DECISION MAKING UNDER UNCERTAINTY IN PETROLEUM EXPLORATION

by

David R. McCullough

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Golden, Colorado

Date: November 6, 1989

Signed: [signature]
David R. McCullough

Approved: [signature]
Donald G. Davis
Thesis Advisor

Golden, Colorado

Date: November 6, 1989

[signature]
Dr. Craig W. Van Kirk
Head, Department of
Petroleum Engineering

ABSTRACT

This paper discusses decision making under uncertainty, and demonstrates that a consistent policy of maximizing expected monetary value leads to overall higher gains than other decision making criteria under uncertainty. Expected monetary value is defined as the product of the probability of occurrence of the outcome and the conditional value that is received if the outcome occurs, expressed in monetary units of value.[1] To perform the study, several portfolios of drilling prospects were generated randomly by a computer program. The generated data included the most likely and expected monetary value for each prospect. Then using Monte Carlo simulation, one pass determined the truth or final outcome of each prospect. The gains or losses from those prospects selected under the expected monetary value criteria were compared to the gains or losses from the prospects selected using other decision making criteria under uncertainty.

The effect of error in subjective judgement or information on the expected value criteria was evaluated by introducing bias. The gains or losses from projects selected under biased expected monetary value were compared to the

gains or losses from the other decision making criteria under uncertainty, also subject to the same degree of bias, where applicable.

To determine if investment environment affects the choice of decision criteria, the procedure was repeated over several ranges of potential profit to investment ratios. Wider ranges of profit to investment ratios represent a more favorable investment climate, while narrower ranges represent a more severe investment climate.

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INTRODUCTION

> If the decision maker consistently selects the alternative having the highest positive expected monetary value his total net gain from all decisions will be higher than his gain from any alternative strategy for selecting decisions under uncertainty. This statement is true even though each specific decision is a different drilling prospect with different probabilities and conditional profitabilities.[2]

The purpose of this paper is to demonstrate the validity of this statement by Paul Newendorp. In his book, Newendorp proposes that to prove or disprove the statement, a series of hypothetical drilling decisions be set up in which the decision maker could use various value systems to choose drilling prospects.[3] That is exactly what was done in this instance. Newendorp's statement is very bold. It says, "will be higher", not generally or on average. If this is true, why are other decision making criteria being used? Why is the success rate in making correct decisions only 53% in successful companies?[4] This can best be explained by people's perception of probability, and their willingness to assume risk. There are always people that believe that they can beat the odds. In fact, there are several cities in Nevada devoted to this kind of personality. Conversely there are people that always assume the worst outcome will happen to them. Later in this paper two decision criteria specifically attuned to these two situations will be discussed.

Newendorp goes further in his assertion, stating that expected monetary value will prevail whether the decision is made using discounted or undiscounted cash flow.[5] This is intuitive. If the criteria triumphs under undiscounted cash flow, it should triumph under discounted cash flow, so long as the values are expressed in consistent fashion. That is discounted results are used in the discounted expected cash flow case and undiscounted results are used in the undiscounted expected cash flow case. By discounting the cash flow, the effect is to decrease the dollar amount. It may also alter the ranking of the portfolio of prospects due to the timing of the cash flow. But as all decision criteria under uncertainty are so affected, it should not effect the outcome as to which criteria triumphs. The units of measurement are unimportant. It is the concept of probability that is important.

UNCERTAINTY

> So far as the theorems of mathematics are about reality, they are not certain; so far as they are certain they are not about reality.[6]

This quote by Albert Einstein illustrates the difficulty in dealing with uncertainty. What is uncertainty? Websters' defines uncertainty as "lack of sureness about someone or something."[7] However for the purpose of this paper the following definition will be used:

> A decision is made under uncertainty if each course of action leads to an array of possible outcomes, each with its own probability of occurrence. The decision maker knows there is no opponent (other than chance or nature) to affect the outcome for a decision. For example, the decision whether to place a dollar on "red" in roulette is a decision problem under uncertainty. I know the probability of winning a dollar is 18/38 and the probability of losing a dollar is 20/38. Thus if the selected course of action is to play the game, I do not know for certain what will happen. But I do know the probabilities for what might happen.[8]

Probability

The probability of an event is the chance of it occurring. Probability can be divided into two types. Objective or classical probability and subjective probability. The example of the roulette wheel is an example of objective probability. The number of occurrences of an event are divided by the total number of attempts. This is also sometimes thought of as relative frequency of occurrence. A subjective probability, on the other hand is based on the decision maker's judgement or degree of belief concerning the chance that a given event will occur.[9] Probability is measured on a scale of zero to one, with zero having no chance of occurring, and one being certain to occur.

Probability is important because it gives us a means to communicate our degree of uncertainty. It tells one how much an estimate may be in error, and the chances that it may be that much in error.[10] To borrow a quote from Lord Kelvin:

> ...when you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the state of science, whatever the matter may be.[11]

Categories of Uncertainty

Uncertainty in petroleum exploration can be divided into three categories: 1) Technical uncertainty, 2) Economic uncertainty, and 3) Political uncertainty. Technical uncertainty relates to the hydrocarbon volumes estimated to be present and the rate at which they may be produced. Do hydrocarbons exist, and if so, are they at the volumes estimated by the engineers and geologists, and if they are present, will the wells produce at the rates projected? Economic uncertainty relates to the price structure for the hydrocarbons. The actions of OPEC and the rapid decline in oil prices in 1986 has brought home this type of uncertainty. It is extremely difficult to project oil and gas prices at this time with any degree of confidence. The third type of uncertainty is political uncertainty. This deals with the stability of governments and tax structures. This can best be illustrated by the Alaska North Slope and the economic limit factor (ELF), which is discouraging development of many, small by Alaska's standards, oil fields.[12] Political uncertainty can change the economic criteria for a project from highest value to quickest payout.[13] This paper will consider only technical uncertainty.

Risk versus Uncertainty

There is confusion in the literature as to whether or not uncertainty and risk are the same thing. Newendorp states that risk and uncertainty can be considered synonymous.[14] Felsen considers a decision to be made under uncertainty if one knows absolutely nothing about the probability of the outcome. He considers a decision made under risk if one has some knowledge about the probability of the outcomes.[15] While there are cases in which one cannot assign some probability to a project, they are rare. Everyone has some feeling or opinion as to the probability of events. For example twenty people outside the oil industry at the University of Colorado were asked for the probability of finding oil in Colorado. Each person had an opinion as to the probability, even though they had no direct knowledge on which to formulate an answer. Garb considers the decision to drill a well to be made under risk. The decision on how much reserves to book if oil is discovered is considered to be made under uncertainty.[16] The only difference appears to be in the decision to drill; money is directly at risk. One is still uncertain as to whether there is oil present in economic quantity. However in the case of booking reserves, money is also at risk, admittedly indirectly, because this can affect a firm's or individual's financial position with regard to ability to access capital funds. So are risk and uncertainty one and the same? It appears if you want it to be, it is. As a decision under uncertainty always involves risk, I believe they should be considered synonymous.

DECISION MAKING CRITERIA UNDER UNCERTAINTY

There are five generally recognized criteria for decision making under uncertainty. These are: 1) Maximin, 2) Maximax, 3) Minimize Regret, 4) Laplace criteria, and 5) Baysian criteria (expected value). In this analysis, a sixth method, random selection will be used also.

Maximin

Maximin (Wald) is the criteria of pessimism referred to in the introduction. This criteria directs the decision maker to act as though the least favorable state will occur. He or she then selects the event which has the highest minimum.[17] This is the same as assigning a probability of one to the least favorable state. In petroleum exploration, the least favorable state or outcome is a dry hole, or possible setting casing on a dry hole. The event in this state with the highest minimum would be the project with the lowest drilling cost. Consider the four prospects described by Example 1. The pessimistic decision maker would select prospect A, even though it has a lower payoff if successful. However it has the lowest dollar amount at risk if it fails.

Example 1

Prospect	Payoff	Cost
A	100	50
B	200	100
C	300	150
D	400	200

Maximax

Maximax (Hurwicz) is the most optimistic decision making criteria. The decision maker determines the best possible state, then selects the highest value event under that state.[18] This is the same as the Las Vegas gambler, mentioned in the introduction. In petroleum exploration, this favorable state would be a producing well. The optimistic decision maker would select the prospect with the highest possible value. Referring to Example 1, the decision maker using maximax criteria would choose prospect D. He or she implicitly assigns a probability of one to the successful state and a probability of zero to the failure case.

Minimize Regret

Minimize regret (Savage) realizes the decision maker may experience regret after the decision and payoff has occurred. This method would attempt to minimize this regret before selecting the action to be taken.[19] This method is impossible to evaluate as there is no way to quantify regret between two people. This enters into utility theory, in which a dollar or something of value, may have different utility or value to different individuals. This utility value may change with time depending on the fortunes of the decision maker. For this reason, this method was not used in the comparison.

Random Selection

Random selection is much like being blindfolded and picking prospects from a hat. Each prospect has an equally likely chance of being picked. There is nothing to cause one prospect to be selected over another. In this experiment, the selection process will be that of sampling without replacement. There are only 100 prospects generated each year. If one prospect is selected, then there are 99 prospects remaining to be considered.

In the above methods, probabilities have not been assigned to an outcome, or have been assigned implicitly. In the next two criteria probabilities will be explicitly assigned. This distinction is sometimes referred to as nonprobabilistic and probabilistic methods.

Laplace Criteria

The Laplace criteria assigns equal probability to every outcome and event.[20] Using the scenario in Example 1 and the Laplace criteria, Example 2 would result. To get the expected values, each payoff and cost in Example 2 is multiplied by 0.5, the probability, and summed. A decision maker using the Laplace criteria would select Prospect D, due to the higher Laplace expected value. R. M. Rao Tummala

Example 2

Prospect	Payoff	Cost	Probability Success	Expected Value
A	100	50	.5	25
B	200	100	.5	50
C	300	150	.5	75
D	400	200	.5	100

asserts that this method should only be used when a decision maker is completely ignorant about the occurrence of the states of nature.[21] However people use this method when clearly the prospect of decision is not a 50-50 proposition. As an example, how many times have you heard the statement, "The well will either be dry or productive, therefore the chance factor is 50%." Surprisingly this attitude is expressed by some very educated people. To quote Dr. Martinson of the University of Colorado Business School:

> This concept of expected value is nonsense. A business venture will either succeed of fail. The probability is always 50%.[22]

This ignores repeated trials and a look at statistics for successful wells quickly shows this to be misleading. The success rate for new field exploratory drilling was only

14.7% during 1987.[23] While it is true there are only two states of outcome, to assume they are equally likely is incorrect. This is a Bernoulli distribution, or to be more precise, a binomial distribution, which is a special type of Bernoulli distribution. A Bernoulli process is an experiment or process in which only two outcomes are possible. A binomial distribution arises when one counts the number of successes in n trials of a constant Bernoulli process.[24] To truly have a 50% chance of occurring, the successful efforts divided by all efforts would have to equal 0.50. Using this criteria for petroleum exploration, a dry hole would have the same probability as a productive well. The same probability for reserves would be assigned to zero as to the highest value possible, and to any value in between.

