MATHEMATICAL MODELING OF A PETROLEUM REFINERY FOR OPTIMIZATION BY LINEAR PROGRAMMING TECHNIQUES

by

Trakarn Chairat

ER-1340

A Thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfullment of the requirements for the degree of Master of Engineering.

Signed: Trakarn Chairat
Trakarn Chairat

Golden, Colorado

Date: 14 January 1971

Approved: James H. Gary
Dr. James H. Gary
Thesis Advisor

James H. Gary
Dr. James H. Gary
Head, Department of
Chemical and Petroleum
Refining Engineering

Golden, Colorado

Date: 14 January, 1971

ABSTRACT

Mathematical modeling of a petroleum refinery is the set of linear equations which is formulated from the unit capacities, material balances, and product requirements of the integrated processes of the whole refinery.

The linear programming code was built by pivot operations technique to solve this set of equations for optimum profit. The optimal solution for the 110,000 BPD of a petroleum refinery is $159,101.52 per day by producing the products of fuel gas, propane, butane, jet fuel, kerosine, fuel oil, economy diesel, and coke.

ACKNOWLEDGMENTS

The writer wishes to express his sincere gratitude to his major professor, Dr. James H. Gary, for his guidance and much valuable advice during this study.

Appreciation is extended to Dr. Ardel J. Boes and Dr. John O. Golden for serving as committee members.

Special thanks are given to Mr. Keith R. Linck for his assistance in computer programming and techniques.

Financial support provided by the Thai Oil Refinery Company under supervision of the Ministry of Industry Bangkok, Thailand, is gratefully acknowledged.

CONTENTS

TABLES

FIGURES

INTRODUCTION

The main purpose of this study is to demonstrate the mathematical modeling of a petroleum refinery by using linear programming techniques. The example of a 100,000-barrel-per-day refinery is used to find the optimal solution by this method.

The crude oil from the Salt Creek Field, Natrona County, Wyoming, is used as refinery feed; and the refinery includes units of crude distillation, delayed coking, fluid catalytic cracking, platinum catalytic reformer, alkylation, and distillate hydrodesulfurization.

The volume percent yields(3,4,6,9) are formulated to the linear equations as the number of equality constraints. The material balances on the units are written by balancing the inputs and outputs from the unit. The formulations are set in between the limits as inequality constraints. The upper bounds are capacity limits on the units, and the lower bounds are the economic minimum for the operation of the units. The operating costs are approximated as linear function of the flow through the units for this model.

The addition of slack and artificial variables(8) are introduced by the computer program into the constraints. The problem uses Phase I to obtain the initial basic feasible solution in terms of slack and real variables (no artificial variables) and as Phase II the obtaining of the optimal solution in terms of those same variables. A series of pivot operations are used to solve the set of linear equations. The optimal solution is obtained from the Linear Programming (LP) techniques by maximizing the profit as the objective function. The validity of the optimality is tested by the sensitivity analysis to the non-basic elements of the cost row, right-hand side, and non-basic constraint coefficients.

FIGURE 1

Simplified Refinery Operations

(1) Crude Distillation Unit

(2) Delayed Coking Unit

(3) Fluid Catalytic Cracking

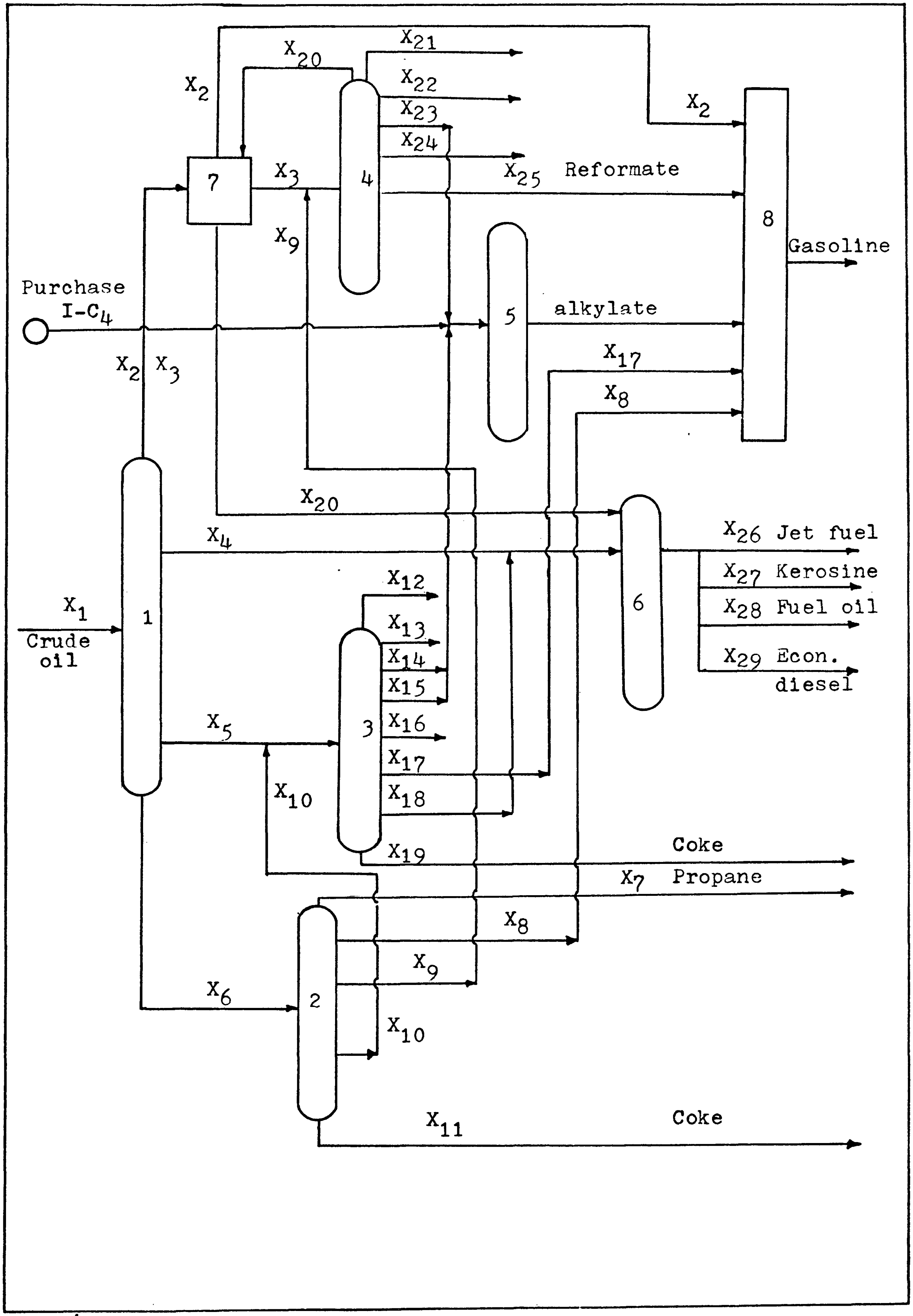
(4) Platinum Catalytic Reformer Unit

(5) Alkylation Unit

(6) Distillate Hydrodesulfurization Unit

(7) Pre-treating Unit

(8) Gasoline Pool


X2
X20
X21
X22
X23
X24
X25 Reformate
X3
7
4
8
Gasoline
X9
Purchase I-C4
5
alkylate
X17
X8
X2 X3
X20
X4
6
X26 Jet fuel
X27 Kerosine
X28 Fuel oil
X29 Econ. diesel
X1
Crude oil
1
X12
X13
X14
X15
3
X16
X5
X17
X10
X18
X19
Coke
X7 Propane
X8
X9
X6
2
X10
X11
Coke

PROCESS APPROACH

The refining of crude petroleum for the production of specification products is a complex process. A simplified diagram of the usual refinery operations for the fuel products is shown in figure 1.

One hundred thousand barrels per day of crude oil is fractionated by the crude distillation unit, into a number of intermediate products requiring further processing. The light naphtha requires only treating and is carried to the gasoline pool. The heavy naphtha is also treated and then sent to the platinum catalytic reformer where it is included with naphtha from delayed coking as feed stock to the platinum catalytic reformer. Here its octane rating is improved. The reformate is then sent to the gasoline pool. The production of 350-400°F., kerosine distillate, diesel, and the production of 550-650°F. is sent to the distillate hydrodesulfurization unit to remove sulfur, nitrogen, unsaturated hydrocarbons, and metals. The products are jet fuel, kerosine, fuel oil, and economy diesel.

Since a large amount of residium is drawn from the crude tower, a delayed coking unit is included. A delayed coking unit is used because it favors maximum yields of gas oils. The products from the delayed coking unit are propane, gasoline, naphtha, coker gas oil and coke.

Atmospheric gas oil and vacuum gas oil from crude distillation unit and coker gas oil from the delayed coking unit go to the fluid catalytic cracking (FCC) unit for conversion into catalytic cracked gasoline. Light and heavy cycle oils from the FCC are sent to the distillate hydrodesulfurization unit.

The propene, butene, and iso-butane from the FCC and platinum catalytic reformer is sent to the alkylation unit to produce a large iso-paraffin. Alkylate goes to gasoline pool. The n-butane is used to blend the gasoline to increase the octane number. The final products are fuel gas, propane, gasoline, jet fuel, kerosine, fuel oil, economy diesel, and coke.

LITERATURE SURVEY

CRUDE DISTILLATION

Crude petroleum is refined to increase its value. The whole crude is charged to a distillation tower commonly referred to as the "atmospheric tower" due to it's operating pressure of 0 to 5 psig. In the early stages of petroleum refining, products were separated into boiling ranges by distillation or fractionation. Fractionation is still the basic process in refining even though such processes as thermal and catalytic cracking, alkylation, reforming, and hydrotreating have been developed to upgrade low priced petroleum fractions into more valuable products.

DELAYED COKING

Delayed coking is the process for converting residuals to lighter, more valuable stocks. The heavy residue passes from the bottom of fractionator to a furnace where it acquires the heat of the cracking. The heated residue is introduced into an insulated drum where the residue time is sufficient for coke to form.

The coke builts up in the coke drums. A full coke drum is removed from the process flow, steamed to strip light hydrocarbons from the coke, and cooled by water injection. More recent designs use high pressure (over 1,000 psig) water jets to cut the coke from the drum.

