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OBTAINING A BOUNDED NEAR-OPTIMUM SOLUTION TO LINEAR INTEGER PROGRAMMING PROBLEMS

By

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ABSTRACT

A digital computer program is developed to solve zero-one mathematical programming problems. The theoretical development of the algorithm comes from a particular application (Everett, 1963) of the method of Lagrange Multipliers. Three example problems are included to show the broad applications of the proposed algorithm.

TABLE OF CONTENTS

TABLE OF CONTENTS - Continued

LIST OF ILLUSTRATIONS AND FIGURES

INTRODUCTION

The purpose of this thesis is to describe a method of solving linear integer programming problems. Even though quite a number of algorithms have been proposed for linear integer problems, only a few have been demonstrated to be effective when applied to large-scale problems. For example, when the branch and bound algorithm is applied to a linear integer problem the solution procedure usually results in solving several continuous linear programming problems of the same magnitude as the original integer problem. This technique can keep a large-sized high-speed digital computer occupied for a considerable time period. Even the additive algorithm (Balas, 1965) for solving zero-one linear programming problems, which uses a superior strategy of explicit and implicit enumeration of the solutions, is an extremely time-consuming method. The algorithm proposed in this thesis is fast and effective for many large-scale problems.

The proposed method requires that an integer linear programming problem be expressed as a zero-one programming problem. One of the problem applications presented in this thesis will show that the zero-one reduction procedure can be done for any linear integer problem. An application of Everett's method of Lagrange Multipliers is used to rank the importance of each of the decision variables. The decision variables will be brought into the basis in their ranked order. However, a decision variable will not be allowed to enter the basis if a constraint equation is violated. In general, there is no guarantee that the solution obtained from the proposed algorithm is an optimum solution. Therefore an upper bound will be evaluated by relaxing the integer constraints and solving the continuous linear programming problems by the simplex algorithm. Then the true optimum value, Z*, will be bracketed.

This thesis is divided into three parts. The first part describes the algorithm for obtaining a near-optimum solution to a linear integer programming problem. In this section the theory behind the method is described. The second part presents detailed worked examples illustrating the computational aspects of the algorithm. Finally, a summary is presented which discusses the most important points in the theoretical development.

EVERETT'S METHOD OF LAGRANGE MULTIPLIERS

Lagrange multipliers [Everett, 1962] have been used to seek a global maximum to a function subject to constraint equations. Everett's main theorem will be applied in this thesis and, therefore, the main points of that theorem will be covered in this chapter. Let a function to be maximized be represented by Z(X) where Z is a real valued function and X the vector of decision variables that are members of some arbitrary set S. Let Z(X) be subjected to constraint inequalities of the form

$$\sum_{j=1}^{n} D_j^i \,(X_j) \leq E^i, \; i = 1, \ldots , m.$$

The Lagrange function, or the Lagrangian, will be defined as

$$Z(X) - \sum_{i=1}^{m} \lambda^i \, D^i(X)$$

where the λ^i are called Lagrange multipliers and they are constrained to be nonnegative real numbers. The main theorem states that a solution, X*, that yields a global maximum to the Lagrangian with a given set of values for λ^i will yield a global maximum to Z(X) subject to the set of constraints with the values of the right-hand side given by $E^i = D^i(X^*)$.

The proof of the main theorem is straightforward and it will not be repeated here. The approach is quite general since Z(X), $D^i(X)$, and the set X are completely arbitrary.

There is a limitation in satisfying Everett's main theorem for linear integer programming problems. Consider any linear integer objective function subject to a particular constraint equation given as follows:

$5X_1 + 10X_2 + 15X_3 \leq 23 \quad X_1, X_2, X_3 = 0, 1.$

It is obvious from inspection that any feasible solution will yield a right-hand side value for the constraint equation, $E = D(X)$, one of the following values: $E = 0, 5, 10, 15,$ or 20. The value $E = 23$ is an inaccessible value.

In general, linear integer programming problems will have inaccessible regions consisting of b_i vectors that are not generated by any λ vectors. This limitation means that Everett's main theorem will not indicate a guaranteed optimum solution to many linear integer problems. However, the procedure is fail-safe in the sense that if the right-hand side values of the constraint equations are equal to the values given by $E^i = D^i(X^*)$, then (X^*) is an optimum value to this problem.

A particular application of Everett's method of Lagrange multipliers to linear integer programming problems will be explained in the next chapter. It will be shown that if λ^i is restricted to a constant value, say λ, for each of the constraint equations, then it will be possible to rank the decision variables for their importance in maximizing the Lagrangian. This technique will eliminate adjusting the λ^i values by an iteration procedure. Since the λ^i values will all be identical and since the Lagrangian can always be maximized, Everett's Lambda theorem and Epsilon theorem will not be discussed in this thesis.

THEORY

Frequently in engineering, an analyst is faced with the problem of optimizing some process or operation. He wishes to minimize or maximize some function, termed the objective function, subject to certain constraints. When formulated as mathematical statements, a broad class of these optimization problems can be grouped together into a category termed linear integer programming problems. A linear integer programming problem is one in which a linear function is the criterion to be minimized or maximized, a criterion subject to constraints that are also linear functions. Also, the decision variables are restricted to non-negative integer values. The material that follows will describe an algorithm that will give a near-optimum solution to linear integer programming problems.

Objective Function

The objective function can be assembled in the form

$$Z = C_1X_1 + C_2X_2 + \dots + C_nX_n = \sum_{j=1}^{n} C_jX_j$$

where the C_j's are constants and the X_j's are the decision variables. It will be shown in one of the later problem applications that any linear integer programming problem can, by transformations of the decision variables, be presented as a linear zero-one programming problem. Hence, the X_j's in this thesis will be constrained to have values of 0 or 1. Also Z will be defined as the value of the objective function.

In the optimization of an operation, the analyst either wants to maximize the benefits or minimize the detriments of the process. Hence, the objective function can take one of the following two forms:

1. Maximize $Z = C_1X_1 + C_2X_2 + \ldots + C_nX_n = \sum_{j=1}^{n} C_jX_j$

2. Minimize $Z = C_1X_1 + C_2X_2 + \ldots + C_nX_n = \sum_{j=1}^{n} C_jX_j$

The algorithm will require that the decision variables be transformed such that the objective function is to be maximized and that there be positive coefficients preceding each of the decision variables. To see how this may be accomplished consider the following objective function:

$$\text{Minimize } Z = 5X_1 + 3X_2 - 2X_3.$$

The maximum of the negative of the function is the minimum of the function. Hence, our objective function can be transformed as follows:

$$\text{Maximize } (-Z) = -5X_1 - 3X_2 + 2X_3.$$

For any general problem let the following transformations be defined:

$\overline{X}_k = 1 - X_k$, $k \in P$, where P is the integer set $(1,2,\ldots,n)$, and

$\overline{Z} = -Z.$

Thus the objective function for this particular problem will be:

$$\text{Maximize } \overline{Z} = 5(1-X_1) + 3(1-X_2) + 2X_3 - 8, \text{ or}$$

$$\text{Maximize } \overline{Z} = 5\overline{X}_1 + 3\overline{X}_2 + 2X_3 - 8.$$

In summary, the algorithm requires that the analyst must first express a linear integer programming problem as a linear zero-one programming problem. Then the analyst must express the objective function in the following form:

$$\text{Maximize } \tilde{Z} = \left(\sum_{j=1}^{n} \bar{C}_j \tilde{X}_j \right) - K, \text{ where}$$

$\bar{C}_j$ = a positive constant,

$\tilde{Z}$ = Z or $\bar{Z}$,

$\tilde{X}_j$ = X_j or $\bar{X}_j$,

K = Zero or a positive constant.

Constraint Equations

As previously mentioned, the constraint equations are linear functions. Once the integer programming problem is transformed into a zero-one problem the constraint equations take the form

$$\sum_{j=1}^{n} a_{ij} X_j \; \tau_i \; b_i, \quad i = 1, \ldots, m,$$

$$X_j = 0,1, \quad j = 1, \ldots, n,$$

where τ_i is either $\geq$, or $\leq$, for each $i = 1, \ldots, m$.

When the analyst sets up the objective function in its required form, he must transform the decision variables in the constraint equations such that they are compatible with the variables in the objective function. For example, if $\bar{X}_4$ and X_5 appear in an objective function, then $\bar{X}_4$ and X_5 should appear in each of the associated constraint equations. With this adjustment the constraint equations take the form

$$\sum_{j=1}^{n} a_{ij}\, \tilde{X}_j\, \tau_i\, \bar{b}_i\,, \quad i = 1, \ldots, m,$$

$$\tilde{X}_j = 0, 1, \; j = 1, \ldots, n,$$

$$\tau_i \text{ is } \geq \text{ or } \leq, \; i = 1, \ldots, m.$$

It is required that each $\bar{b}_i$ be a non-negative value. If $\bar{b}_i$ is negative, the analyst can always multiply both sides of the equation by a (-1) and reverse the sign of the inequality.

The constraint equations will be classified into two sets. Each constraint equation will either be a member of Set A or a member of Set B. Set A will be the set of constraint equations where τ_i is $\leq$ and $b_i \neq 0$ for $i = 1, \ldots, m$. Set B will be the set of the remaining constraint equations that are not members of Set A.

Now it is possible to normalize the constraint equations that are members of Set A. This can be done by defining $\bar{a}_{ij} = a_{ij} / \bar{b}_i$, $i \epsilon A$ then $\sum_{j=1}^{n} \bar{a}_{ij}\, \tilde{X}_j \leq 1$, $i \epsilon A$, and $\sum_{j=1}^{n} a_{ij}\, \tilde{X}_j\, \tau_i\, \bar{b}_i$, $i \epsilon B$.

The justification for normalization comes from the fact that if λ^i is restricted to a constant value λ for each of the constraint equations, then it makes sense to equally weight these constraint equations.

Lagrange Multipliers

The Lagrangian for a linear integer programming problem can be expressed as:

$$1.\quad \left(\left(\sum_{j=1}^{n} \bar{c}_j \tilde{X}_j \right) - K - \sum_{i=1}^{m} \lambda^i \sum_{j=1}^{n} a_{ij} \tilde{X}_j \right)$$

or if the constraint equations are normalized, then,

$$2.\quad \left(\left(\sum_{j=1}^{n} \bar{c}_j \tilde{X}_j \right) - K - \sum_{i=1}^{m} \lambda^i \sum_{j=1}^{n} \bar{a}_{ij} \tilde{X}_j \right).$$

It should be noted that Everett's main theorem was proven for a set of constraint equations with inequalities of the less than or equal type; hence, only the constraint equations that are a member of Set A will apply in this discussion. Normally, the goal would be to select a set of non-negative λ^i values such that the Lagrangian is maximized by X* and the right-hand side of the constraint equations is equal to $\bar{b}_i = D^i(X^*)$.

