LONG- AND INTERMEDIATE-RANGE PRODUCTION PLANNING FOR BLOCK-CAVING MINES

by

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Dedication

To my wife, Carmen Patricia, for her help, patience, and understanding, throughout the development of this thesis.

T 1424

A Thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science.

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ABSTRACT

An opportunity exists to improve the profitability of block-caving mines by the use of operations research techniques. In this thesis linear programming has been applied to the problem of production scheduling in block-caving mines with the objective of maximizing the total revenue of the operation, subject to requirements on production, uniformity of the grade of the ore produced and limitations concerning the dilution of the ore.

Optimum solutions were obtained in the application of a theoretical model to synthetic data, and in the application of a different model to the Ceresco Level of Climax Molybdenum Mine. The total revenue obtained by the application of the model was found to be 4.25% greater than the one obtained by the Mining Company, with the same production.

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INTRODUCTION

Mines that are operated by block caving, are in most cases large mines; hence to run these mines large sums of money are needed. Consequently, if we could provide a mining sequence or a production schedule that could make the operation more profitable, i.e. 1% or so, this would mean a great sum of money for the Company.

The purpose of making this study was to develop optimizing models of long- and intermediate-range production planning for two cases of block caving, namely Total Block Caving and Progressive Block Caving.

By Total Block Caving is meant that a large area of a mine is undercut at one time and the contact plane between the sinking ore and the waste above is drawn essentially horizontal. A theoretical model will be presented and an application of this model to fictitious data will be solved. The data will be fictitious because this method, although optimum with respect to dilution problems, requires a very large investment with no production in the development stage. Because of this last reason, no mines that use this method were found, at least here in Colorado.

Progressive Block Caving involves the exploitation of a mining area including development and ore drawing at the same time, in this way the initial investment is not so large and a revenue is obtained at once from the ore extracted. For this mining method, a model will be presented and an application of the model to the Ceresco level of the Climax Molybdenum mine will be made. Also the effect on managerial decisions to be made, i.e. cut-off grade, angle of drawing plane, etc., will be discussed.

METHODS OF ANALYSIS

The following analytical methods require consideration in attacking the problem of optimizing production from block-caving mines:

Network Methods

Methods like Program Evaluation and Review Technique (PERT) and Critical Path Methods (CPM) have been useful in production scheduling. However, these techniques are not suitable for making decisions based on optimality. In this case, the optimum solution of the problem has to be chosen from among numerous possibilities. For this reason, these techniques are not directly applicable.

Simulation

Simulation is a useful technique that can be applied to find the best solution of a problem from among a reasonable number of alternatives. In this particular case this method cannot be successfully applied because there is an infinitely large number of different possibilities to study. This would make the analysis very long and tedious.

Dynamic Programming

Dynamic Programming is a technique that can be used to optimize a function subject to a few constraints, i.e. two or three.

In this case, the number of constraints exceeds 500. Therefore, this technique cannot be used.

Mixed Linear-Integer Programming

This technique can be used to optimize a function subject to linear and integer constraints. The integer constraints can include 0 or 1 variables. A model of this kind was proposed for the solution of the problem. However, its application was impossible due to the fact that no large codes which can solve this type of mathematical model are available.

Linear Programming

Linear programming was chosen in this case in view of the following factors:

1. It does provide an optimizing model

2. The objective function and the constraints for this problem can be written as linear functions without destroying the practical meaning of them

3. Large linear-programming codes are available.

TOTAL BLOCK-CAVING LINEAR-PROGRAMMING MODEL

Statement of the Problem

Let us consider that we have an ore body like the one shown in Fig. 1. After geological and rock mechanics studies of the ore body, it was determined that the best mining method to be used is block caving. From the above-mentioned studies the characteristics of the blocks, that is to say, length, width, and height, were also obtained.

From economical feasibility studies, the capacity of the concentrating plant and the capacity of all the other facilities were deduced, hence, the life of the operation can be determined.

So far we have an ore body with given reserves, with a given life, and with a kind of rock suitable for Block Caving. According to this rock type, the ore body has been divided into elements or blocks. Now we can divide the life of the ore body into several periods of time.

The problem is to determine the quantity of ore to be extracted from each block in each period of time in order to maximize the total profit of the operation subject to certain limitations such as the capacity of the concentrating plant, uniformity of the grade of the ore produced, intermediate haulage capacity (if an intermediate haulage system exists, as in the case of the Climax mine), and restrictions concerning the dilution of the ore.

This production-planning study can be a long-, intermediate-, or short-range production planning, depending only on the number of periods of the time into which we divide the life of the ore body. That is, if we divide it into periods of several years each, it would be a long-range production plan. On the other hand, it can be divided into periods of only a week each; this would be a short-range production plan and it would be more useful for the everyday operation of the mine. However, this decision is up to the manager of the mining company.

Assumptions

1. The ore body has been previously divided into levels, and each level was divided into elements or blocks. In this study the production planning of one level only will be considered, since the study of the others is similar (Fig. 2).

2. It will be assumed that a large area of the level has been undercut at one time. This study will start from this point to determine the optimum-production schedule of each block.

3. It will make no difference in this study whether a block has one draw point or several draw points since one variable will be defined for each draw point; consequently,

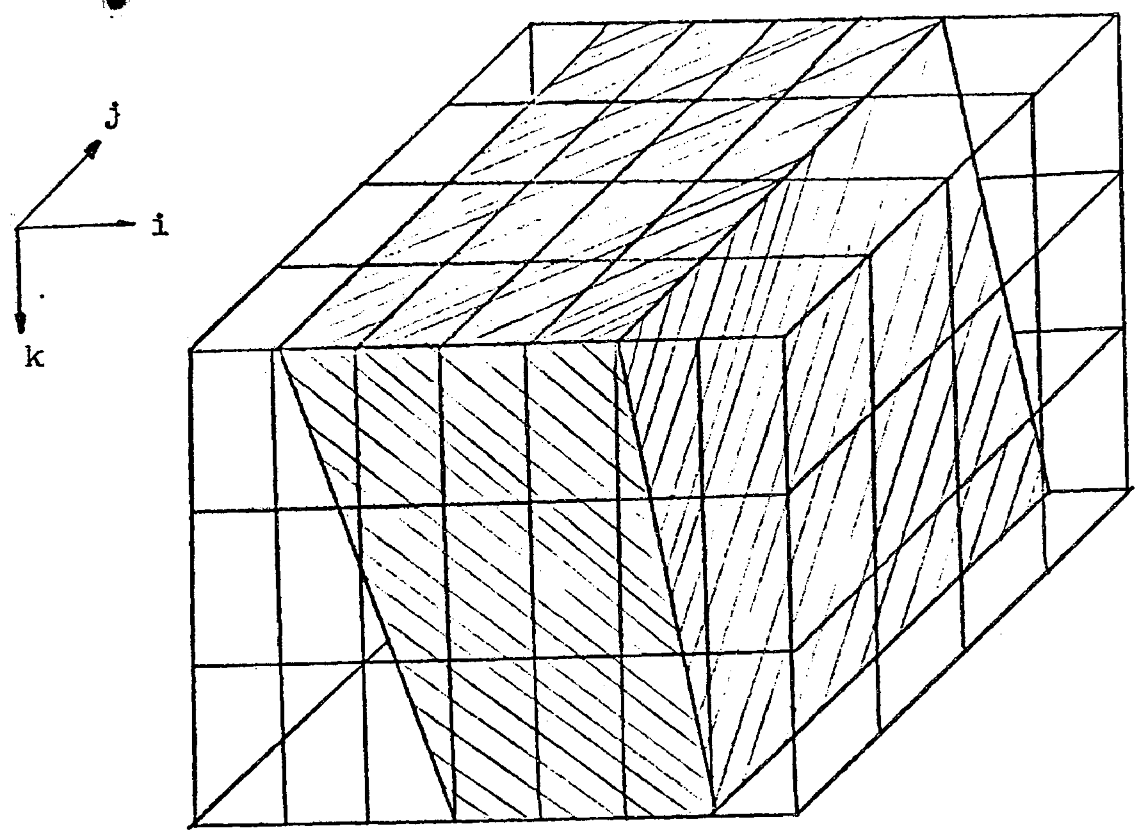


Figure 1. Fictitious ore body. The shaded area indicates the ore.

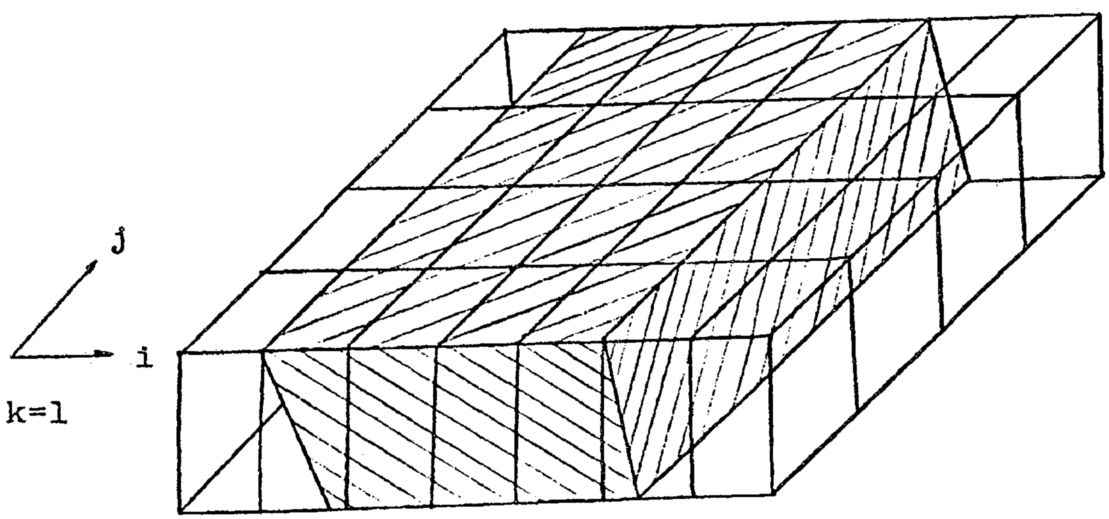


Figure 2. First layer of blocks of the fictitious ore body. The shaded area indicates the ore.

the number of draw points will determine only the size of the problem.

4. It can be assumed that the production of each draw point in a certain period of time is linearly proportional to the height of the ore column that caved in that period of time. In other words, there is a column of even cross section that caves down when ore is drawn from a given draw point.

Gravity flow of granular materials has been studied by Kvapil (1964, p. 34). Fig. 2a, b, c, and d of Kvapil's publication have been included in this thesis as Fig. 3, and they show that this assumption will not lead to greater errors.

In the upper part of Fig. 2c and d it can be seen that a cone is starting to develop; this would make the third assumption invalid. However, this cone, in the case of an underground mine will start to develop in the overburden. Consequently, this will not affect the flow of ore; therefore, this assumption is still valid.

5. It can be assumed that the contact plane between the ore and the waste above of the mining area is horizontal. In other words, every block has the same height; however, this assumption will not be valid in the Progressive Block Caving model.

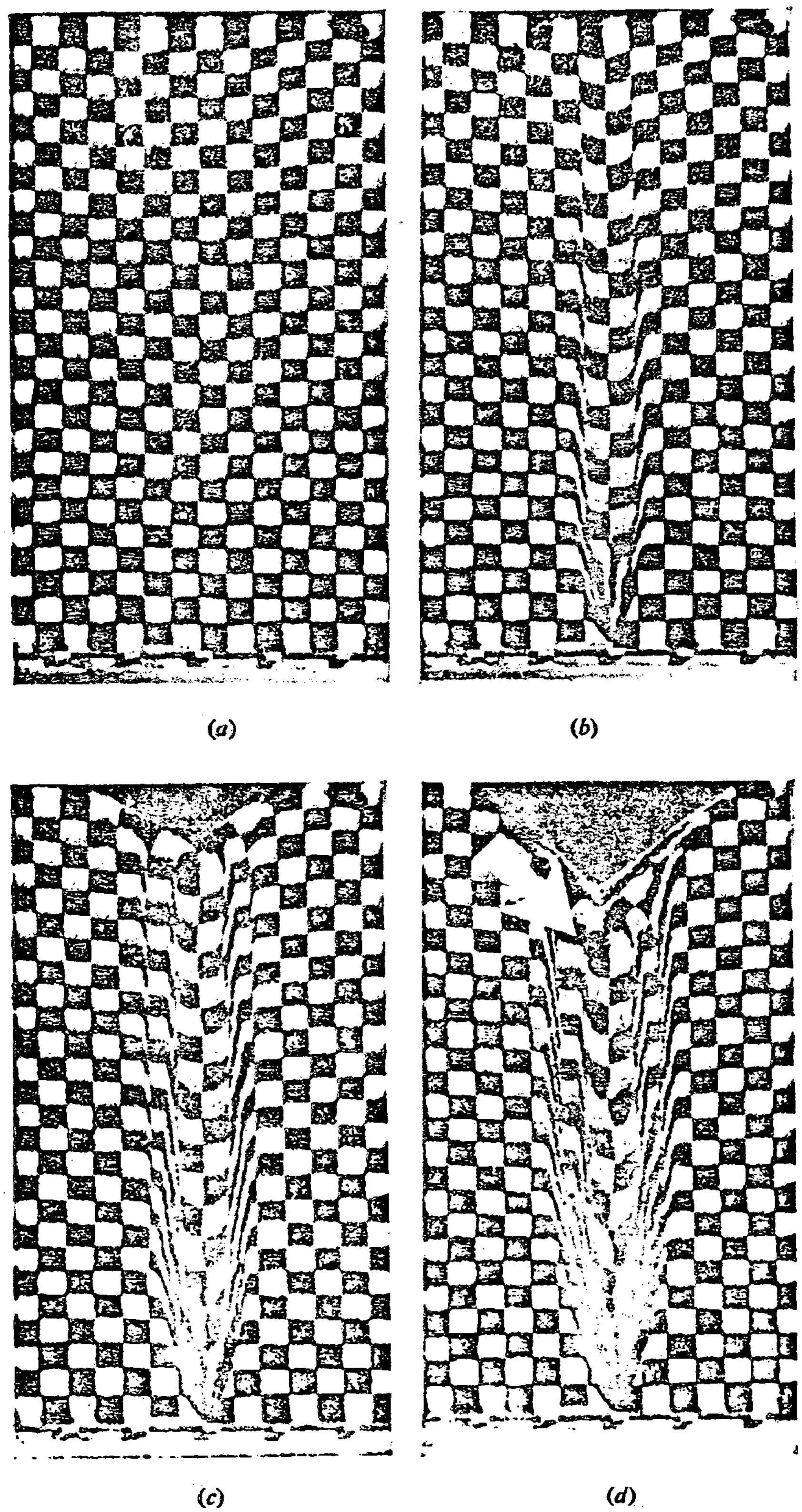


Figure 3. Model illustrating gravity flow of granular materials (by Kvapil, 1964, p. 34).

Mathematical Formulation

The mathematical formulation of the model will be given for the general case in order to show the flexibility of the model.

A numerical example will be included as an illustration of the use of the model and the conclusions that can be obtained.

Specifications of Parameters and Variables

Subscripts: All the variables involved in the model will contain the three following subscripts:

i - indicates the number of a given block in the direction of the X coordinate axis.

j - indicates the number of a given block in the direction of the Y coordinate axis.

t - indicates the number of the period of time.

Variables: The variable X_{ijt} will indicate the number of tons extracted from a block that has a location i,j in a time period t.

Specifications of Mine Parameters: G(ij) - grade of the ore contained in a block of location ij. It is assumed here that the grade of the ore contained in a block is constant for the production planning time. However, the ore grade decreases towards the end of the life of the block, down to the point of reaching the cut-off grade. This problem can be solved by changing the input data when the real world conditions change.

T(ij) - tonnage of ore originally contained in a block of location (ij).

B(ijt) - profit per ton of ore contained in a block at location (ij), in a period of time t.

The profit per ton depends on the time because of the capital interest.

L(t) - lower bound on the production of the period of time t. This lower bound is determined by the mining company according to their minimum production requirements.

U(t) - upper bound on the production of the period of time t. This upper bound is usually given by the capacity of the concentrating plant.

g(t) - lower bound on the average grade of the ore produced in period of time t.

G(t) - upper bound on the average grade of the ore produced. The upper and lower bound on the average grade of ore produced in any period of time is usually given by the operating conditions of the concentrating plant; i.e. if it were a flotation plant, large variations on the ore received by the plant are not desirable because it causes trouble with the amount of reagents used in the plant.

Skt - capacity of the k^{th} intermediate haulage system in the period of time t.

Specifications of Geometric Parameters:

a - Width of a block

b - Length of a block

p - Conversion factor. This conversion factor gives a relation between a given number of tons drawn from a block and the height of the block that caved by drawing that number of tons.

ρ - Weight per unit volume of the ore. It will be assumed that this parameter is constant throughout the ore body. In the case of an ore body suitable for block caving, this assumption is quite true.

The conversion factor p is calculated thus: The number of tons contained in one unit of height of a block of cross-section a x b is p:

$$p = a \times b \times 1 \times \rho \qquad \text{(tons/ft)}$$

α_1 - Upper bound on the angle that the plane between the ore and the waste above forms with an horizontal plane.

α_2 - Lower bound on the angle that the plane between the ore and the waste above forms with a horizontal plane.

Objective Function

The objective function will be to maximize the total profit of the mining operation; analytically the objective function will be:

$$\text{Max } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} X_{ijt} \cdot B_{ijt}$$

<u>General Constraints</u>

1 - Lower bound on the production of each period of time:

$$\sum_i \sum_j X_{ijt} \geq L(t) \qquad \text{for } t = 1,\ldots\ldots r$$

2 - Upper bound on the production of each period of time:

$$\sum_i \sum_j X_{ijt} \leq U(t) \qquad \text{for } t = 1,\ldots\ldots r$$

3 - Each block contains a certain number of tons of ore; therefore the number of tons drawn from each block cannot exceed this number:

$$\sum_{t=1}^{r} X_{ijt} \leq T_{ij} \qquad \text{for } i = 1,2\ldots\ldots m \quad j = 1,2\ldots\ldots n$$

4 - The average grade of the ore produced in a certain period of time will be restricted to vary within certain limits, upper and lower bound:

$$\frac{\sum_i \sum_j X_{ijt} G_{ij}}{\sum_i \sum_j X_{ijt}} \leq G(t)$$

$$\frac{\sum_i \sum_j X_{ijt} G_{ij}}{\sum_i \sum_j X_{ijt}} \geq g(t)$$

5 - The number of tons drawn from a certain group of blocks that is served by a given intermediate haulage system cannot exceed the capacity of this intermediate haulage system.

$$\sum_i \sum_j X_{ijt} \leq S_{kt} \qquad \text{for } t = 1,2,\ldots\ldots r$$
$$k = 1,2,\ldots\ldots K$$

i and j will vary within the group of blocks served by a particular intermediate haulage system.

Geometric Constraints

The purpose of this group of constraints is to avoid as much as possible dilution of the ore.

The contact plane will be referred to as the plane or surface that lies between the ore and the waste above

It was stated before that the contact surface was originally flat. From a dilution point of view, it would be optimum to maintain the contact surface horizontal throughout the entire life of the mining area; however, this is difficult to achieve practically. Therefore, the angle that the contact plane forms with a horizontal plane will be allowed to vary between two limits: α_1 and α_2.

As the optimum situation is to maintain the contact plane horizontal, α_1 and α_2 will be small angles, i.e. +10° and -10°.

Fig. 4 illustrates how to obtain the analytical expression of this constraint for the case of two blocks.

Upper bound on α:

$$\frac{X_{111}-X_{211}}{P\ s} \leq \tan \alpha_1$$

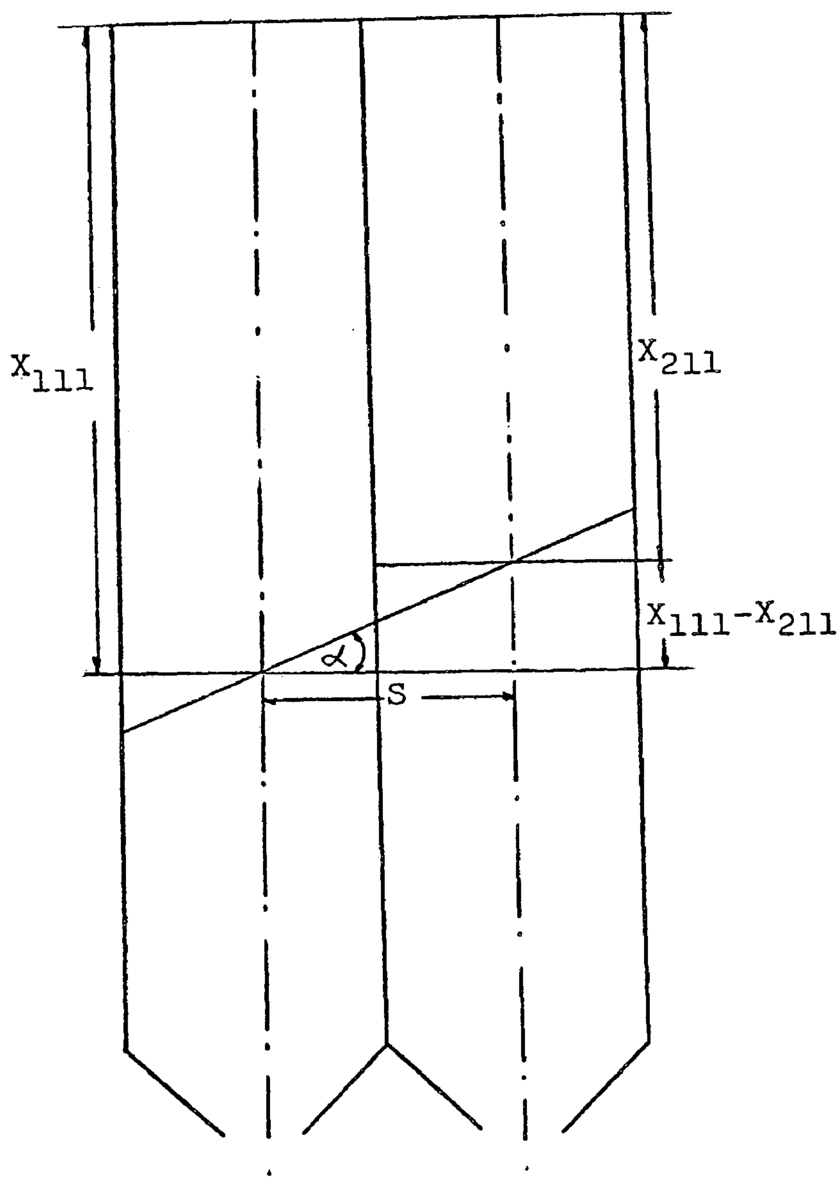


Figure 4. Illustration of the geometric constraints for one period of time, for the total block-caving model.

Lower bound on α:

$$\frac{X_{111}-X_{211}}{P\ s} \geq \tan(-\alpha_2)$$

Notice that $X_{111}-X_{211}$ is measured in tons; therefore, it was divided by the conversion factor P to obtain longitude units.

This constraint can be expressed analytically for the general case as follows:

Upper bound on α:

$$X_{i11}+X_{i12}+\ldots+\ldots X_{i1r}-(X_{i+1,1,1}+X_{i+1,1,2}+\ldots X_{i+1,1,r})$$

$$\leq sP\tan\alpha_1$$

Lower bound on α:

$$X_{i11}+X_{i12}+\ldots X_{i1r}-(X_{i+1,1,1}+X_{i+1,1,2}+\ldots X_{i+1,1,r})$$

$$\geq sP\tan\alpha_2$$

for $i = 1,2,\ldots\ldots m-1$

and

$$X_{mj1}+X_{mj2}+\ldots X_{mjr}-(X_{m,j+1,1}+X_{m,j+1,2}+\ldots X_{m,j+1,r})$$

$$\leq sP\tan\alpha_1$$

$$X_{mj1}+X_{mj2}+\ldots X_{mjr}-(X_{m,j+1,1}+X_{m,j+1,2}+\ldots X_{m,j+1,r})$$

$$\geq sP\tan\alpha_2$$

for $j = 1,2,\ldots\ldots n-1$

Principal direction will be referred to as the direction in the mining area (in plan view) in which the angle α can vary.

Secondary directions will be referred to as any line perpendicular to the principal direction.

The angle that the contact plane makes with an horizontal plane will be restricted to vary in the direction of the principal direction only; therefore, the amount drawn from every block contained in a secondary direction must be the same; analytically this constraint is expressed by

$$X_{ijt} = X_{i+1,j-1,t} = X_{i+2,j-2,t} = \ldots\ldots$$

This constraint was included because it is quite true in the real world.

Application of the Model

The Total Block-Caving Model will be applied to synthetic data in order to show the way in which the model is operated and the conclusions that can be obtained from it.

As stated, the data are fictitious; however, it is typical of what one might find at the "El Salvador" mine in northern Chile.

The model will be applied to the following set of values for the parameters previously defined:

$m = 2$ so $i = 1,2$

$n = 3$ so $j = 1,2,3$

$r = 3$ so $t = 1,2,3$

(grade of the ore contained in each block)

$G(1,1) = 2.1$

$G(2,1) = 0.9$

$G(1,2) = 1.1$

$G(2,2) = 1.8$

$G(1,3) = 0.7$

$G(2,3) = 0.8$

(Profit per ton of ore contained in each block)

$B(1,1) = 5.0$

$B(2,1) = 3.8$

$B(1,2) = 4.2$

$B(2,2) = 4.5$

$B(1,3) = 3.9$

$B(2,3) = 4.0$

Fig. 5 illustrates the characteristics of the fictitious ore body.

height of each block = 100m

length of each block = a = 50m

width of each block = b = 50m

volume of each block = 250,000m^3

density of the ore = 2.8 (tons/m^3)

tonnage of each block = T_{ij} = 700,000 tons ✓

conversion factor P = 7000 T/m

distance AB = BC = CD = s = 36 m (Fig. 5) ✓

upper bound on $\alpha = \alpha_1 = 10°$

lower bound on $\alpha = \alpha_2 = -10°$

U(t) = 170,000 tons/period of time

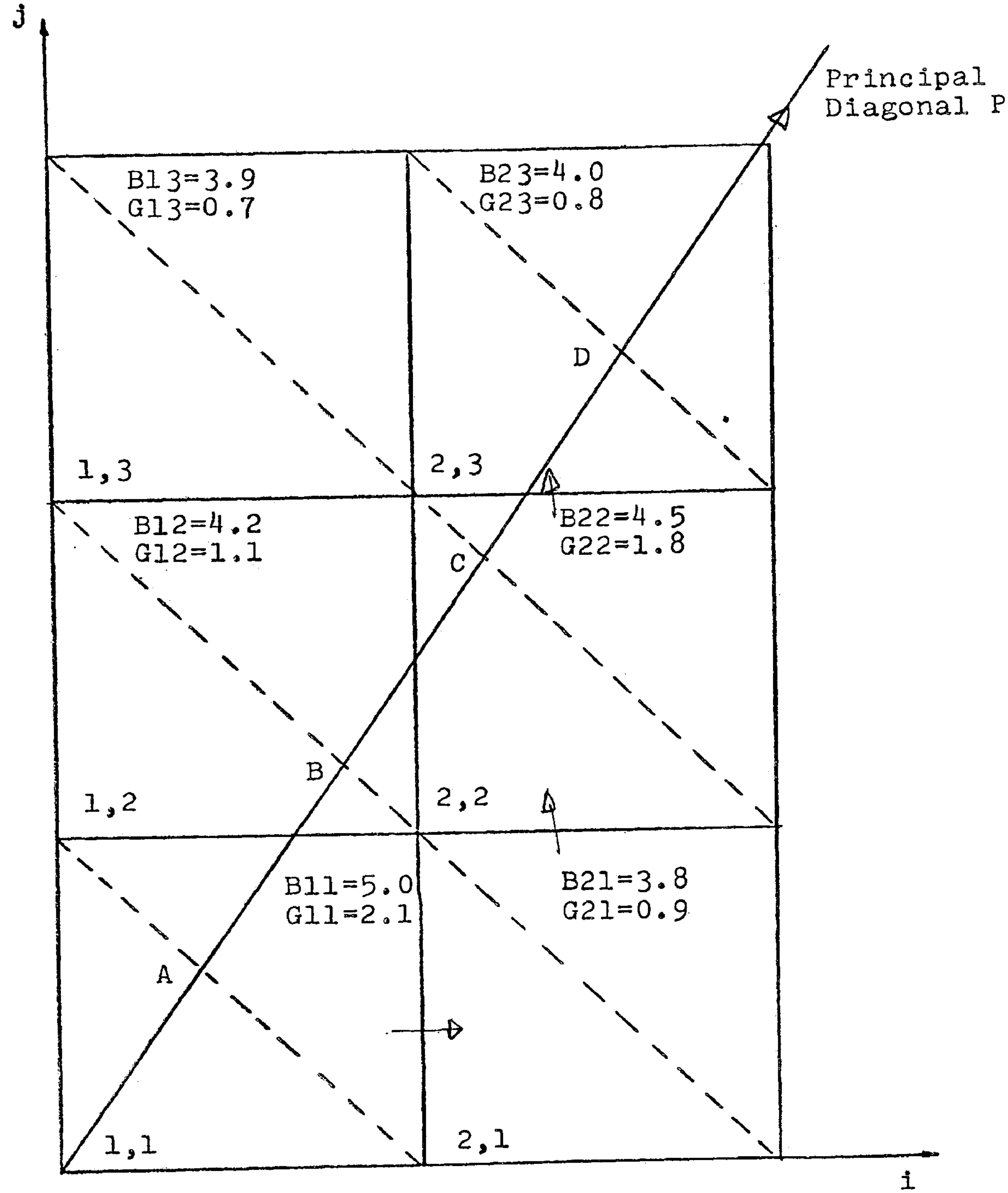


Figure 5. Distribution and characteristics of blocks for the application of the total block-caving model.

$L(t) = 140{,}000$ tons/period of time

$G(t) = 1.4\%$ Cu

$g(t) = 1.0\%$ Cu

For the simplicity of the analysis it was assumed that:

The profit per ton of ore in each block is constant for every period of time; in other words, the interest on the capital was not considered.

The grade of the ore contained in each block does not change with the time; this assumption is quite reasonable in this case because the exploitation of the area is just beginning.

The dimensions of each block are the same. This is true in most block-caving mines, unless the blocks dealt with are on the boundary of the ore body.

Intermediate haulage does not exist; this assumption is true in block-caving mines that use the Long Raise System; this is the case of the El Salvador mine.

Mathematical Formulation

Objective Function: As the linear programming code minimizes, the objective function can be written as

$$\begin{aligned}\text{Min } Z = & -5.0\ X_{111} - 3.8\ X_{211} - 4.2\ X_{121} - 4.5\ X_{221} - 3.9\ X_{131} - 4.0\ X_{231} \\ & -5.0\ X_{112} - 3.8\ X_{212} - 4.2\ X_{122} - 4.5\ X_{222} - 3.9\ X_{132} - 4.0\ X_{232} \\ & -5.0\ X_{113} - 3.8\ X_{213} - 4.2\ X_{123} - 4.5\ X_{223} - 3.9\ X_{133} - 4.0\ X_{233}\end{aligned}$$

<u>Constraints</u>: As this numerical example is small, all the constraints will be written without using abbreviating symbols, in order to make the model easier to understand.