Table 1 Crude Distillation Material Balances

Feed: Crude oil 100,000 BPD* = X_1

COMPONENT	BOILING RANGE °F.	VOL. PERCENT	BPD
Gas	-	-	-
Lt. Gasoline	100-212	7.8	X_2
Hvy. Gasoline	212-350	16.4	X_3
	350-400	5.8	X_4
Kerosine Dist.	400-500	11.0	
Diesel	500-550	5.5	
	550-650	2.6	
Atm. Gas Oil	550-650	7.9	X_5
Vac. Gas Oil	650-850	21.8	
Residuum	850-up	21.2	X_6

Table 2 Delayed Coking Material Balances

Feed: Residuum X_6 BPD

COMPONENT	VOL. PERCENT	BPD
Propane	10.11	X_7
C_4-400 F Gasoline	24.64	X_8
Naphtha	19.55	X_9
Coker Gas Oil	70.80	X_{10}
Coke	9.84	X_{11}

FLUID CATALYTIC CRACKING

Fluid catalytic cracking (FCC) is the major unit used for converting high-boiling fractions into gasoline. A moving bed of catalyst is used to convert heavy gas oil to gasoline, light olefins and liquified petroleum gases. In a FCC unit, paraffin and naphthene molecules are cracked, and condensed aromatic molecules either form coke or are unconverted and go into cycle oils. The catalyst continually circulates between the reactor and a regenerator where the coke is burned off. Gasoline from FCC is high in olefin content, which are results in high sensitivity and a good response to the addition of lead. The gasoline will also contain a reasonably high concentration of aromatics, resulting from single-ring and double-ring compounds that have had the side chain cracked off. The catalyst also promotes isomerization of paraffins for octane improvement.

PLATINUM CATALYTIC REFORMER

Platinum catalytic reformer is used to increase octane numbers or to produce aromatics from straight run, hydrocracked naphthas and naphtha gasoline from delayed coking. The catalyst, which contains platinum, promotes ring formation, isomerization, and dehydrogenation. Products from catalytic reforming operations are excellent stocks (reformate) for blending into gasoline, usually without any further treatment. They are stable, high octane, and low in sulfur and gum. They respond well to the addition of tetraethyl-lead.

Table 3 FCC Material Balances

Feed: Gas Oil(from crude unit) X_5 BPD

Coker Gas Oil X_{10} BPD

COMPONENT	VOL. PERCENT	BPD
Dry gas	5.42	X_{12}
Propane	2.60	X_{13}
Propene	7.00	X_{14}
Butene	11.00	
Iso-butane	8.10	X_{15}
N-butane	1.40	X_{16}
Cracked Gasoline	71.10	X_{17}
Lt. Cycle Oil	7.00	X_{18}
Hvy. Cycle Oil	3.00	
Coke	5.50	X_{19}

Table 4 Platinum Catalytic Reformer Material Balances

Feed: Heavy Gasoline (from crude unit) X_3 BPD

Naphtha (from coking unit) X_9 BPD

COMPONENT	VOL. PERCENT	BPD
Hydrogen	11.85	X_{20}
Dry gas	3.70	X_{21}
Propane	3.81	X_{22}
Iso-butane	2.30	X_{23}

(Table 4 continued)

COMPONENT	VOL. PERCENT	BPD
N-butane	3.30	X_{24}
Reformate	87.20	X_{25}

ALKYLATION

Alkylation is any reaction in which an alkyl group is added to a compound; in petroleum refining, it normally means reaction of an olefin with an iso-paraffins to produce a larger iso-paraffin.

Alkylation improves the gasoline yield structure of a refinery by combining gaseous feeds into liquid products. The process combines an iso-paraffin, usually iso-butane, with olefins such as propene, butene, or amylene. The resulting product is a gasoline component with very good stability in addition to its high octane ratings.

Table 5 Alkylation Material Balances

Feed:	Propene and Butene (from FCC)	X_{14} BPD
	Iso-butane (from FCC)	X_{15} BPD
	Iso-butane (from Reformer)	X_{23} BPD

Propene and butene mixed feed per barrel olefin converted: 1.76 barrels for alkylate produced and 1.24 barrels for iso-butane consumed(4). Therefore:

$$\text{Alkylate produced} = 1.76X_{14} \quad \text{BPD}$$

$$\text{Iso-butane consumed} = 1.24(X_{14}) \quad \text{BPD}$$

DISTILLATE HYDRODESULFURIZATION

Distillate hydrodesulfurization is used to improve qualities of a wide range of petroleum stocks by removal of sulfur, nitrogen, and heavy metallic contaminants. The process hydrogenates olefinic hydrocarbons, improves color, odor, and stability of these petroleum cuts. Reaction products, such as hydrogen sulfide, ammonia, water, and light hydrocarbons build up to an equilibrium concentration in the hydrogen recycle gas.

Table 6 Distillate Hydrodesulfurization Material Balances

Feed: Products of 350-650°F.(from crude unit) X_4 BPD*

Cycle Oils (from FCC) X_{18} BPD

COMPONENT	VOL. PERCENT	BPD
Jet fuel	42.4	X_{26}
Kerosine	20.0	X_{27}
Fuel oil	20.5	X_{28}
Economy diesel	17.1	X_{29}

* barrel per day

STUDY PROBLEM

The study of the petroleum refinery is concerned with maximizing profit. The profit is composed of the following items:

Profit = Sale of products - Cost of crude oil - operating expenses.

The limitations to the operations are set by the requirements for the products and the capacities for the processing. Thus:

Minimum Capacity	$\leq$	Crude rate	$\leq$	Maximum Capacity
Minimum Capacity	$\leq$	Delayed Coking	$\leq$	Maximum Capacity
Minimum Capacity	$\leq$	FCC	$\leq$	Maximum Capacity
Minimum Capacity	$\leq$	Reformer	$\leq$	Maximum Capacity
Minimum Capacity	$\leq$	Alkylation	$\leq$	Maximum Capacity
Minimum Capacity	$\leq$	Dist. Hydrodesulfurization	$\leq$	Maximum Capacity
Minimum Requirement	$\leq$	Gasoline	$\leq$	Maximum Requirement
Minimum Requirement	$\leq$	Jet fuel	$\leq$	Maximum Requirement

The crude rate is limited between the minimum practical operation without shutting down and the maximum operation as limited by equipment capacity. The delayed coking, FCC, platinum catalytic reformer, alkylation, and distillate hydrodesulfurization, units are limited by the maximum and minimum capacities. The products, gasoline and jet fuel, are limited by the maximum and minimum requirements.

The constraints consist of the individual component balances stated for the units and the overall balances over each unit are included as the equality constraints.

MATHEMATICAL MODELING

ILLUSTRATIVE PROBLEM

The actual refinery processing involves the following constraints for quantities:

(1) Crude Distillation Unit

From Table 1, "Crude Distillation Material Balances", the crude rate is set into the maximum and minimum capacities of the unit as:

$$X_1 \leq 110{,}000 \qquad (1)$$

$$X_1 \geq 100{,}000 \qquad (2)$$

X_1 is represent the crude rate. The equality equations are formulated into the linear equations from the material balances as:

$$-0.078X_1 + X_2 = 0 \qquad (3)$$

$$-0.164X_1 + X_3 = 0 \qquad (4)$$

$$-0.249X_1 + X_4 = 0 \qquad (5)$$

$$-0.297X_1 + X_5 = 0 \qquad (6)$$

$$-0.212X_1 + X_6 = 0 \qquad (7)$$

This set of equations condense into one equation as overall balances:

overall balances:

$$- X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 0 \quad (8)$$

X_2 to X_6 represent the products from the crude distillation unit.

(2) Delayed Coking Unit

From Table 2, "Delayed Coking Material Balances", the feed stock is set into the maximum and minimum capacities of the unit as:

$$X_6 \leq 25{,}000 \quad (9)$$

$$X_6 \geq 21{,}200 \quad (10)$$

X_6 represents the feed stock to the delayed coking unit. The equality equations are:

$$-0.1011X_6 + X_7 = 0 \quad (11)$$

$$-0.2464X_6 + X_8 = 0 \quad (12)$$

$$-0.1955X_6 + X_9 = 0 \quad (13)$$

$$-0.7080X_6 + X_{10} = 0 \quad (14)$$

$$-0.0984X_6 + X_{11} = 0 \quad (15)$$

Therefore, the overall material balances equation is:

$$-1.3494X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} = 0 \quad (16)$$

X_7 to X_{11} represent the products from the delayed coking unit.

(3) Fluid Catalytic Cracking

From Table 3, "FCC Material Balances", the feed stocks obtained from crude unit and delayed coking unit are set into the maximum and minimum capacities of the unit as:

$$X_5 + X_{10} \leq 60{,}000 \qquad (17)$$

$$X_5 + X_{10} \geq 40{,}000 \qquad (18)$$

X_5 and X_{10} represent the feed stocks to the FCC unit. The equality equations are:

$$-0.0542(X_5 + X_{10}) + X_{12} = 0 \qquad (19)$$

$$-0.0260(X_5 + X_{10}) + X_{13} = 0 \qquad (20)$$

$$-0.1800(X_5 + X_{10}) + X_{14} = 0 \qquad (21)$$

$$-0.0810(X_5 + X_{10}) + X_{15} = 0 \qquad (22)$$

$$-0.0140(X_5 + X_{10}) + X_{16} = 0 \qquad (23)$$

$$-0.7110(X_5 + X_{10}) + X_{17} = 0 \qquad (24)$$

$$-0.1000(X_5 + X_{10}) + X_{18} = 0 \qquad (25)$$

$$-0.0550(X_5 + X_{10}) + X_{19} = 0 \qquad (26)$$

Therefore, the overall material balances equation is:

$$-1.2212(X_5 + X_{10}) + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} + X_{18} + X_{19} = 0 \qquad (27)$$

X_{12} to X_{19} represent the products from the FCC unit.

(4) Platinum Catalytic Reformer Unit

From Table 4, "Platinum Catalytic Reformer Material Balances", the feed stocks obtained from crude unit and delayed coking unit are set into the maximum and minimum capacities of the unit as:

$$X_3 + X_9 \leq 30,000 \qquad (28)$$

$$X_3 + X_9 \geq 20,000 \qquad (29)$$

X_3 and X_9 represent the feed stocks to the platinum catalytic reformer unit. The equality equations are:

$$-0.1185(X_3 + X_9) + X_{20} = 0 \qquad (30)$$

$$-0.0370(X_3 + X_9) + X_{21} = 0 \qquad (31)$$

$$-0.0381(X_3 + X_9) + X_{22} = 0 \qquad (32)$$

$$-0.0230(X_3 + X_9) + X_{23} = 0 \qquad (33)$$

$$-0.0330(X_3 + X_9) + X_{24} = 0 \qquad (34)$$

$$-0.8720(X_3 + X_9) + X_{25} = 0 \qquad (35)$$

Therefore, the overall material balances equation is:

$$-1.1216(X_3 + X_9) + X_{20} + X_{21} + X_{22} + X_{23} + X_{24} + X_{25} = 0 \qquad (36)$$

X_{20} to X_{25} represent the products from the platinum catalytic reformer unit.