However, in this thesis the goal will be to determine a means of ranking the importance of each decision variable in maximizing the Lagrangian without going through an iteration process. This goal would be highly desirable for large-scale integer problems where the amount of digital computer time used for the solution is of prime importance. If λ^i is restricted to a constant value, say λ, for all of the constraint equations, then since λ is independent of i, the Lagrangian becomes:

$$1.\quad \left(\left(\sum_{j=1}^{n} \bar{c}_j \tilde{X}_j \right) - K - \lambda \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \tilde{X}_j \right) \text{ or}$$

$$2.\quad \left(\left(\sum_{j=1}^{n} \bar{c}_j \tilde{X}_j \right) - K - \lambda \sum_{i=1}^{m} \sum_{j=1}^{n} \bar{a}_{ij} \tilde{X}_j \right)$$

Now the Lagrangian can be expressed differently, as follows:

$$\left(\left(\sum_{j=1}^{n} \bar{c}_j \tilde{x}_j\right) - \lambda \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \tilde{x}_j\right) - K, \text{ or}$$

$$\left(\sum_{j=1}^{n} \left(\bar{c}_j - \lambda \sum_{i=1}^{m} a_{ij}\right) \tilde{x}_j\right) - K .$$

Hence, the Lagrangian is now expressed in terms of each decision variable. A pivotal lambda value will be defined for each decision variable as that value of lambda such that the coefficient in the Lagrangian for that variable is equal to zero. Then

$$\bar{c}_j - \lambda_j \sum_{i=1}^{m} a_{ij} = 0, \qquad \text{or}$$

$$\lambda_j = \bar{c}_j / \sum_{i=1}^{m} a_{ij} \quad , \quad j = 1, 2, \ldots , n .$$

The pivotal lambda values serve to rank the decision variables in the order in which they should enter the basis and maximize the Lagrangian. The Lagrangian has been stated as

$$\left(\sum_{j=1}^{n} \left(\bar{c}_j - \lambda \sum_{i=1}^{m} a_{ij}\right) \tilde{x}_j\right) - K .$$

For a particular problem assume $K = 0$, and let $\lambda = \lambda_{\bar{j}}$, where $\lambda_{\bar{j}}$ is the largest pivotal lambda value of all the associated decision variables. Then the values $\left(\bar{c}_j - \lambda \sum_{i=1}^{m} a_{ij}\right)$ in the Lagrangian would be negative for each decision variable where $j \neq \bar{j}$, and it would be equal to zero for $j = \bar{j}$. Hence $X_{\bar{j}}$ should be the first decision variable to enter the basis. The other decision variables can be ranked in relationship to their pivotal lambda values. Since usually there is no guarantee that the solution obtained by Everett's method will be the optimum solution to the original stated problem, two sets of pivotal lambda values will be evaluated. One set will be calculated from the Lagrangian with the original constraints equations. The other set will be calculated from the Lagrangian with the normalized constraint equations. Of the two sets, the ranking which gives the maximum value for the objective function will be used as the near-optimum solution to the original linear programming problem. A computer program has been written to solve zero-one programming problems by the proposed algorithm. It is presented in the Appendix.

Upper Bound Solution

Integer linear programming problems differ from linear programming problems only in that in integer problems all variables are constrained to have nonnegative integer values. Hence, if the integer constraints are relaxed, the system becomes a linear programming problem. The simplex algorithm can be used to solve integer linear programming problems. If it produces an all-integer solution, the optimum solution has been found. If it produces a noninteger solution an upper bound to the integer solution has been found.

Limitations

The techniques described in this thesis will not apply to all linear integer problems. One of the requirements of Everett's main theorem was that the λ^i are non-negative real numbers. If negative pivotal lambda values are calculated for any problem, then the solution technique described in this thesis is not applicable to that particular problem. Also, if the solution vector violates any of the constraint equations which are a member of Set B, then the proposed solution technique is no longer valid.

The first two example problems presented in the next section will demonstrate the need for partitioning the constraints into Set A and Set B.

PROBLEM APPLICATIONS

Three separate examples of integer programming problems have been selected. Two of the problems are zero-one programming problems and the third is a general integer programming problem. The descriptions of the first two problems are taken quite literally from McMillan (1970, p. 271 and p. 387). Although the three example problems are small problems, this algorithm was developed to handle large problems. The present program is set up to handle 50 constraint equations with 100 decision variables. However, the only limitation on problem size is the storage capacity of the digital computer the analyst is using.

The Appendix contains a computer printout of the solution and the run time for each of the three problems.

The Capital Budgeting Problem

The capital budgeting problem is a zero-one programming problem. This is representative of many managerial-type problems, where multiple schemes are proposed and management must select an optimum or near-optimum plan.

Problem Description

A firm is obligated to choose one alternative from each of 3 sets of alternative proposals in a budgeting problem. The first alternative concerns the production of parts for a new line of radios. The second concerns the assembly of the parts, and the third concerns their storage. The alternative proposals, the forecasts of the net present value of the returns associated with the alternatives, the number of men required for each alternative, and the cash outflow required over the next 5 years are given in Table 1.

TABLE 1

Proposal	Nature of Investment	NPV	Employees Required	Cash Outflows				
				Year 1	Year 2	Year 3	Year 4	Year 5
1	Subcontract the production of parts	757	7	$ 5	$ 5	$ 5	$ 5	$ 2
2	Produce parts in existing facilities	825	35	15	12	4	4	4
3	Produce parts in new facilities	987	20	30	2	0	0	8
4	Subcontract assemblage of parts	350	12	10	10	10	6	3
5	Assemble parts in existing facilities	596	65	7	4	4	4	4
6	Assemble parts in new facilities	650	60	15	2	2	2	2
7	Store radios in existing warehouse	1,420	20	50	10	5	0	0
8	Store radios in leased new warehouse	1,425	5	7	7	7	7	7

The firm wishes to select that set of 3 proposals which will maximize the net present value of its total expected return. It has been decided that no more than 100 men will be committed to the radio project and that the cash outflows should not exceed the following:

Year	Maximum cash outflow
1	70
2	30
3	15
4	15
5	15

The problem is made somewhat more complicated by the fact that the subcontractor will produce the parts without assembling them, but he will not assemble them unless he also produces them.

Problem Formulation

The following decision variables will be employed:

$$X_j = \begin{cases} 1 & \text{if the jth proposal is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

$$C_j = \text{the net present value of the jth proposal.}$$

The objective function is given as:

$$\text{Maximize } Z = 757X_1 + 825X_2 + 987X_3 + 350X_4 + 596X_5 + 650X_6 + 1420X_7 + 1425X_8 ,$$

$$X_j = 0,1 \quad j = 1, \ldots , 8 .$$

Clearly, no transformations of the variables are required to set the objective function in the form required for the proposed algorithm. This is true because the two criteria for the objective function are already satisfied, i.e.:

1. The objective function is to be maximized, and
2. The coefficients for each decision variable are positive.

The constraint equations can be stated as follows:

$$7X_1 + 35X_2 + 20X_3 + 12X_4 + 65X_5 + 60X_6 + 20X_7 + 5X_8 \leq 100$$

$$5X_1 + 15X_2 + 30X_3 + 10X_4 + 7X_5 + 15X_6 + 50X_7 + 7X_8 \leq 70$$

$$5X_1 + 12X_2 + 2X_3 + 10X_4 + 4X_5 + 2X_6 + 10X_7 + 7X_8 \leq 30$$

$$5X_1 + 4X_2 + 10X_4 + 4X_5 + 2X_6 + 5X_7 + 7X_8 \leq 15$$

$$5X_1 + 4X_2 + 6X_4 + 4X_5 + 2X_6 + 7X_8 \leq 15$$

$$2X_1 + 4X_2 + 8X_3 + 3X_4 + 4X_5 + 2X_6 + 7X_8 \leq 15$$

$$X_1 - X_4 \geq 0$$

$$X_1 + X_2 + X_3 \leq 1$$

$$X_1 + X_2 + X_3 \geq 1$$

$$X_4 + X_5 + X_6 \leq 1$$

$$X_4 + X_5 + X_6 \geq 1$$

$$X_7 + X_8 \geq 1$$

$$X_7 + X_8 \leq 1$$

The first constraint assures that the number of men assigned will not exceed 100. The next 5 assure that the maximum allowable cash outflows will not be exceeded. The final 6 constraints assure that one and only one proposal will be chosen for each of the 3 sets of alternative proposals. The seventh constraint portrays the subcontractor's requirement. If $X_4 = 1$, then $X_1 = 1$ and the subcontractor both produces and assembles the parts. If $X_4 = 0$, then X_1 can be 0 or 1, meaning that the subcontractor will not assemble the parts although he may or may not produce them.

Since the objective function variables are the same variables used in the constraint equations, and since b_i is nonnegative for all 13 constraints, the constraint equations do not have to be modified. The constraint equations that are members of Set A are as follows:

$$7X_1 + 35X_2 + 20X_3 + 12X_4 + 65X_5 + 60X_6 + 20X_7 + 5X_8 \leq 100$$

$$5X_1 + 15X_2 + 30X_3 + 10X_4 + 7X_5 + 15X_6 + 50X_7 + 7X_8 \leq 70$$

$$5X_1 + 12X_2 + 2X_3 + 10X_4 + 4X_5 + 2X_6 + 10X_7 + 7X_8 \leq 30$$

$$5X_1 + 4X_2 + 10X_4 + 4X_5 + 2X_6 + 5X_7 + 7X_8 \leq 15$$

$$5X_1 + 4X_2 + 6X_4 + 4X_5 + 2X_6 + 7X_8 \leq 15$$

$$2X_1 + 4X_2 + 8X_3 + 3X_4 + 4X_5 + 2X_6 + 7X_8 \leq 15$$

$$X_1 + X_2 + X_3 \leq 1$$

$$X_4 + X_5 + X_6 \leq 1$$

$$X_7 + X_8 \leq 1$$

The constraint equations that are a member of Set B are as follows:

$$X_1 - X_4 \geq 0$$

$$X_1 + X_2 + X_3 \geq 1$$

$$X_4 + X_5 + X_6 \geq 1$$

$$X_7 + X_8 \geq 1$$

Finally, the normalized constraint equations that are members of Set A are as follows:

$$0.070X_1+0.350X_2+0.200X_3+0.120X_4+0.650X_5+0.600X_6+0.200X_7+0.050X_8 \leq 1$$

$$0.071X_1+0.214X_2+0.428X_3+0.142X_4+0.100X_5+0.214X_6+0.714X_7+0.100X_8 \leq 1$$

$$0.167X_1+0.400X_2+0.067X_3+0.333X_4+0.133X_5+0.067X_6+0.333X_7+0.233X_8 \leq 1$$

$$0.333X_1+0.267X_2 \qquad +0.667X_4+0.267X_5+0.133X_6+0.333X_7+0.467X_8 \leq 1$$

$$0.333X_1+0.267X_2 \qquad +0.400X_4+0.267X_5+0.133X_6 \qquad +0.467X_8 \leq 1$$

$$0.133X_1+0.267X_2+0.533X_3+0.200X_4+0.267X_5+0.133X_6 \qquad +0.467X_8 \leq 1$$

$$X_1 + X_2 + X_3 \leq 1$$

$$X_4 + X_5 + X_6 \leq 1$$

$$X_7 + X_8 \leq 1$$

Problem Solution

The appendix contains the digital computer printout for the solution of the objective function with the unmodified constraint equations. The solution shows that proposals 1, 6, and 8 should be accepted and the others should be rejected. If this plan is adopted the objective function value is 2,832.

