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OPTIMUM HEAT-EXCHANGER DESIGN
BY GEOMETRIC PROGRAMMING

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A Thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science in Mathematics.

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ABSTRACT

The logarithmic mean temperature difference is a common factor in chemical engineering problems. It is also a difficult factor to work with. For this reason the arithmetic mean is often substituted for the log mean in hand calculations. For problems to be solved by geometric programming, approximating the logarithmic mean by the geometric mean is the optimal thing to do. Substituting the geometric mean will not only make the problem more straightforward, but will usually reduce the number of degrees of difficulty by one. The geometric mean is also twice as good an approximation of the logarithmic mean as the arithmetic mean.

Three problems are formulated as geometric programming problems. The first finds the optimal flow rate of cooling water in a condenser (from Peters and Timmerhaus, 1968, p. 308). The problem is fairly simple but getting the final answer is tedious work. Using the geometric mean and geometric programming makes the problem mechanical. Once programmed, it is only necessary to input two constants to get the optimum flow rate.

The second problem is optimum air-cooled heat-exchanger design (from Lohrisch, 1966). Again using geometric programming and the geometric mean simplifies the problem. Geometric programming also provides information not available from other

methods, giving bounds on the contributions toward cost. The cost of power for running the blower must contribute less than or equal 25.5 per cent of the total annual cost and the fixed charges must contribute at least 74.5 per cent of the total annual cost for the heat exchanger to be operating optimally.

The last problem is optimum design of a shell-and-tube heat exchanger (from Peters and Timmerhaus, 1968, p. 574). This is a difficult problem made straight-forward by the use of geometric programming and the geometric mean. Here too, geometric programming gives bounds on the contributions toward cost. The cost of pumping the fluid through the tubes must be less than or equal to 28.6 per cent. The cost of pumping the fluid through the shell and around the tubes must be less than or equal to 21 per cent of the total annual cost. The fixed charges on equipment and the cost of the utility fluid must contribute at least 50.4 per cent of the total annual cost if the heat-exchanger is to be operated at optimum.

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LOGARITHMIC MEAN TEMPERATURE DIFFERENCE

A common factor in chemical engineering problems is the logarithmic mean temperature difference. Because it is necessary to use an expansion to approximate the logarithm, the use of logarithms in geometric programming is inconvenient at the least. Substituting the arithmetic mean is a common practice for hand calculations. Substituting the geometric mean is an excellent way to simplify a problem to be solved by geometric programming.

Logarithmic Mean

To illustrate, find the logarithmic mean temperature difference between t_1 and t_2 . The log mean temperature difference is

$$\text{LMTD} = (t_1 - t_2) / \text{LOG}(t_1/t_2) \quad (1)$$

To put this in the proper form for geometric programming, two artificial constraints must be created. The first is to let

$$v_1 \geq t_1 - t_2 \quad (2)$$

To put this into the necessary form for geometric programming first add t_2 to both sides

$$v_1 + t_2 \geq t_1 \quad (3)$$

Now divide by the quantity on the left to get the desired form

$$\frac{t_1}{v_1 + t_2} \leq 1 \quad (4)$$

But the quantity must be approximated by the method of Avriel and Williams (1971), and solved by a series of successive approximations to the variable values. The approximation is

$$t_1 \left(\frac{\bar{v}_1 + \bar{t}_2}{\bar{v}_1} v_1 \right)^{\frac{-\bar{v}_1}{\bar{v}_1 + \bar{t}_2}} \left(\frac{\bar{v}_1 + \bar{t}_2}{\bar{t}_2} t_2 \right)^{\frac{-\bar{t}_2}{\bar{v}_1 + \bar{t}_2}} \leq 1 \quad (5)$$

where \bar{v}_1 and \bar{t}_2 are the values of the variables at the current approximation.

The expansion is not only messy, but also makes the problem more difficult to solve because successive iterations must be used. The expansion essentially eliminates the orthogonality constraint from being able to be used to get bounds on the contributions toward cost.

The second artificial constraint needed replaces the log term. Let

$$v_2 \leq \text{LOG}(t_1/t_2) \quad (6)$$

The log term is approximated by the expansion suggested by Duffin and others (1967, p 100)

$$V_2 \leq \epsilon^{-1} t_1^\epsilon t_2^{-\epsilon} \epsilon^{-1} \quad (7)$$

for ϵ small. To get this into the proper form add ϵ^{-1} to both sides giving

$$V_2 + \epsilon^{-1} \leq \epsilon^{-1} t_1^\epsilon t_2^{-\epsilon} \quad (8)$$

Multiply both sides by $\epsilon^{+1} t_1^{-\epsilon} t_2^\epsilon$ to get

$$V_2 \epsilon t_1^{-\epsilon} t_2^\epsilon + t_1^{-\epsilon} t_2^\epsilon \leq 1 \quad (9)$$

The trouble here is in evaluating ϵ . The expansion is supposed to use ϵ small, but how small? As ϵ gets very small, both $t_1^{-\epsilon}$ and t_2^ϵ become "...a uniform approximation to unity..." (Duffin and others, 1967, p 100). Note also that as ϵ goes to zero, the approximation also approaches zero. Here the orthogonality constraint involving ϵ is not very useful in finding bounds.

Hence the log mean temperature difference when formulated as a geometric programming problem is

$$\text{Min } C_t = \dots V_1/V_2 \dots \quad (10)$$

ST

$$t_1 \left(\frac{\bar{v}_1 + \bar{t}_2}{\bar{v}_1} v_1 \right)^{\frac{-\bar{v}_1}{\bar{v}_1 + \bar{t}_2}} \left(\frac{\bar{v}_1 + \bar{t}_2}{\bar{t}_2} t_2 \right)^{\frac{-\bar{t}_2}{\bar{v}_1 + \bar{t}_2}} \leq 1 \quad (5)$$

$$v_2^\epsilon t_1^{-\epsilon} t_2^\epsilon + t_1^{-\epsilon} t_2^\epsilon \leq 1 \quad (9)$$

Here there are four terms required. Assuming, as is true in the problems to follow, that t_2 is the variable, then the four terms and three variables (t_2, v_1, v_2) result in contributing one degree of difficulty to the total number of degrees of difficulty.

Because of the condensation factor, the ϵ in the orthogonality constraint, and the additional degree of difficulty, the use of the logarithmic mean temperature difference is undesirable.

