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NORMALIZATION OF POROSITY WELL LOGS

by

Andrew L. Prestridge

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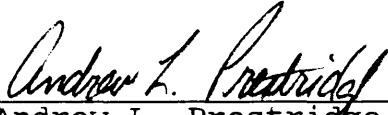
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Golden, Colorado

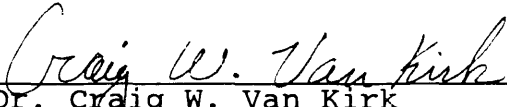
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ABSTRACT

This thesis proposes a method to correct erroneous porosity logs for specific conditions. Porosity is a very important property of hydrocarbon-bearing formations. Knowledge of porosity aids in decisions to complete or plug and abandon wells. If production is found, volumetric reserve estimates can be made with porosity information. Determination of porosity in wells is commonly done by means of wireline logs that are run across formations of interest. Different logs are available for porosity determination, and each have inherent advantages as well as disadvantages. Problems arise when porosity logs give erroneous results.

The three most widely used porosity devices are the Density, Neutron, and Sonic. Each porosity device independently monitors information such as secondary gamma rays detected from gamma bombardment (Density device), hydrogen detection as a result of neutron bombardment (Neutron device), and acoustic travel time (Sonic device) and transforms this data into porosity information. Additional processing such as environmental corrections, filtering and depth shifting are necessary to properly compare data from each device. Comparison and correction of this information within the same wellbore is defined as vertical normalization.

Due to the construction of the Density device, an assumption of the type of error allows the systematic correction of density data. Neutron and Sonic data are then vertically normalized with respect to the corrected density data. Correlations of data are made using Reduced Major Axis, sometimes called RMA. This allows independently measured tool responses to be mathematically correlated as such.

Normalized porosity information was also compared to core information. Although core information and log information are difficult to compare, a good correlation was found in most cases.

Normalized porosity data should allow improved wellsite decisions, reserve estimates, and reservoir characterization. Better use of existing data, by means of normalization, should also aid in unitization proceedings where volumetric reserve estimates are contested. Normalization of porosity data is a useful technique that may save information that was previously believed to be "miscalibrated" or "bad".

Cored wells with sufficient porosity log data from the Anschutz Ranch East Field, Summit Co., Utah were normalized and used as examples.

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The freedom with which he was allowed to work, and liberality with which Powell gave away his most illuminating ideas, cemented a personal friendship that was as close as any...

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Chapter 1

Introduction

Normalization is a very generic term and can have different meanings within the field of log analysis. The meaning of normalization as it pertains to porosity logs could be twofold, either vertical normalization or horizontal normalization. Horizontal normalization is the comparison of like well log traces between wells within a field, or within close proximity. Vertical normalization is the comparison of unlike well log traces against others in the same well. Past researchers have dealt with both types of normalization to remove errors in log responses.

1.1 Scope and Purpose

In order to build a solid basis for normalization a simple case was solved. Factors, other than porosity, that affect most porosity devices to some degree include lithology, formation fluids, mud and mud filtrates, borehole, depth shifting, vertical resolution, and other more common environmental corrections. From Table 1 it is apparent that shale and gas can cause significant variations in log responses. Since log error adds to this list,

Table 1
Tool Response Chart

Lithology and Saturating Fluid	Density Tool Response (p.u.)	Neutron Tool Response (p.u.)	Sonic Tool Response (p.u.)
Clean Sand $S_1 = 100\%$	← OK →	← OK →	← OK →
Shaley Sand $S_1 = 100\%$	Small ↑	Very High ↑	Very High ↑
Clean Sand $S_g > 0\%$	Small ↑	Very Low ↓	← OK →
Shaley Sand $S_g > 0\%$	High ↑	← ? → ↑ ↓	Very High ↑

selection of a clean sand containing no light hydrocarbons (gas) will reduce the variables to be determined and allow vertical normalization. Log errors are related to the difficulty of in-situ calibration and can be a gain (sensitivity or slope) error and/or a shift error, as described in more detail in Chapter 4. This thesis investigates the simple case of a single lithologic rock type (sandstone), where formation fluids are water or liquid hydrocarbon, borehole conditions are favorable, and environmental corrections can be made.

The purpose of this investigation is to remove any log

errors and show that log data can often be normalized to accurately measure porosity. Detection of error is difficult since each porosity tool may have a gain (sensitivity or slope) and a shift error (Schlumberger 1984). If lithology, saturating fluid properties, and environmental factors are corrected for, only log error remains. Thus for this base case the following equations apply;

$$\phi_{Density\ Normalized} = (\phi_{Density_{Log}} \times GF_D) + SF_D$$

$$\phi_{Neutron\ Normalized} = (\phi_{Neutron_{Log}} \times GF_N) + SF_N$$

$$\phi_{Sonic\ Normalized} = (\phi_{Sonic_{Log}} \times GF_S) + SF_S$$

where the corrections for the gain (sensitivity or slope) error are denoted by GF and are unitless, the corrections for the shift error are denoted by SF and are in porosity units (p.u.), each with a subscript descriptive of the porosity device (D, N or S).

From the three equations above there are 9 unknowns, since only log porosity values are known. This thesis is based on the assumption that the density tool responds with a shift error only (Davis 1984, Davis 1991) which reduces

the number of variables in the first equation above by eliminating the gain (sensitivity or slope) error;

$$GF_D = 1$$

The remaining shift error may be resolved through interpretation of delta rho ($\Delta\rho$), caliper information, and general mud system information, as these are all checks on the performance of the density tool. Chapter 6 describes the analysis of this data to determine the density shift error. Normalized density porosity may then be calculated. The ensuing gain (sensitivity or slope) and shift errors for the sonic and neutron devices are resolved through crossplotting against the normalized density porosity. Schlumberger CSU (1984) calibrates logs before and after the job, however, this may not always eliminate errors. Gain (sensitivity or slope), shift, and intermittent types of errors are discussed in Chapter 4.

1.2 Regression of Log Information

Reduced Major Axis (RMA) or Standard Deviation Line (as discussed by Freedman et al. 1980) is used to properly interpret relationships between independent variables. When dealing with information from well logs it is necessary to

compare data and draw conclusions about how variables relate to each other. Linear regression is a mathematical method that is commonly used in engineering, however, should not be used when comparing values measured by logging tools.

Linear regression assumes that one variable is correct and that the other variable is dependent on the first. In the case of log measured quantities, this is not the case.

Porosity values from the Density device do not in any way depend on porosity values from the Sonic device or the Neutron device! Each of these tools independently monitor data that is transformed into porosity information.

Therefore a method for comparison of data, which allows each tools response to be correlated independently was used.

More detailed information concerning Linear Regression and RMA can be found in Appendix A.

Unless noted otherwise all correlation of log measured information and the values resulting from crossplotting for normalization are made using the RMA method.

Chapter 2

Literature Review

2.1 Horizontal Normalization

Nienast and Knox (1973) were the first authors to publish information concerning normalization of well log information. These authors compared a histogram of bulk density from a composite of many wells against individual well histograms. Wells chosen for the composite were subjectively believed to be reading properly. This method normalizes porosity in all the wells in a field to have the same distribution of porosity. Although their work does point to problems with well log information, their recommended procedure did not work in some cases. Figure 1 is from their original work and shows the use of histograms to shift porosity distributions. Note that the histogram shown includes bulk density information from the shales ($\rho_b \approx 2.65$ g/cc) as well as the sandstone producing formation ($\rho_b \approx 2.50$ g/cc). Each has a mode making the histogram bimodal. In Chapter 3, the composite method is modified to include the proposed vertical normalization technique, and remove the subjectivity of choosing the composite wells and dispel the use of bimodal histograms. The upper histogram

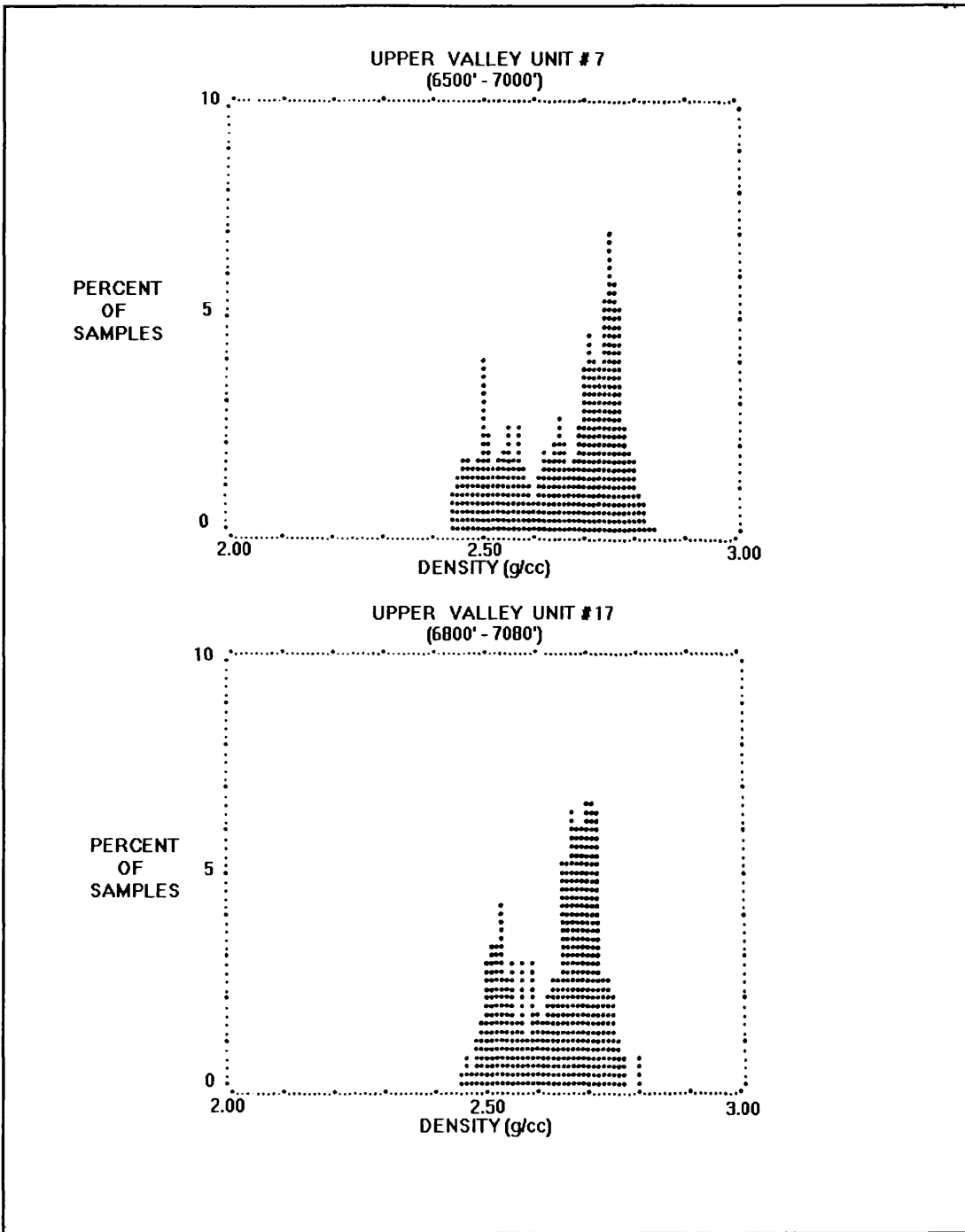


Figure 1

Normalizing with Histograms (After Nienast & Knox 1973)

of Well #7 was assumed to be reading correctly. When compared to other wells such as Well #17 modes of both the shale and sand are matched to Well #7. The shale mode has a higher ρ_b than the sand mode. Knienast and Knox proposed shifting the shale mode of Well #17 by 0.05 g/cc while also shifting the sand mode by -0.02 g/cc. Shifting the shale mode is conjectural at this point, but the shift of the sand mode is clearly incorrect. If sand modes for all wells were corrected to read the same as Well #7, then all wells in the field would have the same porosity distribution. Any geologist would almost certainly agree that there are lithologic changes across a field!

Hovarth (1973) looked at errors and their propagation through log analysis calculations, while only briefly touching on the term recalibration procedure. This term, recalibration procedure, is the subject of this thesis and the definition, as defined previously, is really vertical normalization.

Knox (1974), Holt (1975) and Connolly (1974) discussed operational field procedures to increase log quality. Although their recommendations should be considered, Connolly (1974) recognized from his experience that;

All these examples indicate logs that have failed to record properly in the hole, yet have calibrations before and after survey that appear valid.

Farnan and McHattie (1984) used crossplotting of the repeat and main sections of a log in order to determine log quality;

Digitally plotted overlays of the repeat and main section of each individual log have proven an excellent tool for detecting poor repeatability and hence bad logs.

Patchett and Coalson (1979) arrived at a much more reasonable horizontal normalization procedure by comparing histograms of shale bulk densities. Figure 2 shows the composite histogram approach used by Patchett and Coalson (1979). This procedure is based on the hopefully consistent log response of shales from well to well. It should be noted that, like Nienast and Knox (1973), the selection of wells used in building a composite histogram is subjective and that lithological changes in shale are also possible. Comparison of log and core information are noted in the examples from this paper.

Lang (1980) took a step backwards from the Patchett and Coalson (1979) procedure and used almost the same methodology as Nienast and Knox (1973). Lang (1980) did note the danger in his normalization procedure is that all of the meaningful lithologic changes will be normalized away.

Doveton and Bornemann (1981), Doveton (1986), and Ellis (1987) took the lithologic changes into account with the

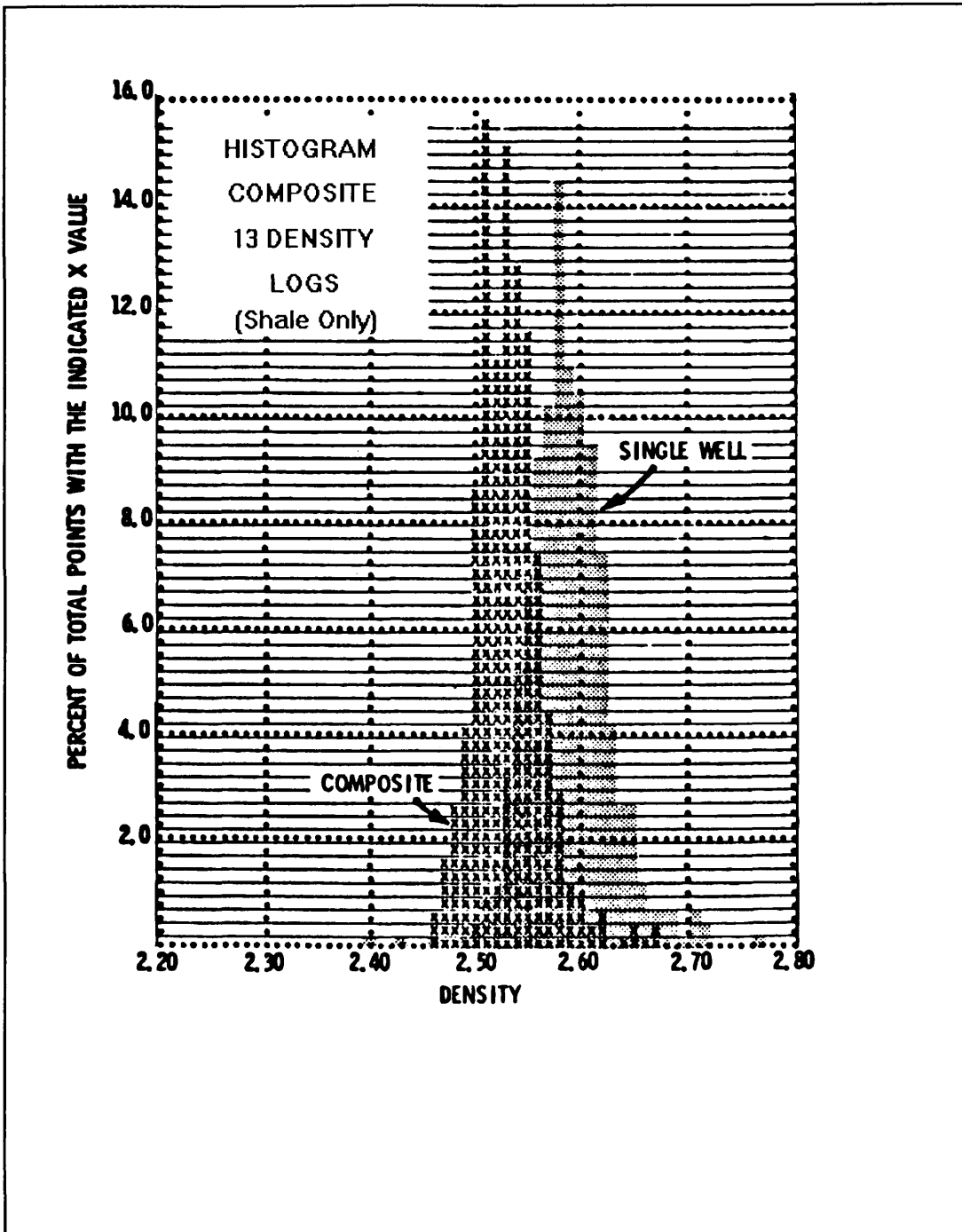


Figure 2

Composite Histogram (After Patchett and Coalson 1979)

concepts of trend surface analysis. This horizontal normalization technique, while quite insightful, requires the presence of a calibrator bed which must be present across the field. In general, trend surface analysis is a method that fits a surface through a map of errors to determine how much of the error is due to lithologic changes, and how much is the residual error. Doveton (1986) and Davis (1986) have documented excellent examples of this in their work, and it is suggested that the reader use these references for more detailed information. Davis (1986) describes trend surface analysis;

Trend analysis is the geology profession's name for a mathematical method of separating map data into two components - that of a regional nature, and local fluctuations. This has been done intuitively or graphically by geologists for years. Petroleum geologist, for example, refer to 'regional dip' or 'basinal configuration' as opposed to 'local structures.' Petrologist may speak of the 'regional grain' of a metamorphic terrain. Geophysicists have long been accustomed to the concept of 'regional trends' and 'local anomalies.' All of these expressions imply a belief that any given observation is the outcome of two interacting geologic forces or sets of forces - that which shaped the region or general geologic setting, and that which caused small areas to deviate from the regional pattern.

This method does not force the porosity in all wells in the field to be the same, and is the most desirable method of normalization since marker beds act as in-situ calibration devices.

2.2 Comparison of Log and Core Data

Cooke-Yarborough (1987) pointed out the many possible errors and corrections that are inherent in both log and core data. Vertical resolution of porosity logs is mentioned as a problem when comparing to core data. It is very interesting to note that a correlation between heterogeneity of the formation and the degree of fit of the core and log data is mentioned. No attempt was made to quantitatively match geological environment or heterogeneity with a correlation of fit of the log vs. core data.

Helander (1983) expands on a root problem with cores which he describes so well;

The ideal in core recovery would be, of course, to obtain a sample of the rock as it exists in-situ in the undisturbed state. This, however, is impossible since, during the drilling process and the subsequent removal to the surface, the core and its contained fluids are irretrievably altered.

This is not to say that core data is not correct but, rather to show that core data is also subject to corrections that may not be fully understood at this point.

2.3 Density Tool Response

The Density tool response was studied in order to

formulate a method of correcting or normalizing logs under certain conditions. The two most common types of density tools found in use today are the Compensated Formation Density tool or FDC, and the Litho-Density Tool or LDT. The FDC was introduced to the industry in approximately 1963, and replaced the single detector density tool not discussed in this thesis. The LDT is a more recent density tool with some new capabilities. The LDT tool has a self calibrating feature that was added to help eliminate calibration problems with the FDC tool. The LDT also has a more powerful source, and more sensitive far detector. The far detector measures three pairs (pairs effect) of electrons, those returning from the source due to compton scattering (FDC also measures this - ρ_b information) and the photoelectric effect P_e . P_e information cannot be captured by the older FDC tool. The P_e is useful because it is not very sensitive to porosity and can help with lithology interpretation. P_e information was not used in this thesis.

Wahl, Tittman, Johnstone, and Alger (1964) introduced the dual spaced compensated density tool or FDC. This paper went into detail concerning the Spine and Ribs plot that is used to show how the tool corrects for mudcakes of varying thicknesses and densities. The Spine and Ribs plot and the

correction for borehole size plot have been modified slightly but are still reproduced in current Schlumberger chartbooks.

Tittman and Wahl (1965) described the fundamentals of formation density logging. The paper goes into detail about gamma-gamma interactions and the equations that model them. Converting the returning gamma counts into electron density and then into bulk density is also described.

Ellis (1987) talks about density tools in his book, and explores the newer LDT tool that was not available in 1964-65 when previous authors were writing on the subject.

Chapter 3

Normalization Models

Normalization is closely related to geology. If the stratigraphic section of the field to be studied has a marker bed near the formation of interest, then horizontal normalization may be utilized as proposed by Doveton (1986). Horizontal normalization along with geological acumen for an area should be used whenever possible as the marker bed acts as an in-situ calibrator. Vertical normalization in the strictest sense should be used in all cases as a check.

3.1 Horizontal Normalization

Horizontal Normalization using a marker bed is the most desirable normalization case when this geological condition exists. The fact that a lithologic unit which is consistent across the field to be studied allows additional geological information to be used. Doveton (1986) has documented an excellent example of this in his book. Marker beds which are most desirable, and in order of preference; anhydrite and dense lime without porosity. A marker bed is a superior case because it allows in-situ calibration of log responses. Trend surface analysis (Doveton 1986) allows the error in

log response to be properly interpreted. This is necessary because even though a marker bed may be consistently present in all the wells in a field, lithologic properties of the marker bed may not be exactly constant. Figure 3 is a general process diagram of normalizing with a marker bed, it has been modified to add vertical normalization as proposed by the author. Note that vertical normalization has not been recorded in the literature and that any mixed lithology, including gas bearing formations can be normalized.

