THE OPTIMIZATION OF THROUGHPUT AND BRAKE HORSEPOWER FOR RECIPROCATING NATURAL GAS COMPRESSORS

BY

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ABSTRACT

Compressor clearance typically is set to satisfy some particular flow rate and/or compression ratio. A new method is presented here by which multi-ended, multi-stage compression ratios which optimize throughput and/or brake horsepower can be calculated with optimum clearances. A substantial amount of money can be saved by calculating displacements and clearances required to best satisfy a range of anticipated operating conditions. This minimizes the costly practice of subsequently changing cylinders or performing trial and error field tests. A discussion of the application of the method is included. As well, an actual analysis is made with measured data. The optimum calculated clearances were compared with normal field operating clearances. This revealed fundamental flaws in certain current field procedures. Finally the sensitivity to several input and calculated parameters is examined.

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I. INTRODUCTION

As the wellhead cost of gas rises, the need for more conservative pipeline operating practices increases. One area which has received only limited attention is improvement in compressor performance and design procedures. Although the ultimate goal is to minimize fuel consumption for a given compressor configuration, rate and pressure ratio, an easier intermediate step seems appropriate. This investigation treats minimizing horsepower for a given operating condition of specified rate and pressure by varying the stage clearances in a predeterminable manner. It provides a similar analysis for maximum throughput at fixed horsepower. These procedures are mathematically predictable when compared to the normal empirical field tests.

The compressor cylinder clearance is the fraction of the total cylinder volume which is unswept. Varying the clearance affects the throughput, stage compression ratio, and thus the required horsepower. The required clearance volume may either be fixed or continuously variable over a pre-set range. In this work, the assumption will be made that clearance is continuously variable. The compression ratio is defined as the ratio of the discharge pressure to the suction pressure. The derivation of the optimum throughput or minimum brake horsepower is predicated upon the existence of an optimum sequence of clearance settings with which operating conditions could be met.

This study commenced by developing the mathematical equations descriptive of the optimum conditions. The next step of the study required that the newly found equations be tested on actual data. Because field units are frequently misdesigned, a method for applying the equations had to be developed to handle constrained optimization. The resulting method proved to be useful in determining clearances which minimize required horsepower. The method, however, was found to be of even greater value for designing a multi-stage configuration.

For a given flow rate, suction pressure and temperature, discharge pressure and gas composition, an optimum set of compression ratios and hence clearances can be calculated which minimize horsepower. Furthermore, displacements as well as a range of clearances can be calculated which will satisfy present as well as the anticipated future conditions (i.e. flow rate may change). The relatively small investment required in performing these calculations will provide substantial savings by minimizing the need to later change out cylinders. The procedure also permits the selection of replacement cylinders which are consistent with or match the overall performance. This method provides a means for calculating displacements and clearances at a fraction of the cost of trial and error field tests which are the only current available method for determining these values.

I.A. Current Method for Determining Clearance

Normally, when a compressor is purchased, the minimum and maximum clearances are pre-determined by the manufacturer. Often when the unit is being rebuilt, mismatched cylinders are installed. The minimum clearance is mechanically limited by the diameter of the valves and the minimum allowable distance between the piston and the cylinder head. Volumetric efficiency, which is defined as the ratio of pumped gas to cylinder displacement, decreases as clearance increases. Usually compressors are designed to handle some volumetric throughput at a certain compression ratio (i.e. 5 MMSCFD at a compression ratio of 2) at a given minimum clearance. The clearance can then be increased when the cylinder displacement is too large to operate at other than design conditions.

The maximum operating clearance is usually limited by practical considerations such as cylinder mismatch or rapidly declining volumetric efficiency with increasing clearance. Usually, clearance can be varied between the minimum and maximum values to alter the flow rate or change the stage compression ratio. This naturally leads to the possibility that an optimum clearance exists for a given set of suction and discharge conditions.

I.B. Review of Existing Literature

A large body of literature exists regarding the operating procedures for compressors from a mechanical viewpoint. Similarly, a wide variety of authors have dealt with the modelling of their performance, both from a mechanical standpoint and a thermodynamic perspective. However, in the literature search performed in preparation for this study, no works were found dealing with the treatment of optimization of compressor operations, performance, horsepower of throughput. This review included an examination of the COMPENDEX computer database which incorporates technical literature from approximately 3,500 journals worldwide such as the Engineering Index, Journal of

<u>Petroleum Technology</u>, etc.

Several of the papers were abstracted, but were found to be lacking either in technical accuracy or pertinence to the problem at hand. Since this is apparently a new direction of investigation, only three technical treatises were routinely referenced. The mathematical treatment of the equations describing the behavior of multi-stage compressors operating under assumptions of isentropic thermodynamic behavior are lucidly presented by Kuss.(1) As an introduction to that theory, and a presentation of the mechanical aspects of the problem, Scheel's work(2) is informative. Finally, in order to adequately treat the thermodynamic phenomena associated with real gases, an in-depth industry report(3) and associated computer program using the Benedict-Webb-Rubin Equation of State model provides the needed basis for calculating thermodynamic efficiency.

II. OPTIMIZATION OF THROUGHPUT AND BRAKE HORSEPOWER

For a single stage compressor, the problem of determining clearance is relatively easy. For a given flow rate and compression ratio, the clearance can be calculated directly. However, for multi-stage compressors, there are many feasible solutions. Within each single stage, there may also be multiple compression "ends". The stages are assumed to be in series while the ends are parallel. In a parallel end configuration, each end contributes to the total flow rate through the stage, i.e. the total rate is the sum of the individual ends. Stages in series, on the other hand, have the same throughput for each stage since gas from one stage passes into the next stage.

The total compression ratio is the product of the compression ratios for the individual stages usually adjusted for valve losses, interstage cooling, surge bottles, etc. For example, as a normal design procedure the stage ratio is computed from the total ratio by,

$$R_i = R_T^{1/NS}$$

where

R_T = Total compression ratio

NS = Number of stages

Except for physical constraints virtually any combination of ratios is possible. This is where the real problem occurs.

The existance of an optimum operating point is assured by the laws of thermodynamics. A corollary to the Second Law of Thermodynamics requires that physical systems function at minimum energy conditions. That is the fundamental concept behind variational principles. Thus, for a given flow rate and total compression ratio, some optimum combination of individual stage compression ratios exists in the minimum energy sense. The optimization problem can be posed as one of finding the combination of individual stage compression ratios that yields the largest throughput for a fixed amount of brake horsepower, or conversely the lowest horsepower consumption for a fixed rate.

Therefore, the optimization problem becomes one of solving for the clearance required to obtain the optimum set of ratios. As the clearance increases, the volumetric efficiency and flow rate decrease for a fixed compression ratio. On the other hand, for a fixed flow rate, increasing the clearance decreases the ratio of compression for fixed horsepower. For a properly designed unit within the range of permissible values for clearance, there should exist a value which satisfies the required flow rate and causes the gas to be discharged at the appropriate pressure.

The optimization consists of maximizing the throughput or flow rate. From the equation for throughput,

$$Q_T = 0.00144 \frac{D_i P_{si} T_B}{P_B z_{si} T_{si}} (1 - c_i(R_i^m - 1))$$

where

Q_T = Throughput, MMSCFD

D_i = Stage piston displacement, CFM, (ft^3/min)

P_{si} = Stage suction pressure, psia

T_B = Base temperature, ^oR

P_B = Base pressure, psia

z_{si} = Z-factor at T_{si} and P_{si}

T_{si} = Stage suction temperature, ^oR

c_i = Stage clearance, fraction

R_i = Stage compression ratio

m = 1/n where n is the polytropic exponent

it is readily apparant that the group

$$X = \delta_i c_i (R_i^m - 1)$$

where

$$\delta_i = 0.00144 \frac{D_i P_{si} T_B}{P_B z_{si} T_{si}}$$

must be minimized for a maximum flow rate (See Appendix I). With this, the problem is formulated in to a minimization problem. Proving that the second partial derivative with respect to the stage compression ratio, R_i, is positive guarantees a unique minimum. Setting the first partial derivative equal to zero yields the optimum stage ratio of compression for maximum throughput which is given by,

$$R_i = \left(\frac{R_T^{\,m} \prod_{j=1}^{NS} \delta_j c_j}{(\delta_i c_i)^{NS}} \right)^{\frac{1}{mNS}} \qquad i = 1,2,\ldots,NS \qquad (1)$$

for a compressor with one compression-end per stage.