Baysian Criteria

Baysian or expected value criteria is a weighted average of the conditional consequences, where the weights correspond to the probability assigned to the various states. This expected value is then used as the representative value of the act.[25] To use this criteria the decision maker must assign probabilities to the individual states of nature that reflect his or her subjective judgement or objective measurement regarding the likelihood of their occurrence. In petroleum exploration, this ranges from a dry hole to a well at the highest possible value of reserves. Somewhere in between lies a value determined either objectively or subjectively that represents the expected value. This should not be confused with the most likely value. If the distribution of probability is normal, the expected value and most likely value will coincide. If however as is most often the case, the distribution is skewed; the mode value is the most likely value and the mean value is the expected value.[26] Baysian criteria can be used with either the binomial, or any other discrete distribution, or with a continuous distribution, that would measure every possible value within a state of nature. For example; A binomial distribution could be used to measure success/failure such as dry hole/productive well, while a continuous distribution could be used to measure the likelihood of moderate to highly successful, such as the reserve distribution. Referring to Example 1, if the decision maker either objectively or subjectively determined that the chance of success for prospect A was 30%, prospect B 40%, prospect C 60%, and prospect D 10%, the chance of failure would be 70%, 60%, 40% and 90% respectively. This is shown in Example 3, along with the resulting expected value. This is because this is a Bernoulli process and in a Bernoulli process the complement must equal one minus the probability of success.

Using expected value, the decision maker would pick prospect C.

Example 3

Prospect	Payoff	Cost	Probability Success	Expected Value
A	100	50	.3	-5
B	200	100	.4	20
C	300	150	.6	120
D	400	200	.1	-140

BIAS

If one throws a fair die, one expects a one in six chance of rolling any spot on the die. What happens if the die is not fair? In that case the die is biased for or against certain numbers. Depending on the payoff and cost, using expected value, profitable events could be passed up, or unprofitable events chosen under bias. For example; Consider a wager with the following consequences: Receive $100 for each five rolled, and pay $5 for each roll. The expected value of one roll is $11.65 (1/6x$95 + -5/6x$5). Using expected monetary value criteria, a decision maker would take this wager because it has a positive expected value. If the die were loaded or biased such that no five could be rolled, the true expected monetary value would be -$5.00 (0/6x$95 + -6/6x$5). In this case the decision maker has been lured into a losing proposition because the actual expected value is -$5.00. The opposite could just as well happen. Consider the original wager, but with $10.00 received for every five rolled and the original cost of $5.00 per roll. The expected value would be -$3.33. The decision maker using expected value would pass up this wager. If the die were biased such that every six rolls resulted in four fives, the true expected value would be $1.67. The decision maker would have passed up a profitable opportunity.

Bias can just as easily be introduced by ignorance or imperfect information. Suppose our player witnessed six rolls of a die, two of which were fives. He or she might conclude the true odds were two in six of rolling a five. Basing decisions on incorrect or incomplete probability can be potentially dangerous. One objective of this paper is to determine how much bias can be tolerated before the expected value criteria is worse than simple random guessing.

In this paper bias means the actual distribution is different than the distribution assumed by the decision maker.

METHODOLOGY

To begin the trial, each decision making criteria is given equal initial budgets. Using a Monte Carlo computer program, 100 prospects are generated per year for twenty years. The generated values include expected net dry hole cost, expected net completion cost, discounted expected net operating income, and the chance factor. Net operating income being defined as your share of gross revenue, less your share of direct cost.[27] Along with these, for each prospect, actual values of net dry hole cost, net completion cost, and discounted net operating income are determined using one pass of the Monte Carlo simulator. For simplicity, taxes are ignored. Each year the 100 prospects are ranked by expected monetary value, maximax, maximin, Laplace, and random criteria, and selected as the budgets allow. The profit or loss for each criteria, as determined from the actual values, or "truth", is the increase or decrease in that criteria's budget for the following year. This is analogous to selling the properties to raise capital for expansion. Thus all values are assumed to be discounted. The results from each criteria are compared at the end of twenty years, to determine which criteria yields the highest cumulative discounted profit.

In this process the input values set by the user are initial budgets, minimum and maximum net dry hole cost, and minimum and maximum net operating income. Triangular distributions were used for all uncertain parameters. The computer program randomly selects the most likely values for net operating income and net dry hole cost, between the minimum and maximum input values. Net completion cost is set equal to net dry hole cost. The process is outlined below in step fashion. Should anyone care to replicate the process, the computer program and subroutines are included in the appendix.

Step 1. Set values for initial budget, minimum dry hole cost, maximum dry hole cost, minimum and maximum net operating income. For this test initial values selected were $3,000,000, $200,000, $1,500,000, $0, and $4,000,000 respectively. The reason for setting minimum net operating income equal to zero is to duplicate the condition of setting pipe on a dry hole.

Step 2. For each prospect the most likely value for the triangular distribution for dry hole cost is generated using a uniform cumulative frequency function, (Subroutine Uniform in the appendix), and the intrinsic random number function of Vax 8600 VMS v5.0 Fortran to solve the equation:[28]

$$X = ((CF)(XMAX-XMIN)) + XMIN$$

In the equation, CF is cumulative frequency, between 0 and 1, represented by the random number obtained from the intrinsic random number function. XMAX is the maximum value of net dry hole cost and XMIN is the minimum value of net dry hole cost. This is illustrated by the following example. XMIN is set equal to $200,000, the minimum value of net dry hole cost. XMAX is set equal to $1,500,000, the maximum value for net dry hole cost. The random number obtained from the intrinsic random number function is 0.8010457. Inserting into the equation and solving shows the most likely value of net dry hole cost to be $1,241,359. This is depicted in Figures 1 and 2. Figure 1 shows the uniform distribution. Figure 2 shows the resulting cumulative frequency curve. Note the above equation is the equation for the cumulative frequency curve. In this example, entering the curve on the y axis at 0.801457, returns the x value or most likely net dry hole cost of $1,241,359. The mean or conditional expected value for net dry hole cost

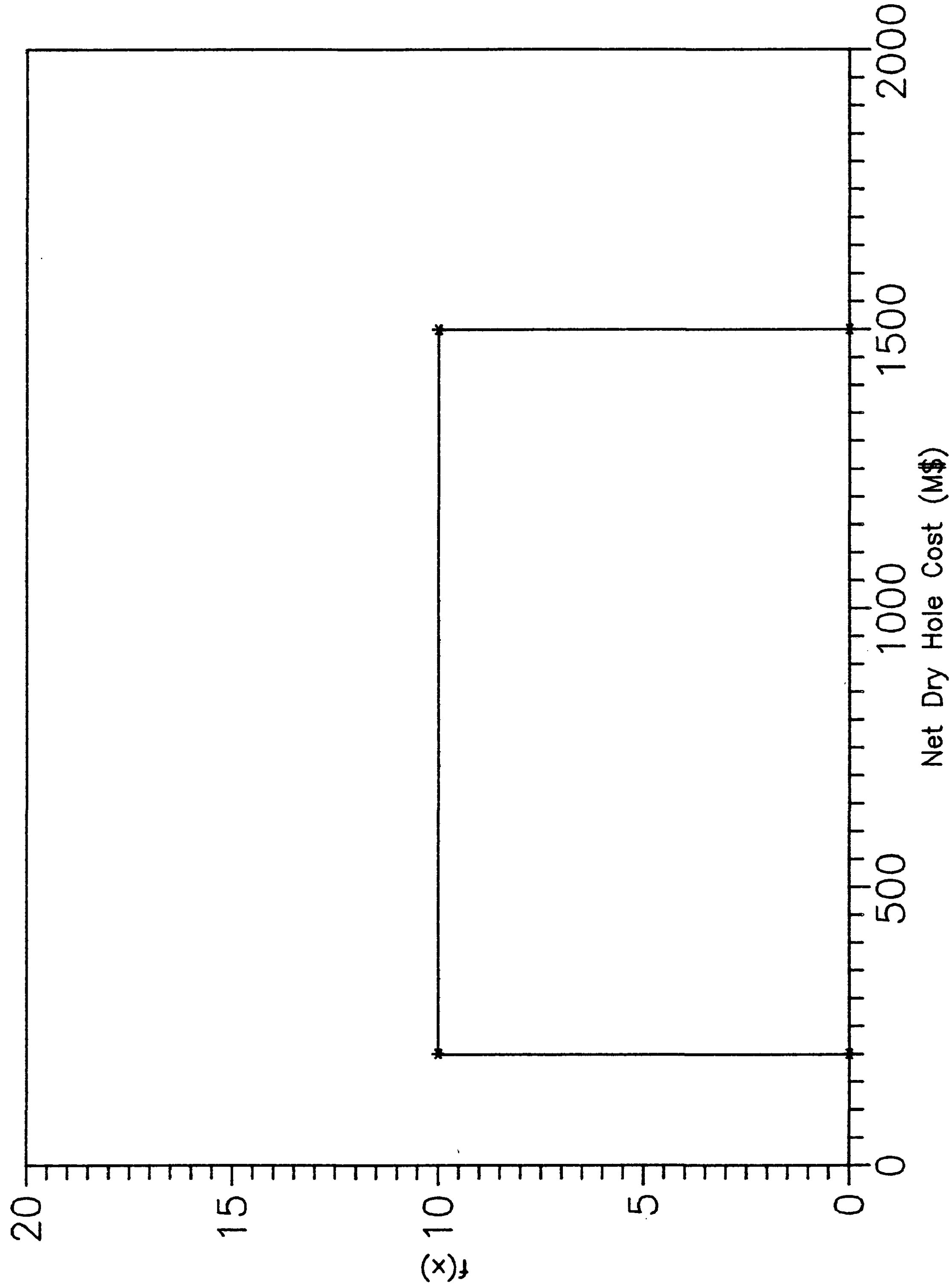


Figure 1. Uniform Probability Distribution

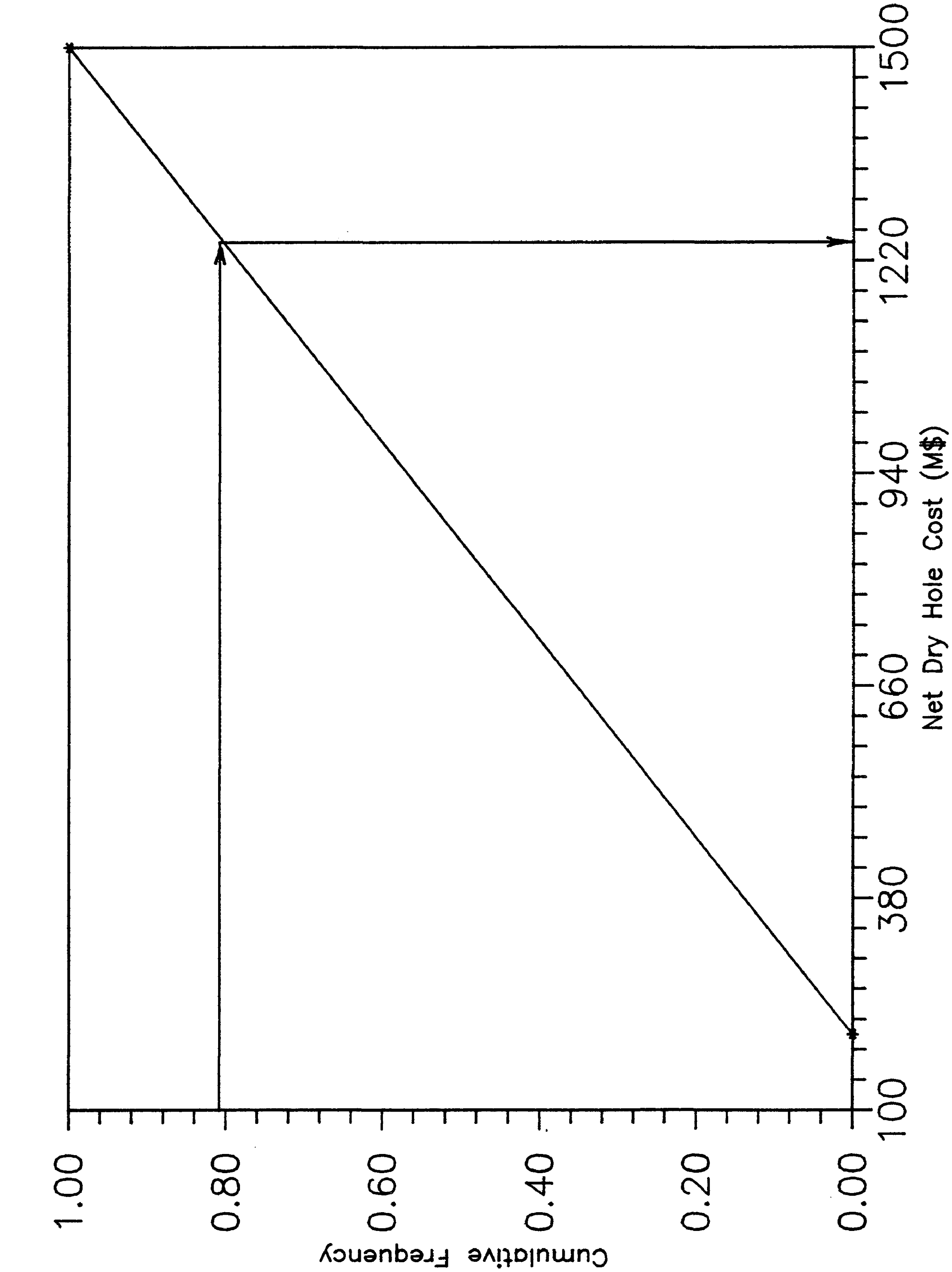


Figure 2. Cumulative Frequency Distribution

is the arithmetic average of minimum, maximum, and most likely net dry hole cost, $980,453.

