MODELING A NATURAL RESOURCE PROBLEM USING AN OPTIMAL LINEAR FEEDBACK CONTROL: AN APPLICATION IN THE PETROLEUM INDUSTRY

by

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ABSTRACT

This study analyzes a natural resource (petroleum reservoir management) optimal control problem. It is the case where decision makers, after receiving new information about a petroleum reservoir, want to act optimally to increase the value of the proved developed reserves. The problem structure is characteristic of certain natural resource investment problems where the control policy influences the rate of production and depletion of a reserve.

The research question is what would be an optimal depletion path that proved developed and undeveloped oil reserves should follow while minimizing cost (effort). The optimal control solution will be a function of the reserve itself and is known in the literature as a closed-loop solution. The feedback solution is obtained by using a format known as the linear-quadratic problem. The format received its name by the use of linear state equations and a quadratic objective function, which will be minimized.

The base case model results are compared to production data from the Rodessa oilfield in Louisiana. The study also simulates three other management cases, a change in terminal time to depletion, an increase in drilling technology and a reversion in the proved developed reserves. All of the cases are compared to the response of the reserve depletion

path as if it were modeled as an open loop optimal control problem. The conclusions indicate a production tilting toward the present even when using a zero discount rate. There is also an increase in drilling effort based on new reservoir information. The class of problem introduced in this paper appears to offer an attractive format for modeling a diversity of natural resource problems.

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LIST OF VARIABLES AND DEFINTIONS

OBJECTIVE FUNCTION

x_1 = proved developed reserves (mmbbls)

x_2 = proved undeveloped reserves (mmbbls)

u_1 = a production factor quantity index of capital, labor, energy, materials

u_2 = a drilling factor quantity index of capital, labor, energy, materials

Q = a weighting square matrix on the state variables, i.e. $Q = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 4x_1 + x_2$

Note: the x_1 variable is weighted more than the x_2 variable

R = a weighting square matrix on the input factors u_1 and u_2, $R = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = u_1 + 10u_2$

Note: the u_2 variable is weighted more than the u_1 variable

J = a scalar value for the objective function, for this problem it is to be minimized

LIST OF PARAMETERS, SYSTEM STATE AND CO STATE VARIABLES

A_1 = an exponential decline parameter that is site specific

c_1 = a parameter that gives the form for the concave production maintenance function q

c_2 = a parameter that gives the form for the concave drilling function w

$\dot{x}_1$ = the change in proved developed reserves with respect to time

$\dot{x}_2$ = the change in proved undeveloped reserves with respect to time

q(•) = a production maintenance function of the state variable x_1, the control variable u_1 and time. It describes the effect of production maintenance on proved developed reserves

w(•) = a drilling function of the state variable x_2, the control variable u_2 and time. It describes the effect of the drilling program on undeveloped reserves

λ_1 = the co state variable of proved developed reserves. The shadow price of proved developed reserves.

λ_2 = the co state variable of proved undeveloped reserves. The shadow price of proved undeveloped reserves.

$\dot{\lambda}_1$ = the change in the co state variable with respect to time.

$\dot{\lambda}_2$ = the change in the co state variable with respect to time.

P = the solution to the Riccati equation

f_x = is the partial derivative of the system state equation with respect to the state

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Chapter 1

INTRODUCTION

One of the numerous objectives faced by extractive firms is the maximization of shareholder value. Extractive firms at the headquarters level of management must meet that objective by optimizing profits within a different economic cost environment, i.e., by including user cost in their cost function, a factor generally not considered by manufacturing firms. At the regional management level, the economic environment is different.

For instance, a typical scenario and objective for a regional petroleum manager is to drill and produce proven reserves from an oil field while minimizing the operating and drilling costs. This is what the management team has control over, not the price of oil, determined by the market, not risk management tools, nor the cost of capital, or any other input which is given from headquarters. The management team is in the situation of trying to act "optimally" within its economic environment. A modeling and simulation tool is proposed to assist cross-functional management teams of the extractive firms in achieving the optimal depletion of proved developed and undeveloped reserves while contributing to shareholder value.

Optimization is a predominant theme in economic analysis and in particular dynamic optimization problems. This poses the question of what is the optimal magnitude of a choice variable in each period of time or at each point of time in a given time interval. It is only natural to want to control a given system in some "best" or optimal matter. This intuitive idea will be formalized by mathematically formulating the problem of optimal control and then illustrating some of the concepts through an applied model.

Assume that an oil company has a portfolio of reserves. Some of the reserves are proved developed and some are proved undeveloped. Proved developed reserves are reserves that can be expected to be recovered through existing wells and operating methods at the current price and technology. Proved undeveloped reserves are reserves that are expected to be recovered from new wells on undrilled acreage, or from existing wells where a relatively major expenditure is required for recompletion (Thompson and Wright 1987).

Management has been informed by its reservoir staff that a certain oil field with undeveloped reserves has a reservoir drive mechanism that is not production rate-sensitive. That is to say, the firm can increase the per well or reservoir production rate and not hurt ultimate reserve recovery as a percentage of original oil in place (Craft and Hawkins 1991). Management wants to exploit this situation and has decided to increase the proved developed reserves by increasing the drilling of wells in the undeveloped portion of the reservoir. This would allow the firm to account for or "book" these undeveloped reserves to developed reserves thereby increasing the firm market value. The market places a higher value on proved developed reserves. By implementing a cost-effective production and drilling maintenance effort, the company may achieve this management objective. This is equal to the task of requesting capital budget approval from senior management (Authority for Expenditure) to make a drilling investment, i.e., to convert proved undeveloped reserves to proved developed reserves. The investment decision will achieve the regional management objective. To assist the regional management in quantifying the investment expenditure, a modeling and simulation tool is required to indicate where the money will be spent over time and until abandonment.

The research question is, what is the optimal control policy and reserve depletion path that developed and undeveloped reserves should follow while minimizing the total cost (effort)? The method used to determine the optimal policy would contribute to the natural resource literature by using or having the control as a function of the state variable. This allows for disturbances or revisions to enter the dynamics of the optimal path of depletion.

When studying a nonrenewable natural resource problem, the management issue is typically how to allocate the intertemporal product flows and resource stocks so as to optimize an objective such as maximizing of profits. There may be many solutions to such a problem when the constraints vary or when the objectives of various decision makers differ. There is a competing family of solutions, but which one is optimal?

One of the first tasks in modeling such a problem is obtaining differential (difference) equations that describe the dynamics of the natural resource economic system.

By coupling the mathematical modeling with the developed discipline of optimal control theory, management can apply a methodology, known as dynamic optimization, to solve the problem.

The basic problem of mathematical dynamic theory is to find the system response for the given input function. It is basically the straightforward problem of solving the corresponding differential equation. The problem can be solved by using standard knowledge from the mathematical theory of differential equations. However, the control problem is much more challenging; namely, the input function has to be found such that the system response or state variable has the optimal behavior. The optimal control problem studied in the time domain is known in the literature as the state space form. A general version of the control problem in state space form will be introduced in chapter 3.

When a control function, through analysis or simulation, can be found such that the corresponding system is observed as having some optimal response, then the control problem of interest is solved. But if this is true, is it all we need? Assume for a moment there exists an optimal control solution, which is a function of time. Also imagine that there are some slight parameter variations in the dynamic equations or that some disturbances enter the state variable path. Is the control still optimal? The original control function was given as a precomputed time function and also based on the initial conditions of the state variable, but given the changing conditions above will it still produce a satisfactory solution? The answer is no. One way to solve the problem is to solve the problem again given the new conditions from that point in time forward and to get a new expression for the control function. But this approach is at the expense of additional computation time, which is fine if the parameters of the system dynamics are exactly known. The original control solution, before the disturbances, is known in the literature as the "open-loop" control solution.

For decades, in the economic literature, the tool of choice for dynamic optimization of natural resource problems has been a subclass of optimal control theory known as "open loop" control (Conrad & Clark 1987). This subclass of control theory problems will be developed in chapter 3 and compared to another sub-class of optimal control problems known in the literature as "closed-loop" control. The focus of this research work will be on "closed loop" control. But what is " closed loop control ?"

Imagine that one desires, in an attempt to solve the control problem, to obtain the input control as a function of some essential system variables that completely determine the system dynamics. These essential system variables are called the state space variables. The issue is: can the required control variable be obtained as a function of the state space variables? If the answer is yes, then the existence of such a control function indicates the existence of a state feedback control, known in the literature as "closed loop" control. The control is a function of the state variable at every point in time not just the initial time. Even if some parameters of the system are not precisely known, or there is a disturbance to the system, the state variables will reflect exactly the state of the system at any given time.

Another class of optimal control problems is known in the control literature as linear quadratic systems (LQ). This class represents an important special family of optimal control problems. These problems are defined as a linear dynamic system with the objective function of quadratic form. The importance stems in part from the fact that in many situations this structure represents a natural formulation; but actually the main source of importance derives from the strong analytic results available for this type of formulation. The primary feature of these problems is that the optimal control can be expressed in linear feedback form. Thus, the resulting closed-loop system is also a linear dynamic system (Luenberger 1979). However, there has been minimal use of this system or format in the nonrenewable natural resource literature. The system has been applied mostly in macroeconomics and also in the renewable resources field of agriculture and forestry management. The motivation for using this problem-solving format is that it would provide more flexibility in resource modeling and increase the robustness of the systems response to any disturbances entering the system.

Most of the LQ problems in economics have been formulated as "tracking problems," where a target or desired path is used as a reference and the actual path is compared, thus producing an error or deviation. The objective of the control function for this optimization process is to minimize the sum of squared deviations. Should the desired path or reference point be inserted in the problem as a set point the system now acts as a "regulator." The specific problem addressed in this paper is to model the dynamics and objective function not as a "tracking problem" but as a "regulator" problem, hence the name LQR (linear-quadratic-regulator).

A statement was made in the natural resource literature (Conrad & Clark 1987) that the LQR format does not apply to the typical economic objective of maximization of net benefits. However, the minimization of an objective function is the same as the maximization of the negative of the objective function as will be shown in chapter 3. In modeling the problem dynamics as a regulator problem, the objective will be to move a state variable (reserves) to some set point in state space, while minimizing the cost (effort) inside the objective function. In other words, produce the reserves until it is uneconomic. This point is known in the petroleum literature (Thompson and Wright 1984) as economic abandonment. Therefore, such a formulation of a natural resource problem in the LQR format, that produces an optimal response with an economic interpretation and uses that response to aid in the formulation of an optimal policy or decision, will contribute to the natural resource economic literature.

Some advantages for resource management that this modeling format will provide and which differ from the current literature are as follows:

- The control solution or policy is expressed as a function of the state variables. This means that should a parameter of the dynamic process change or drift though time or should there be a disturbance to the process, the optimal control policy is provided as a function of the changed state variables.

- The closed loop optimal control problem has a closed-form analytical solution. This advantage will be expanded upon in chapter 3.
- For modeling and simulation purposes, the closed-loop linear-quadratic format has additional features which are desirable for resource management.

Inside the quadratic objective function there are weighting parameters assigned to the state and control variables that can be varied. When these factors are varied it brings a changing response to the system and this changed response will have economic meaning for management. The economic meaning could range from modeling a technology shock to a change in fiscal policy, such as a royalty tax change. Therefore, management can simulate various "what-if" designs and evaluate the system response.

Once the decision makers have developed a simulation tool based on optimal control theory, what would be the management implications of that tool? One of the crucial areas for the petroleum management team is to engage in defining the implications as well as the issues of reservoir management. What is the definition of reservoir management? Reservoir management has been defined by many authors (Satter 1992). This study will apply the definition of reservoir management by saying it is the judicious use of various means available to a manager to maximize the net benefits from a reservoir.

The reservoir management process involves setting strategy, developing a plan, implementing and monitoring the plan and, after evaluating the results, possibly revising the plan as more information and data become available. This process is dynamic and the plan requires flexibility. The success of the reservoir management plan depends on the technology and the technology toolbox. This research brings a management tool to assist in the optimum reservoir management process.

By organizing an integrated petroleum management team comprised of geoscientists from reservoir, production, drilling, and economic personnel and utilizing the latest modeling and simulation tools, a solution for almost any dynamic natural resource process can be found. The modeling and simulation of the problem will assist the management in their choices to provide a more cost efficient reservoir development design and will provide guidance on the implementation of that design.

Chapter 2 will present a literature review that includes topics about nonrenewable resources, optimal control theory, and economic applications of optimal control theory. That chapter will be followed by a modeling and concept chapter 3, developing state-space optimal control theory including first-order conditions and will also include the derivation of the optimal control law based on the maximum principle. Chapter 4 will include an applied model that also includes modeling results based on data from the Rodessa oil field in Louisiana. This chapter will present modeling parameter definitions, the quantitative equations and their economic meaning. Chapter 5 discusses the conclusions of the model, it will also include model extensions, for example, the use of difference equations to model delays in the investment decision and areas for further research.

Chapter 2

LITERATURE REVIEW

The theoretical literature applicable to this study is from nonrenewable resources, optimal control theory and optimal control theory with economic applications. This literature provides a guideline for the modeling in chapter 3 and the management conclusions in chapter 5. The nonrenewable resources literature contains several principles borrowed from capital theory because the extraction of petroleum is traditionally managed as a capital stock. In this study, an additional capital stock, proved undeveloped reserves, must be considered, and the optimal management of these two stocks, proved developed and proved undeveloped reserves, gives a different perspective to the traditional capital theoretic results of optimal capital theory. The optimal control literature establishes the necessary conditions for the maximum principle and the different forms of the objective function. The optimal control with economic applications is rich in complex examples of models developed for specific sites. However, the majority of the literature emphasizes open loop control modeling. This study will present an example of closed loop control modeling. The first place to start the literature summary is with the nonrenewable resources literature.

Nonrenewable Resources

Typically nonrenewable resources pose a unique economic problem because they include (by definition) some limiting inputs. The problem can be analyzed in a static microeconomic context. However, this study problem is fundamentally a dynamic problem with the focus on how best to manage a petroleum reserve. Petroleum reserve management lends itself to a capital theoretic treatment since the problem of optimal depletion of the reserves is closely related to the problem of optimal accumulation of capital. Several papers provide the necessary theory.

Harold Hotelling's 1931 article " The Economics of Exhaustible Resources," is always a good starting point when addressing non-renewable natural resource problems. Hotelling's profound mathematical treatment of the management problem for a fixed stock of ore is addressed in that paper. The features of this article had been considered earlier by L. C. Gray, "Rent Under the Assumption of Exhaustibility" in 1914. However, Hotelling used calculus of variations to model the dynamics of this mining problem in a pure depletion model for the mining industry as a whole. He looked at three cases of a mine: price taker, monopolist, and social optimizer. Hotelling did not consider the opportunity of discoveries through exploration. He also developed the idea of "user cost," incurred by extracting today and not having the resource available for the future. Anthony Scott's (1967) " The Theory of the Mine Under Conditions of Certainty," used microeconomics to analyze a single mine. Scott's article was different from Hotelling's in that it was graphical and intuitive rather than mathematical. Scott demonstrated it was in fact optimal for managers to tilt production toward the present as their rate of time preference increases. He demonstrated that without discounting or the future counts the same as today the manager will try to maximize profits by operating at the minimum cost point, while discounting will encourage the manager to operate at a point closer to current maximum profits where P = MC. These two points are distinct and that cost is minimized at a lower rate of extraction than that which maximizes profits. Scott's article focused on the economics of a single mine rather than Hotelling's focus as a social optimizer on the entire mining industry. This study will focus on a single oil field. Roan and Martin (1996) discussed Scott's article and concluded that once a mine is analyzed as a two-product firm, i. e., two state variables, tilting can occur even at a zero discount rate. This is contrary to Scott's conclusions.

Pindyck (1978) introduced a model that extended Hotelling's by adding an exploration function. Now reserves could change not only by depletion but also by discovery of new reserves. Like Hotelling, Pindyck used dynamic optimization to solve this problem. However, by this time, the most widely used method of dynamic analysis was optimal control theory, introduced in the 1950s as an improvement of the calculus of variation. Pindyck used two state variables (reserves, cumulative discoveries) and two control variables (production, exploratory effort). His result was an optimal open loop control policy solution, most papers written by the researchers obtain an open loop optimal control solution.

Two examples of the open-loop optimal control format are Kumar (1997) and Toman (1986). Kumar addresses the issue of optimal capacity expansion for domestic processing of exhaustible, natural resource exports in a small open economy. The solution expresses qualitative optimal dynamic paths for resource extraction. Toman addresses optimal paths by using open-loop optimal control methodology for a competitive mining firm with the possibility of either a finite or an infinite terminal time and a non-zero terminal (salvage) valuation if the terminal time is finite.
Another relevant article by Burness and Martin (1988) is about the management of a tributary aquifer and includes a hydrologically connected groundwater stock to a surface water source. The study uses a similar model structure, which includes an initial stock of water in the aquifer, a historical pattern of groundwater pumping and a pattern of aquifer recharge via river effects.

Of the many excellent papers written in this field, the common thread, notwithstanding their contribution to the literature, is that they were open-loop optimal control solutions. The optimal control was a function of the initial conditions of the state or determined outside the model. As stated earlier, this paper proposes an optimal control as a function of the state variable other than only at initial conditions, a closed-loop solution. The author will also use the Pindyck model in concept, i.e., two state variables,

x_1, proved developed reserves and x_2, proved undeveloped reserves. The two control variables are u_1, a quantity index for production maintenance and u_2 a quantity index for the drilling function. The problem will use a different optimal control format. The format to be used is a linear quadratic system comprising linearized model equation dynamics with a quadratic objective function. For a general development and presentation of most of the results of nonrenewable resources see Dasgupta and Heal (1978).

Optimal Control

How can a person tell whether a given control strategy is best or optimal? Associated with each optimal control problem is an integral called an objective function where the value J, a scalar function, is a measure of how well a particular control variable (u) causes the system state variable(s) (x) to perform with respect to specified criteria. An optimal control input may be one which either minimizes some cost function or maximizes some payoff function. The minimum of J is the maximum of - J, so that no loss of generality results in our studying either the maximization of benefit or "payoff" problems or the minimization of costs Padulo and Arbib (1974).

