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A STUDY ON HELICAL BUCKLING OF DRILL STRINGS IN INCLINED HOLES

by

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Dedicated to:

The memory of my dear brother,

Emin Kutan

ABSTRACT

During drilling, drill strings buckle under small axial loads. Plane form buckling occurs first, followed by helical buckling of varying pitch lengths. Helical buckling can have a harmful effect on the drill string; however, in some cases these loads can be acceptable.(1)

This thesis investigates the post buckling condition of drill strings. Namely, it assumes that the drill string has attained a helically buckled form. Analysis follows two approaches: first, discrete strain energies are summed over the length of the drill string, and second, strain energy is found with combined loading. The approaches differ only in the method of determination of the strain energy of the drill string.

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INTRODUCTION

The stability of long, slender columns which are subjected to axial loading, has been an interesting topic in the theory of elasticity for years. The first reported study was by Euler in 1744.

When a small axial load is applied to a rigid column, it causes the column to slightly bend, and when the load is removed, the column regains its straightness. In either of these cases, bent or straight, the column is said to be in stable equilibrium. Additional axial load will buckle the column. This is said to be a position of unstable equilibrium. The transition between these two equilibriums is called neutral equilibrium.

Euler defined buckling as continued deflection of a column without additional axial load. He showed that critical axial load causing the onset of buckling rose with Young's modulus and moment of inertia, and diminished with the square of the length of the column. His critical buckling load for a free column is given precisely by the formula:

$$F_{cr} = \frac{\pi^2 EI}{L^2} \qquad (1)$$

In his book, "A Treatise on the Mathematical Theory of Elasticity"[2], Love developed a theory of bending and twisting of long, slender rods. In the bent and twisted state, rods have three deflection components. Two of the components rise because of the curvature of its centerline, and the other component is caused by angular twist of the column. He writes the requisite strain energies for these deflections. Mitchell[3] applied this method. He developed the strain energy for a three-dimensionally buckled column and he derived the differential equations of buckling, then solved for helically buckling.

The second method of stability analysis is to find the "equations of equilibrium" for deflected rods. Love[2] also writes the "equations of equilibrium" for bent and twisted rods. This method was utilized by Mitchell[4] and Walker, et al.[5] to predict drill string or tubing buckling.

ENERGY METHODS

Energy methods are basically the application of the first law of Thermodynamics[2]. According to this law, the increment of the total energy of the body is equal to the net of the work done by the external forces and the quantity of heat supplied. Heat is neglected by most authors. The virtual work principle states that the virtual work of the internal forces is equal to the negative of the virtual change in the elastic strain energy[6]. That is,

$$\delta W_i = -\delta U \tag{2}$$

or,

$$\delta U + \delta W_i = 0$$

The applications of this principle to stability problems may be found widely in reference 7.

For linearly elastic materials, strain energy equations in terms of stress and strain components are as given:[6]

$$U = \int_V \left[\frac{1}{2E}\left(S_{xx}^2 + S_{yy}^2 + S_{zz}^2\right)\right.$$

$$\left. - \frac{\nu}{E}\left(S_{xx}\,S_{yy} + S_{yy}\,S_{zz} + S_{zz}\,S_{xx}\right)\right. \qquad \text{(continued)}$$

$$+ \frac{1}{2G} (S_{yz}^2 + S_{zx}^2 + S_{xy}^2)] \, dV \qquad (3)$$

$$U = \int_V [\frac{1}{2} \lambda (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})^2 + G(\varepsilon_{xx}^2 + \varepsilon_{yy}^2 + \varepsilon_{zz}^2$$

$$+ 2\varepsilon_{xy}^2 + 2\varepsilon_{xz}^2 + 2\varepsilon_{yz}^2)] \, dV \qquad (4)$$

In these equations, heat is not considered; i.e., isothermal changes in state.

A more practical way of obtaining the total strain energy of the column is through the separate consideration of strain energies generated by axial loading, bending, shearing, and torsion. This method was utilized by Lubinski, et al.[8]

Their total energy of a column was found by summing the strain energies and potential energy. Thereafter, they minimized the total energy to find the allowable axial load. On the other hand, Paslay and Bogy utilized the variational calculus to obtain the differential equation of the deflected center-line of a column.[9,10,11]

HELICAL BUCKLING STUDIES

During normal drilling within a hole filled with fluid, a load is placed on the bit. This load is arbitrarily selected. It may be less, equal, or more than the critical buckling load for the bottom hole assembly. This load is given by the following equation:

$$W_B = W_{BHA} \cos \alpha + F \tag{5}$$

If the weight on the bit, W_B, is equal to buoyed weight of the bottom hole assembly in the direction of the hole, the neutral plane of buckling will be at the top of the drill-collars. In this case, there will be no buckling moments transmitted to the drill-pipe. But, if $W_B > W_{BHA} \cos \alpha$, there occurs a load at the bottom of the drill-pipe string, and this load could cause the drill-pipe to buckle.

For low values of load F, drill pipe buckles in a plane. This is called "plane form buckling". But as the load F increases, the buckled shape changes to that of a helix with varying pitches.

The plane buckling problem has been solved by Lubinski[12] for a vertical hole. He has found the critical load for the buckling of bottom hole assembly for several buckled

configurations. According to his study[12], bottom hole assemblies buckle at low loads, and it is reasonable to drill with buckled bottom hole assemblies.

The same problem was studied by Paslay and Bogy[10] for inclined holes. They gave a relationship between the deflection and load necessary to induce an instability.

Later, Lubinski et al.[8] studied helical buckling of tubings and drill pipes in vertical holes. Their work predicted acceptable buckling loads rather than critical buckling loads. Their analysis can be accepted as a post buckling analysis.

Critical helical buckling load for drill strings in inclined wells was delineated by Paslay and Bogy[11]. It is interesting to note the similarity of their equation and the equation for beams on elastic foundations, as given by Timoshenko[7]. Their equation applies only in inclined wells and gives higher buckling loads than Euler's critical buckling load. They state that the reason for the higher buckling loads is because the wall of the hole supports some of the weight of the drill string.[1]

In the next sections, both the Lubinski et al. analysis[8] and the Paslay and Bogy analysis[11] has been expanded to post buckling analysis.

POST BUCKLING ANALYSIS - I

In an inclined bore, the drill string normally rests on the low side of the hole and friction is produced at the points of contacts. Therefore, upon rotation of the string, torsion exists and diminishes with depth. It may not diminish to a value of zero at the bottom, if a bit is attached. These effects are considered in addition to those considered in reference 8.

The assumptions made during the derivation of the author's model are the following:

1. The drill string is continuous. Presence of tool joints is neglected.

2. The wellbore is inclined and straight, i.e. normal and bending stresses caused by hole curvature itself are not present.

3. Hole diameter is constant in value.

4. All types of friction and mud flow effects are neglected, except for torsion.

5. The drill string has a helical configuration.

6. Only torsion and axial loads are considered; i.e. no wall forces.

7. There is a transition region from plane buckling to helical buckling. It is not investigated.

8. Because the drill collars are stiffer than drill pipes, they have a straightening effect that prevents the helical shape along a certain length of drill pipe. It is just like the packer effect on tubing and it is not investigated.

9. String cross-section is circular.

In light of these assumptions, the model can be developed.

Figure 1 shows the loads in a helically buckled string. Without friction, F_z diminishes linearly along the axis of the string. The neutral plane is defined at the location in the string where the force, F_z, diminishes to zero.

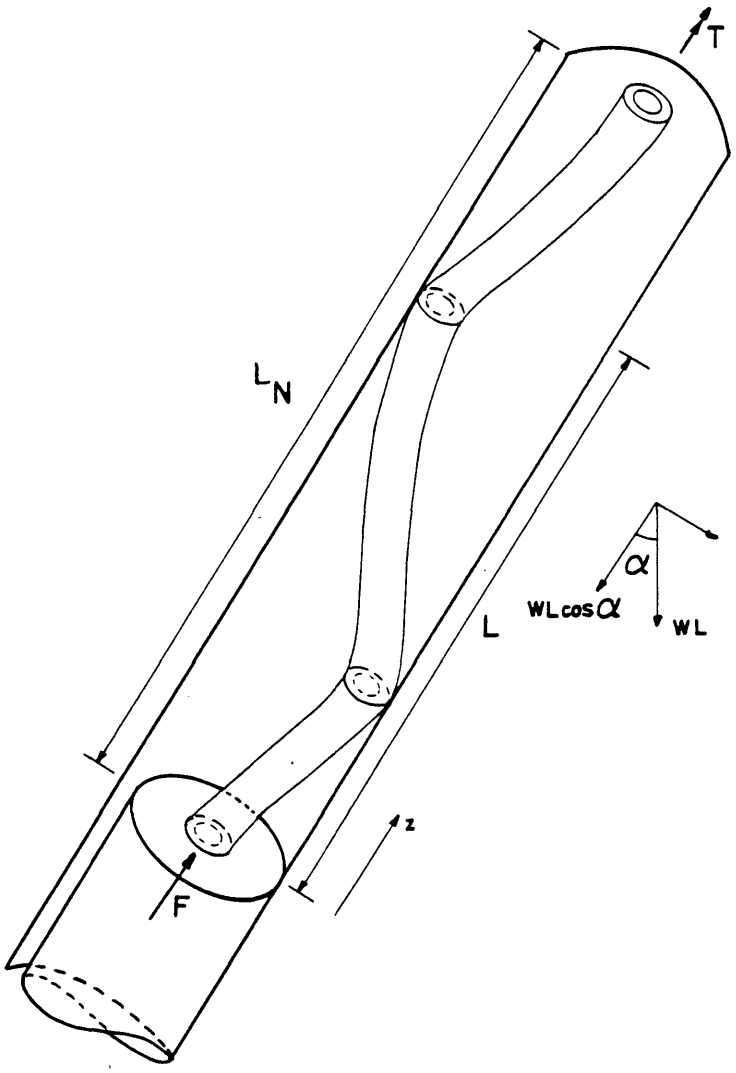

Figure 1. Helically Buckled Drill String

$$F = F_z \quad \text{at} \quad z = 0$$

$$F = 0 \quad \text{at} \quad z = L_N$$

$$F_z = F \frac{L_N - z}{L_N} \tag{6}$$

W_z, which is the buoyed weight of the string at any location is likewise given by,

$$W_z = W L_N * \cos \alpha * \frac{z}{L_N} \tag{7}$$

where,

$$W = W_s + W_i - W_o \tag{8}$$

So, the axial load on the string is,

$$P_z = F \frac{L_N - z}{L_N} + W L_N (\cos \alpha) \frac{z}{L_N} \tag{9}$$

Strain energy for axial loading is given by,

$$U_A = \int_0^L \left(\frac{P_z^2}{2 AE}\right) dz \tag{10}$$

Substituting P_z from Equation 9, Equation 10 becomes,

$$U_A = \int_{z=0}^{L} \frac{\left(F \frac{L_N - z}{L_N} + W L_N (\cos \alpha) \frac{z}{L_N}\right)^2}{2 AE} dz \qquad (11)$$

After the integration,

$$U_A = \frac{1}{2 AE} \left[F^2 \left(L - \frac{L^2}{L_N} + \frac{L^3}{3L_N^2}\right) + F W L_N \cos \alpha \left(\frac{L^2}{L_N} - \frac{2}{3} \frac{L^3}{L_N^2}\right)\right.$$

$$\left. + \frac{1}{3} (W L_N \cos \alpha)^2 \frac{L^3}{L_N^2}\right] \qquad (12)$$

As shown in Figure 2, loads of Equation 12 loads are along the axis of the string. If they are converted to the axis of the helix, the following equation occurs:

$$U_A = \frac{1}{2 AE} \left[F^2 \left(L - \frac{L^2}{L_N} + \frac{L^3}{3L_N^2}\right) + F W L_N \cos \alpha \left(\frac{L^2}{L_N} - \frac{2}{3} \frac{L^3}{L_N^2}\right)\right.$$

$$\left. + \frac{1}{3} (W L_N \cos \alpha)^2 \frac{L^3}{L_N^2}\right] \frac{P^2}{P^2 + 4\pi^2 R^2} \qquad (13)$$

Where p is the length of pitch of the helix and R is the radial clearance.

