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ECONOMIC ANALYSIS OF INVESTMENT PROPOSALS WITH DUAL RATES OF RETURN

by

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ABSTRACT

Rate of return analysis of project proposals with dual rates of return often leads to vague conclusions. This thesis presents an empirical approach to investment situations which show two reversals in sign of the cash flow sequence. In particular, the alternating cost, income, cost investment situation which oftern produce dual rates of return is analyzed.

A computer program was developed in order to determine when the final cost or costs in the alternating cost, income, cost investment situation can be considered insignificant. When the final cost or costs are found to be negligible, then the rate of return (ROR) assuming a final cost(s) of zero can be taken as the ROR of the particular investment proposal under consideration.

The above procedure, called a Valid ROR Method, was applied to actual investment proposals and the results show the method to be adequate under certain conditions. The effectiveness of the Valid ROR Method is also discussed.

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CHAPTER I

INTRODUCTION

The purpose of this thesis is to present an economic evaluation method that can be used to deal with project proposals having dual rates of return. Most authors in engineering economy recommend the use of methods other than the rate of return criterion with the dual rate of return situation. This thesis discusses not only the reasons for such a recommendation but also the conditions when rate of return can be valid. Thus, the proposed technique is called a Valid Rate of Return Method.

Project proposals characterized by two sign reversals in the sequence of cash flows often produce dual rates of return. Sign reversal refers to the change from a cash inflow to a cash outflow or vice-versa. Thus, projects of this nature start with a disbursement or cost followed by income and finally by another cost term. This alternating cost, income, cost situation can occur directly or indirectly such as in incremental rate of return analysis. Both occurrences abound in mineral and petroleum related projects. The proposed Valid Rate of Return offers a simple but effective approach to the evaluation of the aforementioned type of investment proposals under certain conditions.

The first part of this thesis defines the rate of return (ROR) criterion. The calculation process as well as misconceptions about the ROR approach is also presented. Next, the concept of reinvestment is introduced in order to lead to the discussion on the dual rates of return. The term "dual rates of return" is actually a misnomer since each of these dual rates of return combines the rate of return and rate of reinvestment meaning at different stages of the project. ROR Analysis methods to deal with this dual ROR situation include the Present Worth Cost Modified ROR Analysis and Growth ROR Analysis (Stermole, 1982). Both of these methods eliminate the alternating cost, income, cost situation by modifying either the final cost or the incomes. Consequently, application of these methods yields a simple type of investment proposal where cost precedes income.

With the objective of converting the dual ROR situation into a simple type of project, under certain specified conditions, the Valid ROR method was developed. This method is purely empirical in nature, It makes use of plots which provide a straight forward way of determining the conditions when the Valid ROR may be used. Valid ROR is the rate of return calculated for cost, income, cost situations by assuming a final cost of zero. If the final cost in a project with alternating cost, income, cost can be considered insignificant, then this Valid ROR may be used as the project's rate of return. A major objective of this study was to determine cost, income, cost evaluation situations where the final cost has insignificant effect on economic evaluation ROR results and therefore can be neglected.

In order to assess its acceptability, the Valid ROR Method was applied to various cases arising in investment situations. Comparison of this approach with the Present Worth Cost Modified ROR Analysis and Growth ROR Analysis was also made. Finally, the Valid ROR Method was used in actual case studies involving incremental rate of return analysis as well as a direct occurrence of dual ROR's in a surface mine project. Both applications of the Valid ROR Method show the effectiveness of the proposed technique.

CHAPTER II

THE RATE OF RETURN CRITERION

Considered as one of the fundamental approaches in economic evaluation, the rate of return criterion is based on a breakeven type of calculation involving the time value of money. The rate of return for a project being evaluated is that interest rate at which the project's costs are equated on an equivalent basis with the project's income.

Costs refer to disbursements or cash outflows, while income connotes receipts or cash inflows. These cash flows are accrued over the life of the project on a regular periodic arrangement, usually end of year periods. Equating costs and income on an equivalent basis simply means that a specific point in time is chosen as reference and the cash flows are transformed accordingly. Although any reference point can be selected, the time frames most commonly used are present (time zero), future (last period of the project life) and annual (or uniform end of period if cash flows are not accounted for on a yearly basis). The equations resulting from the use of these time frames are called present worth, future worth and annual worth respectively. Hence, time value of money is involved in these calculations.

The unknown in all of these equations is the interest rate. Although all of these equations yield the same result, the present worth equation is perhaps the simplest. To illustrate how rate of return is calculated employing the present worth equation, consider the following example. An investment proposal entailing an initial cost of $1,000 will provide annual cash inflows of $400 over a 4-year project life. The time diagram for this simple case is shown below (C = Cost and I = Income).

	C=1000	I=400	I=400	I=400	I=400
Year	0	1	2	3	4

Present Worth Equation:

Present Worth Cost = Present Worth (PW) Income

$$1{,}000 = \sum_{n=1}^{4} \frac{1}{(1+i)^n} (400)$$

Where i equals unknown interest rate and n represents periods of accrual of cash flows.

The reference point is zero on the time diagram. The expression $1/(1+i)^n$, called the "single payment present worth factor," transforms a future single sum to a present single sum. Values for this factor can be computed for any interest rate or a Table of Compound Interest Factors may be consulted. Most solutions to the present worth factor involve trial and error. Letting i = 20 percent, and looking up the single payment present worth factor in a Table of Compound Interest Factors.

$$\text{PW Income} = \sum_{n=1}^{4} \frac{1}{(1+i)^n} (400)$$

$$= \frac{1}{(1+i)^1} (400) + \frac{1}{(1+i)^2} (400) + \frac{1}{(1+i)^3} (400)$$

$$+ \frac{1}{(1+i)^4} (400)$$

$$= (0.8333)(400) + (0.6944)(400) + (0.5787)(400)$$

$$+ (0.4823)(400)$$

$$= 1{,}035.48$$

Since this value is greater than the Present Worth Cost of \$1,000, a larger i value is now tried.

Let i = 25%:

$$\text{PW Income} = (0.8000)(400) + (0.6400)(400)$$

$$+ (0.5120)(400) + (0.4096)\ (400)$$

$$= 944.64$$

Interpolating between 20% and 25%:

at i = 20%	PW Income = 1,035.48
at i = ?	= 1,000.00
at i = 25%	= 944.64

$$\frac{i - 20}{25 - 20} = \frac{1{,}000.00 - 1{,}035.48}{944.64 - 1{,}035.48}$$

$$i = 21.95\% \simeq 22.0\%$$

Hence, the rate of return is 22.0 percent. A more accurate interpolation would result if the Table of Compound Interest Factors gave values for smaller increments of the interest rate. One way to overcome this shortcoming of the Table is to compute the factor itself.

Of course, there are other ways of determining the unknown i. One which is commonly included in computer software for financial decision making involves extracting the roots of the present worth equation written in polynomial form. In the above example, let $x = 1/(1+i)$ and the present worth equation becomes:

$$-1,000 + 400x + 400x^2 + 400x^3 + 400x^4 = 0$$

In general:

$$CF_o + CF_1x + CF_2x^2 + CF_3x^3 + \ldots + CF_nx^n = 0$$

where the CF's are the periodic cash flows and n is the project life. In this thesis, a computer routine, RORFOR, makes use of the IMSL (International Mathematical and Statistical Library) Subroutine called ZPOLR. The subroutine ZPOLR computes real as well as complex roots by the Laguerre method. Of course only the real roots are of interest in this thesis. For the preceding example, the routine gives a rate of return of 21.9 percent. Again, there are other numerical analysis methods for extracting the roots of a polynomial equation.

Once the interest rate i is determined, this i now represents the project's rate of return. The decision maker or investor would then compare the project's rate of return to a so-called minimum rate of return. This minimum rate of return (Min ROR) is the lower limit on the rate of return the investor would be willing to receive on his investment In other words, the Min ROR reflects the opportunity cost of capital or the return the investor can get for using the same investment capital on alternative projects. A project rate of return lower than the Min ROR would be unsatisfactory.

The Reinvestment Assumption Refuted

"Discounting" has become synonymous to "present worthing" or employing the present worth equation in calculating rate of return. Hence, another name for rate of return is Discounted Cash Flow Rate of Return (DCFROR). There is a misconception regarding the rate of return approach. One of the supposed weaknesses of the ROR analysis is the "inherent assumption of reinvestment." (Blankenship, 1978). It is argued that it is inherent in the calculations of the ROR for the cash inflows to be reinvested at that same ROR. This fallacy may have originated from the fact that the discounting procedure is the reciprocal of compounding, which clearly requires reinvestment of earnings at the computed ROR.

First of all, the rate of return criteria as a measure of profitability defines the interest rate which could be paid each interest compounding period on the unamortized investment. To explain this in graphical form, a cumulative cash position diagram of the previous example is shown in Figure 1. A cumulative cash position is defined as the project cost plus accrued interest which has not been recovered through any cash inflows. In the diagram, the sign convention is negative for costs and positive for income.

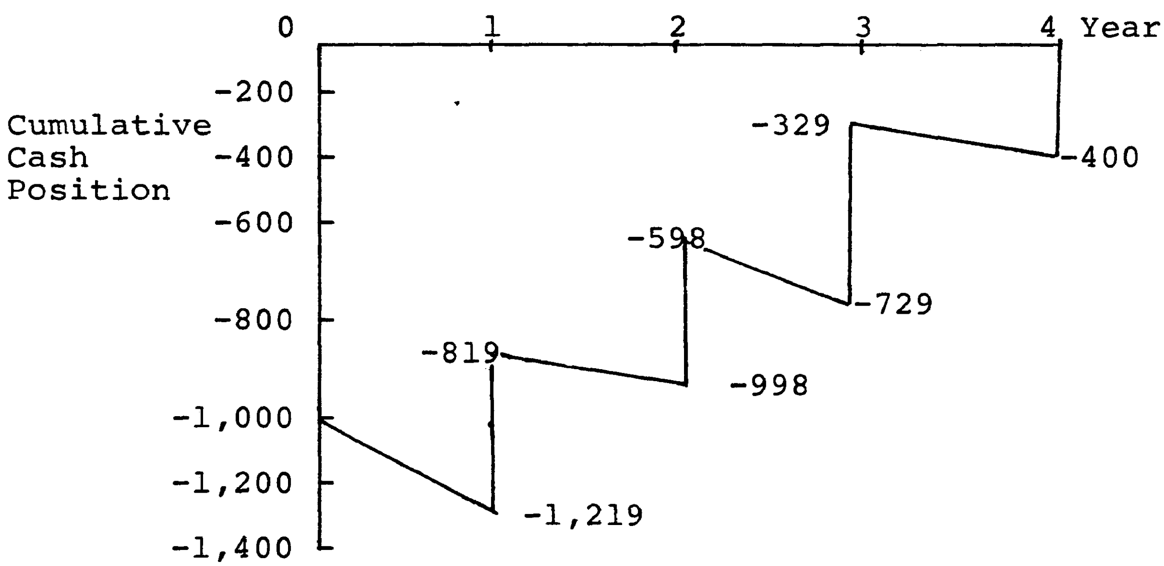

Figure 1. Cumulative Cash Position Diagram

It is clear from the cumulative cash position diagram that at the end of each interest compounding period, the cumulative cash position is adjusted for interest on the unrecovered project cost. There is no reinvestment of the interim cash flows involved in the adjustment of the cumulative cash position. From the investor's point of view, however, reinvestment of cash inflows will necessarily influence the growth rate of available capital. Nevertheless, reinvestment of cash inflows is independent of the way rate of return was calculated for the particular project the investor is contemplating.

<u>The True Reinvestment Situation</u>

Reinvestment of interim cash flows occurs in economic analysis situations where income precedes cost. This situation occurs for example in equipment replacement proposals with savings being realized but with a major cost at the end. In such cases, however, the i calculated is the interest rate at which the cash inflows would have to be reinvested in order to compensate for the cost at the end of the project. To illustrate, consider the following example. Assume a project has annual income of $400 over 4 years and a cost of $2,200 at the end of year 4. The time diagram is a follows:

```
      I=400    I=400    I=400    I=400
|--------+--------+--------+--------|   C = 2,200
0        1        2        3        4
```

Present Worth Equation:

PW Income = PW Cost

$$\sum_{n=1}^{4} \frac{1}{(1+i)^n}(400) = \frac{1}{(1+i)^4}(2{,}200)$$

or PW Income - PW Cost = 0

$$\sum_{n=1}^{3} \frac{1}{(1+i)^n}(400) - \frac{1}{(1+i)^4}(1{,}800) = 0$$

The last two equations employ the concept of Net Present Value (NPV). The difference between Present Worth Income and Present Worth Cost represents the Net Present Value. When NPV is set to zero, the unknown interest rate i is the rate of return.

Using the routine RORFOR, the solution yields an interest rate of 21.7 percent. The cumulative cash position diagram for this example in shown in Figure 2.

