

UNDERGROUND PRODUCTION SCHEDULING
OPTIMIZATION WITH VENTILATION
CONSTRAINTS

by
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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Mining and Earth Systems Engineering).

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ABSTRACT

Underground production scheduling has traditionally been a time-consuming, manual task in which the goal is to achieve corporate- or operation-defined production targets. This dissertation presents a generalized, mathematical formulation that results in a large-scale integer optimization model. The model maximizes discounted gold ounces mined and determines the optimal or near-optimal sequence of activities related to the development, extraction and backfilling of an underground mine. Constraints include physical precedence and resource capacities. The research uses data from an existing underground mine; however, the model formulation has the ability to serve other underground mines with similarly structured data and provides the ability to customize constraints. Additionally, the model includes a constraint that treats available mine ventilation as a consumable resource.

Diesel particulate matter, DPM, is a primary contaminant found in underground mining. Ventilation is used to dilute DPM below regulatory levels; however, mine ventilation is a limited resource, meaning that airflow through a mine cannot easily be increased once a ventilation system has been implemented.

Large integer optimization models are traditionally solved using the branch-and-bound algorithm; however, results show the benefit of using a solver called OMP, originally designed for open pit mining applications, which integrates a specialized linear programming algorithm and a heuristic to induce integrality. The author evaluates two solution methods, the CPLEX optimization package and the OMP Solver, and compares solution time. The OMP Solver schedules are compared to manual production schedules, with and without ventilation constraints. These comparisons show that with the OMP solver, it is possible to produce higher quality, implementable schedules in much less time relative to the manually generated schedules. Furthermore, the OMP solver outperforms CPLEX at this task.

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LIST OF ABBREVIATIONS

Backfill Tons per Day	FiT/day
Bienstock-Zuckerberg Algorithm	BZ
Cubic Feet per Minute	cfm
Diesel Particulate Matter	DPM
Integer Programming	IP
Linear Feet per Day	ln ft/day
Linear Programming	LP
Mine Safety and Health Administration	MSHA
Mined Tons per Day	miT/day
Mixed Integer Nonlinear Programming	MINLP
Mixed Integer Programming	MIP
Net Present Value	NPV
Nonlinear Programming	NLP
Open Mine Planning Solver	OMP Solver
Resource Constrained Project Scheduling Problem	RCPSP
Troy Ounces	t.oz.
Ventilation On Demand	VOD

ACKNOWLEDGMENTS

I would like to sincerely thank the many people who have generously supported me in this journey. This project would never have succeeded without the guidance and advice of my dedicated advisers, Dr. Alexandra Newman and Dr. Mark Kuchta. I would probably still be working on this problem if not for the help of Dr. Marcos Goycoolea, Dr. Daniel Espinoza, Dr. Eduardo Moreno, and Orlando Riveria, to whom I am forever indebted.

I would like to thank my committee, Dr. Hugh Miller, Dr. John Grubb, Dr. Jurgen Brune, and Dr. Panos Kusious. A special thanks to Dr. Priscilla Nelson, Dr. Kadri Dagdelen, and the entire mining engineering department at Colorado School of Mines.

I sincerely thank the Society of Mining, Metallurgy and Exploration, Inc., and the Seeley W. Mudd Memorial Fund for providing me with generous financial support during this endeavor.

This project would not have been conceived without the assistance and support of Monica Dodd, Xiaolin Wu, Bryan Evans, Ralf Kinsel, Henry Wang, Conor Meagher, Terry Bush, Chris Johnson, Chris Roos, Sue Liesveld, Veronica Tough, Kathryn Reed, Peter Haarala, Scott Rosenthal, Sven Leyffer, and Chris Alford. A special thanks to Deswik Mining Consultants, especially Russel Vance and Wayne Romer.

I would like to thank my colleagues who have helped me along the way specifically, Barry King, JD Baugher, Chotipong Somrit, Brian Lambert, Donal O'Sullivan, Chris Wyatt, and all my fellow graduate students. For helping keep me sane after months in the deprivation chamber, a.k.a. my office, I thank Shannon Mann, Bruce Yoshioka, Sara Schwarz, and Christine Monroe.

I would like to thank my mentor, Dr. Charles Kliche, who has been encouraging me for over 15 years to pursue a PhD. Finally, the person to whom I am most thankful, my husband, Chris.

To my parents and my husband.

CHAPTER 1

INTRODUCTION

Metal and mineral extraction using underground mining methods requires detailed engineering and planning to ensure the safe and economic operation of a mine. Mine planning is an iterative process and involves the evaluation of numerous options and scenarios [1]. Currently, most underground mine plans are developed using manual scheduling techniques, i.e., an engineer selects the sequence of activities that attempts to meet a desired production goal. These labor-intensive manual schedules tend to only satisfy a few constraints and may or may not be feasible in application. Mathematical modeling can incorporate a greater number of constraints while producing an optimal or near-optimal schedule in less time than a manual schedule.

This dissertation presents an underground production scheduling model, (\mathcal{Z}), which is a variation of a resource constrained project scheduling problem (RCPSP). The RCPSP consists of an objective function, resource constraints, and precedence constraints [2]. Using the RCPSP formulation as a basis, the author formulates (\mathcal{Z}) as an integer optimization model that schedules underground mining activities for a two-year time horizon. Model (\mathcal{Z}) expands upon the basic RCPSP formulation by incorporating features that provide an operationally implementable solution that better reflects the actual mining environment.

A novel approach to scheduling is evaluated with the introduction of a ventilation constraint into the production scheduling model. Additionally, the ventilation constraint is used to evaluate three estimation methods that are based on the required airflow needed to dilute diesel particulate matter below regulatory limits.

Two solution methods are used to solve (\mathcal{Z}). First, the branch-and-bound algorithm is evaluated using the commercially available software, CPLEX. The second solution method uses the academic research software, OMP Solver, which implements an unconventional

algorithm and heuristic. Both solution methods determine an integer solution for model (\mathcal{Z}).

1.1 Objective of the Research

On May 20, 2008, the United States Mine Safety and Health Administration (MSHA) implemented regulations reducing the allowable exposure concentration of diesel particulate matter (DPM) for underground miners. DPM is recognized by the World Health Organization as a carcinogen [3]. MSHA has recommended a number of control methods to help reduce DPM concentrations; however, some mines still struggle to remain compliant. The majority of MSHA's recommendations focus on reducing the amount of DPM that is being created or exhausted. These include purchasing low-emission engines, improving engine maintenance, using biofuels, and implementing after-treatments, such as passive regenerating ceramic filters. Other suggestions focus on reducing exposure through *(i)* increased airflows, *(ii)* improved barriers, e.g., environmentally controlled equipment cabs, and *(iii)* added administrative controls to limit a miner's exposure to DPM, e.g., reduced work hours in high-concentration areas or procedures to limit idling of equipment [4].

The primary objective of this research is to determine a mid-term production schedule for an underground mine that includes ventilation constraints. This dissertation focuses on the development and evaluation of the integer program, (\mathcal{Z}), and solution methods. Additionally, the model includes features that better reflect the operational environment at underground mines using stoping methods.

The following objectives are the focus of the dissertation:

- Develop an integer program to determine a mid-term production schedule for an underground mine.
- Formulate model to be easily applied to other underground mines.
- Incorporate ventilation constraints into the formulation.
- Determine methods of airflow estimation based on diesel particulate matter production for scheduled activities.

- Incorporate features into the model that reflect the operational environment.
- Investigate solution methods for large integer programs.
- Implement and evaluate solution methods, both commercial and academic software, and compare solution times and quality.
- Compare resulting schedules to the manual schedule from MineX.

1.2 Research Conducted and Methodology

Research for this dissertation consists of three distinct components; model formulation, solution methods, and inclusion of a ventilation component into the mine scheduling process.

The resource constrained project scheduling problem (RCPSP) is the foundation of this dissertation. The RCPSP consists of a linear objective, resource constraints, and predecessor constraints [2]. Using the RCPSP structure, this dissertation presents a large-scale integer programming production scheduling model, (\mathcal{Z}), formulated to maximize the gold ounces with an applied discount factor, assuming an operational mine with a given design. The formulation is applicable to any underground mine with similar data structures and provides the ability to be easily adjusted to accommodate unique resource constraints.

Constraints for resources, such as mined tons and linear feet developed, are capacity constraints with upper bounds. Ventilation is represented by a capacity constraint that treats available mine airflow as a consumed resource. A physical precedence constraint ensures the required sequencing for activities is not violated. The final constraints ensure that an activity occurs at most once and that the variable is binary.

Model (\mathcal{Z}) incorporates the ability to delay or overlap activities based on specified criteria. This provides added flexibility and functionality to the resulting schedules. Many activities require a delay, i.e., a certain number of time periods must pass before successor activities could begin. For example, when a stope is backfilled with cemented rock fill, the cement must set before adjacent stopes can be mined. A delay provides a transitional pause between activities during which resources are not consumed. This feature is more representative of how the schedule would be implemented in practice. Additionally, some activities can begin

once a preceding activity reaches a specified level of completion, i.e., some stopes require backfill prior to the completion of the stoping activity. For example, once a stope is 50 percent complete, backfilling can begin, creating two activities occurring simultaneously while still having a predecessor-successor relationship.

For model (\mathcal{Z}) , the following assumptions hold:

- The mine is currently or is very near operational, i.e., all systems are in place and mining can begin immediately.
- The location and dimensions of all stopes, access, and infrastructure activities are known.
- The tonnage, grade, and precedence are known for all activities.
- The mine systems, such as equipment, haulage systems and ventilation systems are known and operational.
- The mining method and production rates have been assigned to all activities.

Integer programs like (\mathcal{Z}) are difficult to solve and can become intractable if the model is large. The number of activities being scheduled by MineX results in a large model, i.e., taking hours to solve. The author evaluates two distinct solution methods to determine the best approach to solve (\mathcal{Z}) , both of which attempt to determine the globally optimal solution. The intelligent enumeration method, branch-and-bound, is evaluated; however, results show that this method requires an impractical amount of computation time for large integer models such as (\mathcal{Z}) .

This dissertation presents an unconventional and novel algorithm to solve (\mathcal{Z}) . The Bienstock-Zuckerberg algorithm (BZ) is a linear programming relaxation algorithm that has successfully been applied to open pit scheduling problems [5]. The BZ algorithm does not provide an integer feasible solution because the decision variables are treated as continuous. The resulting fractional solution is translated into an integer solution with the TopoSort heuristic [6].

A ventilation component is incorporated into model (\mathcal{Z}) as a resource constraint to evaluate the effects of ventilation on the production schedule. The reductions in the allowable

exposure of diesel particulates to those working in underground mines have forced mine engineers and management to become more focused on developing ways in which to reduce levels and exposure. Mining companies are actively implementing controls to help reduce the quantity of DPM underground. Current initiatives include:

- Incorporate more electric equipment
- Monitor equipment in working areas using the Mine Control System
- Increase DPM filter or catalytic converter usage
- Evaluate contractor equipment performance
- Reduce re-handle and haul distances
- Increase ventilation air quantities
- Incorporate ventilation requirements into scheduling functions

The work presented focuses on the last initiative by incorporating ventilation into (\mathcal{Z}) and evaluating three methods of airflow assignment. It is difficult to determine the exact airflow required to dilute DPM for two primary reasons: (*i*) the MSHA regulation limits DPM exposure to a time-weighted average of 160 micrograms per cubic meter for an individual miner and (*ii*) both miners and DPM-producing equipment continually move throughout the mine, creating difficulties in quantifying the exact exposure and DPM produced, respectively.

By incorporating the ventilation requirements and capacity of the ventilation system into the scheduling process, it is expected that the available ventilation resources will be better utilized, thereby resulting in increased production. In addition, the incorporation of ventilation constraints may offer possible safety and health benefits, and information obtained from the schedule could be used to justify and plan any necessary expansion or modification to the ventilation system.

The resulting schedule provides a strong basis for mid- and short-term planning of the operation and improves the scheduling process when compared to current practice. Another benefit is production schedule “leveling,” resulting in more effective resource utilization

through reasonably consistent production rates, i.e., avoiding peaks and dips often found in a manual production schedule.

The schedules obtained from model (\mathcal{Z}) are meant for use as a tool to assist in the short-term planning process, scenario analysis, and limitation prediction. The schedules obtained are not meant to replace an engineer's judgment and knowledge.

Research Hypothesis

Solutions from the RCPSP can be used as a mid-term production schedule for underground stoping mining operations. Additionally, the inclusion of ventilation constraints with estimated activity airflow quantities to dilute diesel particulate matter will alter the resulting production schedule. Accounting for activity duration delay and overlap in the formulation provides a more operationally implementable schedule. The precedence constrained underground mine scheduling problem can be solved using the Bienstock-Zuckerberg algorithm and the TopoSort heuristic. This dissertation will address the validity of this hypothesis.

1.3 Case Study

MineX, the case study mine for this dissertation, is a large-scale underground gold mining operation located in the United States. Annual production for MineX is approximately 1.8 million tons of material (ore and waste), 1.3 million tons of ore and 370,000 troy ounces of gold [7]. The operation uses various stoping mining methods for the three distinct deposits currently being mined.

The mine consists of a production shaft, decline, and a ventilation shaft. The production shaft and decline serve as air intakes and the ventilation shaft provides air exhaust for ventilation. Two, 11 ft. diameter, 2,000 horsepower (hp) fans, located underground, have a combined capacity of 1.25 million cubic feet per minute [8].

MineX utilizes mechanized mining equipment powered by diesel engines that expel diesel particulate matter through exhaust into the underground environment. The mine is currently facing challenges regarding remaining compliant with diesel particulate matter regulations. MineX is taking action to meet the new DPM regulations through the implementation of

DPM reduction and control methods, but these controls often result in reduced productivity or increased operational costs. Ventilation infrastructure is expensive; therefore, it is important to use available ventilation as effectively as possible [9]. The MineX scheduling process currently involves manually scheduling and subsequently using software that contains a heuristic to determine the life-of-mine plan. Ventilation considerations are not included in the manual mid-term plan.

The author proposes three estimation methods based on information provided by MineX. The first method assigns a three-tiered classification based on the equipment used for the activity and the experience of MineX's ventilation engineer. The second method assigns a fixed air quantity of 25,000 cfm to all activities. The final method utilizes equipment manufacturer information to determine the airflow required for individual equipment. Additionally, MineX sets a minimum of 15,000 cfm for all active headings. The resulting airflows are assigned to all activities.

Comparisons between the manual schedule, the optimized schedule without ventilation, and the optimized schedule with ventilation constraints are presented herein. The results show a significant improvement in metal production compared to the current manual scheduling method employed at MineX. The evaluation of the optimized schedules, with and without the ventilation constraints, will show the impact of ventilation on metal production.

1.4 Original Contribution

The original contributions of the research include:

1. Development of a generalized underground mining production schedule optimization model for stoping operations
2. Integration of delay and overlap duration between an activity and associated predecessor
3. Incorporation of ventilation capacities and requirements into the scheduling process, creating a more holistic view of the operation
4. Evaluation of various air quantity requirements based on DPM estimations
5. Application of non-traditional solution methods to underground mine schedule optimization

The solutions presented herein offer value to the underground mining industry by providing an optimized sequence of activities within the limits set forth in the model.

1.5 Contents of this Dissertation

Chapter 2 provides a basic background for underground mining, mine ventilation, and operations research and methods. Chapter 3 reviews previous work in the areas of underground mine plan optimization and mine ventilation. Chapter 4 presents the mathematical formulation of the RCPSP and how it is solved using the Bienstock-Zuckerberg algorithm and TopoSort heuristic, as well as the detailed formulation of model (\mathcal{Z}). Chapter 5 reviews the solution methods used to solve model (\mathcal{Z}). Chapter 6 describes the data used in model (\mathcal{Z}) and the ventilation airflow estimation methods. Chapter 7 presents the results and comparisons between the solution methods and the quality of the solutions. Chapter 8 includes the conclusions and presents ideas for future work.

CHAPTER 2

BACKGROUND

This chapter provides a basic overview of underground mining, mine planning, mine ventilation, operations research and operations research methods.

2.1 Underground Mining

Underground mining is a method of extracting rock containing economic quantities of metal or minerals, called ore, from the Earth's crust. Non-economic rock is referred to as waste. Underground mining is most often used when costs or safety factors, e.g., slope stability, prohibit accessing the ore deposit from the surface [10]. Although more expensive per ton of material extracted than surface mining, most underground mining methods provide greater selectivity, thereby reducing the mining of waste. Underground mine ore deposits are accessed via a shaft or decline, i.e., a ramp. From these entry points, drifts and haulage ways are constructed to provide access to the ore deposit. The orebody is mined, with the chosen mining method, and the ore is transported, i.e., via trucks, trains, or conveyor belts, to the shaft or decline and then brought to the surface. Most ores require some processing to remove contaminants, i.e., unwanted minerals or metals contained in the rock, or to extract the final product before being sold. Figure 2.1 shows the major underground mine design elements, such as deposit access, production stopes, haulage levels, and backfilled stopes.

Underground mines are designed and planned based on various factors including: geology, deposit type, geotechnical characteristics, and economics. Decisions include how the deposit will be: *(i)* accessed, e.g., vertical shaft or decline, *(ii)* mined, e.g., stope and/or cut-and-fill, *(iii)* transported, e.g., trucks, rail, conveyor, and/or hoist, and *(iv)* remediated, e.g., paste or rock backfill. These decisions are based on economic factors, physical limitations, operational parameters, and safety [11].