Step 3. The conditional expected value for net operating income is calculated by the same process outlined in Step 2, using the input values for minimum and maximum net operating income to generate the probability distributions. It should be noted that with the values of minimum, maximum, and most likely, we now have triangular probability distributions for net dry hole cost and net operating income.

Step 4. The conditional expected value for the whole project is now calculated by subtracting conditional expected net drilling and completion cost, (net dry hole cost + net completion cost), from conditional expected net operating income. This gives the conditional expected net present value if successful. The failure mode is simply the conditional expected net dry hole cost. Again, using the intrinsic random number function, a number between one and zero is generated. This is the chance factor of success. The number is multiplied by the conditional expected value of success. One minus the chance factor of success is multiplied by the conditional expected value of failure. The two are summed and the resultant value is the expected monetary value for the project.

Step 5. The Laplace criteria is calculated in the same manner as the expected monetary value in step 4, with the exception that 0.5 is always used as the chance factor for success.

Step 6. Maximax criteria is set equal to the conditional value of success described in step 4.

Step 7. Maximin criteria is set equal to the conditional value of failure described in step 4.

Step 8. Random criteria is accomplished by assigning a random number to each project, using the intrinsic random number generator. The projects are then ranked in ascending order.

Step 9. The actual value for net dry hole cost is determined using a triangular cumulative frequency function, (Subroutine Triangle in the appendix), and the intrinsic random number function, to solve the equations:[29]

For $X \leq X_2$

$$X = X_1+(X_3-X_1)((CF)(X_2-X_1)/(X_3-X_1))^{.5}$$

For $X \geq X_2$

$$X = X_1+(X_3-X_1)((1-((1-CF)(X_2-X_1)/(X_3-X_1))^{.5}$$

Where X_1 = minimum value of net dry hole cost, X_2 = most likely value of net dry hole cost, X_3 = the maximum value of net dry hole cost, and CF = cumulative frequency, represented by a random number between zero and one.[29] This is illustrated by continuing the example in step 2. X_1 = \$200,000, X_2 = \$1,241,359, X_3 = \$1,500,000, and CF = 0.7402042. Inserting these values in the above equations, yields an actual value for net dry hole cost of \$1,204,447, which is close to the expected value of \$1,241,359, determined in step 2. This process is illustrated graphically by Figures 3 and 4. Figure 3 is the triangular distribution determined in step 2. The end points were set by the user, and the most likely value or mode was determined by the computer program. Figure 4 is the associated cumulative frequency curve. The above equations are the equations for the cumulative frequency curve on either side of the inflection point. Entering the y axis of Figure 4 at a cumulative frequency of 0.7402042, returns an X value or "truth" for net dry hole cost of \$1,204,447. Actual drilling and completion cost is simply two times this number.

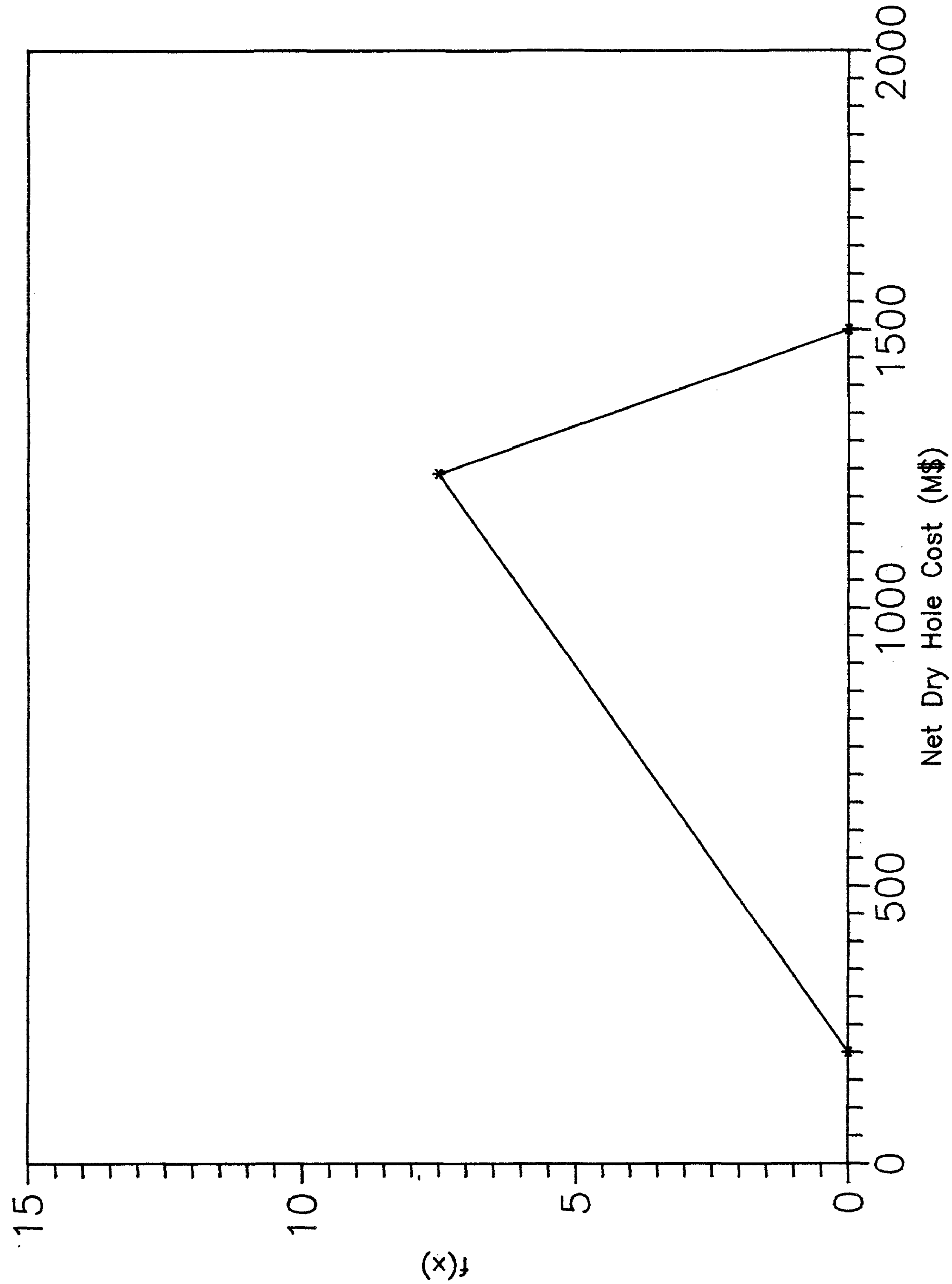


Figure 3. Triangular Probability Distribution

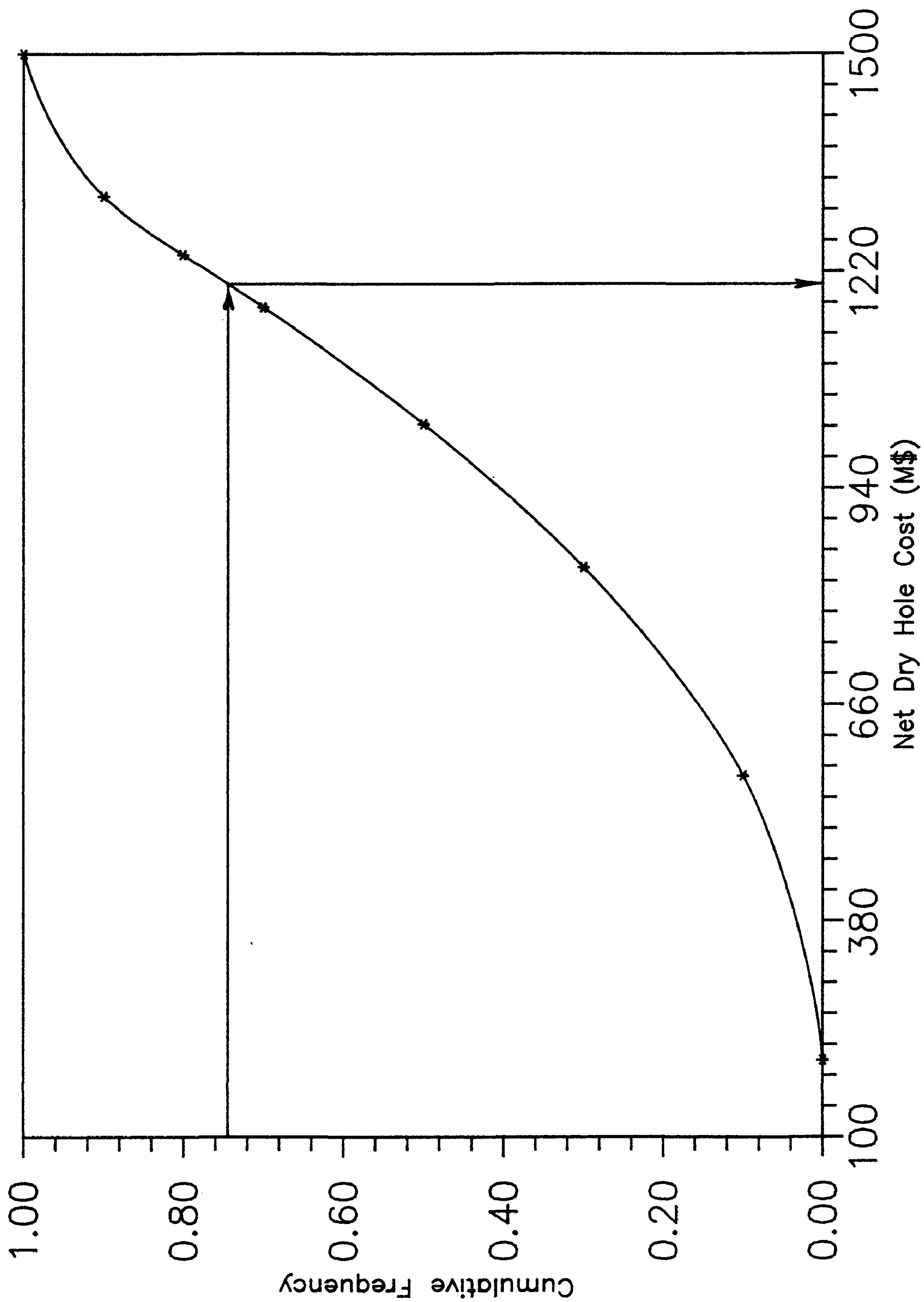


Figure 4. Cumulative Frequency Distribution

Step 10. The actual values for net operating revenue are determined in the same fashion as in step 9, with the corner values of the triangular distribution, the values of minimum, maximum, and most likely net operating revenue, determined in step 3.

