OPTIMIZATION-BASED PROCEDURES FOR UNDERGROUND MINE PLANNING

by

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ABSTRACT

There is need for greater collaboration across the disciplines of mining, geology, operations research, statistics, and computer science to improve underground mine planning. Active areas of research include, *inter alia*: (i) integrating design and scheduling – including improved geotechnical modeling, and (ii) addressing the volatility of real-time operations through more robust schedules. We address (i) through examination of a strategic underground mine design and scheduling problem by considering an ore body partitioned into panels, each of which is extracted by a specific method. An integer programming model prescribes an optimal set of methods with which to extract each panel and the corresponding schedule to maximize the net present value. The solution we provide for a base-case industry data set results in a design and corresponding schedule with 44% scaled additional value, compared to the best industry-derived solution for this strategic planning model. Related specifically to (ii), we relax the assumption of perfect knowledge regarding value and duration of each activity in an underground mining operation and present a stochastic programming model whose tractability is questionable for realistic-sized instances, and demonstrate that by relaxing certain constraints and developing a heuristic that exploits the resulting mathematical structure, we can obtain good-quality solutions, feasible for practical time horizon lengths, even in the presence of the relaxed constraints, within several hours, at most. We further demonstrate empirically that the solution quality improves relative to solving a deterministic equivalent based on point estimates of value and duration data.

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CHAPTER 1
INTRODUCTION

Mining is a critical global industry. For example, at the time of this writing, the United States Geological Survey estimates that the average American-born human will need millions of pounds of fuels, minerals and other extracted resources in their lifetime. The top ten gold-producing countries span five continents (Africa, Asia, Australia, North and South America), and the top five mining countries as a percent of gross domestic product cover four continents (Africa, Asia, Australia, and Europe). Increasingly, this demand is driving mining companies to explore and pursue deeper mineral deposits as near-surface deposits deplete. Correspondingly, there has been a significant rise in industry interest in applying operations research techniques to improve underground mine planning. Newman et al. [10] present a review of such techniques, applied both to open pit and underground mining operations. We begin this dissertation with a paper that focuses on the advancements in the mining literature since that publication, concentrating our efforts on underground applications in metalliferous deposits, e.g., copper, iron, and gold. The resulting paper, *Operations Research in Underground Mine Planning: A Review*, has been submitted to *INFORMS Journal on Applied Analytics*. The contributions to this paper specific to the dissertation writer include the technical appraisal of reviewed literature and description of reviewed methods.

Underground mine schedules seek to determine start dates for activities related to the extraction of ore. A common objective is to maximize net present value, though other objectives, such as to minimize the deviation from long-term contracts, may also be applicable. Constraints enforce geotechnical precedence between activities, and restrict resource consumption on a per-time-period basis, e.g., development footage and extracted tons. Underground mine planning optimization models can be categorized as deterministic or stochastic. Often, deterministic models are sufficiently difficult to formulate and solve, especially when they combine both strategic (or design) and operational (or scheduling) decisions. The second paper in this dissertation addresses an underground mine design and scheduling problem, in which ore extraction methods are determined and resulting mining activities are scheduled. The mining method influences necessary infrastructure, and the activities and their timing. We divide the ore body into partitions (i.e., panels), each of which is extracted using a specific method, if at all. We consider two extraction methods, namely open-stope mining and bottom-up stoping with backfill, as well as an option of doing nothing. The myriad decisions presents a challenging integer-programming problem for which we propose an optimization-based heuristic to generate an initial feasible solution. We further expedite solutions to the monolith by (i) eliminating unnecessary variables and (ii) strengthening the formulation. Our empirical study, conducted using 36 instances – four of which are directly obtained from our industry partner, demonstrate that the proposed model provides good-quality solutions (with gaps averaging less than 7%) within CPU times considered to be reasonable for long-term planning purposes, i.e., two hours or fewer. Our results also show that instances with irregular disposition and lower development rates are more tractable. Solving an industry-partner provided instance results in a design with 44% additional value compared to that obtained via industry practice. The resulting paper, *Optimizing Underground Mine Design with Method-Dependent Precedences*, has been submitted to *IISE Transactions*. The contributions to this paper specific to the dissertation writer include formulation of ($\mathcal{HDS}$) and ($\mathcal{HDS}'$), development of instances, application of an optimization-based heurisitc to generate an initial feasible solution, integration of model enhancements, and the synthesis of results.

Finally, we present a non-deterministic model, i.e., one that captures uncertainty in the input data. Taking the design as fixed (an assumption relaxed in the previous paper), we seek to improve upon strategic schedules that address activity start dates at a coarse level. Specifically, in the operational context, schedules must account for the day-to-day variability of underground mine operations, such as unanticipated equipment breakdowns and ground conditions, both of which might slow production. Additionally, ore content associated with extraction activities may not be apparent from borehole samples that generate coarse block models, resulting in miscalculated profits. At the time of this writing, the underground mine scheduling literature is dominated by a deterministic treatment of the problem, which precludes mine operators from reacting to unforeseen circumstances. We therefore propose a model that: (i) characterizes uncertainty in duration and economic value for each underground mining activity; (ii) presents a corresponding stochastic program; (iii) suggests an optimization-based heuristic; and, (iv) provides managerial insights. We show that a stochastic integer program can produce feasible schedules in an operationally feasible amount of time, while improving the implementability of said schedules. The resulting paper, *Underground Mine Scheduling Under Uncertainty*, is planned for submission to the *European Journal of Operational Research*. The contributions to this paper specific to the dissertation writer include the formulation of $(\mathcal{S})$ as a stochastic variation of a resource constrained production scheduling problem applicable to tactical underground mine scheduling, a method to represent uncertainty in duration, the construction of the set of scenarios from the deterministic MineX source of data, measures of schedule utility, and the synthesis of results.

Finally, Chapter 5 summarizes our contributions and suggests ideas for future work.

CHAPTER 2

OPERATIONS RESEARCH IN UNDERGROUND MINE PLANNING: A REVIEW

This paper has been submitted to the journal *INFORMS Journal on Applied Analytics*.

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2.1 Abstract

Mining is a critical global industry. For example, at the time of this writing, the United States Geological Survey estimates that the average American-born human will need millions of pounds of fuels, minerals and other extracted resources in their lifetime. The top ten gold-producing countries span five continents (Africa, Asia, Australia, North and South America), and the top five mining countries as a percent of gross domestic product cover four continents (Africa, Asia, Australia, and Europe). Increasingly, this demand is driving mining companies to explore and pursue deeper mineral deposits as near-surface deposits deplete. Correspondingly, there has been a significant rise in industry interest in applying operations research techniques to improve underground mine planning. Newman et al. [10] present a review of such techniques, applied both to open pit and underground mining operations. We focus here on the advancements since that publication, and concentrate on underground applications in metalliferous deposits, e.g., copper, iron, and gold.

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2.2 Introduction

Mining is a critical industry. An average person in the United States uses approximately 39,000 pounds of minerals per annum (USGS [11]). Mining is the process of extracting naturally occurring minerals from the earth through various methods which can be simply categorized as surface or underground, depending on whether the deposit is shallow or deeper-lying, respectively. Both of these broad categorizations require a mine plan to extract ore profitably and safely. Open pit, i.e., surface, mining operations have certain advantages over those related to underground mines, such as: (i) faster access to the orebody, thereby creating revenue earlier in the mine's life; and (ii) less capital up-front and shorter construction time to build the necessary infrastructure needed to reach the deposit. However, the declining discovery rate of large near-surface deposits, increasing environmental activism, and the need for a comprehensive social license to mine have been outweighing the benefits of open pit mining (Morrison [12]). Advancements in underground mining equipment, mineral processing, and safety technology have improved the economic margins of underground mineral deposits. The mining industry has, in turn, called for new analytical methods and tools to improve the mine planning process. Newman et al. [10] review such operations research techniques applied to open pit and underground mining operations. At the time of their review, underground mine optimization techniques lagged significantly behind those applied to open pit operations. In the decade since, a number of researchers have addressed this shortcoming; we review these works here, confining ourselves to the area of underground mine design and production scheduling, primarily in metalliferous deposits, e.g., copper, iron, and gold.

Mine planning determines strategic and tactical decisions for the exploitation of mineral reserves to maximize value, while considering regulatory requirements (such as safety measures, and social license to operate), and production scheduling constraints (such as precedence between extractable areas of an orebody, and mining capacity). Open pit metalliferous deposits utilize modeling structures that are relatively consistent in their operations and in the corresponding modeling decisions related to design and extraction. For example, an open pit mine must be extracted from the surface downwards, whereas underground mines display more degrees of freedom with respect to mining direction. The mine plan adopted to economically exploit a mineral deposit can vary, and is dependent on factors such as geology, the position of the orebody underground, and the strength of the host rock [13].

An underground mine consists of a series of openings needed to access minerals for economic benefit. The size, shape, location, and number of openings depend on the mining method, geological characteristics of the deposit, and operational requirements [14]. For example, **??** depicts the general layout of a sublevel stoping operation. The illustration is generic but indicates infrastructure features common to underground mines. First, all underground mines need primary access to the underground deposit, which is established through permanent infrastructure such as a vertical shaft and/or decline (ramp) system. This permanent infrastructure, called primary development, is used to transport miners, materials, ore, and waste to and from the surface. Secondary development emanates from the shaft or decline to access the orebody, i.e., the currently identified profitable extents of the mineral deposit. Examples of secondary development include production levels to access different areas of the orebody, ore passes to transport broken ore via gravity between levels, haulage levels to prepare and transport ore to the surface, and vertical raises to create a ventilation network. In addition to serving as roadways for equipment and miners, these developments also carry essential utilities such as electricity, water, and air (ventilation) to the working areas. Ore production occurs by mining stopes, which are profitable sections of the orebody identified during the design process. Once a stope has been mined, it may be backfilled with waste material to provide ground support or to facilitate mining of adjacent stopes. Mined-out working areas, i.e., parts of the orebody which have been been exhausted of ore, and the respective secondary developments leading to them, can be closed.

The scope of mine planning decisions moves from strategic to tactical as the mining project evolves. Prior to production, a mineral deposit is explored, and pre-feasibility and

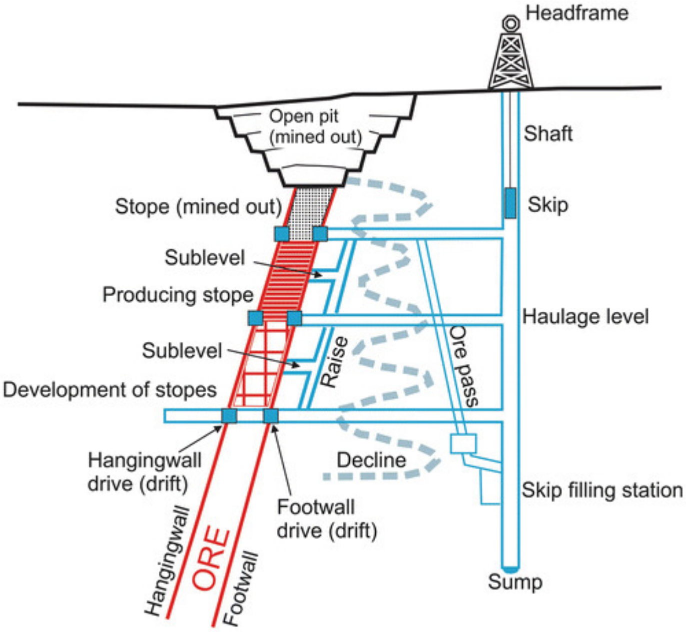


Figure 2.1: Underground development is required for access to working areas, ore transportation, ventilation, drainage, utility distribution, and equipment maintenance. (Source: Abzalov [1])

feasibility studies are conducted to assess the economic potential and technical viability. These studies are used to define the extents of the orebody and secure the capital investment necessary for initial infrastructure and development. If successful, the planning process moves into the engineering phase which specifies details regarding the implementation of the plan, including: (i) geological modeling; (ii) mine design; and (iii) production scheduling. Geological modeling discretizes the underground volume of interest into manageable units, mapping grade estimations to blocks according to sampling and statistical simulation, then organizing the result into a block model. Grade is defined as the average amount or percentage of valuable commodity contained in a unit mass of rock. Non-uniform mineralization is a common feature in many metalliferous deposits and is a consequence of inherent geological variability.

A detailed mine design for a given mining method(s) is developed based on strategic inputs such as production rate, cut-off grade, geotechnical data, and geological information (from the block model). *Production rate* is a measure of the mass or volume of material mined per time period and is dependent on the equipment employed at the operation. Mine planners distinguish ore from waste rock with a *cut-off grade*, which is determined by factors such as commodity price, and mining and milling costs [15]. Multiple cut-off grade values may be applied, depending on intended use, e.g., ore destined for stockpiling versus processing. Multiple mine designs may need to be evaluated based on the range of non-dominated strategic inputs available. *Geotechnical data* and data from other ground control studies are conducted to estimate the rock strength and stress behavior of the rock mass in the presence of underground excavations. The last step consists of production scheduling, wherein the planner defines the extraction sequence of the orebody while considering *geological data* and adhering to technical rules associated with design and resource availability. Steps (ii) and (iii) lend themselves to operations research approaches, most commonly through optimization, although simulation and queuing theory are also employed. The mine design must enable the extraction of material from the mine to make a profit or to meet a contract while adhering to constraints on ground stability and safety. The myriad of factors that must be considered in the creation and evaluation of a production schedule include: (i) economics, (ii) precedence constraints, and (iii) resource limitations. Economic factors encompass the (estimated or current) price of the commodity, as well as fixed capital and variable operational costs associated with the removal of ore and waste. Precedence constraints preclude certain blocks of rock from being extracted before others, and, in the underground mine planning setting, often include considerations involving preparation of an area to be mined and, subsequently, backfilled to create stability associated with a void [16]. Resources are connected with availability and capacity of equipment, such as haul trucks and infrastructure (e.g., processing plants), as well as with factors that help mitigate a hazardous work environment. Watson [17] emphasizes the importance of a proper mine design process for achieving production targets.

The inherent geologic variability of mineral deposits requires accounting for site-specific conditions, evaluating different operational scenarios, and identifying appropriate best prac-

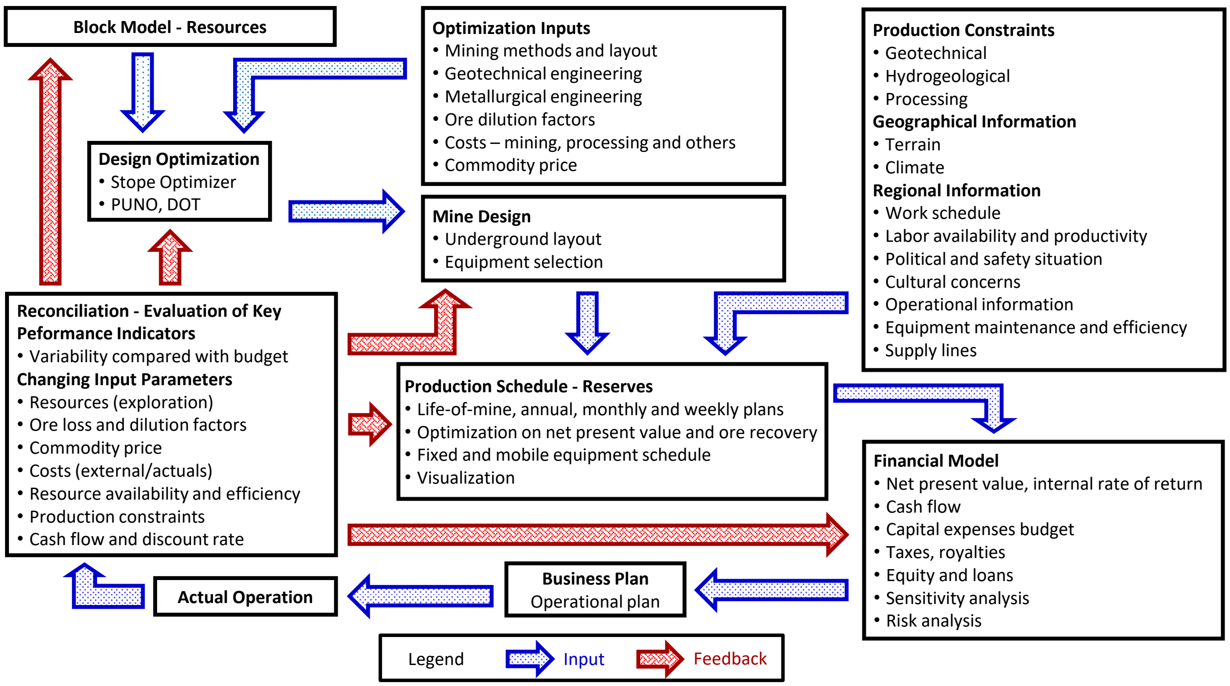


Figure 2.2: Iterative nature of underground mine planning starting with the block model, depicted in the upper left-hand corner. (Adapted from: SRK Mining)

tices. Decisions associated with developing a mine plan are difficult due to the plurality of feasible options [18]. Mine design and scheduling are part of the iterative and cyclical mine planning process (see **??**), in which multiple scenarios must be considered and evaluated. Feedback from previous decisions, completed operations, and observations can assist future decision making [19].

Less than a decade before the time of this writing, authors assumed that there was no single, uniform way in which underground mine planning could be expressed as an integer program. While underground operations are typically much more heterogenous than their open pit counterparts [13], a general formulation applicable to many operations, regardless of method, exists. We use the notation that $a \in \mathcal{A}$ denotes the set of activities to be executed under a given design, the predecessors to each activity a populate the set $\mathcal{P}_a$, and each activity is associated with a net present value c_{at} and a resource consumption q_{ra} whose maximum value for a given resource r in each time period t is $\bar{r}_{rt}$. We then determine whether or not activity a starts in time period t, X_{at}, to maximize the value of the schedule of activities subject to resource and precedence constraints. Other considerations are the duration of each activity, the delay associated with a pair of activities, and resource constraints that may be aggregated over multiple time periods. We present this binary program, known as a *Resource-Constrained Production Scheduling Problem* (RCPSP) in the appendix and note that it can be solved efficiently with the academic software OMP [20], software based on a specialized algorithm [21] that provides the linear-programming relaxation solution of the integer scheduling model.

2.3 Underground Mine Design

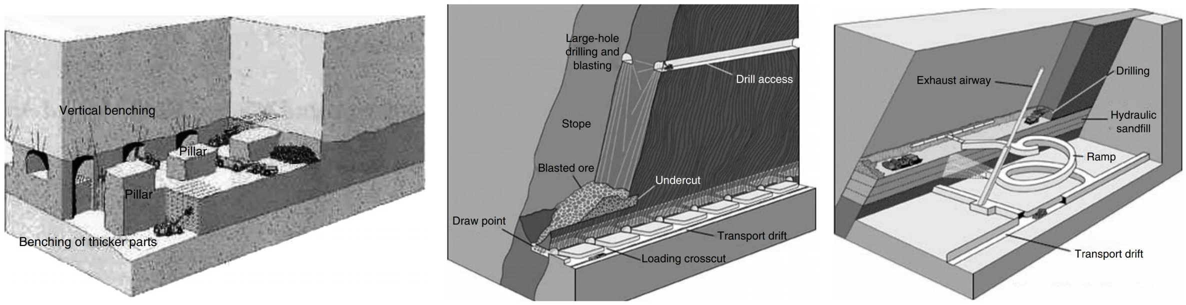
Hall [22] documents how traditional methods that include cost reduction policies to meet production targets do not achieve strategic goals such as maximizing net present value, maximizing return on investment or minimizing deviation from production goals. The author demonstrates this by treating cut-off grade as time-varying and accounting for development costs and production. Strategic decisions are typically associated with a 10-to-20 year, or life-of-mine, scope [23]. The corresponding analysis often considers insights from several models at a tactical level to create a global mine plan, which considers the interactions and influences between individual systems, omitting detail to maintain tractability [24, 25]. Many strategic decisions are made during the initial planning stages of a mining operation and are supported by a *feasibility study* [15]. Five primary, strategic decisions include: (i) mining method selection, (ii) cut-off grade, (iii) layout and infrastructure, (iv) open pit-underground transition, and (v) equipment selection. We address each of these in turn.

2.3.1 Mining Method Selection

The optimal mining method is the means by which to most profitably extract ore given geological, economic, and engineering conditions. There are numerous underground mining methods based on how the rock mass is supported during mining; broadly speaking, these are classified as (i) unsupported, e.g., room-and-pillar mining, sublevel stope mining; (ii) supported, e.g., cut-and-fill mining, longwall mining; and (iii) caving, e.g., sublevel caving, block caving [3, 26].

For horizontally-bedded deposits with a strong host rock, *room-and-pillar* mining (Figure 2.3(a)) extracts a series of rooms within the orebody and leaves pillars of ore in place to support the overlying strata. This results in a repeating pattern, with room and pillar sizes determined by the strength of the pillar material and depth of mining; lower strength material and greater depth result in bigger pillars and smaller rooms. It may be possible to extract a subset of the pillars if geotechnically feasible, i.e., the remaining pillars are able to sustain the increased vertical stresses. This is known as retreat mining, and begins at the pillar furthest from the exit. Steeply dipping deposits, where the orebody is particularly thick and the host rock is strong, are suited to the large-scale and economically efficient *long-hole stoping* process (Figure 2.3(b)), in which a sublevel is developed below the ore to be excavated. The ore is then drilled and blasted, after which is falls to the sublevel where a loader scoops up the broken rock and carries it to a truck or ore pass to be transported to the surface. Where the mine has poor host rock strength, *drift-and-fill,* also known as *cut-and-fill*, mining is used (Figure 2.3(c)), in which a corridor of ore, known as a drift, is removed from the orebody. The resultant void is then backfilled with either waste rock or processing tailings, both of which would be combined with a binding agent, i.e., cement, to provide structural support before mining an adjacent drift through the ore [2]. Sometimes, more than one method is employed to extract ore in a single mine [5].

