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PHYSICAL COMPARISON OF THE BUCKLEY-LEVERETT AND DIETZ FRONTAL ADVANCE THEORIES FOR A LINEAR, HORIZONTAL WATERFLOOD

By

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ABSTRACT

An experimental apparatus was set up to determine which theory either Buckley-Leverett or Dietz was most accurate in predicting oil recoveries in a linear horizontal waterflood for thick (i.e. the transition zone is several times smaller than the formation thickness) homogeneous sands.

The Dietz theory assumes a distinct interface between oil and water due to gravity segregation, whereas, the Buckley-Leverett theory assumes that there is no movement perpendicular to the direction of flow and assumes uniform water saturation in the cross-section perpendicular to the driving force.

Waterfloods were carried out at different rates in a 6 ft long, 9 in. by 6 in. rectangular tube packed with 10-20 mesh sand. The water saturation throughout the sand was recorded continuously during the flood by electrical resistance measurements across 55 pairs of electrodes.

The experimental results show that gravity tonguing does occur and that neither the Dietz nor the Buckley-Leverett theories were satisfactory under the reservoir conditions simulated in the experiments. A theory has been developed which describes the experimental results satisfactorily.

The theory consists of two assumptions:

(1) A portion of the water tongue assumes that gravity forces are negligible and are not rate sensitive, and

(2) The other portion of the tongue assumes gravity forces are dominant and hence rate sensitive.
This theory explains why as the injection rate increased the initial oil recovery (per unit pore volume injected) increased.

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INTRODUCTION

Waterflooding is a popular secondary recovery method. In order that a realistic economic evaluation of a waterflood project can be carried out it is important that the correct theory is used to calculate oil displacement efficiency. At present, two frontal advance theories are available, namely, the Buckley-Leverett theory and the Dietz theory.

1. Buckley-Leverett Frontal Advance Theory

Leverett (1941) developed the fractional flow equation. Starting with Darcy's law for oil and water he obtained:

$$f_w = \frac{1 + \frac{K\,K_{ro}}{U_t\,\mu_o}\left(\frac{\delta P_c}{\delta L} - g\Delta\rho\,\mathrm{Sin}\,\alpha\right)}{1 + \frac{\mu_w}{\mu_o}\,\frac{K_o}{K_w}} \tag{3}$$

where f_w = fraction of water in the flowing stream (i.e watercut)

K = formation permeability.

K_{ro} = relative permeability

K_o = effective permeability to oil

K_w = effective permeability to water

μ_o = oil viscosity

μ_w = water viscosity

U_t = total fluid velocity (q_t/A)

P_c = capillary pressure = $P_o - P_w$

L = distance along direction of movement

g = acceleration due to gravity

$\Delta\rho$ = water-oil density difference = $\rho_w - \rho_o$

α = angle of formation dip to the horizontal

All the factors necessary to calculate the value of f_w are available except one, the capillary pressure gradient, now

$$\frac{\delta P_c}{\delta L} = \frac{\delta P_c}{\delta S_w} \frac{\delta S_w}{\delta L}$$

Although $\frac{\delta P_c}{\delta S_w}$ can be determined, $\frac{\delta S_w}{\delta L}$ is not available. In practice the capillary pressure term is neglected. This simplifies equation 3 to

$$f_w = \frac{1 - \frac{K}{U_t} \frac{K_{ro}}{\mu_o} g \Delta \rho \, \mathrm{Sin}\,\alpha}{1 + \frac{\mu_w}{\mu_o} \frac{K_o}{K_w}}$$

For a horizontal system:

$$f_w = \frac{1}{1 + \frac{\mu_w}{\mu_o} \frac{K_{ro}}{K_{rw}}} \qquad (4)$$

Buckley and Leverett (1942) derived the frontal advance equation. From the material balance equation they derived:

$$L = \frac{W_i}{A\phi} \left(\frac{df_w}{dS_w}\right) \qquad (5)$$

where

W_i = cumulative fluid injected

L = the total distance that a plane of constant S_w has moved.

A = cross-sectional area

ϕ = porosity

Using equation (5) the saturation distribution during a flood can be calculated. The fractional flow curves (equation 4) show that there are two water saturations having the same value of df_w/dS_w. The result is the well-known triple-valued saturation distribution.

Terwilliger et al (1951) showed that by using equation (3) the experimental results agreed with theory. They also showed that by laying a tangent to the fractional flow curve from S_w original and f_w equal zero they could define the saturation at the upstream end of the stabilized zone. As a result they developed the concept of stabilized and non-stabilized zones.

Stabilized zone - saturation interval where all points of saturation move at the same rate.

Non-Stabilized zone - saturation interval where all points of saturation continue to get further apart.

Welge (1952) developed the Buckley-Leverett work and derived the following equation:

$$S_w - S_{w2} = Q_i \, f_{o2} \tag{6}$$

where

S_w = average water saturation (and hence cumulative oil production can be calculated), fraction pore volume.

S_{w2} = water saturation at the producing end.

Q_i = pore volumes of cumulative injected fluid.

f_{o2} = fraction of oil flowing at the producing end.

Welge also determined that:

$$Q_i = \frac{1}{\left(\frac{df_w}{dS_w}\right)_{S_{w2}}} \qquad (7)$$

A fractional flow curve (fig. 7) was constructed using equation (4) employing the model properties and the experimentally derived relative permeability curve (fig. 8). From equations (6) and (7) plus the fractional flow curve cumulative oil recovered (percent pore volume) versus cumulative fluid injected (percent pore volume) curves were constructed (figs. 10, 11, 12 and 13).

The Buckley-Leverett predictions did not agree with the experimental results (figs. 10, 11, 12 and 13) because with the formation of a gravity tongue assumption no. 3 below was invalidated. The assumptions for the Buckley-Leverett frontal advance theory are outlined below:

1. Incompressible fluids.
2. Restricted to two mobile immiscible fluids.
3. Net flux in one direction parallel to bedding plane.
4. Homogeneous formation in one direction.
5. Neglected capillary pressure.
6. Darcy's law applies - requires relative permeability data.

2. Dietz Theory

Whereas the Buckley-Leverett theory is concerned with the growth of the transition zone from the water phase to the oil phase during a waterflood, Dietz was concerned with edge-water drive and considered that the water tongued under the oil. The assumptions made were as follows:

1. Sharp interface - no transition zone from water to oil.
2. Constant absolute permeability.
3. Mobility ratio

$$\frac{K_o}{\mu_o} \Big/ \frac{K_w}{\mu_w} < 1$$

4. Constant injection rate.
5. Incompressible fluids.
6. Constant pressure drop or zero pressure drop across the oil/water interface.
7. Darcy's law applies - relative permeability data is not required for the full range of water saturation; however the permeability to oil at irreducible water saturation and the permeability to water at irreducible oil saturation is required.

In order to solve mathematically the continuity equation, Dietz ignored the effects of gravity.

The equation for the shape of the gravity tongue is:

$$x = \frac{aH}{\left[a(H-h) + h\right]^2} \frac{Q}{\phi s} \qquad (8)$$

where

x = distance along the bedding plane

h = perpendicular distance from the bottom of the bedding plane to the interface

H = formation thickness

Q = cumulative fluid injected/unit width

a = mobility ratio $\left(\frac{\mu_w}{\mu_o} \quad \frac{k_o}{K_w}\right)$

s = difference of saturation in the oil-bearing and waterflooded regions

ϕ = porosity

he = height of the water tongue at the producing end

at $x = L$, $h = h_e$ and where L = length of the box

from equation (8)

$$Q = \frac{L \left[a(H-he) + he\right]^2}{aH} \phi s$$

or Q in pore volumes

$$Q = s \frac{\left[a(H-he)+he\right]^2}{aH^2} \qquad (9)$$

The volume of oil recovered is found by integrating the volume of the water tongue. From Dietz (1953) paper, equation 32:

$$Q_o = \text{(pore volumes)} = \frac{x_{crest}\ he}{HL} s - \frac{Qah}{(1-a)} \left. \frac{1}{a(H-h_e)+he} \right|_{he}^{H}$$

now $x_{crest} = QHL\phi \dfrac{a}{Hfs}$

$$Q_o = Qa\frac{h_e}{H} + \frac{Qa}{(1-a)}\left[\frac{1}{a(1-\frac{h_e}{H}) + \frac{h_e}{H}} - 1\right] \qquad (10)$$

Using equations (9) and (10) for values of h_e ranging from 0 to H cumulative injection versus cumulative oil recovered can be calculated. This was carried out for the model properties. The results are shown in figures 10 through 13. It can be seen that this form of the theory did not fit the experimental results very well. An approximate solution to the continuity equation was developed to take into account the effects of gravity forces.

In summary, the Dietz theory assumes a distinct interface between oil and water due to gravity segregation, whereas the Buckley-Leverett theory assumes that there is no movement perpendicular to the direction of flow and assumes uniform water saturation in the cross-section perpendicular to the driving force.

It is important that the correct theory is used because according to Buckley-Leverett once a well begins to cut water remedial workover operations will not reduce water intrusion. However, should gravity tonguing prove to be correct then remedial workover operations would reduce water production and hence reduce reservoir energy losses and producing costs to a minimum.

Croes, G.A. and Schwarz, N. (1955) carried out a series of experimental waterfloods and evaluated the results using the Buckley-Leverett and Dietz theories. Although the Dietz theory was found to be in agreement with some of the experimental results, the Buckley-Leverett theory was successful in correlating all the experimental results. This can be explained by the fact that they used small diameter cores (transition zone being larger than the height of the core), high flow rates and the fact that the Dietz theory assumed a uniform cross section which was not the case for a tube.

Hele-Shaw models have been employed by researchers and have confirmed the Dietz theory. However this type of model ignores the growth of the transition zone from water to oil which could cause a 'barrier' to the water. Other visual models have been employed and have confirmed gravity tonguing of the water under the oil. There is no mention in the literature of water saturation measurements being taken at the waterflood front during a horizontal oil/waterflood.

The experimental apparatus employed in this thesis was designed to have a small transition zone (approximately 1/5 of the height). A rectangular box 6 ft long, 9 in high and 6 in. wide was constructed with ½ in. plexi-glass (lucite) and packed with 10-20 mesh Ottawa sand. Waterflood experiments were carried out at different flowrates. The waterflood front was tracked by means of measuring the resistance across 55-pairs of electrodes installed in the box and the resistance reading being converted to water saturation. The oil recoveries and the shape of the waterflood fronts were compared as function of cumulative water injected for each flowrate. The waterfloods were also evaluated employing the Buckley-Leverett, Dietz and the author's theories. All tests were conducted with 2 to 3 pore volumes of saturated brine injected.

EXPERIMENTAL WORK

The experimental work carried out in this thesis will be discussed under the headings of Experimental Apparatus, Model Scaling, and Experimental Procedure.

Experimental Apparatus

A rectangular box, 6 ft by 6 in. by 9 in. was made with ½ in. thick plexi-glass (lucite). The joints were glued with epoxy resin and screwed every 4 in. (fig. 1). The end-plates (fig. 2) were screwed to flanges at the ends of the box and sealed with rubber gaskets. The end-plates were constructed with a diffuser plate which consisted of a network of 70 1/20 in. diameter holes, a ¼ in. spacer plate and a ½ in. end-plate, all glued together with epoxy resin. Three 3/8 in. inlets positioned top, center and bottom were inserted into the end-plate to eliminate possible end effects due to the positioning of the inlets and outlets. Under pressure the box leaked - fibre glass was run along the joints and the leaks were stopped. The box has satisfactorily held a pressure of 8 psi (generally experiments were carried out with pressures less than 5 psi).

The box was packed with 10-20 mesh Ottawa sand in saturated brine. Initial experiments, using a short section of the box, indicated that fluid flowed along the sides of the box. Sand was glued to the sides of the box with clear rubber cement. This has satisfactorily stopped flow along the sides of the box.

Fifty-five pairs of electrodes, 5 per column - approximately 2 in. apart - and, 11 columns - 6 in. apart - were installed to track the water saturation profile during the flood. Initially, the electrodes were 1/16 in. copper wire but inconsistent results were obtained due to oil wetting the copper probes. This problem was solved by gluing with silver paint felt pads ¼ in. by ¼ in. to the copper probes. The felt pads were initially soaked in brine thus ensuring capillary continuity from the sand through the electrodes.

An a-c supply was used to measure the resistance across each pair of electrodes. The voltage drop across each pair of electrodes after being rectified to direct current was recorded on a 24-point recorder. An electrical circuit diagram is shown at figure 3. A stepping switch system (fig. 4) operated by a separate 40-v d-c supply, automated the switching from one pair of electrodes to the next. After 20 readings the recorder was automatically calibrated with 4 known resistances. One complete sweep of the box took about 36 minutes.

The box weighed approximately 500 lbs when full of brine and sand. An A-frame and a block-and-tackle arrangement was employed to maneuver the box into position.

The fluid injection rate was controlled by a Reeves Duplex, variable rate, variable stroke pump - the outlet filtered by a 1.2μ filter. The set up of the equipment is illustrated at figure 5.

The properties of the model are listed in table 1.

Model Scaling

Rapoport and Leas (1953) made an investigation of the stablized zone effect. From both theoretical and experimental considerations the term $Lu\mu_w$ was found to be the scaling coefficient to check that laboratory waterflood experiments are stabilized - where L = length, cm, u = flowrate per unit cross-sectional area, cm per sec, μ_w = viscosity of water, cp. A scaling coefficient greater than 1.5 signifies stability.

In the experiments conducted this criteria was met.

If the Dietz assumptions are assumed to fit the reservoir the dimensionless groups that must be the same in the model and reservoir are:

1. Ratio of length to thickness. In the model this was 1/8.
2. Angle of formation dip. In the model this was 0.
3. Fluid mobility ratio $\left[\frac{K_w}{\mu_w} \times \frac{\mu_o}{K_o}\right]$. This was approximately 2.8 - 5.5 in the model.
4. Ratio of gravity to viscous forces $\left[\frac{(\rho_w - \rho_o) \quad K_w}{U \mu_w}\right]$. This ranged from 2.0 to 20.0 in the model.
5. Transition zone must be smaller than the reservoir thickness. In the model the transition zone was approximately 1/5 of the formation thickness (fig. 6).
6. A check employing a Buckley-Leverett type calculation using measured relative permeability curve for the reservoir to ensure that the displacement of oil by

water proceeds with little movement of oil behind the flood front. This was confirmed in the model (see fractional flow curve, fig. 7).

Experimental Procedure

Initial experiments conducted on a small section of the box

These were conducted in order to determine the following:

1. Relative permeability curves.
   (a) Box was cleaned.
   (b) Sand was cemented to the sides of the box with clear rubber cement.
   (c) Box was packed with 10-20 mesh Ottawa sand in brine.
   (d) The porosity was determined by volumetric method.
   (e) Displaced brine with 2 pore volumes of refined crude oil with a viscosity of 520 cp.
   (f) Calculated irreducible water saturation by volumetric method.
   (g) Displaced oil with brine recording volumes of oil and water produced.
   (h) Calculated relative permeability using the method of Croes, G.A. and Schwarz, N. (1955) (fig. 8).

2. Calibration of probes:
   (a) Box was cleaned.
   (b) Sand was cemented to the sides of the box with clear rubber cement.
   (c) Box was packed with 10-20 mesh Ottawa sand in brine.

(d) Measured resistance across each probe at 100 percent S_w.

(e) Displaced brine with 2 pore volumes of refined crude oil with a viscosity of 520 cp.

(f) Measured resistance across each probe at Sw_{irr}.

(g) Displaced oil with brine. At various times recorded resistances across probes corresponding to an average water saturation in the box as calculated volumetrically.

The results of these experiments are shown in figure 9, and indicate that a relationship of the form shown below is correct.

$$\text{Log } S_w = M \text{ Log } R + \text{constant}$$

Initially the electrodes were 1/16 in. copper wire but inconsistent results were obtained due to oil wetting the copper probes. This problem was solved by gluing with silver paint felt pads ¼ in.by ¼ in. to the copper probes. The felt pads were initially soaked in brine thus ensuring capillary continuity from the sand through the electrodes.

Experiments conducted in full size box

The waterflood experiments were conducted as outlined in the following steps:

(a) Box was cleaned.

(b) Sand was cemented to the sides of the box with clear rubber cement.

(c) Box was packed with 10-20 mesh Ottawa sand in brine.

(d) The porosity was determined by volumetric method.

(e) The permeability to brine was measured. This was determined by measuring the flowrate and the pressure drop across the box. The pressure drop was measured with a manometer which utilized the circulating brine as the measuring fluid.

(f) With the box in a vertical position, brine was displaced with 1½ pore volumes of 22 cp mixture of gasoline and 520 cp refined crude oil injected at the top of the box.

(g) Sw_{irr} was determined volumetrically.

(h) Permeability to oil at Sw_{irr} was determined as in step (e).

(i) Resistances of all 55 probes were recorded at Sw_{irr}.

(j) The box was then placed in the horizontal position.

(k) The contacts on the stepping switches were cleaned and the recorder checked.

(l) Commenced waterflood.

(m) Pump was started and adjusted to the desired rate.

(n) Recorded flowrate and volume of oil and water produced.

(o) Recorded inlet and outlet pressures.

(p) Stopped flood after 2½ to 3 pore volumes.

(q) Turned box vertical, waited overnight.

(r) Displaced remaining oil with brine, calculated So_{irr} volumetrically and recorded resistances of all probes.

(s) Measured permeability to brine at So_{irr} as in step (e).

(t) During the flood the voltage drop across the probes was recorded automatically.

The experiments were repeated at different rates omitting the steps concerned with permeability measurements.

Capillary pressure measurements were taken from Thesis T-1575 by D.B. Tippie. This data was adjusted (fig. 6) by a factor of $\sqrt{\frac{K_T}{K_m}}$ as suggested by the Leverett (1941) J-function.

where

K_T = absolute permeability of the sand employed by D.B. Tippie

K_m = absolute permeability of the sand employed in this thesis

DISCUSSION OF EXPERIMENTAL RESULTS

Interpretation of Data

In order to interpret the readings recorded on the 24-point d-c recorder a computer program was written. The recorded data was punched onto cards and the program interpreted the data as outlined below:

Calculation of Constants

(a) Calibrate the recorder using the 4 known resistances after each 20 readings,

$$R = a + b/(\text{reading} + c) \qquad (1)$$

Iterate on c for best straight line.