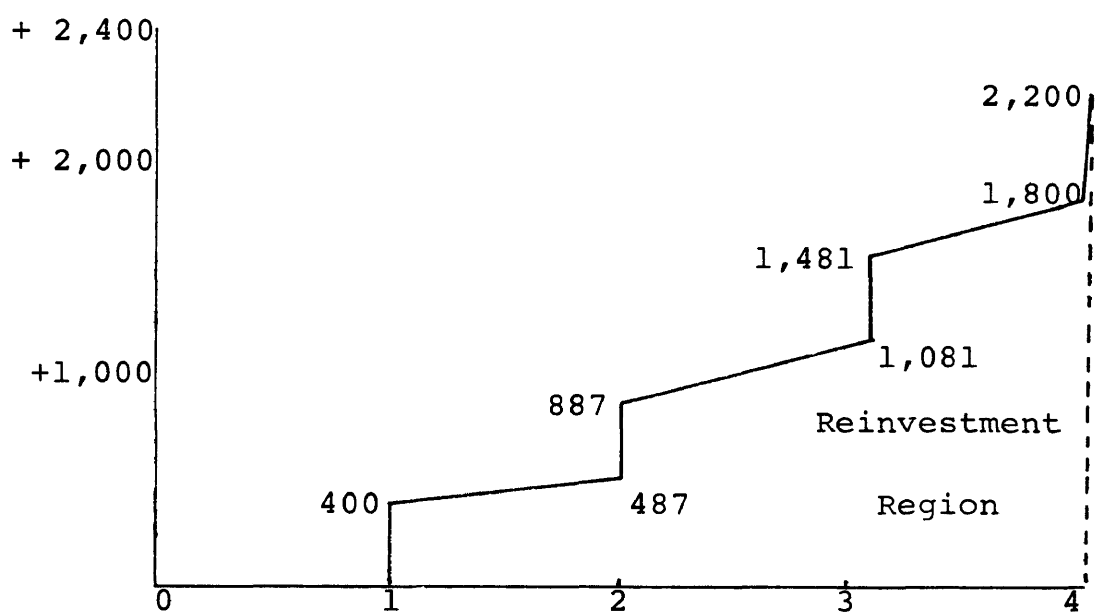

Figure 2. Cumulative Cash Position Diagram for Income Preceding Cost

Comparing Figure 2 with Figure 1, it should be noted that the cumulative cash position at each year remains positive for the case where income precedes cost (Figure 2). Hence, there is no investment or initial project cost involved in Figure 2. It is also clear that at the end of each year, the cumulative cash position in Figure 2 is adjusted for interest on the cash inflow plus whatever accrued interest there is at that point.

Therefore, the i calculated when income precedes cost has an entirely different meaning from rate of return. This i is called the rate of reinvestment. For decision-making purposes, this rate of reinvestment should be compared to a minimum rate of return which represents other opportunities for reinvestment of capital. In this case, however, a rate of reinvestment greater than the minimum ROR would be unsatisfactory. This means that if the rate of reinvestment is greater than the minimum ROR, the project would be unacceptable since alternative projects exist in which capital can be reinvested at an interest rate lower than that of the project under consideration.

CHAPTER III

DUAL RATES OF RETURN

Some authors refer to the situation where cost precedes income as a simple or a conventional type of project proposal (de Faro, 1974). A project proposal is of the simple type when there is only a single cash outflow, as in a lending proposal. Conventional refers to those with several cash outflows at the beginning of the project life. An example of the latter is that of a project with a construction phase lasting for several interest compounding periods.

There is another category of project proposals where the sequence of cash flows involve two or more sign reversals. These are called "non-conventional" projects (de Faro, 1974). A subgroup of this category which can be of special interest in mineral and petroleum projects is that which presents two reversals in sign. In other words, the time diagram shows a situation of alternating cost, income, cost. Income both follows and precedes cost. This situation can arise directly in many kinds of project proposals. In the mining industry, for example, land restoration costs or other types of reclamation costs are required at the end of the productive life of the mine. Another way the alternating cost-income-cost situation can occur is through acceleration proposals where incremental difference between unequal life alternatives are involved. Both of these occurrence of the alternating cost, income, cost situation will be examined in depth later in Chapter V of this thesis.

<u>Combination Rate of Return and Rate of Reinvestment</u>

Whenever income both follows and precedes cost, the interest rate i calculated has a combination of the rate of return meaning and the rate of reinvestment significance. Consider the following example with the time diagram shown below. Assume an initial cost of \$700, a uniform annual income of \$400 over 5 years and a final cost of \$1,400 at the end of year 6.

C=700	I=400	I=400	I=400	I=400	I=400	C=1,400
0	1	2	3	4	5	6

Present Worth Equation:

PW Income - PW Cost = 0

$$\left[\sum_{n=1}^{5} \frac{1}{(1+i)^n}(400)\right] - \left[700 + \frac{1}{(1+i)^6}(1400)\right] = 0$$

The present worth equation expressed as a polynomial function is:

$$f(x) = -700 + 400x + 400x^2 + 400x^3 + 400x^4 + 400x^5 -1,400x^6$$

where $x = 1/(1+i)$ and $f(x)$ is equated to zero in order to get the x roots. Descartes' Rule of Signs says in part that the number of real positive roots of a polynomial is less than or equal to the number of sign changes in the sequence of its coefficients (Thuesen, 1977). In this case, there are two sign changes. One sign change occurs after the initial negative value to positive at the end of year 1. The sequence of signs stays the same until the end of year 6 when the sign changes from positive to negative. Applying Descartes' Rule of Signs, the maximum number of real positive x roots is two.

A plot of the value of the polynomial function, $f(x)$ versus x is shown in Figure 3. It may be observed that $f(x)$ is zero when x is approximately 0.79 and 0.94. Thus, these are the two positive real roots for the polynomial function. The rates of return are consequently derived from the equation:

$$i = \frac{1-x}{x}$$

The computer program developed for this thesis, RORFOR, translates the real x roots to the corresponding rates of return. For the above example, the rates of return are 6.2 percent and 26.9 percent. These are commonly termed "dual rates of return." However, this term is misleading since both are not rates of return in the strict sense. To show this point graphically, the cumulative cash position diagrams for both i values are depicted in Figure 4.

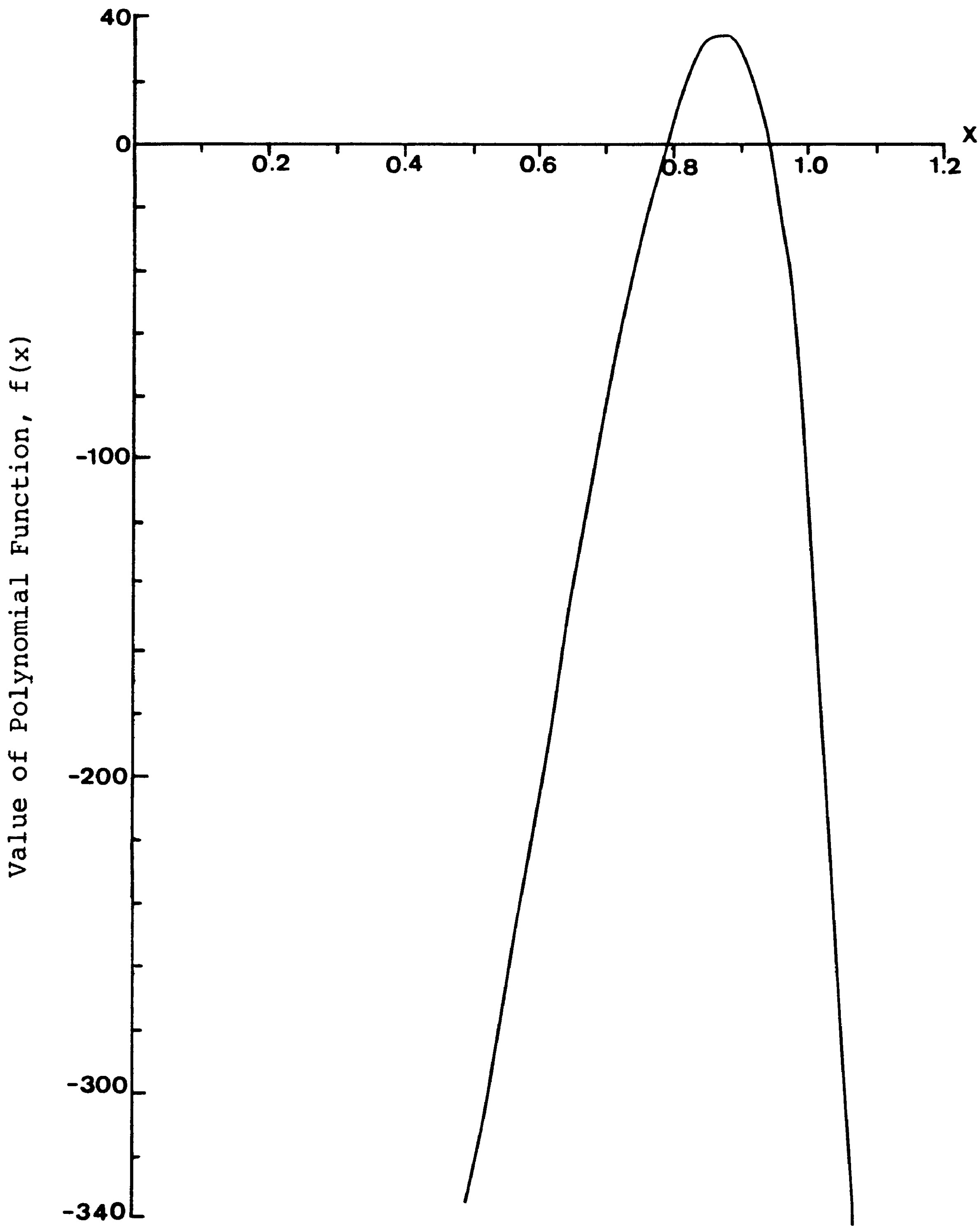

Figure 3. Value of Polynomial Function, f(X), versus X for Dual ROR Example.

The cumulative cash position diagram shown in Figure 4 clearly combines the characteristics of the cumulative cash position diagrams for Figures 1 and 2. As long as the cumulative cash position is negative, i takes on the rate of return meaning. On the other hand, a positive cumulative cash position denotes rate of reinvestment for that same i. It is in this situation where rate of return analysis encounters difficulties. Since these "dual rates of return" do not have any unique meaning, comparison of these interest rates with the minimum ROR for decision making becomes difficult. For the project to be considered satisfactory, should the dual rates of return be greater or less than the minimum ROR? The answer to this question is indefinite although these dual ROR's bracket the range of the minimum ROR for which a project is satisfactory (Stermole, 1980). It is for this reason that most authors of textbooks in engineering economy recommend the use of analysis methods other than ROR analysis for the alternating cost, income, cost situation.

Methods to Eliminate Dual Rates of Return

One method to eliminate dual rates of return is the "Present Worth Cost Modified ROR Analysis" (Stermole, 1982). This method transforms the alternating cost, income, cost

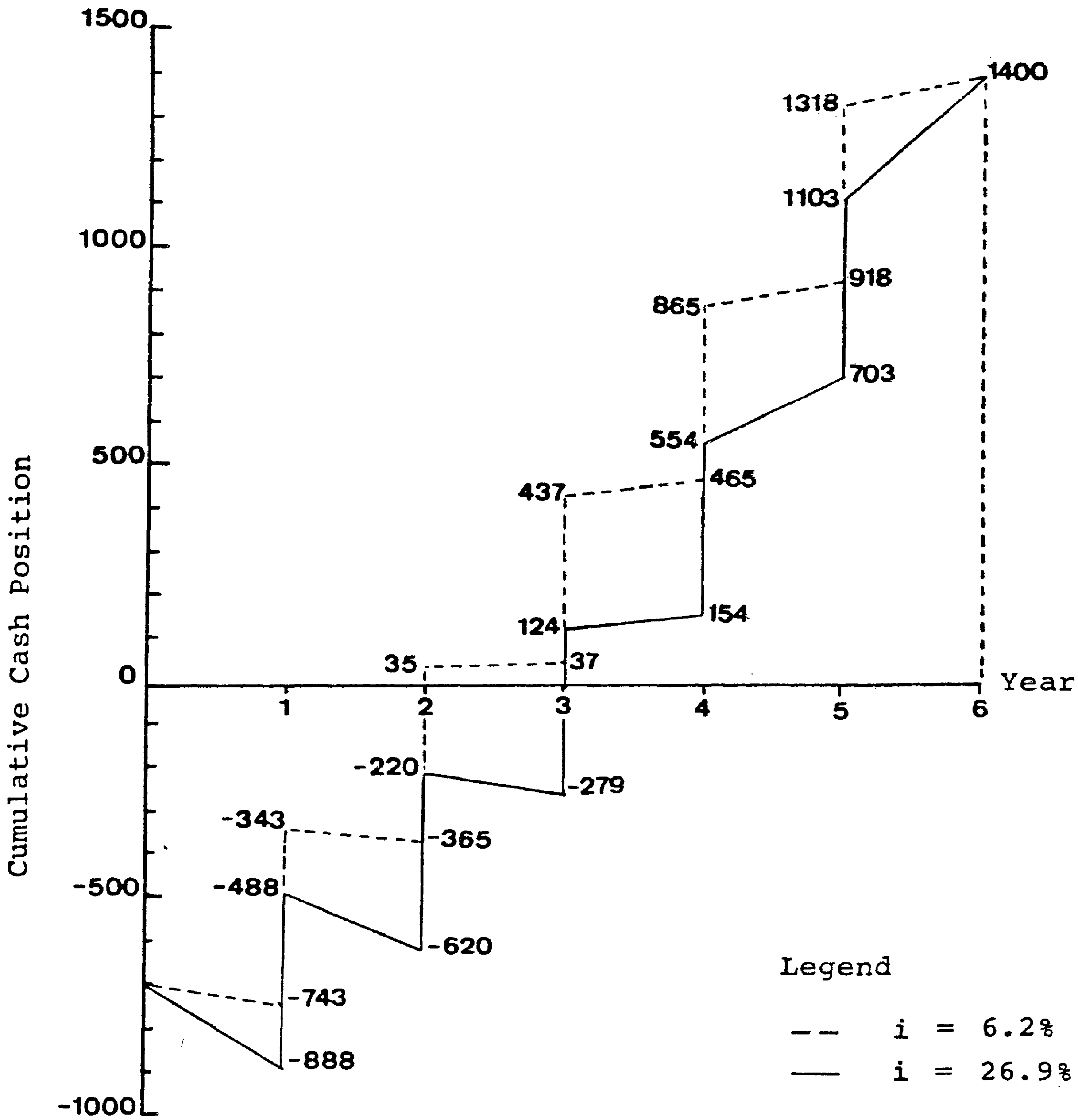

Figure 4. Cumulative Cash Position for Dual ROR's from Alternating Cost, Income, Cost.

situation to a simple or conventional type of project proposal by first present worthing the final cost at the minimum ROR. After which, the rate of return is calculated in the usual manner.