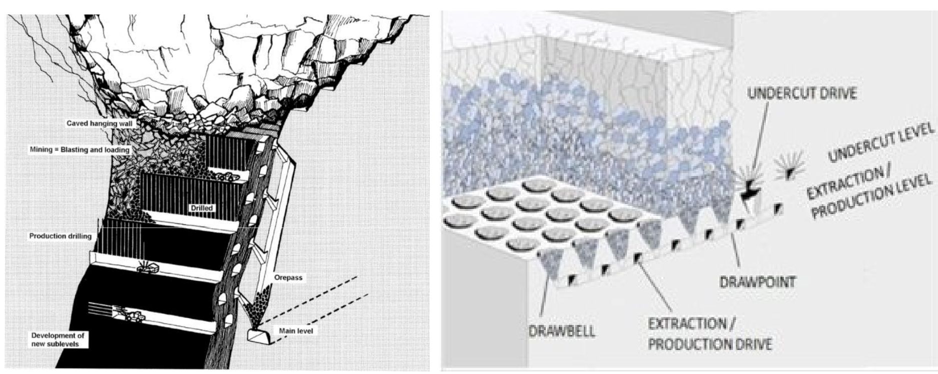
In *sublevel caving* (Figure 2.4(a)), parallel developments run along or across a steeply dipping orebody from the footwall, i.e., the underlying side of the orebody. The ore is then drilled, blasted, and transported. Mining progresses towards the footwall and top-to-bottom along increasingly deeper levels, with the hanging wall (rock overlying the deposit) caving behind. By contrast, *the block caving method* (Figure 2.4(b)) is applied to massive orebodies, where caving itself is used to effect ore breakage, through a combination of internal rock stresses and weight. All development, consisting of drifts, drawbells, and haulage



(a) Room and Pillar (b) Long-hole Stoping (c) Drift-and-Fill

Figure 2.3: Common underground methods include: (Figure 2.3(a)) Room-and-pillar, (Figure 2.3(b)) long-hole stoping, and (Figure 2.3(c)) drift-and-fill. (Source: Hamrin [2])

infrastructure, is located on a single extraction level below the orebody and is constructed prior to production. Once in place, the orebody is undercut, i.e., a slice of the orebody supporting the overlying rock is extracted, thus allowing the orebody to cave. The broken ore is carefully drawn down to prevent the creation of voids in the caved material whose collapse can lead to hazardous conditions such as an air blast, or an inrush of broken material into the working face [1].



(a) Sublevel Caving (b) Block Caving

Figure 2.4: Caving methods are generally used for larger-scale, deep, vertical operations, and are based on fracturing the mineralized and surrounding waste rock under controlled conditions. (Sources: Gertsch and Bullock [3], Sainsbury et al. [4])

Mining method selection has evolved from being based on engineering intuition to various qualitative ranking systems, e.g., (Boshkov and Wright [27], Morrison [28], Laubscher

[29], Hartman [30]). The *Nicholas method* [31] represents the first truly quantitative ranking system. It uses input parameters such as deposit geometry, grade distribution, geotechnical assessments, economics, and risk sources to rank the applicability of different mining methods to a deposit. However, drawbacks to the Nicholas method and its derivatives include imprecise definitions of weights, and only slight differentiation between favorable and unfavorable scores [32].

Subsequently, researchers have attempted to alleviate these concerns by applying multi-criteria decision making methods. For example, Musingwini and Minnitt [33] use the *analytic hierarchy process* to compare four mining methods for the UG2 Reef, a geological formation in South Africa containing platinum-group metals, where the selected mining method provides the best trade-off between factors such as extraction ratio of ore and waste, production rate, and productivity, while simultaneously minimizing dilution, capital costs and operating costs. Bogdanovic et al. [34] and Gupta and Kumar [35] also use the analytic hierarchy process to determine weights of individual selection criteria. The former also implement *preference ranking organization method for enrichment evaluation* (PROMETHEE) to determine a ranking based on all mining methods, while the latter use the weights specifically for stope mining method variants. PROMETHEE offers a prescriptive ranking for complex multi-criteria problems that are characterized by expert judgment (Brans and Triomphe [36]). Ataei et al. add *Monte Carlo* simulation to determine the confidence level of each alternative with respect to the variance of different decision makers' preferences; the conventional analytical hierarchy process determines the ranking of the individual mining methods [37].

Yavuz [38] determines an underground mining method for the Kutahya-Sabuncupinar chromite orebody using *Yager's algorithm* [39], a process of select-prioritize-rank for a finite number of alternatives. Nieto [40] uses indicators that quantify deposit characteristics and maps the advantages and disadvantages of individual mining methods to favorable deposit indicators; the author presents a simple ranking approach, and provides a corresponding analysis. Other authors either treat the problem more generally (e.g., Kenzap and Kazakidis [41], Balusa and Gorai [42]), or use established methods [43]. See Iphar and Alpay [44] for a review of traditional (qualitative) mining method selection processes and multi-criteria decision-making models. With these ranking systems, the science of determining the best mining method has progressed from subjective analysis to a more objective approach, leveraging decision analysis tools based on expert elicitation that consider previous success in similar geological circumstances. Although these tools can eliminate dominated choices, they still sometimes fail to distinguish quantitatively between suitable options and instead rely on organizational familiarity for final selection.

2.3.2 Cut-off Grade

Cut-off grade is one attribute used to determine the extractable shapes of ore (**??**). A lower cut-off grade treats more rock as ore, which increases tonnage extracted at a lower average grade, thereby lengthening a mine's life. Additional development may be required for lower cut-off grades, which can be time consuming and expensive. A higher cut-off grade yields less tonnage extracted and at a higher average grade, and therefore fewer total ounces of metal produced; the reduced extraction effort may improve overall operational economics. An extremely high cut-off grade may not produce enough metal to justify the capital outlays.

Seminal work on cut-off grade selection lies in open pit mining [45], which considers mining as a three-stage operation consisting of extraction, processing, and refining. Each stage's operating capacity determines a possible cut-off grade, with one of these representing the optimum contingent on the status of the operation. Hall [46] describes the importance of cut-off grade in modern strategic mine planning and demonstrates commonly used methods in its determination (e.g., breakeven analysis, Mortimer's definition, and Lane's Methodology). McIsaac [47] develops a Monte Carlo process to set a commodity price combined with a production rate and cut-off grade to create realizations of profitability and recommend a robust long-term strategic plan. Elkington et al. [23] compare simulation-based strategies for representing grade and cost uncertainty in selecting a cut-off grade. Gu et al. [48] use dynamic

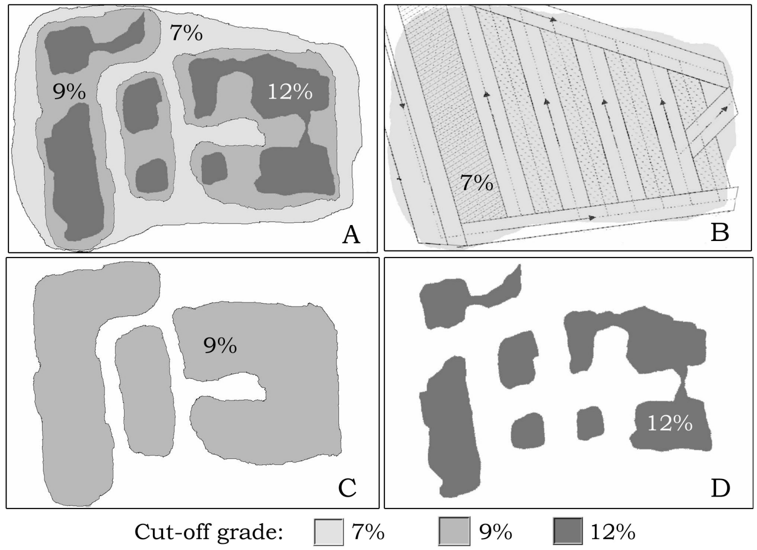


Figure 2.5: For a representative sample of ore grades, shown in Box A, a low cut-off grade (Box B) yields a large area classified as ore, while increasingly higher cut-off grades produce less extractable material, e.g., Boxes C and D. (Source: O'Sullivan and Newman [5])

programming to refine cut-off grades for a given mining method and corresponding design to maximize net present value. The authors assume a sequence of predetermined mineable sections to find the optimal final state and associated cut-off grade policy for each section. To the same end, Roberts and Bloss [49] modify an optimization model configured for open pit mine design to accommodate stope layout for a given set of enumerated characteristics, e.g., cut-off grade and mine development rate; the resulting schedules, although they do not necessarily yield a global optimum, provide strategic planning guidance.

Modern mine planning software allows operators to consider different mine designs and to evaluate a number of possible options. Alford and Hall [50] identify tools to create thousands of stopes in a matter of seconds for any cut-off grade, eliminating the tedious task of manual drawing. Therefore, the mining industry is examining more cut-off grades than ever. However, mine planners are limited in their ability to analyze the net present value of a mine schedule for each cut-off grade for the following reasons: (i) underground mine planning software is not very advanced, with many mines still relying on heuristics [51]; (ii) production schedules are difficult to generate, even for a fixed cut-off grade; (iii) studies are often performed in isolation and (or) by examining a subset of cut-off grades; and (iv) the sheer number of cut-off grades that can exist in an underground mine precludes a thorough manual analysis. It is possible to use a mixed-integer program to improve the productivity of a mine complex by selecting a cut-off grade for each mine [52]. King and Newman [53] leverage the work of Alford et al. [54] to dynamically create a stope layout for a set of viable cut-off grades and evaluate the design for a fixed set of cut-off grades by determining an associated schedule, thereby linking decisions usually made in isolation; their ability to solve large instances is possible through the exploitation of the underlying Rcpsp structure of their model.

2.3.3 Layout and Infrastructure

Underground layout and infrastructure decisions include determining the number and placement of developments and production areas with respect to the orebody. These decisions often follow the selection of a mining method. For stope mining methods such as drift-and-fill and long-hole stoping, the size and location of stopes must be determined to ensure a sufficiently high production rate, i.e., the percentage of mineable reserves extracted, and a geotechnically safe working environment. Analogous constructs in other mining methods are: (i) the size of pillars in room-and-pillar mining; and (ii) drawbell and drawpoint placement in block caving and sublevel caving operations, respectively. The primary criteria influencing room-and-pillar layout decisions are geotechnical, i.e., whether the pillars can withstand the vertical stresses exerted by the overlying strata. The critical design element in block cave layouts is drawpoint spacing because it has a significant impact on production, dilution and extraction [55].

Stope Layout

Bai et al. [56] present a *maximum flow algorithm* in which blocks (represented by nodes), or some subset thereof, constitute a single stope. The blocks chosen are subjected to constraints on stope wall angles, stope width, and stope height with respect to a single raise, i.e., vertical excavation. An optimally sized stope possesses maximum value. Bai et al. [57] extend the previous work to also include the placement and length of vertical slots, i.e., raises, used to create a void to allow for the expansion of rock when blasted. The authors demonstrate the procedure on synthetic and real data. A further extension eliminates unfavorable stopes based on additional geotechnical and shape considerations [58].

Sandanayake et al. [59] implement an enumerative heuristic respecting physical mining and geotechnical constraints to determine from a subset of possible stope layouts that which maximizes the value of an underground copper project comprised of 47,052 blocks. Erdogan et al. [60] apply to an existing mine two industry-practiced stope boundary optimization techniques, including *Floating Stope*, *Maximum Value Neighborhood*, and two heuristics introduced by Topal and Sens [61] and Sandanayake et al. [59]; the authors compare the profits associated with the stope layouts to that given by Bootsma et al. [62] and conclude that none of the resulting layouts yields as high a profit, though all approaches may yield infeasibilities. Sipeki et al. [63] consider a related mining method, top-down open-stoping, but one in which pillars, rather than backfill, provide structural stability. This implies that an important consideration is to minimize the amount of ore left in situ while still satisfying geotechnical stability constraints. The authors use a particular, relaxed integer-programming formulation to enhance model tractability.

Room-and-Pillar Layout

The primary design question for this layout is determining the pillar size and room span. Finite Element Analysis is commonly used to assess and size pillars, given geotechnical parameters such as rock strength, depth of mining, and discontinuities present in the host rock. The location of pillars is commonly defined by a uniform pattern depending on regions with similar geotechnical properties. Zipf [64] poses three pillar configurations to determine what material should be selected to serve as pillars, in this case, massive areas of ore and waste whose size is commensurate with the area that is being supported. Anani [65] determines an optimal sequence of room-and-pillar development using integer programming respecting operational, quality and precedence constraints according to objectives of (i) maximizing net present value and (ii) minimizing the discounted risk to each mining unit. This method is validated on a lead room-and-pillar mine containing 4,722 blocks, each assigned geologic attributes including grade and geotechnical risk. Anani et al. [66] investigates the impact of panel width on the cost and productivity of a room-and-pillar operation through discrete-event simulation. Gligorić et al. [67] consider uncertainty in metal price, the value of extracted ore, and resources required for extraction in scheduling a room-and-pillar mine using a fuzzy optimization model in which their variables represent whether or not a specific collection of blocks (i.e., a mining cut) is extracted in a given time period or not. They argue that the use of multiple instantiations of the objective to capture uncertainty provides a means to determine a schedule under uncertainty, and they illustrate their argument using a small case study.

Caving Layout

Caving layout decisions rely on specialized geotechnical models to ensure long-term stability of infrastructure under high stress stemming from consistent, long-term drawdown of caving material. Accurately predicting crack propagation properties of the orebody is critical in the development of an effective ore drawdown strategy. Gomes et al. [68] provide a geotechnical risk assessment for the design of developments at the Chuquicamata copper mine. They use a Monte Carlo simulation model to account for risks such as seismic activity, ground water infiltration and geologic discontinuities impacting the safety and effectiveness of construction and mining operations to recommend a probabilistically robust set of extraction and support measures. Rafiee et al. [69] combine the *bonded particle model* for intact rock and *discrete fracture network simulation* to investigate the influence of seven rock qualities, changing the value of one parameter while fixing the other six. Zarate et al. [70] inform drawpoint-layout and extraction-level decisions for a block cave mine design under grade uncertainty through enumeration of equally probable ore concentration realizations; they demonstrate the robustness of this approach over the traditional planning method that considers only a single realization.

General Infrastructure Layout

The following authors examine methods for optimally locating shafts and declines, which could be applicable to any underground mining method. Zuo et al. [71] use multi-objective optimization for underground mine design considering interactions between production, safety, and environmental impacts in order to appropriately size infrastructure and meet strategic goals. They introduce an evolutionary algorithm to recommend a strategic design for a real lead and zinc mine. Brazil et al. [72] demonstrate a procedure for optimally placing a decline juncture, allowing for multiple concurrent faces to be developed, primarily for haulage operations, in order to maximize net present value. In a similar vein with respect to the operations and objective function, Sirinanda et al. [73, 74] use a *gradient-constrained discounted Steiner point* algorithm to identify infrastructure junctures and to connect multiple sections of an orebody through declines. Carpentier et al. [75] seek a robust cut-off grade for a cluster of underground nickel mines that use the same labor and material resources; their two-stage stochastic program includes mine opening and closure and precedence and elastic constraints on mining operations (e.g., development and extraction), and maximizes net present value and minimizes deviation from target production and geological risk. Grossman et al. [76] develop a recursive algorithm, and determine its theoretical complexity, to maximize the discounted value of extracted ore by representing the model as one of traversing nodes in a tree on a graph. The problem then is to determine which nodes to visit, representing which areas of the orebody to extract for a profit, and in what order. The single resource constraint precludes more than one mining activity from being executed at a time.

2.3.4 Open Pit - Underground Transition

In addition to mining operations that consider a single broadly categorized method, many mines at the time of this writing are considering the transition between open pit and underground as the depth of the open pit mine becomes too deep to economically and/or safely sustain. Or, alternately, a pre-feasibility or feasibility study may find certain cases in which the orebody limits are amiable to both open pit and underground mining (at different depths), leading to the decision to select a design using both methods with a deliberate transition (see ??). In this case, there is a distinct portion of the orebody that is extracted via surface methods, and another part extracted via one or more underground methods. A *crown pillar* separates and provides structural stability between these two areas. Eventually, the crown pillar may be mined out as well.

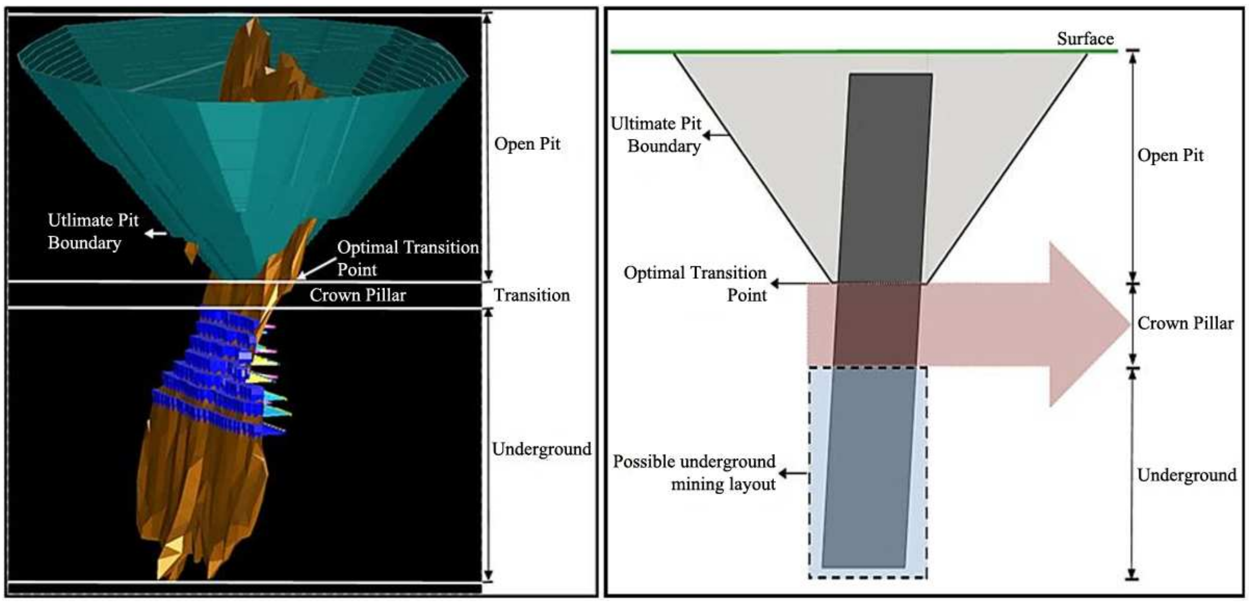


Figure 2.6: The transition between open pit and underground mining is demarcated by a crown pillar. (Source: Chung et al. [6])

The Cripple Creek and Victor gold district in Colorado, United States, is an example of an expansive deposit extracted via both open pit and underground methods. Around the turn of last century, miners sought high-grade, narrow veins through underground mining, which was made economically possible by a high price per ounce of gold at that time. After World War II, with a fixed gold price and inflation, underground mining became economically infeasible [77]. However, in the late 1970s, advancement in processing technologies allowed low-grade ore near the surface to be profitably extracted and processed, which induced economic feasibility of the open pit mining operation at the time of this writing.

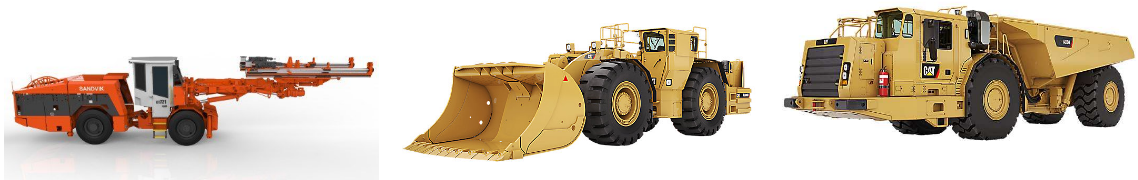
The decisions of when and how deep to install the underground infrastructure and the integration of extraction schedules above and below ground have a sizable impact on profits

[78]. Finch and Elkington [79] provide an introduction to the open pit-underground transition problem and factors associated with determining a good transition point. The author also lists three manual approaches for determining the transition point considering: (i) biggest economic pit, (ii) incremental undiscounted cash flow, and (iii) and automated scenario analysis. Opoku and Musingwini [80] determine the feasibility of an open-pit-to-underground mine transition; they develop mine designs from different realizations of resource distributions based on block models, and establish the validity of a transition based on the following indicators: (i) stripping ratio; (ii) average run-of-mill grade; (iii) profit margin per unit of mineral commodity; (iv) net present value of the operation; and (v) production rate. They apply this methodology to five international mineral deposits. Ben-Awuah et al. [81] examine a mixed-integer program that considers both open pit and underground mining methods in concert. The authors include detailed mathematical stockpiling and blending constraints, but only summarize geospatial requirements pertaining to infrastructure and precedence. The scale of the model is not specified, but instances can be solved as monoliths. The conclusion suggests that open pit and underground methods should be considered in concert to take advantage of the relative economic benefits of both. Chung et al. [6] determine design, weighing the extraction options of open pit-only, open pit-to-underground (considering stoping or block caving with an arbitrary transition point) and underground-only. MacNeil and Dimitrakopoulos [82] calculate the optimal production schedule for each depth's unique open pit and underground operations under geologic uncertainty; this produces tactical schedules that reduce a mining project's susceptibility to geological risk. Whittle et al. [83] consider the transition from open pit to underground as a maximum closure problem, and include a decision to determine the shape of the crown pillar from a set of prescribed shapes. Dagdelen and Traore [84] present an iterative procedure to determine the location of a transition using available software, e.g., Whittle, OptiMine, Studio 5D; they consider various interconnected factors such as geology, production sequencing, cost of mining for open pit and underground, cost of processing, mining rate, discount rate and revenue. Finally, King et al. [85] provide

a mixed-integer program that, when reformulated as an RCPSP, determines solutions that contain not only the depth of the transition, but also schedules at the block and stope levels (approximately 1,300 open pit bin-block combinations and 2,600 underground stoping– or stope-related – activities, respectively). These solutions are provable to within about 5% of optimality and demonstrate that a hybrid open pit-underground strategy is most economically beneficial relative to a strategy that does not combine mining methods. Over the past decade, the problem of determining the transition between open pit and underground mining has helped plan open pit operations such that the resulting revenue partially offsets the cost of the initial development of underground operations. Further research will help to determine greater synergies from the feasibility stage onwards, promoting less reactive policies.

2.3.5 Equipment

Mining equipment is required to be rugged, specialized, and reliable, thereby constituting a significant portion of the strategic capital outlay at a proposed site. In mining, rock breakage is accomplished by either blasting, i.e., drilling holes in the rock, loading them with explosives, and blasting, or by mechanical means, i.e., breaking rock using a mechanical cutter (e.g., continuous miner). The broken rock is then loaded onto haulage equipment by excavators for further transport. Open pit mines commonly use a combination of surface drill rigs, excavators, and trucks to mine both ore and waste rock. Underground mines, on the other hand, require a more varied set of equipment, each suited to a specific type of activity. For example, underground developments are excavated in two-to-three-meter increments with a traditional multi-boom jumbo (Figure 2.7(a)) to drill the holes required for the blast. Stope extraction requires the use of blast-hole drill rigs. A load-haul-dump, or LHD, (Figure 2.7(b)) is a front-end loader that loads the broken rock, and transports it over short distances to the next step in the haulage chain, e.g., to an underground haul truck, a conveyor, or an ore pass. An underground haul truck (Figure 2.7(c)) is an articulated low-profile truck to transport ore, waste rock, and backfill over long distances, potentially to and from the surface.