(5) Alkylation Unit

From Table 5, "Alkylation Material Balances", the feed stocks obtained from the FCC and platinum catalytic reformer, units are set into the maximum and minimum capacities of the unit as:

$$X_{14} \leq 9,000 \quad (37)$$

$$X_{14} \geq 8,000 \quad (38)$$

$$X_{15} + X_{23} \leq 5,000 \quad (39)$$

$$X_{15} + X_{23} \geq 4,000 \quad (40)$$

X_{14}, X_{15}, and X_{23} represent the feed stocks to the alkylation unit. The product is gasoline (alkylate) which will be calculated after knowing the value of X_{14}, X_{15}, and X_{23}.

(6) Distillate Hydrodesulfurization Unit

From Table 6 "Distillate Hydrodesulfurization Material Balances", the feed stocks obtained from the crude unit and the FCC unit are set into the maximum and minimum capacities of the unit as:

$$X_4 + X_{18} \leq 35,000 \quad (41)$$

$$X_4 + X_{18} \geq 29,000 \quad (42)$$

X_4 and X_{18} represent the feed stocks to the distillate hydrodesulfurization unit. The equality equations are:

$$-0.424(X_4 + X_{18}) + X_{26} = 0 \quad (43)$$

$$-0.200(X_4 + X_{18}) + X_{27} = 0 \quad (44)$$

$$-0.205(X_4 + X_{18}) + X_{28} = 0 \quad (45)$$

$$-0.171(X_4 + X_{18}) + X_{29} = 0 \quad (46)$$

Therefore, the overall material balances equation is:

$$-(X_4 + X_{18}) + X_{26} + X_{27} + X_{28} + X_{29} = 0 \qquad (47)$$

X_{26} to X_{29} represent the products from the distillate hydrodesulfurization unit.

The gasoline and the jet fuel are considered to meet the maximum and minimum requirements of the market, so that the constraints are included:

$$X_2 + X_8 + X_{17} + X_{25} \leq 72{,}000 \qquad (48)$$

$$X_2 + X_8 + X_{17} + X_{25} \geq 62{,}000 \qquad (49)$$

$$X_{26} \leq 20{,}000 \qquad (50)$$

$$X_{26} \geq 12{,}500 \qquad (51)$$

In this problem there are fifty-one constraints and twenty-nine variables. It is next necessary to develop the objective function. In linear programming, the cost row (C_j) values are the unit profits associated with each X_j and these value are determined for the problem as follows:

DEVELOPMENT OF THE OBJECTIVE FUNCTION

It is advisable to set up a profit and loss balances in accordance with the general procedure:

Profit(Z) = Sale of Products - Cost of Crude Oil

- Operating Expenses.

Table 7 LIST PRICES OF A PETROLEUM REFINERY

OPERATING COSTS		PRODUCT PRICES	
UNIT	$/BBL	STOCK	$/BBL
Crude	0.3082	Fuel gas	1.42
Delayed Coking	0.2200	Propane	1.70
FCC	0.3818	Propene	1.55
Reformer	0.3362	Iso-butane	2.52
Alkylation	0.5017	N-butane	1.95
Distillate Hydro-		Gasoline	5.04
desulfurization	0.2800	Jet fuel	4.326
		Kerosine	4.116
		Fuel oil	3.864
		Economy diesel	3.54
		Coke	1.26
Crude oil cost	2.9200	Hydrogen	0.50

Note: The sources of these prices are the preliminary papers (9,10) and the Oil & Gas Journal (7). In the linear program, these prices can be changed very easily.

Sale of Products

Fuel gas $= 1.42(X_{12} + X_{21})$

Propane $= 1.70(X_7 + X_{13} + X_{22})$

Propene,Butene $= 1.55X_{14}$

Iso-butane $= 2.52(X_{15} + X_{23})$

N-butane $= 1.95(X_{16} + X_{24})$

Sale of Products

Gasoline	=	$5.04(X_2 + X_8 + X_{17} + X_{25})$
Jet fuel	=	$4.326X_{26}$
Kerosine	=	$4.116X_{27}$
Fuel oil	=	$3.864X_{28}$
Economy diesel	=	$3.54X_{29}$
Coke	=	$1.26(X_{11} + X_{19})$
Hydrogen	=	$0.50X_{20}$

Cost of Crude Oil

Crude oil	=	$2.92X_1$

Operating Expenses

Crude unit	=	$0.3082X_1$
Delayed Coking	=	$0.22X_6$
FCC	=	$0.3818(X_5 + X_{10})$
Reformer	=	$0.3362(X_3 + X_9)$
Distillate Hydro-desulfurization	=	$0.28(X_4 + X_{18})$

The profit function may now be rearranged in term of the problem variables:

Profit(Z) = Sale of Products - Cost of Crude Oil
- Operating Expenses.

$$\begin{aligned}\text{Profit}(Z) = & -3.2282X_1 + 5.04X_2 - 0.3362X_3 - 0.28X_4 \\ & -0.3818X_5 - 0.22X_6 + 1.70X_7 + 5.04X_8 - 0.3362X_9 \\ & -0.3818X_{10} + 1.26X_{11} + 1.42X_{12} + 1.70X_{13} \\ & +1.55X_{14} + 2.52X_{15} + 1.95X_{16} + 5.04X_{17} - 0.28X_{18} \\ & + 1.26X_{19} + 0.50X_{20} + 1.42X_{21} + 1.70X_{22} \\ & + 2.52X_{23} + 1.95X_{24} + 5.04X_{25} + 4.326X_{26} \\ & + 4.116X_{27} + 3.864X_{28} + 3.52X_{29}\end{aligned}$$

In the above expression the coefficients of variables X_1 to X_{29} are the cost row (C_j) value in the objective function for the problem.

THE LINEAR PROGRAMMING OPERATIONS

A series of pivot operations are used to solve a set of linear equations problem.

THE PIVOT OPERATION

A pivot in Row i Column j is a sequence of elementary operations where the i th equation is divided by the coefficient of the j th variable in that equation and then multiples of the resulting equation are added to the other equations to yield zero coefficients for the j th variable.

Example Suppose the system is started with:

$$2X_1 + 4X_2 - 18X_3 = 8$$
$$4X_1 - 4X_2 + 24X_3 = 4$$

First, pivot in row 1, column 1

(1) Multiply equation 1 by $\frac{1}{2}$

$$X_1 + 2X_2 - 9X_3 = 4$$
$$4X_1 - 4X_2 + 24X_3 = 4$$

(2) Add -4 times first equation to the second

$$X_1 + 2X_2 - 9X_3 = 4$$
$$-12X_2 + 60X_3 = -12$$

Second, pivot in row 2, column 2

(1) Multiply equation 2 by -1/12

$$X_1 + 2X_2 - 9X_3 = 4$$
$$X_2 - 5X_3 = 1$$

(2) Add -2 times second equation to first

$$X_1 + X_3 = 2$$
$$X_2 - 5X_3 = 1$$

This can have infinite number of solutions, one for each value of X_3.

THE CANONICAL FORM

The example of the system:

$$X_1 + X_3 = 2$$
$$X_2 - 5X_3 = 1$$

is in the Canonical Form, with X_1 and X_2 as BASIC VARIABLES, and X_3 as a NONBASIC VARIABLE.

A basic solution is obtained when the nonbasic variable X_3 is set equal to zero.

X_1	=	2	Basic Variable
X_2	=	1	Basic Variable
X_3	=	0	Nonbasic Variable

INEQUALITIES AND SLACK VARIABLES

There are two kinds of inequalities:-

(1) "Less Than" constraint

(2) "Greater Than" constraint

"Less Than" Constraints: For example

$$X_1 + X_2 \leq 3$$
$$-X_1 + X_2 \leq 2$$

Turning these equations into equalities by noting that the left-hand-side is always "less than" the right by a "positive amount" or at least by a non-negative amount.

Thus:

$$X_1 + X_2 + X_3 = 3$$

$$-X_1 + X_2 + X_4 = 2$$

Where X_3 and X_4 have been introduced into the problem as "slack" variables. Requirements are that $X_3 \geq 0$ and $X_4 \geq 0$, which is nothing new for the linear programming formulation.

"Greater Than" Constraints: For example

$$X_1 + 2X_2 \geq 2$$

Same idea here, the right-hand-side which is always smaller by a non-negative amount.

Thus:

$$X_1 + 2X_2 = 2 + X_5$$

or

$$X_1 + 2X_2 - X_5 = 2$$

Where X_5 is another slack variable. We demand only that $X_5 \geq 0$.

ADDITION OF SLACK AND ARTIFICIAL VARIABLES

(1) Number of "less than" constraints add only slack variables.

(2) Number of "equalities" constraints add only artificial variables.

(3) Number of "greater than" constraints substract slack variables and add artificial variables.

LINEAR PROGRAMMING IN MATRIX NOTATION

The Canonical Form for Phase I can be expressed in matrix notation as an example:

Canonical Form

$$
\begin{array}{llllll}
a_{11}X_1 + \cdots\cdots & + a_{1n}X_n & + X_{n+1} & & & = b_1 \\
a_{21}X_1 + \cdots\cdots & + a_{2n}X_n & \quad + X_{n+2} & & & = b_2 \\
\vdots & \vdots & \quad\ddots & & & \vdots \\
a_{m1}X_1 + \cdots\cdots & + a_{mn}X_n & & + X_{n+m} & & = b_m \\
C_1X_1 + \cdots\cdots & + C_nX_n & & & (-Z) & = 0 \\
d_1X_1 + \cdots\cdots & + d_nX_n & & & (-W) & = -W^o
\end{array}
$$

Matrix Notation

$$
\left[\begin{array}{lll|llllll}
a_{11} & a_{12}\cdots & a_{1n} & 1 & & & & & \\
a_{21} & a_{22}\cdots & a_{2n} & & 1 & & & & \\
\vdots & \vdots & \vdots & & & \ddots & & & \\
a_{m1} & a_{m2}\cdots & a_{mn} & & & & 1 & & \\
C_1 & C_2 \cdots & C_n & & & & & 1 & \\
d_1 & d_2 \cdots & d_n & & & & & & 1
\end{array}\right]
\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{m+n} \\ (-Z) \\ (-W) \end{bmatrix}
=
\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \\ 0 \\ -W^o \end{bmatrix}
$$

HOW TO GET THE INITIAL BASIC FEASIBLE SOLUTION

There are two phases set up and the initial basic feasible solution can be obtained from each phase as:

The Phase I Set Up: Suppose the original system of the problem is

$$
\begin{array}{lllll}
a_{11}X_1 & + a_{12}X_2 & + \ldots\ldots\ldots\ldots + a_{1n}X_n & = & b_1 \\
a_{21}X_1 & + a_{22}X_2 & + \ldots\ldots\ldots\ldots + a_{2n}X_n & = & b_2 \\
\vdots & \vdots & \vdots & & \vdots \\
a_{m1}X_1 & + a_{m2} & + \ldots\ldots\ldots\ldots + a_{mn}X_n & = & b_m \\
C_1X_1 & + C_2X_2 & + \ldots\ldots\ldots\ldots + C_nX_n & = & Z
\end{array}
$$

The original system is expanded to include an "artificial" variable for each equation and a new objective W. Therefore:

$$
\begin{array}{llllll}
a_{11}X_1 + \ldots\ldots + a_{1n}X_n & +X_{n+1} & & & = & b_1 \\
a_{21}X_1 + \ldots\ldots + a_{2n}X_n & & +X_{n+2} & & = & b_2 \\
\vdots & & & \ddots & & \vdots \\
a_{m1}X_1 + \ldots\ldots + a_{mn}X_n & & & +X_{n+m} & = & b_m \\
C_1X_1 + \ldots\ldots + C_nX_n & & & & = & Z \\
 & X_{n+1} & +X_{n+2} \ldots\ldots & +X_{n+m} & = & W
\end{array}
$$

W is called Phase I objective row.