In the previous example for dual rates of return, discounting the final cost of \$1,400 at an assumed minimum ROR of 15 percent yields the following time diagram.

$C=700+\frac{1}{(1+0.15)^6}(1{,}400)$	I=400	I=400	I=400	I=400	I=400	-
0	1	2	3	4	5	6

Now the time diagram above demonstrates a simple case of cost followed by income. The final cost term is cancelled as a result of present worthing it at the minimum ROR. Hence, calculation of the rate of return can be done in a straightforward way.

Present Worth Equation:

PW Income - PW Cost = 0

$$\left(\sum_{n=1}^{5}\frac{1}{(1+i)^n}(400)\right)-\left(700+\frac{1}{(1+0.15)^6}(1{,}400)\right)=0$$

Using the routine RORFOR, the PW Modified ROR = i = 16.2%

Another method which also transforms the alternating cost, income, cost situation to one where cost precedes income is the "Growth ROR Analysis" (Stermole, 1982). Another term for this method is "Wealth Growth Rate" (Berry 1972). Growth ROR Analysis uses the Present Worth Equation while the Wealth Growth Rate Method employes the Future Worth Equation. In this analysis method, the cash inflows or incomes are assumed to be reinvested at the minimum ROR. This modification of the cash inflows involves a future worthing of these incomes at the minimum ROR. The time diagram for the reinvestment of the cash inflows is shown below.

-	C=400	C=400	C=400	C=400	C=400	I=?
0	1	2	3	4	5	6

Future Worth Equation:

Future Worth Income = Future Worth Cost

$$I = \sum_{n=1}^{5} (1+i)^{n}(400)$$

$$= 3{,}101.5 \quad \text{for } i = 15\%$$

where n is the number of interest compounding periods to the future time reference point at period 6 and i equals the minimum ROR of 15 percent. In the above calculations, $(1+i)^n$ is called the "single payment compound amount factor." Each of the reinvested incomes is translated to a future single sum of money. Because the incomes were modified to a single future value, these income terms are cancelled and the time diagram for the same project proposal is now as follows:

C=700	-	-	-	-	-	I=3,101.5
0	1	2	3	4	5	6 C=1,400

Present Worth Equation:

PW Income - PW Cost = 0

$$\left(\frac{1}{(1+i)^6}(3{,}101.5)\right) - \left\{\left(\frac{1}{(1+i)^6}(1{,}400)\right) + 700\right\} = 0$$

$$\frac{1}{(1+i)^6} = 0.41140$$

$$1+i = \sqrt[6]{2.43071}$$

Growth ROR = i = 16.0%

Therefore, the rate of return using Growth ROR analysis is 16.0 percent. Obviously, the value chosen for the assumed minimum ROR would affect the calculations. The nature of these effects on Growth ROR analysis of the rate of return as well as on Present Worth Cost Modified ROR analysis will be studied in detail in Chapter IV of this thesis.

CHAPTER IV

PROPOSED VALID ROR METHOD FOR DEALING WITH DUAL RATES OF RETURN

It may be observed that the methods to eliminate dual rates of return require calculations using an assumed minimum ROR to modify either the final cost or the incomes. These modifications result in transforming the alternating cost, income, cost situation to a simple type of investment proposal where cost is followed by income.

Since the above transformation is the ultimate objective of the aforementioned methods, it would seem that there is some magnitude of the final cost such that its effect on the rate of return meaning will be insignificant. In other words, given a certain initial cost and income stream, how small should the final cost be for it to be considered insignificant? Furthermore, if the final cost can be ignored, then a rate of return can be calculated in the usual manner for the situation where cost precedes income. The rate of return calculated will therefore be valid for economic evaluation. The foregoing argument would provide a procedure for dealing with dual rates of return without the use of a minimum ROR in the calculation of the rate of return. Another advantage would be the simplicity and ease of such a procedure.

A Sensitivity Analysis for the Case of Cost, Uniform Income, Cost Situation

Using the example for the dual rate of return situation, a sensitivity analysis was conducted on the effect of the magnitude of the final cost on the rate of return. To recapitulate, the example assumes an initial cost of $700, a uniform annual income of $400 for 5 years and a final cost of $1,400. The time diagram is shown below.

$C_0 = 700$	$I=400$	$I=400$	$I=400$	$I=400$	$I=400$	$C_F = 1,400$
0	1	2	3	4	5	6

The final cost, C_F, is varied and the computer program RORFOR is used to calculate rate of return. The resulting dual rates of return for different C_F values are listed in Table I.

TABLE I
DUAL RATES OF RETURN

Project	Final Cost, CF (dollars)	Dual Rates of Return (percent)	
A	1,800	-*	-*
B	1,400	6.2	26.9
C	1,300	0.0	30.8
D	1,000	-14.8	37.7
E	600	-35.2	43.5
F	500	-41.3	44.6
G	200	-66.4	47.7

* Complex Roots

When the final cost is zero, the project proposal becomes a simple type of project with cost preceding income. The routine RORFOR calculates a single rate of return of 49.5 percent. Let this investment situation be labeled Project Z.

Graphical analysis of the foregoing results was also conducted. Since the present worth equation can be expressed as a polynomial, a plot of the Net Present Value (PW Income - PW Cost) as a function of interest rate can reveal substantial information. The computer program NPVFOR was developed in order to calculate the Net Present Value for interest rates ranging from -98 to 100 percent in increments of 2 percent. A plot for each of the cash flow streams with different final costs is shown in Figure 5. It is clear that the curves intersect the horizontal axis when net present value is zero. The value of interest rate at this intercept is the rate of return.

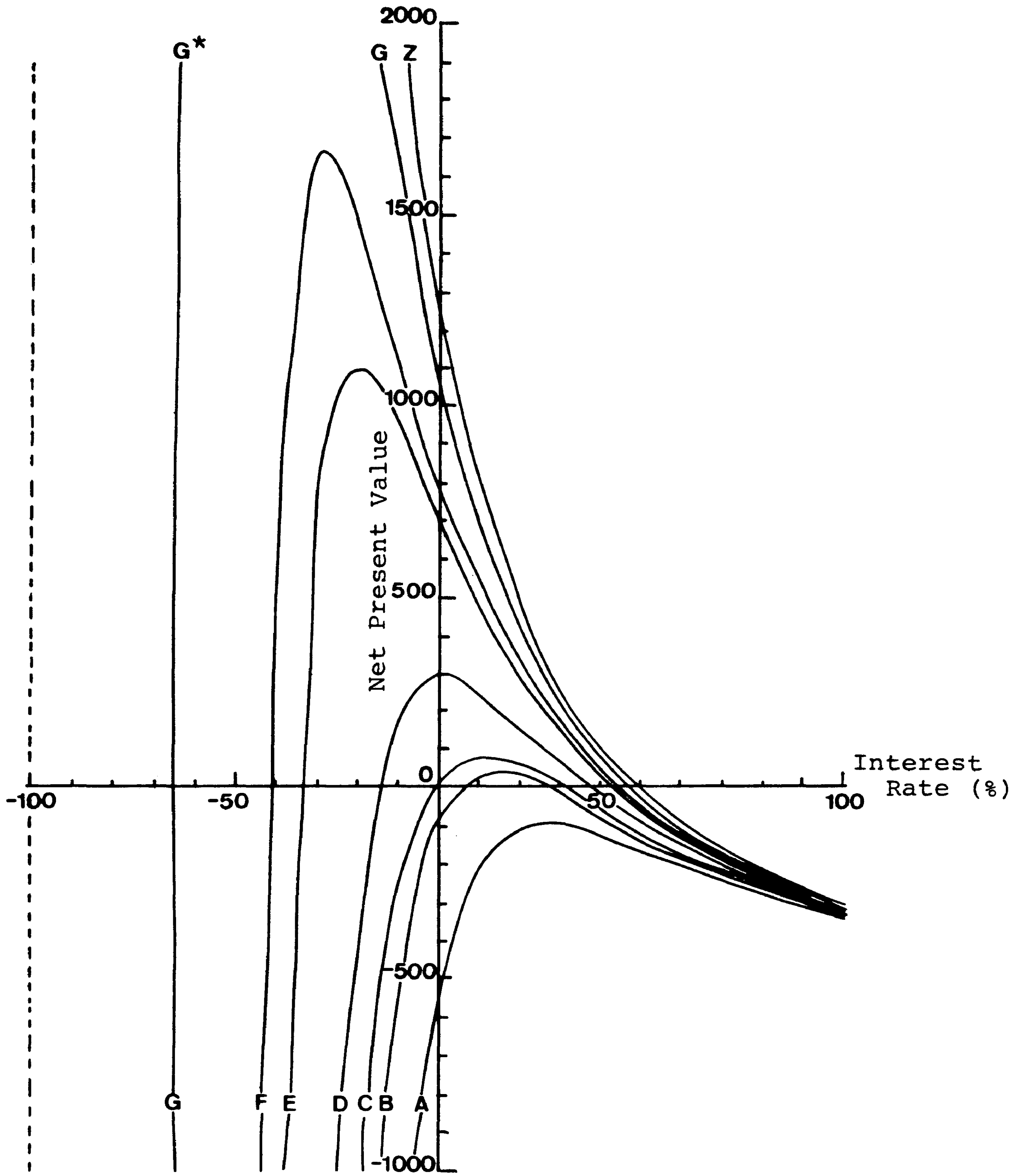

Figure 5. Net Present Value as a Function of Interest Rate.

* Note: Break in plot of G due to scale of the graph.

Examination of the curves reveal that all the investment situations with a final cost term, Projects A-G, yield plots which are concave upward. All are assymptotic to a vertical line passing through i equal to -100%. This is understandable since the single payment present worth factor, $1/(1+i)^n$, is equal to infinity for i equal to -100%. The plot for the investment situation with a zero final cost, Project Z, is also assymptotic to i equal to -100%. However, this curve representing a simple type of project proposal (with cost preceding income) is monotonically decreasing. The curve for Project Z is of course a limiting case since the final cost term disappears.

It can also be observed from the curves that the plots of Projects A to G get progressively more concave and shift upwards. Some positive values for Net Present Value cannot even be accommodated in the graph. It is, however, noteworthy that the curves for Projects E to G appear to be closer to the curve for Project Z than those for Projects A to D. Looking at the dual rates of return for Projects E to G, the maximum positive dual rate of return for these projects are within approximately 10 percent of the rate of return for Project Z.

Graphical analysis using cumulative cash position diagrams was also done for this sensitivity analysis. The cumulative cash position diagrams for Projects B to G are shown in Figure 6. Only the diagrams for the maximum dual ROR are plotted. Looking at the diagram for Project F where final cost is equal to $500, it may be observed that the reinvestment region covers only the last period or the sixth year. It is clear that the contribution of reinvestment applies only to part of the last cash inflow. The reinvestment region in the diagram for Project G is even smaller. Hence, cases where the final cost is less than $500 show a relatively insignificant contribution attributable to reinvestment. Again, Project G which has the final cost of $500 has a maximum dual rate of return within 10 percent of the rate of return for Project Z where the final cost is zero. Project G has a maximum dual rate of return equal to 44.6 percent while Project Z has rate of return of 49.5 percent.