Equation 1 is valid for stages in series. The complete derivation of Eqn. 1 and the equations to follow are included as Appendix I. Appendix II treats the procedure needed to evaluate a multi-end stage.

In a similar fashion, starting with an equation for horsepower,

$$H_{PT} = \sum_{i=1}^{NS} 43.86 \frac{k}{k-1} (R_i^{(k-1)/k} - 1) \frac{z_{si} T_{si} P_B}{14.47 T_B c_{ei}} Q_i$$

where

H_{PT} = Total horsepower, hp

k = Specific heat ratio

c_{ei} = Combined efficiency, fraction

Q_i = Stage flow rate, MMSCFD

and all other variables are as previously defined, an equation for stage compression ratio which minimizes brake horsepower for a given rate was derived as,

$$R_i = \left(\frac{R_T^{\sigma} \prod_{j=1}^{NS} \beta_j}{\beta_i^{NS}} \right)^{\frac{1}{\sigma NS}} \qquad i = 1,2,\ldots,NS \qquad (2)$$

with

$$\beta_i = \frac{43.86}{14.47} \frac{z_{si} T_{si} P_B}{\sigma \, T_B c_{ei}}$$

and where $\sigma = (k-1)/k$.

Note that k, the specific heat ratio, and n, the polytropic exponent, are equal for isentropic compression. Also, Equations 1 and 2 are for multi-stage units, i.e. $NS \neq 1$.

Assuming that within a multi-stage unit, a single stage compression ratio exists which would both maximize throughput and minimize horsepower, Eqn. 1 can be set equal to Eqn. 2 with the result,

$$\frac{(\delta_i c_i)^{\sigma}}{\beta_i^{m}} = \xi = \text{constant} \tag{3}$$

The left hand side of Eqn. 3 may be evaluated for one stage and the clearances for the remaining stages may be found from,

$$c_i = \left[\frac{Q_T}{(\xi \ \beta_i^{m})^{1/\sigma}} + R_i^{m} - 1 \right]^{-1} \tag{4}$$

III. APPLICATION

III.A. Unconstrained Versus Constrained

Equations 1 through 4 were derived without concern for minimization constraints on the value of the clearance, c. Thus, the use of Eqn. 3 and 4 represent an unconstrained optimum. Clearances can be calculated if the displacements are known. Equation 3 can also be used to determine the displacement required if a particular clearance is used.

These equations may yield values for clearance which are not within an allowable range. Clearances may still be calculated but constraints or range limitations must be placed on the values of clearance. This constrained case arises from physical limitations within the compressor.

III.A.1 Use of the Unconstrained Case When Designing Compressor Configuration

Clearance for any stage may be found from Eqn. 4. First the optimum compression ratios are determined using Eqn. 2. The constant, ξ, may be calculated for any one stage using Eqn. 3. Since suction pressure is required for the δ_i term in Eqn. 3, the first stage will generally be used to determine ξ. Clearances for all other stages are then calculated using Eqn. 4 and the compression ratios obtained from Eqn. 2. If displacements rather than clearances are required, Eqn. 3 may be solved for the δ_i term which in turn can be solved for displacement. The suction pressures needed for the δ_i term are calculated using the compression ratios found from Eqn. 2.

III.A.2. Use of the Constrained Case When Optimizing An Existing Operational Configuration

Optimizing the operation of a unit with a fixed configuration is a constrained minimization problem. There are practical physical limits to the minimum and maximum values for stage compression ratio and clearance. Although Eqns. 1-4 are unconstrained with respect to stage compression ratio and clearance, Eqns. 1 and 2 may still be used if the constraints are applied at the time of calculation. However, the resulting stage compression ratios are no longer a global optimum but represent a local optimum within the constraints of feasible ratios and clearances. This describes a 'best possible' operation with a given configuration for a given throughput and total compression ratio. Appendix III presents a step by step procedure for determining optimum compression ratios and clearances.

III.B. A Graphical Demonstration of the Existence of an Optimum Set of Operating Conditions

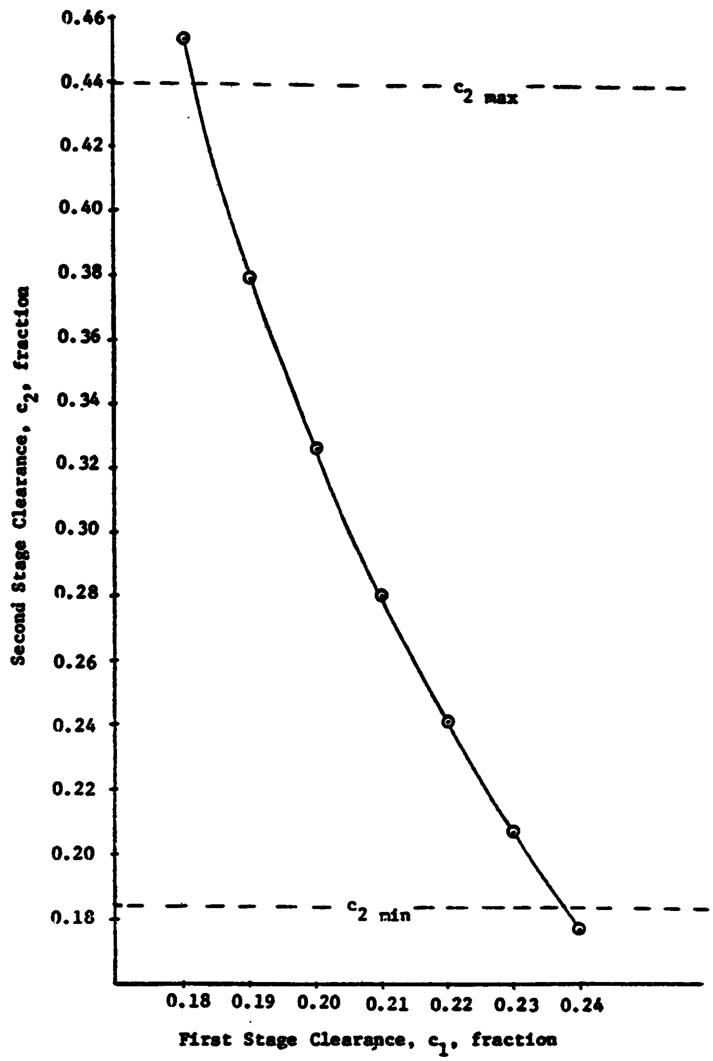
As stated before, the real problem with determining clearance for a multi-stage unit is the multitude of possible clearance combinations. A set of operating conditions which minimize required horsepower is known to exist. Thus, if required brake horsepower is calculated for the possible clearance combinations, a minimum point should be found by which the method could be tested.

The data in Table 1 were taken from a two stage four cylinder compressor. The first stage contains cylinders 1 and 2 while the second stage is composed of cylinders 3 and 4. Calculations for the required second stage displacement and the optimum compression ratios are included as Appendix IV to illustrate the use of Appendix III. Figure 1 contains all possible clearance combinations. It should be noticed

TABLE 1: Compressor Data

Number of Stages, NS	2
Number of Cylinders	4
Total Displacement for Cylinders 1 and 2, D_2	2817.6 CFM
Flow Rate, Q_T	21.27 MMSCFD
Ratio of Specific Heats, k	1.26
Suction Pressure, P_s	75.0 psig
Discharge Pressure, P_p	350.0 psig
Base Pressure, P_b	14.65 psi
Suction Temperature, T_s	60^oF
Base Temperature, T_b	60^oF
Combined Efficiency for Stages 1 and 2, C_e	0.80
Specific Gas Gravity, γ_g	0.65
Percent CO_2	0.01%
Percent N_2	0.02%
Maximum Clearance for Stage 1, C_{1max}	0.427
Minimum Clearance for Stage 1, C_{1min}	0.175
Maximum Clearance for Stage 2, C_{2max}	0.439
Minimum Clearance for Stage 2, C_{2min}	0.185