1 - Upper bound on the production of each period of time:

Period I: (UBPP1)

$$X_{111} + X_{211} + X_{121} + X_{221} + X_{131} + X_{231} \leq 170{,}000 \text{ tons.}$$

Period II: (UBPP2)

$$X_{112} + X_{212} + X_{122} + X_{222} + X_{132} + X_{232} \leq 170{,}000 \text{ tons.}$$

Period III: (UBPP3)

$$X_{113} + X_{213} + X_{123} + X_{223} + X_{133} + X_{233} \leq 170{,}000 \text{ tons.}$$

2 - Lower bound on the production of every period of time:

Period I: (LBPP1)

$$X_{111} + X_{211} + X_{121} + X_{221} + X_{131} + X_{231} \geq 140{,}000$$

Period II: (LBPP2)

$$X_{112} + X_{212} + X_{122} + X_{222} + X_{132} + X_{232} \geq 140{,}000$$

Period III: (LBPP3)

$$X_{113} + X_{213} + X_{123} + X_{223} + X_{133} + X_{233} \geq 140{,}000$$

3 - Ore reserves of each block:

Block 11: (ORB11)

$$X_{111} + X_{112} + X_{113} \leq 700{,}000$$

Block 21: (ORB21)

$$X_{211} + X_{212} + X_{213} \leq 700{,}000$$

Block 12: (ORB12)

$$X_{121} + X_{122} + X_{123} \leq 700{,}000$$

Block 22: (ORB22)

$$X_{221} + X_{222} + X_{223} \leq 700{,}000$$

Block 13: (ORB13)

$$X_{131} + X_{132} + X_{133} \leq 700{,}000$$

Block 23: (ORB23)

$$X_{231} + X_{232} + X_{233} \leq 700{,}000$$

4 - Upper bound on the average grade of the ore produced in each period of time:

Period I: (UBGP1)

$$\frac{2.1X_{111} + 0.9X_{211} + 1.1X_{121} + 1.8X_{221} + 0.7X_{131} + 0.8X_{231}}{X_{111} + X_{211} + X_{121} + X_{221} + X_{131} + X_{231}} \leq 1.4$$

The result after reducing this equation is

$$0.7X_{111} - 0.5X_{211} - 0.3X_{121} + 0.4X_{221} - 0.7X_{131} - 0.6X_{231} \leq 0$$

Following the same procedure, the equations for the other periods can be obtained:

Period II: (UBGP2)

$$0.7X_{112} - 0.5X_{212} - 0.3X_{122} + 0.4X_{222} - 0.7X_{132} - 0.6X_{232} \leq 0$$

Period III: (UBGP3)

$$0.7X_{113} - 0.5X_{213} - 0.3X_{123} + 0.4X_{223} - 0.7X_{133} - 0.6X_{233} \leq 0$$

Lower bound on the average grade of the ore produced in each period of time:

Period I: (LBGP1)

$$1.1X_{111} - 0.1X_{211} + 0.1X_{121} + 0.8X_{221} - 0.3X_{131} - 0.2X_{231} \geq 0$$

Period II: (LBGP2)

$$1.1X_{112} - 0.1X_{212} + 0.1X_{122} + 0.8X_{222} - 0.3X_{132} - 0.2X_{232} \geq 0$$

Period III: (LBGP3)

$$1.1X_{113} - 0.1X_{213} + 0.1X_{123} + 0.8X_{223} - 0.3X_{133} - 0.2X_{233} \geq 0$$

5 - In this case no intermediate haulage system exists. ✓

6 - Geometric constraints for period I (t=1)

Upper bound on α

$$X_{111} - X_{211} \leq 36 \cdot 7{,}000 \cdot \tan 10° \quad (i=m-1=1) \quad \text{(ANG1P1)}$$

$$X_{211} - X_{221} \leq 36 \cdot 7{,}000 \cdot \tan 10° \quad (j=1) \quad \text{(ANG2P1)}$$

$$X_{221} - X_{231} \leq 36 \cdot 7{,}000 \cdot \tan 10° \quad (j=n-1=2) \quad \text{(ANG3P1)}$$

Lower bound on α

$$X_{111} - X_{211} \geq 36 \cdot 7{,}000 \cdot \tan(-10°)$$

this can be written as

$$X_{111} - X_{211} \geq -36 \cdot 7{,}000 \cdot \tan(+10°)$$

but as the minus sign is not acceptable in the right hand side, the equation can be written as follows:

$$X_{211} - X_{111} \leq 36 \cdot 7{,}000 \cdot \tan 10° \quad (i=m-1=1) \quad \text{(A1P1)}$$

similarly:

$$X_{221} - X_{211} \leq 36 \cdot 7{,}000 \cdot \tan 10° \quad (j=1) \quad \text{(A2P1)}$$

$$X_{231} - X_{221} \leq 36 \cdot 7{,}000 \cdot \tan 10° \quad (j=n-1=2) \quad \text{(A3P1)}$$

The production of the blocks contained in secondary directions must be equal to each other in every period of time:

$X_{221} = X_{131}$ (MROBP1)

$X_{211} = X_{121}$ (MRBLP1)

These blocks are shown in Fig. 4.

Geometric Constraints for Period II (t=2)

Upper bound on α

Fig. 6 shows how the equations can be obtained for more than one period of time.

$X_{111} + X_{112} - (X_{211} - X_{212}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$ (i=1)(ANG1P2)

$X_{211} + X_{212} - (X_{221} + X_{222}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$ (j=1)(ANG2P2)

$X_{221} + X_{222} - (X_{231} + X_{232}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$ (j=2)(ANG3P2)

Lower bound on α

$X_{211} + X_{212} - (X_{111} - X_{112}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$ (i=1)(A1P2)

$X_{221} + X_{222} - (X_{211} + X_{212}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$ (j=1)(A2P2)

$X_{231} + X_{232} - (X_{221} + X_{222}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$ (j=2)(A3P2)

$X_{222} = X_{132}$ (MROBP2)

$X_{212} = X_{122}$ (MRBLP2)

Geometric Constraints for Period III (t=3)

Upper bound on α (ANG1P3, ANG2P3, ANG3P3)

$X_{111} + X_{112} + X_{113} - (X_{211} + X_{212} + X_{213}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$

(i=1)

$X_{211} + X_{212} + X_{213} - (X_{221} + X_{222} + X_{223}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$

(j=1)

$X_{221} + X_{222} + X_{223} - (X_{231} + X_{232} + X_{233}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$

(j=2)

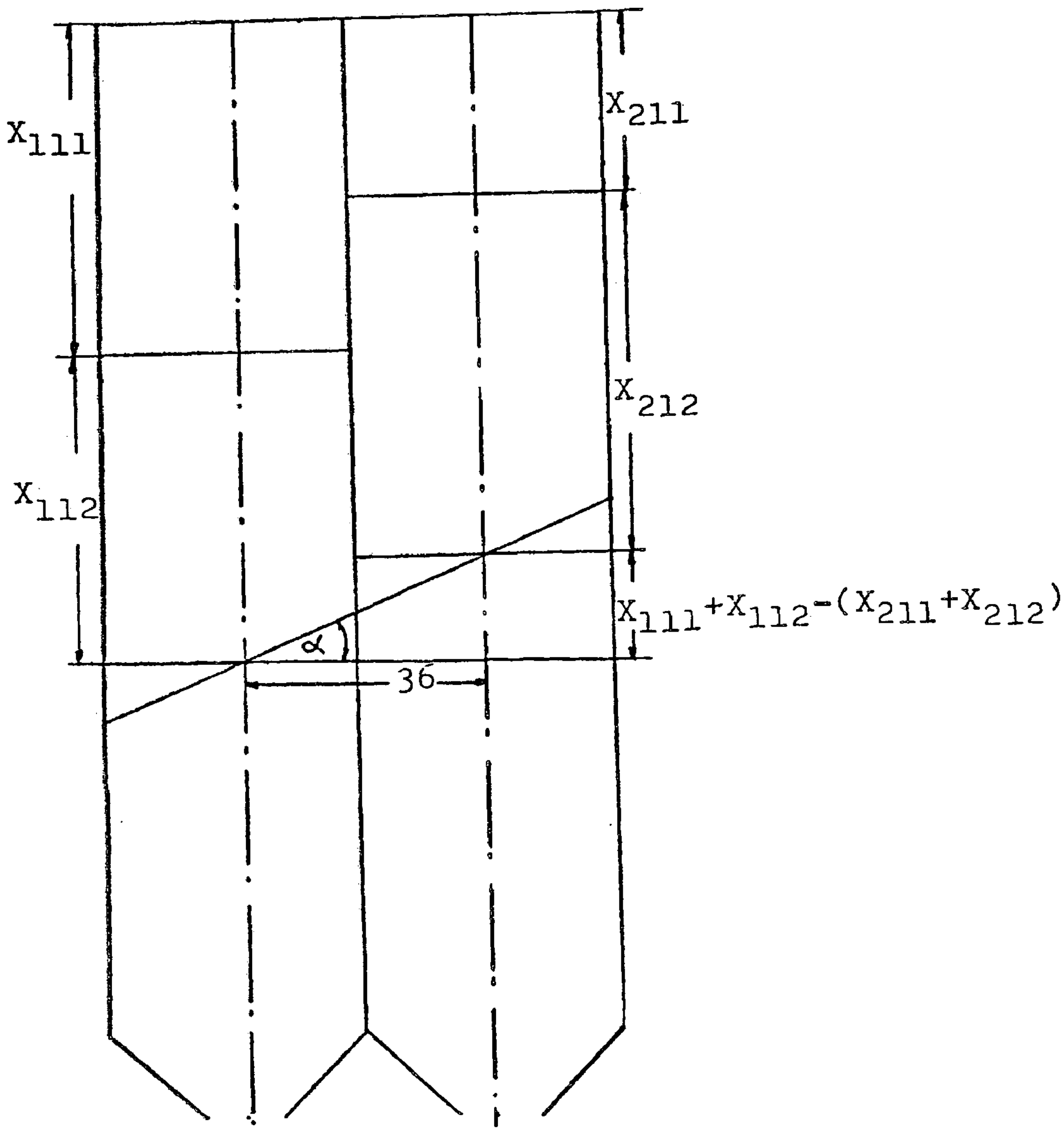


Figure 6. Illustration of geometric constraints for more than one period of time, for the total block-caving model.

Lower bound on α (A1P3, A2P3, A3P3)

$$X_{211} + X_{212} + X_{213} - (X_{111} + X_{112} + X_{113}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$$
(i=1)

$$X_{221} + X_{222} + X_{223} - (X_{211} + X_{212} + X_{213}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$$
(j=1)

$$X_{231} + X_{232} + X_{233} - (X_{221} + X_{222} + X_{223}) \leq 36 \cdot 7{,}000 \cdot \tan \cdot 10^\circ$$
(j=2)

$X_{223} = X_{133}$ (MROBP3)

$X_{213} = X_{123}$ (MRBLP3)

All these variables and constraints form a system of 18 variables and 42 constraints. This system is shown in matrix form in Table 1.

Solution of the Problem

General: A linear-programming code called ALPS I was used to solve the problem with a Burroughs B-5500 computer, that uses ALGOL language. The capacity of ALPS I is for problems that have up to 1022 constraints, 1022 variables, including right-hand side, slack and artificial vectors, 16,350 non-zero matrix entries, and 31,682 non-zero transformation entries. The linear-programming technique used by this program is the revised simplex method with product form of obtaining the inverse.

This problem was feasible at the first run.

The Computer times needed were

Tableau Form of the Total Block-Caving Model

X_{111}	X_{211}	X_{121}	X_{221}	X_{131}	X_{231}	X_{112}	X_{212}	X_{122}	X_{222}	X_{132}	X_{232}	X_{113}	X_{213}	X_{123}	X_{223}	X_{133}	X_{233}	RHS
-5	-3.8	-4.2	-4.5	-3.9	-4.0	-5	-3.8	-4.2	-4.5	-3.9	-4.0	-5	-3.8	-4.2	-4.5	-3.9	-4.0	
1	1	1	1	1	1													≥ 140,000
1	1	1	1	1	1													≤ 170,000
						1	1	1	1	1	1							≥ 140,000
						1	1	1	1	1	1							≤ 170,000
												1	1	1	1	1	1	≥ 140,000
												1	1	1	1	1	1	≤ 170,000
1						1						1						≤ 700,000
	1						1						1					≤ 700,000
		1						1						1				≤ 700,000
			1						1						1			≤ 700,000
				1						1						1		≤ 700,000
					1						1						1	≤ 700,000
0.7	-0.5	-0.3	0.4	-0.7	-0.6													≤ 0
1.1	-0.1	0.1	0.2	-0.3	-0.2													≥ 0
						0.7	-0.5	-0.3	0.4	-0.7	-0.6							≤ 0
						1.1	-0.1	0.1	0.2	-0.3	-0.2							≥ 0
												0.7	-0.5	-0.3	0.4	-0.7	-0.6	≤ 0
												1.1	-0.1	0.1	0.8	-0.3	-0.2	≥ 0
1	-1																	≤ 44,500
-1	1																	≤ 44,500
	1		-1															≤ 44,500
	-1		1															≤ 44,500
			1		-1													≤ 44,500
			-1		1													≤ 44,500
			1	-1														= 0
	1	-1																= 0
1	-1					1	-1											≤ 44,500
-1	1					-1	1											≤ 44,500
	1		-1				1		-1									≤ 44,500
	-1		1				-1		1									≤ 44,500
			1		-1				1		-1							≤ 44,500
			-1		1				-1		1							≤ 44,500
									1	-1								= 0
							1	-1										= 0
1	-1					1	-1					1	-1					≤ 44,500
-1	1					-1	1					-1	1					≤ 44,500
	1		-1				1		-1				1		-1			≤ 44,500
	-1		1				-1		1				-1		1			≤ 44,500
			1		-1				1		-1				1		-1	≤ 44,500
			-1		1				-1		1				-1		1	≤ 44,500
															1	-1		= 0
													1	-1				= 0

for Basis:

Processing time: 19.8 sec

Elapsed: 42.2 sec

for Input:

Processing time: 20.4 sec

Elapsed: 62.5 sec

Linear Programming Results: Value of the objective function was

Z = $2,216,850.

Table 2 shows the production schedule of each block, the total number of tons drawn from each block, the ore left in each block after the whole operation - and the average grade of the original reserves, the production of each period, the total production, and the ore left in the blocks.

The results shown in Table 2 appear graphically in Fig. 7. From this figure it is possible to appreciate the effect of the geometric constraints.

Sensitivity Analysis: As was previously stated, an appendix dealing with sensitivity analysis will be presented; in it all the formulas will be derived and explained.

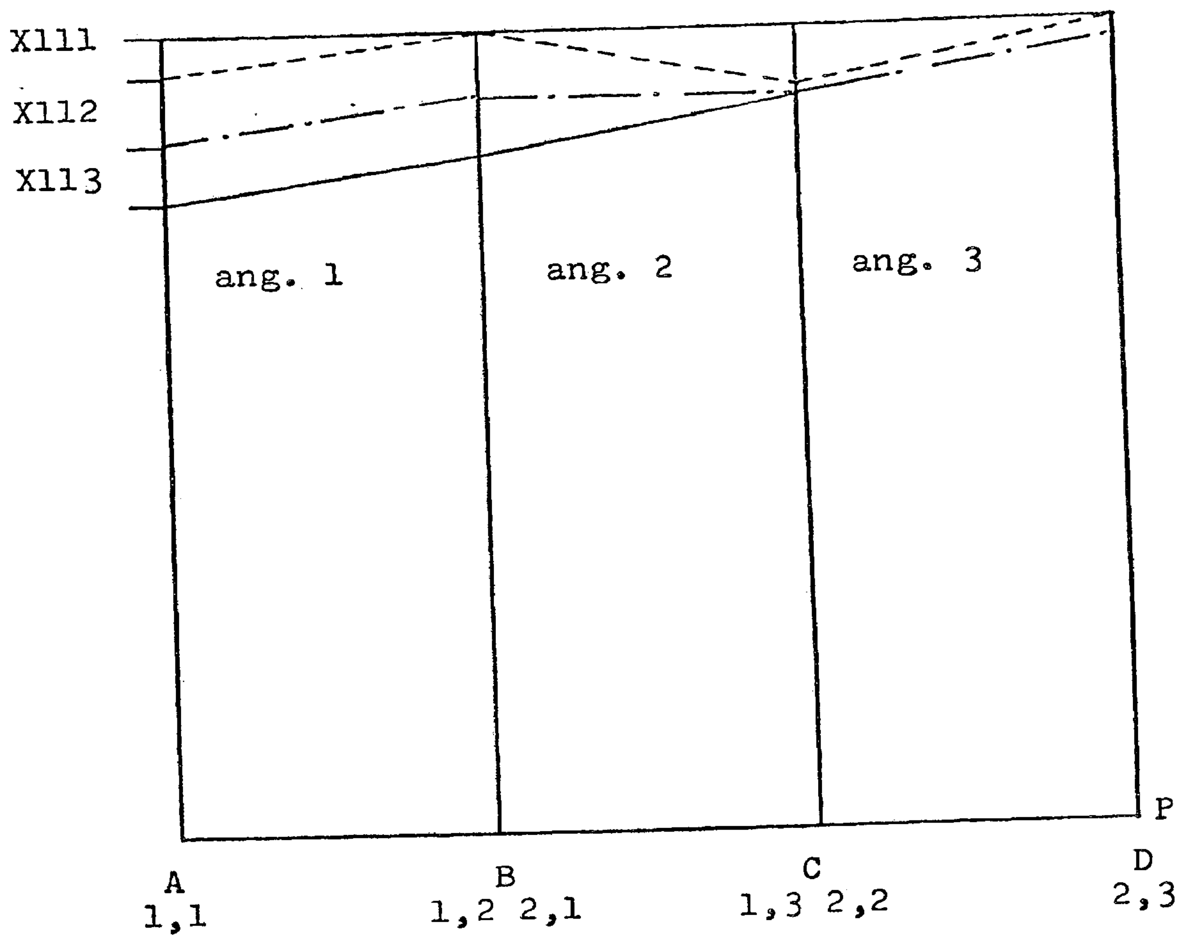
Here only the facts derived from sensitivity analysis will be presented in table form, in order to show the usefulness of this technique to managerial decision-making.

Table 3 shows the actual profit that is being obtained by the production of one ton in each block. The profit per

Table 2

Production Schedule for the Total Block-Caving Model

Block i,j	Grade % Cu	Profit per ton	Reserves x 10^3 tons	Production Period 1	Production Period 2	Production Period 3	Total Produc. of Block	Ore Left in Block
1,1	2.1	5.0	700	39,317	50,615	61,818	151,750	548,250
2,1	0.9	3.8	700	8,337	44,823	54,091	107,250	592,750
1,2	1.1	4.2	700	8,337	44,823	54,091	107,250	592,750
2,2	1.8	4.5	700	52,837	9,913	-	62,750	637,250
1,3	0.7	3.9	700	52,837	9,913	-	62,750	637,250
2,3	0.8	4.0	700	8,337	9,913	-	18,250	681,750
Total			4200	170,000	170,000	170,000	510.10^3	369.10^4
Average grade			1.23	1.40	1.36	1.40	1.386	1.213



---- Shape of the ore body after Period I
—·— Shape of the ore body after Period II
—— Shape of the ore body after Period III

Figure 7. Principal direction profile, showing the shape of the ore body after the different periods of time, for the total block-caving model.

Table 3

Sensitivity of the Cost Coefficients of the Total Block Caving-Model

	Block	Actual Profit	Max. Profit	Entering Variable	Exiting Variable	Min. Profit	Entering Variable	Exiting Variable
PERIOD I	1,1	5.0	5.0	A2P1	X231	5.0	UBGP1	UBGP2
	2,1	3.8	3.8	UBGP1	UBGP2	3.8	ANG3P1	X232
	1,2	4.2	4.2	UBGP1	UBGP2	4.2	ANG3P1	X232
	2,2	4.5	4.5	UBGP1	UBGP2	4.5	A2P1	X231
	1,3	3.9	3.9	UBGP1	UBGP2	3.9	A2P1	X231
	2,3	4.0	4.0	UBGP1	UBGP2	4.0	A2P1	X231
PERIOD II	1,1	5.0	5.0	UBGP1	UBGP2	5.0	A2P1	X231
	2,1	3.8	3.8	ANG3P1	X232	3.8	UBGP1	UBGP2
	1,2	4.2	4.2	ANG3P1	X232	4.2	UBGP1	UBGP2
	2,2	4.5	4.5	A2P1	X231	4.5	UBGP1	UBGP2
	1,3	3.9	3.9	A2P1	X231	3.9	UBGP1	UBGP2
	2,3	4.0	4.0	A2P1	X231	4.0	UBGP1	UBGP2
PERIOD III	1,1	5.0	6.3	ANG3P3	X132	5.0	UBGP3	ANG1P2
	2,1	3.8	3.8	UBGP3	ANG1P2	3.8	X133	X132
	1,2	4.2	4.2	UBGP3	ANG1P2	4.2	X133	X132
	2,2	4.5	4.5	X133	X132	OPEN		
	2,3	4.0	4.0	X133	X132	4.0	ANG3P2	X233

ton can reach a minimum or a maximum before a change occurs in the optimum production schedule. If these limits are exceeded, a change will occur; therefore, a variable will leave the optimum plan, and a new variable will enter into the new plan. Example: when the profit per ton extracted from block 1,3 in period I increases a little (i.e. 1%), the slack variable associated with the upper-bound constraint on the average grade of period I (UBGP1) will enter the basis. It means that this slack variable will have a positive value. Hence, the corresponding constraint will not be met exactly. Therefore, the average grade of period I will decrease. This decrease in grade can be explained as follows: As block 1,3 is more profitable in period I, more tons will be extracted from blocks 1,3 and 2,2 in the time period I (note that there is a constraint which makes the production of these two blocks the same). The average grade of these two blocks is (1.8 + 0.7)/2 = 1.25%; consequently, if more ore is extracted from these blocks, the average grade of the ore extracted in period I (that was 1.4) decreases.

The fact that the exiting variable is UBGP2 (slack associated to the constraints on the upper bound on the grade of period II) means that this slack will have a value of zero; therefore, the average grade of period II will be 1.4 instead of 1.36.

When the profit per ton mined from block 1,3 in period I decreases, the slack variable associated with the angle shown in Fig. 7 between B and C for the first period of time (A2P1) will enter the basis. Therefore, this slack variable will have a positive value, and the associated constraint will not be met exactly. Consequently, the angle mentioned above will decrease from 10° to a lower value. This angle decrease implies that less tons will be mined from blocks 1,3 and 2,2 in period I.

The fact that the exiting variable is X_{231} means that no production will be obtained from block 23 in period I.

Table 4 shows the stability limits for the constraints requirements, the exiting variables when these limits are exceeded, and the extra profit per unit that can be obtained increasing the requirements up to the upper bounds indicated. Of course, if the requirements are decreased to the lower bounds indicated, the profit will decrease by the amount indicated in the last column. (This last column indicates increase or decrease in revenue per unit of requirement.) For example, the upper bound on the production of period I (UBPP1) is 170,000 tons. If this upper bound is decreased to 140,000 tons, then the lower bound on the production of period I will be met exactly; hence, the slack variable of this last constraint (LBPP1) will be equal to zero. Therefore, this slack variable will leave the basis.

If the upper bound is increased to 240,299 tons, the geometric constraint (ANG1P1) will be met exactly; therefore its corresponding slack will leave the basis.

The last column of this table is the most important from the managerial point of view because it indicates the way in which profit could be increased.

In this example, if the upper bound on the production of period I could be increased, to the upper range indicated in Table 4, an extra profit of 42.3¢ per ton would be obtained.

Table 4 shows that the geometric constraints, especially the ones labeled as ANG1P3, ANG2P3, ANG3P3, MROBP3 and MRBLP3 have a great influence in the total profit. (The first three of these constraints deal with angles 1, 2, and 3 shown in Fig. 7 for period III, respectively. The constraint labeled as MROBP3 states that the production of block 1,2 and block 2,1 in period III have to be the same. The constraint MRBLP3 shows that the production of blocks 1,3 and 2,2 in period III have to be the same.) For example, the requirement of constraint ANG1P3 is 10°. However, if the requirement is increased to 11° an extra profit of $34,300 would be obtained. The bounds on this requirement are -4° and 12°; consequently, the present solution is valid only in this range; otherwise the problem has to be rerun.

From this table it can be concluded that the profitability of the mining operation could be substantially increased if the upper bounds on the angle requirements could be implemented.

The solution is quite sensitive to variations in the angles. This can be concluded because in most cases the upper and lower bounds on the angle constraints are close to each other.

Table 4

Sensitivity and Economic Influence of the Requirement Vector for the Total Block-Caving Model

Row Name	Requirement	Lower Range	Exiting Variable	Upper Range	Exiting	Extra Profit per unit x 10
LBPP1	140,000	open		170,000	LBPP1	0.0
UBPP1	170,000	140,000	LBPP1	240,230	ANG1P1	4.23
LBPP2	140,000	open		170,000	LBPP2	0.0
UBPP2	170,000	140,000	LBPP2	open		4.23
LBPP3	140,000	open		170,000	LBPP3	0.0
UBPP3	170,000	140,000	LBPP3	279,724	A2P2	4.23
ORB11	700,000	151,750	ORB11	open		0.0
ORB21	700,000	107,250	ORB21	open		0.0
ORB12	700,000	107,250	ORB12	open		0.0
ORB22	700,000	62,750	ORB22	open		0.0
ORB13	700,000	62,750	ORB13	open		0.0
ORB23	700,000	18,250	ORB23	open		0.0
UBGP1	0	-9,350	UBGP2	11,716	ANG1P1	0.0
LBGP1	0	open		24,751	LBGP1	0.0
UBGP2	0	-9,350	UBGP2	open		0.0
LBGP2	0	open		58,650		0.0
UBGP3	0	5,667	ANG1P2	55,676	X112	0.0
LBGP3	0	open		67,999	LBGP3	0.0
ANG1P1	10	5.7	ANG1P1	open		0.0
A1P1	10	-5.7	A1P1	open		0.0
ANG2P1	10	-10.0	ANG2P1	open		0.0
A2P1	10	5.5	X231	13.3	X211	0.0
ANG3P1	10	5.7	X232	12.4	X231	0.0
A3P1	10	-10.0	A3P1	open		0.0
MROBP1	0	-21,968	ANG1P1	17,531	UBGP2	0.33
MRBLP1	0	-43,350	X231	10,321	X121	0.03
ANG1P2	10	5.8	ANG1P2	open		0.0
A1P2	10	-5.8	A1P2	open		0.0
ANG2P2	10	-2.0	ANG2P2	open		0.0
A2P2	10	2.0	A2P2	open		0.0
ANG3P2	10	10.0	X233	12.0	X232	0.0

Table 4 (Continued)

Row Name	Requirement		Lower Range	Exiting Variable	Upper Range	Exiting	Extra Profit per unit x 10
A3P2	10		-10.0	A3P2	open		0.0
MROBP2		0	-59,481	X222	11,896	X132	0.33
MRBLP2		0	-59,481	X132	53,788	X122	0.03
ANG1P3	10		-4.0	X112	12.0	ANG1P2	3430.0
A1P3	10		-10.0	A1P3	open		0.0
ANG2P3	10		2.0	A2P2	15.0	X132	1340.0
A2P3	10		-10.0	A2P3	open		1020.0
ANG3P3	10		-4.0	X132	10.0	X233	1020.0
A3P3	10		-10.0	A3P3	open		0.0
MROBP3		0	0.0	X233	11,896	X132	1470.0
MRBLP3		0	-59,481	X132	21,250	ANG1P2	134.0

PROGRESSIVE BLOCK-CAVING MODEL

This model is very similar to the one previously presented, except for the geometric constraints. Therefore, the assumptions and equations that are used in the two models will be presented briefly in this one.

Statement of the Problem

As stated in the Total Block-Caving Model, the problem is: given an ore body, that has been divided into elements, determine the production schedule of each element such that the total profit obtained from the Block-Caving mining operation is maximum, subject to the following group of constraints:

1. Capacity of the mining facilities
2. Uniformity of the grade of ore produced
3. Constraints concerning the dilution of the ore.

Assumptions

1. The ore body has been divided into levels, and each level into elements. This study shall consider only the production schedules of the elements contained in one level, since all the levels are similar.

2. It will be assumed that a certain area of the level has been undercut and is ready to go into production.

Production will start in this area and progress towards new areas that will be put into production when the old ones cannot meet the production requirements, subject to the model constraints.

3. As in the previous model, the production of each draw point in a certain period of time will be linearly proportional to the height of the ore column that caved in that period of time.

4. The contact surface between the ore and the waste above can have any shape; in other words, the height of the different elements is not necessarily the same.

Mathematical Formulation

The mathematical formulation will be given in general.

A real world example will be included to show the application of the model in practical situations.

Specifications of Variables

As in the previous model, X_{ijt} will be the number of tons drawn from an element of location i,j in time period t.

Specifications of Mine Parameters

G(ij) - grade of the ore contained in element ij; it is assumed here that the ore grade is constant within the production planning period of time.

T(ijt) - tonnage contained in element ij, in period of time t. For t=0, T(ijt) will be the original reserves of element ij.

B(ijt) - profit per ton of ore contained in element ij, in period of time t.

L(t) - lower bound on the production of period t.

U(t) - upper bound on the production of period t.

g(t) - lower bound on the average grade of the ore produced in period t.

G(t) - upper bound on the average grade of the ore produced in period t.

S(kt) - capacity of the k^{th} intermediate haulage system in period of time t.

Specifications of Geometric Parameters

a - Width of an element

b - Length of an element

ρ - Weight per unit volume of the ore

P - Conversion factor that gives a relation between the number of tons drawn from a given element, and the height of ore that caved on the drawing of that number of tons.

$P = a \cdot b \cdot \rho$ (tons/ft)

Principal Direction: direction in which the mining progresses, i.e. from one zone of the ore body to another.

Secondary Direction: any direction perpendicular to the principal direction.

α - angle formed between the contact surface and a horizontal plane, in the direction of the principal direction.

α_1 - upper bound on α

α_2 - lower bound on α

β_{lqt} - angle formed between the contact plane and a horizontal plane in the ℓ^{th} secondary direction, between elements q and q+1, in time period t (Fig. 9).

β_{lqt1} - upper bound on β_{lqt}

β_{lqt2} - lower bound on β_{lqt}

δ_{ijt} - difference in ore reserves between the highest element and an element ij in period t.

It is assumed in this definition that all the elements have the same cross-section.

$$\delta_{ijt} = (T_{ijt})\ \text{Max} - T_{ijt}$$

Objective Function

Maximize the total profit of the mining operation:

$$\text{Max } z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{r} X_{ijt} \cdot B_{ijt}$$

General Constraints

1. Lower bound on the production of each period of time:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ijt} \geq L(t) \qquad \text{for } t = 1, \ldots, r$$

2. Upper bound on the production of each period of time:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ijt} \leq U(t) \qquad \text{for } t = 1,\ldots\ldots r$$

3. Ore reserves of each element:

$$\sum_{t=1}^{r} X_{ijt} \leq T_{ijo} \qquad \text{for } i = 1\ldots\ldots m$$

$$\text{for } j = 1\ldots\ldots n$$

4. Upper and lower bound on the average grade of the ore produced in each period of time:

$$\frac{\sum_i \sum_j X_{ijt} \, G_{ij}}{\sum_i \sum_j X_{ijt}} \leq G(t) \qquad \text{for } t = 1\ldots\ldots r$$

$$\frac{\sum_i \sum_j X_{ijt} \, G_{ij}}{\sum_i \sum_j X_{ijt}} \geq g(t)$$

5. Capacity of the intermediate haulage systems:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ijt} \leq S_{kt} \qquad \text{for } t = 1\ldots\ldots r$$

$$k = 1\ldots\ldots k$$

i and j will vary within the group of elements served by a particular intermediate haulage system.

Geometric Constraints

Upper and Lower Bounds on Angles Along the Principal Direction: The contact surface will be allowed to move along the principal direction, forming an angle with a horizontal plane that can vary within given limits.

$\overline{\delta}_{\ell t}$ will be referred to as the average of all the δ_{ijt} contained in the ℓ^{th} secondary direction in period t.

$\overline{X}_{\ell t}$ will be referred to as the average production of all the elements contained in the ℓ^{th} secondary direction, in period of time t.

$$\overline{X}_{\ell t} = \frac{X_{\ell 1} + X_{\ell-1,2} + X_{\ell-2,3} + \cdots\cdots X_{\ell-p,p}}{p}$$

for $\ell = 1\ldots.m$ $\quad p = 1\ldots.n$

$$\overline{X}_{\ell t} = \frac{X_{mj} + X_{m-1,j+1} + X_{m-2,j+2} + \cdots\cdots X_{m-p,j+p}}{p}$$

for $\ell = m + j$ $\quad j = 2\ldots.n$

Fig. 8 shows a profile of the principal direction and only two secondary directions to illustrate how this geometric constraint can be expressed analytically.

Upper bound on α

$$\frac{\sum_{k=1}^{t} \overline{X}_{\ell k} + \overline{\delta}_{\ell t} - (\sum_{k=1}^{t} \overline{X}_{\ell+1k} + \overline{\delta}_{\ell+1t})}{Sp} \leq \tan \alpha_1$$

Lower bound on α

$$\frac{\sum_{k=1}^{t} \overline{X}_{\ell k} + \overline{\delta}_{\ell t} - (\sum_{k=1}^{t} \overline{X}_{\ell+1k} + \overline{\delta}_{\ell+1t})}{Sp} \geq \tan \alpha_2$$

In this model α will vary approximately between 0° and 45°.