Arithmetic Mean

The log mean temperature difference can be approximated by the arithmetic mean. The arithmetic mean temperature difference is

$$\text{AMTD} = (t_1 + t_2)/2 \quad (11)$$

To formulate this in a manner amenable to treatment by

geometric programming, replace the numerator by the artificial variable V_3 and add the constraint

$$V_3 \geq t_1 + t_2 \quad (12)$$

Dividing by V_3 gives the required form

$$t_1 V_3^{-1} + t_2 V_3^{-1} \leq 1 \quad (13)$$

Therefore the arithmetic mean temperature difference as formulated for geometric programming is

$$\min C_t = \dots V_3 / 2 \dots \quad (14)$$

$$\text{ST } t_1 V_3^{-1} + t_2 V_3^{-1} \leq 1 \quad (13)$$

There are three terms involved. If only t_2 and V_3 are variables, this formulation results in one degree of difficulty from the mean temperature difference term.

Geometric Mean

The geometric mean temperature difference can also be used to approximate the log mean temperature difference.

The geometric mean temperature difference is

$$\text{GMTD} = t_1^{1/2} t_2^{1/2} \quad (15)$$

This is already in the proper geometric programming form.
The problem would simply read

$$\text{Min } C_t = \dots t_1^{1/2} t_2^{1/2} \dots \quad (16)$$

Assuming that t_2 is the only variable, then the temperature difference has one term and one variable thereby contributing zero to the number of degrees of difficulty.

Inverse Logarithmic Mean

The situation is similar if the problem calls for the inverse of the logarithmic mean temperature difference as do the problems to follow. The inverse of the log mean temperature difference is

$$\text{LMTD}^{-1} = \text{LOG}(t_1/t_2)/(t_1 - t_2) \quad (17)$$

Again two artificial constraints must be employed. The easier of the two is to replace the denominator by V_4 and add the constraint

$$V_4 \leq t_1 - t_2 \quad (18)$$

To get the proper form add t_2 to both sides

$$V_4 + t_2 \leq t_1 \quad (19)$$

and divide by t_1

$$v_4 t_1^{-1} + t_2 t_1^{-1} \leq 1 \quad (20)$$

The other constraint involves replacing the log term by v_5 and adding the constraint

$$v_5 \geq \text{LOG}(t_1/t_2) \quad (21)$$

Again using the approximation of Duffin and others (1967, p 100), this becomes

$$v_5 \geq \epsilon^{-1} t_1^\epsilon t_2^{-\epsilon} - \epsilon^{-1} \quad (22)$$

for ϵ small. Add ϵ^{-1} to both sides

$$v_5 + \epsilon^{-1} \geq \epsilon^{-1} t_1^\epsilon t_2^{-\epsilon} \quad (25)$$

Multiply by ϵ to give

$$v_5 \epsilon + 1 \geq t_1^\epsilon t_2^{-\epsilon} \quad (24)$$

Divide by the left side and the proper geometric programming form resulting is

$$\frac{t_1^\epsilon t_2^{-\epsilon}}{v_5 \epsilon + 1} \leq 1 \quad (25)$$

This constraint must be condensed by the method of Avriel and Williams (1971). By this method the constraint becomes

$$t_1^\epsilon t_2^{-\epsilon} \left(\frac{\bar{v}_5^\epsilon + 1}{\bar{v}_5^\epsilon} v_5^\epsilon \right)^{\frac{-\bar{v}_5^\epsilon}{\bar{v}_5^\epsilon + 1}} \left(\frac{1}{\bar{v}_5^\epsilon + 1} \right)^{\frac{-1}{\bar{v}_5^\epsilon + 1}} \leq 1 \quad (26)$$

Note that the third factor is a constant.

Again because of ϵ in the orthogonality constraints and the necessity of taking a series of successive approximations to find the exponent of v_5 , this method is not very useful.

The entire formulation of the inverse of the logarithmic mean temperature difference is

$$\text{Min } C_t = \dots v_5 / v_4 \dots \quad (27)$$

$$\text{ST } v_4 t_1^{-1} + t_2 t_1^{-1} \leq 1 \quad (20)$$

$$t_1^\epsilon t_2^{-\epsilon} \left(\frac{\bar{v}_5^\epsilon + 1}{\bar{v}_5^\epsilon} v_5^\epsilon \right)^{\frac{-\bar{v}_5^\epsilon}{\bar{v}_5^\epsilon + 1}} \left(\frac{1}{\bar{v}_5^\epsilon + 1} \right)^{\frac{-1}{\bar{v}_5^\epsilon + 1}} \leq 1 \quad (26)$$

Again there are 4 terms and 3 variables meaning one degree of difficulty added to the problem.

Inverse Arithmetic Mean

Using the inverse of the arithmetic mean temperature difference is not very useful because Avriel and Williams' (1971) technique must be used. The inverse arithmetic mean is

$$\text{AMTD}^{-1} = 2/(t_1 + t_2) \quad (27)$$

One artificial constraint is needed. Replace the denominator by v_6 and add the constraint

$$v_6 \leq t_1 + t_2 \quad (28)$$

To put this into proper form, divide by the right-hand side

$$\frac{v_6}{t_1 + t_2} \leq 1 \quad (29)$$

Using the method of Avriel and Williams (1971) this becomes

$$v_6 \left(\frac{\bar{t}_1 + \bar{t}_2}{\bar{t}_1} t_1 \right)^{\frac{-\bar{t}_1}{\bar{t}_1 + \bar{t}_2}} \left(\frac{\bar{t}_1 + \bar{t}_2}{\bar{t}_2} t_2 \right)^{\frac{-\bar{t}_2}{\bar{t}_1 + \bar{t}_2}} \leq 1 \quad (30)$$

The second factor is a constant if t_1 is a constant.

Hence the total formulation for the inverse of the arithmetic mean temperature difference is

$$\text{Min } C_t = \dots 2/v_6 \dots \quad (31)$$

$$ST \quad v_6 \left(\frac{\bar{t}_1 + \bar{t}_2}{\bar{t}_1} t_1 \right)^{\frac{-\bar{t}_1}{\bar{t}_1 + \bar{t}_2}} \left(\frac{\bar{t}_1 + \bar{t}_2}{\bar{t}_2} t_2 \right)^{\frac{-\bar{t}_2}{\bar{t}_1 + \bar{t}_2}} \leq 1 \quad (30)$$

The inconvenience of Avriel and Williams' (1971) method is not compensated by the fact that there are two terms and two variables contributing zero to the degrees of difficulty.