Figure 1: Possible First and Second Stage Clearance Combinations

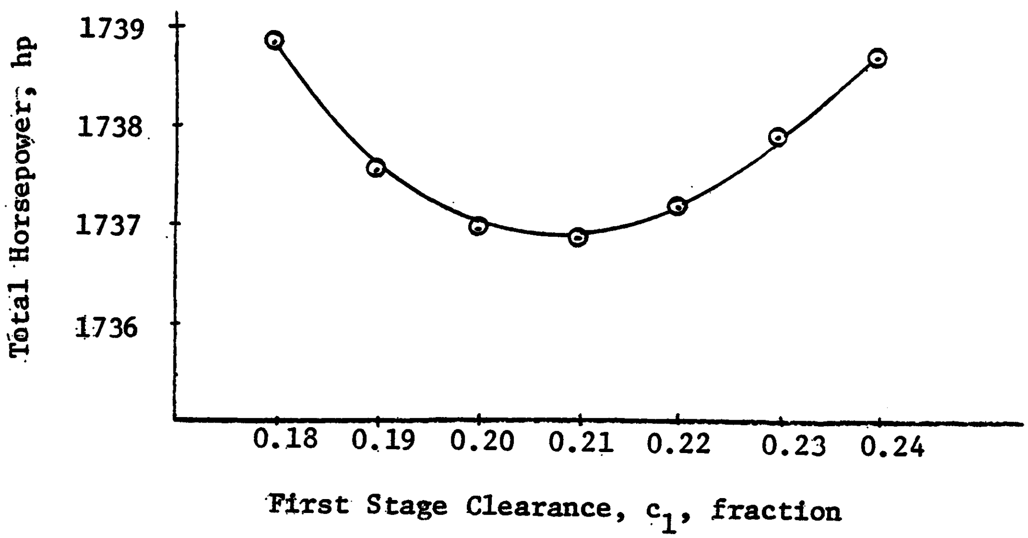


that the first stage clearance need not be so variable since the first stage clearance must be greatly restricted to keep the second stage clearance within the required range.

The compression ratios and total horsepower were calculated for various clearance combinations spanning the feasible range. Figure 2 illustrates total horsepower required by various clearance combinations. This figure clearly illustrates that a unique minimum does indeed exist somewhere around a first stage clearance of 21 %.

The calculations included as Appendix IV indicate an optimum first stage clearance of 22.8 %. This is probably due to the fact that the derivation does not consider the Z-factor to be a function of compression ratio, i.e. both inlet and outlet pressure. The Z-factor is the only variable affecting horsepower since temperature and combined efficiency were held constant. This small difference in calculated versus true optimum would normally be offset by using combined efficiency as a function of suction pressure. Combined efficiency increases with pressure. This has the effect of decreasing the first stage calculated optimum compression ratio (increasing c_1) to compensate for the less efficient first stage.

Figure 2: Total Horsepower Required For Feasible Clearance Combinations



IV. TESTING THE METHOD

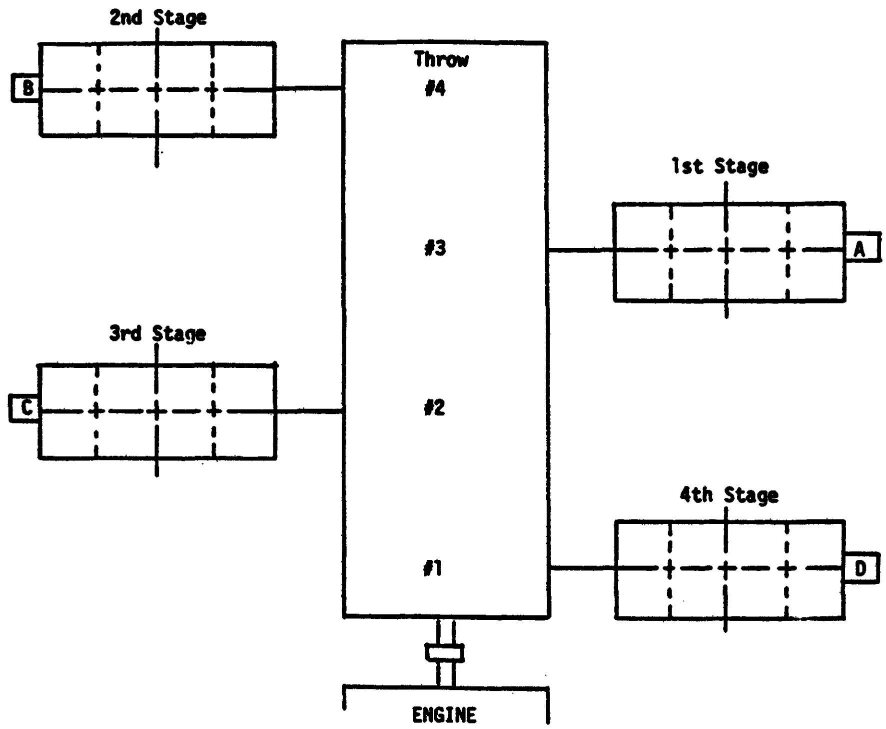
IV.A. Source of Data

Data were compiled by Panhandle Eastern Pipeline Company from tests run on a Worthington Super Cub compressor. The cylinder sketch is included as Figure 3. The tests run on the unit consisted of varying the clearances for each stage and measuring the required brake horsepower. These tests represent a trial and error procedure for determining the best clearance for each stage with certain input conditions and overall conpression ratio requirements. Selected measured and calculated parameters are included as Table 2. Comparing the nine runs on a horsepower per million standard cubic feet (MMscf) basis, Run 6 appears to be the best run since it requires the least horsepower per unit volume of gas.

IV.B. Method of Data Analysis

If the brake horsepower per MMscf calculated by this method from predicted ratios of compression is less than or equal to the brake horsepower per MMscf observed for

Figure 3: Worthington Super Cub Compressor Configuration
(Courtesy of Panhandle Eastern Pipeline Company)



A = 150.80 Cu. In. Variable Volume Clearance Pocket
B = 235.60 Cu. In. " " " "
C = 68.75 Cu. In. " " " "
D = 29.10 Cu. In. " " " "

COMPRESSOR DATA:

	1st Stage	2nd Stage	3rd Stage	4th Stage
Cylinder Diameter (In.)	21.00	13.75	8.00	5.00
Stroke (In.)	6.00	6.00	6.00	6.00
Piston Rod Diameter (In.)	2.50	2.50	2.50	2.50
Total Piston Displacement (CFM)	2388.24	1014.13	332.02	119.31
Minimum Clearance (%)	11.7	15.4	16.6	19.0
Maximum Clearance (%)	15.3	28.6	28.0	31.4

TABLE 2: TEST DATA FROM A 4-STAGE WORTHINGTON SUPER CUB COMPRESSOR (Courtesy of Panhandle Eastern Pipeline Company)

Run #	Initial Suction Pressure (psia)	Total Compression Ratio	Total Unit Horsepower Measured per MMSCF	1st Stage Measured Horsepower	1st Stage Clearance %	2nd Stage Measured Horsepower	2nd Stage Clearance %	3rd Stage Measured Horsepower	3rd Stage Clearance %	4th Stage Measured Horsepower	4th Stage Clearance %
1	16.11	35.12101	223.3	192.5	11.7	201.4	15.4	157.5	16.6	109.3	19.0
2	19.11	30.28239	219.0	223.4	11.7	233.5	15.4	179.0	16.6	115.3	19.0
3	19.41	30.82922	216.3	222.4	12.7	236.8	15.4	178.8	16.6	117.1	19.0
4	19.81	31.16060	217.8	220.0	13.8	218.8	19.8	177.0	16.6	118.4	19.0
5	19.81	30.70201	215.4	219.6	15.3	206.5	24.2	168.7	19.9	118.1	19.0
6	19.81	30.442[illegible]8	214.7	217.3	15.3	194.0	28.6	160.5	23.	115.1	22.6
7	18.11	33.172[illegible]3	221.1	215.8	15.3	195.9	28.6	155.2	28.0	105.0	31.4
8	16.71	36.256[illegible]0	228.4	192.0	15.3	175.8	28.6	140.8	28.0	103.0	31.4
9	16.71	36.285[illegible]4	228.9	173.3	15.3	156.8	28.6	128.8	28.0	88.4	31.4

Run 6 then this procedure can be of use in improving compressor operations. This would suggest that by knowing only the inlet an outlet conditions, compressor configuration and flow rate, the unit's performance can be maximized without trial and error field tests. Since the compressor configuration is predetermined, the "constrained" method of analysis was used. The displacements required to achieve the optimum compression ratios were also calculated.