3.2 Vertical Normalization

When a marker bed does not exist in the field to be studied, then the only way to normalize porosity log responses is with vertical normalization. This method requires the interpretation of factors that affect the density log response, and arriving at reasonable corrections to raw log responses when necessary. Trend surface analysis is difficult and sometimes not possible without a marker bed.

This thesis has studied the vertical normalization process in detail. This should aid in the evaluation of porosity logs on an individual well basis. In the case of

HORIZONTAL NORMALIZATION

General Process Diagram

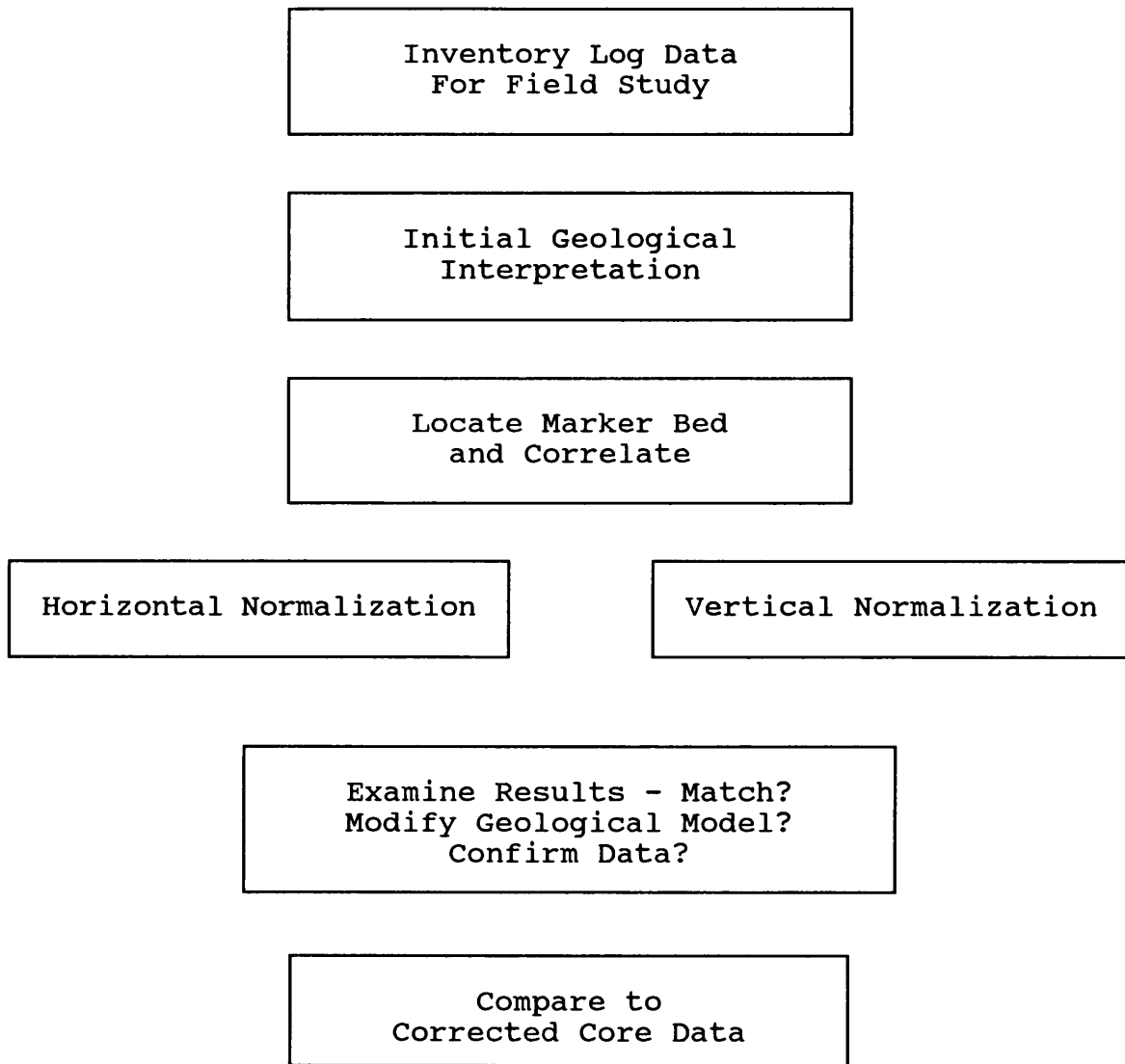


Figure 3

Normalization with a Marker Bed

an exploratory well with a marker bed, vertical normalization should be used as a check because trend surface analysis of the marker bed is not possible without neighboring wells. The proper evaluation of a vertical normalization bed should be combined with geological interpretation to make sure the same formation is used in all wells if possible. This process can normalize complex lithology containing gas if the vertical normalization bed exists. Figure 4 is a general process diagram of vertical normalization without a marker bed, as proposed by the author. Chapter 6 explains the process used in developing and giving credence to vertical normalization.

3.3 Normalization - Gulf Coast Method

The Gulf Coast presents a special case of normalization. Since the dominant lithology in the gulf coast region is sand-shale sequences, marker beds perse are not available for use. But if the shale immediately above the formation of interest is used as a calibration bed then it is possible to horizontally normalize porosity log responses. A composite histogram of shale density from correctly reading wells is constructed for comparison to individual erroneous well histograms. Correlation of this

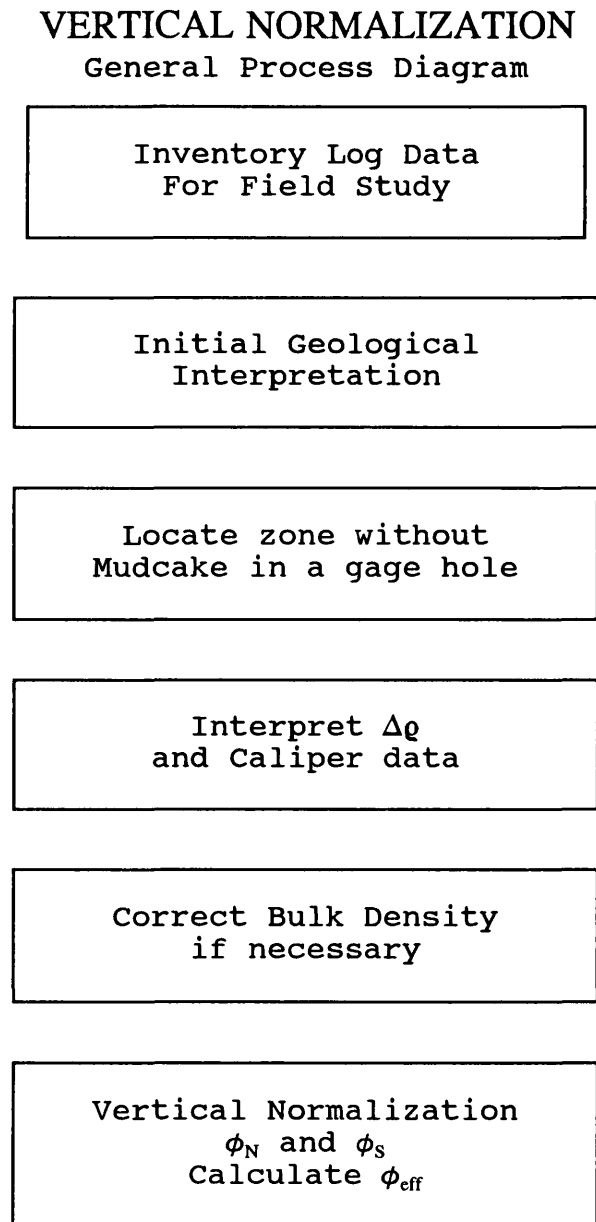


Figure 4

Vertical Normalization (Without Marker Bed)

shale bed and the selection of the formation top and bottom should be done with the assistance of a geologist to insure consistency. Histograms of the density of the overlying shale beds are then made for all wells in question and compared to the composite. Trend surface analysis may be used to show the validity of normalization and give insight into the geologic setting of the shale deposition and compaction. Figure 5 is a general process diagram of normalization using the composite histogram method, modified with the addition of vertical normalization. Although the composite method is being practiced, the author feels that vertical normalization will be helpful in determining selection of wells for the composite instead of subjectively choosing these wells, and as a check for correction of other wells.

NORMALIZATION - GULF COAST METHOD General Process Diagram

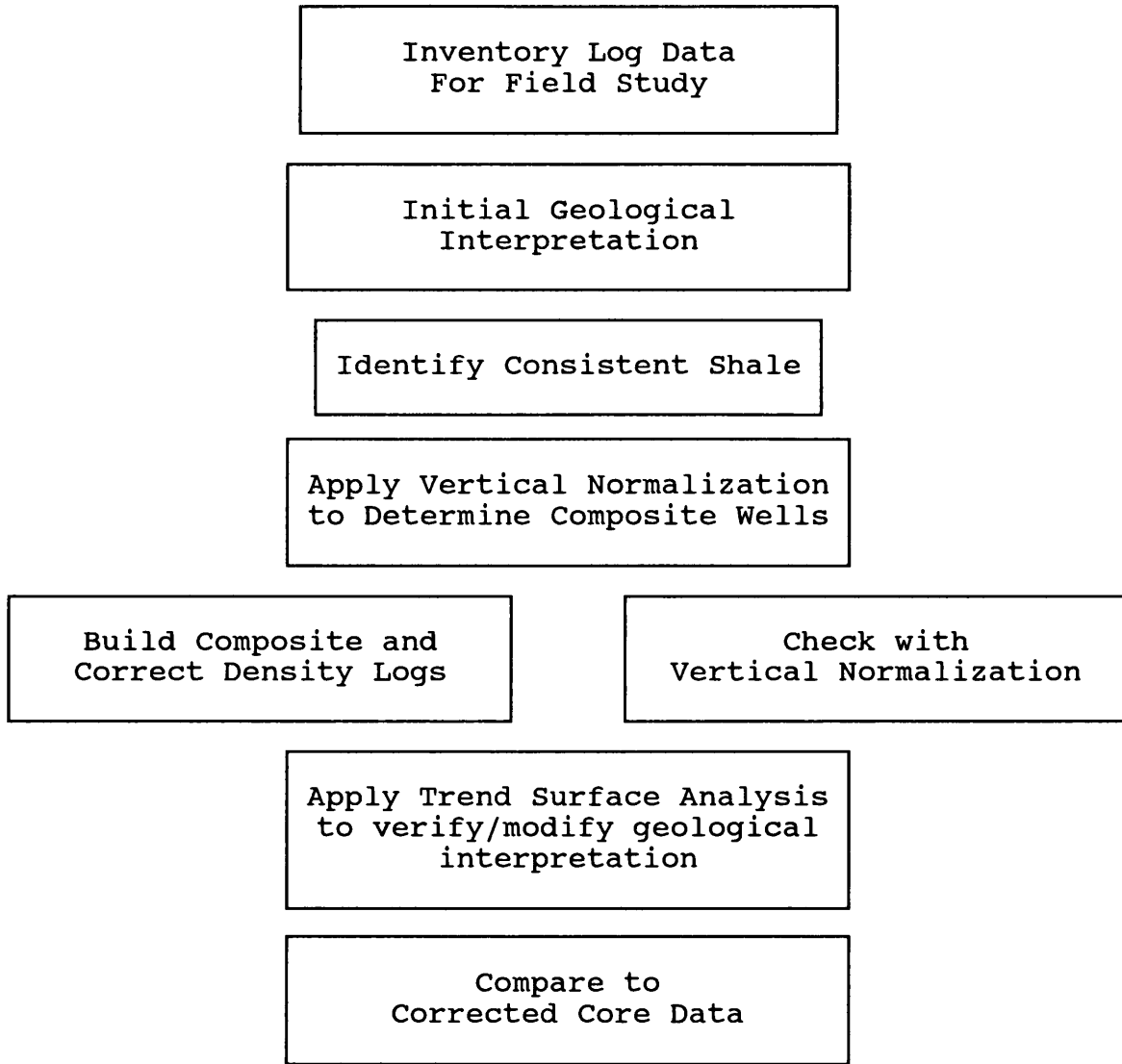


Figure 5

Normalizing with Composite Histogram

Chapter 4

Types of Log Errors

Log errors will probably always exist to some degree. It is not the intent of the author to point the finger at service companies as the source of this error, yet hopefully provide a tool to interpret these errors. To this end the author hopes to be a part of the solution and not a part of the problem.

4.1 Intermittent Errors

Intermittent or erratic tool response should be recognized as such in the field, and new tools should be sent to the wellsite. If this type of error does manage to get by in the field, then a note that the log is in error should be recorded on the final print. This type of error holds no hope of being corrected or normalized. The possible causes of intermittent errors are so large and varied that it would not be possible to characterize them. Any tool is subject to intermittent error.

4.2 Shift Errors

Shift errors are probably due to a change in calibration at downhole conditions. A shift error moves the appropriate tool response by a constant magnitude. Thus at any depth in a well, if the shift error can be determined, this amount can be added to the erroneous log response to arrive at a vertically normalized log response. As noted previously by Connolly (1974), this type of error may exist even if the calibrations before and after survey appear valid.

Normalizing Density logs with shift type errors is possible with the proper interpretation of $\Delta\rho$ and caliper information. This may allow data initially believed to be miscalibrated or incorrect to be normalized.

Density tools, especially those prior to the LDT, have this type of error. The bulk density is usually off by a constant amount which could be positive or negative. Nienast and Knox (1973) suspect that more than 70% of the Density logs are reading erroneously. Table 2 summarizes their perceived accuracy of porosity logs as of 1973.

Shift errors (as defined with ξ) are corrected by adding the shift factor (SF) to the log as will be discussed in Chapters 5 and 6. Induction tools, may also have this

type of error.

Table 2
Log Error Chart (After Nienast and Knox 1973)

Porosity Tool	Percent of Time in Error	Amount of Error
Density	70	Small
Neutron	30	Medium
Sonic	15	Large

4.3 Gain, Sensitivity or Slope Error

Gain, sensitivity and slope are used interchangeable in this thesis and are described here. This is a sensitivity type of error. One example would be a sonic log that reads too high in the higher range of porosities and too low in the lower range of porosities. Figure 6 shows RMA results of normalized density porosity values vs. raw neutron or sonic values. The previous example would fall into the Type II (b) category.

Gain errors may be accompanied by the shift error, but not necessarily as in the case of the Density tool. This type of error can be normalized but requires comparison to unlike data in the same well, or vertical normalization.

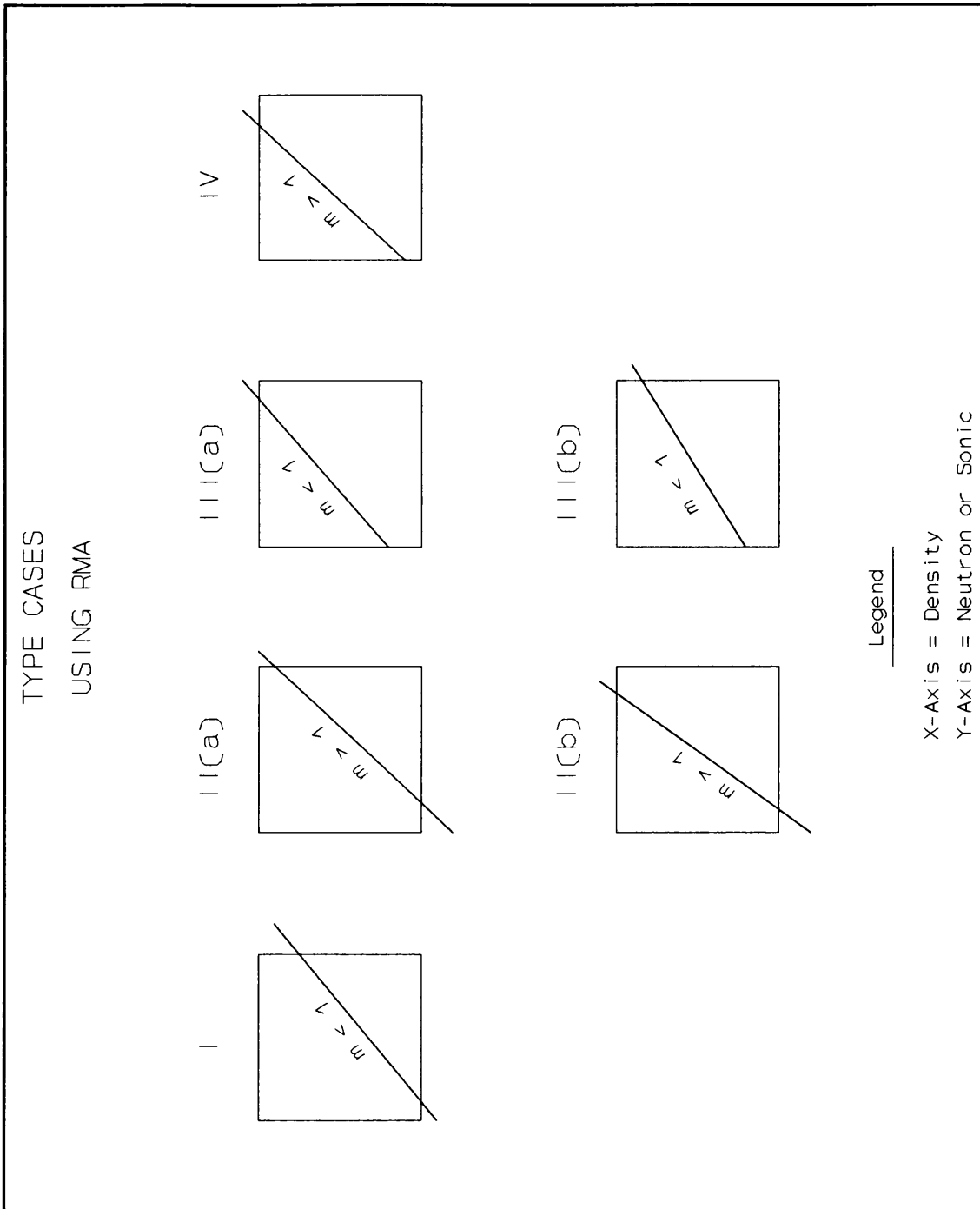


Figure 6

Type Cases of Normalization

This can be caused by an erroneous scaling of the log on the printout. Logs that usually exhibit this type of error are the sonic and neutron logs.

Gain errors are corrected by multiplying the log response by the gain factor (GF). The gain factor is the reciprocal of the gain error. These calculations are discussed in more detail in Chapters 5 and 6.

Chapter 5

Correction of Density Tool Errors

The assumption that the Density tool will be in error with a shift type error only, and that the gain (sensitivity or slope) error is zero allows the interpretation of $\Delta\rho$ and caliper information to resolve the shift error. Since this leads to the correction of log responses from the Density tool, normalized density porosity information can be attained.