A continuous time optimal control problem is sometimes expressed by the form of its objective function. This study will use the following form.

$$J = \int_0^T l(x(t), u(t))\,dt \qquad \text{(problem of Lagrange)}$$

Where the function has the state variables and control variables included. Traditional optimal problems were originally formulated using the calculus of variations, which initiated with a study by the Bernoulli brothers in 1696 (in Luenberger 1979). The classical calculus of variation approach was extended to problems with inequality constraints around 1930, mainly by McShane (in Luenberger1979). The classical formulation contains no explicit control variable, but the substitution $\dot{x} = u$ can be used to convert these problems to control form, where $\dot{x}$ = dx/dt. The application was greatly broadened and the notation improved by the explicit introduction of the control variable. The control variable was used in the development of the general maximum principle of Pontryagin (1962) (in Luenberger 1979) but also developed about the same time by Hestenes (1963) (in Chow 1992). The statement of the maximum principle involves the concepts of the Hamiltonian function and the costate variable. In a problem statement the variables t(time), x(state) and u(control) will have been identified. In the solution process, another variable will emerge called the co-state variable, to be denoted by λ (t). This variable is akin to a Lagrange multiplier and, as such, it is in the nature of a valuation variable, measuring the shadow price of an associated state variable. Like x and u the co-state variable can take on different values at different points of time.

The vehicle through which the co-state variable gains entry into the optimal control problem is the Hamiltonian function that figures very prominently in the solution process. Denoted by H, the Hamiltonian is defined as

$H(x,u,t,\lambda)= \lambda_0 (J(x,u,t) + \lambda(t) f(x,u,t);$

where (J) is the objective function and (f) is the system dynamics. J can represent a profit function, cost function or another type of economic objective. If it is a profit function, then based on the current reserves, x_1 and the current policy decision taken at that time that component part is the "current profit corresponding to policy u" where u is the control variable. Since H consists of the integrand function J plus the product of the costate variable and the function f, it should naturally be a function with four arguments: t,x,u as well as λ. Note that in the above Hamiltonian there has been assigned a λ_0 coefficient to J, which is in contrast to the yet undetermined $\lambda(t)$ coefficient for f. Strictly speaking, the Hamiltonian has λ_0 in front of the J function, here λ_0 is strictly positive; and can therefore be normalized to a unit value. Since optimal control problems in economics are those where the objective function (J) matters, the prevalent practice among economists is simply to assume $\lambda_0 > 0$, then normalize it to unity and use the Hamiltonian as stated above.

The second component of the Hamiltonian, f(x,u,t) indicates the rate of change of (physical) reserves, $\dot{x}$, corresponding to policy or control u, but when the f function is multiplied by the shadow price λ (t), in the case of a non-renewable reserve, it is converted to a monetary value. Therefore, the second component of the Hamiltonian represents the rate of change of reserve value corresponding to policy u. Unlike the first term, which relates to the current-profit effect of u, the second term can be viewed as the future-profit effect of u, since the objective of reserve accumulation is to provide a way for production of profits for the firm in the future. If a particular policy decision u is favorable to the current profit, then it will normally involve a sacrifice in the future profit. In sum, then, the Hamiltonian represents the overall profit prospect of the various policy decisions, with both the immediate and the future effects taken into account. (Chow 1992)

The maximum principle involves two first-order differential equations in the state variable and the costate variable plus initial conditions on the state variable and a transversality condition on the costate variable. There is a requirement that the Hamiltonian be maximized with respect to the control variable u at every point of time. The maximum principle conditions for this study's free end point problem are

Max H (x,u,t, λ) for all t from 0 to T.

$$\dot{x} = \frac{\partial H}{\partial \lambda}; \; x(0) = \bar{x}$$

$$-\dot{\lambda} = \frac{\partial H}{\partial x}; \; \lambda(T) = 0 \quad \text{Transversality condition}$$

$$\text{Max}_u \; \frac{\partial H}{\partial u} = 0$$

The symbol Max H means that the Hamiltonian is to be maximized with respect to u alone as the choice variable. Note that it is this requirement of maximizing H with respect to u that gives rise to the name "the maximum principle." The next expression is the equation dynamics for the state variable and the last expression is the equation dynamics for the co-state variable. The state variable is free and the co-state variable is fixed to a value of zero at terminal time T.

The next section will provide several economic examples using optimal control theory and will introduce the principle of optimality that is based upon Bellman's dynamic programming equation.

Closed Loop Optimal Control With Renewable Economic Applications

The resource economics literature that applies to optimal control theory includes Bruce Dixon and Richard Howitt(1979) who researched and modeled a forest management problem by applying a Linear-Quadratic-Gaussian system. The empirical problem of forest resource management is typically the time allocation of product flows and resource stocks under uncertainty. National forest harvest scheduling is conceptualized in their study as a stochastic optimal control problem. In order to solve this problem, certain solution techniques were required; their study employs one such technique called the Linear-Quadratic-Gaussian (LQG) control method. The LQG model is both an estimation and a control problem. This is the stochastic version of the LQR (deterministic) model. Their problem is modeled as a typical "tracking problem," where given a set of desired or target levels for stocks through time, the LQG optimization criterion is to keep the actual evolution of the system as close as possible to the target levels.

Additional literature on tracking problems is in a textbook by Rauisser and Hochman (1979) that provides guidelines to empirical research on how to apply the methods of dynamic control analysis, stochastic or otherwise, to economic systems, especially agriculture. There are also several chapters that appeared in Giannini's Foundation Monograph No. 29 (1972) on this same subject. Modeling the dynamics of a tracking problem is different from modeling the dynamics of a system as a regulator problem, because there is no desired target tracking path but rather a desired point in state-space that the system dynamics drive the state variable(s). This study will model a petroleum reservoir management problem as an optimal control regulator. The model and the economic meaning of the variables will be explained in chapter 3.

Several optimal control studies deal directly with dynamic economic analysis. Two of these, Aoki (1976) and Chow (1975) emphasize the conceptual and technical aspects of control theory in the context of dynamic economic analysis. However, only a few illustrative examples or potential applications are provided. Aoki (1976) discussed the linear-quadratic system but addressed the format as a "tracking problem" not as a regulator problem. The economic applications used in his text were in macroeconomics.

Seierstad and Sydsacter, 1987, introduced the theory of deterministic optimal control problems (open-loop) but only briefly mentioned feedback control theory and with very limited applications. A monograph, "Control Theory Methods in Economics" (Sengupta and Fanchon 1997) has an application of the LQR format, the first seen in this literature review. It was applied in the macroeconomics field of expected inflation. The optimal control solution described the optimal time path of the state variable, expected inflation. The state variable was to be minimized, i.e., driven to zero in state space with time free and the control variable was the real money supply. The dynamics involved modeling a Phillips curve, where actual inflation was a function of expected inflation, unemployment, and labor productivity. It was an excellent example of how to model a dynamic problem using the LQR methodology.

Before the discussion of a special and extremely important family of optimal control problems, namely linear systems with a quadratic objective function, let us present an alternative approach to solving optimal control problems: dynamic programming. An approach that in fact exploits the dynamic structure of these problems more directly than the variational approach. The concept has a long history, but its importance was developed by Bellman in the 1950s. The basic concept behind dynamic programming is the "principle of optimality" Bellman (1950), which points out a fundamental relation between a given optimal problem and various other subproblems. The "principle of optimality" says: from any point on an optimal trajectory, the remaining trajectory is optimal for the corresponding problem initiated at that point (Luenberger 1979) . This principle allows one to obtain solutions by progressing backward in time.

Problem solutions are obtained by solving a partial differential equation known in the literature as the Hamilton-Jacobi-Bellman equation. However, what is important is the relationship between the calculus of variation, optimal control theory and dynamic programming. They all have the common thread of satisfying the necessary conditions of the maximum principle and the principle of optimality. An advantage of the dynamic programming approach is that it determines the optimal control in feedback form. The disadvantage is what Bellman called the "curse of dimensionality" which means the optimal solution is embedded in a larger family of solutions and each sub-solution must be checked against the others to minimize or maximize the objective function. This procedure requires a large memory computer.

Also, when solving the H-J-B equation, numerical techniques are required which obtain an approximate solution to the exact optimization equation. It can be shown that the equations that constitute Pontryagin's maximum principle can be derived from the H-J-B functional equation (Kirk 1970). There is also an important relationship between Pontryagin's maximum principle, dynamic programming and the LQR of optimal control. The literature (Luenberger 1979) has developed methods for solving the LQR problem by using the maximum principle or the H-J-B equation of dynamic programming. Nevertheless, when satisfying the necessary conditions of the maximum principle there will be 2n differential equations and 2n end points with 2n unknown functions. The difficulty is that the 2n endpoint conditions are not at the same point but at different ends. This is a boundary value problem. However, it can be shown (Luenberger 1979) that a closed-form solution is possible if a certain type of auxiliary equation is used. When this equation is developed and then integrated backwards it solves what is known as the Ricatti equation (Kalman 1960). By knowing its solution, we are able to develop the feedback control law (u(x(t)) which is a function of the state variables (x).

In summary, the optimal control law can be derived while satisfying the necessary conditions of the maximum principle, or the H-J-B equation of dynamic programming, the results are the same (Luenberger1979). But determining the optimal control function by satisfying the necessary conditions of the maximum principle leads to a non linear two-point boundary-value problem that requires the use of iterative numerical techniques for solutions. These iterative algorithms determine optimal control in open loop form. That is, the control law is a function of the initial conditions of the state variable. However, if the state equations are linear or have been linearized, and the objective function is of quadratic form, the optimal control law can be determined by numerically integrating an auxiliary differential equation with a single point boundary condition. This gives the optimal control in closed loop form. The control law is now a function of the state variables at that specific time. This is an improved system, because if the state variable should change because of disturbances, the optimal control keeps the state variable on its optimal path. It is now a more "robust" system to disturbances.

The literature discussed in this chapter will be used to develop modeling concepts as well as the qualitative and quantitative models of this study. In particular, the maximum principle will be used to develop the feedback control rule, in lieu of dynamic programming because the computer aided software to be used has an application for the maximum principle. The concepts from Burness and Martin p.14 about the management of a tributary aquifer will be used as the seed to structure the qualitative petroleum model.

In the context of this study, the particular application chosen to model is perhaps less significant than the abstract problem and its potential for generalization and further application. The results that are derived for this problem suggest that this format may be an addition to our collection of methods to model natural resource problems.

Chapter 3

METHOD AND MODEL

Method

This chapter will develop the general solution for a linear quadratic problem, it will define the variables, parameters and their economic meaning. The second part of the chapter will express the qualitative model including the economic properties required of the model.

The link between the petroleum problem to be modeled and the method of finding the optimal control solution will be the application of the maximum principle based on the principle of optimality. First, let us develop a general solution for the standard linear quadratic problem and then later develop the qualitative petroleum model. The applied quantitative petroleum model will be addressed in chapter 4 and the optimal control solution will be used to simulate various management scenarios.

The standard linear quadratic problem for a given linear n order state system is:

$\mathrm{J} = \frac{1}{2}\int (x^T Q x + u^T R u)dt$ Objective Function

$\dot{x} = A\,x(t) + B\,u(t),$ State Equation (3-1)

where

$x(0) = x_0$ Initial Condition

$\lambda(T) = 0$ Transversality Condition

$y = C\,x(t) + Du(t)$ Output Equation (3-2)

where x, u, y are, vectors of system states, control inputs, and system outputs, respectively.[1] The matrix A (nxn) describes the internal behavior of the system, while matrices B(nxm), C(pxn) and D(pxr) represent connections between the external world and the system. If there are no direct paths in the forward loop between inputs and outputs, this means there are no direct connections between the inputs u and the outputs y which is this case, the matrix D is zero. Refer to the flow diagram A3 in the appendix. The objective function for this studies problem is of the following form:

$$\min \; J = \frac{1}{2}\int (x^T Q x + u^T R u)dt$$

In this problem, (u(x(t)) is an m-dimensional input function and it is not subject to any constraints other than to be non-negative. The objective function is quadratic in both the state (x) and the control (u). The quadratic functions are defined by the matrices Q(t) and R(t) that are symmetric square matrices of dimension nxn and mxm, respectively, and where n is equal to m for this problem. The matrix Q(t) is assumed to represent a positive semidefinite quadratic form so that $x(t)^T Q(t) x(t) \geq 0$ for all x(t). The matrix R(t) is

[1] Refer to vi and vii for a listing and meaning of the variables, if required.

assumed to represent a positive definite quadratic form so that R> 0. All functions are assumed to be continuous with respect to time. Note that in the objective function Q and R are matrices chosen to apply the desired weights to the various states and inputs. Q must be at least positive semidefinite, and R must be positive definite so as to always impose a cost on the input factors and to guarantee a well defined minimum for J. Another purpose for the Q matrix has to do with the asymptotic properties and stability of the system dynamics. This is demonstrated by defining a Lyapunov function (Luenburger 1979). The Lyapunov function exists if Q is positive definite and is related by the expression $-Q = A^T P + PA$, where P is the solution to the Riccati equation (developed below in the method section). Should this Lyapunov function exist, the system dynamics are stable.[2]

This problem is a special case of the basic free endpoint problem using the maximum principle. The co-state variable is zero at terminal time. However, the necessary conditions are still satisfied even for a fixed point, free time problem but with additional transversitility conditions. The general rule is if the state variable is free the co-state variable is fixed and if the state variable is fixed at terminal time, the co-state variable is free. The maximum principle conditions for this problem is as follows. The co-state equation is:

$$-\dot{\lambda}(t)^T = \lambda(t)^T f_x(x(t),u(t)) + J_x(x(t),u(t))$$

[2] This is not of great importance for the applied petroleum model in this study.

where $\lambda(t)$ is the co-state variable, it is interpreted as the value of the reserves in the ground, the co-state variable has the nature of a LaGrange multiplier or a shadow price of the reserves. The f_x and the J_x, respectfully, are the partial derivatives of the state equation dynamics and the objective function with respect to the state variable. Therefore, for the linear system with a quadratic objective function, the co-state equation is:

$$-\dot{\lambda}^T = \lambda(t)^T A(t) - x(t)^T Q(t) \tag{3-3}$$

with terminal condition λ (T) = 0.

The negative term on the right-hand side of the equal sign is due to the fact that we are maximizing -J. The Hamiltonian is:

$$H = \lambda^T(t)\, A(t)\, x(t) + \lambda^T\, B(t)u(t) - \frac{1}{2}x(t)^T\, Q(t)x(t) - \frac{1}{2}u(t)^T\, R(t)u(t) \tag{3-4}$$

The condition for maximizing the Hamiltonian with respect to u(x(t)) is $H_u = 0$ or,

$$\lambda(t)^T\, B(t) - u(t)^T\, R(t) = 0 \tag{3-5}$$

Therefore:

$$u(t) = R(t)^{-1}\, B(t)^T\, \lambda(t) \tag{3-6}$$

This expression can be substituted into the original system equation. If this substitution is made, and if the co-state equation (3-3) is written in transposed form, we obtain the equations:

$$\dot{x}(t)=A(t)x(t)+B(t)R(t)^{-1}B(t)^{T}\lambda(t) \tag{3-7}$$

$$\dot{\lambda}(t) = Q(t)x(t) - A(t)^{T}\lambda(t) \tag{3-8}$$

with conditions

$$x(0) = x_0 \; ; \; \lambda(T) = 0.$$

This is the same first order conditions found in the economics literature for an open loop solution for this particular objective function. Should a different objective function be used the same mathematical logic and procedure would be applied. The economic meaning of equation 3-7 expresses that the change in proved developed reserves over time is not only a function of itself but also of the shadow price of the proved developed reserves, the co-state variable λ. Equation 3-8 expresses the change in the shadow price of the proved developed reserves over time which is proportional to the proved developed reserves and inversely related to the shadow price of the reserves. This makes economic sense in that as the reserves are depleted the shadow price of the reserves fall.

Referring to 3-7 and 3-8 there are 2n differential equations, 2n endpoint conditions, and 2n unknown functions in this open loop problem. The difficulty, of course, is that the 2n conditions are not all at the same endpoint. If they were all initial conditions, the system could be solved by integrating forward in time. Since they are not we would solve by numerical methods. However, in the real world the state variable x_1, or in my problem proved developed reserves changes with new information. What would be the new optimal path? The open loop optimal control (u(t)) solution does not allow for these changes or feedback, the new control variable must be written as a function of the state (u(x(t). In order to solve for the optimal control as a function of the state variable a different approach must be taken.

Since the system is linear, it can be said that both $x(t)$ and $\lambda(t)$ depend linearly on x_0. Accordingly, $\lambda(t)$ depends linearly on $x(t)$. This motivates one to try a solution of the form

$$\lambda(t) = -P(t)x(t) \tag{3-9}$$

where P(t) (determined below) is the solution to the Riccati equation (Luenberger 1979). P(t) is an nxn matrix.

Substituting equation (3-9) in the system (3-7)and (3-8) yields the two equations:

$$\dot{x}(t) = [A(t) - B(t)R(t)^{-1}B(t)^T P(t)]x(t) \tag{3-10}$$

$$-P(t)\dot{x}(t) - \dot{P}(t)x(t) = [Q(t) + A(t)^T P(t)]x(t) \tag{3-11}$$

Multiplication of (3-10) by P(t) and addition to (3-11) then yields

$$0 = [\dot{P}(t) + P(t)A(t)^T P(t) - P(t)B(t)R(t)^{-1}B(t)^T P(t) + Q(t)]x(t) \tag{3-12}$$

This will be satisfied for any x(t) if P(t) is chosen so as to satisfy the matrix differential equation

$$-\dot{P}(t) = P(t)A(t) + A(t)^T P(t) - P(t)B(t)R(t)^{-1}B(t)^T P(t) + Q(t) \tag{3-13}$$

From the transversality condition $\lambda(T) = 0$, we derive the corresponding condition $P(T) = 0$. The differential equation (3-13) which is quadratic in the unknown P(t), is called a Riccati equation, (Luenberger 1979). The solution $P(t)$ is symmetric, since $\dot{P}(t)$ is symmetric for all t. It can also be shown that $P(t)$ is positive semidefinite.

The solution to the control problem (3-1) is now obtained as follows. One first solves the matrix Riccati equation (3-13) by backward integration starting at t=T with the condition P(T) = 0. Once P(t) is known we can substitute P(t) into equation 3-9 to obtain λ. The control is found by combining equation (3-6) and equation (3-9) to obtain:

$$u(t) = -R(t)^{-1}B(t)^{T}P(t)x(t) \qquad (3\text{-}14)$$

or, equivalently,

$$u(t) = K(t)x(t)$$

where:

$$K(t) = -R(t)^{-1}B(t)^{T}P(t)$$

If I substitute u[x(t)] (control variable) into 3.1, I can solve for the optimal x (state variable).