Although the unit has 4 stages, there are really 8 ends since each stage is double acting or has 2 ends. To simplify the analysis, the two ends were combined into one and the analysis was made on an equivalent 4 stage unit.

A computer program, which performs the calculations for the method outlined in Appendix III, was used with the following inclusions. Suction temperature was held constant for all stages even though measured suction temperatures varied ± 10 °F. This was done so that the calculations could be made with a procedure similar to that used during design when the temperatures are unknown. For the same reason measured discharge temperatures were not used but were calculated from the procedure recommended in Ref. 2,

$$T_{Di} = T_{si} \frac{z_{si} P_{Di}}{z_{Di} P_{si}} \qquad i = 1,2,\ldots,NS$$

Discharge temperature is required in order to calculate the combined efficiency. The combined efficiency, c_e, is the product of the mechanical efficiency and the isentropic efficiency. A value of 0.95 was used for the mechanical efficiency. A detailed explanation of the method for calculating isentropic efficiency is contained in Reference 3. The Z-factors were calculated using the Hall and Yarborough method with corrections made on the critical properties for carbon dioxide and nitrogen.

IV.C. Results of the Analysis

The compressor tests had to be interpreted to provide a basis of comparison for the optimization study. Flow rates and required horsepowers were calculated for the compressor tests and are listed in Table 3. Calculations were made using actual stage compression ratios and actual suction and discharge temperatures. The calculated flow rates vary for each of the four stages as a consequence of combining the ends for each stage, and interstage losses being neglected.

The flow rates as presented in Table 2 are, in turn, used to calculate the optimum stage compression ratios and

TABLE 3: Calculated Throughput and Brake Horsepower for the 9 Runs

Run	Stage 1 Flow Rate MMSCFD	Stage 2 Flow Rate MMSCFD	Stage 3 Flow Rate MMSCFD	Stage 4 Flow Rate MMSCFD	Total Horsepower hp
1	3.237054	3.070702	2.987266	3.070948	605.3597
2	3.965857	3.549045	3.574985	3.532884	672.9506
3	3.926823	3.784240	3.655227	3.572185	696.5084
4	3.894680	3.606210	3.567971	3.531199	682.6147
5	3.847377	3.463914	3.395889	3.405634	656.5875
6	3.897393	3.387301	3.260124	3.321731	635.8980
7	3.460011	3.172179	3.143508	3.143946	613.3429
8	2.976476	2.903683	2.836764	2.873261	689.8069
9	2.687821	2.542783	2.530413	2.670403	529.7939

clearances. Optimum compression ratios were first calculated using the data from Run 6 to determine if the performance of the best run could be improved upon. Since the optimum compression ratios require values for clearance which are greater than maximum, the maximum clearance was used for the first stage to calculate the displacements required. These calculated displacements are included as Table 4. Comparing the calculated displacements to the actual installed cylinders, which are listed in Figure 3, it is apparent that the unit's cylinders are mismatched. The displacements for all stages could be altered for a more efficient unit.

An attempt was made to calculate optimum stage ratios of compression for each of the nine runs listed in Table 2 since the overall compression ratio varied for each case. Tables 5-13 contain the results of the nine runs. Feasible clearances could only be calculated for the data from Run 6. It should be noticed in Tables 11-13, that although clearances were calculated, the total compression ratio of the individual stages is greater than the total required compression ratio. This supports the premise that the cylinders are mismatched and the unit is better suited for either a higher flow rate or a larger total compression ratio. Results for the other runs could not be obtained

TABLE 4: Optimum Ratios of Compression and Clearances for Run 3

Methane	0.737	I-Pentane	0.006	Carbon Dioxide	0.006
Ethane	0.067	N-Pentane	0.006	Ethylene	0.000
Propane	0.053	N-Hexane	0.004	Propylene	0.000
I-Butane	0.010	N-Heptane	0.001	I-Butylene	0.000
N-Butane	0.023	Nitrogen	0.088	Oxygen	0.000

Suction Pressure, psia	19.41	Base Pressure, psia	14.47
Suction Temperature, deg F	70.10	Base Temperature, deg F	60.00
Total Ratio of Compression	30.82922	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2388.24	11.7	15.3	3.9268
2	1014.13	15.4	28.6	3.7842
3	332.02	16.6	28.0	3.6552
4	119.31	19.0	31.4	3.5722

Results cannot be obtained for given input.
Flow rate may be too high.

TABLE 5: Optimum Ratios of Compression and Clearances for Run 1

Methane	73.650	I-Pentane	0.580	Carbon Dioxide	0.580
Ethane	6.700	N-Pentane	0.600	Ethylene	0.000
Propane	5.250	N-Hexane	0.428	Propylene	0.000
I-Butane	0.990	N-Heptane	0.142	I-Butylene	0.000
N-Butane	2.310	Nitrogen	8.770	Oxygen	0.000

Suction Pressure, psia	16.11	Base Pressure, psia	14.47
Suction Temperature, deg F	67.20	Base Temperature, deg F	60.00
Total Compression Ratio	35.12101	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2388.24	11.7	15.3	3.2371
2	1014.13	15.4	28.6	3.0707
3	332.02	16.6	28.0	2.9873
4	119.31	19.0	31.4	3.0709

Results cannot be obtained for given input.
(1 - Q/Delta) is negative.

TABLE 6: Optimum Ratios of Compression and Clearances for Run 2

Methane	73.650	I-Pentane	0.580	Carbon Dioxide	0.580
Ethane	6.700	N-Pentane	0.600	Ethylene	0.000
Propane	5.250	N-Hexane	0.428	Propylene	0.000
I-Butane	0.990	N-Heptane	0.142	I-Butylene	0.000
N-Butane	2.310	Nitrogen	8.770	Oxygen	0.000

Suction Pressure, psia	19.11	Base Pressure, psia	14.47
Suction Temperature, deg F	68.70	Base Temperature, deg F	60.00
Total Compression Ratio	30.28239	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2388.24	11.7	15.3	3.9659
2	1014.13	15.4	28.6	3.5490
3	332.02	16.6	28.0	3.5750
4	119.31	19.0	31.4	3.5329

Results cannot be obtained for given input.
(1 - Q/Delta) is negative.

TABLE 7: Optimum Ratios of Compression and Clearances for Run 3

Methane	73.650	I-Pentane	0.580	Carbon Dioxide	0.580
Ethane	6.700	N-Pentane	0.600	Ethylene	0.000
Propane	5.250	N-Hexane	0.428	Propylene	0.000
I-Butane	0.990	N-Heptane	0.142	I-Butylene	0.000
N-Butane	2.310	Nitrogen	8.770	Oxygen	0.000

Suction Pressure, psia	19.41	Base Pressure, psia	14.47
Suction Temperature, deg F	70.10	Base Temperature, deg F	60.00
Total Compression Ratio	30.82922	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2388.24	11.7	15.3	3.9268
2	1014.13	15.4	28.6	3.7842
3	332.02	16.6	28.0	3.6552
4	119.31	19.0	31.4	3.5722

Results cannot be obtained for given input.
(1 - Q/Delta) is negative.

TABLE 8: Optimum Ratios of Compression and Clearances for Run 4

Methane	73.650	I-Pentane	0.580	Carbon Dioxide	0.580
Ethane	6.700	N-Pentane	0.600	Ethylene	0.000
Propane	5.250	N-Hexane	0.428	Propylene	0.000
I-Butane	0.990	N-Heptane	0.142	I-Butylene	0.000
N-Butane	2.310	Nitrogen	8.770	Oxygen	0.000

Suction Pressure, psia	19.61	Base Pressure, psia	14.47
Suction Temperature, deg F	71.50	Base Temperature, deg F	60.00
Total Compression Ratio	31.16060	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2388.24	11.7	15.3	3.8985
2	1014.13	15.4	28.6	3.6062
3	332.02	16.6	28.0	3.5680
4	119.31	19.0	31.4	3.5312

Results cannot be obtained for given input.
(1 - Q/Delta) is negative.