The strain energy of bending is modelled with the same approach as was used by Lubinski et al.[8] Curvature

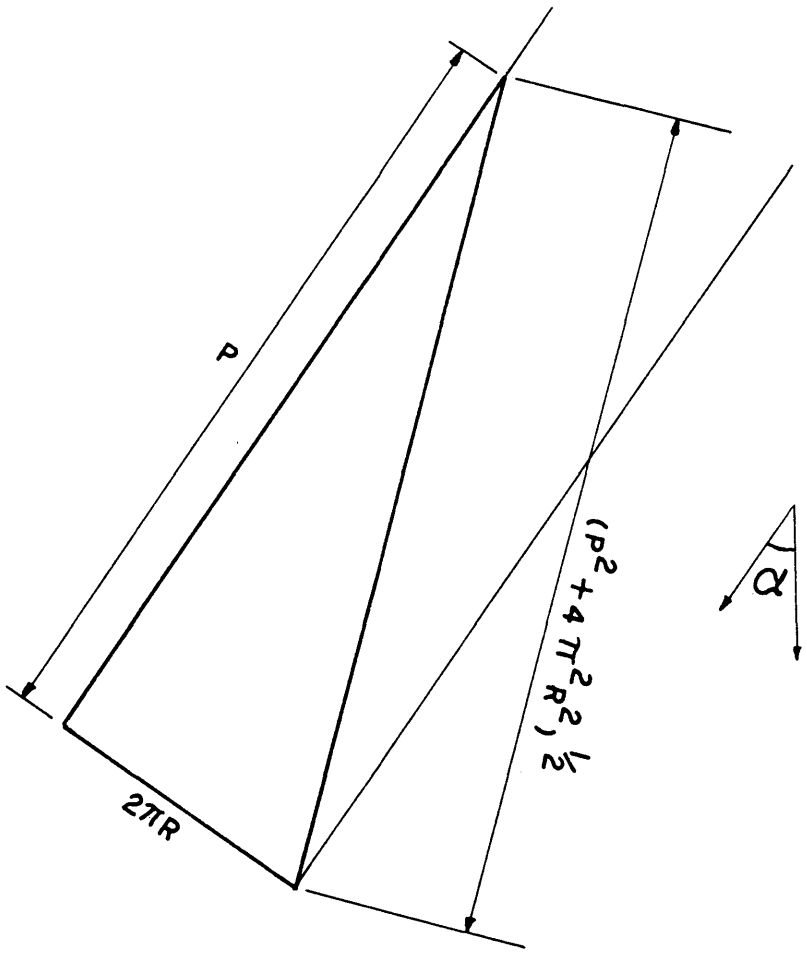

Figure 2. Development of Helix

changes along the axis of the helix. The strain energy of bending is expressed in the following equation:

$$U_B = \int_o^L \left(\frac{M_b^2}{2EI}\right) dz \tag{14}$$

Because,

$$M_b = EIC_z \tag{15}$$

then,

$$U_B = \frac{EI\ L\ C_z^2}{2} \tag{16}$$

Substituting C_z from Equation 45 of reference 8,

$$U_b = \frac{8\pi^2\ R^2\ EIL}{(P^2 + 4\pi^2\ R^2)^2} \tag{17}$$

The strain energy of shearing is given by the following equation:

$$U_s = \int_o^L \frac{k\ V_z^2}{2\ GA}\ dz \tag{18}$$

Since

$$\tau = \frac{V}{A} = \frac{Tr}{J} \tag{19}$$

then

$$V_z = \frac{T_z A\ (OD-ID)}{2J} \tag{20}$$

Therefore,

$$U_s = \int_o^L \frac{k\ T_z^2\ (OD-ID)^2}{GJ\ (OD^2 + ID^2)}\ dz \tag{21}$$

Torsion, T is assumed to be a maximum at the top of the investigated portion of the string and diminishes to zero at the top of the drill collars. This may not be precisely true in practice, but it will be shown that torsion is not a dominant factor in the buckling of drill strings. With the above considerations, the following equations occur:

$$T_z = T \quad \text{at} \quad z = L_N$$

$$T_z = 0 \quad \text{at} \quad z = 0$$

$$T_z = T \frac{z}{L_N} \tag{22}$$

The strain energy of shearing, after substitution and integration of Equation 18, becomes the following equation:

$$U_s = k \frac{T^2}{GJ} \frac{(OD-ID)^2}{OD^2 + ID^2} \frac{L^3}{3L_N^2} \tag{23}$$

Here, k is a factor for columns of different cross-sectional shapes. For thin wall circular cross-sections k has a value of two. Changing from the axis of the string to the axis of the helix, Equation 23 becomes the following:

$$U_s = \frac{2}{3} \frac{T^2}{GJ} \frac{(OD - ID)^2}{(OD^2 + ID)^2} \frac{L^3}{L_N^2} \frac{P^2}{P^2 + 4\pi^2 R^2} \tag{24}$$

Finally, strain energy of torsion is,

$$U_T = \int_0^L \frac{T^2 \frac{z^2}{L_N^2}}{2 GJ} dz \tag{25}$$

Along the axis of the helix and after integration, Equation 25 becomes the following equation:

$$U_T = \frac{1}{6} \frac{T^2}{GJ} \frac{L^3}{L_N^2} \frac{P^2}{P^2 + 4\pi^2 R^2} \tag{26}$$

The potential energy in the string, caused by the load F_z, is the following:

$$PE_1 = \int_0^L F\left(\frac{L_N - z}{L_N}\right) \frac{P}{\left(P^2 + 4\pi^2 R^2\right)^{1/2}}$$

$$\left[1 - \frac{F\left(\frac{L_N - z}{L_N}\right)}{AE} \frac{P}{\left(P^2 + 4\pi^2 R^2\right)^{1/2}}\right] dz \qquad (27)$$

Upon integration Equation 27 becomes the following:

$$PE_1 = \frac{P}{\left(P^2 + 4\pi^2 R^2\right)^{1/2}} \left| Fz - \frac{Fz^2}{2L_N} - \left[F^2 z - \frac{F^2 z^2}{L_N}\right.\right.$$

$$\left.\left. + \frac{F^2 z^3}{3L_N^2}\right] \frac{P}{AE\left(P^2 + 4\pi^2 R^2\right)^{1/2}} \right|_{z=0}^{L} \qquad (28)$$

Then, after introducing the integration limits, the potential energy becomes:

$$PE_1 = \frac{P}{\left(P^2 + 4\pi^2 R^2\right)^{1/2}} \left[FL\left(1 - \frac{L}{2L_N}\right)\right. \qquad \text{(continued)}$$

$$- \frac{F^2 L}{AE} \frac{P}{(P^2 + 4\pi^2 R^2)^{1/2}} \left(1 - \frac{L}{L_N} + \frac{L^2}{3 L_N^2}\right)] \qquad (29)$$

The potential energy in the string, caused by the load W_z, is the following:

$$PE_2 = \int_0^L W L_N \cos \alpha \frac{z}{L_N} \frac{P}{(P^2 + 4\pi^2 R^2)^{1/2}}$$

$$\left[1 - \frac{W L_N \cos \alpha}{AE} \frac{z}{L_N} \frac{P}{(P^2 + 4\pi^2 R^2)^{1/2}}\right] dz \qquad (30)$$

Integration yields the following equation:

$$PE_2 = \frac{P}{(P^2 + 4\pi^2 R^2)^{1/2}} \left| W L_N \cos \alpha \frac{z^2}{2L_N} \right.$$

$$\left. - \frac{(W L_N \cos \alpha)^2}{AE} \frac{z^3}{3L_N^2} \frac{P}{(P^2 + 4\pi^2 R^2)^{1/2}} \right|_{z=0}^{L} \qquad (31)$$

Then, after introducing the integration limits, the potential energy becomes,

$$PE_2 = W L_N \cos \alpha \frac{L^2}{2L_N} \frac{P}{(P^2 + 4\pi^2 R^2)^{1/2}}$$

$$- \frac{(W L_N \cos \alpha)^2}{AE} \frac{L^3}{3L_N^2} \frac{P^2}{P^2 + 4\pi^2 R^2} \qquad (32)$$

The total energy of the string is the sum of all the energies. Thus, the following equation is given:

$$E_T = U_A + U_B + U_S + U_T + PE_1 + PE_2 \qquad (33)$$

Substitution of member terms yields the following equation:

$$E_T = \frac{1}{2AE} \left[F^2 \left(L - \frac{L^2}{L_N} + \frac{L^3}{3L_N^2}\right) + F W L_N \cos \alpha \left(\frac{L^2}{L_N} - \frac{2}{3} \frac{L^3}{L_N^2}\right)\right.$$

$$\left. + \frac{1}{3} (W L_N \cos \alpha)^2 \frac{L^3}{L_N^2}\right] c^2 + \frac{8\pi^2 R^2 EIL}{(P^2 + 4\pi^2 R^2)^2}$$

$$+ \frac{2}{3} \frac{T^2}{GJ} \frac{(OD - ID)^2}{(OD^2 + ID^2)} \frac{L^3}{L_N^2} c^2 \qquad \text{(continued)}$$

$$+ \frac{1}{6} \frac{T^2}{GJ} \frac{L^3}{L_N^2} c^2 + c \left[FL\left(1 - \frac{L}{2L_N}\right)\right.$$

$$\left. - \frac{F^2 L}{AE} c \left(1 - \frac{L}{L_N} + \frac{L^2}{3L_N^2}\right)\right]$$

$$+ W L_N \cos \alpha \frac{L^2}{2L_N} c - \frac{(W L_N \cos \alpha)^2}{AE} \frac{L^3}{3L_N^2} c^2 \qquad (34)$$

where, for convenience,

$$c = \frac{P}{(P^2 + 4\pi^2 R^2)^{1/2}}$$

The condition of stable equilibrium of the string is found by minimizing the total energy of the string. Calculus is used to find the minimum:

$$\frac{dE_T}{dP} = 0 \qquad (35)$$

Then substitution of Equation 34 into Equation 35, and differentiation gives the following equation:

$$\frac{4\pi^2R^2P}{AE(P^2+4\pi^2R^2)^2}\left[F^2\left(L-\frac{L^2}{L_N}+\frac{L^3}{3L_N^2}\right)\right.$$

$$+FWL_N\cos\alpha\left(\frac{L^2}{L_N}-\frac{2}{3}\frac{L^3}{L_N^2}\right)$$

$$\left.+\frac{1}{3}(WL_N\cos\alpha)^2\frac{L^3}{L_N^2}\right]-\frac{32\pi^4R^2EILP}{(P^2+4\pi^2R^2)^3}+\frac{4}{3}\pi^2R^2\frac{T^2}{GJ}\frac{L^3}{L_N^2}$$

$$\frac{P}{(P^2+4\pi^2R^2)^2}\left[4\frac{(OD-ID)^2}{(OD^2+ID^2)}+1\right]$$

$$+\frac{4\pi^2R^2FL}{(P^2+4\pi^2R^2)^{3/2}}\left(1-\frac{L}{2L_N}\right)$$

$$-\frac{8\pi^2R^2F^2LP}{AE(P+4\pi^2R^2)^2}\left(1-\frac{L}{L_N}+\frac{L^2}{3L_N^2}\right)+\frac{2\pi^2R^2WL_N\cos\alpha}{(P^2+4\pi^2R^2)^{3/2}}\frac{L^2}{L_N}$$

$$-\frac{8}{3}\frac{\pi^2R^2}{AE}(WL_N\cos\alpha)^2\frac{L^3}{L_N^2}\frac{P}{(P^2+4\pi^2R^2)^2}=0 \qquad (36)$$

If $P^2 >> 4\pi^2R^2$, then the approximation, $P \approx (P^2+4\pi^2R^2)^{1/2}$, may be made. By this approximation and

multiplying each term by

$$\frac{P^3}{\pi^2 R^2 L}$$, the equation simplifies to the following:

$$\frac{4F^2}{AE}\left(1-\frac{L}{L_N}+\frac{L^2}{3L_N^2}\right)+\frac{4F\ W\ L_N\ \cos\ \alpha}{AE}\left(\frac{L}{L_N}-\frac{2}{3}\frac{L^2}{L_N^2}\right)$$

$$+\frac{4}{3}\frac{(W\ L_N\ \cos\ \alpha)^2}{AE}\frac{L^2}{L_N^2}-\frac{32\pi^2\ EI}{P^2}$$

$$+\frac{4}{3}\frac{T^2}{GJ}\frac{L^2}{L_N^2}\left[\frac{4(OD-ID)^2}{(OD^2+ID^2)}+1\right]+4F\left(1-\frac{L}{2L_N}\right)$$

$$-\frac{8F^2}{AE}\left(1-\frac{L}{L_N}+\frac{L^2}{3L_N^2}\right)+2\frac{L}{L_N}W\ L_N\ \cos\ \alpha$$

$$-\frac{8}{3}\frac{(W\ L_N\ \cos\ \alpha)^2}{AE}\frac{L^2}{L_N^2}=0 \qquad (37)$$

Without friction and externally applied axial loads, the value of F can only be the weight of the drill string itself. The following equation applies to this case:

$$F = W L_N \cos \alpha \tag{38}$$

Substituting Equation 38 into Equation 37 gives the second degree equation in F.

$$- \frac{F^2}{AE} \left(1 - 2 \frac{L}{L_N} + \frac{4}{3} \frac{L^2}{L_N^2}\right) + F - \frac{8\pi^2 EI}{P^2}$$

$$+ \frac{1}{3} \frac{T^2}{GJ} \frac{L^2}{L_N^2} \left[4 \frac{(OD-ID)^2}{(OD+ID^2)} + 1\right] = 0 \tag{39}$$