(b) Calculate the constants for each set of electrodes in the following equation using the resistances/ water saturations at So_{irr} and Sw_{irr}.

$$\text{Log } S_w = d \text{ Log } R + e \qquad (2)$$

where

R = resistance

S_w = water saturation

Initial experiments on a small section of the box showed that equation (2) was correct (see fig. 9).

Calculation of Water Saturation

(a) Calibrate recorder (equation 1).

(b) Calculate resistance and then S_w (equation 2).

(c) Store onto disc files S_w and the time the reading was taken.

(d) Interpolate water saturation every ½-hour after start of flood.

(e) Print out water saturation maps every ½-hour.

The print-outs are shown at plots 1 - 16. The decimal point represents a pair of probes. The figures are the percent water saturation calculated for each pair of probes. Fluid injection is from left to right.

Author's Theory - Approximate Solution for the Case Where Gravity Forces are Significant in a Horizontal Waterflood

Whereas the Buckley-Leverett theory is concerned with the growth of the transition zone from the water phase to the oil phase during a waterflood, the author was concerned with water tonguing under the oil.

The assumptions employed in the theory were after Dietz, as follows:

1. Sharp interface - no transition zone from water to oil.
2. Constant absolute permeability.
3. Mobility ratio

$$\frac{K_o}{\mu_o} / \frac{K_w}{\mu_w} < 1$$

4. Constant injection rate.
5. Incompressible fluids.
6. Constant pressure drop or zero pressure drop across the oil/water interface.
7. Darcy's law applies - relative permeability data is not required for the full range of water saturation; however the permeability to oil at irreducible water saturation is required.

It can be shown that:

$$\frac{\delta P_w}{\delta x} = \frac{\delta P_o}{\delta x} + \Delta \rho \frac{\delta h}{\delta x} \tag{11}$$

where

P_w = pressure in the water phase

P_o = " " " oil "

$\Delta \rho$ = density difference (ρ_w - ρ_o)

$\delta h/\delta x$ = gradient of the oil/water interface

ρ_w = density of the water

ρ_o = density of the oil

For oil to flow

$$\frac{\delta P_w}{\delta x} > \Delta \rho \frac{\delta h}{\delta x}$$

$$q_o = W(H-h) \frac{K_o}{\mu_o} \frac{\delta P_o}{\delta x} \tag{12}$$

$$q_w = Wh \frac{K_w}{\mu_w} \frac{\delta P_w}{\delta x}$$

where

q_o = oil flow rate

q_w = water flow rate

W = width of the formation

from equation (11) substituting for $\delta P_w/\delta x$

$$q_w = Wh \frac{K_w}{\mu_w} \left(\frac{\delta P_o}{\delta x} + \Delta \rho \frac{\delta h}{\delta x} \right)$$

from equation (12) substituting for $\frac{\delta P_o}{\delta x}$

$$q_w = Wh \frac{K_w}{\mu_w} \left(\frac{q_o \, \mu_o}{(H-h) K_o W} + \Delta\rho \frac{\delta h}{\delta x} \right) \qquad (13)$$

the continuity equation is

$$\frac{\delta q_w}{\delta x} = W \phi s \frac{\delta h}{\delta t} \qquad (14)$$

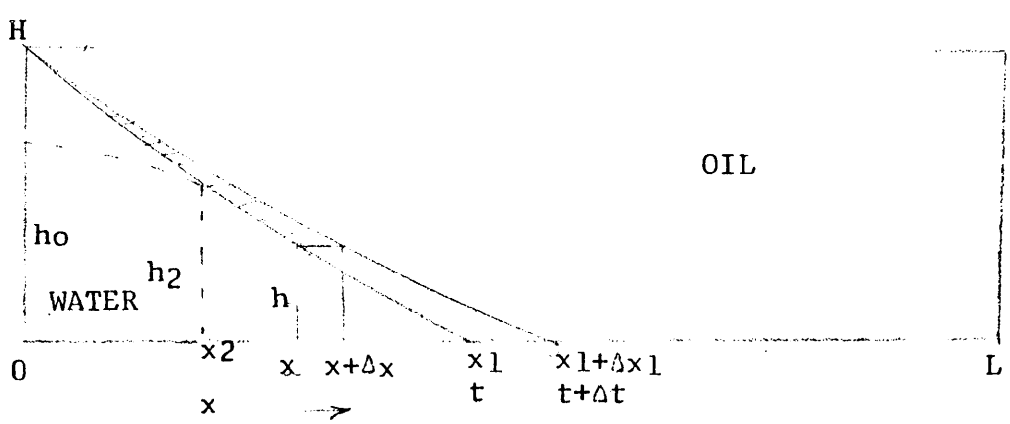
substituting equation (13) into equation (14)

$$\frac{\delta}{\delta x} \left(\frac{1/a \; h \; q_o}{W \; (H-h)} + \frac{K_w}{\mu_w} \Delta\rho \; h \; \frac{\delta h}{\delta x} \right) = \phi s \frac{\delta h}{\delta t} \qquad (15)$$

Equation (15) has been solved by Dietz for the case where $\frac{\delta h}{\delta x} \longrightarrow 0$ i.e. for long thin tongues and is probably correct where $\frac{\delta h}{\delta x}$ is small and $h \longrightarrow 0$. However, where $q_o \longrightarrow 0$ or where $1/a \longrightarrow 0$ (such as in the case of a gas) gravity forces are dominant and equation (15) reduces to

$$\frac{\delta}{\delta x} \left(\frac{K_w}{\mu_w} \Delta\rho \, h \; \frac{\delta h}{\delta x} \right) = \phi s \frac{\delta h}{\delta t} \qquad (16)$$

Equation (16) has been solved by hydrologist Polubarinova-Kochina (1962) for water flow into an empty reservoir. It would be interesting to carry out some experiments where a is greater than one. However, in order to solve equation (15) for the case where gravity forces become dominant near the fluid injection end of the tongue let us assume that gravity forces are negligible near the producing end of the tongue.



Consider a position h, x on the oil/water interface

$$q_w, \text{ Water rate } = hW\phi s \left(\frac{\delta x}{\delta t}\right)_h$$

also

$$q_w = hW \frac{K_w}{\mu_w} \frac{\delta p}{\delta x}$$

the oil rate is equal to the shaded volume swept out in time Δt.

On inspection of the water saturation maps (see plots 1 - 16 and photograph) the shape of the water tongue appears to be of the form:

$$x = B(H-h)^A \tag{17}$$

where A, B are constants.

Boundary conditions are

at $x = 0, \quad h = H$

at $x = x_1, \quad h = 0$

hence equation (17) becomes

$$x = \frac{x_1}{H^A} (H-h)^A \tag{18}$$

volume of oil swept out due to a change Δx_1

$$\frac{\Delta Q_o}{W\phi s} = \left|\int_h^H x \, dh \right|_{x + \Delta x} - \left|\int_h^H x \, dh\right|_x$$

i.e. $$\frac{1}{W\phi s} \frac{dQ_o}{dx} = \frac{d}{dx}\left(\int_h^H x \, dh\right)$$

from equation (18)

$$\int_h^H x \, dh = \int_h^H \frac{x_1}{H^A} (H-h)^A \, dh = \frac{x_1}{H^A} \frac{(H-h)^{A+1}}{A+1}$$

now $$\frac{1}{W\phi s}\left(\frac{dQ_o}{dx}\right)_h = \frac{dx_1}{dx}\;\frac{(H-h)^{A+1}}{H^A\,A+1} \qquad (19)$$

from equation (18)

$$\frac{dx_1}{dx} = \frac{H^A}{(H-h)^A}$$

substitute into equation (19)

$$\frac{1}{W\phi s}\left(\frac{dQ_o}{dx}\right)_h = \frac{(H-h)}{A+1} \qquad (20)$$

$$\text{now } q_o = \left(\frac{dQ_o}{dx}\right)_h \times \left(\frac{dx}{dt}\right)_h = W\phi s\,\frac{(H-h)}{A+1}\left(\frac{dx}{dt}\right)_h =$$

$$W(H-h)\;\frac{K_o}{\mu_o}\;\frac{\delta P}{\delta x}$$

$$\text{also } q_w = \left(\frac{\delta x}{\delta t}\right)_h W\phi sh = W\,h\,\frac{K_w}{\mu_w}\;\frac{\delta P}{\delta x}$$

dividing and rearranging $A+1 = M$

$$\text{where } M = \frac{\mu_o}{K_o} \times \frac{K_w}{\mu_w} = \frac{1}{a}$$

Hence a solution for the case where gravity forces are neglected is:

$$x = \frac{x_1}{H^A}\;(H-h)^{M-1} \qquad (21)$$

At the point where

$$\frac{\delta P_w}{\delta x} = \Delta\rho\frac{\delta h}{\delta x}$$, the oil rate is zero

Let us assume that the steady state solution of equation (16) is sufficient, which is

$$h\,\frac{K_w}{\mu_w}\,\Delta\rho\frac{\delta h}{\delta x} = \text{Constant} = \frac{q_w}{W} \text{ (equ. (13) with } q_o = 0)$$

on integrating since $q = q_w$

$$(h_o{}^2 - h^2)\,\frac{K_w}{\mu_w}\,\Delta\rho = \frac{qx}{W} \qquad (22)$$

where $h = h_o$ at $x = 0$, q = the injection rate.

Equation (18) is employed until

$$\frac{\delta P_w}{\delta x} = \Delta \rho \frac{\delta h}{\delta x}$$

and

$$\frac{q \mu_w}{W h K_w} = \Delta \rho \frac{\delta h}{\delta x} \qquad (23)$$

from thereon equation (22) is assumed

now from equation (18)

$$\frac{\delta h}{\delta x} = \frac{H^{M-1}}{(M-1)\ x_1\ (H-h)^{M-2}}$$

substituting into equation (23)

$$\frac{q \mu_w}{W h \Delta \rho K_w} = \frac{H^{M-1}}{(M-1)\ x_1\ (H-h)^{M-2}} \qquad (24)$$

Equation (24) is solved for $h = h_2$, $x = x_2$ (see diagram).

from equation (18)

$$x_2 = \frac{x_1\ (H-h_2)^{M-1}}{H^{M-1}}$$

from equation (22)

$$h_o = \left(h_2^2 + x_2 \frac{q \mu_w}{W \Delta \rho K_o}\right)^{\frac{1}{2}}$$

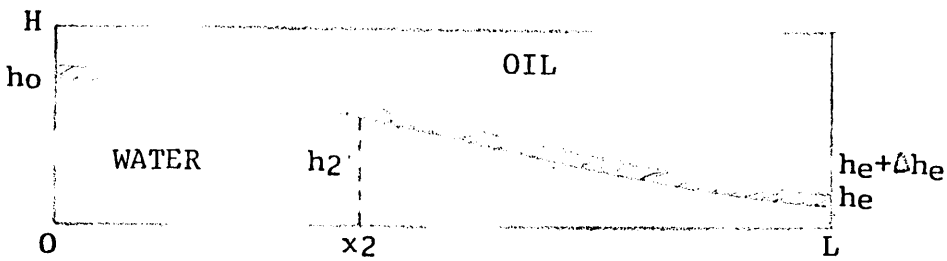
At breakthrough $x_1 = L$ and the volume of oil recovered is found by integrating the volume of the water tongue.

$$\frac{Q_o}{W \phi s} = \int_0^{x_2} h\, dx + \int_{x_2}^{L} h\, dx$$

$$= \frac{2}{3} \frac{\Delta \rho K_w W}{q \mu_w} \left[h_o^3 - \left(h_o^2 - \frac{x_2\, q\, \mu_w}{\Delta \rho K_w W} \right)^{3/2} \right] +$$

$$H(L-x_2) + H \left[\frac{x_2^{M/M-1}}{L^{1/M-1}} - L \right] \left(\frac{M-1}{M} \right) \qquad (25)$$

After breakthrough



Oil swept out (ΔQ_o) due to a change (Δh_e) in h_e.

$$\frac{\Delta Q_o}{W\phi s} = \left| \int_0^L h\,dx \right|_{h+\Delta h_e} - \left| \int_0^L h\,dx \right|_{h_e}$$

$$\frac{1}{W\phi s} \frac{\partial Q_o}{\partial h_e} = \frac{\partial}{\partial h_e} \left| \int_0^L h\,dx \right|$$

Oil production rate at $x = L$ is $= \left(\frac{\partial Q_o}{\partial h_e}\right)_{x=L} \times \left(\frac{\partial h_e}{\partial t}\right)_{x=L}$

$$= W(H-h_e) \frac{K_o}{\mu_o} \frac{\partial P}{\partial x} \qquad (26)$$

Water production rate at $x = L$ is $q - \left(\frac{\partial Q_o}{\partial h_e}\right)_{x=L} \times \left(\frac{\partial h_e}{\partial t}\right)_{x=L}$

$$= h_e \frac{K_w}{\mu_w} \frac{\partial P}{\partial x} \qquad (27)$$

dividing equation (27) by equation (26)

$$\frac{q - \frac{\partial Q_o}{\partial h_e} \times \frac{\partial h_e}{\partial t}}{\frac{\partial Q_o}{\partial h_e} \times \frac{\partial h_e}{\partial t}} = \frac{h_e M}{(H-h_e)}$$

and rearranging

$$q = \frac{\partial Q_o}{\partial h_e} \times \frac{\partial h_e}{\partial t} \left(\frac{M h_e}{(H-h_e)} + 1 \right)$$

$$\int_0^t q\,dt = \int_0^{h_e} \frac{d Q_o}{dh_e} \left(\frac{M h_e}{(H-h_e)} + 1 \right) dh_e$$

$$Q_t = \Big| _o^{h_e} Q_o \frac{(h_e M + (H-h_e))}{(H-h_e)} \Big| - \int_o^{h_e} Q_o \frac{MH}{(H-h_e)^2} dh_e \qquad (28)$$

Q_o is calculated as follows:

from equation (18)

$$x = \frac{L\ (H-h)^{M-1}}{(H-h_e)^{M-1}} \qquad (29)$$

Since the new boundary conditions are

at $x = 0, \quad h = H$

at $x = L, \quad h = h_e$

as before h_2 is determined by solving

$$\frac{q}{h\ W} \frac{\mu_w}{\Delta\rho K_w} = \frac{(H-h_e)^{M-1}}{L(M-1)\ (H-h)^{M-2}} \qquad (30)$$

and

$$x_2 = \frac{L(H-h_2)^{M-1}}{(H-h_e)^{M-1}} \qquad (31)$$

$$h_o = \left(h_2^2 + x_2 \frac{q\ \mu_w}{\Delta\rho K_w} \right)^{\frac{1}{2}} \qquad (32)$$

Q_o is determined by integrating the volume of the water tongue and

$$\frac{Q_o}{W\phi s} = \frac{2}{3} W \frac{\Delta\rho K_w}{q\ \mu_w} \left[h_o^3 - \left(h_o^2 - \frac{x_2\ q\mu_w}{K_w\ W} \right)^{3/2} \right] + H(L-x_2)$$
$$+ (H-h_e) \left[\frac{x_2^{M/M-1}}{L^{1/M-1}} - L \right] \frac{M-1}{M} \qquad (33)$$

The integral in equation (28) can be evaluated employing the trapezoidal rule and Q_o is determined from equation (33).

Total fluid injected $= Q_t + Q_{ob}$

where Q_{ob} = oil recovered at breakthrough (equation 25).

Appendix A further develops the theory to correct for gravity at the producing end of the tongue, and it is shown that it has a negligible effect on its shape. However, at low rates, the inclusion of the gravity forces is required in order to predict the waterflood performance accurately.

A computer program was written to calculate Q_o and Q_t from equations (33) and (A6) for values of h_e ranging from 0 to H for the model. The results are shown at figures 10 to 13.

Appendix B illustrates how an engineer can use these equations to predict waterflood performance.

Discussion of the Shape of the Water Tongue

It can be seen that the author's theory fits the observed data satisfactorily. The calculated shapes of the water tongues are plotted in plots 1, 4, 7, 10, 12, 13, 15 and 16. The agreement between the calculated shapes and the observed as recorded on the water saturation maps is good.

The first waterflood experiment (4/5th April 1973) was used to determine the experimental procedure and to familiarize the author with the equipment. This flood was carried out with the inlet and outlet positioned in the center of the end-plates. There was a distinct oil/water interface in the end-plates which corresponded to the oil/water interface in the sand pack. On observing the oil/water interface in the inlet end-plate, the box was turned vertical and left overnight in an attempt to obtain an initial vertical oil/water interface. However, shortly after the start of injection with the box horizontal, an oil/water interface appeared again in the injection end-plate. The subsequent waterflood experiments were carried out with zero initial oil/water interface. It was thought that the appearance of an oil/water interface in the end-plates was due to the position of the inlet and outlet. All subsequent floods were therefore carried out with the inlet positioned at the top of the end-plate and the outlets positioned top, center and bottom of the end-plate. An oil/water interface in both end-plates was still observed, as would be expected from the author's theory.

The effect of rate on the shape of the water tongue can be seen from the results of the first flood. On reducing the injection rate from 1.5 to 0.69 cu ft/day, the gravity controlled (i.e. injection end) portion of the tongue reduced in height and increased in length and the tongue became essentially horizontal (see plot 5) and oil recovery was reduced (see fig. 10). This was in agreement with the author's theory. On increasing the rate to 9.7 cu ft/day the injection end of the tongue increased in height and reduced in length (see plot 6) and the oil recovery was consequently increased (see fig. 10). The shape of the tongue at the producing end appeared to be independent of rate and in agreement with the theory. These phenomena, discussed above, were also observed in the remaining waterflood experiments.

The contacts on the stepping switches were cleaned very carefully before each run since a change in resistance of 2 ohms at saturations of about 80 percent corresponds to a change in saturation of approximately 4 percent (see fig. 9). The measured water saturation values are accurate to within ± ½ percent for low water saturation readings and increase to approximately ± 6 percent at the high water saturation readings.

A closer study of the water saturation maps shows that the transition zone does not develop at all, indicating that after breakthrough oil is displaced essentially vertically near the oil/water contact and with almost piston-like displacement. Since the transition zone is not developed it is possible that the streamlines in the water tongue are bent parallel with the oil/water contact due to the permeability reduction of both oil and water.

A point of interest was that just before a pair of probes changed from low saturation to a high water saturation, the water saturation as measured by the probes reduced slightly which could be due to oil moving vertically away from the oil/ water contact.