The mxn matrix K(t) can be computed before operation of the system. Then as the system evolves, the control is computed at each instant on the basis of the current state. This is a feedback solution, in this case it is linear feedback. In the finite horizon (time) regulator case with n = 0,1,2...n-1, the final values in K are zero. A flow diagram model representing the closed loop solution is shown in the appendix A3.

As all the variables in the system have been defined, the concluding definition has to do with a view of the meaning of K in the optimal control law. The optimal control literature for physical systems uses the word "gain" as a meaning for K; in the sense of an amplifier gain. That is, in order to increase the signal to noise ratio of a waveform and to increase the systems performance the gain (K) must increase. That is how an amplifier works; there is an input signal and via the amplifier gain (K) there is a change in the output signal. However the economic optimal control literature has not yet expressed the meaning of the K matrix even though the format is derived and used (macroeconomics context) in several papers and textbooks (Akoi 1975 Chow 1976). The K matrix in the feedback loop of the optimal control law, is the reciprocal of the marginal product coefficients of the factors of production that relate changes in the state variables to the changes of the factors of production, which serve as inputs to the model. The optimal control law expresses a relationship through (K(t)) between changes in state variables x(t) and changes in the inputs u[x(t)].

Earlier it was mentioned that the Hamilton-Jacobi-Bellman equation from dynamic programming[3] can be applied to the LQR problem and the results for a feedback control law would be the same as has been shown using the maximum principle and the Riccati

[3] It will not be derived here but the reader may refer to Kirk 1978 or Luenberger 1979.

equation. The net result of an optimal control problem is to transform a system with inputs to a free system since the inputs are specified by the optimization process (Luenberger 1979).

The general solution for the LQR problem has been derived and the objective of the rest of this chapter is to develop the qualitative petroleum model. The optimal control law will be applied on a linear quadratic problem using production data from the Rodessa field in Louisiana as a base case and then developing various modeling assumptions and simulations. For example, changing the weighting factor (R) in the objective function to simulate a change in drilling technology or inserting a disturbance (reversion) for proved reserves.

Model

The model developed in this chapter takes the perspective of a regional petroleum manager who wishes to optimally deplete proved developed and undeveloped reserves by applying the tools of integrated petroleum reservoir management and optimal control theory. It is assumed that the only controls of reservoir management are production maintenance (q) and a drilling development program (w). Both of these policies affect expenses in the form of maintenance and development costs. Profit is affected in the form of increased revenue from a more developed and maintained reservoir. In reality, there would be several other aspects to the manager's problem aside from production maintenance and drilling development costs. There would be the possibility of tax considerations, uncertainty in prices, reservoir uncertainty, and so on. These aspects of the problem is assumed to be fixed in order to obtain a model that captures the essential features of this deterministic optimal control problem.

Without increasing production and drilling costs, the quality/quantity level of the reserves would gradually decline through production and equipment aging. Production and drilling cost, and this is where the control aspects enter, can increase or at least minimize the rate of production decline. The effect of these maintenance costs, however, is subject to certain economic properties including decreasing marginal products for the inputs. The optimizing manager will want to select a policy in such a way so as to deplete the proved undeveloped and developed reserves over time while minimizing the costs.

How can this conflicting problem be formulated and modeled while maintaining the economic properties desired? As was explained earlier, dynamic optimization methods have been developed in the economic control literature in which this problem will fit. It is called the linear-quadratic regulator problem (LQR), consisting of linear state equation dynamics and a quadratic objective function. The optimal control solution for this problem was developed in the method section of this chapter. The objective function of this LQR problem is:

$$\min \ J = \frac{1}{2}\int (x^T Q x + u^T R u) dt \tag{3-15}$$

The format and object of J; the objective function to be minimized, is a scalar function. It may take on many different schemes for the same system dynamics to give quite different results. It would therefore be expected that different objective functions would look very different for different control schemes. For example, in the economics control literature, the objective function is usually expressed as the integral of revenue minus total cost, i.e., profit. But there could be alternative formats for the objective function, such as the problem of producing a reserve from an initial condition to depletion while minimizing the cost. Such an objective function would look different from the profit objective function. The objective function used here has two terms: first, the state variables, i.e., reserves which are to be minimized from some initial condition. The second term in the objective function is the controls, i.e. the costs inputs for production and for development, that are also to be minimized over time. Although the function may look unfamiliar, in reality its interpretation is similar to the negative of a profit function. In a profit function revenues are price times quantity produced. However, quantity produced depletes reserves where $-y = \dot{x}$ where I choose $\dot{x}$ to be a quadratic function of x and I normalize price to 1. I choose a quadratic cost function since it allows for rising marginal cost and for model tractability. Thus the positive value on reserves and the positive value on cost makes this similar to a negative profit function, which is to be minimized.

Normally we would discount profits over the planning horizon. However, in a Hotelling world, price should go up over time by the discount rate, thus the price increase parameter cancels the discount parameter. Assuming a similar increase for cost over time eliminates the discount parameter for cost as well.

The relative weightings chosen for Q and R determine the relative emphasis placed upon reducing errors and saving control energy. Management has a choice in the weighting of matrices Q and R in the objective function. This will have an important impact on the optimal path of the state and control variables and of the modeling of this problem. Because the state variable(s) (reserves) are to be produced to a desired point, this is the regulator concept, management can weight the importance of each state in reaching that point. For example, for political reasons, drilling undeveloped reserves may be preferred to any production decision, and could be modeled by weighting undeveloped reserves more in the Q matrix.

The weighting matrix R imposes a higher or lower value on the control inputs u[x(t)]. For example, if control variables u_1 and $3u_2$ are weighted accordingly, then u_2 contributes three times as much as u_1 to the scalar value J in the objective function. This means the u_2 costs are more heavily weighted, it is more of a penalty on the objective function. Therefore, what is important is the relative weight of matrix R with respect to Q in the objective function. Moving a state variable (reserves) from one point in state space to another point imposes a cost or penalty. Unless a cost is imposed for the use of factor resources the reserves would be immediately exhausted, besides economic factors of production do in fact have a real cost. Reducing the weighting of R in the objective function has the same effect as increasing u[x(t)] but without changing the value of the objective function (Schultz 1967). This causes the production of the proved reserves to increase toward the present, i.e., a production shift, while the area under the objective function J is the same. Varying the weights of these matrices allows management to simulate various fiscal or operating conditions, all of which can impact managerial decisions.

The rationale for the qualitative petroleum model system constraints refers to a paper written by Burness and Martin (1988) about the management of a tributary aquifer. The first step in the modeling process will be to describe the state of the system at some time t as characterized by the stock of proved developed reserves and the cumulative drilling effects on proved undeveloped reserves. The stock of oil x_1 (t)) in the reservoir at time t depends on (1) the initial oil reserves (2) a pattern of producing the oil from the reservoir y (t), (3) a production maintenance term q(t) and (4) a pattern of reserve replacement via the drilling effects w(t). The stock of oil in the reservoir at time t equals the stock at the beginning of the planning horizon minus all oil that has been produced from the reservoir to time t plus the effects of production maintenance plus the cumulative drilling effects at time t. Therefore, from the above definition

$$x_1(t) = x_1(0) - \int_0^t y(z)dz + \int_0^t q(r)dr + \int_0^t w(s)ds \qquad (3\text{-}16)$$

where z, r and s are dummy variables of integration.

Differentiating the above with respect to time yields (where $\dot{x} = dx/dt$)

$$\dot{x}_1 = -y(t) + q(t) + w(t) \tag{3-17}$$

or, the change over time in the stock of oil in the reservoir is reduced by production from the reservoir -y(t) and enhanced by the production maintenance term q(t) and the drilling effects or flows w(t). The production maintenance term q(t) is added to the system dynamics as an enhancement for production and as an offset for any corrosive effects on the pipe, valves or fittings. The w(t) is the drilling process, by utilizing this process proved undeveloped reserves are converted to proved developed reserves. Therefore equation 3-17 is transformed to a state space format as

$$\dot{x}_1 = -A_1 x_1 + q(x_1 u_1 t) + w(x_2 u_2 t) \tag{3-18}$$

$$w(x_2 u_2 t) = -\dot{x}_2 \tag{3-19}$$

These state space equations relate to the previous resultant equation 3-17 as follows. From the general LQR problem, the output equation in state space form is y = Cx. Therefore, -y(t) is equal to - A_1x_1, where $-A_1$ is equal to -C, the term for production maintenance q(t) is $q(x_1,u_1,t)$ and $w(x_2u_2t)$ is equal to w(t). Therefore, equation 3-17 is identical to equation 3.18.

The equations for q(t) and w(t) are concave functions with respect to the control variable. The first term on the right side of equation 3-18 represents the rate that the reservoir would decline by producing in the absence of production or drilling maintenance.

If A were a parameter and constant where A(t) = A_1, then the normal decline would be exponential (with a half-life of (ln2/ A_1). This parameter is a technical factor that is site specific. The second term and third terms of equation 3-18 have a specific quantitative form that satisfies certain economic properties that will be specified in chapter 4. The qualitative economic properties for q and w are as follows:

$q(u_{1,}x_{1,})$: 1) $q(0,x_1)=0$ 2) $q_{u1}>0$ 3) $q_{x1}>0$ 4) $q_{uu}<0$

$w(u_{2,}x_2)$: 1) $w(0,x_2)=0$ 2) $w_{u2}>0$ 3) $w_{x2}>0$ 4) $w_{uu}<0$

For q these conditions have the following economic interpretations: Condition 1) for zero production maintenance the quantity of proved developed reserves is governed by the linear term in equation (2) which means that proved developed reserves will decline exponentially. Condition 2) maintenance has benefit in that it decreases the rate of reserve decline from external environmental effects, so that increasing the level of maintenance increases the positive effects of maintenance; environmental effects could mean pipe scaling, equipment corrosion, etc. all which affect production rates and changes in reserve levels. Condition 3) a given rate of production maintenance is more effective at higher proved developed reserve levels than at lower levels, meaning maintenance is more cost effective for larger proved developed reserves than on lower value of reserves, implying that the range of feasible maintenance rates increases as the quantity level of developed reserves moves closer to an upper limit. Condition 4) the effectiveness of production maintenance is subject to decreasing marginal products.

For w these conditions have the following economic interpretation. Condition 1) the function w represents the effects of drilling maintenance, and is zero when u[x(t)] is zero. Condition 2) the drilling process is beneficial and increasing the level of drilling increases the effect of drilling. Condition 3) the drilling process is more effective at higher quantities of undeveloped reserves than at lower levels; in this sense it is far easier to increase the quantity of reserves from a field with larger undeveloped reserves than one which is already developed. Condition 4) the effectiveness of the drilling process is subject to decreasing marginal products from the input u[x(t)].

In summary, remember the object is to choose the control vector u[x(t)] such that a given objective function is to be minimized. It has been shown in the method section that the control vector can be obtained by using Pontryagin's maximum principle of optimal control theory (open-loop solution) coupled with solving the Riccati equation (closed loop solution). The necessary conditions of optimality will be satisfied. The optimal solution will also yield a linear feedback control law such that u[x(t)] = K(t)x(t).

The next chapter will apply this optimal solution to the production data of the Rodessa field in Louisiana and simulate various management scenarios, including a base case. The management scenarios will simulate a change in terminal time which will be shorter than the base case time. There will be a simulation of a change in drilling technology, i.e. better bit technology. What will be the management implications? The last management scenario to be simulated is a change in proved developed reserves. The reservoir engineering department has received new information about the Rodessa field and has advised management of a significant increase in proved developed reserves. What will be the management implications? These questions will be answered in chapter 4 with the conclusions stated in chapter 5.

Chapter 4

APPLICATION OF THE MODEL

This chapter will introduce the variables and their relationships to one another. It will present a base case along with three different management scenarios and conclude with the advantages of closed loop optimal control modeling. But first some history and reservoir details of the Rodessa Field.

The Rodessa Field

Before developing and then applying the quantitative petroleum model based on the concepts of chapter 3, lets first review some energy mechanisms and reservoir details of the Rodessa oil field.

Many reservoirs have been discovered which are of the volumetric undersaturated type and whose production, therefore, is controlled largely by the solution gas drive mechanism. In many cases the mechanism is altered to a greater or lesser extent by gravitational segregation of the gas and oil, by small water drives, and by pressure maintenance, all of which improve recovery. The important characteristics of this type of production may be summarized as follows and observed in the appendix A1 for the Rodessa field. Above the bubble point pressure the reservoir is produced by liquid expansion and there is a rapid decline in reservoir pressure which accompanies the recovery of a fraction of 1 percent to a few percentage points of the initial oil in place. The gas-oil ratios remain low, and generally near the value of the initial solution gas-oil ratio. Below the bubble point, a gas phase develops, which in most cases is immobile until the gas saturation reaches the critical gas saturation in the range of up to 20 percent. During this period the reservoir produces by gas expansion, which is characterized by a much slower decline in pressure and gas-oil ratios near, or in some cases even below, the initial solution gas-oil ratio. After the critical gas saturation is reached, free gas begins to flow, reducing the oil flow rate, and depleting the reservoir of its main source of energy. By the time the gas saturation reaches a value usually in the range of 15 to 20 percent, the flow of oil is small compared with gas and the reservoir gas rapidly depletes. At abandonment, the recoveries are usually in the range of 10 to 25 percent by the solution gas-drive mechanism alone, but may be improved by gravitational segregation.

The production of the Rodessa field in Louisiana is a good example of a reservoir that produced during the major portion of its life by the dissolved gas drive mechanism. The availability of reasonably accurate data on this reservoir relating to oil and gas production, reservoir pressure decline, sand thickness, and the number of producing wells, provide an excellent example of the theoretical features of the dissolved gas drive mechanism. The Rodessa field has an oil of 42.8 degrees API gravity, an original bottomhole pressure of 2700 psig and a solution gas-oil ratio of 627 SCF/STB. The wells were produced at high rates and rapidly declined in production.

Many unsuccessful attempts were made to decrease the gas-oil ratios, by blanking off upper portions of the formation in producing wells and by perforating only the lowest sand members. The dip in the actual production profile indicates a management decision of shutting in some wells because of this high gas-oil ratio problem. The strategy did not work, and the wells were put back on production from period 13 to 15. This negative shock to the Rodessa field was not simulated in the problem model, but could have been using ramp functions. The failure to reduce the gas-oil ratio is typical of the dissolved gas drive mechanism, because when the critical gas saturation is reached, the gas-oil ratio is a function of the decline in reservoir pressure or depletion, and is not materially changed by production rate or completion methods. Hence, the double hump on the production curve (appendix A1) as wells were shut in and later placed back in production. As the controlling factor in this type of mechanism is gas flow in the reservoir, the rate of production will have no material effect on the ultimate recovery. Likewise, well spacing has no proven effect on recovery; however, well spacing and production rate affect economic return.

The actual production field data used to compare with the idealized computer model is found in the appendix. The plotting of the data is appendix A2. These data were obtained from the textbook, <u>Applied Petroleum Reservoir Engineering,</u> 2nd edition by Craft and Hawkins 1991. The data gives the number of wells drilled, the monthly production in thousands of barrels of oil, daily oil production and cumulative oil production.

The qualitative model developed in chapter 3 will be applied to a quantitative model developed in this chapter with the following objective function.

$$\text{Min } J = \frac{1}{2}\int_0^{60} (xQx^T + uRu^T)dt \tag{4-1}$$

subject to:

$$\dot{x}_1 = -.1x_1(t) + q(u_1, x_1, t) + (-\dot{x}_2) \tag{4-2}$$

$$-\dot{x}_2 = w(u_2, x_2, t) \tag{4-3}$$

where:

$$q = u_1 - \frac{1}{2}c_1u_1^2 / x_1;$$

$$-\dot{x}_2 = w = u_2 - \frac{1}{2}c_2u^2 / x_2; \; c_1, c_2, \; x_1, x_2 > 0 \text{ and } u_1, u_2 \geq 0$$

$$x_1(0) = 100; \; x_2(0) = 1000; \; \lambda_1(T) = 0; \lambda_2(T) = 0$$

replacing w for $-\dot{x}_2$ in equation 4.2 and for ease of tractability, therefore:

$$\dot{x}_1 = -.1x_1(0) + u_1 - \frac{1}{2}c_1u_1^2 / x_1 + u_2 - \frac{1}{2}c_2u_2^2 / x_2 \tag{4-4}$$

These equations satisfy the qualitative properties mentioned at the end of chapter 3 and are the same as equations 3-17, 3-18 and 3-19. The Q and R matrix in the objective function are weighting functions as explained in chapter 3 but will be discussed in more detail later in this chapter under the base case scenario. The first term of equation 4-4 explains how proved developed reserves would change without any production maintenance or a drilling program in place. Remember the output equation $y = Cx$ from chapter 3 of the method section, by substituting Cx for y(t) in the state space format gives the term $-.1x_1$. Management had decided, based on the initial reservoir information and the recommendation of the reservoir engineers, to deplete the reservoir at the rate of 10% per year.

The second term of equation 4-4 is the production maintenance term; it is a concave function with respect to the input u_1. This term includes u_1, x_1 and c_1. The value of c_1 depends on the slope of the production maintenance function being used and is site specific. The third term of equation 4-4 is the drilling process term. This term includes u_2, x_2 and c_2. The value of c_2 depends on the slope of the drilling process being used and is also site specific. The higher the value, the flatter the concave function.

The model uses c_1 as being equal to 1.3 and c_2 as being equal to 1. The reason for these values is that there are more diminishing returns for the production maintenance function than the drilling program function after c_2 has been normalized to unity. These values also give a reasonable historical match for the Rodessa field production data.

Based on new information, management has learned that the oil field is not rate sensitive and has decided to increase the drilling program. In an attempt to act "optimally", management has decided to reassess the quantities of the factors of production going into the production and drilling process. The tools of modeling and simulation will assist the management in this objective. The model above is a nonlinear open-loop control system with a quadratic objective function. The vast majority of systems are nonlinear. When compared to linear systems, nonlinear systems are generally more difficult to control because models cannot capture all the nonlinear dynamics. Applying the maximum principle to the above system may yield an untractable system of equations. Assuming the problem has an open loop control solution, which is solved by the use of numerical techniques because of the boundary value conditions, can this nonlinear problem behave optimality?

To solve this problem several methods have been developed to fit the various characteristics of the system. One often used method is to approximate nonlinear systems by linear ones. A viable approach is to derive a control that cancels the nonlinear element of the system while minimizing the objective function using linear control techniques. This is called feedback linearization. Applying the maximum principle coupled with the Riccati equation solution, an optimal control can be found. Inserting this optimal solution in the nonlinear system forces the problem to behave linearly in the state variable.[4] Lets now apply this control solution to a base case and several alternative management scenarios.