TABLE 9: Optimum Ratios of Compression and Clearances for Run 5

Methane	73.650	I-Pentane	0.580	Carbon Dioxide	0.580
Ethane	6.700	N-Pentane	0.600	Ethylene	0.000
Propane	5.250	N-Hexane	0.428	Propylene	0.000
I-Butane	0.990	N-Heptane	0.142	I-Butylene	0.000
N-Butane	2.310	Nitrogen	8.770	Oxygen	0.000

Suction Pressure, psia	19.81	Base Pressure, psia	14.47
Suction Temperature, deg F	72.10	Base Temperature, deg F	60.00
Total Compression Ratio	30.70201	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2388.24	11.7	15.3	3.8474
2	1014.13	15.4	28.6	3.4639
3	332.02	16.6	28.0	3.3959
4	119.31	19.0	31.4	3.4086

Results cannot be obtained for given input.
(1 - Q/Delta) is negative.

TABLE 10: Optimum Ratios of Compression and Clearances for Run 6

Methane	73.650	I-Pentane	0.580	Carbon Dioxide	0.580
Ethane	6.700	N-Pentane	0.600	Ethylene	0.000
Propane	5.250	N-Hexane	0.428	Propylene	0.000
I-Butane	0.990	N-Heptane	0.142	I-Butylene	0.000
N-Butane	2.310	Nitrogen	8.770	Oxygen	0.000

Suction Pressure, psia	19.81	Base Pressure, psia	14.47
Suction Temperature, deg F	72.70	Base Temperature, deg F	60.00
Total Compression Ratio	30.44258	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2388.24	11.7	15.3	3.8074
2	1014.13	15.4	28.6	3.3873
3	332.02	16.6	28.0	3.2601
4	119.31	19.0	31.4	3.3217

RESULTS:

Stage	Optimum Ratio of Compression	Feasible Ratio of Compression	Clearance %	Horsepower hp/MMSCF
1	2.20620	2.59013	15.30	51.816
2	2.21451	2.63261	28.60	52.590
3	2.43018	2.35670	28.00	45.989
4	2.55505	1.89439	24.67	33.257

Total Horsepower: 635.822 hp
Total Compression Ratio: 30.44258

TABLE 11: Optimum Ratios of Compression and Clearances for Run 7

Methane	73.650	I-Pentane	0.580	Carbon Dioxide	0.580
Ethane	6.700	N-Pentane	0.600	Ethylene	0.000
Propane	5.250	N-Hexane	0.428	Propylene	0.000
I-Butane	0.990	N-Heptane	0.142	I-Butylene	0.000
N-Butane	2.310	Nitrogen	8.770	Oxygen	0.000

Suction Pressure, psia	18.11	Base Pressure, psia	14.47
Suction Temperature, deg F	67.30	Base Temperature, deg F	60.00
Total Compression Ratio	33.17303	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2388.24	11.7	15.3	3.4600
2	1014.13	15.4	28.6	3.1722
3	332.02	16.6	28.0	3.1435
4	119.31	19.0	31.4	3.1439

RESULTS:

Stage	Optimum Ratio of Compression	Feasible Ratio of Compression	Clearance %	Horsepower hp/MMSCF
1	2.19636	2.72283	15.30	54.563
2	2.27630	2.76172	28.60	54.985
3	2.51030	2.57405	28.00	50.724
4	2.63228	2.32504	31.40	43.943

Total Horsepower: 660.814 hp
Total Compression Ratio: 45.00357

TABLE 12: Optimum Ratios of Compression and Clearances for Run 8

Methane	73.650	I-Pentane	0.580	Carbon Dioxide	0.580
Ethane	6.700	N-Pentane	0.600	Ethylene	0.000
Propane	5.250	N-Hexane	0.428	Propylene	0.000
I-Butane	0.990	N-Heptane	0.142	I-Butylene	0.000
N-Butane	2.310	Nitrogen	8.770	Oxygen	0.000

Suction Pressure, psia	16.71	Base Pressure, psia	14.47
Suction Temperature, deg F	69.10	Base Temperature, deg F	60.00
Total Compression Ratio	36.25600	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2388.24	11.7	15.3	2.9765
2	1014.13	15.4	28.6	2.9037
3	332.02	16.6	28.0	2.8368
4	119.31	19.0	31.4	2.8733

RESULTS:

Stage	Optimum Ratio of Compression	Feasible Ratio of Compression	Clearance %	Horsepower hp/MMSCF
1	1.39700	2.74150	15.30	68.674
2	3.05142	2.79885	28.60	56.262
3	2.97921	2.70862	28.00	53.900
4	3.33575	2.57932	31.40	50.258

Total Horsepower: 665.080 hp
Total Compression Ratio: 53.60698

TABLE 13: Optimum Ratios of Compression and Clearances for Run 9

Methane	73.650	I-Pentane	0.580	Carbon Dioxide	0.580
Ethane	6.700	N-Pentane	0.600	Ethylene	0.000
Propane	5.250	N-Hexane	0.428	Propylene	0.000
I-Butane	0.990	N-Heptane	0.142	I-Butylene	0.000
N-Butane	2.310	Nitrogen	8.770	Oxygen	0.000

Suction Pressure, psia	16.71	Base Pressure, psia	14.47
Suction Temperature, deg F	71.40	Base Temperature, deg F	60.00
Total Compression Ratio	36.28574	Specific Gravity	0.769

Stage	Total Piston Displacement CFM	Minimum Clearance %	Maximum Clearance %	Flow Rate MMSCFD
1	2149.42	11.7	15.3	2.6878
2	912.72	15.4	28.6	2.5428
3	298.82	16.6	28.0	2.5304
4	107.38	19.0	31.4	2.6704

RESULTS:

Stage	Optimum Ratio of Compression	Feasible Ratio of Compression	Clearance %	Horsepower hp/MMSCF
1	1.28545	2.72953	15.30	69.733
2	3.07435	2.86067	28.60	57.807
3	2.97350	2.78820	28.00	55.808
4	3.52684	2.62222	31.40	51.480

Total Horsepower: 613.110 hp
Total Compression Ratio: 57.08850

because the term $(1 - Q_T/\delta_i)$ was calculated to be negative for one or more stages. The calculated brake horsepowers are used as a basis for comparison because the theoretical horsepower is generally less than the measured horsepower due to interstage losses, valve losses, etc..

As stated in Section IV.B., one of the objectives of the method is to determine the best compression ratios without trial and error field tests. It is apparent that Run 6 approaches the local optimum. Using the actual compression ratios and clearances, the total horsepower calculated for Run 6 is 642.26 hp. The calculated optimum comparable to Run 6 required slight changes in clearance for the last two stages which lowered the required horsepower to 635.82 hp. This represents a 1% savings.

IV.D. Sensitivity of the Results to Several Parameters

Several words of warning are in order here regarding the values used for several input and calculated parameters. The Z-factor must be corrected for impurities such as carbon dioxide and nitrogen. Without these corrections the Z-factor is too low which has the effect of making the calculated minimum compression ratio too high. A high minimum compression ratio may also be caused by too large a value for the ratio of specific heats, k.

In step 7 of the calculation method, Appendix III, a sufficiently large tolerance for the iterated pressures must be used in the calculation since certain cases may tend to oscillate. This is due to the function being more similar in shape to a U rather than a V. Thus, the minimum does not converge rapidly to a point.

If the overall compression ratio cannot be met because the product of the minimum ratios for each stage exceeds the total ratio desired, one alternative is to increase the flow rate. However, an error will occur when the following group, which arises from solving the throughput equation for R_i, is less than zero

$$\left(\frac{1 - Q_T/\delta_i}{c_i} + 1 \right) < 0$$

and is raised to the n power. Although δ_i is unknown until R_i is determined, a new flow rate may be found by trial and error.

