INPUT - OUTPUT MARKOV ANALYSIS OF GOVERNMENT STIMULUS

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Abstract

The purpose of this thesis is to formulate a method to analyze the effects and rate of return of governmental stimulus in an economy. The method is based on Input-Output analysis and Markov theory. Input-Output analysis projects changes in the level of economic activity and Markov analysis projects the associated change in the relative number of transactions in the economy. Combined, the two projections allow expected tax revenues and a subsequent rates-of-return to be projected.

Introduction

There are few things more dear to a citizen than how his government spends the tax revenues generated from his income. Karl Marx stated:

> The English Established Church...will more readily pardon an attack on 38 of its 39 articles than 1/39 of its income [8].

The philosophy of Marx's statement is one of the underlying philosophies of this thesis.

The purpose of the thesis is to formulate a method to analyze the effects and rate of return of governmental stimulus in an economy. The method is devised for a Keynesian environment where the key to achievement is to maintain a consistantly high level of aggregate demand. In a Keynesian economy, if high levels of aggregate demand cannot be maintained by the private sector of the economy, the government must assume the responsibility to stimulate demand. Most citizens have an inborn distrust of government when it attempts to alter the economy. Adam Smith's views on the relationship between government and the economy were that politicians' goals on regulating the economy were to "deceive and oppress" and that "monopoly is a great enemy to good management" [18]. Since government is a monopoly composed of politicians, it is no wonder there is a natural distrust of government meddling with the economy.

The method formulated in the thesis allows the overall effects of changes in the economy to be investigated. In this manner, the effects of a governmental proposal to stimulate or direct the economy may be predicted and compared with alternate proposals. The key part of the method is its ability to determine a rate of return for the proposals. The rate of return would be derived from the tax revenues attributed to the stimulus.

With recent developments in the Government's policies to combat the energy crisis, this method of analysis takes on additional importance. Governmental proposals aimed at changing the established consumption patterns of the American public, and the overall effects of imposing stiff taxes to deter the purchase of energy inefficient products can be more thoroughly evaluated.

Development of the analysis method is based on Input-Output (I-O) analysis and Markov theory. Traditional I-O analysis is used to predict the change in the level of economic activity (GNP) resulting from changes in aggregate demand. The results of the I-O analysis are then used to determine the relative change in the number of transactions in the economy. Knowing the change in the level of economic activity and the change in the expected volume of transactions provides valid predictions of expected tax revenues. The relationship between cost of stimulus and expected tax revenues associated with the stimulus establish the rate of return.

Two major assumptions are made in the thesis. First, it is assumed tax revenues are directly proportional to the number of transactions (taxable situations) occurring in the economy. Second, it is assumed that the probable dollar-flow from sector to sector in the economy is proportional to the monetary volume of inter-sector transactions. No attempt is made in the thesis to validate or discredit these assumptions.

The thesis is presented in three major sections. Chapter I deals with a review of the formulation and validity of I-O analysis. Chapter II develops Markov theory and its application to determining the relative number of transactions in an economy. The third chapter combines I-O analysis and Markov theory to establish the method to analyze the effects and rate of return of governmental stimulus. Throughout the thesis a 6-sector model of the U.S. economy will be used to clarify and demonstrate the methods discussed. Following the third chapter and conclusion is an appendix that will further investigate the analytic relationship between stimulus and transactions.

Input-Output Analysis

Classical economic doctrine prescribes that no one should tamper with the economy. Adam Smith concurred with this philosophy in his belief that an economy should be based on individualism and not a paternal government [19]. Input-Output (I-O) analysis is a key tool for those who oppose the classical doctrine.

I-O analysis provides the modern economist with a reliable and systematic method of predicting the effects of tampering with an economy. The primary objective of I-O is to relate the interdependencies of the various sectors of an economy. By relating these interdependencies, I-O analysis offers an acceptable means to forecast economic activity, identify possible economic bottlenecks, and compare different economies.

Modification of I-O analysis increases its versatility. With slight modifications, I-O has been used for analyzing the concept of optimal appropriating of economic rent in the field of international economics [20]. More extensive modifications and the use of linear programming make it possible for I-O analysis to be applied to dynamic applications. For further information on these time-phased dynamic applications consult Gass [4].

One of the advantages of I-O analysis is that it adequately deals with the multiplier effect that is present in a multi-sector economy. Since the multiplier effect is a summable converging infinite series, I-O analysis can determine the outcome of the infinitely many of adjustments an economy goes through when there is a change in consumption or production.

The importance of I-O to this thesis is that it will be used in conjunction with Markov theory to establish the rate-of-return and transaction concepts. Fundamental to the establishment of these concepts is to demonstrate the validity of forecasts based on I-O analysis.

Most of the underlying theory and development of I-O analysis is attributed to Wassily W. Leontief of Harvard. In the 1930's, Leontief formulated a 45-sector I-O model of the 1919-1920 U.S. economy. Leontief published this work in 1941, and since that time, the field of I-O analysis has been growing steadily. Today, with the aid of computers, there are more than 50 national I-O models [18].

I-O analysis is a very powerful tool, but it also has its drawbacks. One of the more prohibitive drawbacks is the fact that I-O analysis is costly and sophisticated. The volumes of data that must be obtained for a multiple-sector model are expensive and very time consuming to accumulate. In many cases a lapse of 2 to 5 years is required to accumulate and organize the data.

Assuming data is available for an I-O analysis, there are two simplifying assumptions that must be made. The first of these assumptions is that of proportionality within the economy. Proportionality requires that a 10% increase in production in the agriculture sector demands 10% more input from the other sectors that contribute to agricultural production. Secondly, I-O analysis assumes that the ratios of sector interaction will not change during the period of analysis. According to [11] and [4], this static condition is not extremely critical to the accuracy of short term I-O analysis. This point will be discussed in greater detail later.

The remainder of this chapter will deal with two major areas. The first area will be the formulation of an I-O model. The second area will be concerned with establishing the validity of I-O analysis. I-O analysis will be compared to alternative forms of analysis based primarily on a study by the Bureau of Economic Analysis [1].

Formulation of I-O Model

Provided all required data are available, the primary step in formulating an I-O model is to organize an I-O table or transaction table. The table partitions the economy into sectors, and the entries in the table represent the monetary volume of sales between the different sectors. In addition, a column is incorporated into the table to denote the dollar-consumption of each sector's products. The next step in formulating an I-O model is to develop the technical coefficients and corresponding technical coefficient matrix. Technical coefficients define the proportional interdependencies between sectors. Following this step, the Leontief matrix is calculated, and the model is ready for its role as an aid to economic analysis. Each of the steps will be clarified and discussed in greater detail later.

Matrix notation makes the derivation and formulation of I-O analysis very straightforward. However, a step-by-step example will further clarify the mathematics of I-O analysis. A 6-sector economic model will serve as the example. This 6-sector model will be referred to throughout the thesis. The data for the model is extracted from Summary of Input-Output Tables of the U.S. Economy: 1968, 1969, 1970 [23].

This summary contains the data for an 85-sector I-0 model. The 85-sector model was condensed to the 6-sector example. By condensing the larger model, the relative proportions of the six aggregate sectors may have some relation to the actual U.S. economy. The six sectors and their abbreviations are:

Agr. = Agriculture

Min. = Mining

Cnst. = Construction

Man. = Manufacturing

Fin. = Finance

Serv. = Services

The standard form of the transaction table is illustrated in Table I. For example, an entry (x_{ij}) in row "i" and column "j" denotes the amount of sales from sector "i" to sector "j" during the period of interest. The consumption or final demand column represents those elements in the economy which consume the various commodities but do not contribute or feed back to the economy a product of their own. Examples of these elements include foreign trade, government operations, and household consumption. (In the next chapter, elements that consume, but do not produce, will be represented by absorbing states of a Markov chain). Finally, the total output column displays the entire production for each of the sectors. The total output of sector "j", X_j, is the sum of elements x_{ij} corresponding to sector "j" ($X_j = \sum_j x_{ij}$) and the consumption for sector "j" is denoted by Y_j.

TABLE I

Transaction Table

From Sector \ To Sector	1	2	3	. . .	n	Consumption	Total Output
1	x_{11}	x_{12}	x_{13}	. . .	x_{1n}	Y_1	X_1
2	x_{21}	x_{22}	x_{23}		x_{2n}	Y_2	X_2
3	x_{31}	x_{32}	x_{33}		x_{3n}	Y_3	X_3
.	.				.	.	.
.	.				.	.	.
.	.				.	.	.
n	x_{n1}	x_{n2}	x_{n3}		x_{nn}	Y_n	X_n

where

$x_{ij} \geq 0$: total sales (in dollars) of sector i to sector j.

$Y_i \geq 0$: final demand or consumption (in dollars) of products produced by sector i.

$X_i > 0$: total output (in dollars) of industry i.

Table II shows the transaction table for the 6-sector economic model. Recall the entries in the table were extracted from actual U.S. economic data and their values have relative importance. When $x_{ij} = 0$ there may still be interactions between these sectors, but not a significant amount as determined by the decimal place accuracy of the table.

TABLE II

Transaction Table for 6-Sector Economy

(Billions of Dollars)

	Agr.	Min.	Cnst.	Man.	Fin.	Ser.	Consum.	Tot.Out.
Agr.	22.3	0.0	.3	26.6	2.9	.4	11.0	63.5
Min.	.1	1.7	10.3	15.1	.4	0.0	3.0	30.6
Cnst.	.6	.7	.1	10.7	10.2	1.9	103.4	127.6
Man.	8.8	2.5	38.4	247.5	6.0	27.0	318.4	648.6
Fin.	3.9	4.1	1.8	31.3	25.9	12.0	126.9	205.9
Ser.	2.0	.7	6.2	40.4	10.5	13.3	113.6	186.7

The next step in the I-O process is to develop the technical coefficients and the technical coefficient matrix. It is through the technical coefficients that the assumption of proportionality is invoked. Technical coefficients, a_{ij}, state the amount of input from sector i that is necessary to produce one unit of output from sector j.(see definition, below).

The assumption that the ratio of sector interactions is independent of time is implied by the use of the technical coefficients. The set of technical coefficients is produced from the transaction table, and it is assumed these coefficients are applicable through the entire forecasting horizon. Technical coefficients shall be denoted by "a_{ij}'s" and defined as follows:

$$a_{ij} = \frac{\hat{x}_{ij}}{\hat{X}_j} \geq 0$$

is the dollar amount of input from sector i necessary to produce one unit of output from sector j.

A "tilde" over an element will be used to signify the known data values for x_{ij}, X_j, and Y_j for the given base period.

Given data (such as Table II) for a base period, the n-sector transaction table can be rewritten in terms of the technical coefficients.

$$
\begin{array}{l}
a_{11}\hat{X}_1 + a_{12}\hat{X}_2 + \cdots + a_{1n}\hat{X}_n + \hat{Y}_1 = \hat{X}_1 \\
a_{21}\hat{X}_1 + a_{22}\hat{X}_2 + \cdots + a_{2n}\hat{X}_n + \hat{Y}_2 = \hat{X}_2 \\
\vdots \\
a_{n1}\hat{X}_1 + a_{n2}\hat{X}_2 + \cdots + a_{nn}\hat{X}_n + \hat{Y}_n = \hat{X}_n
\end{array} \qquad (1)
$$

This system of equations (1) can be rewritten.

$$
\begin{array}{l}
\hat{X}_1 - a_{11}\hat{X}_1 - a_{12}\hat{X}_2 - \cdots - a_{1n}\hat{X}_n = \hat{Y}_1 \\
\hat{X}_2 - a_{21}\hat{X}_1 - a_{22}\hat{X}_2 - \cdots - a_{2n}\hat{X}_n = \hat{Y}_2 \\
\vdots \\
\hat{X}_n - a_{n1}\hat{X}_1 - a_{n2}\hat{X}_2 - \cdots - a_{nn}\hat{X}_n = \hat{Y}_n
\end{array} \qquad (2)
$$

Collecting like terms, equations (2) can be conveniently written in matrix notation.

$$(I - A)\hat{X} = \hat{Y} \tag{3}$$

where

$$I = \begin{bmatrix} 1 & 0 & . & . & . & 0 \\ 0 & 1 & & & & 0 \\ . & & & & & . \\ . & & & & & . \\ . & & & & & . \\ 0 & 0 & . & . & . & 1 \end{bmatrix} \qquad A = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & & & & a_{2n} \\ . & & & & & . \\ . & & & & & . \\ . & & & & & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{bmatrix}$$

$$\hat{X} = \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ . \\ . \\ . \\ \hat{X}_n \end{bmatrix} \qquad \hat{Y} = \begin{bmatrix} \hat{Y}_1 \\ \hat{Y}_2 \\ . \\ . \\ . \\ \hat{Y}_n \end{bmatrix}$$

The (I-A) matrix is appropriately termed the Leontief matrix. By accepting the assumption that the economy is "linear" (proportionality assumption) and technical coefficients are static, the Leontief matrix can be used to describe the activity of the economy during current and future periods.

Premultiplying both sides of equation (3) by the inverse of the Leontief matrix yields:

$$(I - A)^{-1}(I - A)\hat{X} = (I - A)^{-1}\hat{Y}$$

or

$$\hat{X} = (I - A)^{-1}\hat{Y} \tag{4}$$

Thus, by multiplying the inverse of the Leontief matrix by a desired consumption matrix (Y) the required total output matrix (X) can be estimated.

To summarize, the crux of I-O analysis is to determine the total output vector (X) which satisfies for a given Y,

$$(I - A) \quad X = Y \qquad (5)$$

$$X \geq 0.$$

The solution of equation (5) is based on the existence of the inverse of the Leontief matrix. Morgenstern [12] has shown that the inverse does exist in all cases of non-zero consumption. His proof that the Leontief matrix is nonsingular and thus has an inverse is based on

$a_{ij} \geq 0$ and

$\sum_i a_{ij} < 1$ for all $j = 1,2,\ldots,n.$

Computing the Leontief matrix for the 6-sector model will help clarify the I-O procedure. Based on the transaction table, Table II, the technical coefficient matrix for the I-O model is

A =

	Agr.	Min.	Cnst.	Man.	Fin.	Ser.
Agr.	.351	0.0	.002	.041	.014	.002
Min.	.002	.056	.081	.023	.002	0.0
Cnst.	.009	.023	.001	.016	.050	.010
Man.	.139	.082	.301	.382	.029	.145
Fin.	.061	.134	.014	.048	.126	.064
Ser.	.031	.023	.048	.062	.051	.071

The corresponding Leontief matrix, (I-A), is

(I-A) =

	Agr.	Min.	Cnst.	Man.	Fin.	Ser.
Agr.	.649	0.0	-.002	-.041	-.014	.002
Min.	-.002	.944	-.081	-.023	-.002	0.0
Cnst.	-.009	-.023	.999	-.016	-.050	-.010
Man.	-.139	-.082	-.301	.618	-.029	-.145
Fin.	-.061	-.134	-.014	-.048	.874	-.064
Ser.	-.031	-.023	-.049	-.062	-.051	.929

Determination of the inverse of the Leontief matrix will make it possible to predict the effect on the economy associated with altering the consumption vector (Y). Inverting the Leontief matrix yields

$(I-A)^{-1}$ =

1.569	.016	.039	.111	.032	.023
.016	1.067	.101	.045	.010	.009
.029	.037	1.017	.036	.061	.021
.396	.184	.544	1.702	.111	.280
.141	.179	.071	.118	1.161	.099
.088	.051	.098	.127	.076	1.103

To see the consequences of altering the consumption vector, consider a Governmental program that will increase the total demand (consumption) in the construction sector. Table III shows the changes in the total output (X) required by 5% increments in demand in the construction sector. For example, the first column of Table III displays the total output required when construction demand is $103.4 billion (this column corresponds to the total output column of Table II). Increasing the demand for construction by 5% to $108.6 billion, requires the output shown in the second column of Table III. Output for a 10%

increase is shown in the third column, and so on.

TABLE III

Effects on Total Output due to Increased Demand for Construction

	Level of Construction Demand ($ billions)				
	base	base+5%	base+10%	base+15%	base+20%
	103.4	108.6	113.7	118.9	124.0
Agr.	63.5	63.6	63.8	64.0	64.2
Min.	30.6	31.0	31.5	32.0	32.6
Cnst.	127.6	132.6	137.8	143.1	148.3
Man.	648.6	651.7	654.5	657.3	660.1
Fin.	205.9	206.0	206.4	206.7	207.1
Ser.	186.7	187.0	187.5	188.0	188.5
Total or GNP	1262.9	1271.9	1281.5	1291.1	1300.8

It is apparent from Table III that a change in one sector of the economy will initiate the multiplier effect and have repercussions throughout the entire economy. Referring to the technical coefficient matrix, A, it can be seen that each dollar increase in demand for construction fosters a $.002 increase in agricultural production, an $.081 increase in mining production, and so on. In turn, each of these increases created additional increases, and the cycle continues until the increases become negligible. When the demand of several sectors is altered, the total effect is the sum of the individual effects.

Validity of I-O Analysis

Since much of this thesis is based on I-O analysis, it is worthwhile to investigate the accuracy and validity of I-O forecasts. Many facets of economic analysis have been aided by the everincreasing interest in I-O analysis. I-O tables provide a means to organize and classify data. Also, I-O has emphasized the importance of interindustry analysis and has caused an upgrading in the quantity and quality of interindustry data.

The main controversy confronting I-O analysis is whether or not it is a valid and efficient economic forecasting device. It is hard to judge the "efficiency" of I-O analysis, but the efficiency has improved since Leontief's first attempts. Leontief's first I-O table was published in 1936 and was composed of data relating to the 1919-1921 U.S. economy. Today, that 15-year lapse in collecting data has been reduced to about 3 years [23] by the Department of Commerce.

Most arguments against the validity of I-O analysis arise from the restrictive assumptions placed on the technical coefficients. I-O analysis approximates the complex production functions that govern interindustry transactions with a single constant coefficient. Approximating a complex production function by one coefficient may be the only way to keep large I-O tables manageable and workable. However, the requirement that the technical coefficients remain "acceptably" static for a reasonable period of time can be examined by searching historical data for comparison.

Direct tests consisting of observations of technical coefficients and interindustry relationships at different times can be used to see if the coefficients vary with time. The general consensus of these tests seems to agree that technical coefficients are not totally static, but the degree of time dependence is not critical for short term forecasts.

Leontief performed one of the first studies on the variance of technical coefficients [9]. His study was based on data from 1919, 1929, and 1939. The conclusion of the study disagrees with many more recent studies. Leontief found considerable change in the values of technical coefficients over this 20-year time period.

Burgess Cameron conducted a test based on a series of data from Australian manufacturing industries [2]. In his 1952 study he found the technical coefficients to be fairly constant over a short period of time. A short period of time was defined to be about 3 years.

In a more recent study [1], B. N. Vaccara examined changes that occurred in the technical coefficients of the U.S. economy between 1947 and 1958. The findings of the study were that the coefficients changed, but the changes were unpatterned.

Separate studies conducted by A. Ghosh [5] and W. Miernyk [11] found similar results. Both studies concluded the static requirement placed on technical coefficients is not critical to the accuracy of short term analysis. Here again, short term is in the neighborhood of 3 years.

Concluding from these studies that the static condition imposed on the technical coefficients is not critical, it is now appropriate to investigate the accuracy of an I-O forecast. Two methods are available for testing I-O forecasts. One method is backcasting, and the other is merely comparing previous forecasts with actual occurrences.

The Bureau of Economic Analysis (BEA) issued a report [1] in 1975 that thoroughly analyzes the validity of I-O forecasts. Included in the report are comparisons between the forecasting results of 16 I-O models and other techniques of forecasting. The 16 I-O models are both foreign and domestic, and have been formulated in the past quarter century. The other forecasting methods used in the BEA report include variation of GNP blowup, final demand blowup, and regression analysis. GNP blowup predicts an industry's output by assuming its output will remain the same proportion of GNP as was displayed in a base year . Final demand blowup is similar to GNP blowup except an industry's output is assumed to remain proportional to the final demand of a base year. Regression analysis is a data fitting procedure that is usually limited to a few variables that are believed to have a strong influence on the variable to be predicted.

Table IV and Table V list the results of the BEA study. In Table IV the direct results of the comparisons between the different techniques are presented. Table V, compiled by A. K. Shapiro [1], shows how the accuracy of the techniques vary with respect to the length of the time period involved in the forecast.

The general conclusion of the BEA study is that there are no simple alternative forecasting methods which are consistently as good as I-O. The adjective "simple" may be slightly ambiguous, but A. Ghosh's Experiments with Input-Output Analysis agrees with the conclusion of acceptable accuracy.

TABLE IV

Major Empirical Tests of Input-Output Forecasts

Researcher	Year[a]	Country	Procedure	Results
Leontief	1951	U.S.	Used 13-by-13 1939 matrix to estimate 1919 and 1929 outputs and then compared these against final demand blowup and GNP blowup estimates.	Input-output estimates were superior to both alternatives.
Arrow	1951	U.S.	Used 38-by-38 1939 matrix to estimate outputs for 22 industries for odd years 1929-1939 and then compared these against estimates derived from final demand blowup, GNP blowup, and regression equations.	Regression model performed best, input-output and final demand blowups estimates were of about equal accuracy, GNP blowup estimates were the poorest.
Barnett	1951	U.S.	Used revised 38-by-38 1939 matrix to estimate 1950 outputs for 28 industries and then compared these to final demand blowup, GNP blowup and regression estimates.	Input-output and regression estimates were of about equal accuracy and were superior to final demand and GNP blowup estimates.
Hoffenberg-BLS	1954	U.S.	Used 38-by-38 1939 matrix to estimate outputs of 25 industries for odd years 1929-1937 and compared these to final demand blowup and GNP blowup estimates.	Input-output and final demand blowup estimates were of similar accuracy and were superior to the GNP blowup estimates.
Vaccara	1953	U.S.	Reanalyzed Arrow, Barnett and Leontief findings using different error measures and production indices.	Results of all three studies were sensitive to the error measures and production indices utilized.
Clark	1953	Italy	Used 180-by-180 1950 matrix to estimate 1951 outputs and compared these to GNP blowup estimates.	Input-output estimates were superior.

TABLE IV(Continued)

Researcher	Year[a]	Country	Procedure	Results
Hoffenberg-BLS	1955	U.S.	Used 190-by-190 1947 matrix to estimate 1951 outputs of 163 industries and compared these to final demand and GNP estimates.	Input-output estimates were the most accurate; 1947 table was statistically superior to 1939 table.
Adams and Stewart	1956	Britain	Used 36-by-36 1935 matrix to estimate outputs for 1924, 1930, 1933 and 1934 and compared these to final demand blowup and GNP blowup estimates.	Input-output model superior to both alternatives.
Sevaldson	1956	Norway	Used 30-by-30 1948 matrix to estimate 1947 outputs of industries and compared these to two types of final demand blowup estimates.	Input-output forecasts were superior to both alternatives.
Hatanaka	1960	U.S.	Used 30-by-70 version of 1947 matrix to estimate outputs of industries for 1950, 1949, 1940, 1939 and 1937 and compared these with final demand blowup, intermediate demand blowup and regression estimates.	Input-output forecasts were superior to final demand blowup and regression estimates but not to intermediate demand blowup estimates.
Ghosh	1964	Britain	Used 50-by-50 1948 matrix to estimate outputs for industries for years 1950-55 and compared these to final demand blowup, GNP blowup and three kinds of regression estimates.	Input-output forecasts superior to all alternatives.