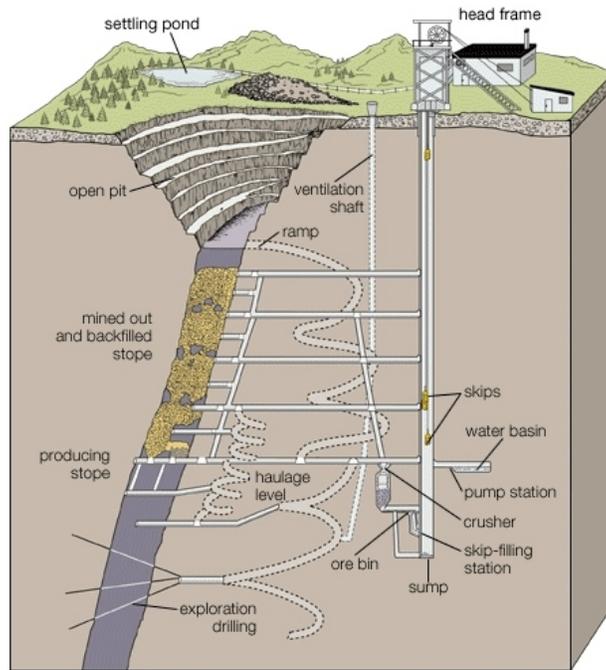


Figure 2.1: Diagram of Open Pit and Underground Mine (Source: Atlas Copco, 1997)

Underground mines utilize various methods which are often determined by the spatial and physical characteristics of the deposit. Mining methods are frequently generalized when compared to actual implementation, which can differ from mine to mine. Some common mining methods are:

- Room-and-Pillar
- Stoping
- Cut-and-Fill
- Longwall
- Block Caving

The room-and-pillar mining method is used for relatively flat lying deposits. Ore is removed from rooms in the deposit with significant portions left unmined (pillars) to provide support. When the deposit is steeply dipping and the rock is competent, stoping methods are often implemented. Stopes, defined as areas where ore is removed from the surrounding rock [12], range in size based on the stability of the host rock. The size of the stope can

impact production, with larger stopes providing higher production rates. Stopes can be left open, i.e., void, or they can be backfilled with loose rock, tailings, or cemented rock which can provide support for neighboring stopes. Cut-and-fill is similar to stoping methods. With cut-and-fill, ore is removed in horizontal slices starting at the bottom and moving upwards. Voids are backfilled to provide support and a working platform for the next slice. Longwall mining is used when the ore deposit is horizontal, thin-bedded, and does not require blasting. Mining is conducted using a mechanized longwall machine that takes long horizontal slices along the ore face. Once mined, the area may be allowed to collapse or is supported with man-made structures. Block caving is used in massive, friable deposits. Gravity is used to break the rock and extraction is performed at or near the bottom of the deposit. Caving is a highly productive mining method, but creates surface subsidence as the deposit is extracted.

2.2 Mine Planning

Mine planning is a broad term that covers many aspects including ore-deposit modeling, resource and reserve determination, mining method selection, mine layout, infrastructure design, costs and revenue economics, and production scheduling.

Mine planning is an iterative process that begins prior to the construction of a mine and continues throughout its life. The process begins with feasibility analysis to determine the technical and economic viability of the deposit. Once deemed viable, the next phase involves designing the mine layout and infrastructure. Using the mine design, the final phase is defined by the creation of long-, mid-, and short-term production schedules. These schedules are continually evaluated and modified throughout the life-of-mine.

Production schedules for underground mines are developed manually or with the assistance of scheduling software such as MineRP's EPS Schedule Optimisation Tool (EPSOT) [13] and Datamine's Enhanced Production Scheduler (EPS) [14]. Both of these programs use genetic algorithms and heuristic methods to determine near-optimal solutions with given parameters and constraints. The results provide a series of feasible solutions which are ranked according to the desired metric, i.e., NPV or metal production.

2.3 Mine Ventilation

Mine ventilation is the movement of air within a mine via mechanical fans. Ventilation is an important aspect of any underground mining operation and is used to create a safe and comfortable working environment for miners. Ventilation systems are designed to provide essential gases, i.e., oxygen and nitrogen, and to flush out harmful compounds and particulates, i.e., methane, carbon monoxide, dust, and diesel particulate matter. Ventilation must also be effective at reducing concentrations of these gases and particulates below harmful levels [15]. Over the last few decades, engineers have begun to employ operations research to optimize ventilation systems to assist in achieving regulatory compliance, reduce ventilation costs, and improve efficiency.

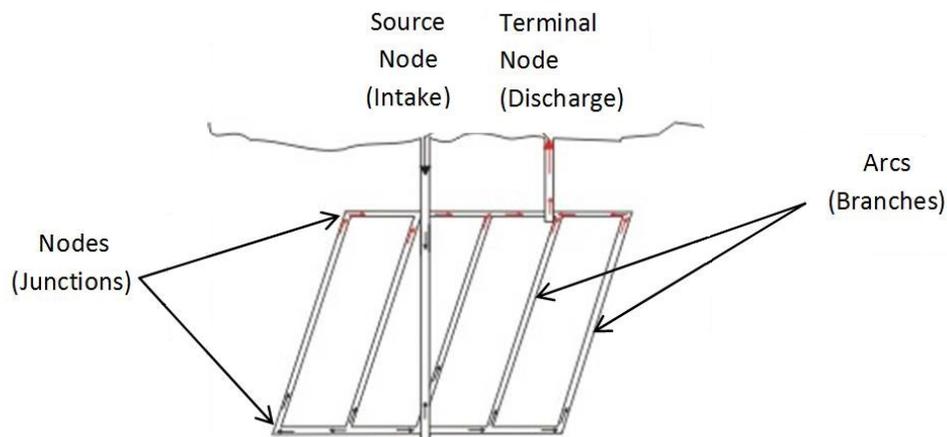


Figure 2.2: Diagram of a Basic Ventilation Network (Source: Mechanicalbook.com)

Ventilation in underground mines can be modeled with an underlying network structure. The source node is represented by the intake and the terminal node by the discharge. The workings of the underground mine are referred to as branches and can be converted to arcs in a network diagram. Nodes of the network represent the junctions. Figure 2.2 provides a basic diagram of a ventilation network for an underground mine.

Natural ventilation is airflow created by pressure and temperature differences between two openings at the surface; however, most underground mines utilize large mechanical fans

that push or pull air through a mine. For all underground mines, there must be at least one point at the surface for air intake and another for discharge, as shown in Figure 2.3.

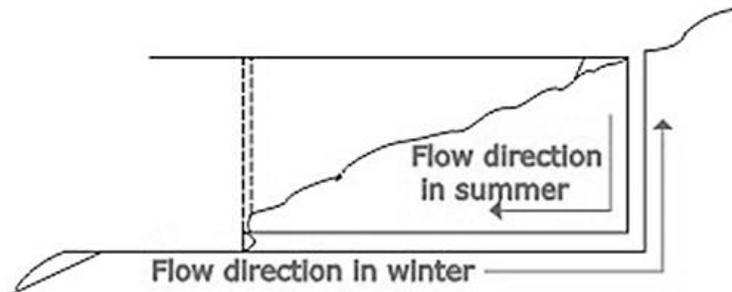


Figure 2.3: Diagram of Natural Ventilation (Source: Mechanicalbook.com)

Mine ventilation systems typically employ mechanical fans to move air throughout the mine. The systems are designed to provide a quantity of air based on various factors such as equipment used, hazardous gases, and contaminants. Once a ventilation system is constructed and operational, it becomes challenging to modify. Achieving additional airflow can require large capital investments to the ventilation system and may include construction of additional infrastructure, e.g., additional fans, ventilation shafts.

2.4 Operations Research and Methods

Operations Research “is the study of how to form mathematical models of complex engineering and management problems and how to analyze them to gain insight about possible solutions” [16]. Operations research was developed during World War II to assist British and American military leaders to better deploy and manage radar, convoy, bombing, anti-submarine, and metal and non-metal mining operations. From these initial applications, researchers have created, incorporated, and improved tools for use in many other fields such as the computer, manufacturing, transportation, chemical, and energy industries.

The first step in implementing operations research methods is to formulate the problem. It is necessary to define the parameters, decision variables, objective function, and constraints. Parameters provide coefficients for the objective function, constraints, and exponents in non-linear formulations. The decision variables are unknown values that the

mathematical model determines. The objective is a function that is either maximized and/or minimized, while constraints provide bounds on the solution such as production and/or milling capacities and production and/or milling rates.

Common model types found in operations research and applied to underground mining include: linear programming, integer programming, nonlinear programming, and network theory.

Linear programming (LP) is used for mine production scheduling and involves a mathematical model consisting of a linear objective function and linear constraints [17]. The variables assume non-negative, continuous values. The general form of a linear program, often referred to as standard form, can be portrayed as follows:

$$\text{maximize } cx \tag{2.1}$$

$$\text{subject to } Ax \leq b \tag{2.2}$$

$$x \geq 0 \tag{2.3}$$

where x represents the decision variables, c and b are both vectors of known coefficients, and A is a matrix of coefficients. The objective function maximizes cx , (2.1), subject to inequality constraints, (2.2) and (2.3). The special structure of linear programming models guarantees that a local maximum is also the global maximum. Solution algorithms for linear programs include both the primal and dual Simplex method, and the interior-point method. These methods capitalize on the distinct properties, i.e., convexity, of linear programs [16].

Networks constitute a special class of linear programs [18], and are unique in that it is possible to graphically represent the problem using nodes and arcs. Network algorithms can be used to determine the shortest path from one node to another or to determine the maximum production rate, or flow, given restrictions on the network. For linear programs exhibiting network structures with side constraints, the Bienstock-Zuckerberg algorithm can be applied [19].

The form of an integer programming model (IP) is almost identical to that of a linear program with the exception of the decision variable. In LP, the decision variable x is defined, without a loss of generality, as being any value greater than or equal to 0. In IP, the decision variable is defined as being greater than or equal to 0 but also integer. The decision variable can also be defined as binary, i.e., equaling 1 or 0 [20], and can represent whether a designated activity occurs or not.

Integer programs are not as easy to solve relative to linear programs, despite being similar in form. Integer programs can be solved with a systematic enumeration algorithm called branch-and-bound. Unlike the simplex or interior point methods which use directional search logic, branch-and-bound enumerates feasible solutions. To improve the speed and reduce the likelihood of having to enumerate all possible solutions, heuristic methods and cuts can be applied.

A formulation with a nonlinear objective function and/or any nonlinear constraints is considered to be a nonlinear program (NLP). Variables can assume discrete, non-negative values. Sometimes the nonlinear objective or constraints can be linearized, thereby transforming the problem into a linear program. This transformation makes the problem easier to solve, but increases the potential for a loss of accuracy. Solution methods for unconstrained nonlinear programs include steepest decent and Newton's method. Constrained nonlinear programs can be solved using penalty and barrier algorithms [21].

When all variables in a model are defined as integer, the formulation is said to be a pure integer program; however, integer programs can include continuous variables and/or nonlinear constraints. These combined formulations are described as mixed integer programs (MIP) or mixed integer nonlinear programs (MINLP), respectively.

The complexity of a mathematical model can be classified as P or NP. Problems that are solved in polynomial time are classified as P and are considered relatively easy to solve in that the solution time of the corresponding instances grows as a polynomial function of problem size. It is "easy," i.e., can be solved in polynomial time, to check the correctness

of the solutions for problems classified as NP, but not necessarily easy to solve them [21]. NP problems can be further classified as either NP-complete or NP-hard problems. NP-complete problems are all problems that cannot be solved with any known polynomial-time algorithm [22]. NP-hard problems are as hard or harder to solve than any NP problem. In general, these problems require exponential-time algorithms to determine a provably optimal solution.

Engineers and management may use simulation to evaluate potential scenarios under complex uncertainties, e.g., multiple mines, complex blending requirements or multiple processes. Simulation is often used in a mine scheduling setting to determine equipment fleet size based on haulage routes and production requirements or to size a piece of equipment based on material densities and production rates. Optimization techniques can also be applied to these types of problems.

When problems are difficult to express explicitly and/or determining a solution is time consuming, heuristic methods can provide practical and logical solutions. Heuristic methods can be used in conjunction with an optimization algorithm to assist in providing an initial solution which may lead to faster convergence.

CHAPTER 3

PREVIOUS WORK

This chapter discusses previous work with respect to underground mine production schedule optimization and ventilation optimization.

3.1 Underground Mine Production Schedule Optimization

Operations research has been used for mine scheduling since the early 1960's; however, the majority of the early research was focused on open pit mining applications [23, 24].

Early open pit models made great progress but fell short in some ways. Most of the initial models were site-specific and lacked a level of generality that would allow engineers to apply them to multiple mines. In addition, they typically focused on one specific aspect of the operations. Using linear programming, researchers determined optimal blending or production schedules; however, these types of mathematical models did not allow the use of binary decision variables that are required to determine plant location or meet precedence needs.

Gershon [25] formulates a generalized mine scheduling optimization model using mixed integer programming, called Mine Scheduling Optimization (MSO). The model serves virtually all open pit mining methods and is successfully applied on copper, coal, and cement operations. MSO maximizes NPV and provides optimal long-range plans for each operation.

Barbaro and Ramani [26] develop a generalized MIP that optimizes the production schedule of multiple mines while selecting processing plant types and locations necessary to meet specified market demand. The model contains 32 continuous and 10 integer variables. While this model accommodates the decisions associated with a coal mine where the extent and quality of the deposit is known with greater certainty, the results of the model would be difficult to apply to a metal mine where grades and quantities are more challenging to predict. Gershon mentions expanding his model to include underground and strip mine scheduling;

however, neither Gershon nor Barbaro and Ramani mention any implementation of their respective models at an underground mining operation.

Using the Mt. Isa and Cannington mines as case studies, Trout [27] implements a mixed integer program to schedule ore extraction and backfilling activities associated with an underground mine. The formulation includes four integer variable sets used to define the time periods for extraction and backfilling activities. A fifth integer variable set indicates the time period during which a stope is void, i.e., extraction is complete but backfilling has not begun. The objective function maximizes NPV and the linear constraints include: extraction duration, backfill duration, activity, capacity, and precedence constraints.

Trout's model consists of approximately 3500 variables and 7000 constraints, compared to today's models with hundreds of thousands of variables and millions of constraints, which contributed to the inability to determine an optimal solution for a model of this size with available computational power. To reduce the number of variables, Trout applies early and late start dates for stope extraction and backfilling, meaning that the stopes are bounded by the time period in which they must be mined, could be mined, or when they must be completed. Variables that could not be mined within a given time period were not evaluated, thereby reducing the model size. With the variable reduction techniques, Trout determines a feasible, yet non-optimal, solution that provides a 123 percent increase in NPV over the manual schedule. Trout's work shows the potential of applied operations research techniques to underground production scheduling.

Carlyle and Eaves [28] expand on Trout's work by creating a model for the Stillwater Mine that includes extraction and backfilling activities but adds development activities to the formulation. Two objectives were evaluated: maximum discounted metal ounces produced and maximum discounted profit. The model conclusions are similar for both objectives. Modifications to the model evaluate various planning time horizons, stope access development spacing, and production ramp-up and labor scenarios. The model implementation at the mine proved critical in the decision making process for a large mine expansion.

Smith et al. [29] use aggregation at the Mt. Isa mine to reduce the number of variables needed to solve the life-of-mine (approximately 13 year) production schedule for the multi-mine, multi-metal, multi-process operation. The model contains continuous, semi-continuous, and integer decision variable sets. The integer variable set represents the active mining areas in a given time period. The continuous variables indicate the quantities mined from the areas. The model objective is to maximize NPV. Constraints limit production by the physical tonnage, metal quantity, capacity of processing facilities, metal market demands, infrastructure capacities, number of active mining areas, average grade delivered to the processing facility, and physical precedence. The results, when compared to the Mt. Isa manual schedule, show that the MIP provides a schedule with more consistent grade delivery to the processing facilities, a smoother production tonnage for the life of mine, and a more balanced delivery of ore to the various processing facilities.

Kuchta et al. [30] incorporate variable reduction methods in an effort to improve solution times using data from the LKAB's Kiruna Mine, located in Sweden. The model focuses on machine placement for ore extraction with an objective of minimizing deviation from a desired production target. Constraints include production requirements and capacities for different ore types, vertical and horizontal precedence, and the number of machine placements within specified areas. The authors reduce the number of integer variables with early and late starts. Early starts evaluate only those variables which could occur at or before a given time period while late starts ensure that an activity begins by a specified time period.

Continued improvements to the Kiruna model include aggregation and decomposition [31, 32]. Martinez and Newman [33] use a decomposition heuristic in which machine placements are scheduled in the long term while the smaller production blocks they contain are scheduled in the short term. The heuristic solves subproblems and, using information obtained from the subproblem solutions, solves a modified version of the original model. The heuristic produces better and faster solutions in most instances, when compared to solving the original model as a monolith, i.e., the entire model.

Using an underground coal mine as a case study, Sarin and West-Hansen [34] develop a complex model to determine maximum NPV by scheduling equipment groups working in a given section of the mine. This formulation includes a linearized production leveling constraint that captures the deviation of coal quality between time periods. The complexity of the problem makes it difficult to solve using the branch-and-bound algorithm, even with constraint relaxations using Benders' cuts [35]. Using Benders' Decomposition, the authors solve the model and produce near-optimal solutions resulting in a successful implementation at the mine.

McIsaac [36] schedules an underground mine using mixed-integer linear programming for both development and production within individual zones. McIsaac evaluates two objective functions: (i) maximizing cash flow and (ii) minimizing development time. The constraints include upper and lower bounds on production and development, as well as restrictions on individual zones. The author also places precedence constraints on the various activities, e.g., pre-production development must be completed before production development can begin. An additional constraint dictates that once an activity begins in a zone, it must be completed in consecutive time periods. The author uses Frontline's Xpress Solver [37], an Excel add-on, to solve the problem.