V. CONCLUSIONS AND RECOMMENDATIONS

Optimum stage compression ratios and thus optimum clearances for each stage within a multi-stage unit can be determined for a fixed flow rate or fixed horsepower and a given total compression ratio. The means by which individual ends within a stage can be matched is also presented. This same procedure is amenable to designing a matched or "tuned" configuration. When designing the configuration of a unit, the piston displacement that is required to keep the optimum clearance within the feasible boundaries can be calculated. Finally, since flow rate and/or initial suction pressure may change over time, an envelope of optimum sets of operating conditions can be readily determined. When the envelope of optimum operating conditions is exceeded, the analysis can be used to select matched replacement cylinders.

The primary weakness with the method arises from the simplifying assumptions made with regard to the initial equations. Calculated brake horsepower is lower than measured data, as expected, since in actual systems losses occur due to valve losses, interstage pipe loss, etc.. Another problem arises with the method if insufficient interstage cooling is used since the results will be affected by the predicted suction temperatures.

Further research in these areas would greatly advance this work. The aim of those studies should be to

1. Test the model on more compressors.
2. Incorporate more sophisticated equations which provide a better thermodynamic treatment for calculating discharge temperature, throughput, and horsepower.
3. Determine the physical significance of the term $(1 - Q_T/\delta_i)$ being negative.
4. Determine if a similar optimization technique exists for centrifugal compressors.

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APPENDIX I

DERIVATION OF OPTIMUM COMPRESSION RATIO EQUATIONS

APPENDIX I. DERIVATION OF OPTIMUM COMPRESSION RATIO EQUATIONS

For stages in series, the total throughput, Q_T, neglecting slippage, can be written as (Ref. 1),

$$Q_T = 0.00144 \frac{D_i P_{si} T_B}{P_B z_{si} T_{si}} (1 - c_i(R_i^{1/n} - 1)) \qquad \text{(A-1)}$$

while noting that for each stage with multiple ends,

$$Q_T = \sum_{j=1}^{N_{ei}} 0.00144 \frac{D_{ji} P_{si} T_B}{P_B z_{si} T_{si}} (1 - c_{ji}(R_i^{1/n} - 1))$$

$$i = 1,2,\ldots,NS$$

where,

Q_T = Total flow rate, MMSCFD

D_i = Stage displacement, CFM (ft^3/min)

D_{ji} = End displacement, CFM

P_{si} = Stage suction pressure, psia

T_B = Base temperature, ^oR

P_B = Base pressure, psia

z_{si} = Stage Z-factor

T_{si} = Stage suction temperature, ^{o}R

c_i = Stage clearance, fraction

c_{ji} = End clearance, fraction

R_i = Stage compression ratio

n = Polytropic exponent

N_{ei} = Number of ends on the i^{th} stage

NS = Number of stages

Since any maximization problem can be rewritten as a minimization problem, the initial task is to formulate a minimization problem from Eqn. A-1. Letting,

$$\delta_i = 0.00144 \frac{D_i P_{si} T_B}{P_B z_{si} T_{si}}$$

and

$$m = 1/n$$

Eqn. A-1 becomes a more tractable equation as,

$$Q_T = \delta_i(1 - c_i(R_i{}^m - 1)) \qquad \text{(A-2)}$$

Noticing that as the term,

$$X_T = \delta_i c_i (R_i - 1)$$

in Eqn. A-2 approaches some minimum value, Q_T approaches some maximum value, an objective function can be cast as,

$$X_T = \sum_{j=1}^{NS} \delta_i c_i (R_i^m - 1) \qquad \text{(A-3)}$$

Thus, the minimization problem is simply one of minimizing X_T as given by Eqn. A-3.

Obviously X_T is minimized for $R_i = 1$ but this represents the trivial solution for the case where no work is being done. Thus, to have a useful solution a restriction must be placed on the values of R_i such that the individual stage compression ratios satisfy some total compression ratio, R_T. This can be written as,

$$R_T = \prod_{j=1}^{NS} R_i$$

which can be rearranged to,

$$R_{NS} = \frac{R_T}{\prod_{i=1}^{NS-1} R_i} \qquad \text{(A-4)}$$

Substituting Eqn. A-4 into Eqn. A-3 gives,

$$X_T = \sum_{i=1}^{NS-1} \delta_i c_i (R_i^m - 1) + \delta_{NS} c_{NS} \left(\frac{R_T^m}{\prod_{i=1}^{NS-1} R_i^m} - 1 \right) \quad \text{(A-5)}$$

Taking the first partial derivative of Eqn. A-5 with respect to each R_i yields,

$$\frac{\partial X_T}{\partial R_i} = \delta_i c_i m R_i^{m-1} - \frac{\delta_{NS} c_{NS} \; m \; R_T^m}{R_i^{m+1} \prod_{j=1}^{i-1} R_j^m \prod_{j=i+1}^{NS-1} R_j^m} \quad \text{(A-6)}$$

The second partial derivative with respect to R_i is,

$$\frac{\partial^2 X_T}{\partial R_i^2} = \delta_i c_i m(m-1) R_i^{m-1} + \frac{\delta_{NS} c_{NS} \; m(m+1) \; R_T^m}{R_i^{m+2} \prod_{j=1}^{i-1} R_j^m \prod_{j=i+1}^{NS-1} R_j^m} \quad \text{(A-7)}$$

If Eqn. A-7 is unconditionally positive, then Eqn. A-6 is the desired minimum.

Since m is less than 1, the first term in Eqn. A-7 is negative. For Eqn. A-7 to be positive the following must be true:

$$|\delta_i c_i m(m-1) R_i^{m-1}| < \frac{\delta_{NS} c_{NS} (m+1) R_T^m}{R_i^{m+2} \prod_{j=1}^{i-1} R_j^m \prod_{j=i+1}^{NS-1} R_j^m}$$

or,

$$|\delta_i c_i (m-1)| < \frac{\delta_{NS} c_{NS} (m+1) R_{NS}}{R_i^{m+1}} \qquad \text{(A-8)}$$

Equation A-8 can be satisfied by labeling the stage with the largest ($\delta_{NS} c_{NS} R_{NS}$) product the last stage since no restrictions on stage numbering have thus far been imposed. Now that a minimum is known to exist at the inflection point described by Eqn. A-6, Eqn. A-6 may be set equal to zero and rearranged as,

$$\delta_i c_i R_i^{m-1} = \frac{\delta_{NS} \ c_{NS} \ R_T^m}{R_i^{m+1} \prod_{j=1}^{i-1} R_j^m \prod_{j=i+1}^{NS-1} R_j^m} = \frac{\delta_{NS} \ c_{NS} \ R_T^m}{R_i \prod_{j=1}^{NS-1} R_j^m}$$

or,

$$\delta_i c_i R_i^m = \frac{\delta_{NS} \ c_{NS} \ R_T^m}{\prod_{j=1}^{NS-1} R_j^m} \qquad \text{(A-9)}$$

Writing Eqn. A-9 for some other ratio such as i+1 yields,

$$\delta_{i+1}\ c_{i+1}\ R_{i+1}{}^{m} = \frac{\delta_{NS}\ c_{NS}\ R_T{}^m}{\prod_{j=1}^{NS-1} R_j{}^m}$$

An expression for R_j can be found by relating the two ratios as,

$$\delta_i c_i R_i{}^m = \frac{\delta_{NS}\ C_{NS}\ R_T{}^m}{\prod_{j=1}^{NS-1} R_j{}^m} = \delta_{i+1}\ c_{i+1}\ R_{i+1}{}^m$$

or

$$R_j{}^m = \frac{\delta_i c_i R_i{}^m}{\delta_j c_j} \tag{A-10}$$

Substituting Eqn. A-10 into Eqn. A-9 gives,

$$\delta_i c_i R_i{}^m = \frac{\delta_{NS}\ c_{NS}\ R_T{}^m}{\prod_{j=1}^{NS-1} \dfrac{\delta_i c_i R_i{}^m}{\delta_j c_j}} = \frac{\delta_{NS}\ c_{NS}\ R_T{}^m}{(\delta_i c_i R_i{}^m)^{NS-1}} \prod_{j=1}^{NS-1} \delta_j c_j$$

This can be further simplified as,

$$(\delta_i c_i R_i{}^m)^{NS} = R_T{}^m \prod_{j=1}^{NS} \delta_j c_j \tag{A-11}$$