Inverse Geometric Mean

Again approximating the inverse of the log mean temperature difference by the geometric mean temperature difference is the simplest method. The inverse of the geometric mean temperature difference is

$$GMTD^{-1} = t_1^{-1/2} t_2^{-1/2} \quad (32)$$

Again the term is already in the proper form for geometric programming and again the term contributes zero to the total number of degrees of difficulty.

For geometric programming, approximating the logarithmic mean temperature difference by the geometric mean temperature difference is the optimal thing to do.

Error

Peters and Timmerhaus (1968, p 537), who are more interested in using the arithmetic mean for hand solutions, state that the arithmetic mean is within 4 percent of the logarithmic mean and is within 6 percent of the geometric mean if the ratio of t_1 to t_2 does not exceed 2.0. They further state that this accuracy is adequate for most calculations.

They further state that the arithmetic mean is always greater than the logarithmic mean or the geometric mean. This point can be extended to the logarithmic mean always being greater than the geometric mean. This means that it is more nearly correct to replace the inverse of the logarithmic mean by the inverse of the geometric mean because the direction of the function will be reinforced (assuming it is in the proper form of a minimization problem or less than or equal to constraint).

My own research has shown that, for ratios greater than one, the geometric mean gives a closer approximation to the logarithmic mean than does the arithmetic mean. For the ratio of t_1 to t_2 less than 2.0, the arithmetic mean is within 4 percent of the log mean but the geometric mean is within 2 percent. The geometric mean is within 5 percent of the log mean when the ratio is less than 3. Since the ratios involved in the logarithmic mean tem.

perature difference are usually small, the error induced by using the geometric mean will be small.

It should be noted that it is better to have a ratio of 3 rather than $1/3$, if possible. The approximation is more accurate and the percentage error, because of the larger denominator, is smaller.

LOGARITHMS IN GEOMETRIC PROGRAMMING

The use of the approximation of the logarithmic mean by the geometric mean is, of course, not limited to temperature difference. Temperature difference was used as an example because the logarithmic mean temperature difference is a common factor in chemical engineering problems, as evidenced by the three problems that follow. Peters and Timmerhaus (1968, p 536) discuss the various means in the context of mean area. Whenever the mean is required in geometric programming, the geometric mean is the best one to use. Also, the geometric mean is not limited to two items, it can handle any number like the arithmetic mean. That is, after all, the basis for geometric programming.

The Approximation of Duffin and Others (1967)

The criticisms of the logarithmic expansion discussed under the context of the logarithmic mean will be expanded upon here. The expansion advocated by Duffin and others (1967, p 100) is

$$\text{LOG}(u(t)) = \epsilon^{-1}(u(t))^\epsilon - \epsilon^{-1} \quad (33)$$

for ϵ small.

The first question is how small an ϵ . Is there an optimal ϵ ? My own research has shown that 10^{-4} is a good value for ϵ . Within the accuracy of our computer,

the approximation will go to zero as ϵ gets very small, less than 10^{-10} . This approximation assumes that the value of the logarithm is zero. In Table I are the values calculated on the PDP-10 using the approximation of Duffin and others (1967, p 100).

TABLE I

Approximation of Log X

X	LOG X	ϵ	$\epsilon^{-1}X^\epsilon - \epsilon^{-1}$
1.5	0.405465108	0.01	0.4137974
1.5	0.405465108	10^{-4}	0.4055
1.5	0.405465108	10^{-8}	0.0
10.	2.30258509	0.01	2.5892541
10.	2.30258509	10^{-4}	2.3028
10.	2.30258509	10^{-8}	2.0
10.	2.30258509	10^{-10}	0.0

Having the value of ϵ in the orthogonality constraint is not helpful either. Even if ϵ is fixed at 10^{-4} , that number is so small it is difficult to use.

From the formulations of log mean temperature difference and inverse log mean temperature difference it is obvious that the log term is easier to handle if the log is in the

denominator. If the log term is in the numerator, the condensation procedure of Avriel and Williams (1971) must be used. This makes the orthogonality constraint for the artificial variable of no help in trying to find bounds on the contributions towards cost. Together with the impracticality of ϵ in another orthogonality constraint, the use of the method of Duffin and others (1967, p 100) essentially eliminates two orthogonality constraints from effective use. This is equivalent to adding two degrees of difficulty to the problem if the log term is in the denominator and one degree of difficulty if the log term is in the numerator.

The Approximation of Logarithms by the Geometric Mean

The geometric mean can be used to approximate a pure logarithmic term. This works out especially well if the logarithmic term is in the denominator. To illustrate, assume the log is a ratio of two quantities and is in the denominator as

$$\frac{K}{\text{LOG}(\Delta t_2 / \Delta t_1)} \quad (34)$$

The logarithm can be made into the form of a logarithmic mean by multiplying and dividing by the difference of the two quantities. This gives

$$\frac{(\Delta t_2 - \Delta t_1) K}{(\Delta t_2 - \Delta t_1) \text{LOG}(\Delta t_2 / \Delta t_1)} \quad (35)$$

Replace the log mean by the geometric mean to give

$$\frac{K}{(\Delta t_2 - \Delta t_1) \Delta t_2^{1/2} \Delta t_1^{1/2}} \quad (36)$$

Let

$$v_7 \leq \Delta t_2 - \Delta t_1 \quad (37)$$

Add Δt_1 to both sides

$$v_7 + \Delta t_1 \leq \Delta t_2 \quad (38)$$

Divide by Δt_2

$$v_7 \Delta t_2^{-1} + \Delta t_1 \Delta t_2^{-1} \leq 1 \quad (39)$$

So the problem now is

$$\text{Min } C_t = \dots \frac{K}{v_7 \Delta t_2^{1/2} \Delta t_1^{1/2}} \dots \quad (40)$$

$$\text{ST } v_7 \Delta t_2^{-1} + \Delta t_1 \Delta t_2^{-1} \leq 1 \quad (39)$$

There are 3 terms and 2 variables resulting in one degree of difficulty.

This can also be done if the log term is in the numerator but then the method of Avriel and Williams (1971) must be used on the artificial constraint.

If the logarithm is only of a single number, the method is the same except Δt_1 is 1.

The error discussion also applies here. That is, if the ratio is very large the approximation will not be very good.

It should also be obvious that the geometric mean can be used to approximate the arithmetic mean. This eliminates the need for Avriel and Williams (1971) method for the inverse arithmetic mean and reduces the degree of difficulty by one for the normal arithmetic mean. Care should be taken before using this approximation for the arithmetic mean because the error is greater than that involved in approximating the log mean.