A deviation of 10 percent from the rate of return when final cost is zero appears to be an insignificant departure. This deviation applies to the maximum positive dual rate of return of an investment situation with alternating cost, income, cost. This 10 percent deviation is arbitrarily chosen as guideline to determine when the final cost in a dual rate of return situation can be considered insignifi-

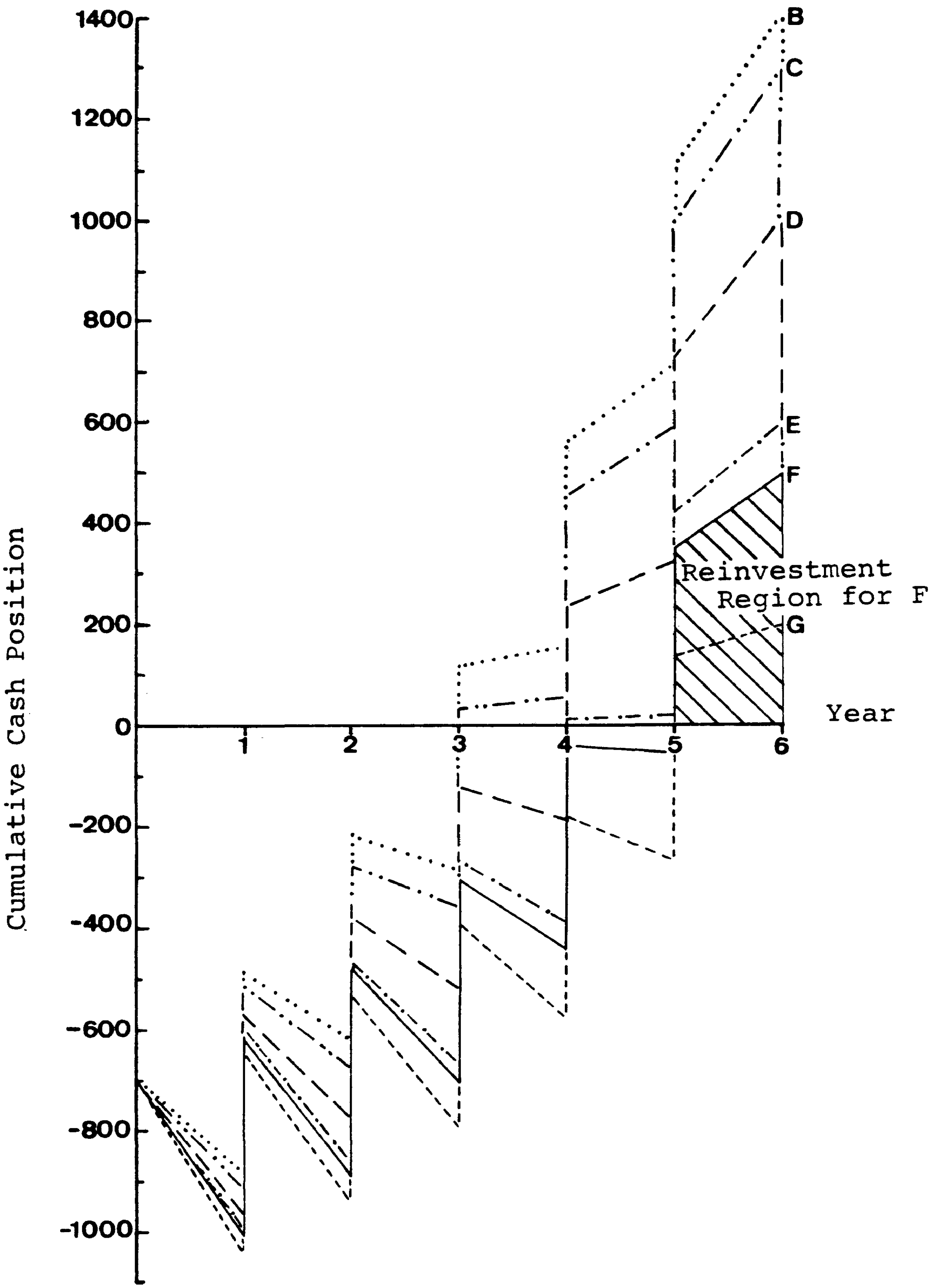

Figure 6. Cumulative Cash Position Diagram for the Final Cost Sensitivity Analysis.

cant. If the final cost is dropped, the project proposal becomes a simple type proposal where cost precedes income. The rate of return calculated for a simple type of proposal is of course valid for economic evaluation. This proposed method for dealing with dual rates of return can be called a "Valid ROR" method. Such a method is mainly empirical in nature and is not derived from any mathematical formulation.

<u>Runs Using the Valid ROR Method for the Case of Cost, Uniform Income, Cost Situation with Different Project Lives</u>.

Since the proposed Valid ROR method is an empirical technique, the method will have to be applied to different combinations of initial cost, income and final cost with different project lives in order for the method to provide meaningful guidelines for economic evaluation. Runs were conducted for project lives of 5, 10, 15 and 20 interest compounding periods. Because the combinations of initial cost, income and final cost are literally infinite, income per interest compounding period was assumed to be uniform. Also, combinations of initial cost and uniform income per interest compounding period were established such that ratios ranging from 1.0 to 20.0 were employed in the runs. Finally, the only step left is that of finding the magnitude of the final cost. The end result of the above procedure is expressed in terms of a ratio of final cost to initial cost.

To illustrate the foregoing procedure for one particular run, consider a project life of 5 periods and a ratio of initial cost to uniform income per period of 2.5. Assuming an initial cost of $1,000, the uniform income will be $400 over 4 periods and the unknown would be the final cost at period 5. The time diagram for this example is shown below.

	C_O=1000	I=400	I=400	I=400	I=400	C_F=?
Period	0	1	2	3	4	5

Simulating the calculations performed by the computer programs, the first computation to be done is that of determining the rate of return assuming a final cost of zero. This rate of return calculated by the routine RORFOR is 21.9 percent. A deviation of 10 percent from 21.9 percent yields 19.7 percent. Shifting now to the original problem, determine the final cost which will give a maximum dual ROR of 19.7 percent. A subroutine, VALID, called by the main program RORFOR performs all of these steps. The present worth equation for the above time diagram at i equal to 19.7 percent is:

PW Income = PW Cost

$$\sum_{n=1}^{4} \frac{1}{(1+0.197)^n} (400) = 1{,}000 + \frac{1}{(1+0.197)^5} (C_F)$$

$$C_F = 103$$

therefore,

$$\frac{C_F}{C_O} = \frac{103}{1,000} = 0.103 \approx 0.1$$

Hence, the result of this particular run is a final cost to initial cost ratio of 0.1. Results for all the runs made is given in Table II. Again, the output of each run is the ratio of final cost to initial cost.

It should be noted that when the ratio of initial cost to uniform income, C_O/I, is equal to the project life minus one, N-1, then the result for ratio of final cost to initial cost, C_F/C_O, is zero. For instance, when N is equal to 5, a value of C_O/I of 4.00 gives a C_F/C_O value of zero. This is understandable since the Valid ROR for the aforementioned combination of C_O/I and N is always zero. Valid ROR refers to the rate of return assuming the final cost is zero. The maximum dual rate of return within 10 percent of the Valid ROR is also zero. Hence, the ratio of the final cost to initial cost, C_F/C_O, would also end up to be zero. Also, any C_O/I ratio greater than the project life minus one gives negative ROR values.

To examine the ramifications of these results, a plot of C_F/C_O as a function of C_O/I was made for project lives of 5, 10, 15, and 20. These curves are shown in

TABLE II

FINAL COST TO INITIAL COST RATIOS RESULTING IN A MAXIMUM DUAL ROR WITHIN 10 PERCENT OF THE ROR WHEN FINAL COST IS ZERO (UNIFORM INCOME SITUATION)

Ratio of Initial Cost to Uniform Income per Period, C_o/I	Ratio of Final Cost to Initial Cost, C_F/C_o, for Different Project Lives, N			
	N = 5	N = 10	N = 15	N = 20
0.50	17.60	3,228.56	566,403.10	97,480,500.00
1.00	1.92	66.71	1,684.98	41,764.56
1.25	0.98	24.00	377.50	5,704.15
1.50	0.56	11.20	126.76	1,341.68
1.75	0.35	6.14	54.89	445.18
2.00	0.23	3.75	28.18	186.26
2.25	0.15	2.47	16.30	91.78
2.50	0.10	1.72	10.28	50.96
2.75	0.07	1.25	6.92	30.95
3.00	0.05	0.94	4.89	20.15

TABLE II, Continued

Ratio of Initial Cost to Uniform Income per Period, C_o/I	Ratio of Final Cost to Initial Cost, C_F/C_o, for Different Project Lives, N			
	N = 5	N = 10	N = 15	N = 20
3.50	0.02	0.56	2.73	9.93
4.00	0.00	0.36	1.69	5.65
4.50	-	0.24	1.12	3.54
5.00	-	0.17	0.78	2.38
5.50	-	0.11	0.57	1.68
6.00	-	0.08	0.42	1.23
6.50	-	0.06	0.32	0.93
7.00	-	0.04	0.25	0.72
7.50	-	0.02	0.19	0.57
8.00	-	0.01	0.15	0.46
8.50	-	0.01	0.12	0.37
9.00	-	0.00	0.10	0.30

TABLE II, Continued

Ratio of Initial Cost to Uniform Income per Period, C_o/I	Ratio of Final Cost to Initial Cost, C_F/C_o, for Different Project Lives, N			
	N = 5	N = 10	N = 15	N = 20
9.50	-	-	0.08	0.25
10.00	-	-	0.06	0.21
10.50	-	-	0.05	0.18
11.00	-	-	0.04	0.15
11.50	-	-	0.03	0.12
12.00	-	-	0.02	0.10
12.50	-	-	0.01	0.09
13.00	-	-	0.01	0.07
13.50	-	-	0.00	0.06
14.00	-	-	0.00	0.05
14.50	-	-	-	0.04
15.00	-	-	-	0.04
15.50	-	-	-	0.03

TABLE II, Continued

Ratio of Initial Cost to Uniform Income per Period, C_o/I	Ratio of Final Cost to Initial Cost, C_F/C_o, for Different Project Lives, N			
	N = 5	N = 10	N = 15	N = 20
16.00	-	-	-	0.02
16.50	-	-	-	0.02
17.00	-	-	-	0.01
17.50	-	-	-	0.01
18.00	-	-	-	0.01
18.50	-	-	-	0.00
19.00	-	-	-	0.00

Figures 7, 8, 9, and 10 respectively. The left hand portion of the curves had to be truncated in order to show more detail. The cut-off point was a C_O/I ratio which gave an ROR of less than 50 percent when final cost is zero. Each of these curves can provide a quick way of determining the final cost where the Valid ROR Method would be applicable. For example, given a project proposal with a project life of 15, an initial cost of \$100,000 and a uniform income of \$25,000 for 14 periods. The ratio of initial cost to uniform income per period is \$100,000/\$25,000 which is equal to 4. Looking up Figure 9, a C_F/I value of 4 gives a C_F/C_O value of 1.7. Therefore, the final cost should be \$42,500 or less. In other words, for the given alternating cost, income, cost situation, the rate of return of 23.7 percent when final cost is assumed to be zero can be used as the ROR for economic evaluation as long as the actual final cost is less than or equal to \$42,500.

A summary graph of the plots in Figures 7, 8, 9, and 10 is given in Figure 11. There are several things that can be observed from this summary. First of all, the longer the project, the less significant the final cost is. Therefore, a longer project life allows for a larger final cost in general. For example, the final cost to the initial cost ratios are 0.02, 0.56, 2.73 and 9.93 corresponding to project lives of 5, 10, 15, and 20 respectively for an