Solving Eqn. A-11 for the stage compression ratio, R_i, gives an expression for the R_i which maximizes throughput,

$$R_i = \left(\frac{R_T{}^m \prod_{j=1}^{NS} \delta_j c_j}{(\delta_i c_i)^{NS}} \right)^{\frac{1}{mNS}} \qquad \text{(A-12)}$$

A similar analysis can be performed for the minimization of brake horsepower. Assuming that the stage is composed of only one compression end and that a multi-stage unit has no interstage losses, the total horsepower is given by,

$$H_{PT} = \sum_{i=1}^{NS} 43.86 \frac{k}{k-1} (R_i{}^{(k-1)/k} - 1) \frac{z_{si} T_{si} P_B}{14.47 T_B c_{ei}} Q_i \qquad \text{(A-13)}$$

$$i = 1,2,\ldots,NS$$

where,

H_{PT} = Total horsepower, hp

k = Specific heat ratio

c_{ei} = Stage combined efficiency, fraction

Q_i = Flow rate, MMSCFD

and all other variables are as previously defined. Eqn. A-13 can be simplified by defining σ and β as,

$$\sigma = (k-1)/k$$

and

$$\beta = \frac{43.86}{14.47} \frac{z_{si} T_{si} P_B}{\sigma \, T_B \, c_{ei}}$$

Equation A-13 becomes,

$$H_{PT} = \sum_{i=1}^{NS} \beta_i \, (R_i^{\sigma} - 1) \, Q_i \qquad \text{(A-14)}$$

For a fixed throughput, Q_i, the total horsepower can be minimized by finding the minimum of

$$X = \prod_{i=1}^{NS} \beta_i Q_i \, (R_i^{\sigma} - 1) \qquad \text{(A-15)}$$

Noting that Eqn. A-15 has the same form as Eqn. A-3 with different coefficients and exponents, the solution has the same form or,

$$R_i = \left(\frac{R_T^{\sigma} \prod_{j=1}^{NS} \beta_j Q_j}{(\beta_i Q_i)^{NS}} \right)^{\frac{1}{\sigma NS}} \qquad \text{(A-16)}$$

Since for a series configuration

$$Q_i = Q_{i+1}$$

or

$$\prod_{j=1}^{NS} Q_i = Q_i^{NS}$$

Eqn. A-16 reduces to,

$$R_i = \left(\frac{R_T^{\sigma} \prod_{j=1}^{NS} \beta_j}{\beta_i^{NS}} \right)^{\frac{1}{\sigma NS}} \qquad \text{(A-17)}$$

Equation A-12 provides a means for determining the compression ratio which maximizes throughput while Eqn. A-17 yields the compression ratio which minimizes total required horsepower. Since clearance must be known to use Eqn. A-12 and the compression ratio must be known for the stage to determine clearance, an expression by which clearance can be directly calculated is desirable. If it is assumed that a compression ratio exists that would both maximize throughput and minimize horsepower, the two solutions can be set equal to each other and the clearance may be solved for. This assumption implies that the two solutions represented by Eqn. A-12 and Eqn. A-17 intersect at some value of compression ratio at a point where the horsepower per MMSCF is a minimum. This point could be more formally arrived at by taking the following partial derivative

$$\frac{\partial\left(\frac{H_{PT}}{Q_T}\right)}{\partial R_i}$$

Since the mathmatics rapidly get out of hand for this partial derivative, the previous assumption is made and the solutions represented by Eqn. A-12 and A-17 are set equal as,

$$R_i = \left(\frac{R_T^{\,m} \prod_{j=1}^{NS} \delta_j c_j}{(\delta_i c_i)^{NS}} \right)^{\frac{1}{mNS}} \doteq \left(\frac{R_T^{\,\sigma} \prod_{j=1}^{NS} \beta_j}{\beta_i^{\,NS}} \right)^{\frac{1}{\sigma NS}}$$

which can be simplified to

$$\frac{(\delta_i c_i)^{\sigma}}{\beta_i^{\,m}} = \frac{\left(\prod_{j=1}^{NS} \delta_j c_j\right)^{\frac{\sigma}{NS}}}{\left(\prod_{j=1}^{NS} \beta_j\right)^{\frac{m}{NS}}} = \xi \qquad \text{(A-18)}$$

The right hand side of Eqn. A-18 is a constant and has been labeled ξ. Thus, the left hand side may be calculated for any single stage and the resulting constant used for determining the clearance for all other stages.

Solving Eqn. A-2 for δ_i,

$$\delta_i = \frac{Q_T}{1 - c_i \ (R_i{}^m - 1)}$$

and substituting into Eqn. A-18 yields an expression which can be rearranged to,

$$c_i = \left[\frac{Q_T}{(\xi \ \beta_i{}^m)^{1/\sigma}} + R_i{}^m - 1 \right]^{-1} \qquad \text{(A-19)}$$

Eqn. A-19 may be used to calculate the clearance when the cylinder displacement is unknown. After the required clearance is known, the cylinder displacement can be calculated.

APPENDIX II

DERIVATION FOR MULTI-END SINGLE STAGE

APPENDIX II. DERIVATION FOR MULTI-END SINGLE STAGE

The flow rate for any single stage is given by,

$$Q_T = 0.00144 \frac{D_i P_{si} T_B}{P_B z_{si} T_{si}} (1 - c_i (R_i^{1/n} - 1))$$

where all variables are as defined in Appendix I. For a stage with multiple ends, the flow rate for the stage is given by,

$$Q_T = \sum_{j=1}^{N_{ei}} Q_j$$

or

$$Q_T = \sum_{j=1}^{N_{ei}} 0.00144 \frac{D_{ji} P_{si} T_B}{P_B z_{si} T_{si}} (1 - c_{ji} (R_i^{1/n} - 1))$$

where R_i is a constant for all "j" ends in stage "i". Now let

$$\delta_{ji} = \frac{D_{ji} P_{si} T_B}{P_B z_{si} T_{si}}$$

then

$$Q_T = \sum_{j=1}^{N_{ei}} \delta_{ji} - c_{ji}\delta_{ji}(R_i^{1/n} - 1)$$

$$= \sum_{j=1}^{N_{ei}} \delta_{ji} - (R_i^{1/n} - 1) \sum_{j=1}^{N_{ei}} c_{ji}\delta_{ji}$$

So

$$Q_T = \sum_{j=1}^{N_{ei}} \delta_{ji}\left(1 - \frac{\sum_{j=1}^{N_{ei}} c_{ji}\delta_{ji}}{\sum_{j=1}^{N_{ei}} \delta_{ji}} (R_i^{1/n} - 1)\right)$$

Then if

$$\delta_{Ti} = \sum_{j=1}^{N_{ei}} \delta_{ji}$$

and

$$\bar{c} = \frac{\sum_{j=1}^{N_{ei}} c_{ji}\delta_{ji}}{\sum_{j=1}^{N_{ei}} \delta_{ji}}$$

the flow rate for any single stage can be found from,

$$Q_T = \delta_{Ti} \left(1 - \bar{c}_i (R_i^{1/n} - 1)\right)$$

APPENDIX III

CALCULATING CLEARANCE AND/OR DISPLACEMENT

APPENDIX III. CALCULATING CLEARANCE AND/OR DISPLACEMENT

The step-by-step procedure for calculating clearances and/or displacements for a fixed flow rate must begin with either the first stage where suction pressure is known or the last stage where the discharge pressure is known. The procedure outlined here assumes that the calculations begin with the first stage, i.e. P_{s1} is given. Reference to the next stage refers to the stage immediately following the stage for which calculations were made in the direction of flow.

1. Assume a ratio of compression for each stage,

$$R_i = R_T^{1/NS}$$

2. Calculate suction pressure, P_{si}, for all but the first stage and discharge pressure, P_D, for all stages where,

$$P_{s_{i+1}} = P_{Di} \qquad i = 1,2,\ldots,NS$$

and

$$P_{Di} = P_{si}R_i$$

3. Obtain a Z-factor for each stage using T_{si} and

P_{si}. Suction temperature may be assumed constant if sufficient interstage cooling is used.

4. Calculate the specific heat ratio, k, and polytropic exponent, n, at T_s, P_s for each stage. Obtain the combined efficiency for all stages from the product of the mechanical efficiency and the isentropic efficiency. The method for calculating isentropic efficiency is detailed in Ref. 3.