The first case to be modeled is the base case or case1 followed by three other cases. The motivating reason for the base case is to model the output and reserve response of an optimal solution and compare that response to an actual oil field, the Rodessa field. Even though this study uses a closed loop control, the modeled solution gives the same result as an open loop control solution. However, as mentioned earlier in this study, the shutting in of high GOR wells in the Rodessa field was not modeled. Both of these formats do not show a disturbance to the base case because the closed loop solution is only an idealized computer simulation and does not use the actual data. If the actual data would have been used, the closed loop solution would have been able to " see" the shut-in wells from a change in the state variables and would have produced a corresponding change in factor inputs u[x(t)]. Only by remodeling at the specific times when the disturbance occurred would an open loop control give the same result. Having to remodel the problem gives an advantage to closed loop modeling and indicates to management the relevance of this modeling tool.

A change in terminal time to deplete the proved developed reserves is named case 2. In this case management desires to deplete the field in 40 time periods rather than 60 as

[4] The computer coding used to linearize the system and find the optimal feedback control is Matlab 5.1 and Simulink 2.2.

in the base case. Again since there is no disturbance to the state variables the open loop and closed loop solutions are the same. The co-state variable drops to zero in 40 time periods rather than in 60.

Another case to be modeled is a change in drilling technology (case 3). This case involves a change in the weighting matrix R on the input variables u. Again the open loop and closed loop solutions are the same, provided the open loop format has a weighting function on the input factors. The relevance to management is that a similar scenario brought about by changes in technology may be of interest in the near future and a tool is available to model such an event.

The last case to be modeled (case 4) is an upward change or reversion in proved developed reserves, which has relevance to management because it is a fairly typical scenario. Reservoir studies periodically update reserve figures. This case explicitly shows the advantage of closed loop over open loop control modeling of a disturbance to the system.

Case 1: Base Case

As mentioned above several scenarios will be modeled in conjunction with a base case. The solution to the base case, or case 1, is depicted in figures 1- 4. The

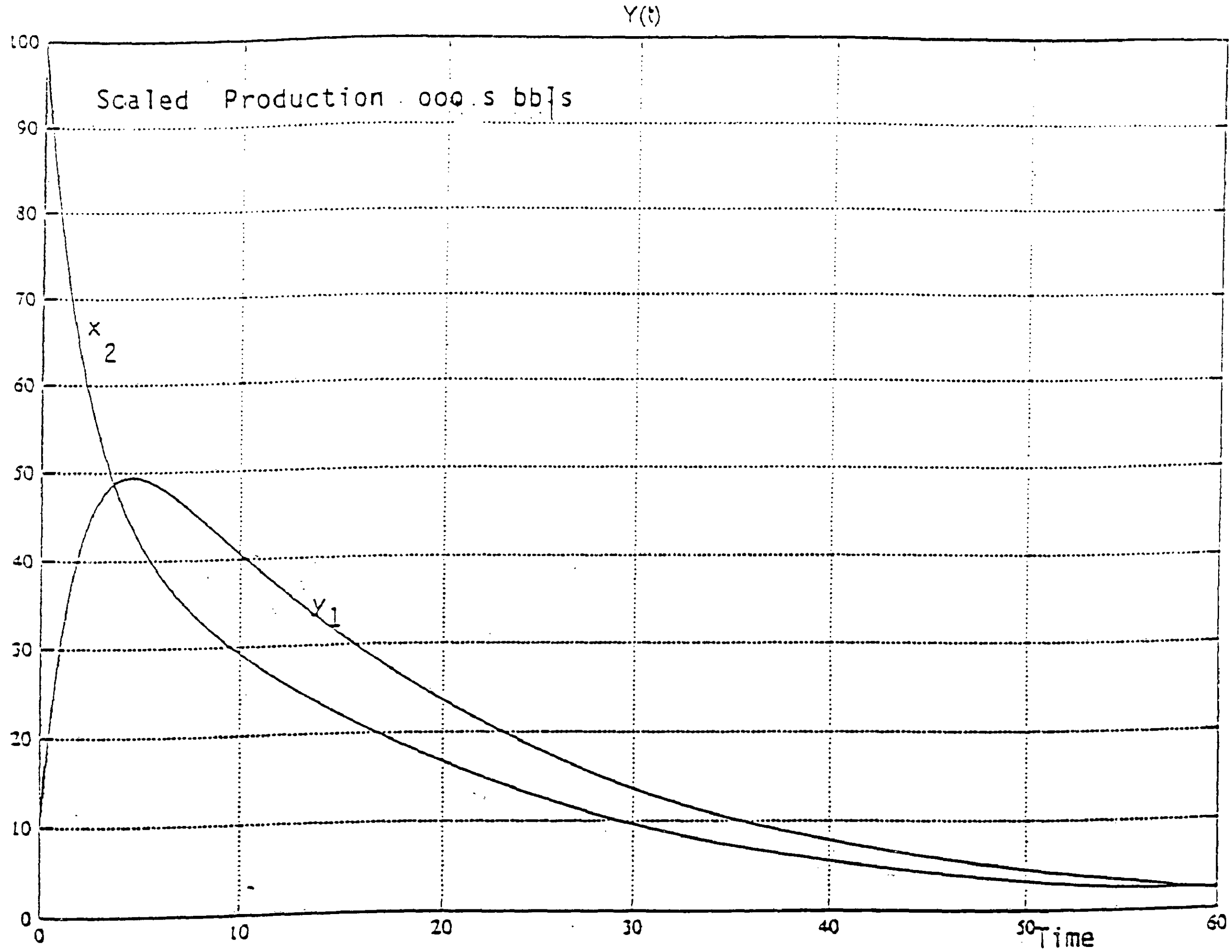


Figure 1.Optimal Production Paths: Base Case

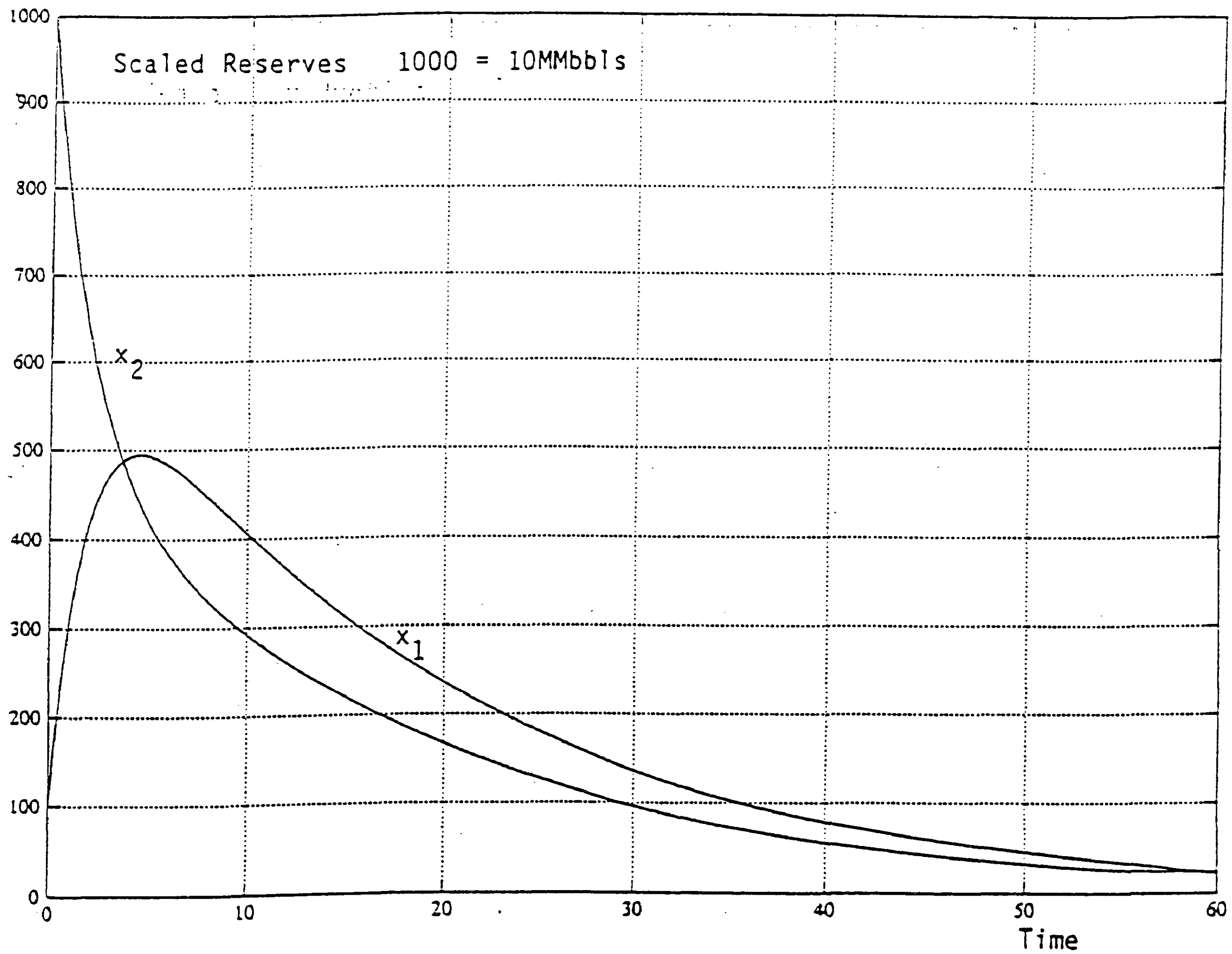


Figure 2.Optimal Reserve Paths: Base Case

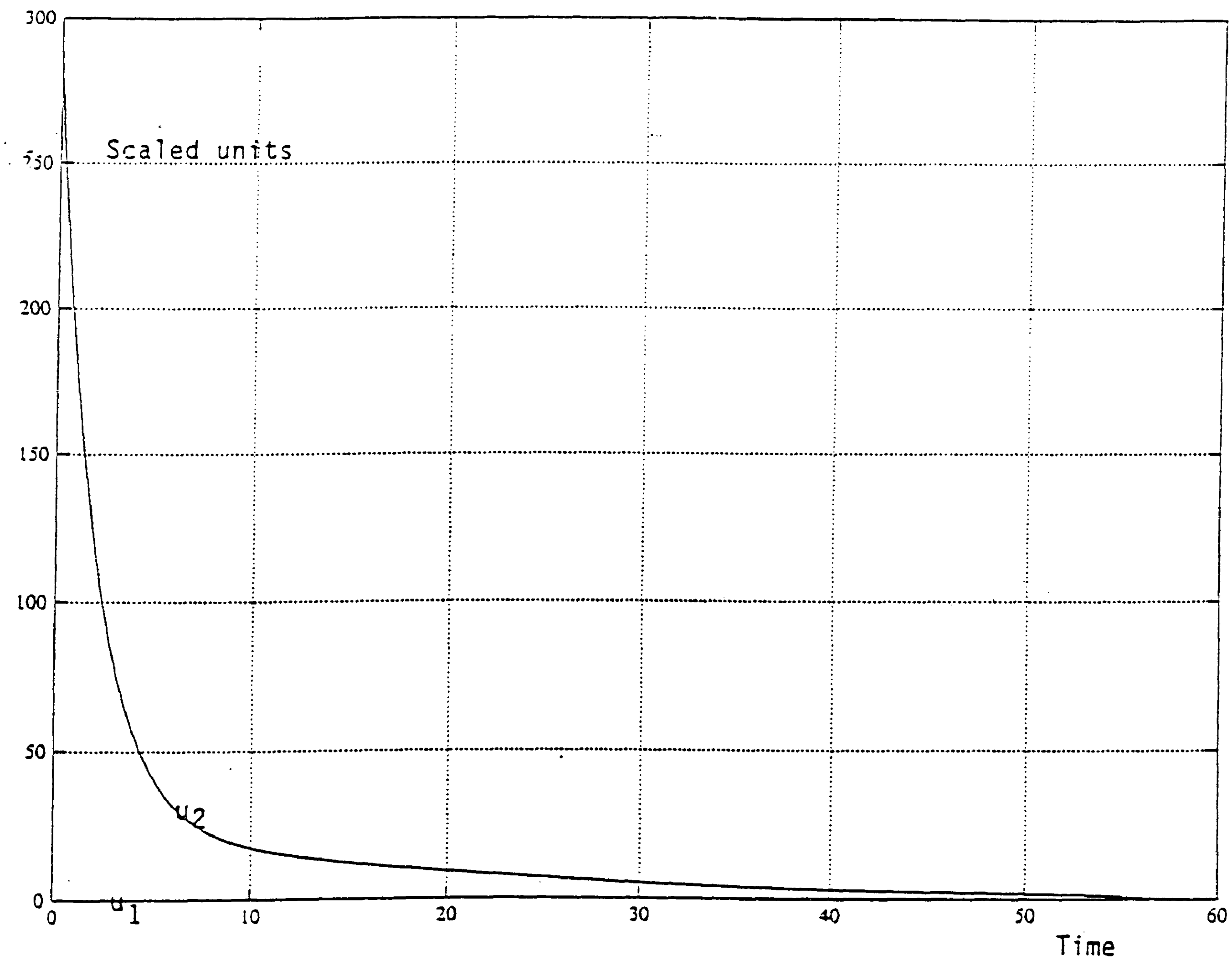


Figure 3.Optimal Input Paths: Base Case

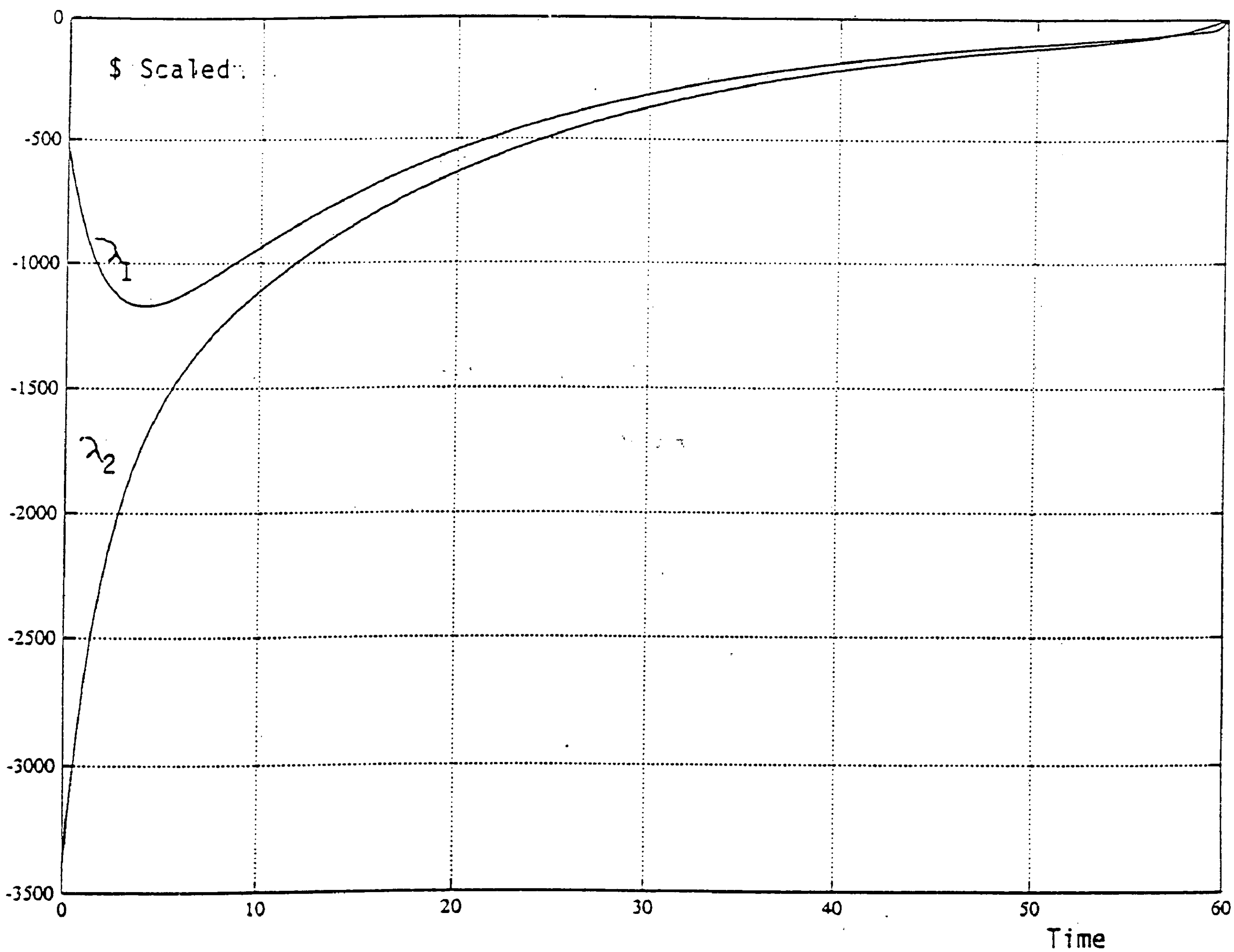


Figure 4. Optimal Co State Variable Paths: Base Case

initialization of the model comprises c_1 equal to 1.3, c_2 equal to 1, Q is $\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$,

R is $\begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}$, initial reserves are x_1 is 100 and x_2 is 1000. The Q matrix is chosen because management has decided it is more important that the proved developed reserves be depleted at T and therefore x_1 is more heavily weighted. There is more of a penalty applied to that portion of the state vector inside the objective function. Another reason for the Q matrix values is that this Q matrix provides dynamic stability as was discussed in the model section of chapter 3. The R matrix is chosen because the drilling cost is much higher than the production cost. The u_2 cost has ten times the affect on the objective function J as does u_1. The chosen parameters also give a reasonable historical match of the simulated production profile verses the actual production profile. In my model I increase production by increasing the drilling intensity (u_2) on the proved undeveloped reserves transforming them into proved developed reserves. The production profile is tilted to the left on the simulated idealized graph versus the actual data. The values for x_1 and x_2 are scaled in the computer simulation. The actual values of x_1, proved developed reserves and x_2, proved undeveloped reserves is 1MMbbl and 10MMbbl, respectfully. Based on an ultimate recovery of 20% of original oil in place, this gives about 50MMbbl of original oil in place.