TABLE IV(Continued)

Researcher	Year[a]	Country	Procedure	Results
Chakravarti	1965	India	Used 36-by-36 1954 matrix to estimate 1956 value added for industries and compared these with modified final demand and GNP blowup estimates.	Input-output value added estimates were superior to both alternatives.
Tilanus	1966	Netherlands	Used 35-by-35 matrices from years 1948-1961 to make forecasts and compared these with final demand blowup estimates.	Input-output forecasts were superior as long as inter-industry matrix was not more than four years out of date.
Theil	1966	Netherlands	Used 35-by-35 matrices from years 1948-1958 to estimate value added and compared these to final demand blowup estimates.	Input-output value added estimates were superior.
Vaccara	1971	U.S.	Used 80-by-80 1958 matrix intact and modified to estimate 1968 outputs of 51 industries and compared these to GNP blowup estimates.	Input-output estimates were superior; adjusting inter-industry coefficients for certain changes increased margin of superiority.
Shapiro	1972	U.S.	Used modified 1958 80-by-80 matrix to estimate 1964-1968 outputs of 65 industries and compared these to estimates derived from final demand blowup and two regression models.	Input-output estimates were as good as and usually better than any alternative estimates.

[a] Refers to year in which the research was published.

TABLE V

Summary of the Results of Shapiro's Tests

Projection Period	Forecast Method: Input-Output	GNP Blowup	2 Variable Regression	5 Variable Regression	Final Demand Blowup	Combined Blowup
One-Year Ahead						
Weighted Percent Error	5.3	6.1	8.6	6.6	17.7	5.5
Unweighted Percent Error	7.0	8.3	9.7	9.4	17.7	7.2
Two-Years Ahead						
Weighted Percent Error	6.5	7.4	11.0	7.3	14.2	6.6
Unweighted Percent Error	8.4	9.8	12.1	10.9	18.0	8.5
Three-Years Ahead						
Weighted Percent Error	7.2	8.3	14.0	8.5	15.5	7.5
Unweighted Percent Error	9.7	11.3	14.9	12.7	20.2	10.0
Four-Years Ahead						
Weighted Percent Error	8.1	8.8	16.0	9.0	16.7	8.0
Unweighted Percent Error	11.2	12.2	17.3	13.8	22.9	10.9
Five-Years Ahead						
Weighted Percent Error	8.7	10.3	18.2	8.9	18.2	9.1
Unweighted Percent Error	12.2	14.1	20.2	14.0	27.5	12.6

Transactions and Markov Analysis

As seen in the previous section, Input-Output analysis provides an efficient means of analyzing the effects of Governmental policy changes on the economy. However, the primary concern of I-O analysis is to determine the new sector total outputs created by specified changes in consumption. When changes in consumption are derived from a Governmental package intended to alter aggregate demand, it is extremely important to know what repercussions the package will have on the Governments financial picture. For example, if the government is faced with choosing between two different economic stimulus programs that seem to have comparable effects on the economy (I-O analysis), how should the decision be made as to which program to support.

The decision of which programs to support should, at least in part, be based on general business practices. Few successful business organizations will fund a proposal without a detailed forecast of the consequences of the proposal on the financial standing of the business. A primary measure used by private enterprise to evaluate a proposed venture is rate of return analysis. In the same manner, a government should predict an expected rate of return in order to properly value the impact of a program, or expected rates of return to properly select the better program from many alternatives.

What is needed is a systematic method to determine expected rates of return (ROR) for governmental programs that are aimed at improving the economy by stimulating or creating artificial demand. Since taxes are the Government's source of income, a method to project a rate of return must be concerned with the level of tax revenues.

Input-Output analysis does not directly address the problem of rate of return on economic intervention. One possible way in which I-O analysis could be utilized to give insight to ROR questions would be to assume the overall change in total output will be taxed at a given percent. In this manner a type of ROR projection could be established. However, it seems more appropriate and more accurate that the ROR should consider the flow of money and number of taxable situations occurring in the economy. Economic stimulation would most likely alter money flow as well as total production.

Markov Chains

The theory of Markov chains originated with A. A. Markov in 1907. Since its introduction, the theory has been widely developed. Today many sciences, such as sociology and biology, recognize Markov chain theory as an extremely useful aid to analyze theoretical and observed happenings. There is no reason why the theory cannot be utilized in the field of economics.

Incorporating Markov chain theory into the answering of the ROR question provides three basic advantages. The first advantage is that Markov chain theory is very systematic. Results from the I-O analysis are inputs for the Markov analysis. Each set of data is treated and analyzed in an impartial and objective manner.

The second advantage to the Markov analysis is that it produces quantified results. This quantification helps to remove the problem of ambiguous conclusions that are likely to be misinterpreted. Quantification also provides a straightforward means of comparing the effects of differing economic strategies.

The third advantage of the Markov analysis is that it is developed in matrix form similar to that used in I-O analysis. Matrix notation allows masses of data to be presented or displayed in a way that is easily manipulated, understood, and digested.

At this time, it is beneficial to state precisely what the Markov analysis is expected to accomplish. Through Markov chain theory, a matrix will be derived that forecasts the relative number of transactions that will occur in the economy during a given time period. The expected number of transactions in an economy corresponds to the number of opportunities a government has to recoup its investment (stimulation program) through taxation.

The matrix containing the relative number of transactions to be expected is called the fundamental matrix or N-matrix. This N-matrix is derived from a transition matrix whose elements are the probabilities that a dollar circulating in the economy and presently in sector A will be involved in a transaction with sector B. Determination of the transaction matrix probabilities are based on the results of the I-O analysis.

Thus, a proposed economic program will first undergo an I-O analysis to determine the overall effects to the economy. Using the results from the I-O analysis, the Markov analysis will project the expected number of taxing opportunities generated by the economic program. Knowing the relative number of transactions (taxing opportunities), the corresponding magnitude of these transactions, and an applicable figure to represent the tax rate, enables a ROR to be calculated for a proposed economic package.

One point should be clarified. The N-matrix is not intended to be interpreted as a representation of the exact number of transactions to be expected in the economy. Proper interpretation of the N-matrix is the relative number of transactions in relation to different economic policies. For example, one proposed economic program may increase the expected number of transactions in the economy from 12.0 to 13.2. The importance of this increase is not the 1.2 increase in transactions. The importance of the increase is the fact that the proposed program will increase the taxable opportunities by 10%.

The remaining portion of this chapter will deal more explicitly with the development of Markov chain theory. First the basic definitions, notation, and construction of the transition matrix will be covered. Following the basics will be the development of the N-matrix, and an introduction to other useful matrices, derived from Markov theory.

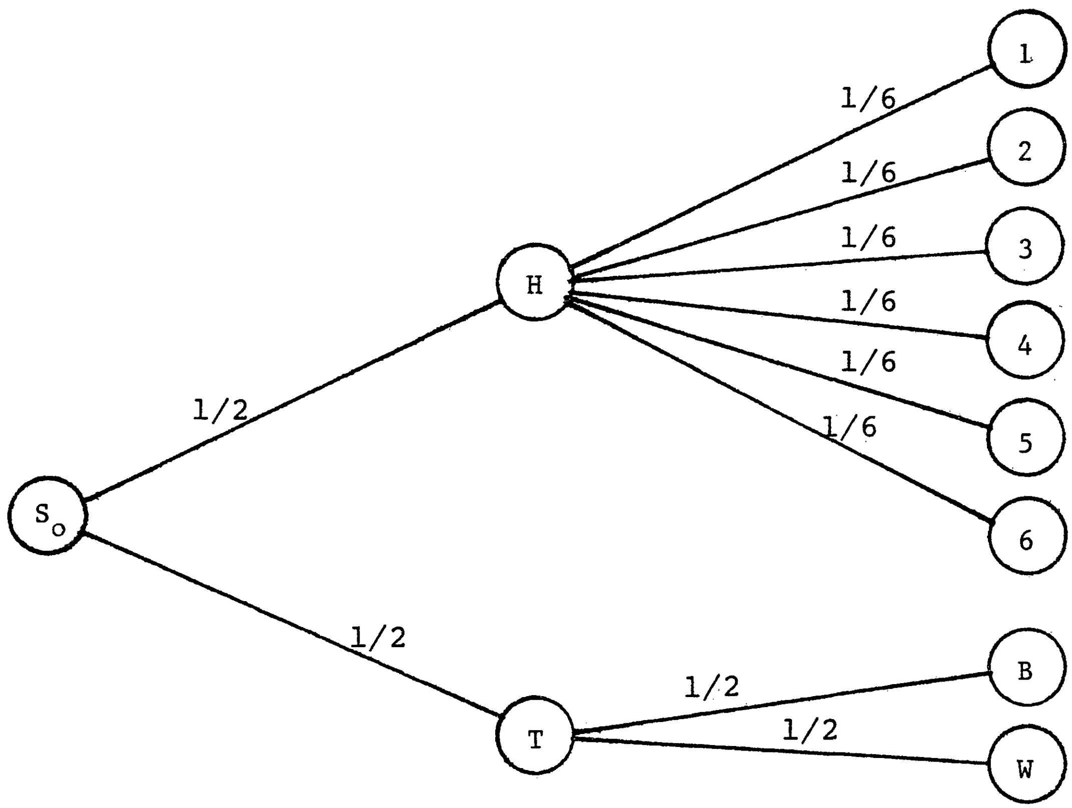
Markov Process Formulation

In the next few pages, the Markov chain theory that is required for Markov ROR analysis will be developed. The first objective will be to present a simple example that illustrates the concept of a Markov process.

Consider the probability tree diagrammed in Figure I. This tree corresponds to the possible outcomes of a simple 2-stage game. The first stage of the game involves the flipping of an unbiased coin. If the results of the coin toss is heads (H), the second stage of the game consists of a roll of an unbiased die. If the results of the coin toss in the first stage of the game is tails (T), the second stage of the game will consist of another tossing of an unbiased disc that has a black face (B) and a white face (W).

FIGURE I

Coin Toss & Die Roll Illustration



Thus, Figure I diagrams all of the possible outcomes of the simple game. Note that each branch in the diagram has attached to it the corresponding probabilities of the game following that specific path.

As simple as this game appears, it does represent a stochastic process. That is, the game constitutes a sequence of experiments (coin toss, die roll, disc toss) such that each experiment has a finite number of outcomes (2 for the disc and coin tosses, 6 for the die roll) with given probabilities (1/2 and 1/6). The concept of a stochastic process is the foundation of a Markov process and a Markov chain.

A Markov chain is a stochastic process such that the outcome of any experiment or trial in the sequence depends, at most, on the outcome of the immediately preceding experiment. Each outcome of an experment will be referred to as a state (s_i). Corresponding to each pair of states (s_i,s_j) is a given transition probability p_{ij}. This probability relates the chance that the outcome of the n^{th} experiment will be s_j given the outcome of experiment n-1 is s_i.

Returning to the simple 2-stage game for clarification, define:

s_0 = state corresponding to the beginning state of the game.

s_1 = state corresponding to the outcome of heads on the coin toss.

s_2 = state corresponding to the outcome of tails on the coin toss.

s_3 = state corresponding to the outcome of 1 on the die roll.

s_4 = state corresponding to the outcome of 2 on the die roll.

s_5 = state corresponding to the outcome of 3 on the die roll.

s_6 = state corresponding to the outcome of 4 on the die roll.

s_7 = state corresponding to the outcome of 5 on the die roll.

s_8 = state corresponding to the outcome of 6 on the die roll.

s_9 = state corresponding to the outcome of black on the disc toss.

s_{10}= state corresponding to the outcome of white on the disc toss.

The corresponding non-zero transition probabilities would be:

$p_{01} = 1/2$	$p_{15} = 1/6$	$p_{29} = 1/2$
$p_{02} = 1/2$	$p_{16} = 1/6$	$p_{210} = 1/2$
$p_{13} = 1/6$	$p_{17} = 1/6$	
$p_{14} = 1/6$	$p_{18} = 1/6$	

Let f_j be a function with the domain the set of paths in the tree and value the outcome at the j^{th} stage. It is evident that the probability of being in s_5 in the second stage of the game given the first stage results, $f_1 = s_1$, may be denoted.

$$p_{18} = P[f_2 = s_8 | f_1 = s_1] = 1/6$$

In general, the transition probabilities will be defined:

$$p_{ij} = P[f_j = s_j | f_i = s_i].$$

Note that the transition probabilities are static. The probability of going from state i to state j is not altered by when this transition will occur or how many times it has previously occurred. There will always be the same probability, p_{ij}, of the occurrance of outcome s_j given the preceding outcome is s_i.

The fact that the transition probabilities are static and have no "memory" allows them to be displayed and dealt with in efficient matrix notation. A transition matrix, Q, for a finite Markov chain is a square matrix whose elements are transition probabilities. (The term finite Markov chain refers to a Markov chain with a finite number of states). For example, the 2 x 2 transition matrix,

$$
\begin{array}{cc} & \text{(To)} \\ & \begin{array}{cc} s_1 & s_2 \end{array} \\ \text{(From)} \begin{array}{c} s_1 \\ s_2 \end{array} & \begin{bmatrix} p_{12} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = Q \end{array}
$$

designates there is a probability p_{21} of being in state s_1 in the next stage of the process given the current state is s_2. There is also a probability p_{22} that of remaining in state s_2 in the next stage of the process.

It is now advantageous to begin to relate the theory of Markov chains and processes to the realm of economics. In the previous chapter, an economic model was developed in conjunction with I-O analysis. That same model will serve as an example to demonstrate how Markov theory can be applied to estimate dollar-flows within the economy.

Recall in the 6-sector I-O table (Table II) that the entries in the table represent the dollar-volume of transactions between the sectors of the economy. If it is assumed that the magnitude of an entry, x_{ij}, relative to its sector output, X_j, is proportional to the probability of a dollar flowing from sector "i" to sector "j" during a transaction, then a transition matrix can be developed to describe monetary flows.

A closer look at the agricultural sector of the I-O model will help clarify the probable-dollar-flow concept. The I-O table gives the dollar volume of transactions with the agricultural sector as follows:

Agriculture to Agriculture	\$22.3 billion/yr
Agriculture to Mining	0 billion/yr
Agriculture to Construction	.3 billion/yr
Agriculture to Manufacturing	26.6 billion/yr
Agriculture to Finance	2.9 billion/yr
Agriculture to Services	.4 billion/yr
Agriculture to Consumption	11.0 billion/yr
Total Output	\$63.5 billion/yr

The above assumption requires that a dollar involved in a transaction originating with the agricultural sector will end up in a new sector with a probability proportional to that sectors dollar-volume of agricultural transactions. In the notation developed for I-O analysis and Markov chains the probability of a dollar flowing from sector i to sector j during a transaction is

$$p_{ij} = \frac{x_{ij}}{X_i} \quad .$$

So for agriculture, the transition probabilities are obtained by dividing the first row elements of Table II by 63.5.

		(To)						
		Agr.	Min.	Cnst.	Man.	Fin.	Ser.	Consum.
(From)	Agr.	.351	0	.005	.419	.046	.006	.173

During a given transaction a dollar has the probability of .351 that it will remain in the agricultural sector, a probability of 0 that it will flow into the mining sector, a probability of .005 that it will flow into the construction sector, and so on.

Extending this theory to all sectors of the economy, a complete transition matrix can be formulated. Table VI displays the entire transition matrix for the 6-sector economic model.

TABLE VI

Transition Matrix for 6-Sector Economy

(To)

State	1 Agr.	2 Min.	3 Const.	4 Man.	5 Fin.	6 Ser.	7 Consum.
1 Agr.	.351	0	.005	.419	.046	.006	.173
2 Min.	.003	.056	.337	.493	.013	0	.098
3 Cnst.	.005	.005	.001	.084	.080	.015	.811
4 Man.	.014	.004	.059	.381	.009	.042	.491
5 Fin.	.019	.020	.009	.152	.126	.058	.616
6 Ser.	.011	.004	.033	.216	.056	.071	.608
7 Consum.	.000	.000	.000	.000	.000	.000	1.000

Close examination of Table VI points out an interesting situation. A dollar can flow forever between sectors of the economy. However, there is always the possibility the dollar will enter the consumption state. When a dollar does enter into the consumption state, it is "consumed". The dollar is consumed in terms of never being able to leave this state and re-enter the sector-to-sector flow. A state with this property ($p_{ii} = 1$) is known as an absorbing state. A Markov chain including such states is an absorbing Markov chain .

There are some interesting properties associated with absorbing Markov chains. One of the more important properties of an absorbing Markov chain is that no matter where the process starts, the probability after n steps that the process will be in an absorbing state tends to 1 as n tends to infinity [4]. In other words, wherever a dollar enters the economy it will eventually enter the consumption state and be removed from the economy. (The number of taxable transactions a dollar will be involved in before consumption is the point of interest of this thesis).

Incorporating absorbing states into the Markov process can be conveniently displayed in canonical form. The canonical form of an absorbing Markov process with r absorbing states and s transient states is the (r + s) x (r + s) matrix.

$$\begin{array}{c} \\ r \\ s \end{array} \begin{array}{c} \begin{array}{cc} r & s \end{array} \\ \left[\begin{array}{c|c} I & 0 \\ \hline R & Q \end{array}\right] \end{array}$$

In this notation,

I = a r x r identity matrix

0 = a r x s matrix with all elements equalling 0.

R = a s x r matrix whose elements are the probabilities of transferring from a transient to an absorbing state.

Q = a s x s submatrix of transition matrix concerned with transient states.

Putting the 6-sector economic model and consumption state in canonical form yields the matrix displayed in Table VII.

TABLE VII

Canonical Form for 6-Sector Economy with Consumption State

(To)

(From)	Consume	Agr.	Min.	Cnst.	Man.	Fin.	Ser.
Consume	1	0	0	0	0	0	0
Agr.	.173	.351	0	.005	.419	.046	.006
Min.	.098	.003	.056	.337	.493	.013	0
Cnst.	.811	.005	.005	.001	.084	.080	.015
Man.	.491	.014	.004	.059	.381	.009	.042
Fin.	.616	.019	.020	.009	.152	.126	.058
Ser.	.608	.011	.004	.033	.216	.056	.071

The matrix in Table VII completely establishes the rules for a dollar flowing in the economy. What is of interest now is to develop a means to predict the expected number of transactions a dollar will be involved in before it is absorbed by the consumption state. Each transaction corresponds to an opportunity for taxation.

The means developed for predicting the expected number of transactions is derived directly from Markov theory. Using the transition matrix as input, it is possible to calculate the fundamental matrix, also known as the N-matrix. The elements of the N-matrix (n_{ij}) are the expected number of times the process will be in state s_j given the process began in state s_i.

Thus, for a process beginning in s_2 and having the N-matrix,

$$N = \begin{array}{c c} & \begin{array}{cc} s_1 & s_2 \end{array} \\ \begin{array}{c} s_1 \\ s_2 \end{array} & \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} \end{array}$$

the process is expected to be in state s_1 n_{21} times before being absorbed and n_{22} times in state s_2 before being absorbed. For a dollar entering the 6-sector model, the corresponding fundamental matrix will relate the relative expected number of transactions the dollar will be involved in before it enters the consumption state. The elements of the N-matrix are not intended to predict the actual number of transactions that will occur in the real economy. However, the elements provide a relative number of transactions to be expected from differing cases of the model. It is assumed that there is a strong correlation between the predicted values for the cases examined in the model and the volume of transactions in the modeled economy.

The formula for calculating the fundamental matrix is

$$N = (I - Q)^{-1}$$

where Q relates to the transient states of the transition matrix and I is an identity matrix of the same dimensions as Q. Proof of this formula is brief and will be presented. The proof is taken from Finite Markov Chains. [6].

Proof:

Define n_j to be the function giving the total number of times that the process is in transient state s_j. Define u_j^k to be the function that is 1 if the process is in s_j after k steps and 0 otherwise. Further, define $\{ M_i[n_j] \}$ to be the matrix in which

M_i denotes the mean value function of entering state j from state i.

Thus, for the transient states, the proof consists of showing

$$\{M_i[n_j]\} = N.$$

Clearly by definition

$$n_j = \sum_{k=0}^{\infty} u_j^k$$

Hence

$$\{M_i[n_j]\} = \{M_i[\sum_{k=0}^{\infty} u_j^k]\}$$

$$= \{\sum_{k=0}^{\infty} [M_i\ u_j^k]\}$$

$$= \{\sum_{k=0}^{\infty} ((1 - p_{ij}^k) \cdot 0 + p_{ij}^k \cdot 1)\}$$

where

p_{ij}^k is the transition probability at the k^{th} step.

Continuing

$$\{\sum_{k=0}^{\infty} ((1 - p_{ij}^k) \cdot 0 + p_{ij}^k \cdot 1\} = \sum_{k=0}^{\infty} \left\{p_{ij}^k\right\}$$

$$= \sum_{k=0}^{\infty} Q^k$$

$$= I + Q + Q^2 + \ \ldots$$

$$= (I - Q)^{-1}$$

$$= N$$

which completes the proof.

The proof also establishes the fact that the mean of the total number of times the process is in a given transient state is finite. This fact enables the N-matrix to be multiplied by a column vector whose elements are all 1 and determines the total expected number of states the process will be in if entered in state s_i. Define T to be a matrix which is the product of N and the column vector (C). For a 2 x 2 situation

$$T = NC$$

$$= \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_{11} + n_{12} \\ n_{21} + n_{22} \end{bmatrix}$$

Returning to the 6-sector economic model, the development of it's N and T matrix are as follows:

$$N = (I - Q)^{-1}$$

N =

	Agr.	Min.	Const.	Man.	Fin.	Ser.	
Agr.	.649	0	-.005	-.419	-.046	-.006	-1
Min.	-.003	.944	-.337	-.493	-.013	0	
Cnst.	-.005	-.005	.999	-.084	-.080	-.015	
Man.	-.014	-.004	-.059	.619	-.009	-.042	
Fin.	-.019	-.020	-.009	-.152	.126	-.058	
Ser.	-.011	-.004	-.033	-.216	-.056	.929	

	Agr.	Min.	Cnst.	Man.	Fin.	Ser.
Agr.	1.570	.008	.080	1.130	.106	.069
Min.	.032	1.067	.420	.965	.069	.055
Cnst.	.015	.008	1.017	.190	.098	.031
Man.	.040	.009	.107	1.701	.035	.081
Fin.	.044	.027	.448	.374	1.161	.090
Ser.	.031	.009	.066	.442	.083	1.103

T = NC

	Agr.	Min.	Cnst.	Man.	Fin.	Ser.	
Agr.	1.570	.008	.080	1.130	.106	.069	1
Min.	.032	1.067	.420	.065	.069	.055	1
Cnst.	.015	.008	1.017	.190	.098	.031	1
Man.	.040	.009	.107	1.701	.035	.081	1
Fin.	.044	.027	.448	.374	1.161	.090	1
Ser.	.031	.009	.066	.442	.083	1.103	1

Agr.	2.963
Min.	2.608
Cnst.	1.359
Man.	1.973
Fin.	2.144
Ser.	1.734

Thus, a dollar inserted into the agricultural sector would be involved in the most transactions.

One point of controversy could arise in applying the N-matrix to an economic model. Recall the proof of the N-matrix formula assumed the process would continue for infinitely many steps. The economic model must then be assuming there are an infinite number transaction opportunities (steps) arising annually. Obviously this is an invalid assumption.

