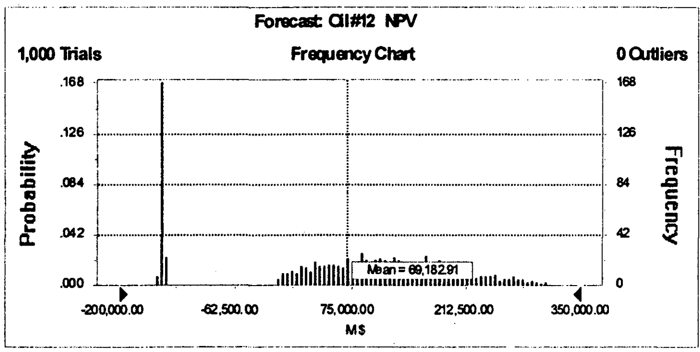

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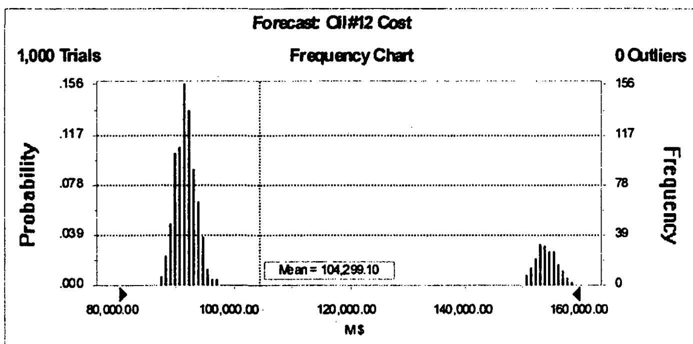

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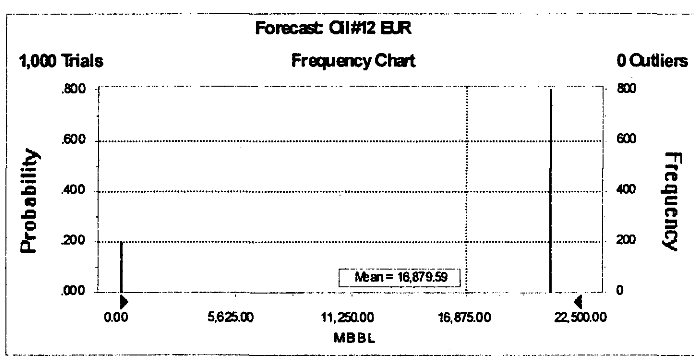

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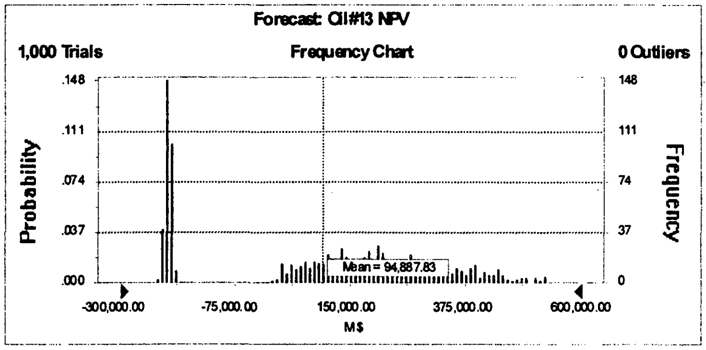

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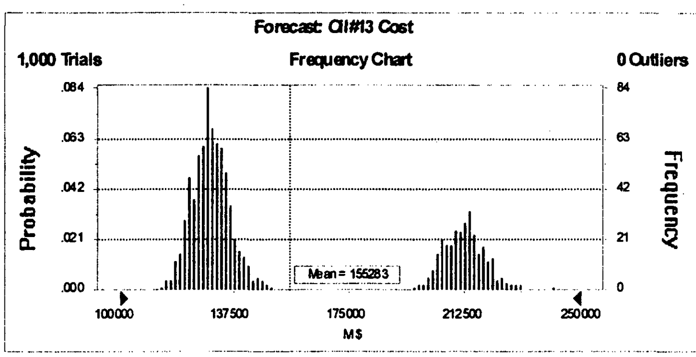

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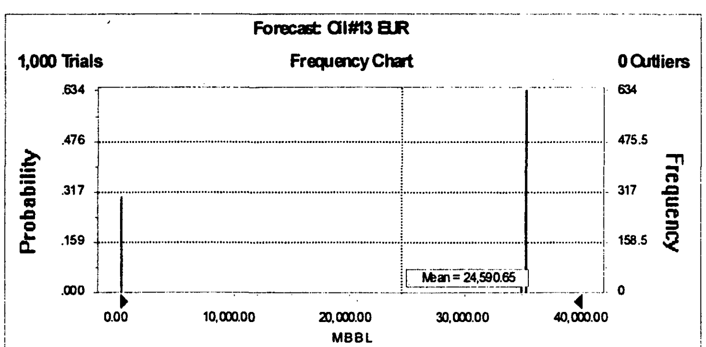
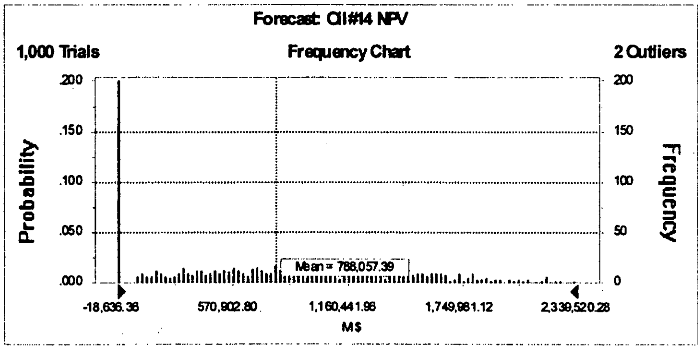
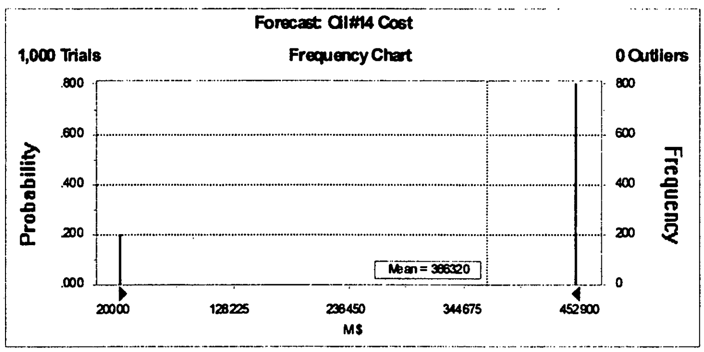