Step 11. A random number between zero and one is generated and compared to the chance factor of success, generated in step 4. If the new random number is less than the chance factor of success, then the well is deemed a success; that is hydrocarbons are present. This does not necessarily mean the well will make a profit. It could represent the state of a well, that while hydrocarbons are present, the well does not payout. It could even represent the extreme case of setting pipe on a dry hole. Profit is equal to discounted net operating income less the drilling and completion cost. If the newly generated random number is greater than the chance factor for success, then the well is deemed a "duster", and cost is equal to the net dry hole cost determined in step 9. The net operating income is of course zero.

Step 12. The process is repeated an additional 99 times to complete the 100 prospects for the year, with the values stored in an array.

Step 13. The 100 values for each criteria are sorted in descending order using a bubble sort routine, (Subroutine Sort in the appendix), with the exception of random criteria, which is sorted in ascending order. (Subroutine Sorta in Appendix A)

Step 14. Using a selection routine, (Subroutines Sum and Mus in the appendix), the prospects are selected until the budget for the year is exhausted. Should a project result in a dry hole, the budgeted completion cost is returned to the budget for the year. If the last project selected is not 100% funded, a portion of the project is undertaken, and net operating income and cost are allocated based on the ratio of remaining budget to actual net drilling cost or net dry hole cost. The net operating income is used as the budget for the following year. There is a provision for returning any unexpended funds, should the total budget not be spent, such as the case of running out of acceptable projects before running out of money. In the case of random criteria and maximin, the selection of projects with values of less than zero is permitted, in all other cases it is not.

Step 15. The actual values for the projects selected under each criteria are presented, and the process is repeated an additional 19 times to complete the 20 year period, making a total of 2000 individual projects evaluated.

RESULTS

Perfect Information

The results of thirty such computer runs are summarized in Table 1. Each computer run began with a unique random number function seed. For those wishing to reproduce this experiment, the original seed was 5584163. For each succeeding computer run, the seed was increased by two, so that the runs are separate and distinct.

To test the hypothesis that expected monetary value will always outperform the other criteria, statistical inference is used. Statistical inference is the set of statistical techniques used to make statements about a population, based on information contained in a sample, randomly obtained from the general population.[30] The use of the intrinsic random number function allows the criteria of randomness to be met. The procedure is relatively simple:

1. State a belief.
2. Perform an experiment and observe the results.
3. Consider the probability of this experimental result if the belief is true.
4. If the result is unusual, discard the belief, otherwise hold to the belief.

This belief is also referred to as the null hypothesis in

Table 1
Criteria Performance
Perfect Information

Run #	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
1	181.29	87.51	41.37	37.09	0
2	174.93	117.635	58.44	59.17	0
3	190.21	98.26	35.25	54.36	0
4	260.55	128.64	71.55	71.29	0
5	282.03	148.63	76.58	65.31	0
6	147.54	101.35	45.33	52.22	0
7	229.72	158.73	78.56	76.82	2.5
8	227.87	161.56	59.43	55.56	0
9	202.65	122.19	48.39	57.68	0
10	183.39	121.37	43.64	39.68	0
11	198.77	94.97	46.31	40.36	0
12	238.52	133.42	58.12	65.07	0
13	203.90	125.71	59.97	56.86	0
14	198.92	108.75	44.48	53.12	0
15	241.12	138.20	72.27	69.39	0
16	217.76	0	0	50.97	0
17	246.43	144.02	74.07	70.42	0
18	184.37	101.47	60.16	48.70	0
19	199.19	124.92	55.73	0	0
20	0	0	39.55	50.32	0
21	251.43	99.82	81.41	76.51	0
22	240.23	122.41	69.35	75.61	0
23	208.88	112.53	54.77	49.27	0
24	227.25	124.59	63.05	68.06	0
25	199.45	62.87	41.62	34.08	0
26	230.94	107.20	50.84	53.67	0
27	195.22	77.83	50.16	57.60	0
28	210.08	119.931	57.54	47.82	0
29	183.99	0	0	56.31	0
30	225.79	102.48	59.96	68.15	0

Initial Budget = $3,000,000
Dry Hole Costs = $200,000 - $1,500,000
Net Operating Income = $0 - $4,000,000

statistical analysis research. The object is to decide whether or not the difference in the experimental results and the null hypothesis is due to more than chance; if so the null hypothesis is rejected. In our experiment the belief is:

> If the decision maker consistently selects the alternative having the highest positive expected monetary value his total net gain from <u>all</u> (emphasis added) decisions will be higher than his gain from any alternative strategy for selecting decisions under uncertainty. This statement is true even though each specific decision is a different drilling prospect with different probabilities.

With the use of the computer program described in the Methodology section of this paper, the requirements of the last sentence have been satisfied.

Looking at run number twenty in Table 1 shows this hypothesis to be false. Expected monetary value was beaten by both maximax and maximin criteria in this particular run. What then can we say about our results? First restate the belief as: Expected monetary value will result in a higher net gain than any alternative decision making criteria under uncertainty 99% of the time. Conversely 1% of the time some alternative strategy for decision making under uncertainty will result in a higher net gain. The binomial distribution right tail (cumulative) term, is used to determine the probability value, P, described by the equation:[31]

$$\sum_{X}^{n} \frac{n!}{X!\,(n-X)} \pi^{X} (1-\pi)^{n-X}$$

Where π is the probability that some other criteria will defeat expected monetary value, n is the number of samples in the experiment, and X is the actual number of times expected monetary value was defeated in the experiment. We see that setting the probability (π) that some other decision criteria under uncertainty will result in a higher net gain than expected monetary value 1% of the time, with thirty samples (n) and one time in which the expected monetary value criteria resulted in a lower net gain than another criteria (X), our probability value or "p-value" is 0.2603.

$$P(X \geq 1 | n=30, \pi=0.01) = 0.2603$$

In simple terms, in any sample of size thirty, there is a 26.03% chance that the expected monetary value criteria will result in lower net gain than some other decision criteria under uncertainty, simply due to chance, if in fact it is true that expected monetary value will result in higher net gains 99% of the time.

Next this must be compared to the level of significance, α. The level of significance tells if we can accept or reject the hypothesis. In inference testing this level is usually set at .1 or .15. If $P \geq \alpha$, then the result is considered significant or usual and the hypothesis stands. Otherwise the hypothesis is rejected. As P = .2603, we cannot reject the null hypothesis that expected monetary value will beat the other decision criteria tested 99% of the time. Note that this does not prove or disprove the hypothesis, it only gives us assurance that it is true. If the expected monetary value criteria had been defeated two times, the p-value would have been .0361 and we would have had to reject our hypothesis. Thus we can stand one defeat but not two and still retain our hypothesis. What is of particular interest, in addition to expected monetary value winning 97% of the actual cases, is the fact that random selection managed only once to survive the full twenty years, and in that case still lost $500,000.

Bias and P/I Ratio

To look at the effect of bias or error, the conditional expected value for net operating income is multiplied by a bias factor. This will either increase or decrease expected net operating income, and thus expected monetary value, dependent on the bias factor. Only the perceived expected values are changed. The actual results of each project will remain the same as under perfect information, but the order in which projects are selected can change. To determine the effects of bias, cases were run from -50% bias to 100% bias. That is the expected net operating income was calculated, underestimated by 50%, to overestimated by 100%

In addition to bias the profit to investment ratio was also varied. To vary the potential profit to investment ratio, (P/I=(Net Operating Income-Cost)/Cost)), of the projects, the minimum net dry hole cost is increased or decreased. The minimum value of P/I is always -1. Zero net operating income minus dry hole cost divided by dry hole cost. The maximum P/I ratio is the maximum value of net operating income less the minimum value of drilling cost divided by the minimum value of drill cost. For example:

Maximum P/I = ($4,000,000 - $400,000)/$400,000 = 9

The P/I ratio for any project can range from the minimum value to the maximum value of P/I ratio. To observe the effect of P/I ratio, the maximum value was set at 9, 7, 5, 3, 2, and 1.

The effects of systematic bias (bias in the same direction) and P/I ratios are presented in Tables 2,4,6,8,10, and 12. Random bias is observed by allowing bias in either direction. That is, + or - 10% error, for example. To do this a random variable decides if the error is positive or negative. This effect is presented in Tables 3, 5, 7, 9, 11, and 13.

Table 2
Criteria Performance
P/I = -1 to 9
Systematic Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
-50%	3.00	3.00	8.04	37.09	0
-40%	26.00	3.00	54.29	37.09	0
-30%	86.60	3.00	90.15	37.09	0
-20%	151.09	14.79	55.46	37.09	0
-10%	179.26	54.29	42.03	37.09	0
0%	181.29	87.51	41.37	37.09	0
10%	163.41	79.60	41.33	37.09	0
20%	155.74	55.46	41.33	37.09	0
30%	120.93	42.39	41.33	37.09	0
40%	101.20	41.33	39.03	37.09	0
50%	92.27	41.37	38.23	37.09	0
60%	83.27	41.57	37.20	37.09	0
70%	79.46	41.33	37.20	37.09	0
80%	78.13	41.33	34.51	37.09	0
90%	77.40	41.33	30.83	37.09	0
100%	77.10	41.33	27.55	37.09	0

Table 3
Criteria Performance
P/I = -1 to 9
Random Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
10%	162.18	70.67	39.21	37.09	0
20%	127.34	49.05	39.63	37.09	0
30%	105.93	40.37	39.94	37.09	0
40%	93.81	38.48	39.03	37.09	0
50%	87.12	39.21	41.02	37.09	0
60%	83.48	41.33	37.20	37.09	0
70%	79.99	40.57	37.20	37.09	0
80%	72.01	39.63	34.51	37.09	0
90%	65.13	39.94	30.83	37.09	0
100%	63.27	38.29	27.55	37.09	0

Table 4
Criteria Performance
P/I = -1 to 7
Systematic Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
-50%	3.00	3.00	3.00	30.61	0
-40%	14.89	3.00	35.52	30.61	0
-30%	60.15	3.00	72.26	30.61	0
-20%	117.18	9.55	42.03	30.61	0
-10%	150.03	35.52	38.07	30.61	0
0%	159.02	62.42	38.66	30.61	0
10%	142.25	71.11	36.47	30.61	0
20%	132.99	42.03	32.61	30.61	0
30%	113.37	38.07	37.85	30.61	0
40%	84.72	38.07	32.11	30.61	0
50%	73.79	38.66	28.90	30.61	0
60%	74.86	36.47	24.42	30.61	0
70%	75.15	34.63	24.15	30.61	0
80%	75.15	32.11	21.15	30.61	0
90%	73.79	37.85	22.84	30.61	0
100%	74.44	32.04	7.39	30.61	0