The criticality of the non-infinite step assumption can be investigated by noting the rate at which Q^k tends to 0 as k tends to infinity. Since

$$N = (I - Q)^{-1}$$
$$= I + Q + Q^2 + Q^3 + \dots$$

a rapid convergence will cause slight or unnoticeable inaccuracies in the application of the N matrix. Referring again to the 6-sector model, notice

$$Q = \begin{bmatrix} .351 & 0 & .005 & .419 & .046 & .006 \\ .003 & .056 & .337 & .493 & .013 & 0 \\ .005 & .005 & .001 & .084 & .080 & .015 \\ .014 & .004 & .059 & .381 & .009 & .042 \\ .019 & .020 & .009 & .152 & .126 & .058 \\ .011 & .004 & .033 & .216 & .056 & .071 \end{bmatrix}$$

$$Q^2 = \begin{bmatrix} .130 & .003 & .027 & .315 & .026 & .023 \\ .010 & .007 & .048 & .247 & .033 & .027 \\ .004 & .002 & .008 & .052 & .012 & .009 \\ .011 & .002 & .025 & .168 & .012 & .020 \\ .012 & .005 & .019 & .108 & .022 & .018 \\ .009 & .003 & .017 & .115 & .016 & .018 \end{bmatrix}$$

$$Q^4 = \begin{bmatrix} .021 & .001 & .013 & .102 & .009 & .011 \\ .005 & .001 & .008 & .056 & .005 & .007 \\ .001 & 0 & .002 & .014 & .001 & .002 \\ .004 & .001 & .005 & .037 & .003 & .005 \\ .003 & .001 & .004 & .029 & .003 & .004 \\ .003 & 0 & .004 & .028 & .003 & .003 \end{bmatrix}$$

$$\vdots$$

$$Q^{10} \begin{bmatrix} 2x10^{-4} & .3x10^{-4} & 2x10^{-4} & 16x10^{-4} & 1x10^{-4} & 2x10^{-4} \\ .9x10^{-4} & .1x10^{-4} & 1x10^{-4} & 8x10^{-4} & .7x10^{-4} & .9x10^{-4} \\ .2x10^{-4} & .03x10^{-4} & .3x10^{-4} & 2x10^{-4} & .2x10^{-4} & .2x10^{-4} \\ .6x10^{-4} & .08x10^{-4} & .7x10^{-4} & 5x10^{-4} & .5x10^{-4} & .6x10^{-4} \\ .5x10^{-4} & .06x10^{-4} & .6x10^{-4} & 4x10^{-4} & .4x10^{-4} & .5x10^{-4} \\ .5x10^{-4} & .06x10^{-4} & .5x10^{-4} & 4x10^{-4} & .3x10^{-4} & .5x10^{-4} \end{bmatrix}$$

Thus it seems that if only one transaction opportunity arises each business day of the year no accuracy is lost, since Q^{250} is essentially 0.

Before leaving the Markov process, it is worthwhile to mention other applications of the process that can be used in conjunction with I-O analysis to gain insight into the effects of economic policies. The following matrices could be very valuable:

a. the matrix whose elements are the variances of the n_{ij}'s.

b. the matrix whose elements are the variances of the elements of the T-matrix.

c. the matrix whose elements are the probabilities of a dollar ending in a particular absorbing state. (Multiple absorbing state could be used to subdivide consumption into Government, household, etc.)

d. the matrix whose entries correspond to the number of changes of state before absorbtion given an entry state.

e. the matrix whose entries corresponding to the probability a process will ever be in the transient state s_j given the starting transient state s_i.

These matrices and others could be very useful in forecasting the flow of stimulating dollars in an economy and the possibilities of causing the results desired. For further reference consult [6].

Input-Output Markov Analysis

In the previous two chapters, the functions of I-0 analysis and Markov processes were developed. In this chapter, the two will be meshed together into one thorough analytic technique. The I-0 Markov analysis combines the ability to predict the level of economic activity (I-0 analysis) with the ability to forecast the relative level of transactions (Markov analysis). Combining the two processes provides a means of evaluating the effects of economic stimulus proposals and their corresponding rates of return.

The rate of return (ROR) aspect of the I-0 Markov method is one of its more intriguing attributes. ROR analysis becomes dependent upon both the change in level of economic activity and the change in the number of transactions. The number of transactions is not in the sphere of I-0 analysis. I-0 Markov analysis provides the Government with a means to evaluate economic stimulus in a more business-like manner.

This chapter will investigate I-0 Markov analysis further and apply it to the 6-sector model of the economy. In addition, a section will be focused on the need for ROR analysis for Governmental proposals and a section will focus on the implications of this technique in the areas of inflation and unemployment.

Need for ROR Analysis to Evaluate Economic Stimulus

As early as the 19th century the hypothesis was purposed that economic progress creates a relative increase in economic importance of government activity [15]. The 19th century German fiscal theorist, Adolf Wagner, stated this principle in his law:

> Comprehensive comparison of different countries and different times shows that, among progressive people, with which we alone are concerned, an increase regularly takes place in the activity of both the central and local governments. This increase is both extensive and intensive; the central and local governments constantly undertake new functions, while they perform both old and new functions more efficiently and completely. In this way the economic needs of the people, to an increasing extent and in a more satisfactory fashion, are satisfied by the central and local governments. The clear proof of this is found in the statistics which show the increased needs of central and local political units [21].

Wagner's law is substantiated throughout recent history. The central and local governments in the U.S. have continued to undertake new functions and these functions have clearly increased in economic importance.

Professor Richard A. Musgrave has grouped the objectives of the ever-increasing economic functions of a modern government into 3 classifications [13]. The first classification includes those functions required to satisfy the social wants of society. The second classification is composed of those functions that allow the government to redistribute income. The final classification constitutes those functions needed to stabilize the economy and the value of money while creating high levels of income and employment. To achieve these objectives, governments are armed with the power of taxation.

Thus, governments are faced with the problem of how to use taxation to accomplish their objectives. Integral to the problem of using taxation is the problem of how much taxation should be levied and what should be the optimal size of government. Ideally, the optimal size of government and the amount of taxation would be that which corresponds to the situation where marginal social benefits equal marginal social costs. Broad agreement on the point at which marginal social benefits equal martinal social cost is, for practical purposes, impossible.

Musgrave purposed a theory that could theoretically define the point of optimal government size and level of taxation. In his Voluntary Exchange Theory [13] concerning public economies, Musgrave defined taxes as voluntary payments made in hopes of exchange for services. The key word in the definition is "voluntary". If taxes were voluntary, a market situation would prevail and marginal social benefits would effectively equal marginal social costs. Though theoretically sound, the theory is of little practical value. Thus, since the size of governments and levels of taxation are not directly regulated by the society, standard practices should be established (ROR analysis) in an attempt to provide the society with more social benefit for less social cost.

The U.S. is a prime example of a country whose government needs standard practices to evaluate social costs and benefits. Since the first peacetime exise tax was invoked on playing cards in the 1890's, the size and budget of the Government have grown dramatically. Federal Government expenditures as a percent of GNP and total transfer payments as a percent of national income have increased more than sixfold in recent years [15]. It is questionable whether the rise in social benefits has kept pace with the rise in social costs.

To evaluate the balance between social costs and benefits, ROR analysis could be used to inform the taxpayer of the effectiveness of his tax dollar. Examining the current breakdown of the U.S. tax dollar and the inflation and unemployment rates, an argument could be made that the Government has not followed the wisest path in its quests to achieve its economic objectives (satisfy social wants, redistribute income, stabilize the economy). Currently, 8.9% of the average tax dollar is needed to pay the interest on the public debt as opposed to 9.2% of the average tax dollar directed to public health [16].

By employing the I-O Markov analysis technique to investigate ROR of economic stimulus, the Government would be better able to gauge the effectiveness of their economic functions. The I-O Markov technique integrates the level of economic activity with the volume of economic transactions to give an analytic method that conforms to the Government's means of revenue generated from these taxes. The evaluation of economic proposals should be concerned with the corresponding effect on the relative number of transactions in the economy since, most transactions represent an opportunity to impose either an income or exise tax.

The Government can enhance its financial position by stimulating sectors of the economy that tend to cause an increase in the number of transactions and consequently a higher ROR. I-O Markov analysis provides the means to determine the sectors that are most favorable for stimulation.

To summarize, there is a great need for an ROR analysis technique such as the I-O Markov technique. The major reasons this technique is needed are: national economic progress has created an increase in economic importance of Government activity, an attempt must be made to equate marginal social cost to marginal social benefit, the immense growth of the federal budget requires a standardized process to allocate funds, and the I-O Markov method is based on both level of economic activity and volume of transactions (taxable situations).

Input Output Analysis and the 6-Sector Model

Input-Output, as described in Chapter I shall now be applied to the 6-sector economic model. The consumption for each sector will be increased parametrically to investigate the effects on total economic activity. To increase consumption, the Government can increase the aggregate demand for a sector's production or reduce the level of taxation in the economy. A reduction in taxes can be translated to increased purchasing potential and a subsequent increase in consumption. Historically, 92%-98% of all additional income is spent by the American society [14], so a reduction in taxes is a prime means of increasing consumption.

The key to I-O analysis is calculating the inverse of the Leontief matrix in order to determine the new total production vectors for the economic model. Once the inverse is calculated, it is a simple process of matrix multiplication to determine the new total production vector (X). For the 6-sector model the inverse of the Leontief matrix was determined to be

$$(I-A)^{-1} = \begin{bmatrix} 1.569 & 0.016 & 0.039 & 0.111 & 0.032 & 0.023 \\ 0.016 & 1.067 & 0.101 & 0.045 & 0.010 & 0.009 \\ 0.029 & 0.037 & 1.017 & 0.036 & 0.061 & 0.021 \\ 0.396 & 0.184 & 0.544 & 1.702 & 0.111 & 0.280 \\ 0.141 & 0.179 & 0.071 & 0.118 & 1.161 & 0.099 \\ 0.088 & 0.051 & 0.098 & 0.127 & 0.076 & 1.103 \end{bmatrix}$$

To determine the new X vector, the only operation required is to multiply $(I-A)^{-1}$ by Y, where Y is the new consumption vector. By summing the elements of the total production vector, X, the GNP for the economy can be projected. This definition of GNP will be used interchangeably with level of economic activity.

In Figure II through Figure VII, the effect on the level of economic activity (GNP) caused by increasing consumption is plotted for each sector. The horizontal scale on each plot represents an increase in consumption from 0 to 100%.

Note the relationship between an increase in a sector's consumption and the level of economic activity is linear (deviations from the linear relationship are attributed to roundoff error). This is a direct result of the fact the new X vector is merely the product of a constant, $(I-A)^{-1}$, and the new Y vector.

An interesting aspect to notice when observing the plots is that the slopes of the lines on the graphs differ for each sector of the economy. Thus, a dollar of stimulus will increase the GNP at different rates depending on the sector in which the stimulus is injected.

The differing effects on GNP corresponding to the sector stimulated

FIGURE II

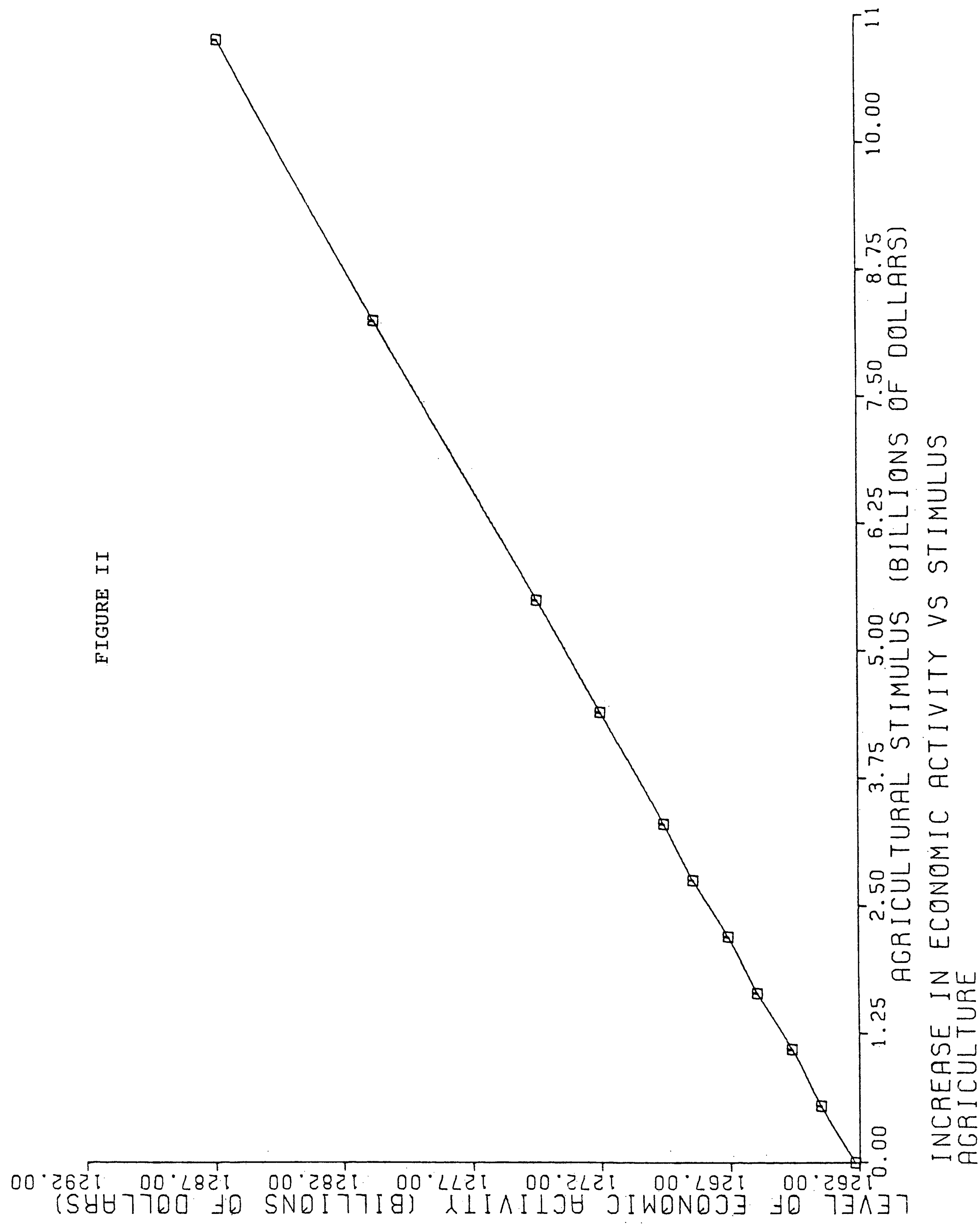

INCREASE IN ECONOMIC ACTIVITY VS STIMULUS
AGRICULTURE
AGRICULTURAL STIMULUS (BILLIONS OF DOLLARS)
0.00
1.25
2.50
3.75
5.00
6.25
7.50
8.75
10.00
11.25
LEVEL OF ECONOMIC ACTIVITY (BILLIONS OF DOLLARS)
1262.00
1267.00
1272.00
1277.00
1282.00
1287.00
1292.00

FIGURE III

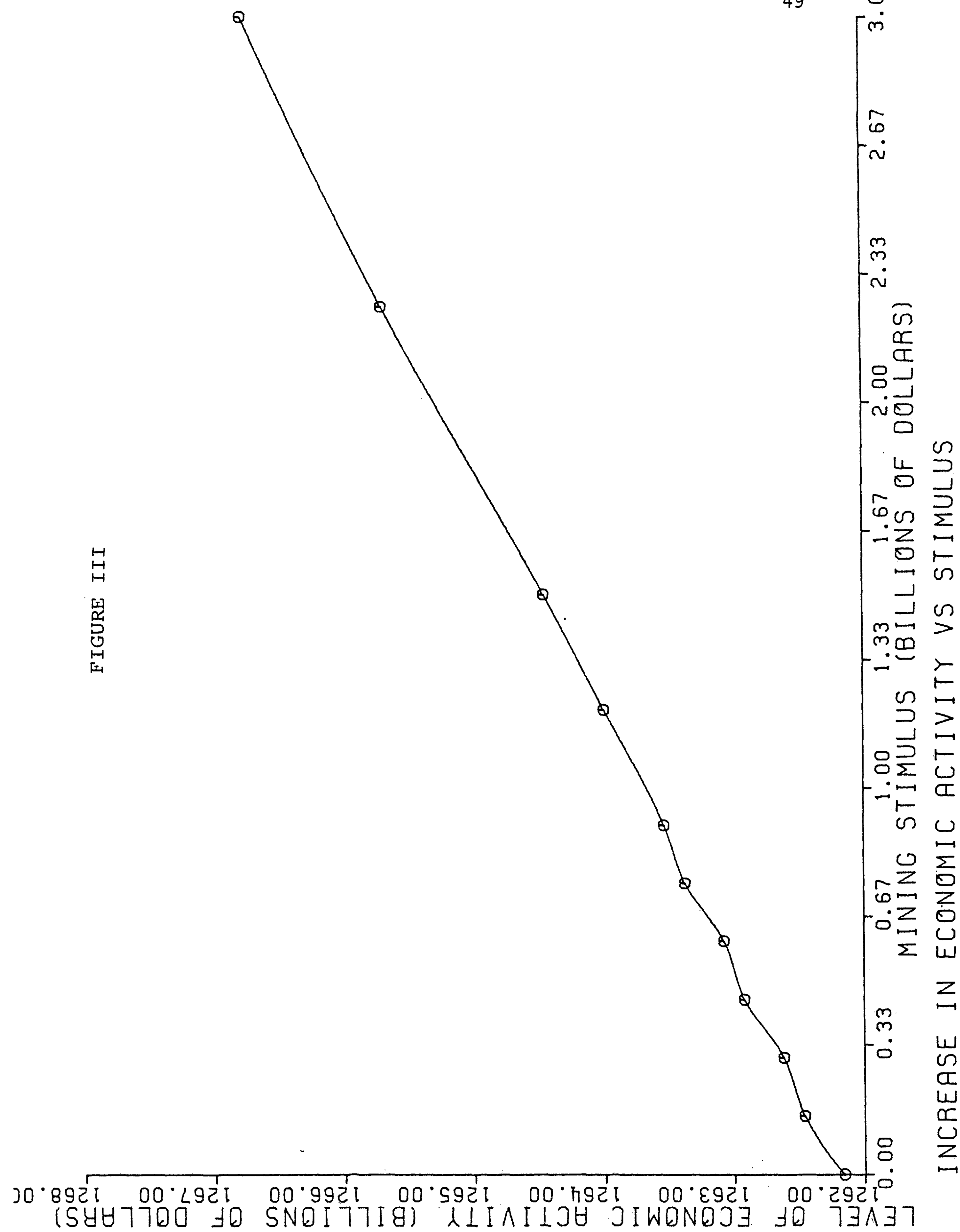
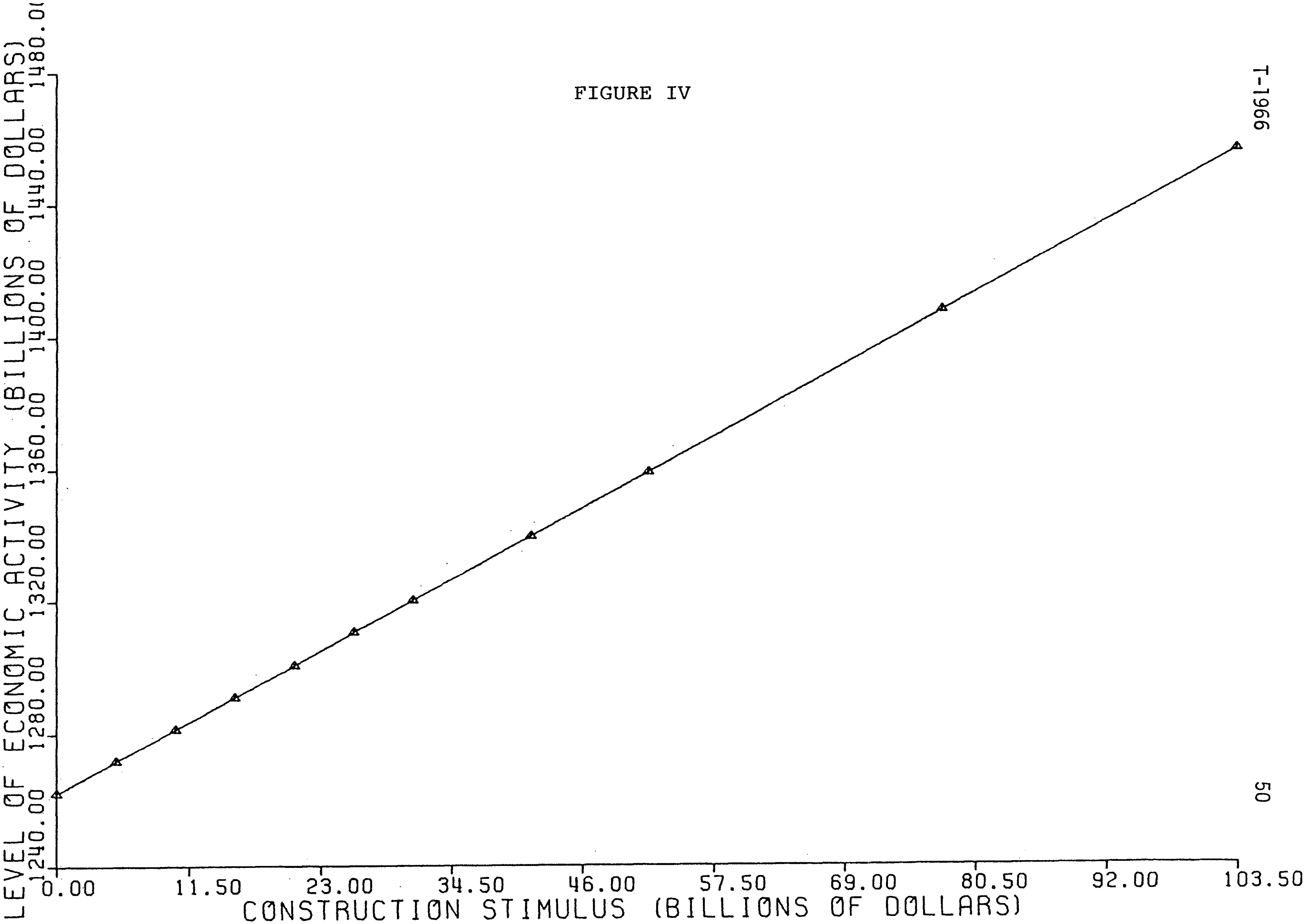
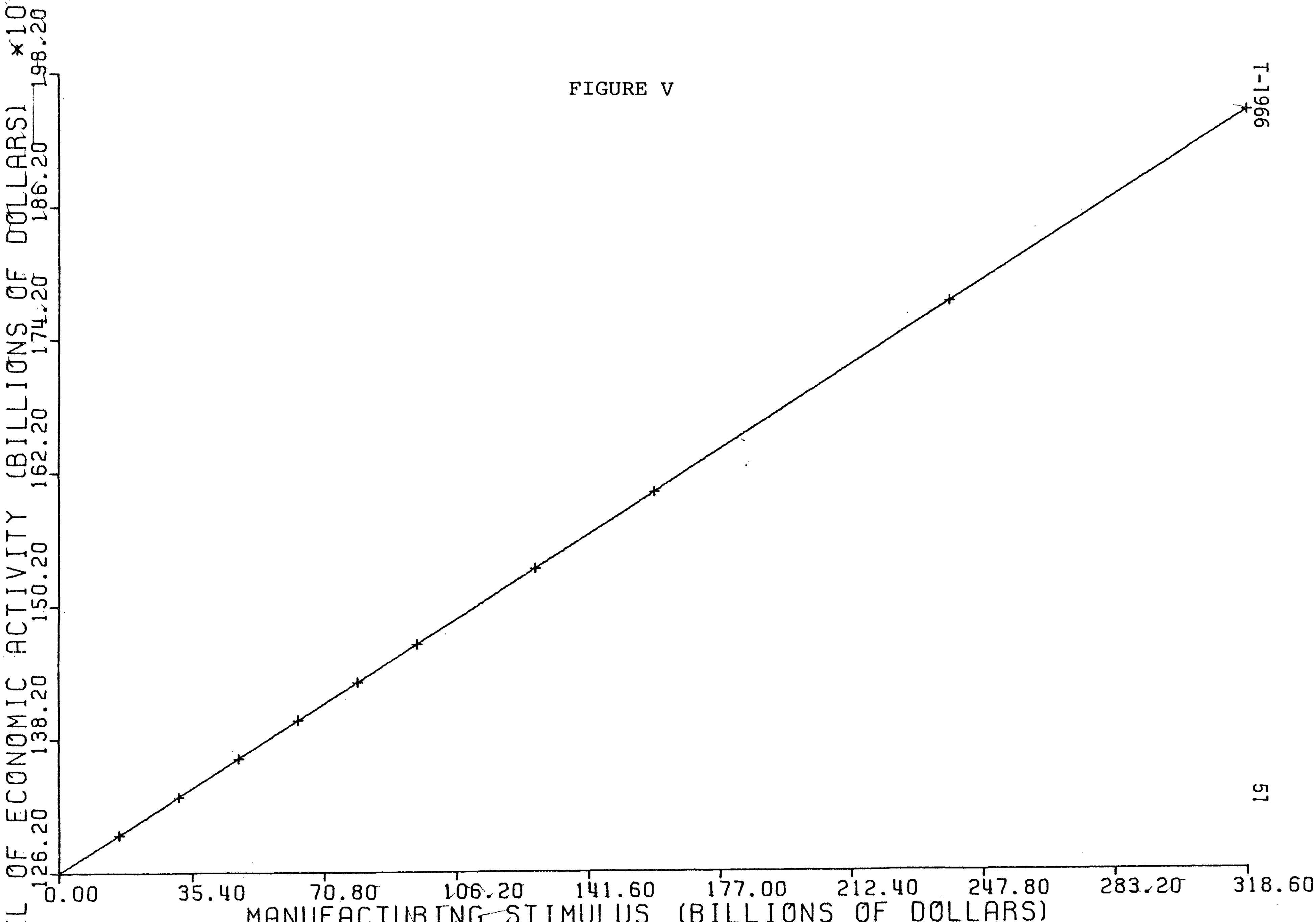