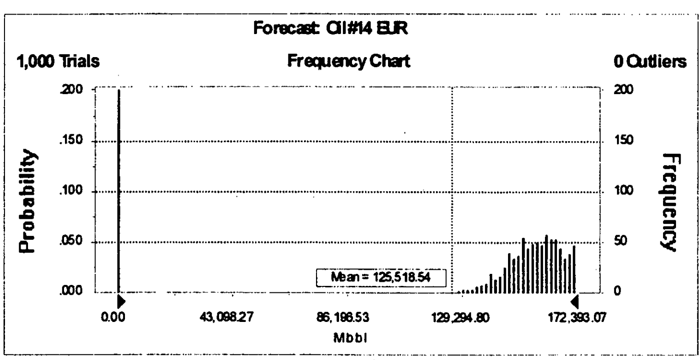
Forecast: Oil#14 NPV



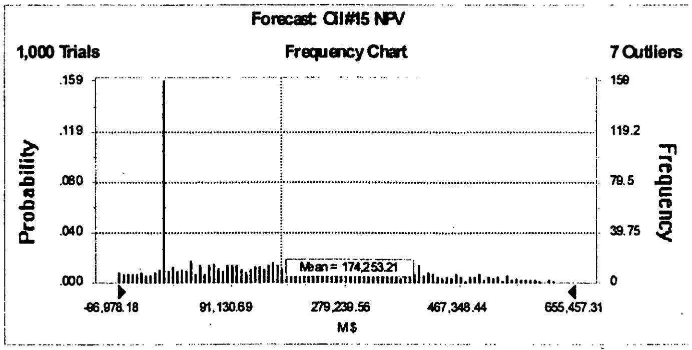
Forecast: Oil#14 Cost



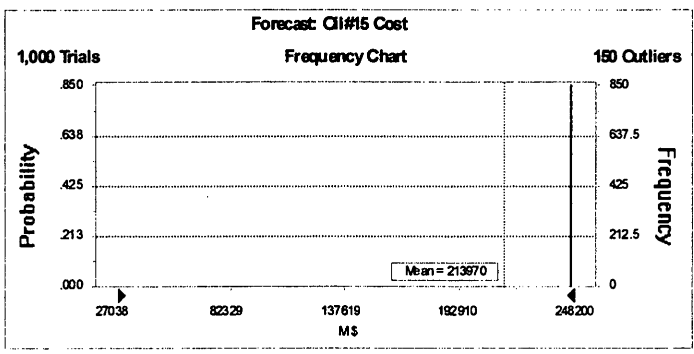
Forecast: Oil#14 EUR



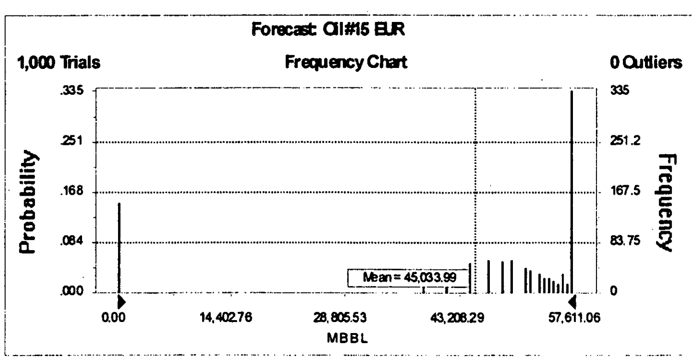
Forecast: Oil#15 NPV



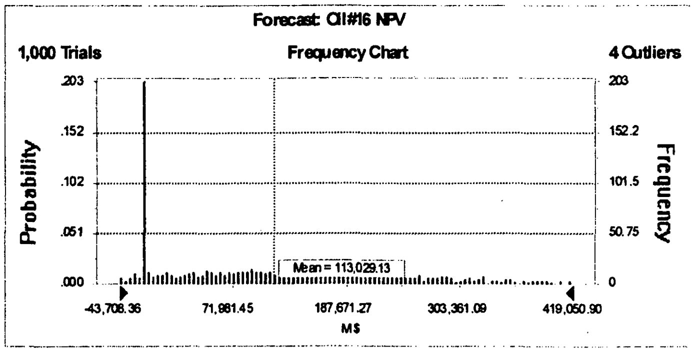
Forecast: Oil#15 Cost



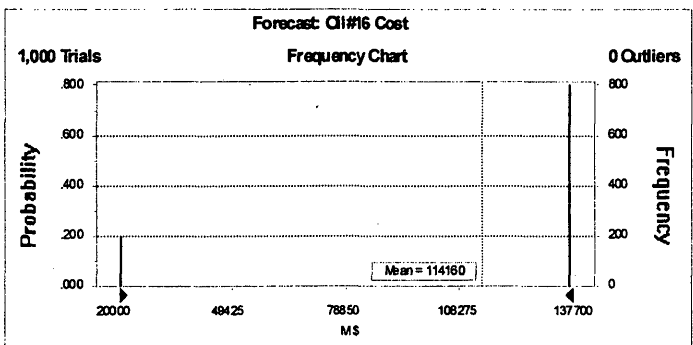
Forecast: Oil#15 EUR



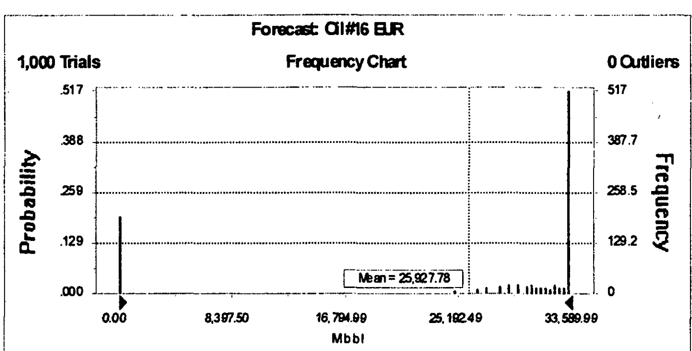
Forecast: Oil#16 NPV



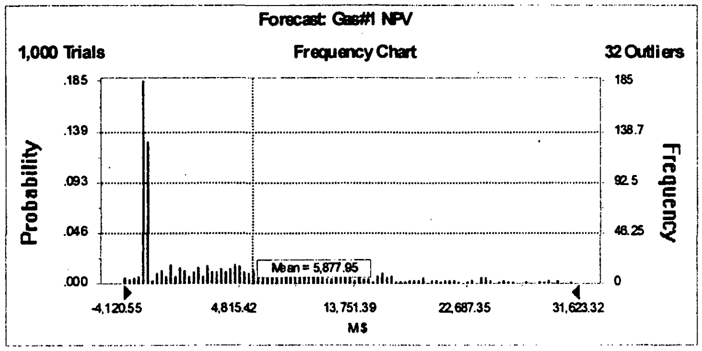
Forecast: Oil#16 Cost



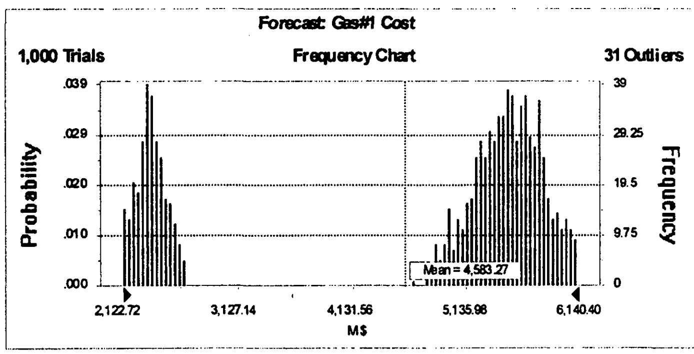
Forecast: Oil#16 EUR



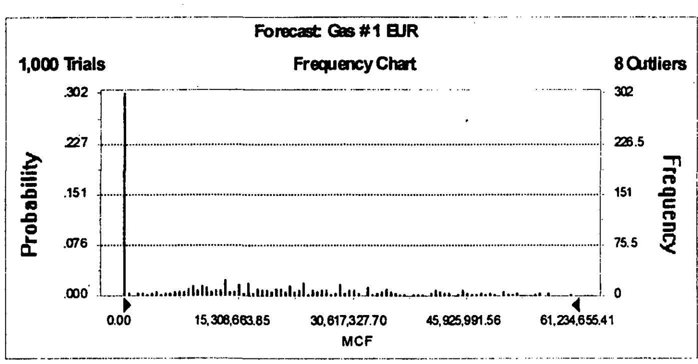
Forecast: Gas#1 NPV



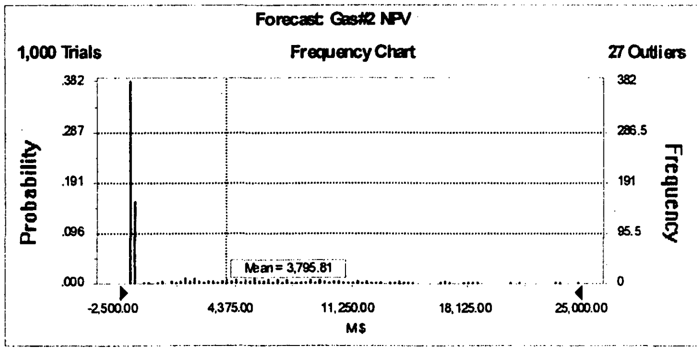
Forecast: Gas#1 Cost



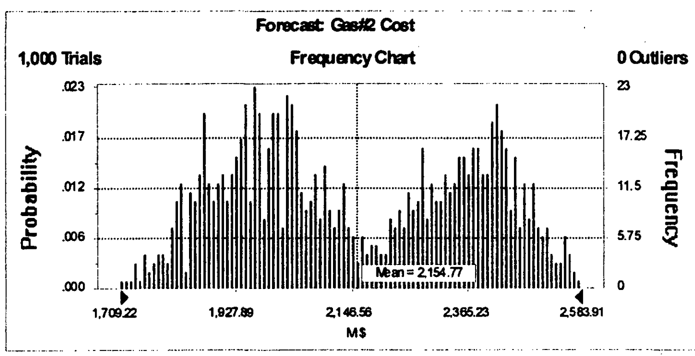
Forecast: Gas # 1 EUR



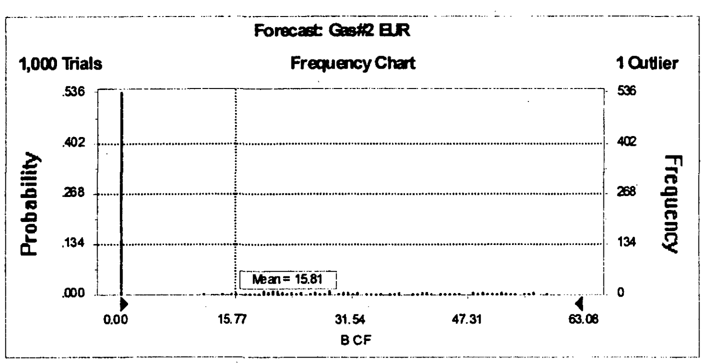
Forecast: Gas#2 NPV



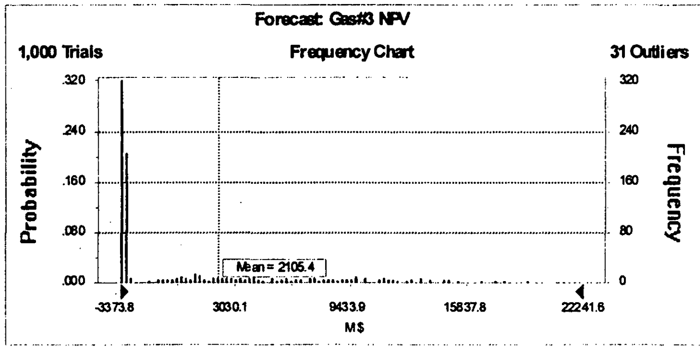
Forecast: Gas#2 Cost



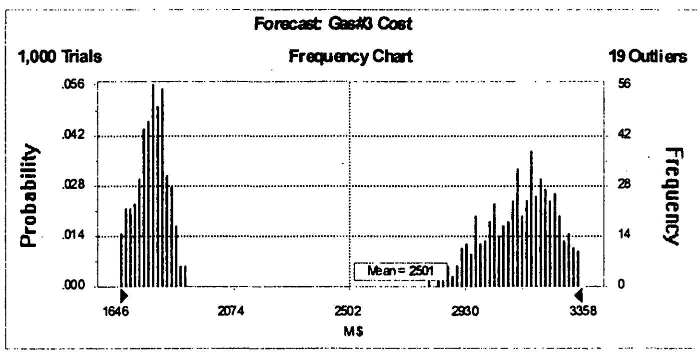
Forecast: Gas#2 EUR



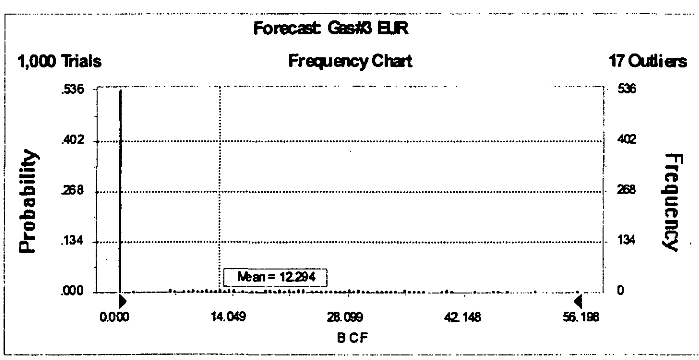
Forecast: Gas#3 NPV



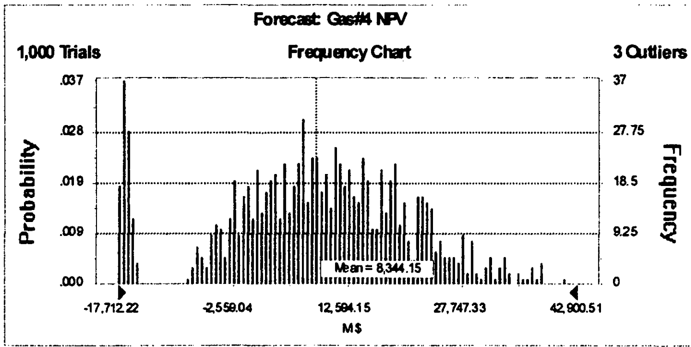
Forecast: Gas#3 Cost