Table 5
Criteria Performance
P/I = -1 to 7
Random Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
10%	130.54	63.57	35.73	30.61	0
20%	112.93	40.39	32.11	30.61	0
30%	97.38	36.75	37.85	30.61	0
40%	81.95	37.30	32.11	30.61	0
50%	73.60	35.88	28.90	30.61	0
60%	65.30	36.11	24.42	30.61	0
70%	61.13	34.76	24.15	30.61	0
80%	58.80	32.11	21.15	30.61	0
90%	58.80	37.85	22.84	30.61	0
100%	58.80	32.04	7.39	30.61	0

Table 6
Criteria Performance
P/I = -1 to 5
Systematic Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
-50%	3.00	3.00	3.00	19.43	0
-40%	3.00	3.00	6.04	19.43	0
-30%	28.88	3.00	33.33	19.43	0
-20%	84.25	3.00	21.73	19.43	0
-10%	117.17	6.04	25.88	19.43	0
0%	123.42	28.69	25.88	19.43	0
10%	112.87	35.16	22.84	19.43	0
20%	98.21	21.73	25.88	19.43	0
30%	81.73	22.04	4.13	19.43	0
40%	68.73	25.88	0	19.43	0
50%	68.37	25.88	0	19.43	0
60%	68.37	25.88	0	19.43	0
70%	65.97	22.84	0	19.43	0
80%	65.97	25.88	0	19.43	0
90%	65.97	25.88	0	19.43	0
100%	65.97	3.92	0	19.43	0

Table 7
Criteria Performance
P/I = -1 to 5
Random Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
10%	99.34	35.65	22.84	19.43	0
20%	84.97	23.05	25.88	19.43	0
30%	74.61	23.71	4.13	19.43	0
40%	59.97	25.88	0	19.43	0
50%	58.80	25.88	0	19.43	0
60%	58.80	25.88	0	19.43	0
70%	56.71	22.84	0	19.43	0
80%	54.11	25.88	0	19.43	0
90%	53.94	25.88	0	19.43	0
100%	53.94	3.92	0	19.43	0

Table 8
Criteria Performance
P/I = -1 to 3
Systematic Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
-50%	3.00	3.00	3.00	10.69	0
-40%	3.00	3.00	3.00	10.69	0
-30%	3.00	3.00	0	10.69	0
-20%	31.92	3.00	0	10.69	0
-10%	58.49	3.00	0	10.69	0
0%	60.84	6.44	0	10.69	0
10%	57.43	0	0	10.69	0
20%	54.53	0	0	10.69	0
30%	48.88	0	0	10.69	0
40%	48.41	0	0	10.69	0
50%	48.84	0	0	10.69	0
60%	47.57	0	0	10.69	0
70%	47.57	0	0	10.69	0
80%	47.57	0	0	10.69	0
90%	46.17	0	0	10.69	0
100%	46.80	0	0	10.69	0

Table 9
Criteria Performance
P/I = -1 to 3
Random Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
10%	55.08	0	0	10.69	0
20%	48.80	0	0	10.69	0
30%	48.89	0	0	10.69	0
40%	47.85	0	0	10.69	0
50%	44.75	0	0	10.69	0
60%	44.51	0	0	10.69	0
70%	44.69	0	0	10.69	0
80%	44.74	0	0	10.69	0
90%	44.74	0	0	10.69	0
100%	45.87	0	0	10.69	0

Table 10
Criteria Performance
P/I = -1 to 2
Systematic Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
-50%	3.00	3.00	3.00	0	0
-40%	3.00	3.00	3.00	0	0
-30%	3.00	3.00	3.00	0	0
-20%	5.06	3.00	0	0	0
-10%	26.01	3.00	0	0	0
0%	34.75	3.00	0	0	0
10%	30.01	5.65	0	0	0
20%	28.60	0	0	0	0
30%	28.31	0	0	0	0
40%	27.39	0	0	0	0
50%	22.85	0	0	0	0
60%	22.85	0	0	0	0
70%	22.84	0	0	0	0
80%	22.84	0	0	0	0
90%	22.39	0	0	0	0
100%	22.39	0	0	0	0

Table 11
Criteria Performance
P/I = -1 to 2
Random Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
10%	28.94	5.65	0	0	0
20%	28.24	0	0	0	0
30%	28.31	0	0	0	0
40%	27.31	0	0	0	0
50%	22.85	0	0	0	0
60%	22.85	0	0	0	0
70%	22.84	0	0	0	0
80%	22.84	0	0	0	0
90%	22.39	0	0	0	0
100%	22.39	0	0	0	0

Table 12
Criteria Performance
P/I = -1 to 1
Systematic Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
-50%	3.00	3.00	3.00	0	0
-40%	3.00	3.00	3.00	0	0
-30%	3.00	3.00	3.00	0	0
-20%	3.00	3.00	3.00	0	0
-10%	3.00	3.00	1.84	0	0
0%	3.87	3.00	0	0	0
10%	0.43	3.00	0	0	0
20%	0.58	3.00	0	0	0
30%	0.49	3.00	0	0	0
40%	0.25	0	0	0	0
50%	0.10	0	0	0	0
60%	0.07	0	0	0	0
70%	0.07	0	0	0	0
80%	0.07	0	0	0	0
90%	0.07	0	0	0	0
100%	0.07	0	0	0	0

Table 13
Criteria Performance
P/I = -1 to 1
Random Bias

Bias	Expected Monetary Value MM$	Laplace Criteria MM$	Maximax Criteria MM$	Maximin Criteria MM$	Random Criteria MM$
10%	0.43	3.00	0	0	0
20%	0.58	3.00	0	0	0
30%	0.49	3.00	0	0	0
40%	0.24	0	0	0	0
50%	0.10	0	0	0	0
60%	0.07	0	0	0	0
70%	0.07	0	0	0	0
80%	0.07	0	0	0	0
90%	0.07	0	0	0	0
100%	0.07	0	0	0	0

Bias. Negative bias reduces the results from expected monetary value much more rapidly than does positive bias. This is shown in Figure 5. The reason for this is that under expected monetary value criteria, the projects are ranked by their expected net present value. Under negative bias, the net present value is underestimated, eliminating some of these higher ranked projects, that are more likely to succeed. As more negative bias is added the results from other criteria meet or exceed those from expected monetary value. This is to be expected; one cannot win a race if one does not run. This occurs in the area of -30% bias.

Positive bias results in the addition of projects, as the budget allows. While these projects have lower potential than the original projects, some will result in net operating income. Also if the project is a failure it will only lose the dry hole cost. Also noteworthy is the fact that as the bias approaches 100% , the decline in the results stabilizes. This is a result of two interactions. From zero to 60% bias, not all the drilling budget is spent each year. This is illustrated in Figure 6. In this example, after year six, there is more money each year than there are positive expected value projects. As such this money is banked. At a bias of less than 70%, the additional projects, caused by positive bias are funded from this

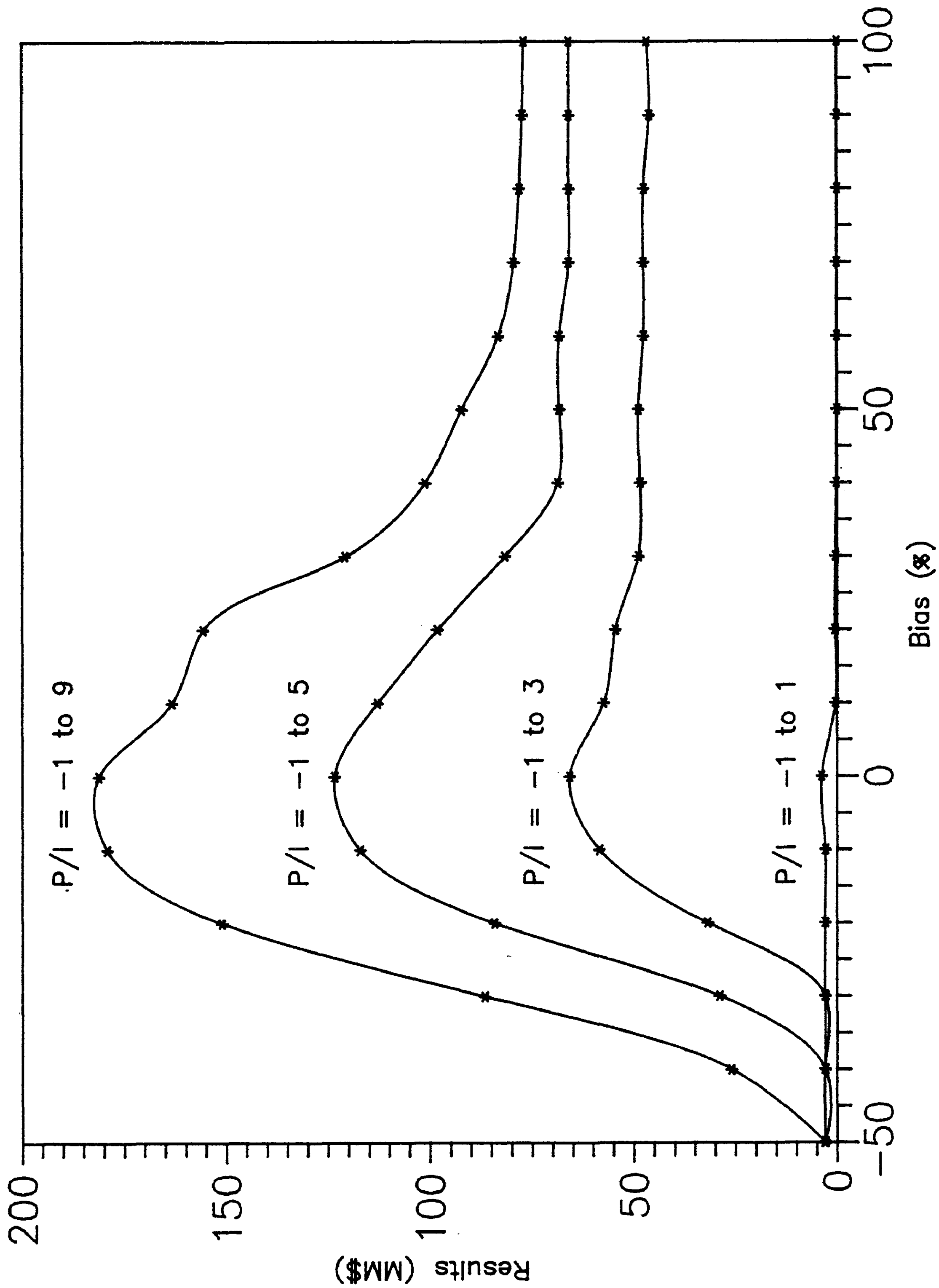
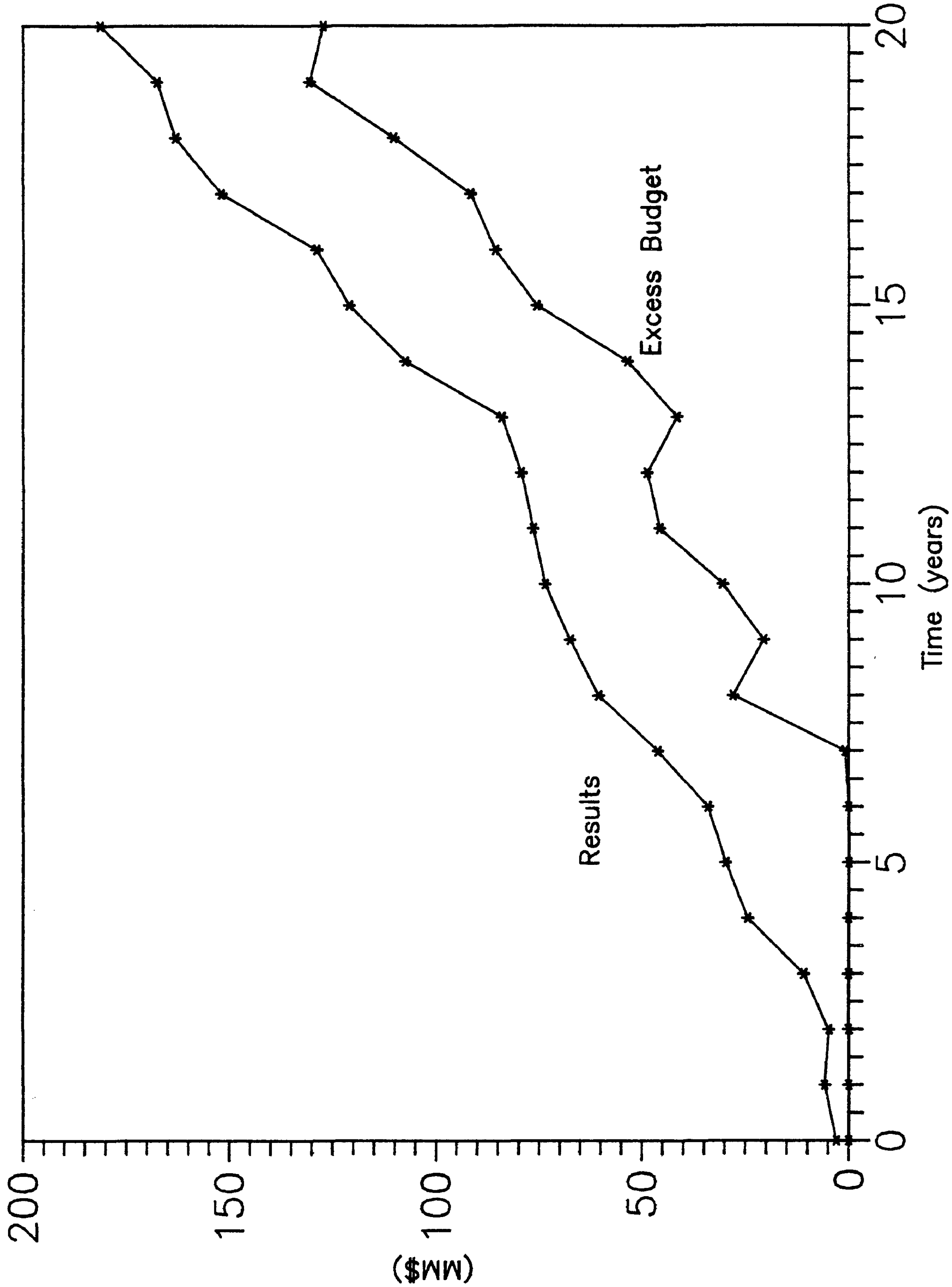