(a) Drill Jumbo (b) Load-Haul-Dump (c) Underground Haul Truck

Figure 2.7: Specialized heavy equipment allows for efficient underground mining. (Sources: Sandvik [7], Caterpillar [8])

The design (i.e., width, height, and length) of mineable shapes is bounded by minimum and maximum values based on the extraction equipment used and geotechnical criteria, respectively, on the economics of the operation, and on orebody characteristics [86]. Anjomshoa et al. [87] minimize the time required to achieve production targets by regulating vehicle departures from the active working area, and by placing truck passing bays in locations determined by a mixed-integer program. Operational analysis is based on mine design, available resources and production goals. Identifying differences between actual and projected production gives insight into improving operational efficiency, and refines tactical modeling assumptions [88]. The most important factors for mine equipment selection are: (i) site conditions, (ii) correct pass-matching between loaders and trucks, and (iii) proper machine configuration for optimal safety, reliability and maintenance [89].

Botin et al. [90] use simulation modeling to assess the financial risk associated with various equipment fleets in the pre-production stage of a block caving operation, and, hence, to guide investment decisions. Salama [91] contrast two differently sized haulage units, paired with a single LHD. The authors use discrete event simulation to determine the feasibility of meeting production targets, and evaluate operational conditions, i.e., truck utilization, and cycle time, based on tramming distances, mining depth, and an increased number of trucks. Salama et al. [92] utilize a similar approach to compare two loader-truck combinations and to determine each fleet's ability to achieve planned production levels, and presents a comparison based on energy utilization of four haulage systems: diesel trucks, electric trucks, shaft haulage, and conveyor belts. Energy costs are determined using discrete event simulation, after which a mixed-integer program yields schedules. Each instance possesses a gap of less than 5% after 10 hours of computation time. Research supporting the decisions concerning reliability assume a fleet configuration and use a genetic algorithm to determine a maintenance and replacement schedule [93].

Ozfirat et al. [94] use a multi-objective fuzzy goal program to select critical equipment. Simulation software, e.g., General Purpose Simulation System and Arena, improve estimates of equipment production rates by approximating operational processes under typical underground mining conditions [95].

Park et al. [96] simulate a loader-truck haulage system using cycle time as a metric, and investigate allocation options of trucks to working areas to improve daily production and to shorten average delay. Pérez et al. [97] investigate fleet configuration through an iterative methodology, producing schedules with integer programming and evaluating resulting equipment performance via simulation. Consideration of equipment maintenance is often done after acquisition. Bouffard et al. [98] apply discrete event simulation to evaluate potential designs for a potash mine as part of a prefeasibility study. Trade-off analysis compares expenditures and production values for different strategies of mine haulage, equipment maintenance, and automation to improve capital outlay.

Selecting a design requires many strategic decisions to achieve goals, often considering multiple criteria such as economics, safety, production and public perception. No published model has yet demonstrated a comprehensive treatment. There is little evidence that the mine design process can be successfully decomposed to allow for an optimal solution via a series of short-sighted goals. Likely, there is a need to increase the scope of current strategic tools to identify and inform operations within the framework of an optimal strategic plan.

2.3.6 Combination of Design and Scheduling

Some models possess a sufficiently wide scope that decisions affecting both mine design and production scheduling are made in concert. For example, Nehring et al. [99, 100] identify potential scheduling infeasibilities for a case study involving 30 stopes. The results show that the net present value only increases marginally for an approach that considers both strategic and tactical decisions, but the authors contend that their method is still superior in that it identifies infeasbilities in tactical schedules.

Epstein et al. [101] present a supply chain model for Codelco. The authors evaluate underground and open pit production schedules in tandem with downstream mineral processing operations, using a capacitated multi-commodity network flow model with inputs derived from upstream production schedules. The linear programming (LP) relaxations of the production scheduling models are solved and a rounding heuristic is used to find integer-feasible solutions. This unified approach has been implemented at multiple Codelco operations to improve individual mine production schedules and to enable planners to better evaluate strategic decisions such as the simultaneous planning of underground and open pit operations.

Copland [102] determine a sub-level stoping layout and associated production schedule using an integer program that maximizes net present value subject to mining capacities, mill requirements, stope shape geotechnics, and stope sequencing constraints. Their case study using synthetic data demonstrates a reduction in solution time and a more detailed answer than that found manually. Foroughi et al. [103] use a multi-objective integer-programming formulation to simultaneously determine stope boundaries and the production schedule; the authors use a *Non-Dominated Sorting Genetic Algorithm* to obtain a solution. Martinelli et al. [104] formulate a mixed-integer program to determine cut-off grade production scheduling decisions for a long-hole stoping and cut-and-fill mining operation. Their model possesses additional decisions associated with capital investment for development of a given mining area, which must then be closed when operation ceases. They construct a variant of a fix-and-relax heuristic to generate solutions that are within about 15% of optimality, as determined by the bound from the monolith.

Some work cited in both the **Transition** and **Cut-off Grade** sections combine design and scheduling; see King et al. [85], MacNeil and Dimitrakopoulos [82], and King and Newman [53]. While MacNeil and Dimitrakopoulos [82] use enumeration without a special algorithm to solve the resulting model, both King et al. [85] and King and Newman [53] exploit mathematical structure, leading to efficient solutions to large-scale models. Because underground mining decisions often prove interdependent, the ability to solve large-scale, integrated, design-and-scheduling models allows for a comparison of tactical and strategic options. However, at the time of this writing, the application of algorithms, e.g., Rivera et al. [20], Bienstock and Zuckerberg [21], that can be used to exploit (underlying) mathematical structure is sparse. More commonly appearing models that determine dynamic decisions take the design as fixed, and perform production scheduling in isolation. We discuss these models next.

2.4 Production Scheduling

Underground mines differ from their open pit counterparts in that the nature of the operation is much less homogeneous. As mining progresses, the open pit expands outwards and goes deeper; in underground mining, contingent on geotechnical considerations and the selected mining method, decisions have to be made regarding which section of the orebody to exploit next, and these decisions offer greater degrees of freedom. Moreover, the extractable shapes may differ greatly. Conversely, many types of activities must precede, e.g., blasting, and follow, e.g., backfilling, the sequence of underground extraction (see Figure 2.8).

Strategic schedules in mining inform the life-of-mine production targets by determining annual mining and processing capacities, the high-level feasibility of the operation, and the management of multi-mine complexes. Tactical schedules define an extraction sequence for a three- to five-year time horizon at monthly or quarterly fidelity. At the tactical level, a mine planner focuses on guiding operations to meet overall strategic goals, and seeks to determine activity start dates, typically to meet an economic objective, while adhering to restrictions regarding geospatial precedences on the mining activities and resource restrictions that govern the extent to which groups of activities can be executed within any given time period. Planners commonly attempt to create schedules to maximize the net present value of the operation, to minimize deviation of produced ore tons from those promised on long-term contracts, or to optimize a combination of other key performance indicators for the operation. The activities are derived from the mine design, with each individual activity tied to an excavation or mining task required to conduct safe operations. Resources consist of personnel with requisite skills, equipment necessary to provide the capability of executing mining activities, and availability of ventilation and refrigeration to preserve a safe operating environment. Mathematically, posing the *production scheduling problem* as an integer program requires durations for each activity, and delays between pairs of activities that directly follow each other. The remaining mathematical structure is unchanged from that of a basic open pit production scheduling problem without stockpiling, i.e., the (CPIT) problem described in Espinoza et al. [105]. As of less than a decade before the time of this writing, researchers did not recognize the existence of a general framework for underground production scheduling. As a result, they tended to simplify the model to reduce problem size or relied on heuristics tailored to a specific instance. Improved tactical scheduling also offers insight into strategic decision making and inform shift-to-shift decision making, respectively.

2.4.1 Stoping Methods

The Kittilä gold Mine uses a long-hole stoping method with backfilling and serves as a research platform for the following three papers. Song et al. [106] models underground mine scheduling as a flow shop problem to provide a tactical schedule with a set of heuristics to assign three machine sets to 35 working areas, each consisting of seven unit operations. Song et al. [107] then consider uncertainty in activity duration for a fixed production schedule and re-allocate resources in a simulation to minimize the expected critical path. Åstrand [108] solves a similarly defined flow shop problem with ten faces and two cycles at the Kittilä mine

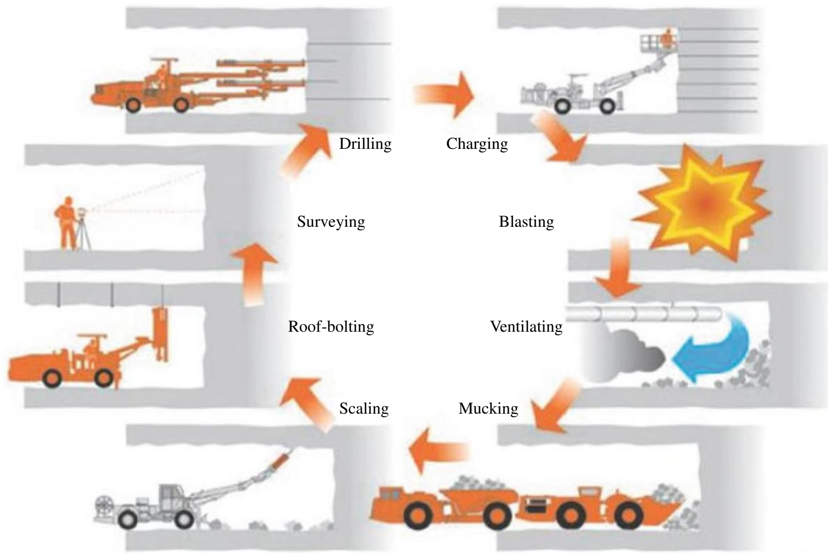


Figure 2.8: A typical underground production cycle includes a combination of the following unit operations: (i) surveying and marking drill holes on the face, (ii) collaring and drilling blast holes, (iii) charging the blast holes with explosives, (iv) blasting at the end of shift, (v) ventilating the blasted face, (vi) mucking blasted rock, (vii) scaling walls to remove loose pieces of rock, and (viii) installing roof support. (Source: Heiniö [9])

using constraint programming and a relaxation-based search heuristic.

Terblanche and Bley [109] describe a formulation, in Rcpsp format, for a notional underground stoping mine to maximize net present value while adhering to precedence and resource constraints. By using the concept of earliest and latest activity start times, as well as both spatial and temporal aggregation, they expedite solutions. Brickey [110] exploits the Rcpsp structure present in an underground stoping mine to maximize net present value subject to precedence and resource constraints, where the latter include ventilation requirements. Results from a real-world instance demonstrate a gain in net present value even when considering ventilation requirements, relative to the modus operandi. Lopes [111] formulates a detailed production scheduling model for an underground stoping and cut-and-fill mine as an Rcpsp, and determines the effectiveness of using a sliding time window heuristic to expedite solutions for large (e.g., 31,000 activities and 730 time periods), real-world instances of the problem. Wang et al. [112] formulate a multi-objective optimization model that determines grades at which to extract and process ore to maximize both profit and efficient resource utilization for a stoping mine; constraints bound the ore grades. The resulting Pareto-optimal solutions are found using a genetic algorithm.

2.4.2 Cut-and-Fill Methods

Given a design for a cut-and-fill mine, Huang et al. [113] formulate a mixed-integer program to determine the timing of extraction and backfilling decisions to maximize net present value subject to a series of operational constraints including precedence, resource capacity and blending. Using case studies containing 12 time periods and between 40 and 120 extractable units (stopes), the authors show, on average, more than 10% improvement over the net present value resulting from current operations. Brickey et al. [114] present an application of the RCPSP in determining five-year tactical schedules at daily fidelity for Barrick's Turquoise Ridge cut-and-fill mine. These models incorporate greater operational details than their predecessors from a decade or more earlier, allowing for the creation of more realistic and adoptable schedules. In turn, this has been achieved through improved formulations and solution techniques.

2.4.3 Caving Methods

Block caving operations differ significantly from other underground mining methods, in that most development activities are completed prior to full-scale production commencement. At block and panel cave operations, scheduling consists of dividing the ore columns directly above each drawpoint into slices and determining the sequence of extraction for individual slices [115]. Typically, the schedules are strategic, and serve to minimize dilution (waste mixed with ore, which reduces value) and to maintain the desired draw profile. Pourrahimian et al. [116] address production scheduling in block caving operations using a mixed-integer program, modeling the ore column above each drawpoint as a collection of slices and then determining the sequence of extraction of the individual slices. Khodayari and Pourrahimian [117] present a review of optimization applications in block-cave scheduling, identifying common mine design considerations and generally describing the scheduling problem as the selection and extraction of columns of ore over time, considering geomechanical, operational, economic and environmental constraints. Nezhadshahmohammad et al. [118] develop a mixed-integer linear program to optimize the extraction sequence of drawpoints with respect to mining capacity, continuous extraction, production grade, limits on active drawpoints and precedence between them while demonstrating results with an instance of 325 drawpoints and 15 time periods. Alonso-Ayuso et al. [119] incorporate uncertainty of commodity price in the block caving scheduling problem with a stochastic mixed-integer program that considers block clustering and precedence, flow constraints and limits for the processing stream, and extraction capacity to maximize expected net present value. They compare their results to a deterministic approach that uses the expected value of the uncertain parameters in a case study modeled after the El Teniente copper mine in Chile. Khodayari and Pourrahimian [120] present a stochastic mixed-integer program that introduces uncertainty in ore mixing amongst adjacent drawpoint columns, showing up to a 4% improvement in net present value over that provided by standard industry software for a real block caving operation of 424 drawpoints. Dirkx et al. [121] account for uncertainty in grade and drawdown rate in determining feasibility of strategic production targets for a potential mineral deposit using block cave mining. The authors use stochastic mixed-integer programming to maximize the net present value and minimize production target deviation with respect to mining capacity, continuous extraction, production grade, inter-drawpoint precedence, and milling operations.

2.4.4 Combined Methods

O'Sullivan and Newman [5] use optimization-based heuristics to develop production schedules for an underground lead and zinc mine that uses three different mining methods in Ireland. Their optimization-based heuristic yields production schedules that maintain feasibility and that can be evaluated for several end-of-life-of-mine scenarios. Campeau and Gamache [122] maximize discounted ore tonnes extracted from a Canadian mine using cut-and-fill and long-hole stoping. The authors determine the start of an activity and the corresponding tonnes extracted while accounting for unit operations and geotechnical requirements. They produce a near-optimal weekly schedule for half a year using a straight-forward implementation a commercial solver on their monolith (after some standard preprocessing techniques) that prioritizes ore tonnage over future development.

There are many systems that impact an underground mining schedule that are either not incorporated and/or not considered in concert at the time of this writing. For example, ground control systems and ventilation networks are notably absent, but lack of accurate models that can be coordinated within an optimization framework, combined with complex mathematical structures, currently preclude this level of sophistication.

2.5 Commercial Software

In practice, underground mine planning is accomplished using a number of commercial software suites offered by companies such as Deswik [123], Maptek [124], and Datamine [125], which combine a geological model and a 3-dimensional CAD platform with a Gantt-chart style scheduling tool. This software helps to define parameters and constraints necessary for the optimization models described in this paper (see **??**). For example, the Mineable Shape Optimizer [62, 126, 127] prescribes stope shapes from the geological block model in the CAD module, subject to expected geotechnical and operational limitations such as maximum safe stope size, development needed for stope access, and cut-off grade. The stope shapes are then used to delineate activities and their precedence in the scheduler according to the selected mining method. Smaller mining operations may just use the CAD functionality and forego the scheduling module for a simpler spreadsheet-based approach. Other design tools include the Planar Underground Network Optimizer (PUNO) that selects a layout for each development level while minimizing costs, and the Decline Optimization Tool (DOT) that determines optimal placement for a network of declines (both of which are owned by RPMGlobal [128]).

Relative to manual scheduling, heuristics create better schedules, despite their main disadvantage of a lack of an optimality guarantee. Heuristics use the list of activities and their precedences, as well as resource and other operational constraints, to estimate approximate activity start times, track production, and project key performance indicators. The mining industry has started to adopt corresponding software, such as the heuristic-based Revolution Mining's Schedule Optimization Tool [129], RPMGlobal's XPAC, and Datamine's Enhanced Production Scheduler. Maybee and Fava [130] and Fava et al. [131] describe a genetic algorithm-based scheduling approach that incorporates ventilation requirements and price uncertainty through high-level approximations; Sharma [132] builds on this work by considering geotechnical risk. Geovia's Personal Computer Block Caving (PC-BC), a block caving planning and scheduling software package [133, 134], is the most commonly used tool to determine schedules for a block or panel cave operation. Underground Development Sequencer and Scheduler (UDESS) [135] and Open Mine Planner (Omp) [20] are notable optimization-based scheduling solvers. UDESS is implemented as a callable Python library and coded to function with the CPLEX [136] and Gurobi [137] solvers. OMP exploits the mathematical structure in a resource-constrained production scheduling problem, capitalizing on ideas from the *Bienstock-Zuckerberg* algorithm [21]. Despite significant progress in software development over the past decade, general tools may need to be adapted to suit a specific mine. For example, the Grasberg block caving mine in Indonesia requires a more tailored approach than what PC-BC can provide to determine an operationally feasible undercut layout and corresponding schedule [138].

In practice, the mine planning process follows a sequential and iterative approach, often requiring a refinement of plans as models with updated economic, geological and geotechnical data become available. Commercial software packages are developed from successful analytical procedures addressing a step or sub-problem of the mine planning process. For example, the Deswik Resource Leveler examines resource under-utilization or over-allocation in a production schedule. Although platforms that integrate these software suites are becoming more commonplace, there is a need for better coordination among different aspects of the mine planning process, and greater standardization across the industry.

2.6 Conclusions

Over the past decade, a number of advances have been made in operations research techniques to enhance underground mine plans. Data-driven techniques increasingly supplement expert judgment. Improved representation of operational procedures, and the inclusion of interdependent decisions, have allowed for objective analysis of trade-offs between mine design and scheduling decisions. Additionally, integrated models may determine fleet configuration decisions, and maintenance and capital expenditures; they could also simultaneously inform designs for multi-mine complexes. Nonetheless, mathematical tools available at the time of this writing may require significant adjustments or modifications to obtain results for specific cases, such as multi-method operations. Commercial software platforms will continue to define data standards and facilitate technology transfer amongst academia and industry, expediting industry adoption of modeling techniques.

There is need for greater collaboration across the disciplines of mining, geology, operations research, statistics, and computer science. Active areas of research include addressing the volatility of real-time operations through more robust schedules, and integrating design and scheduling – including improved geotechnical modeling, and supply chain management. Data management may also improve confidence in decision making, allowing for a reduction in perceived economic and operational risk, thus easing the implementation of underground mine plans [139].

CHAPTER 3

OPTIMIZING UNDERGROUND MINE DESIGN WITH METHOD-DEPENDENT PRECEDENCES

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3.1 Abstract

This paper addresses an underground mine design and scheduling problem, in which ore extraction methods are determined and resulting mining activities are scheduled. The mining method influences necessary infrastructure, and the activities and their timing. We divide the ore body into partitions (i.e., panels), each of which is extracted using a specific method, if at all. We consider two extraction methods, namely open-stope mining and bottom-up stoping with backfill, as well as an option of doing nothing. The myriad decisions presents a challenging integer-programming problem for which we propose an optimization-based heuristic to generate an initial feasible solution. We further expedite solutions to the monolith by (i) eliminating unnecessary variables and (ii) strengthening the formulation. Our empirical study, conducted using 36 instances – four of which are directly obtained from our industry partner, demonstrate that the proposed model provides good-quality solutions (with gaps averaging less than 7%) within CPU times considered to be reasonable for long-term planning purposes, i.e., two hours or fewer. Our results also show that instances with

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irregular disposition and lower development rates are more tractable. Solving an industry-partner provided instance results in a design with 44% additional value compared to that obtained via industry practice.

3.2 Introduction

Underground mining seeks to extract valuable minerals, which are not directly accessible from the surface. Deposits considered for underground extraction operations often start hundreds of meters below ground and extend for several hundred more. Mine planners identify a section of natural rock with a minimal content of desirable minerals, or *cut-off grade*, as an ore body [15]. Following geotechnical engineering principles of rock mechanics, they design and operate mines to safely gain access to the ore body, and separate valuable material that generates profit from waste.

Underground mines are operated through the repeated application of mining procedures (i.e., drilling, blasting and removal of rock). A single completed cycle of these procedures is called an activity. We consider two types of activities, namely development and extraction, which consume resources and generate an economic value. Development activities create infrastructure that provides access to the ore body: declines form a system of ramps outside the vertical edge to gain vertical access; ore drives horizontally proceed through a specific level of the mine. With access, extraction activities free material and transport it to the surface for processing. We consider the ore body to be represented by subdivided partitions (i.e., panels), each to be extracted using a specific method. Standard financial analysis procedures discount the economic value of activities over time by taking into account risks, e.g., political instability, commodity prices, natural disasters.

Selecting the best combination of mining methods and corresponding activities is a design problem, while coordinating all activities of the selected design over the life of the mine is a scheduling problem. The complexities inherent to underground mine design and scheduling reveal challenges in formulating and solving the corresponding mixed-integer programming models. Therefore, design and scheduling decisions have typically been determined independently. Our contribution lies in *jointly* determining a set of methods for ore extraction and a schedule of associated activities in order to maximize the net present value obtained over the life of mine. Our proposed model could be considered as an extension of the resource-constrained project scheduling problem (RCPSP). While the standard RCPSP determines a schedule for a given set of activities with a fixed set of precedence relationships, our model considers the additional decisions of how an activity is executed, or its *mode*, which, in turn, determines resource consumption of and precedence relationships between activities.

The remainder of this paper is organized as follows. Section 3.3 provides relevant literature. Section 3.4 defines the underground design and scheduling problem. We outline the mathematical program in Section 3.5, and provide model enhancements in Section 3.6. Section 3.7 presents computational experiments. Section 3.8 concludes the paper.

3.3 Literature Review

Operations research applications inform strategic and tactical mining operations throughout the life of a mine [10, 140]. Strategic decisions affect the overall scope of a mining project, and often include *inter alia*, (i) boundaries of the ore body; (ii) cut-off grade; (iii) infrastructure (e.g., the number and placement of declines, drives, shafts and roads); (iv) mining methods; and, (v) production targets. Many of these decisions are made in isolation, with little or no assessment of outcomes at the operational level.

Alford [141] uses optimization to determine the outer boundary of an ore body for underground mine design. Alford et al. [54] notes that underground mine design is conceptually more difficult and less constrained than open pit design. King et al. [85] addresses the transition from open pit to underground. The authors account for consequences at the tactical level by successively fixing a transition point and solving for an optimal schedule, considering above-ground and underground elements, as appropriate. King and Newman [53] evaluate the strategic decision of cut-off grade using an optimization-based heuristic without accounting for method-dependent precedence between mining activities.

Operations research advancements informing strategic mining decisions are more recent than tactical mining studies. Tactical decisions include short-term production scheduling, as well as acquisition and utilization of major equipment. Williams et al. [142] use a linear programming model to determine the amount of ore to be mined each time period in order to minimize production fluctuations, subject to meeting demand and production constraints. Chanda [143] proposes an optimization-based heuristic to determine work shift schedules. Trout [144] uses a mixed-integer program (MIP) to schedule ore extraction and backfilling activities.