OPTIMUM FLOW RATE OF COOLING WATER IN A CONDENSER

This is the first of three chemical engineering problems. These problems all use the geometric mean temperature difference as an approximation for the logarithmic one. The problems are not here to illustrate how well this technique works, but are formulated because a simple mechanical method is needed to solve the problem. The problems are usually solved by the tedious method of trial and error or by a computer program to solve that specific problem (Tarrer and others, 1971). Geometric programming offers a simple, straight-forward method of solving these problems, made even simpler and more straight-forward by the use of the geometric mean.

The first problem is presented in Peters and Timmerhaus (1968, p 308). It is for finding the optimum flow rate of cooling water through a condenser. They take a derivative and use trial and error or a graph to determine the optimum. Geometric programming is much simpler.

The equation for total annual variable cost as derived in Peters and Timmerhaus (1968, p 310) is

$$C_t = \frac{qH_y C_u}{c_p(t_2 - t_1)} + \frac{qK_F C_{A_0} \text{LOG}((t' - t_1)/(t' - t_2))}{U(t_2 - t_1)} \quad (41)$$

where t_2 , the exit temperature of the cooling water, is the only variable (the rest of the nomenclature is in the appendix).

Simplify the problem by adding and subtracting t' to the two $t_2 - t_1$ factors to get

$$C_t = \frac{qH_y C_u}{c_p((t' - t_1) - (t' - t_2))} + \frac{qK_F C_{A_0} \text{LOG}((t' - t_1)/(t' - t_2))}{U((t' - t_1) - (t' - t_2))}$$

Let

$$\Delta t_1 = t' - t_1 \quad (43)$$

$$\Delta t_2 = t' - t_2 \quad (44)$$

Then the problem is

$$\text{Min } C_t = \frac{qH_y C_u}{c_p(\Delta t_1 - \Delta t_2)} + \frac{qK_F C_{A_0} \text{LOG}(\Delta t_1/\Delta t_2)}{U(\Delta t_1 - \Delta t_2)} \quad (45)$$

Now replace the logarithmic mean temperature difference by the geometric mean temperature difference, giving

$$\text{Min } C_t = \frac{qH_y C_u}{c_p(\Delta t_1 - \Delta t_2)} + \frac{qK_F C_{A_0}}{U\Delta t_1^{1/2}\Delta t_2^{1/2}} \quad (46)$$

Factor out Δt_1 , a constant, and let

$$\theta = \frac{\Delta t_2}{\Delta t_1} \quad (47)$$

Then the problem as a function of θ is

$$\text{Min } C_t = \frac{qH_y C_u}{c_p \Delta t_1 (1 - \theta)} + \frac{qK_F C_A}{U \Delta t_1 \theta^{1/2}} \quad (48)$$

Replace $1 - \theta$ in the first term by V and add the artificial constraint

$$V \leq 1 - \theta \quad (49)$$

Adding θ to both sides gives the constraint in the proper form (eq 51). The problem as formulated as a geometric programming problem in θ is

$$\text{Min } C_t = \frac{qH_y C_u}{c_p \Delta t_1 V} + \frac{qK_F C_A}{U \Delta t_1 \theta^{1/2}} \quad (50)$$

$$\text{ST } V + \theta \leq 1 \quad (51)$$

The problem is easier to see if the constants are replaced.

Let

$$K_1 = \frac{qH_y C_u}{c_p \Delta t_1} \quad (52)$$

and

$$K_2 = \frac{qK_F C_A}{U \Delta t_1} \quad (53)$$

Now the problem is

$$\text{Min } C_t = K_1 V^{-1} + K_2 \theta^{-1/2} \quad (54)$$

$$\text{ST } V + \theta \leq 1 \quad (51)$$

The dual geometric programming problem is

$$\text{Max } \phi = \left(\frac{K_1}{\delta_1}\right)^{\delta_1} \left(\frac{K_2}{\delta_2}\right)^{\delta_2} \left(\frac{1}{\delta_3}\right)^{\delta_3} \left(\frac{1}{\delta_4}\right)^{\delta_4} (\delta_3 + \delta_4)^{\delta_3 + \delta_4} \quad (55)$$

Subject to the normality and orthogonality constraints

$$\delta_1 + \delta_2 = 1 \quad (56)$$

$$V: \quad -\delta_1 + \delta_3 = 0 \quad (57)$$

$$\theta: \quad -1/2\delta_2 + \delta_4 = 0 \quad (58)$$

There are four terms and two variables giving one degree of difficulty. Because the quantity $\Delta t_1 - \Delta t_2$ was in the first term as well as in the log mean temperature difference, using the geometric mean did not reduce the degree of difficulty.

Unfortunately the normality and orthogonality constraints do not give bounds on the contributions toward cost of the terms of the objective function. The only bound available is that δ_4 is upper bound by $1/2$.

The variables can be found from the relation

$$\phi^* = \frac{K_1}{\delta_1 V} = \frac{K_2}{\delta_2 \theta^{1/2}} \quad (59)$$

where ϕ^* is the minimum annual variable cost. What is desired is the flow rate of cooling water, w , which is

found from the following equation

$$w = \frac{q}{c_p \Delta t_1 (1 - \theta^*)} \quad (60)$$

where θ^* is the optimum ratio of temperature difference.

The substitution of the geometric mean temperature difference and the use of geometric programming result in a problem that is very simple to solve, although a computer is necessary. Note that once programmed, it is only necessary to input the constants, K_1 and K_2 , to solve a problem. For comparison, the equation to be solved by trial and error to find the optimum t_2 if a derivative had been taken is (Peters and Timmerhaus, 1968, p 310)

$$\frac{t' - t_1}{t' - t_{2,opt}} - 1 + \text{LOG} \left[\frac{t' - t_{2,opt}}{t' - t_1} \right] = \frac{U_H y C_u}{K_F c_p C_{A_0}} \quad (61)$$

OPTIMUM AIR-COOLED HEAT EXCHANGER DESIGN

A model for the minimum annual cost of an air-cooled heat exchanger is given in Lohrisch (1966). The equation for minimum annual cost as a function of pressure drop, Δp , and air exit temperature, t_2 , is

$$\begin{aligned}
 C_t = & \frac{qaH_y j \Delta p}{c_p (\Delta t_1 - \Delta t_2 + t_1' - t_2')} \\
 & + \frac{qSu^{0.356} n^{0.343} d^{0.4} \text{LOG}(\Delta t_2 / \Delta t_1)}{A_T P^{0.343} K^{0.67} c_p^{0.33} (\Delta t_2 - \Delta t_1) \Delta P^{0.343} (207.6)} \\
 & + \frac{qSC \text{ LOG}(\Delta t_2 / \Delta t_1)}{(\Delta t_2 - \Delta t_1)} \tag{62}
 \end{aligned}$$