Figure 5. Expected Monetary Value Results v. Bias



P/I = −1 to 9

Figure 6. Expected Monetary Value Results & Excess Budget v. Time

"bankroll". At and above 70% bias, the "bankroll" does not exist, and all projects are funded from current net operating income. This forces the high grading of prospects. As the budget decreases, the quality of projects selected improves. A perusal of Tables 2 through 12 shows that under positive bias, expected monetary value achieves higher results than any of the other criteria tested.

Under random bias, the results from using the expected monetary value criteria decline as a function of bias, as shown in Figure 7. This decline appears more rapid than under systematic bias.

<u>P/I Ratio.</u> Profit to investment ratio does not appear to influence the choice of criteria, other than to reduce the results from all criteria. The effect on expected monetary value is shown in Figure 8. The relationship appear to be logarithmic, described by the equation:

$$Y = 84.908037 * \ln(X) - 12.897405$$

In evaluating Tables 2 through 13, it is evident that as P/I decreases, it becomes more critical that one used expected monetary value as a decision tool. As the P/I ratio decreases, the other decision criteria either go bankrupt or refuse to invest. Expected monetary criteria is the only criteria to consistently show a profit. At the lower P/I ratios, the effect of bias is particularly harsh on the

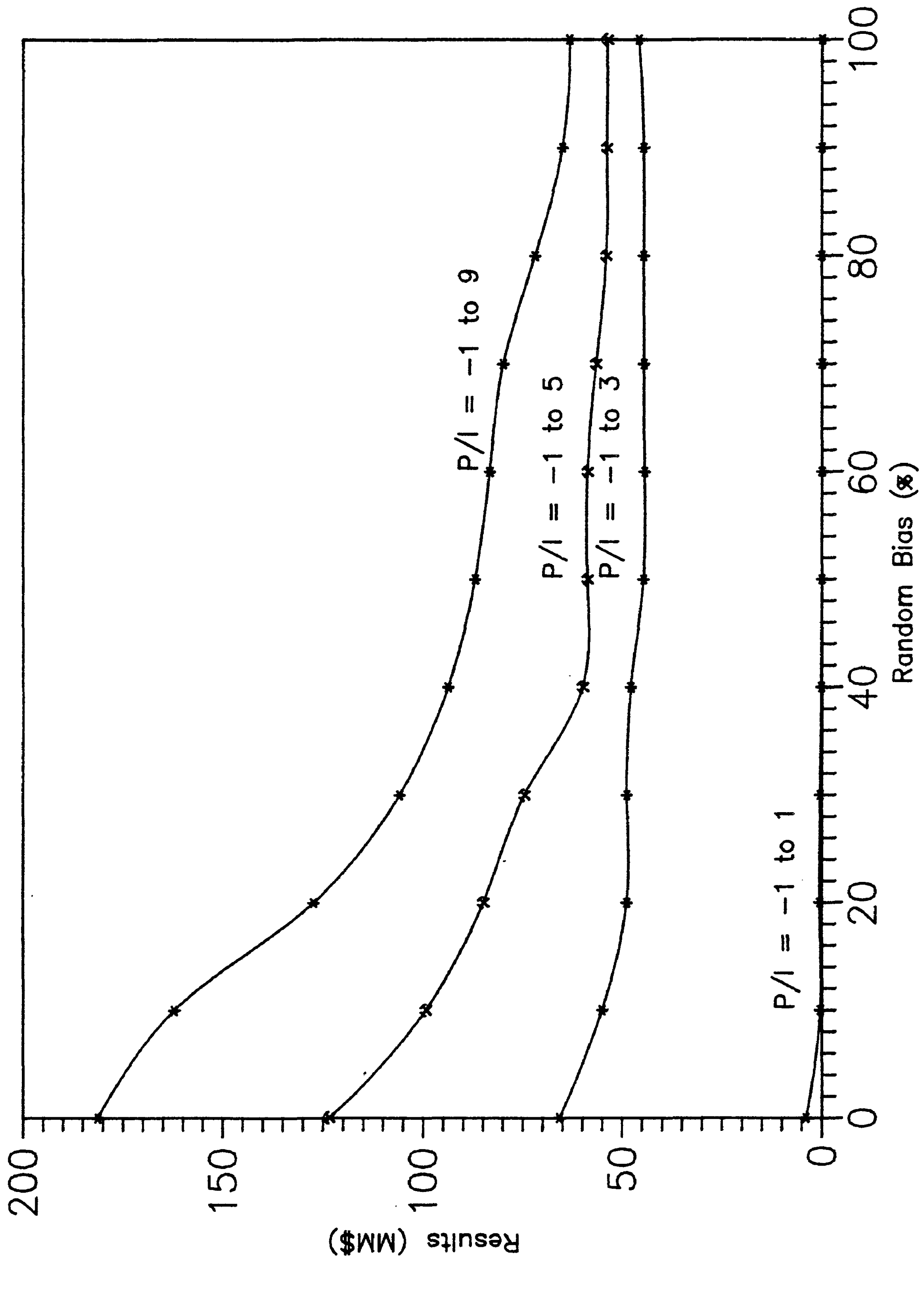


Figure 7. Expected Monetary Value Results v. Random Bias

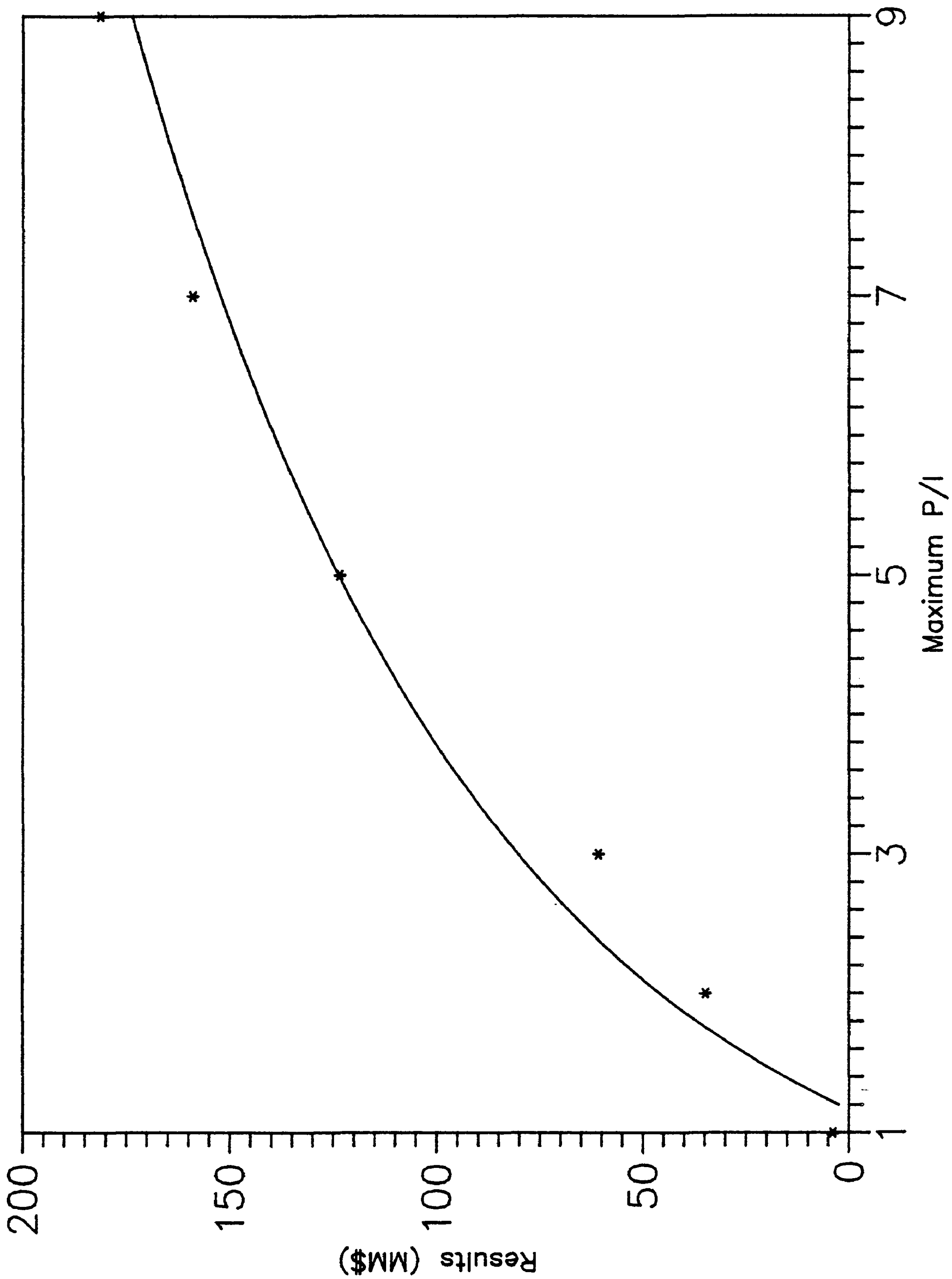


Figure 8. Expected Monetary Value Results v. Maximum P/I

alternative decision criteria.