Upper and Lower Bounds on Angles Along the Secondary Direction: The first geometric constraint allows the contact surface to move along the principal direction. However, in the secondary directions there can exist a great difference

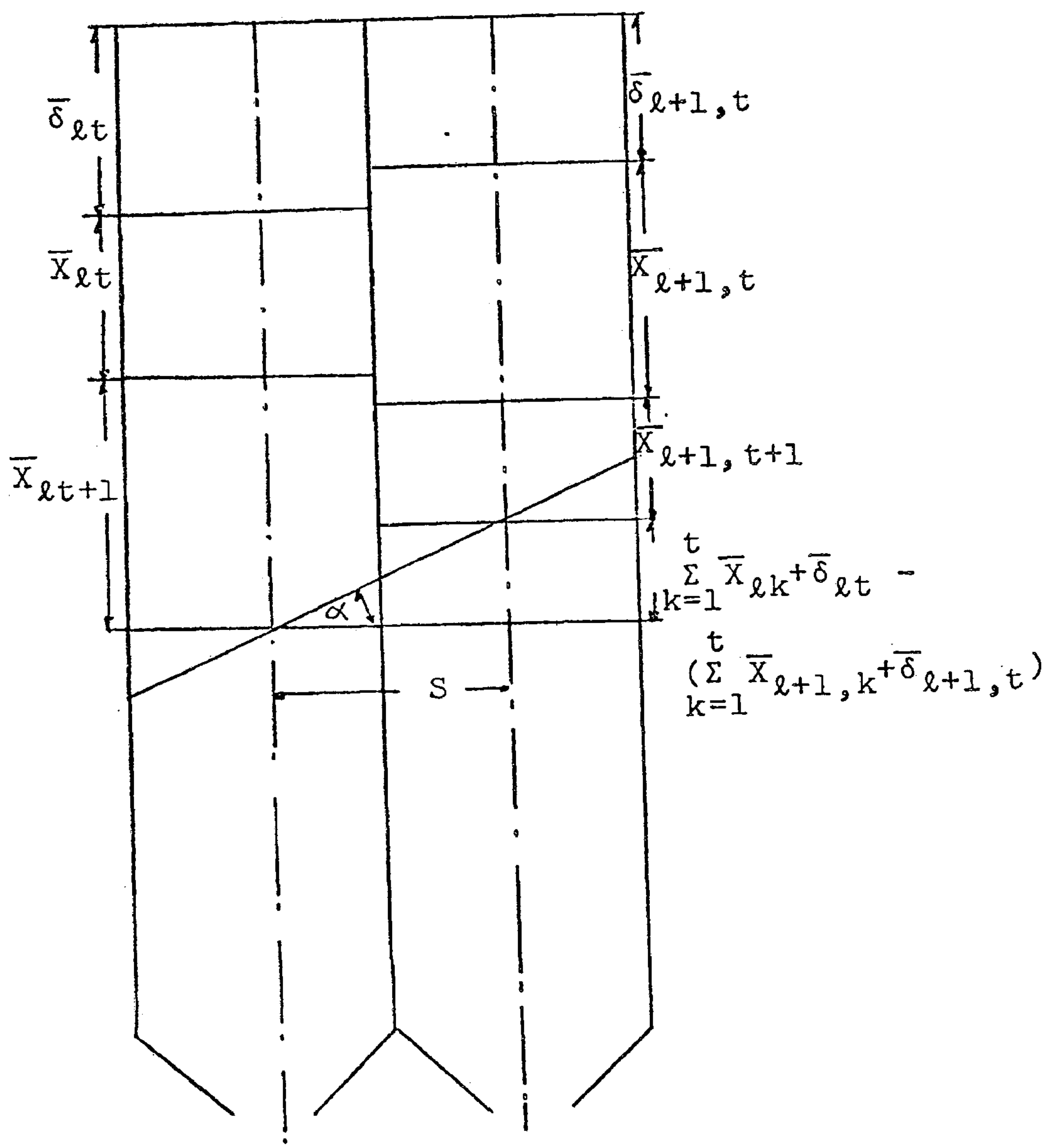


Figure 8. Illustration of geometric constraints along the principal direction, for the progressive block-caving model.

in height between two adjacent elements because the original contact surface can have any shape.

This constraint will smooth the secondary direction contact lines progressively, i.e., if originally the contact lines form an angle of 80° between two adjacent elements, in the first period this angle will decrease to 50°, in the second period to 30°, etc., finally to a variation between plus or minus 15°.

This is the reason why the angle β has three sub-indexes: ℓ indicates the secondary direction number; q indicates the position in that secondary direction; and t, the period of time.

Fig. 9 illustrates this type of constraint. Analytically this constraint can be expressed as:

$$\frac{\sum_{k=1}^{t} X_{ijk} + \delta_{ijo} - (\sum_{k=1}^{t} X_{i-1,j+1k} + \delta_{i-1,j+1,o})}{sp} \leq \tan\beta_{lqt1}$$

$$\frac{\sum_{k=1}^{t} X_{ijk} + \delta_{ijo} - (\sum_{k=1}^{t} X_{i-1,j+1,k} + \delta_{i-1,j+1,o})}{sp} \geq \tan\beta_{lqt2}$$

for $t = 1,r$

$i = 2,m$

$j = 1,n$

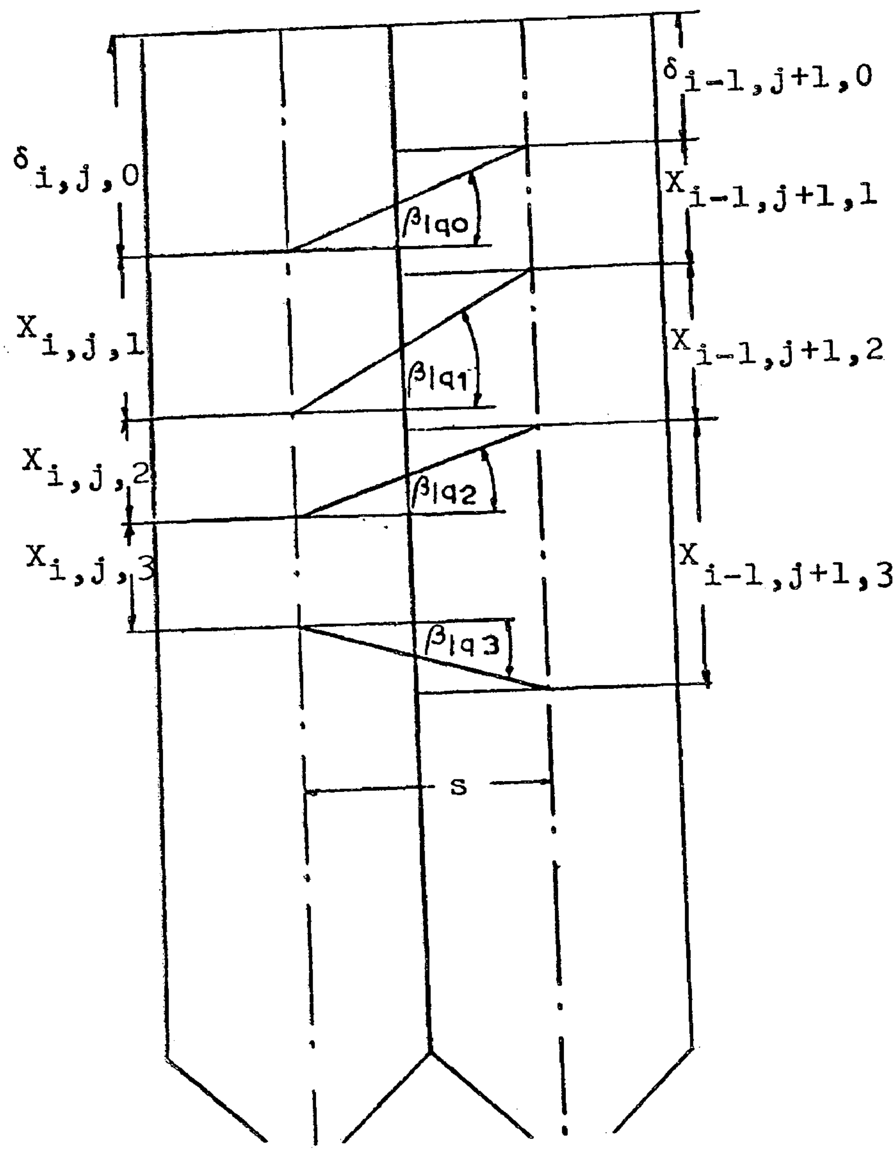


Figure 9. Illustration of geometric constraints along the secondary directions for the progressive block-caving model.

First Application of the Model

The Progressive Block-Caving model will be applied to intermediate and long-range production planning of a part of the Ceresco level of Climax Molybdenum Mine.

Climax Molybdenum Mine

Location: The Climax Molybdenum Mine is located in the state of Colorado, 12 miles NE from Leadville by Highway 91.

Ore Body: The ore body is located on the west slope of the Ten-Mile Range in central Colorado. The Climax Molybdenite deposit is the largest known to man at present. Its ore reserves are estimated to provide a 30 to 40 year life for the mining operation.

The shape of the ore deposit is like the top half of a massive cantalope. This formation, located near the center of Bartlett Mountain, is covered by 100 to 200 ft of glacial debris.

The Mine: The mine is divided into four main levels:

Phillipson level	(elevation 11,463)
Storke level	(elevation 11,168)
600 level	(elevation 10,868)
Ceresco level	(elevation 11,435)

The production of the mine is 43,000 tons per day. This ore is produced mainly at the Phillipson and Storke levels. The Ceresco level does not work at full capacity. The 600 level is still in the development stage.

The mining method is progressive block caving, and it is standardized throughout the mine.

A set of drawings included in the pocket of this thesis show the mine levels, the method and the procedure used for undercutting the production areas.

The linear programming technique for determining optimum production schedules will be applied to the part of the Ceresco level which was in production when this thesis was started.

Ceresco Level: The drawing number 10 Q-2 (included in the pocket) shows the production blocks (406, 408, etc.). Each block contains a number of slusher drifts (406-22, 408-10, etc.). Each slusher drift contains normally six fingers (406-22A, 406-22B, etc.). The slusher takes the ore from the fingers and dumps it into the train which runs through the haulage drifts (406, 408, etc.).

The application of the model will include the fingers contained in the following slusher drifts: 406-10, 406-12, 406-14, 406-16, 406-18, 406-20, 406-22, 406-24, and 408-9 through 408-25.

A rectangular coordinate system (i,j) was used in order to simplify the notation. Axis i coincides with the haulage drift 406 and axis j is perpendicular to the former. The origin (1,1) coincides with the finger 406-24A.

The numbers adjacent to the letters A, B, C, etc. indicate the tonnage in thousands of tons contained in each finger.

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The circled numbers indicate percentage drawn from each finger.

Mathematical Formulation

Evaluation of the Mine Parameters: In this section all the parameters defined in the mathematical formulation of the model will be evaluated.

The values given to these parameters were provided by Climax Molybdenum Co.

i - will vary from 1 to 16

j - will vary from 1 to 9

There are only 143 fingers in production because finger (16,1) does not exist (Ceresco level drawing).

t - will vary from 1 to 2 (there are only 2 periods of time).

U(t) = 831,983 tons per period. This figure was determined thus. As can be seen from the drawing of Ceresco level, a certain number of tons has been drawn from each finger. The total production of the area of interest was 1,663,966 tons up to the day in which this thesis was started.

As the model will consider two periods of time, U(t) was chosen to be one-half of the actual production of the area, so as to be able to compare the revenue actually obtained by Climax Molybdenum Co. with the one obtained by the model at the end of the second period of time.

$L(t)$ = 600,000 tons

$g(t)$ = 0.2%

$G(t)$ = 0.3%

$s(kt)$ = 250,000 tons (constant for all k and t)

This represents the capacity of each slusher in each period of time. This figure was obtained as follows: the capacity of a slusher is 500 T/day or 15,000 T/month.

The production of the whole level is about 50,000 T/month, consequently the production of 831,983 tons was obtained in 16.7 months. Therefore, the capacity of a slusher in 16.7 months is 250,000 tons. Table 1B (included in Appendix B) shows

- The name of each finger
- The location (ij) of each finger
- The tonnage contained in each finger (T_{ij})
- The grade of ore contained in each finger (G_{ij})
- The revenue per ton obtainable from each finger (B_{ij})
- The value of δ_{ij} of each finger

$$\delta_{ij} = 39 \cdot 10^3 \text{ tons} - T_{ij}$$

<u>Evaluation of Geometric Parameters</u>:

a = 32 ft (width of a finger)

b = 32 ft (length of a finger)

ρ = 160 lb/ft^3 (density)

p = 82 T/ft = 0.082 T 10^3/ft

$\alpha_1 = 45°$ upper bound on α (Fig. 10)

$\alpha_2 = 0°$ lower bound on α (Fig. 10)

Table 5 shows how to determine the angles β_{1qt1} and β_{1qt2}:

Table 5

Upper and Lower Bounds on the Angles of the Geometric Constraints Along Secondary Directions

Angle Between 2 Adjacent Fingers in the Same Secondary Direction	Angle After Period I		Angle After Period II	
	β_{1q11}	β_{1q12}	β_{1q21}	β_{1q22}
Over 40°	+40°	−40°	+15°	−15°
Equal 40°	+15°	−15°	0°	0°
Less than 40°	0°	0°	0°	0°

Table 6 shows the same angles as Table 5, but expressed in thousands of tons of difference between two adjacent fingers.

Table 6

Upper and Lower Bounds on the Angles of the Geometric Constraints Along the Secondary Directions, Expressed in Thousands of Tons

Difference in 10^3 Tons Between 2 Adjacent Fingers in Same Secondary Direction	Difference in Height After Period I		Difference in Height After Period II	
	Upper Bound $\cdot 10^3$ Tons	Lower Bound $\cdot 10^3$ Tons	Upper Bound $\cdot 10^3$ Tons	Lower Bound $\cdot 10^3$ Tons
Over 3	+3	−3	+1	−1
Equal 3	+1	−1	0	0
Less than 3	0	0	0	0

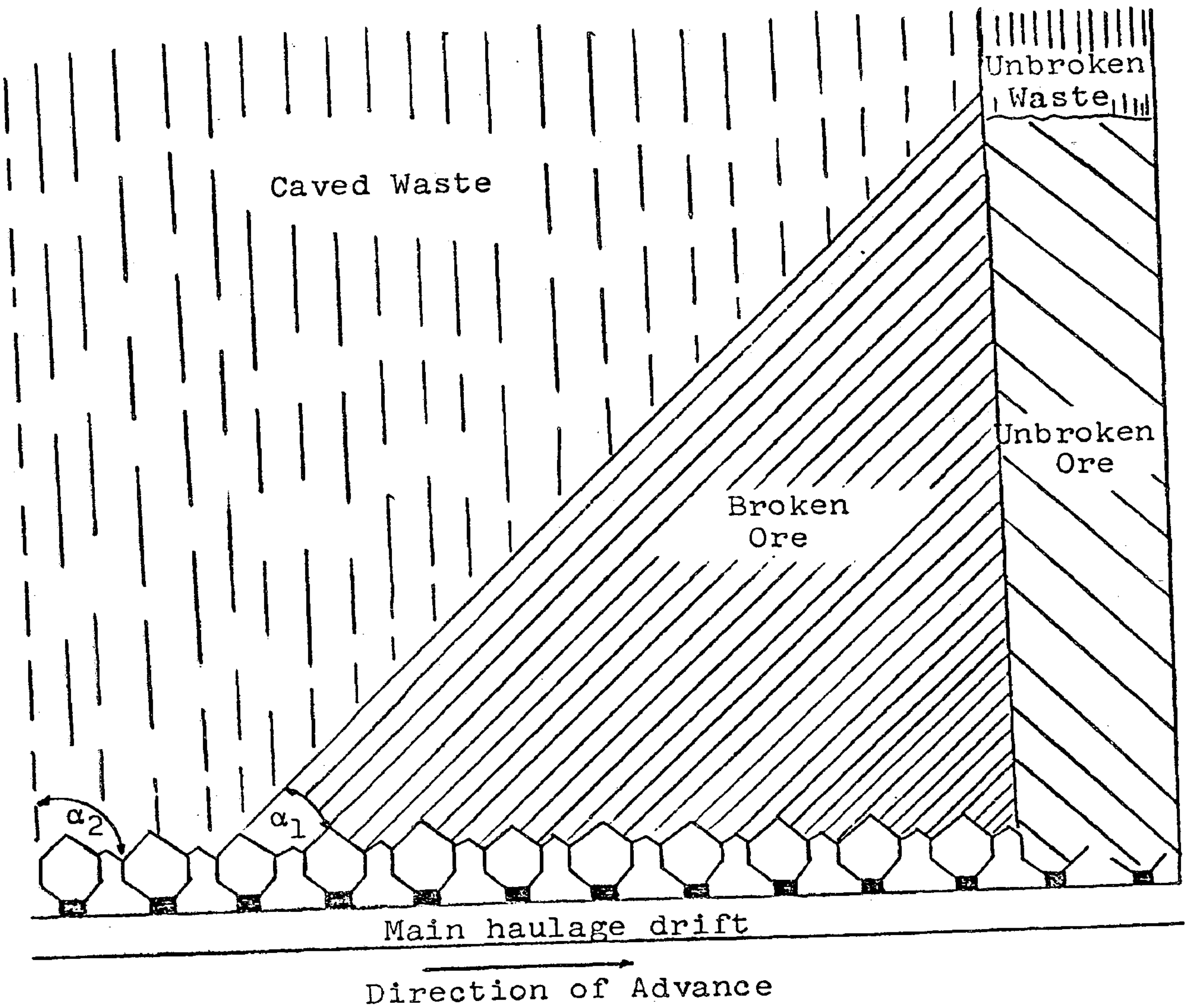

Figure 10. Principal direction profile, showing upper and lower bounds on the angle α for the progressive block-caving model (by Adrian J. Mathias, 1967, p. 4).

Table 7 shows the fingers contained in each secondary direction and the average $\bar{\delta}$ of each secondary direction.

Solution of the Problem

When all the parameters previously defined are introduced in the corresponding equations, a system of 571 constraints and 705 columns is formed. The number of matrix entries is 5159.

The problem was run in a Burroughs B-5500 computer. The linear-programming code used was ALPS-I.

The problem was found to be feasible after the 356th iteration.

The optimal solution was reached at the 446th iteration.

Computer times: processing time - 1 hr, 46 min, 22 sec
input-output time - 3 hr, 54 min, 38 sec

Linear-Programming Results: The value of the objective function was $15,625,058, for the two periods of time. The total revenue for the first period was $7,790,301, and for the second period $7,834,757.

Table 2B (included in Appendix B) shows

- the location of each finger (ij)
- the original grade of the ore contained in each finger.
- the revenue per ton of ore of each finger
- the original reserves of each finger
- the number of tons extracted from each finger in period of time.

Table 7

Fingers Contained in Each Secondary Direction of the Progressive Block-Caving Model

econdary irection N°	Fingers Contained in Secondary Direction	Average ∂
1	1,1	22
2	2,1 1,2	20
3	3,1 2,2 1,3	18
4	4,1 3,2 2,3 1,4	17
5	5,1 4,2 3,3 2,4 1,5	15
6	6,1 5,2 4,3 3,4 2,5 1,6	14
7	7,1 6,2 5,3 4,4 3,5 2,6 1,7	11
8	8,1 7,2 6,3 5,4 4,5 3,6 2,7 1,8	10
9	9,1 8,2 7,3 6,4 5,5 4,6 3,7 2,8 1,9	8
10	10,1 9,2 8,3 7,4 6,5 5,6 4,7 3,8 2,9	7
11	11,1 10,2 9,3 8,4 7,5 6,6 5,7 4,8 3,9	6
12	12,1 11,2 10,3 9,4 8,5 7,6 6,7 5,8 4,9	6
13	13,1 12,2 11,3 10,4 9,5 8,6 7,7 6,8 5,9	7
14	14,1 13,2 12,3 11,4 10,5 9,6 8,7 7,8 6,9	7
15	15,1 14,2 13,3 12,4 11,5 10,6 9,7 8,8 7,9	8
16	15,2 14,3 13,4 12,5 11,6 10,7 9,8 8,9	7
17	16,2 15,3 14,4 13,5 12,6 11,7 10,8 9,9	8
18	16,3 15,4 14,5 13,6 12,7 11,8 10,9	8
19	16,4 15,5 14,6 13,7 12,8 11,9	8
20	16,5 15,6 14,7 13,8 12,9	7
21	16,6 15,7 14,8 13,9	8
22	16,7 15,8 14,9	8
23	16,8 15,9	10

- the total production of each finger
- the ore reserves left in each finger
- the total reserves of the mining area
- the average grade of the total reserves
- the total production of each period with its corresponding grade.
- the total production of the whole operation with its average grade.
- the total reserves left after the mining operation with its average grade.

The production schedules of each finger are shown graphically in Figs. 1A through 22A. Fig. 11 is a profile of the principal direction. Figs. 1A through 22A are profiles of the secondary directions and are included in Appendix A.

Sensitivity Analysis: In the application of the progressive block-caving model, the sensitivity analysis will be presented in the form of questions and answers. These questions might occur to a mine manager that has received a linear-programming production schedule.

1. There are some fingers that are not in the optimum plan; in other words, their production is equal to zero in one or both periods of time. For these fingers, how much does the revenue per ton have to increase before they are considered in the optimum plan?

The answer to this question can be found in the column of the output under "Reduced Costs."

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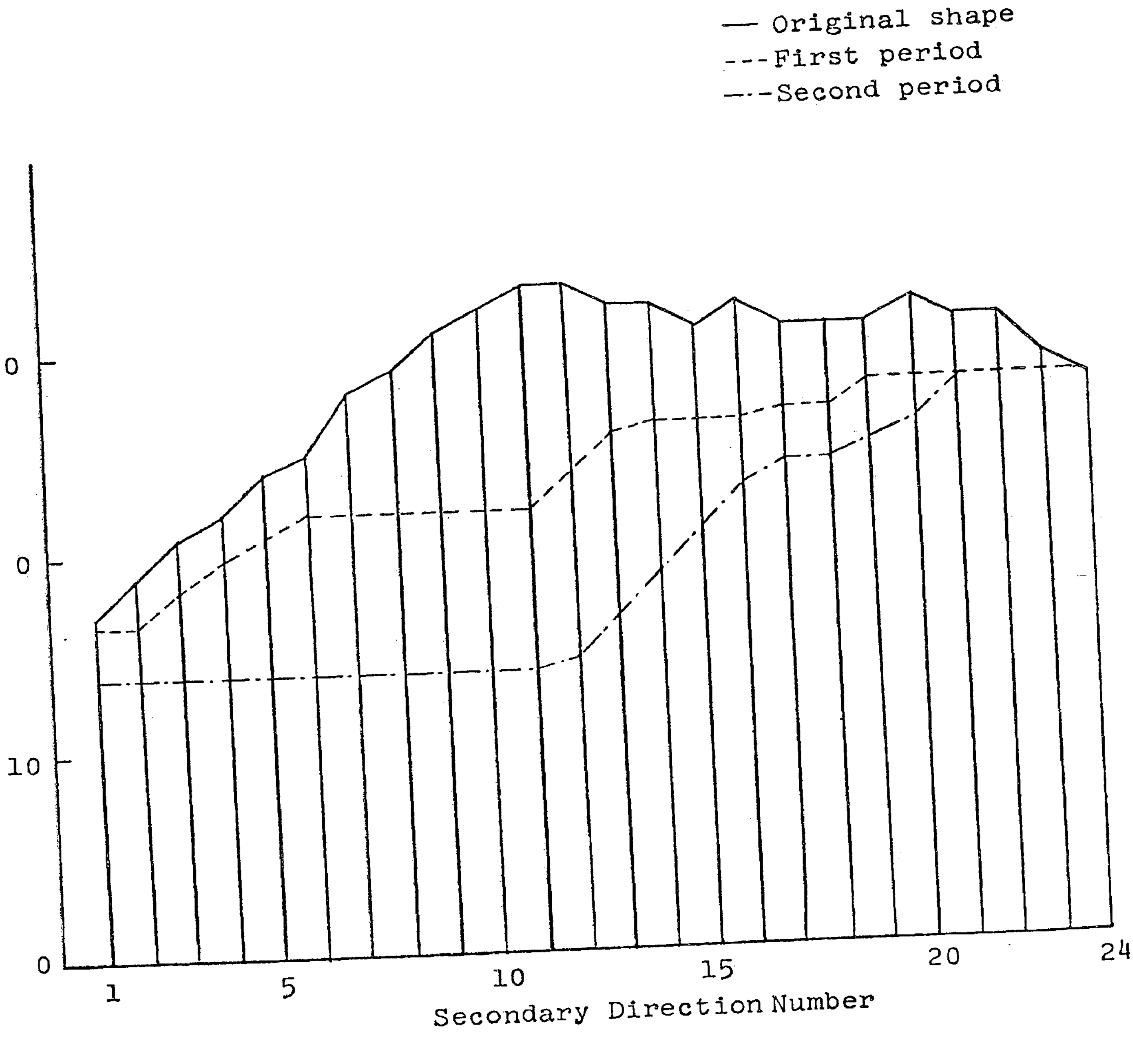

Figure 11. Principal Direction Profile (1st application).
Horizontal Scale 1/4" = 24 ft.

Table 8 shows the fingers that are not considered in the optimum solution, their actual revenue per ton, and the necessary increment needed for them to come into the optimum plan. For example, finger 1,2 has an actual revenue of $7.568 per ton. This finger was not considered for production in either time period I or II. However, if the revenue per ton in period I had been 7.568 + 1.536 = $9.104, this finger would have been considered for production in period I. If the revenue per ton in period II had been $7.568 + $0.757 = $8.325, finger 1,2 would have been considered for production in period II.

It can be seen from Table 8 that some fingers like finger 2,6 in period I have a necessary revenue increment of zero to be able to go into production. This means that the optimum solution is not unique.

2. There is a group of fingers which are in production according to the optimum plan. The grade of the ore contained in the fingers is not constant; therefore, the revenue per ton is not constant. The question is, "What are the limits within which the revenue per ton and the grade can vary before there is a change in the optimum solution?"

Table 3B (included in Appendix B) shows the fingers that are actually in production, their actual revenue per ton and grade. It also shows the limits within which these two quantities can vary in each period of time before there

Table 8

Revenue Increment Required to Make Non-Productive Fingers Economical for Mining, for the First Application of the Progressive Block-Caving Model

Finger i,j	Actual Revenue per ton	Necessary Increment per ton Period I	Period II
1,2	7.568	1.536	0.757
13,2	7.293	0.000	
15,2	7.293	9.120	
16,2	6.605	8.943	8.943
1,3	8.428	0.000	
16,3	6.605	16.393	14.445
1,4	8.600	0.519	
16,4	6.467	28.805	27.376
1,5	8.978	2.948	
16,5	5.917	1.264	1.264
1,6	8.256	0.612	0.612
2,6	10.836	0.000	
2,7	8.531	0.000	
13,8	7.052	0.000	
1,9	8.910	0.000	
16,9	5.882	28.645	9.694
14,3	7.499		5.743
15,3	7.121		0.000
15,7	6.467		8.234
16,7	6.054		3.140
15,8	6.605		3.140
14,9	6.054		3.140

is a change in the optimum plan. For example, finger 1,1 has an actual revenue per ton of $7.912, which corresponds to a grade of 0.230%. (Note that the revenue per ton is linearly proportional to the grade.) This finger was considered in the production schedule for time periods I and II. Finger 1,1 will be considered in period I as long as its grade in this period remains between 0.234% and 0%, which corresponds to a revenue per ton of $8.042 and $0.0, respectively. Finger 1,1 will be considered in time period II as long as its grade in this period remains between 0.268% and 0.226%, which corresponds to a revenue per ton of $9.230 and $7.782, respectively.

Of course, these changes in revenue per ton and grade mean an increase or decrease in the value of the objective function; however, they do not mean a change in the number of tons produced from the relevant finger.

In general, it can be concluded from this table that the upper and lower bounds on the grade or revenue per ton are either widely separated (cases in which one of the bounds is open) or they are very close to each other (cases in which the upper and lower bounds are the same). In the first cases, the solution is not sensitive to grade changes in the fingers. In the second cases, the optimum solution is extremely sensitive to grade changes; this means that if the grade varies a little in these fingers, the solution will no longer be valid and the problem would have to be rerun.

3. If the production requirement cannot be met exactly, what are the limits within which the production of each period of time can vary before there is a change in the basis? What influence in the total revenue would a change in the production have (as long as this change is within the limits mentioned above)?

The answers to these questions can be found in Table 9. For example, the production requirement for period I is 831,983 tons. The solution will not change so long as the production of period I remains between 820,647 tons and 846,505 tons.

The column "Extra Profit per Ton" has an important meaning from the managerial point of view. For example, if the production requirement in period I is decreased or increased, a loss or extra profit of $9.507 per ton, respectively, will be obtained.

Table 9

Bounds and Economical Influence of the Production Requirements of the First Application of the Progressive Block-Caving Model

	Optimum Production	Lower Bound	Upper Bound	Extra Profit (per ton)
P I	831,983	820,647	846,505	9.507
P II	831,983	764,860	947,877	9.377

4. The grade constraints for each period of time show that the average grade of the ore produced has to be between 0.2% and 0.3% molybdenum.

What influence did this constraint have in the linear programming system?

Can anything be said about the economical influence of changing the upper or lower bound of the average grade desired?

In this particular case, the grade constraints were not binding. This is evidenced by the fact that in both periods of time the slack variables associated with each constraint had positive values. Hence, none of the constraints were met exactly. Therefore, in this case this group of constraints could have been left out.

In other words, by the satisfying of the geometric constraints and the production constraints, the grade constraints were satisfied automatically.

The geometric constraints could become binding if the lower bound is higher than the average grade of the ore produced, or the upper bound is lower than the average grade produced. In this case, whether the lower bound had been higher than 0.289% for period I, and higher than 0.283% for period II, or whether the upper bound had been lower than 0.289% for period I and lower than 0.283% for period II. It can be concluded from this that the most important constraints are the geometric constraints. This group will be analyzed in the next section.

5. The geometric constraints will be analyzed in the same way as the production-requirement constraints were analyzed in section 3.

In this case, the questions are:

How much could the management change the angles that the contact surface makes with a horizontal plane before a change occurs in the optimum plan?

What influence would these changes have in the total revenue?

Table 10 shows the answers to these questions. The constraint names beginning with the letter U refer to upper bounds, the ones beginning with the letter L refer to lower bounds. The subscripts i and j indicate the number of the two secondary directions involved in the constraint. Table 10 also shows the angles required, their upper and lower bounds, and the extra profit that can be made by increasing or decreasing the required angles in one degree, provided that this change lies between the upper and lower bounds indicated.

For example, U23 is the constraint associated with the upper bound on the angle formed between secondary directions II and III (Fig. 11). The angle required for this constraint is 45° for time periods I and II. Table 10 shows that if this required angle lies between 14° and 66° in time period I, there will be no change in the optimum solution.

Table 10

Bounds and Economic Influence of Geometric Constraints for the 1st Application of the Progressive Block-Caving Model

Constraint Name	Angle Required	Period I Lower Bound	Period I Upper Bound	Period I Economical Influence 10^3 \$/Deg.	Period II Lower Bound	Period II Upper Bound	Period II Economical Influence 10^3 \$/Deg.
U12	45°	0°	Open	0.0000	0°	Open	0.0000
L12	0°	45°	-14°	0.0054	45°	-56°	0.0680
U23	45°	14°	66°	0.0077	20°	Open	0.0000
L23	0°	45°	Open	0.0000	5°	-42°	0.1580
U34	45°	32°	Open	0.0000	0°	Open	0.0000
L34	0°	32°	Open	0.0000	7°	-45°	0.2775
U45	45°	27°	Open	0.0000	0°	Open	0.0000
L45	0°	27°	Open	0.0000	7°	-45°	0.5200
U56	45°	22°	Open	0.0000	0°	Open	0.0000
L56	0°	22°	Open	0.0000	9°	-56°	0.3360
U67	45°	Open	0°	0.0000	Open	0°	0.0000
L67	0°	27°	-27°	0.0320	14°	-45°	0.4866
U78	45°	0°	Open	0.0000	0°	Open	0.0000
L78	0°	24°	-37°	0.0650	14°	-45°	0.3570
U89	45°	0°	Open	0.0000	0°	Open	0.0000
L89	0°	14°	-42°	0.1170	13°	-40°	0.2300
U910	45°	0°	Open	0.0000	0°	Open	0.0000
L910	0°	30°	-9°	0.1420	25°	-31°	0.3000
U1011	45°	0°	Open	0.0000	0°	Open	0.0000
L1011	0°	25°	-14°	0.2025	31°	-27°	0.2240
U1112	45°	36°	55°	0.2325	29°	Open	0.0000
L1112	0°	Open	45°	0.0000	Open	27°	0.0000
U1213	45°	20°	54°	0.1150	31°	56°	0.2350
L1213	0°	Open	45°	0.0000	Open	45°	0.0000
U1314	45°	14°	Open	0.0000	37°	53°	0.2960
L1314	0°	Open	14°	0.0000	Open	68°	0.0000
U1415	45°	0°	Open	0.0000	37°	51°	0.0443
L1415	0°	-33°	14°	-0.0439	Open	45°	0.0000
U1515	45°	0°	Open	0.0000	0°	Open	0.0000
L1515	0°	9°	-14°	0.0780	14°	-9°	0.1223

Table 10 (Continued)

Constraint Name	Angle Required	Lower Bound	Upper Bound	Economical Influence 10^3 \$/Deg.	Lower Bound	Upper Bound	Economical Influence 10^3 \$/Deg.
U1516	45°	14°	Open	0.0000	37°	Open	0.0000
L1516	0°	Open	14°	0.0000	Open	37°	0.0000
UH23	45°	0°	Open	0.0000	0°	Open	0.0000
LH23	0°	-9°	17°	-0.0350	-11°	27°	-0.9090
UH34	45°	37°	Open	0.0000	25°	Open	0.0000
LH34	0°	Open	42°	0.0000	Open	-25°	0.0000
UH45	45°	39°	48°	0.0457	45°	51°	0.9130
LH45	0°	0°	Open	0.0000	0°	Open	0.0000
UH56	45°	-46°	Open	0.0000	14°	Open	0.0000
LH56	0°	-46°	Open	0.0000	0°	Open	0.0000
UH67	45°	0°	Open	0.0000	0°	Open	0.0000
LH67	0°	0°	45°	-0.0386	Open	0°	0.0000
UH78	45°	0°	Open	0.0000	0°	Open	0.0000
LH78	0°	0°	37°	-1.1100	Open	0°	0.0000
UH89	45°	0°	Open	0.0000	0°	Open	0.0000
LH89	0°	0°	0°	-9.4110	0°	0°	-3.0990

Furthermore, the mining company will incur a loss or profit of $7.7 per degree if this requirement is decreased or increased, respectively.