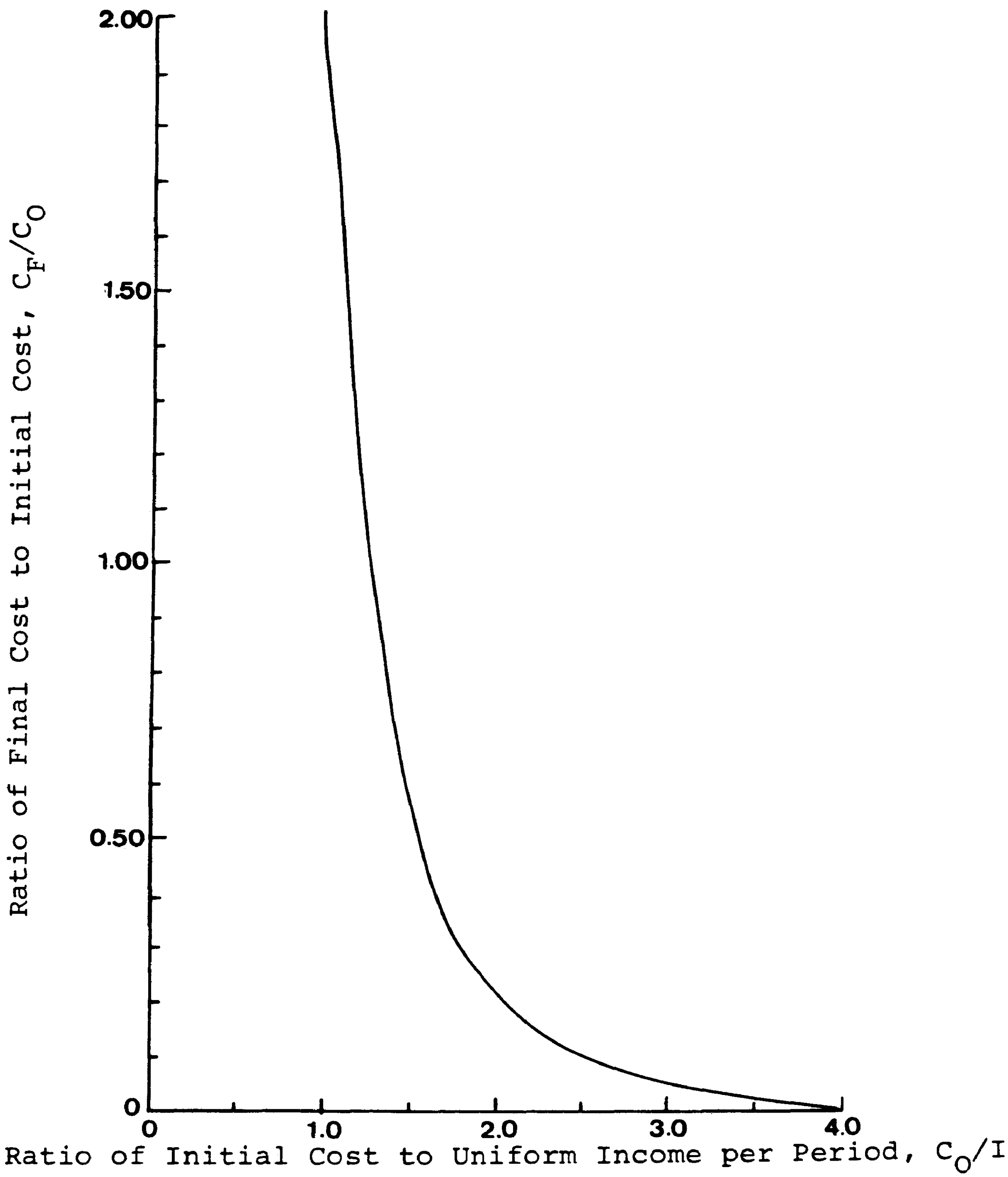

Figure 7. Ratio of Final Cost to Initial Cost (which Yields a Maximum Dual ROR within 10 Percent of the ROR when Final Cost is Zero) as a Function of the Ratio of Initial Cost to Uniform Income per Period for Project Life of N = 5.

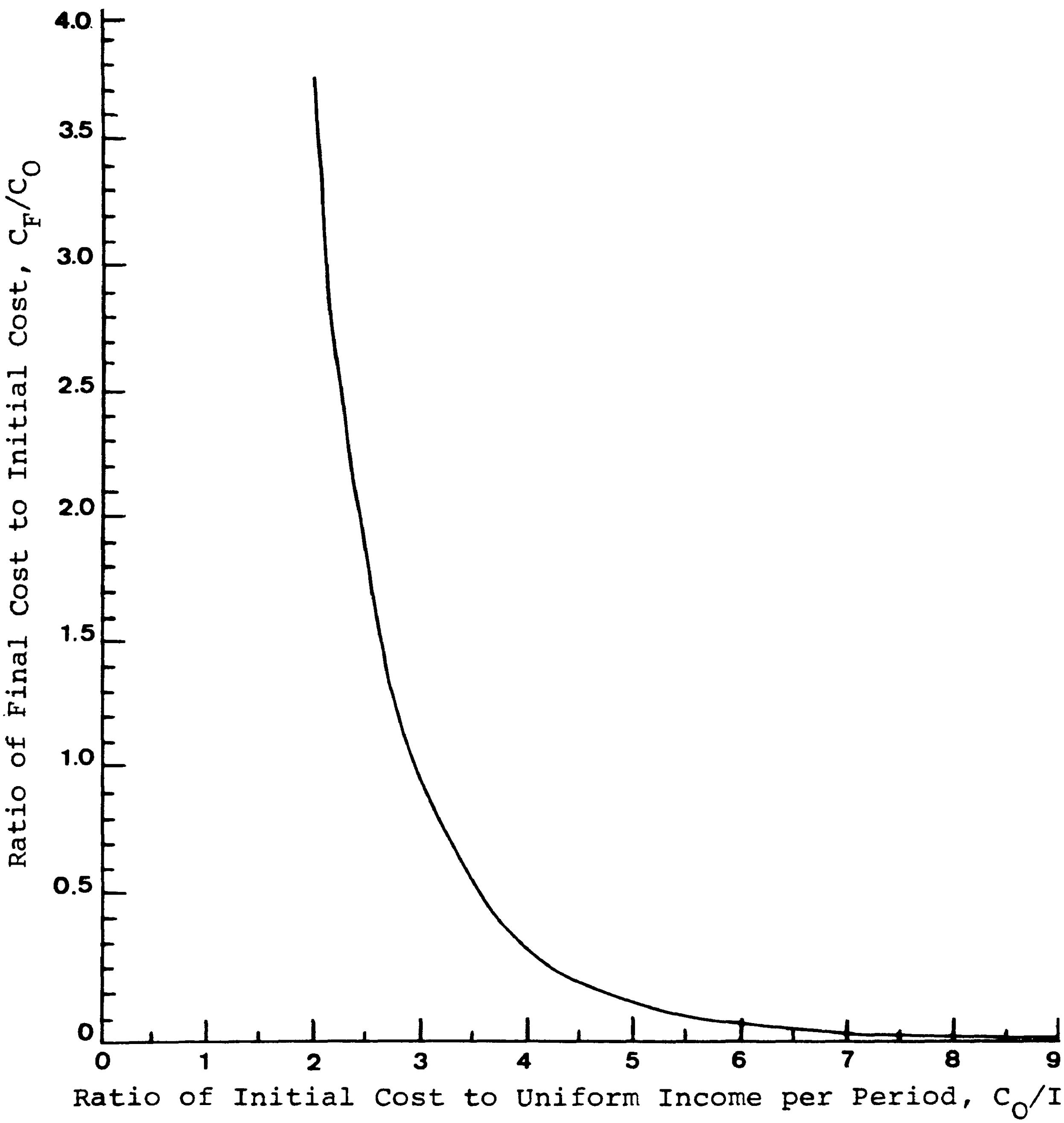

Figure 8. Ratio of Final Cost to Initial Cost (which Yields a Maximum Dual ROR within 10 Percent of the ROR when Final Cost is Zero) as a Function of the Ratio of Initial Cost to Uniform Income per Period for Project Life of N = 10.

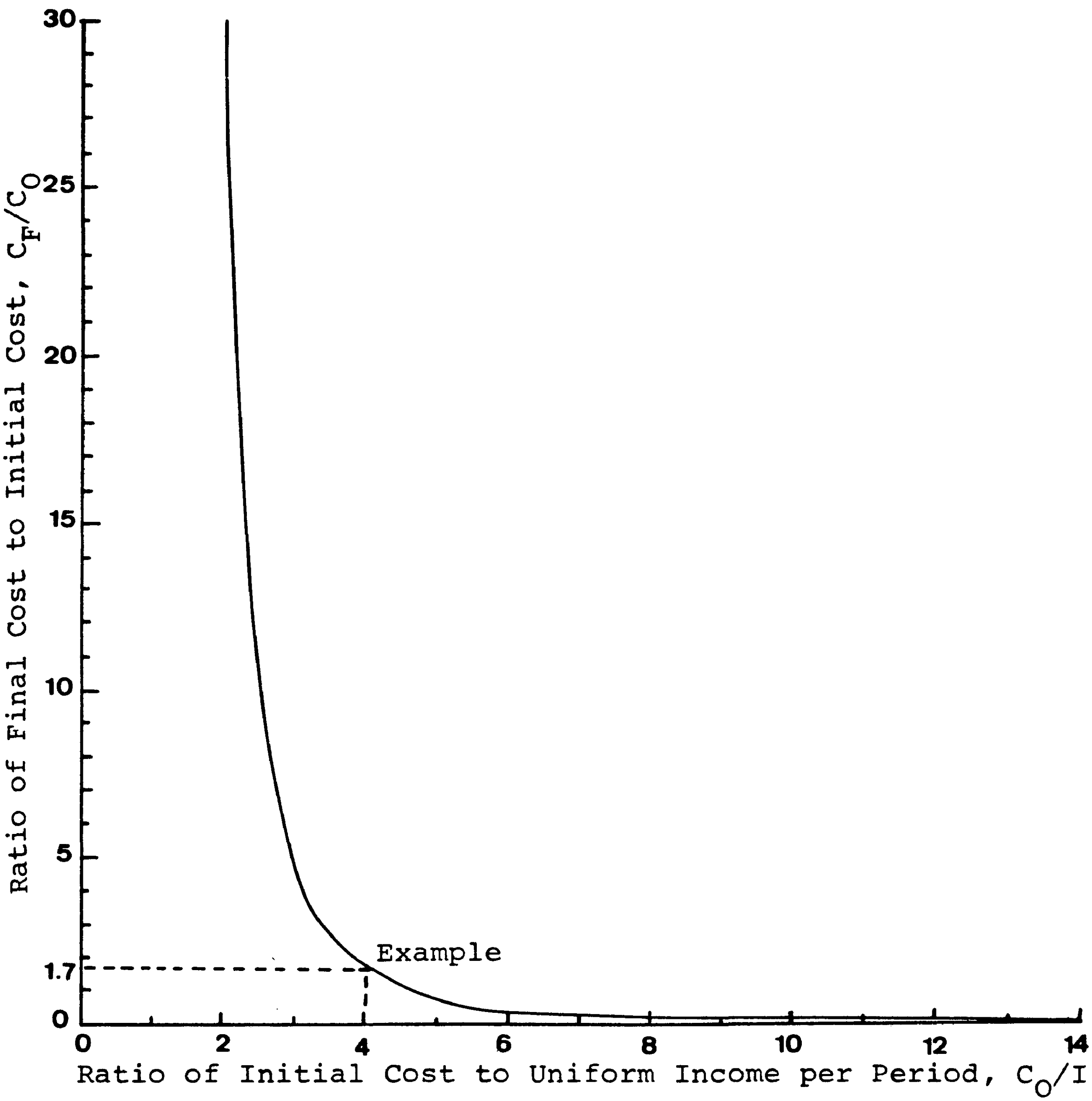

Figure 9. Ratio of Final Cost to Initial Cost (which Yields a Maximum Dual ROR within 10 Percent of the ROR when Final Cost is Zero) as a Function of the Ratio of Initial Cost to Uniform Income per Period for Project Life of N = 15.

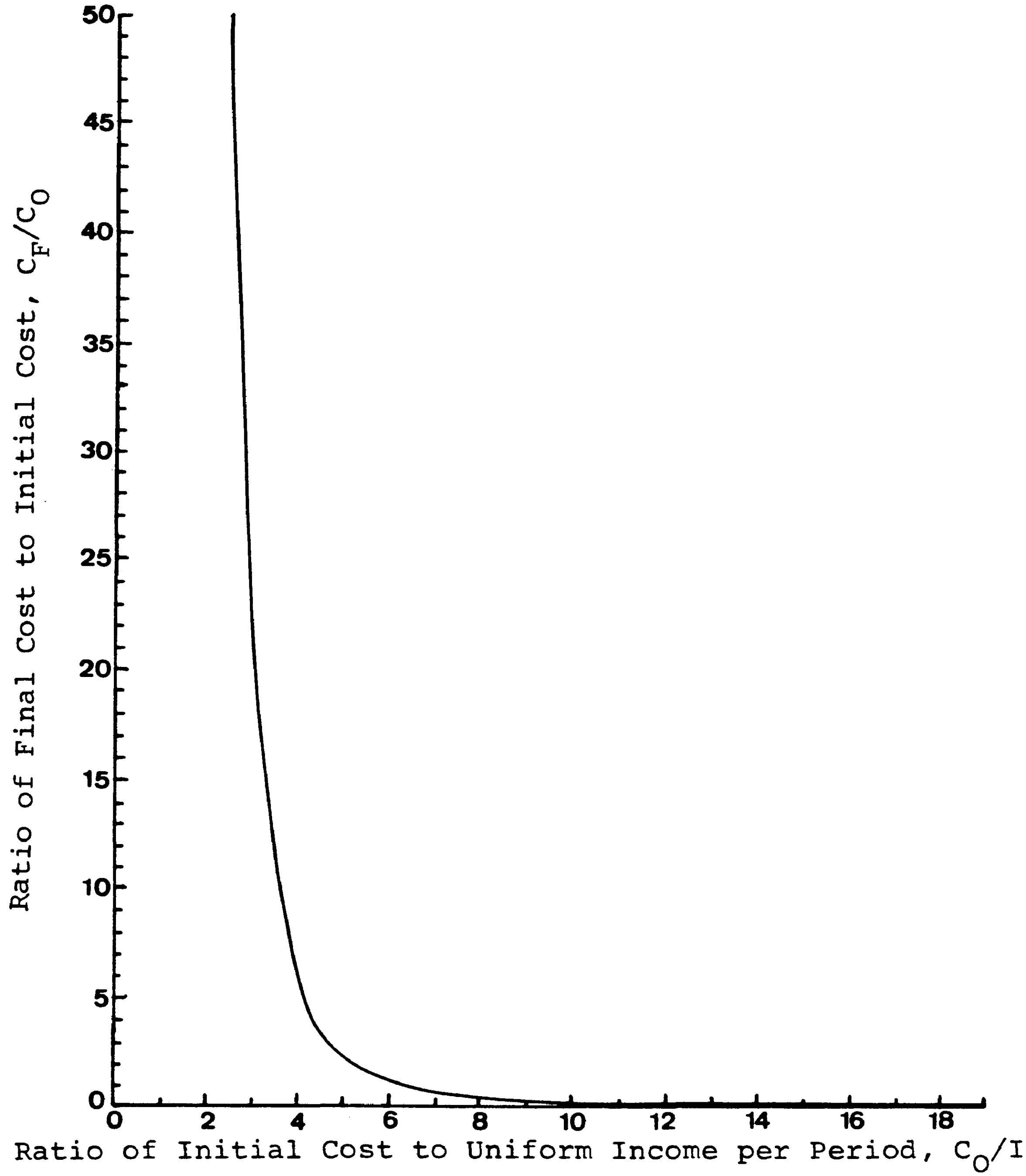

Figure 10. Ratio of Final Cost to Initial Cost (which Yields a Maximum Dual ROR within 10 Percent of the ROR when Final Cost is Zero) as a Function of the Ratio of Initial Cost to Uniform Income per Period for Project Life of N = 20.