Substitute the geometric mean temperature difference for the log mean temperature difference

$$\begin{aligned}
 C_t = & \frac{qaH_y j \Delta P}{c_p (\Delta t_1 - \Delta t_2 + t_1' - t_2')} \\
 & + \frac{qSu^{0.356} n^{0.343} d^{0.4}}{A_T P^{0.343} K^{0.67} c_p^{0.33} (207.6) \Delta P^{0.343} \Delta t_2^{1/2} \Delta t_1^{1/2}} \\
 & + \frac{qSC}{\Delta t_2^{1/2} \Delta t_1^{1/2}} \tag{63}
 \end{aligned}$$

Factor out Δt_1 and let

$$\theta = \frac{\Delta t_2}{\Delta t_1} \tag{64}$$

Then the problem as a function of θ and ΔP is

$$\begin{aligned}
 \text{Min } C_t = & \frac{q a H_y j \Delta P}{c_p \Delta t_1 (1 - \theta + (t'_1 - t'_2) / \Delta t_1)} \\
 & + \frac{q S u_n^{0.356} d^{0.343} \mu^{0.4}}{A_T P^{0.343} K^{0.67} (207.6) \Delta P^{0.343} \Delta t_1 \theta^{1/2} c_p^{0.33}} \\
 & + \frac{q S C}{\Delta t_1 \theta^{1/2}}
 \end{aligned} \tag{65}$$

To simplify the first term, let

$$U \leq 1 - \theta + (t'_1 - t'_2) / \Delta t_1 \tag{66}$$

Add θ to both sides, giving

$$U + \theta \leq 1 + (t'_1 - t'_2) / \Delta t_1 \tag{67}$$

Rearranging, the right-hand side can be written

$$\frac{\Delta t_1 + t'_1 - t'_2}{\Delta t_1} \tag{68}$$

But

$$\Delta t_1 = t'_2 - t_1 \tag{69}$$

Combining 68 and 69 gives the right-hand side of the constraint (67) in simplest terms

$$\frac{t'_1 - t_1}{t'_2 - t_1} \tag{70}$$

This is the process fluid entering temperature minus the utility fluid entering temperature divided by the process fluid exit temperature minus the utility fluid entering temperature. If the heat exchanger is to heat the process fluid, then the process fluid entering temperature will be less than the utility fluid entering temperature so the numerator will be negative. Since the process fluid cannot be heated to a temperature higher than the utility fluid entering temperature, the process fluid exit temperature will be less than the utility fluid entering temperature so the denominator will be negative. Thus, on heating, both numerator and denominator are negative so the term will be positive.

If the heat exchanger is to cool the process fluid, then the process fluid entering temperature will be greater than the utility fluid entering temperature meaning a positive numerator. Since the process fluid can be cooled to at most the utility fluid entering temperature, then the process fluid exit temperature will be greater than the utility fluid entering temperature making the denominator positive. Therefore on cooling, both numerator and denominator are positive giving a positive term. Hence the right-hand side is always positive and can be divided across the inequality without changing the sense of the inequality. Doing this to eq 67 gives

$$U(1 + \frac{t'_1 - t'_2}{\Delta t_1})^{-1} + \theta(1 + \frac{t'_1 - t'_2}{\Delta t_1})^{-1} \leq 1 \quad (71)$$

Inserting the artificial variable U into the objective function and adding constraint (71) gives the problem in the desired form of a geometric programming problem

$$\begin{aligned} \text{Min } C_t &= \frac{qaH_y j \Delta P}{c_p \Delta t_1 U} \\ &+ \frac{qSu^{0.356} n^{0.343} d^{0.4}}{A_T P^{0.343} K^{0.67} c_p^{0.33} (207.6) \Delta P^{0.343} \Delta t_1 \theta^{1/2}} \\ &+ \frac{qSC}{\Delta t_1 \theta^{1/2}} \end{aligned} \quad (72)$$

ST

$$U(1 + \frac{t'_1 - t'_2}{\Delta t_1})^{-1} + \theta(1 + \frac{t'_1 - t'_2}{\Delta t_1})^{-1} \leq 1 \quad (71)$$

The problem would be easier to understand if the constants were replaced. Let

$$K_1 = \frac{qaH_y j}{c_p \Delta t_1}, \quad (73)$$

$$K_2 = \frac{qSu^{0.356} n^{0.343} d^{0.4}}{A_T P^{0.343} K^{0.67} c_p^{0.33} (207.6) \Delta t_1}, \quad (74)$$

$$K_3 = \frac{qSC}{\Delta t_1} \quad , \quad (75)$$

and

$$K_4 = \left(1 + \frac{t'_1 - t'_2}{\Delta t_1}\right)^{-1} \quad (76)$$

Then the problem is

$$\text{Min } C_t = K_1 \Delta p U^{-1} + K_2 \Delta p^{-0.343} \theta^{-1/2} + K_3 \theta^{-1/2} \quad (77)$$

$$\text{ST } UK_4 + \theta K_4 \leq 1 \quad (78)$$

The dual geometric programming problem is

$$\text{Max } C_t = \left(\frac{K_1}{\delta_1}\right)^{\delta_1} \left(\frac{K_2}{\delta_2}\right)^{\delta_2} \left(\frac{K_3}{\delta_3}\right)^{\delta_3} \left(\frac{K_4}{\delta_4}\right)^{\delta_4} \left(\frac{K_4}{\delta_5}\right)^{\delta_5} (\delta_4 + \delta_5)^{\delta_4 + \delta_5} \quad (79)$$

Subject to the normality and orthogonality constraints

$$\delta_1 + \delta_2 + \delta_3 = 1 \quad (80)$$

$$P: \quad \delta_1 - 0.343 \delta_2 = 0 \quad (81)$$

$$U: \quad -\delta_1 + \delta_4 = 0 \quad (82)$$

$$\theta: \quad -1/2 \delta_2 - 1/2 \delta_3 + \delta_5 = 0 \quad (83)$$

There are 5 terms and 3 variables for 1 degree of difficulty.