It is shown in Tables 1 and 2 that neglecting the second degree term creates very minor error. Therefore, for convenience purposes, the following equation is written:

$$F = \frac{8\pi^2 EI}{P^2} - \frac{T^2}{GJ} \frac{L^2}{3L_N^2} \left[4 \frac{(OD-ID)^2}{(OD^2 + ID^2)} + 1\right] \tag{40}$$

If axial load F is set equal to zero, pure torsional buckling conditions can be found for a given configuration. For any given pitch length the required torque for buckling is given by,

TABLE 1

COMPARISON OF RESULTS FROM EQ. 39 AND EQ. 40
(5 in. , 19.5 ppf Drill-pipe)

RADIAL CLEARANCE (INCHES)	RESULTS FROM EQ. 39 (POUNDS)	RESULTS FROM EQ. 40 (POUNDS)	ERROR (%)
.50	124438.0	124406.5	.2532E-01
1.00	62202.0	62193.0	.1454E-01
1.50	41460.0	41455.1	.1178E-01
2.00	31090.0	31086.2	.1225E-01
2.50	24866.0	24864.8	.4669E-02
3.00	20720.0	20717.3	.1318E-01
3.50	17756.0	17754.7	.7199E-02
4.00	15534.0	15532.8	.7665E-02
4.50	13804.0	13804.7	-.4751E-02
5.00	12424.0	12422.1	.1503E-01
5.50	11292.0	11291.0	.9051E-02
6.00	10348.0	10348.3	-.3369E-02
6.50	9552.0	9550.7	.1320E-01
7.00	8866.0	8867.1	-.1211E-01
7.50	8274.0	8274.6	-.6818E-02

TABLE 2

COMPARISON OF RESULTS FROM EQ. 39 AND EQ. 40
(8 in. , 147 ppf Drill-collar)

RADIAL CLEARANCE (INCHES)	RESULTS FROM EQ. 39 (POUNDS)	RESULTS FROM EQ. 40 (POUNDS)	ERROR (%)
.50	1719344.0	1718585.3	.4413E-01
1.00	859488.0	859290.9	.2293E-01
1.50	572944.0	572859.5	.1474E-01
2.00	429696.0	429643.8	.1215E-01
2.50	343744.0	343714.4	.8622E-02
3.00	286448.0	286428.1	.6958E-02
3.50	245536.0	245509.3	.1087E-01
4.00	214832.0	214820.2	.5488E-02
4.50	190960.0	190950.9	.4750E-02
5.00	171872.0	171855.5	.9602E-02
5.50	156240.0	156232.0	.5142E-02
6.00	143216.0	143212.4	.2545E-02
6.50	132192.0	132195.8	-.2844E-02
7.00	122752.0	122753.0	-.7868E-03
7.50	114576.0	114569.2	.5926E-02

$$T = \frac{3.7\ \pi\ EI\ L_N}{PL} \tag{41}$$

or for a given torque value, the pitch length is,

$$P = \frac{3.7\ \pi^2\ EI\ L_N}{TL} \tag{42}$$

POST BUCKLING ANALYSIS - II

In this section, a different approach to post buckling analysis is given. It is basically similar to that presented in reference 11.

The assumptions made in the analysis given in the preceding section are valid in this analysis also. Additionally, in this case the drill string is constrained to be in contact with the wall of the hole.

A drill string in an inclined hole normally lies on the low side of the hole. Its initial and displaced configurations are shown in Figure 3. Initial coordinates of a particle, in cylindrical coordinates system are given by[11]:

$$r_{initial} = \left[(R + r\cos\theta)^2 + (r\sin\theta)^2\right]^{1/2}$$

$$\theta_{initial} = \text{Arctan}\left[\frac{r\sin\theta}{R + r\cos\theta}\right] \qquad (43)$$

$$z_{initial} = z$$

After displacement the coordinates of this particle become,

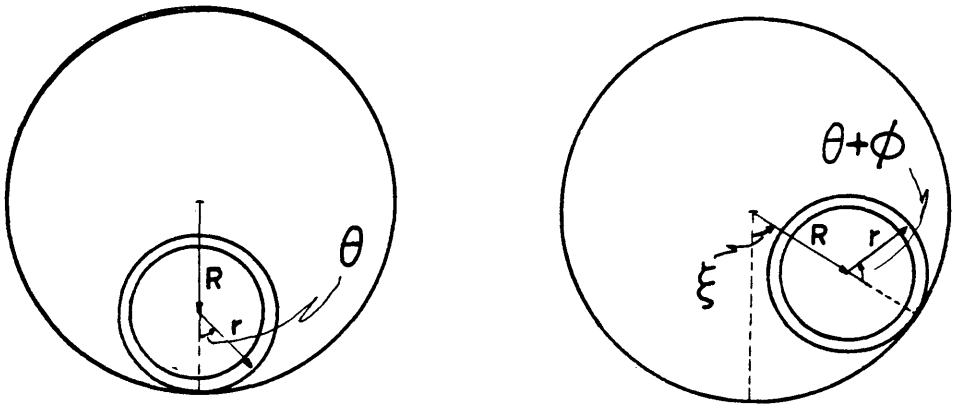

Figure 3. Initial and Displaced Configuration of Drill Pipe Cross-Section

$$r_{displaced} = \{[R + r\cos(\theta+\phi)]^2 + [r\sin(\theta+\phi)]^2\}^{1/2}$$

$$\theta_{displaced} = \xi + \text{Arctan}\left[\frac{r\sin(\theta+\phi)}{R + r\cos(\theta+\phi)}\right] \qquad (44)$$

$$z_{displaced} = (1 + \varepsilon)\, z - Rr\, \xi' \sin(\theta+\phi)$$

where prime denotes the differentiation with respect to z. ε is the axial strain due to load F. Load F is shown on Figure 4. Other parameters are shown on Figure 3.

Strain components of displacement shown as Equation 11 of reference 11 hold true for this case with slight modifications. These modifications are explained in the following arguments.

It is clear that the value of ξ is given by,

$$\xi = \frac{2\pi z}{P} \qquad (45)$$

assuming that the drill string attained its helical form.

Therefore $\xi'' = 0$. Double prime denotes the second derivative with respect to z. Additionally, for instability to occur it is necessary

$$\phi' = -\xi' \qquad (46)$$

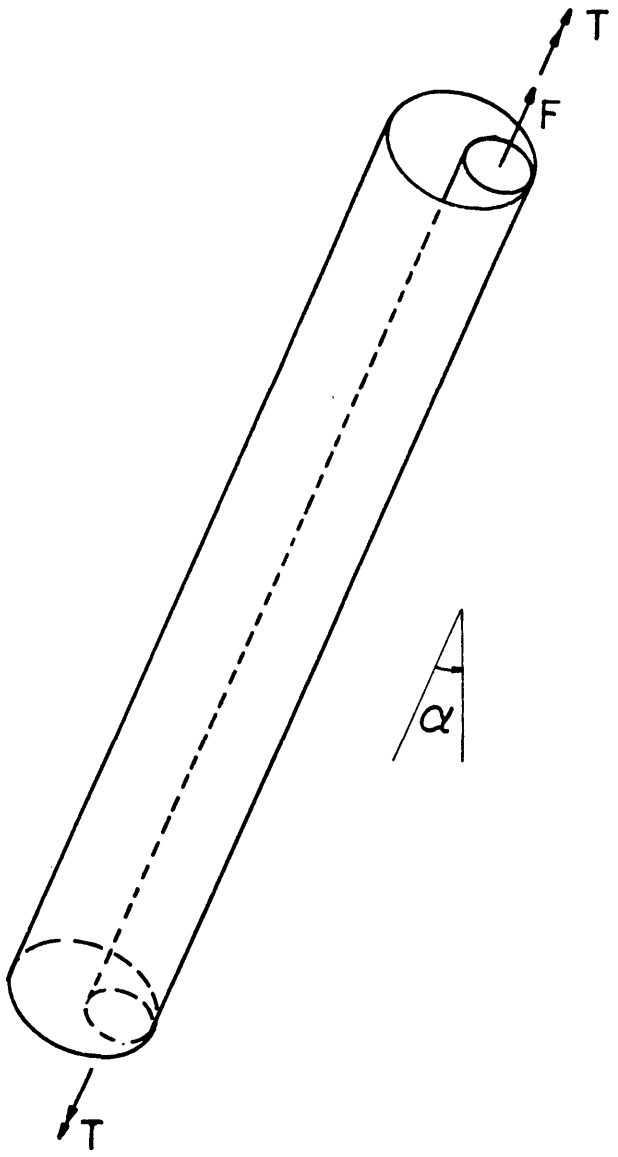

Figure 4. Loading Conditions for Analysis-II.

as was done in Equation 19 of reference 11. "The physical interpretation of Equation 46 is that the sections of the rod perpendicular to its axis must translate, that is they must not rotate relative to the hole, as they move around in contact with the hole."

Then the strain components are:

$$\varepsilon_{rr} = \frac{1}{2} r^2 \xi'^2 \sin^2(\theta+\phi)$$

$$\varepsilon_{\theta\theta} = \frac{1}{2} R^2 r^2 \xi'^2 \cos^2(\theta+\phi)$$

$$\varepsilon_{zz} = \varepsilon + \frac{1}{2}\varepsilon^2 + \frac{1}{2} r^2 \phi'^2 + \frac{1}{2}[R^2 + 2Rr\cos(\theta+\phi) + r^2]\xi'^2$$

$$+ r^2\xi'\phi' - Rr\varepsilon\xi'\phi'\cos(\theta+\phi) + \frac{1}{2} R^2 r^2 \phi'^2 \xi'^2 \cos(\theta+\phi)$$

$$\varepsilon_{r\theta} = \frac{1}{2} R^2 r\, \xi'^2 \cos(\theta+\phi)\sin(\theta+\phi) \qquad (47)$$

$$\varepsilon_{\theta z} = \frac{1}{2} R^2 r^2 \xi'^2 \phi' \cos^2(\theta+\phi) - \frac{1}{2} Rr\, \xi'\varepsilon\cos(\theta+\phi)$$

$$\varepsilon_{zr} = \frac{1}{2} R^2 r\, \xi'^2 \phi' \sin(\theta+\phi)\cos(\theta+\phi) - \frac{1}{2} R\xi'\varepsilon\sin(\theta+\phi)$$

The strain energy for elastic material, in the cylindrical coordinate system, is given by,(11)

$$U = \int_{r=r_i}^{r_o} \int_{\theta=o}^{2\pi} \int_{z=o}^{L} \left[\frac{G\nu}{1-2\nu}\left(\varepsilon_{rr}+\frac{1}{r^2}\varepsilon_{\theta\theta}+\varepsilon_{zz}\right)^2\right.$$

$$+ G\left(\varepsilon_{rr}^2 + \frac{1}{r^4}\varepsilon_{\theta\theta}^2 + \varepsilon_{zz}^2\right.$$

$$\left.\left. + \frac{2}{r^2}\varepsilon_{\theta r}^2 + \frac{2}{r^2}\varepsilon_{\theta z}^2 + 2\varepsilon_{rz}^2\right)\right] r\,dr\,d\theta\,dz \qquad (48)$$