One possible source of error is end effects caused by a water saturation discontinuity at the injection and producing ends of the model. However since a distinct interface was observed in the end-plates (regardless of the positioning of the inlets and outlets) corresponding to the oil/water contacts in the sand pack no such discontinuity arises and errors due to end effects can be eliminated.

CONCLUSIONS

The purpose of this thesis was to compare experimentally the Dietz and Buckley-Leverett frontal advance theories. The following conclusions can be drawn from the experimental results.

1. Where gravity forces are significant during a horizontal or nearly horizontal waterflood the normal Dietz theory is unsatisfactory. By assuming that the oil rate was equal to the rate at which the oil/water contact swept out oil, a different theory was developed taking into account gravity forces. Although this was an approximate solution to the continuity equation, the solution agreed satisfactorily with the experimental data.

2. If water tonguing in horizontal reservoirs occurs the Buckley-Leverett theory is unsatisfactory in predicting waterflood performance.

3. Once a tongue has formed the transition zone does not develop and oil is displaced vertically near the oil/water contact with piston-like displacement.

4. As the injection rate increased the effects of gravity decreased and the initial oil recovery increased.

LITERATURE CITED

1. Buckley, S.E., and Leverett, M.C., 1942, Mechanism of fluid displacement in sands: Trans. AIME, v. 146 p. 107-116.

2. Croes, G.A., and Schwarz, N., 1955, Dimensionally scaled experiments and the theories on the water-drive process: Trans. AIME, v. 204, p. 35-42.

3. Dietz, D.N., 1953, A theoretical approach to the problem of encroaching and bypassing edge water: Proc. Kominkl. Ned. AKad. Wetenschap, 1953, B56, p. 38.

4. Leverett, M.C., 1941, Capillary behaviour in porous solids: Trans. AIME, v. 142, p. 152-169.

5. Polubarinova-Kochina, P.YA., 1962, The theory of ground water movement: New Jersey, Princeton University Press, p. 481-517.

6. Rapoport, L.A., and Leas, W.J., 1953, Properties of linear waterfloods: Trans. AIME, v. 189, p. 139-148.

7. Terwilliger, P.L., Wilsey, L.E., Hall, H.N., Bridges, P.M., and Morse, R.A., 1951, An experimental and theoretical investigation of gravity drainage performance: Trans AIME, v. 192, p. 285-296.

8. Tippie, D.B., 1973, The effect of producing rate on the formation of arches in unconsolidated sands: Colorado School of Mines Thesis, T-1575, p. 199.

9. Welge, H.J., 1952, A simplified method for computing oil recovery by gas or water-drive: Trans. AIME, v. 195, p. 91-98.

APPENDIX A

Correction for the effects of gravity at the producing end of the tongue.

at $x = L, \; h = h_e$

$$q_o = \frac{dQ_o}{dh_e} \frac{dh_e}{dt} = W(H-h_e) \frac{K_o}{\mu_o} \left(\frac{\delta P_w}{\delta x} - \Delta\rho \frac{\delta h_e}{\delta x} \right)$$

rearranging

$$\frac{dQ_o}{dh_e} \frac{dh_e}{dt} + \Delta\rho \frac{\delta h}{\delta x} \; W \; (H-h_e) \frac{K_o}{\mu_o} = W \; (H-h_e) \frac{K_o}{\mu_o} \frac{\delta P_w}{\delta x} \qquad (A1)$$

also from equation (27) in the text

$$q - \frac{dQ_o}{dh_e} \frac{dh_e}{dt} = W \, h_e \frac{K_w}{\mu_w} \frac{\delta P_w}{\delta x} \qquad (A2)$$

dividing equation (A2) by equation (A1) and rearranging

$$\int_0^t \left(q - WM \Delta\rho \frac{\delta h_e}{\delta x} \frac{K_o}{\mu_o} \right) dt = \int_0^{h_e} \left[\frac{h_e M + H - h_e}{(H-h_e)} \right] \frac{dQ_o}{dh_e} \; dh_e$$

$$\text{i.e.} \quad Q_t - \int_0^t h_e M \Delta\rho \frac{\delta h_e}{\delta x} \frac{K_o}{\mu_o} W dt = \left| Q_o \frac{(h_e M + (H-h_e)}{(H-h_e)} \right|_0^{h_e} - \int_0^{h_e} Q_o \frac{MH}{(H-h_e)^2} \; dh \qquad (A3)$$

now $q = \dfrac{dQ_t}{dt}$

substituting into equation (A3)

$$Q_t - \int_0^{Q_t} \frac{h_e M \Delta\rho}{q} \frac{\delta h_e}{\delta x} \frac{K_o}{\mu_o} W dQ_t = \text{R.H.S.} \qquad (A4)$$

using the trapezoidal rule

$$\int_0^{Qt} h_e \frac{M\Delta\rho}{q} \frac{\delta h_e}{\delta x} \frac{K_o}{\mu_o} W dQt = \frac{(Qt_n - Qt_{n-1})}{2} (B_n + B_{n-1}) +$$

$$\sum_{i=1}^{n-1} \frac{(Qt_i - Qt_{i-1})(B_i + B_{i-1})}{2} \qquad \text{(A5)}$$

where $B_n = WM\Delta\rho\, h_e \frac{K_o}{\mu_o q} \left(\frac{\delta h_e}{\delta x}\right)_n$

from equation (29) in the text

$$\left(\frac{\delta h_e}{\delta x}\right)_n = \frac{(H-h_e)}{L\ (M-1)}$$

substituting equation (A5) into equation (A4) and rearranging

$$Qt\ (1-AB) = R.H.S. + \sum_{i=1}^{n-1} (Qt_i - Qt_{i-1})(B_i + B_{i-1}) -$$

$$Qt_{n-1}\ AB \qquad \text{(A6)}$$

where $AB = (B_n + B_{n-1})/2$, R.H.S. = right hand side of (A3)

Total fluid injected = $Q_t + Q_{ob}$

where Q_{ob} = oil recovered at breakthrough

It can be seen that the correction for gravity effects depends mainly on AB.

$$\text{now } AB \approx WM \frac{\Delta\rho K_o}{L \mu_o\ q\ (M-1)} h_e\ (H-h_e) \qquad \text{(A7)}$$

AB is 0 at both $h_e = 0$ and $h_e = H$, the maximum value of AB is at $h_e = H/2$.

For field units

all distances in ft

q in cu ft/day

$\Delta\rho$ in gms/cc

K_o in darcys

μ_o in cp

$$AB \approx 2.75 \ \frac{WM\Delta\rho}{L} \ \frac{K_o}{\mu_o q \ (M-1)} \ h_e(H-h_e) \tag{A8}$$

using the model properties and for M = 2.8, q = 1.5 cu ft/day.

(1st flood) $AB_{max} \approx 0.36$

for M = 5.5 and q = 5, 10, 15 cu ft/day respectively

$AB_{max} \approx 0.043,\ 0.022,\ 0.014$ respectively

It can be seen that at a high value of M and q the effect of gravity is negligible but at low values of M and q the correction must be made.

A computer program was written to calculate Q_o and Q_t using equations (A6) and (33). The results compare very well with the observed data (figs. 10 - 13) showing that the a priori assumption, that the effect of gravity does not significantly change the shape of the water tongue at the producing end, was correct.

APPENDIX B

Application of the theory.

1. Breakthrough

The time when breakthrough of water occurs is found by dividing the cumulative oil produced at breakthrough by the injection rate. The cumulative oil produced is estimated from equation (25).

inserting field units where:

q = injection rate, cu ft/day

heights, width, and lengths are in ft

K_w = effective permeability to water, darcys

μ_w = viscosity of water, cp

$\Delta\rho$ = density difference, gms/cc

s = $1 - So_{irr} - Sw_{irr}$, fraction

put $C = \dfrac{q\,\mu_w}{2.75\,\Delta\rho\,K_w W}$

we obtain

$$Q_o = W\phi s \left\{ \frac{2}{3}\,\frac{1}{C}\left[h_o^3 - (h_o^2 - x_2 C)\right]^{3/2} + H(L - x_2) + H\left[\frac{x_2^{M/M-1}}{1/M-1} - L\right]\left(\frac{M-1}{M}\right)\right\} \quad (B1)$$

h_2 is determined from equation (24) putting $x_1 = L$ and inserting field units. h_2 is solved in equation (B2)

$$\frac{C}{h_2} = \frac{H^{M-1}}{(M-1)\;L\;(H-h_2)^{M-2}} \quad (B2)$$

$$x_2 = \frac{L\;(H-h_2)^{M-1}}{H^{M-1}} \quad (B3)$$

$$h_o = (h_2^2 + x_2 C)^{\frac{1}{2}} \quad (B4)$$

knowing x_2 (equation B3) and h_o (equation B4), Q_o is calculated for equation (B1).

Time for breakthrough = Q_o/q days.

Example

for $M = 5.5$

$q = 5$ cu ft/day

$H = 0.75$ ft

$W = 0.5$ ft

$s = 0.76$

$L = 6$ ft

$\phi = 0.32$

$\Delta\rho = 0.36$ gms/cc

$K_w = 170$ darcys

$\mu_w = 2.0$ cp

from equation (B2)

$h_2 = 0.376$ ft

from equation (B3)

$x_2 = 0.26$ ft.

from equation (B4)

$h_o = 0.415$ ft.

from equation (B1)

$Q_o = 0.0935$ cu ft

Time of breakthrough = 0.0935/5

= 0.0187 days

= 28 min

2. After Breakthrough

In a similar manner as the breakthrough calculations were carried out, cumulative oil recovered as a function of cumulative water injected can be estimated using equation (A6) and equation (33) (in the text). This has been carried out for the waterflood 26th April 1973 and is plotted in figure 11 in terms of pore volumes.

The following can now be estimated:

(a) Oil production rate as a function of time

$$q_o = (1-f_w)q \tag{B5}$$

where f_w is the water cut

now $$f_w = 1 - \frac{dQ_o}{dQ_t} \tag{B6}$$

$\frac{dQ_o}{dQ_t}$ is the gradient of the curve in figure 11.

substituting into equation (B5)

$$q_o = \frac{dQ_o}{dQ_t}\, q \tag{B7}$$

The oil production rate for various values of Q_t can be estimated from equation (B7) and the time is found

$$t(\text{days}) = Q_t/q$$

(b) Water production rate as a function of time

$$q_w = f_w q$$

from equation (B6)

$$q_w = \left(1-\frac{dQ_o}{dQ_t}\right) q \tag{B8}$$

The water production rate for various values of Q_t can be estimated from equation (B8) and the time is

$$t(\text{days}) = Q_t/q \tag{B9}$$

(c) Cumulative oil production as a function of time. From figure 11, Q_o is determined as a function of Q_t and time is found from equation (B9).

(d) Cumulative water production as a function of time

$$Q_w = Q_t - Q_o \tag{B10}$$

From figure 11, Q_o and hence Q_w (equation B10) is determined as a function of Q_t and time is found from equation (B9).

<u>Example</u>

For the conditions of waterflood, 26th April 1973, the oil production rate, the water production rate, the cumulative oil production, and the cumulative water production were found as functions of time. The results are tabulated below:

Q_t cu ft	Time min	$\frac{dQ_o}{dQ_t}$	q_o cu ft/d	q_w cu ft/d	Q_o cu ft	Q_w cu ft	Water Cut
0.0913	31	1.0	4.25	0.0	0.0913	0.0	0.0
0.281	95	0.34	1.44	2.81	0.183	0.098	0.66
0.562	191	0.205	0.87	3.38	0.26	0.302	0.795
0.843	287	0.15	0.64	3.61	0.31	0.533	0.85
1.13	384	0.124	0.53	3.72	0.344	0.786	0.875
1.4	476	0.0795	0.34	3.91	0.372	1.128	0.9205
1.69	574	0.0456	0.19	4.06	0.386	1.304	0.9544
1.97	670	0.0515	0.22	4.03	0.400	1.57	0.9485

APPENDIX C - NOMENCLATURE

A	=	cross-sectional area, sq cm
a	=	mobility ratio $\frac{K_o}{\mu_o}\frac{\mu_w}{K_w}$
f_{o2}	=	fraction of water flowing at producing face
f_w	=	fraction of water in the flowing stream (i.e. watercut)
g	=	acceleration due to gravity, cm per sec per sec
H	=	formation thickness, cm
h	=	perpendicular distance from the bottom of the bedding plane to the oil/water interface, cm
h_2		height of the water tongue at the point where $\frac{\delta P_w}{\delta x} = \Delta\rho\frac{\delta h}{\delta x}$, cm
h_e	=	height of the water tongue at the producing face, cm
K	=	absolute permeability, darcy
K_o	=	effective permeability to oil at irreducible water saturation, darcy
K_w	=	effective permeability to water at irreducible oil saturation, darcy
K_{ro}	=	relative permeability to oil
L	=	length of the box, cm
M	=	mobility ratio $\frac{\mu_o}{K_o}\frac{K_w}{\mu_w} = \frac{1}{a}$
P_c	=	capillary pressure, atm
P_o	=	pressure in the oil phase, atm
P_w	=	pressure in the water phase, atm

Q_i	=	cumulative pore volumes of fluid injected
Q_o	=	cumulative oil production, cc
Q_{ob}	=	cumulative oil produced at breakthrough, cc
Q_w	=	cumulative water production, cc
Q_t	=	cumulative fluid injected, cc
Q	=	cumulative fluid injected per unit width, sq cm
q	=	injection rate, cc per sec
q_o	=	oil production rate, cc per sec
q_w	=	water production rate. cc per sec
R	=	electrical resistance in ohms
R.H.S.	=	right hand side of previous equation
s	=	$1 - S_{w_{irr}} - S_{o_{irr}}$
S_w	=	water saturation, fraction
S_{w2}	=	water saturation at producing face, fraction
$S_{w_{irr}}$	=	irreducible water saturation, fraction
$S_{o_{irr}}$	=	irreducible oil saturation, fraction
U	=	total fluid velocity (q/A), cm per sec
W	=	width of formation, cm
α	=	formation dip, deg
δ	=	partial derivative
$\Delta\rho$	=	density difference $(\rho_w - \rho_o) \times 0.000968$, atm per cm
μ_o	=	oil viscosity, cp
μ_w	=	water viscosity, cp
ρ_o	=	oil density, gm per cc
ρ_w	=	water density, gm per cc
ϕ	=	porosity, fraction