Nehring and Topal [38] expand Trout's [27] model to include multiple activities including: stope preparation, extraction, void, i.e., when a stope has been mined and remains open before being backfilled, and backfilling. The data represents nine stopes arranged in a 3×3 matrix. Each stope contains more material than can be mined in a single time period. The formulation includes a constraint to limit multiple fill mass exposures, i.e., any form of production around a backfilled stope, to one adjacent stope. The optimized schedule provides a trivial increase in NPV over the manual schedule. The most recent formulation of this model [39] reduce the number of decision variables through the assumption that defined groups of mining activities are continuous. For example, once production drilling is completed, production of the stope begins immediately, followed by backfilling. This

assumption significantly reduces the number of decision variables, leading to faster solution times, especially when using larger datasets.

O’Sullivan [40] schedules the underground Lisheen Mine, located in Ireland, using an integer programming model. The objective maximizes discounted metal (zinc and lead) with complex precedence constraints. The resulting schedule took fewer than 20 hours to solve to near-optimality using commercially available optimization software and a high powered computer, i.e., two dual Intel Xeon X5570 quad core processors and 48 GB of RAM. O’Sullivan utilizes aggregation techniques to reduce variables and developed methods for assigning precedence to the various activities; however, aggregation is limited as a result of the complicated precedence structures. Lisheen used the model to make end-of-mine-life production scheduling decisions.

3.2 Mine Ventilation Optimization

Literature pertaining to mine ventilation optimization is primarily focused on approximating airflow within a ventilation network [41] and minimizing fan power [42, 43]. Many ventilation equations used to determine airflow within a network are non-linear. Traditionally, the most common method of solving these ventilation networks is with the Hardy Cross algorithm. Similar to the Gauss-Seidel method of solving a set of linear equations, this is an iterative method in which an approximate solution for airflow is continually improved until the error is below an allowable limit [41].

Engineers and academics use other nonlinear programming methods to evaluate mine ventilation networks, in addition to the Hardy Cross method. One of the simplest nonlinear programming techniques, Steepest Descent, is used by Sarac and Sensogut [44] in conjunction with the Hardy Cross method, resulting in improved solution time. The Hardy Cross method typically starts by setting all branch air quantities to zero; however, it has been shown that by using the Steepest Descent method for the first few iterations, it is possible to obtain air quantities that are nearly optimal. Using the initial values determined with Steepest Descent, the Hardy Cross algorithm can be used to converge to the optimal solution in

approximately half the time relative to the Hardy Cross method.

The general ventilation network problem is defined as a system of n equations and n unknowns, where n is the number of branches in the ventilation network [45, 46]. Using Kirchoff's current and voltage laws, the network problem is formulated as a non-convex nonlinear programming model. Nonlinear programs, especially nonconvex nonlinear programs, are difficult to solve. Many researchers use linear approximations [47] and special ordered sets to allow the problems to be solved using linear models [48]. With the problem formulated as a linear optimization, common linear techniques, such as Simplex and branch-and-bound, can be applied.

Various commercial software applications are available to help engineers optimize ventilation networks. The two most commonly used software packages are Ventsim [49] and VnetPC [50]. Both software packages utilize the Hardy Cross method to simulate and optimize ventilation networks. Other commercial software packages use a more analytic approach to solving ventilation network problems.

Allocation of ventilation typically occurs after a production schedule has been developed. Because ventilation systems are very expensive and require a significant amount of fixed infrastructure, it is sometimes challenging to meet the ventilation requirements of a production schedule without investing additional capital. Current industry focus is on designing systems in which ventilation can be adjusted based on immediate need. This concept is referred to as Ventilation on Demand (VOD) which reduces ventilation costs by only using air where and when it is needed [51]. By adding variable frequency drives to auxiliary fans, it becomes possible to adjust the speed of the fan blades to produce the required airflow. If a mine has implemented a VOD system, the ability of the ventilation system to adapt to the needs of a production schedule can be significantly improved.

To date, ventilation optimization has focused primarily on ventilation networks, without incorporating this knowledge into the production scheduling process. With the ability to solve larger and more complex scheduling problems, there is great opportunity for incorpo-

rating ventilation constraints with production scheduling optimization.

3.3 Discussion

Underground production scheduling has evolved over the last 20 years, but has still proven to be a complicated and challenging task. Computational advances have allowed researchers to solve much larger problems in less time, but challenges remain as the problems become more complex. For further information regarding mine planning optimization and ventilation optimization, see Newman et al. [52] and Acuña and Lowndes [46], who provide a detailed overview of these topics, respectively.

While the model presented in this dissertation might not be as complicated as some of the underground production scheduling models developed by other researchers, its special structure makes it possible to solve much larger instances of the problem. Additionally, the incorporation of a ventilation component provides insight into the potential for including more aspects of the mining operation into the planning process.

CHAPTER 4

MODEL FORMULATION

The author presents a deterministic formulation of the resource constrained project scheduling problem (RCPSP). First, the specifics of the RCPSP are presented, followed by the author's formulation.

The RCPSP falls into the class of NP-hard combinatorial optimization problems [53, 54] that consist of an objective, resource capacity constraints, and precedence constraints [2]. Similar formulations, as described in [55], have been applied to open pit mine scheduling; however, this dissertation shows that a comparable formulation structure can also be applied to underground production scheduling. A comparison of the differences and challenges of underground and open pit mine production schedule optimization is given in [56].

4.1 The Resource Constrained Project Scheduling Problem (RCPSP)

The generalized mathematical formulation for the RCPSP is as follows. A detailed formulation is defined in Section 4.4.

Sets:

- \mathcal{A} : set of all activities.
- \mathcal{P}_a : set of all activity precedence relationships. That is, if $a' \in \mathcal{P}_a$, then activity a' must be completed prior to the initiation of a .
- \mathcal{R} : set of all resources that are consumed in order to carry out the activities. During each period an activity is carried out, resources are consumed.
- \mathcal{T} : set of all time periods in which it is possible to initiate activities, $t = 1 \dots T$.

Parameters:

- \hat{d}_a : vector of activity duration. For each $a \in \mathcal{A}$, the value \hat{d}_a represents the number of time periods required to complete activity a . That is, if activity a is initiated in time period $t \in \mathcal{T}$, then activity a will be completed at the end of time period $t + \hat{d}_a - 1$.

- c_{at} : vector of activity objective values. For each $a \in \mathcal{A}$ and for each $t \in \mathcal{T}$, the parameter c_{at} represents the objective value that will be obtained if activity a is initiated in time period t . Parameter c can assume positive or negative values.
- $\bar{d}_{aa'}$: vector of activity precedence relationship delays. For each $a' \in \mathcal{P}_a$, the value $\bar{d}_{aa'}$ represents the number of time periods that must pass after activity a' is initiated and before activity a can be initiated. That is, if activity a is initiated in time t , then activity a' must be initiated no later than in time $t - \bar{d}_{aa'}$.
- \hat{r}_{rt} : vector of resource availability per time period. For each $r \in \mathcal{R}$, there is an amount \hat{r}_{rt} of resource r in time period and t . The vector \hat{r}_{rt} is positive.
- q_{ar} : vector of activity-resource consumption. For each $a \in \mathcal{A}$ and $r \in \mathcal{R}$, the positive vector q_{ar} represents the amount of resource r consumed by carrying out activity a each time period.

Variables:

- y_{at} : binary variable for each $a \in \mathcal{A}$ and $t \in \mathcal{T}$.

$$\text{maximize } \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} c_{at} y_{at} \quad (4.1a)$$

$$\text{subject to } \sum_{t \in \mathcal{T}} y_{at} \leq 1 \quad \forall a \in \mathcal{A} \quad (4.1b)$$

$$\sum_{t' \leq t} y_{at'} \leq \sum_{t'=1}^{t-\bar{d}_{aa'}+1} y_{a't'} \quad \forall a \in \mathcal{A}, a' \in \mathcal{P}_a, t \in \mathcal{T} \quad (4.1c)$$

$$\sum_{a \in \mathcal{A}} \sum_{t'=t-\hat{d}_a+1}^t q_{ar} y_{at'} \leq \hat{r}_{rt} \quad \forall r \in \mathcal{R}, t \in \mathcal{T} \quad (4.1d)$$

$$y_{at} \in \{0, 1\} \quad \forall a \in \mathcal{A}, t \in \mathcal{T} \quad (4.1e)$$

In the generalized formulation, the objective function (4.1a) maximizes the net present value (NPV). The objective may be altered to maximize profit or metal produced or minimize costs. Constraints (4.1b) impose that activities are initiated at most once; constraints (4.1c) impose the precedence relationships; constraints (4.1d) represent resource consumption limits; and constraints (4.1e) impose integrality conditions.

4.2 Solving the RCPSP Instance

The methodology for solving the RCPSP includes four basic steps, given as follows:

1. Convert the problem from “AT” format to “BY” format.
2. Solve the linear programming relaxation of the “BY” problem using the Bienstock-Zuckerberg (BZ) algorithm [5].
3. Convert the solution from “BY” format back to “AT” format.
4. Use the TopoSort heuristic [6] to round the fractional LP solution to obtain an integral one.

Solutions are determined using the OMP solver. OMP automatically executes Steps 1-4, and additionally introduces a number of important preprocessing and speed-up techniques. For more information on OMP, see [57].

There are two important details concerning Steps 1 – 4 that are unique to the problem application. The model (4.1) is slightly different when compared to other RCPSP problems whose LP relaxations have previously been solved with the BZ algorithm in that this model incorporates activity duration and precedence lags (given by vectors \hat{d}_a and $\bar{d}_{aa'}$, respectively) and requires some special implementations. Section 4.3 properly defines the “AT” and “BY” formats of problem (4.1), and describes how to convert (4.1) from “AT” format to “BY” format and back. Section 4.3.1 describes the TopoSort algorithm that computes the integer feasible solutions.

4.3 Converting the RCPSP Instance from “AT” to “BY” Format, and Back.

The OMP solver begins by reformulating the problem using a variable substitution scheme, as explained by Lambert et al. [58] who provides a tutorial on efficient formulation of the open pit block scheduling problem, i.e., a problem with a mathematical structure very similar to that of (4.2). For this, the “AT” variables y are replaced with new “BY” variables z , defined as follows.

To obtain the z variables from the y variables, simply define $z_{at} = \sum_{t'=1}^t y_{at'}$. To obtain y variable values from the z variable values, let

$$y_{at} = \begin{cases} z_{at} & \text{if } t = 1 \\ z_{at} - z_{a,t-1} & \text{otherwise.} \end{cases}$$

When the “AT” variables are substituted with the “BY” variables, z_{at} becomes a binary variable equal to 1 if and only if activity a is initiated “by” time t (i.e., no later than t). Using the substitutions described in formulation (4.1), the equivalent formulation of RCPSP is obtained. This formulation has the objective function:

$$\text{maximize } \sum_{a \in \mathcal{A}} \sum_{t=2}^T c_{at} (z_{at} - z_{a,t-1}),$$

activity single occurrence constraints:

$$z_{at} \leq z_{a,t+1} \quad \forall a \in \mathcal{A}, t = 1, \dots, T-1,$$

precedence constraints:

$$z_{at} \leq z_{a't} \quad \forall a \in \mathcal{A}, a' \in \mathcal{P}_a, t \in \mathcal{T},$$

resource consumption constraints:

$$\sum_{a \in \mathcal{A}} q_{ar} (z_{at} - z_{a,t-d_a+1}) \leq \hat{r}_{rt}, \quad \forall r \in \mathcal{R}, t = \max\{2, d_a\}, \dots, T,$$

and integrality constraints:

$$z_{at} \in \{0, 1\} \quad \forall a \in \mathcal{A}, t \in \mathcal{T}.$$

That is, the RCPSP formulation reduces to a problem of the form:

$$\text{maximize } c'z \tag{4.2a}$$

$$\text{subject to } z_a \leq z_{a'}, \quad \forall a \in \mathcal{A}, a' \in \mathcal{P}_a \tag{4.2b}$$

$$Hz \leq h, \tag{4.2c}$$

$$z \in \{0, 1\} \tag{4.2d}$$

for a suitably defined matrix H and index set \mathcal{P}_a . This is precisely the class of problems that can be solved with the BZ algorithm.

4.3.1 The TopoSort Algorithm.

Figure 4.1 describes a modification of the TopoSort algorithm, as originally presented by Chicoisne et al. [6] and later modified by Muñoz et al. [57], that is adapted to obtain

an integer feasible solution of the RCPSP instance, as defined in (4.1), given a feasible solution to the LP relaxation of the problem. After the LP relaxation solution is determined by BZ using the “BY” formulation, the solution is converted back to the “AT” formulation. Then, the “AT” solution is used by the TopoSort algorithm to determine the integer solution.

Algorithm 1 Notation

Inputs:

- \mathcal{S}_a = set of remaining predecessor activities for a
- \hat{p}_a = count of remaining predecessor activities that have yet to be scheduled in the IP solution
- δ_a = earliest expected start for activity a
- \hat{e}_a = expected start of a , i.e., the weighted sum of start times for a in the LP relaxation
- T = maximum number of time periods in \mathcal{T}
- y_{at}^* = LP relaxation from the “AT” solution

Outputs:

- \bar{y}_{at} = IP solution

4.3.2 Explanation

TopoSort converts the LP relaxation solution of model (4.1) obtained from the OMP Solver to a feasible integer solution through the iterative process outlined in Figure 4.1. To begin, a series of sets and parameters are defined, as shown in lines (1)-(3), which include the count of remaining predecessor activities for a given activity, set of predecessor activities for a given activity, earliest expected start time period and weighted expected value. Next, The algorithm selects an activity that has yet to be scheduled among the set of \mathcal{A} that meets specified criteria (lines 4 and 6). The algorithm selects the activity with the minimum expected start, \hat{e}_a , where the activity has zero remaining predecessors, \hat{p}_a , and the earliest expected start, δ_a , for the activity is less than the maximum number of time periods. For the selected activity, \bar{y}_{at} is assigned a value of 1 (lines 7 – 8). Line 9 determines if the activity

Algorithm 1: The TopoSort Heuristic.

Input: An instance of RCPSP, as defined in Problem (1.1), and a feasible solution y_{at}^* of its LP relaxation.

Output: A solution \bar{y} of the RCPSP instance satisfying the integrality constraint.

- 1 Let $\mathcal{T}^* = \mathcal{T} \cup \{T + 1\}$.
- 2 For each $a \in \mathcal{A}$ define:
- 3

$$\begin{aligned}\hat{p}_a &= |\mathcal{P}_{a'}|, \\ \mathcal{S}_a &= \{a' \in \mathcal{A} : a \in \mathcal{P}_{a'}\}, \\ y_{a,t'+1}^* &= 1 - \sum_{t \in \mathcal{T}} y_{at}^*, \\ \delta_a &= \min\{t \in \mathcal{T}^* : y_{at}^* > 0\}, \\ \hat{e}_a &= \sum_{t \in \mathcal{T}^*} t y_{at}^*.\end{aligned}$$

- 4 $\bar{y}_{\bar{a}\bar{t}} \leftarrow 0$.
 - 5 **while** $\mathcal{A} \neq \emptyset$ **do**
 - 6 Choose $\bar{a} \in \mathcal{A}$ that solves $\min\{\hat{e}_a : \hat{p}_a = 0, \delta_a \leq T\}$.
 - 7 **for** $\bar{t} = 1, \dots, T$ **do**
 - 8 $\bar{y}_{\bar{a}\bar{t}} \leftarrow 1$.
 - 9 **if** $\left(\sum_{a \in \mathcal{A}} \sum_{t=t'-\bar{d}_a+1}^{t'} q_{ar} \bar{y}_{at} \leq l_{rt'} \quad \forall r \in \mathcal{R}, t' \in \mathcal{T} \right)$ **then break**.
 - 10 **else** $\bar{y}_{\bar{a}\bar{t}} \leftarrow 0$.
 - 11 **if** $\bar{y}_{\bar{a}\bar{t}} = 1$ **then**
 - 12 **for** $a' \in \mathcal{S}_{\bar{a}}$ **do**
 - 13 $\hat{p}_{a'} \leftarrow \hat{p}_{a'} - 1$.
 - 14 $\delta_{a'} \leftarrow \max\{\delta_{a', \bar{t} + \bar{d}_{aa'}}\}$.
 - 15 $\mathcal{A} \leftarrow \mathcal{A} \setminus \{\bar{a}\}$. ;
-

Figure 4.1: The TopoSort Algorithm

violates the resource constraint. If violated, the variable \bar{y}_{at} is assigned the value of 0 (line 10), and the algorithm breaks and returns to line 7 and it is assigned in the next available time period. Once the resource constraints are not violated, \bar{y}_{at} is set to 1 and the algorithm continues (lines 11 – 15), which updates the predecessor count $\hat{p}_{a'}$ and the earliest expected start $\delta_{a'}$. The activity is removed from the set of remaining activities that have not been scheduled and the algorithm continues until all activities have been scheduled or all variables with a positive value in the LP relaxation have been evaluated.

4.4 Detailed Formulation of (\mathcal{Z})

The underground mine production scheduling model, (\mathcal{Z}) , contains one set of variables: a binary variable that indicates when an activity starts. The objective maximizes discounted gold production; however, it can be modified to maximize NPV. The model is constrained by resource capacities, ventilation capacity, and physical precedence. Resource constraints provide lower and upper bounds on the capacities for an activity type, or types and ventilation quantities per time period. An additional resource constraint limits the number of specified activities that can occur during a given time period.

The data consist of spatially defined activities. The details of the data used in the parameters are provided in Chapter 6. The following assumptions hold for (\mathcal{Z}) :

- Activities are completed continuously once begun.
- Equipment fleet specifications are consistent throughout the schedule.
- Ore is not stockpiled.

