TRIAXIAL CASING DESIGN

by

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Dedicated to:

My Father and Mother

ABSTRACT

Casing-string design is an engineering problem which requires understanding and careful analysis of many factors to achieve the result of specifying casing which will meet well conditions at the minimum cost consistent with a desired degree of safety (Hills, 1951).

This thesis is concerned with a deep-well casing design considering triaxial loading conditions. Casing-string weight and grade were determined at each depth utilizing the maximum-distortion-energy theory as a criterion for failure. Reduced collapse and burst pressures were calculated using Equations 42 and 43. These equations are based on the Hencky-Von Mises maximum energy of distortion theory of yielding, and they are applicable where the collapse and burst pressures are directly proportional to yield strength (API Bulletin 5C3, 1980). Since this condition is met by the API collapse formulas for yield collapse and transition collapse and the API burst formula, and approximately by the API formula for plastic collapse, these formulas are used to determine simple collapse and burst pressures.

Lame's equations were used to calculate the tangential (hoop) and the radial stresses at the inner wall of the casing-string.

The difference in deep-well casing design when consider-

ing triaxial loading conditions rather than biaxial loading conditions is shown in terms of casing-string thickness variation with depth and with the ratio of (P_i/P_e). Buoyancy is accounted for in the design. Safety factors of 2, 1.125, and 1.1 were used for axial load, collapse, and burst, respectively. A computer program was written to calculate casing-string weight and grade at each depth when considering the conditions of triaxial loading.

Axial load was determined using Newton's Law, where the axial load is the resultant force after summing the forces in the vertical direction. The vertical acting forces are (1) total weight of the casing-string and (2) force caused by the mud pressure acting on its lower end (Mitchell, 1980).

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INTRODUCTION

An integral part of the completion of an oil well is the casing used as a protective conduit for the various downhole production assemblies. In designing oil well casing one is confronted with a circular cylindrical tubular structure exposed to internal and external pressure, axial load, and possibly flexure (not considered in this thesis).

The actual behavior of commercial casing-strings by no means can be considered as simple, due to the various factors influencing the plastic mode of failure. Some careful experiments, which were made by various investigators on the pure yielding of the ductile metals under combined stress, allow a few conclusions in this respect. From these tests, it appeared that a ductile metal starts to flow when:

$$(S_r - S_h)^2 + (S_h - S_z)^2 + (S_z - S_r)^2 = 2S_o^2 = \text{constant}$$

S_r, S_h, and S_z are the radial, hoop, and axial stresses. S_o is the yield strength in pure tension (Holmquist, et al, 1939).

Deep-well casing design under triaxial loading is based on the theory of maximum-distortion-energy which states that plastic yielding will occur first at the portion of the casing wall for which:

$$(S_r - S_h)^2 + (S_h - S_z)^2 + (S_z - S_r)^2 > 2S_o^2$$

The term $(S_r - S_h)^2 + (S_h - S_z)^2 + (S_z - S_r)^2$ has its maximum absolute value at the inner wall of the casing-string (Nester, et. al., 1955). This value has been set equal to $(2S_o^2)$ as the criterion for casing-string failure.

The problem of determining the tangential (hoop) stress and the radial stress at any point in the wall of a thick-walled cylinder, in terms of the pressures applied to the cylinder, was solved by the French elastician Lame' in 1833 (Higdon, et. al., 1976). The maximum value for hoop and radial stresses caused by internal or by external pressure will be at the inner surface of the cylinder.

Design Factors

It is seldom desirable to subject any material to its maximum allowable stress. This is particularly true for material of which casing is constructed, since minimum physical properties are determined statistically, and the properties of any individual length of casing may deviate considerably from the statistical average.

Accordingly, we must introduce the concept of the design factor, which is the ratio of maximum allowable stress to the actual working stress. Many authorities point out that the term safety factor is a misnomer in view of the fact that actual physical properties are not generally known. They prefer to use the term design factor, which is more descriptive

of the use made of the factor.

In 1955, the API Mid-Continent District study committee on casing programs reported the results of an investigation of design factors being applied in casing programs. The report presented a summary of replies to questionnaires sent to all members of the committee. An analysis of 50 replies received from 38 companies indicates the following practices concerning design factors (Craft, Holden, and Graves, 1962).

1. Design factors for collapse vary from 1.0 to 1.5. A design factor of 1.125 was used on 68 per cent of the casing-strings reported.
2. Design factors for tension vary from 1.5 to 2.0, the factors 1.6 (on 29 per cent of the casing-strings), 1.75 (on 21 per cent), 1.8 (on 25 per cent) being most commonly used. In this connection, about one half the replies indicated use of the same tension design factor for ensuring against failure in the pipe body and in the joints. Where a distinction was made between the two types of tension failure, the design factor for failure in the pipe body was generally somewhat lower than for joint strength. For example, many replies showed that tensile yield load design factor of 1.6 and 1.8 were used with joint strength design factors of 1.8 and 2.0, respectively.

3. Design factors for internal yield vary from 1.0 to 1.75, with a factor of 1.1 being used on 32 per cent of the casing-strings reported.

Lame's Equation

The problem of determining the tangential stress S_h and the radial stress S_r at any point in the wall of a thick-walled cylinder, in terms of the pressures applied to the cylinder, was solved by the French elastician G. Lame' in 1833. The results can be applied to a wide variety of design situations involving cylindrical pressure vessels, hydraulic cylinders, piping systems, shrink and press-fit applications, etc. (Higdon, et. al., 1976).

Consider a thick-walled cylinder having inner radius a and outer Radius b, as shown in Figure 1a. The cylinder is subjected to an internal pressure P_i and an external pressure P_o. For purposes of analysis, the thick-walled cylinder can be considered to consist of a series of thin rings. A typical ring located at a radial distance r from the axis of the cylinder and having a thickness dr is shown by the dashed lines in Figure 1a. As a result of the internal and external pressure loadings, a radial stress S_r would develop at the interface between rings located at radial position r, while a slightly different radial stress $(S_r + dS_r)$ would develop at radial position $(r + dr)$. These stresses would be uni-

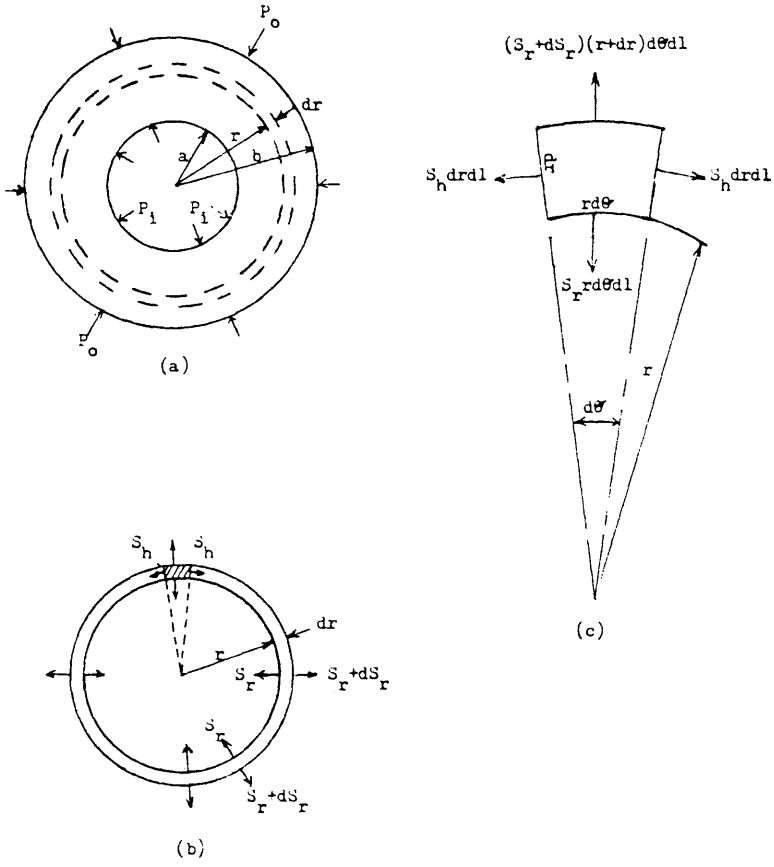

P_o
dr
r
a
b
P_i
P_i
P_o
(a)
S_h
S_h
dr
r
S_r
S_r+dS_r
S_r
S_r+dS_r
(b)
$(S_r+dS_r)(r+dr)d\theta dl$
$S_h drdl$
P_i
$S_h drdl$
$rd\theta$
$S_r rd\theta dl$
r
$d\theta$
(c)

Figure 1

formly distributed over the inner and outer surfaces of the ring, as shown in Figure 1b. Shearing stresses would not develop on the inner and outer surfaces of the ring, since the pressure loadings do not tend to force the rings to rotate with respect to one another. Since the ring is assumed to be thin, the tangential (hoop) stress S_h can be considered to be uniformly distributed through the thickness of the ring. A relationship between radial stress S_r and tangential stress S_h can be obtained from equilibrium considerations. A free-body diagram of a small part of ring, such as the one shown in Figure 1c for the shaded part of Figure 1b, is useful for this determination. The axial stress S_z, which may be presented in the cylinder, has been omitted from this diagram since it does not contribute to equilibrium in the radial or tangential directions. The free-body is assumed to have a length dl along the axis of the cylinder.

From a summation of forces in the radial direction:

$$(S_r + dS_r)(r + dr)\, d\theta\, dl - S_r r\, d\theta\, dl - 2\, S_h dr\, dl \sin \frac{d\theta}{2} = 0 \qquad (1)$$

By neglecting higher-order terms and noting that for small angles $\sin d\theta/2 = d\theta/2$, Equation (1) can be reduced to:

$$r\, dS_r/dr + S_r - S_h = 0$$

Equation (2) cannot be integrated directly, since both S_r

and S_h are functions of radial position r. In previous instances, when such statically indeterminate situations were encountered, the problem was solved by considering deformations of the structure.

For the case of a thick-walled cylinder, the axial strain ε_a at any point in the wall of the cylinder can be expressed in terms of S_z, S_r, and S_h by using generalized Hooke's Law. Thus,

$$\varepsilon_a = \{S_z - \upsilon (S_r + S_h)\}/E \qquad (3)$$

The assumption normally made concerning strains in the thick-walled cylinder, which has been verified by careful measurement, is that axial strain is uniform. This means that plane transverse cross sections before loading remain plane and parallel after the internal and external pressures are applied. So far as the axial stress S_z is concerned, two cases are of interest in a wide variety of design applications. In the first case, the axial loads induced by the pressure are not carried by the walls of the cylinder ($S_z = 0$). This situation arises in gun barrels and in many types of hydraulic cylinders where pistons carry the axial loads. In the second case, the wall of the cylinder carry the loads. This situation occurs in pressure vessels with various types of end closures or heads. In this second case, in regions of the cylinder away from the ends, it has been found that the axial

stress is uniformly distributed over the cross section. Hence, ε_a, S_z, E, and υ are constant for the two cases being considered; therefore, it follows from Equation (3) that:

$$S_r + S_h = (S_z - E\varepsilon_a)/\upsilon = 2C_1 \tag{4}$$

The constant is taken as $2C_1$ for convenience in the following derivation.

When the value for S_h from Equation (4) is substituted into Equation (2), this latter equation may be written as:

$$r \frac{dS_r}{dr} + 2S_r = 2C_1 \tag{5}$$

If Equation (5) is multiplied by r, the terms before the equal sign can be expressed as:

$$\frac{d}{dr} (r^2 S_r) = 2C_1 r$$

Integrating yields:

$$r^2 S_r = C_1 r^2 + C_2$$

where C_2 is a constant of integration. Thus,

$$S_r = C_1 + C_2/r^2 \tag{6}$$

The tangential stress S_h is then obtained from Equation (4) as:

$$S_h = C_1 - C_2/r^2 \tag{7}$$

Values for the constants C_1 and C_2 in Equations (6) and (7) can be determined by using the known values for the pressures

at the inside and outside surfaces of the cylinder. These values, commonly referred to as boundary conditions, are:

$$S_r = -P_i \quad \text{at } r = a$$

$$S_r = -P_o \quad \text{at } r = b$$

The minus signs indicate that the pressures (normally considered as positive quantities) produce compressive normal stresses at the surfaces on which they are applied. Substituting the boundary conditions into Equation (6) yields:

$$C_1 = (a^2P_i - b^2P_o)/(b^2 - a^2)$$

$$C_2 = -a^2\, b^2(P_i - P_o)/(b^2 - a^2)$$

The desired expressions for S_r and S_h are obtained by substituting these values for C_1 and C_2 into Equations (6) and (7). Thus,

$$S_r = \frac{a^2P_i - b^2P_o}{(b^2 - a^2)} - \frac{a^2b^2\,(P_i - P_o)}{(b^2 - a^2)\,r^2} \tag{8}$$

$$S_h = \frac{a^2P_i - b^2P_o}{(b^2 - a^2)} + \frac{a^2b^2\,(P_i - P_o)}{(b^2 - a^2)\,r^2} \tag{9}$$

But the maximum stress (hoop or radial) caused by either internal pressure or by external pressure will be at the inner surface. i.e. r = d. So:

$$S_r = \frac{a^2 P_i - b^2 P_o - b^2 (P_i - P_o)}{(b^2 - a^2)}$$

$$S_r = -P_i \tag{10}$$

And:

$$S_h = \frac{P_i (b^2 + a^2)}{(b^2 - a^2)} - P_o \frac{2\, b^2}{(b^2 - a^2)} \tag{11}$$

THEORIES OF FAILURE

A tension test of an axially loaded member is easy to conduct, and the results for many types of materials are well known. When such a member fails, the failure occurs at a specific principal (axial) stress, a definite axial strain, a maximum shearing stress of one-half the axial stress, and a specific amount of strain energy per unit volume of stressed material. Since all of these limits are reached simultaneously for an axial load, it makes no difference which criterion (stress, strain, or energy) is used for predicting failure in another axially loaded member of the same material (Higdon, et. al., 1976).

For an element subjected to biaxial or triaxial loading, however, the situation is more complicated because the limits of normal stress, normal strain, shearing stress, and strain energy existing at failure for an axial load are not all reached simultaneously. In other words, the cause of failure, in general, is unknown. In such cases it becomes important to determine the best criterion for predicting failure, because test results are difficult to obtain and combinations of loads are endless. Several theories have been proposed for predicting failure of various types of material subjected to many combinations of loads. Unfortunately, none of the theories agree with test data for all types of materials and

combinations of loads (Higdon, et. al., 1976). Several of the more common theories of failure are presented and briefly explained in the following paragraphs.