The digital computer printout for the solution of the objective function with the normalized constraint equations shows that proposals 1, 6, and 7 should be accepted and the others should be rejected. If this plan is adopted the objective function value is 2,827, which is five points lower than the solution calculated for the unmodified constraint equations.

However, both solutions should be presented to management as proposal 7 may be adopted over proposal 8 despite the loss of five points. A linear programming solution shows that the upper bound for this problem is 2832.0.

The Traveling Salesman Problem

The traveling salesman problem is a zero-one programming problem which can be represented as a network problem. The branch and bound algorithm was developed to solve this problem. The algorithm developed in this thesis will also solve this problem.

Problem Description

Cities 1, 2, 3, and 4 are so connected that one can travel directly from any one of them to any of the others. Their configuration is portrayed in Figure 1, with the cost associated with traveling from each to the others so labeled. Note that the network is not symmetric in that the cost of going from any city i to any city j is not necessary the same as the cost of going from j to i. This want of symmetry may not be entirely representative of reality but it is not entirely implausible.

The objective is to find that four-city, round-trip tour that includes every city once and only once and has the lowest cost. If there are n cities in a fully connected network of this sort, there will be (n-1)! round-trip tours. Since we have only four cities, exhaustive enumeration is doubtless the appropriate solution method.

We shall represent a tour by listing, separated by commas, the sequence of cities to be visited. Thus (3, 1, 2, 4, 3) represents the round trip 3 to 1 to 2 to 4 to 3, and also the tour 1 to 2 to 4 to 3 to 1, etc., without regard to the starting and terminating

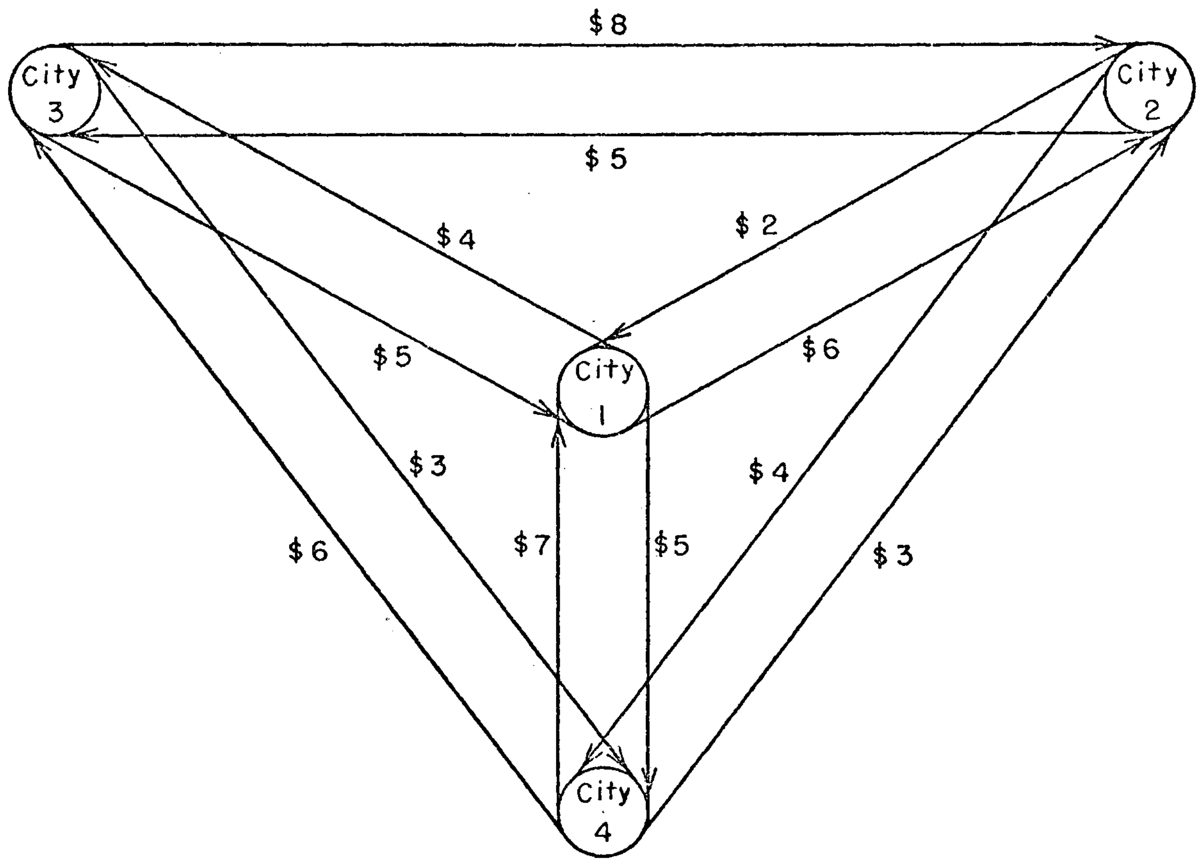

Figure 1 City Configuration for the Travelling Salesman Program

cities. The possible tours and their associated costs are as follows:

Tour	Cost
1, 2, 3, 4, 1	$21
1, 2, 4, 3, 1	21
1, 4, 2, 3, 1	18
1, 4, 3, 2, 1	21
1, 3, 2, 4, 1	23
1, 3, 4, 2, 1	12

The least cost tour is (1, 3, 4, 2, 1) at a cost of $12. Even a problem involving as few as 10 cities assumes such proportions as to make exhaustive enumeration unattractive and we would therefore like a superior search method.

Problem Formulation

The decision variables for this problem will be defined as follows:

X_1 - the tour from city 1 to city 2,

X_2 - the tour from city 1 to city 3,

X_3 - the tour from city 1 to city 4,

X_4 - the tour from city 2 to city 1,

X_5 - the tour from city 2 to city 3,

X_6 - the tour from city 2 to city 4,

X_7 - the tour from city 3 to city 1,

X_8 - the tour from city 3 to city 2,

X_9 - the tour from city 3 to city 4,

X_{10} - the tour from city 4 to city 1,

X_{11} - the tour from city 4 to city 2,

X_{12} - the tour from city 4 to city 3.

C_j - the cost of tour X_j, j = 1, 12.

The objective function is to

$$\text{minimize } Z = 6X_1 + 4X_2 + 5X_3 + 2X_4 + 5X_5 + 4X_6 + 5X_7 + 8X_8 + 3X_9 + 7X_{10} + 3X_{11} + 6X_{12} ,$$

$$X_j = 0,1 \quad j = 1, \ldots , 12$$

Clearly, the decision variables must be transformed to express the objective function as a maximization problem. Hence, the transformed objective function is

$$\text{maximize } \overline{Z} = 6\overline{X}_1 + 4\overline{X}_2 + 5\overline{X}_3 + 2\overline{X}_4 + 5\overline{X}_5 + 4\overline{X}_6 + 5\overline{X}_7 + 8\overline{X}_8 + 3\overline{X}_9 + 7\overline{X}_{10} + 3\overline{X}_{11} + 6\overline{X}_{12} - 58 ,$$

$$\overline{X}_j = 0, 1 \quad j = 1, \ldots , 12$$

The constraint equations can be stated as follows:

$$
\begin{aligned}
&X_1 + X_2 + X_3 \le 1 \\
&X_1 + X_2 + X_3 \ge 1 \\
&X_4 + X_5 + X_6 \le 1 \\
&X_4 + X_5 + X_6 \ge 1 \\
&X_7 + X_8 + X_9 \le 1 \\
&X_7 + X_8 + X_9 \ge 1 \\
&X_{10} + X_{11} + X_{12} \le 1 \\
&X_{10} + X_{11} + X_{12} \ge 1 \\
&X_4 + X_7 + X_{10} \ge 1 \\
&X_4 + X_7 + X_{10} \le 1 \\
&X_1 + X_8 + X_{11} \ge 1 \\
&X_1 + X_8 + X_{11} \le 1 \\
&X_2 + X_5 + X_{12} \le 1 \\
&X_2 + X_5 + X_{12} \ge 1 \\
&X_3 + X_6 + X_9 \le 1 \\
&X_3 + X_6 + X_9 \ge 1 \\
&X_1 + X_4 \le 1 \\
&X_2 + X_7 \le 1 \\
&X_3 + X_{10} \le 1 \\
&X_5 + X_8 \le 1 \\
&X_6 + X_{11} \le 1 \\
&X_9 + X_{12} \le 1
\end{aligned}
$$

The first 8 constraint equations assure that each city is traveled into once and only once. The next 8 constraint equations assure that each city is traveled out of once and only once. It is necessary to prevent more than one closed loop within the system. If n represents the number of cities, then the closed loops determined by the integer calculation n/2, n/2-1, n/2-2, etc., must be prevented from occurring. Hence, the last 6 constraints prevent closed loops between any 2 of the 4 cities.