5. Calculate β_i for all stages where,

$$\beta_i = \frac{43.86}{14.47} \frac{z_{si} T_{si} P_B}{\sigma \, T_B c_{ei}}$$

with $\sigma = (k-1)/k$

6. Calculate optimum compression ratios from

$$R_i = \left(\frac{R_T{}^{\sigma} \prod_{j=1}^{NS} \beta_j}{\beta_i{}^{NS}} \right)^{\frac{1}{\sigma NS}}$$

7. Repeat stops 2-6 until the new pressures calculated are within some tolerance of the pressures used to make the calculation. To calculate required clearances which are constrained by

minimum and maximum values, go to Step 9. To calculate displacements, use Step 8.

8. If the first stage displacement is known, calculate δ_1 from,

$$\delta_1 = 0.00144 \frac{D_1 P_{s1} T_B}{P_B z_{s1} T_{s1}}$$

Then calculate the required clearance

$$c_1 = \frac{1 - Q_T / \delta_1}{R_1^m - 1}$$

and determine the constant, ξ, from

$$\frac{(\delta_1 e_1)^\sigma}{\beta_1^m} = \xi$$

If the first stage displacement is unknown, a clearance should be assumed for the first stage and δ_1 can be calculated from

$$\delta_1 = Q_T / (1 - c_1 (R_1^m - 1))$$

and the constant can be calculated as before.

Calculate the clearance for all other stages from

$$c_i = \left[\frac{Q_T}{(\xi \ \beta_i^m)^{1/\sigma}} + R_i^m - 1 \right]^{-1}$$

and calculate δ_i for all other stages from

$$\delta_i = \frac{(\xi \ \beta_i^m)^{1/\sigma}}{c_i}$$

Displacement can then be found from

$$D_i = \frac{\delta_i \ P_B \ z_{si} \ T_{si}}{0.00144 \ P_{si} T_B}$$

9. Calculate the minimum ratio of compression for the first stage using the maximum clearance

$$R_{min} = \left[\frac{(1 - Q_T/\delta_1)}{c_{max}} + 1 \right]^n$$

10. If R_i calculated in Step 6 is less than R_{min}, set R_i equal to R_{min}. If R_i is greater than R_{min} calculate R_{max} using the minimum allowable clearance.

$$R_{max} = \left[\frac{(1 - Q_T/\delta_i)}{c_{min}} + 1 \right]^n$$

If R_i is greater than R_{max} set R_i equal to R_{max}. If R_i is between R_{min} and R_{max}, calculate the clearance where,

$$c_i = \frac{1 - Q_T/\delta_i}{R_i^{\,m} - 1}$$

and repeat Steps 9 and 10 for the next stage.

11. If R_i was constrained by R_{min} or R_{max} calculate a new discharge pressure,

$$P_{Di} = P_{si} R_i$$

and the remaining overall compression ratio, R_{Trem}, from

$$R_{Trem} = \frac{R_T}{R_{iset}}$$

where R_{iset} is the product of the ratio of compression for all set stages.

12. Repeat Steps 3-6 to determine new R_i's for the remaining stages.
13. Repeat Steps 10 and 11 for the next stage.
14. Continue until all clearances are calculated.

APPENDIX IV

SAMPLE CALCULATIONS

APPENDIX IV. SAMPLE CALCULATIONS

Using the method outlined in Appendix III and the compressor data from Table 1, the displacement required for the second stage and the optimum compression ratios are calculated as follows.

1. The overall compression ratio is found as,

$$R_T = \frac{350 + 14.65}{75 + 14.65} = \frac{364.65}{89.65} = 4.0675$$

The assumed compression ratio for each stage is,

$$R_i = R_T^{1/NS} = (4.0675)^{\frac{1}{2}} = 2.0168$$

2. The suction pressure for the second stage is,

$$P_{s2} = P_{s1}R_1 = (89.65)(2.0168) = 180.81 \text{ psia}$$

3. The Z-factors, corrected for carbon dioxide and nitrogen are found to be,

$$z_{s1} = 0.9830 \; (P_{s1} = 89.65 \text{ psia}, \; T_{s1} = 520^{\circ}R)$$
$$z_{s2} = 0.9656 \; (P_{s2} = 180.61 \text{ psia}, \; T_{s2} = 520^{\circ}R)$$

4. The specific heat ratio is given as 1.26 and the polytropic exponent, n, is assumed to equal the specific heat ratio.

5. The β term is calculated for both stages as,

$$\beta_i = \frac{43.86}{14.47} \frac{z_{si} T_{si} P_B}{\sigma \, T_B c_{ei}}$$

where

$$\sigma = (k-1)/k = (1.26 - 1)/1.26 = 0.2063$$

Thus,

$$\beta_1 \quad \beta_1 = \frac{(43.86)(0.9830)(520)(14.65)}{(14.47)(0.2063)(520)(0.80)} = 264.4856$$

$$\beta_2 \quad \beta_2 = \frac{(43.86)(0.9656)(520)(14.65)}{(14.47)(0.2063)(520)(0.80)} = 259.8040$$

Note that the combined efficiency has been held constant for both stages although it typically increases as the suction pressure increases. However, since the gas components are unknown, the combined efficiency cannot be calculated.

6. The optimum compression ratios are calculated from,

$$R_i = \left(\frac{R_T^{\sigma} \prod_{j=1}^{NS} \beta_j}{\beta_i^{NS}} \right)^{\frac{1}{\sigma NS}}$$

or

$$R_1 = \left(\frac{(4.0675)^{0.2063} (264.4856)(259.8040)}{(264.4856)^2} \right)^{\frac{1}{2(0.2063)}}$$

$$= 1.9314$$

7. The new suction pressure for the second stage is

$$P_{s2} = (89.65)(1.9314) = 173.15 \text{ psia}$$

The Z-factor for this pressure is 0.9671. Since the specific heat ratio is assumed constant as k = 1.26, β_2 is

$$\beta_2 = \frac{(43.86)(0.9671)(520)(14.65)}{(14.47)(0.2063)(520)(0.80)} = 260.2075$$

The new optimum compression ratio for the first stage is,

$$R_1 = \left(\frac{(4.0675)^{0.2063} \ (264.4856)(260.2095)}{(264.4856)^2}\right)^{\frac{1}{2(0.2063)}}$$

$$= 1.9386$$

This yields a suction pressure for the second stage of 173.80 psia and a Z-factor of 0.9669. Subsequent iterations yield:

β_2	R_1	P_{s2}	z
260.1537	1.9377	173.71	0.9670
260.1806	1.9382	173.76	0.9670

Thus, R_1 = 1.9382, P_{s2} = 173.76 psia, and β_2 = 260.1806. The optimum compression ratio for the second stage is,

$$R_2 = \frac{4.0675}{1.9382} = 2.0986$$

8. Since the displacement for the first stage is known, δ_1, is calculated as,

$$\delta_1 = 0.00144 \frac{(2817.6)(89.65)(520)}{(14.65)(0.9830)(520)}$$

$$= 25.2581$$

The clearance required for the first stage is

$$c_1 = \frac{1 - Q_T/\delta_1}{R_1^{\,m} - 1} = \frac{1 - 21.27/25.2581}{1.9382^{1/1.26} - 1} = 0.229$$

which is within the allowable clearances. If the clearance had been outside the allowable values, a new displacement for the first stage would have been required or the compression ratio for the first stage could be set to R_{min} or R_{max}. The constant, ξ, is calculated from,

$$\frac{(\delta_1 c_1)^{\sigma}}{\beta_1^{\,m}} = \frac{((25.2581)(0.229))^{0.2063}}{264.4856^{1/1.26}} = 0.0172$$

The clearance for the second stage is,

$$c_2 = \left[\frac{21.27}{((0.0172)(260.1806)^{1/1.26})^{1/0.2063}} + 2.0986^{1/1.26} - 1 \right]^{-1} = 0.214$$

The δ_2 term is calculated from

$$\delta_2 = ((0.0172)(260.1806)^{1/1.26})^{1/0.2063} = 25.6083$$

and the required displacement is

$$D_2 = \frac{(25.6083)(14.65)(0.9670)(520)}{(0.00144)(173.76)(520)}$$

$$= 1449.9 \text{ CFM}$$