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ELECTRICAL RESISTIVITY
AND DIELECTRIC CONSTANT
OF FROZEN ROCKS

by
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A Thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science in Geophysical Engineering.

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ABSTRACT

Laboratory experiments traced the effect of temperature variations on the electrical resistivity and dielectric constant of rocks in the range from + 22 to - 40°C. Six sandstone cores of three different porosities were measured while saturated with 0.02, 0.1, 0.2, and 0.5 molar sodium chloride solutions. The resistance and capacitance of the cores were measured at nine temperatures with a conductance-capacitance bridge. For each temperature, the frequency of the applied voltage was varied from 200 to 100,000 cps. Above the freezing point, the change in resistivity of the cores with temperature was due mainly to the change in resistivity of the solution. As the water started to freeze, the rate of increase in resistivity was larger in a rock saturated with a less concentrated solution or in a rock of medium porosity. Below - 10°C, the salinity of the solution being nearly the same in all the samples, the resistivity

depended almost entirely on the amount of unfrozen solution. The resistivity continued to increase uniformly with decreasing temperature unless the remaining solution became saturated and crystallized. The dielectric constant, due mostly to interfacial polarization, varied with temperature in about the same manner as the reciprocal of the resistivity. The change in resistivity with frequency was more pronounced at low temperatures and was almost nonexistent above the freezing point, whereas the largest variation of dielectric constant with frequency took place at room temperature.

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INTRODUCTION

The electrical properties of rocks and soils have been measured in the field and in the laboratory by several investigators (Smith-Rose, 1934; Keller and Licastro, 1959). Their results show that the electrical resistivity and dielectric constant are much affected by the amount of water in the pores and the concentration of ions in solution. If all the "free" water in a rock were removed by evaporation, the resistivity of the rock may increase by a factor of 10^5 , and, at the same time, the dielectric constant may decrease by a factor of 10^4 . Consequently, when the water freezes in the pores of a rock, very large variations in the electrical properties may also be expected. A knowledge of these electrical properties of frozen rocks is necessary if geoelectrical methods are to be applied efficiently in areas of permafrost. According to Muller (1947), approximately one-fifth of the entire land surface of the earth is underlain

by permafrost, most of which is still unexplored.

Very little information has been published on the electrical properties of rocks below 0°C . Higgs (1930) studied the relation between the resistivity and temperature for a loam, in the range from $+20$ to -15°C . Smith-Rose (1934) measured the dielectric constant and electrical resistivity of a soil sample at a frequency of 1,200 kcps, varying the temperature from $+30$ to -3°C . Nesterova and Nesterov (1947) measured the electrical resistivity of rock samples, partially saturated with sodium chloride solutions, from $+20$ to -20°C . Ananyan (1958) measured the electrical resistivity of fine-grained rocks, for various moisture contents, in the range from $+20$ to -40°C . In most cases, the resistivity was found to increase slightly with decreasing temperature down to 0°C ; at this point, the resistivity increased by a factor of 10 to 100 in a range of only a few degrees, after which it continued to increase steadily with a further decrease in temperature. The only measurement made of the dielectric constant showed that the effect of temperature was negligible except at the freezing point where the dielectric constant decreased sharply from 45 to 13.

The purpose of the present research was to study the effect of three factors on the electrical properties of frozen rocks: the porosity of the rock, the concentration

of the electrolyte, and the frequency of the applied voltage. Sandstone cores, of three different porosities, were saturated with 0.02, 0.1, 0.2, and 0.5 molar sodium chloride solutions. The resistivity and dielectric constant of the cores were measured at nine temperatures in the range from + 22 to - 40°C and for voltages of various frequencies in the audio range.

EXPERIMENTAL WORK

The resistance and capacitance across the sandstone cores were measured by comparing the sample in one arm of a capacitance-conductance bridge with a resistor and capacitor in parallel in the opposite arm. These measurements were taken at nine temperatures between - 40 and + 22°C. For each temperature, the frequency of the applied voltage was varied from 200 to 100,000 cycles per second.

The following sections describe the preparation of the samples, the laboratory apparatus and equipment used to record the data, and the procedure followed; in the last section, the calculated results are presented in tables and graphs.

Preparation of Samples

Sandstone of the Fountain formation (Pennsylvanian) was used in all the experiments as sample material. Three

blocks of sandstone, each of different porosity but similar in composition, were obtained from an outcrop in the Golden area. From these blocks, several disk-shaped samples were cut with a rotary drill and a diamond saw. The disks had a uniform diameter of 32 mm and varied in thickness from eight to ten mm.

The porosity of the samples was determined by allowing the cores to dry in an oven for 24 hours, after which they were weighed on a precision balance. The cores were then saturated with distilled water in a vacuum dessicator and weighed again, first in air and, then, suspended in water. The effective porosity was found by dividing the volume of water that could be forced into the pores of the rock by the volume of water displaced by the sample.

Next, the resistance across the cores saturated with distilled water was measured at a frequency of 1,000 cps and at room temperature, with a General Radio Z-Y Bridge, type No. 1603-A, serial No. 335.

From these preliminary measurements, 42 samples were selected; the ones that showed an anomalous porosity or resistivity were rejected. The cores were then dried a second time, and to prevent evaporation of the water while the measurements were being taken, their cylindrical surface was coated with an insulating, water-repellent lacquer.

Finally, the samples were divided into six groups and saturated with sodium chloride solutions of different concentration. The resistivity of the solutions was measured with a conductance cell. The porosity, resistivity, and type of solution for each group of samples are given in table I.

Apparatus and Equipment

A General Radio Capacitance Bridge, type No. 716-C, serial No. 1819, was used to measure the capacitance and resistance across the samples. A schematic diagram of the bridge and accessories is shown in figure 2.

The bridge consists of two equal, shielded ratio arms Q-S and R-S and a variable precision capacitor C_n . A decade resistance box R_n is inserted in parallel with C_n . The capacitor C_p and resistor R_p are inserted in the opposite arm to balance the effect of C_n and R_n . An oscillator (OSC), generating a voltage of known frequency, is connected between the points Q-R; and the voltage between the terminals S-T is detected on the screen of a cathode-ray oscillograph (CRO).

If the oscillator is also connected to the horizontal plates of the oscilloscope, a Lissajous figure (an ellipse in this case) is displayed on the screen. When the arms Q-T and R-T are equal in phase and amplitude, the bridge is

TABLE I

Sample No.	Porosity in percent	Resistivity * of samples (ohm-meters)	Molarity of NaCl solution	Resistivity of solution (ohm-meters)
1	3.3 to 3.7	389 to 448	0.1	0.97
2	6.3 to 6.9	256 to 322	0.1	0.97
3	17.3 to 17.4	91 to 126	0.02	4.45
4	17.5 to 17.7	107 to 115	0.1	0.97
5	16.8 to 17.3	97 to 118	0.2	0.45
6	17.9 to 18.4	91 to 120	0.5	0.23

* Measured at + 22°C and a frequency of 1,000 cps on cores saturated with distilled water.

Figure 1
Some properties of the prepared samples.

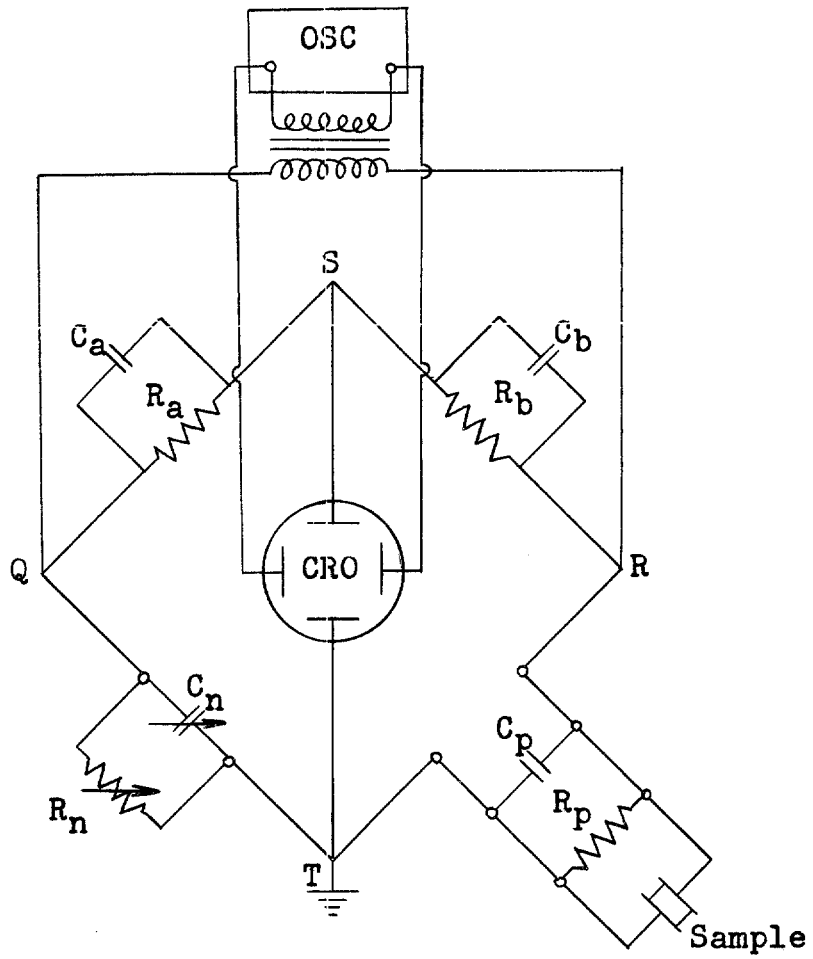


Figure 2

Schematic diagram of bridge arrangement.

balanced and the voltage between the terminals S-T is zero; in this case, the vertical component on the screen is zero and the figure becomes a horizontal straight line.

A Statham temperature chamber, Model TC-2B, operated with solid CO₂, was used to bring the samples to the desired temperatures and maintain them there. The chamber consists of an insulated box divided into two compartments: one contains a six-cell electrode tray, space for approximately 20 lb of dry ice, a thermostat and a thermometer; the other compartment contains a blower and an electric-resistance heater. The blower provides a continuous circulation of the carbon dioxide vapors in the chamber, and the heater, controlled by the thermostat, maintains a constant temperature with an accuracy of $\pm 1.1^{\circ}\text{C}$.