Maximum-Normal-Stress Theory

The Maximum-Normal-Stress Theory, credited to W.J.M. Rankine (1802 - 1872), is based on the assumption that failure will occur when the maximum normal stress at any point attains the value of the axial failure stress as determined by an axial tensile or compressive test regardless of the other stresses acting at the point (Olsen, 1982). Failure may be considered to take place by excessive yielding or by fracture.

The Maximum-Normal-Stress Theory is presented graphically in Figure 2b for an element subjected to biaxial principal stresses in the x and y directions, as shown in Figure 2a. The limiting stress S_o is the failure stress for this material when loaded axially. Any combination of biaxial principal stresses S_x and S_y represented by a point inside the square of Figure 2b is safe according to this theory, whereas any combination of stresses represented by a point outside of the square will cause failure of the element on the basis of this theory (Higdon, et. al., 1976).

Maximum-Shearing-Stress Theory

The maximum shearing stress theory is an outgrowth of

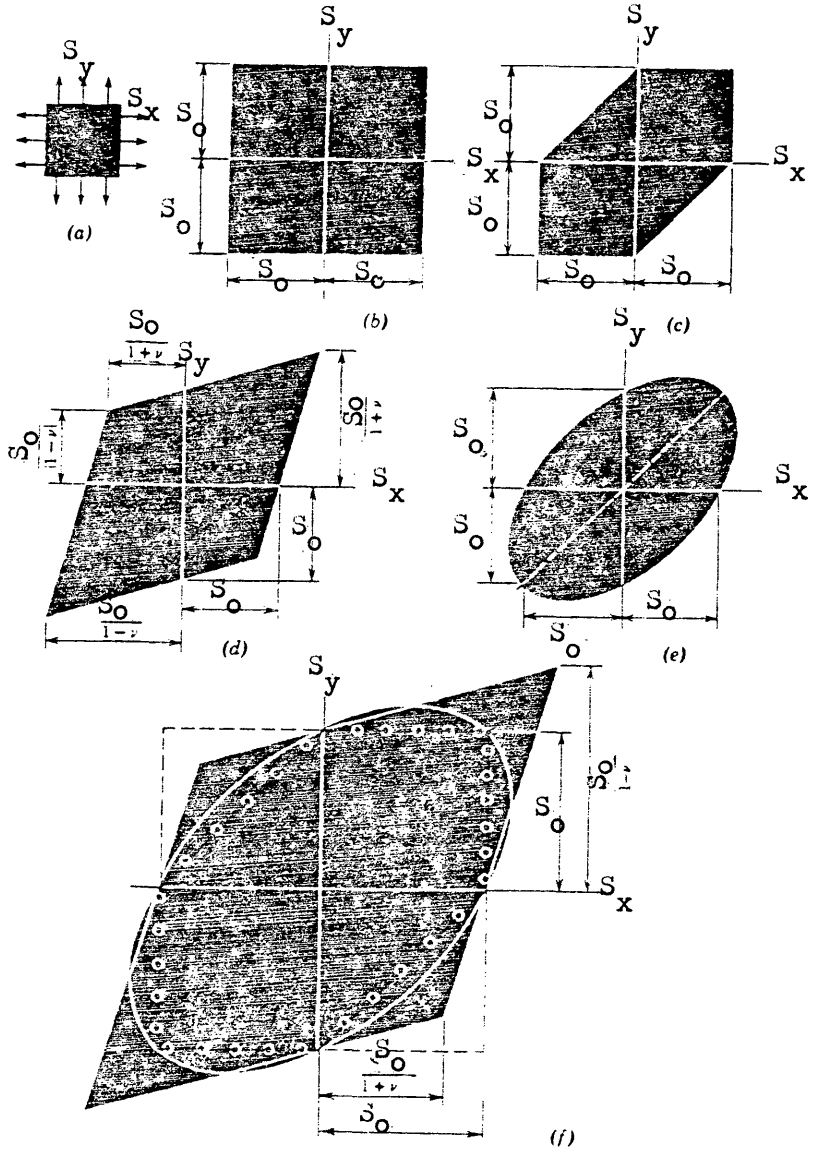


Figure 2

the experimental observation that a ductile material yields as a result of slip or shear along crystalline planes. Proposed by C.A. Coulomb (1736 - 1806) it is also referred to as the Tresca or Guest Theory in recognition of the contribution of these men to its application (Ugural, et. al., 1981). This theory predicts failure of a specimen subjected to any combination of loads when the maximum shearing stress at any point reaches the failure stress τ_f equal to $S_o/2$, as determined by an axial tensile or compressive test of the same material. For ductile materials, the shearing elastic limit, as determined from a torsion test (pure shear), is greater than one-half the tensile elastic limit (with an average value of τ_f about 0.57 S_o). This means that the Maximum-Shearing-Stress Theory errs on the conservative side by being based on the limit obtained from an axial test (Higdon, et. al., 1976).

The Maximum-Shearing-Stress Theory is presented graphically in Figure 2c for an element subjected to biaxial (S_z is equal to zero) principal stresses, as shown in Figure 2a. In the first and third quadrants, S_x and S_y have the same sign, and the maximum shearing stress is half of the numerically larger principal stress S_x or S_y. In the second and fourth quadrants, where S_x and S_y are of opposite sign, the maximum shearing stress is half of the arithmetical sum of the two principal stresses. In the fourth quadrant, the equation of

the boundary or limit stress, line is:

$$S_x - S_y = S_o \tag{12}$$

and in the second quadrant the relation is:

$$S_x - S_y = -S_o \tag{13}$$

Maximum-Normal-Strain Theory

This theory is originated by St. Venant (1797 - 1866). It predicts failure of a specimen by yielding when the maximum-normal-strain reaches the tensile yield strain S_o/E at the proportional limit, as determined by an axial tensile or compressive test of the same material.

The Maximum-Normal-Strain Theory is presented graphically in Figure 2d for an element subjected to biaxial principal stresses (not greater than the proportional limit), as shown in Figure 2a. The limiting strain in the positive x direction is:

$$\varepsilon_x = \frac{S_o}{E} = \frac{S_x - \upsilon S_y}{E} \tag{14}$$

from which:

$$S_y = \frac{1}{\upsilon}(S_x - S_o) \tag{15}$$

which is observed to be a straight line through the point $(S_o, 0)$ with a slope of $1/\upsilon$. The limiting strain in the positive y direction is:

$$\varepsilon_y = \frac{S_o}{E} = \frac{S_y - \upsilon S_x}{E} \tag{16}$$

from which:

$$S_y = \upsilon S_x + S_o \tag{17}$$

which is observed to be a straight line through the point $(0, S_o)$ with a slope equal to υ (Higdon, et. al., 1976).

Maximum-Strain-Energy Theory

The Maximum-Strain-Energy Theory, proposed by Beltrami (1885) predicts failure of a specimen subjected to any combination of loads when the strain energy per unit volume of any portion of the stressed member reaches the failure value of the strain energy per unit volume as determined by an axial tensile or compressive test of the same material (Higdon, et. al., 1976). A specimen subjected to a biaxial state of stress produces a unit maximum energy absorption equal to:

$$u = \tfrac{1}{2} S_x \varepsilon_x + \tfrac{1}{2} S_y \varepsilon_y \tag{18}$$

$$= \tfrac{1}{2} S_x \left(\frac{S_x}{E} - \upsilon \frac{S_y}{E}\right) + \tfrac{1}{2} S_y \left(\frac{S_y}{E} - \upsilon \frac{S_x}{E}\right) \tag{19}$$

$$= \tfrac{1}{2} \frac{S_x^2}{E} + \tfrac{1}{2} \frac{S_y^2}{E} - \upsilon \frac{S_x S_y}{E} \tag{20}$$

according to the theory, this energy produces failure when equal to that obtained in a tension or compression test. Thus,

$$\frac{S_x^2}{2E} + \frac{S_y^2}{2E} - \upsilon \frac{S_x S_y}{E} = \frac{S_o^2}{2E} \tag{21}$$

or:

$$S_x^2 + S_y^2 - 2\upsilon S_x S_y = S_o^2 \tag{22}$$

Dividing through by S_o^2 gives:

$$\left(\frac{S_x}{S_o}\right)^2 + \left(\frac{S_y}{S_o}\right)^2 - 2\upsilon \left(\frac{S_x}{S_o}\right)\left(\frac{S_y}{S_o}\right) = 1 \tag{23}$$

If the proportional limit stresses in tension and compression are equal, this equation will form, when plotted graphically, an ellipse, shown as Curve e on Figure 2.

This theory has been largely replaced by the Maximum-Distortion-Energy Theory.

Maximum-Distortion-Energy Theory

This theory, formulated independently by Von Mises and Hencky, and later supplemented by Huber (Olsen, 1982). This theory differs from the Maximum-Strain-Energy Theory in that the portion of the strain energy producing volume change is considered ineffective in causing failure by yielding. Supporting evidence comes from experiments showing that homo-

genous materials can withstand very high hydrostatic stresses without yielding. The portion of the strain energy producing change of shape of the element is assumed to be completely responsible for failure of the material by inelastic action (Higdon, et. al., 1976).

The strain energy of distortion is most readily computed by determining the total strain energy of the stressed material and subtracting the strain energy corresponding to the volume change. The quantity $S^2/(2E)$ is defined as the strain energy per unit volume for a member subjected to a slowly applied axial load. This expression can also be written as:

$$u = \frac{S^2}{2E} = \frac{S\epsilon}{2} \tag{24}$$

where u is the strain energy per unit volume and S and ϵ are the slowly applied axial stress and strain. This equation assumes that the stress does not exceed the proportional limit.

When an elastic element is subjected to triaxial loading, the stresses can be resolved into three principal stresses such as S_x, S_y, and S_z, where x, y, and z are the principal axes. These stresses will be accompanied by three principal strains related to the stresses by the equation:

$$\varepsilon_x = \{S_x - \upsilon (S_y + S_z)\}/E$$

$$\varepsilon_y = \{S_y - \upsilon (S_x + S_z)\}/E$$

$$\varepsilon_z = \{S_z - \upsilon (S_x + S_y)\}/E \qquad (25)$$

If it is assumed that the loads are applied simultaneously and gradually the stresses and strains will increase in the same manner. The total strain energy per unit volume is the sum of the energies produced by each of the stresses (energy is a scalar quantity and can be added algebraically regardless of the direction of the individual stresses); thus,

$$u = (\tfrac{1}{2})(S_x \varepsilon_x + S_y \varepsilon_y + S_z \varepsilon_z) \qquad (26)$$

When the strains are expressed in terms of the stresses, this equation becomes:

$$u = \{1/(2E)\} \{(S_x^2 + S_y^2 + S_z^2) - 2\upsilon (S_xS_y + S_yS_z + S_zS_x)\} \qquad (27)$$

The strain energy can be resolved into two components u_v and u_d, resulting from a volume change and a distortion, respectively, by considering the principal stresses to be made up of two sets of stresses as indicated in Figure 3a, b, and c. The state of stress in Figure 4C will result in dis-

tortion only (no volume change) if the sum of the three normal strains is zero. That is,

$$E(\varepsilon_x + \varepsilon_y + \varepsilon_z)_d = \{(S_x - P) - \upsilon(S_y + S_z - 2P)\} + \{(S_y - P) - \upsilon(S_z + S_x - 2P)\} + \{(S_z - P) - \upsilon(S_x + S_y - 2P)\} = 0 \quad (28)$$

which reduces to:

$$(1 - 2\upsilon)(S_x + S_y + S_z - 3P) = 0 \quad (29)$$

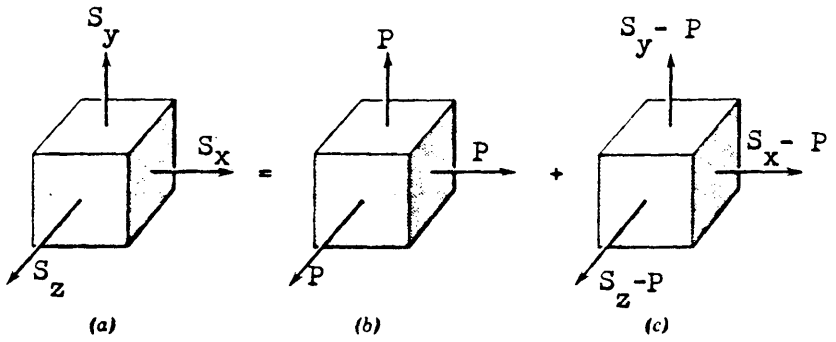


Figure 3

Therefore,

$$P = (S_x + S_y + S_z)/3 \tag{30}$$

The three normal strains due to P are, from Equation (25):

$$\varepsilon_v = (1 - 2\upsilon)\, P/E \tag{31}$$

and the energy resulting from the hydrostatic stress (the volume change) is:

$$u_v = 3\left(\frac{P\varepsilon_v}{2}\right) = \frac{3}{2}\,\frac{1 - 2\upsilon}{E}\,P^2$$

$$= \frac{1 - 2\upsilon}{6E}\,(S_x + S_y + S_z)^2 \tag{32}$$

The energy resulting from the distortion (change of shape) is:

$$u_d = u - u_v$$

$$= \frac{1}{6E}\,\{3(S_x^2 + S_y^2 + S_z^2) - 6\upsilon(S_xS_y + S_yS_z + S_zS_x)$$

$$- (1 - 2\upsilon)(S_x + S_y + S_z)^2\} \tag{33}$$

When the third term in the brackets is expanded, the expression can be rearranged to give:

$$u_d = \frac{1 + \upsilon}{6E} \{(S_x^2 - 2S_xS_y + S_y^2) + (S_y^2 - 2S_yS_z + S_z^2) + (S_z^2 - 2S_zS_x + S_x^2)\}$$

$$= \frac{1 + \upsilon}{6E} \{(S_x - S_y)^2 + (S_y - S_z)^2 + (S_z - S_x)^2\} \quad (34)$$

The maximum-distortion-energy theory of failure assumes that inelastic action will occur whenever the energy given by Equation (34) exceeds the limiting value obtained from a tensile test. For this test, only one of the principal stresses will be nonzero. If this stress is called S_o, the value of u_d becomes:

$$(u_d)_o = \frac{1 + \upsilon}{3E} S_o^2 \quad (35)$$

and when this value is substituted in Equation (34) it becomes:

$$2 S_o^2 = (S_x - S_y)^2 + (S_y - S_z)^2 + (S_z - S_x)^2 \quad (36)$$

for failure by slip.

Comparison of Theories

On Figure 4 where the graphs of the five theories are shown superimposed on the same set of axes, the typical results of combined stress tests of various materials are also plotted. Note that the maximum normal stress theory appears

to be most correct for brittle materials and the maximum distortion energy theory for ductile materials (Olsen, 1982). Moreover, experiments involving yielding failure in ductile materials show good agreement with the maximum distortion energy criterion. This criterion is widely accepted as being the best available for predicting the onset of yield in ductile materials subjected to combined stresses (Willeml, et. al., 1981).