Viewing the base case figures 1-4, figure 1 is the output or oil production (y_1) versus time, figure 2 is the state variables proved developed reserves (x_1) and proved undeveloped reserves (x_2) versus time, figure 3 are the two inputs (u_1) production index quantities and (u_2) drilling index quantities and figure 4 are the co-state variables λ_1 and λ_2. The path of - $\dot{\lambda}$, equation 3-3, approaches zero as the transversility condition states. But why is this $\dot{\lambda}$ negative? To a mathematician it is the negative rate at which the state variable is changing. To a mineral economist, it is the rate at which the shadow price of a unit of proved reserves depletes at time t. λ_1 is the shadow price of proved developed reserves, λ_2 is the value or shadow price of proved undeveloped reserves. The maximum principle requires that the shadow price of reserves depreciate at the rate at which reserves contribute to the current and future profits of the firm. The petroleum industry usually ignores the negative sign in front of λ and speaks of an oil field having positive reserve values. However, what is important is that the reserve values approach zero at terminal time, i.e. as the field is produced and depleted.

When the inputs are limited to being positive only (figure 3), u_2 indicates positive values. However, the same plot indicates that u_1 is zero or some minimal value. The interpretation of this situation is that management expend inputs into the drilling process and to minimize the expenditures on the production process.

The remaining part of this chapter will provide various management scenarios and will compare the results between open loop and closed loop modeling. It will conclude with a summary of the advantages of closed loop optimal control modeling.

Case 2: Terminal Time Change

The second case of this optimal simulation is a change in the terminal time only from the base case of 60 to a time period of 40. The results are shown in figures 5-8. This closed loop solution would be the same as an open loop solution. There have been no disturbances to the system and no change of the Q and R weighting matrixes. The advantage of the closed loop approach is if there are any stability problems with the system dynamics they can be remedied by closing the feedback loop (Luenberger 1979). The shapes of the curves in figures 5-8 are similar to the curve shapes of the base case except the co-state variables, λ 's, approach zero sooner, a transversality condition which must be satisfied, and the values in the figures are spread over 40 time periods, not 60. This condition indicates the shadow price of reserves is driven to zero at terminal time. The reason for this is that the value in reserves to the firm emanates from its potential for production to be sold. The understanding is that production from 0 to T can be sold and any reserves left would have no economic value to the firm. The firm should therefore try to produce most of its reserves by terminal time. This terminal time is constrained by the rock and fluid properties of the reservoir. Depletion rates for the reserves typically allowed are between 5 and 20% per annum.

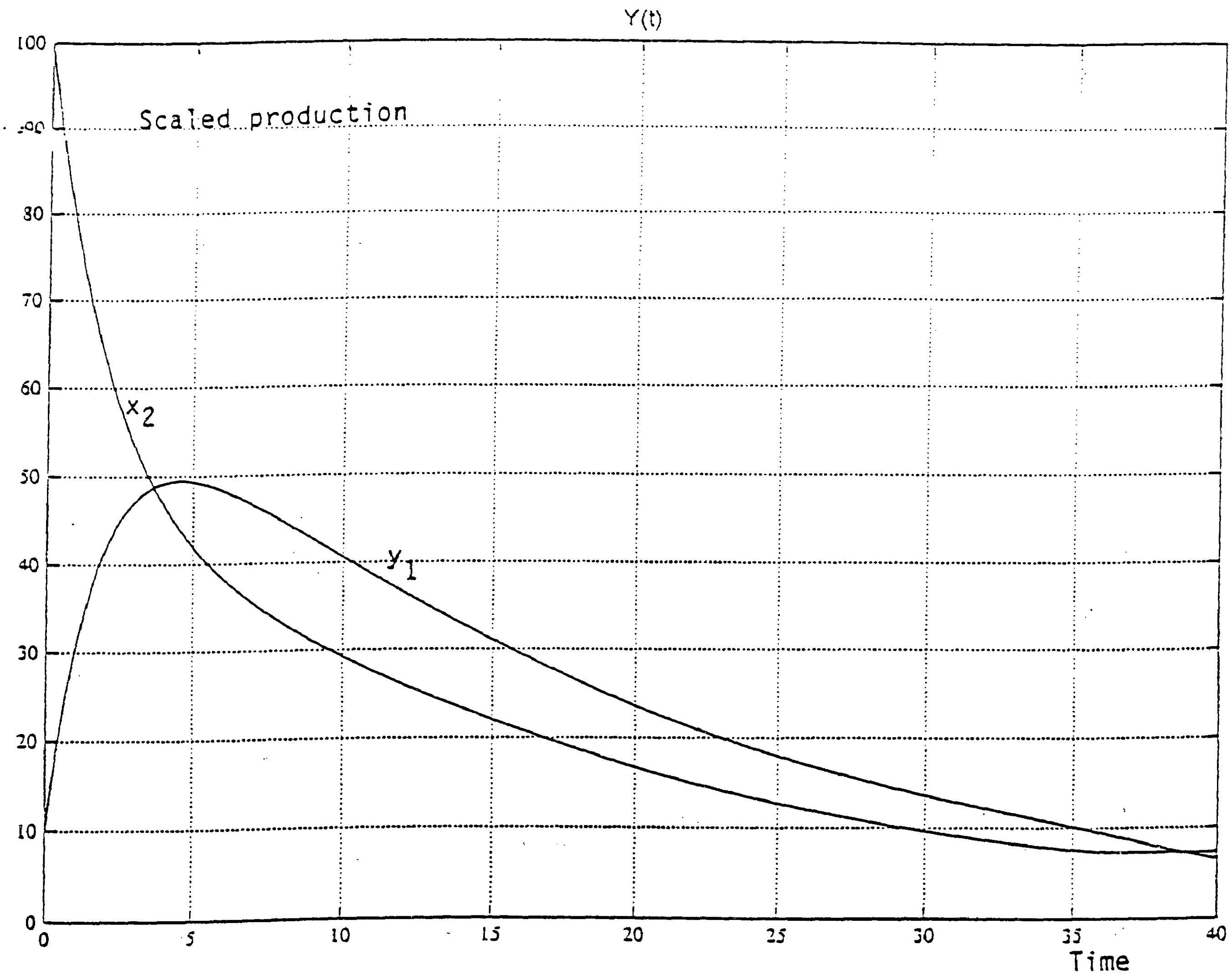


Figure 5. Optimal Production Paths: Change in Terminal Time

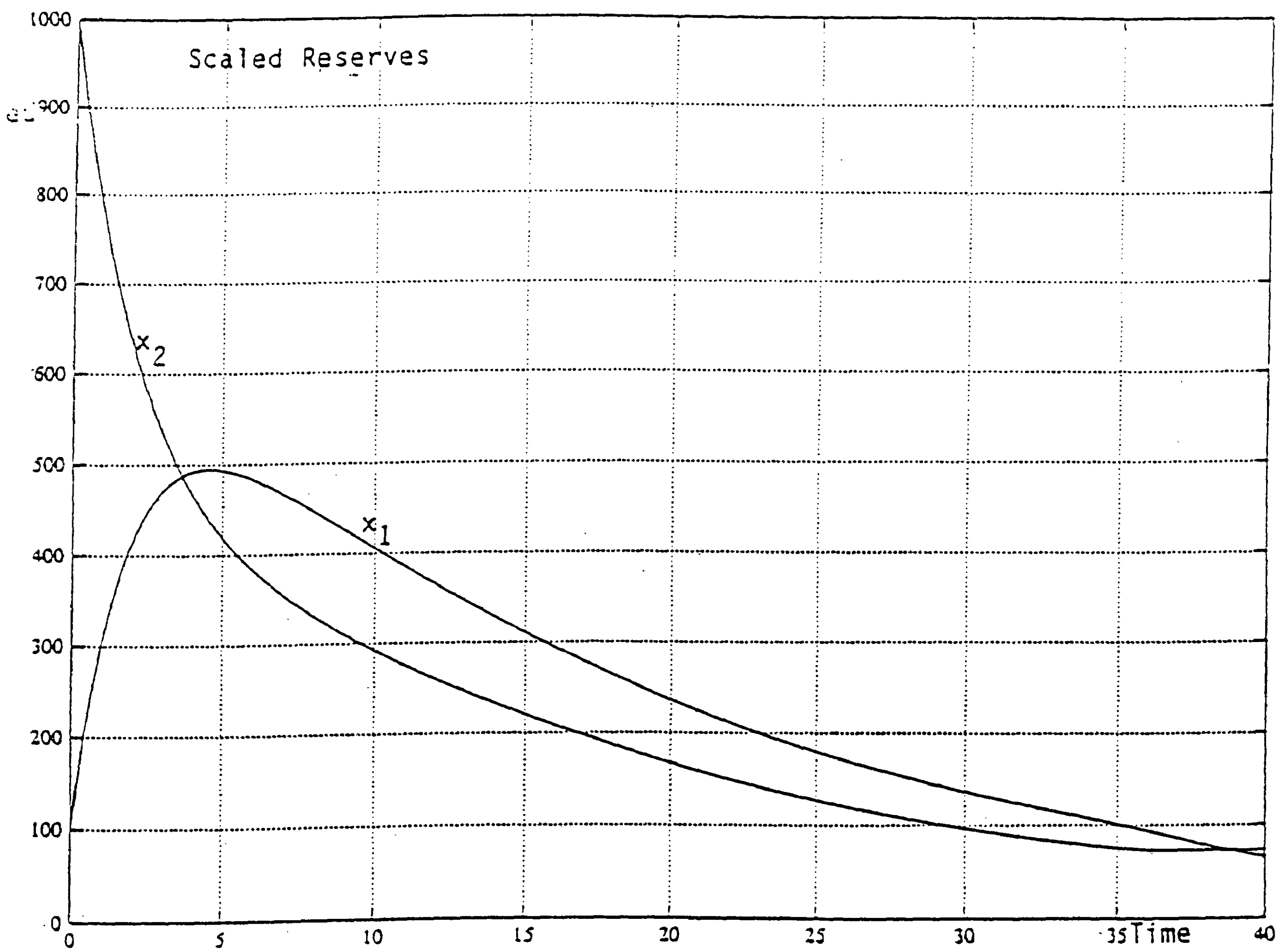


Figure 6. Optimal Reserve Paths: Change in Terminal Time

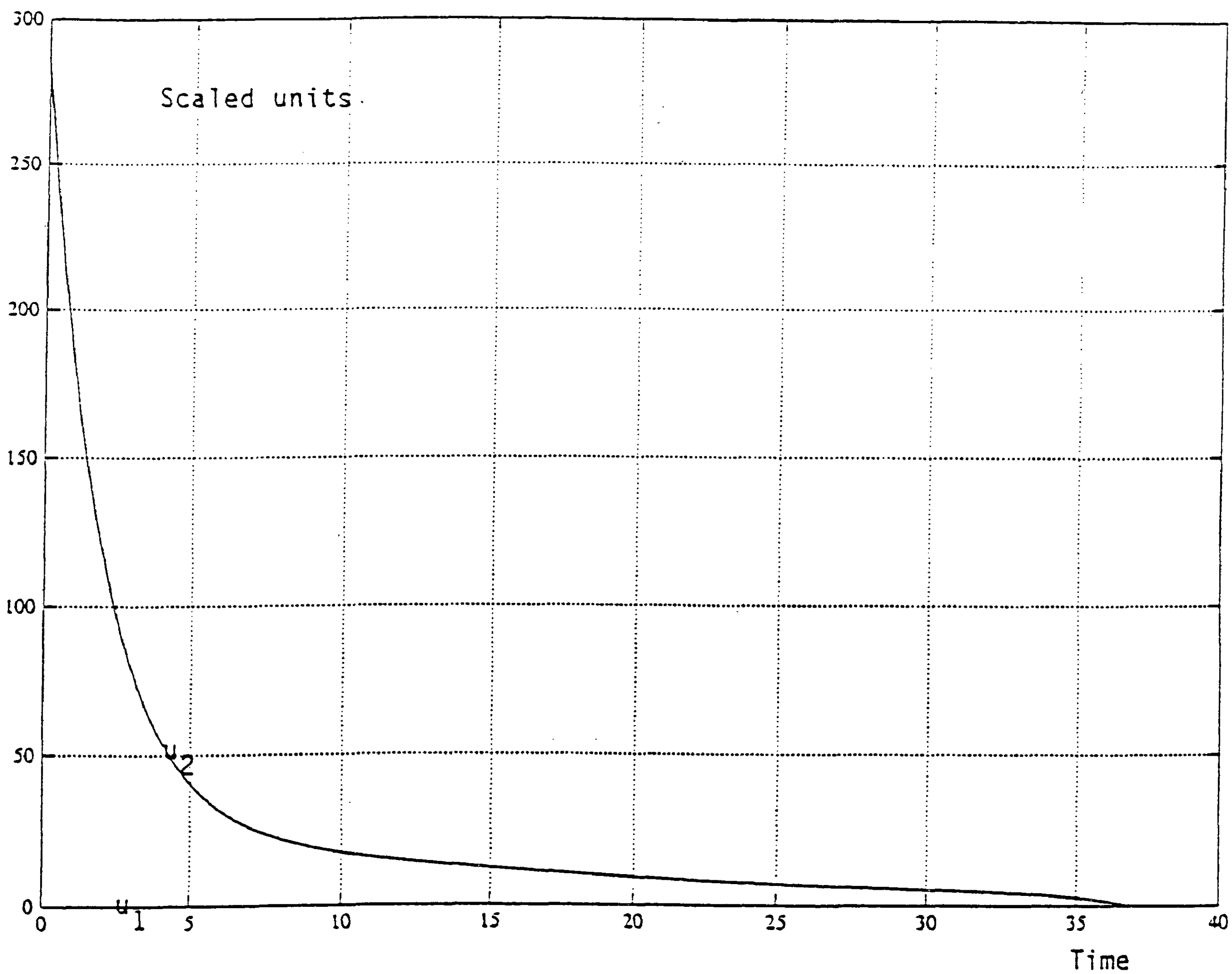


Figure 7. Optimal Input Variable Paths: Change in Terminal Time

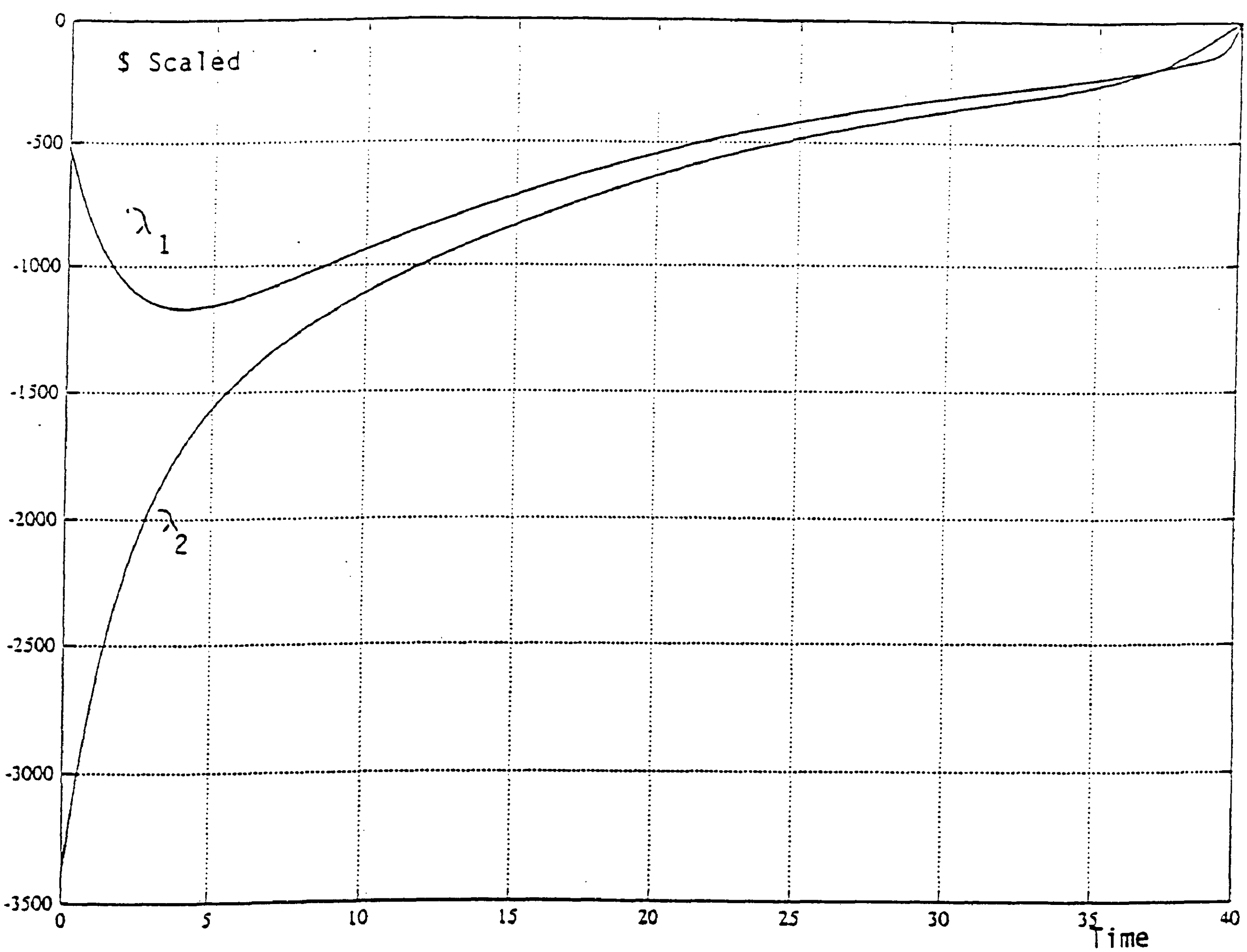


Figure 8. Optimal Co State Variable Paths: Change in Terminal Time

Case 3: Drilling Technology Change

The third management case is to assume a change in the drilling technology, an increase in drilling productivity. The results are presented in figures 9-12. This scenario indicates that management should increase the drilling intensity beyond the base case figure 1. The technology change causes production to be accelerated, i.e. shifted to the left on figure 9. The reserve values figure10 also seem to have a steeper decline than the base case reserve values in figure 2. What is driving these production/reserve changes is the increase in drilling inputs from about 280 units of u_2 in figure 3 to almost 380 units of u_2 in figure 11. The change in drilling technology is modeled by lowering the weighting factor R from the base case to a matrix $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ in the objective function. As I explained in chapter 3 how to model a technology change, the lowering of the weighting on u_2 has the same effect as increasing u_2 but without changing the objective function scalar value J. This closed loop solution would be the same as an open loop solution, but only if in the objective function of the open loop problem, there is a weighting factor variable that can be changed on the cost variables. Should management be motivated to simulate a fifth management case such as political risk because of new information, or because they want to move oil production forward in time, an increase in the weighing factor Q on the state variables would accomplish that objective. The response of the political risk case is similar