5.1 The Spine and Ribs Correction

A general schematic of the density tool is shown in Figure 7. The tool is composed of a radioactive source and two detectors (near and far). The tool is self compensating for mudcake and minor borehole irregularities. The equation that all density tools use to give bulk density is;

$$\rho_b = \rho_{b_{LS}} + \Delta\rho$$

$\rho_{b_{LS}}$ is the long spaced detector counting rate, which is corrected for mudcake by the computation of $\Delta\rho$ from a spine

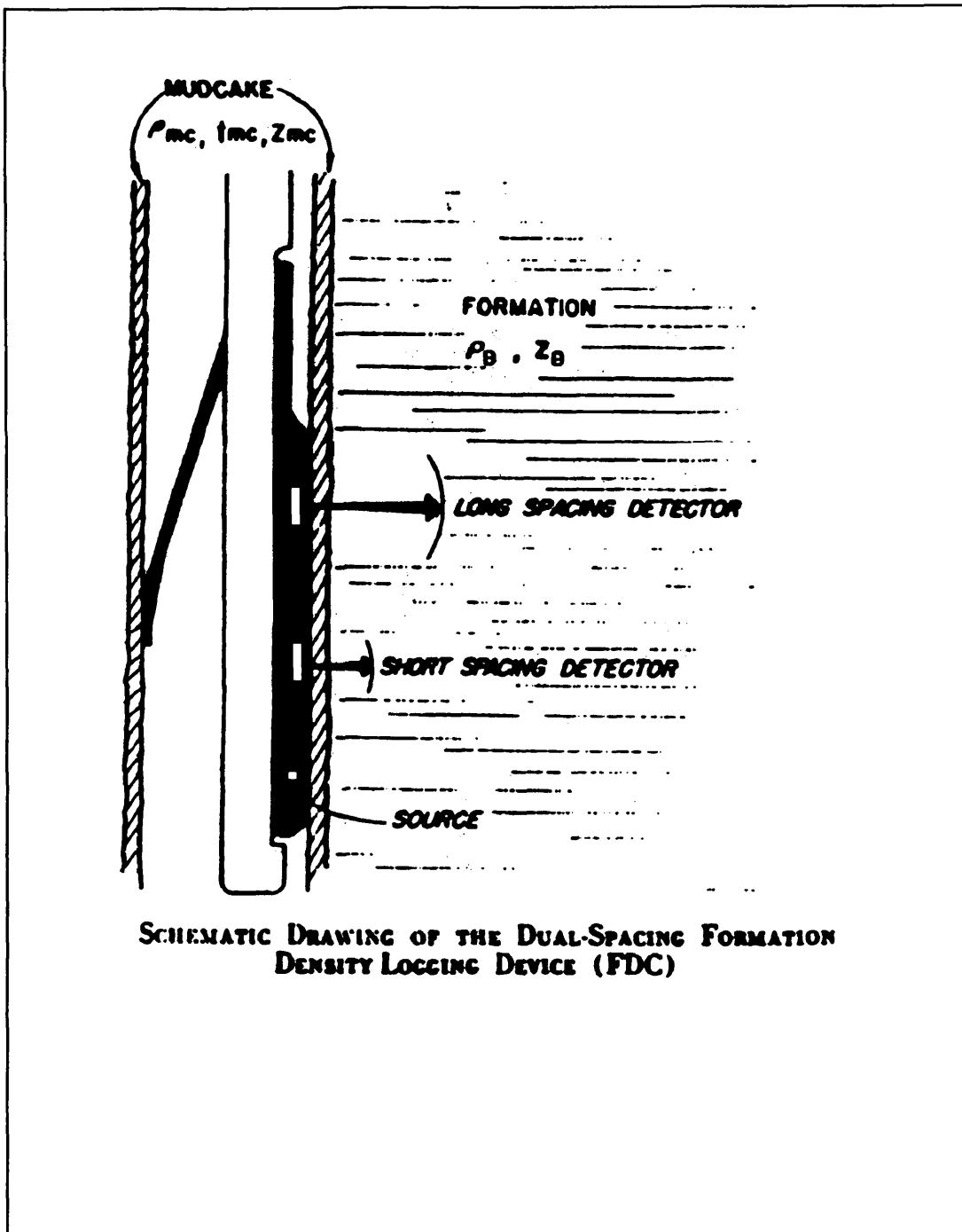


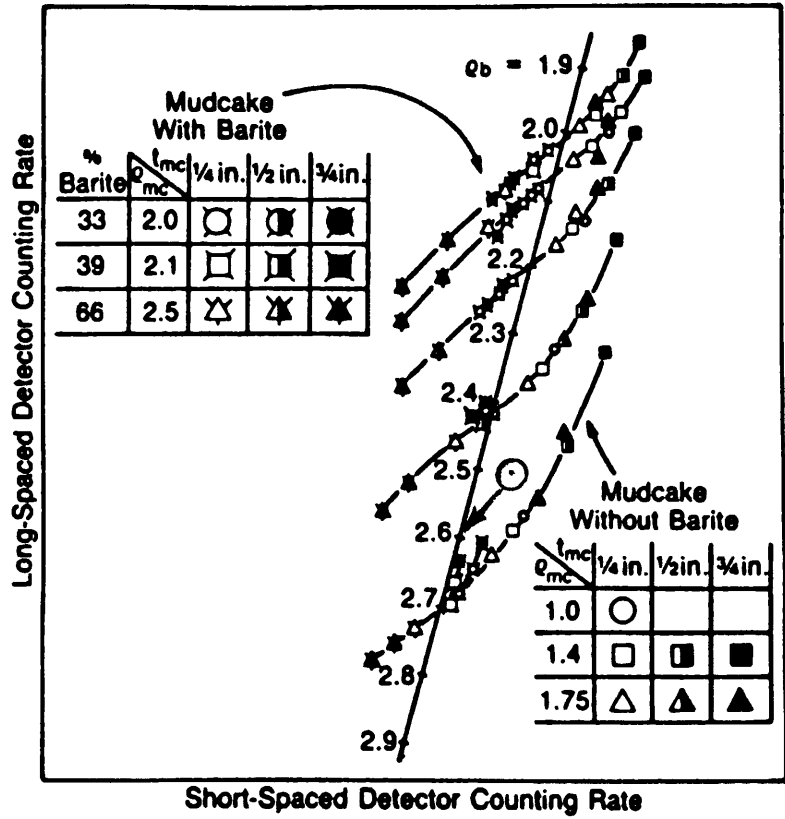
Figure 7

Density Tool Schematic (After Wahl et al. 1964)

and rib chart for the FDC density tools. A spine and rib chart for the FDC density tools is in the Schlumberger(1989) Log Interpretation Principles/Applications book on page 5-10 and included here as Figure 8. The short-spaced detector counting rate may not intersect the long-spaced detector counting rate on the straight line (spine) or no mudcake spine. If the intersection point lies to the right of the spine then the correction is added to the long spaced detector and is the distance from the intersection point to the spine, along the ribs. An example data point and subsequent correction to the spine is shown in Figure 8. Natural mud and mudcake is to the right of the line and barite mud and mudcake is to the left of the line. Since the true electrical density as measured in gamma counts is not exactly linearly related to true density Tittman and Wahl (1965) proposed Figure 9. It should be noted that the correction in sandstone without gas is quite small and in the range of 0.003 to 0.004 g/cc. This correction is added to the bulk density value from the log and decreases porosity by about 0.2 porosity units (p.u.).

5.2 Calibration

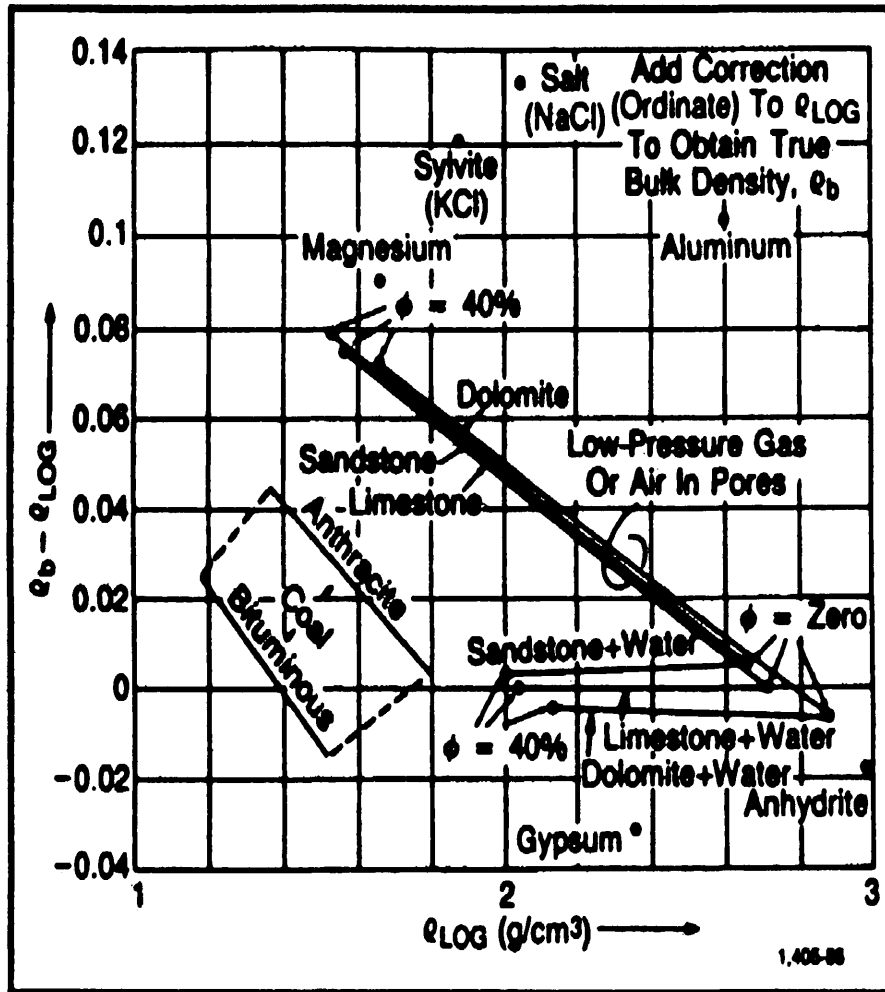
In order to calibrate the Density tool, large



"Spine-and-ribs" plot, showing response of FDC counting rates to mudcake.

Figure 8

Spine and Ribs Plot (After Schlumberger 1989)



Correction needed to get true bulk density from log density.

Figure 9

Bulk Density Correction (After Tittman 1965)

homogeneous blocks of sulfur or aluminum are used. Since the density of the material is known, counts are monitored and the results converted to bulk density. Since these calibration blocks were so large and heavy they may not have been used at the wellsite. So a small jig was made that would hold a radioactive test pill so that when properly positioned on the tool, would mimic the response that the tool would have with the large sulfur block. Calibration is necessary and should always be done but this is no guarantee that any tool will give correct results.

5.3 Possible Types of Density Tool Errors

5.3.1 Detectors

The detectors in the FDC tool are subject to calibration (usually monthly shop tests) and have a finite life for dependable readings. The detectors are built such that they use energy in the appropriate plateau or window where the current is flat. Another interesting note is the detector measurements. Figure 10 shows the detector response as a function of input current (I) and measured voltage (E). The area of almost constant (I) in a small range of (E) is the area where the detection should take

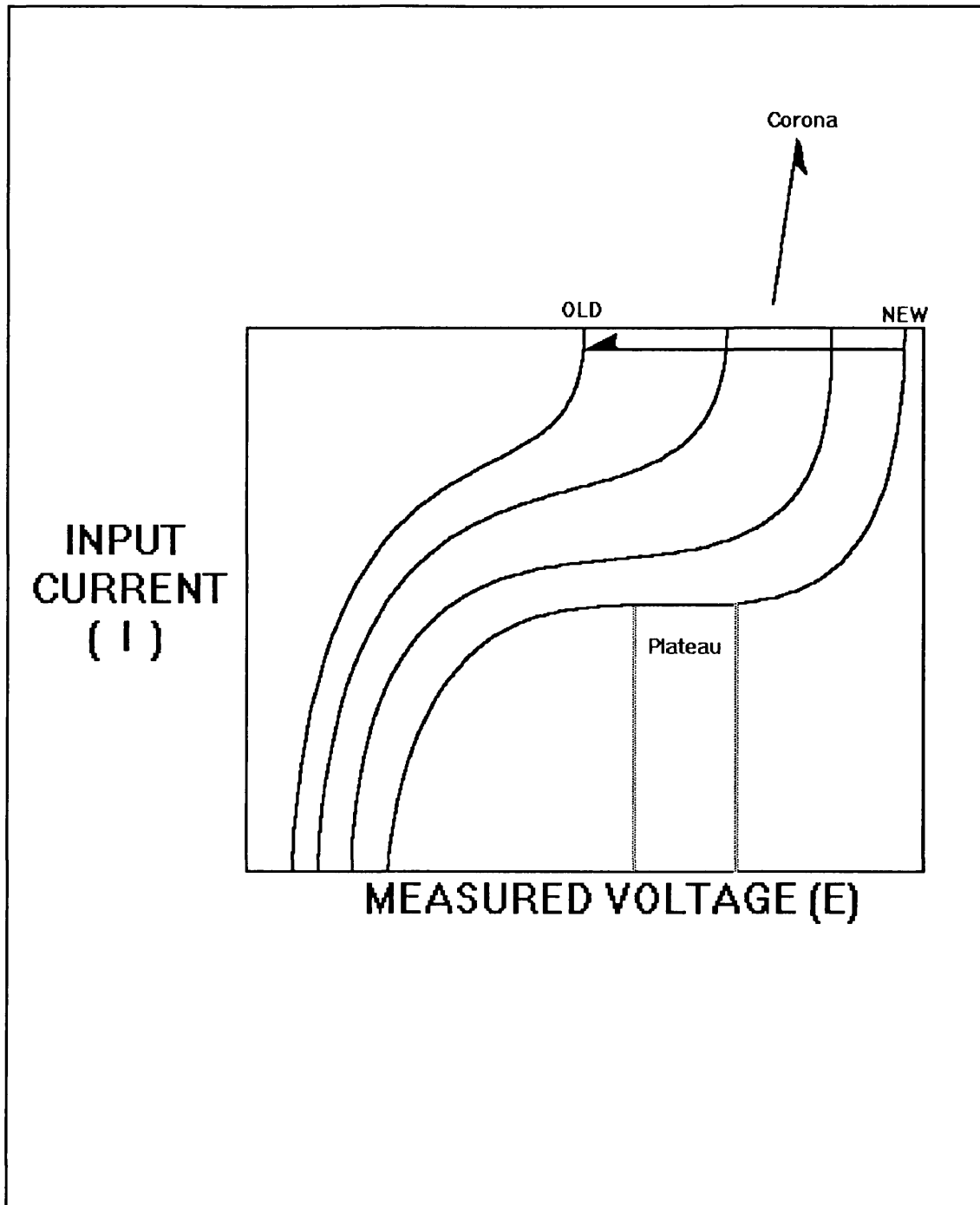


Figure 10

Detector Window of Operation

place. Since the detector has a changing shape with age the (E) range gets smaller and then dies in a complete corona. The life of a detector is analogous to the life of a light bulb. Sometimes they last years, and sometimes just a month or so. It is difficult to infer how many different variations in shift error this may cause but hopefully the error is also a constant shift and could be either positive or negative.

5.3.2 Pad Wear

Since the Density tool is a contact device, the associated pad is subject to wear. The near detector is probably most affected by this wear and gives a reading that the formation is too light (not dense enough). This in turn makes the algorithm of the Spine and Ribs plot believe that there is mudcake and the $\Delta\rho$ correction is made unnecessarily. Thus as seen in Figure 8, for a well drilled with native or low solids mud without barite, a reading in the case of pad wear would be to the right of the tilted spine. The corresponding histogram of $\Delta\rho$ for pad wear would look like Figure 11 which has a mode of approximately +0.020 g/cc. This is also what a histogram of a properly reading log would look like in a zone with some mudcake. In order

to differentiate between the two, a closer inspection of the caliper from the density tool run should be analyzed especially in dense zero porosity rock and also in permeable rocks. If mudcake does exist, this data should be removed from the histogram and the remaining correction should be applied to the raw log response. Pad wear is a positive shift in the $\Delta\rho$ histogram and should therefore be subtracted from the bulk density trace.

5.3.3 Hole Conditions

Another type of error that has been noticed in analysis of density log data is the error due to the lack of contact with the borehole in small holes. According to the Schlumberger chartbook holes between 6 and 9 inches should not require borehole correction. Figure 12 shows a density tool in a standard 8 inch hole making perfect pad contact. This does not always appear to be the case in boreholes 6 inches or less in diameter. Partial pad contact will occur and the central part of the pad will not be in contact with the borehole as seen in Figure 13.

If a keyseat or slot exists on one side of the borehole the error is worse since the density tool will probably follow the long axis of the borehole. Both detectors are

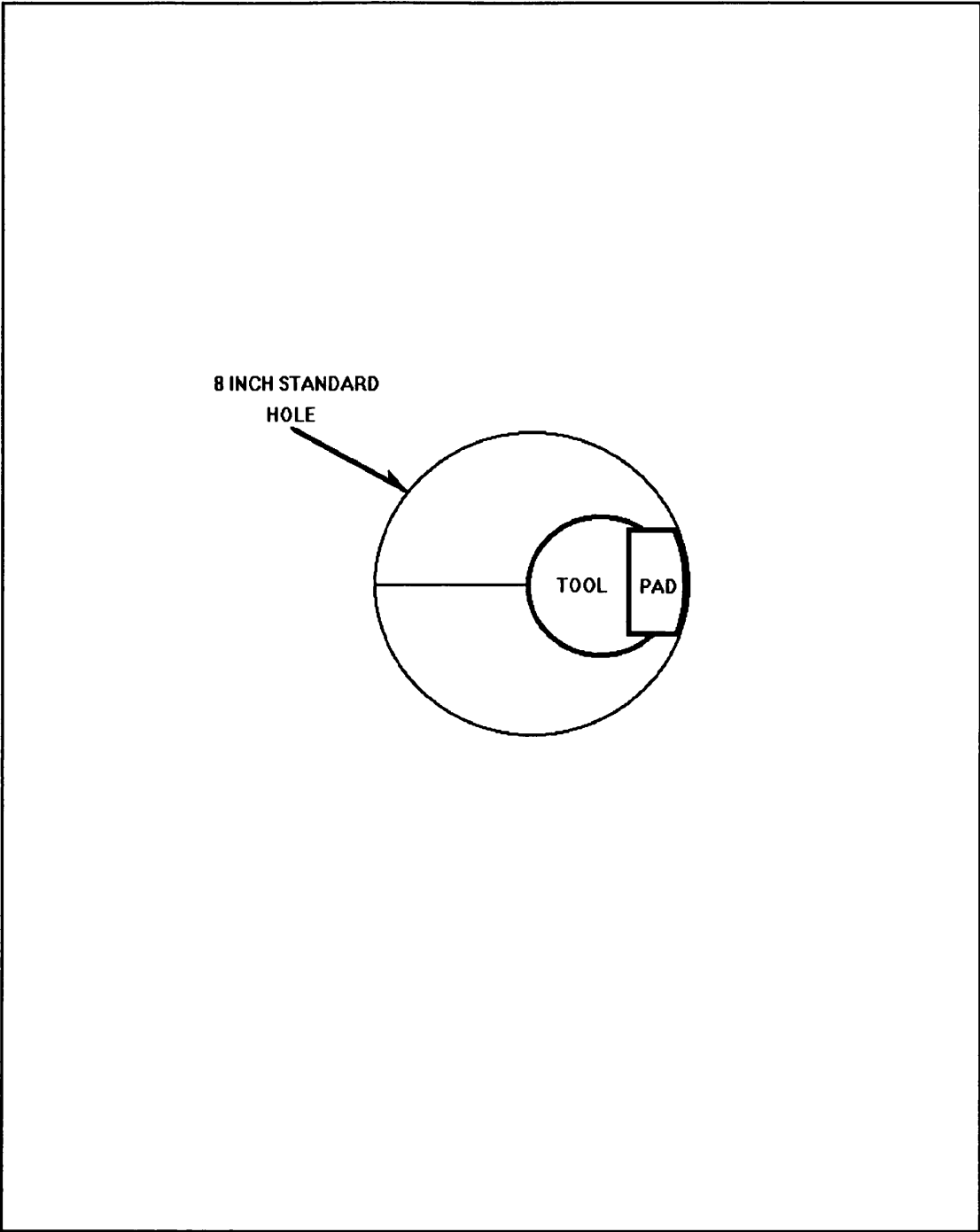


Figure 12

Density Tool in Standard 8" Hole

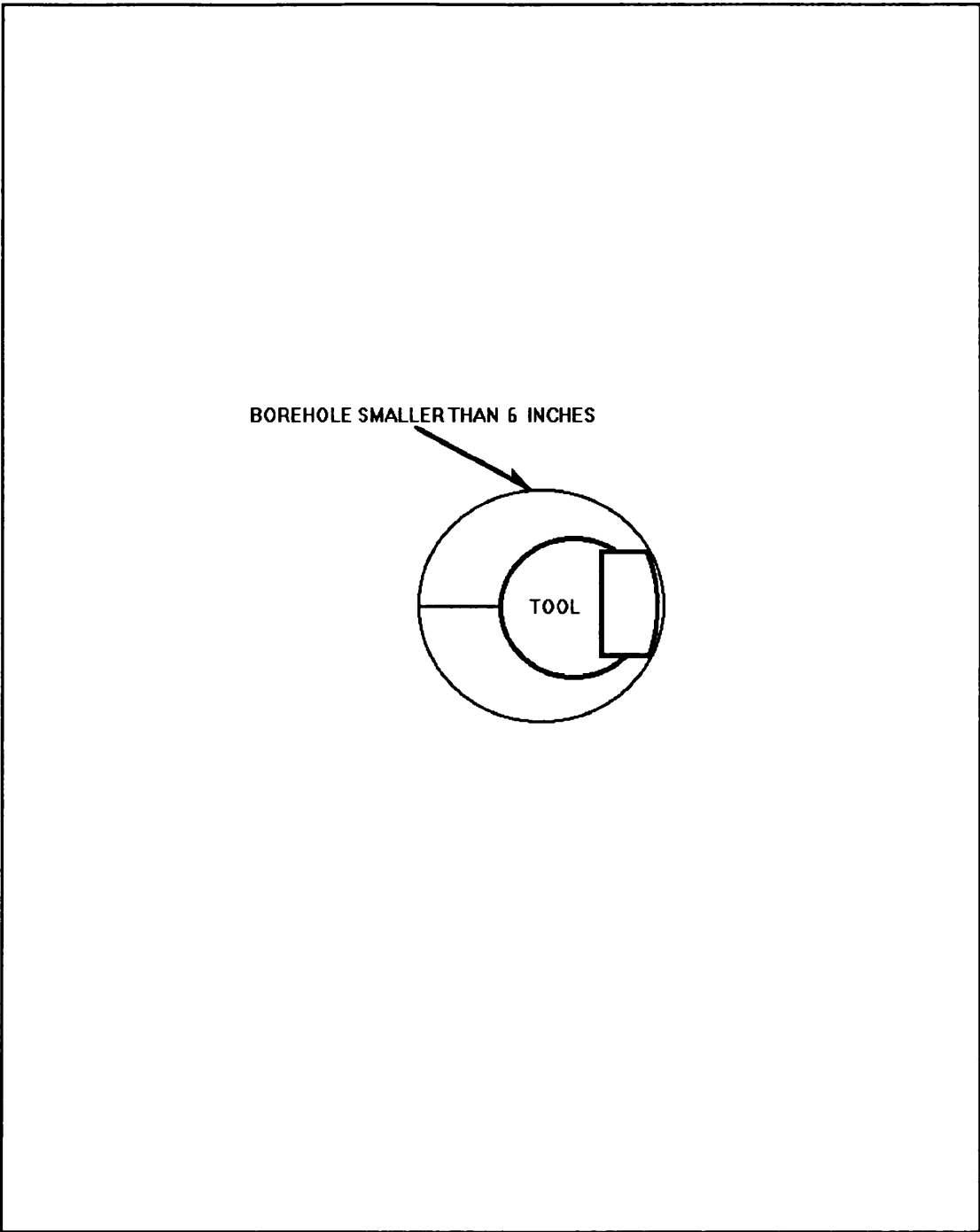


Figure 13

Density Tool in Hole Smaller than 6 inches

adversely affected which could possibly give a positive or negative correction from the $\Delta\rho$ histogram.

Even when the borehole is at the standardized 8 inches according to the caliper, if the shape is elliptical, small hole affects can occur as seen in Figure 14.