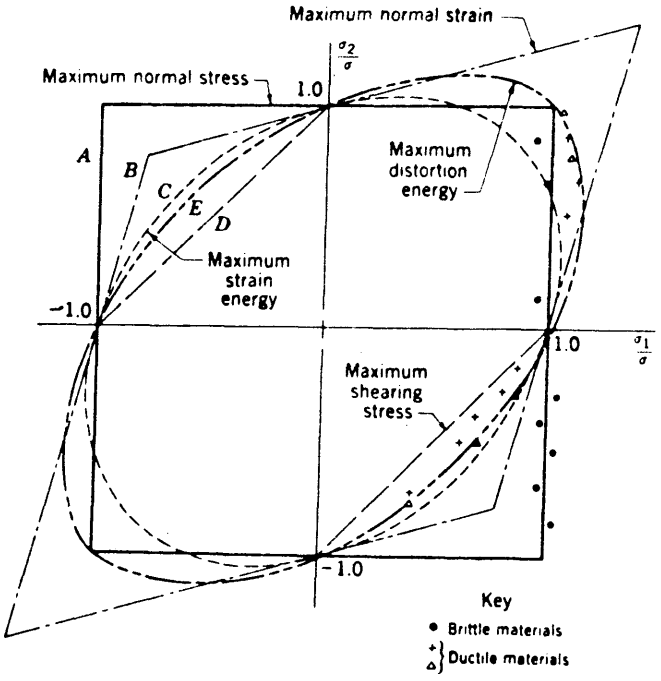


Figure 4. Comparison of Theories

LOADS ON CASING STRING

An oil well casing is exposed to three principal types of loading. These are collapse, axial load, and bursting. Casing string must be designed to withstand these types of loading with a dependable degree of safety and economy. The sources and characteristics of these loads are illustrated and discussed later. However, there are other considerations influencing casing design but they are not readily subject to calculation, and are met by applying the lessons of experience. These other considerations include wear, corrosion, vibration and pounding by drill pipe, effects of gun perforating, erosion, and general abuse by tools and equipment run in the well. Hills (1951) does not consider bending load in casing design because modern drilling practice does not permit bends in wells sharp enough to cause critical bending loads, even in directional drilled holes.

Collapse

Collapse loading on casing-string is generated by an external pressure higher than that inside the casing, and when the difference is of sufficient magnitude, there is a tendency for the casing to collapse. The most severe situation is having an empty casing and a full column of drilling fluid or formation pressure outside the casing (Craft, et. al., 1962). Any internal pressure that will be maintained

for the life of the well will allow a reduction of the collapse pressure (Hills, 1951). The ability of casing to withstand external pressure without experiencing either plastic or elastic failure is called collapse resistance (Craft, et. al., 1962). This resistance depends on the grade of casing and the ratio of outside diameter to casing thickness.

Axial Load

Two forces cause the axial load on casing-string; the weight of pipe when hanging free just before cementing (axial tension) and buoyancy which exerts axial compression on the lower part of the casing string. The effect of axial tension is twofold. First, it tends to cause the casing to fail by longitudinal deformation or yielding, and second, it lowers the resistance of casing to collapse and increases the resistance to burst. The stress at which permanent deformation occurs is called the minimum yield strength (Craft, et. al., 1962).

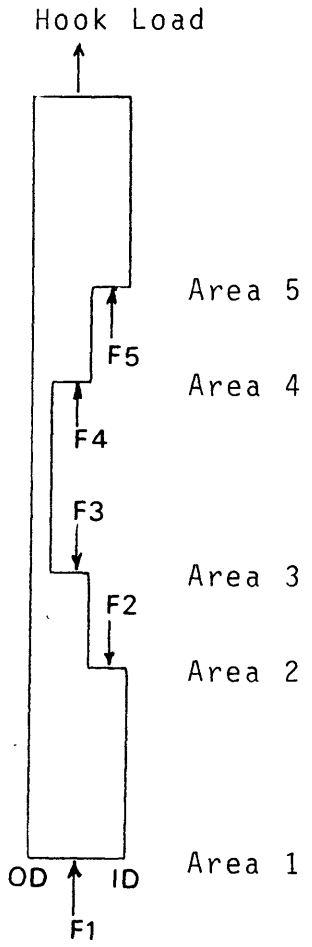
Buoyancy has been omitted in some casing designs in the past for various reasons. Due to the actual manner in which buoyancy is applied to a casing string, burst and collapse resistances are altered by the effect of combined stresses. For this reason buoyancy cannot be overlooked in a maximum load design (Adams, 1980). The effect of buoyancy is commonly thought of as the reduction in string weight when run in

liquid as compared to the string in air. However, no allowance is made regarding the manner in which the buoyant force is applied to the casing. The buoyancy, or reduction in string weight, as noted on the surface is actually the resultant of forces acting on all the exposed, horizontally oriented areas of the casing string (Adams, 1980).

The forces are equal to the hydrostatic pressures at each respective depth times the exposed areas, and defined as negative if acting upward. The area in reference are the tube end areas, the shoulders at changing casing weights, and to a small degree the shoulders on collars. Figure 5 shows the reference forces acting at each exposed area of a casing string, with the resultant loading indicated as negative tension (compression).

The forces acting on the areas of collar shoulders are sufficiently small as to be considered negligible in a practical casing design. The reduction in hook load observed at the surface is the same as that calculated using either the "buoyancy factor" method, or by calculating the weight of the volume of known density liquid displaced and subtracting it from the dry weight. However, the tension loadings differ greatly.

Mitchell (1980) listed two rules to calculate axial load. The first is using Newton's Law to calculate stresses and loads within the walls of the casing string, and the



$$\text{Buoyancy} = \sum_{o}^{i} \text{Forces}$$

$$B = - F_1 + F_2 + F_3 - F_4 - F_5$$

$$B = -P_1A_1 + P_2A_2 + P_3A_3 - P_4A_4 - P_5A_5$$

Buoyancy = - (lbs.)

Figure 5. Buoyancy Effect

second is calculating hook loads with Archimedes' principle.

Bursting

Casing string is exposed to high internal pressure during the entry of formation fluid or in such operations as squeezing and fracturing. In the upper portions of the string, external pressure is negligible, since near the surface there is neither formation pressure nor significant fluid column pressure opposite the casing. Therefore, any appreciable internal pressure (whether resulting from entry of fluid or from surface pump pressure) would cause an excess of internal pressure over external pressure in the upper portions, with a resulting tendency of the casing to fail by longitudinal splitting (Craft, et. al., 1962). The excess internal pressure at which this type of failure takes place is called bursting pressure.

Effect of Combined Stresses on Casing Strength

It has been noted that the exposure of casing to combined stresses alters the yield strength of casing. For example, an axial tensile stress reduces the resistance of casing to external pressures (collapse) and increases its resistance to internal pressures (burst). While an axial compressive or radial stress increases the resistance of casing to external pressure and reduces the resistance to internal pressure. In other words, under combined stresses, the effect is the same

as if the casing was of originally lower or higher-strength material (Edwards, et. al., 1939).

The strength of casing while acted upon by combined stresses is based on the Hencky-Von Mises maximum strain energy of distortion theory of failure (Mitchell, 1980). The theory does not apply to the elastic collapse of casing. Therefore, elastic collapse pressures should not be reduced because of combined stresses.

Formulas Used in Calculating Adjusted and Simple Collapse and Burst Pressures

Four equations for calculating simple collapse pressure were listed in API Bulletin 5C3. Each of these equations is applicable in certain range of outside diameter to casing thickness ratio (Table 1). These equations are:

$$P_{yp} = 2\,Y_p\,\left\{\frac{(D/t) - 1}{(D/t)^2}\right\} \qquad D/t \leq \text{Row 1} \qquad (37)$$

$$P_p = Y_p\,\left\{\frac{A}{D/t} - B\right\} - C \qquad \text{Row 1} < D/t \leq \text{Row 2} \qquad (38)$$

$$P_T = Y_p\,\left\{\frac{F}{D/t} - G\right\} \qquad \text{Row 2} < D/t < \text{Row 3} \qquad (39)$$

$$P_E = \frac{46.95 \times 10^6}{(D/t)\,\{(D/t) - 1)\}^2} \qquad (40)$$

Equation 37 is a theoretically derived equation which generates the minimum yield stress of the steel, Y_p, on the inside wall of the pipe by means of the Lame's equation. Equation 38 for plastic collapse pressure P_p and factors A, B, and C were derived by statistical regression analysis from 2488 collapse test on different grade of casing. Equation 39 was developed graphically to calculate the transition collapse pressure of the pipe. It starts from the value of D/t where the average plastic collapse pressure formula gives a collapse pressure of zero and is tangent to the minimum elastic collapse pressure formula. Then the generated formula used to calculate minimum collapse pressures between its tangency to the elastic collapse pressure curve and its intersection with the plastic collapse pressure curve. This is shown on Figure 6 for grade N-80 casing. The Factors F and G were determined by graphical methods for API grades. Equation 40 gives the elastic collapse pressure and it is derived theoretically. This equation was excluded because it is based on the elastic mode of failure which is not considered in the design procedure proposed by this thesis.

To calculate minimum burst pressure, the following equation is used:

$$P_b = 0.875 \frac{2\ Yp}{D/t} \qquad D/t \leq \text{Row 3} \qquad (41)$$

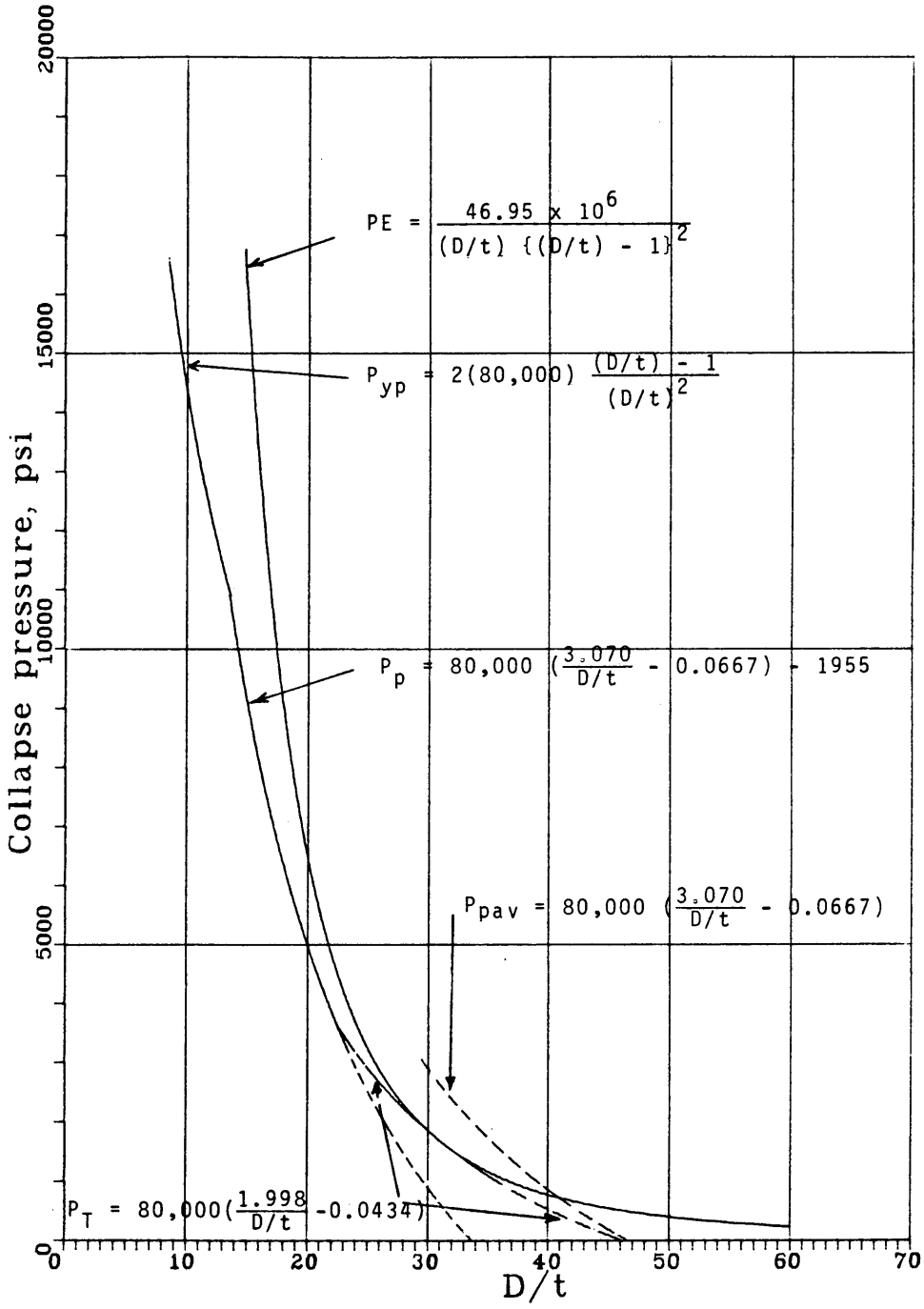


Figure 6. Grade N-80 Transition Collapse Formula Derivation

The factor 0.875 appearing in the equation allows for minimum wall tolerances. This equation was derived from theoretical considerations.

The adjusted collapse and burst pressures were calculated using the formulas listed in the API Bulletin 5C3:

$$P_{ca} = P_{co} \left\{ \left\{1 - 0.75 \left(\frac{S_z + P_i}{Y_p}\right)^2 \right\}^{\frac{1}{2}} - 0.5 \left(\frac{S_z + P_i}{Y_p}\right)\right\} \quad (42)$$

$$P_{ba} = P_b \left\{ \left\{1 - 0.75 \left(\frac{S_z + P_i}{Y_p}\right)^2 \right\}^{\frac{1}{2}} + 0.5 \left(\frac{S_z + P_i}{Y_p}\right)\right\} \quad (43)$$

These formulas are based on the maximum distortion energy theory of yielding. They are applicable where the collapse and burst pressures are directly proportional to yield strength. This condition is met by the API mentioned Equations 37, 38, 41, and approximately by Equation 39. It is not applicable to the elastic collapse formula (Equation 40 was not considered). The derivation of Equations 42 and 43 is shown in Appendix A.

COMPUTER PROGRAM

The computer program is written in FORTRAN IV language, for a PDP-10 computing system. Listing of the program and the subroutines are in the appendix. The program will now be discussed with the subroutines.