Substituting Equation 47 into Equation 48 and carrying out integrals over r and θ,

$$U = \int_{z=o}^{L} \left\{\frac{G\nu}{1-2\nu}\left[\frac{\pi}{4}(r_o^2 - r_i^2)\varepsilon^4 + \pi(r_o^2 - r_i^2)\varepsilon^3 + \pi(r_o^2 - r_i^2)\varepsilon^2\right.\right.$$

$$+ \pi\left[R^2(r_o^2 - r_i^2) + \frac{1}{4}(r_o^4 - r_i^4)\right]\varepsilon^2\xi'^2 + \frac{\pi}{4}(r_o^4 - r_i^4)\varepsilon^2\phi'^2$$

$$\left.\left. + \frac{\pi}{2}(r_o^4 - r_i^4)\varepsilon^2\xi'\phi' + \frac{3}{8}\pi R^2(r_o^4 - r_i^4)\varepsilon^2\xi'^2\phi'^2\right. \text{(continued)}$$

$$+ \pi \left[2R^2(r_o^2 - r_i^2) + \frac{1}{2}(r_o^4 - r_i^4)\right]\varepsilon\xi'^2 + \frac{\pi}{2}(r_o^4 - r_i^4)\varepsilon\phi'^2$$

$$+ \pi (r_o^4 - r_i^4)\varepsilon\xi'\phi' + \frac{\pi}{4} R^2(r_o^4 - r_i^4)\varepsilon\xi'^2\phi'^2$$

$$- \frac{\pi}{2} R^2(r_o^4 - r_i^4)\varepsilon\xi'^3\phi'$$

$$+ \pi \left[R^4(r_o^2 - r_i^2) + \frac{3}{4} R^2(r_o^4 - r_i^4) + \frac{1}{12}(r_o^6 - r_i^6)\right]\xi'^4$$

$$+ \pi \left[\frac{R^4}{4}(r_o^4 - r_i^4) + \frac{R^2}{12}(r_o^6 - r_i^6)\right]\xi'^4\phi'^2$$

$$+ \frac{\pi}{32} R^4(r_o^6 - r_i^6)\xi'^4\phi'^4$$

$$+ \pi \left[R^2(r_o^4 - r_i^4) + \frac{1}{3}(r_o^6 - r_i^6)\right]\xi'^3\phi'$$

$$+ \frac{\pi}{6} R^2(r_o^6 - r_i^6)\xi'^3\phi'^3$$

$$+ \pi \left[\frac{R^2}{2}(r_o^4 - r_i^4) + \frac{1}{2}(r_o^6 - r_i^6)\right]\xi'^2\phi'^2$$

$$+ \frac{\pi}{12} R^2(r_o^6 - r_i^6)\xi'^2\phi'^4$$

$$+ \frac{\pi}{3}(r_o^6 - r_i^6)\xi'\phi'^3 + \frac{\pi}{12}(r_o^6 - r_i^6)\phi'^4\Big] + G\left[\frac{\pi}{4}(r_o^2 - r_i^2)\varepsilon^4\right.$$

$$+ \pi(r_o^2 - r_i^2)\varepsilon^3 + \pi(r_o^2 - r_i^2)\varepsilon^2 + \pi\left[R^2(r_o^2 - r_i^2)\right. \quad \text{(continued)}$$

$$+ \frac{1}{4}(r_o^4 - r_i^4)]\varepsilon^2\xi'^2$$

$$+ \frac{\pi}{2}(r_o^4 - r_i^4)\varepsilon^2\xi'\phi' + \frac{3}{8}\pi R^2(r_o^4 - r_i^4)\varepsilon^2\xi'^2\phi'^2$$

$$+ \frac{\pi}{4}(r_o^4 - r_i^4)\varepsilon^2\phi'^2$$

$$+ \pi\left[R^2(r_o^2 - r_i^2) + \frac{1}{2}(r_o^4 - r_i^4)\right]\varepsilon\xi'^2 + \frac{\pi}{2}(r_o^4 - r_i^4)\varepsilon\phi'^2$$

$$+ \pi(r_o^4 - r_i^4)\varepsilon\xi'\phi'$$

$$+ \frac{\pi}{4}R^2(r_o^4 - r_i^4)\varepsilon\xi'^2\phi'^2 - \frac{\pi}{2}R^2(r_o^4 - r_i^4)\varepsilon\xi'^3\phi'$$

$$+ \frac{\pi}{2}\left[R^4(r_o^2 - r_i^2) + R^2(r_o^4 - r_i^4) + \frac{1}{6}(r_o^6 - r_i^6)\right]\xi'^4$$

$$+ \frac{\pi}{4}\left[R^4(r_o^4 - r_i^4) + \frac{R^2}{3}(r_o^6 - r_i^6)\right]\xi'^4\phi'^2$$

$$+ \frac{\pi}{32}R^2(r_o^6 - r_i^6)\xi'^4\phi'^4 + \frac{\pi}{6}R^2(r_o^6 - r_i^6)\xi'^3\phi'^3$$

$$+ \frac{\pi}{2}\left[R^2(r_o^4 - r_i^4) + \frac{1}{3}(r_o^6 - r_i^6)\right]\xi'^3\phi'$$

$$+ \frac{\pi}{2}\left[\frac{R^2}{2}(r_o^4 - r_i^4) + (r_o^6 - r_i^6)\right]\xi'^2\phi'^2$$

$$+ \frac{\pi}{12}R^2(r_o^6 - r_i^6)\xi'^2\phi'^4$$

(continued)

$$+ \frac{\pi}{12}(r_o^6 - r_i^6)\phi'^4 + \frac{\pi}{3}(r_o^6 - r_i^6)\xi'\phi'^3]\} \, dz \qquad (49)$$

Equation (49) may be reduced by substituting the following recognized parameters.

Area:

$$A = \pi(r_o^2 - r_i^2)$$

Cross-sectional moment of inertia:

$$I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

Poisson's ratio:

$$\nu = .3 \text{ for steel} \qquad (50)$$

Modulus of shear:

$$G = \frac{E(1-2\nu)}{2(1-\nu)} = \frac{2E}{7}$$

Substituting Equations 50 and carrying out the integration over z, Equation 49 becomes,

$$
\begin{aligned}
U = EL \{ & \frac{1}{8} A\varepsilon^4 + \frac{1}{2} A\varepsilon^3 + \frac{1}{2} A\varepsilon^2 + \frac{1}{2} (R^2A+I)\varepsilon^2\xi'^2 + I\varepsilon^2\xi'\phi' \\
& + \frac{1}{2} I\varepsilon^2\phi'^2 + \frac{3}{4} R^2I\varepsilon^2\xi'^2\phi'^2 + (\frac{5}{7} R^2A+I)\varepsilon\xi'^2 + I\varepsilon\phi'^2 \\
& + 2I\varepsilon\xi'\phi' + \frac{1}{2} R^2I\varepsilon\xi'^2\phi'^2 - R^2I\varepsilon\xi'^3\phi' \\
& + [\frac{5}{14} R^4A + \frac{17}{14} R^2I + \frac{\pi}{24} (r_o^6 - r_i^6)]\xi'^4 \\
& + [\frac{1}{2} R^4I + \frac{\pi}{24} R^2(r_o^6 - r_i^6)]\xi'^4\phi'^2 \\
& + \frac{1}{64} \pi R^4(r_o^6 - r_i^6)\xi'^4\phi'^4 + [\frac{10}{7} R^2I \\
& + \frac{5}{42} \pi (r_o^6 - r_i^6)]\xi'^3\phi' \\
& + \frac{\pi}{12} R^2(r_o^6 - r_i^6)\xi'^3\phi'^3 + [\frac{5}{7} R^2I + \frac{\pi}{4} (r_o^6 - r_i^6)]\xi'^2\phi'^2 \\
& + \frac{\pi}{24} R^2(r_o^6 - r_i^6)\xi'^2\phi'^4 + \frac{\pi}{6} (r_o^6 - r_i^6)\xi'\phi'^3 \\
& + \frac{\pi}{24} (r_o^6 - r_i^6)\phi'^4 \} \qquad (51)
\end{aligned}
$$

Potential energy of the load F is the following:

$$PE = FL\ c - \frac{F^2 L\ c^2}{AE} \tag{52}$$

Then the total energy of the drill string is the sum of Equations (51) and (52) and is given in Equation (53).

$$E_T = U + PE \tag{53}$$

The variables, ε, ξ', ϕ', within Equations (51) and (52) may now be replaced by their equivalents.

From Hooke's Law

$$\varepsilon = \frac{F}{AE}\ c \tag{54}$$

From the helically buckled shape of the drill string

$$\xi' = \frac{2\pi}{P}$$

From the angle of the twist-torsion relationship

$$\phi' = \frac{Tc}{GJ}\ .$$

For equilibrium of buckled drill string, total energy is minimized by using differential calculus,

$$\frac{d\,E_T}{d\,P} = 0$$

If the variables ε^4, ε^3, ϕ'^4, ϕ'^3 are neglected and each term of Equation 53 is divided by L, and the following approximation is made, after differentiation, Equation 53 then becomes Equation 55:

$$P \approx (P^2 \mp 4\pi^2 R^2)^{1/2} \approx (P^2 \mp 8\pi^2 R^2)^{1/2} \approx \ldots$$

$$\frac{4\pi^2 R^2}{P^3}\frac{F^2}{AE} - \frac{8\pi^2 R^2 F^2}{P^3\,AE} - \frac{4\pi^2}{P^3}(R^2 A+I)\frac{F^2}{A^2 E} - \frac{2\pi I}{P^2}\frac{T}{GJ}\frac{F^2}{A^2 E}$$

$$+ \frac{8I}{P^3}\frac{T^2}{G^2 J^2}\frac{F^2}{A^2 E} - \frac{6\pi^2 R^2 I}{P^3}\frac{T^2}{G^2 J^2}\frac{F^2}{A^2 E} - \frac{8\pi^3}{P^3}\left(\frac{5}{7}R^2 A+I\right)\frac{F}{A}$$

$$+ \frac{12\pi^2 R^2 I}{P^3}\frac{T^2}{G^2 J^2}\frac{F}{A} - \frac{4\pi I}{P^2}\frac{T}{GJ}\frac{F}{A} - \frac{4\pi^2 R^2 I}{P^3}\frac{T^2}{G^2 J^2}\frac{F}{A}$$

(Continued)

$$+ \frac{24\pi^3 R^2 I}{P^4} \frac{T}{GJ} \frac{F}{A} + \frac{4\pi^2 R^2}{P^3} F - \frac{64\pi^4}{P^5} \left[\frac{5}{14} R^4 A + \frac{17}{14} R^2 I \right.$$

$$\left. + \frac{\pi}{24} (r_o^6 - r_i^6)\right]$$

$$- \frac{64\pi^4}{P^5} \left[\frac{R^4 I}{2} + \frac{\pi}{24} R^2 (r_o^6 - r_i^6)\right] \frac{T^2}{G^2 J^2}$$

$$- \frac{24\pi^3}{P^4} \left[\frac{10}{7} R^2 I + \frac{5}{24} \pi (r_o^6 - r_i^6)\right] \frac{T}{GJ}$$

$$- \frac{8\pi^2}{P^3} \left[\frac{5}{7} R^2 I + \frac{\pi}{4} (r_o^6 - r_i^6)\right] \frac{T^2}{G^2 J^2} = 0 \qquad (55)$$

Finally, neglecting second order torsion terms and multiplying each term by $\frac{P^3}{4\pi^2}$, the minimized potential energy equation results.

$$\left[-2R^2 - \frac{I}{A} - \frac{IP}{2\pi A} \frac{T}{GJ}\right] \frac{F^2}{AE} + \left[R^2 \left(1 - \frac{10}{7} \pi\right) - \frac{2\pi I}{A}\right.$$

$$\left. - \frac{T}{GJ} \frac{I}{A} \left(6\pi R^2 - \frac{P}{A}\right)\right] F \qquad \text{(Continued)}$$

$$- \frac{8\pi^2 E}{P^2} \left[\frac{5}{7} R^4 A + \frac{17}{7} R^2 I + \frac{\pi}{12} (r_o^6 - r_i^6)\right]$$

$$- \frac{6\pi E}{P} \left[\frac{10}{7} R^2 I + \frac{5}{24} \pi (r_o^6 - r_i^6)\right] \frac{T}{GJ} = 0 \qquad (56)$$

It is shown on Tables 3 and 4 that neglecting F^2 term does not create any serious error. Then,

$$F = - \frac{\frac{8\pi^2 E}{P^2} \left[\frac{5}{7} R^4 A + \frac{17}{7} R^2 I + \frac{\pi}{12} (r_o^6 - r_i^6)\right] + \frac{6\pi E}{P} \left[\frac{10}{7} R^2 I + \frac{5}{24} \pi (r_o^6 - r_i^6)\right] \frac{T}{GJ}}{\left[R^2 (1 - \frac{10}{7} \pi) - \frac{2\pi I}{A} - \frac{T}{AG} (\frac{P}{2\pi} - \frac{3\pi R^2}{P})\right]} \qquad (57)$$

The minus sign is given, because of the direction of the load F.

TABLE 3

COMPARISON OF RESULTS FROM EQ. 56 AND EQ. 57
(5 in. , 19.5 ppf Drill-pipe)

RADIAL CLEARANCE (INCHES)	RESULTS FROM EQ. 56 (POUNDS)	RESULTS FROM EQ. 57 (POUNDS)	ERROR (%)
.50	24692.3	24688.5	.1512E-01
1.00	17521.4	17517.8	.2061E-01
1.50	16837.9	16837.0	.4917E-02
2.00	17582.8	17581.7	.6130E-02
2.50	18833.7	18835.4	-.8982E-02
3.00	20343.4	20342.7	.3191E-02
3.50	22011.1	22009.2	.8948E-02
4.00	23790.1	23789.1	.4193E-02
4.50	25657.8	25655.0	.1070E-01
5.00	27590.3	27588.3	.7246E-02
5.50	29577.9	29575.4	.8328E-02
6.00	31608.3	31605.9	.7464E-02
6.50	33676.2	33672.1	.1220E-01
7.00	35772.1	35767.8	.1200E-01
7.50	37893.0	37888.0	.1317E-01

TABLE 4

COMPARISON OF RESULTS FROM EQ. 56 AND EQ. 57
(8 in. , 147 ppf Drill-collar)

RADIAL CLEARANCE (INCHES)	RESULTS FROM EQ. 56 (POUNDS)	RESULTS FROM EQ. 57 (POUNDS)	ERROR (%)
.50	349716.7	349684.8	.9110E-02
1.00	212817.2	212836.2	-.8929E-02
1.50	181861.5	181877.9	-.9032E-02
2.00	176125.8	176099.5	.1494E-01
2.50	179744.3	179731.2	.7269E-02
3.00	187845.5	187832.3	.7013E-02
3.50	198463.1	198456.6	.3303E-02
4.00	210728.2	210700.7	.1305E-01
4.50	224099.1	224082.7	.7328E-02
5.00	238332.3	238313.2	.8020E-02
5.50	253220.4	253201.8	.7346E-02
6.00	268629.2	268614.7	.5396E-02
6.50	284490.5	284453.5	.1302E-01
7.00	300669.2	300643.2	.8632E-02
7.50	317168.8	317125.6	.1363E-01

NUMERICAL EXAMPLE

The following practical numerical example is presented for the purpose of demonstrating the two distinct analyses pertinent to post buckling. This example also gives a comparison between Analysis-I, which is consideration of strain energies because of different loadings separately, and Analysis-II, which is strain energy of combined loading.