For enlarged boreholes (>9") chart Por-15a is available in the Schlumberger (1989) chartbook to correct the bulk density values. The pad does not contact the borehole on the edges as seen in Figure 15.

5.4 FDC vs. LDT Algorithm

On the current LDT there is a small source at the bottom of the pad which allows self calibration since the gamma rays travel through a constant amount of steel regardless of pad wear. While this is a calibration improvement, the $\Delta\rho$ correction and the spine-rib style of computing it has been changed. The algorithm is internal to service company computing processes and not available to industry. The real problem with the unknown algorithm is the answers that it gives. When the density pad does not touch the borehole and is essentially measuring mud density, the LDT delta rho algorithm reads **zero**. Therefore, there must be some maximum delta rho correction that the algorithm

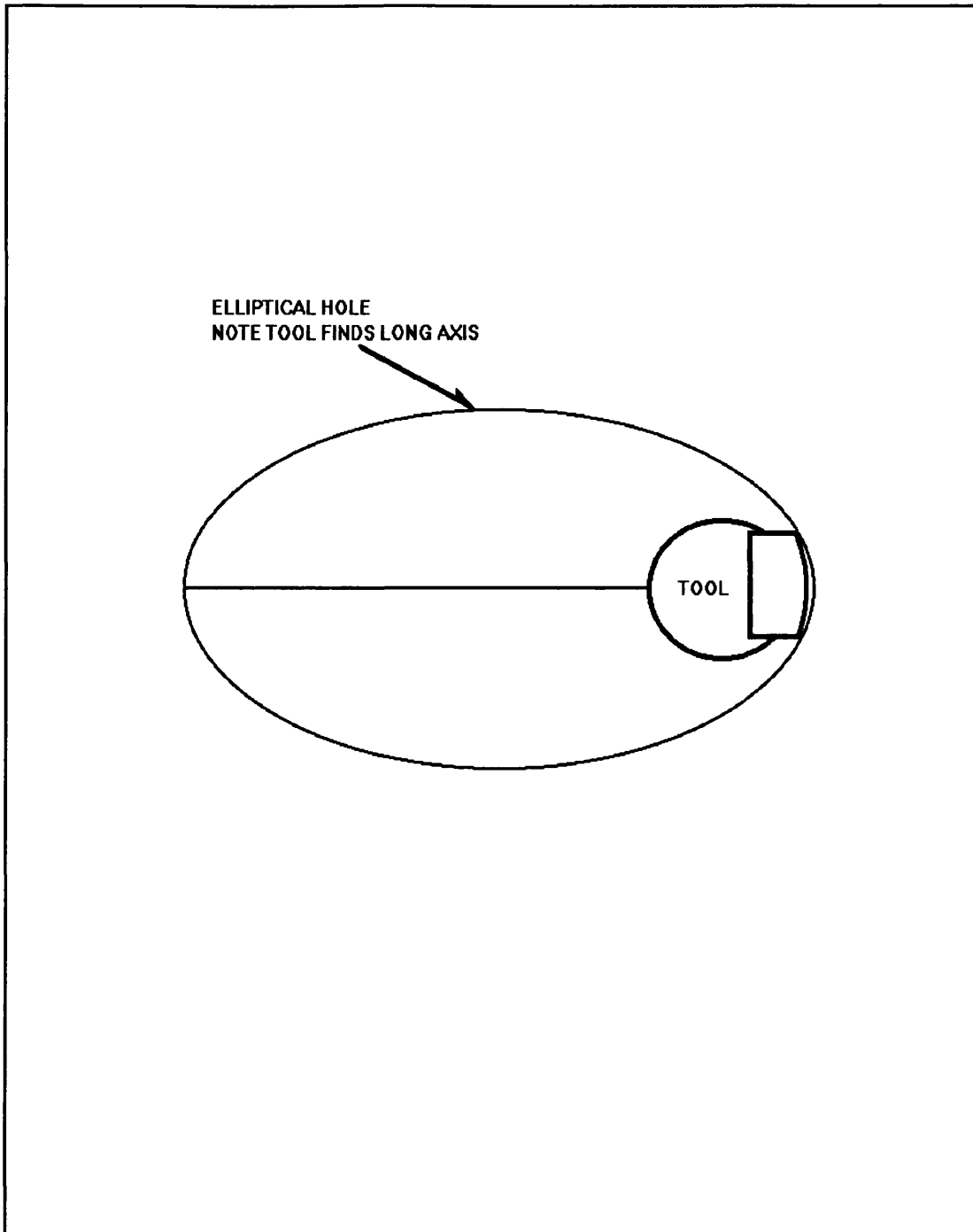


Figure 14

Density Tool in Elliptical Hole

will calculate before starting back to **zero** when the tool is not touching the borehole at all. In the previously referenced Schlumberger book it states;

The procedure for mudcake and borehole rugosity compensation with Litho-Density tool uses 'spine and rib' as done with the FDC tool. Because of the fixed radius of curvature of the measuring device surface, borehole size also influences the measurement. The borehole-size correction is shown in Chart Por-5.

On a first reading, one might not notice that the correction algorithm for $\Delta\rho$ is not given. Also the correction for borehole is not on Chart Por-5, but rather on the opposing page in Chart Por-15a.

5.5 Conclusion

Finally after all the types of errors that could cause the density tool to respond improperly, if mudcake and rugose borehole are discriminated from the data set then the sum of the errors is reflected by the mode of the remaining histogram. This delta rho histogram is made from the delta rho trace which is plotted as a continuous trace with bulk density information on the original hard copy of the log. Once the data is properly discriminated, the error is added or subtracted from the raw log bulk density trace. For example, in Figure 11 the mode is +0.020 g/cc thus the shift

error (ξ) is -0.020 g/cc. The shift error (ξ) is then added to the bulk density values to normalize the density tool response a constant amount for all depths of that log run. This normalizes the density data so that it can be transformed to porosity with the well known equation:

$$\phi_D = \frac{\rho_{ma} - \rho_{bNormalized}}{\rho_{ma} - \rho_{fl}}$$

Where

ϕ_D	= Density porosity
$\rho_{bNormalized}$	= Normalized Bulk Density in g/cc
ρ_{ma}	= Matrix Density in g/cc
ρ_{fl}	= Fluid Density in g/cc

Chapter 6

Procedure for Vertical Normalization

Although this is the procedure that was used on the Anschutz Ranch East field log data, it is meant as a guide only, since tool types, mud programs, line speeds, etc may vary in infinite combinations. The Anschutz Ranch East field produces from Eolian Nugget sandstone. The reservoir is found in the hanging wall of a thrust fault at depths ranging from 11,000 to 14,000 feet true vertical depth. Eleven wells were cored in this field and select wells are used throughout this chapter for examples.

6.1 Inventory of Logs and Digital Database

This may appear to be trite when first approaching a field study of logs, but this first step will help minimize time spent analyzing the large database of information. For example, each of the Anschutz Ranch East wells has a pay zone around 1000 feet thick. L.I.S. tapes were available for each well for all the traces from the porosity and resistivity devices. Each well has on the average about 25 traces and after interpretation of data may have as many as 64 traces. The data was in increments of one data point for

every 1/2 foot of depth.

$$1000 \times 64 \times 2 = 128,000 \text{ LOG VALUES PER WELL}$$

More than eight wells were analyzed, which raised the number of log values that needed to be calculated, interpreted and plotted, for the project to over one million! This does not include the data that is on the log header and calibration surveys. An inventory of this data was generated that helped in formulating an approach to solving the problem. The inventory process should also include a visual inspection of the hard copy of the logs and a check to verify the validity of the digital database on tape.

6.2 Initial Geological Interpretation

The geological interpretation should involve a Petroleum Engineer, Geologist, Log Analyst and whoever may have information to add. The general character of the reservoir, age, depositional environment, lithology, mud system and fluids, reservoir liquids and gases should be discussed and documented. This process, along with another visual inspection of the logs, will help formulate a solution to the problem and identify logs and depths that

should receive a first interpretation. If digital log tapes are not available, at this point it will be possible to specify what log intervals need to be digitized which returns the user (with excruciating pain) to Section 6.1.

6.3 Detailed Interpretation

For the Anschutz Ranch East field it was concluded that there was not a consistent marker bed. Therefore clean sections of sandstone were located in each well, and these zones were used for the vertical normalization model (the reader is referred back to Figure 4). The porosity devices looked within the flushed zone where invasion of the mud filtrate caused the porosity devices to see fresh water as the saturating fluid. Crossplots (Density porosity vs. Neutron porosity and Density porosity vs. Sonic porosity) were generated and there was little or no light hydrocarbon effect where flushing did not occur. When noticeable light hydrocarbon was detected it was discriminated (removed) from the data set along with all the other log parameters at those particular depths.

6.3.1 The Caliper vs. $\Delta\rho$ Crossplot

This is the first plot that will offer some insight into the performance of the density tool. Figure 16 is a caliper vs. $\Delta\rho$ crossplot showing an example of a properly performing LDT density tool from the Anschutz Ranch East Well 30-14. Note that the majority of the data lies along the zero $\Delta\rho$ (DRHO) value or are positive. For properly reading barite mud systems the data would lie along the zero $\Delta\rho$ value or negative. The Gamma Ray from the density tool (GRDN) is used as the color axis on this plot. The darker blue color spectrum indicates mudcake is apparent where the caliper (CLDN) readings are less than 8.5" which is the bit size. There has not been any data discriminated (removed) from this plot. Caution should be used when interpreting caliper information as this device does not always accurately measure the hole size but is a good indicator of the relative changes occurring in the borehole diameter. The caliper vs. $\Delta\rho$ crossplot is a qualitative quick look to determine the general performance of the density tool.

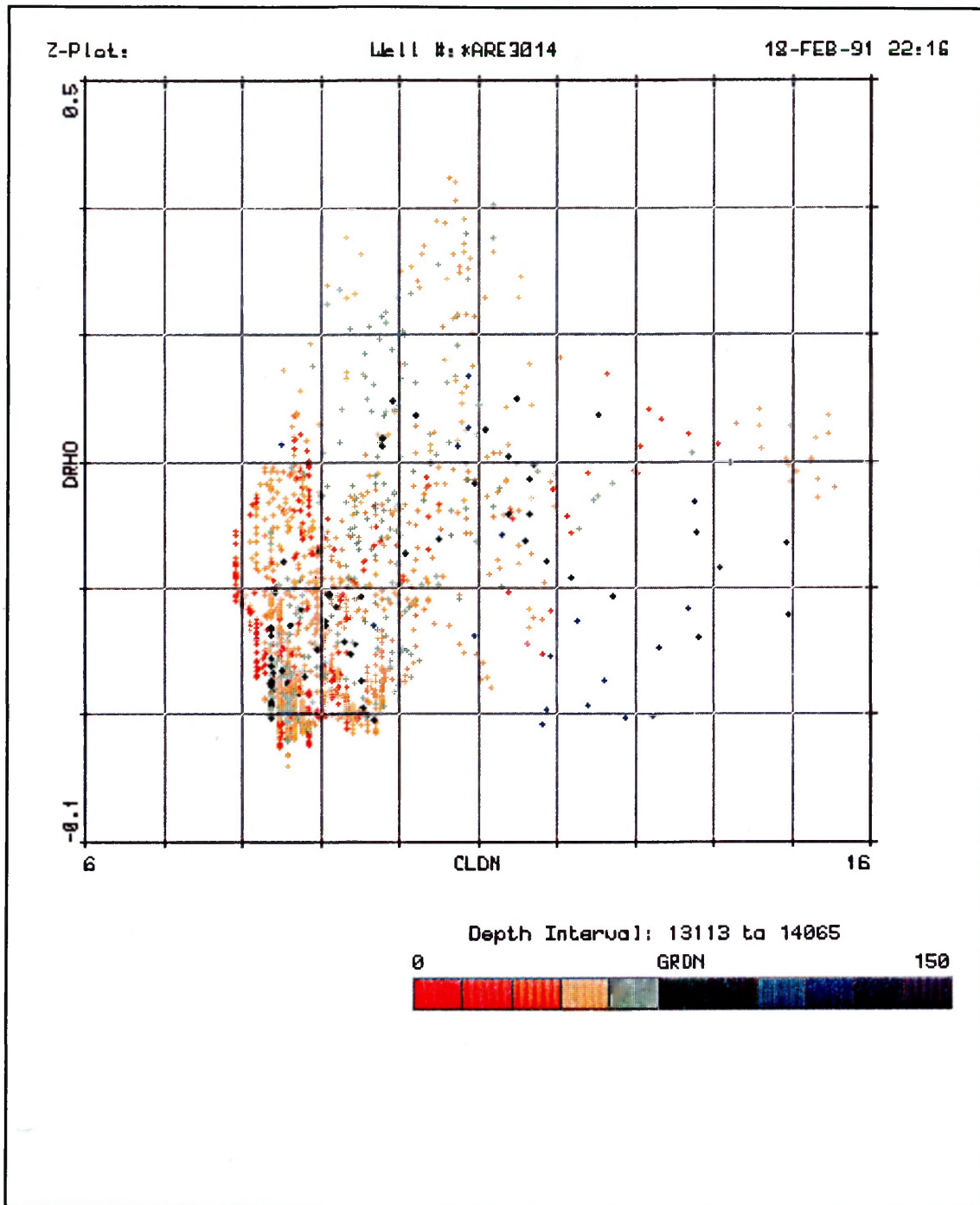


Figure 16

Caliper vs. $\Delta\rho$ Crossplot

6.3.2 The $\Delta\rho$ Histogram

Figure 17 is the $\Delta\rho$ histogram for the same well. The shape is generally lognormal, with a mode of 0.000 g/cc. If there had been anomalous readings the histogram would look like Figure 18 which is from Anschutz Ranch East field well 29-14. The mode of this $\Delta\rho$ histogram is 0.020 g/cc. This histogram does not include any obvious zones with mudcake and should have a mode of 0.000 g/cc. Since the $\Delta\rho$ correction that the tool made was high by 0.020 g/cc, porosities on the log are reported too low since;

$$\rho_b = \rho_{b_{LS}} + \Delta\rho$$

Error could be caused by many factors with pad wear being most likely, $\xi_{PW} = -0.0200$ g/cc. Adding ξ_{PW} to the original bulk density information increases porosity by $0.0200 / (2.65 - 1.00) = 1.212$ p.u.. This amount of error is not large or uncommon as seen in Table 3. In general it should be noted that the wells with better hole conditions, slower line speeds, and better mud properties had lower ξ values.

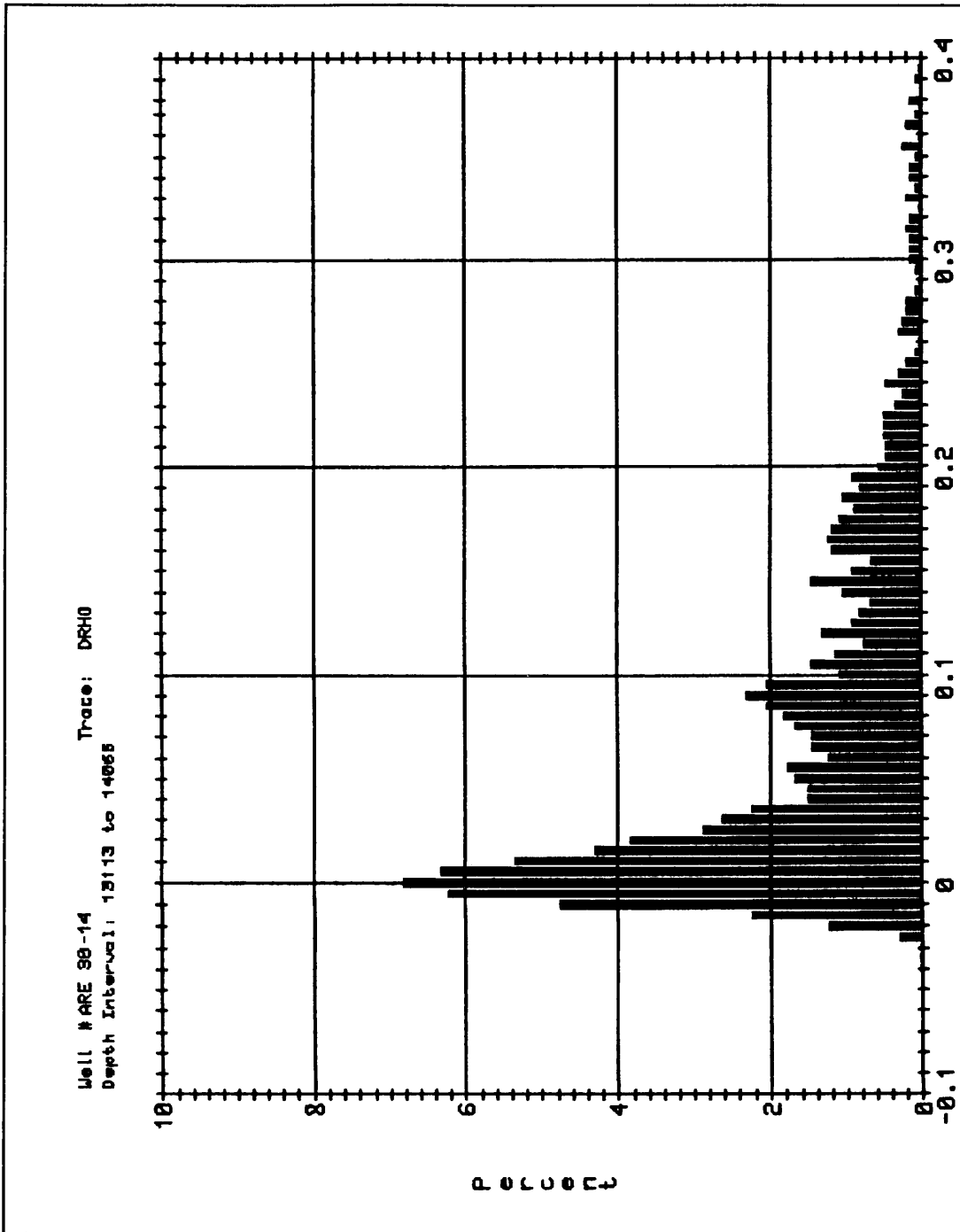


Figure 17

Histogram of $\Delta\rho$ A.R.E. # 30-14

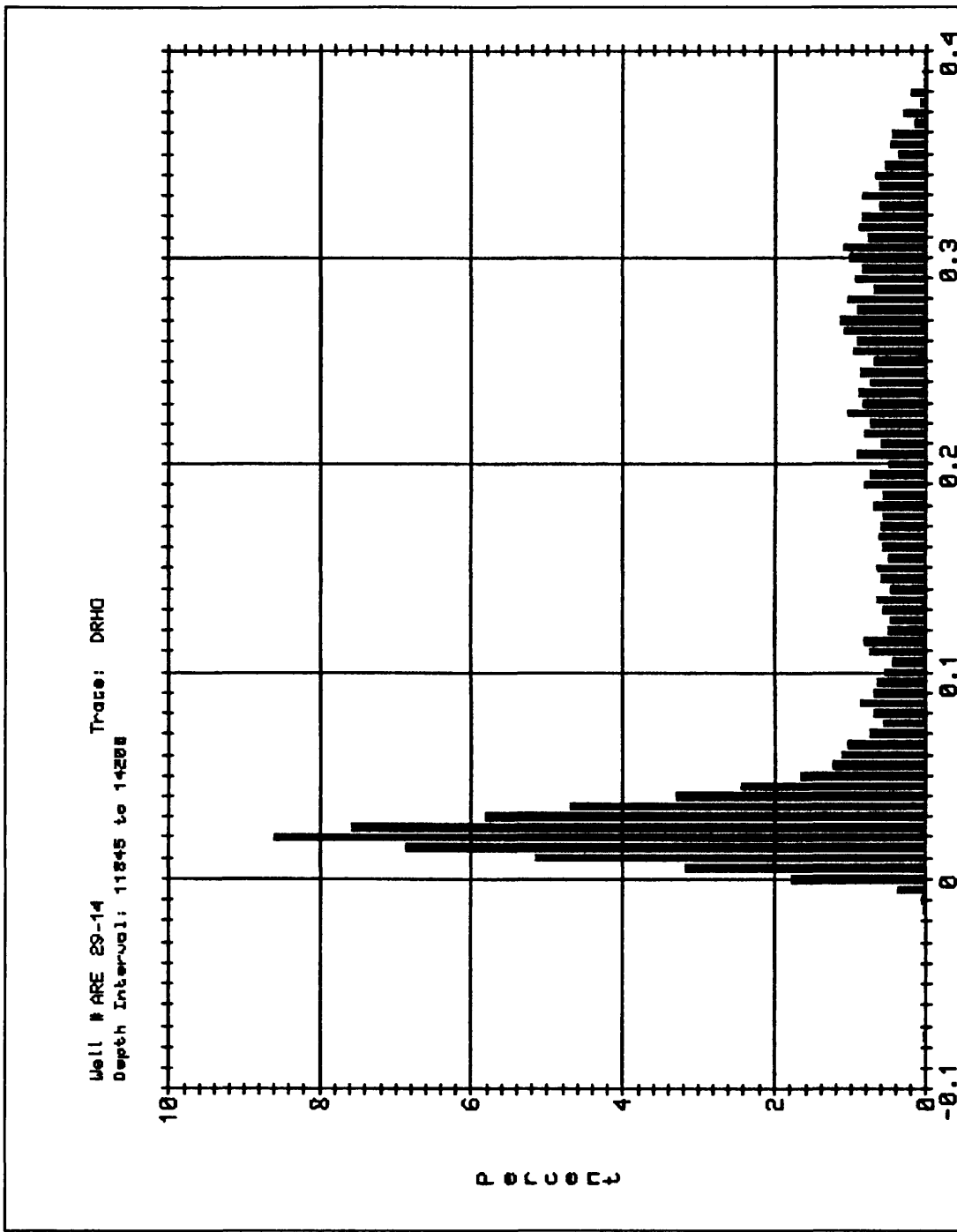


Figure 18

Histogram of $\Delta\rho$ A.R.E. # 29-14

Table 3
Density Shift Correction Data

Well Number	Tool Type	Shift Error (ξ_{PW})	Amount of Correction in p.u.	Possible Cause of Error
1101	FDC	-0.000	0.000	NONE
2016	FDC	-0.040	2.424	Pad Wear
2904	FDC	-0.021	1.273	Pad Wear
2912	FDC	-0.014	0.848	Pad Wear
2914	FDC	-0.020	1.212	Pad Wear
3014	LDT	-0.000	0.000	NONE
3608	FDC	Erratic Error	Erratic Error	Caliper Erratic
3616	LDT	0.005	-0.303	Small Hole

6.3.3 The Δq Histogram and Mudcake

Note the Anschutz Ranch East well 29-12 Δq histogram in Figure 19. Interpretation of the Δq histogram showed a fairly high shift error of $\xi = -0.034$ g/cc for the interval. When the caliper was investigated further it was observed that there was a considerable amount of mudcake in the well. When this data was discriminated (removed), the resultant histogram had a much lower shift error of $\xi = -0.014$ g/cc.