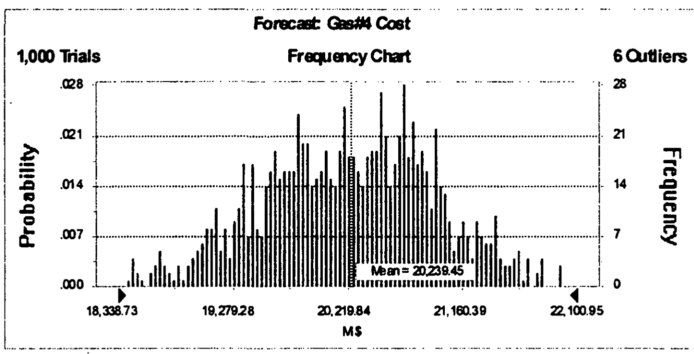
Forecast: Gas#3 EUR



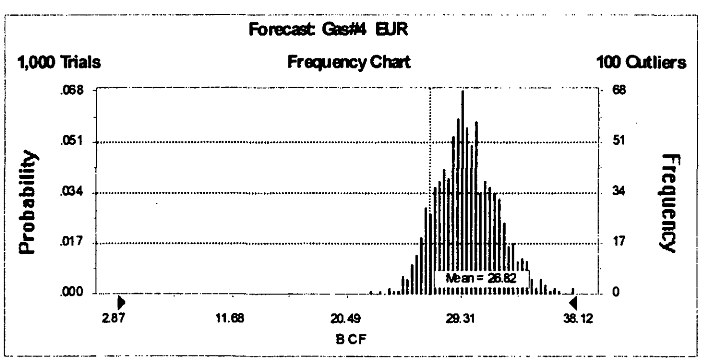
Forecast: Gas#4 NPV



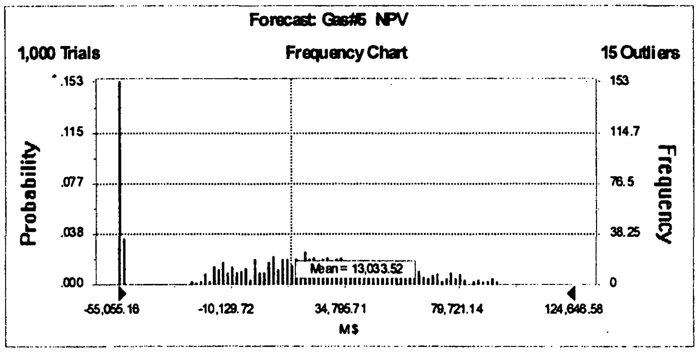
Forecast: Gas#4 Cost



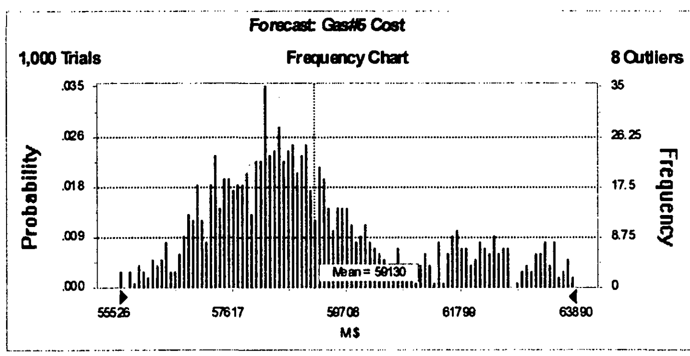
Forecast: Gas#4 EUR



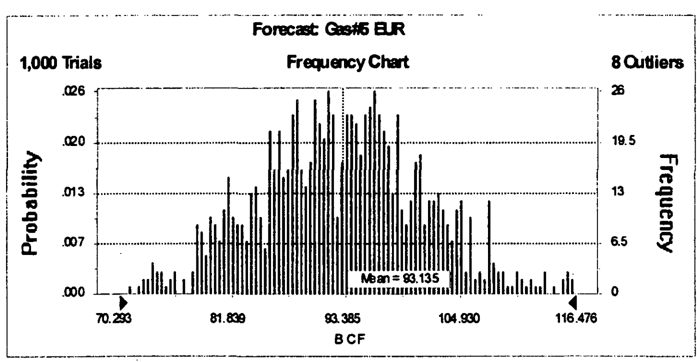
Forecast: Gas#5 NPV



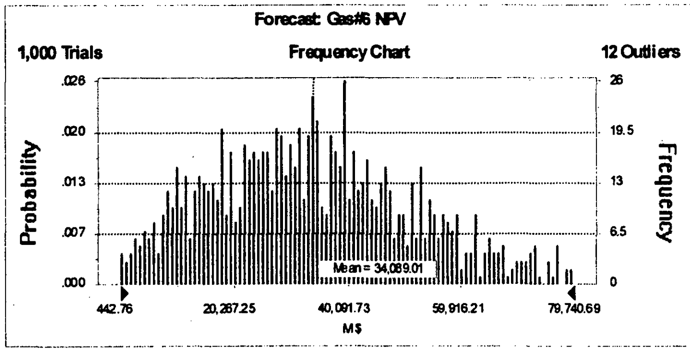
Forecast: Gas#5 Cost



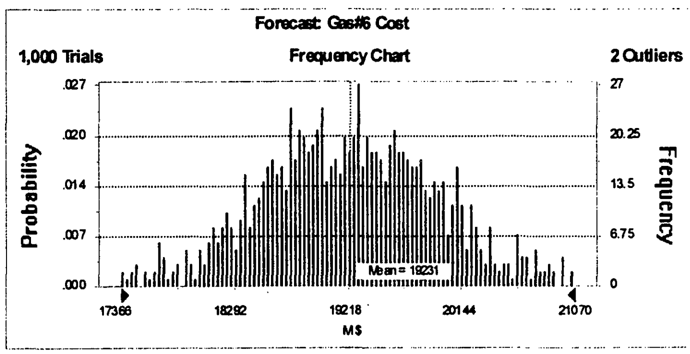
Forecast: Gas#5 EUR



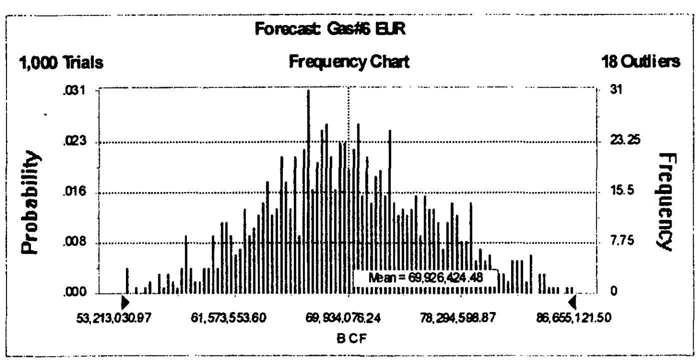
Forecast: Gas#6 NPV



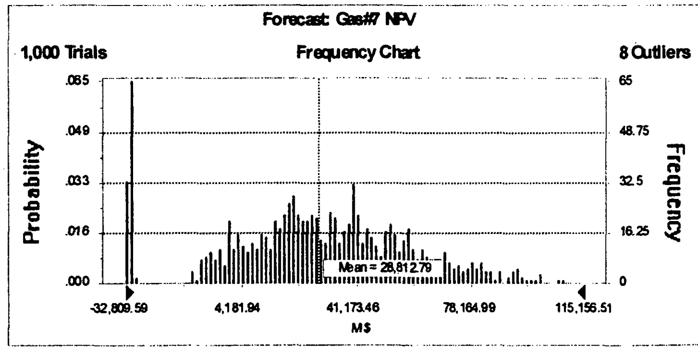
Forecast: Gas#6 Cost



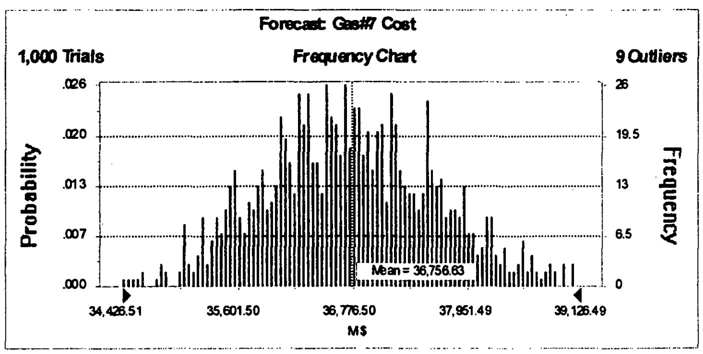
Forecast: Gas#6 EUR



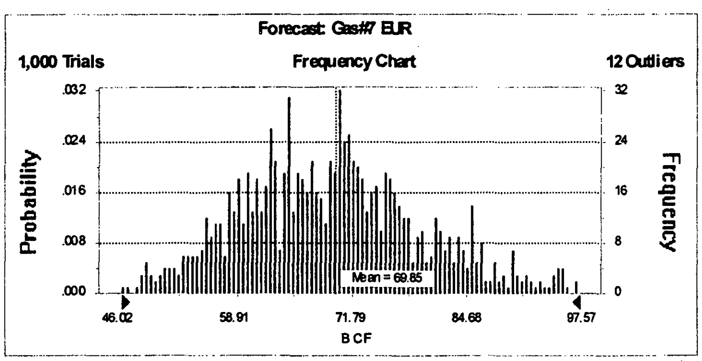
Forecast: Gas#7 NPV



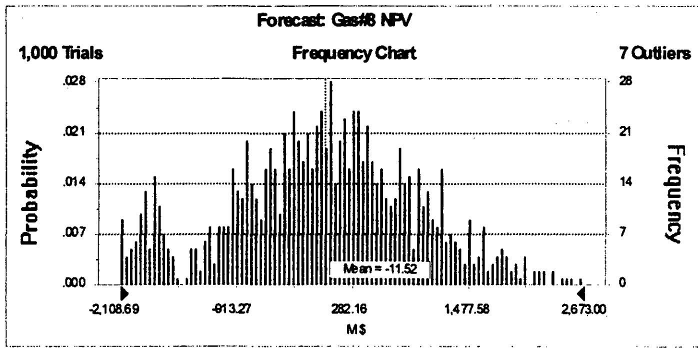
Forecast: Gas#7 Cost



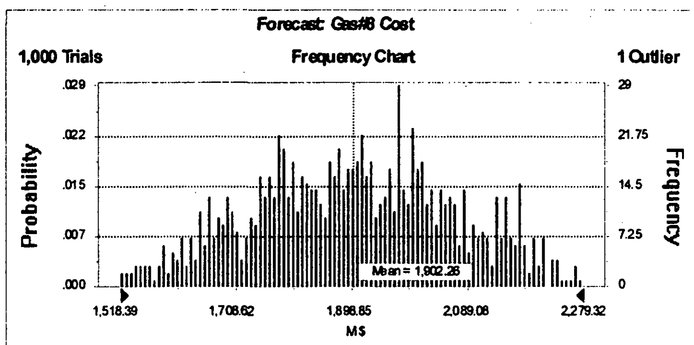
Forecast: Gas#7 EUR



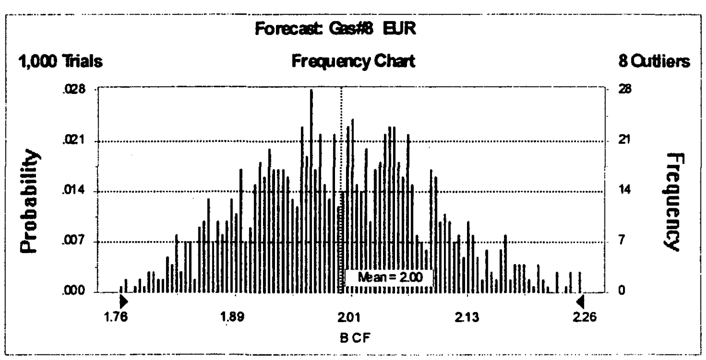
Forecast: Gas#8 NPV



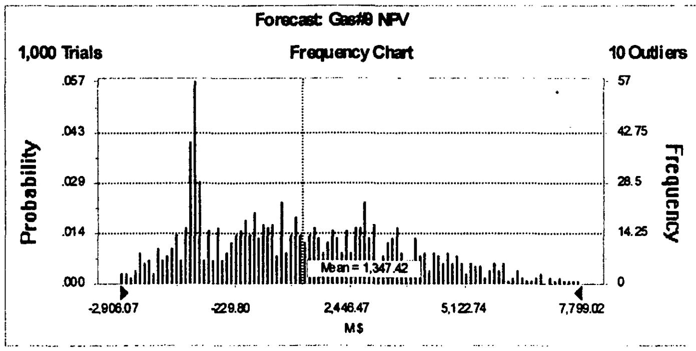
Forecast: Gas#8 Cost



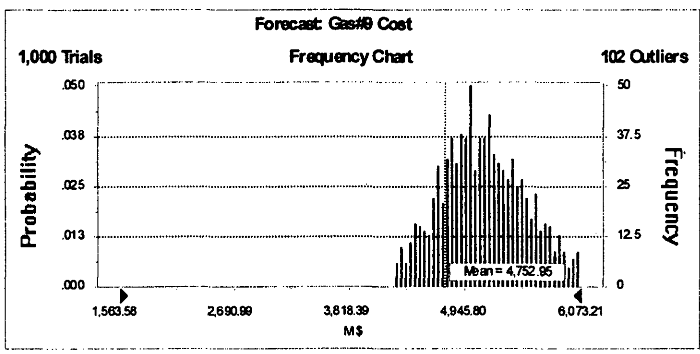
Forecast: Gas#8 EUR



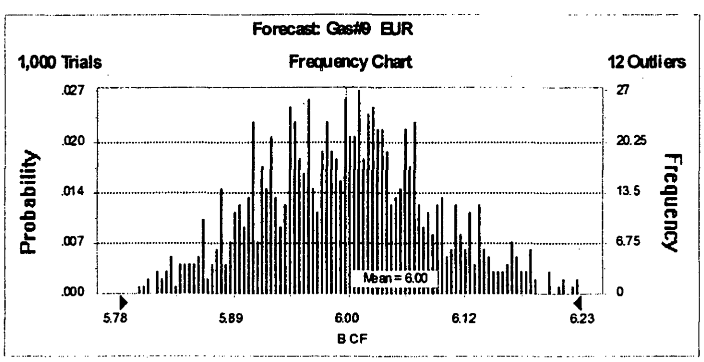
Forecast: Gas#9 NPV



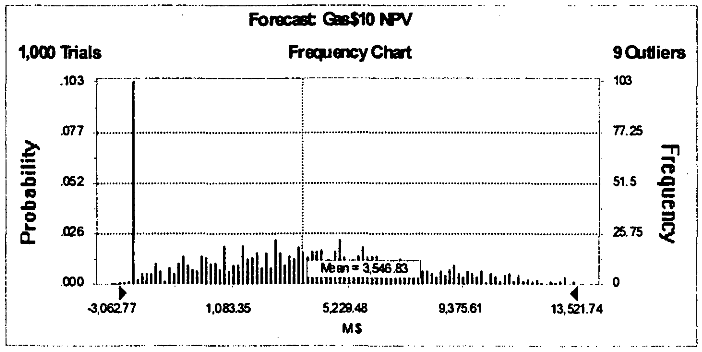