Carlyle and Eaves [145] determine the set of scheduled activities, including development, to inform operations at the Stillwater mine. Kuchta et al. [146] and Newman et al. [147] use a mixed-integer program to schedule iron ore production at the Kiruna Mine in Sweden. Sarin and West-Hansen [148] describe the production scheduling problem in an underground coal mine as a problem of non-preemptive scheduling of precedence-related jobs on parallel processors, model it with a mixed-integer program, and solve it via Bender's decomposition. Nehring et al. [99] give an overview of different types of MIP formulations for underground mine scheduling. O'Sullivan et al. [51] use optimization-based heuristics to develop production schedules for a complex underground mine in Ireland. Brickey [110] uses the general framework of a resource-constrained project scheduling problem for underground mine scheduling, and demonstrates its capability to model development and extraction operations of a large underground mine at daily fidelity for a two-year time horizon; an optimization-based heuristic significantly decreases the time required to produce a near-optimal schedule. Sipeki et al. [63] design a geologically similar ore body via top-down open-stoping only. The authors evaluate column placement, balancing economic extraction considerations with leaving columns of ore for geotechnical stability, and use a particular, relaxed integer-programming formulation to enhance model tractability.

The main contributions of this paper are as follows: (i) we propose an integer-programming model that jointly determines the mine design *and* schedule of resulting activities in order to maximize net present value; (ii) to tackle the computational challenges due to the combinatorial nature of this model, we enhance the model by incorporating variable elimination, modifying the model to tighten the dual bound, and developing an optimization-based heuristic in order to obtain an initial feasible solution with which to provide the solver; and, (iii) our model prescribes decisions that result in net present value which is superior to that obtained using classical industry techinques.

3.4 Problem Statement

This section provides details regarding the underground mine design and scheduling problem. We seek to design an underground mine by determining a set of ore extraction methods and a schedule of development and extraction activities in order to maximize the net present value (i.e., profitability) over a mine's lifetime.

We examine an ore body, which is defined by a minimum cut-off grade and is formed similar to a buried brick wall; a block model, consisting of mineable shapes as determined by mining engineers, represents this ore body, as shown in Figure 3.1. As a result of the block model, we can partition the ore body into regularly spaced horizontal levels and further subdivide these levels into *panels*, analogous to layers of bricks and individual bricks within a layer. In the figure, the x-, y- and z-axes represent the length of any level, its depth from the surface, and its width, respectively. To gain access to any panel, sequence-dependent development activities (i.e., declines and ore-drives) that provide infrastructure must be executed. Specifically, in order to access any level of the ore drive, a decline must be excavated as a tunnel from the surface downwards. Once a level is reached by a decline, to horizontally proceed throughout this level in order to access a specific panel, ore drives must be excavated. Some development activities may not be needed and scheduled. For example, if the entire bottom level of the ore drive shown in Figure 3.1 is not extracted, then there is no need to extend the excavation of the decline to this level. Similarly, there is no need to excavate any ore drives on this level since there is no panel to remove. Once a panel is accessed by employing development activities, it is removed and transported to the surface using a set of extraction activities. These activities are specific to the selected method for each panel. Regarding these activities, precedence requirements, resource usage, physical location and value of completion are known *a priori*.

The value (profit) of each activity comprises the difference between expected revenue and expected cost. It can be negative for activities that do not produce valuable material, and is determined by: (i) extraction cost per ton (i.e., cost of labor and equipment); (ii) tons of ore available for extraction within the space represented by an activity; (iii) extraction ratio (i.e., actual tons of ore extracted by an activity via a certain method); (iv) dilution ratio (i.e., ratio of waste rock to ore extracted per extracted waste ton); (v) backfill cost per ton; and, (vi) the price of the mineral. The economic return for a development activity (in which no ore is produced) is considered a loss, and the specific numerical value is determined by the labor, equipment and infrastructure costs per ton. The portion of a panel extracted by an activity per time period is determined by: (i) development rate; (ii) extraction rate; (iii) extraction ratio; (iv) dilution ratio; and, (v) backfill rate. Once an activity has been started, it must be completed before starting a successor activity. We use yearly time fidelity and assume that all activities, if executed, can be completed in one time period and are completed by the end of the time horizon.

Current industry practice starts by selecting a single method based on high-level ore body characteristics (e.g., grade, depth, and rock hardness), after which engineers generate a schedule of extraction activities to maximize net present value, using standard mine extraction scheduling software (e.g., Deswik Mining Consultants Pty Ltd [123], Datamine International [125]). The process is labor intensive, often requiring a month per design, and is markedly longer in atypical cases in which geologically different regions of the ore body prevent uniform application of any one method, requiring engineers to consider alternates. We consider two industry-accepted methods that could be used to extract panels conducive to this mining environment: ***top-down open-stope mining*** and ***bottom-up stoping with backfill***. In addition, we consider an option of do-nothing (which consumes no resources or

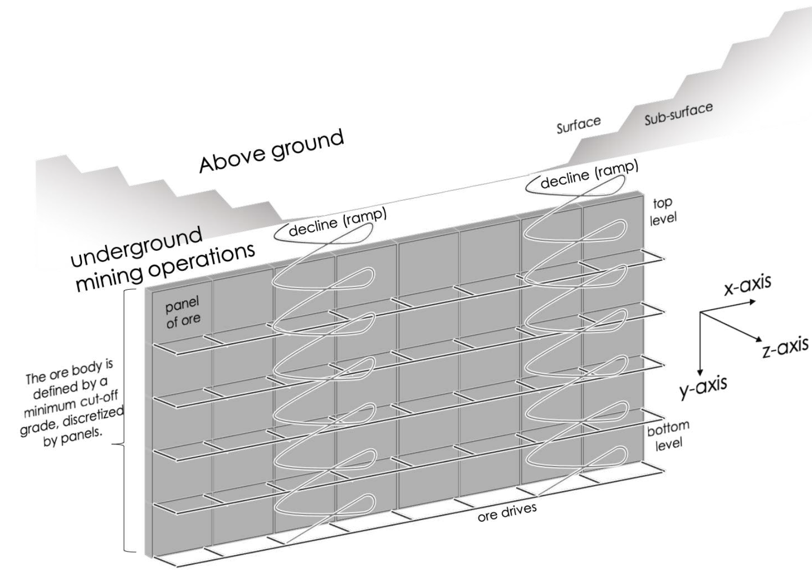

Figure 3.1: Underground mining depends on access from the surface. Depicted are two declines leading to ore drives, and, ultimately, to panels of rock to be extracted via underground mining methods.

time), by leaving a panel *in situ* as a "sill."

Top-down open-stope mining begins at the top of the ore deposit and requires minimal initial infrastructure. As the top level of the ore body is accessed by a decline, top-down mining begins and gradually progresses through lower levels (see the left panel of Figure 3.2). For cases in which higher-grade material is located at the top of the ore body, top-down mining allows access to said ore without the need to create a sterilizing sill pillar. To achieve geotechnical stability, panels are partially extracted by leaving some parts *in situ* as structural columns. As lateral stress on the walls of excavated spaces build up, each descending level requires increasingly larger structural columns, which reduces the extraction ratio (available ore for extraction).

The right panel of Figure 3.2 illustrates bottom-up stoping with backfill. This method begins by extracting panels at the bottom level of the ore body and gradually progresses towards upper levels. The large void left after every extraction is filled with "backfill" that restores geotechnical stability, allowing extraction activities to resume in adjacent panels.

While employing the bottom-up method to extract a panel, it may not be possible to extract a panel completely. Rather, some part of the panel may be required to be left *in situ* as a barrier. For example, in Figure 3.2, using the bottom-up method for the right panel requires leaving some part of the panel as a barrier, since the left panel is extracted using the top-down method. Without a barrier, backfill paste would spill over into the left panel. A similar barrier structure can be designed on the end of a panel mined via top-down open-stope mining; a barrier relaxes precedence requirements between the adjacent panels. To account for this specific situation, we consider them to be different methods, i.e., ***bottom-up method with a barrier*** and ***top-down method with a barrier***.

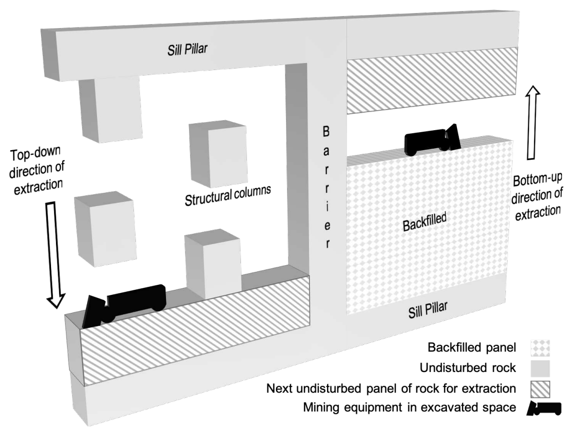

Figure 3.2: Rock left *in situ* can relax vertical (in the case of sills) or horizontal (in the case of barriers) precedence. Undisturbed rock also maintains structural integrity.

Panel groups are contiguous panels on the same level and same side of a decline, as shown in Figure 3.3. Retreat mining, or the practice of completing panels furthest from the decline first, occurs within panel groups. Panels mined within panel groups are not required to be executed using the same method. Employing more than one method to extract a group of adjacent panels requires adherence to compatibility rules; the set of methods by which a panel can be extracted depends on the methods used to extract adjacent panels. Aside from the method, the relative placement of a panel to a given panel (e.g., above, below, right, left) also affects the allowable methods in each panel and the precedence requirements (if any) between them. Figure 3.3 illustrates an example of a feasible configuration, along with the order of activity completion. Figure 3.4 illustrates all compatible (and incompatible) methods for adjacent panels (above, below, left and right) in detail. In the figure, BU LB, BU RB, TD LB, TD RB represent the modified variants of the bottom-up and top-down methods, respectively, where some portions of panels are left as barriers to the left or right. As detailed in Figure 3.4, to be able to extract a panel using the top-down method (TD), the panel should be: (i) on the level at which initial extraction commences; (ii) below a completely extracted top-down panel; or, (iii) below a sill. A panel cannot be extracted via the top-down method if the panel above is mined using the bottom-up method. To be able to extract a panel using the bottom-up method (BU), the panel should be: (i) on the lowest level of the ore deposit; (ii) above an extracted bottom-up panel; or, (iii) above a sill. Extraction activities cannot occur above a finished top-down panel or below a bottom-up panel. Conversely, follow-on activities below a top-down panel or above a bottom-up panel are not restricted.

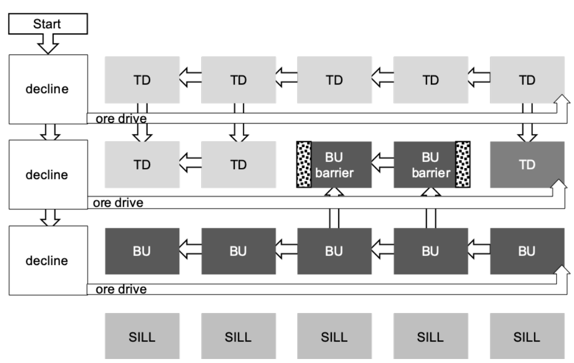

Figure 3.3: Business rules define the feasible configuration of panel-method planning, and determine the precedence order within the configuration.

Top-down and bottom-up mining possess relative advantages and disadvantages. Specifically, top-down extraction is favorably fast, and its selective nature reduces dilution, or unintended extraction of low-grade ore. Schedules derived from top-down-only mine designs generally extract ore early at a high rate. Using the bottom-up method generally yields more available ore for extraction at comparatively higher extraction ratios and rates; however, its

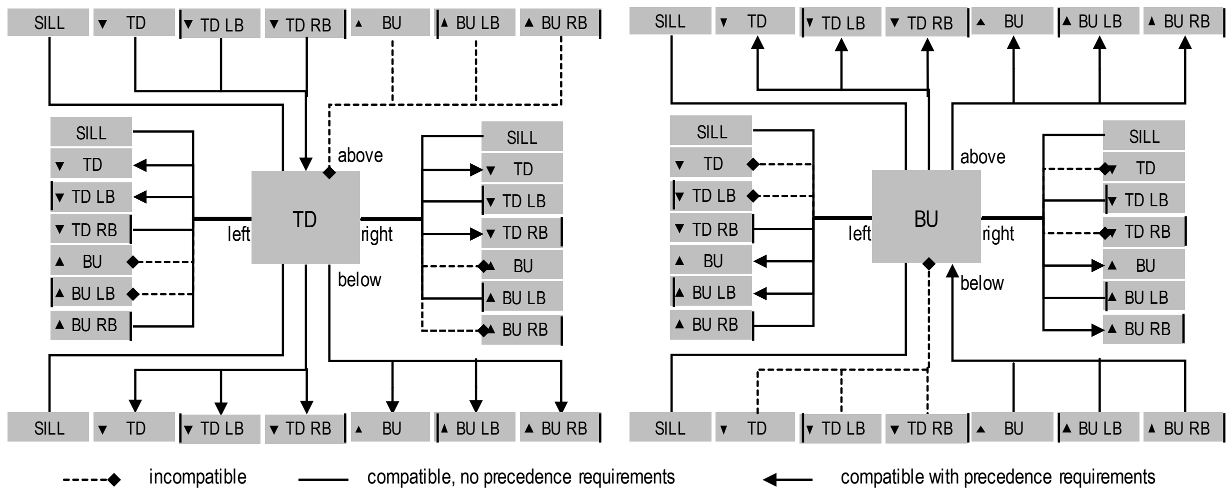

Figure 3.4: Sills allow compatibility in all directions with precedence requirements. Arrows pointing from panel p to panel p' represent panel p requiring completion before p' can start.

advantages are offset by backfill requirements. The higher extraction volume of the bottom-up method also results in higher dilution rates. The bottom-up method is slower than the top-down method partially due to the requirement of backfilling before starting to extract another panel.

Mining resources required to extract a panel by a method are listed in Table 3.1, along with their descriptions and units. These resources are limited and uniformly allocated to the panel's activities for that method. Panels that are used as sills, and the panel portions that are used as structural columns and barriers cannot be extracted, losing valuable material. The duration of each extraction activity is calculated based on the number of tons to be extracted and the steady state extraction rate.

The underground mine design and scheduling problem can be defined as follows: We seek a geotechnically-stable design by determining a method for each panel of the ore body, as well as a schedule of corresponding development and extraction activities, in order to maximize profit. In the following section, we propose a mathematical programming model for this problem, which prescribes optimal decisions by taking into account intertwined relationships between the mine design and activity schedule.

Table 3.1: Limited mining resources are required to extract a panel by a method.

Resource Type	Description	Units
single drive	single drive length	[feet]
total drive	total drive lengths	[feet]
decline	total decline development	[feet]
footage	total ore drive development	[feet]
extraction	material extracted	[tons]
tonnage	total material hauled	[tons]
ore	total ore hauled	[tons]
level limit	total activities on a level	[activities]
mine limit	total activities	[activities]

3.5 Mathematical Formulation and Complexity

For the aforementioned underground mine design and scheduling problem, we consider an integer programming model, $(\mathcal{HDS})$, which prescribes the optimal methods for panels and a schedule of resulting activities in a way that maximizes net present value. We define the notation below. Upper case letters in roman font are variables, upper case letters in calligraphic font are sets, and lower case letters are parameters and indices. Hats and overbars differentiate sets that represent similar entities.

Indices and Sets

$a \in \mathcal{A}$	activities
$p \in \mathcal{P}$	panels in a regularly spaced ore body partition
$r \in \mathcal{R}$	resources required for activity completion
$m \in \mathcal{M}$	mining methods
$d \in \mathcal{D}$	direction from panel
$t \in \mathcal{T}$	time periods
$\mathcal{F}_p \subset \mathcal{A}$	first activities of panel p
$\mathcal{L}_p \subset \mathcal{A}$	last activities of panel p
$\mathcal{I}_a \subset \mathcal{A}$	activities which precede a within same panel and method
$\mathcal{U}_r \subset \mathcal{A}$	activities that use resource r
$\mathcal{E}_{pm} \subset \mathcal{A}$	extraction activities required to mine panel p using method m
$\hat{\mathcal{P}}_{pd} \subset \mathcal{P}$	neighboring panels in direction d from panel p
$\hat{\mathcal{M}}_{md} \subset \mathcal{M}$	compatible mining methods in direction d from a method m
$\bar{\mathcal{M}}_{p'p} \subset \mathcal{M}$	methods that do not require precedence enforcement between p' and p

Parameters

v_a	value of completing activity a	[$]
c_{ar}	consumption of resource r by activity a	[units]
b_r	budget for resource r	[units/time period]
δ_t	discount factor for time period t	[fraction]

Variables

$$X_{at} = \begin{cases} 1 & \text{if activity } a \text{ starts in time } t \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{pm} = \begin{cases} 1 & \text{if panel } p \text{ is classified as using method } m \\ 0 & \text{otherwise} \end{cases}$$

Formulation

$$(\mathcal{HDS}) \quad \max_{X_{at},\, Y_{pm}} \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} \delta_t \, v_a \, X_{at} \tag{3.1}$$

$$\text{s.t.} \sum_{a \in \mathcal{U}_r} c_{ar} \, X_{at} \le b_r \qquad \forall\, r \in \mathcal{R},\ t \in \mathcal{T} \tag{3.2}$$

$$\sum_{t' \le t} X_{at'} \le \sum_{t' \le t} X_{a't'} \qquad \forall\, a \in \mathcal{A},\ a' \in \mathcal{I}_a,\ t \in \mathcal{T} \tag{3.3}$$

$$\sum_{t \in \mathcal{T}} X_{at} = Y_{pm} \qquad \forall\, p \in \mathcal{P},\ m \in \mathcal{M},\ a \in \mathcal{E}_{pm} \tag{3.4}$$

$$\sum_{m \in \mathcal{M}} Y_{pm} \le 1 \qquad \forall\, p \in \mathcal{P} \tag{3.5}$$

$$Y_{pm} \le \sum_{m' \in \hat{\mathcal{M}}_{md}} Y_{p'm'} \qquad \forall\, m \in \mathcal{M},\ d \in \mathcal{D},\ p \in \mathcal{P},\ p' \in \hat{\mathcal{P}}_{pd} \tag{3.6}$$

$$\sum_{t' \le t} \sum_{a \in \mathcal{F}_p} X_{at'} \le \sum_{t' \le t} \sum_{a' \in \mathcal{L}_{p'}} X_{a't'} + \sum_{m \in \bar{\mathcal{M}}_{p'p}} Y_{p'm}$$
$$\forall\, p \in \mathcal{P},\ d \in \mathcal{D},\ p' \in \hat{\mathcal{P}}_{pd},\ t \in \mathcal{T} \tag{3.7}$$

$$X_{at} \ \text{binary} \ \forall\, a, t \ ; \quad Y_{pm} \ \text{binary} \ \forall\, p, m \tag{3.8}$$

The objective function (3.1) maximizes the discounted value of scheduled activities, i.e., net present value. Constraint (3.2) is a knapsack that enforces capacity restrictions of mining resources for each time period. Constraint (3.3) implements precedence requirements for all extraction activities within the same panel and method to ensure geotechnical stability requirements. Constraint (3.4) requires that all activities executed via method m in panel p are completed, if method m is selected for panel p. Constraint (3.5) dictates that at most one method can be selected for each panel p. Constraint (3.6) ensures that if a method m is selected for panel p, then compatible methods must be selected for its neighboring panels in direction d. Constraint (3.7) guarantees that the last activity a' of panel p' precedes the first activity a of panel p in each direction d, if the methods of panels p and p' require precedence enforcement between p and p'. Finally, Constraint (3.8) enforces binary restrictions on the decision variables.

Proposition 1 *($\mathcal{HDS}$) is $\mathcal{NP}$-hard.*

Proof. This proof is based upon reduction from the resource-constrained project scheduling problem (RCPSP), which is notoriously known to be $\mathcal{NP}$-hard [149]. ($\mathcal{HDS}$) can be reduced to an RCPSP by fixing the design, i.e., the values of variable $Y_{p'm}$. Therefore, ($\mathcal{HDS}$) is also $\mathcal{NP}$-hard, because it is at least as difficult as an RCPSP. ■

To address the computational challenges that ($\mathcal{HDS}$) reveals due to its complexity, we propose an enhanced formulation and a solution methodology in the next section.

3.6 Model Enhancement and Solution Strategies

We propose three strategies to expedite solutions: (i) variable elimination; (ii) dual bound tightening and, (iii) an optimization-based heuristic to generate an initial feasible solution. The first two enhance the model formulation, while the third is a heuristic. We also discuss the possible addition of cuts to strengthen the formulation.

3.6.1 Variable Elimination

Eliminating decision variables has been shown to improve model tractability. For example, Basu et al. [150] applies a sequential variable elimination scheme for both integer and continuous variables. Lambert et al. [151] offers a tutorial that includes examples for variable elimination in an open pit mining problem without compromising optimality. Newman and Kuchta [152] demonstrate a form of heuristic variable elimination based on aggregating time periods, and then solving the original model using information gained from the aggregated model to reduce problem size in a large-instance underground mining application. We suggest using the precedence structure in order to reduce the set of variables. In the optimization model, there exist certain X_{at} variables which must assume a value of 0 in any feasible solution due to precedence restrictions for an associated design. Such variables can be identified and eliminated from the model without compromising optimality.

3.6.2 Enhanced Formulation

Martin [153] provides examples of enhanced (extended) formulations for tighter linear programming relaxations to improve tractability. In a similar vein, we enhance our formulation by introducing binary variables $Z_{p'p}$, which indicate whether the extraction of panel p' is completed before panel p.

$$Z_{p'p} = \begin{cases} 1 & \text{if panel } p' \text{ is extracted before } p \\ 0 & \text{otherwise} \end{cases}$$

Since these new variables yield an alternate means to account for panel extraction precedence, the set $\bar{\mathcal{M}}_{p'p}$ is no longer necessary, creating an enhanced formulation by enabling variable elimination and tightening the dual bound.

Our modified model ($\mathcal{HDS}'$) maximizes (3.1), subject to (3.2), (3.3), (3.4), (3.5), (3.6), (3.8) and the following new constraints:

$$Z_{p'p} \geq Y_{pm} + \sum_{m' \in \hat{\mathcal{M}}_{md}} Y_{p'm'} - 1 \qquad \forall\, m \in \mathcal{M},\ d \in \mathcal{D},\ \ p \in \mathcal{P},\ p' \in \hat{\mathcal{P}}_{pd} \tag{3.9}$$

$$\sum_{t' \leq t, a \in \mathcal{F}_p} X_{at'} \leq \sum_{t' \leq t, a' \in \mathcal{L}_{p'}} X_{a't'} + (1 - Z_{p'p}) \qquad \forall\, p \in \mathcal{P},\ d \in \mathcal{D},\ p' \in \hat{\mathcal{P}}_{pd}, t \in \mathcal{T} \tag{3.10}$$

$$Z_{p'p} \text{ binary } \forall\, p', p \tag{3.11}$$

Constraints (3.9) and (3.10) together replace Constraint (3.7). Specifically, Constraint (3.9) recognizes whether the extraction of panel p' is completed before panel p due to the selected methods of the panels. Constraint (3.10) guarantees that the last activity a' of panel

p' precedes the first activity a of panel p in each direction d, if the extraction of panel p' is completed before panel p. Finally, Constraint (3.11) enforces binary restrictions on the additional decision variables.