Main Program

The input data for the main program are:

1. Mud weight inside and outside casing (WMI, WMO)
2. Cement weight and height (CWT, HCO)
3. Surface pressure inside and outside casing (PIS, PES)
4. Total depth of the well (TD)
5. Casing yield strength if one grade is required to be used (YP)
6. Design factors for axial, collapse, and burst (DFA, DFC, DFB).

The program calculates:

1. Mud height outside casing (HMO)
2. Number of depth intervals (IN)
3. Pressure inside and outside casing (PI, PE)
4. Casing thickness (T)
5. Casing inside diameter (DI)

SUBROUTINE INPUTD:

This subroutine is used to input the required data to the main program.

SUBROUTINE RADIUS:

The input data for this subroutine are:

1. Casing outside radius (B)
2. Total depth of the well (TD)
3. Casing yield strength (YP)
4. Pressure inside and outside casing (PI, PE)
5. Depth of each interval (DEP)
6. Average weight of the casing string (AVWP)

At each depth interval, the subroutine selects a certain casing weight and grade (WP, GR) starting at the least weight for that grade and size. Then the subroutine calls subroutine BOYNCY to calculate buoyancy. Afterward, the subroutine calculates the axial stress (Z). The subroutine calls subroutine SIMPLE to determine the simple collapse and burst pressures (PCO, PBO) in order to use them in Equations (5A) and (6A), respectively to calculate the adjusted collapse and burst pressures (PCA, PBA). These calculated values for (PCA) and (PBO) are compared to the pressures inside and outside casing (PE, PI). If (PCA) and (PBA) are greater than PE and PI by the design factors for collapse and Burst (DFC, DFB), the subroutine will output casing weight (Wp), yield strength (Yp), and inside radius (A) to the main program, and goes to the next depth interval. But if PCA and PBA are less than PE and PI, respectively, then the subroutine will go back and selects the next larger weight or grade of casing and checks for

failure again until reaching the satisfying casing weight and grade to withstand the actual applied loads.

The right values for casing weight (WP), grade (GR), and inside radius (A) will be output to the main program and switch to another depth interval and so on.

SUBROUTINE SIMPLE:

This subroutine is used to calculate the simple collapse and burst pressures of casing using Equations (37), (38), (39), and (41), for the grades N-80, C-95, P-110, and V-150.

The input data for this subroutine are:

1. Casing inside and outside radii (A, B)
2. Yield strength of casing (Y_p)
3. The output of this subroutine is the simple collapse and burst pressures of casing (PCO, PBO)

SUBROUTINE OUTPUT:

The use of this subroutine is to output the following results:

1. Interval depth (DEP)
2. Weight of casing (WP)
3. Grade of casing (GR)
4. Inside diameter of casing (DI)
5. Casing thickness (T)

SUBROUTINE BOYNCY:

This subroutine calculates buoyancy using the method explained previously. The input data for this subroutine are:

1. Casing inside and outside radii, A and B
2. Pressure inside and outside casing, PI and PE

And the output of this subroutine is the buoyancy, BF

DISCUSSION OF RESULTS

Consideration of the radial stress in deep oil well casing design has a significant effect on the selection of the casing string weight and grade that is sufficient to withstand hydrostatic loading conditions which induce the three types of stresses, hoop, radial, and axial. Therefore considering only biaxial stresses (hoop and axial) in casing design of deep oil wells will not account for the actual loading conditions that a casing string may face in such wells.

The difference between biaxial and triaxial casing design is shown in Figures 7 through 15 in terms of casing thickness variation with depth as the ratio of internal pressure to external pressure (P_i/P_e) changes from zero to infinity. In these figures, it could be noticed that when (P_i/P_e) is between 0.5 and 1, casing thickness calculated by biaxial casing design is greater than that calculated by triaxial casing design and when (P_i/P_e) is between 1 and infinity, casing thickness calculated by biaxial casing design is less than that calculated by triaxial casing design, i.e., casing design that ignores radial stress will give casing weight and/or grade greater than that required when (P_i/P_e) ratio is between 0.5 and 1, and less than required when (P_i/P_e) ratio is between 1 and infinity.

Buoyancy effect on casing design is shown in Figures 16, 17, and 18. It is clear on these figures that thickness of the lower section of the casing string increases when buoyancy

effect is considered in the design. Due to buoyancy, the lower part of the casing will be subjected to a compressive axial stress and this will reduce the resistance of the casing to burst and increase its resistance to collapse. However, consideration of buoyancy will reduce the axial tension on the upper section of the casing string. The data in Table 5 are used to plot Figures 16, 17, and 18. According to these data (P_i/P_e) ratio is equal to 1.

Tables 2 and 3 show multi-grade casing design using biaxial and triaxial stresses, respectively.

CONCLUSIONS AND RECOMMENDATIONS

1. A procedure for deep oil wells casing design utilizing the theory of maximum-distortion-energy as the criterion for failure was developed.
2. Large differences in casing wall thickness were noted in casing design between triaxial and biaxial theory.
3. Biaxial casing design gives casing string weight and/or grade greater than that obtained by triaxial casing design when the ratio of P_i/P_e is between 0.5 and 1, and less than that obtained by triaxial casing design when the ratio of P_i/P_e is greater than 1.
4. This work could be extended to include buckling in casing design.

TABLE 1

BURST AND COLLAPSE TABLE

Grade		H-40	J, K-55	C-75	L, N-80	C-95	P-110	V-150
Y_p		40,000	55,000	75,000	80,000	95,000	110,000	150,000
A		2.95	2.99	3.060	3.070	3.125	3.180	3.335
B		.0463	.0541	.0642	.0667	.0745	.0820	.1020
C		755	1205	1805	1955	2405	2855	4055
F		2.047	1.990	1.985	1.998	2.047	2.075	2.170
G		.0313	.036	.0417	.0434	.0490	.0535	.0663
Critical D/t Ratios	Row 1	16.44	14.80	13.67	13.38	12.83	12.42	11.67
	Row 2	26.62	24.99	23.09	22.46	21.21	20.29	18.57
	Row 3	42.70	37.20	32.05	31.05	28.25	26.20	22.12

TABLE 2. MULTIGRADE BIAXIAL CASING DESIGN

Depth	Casing wt.	Grade	Inside diam.	Casing thick.
25000.	44.0	V-150	5.717	.641
24000.	41.0	V-150	5.814	.593
23000.	38.0	V-150	5.909	.546
22000.	38.0	V-150	5.909	.546
21000.	38.0	V-150	5.909	.546
20000.	38.0	V-150	5.909	.546
19000.	35.0	V-150	6.002	.499
18000.	35.0	V-150	6.002	.499
17000.	35.0	V-150	6.002	.499
16000.	35.0	V-150	6.002	.499
15000.	35.0	V-150	6.002	.499
14000.	32.0	V-150	6.094	.453
13000.	32.0	V-150	6.094	.453
12000.	32.0	V-150	6.094	.453
11000.	32.0	V-150	6.094	.453
10000.	32.0	P-110	6.094	.453
9000.	29.0	V-150	6.185	.408
8000.	29.0	V-150	6.185	.408
7000.	29.0	V-150	6.185	.408
6000.	29.0	V-150	6.185	.408
5000.	26.0	V-150	6.274	.363
4000.	26.0	V-150	6.274	.363
3000.	26.0	V-150	6.274	.363
2000.	29.0	V-150	6.185	.408
1000.	29.0	V-150	6.185	.408
0.	32.0	V-150	6.094	.453

TABLE 3. MULTIGRADE TRIAXIAL CASING DESIGN

Depth	Casing wt.	Grade	Inside diam.	Casing thick.
25000.	49.5	V-150	5.536	.732
24000.	44.0	V-150	5.717	.641
23000.	41.0	V-150	5.814	.593
22000.	41.0	V-150	5.814	.593
21000.	38.0	V-150	5.909	.546
20000.	35.0	V-150	6.002	.499
19000.	35.0	V-150	6.002	.499
18000.	35.0	V-150	6.002	.499
17000.	35.0	V-150	6.002	.499
16000.	35.0	V-150	6.002	.499
15000.	32.0	V-150	6.094	.453
14000.	32.0	V-150	6.094	.453
13000.	32.0	V-150	6.094	.453
12000.	32.0	V-150	6.094	.453
11000.	32.0	P-110	6.094	.453
10000.	29.0	V-150	6.185	.408
9000.	29.0	V-150	6.185	.408
8000.	29.0	V-150	6.185	.408
7000.	29.0	P-110	6.185	.408
6000.	26.0	V-150	6.274	.363
5000.	26.0	V-150	6.274	.363
4000.	26.0	V-150	6.274	.363
3000.	26.0	V-150	6.274	.363
2000.	29.0	V-150	6.185	.408
1000.	29.0	V-150	6.185	.408
0.	32.0	V-150	6.094	.453

TABLE 4

DATA USED FOR TEST RUN IN FIGURES 7 TO 15

Figure No.	(P_i/P_e) Ratio	Mud Weight Inside Casing, ppg	Mud Weight Outside Casing, ppg	Total Depth, Ft.	Grade of Casing
7	0	0	12	20,000	P-110
7	.25	3	12	20,000	P-110
8	.5	6	12	20,000	P-110
9	.75	9	12	20,000	P-110
10	1	12	12	20,000	P-110
11	1.5	15	10	20,000	P-110
12	2	15	7.5	20,000	P-110
13	3	15	5	20,000	P-110
14	4	15	3.75	20,000	P-110
15	∞	15	0	20,000	P-110

TABLE 5

DATA USED FOR TEST RUN IN FIGURES 16, 17, AND 18

Well depth = 25,000 ft.

Casing grade = V-150

Mud weight inside casing = 12 ppg

Mud weight outside casing = 14 ppg

Cement weight = 16 ppg

Cement height outside casing = 3,500 ft.