Forecast: Gas#9 Cost



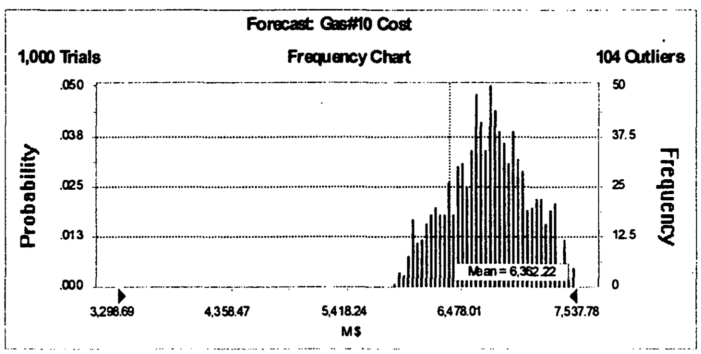
Forecast: Gas#9 EUR



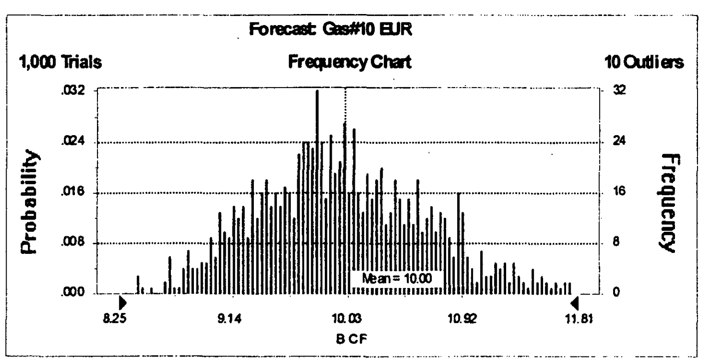
Forecast: Gas$10 NPV



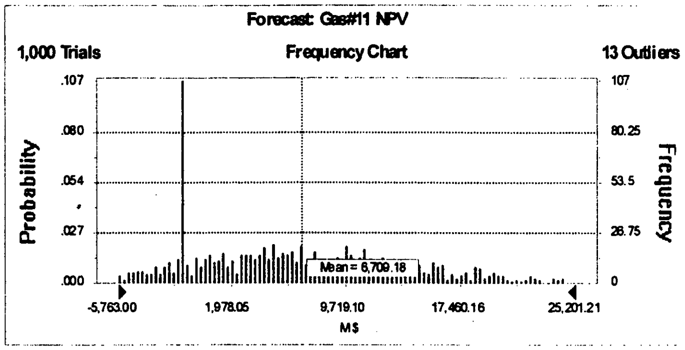
Forecast: Gas#10 Cost



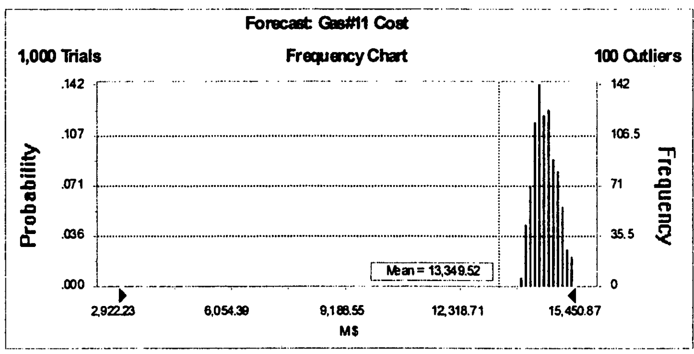
Forecast: Gas#10 EUR



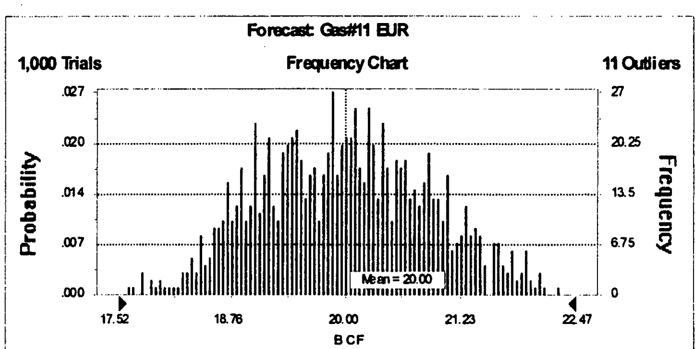
Forecast: Gas#11 NPV



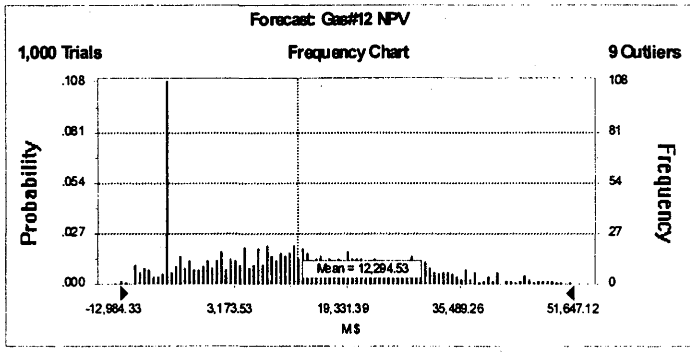
Forecast: Gas#11 Cost



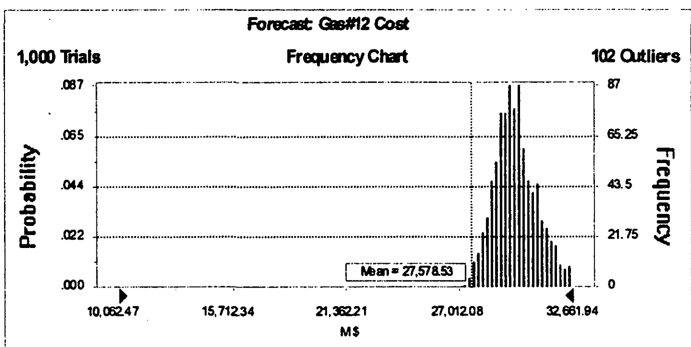
Forecast: Gas#11 EUR



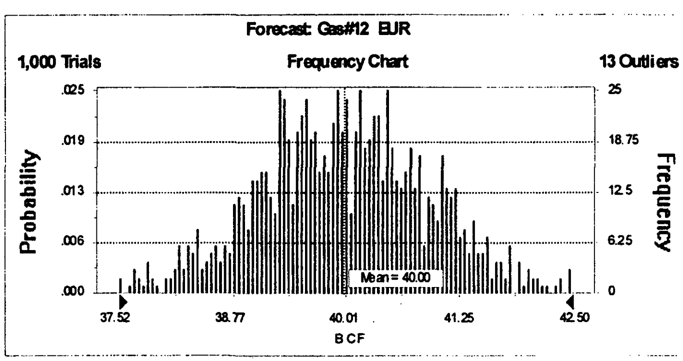
Forecast: Gas#12 NPV



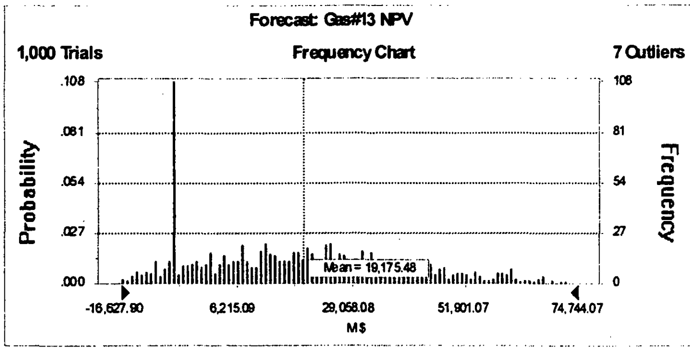
Forecast: Gas#12 Cost



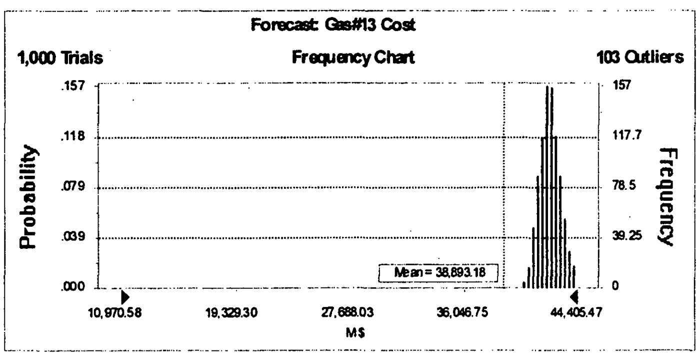
Forecast: Gas#12 EUR



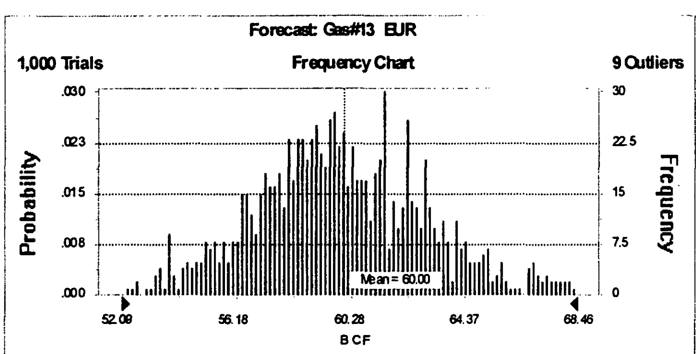
Forecast: Gas#13 NPV



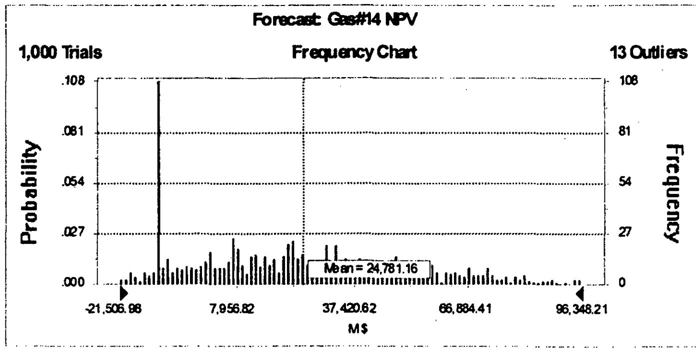
Forecast: Gas#13 Cost



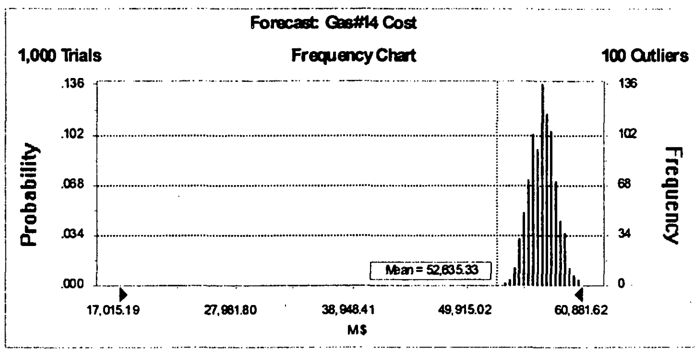
Forecast: Gas#13 EUR



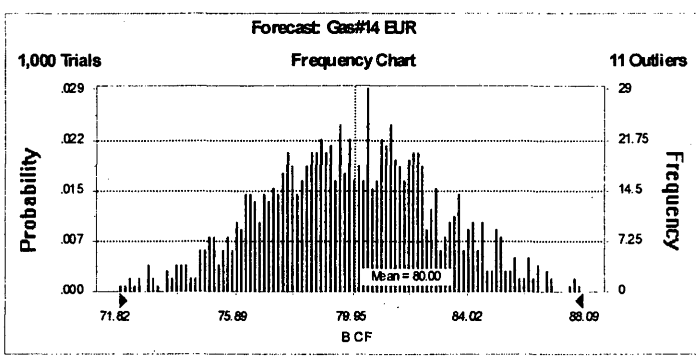
Forecast: Gas#14 NPV



Forecast: Gas#14 Cost



Forecast: Gas#14 EUR



APPENDIX B
OPTIMIZATION ANALYSIS RESULTS

This Appendix includes the efficient frontier plots and the portfolio compositions for the scenarios below.