The RCPSP model shown in Section 4.1 is the generalized formulation used to develop the detailed model for determining the optimal production schedule for MineX. All sets, parameters, the objective function, and the constraints follow:

Simple Sets:

- \mathcal{A} : set of all mining activities ($a \in \mathcal{A}$)
- \mathcal{B} : set of all activity types ($b \in \mathcal{B}$)

- \mathcal{I} : set of tonnage resource constraints ($i \in \mathcal{I}$)
- \mathcal{J} : set of footage resource constraints ($j \in \mathcal{J}$)
- \mathcal{K} : set of concurrent activity constraints ($k \in \mathcal{K}$)
- \mathcal{L} : set of ore tonnage resource constraints ($l \in \mathcal{L}$)
- \mathcal{N} : set of ventilation *Domains* ($n \in \mathcal{N}$)
- \mathcal{T} : set of periods in the time horizon ($t = 1 \dots |\mathcal{T}|$)

Subsets:

- $\mathcal{PD} \subseteq \mathcal{A}$: set of all primary development activities
- $\mathcal{ED} \subseteq \mathcal{A}$: set of all exploration development activities
- $\mathcal{SD} \subseteq \mathcal{A}$: set of all secondary development activities
- $\mathcal{SM} \subseteq \mathcal{A}$: set of all stope mining activities
- $\mathcal{SB} \subseteq \mathcal{A}$: set of all slurry backfill activities
- $\mathcal{PB} \subseteq \mathcal{A}$: set of all paste backfill activities
- $\mathcal{CB} \subseteq \mathcal{A}$: set of all cemented rock backfill activities
- $\mathcal{RB} \subseteq \mathcal{A}$: set of all unconsolidated rock backfill activities
- $\mathcal{VD} \subseteq \mathcal{A}$: set of all vertical development activities
- $\mathcal{DL} \subseteq \mathcal{A}$: set of all drilling activities
- $\mathcal{A}^c \subseteq \mathcal{A}$: set of all ore activities

Indexed sets:

- $\hat{\mathcal{L}}_i \subseteq \mathcal{B}$: set of activity types b measured in tons for every constraint $i \in \mathcal{I}$
- $\hat{\mathcal{J}}_j \subseteq \mathcal{B}$: set of activity types b measured in feet for every constraint $j \in \mathcal{J}$
- $\hat{\mathcal{K}}_k \subseteq \mathcal{B}$: set of activity types b measured in concurrent activities for every constraint $k \in \mathcal{K}$
- $\hat{\mathcal{L}}_l \subseteq \mathcal{B}$: set of activity types b measured in ore tons for every constraint $l \in \mathcal{L}$
- $\hat{\mathcal{A}}_b \subseteq \mathcal{A}$: set of all activities for activity type $b \in \mathcal{B}$
- $\hat{\mathcal{A}}_b^c \subseteq \hat{\mathcal{A}}_b$: set of activities containing ore for activity type $b \in \mathcal{B}$

- $\mathcal{P}_a \subseteq \mathcal{A}$: set of predecessors for activity $a \in \mathcal{A}$
- $\hat{\mathcal{N}}_n \subseteq \mathcal{A}$: set of activities $a \in \mathcal{A}$ in ventilation domain n

Parameters:

- c_a^m : material tonnage for activity a (tons/day)
- c_a^f : linear feet advance for activity a (linear ft/day)
- c_a^g : recoverable grade per ton for activity a (oz/ton)
- \bar{m}_{it}^m : maximum tonnage for resource constraint i in time t (tons/day)
- \bar{m}_{jt}^f : maximum linear feet for resource constraint j in time t (linear ft/day)
- \bar{m}_{kt}^n : maximum number of concurrent activities for resource constraint k in time t (# of activities/day)
- \bar{m}_{lt}^o : maximum ore tonnage for resource constraint l in time t (tons/day)
- es_a : early start time for activity a
- q_a : air quantity needed to complete activity a (cfm/day)
- v_t : available airflow quantity for the mine in cubic feet at time t (ft³/day)
- \hat{v}_{nt} : available airflow quantity for ventilation domain n in cubic feet at time t (ft³/day)
- f_t : fixed consumed airflow quantity at time t (cfm/day)
- \hat{d}_a : number of days to complete activity a
- $\hat{l}_{aa'}$: number of days of lag or overlap time for activity a and predecessor activity a' . Lag is represented by a positive scalar while overlap is a negative scalar.
- $\tilde{d}_{aa'}$: sum of the activity duration \hat{d}_a and activity lag or overlap $\hat{l}_{aa'}$
- δ : discount factor for objective function
- \mathcal{V}_a^{mg} : $c_a^m \cdot c_a^g \cdot \frac{(1+\delta)^{\hat{d}_a} - 1}{\delta \cdot (1+\delta)^{\hat{d}_a - 1}}$: uniform series present-worth factor is applied to calculate gold ounces at the end of the first time period for activity a . The derivation of \mathcal{V}_a^{mg} is shown in Appendix B.

Decision Variables:

- $y_{at} : \begin{cases} 1 & \text{if activity } a \text{ starts at time period } t \in \mathcal{T} : t \geq es_a \\ 0 & \text{otherwise} \end{cases}$

Objective Function:

$$(\mathcal{Z}) : \text{maximize} \quad \sum_{a \in \mathcal{A}^{\mathcal{L}}} \sum_{t \in \mathcal{T}} v_a^{mg} \cdot \frac{1}{(1 + \delta)^t} \cdot y_{at} \quad (4.3)$$

Constraints:

Tonnage:

$$\sum_{b \in \hat{\mathcal{I}}_i} \sum_{a \in \hat{\mathcal{A}}_b} c_a^m \cdot \left(\sum_{t' = \max\{t - \hat{d}_a + 1, es_a\}}^t y_{at'} \right) \leq \bar{m}_{it}^m \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (4.4)$$

Footage:

$$\sum_{b \in \hat{\mathcal{J}}_j} \sum_{a \in \hat{\mathcal{A}}_b} c_a^f \cdot \left(\sum_{t' = \max\{t - \hat{d}_a + 1, es_a\}}^t y_{at'} \right) \leq \bar{m}_{jt}^f \quad j \in \mathcal{J}, t \in \mathcal{T} \quad (4.5)$$

Ore Tonnage:

$$\sum_{b \in \hat{\mathcal{L}}_l} \sum_{a \in \hat{\mathcal{A}}_b} c_a^m \cdot \left(\sum_{t' = \max\{t - \hat{d}_a + 1, es_a\}}^t y_{at'} \right) \leq \bar{m}_{lt}^o \quad \forall l \in \mathcal{L}, t \in \mathcal{T} \quad (4.6)$$

Concurrent Activities:

$$\sum_{b \in \hat{\mathcal{K}}_k} \sum_{a \in \hat{\mathcal{A}}_b} \sum_{t' = \max\{t - \hat{d}_a + 1, es_a\}}^t y_{at'} \leq \bar{m}_{kt}^n \quad k \in \mathcal{K}, t \in \mathcal{T} \quad (4.7)$$

Allowable Mine ventilation:

$$\sum_{a \in \mathcal{A}} q_a \cdot \left(\sum_{t' = \max\{t - \hat{d}_a + 1, es_a\}}^t y_{at'} \right) \leq (v_t - f_t) \quad \forall t \in \mathcal{T} \quad (4.8)$$

Allowable Domain Ventilation:

$$\sum_{a \in \hat{\mathcal{N}}_n} q_a \cdot \left(\sum_{t' = \max\{t - \hat{d}_a + 1, es_a\}}^t y_{at'} \right) \leq (\hat{v}_{nt} - f_t) \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \quad (4.9)$$

Mining Activity Predecessors:

$$\sum_{t' = es_a}^t y_{at'} \leq \sum_{t' = es_{a'}}^{t - \bar{d}_{aa'}} y_{a't'} \quad \forall a \in \mathcal{A}, a' \in \mathcal{P}_a, t \in \mathcal{T} : t \geq es_a \quad (4.10)$$

Mining Activity Single Occurrence:

$$\sum_{t \in \mathcal{T} : t \geq es_a} y_{at} \leq 1 \quad \forall a \in \mathcal{A} \quad (4.11)$$

Variable Restrictions:

$$y_{at} \text{ binary} \quad \forall a \in \mathcal{A}, t \in \mathcal{T} \quad (4.12)$$

4.4.1 Explanation

Detailed descriptions of the formulation follow:

- The objective function (4.3) maximizes gold ounces produced throughout the time horizon, evaluated with a discount factor.
- Constraints (4.4), (4.5), (4.6), and (4.7) are resource constraints on the available resources. These inequalities constrain the number of activities in each of sets \mathcal{I} , \mathcal{J} , \mathcal{K} ,

and \mathcal{L} that can occur each time period.

- The ventilation resource constraint (4.8) guarantees that the ventilation capacity of the mine is not exceeded.
- Constraint (4.9) guarantees that the ventilation capacity of the domain is not exceeded.
- Constraint (4.10) ensures the physical precedence for the activities is not violated and allows for lag and overlap between an activity and its predecessor.
- The final constraints, (4.11) and (4.12), ensure that the activity only occurs at most once and that the variables are binary, respectively.

CHAPTER 5

SOLUTION METHODS

The author presents two primary solution methods for solving (\mathcal{Z}). The first method utilizes the IBM ILOG CPLEX Optimizer (CPLEX) [59]. The second method is the OMP Solver, developed by Dr. Marcos Goycoolea, Dr. Daniel Espinoza, Dr. Eduardo Moreno, and Orlando Rivera. Both methods attempt to determine the global optimal solution.

5.1 CPLEX

Among other model types, CPLEX is capable of solving integer, linear, and mixed integer programming problems using the simplex method, both primal and dual, and the interior point method. The software also employs a variety of proprietary heuristics and preprocessing methods. The code for model (\mathcal{Z}) is written in the AMPL programming language [60] for use with CPLEX.

5.2 OMP Solver

The OMP Solver is an academic endeavor utilizing the Bienstock-Zuckerberg algorithm as a column generation method [61] developed by researchers at the University of Chile and Adolfo Ibáñez University. Five data files are required to utilize the OMP Solver:

- Block - contains activity ID, duration, objective function value, and tonnage, footage, and ventilation parameter values
- Precedence - contains all activity IDs and associated predecessor activity IDs
- Delay - contains overlap and lag duration between activities and predecessor activities
- Parameter - contains the setting to be used by the OMP Solver

- Problem - contains the number of time periods, column location, i.e., field, for resource constraint parameters within the Block file, and resource constraint bounds for each time period

Additional information regarding the file formats can be found in Espinoza et al. [62].

5.3 Early Starts

To improve solution times for (\mathcal{Z}), the use of early starts is evaluated. Early starts is a variable reduction method used to determine whether an activity can occur during a given time period based on precedence and resource consumption necessary to access that activity. With early starts, those activities which cannot physically occur in a time period are not evaluated, thereby reducing the solution time [27, 58, 63].

5.4 Activity Aggregation

In an attempt to reduce problem size, the author aggregates activities using two different schemes: (*i*) waste activities only and (*ii*) ore and waste activities. The first scheme only aggregates waste activities that are immediate predecessors to one another. The second scenario is the same as the first, but adds the aggregation of ore activities, which are based on ore grade with activities falling into a specified low-, medium-, and high-grade classification. Ore activities with the same grade classification are aggregated to maintain data integrity. A maximum of four activities are aggregated to prevent the duration of the aggregated activities from being excessive.

The difficulties that these schemes pose are the loss of data fidelity, potentially having a negative effect on the objective function. Flexibility could be decreased because all activities contained within the aggregated activity must be completed continuously. Additionally, underground activities are not homogeneous but rather vary by size, relative location, duration, and physical characteristics, i.e., tonnage and footage.

Figure 5.1 provides an example of like activities that could be aggregated.

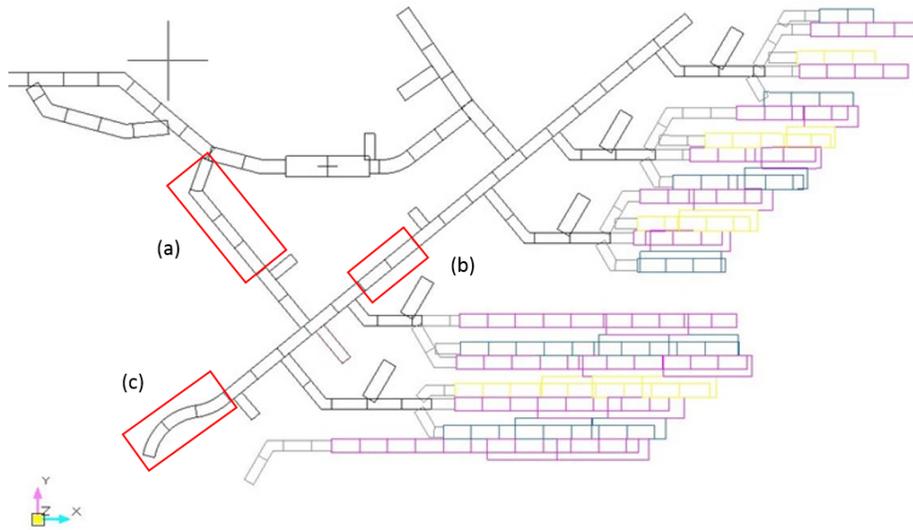


Figure 5.1: Example of Activity Aggregation: (a) shows four, the maximum allowable, (b) shows two like activities, and (c) shows three like activities that could be aggregated.

In fact, although aggregation has been shown to be beneficial in some cases [32, 64, 65], no observable improvement is seen when solving model (\mathcal{Z}) because of the lack of homogeneity within the data set. Figure 5.2 illustrates the scheduling limitations that can be created when aggregating activities.

In an attempt to improve the aggregation results, a smaller data set is evaluated which is limited to those activities that could possibly be scheduled during the first two years. The percentage of activities being aggregated increases from 12 percent to 24 percent when using the reduced data set. The aggregation results for both scenarios and data sets are shown in Table 5.1. Despite the increase in aggregation, there were no noticeable decreases in solution times and, therefore, aggregation is omitted in the schedules presented in Chapter 7.

5.5 Computer Details

Model (\mathcal{Z}) is coded with the AMPL programming language, Version 20140908 [60], and solved with the CPLEX solver, Version 12.6.0.1 [59]. The OMP Solver, programmed in C++, is currently only available for academic use. All models were run on a Dell PowerEdge R410 machine with 16 processors (2.72 GHz each) and 12 GB of RAM.

Table 5.1: Comparison of Aggregation Results

Data Set	# of Original Activities	# of Aggregated Activities	Difference	Change %
Waste Only	24,016	22,382	-1,634	-7%
	24,016	21,167	-2,849	-12%
Reduced Data Set*	10,779	9,651	-1,128	-10%
	10,779	8,221	-2,558	-24%

* Data set is reduced to only activities that could occur within the first two years using the early start for each activity.

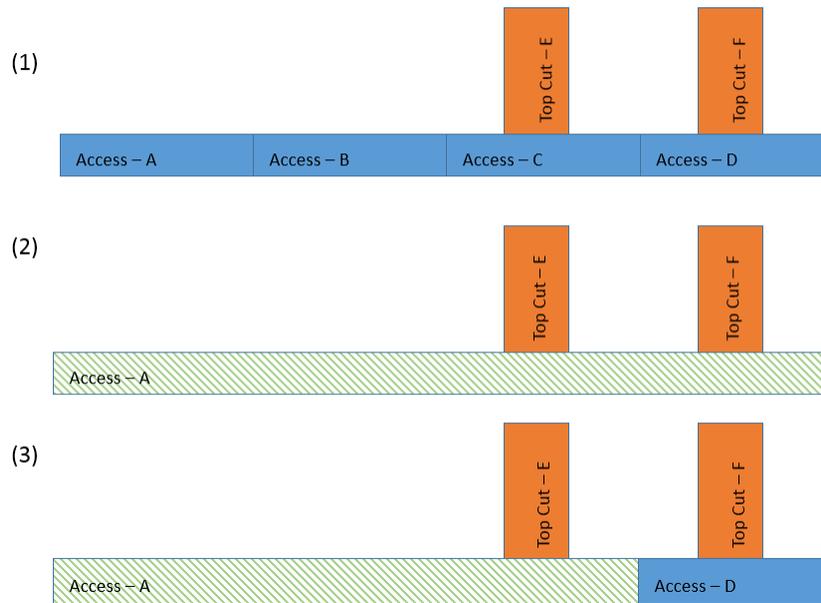


Figure 5.2: Effects of Aggregation: (1) shows the unaggregated Access and Top Cut activities. With the unaggregated data, activity *E* can begin after activity *C* is completed. (2) illustrates the aggregation of all four Access activities. With this aggregation, activity *E* cannot begin until all access activities are completed. (3) shows the maximum effective aggregation than can be completed without limiting the sequence of activities.

CHAPTER 6

DATA

This chapter provides an explanation of the data obtained from MineX and its use in model (\mathcal{Z}). Data includes a variety of physical characteristics and resource capacity limits.

6.1 Mine Description

The data used for this research was obtained from a large-scale underground hard rock mining operation, MineX. The mine has one production shaft, one decline, and one ventilation shaft. The production shaft serves as the air intake and the ventilation shaft provides air exhaust. The operation has three active mining areas and is connected to an adjacent mine, referred to as Area 4. Mining activities from Area 4 are not scheduled in (\mathcal{Z}). The relative location of the areas is shown in Figure 6.1.

The operation utilizes various mining methods such as conventional long-hole stoping, up-hole stoping, and cut-and-fill [8]. The various methods are implemented throughout the mine depending on the deposit geometry and other factors regarding safety and overall mine layout.