The Anschutz Ranch East well 36-08 Δq histogram is seen in Figure 20. The Δq histogram showed a fairly high shift

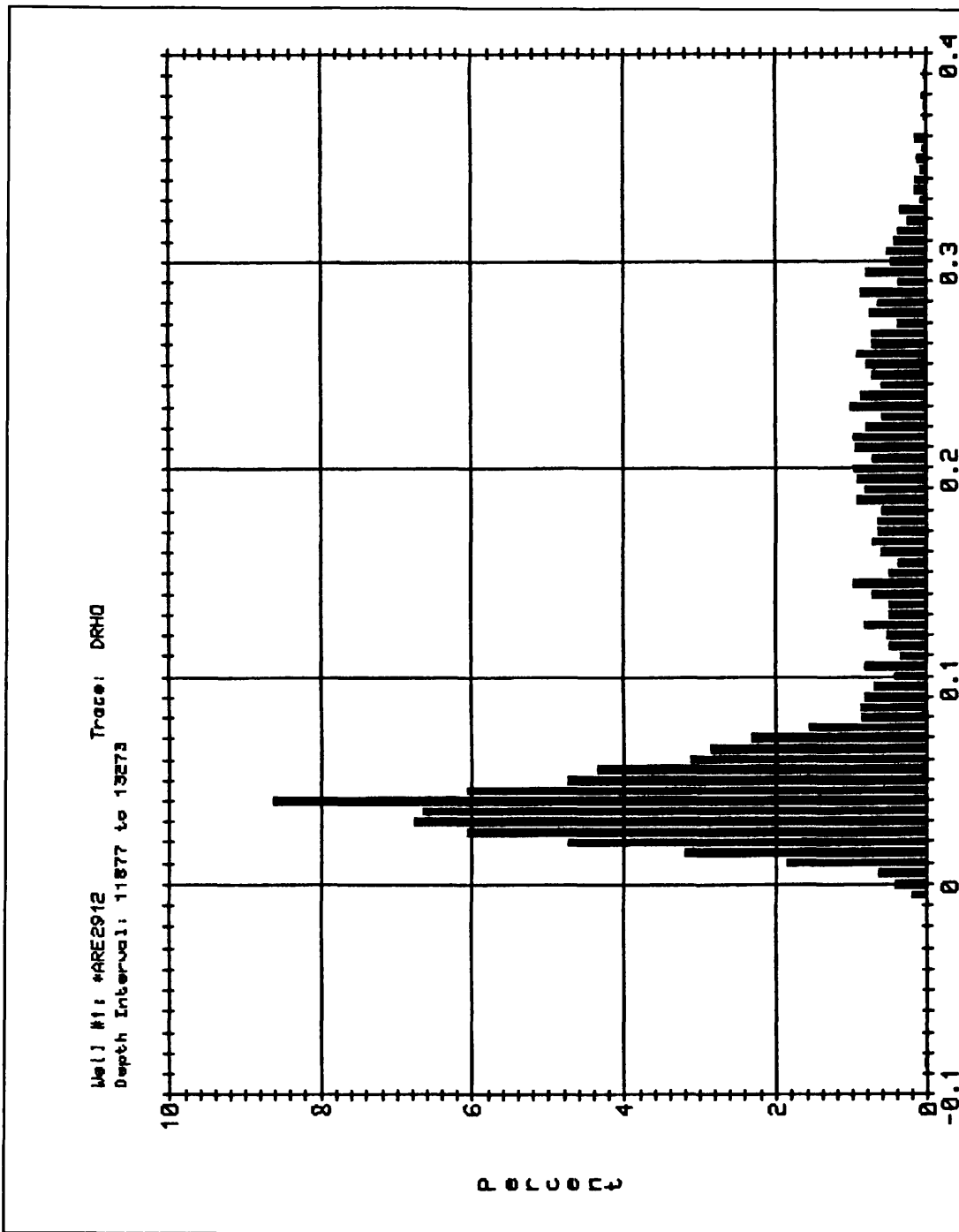


Figure 19

Histogram of $\Delta\rho$ A.R.E. # 29-12

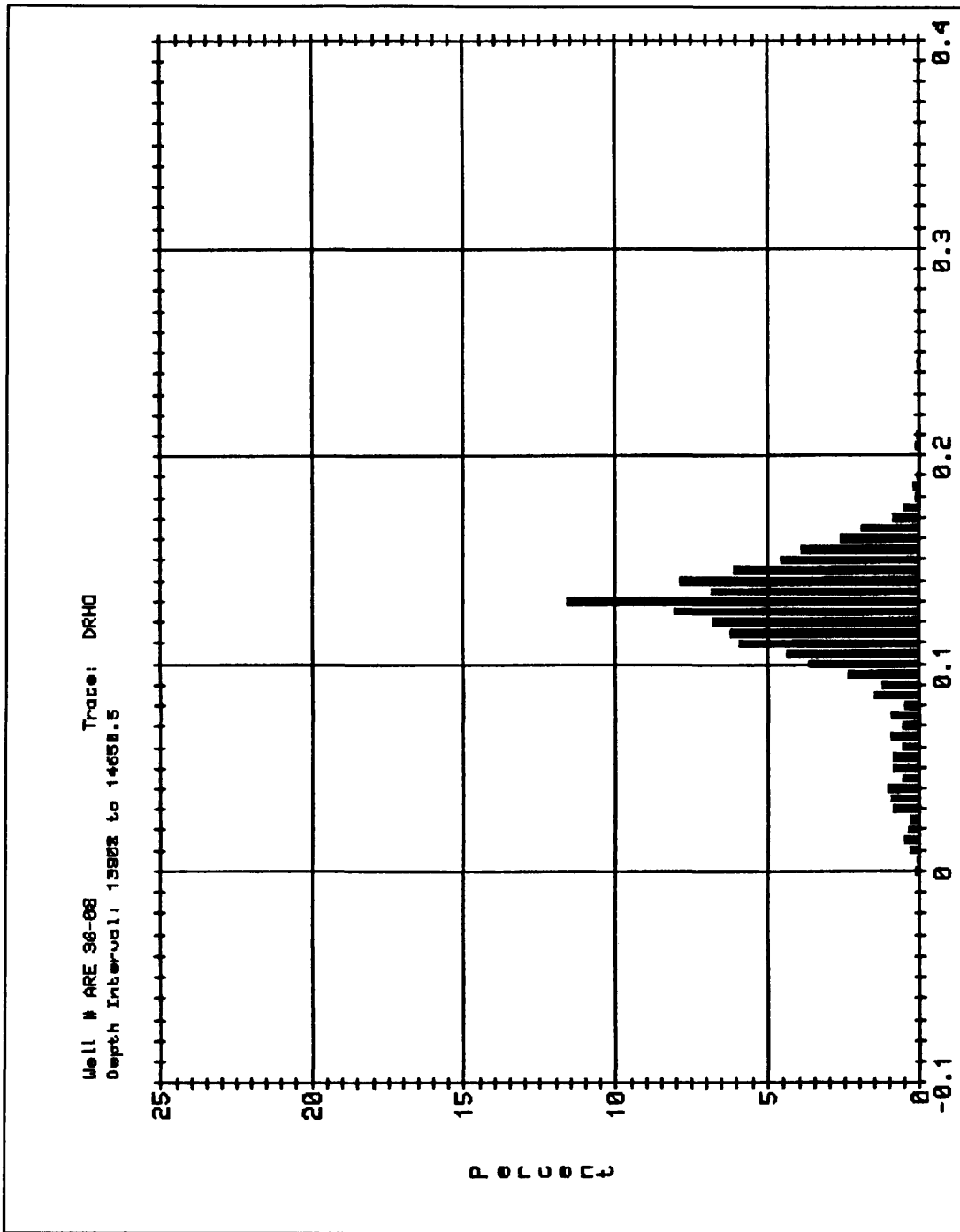


Figure 20

Histogram of $\Delta\rho$ A.R.E. # 36-08

error of $\xi = -0.130$ g/cc for the interval. When the hard copy was checked the $\Delta\rho$ information was confirmed, and the log heading noted trouble with the caliper. There were many repeat runs on the well attempting to get the caliper to function properly. From this information it is interpreted that the caliper was reading improperly and showing the hole to be smaller than it actually was. The caliper measures the hole size but also supplies the force to push the pad against the formation. Since there was no such force the tool was drifting in the hole and reading through mud. The tool responded erratically since pad location in the borehole was not consistently forced against the formation face. An erratic type of error is not correctable using normalization. Another tool should have been called to location since the logging engineer was aware of the problem. Rig costs, and hole conditions may not have allowed this option.

6.4 Depth Shift of Traces

Several trips into the hole are usually made to acquire a full log suite. For example, one trip for the resistivity tools, one trip for the combination neutron-density tools, and one trip for the sonic tool. Each of these trips will

probably be off depth slightly, due to line stretch, hole conditions, and just plain repeatability. So that a meaningful depth match between information from each trip can be made, depth shifting is required. The amount of depth shift is often small, however, the effects can cause a large error (Davis 1990). The Anschutz Ranch East data supplied for this thesis was already depth shifted by Schlumberger. Playbacks of raw log data confirmed the validity of this preprocessing.

Data from a cored interval will also require depth shifting when compared to log data. Cross correlation can determine the depth shift (or lag shift as proposed by Doveton 1986). Cross correlation is borrowed from dipmeter analysis and is an alternative to visual examination and depth shifting of traces.

6.5 Environmental Corrections

Environmental corrections are presented in service company publications such as the Schlumberger Log Interpretation Chartbook (1989). These corrections are service company and tool specific. For example, to correct a Schlumberger Litho-Density tool (LDT), a specific chart (Por-15a) from the Schlumberger Log Interpretation Charbook

is used. Tools built by Western Atlas International or Halliburton Logging services require these service companies appropriate chartbooks or algorithms.

Environmental corrections depend on the tool and factors that effect responses not easily removed by tool design. The following is a list of some of these possible corrections;

Borehole Size

Mudcake Thickness

Tool Position in Hole (Centered vs. Eccentered)

Formation Fluid Salinity

Borehole Fluid Salinity

Mud Weight

Borehole Temperature

Pressure

Mud Type (Water Based vs. Oil Based)

Standoff

6.6 Vertical Resolution and Filtering

In order to properly compare porosity as measured by the neutron, density and the sonic, filtering is necessary. Each tool looks at a volume of rock and reports a value. Depending on the tool, this volume will have a specific

vertical component or resolution. Vertical resolution for the logs run in the Anschutz Ranch East field are approximately;

Neutron	-	5 feet
Density	-	1.5 feet
Sonic	-	2.0 feet

It would be nice if the neutron and sonic had vertical resolutions as low as the density in order to help define thin beds and average a smaller volume of rock. Since deconvolution (making 5 feet data look like 1.5 feet data) is not simple or without conjecture, measurement of density and sonic must be filtered or averaged so that their vertical resolution is forced to be similar to that of the neutron.

Figure 21 shows a moving average operator as proposed by Doveton (1986). Figure 22 shows the effects of receiver separation on vertical resolution (Lyle and Williams 1987). The trace labelled 'profile' is analogous to the density with 1.5 foot vertical resolution, while the trace labelled ' 2 foot log ' is approximately analogous to the sonic with a 2.0 foot vertical resolution. The neutron is approximately analogous to the ' 5 foot log '. There is a noticeable difference between these three curves.

Because neutron tool is more sensitive to the hydrogen

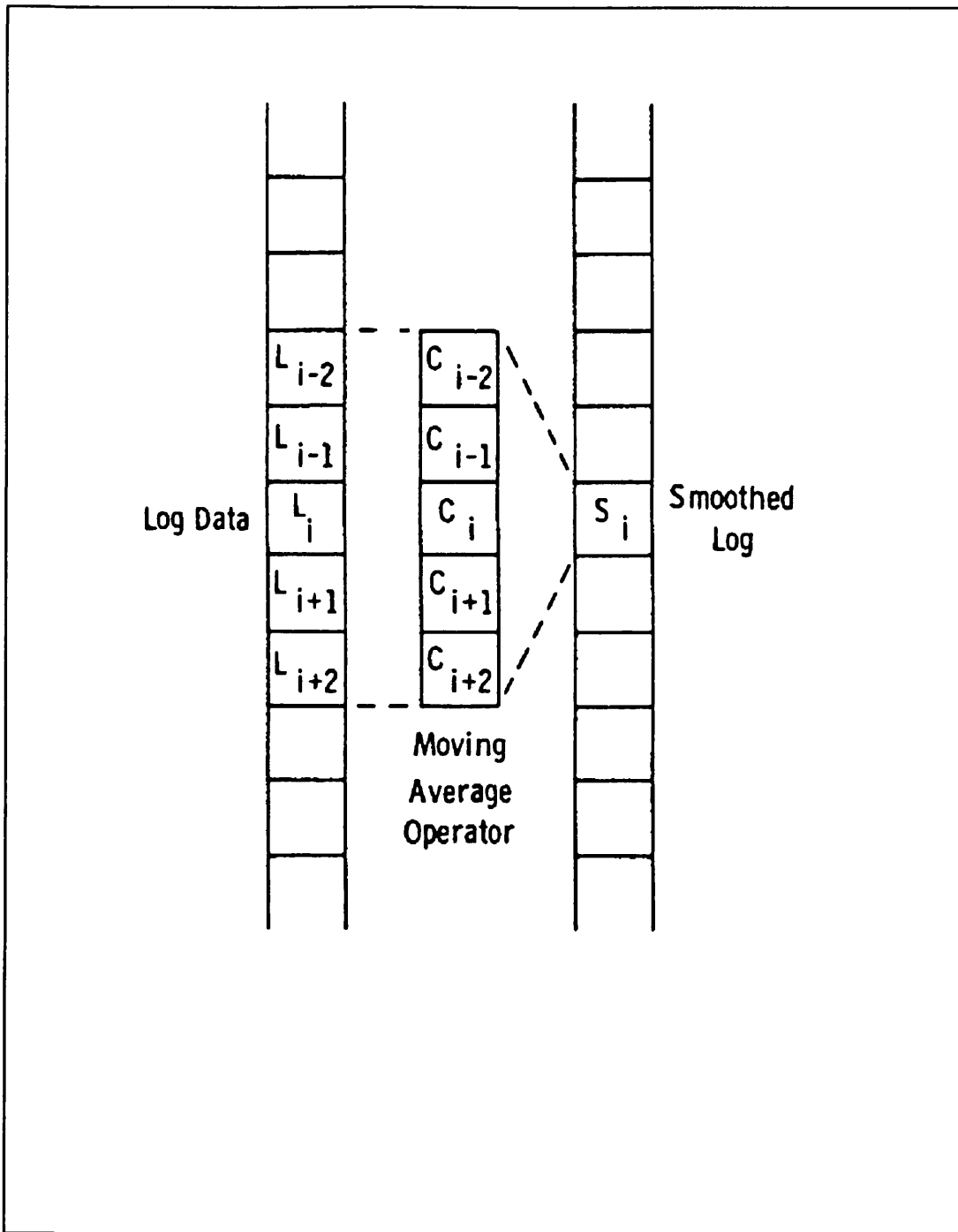


Figure 21

Operation of a Moving Average Filter(After Doveton 1986)

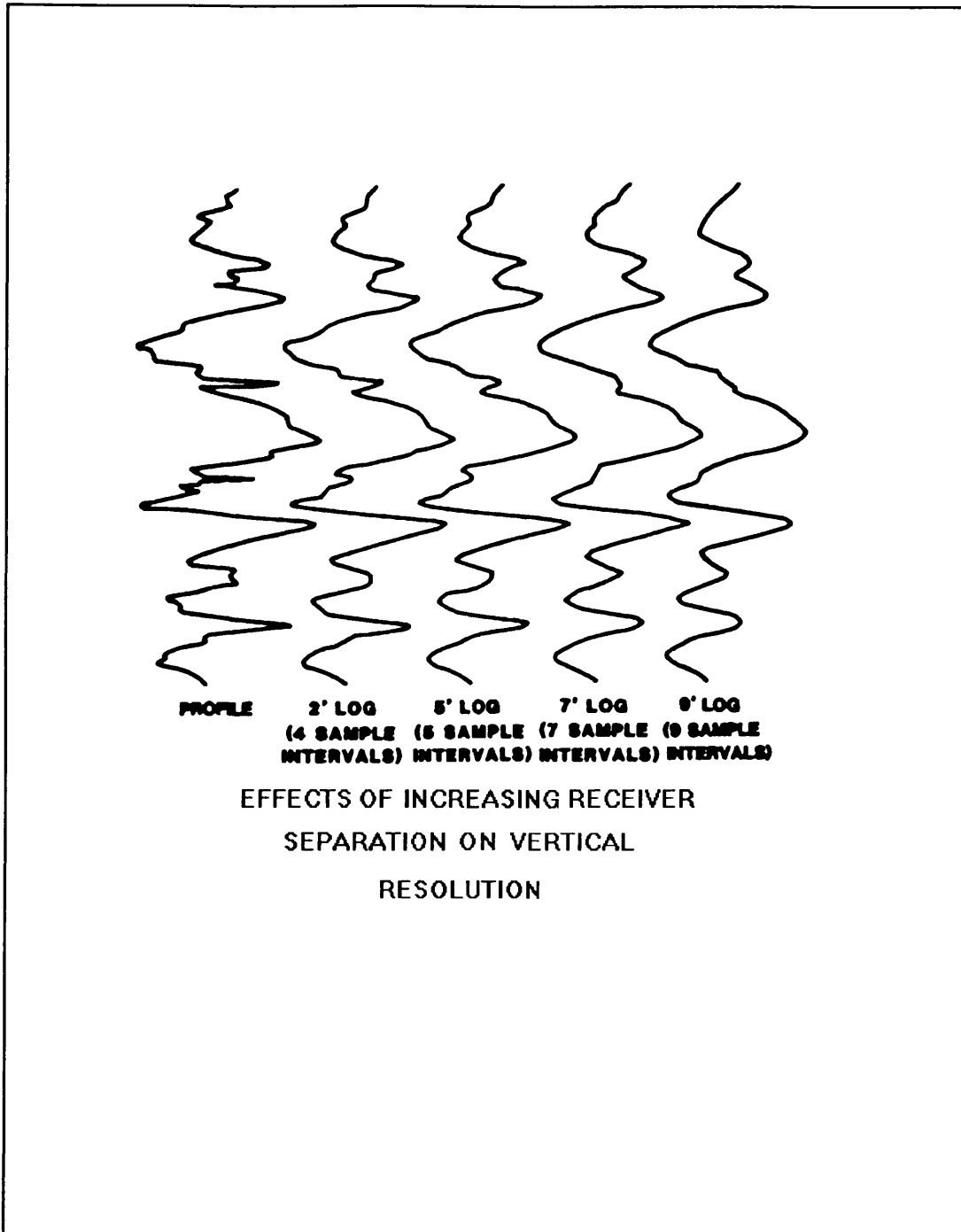


Figure 22

Filtering and Vertical Resolution (After Lyle 1987)

in shale it may look like the 'profile' trace even though its vertical resolution is approximately 5 feet. In laminated sand shale sequences the neutron will respond to thin shale beds due to their hydrogen content. In carbonates with liquid filled pores the neutron log will tend to be more like the '5 foot' log due to the lack of hydrogen contrast between beds.

Simple filters were used to remove the effect of different tools' vertical resolution for the Anschutz Ranch East field. The digital data was on 1/2 foot basis or one data point every 1/2 foot. Nine depths, on 1/2 foot basis (for the density) were filtered (4 above, 1 @ depth, and 4 below) to yield a vertical resolution of approximately 5 feet. For the sonic 9 depths on 1/2 foot basis were filtered in the same manner as the density to yield a vertical resolution of approximately 5 feet. Depth of investigation was not compensated for in this study, and probably has a very minor effect when compared to vertical resolution. D.V. Ellis (1987) has studied depth of investigation for the density and neutron tools, and the reader is referred to this work for more detailed information.

6.7 Discrimination of Data

Although mentioned previously, a detailed discussion of discrimination used for the Anschutz Ranch East field is presented. Discrimination is the selective removal of various types of data. Since the density device is affected by mudcake and rugose boreholes, data from these portions of the well need to be removed prior to using RMA as described in Appendix A. If not removed this data could cause large variations in the RMA results. $\Delta\rho$ is commonly discriminated because it is built as an indicator of both of these factors. Different values of $\Delta\rho$ are used in discrimination in the industry, however, no known method to determine this value is published. Initial difficulty in getting stable and repeatable results from RMA led to the following investigation of discriminating with $\Delta\rho$. Discrimination of data removes values from the original set of data as seen in Figure 23, showing a filter plot of $\Delta\rho$ vs. the percent of original data remaining after discrimination. As the tolerance of $\Delta\rho$ decreases, more data is removed.