The required angle of 45° can vary between 20 and 90° in time period II without any change in the optimum solution. However, an angle of 90° is theoretically possible, but practically it is not recommended because it will involve dilution of the ore. In this case, no loss or extra profit will be the consequence of changing the angle requirement.

As a general conclusion drawn from Table 10, we can say that the problem is not highly sensitive to changes in the angle policy. The reason for this is that the intervals between the upper and lower bounds on the required angles are usually large. In most cases, one of the bounds is open, a fact which means that the limiting angle can be +90° or -90°, depending on the case. This property of the optimum solution can be used advantageously by the management of the company in two ways:

1. More profit can be obtained by setting the required angles in an appropriate way.

2. The mining operation can be accomplished with more elasticity in the ore-drawing policy.

<u>Comparison Between the Total Revenue Obtained by Climax Molybdenum Co. and the Application of the Model</u>: Climax Molybdenum Co. has produced 1,663,968 tons of ore from the mining area that concerns this study up to the initiation

date of the study. The total revenue obtained by Climax Molybdenum Co. was calculated in the same way as the one obtained by the application of the model, that is:

$$\text{Revenue} = \Sigma \text{ Production} \cdot \text{Grade} \cdot \text{Price of Molybdenum}$$

Table 11 shows the revenue comparison.

Table 11

Revenue Comparison Between the First Application of the Model and the Climax Molybdenum Mine

	Model			Climax Mo. Co.
	Period I	Period II	Total	
Production	831,984	831,984	1,663,968	1,663,968
Revenue	7,790,301	7,834,757	15,625,058	15,511,867

It can be seen that the increase of revenue was not very substantial; in fact, it amounts to only 0.73%.

Table 10 shows the influence of the geometry of the system in the total revenue. Therefore, by using the results of this table in an appropriate way, it is possible to change those constraints that are harming the revenue. The geometry of the system is the only thing that a manager could change in order to obtain a larger value of the objective function with the same production and grade requirements. This is what is intended in the second application of the model.

Second Application of the Model

In the first application of the model to the Ceresco level of Climax Molybdenum Mine the total revenue obtained was only 0.73% higher than the one obtained by the mining company (with the same production). The intention of this application of the model is to increase the value of the objective function with the same production and grade requirements.

In this model, four periods of time will be included. The first two periods will have a production of one-half that of either of the two periods of the first application. The second two periods will have the same production as either of the ones included in the first application.

Table 12 shows the production requirements of the four periods.

Table 12

Production Requirements for the Second Application of the Progressive Block-Caving Model

	Period I	Period II	Period III	Period IV
Production Requirement (in tons)	415,992	415,992	831,983	831,983

The intention in putting small production requirements on the first two periods is to show that the model can be applied to long- or short-range production planning.

The rest of the model is exactly the same as the one

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used in the first application, except for some geometric constraints.

Geometric Constraints

In Table 10 it can be seen that the following constraints have a negative or neutral influence in the revenue of period I: L1415, L1516, LH23, LH56, LH67, LH78, LH89. These constraints govern the lower bound on the angle formed between the following secondary directions: 14-15, 16-17, 17-18, 20-21, 21-22, 22-23, 23-24, respectively.

It can be seen in Fig. 11 that the model forced the production of the secondary directions 19 through 24 in period I, because the lower bound on the angles formed between these secondary directions was equal to zero. Negative angles were not allowed. This decreased the total revenue, as revealed by the sensitivity analysis.

In this application of the model, negative angles will be allowed as lower bounds on the angles formed between the above mentioned secondary directions, in the first 2 periods of time. The reason for this can be seen in Table 10.

Example: LH23 in period I has an economical influence of -0.035×10^3 \$/Deg. This means that if the lower bound on the angle formed between secondary directions 17-18 is increased by one degree, that is from 0° to 1°, the revenue will decrease by 35 dollars. Hence, if the lower bound on that constraint is decreased, the total revenue will increase.

The new values of the lower bounds on those constraints can be seen in Table 13. These values were chosen so as to allow the ore body to maintain its original shape in secondary directions 16 through 24. In this way no production will be forced in this area unless it is highly profitable in periods I and II.

Table 13

Lower Bounds on Angles of Geometric Constraints for the 2nd Application of the Progressive Block-Caving Model.

Constraint Name	Angle Required
L1516	-27°
LH56	-27°
LH78	-45°
LH89	-27°

The geometric and production constraints for the last two periods are the same as the ones used in the first application of the model.

<u>Solution of the Problem</u>

The first two periods of time form a system of 571 constraints and 705 variables, with the exclusion of artificial variables and 5073 matrix entries. The problem was found to be feasible after 399 iterations. The optimal solution was reached at the 489^{th} iteration. The computer times needed were:

Processing time: 1 hr, 25 min, 19 sec

Input-output time: 2 hr, 13 min, 2 sec

The second two periods of time form a system of 571 constraints, 705 variables excluding artificials and 5041 matrix entries. The problem was feasible after 347 iterations. The optimal solution was reached after 466 iterations. The computer times needed were:

Processing time: 2 hr, 12 min, 41 sec

Input-output time: 3 hr, 56 min, 10 sec

The models for the four periods were run in a B-5500 Burroughs computer. The linear-programming code used was ALPS I.

Linear-Programming Results: The values of the objective function for the four periods of time are shown in Table 14.

Table 14

Revenues Obtained in Each Period of Time of the Second Application of the Progressive Block-Caving Model

	Periods I and II		Periods III and IV	
Total Revenue	7,968,005		15,564,689	
Revenue	Period I	Period II	Period III	Period IV
	4,091,620	3,876,385	8,203,593	7,361,096

The optimum production schedule is shown in Table 4B (included in Appendix B). This table shows the grade, revenue per ton, original reserves, production schedule and the total production of each finger. It also shows the ore left in each finger, the total production of each period of time, the total production of the whole operation, and the total reserves left with their corresponding average grades.

The production schedules of each finger are shown graphically in Figs. 23A through 44A. Fig. 12 is a profile of the principal direction, and the rest are profiles of each secondary direction. Fig. 23A through 44A are included in Appendix A.

<u>Sensitivity Analysis</u>: The sensitivity analysis for this application of the model will be presented in the same way as for the first application.

1. Table 5B (included in Appendix B) shows the fingers that are not in production in one or more periods of time, their actual revenue per ton, and the necessary revenue increment per ton for these fingers to be considered in the optimum plan. For example, finger 13,2 was not considered for production in time period I; however, if its revenue per ton had been \$7.293 + \$15.599 = \$22.892 it would have been considered in the optimum production schedule.

It can be seen from Table 5B that some fingers have a necessary revenue increment of zero to be economical for mining. This implies that the optimum solution is not unique. The practical meaning of this is that if for any reason we cannot mine from a finger that was part of the optimum plan, we can mine from a finger whose "Reduced Cost" is equal to zero. By doing this, we do not decrease the value of the objective function. Therefore, this gives more flexibility

Shape after Period I
Shape after Period II
Shape after Period III
Shape after Period IV

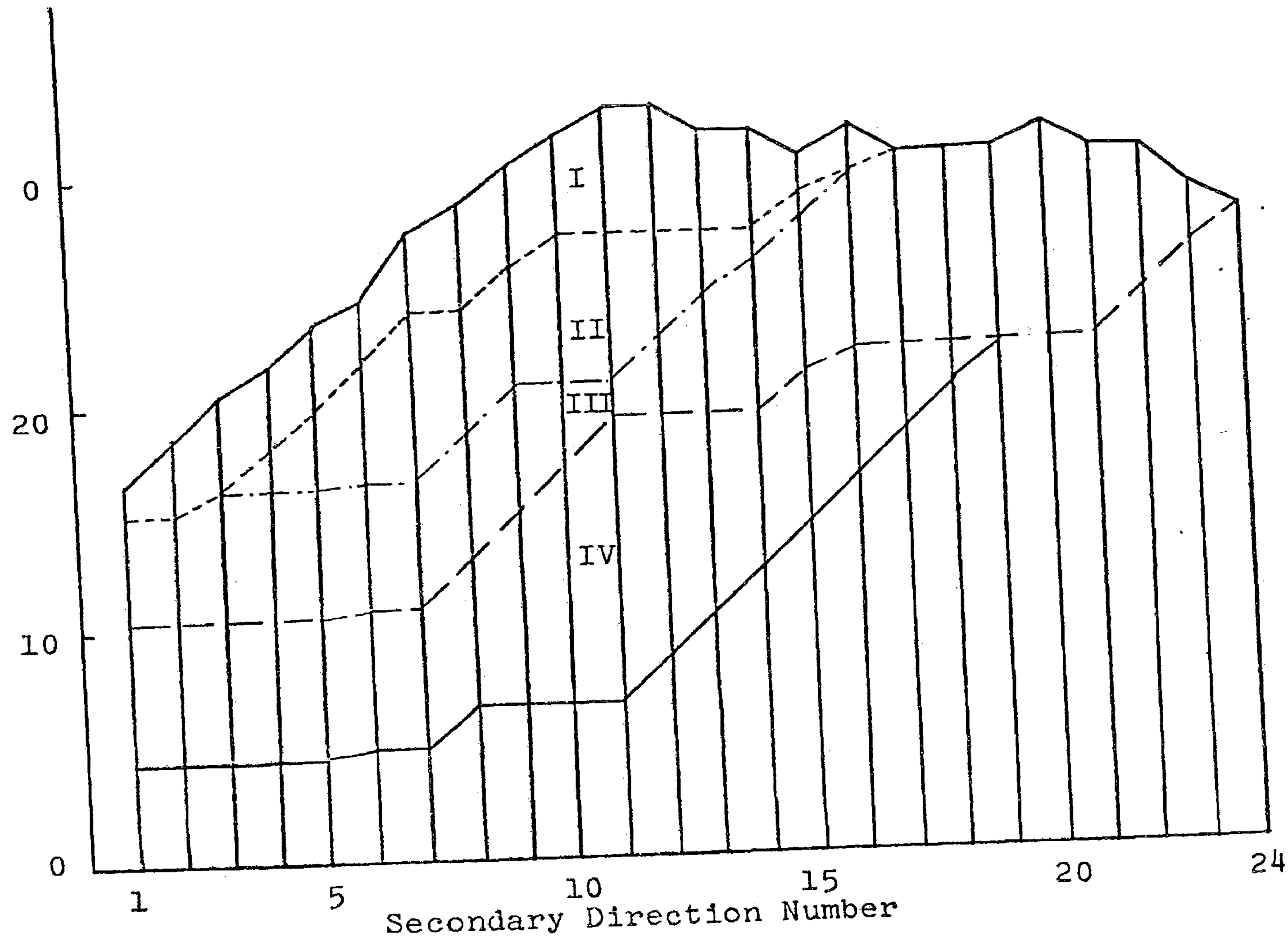

Figure 12. Principal Direction Profile (2nd application).
Horizontal Scale 1/4" = 24 ft

to the optimum solution.

2. Table 6B (included in Appendix B) shows the fingers that are in production, their actual revenue per ton and grade. Table 6B also shows the limits within which these quantities can vary before there is a change in the optimum plan.

For example, finger 1,1 has an actual grade of 0.230%, which implies an actual revenue of 7.912 $/ton. According to Table 4B the production of this finger in time period I will be 1500 tons. This finger will go on producing 1500 tons in period I so long as the grade remains between 0.278% and 0.230% or, which is the same, as long as the revenue per ton remains between $9.594 and $7.912. If the grade changes within these limits there will be a change only in the value of the objective function, there will be no change in the fingers that are in production or the quantities of ore extracted from them.

As a general conclusion drawn from Table 6B, we can say that the optimum solution is sensitive to changes in grade because the upper and lower bounds on the acceptable grade are quite close to each other; in many cases the two bounds coincide, with the result that small changes in the grade of these fingers will make the solution invalid (as long as the change exceeds one of the bounds indicated). In this case the problem can be rerun with the new grade to determine the new coefficient of the objective function as well as the new coefficients of the grade constraints. In order to avoid running the problem again, which is time-consuming, another iteration in the Simplex Algorithm can be done to determine the exiting variable, the entering variable, and its level.

As a result of the figures shown in Table 6B the optimum solution is much more elastic and easy to implement because in practice, grade changes will occur and through the sensitivity analysis shown in this table management can find a way to handle them.

3. Table 15 shows the sensitivity analysis of the production requirements and their economical influence. For example, the production requirement for time period I is 415,992 tons. The optimum solution will not change so long as the production of period I remains between 414,509 tons and 433,000 tons.

The mining company will incur an extra profit or loss of $9.594 per ton if the requirement is increased or decreased, respectively. This makes the optimum solution implementable because meeting a requirement of 415,992 tons exactly is practically impossible.

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Table 15

Bounds and Economical Influence of the Production Requirements of the Second Application of the Progressive Block-Caving Model

	Production Requirement	Lower Bound	Upper Bound	Extra Profit (per ton)
Period I	415,992	414,509	433,000	$9.594
Period II	415,992	414,509	440,468	9.594
Period III	831,983	811,080	847,967	9.310
Period IV	831,983	806,745	919,323	9.248

4. Table 7B (included in Appendix B) shows the sensitivity analysis of the geometric constraints and their economical influence.

For example, L2021 in time period III represents the lower bound on the angle formed between secondary directions 20 and 21 in period III (Fig. 12). The required angle is zero degree; however, according to Table 7B the required angle can lie anywhere between -6° and zero degree before any change in the optimum solution occurs. Furthermore, for every degree that the requirement is decreased, an extra profit of $22.5 is obtained.

The following general conclusions can be drawn from Table 7B:

In many cases the bounds on the required angles along the principal direction are open. For example, U12 in time period I represents the angle formed between secondary directions 1 and 2 in period I (Fig. 12). The lower bound on the requirement of this constraint is zero degree and the upper bound is open; in other words, the upper bound is 90°. As far as a mine manager is concerned, this means that he can modify the requirement (45°) substantially and thus, which is more important, the value of the objective function is not affected.

In the cases in which either bounds are open, the upper and lower bounds are quite separated. In most of these cases, the economical influence of changing the required angles is not zero. Therefore, management can use this information to improve the value of the objective function by changing in an appropriate way the constraint requirement.

5. The grade constraints, as in the first application of the model, show that the average grade of periods I, II, III, and IV must be between 0.2% and 0.3% molybdenum.

In this case, the grade constraints were not binding. This is evidenced by the fact that in the four periods of time the slack variables associated with each constraint had positive values. Hence, none of these constraints were met exactly. Therefore, this group of constraints could have been left out. In other words, the grade constraints were satisfied automatically by the satisfying of the geometric constraints and the production constraints. Evidently, these constraints have an economical influence of zero; therefore, the value of the objective function will not be affected if these constraint requirements are changed marginally.

Comparison of the Total Revenue Obtained by Climax Molybdenum Co. and the Second Application of the Model: Table 16 shows this comparison. Only the first three periods were considered because their production is equivalent to the one of Climax.

Table 16

Revenue Comparison Between the Second Application of the Model and the Climax Molybdenum Mine

	Model				Climax Mo. Co.
	Total	Period I	Period II	Period III	
Prod.	1,663,967	415,992	415,992	831,983	1,663,968
Revenue	16,171,598	4,091,620	3,876,385	8,203,593	15,511,867

It can be seen from this table that the difference in revenue between Climax Molybdenum Co. and the second application of the model is more substantial than the previous one. The percent increase over Climax is expressed thus:

$$(16,171,598 - 15,511,867)/15,511,867 =$$

$$\% \text{ increase} = 659,731/15,511,867 = 4.25\%$$

Comparison Between the Principal Direction Profiles of the First and Second Applications of the Progressive Block-Caving Model

The principal direction profiles to which we will refer in this section are shown graphically in Figs. 11 and 12. As far as the production requirements are concerned, the broken line in Fig. 11 corresponds to the dash-dotted line in Fig. 12.

As we said before, the model forced production from secondary directions 19 through 24 in the first period of time of the first application of the model. This is evidenced in Fig. 11 by the horizontal line between the abovementioned secondary directions. Note that the lower bounds on the angle requirements were of zero degree (negative angles were not allowed). It can be seen that in the second application of the model (Fig. 12) this did not occur due to the fact that in this model negative angles as lower bounds on the angle requirements were allowed.

The fact that the model did not force production from the above mentioned area explains the increase of the objective function value in the second application of the model because production was obtained from a more profitable area.

SUMMARY AND CONCLUSIONS

In summary, linear programming with sensitivity analysis of the optimum solution has been successfully applied to the following cases:

1. Total Block Caving operations. This mining method involves undercutting a large area with the surface formed between the ore and the waste remaining essentially horizontal.

2. Progressive Block Caving operations. A mining method that allows development work and production to occur simultaneously within a given area. The data used was provided by Climax Molybdenum Co.

For the latter case, two applications of a linear programming model were completed. The total revenue for the first application was found to be 0.73% larger than that obtained by the Climax Molybdenum Co. operation. The model was improved by the use of sensitivity analysis. The application of the improved model resulted in 4.25% more revenue than obtained by the mining company, with the same production.

The following conclusions can be drawn from this study:

1. Linear programming can be successfully applied for mine-production planning in operating Block-Caving mines.

2. Sensitivity analysis can be very useful in managerial decision-making. It is also a very useful tool for making the linear programming solution more implementable in the

real world: it gives a range where the linear-programming solution is valid. In this way, small changes in the real world do not affect the optimum solution.

3. By relating constraints to policies, for example cutoff grade or drawing angle policies, management can see the economic effect of such decisions through sensitivity analysis.

One caution is necessary. The user of any linear programming code should be sure that excessive roundoff error does not occur on the computer on which the program is run. Computers do not all have the same precision in handling numbers and when many manipulations of the data are required, as in the Simplex Algorithm, the loss of accuracy due to roundoff may become serious.

The ALPS I program used in this work avoids this problem by the use of a data refreshing technique.

SUGGESTIONS FOR FUTURE RESEARCH

In order to use a linear-programming model consistently in mine-production planning for block-caving mines, the following improvements would be needed:

1. A code that can be used to update the matrix every time there is a substantial change in the mine parameters.

2. An economic study that compares the extra profit that can be made by using a linear-programming model versus the expenses that the company would incur in the application of the model. These expenses could be for extra sampling, and extra supervision of the practical implementation of the model, etc.

3. It has been observed that the grade of a finger decreases towards the end of the life of the finger. If enough data are collected, it would be possible to obtain a graph that represents the grade of the finger versus the amount of ore drawn from it. This would improve the model if we could feed the new-grade data to the linear-programming code as the production of the finger increases. We can suggest one way of solving this problem:

Divide the finger horizontally into sections of different grades; associate a different variable to each section. Build constraints that oblige the model to start drawing from one section after the one below it is finished. This can be accomplished by the use of separable programming.

4. The total profit of a mine operation could be known before starting the development stage (before doing any investments), by building a model that includes the whole mine. If the mine is too large and there are no large-enough codes to solve the model, a small model can be built and rerun several times. In each different run the variables corresponding to fingers that are completely exhausted are left out and variables that correspond to new fingers can be incorporated.

APPENDIX A

Secondary Direction Profiles for the First and Second Application of the Progressive Block-Caving Model

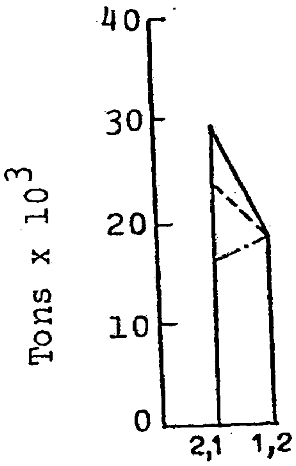

Fig. 1A. 2nd Secondary Direction Profile.

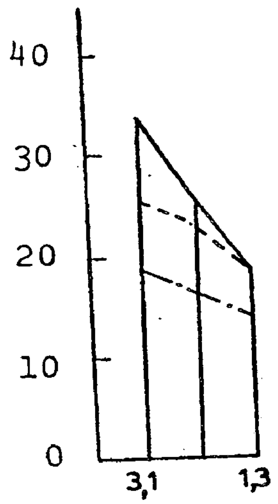

Fig. 2A. 3rd Secondary Direction Profile.

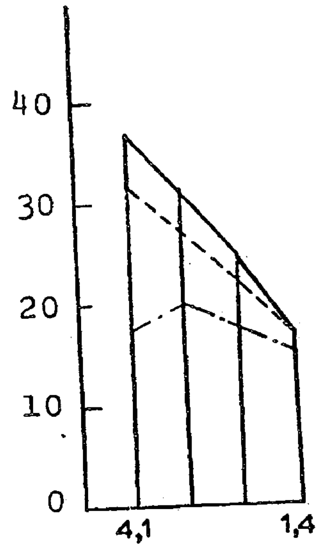

Fig. 3A. 4th Secondary Direction Profile.

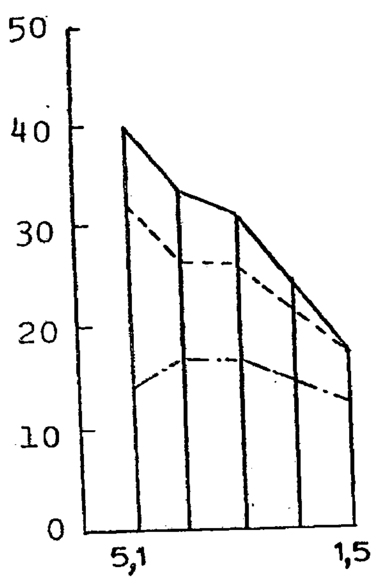

Fig. 4A. 5th Secondary Direction Profile.

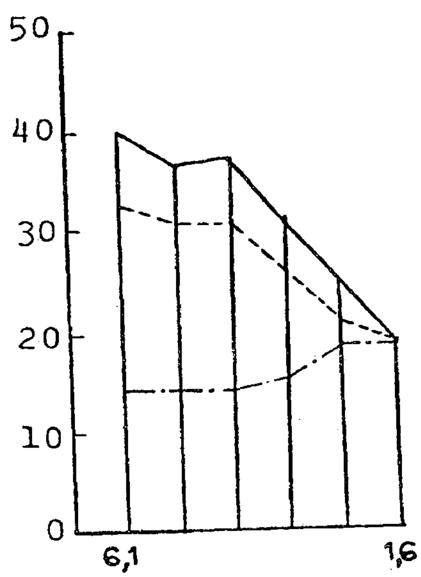

Fig. 5A. 6th Secondary Direction Profile.

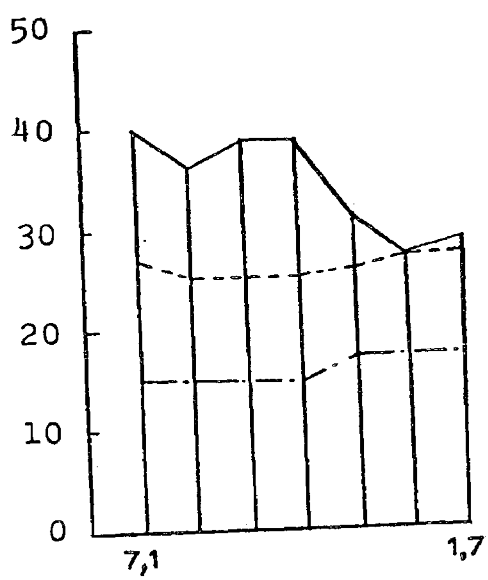

Fig. 6A. 7th Secondary Direction Profile.

Horizontal Scale: 1/4" = 45'

Vertical Scale: 1/2" = 10,000 tons

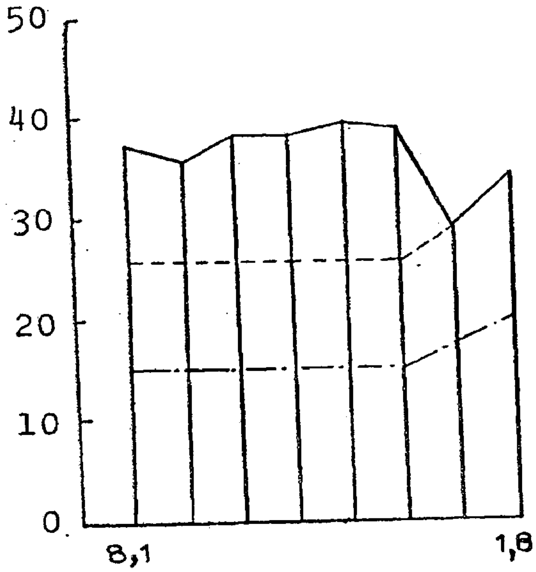

Fig. 7A. Eighth Secondary Direction Profile.

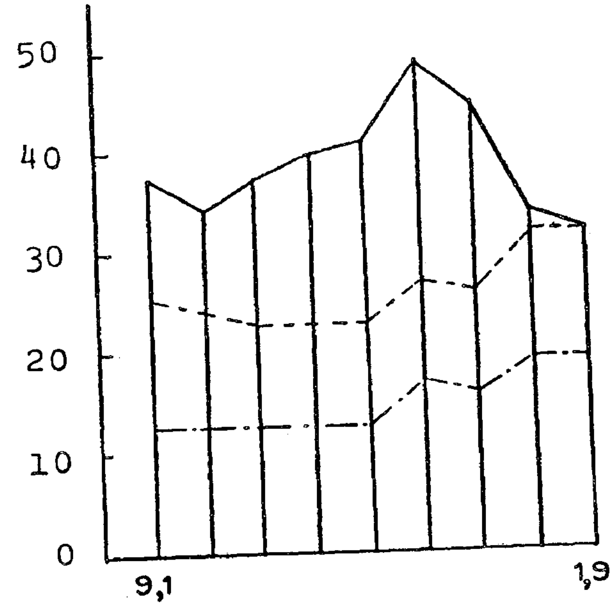

Fig. 8A. Ninth Secondary Direction Profile.

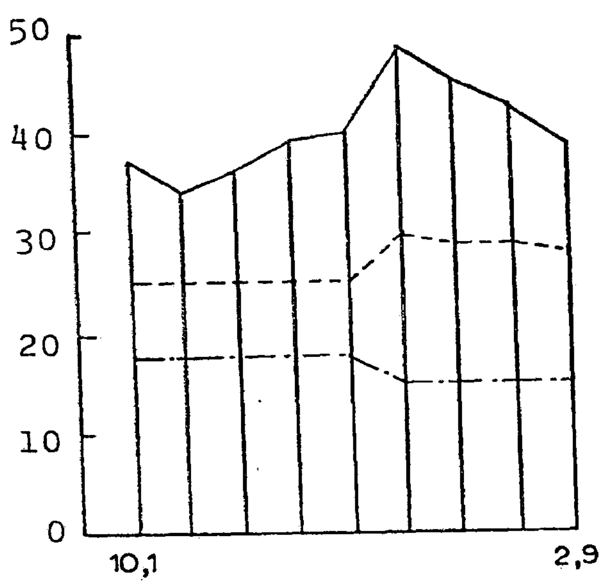

Fig. 9A. Tenth Secondary Direction Profile.

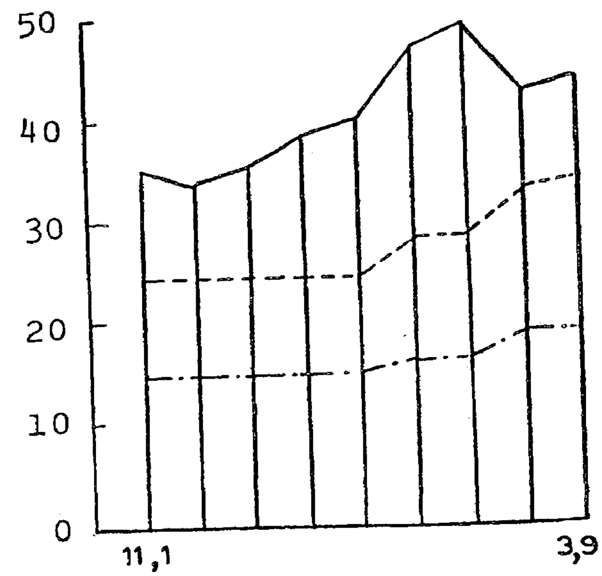

Fig. 10A. Eleventh Secondary Direction Profile.

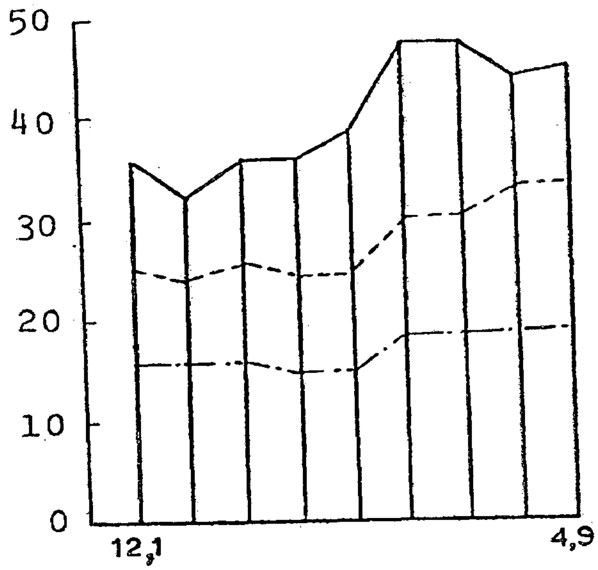

Fig. 11A. Twelfth Secondary Direction Profile

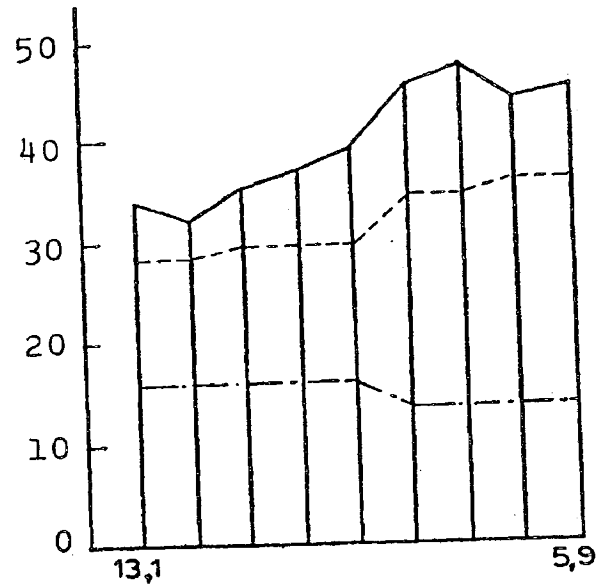

Fig. 12A. Thirteenth Secondary Direction Profile.

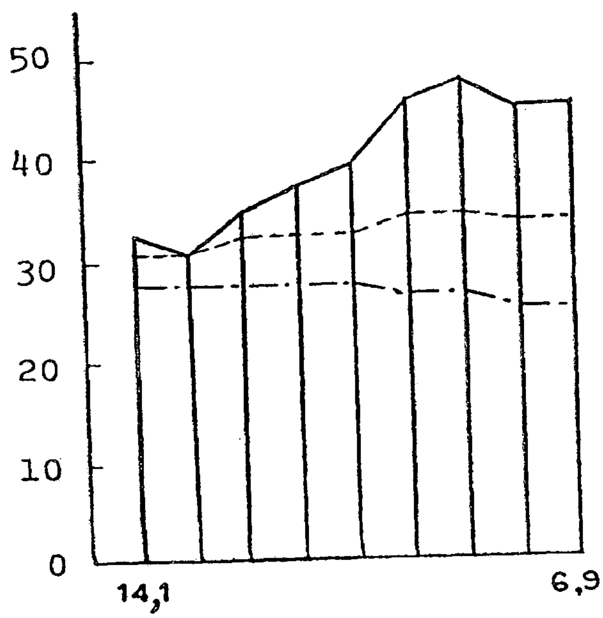

Fig. 13A. Fourteenth Secondary Direction Profile.

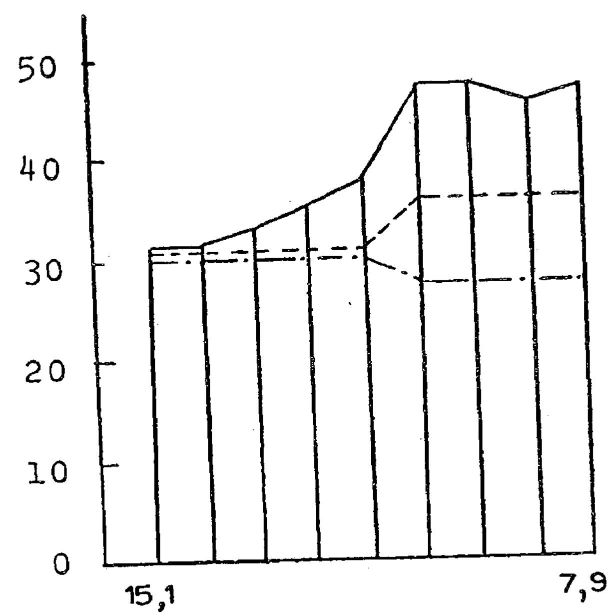

Fig. 14A. Fifteenth Secondary Direction Profile.