The constraint decision variables are transformed to conform to the objective function variables with the following results:

$$
\begin{array}{llllllllllllll}
\bar{X}_1 & + \bar{X}_2 & + \bar{X}_3 & & & & & & & & & & \geq & 2 \\
\bar{X}_1 & + \bar{X}_2 & + \bar{X}_3 & & & & & & & & & & \leq & 2 \\
 & & & \bar{X}_4 & + \bar{X}_5 & + \bar{X}_6 & & & & & & & \geq & 2 \\
 & & & \bar{X}_4 & + \bar{X}_5 & + \bar{X}_6 & & & & & & & \leq & 2 \\
 & & & & & & \bar{X}_7 & + \bar{X}_8 & + \bar{X}_9 & & & & \geq & 2 \\
 & & & & & & \bar{X}_7 & + \bar{X}_8 & + \bar{X}_9 & & & & \leq & 2 \\
 & & & & & & & & & \bar{X}_{10} & + \bar{X}_{11} & + \bar{X}_{12} & \geq & 2 \\
 & & & & & & & & & \bar{X}_{10} & + \bar{X}_{11} & + \bar{X}_{12} & \leq & 2 \\
 & & & \bar{X}_4 & & & + \bar{X}_7 & & & + \bar{X}_{10} & & & \geq & 2 \\
 & & & \bar{X}_4 & & & + \bar{X}_7 & & & + \bar{X}_{10} & & & \leq & 2 \\
\bar{X}_1 & & & & & & & + \bar{X}_8 & & & + \bar{X}_{11} & & \geq & 2 \\
\bar{X}_1 & & & & & & & + \bar{X}_8 & & & + \bar{X}_{11} & & \leq & 2 \\
 & \bar{X}_2 & & & + \bar{X}_5 & & & & & & & + \bar{X}_{12} & \geq & 2 \\
 & \bar{X}_2 & & & + \bar{X}_5 & & & & & & & + \bar{X}_{12} & \leq & 2 \\
 & & \bar{X}_3 & & & + \bar{X}_6 & & & + \bar{X}_9 & & & & \geq & 2 \\
 & & \bar{X}_3 & & & + \bar{X}_6 & & & + \bar{X}_9 & & & & \leq & 2 \\
\bar{X}_1 & & & \bar{X}_4 & & & & & & & & & \geq & 1 \\
 & \bar{X}_2 & & & & & + \bar{X}_7 & & & & & & \geq & 1 \\
 & & \bar{X}_3 & & & & & & & + \bar{X}_{10} & & & \geq & 1 \\
 & & & & \bar{X}_5 & & & + \bar{X}_8 & & & & & \geq & 1 \\
 & & & & & \bar{X}_6 & & & & & + \bar{X}_{11} & & \geq & 1 \\
 & & & & & & & & \bar{X}_9 & & & + \bar{X}_{12} & \geq & 1
\end{array}
$$

The modified constraint equations that are members of Set A are as follows:

$$\bar{X}_1 + \bar{X}_2 + \bar{X}_3 \leq 2$$
$$\bar{X}_4 + \bar{X}_5 + \bar{X}_6 \leq 2$$
$$\bar{X}_7 + \bar{X}_8 + \bar{X}_9 \leq 2$$
$$\bar{X}_{10} + \bar{X}_{11} + \bar{X}_{12} \leq 2$$
$$\bar{X}_4 \quad \bar{X}_7 \quad \bar{X}_{10} \leq 2$$
$$\bar{X}_1 + \bar{X}_8 + \bar{X}_{11} \leq 2$$
$$\bar{X}_2 \quad \bar{X}_5 \quad \bar{X}_{12} \leq 2$$
$$\bar{X}_2 + \bar{X}_6 + \bar{X}_9 \leq 2$$

The constraint equations that are members of Set B are as follows:

$$\bar{X}_1 + \bar{X}_2 + \bar{X}_3 \geq 2$$
$$\bar{X}_4 + \bar{X}_5 + \bar{X}_6 \geq 2$$
$$\bar{X}_7 + \bar{X}_8 + \bar{X}_9 \geq 2$$
$$\bar{X}_{10} + \bar{X}_{11} + \bar{X}_{12} \geq 2$$
$$\bar{X}_4 + \bar{X}_7 + \bar{X}_{10} \geq 2$$
$$\bar{X}_1 + \bar{X}_8 + \bar{X}_{11} \geq 2$$
$$\bar{X}_2 + \bar{X}_5 + \bar{X}_{12} \geq 2$$
$$\bar{X}_3 + \bar{X}_6 + \bar{X}_9 \geq 2$$
$$\bar{X}_1 + \bar{X}_4 \geq 1$$
$$\bar{X}_2 \quad \bar{X}_7 \geq 1$$
$$\bar{X}_3 + \bar{X}_{10} \geq 1$$
$$\bar{X}_5 + \bar{X}_8 \geq 1$$
$$\bar{X}_6 + \bar{X}_{11} \geq 1$$
$$\bar{X}_9 + \bar{X}_{12} \geq 1$$

The normalized constraint equations for Set A will not be listed since they are self-evident.

Problem Solution

The appendix contains the digital computer printouts for the solution to this problem. The solution for the objective function with the unnormalized constraint equations shows that decision variables X_2, X_4, X_9, and X_{11} should be selected. If this plan is adopted the minimum cost is the optimum solution of $12. The solution for the objective function with the normalized constraint equations also gives the optimum solution. The upper bound was calculated to be 12.0.

A Linear Integer Programming Problem

Many important decision problems can be formulated as linear integer programming problems. An example of such a problem will be discussed.

Problem Description

A company produces 3 similar products. For identification purposes those products will be labeled product Y_1, product Y_2, and product Y_3. In the manufacturing process each product must pass through 3 separate assembly-line devices. The company has 3 work crews; one work crew for each assembly line. The first work crew works on the first assembly line, and they are responsible for the initial preparation of the product. The second work crew is responsible for the intermediate preparation, while the third crew is responsible for the product-finishing process.

The company makes \$300 profit for each Y_1 produced, \$200 for each Y_2, and \$400 for each Y_3. A time study has shown that the initial-preparation crew will work at full potential an equivalent 25 hours out of a 40-hour week. The intermediate crew and the finishing crew will work an equivalent 30 and 32 hours, respectively.

Each product Y_1 requires 4 hours of the initial-preparation crew's time, 2 hours of the intermediate crew's time, and 3 hours of the finishing crew's time. Each product Y_2 requires 3 hours of the initial-preparation crew's time, 4 hours of the intermediate crew's time, and 2 hours of the finishing crew's time. Each product Y_3 requires 3 hours of the initial-preparation crew's time, 5 hours of the intermediate crew's time, and 2 hours of the finishing crew's time.

The company's management wants to determine how many of each of three products it should manufacture each week to maximize the profit. Also for the optimum scheme they want to know the work shift schedule for each work crew. If possible, they would like a second near-optimum proposal to occasionally provide some variation to the work crews' schedule. This would help stimulate the workers' morale.

Problem Formulation

The problem can be formulated as follows:

Maximize $Z = 300\ Y_1 + 200\ Y_2 + 400\ Y_3$,

subject to:

$$4\ Y_1 + 3\ Y_2 + 3\ Y_3 \leq 25,$$
$$2\ Y_1 + 4\ Y_2 + 5\ Y_3 \leq 30,$$
$$3\ Y_1 + 2\ Y_2 + 2\ Y_3 \leq 32,$$
$$Y_1,\ Y_2,\ Y_3 = 0,\ 1,\ 2,\ \ldots$$

The decision variable Y_j represents the number of product j produced, C_j represents the unit profit for product j. The first task with an integer programming problem is to reduce it to a zero-one mathematical programming problem. This is accomplished by examining the constraint equations to determine the maximum value of each of the integer variables. First let Y_2, $Y_3 = 0$, then

$$4\ Y_1 \leq 25,$$

$$2\ Y_1 \leq 30,$$

$$3\ Y_1 \leq 32,$$

or Y_1's maximum value is 6, because if $Y_1 = 7$, the first constraint will be violated. Next, let Y_1, $Y_3 = 0$, then Y_2's maximum value is 7. Finally, let Y_1, $Y_2 = 0$, then Y_3's maximum value is 6.

To reduce an integer problem to a zero-one problem, it will be necessary to convert the problem into a binary code. Hence, the following substitutions will be made:

$$Y_1 = 4\ X_1 + 2\ X_2 + X_3,$$

$$Y_2 = 4\ X_4 + 2\ X_5 + X_6,$$

$$Y_3 = 4\ X_7 + 2\ X_8 + X_9,$$

where $X_1, \ldots, X_9 = 0,1$.

Now it is obvious that any integer problem can be converted into a zero-one problem by introducing a new set of variables associated with the binary numbering system. For this particular problem it was necessary to introduce three new X variables for each of the Y variables. The final objective function becomes

$$\text{Maximize } Z = 1200\,X_1 + 600\,X_2 + 300\,X_3 + 800\,X_4 + 400\,X_5 + 200\,X_6 + 1600\,X_7 + 800\,X_8 + 400\,X_9,$$

$$X_1, \ldots, X_9 = 0,1.$$

The transformed constraint equations are given as follows:

$$16X_1 + 8X_2 + 4X_3 + 12X_4 + 6X_5 + 3X_6 + 12X_7 + 6X_8 + 3X_9 \leq 25,$$

$$8X_1 + 4X_2 + 2X_3 + 16X_4 + 8X_5 + 4X_6 + 20X_7 + 10X_8 + 5X_9 \leq 30,$$

$$12X_1 + 6X_2 + 3X_3 + 4X_4 + 2X_5 + X_6 + 8X_7 + 4X_8 + 2X_9 \leq 32,$$

$$X_1, \ldots, X_9 = 0,1.$$

The normalized constraint equations are as follows:

$$0.640X_1 + 0.320X_2 + 0.160X_3 + 0.480X_4 + 0.240X_5 + 0.120X_6 + 0.480X_7 + 0.240X_8 + 0.120X_9 \leq 1$$

$$0.267X_1 + 0.133X_2 + 0.067X_3 + 0.533X_4 + 0.267X_5 + 0.133X_6 + 0.667X_7 + 0.333X_8 + 0.167X_9 \leq 1$$

$$0.375X_1 + 0.188X_2 + 0.094X_3 + 0.125X_4 + 0.062X_5 + 0.031X_6 + 0.250X_7 + 0.125X_8 + 0.062X_9 \leq 1$$

All 3 of the constraint equations are members of Set A.

Problem Solution

The appendix contains the digital computer printouts for the solution for this problem. One solution shows that X_7 and X_8 should be equal to 1 and all other variables should be equal to 0. The original integer variables evaluate with Y_1 and Y_2 equal to 0 and Y_3 equal to 6. Hence, the optimum scheme would be to produce 6 Y_3 products per week. This will result in a weekly profit of $2,400.

The intermediate crew will be fully occupied with this schedule. The initial-preparation crew will have 7 hours of free time. The finishing crew will have 20 hours of free time.

An alternate scheme shows that X_2, X_3, X_6, X_8, and X_9 should be equal to 1 and all other variables should be equal to 0. This is equivalent to Y_1 equal to 3, Y_2 equal to 1, and Y_3 equal to 3. Hence this solution would be to produce 3 Y_1 products, 1 Y_2 product, and 3 Y_3 products per week. This would result in a weekly profit of $2,300. The initial preparation crew would have 1 hour of free time per week. The intermediate crew would have 5 hours of free time, while the finishing crew would have 5 hours of free time per week.

A linear programming solution shows that the upper bound for this problem is a weekly profit of $2400.0.

CONCLUSION

It has been shown that any linear integer programming problem can be reduced to a linear zero-one programming problem by a binary transformation of the decision variables. Also, any problem involving minimization of the objective function can be presented as a maximization problem by a linear transformation of the decision variables. The decision variables are ranked by means of the pivotal lambda values. An optimum or near-optimum solution is found by bringing the decision variables into the basis in their ranked order until a constraint equation will be violated. An upper bound to the problem will be found by solving the associated linear programming problem with real variables.