Figure 24 shows results of lab data and calculated results for the FDC $\Delta\rho$ algorithm (Wahl et al. 1964). For natural muds the algorithm starts to deviate from the 45° line at about 0.010 g/cc. This should be an appropriate

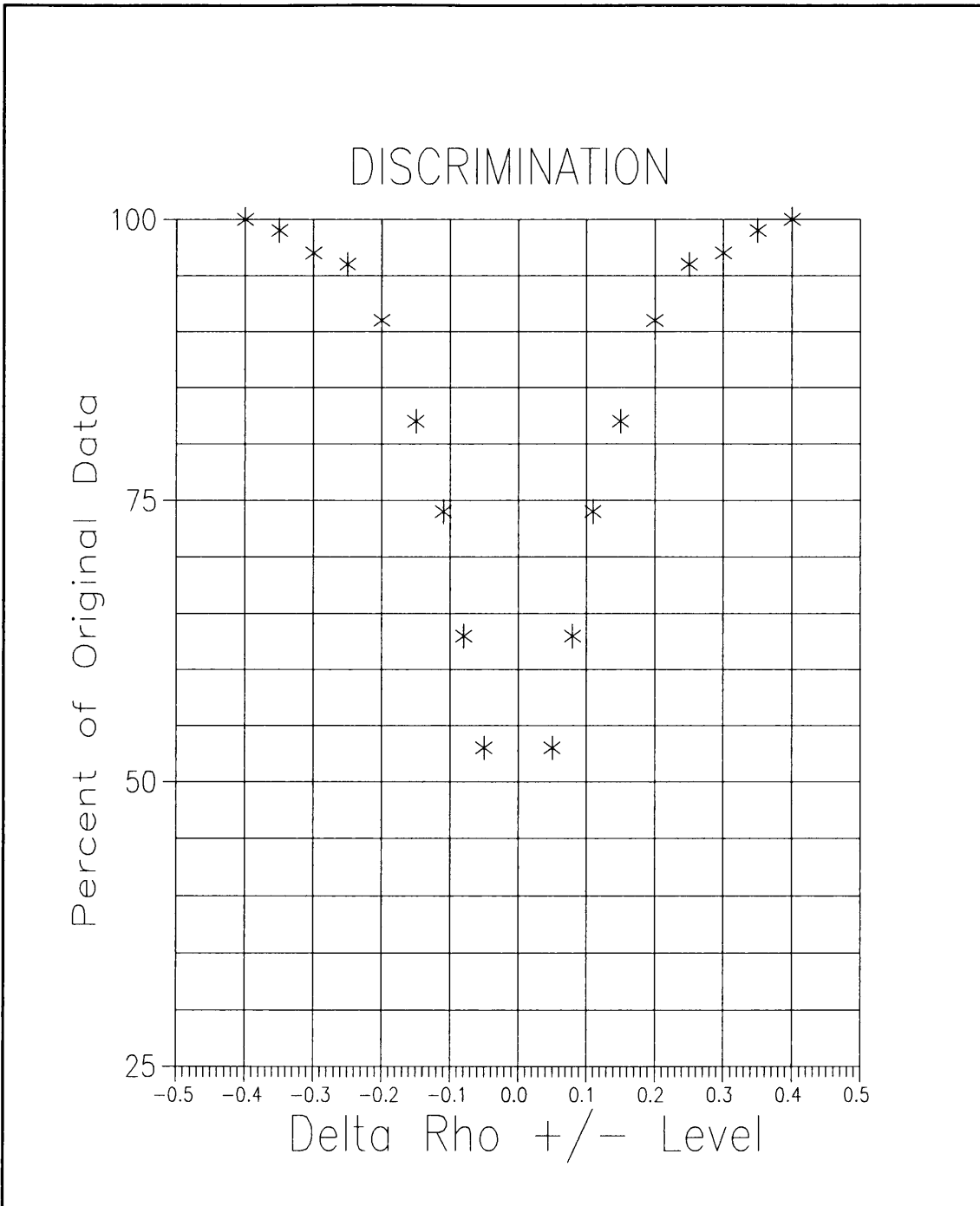


Figure 23

Delta Rho vs. % of Original Data

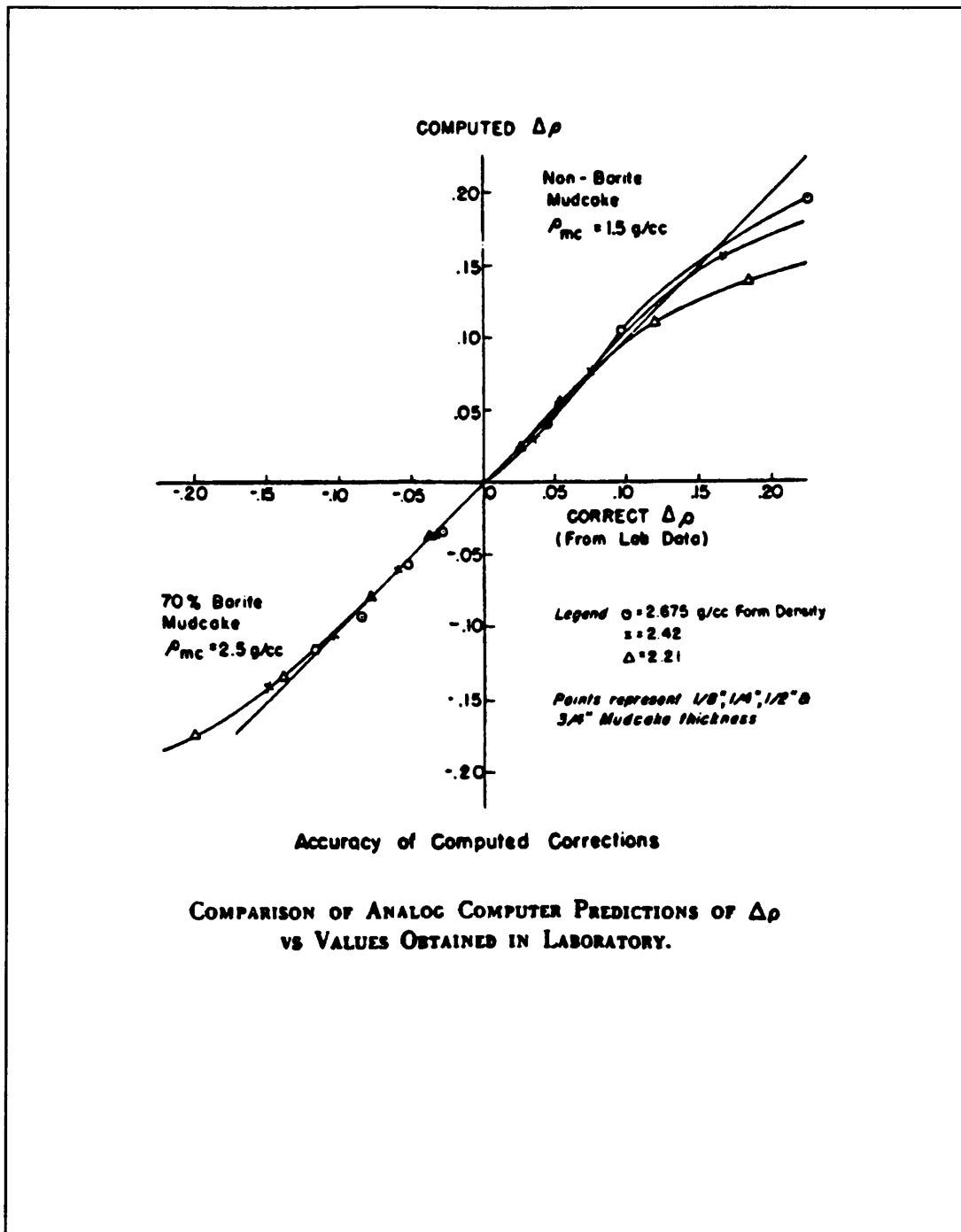


Figure 24

Accuracy of $\Delta\rho$ (After Wahl et al. 1964)

value to discriminate (remove) poor data to enhance crossplot results.

A crossplot of the Density vs. Neutron data was made for different $\Delta\rho$ filter values. The corresponding $\Delta\rho$ filter value was plotted vs. the correlation of fit or R from RMA in Figure 25. This plot shows that as the filter value approaches $\Delta\rho$ of ± 0.250 g/cc data starts to be discriminated. In the region from ± 0.250 to ± 0.100 g/cc bad data is being removed from the data, and there is a noticeable change in slope at 0.0100 g/cc. From ± 0.100 to ± 0.000 g/cc the slope of the data increases which is interpreted as meaning good data removal from the data set. Therefore a cutoff of ± 0.100 g/cc is recommended as a standard discrimination value. Referring to Figure 16 while thinking about discrimination may help the reader visualize the discrimination process.

Warning: Do not use this without first looking at the $\Delta\rho$ histogram and locating the mode. Discrimination should be used on either side of the $\Delta\rho$ mode! Therefore if the well being analyzed has a mode of $+0.020$ g/cc then the actual discriminators used would be $+0.010$ to $+0.030$ g/cc. Most of the density logs analyzed had a positive mode and from Figures 18, 19 or 20, the reader can see the problems in using a $\Delta\rho$ discriminator without the benefit of the $\Delta\rho$

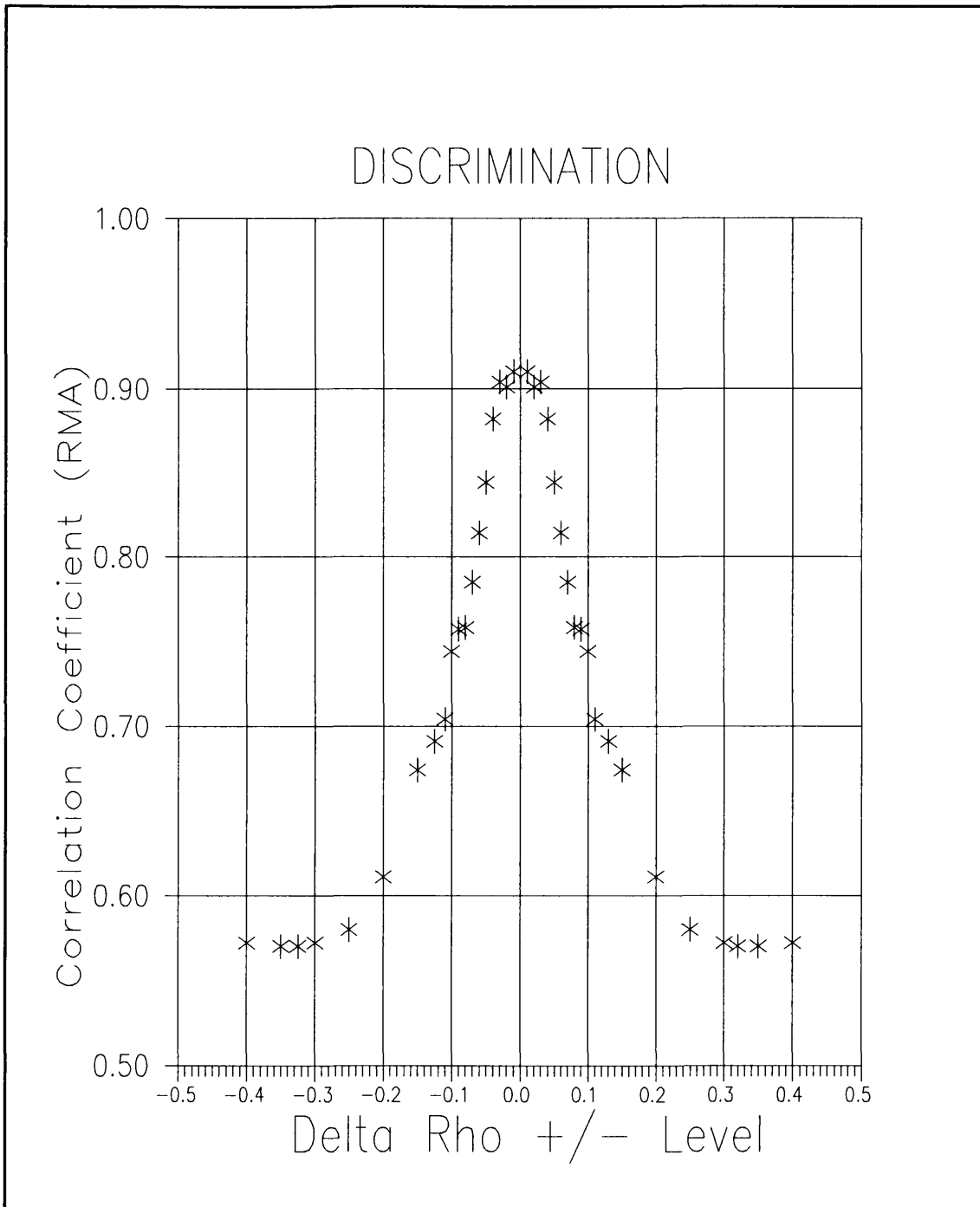


Figure 25

Delta Rho vs. Correlation Coefficient

histogram.

Proper use of the $\Delta\rho$ discriminator reduced the scatter of data to nice compact clouds of data. The RMA correlation of fit usually increased to above 0.90 from a prediscrimination value of 0.65.

6.8 Normalization of Neutron and Sonic

The methodology for normalizing the neutron and the sonic is as follows. The normalized density porosity is used as a standard to normalize these other tools. The neutron information is converted to a sandstone matrix and then crossplotted against the density also in a sandstone matrix. The fit of the plot is interpreted from the RMA method. The slope and intercept of the RMA line are used to calculate correction factors. The correction factors are then applied to the neutron values and a normalized neutron trace is calculated. For example, if the RMA fit of a crossplot has a slope of 1.1 and an intercept of 0.007 p.u., then the gain (sensitivity or slope) or gain factor is the reciprocal of the slope or $(GF_N) = 0.909091$ and the shift factor is the intercept multiplied by -1 or $(SF_N) = -0.007$ p.u.. Using the equation to correct the neutron as stated earlier in Chapter 1;

$$\phi_{Neutron\ Normalized} = (\phi_{Neutron\ LOG} \times GF_N) + SF_N$$

would yield the normalized neutron porosity value. For example if the neutron log reported 12% porosity then the normalized neutron porosity would be 10.2%. This is a very dramatic example.

The procedure for normalization of the sonic porosity information is identical.

6.8.1 Neutron Normalization Results

As noted previously (Davis 1984 and 1989), the Neutron tool commonly exhibits a gain error. From the examples that the author has seen, this is confirmed. Wells from the Anschutz Ranch East were studied, and the normalization results for the neutron are as follows in Table 4.

Note that these wells contain shift errors as well as gain errors. Wells with gain (sensitivity or slope) errors greater than one, and negative shift error are more common in this set of data and show the type III(b) case of error. Type-Cases of errors can be seen in Figure 6. The type III(b) case is probably indicative of most Neutron tool responses for the Anschutz Ranch East field. Shift greater than one combined with a gain (sensitivity or slope) error

Table 4
Neutron Normalization Results
Anschutz Ranch East

Well Number	Case*	Density Shift Error (ξ)	Neutron Gain Factor (GF_N)	Neutron Shift Factor (SF_N)	Neutron Tool Type
11-01	III(b)	-0.000	1.05800	-0.0040	CNT-A
20-16	III(b)	-0.040	1.30300	-0.0060	CNT-A
29-04	IV	-0.021	0.99268	-0.0050	CNT-A
29-12	III(b)	-0.014	1.24270	-0.0205	CNT-A
29-14	III(b)	-0.020	1.13800	-0.0070	CNT-A
30-14	I	-0.000	1.07350	0.0100	CNT-H
36-08	II(ab)	-0.130	0.88600	0.0125	CNT-H
36-16	III(b)	0.005	1.66100	-0.0345	CNT-H

*See Figure 6 on page 25

that is negative are opposing corrections, and have a canceling effect. For example, a Neutron porosity value of 12% corrected by a negative shift error of -0.0100 decreases porosity, and a gain error of 1.1000 increases porosity to a normalized value of 12.1% porosity. The degree of correction depends on the range of porosity that is being corrected.

6.8.2 Sonic Normalization Results

As noted previously (Davis 1984 and 1990), the Sonic tool commonly exhibits a gain error. This is confirmed in the results from a study of wells at the Anschutz Ranch Field. The gain (sensitivity or slope) error is large and on the order of 3% to 40% or 1.03 to 1.40. Sonic porosities were calculated based on the Wylie time travel equation with a matrix travel time of 51.3 $\mu\text{sec}/\text{ft}$ and a fluid travel time of 189 $\mu\text{sec}/\text{ft}$. The normalization results for all of the sonic logs studied is included below in Table 5.

Table 5
Sonic Normalization Results
Anschutz Ranch East

Well Number	Case*	Density Shift Error (ξ)	Sonic Gain Error (GF_s)	Sonic Shift Error (SF_N)	Matrix Est.
11-01	IV	-0.000	0.94620	-0.0060	50.5
20-16	III(ab)	-0.040	1.40600	-0.0026	56.1
29-04	I	-0.021	1.39700	0.0005	57.9
29-12	III(ab)	-0.014	1.26400	-0.0015	56.5
29-14	I	-0.020	1.11350	0.0090	53.05
30-14	I	-0.000	1.02600	0.0010	54.6
36-08	III(ab)	-0.130	1.09850	-0.0070	53.25
36-16	No	Sonic	Log		

*See Figure 6 on page 25

The fact that the type of error and the variation in results are unique for each well may indicate that there is some variable that is not tool related.

The estimated matrix travel time from a correlation to the density porosity varied from 50.5 to 57.9 $\mu\text{sec}/\text{ft}$. Figure 26 (Schlumberger 1989) may give some insight to the variance of these values. As seen on this chart there are two transforms presented. The well known Wyllie time average equation is represented by the straight lines, and the empirical relationship (Field Observation) represented by the curved lines. The empirical transform was proposed by Raymer in 1980. For the Anschutz Ranch East field, rocks are consolidated and the appropriate Wyllie matrix value should be 51.3 $\mu\text{sec}/\text{ft}$. This does not appear to be true from the previous correlation. It is surmised that the Anschutz Ranch East wells follow some variation of the empirical transform, and depending on the range of porosities present in the well, different matrix values will result. For example, if a well has a high range of porosity values from 25 - 30 p.u., then the tangent to the empirical line, in this porosity range, will project a very low value of matrix travel time. The converse is true of low porosity range wells whose tangent would project a high value of matrix travel time. Anschutz Ranch East well 29-04 had an

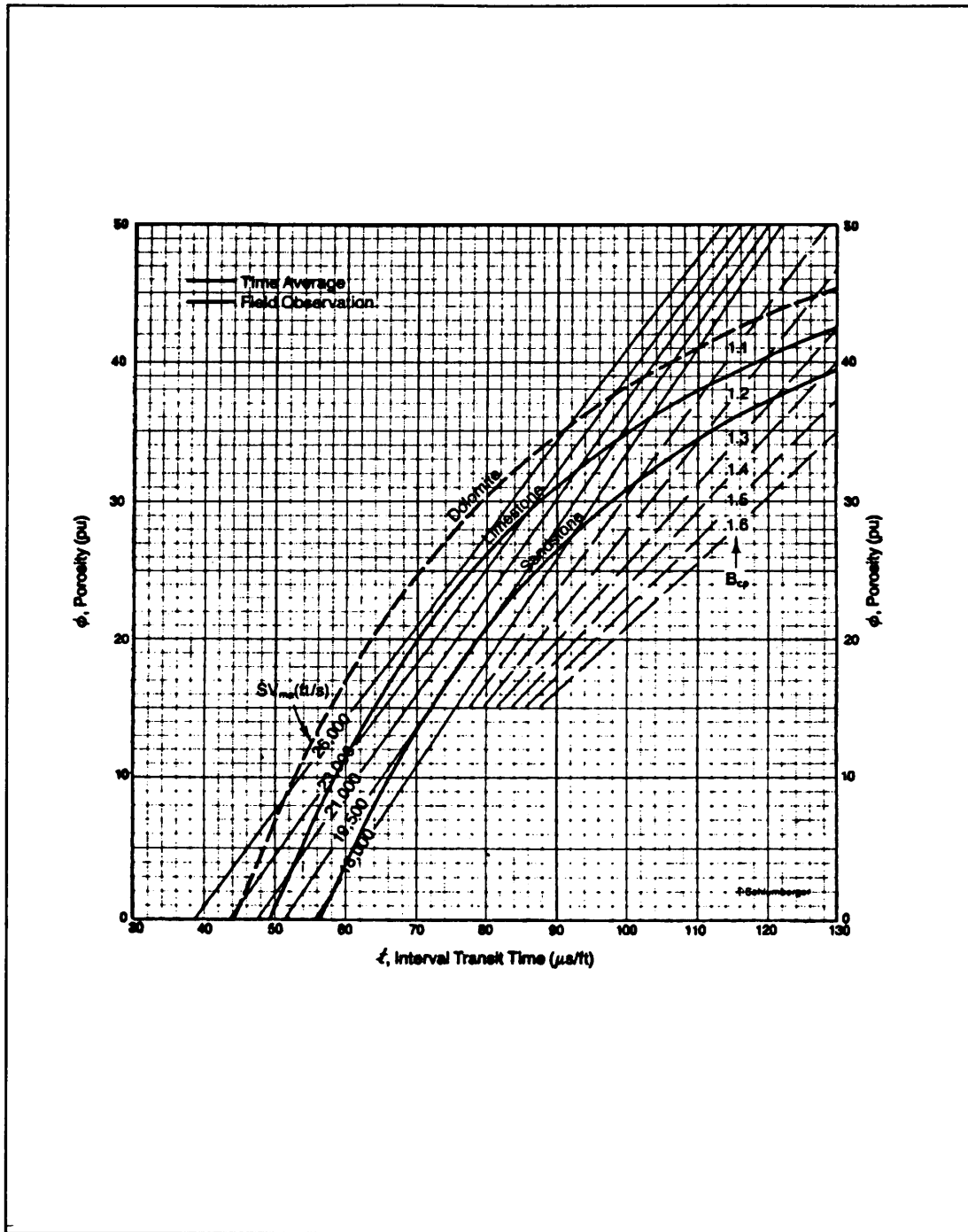


Figure 26

Travel Time to Porosity Transform (After Schlumberger 1989)

anomalously high projection of matrix travel time. The reason this value is high may be because the raw sonic values were compared to normalized density porosity. After the gain (sensitivity or slope) corrections are made the resultant sonic porosities depend to a much lesser extent on matrix travel times used. This is one of the advantages of normalization, if applied correctly, most error or model deficiency is removed.