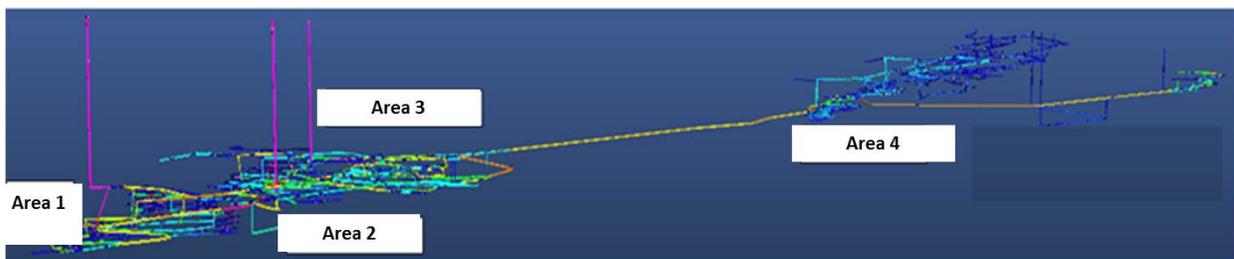


Figure 6.1: 3-Dimensional orthogonal view of the underground mine

MineX activity layout is created in the mine planning software, Vulcan [66]. The activity information created in Vulcan is imported into Deswik mine design software [67, 68], where the activities are assigned predecessors and a schedule can be created manually and viewed

as a Gantt chart. The data produced in Deswik represent the activities to be scheduled (Figure 6.2). Information regarding the *(i)* resource, e.g., grade, tons, linear feet, *(ii)* precedence, e.g., activities that must occur prior, and *(iii)* location, e.g., coordinates, of each individual activity is exported for use in (\mathcal{Z}) .

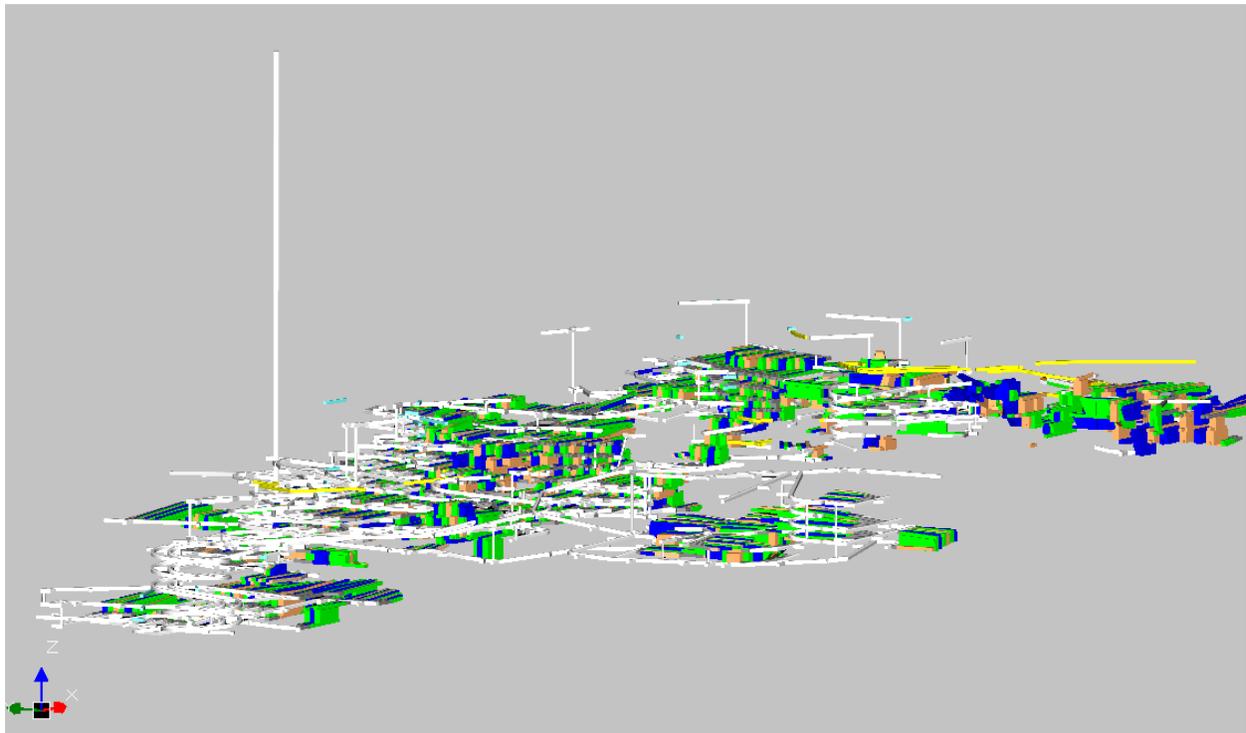


Figure 6.2: View of MineX underground operations in Deswik showing development (white) and stopes with associated secondary development (green, blue, and orange)

6.2 Model Data and Specifications

The RCPSP formulation of the underground mine production scheduling problem has previously been intractable with the number of variables presented herein. The author solves and reports the results of three versions of model (\mathcal{Z}^i) using the OMP Solver.

- No Ventilation (*No Vent* or \mathcal{Z}^1)
- Mine Ventilation (*Mine Vent* or \mathcal{Z}^2)
- Ventilation Domains (*Domains* or \mathcal{Z}^3)

The No Ventilation (*No Vent*) model (\mathcal{Z}^1) provides the production schedule without the ventilation constraints (4.8) and (4.9) from Section 4.4. The second model, (\mathcal{Z}^2), incorporates the ventilation constraint (4.8) which provides an upper bound on the amount of airflow through the mine. The final model (*Domains*), (\mathcal{Z}^3), use the ventilation constraint (4.8) and also provides an upper bound on the airflow to each of the three distinct *Domains* within the mine using constraint (4.9). The ventilation domain constraint limits the amount of airflow that can be directed to specific areas within the mine.

All data used in the three versions of model (\mathcal{Z}^i) are the same with the exception of the ventilation parameters for (\mathcal{Z}^2) and (\mathcal{Z}^3). Within *Mine Vent* and *Domains*, the author modifies the ventilation parameters to evaluate three different required airflow estimation methods. These methods assign the required airflow needed to dilute diesel particulate matter, measured in cubic feet per minute (cfm), for each activity. These methods are:

- *Fixed* - all activities are assigned 25,000 cfm
- *Class* - all activities are assigned airflow values ranging from 0 to 35,000 cfm based on an assigned classification, as shown in Table 6.2
- *Max Equip* - all activities are assigned airflow values ranging from 0 to 41,000 cfm based on the equipment manufacturer-estimated DPM. The maximum airflow value assigned to a piece of equipment used in a designated activity is assigned to the activity, as shown in Table 6.3

The Fixed method is the least restrictive of the methods, while Max Equip is the most restrictive. Class is most realistic; however, validation of the estimation methods is difficult. Opportunities to validate and improve the estimation methods are discussed in Section 8.1.

The results focus on the seven scenarios outlined below and consist of two parts: a comparison of the solution times for CPLEX and OMP Solver, and comparison of the OMP Solver schedules to the manual schedule. Section 7.1 discusses the results regarding how the solvers performed with the detailed model, presents the results obtained using (\mathcal{Z}^i) compared with the manual schedule developed by MineX, and discusses the findings. The seven scenarios are:

- *No Vent*
- *Mine Vent - Fixed*
- *Mine Vent - Class*
- *Mine Vent - Max Equip*
- *Domains - Fixed*
- *Domains - Class*
- *Domains - Max Equip*

The data and model specifications for (\mathcal{Z}) are outlined in the proceeding sections. The data, provided by MineX, includes resource information for all scheduled activities and upper bounds for all resource constraints.

6.2.1 Time Horizon and Fidelity

The time horizon for this research is limited to a maximum of two years. Shorter time horizons were also evaluated for solve times comparisons between CPLEX and the OMP Solver. The time fidelity is daily. Therefore, for a time horizon of two years, there are 730 days of planning.

6.2.2 Activities

The data set for MineX consists of 24,016 unique activities. Each activity represents an area of the mine that is to be developed, mined, or backfilled. The activities found in areas 1, 2, and 3 are listed in Appendix A in Table A.1, Table A.2, and Table A.3, respectively.

6.2.3 Activity Types

Each activity is identified as belonging to an activity type, which, in turn, maps it to a specific usage of each resource. The activity types consist of the following:

- \mathcal{PD} : primary development activities
- \mathcal{ED} : exploration development activities
- \mathcal{SD} : secondary development activities

- SM : stope mining activities (i.e., stopes, cut-fill, up-hole, floor pull)
- PB : paste backfill activities
- CB : cemented rock backfill activities
- RB : unconsolidated rock backfill activities
- VD : vertical development activities
- DL : drilling activities
- NA : activities that are scheduled but do not consume resources

6.2.4 Design Elements

The activities are assigned one of 46 design elements that represent the mining method or infrastructure type. Table 6.1 shows the design elements used in the schedule.

6.2.5 Resource Data

The resource data provided by MineX consists of the physical characteristics for each activity. These include:

- Grade
- Tons
- Footage
- Activity duration
- Physical precedence
- Delay or overlap time

Grade, Tons, Footage

The average grade for the activity is reported in ounces per ton. The grade represents the entire material tonnage associated with the activity. A unique cutoff grade is applied to both development and stope mining activities. Activities below cutoff are treated as containing no metal and the grade is set to zero. Activities that involve the removal or placement of material

are measured in short tons. Development activities, including secondary, exploration, access, etc., are measured in linear feet.

Cutoff grades were applied to the data to separate ore and waste. The cutoff grades of 0.12 and 0.085 ounces per ton were applied to mining and development activities, respectively.

Activity Duration

Each activity is assigned a duration, which represents the number of days needed to complete the activity. The duration is based on the production rate of each activity. All calculated activity durations are rounded up to the nearest integer value.

Precedence

The activity precedence in Deswik provides the necessary sequencing of activities. For example, the top and bottom cuts must be constructed prior to the vent raise, and the vent raise must be completed before mining the stope. The precedence structure ensures that the required infrastructure is in place before an activity can be scheduled. For example, a stope cannot be mined prior to the completion of preceding secondary development or a stope must be backfilled before an adjacent stope can be mined.

Delay and Overlap

Some activities require a time delay before the successor activity can begin. Delays allow backfill time to set up (when cement or paste backfill is used) before mining nearby stopes. The delay time does not utilize resources, i.e., backfilling capacity. Overlap provides the ability for activities to occur simultaneously once the predecessor activity has reached a specified stage of completion. For example, a stope can be mined to 70 percent completion at which point backfilling may begin. This functionality allows the model to better represent the true operational environment.

Modifications to Resource Data

The data must be formatted according to CPLEX and OMP solver specifications. Specifically, each activity must have associated with it the quantity that is developed, mined, or backfilled for a single day.

6.2.6 Ventilation

The ventilation constraint consists of the total quantity of air within the mine or domain, a fixed quantity of air to incorporate nonscheduled needs, and the ventilation consumed by each activity. The total quantity of air pulled through the mine via the two underground fans is 1.25 million cfm.

The fixed air quantity accounts for other areas of the mine that have ventilation needs but are not scheduled. These include maintenance areas, offices, and other non-scheduled activities. The fixed ventilation factor for the schedules presented in this dissertation are assumed to be 10 percent of total air for the mine. Recirculation of air can be incorporated into the ventilation constraint; however, determining the amount of recirculation for a schedule is an approximation at best, and is not considered in (\mathcal{Z}).

The model does not determine the airflows for the mine. Ventilation systems are networks which are continuously changing based on mining activities, equipment movement, and the opening and closing of doors and regulators at any given time. This reality makes modeling the ventilation system in conjunction with the development of a long- or mid-term production schedule difficult. Ventilation issues such as recirculation, booster fan placement and sizing, regulator placement and sizing, and/or exact ventilation airflow are not evaluated in model (\mathcal{Z}).

Two ventilation models were evaluated for this project in addition to the model without ventilation considerations. The first ventilation model applies an upper bound to the amount of ventilation available to the entire mine. The second model utilizes three ventilation *Domains* that are assigned by zones within the deposit. The upper bounds for the ventilation domain constraints are shown in Table 6.5.

Diesel Particulate Matter

DPM is recognized by various health and safety regulatory agencies as a potential carcinogen. Presented are three methods for estimating the required airflow to dilute the diesel particulate matter below the regulatory limit. Challenges associated with measuring DPM

production have led to the utilization of three estimation methods: class, fixed, and max equipment. Table 6.4 shows the airflow assignment by activity for the three airflow estimation methods.

Class

All activities are assigned one of three classifications based on the historical ventilation needs provided by MineX engineers. The classifications correspond to the estimated quantity of airflow required for the activity, as shown in Table 6.2.

Fixed

The second method uses a fixed airflow quantity of 25,000 cfm for all activities that utilize diesel-powered equipment.

Max Equipment

The final airflow estimation method utilizes equipment manufacturer data. The average estimated air quantity provided by manufacturers is listed in Table 6.3. The airflow assigned to each activity is based on the equipment used during the activity. For many activities, the loader requires the highest cfm; therefore, activities that utilize a loader are assigned the value of 41,000 cfm.

6.2.7 Capacity Constraints

MineX's resource constraints provide upper bounds regarding the amount of tonnage, length of footage, number of concurrent activities, and ventilation available for each time period. Using the activity types, the constraints provide the necessary bounds on (\mathcal{Z}) . Table 6.5 provides lower and upper bounds based on the activity types. These bounds can be adjusted by time period.

Table 6.1: Design Elements and Activity Types

Design Element	Abbreviation	Description	Activity Type
AXS		Access x-cut to stope	<i>PD</i>
BF TRANSFER		Backfill transfer	<i>PD</i>
BOT-CUT		Bottom cut	<i>SD</i>
BYPASS		Haulage bypass	<i>PD</i>
CUT-FILL		Cut and fill mining method	<i>SD</i>
DDS		Diamond Drilling station	<i>ED</i>
DECLINE		Decline ramp	<i>PD</i>
DEWATERING		Dewatering station	<i>PD</i>
DRILLING		Drilling station	<i>DL</i>
ESCAPE RAISE		Escape raisebore	<i>VD</i>
ESCAPEWAY		Escapeway access	<i>PD</i>
EXPLORATION		Exploration drift	<i>ED</i>
FLOOR-PULL		Floor pull - mining	<i>SD</i>
FUEL BAY		Fuel bay	<i>PD</i>
GOB-FILLING		GOB-filling	<i>RB</i>
GOB-JAMMING		GOB-jamming	<i>RB</i>
INT MUCKBAY		Internal muckbay	<i>PD</i>
JAMMING		Jamming	<i>CB</i>
LEVEL		Level access	<i>PD</i>
LOAD CENTER		Load center	<i>PD</i>
MCC		Motor control center	<i>PD</i>
MID-CUT		Middle cut	<i>SD</i>
MOS		Mobile operator station	<i>PD</i>
MUCKBAY		Muckbay	<i>PD</i>
OP RAISE		Ore pass raisebore	<i>VD</i>
ORE PASS		Ore pass access	<i>PD</i>
PASTE		Paste access	<i>PD</i>
PASTE-FILLING		Paste filling	<i>PB</i>
RAISE-LINING		Raisebore lining	<i>NA</i>
RAMP		Ramp	<i>PD</i>
REFUGE		Refuge cutout	<i>PD</i>
RUN AROUND		Run around	<i>PD</i>
SHAFT STATION		Shaft station	<i>PD</i>
SHOP		Shop	<i>PD</i>
SLABBING		Stope slabbing	<i>SD</i>
SLURRY-FILLING		Slurry backfill	<i>SB</i>
STOPE-FILLING		Stope filling	<i>CB</i>
STOPE-MINING		Stope mining	<i>SM</i>
SUMP		Sump	<i>PD</i>
TOP-CUT		Top cut	<i>SD</i>
TRK LOAD-OUT		Truck loadout	<i>PD</i>
UP-HOLE		Up hole mining	<i>SM</i>
UTILITY		Utility cutout	<i>PD</i>
VENT		Vent access	<i>PD</i>
VENT RAISE		Vent raisebore	<i>VD</i>
VENT SHAFT		Vent shaft	<i>NA</i>

Table 6.2: Classification

Classification	Air Quantity, cfm
1	15,000
2	25,000
3	35,000

Table 6.3: Equipment Manufacturer Average Airflow Requirement for DPM Dilution below $160 \mu\text{g}/\text{m}^3$

Equipment Type	Average Airflow (cfm)
3yd loaders	34,000
6yd loaders	41,000
9yd loaders	41,000*
Batch Trucks	32,000
Bolters	32,000
Charging	32,000
Haul Trucks	32,000
Jammers	29,000
Jumbos	25,000
Light Vehicle	25,000
Longhole Drill	19,000
Lube Truck	30,000
Scissors	25,000
Service Equip	23,000
Shotcrete	32,000
Utility Trucks	31,000
Water Truck	28,000

* No information available; the 6yd loader value is assumed.