The six electrodes, used as sample holders, shown in figure 3, are two-terminal, solid-silver electrodes: the upper electrode is a circular disk, 51 mm in diameter, pressed firmly on the sample by means of a spring; the lower one is a rectangular plate grounded to the temperature chamber.

Procedure

The measurements were taken at -40, -30, -20, -11, -5, 0, +3, +11, and +22°C. In a preliminary series of experiments,

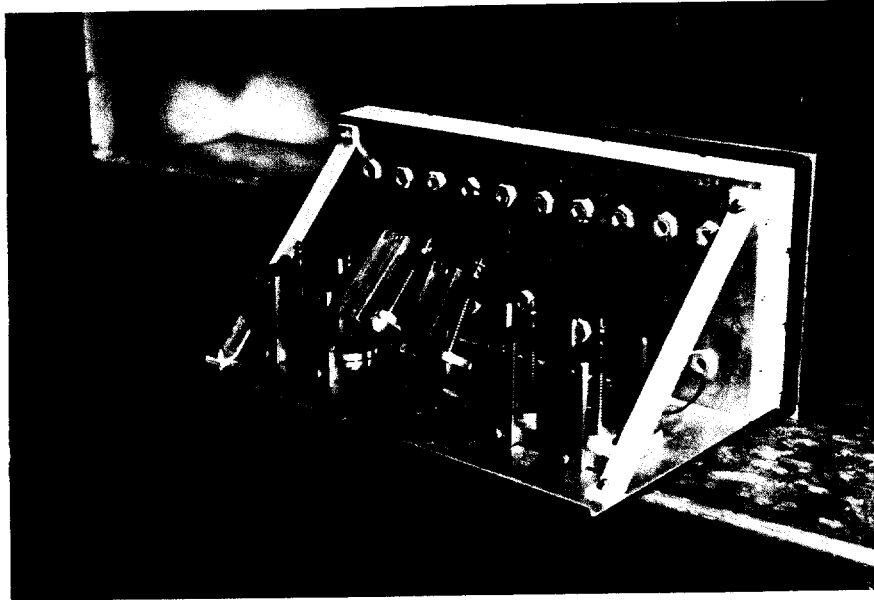


Figure 3
Electrodes used as sample holders

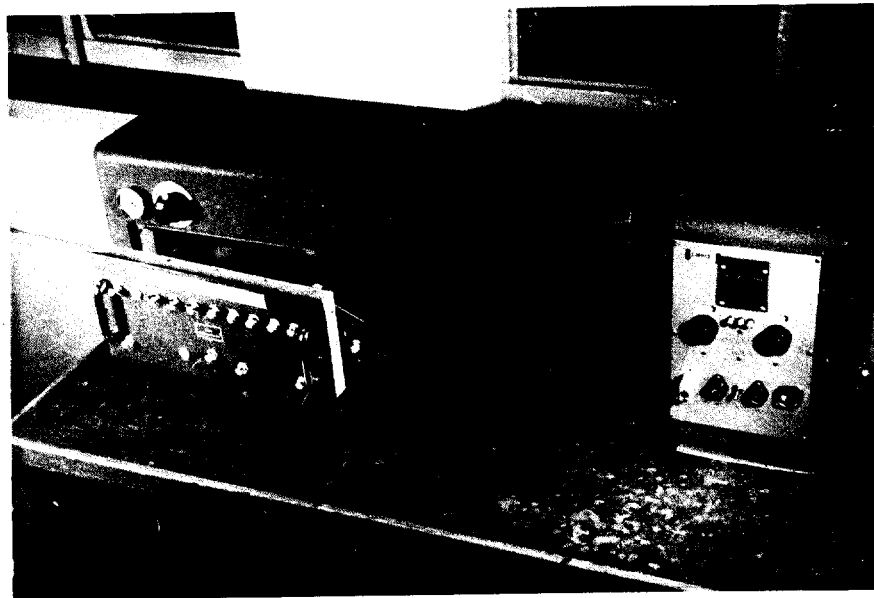


Figure 4
Temperature test chamber

the resistance and capacitance of six samples were measured at -40 and -20°C , after which the samples were put back into their respective solution. When the same measurements were repeated a few days later, a wide discrepancy was noticed in the results: the resistivity was lower, and the dielectric constant higher in the second measurements. It was assumed that the expansion of water upon freezing somewhat affected the structure of the rock, by breaking some of the pores, and at the same time increased the effective porosity of the rock. A different set of samples was used for each temperature to overcome this effect, except above 0°C ($+3$, $+11$, $+22^{\circ}\text{C}$); at these temperatures, the measurements were taken on the same six samples.

Before the samples were inserted in the temperature chamber, their flat surfaces were covered with silver paint to provide a good electrical contact between the samples and the rigid electrodes of the cell. The tray containing the electrodes was then sealed in the chamber and about one hour was allowed for the cell and the samples to reach an equilibrium temperature. For each set of experiments, a thermistor thermometer, more accurate than the one in the chamber, was used to measure the temperature at a short distance from the sample; in all cases, the readings from both thermometers did not differ by more than 1°C .

A substitution method was used to measure the resistance and capacitance of the samples on the bridge. With the oscillator adjusted to the desired frequency, and the high lead from the bridge to the sample disconnected, a first balance was obtained by adjusting C_n and R_n until a perfect null was displayed on the oscilloscope. The bridge was then connected to the sample and rebalanced by decreasing R_n and increasing C_n . The unknown capacitance (C_x) and resistance (R_x) of the samples were given by the relations:

$$C_x = C_{n2} - C_{n1}$$

$$R_x = \frac{R_{n1} \cdot R_{n2}}{R_{n1} - R_{n2}}$$

where the indices 1 and 2 indicate the initial and final values of C_n and R_n , respectively. The same procedure was followed for each sample at frequencies of 200, 500, 1,000, 2,000, 5,000, 10,000, 20,000, 50,000, and 100,000 cps.

Results

Values of the electrical resistivity and dielectric constant, calculated for each sample, are tabulated in the appendix. The most important features of these results are illustrated in figures 5 to 12. Figure 5 shows the electrical

resistivity of the samples as a function of temperature, for different electrolyte concentrations and different porosities, at a frequency of 50,000 cps. The influence of frequency on the resistivity is indicated in figures 6, 7, and 8. Figures 9 to 12 show the variations of dielectric constant with frequency at different temperatures.

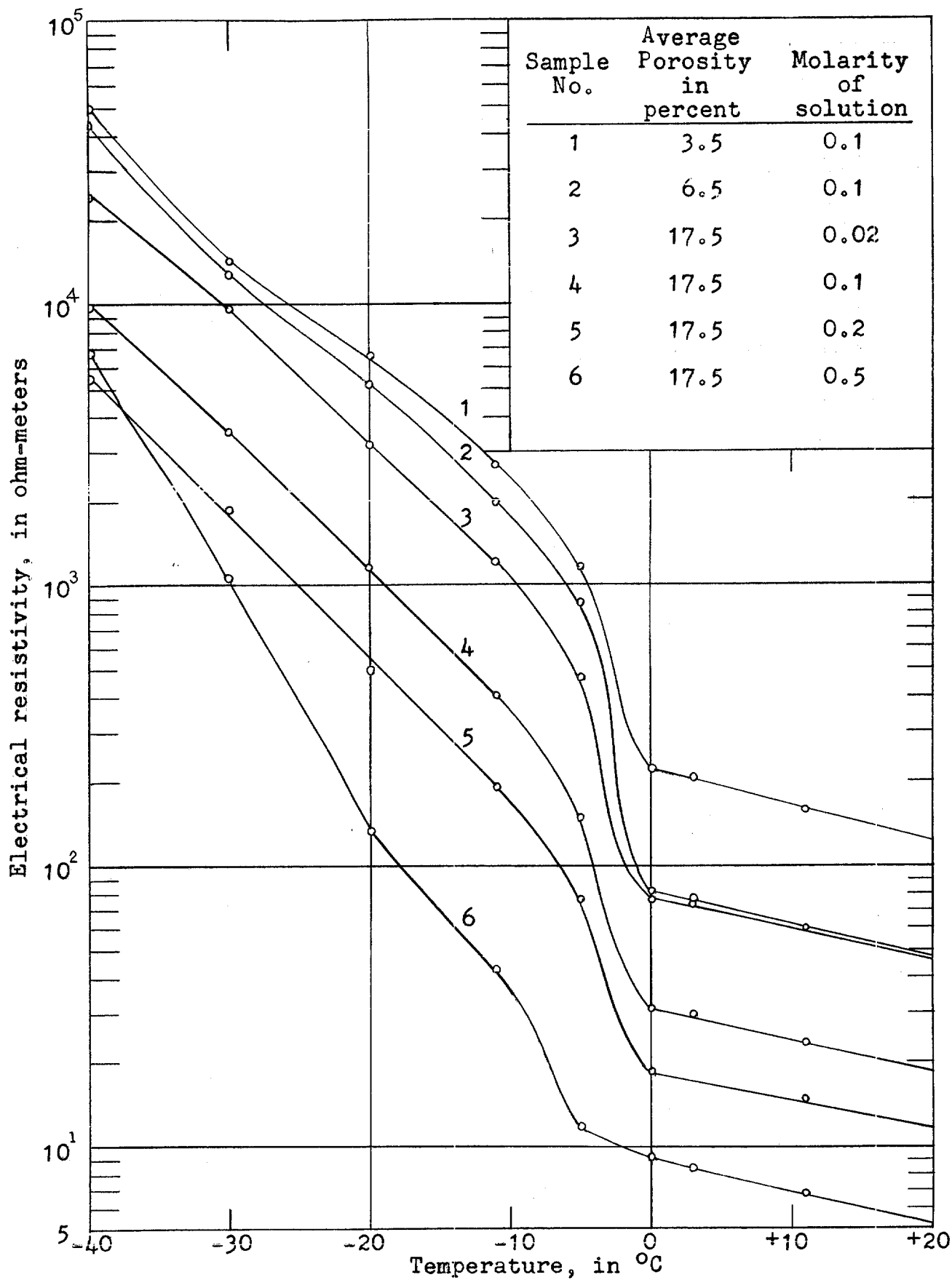


Figure 5
Electrical resistivity as a function of temperature
at a frequency of 50,000 cps.