6.9 Normalization Check

A good check on vertical normalization is the plot of normalized sonic (PSSN) vs. normalized neutron (PNEUTN). If properly normalized, these two traces should have an RMA line with a slope of 1.000 and an intercept of 0.000. As seen in Figure 27 for the Anschutz Ranch East #30-14 the slope is 0.988 and the intercept is 0.002 which are very close to the desired parameters.

6.10 Comparison to Core

Comparison of log and core data is much like comparing apples and oranges. Even when corrections to each type of

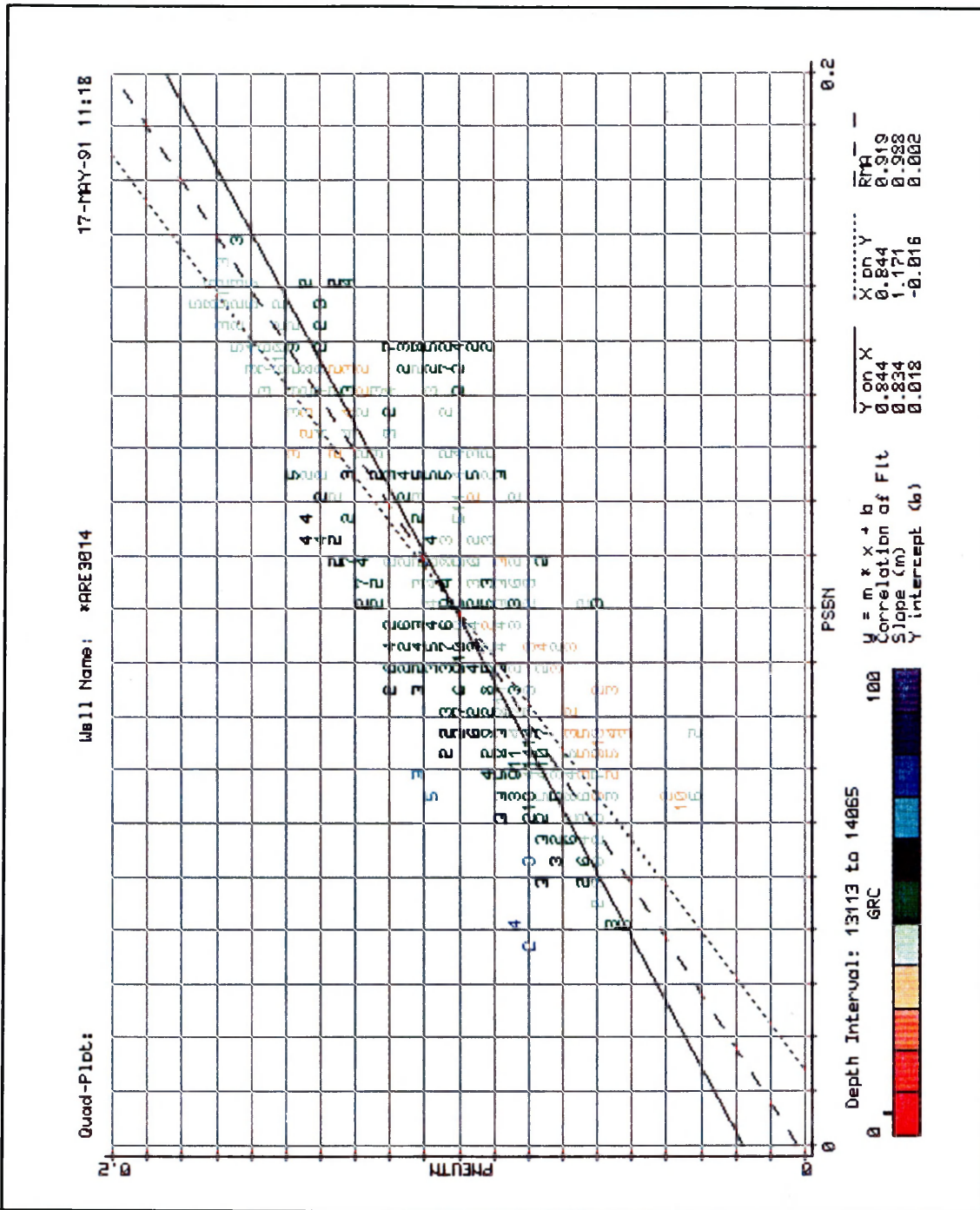


Figure 27

Normalization Check by Crossplotting

data are made in painstaking detail, variations still remain. Log and core information are two distinctly different types of data measured with two distinctly different sets of equipment. Porosity log processing has been briefly discussed by the author in this thesis. Core data from Anschutz Ranch East was processed before comparison to porosity logs by correction for overburden and filtering. Overburden corrections were made by The Anschutz Corporation and were not changed, while filtering was done by the author.

Core porosity is measured at atmospheric conditions for standard core analysis. One correction of this type of data is overburden or net effective stress. For example, the Anschutz Ranch East has an initial reservoir pressure of 5300 psi at a depth of 13,500'. Assuming an overburden gradient of 1 psi/ft the net effective stress is 13,500 psi - 5300 psi = 8200 psi. Cores have been removed from this stress and consequently expand. Thus core porosity values are optimistic. If rock compressibility is known, then the reduction of porosity can be estimated by;

$$C_f = \left[\frac{2 \times (\phi_1 - \phi_2)}{(\phi_1 + \phi_2 - 2 \times \phi_1 \times \phi_2)} \right] \times \left[\frac{1}{(\Delta P)} \right]$$

Where:

- ϕ_1 - Initial porosity (Decimal)
- ϕ_2 - Porosity at Pore Pressure (Decimal)
- ΔP - Change in Pore Pressure (psi)
- C_f - Rock Compressibility (1/psi)

For the Anschutz Ranch East field, where the reservoir rock is consolidated and at great depth compressibility will be low and in the range of 2×10^{-6} 1/psi to 15×10^{-6} 1/psi. If the porosity is 15% then the corrected porosity value will be 14.8 p.u. which is small. Larger compressibility values will cause a larger decrease in core porosity when corrected for overburden. Compressibility as high as 300×10^{-6} 1/psi occurs in some California reservoirs and can have a drastic effect on core porosity measurement, on the order of 5 to 10 p.u. (Scorer 1974).

Core data also needs to be corrected for vertical resolution. Since core plugs are about 1 inch in diameter and usually taken along the bedding planes, they represent this much of the reservoir vertically. Filtering of core data is necessary to make data similar to measurements that are made with porosity logs. Core data has a vertical resolution of 1/12 feet. Since core measurements are typically made on 1 foot intervals, the filter method used weighted averaging of core measurement above and below a

certain depth by 25% and the measurement at that depth by 50%. This results in a core vertical resolution of about 1 1/2 feet. Filtering to the same 5 1/4 foot vertical resolution of the logs removes most meaningful character in the core data.

Log and core data are often crossplotted to determine a relationship or correlation. Without detailed processing of log and core data, a poor fit may occur. Figure 28 shows a playback of normalized porosity traces along with core data. Log and core data are generally in good agreement.

The interval from 13,700' to 13,720' is washed out and rugose as seen from the caliper trace (CLDN). This is the cause of anomalously high porosities reported by the density tool due to poor pad contact. A discriminator (DISC21) was used to remove this data before comparison with other porosity traces. The discriminator trace is plotted in track 1 of the playback, a zero value means data passes discrimination criterion discussed earlier, and any other value greater than zero means failure of density data to pass the $\Delta\rho$ and hole rugosity tests.

The interval from 13,720' to 13,734' shows discrepancy between log and core data; core porosity being lower than log porosity. This response may infer shale, however the

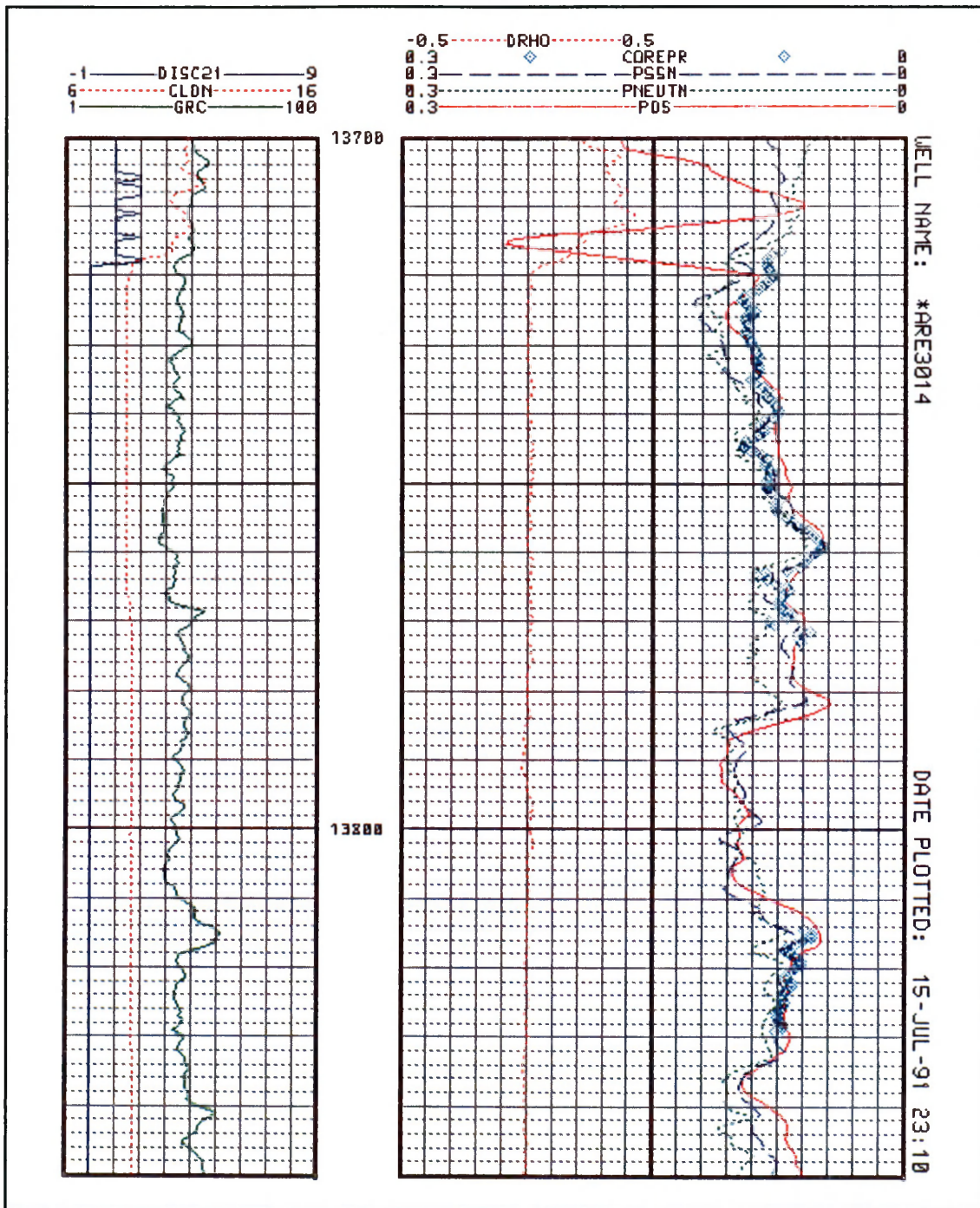


Figure 28

Playback of Normalized Porosity Traces

gamma ray trace (GRC) does not indicate more shale than is present in other intervals where log and core data are in agreement.

A more reasonable explanation for the discrepancy may be the hole geometry and standoff near the washout interval. It is possible that standoff of the tools is not corrected for appropriately. Since the tools are near the washout section above, their position in the borehole may be suspect and the standoff corrections may not apply to the actual case.

Figure 29 is a playback from the Anschutz Ranch East #20-16. The traces show the difference normalization makes in density porosity values when compared to core. Track 2 shows raw density porosity and raw core data. Notice the difference in these as compared to the normalized density and processed core data in track 3. Depth shifting of core data was not done, but the merits of this additional processing are apparent in this playback. Note the depth shift discrepancy at 12,720'. A depth shift made by simply shifting the core data up or down may not be appropriate, while sets of accordion shifts would more accurately depth shift core data. The core analyst is invaluable in this process and should be included when making core depth shifts.

Also note on Figure 29 the apparent discrepancy at 12,767'. This is caused by the difference in vertical resolution of the logging tools and core data. A clean, and evidently very porous thin streak of sandstone at this depth is averaged with the lower porosity rocks nearby to arrive at the log porosity value. Core data can sample at this small scale while the logs cannot.

A crossplot of log vs. core data was made for the same well #20-16. Raw porosity (PHIRAW) was compared to raw core porosity (CORERA) in Figure 30. The resultant RMA fit has a slope of 1.149 and an intercept of -0.007.

Figure 31 shows the normalized porosity (PHINOR) vs. the processed core porosity (COREPR). The RMA fit has a slope of 1.005 and an intercept of 0.007 which is in better statistical agreement than the raw data of Figure 30. As a reminder, ideally the slope should be 1.000 and the intercept should be 0.000.

Figure 32 shows the difference that normalization makes on the porosity traces. Track 2 shows the before normalization traces; raw density porosity (PDRAW), raw sonic porosity (PSS), and raw neutron porosity (PNSC). Track 3 shows the after normalization traces; density porosity (PDS), sonic porosity (PSS), and neutron porosity (PNEUTN). Caution should be used when looking at the