FIGURE 1

SIDE VIEW OF BOX

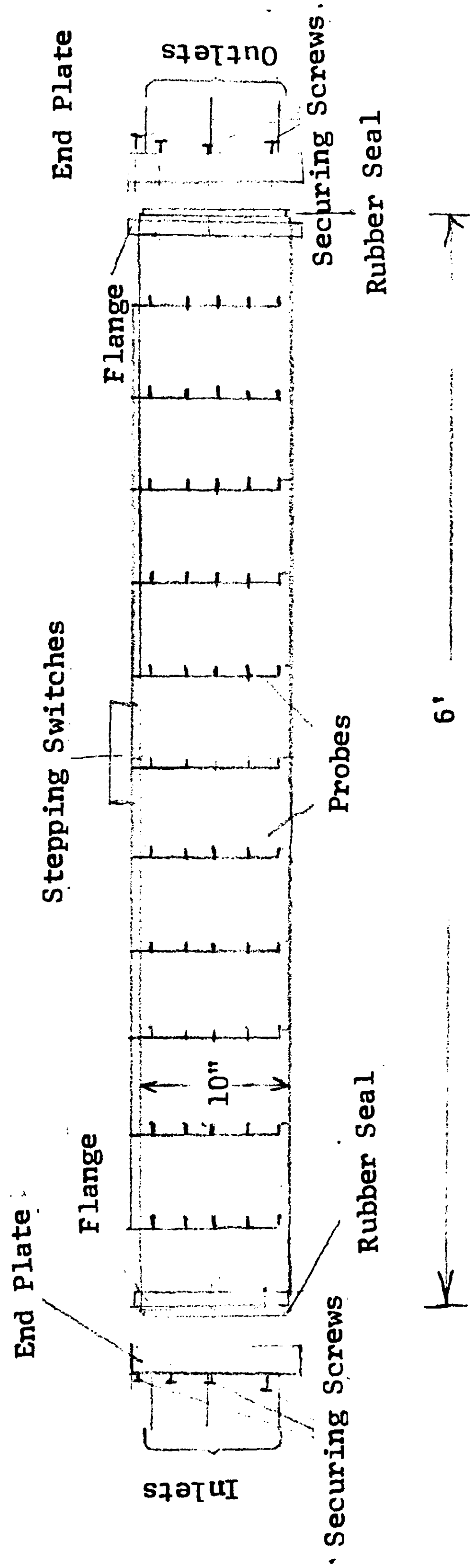


FIGURE 2

DIAGRAM OF END PLATES

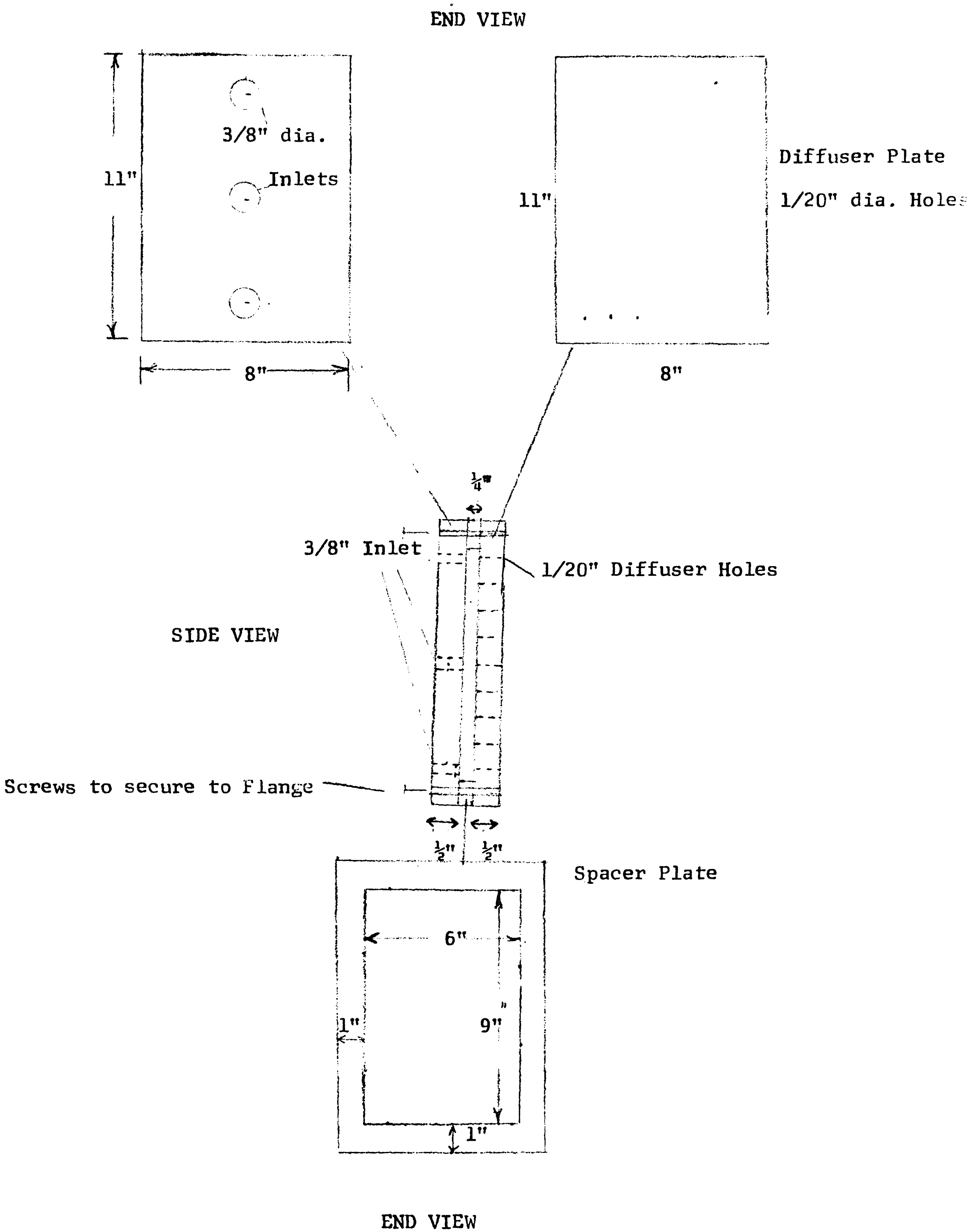


FIGURE 3

ELECTRICAL CIRCUIT DIAGRAM

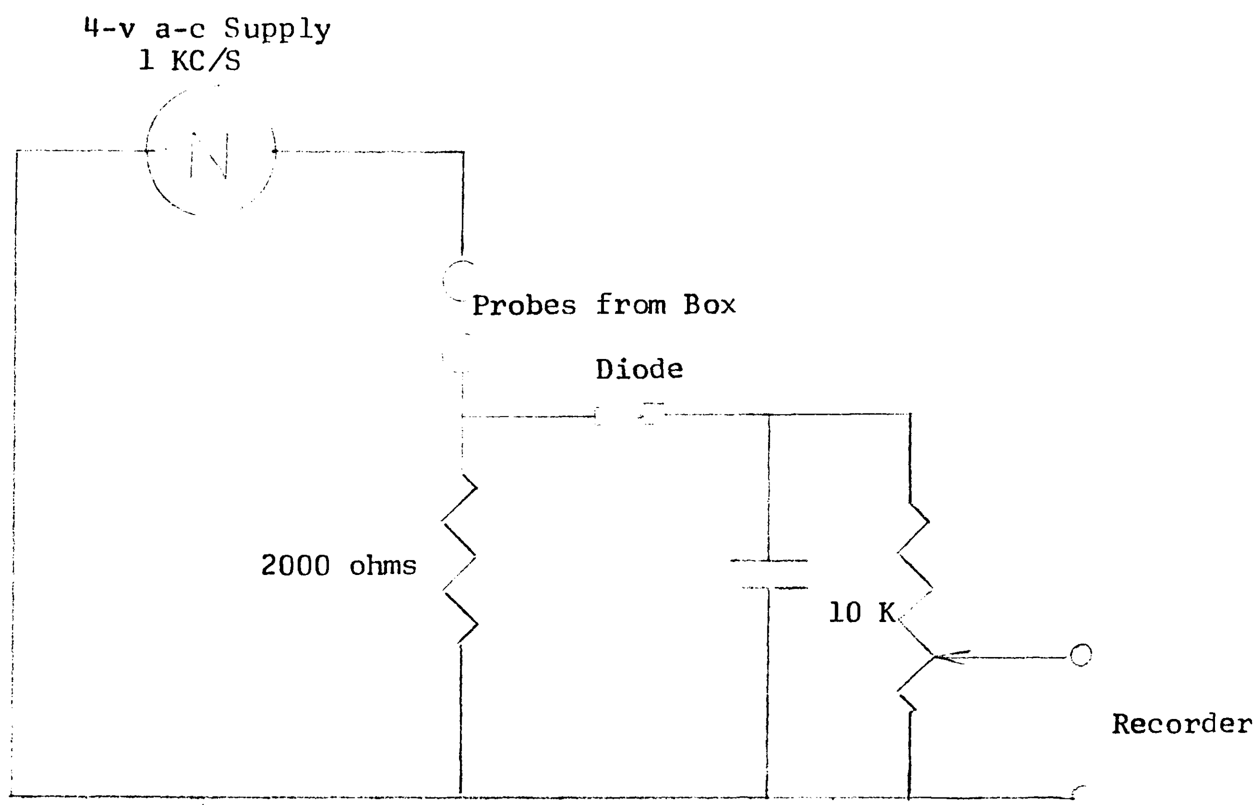
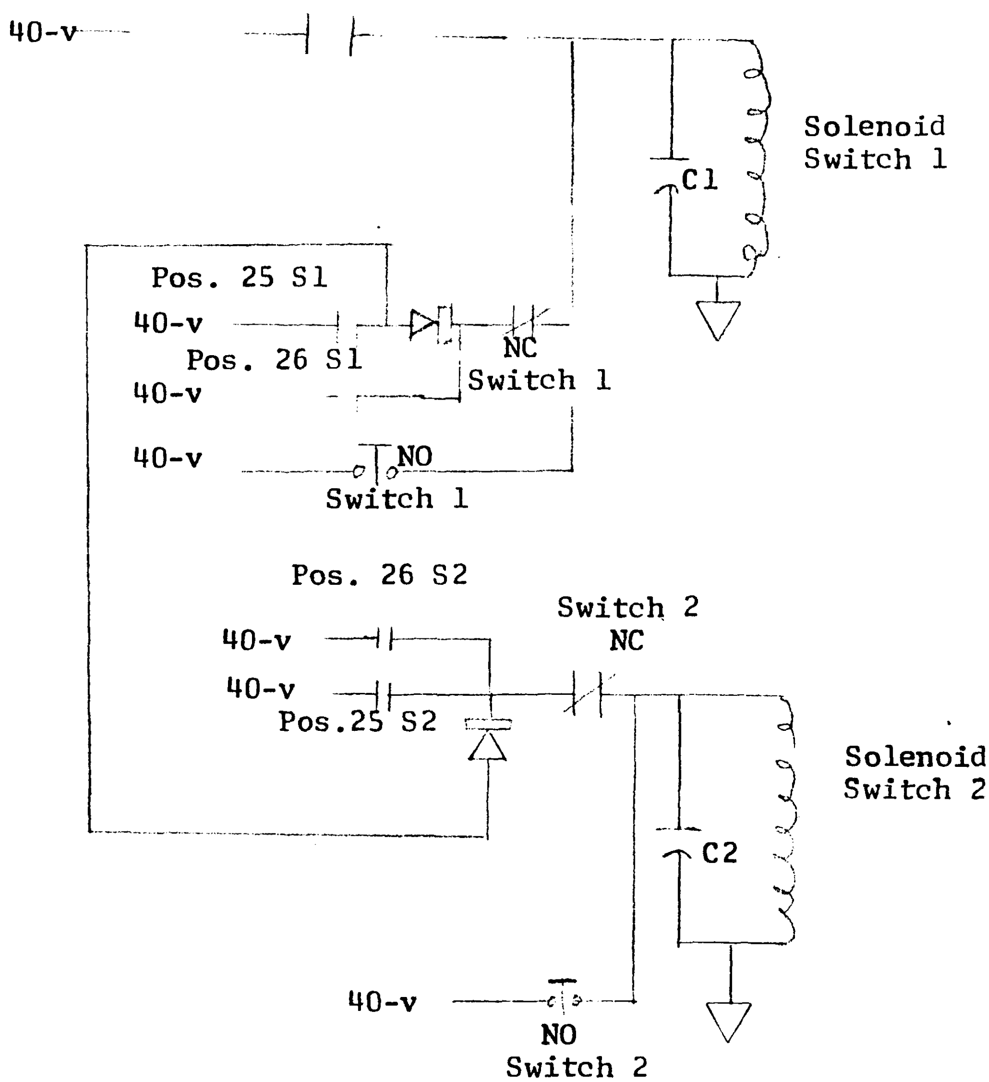


FIGURE 4

STEPPING SWITCHES

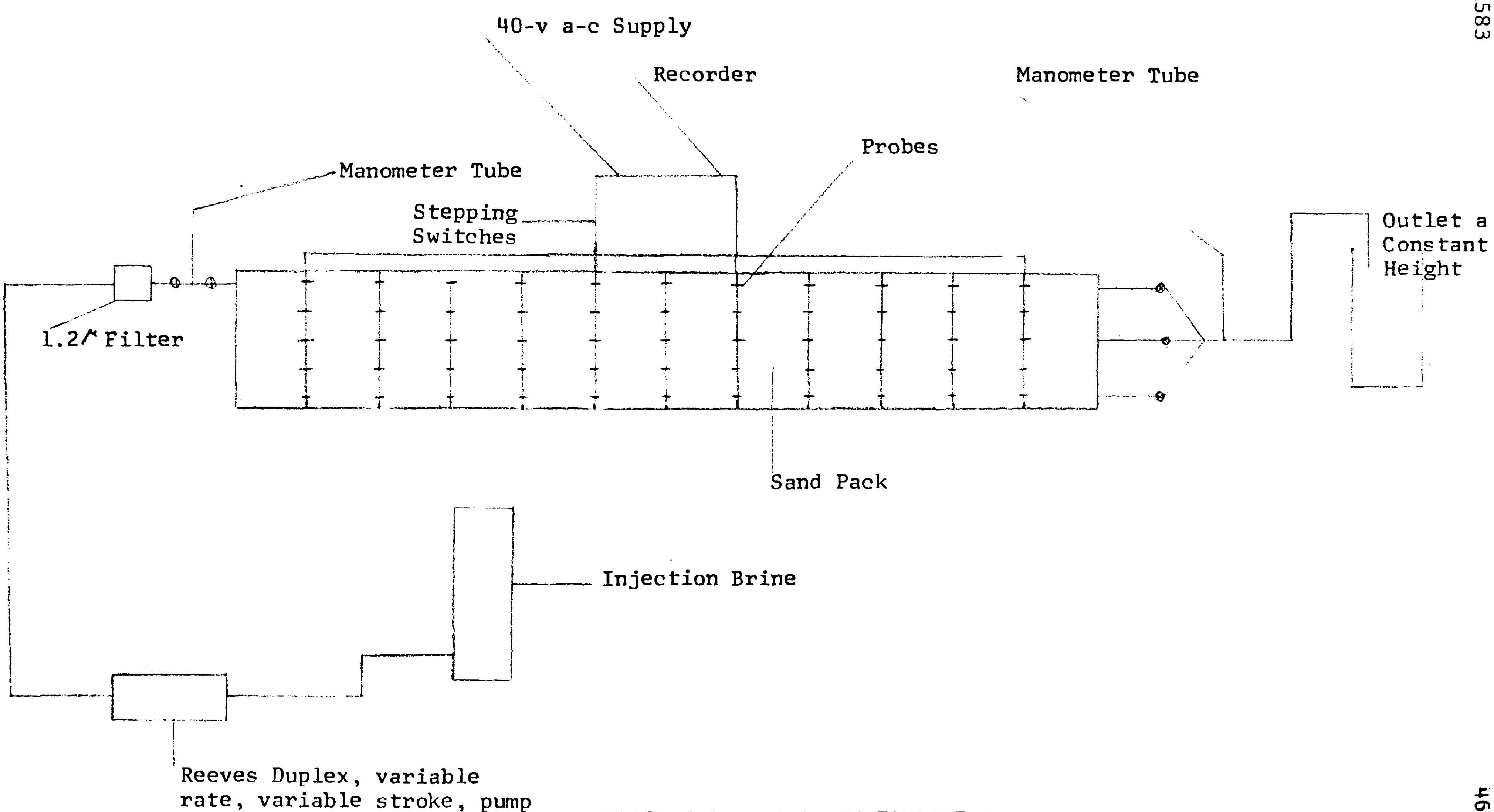
CIRCUIT DIAGRAM

Positions 1 to 24 on Recorder operated by Microswitch



S1 Switch No. 1
S2 Switch No. 2
NC Normally Closed
NO Normally Open

FIGURE 5



SCHEMATIC DIAGRAM OF EQUIPMENT

TABLE 1

ROCK AND FLUID PROPERTIES

Oil:	50 SAE refined crude diluted with gasoline.		
1st Flood	sp gr	=	0.837 gms/cc
	Viscosity	=	9.7 cp
All other Floods	sp gr	=	0.857 gms/cc
	Viscosity	=	22 cp
Brine:	Fully saturated with salt.		
	sp gr	=	1.195 gms/cc
	Viscosity	=	2.0 cp
	Salinity	=	330,000 ppm
Sand:	10 - 20 mesh Ottawa sand		
	Grain density	=	2.66 gms/cc
	Porosity	=	32%
	Absolute permeability to brine	=	390 darcies
	Capillary pressure		see Fig. 6
	The transition zone was estimated to be approximately 2 in.		

Table 1 (continued)

Estimated total weight of box, plus sand, plus brine	=	500 lbs.
Irreducible water saturation	=	3 - 11%
Irreducible oil saturation	=	21 - 23%
Pore volume	=	19,865 cc.

All measurements were taken at a room temperature of 72°F.

FIGURE 6

AIR/WATER CAPILLARY PRESSURE DATA

(ADJUSTED DATA - FROM D.B. TIPPIE T-1575)

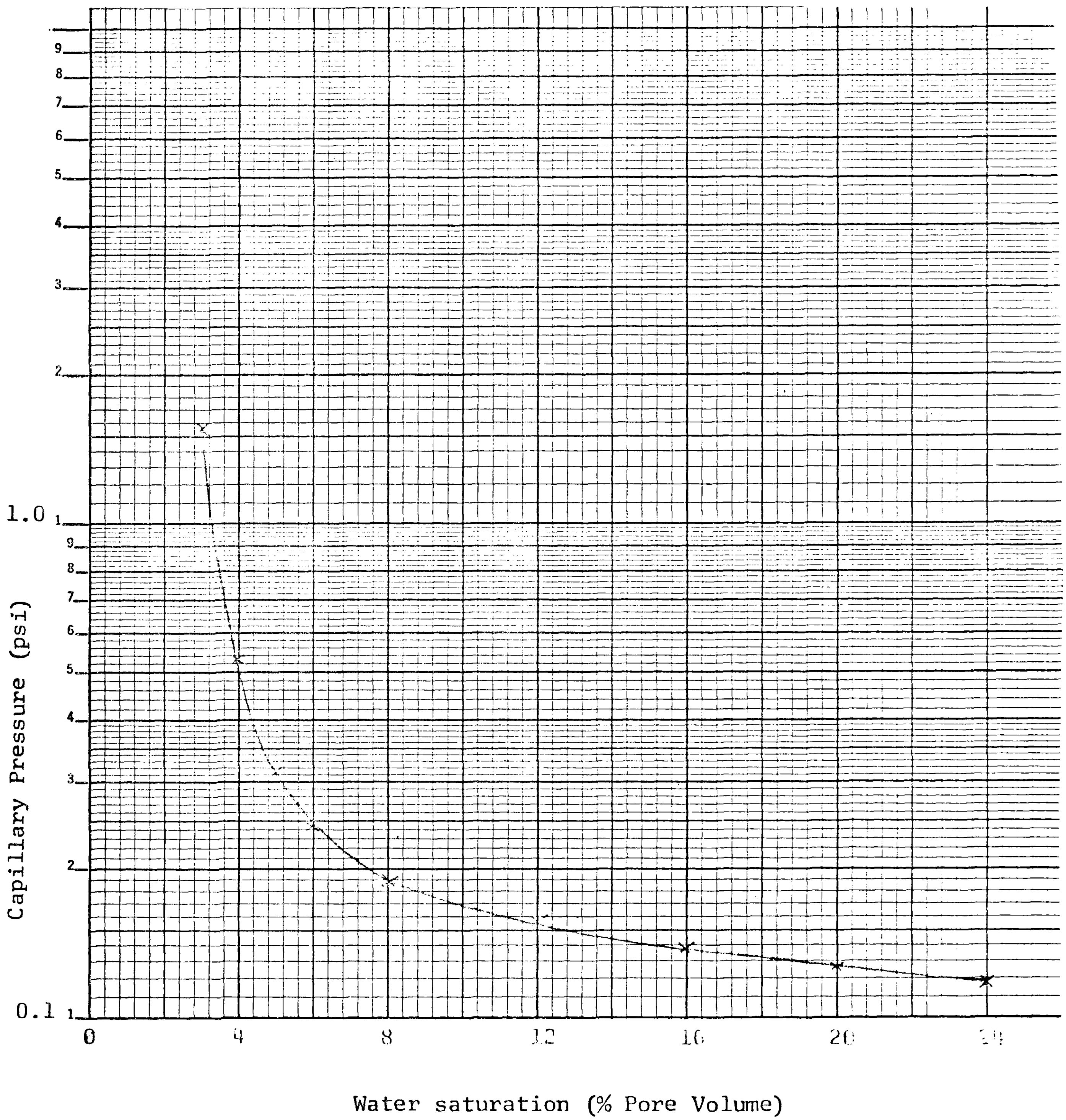
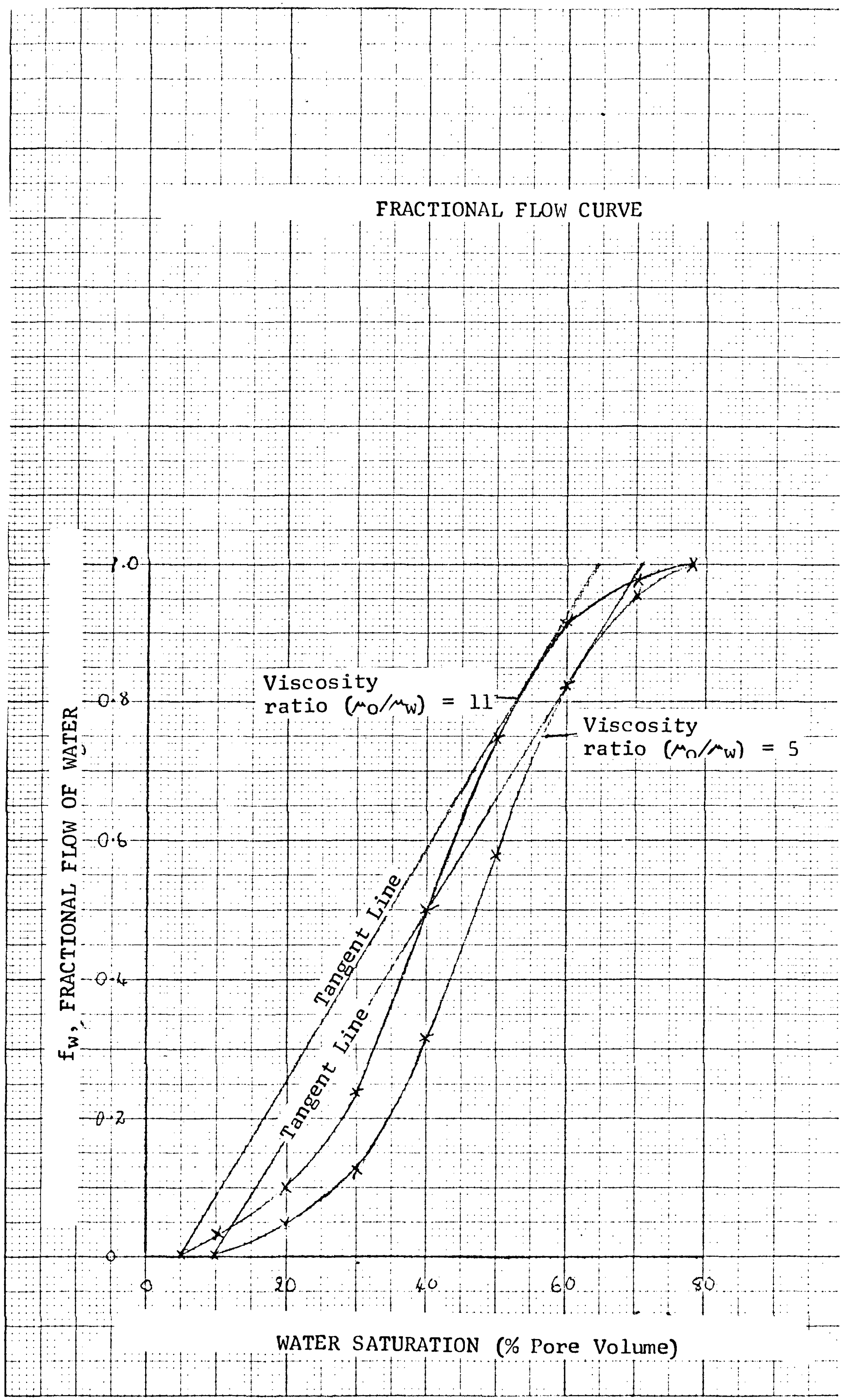