The normality and orthogonality constraints can be used to get bounds on the contribution toward cost of the terms of the objective function. From eq 80

$$\delta_3 = 1 - \delta_1 - \delta_2 \quad (84)$$

But from eq 81

$$\delta_2 = \frac{1}{0.343} \delta_1 = 2.92\delta_1 \quad (85)$$

So

$$\delta_3 = 1 - 3.92\delta_1 \quad (86)$$

But

$$0 \leq \delta_3 \leq 1 \quad (87)$$

So

$$0 \leq 1 - 3.92\delta_1 \leq 1 \quad (88)$$

Thus

$$0 \leq \delta_1 \leq 1/3.92 = .255 \quad (89)$$

Since δ_1 represents the contribution toward cost of the cost of power to run the blower, eq 89 says that the cost of power must be less than or equal to 25.5 percent of the total cost to be operating the equipment optimally.

Eq 80 gives

$$\delta_2 + \delta_3 = 1 - \delta_1 \quad (90)$$

Using eq 89 and evaluating eq 90 at upper and lower bounds gives

$$.745 \leq \delta_2 + \delta_3 \leq 1 \quad (91)$$

Realizing that δ_2 and δ_3 together represent the contribution toward cost of the fixed charges on the equipment, then eq 91 says that the contribution toward cost of the fixed costs must be greater than or equal to 74.5 per cent to be at optimum.

In this case geometric programming not only gives a fairly easy way to solve the problem, but also gives bounds on the contributions toward cost. The use of the geometric mean temperature difference also reduced the degree of difficulty by one.

The variable can be found from the following relation

$$C_t^* = \frac{K_1 \Delta p}{U \delta_1} = \frac{K_2}{\Delta p^{0.343} \theta^{1/2} \delta_2} = \frac{K_3}{\theta^{1/2} \delta_3} \quad (92)$$

where C_t^* is the minimum annual cost of operating an air-cooled heat exchanger.

OPTIMUM HEAT-EXCHANGER DESIGN

A general equation for the minimum annual cost of a shell and tube heat exchanger was derived by Cichelli and Brinn (1956) and is presented in Peters and Timmerhaus (1968, p 574, which corresponds to Peters, 1958, p 344). To find the optimum, they used Lagrange multipliers which, upon differentiation, yield nonlinear equations that must be solved by trial and error or graphical methods. The use of geometric programming, especially after substituting the geometric mean temperature difference, offers a much more straight-forward means of solution.

The variable annual cost is dependent on the fixed charges on the equipment, cost of the utility fluid, cost of pumping the process fluid through the exchanger, and the cost of pumping the utility fluid through the exchanger. The total annual cost to be minimized is represented by the following equation

$$C_t = A_O K_F C_{A_O} + w_u H_Y C_u + A_O E_i H_Y C_i + A_O E_O H_Y C_O \quad (93)$$

This equation was developed by Cichelli and Brinn (1956) into one involving four primary variables. The equation is

$$\begin{aligned} C_t = A_O K_F C_{A_O} &+ \frac{q H_Y C_u}{c_{P_u} (\Delta t_1 - \Delta t_2 + t_1' - t_2')} + A_O Y_i h_i^{3.5} H_Y C_i \\ &+ A_O Y_O h_O^{4.75} H_Y C_O \end{aligned} \quad (94)$$

where the variables are the outside tube area, A_O , Δt_2 , the utility fluid exit temperature (the real unknown) minus the process fluid entering temperature (given in the problem), h_i , the inside film coefficient of heat transfer, and h_o , the outside film coefficient of heat transfer.

Only three of the four variables are independent so the following relation holds

$$\frac{F_T(\Delta t_2 - \Delta t_1)}{q \text{ LOG}(\Delta t_2/\Delta t_1)} = \frac{1}{A_O} \left(\frac{D_O}{D_i h_i} + \frac{1}{h_o} + R_{dw} \right) \quad (95)$$

To get the problem into the form required by geometric programming, solve eq 95 for A_O

$$\frac{q \text{ LOG}(\Delta t_2/\Delta t_1)}{F_T(\Delta t_2 - \Delta t_1)} \left(\frac{D_O}{D_i h_i} + \frac{1}{h_o} + R_{dw} \right) = A_O \quad (96)$$

Notice that in the objective function, eq 94, A_O always has an exponent of plus one. By saying A_O is greater than the quantity on the left, the objective function is strengthened and the constraint is in the desired form of an inequality. It should be realized that the constraint will be an equality at optimality, caused by treating A_O as an artificial variable and the constraint as its new constraint derived from the objective function. Therefore the inequality in the desired form is

$$\frac{q \text{ LOG}(\Delta t_2/\Delta t_1)}{F_{T^0} A_0 (\Delta t_2 - \Delta t_1)} \left(\frac{D_0}{D_i h_i} + \frac{1}{h_0} + R_{dw} \right) \leq 1 \quad (97)$$

A_0 is the heat transfer area of the heat exchanger and is therefore always positive so there is no problem in dividing it across the inequality.

The problem now is to minimize eq 94 subject to eq 97. Replace the logarithmic mean temperature difference by the geometric mean temperature difference in the constraint to get

$$\frac{q}{F_{T^0} A_0 \Delta t_2^{1/2} \Delta t_1^{1/2}} \left(\frac{D_0}{D_i h_i} + \frac{1}{h_0} + R_{dw} \right) \leq 1 \quad (98)$$

Factor out Δt_1 and let

$$\theta = \frac{\Delta t_2}{\Delta t_1} \quad (99)$$

Then the problem is

$$\begin{aligned} \text{Min } C_t = & A_0 K_F C_{A_0} + \frac{q H_Y C_u}{c_{P_u} \Delta t_1 (1 - \theta + (t_1' - t_2')/\Delta t_1)} \\ & + A_0 Y_i h_i^{3.5} H_Y C_i + A_0 Y_0 h_0^{4.75} H_Y C_0 \end{aligned} \quad (100)$$

$$\text{ST } \frac{q D_0}{F_{T^0} A_0 \theta^{1/2} \Delta t_1 D_i h_i} + \frac{q}{F_{T^0} A_0 \theta^{1/2} \Delta t_1 h_0} + \frac{q R_{dw}}{F_{T^0} A_0 \theta^{1/2} \Delta t_1} \leq 1 \quad (101)$$