The techniques described in this thesis will not be applicable to all linear integer problems. Some problems will have negative pivotal lambda values and these problems cannot necessarily be solved by the proposed techniques. Also, the solution vector may violate those constraints which are not a member of Set A. Then another solution algorithm must be sought. It must be stressed that the solution derived from the proposed algorithm is not necessarily an optimum solution. However, it will be an optimum solution if the right-hand side of the constraint equations, $\bar{b}_i$ -terms, are changed to the values calculated by the solution vector.

APPENDIX

This appendix contains a flow chart for program 3BMP, a definition of the terms used in the program, a user's manual, and a TIMESHARING FORTRAN listing of the program. The program was written for a UNIVAC 1108 computer. It is a TIMESHARING PROGRAM written in FORTRAN IV. Also included in this appendix are the data files and the computer printouts for each of the three example problems.

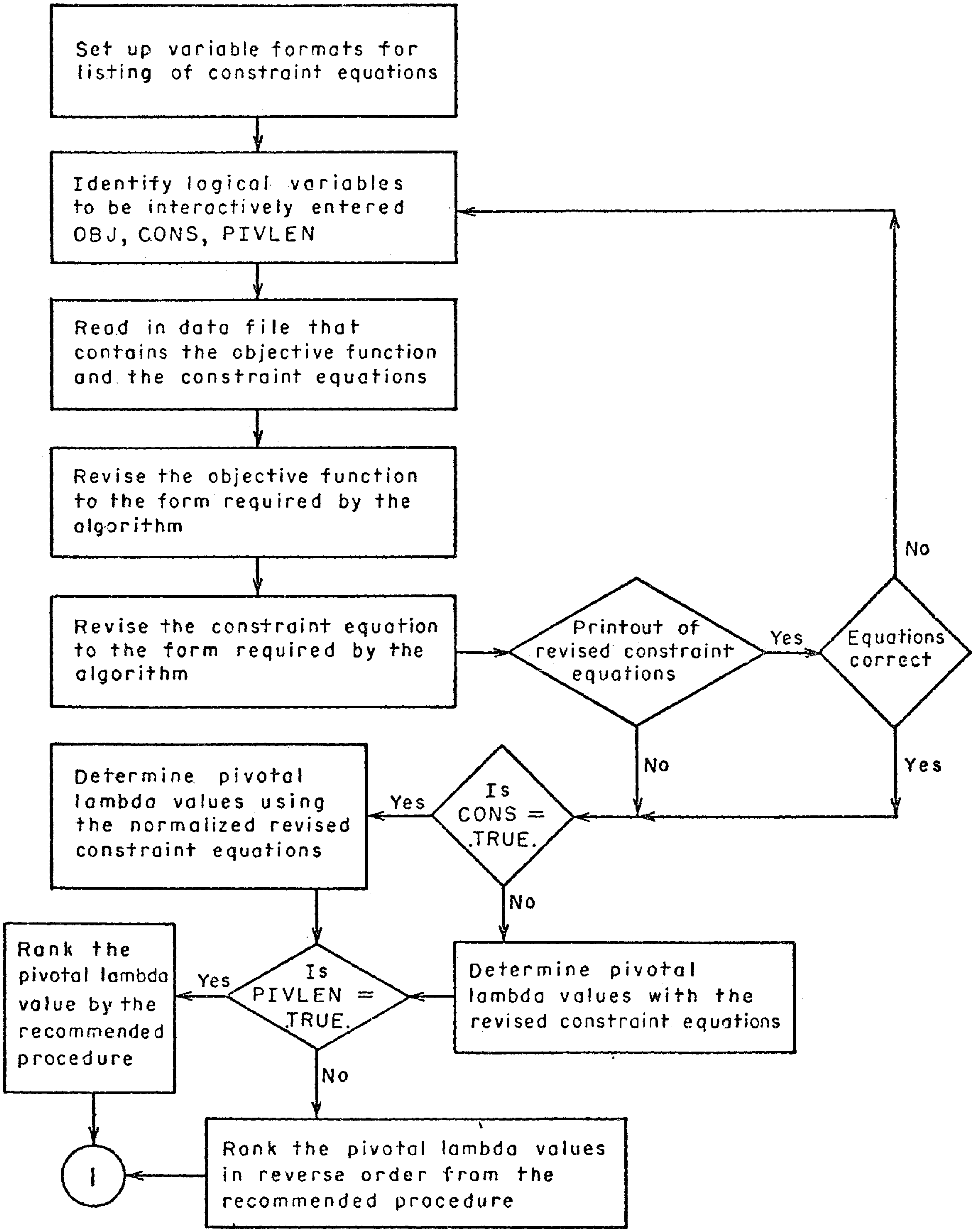

Figure 2 A Flow Chart for the Digital Computer Program

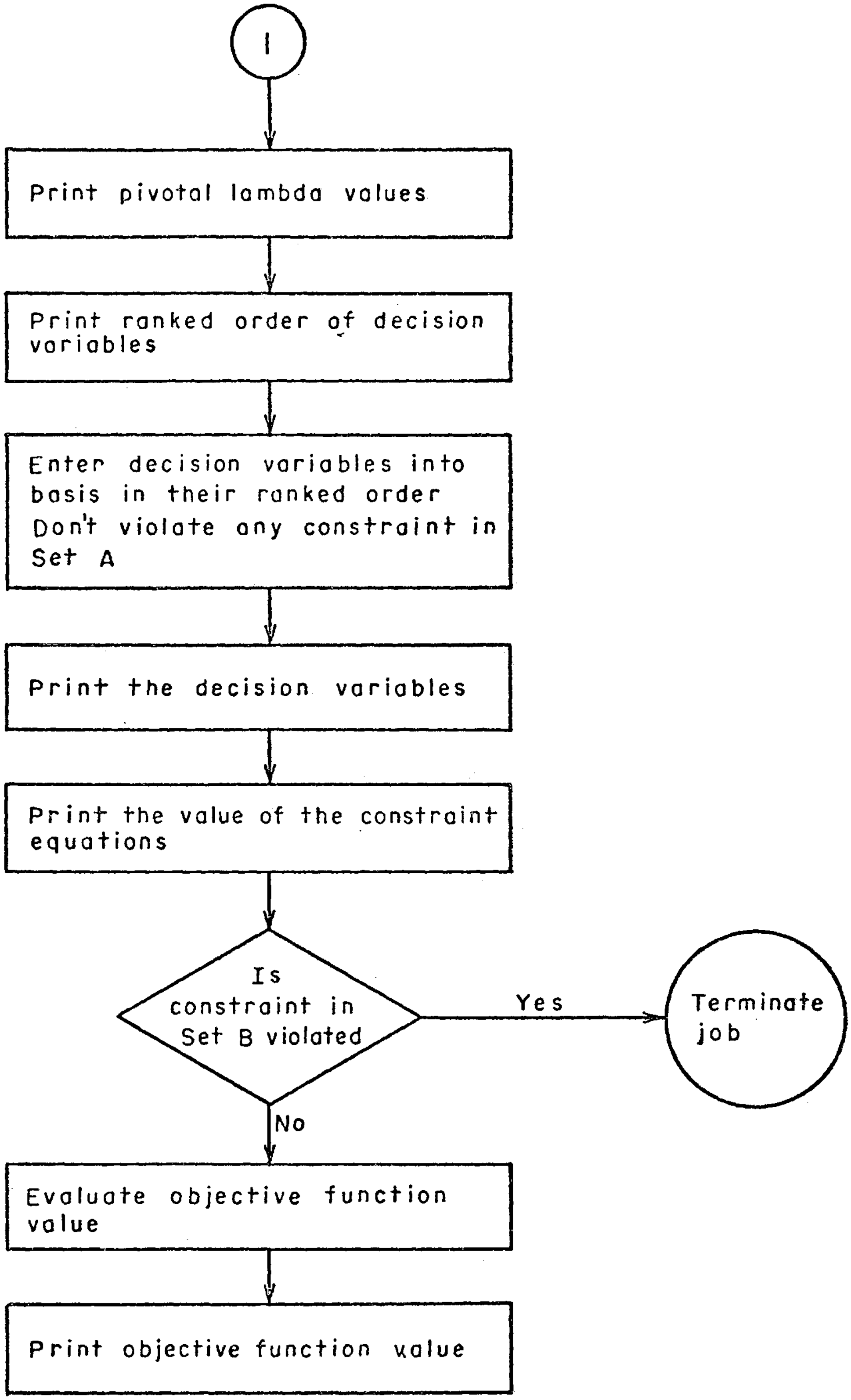

Figure 2 (Continued)

Program Term Definitions

A(I,J) - coefficients of the constraint equations, a_{ij}.

AB(I,J) - normalized coefficients of the constraint equations that are members of Set A.

B(I) - the values of the constants on the right-hand side of the constraint equations.

C(J) - the coefficients of the objective function.

CB(J) - the revised coefficients of the objective function.

CS(I) - alphanumeric terms representing the four types of constraint inequalities.

I - a particular constraint equation.

J - a particular decision variable.

M - the total number of constraint equations.

N - the total number of decision variables.

NCLT(I) = 1 - constraint equation I is a member of Set A.

NCLT(I) = 0 - constraint equation I is a member of Set B.

If NCV(I) = 1, then CS(I) = LE and CSC(I) = GE.

If NCV(I) = 2, then CS(I) = LT and CSC(I) = GT.

If NCV(I) = 3, then CS(I) = GE and CSC(I) = LE.

If NCV(I) = 4, then CS(I) = GT and CSC(I) = LT.

NR(J) - the ranks of the decision variables.

R(J) - the pivotal lambda values for the decision variables.

X(J) - the value of the decision variable J.

XB(J) = X(J) - 1.

Program User's Manual

The user is required to set up a data file that will contain information about the objective function and the constraint equations. The data file requires line numbering on the UNIVAC system. On the first line the two integer variables N and M are entered. A blank space serves as a delimiter. The coefficients of the objective function, C(J) are entered on the next line or lines. Then the coefficients for the first constraint equations A(1,J) are entered, followed by the inequality integer value, NCV(1) and lastly the B(1) value. In a like manner the values are entered for constraint Equation 2 through constraint Equation M.

A listing of the data files for each of the three problem sets can be found in this appendix under the program solution printouts.