Scenario A3	page 142
Scenario A5	page 144
Scenario A6	page 145
Scenario A7	page 146
Scenario A9	page 147
Scenario A10	page 148
Scenario A12	page 149
Scenario A13	page 150
Scenario A14	page 151

Scenario A3

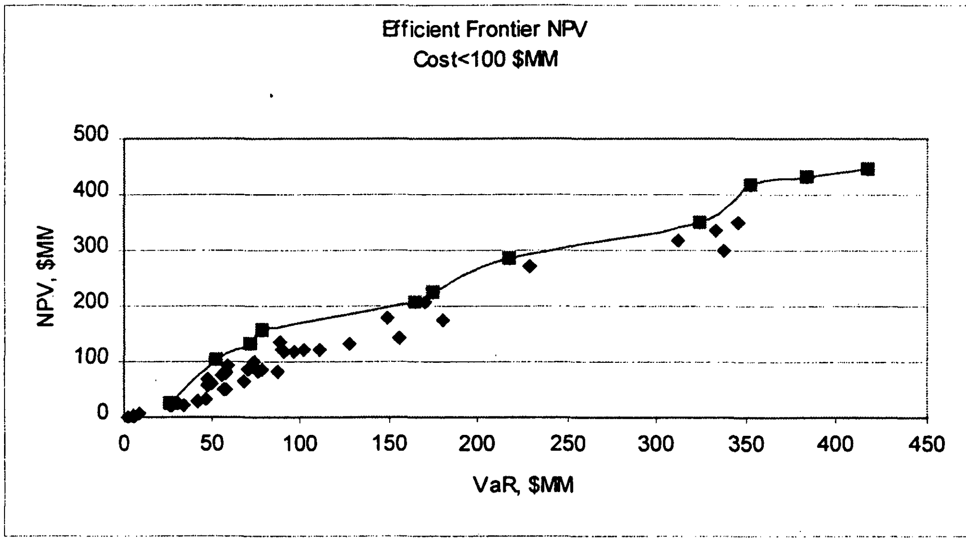


Figure B-1 Scenario A3 Efficient Frontier.

Table B-1 Scenario A3 Portfolio Compositions.

NPV $M	Cost $M	VaR $M	Oil#3 WI	Oil#2 WI	Oil#11 WI	Oil#1 WI	Gas#9 WI	Gas#8 WI	Gas#7 WI	Gas#6 WI	Oil#9 WI	Gas#5 WI	Gas#4 WI	Oil#7 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Gas#1 WI	Oil#8 WI	Oil#4 WI	Oil#10 WI
445464	96849	417147	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	1	1	0
431573	96309	383548	1	1	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0
416376	88516	351896	0	1	0	1	1	0	0	0	0	0	0	1	1	1	1	0	1	1	1	0
350625	93983	324601	0	0	0	0	0	0	0	1	0	0	1	0	0	1	1	0	1	1	1	0
349193	99273	345418	0	0	0	0	0	1	1	0	0	0	0	1	1	1	1	0	1	1	1	0
334105	92845	332842	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	0	1	0	0
318890	80089	311692	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	1	1	0
300080	99024	338321	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
285291	86675	218072	1	1	0	1	1	1	0	1	0	0	0	0	1	1	0	1	0	0	0	0
271586	96862	228514	1	0	0	1	1	1	0	1	0	0	1	1	0	1	1	1	0	0	0	0
224798	70426	175057	1	1	0	1	1	1	0	0	0	0	0	0	1	1	0	1	0	0	0	0
208246	98871	164768	1	1	0	0	1	0	0	1	0	0	1	1	1	1	0	1	0	0	1	0
207644	78157	170687	1	1	0	0	1	1	0	1	0	0	0	0	1	1	0	1	0	0	0	0
177906	72056	149062	1	1	0	0	1	1	0	1	0	0	0	0	1	0	0	1	0	0	0	0
174712	94982	180314	1	1	0	0	1	1	0	0	0	0	0	1	1	1	0	0	0	0	1	1
158030	54854	78689	0	1	0	1	1	1	0	1	0	0	0	0	1	1	0	1	0	0	0	0
144042	99705	155680	1	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	1	0	0	1

NPV $M	Cost $M	VaR $M	Oil#3 WI	Oil#2 WI	Oil#11 WI	Oil#1 WI	Gas#9 WI	Gas#8 WI	Gas#7 WI	Gas#6 WI	Oil#9 WI	Gas#5 WI	Gas#4 WI	Oil#7 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Gas#1 WI	Oil#8 WI	Oil#4 WI	Oil#10 WI
134722	45486	89072	0	0	0	1	1	1	0	1	0	0	0	0	1	1	0	1	0	0	0	0
133632	45840	71695	0	1	0	1	1	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0
131402	54543	128471	1	1	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0
121605	62188	89503	0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	1	1	0	0	0
120153	97524	101806	0	0	0	1	1	0	1	0	0	0	0	1	1	0	1	1	1	0	0	1
119742	94546	110654	0	0	0	1	1	0	1	0	0	0	0	0	1	0	1	1	1	0	0	1
118743	61513	96608	0	0	0	1	1	1	1	0	0	0	0	0	1	0	1	1	1	0	0	0
116701	60546	90995	0	0	0	1	1	0	1	0	0	0	0	1	1	0	0	1	1	0	0	0
102096	47150	53247	0	1	0	0	1	1	0	1	0	0	0	0	1	1	0	1	0	0	0	0
100444	62536	74062	0	0	0	1	1	0	1	0	0	0	0	1	1	0	1	1	1	0	0	0
94315	59703	59697	0	1	0	0	1	0	0	1	0	0	1	1	0	0	1	0	1	0	1	0
87413	31163	70577	0	0	0	1	1	0	0	0	0	0	0	1	1	0	1	1	1	0	0	0
87057	32956	79056	0	0	0	1	1	1	0	0	0	0	0	1	1	0	1	1	1	0	0	0
83745	80794	76355	0	0	0	0	0	0	1	0	0	0	0	1	0	1	1	0	1	0	0	1
82569	30792	57971	0	0	0	1	1	1	0	0	0	0	0	1	0	0	1	1	1	0	0	0
81836	59122	87910	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	1	1	0	0	1
77070	35766	57795	0	0	0	0	1	1	0	1	0	0	0	1	0	1	1	0	0	0	1	0
75683	60816	55754	0	1	0	0	0	1	0	0	0	0	0	1	0	1	1	0	1	0	0	1
55754	38726	48442	0	0	0	0	0	1	0	0	1	0	0	1	1	0	1	0	1	0	1	0
67010	38726	48442	0	0	0	0	1	0	0	1	0	0	0	1	1	0	1	1	0	0	1	0
65471	61021	68764	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	0	1	1
61219	33379	50524	0	0	0	0	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0
51772	53209	56718	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	1	1
50937	50915	57884	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	1
33550	42123	47173	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1
28563	47400	43071	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	1	1
25895	9956	27314	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0
23723	9335	31529	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0
22886	7026	27735	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
22123	5300	27208	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
19902	41039	34943	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1
5879	5967	8934	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
3034	4653	5355	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
1749	4007	5848	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
91	1723	1775	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table B-1 Continued

Scenario A5

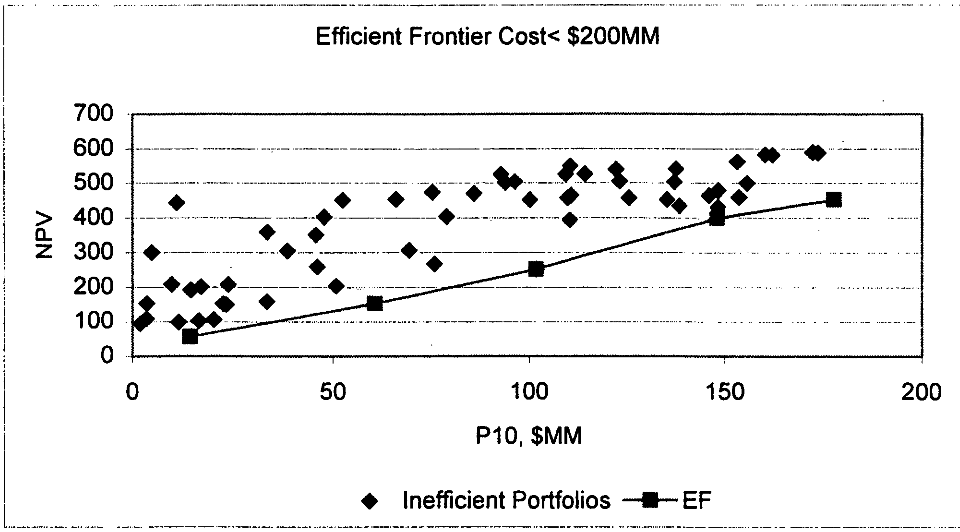


Figure B-2 Scenario A5 Efficient Frontier.

NPV $M	Risk $M	Oil#3 WI	Oil#2 WI	Oil#11 WI	Oil#10 WI	Oil#1 WI	Oil#4 WI	Gas#8 WI	Gas#9 WI	Gas#7 WI	Gas#6 WI	Oil#9 WI	Gas#4 WI	Gas#5 WI	Oil#7 WI	Oil#8 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Gas#1 WI
56937	14432	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	1	0	0	1	0
153218	60675	0	0	1	0	1	0	1	1	0	0	0	1	0	1	0	1	1	0	1	0
252238	101843	0	1	1	0	1	1	0	1	0	1	1	0	0	1	0	1	1	0	0	1
399744	148283	0	1	0	1	1	0	1	1	0	0	0	1	1	1	1	1	1	1	0	0
453902	177954	0	0	1	0	0	0	1	0	1	1	0	1	0	0	1	0	1	1	1	0

Table B-2 Scenario A5 Portfolio Compositions.