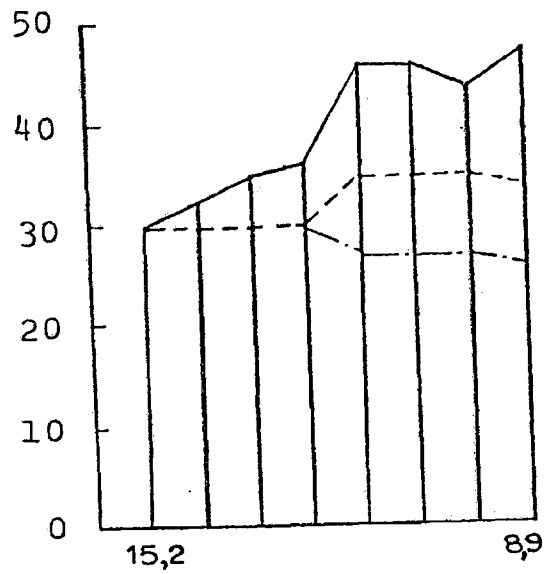

Fig. 15A. Sixteenth Secondary Direction Profile.

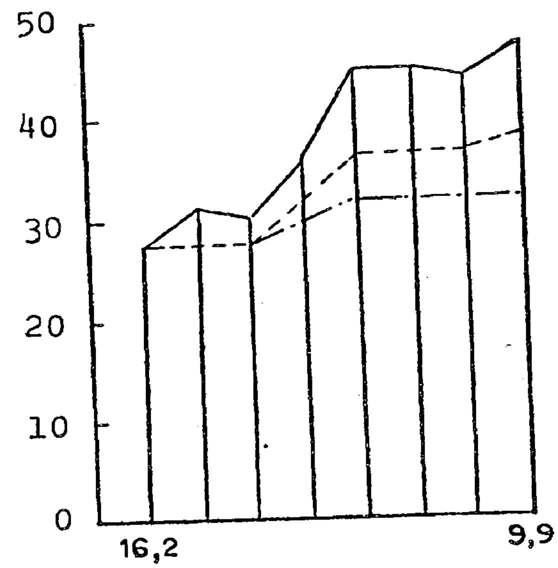

Fig. 16A. Seventeenth Secondary Direction Profile.

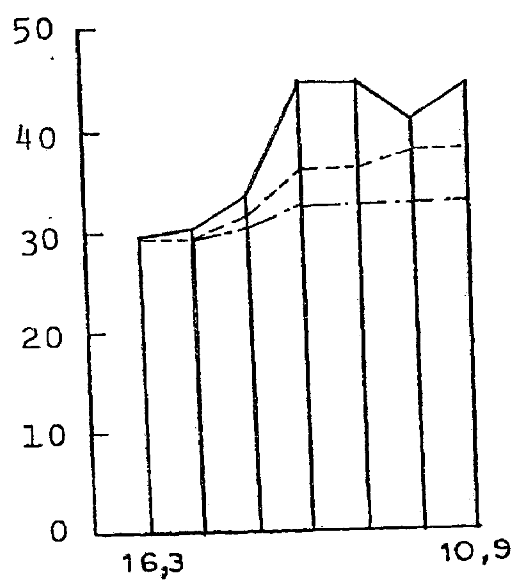

Fig. 17A. Eighteenth Secondary Direction Profile.

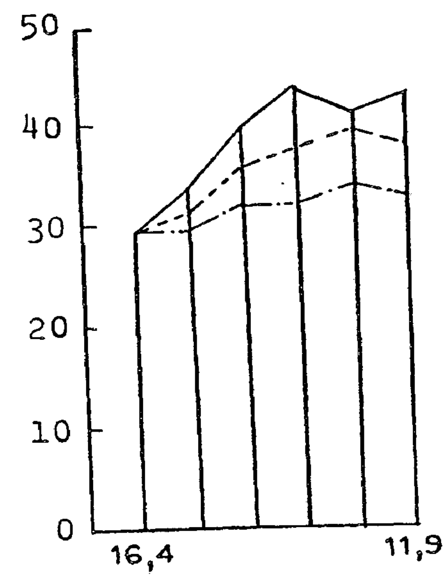

Fig. 18A. Nineteenth Secondary Direction Profile.

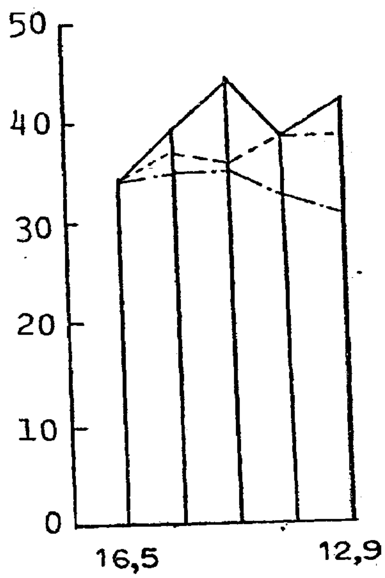

Fig. 19A. Twentieth Secondary Direction Profile.

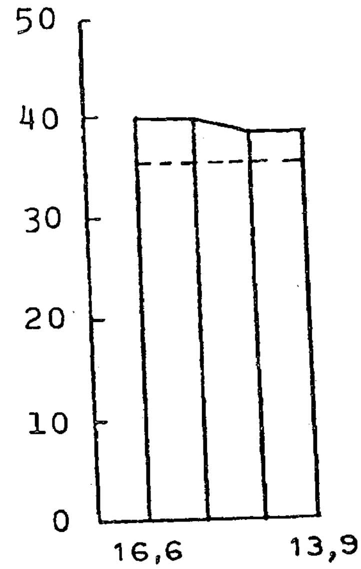

Fig. 20A. Twenty-first Secondary Direction Profile.

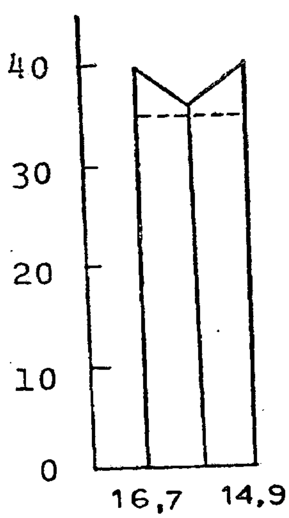

Fig. 21A. Twenty-second Secondary Direction Profile.

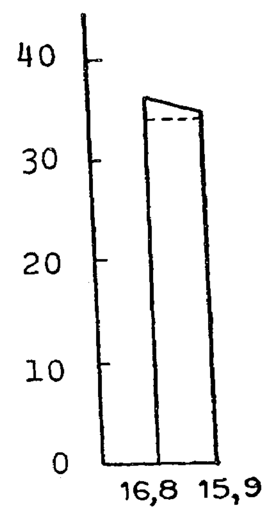

Fig. 22A. Twenty-third Secondary Direction Profile.

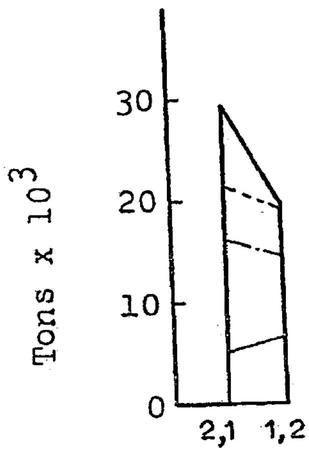

Fig. 23A. 2nd Secondary Direction Profile.

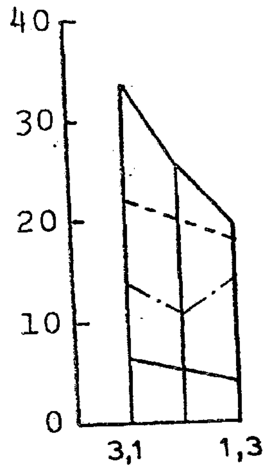

Fig. 24A. 3rd Secondary Direction Profile.

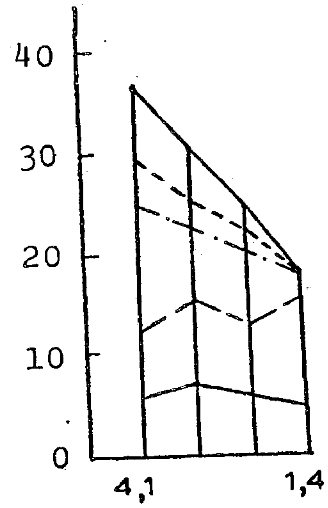

Fig. 25A. 4th Secondary Direction Profile.

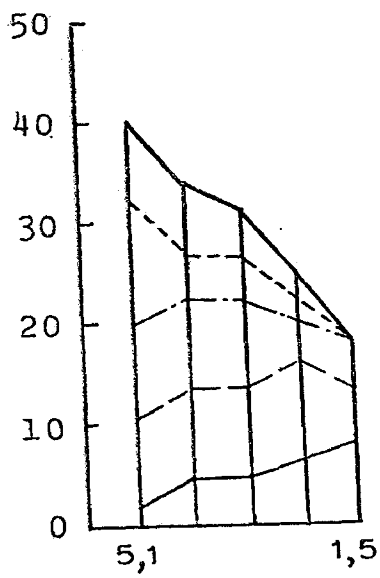

Fig. 26A. 5th Secondary Direction Profile.

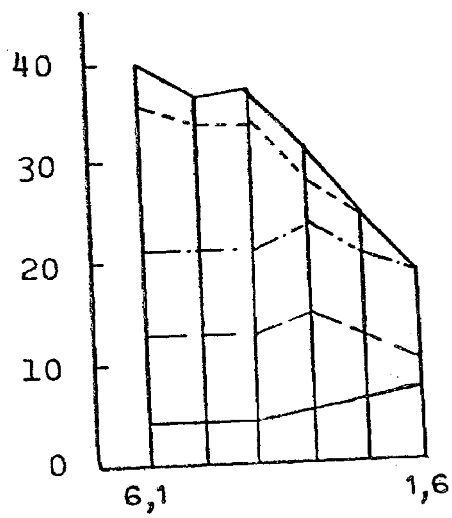

Fig. 27A. 6th Secondary Direction Profile.

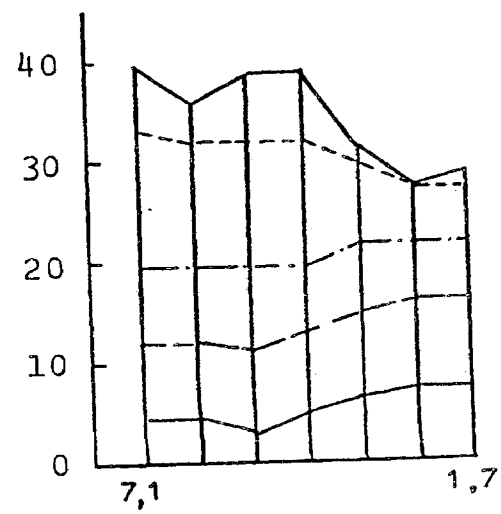

Fig. 28A. 7th Secondary Direction Profile.

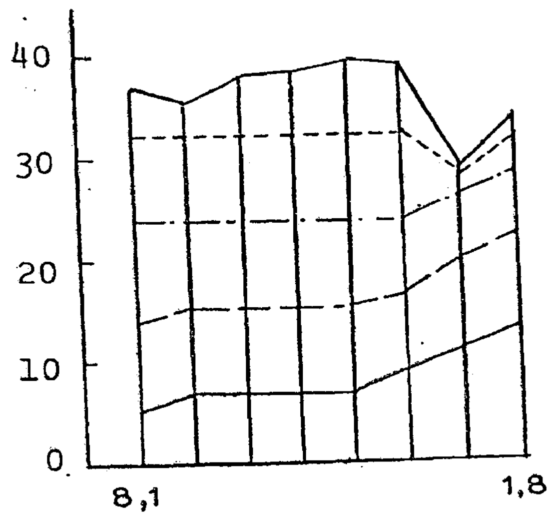

Fig. 29A. Eighth Secondary Direction Profile.

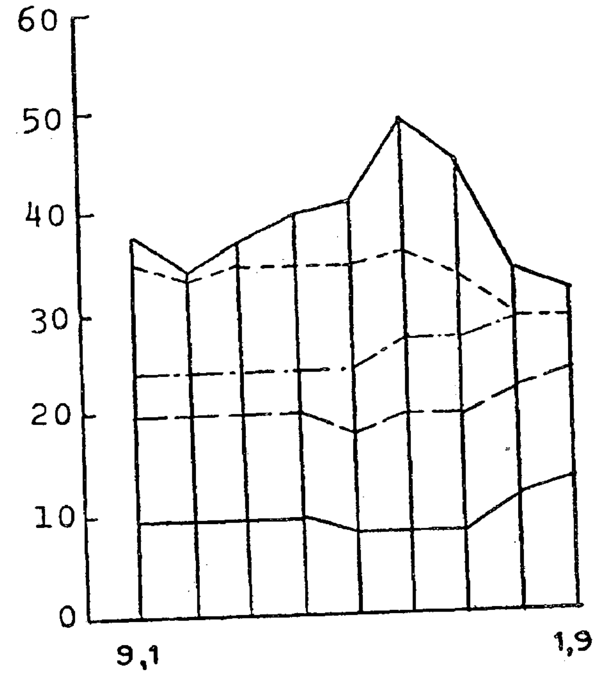

Fig. 30A. Ninth Secondary Direction Profile.

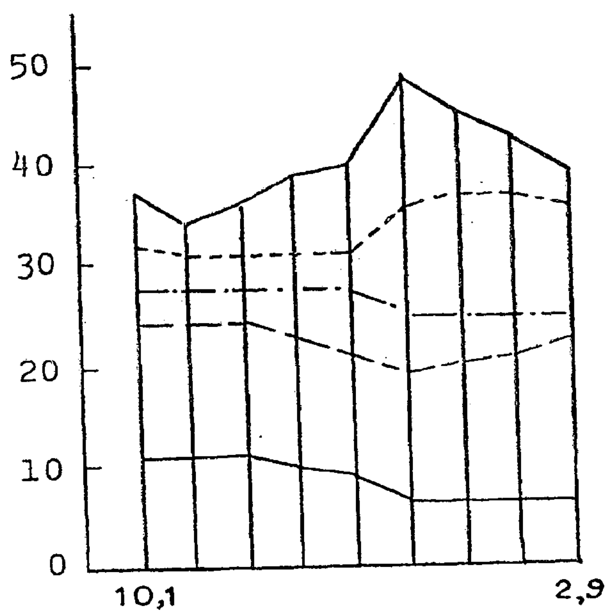

Fig. 31A. Tenth Secondary Direction Profile.

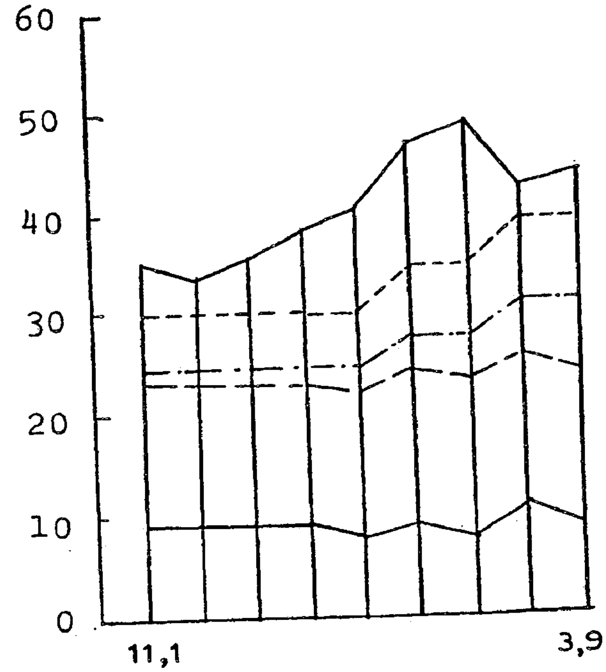

Fig. 32A. Eleventh Secondary Direction Profile.

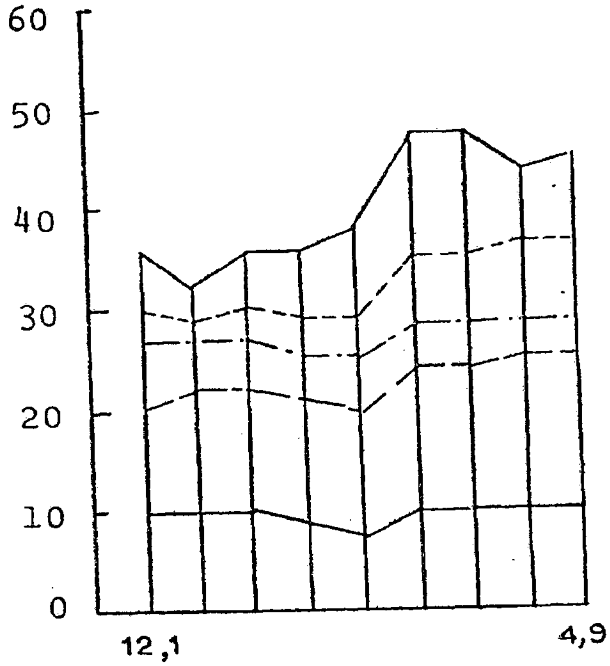

Fig. 33A. Twelfth Secondary Direction Profile.

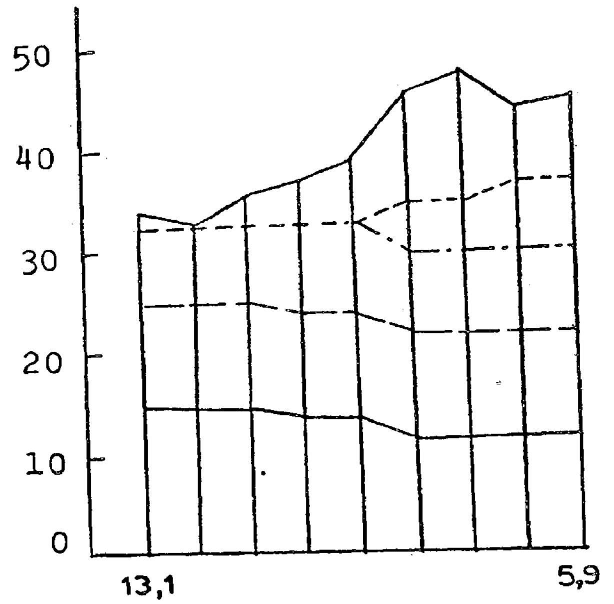

Fig. 34A. Thirteenth Secondary Direction Profile.

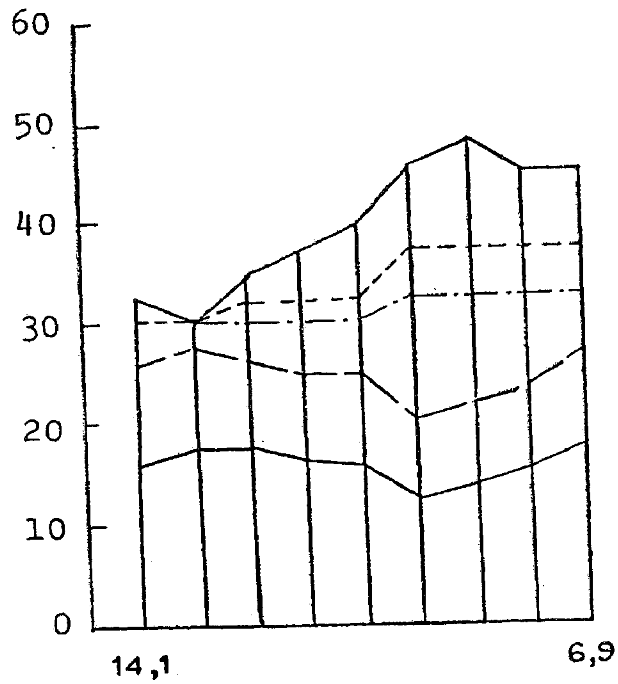

Fig. 35A. Fourteenth Secondary Direction Profile.

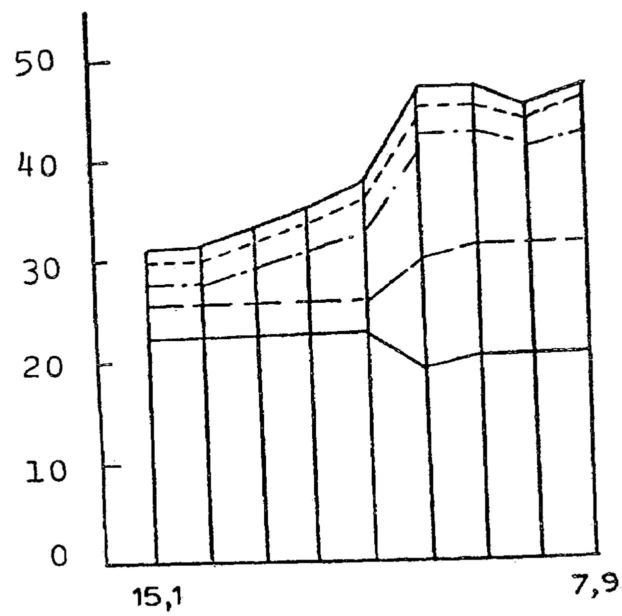

Fig. 36A. Fifteenth Secondary Direction Profile.

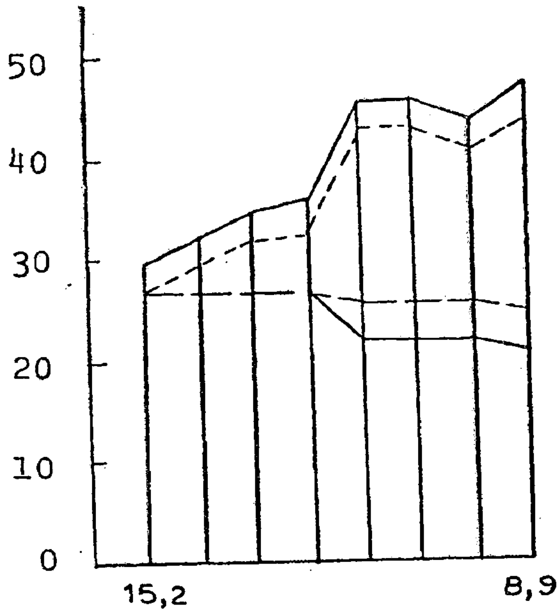

Fig. 37A. Sixteenth Secondary Direction Profile.

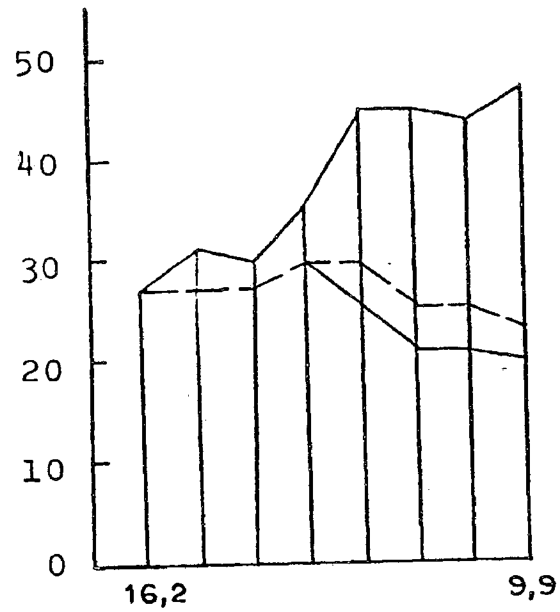

Fig. 38A. Seventeenth Secondary Direction Profile.

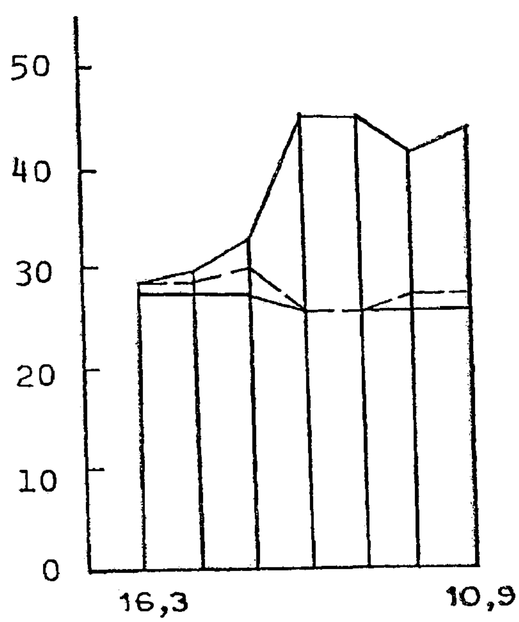

Fig. 39A. Eighteenth Secondary Direction Profile.

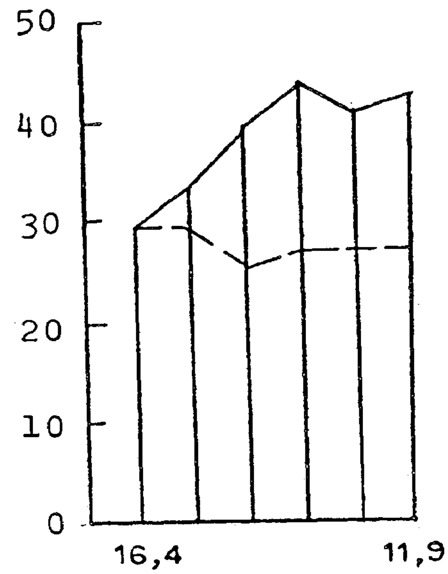

Fig. 40A. Nineteenth Secondary Direction Profile.

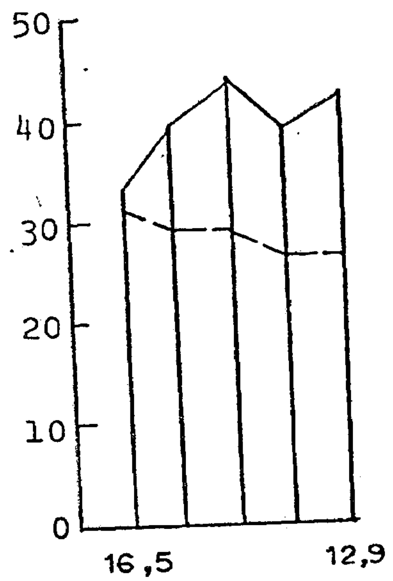

Fig. 41A. Twentieth Secondary Direction Profile.

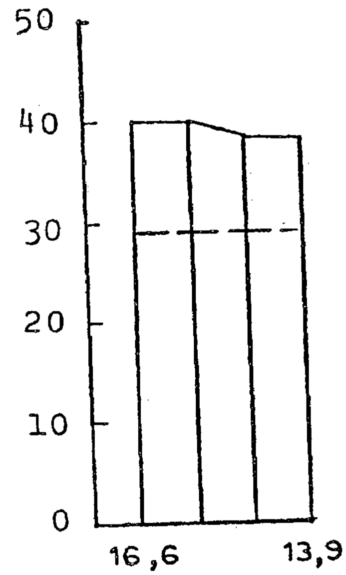

Fig. 42A. Twenty-first Secondary Direction Profile.

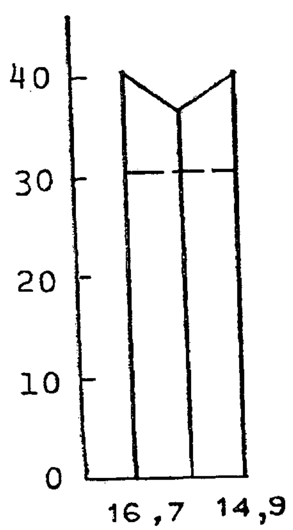

Fig. 43A. Twenty-second Secondary Direction Profile.

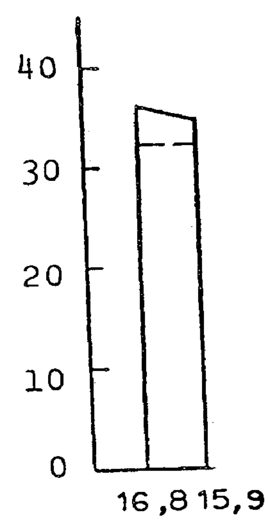

Fig. 44A. Twenty-third Secondary Direction Profile.