Table 6.4: Air Quantity Assignment by Design Element

Design Element	Class (cfm)	Fixed (cfm)	Max Equip. (cfm)
AXS	25,000	25,000	41,000
BF TRANSFER	25,000	25,000	41,000
BOT-CUT	25,000	25,000	41,000
BYPASS	25,000	25,000	41,000
CUT-FILL	25,000	25,000	41,000
DDS	25,000	25,000	41,000
DECLINE	25,000	25,000	41,000
DEWATERING	25,000	25,000	41,000
DRILLING	35,000	25,000	19,000
ESCAPE RAISE	15,000	25,000	15,000
ESCAPEWAY	25,000	25,000	41,000
EXPLORATION	25,000	25,000	41,000
FLOOR-PULL	35,000	25,000	41,000
FUEL BAY	25,000	25,000	41,000
GOB-FILLING	35,000	25,000	41,000
GOB-JAMMING	35,000	25,000	29,000
INT MUCKBAY	25,000	25,000	41,000
JAMMING	35,000	25,000	29,000
LEVEL	25,000	25,000	41,000
LOAD CENTER	25,000	25,000	41,000
MCC	25,000	25,000	41,000
MID-CUT	25,000	25,000	41,000
MOS	25,000	25,000	41,000
MUCKBAY	25,000	25,000	41,000
OP RAISE	15,000	25,000	15,000
ORE PASS	25,000	25,000	41,000
PASTE	25,000	25,000	32,000
PASTE-FILLING	15,000	25,000	32,000
RAMP	25,000	25,000	41,000
REFUGE	25,000	25,000	41,000
RUN AROUND	25,000	25,000	41,000
SHAFT STATION	25,000	25,000	41,000
SHOP	25,000	25,000	41,000
SLABBING	25,000	25,000	41,000
SLURRY-FILLING	15,000	25,000	32,000
STOPE-FILLING	35,000	25,000	32,000
STOPE-MINING	35,000	25,000	41,000
SUMP	25,000	25,000	41,000
TOP-CUT	25,000	25,000	41,000
TRK LOAD-OUT	25,000	25,000	41,000
UP-HOLE	35,000	25,000	19,000
UTILITY	25,000	25,000	41,000
VENT	25,000	25,000	41,000
VENT RAISE	15,000	25,000	15,000

Table 6.5: Resource Constraint Bounds

Constraint	Activity Types	Upper Bound	Units
Tonnage Constraints	$\mathcal{PD} \ \mathcal{ED} \ \mathcal{SD} \ \mathcal{SM} \ \mathcal{CB} \ \mathcal{RB} \ \mathcal{PB}$	11,000	tons/day
	$\mathcal{CB} \ \mathcal{PB}$	5,000	tons/day
	\mathcal{RB}	2,500	tons/day
Ore Tonnage	$\mathcal{PD} \ \mathcal{SD} \ \mathcal{SM}$	6,000	tons/day
Footage	$\mathcal{PD} \ \mathcal{ED} \ \mathcal{SD}$	155	feet/day
	\mathcal{DL}	1,500	feet/day
Concurrent Activities	\mathcal{VD}	1	activity/day
Ventilation	Mine	1,250	kcfm/day
	Domain 1	550	kcfm/day
	Domain 2	300	kcfm/day
	Domain 3	450	kcfm/day

CHAPTER 7

RESULTS AND DISCUSSION

In this chapter, the two solution methods are compared and evaluated based on solution time and quality. The solutions from the three versions of model (\mathcal{Z}) are compared to the manual schedule. Analysis of the ventilation airflow estimation methods based on DPM production are reviewed and discussed.

7.1 Solvers

This section compares the solution time and solution quality for CPLEX and the OMP Solver solutions using the *No Vent* model and various time horizon lengths. This comparison shows the solutions times for both the linear programming (LP) solution and integer programming (IP) solution of model (\mathcal{Z}). The solution quality represents the percent difference, or gap, between the LP and IP solutions. The gap represents how close the integer programming solution is to the optimal linear programming solution. The linear solution is not feasible or implementable for the mine because of the relative timing of certain activities; however, the comparison does give some understanding of how close the integer solution is to optimal.

7.1.1 Solution Time Comparison for CPLEX and OMP Solver

CPLEX is unable to solve (\mathcal{Z}^1) for time horizons greater than 180 days due to memory issues associated with the large number of variables and constraints. Table 7.1 shows the CPLEX and OMP solution times for various time horizons. While the OMP Solver is able to solve the 730-day time horizon in less time than CPLEX can solve the 90-day option, this increase in speed does come with some loss in precision. The gap between the LP and IP solutions for the OMP Solver is greater than the ≤ 1 percent gap with CPLEX.

The increased gap is attributed to the fidelity of the data and potential inefficiencies in the TopoSort heuristic.

Table 7.1: Comparison of Results: CPLEX vs. OMP Solver Solution Times

Time Horizon	Solver	LP Solution Time (sec)	IP Solution Time (sec)	Total Time (sec)
30-day	CPLEX	1	5	6
90-day	CPLEX	55	24,061	24,116
180-day	CPLEX	– **	–	–
30-day	OMP	1	<1*	2
90-day	OMP	15	< 1*	16
180-day	OMP	107	< 1*	118
365-day	OMP	1,878	1*	1,904
730-day	OMP	23,536	4*	23,592

* Toposort heuristic

** Unable to solve due to memory issues

7.1.2 Solution Quality Comparison for CPLEX and OMP Solver

The author evaluates the solution quality by comparing the solutions from CPLEX and the OMP Solver to determine how close the integer solution is to the solution of the LP relaxation of the model. The CPLEX solver uses the Simplex algorithm for the LP and the branch-and-bound algorithm for the IP. OMP Solver uses a variation of the Bienstock-Zuckerberg algorithm [61] to determine the LP relaxation, and heuristics to determine an integer solution.

The comparison shows that while CPLEX is able to find an integer solution that is much closer to the linear programming relaxation solution for time horizons of fewer than 180 days, the required solution time is significantly greater than that required by the OMP Solver, Table 7.2.

The OMP Solver produces “good” solutions in a much shorter amount of time and is able to solve much larger instances of the model. This decrease in solution time and ability to solve longer time horizons provides a great deal of benefit to mine planners, allowing them

to evaluate numerous feasible planning options in a shorter amount of time.

Table 7.2: Comparison of Results: CPLEX vs. OMP Solver Objectives and Gap

Time Horizon	Solver	LP Objective (discounted metal)	IP Objective (discounted metal)	LP/IP Gap
30-day	CPLEX	58,522	57,992	0.9%
90-day	CPLEX	155,517*	152,082	2.3%
180-day	CPLEX	– **	–	–
30-day	OMP	59,129	49,946	18.4%
90-day	OMP	155,181	133,145	16.6%
180-day	OMP	304,180	257,358	18.2%
365-day	OMP	626,383	539,429	16.1%
730-day	OMP	1,179,941	1,034,275	14.1%

* Unable to solve to <1% optimality due to memory issues

** Unable to solve due to memory issues

7.1.3 Solution Time Comparison for 730-day Schedules

Table 7.3 provides solution times for the three versions of model (\mathcal{Z}), i.e., *No Vent*, *Mine Vent*, and *Domains* and three ventilation airflow estimation methods solved using the OMP Solver. The solution times range from approximately 3 to 12 hours. The shortest solution time corresponds to that of *Mine Vent - Max Equip* schedule, which is highly constrained and the ventilation estimation method is the most conservative. The longest solution time manifests itself in the *Domains - Fixed* schedule, which possess the most degrees of freedom in its choice of selection, and where there are non-obvious trade-offs.

7.1.4 Solution Quality Comparison for Schedule Scenarios

Solution quality for the seven scenarios is presented in Table 7.4. The LP objectives show similar results for the 730-day time horizon. The gap is suspected to be the result of lumpy knapsacks, i.e., when the IP solution is determined, the fidelity of the data creates slack within the resource constraints. The resulting solutions are better than what manual methods or CPLEX can yield.

Table 7.3: Solution Times for 730-day Schedules

Time Horizon	Model	Airflow Estimation Method	LP Solution Time (sec)	IP Solution * Time (sec)	Total Time (sec)
730-day	<i>No Vent</i>		23,536	4	23,592
730-day	<i>Mine Vent</i>	<i>Fixed</i>	25,944	7	26,062
730-day	<i>Mine Vent</i>	<i>Class</i>	13,943	4	14,006
730-day	<i>Mine Vent</i>	<i>Max Equip</i>	11,496	5	11,561
730-day	<i>Domains</i>	<i>Fixed</i>	42,752	6	42,838
730-day	<i>Domains</i>	<i>Class</i>	29,327	6	29,414
730-day	<i>Domains</i>	<i>Max Equip</i>	21,227	6	21,314

* Toposort heuristic

7.2 Significance of Solutions

The solutions obtained from the OMP Solver are not easily understood and must be imported into visualization software. Deswik ([67, 68]) is a commercially available mine design and scheduling software package that can visually display three-dimensional activity layouts or a mine production schedule in Gantt chart format. Evaluating the solutions in Deswik allows for the undiscounted metal production, in addition to other resources, to be evaluated on a monthly basis. The manual schedule provided by MineX was developed manually using Deswik; therefore, solution evaluations were done using the same software. The daily solutions obtained from (\mathcal{Z}) were imported into Deswik and displayed on a monthly basis for ease of comparison.

7.2.1 Manual vs. *No Vent*

Solutions are presented first by comparing the manual MineX schedule to that obtained from OMP using the *No Vent* schedule. MineX did not use any ventilation considerations when developing the manual schedule; therefore, no ventilation considerations were included in *No Vent*.

Table 7.4: Solution Objectives for 730-day Schedules

Time Horizon	Model	Airflow Estimation Method	LP Objective (discounted metal)	IP Objective * (discounted metal)	LP/IP
730-day	<i>No Vent</i>		1,179,941	1,034,275	14%
730-day	<i>Mine Vent</i>	<i>Fixed</i>	1,160,482	1,030,200	13%
730-day	<i>Mine Vent</i>	<i>Class</i>	1,105,413	977,463	13%
730-day	<i>Mine Vent</i>	<i>Max Equip</i>	925,696	837,381	11%
730-day	<i>Domains</i>	<i>Fixed</i>	1,135,280	984,727	15%
730-day	<i>Domains</i>	<i>Class</i>	1,078,285	943,382	14%
730-day	<i>Domains</i>	<i>Max Equip</i>	904,695	803,757	13%

* Toposort heuristic

Comparing MineX’s manual schedule to the *No Vent* schedule obtained with the OMP Solver shows a significant increase in metal production. Over the 730-day time horizon, the cumulative metal production for the *No Vent* solution is 23 percent greater than that of the manual schedule, as shown in Figure 7.1.

In addition to an increase in metal production, the monthly ore tonnage removed from month to month is more consistent when compared to the manual schedule (Figure 7.2). In fact, the ore production in the manual schedule exceeds the monthly maximum allowable on numerous occasions. Figure 7.3 shows that the manual schedule also exceeds the development footage capacity, while that given in the *No Vent* solution remains fairly level and consistent throughout the time horizon. The leveling of resource consumption or production consistency allows the mine to more efficiently utilize labor and equipment resources.

The comparisons in Figure 7.1, Figure 7.2, and Figure 7.3 show that the solutions obtained using (\mathcal{Z}) and the OMP Solver provide significant improvement over the manual schedule. The increase in metal production demonstrates the potential for significant economic benefit of optimization techniques on underground mine scheduling.

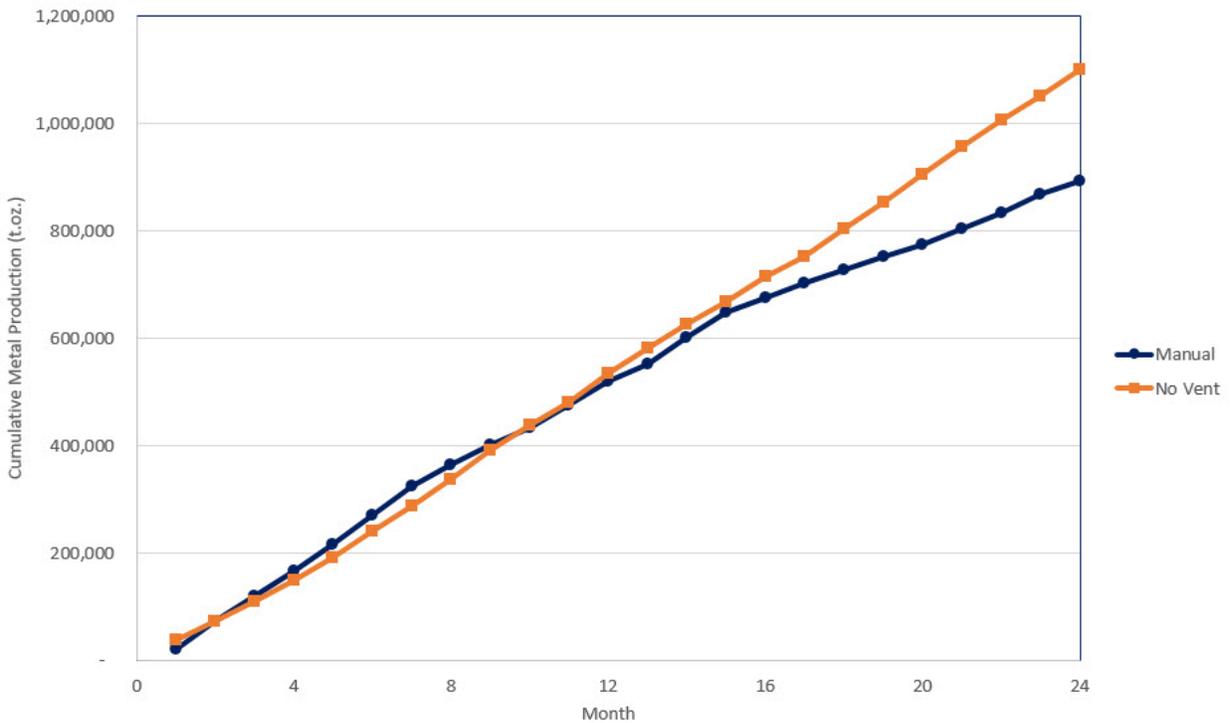


Figure 7.1: Cumulative Metal Production for *No Vent* Schedule Compared to the Manual Schedule

7.2.2 Solutions with Ventilation

Two versions of model (\mathcal{Z}), *Mine Vent* and *Domains*, are compared using the three airflow estimation methods: *Fixed*, *Class*, and *Domains*.

First, the author post-processes the *Class* ventilation parameter to the manual and *No Vent* schedules. Applying the *Class* ventilation parameters to the manual schedule, Figure 7.4 shows that ventilation capacities would be exceeded for more than half of the two-year time horizon. Although the *No Vent* schedule does not include any ventilation constraints, when the *Class* ventilation parameter is applied, the results show that the maximum ventilation capacity is rarely exceeded.

Next, the author compares the manual schedule to the three ventilation parameters for the *Mine Vent* model. Ore production values, shown in Figure 7.6, remain fairly consistent throughout the time horizon for all of the *Mine Vent* models.

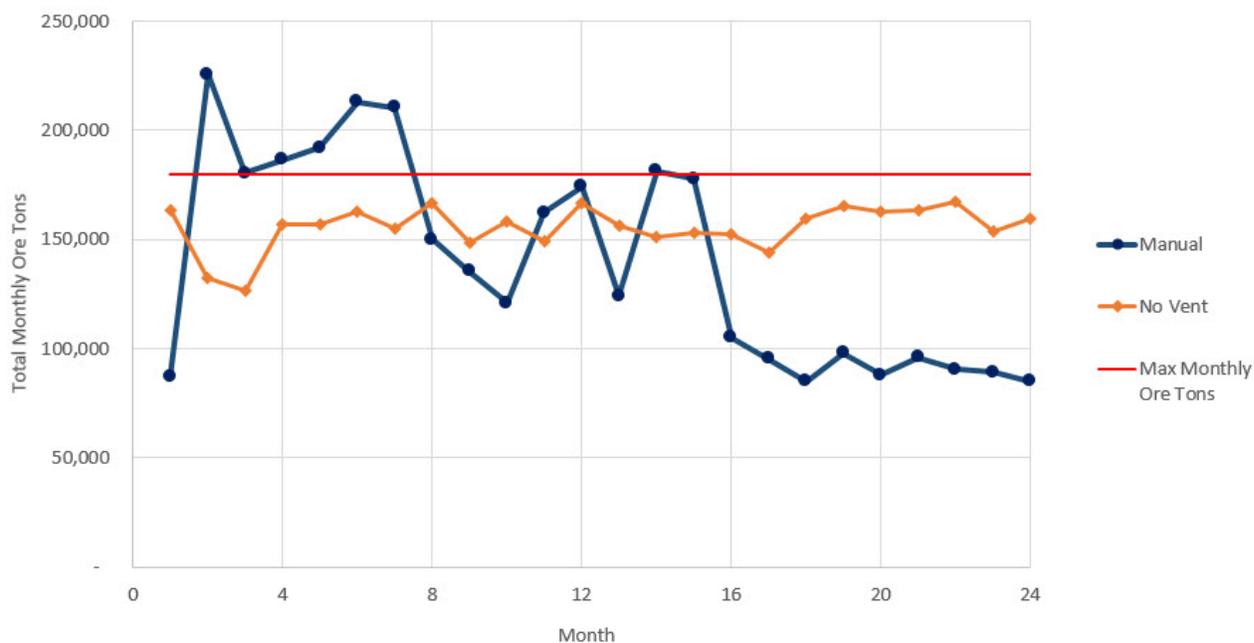


Figure 7.2: Monthly Ore Tonnage for *No Vent* Schedule Compared to Manual Schedule

Figure 7.5 shows that the ventilation resource constraint for the *Mine Vent* schedules provides fairly consistent ventilation consumption for the three ventilation estimation parameters. In Figure 7.6, the effects of the different ventilation estimations on ore tonnage production are apparent. *Max Equip* is the most restrictive of the ventilation airflow estimation methods and produces the least amount of ore tonnage, as expected.

Finally, the author compares the manual schedule to the *Domains* model results. Figure 7.7 shows a fairly level monthly ore production with slightly lower production values for the various airflow estimation methods when compared to the *Mine Vent* model. This reduction is attributed to the more restrictive domain ventilation constraints. The ventilation usage for the *Domains* scenarios, shown in Figure 7.8, is very similar to the ventilation consumption for the *Mine Vent* scenarios which indicates that the ventilation domain constraints are further constraining the production schedule resulting in decreased ore production.