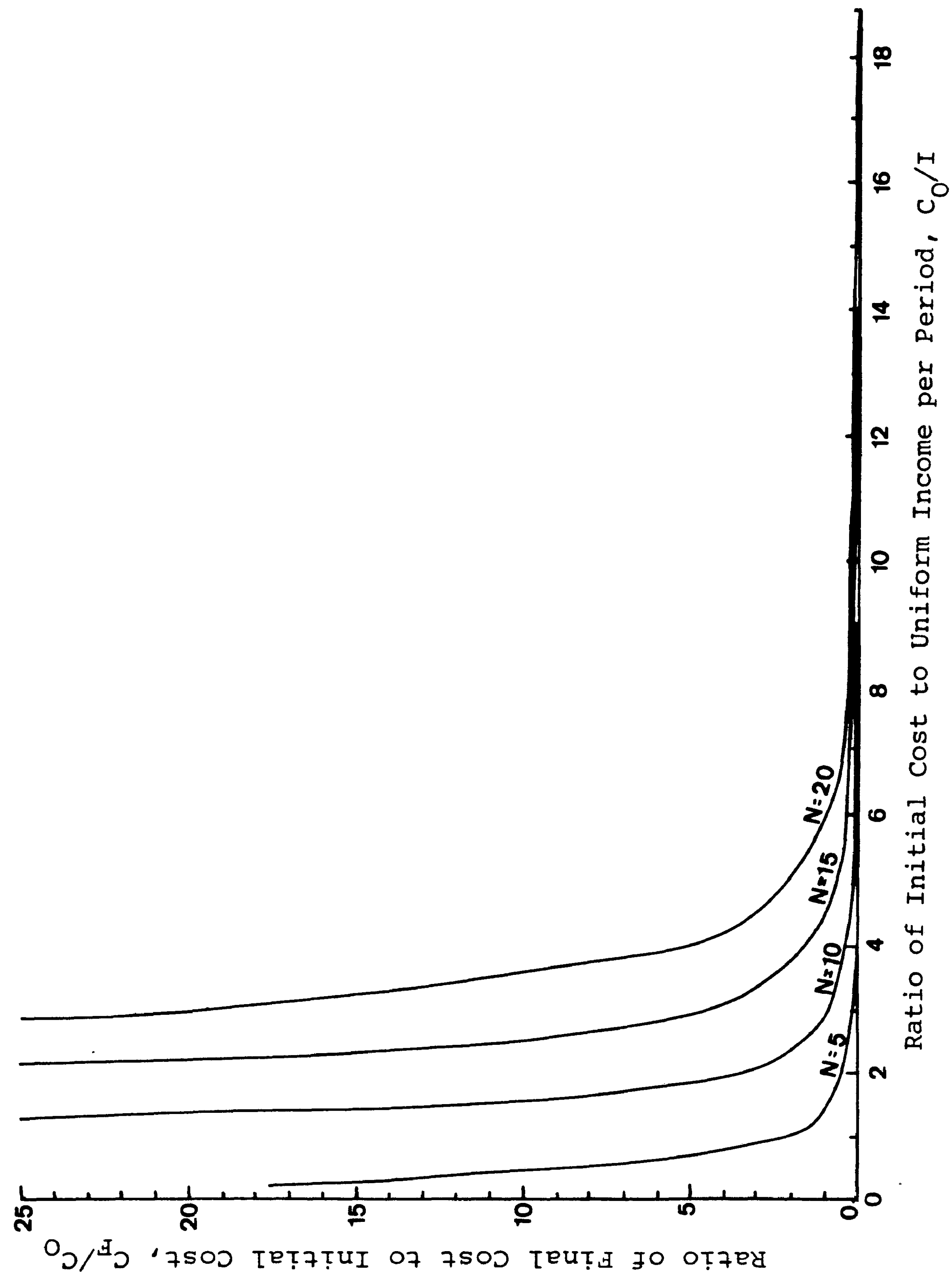

Figure 11. Summary Graph.

initial cost to income ratio of 3.5. Also, the curves dampen out drastically and tend to be assymptotic to the horizontal axis. Values of the ratio of final cost to initial cost are less than one over a larger range of C_o/I values for longer life projects. For N equals 20, the ratio of final cost to initial cost is less than one for C_o/I values of 6 to 19.

To compress all of the above results into a single graph, Appendix C shows the same output but with the horizontal axis being project life. With this graph, final cost to initial cost ratios can be derived for project lives other than 5, 10, 15, and 20. Also, ratios of initial cost to uniform income per period can be interpolated between the curves.

Runs Using the Valid ROR Method for Cases with Non-Uniform Income

What would happen to the final cost to initial cost ratio which gives a maximum dual ROR within 10 percent of the ROR when final cost is zero if the income stream was not uniform? It would seem that if income was greater at the start of a project, final cost can be larger than if the income stream was uniform.

To test the above hypothesis, runs were conducted for increasing as well as decreasing income streams with project lives of 5 and 10 interest compounding periods. An initial cost to average income ratio of 2.5 was used for all four runs. Assuming an initial cost of \$1,000, the average income would be \$400.

The time diagrams for all four runs are as follows:

Project I

C_O=1,000	I=160	I=320	I=480	I=640	C_F=?
0	1	2	3	4	5

C_F/C_O= 0.09

Project J

C_O=1,000	I=640	I=480	I=320	I=160	C_F=?
0	1	2	3	4	5

C_F/C_O= 0.13

Project K

C_O=1,000	I=80	I=160	I=240	I=320	I=400
0	1	2	3	4	5

I=480	I=560	I=640	I=720	C_F=?
6	7	8	9	10

C_F/C_O= 0.84

Project L

C_O=1000	I=720	I=640	I=560	I=480	I=400
0	1	2	3	4	5

I=320	I=240	I=160	I=80	C_F=?
6	7	8	9	10

C_F/C_O= 5.83

Looking up Figures 7 and 8, the C_F/C_O ratios are 0.10 and 1.72 for 5 and 10-period project lives respectively when income is uniform per period at a C_O/I ratio of 2.5. A tabulation of the foregoing results is shown in Table III. Runs were also conducted for 15 and 20 periods and for an initial cost to average income ratio of 3.5. The results of these runs are also shown in Table III. In all of these runs, the gradient of the income stream is always equal to the initial income term for increasing income streams or the final income term for decreasing income streams.

It is clear from Table III that a decreasing income stream allows for a larger final cost to initial cost ratio than if the income stream was uniform. This becomes even more pronounced as project life lengthens. The opposite trend is true for an increasing income stream; a longer project life implies a vastly lower final cost to initial cost ratio. However, as the ratio of initial cost to average income approaches the magnitude of the project life, both increasing and decreasing income streams give ratios for final cost to initial cost closer to the ratio of final cost to initial cost for the uniform income situation. Thus, for a C_O/I value of 3.5, a project life of N equal to 5 gives identical C_F/C_O ratios.

TABLE III

FINAL COST TO INITIAL COST RATIOS RESULTING IN A MAXIMUM DUAL ROR WITHIN 10 PERCENT OF THE ROR WHEN FINAL COST IS ZERO (UNIFORM AND NON-UNIFORM CASE)

Ratio of Initial Cost to Average Income, C_o/I	Income Stream	Ratio of Final Cost to Initial Cost, C_F/C_o for Project Life, N			
		N=5	N=10	N=15	N=20
2.5	Uniform	0.10	1.72	10.30	51.00
	Increasing*	0.09	0.84	2.37	4.91
	Decreasing**	0.13	5.83	110.75	1834.57
3.5	Uniform	0.02	0.56	2.73	9.93
	Increasing*	0.02	0.36	1.08	2.20
	Decreasing**	0.02	1.24	14.38	136.62

* Strictly increasing income stream with gradient equal to initial income term.

** Strictly decreasing income with gradient equal to final income term.

Of course, actual income streams do not behave in strictly increasing or strictly decreasing fashions. The preceding runs were made for the sole purpose of determining the effect of a non-uniform trend in income. Since the effect of a decreasing income stream is exactly the opposite that of an increasing income stream, cash inflows which show, as they generally do, a fluctuation around or insignificant departure from a mean can be treated as being a uniform income situation. In conclusion, the Valid ROR Method can be used for the alternating cost, non-uniform income, cost situation as long as the foregoing considerations are recognized.

Comparison of Proposed Valid ROR Method with Other Methods of Dealing with Dual Rates of Return

In order to see how the Valid ROR method fares in comparison to the Present Worth Cost Modified ROR Analysis and the Growth ROR Method, a dual ROR situation was subjected to all three techniques. Consider the following dual ROR situation.

C_o=1000	I=400	I=400	I=400	I=400	C_F=?
0	1	2	3	4	5

The Valid ROR Method yields a C_F/C_o ratio of 0.013. Therefore, the final cost which gives a maximum dual ROR within 10 percent of the ROR when the final cost is assumed to be zero is \$103. The rate of return when final cost is assumed to be zero is 21.9 percent. Thus, when the final cost for the given alternating cost, income, cost situation is \$103 or less the Valid ROR Method gives a rate of return equal to 21.9 percent. The time diagram is now:

C_o=1000	I=400	I=400	I=400	I=400	C_F=103
0	1	2	3	4	5

As previously discussed, both the Present Worth Cost Modified ROR Analysis and the Growth ROR Method require the use of the Minimum ROR. In Growth ROR Analysis, the Minimum ROR is the rate at which the incomes are reinvested. On the other hand, present worthing of the final cost is done at the minimum ROR when the Present Worth Cost Modified ROR Analysis is applied. Both analysis methods are applied to the above example at three different levels of the minimum ROR. These are at 10 percent, 15 percent and 20 percent. Using the Present Worth Cost Modified Analysis, the time diagrams and the rate of return at these minimum ROR's are as follows:

<u>10 Percent Minimum ROR</u>

$C=1{,}000 + 103\left(\frac{1}{(1+0.10)^5}\right)$	I=400	I=400	I=400	I=400	-
0	1	2	3	4	5

Modified Cost Present Worth Equation:

PW Income - PW Cost = 0

$$\left(\sum_{n=1}^{4} \frac{1}{(1+i)^n}(400)\right) - \left\{1{,}000 + 103\left(\frac{1}{(1+0.10)^5}\right)\right\} = 0$$

PW Modified ROR = i = 18.58 percent

<u>15 Percent Minimum ROR</u>

$C=1{,}000 + 103\left(\frac{1}{(1+0.15)^5}\right)$	I=400	I=400	I=400	I=400	-
0	1	2	3	4	5

Modified Cost Present Worth Equation:

PW Income - PW Cost = 0

$$\left(\sum_{n=1}^{4} \frac{1}{(1+i)^n}(400)\right) - \left[1{,}000 + 103\left(\frac{1}{(1+0.15)^5}\right)\right] = 0$$

PW Modified ROR = i = 19.21 percent

<u>20 Percent Minimum ROR</u>

$$C=1{,}000 + 103\left(\frac{1}{(1+0.20)^5}\right)$$

C=1,000 + 103 $\left(\frac{1}{(1+0.20)^5}\right)$	I=400	I=400	I=400	I=400	-
0	1	2	3	4	5

Modified Cost Present Worth Equation:

PW Income - PW Cost = 0

$$\left(\sum_{n=1}^{4} \frac{1}{(1+i)^n}(400)\right) - \left[1{,}000 + 103\left(\frac{1}{(1+0.20)^5}\right)\right] = 0$$

PW Cost Modified ROR = i = 19.70 percent

As the reader may discern, a lower minimum ROR tends to pull the PW Modified ROR even lower. This is generally the effect of present worthing the final cost since the minimum ROR acts as a weighing factor. It may be observed that at 10 percent and 15 percent minimum ROR, the Present Worth Modified ROR's indicate a satisfactory project proposal since they are greater than the corresponding minimum ROR's. The Valid ROR is also greater than the minimum ROR's for either case. In the case of a minimum ROR of 20 percent, the PW Cost Modified ROR Analysis would imply an indifferent decision since the PW Modified ROR of 19.70 percent is almost equal to the minimum ROR of 20 percent. The Valid ROR, however, indicates a satisfactory project since it is greater than the minimum ROR of 20 percent. This Valid ROR of 21.86 percent suggests an accept decision but considering the margin of error of 10 percent for the Valid ROR method, the investor would still consider this investment situation as marginal.