Timesharing FORTRAN Listing of Program

What follows is a FORTRAN listing of the digital computer program which solves zero-one programming problems via the proposed algorithm.


```
% THIS IS A CØMPUTER PRØGRAM THAT UTILIZES A 0-1%
% MATHEMATICAL PRØGRAMMING PRØCEDURE.
DIMENSIØN A(50,100),AB(50,100),B(50),C(100),CB(100),CS(50),%
CSC(50),FMB2(7),FMB3(7),FMB4(7),FMB5(3),FMC(3),FMF1(3),FMF2(4),%
FMF3(7),FMF4(7),FMF5(8),FMG1(7),NCLT(50),NCV(50),NR(100),%
R(100),T(50),X(100),XB(100)
LØGICAL ØBJ,ANSWER,CØNS,PIVLEN
DATA XX,BX/2HXX,2HXB/
DATA SETA,SETB/2H A,2H B/
 DATA FMB2/'(I4,2E11.4,3X,A4,3X,E11.4)'/
 DATA FMB3/'(I4,3E11.4,3X,A4,3X,E11.4)'/
 DATA FMB4/'(I4,4E11.4,3X,A4)'/
 DATA FMB5/'(I4,5E11.4)'/
 DATA FMC/'(4X,5E11.4)'/
 DATA FMF1/'(4X,E11.4)'/
 DATA FMF2/'(7X,A4,3X,E11.4)'/
 DATA FMF3/'(4X,E11.4,3X,A4,3X,E11.4)'/
 DATA FMF4/'(4X,2E11.4,3X,A4,3X,E11.4)'/
 DATA FMF5/'(4X,3E11.4,3X,A4,3X,E11.4)'/
 DATA FMG1/'(4X,4E11.4,3X,A4,4X)'/
 1 PRINT *,'FØR ALL QUESTIØNS, TYPE T FØR YES AND TYPE%
  F  FØR NØ.'
 PRINT *,'IS THE ØBJECTIVE FUNCTIØN TØ BE MAXIMIZED?'
 READ *,ØBJ
 PRINT *,' ARE THE PIVØTAL LAMBDA VALUES TØ BE CALCULATED'
 PRINT *,' WITH THE NØRMALIZED CØNSTRAINTS?'
 READ *,CØNS
 PRINT *,'IF ANY PIVØTAL LAMBDA VALUES ARE EQUAL, DØ YØU'
 PRINT *,' WANT THE RANKING BY THE NØRMAL PRØCEDURE?'
 READ *,PIVLEN
 CALL ØBEY('!EQUATE 11 3BMPDF',5)
 READ(11,*)N,M,(C(J),J=1,N),((A(I,J),J=1,N),NCV(I),B(I),I=1,M)
```

```
      DØ 10 I=1,N
IF(C(I).LT.0.0) XB(I)=1
IF(ØBJ) GØ TØ 10
IF(XB(I).EQ.1) GØ TØ 9
XB(I)=1
GØ TØ 10
9 XB(I)=0
10 CØNTINUE
PRINT *,'REVISED ØBJECTIVE FUNCTIØN'
PRINT *,' CØEFFICIENT  VARIABLE-XX=X,XB=1-X'
CØNST=0.
      DØ 15 I=1,N
IF(C(I).LT.0.0)  AA=-C(I)
IF(C(I).LT.0.0) GØ TØ 14
AA=C(I)
14 IF(XB(I).EQ.1) AC=BX
IF(XB(I).EQ.0) AC=XX
WRITE(6,900) AA,AC
900 FØRMAT(2X,F10.2,5X,A2)
CØNST=CØNST+XB(I)*AA
CB(I)=AA
15 CØNTINUE
CØNST=-CØNST
PRINT *,'ØBJ. FUNCTIØN CØNSTANT TERM=',CØNST
DATA CØ1/4H LE /
DATA CØ2/4H LT /
DATA CØ3/4H GE /
DATA CØ4/4H GT /
      DØ 20 I=1,M
K1=NCV(I)
GØ TØ (16,17,18,19),K1
16 CS(I)=CØ1
CSC(I)=CØ3
GØ TØ 20
17 CS(I)=CØ2
CSC(I)=CØ4
GØ TØ 20
18 CS(I)=CØ3
CSC(I)=CØ1
GØ TØ 20
19 CS(I)=CØ4
CSC(I)=CØ2
20 CØNTINUE
% EXAMINE B(I) TØ SEE IF ANY ARE .LE. ZERØ.
     DØ 22 J=1,N
IF(XB(J).EQ.0.)  GØ TØ 22
```

```
      DØ 21 I=1,M
A(I,J)=-A(I,J)
B(I)=B(I)+A(I,J)
21 CØNTINUE
22 CØNTINUE
      DØ 35 I=1,M
IF(B(I).LT.0.) GØ TØ 30
IF(B(I).EQ.0.) GØ TØ 35
28     DØ 29 J=1,N
AB(I,J)=A(I,J)/B(I)
29 CØNTINUE
GØ TØ 35
30 B(I)=-B(I)
IF(NCV(I).EQ.1) K1=3
IF(NCV(I).EQ.2) K1=4
IF(NCV(I).EQ.3) K1=1
IF(NCV(I).EQ.4) K1=2
NCV(I)=K1
CT=CS(I)
CS(I)=CSC(I)
CSC(I)=CT
     DØ 31 J=1,N
A(I,J)=-A(I,J)
31 CØNTINUE
32 IF(NCV(I).EQ.1.ØR.NCV(I).EQ.2)  GØ TØ 28
35 CØNTINUE
PRINT *,'DØ YØU WANT TØ SEE A PRINTØUT ØF THE%
 CØNSTRAINT EQUATIØNS?'
READ *,ANSWER
IF(.NØT.ANSWER) GØ TØ 70
PRINT *,'     REVISED CØNSTRAINT EQUATIØNS'
PRINT *,'  I     1          2          3          4          5'
JB=N/5
JE=(N+1)/5
JR=N-5*JB+2
      DØ 60 I=1,M
IF(JB.NE.0) GØ TØ 49
K=N-1
GØ TØ (46,47,48),K
46 WRITE(6,FMB2) I,(A(I,J),J=1,N),CS(I),B(I)
GØ TØ 60
47 WRITE(4,DM@3) I,(A(I,J),J=1,N),CS(I),B(I)
GØ TØ 60
48 WRITE(6,FMB4) I,(A(I,J),J=1,N),CS(I)
GØ TØ 52
49 WRITE(6,FMB5) I,(A(I,J),J=1,5)
IF(JB.LT.2) GØ TØ 51
```

```
K=JB-1
      DØ 50 L=1,K
L1=5*L+1
L2=L1+4
WRITE(6,FMC) (A(I,M1),M1=L1,L2)
50 CØNTINUE
51 GØ TØ (52,53,54,55,56,57),JR
52 WRITE(6,FMF1) B(I)
GØ TØ 60
53 WRITE(6,FMF2) CS(I),B(I)
GØ TØ 60
54 WRITE(6,FMF3) A(I,N),CS(I),B(I)
GØ TØ 60
55 N1=N-1
WRITE(6,FMF4) (A(I,J),J=N1,N),CS(I),B(I)
GØ TØ 60
56 N2=N-2
WRITE(6,FMF5) (A(I,J),J=N2,N),CS(I),B(I)
GØ TØ 60
57 N3=N-3
WRITE(6,FMG1) (A(I,J),J=N3,N),CS(I)
GØ TØ 52
60 CØNTINUE
PRINT *,'IS THIS DATA CØRRECT?'
READ *,ANSWER
IF(ANSWER) GØ TØ 70
PRINT *,'DØ YØU WANT TØ EXIT?'
READ *,ANSWER
IF(.NØT.ANSWER) GØ TØ 1
GØ TØ 500
70    DØ 80 I=1,M
IF(NCV(I).EQ.1.ØR.NCV(I).EQ.2)  GØ TØ 75
NCLT(I)=0
GØ TØ 80
75 IF(B(I).NE.0.) NCLT(I)=1
80 CØNTINUE
      DØ 90 J=1,N
R(J)=0.
     DØ 85 I=1,M
IF(NCLT(I).EQ.0)  GØ TØ 85
IF(CØNS) R(J)=R(J)+AB(I,J)
IF(.NØT.CØNS) R(J)=R(J)+A(I,J)
85 CØNTINUE
IF(R(J).EQ.0.) GØ TØ 90
R(J)=CB(J)/R(J)
90 CØNTINUE
      DØ 100 J=1,N
```

```
NR(J)=1
      DØ 100 K=1,N
IF(J.EQ.K)  GØ TØ 100
IF(R(J).EQ.R(K))  GØ TØ 94
IF(R(J).LT.0.)  GØ TØ 93
IF(R(J).GT.R(K))  GØ TØ 100
91 NR(J)=NR(J)+1
GØ TØ 100
92 IF(J.LT.K)  GØ TØ 91
GØ TØ 100
93 PRINT *,' PIVØTAL LAMBDA VALUE IS NEGATIVE'
PRINT *,' THIS SØLUTIØN TECHNIQUE WILL NØT WØRK.'
GØ TØ 500
94 U1=0.
U2=0.
U3=0.
      DØ 98 I=1,M
IF(NCLT(I).EQ.0)  GØ TØ 98
U1=1.-AB(I,J)
IF(U1.LT.0.)  GØ TØ 92
U2=1.-AB(I,K)
IF(U2.LT.0.) GØ TØ 100
U3=U3+U1-U2
98 CØNTINUE
IF(PIVLEN) GØ TØ 99
86 IF(U3) 91,92,100
99 IF(U3) 100,92,91
100 CØNTINUE
PRINT *,' VARIABLE  PIVØTAL LAMBDA VALUE'
      DØ 110 J=1,N
WRITE(6,903) J,R(J)
903 FØRMAT (I6,2X,F10.2)
110 CØNTINUE
PRINT *,' VARIABLE  ØRDER'
      DØ 120 J=1,N
WRITE(6,904) J,NR(J)
904 FØRMAT(I6,I8)
120 CØNTINUE
      DØ 130 J=1,N
      DØ 130 K=1,N
124 IF(NR(K).EQ.J)  GØ TØ 125
GØ TØ 130
125 X(K)=1.
      DØ 129 L=1,M
IF(NCLT(L).EQ.0)  GØ TØ 129
T(L)=T(L)+AB(L,K)
IF(T(L).GT.1.)  GØ TØ 135
```

```
IF(T(L).EQ.1.0.AND.NCV(L).EQ.2)  GØ TØ 135
129 CØNTINUE
GØ TØ 130
135 X(K)=0.
     DØ 136 I2=1,L
IF(NCLT(I2).EQ.0) GØ TØ 136
T(I2)=T(I2)-AB(I2,K)
136 CØNTINUE
130 CØNTINUE
PRINT *,' VARIA@LE  TALUE%
      DØ 150 J=1,N
WRITE(6,903) J,X(J)
150 CØNTINUE
Z=0.
      DØ 160 J=1,N
Z=Z+CB(J)*X(J)
160 CØNTINUE
PRINT *,'    ALL CØNSTRAINTS'
PRINT *,' NUMBER  SET   VALUE INEQUALITY B(I)'
      DØ 145 I=1,M
CØN=0.
      DØ 144 J=1,N
CØN=CØN+A(I,J)*X(J)
144 CØNTINUE
IF(NCLT(I).EQ.1) GØ TØ 146
IF(B(I).GT.CØN.AND.NCV(I).EQ.3) NFAULT=1
IF(B(I).GE.CØN.AND.NCV(I).GE.4) NFAULT=1
IF(B(I).LT.CØN.AND.NCV(I).EQ.1) NFAULT=1
IF(B(I).LE.CØN.AND.NCV(I).EQ.2) NFAULT=1
WRITE(6,905) I,SETB,CØN,CS(I),B(I)
905 FØRMAT(I5,4X,A0,D10,0,0X,A4,D10,2)
IF(NFAULT.EQ.1) PRINT *,' SET B CØNSTRAINT IS VIØLATED,'
IF(NFAULT.EQ.1) PRINT *,'THIS SØLUTIØN PRØCEDURE WILL NØT WØRK.'
IF(NFAULT.EQ.1) GØ TØ 500
GØ TØ 145
146 WRITE(6,905) I,SETA,CØN,CS(I),B(I)
145 CØNTINUE
Z=Z+CØNST
IF(Z.LT.0.) GØ TØ 147
PRINT *,' ØBJECTIVE FUNCTIØN VALUE Z=',Z
GØ TØ 143
147 PRINT *,' ØBJECTIVE FUNCTIØN VALUE ZB=',Z
143 CØNTINUE
500 CØNTINUE
STØP
END
```