Scenario A6

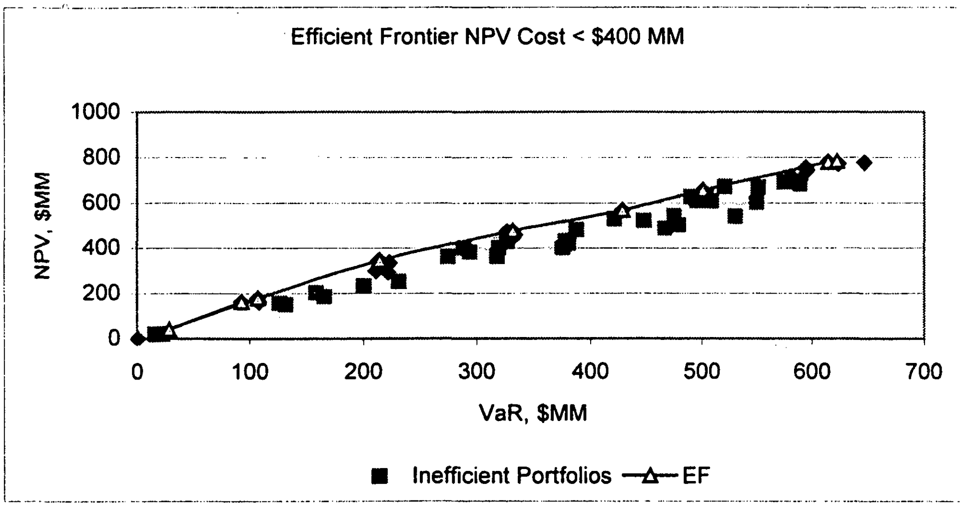


Figure B-3 Scenario A6 Efficient Frontier.

NPV $M	40394	178611	345748	477743	568628	657864	780734	783930
Cost $M	32614	116829	161369	220724	238978	341119	368045	396165
Risk $M	28670	107175	214175	331793	428835	501037	614150	621830
Oil#7 WI	0.75	1	1	0	0.5	0	1	1
Oil#2 WI	1	1	1	1	1	1	1	1
Oil#3 WI	0.25	0.25	0.5	1	1	1	1	1
Oil#11 WI	0.25	0	0	0.5	0.5	1	1	1
Oil#10 WI	0.25	0.75	0	0.25	0.25	1	0	1
Oil#1 WI	1	0	1	1	1	1	1	1
Oil#4 WI	0	1	0	0.25	0.75	1	0.25	1
Gas#9 WI	0.25	0.25	1	0	0.5	1	0.5	0.5
Gas#8 WI	0.25	0.75	0.5	0	0.5	0	0.75	0.5
Gas#7 WI	0.75	1	0.75	1	1	1	1	1
Gas#6 WI	0.5	1	1	1	0.75	0.25	1	1
Oil#9 WI	0.5	0	0.5	0	0.25	1	1	1
Gas#5 WI	0.25	0	0.25	1	0.5	0	1	1
Oil#8 WI	0.75	0	0.25	0.5	0.75	1	1	1
Gas#4 WI	1	0.5	0.75	0	0.5	1	0.75	0.25
Gas#1 WI	0	0.75	0	0.25	0.25	1	1	1
Gas#3 WI	1	0	0	0.25	0.75	1	1	0.25
Oil#6 WI	0.25	1	1	1	1	1	1	1
Gas#2 WI	0.75	0	0.25	0.25	0.5	0	1	0.75
Oil#5 WI	0.25	0	0.5	0.25	0.5	1	0.75	1

Table B-3 Scenario A6 Portfolio Compositions.

Scenario A7

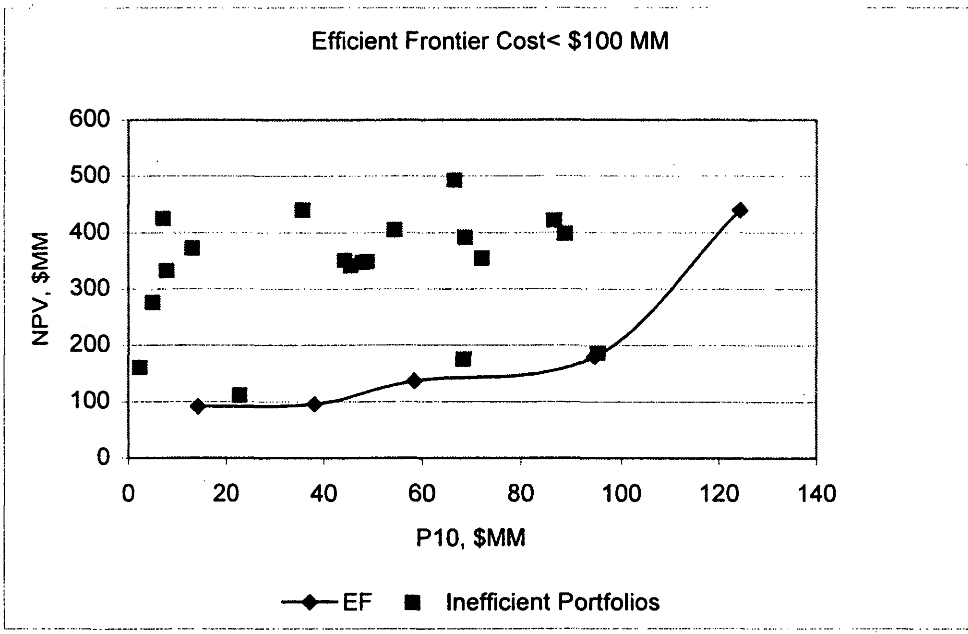


Figure B-4 Scenario A7 Efficient Frontier.

NPV $M	Cost $M	Risk $M	Oil#3 WI	Oil#5 WI	Oil#11 WI	Oil#10 WI	Oil#1 WI	Oil#4 WI	Gas#9 WI	Gas#8 WI	Gas#7 WI	Gas#6 WI	Oil#9 WI	Gas#5 WI	Oil#8 WI	Gas#4 WI	Gas#1 WI	Oil#7 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#2 WI
91596	94413	14230	0	1	0	0	0	1	0	0	0	0	0	1	0	1	1	0	1	0	0	1
96120	28306	38128	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	1	0	1	0	0
136519	95404	58469	0	0	0	1	1	0	1	1	1	0	0	0	0	0	0	1	1	0	0	1
179909	179909	94729	0	1	0	1	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1
439576	97107	124247	0	0	0	0	1	1	0	1	0	1	0	0	1	0	1	1	0	1	0	1

Table B-4 Scenario A7 Portfolio Compositions.

Scenario A9

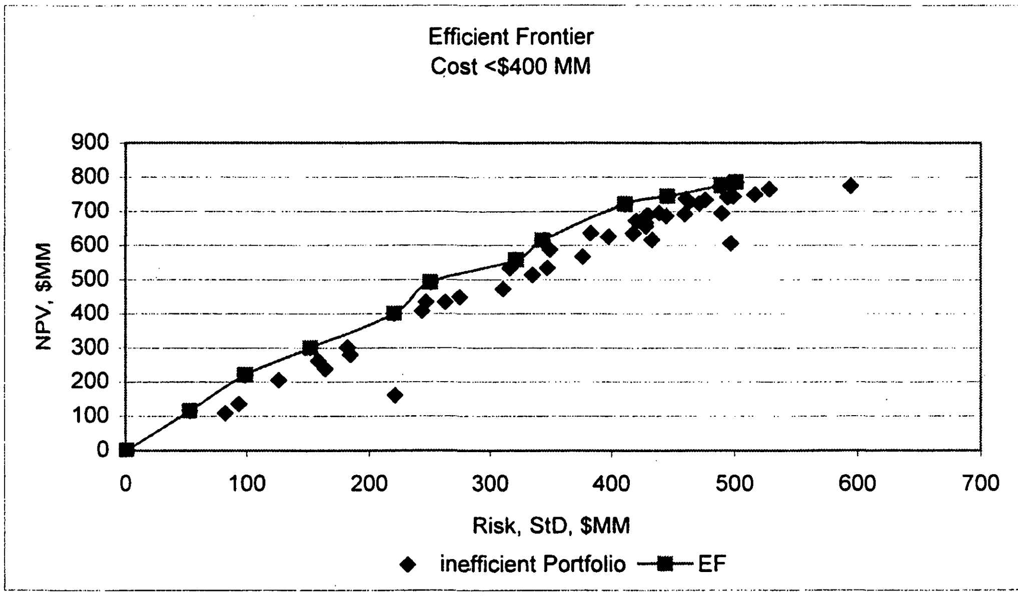


Figure B-5 Scenario A9 Efficient Frontier.

NPV $MM	1	116	221	301	400	493	557	616	722	745	776	786
Cost $MM	2	89	139	170	192	232	314	370	392	386	394	395
Risk $MM	1	54	99	152	221	251	322	343	411	445	490	501
Oil#2 WI	0	0.5	1	0.75	1	1	0.75	1	0.75	1	1	1
Oil#3 WI	0	0	0	0.25	0.5	0.25	0.75	0.75	1	1	1	1
Oil#10 WI	0	0	0	0.5	0	0.75	0.75	1	1	1	1	1
Oil#1 WI	0	0.25	1	0.75	1	0.75	0.75	1	0.75	1	1	1
Gas#9 WI	0.25	0.25	1	0	0	0.75	0.75	0.25	0.75	0.75	0.75	0.5
Gas#8 WI	0	0.75	0	0	0.75	0	0.75	1	1	1	0.75	0.75
Gas#7 WI	0	0.25	0	0.5	1	1	0.75	0.5	1	1	1	1
Gas#6 WI	0	0.75	1	0.75	0.75	1	0.75	1	0.75	1	1	1
Oil#9 WI	0	0.5	1	0.25	0.75	0	0.75	0.25	1	1	1	1
Gas#5 WI	0	0.25	0	0.5	0	0	0.75	0.75	1	1	1	1
Oil#8 WI	0	0	0	0.25	0.5	0.75	0.75	1	1	1	1	1
Gas#4 WI	0	0	1	0	0.75	1	1	0	0.75	0.25	0.25	0.5
Oil#7 WI	0.25	0.75	0	0.5	0	0.25	1	1	1	1	1	0.25
Gas#3 WI	0	0.25	1	0	1	0	1	0.5	1	0	0.5	1
Oil#6 WI	0	0.25	1	1	0.75	0.75	1	0.5	1	1	1	1
Gas#2 WI	0	0.25	1	1	0	1	0.75	1	0.75	1	1	0.75
Oil#5 WI	0	0.75	0	1	0.25	1	0.75	0.5	0.75	1	1	1
Gas#1 WI	0	0.25	1	0	0.25	0	0.75	1	1	1	1	1
Oil#4 WI	0	0.25	1	0.25	0.25	0.75	0.75	0.75	1	1	0.75	0.5
Oil#11 WI	0	0	0	0.5	0.25	1	0.75	1	1	1	1	1

Table B-5 Scenario A9 Portfolio Compositions.

Scenario A10

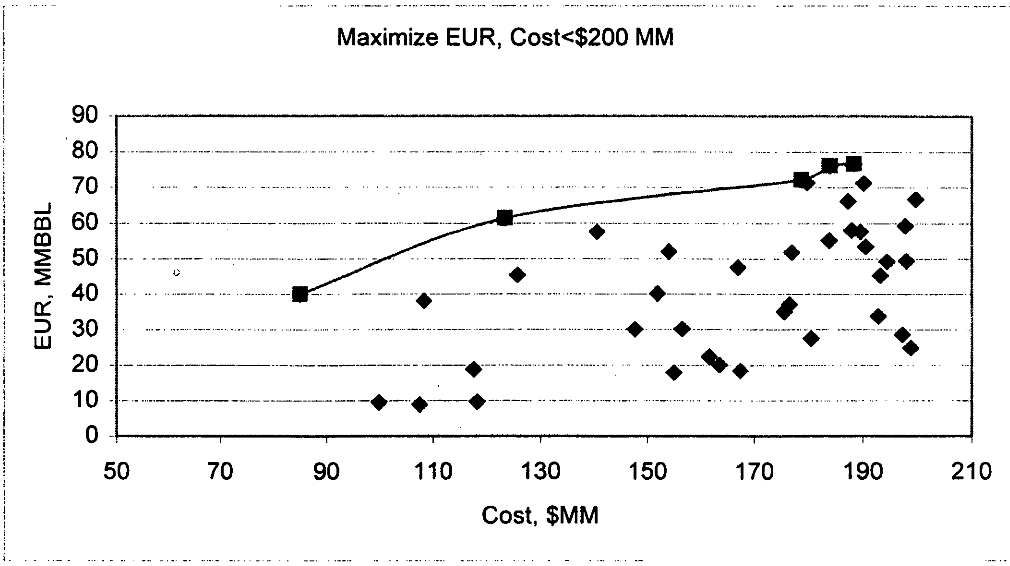


Figure B-6 Scenario A10 Efficient Frontier.