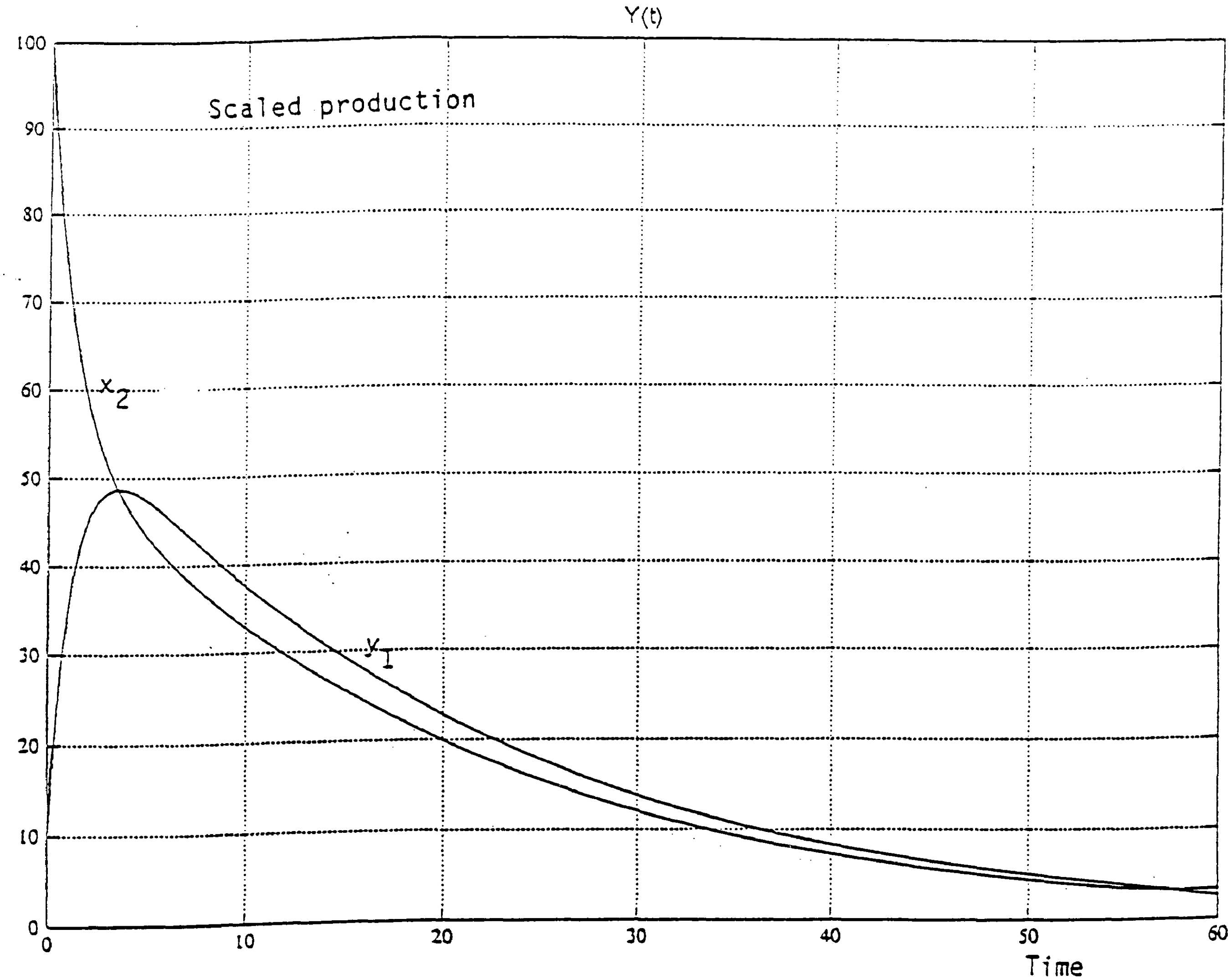


Figure 9. Optimal Production Path: Drilling Technology Case

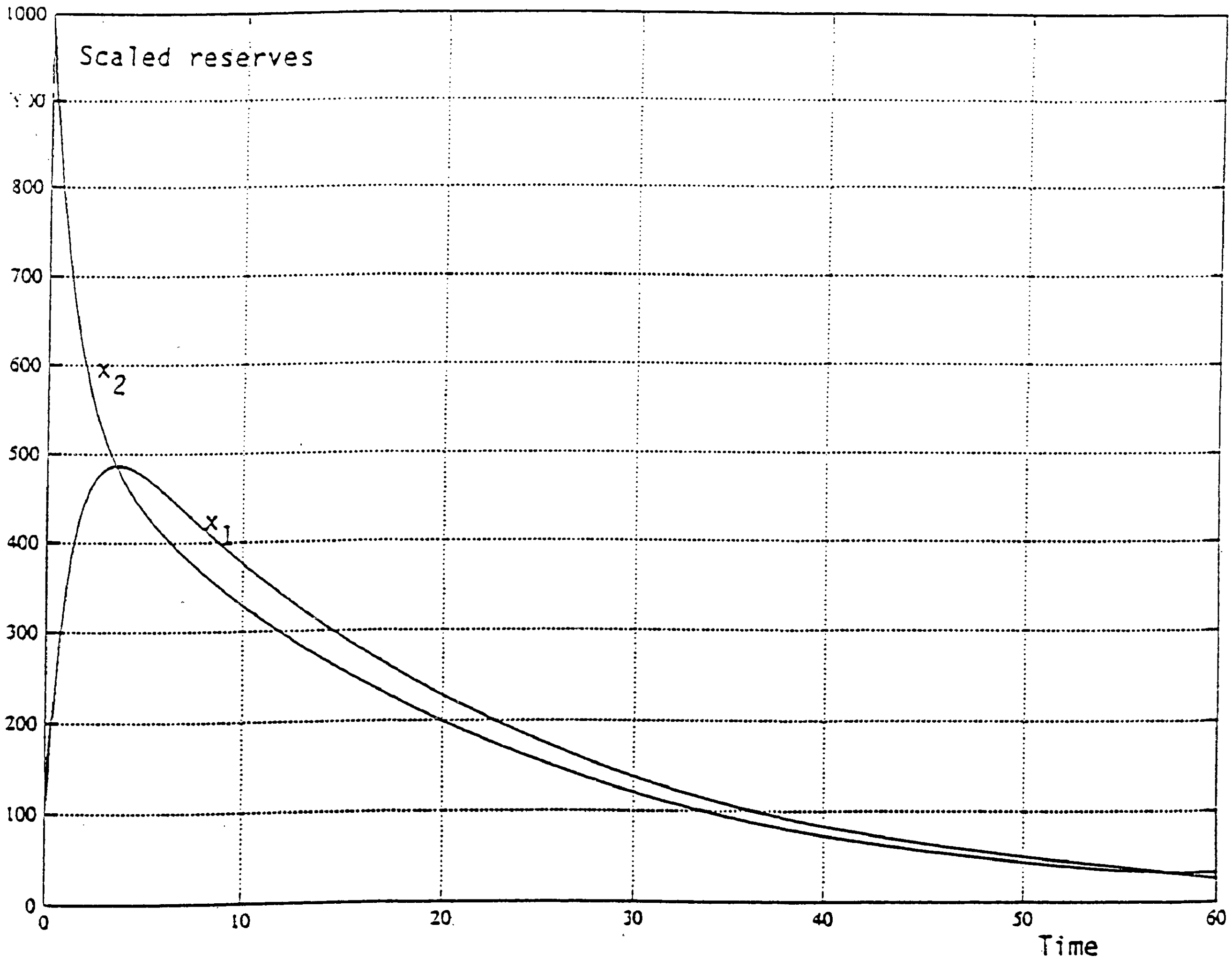


Figure 10. Optimal Reserve Paths: Drilling Technology Case

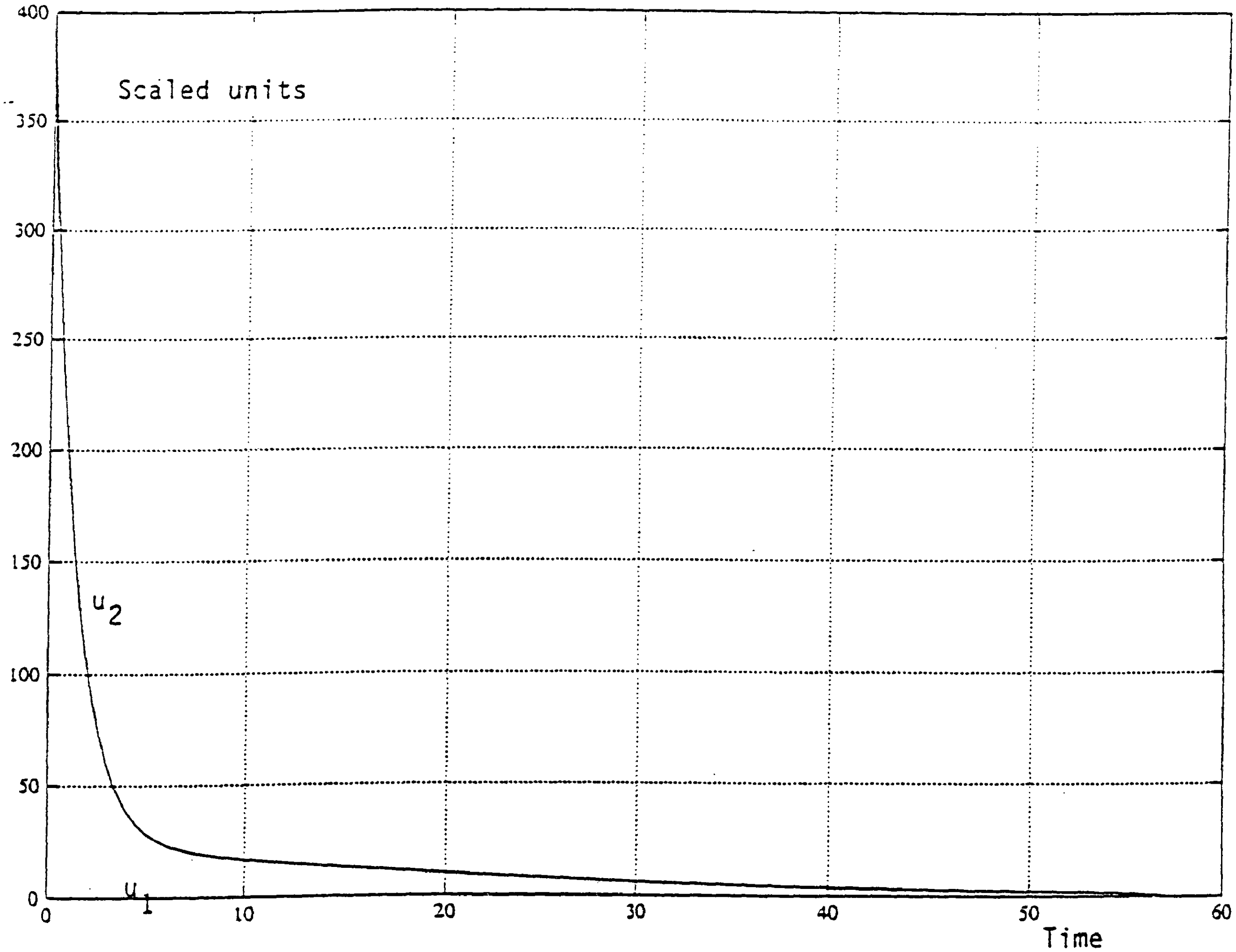


Figure 11. Optimal Input Paths: Drilling Technology Case

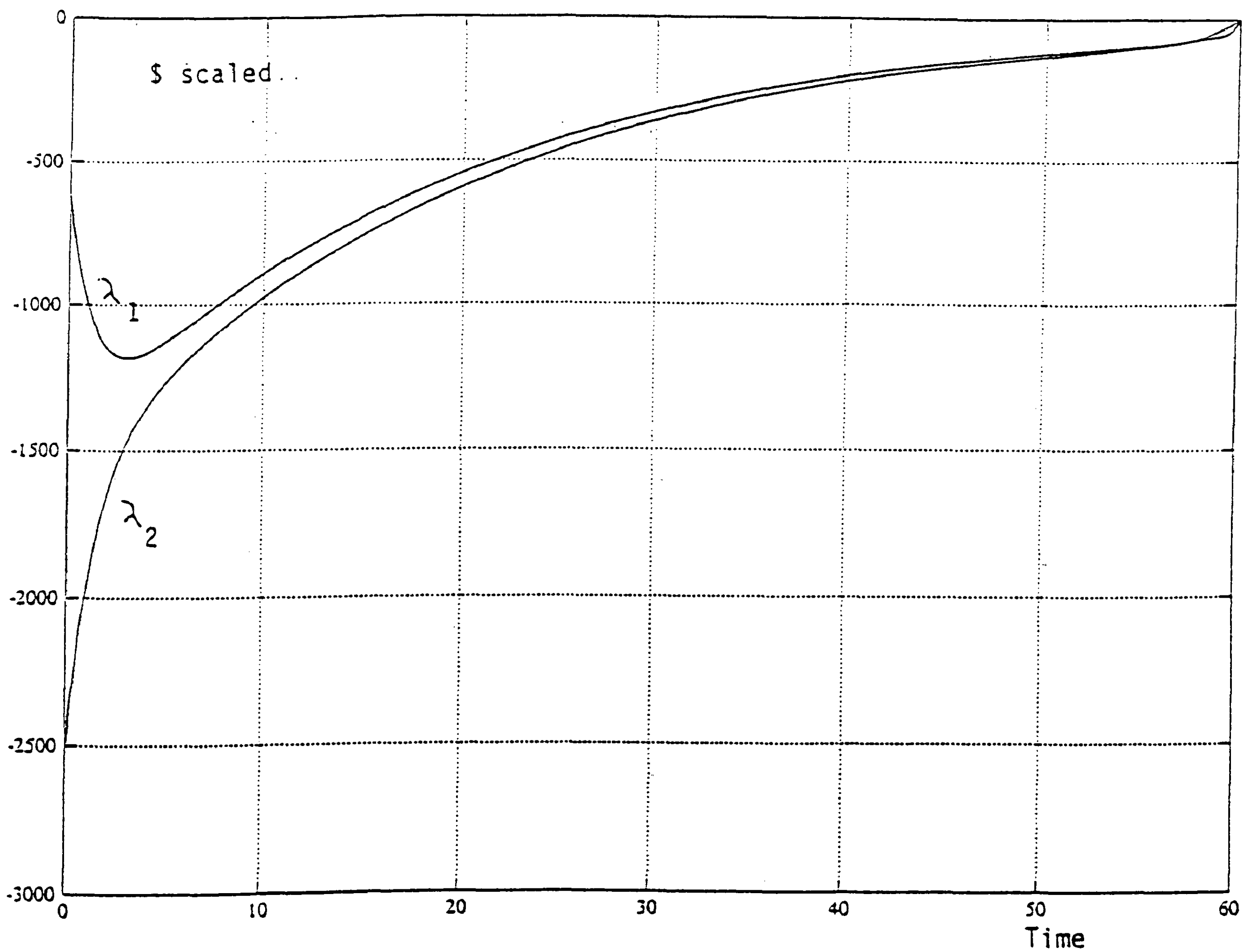


Figure 12. Optimal Co State Variable Paths: Drilling Technology Case

to case 2; however, there is a difference in the response of the output curve. In the case of political risk the production profile is tilted more to the present and has a longer and thinner tail toward terminal time. In case 2, i.e., a change in terminal time, the production profile has a peak production at a later time and with a fatter but shorter tail toward terminal time. The increase in Q would shift the reserve depletion and production forward and indicate to management of a need to increase the drilling rate intensity.