INCREASE IN ECONØMIC ACTIVITY VS STIMULUS
MINING
MINING STIMULUS (BILLIØNS ØF DØLLARS)
0.00
0.33
0.67
1.00
1.33
1.67
2.00
2.33
2.67
3.00
LEVEL ØF ECØNØMIC ACTIVITY (BILLIØNS ØF DØLLARS)
1262.00
1263.00
1264.00
1265.00
1266.00
1267.00
1268.0C

FIGURE IV



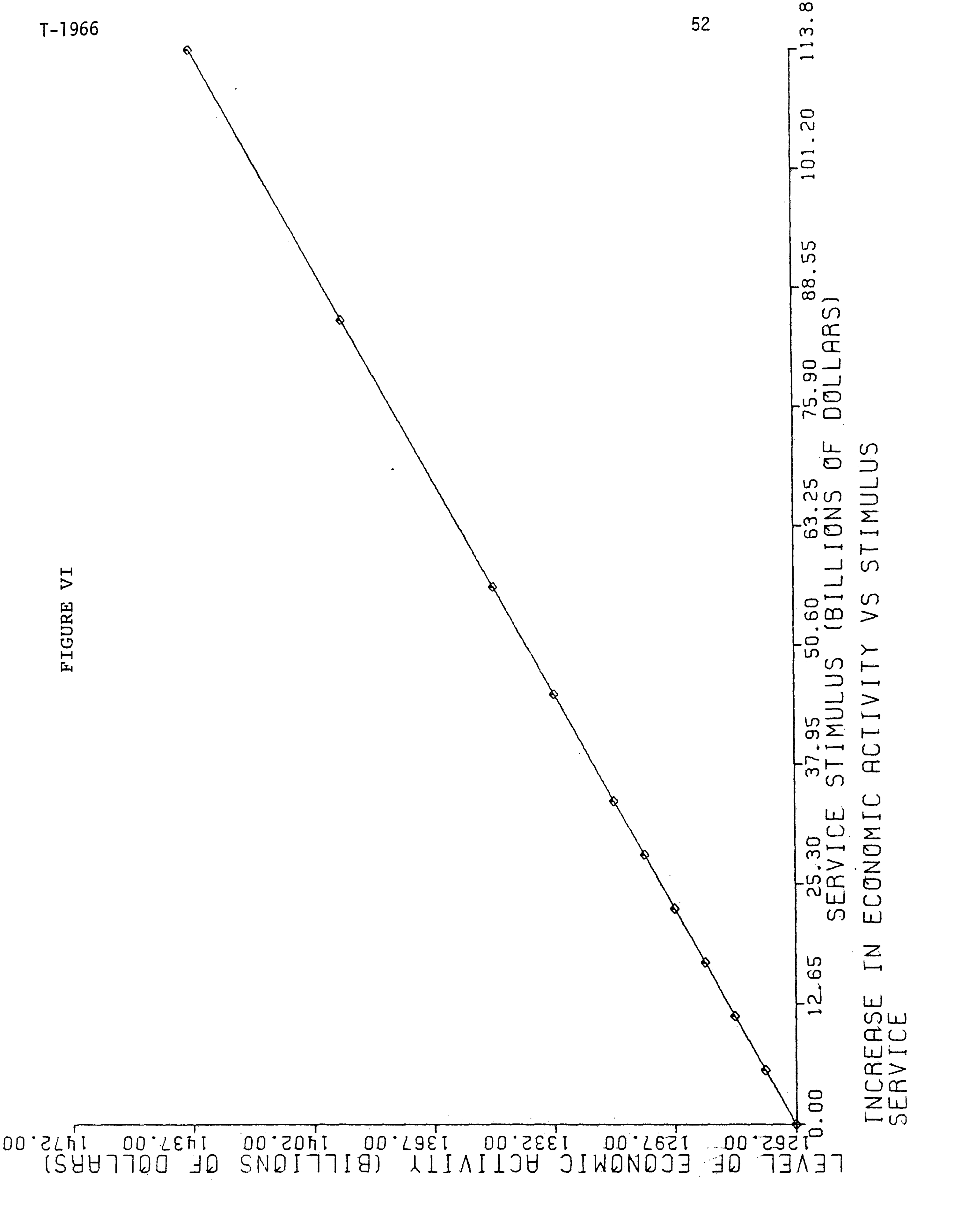
INCREASE IN ECONOMIC ACTIVITY VS STIMULUS
CONSTRUCTION

FIGURE V



INCREASE IN ECONOMIC ACTIVITY VS STIMULUS
MANUFACTURING

FIGURE VI


0.00
12.65
25.30
37.95
50.60
63.25
75.90
88.55
101.20
113.85
SERVICE STIMULUS (BILLIONS OF DOLLARS)
1262.00
1297.00
1332.00
1367.00
1402.00
1437.00
1472.00
LEVEL OF ECONOMIC ACTIVITY (BILLIONS OF DOLLARS)
INCREASE IN ECONOMIC ACTIVITY VS STIMULUS
SERVICE

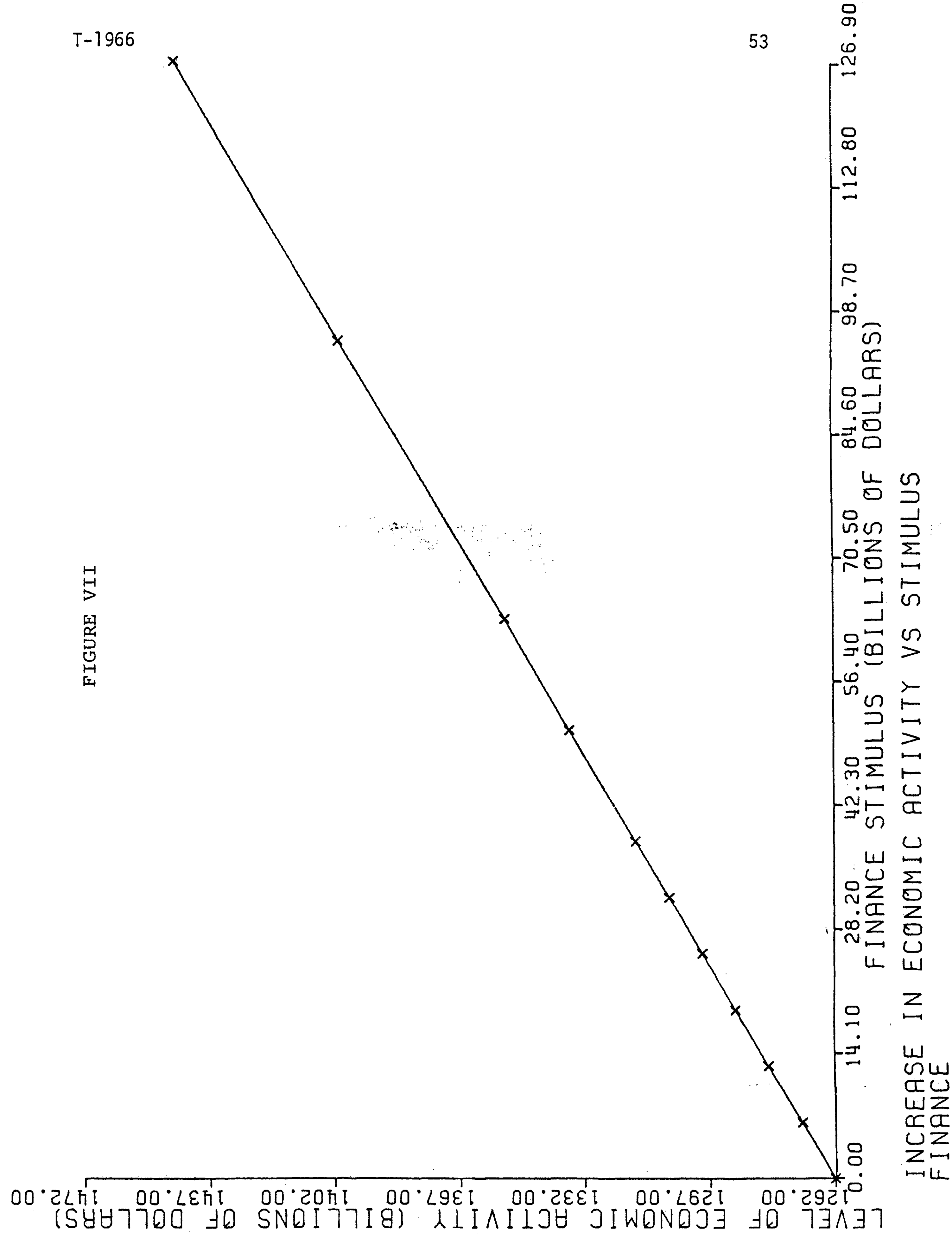

LEVEL OF ECONOMIC ACTIVITY (BILLIONS OF DOLLARS)
1262.00
1297.00
1332.00
1367.00
1402.00
1437.00
1472.00
0.00
14.10
28.20
42.30
56.40
70.50
84.60
98.70
112.80
126.90
FINANCE STIMULUS (BILLIONS OF DOLLARS)
INCREASE IN ECONOMIC ACTIVITY VS STIMULUS
FINANCE

FIGURE VII

relates back to the multiplier effect. If the assumption was made that tax revenues are a constant proportion of GNP, then the logical sector of the economy to stimulate would be the sector with the greater multiplier effect. Theoretically, if the tax rate is greater than or equal to the reciprocal of the multiplier, the cost of stimulating the economy could be recouped through tax revenue levied on the increase in economic activity.

The multiplier for each sector of the 6-sector model is listed in Table VII. Based only on I-O analysis, the agricultural sector is the most attractive for stimulation. Each dollar of stimulus injected in agriculture will increase the GNP by $2.23. A multiplier of this magnitude requires a tax rate of at least 45% if the stimulus is to be fully financed by the increase in tax revenues.

TABLE VIII

The Multiplier Effect for the Sectors of the 6-Sector Model

Sector	Multiplier
Agr.	2.23
Min.	1.53
Cnst.	1.87
Man.	2.14
Fin.	1.45
Ser.	1.54

Markov Analysis and the 6-Sector Model

Markov analysis is intended to complement I-O analysis by bringing into account the second key to ROR analysis. I-O analysis deals only with projecting changes in total economic activity. Markov analysis concentrates on the effects stimulus may have on the flow of money within the economy and the relative number of taxable situations. The flow of money is a very important concept in ROR analysis. Adam Smith recognized the importance of the flow of money by distinguishing capital from revenue: wherever capital predominates, industry previals; wherever revenue, idleness [19]. Markov analysis can be very useful in gauging the degree that money is flowing in the economy.

Recall from Chapter II that the transition probabilities for the transition matrix (Q) in the Markov process are derived by dividing sector output into sector transactions,

$$p_{ij} = \frac{x_{ij}}{X_i}$$

To determine the new transition probabilities generated by stimulating the consumption vector, the new values of sector transactions (x_{ij}') are divided by the new value of sector output (X_i'),

$$p_{ij}' = \frac{x_{ij}'}{X_i'}$$

The proportionality assumptions of I-O analysis implies that the value of sector transactions will increase proportionately with the increase in total sector output. Thus, x_{ij}' can be expressed in terms of x_{ij}, and X_j.

$$x_{ij}' = x_{ij} \cdot \frac{X_j'}{X_j}$$

In turn, the new transition probabilities may be expressed in terms of x_{ij}, X_i', and X_j.

$$p_{ij}' = \frac{x_{ij}'}{X_i'}$$

$$= (x_{ij} \frac{X_j'}{X_j}) / X'_i$$

$$= \frac{x_{ij}}{X_j} \cdot \frac{X_j'}{X_i'}$$

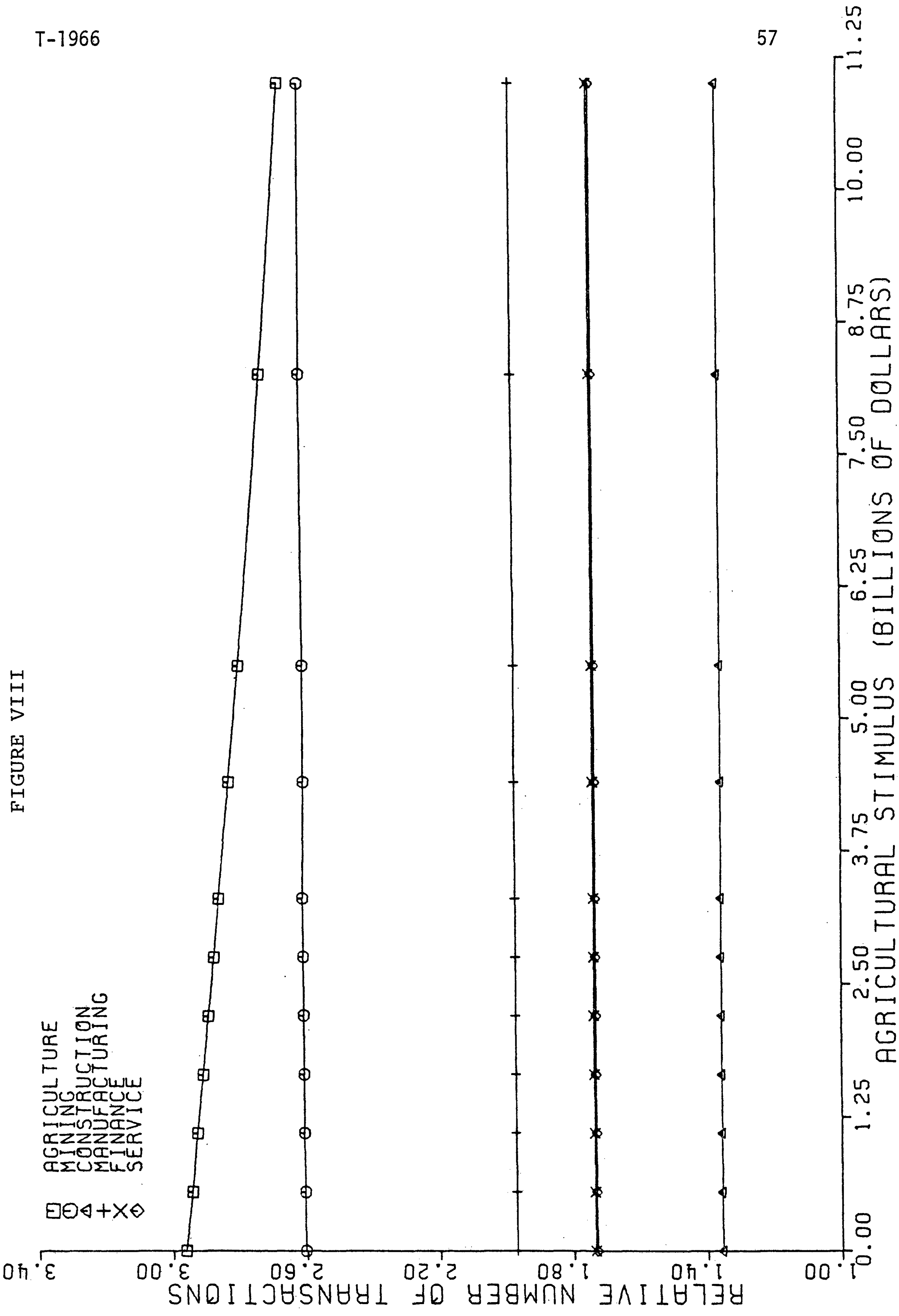
Noticing $\left(\frac{x_{ij}}{X_j}\right) = a_{ij}$,

the new transition probabilities are determined by multiplying the technical coefficients (a_{ij}) by the ratio of X_j'/X_i'.

Subtracting the new transition matrix from an appropriately dimensioned identity matrix and calculating the inverse of the resulting matrix, produces the fundamental matrix. By summing the elements of each row of the fundamental matrix, the relative number of transactions a dollar, in the respective sector, will be involved in before consumption is obtained. Figure VIII through Figure XIX display the relationships between sector stimulation and transactions. The figures should be viewed in consecutive pairs. The first graph shows the sector by sector effect on transactions given the stimulus in one of the sectors. The second graph of the pair shows the accumulative effect of all sectors in relation to a sector's stimulus.

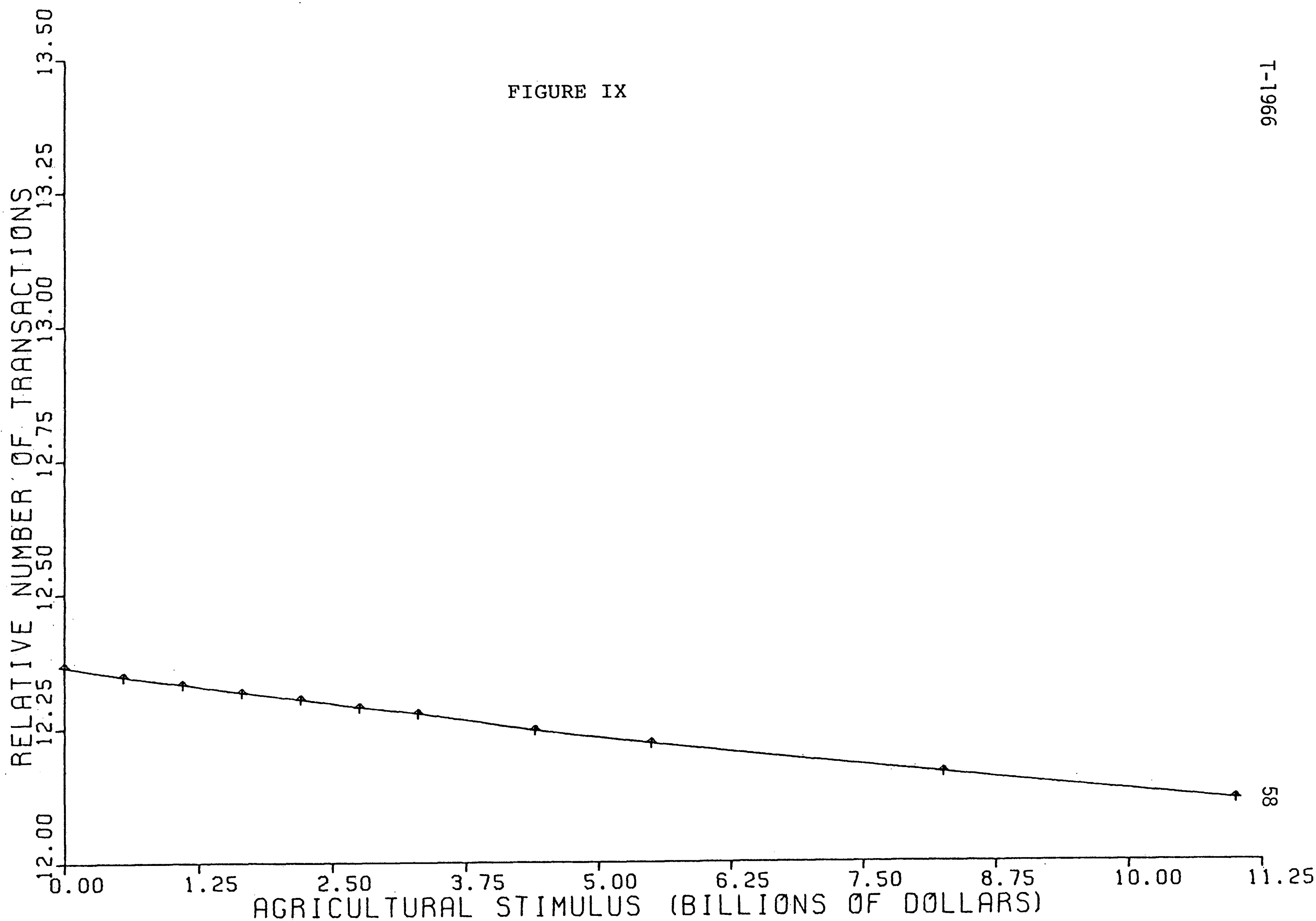
The most startling conclusion to be drawn from the graphs is that the relationship between stimulation and transactions is seemingly linear in most cases. An analytic investigation of the relationship is presented in appendix A.