To eliminate the artificial variables from the Phase I objective row subtract each equation from it in turn.

Therefore:

$$
\begin{array}{llllll}
a_{11}X_1 & +\ldots\ldots+a_{1n}X_n & +X_{n+1} & & & = b_1 \\
a_{21}X_1 & +\ldots\ldots+a_{2n}X_n & & +X_{n+2} & & = b_2 \\
\vdots & \vdots & & \ddots & & \vdots \\
a_{m1}X_1 & +\ldots\ldots+a_{mn}X_n & & & +X_{m+n} & = b_m \\
C_1X_1 & +\ldots\ldots+C_nX_n & & & & = Z \\
-\sum a_{11}X_1 & +\ldots\ldots-\sum a_{1n}X_n & & & & = W-\sum b_1
\end{array}
$$

This is in canonical form with the initial basic feasible solution.

$$X_1 = X_2 = \ldots\ldots\ldots\ldots = X_n = 0$$

and $X_{n+1} = b_1, \ldots\ldots, \ldots\ldots, X_{n+m} = b_m$

with $W = \sum b_1$

The sum of non-negative "artificial" is known as W.

There are two possibilities of W:

(1) Minimum $W > 0$, This means the original problem has no initial basic feasible solution.

(2) Minimum $W = 0$, At this point every artificial variable has value zero. Therefore, the original problem has a basic feasible solution not involving artificials. So Phase II can proceed.

<u>The Phase II Set Up</u>: The phase I objective row and all traces of the artificial can be dropped and the pivot operations, performed until the problem has reached to the optimal solution.

REVIEW OF THE PHASE I PROCEDURE

Step 1 Pivot Column Selection

The Phase I cost row coefficients are searched until we find the most negative. This determines the pivot column. This step requires only the cost row coefficients.

Step 2 Pivot Row Selection

For each positive column, we compute the ratio of right-hand-side column element and selected column element(from step 1). The pivot row is chosen as that having the minimum ratio and greater than zero. (Note that step 2 requires only the pivot column and right-hand-side.)

Step 3 Pivot on Chosen Element.

REVIEW OF THE PHASE II PROCEDURE

Phase II is just like Phase I exept that:

(1) A different cost row is used.

(2) There are no artificials.

THE REVISED SIMPLEX METHOD TO SOLVE THE PROBLEM

The system is started with the initial basic feasible solution and the equations can be written in the matrix notation in the form of:

$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} b \end{bmatrix}$$

$$Ax = b$$

where A = matrix coefficients including the cost row coefficients x = the variables vector included Z

b = the current right-hand-side.

After pivoting the system can be represented by:

(1) $\bar{A}x = \bar{b}$

(2) $P_n \ldots P_3 P_2 P_1 Ax = P_n \ldots P_3 P_2 P_1 b$ (after n pivot)

(3) $A_B^{-1} Ax = \bar{b}$

where $\bar{A} = P_n \ldots P_3 P_2 P_1 A = A_B^{-1} A$

$\bar{b} = P_n \ldots P_3 P_2 P_1 b = A_B^{-1} b$

$n = 1, 2, 3, \ldots\ldots\ldots, n.$

So, the system set up in the form of:

$$A_B^{-1} Ax = \bar{b}$$

where A = original matrix coefficients

A_B^{-1} = inverse matrix of the basis

$\bar{b}$ = the current right-hand-side vector.

<u>THE PIVOT MATRIX(P)</u>

A pivot on the r <u>th</u> row and s <u>th</u> column is accomplished by premultiplying both sides by the selected column elements and minus sign, such as:

$$P = \begin{bmatrix} 1 & & & -\frac{a_{1s}}{a_{rs}} & & & \\ & \cdot & & \vdots & & & \\ & & \cdot & -\frac{a_{r-1,s}}{a_{rs}} & & & \\ & & & \frac{1}{a_{rs}} & & & \\ & & & -\frac{a_{r+1,s}}{a_{rs}} & \cdot & & \\ & & & \vdots & & \cdot & \\ & & & -\frac{c_s}{a_{rs}} & & & \cdot \\ & & & -\frac{d_s}{a_{rs}} & & & & 1 \end{bmatrix}$$

UPDATE THE INVERSE AND RIGHT-HAND-SIDE

Suppose we have the original identity matrix as an inverse matrix(A_B^{-1}) and has the value of zero pivots. After the first pivot matrix, the pivot P_1 corresponds to the first pivot operation. The update A_B^{-1} and $\bar{b}$ are:

$$A_B^{-1} = \left[P_1 \right] \left[\text{old } A_B^{-1} \right] = \text{new } A_B^{-1}$$

$$\bar{b} = \left[P_1 \right] \left[\text{old } \bar{b} \right] = \text{new } \bar{b}$$

As mention above, the system is set up in the form of $A_B^{-1} Ax = \bar{b}$. The further update of the inverse matrix and right-hand-side do the same fashion until the optimal solution is obtained.

GENERATION OF THE COST ROW

A new variable must be chosen to come into the basis. This means we need to know the cost row of the transformed matrix $\bar{A}$ where

$$\bar{A} = A_B^{-1} A$$

(The entire $\bar{A}$-matrix does not need to be known; just the cost row.)

Pick up the last row of the inverse matrix (A_B^{-1}) and multiply by the entire original A-matrix. The results are the cost row coefficients. That is, the new cost row is generated. The new cost row can set up in the form of:

$$\bar{A}_n = (A_B^{-1})_n A$$

where n = 1,2,3,.....,n.

We continue in the same fashion until no negative value appears in the cost row coefficients. This means, the optimal solution is reached.

SENSITIVITY ANALYSIS

To find the validity of the optimal solution, it is necessary to have the sensitivity analysis. The effect of the nonbasic elements of the cost row, right-hand-side, and non-basic constraint coefficients, are considered.

Knowing how sensitive the "optimal" solution is to uncertainties and changes in various parameters is often as valuable as the LP solution itself.

A shadow price is valid only until a basic change occurs. A shadow price tells that how much the profit (Z) would change if one of the right-hand-side were change.

The effect of uncertain costs on the non-basic variables, we want to know how much a cost coefficient on a non-basic variable has to change before it affects the optimal solution.

The effect of uncertain constraint coefficients for non-basic variables, we want to know how much constraint coefficients for non-basic variable has to change before it affects the optimal solution.

LINEAR PROGRAMMING TEST

Subroutine pivot is one of the most important parts. It is the working portion of the linear programming techniques. The LP code was tested by the simple problem which has already solved. The system consisted of:

$$\begin{aligned} 4.5X_1 + 8.5X_2 + 6X_3 + 20X_4 + X_5 &= 6{,}000 \\ X_1 + X_2 + 4X_3 + 40X_4 + X_6 &= 4{,}000 \\ -14X_1 - 18X_2 - 16X_3 - 80X_4 \quad (-Z) &= 0 \end{aligned}$$

Z = objective function

To start with the basic feasible solution:

$$X_1 = X_2 = X_3 = X_4 = 0$$
$$X_5 = 6{,}000, \; X_6 = 4{,}000, \; Z = 0$$

After the third pivot operation, there are no negative cost row coefficients. So that, the solution is optimal with:

$$X_1 = 1{,}000, \; X_4 = 75, \; Z = -20{,}000 \text{ (minimize Z)}$$
$$X_2 = X_3 = X_5 = X_6 = 0$$

The main program,subroutine DAATA, and subroutine SENSE was tested by the system:

$$\begin{aligned} 4.5X_1 + X_2 - X_3 &= 14 \\ 8.5X_1 + X_2 - X_4 &= 18 \\ 6X_1 + 4X_2 &\geq 16 \\ 20X_1 + 40X_2 &\geq 80 \\ 6000X_1 + 4000X_2 &= Z \end{aligned}$$

After the fourth pivot operation, the optimal solution is found as:

$$X_1 = 1{,}000, \; X_4 = 75, \; Z = -20{,}000$$

$$X_2 = X_3 = X_5 = X_6 = 0$$

Therefore, this Linear Programming Code can be used to solve the set of linear equations to obtain the optimal solution.

LINEAR PROGRAMMING SOLUTION

By using the set of linear equations which consist of nine numbers of less-than constraints, thirty-three numbers of equality constraints, nine numbers of greater than constraints, and the objective function (cost row); and there are twenty-nine variables and one hundred forty-two non-zero variables. These were fed to the linear programming code as the input data. The output came out as profit of dollars per day including the value of variables which are the optimal solution of this problem.

HOW TO READ THE LP COMPUTER SOLUTIONS

The problem was set as Phase I. The pivot operated forty-five times to get out all artificial variables to obtain the initial basic feasible solution. The Phase II was continued at this point. At the first pivot, the optimal solution was found and the optimal objective function value is $131,115.65 per day.

The shadow prices are the increment increase of the products which appear in the last row of the inverse matrix when the solution is optimum. It is valid only if a basic change occurs.