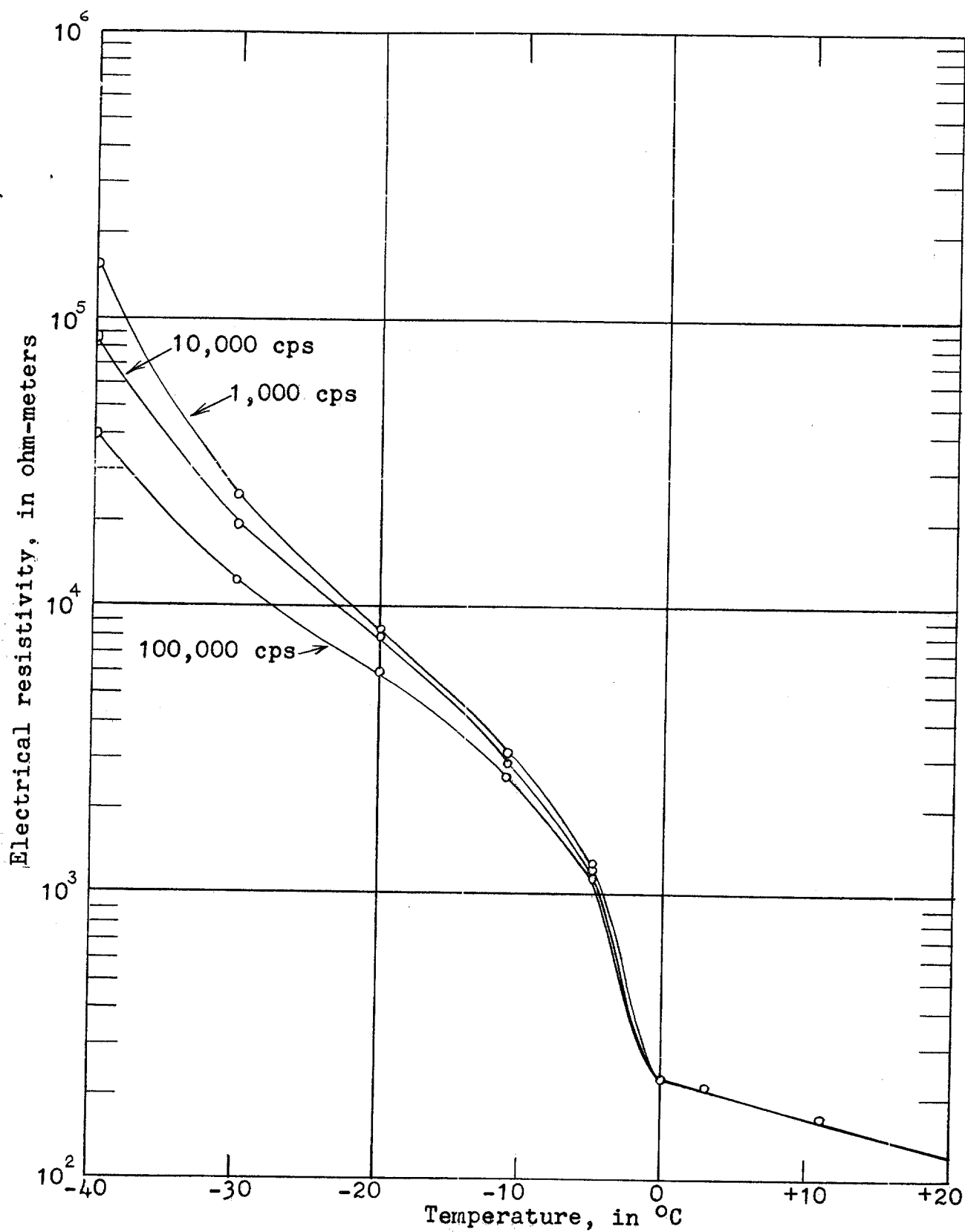


Figure 6
Electrical resistivity of sample 1 as a
function of temperature and frequency.

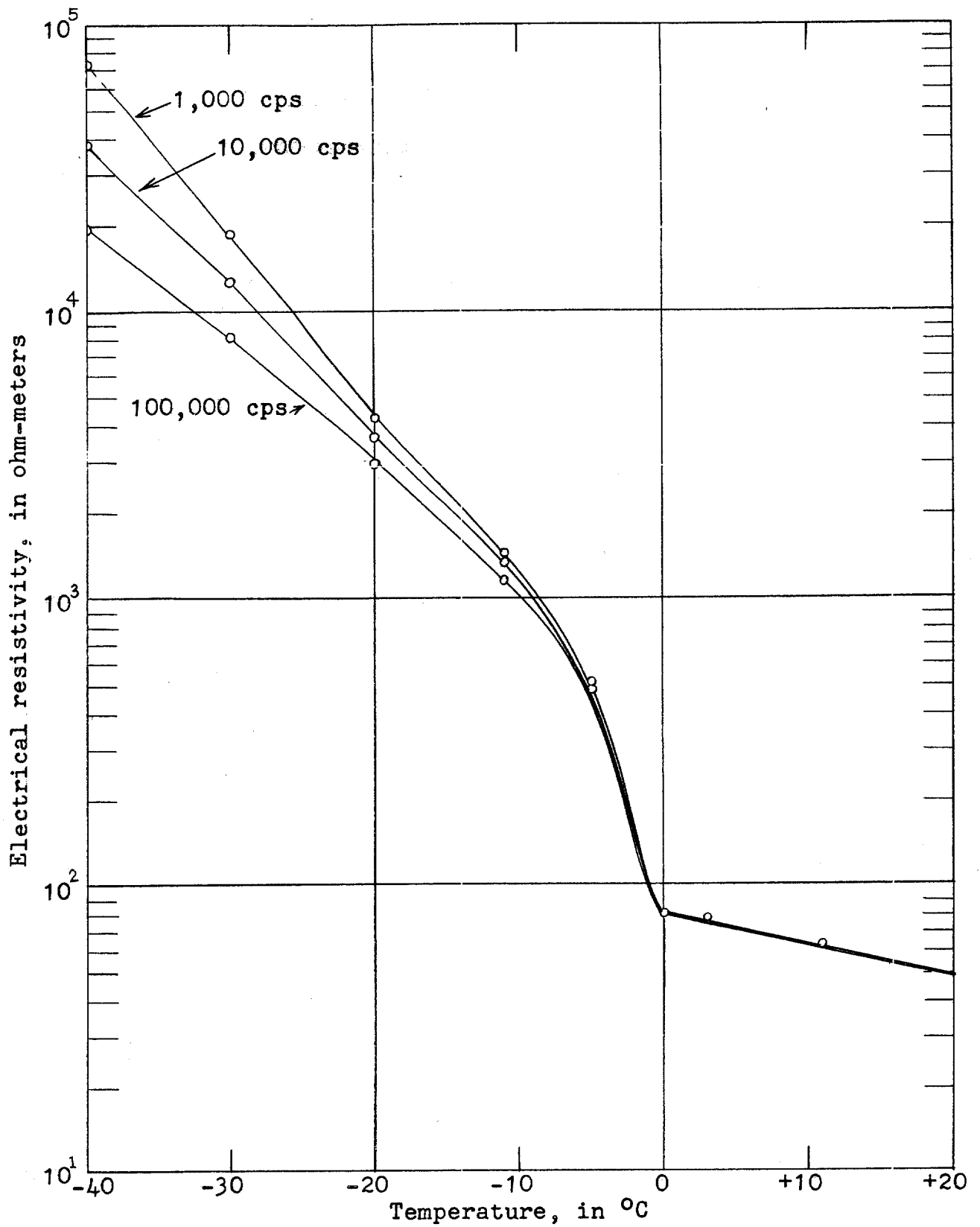


Figure 7
Electrical resistivity of sample 3 as a
function of temperature and frequency.