FIGURE VIII



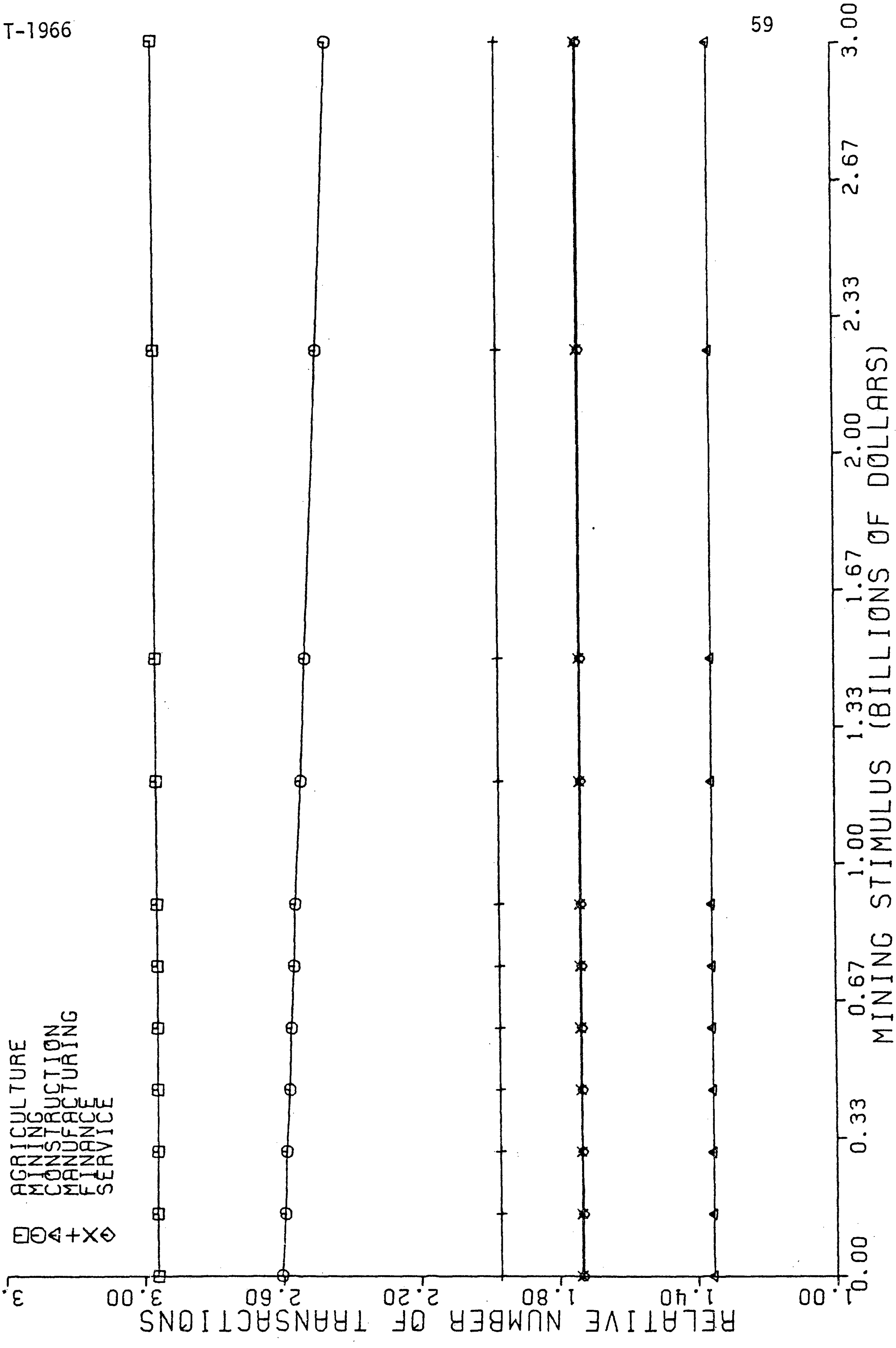
NUMBER OF TRANSACTIONS VS AGRICULTURAL STIMULATION
EFFECTS OF AGRICULTURAL STIMULUS ON EACH SECTOR

FIGURE IX



NUMBER OF TRANSACTIONS VS AGRICULTURAL STIMULATION
ACCUMULATED EFFECT OF AGRICULTURAL STIMULUS

FIGURE X



NUMBER OF TRANSACTIONS VS MINING STIMULATION
EFFECTS OF MINING STIMULUS ON EACH SECTOR

FIGURE XI

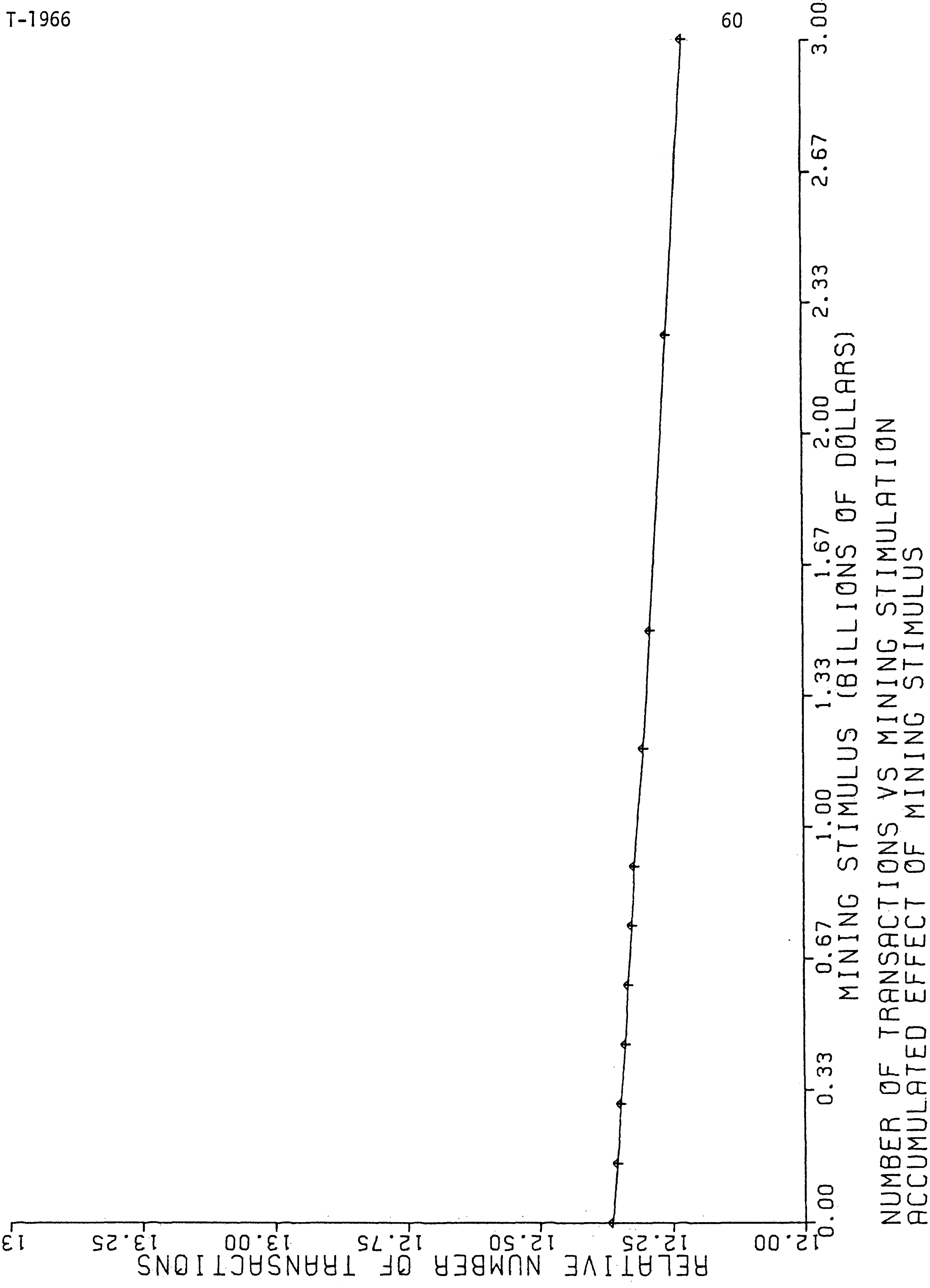
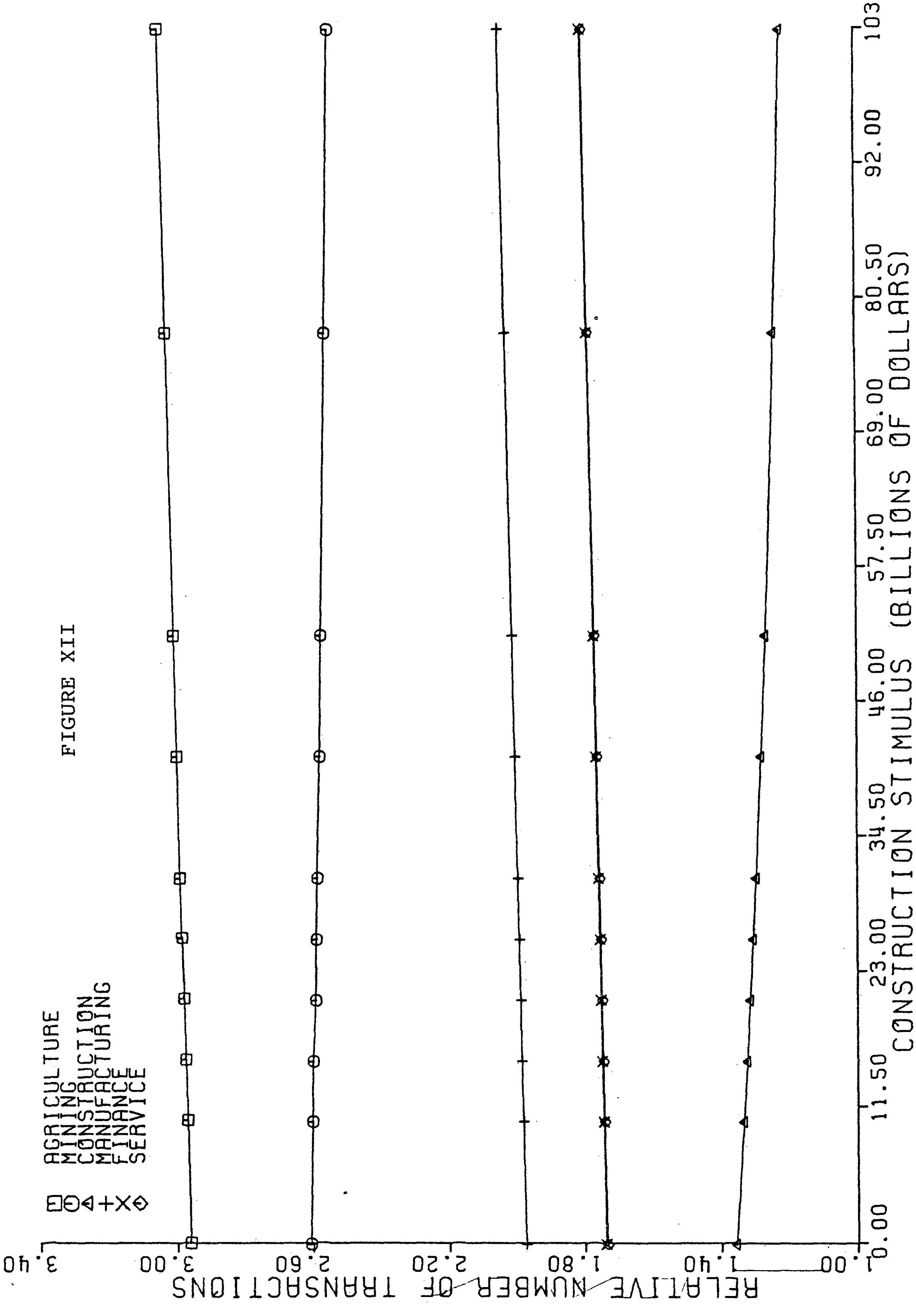

NUMBER OF TRANSACTIONS VS MINING STIMULATION
ACCUMULATED EFFECT OF MINING STIMULUS
MINING STIMULUS (BILLIONS OF DOLLARS)
RELATIVE NUMBER OF TRANSACTIONS
0.00
0.33
0.67
1.00
1.33
1.67
2.00
2.33
2.67
3.00
12.00
12.25
12.50
12.75
13.00
13.25
13.50

FIGURE XII



NUMBER OF TRANSACTIONS VS CONSTRUCTION STIMULATION
EFFECTS OF CONSTRUCTION STIMULUS ON EACH SECTOR

FIGURE XIII

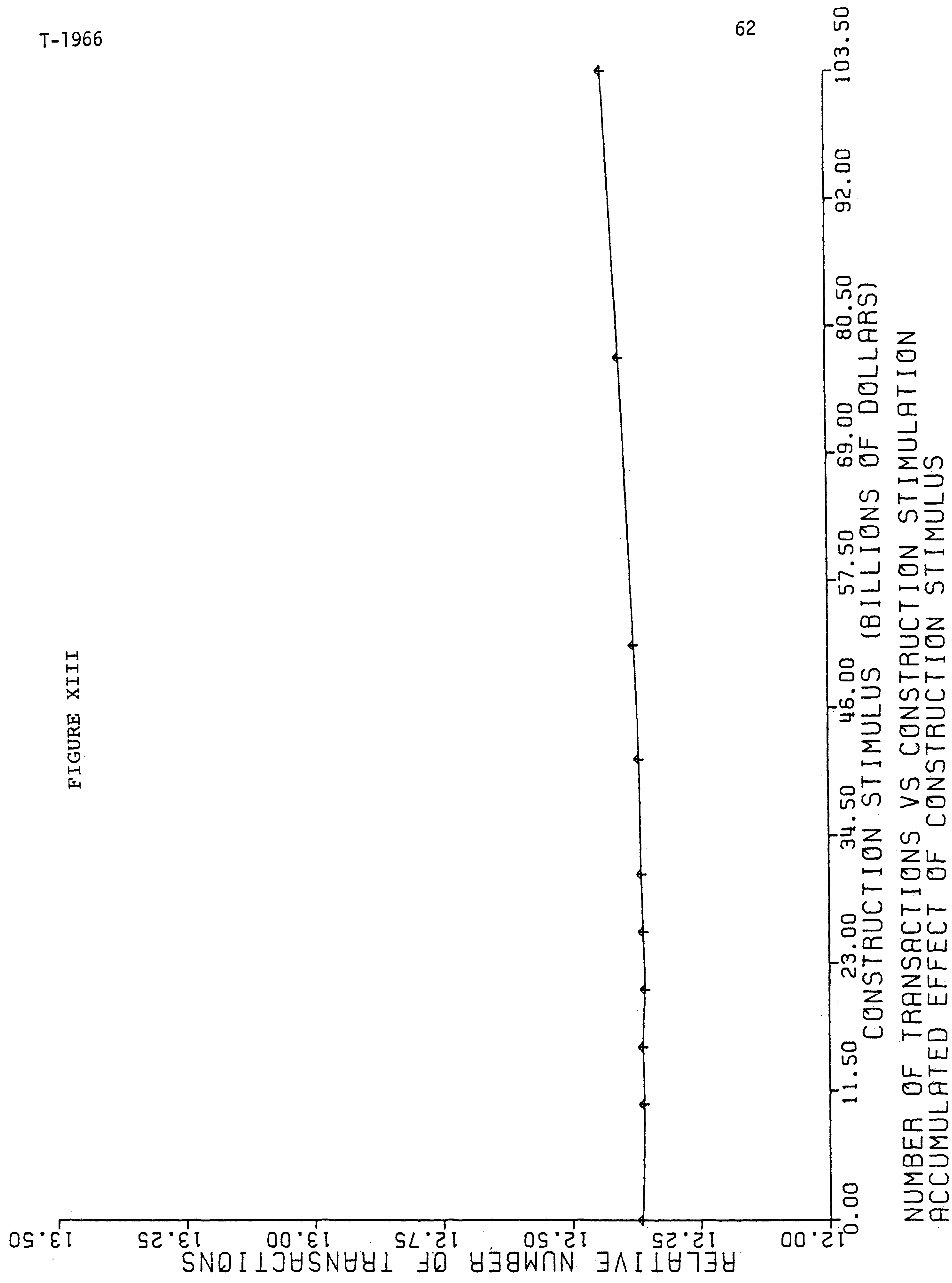
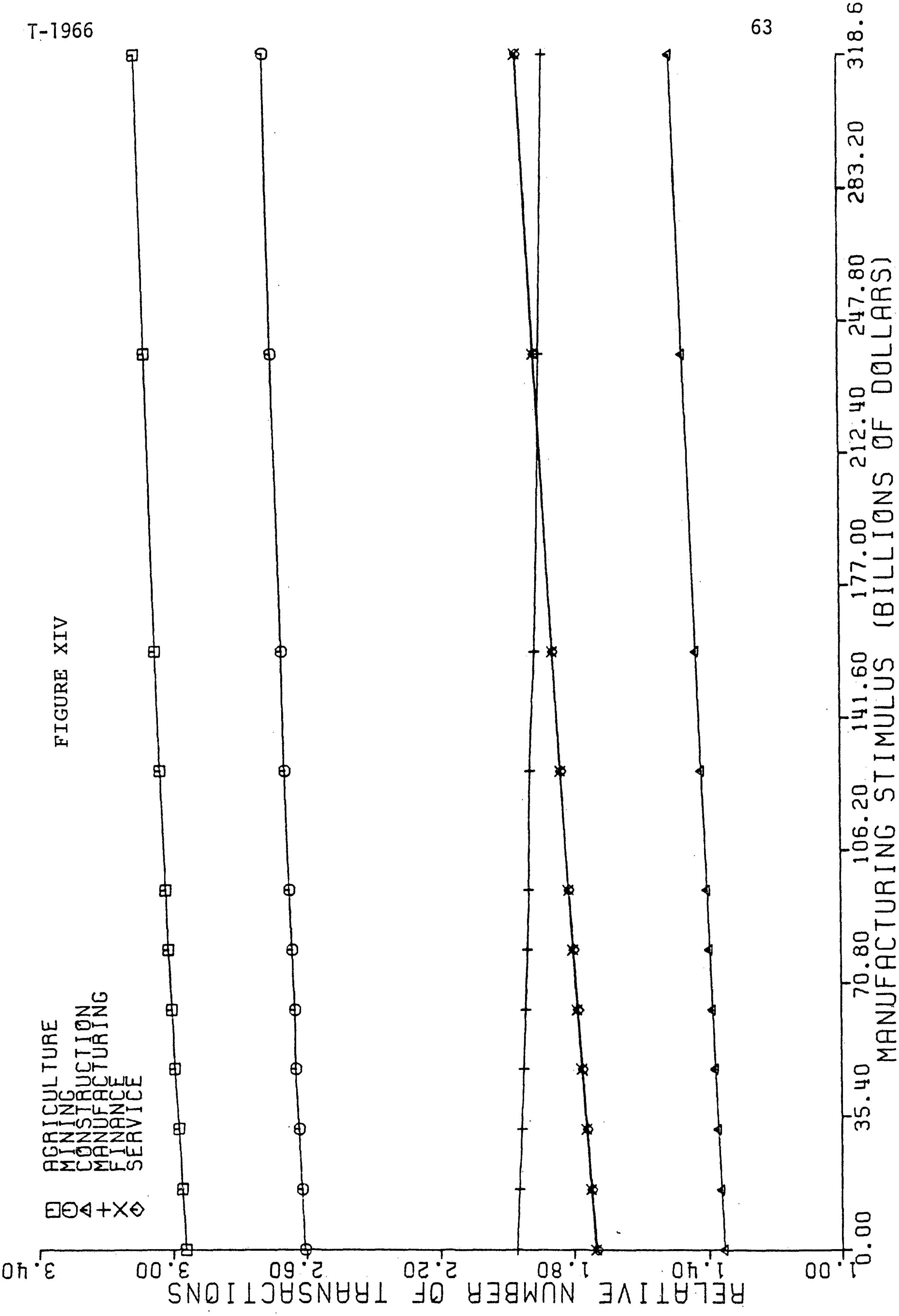

NUMBER OF TRANSACTIONS VS CONSTRUCTION STIMULATION
ACCUMULATED EFFECT OF CONSTRUCTION STIMULUS
CONSTRUCTION STIMULUS (BILLIONS OF DOLLARS)
0.00
11.50
23.00
34.50
46.00
57.50
69.00
80.50
92.00
103.50
RELATIVE NUMBER OF TRANSACTIONS
12.00
12.25
12.50
12.75
13.00
13.25
13.50