3.6.3 Cuts

Recognizing that there exists more mathematical structure between the scheduling variables, X_{at} and the variables connecting a panel and a method, Y_{pm}, than explicitly given in ($\mathcal{HDS}'$), we develop cuts to tighten the relationship. Specifically, under the assumption that each activity requires one time period to execute, we note that:

- (i) the sum of all executed extraction activities required to complete panel p using method m up until the time at which the last activity in the sequence is executed must be less than or equal to the number of such activities, if indeed the first activity in the panel has been started at time t, and 0 otherwise.

- (ii) the sum of all executed extraction activities required to complete panel p using method m up until time period $t-1$ is 0 if neither the first panel nor the last panel is executed at time period t; otherwise, this quantity is bounded above by the product of the number of activities required to complete panel p using method m and the current time period less 1 if the first panel is not mined until time period t, and is bounded above by the product of the number of activities required to complete panel p using method m and the cardinality of the set of time periods if the last panel is not mined until time period t.

- (iii) if first and last activities required to complete panel p using method m are executed in time periods t and t', respectively, then any other activity in that same panel being executed by that same method must occur between time periods t and t'.

We provide the mathematical formulation of cut (iii) below:

$$\sum_{t'=t_1}^{t_3} X_{a_2 t'} \geq X_{a_1 t_1} + X_{a_3 t_3} - 1$$

$$\forall\, a_1 \in \mathcal{E}_{pm} \cap \mathcal{F}_p, a_3 \in \mathcal{E}_{pm} \cap \mathcal{L}_p,\ t_1,\ t_3 \in \mathcal{T} : t_1 \leq t_3, a_2 \in \mathcal{E}_{pm} : a_2 \neq a_1, a_3$$

3.6.4 Model Initialization – An Optimization-based Heuristic

We propose an optimization-based heuristic that creates an initial feasible solution for ($\mathcal{HDS}'$), thus further expediting solutions. Recall that ($\mathcal{HDS}'$) can be reduced to an RCPSP for a fixed design. We propose an optimization-based heuristic in which myriad designs are generated, and, for each design, an RCPSP is solved in order to prescribe a corresponding optimal schedule of activities; based on this (partial) set of optimal schedules, the best design is selected. OMP Solver [154], an academic piece of software which is capable of quickly determining an optimal schedule for real-world instances by using the Bienstock-Zuckerberg algorithm [155], solves each RCPSP.

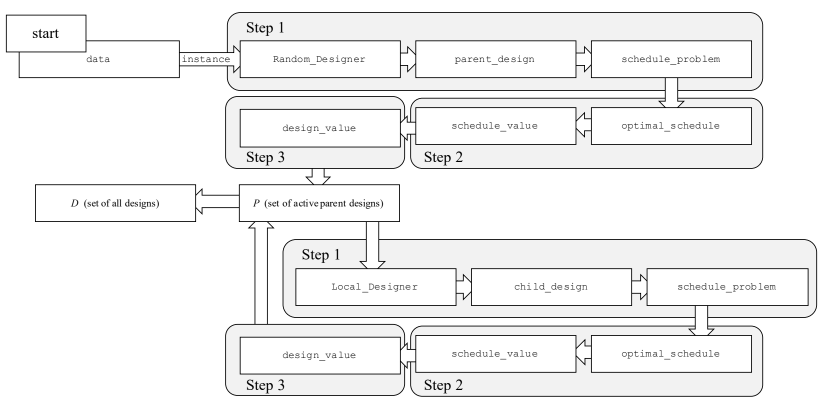

Figure 3.5: The heuristic follows a three-step procedure.

Because a complete enumeration over all possible designs is computationally intractable, the proposed heuristic efficiently searches the solution space for a “reasonable set” of designs using three steps: (i) initialization, which consists of selecting a feasible design (i.e., fixing the Y_{pm} and $Z_{p'p}$ design variables) to produce an RCPSP, and solving the RCPSP with OMP to prescribe the corresponding optimal schedule of activities; (ii) parent nomination and child production; and, (iii) child evaluation and parent replacement (Figure 3.5). Steps (ii) and (iii) iteratively evaluate the design and schedule to gain insights in order to generate improved designs that would provide better net present values.

Algorithm 1: Pseudocode for the optimization-based heuristic

```
D ← Ø   (all found feasible designs and corresponding value)
P ← Ø   (set of parent design and corresponding value)
repeat
    while (|P| ≤ min_required_designs) do
        Random_Designer produces parent_design defining schedule_problem // Step 1
        Calculate optimal_schedule and corresponding schedule_value // Step 2
        parent_design ← design_value ← schedule_value // Step 3
        D ≡ D ∪ parent_design
        P ≡ P ∪ parent_design
    end
    for each parent_design in P do
        parent_design descendent_number += 1
        for the given number of iterations do
            Local_Designer produces child_design defining schedule_problem // Step 1
            Calculate optimal_schedule and corresponding schedule_value // Step 2
            child_design ← design_value ← schedule_value // Step 3
            D ≡ D ∪ parent_design
        end
        if design_value < child_design_value then
            P ← (P \ {parent_design }) ∪ {child_design }
        end
        if parent_design descendent_number > max_allowable then
            P ← P \ {parent_design }
        end
    end
until maximum iteration
```

The heuristic is initialized by a subroutine that can randomly generate designs (Algorithm 1). This subroutine selects a transition level within each vertical stack of panels, with all panels above this transition classified as top-down and those below as bottom-up. It also randomly selects up to three sill pillars in the bottom-up section of the same vertical stack. Initialization establishes a set of parent designs and schedules, P, from which to create child designs. A local search uses a given design with its optimal schedule as a parent, and produces child designs by applying local changes. A successful child that has a higher corresponding net present value than its parent, replaces the parent in the active parent set.

Parent designs and schedules that do not produce successful children are pruned without replacement. Parents failing to produce successful children are replaced with randomly generated designs. All designs are retained in set D for post analysis. The search continues until a given number of designs with corresponding schedules are generated. Then, the design, with its associated schedule which results the highest net present value, is selected. Therefore, by using this methodology, a feasible mine design and corresponding optimal schedule is obtained.

3.7 Data and Numerical Results

This section introduces industry data, as well as the data generation scheme that uses industry data to randomly generate more instances; it also discusses our computational experiments and their results.

3.7.1 Data

From our industry partner, we obtain four base instances, (A,B,C,D), each including data of a different ore body for a large prospective mine. Each instance includes compatibility rules, precedence requirements, and mining resources. In addition, grade and cost data are provided confidentially.

These instances are obtained as block models, where each ore body is first represented as a set of blocks, which are then aggregated into activities and panels. Table 3.2 shows the number of panels, the life time (in years) of the ore body and the number of blocks for each instance. According to Table 3.2, ore bodies A and C are relatively larger than average industry-size ore bodies, whereas B and D are of typical size. For example, instance A includes 190 panels and seven methods, resulting in $\approx 8.9 \times 10^{15}$ permutations of the Y_{pm} variable alone.

Eight additional variants are developed for each of the four base instances, resulting in a total of 36 instances with modifications to the conditions of ore disposition and development rate. Ore disposition represents how the grade is dispersed in the block model in terms of

Table 3.2: Four ore bodies of varying size are represented by block models developed for planning a real mine complex.

Ore Body	$\|\mathcal{P}\|$ [panels]	$\|\mathcal{T}\|$ [years]	5×5×5 Blocks [meters]
A	190	12	5,299
B	68	5	1,470
C	152	10	4,522
D	105	9	2,382

Table 3.3: Eight variants for each ore body represent modifications to the conditions of ore disposition and development rate.

Differentiating Factors	Symbol	Description
ore disposition	.	industry selected
	H	more homogeneous than industry
	I	more irregular than industry
development rate	.	industry selected
	+	15% higher than industry
	-	15% lower than industry

expected grade per block, and is obtained by the statistical models that estimate spatial characteristics of the ore body. Our instances include the selected disposition for planning the mine, as well as block models that represent the results of a more irregular and homogeneous ore disposition. An underground mine design requires an achievable development rate of decline and ore drives. Our base instances include these development rates, which are directly obtained from the mine planner. Generated instances include development rates 15% greater and less than average. The number of development activities varies between instances depending on this rate.

3.7.2 Computational Experiments and Numerical Results

We conduct computational experiments on 36 instances: four base instances and 32 generated instances. We model our integer program using the AMPL programming language [156] execute the runs with the Version 12.7 [136] of the CPLEX solver, using a Dell PowerEdge R430 server with two Intel Xeon E5-2620 v4s (2.1 gigahertz each), 32 gigabytes of RAM, and 1 terabyte of HDD.

Industry provided two designs for base instance A, representing mining the ore body via a single method, either with bottom-up stoping with backfill or top-down open-stoping. Note that all design values are scaled to protect proprietary information. The top-down and bottom-up designs have industry-partner-verified scaled net present values of \$45M and \$52M, respectively. These two designs are compared with all of the heuristically enumerated designs and with the "optimized" (i.e., determined to be less than 10% from optimality) design in Figure 3.6. After approximately 20 minutes of run time, the heuristic found a best hybrid design, which provides a scaled net present value of \$70M, a 34% improvement over all bottom-up. Starting with this solution, and after two hours of run time, ($\mathcal{HDS}$) found a hybrid design worth a scaled net present value of \$75M, 44% better than all bottom-up.

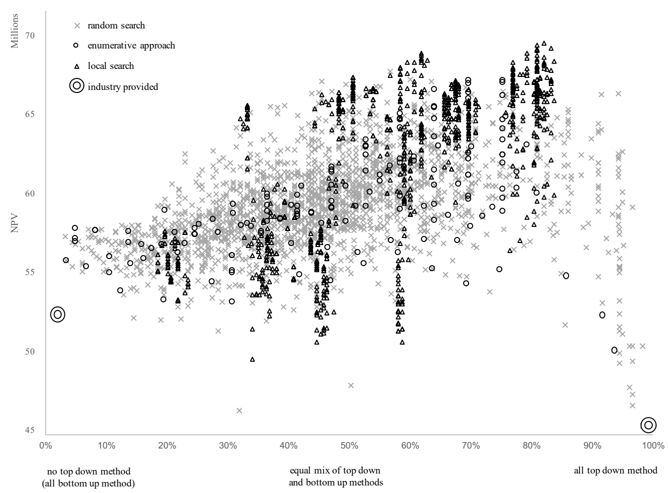

Figure 3.6: Heuristically produced feasible mine designs are evaluated by the scaled net present value of the associated extraction schedule. Designs constructed with random method arrangements define the general shape of the solution space while designs resulting from a local search for schedules with higher resultant scaled net present value better define the upper boundary. The two solutions determined via classical industry techniques, and depicted on the boundaries of the graph, correspond to extremes of method mix (\$52M for bottom-up stoping with backfill and \$45M for top-down open stope).

We use all 36 instances to test the usefulness of enhanced formulation, ($\mathcal{HDS}'$), which includes variable elimination and dual bound tightening, by comparing it to the base formulation ($\mathcal{HDS}$). We also compare the results of the enhanced formulation ($\mathcal{HDS}'$) with and without an initial feasible solution, obtained by the proposed optimization-based heuristic.

Table 3.4 compares the gap between the objective function value and lower bound of each instance, under a stopping criteria of a 1% optimality gap and a two-hour time limit–which ever is reached first, using ($\mathcal{HDS}$), ($\mathcal{HDS}'$) without an initial feasible solution, and ($\mathcal{HDS}'$) with an initial feasible solution. Results show that ($\mathcal{HDS}'$) provides an enhancement to model tractability over ($\mathcal{HDS}$). Particularly, solving the instances using ($\mathcal{HDS}$) yields a 14.8% optimality gap upon termination via the stopping criteria, on average, while using ($\mathcal{HDS}'$) decreases the average optimality gap to 9.3%. Some of this improvement is certainly attributable to the addition of the $Z_{p'p}$ variables, which decreases the density of the A matrix by a factor of three during initial testing. The initial feasible solution further reduces the average optimality gap to 7.9%. Not reported in the table are the times required to obtain the initial feasible solution, which average 20 minutes per instance. In practice, one may elect not to use the heuristic for the faster-running instances; for the more difficult cases, the additional 20 minutes has a negligible effect on solution time because the gaps tend to stagnate long before the two-hour time limit is reached.

We also examine the reduction in optimality gaps for the 16 instances which have gaps greater than 10% (specifically, 27%, on average) when modeled as ($\mathcal{HDS}$). Solving the same instances using the enhanced model ($\mathcal{HDS}'$) versus the original ($\mathcal{HDS}$) yields a 15.4% average optimality gap; the initial feasible solution further reduces the average optimality gap to 12.4%. We conclude that, for our computation, model enhancements and the initial feasible solution reduce optimality gaps to a greater extent on hard instances.

Table 3.4: Comparison of results associated with solving: (i) $(\mathcal{HDS})$ with no model enhancements or improved solution strategies; (ii) $(\mathcal{HDS})'$ using variable elimination and dual bound tightening, titled $(\mathcal{HDS})'$; and, (iii) $(\mathcal{HDS})'$ using variable elimination, dual bound tightening and a heuristically provided initial feasible solution, titled $(\mathcal{HDS})'$ With IFS. Differentiating factors in the data sets include: ore disposition, and decline and ore drive development rates. Three levels of ore disposition are: industry selected (.), more homogeneous than industry (H) and more irregular than industry (I). Three levels of development rate are: industry selected (.), 15% higher than industry (+) and 15% lower than industry (-).

Instance					$(\mathcal{HDS})$		$(\mathcal{HDS}')$ without IFS		$(\mathcal{HDS}')$ with IFS	
	Ore Body	Rate	Disp	$\lvert\mathcal{A}\rvert$	[%]	[sec]	[%]	[sec]	[%]	[sec]
1	A	.	.	4,820	44.55	7,200	11.44	7,200	11.99	7,200
2	A	.	I	4,820	6.65	7,200	5.56	7,200	4.15	7,200
3	A	.	H	4,820	39.11	7,200	12.95	7,200	9.56	7,200
4	B	.	.	1,053	12.33	7,200	11.21	7,200	3.42	7,200
5	B	.	I	1,053	0.01	355	0.01	59	0.01	1,453
6	B	.	H	1,053	5.49	7,200	4.99	7,200	4.18	7,200
7	C	.	.	3,090	22.82	7,200	17.02	7,200	16.26	7,200
8	C	.	I	3,090	9.02	7,200	8.20	7,200	6.83	7,200
9	C	.	H	3,090	21.19	7,200	19.26	7,200	15.73	7,200
10	D	.	.	1,787	6.67	7,200	6.06	7,200	5.76	7,200
11	D	.	I	1,787	0.01	2,399	0.01	66	0.01	1,336
12	D	.	H	1,787	5.76	7,200	5.24	7,200	5.22	7,200
13	A	+	.	4,802	51.31	7,200	13.15	7,200	12.43	7,200
14	A	+	I	4,802	9.92	7,200	9.02	7,200	8.63	7,200
15	A	+	H	4,802	46.80	7,200	13.35	7,200	15.31	7,200
16	B	+	.	1,044	6.54	7,200	5.55	7,200	6.11	7,200
17	B	+	I	1,044	0.01	236	0.01	230	0.01	1,206
18	B	+	H	1,044	6.01	7,200	5.46	7,200	5.43	7,200
19	C	+	.	3,087	24.39	7,200	22.17	7,200	16.83	7,200
20	C	+	I	3,087	10.76	7,200	9.64	7,200	9.08	7,200
21	C	+	H	3,087	28.33	7,200	25.75	7,200	16.63	7,200
22	D	+	.	1,782	8.59	7,200	7.81	7,200	7.62	7,200
23	D	+	I	1,782	1.23	7,200	1.12	7,200	0.01	5,966
24	D	+	H	1,782	9.70	7,200	8.82	7,200	8.64	7,200
25	A	-	.	4,867	13.26	7,200	10.84	7,200	12.35	7,200
26	A	-	I	4,867	5.64	7,200	4.98	7,200	5.13	7,200
27	A	-	H	4,867	45.71	7,200	14.16	7,200	11.77	7,200
28	B	-	.	1,075	5.27	7,200	4.46	7,200	4.22	7,200
29	B	-	I	1,075	0.43	678	0.01	135	0.01	1,297
30	B	-	H	1,075	4.98	7,200	4.53	7,200	4.60	7,200
31	C	-	.	3,139	18.18	7,200	16.53	7,200	15.4	7,200
32	C	-	I	3,139	9.50	7,200	8.16	7,200	8.68	7,200
33	C	-	H	3,139	30.35	7,200	27.59	7,200	16.2	7,200
34	D	-	.	1,814	12.87	7,200	11.70	7,200	7.91	7,200
35	D	-	I	1,814	0.14	3,500	0.13	5,857	0.01	2,127
36	D	-	H	1,814	10.63	7,200	9.66	7,200	8.12	7,200

We also conclude that model enhancements and the initial feasible solution are more helpful for the instances with a homogeneous disposition relative to those with an irregular disposition. Specifically, the initial optimality gap for $(\mathcal{HDS})$ is 21.2%, whereas that for

($\mathcal{HDS}'$) alone is 12.6% and that for ($\mathcal{HDS}'$) with an initial feasible solution is 10.1%. Solving instances with irregular disposition is more tractable than solving those with homogeneous disposition; considering all feasible designs and a corresponding set of methods that could be implemented for each specific panel, an ore body with irregular disposition may include fewer possible designs than one with homogeneous disposition. As an extreme case, consider an ore body with purely homogeneous disposition, in which all panels are identical. In this case, many symmetrical designs exist, confounding the solver from eliminating dominated or dominating solutions.

Results also show that while the usefulness of an initial feasible solution is not affected by different development rates, model enhancements have a greater impact on the instances with higher development rates. In particular, the initial optimality gap for ($\mathcal{HDS}$) is 17.0%, whereas that for ($\mathcal{HDS}'$) alone is 10.2% and that for ($\mathcal{HDS}'$) with an initial feasible solution is 8.9%. Determining dominating designs is more difficult with high development rates, because the inherent advantage of starting earlier with the top-down method is less pronounced relative to the bottom-up method, the latter of which can be executed earlier with higher development rates.

Finally, we note that the cuts we describe in §3.6.3 have a negligible benefit on the solution time and quality; in the majority of cases, in fact, the cuts have a slightly deleterious effect. However, under different algorithmic parameter settings, with a modified heuristic, or with different problem instances, they may play a role in enhancing solutions for this type of mining operation.

3.8 Conclusions and Future Directions

We address a strategic underground mine design and scheduling problem by considering an ore body partitioned into panels, each of which is extracted by a specific method, namely, top-down or bottom-up; there is also an option of leaving the panel *in situ* as a sill. Our instances consider two types of activities with precedence relationships: development and extraction. An integer programming model prescribes an optimal set of methods with which to extract each panel and the corresponding schedule to maximize the net present value. Computational results show that variable elimination, dual bound tightening and an initial feasible solution enhance the quality of the solution and the speed with which it is obtained. In particular, final gaps average 6.9% for our 36 test instances within a 2-hour time limit, where average optimality gaps are reduced to a greater extent for "difficult" instances with initial gaps of 10% of higher. The solution we provide for the base-case industry data set results in a design and corresponding schedule with 44% scaled additional value, compared to the best industry-derived solution for this strategic planning model.

Future work could focus on several enhancements resulting from a better understanding of the model's exploitable structure: This model yields loose dual bounds, in which a good integer solution is found at the early stages of the branch-and-bound tree, and most of the solver's efforts are spent on the generation of cuts to tighten the dual bound in order to prove optimality. A solution methodology that decomposes the problem might lead to early improvement of the integer solution by fixing and relaxing variables. Two promising decompositions: (i) separate the problem based on the maximum (or minimum) number of sill pillars in the design, and (ii) divide the problem into contiguous groups of panels. This decomposition might more effectively take advantage of the cuts we recommend in §3.6.3.

CHAPTER 4

UNDERGROUND MINE SCHEDULING UNDER UNCERTAINTY

This paper is planned for submission to the *European Journal of Operational Research.*

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4.1 Abstract

Underground mine schedules seek to determine start dates for activities related to the extraction of ore. A common objective is to maximize net present value. Constraints enforce geotechnical precedence between activities, and restrict resource consumption on a per-time-period basis, e.g., development footage and extracted tons. Strategic schedules address these start dates at a coarse level, whereas operational schedules must account for the day-to-day variability of underground mine operations, such as unanticipated equipment breakdowns and ground conditions, both of which might slow production. Additionally, ore content associated with extraction activities may not be apparent from borehole samples that generate coarse block models, resulting in miscalculated profits. At the time of this writing, the underground mine scheduling literature is dominated by a deterministic treatment of the problem, which precludes mine operators from reacting to unforeseen circumstances. We propose a model that: (i) characterizes uncertainty in duration and economic value for each underground mining activity; (ii) presents a corresponding stochastic program; (iii) suggests an optimization-based heuristic; and, (iv) provides managerial insights. We show that a

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stochastic integer program can produce implementable schedules in an operationally feasible amount of time.

4.2 Introduction

Underground mining seeks to extract ore from deep underground through constructed passageways, or tunnels. An underground mine design defines the infrastructure necessary to efficiently gain access to this ore, and a production schedule informs the timing of operational decisions, or the execution of activities, given a design; common objectives include maximizing net present value or minimizing deviations from contracts. Constraints: (i) enforce geotechnical precedence between activities, e.g., development in an area before extraction; and, (ii) restrict resource consumption on a per-time-period basis, e.g., development footage and extracted tons. Brickey [110] presents a generalized underground mine scheduling model as a *resource-constrained project scheduling problem* (RCPSP) in which: (i) the duration of an activity; (ii) lag, or required delay between activities; and, (iii) economic value of completing each activity are known. Scheduling synchronizes allocation of labor and mechanical resources within the production process; in practice, schedules often fall short of providing an achievable operational plan because of uncertainty associated with the parameters. We therefore propose to include uncertainty for operational decision making.

Production scheduling is used by mine management to make large financial decisions (e.g., the size and quantity of equipment to purchase) and to meet production goals (e.g., maximize net present value) [25]. A mining company operating at a specific site is subject to the oversight of a corporation, which can influence the mine design, the equipment on hand, the extraction schedules, and downstream operations. In this sense, the corresponding strategic and operational policies are driven and governed by a centralized operator who acts in the best interest of a single entity, here, the mine. In this paper, we develop policies associated with enterprises that are under the control of a single planner. We take exogenous factors (such as the price of a commodity) as given, thereby omitting market influences. However, this centralized operator must still contend with endogenous uncertainty. Operational performance does not always match expectations, and policy decisions can influence inherent stochastic processes through the timing of activities and outcome revelation. We assume that all uncertainty associated with an underground mining activity is resolved completely as soon as the activity is completed.