FIGURE 7



FRACTIONAL FLOW CURVE
1.0
0.8
0.6
0.4
0.2
0
0
20
40
60
80
Viscosity ratio (μ_o/μ_w) = 11
Viscosity ratio (μ_o/μ_w) = 5
Tangent Line
Tangent Line
f_w, FRACTIONAL FLOW OF WATER
WATER SATURATION (% Pore Volume)

FIGURE 8

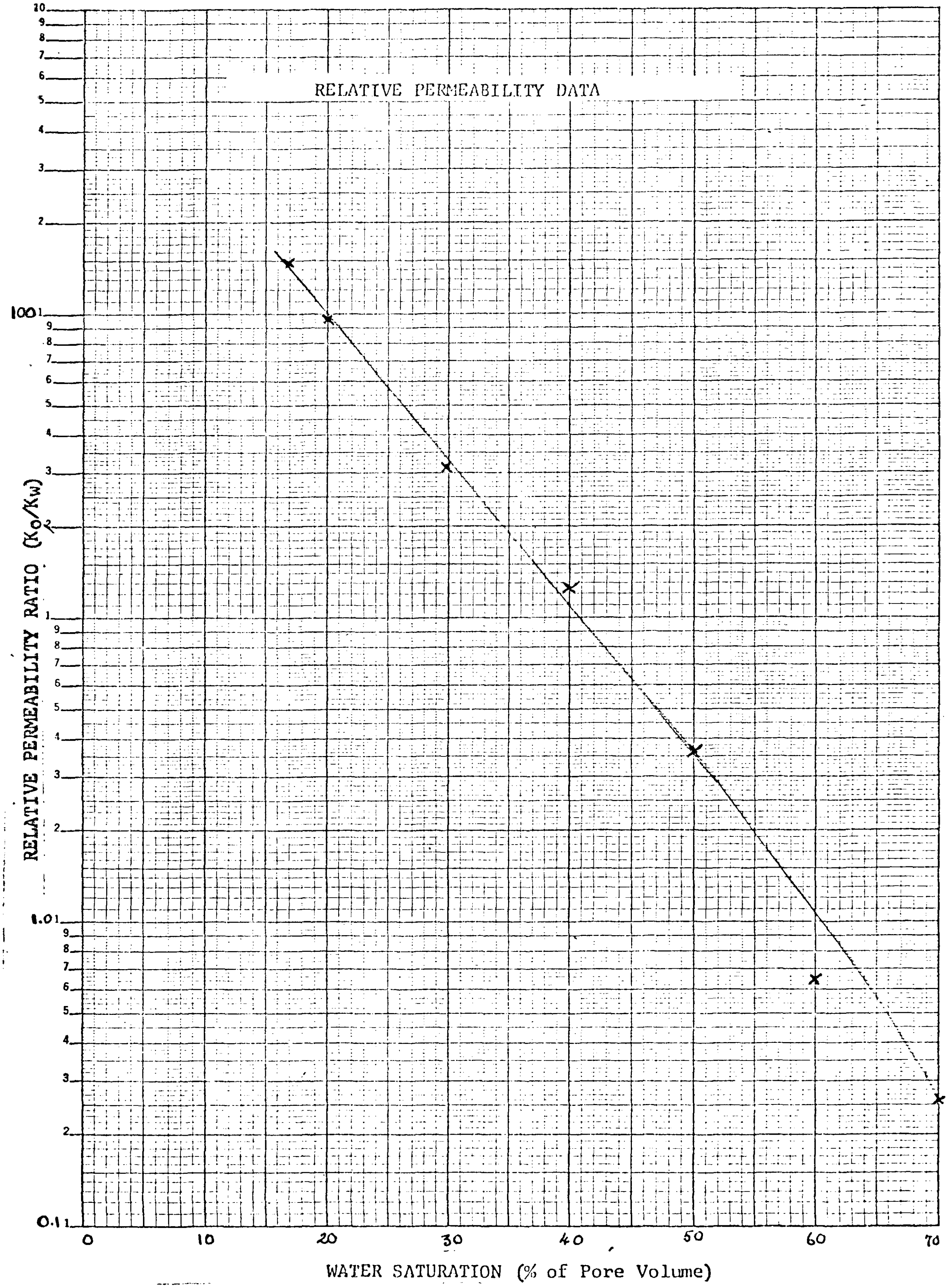

RELATIVE PERMEABILITY DATA
RELATIVE PERMEABILITY RATIO (K_o/K_w)
100
10
1
0.1
0
10
20
30
40
50
60
70
WATER SATURATION (% of Pore Volume)

FIGURE 9

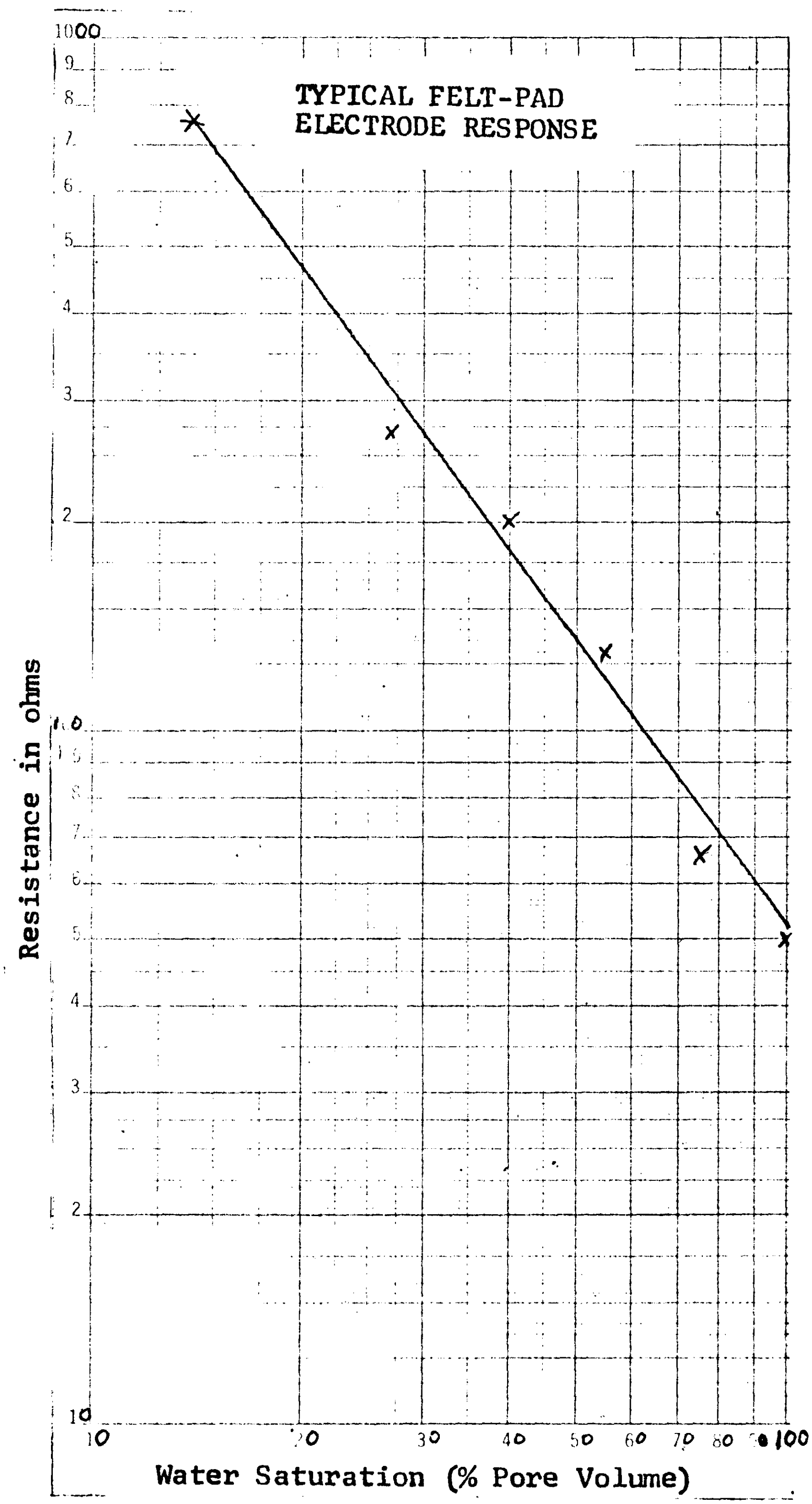

TYPICAL FELT-PAD
ELECTRODE RESPONSE
Resistance in ohms
Water Saturation (% Pore Volume)
1000
100
10
10
20
30
40
50
60
70
80
100

TABLE 2

HORIZONTAL FLOOD 4/5TH APRIL 1973

Date	Time Hrs :Mins	Cumulative Oil Produced % Pore Vol	Cumulative Brine Injected % Pore Vol	Rate cu ft/ day	*Inlet Pressure psi	*Outlet Pressure psi
4/4	14.35	13.8	13.8		Start of Flood	
"	15.49	27.9	28.08	1.97	1.22	1.0
"	16.35	32.2	35.9	1.60	1.32	1.1
"	17.13	34.8	41.5	1.61	-	-
"	18.30	38.1	52.95	1.55	1.22	1.25
"	19.41	40.5	63.5	1.31		
"	20.51	42.3	73.6	1.52		
"	21.30	43.2	79.2	1.45		
5/4	10.30	45.9	131.9	0.69		
"	10.50	47.6	151.6	9.7	1.82	1.62
"	12.20	54.0	236.5	9.6		

(See Figure 10)

Initial oil displaced by brine with box in vertical position = 13.8% PV

(to obtain initial vertical oil/water interface)

Porosity	=	32%
Pore Volume	=	19,865 ccs
Absolute Permeability	=	390 darcys
K_w at So_{irr}	=	170 darcys
K_o at Sw_{irr}	=	300 darcys
Sw_{irr}	=	11%
So_{irr}	=	23%
Viscosity of oil	=	9.7 cp
Viscosity of brine	=	2.0 cp
sp gr of brine	=	1.195 gms/cc
sp gr of oil	=	0.837 gms/cc

* Measured with a pressure gauge, accurate to $\pm$ 0.02 psi

Viscosity measurements accurate to $\pm$ 3%
Specific gravity measurements accurate to $\pm$ 0.002 gms/cc

FIGURE 10

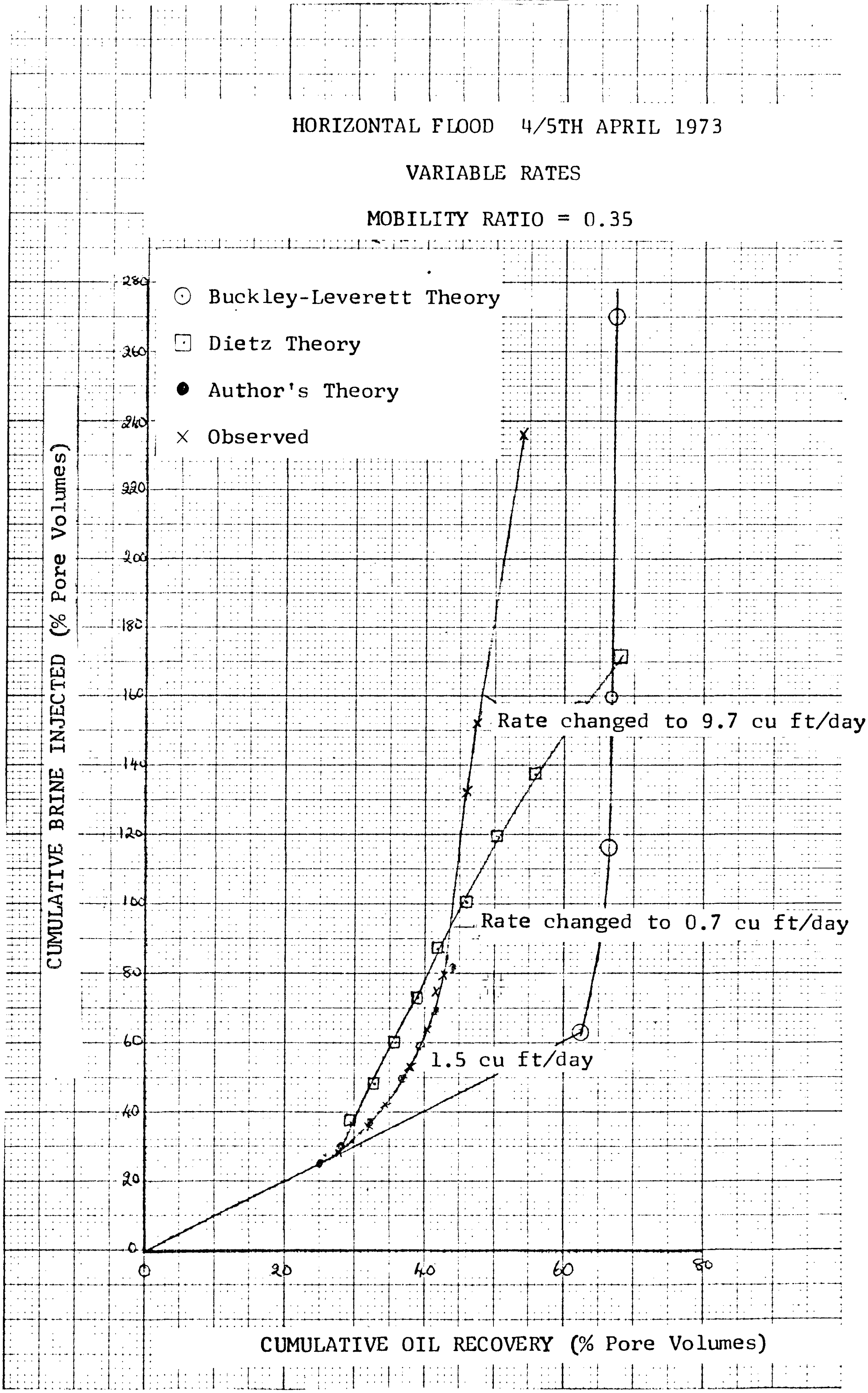

HORIZONTAL FLOOD 4/5TH APRIL 1973
VARIABLE RATES
MOBILITY RATIO = 0.35
Buckley-Leverett Theory
Dietz Theory
Author's Theory
Observed
Rate changed to 9.7 cu ft/day
Rate changed to 0.7 cu ft/day
1.5 cu ft/day
CUMULATIVE BRINE INJECTED (% Pore Volumes)
CUMULATIVE OIL RECOVERY (% Pore Volumes)
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280

TABLE 3

HORIZONTAL FLOOD 26TH APRIL 1973

Date	Time Hrs:Mins	Cumulative Oil Produced % Pore Vol.	Cumulative Brine Injected % Pore Vol.	Rate cu ft/ day	*Inlet Pressure psi	*Outlet Pressure psi
26/4	15.50	-	-	-	-	-
"	16.22	14.0	14.0	4.55	1.35	0.725
"	16.50	19.55	24.05	3.68	1.34	0.93
"	17.50	30.0	50.7	4.5	1.26	0.82
"	18.50	36.6	76.4	4.3	1.175	0.745
"	19.41	40.6	98.6	4.4	1.05	0.84
"	20.49	44.7	127.1	4.26	0.965	0.69
"	21.40	47.2	148.6	4.27		
"	22.55	50.25	180.0	4.25	0.952	0.683
"	23.45	51.8	201.5	4.33		
27/4	00.30	52.9	218.5	3.92		