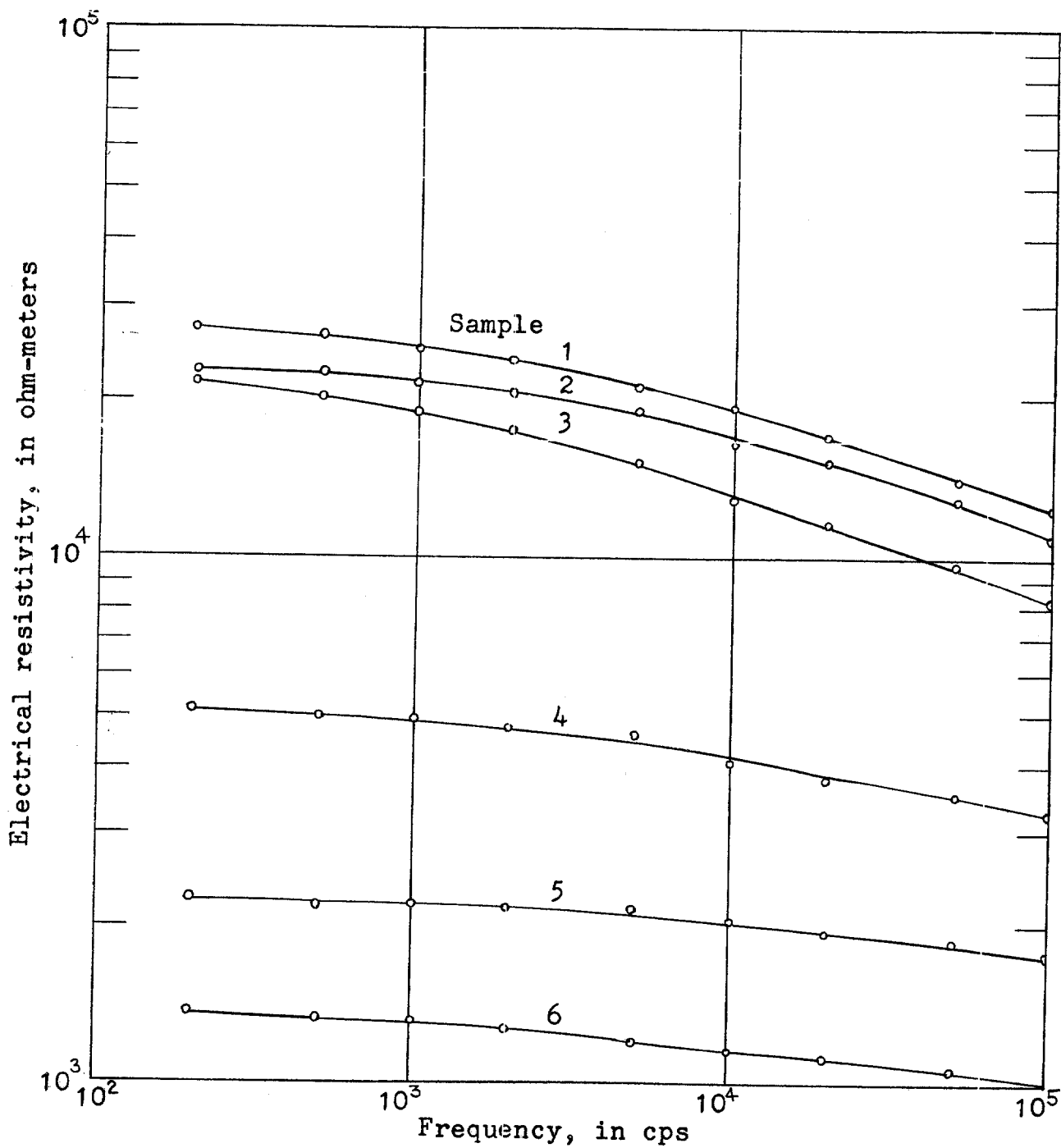


Figure 8

Electrical resistivity as a function of frequency at -30°C .

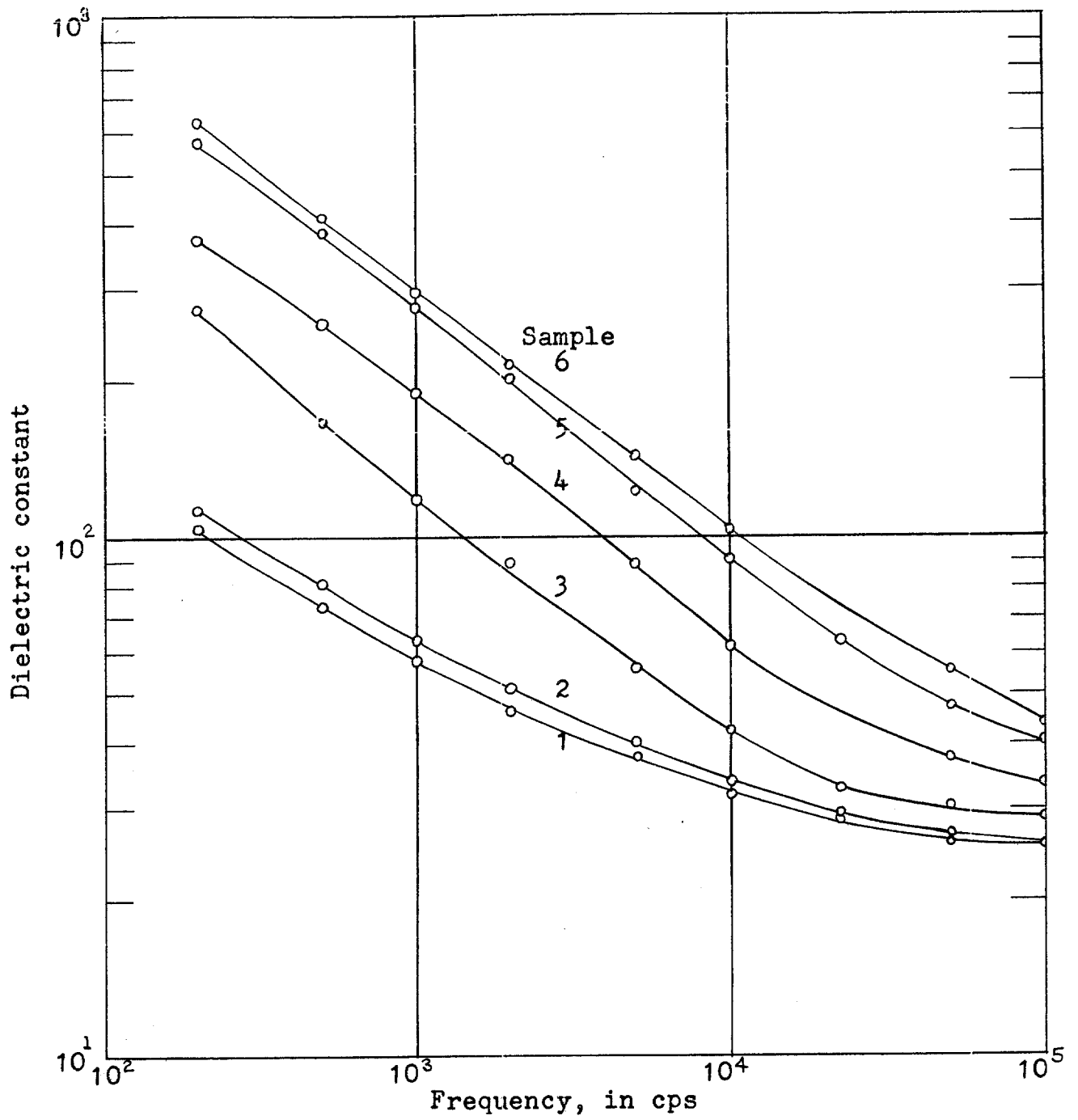


Figure 9

Dielectric constant as a function of frequency at -40°C .

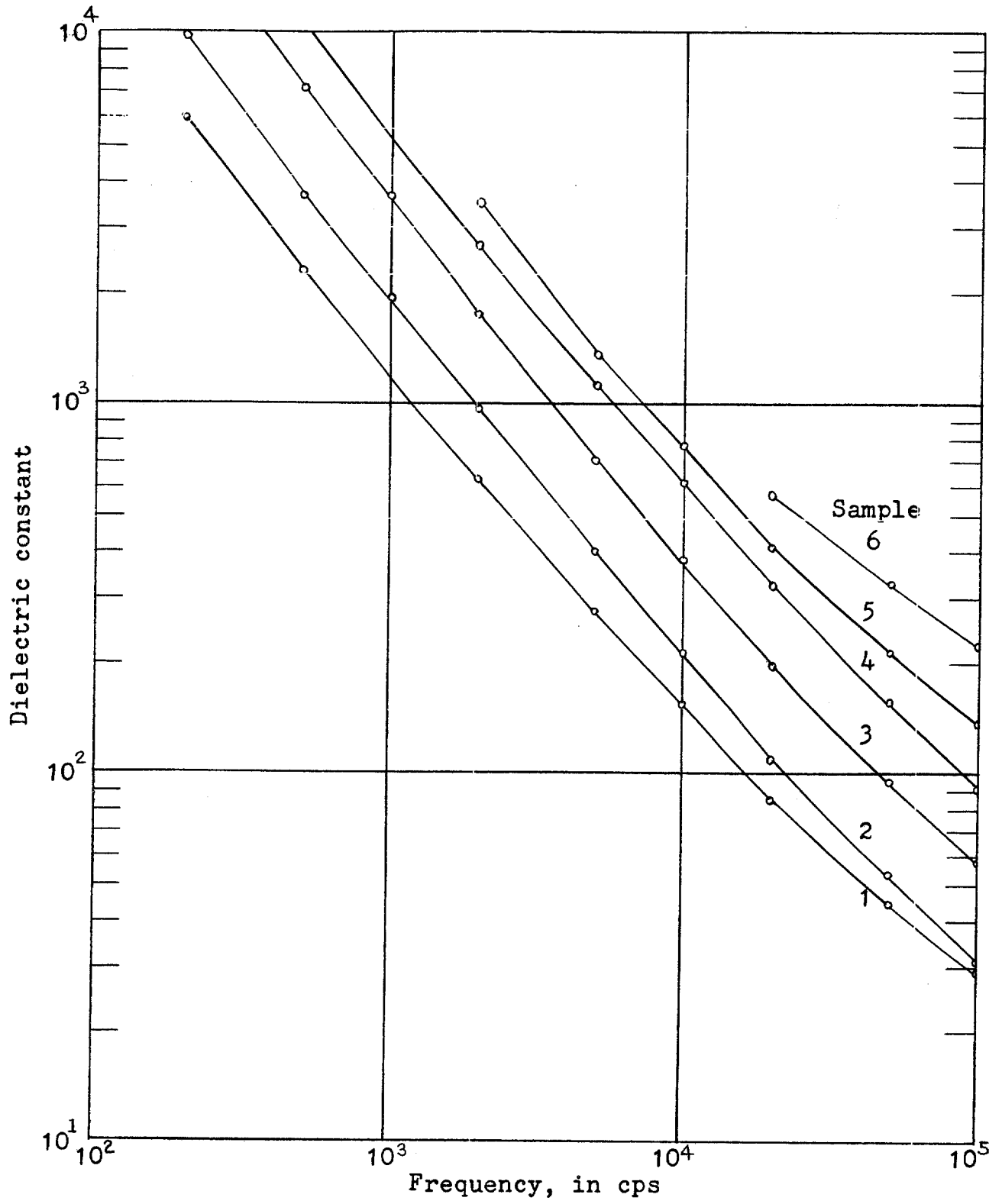


Figure 10

Dielectric constant as a function of frequency at + 3°C.