The variables and variable values which are in the basis, correspond to the feed stocks and products of a petroleum refining design as:

		VARIABLES ARE IN THE BASIS		
UNIT	COMPONENT	ROW NO.	VARIABLE	VARIABLE VALUE BPD
Crude	Crude rate	8	X_1	110,000.00
	Lt. Gasoline	3	X_2	8,579.9999
	Hvy. Gasoline	4	X_3	18,040.00
	Prod. 350-650°F	5	X_4	27,390.00
	Prod. 350-400°F			6,370.00
	Kerosine Dist.			12,030.00
	Diesel			6.070.00
	Prod. 550-650°F			2,870.00
	Gas Oils	6	X_5	32,670.00
	Atm. Gas Oil			8,680.00
	Vac. Gas Oil			23,990.00
	Residuum	7	X_6	23,320.00
Delayed-Coking	Feed, Residuum	7	X_6	23,320.00
	Propane	11	X_7	2,357.652
	C_4-400°F.Gaso.	12	X_8	5,746.0479
	Naphtha	13	X_9	4,559.06
	Coker Gas Oil	14	X_{10}	16,510.560
	Coke	16	X_{11}	2,294.6880

VARIABLES ARE IN THE BASIS

UNIT	COMPONENT	ROW NO.	VARIABLE	VARIABLE VALUE BPD
FCC	Feed, Gas Oil	6	X_5	32,670.00
	Coker G.O.	14	X_{10}	16,510.560
	Dry Gas	19	X_{12}	2,665.5863
	Propane	20	X_{13}	1,288.5306
	Propene+Butene	21	X_{14}	8,852.5006
	Propene			3,442.6392
	Butene			5,409.8714
	Iso-butane	22	X_{15}	3,973.7892
	N-butane	23	X_{16}	688.5278
	Cracked Gaso.	24	X_{17}	34,967.378
	Cycle Oils	42	X_{18}	4,918.0559
	Lt. Cycle Oil			3,442.6391
	Hvy. Cycle Oil			1,475.4168
	Coke	26	X_{19}	2,704.9308
Reformer	Feed, Hvy. Gaso.	4	X_3	18,040.00
	Naphtha	13	X_9	4,559.06
	Hydrogen	30	X_{20}	2,677.9886
	Dry Gas	31	X_{21}	836.1652
	Propane	32	X_{22}	861.0242
	Iso-butane	33	X_{23}	519.7784
	N-butane	36	X_{24}	745.7690
	Reformate	35	X_{25}	99,320.1552

VARIABLES ARE IN THE BASIS

UNIT	COMPONENT	ROW NO.	VARIABLE	VARIABLE VALUE BPD
Alkylation	Feed,			
	Propene+Butene	21	X_{14}	8,852.5006
	Iso-butane	22	X_{15}	3,973.7892
	Iso-butane	33	X_{23}	519.7784
Dist. Hydrodesul-furization	Feed,			
	Prod. 350-650°F	5	X_4	27,390.00
	Cycle Oils	42	X_{18}	4,918.0559
	Jet fuel	43	X_{26}	13,698.616
	Kerosine	44	X_{27}	6,461.6111
	Fuel oil	45	X_{28}	6,623.1515
	Economy diesel	47	X_{29}	5,524.6773

The slack variables tell that the requirements or capacities are not reached to the upper boundary (maximum amount) by that amount.

The sensitivity analysis on the non-basic variables show that $X_{30} = 0$ and its cost can vary from 0.00 t0 -1.1919605 without the change of this solution.

The sensitivity analysis of the solution to the non-basic constraints show that X_{30} in constraint number 1 can vary from 1.00 to 0.00 without the change of solution.

Objective Function in the Optimal Solution

The profit for the optimal case can be calculated as follow:

Income and Credits

Lt. Gasoline	X_2	8,579.9999	x 5.04	=	43,243.1995
Propane	X_7	2,357.652	x 1.70	=	4,008.0084
Gasoline	X_8	5,746.0479	x 5.04	=	28,960.0814
Coke	X_{11}	2,294.6880	x 1.26	=	2,891.3069
Fuel gas	X_{12}	2,665.5863	x 1.42	=	3,785.1325
Propane	X_{13}	1,288.5306	x 1.70	=	2,190.5020
Propene+Butene	X_{14}	8,852.5006	x 1.55	=	13,721.3759
Iso-butane	X_{15}	3,973.7892	x 2.52	=	10,013.9488
N-butane	X_{16}	688.5278	x 1.95	=	1,342.6292
Gasoline	X_{17}	34,967.378	x 5.04	=	176,235.5851
Coke	X_{19}	2,704.9308	x 1.26	=	3,408.2128
Hydrogen	X_{20}	2,677.9886	x 0.50	=	1,338.9943
Fuel gas	X_{21}	836.1652	x 1.42	=	1,187.3546
Propane	X_{22}	861.0242	x 1.70	=	1,463.7411
Iso-butane	X_{23}	519.7784	x 2.52	=	1,309.8415
N-butane	X_{24}	745.7690	x 1.95	=	1,454.2495
Gasoline	X_{25}	19,706.380	x 5.04	=	99,320.1552
Jet fuel	X_{26}	13,698.616	x 4.326	=	59,260.2128
Kerosine	X_{27}	6,461.6111	x 4.116	=	26,595.9913
Fuel oil	X_{28}	6,623.1515	x 3.864	=	25,591.8574
Economy diesel	X_{29}	5,524.6773	x 3.52	=	19,446.8641

Income and Credits = $526,769.2444 per day

Operating Costs

Crude Oil	X_1	110,000.0000 x 3.2282 =	355,102.0000
Hvy. Gasoline	X_3	18,040.0000 x 0.3362 =	6,065.0480
Prod. 350-650 °F	X_4	27,390.0000 x 0.2800 =	7,669.2000
Gas Oils	X_5	32,670.0000 x 0.3818 =	12,473.4060
Residuum	X_6	23,320.0000 x 0.2200 =	5,130.4000
Naphtha	X_9	4,559.0600 x 0.3362 =	1,532.7560
Coker Gas Oil	X_{10}	16,510.5600 x 0.3818 =	6,303.7318
Cycle Oils	X_{18}	4,918.0559 x 0.2800 =	1,377.0556

Operating Costs = $395,653.5974 per day

Net Profit = Income and Credits - Operating Costs

= 526,769.2444 - 395,653.5974

= $131,115.65 per day

This calculated profit checks the profit derived by the revised simplex solution of the linear programming problem.

In the optimal solution all of the coefficients in the objective function are either zero or positive. This means that the profit can not be increased by replacing any variable.

CALCULATION ON ALKYLATION UNIT

From Table 5."Alkylation Material Balances",

Alkylate produced	=	$1.76X_{14}$	BPD
	=	1.76 x 8,852.5006	
	=	15,580.4011	BPD
Iso-butane consumed	=	$1.24X_{14}$	BPD
	=	1.24 x 8,852.5006	
	=	10,977.1007	BPD
Iso-butane produced	=	$X_{15} + X_{23}$	BPD
	=	3,973.7892 + 519.7784	
	=	4,493.5676	BPD
Iso-butane purchase required	=	10,977.1007 - 4,493.5676	
	=	6,483.5332	BPD

Price of iso-butane $2.52 per bbl.

Iso-butane cost	=	6,483.5332 x 2.52	
	=	$16,338.5036	per day
Alkylate produced	=	15,580.4011 x 5.04	
	=	$78,525.2213	per day

Operating cost of alkylation unit per output $0.5017 per bbl.

Operating cost	=	15,580.4011 x 0.5017	
	=	$7,816.6872	per day

Since X_{14}, X_{15}, and X_{23} was used up in the alkylation process to produce alkylate, therefore the income of X_{14}, X_{15}, and X_{23} must be subtracted from the net profit.

Income of X_{14} = $13,721.3759 per day

Income of X_{15}	=	\$10,013.9488	per day
Income of X_{23}	=	\$ 1,309.8415	per day
Total expenses of unit	=	\$49,200.3570	per day
Alkylate Produced	=	\$78,525.2213	per day
Credit for alkylation unit	=	78,525.2213 - 49,200.3570	
	=	\$29,324.8643	per day

Net profit from computer solution \$131,115.65 per day

Therefore, the net profit which included the alkylation unit

= 131,115.65 + 29,324.8643

= \$160,440.51 per day

Because of the hydrogen was used up in treating unit and distillate hydrodesulfurization, therefore the income of hydrogen (X_{20}) equal to \$1,338.9943 per day must be substracted from the net profit.

Final net profit = \$160,440.51 - 1,338.9943

= \$159,101.52 per day

CONCLUSIONS

The refining of crude petroleum is a complex process, and there are many ways to formulate the system of linear equations. Mathematical modeling of this petroleum refinery is the set of linear equations which are formulated from the unit capacities, material balances of the processes, and required products of the integrated processes of the whole refinery.

The linear programming code was built by the techniques of pivot operations to solve this problem for optimizing the profit. This LP Code will work for any set of linear equations problem within this dimension. It is necessary to expand the dimension to use it with a large set of linear equations problem.

By using the example of basic design of the 100,000 BPD petroleum refinery to formulate the system of the problem, the optimal solution (maximizing the profit) is $159,101.52 per day by producing the products of fuel gas, gasoline, jet fuel, kerosine, fuel oil, economy diesel, propane, butane, and coke for the credits. This is the only one of many possible processes.

The validity of the optimality was tested by the sensitivity analysises of the solution on the non-basic variables on cost row (C_j), right-hand side, and non-basic constraint coefficients.

The subroutine pivot was tested by the Standard IC-4000 computer, and the solution of the problem was carried out by the Revised Simplex Linear Programming Method on a remote time-sharing teletype terminal, Digital Equipment Corporation PDP-10 computer.

APPENDIX I

CRUDE PETROLEUM ANALYSIS(1)

Field Salt Creek Field, Natrona County Wyoming.(Third Wall Creek, Upper Cretaceous 1683-1693).

General Characteristics

Gravity, specific : 0.842

Gravity, °API : 36.6

Pourpoint, °F : 55

Sulfur, percent : 0.12

Color, brownish green

Viscosity, Saybolt Universal at 100°F : 43 sec

Nitrogen, percent : ....

Distillation, Bureau of Mines Routine Method

Stage 1 Distillation at atmospheric pressure, 760 mmHg.

First drop : 77 °F.

Fraction No.	Cut Temp °F	Percent	Sum %	sp.gr. 60/60 °F	°API	CI
1	122	-	-	-	-	-
2	167	0.2	2.0	0.660	82.9	-
3	212	4.2	6.2	0.714	66.9	18
4	257	6.3	12.5	0.742	59.2	23
5	320	5.4	17.9	0.762	54.2	25

Fraction No.	Cut Temp °F	Percent	Sum %	sp.gr. 60/60 °F	°API	CI
6	347	5.5	23.4	0.779	50.1	26
7	392	4.2	27.6	0.793	46.9	26
8	437	5.5	33.1	0.807	43.8	27
9	482	4.6	37.7	0.819	41.3	28
10	527	7.1	44.8	0.830	39.0	28
Stage 2	Distillation continues at 40 mm Hg.					
11	392	3.7	48.5	0.846	35.8	32
12	437	6.6	55.1	0.850	35.0	30
13	482	6.1	61.2	0.863	32.5	33
14	527	5.3	66.5	0.875	30.2	35
15	572	6.5	73.0	0.886	28.2	37
Residuum	-	26.1	99.1	0.939	19.2	-