(See Figure 11)

Average Injection Rate = 4.25 cu ft/day

Porosity = 32%

Pore Volume = 19,865 ccs

Absolute Permeability = 390 darcys

K_w at So_{irr} = 170 darcys

K_o at Sw_{irr} = 340 darcys

Sw_{irr} = 4%

So_{irr} = 23%

Viscosity of oil = 22 cp

Viscosity of brine = 2 cp

sp gr of brine = 1.195 gms/cc

sp gr of oil = 0.857 gms/cc

* Measured with a manometer, accurate to ± 0.005 psi

Viscosity measurements accurate to ± 3%
Specific gravity measurements accurate to ± 0.002 gms/cc

FIGURE 11

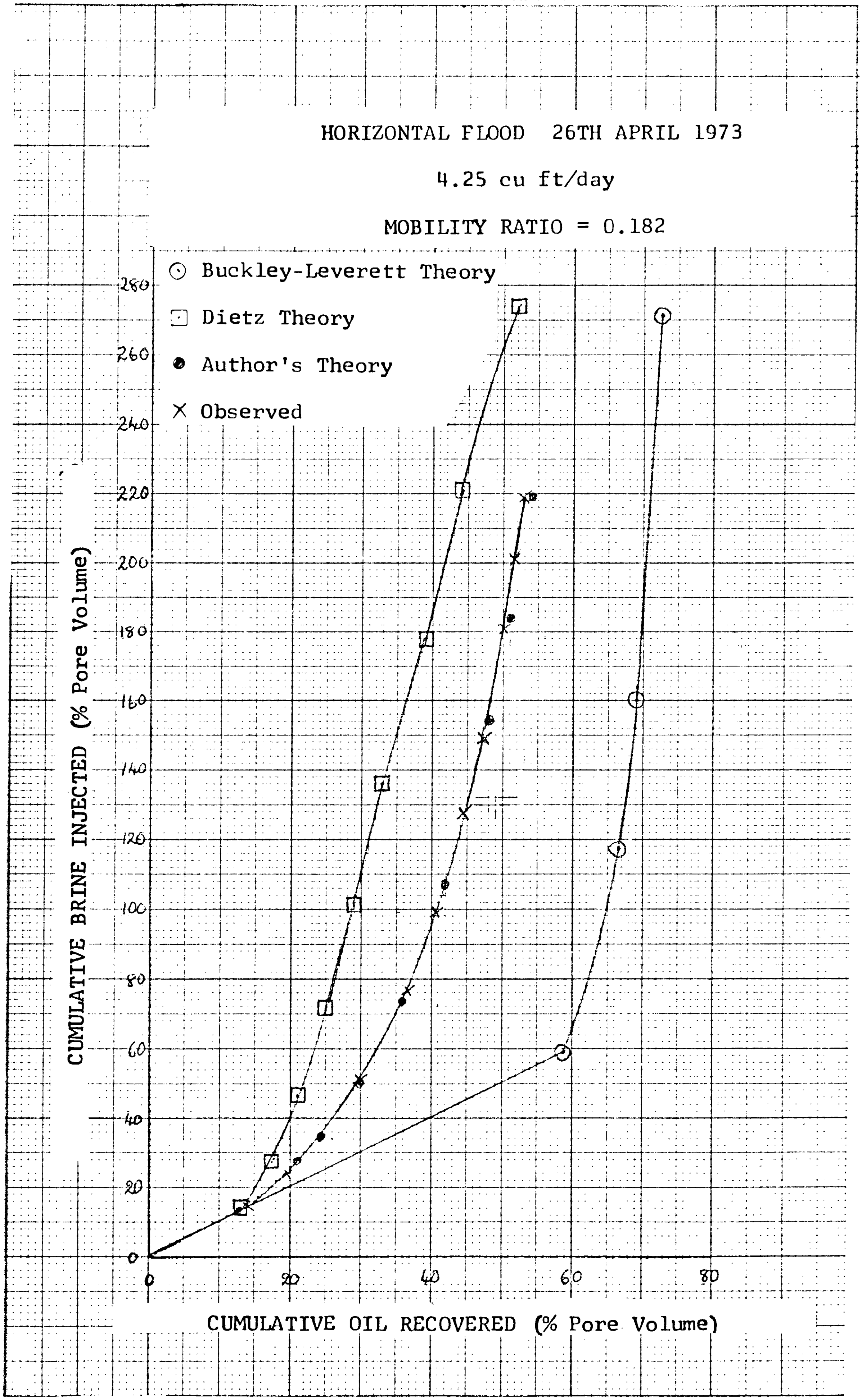

HORIZONTAL FLOOD 26TH APRIL 1973
4.25 cu ft/day
MOBILITY RATIO = 0.182
Buckley-Leverett Theory
Dietz Theory
Author's Theory
Observed
CUMULATIVE BRINE INJECTED (% Pore Volume)
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
20
40
60
80
CUMULATIVE OIL RECOVERED (% Pore Volume)

TABLE 4

HORIZONTAL FLOOD 30TH APRIL 1973

Date	Time Hrs:Mins	Cumulative Oil Produced % Pore Vol.	Cumulative Brine Injected % Pore Vol.	Rate cu ft/ day	*Inlet Pressure psi	*Outlet Pressure psi
30/4	17.30	-	-	-	-	-
"	17.45	13.25	13.25	8.95	2.06	1.21
"	18.30	32.2	58.8	10.2		
"	19.15	-	-	-	1.51	1.1
"	19.30	44.6	117.5	9.9		
"	20.30	51.4	173.0	9.4	1.39	1.02
"	21.30	-	-	-	1.25	0.984
"	22.00	58.4	250.0	8.5		

(See Figure 12)

Average Injection Rate = 9.3 cu ft/day

Porosity	=	32%
Pore Volume	=	19,865 ccs
Absolute Permeability	=	390 darcys
K_w at So_{irr}	=	170 darcys
K_o at Sw_{irr}	=	340 darcys
Sw_{irr}	=	3%
So_{irr}	=	23%
Viscosity of oil	=	22 cp
Viscosity of brine	=	2.0 cp
sp gr of brine	=	1.195 gms/cc
sp gr of oil	=	00.857 gms/cc

* Measured with a manometer, accurate to ± 0.005 psi

Viscosity measurements accurate to ± 3%
Specific gravity measurements accurate to ± 0.002 gms/cc

FIGURE 12

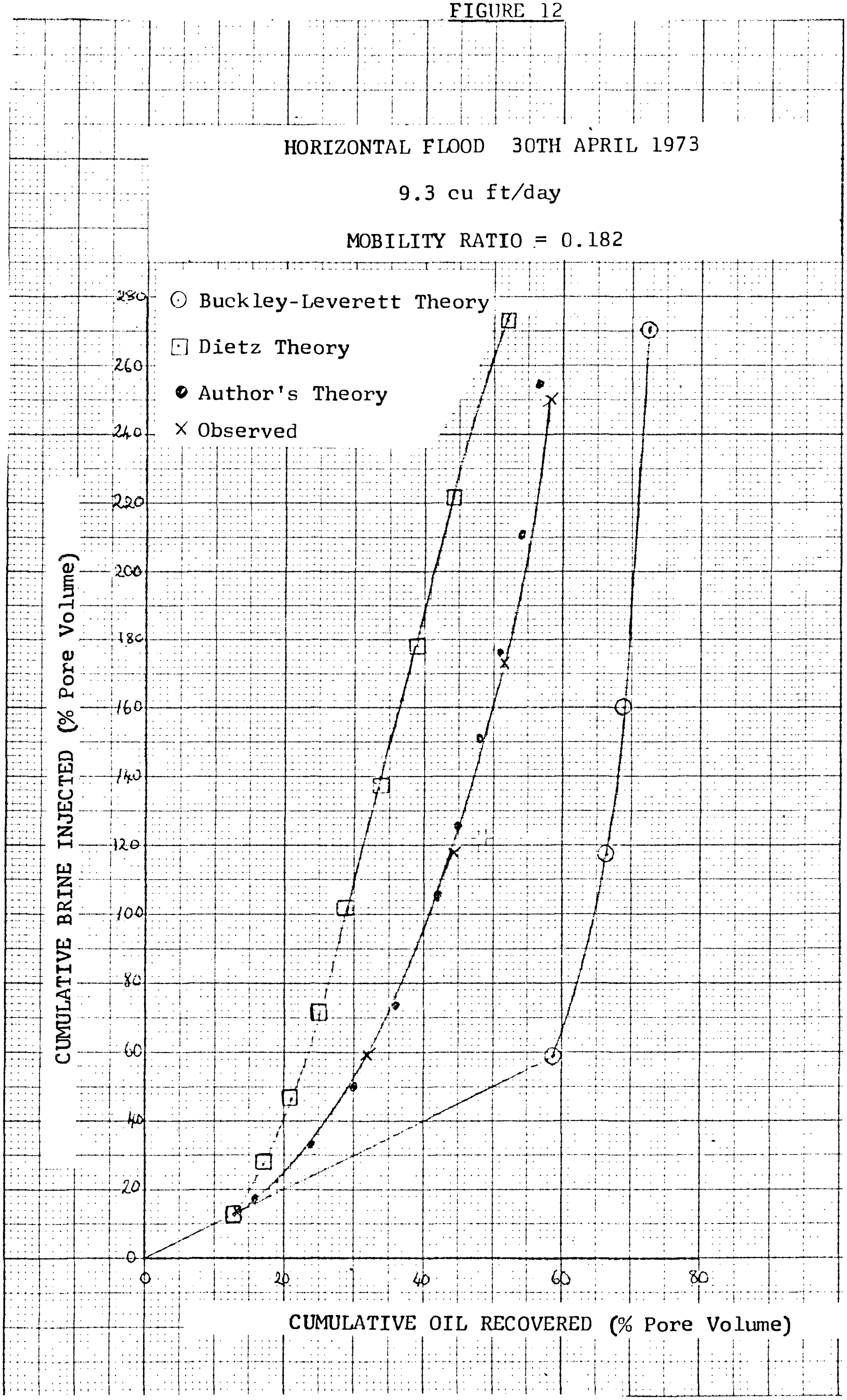

HORIZONTAL FLOOD 30TH APRIL 1973
9.3 cu ft/day
MOBILITY RATIO = 0.182
Buckley-Leverett Theory
Dietz Theory
Author's Theory
Observed
CUMULATIVE BRINE INJECTED (% Pore Volume)
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
0
20
40
60
80
CUMULATIVE OIL RECOVERED (% Pore Volume)

TABLE 5

HORIZONTAL FLOOD 3RD MAY 1973

Date	Time Hrs:Mins	Cumulative Oil Produced % Pore Vol.	Cumulative Brine Injected % Pore Vol.	Rate cu ft/ day	*Inlet Pressure psi	*Outlet Pressure psi
3/5	12.47	-	-	-		
"	12.58	15.0	15.0	13.8		
"	13.18	27.3	42.9	19.3		
"	13.43	40.3	84.0	16.6		
"	14.13	48.5	131.6	16.2		
"	15.00	52.4	207.5	13.4	1.59	0.87
"	15.47	62.4	295.0	16.6		

(See Figure 13)

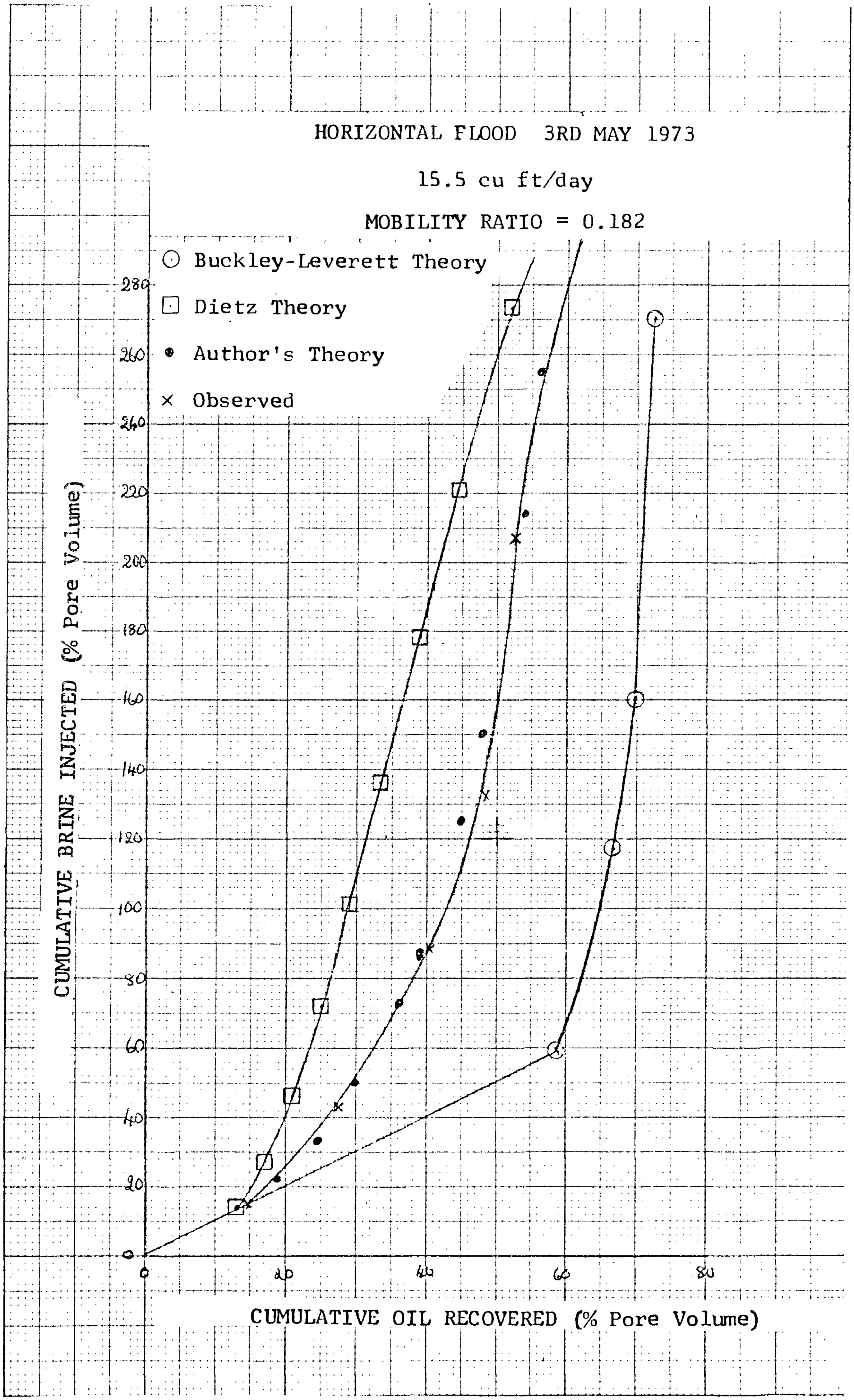
Average Injection Rate = 15.5 cu ft/day

Porosity	=	32%
Pore Volume	=	19,865 ccs
Absolute Permeability	=	390 darcys
K_w at So_{irr}	=	170 darcys
K_o at Sw_{irr}	=	340 darcys
Sw_{irr}	=	3%
So_{irr}	=	21%
Viscosity of oil	=	22 cp
Viscosity of brine	=	2.0 cp
sp gr of brine	=	1.195 gms/cc
sp gr of oil	=	0.857 gms/cc

* Measured with a Manometer, accurate to ± 0.005 psi

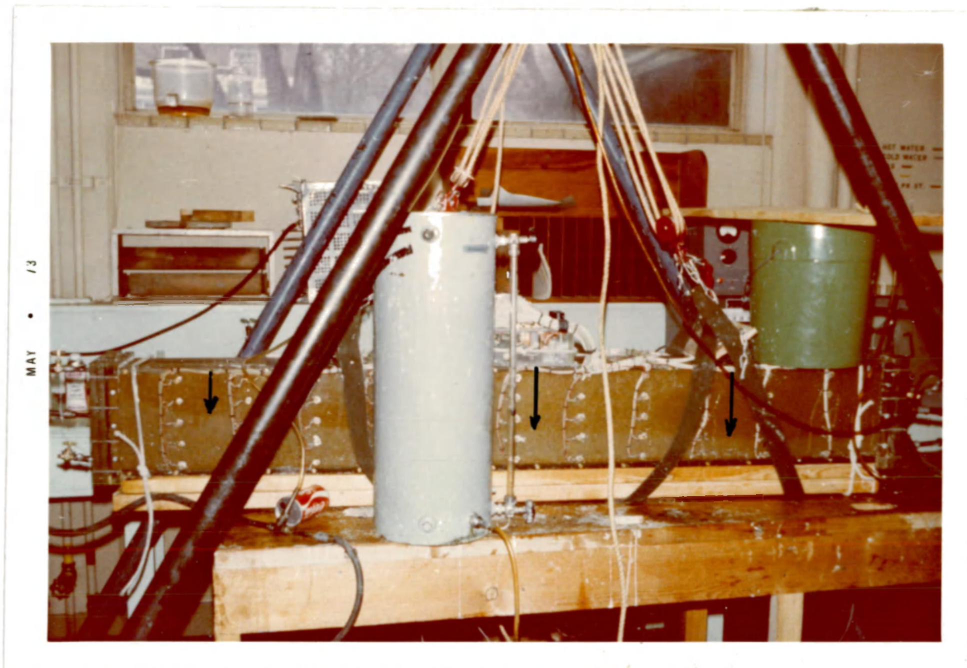
Viscosity measurements accurate to ± 3%
Specific gravity measurements accurate to ± 0.002 gms/cc

FIGURE 13



HORIZONTAL FLOOD 3RD MAY 1973
15.5 cu ft/day
MOBILITY RATIO = 0.182
Buckley-Leverett Theory
Dietz Theory
Author's Theory
Observed
CUMULATIVE BRINE INJECTED (% Pore Volume)
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
0
20
40
60
80
CUMULATIVE OIL RECOVERED (% Pore Volume)

FIGURE 14



PHOTOGRAPH OF GRAVITY TONGUE

DURING WATERFLOOD

26th April, 1973

WATER SATURATION MAPS EVERY 30 MINS,

HORIZONTAL FLOOD 4/5TH APRIL 1973 VARIABLE RATE

Plot 1

0.2 HRS. AFTER START OF FLOOD

15.16	14.66	10.99	10.49	10.55	11.14	10.39	10.65	11.29	10.84	10.90	
55.43	15.38	10.21	11.10	10.33	10.88	10.53	10.63	10.88	11.23	10.56	
74.46	17.03	11.07	10.98	10.83	10.65	10.53	11.26	11.07	10.87	10.77	Outl
86.94	29.25	0.00*	11.01	10.81	10.91	10.41	10.71	11.06	10.94	11.08	
99.14	29.02	17.08	16.03	13.62	12.93	12.57	12.24	11.54	10.63	10.57	

0.5 HRS. AFTER START OF FLOOD

15.22	12.40	10.44	10.96	10.79	10.63	10.51	10.86	11.49	11.34	11.38	
28.76	12.94	10.57	11.84	10.54	10.48	10.26	10.87	11.38	11.56	11.08	
78.43	33.24	11.88	12.46	11.02	10.53	10.37	11.16	11.69	11.01	11.33	Outle
73.49	74.93	0.00*	53.49	18.74	15.59	11.52	11.06	11.10	11.33	11.59	
80.36	75.26	71.63	77.37	53.78	56.99	48.90	46.82	35.03	14.97	11.42	

1.2 HRS. AFTER START OF FLOOD

14.84	12.36	10.42	10.99	10.89	10.95	10.65	10.93	11.82	11.59	11.82	
19.62	13.03	10.74	12.02	10.58	10.61	9.77	10.49	11.67	11.89	11.28	
83.55	46.26	14.07	13.44	11.18	10.22	9.82	10.29	11.86	11.37	11.42	Outle
76.25	73.76	0.00*	79.33	43.27	36.42	23.37	17.55	11.31	11.80	12.02	
82.27	77.85	72.55	79.01	48.86	43.78	39.01	34.67	69.09	65.77	62.23	

---- CALCULATED WATER TONGUE AT BREAKTHROUGH

* Probe disconnected - refers to Plots 1 through 6

WATER SATURATION MAPS EVERY 30 MINS.