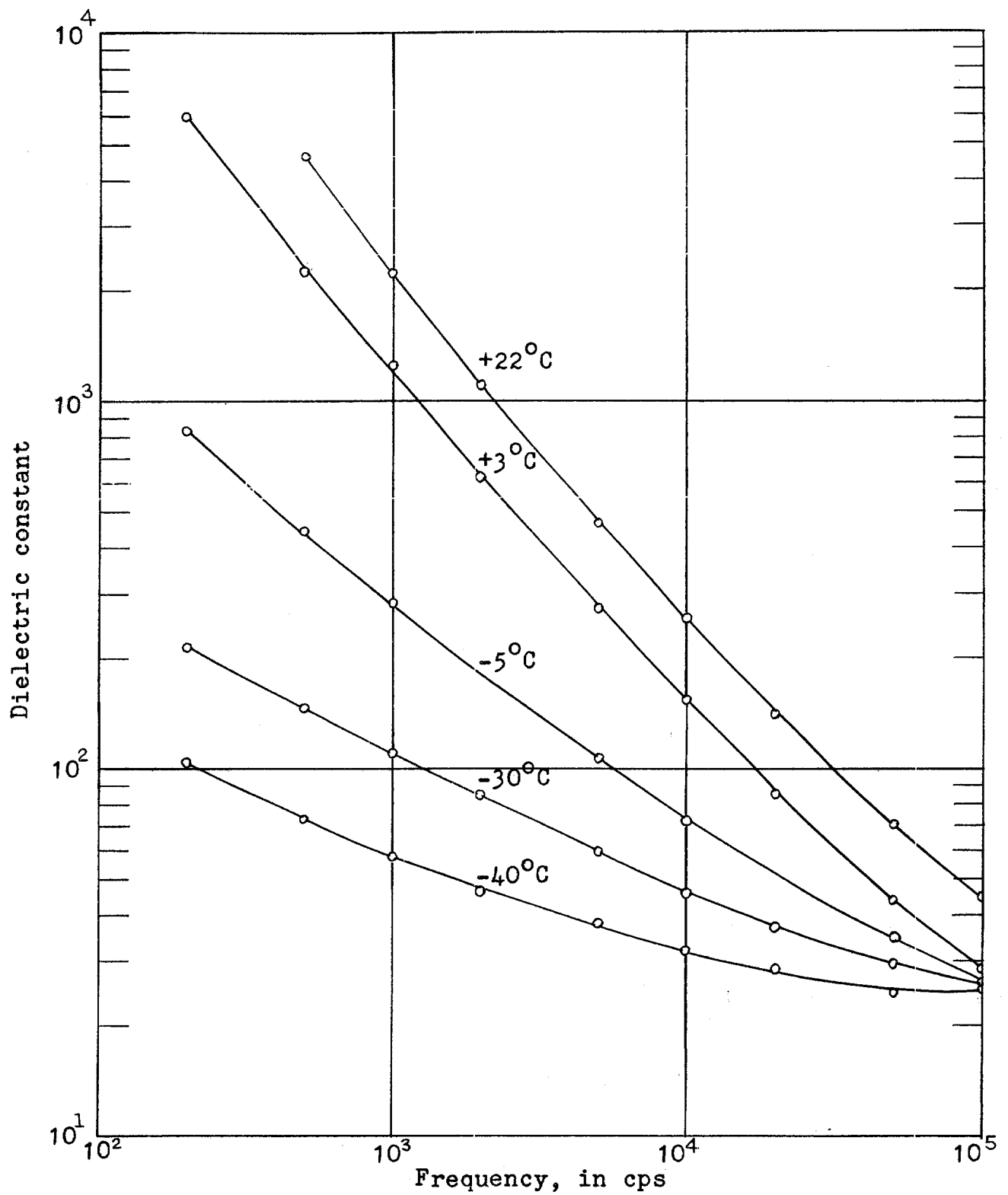


Figure 11

Dielectric constant of sample 1 as a function of temperature and frequency.

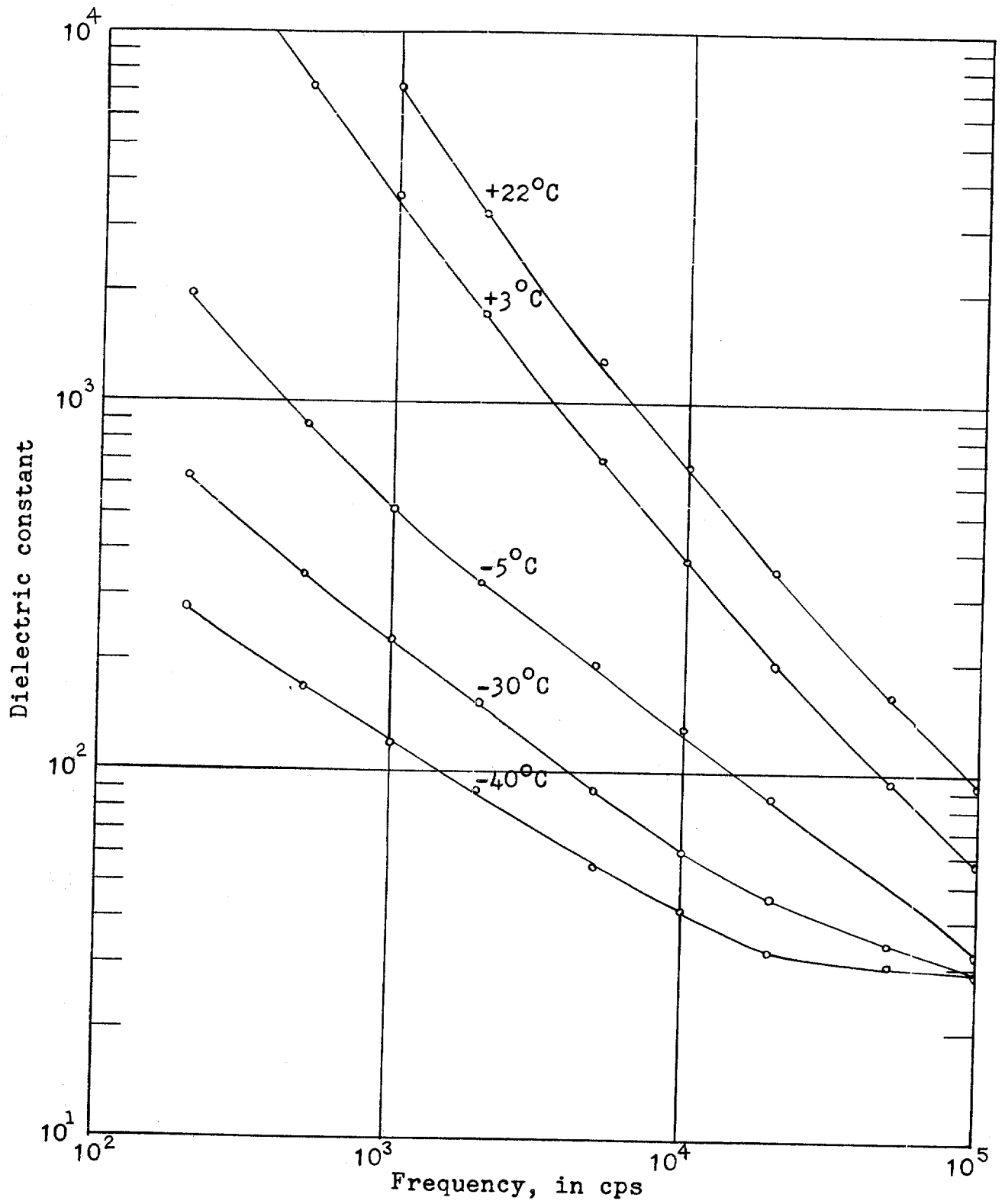


Figure 12

Dielectric constant of sample 3 as a function of temperature and frequency.

DISCUSSION

The changes in electrical resistivity and dielectric constant of the sandstone cores, for variations in temperature and frequency, are summarized under the first two headings of this chapter. The last section is an analysis of the errors in the data and the validity of the results.

Electrical Resistivity

As shown in figure 5, the electrical resistivity of all the samples follows the same trend with the decrease in temperature: the resistivity first increases uniformly until the water in the pores starts to freeze; at this point, a sharp increase in resistivity occurs in a range of five or six degrees, after which it continues to increase uniformly but at a much greater rate than it had above the freezing point. This general pattern is consistent with previous measurements made by Higgs (1930), Smith-Rose (1934),

and Ananyan (1958). Above 0°C and below -10°C , the rate of change in resistivity is approximately the same for all the samples, except 6, and, seemingly, the porosity of the rock or the salinity of the saturating solution has little or no effect in this case.

Because of the sodium chloride in solution, the water in the pores of the rock starts to freeze slightly below 0°C . Data on the freezing points of the solutions, obtained from Hall and Sherrill (1928), are given in table II. In most of the samples, the large increase in resistivity occurs between 0°C and -10°C . The amount of increase in this temperature range appears to depend on the porosity of the rock and also on the salinity of the solution. Electrolytes with a higher concentration in salts tend to diminish the amount of increase and flatten out the curve (samples 3, 4, 5, and 6). In samples saturated with the same electrolyte (1, 2, and 4), the largest increase occurs in the sample of medium porosity (sample 2), and the smallest increase in sample 1, which has a lower porosity than the other two.

The change in resistivity with frequency is illustrated in figures 6, 7, and 8. In most cases, the resistivity decreases with an increase in frequency. This frequency dependence of the resistivity, more pronounced at lower temperatures, is practically nonexistent above -5°C ; it is

also more pronounced in rocks saturated with a solution of lower salinity and in rocks having a lower porosity.

TABLE II

Sample No.	Molarity of solution	Freezing point of solution (°C)	$\frac{\rho_{-10}}{\rho_0}$
1	0.1	- 0.35	11.0
2	0.1	- 0.35	22.3
3	0.02	- 0.07	14.5
4	0.1	- 0.35	11.5
5	0.2	- 0.68	9.2
6	0.5	- 1.69	4.1

Figure 13

Increase in resistivity from 0 to - 10°C.

Dielectric Constant

The very large values of the dielectric constant, especially when measured at low frequencies, are rather the rule for a material having a solid-liquid interface. Smith-Rose (1934) measured a dielectric constant of 25,000 at 200 cps and 95 at 100,000 cps across a soil sample with a 25 percent moisture content. Keller and Licastro (1959) measured dielectric constants as high as 100,000 at 200 cps across natural-state sandstone cores.