FIGURE 2

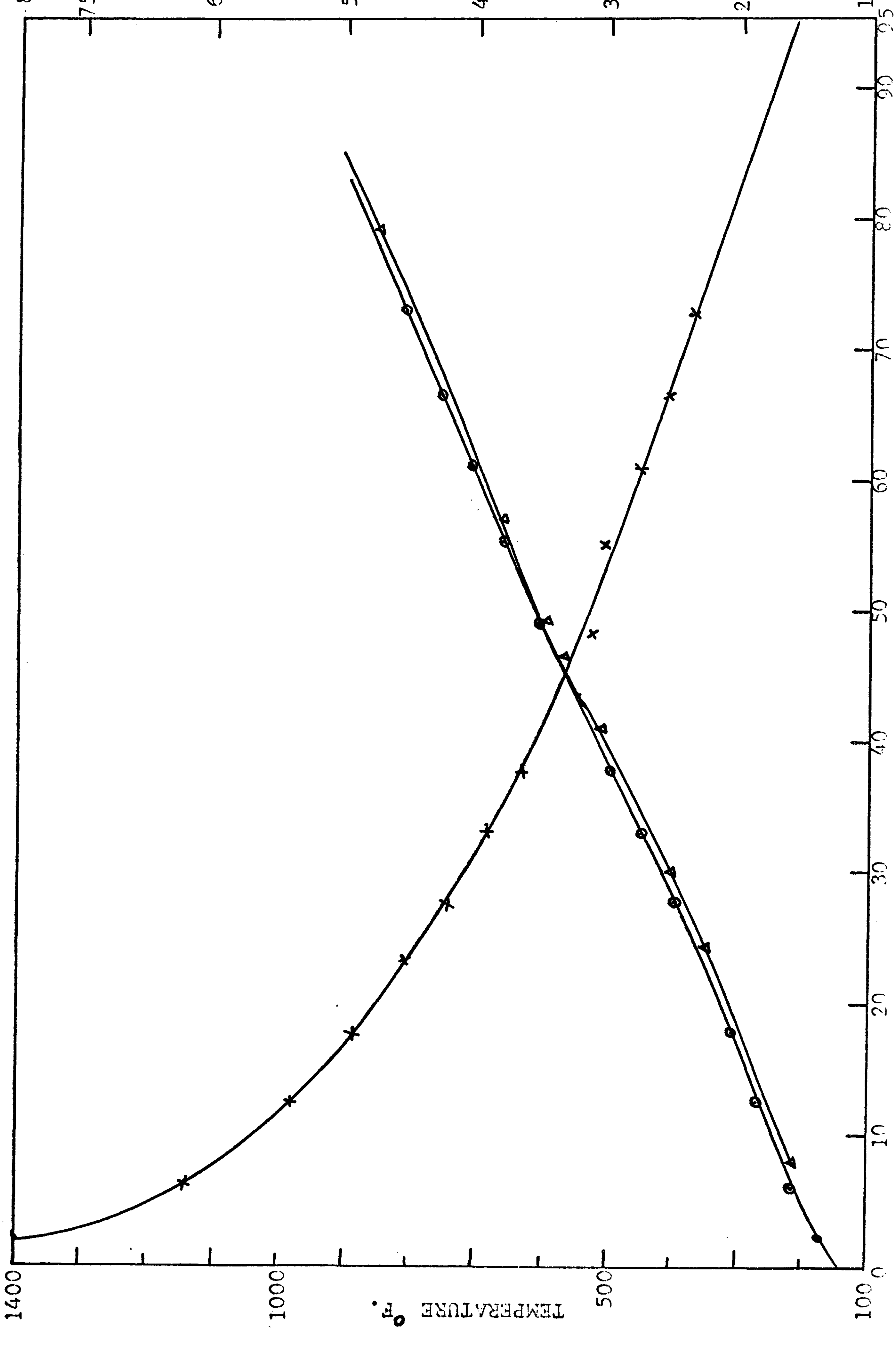
Hempel Distillation, API and

True-Boiling-Point Distillation Curves

⊙ Hempel Distillation Curve

× API Curve

Δ True-Boiling-Point Distillation Curve


API
80
75
65
55
45
35
25
15
TEMPERATURE °F.
1400
1000
500
100
VOLUME PERCENT
0
10
20
30
40
50
60
70
80
90
95

CALCULATION OF TBP CURVE

Conversion of Hempel 40 mm Hg. point to 760 mm Hg. (9).

Fraction No.	°F at 40mmHg.	Sum %	°API	°F at 760 mm Hg
11	392	48.5	35.8	590
12	437	55.1	35.0	640
13	482	61.2	32.5	691
14	527	66.5	30.2	742
15	572	73.0	28.2	800

From Fig. 2 Hempel 50 % point = 591 °F

TBP 50 % point by interpolation (5)

TBP	Hempel Correction		
	500	591	600
500	5	4.09	4
$T_{50\%}$		ΔT	
600	5.5	3.68	3.5
700	7.0	4.73	4.5

Secondary interpolation:

$$\frac{T_{50\%} - 500}{600 - 500} = \frac{\Delta T - 4.09}{3.68 - 4.09}$$

and $T_{50\%} = 591 - \Delta T$

$$\frac{591 - \Delta T - 500}{600 - 500} = \frac{\Delta T - 4.09}{3.68 - 4.09}$$

$$\Delta T = 3.64\ °F$$

$$T_{50\%} = 591 - 3.64$$

$$= 587.36\ °F$$

Calculation of the remaining points:

TBP	HEMPEL 200	300	400	500	600	700
500	10	9.0	8.0	5.0	4.0	4.0
587	10	9.4	8.9	5.4	3.6	2.7
600	10	9.5	9.0	5.5	3.5	2.5

Hempel Temp. - ΔT = TBP

HEMPEL	T	TBP
200	10.0	190
300	9.4	290.6
400	8.9	391.6
500	5.4	494.6
600	3.6	596.4
700	2.7	697.3

Volume Percent vs TBP

TBP	VOLUMN %
190	6.0
290.6	17.5
391.1	30.0
494.6	43.0
596.4	52.0
697.3	65.0

The above TBP cut pionts are drawn on Fig. 2 to obtain the volume percent of each component as product of the crude distillation unit.