APPENDIX B

Tables Concerning Data, Results, and Sensitivity Analysis of the First and Second Application of the Progressive Block-Caving Model

Table 1B

Block Data for the Progressive Block-Caving Model

Name	Location (i,j)	Tonnage x 10^3	Grade	Revenue	$\partial(i,j)$
406-24A	1,1	17	0.230	7.912	22
406-24B	2,1	24	0.242	8.325	15
406-22A	3,1	27	0.230	7.912	12
406-22B	4,1	30	0.246	8.462	9
406-20A	5,1	32	0.300	10.320	7
406-20B	6,1	32	0.306	10.526	7
406-18A	7,1	32	0.310	10.664	7
406-18B	8,1	30	0.314	10.802	9
406-16A	9,1	30	0.324	11.146	9
406-16B	10,1	30	0.288	9.907	9
406-14A	11,1	29	0.223	7.671	10
406-14B	12,1	29	0.263	9.047	10
406-12A	13,1	27	0.210	7.224	12
406-12B	14,1	26	0.236	8.118	13
406-10A	15,1	25	0.200	6.880	14
406-24C	1,2	15	0.220	7.568	24
406-24D	2,2	20	0.252	8.669	19
406-22C	3,2	25	0.227	7.809	14
406-22D	4,2	27	0.249	8.566	12
406-20C	5,2	29	0.314	10.802	10
406-20D	6,2	29	0.292	10.045	10
406-18C	7,2	29	0.300	10.320	10
406-18D	8,2	27	0.244	8.394	12
406-16C	9,2	27	0.298	10.251	12
406-16D	10,2	27	0.349	12.006	12
406-14C	11,2	26	0.230	7.912	13
406-14D	12,2	26	0.256	8.806	13
406-12C	13,2	25	0.212	7.293	14
406-12D	14,2	25	0.234	8.050	14
406-10C	15,2	24	0.212	7.293	15
406-10D	16,2	22	0.192	6.605	17
406-24E	1,3	15	0.245	8.428	24
406-24F	2,3	20	0.227	7.809	19
406-22E	3,3	25	0.288	9.907	14
406-22F	4,3	30	0.188	6.467	9
406-20E	5,3	31	0.309	10.630	8
406-20F	6,3	31	0.297	10.217	8
406-18E	7,3	30	0.223	7.671	9
406-18F	8,3	29	0.201	6.914	10
406-16E	9,3	29	0.366	11.558	10
406-16F	10,3	29	0.363	12.487	10
406-14E	11,3	29	0.254	8.738	10
406-14F	12,3	28	0.232	7.981	11
406-12E	13,3	26	0.238	8.187	13
406-12F	14,3	26	0.218	7.499	13
406-10E	15,3	25	0.207	7.121	14
406-10F	16,3	23	0.192	6.605	17

Table 1B (Continued)

Name	Location (i,j)	Tonnage x 10^3	Grade	Revenue	$\partial(i,j)$
408-25E	1,4	14	0.250	8.600	25
408-23F	2,4	20	0.271	9.322	19
408-23E	3,4	25	0.224	7.706	14
408-21F	4,4	31	0.294	10.114	8
408-21E	5,4	31	0.318	10.939	8
408-19F	6,4	31	0.327	11.249	8
408-19E	7,4	31	0.267	9.185	8
408-17F	8,4	31	0.267	9.185	8
408-17E	9,4	30	0.264	9.082	9
408-15F	10,4	30	0.287	9.873	9
408-15F	11,4	30	0.299	10.286	9
408-15F	12,4	28	0.248	8.531	11
408-13E	13,4	28	0.244	8.394	11
408-11F	14,4	24	0.199	6.846	15
408-11E	15,4	24	0.191	6.570	15
408-9F	16,4	24	0.188	6.467	15
408-25C	1,5	14	0.261	8.978	25
408-23D	2,5	19	0.257	8.841	20
408-23C	3,5	25	0.262	9.013	14
408-21D	4,5	32	0.284	9.770	7
408-21C	5,5	32	0.231	7.946	7
408-19D	6,5	32	0.280	9.632	7
408-19C	7,5	32	0.280	9.632	7
408-17D	8,5	32	0.318	10.939	7
408-17C	9,5	31	0.218	9.666	8
408-15D	10,5	31	0.290	9.976	8
408-15C	11,5	30	0.268	9.219	9
408-13D	12,5	29	0.230	7.912	10
408-13C	13,5	29	0.187	6.433	10
408-11D	14,5	27	0.184	6.330	12
408-11C	15,5	27	0.206	7.086	12
408-9D	16,5	27	0.172	5.917	12
408-25A	1,6	15	0.240	8.256	24
408-23B	2,6	22	0.315	10.836	17
408-23A	3,6	31	0.264	9.082	8
408-21B	4,6	39	0.252	8.669	0
408-21A	5,6	39	0.243	8.359	0
408-19B	6,6	38	0.279	9.598	1
408-19A	7,6	38	0.255	8.772	1
408-17B	8,6	37	0.311	10.698	2
408-17A	9,6	37	0.274	9.426	2
408-15B	10,6	37	0.300	10.320	2
408-15A	11,6	37	0.258	9.804	2
408-13B	12,6	36	0.287	9.873	3
408-13A	13,6	36	0.226	7.774	3
408-11B	14,6	32	0.190	6.536	7
408-11A	15,6	32	0.200	6.880	7
408-9B	16,6	32	0.204	7.018	7

Table 1B (Continued)

Name	Location (i,j)	Tonnage x 10^3	Grade	Revenue	$\partial(i,j)$
408-24A	1,7	23	0.237	8.153	16
408-24B	2,7	23	0.248	8.531	16
408-22A	3,7	36	0.256	8.806	3
408-22B	4,7	36	0.268	9.219	3
408-20A	5,7	39	0.308	10.595	0
408-20B	6,7	38	0.285	9.804	1
408-18A	7,7	38	0.274	9.426	1
408-18B	8,7	38	0.277	9.529	1
408-16A	9,7	37	0.299	10.286	2
408-16B	10,7	37	0.259	8.910	2
408-14A	11,7	36	0.281	7.499	3
408-14B	12,7	36	0.235	8.084	3
408-12A	13,7	35	0.180	6.192	4
408-12B	14,7	35	0.190	6.536	4
408-10A	15,7	32	0.188	6.467	7
408-10B	16,7	32	0.176	6.054	7
408-24C	1,8	27	0.236	8.118	12
408-24D	2,8	27	0.270	9.288	12
408-22C	3,8	34	0.360	12.348	5
408-22D	4,8	34	0.274	9.426	5
408-20C	5,8	35	0.294	10.079	4
408-20D	6,8	35	0.308	10.595	4
408-18C	7,8	36	0.268	9.219	3
408-18D	8,8	36	0.253	8.703	3
408-16C	9,8	35	0.290	9.976	4
408-16D	10,8	35	0.268	9.219	4
408-14C	11,8	33	0.253	8.703	6
408-14D	12,8	33	0.186	6.398	6
408-12C	13,8	31	0.205	7.052	8
408-12D	14,8	31	0.265	9.116	8
408-10C	15,8	29	0.192	6.605	10
408-10D	16,8	29	0.172	5.917	10
408-24E	1,9	26	0.259	8.910	13
408-24F	2,9	31	0.313	10.767	8
408-22E	3,9	35	0.299	10.286	4
408-22A	4,9	36	0.280	9.632	3
408-20E	5,9	36	0.270	9.288	3
408-20F	6,9	36	0.259	8.910	3
408-18E	7,9	37	0.306	10.526	2
408-18F	8,9	38	0.367	12.625	1
408-16E	9,9	38	0.284	9.770	1
408-16F	10,9	35	0.274	9.426	4
408-14E	11,9	34	0.209	7.190	5
408-14F	12,9	34	0.240	8.256	5
408-12E	13,9	31	0.194	6.674	8
408-12F	14,9	32	0.176	6.054	7
408-10E	15,9	28	0.193	6.639	11
408-10F	16,9	28	0.171	5.882	11

Table 2B

Production Schedule for the First Application of the Progressive Block-Caving Model

Finger i,j	Grade % Mo	Revenue per ton	Reserves x 10^3 tons	Production (tons) Period I	Production (tons) Period II	Total Prod. of Finger	Ore Left in Finger
1,1	0.230	7.912	17	500	2792	3292	13708
2,1	0.242	8.325	24	5000	5583	10583	13417
3,1	0.230	7.912	27	6500	5125	11625	15375
4,1	0.246	8.462	30	4000	12042	16042	13958
5,1	0.300	10.320	32	6000	13864	19864	12126
6,1	0.306	10.526	32	5876	14801	20677	11323
7,1	0.310	10.664	32	10444	9906	20350	11650
8,1	0.314	10.802	30	9195	8369	17564	12436
9,1	0.324	11.146	30	9262	9247	18509	11491
10,1	0.288	8.907	30	8730	7334	16064	13936
11,1	0.223	7.671	29	9397	7334	16731	12269
12,1	0.263	9.047	29	7730	8567	16297	12703
13,1	0.210	7.224	27	4063	5901	9964	17036
14,1	0.236	8.118	26	1000	3097	4097	21903
15,1	0.200	6.880	25	431	333	764	24236
1,2	0.220	7.568	15	--	--	--	15000
2,2	0.252	8.669	20	1000	5625	6625	13375
3,2	0.227	7.809	25	3000	6042	9042	15958
4,2	0.249	8.566	27	5000	7864	12864	14136
5,2	0.314	10.802	29	3876	13801	17677	11323
6,2	0.292	10.045	29	8444	8906	17350	11650
7,2	0.300	10.320	29	8195	8369	16564	12436
8,2	0.244	8.394	27	7262	8247	15509	11491
9,2	0.298	10.251	27	6730	6334	13064	13936
10,2	0.349	12.006	27	7397	7334	14731	12269
11,2	0.230	7.912	26	5730	7567	13297	12703
12,2	0.256	8.806	26	3063	5901	8964	17036
13,2	0.212	7.293	25	--	3097	3097	21903
14,2	0.234	8.050	25	431	333	764	24236

Table 2B

Finger i,j	Grade % Mo	Revenue per ton	Reserves x 10^3 tons	Production (tons) Period I	Production (tons) Period II	Total Prod. of Finger	Ore Left in Finger
15,2	0.212	7.293	24	--	--	--	24000
16,2	0.192	6.605	22	--	--	--	22000
1,3	0.245	8.428	15	--	3625	3625	11375
2,3	0.227	7.809	20	2000	4042	6042	13958
3,3	0.288	9.907	25	3000	7864	10864	14136
4,3	0.188	6.467	30	4876	13801	18677	11323
5,3	0.309	10.630	31	10444	8906	19350	11650
6,3	0.297	10.217	31	10195	8369	18564	12436
7,3	0.223	7.671	30	11262	7247	18509	11491
8,3	0.201	6.914	29	8730	6334	15064	13936
9,3	0.366	11.558	29	9397	7334	16731	12269
10,3	0.363	12.487	29	7730	8567	16297	12703
11,3	0.254	8.738	29	5063	6901	11964	17036
12,3	0.232	7.981	28	2000	4097	6097	21903
13,3	0.238	8.187	26	1431	333	1764	24236
14,3	0.218	7.499	26	2000	--	2000	24000
15,3	0.207	7.121	25	3000	--	3000	22000
16,3	0.192	6.605	23	--	--	--	23000
1,4	0.250	8.600	14	--	2042	2042	11958
2,4	0.271	9.322	20	2000	5864	7864	12136
3,4	0.224	7.706	25	3876	7801	11677	13323
4,4	0.294	10.114	31	10444	8906	19350	11650
5,4	0.318	10.939	31	10195	8369	18564	12436
6,4	0.327	11.249	31	12262	7247	19509	11491
7,4	0.267	9.185	31	10730	6334	17064	13936
8,4	0.267	9.185	31	11397	7334	18731	12269
9,4	0.264	9.082	30	8730	8567	17297	12703
10,4	0.278	9.873	30	6063	6901	12964	17036
11,4	0.299	10.286	30	4000	4097	8097	21903
12,4	0.248	8.531	28	3431	333	3764	24236
13,4	0.244	8.394	28	4000	--	4000	24000
14,4	0.199	6.846	24	2000	--	2000	22000
15,4	0.191	6.570	24	1000	--	1000	23000
16,4	0.188	6.467	24	--	--	--	24000

Table 2B (Continued)

Finger i,j	Grade % Mo	Revenue per ton	Reserves x 10^3 tons	Production (tons) Period I	Period II	Total Prod. of Finger	Ore Left in Finger
1,5	0.261	8.978	14	--	3864	3864	10136
2,5	0.257	8.841	19	1876	1801	3677	15323
3,5	0.262	9.013	25	4000	7350	11350	13650
4,5	0.284	9.770	32	11195	8369	19564	12436
5,5	0.231	7.946	32	13262	7247	20509	11491
6,5	0.280	9.632	32	11730	6334	18064	13936
7,5	0.280	9.632	32	12397	7334	19731	12269
8,5	0.318	10.939	32	10730	8567	19297	12703
9,5	0.218	9.666	31	7063	6901	13964	17036
10,5	0.290	9.976	31	5000	4097	9097	21903
11,5	0.268	9.219	30	5431	333	5764	24236
12,5	0.230	7.912	29	5000	--	5000	24000
13,5	0.187	6.433	29	3457	1543	5000	24000
14,5	0.184	6.330	27	2000	1000	3000	24000
15,5	0.206	7.068	27	2000	1000	3000	24000
16,5	0.172	5.917	27	--	--	--	27000
1,6	0.240	8.256	15	--	--	--	15000
2,6	0.315	10.836	22	--	8350	8350	13650
3,6	0.264	9.083	31	10195	8369	18564	12436
4,6	0.252	8.669	39	16262	9247	25509	13491
5,6	0.243	8.359	39	14730	12334	27064	11936
6,6	0.279	9.598	38	15397	9334	24731	14269
7,6	0.255	8.772	38	12730	10567	23297	14703
8,6	0.311	10.698	37	9063	12901	21964	15036
9,6	0.274	9.426	37	10469	6628	17097	19903
10,6	0.300	10.320	37	8431	6333	14764	22236
11,6	0.258	9.804	37	9000	6000	15000	22000
12,6	0.287	9.873	36	6457	3650	10107	25893
13,6	0.226	7.774	36	7000	3000	10000	26000
14,6	0.190	6.536	32	3000	3000	6000	26000
15,6	0.200	6.880	32	1583	2042	3625	28375
16,6	0.204	7.018	32	3500	--	3500	28500
1,7	0.237	8.153	23	1000	8350	9350	13650

Table 2B (Continued)

Finger i,j	Grade % Mo	Revenue per ton	Reserves x 10^3 tons	Production (tons) Period I	Period II	Total Prod. of Finger	Ore Left in Finger
2,7	0.248	8.531	23	--	8564	8564	14436
3,7	0.256	8.806	36	14000	8509	22509	134912
4,7	0.268	9.219	36	12730	11334	24064	11936
5,7	0.308	10.595	39	16397	9334	25731	13269
6,7	0.285	9.804	38	12730	10567	23297	14703
7,7	0.274	9.426	38	10063	12901	22964	15036
8,7	0.277	9.529	38	11469	6628	18097	19903
9,7	0.299	10.286	37	8431	6333	14764	22236
10,7	0.259	8.910	37	9000	6000	15000	22000
11,7	0.281	7.499	36	6457	3650	10107	25893
12,7	0.235	8.084	36	7000	3000	10000	26000
13,7	0.180	6.192	35	5000	4000	9000	26000
14,7	0.190	6.536	35	5583	1042	6625	28375
15,7	0.188	6.467	32	3500	--	3500	28500
16,7	0.176	6.054	32	4000	--	4000	28000
1,8	0.236	8.118	27	--	10564	10564	16436
2,8	0.270	9.288	27	1000	10509	11509	15491
3,8	0.360	12.348	34	10730	11334	22064	11936
4,8	0.274	9.426	34	7397	11334	18731	15269
5,8	0.294	10.074	35	8730	11567	20297	14703
6,8	0.308	10.595	35	6063	13901	19964	15036
7,8	0.268	9.219	36	9469	6628	16097	19903
8,8	0.253	8.703	36	7431	6333	13764	22236
9,8	0.290	9.976	35	7000	6000	13000	22000
10,8	0.268	9.219	35	5457	3650	9107	25893
11,8	0.253	8.703	33	3000	4000	7000	26000
12,8	0.186	6.398	33	3000	4000	7000	26000
13,8	0.205	7.052	31	--	4625	4625	26375
14,8	0.265	9.116	31	2500	--	2500	28500
15,8	0.192	6.605	29	1000	--	1000	28000
16,8	0.172	5.917	29	1500	--	1500	27500
1,9	0.259	8.910	26	--	10509	10509	15491
2,9	0.313	10.767	31	8730	10334	19064	11936
3,9	0.299	10.286	35	8397	11334	19731	15269

Table 2B (Continued)

Finger i,j	Grade % Mo	Revenue per ton	Reserves x 10^3 tons	Production (tons) Period I	Production (tons) Period II	Total Prod. of Finger	Ore Left in Finger
4,9	0.280	9.632	36	9730	11567	21297	14703
5,9	0.270	9.288	36	7063	13901	20964	15036
6,9	0.259	8.910	36	9469	6628	16097	19903
7,9	0.306	10.526	37	8431	6333	14764	22236
8,9	0.367	12.625	38	11000	6000	17000	21000
9,9	0.284	9.770	38	7457	4650	12107	25893
10,9	0.274	9.426	35	5000	4000	9000	26000
11,9	0.209	7.190	34	4000	4000	8000	26000
12,9	0.240	8.256	34	2000	5625	7625	26375
13,9	0.194	6.674	31	2500	--	2500	28500
14,9	0.176	6.054	32	4000	--	4000	28000
15,9	0.193	6.639	28	500	--	500	27500
16,9	0.171	5.882	28	--	--	--	28000
Total			4264	831,985	831,983	1663,968	2600,032
Average Grade			0.268	0.289	0.283	0.286	0.257

Table 3B

Sensitivity of the Cost Coefficients of the First Application of the Progressive Block-Caving Model

Finger i,j	Actual Revenue	Actual Grade	Limits Period I Revenue Upper	Limits Period I Revenue Lower	Limits Period I Grade Upper	Limits Period I Grade Lower	Limits Period II Revenue Upper	Limits Period II Revenue Lower	Limits Period II Grade Upper	Limits Period II Grade Lower
1,1	7.912	0.230	8.042	Open	0.234	--	9.230	7.782	0.268	0.226
2,1	8.325	0.242	8.715	Open	0.254	--	8.325	7.938	0.242	0.230
3,1	7.912	0.230	8.042	7.912	0.233	0.230	7.912	7.782	0.230	0.226
4,1	8.462	0.246	8.592	Open	0.250	--	8.600	8.332	0.250	0.242
5,1	10.320	0.300	10.450	Open	0.302	--	11.374	10.190	0.330	0.294
6,1	10.526	0.306	10.526	7.464	0.306	0.217	11.405	10.526	0.332	0.306
7,1	10.664	0.310	10.664	10.664	0.310	0.310	10.664	10.664	0.310	0.310
8,1	10.802	0.314	13.824	10.802	0.402	0.314	10.802	7.886	0.314	0.229
9,1	11.146	0.324	11.146	11.146	0.324	0.324	11.146	11.146	0.324	0.324
10,1	9.907	0.288	9.907	9.907	0.288	0.288	9.907	9.907	0.288	0.288
11,1	7.671	0.223	56.340	0.770	1.680	0.022	8.725	7.500	0.254	0.218
12,1	9.047	0.263	9.047	9.047	0.263	0.263	9.047	9.047	0.263	0.263
13,1	7.224	0.210	9.677	6.957	0.282	0.203	7.515	6.792	0.218	0.197
14,1	8.118	0.236	8.118	Open	0.236	--	10.819	8.118	0.315	0.236
15,1	6.880	0.200	9.218	6.880	0.268	0.200	6.880	4.542	0.200	0.132
1,2	7.568	0.220	--	--	--	--	--	--	--	--
2,2	8.699	0.252	8.699	Open	0.252	--	8.944	8.699	0.260	0.252
3,2	7.809	0.227	8.069	Open	0.234	--	7.947	7.550	0.229	0.220
4,2	8.566	0.249	8.956	Open	0.260	--	9.621	8.178	0.280	0.238
5,2	10.802	0.314	10.802	10.802	0.314	0.314	10.802	10.802	0.314	0.314
6,2	10.045	0.292	10.045	10.045	0.292	0.292	10.045	10.045	0.292	0.292
7,2	10.320	0.300	13.342	10.320	0.388	0.300	10.320	7.404	0.300	0.216
8,2	8.394	0.244	8.394	8.394	0.244	0.244	8.394	8.394	0.244	0.244
9,2	10.251	0.298	10.251	10.251	0.298	0.298	10.251	10.251	0.298	0.298
10,2	12.006	0.349	60.675	3.565	1.760	0.140	13.061	11.835	0.380	0.342
11,2	7.912	0.230	12.440	7.912	0.362	0.230	7.912	7.480	0.230	0.218
12,2	8.806	0.256	11.249	8.539	0.327	0.248	9.097	8.374	0.262	0.242
13,2	7.293	0.212	--	--	--	--	9.994	7.293	0.290	0.212
14,2	8.050	0.234	10.388	8.050	0.301	0.234	8.050	5.712	0.234	0.166

Finger i,j	Actual Revenue	Actual Grade	Limits Period I Revenue Upper	Limits Period I Revenue Lower	Limits Period I Grade Upper	Limits Period I Grade Lower	Limits Period II Revenue Upper	Limits Period II Revenue Lower	Limits Period II Grade Upper	Limits Period II Grade Lower
15,2	7.293	0.212	--	--	--	--	13.036	Open	0.379	--
16,2	6.605	0.192	--	--	--	--	--	--	--	--
1,3	8.428	0.245	--	--	--	--	9.746	8.428	0.282	0.245
2,3	7.809	0.227	8.199	Open	0.238	--	8.977	7.671	0.261	0.223
3,3	9.907	0.288	10.297	Open	0.300	--	10.962	9.519	0.318	0.276
4,3	6.467	0.188	6.467	6.467	0.188	0.188	6.467	6.467	0.188	0.188
5,3	10.630	0.309	10.630	10.630	0.309	0.309	10.630	10.630	0.309	0.309
6,3	10.217	0.297	13.239	10.217	0.385	0.297	10.217	7.301	0.297	0.212
7,3	7.671	0.223	7.671	7.671	0.223	0.223	7.671	7.671	0.223	0.223
8,3	6.914	0.201	6.941	6.941	0.201	0.201	6.941	6.941	0.201	0.201
9,3	11.558	0.336	60.227	3.117	1.750	0.090	12.613	11.387	0.367	0.330
10,3	12.487	0.363	12.487	12.487	0.363	0.363	12.487	12.487	0.363	0.363
11,3	8.738	0.254	9.674	7.568	0.281	0.220	10.559	8.306	0.306	0.242
12,3	7.981	0.232	7.981	Open	0.232	--	10.682	7.981	0.308	0.232
13,3	8.178	0.238	10.525	8.187	0.306	0.238	8.187	5.849	0.238	0.170
14,3	7.499	0.218	16.619	Open	0.483	--	--	--	--	--
15,3	7.121	0.207	16.064	7.121	0.466	0.207	--	--	--	--
16,3	6.605	0.192	--	--	--	--	--	--	--	--
1,4	8.600	0.250	--	--	--	--	9.768	8.462	0.284	0.246
2,4	9.322	0.271	9.841	Open	0.286	--	10.377	8.806	0.301	0.256
3,4	7.706	0.224	7.706	7.706	0.224	0.224	7.706	7.706	0.224	0.224
4,4	10.114	0.294	10.114	10.114	0.294	0.294	10.114	10.114	0.294	0.294
5,4	10.939	0.318	13.961	10.939	0.405	0.318	10.939	8.023	0.318	0.234
6,4	11.249	0.327	11.249	11.249	0.327	0.327	11.249	11.249	0.327	0.327
7,4	9.185	0.267	9.185	9.185	0.267	0.267	9.185	9.185	0.267	0.267
8,4	9.185	0.267	57.854	0.744	1.680	0.022	10.240	9.014	0.295	0.262
9,4	9.082	0.264	9.082	9.082	0.264	0.264	9.082	9.082	0.264	0.264
10,4	9.873	0.278	10.809	8.703	0.314	0.253	11.694	9.441	0.340	0.274
11,4	10.286	0.299	10.286	Open	0.299	--	12.987	10.286	0.378	0.299
12,4	8.531	0.248	10.869	8.531	0.316	0.248	8.531	6.193	0.248	0.180
13,4	8.394	0.244	17.514	Open	0.510	--	14.137	Open	0.412	--
14,4	6.846	0.199	15.789	6.846	0.459	0.199	6.846	Open	0.199	--
15,4	6.570	0.191	22.963	Open	0.667	--	21.015	Open	0.611	--
16,4	6.467	0.188	--	--	--	--	--	--	--	--

Finger i,j	Actual Revenue	Actual Grade	Limits Period I Revenue Upper	Limits Period I Revenue Lower	Limits Period I Grade Upper	Limits Period I Grade Lower	Limits Period II Revenue Upper	Limits Period II Revenue Lower	Limits Period II Grade Upper	Limits Period II Grade Lower
1,5	8.978	0.261	--	--	--	--	10.033	7.620	0.300	0.222
2,5	8.841	0.257	9.556	8.841	0.278	0.257	8.841	8.133	0.257	0.236
3,5	9.013	0.262	9.013	9.013	0.262	0.262	9.013	9.013	0.262	0.262
4,5	9.770	0.284	12.792	9.770	0.372	0.284	9.770	6.854	0.284	0.199
5,5	7.946	0.231	7.946	7.946	0.231	0.231	7.946	7.946	0.231	0.231
6,5	9.632	0.280	9.632	9.632	0.280	0.280	9.632	9.632	0.280	0.280
7,5	9.632	0.280	58.301	1.191	0.694	0.033	10.687	9.461	0.310	0.275
8,5	10.939	0.318	10.939	10.939	0.318	0.318	10.939	10.939	0.318	0.318
9,5	9.666	0.281	10.602	8.496	0.309	0.247	11.487	9.233	0.334	0.268
10,5	9.976	0.290	9.976	Open	0.290	--	12.677	9.976	0.368	0.290
11,5	9.219	0.268	11.557	9.219	0.336	0.268	9.219	6.881	0.268	0.200
12,5	7.912	0.230	17.012	Open	0.495	--	13.655	Open	0.397	--
13,5	6.433	0.187	6.433	6.433	0.187	0.187	6.433	6.433	0.187	0.187
14,5	6.330	0.184	7.721	Open	0.225	--	20.775	4.939	0.602	0.143
15,5	7.068	0.206	8.276	Open	0.240	--	34.462	5.896	1.000	0.172
16,5	5.917	0.172	--	--	--	--	--	--	--	--
1,6	8.256	0.240	--	--	--	--	--	--	--	--
2,6	10.836	0.315	--	--	--	--	11.891	10.836	0.345	0.315
3,6	9.082	0.264	12.104	9.082	0.353	0.264	9.802	6.166	0.264	0.179
4,6	8.667	0.252	8.667	8.667	0.252	0.252	8.667	8.667	0.252	0.252
5,6	8.359	0.243	8.359	8.359	0.243	0.243	8.359	8.359	0.243	0.243
6,6	9.598	0.279	17.298	1.157	0.502	0.032	9.735	8.345	0.283	0.242
7,6	8.772	0.255	8.772	8.772	0.255	0.255	8.772	8.772	0.255	0.255
8,6	10.698	0.311	11.634	7.421	0.338	0.216	13.139	10.266	0.382	0.296
9,6	9.426	0.274	9.945	9.426	0.288	0.274	9.426	8.907	0.274	0.259
10,6	10.320	0.300	10.320	9.147	0.300	0.266	13.458	10.320	0.391	0.300
11,6	9.804	0.258	11.492	Open	0.334	--	9.804	8.112	0.258	0.236
12,6	9.873	0.287	9.873	9.873	0.287	0.287	9.873	9.873	0.287	0.287
13,6	7.774	0.226	8.887	Open	0.258	--	12.608	6.661	0.368	0.194
14,6	6.536	0.190	7.488	Open	0.218	--	25.050	5.584	0.730	0.162
15,6	6.880	0.200	6.880	6.880	0.200	0.200	6.880	6.880	0.200	0.200
16,6	7.018	0.204	15.772	Open	0.458	--	15.252	Open	0.444	--
1,7	8.153	0.237	8.153	Open	0.237	--	9.208	8.153	0.268	0.237
2,7	8.531	0.248	--	--	--	--	9.586	8.531	0.279	0.248

Finger i,j	Actual Revenue	Actual Grade	Limits Period I Revenue Upper	Limits Period I Revenue Lower	Limits Period I Grade Upper	Limits Period I Grade Lower	Limits Period II Revenue Upper	Limits Period II Revenue Lower	Limits Period II Grade Upper	Limits Period II Grade Lower
3,7	8.806	0.256	8.806	8.806	0.256	0.256	8.806	8.806	0.256	0.256
4,7	9.219	0.268	9.219	9.219	0.268	0.268	9.219	9.219	0.268	0.268
5,7	10.595	0.308	18.295	2.154	0.532	0.063	10.732	9.342	0.312	0.271
6,7	9.804	0.285	9.804	9.804	0.285	0.285	9.804	9.804	0.285	0.285
7,7	9.426	0.274	10.362	6.149	0.301	0.179	11.867	8.994	0.345	0.261
8,7	9.529	0.277	10.048	9.529	0.291	0.277	9.529	9.010	0.277	0.262
9,7	10.286	0.299	10.286	9.181	0.299	0.267	13.424	10.286	0.391	0.299
10,7	8.910	0.259	10.598	Open	0.308	--	14.653	7.222	0.426	0.210
11,7	7.499	0.281	7.499	7.499	0.281	0.281	7.499	7.499	0.281	0.281
12,7	8.084	0.235	9.197	Open	0.267	--	12.918	6.971	0.376	0.202
13,7	6.192	0.180	6.906	Open	0.201	--	24.706	5.478	0.720	0.159
14,7	6.356	0.190	6.796	6.536	0.197	0.190	6.536	6.276	0.190	0.182
15,7	6.467	0.188	15.221	Open	0.442	--	--	--	--	--
16,7	6.054	0.176	15.473	1.345	0.450	0.039	--	--	--	--
1,8	8.118	0.236	8.118	Open	0.236	--	9.173	8.118	0.266	0.236
2,8	9.288	0.270	9.288	Open	0.270	--	9.341	9.288	0.271	0.270
3,8	12.324	0.360	12.384	12.384	0.360	0.360	12.384	12.384	0.360	0.360
4,8	9.426	0.274	12.154	0.985	0.352	0.029	9.563	6.712	0.278	0.195
5,8	10.079	0.294	10.079	10.079	0.294	0.294	10.079	10.079	0.294	0.294
6,8	10.595	0.308	11.531	2.154	0.336	0.063	13.036	10.163	0.380	0.293
7,8	9.219	0.268	9.738	9.219	0.283	0.268	9.219	8.700	0.268	0.253
8,8	8.703	0.250	8.703	-10.764	0.250	--	11.801	8.703	0.343	0.250
9,8	9.976	0.290	11.664	Open	0.339	--	15.719	8.288	0.457	0.240
10,8	9.219	0.268	9.219	9.219	0.268	0.268	9.219	9.219	0.268	0.268
11,8	8.703	0.253	9.260	Open	0.269	--	13.537	8.146	0.394	0.236
12,8	6.398	0.186	7.112	Open	0.207	--	24.912	5.684	0.725	0.165
13,8	7.052	0.250	--	--	--	--	15.837	7.052	0.461	0.205
14,8	9.116	0.265	17.870	Open	0.519	--	17.350	Open	0.505	--
15,8	6.605	0.192	16.024	1.896	0.466	0.550	--	--	--	--
16,8	5.917	0.172	24.739	Open	0.719	--	12.116	-12.905	0.352	--
1,9	8.910	0.259	--	--	--	--	8.963	8.910	0.261	0.259
2,9	10.767	0.313	10.767	10.767	0.313	0.313	10.767	10.767	0.313	0.313
3,9	10.286	0.299	13.014	1.845	0.379	0.054	10.423	7.572	0.304	0.220

Finger i,j	Actual Revenue	Actual Grade	Limits Period I Revenue Upper	Limits Period I Revenue Lower	Limits Period I Grade Upper	Limits Period I Grade Lower	Limits Period II Revenue Upper	Limits Period II Revenue Lower	Limits Period II Grade Upper	Limits Period II Grade Lower
4,9	9.632	0.280	9.632	9.632	0.280	0.280	9.632	9.632	0.280	0.280
5,9	9.288	0.270	10.224	0.847	0.295	0.025	11.729	8.856	0.341	0.268
6,9	8.910	0.259	9.429	8.910	0.274	0.259	8.910	8.391	0.259	0.244
7,9	10.526	0.306	10.526	8.941	0.306	0.260	13.664	10.526	0.397	0.306
8,9	12.625	0.367	14.313	10.192	0.420	0.293	12.625	10.937	0.367	0.318
9,9	9.770	0.284	9.770	5.135	0.284	0.149	10.918	9.770	0.318	0.284
10,9	9.426	0.274	9.983	Open	0.290	--	14.260	8.869	0.415	0.258
11,9	7.190	0.209	7.904	Open	0.230	--	25.704	6.476	0.747	0.188
12,9	8.256	0.240	8.256	Open	0.240	--	17.041	8.256	0.495	0.240
13,9	6.674	0.194	15.428	Open	0.449	--	14.908	Open	0.434	--
14,9	6.054	0.176	15.473	1.345	0.450	0.039	--	--	--	--
15,9	6.639	0.193	25.461	Open	0.740	--	12.838	-12.183	0.373	--
16,9	5.882	0.171	--	--	--	--	--	--	--	--

Table B

Production Schedule for 2nd Application for the Progressive Block-Caving Model

Finger i,j	Grade Mo	Revenue per ton	Reserves x10^{3}tons	Production Period I	Production Period II	Production Period III	Production Period IV	Tot. Prod. of Finger	Ore Left in Finger
1,1	0.230	7.912	17	1500	--	4661	6459	12,635	4,365
2,1	0.242	8.325	24	7000	--	4661	7959	19,620	4,380
3,1	0.230	7.912	27	8583	--	6994	6126	21,703	5,297
4,1	0.246	8.462	30	6000	4000	9661	5459	25,120	4,880
5,1	0.300	10.320	32	6063	10021	7261	6659	30,004	1,996
6,1	0.306	10.526	32	4625	10375	5661	7959	28,620	3,380
7,1	0.310	10.664	32	5438	10848	5804	6173	28,263	3,737
8,1	0.314	10.802	30	4500	7028	7161	6445	25,134	4,866
9,1	0.324	11.146	30	2472	8000	4550	8251	23,273	6,727
10,1	0.288	9.907	30	3778	4250	2994	10140	21,162	8,838
11,1	0.223	7.671	29	4778	4917	881	12030	22,606	6,394
12,1	0.263	9.047	29	4111	3250	5326	8030	20,717	8,283
13,1	0.210	7.224	27	1000	28	5215	8475	14,718	12,282
14,1	0.236	8.118	26	1000	--	4000	7677	12,677	13,323
15,1	0.200	6.880	25	1333	1666	955	3026	6,980	18,020
1,2	0.220	7.568	15	--	--	4661	4959	9,620	5,380
2,2	0.252	8.669	20	3583	--	6994	5126	15,703	4,297
3,2	0.227	7.809	25	5000	2000	5661	6459	19,120	5,880
4,2	0.249	8.566	27	5063	4021	7261	6659	23,004	3,996
5,2	0.314	10.802	29	2625	9375	5661	7959	25,620	3,380
6,2	0.292	10.045	29	3438	9848	5804	6173	25,263	3,737
7,2	0.300	10.320	29	3500	7028	6161	6445	23,143	5,857
8,2	0.244	8.394	27	472	7000	4550	8251	20,273	6,727
9,2	0.298	10.251	27	1778	3250	2994	10140	18,162	8,838
10,2	0.349	12.006	27	2778	4917	881	12030	20,606	6,394
11,2	0.230	7.912	26	2111	2250	3326	10030	17,717	8,283
12,2	0.256	8.806	26	--	28	5215	8475	13,718	12,282
13,2	0.212	7.293	25	--	--	3000	7677	10,677	14,323
14,2	0.234	8.050	25	1333	1666	955	3025	6,979	18,021
15,2	0.212	7.293	24	2000	--	--	--	2,000	22,000
16,2	0.192	6.605	22	--	--	--	--	--	22,000