Given:

Drill-pipe OD = 5"

Drill-pipe ID = 4.276"

Drill-pipe Weight = 19.5 ppf

Drill-collar OD = 8"

Drill-collar ID = 3"

Drill-collar Weight = 147 ppf

Hole Size = 12.5"

Curvature = 5 deg./100 ft.

Torque = 5000 lb-lt = 60000 lb-inch

Inclination Angle = 35°

1. Analysis - I (Extension of Lubinski et al.[6] work)

   a) Find the bending stress, σ_b

$\sigma_b = 218 * OD * C$

$\sigma_b = 218 * 5 * 5$

$\sigma_b = 5450$ psi

From the modified Goodman diagram[13], for zero axial tensile stress maximum bending stress of pipe is 20000 psi. Thus $\sigma_b = 5450$ psi is safe and there will be no danger for cyclic loading fatigue.

b) Find the length of pitch from Equation 56 of reference 8.

$$P = \left(\frac{2\pi^2 \, EDR}{\sigma_b}\right)^{1/2}$$

$$P = \left(\frac{2*\pi^2*30*10^6*5*3.75}{5450}\right)^{1/2}$$

$P = 1427.34" = 118.95$ ft.

c) Find the axial load, F (assume $L = L_N$) using Equation 40.

$$F = \frac{8\pi^2 EI}{P^2} - \frac{T^2}{GJ}\frac{L^2}{3L_N^2}\left[4\frac{(OD-ID)^2}{OD^2+ID^2} + 1\right]$$

$$F = \frac{8*\pi^2*30*10^6*14.27}{1427.34^2}$$

$$- \frac{6000^2}{8.57*10^6*28.54*3}\left[4\frac{(5-4.276)^2}{5^2+4.276^2}+1\right]$$

$F = 16586$ lb.

d) Find the contact load between the buckled pipe and the hole wall (neglect torsional term) using Equation 5 of reference 1.

$$F_{cont.} = \frac{RF^2}{2EI}$$

$$F_{cont.} = \frac{3.75 * 16586^2}{2*30*10^6*14.27}$$

$F_{cont.} = 1.204$ lb/in.

e) Find the curvature of the buckled pipe (neglect torsional term) using Equation 6 of reference 1.

$$C_z = \frac{RF}{2EI}$$

$$C_z = \frac{3.75*16586}{2*30*10^6*14.27}$$

$$C_z = 7.264 *10^{-5} \text{ rad/in.} = 4.995 \text{ deg/100 ft.}$$

f) Find the length of pipe between the top of the drill-collars and neutral plane.

$$F = W L_N \cos \alpha$$

$$L_N = \frac{F}{W \cos \alpha}$$

$$W = W_s + W_i - W_o$$

$$W = \frac{W_P}{12} + \frac{\delta i \ \pi \ ID^2}{4 * 231} - \frac{\delta o \ \pi \ OD^2}{4 * 231}$$

$$W = \frac{19.5}{12} + \frac{12.2*\pi*4.276^2}{4*231} - \frac{12.2*\pi*5^2}{4*231}$$

$$W = 1.346 \text{ lb./in.}$$

$$L_N = \frac{16586}{1.346 * \cos 35}$$

L_N = 15042.9 inches = 1253.58 ft.

g) Find the additional length of drill-collars to eliminate buckling.

$$L = \frac{F}{B * \cos \alpha * W_{oc}}$$

$$L = \frac{16586}{\left(1 - \frac{12.2}{65.44}\right) \cos 35 * 147}$$

L = 169.3 ft.

which means 6 more 8" x 3" drill collars must be added to prevent buckling.

h) Find the length change due to helical buckling and Hooke's Law effect.

Hooke's Law effect:

$$\Delta e_{HL} = \frac{L*F}{EA}$$

$$\Delta e_{HL} = \frac{15042.9*16586}{30*10^6*5.27}$$

$$\Delta e_{HL} = 1.58"$$

Helical buckling effect:

$$\Delta e_{HB} = \frac{r^2F^2}{8EIW} = \frac{3.75^2*16586^2}{8*30*10^6*14.27*1.346}$$

$$\Delta e_{HB} = .839"$$

Total length change

$$\Delta e_T = 1.58 + .839$$

$\Delta e_T = 2.42"$ shortening, which is negligible.

i) Find the torque necessary to create the same buckled shape without axial loading (assume $L = L_N$) using Equation 41.

$$T = \frac{3.7\ \pi EI}{P}$$

$$T = \frac{3.7*\pi*30*10^{6}*14.27}{1427.34}$$

$T = 3.486*10^{6}$ lb-in. $= 2.905*10^{5}$ lb-ft.

which is impossible in normal drilling and shows the torsional effect is negligible.

2. Analysis - II. (Extension of Paslay and Bogy[11] work)

a) Bending stress, σ_b = 5450 psi, same as Analysis I.

b) Pitch length, P = 1427.34" = 118.95 ft., same as Analysis I.

c) Axial load, F, is found by Eq. 57

$$F = -\frac{\frac{8\pi^2 E}{P^2}\left[\frac{5}{7}R^4A + \frac{17}{7}R^2I + \frac{\pi}{12}(r_o^6 - r_i^6)\right] + \frac{6\pi E}{P}\left[\frac{10}{7}R^2I + \frac{5}{24}\pi(r_o^6 - r_i^6)\right]\frac{T}{GJ}}{\left[R^2(1 - \frac{10}{7}\pi) - \frac{2\pi I}{A} - \frac{T}{AG}\left(\frac{P}{2\pi} - \frac{3\pi R^2}{P}\right)\right]}$$

F = 22822.72 lb.

$$L_N = \frac{F}{W \cos \alpha}$$

$$L_N = \frac{22822.72}{1.346 * \cos 35}$$

$$L_N = 20699.4 \text{ inches}$$

$$L_N = 1725 \text{ ft.}$$

g) Additional length of drill-collars to eliminate buckling.

$$L = \frac{F}{B^* \cos \alpha * W_{DC}}$$

$$L = \frac{228822.72}{1 - \frac{12.2}{65.45} \cos 35 * 147}$$

$$L = 232.97 \text{ ft.}$$

which means 8 more 8"*3" drill collars must be added to the bottom-hole assembly.

DISCUSSION OF RESULTS

The results from two distinct analyses of post buckling behavior of drill strings are presented in Figures 5 to 14. Figures 5-9 are plots of the axial load, required to keep the drill string in helically buckled form versus the radial clearance between the radii of the drill string and the hole. Required axial load decreases with the increase of radial clearance in Analysis-I. This behavior is similar to the behavior of critical buckling load given in reference 1. On the other hand, in Analysis-II, required axial load decreases up to a certain value of radial clearance, then begins to increase. This certain radial clearance value depends on the pipe geometry; cross-sectional area, and second and third cross-sectional moment of inertia of the strings. It varies from 1.25 in. to 2.25 in. for the strings investigated. This somewhat unusual behavior may be explained under the consideration of the constraint that the drill string is in contact with the wall of the hole during helical buckling. If this behavior is confirmed by an experiment, it will be possible to apply more weight on the bit than the weight calculated from Lubinski, et al. work.

In practical drilling, radial clearance range for 5" drill pipe is 2" - 7.5". As may be seen in Figure 5,

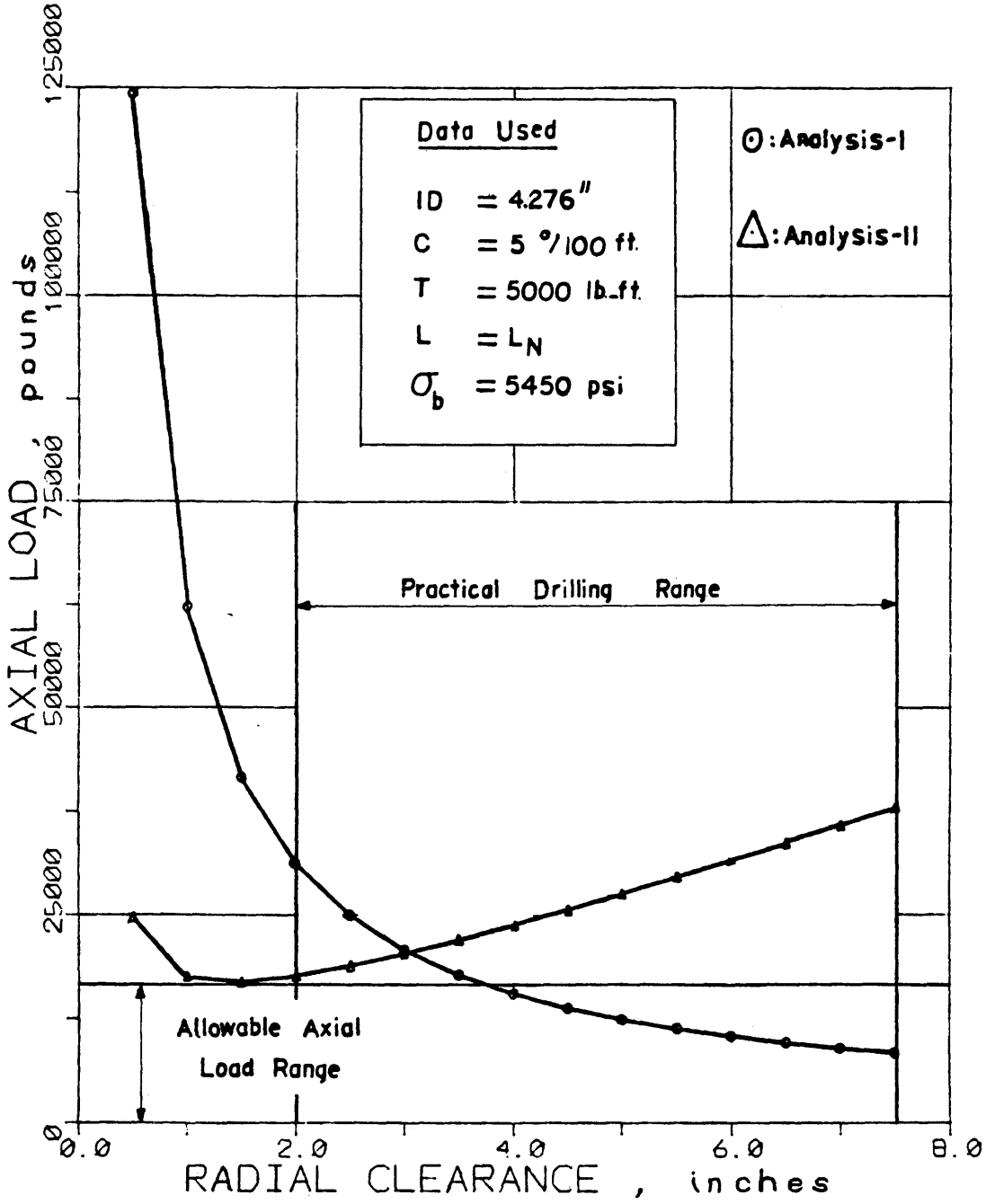

Figure 5. Allowable Axial Load Range - Analysis II 5 in., 19.50 ppf Drill-Pipe.