HORIZONTAL FLOOD 4/5TH APRIL,1973 VARIABLE RATE

Plot 2

1.5 HRS. AFTER START OF FLOOD

14.56	12.19	10.33	10.63	10.93	11.05	11.04	11.20	11.64	11.53	11.62	
16.98	12.97	10.81	12.01	11.02	10.91	11.24	11.48	11.64	11.89	11.21	
84.77	55.57	24.70	16.51	12.19	11.76	11.48	12.03	11.92	11.10	11.39	Outle
75.67	80.88	0.00*	78.09	77.19	76.19	70.64	68.15	21.86	12.93	16.14	
81.90	78.76	72.22	78.28	86.26	81.98	74.64	71.22	80.41	74.40	72.94	

2.0 HRS. AFTER START OF FLOOD

14.27	12.06	10.23	10.75	10.80	10.87	10.76	10.97	11.45	11.49	11.39	
16.26	12.85	10.80	11.99	10.92	10.89	11.28	11.36	11.77	12.22	11.22	
85.21	57.14	44.22	27.23	13.04	12.13	11.79	12.09	12.08	11.35	11.48	Outle
76.29	80.51	0.00*	76.24	78.23	79.80	72.18	81.91	46.81	37.48	44.38	
81.52	77.69	71.64	76.74	77.21	83.29	78.54	73.39	82.55	74.38	74.43	

2.5 HRS. AFTER START OF FLOOD

13.97	11.98	10.30	10.64	10.73	10.68	10.64	10.81	11.38	11.37	11.24	
16.27	12.73	10.83	11.87	10.93	10.87	11.17	11.25	11.71	11.99	11.18	
84.77	58.44	62.88	55.95	19.58	13.16	12.30	12.21	11.74	11.56	11.57	Outle
80.62	80.62	0.00*	79.10	82.49	85.33	73.35	85.00	65.47	61.82	69.24	
83.56	79.94	72.01	78.55	78.96	85.38	79.58	71.55	81.36	74.03	75.31	

3.0 HRS. AFTER START OF FLOOD

13.79	11.93	10.72	10.68	10.66	10.58	10.56	10.69	11.37	11.28	11.16	
16.23	12.59	10.92	11.68	10.86	10.80	11.22	11.16	11.62	11.88	11.38	
78.43	62.63	73.03	69.07	41.23	22.00	15.34	12.53	11.58	11.67	11.64	Outle
85.35	89.54	0.00*	82.31	81.01	84.73	74.76	83.66	74.49	73.72	74.22	
88.[illegible]	86.83	77.89	83.33	77.47	86.99	83.83	72.[illegible]	79.75	75.31	75.99	

Plot 3

WATER SATURATION MAPS EVERY 30 MINS,

HORIZONTAL FLOOD 4/5TH APRIL,1973 VARIABLE RATE

3.5 HRS. AFTER START OF FLOOD

13.64	11.83	10.62	10.62	10.72	10.60	10.54	10.68	11.35	11.27	11.11	
16.28	12.52	10.84	11.77	10.98	12.87	11.24	11.17	11.63	11.88	11.09	
82.53	56.72	73.16	71.56	57.90	36.14	24.66	13.34	12.22	11.89	11.92	Outlet
79.65	85.32	0.00*	79.68	81.48	82.88	76.60	83.95	77.83	76.71	77.32	
83.28	80.63	74.80	79.65	77.94	87.02	82.06	73.38	78.83	75.58	77.13	

4.0 HRS. AFTER START OF FLOOD

13.51	11.73	10.50	10.62	10.69	10.51	10.51	10.68	11.28	11.16	11.22	
16.23	12.48	10.80	11.80	10.86	10.94	11.19	11.20	11.59	11.84	11.25	
84.84	63.25	76.50	69.67	69.09	53.41	39.44	15.83	12.74	12.10	12.16	Outle
79.22	83.50	0.00*	80.52	82.42	80.63	78.95	87.31	78.21	75.79	75.55	
84.33	81.51	74.72	77.97	80.28	84.07	82.69	72.47	78.76	76.70	76.51	

4.5 HRS. AFTER START OF FLOOD

13.39	11.66	10.57	10.58	10.53	10.22	10.33	10.49	11.23	11.19	10.96	
16.42	12.45	10.76	11.85	10.76	10.84	11.19	11.03	11.58	11.85	11.09	
81.86	65.65	78.58	68.17	75.72	68.43	53.82	22.91	14.97	12.61	13.18	Outle
75.67	80.18	0.00*	80.69	81.93	78.16	81.34	86.22	80.76	75.86	76.20	
81.53	80.93	74.57	77.53	81.30	80.32	82.73	76.19	82.18	76.75	77.86	

5.0 HRS. AFTER START OF FLOOD

13.29	11.79	10.57	10.53	10.40	10.30	10.09	10.34	11.22	11.17	10.92	
17.11	12.47	10.73	11.83	10.65	10.72	11.02	10.97	11.53	11.62	11.24	
79.94	69.21	79.32	69.11	75.90	69.90	62.42	42.94	25.38	13.92	15.49	Outlet
75.86	80.51	0.00*	78.21	80.92	80.33	79.16	85.39	81.66	77.12	77.74	
82.29	81.51	74.69	78.99	78.18	87.48	81.16	75.54	81.98	79.52	79.29	

WATER SATURATION MAPS EVERY 30 MINS,

<u>Plot 4</u>

HORIZONTAL FLOOD 4/5TH APRIL,1973 VARIABLE RATE

5.5 HRS. AFTER START OF FLOOD

13.19	11.59	10.56	10.50	10.39	9.96	10.15	10.34	11.26	11.21	10.94	
17.76	12.53	10.72	11.80	10.69	10.80	11.19	11.03	11.67	11.77	11.10	
81.25	75.93	79.83	71.25	72.78	73.72	66.90	62.32	37.88	18.16	19.97	Outle
80.43	84.61	0.00*	81.11	80.23	82.26	78.08	85.43	81.70	76.82	77.82	
85.28	84.52	76.23	79.77	78.25	89.96	83.48	76.78	83.53	79.65	77.68	

6.0 HRS. AFTER START OF FLOOD

13.26	11.57	10.56	10.46	10.58	10.37	10.31	10.49	11.23	11.17	10.93	
17.77	12.59	10.73	11.72	10.73	10.81	11.01	11.10	11.64	11.91	11.14	
82.51	72.04	70.03	69.79	85.38	80.37	72.15	77.01	52.72	28.29	26.87	Outlet
62.24	78.79	0.00*	80.00	87.06	78.43	83.45	82.16	81.46	71.74	76.68	
80.54	77.60	74.93	79.58	82.04	88.74	82.18	70.68	83.87	79.29	76.32	

6.5 HRS. AFTER START OF FLOOD

13.20	12.42	10.55	10.43	10.54	10.19	10.27	10.42	11.21	11.03	10.87	
18.68	12.77	10.77	11.74	10.78	10.80	11.03	11.09	11.60	11.81	11.04	
81.26	75.65	79.55	71.11	84.30	75.60	73.04	79.75	61.07	42.41	36.26	Outlet
74.89	80.17	0.00*	83.23	84.71	78.58	80.16	80.17	81.80	67.91	76.98	
80.56	80.87	76.35	76.95	80.00	86.51	81.28	69.38	82.59	78.42	79.13	

7.0 HRS. AFTER START OF FLOOD

12.92	11.64	10.49	10.39	10.57	10.15	10.23	13.31	11.26	11.12	10.91	
20.47	13.13	10.72	11.76	10.86	10.84	11.17	11.07	11.68	11.79	11.13	
78.58	74.53	80.99	72.48	80.98	73.74	71.76	76.25	67.28	53.69	47.42	Outle
72.24	79.37	0.00*	79.82	83.81	81.48	77.10	82.24	82.87	79.80	82.16	
80.54	79.52	76.12	78.55	79.47	85.36	81.34	73.98	82.24	69.56	79.91	

INJECTION RATE CHANGED TO 0.69 CU FT/DAY.

WATER SATURATION MAPS EVERY 30 MINS.

HORIZONTAL FLOOD 4/5TH APRIL,1973 VARIABLE RATE

Plot 5

17.5 HRS. AFTER START OF FLOOD

12.22	10.91	10.80	10.25	10.24	9.35	9.73	10.01	10.56	10.48	10.25	
40.66	27.21	14.22	14.16	12.28	11.72	11.85	11.19	11.49	11.88	10.96	
77.55	80.26	80.73	76.22	79.35	83.98	79.49	81.98	80.71	76.74	79.94	Outlet
76.96	80.54	0.00*	72.85	79.22	76.26	84.50	78.31	76.33	78.74	81.35	
81.41	77.62	75.30	80.24	80.47	80.63	72.92	75.54	80.69	84.11	79.28	

18.0 HRS. AFTER START OF FLOOD

12.19	10.69	10.72	10.19	10.29	9.44	9.83	10.15	10.61	10.45	10.25	
41.63	28.73	14.59	14.34	12.29	11.94	11.93	11.28	11.57	11.87	10.89	
77.12	79.32	80.72	77.24	78.94	83.55	80.91	88.56	81.54	69.32	78.67	Outlet
75.70	83.09	0.20*	76.10	81.12	78.54	84.85	82.13	77.16	77.37	78.94	
81.88	75.56	74.20	79.18	82.05	82.95	75.47	76.81	75.49	83.79	77.96	

18.5 HRS. AFTER START OF FLOOD

12.22	10.95	10.51	10.25	10.00	8.90	9.53	9.78	10.58	10.47	10.23	
42.46	30.15	15.38	14.41	13.23	11.79	12.18	11.30	11.61	11.86	10.94	
77.89	82.03	82.14	79.06	75.82	81.19	74.08	79.35	79.68	69.58	79.81	Outlet
76.60	82.37	0.00*	76.05	74.32	72.49	82.37	80.85	79.24	83.61	90.91	
82.27	78.04	74.81	78.22	77.61	84.97	79.20	78.01	75.00	85.72	79.66	

19.0 HRS. AFTER START OF FLOOD

12.24	17.35	10.45	10.11	10.07	9.04	9.51	9.81	10.62	10.50	10.24	
47.26	32.84	17.32	14.37	22.25	18.24	26.78	14.57	12.20	11.81	11.25	
77.36	83.02	79.89	70.63	78.20	79.11	71.07	73.31	79.79	74.18	81.46	Outlet
75.19	81.17	0.00*	79.00	75.39	72.26	81.52	80.24	79.24	86.80	83.74	
80.89	76.73	75.46	75.37	79.41	82.00	78.85	76.75	75.72	87.30	82.11	

INJECTION RATE CHANGED TO 0. [illegible]

WATER SATURATION MAPS EVERY 30 MINS,

HORIZONTAL FLOOD 4/5TH APRIL,1973 VARIABLE RATE

Plot 6

19.5 HRS. AFTER START OF FLOOD

12.74	11.15	11.00	10.33	10.15	9.17	9.61	9.86	10.71	10.52	10.30	
82.30	78.76	47.83	50.72	45.50	36.63	52.25	34.91	19.00	11.81	11.04	
86.80	81.26	82.31	74.19	79.73	82.20	75.30	75.68	82.87	77.01	83.25	Outlet
83.18	86.86	0.00 *	74.64	75.79	74.29	81.47	82.54	78.04	81.41	82.52	
87.67	79.53	77.69	78.15	78.51	82.96	78.16	76.19	75.27	87.21	84.66	

20.0 HRS. AFTER START OF FLOOD

15.51	11.25	10.63	10.43	10.30	9.44	9.80	9.90	10.72	10.60	10.28	
80.23	76.06	51.82	73.62	65.96	64.24	75.31	65.86	39.16	12.21	11.14	
79.29	80.27	80.87	76.49	82.78	83.01	80.46	79.94	84.83	76.79	82.76	Outlet
80.29	77.51	0.00 *	78.40	74.79	77.82	81.22	84.54	80.02	81.32	80.63	
82.67	79.58	76.89	78.15	77.70	84.61	78.70	77.02	78.13	85.51	82.80	

20.5 HRS. AFTER START OF FLOOD

15.92	11.32	10.40	10.47	10.51	9.90	10.11	10.26	10.79	10.71	10.32	
76.62	66.72	49.02	89.03	80.51	95.86	87.83	85.30	71.40	13.14	11.17	
70.69	79.42	78.29	77.55	81.92	77.79	79.88	75.36	88.48	86.12	85.32	Outlet
76.45	65.30	0.00 *	86.56	73.11	82.65	80.52	80.08	83.98	85.26	90.16	
76.35	78.89	74.96	78.29	78.51	82.48	79.03	73.84	79.69	92.74	84.95	

WATER SATURATION MAPS EVERY 30 MINS.

Plot 7

HORIZONTAL FLOOD 26 APRIL 5 CU FT/DAY

2.0 HRS. AFTER START OF FLOOD

4.15	4.17	4.13	4.16	4.17	4.10	4.17	4.13	4.24	4.23	4.21	
4.22	4.19	4.15	4.15	4.17	4.22	4.21	4.16	4.25	4.24	4.26	
4.17	4.16	4.16	4.17	4.19	4.23	4.21	4.18	4.23	4.20	4.18	Outlet
4.19	4.20	4.19	4.19	4.18	4.25	4.22	4.19	4.53	4.31	4.23	
4.29	4.29	4.28	4.23	4.31	4.39	4.09	4.22	4.39	4.28	4.23	

2.5 HRS. AFTER START OF FLOOD

4.22	4.27	3.99	4.02	4.06	4.00	4.27	4.07	4.13	4.16	4.13	
4.16	4.19	4.09	4.12	4.06	4.12	4.18	4.06	4.14	4.18	4.24	
4.21	4.22	4.12	4.13	4.11	4.16	4.17	4.21	4.03	4.15	4.27	Outle
61.64	31.27	5.14	4.48	4.34	4.66	4.39	4.24	4.54	4.24	4.31	
61.20	57.94	51.94	47.27	33.14	36.58	27.65	21.25	13.72	7.22	8.86	

1.0 HRS. AFTER START OF FLOOD

4.22	4.24	4.24	4.11	4.06	4.06	4.05	4.10	4.26	4.23	4.27	
4.13	4.17	4.09	4.14	4.08	4.12	4.14	4.03	4.15	4.18	4.26	
4.51	4.31	4.16	4.18	4.15	4.17	4.18	4.21	4.19	4.32	4.17	Outlet
74.35	59.15	42.39	35.68	19.32	15.32	11.88	5.31	5.71	4.52	4.29	
77.33	79.73	82.47	79.26	65.33	66.71	53.45	63.42	51.21	[illegible]	[illegible]	

CALCULATED WATER TONGUE AT BREAKTHROUGH

Plot 8

WATER SATURATION MAPS EVERY 30 MINS.

HORIZONTAL FLOOD 26 APRIL 5CU. FT/DAY

1.5 HRS. AFTER START OF FLOOD

4.24	4.23	4.09	4.12	4.02	3.97	4.00	4.07	4.34	4.30	4.26	
4.17	4.20	4.09	4.17	4.14	4.04	4.12	4.09	4.17	4.12	4.23	
19.75	5.16	4.26	4.25	4.21	4.19	4.20	4.26	4.10	4.27	4.27	Outlet
73.96	76.37	78.55	73.48	51.32	47.76	46.22	13.59	5.86	4.76	5.03	
76.92	81.88	86.87	82.14	76.71	65.73	66.92	63.21	52.28	42.98	40.73	

2.0 HRS. AFTER START OF FLOOD

4.02	4.02	4.07	4.10	4.02	3.98	4.02	3.99	4.11	4.23	4.22	
4.23	4.25	4.09	4.17	4.65	3.94	4.17	4.11	4.13	4.20	4.22	
44.81	17.11	4.99	4.41	4.26	4.32	4.30	4.28	4.10	4.28	4.27	Outlet
75.51	82.29	78.27	82.98	48.36	64.35	56.92	54.97	42.32	9.58	12.04	
76.34	81.16	82.57	80.80	75.85	75.25	78.40	60.20	65.13	50.98	56.15	

2.5 HRS. AFTER START OF FLOOD

3.99	3.99	3.96	4.08	4.01	3.97	3.98	4.02	4.05	4.22	4.20	
4.27	4.25	4.07	4.14	4.14	4.08	4.16	4.11	4.13	4.19	4.22	
65.39	48.42	9.67	4.97	4.64	4.72	4.52	4.37	4.08	4.25	4.27	Outlet
77.26	82.38	74.27	81.17	70.54	71.54	72.25	71.75	59.59	30.01	37.56	
76.71	74.18	81.37	79.34	77.98	75.34	78.36	64.51	72.76	56.42	62.17	

3.0 HRS. AFTER START OF FLOOD

3.97	4.02	[illegible]	4.38	4.71	4.00	3.99	4.06	3.97	4.23	4.20	
4.34	4.28	4.25	4.16	4.08	4.19	4.15	4.38	4.12	4.29	4.21	
69.47	68.69	44.07	10.39	10.97	7.35	5.76	4.56	4.20	4.26	4.28	Outlet
75.75	79.87	74.22	72.48	74.59	78.71	77.94	72.21	60.04	57.96	71.55	
75.14	[illegible]	[illegible]	76.17	75.27	77.46	77.86	67.47	72.69	67.08	72.97	

WATER SATURATION MAPS EVERY 30 MINS.