The second term of the objective function can be simplified by introducing the artificial variable U and adding the constraint (102) to the set of constraints

$$U \leq 1 - \theta + \frac{t'_1 - t'_2}{\Delta t_1} \quad (102)$$

Add θ to both sides

$$U + \theta \leq 1 + \frac{t'_1 - t'_2}{\Delta t_1} \quad (103)$$

Rearranging the right-hand side, the constraint becomes

$$U + \theta \leq \frac{\Delta t_1 + t'_1 - t'_2}{\Delta t_1} \quad (104)$$

But (Peters and Timmerhaus, 1968, p 576)

$$\Delta t_1 = t'_2 - t_1 \quad (105)$$

Substituting this in eq 104 gives

$$U + \theta \leq \frac{t'_1 - t_1}{t'_2 - t_1} \quad (106)$$

The right-hand side is the process fluid entering temperature, t'_1 , minus the utility fluid entering temperature, t_1 , divided by the process fluid exit temperature, t'_2 , minus the utility fluid entering temperature, t_1 again. If the heat exchanger is to heat the process fluid, then t'_1 will be less than t_1

so the numerator will be negative. Because the process fluid cannot be heated to a temperature higher than the utility fluid entering temperature, t_2' will be less than t_1 so the denominator will be negative. Hence on heating, both numerator and denominator will be negative so the right-hand side will be positive.

If the heat exchanger is to cool the process fluid, then t_1' will be greater than t_1 , meaning a positive numerator. Since the process fluid can be cooled by the utility fluid to at most the utility fluid entering temperature, then t_2' will be greater than t_1 so the denominator will be positive. Actually because some of the heat is given off to the air surrounding the heat exchanger, t_2' could be less than t_1 . That case would be unusual and therefore is not treated here. Therefore for cooling, the numerator and the denominator will usually be positive giving a positive term. Hence the right-hand side is almost always positive and can be divided across the inequality sign without changing the sense of the inequality. Doing this to eq 103 puts the constraint in the desired form for geometric programming

$$U \left(1 + \frac{t_1' - t_2'}{\Delta t_1}\right)^{-1} + \theta \left(1 + \frac{t_1' - t_2'}{\Delta t_1}\right)^{-1} \leq 1 \quad (107)$$

The problem is now in the required form for geometric programming. The problem is

$$\begin{aligned} \text{Min } C_t = & A_O K_F C_{A_O} + \frac{q_H Y C_u}{c_{P_u} \Delta t_1 U} + A_O Y_i h_i^{3.5} H_Y C_i \\ & + A_O Y_O h_O^{4.75} H_Y C_O \end{aligned} \quad (108)$$

$$\text{ST } \frac{q^D O}{F_T A_O \theta^{1/2} \Delta t_1 D_i h_i} + \frac{q}{F_T A_O \theta^{1/2} \Delta t_1 h_O} + \frac{q R_{dw}}{F_T A_O \theta^{1/2} \Delta t_1} \leq 1 \quad (101)$$

$$U \left(1 + \frac{t_1' - t_2' - 1}{\Delta t_1} \right) + \theta \left(1 + \frac{t_1' - t_2' - 1}{\Delta t_1} \right) = 1 \quad (107)$$

The problem would be easier to see if the constants were simplified. Therefore let

$$K_1 = K_F C_{A_O} , \quad (109)$$

$$K_2 = \frac{q_H Y C_u}{c_{P_u} \Delta t_1} , \quad (110)$$

$$K_3 = Y_i H_Y C_i , \quad (111)$$

$$K_4 = Y_O H_Y C_O , \quad (112)$$

$$K_5 = \frac{q^D O}{F_T \Delta t_1 D_i} , \quad (113)$$

$$K_6 = \frac{q}{F_T \Delta t_1} , \quad (114)$$

$$K_7 = \frac{qR_{dw} F}{F_T \Delta t_1}, \quad (115)$$

and

$$K_8 = \left(1 + \frac{t_1' - t_2'}{\Delta t_1}\right)^{-1} \quad (116)$$

Now the problem is

$$\text{Min } C_t = K_1 A_O + K_2 U^{-1} + K_3 A_O h_i^{3.5} + K_4 A_O h_O^{4.75} \quad (117)$$

$$\begin{aligned} \text{ST } K_5 A_O^{-1} \theta^{-1/2} h_i^{-1} + K_6 A_O^{-1} \theta^{-1/2} h_O^{-1} \\ + K_7 A_O^{-1} \theta^{-1/2} \leq 1 \end{aligned} \quad (118)$$

$$UK_8 + \theta K_8 \leq 1 \quad (119)$$

There are 9 terms and 5 variables meaning 3 degrees of difficulty. The use of the geometric mean temperature difference has reduced the number of degrees of difficulty by one. It has also reduced the number of terms by 3 and the number of variables by 2 producing a much simpler looking problem, as well as actually being simpler which is indicated by the reduction in the degree of difficulty.

The dual geometric programming problem is

$$\begin{aligned} \text{Max } C_t = & \left(\frac{K_1}{\delta_1}\right)^{\delta_1} \left(\frac{K_2}{\delta_2}\right)^{\delta_2} \left(\frac{K_3}{\delta_3}\right)^{\delta_3} \left(\frac{K_4}{\delta_4}\right)^{\delta_4} \left(\frac{K_5}{\delta_5}\right)^{\delta_5} \left(\frac{K_6}{\delta_6}\right)^{\delta_6} \left(\frac{K_7}{\delta_7}\right)^{\delta_7} \\ & \left(\frac{K_8}{\delta_8}\right)^{\delta_8} \left(\frac{K_8}{\delta_9}\right)^{\delta_9} (\delta_5 + \delta_6 + \delta_7)^{\delta_5 + \delta_6 + \delta_7} (\delta_8 + \delta_9)^{\delta_8 + \delta_9} \end{aligned} \quad (120)$$

Subject to the normality and orthogonality constraints

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 = 1 \quad (121)$$

$$A_O: \quad \delta_1 \quad +\delta_3 +\delta_4 \quad -\delta_5 \quad -\delta_6 \quad -\delta_7 \quad = 0 \quad (122)$$

$$U: \quad -\delta_2 \quad \quad \quad +\delta_8 = 0 \quad (123)$$

$$h_i: \quad 3.5\delta_3 \quad -\delta_5 \quad = 0 \quad (124)$$

$$h_o: \quad 4.75\delta_4 \quad -\delta_6 \quad = 0 \quad (125)$$

$$\theta: \quad -1/2\delta_5 - 1/2\delta_6 - 1/2\delta_7 + \delta_9 = 0 \quad (126)$$

The first four deltas represent the contribution toward cost of each of the four terms of the objective function. That is, δ_1 is the contribution toward total cost of the fixed charges. δ_2 represents the contribution toward total cost of the cost of the utility fluid. δ_3 and δ_4 represent the contribution toward total cost of pumping the fluid inside the tubes and outside the tubes respectively.