Finger i,j	Grade Mo	Revenue per ton	Reserves $x10^3$tons	Production Period I	Period II	Period III	Period IV	Tot. Prod. of Finger	Ore Left in Finger
1,3	0.245	8.428	15	583	--	2994	8126	11,703	3,297
2,3	0.227	7.809	20	2000	2000	5661	5459	15,120	4,880
3,3	0.288	9.907	25	3063	4021	7261	6659	21,004	3,996
4,3	0.188	6.467	30	3625	9375	5661	7959	29,320	770
5,3	0.309	10.630	31	5438	9848	6804	6173	28,263	2,737
6,3	0.297	10.217	31	5500	7028	6161	6445	25,134	5,866
7,3	0.223	7.671	30	2472	8000	4550	8251	23,273	6,727
8,3	0.201	6.914	29	3778	3250	2994	10140	20,162	8,838
9,3	0.366	11.558	29	4778	4917	881	12030	22,606	6,394
10,3	0.363	12.487	29	4111	3250	3326	10030	20,717	8,283
11,3	0.254	8.738	29	3028	--	5215	8475	16,718	12,282
12,3	0.232	7.981	28	2000	1000	4000	6677	13,677	14,323
13,3	0.238	8.187	26	1333	1666	1955	3026	7,980	18,020
14,3	0.218	7.499	26	2000	--	2000	--	4,000	22,000
15,3	0.207	7.121	25	--	--	3000	--	3,000	22,000
16,3	0.192	6.605	23	--	--	--	1187	1,187	21,813
1,4	0.250	8.600	14	--	--	1661	8459	10,120	3,880
2,4	0.271	9.322	20	2063	1938	3261	7659	14,921	5,079
3,4	0.244	7.706	25	2625	3375	5661	7959	19,620	5,380
4,4	0.294	10.114	31	5438	9848	5804	6173	27,263	3,737
5,4	0.318	10.939	31	5500	7028	6161	6445	25,134	5,866
6,4	0.327	11.249	31	3472	8000	5550	8251	25,273	5,727
7,4	0.267	9.185	31	5778	3250	3994	10140	23,162	7,838
8,4	0.267	9.185	31	6778	4917	881	12030	24,606	6,394
9,4	0.264	9.082	30	5111	3250	3326	10030	21,717	8,283
10,4	0.278	9.873	30	4028	--	6215	8475	18,718	11,282
11,4	0.299	10.286	30	4000	1000	5000	6677	16,677	13,323
12,4	0.248	8.531	28	1333	1666	3955	3026	9,980	18,020
13,4	0.244	8.394	28	2000	--	--	4000	6,000	22,000
14,4	0.199	6.864	24	--	--	2000	--	2,000	22,000
15,4	0.191	6.570	24	--	--	1000	1187	2,187	21,813
16,4	0.188	6.467	24	--	--	289	--	289	23,711

Finger i,j	Grade Mo	Revenue per ton	Reserves x10^{3}tons	Production Period I	Production Period II	Production Period III	Production Period IV	Tot. Prod. of Finger	Ore Left in Finger
1,5	0.261	8.978	14	--	--	3261	4659	7,920	6,080
2,5	0.257	8.841	19	--	2000	5661	4959	12,620	6,380
3,5	0.262	9.013	25	2000	5286	5804	6173	19,263	5,737
4,5	0.284	9.770	32	6500	7028	6161	6445	26,134	5,866
5,5	0.231	7.946	32	4472	8000	6550	8251	27,273	4,727
6,5	0.280	9.632	32	6778	3250	4994	10140	25,162	6,838
7,5	0.280	9.632	32	7778	4917	1881	12030	26,606	5,394
8,5	0.318	10.939	32	7111	3250	4326	10030	24,717	7,283
9,5	0.218	9.666	31	5028	--	6214	8475	19,719	11,283
10,5	0.290	9.976	31	5000	1000	5000	6677	17,677	13,323
11,5	0.268	9.219	30	1333	1666	5955	3026	11,980	18,020
12,5	0.230	7.912	29	2000	--	5000	--	7,000	2,200
13,5	0.187	6.433	29	--	--	5187	--	5,187	23,813
14,5	0.184	6.330	27	--	--	3187	2000	5,183	21,817
15,5	0.206	7.068	27	--	--	3289	--	3,289	23,711
16,5	0.172	5.917	27	--	--	2255	--	2,255	24,745
1,6	0.240	8.256	15	--	--	5661	1959	7,620	7,380
2,6	0.315	10.836	22	--	4286	4804	7173	16,263	5,737
3,6	0.264	9.083	31	5500	7028	5161	6445	24,134	6,866
4,6	0.252	8.669	39	10472	7000	6550	9251	33,273	5,727
5,6	0.243	8.359	39	9778	9250	4994	10140	34,162	4,808
6,6	0.279	9.598	38	9778	6917	1881	13030	31,606	6,394
7,6	0.255	8.772	38	9111	5250	4326	11030	29,717	8,283
8,6	0.311	10.698	37	8478	4549	6215	8475	27,717	9,283
9,6	0.274	9.426	37	7000	3000	13931	5722	29,653	7,347
10,6	0.300	10.320	37	1333	1666	10289	9026	22,314	14,686
11,6	0.258	9.804	37	2000	--	24955	3043	29,998	7,002
12,6	0.287	9.873	36	--	--	9000	3293	12,293	23,707
13,6	0.226	7.774	36	--	--	16187	--	16,187	19,813
14,6	0.190	6.536	32	--	--	22289	--	22,289	9,711
15,6	0.200	6.880	32	--	--	9255	--	9,255	22,745
16,6	0.204	7.018	32	--	--	8955	--	8,955	23,045

Table 4B (Continued)

Finger i,j	Grade Mo	Revenue per ton	Reserves $x10^3$tons	Production Period I	Period II	Period III	Period IV	Tot. Prod. of Finger	Ore Left in Finger
1,7	0.237	8.153	23	1000	4286	4804	7173	17,263	5,737
2,7	0.248	8.531	23	15000	1028	4161	7445	14,134	8,866
3,7	0.256	8.306	36	8472	6000	6550	9251	30,273	5,727
4,7	0.268	9.219	36	5778	10250	3994	11140	31,162	4,838
5,7	0.308	10.595	39	10778	6917	2881	13030	33,606	5,394
6,7	0.285	9.804	38	9111	5250	4326	11030	29,717	8,283
7,7	0.274	9.426	38	9478	4549	6215	8475	28,717	9,283
8,7	0.277	9.529	38	8000	3000	10000	5722	26,722	11,278
9,7	0.299	10.286	37	1333	1667	8955	9026	20,981	16,019
10,7	0.259	8.910	37	2000	--	13931	3043	18,974	18,026
11,7	0.281	7.499	36	--	--	16187	3293	19,480	16,520
12,7	0.235	8.084	36	--	--	16187	--	16,187	19,813
13,7	0.180	6.192	35	--	--	13289	--	13,289	21,711
14,7	0.190	6.536	35	--	--	12255	--	12,255	22,745
15,7	0.188	6.467	32	--	--	8955	--	8,955	23,045
16,7	0.176	6.054	32	--	--	7455	--	7,455	24,455
1,8	0.236	8.118	27	1500	3028	4161	7445	16,134	10,866
2,8	0.270	9.288	27	3472	--	6550	9251	19,273	7,727
3,8	0.360	12.348	34	3778	10250	3994	11140	29,162	4,838
4,8	0.274	9.426	34	1778	8917	2881	13030	26,606	7,394
5,8	0.294	10.074	35	5111	6250	3326	12030	26,717	8,283
6,8	0.308	10.595	35	5478	5549	6215	8475	25,717	9,283
7,8	0.268	9.219	36	6000	3000	9000	5722	23,722	12,278
8,8	0.253	8.703	36	1333	1667	7955	9026	19,981	16,019
9,8	0.290	9.976	35	2000	--	11931	3043	16,974	18,026
10,8	0.268	9.219	35	--	--	15187	3293	18,480	16,520
11,8	0.253	8.703	33	--	--	12187	1000	13,187	19,813
12,8	0.186	6.398	33	--	--	11289	--	11,289	21,711
13,8	0.205	7.052	31	--	--	10255	--	10,255	20,745
14,8	0.265	9.116	31	--	--	7955	--	7,955	23,045
15,8	0.192	6.605	29	--	--	4455	--	4,455	24,545
16,8	0.172	5.917	29	--	--	2955	--	2,955	26,045

Table 4B (Continued)

Finger i,j	Grade Mo	Revenue per ton	Reserves x10³tons	Production Period I	Period II	Period III	Period IV	Tot. Prod. of Finger	Ore Left in Finger
1,9	0.259	8.910	26	2472	--	5550	9251	17,273	8,727
2,9	0.313	10.767	31	1778	9250	1994	13140	26,162	4,838
3,9	0.299	10.286	35	2778	8917	3881	13030	28,606	6,394
4,9	0.280	9.632	36	6111	6250	3326	12030	27,717	8,283
5,9	0.270	9.288	36	6478	5549	6215	8475	26,717	9,283
6,9	0.259	8.910	36	6000	3000	8000	5722	22,722	13,278
7,9	0.306	10.526	37	1333	1666	8955	9026	20,980	16,020
8,9	0.367	12.625	38	2000	--	15931	3043	20,974	17,026
9,9	0.284	9.770	38	--	--	19187	2293	21,480	16,520
10,9	0.274	9.426	35	--	--	14187	1000	15,187	19,813
11,9	0.209	7.190	34	--	--	12289	--	12,289	21,711
12,9	0.240	8.256	34	--	--	13255	--	13,255	20,745
13,9	0.194	6.674	31	--	--	7955	--	7,955	23,045
14,9	0.176	6.054	32	--	--	7455	--	7,455	24,545
15,9	0.193	6.639	28	--	--	1955	--	1,955	26,045
16,9	0.171	5.882	28	--	--	--	--	--	28,000
Total			4264	415,991	415,991	831,983	831,983	2495,948	1768,052
Average Grade			0.268	0.291	0.285	0.274	0.282	0.281	0.247

Table 5B

Revenue Increment Required to Make Non-Productive Fingers Economical for Mining, for the Second Application of the Progressive Block-Caving Model

Finger i,j	Actual Revenue per ton	Necessary Revenue Increment Per Ton			
		Period I	Period II	Period III	Period IV
1,2	7.568	4.978	4.978		
12,2	8.806	0.000			
13,2	7.293	15.599			
1,4	8.600	9.425	9.425		
1,5	8.978	0.895	0.895		
1,6	8.256	2.673	2.673		
2,6	10.836	0.000			
10,8	9.219	0.000			
11,9	7.190	44.341			
16,9	5.882	42.848	3.712	3.428	3.366
1,1	7.912		0.000		
3,1	7.912		0.000		
14,1	8.118		15.599		
2,2	8.699		0.000		
15,2	7.293		0.000	46.224	10.217
1,3	8.428		0.000		
11,3	8.738		0.000		
16,3	6.605		26.644	0.123	
13,7	6.192		17.697		
16,8	5.917		39.154		5.940
1,9	8.910		0.000		
13,9	6.674		0.000		
16,2	6.605			1.867	3.340
15,3	7.121				2.946
16,4	6.467				0.000
13,5	6.433				2.657
15,5	7.068				0.000
16,5	5.917				0.000
14,6	6.536				5.411
15,6	6.880				0.000
16,6	7.018				29.528
12,7	8.084				2.146
14,7	6.356				0.000
16,7	6.054				3.011
13,8	7.052				0.000
15,8	6.605				3.011
14,9	6.054				3.011

Table 6B

Sensitivity of the Cost Coefficients for the 2nd Application of the Progressive Block-Caving Model

			Limits Period I				Limits Period II			
			Revenue		Grade		Revenue		Grade	
Finger i,j	Actual Revenue	Actual Grade	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower
1,1	7.912	0.230	9.594	7.912	0.278	0.230	--	--	--	--
2,1	8.325	0.242	9.166	8.325	0.266	0.242	8.325	7.484	0.242	0.218
3,1	7.912	0.230	8.157	7.912	0.237	0.230	--	--	--	--
4,1	8.462	0.246	8.462	8.462	0.246	0.246	8.462	8.462	0.246	0.246
5,1	10.320	0.300	10.320	10.320	0.300	0.300	10.320	10.320	0.300	0.300
6,1	10.526	0.306	10.526	10.526	0.306	0.306	10.526	10.526	0.306	0.306
7,1	10.664	0.310	10.664	10.664	0.310	0.310	10.664	10.664	0.310	0.310
8,1	10.802	0.314	10.802	10.802	0.314	0.314	10.802	10.802	0.314	0.314
9,1	11.146	0.324	11.146	11.146	0.324	0.324	11.146	11.146	0.324	0.324
10,1	9.907	0.288	9.907	9.907	0.288	0.288	9.907	9.907	0.288	0.288
11,1	7.671	0.223	7.671	7.671	0.223	0.223	7.671	7.671	0.223	0.229
12,1	9.047	0.263	9.047	9.047	0.263	0.263	9.047	9.047	0.263	0.263
13,1	7.224	0.210	7.224	Open	0.210	Open	7.401	7.224	0.215	0.210
14,1	8.118	0.236	23.717	Open	0.690	Open	--	--	--	--
15,1	6.880	0.200	6.880	6.880	0.200	0.200	6.880	6.880	0.200	0.200
1,2	7.568	0.220	--	--	--	--	--	--	--	--
2,2	8.699	0.252	8.699	8.699	0.252	0.252	--	--	--	--
3,2	7.809	0.227	7.809	7.809	0.227	0.227	7.809	7.809	0.227	0.227
4,2	8.566	0.249	8.566	8.566	0.249	0.249	8.566	8.566	0.249	0.249
5,2	10.802	0.314	10.802	10.802	0.314	0.314	10.802	10.802	0.314	0.314
6,2	10.045	0.292	10.045	10.045	0.292	0.292	10.045	10.045	0.292	0.292
7,2	10.320	0.300	10.320	10.320	0.300	0.300	10.320	10.320	0.300	0.300
8,2	8.394	0.244	8.394	8.394	0.244	0.244	8.394	8.394	0.244	0.244
9,2	10.251	0.298	10.251	10.251	0.298	0.298	10.251	10.251	0.298	0.
10,2	12.006	0.349	12.006	12.006	0.349	0.349	12.006	12.006	0.349	0.349
11,2	7.912	0.230	7.912	7.912	0.230	0.230	7.912	7.912	0.230	0.230
12,2	8.806	0.256	--	--	--	--	8.983	8.886	0.261	0.256
13,2	7.293	0.212	--	--	--	--	22.892	Open	0.665	Open
14,2	8.050	0.234	8.050	8.050	0.234	0.234	8.050	8.050	0.234	0.234
15,2	7.293	0.212	11.635	7.293	0.338	0.212	--	--	--	--
16,2	6.605	0.192	6.605	6.605	0.192	0.192	6.605	6.605	0.192	0.192

Table 6B (Continued)

Finger i,j	Actual Revenue	Actual Grade	Limits Period I Revenue Upper	Limits Period I Revenue Lower	Limits Period I Grade Upper	Limits Period I Grade Lower	Limits Period II Revenue Upper	Limits Period II Revenue Lower	Limits Period II Grade Upper	Limits Period II Grade Lower
1,3	8.428	0.245	12.172	8.428	0.354	0.245	--	--	--	--
2,3	7.809	0.227	7.809	Open	0.227	Open	15.307	7.809	0.445	0.227
3,3	9.907	0.288	9.907	9.907	0.288	0.288	9.907	9.907	0.288	0.288
4,3	6.467	0.188	6.467	6.467	0.188	0.188	6.467	6.467	0.188	0.188
5,3	10.630	0.309	10.630	10.630	0.309	0.309	10.630	10.630	0.309	0.309
6,3	10.217	0.297	10.217	10.217	0.297	0.297	10.217	10.217	0.297	0.297
7,3	7.671	0.223	7.671	7.671	0.223	0.223	7.671	7.671	0.223	0.223
8,3	6.914	0.201	6.914	6.914	0.201	0.201	6.914	6.914	0.201	0.201
9,3	11.558	0.336	11.558	11.558	0.336	0.336	11.558	11.558	0.336	0.336
10,3	12.487	0.363	12.487	12.487	0.363	0.363	12.487	12.487	0.363	0.363
11,3	11.738	0.254	11.738	11.738	0.254	0.254	--	--	--	--
12,3	7.981	0.232	7.981	Open	0.232	Open	23.580	7.981	0.700	0.232
13,3	8.178	0.238	8.178	8.178	0.238	0.238	8.178	8.178	0.238	0.238
14,3	7.499	0.218	11.841	7.499	0.344	0.281	7.499	Open	0.281	Open
15,3	7.121	0.207	7.121	7.121	0.207	0.207	7.121	7.121	0.207	0.207
16,3	6.605	0.192	33.249	Open	0.966	Open	--	--	--	--
1,4	8.600	0.250	--	--	--	--	--	--	--	--
2,4	9.322	0.271	9.322	9.322	0.271	0.271	9.322	9.322	0.271	0.271
3,4	7.706	0.224	7.706	7.706	0.224	0.224	7.706	7.706	0.224	0.224
4,4	10.114	0.294	10.114	10.114	0.294	0.294	10.114	10.114	0.294	0.294
5,4	10.939	0.318	10.939	10.939	0.318	0.318	10.939	10.939	0.318	0.318
6,4	11.249	0.327	11.249	11.249	0.327	0.327	11.249	11.249	0.327	0.327
7,4	9.185	0.267	9.185	9.185	0.267	0.267	9.185	9.185	0.267	0.267
8,4	9.185	0.267	9.185	9.185	0.267	0.267	9.185	9.185	0.267	0.267
9,4	9.082	0.264	9.082	9.082	0.264	0.264	9.082	9.082	0.264	0.264
10,4	9.873	0.278	10.050	9.873	0.291	0.278	9.873	Open	0.278	Open
11,4	10.286	0.299	10.286	Open	0.299	Open	25.885	10.286	0.752	0.299
12,4	8.531	0.248	8.531	8.531	0.248	0.248	8.531	8.531	0.248	0.248
13,4	8.394	0.244	12.736	8.394	0.370	0.244	8.394	Open	0.244	Open
14,4	6.846	0.199	6.846	6.846	0.199	0.199	6.846	6.846	0.199	0.199
15,4	6.570	0.191	33.214	Open	0.965	Open	33.214	Open	0.965	Open
16,4	6.467	0.188	10.908	Open	0.318	Open	12.103	2.026	0.351	0.058

Table 6B (Continued)

Finger i,j	Actual Revenue	Actual Grade	Limits Period I Revenue Upper	Limits Period I Revenue Lower	Limits Period I Grade Upper	Limits Period I Grade Lower	Limits Period II Revenue Upper	Limits Period II Revenue Lower	Limits Period II Grade Upper	Limits Period II Grade Lower
1,5	8.978	0.261	--	--	--	--	--	--	--	--
2,5	8.841	0.257	8.841	Open	0.257	Open	10.558	8.841	0.307	0.257
3,5	9.013	0.262	11.033	9.013	0.321	0.262	9.013	Open	0.262	Open
4,5	9.770	0.284	9.770	9.770	0.284	0.284	9.770	9.770	0.284	0.284
5,5	7.946	0.231	7.946	7.946	0.231	0.231	7.946	7.946	0.231	0.231
6,5	9.632	0.280	9.632	9.632	0.280	0.280	9.632	9.632	0.280	0.280
7,5	9.632	0.280	9.632	9.632	0.280	0.280	9.632	9.632	0.280	0.280
8,5	10.939	0.318	10.939	10.939	0.318	0.318	10.939	10.939	0.318	0.318
9,5	9.666	0.281	9.843	9.666	0.286	0.281	9.666	Open	0.281	Open
10,5	9.976	0.290	9.976	Open	0.290	Open	25.575	9.976	0.742	0.290
11,5	9.219	0.268	9.219	9.219	0.268	0.268	9.219	9.219	0.268	0.268
12,5	7.912	0.230	12.254	7.912	0.356	0.230	7.912	Open	0.230	Open
13,5	6.433	0.187	6.433	6.433	0.187	0.187	6.433	6.433	0.187	0.187
14,5	6.330	0.184	32.974	--	0.958	--	25.361	Open	0.736	Open
15,5	7.068	0.206	15.722	Open	0.457	Open	12.722	--	0.370	--
16,5	5.917	0.172	5.917	5.917	0.172	0.172	5.917	5.917	0.172	0.172
1,6	8.256	0.240	--	--	--	--	--	--	--	--
2,6	10.836	0.315	--	--	--	--	12.856	10.836	0.373	0.315
3,6	9.082	0.264	9.082	9.082	0.264	0.264	9.082	9.082	0.264	0.264
4,6	8.667	0.252	8.667	8.667	0.252	0.252	8.667	8.667	0.252	0.252
5,6	8.359	0.243	8.359	8.359	0.243	0.243	8.359	8.359	0.243	0.243
6,6	9.598	0.279	9.598	9.598	0.279	0.279	9.598	9.598	0.279	0.279
7,6	8.772	0.255	8.772	8.772	0.255	0.255	8.772	8.772	0.255	0.255
8,6	10.698	0.311	10.698	10.698	0.311	0.311	10.698	10.698	0.311	0.311
9.6	9.426	0.274	9.426	Open	0.274	Open	15.159	9.426	0.440	0.274
10,6	10.320	0.300	10.320	Open	0.300	Open	20.308	10.320	0.590	0.300
11,6	9.804	0.258	14.146	9.804	0.411	0.258	9.804	4.696	0.258	0.136
12,6	9.873	0.287	9.873	Open	0.287	Open	22.849	9.873	0.665	0.287
13,6	7.774	0.226	34.418	--	1.000	--	19.579	Open	0.568	Open
14,6	6.536	0.190	19.858	Open	0.577	Open	24.233	--	0.705	--
15,6	6.880	0.200	6.880	6.880	0.200	0.200	6.880	6.880	0.200	0.200
16,6	7.018	0.204	29.451	7.018	0.856	0.204	7.018	Open	0.204	Open

Table 6B (Continued)

Finger i,j	Actual Revenue	Actual Grade	Limits Period I Revenue Upper	Limits Period I Revenue Lower	Limits Period I Grade Upper	Limits Period I Grade Lower	Limits Period II Revenue Upper	Limits Period II Revenue Lower	Limits Period II Grade Upper	Limits Period II Grade Lower
1,7	8.153	0.237	8.153	Open	0.237	Open	10.173	8.153	0.292	0.237
2,7	8.531	0.248	8.531	8.531	0.248	0.248	8.531	8.531	0.248	0.248
3,7	8.806	0.256	8.806	8.806	0.256	0.256	8.806	8.806	0.256	0.256
4,7	9.219	0.268	9.219	9.219	0.268	0.268	9.219	9.219	0.268	0.268
5,7	10.595	0.308	10.595	10.595	0.308	0.308	10.595	10.595	0.308	0.308
6,7	9.804	0.285	9.804	9.804	0.285	0.285	9.804	9.804	0.285	0.285
7,7	9.426	0.274	9.426	9.426	0.274	0.274	9.426	9.426	0.274	0.274
8,7	9.529	0.277	9.529	Open	0.277	Open	15.262	9.529	0.444	0.277
9,7	10.286	0.299	10.286	Open	0.299	Open	20.274	10.286	0.590	0.299
10,7	8.910	0.259	13.252	8.910	0.386	0.259	8.910	3.802	0.259	0.011
11,7	7.499	0.281	7.499	Open	0.281	Open	20.475	7.499	0.595	0.281
12,7	8.084	0.235	34.728	--	1.010	--	19.889	Open	0.684	Open
13,7	6.192	0.180	50.537	Open	1.608	Open	--	--	--	--
14,7	6.356	0.190	6.356	6.356	0.190	0.190	6.356	6.356	0.190	0.190
15,7	6.467	0.188	28.900	6.467	0.840	0.188	6.467	Open	0.188	Open
16,7	6.054	0.176	6.054	6.054	0.176	0.176	6.054	6.054	0.176	0.176
1,8	8.118	0.236	8.118	8.118	0.236	0.236	8.118	8.118	0.236	0.236
2,8	9.288	0.270	9.341	9.288	0.271	0.270	9.288	Open	0.270	Open
3,8	12.384	0.360	12.384	12.384	0.360	0.360	12.384	12.384	0.360	0.360
4,8	9.426	0.274	9.426	9.426	0.274	0.274	9.426	9.426	0.274	0.274
5,8	10.079	0.294	10.079	10.079	0.294	0.294	10.079	10.079	0.294	0.294
6,8	10.595	0.308	10.595	10.595	0.308	0.308	10.595	10.595	0.308	0.308
7,8	9.219	0.268	9.219	Open	0.268	Open	14.952	9.219	0.435	0.268
8,8	8.703	0.250	8.703	Open	0.250	Open	18.691	8.703	0.543	0.250
9,8	9.976	0.290	14.318	9.976	0.416	0.290	9.976	4.868	0.290	0.142
10,8	9.912	0.268	--	--	--	--	22.195	9.219	0.642	0.268
11,8	8.703	0.253	35.347	--	1.050	--	8.703	Open	0.253	Open
12,8	6.398	0.186	50.739	Open	1.472	Open	24.095	Open	0.700	Open
13,8	7.052	0.250	7.052	7.052	0.250	0.250	7.052	7.052	0.250	0.250
14,8	9.116	0.265	31.549	9.116	0.915	0.265	9.116	Open	0.265	Open
15,8	6.605	0.192	6.605	6.605	0.192	0.192	6.605	6.605	0.192	0.192
16,8	5.917	0.172	45.052	Open	1.310	Open	--	--	--	--

Table 6B (Continued)

Finger i,j	Actual Revenue	Actual Grade	Limits Period I Revenue Upper	Limits Period I Revenue Lower	Limits Period I Grade Upper	Limits Period I Grade Lower	Limits Period II Revenue Upper	Limits Period II Revenue Lower	Limits Period II Grade Upper	Limits Period II Grade Lower
1,9	8.910	0.259	8.963	8.910	0.261	0.259	--	--	--	--
2,9	10.767	0.313	10.767	10.767	0.313	0.313	10.767	10.767	0.313	0.313
3,9	10.286	0.299	10.286	10.286	0.299	0.299	10.286	10.286	0.299	0.299
4,9	9.632	0.280	9.632	9.632	0.280	0.280	9.632	9.632	0.280	0.280
5,9	9.288	0.270	9.288	9.288	0.270	0.270	9.288	9.288	0.270	0.270
6,9	8.910	0.259	8.910	Open	0.259	Open	14.643	8.910	0.426	0.259
7,9	10.526	0.306	10.526	Open	0.306	Open	20.274	10.286	0.589	0.306
8,9	12.625	0.367	16.967	12.625	0.493	0.367	12.625	7.517	0.367	0.218
9.9	9.770	0.284	9.770	Open	0.284	Open	22.746	9.770	0.660	0.284
10,9	9.426	0.274	36.070	--	1.070	--	17.039	Open	0.496	Open
11,9	7.190	0.209	--	--	--	--	24.887	Open	0.724	Open
12,9	8.256	0.240	8.256	8.256	0.240	0.240	8.256	8.256	0.240	0.240
13,9	6.674	0.194	29.107	6.674	0.845	0.194	--	--	--	--
14,9	6.054	0.176	28.487	6.054	0.827	0.176	6.054	--	0.176	--
15,9	6.639	0.193	45.774	Open	1.360	Open	45.774	Open	1.360	Open
16,9	5.882	0.171	--	--	--	--	--	--	--	--

Table 6B (Continued)

Finger i,j	Actual Revenue	Actual Grade	Limits Period III Revenue Upper	Limits Period III Revenue Lower	Limits Period III Grade Upper	Limits Period III Grade Lower	Limits Period IV Revenue Upper	Limits Period IV Revenue Lower	Limits Period IV Grade Upper	Limits Period IV Grade Lower
1,1	7.912	0.230	7.974	4.490	0.232	0.130	9.267	7.849	0.269	0.228
2,1	8.325	0.242	8.325	4.903	0.242	0.143	10.990	8.325	0.319	0.242
3,1	7.912	0.230	7.912	7.912	0.230	0.230	7.912	7.912	0.230	0.230
4,1	8.462	0.246	8.462	8.462	0.246	0.246	8.462	8.462	0.246	0.246
5,1	10.320	0.300	10.320	10.320	0.300	0.300	10.320	10.320	0.300	0.300
6,1	10.526	0.306	10.526	7.104	0.306	0.207	13.191	10.526	0.380	0.306
7,1	10.664	0.310	11.213	7.242	0.326	0.210	13.329	10.089	0.389	0.292
8,1	10.802	0.314	13.789	10.802	0.400	0.314	10.802	8.016	0.314	0.233
9,1	11.146	0.324	11.146	11.146	0.324	0.324	11.146	11.146	0.324	0.324
10,1	9.907	0.288	9.907	9.907	0.288	0.288	9.907	9.907	0.288	0.288
11,1	7.671	0.223	7.671	7.671	0.223	0.223	7.671	7.671	0.223	0.223
12,1	9.047	0.263	10.156	9.047	0.293	0.263	9.047	7.967	0.263	0.232
13,1	7.224	0.210	7.224	7.224	0.210	0.210	7.224	7.224	0.210	0.210
14,1	8.118	0.236	18.177	Open	0.528	Open	10.477	1.898	0.304	0.065
15,1	6.880	0.200	21.524	8.353	0.625	0.243	9.373	1.426	0.272	0.049
1,2	7.568	0.220	7.762	7.568	0.226	0.220	7.568	7.370	0.220	0.214
2,2	8.699	0.252	9.111	8.699	0.265	0.252	8.699	8.272	0.252	0.240
3,2	7.809	0.227	7.809	7.809	0.227	0.227	7.809	7.809	0.227	0.227
4,2	8.566	0.249	8.566	8.566	0.249	0.249	8.566	8.566	0.249	0.249
5,2	10.802	0.314	10.802	10.802	0.314	0.314	10.802	10.802	0.314	0.314
6,2	10.045	0.292	14.281	6.623	0.415	0.192	12.710	6.853	0.370	0.199
7,2	10.320	0.300	13.307	10.320	0.387	0.300	10.320	7.534	0.300	0.219
8,2	8.394	0.244	8.394	8.394	0.244	0.244	8.394	8.394	0.244	0.244
9,2	10.251	0.298	10.251	10.251	0.298	0.298	10.251	10.251	0.298	0.298
10,2	12.006	0.349	12.006	12.006	0.349	0.349	12.006	12.006	0.349	0.349
11,2	7.912	0.230	7.912	7.912	0.230	0.230	7.912	7.912	0.230	0.230
12,2	8.806	0.256	8.806	8.806	0.256	0.256	8.806	8.806	0.256	0.256
13,2	7.293	0.212	17.352	Open	0.505	Open	9.652	1.073	0.280	0.034
14,2	8.050	0.234	22.694	7.183	0.660	0.209	10.543	2.596	0.306	0.075
15,2	7.293	0.212	--	--	--	--	--	--	--	--
16,2	6.605	0.192	--	--	--	--	--	--	--	--

Table 6B (Continued)

Finger i,j	Actual Revenue	Actual Grade	Limits Period III Revenue Upper	Limits Period III Revenue Lower	Limits Period III Grade Upper	Limits Period III Grade Lower	Limits Period IV Revenue Upper	Limits Period IV Revenue Lower	Limits Period IV Grade Upper	Limits Period IV Grade Lower
1,3	8.428	0.245	8.428	8.428	0.245	0.245	8.428	8.428	0.245	0.245
2,3	7.809	0.227	7.809	7.809	0.227	0.227	7.809	7.809	0.227	0.227
3,3	9.907	0.288	9.907	9.907	0.288	0.288	9.907	9.907	0.288	0.288
4,3	6.467	0.188	6.467	6.467	0.188	0.188	6.467	6.467	0.188	0.188
5,3	10.630	0.309	14.866	7.208	0.432	0.210	13.295	7.438	0.386	0.216
6,3	10.217	0.297	13.204	10.217	0.384	0.297	10.217	7.431	0.297	0.126
7,3	7.671	0.223	7.671	7.671	0.223	0.223	7.671	7.671	0.223	0.223
8,3	6.941	0.201	6.941	6.941	0.201	0.201	6.941	6.941	0.201	0.201
9,3	11.558	0.336	11.558	11.558	0.336	0.336	11.558	11.558	0.336	0.336
10,3	12.487	0.363	12.487	12.487	0.363	0.363	12.487	12.487	0.363	0.363
11,3	11.738	0.254	11.738	11.738	0.254	0.254	11.738	11.738	0.254	0.254
12,3	7.981	0.232	34.547	Open	1.005	Open	10.340	1.761	0.301	0.052
13,3	8.187	0.238	22.831	7.046	0.659	0.242	10.680	2.733	0.309	0.079
14,3	7.499	0.218	53.723	Open	1.560	Open	17.716	Open	0.515	Open
15,3	7.121	0.207	8.978	4.175	0.261	0.121	--	--	--	--
16,3	6.605	0.192	--	--	--	--	8.697	6.483	0.252	0.189
1,4	8.600	0.250	8.600	8.600	0.250	0.250	8.600	8.600	0.250	0.250
2,4	9.322	0.271	9.322	9.322	0.271	0.271	9.322	9.322	0.271	0.271
3,4	7.706	0.224	7.706	7.706	0.224	0.224	7.706	7.706	0.224	0.224
4,4	10.114	0.294	14.350	6.692	0.417	0.195	12.779	6.922	0.371	0.201
5,4	10.939	0.318	13.926	10.939	0.405	0.318	10.939	8.153	0.318	0.237
6,4	11.249	0.327	11.249	11.249	0.327	0.327	11.249	11.249	0.327	0.327
7,4	9.185	0.267	9.185	9.185	0.267	0.267	9.185	9.185	0.267	0.267
8,4	9.185	0.267	9.185	9.185	0.267	0.267	9.185	9.185	0.267	0.267
9,4	9.082	0.264	9.082	9.082	0.264	0.264	9.082	9.082	0.264	0.264
10,4	9.873	0.278	9.873	9.873	0.278	0.278	9.873	9.873	0.278	0.278
11,4	10.286	0.299	36.852	Open	1.071	Open	12.644	4.066	0.368	0.118
12,4	8.531	0.248	23.175	6.702	0.673	0.195	11.024	3.077	0.320	0.089
13,4	8.394	0.244	54.618	Open	1.589	Open	18.611	Open	0.542	Open
14,4	6.846	0.199	8.713	3.900	0.053	0.115	9.792	Open	0.284	Open
15,4	6.570	0.191	6.693	Open	0.194	Open	8.662	6.448	0.252	0.187
16,4	6.467	0.188	8.677	6.467	0.252	0.188	--	--	--	--