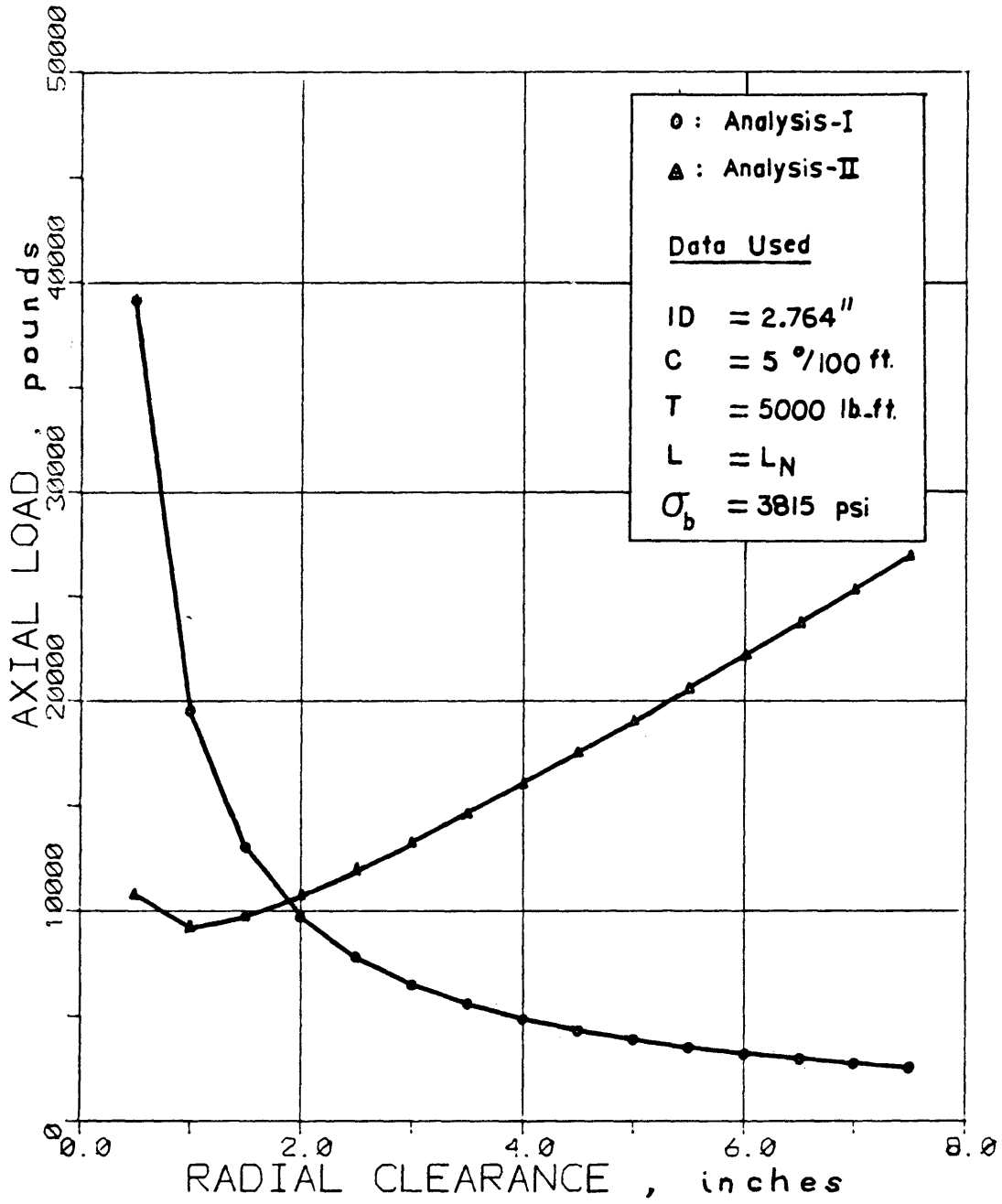

Figure 6. Effect of Radial Clearance on Axial Load 3.5 in., 13.30 ppf Drill-Pipe.

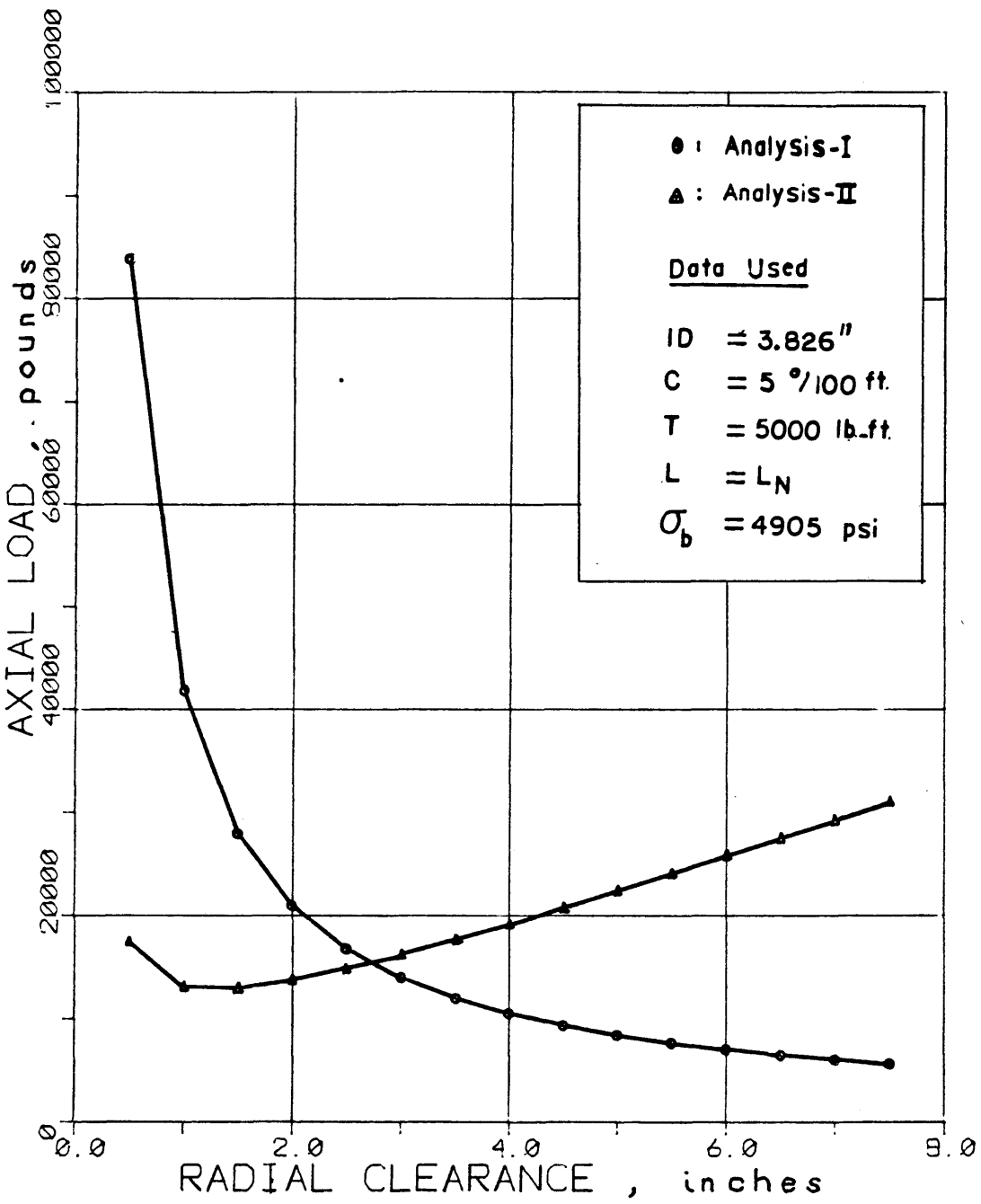

Figure 7. Effect of Radial Clearance on Axial Load 4.5 in., 16.60 ppf Drill-Pipe.

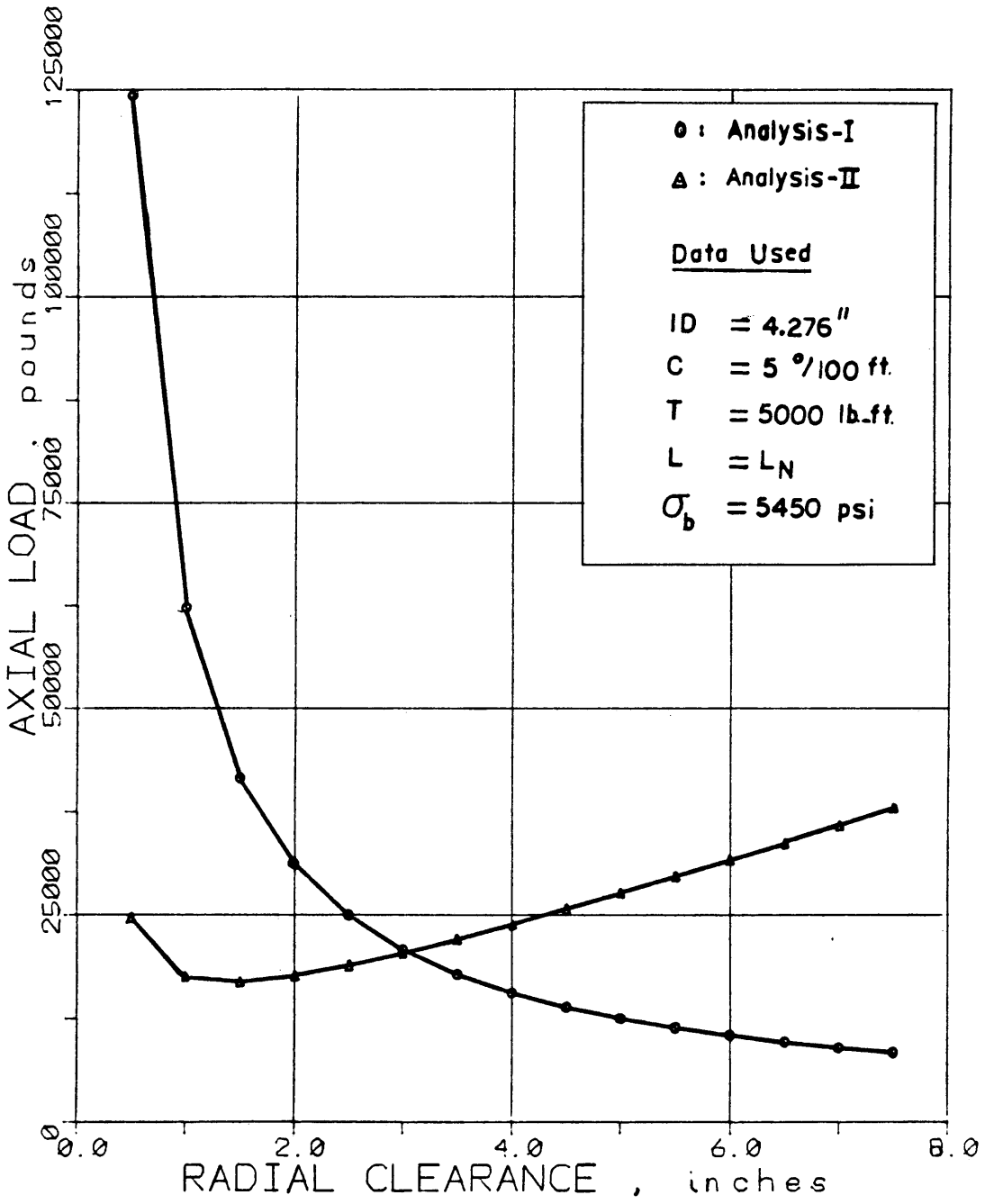

Figure 8. Effect of Radial Clearance on Axial Load 5 in., 19.50 ppf Drill-Pipe.

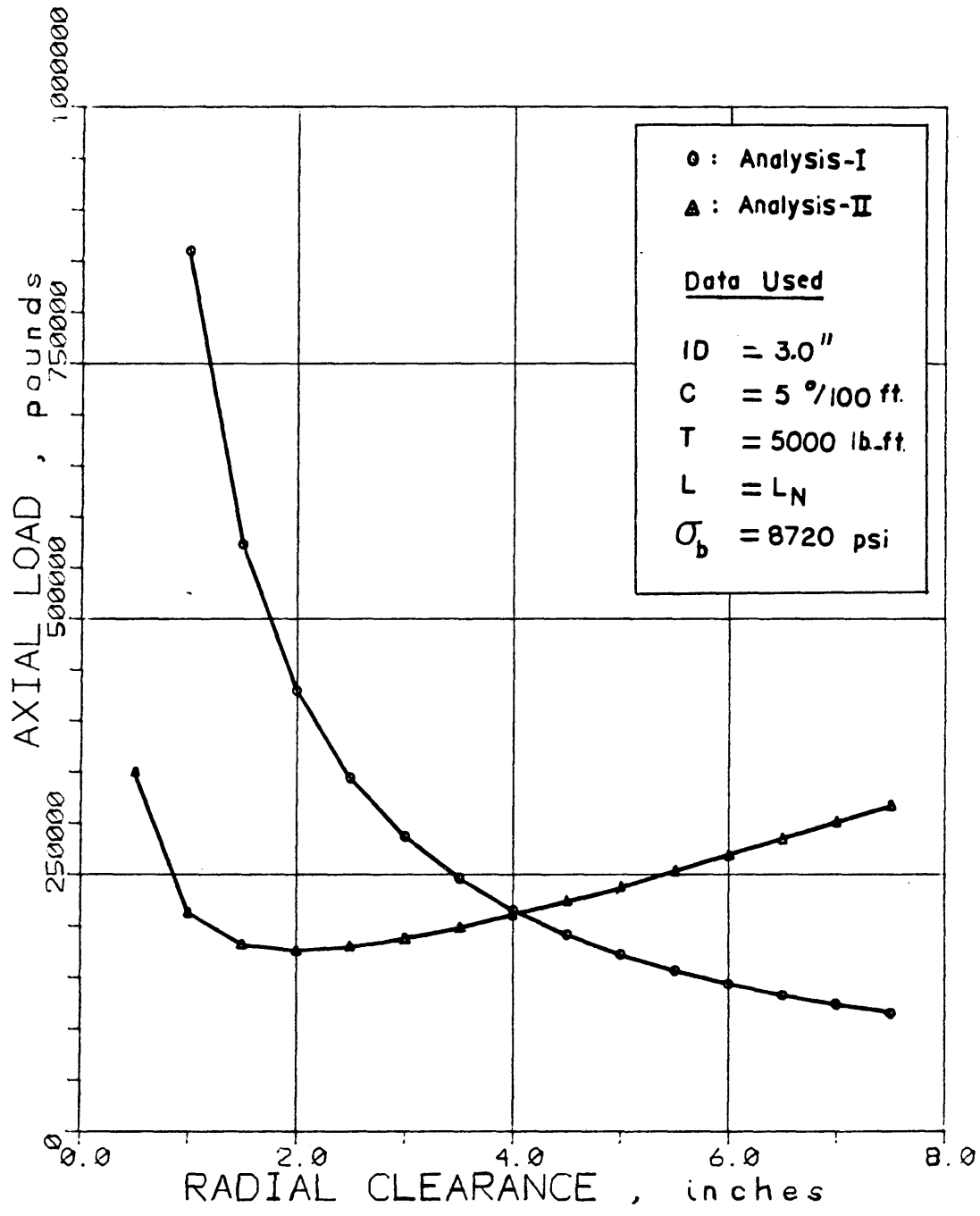

Figure 9. Effect of Radial Clearance on Axial Load 8 in., 147 ppf Drill-Collars.