Figures 9, 10, and 11 show examples of how the individual criteria perform over the twenty year period, at different maximum P/I ratios.

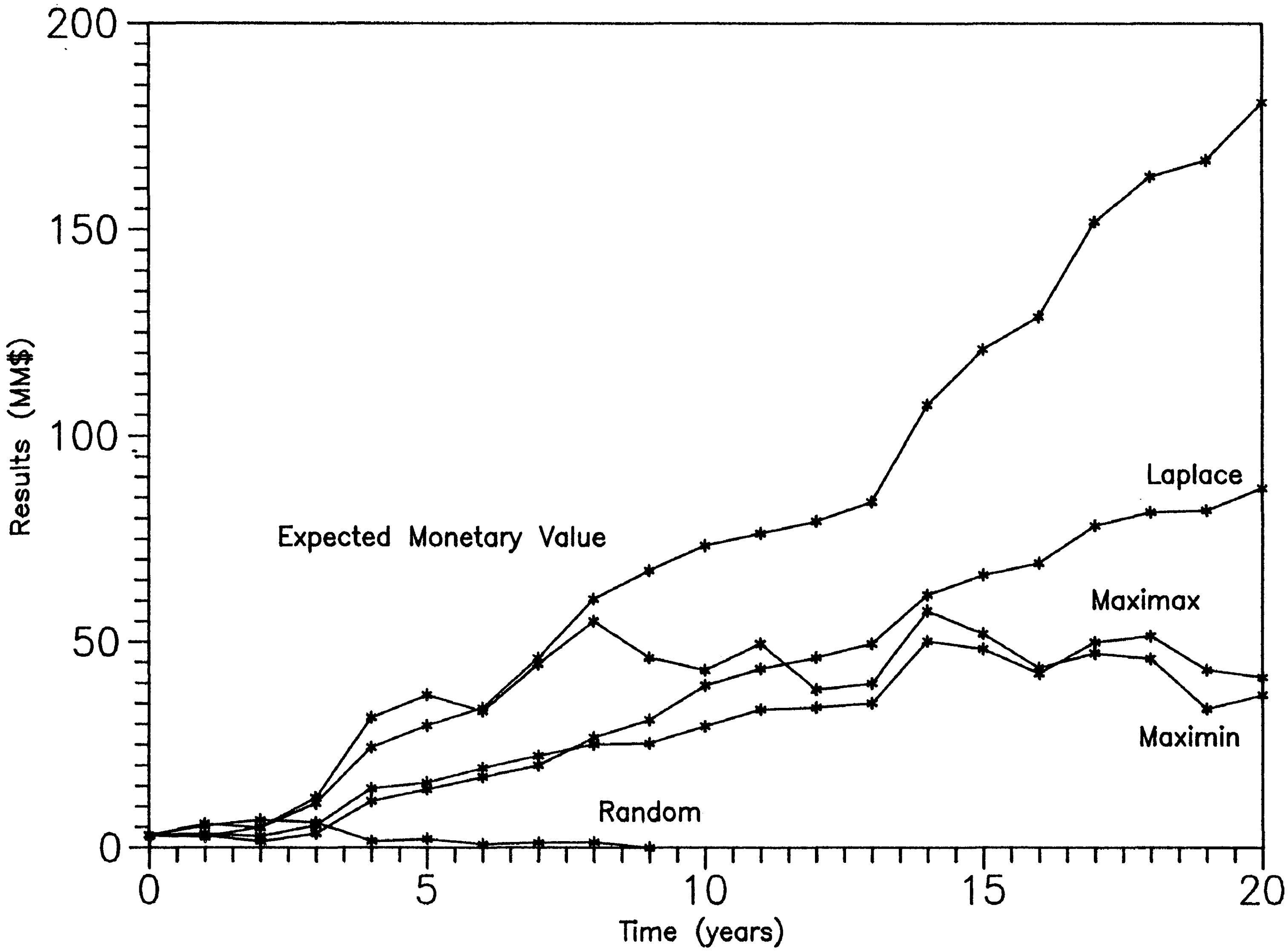
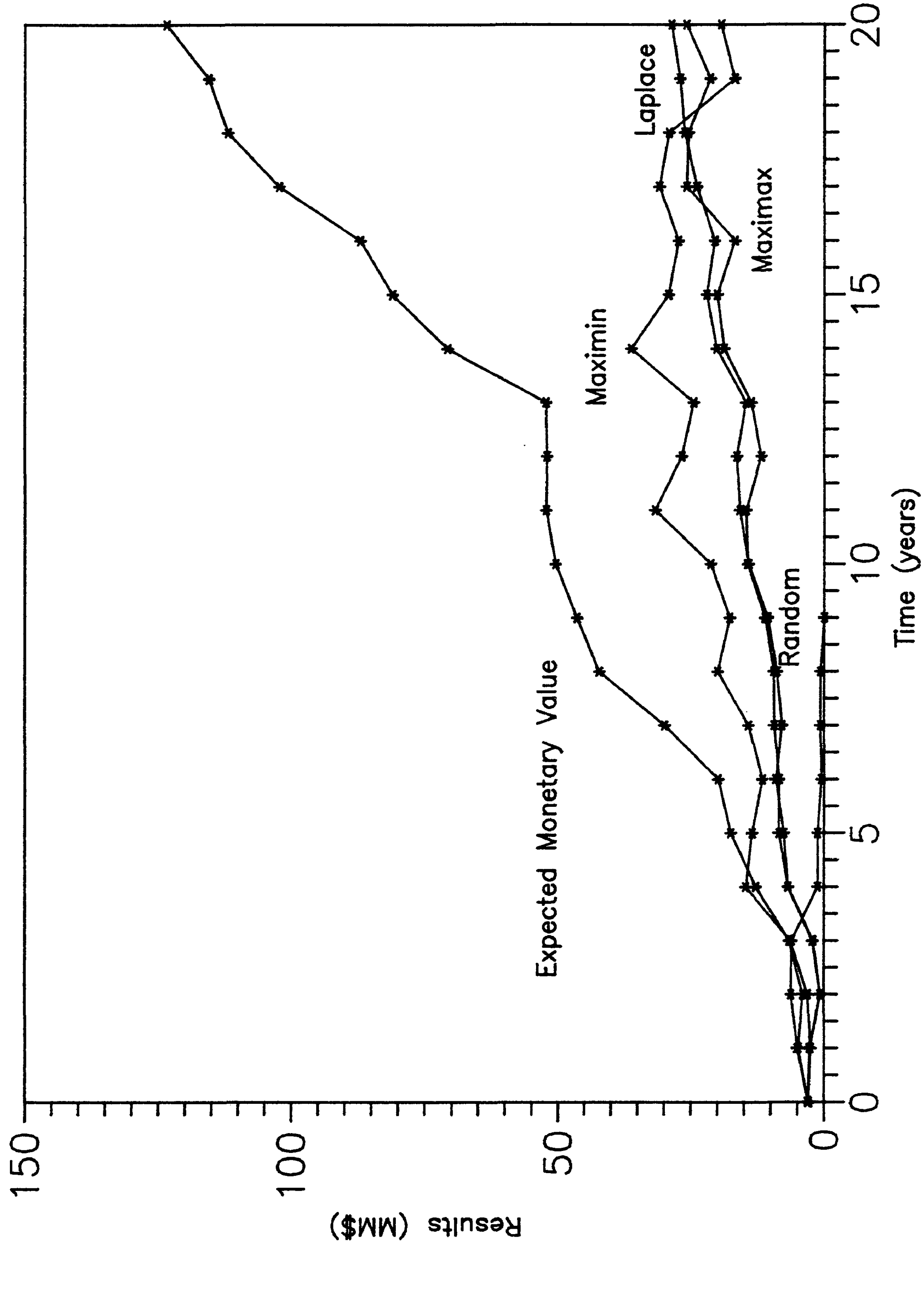
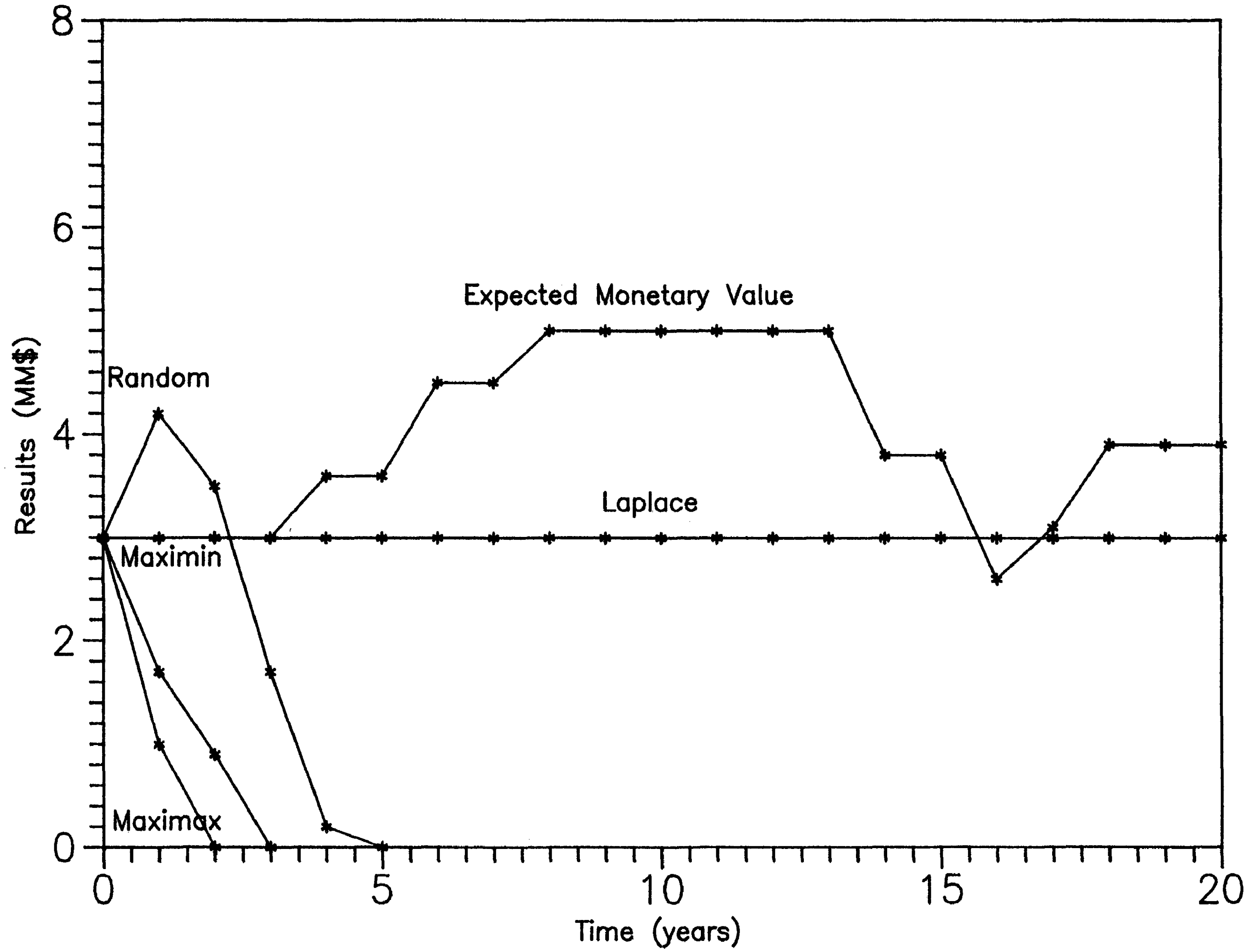


Figure 9. Criteria Performance v. Time



P/I = −1 to 5

Figure 10. Criteria Performance v. Time



P/I = −1 to 1
Figure 11. Criteria Performance v. Time

CONCLUSIONS

1. While the expected monetary value criteria is not a guarantee of making money, it is a valuable decision tool, which consistently out performs the other criteria evaluated. However it does not result in the highest net gain in <u>all</u> cases.
2. The expected monetary value criteria's performance is adversely affected by bias. More so by negative bias than positive bias. The effects of random bias on performance is to reduce profit as a function of bias. Despite the effects of bias, expected monetary value continues to out perform the other decision criteria.
3. The effect of profit to investment ratio is to reduce the revenues of all decision criteria in direct relation to the reduction in profit to investment ratio. As the P/I ratio decreases, it becomes increasingly imperative that expected monetary value be used as the decision criteria.