One of the most significant features appears to be the rapid decrease of the dielectric constant with an increase in frequency, as shown in figures 9 to 12. But, unlike the resistivity, the rate of change with frequency is the largest at room temperature and flattens out as the temperature is lowered.

Variations with temperature are also indicated on the same curves: the dielectric constant decreases approximately linearly with a decrease in temperature except in the range from + 3 to - 5°C, where there is a pronounced discontinuity.

For any temperature, the dielectric constant increases with an increase in the salinity of the solution and also with an increase in the porosity of the samples.

Errors and Reliability of the Measurements

Errors in the results may be due to three sources:

1. inaccuracy of the instruments,
2. inaccuracy in recording the data,
3. variations in the samples.

Because the resistivity and dielectric constant of the frozen rocks vary tremendously with temperature, probably the largest error in the measurements comes from the small cyclic changes of temperature in the test chamber. The deviations from an average temperature can be as large as

1.1°C, which is the accuracy of the thermostatic control.

The accumulation of ions at the electrodes while the measurements are being taken causes a polarization also known as "electrode polarization." This polarization largely contributes to the capacitance across the sample, especially above the freezing point.

The stray capacitances which were always present in the measuring circuit were neglected, partly because of the difficulty to evaluate them, partly because they were considered small in the frequency range in which the experiments were conducted. Such capacitances would include the edge capacitance of the electrodes, and the change in reactance of the decade resistance box when the bridge is rebalanced with the sample in the circuit.

The frequency of the oscillator was measured with a calibrated oscilloscope. Up to 50,000 cps, the waveform displayed on the oscilloscope and the setting of the dial of the generator had a one to one correspondence; at 50,000 and 100,000 cps, the deviation between the two amounted to approximately four percent.

Additional errors have been recorded in measuring the temperature inside the test chamber and also in measuring the dimensions of the samples.

In each of the groups, the samples were assumed to

have the same physical properties and, consequently, the resistivity and dielectric constant of each group should be continuous functions of temperature. However, small variations in the structure and composition of the samples are always present; the solution in the pores of the rock may freeze in a different manner for different temperatures; and most important of all, the growth of ice crystals with the decrease in temperature affects the structure of the sample (the porosity for example) in a different way for different temperatures.

As can be seen from the above considerations, the measurement of the dielectric constant is much more difficult than that of the resistivity. Consequently, the reliability of the dielectric-constant measurements is probably not very high because of such factors as electrode polarization, stray capacitances, and small differences among the samples.

INTERPRETATION

The freezing process in a rock, combined with the effects of freezing on the electrical properties, is a complex phenomenon. Because of the many variables involved and some unknown factors, a qualitative approach rather than quantitative has been used to evaluate the experimental results.

Ice Formation in a Rock

Although the freezing of water in a soil has been a research subject for the past fifty years, some of the processes involved are still obscure or unknown. Experimental evidence shows that ice in a soil starts to form at slightly below 0°C , and freezing continues at a more or less constant temperature until a degree of water saturation is reached where the rate of freezing becomes very small for large decreases in temperature (Parker, 1921; Bodman and

Day, 1943; Cannell and Walter, 1959). The freezing-point depression and the fraction of nonfrozen water seem to depend on at least two factors: the pressure exerted on the solution and the salinity of the solution.

Part of the pressure is due to forces of adsorption at the interface solid-solution. These forces, mainly electrical, are particularly pronounced in clay-water systems because of the exchangeable cations lining the surface of the clay particles. The thickness of the adsorbed water layers and the magnitude of the adsorption forces depend on the nature of the solid and of the solution (Low, 1958). The pressure is assumed to be the greatest at the interface and to decrease logarithmically with increasing distance from the solid.

Winterkorn (1943) reports an adsorption pressure of 25,000 kg per sq cm for methyl alcohol in fuller's earth. According to the same author (p. 111), "the adsorption pressure for water would be at least equal to, if not considerably higher than, that calculated for methyl alcohol." The phase diagram for water-ice, in figure 14, shows that pure water can still be in the liquid state at -22°C . However, there is general agreement that the adsorbed water on the clay particles does not have the physical characteristics of liquid water, and, consequently,

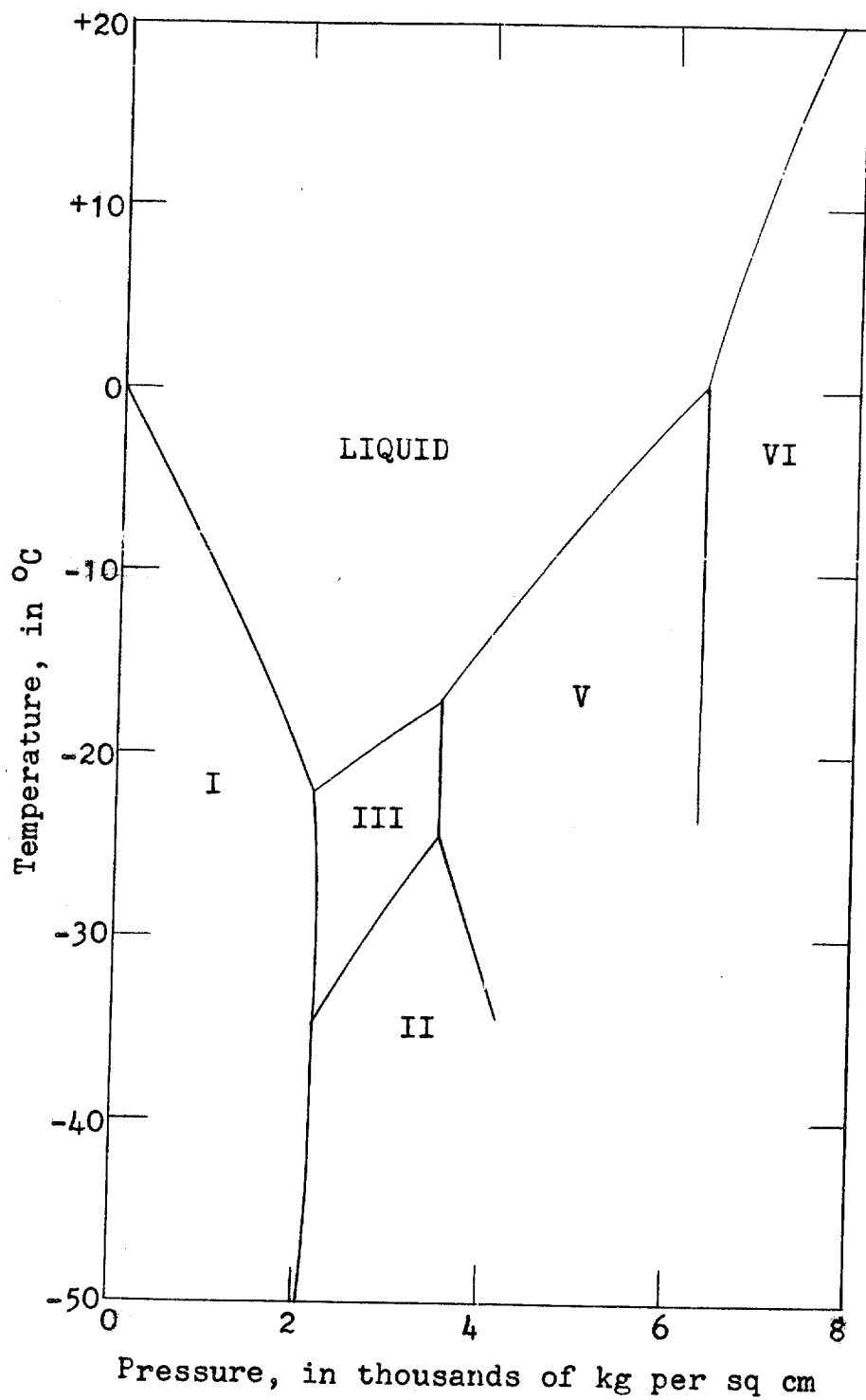


Figure 14
Phase diagram for water and the ices.
(Bridgman, 1928, p. 17)

the effect of the adsorption pressure on the freezing point is not too clear.

The volume expansion of water, when it freezes, causes an added pressure on the unfrozen solution. Because the pressure is proportional to the volume of ice formed, it will be greater in rocks saturated with a more diluted solution, as will be shown later.

The ions in solution also play an important role in the freezing-point depression of water. Figure 15 shows the freezing point of a sodium chloride solution as a function of concentration, at a pressure of one atmosphere. If the initial concentration of the solution is known, the volume of solution remaining at a given temperature can be found from the relation:

$$V_2 = \frac{V_1 M_1}{M_2} \quad (1)$$

where:

V_1 and V_2 are respectively the initial and final volumes of the solution.

M_1 and M_2 are respectively the initial and final molarities of the solution.

At -21.1°C , the solution becomes saturated and crystallizes to form the eutectic mixture $\text{NaCl} \cdot 2\text{H}_2\text{O}$ and ice.