Turning to Growth ROR Analysis, the time diagrams for reinvestment of incomes and the initial project plus reinvestment are shown as follows. Note that computation of Growth ROR is carried out in a mathematically explicit manner.

<u>10 Percent Minimum ROR</u>

Reinvest	-	C=400	C=400	C=400	C=400	I=?
Incomes	0	1	2	3	4	5

$$I = \left[\sum_{n=1}^{4} (1+0.10)^n (400) = 2{,}042\right]$$

Initial Project	C=1,000	-	-	-	-	Net	I = 2,042-103
Plus Reinvest	0	1	2	3	4	5	

Present Worth Equation:

PW Income = PW Cost

$$1{,}000 = (2{,}042-103)\left(\frac{1}{(1+i)^5}\right)$$

Growth ROR = i = 14.16 percent

<u>15 Percent Minimum ROR</u>

Reinvest	-	C=400	C=400	C=400	C=400	I=?
Incomes	0	1	2	3	4	5

$$I = \left[\sum_{n=1}^{4} (1+0.15)^n (400) = 2{,}297\right]$$

Initial Project	C=1,000	-	-	-	-	Net I = 2,297-103
Plus Reinvest	0	1	2	3	4	5

Present Worth Equation:

PW Income = PW Cost

$$1{,}000 = (2{,}297-103)\left(\frac{1}{(1+i)^5}\right)$$

Growth ROR = i = 17.02 percent

<u>20 Percent Minimum ROR</u>

Reinvest Incomes		C=400	C=400	C=400	C=400	I=?
	0	1	2	3	4	5

$$I = \sum_{n=1}^{4} (1+0.20)^n (400) = 2{,}577$$

Initial Project	C=1,000	-	-	-	-	Net I = 2,577-103
Plus Reinvest	0	1	2	3	4	5

Present Worth Equation:

PW Income = PW Cost

$$1{,}000 = (2{,}577-103)\left(\frac{1}{(1+i)^5}\right)$$

Growth ROR = i = 19.86 percent

The same conclusions as those for the Present Worth Cost Modified ROR Analysis can be drawn from the preceding results on Growth ROR. In addition, it should be noted that the minimum ROR in Growth Rate of Return Analysis tends to exert more weight than the minimum ROR in PW Modified ROR Analysis at lower values of the minimum ROR. Thus, the results are more exaggerated for the Growth ROR Analysis.

Overall, the proposed Valid ROR Method proves adequate and simpler than the other two methods under the specific conditions where the Valid ROR applies.

CHAPTER V

APPLICATIONS OF THE VALID ROR METHOD

Incremental Rate of Return Analysis

The most common occurrence of alternating cost, income, cost investment situations which give rise to dual rates of return is in acceleration type of project proposals. In minerals and petroleum related investments, this type of project proposal is the classical acceleration problem whereby a natural resource deposit can be extracted at different rates. The problem is to determine the rate of depletion which is optimal or that which will attain the maximum return on invested capital. Obviously, different rates of extraction will result in different project lives for the same natural resource deposit. Also, a shorter project life may require a bigger initial cost and will simultaneously yield larger period incomes. Thus, this situation can be classified as an evaluation of mutually exclusive investment alternatives with unequal lives. By mutually exclusive is meant the need to select only one among several alternatives.

In general, rate of return analysis of mutually exclusive investment alternatives requires more than just determining the rate of return for each alternative. Although each alternative's total investment rate of return will have to exceed the minimum ROR in order for the alternative to be considered satisfactory, selection of the one optimum project among those which are deemed satisfactory is not clear cut. The project with the highest total investment rate of return will not necessarily yield the maximum return on invested capital. This is because mutually exclusive investment alternatives almost always have varying initial capital requirements or costs. Thus, comparison of the total investment ROR's of the satisfactory projects cannot be made. Marginal analysis of the return on an additional dollar of investment capital must also be carried out. Every dollar increment in cost has to justify itself in comparison with the minimum ROR. In other words, it is taken for granted that any additional capital can be invested at the minimum rate of return (Stermole, 1982).

In mineral projects of the classical acceleration type problem, not only are incremental costs involved but also unequal project lives. To illustrate how mutually exclusive alternatives of this nature should be evaluated, data for a gold-bearing deposit collected by the United States Bureau of Mines was used (Bennett, 1970). The study by the U.S. Bureau of Mines (U.S.B.M.) dealt with sensitivity and probabilistic analysis methods and application of these methods to the gold-bearing deposit which was investigated by the U.S.B.M. (Ageton, 1969). The unbiased nature of the data from this study would preclude any suspicion of developing an investment scenario which would be particular to the use of the Valid ROR Method.

Resources were estimated at 24, 32, and 45 million tons at respectively lower grades. Assuming daily mine production rates of 7,000, 13,000, 19,600 and 29,400 tons, the resulting project lives would be 25, 13, 9 and 6 years. The U.S.B.M. study estimated capital investment (including working capital) and operating costs for different methods of mining and processing. However, this same study concluded that one of the surface mining methods was more flexible than the others and one of the processing methods was more economic. Consequently, the capital investment and operating costs for the best mining and processing methods were used in this thesis. The capital investment costs for the different project lives are shown in Table IV.

Since these costs were estimated at prices prevailing in the early 1970's, revenues and cash flows projected for that time was also used (Hart, 1971). The annual cash inflows from Year 1 to the last year of the productive life of the mine are also shown in Table IV. The working capital is recovered in the last year of the project and is therefore added to the cash flow for that year. Assume that the

TABLE IV

CAPITAL INVESTMENT, WORKING CAPITAL, AND ANNUAL CASH INFLOWS (dollars)

	Project Life (years)			
	25	13	9	6
Capital Investment Including Working Capital	10,637,900	16,866,700	22,563,800	31,940,000
Working Capital	758,900	1,182,600	1,687,500	2,663,800
Annual Cash Inflows	1,574,012	3,858,309	6,059,182	10,010,689

minimum ROR is 15%. The total investment rates of return for the individual alternatives are tabulated in Table V.

TABLE V

RATE OF RETURN

Project Life (Years)	Rate of Return (%)
25	14.0
13	21.1
9	22.9
6	22.7

From Table V, only the alternative with a project life of 25 years has a rate of return below the minimum ROR of 15%. Therefore, the 25-year project life is unsatisfactory and the choice for the optimum economic life is left among the other three alternatives of 13, 9, and 6-year project lives. As previously discussed, it would be erroneous to select the 9-year project life even if it has the highest rate of return without first undertaking incremental rate of return analysis.

Among the remaining three alternatives, the 13-year project life has the lowest level of investment. Consequently, incremental ROR analysis should begin with the 13-year project life versus the next higher level of investment, the 9-year project life. The time diagrams for these alternatives are shown below.

13-year Project Life

```
C=16,866,700 I=3,858,309  I=3,858,309    I=5,040,909
   0             1------------12                  13
```

9-Year Project Life

```
C=22,563,800    I=6,059,182    I=6,059,182  I=7,746,682
   0               1-------------8            9
```

Incremental Time Diagram (9-13)

```
C=5,697,100    I=2,200,873    I=2,200,873    I=3,888,373
   0             1-------------8                9
```

C=3,858,309	C=3,858,309	C=3,858,309	C=5,040,909
10	11	12	13

It should be noted that in the incremental time diagram, the increment of cost is accompanied by increments of income for years 1 to 9. However, for years 10 to 12, the income produced by the 13-year project life is income foregone by the 9-year project life so that they become, in effect, opportunity costs.

Now, to use the Valid ROR Method for incremental ROR analysis, the ratio of initial cost to average income per interest compounding period would have to be first calculated. The average income per year is $2,388,373 and the C_O/I ratio is then 2.38. Using Figure 8, the C_F/C_O ratio, to give a maximum dual ROR within 10 percent of the ROR when the final costs are assumed to be zero, is 2.2. Therefore, the final cost should be $12,533,620 or less in order for it to be considered insignificant. In this case, however, the incremental time diagram presents multiple final cost terms with a combined total of $16,615,836. Because the costs are dispersed over 4 years, it would seem logical that this total approximates the limit of $12,533,620 considering the time value of money. The Valid ROR is 37.0 percent while the dual rates of return are 1.2 percent and 32.5 percent. Thus, the maximum dual ROR is very close to within 10% of the Valid ROR or the ROR when the final costs are assumed to be zero. Because the Valid ROR is greater than the minimum ROR, the 9-year Project Life is indeed more satisfactory than the 13-year Project Life. In other words, the additional investment for the 9-year Project Life is justified. This result checks out with the Present Worth Cost Modified ROR of 20.6 percent computed at the minimum ROR. In fact, the Valid ROR might even be a better measure of profitability since the final costs can be considered insignificant.

Since the 9-year project life proved to be a satisfactory investment proposal, it should be subsequently compared to the next investment level, the 6-year project life. Again, the time diagrams for these project lives as well as the incremental time diagram is shown below.

<u>9-Year Project Life</u>

```
 C=22,563,800    I=6,059,182   I=6,059,182    I=7,746,682
      0                1-------------8              9
```

<u>6-Year Project Life</u>

```
C=31,940,900   I=10,010,689   I=10,010,689   I=12,618,489
      0                1--------------5            6
```

<u>Incremental Time Diagram (6-9)</u>

```
C=9,377,100   I=3,951,507   I=3,951,507   I=6,559,307
      0              1--------------5           6

C=6,059,182    C=6,059,182     C=7,746,682
  7                8               9
```

Once more, the cash flow sequence in the incremental time diagram should be checked to see whether the Valid ROR Method would apply. The average income per year is \$4,386,140 and C_O/I ratio is therefore 2.14. Looking up Appendix C, for N equal to 6 and C_O/I ratio of 2.14, the C_F/C_O ratio is approximately 1.0. The final cost should be \$9,377,100 or less in order for the Valid ROR Method to be applicable. Again, the incremental time diagram presents multiple final costs spread out over three years with a combined total of \$19,865,046. This total, however, is way above the required value of \$9,377,100. The Valid ROR is 37.4 percent while the dual rates of return are 9.2 percent and 20.5 percent. Obviously, the maximum dual ROR is not within 10% of the Valid ROR. Therefore, the Valid ROR Method cannot be used in this situation. Turning to the Present Worth Cost Modified ROR Analysis, the PW Modified ROR at 15 percent minimum ROR is 15.5 percent. Consequently, the extra investment for the 6-year project life is satisfactory. Overall, incremental ROR Analysis indicates that the 6-year project life is the most profitable level of investment.

A Surface Mine Example

Direct occurrence of the alternating cost, income, cost investment situation can manifest itself in a variety of ways. Although examples in other industries abound, one instance in mining project proposals is a surface mine investment that generates income followed by reclamation cost or costs, in general, for the purpose of restoring the mined area to its original condition. The latter is not an uncommon environmental requirement in the United States and other countries.

For this application, the Valid ROR Method takes a more direct role in that the method can detect whether the magnitude of the final cost or costs can be considered significant enough or not. Also, since projections of reclamation costs are invariably rough estimates, the Valid ROR Method can provide a guideline as to which reclamation plan, assuming there would be several, will pose a significant contribution to the investment scenario. However, since environmental costs are peculiar to a mine and its geographical location, the magnitude of the final cost which is proved to be significant would also be mine specific.

Consider the same study by the U.S.B.M. (Bennett, 1970) used in the incremental ROR analysis. Since the 6-year project life was found to be the most profitable level of investment, its cash flow sequence was used. To reiterate, the time diagram for the 6-year project life is as follows:

C_o=31,940,000	I=10,010,689	I=10,010,689	I=12,618,489
0	1----------------	----------------5	6

Assuming environmental regulations dictate the restoration of the open pit area to its approximate original state, how much would the total final costs for reclamation have to be such that the rate of return for the project without reclamation cost would not be substantially changed? It will be recalled that the rate of return for the 6-year project life is 22.7 percent. Appending reclamation cost after the last productive period of the mine would invariably result in dual rates of return. In addition, the maximum dual rate of return will be progressively lower than 22.7 percent as the reclamation cost increases. A maximum dual ROR within 10 percent or less of the 22.7 percent rate of return can be construed as a very insignificant change. This is, of course, the logic behind the Valid ROR Method.

Applying the Valid ROR Method to the 6-year project life, the ratio of initial cost to average income per interest compounding period turns out to be 3.1. Looking up Appendix C for N equal to 7 and C_o/I equal to 3.1, the ratio of final cost to initial cost to give a maximum dual ROR within 10 percent of the 22.7 percent rate of return is approximately 0.2. Hence, a reclamation cost of $6,388,180 or less would not substantially affect the project's rate of return. Reclamation plans should be weighed against such a limit.