Surface pressure inside casing = 3,460 psi

Surface pressure outside casing = 500 psi

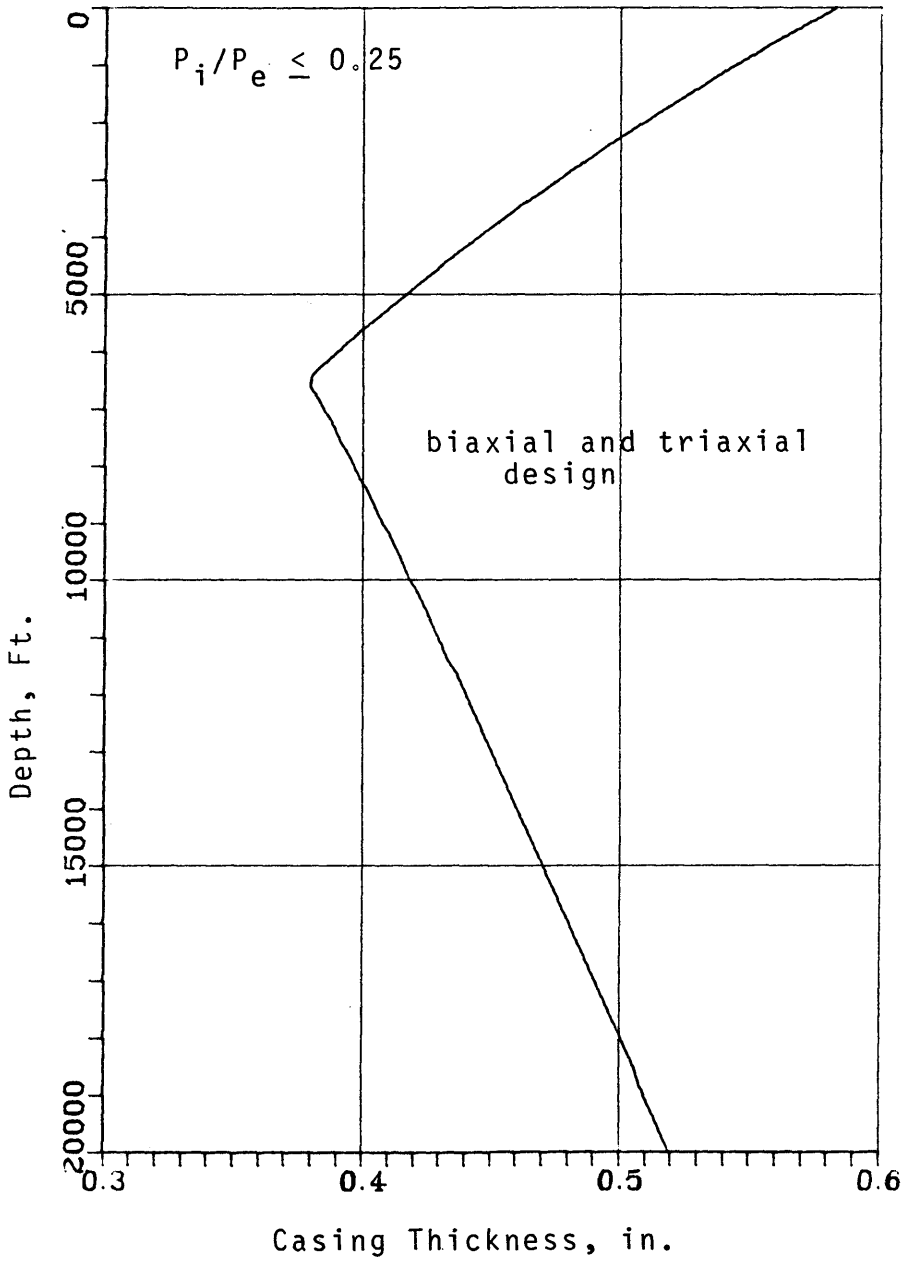


Figure 7. Casing Thickness Variation With Depth
7" P-110 Casing

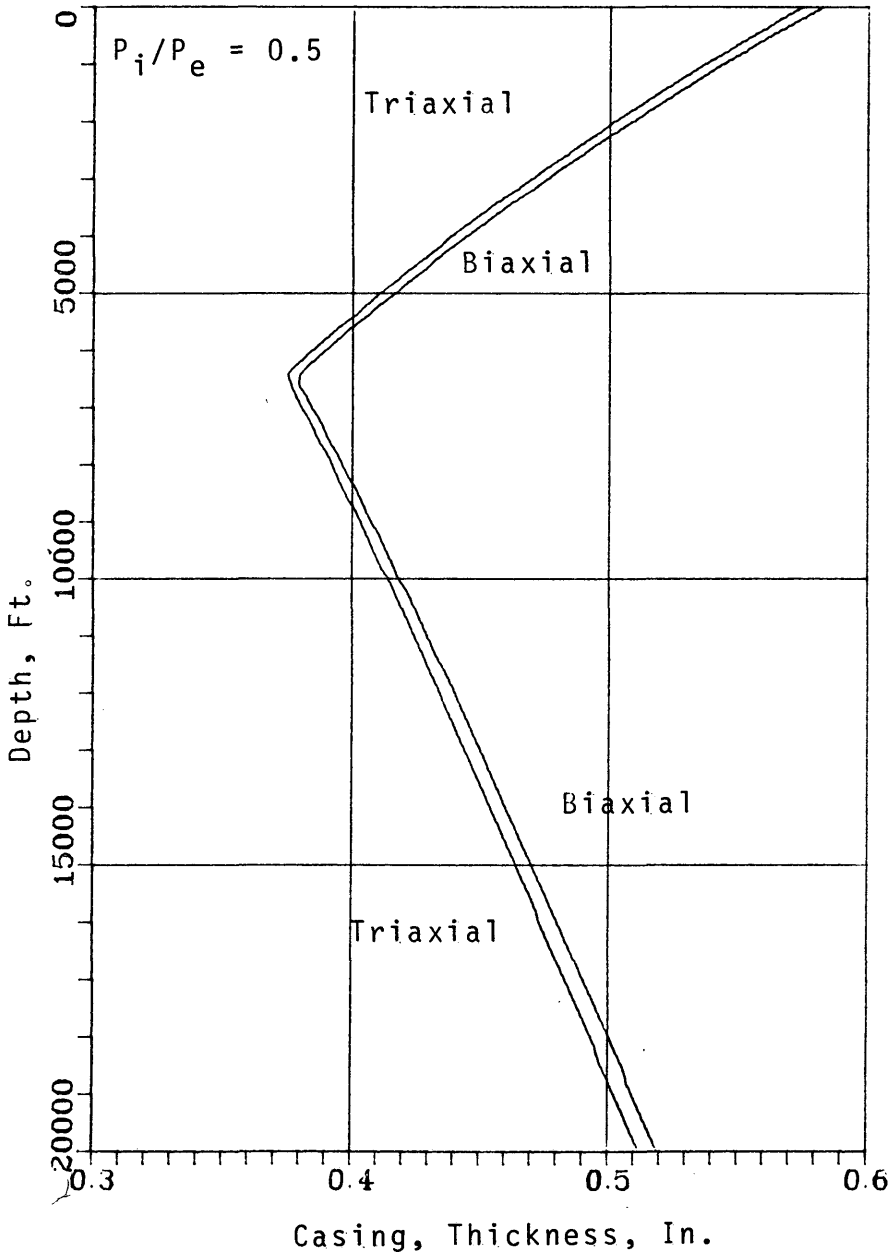


Figure 8. Casing Thickness Variation With Depth
7" P-110 Casing

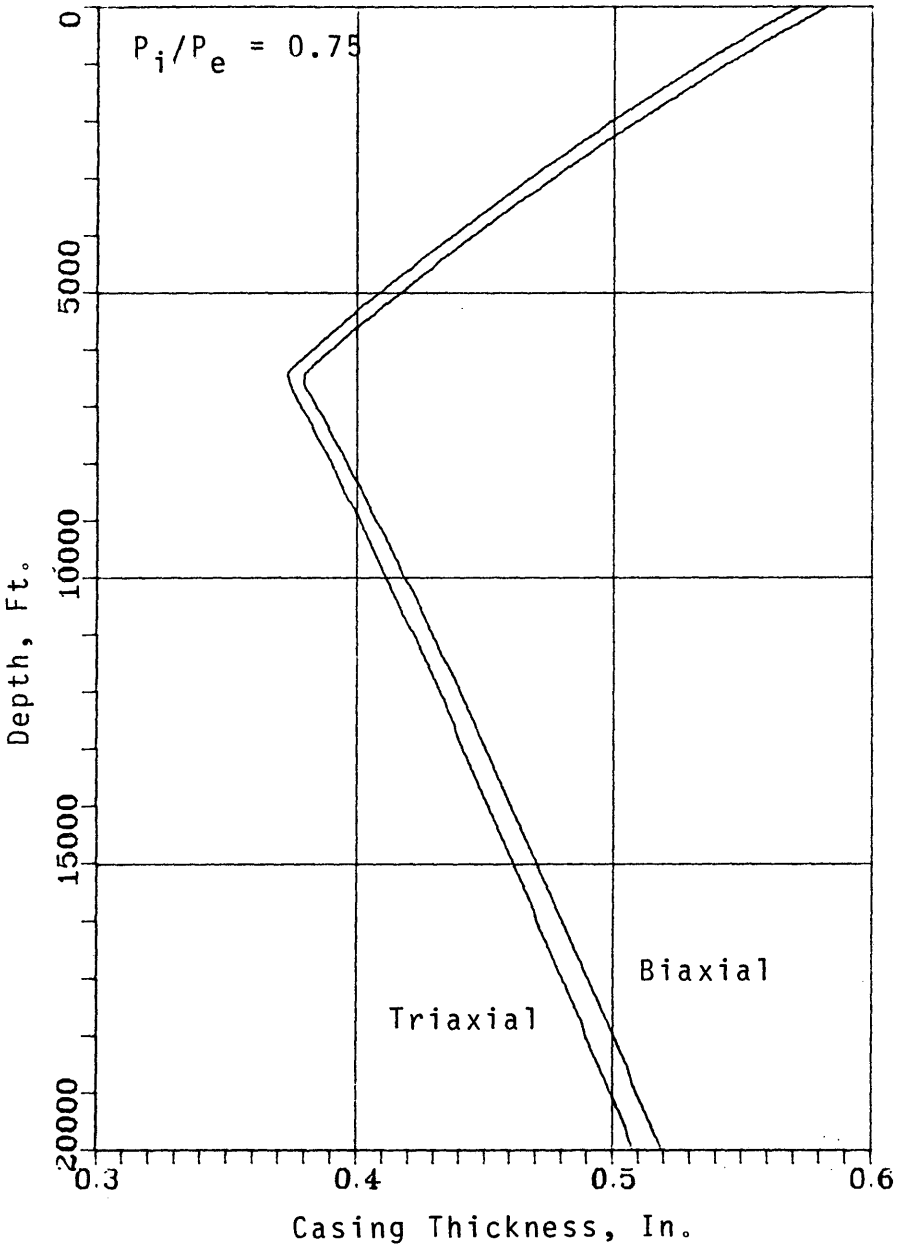


Figure 9. Casing Thickness Variation With Depth
7" P-110 Casing

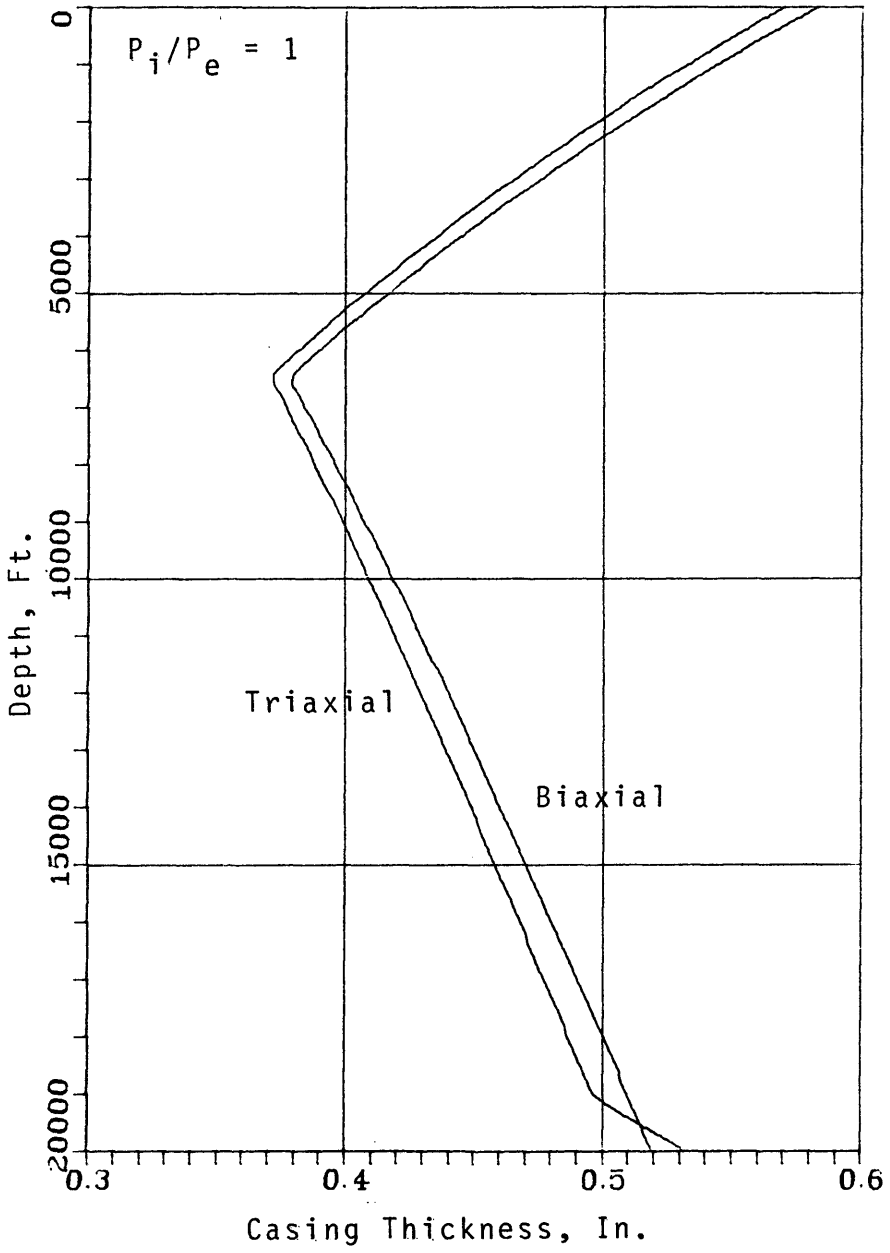


Figure 10. Casing Thickness Variation With Depth
7" P-110 Casing

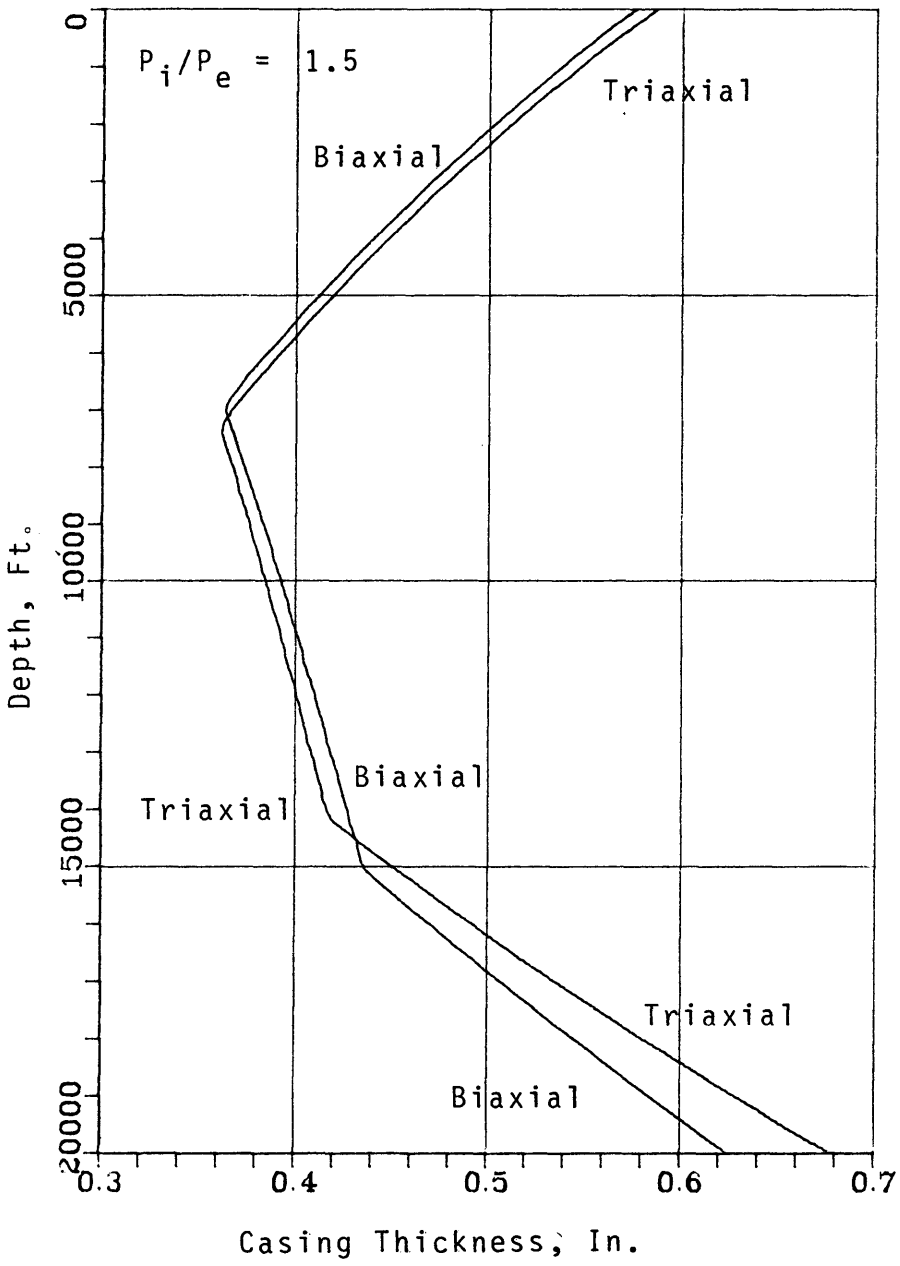


Figure 11. Casing Thickness Variation With Depth
7" P-110 Casing

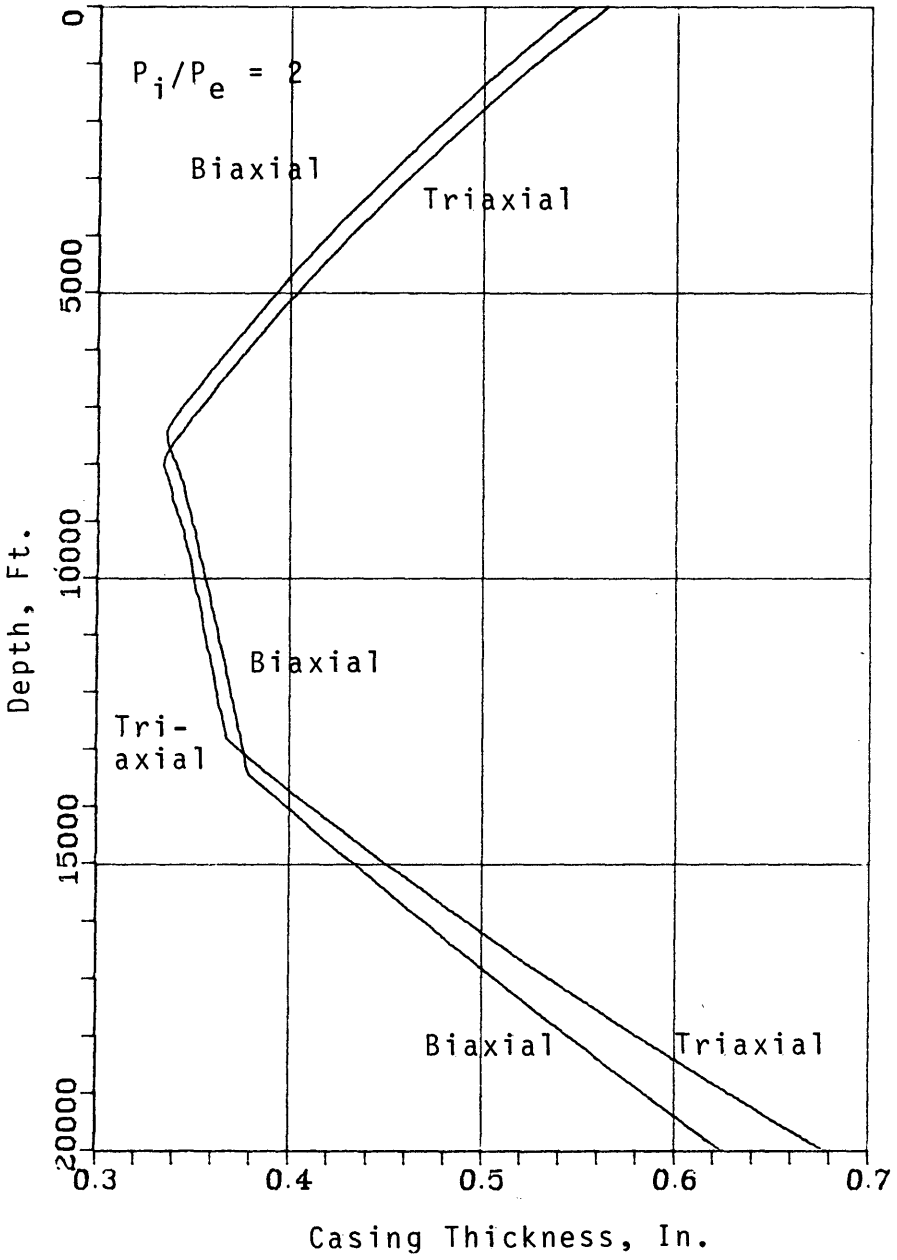


Figure 12. Casing Thickness Variation With Depth
7" P-110 Casing

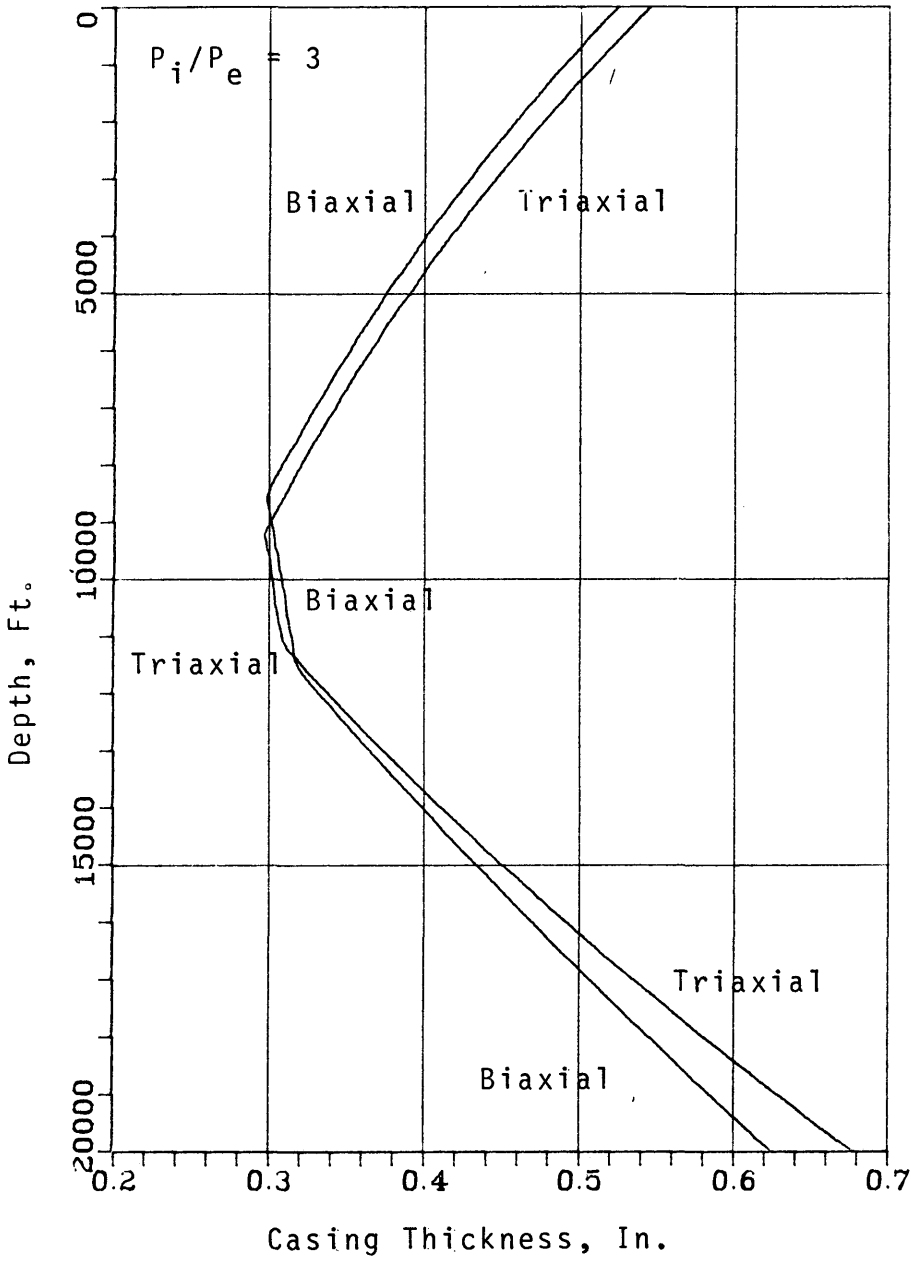


Figure 13. Casing Thickness Variation With Depth
7" P-110 Casing

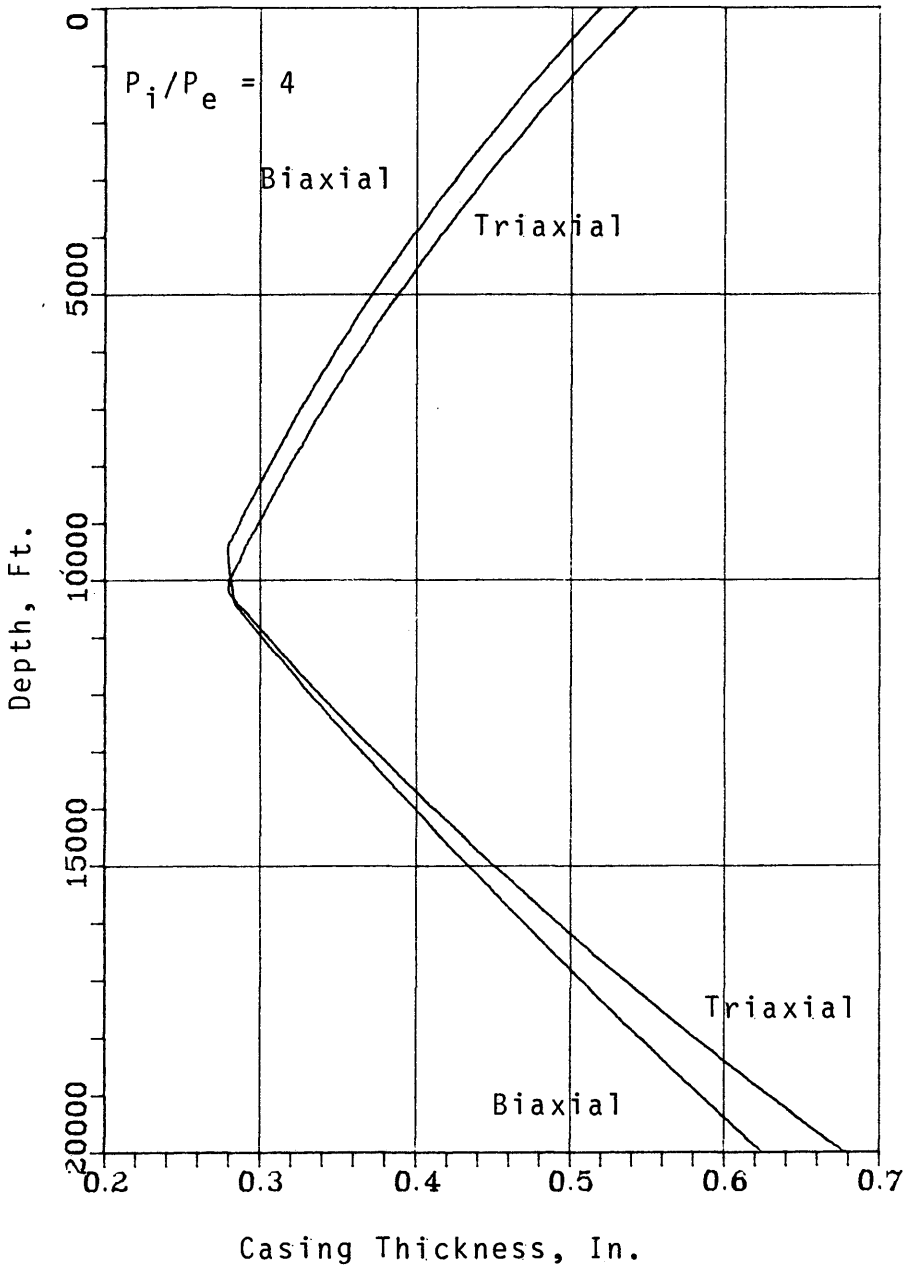


Figure 14. Casing Thickness Variation With Depth
7" P-110 Casing

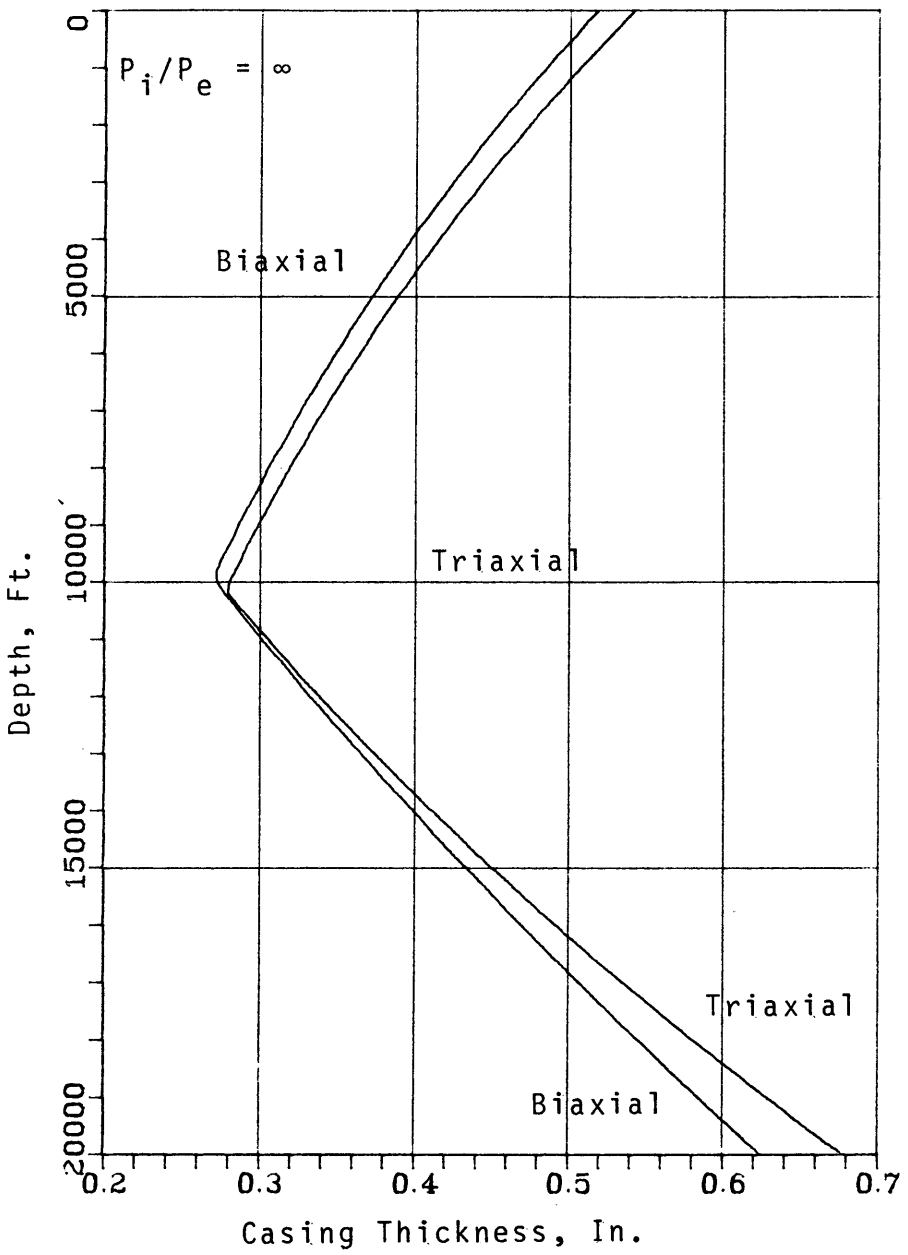


Figure 15. Casing Thickness Variation With Depth
7" P-110 Casing

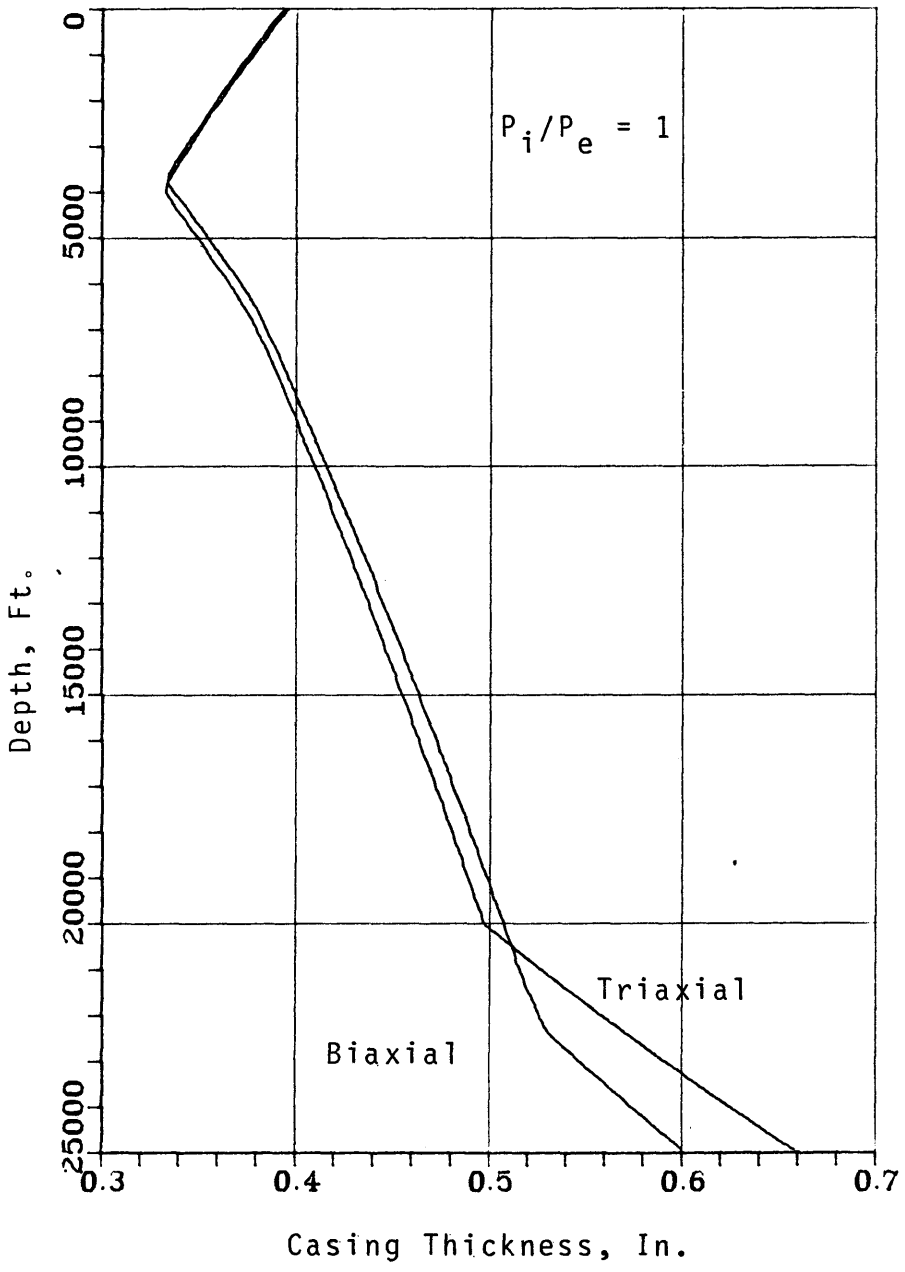


Figure 16. Buoyancy Effect on Casing Design
7" V-150 Casing

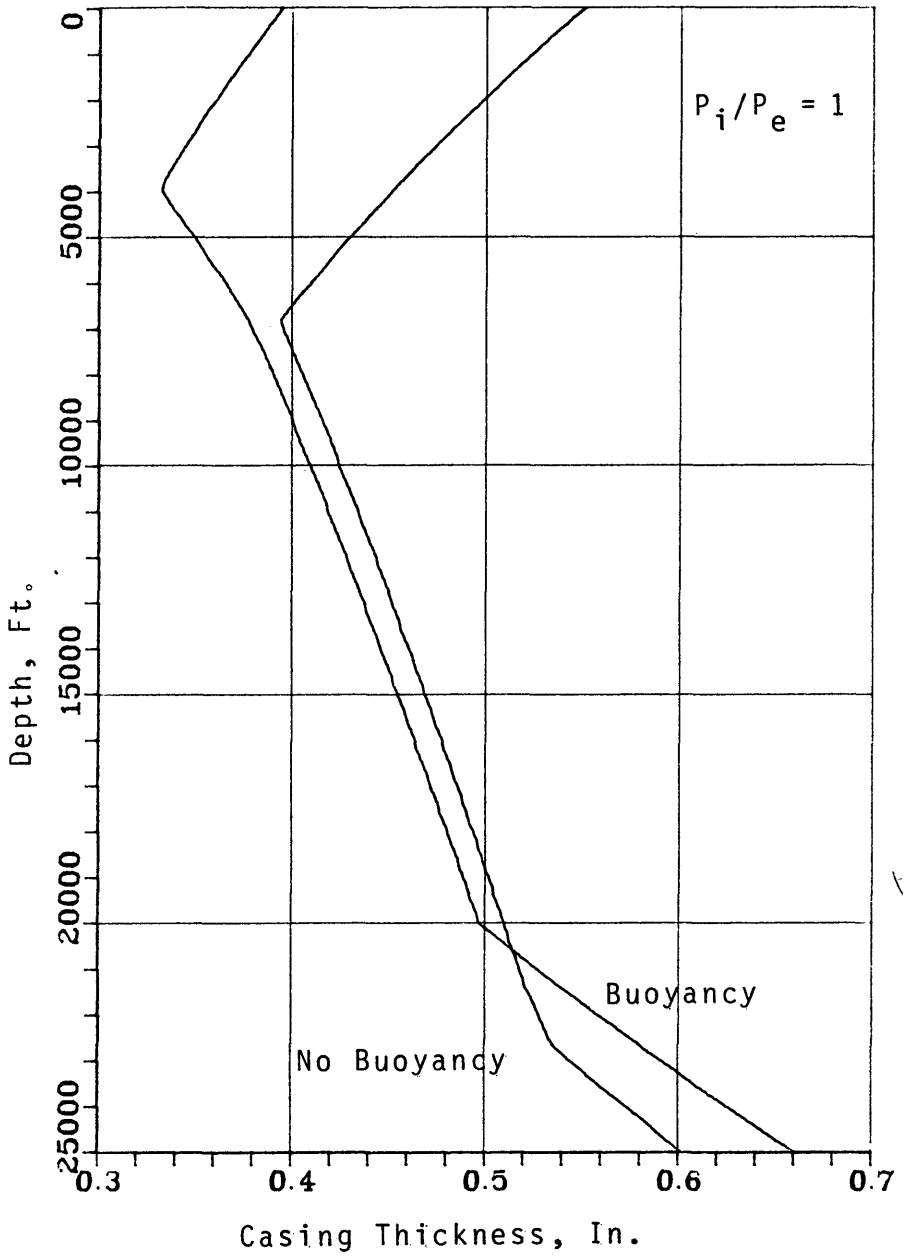


Figure 17. Buoyancy Effect on Triaxial Casing Design
7" V-150 Casing