FIGURE XIV



NUMBER OF TRANSACTIONS VS MANUFACTURING STIMULATION
EFFECTS OF MANUFACTURING STIMULUS ON EACH SECTOR

FIGURE XV

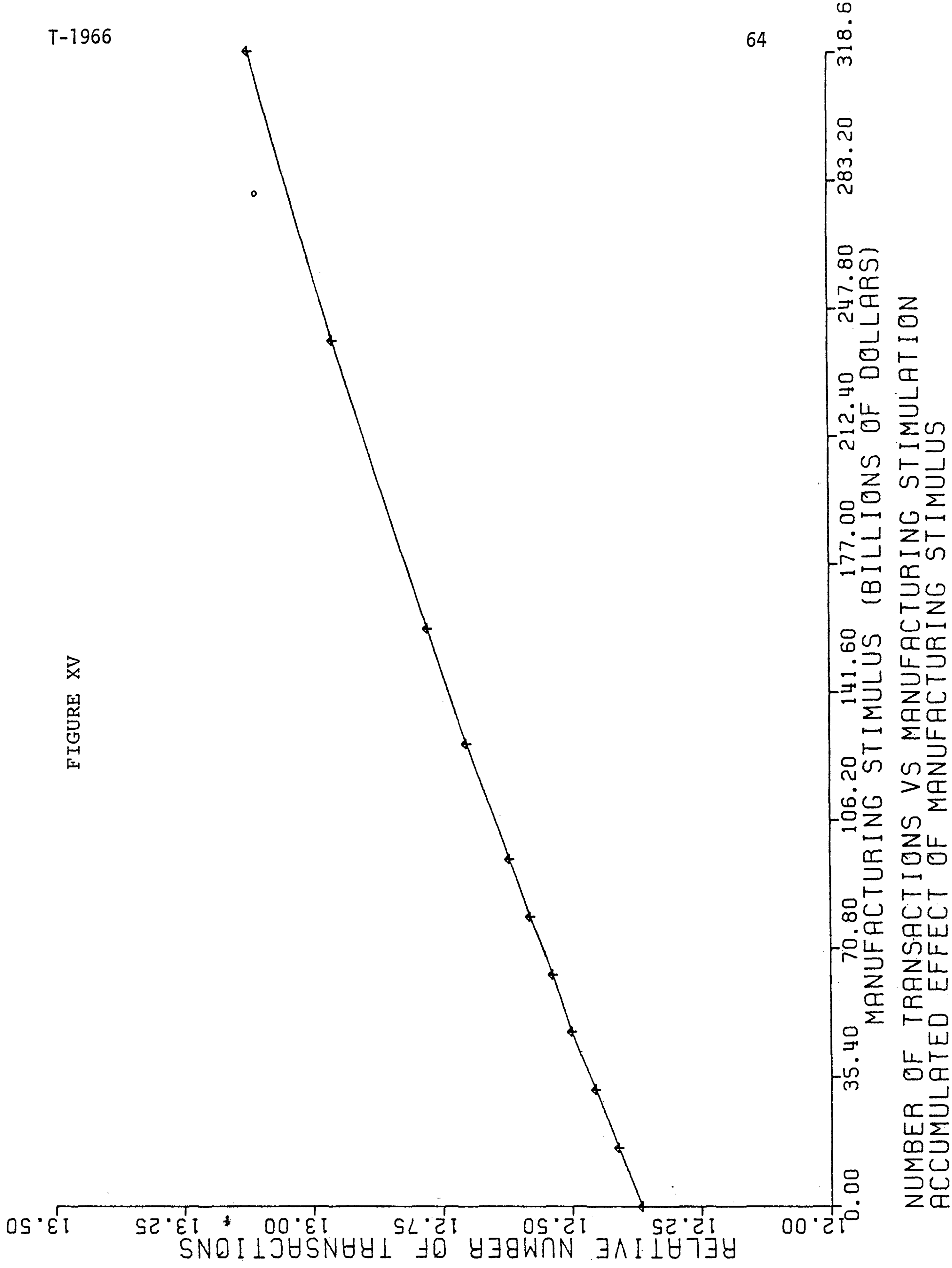
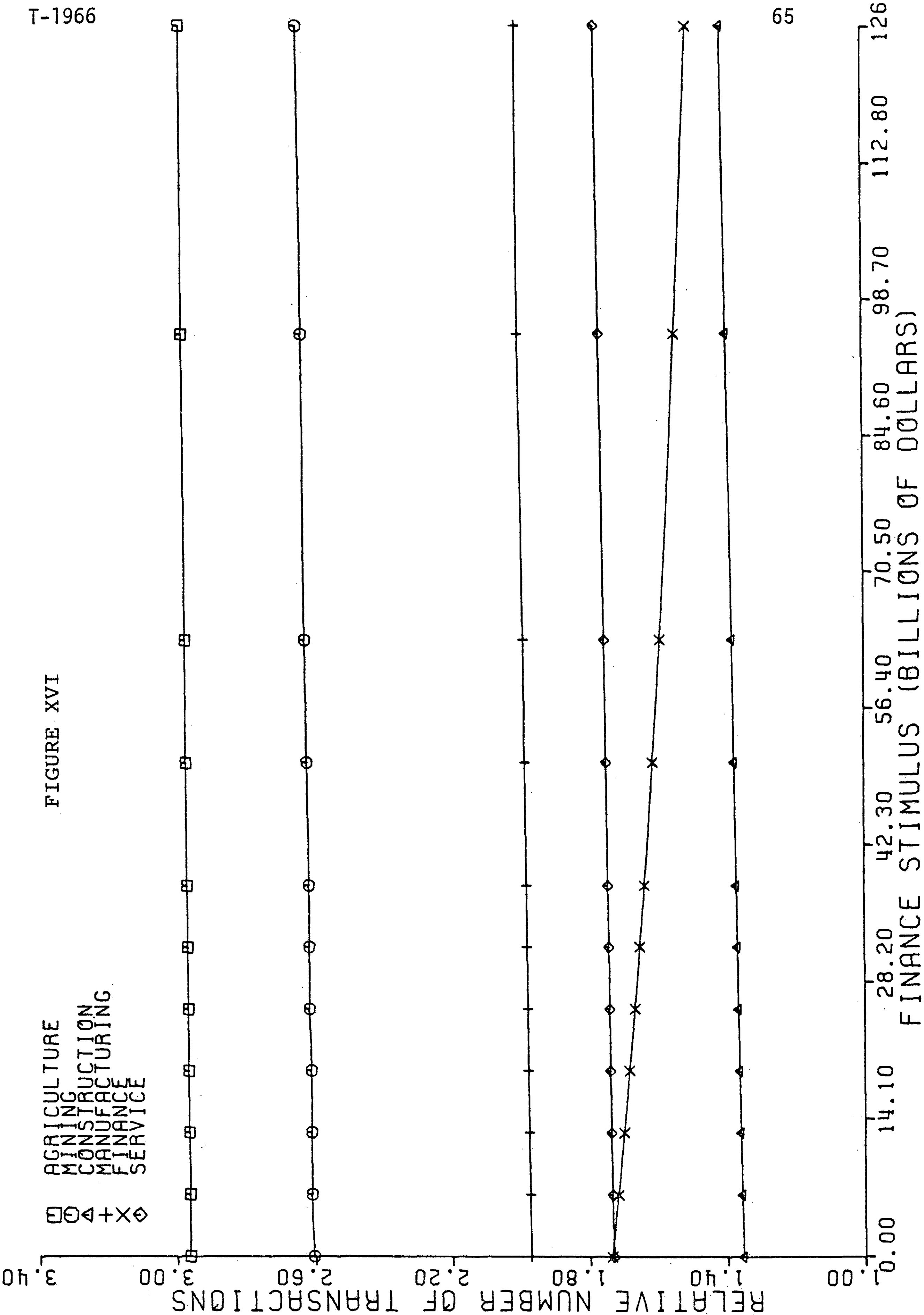
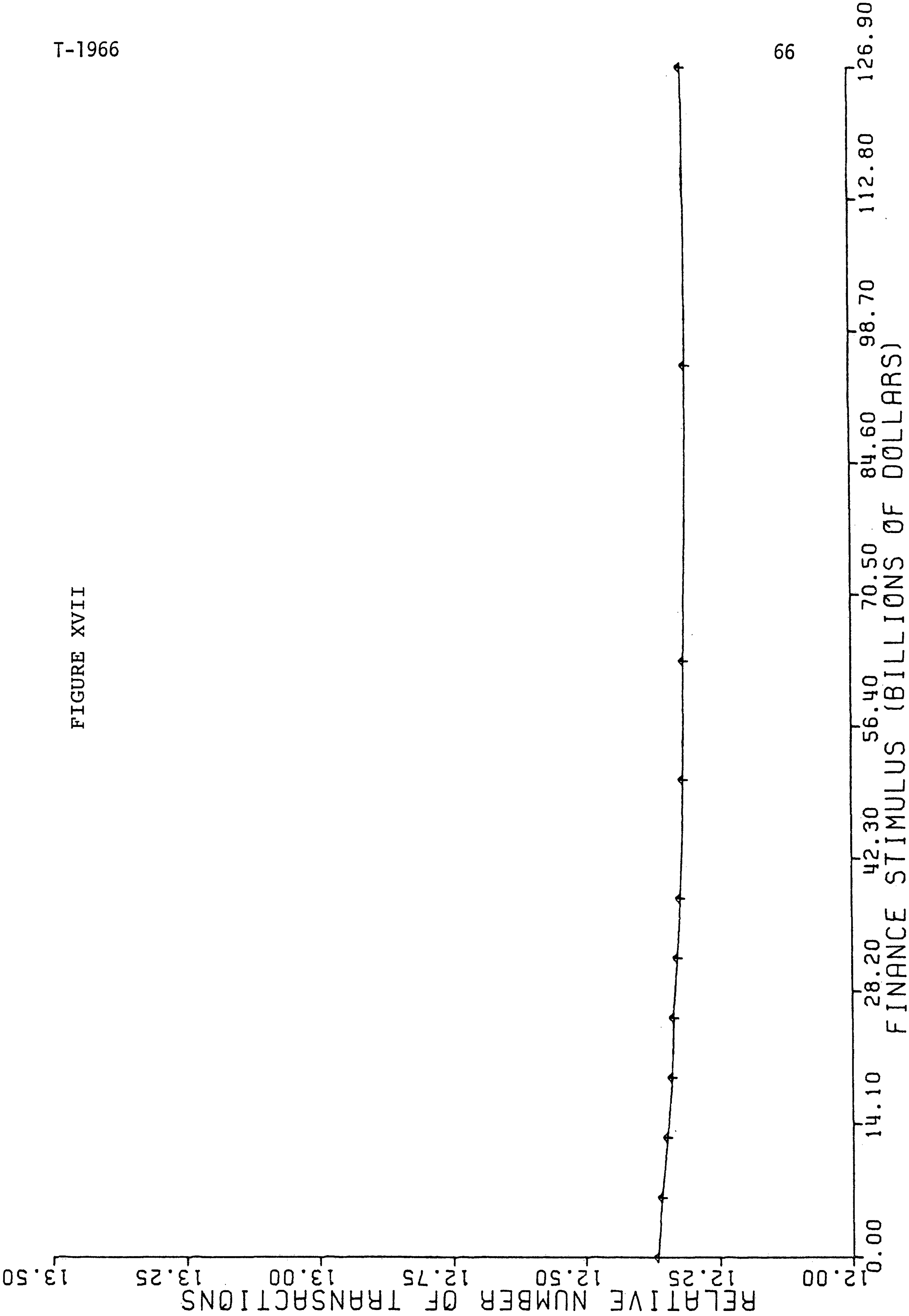

NUMBER OF TRANSACTIONS VS MANUFACTURING STIMULATION
ACCUMULATED EFFECT OF MANUFACTURING STIMULUS
MANUFACTURING STIMULUS (BILLIONS OF DOLLARS)
0.00
35.40
70.80
106.20
141.60
177.00
212.40
247.80
283.20
318.60
RELATIVE NUMBER OF TRANSACTIONS
12.00
12.25
12.50
12.75
13.00
13.25
13.50

FIGURE XVI



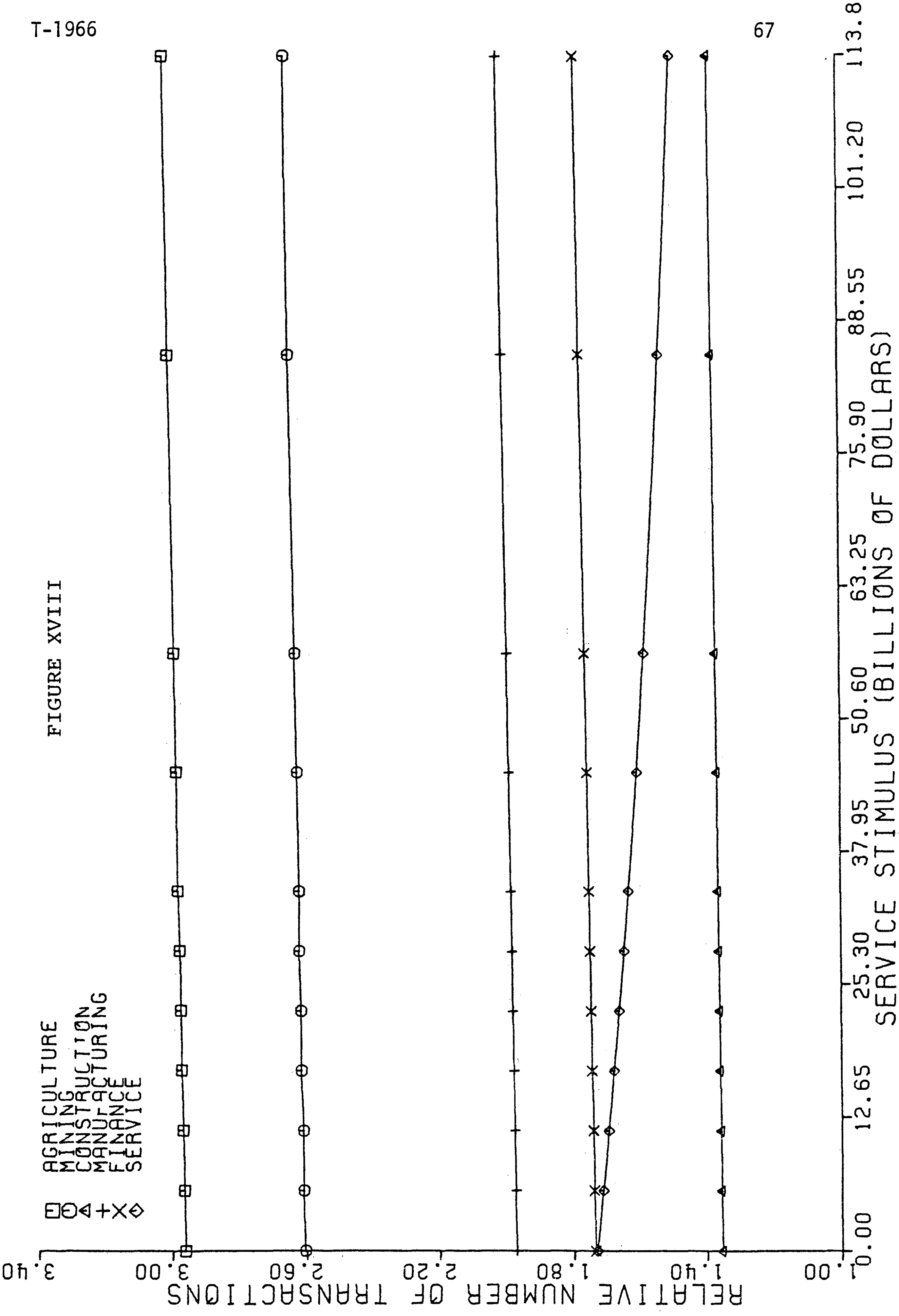
NUMBER OF TRANSACTIONS VS FINANCIAL STIMULATION
EFFECTS OF FINANCIAL STIMULUS ON EACH SECTOR

FIGURE XVII



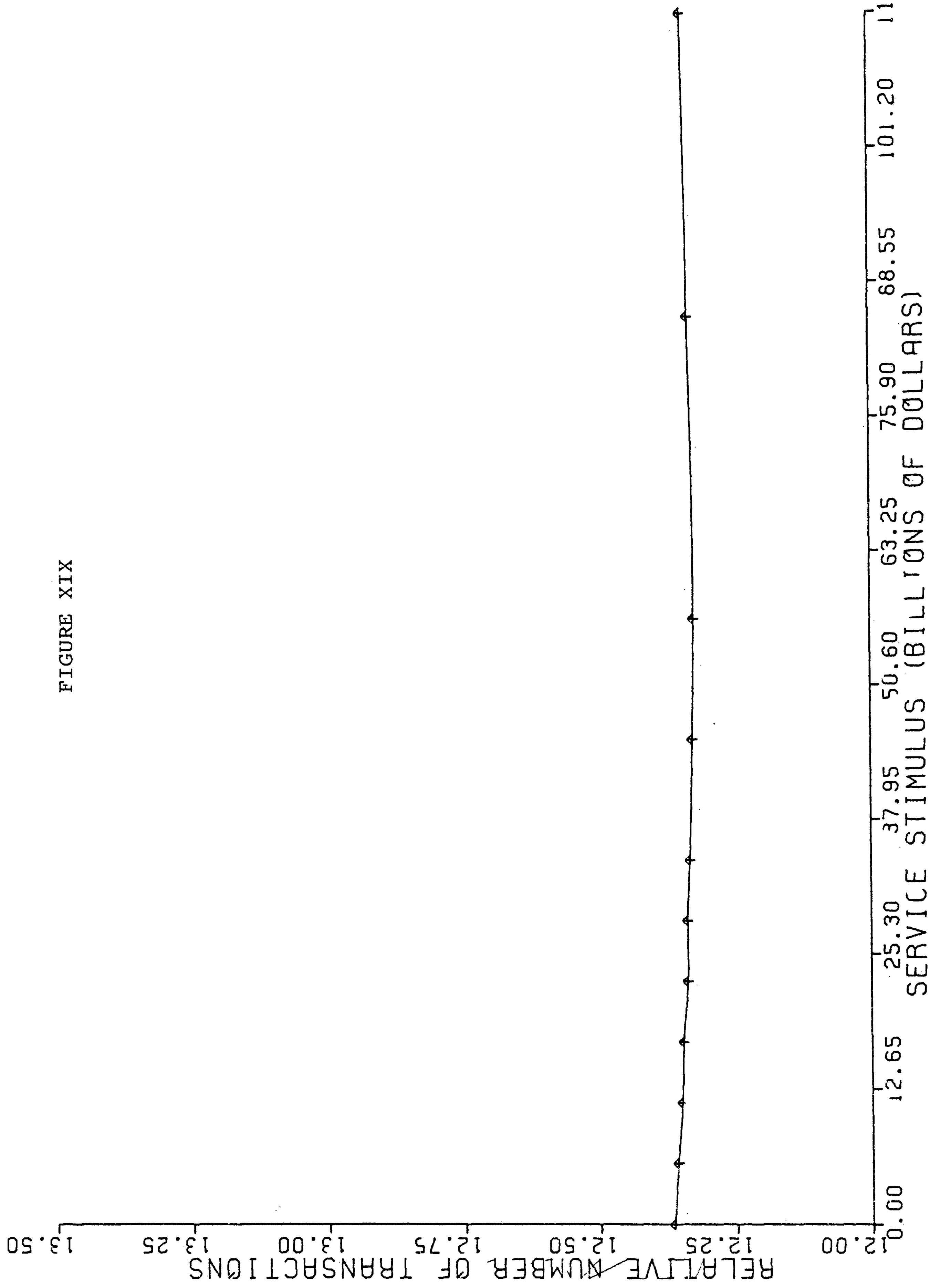
NUMBER OF TRANSACTIONS VS FINANCIAL STIMULATION
ACCUMULATED EFFECT OF FINANCIAL STIMULUS

FIGURE XVIII



NUMBER OF TRANSACTIONS VS SERVICE STIMULATION
EFFECTS OF SERVICE STIMULUS ON EACH SECTOR

FIGURE XIX



NUMBER OF TRANSACTIONS VS SERVICE STIMULATION
ACCUMULATED EFFECT OF SERVICE STIMULUS

There are two other generalities that can be observed in the graphs. The first is that the number of transactions in the stimulated sector steadily decreases. The reason for this decrease is that the probability of consumption has increased while the inter-sector transition probabilities have decreased. Recall the new transition probabilities are given by

$$p_{ij}' = a_{ij}\left(\frac{X_j'}{X_i'}\right)$$

$$= p_{ij}\left(\frac{X_i}{X_j}\right)\left(\frac{X_j'}{X_i'}\right)$$

Assuming sector k is the stimulated sector, the new transition probabilities for sector k are

$$p_{kj}' = p_{kj}\left(\frac{X_k}{X_j}\right)\left(\frac{X_j'}{X_k'}\right)$$

Any stimulus injected in sector k will cause

$$\frac{X_k}{X_j} < \frac{X_k'}{X_j'}$$

since we assumed in the I-O formulation that no sector had 0 consumption. This inequality implies that the inter-sector probabilities will always decrease in the stimulated sector. If the inter-sector probabilities decrease the probability of consumption must increase since the sum of all probabilities in any row (sector) must be unity.

The second generality concerns the change in the number of transactions for the non-stimulated sectors. In most cases there is a small but positive effect on the number of transactions. The degree of the effect can be analyzed by examining the above justification for the change in transition probabilities of the stimulated sector. (See appendix A)

The graphs presenting the accumulative effect show which factor dominates: the relative decrease in the transactions of the stimulated sector or the relative increase in transactions of the non-stimulated sectors.

It is of interest to examine which sectors Markov analysis indicates would be preferable to stimulate. Similar to I-O rankings, the sectors will be evaluated on the basis of transactions created per billion dollars of stimulus. Table IX shows this evaluation using linear approximations for the stimulus-transaction relationships.

TABLE IX

Relative Increase in Transactions per Billion Dollar Stimulus for the 6-Sector Model

Sector	Relative Volume of Transactions/$Billion Stimulus
Agr.	-.0228
Min.	-.0467
Cnst.	.0007
Man.	.0024
Fin.	-.0004
Ser.	-.0002

Using only Markov analysis, manufacturing would appear to be the most favorable sector to stimulate since each dollar of stimulation creates more opportunities for taxation and possibly a higher ROR. Table X compares the I-O preferences for stimulation with those of the Markov analysis.

TABLE X

Comparison of Sector Stimulation Preference for I-O and Markov Analysis

Ranking	I-O	Markov
1	Agr.	Man.
2	Man.	Cnst.
3	Cnst.	Ser.
4	Ser.	Fin.
5	Min.	Agr.
6	Fin.	Min.

I-O Markov Analysis and the 6-Sector Model

I-O Markov analysis combines the Markov analysis process with I-O analysis in order to forecast an expected ROR for economic stimulation. By combining the two methods, a ROR technique is developed that incorporates both the expected level of economic activity and the expected volume of transactions. The combination of I-O and Markov analysis is better adapted to analyze economies where transaction-dependent taxes are the main source of revenue. Since the U.S. Government receives the bulk of its revenues from income and exise taxes, this new technique is quite applicable to the U.S. economy.

The I-O Markov analysis technique allows the Government to evaluate economic proposals in the same manner as the private sector evaluates investment opportunities. Society would be better served if the Government has its own financial interest at heart when tampering with the economy. Rates of return based on I-O Markov analysis for different stimulus proposals establish a good means of comparing the relative expected costs and effects of the proposals.

It was shown through I-O analysis that the relationship between sector stimulus and level of economic activity is linear. The effect of stimulating a sector in I-O analysis is an increase in total output of all sectors of the economy. The results of the Markov analysis are not as consistant as the I-O results. Stimulus in some sectors (construction and manufacturing) created an increase in total transactions wereas stimulus in other sectors (agriculture, mining, finance, and service) decreased the volume of transactions. When I-O and Markov analysis are combined, the increase in the level of economic activity may be dominated by the magnitude of the decrease in transactions to produce an overall negative effect.

One way in which to gauge the total effect of economic stimulus is to use the I-O Markov technique to predict the change in tax revenues arising from the stimulus. Suppose during the base period for the 6-sector model, an arbitrary tax revenue of $420.72 billion was generated. This figure equates to 33% of the GNP of the base period. Assuming the $420.72 billion was generated from the base of 12.365 transactions index and a GNP of $1262.16 billions, a simple division reveals that approximately $.02695 was generated from each transaction-dollar in the economy. Thus, by multiplying the product of the level of economic activity ($) and the relative number of transactions by $.02695/transaction-dollar, a projection of total tax revenue can be obtained.

Figure XX through Figure XXV shows the relationships between sector stimulus and expected tax revenues based on I-O Markov analysis. The horizontal scale on the graphs represent a stimulus ranging from 0 to 100% of base period consumption for the respective sector.

It is interesting to note, in each sector except agriculture and mining, the stimulus spawns an increase in tax revenues. One explanation of these results is that agriculture and mining are primary industries that require relatively little input from the other sectors. Referring to the original transaction table confirms the small amount of reliance of agriculture and mining on the other sectors. Increasing the demand for agricultural and mining products does not create a large enough increase in the level of total economic activity to offset the decrease in total transactions.

To estimate the expected ROR associated with each dollar of stimulus for each particular sector, the slopes of straight-line approximation to the curves in Fugures XX through XXV are calculated. Where appropriate, the straight line approximations are shown as dashed lines on the Figures. These estimated rates of return are tabulated in table XI.