Case 4: Proved Developed Reserve Change

The fourth management case is presented in figures 13-16. This is the case where a disturbance enters the system dynamics, a reversion or a change in proved developed reserves. This disturbance is modeled by a step function. In the actual production data of he Rodessa field there was a downward shock when management decided to shut in several wells because of a high gas to oil ratio problem. In that case production went down for several periods, the strategy did not work, and the wells were reopened and production increased. In this problem, management receives current information from the reservoir engineering department which has revised the proved developed reserves upward. The oil property has more developed reserves than initially thought at t_0. Management as the controller, wants to know how to respond. Is there a new optimal

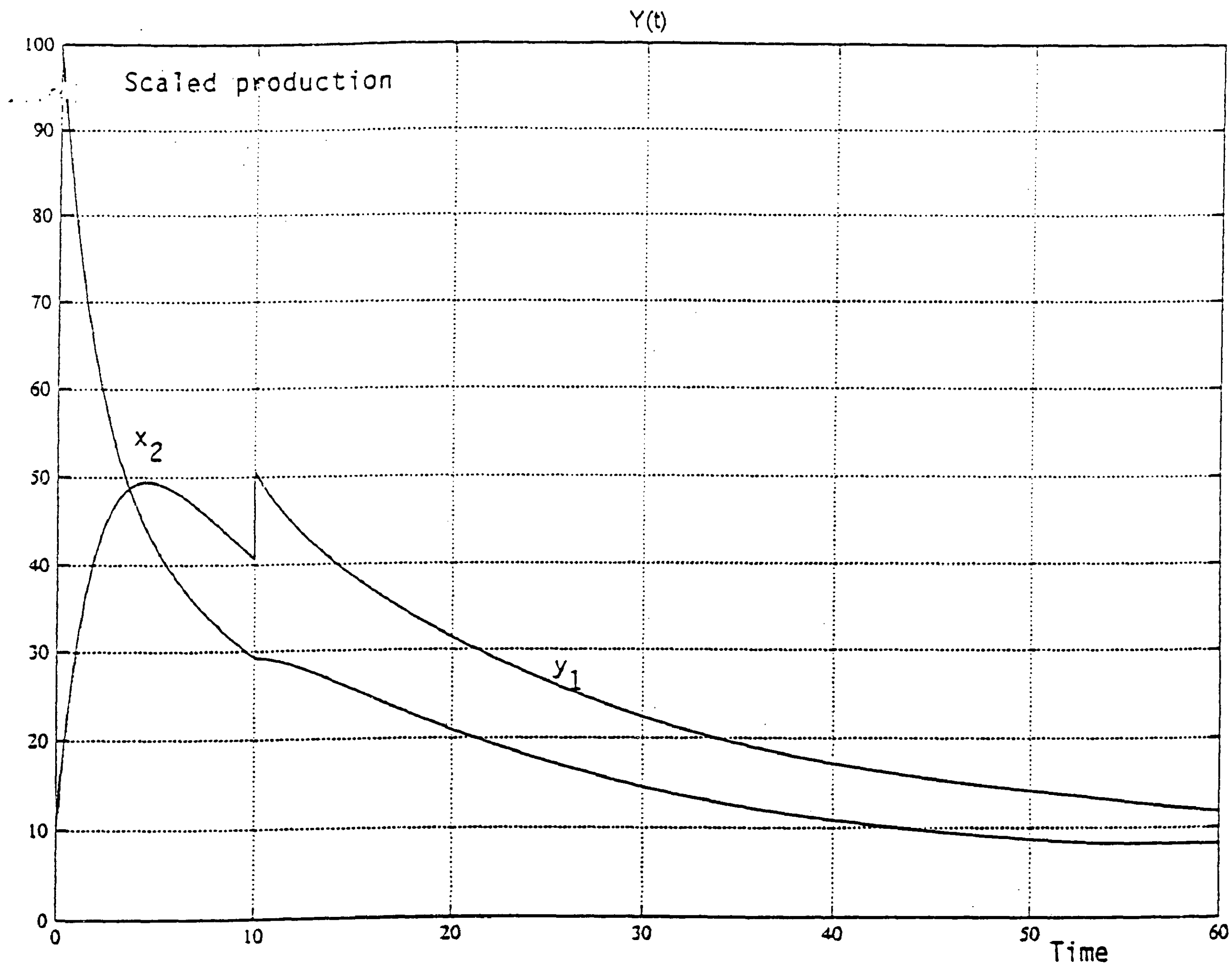


Figure 13. Optimal Production Path: Change in Reserves Case

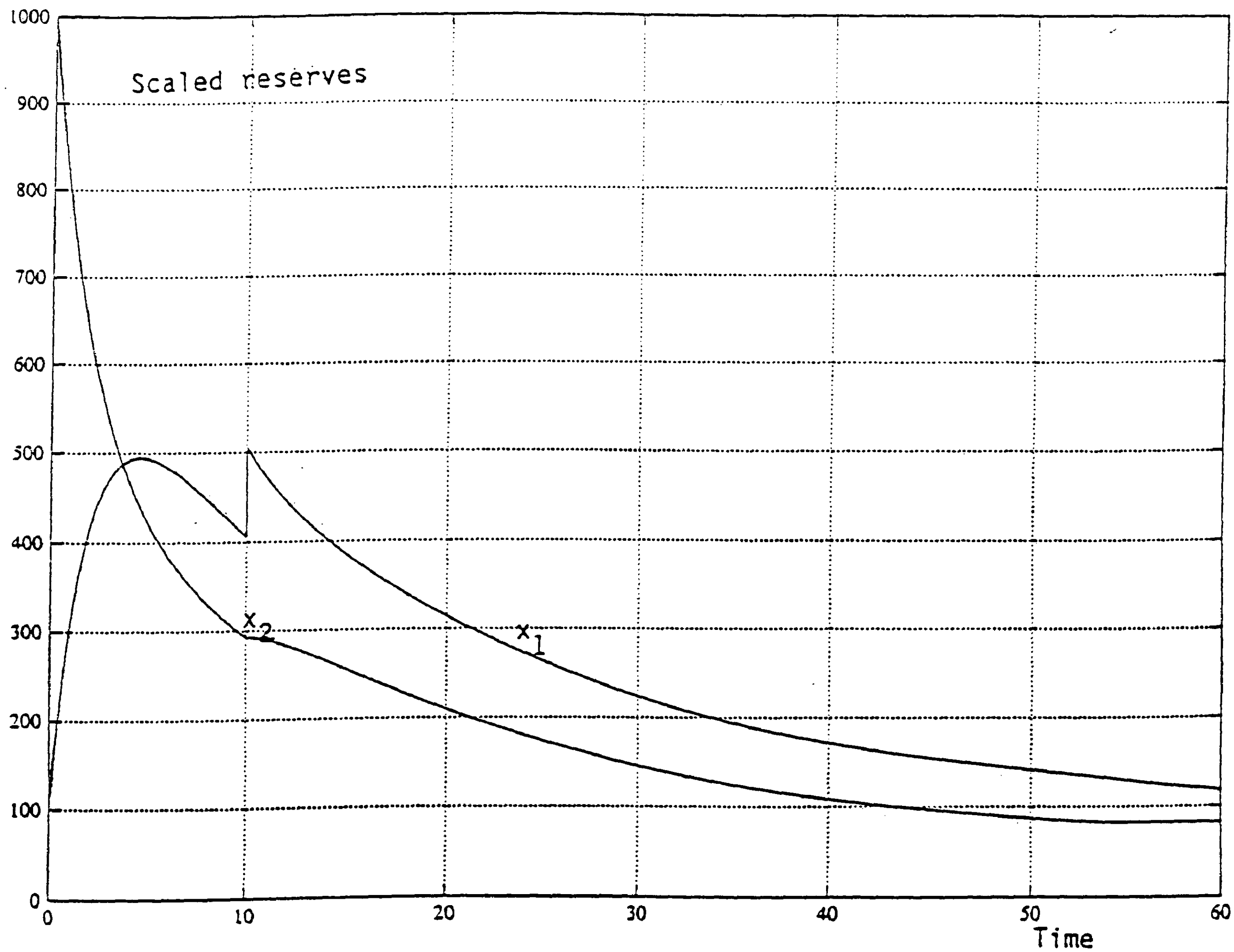


Figure 14. Optimal Reserve Paths: Change in Reserves Case

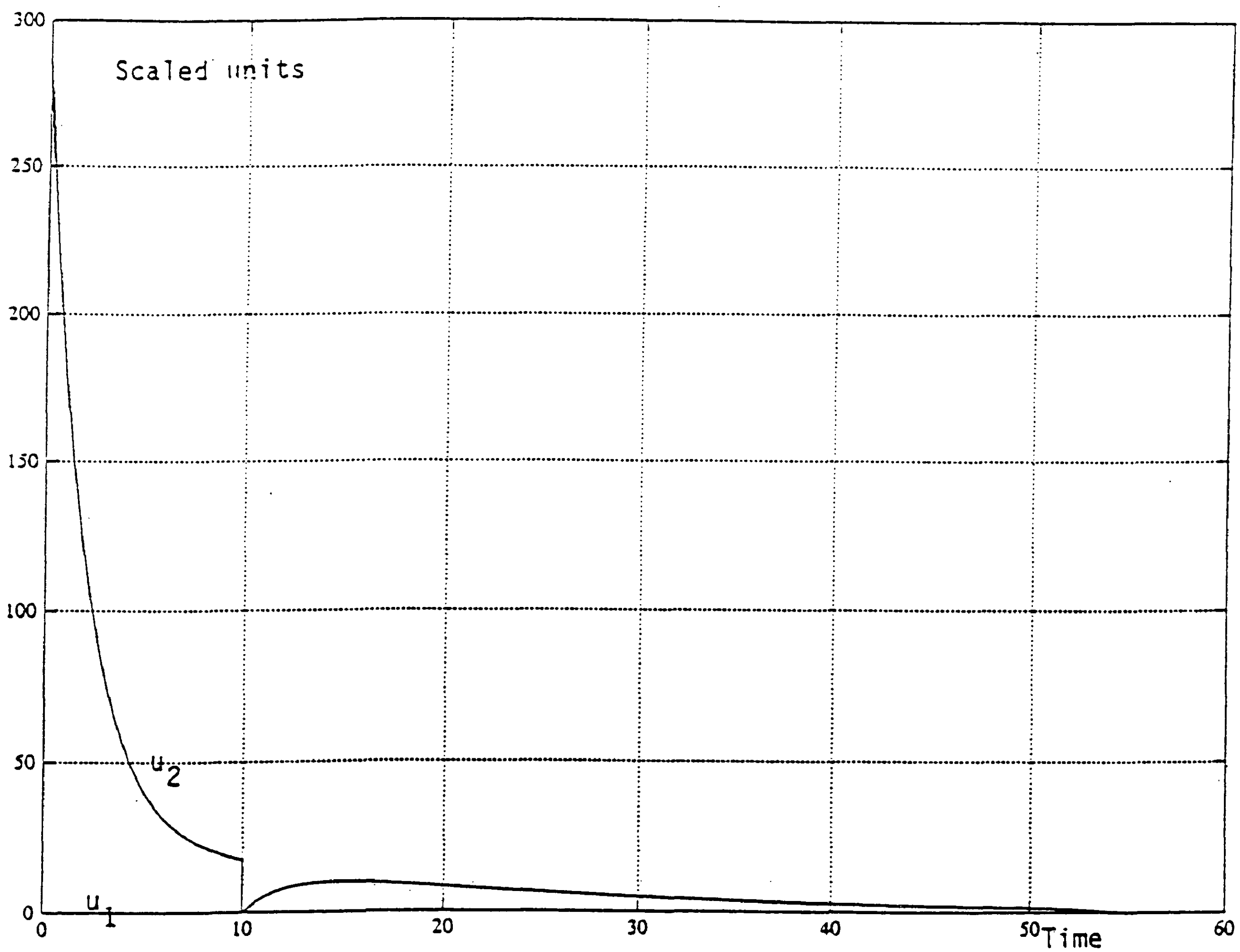


Figure 15. Optimal Input Paths: Change in Reserves Case

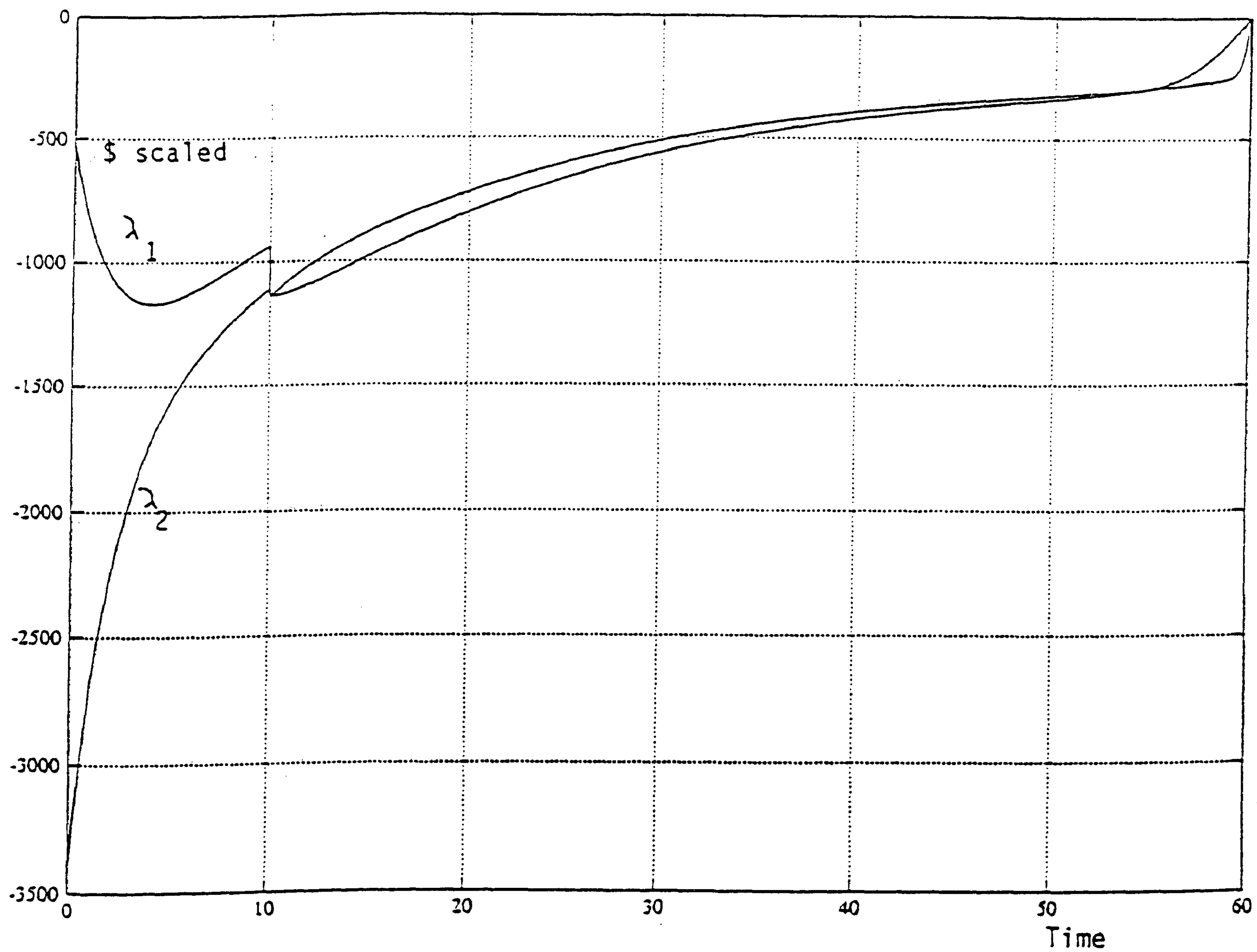


Figure 16. Optimal Co State Variable Paths: Change in Reserves Case

path given this information? Proved developed reserves double at time period 10, management decides to increase depletion x_1 and production y_1. The drilling input factor u_2 drops at period 10 indicating to management a reduction in drilling activity is needed but then slowly increases again. The co-state variables also change and converge to zero at terminal time. Management did not know when this shock would occur, but once the shock had occurred, management could respond quickly with the new controls provided by the model solution and implement them on the system. Because the control is expressed as a function of the now changing states at any time t, the closed loop solution expresses what those new controls should be. If this had been modeled as an open loop problem, management would need to reformulate the problem, calculate new control values when the wells were shut in and production turns down and then again when wells were put back on stream showing an increase in production. This is not needed as indicated on figures 13-16 if modeled as a closed loop control problem.

Summary

There are numerous advantages of closed loop optimal control. In a closed loop control the input is determined on a continuing basis by the behavior of the system itself, as expressed by the behavior of the outputs. This kind of control is feedback control since the outputs are fed back to the input. There are many reasons why feedback or closed loop control is often preferable to open loop control. Feedback control can convert an originally marginally stable system to one that is asymptotically stable. Another reason is that a feedback rule is often simple, while a comparable open loop scheme might require a large amount of calculation, especially if there are more than two state variables (Luenberger 1979). Feedback is often superior to open loop control from a performance standpoint. Feedback can automatically adjust to unforeseen system changes or to unanticipated disturbances such as in scenario 3, a reserve change. The feedback format gives management the new control rules without having to reformulate the problem. In general, a feedback control rule does the required computation as it goes along, and immediately expresses those results for management implementation.

The above discussion demonstrated some of the general advantages of modeling systems in closed loop verses open loop control. Specifically, in the petroleum management base case simulation, the computer model used closed loop optimal control for the problem, but the actual field data from the Rodessa field was not used as an input. If the actual data had been used, the downturn and the upturn in appendix A1 would have been indicated in figure 1. The intent of this study was to demonstrate how to model a petroleum non renewable natural resource problem using closed loop optimal control theory based on the maximum principle. The computerized general shape and the optimal production path derived clearly indicated a similar path to the actual field data of the Rodessa oil field except where production tilting was present, as indicated in figure 1.

Referring to the drilling technology management study, the contribution made by the simulation was in how to model a technology change by using the R matrix weighting factor on the inputs u. By lowering the weighting on the drilling factor u_2, this increased the production tilting, (figure 9), more than in the base case (figure 1). What motivated that response was an increase in the drilling inputs u_2. That makes economic sense because if a lower drilling cost is brought about by a technology change, for example, better bit technology, more wells can be drilled and at a lower total cost than with existing technology. This provides for production tilting toward the present and a steeper fall in reserve decline.

Finally the management case of simulating a change or reversion in proved developed reserves. This result, figures 13-16 indicated a contribution to natural resource modeling. By using closed loop optimal control modeling, $u[x(t)]$, the system "sees" the changes in reserves and immediately solves for a new optimal control value based on the new information. This would not be so using open loop modeling, where based on this new change in state, a reformulation of the problem would be required and new control values calculated. With many reversions or disturbances to the petroleum system this could become quite tedious.

Chapter 5

CONCLUSIONS AND FURTHER RESEARCH

The main conclusions of this study:

1) Production tilting can be modeled using an optimal linear feedback control. The tilting is a conclusion that is consistent with the natural resource literature.

2) The feedback control provides management with information for better reservoir management.

3) This quantitative tool provides management the option to simulate various scenarios and to observe the results.

4) By applying closed loop modeling u[x(t)], the system is able to respond to reserve changes and immediately solves for a new optimal control value. This would not be so using open loop modeling u(t). A reformulation of the problem would be required and new optimal control values calculated. With many reversions or disturbances to the system this could be quite tedious.

This tool should assist petroleum managers in their optimal depletion decisions by making choices in selecting alternative combinations of inputs for the processes involved. It also aided in the optimal choices for the drilling process of undeveloped reserves. Also, three different scenarios with feedback solutions were modeled. The typical scenarios were something a petroleum management team might experience. The feedback solutions were compared with comments to an open loop solutions model. The three management scenarios modeled were, a change in terminal time, a change in drilling technology, and a reversion in the state variable (proved developed reserves); however, other scenarios could have been modeled. The base case indicated to management to tilt oil production to the present (figure 1) by increasing drilling intensity. The first management scenario, a shorter terminal time, indicated similar results to the base case but with the co-state variables converging to zero sooner. The results were similar to an open loop optimal control solution. The second management scenario, an increase in drilling technology, indicated to the management to increase production sooner by increasing the drilling intensity even more than the base case. This closed loop solution would have been similar to an open loop optimal control solution if the open loop objective function would have had a weighting matrix on the control variables. The increase in the depletion and the production profile was modeled by reducing the weighting matrix R in the quadratic objective function. The final scenario, an upward reversion in the proved developed reserves at time period 10 indicated to management to change the production and drilling profiles. The response of management was to decrease the drilling activity at the time of the reserve increase and then to gradually increase the drilling activity. The larger reserves meant management could increase the production profile and the new optimal depletion path rose above the original depletion path. This result from the closed loop optimal control solution is different from the open loop optimal control solution.

This result proved to contribute to the natural resource literature for modeling natural problems because the optimal control response is a function of the state variables and any disturbances to the system that is modeled within the system dynamics will be fed back. The correcting control policy will automatically be conveyed to management for implementation. This is much different when modeling with open loop optimal control. If all variables and constants are well defined and there are no disturbances that will enter the system as specified by the system dynamics, then open loop optimal control modeling is fine. However, should reversions or other disturbances be expected to enter the system dynamics before terminal time, closed loop optimal control modeling is preferred.