The normality and orthogonality constraints do give some bounds on the contributions toward cost. From eq 124

$$3.5\delta_3 = \delta_5 \quad (127)$$

But

$$0 \leq \delta_5 \leq 1 \quad (128)$$

so

$$0 \leq 3.5\delta_3 \leq 1 \quad (129)$$

Hence

$$0 \leq \delta_3 \leq 1/3.5 = .286 \quad (130)$$

Therefore the cost of pumping the fluid through the inside of the tubes, usually the process fluid, must be less than 28.6 percent of the total annual cost of the heat exchanger to be operating it optimally. From eq 125

$$4.75\delta_4 = \delta_6 \quad (131)$$

But

$$0 \leq \delta_6 \leq 1 \quad (132)$$

so

$$0 \leq 4.75\delta_4 \leq 1 \quad (133)$$

Therefore

$$0 \leq \delta_4 \leq 1/4.75 = .21 \quad (134)$$

Thus the cost of pumping the fluid around the outside of the tubes, usually the utility fluid, must be less than 21 percent of the total annual cost if the heat exchanger is to be run optimally. From eq 121, we have

$$\delta_1 + \delta_2 = 1 - \delta_3 - \delta_4 \quad (135)$$

Evaluating at the bounds given in eq 130 and eq 134

$$.504 \leq \delta_1 + \delta_2 \leq 1 \quad (136)$$

This says that the fixed charges and the cost of the utility fluid must contribute at least 50.4 percent of the total annual cost to be operating optimally.

There are some changes that could be made in the model of Peters and Timmerhaus (1968) to make it more realistic. The first is that C_{A_0} , which is the installed cost of the heat exchanger per sq ft of outside area of the tube heat transfer area, is not really a constant. That is the cost of the heat exchanger is not directly proportional to size, there are some economies of scale. The exponent on A_0 to accurately get the cost is determined from a plot of cost for various sizes of heat exchangers. Unfortunately this makes the exponent dependent on the data. As the costs change from year to year, the exponent changes. The graph in Peters and Timmerhaus (1968, p 567) shows that the cost is not a linear relation and an exponent can be determined from that graph. Incorporating the new exponent in the problem is trivial. The exponent of A_0 in the first term in eq 108 and eq 117 is changed from one to whatever fits the data used. The constant of the first term, K_1 , also changes.

This also changes the orthogonality constraint for A_0 . Instead of a one times δ_1 in eq 122, it should be changed to the new exponent of A_0 times δ_1 .

The other oversimplification is that F_T , a correction factor for changing counterflow temperature difference to give mean temperature difference, is not a constant as assumed. F_T is proportional to the variable t_2 but in a very complicated way (Gulley, 1960). For this reason, it was left as Peters and Timmerhaus (1968) presented it. Both changes were also avoided so as not to ruin the simplicity and practical usefulness of Peters and Timmerhaus' (1968) model.

The variables can be found from the following relation

$$C_t^* = \frac{K_1 A_0}{\delta_1} = \frac{K_2}{U \delta_2} = \frac{K_3 A_0 h_i^{3.5}}{\delta_3} = \frac{K_4 A_0 h_o^{4.75}}{\delta_4} \quad (137)$$

where C_t^* is the minimum annual cost of the heat exchanger.

APPENDIX

NOMENCLATURE

- a Temperature factor for air, $T_{abs}(\text{mean})/492$
- A_O Area of outside-tube heat transfer, sq ft.
- A_T Area on outside of tubes between fins and area of both sides of fins, ft^2/ft^2
- c_p Heat capacity, Btu/lb-F
- c_{p_u} Heat capacity of utility fluid, Btu/lb-F
- C_{A_O} Installed cost of heat exchanger per unit of outside-tube heat-transfer area, \$/sq ft
- C $1/h_i$ + fouling factor
- C_i Cost of supplying 1 ft-lb force to pump the fluid through the inside of the tubes, \$/ft-lb force
- C_O Cost for supplying 1 ft-lb force to pump the fluid through the shell side of the heat exchanger, \$/ft-lb force
- C_t Total annual variable cost, \$/year
- C_u Cost of the utility fluid, \$/lb
- d Root diameter of the tubes, ft
- D_i Inside diameter of the tubes, ft
- D_O Outside diameter of the tubes, ft
- E_i Power loss inside the tubes per unit of outside tube area, (ft-lb force)/(hr) (sq ft)

- E_O Power loss outside the tubes per unit of outside tube area, (ft-lb force)/(hr) (sq ft)
- F_T Correction factor on logarithmic mean temperature difference for counterflow to give mean temperature difference, see Peters and Timmerhaus (1968) p 539
- h_i Film coefficient of heat transfer on the inside of the tubes, Btu/(hr) (sq ft) (F)
- h_o Film coefficient of heat transfer on the outside of the tubes, Btu/(hr) (sq ft) (F)
- H_y Numbers of hours of operation per year, hr/year
- j Cost of 1 kilowatt-hour, cents/kwh
- K Thermal conductivity of air, Btu/(ft) (hr) (F)
- K_F Annual fixed charges including maintenance expressed as a fraction of installed cost
- n Number of tube rows
- P Density of air, lb/ft³
- ΔP Pressure drop, in. W.G.
- q Rate of heat transfer, Btu/hr
- R_{dw} Combined resistance of tube wall and scaling or dirt factors, (Btu/((hr) (sq ft) (F)))⁻¹, see Peters and Timmerhaus (1968) p 576
- S Annual capital charges on equipment
- t' Condensation temperature of fluid in condenser, F
- t_1^i Process fluid entering temperature, F
- t_2^i Process fluid exit temperature, F

- t_1 Utility fluid entering temperature (utility fluid in air cooled exchanger is air), F
- t_2 Utility fluid exit temperature, F
- U Overall coefficient of heat transfer, Btu/((hr)(sq ft)(F))
- w Flow rate, lb/hr
- w_u Total flow rate of utility fluid, lb/hr
- Y_i Dimensional factor, see Peters and Timmerhaus (1968) p 577
- Y_0 Dimensional factor, see Peters and Timmerhaus (1968) p 577

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