APPENDIX II

LINEAR PROGRAMMING CODE

```
C     REVERSED SIMPLEX LINEAR PROGRAMMING
      DOUBLE PRECISION ABINVS,P,OPMTRX
      DIMENSION A(65,120),B(65),ABINVS(65,65),C(120),P(65)
      DIMENSION OPMTRX(65,65),ICOL(65),ISLK(65)
      COMMON A,B,ABINVS,C,P,OPMTRX,ICOL,ISLK
      CALL ERRSET(0)
    4 IFEAS = 0
      DO 5 I = 1,65
      B(I)=0.0
      P(I)=0.0
      ISLK(I)=0
      ICOL(I)=0
      DO 6 J = 1,65
      ABINVS(I,J)=0.0
    6 OPMTRX(I,J)=0.0
      DO 5 J = 1,120
    5 A(I,J)=0.0
      DO 7 I = 1,120
    7 C(I)=0.0
      CALL DAATA (II,JJ,JCOL,NVAR,ITMAX,SING)
      DO 1 I=1,II
    1 ABINVS(I,I)=1.0
      CALL PIVOT (II,JJ,1,2,ITMAX,JCOL,IFEAS,SING)
      IF (IFEAS .EQ. 1) GO TO 4
    2 II = II-1
      ITMAX = II-1
      JJ = JCOL
      CALL PIVOT (II,JJ,1,2,ITMAX,JCOL,IFEAS,SING)
      WRITE (2,9)
    9 FORMAT (/23H OPTIMAL SOLUTION FOUND/)
      CALL SENSE (II,JJ,NVAR,SING)
      GO TO 4
      END
```

```
      SUBROUTINE DAATA (II,JJ,JCOL,NVAR,ITMAX,SING)
      DOUBLE PRECISION ABINVS,P,OPMTRX
      DIMENSION A(65,120),B(65),ABINVS(65,65),C(120),P(65)
      DIMENSION OPMTRX(65,65),ICOL(65),ISLK(65)
      COMMON A, B, ABINVS, C, P, OPMTRX, ICOL,ISLK
      DATA MAX/3HMAX/
C     INPUT GENERAL DATA
      READ (1,1,END=13)  NVAR,NCON,MYPE,NLT,NEQ,NGT,NONZRO,NCROW
    1 FORMAT(I2,1X,I2,1X,A3,1X,I2,1X,I2,1X,I2,1X,I5,1X,I2)
      JJ=NVAR+NLT+2*NGT+NEQ+2
      JV=NVAR+NLT+NGT+2
      JCOL=JJ-NEQ-NGT-1
      II=NCON+2
      ITMAX = NEQ+NGT
C     INPUT COST ROW COEFICIENTS
      IC=II-1
      A(IC,JCOL)=1.0
      ICOL(IC)=JCOL
      SING=1.0
      IF (MYPE .EQ. MAX) SING = -1.0
      DO 7 I=1,NCROW
      READ (1,8) NC,COST
    7 A(IC,NC)=COST*SING
    8 FORMAT (I2,1X, E14.7)
C     INPUT NONZERO ACTIVITIES
      DO 9 K=1,NONZRO
      READ (1,10) NROW,NCOL,COEF
    9 A(NROW,NCOL)=COEF
   10 FORMAT (I2,1X,I2,1X,E14.7)
C     INPUT CONSTRAINT KIND AND RHS AND
C     GENERATE SLACKS AND ARTIFICIALS
      BSUM=0.0
      JC=NVAR+1
      DO12 K=1,NCON
      READ  (1,3) M, KIND, RHS
    3 FORMAT (I2,1X,I1,1X,E14.7)
      B(M)=ABS(RHS)
      GO TO (4,5,6),KIND
    4 A(M,JC)=1.0
      ICOL(M) = JC
      GO TO 2
    6 A(M,JC)=-1.0
    5 A(M,JV)=1.0
      ICOL(M) = JV
      A(II,JV)=1.0
      BSUM=BSUM+RHS
      DO 11 J = 1,JJ
   11 A(II,J)=A(II,J) - A(M,J)
      JV=JV+1
    2 IF (KIND .EQ. 2) GO TO 12
      ISLK(M)=JC
```

```
      JC=JC+1
   12 CONTINUE
      B(II) = -BSUM
      A(II,JJ)=1.0
      ICOL(II) = JJ
      RETURN
   13 STOP
      END
      SUBROUTINE PIVOT (II,JJ,IT,IR,ITMAX,JCOL,IFEAS,SING)
      DOUBLE PRECISION ABINVS,P,OPMTRX,TERM
      DIMENSION A(65,120),B(65),ABINVS(65,65),C(120),P(65)
      DIMENSION OPMTRX(65,65),ICOL(65),ISLK(65)
      COMMON A,B,ABINVS,C,P,OPMTRX,ICOL
      IFEAS = 0
      JLV=0
      IT = 1
      NARTF = JJ-JCOL-1
      IA = II-IR
      IF (JCOL .EQ. JJ) SIN = SING
      IF (JJ .NE. JCOL) WRITE (2,101)
  101 FORMAT(1H1,2X,5HPIVOT,3X,10H EXIT VAR.,3X,10HENTER VAR.,2X,
     110HREDUC COST,5X,10HINFEAS.SUM,/)
      IF (JJ .EQ. JCOL) WRITE (2,102)
  102 FORMAT(1H1,2X,5HPIVOT,3X,10H EXIT VAR.,3X,10HENTER VAR.,2X,
     110HREDUC COST,5X,10HOBJ. VALUE,/)
      NLINE=2
      CALL CLEAR (II)
C     GENERATE NEW COST ROW
    9 DO 1 J = 1,JJ
      C(J) = 0
      DO 1 N = 1,II
    1 C(J) = C(J)+ABINVS(II,N)*A(N,J)
C     GENERATE UPDATED B(I) AS OPMTRX(3,I)
      DO 13 I = 1,II
      TERM = 0.0
      DO 14 J = 1,II
   14 TERM = TERM+ABINVS(I,J)*B(J)
   13 OPMTRX(3,I) = TERM
      IF((JJ.GT.JCOL).AND.(OPMTRX(3,II).GE.0.0)) GO TO 10
C     SELECT MOST NEGATIVE C(J)
      MONEG = 0
      DO 2 J=1,JJ
      IF (C(J) .GE.0.0) GO TO 2
      IF(J .EQ. JLV) GO TO 2
      DO 18 N = 1,II
      IF (ICOL(N) .EQ. J) GO TO 2
   18 CONTINUE
      IF (MONEG .EQ. 0) MONEG = J
      IF (C(J) .LT. C(MONEG)) MONEG = J
    2 CONTINUE
      IF (MONEG .EQ. 0) GO TO 10
      IF ((JJ .NE. JCOL).AND.(NARTF .EQ. 0))GO TO 10
      JJJ = MONEG
```

```
C     GENERATE THAT COLUMN
      DO 3 I = 1, II
      OPMTRX(2,I)=0.0
      DO 3 J = 1, II
    3 OPMTRX(2,I)=OPMTRX(2,I)+A(J,JJJ)*ABINVS(I,J)
C     SELECT SMALLEST POSITIVE RATIO OF B(I)/OPMTRX(2,I)
C     FOR PIVOT ROW III
      III = 0
      DO 4 I=1,IA
    4 IF (OPMTRX(2,I) .GT. 0.0) III= I
      IF (III .EQ. 0) GO TO 21
      DO 5 I=1,IA
      IF(OPMTRX(2,I) .LE. 0.0) GO TO 5
      IF (OPMTRX(3,I)/OPMTRX(2,I) .LT. OPMTRX(3,III)/OPMTRX
     1(2,III)) III = I
    5 CONTINUE
C     GENERATE P VECTOR
      DIV=1.0/OPMTRX(2,III)
      DO 6 I=1,II
      P(I)=-OPMTRX(2,I)*DIV
    6 IF(I .EQ. III) P(I)=DIV
   27 IF (ICOL(III)-JCOL) 22 ,22,23
   22 WRITE (2,25)IT,ICOL(III),JJJ,C(JJJ),OPMTRX(3,II)
   25 FORMAT (2X,I5,7X,I3,7X,I3,5X,1PE14.7,5X,1PE14.7)
      GO TO 24
   23 WRITE (2,15) IT,JJJ,C(JJJ),OPMTRX(3,II)
   15 FORMAT (2X,I5,7X,4HARTF,6X,I3,5X,1PE14.7,5X,1PE14.7)
      NARTF = NARTF-1
   24 NLINE=NLINE+1
      IF (NLINE .LT.58) GO TO 28
      IF (JJ .NE. JCOL) WRITE (2,101)
      IF (JJ .EQ. JCOL) WRITE (2,102)
      NLINE=2
   28 IF (JJJ .GT. JCOL) NARTF = NARTF+1
      JLV=ICOL(III)
      ICOL(III) = JJJ
      CALL CLEAR (II)
C     MULTIPLY ABINVS BY PIVOT MATRIX
      DO 7 J = 1, II
      DO 7 I = 1, II
      TERM = 0.0
      DO 8 K = 1, II
      PM = 0.0
      IF (K .EQ. I) PM=1.0
      IF (K .EQ. III) PM = P(I)
    8 TERM = TERM+PM*ABINVS(K,J)
    7 OPMTRX(I,J) = TERM
      DO 11 I = 1,II
      DO 11 J = 1,II
   11 ABINVS(I,J) = OPMTRX(I,J)
      CALL CLEAR (II)
```

```
C       START A NEW PIVOT
        IT = IT+1
        GO TO 9
   10   CONTINUE
        IF(JJ .GT. JCOL) WRITE (2,19) OPMTRX(3,II)
   19   FORMAT (/32H INITIAL FEASIBLE SOLUTION FOUND,5X,
      1 11HIFEAS.SUM=   ,1PE14.7/)
        IF(MONEG .NE. 0) WRITE(2,20)
      1MONEG, C(MONEG)
   20   FORMAT(10X,34HNO ARTIFICIALS IN BASIS, BUT COST(,I3,4H) = ,1PE14.7
      1 /)
        IF(JJ .GT. JCOL)WRITE(2,103)
  103   FORMAT(/27H TYPE 1 FOR GO, 0 FOR NO GO /)
        IF(JJ.GT.JCOL) READ(3,104) IGO
        IF((JJ.GT.JCOL).AND.(IGO.NE.1)) GO TO 21
  104   FORMAT(I1)
        RETURN
   21   IFEAS = 1
        WRITE (2,26)
   26   FORMAT (25H0 PROBLEM IS NOT FEASIBLE)
        RETURN
        END
        SUBROUTINE CLEAR(II)
        DOUBLE PRECISION ABINVS,P,OPMTRX
        DIMENSION A(65,120),B(65),ABINVS(65,65),C(120),P(65)
        DIMENSION OPMTRX(65,65)
        COMMON A, B, ABINVS, C, P, OPMTRX
        DO 15 I = 1,II
        DO 15 J = 1,II
   15   OPMTRX(I,J) = 0.0
        RETURN
        END
        SUBROUTINE SENSE (II,JJ,NVAR,SING)
        DOUBLE PRECISION ABINVS,P,OPMTRX
        DIMENSION A(65,120),B(65),ABINVS(65,65),C(120),P(65)
        DIMENSION OPMTRX(65,65),ICOL(65),ISLK(65)
        COMMON A, B, ABINVS, C, P, OPMTRX, ICOL,ISLK
        JC=JJ-1
        TERM=-OPMTRX(3,II)*SING
        WRITE (2,1) TERM
    1   FORMAT (/37H OPTIMAL OBJECTIVE FUNCTION VALUE IS ,1PE14.7/)
C       PRINT BASIC VARIABLE DATA
        WRITE (2,18)
   18   FORMAT (1H1,5X,3HROW,5X,4HVAR.,5X,14HVARIABLE VALUE,5X,
      112HSHADOW PRICE/)
        NLINE=2
        IR=II-1
        DO 2 I=1,IR
        TERM = -ABINVS(II,I)*SING
        IF (ICOL(I)-NVAR) 3,3,4
    4   DO 21 N=1,IR
   21   IF (ISLK(N) .EQ. ICOL(I)) WRITE(2,6) I,N,OPMTRX(3,I),TERM
    6   FORMAT(6X,I2,2X,6HSLACK(,I2,1H),3X,1PE14.7,5X,1PE14.7)
        GO TO 22
    3   WRITE (2,5)  I,ICOL(I),OPMTRX(3,I),TERM
```

```
    5 FORMAT (6X,I2,5X,I3,6X,1PE14.7,5X,1PE14.7)
   22 NLINE=NLINE+1
      IF (NLINE .LT. 48) GO TO 2
      WRITE (2,18)
      NLINE=2
    2 CONTINUE
C     DO SENSITIVITYANALYSIS ON NONBASIC C(J).
      WRITE (2,19)
   19 FORMAT(1H1,5X,4HVAR.,6X,12HCURRENT COST,7X,12HBOUNDRY COST/)
      NLINE=2
      DO 7 J=1,JC
C     IF X(J) IS BASIC, SKIP TO NEXT COLUMN
      DO 8 I=1,IR
      IF(ICOL(I) .EQ. J) GO TO 7
    8 CONTINUE
      TERM=0.0
      DO 11 I=1,IR
   11 TERM=TERM+ABINVS(II,I)*A(I,J)
      TERM = TERM*SING
      WRITE (2,12) J,A(II,J),TERM
   12 FORMAT (5X,I3,6X,1PE14.7,5X,1PE14.7)
      NLINE=NLINE+1
      IF(NLINE .LT. 48) GO TO 7
      WRITE (2,19)
      NLINE=2
    7 CONTINUE
C     DO SENSITIVITY ANALYSIS ON NONBASIC CONSTRAINT COEF.
      WRITE (2,20)
   20 FORMAT(1H1,5X,4HVAR.,4X,7HCONSTR.,2X,13HCURRENT VALUE,6X,
     113HBOUNDRY VALUE/)
      NLINE=2
      DO 10 J=1,JC
C     IF X(J) IS BASIC, SKIP TO NEXT COLUMN
      DO 16 I=1,IR
      IF(ICOL(I) .EQ. J) GO TO 10
   16 CONTINUE
      TERM=0.0
      DO 17 I=1,IR
   17 TERM=TERM+ABINVS(II,I)*A(I,J)
      DO 13 I=1,IR
      IF(A(I,J) .EQ. 0.0) GO TO 13
      AVAL=TERM-ABINVS(II,I)*A(I,J)
      AVAL=-AVAL/ABINVS(II,I)
      WRITE (2,14) J,I,A(I,J),AVAL
   14 FORMAT (6X,I3,6X,I2,4X,1PE14.7,5X,1PE14.7)
      NLINE=NLINE+1
      IF(NLINE .LT. 48) GO TO 13
      WRITE (2,20)
      NLINE=2
   13 CONTINUE
   10 CONTINUE
      WRITE (2,9)
    9 FORMAT (//10X,14HEND OF PROBLEM)
      RETURN
      END
*
```