The Capital Budgeting Problem

The following is a listing of the data file and the computer solution printout for the capital budgeting problem:

```
8 13
757 825 987 350 596 650 1420 1425
7 35 20 12 65 60 20 5 1 100
5 15 30 10 7 15 50 7 1 70
5 12 2 10 4 2 10 7 1 30
5 4 0 10 4 2 5 7 1 15
5 4 0 6 4 2 0 7 1 15
2 4 8 3 4 2 0 7 1 15
1 0 0 -1 0 0 0 0 3 0
1 1 1 0 0 0 0 0 1 1
1 1 1 0 0 0 0 0 3 1
0 0 0 1 1 1 0 0 1 1
0 0 0 1 1 1 0 0 3 1
0 0 0 0 0 0 1 1 1 1
0 0 0 0 0 0 1 1 3 1
```

```
FOR ALL QUESTIONS, TYPE T FOR YES AND TYPE F  FOR NO.
IS THE OBJECTIVE FUNCTION TO BE MAXIMIZED?
T
 ARE THE PIVOTAL LAMBDA VALUES TO BE CALCULATED
 WITH THE NORMALIZED CONSTRAINTS?
F
IF ANY PIVOTAL LAMBDA VALUES ARE EQUAL, DO YOU
 WANT THE RANKING BY THE NORMAL PROCEDURE?
T
REVISED OBJECTIVE FUNCTION
 COEFFICIENT   VARIABLE-XX=X,XB=1-X
     757.00       XX
     825.00       XX
     937.00       XX
     350.00       XX
     596.00       XX
     650.00       XX
    1420.00       XX
    1425.00       XX
OBJ. FUNCTION CONSTANT TERM=0.
DO YOU WANT TO SEE A PRINTOUT OF THE CONSTRAINT EQUATIONS?
F
 VARIABLE  PIVOTAL LAMBDA VALUE
     1        25.23
     2        11.00
     3        16.13
     4         6.73
     5         6.70
     6         7.74
     7        16.51
     8        34.76
 VARIABLE  ORDER
     1        2
     2        5
     3        4
     4        7
     5        8
     6        6
     7        3
     8        1
 VARIABLE  VALUE
     1         1.00
     2         0.
     3         0.
     4         0.
     5         0.
     6         1.00
     7         0.
     8         1.00
```

ALL CONSTRAINTS

NUMBER	SET	VALUE	INEQUALITY	B(I)
1	A	72.00	LE	100.00
2	A	27.00	LE	70.00
3	A	14.00	LE	30.00
4	A	14.00	LE	15.00
5	A	14.00	LE	15.00
6	A	11.00	LE	15.00
7	B	1.00	GE	0.
8	A	1.00	LE	1.00
9	B	1.00	GE	1.00
10	A	1.00	LE	1.00
11	B	1.00	GE	1.00
12	A	1.00	LE	1.00
13	B	1.00	GE	1.00

ØBJECTIVE FUNCTIØN VALUE Z=2332.

SRU's: 11.9

```
FØR ALL QUESTIØNS, TYPE T FØR YES AND TYPE F  FØR NØ.
IS THE ØBJECTIVE FUNCTIØN TØ BE MAXIMIZED?
T
 ARE THE PIVØTAL LAMBDA VALUES TØ BE CALCULATED
 WITH THE NØRMALIZED CØNSTRAINTS?
T
IF ANY PIVØTAL LAMBDA VALUES ARE EQUAL, DØ YØU
 WANT THE RANKING BY THE NØRMAL PRØCEDURE?
T
REVISED ØBJECTIVE FUNCTIØN
 CØEFFICIENT   VARIABLE-XA=X,XB=1-X
      757.00       XX
      825.00       XX
      937.00       XX
      350.00       XX
      596.00       XX
      650.00       XX
     1420.00       XX
     1425.00       XX
ØBJ. FUNCTIØN CØNSTANT TERM=0.
DØ YØU WANT TØ SEE A PRINTØUT ØF THE CØNSTRAINT EQUATIØNS?
F
 VARIABLE  PIVØTAL LAMBDA VALUE
     1        359.09
     2        293.45
     3        442.33
     4        122.26
     5        222.11
     6        284.97
     7        550.13
     8        511.98
 VARIABLE  ØRDER
     1         4
     2         5
     3         3
     4         8
     5         7
     6         6
     7         1
     8         2
 VARIABLE  VALUE
     1          1.00
     2          0.
     3          0.
     4          0.
     5          0.
     6          1.00
     7          1.00
     8          0.
```

ALL CONSTRAINTS

NUMBER	SET	VALUE	INEQUALITY	B(I)
1	A	87.00	LE	100.00
2	A	70.00	LE	70.00
3	A	17.00	LE	30.00
4	A	12.00	LE	15.00
5	A	7.00	LE	15.00
6	A	4.00	LE	15.00
7	B	1.00	GE	0.
8	A	1.00	LE	1.00
9	B	1.00	GE	1.00
10	A	1.00	LE	1.00
11	B	1.00	GE	1.00
12	A	1.00	LE	1.00
13	B	1.00	GE	1.00

OBJECTIVE FUNCTION VALUE Z=2827.

SRU's: 11.9

The Traveling Salesman Problem

The following is a listing of the data file and the computer solution printout for the traveling salesman problem:

```
12 22
6 4 5 2 5 4 5 8 3 7 3 6
1 1 1 0 0 0 0 0 0 0 0 0 1 1
1 1 1 0 0 0 0 0 0 0 0 0 3 1
0 0 0 1 1 1 0 0 0 0 0 0 1 1
0 0 0 1 1 1 0 0 0 0 0 0 3 1
0 0 0 0 0 0 1 1 1 0 0 0 1 1
0 0 0 0 0 0 1 1 1 0 0 0 3 1
0 0 0 0 0 0 0 0 0 1 1 1 1 1
0 0 0 0 0 0 0 0 0 1 1 1 3 1
0 0 0 1 0 0 1 0 0 1 0 0 1 1
0 0 0 1 0 0 1 0 0 1 0 0 3 1
1 0 0 0 0 0 0 1 0 0 1 0 1 1
1 0 0 0 0 0 0 1 0 0 1 0 3 1
0 1 0 0 1 0 0 0 0 0 0 1 1 1
0 1 0 0 1 0 0 0 0 0 0 1 3 1
0 0 1 0 0 1 0 0 1 0 0 0 1 1
0 0 1 0 0 1 0 0 1 0 0 0 3 1
1 0 0 1 0 0 0 0 0 0 0 0 1 1
0 1 0 0 0 0 1 0 0 0 0 0 1 1
0 0 1 0 0 0 0 0 0 1 0 0 1 1
0 0 0 0 1 0 0 1 0 0 0 0 1 1
0 0 0 0 0 1 0 0 0 0 1 0 1 1
0 0 0 0 0 0 0 0 1 0 0 1 1 1
```

```
FØR ALL QUESTIØNS, TYPE T FØR YES AND TYPE F  FØR NØ.
IS THE ØBJECTIVE FUNCTIØN TØ BE MAXIMIZED?
F
 ARE THE PIVØTAL LAMBDA VALUES TØ BE CALCULATED
 WITH THE NØRMALIZED CØNSTRAINTS?
F
IF ANY PIVØTAL LAMBDA VALUES ARE EQUAL, DØ YØU
 WANT THE RANKING BY THE NØRMAL PRØCEDURE?
T
REVISED ØBJECTIVE FUNCTIØN
 CØEFFICIENT  VARIABLE-XA=X,XB=1-X
        6.00      XB
        4.00      XB
        5.00      XB
        2.00      XB
        5.00      XB
        4.00      XB
        5.00      XB
        8.00      XB
        3.00      XB
        7.00      XB
        3.00      XB
        6.00      XB
ØBJ. FUNCTIØN CØNSTANT TERM=-58.
DØ YØU WANT TØ SEE A PRINTØUT ØF THE CØNSTRAINT EQUATIØNS?
F
 VARIABLE  PIVØTAL LAMBDA VALUE
    1          3.00
    2          2.00
    3          2.50
    4          1.00
    5          2.50
    6          2.00
    7          2.50
    8          4.00
    9          1.50
   10          3.50
   11          1.50
   12          3.00
```

VARIABLE	ORDER
1	4
2	9
3	7
4	12
5	6
6	8
7	5
8	1
9	11
10	2
11	10
12	3

VARIABLE	VALUE
1	1.00
2	0.
3	1.00
4	0.
5	1.00
6	1.00
7	1.00
8	1.00
9	0.
10	1.00
11	0.
12	1.00

ALL CONSTRAINTS

NUMBER	SET	VALUE	INEQUALITY	B(I)
1	B	2.00	GE	2.00
2	A	2.00	LE	2.00
3	B	2.00	GE	2.00
4	A	2.00	LE	2.00
5	B	2.00	GE	2.00
6	A	2.00	LE	2.00
7	B	2.00	GE	2.00
8	A	2.00	LE	2.00
9	B	2.00	GE	2.00
10	A	2.00	LE	2.00
11	B	2.00	GE	2.00
12	A	2.00	LE	2.00
13	B	2.00	GE	2.00
14	A	2.00	LE	2.00
15	B	2.00	GE	2.00
16	A	2.00	LE	2.00
17	B	1.00	GE	1.00
18	B	1.00	GE	1.00
19	B	2.00	GE	1.00
20	B	2.00	GE	1.00
21	B	1.00	GE	1.00
22	B	1.00	GE	1.00

OBJECTIVE FUNCTION VALUE ZE=-12.