FIGURE XX

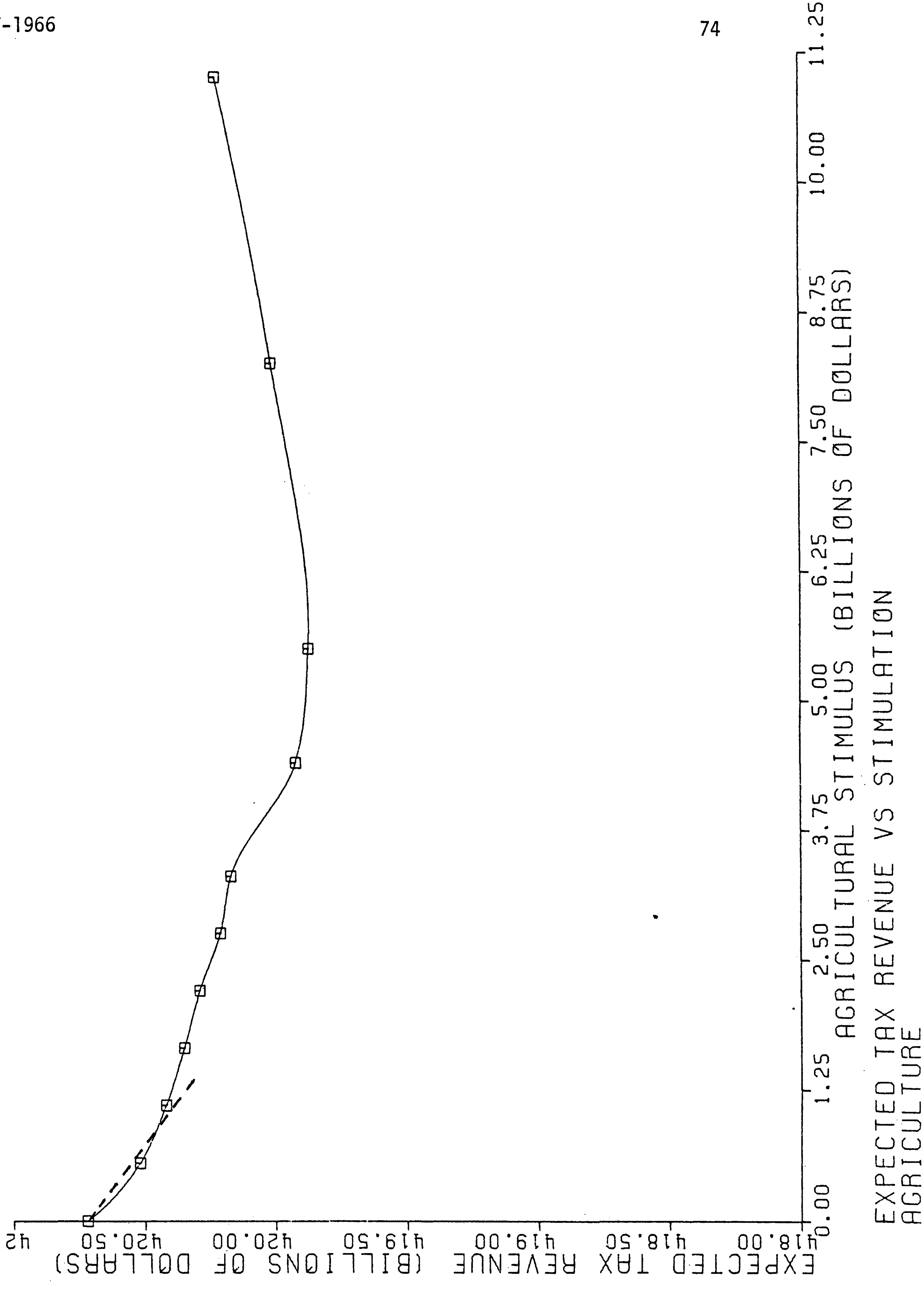
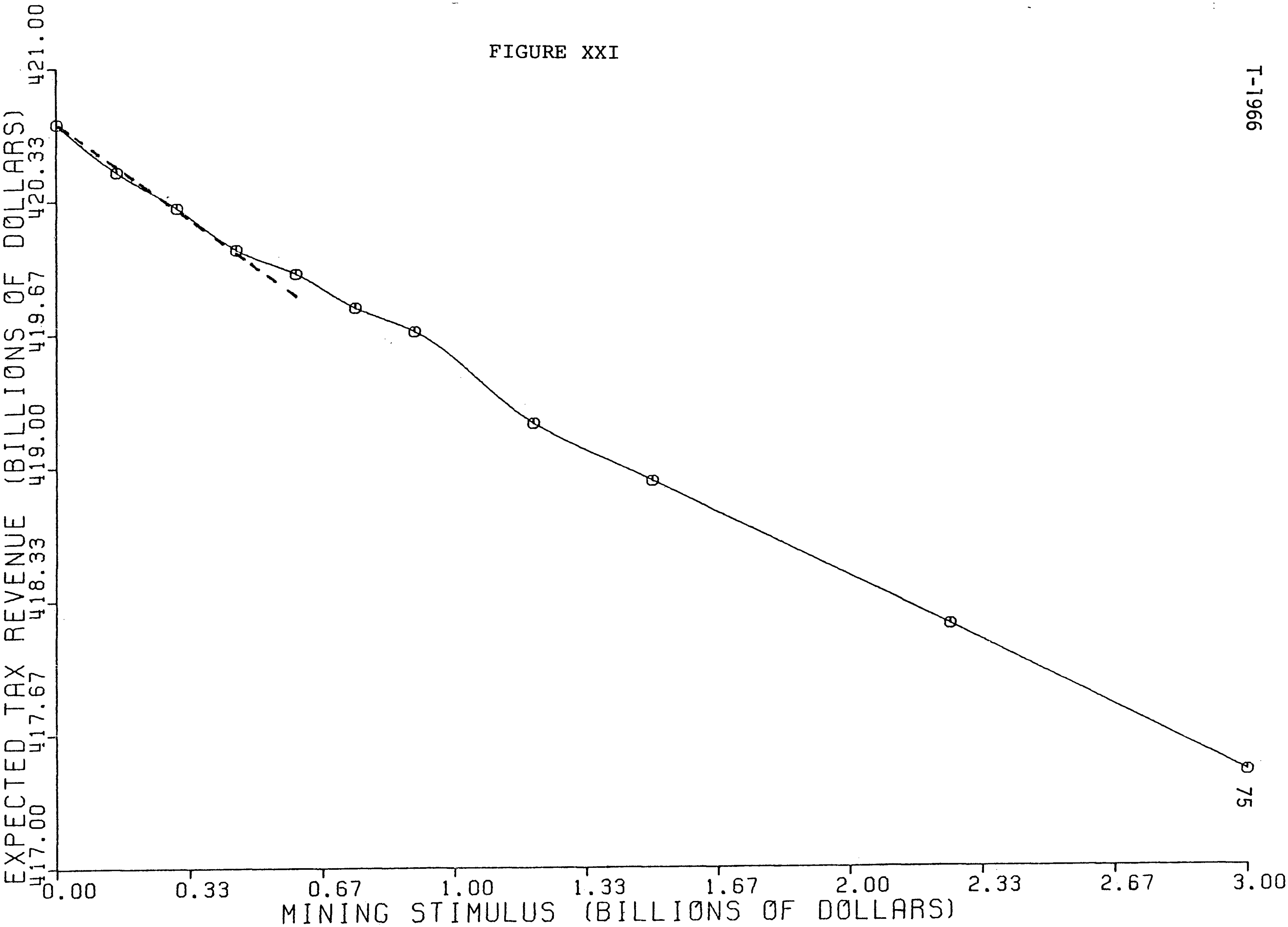

EXPECTED TAX REVENUE VS STIMULATION
AGRICULTURE
AGRICULTURAL STIMULUS (BILLIONS OF DOLLARS)
0.00
1.25
2.50
3.75
5.00
6.25
7.50
8.75
10.00
11.25
EXPECTED TAX REVENUE (BILLIONS OF DOLLARS)
418.00
418.50
419.00
419.50
420.00
420.50
421.00

FIGURE XXI



EXPECTED TAX REVENUE VS STIMULATION
MINING

FIGURE XXII

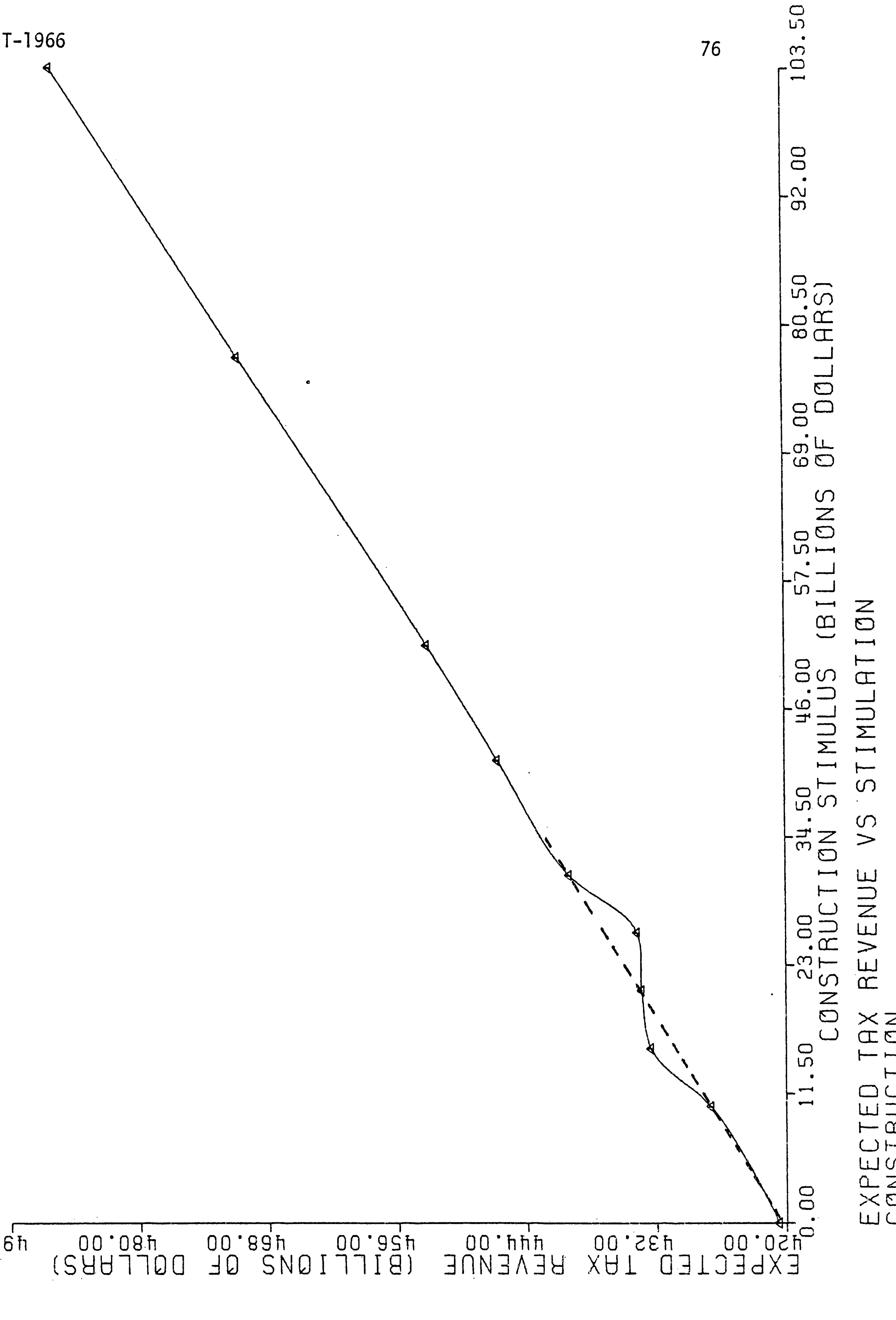

EXPECTED TAX REVENUE VS STIMULATION
CONSTRUCTION
CONSTRUCTION STIMULUS (BILLIONS OF DOLLARS)
0.00
11.50
23.00
34.50
46.00
57.50
69.00
80.50
92.00
103.50
EXPECTED TAX REVENUE (BILLIONS OF DOLLARS)
420.00
432.00
444.00
456.00
468.00
480.00
492.00

FIGURE XXIII

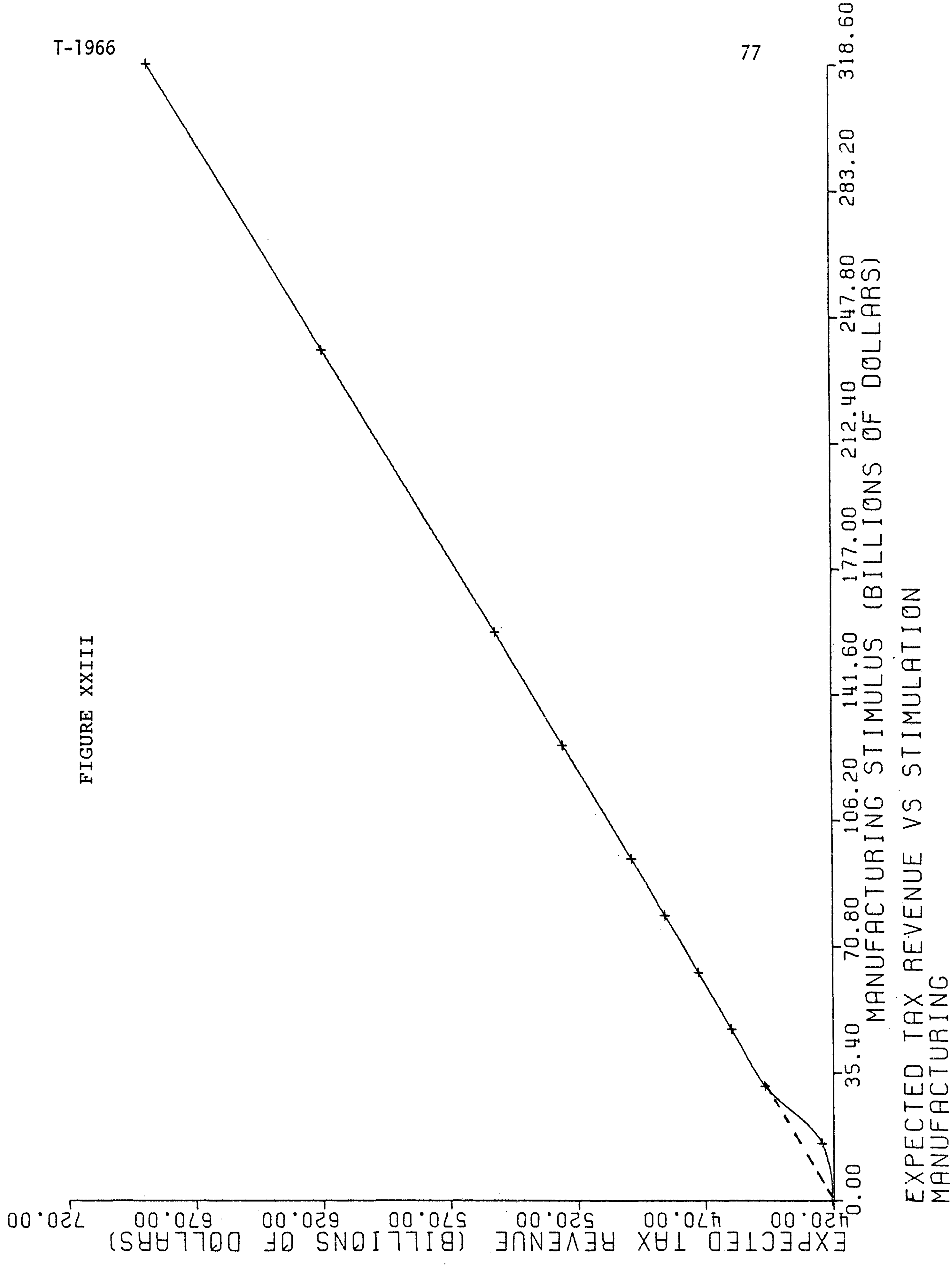
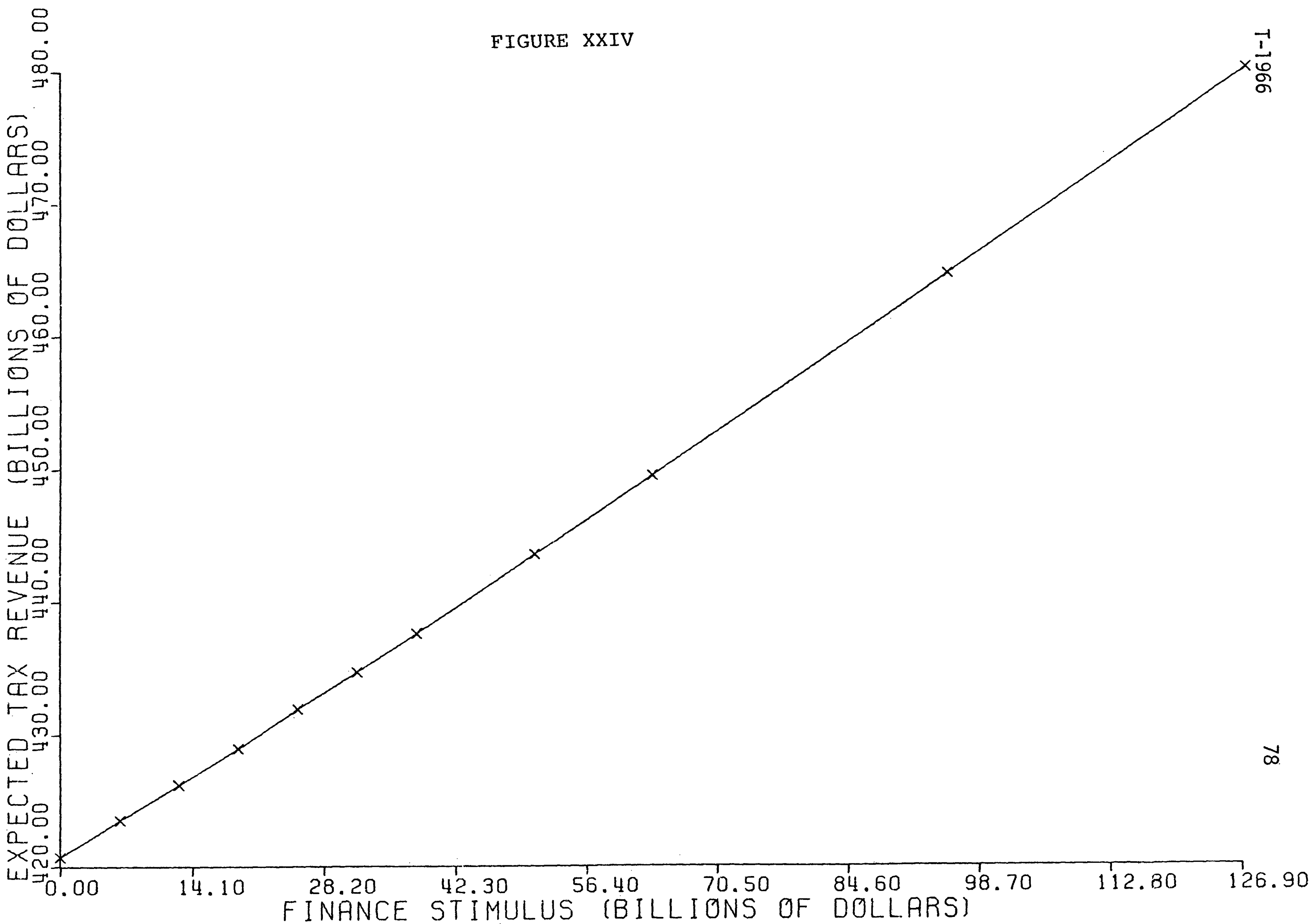
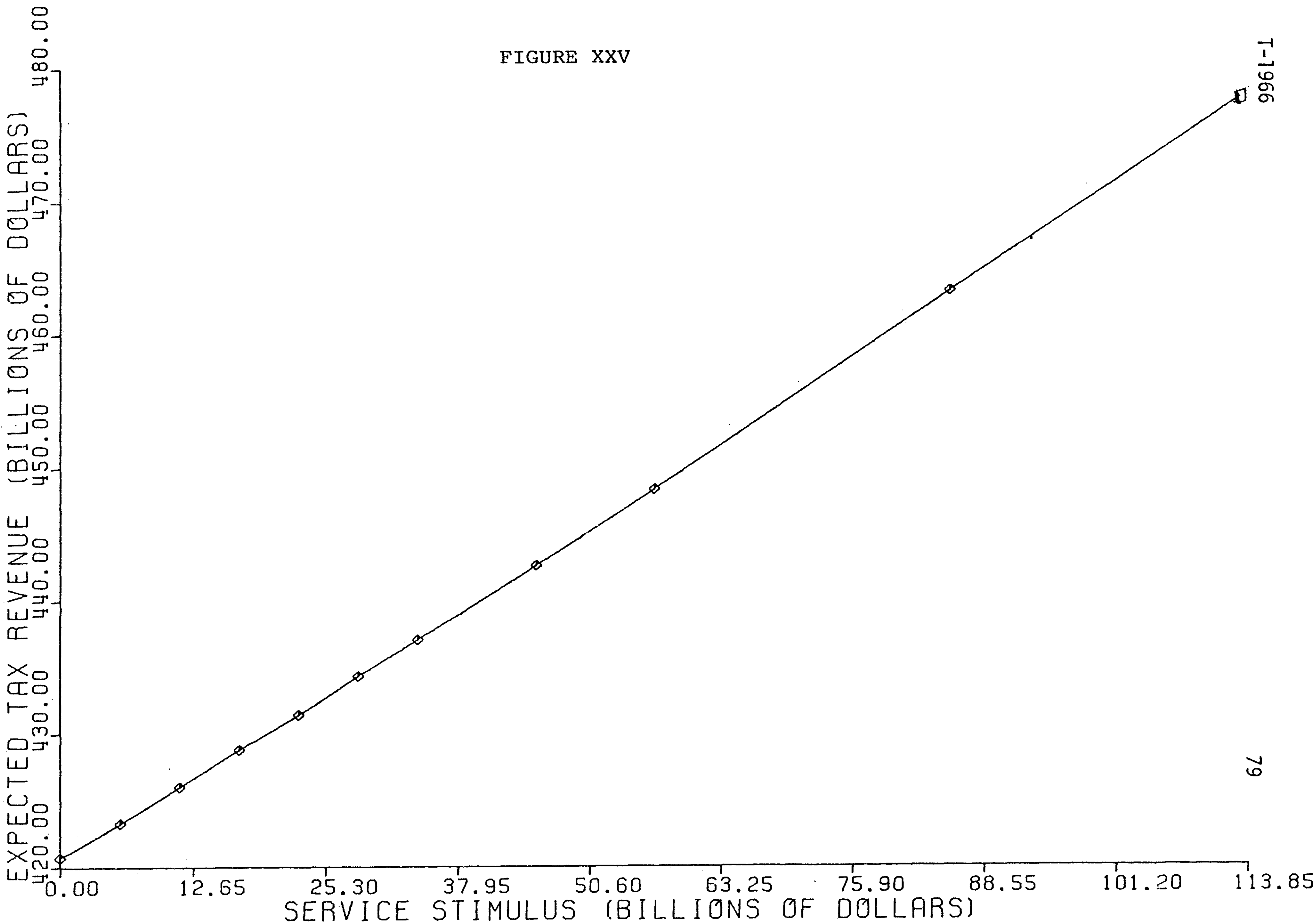


FIGURE XXIV



EXPECTED TAX REVENUE VS STIMULATION
FINANCE

FIGURE XXV



EXPECTED TAX REVENUE VS STIMULATION
SERVICE

TABLE

Sectors and Their Expected ROR per Dollar of Stimulus

Sector	ROR/$ Stimulus
Agr.	-18%
Min.	-119%
Cnst.	64%
Man.	84%
Fin.	47%
Ser.	50%

From Table XI it is seen that a dollars worth of stimulus in the mining sector will cost an additional $1.19 in lost tax revenues, and $ .84 on every dollar of manufacturing stimulus will be recovered through taxation.

Emphasis should be placed on the fact that the transaction volume was derived as a relative figure for the primary purpose of comparison. Thus, the rates of return in Table XI should be taken more for comparison purposes rather than absolute figures.

Implications of I-O Markov Analysis on Inflation and Unemployment

The problems of inflation and unemployment are two of the more serious problems facing the U.S. economy today. Keynes described inflation as the most subtle and sure means of overturning the existing basis of a society [7]. This description of inflation has been supported recently in work done by Warsh and Minard on the causes and effects of inflation [22].

Investigating the causes and effects of inflation with the aid of the I-O markov technique proves to be very interesting. Recall the rates of return generated per dollar of stimulus in the previous section. If these rates are absolute or indicative of the actual rates of return, Government needs inflation to finance its operations. Inflation has the effect of increasing the rates of return. Adequate inflation would cause some rates to become greater than 100% and lead to self-supporting stimulus proposals. Stimulating the sectors of the economy that have the higher rates of return would lessen the magnitude of the required inflation.

Another way of reducing required inflation would be to increase tax levies. Economist Colin Clark argued that taxation in excess of 25% of national income would only serve to fuel disasterous inflation [3]. He may be right in thinking there is a natural limit to effective taxation, but his 25% figure seems arbitrary. Increasing the tax levies initiates a trade-off between decreased aggregate consumption and increased Government revenues through the higher tax. I-O Markov analysis is able to quantify the trade-off process by translating consumption decreases into the appropriate changes in level of economic activity and volume of transactions.

Limits on production and manufacturing capacities are not incorporated into the I-O Markov method, although these limitations can be assimilated into the I-O analysis. It is interesting to note that I-O Markov analysis implies that the economy need not be approaching capacity limits in order to experience high rates of inflation. Government investments in sectors with low rates of return will foster the required inflation rate to "finance" the investment.