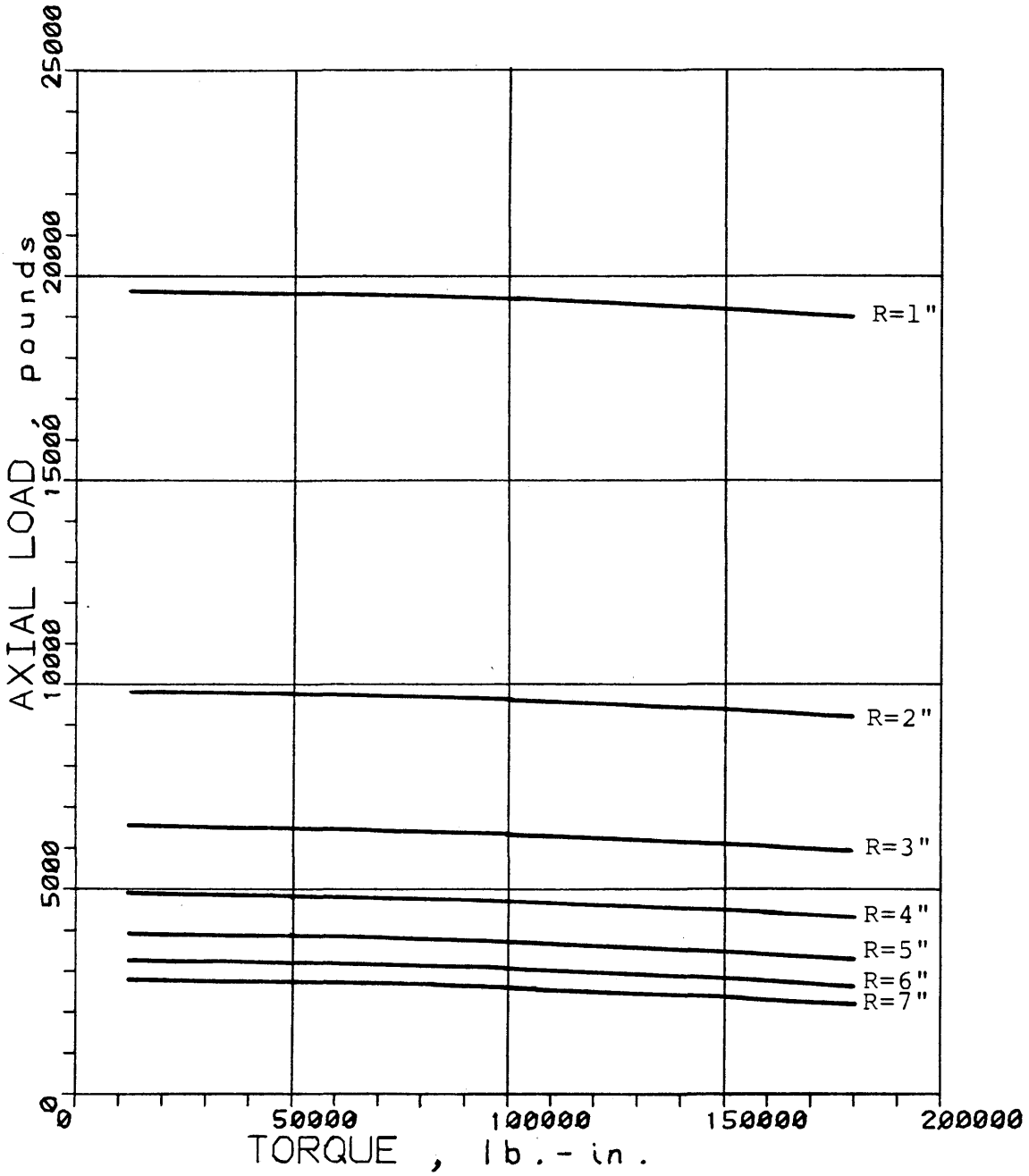

Figure 10. Effect of Torsion on Axial Load Analysis-I - 3.5 in., 13.30 ppf Drill-Pipe.

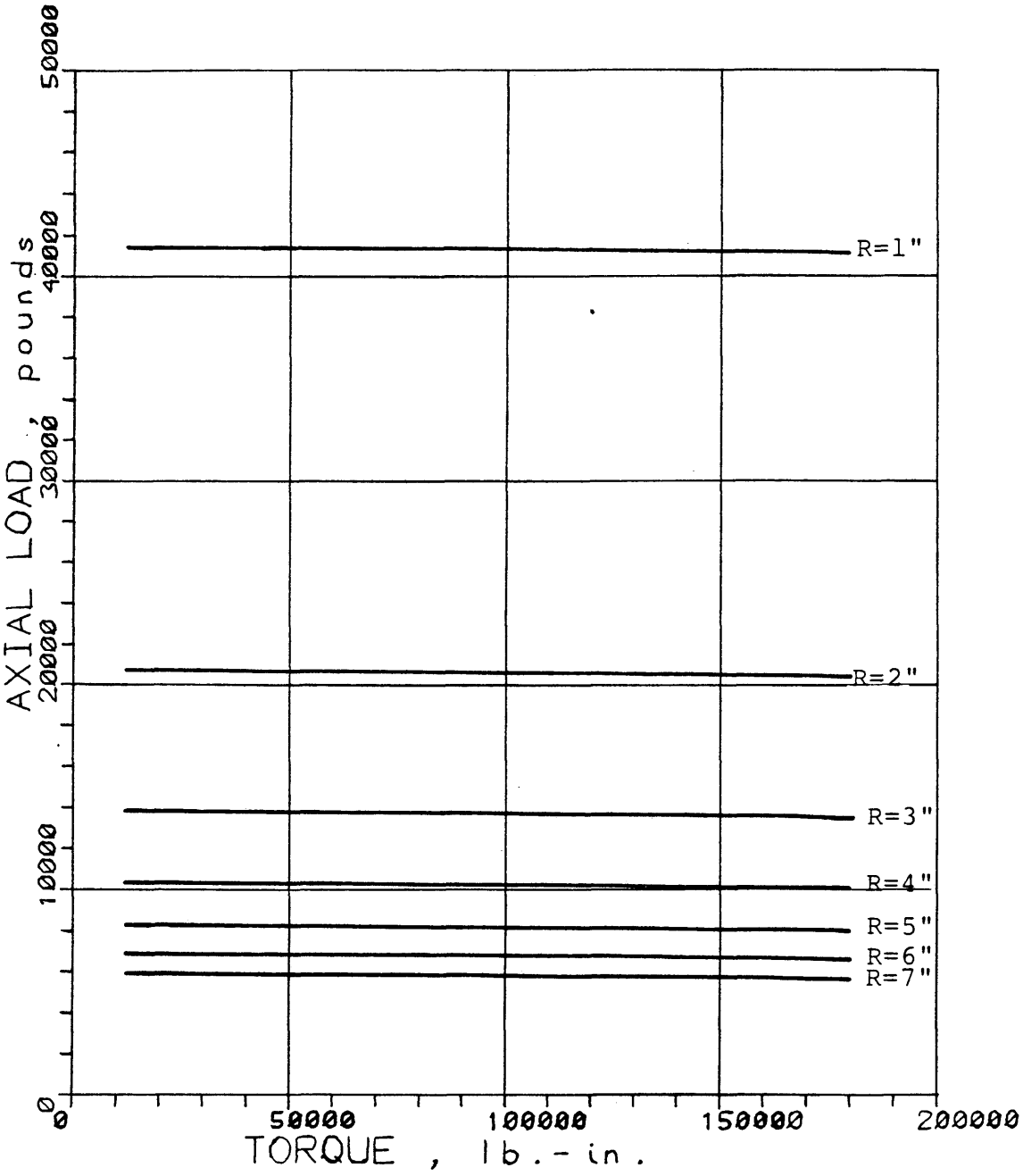

Figure 11. Effect of Torsion on Axial Load Analysis-I - 4.5 in., 16.60 ppf Drill-Pipe.

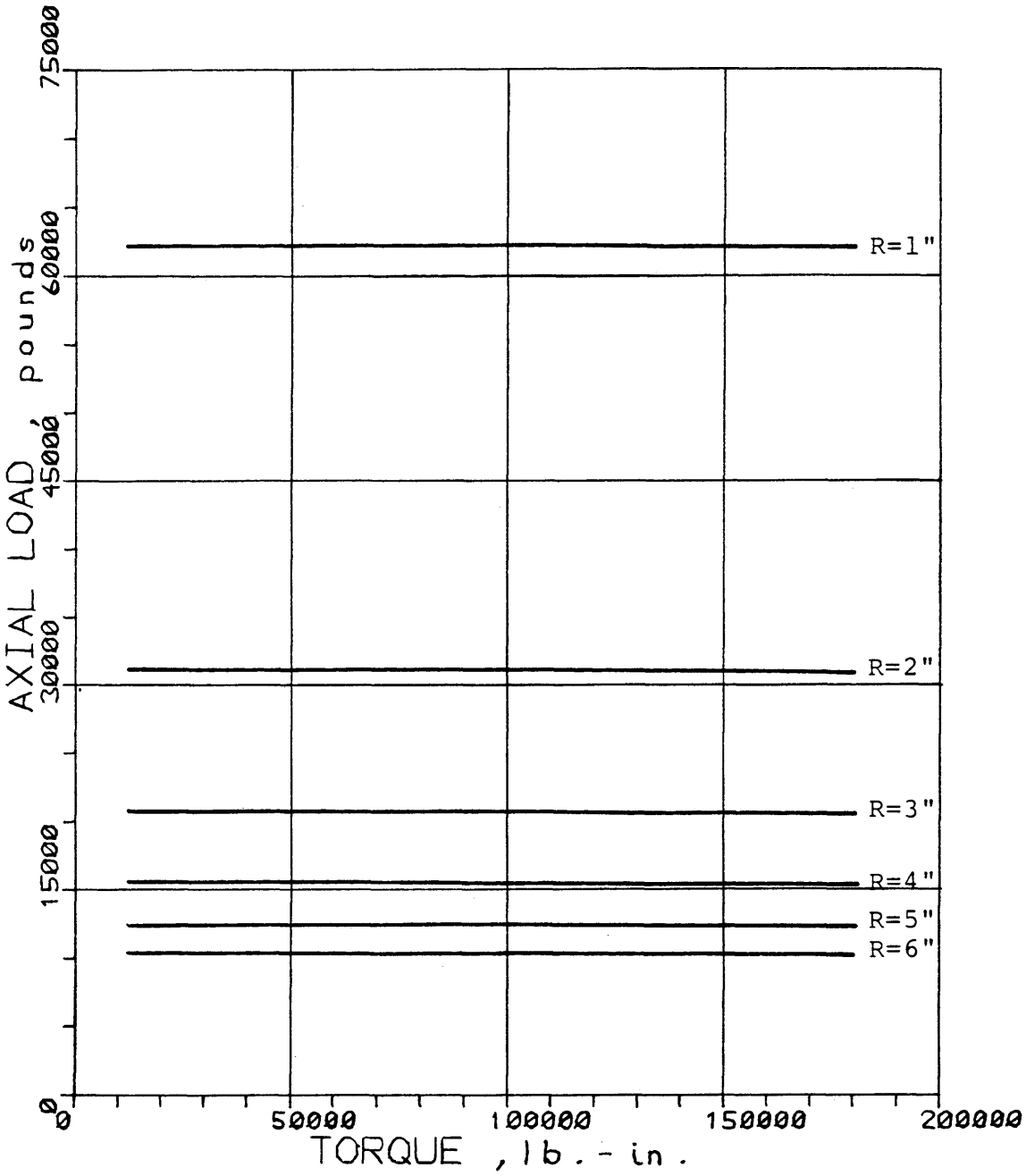

Figure 12. Effect of Torsion on Axial Load Analysis-I - 5 in., 19.50 ppf Drill-Pipe.