SRU's: 12.7

```
FØR ALL QUESTIØNS, TYPE T FØR YES AND TYPE F  FØR NØ.
IS THE ØBJECTIVE FUNCTIØN TØ BE MAXIMIZED?
F
 ARE THE PIVØTAL LAMBDA VALUES TØ BE CALCULATED
 WITH THE NØRMALIZED CØNSTRAINTS?
F
IF ANY PIVØTAL LAMBDA VALUES AREEQUAL, DØ YØU
 WANT THE RANKING BY THE NØRMAL PRØCEDURE?
F
REVISED ØBJECTIVE FUNCTIØN
 CØEFFICIENT   VARIABLE-XX=X,XB=1-X
       6.00        XB
       4.00        XB
       5.00        XB
       2.00        XB
       5.00        XB
       4.00        XB
       5.00        XB
       3.00        XB
       3.00        XB
       7.00        XB
       3.00        XB
       6.00        XB
ØBJ. FUNCTIØN CØNSTANT TERM=-53.
DØ YØU WANT TØ SEE A PRINTØUT ØF THE CØNSTRAINT EQUATIØNS?
F
 VARIABLE  PIVØTAL LAMBDA VALUE
     1          3.00
     2          2.00
     3          2.50
     4          1.00
     5          2.50
     6          2.00
     7          2.50
     8          4.00
     9          1.50
    10          3.50
    11          1.50
    12          3.00
```

VARIABLE	ØRDER
1	4
2	9
3	7
4	12
5	6
6	8
7	5
8	1
9	11
10	2
11	10
12	3

VARIABLE	VALUE
1	1.00
2	0.
3	1.00
4	0.
5	1.00
6	1.00
7	1.00
8	1.00
9	0.
10	1.00
11	0.
12	1.00

ALL CØNSTRAINTS

NUMBER	SET	VALUE	INEQUALITY	B(I)
1	B	2.00	GE	2.00
2	A	2.00	LE	2.00
3	B	2.00	GE	2.00
4	A	2.00	LE	2.00
5	B	2.00	GE	2.00
6	A	2.00	LE	2.00
7	B	2.00	GE	2.00
8	A	2.00	LE	2.00
9	B	2.00	GE	2.00
10	A	2.00	LE	2.00
11	B	2.00	GE	2.00
12	A	2.00	LE	2.00
13	B	2.00	GE	2.00
14	A	2.00	LE	2.00
15	B	2.00	GE	2.00
16	A	2.00	LE	2.00
17	B	1.00	GE	1.00
18	B	1.00	GE	1.00
19	B	2.00	GE	1.00
20	B	2.00	GE	1.00
21	B	1.00	GE	1.00
22	B	1.00	GE	1.00

ØBJECTIVE FUNCTIØN VALUE ZB=-12.

SRU's: 12.7

```
FOR ALL QUESTIONS, TYPE T FOR YES AND TYPE F  FOR NO.
IS THE OBJECTIVE FUNCTION TO BE MAXIMIZED?
F
 ARE THE PIVOTAL LAMBDA VALUES TO BE CALCULATED
 WITH THE NORMALIZED CONSTRAINTS?
T
IF ANY PIVOTAL LAMBDA VALUES ARE EQUAL, DO YOU
 WANT THE RANKING BY THE NORMAL PROCEDURE?
T
REVISED OBJECTIVE FUNCTION
 COEFFICIENT  VARIABLE-XX=X,XB=1-X
       6.00      XB
       4.00      XB
       5.00      XB
       2.00      XB
       5.00      XB
       4.00      XB
       5.00      XB
       8.00      XB
       3.00      XB
       7.00      XB
       3.00      XB
       6.00      XB
OBJ. FUNCTION CONSTANT TERM=-58.
DO YOU WANT TO SEE A PRINTOUT OF THE CONSTRAINT EQUATIONS?
F
 VARIABLE  PIVOTAL LAMBDA VALUE
    1         6.00
    2         4.00
    3         5.00
    4         2.00
    5         5.00
    6         4.00
    7         5.00
    8         8.00
    9         3.00
   10         7.00
   11         3.00
   12         6.00
```

VARIABLE	ORDER
1	4
2	9
3	7
4	12
5	6
6	8
7	5
8	1
9	11
10	2
11	10
12	3

VARIABLE	VALUE
1	1.00
2	0.
3	1.00
4	0.
5	1.00
6	1.00
7	1.00
8	1.00
9	0.
10	1.00
11	0.
12	1.00

ALL CONSTRAINTS

NUMBER	SET	VALUE	INEQUALITY	B(I)
1	B	2.00	GE	2.00
2	A	2.00	LE	2.00
3	B	2.00	GE	2.00
4	A	2.00	LE	2.00
5	B	2.00	GE	2.00
6	A	2.00	LE	2.00
7	B	2.00	GE	2.00
8	A	2.00	LE	2.00
9	B	2.00	GE	2.00
10	A	2.00	LE	2.00
11	B	2.00	GE	2.00
12	A	2.00	LE	2.00
13	B	2.00	GE	2.00
14	A	2.00	LE	2.00
15	B	2.00	GE	2.00
16	A	2.00	LE	2.00
17	B	1.00	GE	1.00
18	B	1.00	GE	1.00
19	B	2.00	GE	1.00
20	B	2.00	[illegible]	1.00
21	B	1.00	GE	1.00
22	B	1.00	GE	1.00

OBJECTIVE FUNCTION VALUE ZB=-12.

SRU's: 12.7

A Linear Integer Programming Problem

The following is a listing of the data file and the computer solution printout for the linear integer programming problem:

```
9 3
1200 600 300 800 400 200 1600 800 400
16 8 4 12 6 3 12 6 3 1 25
8 4 2 16 8 4 20 10 5 1 30
12 6 3 8 4 2 8 4 2 1 32
```

```
FØR ALL QUESTIØNS, TYPE T FØR YES AND TYPE F  FØR NØ.
IS THE ØBJECTIVE FUNCTIØN TØ BE MAXIMIZED?
T
 ARE THE PIVØTAL LAMBDA VALUES TØ BE CALCULATED
 WITH THE NØRMALIZED CØNSTRAINTS?
F
IF ANY PIVØTAL LAMBDA VALUES ARE EQUAL, DØ YØU
 WANT THE RANKING BY THE NØRMAL PRØCEDURE?
T
REVISED ØBJECTIVE FUNCTIØN
 CØEFFICIENT  VARIABLE-XX=X,XB=1-X
    1200.00      XX
     600.00      XX
     300.00      XX
     800.00      XX
     400.00      XX
     200.00      XX
    1600.00      XX
     800.00      XX
     400.00      XX
ØBJ. FUNCTIØN CØNSTANT TERM=0.
DØ YØU WANT TØ SEE A PRINTØUT ØF THE CØNSTRAINT EQUATIØNS?
F
 VARIABLE  PIVØTAL LAMBDA VALUE
     1         33.33
     2         33.33
     3         33.33
     4         22.22
     5         22.22
     6         22.22
     7         40.00
     8         40.00
     9         40.00
 VARIABLE  ØRDER
     1         4
     2         5
     3         6
     4         7
     5         8
     6         9
     7         1
     8         2
     9         3
```

VARIABLE	VALUE
1	0.
2	0.
3	0.
4	0.
5	0.
6	0.
7	1.00
8	1.00
9	0.

ALL CONSTRAINTS

NUMBER	SET	VALUE	INEQUALITY	B(I)
1	A	13.00	LE	25.00
2	A	30.00	LE	30.00
3	A	12.00	LE	32.00

OBJECTIVE FUNCTION VALUE Z=2400.

SRU's: 11.3

```
FOR ALL QUESTIONS, TYPE T FOR YES AND TYPE F  FOR NO.
IS THE OBJECTIVE FUNCTION TO BE MAXIMIZED?
T
 ARE THE PIVOTAL LAMBDA VALUES TO BE CALCULATED
 WITH THE NORMALIZED CONSTRAINTS?
F
IF ANY PIVOTAL LAMBDA VALUES ARE EQUAL, DO YOU
 WANT THE RANKING BY THE NORMAL PROCEDURE?
F
REVISED OBJECTIVE FUNCTION
 COEFFICIENT   VARIABLE-XX=X,XB=1-X
    1200.00       XX
     600.00       XX
     300.00       XX
     800.00       XX
     400.00       XX
     200.00       XX
    1600.00       XX
     800.00       XX
     400.00       XX
OBJ. FUNCTION CONSTANT TERM=0.
DO YOU WANT TO SEE A PRINTOUT OF THE CONSTRAINT EQUATIONS?
F
 VARIABLE  PIVOTAL LAMBDA VALUE
     1         33.33
     2         33.33
     3         33.33
     4         22.22
     5         22.22
     6         22.22
     7         40.00
     8         40.00
     9         40.00
 VARIABLE  ORDER
     1         6
     2         5
     3         4
     4         9
     5         8
     6         7
     7         3
     8         2
     9         1
```

```
VARIABLE   VALUE
    1          0.
    2          1.00
    3          1.00
    4          0.
    5          0.
    6          1.00
    7          0.
    8          1.00
    9          1.00
  ALL CONSTRAINTS
NUMBER   SET    VALUE INEQUALITY B(I)
  1       A      24.00    LE        25.00
  2       A      25.00    LE        30.00
  3       A      17.00    LE        32.00
OBJECTIVE FUNCTION VALUE Z=2300.
```

SRU's: 11.3

```
FOR ALL QUESTIONS, TYPE T FOR YES AND TYPE F  FOR NO.
IS THE OBJECTIVE FUNCTION TO BE MAXIMIZED?
T
 ARE THE PIVOTAL LAMBDA VALUES TO BE CALCULATED
 WITH THE NORMALIZED CONSTRAINTS?
T
IF ANY PIVOTAL LAMBDA VALUES ARE EQUAL, DO YOU
 WANT THE RANKING BY THE NORMAL PROCEDURE?
T
REVISED OBJECTIVE FUNCTION
 CØEFFICIENT  VARIABLE-XX=X,XB=1-X
     1200.00      XX
      600.00      XX
      300.00      XX
      600.00      XX
      400.00      XX
      200.00      XX
     1600.00      XX
      800.00      XX
      400.00      XX
ØBJ. FUNCTION CØNSTANT TERM=0.
DØ YØU WANT TØ SEE A PRINTØUT ØF THE CØNSTRAINT EQUATIONS?
F
 VARIABLE  PIVØTAL LAMBDA VALUE
     1         936.23
     2         936.23
     3         936.23
     4         633.25
     5         633.25
     6         633.25
     7        1145.53
     8        1145.53
     9        1145.53
 VARIABLE  ØRDER
     1         4
     2         5
     3         6
     4         7
     5         8
     6         9
     7         1
     8         2
     9         3
```

```
VARIABLE   VALUE
    1         0.
    2         0.
    3         0.
    4         0.
    5         0.
    6         0.
    7         1.00
    8         1.00
    9         0.
  ALL CØNSTRAINTS
NUMBER   SET    VALUE INEQUALITY B(I)
  1       A      18.00   LE        25.00
  2       A      30.00   LE        30.00
  3       A      12.00   LE        32.00
ØBJECTIVE FUNCTIØN VALUE Z=2400.
```

SRU's: 11.3

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