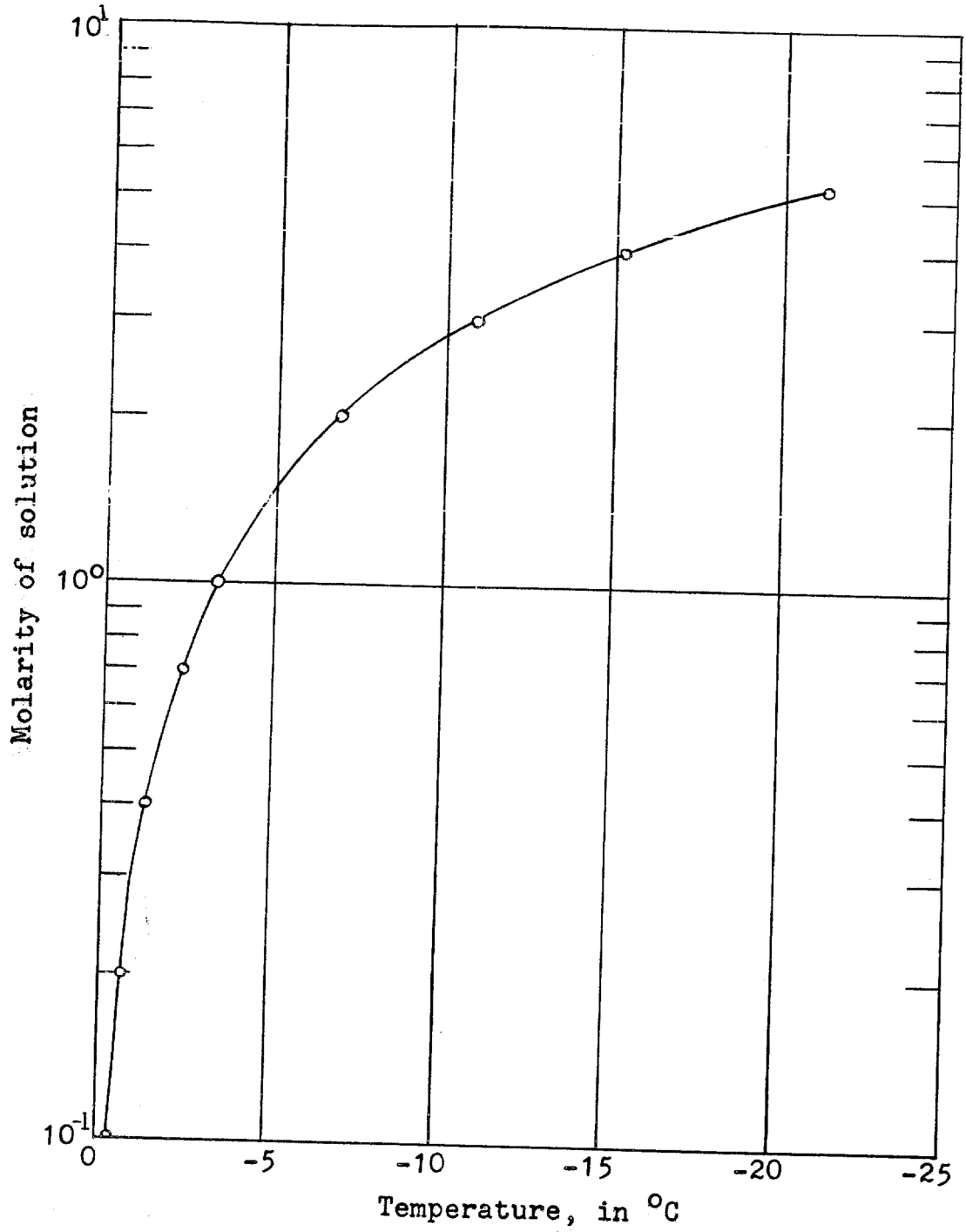


Figure 15

Freezing point of a sodium chloride solution.
(Hall and Sherrill, 1928, p. 258)

It is assumed that the concentration of the solution is approximately the same over the entire volume of the pore and that the solution the farthest from the solid interface is under the least pressure. Then, as the freezing point of the solution is reached, small ice crystals start to form in the center of the pores and larger capillaries. As each small volume of water crystallizes, it rejects the ions and other impurities it originally contained, in such a way that the remaining solution becomes more concentrated as the temperature decreases. The rate of freezing, very large at first, decreases exponentially with the increase in salinity and pressure of the solution. As the temperature further decreases, the ice crystals keep growing only very slowly, until the eutectic point of the solution is reached.

Electrical Resistivity

The nature and mechanisms of electrical conduction in sandstone cores have been discussed extensively by Keller (1959). The passage of an electrical current through a rock is due to the migration of ionic charges under the influence of an electric field, and the current density is proportional to the number of cations moving in the direction of the field and to the number of anions moving in the

opposite direction. Therefore, the resistivity is inversely proportional to the number of ionic charges available and the ease with which these charges can travel through the rock. Often, the electrical resistivity of a rock is controlled mainly by three factors:

1. the amount of solution in the pores,
2. the resistivity of the solution,
3. the size of the pores and interconnecting channels and their relative disposition.

An empirical relation, known as Archie's law, expresses the resistivity of a saturated rock as a function of those three factors:

$$\rho_r = c \rho_w \phi^{-n} \quad (2)$$

where:

- ρ_r is the resistivity of the rock,
 ρ_w is the resistivity of the solution,
 ϕ is the effective porosity.

The parameters c and n depend on the geometrical arrangement of the openings; their approximate values for the various samples are:

- samples 1 and 2 $c = 0.5, n = 1.8$
 samples 3, 4, 5, and 6 $c = 0.7, n = 2.0$

Above the freezing point, the change in resistivity of the rock seems to depend entirely on the change in resistivity of the solution: the lowering of temperature decreases the mobility of the ions, and the result is an increase in the resistivity of the solution. The ratio $\frac{\rho_r}{\rho_w}$, also called the "formation factor," has been calculated for each sample at + 22, + 11, and + 3°C. The values for the resistivity of the solutions were obtained by interpolation from graphs (Schlumberger, 1955). The results are shown in table III.

TABLE III

Sample No.	Formation factor		
	+ 22°C	+ 11°C	+ 3°C
1	113.7	118.5	122.7
2	45.3	43.5	44.2
3	9.6	10.0	9.7
4	17.2	17.0	17.0
5	20.0	21.2	20.6
6	21.3	22.3	22.2

Figure 16

Formation factor of the samples above the freezing point.

When the temperature reaches the freezing point, the formation of ice in the rock causes:

1. a decrease in the amount of solution,
2. a decrease in the mobility of the ions in solution,
3. an increase in the concentration of the solution,
4. a change in the pore structure of the rock.

Partly because of the different salinities of the saturating solution, water in the pores starts to freeze at slightly different temperatures. As the temperature is lowered, the remaining liquid becomes more concentrated in salts until a point is reached where the solution in all the samples is approximately of the same concentration. At this temperature, the samples would differ mainly in the amount of ice formed and the amount of solution remaining.

The apparent porosity of the samples (volume of unfrozen solution per unit volume of solid) has been calculated at -10°C , using equations 1 and 2. At this temperature, the sodium chloride solution has a molarity of 2.8 and a resistivity of 0.11 ohm-meter. In equation 2, the parameters c and n have been replaced by the values computed at $+22^{\circ}\text{C}$, although the effect of temperature on the parameters is not known. The results are given in table IV.

The larger value, obtained from equation 2, probably indicates a slower rate of freezing caused by the pressure exerted on the solution. The pressure caused by the expansion of water would be most effective in rocks saturated

TABLE IV

Sample No.	Molarity at 0°C	Resistivity at -10°C and 50,000 cps (in ohm-meters)	Effective porosity (in percent)	Apparent porosity (in percent)	
				Equation 1	Equation 2
1	0.1	2,430	3.3	0.12	0.27
2	0.1	1,770	6.9	0.25	0.32
3	0.02	1,080	17.4	0.12	0.85
4	0.1	355	17.5	0.63	1.49
5	0.2	168	16.8	1.20	2.16
6	0.5	36.7	18.3	3.27	4.61

Figure 17

Apparent porosity of the samples at -10°C.

with a solution of lower salinity. Effect of adsorption pressure on the freezing-point depression could be significant in a "tight" rock or when the amount of solution remaining in the pores is very small.

The curves in figure 5 show that below -20°C the rate of increase in resistivity is larger in sample 6 than in the other five. This anomalous behavior is, presumably, due to the fact that, because of a smaller pressure in the sample, the eutectic point is reached at a higher temperature.

When the frequency of the applied voltage is increased, the ions in the samples oscillate more rapidly and their energy is increased proportionally. As the temperature decreases, the ions can move only with increasing difficulty, and a larger number of them become immobilized and are unavailable for conduction at low frequencies. However, the extra energy provided by the increase in frequency is sufficient to dislodge some of the ions and, consequently, to increase the conductivity.

Dielectric Constant

The very large values of dielectric constant seem to be caused mainly by interfacial (or space-charge) polarization. This type of polarization occurs in a material when charge carriers, moving under the influence of an electric field,

become trapped in the pores and on the interfaces or cannot be freely discharged at the electrodes (Von Hippel, 1954). Consequently, the dielectric constant of a porous rock saturated with an electrolyte is not so much related to the dielectric constant of the individual constituents as it is to the interaction between the ions of opposite polarity and the interaction between the solution and the solid. At low frequencies, the dielectric constant depends mainly on the ionic concentration and the amount of solution saturating the pores. In this respect, variations with temperature are approximately the same for the dielectric constant as for the conductivity. The effect due to increasing frequency is, however, much more pronounced on the dielectric constant; and the effect increases with temperature: above the freezing point of water, the dielectric constant decreases exponentially with frequency.

CONCLUSIONS

The experimental results have shown that:

1. Above the freezing point of water, the change in resistivity of a rock is due entirely to the change in resistivity of the saturating solution.
2. In the range from 0 to -10°C , the rate of increase in resistivity is proportional to the rate at which ice forms in the pores. The rate of increase is larger (a) in rocks saturated with a less concentrated solution, (b) in rocks of medium porosity.
3. Below -10°C , the resistivity, which depends mainly on the amount of unfrozen solution, increases uniformly with decreasing temperature, unless the solution becomes saturated and crystallizes.
4. The dielectric constant changes with temperature in approximately the same manner as the conductivity.

5. Variations in the electrical properties of rocks with small changes in temperature are much greater below than above the freezing point. This fact might be important if geoelectrical methods are applied in areas of permafrost, where inhomogeneities in ground temperature are rather common.

Although the absolute value of the electrical resistivity and dielectric constant varies greatly, even in a given rock type, the electrical properties of other saturated rocks should show similar changes with temperature. However, the accuracy of the results could be increased considerably by improved measuring techniques.

The amount of unfrozen solution in the pores of a rock, the effects of pressure on the freezing point depression, the eutectic point of an electrolyte under pressure are only a few of the many unanswered questions that deserve more attention in future research. A better knowledge of these factors would certainly help to determine the electrical properties of frozen rocks.

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APPENDIX

Tabulated results.