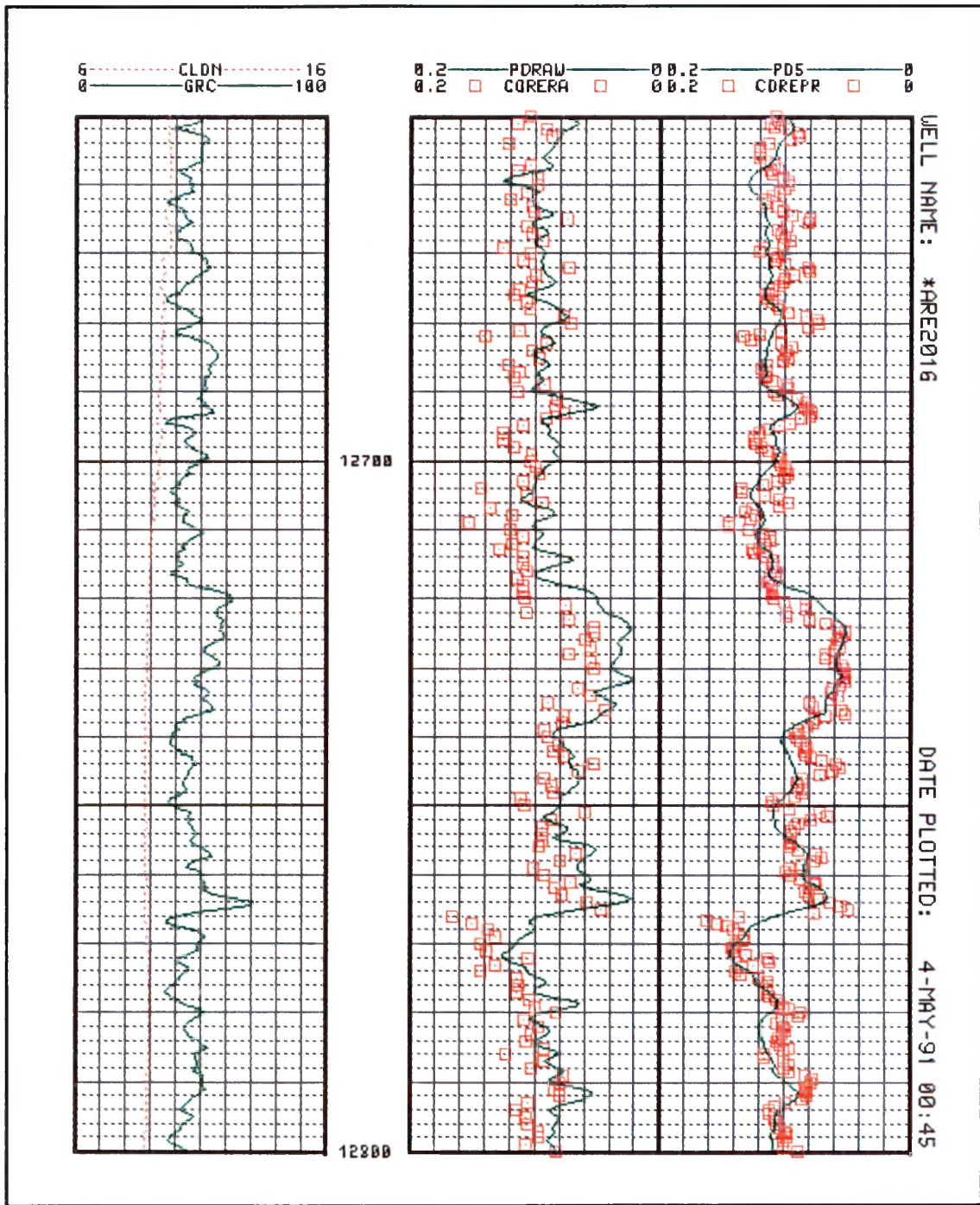


Figure 29

Playback of Log and Core Porosity

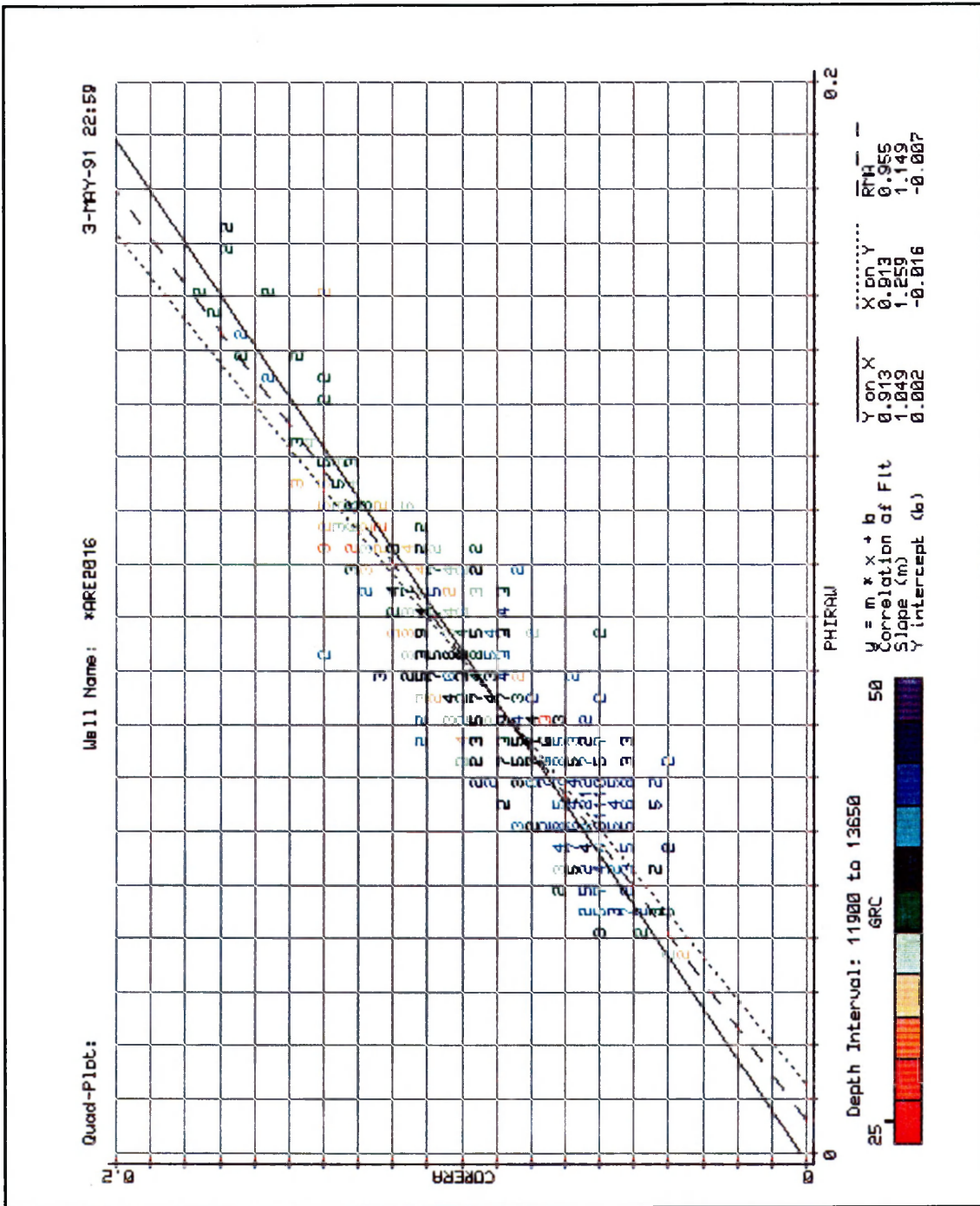


Figure 30

Crossplot of Raw Log and Core Porosity

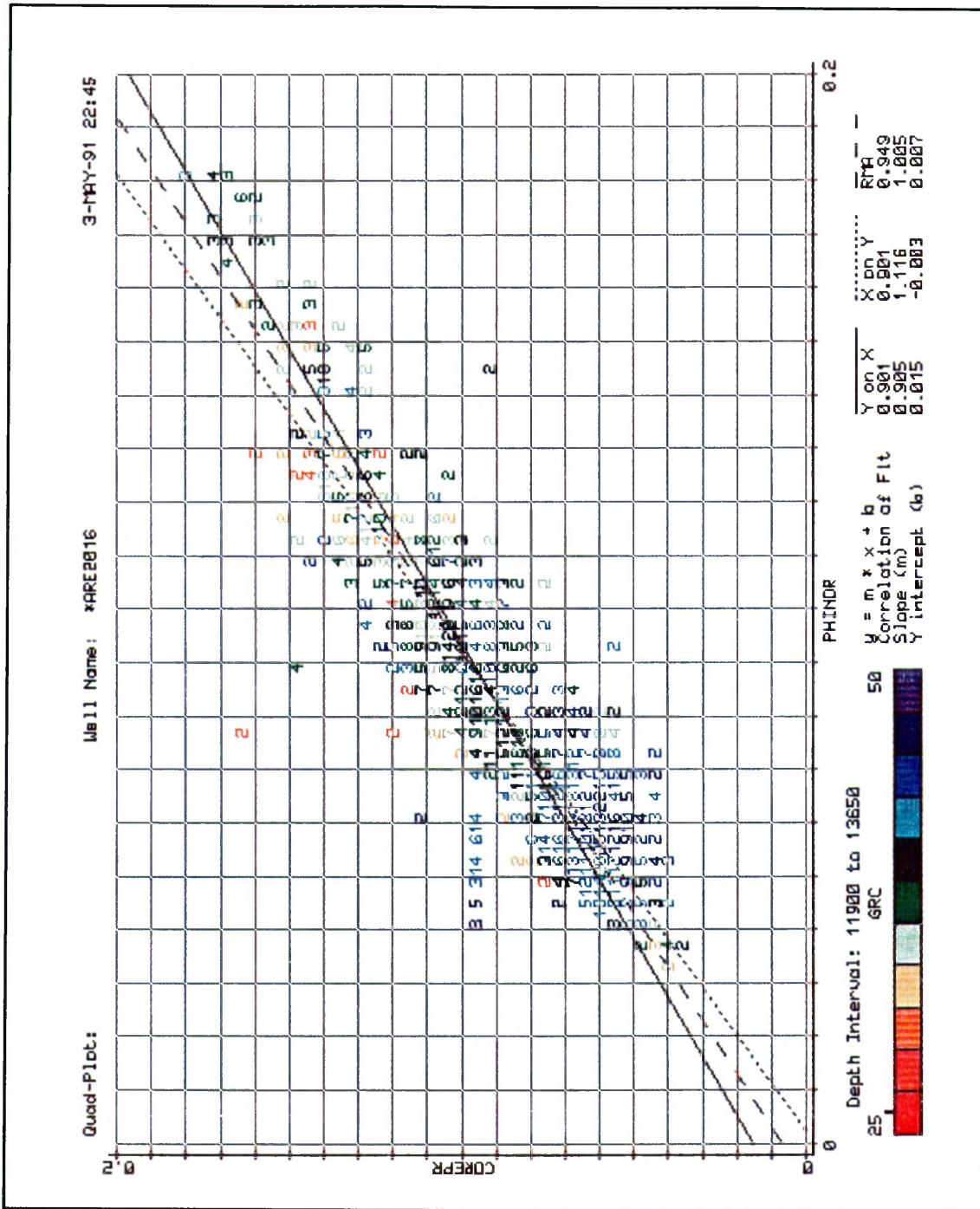


Figure 31

Crossplot of Processed Log and Core Porosity

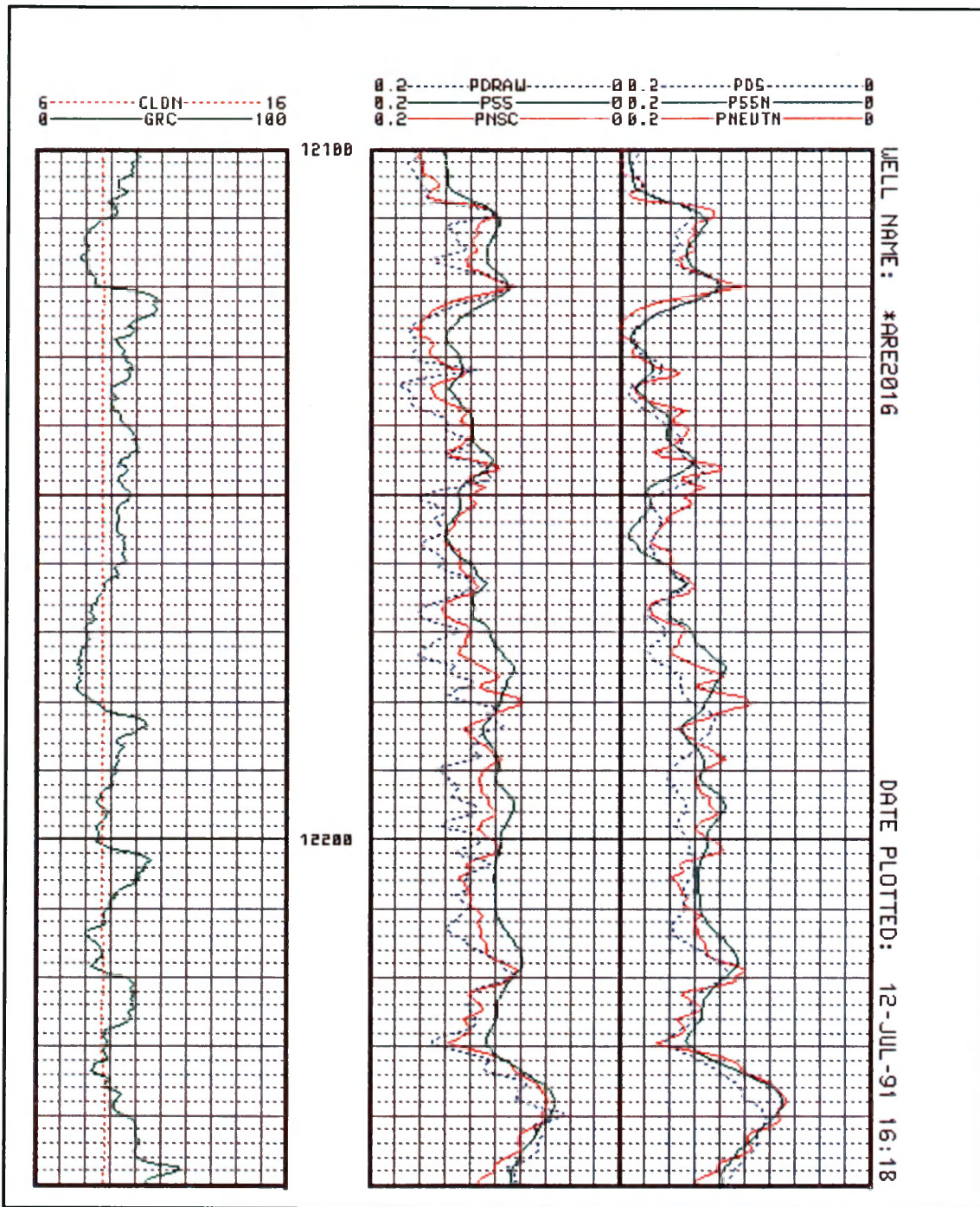


Figure 32

Comparison of Porosity Traces Before and After Normalization

normalized traces. Some differences in the normalized traces should be expected due to lithology and fluid contents that differ from the model. Remember that data has been discriminated for lithology and fluid content in order to solve the model, but playbacks show the entire data set. Notice how the normalized porosity traces in the two clean sand areas (12,110' to 12,120' and 12,170' to 12,180') are in better agreement than the raw porosity traces. Zones containing gas or shale will not show improvement and should not since tool responses will vary depending on the amount of each present. Complex gas corrections and shale corrections were not made as this is beyond scope of this first step in vertical normalization.

Conclusions

1. A Vertical Normalization model for clean sandstones containing gas free liquids is proposed. This model modifies the marker bed and composite methods.
2. Vertical Normalization can stand alone as a method when composite and marker bed methods are not possible.
3. Histograms of delta rho ($\Delta\rho$) proved to be valuable tools in determining the shift error in the density tool (SF_D), and choosing the appropriate discrimination values.
4. The Caliper vs. $\Delta\rho$ crossplot is a qualitative quick look at the performance of the density tool. This plot is also helpful as a visual aid to determine hole conditions and thus the amount of discrimination that will be required.
5. Vertical Normalization yields more refined normalization results and consequently gives porosity information that is closer to truth.

Recommendations for Further Study

1. The more complex case of normalization defined as;
A simple mixed lithology (2) containing gas free liquids should be solved graphically with the envelope technique for certain cases.
2. Shale geology, compaction, fissility, and density should be studied to refine the composite method.
3. Photoelectric Effect (P_e) may be used to refine normalization for LDT tools.
4. Family of curves for error analysis of density porosity ($\partial\phi/\partial\rho_b$), and the various water saturation models ($\partial S_w/\partial\phi$) should be generated.
5. Normalization of Resistivity tools should be investigated, especially the induction tools. A method involving the S_w vs. Bulk Volume Water plot and shapes of data envelopes may help solve this problem. A 100% water bearing zone near the producing formation is required to make this model work.
6. Depth of investigation of the porosity tools should be studied further. Simulated vs. actual playbacks may show problems with current models and lead to a better algorithm. This is PhD level

work.

7. Discrimination should be studied with reference to $\Delta\rho$ and $\Delta\text{Caliper}$ to enhance the work done here and look for trends for barite and native muds.

Nomenclature

$\Delta\rho$	Delta Rho, correction for mudcake and rugosity for the Density tool
ρ	Density in g/cc
t	Compressional Travel Time
ϕ	Porosity
B_{cp}	Compaction Factor
ξ	Shift Error for the Density Log in g/cc
SF	Shift Factor
GF	Gain Factor
p.u.	Porosity Units
SD	Standard Deviation
u,x,v,y	Variables
R	Correlation Coefficient
R	Resistivity
m	Slope

Subscripts

D	Density
N	Neutron
S	Sonic
ma	Matrix
f	Fluid
b	Bulk
w	Water
g	Gas
LOG	Reading from Well Log
PW	Pad Wear
HC	Hole Conditions
t	True
LS	Long Spaced

Trademarks and Acronyms

LDT	Litho Density Tool
FDC	Compensated Formation Density Tool
CNL	Compensated Neutron Log
RMA	Reduced Major Axis
CSU	Cyber Service Unit
LIS	Library Information Standard

List of Trace Names

CLDN - Caliper from the neutron-density run (inches)

COREPR - Core Porosity corrected for overburden and filtered from FCPHI

CPHI - Core porosity from lab also CORERA (Decimal)

DRHO - Correction for mudcake and rugosity (g/cc)

FCPHI - Filtered core porosity (weighted 3 box car on 1' data)

GRC - Environmentally corrected gamma ray (API)

GRDN - Gamma Ray from the neutron-density run (API)

HMC - Thickness of mudcake (inches)

PDRAW - Density porosity from raw bulk density trace

PDS - Normalized density porosity

PHINOR - Effective porosity after normalization

PHIRAW - Effective porosity from raw log traces

PNEUTN - Normalized neutron porosity (Decimal)

PNSC - Neutron porosity (SSPU)

PSS - Filtered sonic porosity (7 box car on .5' data)

PSSN - Normalized sonic porosity (Decimal)

RHBC - Environmentally corrected bulk density (g/cc)

RHBCF - Filtered bulk density (7 box car on .5' data g/cc)

RHOB - Bulk density from Log (g/cc)

RHOBC - Shift (ξ) corrected bulk density (g/cc)

Computer Program References

- ES-LOG - Energy Systems Log Analysis Program, furnished through the generosity of Larry Wells.
- ES2PC - Energy System to Personal Computer, program to convert data formats in either direction. Written and furnished by Don G. Davis.
- DIGITECH - Digitizing Program, written and furnished by Don G. Davis.
- CSE - Chain Saw Editor, program used to edit data files. Written and furnished by the CSM computing center.

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Appendix A

Reduced Major Axis vs. Linear Regression

Reduced Major Axis or SD Line

Freedman (1980) defines the Standard Deviation line as follows;

The points cluster around a line called the SD line. This line goes through the point of averages. When the correlation coefficient (R) is positive, the slope of this line is

$$(SD \text{ of } y) / (SD \text{ of } x).$$

When the correlation coefficient (R) is negative, the slope is

$$(SD \text{ of } x) / (SD \text{ of } y).$$

The point of averages is defined as the coordinate (average of x values, average of y values). The correlation coefficient is the average of x in standard units multiplied by the standard of y in standard units. In equation form, and especially for the computer, the correlation coefficient can be calculated as follows;

$$R = \frac{Cov(x, y)}{(SD \text{ of } x) \times (SD \text{ of } y)}$$

where

$$Cov(x, y) = (\text{ave of products } xy) - (\text{ave of } x) \times (\text{ave of } y)$$

Linear Regression

According to Freedman (1980);

The regression line is to a scatter diagram as the average is to a list. The regression line estimates the average value for the dependent variable corresponding to each value of the independent variable.

The regression line passes through the point of averages, as does the SD line. The slope, however, is not calculated in the same manner. It should be noted that the convention is to plot the independent variable on the abscissa or X-axis and the dependent variable on the ordinate or Y-axis. Holding to this convention, the slope is then calculated as

$$\text{Slope} = \frac{R \times (\text{SD of } y)}{\text{SD of } x}$$

The regression of Y on X is conventional and means that the Y-axis variable is dependent on the X-axis variable. The correction is drawn from data points to the best fit line vertically showing graphically that the error is in Y-axis variable.

Sometimes the regression of X on Y is used. In this case the X-axis variable is dependent on the Y-axis variable. The correction is drawn from the data point to

the best fit line horizontally showing graphically that the error is in the X-axis variable.

The RMA or SD line method mimics an orthogonal correction which is drawn normal to the best fit line. An actual orthogonal fit routine was not used for this work, but is very closely approximated by the RMA or SD line method.

The reader should refer to Freedman (1980) or any other quality statistics book for explanation of these methods.

Summary

The difference in the methods is most noticeable when the correlation coefficient is much different than unity. The closer the two variables are clustered, the closer the answers from the two techniques will be. Correlation is a statistical approach that describes the RMA or SD line method. Regression is referred to as the Linear Regression method. Freedman (1980) wrote more about correlation and these passages may help explain the difference and why the RMA or SD line method should be used for correlation of log porosity values;

ASSOCIATION IS NOT CAUSATION

The Salk vaccine against polio was proposed after the first polio epidemic had claimed many hundreds of thousands of victims from 1916 to 1950. Before the introduction of this vaccine, investigators looked at the relationship between the incidence of polio and the number of soft drinks sold. For each week of the year, they tabulated the number

of soft drinks sold that week, and the number of new cases of polio reported. These data points showed strong positive correlation. During weeks when more soft drinks were sold, there were more new cases of polio; when fewer soft drinks were sold, there were fewer such cases.

Do soft drinks cause polio? If so, prohibiting their sale would have reduced the incidence of the disease. Clearly, the answer was no, and nobody was fooled by the correlation. Polio epidemics were most severe in the summer just when soft-drink sales are at their highest. So there was a third factor driving both variables - season. Thus correlation measures association. But association is not the same as causation.

Porosity measurements from the Density tool are associated with porosity measurement from the Neutron and Sonic tools. The tools independently measure porosity and have no causation effect on each other. Thus to properly compare porosity data a correlation should be made using the RMA or the SD line method.

Appendix B

Error Analysis of Density derived Porosity

The density tool infers bulk density from other measurements. If certain parameters vary then porosity will vary. This is a quick error analysis or sensitivity analysis of the variables in the density porosity transform equation.

Error analysis of $\partial\phi/\partial\rho_b$

The porosity of rocks, in terms of density, is usually stated with the following equation;

$$\rho_b = \phi \times (\rho_f - \rho_{ma}) + \rho_{ma}$$

In order to find out how porosity (ϕ) varies with bulk density (ρ_b) an equation in the form of porosity with respect to bulk density is necessary;

$$\phi = \frac{(\rho_{ma} - \rho_b)}{(\rho_{ma} - \rho_f)}$$

The derivative of this equation with respect to bulk density is;

$$\frac{\partial\phi}{\partial\rho_b} = \frac{1}{\rho_f - \rho_{ma}}$$

The results are linear, and for a sandstone ($\rho_b = 2.65$ g/cc) containing fresh fluid ($\rho_f = 1.00$ g/cc) the equation yields;

$$\partial\phi = -0.60606061 \times \partial\rho_b$$

This says that if ρ_b increases by 0.01 g/cc then the porosity will decrease by 0.00606060 or 0.606060 porosity units. Table 6 shows the sensitivity to ρ_b for different logging scenarios.

Porosity of sandstones containing saltwater are most sensitive to variations in bulk density while porosity of dolomites containing freshwater are least sensitive. As a comparative base if porosity is to be determined within 1 porosity unit (p.u.), then bulk density must be within a tolerance of 0.0165 g/cc. The differential change in porosity varies linearly with bulk density.

Table 6
Error Analysis of $\partial\phi/\partial\rho_b$

ρ_{ma} (g/cc)	ρ_f (g/cc)	$\partial\rho_b$ (g/cc)	$\partial\phi$ (p.u.)
2.65 Sandstone	1.00 Fresh	0.01	0.606
2.65 Sandstone	1.10 Brine	0.01	0.645
2.71 Limestone	1.00 Fresh	0.01	0.585
2.71 Limestone	1.10 Brine	0.01	0.621
2.87 Dolomite	1.00 Fresh	0.01	0.535
2.87 Dolomite	1.10 Brine	0.01	0.565

Error analysis of $\partial\phi/\partial\rho_f$

The differential change in porosity varies with the inverse square of the sum of the fluid density plus the matrix density, and yields a slightly different error analysis equation. Starting with the density porosity transform as before;

$$\phi = \frac{(\rho_{ma} - \rho_b)}{(\rho_{ma} - \rho_f)}$$

The derivative of this equation with respect to fluid density is;

$$\partial\phi = \left[\frac{(\rho_b - \rho_{ma})}{(\rho_{ma} - \rho_f)^2} \right] \times \partial\rho_f$$

If the porosity is to be determined to within 1 p.u. then the fluid density must be known to within 0.136 g/cc in the 12% range of porosity values, and within 0.077 g/cc in the 21% range of porosity values. As you can see the derivative equation is nonlinear and includes bulk density and matrix density as variables. The range of porosities to be studied should be used in error analysis of fluid density.

Error analysis of $\partial\phi/\partial\rho_{ma}$

This differential equation is slightly more complex than the two prior. Since the matrix density is in the numerator and the denominator then solution by parts is required. Once again the density porosity transform is;

$$\phi = \frac{(\rho_{ma} - \rho_b)}{(\rho_{ma} - \rho_f)}$$

If we let the numerator equal $u(x)$ and the denominator equal $v(x)$, then the solution of the partial derivative is of the form;

$$\frac{\partial}{\partial x} \left(\frac{u}{v} \right) = \frac{v \left(\frac{\partial u}{\partial x} \right) - u \left(\frac{\partial v}{\partial x} \right)}{v^2}$$

The derivative then reduces to ;

$$\partial\phi = \left[\frac{\rho_b - \rho_{ma}}{(\rho_f - \rho_{ma})^2} - \frac{1}{\rho_f - \rho_{ma}} \right] \times \partial\rho_{ma}$$

Once again, for a sandstone containing fresh water in the porosity range of 12% the following values apply;

$$\rho_b = 2.45 \text{ g/cc}$$

$$\rho_{ma} = 2.65 \text{ g/cc}$$

$$\rho_f = 1.00 \text{ g/cc}$$

and the evaluation of the error equation yields;

$$\partial\phi = 0.5325 \times \partial\rho_{ma}$$

Thus for the uncertainty in ϕ to be less than 1 p.u. the uncertainty in ρ_{ma} must be less than 0.01878 g/cc.

Error analysis of Archie Water Saturation $\partial S_w / \partial \phi$

Water saturation is calculated as a function of porosity in the simplified clean sand model with Archie;

$$S_w = \sqrt{\frac{R_w}{R_t}} \times \left(\frac{1}{\phi}\right)$$

Since porosity can vary, as is seen by normalization, water saturation will vary. This is shown in the derivative of the Archie equation from above;

$$\partial S_w = \sqrt{\frac{R_w}{R_t}} \times \left(\frac{2}{\phi^2}\right) \times \partial \phi$$

When porosity is low and R_t and R_w are known, a small change in porosity will have a large affect on the calculated water saturation. For example, when R_t is 5 Ohm-m, R_w is 0.017 Ohm-m, and porosity is 6 p.u. the error equation is as follows;

$$\partial S_w = \sqrt{\frac{0.017}{30}} \times \left(\frac{2}{0.06^2}\right) \times \partial \phi$$

$$\partial S_w = 13.22 \times \partial \phi$$

For this scenario, if porosity is in error by only 1 p.u. the resultant water saturation from Archie will be in error by %13.22. Smaller porosity values will have larger errors and larger porosities will have lower errors as seen in Table 7;

Table 7

Archie Water Saturation Error Analysis $\partial S_w / \partial \phi$

ϕ (p.u.)	$\partial \phi$ (p.u.)	R_t	R_w	∂S_w
12	1	30	0.017	% 3.31
6	1	30	0.017	%13.22
5	1	30	0.017	%19.04
4	1	30	0.017	%29.76