Entangled in the problem of inflation is the problem of unemployment.

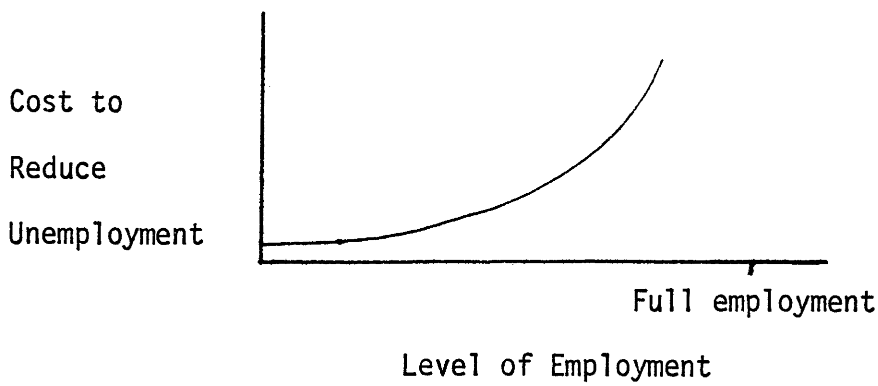
Keynes perceived the prime goal of any economy is to achieve full employment of all factors of production [14]. Due to the inelasticity of wages, a laissez-faire economy does not guarantee full employment. The U.S. Government has intervened in the unemployment problem and with the passage of the Employment Act of 1964 the Government has been effectively given the legal responsibility for creating a prosperous economy.

In Government created jobs used to combat unemployment, the worker should receive a wage that is equal to the marginal product of his labor. As seen in the 6-sector model, the I-O Markov estimates of rate of return in each sector was less than unity. Therefore, artificially creating jobs is a very costly practice. For a mining worker to receive a wage equal to the marginal product of his labor in a created job, the worker would be forced to repay the Government $2.19 ($1.00 for the cost of the stimulus plus $1.19 for the loss of revenues) for every dollar he makes. This example may be trying to say something about the marginal product of Government jobs in general.

A way to express the exponentially increasing cost of reducing unemployment is the Phillips Curve. The general appearance of a Phillips Curve is shown in Figure XXVI. The concept of the curve is that it becomes increasingly more expensive to reduce unemployment as the level of unemployment decreases.

FIGURE XXVI

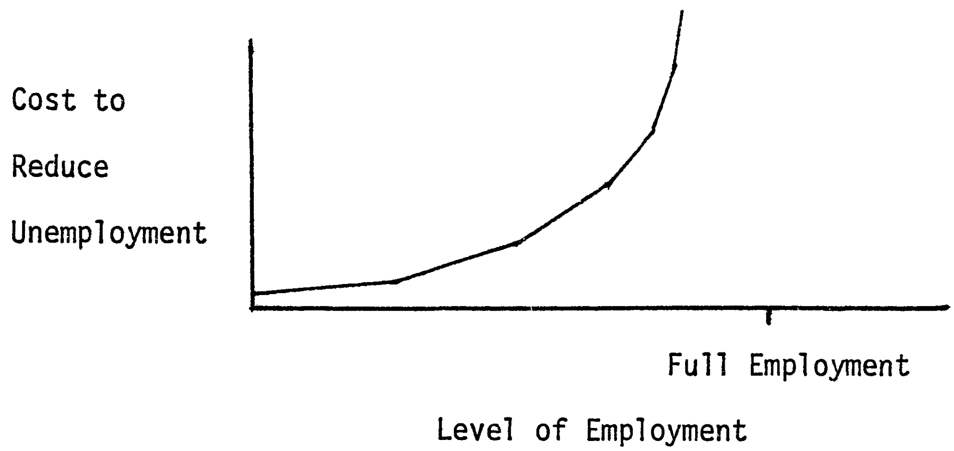
Phillips Curve



I-O Markov analysis conforms to the Phillips Curve concept. The only difference is that the I-O Markov curve is piecewise linear rather than a smooth curve. From the estimates of rate of return in Table XI, the relative cost of creating a job can be determined. For example, a dollar paid to create a job in the manufacturing sector will return $ .84 in revenue. Thus, the cost of the created job will be 16% of every dollar allocated. Similarly, the cost of a created job in the mining sector is 219% of every dollar allocated. Assuming the lower cost jobs will be created and filled first, and there are constraints on the number of jobs that can be created in a sector, a Phillips-type curve can be constructed. Figure XXVII represents the piecewise linear approximation of the Phillips Curve.

Figure XXVII

Piecewise Linear Phillips Curve



(I-O Markov Analysis)

Both inflation and unemployment are serious problems that seem to feed on one another. I-O Markov analysis shows that when Government tries to rectify one problem, it may be adding fuel to the other problem and in turn adversely effecting the original problem.

Suppose Government is trying to fight high unemployment rates. To lower unemployment, Government-sponsored jobs are created in a sector of the economy. Due to the very high cost of creating jobs in certain sectors, required inflation is initiated to finance the created jobs. Now unemployment may begin to fall, but inflation must persist since it is one of the primary means of finance.

To fight inflation, Government may choose to raise taxes or reduce spending. Reducing spending is generally considered poor public relations and may require a cut in the jobs created to fight high unemployment. So, a raise in taxes is initiated. The tax raise has the effect of lowering consumption and may reduce the relative number of transactions in the economy. The reduction in total transactions could signify lower private sector production and an associated reduction in employment. The problem now is to reduce unemployment again. However, this time around the cost of created jobs and higher inflation from previous remedies may still be present. The cycle continues, but each time the problems are more severe and more difficult to solve.

According to I-O Markov analysis, the only recourse the Government has is to invest (stimulate) only in sectors with adequate rates of return. In this manner, required inflation could be minimized and many of the interdependencies of the inflation-unemployment problem eliminated.

Conclusion

The purpose of this thesis is to present the formulation of a method to analyze the effects and rate of return of governmental stimulus in an economy. The major advantage of the method (I-O Markov analysis) is that it is based on both the level of economic activity (GNP) of the economy and the relative number of transactions occurring in the economy.

Input-Output (I-O) analysis is used to project the changes in the level of economic activity while Markov analysis projects the effects on the relative number of transactions caused by economic stimulus. The relative number of transactions is important since it represents the number of opportunities (taxable situations) a government has to recoup the cost of stimulation. Markov analysis lends itself nicely to projecting the number of transactions.

The I-O Markov analysis technique can be used extensively in comparing proposed economic modifications. I-O Markov analysis provides rates of return and projects the effects on both economic activity and transactions for each stimulus proposal. This information enables a government to invest more wisely in its economy.

In developing the method, many interesting aspects come to light. First, the scope of rates of return vary widely with respect to the sector that has been stimulated. Stimulation in certain sectors may even create a negative rate of return. Negative rates of return occur when the reduction in transactions predominates over the stimulus-induced increase in level of economic activity. I-O Markov analysis can determine which sector is the more advantageous to stimulate.

Another aspect of I-O Markov analysis is that it may explain relationship between rates of inflation and unemployment. Analysis shows the high cost of created jobs may initiate a required rate of inflation to finance the jobs. If an unwise stimulation policy is practiced, a decrease in the number of transactions may result and employment in the private sector may be adversely effected. The decrease in employment in the private sector may encourage more governmental action which could intensify the seriousness of the problem. One way to prevent the inflation-unemployment spiral is for the government to stimulate only those sectors with high rates of return and positive effects on the volume of transactions. In other words, a government must invest wisely in its economy.

Through I-O Markov analysis a law of diminishing returns is established for governmental stimulus. The law states that as the amount of stimulation is increased, the stimulation has a lesser effect on the change in the relative number of transactions. Thus, the economy can reach a saturation point where further stimulus will only increase the value of economic activity and not the volume of activity. This situation could instigate cost push inflation [17].

The law of diminishing returns implies there may be an absolute limit to the response of an economy to governmental stimulation I-O Markov analysis is able to define and quantify these limits. One encouraging characteristic concerning the limits is that they can be altered by new coefficients in the technical coefficient matrix of the I-O analysis. Technological advances may be able to raise the limits in those areas where the limits are inhibiting the economies ability to respond to stimulus.

Establishment of I-O Markov analysis reveals that there are many applications of the technique. One of the first areas that should be explored is the validity of I-O Markov analysis. To investigate the validity of I-O Markov analysis, a larger model, such as the 85-sector Bureau of Economic Analysis model, should be used. A larger model would be more representative of the economy and may identify inter-sector relationships that are extremely sensitive to stimulation.

In conjunction with a larger model, I-O Markov analysis should be applied to data acquired from different years. For the U.S., data from the 1960's could be analyzed to see if the results of legislation and stimulus of that time could have been adequately forecasted with I-O Markov analysis.

Assuming the technique is valid, it would be extremely interesting to try to establish where the U.S. economy is in relation to the law of diminishing returns and the natural limits on transactions. One of the major problems foreseen in this determination is the definition of the base year. Since the law of diminishing returns and transaction limits are expressed relative to a base year, proper identification of the base year is very critical.

I-O Markov analysis could also be used to locate those areas of the economy where technological advances would prove most beneficial to the economy. Technology has the power to change the entries in the technical coefficient matrix. The elements of the technical coefficient matrix directly influence the limits on inter-sector transactions. Thus, encouraging efforts to advance technology in certain segments of the economy, may be beneficial to the entire economy.

In summary, I-O Markov analysis provides a new dimension to economic analysis. It provides a much clearer picture of the total effects of governmental stimulus and may help in defining the optimal size of government.

Appendix A

The purpose of this appendix is to analytically investigate the relationship between the amount of stimulus injected in a sector of the economy and the corresponding effect on the relative number of transactions in the economy. It will be shown that the expected number of inter-sector transactions can be written in terms of the base number of inter-sector transactions and the amount of stimulus to be applied. Of extreme importance is the fact that it can be shown that the relative number of transactions is upper bounded and a law of diminishing returns exists between added stimulus and transactions.

The notation used in this appendix is compatible with that developed in Chapters I - III. In addition, bracket notation, $\{e_{ij}\}$, is used to denote an n x n matrix with elements e_{ij}. Unless specifically stated, all matrices are n x n where n is the number of sectors in the model being discussed. A quick review of the dimensions and nomenclature previously developed is as follows:

Total output vector, (n x i)

$X = \{X_i\}$

Consumption vector, (n x 1)

$Y = \{Y_i\}$

Identity matrix, (n x n)

$$I = \begin{bmatrix} 1 & 0 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & & & & . \\ 0 & 0 & 1 & . & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ . & & & & & & . \\ 0 & 0 & 0 & . & . & . & 1 \end{bmatrix}$$

Technical coefficient matrix, (n x n)

$A = \{a_{ij}\} = \{\frac{x_{ij}}{X_j}\}$

Transition matrix, (n x n)

$Q = \{p_{ij}\} = \{\frac{x_{ij}}{X_i}\}$

Fundamental matrix or expected number of transactions, (n x n)

$N = \{n_{ij}\}$

To denote new values of an element, prime notation will be used. For example, $n'_{ij} = f(n_{ij})$ denotes the new expected number of transactions (n'_{ij}), after the stimulus has been applied to the economy, can be represented as a function of the original or base number of transactions (n_{ij}).

First to be investigated are the effects on X caused by a change in an element of Y.

Recall

$X = (I - A)^{-1}Y.$

By defining

$\{\ell_{ij}\} = (I - A)^{-1}$

the effects on X_i of stimulating sector Y_k can be directly evaluated,

$$X'_i = Y_1\ell_{i1} + Y_2\ell_{i2} + \ldots + Y_k\ell_{ik} + \ldots + Y_n\ell_{in} \qquad (1)$$

The first derivative will show how the elements of X will change with respect to stimulus in sector k. Taking the derivative of (1)

$$\frac{dX'_i}{dY_k} = \ell_{ik}$$

Thus, there is a linear relationship between X'_i and stimulus provided through sector k. This conclusion coincides with the conclusion in Chapter III and Figures II through VII. The effects of multisector stimulus would simply be a composite of the individual linear effects.

Knowing the relationship between Y' and X' it is important to investigate the effects of X' on N. N is the fundamental matrix developed in Chapter II whose elements, n_{ij}, denote the expected number of transactions a dollar beginning in sector i will have with sector j. From Chapter II

$$N = \{n_{ij}\}$$
$$= (I - Q)^{-1}$$

The inverse of a matrix can be calculated by using cofactors. (See Lightstone's Fundamentals of Linear Algebra [10]). The cofactor of a square matrix B, cof ij $\{b_{ij}\}$, is the determinant of the matrix obtained by removing the ith row and jth column of B. The sign of the cofactor is positive (negative) if 1 + j is even (odd). Writing N in terms of cofactor yields

$$\{n_{ij}\} = \{cof_{ji}\ (I - Q)/\ |(I - Q)|\}$$

where

$|(I-Q)|$ denotes the determinant of the (I-Q) matrix. Each element of N is

$$n_{ij} = -1^{(i+j)}(cof_{ji}(I-Q)/|(I-Q)|$$

$$n_{ij} = \frac{-1^{(i+j)}}{|(I-Q)|} \begin{bmatrix} 1-p_{11} & -p_{12} & \cdots & -p_{1,i-1} & -p_{1,i+1} & \cdots & -p_{1n} \\ -p_{21} & -p_{22} & \cdots & -p_{2,i-1} & -p_{2,i+1} & \cdots & -p_{2n} \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ -p_{j-1,1} & -p_{j-1,2} & & & & & -p_{j-1\ n} \\ -p_{j+1,1} & & & & & & -p_{j+1\ n} \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ -p_{n,1} & & & & & & -p_{nn} \end{bmatrix}$$

Examining each p_{ij}, the relationship between p_{ij} and p_{ij}' can be determined.

Recall,

$$p_{ij} = \frac{x_{ij}}{X_i} \quad \text{and}$$

$$p'_{ij} = \frac{x'_{ij}}{X'_i} \tag{2}$$

But

$$x'_{ij} = a_{ij}\, X'_j$$

$$= \left(\frac{x_{ij}}{X_j}\right) \cdot X'_j \tag{3}$$

Combining (2) and (3) yields

$$p'_{ij} = \left(\frac{x_{ij}}{X_j} \cdot X'_j\right) / X'_i$$

$$= x_{ij}\left(\frac{X'_j}{X_j}\right) \;/\; X'_i \qquad (4)$$

Multiplying (4) by $\frac{X_i}{X_i}$ gives the relationship between p_{ij} and p'_{ij}

$$p'_{ij} = \left(x_{ij}\left(\frac{X'_j}{X_j}\right) \;/\; X'_i\right)\left(\frac{X_i}{X_i}\right)$$

$$= \left(\frac{x_{ij}}{X_i}\right)\left(\frac{X_j'}{X_j}\right)\left(\frac{X_i}{X_i'}\right)$$

$$= p_{ij}\left(\frac{X'_j}{X_j}\right)\left(\frac{X_i}{X_i'}\right) \qquad (5)$$

Thus, N' may be written in terms of p_{ij}

$$N' = (I - \{p'_{ij}\})^{-1}$$

$$= \left(I - \left\{p_{ij}\left(\frac{X'_j}{X_j}\right)\left(\frac{X_i}{X'_i}\right)\right\}\right)^{-1}$$

and each element can be written in cofactor form

$$
n_{ij}' = \frac{-1^{(i+j)}}{|I-\{p_{ij}'\}|}
\begin{bmatrix}
(1-p_{11}) & \cdots & -\frac{X_1}{X'_1}\frac{X_{i'-1}}{x_{i-1}}\,p_{1,i-1} & -\frac{X_1}{X_1'}\frac{X_{i'+1}}{X_{i+1}}\,p_{1,i+1} & \cdots & \frac{X_1}{X'_1}\frac{X'n}{X_n}\,p_{1n} \\
-\frac{X_2}{X'_2}\frac{X_{i'}}{X_1}\,p_{21} & & & & & \vdots \\
\vdots & & & & & \\
-\frac{x_{j-1}}{x'_{j-1}}\frac{X'_1}{X_1}\,p_{i-1,1} & & & & & \frac{x_{j-1}}{x'_{j-1}}\frac{x_n}{X_n}\,p_{j-1n} \\
-\frac{x_{j+1}}{x'_{j+1}}\frac{X'_i}{X_1}\,p_{i+1,1} & & & & & \frac{x_{j+1}}{x'_{j+1}}\frac{X'_n}{X_n}\,p_{j+1n} \\
\vdots & & & & & \\
-\frac{X_n}{X'_n}\frac{X_{i'}}{X_i}\,p_{n,1} & \cdots & -\frac{X_n}{X'_n}\frac{X_{i'-1}}{X_{i-1}}\,p_{ni-1} & -\frac{X_n}{X'_n}\frac{X'_{i+1}}{X_{i+1}}\,p_{n\,i+1} & \cdots & p_{nn}
\end{bmatrix}
$$

Multiplying each row (column) of a matrix by a constant, γ, has the effect of multiplying the determinant of the matrix by γ[10]. Hence

$$|(I-\{p'_{ij}\})| = \left(\frac{X'_1}{X_1}\right)\left(\frac{X'_2}{X_2}\right)\cdots\left(\frac{X'_n}{X_n}\right)\left(\frac{X_1}{X'_1}\right)\left(\frac{X_2}{X'_2}\right)\cdots\left(\frac{X_n}{X'_n}\right)|(I-\{p_{ij}\})|$$

$$= |(I-\{p_{ij}\}|$$

$$= |(I-Q)| \qquad (6)$$

and similarly

$$n'_{ij} = \left(\frac{X'_1}{X_1}\right)\cdot \;\cdots\; \cdot\left(\frac{X'_{i-1}}{X_{i-1}}\right)\left(\frac{X'_{i+1}}{X_{i+1}}\right)\cdot \;\cdots\; \cdot\left(\frac{X}{X_n}\right)\left(\frac{X'_1}{X_1}\right)\cdot \;\cdots\; \cdot\left(\frac{X_{j-1}}{X'_{j-1}}\right)\left(\frac{X_{j+1}}{X'_{j+1}}\right)\cdot\cdots$$

$$\cdot\left(\frac{X_n}{X'_n}\right)\text{cof}_{ji}\ (I-Q)/|(I-\{p'_{ij}\})|$$

$$= \left(\frac{X_i}{X'_i}\right)\left(\frac{X'_j}{X_j}\right)\text{cof}_{ji}\ (I-Q)/|(I-\ p'_{ij}\})| \qquad (7)$$

Combining (6) and (7)

$$n'_{ij} = \left(\frac{X_i}{X'_i}\right)\left(\frac{X'_j}{X_j}\right) n_{ij} \qquad (8)$$

Thus, N' can be written in terms of X, X', and N.

$$N' = \begin{bmatrix} \frac{X_i}{X'_i}\ \frac{X'_1}{X_1}\ n_{11} & \cdots & \frac{X_1}{X_{1'}}\ \frac{X_{j'}}{X_j}\ n_{ij} & \cdots & \frac{X_1}{X_{1'}}\ \frac{X'_n}{X_n}\ n_{1n} \\ \vdots & & & & \\ \frac{X_n}{X'_n}\ \frac{X_{1'}}{X_1}\ n_{n1} & \cdots & \frac{X_n}{X'_n}\ \frac{X_{j'}}{X_j}\ n_{nj} & \cdots & \frac{X_n}{X'_n}\ \frac{X_{n'}}{X_n}\ n_{nn} \end{bmatrix}$$

Combining the effect of Y' and X' and X' and N' makes it possible to analyze the direct effects of Y' and N'.

. For convenience, let

$$c_{ij} = n_{ij}\left(\frac{X_i}{X_j}\right)$$

so (8) can be condensed

$$n'_{ij} = c_{ij}\left(\frac{X'_j}{X'_i}\right) \qquad (9)$$

From (8) and (9) it is seen that n_{ij}, is a function of the ratio $\frac{X'_j}{X'_i}$.

Writing $\frac{X'_j}{X'_i}$ in terms of (1)

$$\frac{X'_j}{X'_i} = \frac{Y_1\ell_{j1} + \ldots + Y_k\ell_{jk} + \ldots + Y_n\ell_{jn}}{Y_{1\ell i1} + \ldots + Y_{k\ell ik} + \ldots + Y_{n\ell in}} \qquad (10)$$

where k is the stimulated sector. Since y_k is the only variable in (10), it is convenient to define

$$d_i = Y_1\ell_{i1} + \ldots + Y_{k-1}\ell_{ik} + Y_{k+1}\ell i_{k+1} + \ldots + Y_n\ell_{in}$$

So (10) may be rewritten

$$\frac{X'_j}{X'_i} = \frac{Y_k\ell_{jk} + d_j}{Y_k\ell_{ik} + d_i} \qquad (11)$$

Combining (9) and (11) yields

$$n'_{ij} = \frac{Y_k\ell_{jk} + d_j}{Y_k\ell_{ik} + d_i}\, c_{ij} \qquad (12)$$

by taking the derivative of (12) the effect of the change in Y_k can be related to n_{ij}'.

$$\frac{dn'_{ij}}{d\,Y_k} \quad \frac{c_{ij}(\ell_{jk}\,d_i - \ell_{ik}\,d_j)}{(Y_k\ell_{ik} + d_i)^2} \qquad (13)$$

Since $\dfrac{c_{ij}}{(Y_k\ell_{ik} + d_i)^2} > 0$, the sign of (13) is dependent upon the value of $(\ell_{jk}\,d_i - \ell_{ik}\,d_j)$. For n'_{ij} to decrease with respect to stimulus in sector k,

$$\frac{\ell_{ik}}{d_i} > \frac{\ell_{jk}}{d_j}$$

In the same manner,

$$\frac{\ell_{ik}}{d_i} < \frac{\ell_{jk}}{d_j} \quad \text{and} \quad \frac{\ell_{ik}}{d_i} = \frac{\ell_{jk}}{d_j}$$

indicate whether n'_{ij} will increase or remain constant with respect to the stimulus, Y_k.

It is interesting to note that the second derivative,

$$\frac{d^2n_{ij}}{d\,Y^2_k} = \frac{-2c_{ij}(\ell_{jk}di - \ell_{ik}dj)\ell_{ik}}{(Y_k\ell_{ik} + di)^3} \qquad (14)$$

places a comparable importance on the relationship between $\frac{\ell_{ik}}{di}$ and $\frac{\ell_{jk}}{dj}$. Examining (13) and (14) reveals that the second derivative will always have the opposite sign of the first derivative. Thus, an increasing n'_{ij} or decreasing n'_{ij} will tend to level off with additional section-k stimulus.

The leveling-off effect is more readily studied by observing the limits of (9) as Y_k is increased without bound.

$$\lim_{Y_k \to \infty} n'_{ij} = \lim_{Y_k \to \infty} \frac{Y_k \ell_{jk} + d_j}{Y_k \ell_{ik} + d_i} c_{ij}$$

$$= c_{ij} \frac{\ell_{jk}}{\ell_{ik}} \qquad (15)$$

From the definition of c_{ij}, (15) becomes

$$\lim_{Y_k \to \infty} n'_{ij} = n_{ij} \frac{X_i}{X_j} \frac{\ell_{jk}}{\ell_{ik}} \qquad (16)$$

A very important concept is established in (16). The concept is that there is a limit to the number of transactions that stimulus can induce into the economy. Further, examination of (13) and (14) reveals that a law of diminishing returns is present with respect to increased stimulus.

With the establishment of the law of diminishing returns, a government is faced with the problem of determining the degree that the law is currently in effect. The entire I-O Markov analysis compares results relative to a base period. This base period is most likely not the first year of government operations and is therefore already influenced by the law of diminishing returns.

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