SAMPLE NO. 1

Effective porosity: 3.5%

Electrolyte: 0.1 molar NaCl

Electrical resistivity as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22	
Frequency in cps	Electrical resistivity in ohm-meters									
200	183,000	27,000	9,200	3,130	1,340	231	210			
500	182,000	26,100	8,300	3,100	1,290	230	209			121
1,000	157,000	24,800	8,300	3,080	1,290	228	211	166		116
2,000	131,000	23,700	8,200	3,010	1,290	226	211			115
5,000	103,000	21,000	8,100	2,880	1,250	225	208	162		114
10,000	87,000	19,100	7,800	2,860	1,220	224	208	161		113
20,000	70,000	16,900	7,500	2,800	1,220	221	206	161		113
50,000	49,500	14,000	6,520	2,680	1,160	220	204	159		111
100,000	40,200	12,200	5,920	2,540	1,130	218	204	161		111

SAMPLE NO. 2

Effective porosity: 6.5%

Electrolyte: 0.1 molar NaCl

Electrical resistivity as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22	
Frequency in cps	Electrical resistivity in ohm-meters									
200	143,000	22,500	7,600	2,400	990	84	76			
500	138,000	22,300	7,100	2,340	990	84	75		48.1	
1,000	123,000	21,100	6,980	2,390	1,000	83	76	60.9	46.2	
2,000	104,000	20,300	6,830	2,330	980	82	76		46.0	
5,000	83,000	18,900	6,590	2,200	930	82	75	59.4	45.9	
10,000	69,600	16,300	6,280	2,150	910	81	76	59.0	45.7	
20,000	59,000	15,000	5,840	2,100	910	81	75	59.1	45.5	
50,000	43,300	12,800	5,200	1,970	860	80	76	59.1	45.1	
100,000	34,100	10,800	4,800	1,870	870	80	76	60.1	44.9	

SAMPLE NO. 3

Effective porosity: 17.5%

Electrolyte: 0.02 molar NaCl

Electrical resistivity as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22	
Frequency in cps	Electrical resistivity in ohm-meters									
200	94,000	21,300	4,950	1,540	526	80	76			
500	83,000	20,100	4,380	1,450	514	80	76			
1,000	74,000	18,800	4,250	1,410	512	79	76	60.9	45.2	
2,000	58,100	17,200	4,100	1,390	503	78	75		45.1	
5,000	45,300	15,000	4,060	1,330	488	77	75	59.3	44.7	
10,000	38,200	12,800	3,660	1,310	477	76	75	58.6	44.4	
20,000	32,000	11,400	3,500	1,280	479	76	74	58.5	44.1	
50,000	24,100	9,600	3,120	1,200	462	75	74	58.3	43.7	
100,000	19,400	8,200	2,920	1,160	462	75	74	59.2	43.3	

SAMPLE NO. 4

Effective porosity: 17.5%

Electrolyte: 0.1 molar NaCl

Electrical resistivity as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22
Frequency in cps	Electrical resistivity in ohm-meters								
200	21,400	5,100	1,380	442	166	31.9	29.5		
500	20,300	4,990	1,330	429	159	31.8	29.2		
1,000	19,900	4,870	1,310	432	158	31.6	29.4	23.8	17.5
2,000	17,500	4,670	1,300	428	157	31.5	29.5		17.5
5,000	15,100	4,560	1,350	426	154	31.3	29.2	23.3	17.4
10,000	13,300	4,050	1,280	426	155	31.1	29.2	23.1	17.3
20,000	11,800	3,790	1,220	420	155	30.9	29.2	23.1	17.2
50,000	9,700	3,520	1,150	402	149	30.8	29.2	23.1	17.1
100,000	8,200	3,250	1,150	392	150	30.6	29.0	23.6	17.1

SAMPLE NO. 5

Effective porosity: 17.5%

Electrolyte: 0.2 molar NaCl

Electrical resistivity as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22
Frequency in cps	Electrical resistivity in ohm-meters								
200	9,300	2,240	595	194	82	18.9	18.6		
500	9,000	2,190	546	193	82	18.9	18.4		
1,000	8,800	2,190	540	199	82	18.8	18.5	14.9	11.0
2,000	8,200	2,150	541	197	82	18.7	18.7		11.0
5,000	7,400	2,140	553	197	79	18.6	18.5	14.7	11.0
10,000	6,700	2,020	540	197	78	18.5	18.5	14.6	11.0
20,000	6,190	1,920	518	203	77	18.4	18.5	14.6	11.0
50,000	5,470	1,860	497	189	75	18.3	18.5	14.7	10.9
100,000	4,700	1,770	498	186	77	18.3	18.4	14.9	10.9

SAMPLE NO. 6

Effective porosity: 17.5%

Electrolyte: 0.5 molar NaCl

Electrical resistivity as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22	
Frequency in cps	Electrical resistivity in ohm-meters									
200	17,700	1,360	139	51.3	11.9	9.1	8.2			
500	16,500	1,320	146	47.3	11.9	9.1	8.2			
1,000	15,500	1,310	143	45.6	11.7	9.0	8.2	6.7	4.9	
2,000	14,200	1,270	146	43.9	11.8	9.0	8.3		4.9	
5,000	11,800	1,200	141	43.9	11.6	9.0	8.3	6.6	4.9	
10,000	9,900	1,150	122	44.2	11.5	9.0	8.3	6.6	4.9	
20,000	8,400	1,110	137	43.3	11.7	9.0	8.3	6.6	4.9	
50,000	6,700	1,060	133	42.3	11.6	9.0	8.3	6.6	4.9	
100,000	5,110	980	137	41.7	11.7	9.0	8.3	6.7	4.9	

SAMPLE NO. 1

Effective porosity: 3.5%

Electrolyte: 0.1 molar NaCl

Dielectric constant as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22
Frequency in cps	Dielectric constant								
200	103	214	254	435	830	5,910	5,960		
500	73	146	154	277	442	2,220	2,260		4,600
1,000	57.9	110	117	168	280	1,090	1,230	1,370	2,210
2,000	46.3	85	89	128	241	569	620		1,100
5,000	37.9	59.4	72	88	107	252	272	322	466
10,000	32.0	45.7	53.7	72	72	138	153	194	258
20,000	28.5	37.0	44.2	59.4	58.4	79	85	109	140
50,000	24.7	29.4	35.5	47.0	34.9	43.0	44.0	54.9	70
100,000	25.1	25.8	33.4	41.4	26.1	28.7	28.8	33.2	44.7

SAMPLE NO. 2

Effective porosity: 6.5%

Electrolyte: 0.1 molar NaCl

Dielectric constant as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22	
Frequency in cps	Dielectric constant									
200	113	245	309	491	780	12,200	9,800			
500	81	170	235	332	417	4,550	3,640			
1,000	63	126	171	244	279	2,170	1,910	2,210	3,940	
2,000	50.4	94	134	191	211	1,080	960		1,840	
5,000	39.9	64	92	136	130	460	397	480	770	
10,000	33.4	48.0	69	103	95	239	211	266	414	
20,000	29.3	36.6	51.9	80	67	127	109	134	211	
50,000	25.4	30.3	38.6	57.1	37.9	51.3	53.4	64	95	
100,000	25.2	27.8	36.3	50.3	29.8	34.2	30.6	36.2	58.9	

SAMPLE NO. 3

Effective porosity: 17.5%

Electrolyte: 0.02 molar NaCl

Dielectric constant as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22
Frequency in cps	Dielectric constant								
200	275	620	1,120	1,930	1,960	21,200	20,200		
500	168	339	650	1,130	860	7,700	7,100		
1,000	119	225	407	630	513	3,600	3,640	4,350	7,100
2,000	89	151	270	387	324	1,790	1,740		3,250
5,000	55.6	89	150	210	195	760	700	840	1,300
10,000	42.3	61	103	134	130	393	374	461	680
20,000	32.8	45.6	70	96	85	221	196	235	351
50,000	30.3	34.3	56.9	78	42.8	108	95	113	162
100,000	28.7	28.6	42.7	55.9	32.6	64	57.0	65	93

SAMPLE NO. 4

Effective porosity: 17.5%

Electrolyte: 0.1 molar NaCl

Dielectric constant as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22	
Frequency in cps	Dielectric constant									
200	373	541	710	1,990	2,820	32,400	28,200			
500	258	369	598	1,050	1,610	11,900	10,400			
1,000	189	276	309	519	870	5,630	4,110	9,900	10,400	
2,000	140	193	236	356	488	2,720	2,690		5,070	
5,000	88	128	162	235	255	1,200	1,110	1,340	2,080	
10,000	61	85	122	170	166	640	610	750	1,060	
20,000	50.4	57.1	81	119	103	351	323	394	543	
50,000	37.3	38.7	52.6	72	54.6	174	157	193	260	
100,000	33.4	33.2	45.0	56.8	61	100	90	109	145	

SAMPLE NO. 5

Effective porosity: 17.5%

Electrolyte: 0.2 molar NaCl

Dielectric constant as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22	
Frequency in cps	Dielectric constant									
200	578	494	1,340	3,310	8,600	41,200				
500	383	302	640	1,420	3,370					
1,000	278	239	415	670	1,570					
2,000	200	191	296	413	760	3,500	3,470			4,360
5,000	121	134	197	255	376	1,520	1,360	1,830		2,240
10,000	90	93	149	189	228	800	770	910		1,360
20,000	63	63	103	139	154	421	407	457		640
50,000	46.6	41.7	69	85	85	222	215	233		315
100,000	40.4	34.8	58.2	67	49.7	119	137	139		174

SAMPLE NO. 6

Effective porosity: 17.5%

Electrolyte: 0.5 molar NaCl

Dielectric constant as a function of frequency for various temperatures

Temperature in °C	- 40	- 30	- 20	- 11	- 5	0	+ 3	+ 11	+ 22	
Frequency in cps	Dielectric constant									
200	630	1,990	3,420	6,600	31,900					
500	411	1,270	1,050	2,750	14,100					
1,000	295	980	559	1,040	6,300					8,800
2,000	213	610	338	565	2,320					2,830
5,000	142	328	210	334	1,430					1,720
10,000	101	200	157	222	1,150	1,030		1,350		
20,000	70	116	118	159		430	568	680		800
50,000	55.2	70	80	113	351	235	326	423		590
100,000	43.8	46.8	65	83	89	131	220	304		235