The effect of the ventilation constraint on the *Mine Vent-Class* schedule is evident when compared to the cumulative metal production for the *No Vent* and manual schedules in

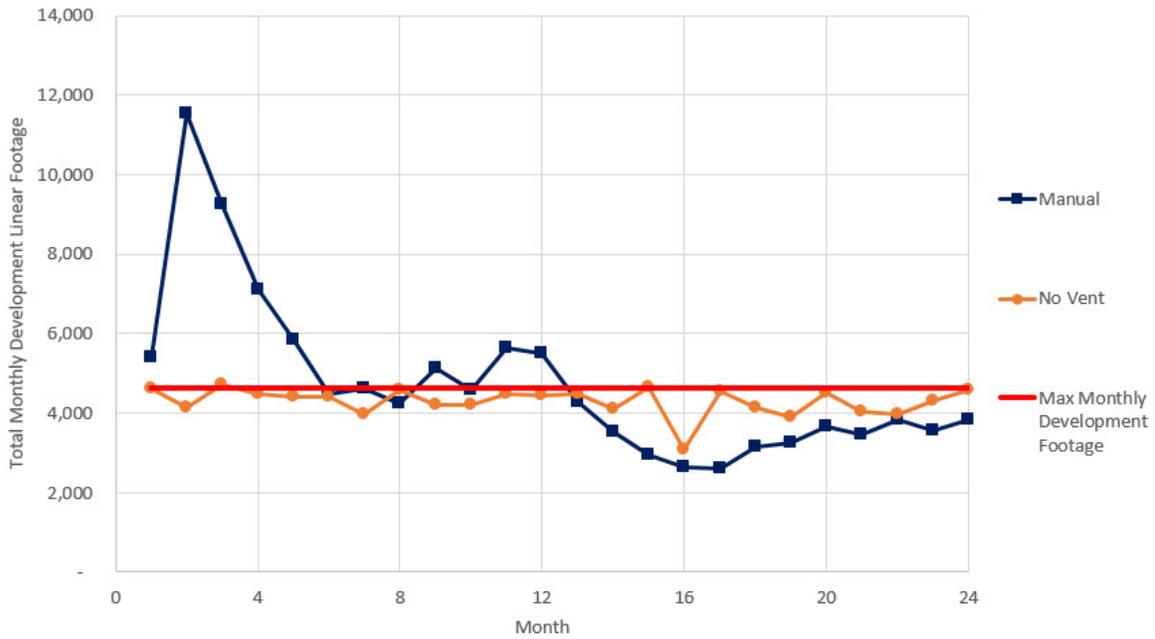


Figure 7.3: Monthly Development Footage for *No Vent* Schedule Compared to Manual Schedule

Figure 7.9. The *Mine Vent-Class* schedule provides more metal production over the two-year time horizon than the manual schedule but less than the *No Vent* schedule, indicating that the ventilation constraint limits ore production when applied to model (\mathcal{Z}).

7.2.3 Validation

Production schedules are routinely analyzed by experienced engineers to determine their validity. Planners at MineX have evaluated the solutions obtained from the OMP Solver and have indicated that the results are implementable. The solutions obtained from the OMP Solver are not meant to replace the experience and knowledge of a mine planner, but to provide the ability to quickly produce and evaluate schedule scenarios. Ultimately, the schedule provides a map or sequence for engineers to follow when producing more detailed schedules.

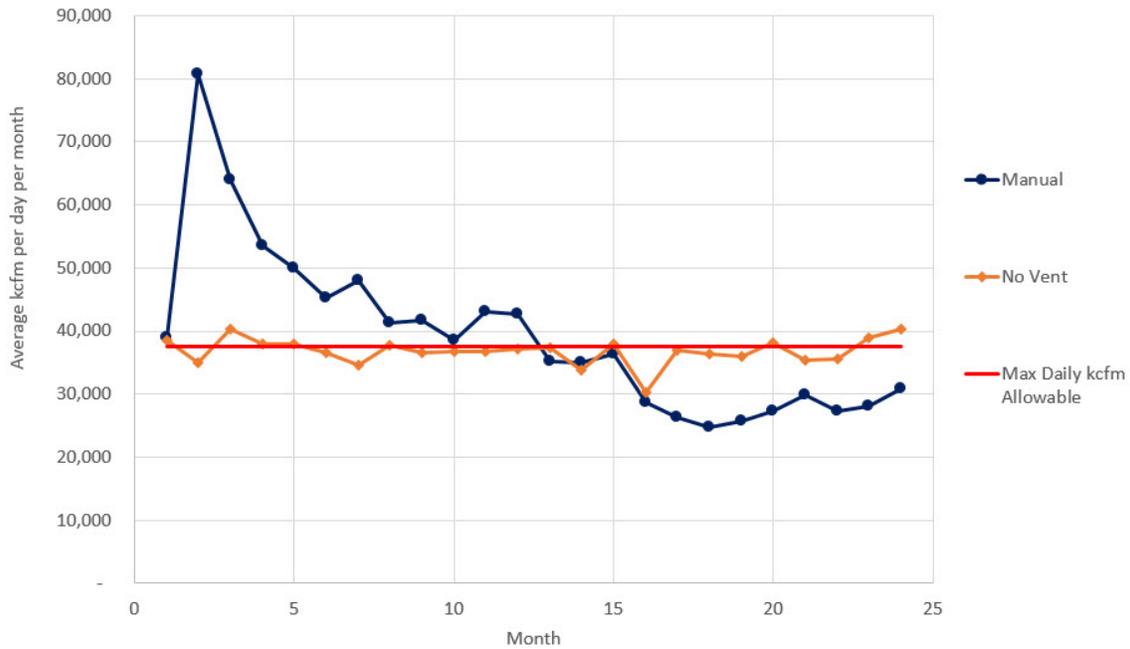


Figure 7.4: Monthly Ventilation Resource Consumption for *No Vent* and the Manual Schedule when the *Class Ventilation Parameter* is assigned *a posteriori*

7.3 Discussion

Comparing all seven scenarios to the manual schedule shows that the solutions obtained using model (\mathcal{Z}) and the OMP Solver provide a significant benefit to metal production over that of the manual schedule. Figure 7.10 shows the discounted objective results for the scenarios as a percentage difference from the manual schedule. The *No Vent* schedule shows a 23 percent increase in metal production, while the *Vent Max* schedule shows a minor increase. The impact of the ventilation constraints is evident in the total metal production. For the *Domain-Max Equip* schedule, the metal production is approximately 4 percent less than the manual schedule.

The results presented show that the OMP Solver is able to provide “good” feasible schedules for the seven scenarios. The ability to solve this large integer programming model using the OMP Solver, with solution times ranging between 3 and 12 hours, makes the application of this formulation to the mining industry realistic. The information obtained

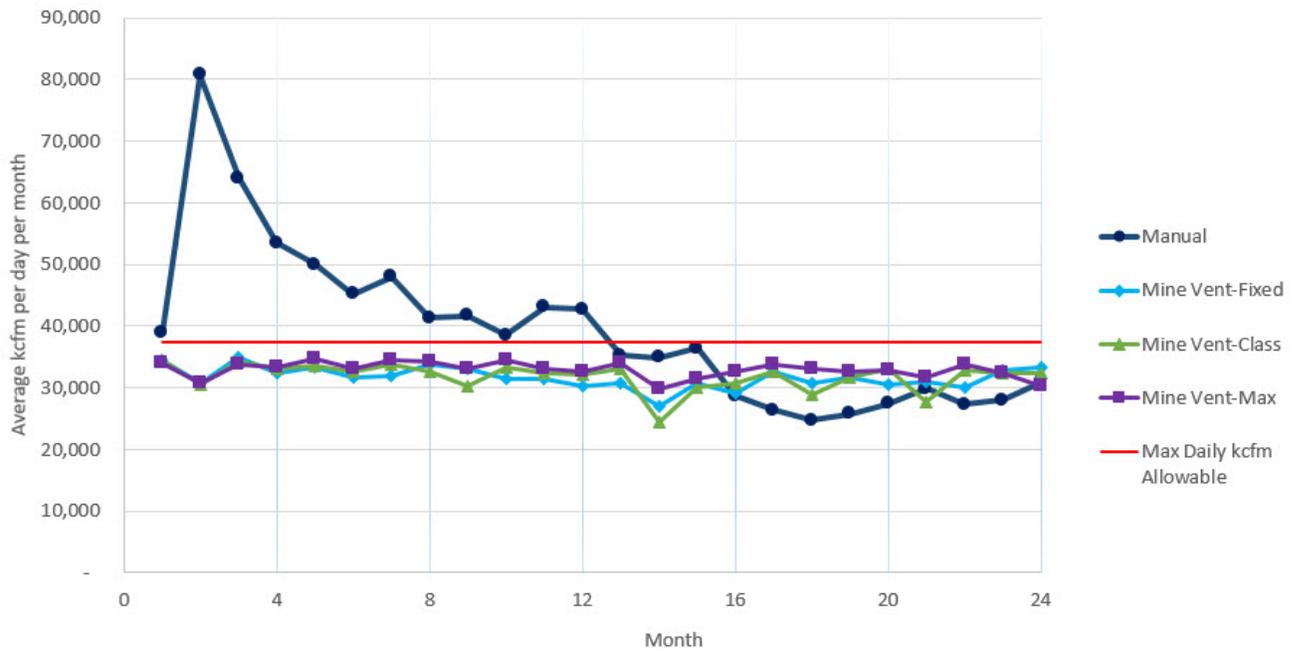


Figure 7.5: Monthly Ventilation Resource Consumption for *Mine Vent* Schedules Compared to Manual Schedule

from the solutions could also be used to determine what additional ventilation resources would need to be acquired and implemented to achieve specific metal production targets. The ability to solve multiple schedule scenarios in days, instead of weeks or months, could have a significant impact on underground mine planning and scheduling.

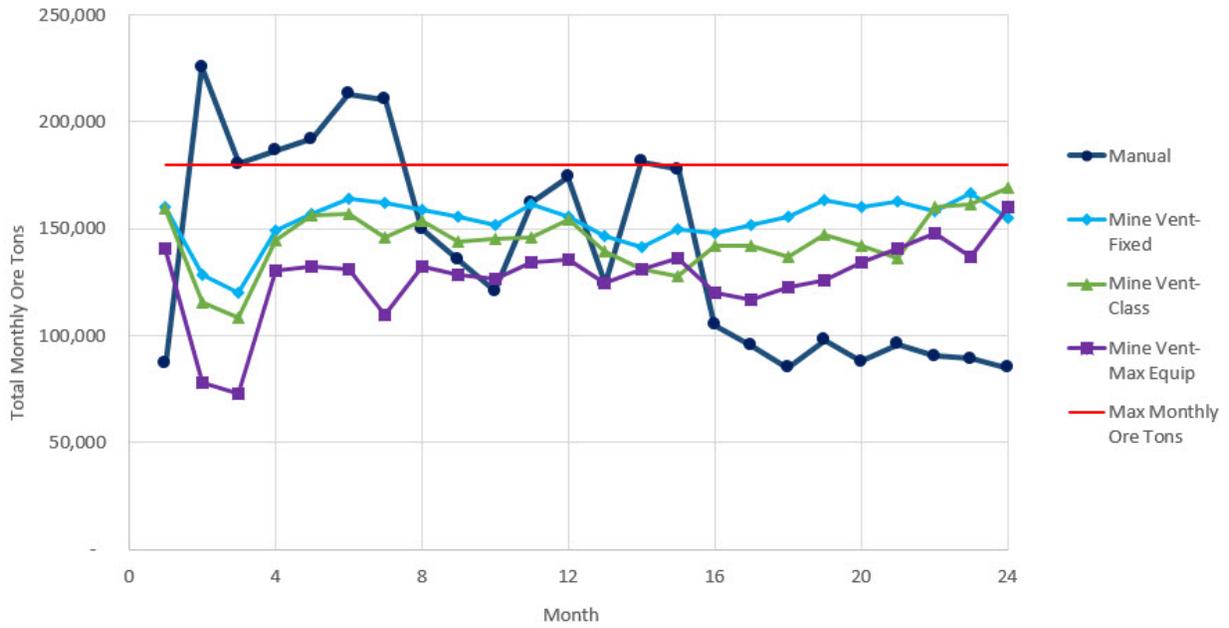


Figure 7.6: Monthly Ore Tonnage for *Mine Vent* Schedules Compared to Manual Schedule

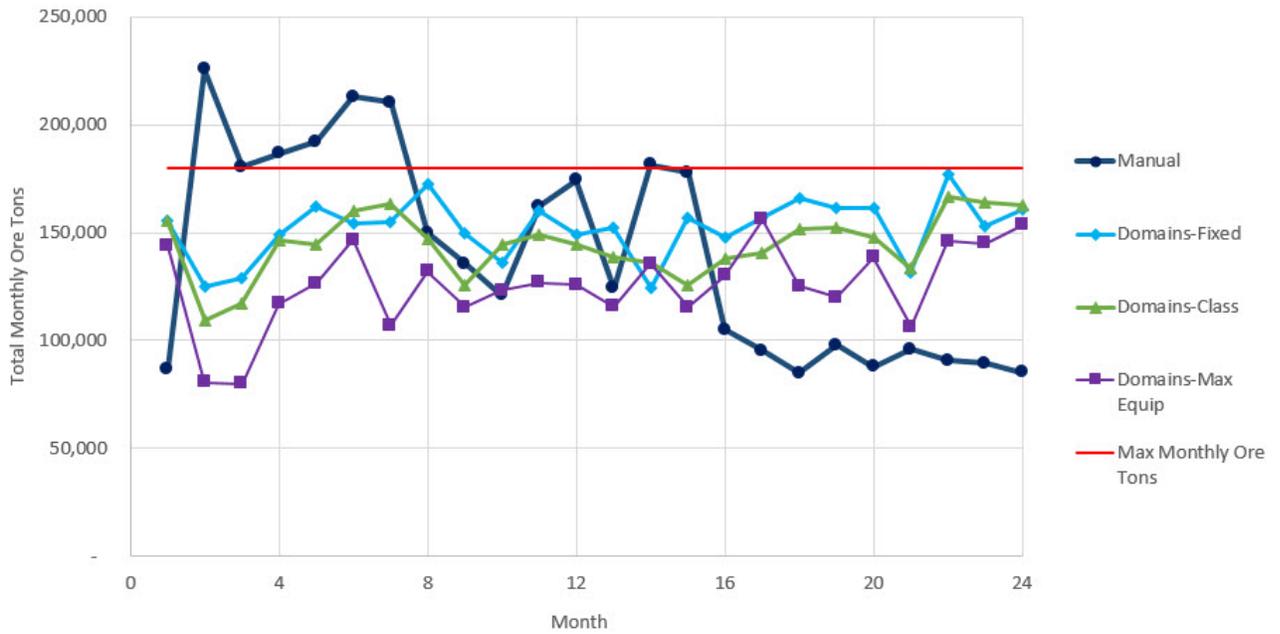


Figure 7.7: Monthly Ore Tonnage for *Domains* Schedules Compared to Manual Schedule

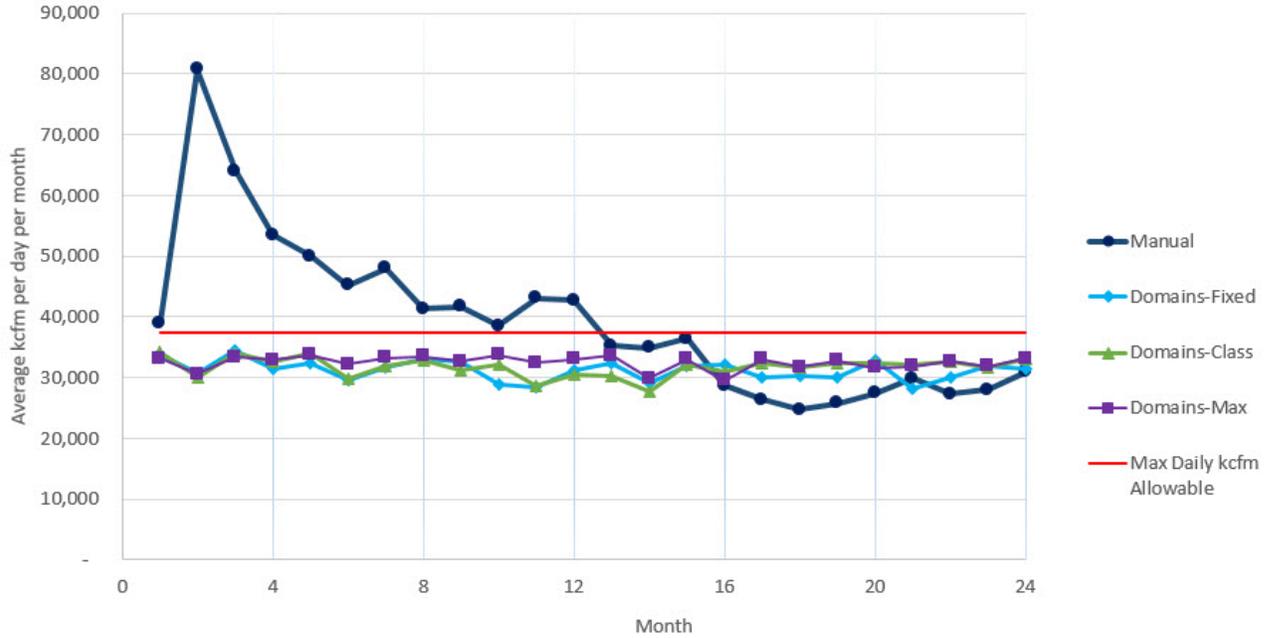


Figure 7.8: Monthly Ventilation Resource Consumption for *Domains* Schedules Compared to Manual Schedule