CHAPTER VI

CONCLUSIONS

Rate of return analysis has been widely accepted as a tool for economic evaluation of investment proposals. It takes into account the time value of money and is easily understood. However, one of its shortcomings is its duality whenever the sequence of cash flows exhibits two sign reversals. Reversal in sign refers to a change from a disbursement to a revenue term or vice-versa. Whenever there are dual rates of return, rate of return analysis is quite vague and can be misleading.

Of special interest particularly in minerals and petroleum development type of project proposals is the alternating cost, income, cost situation. Dual rates of return for this situation can arise directly or indirectly as in incremental analysis. Without the reinvestment term or the final cost in this alternating cost, income, cost sequence of cash flows, it is clear that only a single rate of return is calculated. This rate of return when final cost is assumed to be zero is called the "Valid ROR" in this thesis. It was observed from sensitivity analysis that as the final cost approaches zero, the upper dual rate of return converges to the value of the Valid ROR. Therefore, if the final cost can be considered insignificant because the upper dual rate of return is in close proximity to the Valid ROR, then the final cost may be dropped and the Valid ROR used as the rate of return for the alternating investment, income, reinvestment situation.

The method which was developed, called the Valid ROR Method, provides a quick way of determining the magnitude of the reinvestment term so that the final cost can be deemed negligible. A 10 percent deviation of the maximum dual rate of return from the Valid ROR or the ROR when final cost is zero, is chosen as a reasonable criterion. Thus, if the final cost is of such a magnitude that the maximum dual rate of return is within 10 percent of the Valid ROR, then this final cost can be considered insignificant. The Valid ROR Method makes use of curves developed empirically to determine the limit for the reinvestment term in relation to both the initial investment and the average income.

Comparison of the Valid ROR Method with other methods to deal with dual rates of return shows the Valid ROR to be simpler and easier to use. Nevertheless, the Valid ROR Method is limited by applications which show a final cost term exceeding the limit set for the final cost to be considered insignificant. It is, however, very helpful in cases where a project proposal has yet to specify the limit for the final cost which would not substantially affect the project's rate of return without any reinvestment.

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APPENDIX A
Interactive Computer Program RORFOR and Subroutine Valid

```
C*********************************************************************
C
C
C             Main Program RORFOR:
C
C             This routine computes the rate of return given the
C             cash flows and the life of the project. It uses a
C             routine in the IMSL(International Mathematical and
C             Statistical Library) called ZPOLR.  ZPOLR computes
C             the zeros of a polynomial with real coefficients.
C             Hence, the Present Worth Equation must be expressed
C             as a polynomial with x=1/(1+i), where i is the
C             unknown ROR, and with the cash flows each interest
C             compounding period as the coefficients.
C
C
C*********************************************************************
C
C
C             Input Data Requirements:
C                   A = Real vector of length NDEG+1 containing
C                       the coefficients in decreasing powers of
C                       the variable X.
C
C                   NDEG = Degree of the polynomial which is the
C                          project life.
C                          Note: In the application of the Valid
C                                ROR Method for project proposals
C                                with alternating cost,income,cost,
C                                NDEG should be the period when
C                                income last occurs.
C
C                   OPTION = Option to perform Valid ROR Method
C                            with the value:
C                            0 if Valid ROR Method will not be
C                              performed
C                            1 otherwise.
C
C
C             Intermediate ouput for IMSL Routine ZPOLR:
C                    Z = Complex vector of length NDEG containing
C                        the zeros of the polynomial.
C
C                    IER = Error parameter with value of:
C                        129 when degree of polynomial is not
C                              0 < NDEG < 101
C                        130 when the leading coefficient is zero
C                        131 when there are fewer than NDEG zeros.
C
C
C             Output:
C                   ROOTS = Rate(s) of return in decimal form.
C
C
C*********************************************************************
```

```
C
      DIMENSION A(101),  ROOT(101)
      COMPLEX Z(100)
      WRITE(4,10)
 10   FORMAT(/' READ IN THE DEGREE OF THE POLYNOMIAL:I')
      READ(4,20) NDEG
 20   FORMAT(I)
      WRITE(4,30)
 30   FORMAT(/' READ IN COEF. IN ORDER OF DECREASING POWER:21F')
      NDEG1 = NDEG + 1
      READ(4,40) (A(I),I=1,NDEG1)
 40   FORMAT(21F)
      CALL ZPOLR(A,NDEG,Z,IER)
      WRITE(4,50) IER
 50   FORMAT(/' IER=',I)

      IF(IER.EQ.130) GO TO 60

      IF(IER.EQ.129) GO TO 60

 60   CONTINUE
      WRITE(4,70)
 70   FORMAT(/' THE REAL ROOTS (IN DECIMAL FORM) ARE:')
      DO 90 I=1,NDEG
      IF (AIMAG(Z(I)).NEQ.0.) GO TO 90
      IF (REAL(Z(I)).EQ.0.) GO TO 90
      ROOT(I) = ((1.0 / REAL(Z(I))) - 1.0)
      WRITE(4,80) ROOT(I)
 80   FORMAT(G)
 90   CONTINUE
C
C****************************************************************
C
C              Option to perform Valid ROR Method
C
C****************************************************************
C
      WRITE(4,100)
 100  FORMAT(/' READ IN OPTION')
      READ(4,110) OPTION
 110  FORMAT(I)
      IF(OPTION.EQ.0)  GO TO 120
      CALL VALID(A,NDEG,ROOT,ANSWER)
 120  CONTINUE
      STOP
      END
C
C
C****************************************************************
```

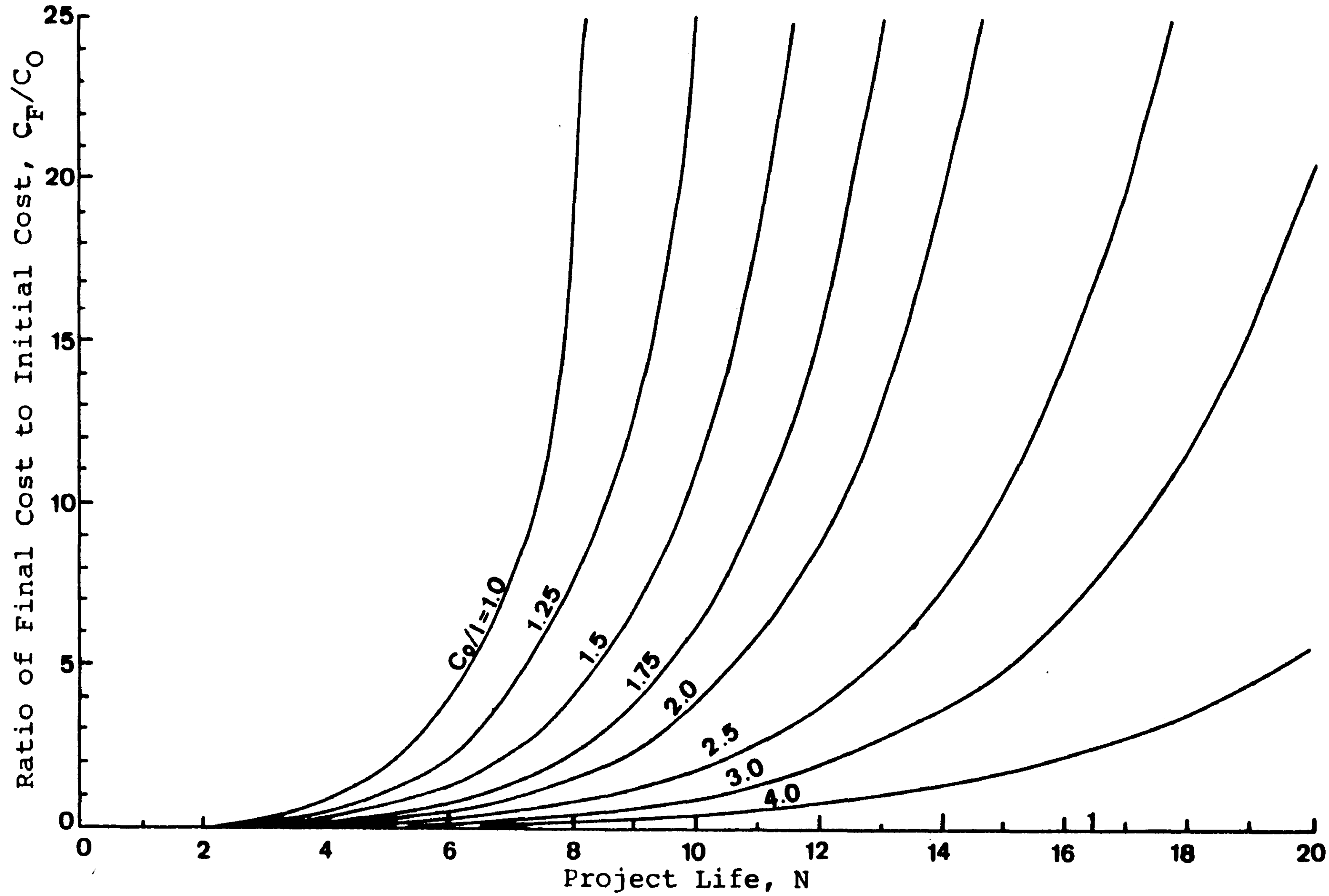
```
C*****************************************************************
C
C
C             Subroutine VALID:
C
C             This subroutine is an option called by the Main
C             program, RORFOR, if the Valid ROR Method is applied
C             to a project proposal with alternating cost,income,
C             cost.
C
C
C-----------------------------------------------------------------
C
C             Output:
C                   ANSWER = Ratio of final cost to initial cost
C                            to give a maximum dual ROR within
C                            10% of the ROR when the final cost is
C                            assumed to be zero.
C
C-----------------------------------------------------------------
C
      SUBROUTINE VALID(A,NDEG,ROOT,ANSWER)
      DIMENSION A(101),ROOT(101)
C
C-----------------------------------------------------------------
C
C     Determine the value of the ROR which is 10% less than the
C      maximum ROR in the array ROOTS (the Valid ROR).
C
C-----------------------------------------------------------------
C
      DO 10 J=2,NDEG
      RORMAX = AMAX1(ROOT(J),ROOT(J-1))
  10  CONTINUE
      REQROR = 0.5 * RORMAX
      WRITE(4,20) REQROR
  20  FORMAT(/' THE REQUIRED ROR WHICH IS 10% LESS THAN THE
     + ROR WHEN FINAL COST IS ASSUMED TO BE ZERO):'/,G)
C
C-----------------------------------------------------------------
C
C     Translate required ROR (REQROR) back to the polynomial
C     variable VARX and evaluate to determine the value of the
C     final cost to get REQROR.
C
C-----------------------------------------------------------------
C
      VARX = 1.0 / (1.0 + REQROR)

      ATOT = 0.
      P = 1.

      DO 30 J=NDEG,1,-1
      ATOT = ATOT + ( A(J) * (VARX**P))
      P = P + 1.
  30  CONTINUE
      NDEG1 = NDEG + 1
      VALUE = ( ABS(A(NDEG1)) - ATOT ) / (VARX ** NDEG1)
      ANSWER = VALUE/A(NDEG1)
      WRITE(4,40) ANSWER
  40  FORMAT(/' FINAL COST/INITIAL COST=',G)
      RETURN
      END
```

APENDIX B
Interactive Computer Program NPVFOR

```
C*****************************************************************
C
C
C              This routine calculates Net Present Value (Present
C              Worth Income - Present Worth Cost) for values of
C              interest rate from -99% to 100%.
C
C
C*****************************************************************
C
C
C              Input Data Requirements:
C
C                    N = Project life
C
C                    A = Real Vector of length N containing the
C                        cash flows from interest compounding
C                        period 0 to period N.
C
C
C              Output:
C
C                    INPV = A two dimensional array containing
C                           pairs of interest rate and the corres-
C                           ponding Net Present Value(NPV).  These
C                           are stored in device number one.
C
C
C*****************************************************************
C
      DIMENSION A(40)
      REAL NPV
      OPEN(UNIT=1,FILE='INPV.DAT')
      WRITE(4,10)
 10   FORMAT(/' INPUT LIFE OF PROJECT,N:I2')
      READ(4,11) N
 11   FORMAT(I2)
      WRITE(4,12)
 12   FORMAT(/' INPUT CASH FLOWS FROM PERIOD 0 TO PERIOD N; G')
      READ(4,13) (A(I),I=1,N+1)
 13   FORMAT(G)
      DO 16 IROR=-99,100,1
      NPV=0.0
      C=0.0
      ROR=IROR
      C = 1.0 / (1.0 + (ROR/100.0))
      NPV = A(N+1)
      DO 14 J=N,1,-1
      NPV = A(J) + (NPV * C)
 14   CONTINUE
      WRITE(1,15) ROR,NPV
 15   FORMAT(2G)
 16   CONTINUE
      CLOSE(UNIT=1, FILE='INPV.DAT')
      END
```



Appendix C. Ratio of Final Cost to Initial Cost as a Function of Project Life for Different Ratios of Initial Cost to Uniform Income to Get a Maximum Dual ROR within 10 Percent of the ROR when Final Cost is Zero.