Uncertainty is inherent variability in a quantified mine planning attribute, and can be characterized according to some (non-)parametric distribution. *Risk*, then, characterizes the significance of uncertainty with respect to performance measures associated with mine operations. We focus on a shorter horizon, in which uncertainty is relevant for operational decision-making. A guaranteed most profitable production plan would require complete knowledge of the orebody and the associated engineering and economic parameters. Lacking this, we use a sample of the following two parameters, both of which are related to geologic uncertainty and correspond to the inability to accurately represent the grade, geologic boundaries, or other conditions of a rock mass [157]: (i) duration of each activity, associated with geotechnical uncertainty in the rock quality, and (ii) grade uncertainty, or quality of mineral per unit of volume or mass. We treat these as independent sources, and introduce a stochastic integer program which yields a production schedule that accounts for them. We develop a discrete set of scenarios via expert interpretation of the limited knowledge of a deposit. The contributions of this paper are as follows: (i) a means to characterize uncertainty in duration and ore grade through multiple scenarios; (ii) a stochastic mathematical programming formulation that accounts for uncertainty, maximizing expected net present value by defining an interval in which activities start; (iii) a corresponding optimization-based heuristic; and, (iv) managerial insights versus those from a deterministic schedule. The remainder of this paper is organized as follows: §4.3 provides a literature review of deterministic and stochastic mine planning models, with an emphasis on underground operations; §4.4 discusses how we create scenarios and formulate our integer program; §4.5 describes our solution techniques, including an optimization-based heuristic and the implications of relaxing certain constraints in our integer programming model; §4.6 introduces results and corresponding analysis, while §4.7 concludes.

4.3 Literature Review

Underground scheduling is more difficult than its open-pit counterpart [13]. The following factors are common sources of complexity: (i) the activity data, e.g., durations, are heterogeneous; (ii) practical instances are particularly large, i.e., they contain many (discrete) variables and constraints; and, (iii) there is an unstylized precedence structure and the graph corresponding to the precedence relationships between activities is dense. Trout [158] first discusses a mixed-integer program to schedule underground ore extraction and backfilling activities. Carlyle and Eaves [145] expand Trout's work by including development activities for a platinum and palladium mine in Stillwater, Montana. Kuchta et al. [146] and Newman and Kuchta [152] demonstrate a means to solve instances of a mixed-integer program that yields lower deviations from contracts compared to manual practice at Kiruna Mine, Sweden. Nehring et al. [99] integrate operational and tactical underground mining schedules into a single mathematical model through minimizing deviation of targeted mill feed grade while maximizing net present value. O'Sullivan and Newman [5] develop optimization-based heuristics that produce schedules for an underground lead and zinc mine in Ireland with a complex set of precedence constraints. Brickey et al. [114] present an RCPSP formulation to determine five-year tactical schedules at daily fidelity for Barrick's Turquoise Ridge cut-and-fill mine. Rivera et al. [20] provide software that vastly expedites solutions for models with this mathematical structure using a tailored algorithm [155]. These latter two underground mining models incorporate greater operational details than earlier work, which, in turn, produces more adoptable schedules. However, none of these references incorporates uncertainty into their scheduling paradigm, and all are therefore more suited to longer term, strategic mining.

In reality, there is uncertainty associated with most inputs, e.g., production rates, costs, and commodity prices, of the mine planning process; point estimates do not necessarily generate feasible *operational* schedules. In practice, the mining industry addresses uncertainty explicitly, but usually not through optimization-based methods. For example, a common practice in strategic decisions extends deterministic analysis by quantifying the effects of uncertainty at multiple, fixed levels of market conditions [159]. Researchers have begun to incorporate uncertainty in their models to produce more realistic operational plans. Rojas et al. [160] formulate an optimal control policy for the extraction of ore in an open pit mine, and demonstrate their methodology on a small example. Sari [161] utilize stochastic modeling to evaluate the potential for accidents and, correspondingly, worker-days lost, in a Turkish coal mine. The authors combine statistical modeling and Monte Carlo simulations. In another safety-related application, Karacan and Luxbacher [162] model the performance of gob gas ventholes, which are used to remove methane in previously mined areas of longwall coal mines; as in Sari [161], their techniques include multi-parameter regression models and Monte Carlo simulations to determine the variability in venthole performance. Caldentey et al. [163] apply real options to address price uncertainty to make capacity expansion decisions in a long-term copper mining project. While these works address uncertainty at an aggregate planning level, other researchers consider uncertainty at the block level in the production planning process. For example, Lamghari and Dimitrakopoulos [164] develop heuristic search techniques to solve an open pit mine production scheduling problem cast as a stochastic integer program that accounts for uncertainty in metal content of the extractable blocks.

There has also been work incorporating uncertainty in underground mining. For example, Alonso-Ayuso et al. [165] include uncertainty of copper price in a block caving (underground) mine scheduling problem; their stochastic program considers many scenarios, and is then transformed into a deterministic equivalent. By testing value-at-risk and conditional-value-at-risk strategies, they conclude that any risk-incorporating strategy yields higher net present value than a risk-neutral one. Carpentier et al. [75] seek a robust cut-off grade for a cluster of underground nickel mines that use the same labor and material resources; their two-stage stochastic program includes mine opening and closure, and includes precedence and elastic constraints on mining operations (e.g., development and extraction); the objective maximizes net present value and minimizes deviation from target production and geological uncertainty. Dirkx and Dimitrakopoulos [166] also account for uncertainty in grade and drawdown rate in determining feasibility of meeting long-term production targets for a potential mineral deposit using block cave mining. The authors use stochastic mixed-integer programming to maximize the net present value and minimize production target deviation with respect to mining capacity, continuous extraction, production grade, inter-drawpoint precedence, and milling operations. Del Castillo and Dimitrakopoulos [167] optimize production planning in the face of price and geologic uncertainty for an open-pit mining complex. Their model considers long-term design and fleet sizing decisions, as well as shorter term operational decisions. They apply their multi-stage model to a copper mine, and contrast their results with a those from a two-stage model.

4.4 Modeling

Our modeling efforts consist first of representing uncertainty in data sets from an industry partner, and then constructing a corresponding integer-programming model in which scenarios are associated with two specific sources of uncertainty. We describe both of these modeling efforts in turn.

4.4.1 Representation of Uncertainty in Activity Value and Duration

Uncertainty is inevitable with widely spaced drill holes from which geological information is gained to construct a block model [168]; correspondingly, this information is used to define activities and their associated characteristics such as ore content and resource requirements for their execution. One type of uncertainty lies in the inability to accurately predict grade, which impacts the value of an activity should it be associated with the extraction and sale of ore. The economic value of completing an activity depends on the mineralogical properties of the rock (such as grade concentration, rock hardness, grain size, and oxidation intensity), the capability of the mining operation, and the metallurgical efficiency of the milling process, *inter alia*. Matheron [169] provides foundations for applying statistical techniques to answer questions in mineral resource reserve estimation and grade control. A natural way to measure the quality of this estimation procedure is to compute the variance of the error it involves. This grade estimation procedure via stochastic or geostatistical simulation is mature and established [170]. Block models, a record of estimated grade for each unit of a spatially discretized orebody, are often a product of a simulation. While these methods are valid, we seek to improve upon them by exploiting all available information.

Another type of uncertainty is geotechnic, which arises from the inability to accurately estimate the quality of the rock, i.e., strength, composition, and structure, and has a direct effect on an activity's duration. Specifically, because rock masses can be unbroken (at one extreme) or highly fractured (at the other), impacting their strength, the amount and type resources needed to develop the necessary underground infrastructure can vary considerably and unpredictably. Ground control mitigates poor rock quality through engineering protocols such as roof bolts, shockcrete, and other supports, and the extent to which this control must be implemented affects the time required to complete various activities [171]. We statistically describe the nature of both grade and geotechnical uncertainty, defining notation using the conventions that lower case letters are parameters and indices; upper case letters in calligraphic font are sets, and upper case letters in roman font are variables. Hats and over-bars differentiate sets that represent similar entities.

Sets

symbol	definition
$a \in \mathcal{A}$	all project activities
$\omega \in \Omega$	scenario within sample space of possible realizations

We treat the scenario-independent values v_a and d_a as independent. A scenario indexes a single realization of geologic conditions in terms of mining requirements (e.g., extraction rates and ground control measures). Exploratory mineral deposit information in the form of proprietary borehole data and simulation produce an informative geologic model, but might

Parameters

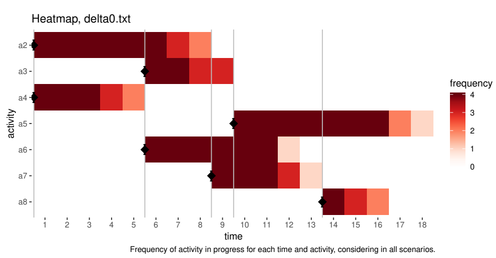
symbol	definition	[units]
v_a^ω	value of completing activity a in scenario ω	[dollars]
d_a^ω	duration of activity a in scenario ω	[time periods]

fail to leverage available borehole data. While it may be appropriate to use a single realization in a deterministic environment, short-term operations call for better estimates of each activity duration $\widehat{d}_a$. Specifically, a greater number of scenarios can improve the representation of uncertainty, yet is computationally onerous. A finite sample space Ω reduces the computational burden in which each realization, expressed as an $|\mathcal{A}|$-dimensional vector of profit-duration pairs, is defined as follows: $((v_a^\omega, d_a^\omega)\ \forall\ a \in \mathcal{A})$, where v_a^ω represents the value obtained by scheduling activity a in scenario ω and realization d_a^ω represents the duration of activity a in scenario ω.

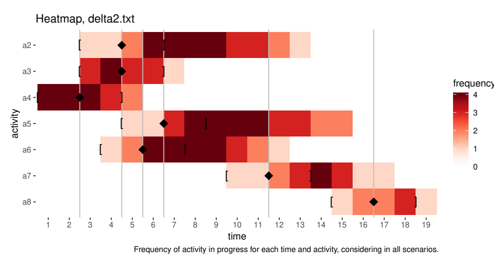
To model value v_a^ω, we use a standard geostatistical approach. We consider a continuously varying quantity over a spatial domain $D \subset \mathbb{R}^3$, and employ a Gaussian Process, defined by the property that any finite combination of observations from D follows a multivariate normal distribution. Within this framework, we use a procedure based on the Cholesky decomposition of the data variance-covariance matrix Σ to simulate values [172]. Modeling duration, d_a^ω, requires an ad-hoc approach given that the available data consists of *estimates* with only one value for each activity. Details regarding simulating both duration and value are provided in §4.6. Other areas of uncertainty include, but are not limited to, market (commodity price), consumable prices (fuel, energy, water), design, and production uncertainty [173]. We focus only on geotechnical and grade uncertainty, although other types of uncertainty could be considered within a stochastic programming environment.

4.4.2 Interval Schedules

We propose to determine a solution for a *baseline schedule* that uses insight gained from a stochastic process, employing the concept of an *interval schedule*, or operational plan consisting of an interval of time during which an activity could start such that the precedence relations and the resource availabilities are respected for all scenarios. The width of the interval can be thought about from two perspectives: (i) From the perspective of a feasible set, the greater the values for Δ_a, the more relaxed the model becomes in that schedules remain feasible even with longer-duration activities. (ii) From the perspective of a decision maker, the greater the values for Δ_a, the more risky the schedule produced by the model becomes in that significant deviations from the baseline are allowed. The higher the risk the decision maker is willing to assume, the higher the potential reward, but the more aggressive the schedule is. Reducing the values of Δ_a creates a more conservative interval schedule, which admits a lower risk tolerance on the part of the decision maker, and often sacrifices objective function value in practice. Figure 4.1 compares two interval schedules. The increase in Δ_a from 0 to 2 for all activities shows that the former subscribes to a plan equivalent to the baseline, while the latter exhibits more flexibility.



(a) Interval schedule, $\Delta_a = 0 \; \forall \; a \in \mathcal{A}$



(b) Interval schedule, $\Delta_a = 2 \; \forall \; a \in \mathcal{A}$

Figure 4.1: Accounting for uncertainty of activity duration in an interval schedule is effected by a risk tolerance measure, represented by Δ_a. Darker shades in the Gantt-style heat map indicate that more scenarios cast an activity as in progress at that time.

An interval schedule communicates uncertainty in starting times, feasibility of scenario-specific schedules, and adaptability to seek a most profitable course of action within a decision maker's risk tolerance. We now incorporate this concept into a stochastic integer program.

4.4.3 Integer Program

We develop an operational schedule that accounts for uncertainty in production estimates. We propose the mathematical model ($\mathcal{S}$), in which the objective maximizes expected net present value ($\mathbf{E}[NPV]$) through the execution of an interval schedule for a given finite sample space and subject to the corresponding restrictions: (i) an activity is completed at most once during the time horizon; (ii) mining activities follow a logical order; (iii) production is limited to availability of mining resources; (iv) the *interval schedule* is a within a maximum deviation of each scenario-specific schedule; and, (v) conditional non-anticipativity (see Goel and Grossmann [174] for the format that inspired our constraints) is maintained.

In a deterministic Rcpsp, the sum of discounted values for completed activities defines the utility of a schedule, and serves as a point estimate. Relaxing the assumption that the value of each activity and its duration are deterministic results in a multi-stage stochastic program in which the distribution of the random variables is known in advance and is independent of the decision variables. In our case, stages are defined a posteriori, i.e., after decisions are made. Specifically, the starting times of activities (i.e., the decision variables) specify the conditions under which uncertainty is realized; the distribution of the random variables depends on the decision variables. We require non-anticipativity constraints that model endogenous uncertainty in underground mine scheduling *conditional* on the information gained through completing activities. For the sake of simplicity, we refer to these as non-anticipativity constraints. However, our *conditional* non-anticipativity constraints should not be confused with the traditional non-anticipativity constraints found in standard stochastic programming handbook (e.g. Birge [175, Chapter 1]).

Set $\tilde{\mathcal{A}}^{\omega\omega'}$ contains activities that have a different duration and/or value for scenarios ω and ω'. This set is defined as follows:

$$\tilde{\mathcal{A}}^{\omega\omega'} := \{a \in \mathcal{A} : v_a^{\omega} \neq v_a^{\omega'} \vee d_a^{\omega} \neq d_a^{\omega'}, \omega < \omega'\}.$$

Sets

symbol	definition
$a' \in \mathcal{P}_a \subset \mathcal{A}$	activities a' preceding activity a
$t \in \mathcal{T}$	horizon of time periods
$r \in \mathcal{R}$	resource required for activity completion
$a \in \tilde{\mathcal{A}}^{\omega\omega'} \subseteq \mathcal{A}$	activities in ω and ω' with different value or duration, where $\omega < \omega'$

Parameters

symbol	definition	
π^{ω}	probability of scenario ω	[fraction]
q_{ar}	quantity of resource r consumed by a while in execution	[units per time period]
$\bar{q}_r$	quota of resource r available	[units per time period]
Δ_a	interval of planned start time of activity a	[time periods]
h	fractional loss of value for each subsequent time period	[fraction]
γ	discount factor, where $\gamma = \frac{1}{1+h}$	[fraction]

The variable X_{at}^{ω} indicates whether or not activity a starts in time t in each scenario. Variable Y_{at} indicates whether or not activity a starts in time t in the interval schedule. Finally, $Z_t^{\omega\omega'}$ assumes a value of zero if no activity in set $\tilde{\mathcal{A}}^{\omega\omega'}$ is finished by time t, and serves as an indication that scenarios ω and ω' are indistinguishable at the beginning of time t.

Variables

symbol	definition
X_{at}^{ω}	$= \begin{cases} 1 & \text{if activity } a \text{ starts at the beginning of time } t \text{ in scenario } \omega \\ 0 & \text{o.w.} \end{cases}$
Y_{at}	$= \begin{cases} 1 & \text{if activity } a \text{ starts at the beginning of time } t \text{ in the interval schedule} \\ 0 & \text{o.w.} \end{cases}$
$Z_t^{\omega\omega'}$	$= \begin{cases} 1 & \text{if any activity } a \text{ in set } \tilde{\mathcal{A}}^{\omega\omega'} \text{ is complete by time } t \text{ in scenario } \omega \text{ or } \omega' \\ 0 & \text{o.w.} \end{cases}$

$$(\mathcal{S}) \quad \max_{X^{\omega}_{at},\, Y_{at},\, Z^{\omega\omega'}_t} \sum_{\omega\in\Omega}\sum_{t\in\mathcal{T}}\sum_{a\in\mathcal{A}} \pi^{\omega}\, \gamma^{t+d^{\omega}_a-1}\, v^{\omega}_a\, X^{\omega}_{at} \tag{4.1}$$

$$\text{s.t.} \sum_{t\in\mathcal{T}} X^{\omega}_{at} \le 1 \qquad \forall\, a\in\mathcal{A}; \omega\in\Omega \tag{4.2}$$

$$\sum_{t'\le t} X^{\omega}_{at'} \le \sum_{t'=1}^{t-d^{\omega}_{a'}} X^{\omega}_{a't'} \qquad \forall\, a\in\mathcal{A};\ a'\in\mathcal{P}_a;\ t\in\mathcal{T};\ \omega\in\Omega \tag{4.3}$$

$$\sum_{a\in\mathcal{A}} \quad \sum_{t'=\max\{1,t-d^{\omega}_a+1\}}^{t} q_{ar}\, X^{\omega}_{at'} \le \bar{q}_r \qquad \forall\, r\in\mathcal{R};\ t\in\mathcal{T};\omega\in\Omega \tag{4.4}$$

$$\sum_{t'\le t} Y_{at'} \le \sum_{t'=1}^{\min\{|\mathcal{T}|,t+\Delta_a\}} X^{\omega}_{at'} \qquad \forall\, a\in\mathcal{A};\ t\in\mathcal{T};\ \omega\in\Omega \tag{4.5}$$

$$\sum_{t'\le t} X^{\omega}_{at'} \le \sum_{t'=1}^{\min\{|\mathcal{T}|,t+\Delta_a\}} Y_{at'} \qquad \forall\, a\in\mathcal{A};\ t\in\mathcal{T};\ \omega\in\Omega \tag{4.6}$$

$$Z^{\omega\omega'}_t \le \sum_{a\in\tilde{\mathcal{A}}^{\omega\omega'}} \left(\sum_{t'=1}^{t-d^{\omega}_a} X^{\omega}_{at'} + \sum_{t'=1}^{t-d^{\omega'}_a} X^{\omega'}_{at'} \right) \qquad \forall\, t\in\mathcal{T};\omega,\omega'<\omega\in\Omega \tag{4.7}$$

$$X^{\omega'}_{at} - Z^{\omega\omega'}_t \le X^{\omega}_{at} \le X^{\omega'}_{at} + Z^{\omega\omega'}_t \qquad \forall\, a\in\mathcal{A};\ t\in\mathcal{T};\omega,\omega'<\omega\in\Omega \tag{4.8}$$

$$X^{\omega}_{at}, Y_{at}, Z^{\omega\omega'}_t \ \text{ binary} \qquad \forall\, a\in\mathcal{A};\ t\in\mathcal{T};\ \omega,\omega'\subseteq\Omega \tag{4.9}$$

The objective, represented by (4.1), is to maximize the discounted value of scheduled activities, probability-weighted by scenario, or **E**[NPV]. Constraints (4.2) state that each activity can start at most once in each scenario. Constraints (4.3) enforce activity completion precedence within each scenario $\omega \in \Omega$. Constraints (4.4) ensure that resource consumption, e.g., tonnage hauled, labor hours used, across all activities does not exceed the quota $\bar{q}_r$ within each scenario for each resource r and time period t. Constraints (4.5) and (4.6) observe a maximum deviation between the interval schedule Y_{at} and all other times scheduled for activity a in each scenario.

Non-anticipativity constraints link decisions by scenario. If two scenarios ω and ω' are indistinguishable up to a given time period t, i.e., each activity in ω and ω' possesses the same value and duration up to time t, then the related decisions (X^{ω}_{at} and $X^{\omega'}_{at}$) up to that period must also be the same. Scheduling decisions in period t are made within the context of information available by $t-1$. Revealed information through activity completion allows schedules with activity a in set $\tilde{\mathcal{A}}^{\omega\omega'}$ to diverge. Constraints (4.7) restricts variable $Z_t^{\omega\omega'}$ if no activity in set $\tilde{\mathcal{A}}^{\omega\omega'}$ is finished by time t. Scheduling decisions in scenarios ω and ω' are equal when $Z_t^{\omega\omega'}$ remains zero, enforced by Constraints (4.8). Constraints (4.9) enforce integrality.

Example

The following example shows how non-anticipativity Constraints (4.7) and (4.8) distinguish scenarios. We say that two scenarios are identical before evidence is revealed to distinguish them. We assume there are two activities (a_1 and a_2) and two scenarios (ω_1 and ω_2). There is only one unit of resource available per time period and activities consume one unit of resource per time period of execution. For simplicity, there are no precedence constraints. Table 4.1 provides value and duration for each activity in each scenario.

Table 4.1: The value, v_a^ω, and duration, d_a^ω, for each activity and scenario in the example.

	v_a^ω		d_a^ω	
	ω_1	ω_2	ω_1	ω_2
a_1	1	1	2	1
a_2	1	1	4	4

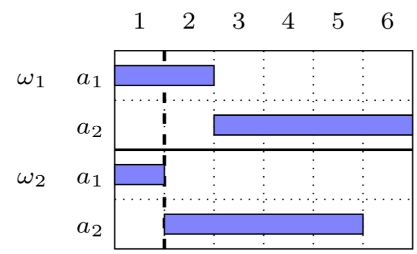
Let us consider two policies: in the first, activity a_1 is executed first, followed by activity a_2; in the second, the reverse. Figure 4.2 represents both policies. For both schedules, Constraint (4.7) forces $Z_1^{\omega_1\omega_2}$ to a value of zero in the first time period, 1, shown explicitly for activity a_1, the only activity in $\tilde{\mathcal{A}}^{\omega\omega'}$:

$$Z_t^{\omega_1\omega_2} \leq \sum_{t'=1}^{t-d_{a_1}^{\omega_1}} X_{a_1t'}^{\omega_1} + \sum_{t'=1}^{t-d_{a_1}^{\omega_2}} X_{a_1t'}^{\omega_2}$$

$$Z_1^{\omega_1\omega_2} \leq \sum_{t'=1}^{1-2} X_{a_1t'}^{\omega_1} + \sum_{t'=1}^{1-1} X_{a_1t'}^{\omega_2}$$

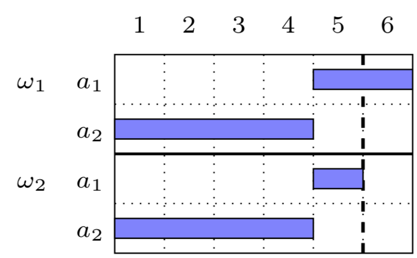
$$Z_1^{\omega_1\omega_2} \leq 0 + 0$$

t		1	2	3	4	5	6
ω_1	a_1	1	0	0	0	0	0
	a_2	0	0	1	0	0	0
ω_2	a_1	1	0	0	0	0	0
	a_2	0	1	0	0	0	0



(a) Policy 1: X_{at}^{ω} values and corresponding Gantt chart

t		1	2	3	4	5	6
ω_1	a_1	0	0	0	0	1	0
	a_2	1	0	0	0	0	0
ω_2	a_1	0	0	0	0	1	0
	a_2	1	0	0	0	0	0



(b) Policy 2: X_{at}^{ω} values and corresponding Gantt chart

Figure 4.2: Gantt charts and tables showing time along the x-axis. Activity a_1 and a_2 share resource r_1, resulting in two possible policies. In a policy, each scenario has its own schedule, and schedules within a policy are identical to the left of the dotted line.