The modeling technique used in this study tracked the production data of an actual oilfield located in Louisiana named the Rodessa field. The reservoir properties and energy mechanism of that field was introduced at the beginning of chapter 4.

Future Research

The model could be extended in several directions, for instance, by adding another term(s) to the system production dynamics. For example, to model waterflooding, (secondary recovery), or enhanced oil recovery, another term or terms could be introduced. This would be developed in a similar fashion to the drilling dynamics term. First, think of the qualitative properties the expression should have in terms of the state and the control variables and then incorporate these properties into a quantitative expression. Figure 17 illustrates some typical results.

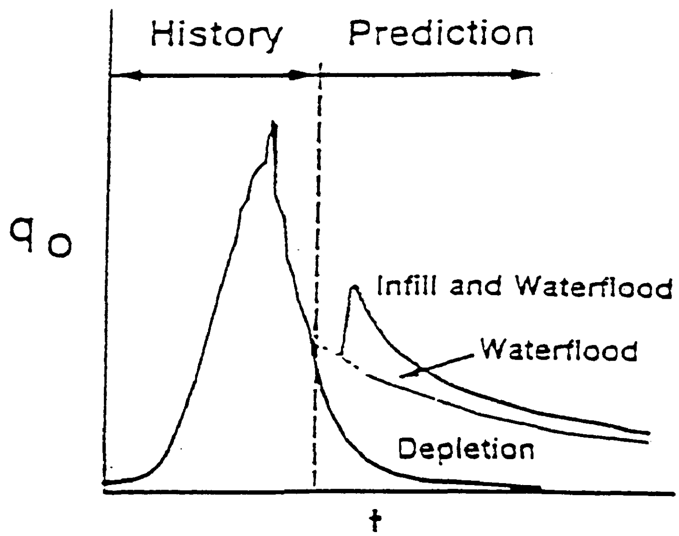


Figure 17 Secondary Recovery

Another area for extending this model would be to have a control variable for each factor of production, (i.e., capital, labor, energy, materials) for each process in the system. In this problem the system processes are production and drilling. Depending upon which factor had the highest productivity, it could be substituted (if possible) for a less productive factor of production. This would enhance the decision-maker's optimal choice for selecting the inputs for that process.

A third method of extending the model would be to incorporate an observer. Many sophisticated procedures for control design are based on the assumption that the full state vector is available for measurement. In large social or economic systems, measurements may require extensive surveys, therefore the entire state vector is not known. An approach to be applied is to construct an approximate to the full state vector on the basis of available measurements. It can be shown in the control literature that a dynamic device known as an observer can be constructed. Its input values are the values of measured outputs from the original system, and its state vector generates missing information about the state of the original system. The observer can be regarded as a dynamic device that, when connected to the system outputs, generates the entire state vector (Luenberger 1979).

The model could be improved by assuming uncertainty of information for the reserve values. This could be modeled by using stochastic variables for the reserves. The management team could gather production/reserve data, find a mean value and standard deviation from that mean and use those parameters for the Kalman filtering technique. The modeling formulation has been developed through the years and the mathematical foundation is based on the theorem of separation. The theorem of separation says: perform first the optimal estimation for x(t), i.e. construct the Kalman filter, and then do the optimal regulation. That is, find the optimal regulator as discussed in this paper with x(t) replaced by $\hat{x}(t)$. The optimal control for this problem would be: $u_{opt}(t) = -K_{opt}\hat{x}$. The K is the same as in the deterministic problem and the $\hat{x}(t)$ comes from the Kalman filter solution. Difference equations could be used to approximate the dynamics when data are available in discrete time such as on a monthly, quarterly, or yearly basis.

Another extension is the insertion of a time delay between when a decision is made and when development and actual production begins. It is possible to simulate this scenario using a time delay term in the modeling process. The dynamics of linear systems containing time-delays is described by delay-differential equations. For example, the state-space form of a time-delay linear system is given by:

$$\dot{x}(t) = Ax(t) + A_D x(t-T) + Bu(t)$$

where T represents the time-delay. This form can be generalized to include state variables delayed by 2T,3T…, time delay periods.

There are several areas for further study using this dynamic state space modeling technique. Modeling a natural resource problem described by non-linear dynamics, similar to this research problem, but having a typical economic objective function such as to maximize profit rather than to minimize cost could be simulated. The key to this optimal control problem is to disconnect the co-state variable λ from being expressed as a function of the state variable and the control variable. Once that is done an optimal feedback control equation can be derived.

Another area for further study would be the modeling of a dynamic natural resource problem by inserting a management utility function in the objective function. Utility theory assumes that every decision maker uses a utility function that translates each of the possible payoffs in a decision problem into a non-monetary measure known as a utility. The utility of a payoff represents total worth, value, or desirability of the outcome of a decision alternative to the decision maker. Different decision makers have different attitudes toward risk and return. Those who are "risk neutral" tend to make decisions using the maximum expected value decision rule. However, some decision makers are risk avoiders or risk averse, and others are risk seekers. The utility functions typically associated with these three types of decision makers can be expressed mathematically.

REFERENCES CITED

Aoki, M. 1976. Optimal Control and System Theory in Dynamic Economic Analysis. New York: North Holland.

Burness, H. S. and W. E. Martin. 1988. Management of a Tributary Aquifer. Water Resources Research 24: 1339-44

Chiang, A. C. 1992. Elements of Dynamic Optimization. New York: McGraw-Hill.

Chow, G. C. 1975. Analysis and Control of Dynamic Economic Systems. New York: Wiley

Chow, G. C. 1992. Dynamic Optimization. New York: McGraw- Hill.

Conrad J. and C. Clark. 1987. Natural Resource Economics, Notes & Problems. New York: Cambridge University Press.

Craft B. and C. Hawkins. 1991. Applied Reservoir Engineering. Englewood Cliffs: Prentice-Hall.

Dasgupta, P.S. and G.M. Heal. 1979. Economic Theory and Exhaustible Resources. Cambridge: Cambridge Univ. Press.

Dixon B. L. and R. E. Howitt. 1979. Uncertainty and Intertemporal Forest Management Of Natural Resources. Giannini Foundation Monograph, No. 38, University of California, Berkley.

Gray, L. C. 1914. "Rent Under the Assumption of Exhaustibility" Quarterly Journal of Economics 28: 466-89.

Hotelling, H. 1931. "The Economics of Exhaustible Resources" Journal of Political Economy 39: 137-75.

Kirk, D. 1970. Optimal Control Theory, An Introduction. Englewood Cliffs: Prentice-Hall.

Kumar,R.C. 1997. "On Optimal Capacity Expansion for Domestic Processing of an Exhaustible, Natural Resource." Journal of Environmental Economics and Management. 32: 154-169

Luenberger, D. 1975. "A Nonlinear Economic Control Problem with a Linear Feedback Solution." IEEE Transaction on Automatic Control. 45: 184-191

Luenberger, D. 1979. Introduction to Dynamic System Theory, Models and Applications. New York: Wiley.

Padulo, L. and M.A. Airbib. 1974. System Theory. Philadelphia: W. B. Saunders.

Pindyck, R. S. 1978. "The Optimal Exploration and Production of Non- Renewable Resources" Journal of Political Economy: 86: 841-62

Rausser, A. and K. Sydsacter. 1987. Optimal Control Theory with Economic Applications Amsterdam: North Holland.

Rausser, G.C. and E. Hochman. 1979 Dynamic Agricultural Systems: Economic Prediction and Control. New York: North Holland.

Roan, P. and Martin W. 1996. "Optimal Production and Reclamation at a Mine Site with an Ecosystem Constraint" Journal of Environmental Economics and Management 30: 186-198

Satter,A. and G. Thakur. 1994. Integrated Petroleum Reservoir Management Tulsa: Pennwell Publishing.

Scott,A.T. 1963. "The Theory of the Mine Under Conditions of Certainty" Extraction, Resources and Taxation Mason Gaffney, ed.

Sengupta, J. and P. Fanchon. 1997. Control Theory Methods in Economics Boston: Kluwer Academic Publishers.

Schultz, D. and J. L. Melsa. 1967. State Functions and Linear ControlSystems New York: McGraw-Hill.

Toman, M. A. 1986. "Existence and Transversality Conditions for a General Unbounded-Horizon Model of the Mining Firm". Journal of Economic Dynamics and Control 10: 395-414.

Takayama, A. 1974. Mathematical Economics. Boston?: Dryden Press.

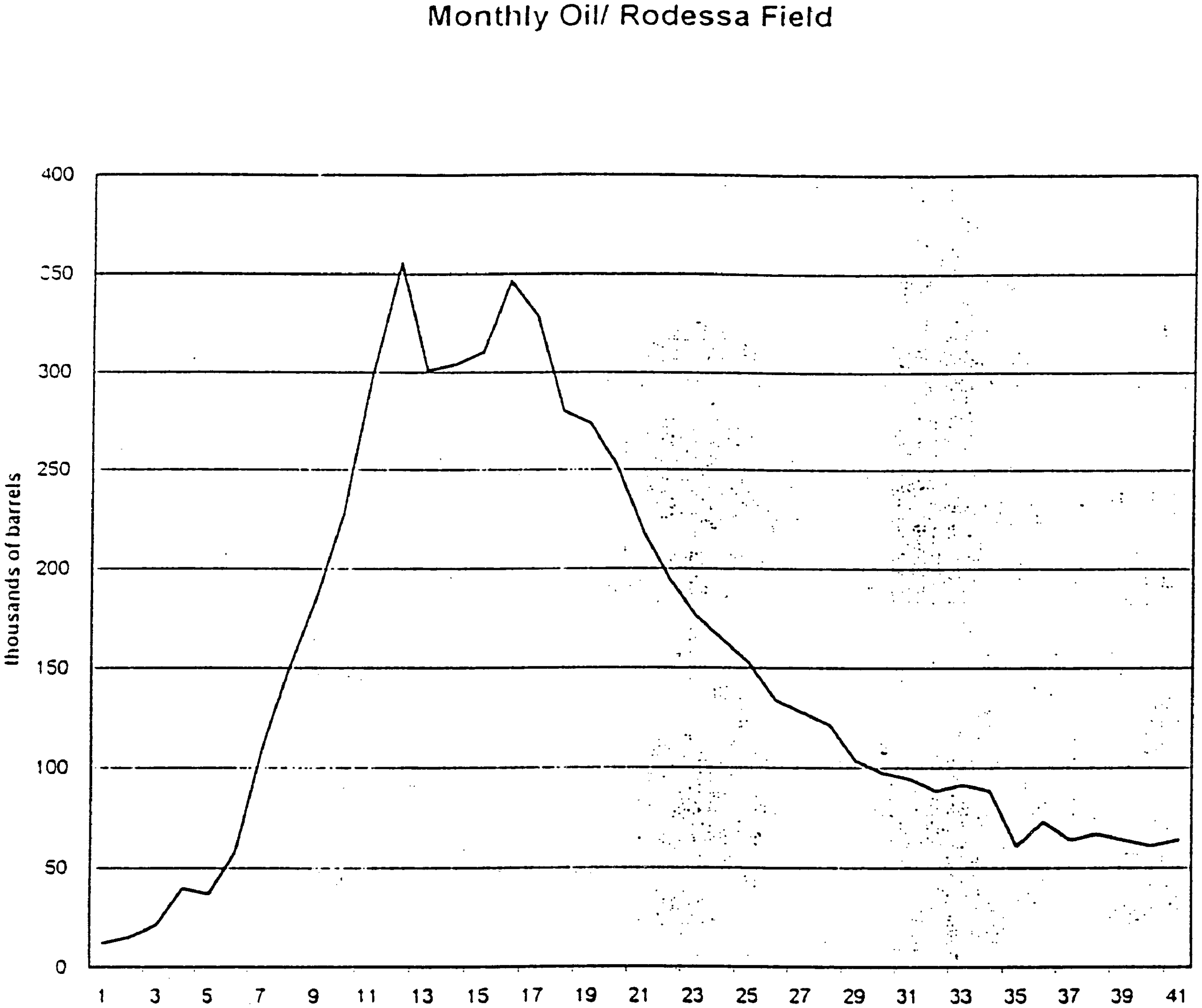
Thompson, R. and J. Wright. 1984. Oil Property Evaluation. Golden: Thompson-Wright Associates.

APPENDIX

A1 Figure 1b

A2 Data 1

A3 Simulation Block Diagram of System

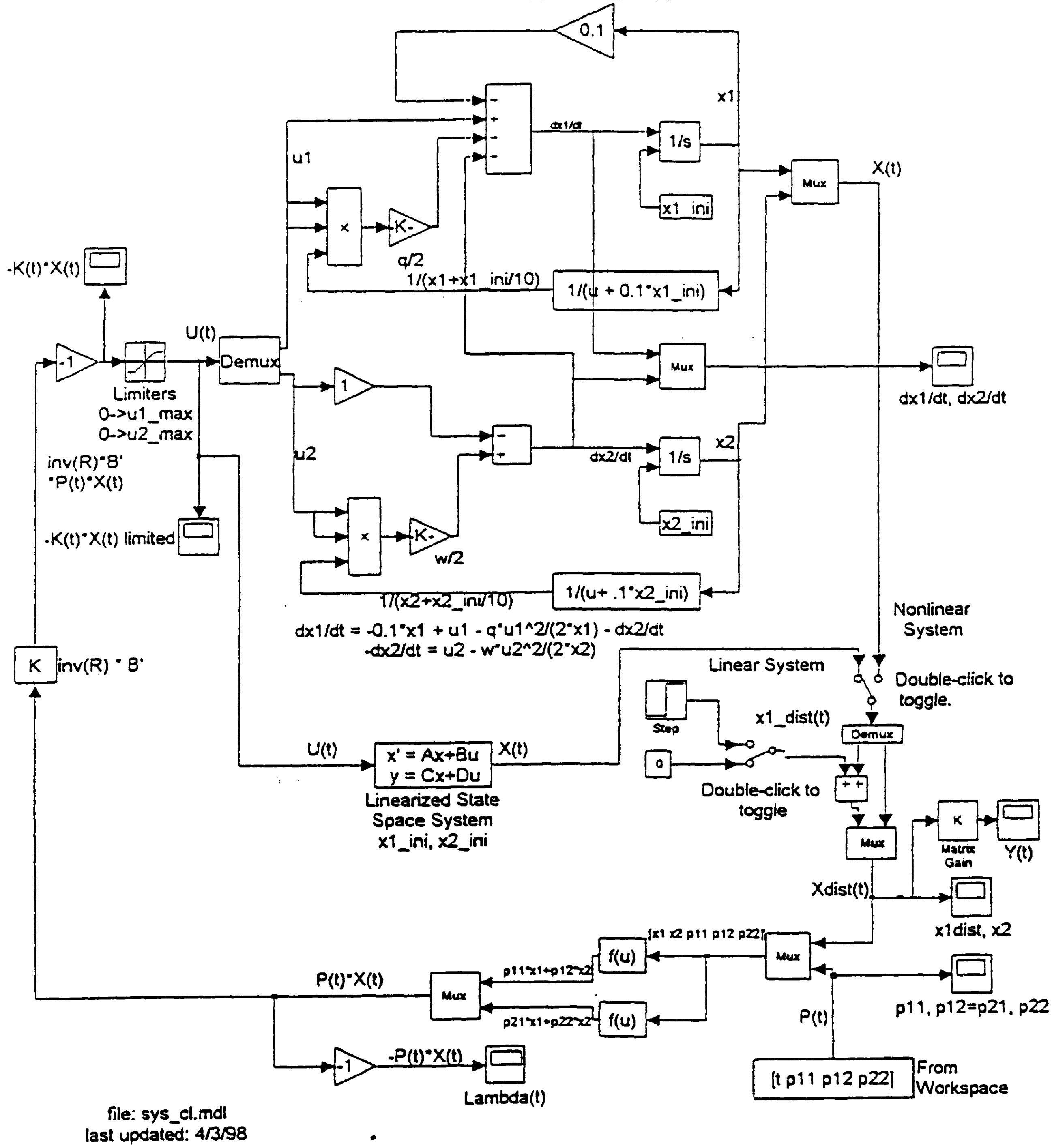

Monthly Oil/ Rodessa Field
thousands of barrels
400
350
300
250
200
150
100
50
0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41

A1 Figure 1b

Time	# wells	cum. Oil	daily oil	monthly oil	monthly oil (thousands)
1	2	12160	400	12160	12.16
2	1	27360	500	15200	15.2
3	3	48640	700	21280	21.28
4	4	88160	1300	39520	39.52
5	4	124640	1200	36480	36.48
6	6	182400	1900	57760	57.76
7	12	291840	3600	109440	109.44
8	16	440800	4900	148960	148.96
9	21	626240	6100	185440	185.44
10	28	854240	7500	228000	228
11	48	1152160	9800	297920	297.92
12	55	1507840	11700	355680	355.68
13	59	1808800	9900	300960	300.96
14	65	2112800	10000	304000	304
15	74	2422880	10200	310080	310.08
16	79	2769440	11400	346560	346.56
17	87	3097760	10800	328320	328.32
18	91	3377440	9200	279680	279.68
19	93	3651040	9000	273600	273.6
20	96	3903360	8300	252320	252.32
21	93	4122240	7200	218880	218.88
22	93	4316800	6400	194560	194.56
23	95	4493120	5800	176320	176.32
24	94	4657280	5400	164160	164.16
25	95	4809280	5000	152000	152
26	92	4943040	4400	133760	133.76
27	94	5070720	4200	127680	127.68
28	94	5192320	4000	121600	121.6
29	93	5295680	3400	103360	103.36
30	95	5392960	3200	97280	97.28
31	91	5487200	3100	94240	94.24
32	93	5575360	2900	88160	88.16
33	92	5666560	3000	91200	91.2
34	88	5754720	2900	88160	88.16
35	87	5815520	2000	60800	60.8
36	90	5888480	2400	72960	72.96
37	88	5952320	2100	63840	63.84
38	88	6019200	2200	66880	66.88
39	87	6083040	2100	63840	63.84
40	82	6143840	2000	60800	60.8
41	85	6207680	2100	63840	63.84

A2 Data1

Simulation of Closed-Loop Non-linear or Linear System
with U(t) = U_fxd-K(t)X(t)
K(t) = inv(R)*B'*P(t)
Lambda(t) = -P(t)*X(t)



A3 Block Diagram of System Simulation