Plot 9

HORIZONTAL FLOOD 26 APRIL 5CU. FT/DAY

3.5 HRS. AFTER START OF FLOOD

3.97	3.97	3.97	4.07	4.01	4.01	3.97	4.06	3.86	4.17	4.13	
4.45	4.36	4.05	4.19	4.09	4.20	4.18	4.08	4.12	4.06	4.19	
69.87	72.88	63.49	48.03	34.88	19.65	18.12	5.09	4.37	4.37	4.15	Outlet
78.21	79.25	75.11	76.53	74.28	78.60	75.93	70.80	62.21	71.69	72.01	
76.93	60.61	82.02	77.18	75.50	78.71	78.22	67.71	73.83	77.17	78.46	

4.7 HRS. AFTER START OF FLOOD

3.98	3.96	3.96	4.06	3.99	4.02	3.93	4.04	4.25	4.19	4.12	
4.75	4.39	4.07	4.22	4.09	4.20	4.21	4.13	4.23	4.20	4.21	
67.32	73.66	70.57	67.00	58.18	39.65	40.42	7.92	4.16	4.34	4.52	Outlet
76.52	77.30	76.51	77.30	74.94	75.32	72.69	73.73	62.83	71.92	71.79	
74.76	76.75	79.43	77.46	77.53	77.75	77.65	58.76	72.80	75.95	77.55	

4.5 HRS. AFTER START OF FLOOD

4.21	3.95	3.94	4.25	3.96	4.24	3.94	4.22	4.21	4.08	4.13	
6.39	4.59	4.11	4.24	4.11	4.24	4.22	4.06	4.12	4.14	4.17	
70.82	77.24	74.86	71.32	68.92	58.29	58.80	18.37	5.72	4.30	4.72	Outlet
76.93	77.10	77.16	76.27	75.46	77.22	72.40	71.71	76.05	72.32	74.52	
75.34	78.51	79.55	78.16	77.75	76.59	76.41	4.81	77.24	77.37	78.72	

5.2 HRS. AFTER START OF FLOOD

4.02	3.96	3.94	4.23	3.95	4.31	3.92	4.00	3.92	4.14	4.13	
13.79	5.27	4.19	4.29	4.14	4.18	4.25	4.15	4.25	4.17	4.13	
75.40	79.16	77.15	76.29	74.90	66.33	66.64	42.01	8.98	5.23	6.67	Outle
79.20	77.71	76.94	77.92	72.66	78.33	76.46	74.77	75.08	72.50	74.55	
78.22	[illegible]	83.81	81.18	75.85	77.59	77.66	62.67	77.3[illegible]	76.83	73.16	

WATER SATURATION MAPS EVERY 30 MINS.

Plot 10

HORIZONTAL FLOOD 26 APRIL 5CU. FT/DAY

5.5 HRS. AFTER START OF FLOOD

3.97	3.97	4.21	4.25	3.95	3.91	3.91	4.22	3.85	4.11	4.11	
27.97	8.15	4.32	4.40	4.17	4.32	4.29	4.16	4.23	4.16	4.18	
75.72	76.55	75.27	74.04	77.81	74.87	72.83	57.29	35.32	11.86	11.78	Outlet
80.22	77.51	76.72	76.27	74.29	81.23	82.04	77.68	72.87	75.37	74.83	
73.82	75.61	81.31	77.22	73.69	80.20	77.92	74.82	79.66	74.68	77.71	

6.0 HRS. AFTER START OF FLOOD

3.95	3.97	4.09	4.08	3.96	3.91	3.88	4.22	4.13	4.17	4.16	
44.74	17.24	4.63	4.51	4.22	4.38	4.32	4.19	4.29	4.17	4.19	
71.40	75.27	78.77	75.43	73.61	77.29	72.81	65.22	55.14	17.98	21.11	Outlet
76.80	78.25	77.94	79.55	75.37	76.98	81.11	77.46	70.18	76.20	75.84	
74.97	74.22	83.20	77.91	77.09	78.38	78.27	74.76	79.97	74.91	78.29	

6.5 HRS. AFTER START OF FLOOD

3.92	3.97	4.21	4.26	3.97	3.93	3.88	3.97	4.27	4.25	4.26	
56.25	36.51	5.76	4.83	4.34	4.49	4.37	4.22	4.34	4.25	4.25	
73.27	80.19	74.76	75.11	78.93	78.84	78.12	70.61	64.14	28.38	32.93	Outlet
79.54	77.72	76.63	82.27	77.10	79.38	78.69	78.77	69.14	72.98	75.42	
75.82	74.39	78.39	79.66	78.19	78.87	77.33	74.14	78.11	78.02	78.25	

7.0 HRS. AFTER START OF FLOOD

3.95	3.96	3.99	4.06	3.99	3.94	3.89	3.96	3.85	4.02	4.09	
65.55	48.28	10.21	5.63	4.64	4.63	4.45	4.26	4.17	4.29	4.16	
77.26	81.26	76.77	76.24	81.95	78.95	81.92	75.40	79.94	65.90	38.15	Outle.
78.47	78.12	77.74	79.05	75.67	80.01	79.11	81.27	73.29	74.67	74.11	
78.24	76.99	78.06	77.92	77.65	78.47	75.96	74.67	79.10	75.40	78.04	

CALCULATED WATER TONGUE AT CUMULATIVE INJECTION OF 180% PV

WATER SATURATION MAPS EVERY 30 MINS. **Plot 11**

HORIZONTAL FLOOD 26 APRIL 5CU. FT/DAY

7.5 HRS. AFTER START OF FLOOD

3.97	3.98	3.97	4.06	3.99	3.92	3.87	4.00	4.02	4.11	4.27
77.11	56.57	16.22	8.50	5.76	4.95	4.55	4.33	4.36	4.15	4.18
78.15	82.52	79.81	76.61	80.38	77.79	75.76	77.73	80.05	72.22	44.24
80.56	79.53	78.42	79.01	72.87	74.98	81.75	78.06	74.33	77.16	75.46
79.99	77.34	79.36	76.90	76.07	76.01	76.17	75.71	74.24	77.36	78.97

Outlet

8.0 HRS. AFTER START OF FLOOD

3.92	3.96	3.92	4.03	3.96	3.65	3.82	3.87	3.90	4.04	4.02
79.02	62.47	22.73	14.83	8.70	5.68	4.70	4.32	4.32	4.30	4.17
78.34	81.15	80.46	76.42	79.06	78.02	74.44	78.00	78.20	74.74	51.32
80.03	77.45	77.95	79.79	73.30	76.73	78.85	78.02	77.29	76.94	76.21
79.47	75.10	79.40	77.90	74.03	79.31	77.04	76.33	78.64	78.04	78.73

Outle

8.5 HRS. AFTER START OF FLOOD

3.52	3.91	3.80	4.01	3.95	3.88	3.87	3.92	3.98	4.04	4.04
77.32	67.05	30.15	24.48	14.25	6.96	5.02	4.49	4.44	4.22	4.17
78.76	76.97	78.28	77.95	75.23	78.03	73.44	78.03	77.96	76.32	64.43
76.64	71.24	77.07	78.28	74.09	74.73	78.30	76.44	78.87	76.43	77.31
77.23	71.60	77.61	77.41	77.04	78.13	80.46	78.79	78.74	78.34	78.25

Outl

WATER SATURATION MAPS EVERY 30 MINS.

Plot 12

HORIZONTAL FLOOD 30 APRIL 10 CU FT/DAY

0.0 HRS. AFTER START OF FLOOD

3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	
3.20	3.00	3.01	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	
3.20	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	Outle
3.19	3.93	3.00	3.00	3.00	3.02	3.00	3.02	2.98	3.00	3.00	
27.36	5.91	3.96	3.00	3.00	3.02	3.00	3.00	3.00	3.00	3.02	

2.5 HRS. AFTER START OF FLOOD

2.84	2.80	2.49	2.94	2.82	2.69	2.83	2.84	2.85	2.59	2.97	
2.93	2.85	2.83	2.72	2.86	2.84	2.92	2.82	3.22	2.97	2.99	
28.34	3.23	2.86	3.01	3.03	3.02	2.96	2.86	3.05	3.02	3.24	Outle
61.53	59.27	46.39	44.73	31.14	29.67	22.85	8.83	5.22	3.18	3.26	
66.42	56.21	53.89	50.34	62.32	63.84	52.96	61.61	43.72	63.48	66.11	

1.2 HRS. AFTER START OF FLOOD

2.52	2.76	2.32	2.66	2.74	2.53	2.72	2.63	2.42	2.38	3.22	
3.42	2.88	2.75	2.59	2.78	2.69	2.82	2.64	2.93	2.72	2.82	
55.52	41.58	33.16	5.00	3.28	3.26	3.11	2.91	3.03	2.91	2.92	Outle
73.64	76.74	69.33	72.75	67.53	71.57	65.65	33.78	22.32	3.47	7.27	
75.75	73.03	76.78	76.79	73.82	70.57	63.46	65.35	54.41	73.09	71.97	

CALCULATED WATER TONGUE AT CUMULATIVE INJECTION OF 59% PV

WATER SATURATION MAPS EVERY 30 MINS.

HORIZONTAL FLOOD 30 APRIL 10 CU.FT/DAY

Plot 13

1.5 HRS. AFTER START OF FLOOD

2.83	2.77	2.37	2.77	2.71	2.43	2.60	2.55	2.49	2.42	2.85	
23.97	3.53	2.83	2.62	2.76	2.66	2.80	2.60	2.94	2.72	2.60	
68.28	69.24	63.75	25.75	16.20	10.85	5.98	2.98	3.03	2.95	2.92	Outlet
72.87	76.93	69.58	73.31	76.22	72.64	72.60	73.32	55.55	54.86	54.54	
75.20	72.33	74.54	76.05	76.13	75.51	64.87	74.25	62.20	76.65	72.51	

2.0 HRS. AFTER START OF FLOOD

2.87	2.79	2.41	2.78	2.69	2.39	2.60	2.52	2.44	2.39	2.79	
52.77	15.34	2.99	2.73	2.73	2.71	2.80	2.58	2.93	2.71	2.80	
69.76	72.01	71.66	71.47	77.19	66.86	57.37	4.21	3.29	3.03	3.47	Outlet
73.72	78.72	71.12	76.64	80.99	79.25	73.81	77.00	67.76	74.08	76.95	
77.44	73.41	72.32	74.93	77.16	79.44	78.63	74.07	70.76	77.56	72.15	

2.5 HRS. AFTER START OF FLOOD

2.99	2.83	2.33	2.77	2.73	2.54	2.64	2.58	2.41	2.37	2.76	
66.61	44.89	3.98	2.92	2.89	2.86	2.89	2.67	2.91	2.71	2.78	
72.52	74.87	81.34	73.98	77.74	78.25	77.16	42.11	31.03	5.61	22.81	Outlet
74.58	75.52	80.29	76.58	79.23	80.73	79.28	77.94	72.00	75.43	77.57	
76.93	73.48	74.57	76.26	80.02	80.38	78.98	76.66	74.65	77.22	73.25	

3.0 HRS. AFTER START OF FLOOD

3.15	2.87	2.37	2.79	2.74	2.54	2.63	2.59	2.37	2.35	2.75	
71.67	65.39	11.84	4.92	3.41	3.20	2.98	2.74	2.88	2.72	2.79	
74.15	77.85	81.56	76.34	78.73	81.29	82.91	70.04	63.66	25.79	56.38	Outlet
77.87	77.57	78.51	77.44	80.34	82.12	82.59	80.61	74.87	75.99	78.03	
78.95	76.02	72.96	76.59	81.54	81.55	81.33	78.68	75.66	76.69	75.52	

CALCULATED WATER TONGUE AT CUMULATIVE INJECTION OF 17.7% PV

WATER SATURATION MAPS EVERY 30 MINS.

HORIZONTAL FLOOD 30 APRIL 10 CU.FT/DAY

Plot 14

3.5 HRS. AFTER START OF FLOOD

3.42	2.95	2.55	2.80	2.74	2.51	2.61	2.57	2.35	2.31	2.73	
74.24	70.32	47.50	25.81	12.29	5.21	3.28	2.81	2.83	2.78	2.88	
74.22	77.99	79.85	75.30	80.63	81.78	83.76	79.37	78.49	68.36	63.06	Outlet
77.91	75.11	73.32	75.68	80.97	82.82	79.78	82.28	76.29	76.92	78.23	
79.21	74.74	74.88	75.41	80.51	80.78	82.09	78.44	77.14	76.75	75.11	

4.2 HRS. AFTER START OF FLOOD

4.11	3.04	2.46	2.83	2.72	2.47	2.59	2.55	2.48	2.38	2.77	
76.63	78.20	70.22	46.82	35.14	14.31	5.89	3.06	2.93	2.87	3.25	
76.97	79.81	78.67	75.34	79.67	80.33	81.24	82.23	84.42	70.62	66.31	Outlet
77.68	76.39	78.16	76.10	79.57	79.95	79.14	79.53	75.51	77.53	82.24	
76.54	76.33	75.65	75.02	77.54	79.42	83.22	77.21	76.63	76.93	80.46	

WATER SATURATION MAPS EVERY 30 MINS, **Plot 15**

HORIZONTAL FLOOD 3 MAY 15 CU FT/DAY

0.0 HRS. AFTER START OF FLOOD

2.99	2.96	3.34	3.02	3.26	4.01	3.86	3.37	3.00	2.98	3.00
3.32	2.97	2.97	3.02	2.99	3.85	3.42	3.68	3.02	3.20	3.20
2.96	3.04	3.00	2.95	3.20	3.20	3.00	3.30	2.96	2.96	2.97
2.98	3.02	2.99	2.97	3.20	3.02	3.00	3.00	3.02	3.02	3.02
3.14	3.02	3.11	3.16	3.55	3.57	3.42	3.51	2.94	3.36	3.35

Outle

0.5 HRS. AFTER START OF FLOOD

2.77	2.76	2.71	2.81	2.96	2.92	2.96	2.97	3.13	2.67	2.72
46.55	2.73	2.95	2.93	2.46	2.97	3.01	2.99	2.55	2.72	2.73
51.65	52.99	31.59	2.65	2.80	2.87	2.83	2.89	2.70	2.43	2.46
54.20	52.93	45.87	40.15	32.62	5.63	28.49	21.90	11.77	7.23	2.78
57.90	51.77	55.28	37.97	56.21	56.91	43.16	63.90	34.85	62.77	64.54

Outle

CALCULATED WATER TONGUE AT CUMULATIVE INJECTION OF 43% PV

1.0 HRS. AFTER START OF FLOOD

12.80	2.74	2.74	2.80	2.69	2.65	2.87	2.91	2.15	2.60	2.66
69.38	41.09	2.99	2.95	2.50	2.96	3.06	2.96	2.55	2.70	2.67
75.26	76.42	65.12	34.24	27.20	19.21	17.77	3.27	2.77	2.48	2.44
78.44	74.88	73.13	67.34	71.93	14.69	81.16	76.80	74.55	73.12	65.92
79.21	71.29	81.63	63.79	72.82	71.01	62.51	75.24	46.47	77.23	76.48

Outlet

WATER SATURATION MAPS EVERY 30 MINS.

Plot 16

HORIZONTAL FLOOD 3 MAY 15 CU.FT/DAY

1.5 HRS. AFTER START OF FLOOD

47.62	2.73	2.79	2.85	2.85	2.96	2.79	2.90	2.24	2.68	2.72	
77.64	65.51	26.32	4.29	2.66	3.04	3.15	3.04	2.63	2.83	2.75	
78.22	79.29	82.67	75.13	75.65	67.02	81.99	6.01	5.13	2.77	2.69	Outlet
82.38	78.17	78.75	71.39	81.38	35.81	80.20	79.50	78.53	73.52	81.49	
79.98	73.13	81.29	74.24	76.31	81.03	75.43	82.85	56.10	83.62	82.18	

2.0 HRS. AFTER START OF FLOOD

74.76	4.27	2.67	2.83	2.87	2.97	2.95	2.95	2.33	2.76	2.75	
78.27	68.17	67.68	18.27	4.66	3.35	3.29	3.11	2.76	2.92	2.82	
74.54	77.00	82.18	79.24	71.25	78.71	83.08	65.19	60.44	49.39	37.58	Outlet
76.27	78.22	79.15	75.21	78.24	83.88	85.11	76.25	82.86	78.75	82.23	
77.21	75.74	79.48	76.71	82.57	75.20	77.17	80.39	68.68	84.14	78.69	

2.5 HRS. AFTER START OF FLOOD

75.16	18.71	2.74	2.82	2.87	3.02	3.01	3.21	2.31	2.76	2.77	
76.74	76.62	77.94	78.17	66.33	10.91	4.36	3.35	2.85	2.95	2.87	
73.81	76.89	78.22	77.49	74.98	78.36	79.06	78.61	81.30	79.12	72.13	Outlet
76.79	76.54	78.03	76.48	78.10	80.17	80.38	78.00	81.45	81.35	80.28	
76.72	76.64	77.92	76.96	76.81	77.92	77.68	78.88	76.22	81.22	77.04	

3.0 HRS. AFTER START OF FLOOD

75.16	77.34	2.74	2.82	2.87	3.26	3.10	3.07	2.26	2.72	2.74	
76.74	76.62	77.94	78.17	77.37	16.29	17.42	4.07	2.90	2.96	2.93	
76.66	76.89	78.22	75.60	77.28	78.03	77.41	77.97	77.12	78.31	78.34	Outlet
76.79	76.54	78.03	76.52	78.26	78.02	77.98	78.09	78.26	78.14	78.21	
76.72	76.64	77.92	76.97	77.24	77.92	77.77	78.10	77.78	77.92	77.93	