RECOMMENDATIONS FOR FUTURE WORK

In this trial, the results or truth were always within the bounds set at the beginning of the trial. In future work it would be interesting to see what effect bias might have should the results not be constrained within the initial bounds. Also, in this trial bias was only introduced in net operating income. Any future work should look at the effect of bias on chance factor. This would affect not only net operating income but drilling and completion cost as well, due to the interaction of the two variables.

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APPENDIX
Computer Program and Subroutines

```
*     PROGRAM TO PERFORM MONTE CARLO SIMULATION
      PARAMETER (NUM=100, NYR=20)
      DIMENSION XMAX(NUM), XMIN(NUM), RANDOM(NUM), DRC(NUM)
      DIMENSION EMV(NUM), EMVLP(NUM), ACTUAL(NUM)
      DIMENSION SMAX(NYR), SMIN(NYR), SEMVLP(NUM), SRAND(NUM)
      DIMENSION SEMV(NYR)
      IRN=5584163
      WRITE(9,100)
100   FORMAT(1X'YR',4X,'EMV',9X,'EMVLP',6X,'MAXIMAX',6X,'
   1  MAXIMIN',6X,'RANDOM')
      BUDGETA=3000
      BUDGETB=3000
      BUDGETC=3000
      BUDGETE=3000
      BUDGETF=3000
      BIAS=1.
      DATA DHCMIN,DHCMAX/-200.,-1500/

      DO 5 L=1,NYR
      DO 10 K=1,NUM
      XX=0.0
      YY=0.0
      EOIL=0.0
      ENOIL=0.0
      RANDOM=0.0
      Z=0.0
      ZZ=0.0

*     CALCULATE EV IF NO OIL

      RN=RAN(IRN)
      CALL UNIFORM(RN,DHCMIN,DHCMAX,DHCML)
      XX=(DHCMIN+DHCML+DHCMAX)/3

*     CALCULATE EV IF OIL

      SUCMIN=0.0
      SUCMAX=4000.
      RN=RAN(IRN)
      CALL UNIFORM(RN,SUCMIN,SUCMAX,SUCML)
      YY=(SUCMIN,SUCML,SUCMAX)/3
      YY=(YY*BIAS)
```

Computer Programs and Subroutines

```
      CALCULATE EXPECTED VALUE

      XMAX(K)=(YY=(2*XX)
      XMIN(K)=XX
      RN=RAN(IRN)
      Z=RN
      EOIL=((YY+(2*XX))*Z)
      ENOIL=XX*(1-Z)
      EMVLP(K)=(((YY+(2*XX))*.5)+(XX*.5))
      EMV(K)=(ENOIL+EOIL)

*     ACTUAL PROJECT VALUE

      RN=RAN(IRN)
      CALL TRIANGLE(RN,DHCMIN,DHCML,DHCMAX,AVNO)
      RN=RAN(IRN)
      CALL TRIANGLE(RN,SUCMIN,SUCML,SUCMAX,AVO)
      RN=RAN(IRN)
      IF(ZZ.LE.Z)THEN
      ACTUAL(K)=AVO
      DRC(K)=2*AVNO
      ELSE
      ACTUAL(K)=AVNO
      DRC(K)=AVNO
      ENDIF
      RN=RAN(IRN)
      RANDOM=RN*100
 10   CONTINUE

*     SORTING AND SUMMING OF CRITERIA

      A=0.0
      B=0.0
      C=0.0
      D=0.0
      E=0.0
      CALL SORT(EMV,EMVLP,XMIN,XMAX,RANDOM,ACTUAL,DRC,NUM)
      IF(BUDGETA.LE.0.0)GOTO 15
      CALL SUM(EMV,ACTUAL,DRC,BUDGETA,NUM,SUMA,A)
      BUDGETA=SUMA+A
      SEMV(L)=SUMA+A
 15   CALL SORT(EMVLP,EMV,XMIN,XMAX,RANDOM,ACTUAL,DRC,NUM)
      IF BUDGETB=SUMB+B
      IF(BUDGETB.LE.0.0)GOTO 25
```

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```
      CALL SUM(EMVLP,ACTUAL,DRC,BUDGETB,NUM,SUMB,B)
      BUDGETB=SUMB+B
      SEMV(L)=SUMB+B
 25   CALL SORT(XMAX,EMV,EMVLP,XMIN,RANDOM,ACTUAL,DRC,NUM)
      IF (BUDGETC.LE.0.0)GOTO 35
      CALL SUM(XMAX,ACTUAL,DRC,BUDGETC,NUM,SUMC,C)
      BUDGETC=SUMC+C
      SMAX(L)=SUMC+C
 35   CALL SORT(XMIN,EMV,EMVLP,XMAX,RANDOM,ACTUAL,DRC,NUM)
      IF(BUDGETD.LE.0.0)GOTO 45
      CALL MUS(XMIN,ACTUAL,DRC,BUDGETD,NUM,SUMD,D)
      BUDGETD=SUMD+D
      SMIN(L)=SUMD+D
 45   CALL SORTA(RANDOM,EMV,EMVLP,XMIN,XMAX,ACTUAL,DRC,NUM)
      IF(BUDGETE.LE.0.0)GOTO 5
      CALL MUS(RANDOM,ACTUAL,DRC,BUDGETE,NUM,SUME,E)
      BUDGETE=SUME+E
      SRAND(L)=SUME+E
  5   CONTINUE
      DO 70 I=1,NYR
      WRITE(9,101)I,SEMV(I),SEMVLP(I),SMAX(I),SMIN(I),
     1 SRAND(I)
 70   CONTINUE
101   FORMAT(I2,1X,F11.3,1X,F11.3,1X,F11.3,1X,F11.3,1X,F11.3)
      STOP
      END

      SUBROUTINE UNIFORM(YY,YMIN,YMAX,Y)
      Y=((YY/1.0)*(YMAX-YMIN))+YMIN
      RETURN
      END

      SUBROUTINE SWITCH(A,B)
      COPY=A
      A=B
      B=COPY
      RETURN
      END
```

Computer Program and Subroutines

```
      SUBROUTINE TRIANGLE(RN,XMIN,XML,XMAX,VALUE)
      CUMPRO=RN
      RANGE=XMAX=XMIN
      IF(RANGE.EQ.0.0) THEN
      XPRIME=XMIN
      VALUE=XMIN
      RETURN
      ELSE
      XM=(XML-XMIN)/RANGE
      XPRIME=SQRT(CUMPRO*XM)
      TEST=XPRIME*RANGE+XMIN
      IF(TEST.LE.XML)THEN
      VALUE=TEST
      ELSE
      XPRIME=1.0-(SQRT((1.0-CUMPRO)*(1.0-XM)))
      VALUE=XPRIME*RANGE+XMIN
      ENDIF
      ENDIF
      RETURN
      END

      SUBROUTINE SORT(KEY,B,C,F,D,E,ACTUAL,DRC,N)

      INTEGER N
      DIMENSION KEY(N),ACTUAL(N),DRC(N),C(N),D(N),E(N),B(N)
      INTEGER K, M, LOCSM
      DO 100 M=N,2,-1
      K=LOCSM(KEY,M)
      CALL SWITCH(KEY(M),DEY(K))
      CALL SWITCH(ACTUAL(M),ACTUAL(K))
      CALL SWITCH(DRC(M),DRC(K))
      CALL SWITCH(B(M),B(K))
      CALL SWITCH(C(M),B(K))
      CALL SWITCH(D(M),D(K))
      CALL SWITCH(E(M),E(K))
100   CONTINUE
      RETURN
      END
```

Computer Program and Subroutines

```
      INTEGER FUNCTION LOCSM(A,N)
      INTEGER N
      DIMENSION A(N)
      INTEGER K
      DIMENSION A(N)
      LOCSM=1
      DO 110 K=2,N
      IF(A(K).LT.A(LOCSM)) THEN
      ENDIF
110   CONTINUE
      RETURN
      END


      SUBROUTINE(SUM(X,ACTUAL,DRC,BUDGET,NUM,XSUM,B)
      DIMENSION X(NUM), ACTUAL(NUM), DRC(NUM)
      XSUM=0.0
      A=0.0
      B=BUDGET
  5   DO 10 I=1,NUM
      A=A+ABS(DRC(I))
      IF(X(I).LE.0.0)GOTO 20
      IF (A.LT.BUDGET) THEN
      B=B-ABS(DRC(I))
      IF(ACTUAL(I).LE.0.0)ACTUAL(I)=0.0
      XSUM=XSUM+ACTUAL(I)
      ELSE
      PER=B/ABS(DRC(I))
      B=B-ABS(PER*DRC(I))
      IF(ACTUAL(I).LE.0.0)ACTUAL(I)=0.0
      XSUM=XSUM+(PER*ACTUAL(I))
      GO TO 20
      ENDIF
 10   CONTINUE
 20   RETURN
      END
```

Computer Program and Subroutines

```
      SUBROUTINE SORTA(KEY,B,C,D,E,ACTUAL)
      INTEGER N
      DIMENSION KEY(N),ACTUAL(N),DRC(N),B(N),C(N),D(N)
      INTEGER K, M, LOCSM
      DO 100 M=N,2,-1
      K=LOCBG(KEY,M)
      CALL SWITCH(KEY(M),KEY(K))
      CALL SWITCH(ACTUAL(M),ACTUAL(K))
      CALL SWITCH(DRC(M),ACTUAL(K))
      CALL SWITCH(B(M),B(K))
      CALL SWITCH(C(M),C(K))
      CALL SWITCH(D(M),D(K))
      CALL SWITCH(E(M),E(K))
100   CONTINUE
      RETURN
      END

      INTEGER FUNCTION LOCBG(A,N)
      INTEGER N
      DIMENSION A(N)
      INTEGER K
      LOCBG=1
      DO 110 K=2,N
      IF (A(K).GT.A(LOCBG)) THEN
      LOCBG=K
      ENDIF
110   CONTINUE
      RETURN
      END


      SUBROUTINE MUS(X,ACTUAL,DRC,BUDGET,NUM,XSUM,B)
      DIMENSION X(NUM),ACTUAL(NUM),DRC(NUM)
      XSUM=0.0
      A=0.0
      B=BUDGET
  5   DO 10 I=1,NUM
      A=A+ABS(DRC(I))
      IF (A.LT.BUDGET)THEN
      XSUM=XSUM+ACTUAL(I)
      B=B-ABS(DRC(I))
      ELSE
      PER=B/ABS(DRC(K))
```

Computer Program and Subroutines

```
      XSUM=XSUM+(PER*ACTUAL(I))
      B=B-ABS(PER*DRC(I))
      GO TO 20
      ENDIF
 10   CONTINUE
 20   RETURN
      END
```