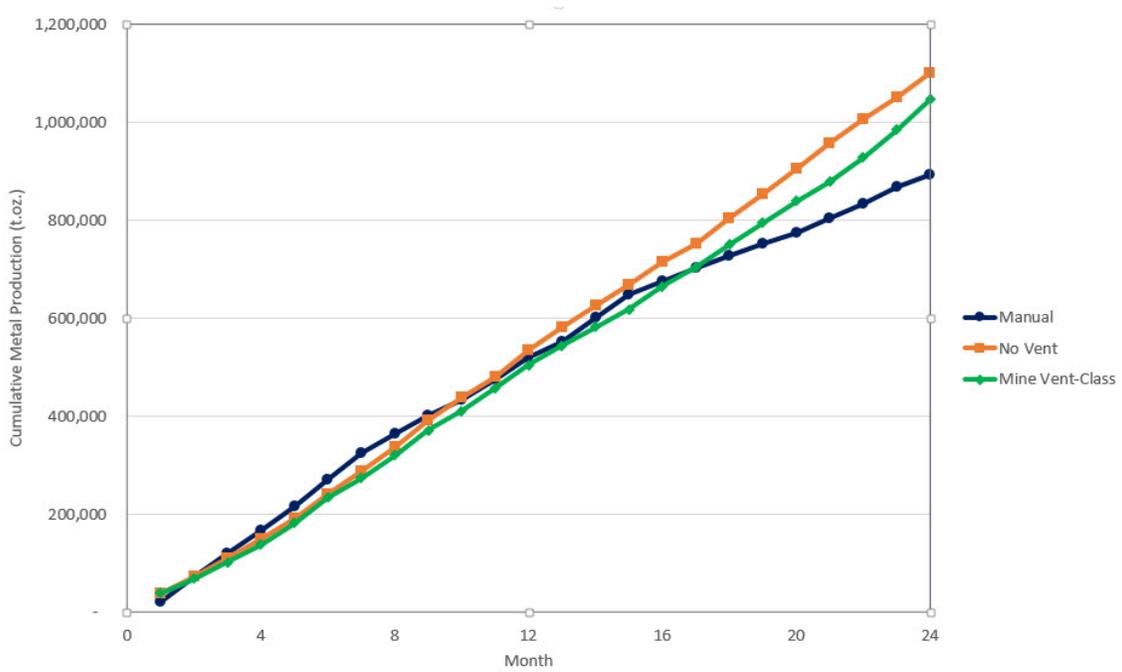


Figure 7.9: Cumulative Metal Production for *No Vent* and *Mine Vent-Class* Schedule Compared to the Manual Schedule

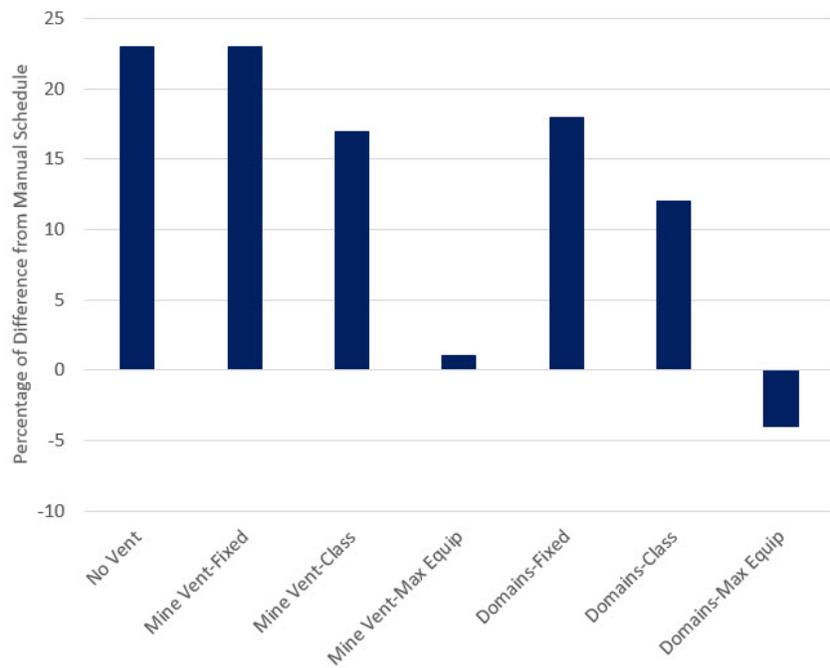


Figure 7.10: Percent Difference of Metal Production Between Model (\mathcal{Z}^i) Scenarios and the Manual Schedule for a Two-Year Time Horizon

CHAPTER 8

CONCLUSION AND FUTURE WORK

The purpose of this dissertation is to develop a mid-term, underground production schedule optimization model that incorporates ventilation constraints. The formulation presented in Chapter 4 is based on the resource constrained production scheduling problem, RCPSP. The author modifies the precedence constraint to incorporate delay and overlap duration between activities, providing a more implementable schedule.

The research hypothesis in 1.2 stated the following:

Solutions from the RCPSP can be used as a mid-term production schedule for underground stoping mining operations. Additionally, the inclusion of ventilation constraints with estimated activity airflow quantities to dilute diesel particulate matter will alter the resulting production schedule. Accounting for activity duration delay and overlap in the formulation provides a more operationally implementable schedule. The precedence constrained underground mine scheduling problem can be solved using the Bienstock-Zuckerberg algorithm and the TopoSort heuristic. This dissertation will address the validity of this hypothesis.

This dissertation shows that model (\mathcal{Z}) provides flexibility that would allow this formulation to be easily modified to accommodate other underground operations. Furthermore, the addition of overlap and delay functionality to the model provides solutions that better mimic the operational environment. The combination of model (\mathcal{Z}) and the OMP Solver produces schedules that contain more metal than the manual schedule. The ability to run scenarios quickly allows for more detailed scenario and sensitivity analysis. Additionally, the solutions show the impacts of the ventilation constraints on the metal production, and this information could be used as a predictive tool to determine when the ventilation system is inadequate.

The OMP Solver, in conjunction with the TopoSort heuristic, is capable of determining near-optimal solutions for problem sizes that are intractable in commercially available optimization software. The dissertation further shows that the OMP solver can produce feasible solutions for MineX with implementable results in a realistic time frame. The solutions present a schedule with increased metal production and significant leveling of resource consumption.

The solutions show that the use of the model (\mathcal{Z}) can provide significant benefit to MineX regarding metal production, scenario analysis, ventilation requirements, and sensitivity analysis.

8.1 Future Work

It is difficult to quantify the vast amount of future research in this area that could benefit the mining industry. Like a tree, the work presented here can provide the roots to help improve and grow the ability to solve complex underground production scheduling problems. The branches of this tree could include developing models that incorporate an even more holistic view of the underground mine, include various time horizons and fidelity, and accommodate uncertainty.

Ideas for future work are outlined below for three distinct areas: formulation, solver, and ventilation.

Formulation

Areas of future work with respect to the formulation of model (\mathcal{Z}) include suggestions on how to further validate and improve the model's capabilities. The author has identified the following:

- Test the model on various underground mines that utilize stoping methods.
- Test the model on various underground mines using non-stoping methods.
- Formulate the model to incorporate variable time fidelity to allow for scheduling to be representative of short-term through life-of-mine planning.

- Add resource balance constraints over a number of time periods to further level resource consumption.
- Incorporate stochastic elements into the formulation, starting with production rates and duration.
- Evaluate further the effects and potential benefit of aggregation and disaggregation.
- Allow variables to be continuous within limits (based on the data). Allow for non-continuous activity completion, e.g., mining a portion of a stope.
- Incorporate late starts on some backfilling activities or those with no predecessor to ensure proper timing of completion. Alter precedence constraints to force one activity to follow another within a certain time span.
- Translate model elements from deterministic to stochastic such as the duration of activities [69].

Ventilation

This dissertation shows that incorporating ventilation into production scheduling alters the solutions; however, ventilation is continuously changing within a mine. The author suggests the following ventilation-focused research:

- Incorporate the concept of air recirculation into ventilation constraints that would better represent the amount of airflow required for each activity.
- Validate and test the DPM production associated with each activity and continue to improve the airflow estimation methods.
- Evaluate the use of the ventilation constraint to determine the effects of air temperature, i.e., heat, on production.
- Develop a mixed non-linear programming model that would utilize a smaller time fidelity and work in conjunction with a Ventilation on Demand (VOD) system.

Solution Method

This dissertation shows that the combination of model (\mathcal{Z}) and the OMP Solver with the TopoSort heuristic provides “good,” feasible schedules for MineX. Throughout the process, areas of desired and/or potential improvement to the OMP Solver and TopoSort heuristic have been identified and are as follows:

- Reduce the gap between the LP relaxation and integer solution.

- Incorporate a sliding time window heuristic to assist with long-term planning.
- Include the ability to assign lower bounds on constraints.
- Evaluate the potential for concurrent aggregation and disaggregation of data during the solution process.

Open pit mining has seen the direct benefit of operations research; however, underground production schedule optimization has been limited due to the complexity of the problems [56]. This dissertation shows that the Bienstock-Zuckerberg algorithm can be used for underground production scheduling and that this application could have a lasting impact on how underground mines are scheduled.

APPENDIX A - ACTIVITY PRODUCTION RATES

Table A.1, Table A.2, and Table A.3 list the activities that are to be scheduled in each of the three areas that constitute MineX. Some activities have multiple rates to account for specific mining conditions; however, individual activities are assigned a single production rate. A description of the activity and the production rates are included.

Table A.1: Activities and Production Rates for Area 1

Activity	Description	Rate	Unit
BOT-CUT	Bottom-cut	4	ft/day
CUT-FILL	Cut and fill mining method	4	ft/day
DDS	Diamond drilling station	4	ft/day
DRILLING	Drilling station	350	ft/day
EXPLORATION	Drilling station	5	ft/day
FLOOR-PULL	Floor pull - mining	1000	miT/day
GOB-FILLING	Unconsolidated rock backfilling	850	FiT/day
GOB-JAMMING	Unconsolidated rock jamming	850	FiT/day
JAMMING	Jamming	850	FiT/day
LEVEL	Level access	5	ft/day
MID-CUT	Middle-cut	4	ft/day
MUCKBAY	Muckbay	5	ft/day
OP RAISE	Ore pass raisebore	8	ft/day
ORE PASS	Ore pass access	5	ft/day
RAISE-LINING	Raisebore lining	20	ft/day
RAMP	Ramp	5	ft/day
REFUGE	Refuge cutout	5	ft/day
SLABBING	Stope slabbing	10	ft/day
STOPE-FILLING	Cemented rock backfilling	850	FiT/day
STOPE-MINING	Stope - mining	1000	miT/day
SUMP	Sump	5	ft/day
TOP-CUT	Top-cut	4	ft/day
VENT	Vent access	5	ft/day
VENT RAISE	Vent raisebore	8	ft/day

Table A.2: Activities and Production Rates for Area 2

Activity	Description	Rate 1	Rate 2	Unit
AXS	Access cross-cut to stope	5		ft/day
BOT-CUT	Bottom-cut	4		ft/day
CUT-FILL	Cut and fill mining method	4		ft/day
DDS	Diamond drilling station	4		ft/day
DECLINE	Decline ramp	5		ft/day
DRILLING	Drilling station	350		ft/day
ESCAPE RAISE	Escape raisebore	8		ft/day
ESCAPEWAY	Escapeway access	5		ft/day
EXPLORATION	Exploration drift	5		ft/day
FLOOR-PULL	Floor pull - mining	1000		miT/day
GOB-FILLING	Unconsolidated rock backfilling	850		FiT/day
GOB-JAMMING	Unconsolidated rock jamming	850		FiT/day
INT MUCKBAY	Internal muckbay	3.5		ft/day
JAMMING	Jamming	850		FiT/day
LEVEL	Level access	5		ft/day
LOAD CENTER	Load center	5		ft/day
MID-CUT	Middle-cut	4		ft/day
MUCKBAY	Muckbay	3.5	5	ft/day
OP RAISE	Ore Pass raisebore	8		ft/day
ORE PASS	Ore Pass access	5		ft/day
PASTE-FILLING	Paste backfilling	850		FiT/day
RAISE-LINING	Raisebore lining	20		ft/day
RAMP	Ramp	5		ft/day
SLABBING	Stope slabbing	10		ft/day
SLURRY-FILLING	Slurry backfilling	850		FiT/day
STOPE-FILLING	Stope cemented rock backfilling	850		FiT/day
STOPE-MINING	Stope - mining	1000		miT/day
SUMP	Sump	5		ft/day
TOP-CUT	Top-cut	4		ft/day
TRK LOAD-OUT	Truck loadout	5		ft/day
UP-HOLE	Up-hole mining	1000		miT/day
VENT	Vent access	5		ft/day
VENT RAISE	Vent raisebore	8		ft/day

Table A.3: Activities and Production Rates for Area 3

Activity	Description	Rate 1	Rate 2	Unit
AXS	Access cross-cut to stope	5		ft/day
BACKFILL-RAISE	Backfill raisebore	8		ft/day
BF TRANSFER	Backfill transfer	5		ft/day
BOT-CUT	Bottom-cut	4		ft/day
BYPASS	Haulage bypass	5		ft/day
CUT-FILL	Cut and fill mining method	4		ft/day
DDS	Diamond drilling station	4		ft/day
DECLINE	Decline ramp	5		ft/day
DEWATERING	Dewatering station	3.5	5	ft/day
DRILLING	Drilling station	350		ft/day
ESCAPE RAISE	Escape raisebore	8		ft/day
ESCAPEWAY	Escapeway access	5		ft/day
EXPLORATION	Exploration drift	5		ft/day
FUEL BAY	Fuel bay	5		ft/day
INT MUCKBAY	Internal muckbay	3.5		ft/day
LEVEL	Level access	5		ft/day
LOAD CENTER	Load center	5		ft/day
MID-CUT	Middle-cut	4		ft/day
MOS	Mobile operator station	5		ft/day
MUCKBAY	Muckbay	3.5	5	ft/day
OP RAISE	Ore pass raisebore	8		ft/day
ORE PASS	Ore pass access	5		ft/day
PASTE	Paste access	5		ft/day
RAISE-LINING	Raisebore Lining	20		ft/day
RAMP	Ramp	5		ft/day
REFUGE	Refuge cutout	5		ft/day
RUN AROUND	Run around	5		ft/day
SHAFT STATION	Shaft station	3.5	5	ft/day
SLABBING	Stope slabbing	10		ft/day
SUMP	Sump	5		ft/day
TOP-CUT	Top-cut	4		ft/day
TRK LOAD-OUT	Truck loadout	5		ft/day
UTILITY	Utility cutout	5		ft/day
VENT	Vent access	3.5	5	ft/day
VENT RAISE	Vent raisebore	8		ft/day
VENT SHAFT	Vent shaft	5		ft/day
FLOOR-PULL	Floor pull - mining	1000	1100	miT/day
STOPE-MINING	Stope - mining	1000	1100	miT/day
UP-HOLE	Up-hole mining	800		miT/day
GOB-FILLING	Unconsolidated rock backfilling	850		FiT/day
GOB-JAMMING	Unconsolidated rock jamming	850		FiT/day
JAMMING	Jamming	850		FiT/day
PASTE-FILLING	Paste backfilling	850		FiT/day
STOPE-FILLING	Stope backfilling	850		FiT/day

APPENDIX B - UNIFORM SERIES PRESENT-WORTH FACTOR DERIVATION

Derivation [70] of parameter \mathcal{V}_a^{mg} for discounted metal production in the objective function.

Parameters:

- c_a^m : material tonnage for activity a (tons/day)
- c_a^g : recoverable grade per ton for activity a (oz/ton)
- \hat{d}_a : number of days to complete activity a
- δ : discount rate for objective function

Let a be an activity, and \hat{d}_a be its duration. If the activity a starts at time period t , then the discounted metal production will be:

$$\mathcal{V}_a^{mg} = \sum_{\tau=t}^{t+\hat{d}_a-1} c_a^m \cdot c_a^g \cdot \frac{1}{(1+\delta)^\tau} \quad (\text{B.1})$$

$$= c_a^m \cdot c_a^g \sum_{\tau=t}^{t+\hat{d}_a-1} \frac{1}{(1+\delta)^\tau} \quad (\text{B.2})$$

$$= c_a^m \cdot c_a^g \sum_{\tau=0}^{\hat{d}_a-1} \frac{1}{(1+\delta)^{\tau+t}} \quad (\text{B.3})$$

$$= c_a^m \cdot c_a^g \cdot \frac{1}{(1+\delta)^t} \cdot \sum_{\tau=0}^{\hat{d}_a-1} \frac{1}{(1+\delta)^\tau} \quad (\text{B.4})$$

It is known that for $n \geq 0$ and $\alpha \neq 1$:

$$\sum_{i=0}^n \alpha^i = \frac{1 - \alpha^{n+1}}{1 - \alpha} \quad (\text{B.5})$$

Taking $n = \hat{d}_a - 1$ and $\alpha = \frac{1}{1+\delta}$, results in the following:

$$\sum_{\tau=0}^{\hat{d}_a-1} = \frac{1 - \frac{1}{(1+\delta)^{\hat{d}_a}}}{1 - \frac{1}{1+\delta}} \quad (\text{B.6})$$

$$= \frac{\frac{(1+\delta)^{\hat{d}_a-1}}{(1+\delta)^{\hat{d}_a}}}{\frac{1+\delta-1}{1+\delta}} \quad (\text{B.7})$$

$$= \frac{(1+\delta)^{\hat{d}_a} - 1}{(1+\delta)^{\hat{d}_a}} \cdot \frac{1+\delta}{\delta} \quad (\text{B.8})$$

$$= \frac{(1+\delta)^{\hat{d}_a} - 1}{\delta(1+\delta)^{\hat{d}_a-1}} \quad (\text{B.9})$$

Therefore, discounted metal production for activity a is defined as:

$$\mathcal{V}_a^{mg} = c_a^m \cdot c_a^g \cdot \frac{(1+\delta)^{\hat{d}_a} - 1}{\delta \cdot (1+\delta)^{\hat{d}_a-1}} \quad (\text{B.10})$$

APPENDIX C - ADDITIONAL INFORMATION

Additional information pertaining to the models and data used in this dissertation are available upon request from the author, Andrea Brickey at andreabrickey@gmail.com. Information includes:

- Data files for solving with AMPL/CPLEX
- Data files for solving with OMP Solver
- Result log files from CPLEX
- Result log files from OMP Solver

Note that IBM CPLEX 12.6 and OMP Solver are required to use the associated model and data files.

Please contact IBM regarding obtaining a CPLEX license. Please contact Dr. Marcos Goycoolea at marcos.goycoolea@uai.cl regarding use of the OMP Solver.

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