The variable $Z_t^{\omega_1\omega_2}$ continues to assume a value of 0 until a decision maker can distinguish between the two scenarios. In policy 1, the completion of a_1 in ω_2 at $t = 2$ signifies a difference between scenarios ω_1 and ω_2. Likewise, the completion of a_1 results in the flexibility of $Z_2^{\omega_1\omega_2}$ to take (although not requiring) a value other than 0, rendering Constraint (4.8) inactive. Constraints (4.3) enforce activity completion precedence for activity a_1 within scenario ω_2:

$$X_{a_1 2}^{\omega_2} - Z_2^{\omega_1\omega_2} \leq X_{a_1 2}^{\omega_1} \leq X_{a_1 2}^{\omega_2} + Z_2^{\omega_1\omega_2}$$

$$X_{a_1 2}^{\omega_2} - 1 \leq X_{a_1 2}^{\omega_1} \leq X_{a_1 2}^{\omega_2} + 1$$

The completion of a_1 results in the flexibility of $Z_2^{\omega_1\omega_2}$ to assume a value other than 0, in turn rendering Constraint (4.8) inactive:

$$X_{a_2 2}^{\omega_2} - Z_2^{\omega_1\omega_2} \leq X_{a_2 2}^{\omega_1} \leq X_{a_2 2}^{\omega_2} + Z_2^{\omega_1\omega_2}$$

$$X^{\omega_2}_{a_2 2} - 1 \leq X^{\omega_1}_{a_2 2} \leq X^{\omega_2}_{a_2 2} + 1$$

For policy 1 in time period 2, non-anticipativity constraints are inactive for scenario pair $\{\omega_1, \omega_2\}$.

Alternatively, policy 2 reverses the order of activity completion and reveals a difference between ω_1 and ω_2 later in time (Figure 4.2). Initiating activity a_1 at time 5 and observing it complete by time 6 offers additional information in the form of $Z^{\omega_1\omega_2}_6$.

$$Z^{\omega_1\omega_2}_t \leq \sum_{t'=1}^{t-d^{\omega_1}_{a_1}} X^{\omega_1}_{a_1 t'} + \sum_{t'=1}^{t-d^{\omega_2}_{a_1}} X^{\omega_2}_{a_1 t'}$$

The decision variable $X^{\omega_2}_{a_1 5} = 1$.

$$Z^{\omega_1\omega_2}_6 \leq \sum_{t'=1}^{6-2} X^{\omega_1}_{a_1 t'} + \sum_{t'=1}^{6-1} X^{\omega_2}_{a_1 t'}$$

$$Z^{\omega_1\omega_2}_6 \leq 0 + 1$$

The non-anticipativity constraints are inactive for scenario pair $\omega_1 < \omega_2$ in time period 6:

$$X^{\omega_2}_{a_1 6} - Z^{\omega_1\omega_2}_6 \leq X^{\omega_1}_{a_1 6} \leq X^{\omega_2}_{a_1 6} + Z^{\omega_1\omega_2}_6$$

$$X^{\omega_2}_{a_1 6} - 1 \leq X^{\omega_1}_{a_1 6} \leq X^{\omega_2}_{a_1 6} + 1$$

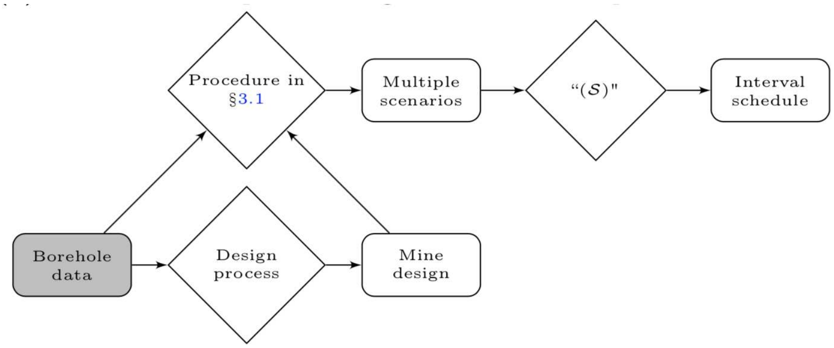
After the first activity in set $\tilde{\mathcal{A}}^{\omega\omega'}$ is complete, the variable $Z^{\omega_1\omega_2}_6$ and all following in time for scenario pair $\omega_1 < \omega_2$ are free to assume 0 or 1, rendering Constraint (4.8) inactive.

4.5 Solution Methodology

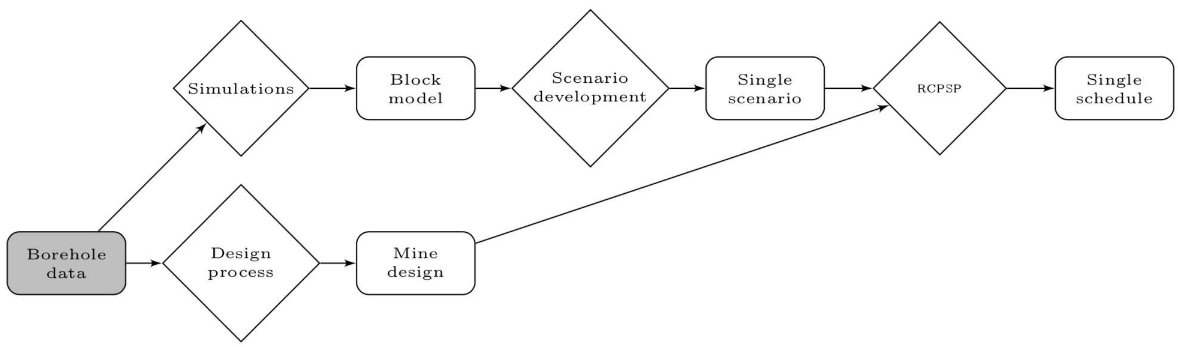
Instances of problem $(\mathcal{S})$ cannot be solved in polynomial time (under the assumption that P $\neq$ NP). The RCPSP is known to be NP-hard (Blazewicz et al. [149]), and reduces to $(\mathcal{S})$ with $|\Omega|$=1 (and, therefore, without Constraints (4.5), (4.6), (4.7) and (4.8)). Realistic instances of operational underground mining problems are large, often including thousands of activities, hundreds of time periods, and multiple scenarios, making it is impossible to solve $(\mathcal{S})$ in an operationally feasible amount of time (e.g., hours) by directly applying some standard mixed-integer programming solver to the monolith. Furthermore, *ad hoc* algorithms designed for scheduling problems with deterministic parameters exploit structure that is absent in our multiple-scenario case.

It is possible to strengthen ($\mathcal{S}$) by reformulating Constraint (4.7); for example, the number of terms on its right-hand side could be reduced by including only the more limiting of the two summations based on activity duration. Another potential formulation enhancement sums Constraint (4.7) over a and its union of predecessors. While valid and potentially useful, numerical results indicate that the linear relaxation of our proposed formulation is tight; at any rate, the first suggestion increases the density of the constraint set. On the other hand, preliminary numerical testing indicates that the RAM storage requirements (which grow with the density of the constraint matrix) are more limiting than the quality of the linear programming relaxation.

Modeling conditional non-anticipativity requires constraints which are theoretically necessary to craft interval schedule solutions given our multi-scenario setting. However, the number and density of these constraints, specifically, Constraints (4.7) and (4.8), contributes significantly to the difficulty of solving ($\mathcal{S}$). We therefore relax these constraints using the justification that the parameter Δ_a we introduce in ($\mathcal{S}$) provides flexibility in determining the interval schedule in order to preserve feasibility of the realized schedule in practice. We call the resulting problem ($\mathcal{S}^-$). Not only does this relaxation remove "difficult" constraints, it reduces the model to one with an Rcpsp-like structure, amenable to solution via an academic research solver. Specifically, Omp Solver [20] is capable of quickly determining near-optimal solutions for realistically sized instances with this mathematical structure by using the Bienstock-Zuckerberg (BZ) algorithm [155] (for the linear programs) and a list-ordering heuristic (to create integer solutions); this combination of techniques has exhibited extraordinary decreases in execution times and memory usage relative to the direct application of traditional commercial solvers [110].



(a) Schedule development using a stochastic optimization model.



(b) Schedule development using a deterministic optimization model.

Figure 4.3: An interval schedule accounts for all uncertainty represented in the multiple schedules developed from borehole data.

The traditional means for creating production schedules takes borehole data obtained from the field and: (i) simulates the geological data to obtain a discretized block model from which we obtain multiple scenarios, selecting one of them, and (ii) uses engineering design principles to determine a mining method. With (i) and (ii) as inputs, we solve an RCPSP model that yields a single schedule. To include this common procedure in our methodological comparison in §4.6, we define a singe-scenario variant titled ($\mathcal{D}$), as ($\mathcal{S}$) with $|\Omega|$=1 and without Constraints (4.5), (4.6), (4.7) and (4.8).

We contrast that with a method that leverages the borehole data to create scenarios using statistical principles (§4.4.1), which, when combined with a given mine design, produce multiple scenarios using the same mean and covariance structure. These serve as inputs to the stochastic programming model, which we solve given the procedure in Table 4.2 to produce an *interval schedule* (see §4.4.2). Figure 4.3 illustrates.

Table 4.2 describes the linear programming-based heuristic $\mathcal{H}$ in four phases. $\mathcal{H}_1$ solves the linear program associated with objective function (4.1) and constraints (4.2)-(4.6). Let (X_{LP}, Y_{LP}) be the corresponding optimal solution. In $\mathcal{H}_2$, we construct a priority list with a simple sort of the solution (X_{LP}, Y_{LP}). Finally, in $\mathcal{H}_3$, given the priority list, we apply a list-scheduling heuristic (§A, App. A, Alg. 3) in order to obtain a feasible interval schedule and a set of feasible schedules for each scenario. Let (X_{IP}, Y_{IP}) denote the corresponding integer-feasible solution.

Table 4.2: Description of heuristic by phases to include inputs, algorithms, and outputs. Phase $\mathcal{H}_3$ produces the robust schedule, Y_{IP}, and an integer-feasible schedule, X^{ω}_{IP}, for each scenario.

Phase	Input →	Algorithm →	Output
$\mathcal{H}_1$	Data from §4.4	OMP	X^{ω}_{LP}, Y_{LP}
$\mathcal{H}_2$	X^{ω}_{LP}, Y_{LP}	SIMPLE SORT (§B, Appendix B, Algorithm 2)	PRIORITY LIST
$\mathcal{H}_3$	PRIORITY LIST	LIST-SCHEDULING (§B, Appendix B, Algorithm 3)	X^{ω}_{IP}, Y_{IP}

Relaxing non-anticipativity has the benefit of algorithmic efficiency through the BZ algorithm. Nevertheless, there is price for relaxing these constraints. Namely, certain solutions representing interval schedules are considered feasible that otherwise would violate non-anticipativity constraints. The Δ_a can provide additional flexibility to the interval schedule in order to keep feasibility in the realized schedule in practice. We accept relaxing these constraints and forming an RCPSP retains sufficient constraints to produce satisfactory solutions.

4.6 Data and Results

The case study for this investigation is a United States-based, large-scale underground mine, identified as MineX at which annual production is approximately 1.8 million tons of material (ore and waste) and 370,000 troy ounces of gold [110]. MineX uses an underground stoping method that consumes five resources (see Table 4.3), associated with development, stope extraction, backfill, or other ancillaries required for the extraction of its 15,773 activities. Each activity has (i) a type, (ii) precedence and resource requirements, (iii) a `value` (which can be negative) and (iv) a `duration`. We describe first how we generate scenarios based on attributes (iii) and (iv) to populate instances of our stochastic programming model, ($\mathcal{S}$), and then how we solve it via the method outlined in Table 4.2.

Table 4.3: Five resources adapted from the case study MineX constrain activity completion.

Constraint	Constrained Activity Types	Upper Bound	Units
Total Tonnage	development, mining and all backfill	11,000	[tons/day]
Total Tonnage	cement and paste backfill	5,000	[tons/day]
Total Tonnage	unconsolidated rock backfill	2,500	[tons/day]
Ore Tonnage	development and mining	6,000	[feet/day]
Footage	development	155	[feet/day]

4.6.1 Scenario Development

Activity grade is derived from simulations of the gold concentration in the orebody given drilling sample data. In our data set, the feature `grade` represents the concentration of gold estimated in troy ounces per ton. This yields a way to compare concentrations of gold over space because, for each activity, the feature accounts for the mass of rock to be mined. We restrict for which activities to model uncertain `grade` and for which to hold their values constant. Grade values used to calculate the revenue component of `value` (from the sale of gold extracted) are adjusted from the block model values, which are based on the physical estimated value of gold in the orebody, and incorporate recovery rates associated with mining and processing.

To model value, we only consider activities associated with mining-specific types, i.e. we do not consider development or ancillary activities. The considered activity types include Stope-mining, Up-hole, Cut-fill, and Floor-pull which represent implementation of specific mining methods directly relating to stope-mining, up-hole mining, cut and fill mining and floor-pull mining operations, respectively [110]. Of the original 15,773 activities, this leaves 1,509. We further limit this number to high-grade activities based on the assumption that the majority of the grade uncertainty lies in this set. This further reduces the set to 159 activities. Let $\{\mathbf{s}_1, ..., \mathbf{s}_{159}\} \in \mathbb{R}^3$ be the locations of the data and $\{\widehat{v}(\mathbf{s}_1), ..., \widehat{v}(\mathbf{s}_{159})\}$ be the values of `grade` observed at those locations.

Figure 4.4 shows that $\{\widehat{v}(\mathbf{s}_1), ..., \widehat{v}(\mathbf{s}_{159})\}$ appears within a tolerance of normality to accept the Gaussian Process assumption as a model for these data. We begin by conducting a formal test for spatial dependence with Moran's I-score [176], a type of correlation coefficient which measures spatial dispersion or correlation present in a data set based on observation proximity. We can formally check for spatial dependence by testing a null hypothesis of purely random spatial observations. Figure 4.5(a) shows Moran's I-score as a function of the number of neighbors, which we determine to be 0.50 with $k = 3$ neighbors, suggesting moderate spatial autocorrelation. For each number of neighbors k, the Moran's I-score tests as significant. We then center the data to form a mean-zero Gaussian Process.

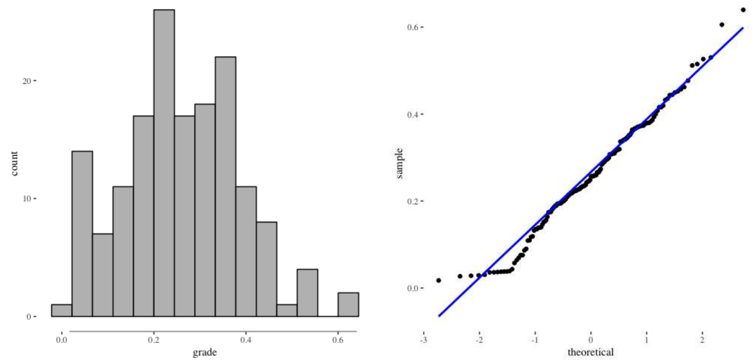
We now determine whether the resulting mean-zero Gaussian Process forms a second-order stationary random field. A spatial field (here, $\{\widehat{v}(\mathbf{s}_1), ..., \widehat{v}(\mathbf{s}_{159})\} \in D$ represent noisy observations of the underlying field) is second-order stationary if $\mathbb{E}(\widehat{v}(\mathbf{s})) = \mu$ and $\text{Cov}(v(\mathbf{s}), v(\mathbf{s}+\mathbf{h})) = C(\mathbf{s}, \mathbf{s}+\mathbf{h}) = C(\mathbf{h})$ for any choice of $\mathbf{h} \in \mathbb{R}^d$ and $\boldsymbol{s} \in D$. That is, the mean is spatially constant and the underlying covariance function depends only on the lag vector $\mathbf{h}$. A random field is *isotropic* if its covariance function depends only on $\|\mathbf{h}\|$.

The stationarity assumption must be checked to validate subsequent analysis and to produce accurate simulations although, in practice, it is almost always an approximation. Bandyopadhyay and Rao [177] provide a method for evaluating the presence of non-stationarity with irregularly spaced spatial data, which uses a Discrete Fourier Transform of the observations. If the resulting Fourier coefficients are "nearly uncorrelated," then the underlying spatial process is second-order stationary; otherwise, this property does not hold. We pose a null hypothesis that $v(\cdot)$ is a second-order stationary random field; tests yield a statistic of 6.60 with a corresponding p-value of 0.22. We therefore fail to reject the null hypothesis, and maintain the stationarity assumption.

We investigate appropriate covariance functions to model the centered data. A classic family are the Matérn covariance functions. While flexible, they depend upon a collection of estimated parameters: The smoothness parameter, ν, is particularly difficult to estimate directly from the data, so instead we evaluate the performance of a set of Matérn covariance functions for a range of chosen values for ν: 0.10, 0.25, 0.50, 0.75, 1.00, 1.25, and 1.50. Figure 4.5(b) shows the log-likelihood of a Matérn covariance function for these values. The maximum log-likelihood occurs where $\nu = 1.25$; however, a close second maximum occurs where $\nu = 1$. In fact, the log-likelihood values for each of these choices of ν agree up to two decimal places, and so, in practice, would perform quite similarly. Given these two options for ν, we select $\nu = 1$ for two reasons: (i) a Matérn covariance function with smoothness ν assumes that that the underlying spatial field is $\lceil \nu - 1 \rceil$ times differentiable, which is a rather strong assumption and difficult to justify in this case, and (ii) taking $\nu = 1$ with a Matérn covariance function is a special case known as a Whittle covariance function [178]. Given our objective to reverse-engineer the simulation process that gave rise to the values of `grade` present in our data, it seems more likely that the engineers who performed this simulation would choose a Whittle covariance function over setting $\nu \approx 1.25$ given its popularity in geostatistical applications. A Whittle covariance function is also dependent upon a range parameter; checking a fine grid yields $\theta = 54$ feet to maximize the likelihood.

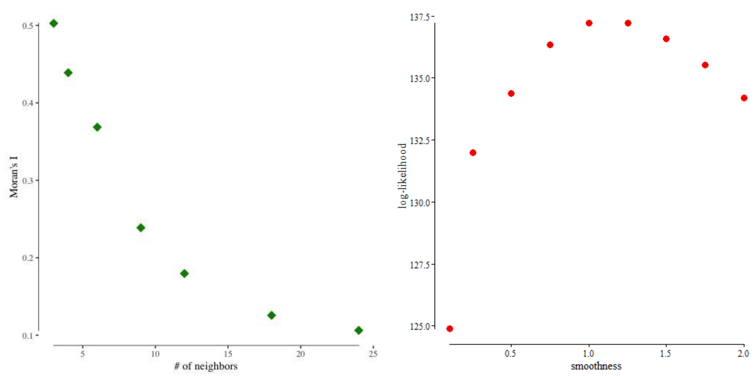
With our chosen covariance function, we construct the variance-covariance matrix Σ. We then use the Cholesky decomposition method to simulate `grade` across the spatial field [172]. This is a fairly general method as it works for general multivariate Gaussian random variables and does not require a stationary or isotropic covariance function.

To simulate $\{v^{\omega}(\mathbf{s}_1), ..., v^{\omega}(\mathbf{s}_{159})\}$, we first calculate the Cholesky factor, L, of the positive definite matrix Σ so that $\Sigma = LL'$ where L is lower triangular. We take $L\varepsilon = L(\varepsilon_1, ..., \varepsilon_{159})'$ where $\varepsilon \sim N_{159}(\mathbf{0}, I)$. This procedure produces exact simulations of a mean zero Gaussian Process with covariance matrix Σ. We then reintroduce the sample mean from the data through summation to achieve a simulation with the same mean and covariance structure as



(a) Histogram of `grade` (b) Normal Q-Q Plot

Figure 4.4: Normality of `grade`. Histogram of 159 `grade` observations used for simulations, displayed in fifteen bins. We also depict the Normal Quantile-Quantile (Q-Q) Plot of `grade` with the theoretical reference line superimposed in blue.

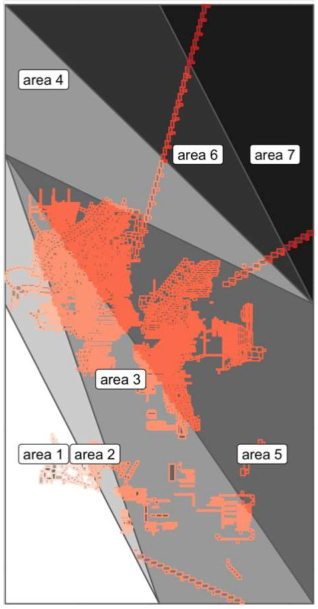


(a) Moran's I-score as a function of the number of neighbors (b) Log-likelihood as a function of smoothness

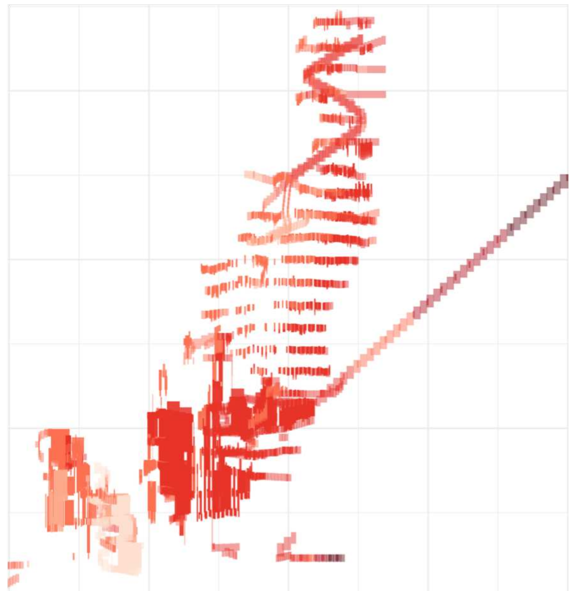
Figure 4.5: Moran's I-score as a function of the number of neighbors. We show log-likelihood as a function of smoothness assuming a Matérn covariance function over the centered `grade` data.

the data $\{\widehat{v}(\mathbf{s}_1), ..., \widehat{v}(\mathbf{s}_{159})\}$.