EUR Oil MBBL	Cost $M	Oil#3 WI	Oil#2 WI	Oil#11 WI	Oil#10 WI	Oil#1 WI	Gas#9 WI	Gas#8 WI	Gas#7 WI	Gas#6 WI	Oil#9 WI	Gas#4 WI	Gas#5 WI	Oil#4 WI	Oil#8 WI	Gas#3 WI	Oil#6 WI	Gas#2 WI	Oil#5 WI	Oil#7 WI	Gas#1 WI
40062	85042	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	1	1
61407	123479	1	0	0	0	1	1	1	0	0	0	1	0	1	1	0	0	0	1	0	1
72257	178847	1	1	1	0	1	1	1	0	0	0	0	0	1	1	0	1	0	1	0	0
76147	184005	1	1	1	0	1	1	1	0	0	0	0	0	1	1	1	0	1	1	1	0
76642	188401	1	1	1	0	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	0

Table B-6 Scenario A10 Portfolio Compositions.

Scenario A12

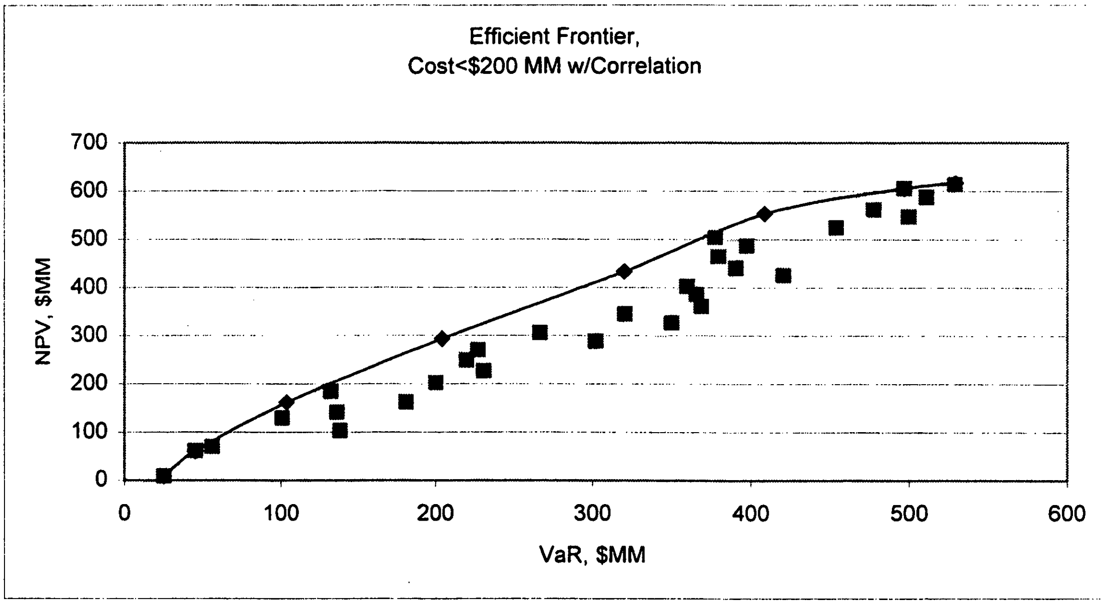


Figure B-7 Scenario A12 Efficient Frontier.

NPV $M	Cost $M	Risk $M	Oil#3 WI	Oil#4 WI	Gas#1 WI	Oil#5 WI	Gas#2 WI	Oil#6 WI	Gas#3 WI	Oil#7 WI	Oil#8 WI	Gas#5 WI	Oil#9 WI	Gas#6 WI	Gas#7 WI	Gas#8 WI	Gas#9 WI	Oil#2 WI	Oil#1 WI	Oil#10 WI	Oil#11 WI	Gas#4 WI
433067	162669	319885	0	0	0	1	1	0	1	1	1	1	0	0	1	1	0	1	1	0	0	0
553692	188672	408718	0	1	0	0	1	1	0	0	1	0	1	1	1	0	0	1	1	0	0	0
605737	178842	496780	1	1	1	1	0	1	0	1	1	0	0	1	0	1	1	1	1	1	0	0
617415	183070	529291	1	0	0	1	1	1	0	0	1	0	0	1	0	0	0	0	1	0	1	0

Table B-7 Scenario A12 Portfolio Compositions.

Scenario A13

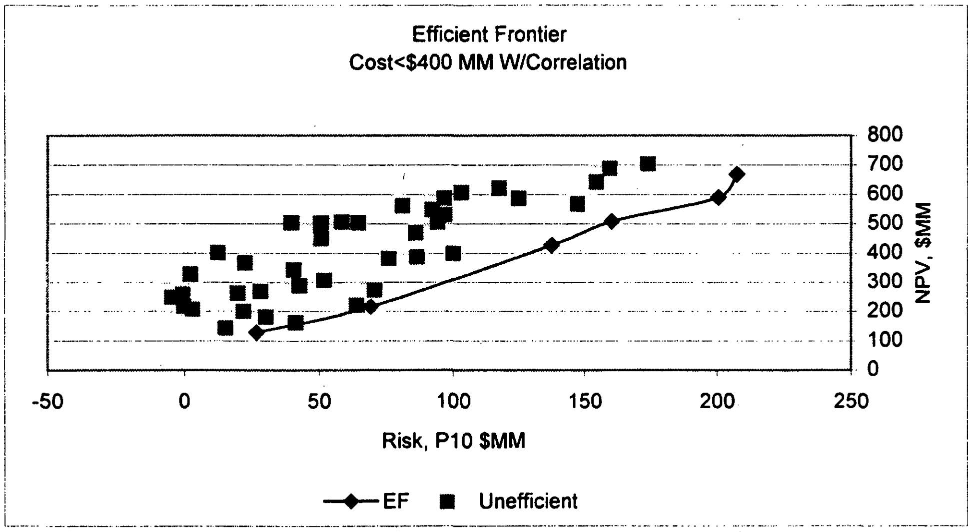


Figure B-8 Scenario A13 Efficient Frontier.

NPV $M	Risk $M	Oil#3 WI	Oil#4 WI	Gas#1 WI	Oil#5 WI	Gas#2 WI	Oil#6 WI	Gas#3 WI	Oil#7 WI	Oil#8 WI	Gas#5 WI	Oil#9 WI	Gas#6 WI	Gas#7 WI	Gas#8 WI	Gas#9 WI	Oil#11 WI	Oil#1 WI	Oil#10 WI	Oil#2 WI	Gas#4 WI
128371	26950	1	0	0	1	0	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1
216623	68935	1	1	1	1	0	0	1	0	0	0	1	0	0	0	1	1	0	1	1	0
428013	137390	1	0	0	1	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	0
590561	200409	1	0	0	1	1	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1
668738	207240	1	0	0	1	0	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1

Figure B-8 Scenario A13 Portfolio Compositions.

Scenario A14

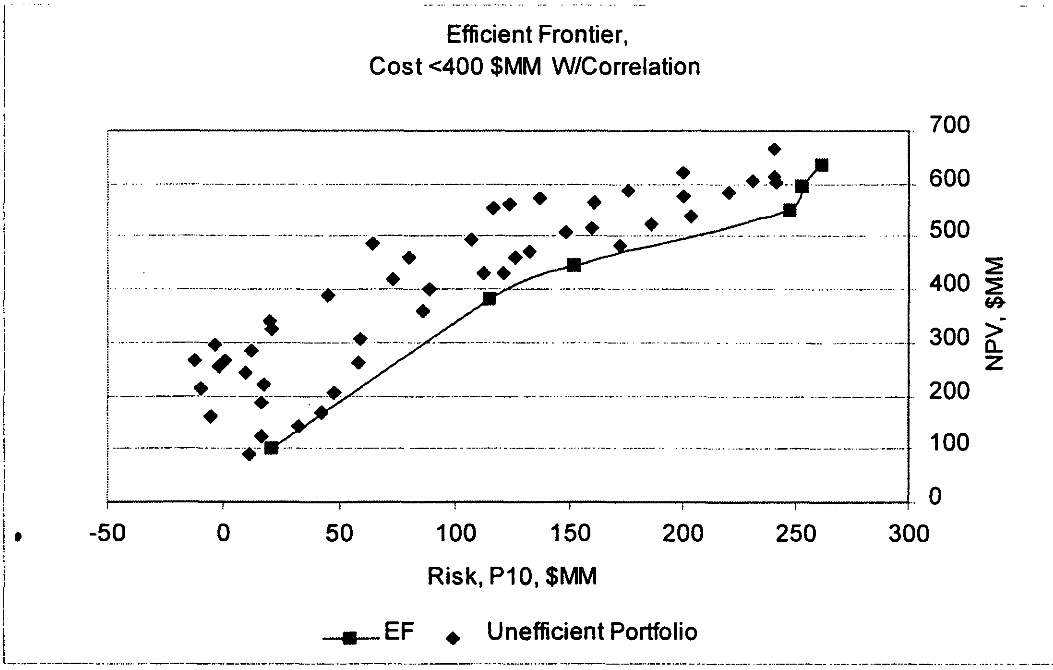


Figure B-9 Scenario A14 Efficient Frontier.

NPV $M	Risk $M	Oil#3 WI	Oil#4 WI	Gas#1 WI	Oil#5 WI	Gas#2 WI	Oil#6 WI	Gas#3 WI	Oil#7 WI	Oil#8 WI	Gas#5 WI	Oil#9 WI	Gas#6 WI	Gas#7 WI	Gas#8 WI	Gas#9 WI	Oil#11 WI	Oil#1 WI	Oil#10 WI	Oil#2 WI	Gas#4 WI
635753	260984	1	0	1	0	1	0	0	0	0	1	1	1	0	0	0	1	0	1	0	0
596876	252609	0	0	1	0	1	0	1	0	1	0	0	0	0	1	1	0	0	1	0	0
549998	247048	1	1	1	0	0	1	1	1	1	0	1	0	0	0	0	1	0	1	0	1
447206	151415	1	1	0	0	1	1	0	1	1	0	1	0	0	0	1	1	0	1	1	1
381497	114740	1	0	0	1	0	1	1	0	1	1	1	1	1	0	0	1	1	0	1	1
101916	20887	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	0

Figure B-9 Scenario A14 Portfolio Compositions.

APPENDIX C
INVESTMENT PROJECTS OIL#2

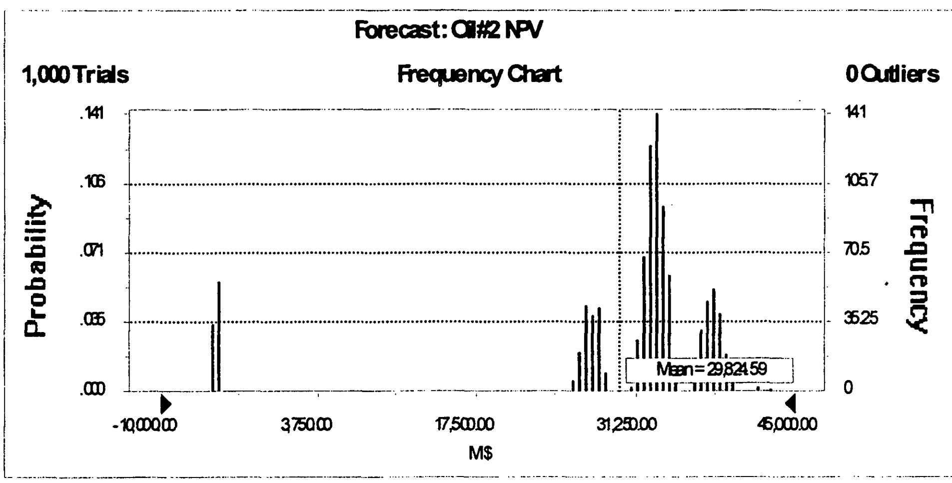
This Appendix includes details of stochastic variables, decision variables, and forecasts for project Oil#2.