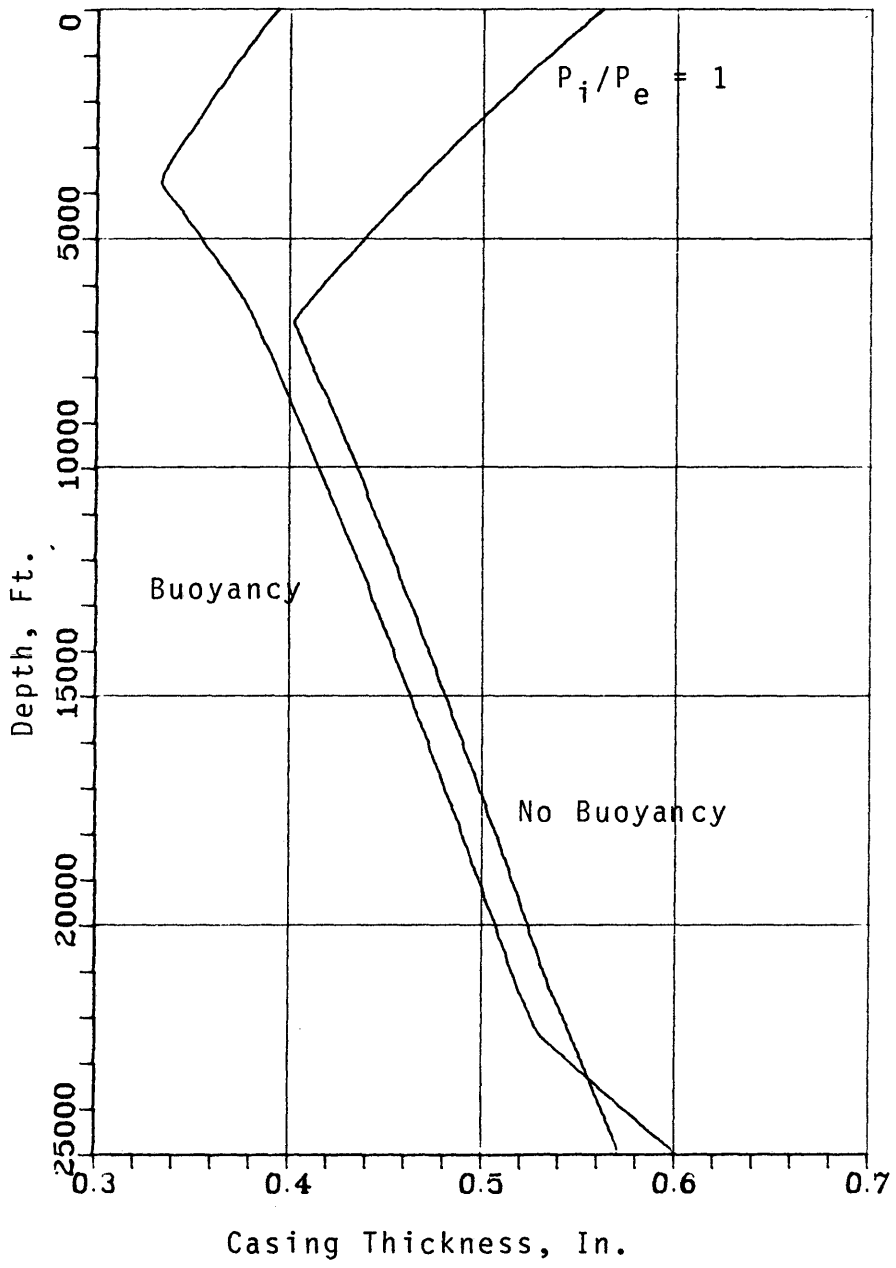


Figure 18. Buoyancy Effect on Biaxial Casing Design
7" V-150 Casing

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APPENDIX A

DERIVATION OF ADJUSTED BURST AND COLLAPSE FORMULAS

From Lame's Equation, the radial and hoop stresses at the inner wall of the casing are:

$$S_r = - P_i \tag{1A}$$

$$S_h = \frac{- 2b^2 P_o + P_i(b^2 + a^2)}{b^2 - a^2} \tag{2A}$$

Plastic yield may be described by the theory of maximum distortion energy as:

$$S_r^2 + S_h^2 + S_z^2 - S_r S_h - S_r S_z - S_h S_z = S_o^2 \tag{3A}$$

Which upon substitution of Equations (1A) and (2A) becomes:

$$(-P_i)^2 + \left(\frac{-2b^2 P_o + P_i(b^2 + a^2)}{b^2 - a^2}\right) + S_z^2 - (-P_i)$$

$$\left(\frac{-2b^2 P_o + P_i(b^2 + a^2)}{b^2 - a^2}\right) - (-P_i) S_z -$$

$$\left(\frac{-2b^2 P_o + P_i(b^2 + a^2)}{b^2 - a^2}\right) S_z = S_o^2 \tag{4A}$$

Assume infinitesimal deformation:

$$\frac{-2b^2 P_o + P_i(b^2 + a^2)}{b^2 - a^2} = \frac{-2b^2 (P_o - P_i)}{b^2 - a^2}$$

Now, rearrange Equation (4A) and divide by S_o^2:

$$\left(\frac{2b^2(P_o - P_i)}{(b^2-a^2) S_o}\right)^2 + \left(\frac{S_z - P_i}{S_o}\right) \left(\frac{2b^2 (P_o - P_i)}{(b^2 - a^2) S_o}\right) +$$

$$\frac{P_i^2 + S_z^2 + P_i S_z}{S_o^2} - 1 = 0.$$