There are six qualitatively different rock densities present in the mine, which is partitioned into seven regions such that each region is labeled a *ground risk area* (see Figure 4.6). Incorporating information regarding these qualitatively different areas of the mine into our duration simulations enables us to account for geotechnical uncertainty. There is a unique observation for each activity in each geological risk area in our data set.



(a) Design top view



(b) Design side view

Figure 4.6: Given a design, the location of activities can be mapped to ground risk areas g through expert analysis of borehole data.

For a scenario $\omega \in \Omega$ and activity $a \in \mathcal{A}$, we model durations as $d_a^\omega := \widehat{d}_a + \beta_a^\omega$, where $\widehat{d}_a$ is the duration of activity a in the data set used in a deterministic model derived from industry standards and β_a^ω accounts for variability associated with geotechnical uncertainty. Let f_g be a scaling factor representing the "worst-case" duration increase resulting from an activity occurring in a ground risk area g. For each $\omega \in \Omega$ and ground risk area g, we generate $U_g^\omega \sim U[-1, 1]$ and define $\beta_a^\omega := f_g \widehat{d}_a U_g^\omega$. We make this modeling decision because, within a window for a given activity duration, we assume all other durations are equiprobable. An additional benefit is that the expected value of β_a^ω is zero, in which case we recover the initial duration estimate $\widehat{d}_a$.

We incorporate only a modest number of scenarios (five), commensurate with the intuition of mine operators, and demonstrate how our solution procedure yields solutions in an operationally feasible amount of time, whereas a straightforward application of a state-of-the-art solver to the monolith solves only the smallest instance. Then, we compare solution quality of the stochastic programming model to that of model ($\mathcal{D}$) with a mean value and duration for activity a over all scenarios, v_a^m and d_a^m, respectively, representing the traditional deterministic approach.

4.6.2 Results

In order to test the efficacy of our heuristic and the quality of the solutions it provides relative to (i) solving the deterministic equivalent and (ii) solving the stochastic program in its monolith form, we present a variety of numerical experiments (Table 4.4).

Table 4.4: We conduct these numerical experiments with the corresponding characteristics.

Method	Solution technique	Treatment of uncertainty	Non-anticipativity	$\lvert\Omega\rvert$	Value	Duration
$(\mathcal{S})$	exact	stochastic	yes	5	v_a^ω	d_a^ω
$(\mathcal{S}^-)$	exact	stochastic	no	5	v_a^ω	d_a^ω
$(\mathcal{S}^-)$: $\mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3$	heuristic	stochastic	no	5	v_a^ω	d_a^ω
$(\mathcal{D})$: $\mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3$	heuristic	deterministic	NA	1	v_a^m	d_a^m

Table 4.5 reports solution times. Solving the monolith directly for $(\mathcal{S})$, and even for $(\mathcal{S}^-)$ – without the non-anticipativity constraints, is only possible for the smallest instance, i.e., that containing 56 activities, and requires an order of magnitude more time than our proposed heuristic (§4.5). As expected, solutions from the stochastic program require longer to obtain than those from the deterministic equivalent, yet still fall below three hours of computation time, within the realm of reason in an operational setting.

Table 4.5: Solution times for the problems given in Table 4.4. All instances contain five scenarios ($|\Omega|$=5) and five resources ($|\mathcal{R}|$=5). Solution times for the linear programming relaxation found via the Bienstock-Zuckerberg algorithm (BZ) possess a duality gap of less than 0.01%.

Instance		Solution Times				
		Exact Solve		$\mathcal{H}_1$+$\mathcal{H}_2$+$\mathcal{H}_3$		BZ
$\lvert\mathcal{A}\rvert$ [activities]	$\lvert\mathcal{T}\rvert$ [days]	$(\mathcal{S})$ [sec]	$(\mathcal{S}^-)$ [sec]	$(\mathcal{D})$ [sec]	$(\mathcal{S}^-)$ [sec]	$(\mathcal{S}^-)^{LP}$ [sec]
56	50	44	31	1	1	13
396	200	†	†	3	6	33
646	300	†	†	11	25	177
1,453	600	†	†	172	273	483
2,323	900	†	†	821	1,142	2,051
3,150	1,200	†	†	1,913	2,856	5,521
3,828	1,500	†	†	3,764	5,784	11,221
4,330	1,800	†	†	5,554	8,488	17,236
4,764	2,100	†	†	7,319	9,781	20,956

†Exceeds available computer memory
[%]: optimality gaps [sec]: solution times

Table 4.6: Expected net present value for the problems given in Table 4.4. All instances contain five scenarios ($|\Omega|$=5) and five resources ($|\mathcal{R}|$=5).

Instance		Objective Function Values					Optimality Gap
		Exact Solve		$\mathcal{H}_1$+$\mathcal{H}_2$+$\mathcal{H}_3$		BZ	$\left(\frac{(\mathcal{S}^-)^{LP}-(\mathcal{S}^-)}{(\mathcal{S}^-)^{LP}}\right)$
$\lvert\mathcal{A}\rvert$ [activities]	$\lvert\mathcal{T}\rvert$ [days]	$(\mathcal{S})$ [\$M]	$(\mathcal{S}^-)$ [\$M]	$(\mathcal{D})$ [\$M]	$(\mathcal{S}^-)$ [\$M]	$(\mathcal{S}^-)^{LP}$ [\$M]	[%]
56	50	0.96	0.96	0.96	0.96	0.96	0.00
396	200	†	†	14.85	14.85	14.85	0.00
646	300	†	†	33.39	33.38	33.38	0.00
1,453	600	†	†	102.8	102.77	102.77	0.00
2,323	900	†	†	194.15	193.30	194.10	0.42
3,150	1,200	†	†	279.29	276.53	279.22	0.96
3,828	1,500	†	†	353.56	352.37	353.47	0.31
4,330	1,800	†	†	401.02	400.92	400.92	0.00
4,764	2,100	†	†	442.14	442.03	442.03	0.00

†Exceeds available computer memory
[%]: optimality gaps [sec]: solution times

Table 4.6 shows the expected net present value **E**[NPV] for each case listed in Table 4.4. For the deterministic equivalent in which $|\Omega| = 1$, we let the parameter $\pi^\omega = 1$. The smallest scenario, which is solvable via both exact and heuristic methods, demonstrates equal objective function values. The heuristic does not impose non-anticipativity constraints yet, for this instance, the omission appears to be irrelevant. Comparing the objective function values for the remaining eight instances shows negligible differences. This might imply that incorporating stochasticity is not important. However, we further analyze the solutions via three metrics: *makespan*, *feasibility*, and *average count of completed activities*, and conclude that the results from the stochastic program are more realistic, and therefore implementable in an operational setting, while not sacrificing significant objective function value.

In order to assess the quality of the solutions, we introduce a variety of metrics, the first of which is the *makespan*, given by τ and defined in Equation (4.10) as the last time period with an activity under execution, as follows:

$$\tau = \max_{a \in \mathcal{A}, \omega \in \Omega} \left\{ \sum_{t \in \mathcal{T}} t \cdot X^{\omega}_{at} + d^{\omega}_{a} - 1 \right\} \tag{4.10}$$

We also measure the *feasibility* of a schedule, which is necessarily satisfied for any solution of the stochastic programming models, $(\mathcal{S})$ and $(\mathcal{S}^{-})$. For the deterministic model $(\mathcal{D})$, feasibility implies the satisfaction of integrality, Constraint (4.2), Constraint (4.3), and Constraint (4.4) for the original five scenarios in Ω:

$$\sum_{t'=1}^{t} X^{\omega}_{at'} \leq \sum_{t'=1}^{t-d^{\omega}_{a'}} X^{\omega}_{a't'} \qquad \forall\, a \in \mathcal{A};\ a' \in \mathcal{P}_a;\ \omega \in \Omega \tag{4.11}$$

$$\sum_{a \in \mathcal{A}} \sum_{t'=\max\{1, t-d^{\omega}_{a}+1\}}^{t} q_{ar} \cdot X^{\omega}_{at'} \leq \bar{q}_r \qquad \forall\, r \in \mathcal{R};\ \omega \in \Omega \tag{4.12}$$

Invariably, there exists some time period(s) in which this is not the case, and our measure ϕ is given as the last feasible time period from the start of the schedule, i.e., the last time period before rescheduling is required:

$$\phi = \max_{\hat{t} \in \mathcal{T}} \left\{ \hat{t} \text{ such that both (4.11) and (4.12) hold for all } t \leq \hat{t} \right\} \tag{4.13}$$

Finally, unlike in typical project scheduling in which all activities are executed, activities are optional in an underground mine. Deterministic models have the clairvoyance not to schedule activities that offer little value. Equation (4.14) defines the measure η as the *average count of completed activities*:

$$\eta = \sum_{\omega \in \Omega} \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}} \pi^\omega \cdot X_{at}^\omega \tag{4.14}$$

We record each of these metrics, τ, ϕ, and η in Table 4.7 for the nine instances given in Table 4.5 and Table 4.6 in both the deterministic, $(\mathcal{D})$, and stochastic, $(\mathcal{S}^-)$, settings. Corresponding to intuition, the makespans are all longer for the stochastic programming solutions; the stochastic models incorporate the corresponding uncertainty from a variety of scenarios, resulting in some duration values that are longer than average, and $(\mathcal{S}^-)$ subscribes to feasibility requirements with respect to these durations. As expected, the deterministic model becomes infeasible in a relatively short amount of time relative to the entire horizon, while the stochastic program maintains feasibility for the entire horizon, as expected. We now see that the small degradation in objective function (Table 4.6) in the stochastic program is more than offset by the gain in feasibility with respect to the five scenarios. Finally, the number of activities executed is reasonably similar for solutions from both the stochastic and deterministic programs, indicating that the real quantitative difference lies in the makespan. This indicates that the uncertainty prolongs the duration of the activities but does not, generally speaking, transform a profitable activity into an unprofitable one.

Table 4.7: We measure τ, ϕ, and η for solutions found via procedures described in Table 4.2. $(\mathcal{D})$ uses a single scenario derived from the mean, while $(\mathcal{S}^-)$ uses five scenarios ($|\Omega|$=5); both models consider five resources ($|\mathcal{R}|$=5).

Instance		Measures of Utility					
		$(\mathcal{D})$			$(\mathcal{S}^-)$		
$\lvert\mathcal{A}\rvert$ [activities]	$\lvert\mathcal{T}\rvert$ [days]	τ [days]	ϕ [days]	η [activities]	τ [days]	ϕ [days]	η [activities]
56	50	6	2	27	7	50	27
396	200	70	2	229	89	200	229
646	300	122	5	423	160	300	423
1,453	600	329	4	1,115	433	600	1,115
2,323	900	743	16	1,950	885	900	1,927
3,150	1,200	1,006	25	2,764	1,197	1,200	2,720
3,828	1,500	1,229	7	3,363	1,498	1,500	3,335
4,330	1,800	1,329	21	3,810	1,720	1,800	3,810
4,764	2,100	1,385	20	4,178	1,789	2,100	4,178

4.7 Conclusions

Assuming perfect knowledge of value and duration for each activity in an underground mining operation may yield inaccurate mine schedules. Often, mine planning decisions require a horizon for which only estimates of certain input parameters are available. We present a stochastic programming model whose tractability is questionable for realistic-sized instances, and demonstrate by relaxing certain constraints and developing a heuristic that exploits the resulting mathematical structure, we can obtain good-quality solutions, feasible for practical time horizon lengths, even in the presence of the relaxed constraints, within several hours, at most. We further demonstrate empirically that the solution quality improves relative to that from a deterministic equivalent based on point estimates of value and duration data.

The intractability of ($\mathcal{S}$) precludes an exhaustive comparison of solution quality of the stochastic-programming monolith to that of our approximation, ($\mathcal{S}^-$), obtained, in part, via heuristics. Nonetheless, solutions are feasible and objective function values appear to be similar to the exact model representation for small instances. Future work would develop alternate comparisons, such as optimistic and pessimistic bounds on the stochastic program. Alternate heuristic solution strategies might incorporate a priority list p_a of activities a for ($\mathcal{S}$) from a mine planner. Finally, we assume that while duration is uncertain, resource consumption is deterministic. Future work might incorporate the ideas of Demeulemeester et al. [179] to relax this assumption, though a corresponding solution technique for large instances remains elusive.

CHAPTER 5
CONCLUSION

Over the past decade, a number of advances have been made in operations research techniques to enhance underground mine plans. Data-driven techniques increasingly supplement expert judgment. Improved representation of operational procedures, and the inclusion of interdependent decisions, have allowed for objective analysis of trade-offs between mine design and scheduling decisions. Additionally, integrated models may determine fleet configuration decisions, and maintenance and capital expenditures; they could also simultaneously inform designs for multi-mine complexes. Nonetheless, mathematical tools available at the time of this writing may require significant adjustments or modifications to obtain results for specific cases, such as multi-method operations. Commercial software platforms will continue to define data standards and facilitate technology transfer amongst academia and industry, expediting industry adoption of modeling techniques.

Our review of the literature shows that there is need for greater collaboration across the disciplines of mining, geology, operations research, statistics, and computer science. Active areas of research include, *inter alia*: (i) integrating design and scheduling – including improved geotechnical modeling, (ii) addressing the volatility of real-time operations through more robust schedules. We have addressed these two topics in the remainder of the dissertation.

Related specifically to (i), we examine a strategic underground mine design and scheduling problem by considering an ore body partitioned into panels, each of which is extracted by a specific method, namely, top-down or bottom-up; there is also an option of leaving the panel *in situ* as a sill. Our instances consider two types of activities with precedence relationships: development and extraction. An integer programming model prescribes an optimal set of methods with which to extract each panel and the corresponding schedule to maximize the net present value. Computational results show that variable elimination, dual bound tightening and an initial feasible solution enhance the quality of the solution and the speed with which it is obtained. In particular, final gaps average 6.9% for our 36 test instances within a 2-hour time limit, where average optimality gaps are reduced to a greater extent for “difficult” instances with initial gaps of 10% of higher. The solution we provide for a base-case industry data set results in a design and corresponding schedule with 44% scaled additional value, compared to the best industry-derived solution for this strategic planning model.

Future work for this design and scheduling model could focus on several enhancements resulting from a better understanding of the model’s exploitable structure: This model yields loose dual bounds, in which a good integer solution is found at the early stages of the branch-and-bound tree, and most of the solver’s efforts are spent on the generation of cuts to tighten the dual bound in order to prove optimality. A solution methodology that decomposes the problem might lead to early improvement of the integer solution by fixing and relaxing variables. Two promising decompositions: (i) separate the problem based on the maximum (or minimum) number of sill pillars in the design, and (ii) divide the problem into contiguous groups of panels. This decomposition might more effectively take advantage of the cuts we recommend in §3.6.3.

Related specifically to (ii), we relax the assumption of perfect knowledge regarding value and duration of each activity in an underground mining operation. Often, mine planning decisions require a horizon for which only estimates of certain input parameters are available. We present a stochastic programming model whose tractability is questionable for realistic-sized instances, and demonstrate that by relaxing certain constraints and developing a heuristic that exploits the resulting mathematical structure, we can obtain good-quality solutions, feasible for practical time horizon lengths, even in the presence of the relaxed constraints, within several hours, at most. We further demonstrate empirically that the solution quality improves relative to solving a deterministic equivalent based on point estimates of value and duration data.

The intractability of $(\mathcal{S})$ precludes an exhaustive comparison of solution quality of the stochastic-programming monolith to that of our approximation, $(\mathcal{S}^{-})$, obtained, in part, via heuristics. Nonetheless, solutions are feasible and objective function values appear to be similar to the exact model representation for small instances. Future work would develop alternate comparisons, such as optimistic and pessimistic bounds on the stochastic program. Alternate heuristic solution strategies might incorporate a priority list of activities for $(\mathcal{S})$ from a mine planner. Finally, for this stochastic program, we assume that while duration is uncertain, resource consumption is deterministic. Future work might incorporate the ideas of Demeulemeester et al. [179] to relax this assumption, though a solution technique for large instances of this extension remains elusive.

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APPENDIX A

RCPSP FORMULATION

A resource-constrained project scheduling problem (RCPSP) possesses two main constraint types: (i) precedence (the majority), and (ii) knapsacks (the minority). These problems can be solved with Open Mine Planner (OMP) [20], which implements several published algorithms, as follows: (i) solution of the linear programming relaxation using a decomposition method (see Muñoz et al. [180] for details), and then (ii) application of a *TopoSort* heuristic to obtain an integer-feasible solution [110, 181]. The latter is a list-ordering heuristic that uses the "expected" completion time of an activity given by the linear programming solution. OMP's variant of an early start algorithm [151] can reduce the number of activity-time period pairs. This formulation is taken from Brickey et al. [114].

Indices and sets:

$a \in \mathcal{A}$	an activity within the set of all activities
$\tilde{a} \in \tilde{\mathcal{A}} \subset \mathcal{A}$	an activity within the set of activities whose start dates have been predetermined
$\bar{a} \in \bar{\mathcal{A}}_a$	an activity $\bar{a}$ within the set of predecessor activities to activity a
$r \in \mathcal{R}$	a resource within the set of resources, such as production and development capacity, whose limits are enforced on a daily basis
$r \in \hat{\mathcal{R}} \subset \mathcal{R}$	a resource within the set of resources, such as production and development capacity, whose limits are enforced on a monthly basis
$t \in \mathcal{T}$	a day within the set of daily time periods
$m \in \mathcal{M}$	a month within the set of monthly time periods
$t \in \hat{\mathcal{T}}_m$	a day within the set of days contained in month m

Parameters:

c_a	monetary value associated with completing activity a [$]
q_{ra}	consumption of resource r associated with completing activity a [tonnes, meters]
$\bar{r}_{rt}$	maximum amount of resource r available on day t [tonnes, meters]
$\hat{r}_{rm}$	maximum amount of resource r available in month m [tonnes, meters]
d_a	duration of activity a [days]
$d_{\bar{a}}$	duration (including mandatory delay) of activity a [days]
δ_t	discount factor for period t [fraction]

Decision variables:

X_{at}	1 if activity a is completed by the end of time t, 0 otherwise

$$
\begin{aligned}
(\mathcal{RCPSP}) \quad \max \quad & \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} \delta_t c_a (X_{at} - X_{a,t-1}) && \text{(A.1a)} \\
\text{s.t.} \quad & X_{a,t-1} \le X_{at} \quad \forall\, a \in \mathcal{A}, t \in \mathcal{T} && \text{(A.1b)} \\
& X_{at} \le X_{\bar{a},t-d_{\bar{a}}} \quad \forall\, a \in \mathcal{A}, \bar{a} \in \bar{\mathcal{A}}_a, t \in \mathcal{T} && \text{(A.1c)} \\
& \sum_{a \in \mathcal{A}} \frac{q_{ra}}{d_a} (X_{at} - X_{a,t-d_a}) \le \bar{r}_{rt} \quad \forall\, r \in \mathcal{R}, t \in \mathcal{T} && \text{(A.1d)} \\
& \sum_{t \in \hat{\mathcal{T}}_m} \sum_{a \in \mathcal{A}} \frac{q_{ra}}{d_a} (X_{at} - X_{a,t-d_a}) \le \hat{r}_{rm} \quad \forall\, r \in \hat{\mathcal{R}}, m \in \mathcal{M} && \text{(A.1e)} \\
& X_{\tilde{a}1} = 1 \;\; \forall \tilde{a} \in \tilde{\mathcal{A}} && \text{(A.1f)} \\
& X_{at} \text{ binary } \;\; \forall a \in \mathcal{A}, t \in \mathcal{T} && \text{(A.1g)}
\end{aligned}
$$

The objective (A.1a) maximizes net present value, which is a discounted function of the monetary value associated with the (on-time) completion of activity a and the time at which said activity is completed. We express the latter as the difference of two variables corresponding to the time by which an activity is completed [151]. Constraints (A.1b) ensure that once an activity is completed at time t-1, it remains completed for all future time periods $t, ..., |\mathcal{T}|$. Constraints (A.1c) enforce precedence between an activity a and its predecessors $\bar{a}$, such that a cannot start unless $\bar{a}$ starts sufficiently early that, when accounting for its duration, it is finished by the time a starts. Constraints (A.1d) constitute knapsacks and ensure that the amount of resource of a particular type consumed by all activities on any given day cannot exceed the availability of said resource. Constraints (A.1e) do the same for a subset of the resources whose consumption must be restricted on a monthly basis. Activities whose start dates have been previously determined to coincide with the beginning of our time horizon must be inserted into the schedule per constraints (A.1f). All variables are required to be binary by constraints (A.1g).

APPENDIX B

SIMPLE SORT AND LIST SCHEDULING PSEUDOCODE

Algorithm 1: SIMPLE SORT $\mathcal{H}_2$

1 **Input:** LP relaxation, Y_{LP}, from having solved the LP in $\mathcal{H}_1$ with OMP.
2 **Output:** Sorted List of activities, SL; Mean Starting times, $MS[a]\ \forall\ a \in \mathcal{A}$.
3 **Assign:** Mean starting time of activity $MS[a] = \sum_{t \in \mathcal{T}} t \cdot Y_{LP\,at}\ \ \forall\ a \in \mathcal{A}$ to list PL.
4 **Assign:** Each activity $a \in \mathcal{A} \ni MS[a] \geq 0.5$ to set $\mathcal{A}'$.
5 **Sort:** All activities $a \in \mathcal{A}'$ non-decreasing by MS[a] and assign to ordered list SL.
6 **Return:** SL and $MS[a]\ \forall\ a \in \mathcal{A}$.

Algorithm 2: LIST-SCHEDULING HEURISTIC $\mathcal{H}_3$

1 **Input:** Sorted List of activities, SL; Mean Starting times, $MS[a]\ \forall\ a \in \mathcal{A}$.
2 **Output:** Integer-feasible solution X^{ω}_{IP}, Y_{IP}.
3 **Assign:** The value 0 to variables X^{ω}_{at} and Y_{at}, $\forall\ a \in \mathcal{A}$, $t \in \mathcal{T}$, $\omega \in \Omega$.
4 **while** *list SL is not empty* **do**
5   Consider the first activity in list SL, a, and delete it from list SL.
6   Assign value of $\lceil MS[a] - 0.5 \rceil$ to t.
7   **while** *period $t \leq T$* **do**
8   Assign the value 1 to variable Y_{at}.
9   **for** $\omega \in \Omega$ **do**
10   **for** $t' \in \{\max\{1, t - \Delta_a\}, \ldots, \min\{T, t + \Delta_a\}\}$ **do**
11   **if** *starting activity a at t' in ω is precedence- and resource-feasible* **then**
12   Assign the value 1 to variable $X^{\omega}_{at'}$.
13   Go to line 9.
14   **end**
15   **end**
16   **end**
17   Assign the value 0 to variable Y_{at}.
18   Assign the value $t + 1$ to t.
19   **end**
20 **end**
21 **Return:** The values of variables X^{ω}_{at} and Y_{at} as X^{ω}_{IP} and Y_{IP}, respectively.