Forecast: Oil#2 NPV

Summary:

Display Range is from -10,000.00 to 45,000.00 M $
Entire Range is from -5,243.85 to 42,973.49 M $
After 1,000 Trials, the Std. Error of the Mean is 362.41

Statistics:	Value
Trials	1000
Mean	29,824.59
Median	32,992.98
Mode	---
Standard Deviation	11,460.28
Variance	131,338,046.28
Skewness	-2.43
Kurtosis	7.76
Coeff. of Variability	0.38
Range Minimum	-5,243.85
Range Maximum	42,973.49
Range Width	48,217.35
Mean Std. Error	362.41



Forecast: Oil#2 NPV (cont'd)

Percentiles:

Percentile	M $
0%	-5,243.85
10%	26,440.55
20%	27,930.92
30%	32,003.34
40%	32,552.38
50%	32,992.98
60%	33,374.39
70%	33,898.45
80%	36,801.05
90%	38,055.44
100%	42,973.49

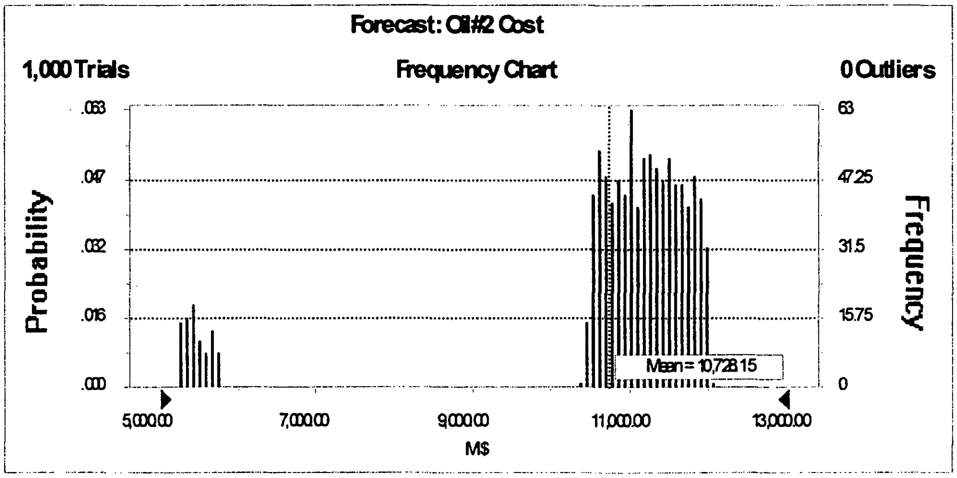
Forecast: Oil#2 Cost

Summary:

Display Range is from 5,000.00 to 13,000.00 M $
Entire Range is from 5,253.04 to 12,042.89 M $
After 1,000 Trials, the Std. Error of the Mean is 53.80

Statistics:	Value
Trials	1000
Mean	10,728.15
Median	11,176.87
Mode	---
Standard Deviation	1,701.46
Variance	2,894,962.91
Skewness	-2.56
Kurtosis	8.21
Coeff. of Variability	0.16
Range Minimum	5,253.04
Range Maximum	12,042.89
Range Width	6,789.85
Mean Std. Error	53.80

Forecast: Oil#2 Cost (cont'd)



Percentiles:

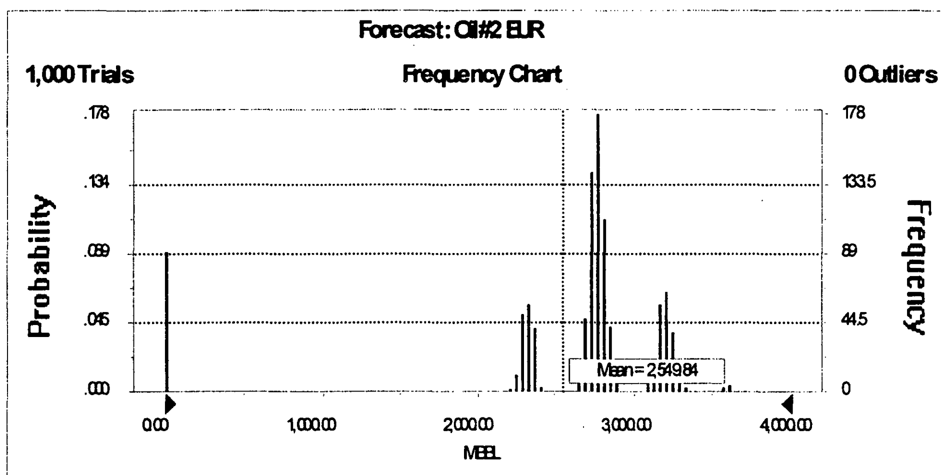
Percentile	M $
0%	5,253.04
10%	10,509.00
20%	10,676.33
30%	10,847.26
40%	11,017.50
50%	11,176.87
60%	11,329.64
70%	11,492.52
80%	11,656.80
90%	11,845.56
100%	12,042.89

Forecast: Oil#2 EUR

Summary:

Display Range is from 0.00 to 4,000.00 MBBL
Entire Range is from 0.00 to 3,645.69 MBBL
After 1,000 Trials, the Std. Error of the Mean is 26.86

Statistics:	Value
Trials	1000
Mean	2,549.84
Median	2,770.87
Mode	0.00
Standard Deviation	849.48
Variance	721,616.60
Skewness	-2.31
Kurtosis	7.41
Coeff. of Variability	0.33
Range Minimum	0.00
Range Maximum	3,645.69
Range Width	3,645.69
Mean Std. Error	26.86



Forecast: Oil#2 EUR (cont'd)

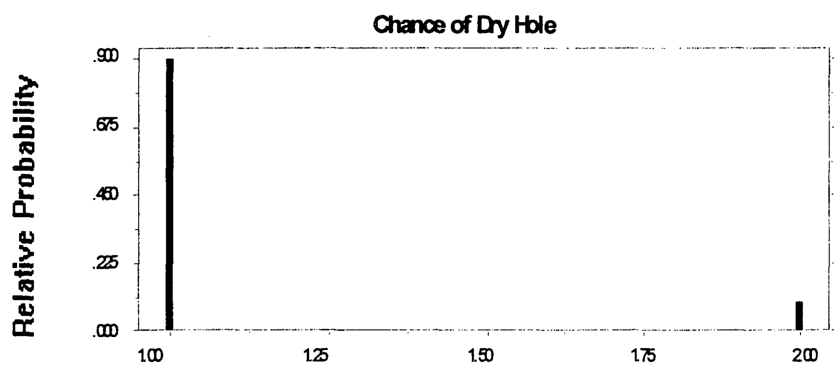
Percentiles:

Percentile	MBBL
0%	0.00
10%	2,264.90
20%	2,351.05
30%	2,714.94
40%	2,747.70
50%	2,770.87
60%	2,792.77
70%	2,820.69
80%	3,139.44
90%	3,215.02
100%	3,645.69

Assumptions

Assumption: Chance of Dry Hole

Custom distribution with parameters:		Relative Prob.
Single point	1.00	0.900000
Single point	2.00	0.100000
Total Relative Probability		1.000000

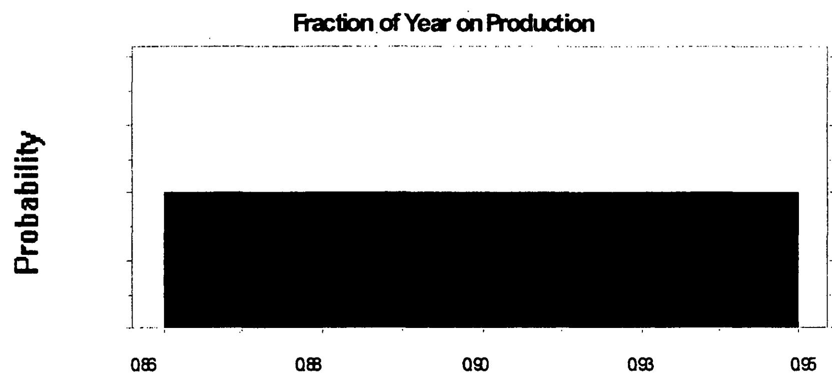


Assumption: Fraction of Year on Production

Uniform distribution with parameters:

Minimum	0.86
Maximum	0.95

Assumption: Fraction of Year on Production (cont'd)

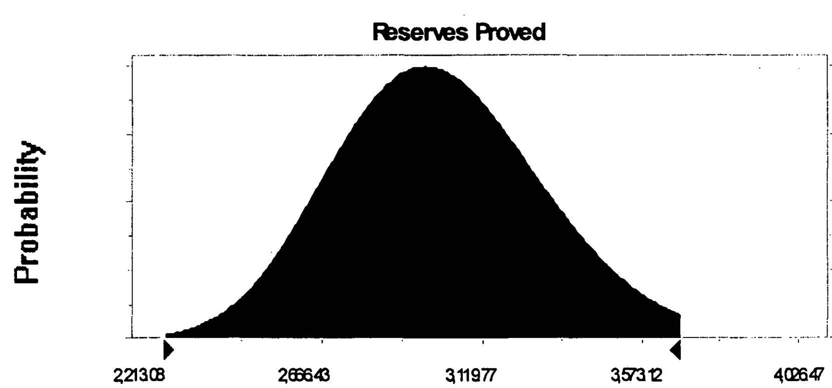


Assumption: Reserves Proved

Lognormal distribution with parameters:

Mean	3,000.00
Standard Dev.	300.00

Selected range is from 2,142.86 to 3,682.05



Assumption: L&WE M$ / Well

Triangular distribution with parameters:

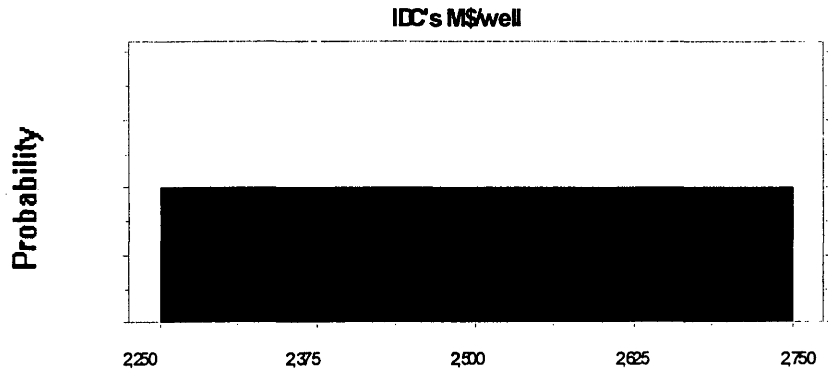
Minimum	225
Likeliest	250
Maximum	275

Selected range is from 225 to 275

Assumption: IDC's M$/well

Uniform distribution with parameters:

Minimum	2,250
Maximum	2,750



Decision Variables

Decision Variable: Oil#2 WI

Variable bounds:

Lower	0
Upper	1
Step	1

APPENDIX D
COMPACT DISK

This Appendix includes a compact disk containing optimization scenarios, investment projects, and input files for the model. Software needed to fully use the data inside this compact disk:

- Microsoft Excel,
- Crystal Ball,
- Optquest.