APPENDIX III

Linear Programming Computer Solutions

PIVOT	EXIT VAR.	ENTER VAR.	REDUC COST	INFEAS.SUM
1	ARTF	2	-3.0000000E+00	-2.9670000E+05
2	ARTF	8	-3.0000000E+00	-2.9670000E+05
3	ARTF	14	-3.0000000E+00	-2.9670000E+05
4	ARTF	15	-3.0000000E+00	-2.9670000E+05
5	ARTF	17	-3.0000000E+00	-2.9670000E+05
6	ARTF	5	-3.4734000E+00	-2.9670000E+05
7	ARTF	10	-3.4734000E+00	-2.9670000E+05
8	ARTF	6	-3.4995672E+00	-2.9670000E+05
9	ARTF	23	-3.0000000E+00	-2.9670000E+05
10	ARTF	25	-3.0000000E+00	-2.9670000E+05
11	ARTF	3	-3.4418000E+00	-2.9670000E+05
12	ARTF	9	-3.4418000E+00	-2.9670000E+05
13	ARTF	26	-3.0000000E+00	-2.9670000E+05
14	ARTF	4	-2.2720000E+00	-2.9670000E+05
15	ARTF	1	-2.2803401E+00	-2.9670000E+05
16	ARTF	50	-6.1212415E+08	-2.9670000E+05
17	ARTF	53	-1.1900807E+01	-9.2686538E+04
18	ARTF	35	-4.1003367E+00	-9.2686534E+04
19	ARTF	51	-7.9548408E+00	-7.8235187E+04
20	ARTF	37	-7.9232410E+00	-7.8235184E+04
21	ARTF	41	-3.8760546E+01	-7.7657816E+04
22	ARTF	18	-2.2720000E+00	-7.5853588E+04
23	ARTF	7	-2.0000000E+00	-6.5882628E+04
24	ARTF	11	-2.0000000E+00	-6.1645676E+04
25	ARTF	12	-2.0000000E+00	-5.7521877E+04
26	ARTF	13	-2.0000000E+00	-5.2731536E+04
27	ARTF	16	-2.0000000E+00	-5.0415908E+04
28	ARTF	19	-2.0000000E+00	-4.9178550E+04
29	ARTF	20	-2.0000000E+00	-4.4317502E+04
30	ARTF	21	-2.0000000E+00	-3.9504871E+04
31	ARTF	22	-2.0000000E+00	-3.8002193E+04
32	ARTF	24	-2.0000000E+00	-3.6454841E+04
33	ARTF	27	-2.0000000E+00	-3.5114615E+04
34	ARTF	28	-2.0000000E+00	-2.3514614E+04
35	ARTF	43	-2.0820000E+00	-1.1624615E+04
36	ARTF	69	-2.1807178E+00	-1.1579497E+04
37	ARTF	45	-2.0990287E+00	-1.1560221E+04
38	ARTF	39	-1.5358307E+01	-1.0833542E+04
39	ARTF	33	-4.8301641E+00	-1.0100521E+04
40	ARTF	29	-2.0000000E+00	-1.0100520E+04
41	ARTF	31	-1.2444349E-01	-5.5652315E+01
42	ARTF	53	-1.6956229E-04	-8.9729442E+00
43	ARTF	60	-1.6952492E-04	-8.9729442E+00
44	ARTF	77	-9.7031330E-08	-8.9729441E+00
45	ARTF	67	-6.6597641E-08	-8.9729441E+00

```
INITIAL FEASIBLE SOLUTION FOUND     IFEAS.SUM= -8.9729435E+00

TYPE 1 FOR GO, 0 FOR NO GO
1


  PIVOT    EXIT VAR.   ENTER VAR.  REDUC COST      OBJ. VALUE

     1        30          47      -9.5714527E+00     1.1964316E+05


OPTIMAL SOLUTION FOUND

OPTIMAL OBJECTIVE FUNCTION VALUE IS  1.3111565E+05
```

ROW	VAR.	VARIABLE VALUE	SHADOW PRICE
1	SLACK(51)	1.1986158E+03	1.1919605E+00
2	SLACK(10)	2.1200001E+03	4.6935511E-09
3	2	8.5799999E+03	7.6049357E-01
4	3	1.8040000E+04	7.8043590E-02
5	4	2.7390000E+04	-5.0804245E-01
6	5	3.2670000E+04	-2.0712427E-09
7	6	2.3320000E+04	9.1999517E-01
8	1	1.1000000E+05	4.2795064E+00
9	SLACK(9)	1.6799998E+03	0.0000000E-01
10	SLACK(38)	8.5250076E+02	-3.3270960E-08
11	7	2.3576520E+03	-2.5795065E+00
12	8	5.7460479E+03	7.6049356E-01
13	9	4.5590600E+03	7.8043579E-02
14	10	1.6510560E+04	-1.6913486E-09
16	11	2.2946880E+03	4.2795064E+00
17	SLACK(17)	1.0819440E+04	0.0000000E-01
19	12	2.6655863E+03	-1.1000000E+00
20	13	1.2885306E+03	-8.1999996E-01
21	14	8.8525006E+03	-9.6999990E-01
22	15	3.9737892E+03	-2.8059568E-09
23	16	6.8852784E+02	-5.6999996E-01
24	17	3.4967378E+04	2.5200000E+00
26	19	2.7049308E+03	-1.2600000E+00
27	SLACK(42)	3.3080560E+03	2.5199999E+00
28	SLACK(28)	7.4009398E+03	0.0000000E-01
29	SLACK(18)	9.1805598E+03	-1.9293941E-09
30	20	2.6779886E+03	-2.0200000E+00
31	21	8.3616522E+02	-1.1000000E+00
32	22	8.6102419E+02	-8.1999995E-01
33	23	5.1977836E+02	1.5776511E-10
35	25	1.9706380E+04	2.5200000E+00
36	24	7.4576899E+02	2.5199999E+00
37	SLACK(37)	1.4749923E+02	0.0000000E-01
38	SLACK(49)	6.9998066E+03	-5.7425521E-08
39	SLACK(39)	5.0643235E+02	0.0000000E-01
40	SLACK(29)	2.5990601E+03	6.5676674E-08
41	SLACK(41)	2.6919440E+03	0.0000000E-01
42	18	4.9180559E+03	-3.1587484E-09
43	26	1.3698616E+04	8.0599995E-01
44	27	6.4616111E+03	5.9599999E-01
45	28	6.6231515E+03	3.4400001E-01

ROW	VAR.	VARIABLE VALUE	SHADOW PRICE
47	29	5.5246773E+03	3.5200000E+00
48	SLACK(48)	3.0001932E+03	0.0000000E-01
49	SLACK(40)	4.9356765E+02	2.8362070E-09
50	SLACK(50)	6.3013841E+03	0.0000000E-01
51	SLACK(2)	9.9999999E+03	6.5669068E-08

VAR.	CURRENT COST	BOUNDRY COST
30	0.0000000E-01	-1.1919605E+00

VAR.	CONSTR.	CURRENT VALUE	BOUNDRY VALUE.
30	1	1.0000000E+00	0.0000000E-01

END OF PROBLEM

EXECUTION TIME: 25 MIN. 25.38 SEC.
TOTAL ELAPSED TIME: 184 MIN. 26.53 SEC.

APPENDIX IV

LINEAR PROGRAMMING COMPUTER SOLUTION

TEST PROBLEM

PIVOT	EXIT VAR.	ENTER VAR.	REDUC COST	INFEAS.SUM
1	ARTF	2	-4.5999999E+01	-1.2800000E+02
2	ARTF	1	-1.6000000E+01	-3.6000001E+01
3	ARTF	4	-1.0000000E+00	-4.0000002E+00
4	ARTF	5	-1.0000001E+00	-4.0000002E+00

INITIAL FEASIBLE SOLUTION FOUND IFEAS.SUM= 4.4703470E-08

NO ARTIFICIALS IN BASIS, BUT COST(5) = 0.0000000E-01

TYPE 1 FOR GO, 0 FOR NO GO
1

PIVOT	EXIT VAR.	ENTER VAR.	REDUC COST	OBJ. VALUE

OPTIMAL SOLUTION FOUND

OPTIMAL OBJECTIVE FUNCTION VALUE IS 2.0000000E+04

ROW	VAR.	VARIABLE VALUE	SHADOW PRICE
1	SLACK(3)	3.9999999E+00	1.0000000E+03
2	1	3.0000000E+00	0.0000000E-01
3	4	8.0000000E+00	0.0000000E-01
4	2	4.9999999E-01	7.4999999E+01

VAR.	CURRENT COST	BOUNDRY COST
3	0.0000000E-01	1.0000000E+03
6	0.0000000E-01	7.5000000E+01

VAR.	CONSTR.	CURRENT VALUE	BOUNDRY VALUE
3	1	-1.0000000E+00	0.0000000E-01
6	4	-1.0000000E+00	-1.2417634E-09

END OF PROBLEM

EXECUTION TIME: 2.07 SEC.
TOTAL ELAPSED TIME: 2 MIN. 24.48 SEC.
NO EXECUTION ERRORS DETECTED

EXIT

.

NOMENCLATURE

LINEAR PROGRAMMING CODE

A(I,J)	A-matrix
ABINVS(I,J)	Inverse matrix
ABS	Absolute
B(II)	Current right-hand-side last row
B(J)	Current right-hand-side column
C(J)	Cost row coefficient
COEF	Coefficient
COST	Cost coefficient
I	Number of rows
II	Last row
III	Selected pivot row
ICOL	Listing elements on the basis
IR	II+1
IT	Iteration
ITMAX	Iterations to maximum
J	Number of columns
JJ	Last column
JJJ	Selected pivot column
JCOL	Number of slack and artificial columns

KIND	Number of NLT, NEQ, and NGT
M	Sequence number of constraints
MYPE	Maximization
MONEG	Most negative value
NC	Sequence number of cost row coefficient
NCOL	Sequence number of column
NCON	Number of constraint
NCROW	Number of element in the cost row
NEQ	Number of equality constraint
NLT	Number of less-than constraint
NGT	Number of greater-than constraint
NONZRO	Number of non-zero coefficient
NVAR	Number of variables
NROW	Sequence number of row
OPMTRX(2,I)	Selected column of C(JJJ)
OPMTRX(2,III)	Current value
OPMTRX(3,I)	Right-hand-side B(I)
OPMTRX(3,II)	Last row of B(I)
OPMTRX(3,III)	B(I) which is selected to pivot
PM	Pivot matrix
RHS	Right-hand-side
SING	Sign
TERM	Update current row B(I)

LITERATURE CITED

(1) Bland,W.F., and Davidson,R.L., 1967, Petroleum processing handbook: New York, McGraw-Hill Book Co. Inc., p. 2-50.

(2) Dosher,R.J., 1970, Trend in petroleum refining: Chem. Eng., v. 77, n. 17, p.96-112.

(3) Hengstebeck,R.J., 1959, Petroleum processing: New York, McGraw-Hill Book Co. Inc.,

(4) Hydrocarbon Processing, 1970, Refining processes handbook: Hydrocarbon Processing, v. 49, n. 9, p. 163-274.

(5) Nelson, W,L., 1952, Question on technology: Oil and Gas Jour., v. 51, n. 17, p. 117.

(6) ------------, 1958, Petroleum refining engineering: New York, McGraw-Hill Book Co. Inc.

(7) ------------, 1967, Crude oil prices,gravity schedule: Oil and Gas Jour., v. 65 n. 6, p. 163.

(8) Paradise,O.W., 1968, A guided tour of linear programming: Control Data Corporation, Data Centers Division.

(9) Preliminary paper, 1970, Petroleum processing course: CPR, Colorado School of Mines.

(10) Preliminary paper, 1968, Northwest Refinery.

(11) Symonds,G.H., 1955, Linear programming is helpful in solving gasoline refining and blending problem: Preliminary paper.