Analysis-II has a minimum of 17,000 pounds required axial load. Therefore loads below 17,000 pounds can be applied safely in practical drilling.

On the other hand for drill collars Analysis-II gives considerably lower amounts of axial loads than Analysis-I, in practical drilling range, which is 1" to 2".

Both analyses indicate that torsion and shear has little effect on post buckling behavior. Figures 10-12 show the torsion effect on the required axial load. Torsion tends to decrease the axial load. This tendency is more dominant in small diameter pipes. But, still, for practical purposes it may be neglected.

Figures 13 and 14 show the effect of torsion in Analysis-II. Torsion tends to increase the required axial load, which is contradictory to Analysis-I. Torsion terms do not contain only pure torsion as in Analysis-I, but combine bending and torsion. It may be concluded that torsion tends to increase required axial load, making the drill string more stable.

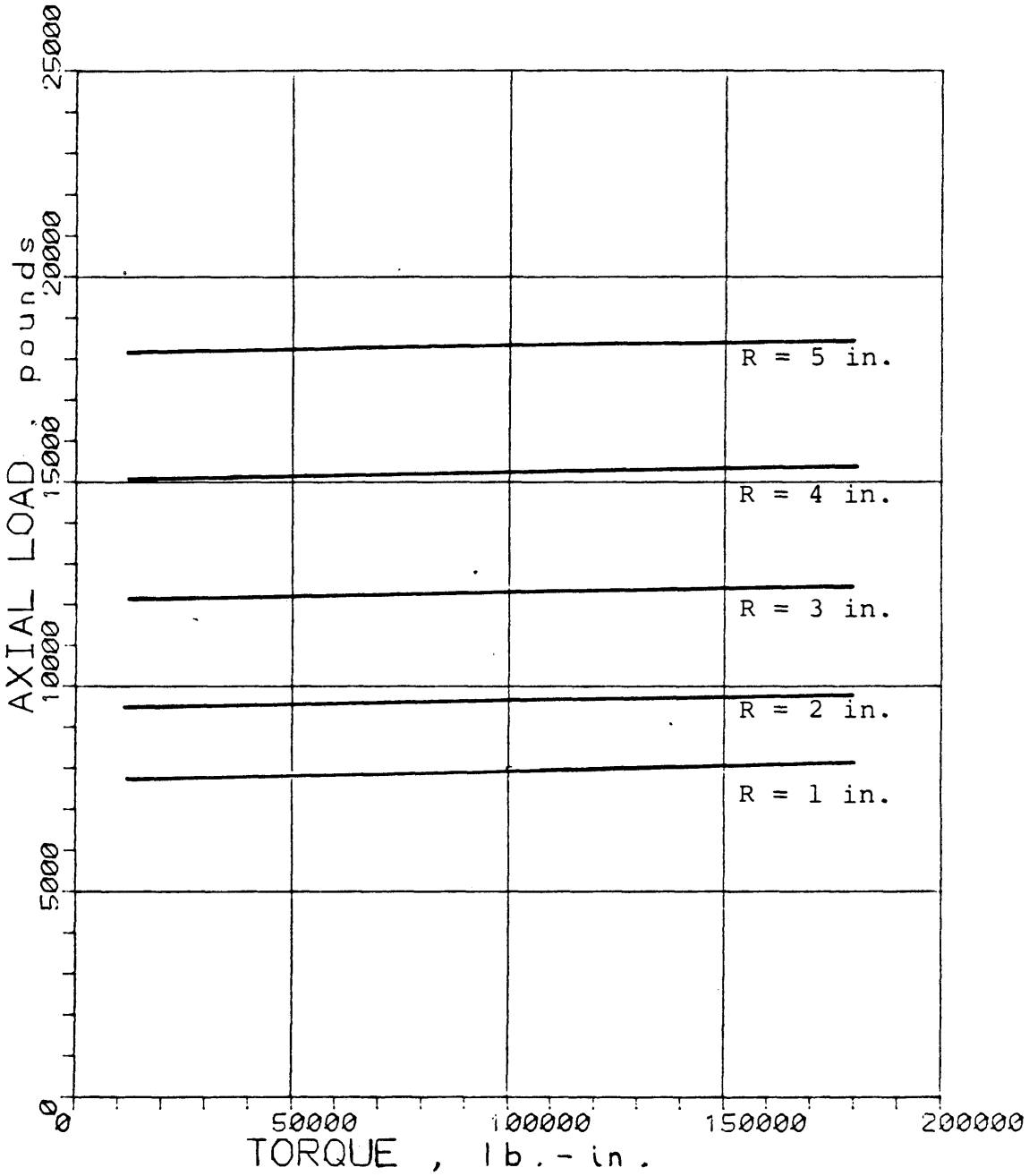

Figure 13. Effect of Torsion on Axial Load Analysis-II - 3.5 in., 13.30 ppf Drill-Pipe.

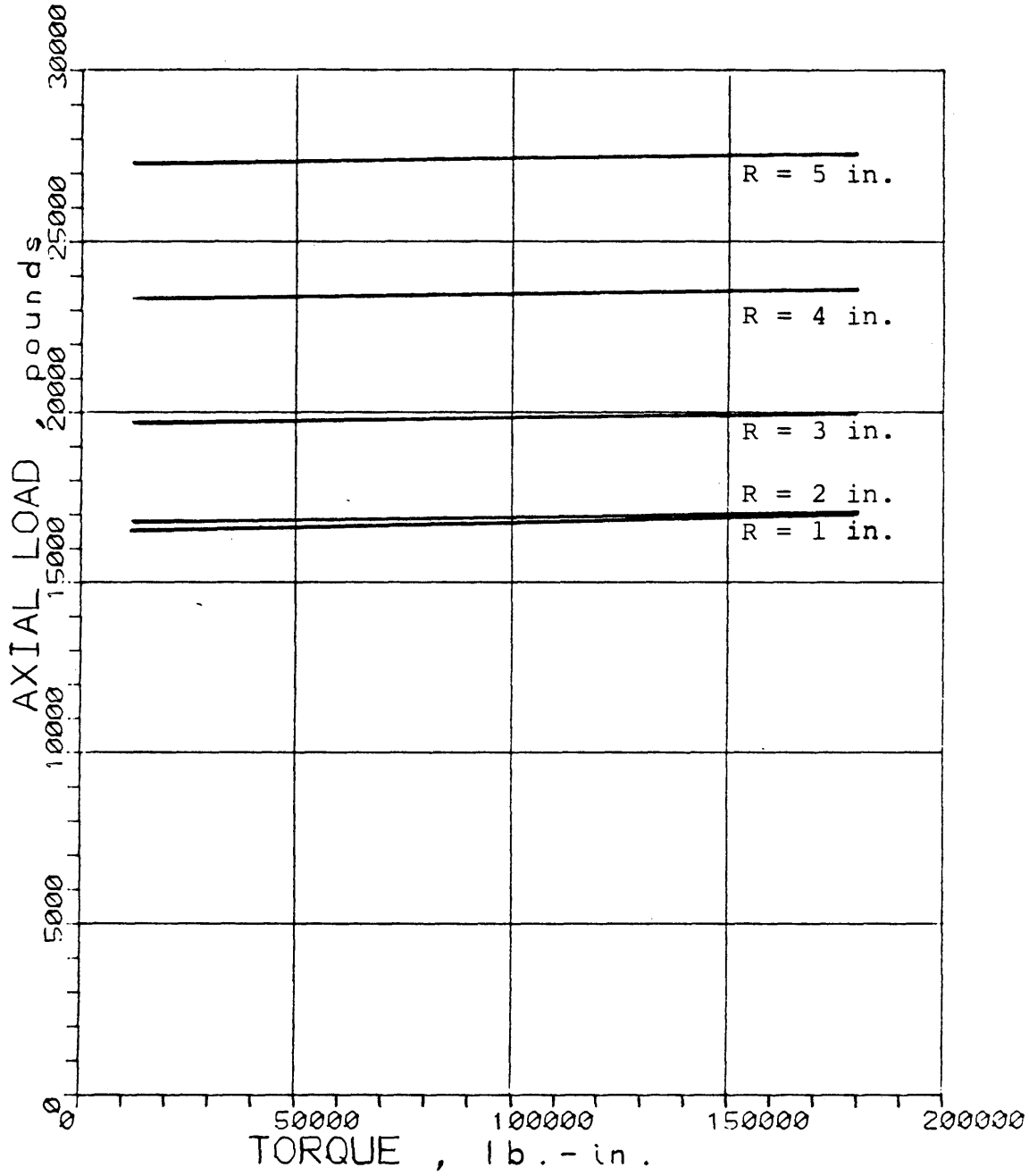

Figure 14. Effect of Torsion on Axial Load Analysis-II - 5 in., 19.50 ppf Drill-Pipe.

CONCLUSION AND RECOMMENDATIONS

1. Two models were successfully developed for the analyses of the post helical buckling of drill strings.

2. Torsion seems to have negligible effect on post helical buckling required loads.

3. For drill collars, Analysis-II predicts conservative results, while for drill pipes Analysis-I predicts smaller loads in practical drilling range. Therefore for a conservative approach, Analysis-II is recommended for drill collars, and Analysis-I is recommended for drill pipe strings.

4. An experiment should be devised to distinguish the validity of each model; i.e., the summing of discrete strain energies, or the combination of loads.

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APPENDIX - NOMENCLATURE

English Symbols

A	:	Cross-sectional area of pipe, sq. in.
B	:	Buoyancy Factor
c	:	Term given by $\frac{P}{(P^2 + 4\pi^2 R^2)^{1/2}}$, unitless
C_z	:	Curvature of helically buckled string, rad./in.
DLS	:	Dog leg severity, degrees/100 ft.
E	:	Young's modulus, $30*10^6$ psi for steel
E_T	:	Total energy, lb.-in.
F	:	Axial load in Analysis-I, pounds; required axial load in Analysis-II, pounds.
F_{cr}	:	Critical buckling load, pounds
G	:	Shear modulus, psi
I	:	Second moment of inertia, $in.^4$
ID	:	Inside diameter of pipe, in.
J	:	Polar moment of inertia, $in.^4$
k	:	Correction factor, unitless
L	:	Any length along the pipe, in.
L_N	:	Length from the top of drill-collars to the neutral plane, in.
M_b	:	Bending moment, lb.-in.
OD	:	Outside diameter of pipe, in.

P	:	Pitch length of helix, in.
PE	:	Potential energy due to external loading, lb.-in.
P_z	:	Total axial load acting on the string
R	:	Radial clearance, in.
r_i, r_o	:	Inside and outside radius, in.
$S_{xx}, S_{yy}, \ldots$	:	Stresses in the xx,yy,... directions, psi
T	:	Torque, lb.-in.
U	:	Strain energy of system, lb.-in.
U_A	:	Strain energy due to axial loading, lb.-in.
U_B	:	Strain energy due to bending, lb.-in.
U_S	:	Strain energy due to shearing, lb.-in.
U_T	:	Strain energy due to torsion, lb.-in.
V	:	Shear load, pounds
W	:	Buoyed weight of pipe, lb./in.
W_B	:	Weight on bit, pounds
W_{BHA}	:	Buoyed weight of bottom hole assembly, pounds
W_i	:	Weight of inside mud, lb./in.
W_o	:	Weight of outside mud, lb./in.
W_s	:	Weight of steel, lb./in.
W_z	:	Buoyed weight of the string at any location z
z	:	Axial coordinate

Greek Symbols

α	:	Inclination angle, degrees
Δe_{HL}	:	Length change caused by Hooke's Law effect, inches
Δe_{HB}	:	Length change caused by helical buckling, inches
δU	:	Virtual change in the elastic strain energy
δW_i	:	Virtual work of the internal forces
$\varepsilon_{xx}, \varepsilon_{yy}, \ldots$	:	Strain components in xx,yy,... directions, unitless.
ϕ	:	Angle of twist, degrees
λ	:	Lame' elastic coefficient, psi
ν	:	Poisson's ratio, unitless, .3 for steel
σ_b	:	Bending stress, psi
τ	:	Shear stress, psi
θ	:	Tangential coordinate
ξ	:	Angular position of helix pitch at any time