$$\frac{2b^2(P_o - P_i)}{(b^2 - a^2) S_o} = \tfrac{1}{2} \left\{-\left(\frac{S_z - P_i}{S_o}\right) \mp \right.$$

$$\left. \left\{\left(\frac{S_z - P_i}{S_o}\right)^2 - \frac{4P_i^2 - 4S_z^2 - 4P_i S_z}{S_o^2} + 4\right\}^{\frac{1}{2}} \right\}$$

$$\frac{2b^2 (P_o - P_i)}{(b^2 - a^2) S_o} = \mp \left\{1 - \frac{3}{4} \left(\frac{S_z + P_i}{S_o}\right)^2 \right\}^{\frac{1}{2}} - \frac{S_z + P_i}{2S_o}$$

Adjusted collapse pressure:

$$P_{ca} = \left\{ \left\{1 - 0.75 \left(\frac{S_z + P_i}{S_o}\right)^2 \right\}^{\frac{1}{2}} - 0.5 \frac{S_z + P_i}{S_o} \right\} P_{co} \qquad (5A)$$

Adjusted burst pressure:

$$Pba = \left\{ \left\{ 1 - 0.75 \left(\frac{S_z + P_i}{S_o} \right)^{2} \right\}^{\frac{1}{2}} + 0.5 \frac{S_z + P_i}{S_o} \right\} Pbo \qquad (6A)$$

APPENDIX B - NOMENCLATURE

ENGLISH SYMBOLS

a : Inner radius

C_1 & C_2 : Constants of Equations (4) and (5)

$d1$: Length of the free body of Equation (1)

E : Modulus of elasticity

P_i : Internal pressure

P_o : External pressure

r : Radius

S_h : Hoop (tangential) stress

S_o : Yield strength of casing string

S_r : Radial stress

S_z : Axial stress

S_x : Stress in the x-direction

S_y : Stress in the y-direction

u : Strain energy per unit volume

GREEK SYMBOLS

υ : Poisson's ratio

ε_a : Axial strain (Equation 3)

ε_x, ε_y, ε_z : Strain in the x, y, and z-directions

θ : Radial angle

τ_f : Shearing failure stress

APPENDIX C - COMPUTER PROGRAM

```
00100 C=============================================================
00200 C
00300 C          DEEP WELL CASING DESIGN
00400 C    CONSIDERING THE TRI-AXIAL STRESSES AFFECTING ON CASING.
00500 C
00600 C=============================================================
00700 C
00800 C  THIS PROGRAM CAN :
00900 C       I. CALCULATE THE REQUIRED CASING WEIGHT & GRADE
01000 C          AT EACH DEPTH.
01100 C      II. CALCULATE THE REQUIRED WEIGHT FOR A DEFINITE
01200 C          GRADE OF CASING AT EACH DEPTH.
01300 C
01400 C=============================================================
01500 C=============================================================
01600 C  A              : CASING RADIUS.
01700 C  ALOD           : LOAD OF CASING STRING.
01800 C  AM             : STEEL AREA FOR BOUNCY FORCE
01900 C                    CALCULATION.
02000 C  AS             : AREA OF STEEL FOR STRESS CALCULATION.
02100 C  AVWP           : AVERAGE WT OF CASING STRING.
02200 C  B              : OUTSIDE RADIAS OF CASING
02300 C  BU             : BUOYANCY .
02400 C  CWT            : CEMENT WEIGHT.
02500 C  D              : CASING OUTSIDE DIAMETER.
02600 C  DEP            : DEPTH OF EACH INTERVAL.
02700 C  DFA            : DESIGN FACTOR FOR AXIAL LOAD.
02800 C  DFB            : DESIGN FACTOR FOR BURST.
02900 C  DFC            : DESIGN FACTOR FOR COLLAPSE.
03000 C  DI             : CASING INSIDE DIAMETER.
03100 C  DL             : INTERVAL LENGTH.
03200 C  GR             : GRADE OF CASING.
03300 C  HMO & HCO      : HEIGHT OF MUD & CEMENT OUTSIDE CASING.
03400 C  IN             : NUMBER OF INTERVALS.
03500 C  J & N          : COUNTERS.
03600 C  PI & PE        : MUD PRESSURE INSIDE &
03700 C                    OUTSIDE CASING.
03800 C  PIS & PES      : SURFACE PRESSURE INSIDE
03900 C                    & OUTSIDE CASING.
04000 C  PCA            : ADJUSTED COLLAPSE PRESSURE.
04100 C  PCO            : SIMPLE COLLAPSE PRESSURE.
04200 C  PBA            : ADJUSTED BURST PRESSURE.
04300 C  PBO            : SIMPLE BURST PRESSURE.
04400 C  T              : CASING THICHNESS.
04500 C  TD             : TOTAL DEPTH.
04600 C  TWP            : TOTAL WT OF CASING STRING.
04700 C  WP             : WEIGHT OF CASING.
04800 C  WMI & WMO      : MUD WEIGHT INSIDE & OUTSIDE CASING.
04900 C  YP             : YIELD STRENGTH OF THE STEEL.
05000 C  Z              : AXIAL STRESS.
05100 C=============================================================
```

```
C=============================================================
C.....INPUT REQUIRED DATA.
   CALL INPUTD(WMI,WMO,CWT,HCO,PIS,PES,TD,DL,N,
     +                 YP,DFA,DFB,DFC)
   J=0
   D=7.
   B=D/2.
   DEP=TD

C.....CALC. MUD HEIGHT OUTSIDE CASING.

   HMO=TD-HCO

C.....CALC. NUMBER OF INTERVALS.

   IN=TD/DL+1




   DO 10 I=1,IN

   YP=YP
   J=J+1


C.....CALC. PRESS INSIDE & OUTSIDE CASING

   PI=PIS+.052*WMI*DEP
   PE=PES+.052*(WMO*HMO+CWT*HCO)

C.....CALL SUBROUTINE TO CALC. CASING INSIDE RADIUS WHEN
C.....CONSIDERING STRESSES ON THE INSIDE WALL OF CASING.

   CALL RADIUS(B,TD,YP,N,PE,PI,DFC,DFA,
     +                    DFB,DEP,J,AVWP,WP,A)

C.....CALC. CASING WALL THICKNESS & DIAMETER.


   T=B-A
   DI=2.*A

   TWP=TWP+WP
   AVWP=TWP/FLOAT(J)

C.....OUTPUT THE RESULTS
```

```
10300
10400     CALL OUTPUT(DI,YP,DEP,J,T,WP)
10500
10600     DEP=DEP-DL
10700     HCO=HCO-DL
10800     IF(HCO.LE.0.0)HCO=0.0
10900     IF(HCO.EQ.0.0)HMO=HMO-DL
11000
11100
11200 10  CONTINUE
11300     STOP
11400     END
```

```
00100 C=============================================================
00200 C          SUBROUTINE INPUT
00300 C=============================================================
00400 C
00500    SUBROUTINE INPUTD(WMI,WMO,CWT,HCO,PIS,PES,TD,
00600       +                                      DL,N,YP,DFA,DFB,DFC)
007J098 WRITE(4,10)
00800    READ(4,20)WMI,WMO
00500    SUBROUTINE INPUTD(WMI,WMO,CWT,HCO,PIS,PES,TD
00600       +                                      DL,N,YP,DFA
007J098 WRITE(4,10)
00800    READ(4,20)WMI,WMO
00900    WRITE(4,30)
01000    READ(4,20)CWT,HCO
01100    WRITE(4,40)
01200    READ(4,20)PIS,PES
01300    WRITE(4,50)
01400    READ(4,20)TD,DL
01900    READ(4,70)YP
02000 99 WRITE(4,90)
02100    READ(4,100)DFA,DFB,DFC
02200    WRITE(4,110)
02300    READ(4,120)ANS
02400    IF(ANS.EQ.'NO ')GO TO 98
02500 10 FORMAT(1X,'ENTER MUD WEIGHT INSIDE & OUTSIDE CASING.')
02600 20 FORMAT(2G)
02700 30 FORMAT(1X,'ENTER CEMENT WEIGHT & HEIGHT.IF NO CEMENT
02800       +    ENTER ZERO.')
02900 40 FORMAT(1X,'ENTER SURFACE PRESS INSIDE & OUTSIDE CASING.')
03000 50 FORMAT(1X,'ENTER DEPTH OF THE WELL & INTERVAL LENGTH.')
03100 60 FORMAT(1X,'ENTER 1 IF YOU WANT TO USE ONE GRADE ONLY,',/,
03200       +    1X,'OTHERWISE ENTER 2 .')
03300 70 FORMAT(G)
03400 80 FORMAT(1X,'ENTER YIELD STRENGTH OF REQUIRED CASING.')
03500 90 FORMAT(1X,'ENTER THE DESIGN FACTORS FOR AXIAL,BURST,',/,
03550       +    1X,'AND COLLAPSE RESISTANCES.')
03600100          FORMAT(3G)
03700110          FORMAT(1X,'WAS ALL DATA ENTERED CORRECTLY ?')
03800120          FORMAT(A3)
04000    RETURN
04100    END
```

```
00100 C=============================================================
00200 C          SUBROUTINE RADIUS
00300 C=============================================================
00400 C
00500 C  THIS SUBROUTINE CALCULATES CASING INSIDE RADIUS & WT.
00600 C
00700 C  THE INPUT DATA FOR THIS SUBROUTINE ARE :
00800 C       1. CASING OUTSIDE RADIUS,B .
00900 C       2. TOTAL DEPTH OF THE WELL,TD .
01000 C       3. YIELD STRENGTH,YB .
01100 C       4. PRESSURE INSIDE & OUTSIDE CASING,PI & PE.
01200 C       5. PRESSURE CAUSING BUOYANCY FORCE ,PBF .
01300 C       6. DEPTH OF EACH INTERVAL , DEP.
01400 C       7. AVERAGE WT OF CASING STRING ,AVWP.
01500 C
01600 C=============================================================
01700
01800
01900      SUBROUTINE RADIUS(B,TD,YP,N,PE,PI,DFC,DFA,
02000     +                  DFB,DEP,J,AVWP,WP,A)
02100      DIMENSION BU(0:200),AB(0:200)
02200
02300      L=0
02400
02500 C.....SET INITIAL WT FOR EACH CASING GRADE.
02600
02700      IF(YP.EQ.80000.0.OR.YP.EQ.95000.0)WP=20.0
02800      IF(YP.EQ.110000.0.OR.YP.EQ.150000.0)WP=23.0
02900
03000020   IF(N.EQ.2) GO TO 40
03100
03200      WP=WP+3.0
03300      IF(WP.EQ.47.0) WP=49.5
03400      IF(N.EQ.1) GO TO 60
03500
03600040   IF(L.EQ.4) L=0
03700      IF(L.EQ.0) YP=65000.
03800      YP=YP+15000.
03900      IF(L.GT.0.OR.WP.GE.23.0) GO TO 80
04000      WP=20.0
04100080   IF(L.EQ.0) WP=WP+3.
04200      IF(WP.EQ.47.0) WP=49.5
04300      L=L+1
04400      IF(L.EQ.4) YP=150000.
04500      IF(L.EQ.2.AND.WP.EQ.23.0) L=4
04600
04700 C.....CALC. CASING INSIDE RADIUS.
04800
04900060   AS=WP/3.434469
05000      A=SQRT(B**2-AS/3.14159)
05100      AB(J)=A
```

```
05200
05300 C.....CALL SUBROUTINE TO CALCULATE BUOYANCY.
05400
05500    CALL BOYNCY(AB,B,J,PI,PE,BU)
05600 C.....CALC. AXIAL LOAD ON CASING.
05700    ALOD=AVWP*(TD-DEP)+BU(J)
05800
05900    IF(J.EQ.1)ALOD=BU(J)
06000
06100
06200 C.....CALC. AXIAL STRESS .
06300    Z=ALOD/AS
06400    IF((YP/DFA).LT.ABS(Z)) GO TO 20
06500
06600 C.....CALL SUBROUTINE TO CALCULATE SIMPLE COLLAPSE &
06700 C.....BURST PRESSURES.
06800
06900    CALL SIMPLE(B,A,YP,PCO,PBO)
07000
07100 C.....CALC. RADIAL STRESS.
07200
07300    R=-PI
07400
07500 C.....CALCULATE ADJUSTED BURST & COLLAPSE PRESSURES
07600 C.....USING MAXIMUM-DISTORTION-ENERGY THEORY.
07700
07800    PBA=PBO*(SQRT(1.-.75*((Z+R)/YP)**2)+.5*(Z+R)/YP)
07900    PCA=PCO*(SQRT(1.-.75*((Z+R)/YP)**2)-.5*(Z+R)/YP)
08000
08100 C.....CHECK FOR CASING FAILURE.
08200
08300    IF((ABS(PI)*DFB).GT.PBA) GO TO 20
08400    IF((PCA/DFC).LT.PE) GO TO 22
08500
08600
08700    A=A
08800    RETURN
08900    END
```

```
00100 C=============================================================
00200 C          SUBROUTINE SIMPLE
00300 C=============================================================
00400 C The input data for this subroutine are:
00500 C      1. Casing inside diameter, A.
00600 C      2. Casing outside diameter, B.
00700 C      3. Yield strength, YP.
00800 C
00900 C The output of this SUBroutine is the simple burst
01000 C and collapse pressures, PBO & PCO.
01100 C
01200 C=============================================================
01300 C
01400       SUBROUTINE SIMPLE(B,A,YP,PCO,PBO)
01500       T=B-A
01600       D=2.*B
01700       DT=D/T
01800       IF(YP.EQ.80000.0) GO TO 10
01900       IF(YP.EQ.95000.0) GO TO 20
02000       IF(YP.EQ.110000.0)GO TO 30
02100       IF(YP.EQ.150000.0)GO TO 40
02200010    C1=13.38 ;C2=22.46 ;C3=31.05
02300       AA=3.07 ;BB=0.0669 ;CC=1955.0
02400       FF=1.998 ;GG=0.0434
02500       GO TO 50
02600020    C1=12.83 ;C2=21.21 ;C3=28.25
02700       AA=3.125 ;BB=0.0745;CC=2405.
02800       FF=2.047 ;GG=0.049
02900       GO TO 50
03000030    C1=12.42 ;C2=20.29 ;C3=26.2
03100       AA=3.18 ;BB=0.082 ;CC=2855.
03200       FF=2.075 ;GG=0.0535
03300       GO TO 50
03400040    C1=11.67 ;C2=18.57 ;C3=22.12
03500       AA=3.335 ;BB=0.102 ;CC=4055.
03600       FF=2.17 ;GG=0.0663
03700050    IF(DT.LE.C1) GO TO 60
03800       IF(DT.GT.C1.AND.DT.LE.C2) GO TO 70
03900       IF(DT.GT.C2.AND.DT.LT.C3) GO TO 80
04000060    PCO=2.*YP*((DT-1.)/DT**2)
04100       GO TO 90
04200070    PCO=YP*(AA/DT-BB)-CC
04300       GO TO 90
04400080    PCO=YP*(FF/DT-GG)
04500090    PBO=0.875*2.*YP/DT
04600       RETURN
04700       END
```

```
00100 C=============================================================
00200 C          SUBROUTINE  BOYNCY
00300 C=============================================================
00400
00500    SUBROUTINE BOYNCY(A,B,N,PI,PE,BF)
00600    DIMENSION A(0:200),BF(0:200),DAR(200),AR(0:200)
00700       BF(0)=0.
00800       AR(N-1)=3.14*(B**2-A(N-1)**2)
00900       AR(0)=0.
01000       AR(N)=3.14*(B**2-A(N)**2)
01100       DAR(N)=AR(N-1)-AR(N)
01200       BF(N)=ABS(PI)*DAR(N)+BF(N-1)
01300       IF(N.EQ.1)BF(N)=PE*DAR(N)
01400    RETURN
01500    END
```

```
00100 C=============================================================
00200 C          SUBROUTINE OUTPUT
00300 C=============================================================
00400 C
00500 C This suboutine output the following data:
00600 C
00700 C    1. Depth at each interval, DEP.
00800 C    2. Casing inside diameter, DI.
00900 C    3. Casing weight, WP.
01000 C    4. Casing grade, GR.
01100 C    5. Casing thickness, T.
01200 C
01300 C=============================================================
01400    SUBROUTINE OUTPUT(A,YP,DEP,I,T,WP1)
01500    IF(I-1)1,2,1
01600 2  WRITE(4,100)
01700100          FORMAT(6X,'Depth',4X,'Casing wt.',4X,'Grade',
01800       +  4X,'Inside diam.',4X,'Casing thick.')
01900    WRITE(4,200)
02000200          FORMAT(1X,/,70('.'))
02100 1  IF(YP.EQ.80000.0) GR='N-80'
02200    IF(YP.EQ.95000.0) GR='C-95'
02300    IF(YP.EQ.110000.0)GR='P-110'
02400    IF(YP.EQ.150000.0)GR='V-150'
02500    WRITE(4,300)DEP,WP1,GR,A,T
02600300          FORMAT(6X,F6.0,6X,F4.1,7X,A5,7X,F5.3,11X,F5.3,/)
02700    RETURN
02800    END
```