Table 6B (Continued)

Finger i,j	Actual Revenue	Actual Grade	Limits Period III Revenue Upper	Limits Period III Revenue Lower	Limits Period III Grade Upper	Limits Period III Grade Lower	Limits Period IV Revenue Upper	Limits Period IV Revenue Lower	Limits Period IV Grade Upper	Limits Period IV Grade Lower
1,5	8.978	0.261	8.978	8.978	0.261	0.261	8.978	8.978	0.261	0.261
2,5	8.841	0.257	8.841	8.841	0.257	0.257	8.841	8.841	0.257	0.257
3,5	9.013	0.262	14.647	5.591	0.426	0.163	11.678	5.821	0.339	0.169
4,5	9.770	0.284	12.757	9.770	0.370	0.284	9.770	6.984	0.284	0.203
5,5	7.946	0.231	7.946	7.946	0.231	0.231	7.946	7.946	0.231	0.231
6,5	9.632	0.280	9.632	9.632	0.280	0.280	9.632	9.632	0.280	0.280
7,5	9.632	0.280	9.632	9.632	0.280	0.280	9.632	9.632	0.280	0.280
8,5	10.939	0.318	10.939	10.939	0.318	0.318	10.939	10.939	0.318	0.318
9,5	9.666	0.281	9.666	9.666	0.281	0.281	9.666	9.666	0.281	0.281
10,5	9.976	0.290	36.532	Open	1.060	Open	12.335	3.756	0.359	0.109
11,5	9.219	0.268	23.863	6.014	0.695	0.175	11.712	3.765	0.341	0.109
12,5	7.912	0.230	54.136	Open	1.572	Open	18.129	Open	0.528	Open
13,5	6.433	0.187	7.619	4.257	0.222	0.126	--	--	--	--
14,5	6.330	0.184	6.391	4.154	0.186	0.121	8.476	6.269	0.246	0.183
15,5	7.068	0.206	9.296	7.086	0.270	0.260	--	--	--	--
16,6	5.917	0.172	7.181	5.917	0.290	0.172	--	--	--	--
1,6	8.256	0.240	10.322	8.256	0.300	0.240	8.256	5.763	0.240	0.167
2,6	10.836	0.315	12.144	7.414	0.353	0.216	13.501	9.368	0.393	0.272
3,6	9.082	0.264	12.069	9.082	0.350	0.264	9.082	6.296	0.264	0.186
4,6	8.667	0.252	8.667	8.667	0.252	0.252	8.667	8.667	0.252	0.252
5,6	8,359	0.243	9.017	8.359	0.262	0.243	8.359	7.711	0.243	0.224
6,6	9.589	0.279	9.598	5.635	0.279	0.164	9.735	9.598	0.283	0.279
7,6	8.772	0.255	8.772	8.772	0.255	0.255	8.772	8.772	0.255	0.255
8,6	10.698	0.311	10.698	--	0.311	--	13.239	10.698	0.385	0.311
9,6	9.426	0.274	11.053	7.733	0.321	0.224	12.238	7.265	0.356	0.211
10,6	10.320	0.300	24.160	5.237	0.700	0.152	12.813	4.866	0.372	0.141
11,6	9.804	0.258	24.448	2.357	0.710	0.068	11.849	7.415	0.342	0.216
12,6	9.873	0.287	39.497	Open	1.149	Open	11.918	7.484	0.346	0.217
13,6	7.774	0.226	9.866	5.598	0.286	0.163	9.920	Open	0.268	Open
14,6	6.536	0.190	21.180	4.507	0.615	0.131	--	--	--	--
15,6	6.880	0.200	10.509	6.880	0.306	0.200	--	--	--	--
16,6	7.018	0.204	21.662	8.215	0.630	0.239	--	--	--	--

Table 6B (Continued)

Finger i,j	Actual Revenue	Actual Grade	Limits Period III Revenue Upper	Limits Period III Revenue Lower	Limits Period III Grade Upper	Limits Period III Grade Lower	Limits Period IV Revenue Upper	Limits Period IV Revenue Lower	Limits Period IV Grade Upper	Limits Period IV Grade Lower
1,7	8.153	0.237	9.461	4.731	0.275	0.137	10.818	6.685	0.314	0.194
2,7	8.531	0.248	8.531	8.531	0.248	0.248	8.531	8.531	0.248	0.248
3,7	8.806	0.256	8.806	8.806	0.256	0.256	8.806	8.806	0.256	0.256
4,7	9.219	0.268	9.219	9.219	0.268	0.268	9.219	9.219	0.268	0.268
5,7	10.595	0.308	10.595	--	0.308	--	10.732	10.595	0.312	0.308
6,7	9.804	0.285	9.804	9.804	0.285	0.285	9.804	9.804	0.285	0.285
7,7	9.426	0.274	9.426	--	0.274	--	11.967	9.426	0.348	0.274
8,7	9.529	0.277	39.153	Open	1.136	Open	12.341	7.368	0.359	0.214
9,7	10.286	0.299	24.930	--	0.710	--	12.779	4.832	0.370	0.140
10,7	8.910	0.259	10.537	7.217	0.306	0.210	10.955	6.521	0.318	0.189
11,7	7.944	0.281	9.591	5.323	0.288	0.155	9.544	5.110	0.287	0.148
12,7	8.084	0.235	10.176	5.908	0.293	0.174	--	--	--	--
13,7	6.192	0.180	6.192	1.109	0.180	0.032	16.751	6.192	0.486	0.180
14,7	6.356	0.190	10.165	6.536	0.294	0.190	--	--	--	--
15,7	6.467	0.188	21.111	--	0.614	--	35.995	Open	1.042	Open
16,7	6.054	0.176	15.114	1.545	0.440	0.045	--	--	--	--
1,8	8.118	0.236	8.118	4.696	0.236	0.136	9.952	8.118	0.289	0.236
2,8	9.288	0.270	9.288	9.288	0.270	0.270	9.288	9.288	0.270	0.270
3,8	12.384	0.360	12.384	12.384	0.360	0.360	12.384	12.384	0.360	0.360
4,8	9.426	0.274	9.426	9.426	0.274	0.274	9.426	9.426	0.274	0.274
5,8	10.079	0.294	10.079	--	0.294	--	11.366	10.079	0.330	0.294
6,8	10.595	0.308	10.595	10.595	0.308	0.308	10.595	10.595	0.308	0.308
7,8	9.219	0.268	38.843	Open	1.110	Open	12.031	7.058	0.350	0.205
8,8	8.703	0.250	23.347	--	0.677	--	11.196	3.249	0.324	0.095
9,8	9.976	0.290	11.603	8.283	0.338	0.241	14.670	10.236	0.426	0.296
10,8	9.912	0.268	11.311	7.043	0.329	0.205	11.264	6.830	0.327	0.199
11,8	8.703	0.253	8.826	6.527	0.256	0.189	10.849	8.580	0.316	0.250
12,8	6.398	0.186	6.398	1.315	0.186	0.038	16.957	6.398	0.493	0.186
13,8	7.052	0.205	21.695	7.052	0.632	0.250	--	--	--	--
14,8	9.116	0.256	23.760	6.117	0.690	0.780	38.644	Open	1.092	Open
15,8	6.605	0.192	15.665	2.096	0.455	0.061	--	--	--	--
16,8	5.917	0.172	11.996	--	0.349	--	--	--	--	--

Table 6B (Continued)

Finger i,j	Actual Revenue	Actual Grade	Limits Period III Revenue Upper	Limits Period III Revenue Lower	Limits Period III Grade Upper	Limits Period III Grade Lower	Limits Period IV Revenue Upper	Limits Period IV Revenue Lower	Limits Period IV Grade Upper	Limits Period IV Grade Lower
1,9	8.910	0.259	8.910	8.910	0.259	0.259	8.910	8.910	0.259	0.259
2,9	10.767	0.310	10.767	10.767	0.310	0.310	10.767	10.767	0.310	0.310
3,9	10.286	0.299	10.286	10.286	0.299	0.299	10.286	10.286	0.299	0.299
4,9	9.632	0.280	9.632	--	0.280	--	10.919	9.632	0.318	0.280
5,9	9.288	0.270	9.288	9.288	0.270	0.270	9.288	9.288	0.270	0.270
6,9	8.910	0.259	38.534	Open	1.120	Open	11.722	6.749	0.342	0.196
7,9	10.526	0.306	25.170	--	0.733	--	13.019	5.072	0.380	0.148
8,9	12.625	0.367	14.252	11.590	0.415	0.338	14.670	10.236	0.426	0.301
9,9	9.770	0.284	11.862	8.294	0.346	0.242	11.227	7.381	0.327	0.215
10,9	9.426	0.274	9.549	7.250	0.278	0.211	11.572	9.303	0.336	0.271
11,9	7.190	0.209	7.190	2.107	0.209	0.061	17.749	7.190	0.517	0.209
12,9	6.256	0.240	8.256	5.837	0.240	0.170	12.618	8.256	0.368	0.240
13,9	6.674	0.194	21.318	--	0.620	--	36.202	Open	1.105	Open
14,9	6.054	0.176	15.114	1.545	0.441	0.045	--	--	--	--
15,9	6.639	0.193	12.718	--	0.371	--	12.580	Open	0.366	Open
16,9	5.882	0.171	--	--	--	--	--	--	--	--

Table 7B

Bounds and Economical Influence of the Geometric Constraints for the Second Application of the Progressive Block-Caving Model

Constraint Name	Angle Required	Period I			Period II		
		Lower Bound	Upper Bound	Economical Influence 10^3 \$/Deg.	Lower Bound	Upper Bound	Economical Influence 10^3 \$/Deg.
U12	45°	0°	Open	-	0°	Open	-
L12	0°	27°	0°	-	0°	-27°	0.0622
U23	45°	32°	Open	-	32°	Open	-
L23	0°	32°	Open	-	32°	Open	-
U34	45°	39°	45°	-	0°	Open	-
L34	0°	0°	Open	-	9°	0°	0.1800
U45	45°	37°	46°	-	0°	Open	-
L45	0°	45°	Open	-	0°	-6°	0.0009
U56	45°	37°	48°	-	41°	Open	-
L56	0°	5°	Open	-	7°	Open	-
U67	45°	48°	33°	-	Open	0°	-
L67	0°	45°	Open	-	14°	-6°	0.0845
U78	45°	0°	Open	-	32°	Open	-
L78	0°	7°	-27°	-	13°	Open	-
U89	45°	22°	48°	-	26°	48°	0.0512
L89	0°	45°	Open	-	45°	Open	-
U910	45°	17°	Open	-	0°	Open	-
L910	0°	17°	Open	-	0°	-26°	0.1540
U1011	45°	0°	Open	-	0°	Open	-
L1011	0°	18°	-45°	-	0°	-37°	0.1296
U1112	45°	0°	Open	-	20°	46°	0.0315
L1112	0°	-56°	27°	-	Open	46°	-
U1213	45°	0°	Open	-	27°	46°	0.1113
L1213	0°	-45°	14°	-	Open	45°	-
U1314	45°	13°	Open	-	27°	Open	-
L1314	0°	Open	13°	-	Open	27°	-
U1415	45°	9°	49°	-	45°	60°	1.0100
L1415	0°	Open	45°	-	Open	45°	-

Table 7B (Continued)

Constraint Name	Angle Required	Period I Lower Bound	Period I Upper Bound	Period I Economical Influence 10^3 \$/Deg.	Period II Lower Bound	Period II Upper Bound	Period II Economical Influence 10^3 \$/Deg.
U1515	45°	9°	Open	-	42°	62°	0.4080
L1515	0°	9°	Open	-	45°	Open	-
U1516	45°	27°	Open	-	27°	Open	-
L1516	-27°	Open	27°	-	Open	27°	-
UH23	45°	0°	Open	-	0°	Open	-
LH23	0°	Open	0°	-	0°	6°	-0.3980
UH34	45°	0°	Open	-	0°	Open	-
LH34	0°	0°	0°	-0.6344	Open	0°	-
UH45	45°	27°	Open	-	27°	Open	-
LH45	0°	27°	Open	-	27°	Open	-
UH56	45°	-27°	Open	-	-27°	Open	-
LH56	-27°	-27°	-27°	-	-27°	-18°	-0.4450
UH67	45°	0°	Open	-	0°	Open	-
LH67	0°	0°	0°	-	0°	0°	-1.8694
UH78	45°	-45°	Open	-	-45°	Open	-
LH78	-45°	Open	-45°	-	-45°	-44°	-0.3785
UH89	45°	-27°	Open	-	-27°	Open	-
LH89	-27°	-27°	-26°	-0.3900	Open	-27°	-

Table 7B (Continued)

Constraint Name	Angle Required	Period III Lower Bound	Period III Upper Bound	Period III Economical Influence 10^3 $/Deg.	Period IV Lower Bound	Period IV Upper Bound	Period IV Economical Influence 10^3 $/Deg.
U12	45°	0°	Open	-	0°	Open	-
L12	0°	45°	-67°	0.0037	45°	-73°	0.0963
U23	45°	0°	Open	-	0°	Open	-
L23	0°	45°	-63°	0.0104	45°	-69°	0.2475
U34	45°	0°	Open	-	0°	Open	-
L34	0°	45°	-45°	0.0176	45°	-70°	0.4240
U45	45°	0°	Open	-	0°	Open	-
L45	0°	45°	-31°	0.0263	45°	-71°	0.7290
U56	45°	0°	Open	-	0°	Open	-
L56	0°	45°	-22°	0.0390	41°	-71°	0.6750
U67	45°	0°	Open	-	0°	Open	-
L67	0°	45°	-17°	0.0540	31°	-55°	0.6000
U78	45°	39°	65°	0.2195	25°	Open	-
L78	0°	45°	Open	-	17°	-48°	0.5130
U89	45°	39°	65°	0.1240	0°	Open	-
L89	0°	45°	Open	-	14°	-43°	0.3260
U910	45°	40°	64°	0.0549	0°	Open	-
L910	0°	45°	Open	-	14°	-38°	0.3460
U1011	45°	42°	Open	-	0°	Open	-
L1011	0°	42°	Open	-	13°	-33°	0.2900
U1112	45°	13°	Open	-	22°	49°	0.1090
L1112	0°	37°	-2°	0.0244	45°	Open	-
U1213	45°	0°	Open	-	27°	48°	0.3620
L1213	0°	33°	-1°	0.4900	45°	Open	-
U1314	45°	0°	Open	-	27°	49°	0.3730
L1314	0°	22°	-1°	0.0970	0°	Open	-
U1415	45°	37°	48°	3.6750	29°	48°	0.2494
L1415	0°	42°	Open	-	65°	Open	-

Table 7B (Continued)

Constraint Name	Angle Required	Period III			Period IV		
		Lower Bound	Upper Bound	Economical Influence 10^3 \$/Deg.	Lower Bound	Upper Bound	Economical Influence 10^3 \$/Deg.
U1516	45°	31°	Open	-	31°	48°	0.1100
L1516	0°	31°	Open	-	63°	Open	-
U1617	45°	14°	60°	4.6900	32°	47°	1.0200
L1617	0°	32°	-7°	2.2650	45°	Open	-
U1718	45°	10°	Open	-	37°	48°	0.4420
L1718	0°	Open	10°	-	Open	9°	-
U1819	45°	14°	Open	-	37°	Open	-
L1819	0°	Open	14°	-	Open	37°	-
U1920	45°	0°	Open	-	39°	49°	0.8340
L1920	0°	0°	-14°	0.0132	9°	0°	0.5400
U2021	45°	0°	Open	-	0°	Open	-
L2021	0°	-6°	0°	-0.0225	0°	6°	-0.8000
U2122	45°	37°	45°	1.1800	45°	Open	-
L2122	0°	Open	45°	-	Open	45°	-
U2223	45°	37°	45°	0.4175	45°	Open	-
L2223	0°	Open	45°	-	Open	45°	-
U2324	45°	37°	Open	-	37°	Open	-
L2324	0°	Open	37°	-	Open	37°	-

APPENDIX C

Sensitivity Analysis

SENSITIVITY ANALYSIS

The importance of the sensitivity analysis of the optimal linear-programming solution has been clearly established throughout this thesis.

In this appendix the sensitivity analysis formulas will be derived and applied to a numerical example.

Uncertainty in the following linear-programming parameters will be considered:

1. Uncertain cost coefficients on the nonbasic variables
2. Uncertain cost coefficients on the basic variables
3. Uncertain constraint values (right hand side)
4. Uncertain input-output coefficients for nonbasic variables
5. Effect of adding a new variable

The following numerical example will be used to illustrate the use of the formulas.

Problem

An open-pit mine has 3 types of trucks: 80-ton, 65-ton, 40-ton. The hauling capacities are 2000 ton/shift, 1400 ton/shift, and 1200 ton/shift, respectively.

The 80-ton trucks use 3 hr of mechanical maintenance per shift, the 65-ton trucks use 2 hr, and the 40-ton trucks use 1 hr.

The resources available are

12 drivers and 24 hrs of mechanical personnel per shift. How many trucks of each kind should operate to maximize the total number of tons hauled per shift?

Mathematical Formulation

Let X_1 be the number of 80-ton trucks in operation

X_2 be the number of 65-ton trucks in operation

X_3 be the number of 40-ton trucks in operation.

$$\text{Max } Z = 2000\ X_1 + 1600\ X_2 + 1200\ X_3$$

Subject to:

$$X_1 + X_2 + X_3 \leq 12$$

$$3X_1 + 2X_2 + X_3 \leq 24$$

In matrix form, after adding slack variables:

		X_1	X_2	X_3	I_1	I_2	Z	b
Drivers	I_1	1	1	1	1	0	0	12
Mechanics	I_2	3	2	1	0	1	0	24
Minimize	Z	-2000	-1400	-1200	0	0	1	0

1st Iteration		X_1	X_2	X_3	I_1	I_2	Z	b
	I_1	0	-1/3	2/3	1	-1/3	0	4
	X_1	1	2/3	1/3	0	1/3	0	8
	Z	0	-200/3	-1600/3	0	2000/3	1	16000

2nd Iter. Optimal		X_1	X_2	X_3	I_1	I_2	Z	b
	X_3	0	1/2	1	3/2	-1/2	0	6
	X_1	1	1/2	0	-1/2	1/2	0	6
	Z	0	200	0	800	400	1	19200

The optimal solution is: X_3 = 6 40-ton trucks

X_1 = 6 80-ton trucks

Notation

B_i^{-1} Inverse of the i^{th} iteration (matrix formed by the vectors of the slack variables and Z), in this case:

Example:

$$B_2 = \begin{vmatrix} 3/2 & -1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 800 & 400 & 1 \end{vmatrix}$$

A_j Vector under variables that are not in the basis.

Example:

$$A_2 \equiv \begin{vmatrix} 1/2 \\ 1/2 \\ 200 \end{vmatrix}$$

b_i Right-hand side in i^{th} iteration.

Example:

$$b_2 = \begin{vmatrix} 6 \\ 6 \\ 19200 \end{vmatrix}$$

C_j Coefficients of objective function, in this case:

$C_1 = -2000$

$C_2 = -1400$

$C_3 = -1200$

C_s Original coefficients of objective function for basic variables; in this case:

$C_1 = -2000$

$C_3 = -1200$

$\overline{C}_j$ Coefficients of objective function in optimal stage, in this case:

$C_1 = 0$

$C_2 = 200$

$C_3 = 0$

X_B Optimum value for the basic variables.

$-\pi_k$ Value of the dual variables correspondent to the k^{th} constraint; these can be found in the last row of the inverse:

Example:

$\pi_1 = -800 \quad \pi_2 = -400 \quad \pi_3 = -1$

From primal dual theory and the revised simplex algorithm, the following equations must hold:

1) The optimum value of the objective function of the primal must be equal to the optimum value of the dual

$$\sum_k \pi_k b_{ok} = \sum_j C_j X_j$$

In this case

$$800 \cdot 12 + 400 \cdot 24 + 1.0 = 6 \cdot 2000 + 6 \cdot 1200 = 19200$$

$$19200 = 19200 = 19200$$

2) Any vector in the optimum tableau can be obtained by multiplying the inverse of the optimal basis times the original vector:

2a) Calculation of any matrix vector in the optimum tableau

$B^{-1} A_j = \overline{A}_j$ ($\overline{A}_j$ expresses A_j in terms of the present basis)

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xample:

$$\bar{B}^1 A_2 = \bar{A}_2$$

$$\begin{vmatrix} 3/2 & -1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 800 & 400 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 2 \\ -1400 \end{vmatrix} = \begin{vmatrix} 1/2 \\ 1/2 \\ 200 \end{vmatrix}$$

2b) Calculation of the optimum value of the variables X_j

$X_B = \bar{B}^1 b = \bar{b}$ (X_B is the optimum value of the variable X_j. The notation X_B is used to indicate that these are the basic variables)

Example:

$$\begin{vmatrix} 3/2 & -1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 800 & 400 & 1 \end{vmatrix} \cdot \begin{vmatrix} 12 \\ 24 \\ 0 \end{vmatrix} = \begin{vmatrix} 6 \\ 6 \\ 19200 \end{vmatrix}$$

2c) Calculation of the value of the coefficients of the objective function at optimality. In section 2a the value of the vector $\bar{A}_2$ was calculated. The last term of this vector (200) corresponds to the coefficient of the objective function $\bar{C}_2$. In other words, if any matrix vector is multiplied by the last row of the inverse, the value of the coefficient of the objective function at optimality is obtained, analytically:

$$\bar{C}_j = C_j - \pi A_j$$

Example:

$$\bar{C}_2 = C_2 - \pi A_2$$

$$\bar{C}_2 = -1400 - (-800 - 400) \begin{vmatrix} 1 \\ 2 \end{vmatrix}$$

$\overline{C}_2 = -1400 + 800 + 800 = 200$

3) Calculation of the values of the dual variables. The values of these variables can be obtained by multiplying the original coefficients of the objective function, for the basic variables times the inverse:

In this case:

$C_B \overline{B}^1 = \pi$

$$(-2000 \quad 1200) \begin{vmatrix} -1/2 & 1/2 \\ 3/2 & -1/2 \end{vmatrix} = (-800 \quad 400)$$

1. <u>Uncertain Cost Coefficients for Nonbasic Variables</u>

The problem is: How much does the cost coefficient of a nonbasic variable have to change before it affects the optimum solution?

In terms of this problem: How much has to be the capacity per shift of the second type of trucks for them to be considered in operation?

Relation 2c

$\overline{C}_j = C_j - \pi A_j$

Suppose that one C_j is increased by θ

$C_j^* = C_j + \theta$ (C_j^* is the new cost coefficient)

$\overline{C}_j^* = C_j - \pi A_j + \theta$

$\overline{C}_j^* = \overline{C}_j + \theta$

if θ increases and $\theta > 0$, then $\overline{C}_j^* > 0$ for all θ; consequently there will be no change in the basis.

It is necessary that $\bar{c}_j^* = 0$ for variable X_j to enter the basis. Therefore, if θ decreases to some negative value, then

$$0 = \bar{c}_j + \theta$$

$$\theta = -\bar{c}_j$$

Consequently, the stability limits of the basis are

$$c_j - \bar{c}_j \leq c_j \leq \infty$$

The exiting variable would be the one with least $\bar{b}_i/\bar{a}_{ij}$ in column j, so that $\bar{a}_{ij} > 0$.

For this example, X_2 is the only nonbasic variable:

$$-1400 - 200 \leq c_j \leq \infty$$

$$-1600 \leq c_j \leq \infty$$

If the capacity of the second type of trucks were 1600 tons/shift, they would be considered in the operational plan.

The exiting variable in this case would be X_3 or X_1 because both have the same $\bar{b}_i/\bar{a}_{ij}$ ratio.

X_3 $\bar{b}_1/\bar{a}_{12} = 6\ 1/2 = 12$

X_1 $\bar{b}_2/\bar{a}_{22} = 6/1/2 = 12$

2. Uncertain Cost Coefficients of Basic Variables

Problem: What are the limits within which the cost coefficients of a basic variable can vary before the optimal solution changes?

In this case: How much could the capacity of truck type 1 and type 3 be changed before they are not considered in the operation plan?

Relation 2c

$$\overline{C}_j = C_j - \pi A_j$$

but

$$C_B \overline{B}^1 = \pi \quad \text{(relation 3)}$$

If relation 3 is multiplied by A_j

$$C_B \overline{B}^1 A_j = \pi A_j$$

but

$$\overline{B}^1 A_j = \overline{A}_j, \text{ therefore}$$

$$C_B \overline{A}_j = \pi A_j$$

so relation 2c can be written as

$$\overline{C}_j = C_j - C_B \overline{A}_j$$

C_B are the cost coefficients of the basic variables. Let m variables be the basic ones:

Basis $(X_1, X_2, \ldots\ldots X_s, \ldots\ldots, X_m)$

on the expanding of the last relation:

$$\overline{C}_j = C_j - C_1\overline{a}_{1j} - C_2\overline{a}_{2j} - \ldots C_s\overline{a}_{sj} - \ldots C_m\overline{a}_{mj}$$

If C_s is changed in θ, the result is:

$$\overline{C}^*_j = C_j - C_1\overline{a}_{1j} - C_2\overline{a}_{2j} - \ldots (C_s + \theta)\overline{a}_{sj} - \ldots - C_m\overline{a}_{mj}$$

$$\overline{C}^*_j = C_j - C_1\overline{a}_{1j} - C_2\overline{a}_{2j} - \ldots -C_s\overline{a}_{sj} - \ldots C_m\overline{a}_{mj} - \overline{a}_{sj}\theta$$

$$\bar{C}_j^* = \bar{C}_j - \theta \bar{a}_{sj}$$

For basic variables, if $s = j$ then $\bar{a}_{sj} = 1$ (it is a unit matrix)
$s \neq j$ then $\bar{a}_{sj} = 0$

If j is basic and $j = r \neq s$, then $\bar{a}_{sr} = 0$ and $\bar{C}_j = 0$ (basic), then in this case $\bar{C}_j^* = 0$. Therefore, there is no change in the basis.

The problem can arise when with the changing of a cost coefficient of a basic variable, changes occur in the optimum cost coefficients of the nonbasic variables. One of these may turn to be negative; therefore, this variable would enter the basis.

Let s be basic and j be nonbasic. For this case we will study what happens with $\bar{C}_j$ when we change C_s:

$$\bar{C}_j^* = \bar{C}_j - \bar{a}_{sj}\theta \geq 0 \quad \text{for all } j$$

a) If θ increases and $\bar{a}_{sj} < 0$,

$\bar{C}_j^*$ will never be < 0, therefore no change will occur in the basis.

b) If θ increases and $\bar{a}_{sj} > 0$, then for some θ, $\bar{C}_j^* = 0$. Therefore, this will be the maximum value of θ.

$$\text{Max } \theta = \text{Min } [\bar{C}_j/\bar{a}_{sj}]$$

$$\bar{a}_{sj} > 0$$

We consider the minimum value of $\bar{C}_j/\bar{a}_{sj}$ because at this point one of the $\bar{C}_j$ goes to zero, and a change in the basis occurs.

If θ decreases and $\bar{a}_{sj} < 0$, then for some θ, $\bar{C}_j^* = 0$. Therefore, the minimum of θ will be

$$\text{Min } \theta = \text{Max } [\bar{C}_j/\bar{a}_{sj}]$$
$$\bar{a}_{sj} < 0$$

The stability limits of the basis are

$$C_s + \underset{\substack{j\ \bar{a}_{sj} < 0 \\ j \text{ basic}}}{\text{Max}} [\bar{C}_j/\bar{a}_{sj}] \leq C_s \leq C_s + \underset{\substack{j\ \bar{a}_{sj} > 0 \\ j \text{ basic}}}{\text{Min}} [\bar{C}_j/\bar{a}_{sj}]$$

Applying this result to the numerical example, we get

Basic	C_s	Lower Bound C_s + Max $\bar{C}_j/\bar{a}_{sj}$ $\bar{a}_{sj} < 0$	Enter. Var.	Exit. Var.	Upper Bound C_s + Min $\bar{C}_j/\bar{a}_{sj}$ $\bar{a}_{sj} > 0$	Enter. Var.	Exit. Var.
X_1	-2000	-2000+800/ -1/2=-3600	I_1	X_3	-2000+200/ 1/2=-1600	X_2	X_3 or X_1
X_3	-2000	-1200+400/ -1/2=-2000	I_2	X_1	-1200+200/ 1/2=-800	X_2	X_3 or X_1

This means in terms of our problem that the capacity of the first type of trucks can vary between 3600 and 1600 T/shift; and the capacity of the third type of trucks can vary between 2000 and 800 T/shift before there is any change in the optimal solution.

The entering variable is the one whose $\bar{C}_j = 0$ when the change is made in the C_j of the basic variable. The exiting variable is found in the column of the entering variable, so that $\bar{b}_i/\bar{a}_{ij}$ is minimum, with $\bar{a}_{ij} > 0$; j indicates the entering variable and i indicates the exiting variable.

3. Uncertain Constraint Values (Right-hand Side)

A change in the R.H.S. (b_i) does not affect the optimum basis because it does not have any effect on the $\bar{C}_j$. (Optimum coefficients of the objective function.) However, it affects the value of the objective function because

$$\text{Min } Z = \sum_j C_j X_j = \sum_i \pi_i b_i = \text{Max } V$$

The problem is to find how much can the b_i be changed before any $\bar{b}_i$ goes negative. The first condition of feasibility of a primal linear-programming problem is that all $X_j \geq 0$. Therefore, the R.H.S. is not allowed to be negative or the problem is infeasible.

We have:

$$X_B = \bar{b} = \bar{B}^1 \cdot b \qquad \text{(relation 2b)}$$

An element of $\bar{b}$ is obtained by multiplying the corresponding row of the inverse ($\bar{B}^1$) times the vector b.

$$X_i = \bar{b}_i = \bar{B}^1_i \, b \qquad (\bar{B}^1_i = i^{th} \text{ row of } \bar{B}^1)$$

An element of b will be changed by θ

$$b'_r = b_r + \theta \qquad (b_r = r^{th} \text{ element of } b)$$

Now the new $\bar{b}_i$ is $\bar{b}^*_i$

$$X^*_i = \bar{b}^*_i = \bar{B}^1_i \cdot b + \bar{B}^1_{ir} \cdot \theta$$

$$X_i^* = \bar{b}_i^* = \bar{b}_i + \bar{B}^1_{ir} \cdot \theta \qquad (r = \text{column of } \bar{B}^1)$$
$$(i = \text{row of } \bar{B}^1)$$

Before any further procedure with the analysis, this last equation will be illustrated in terms of the numerical example:

The vector $\bar{b}$ is obtained as follows:

$$X_B = \bar{b} = \begin{vmatrix} 3/2 & -1/2 \\ -1/2 & 1/2 \end{vmatrix} \cdot \begin{vmatrix} 12 \\ 24 \end{vmatrix} = \begin{vmatrix} 6 \\ 6 \end{vmatrix}$$

Example: What happens to the second element of $\bar{b}$ (6) when the first element of b (12) is changed by θ. We obtain the second element of $\bar{b}$ by doing:

$$(-1/2 \quad 1/2) \cdot \begin{vmatrix} 12 \\ 24 \end{vmatrix} = 6$$

Now:

$$(-1/2 \; 1/2) \begin{vmatrix} 12 + \theta \\ 24 \end{vmatrix} = -1/2 \cdot (12 + \theta) + 1/2 \cdot (24)$$
$$= 6 - 1/2 \cdot \theta$$

The problem arises if θ is increased to $(12 + \varepsilon)$, then the second element of $\bar{b}$ goes negative and the solution is infeasible.

In general:

$$X_i^* = \bar{b}_i^* = \bar{b}_i + \bar{B}^1_{ir} \cdot \theta \quad \geq 0 \text{ for all } i.$$

1) If θ increases and $\bar{B}^1_{ir} > 0$, then $\bar{b}_i^*$ will never be negative. Therefore, there will be no change in the basis.

2) If $\bar{B}^1_{ir} < 0$ and θ increases, then for some θ, $\bar{b}^*_i = 0$. This gives the maximum value of θ.

$$\text{Max } \theta = \text{Min } [-\bar{b}_i/\bar{B}^1_{ir}] \quad (\text{i such that } \bar{B}^1_{ir} < 0)$$

We take the minimum value of $[-\bar{b}_i/\bar{B}^1_{ir}]$ because for this value of θ some $\bar{b}_i$ goes to zero; therefore, there is a change in the basis.

If $\bar{B}^1_{ir} > 0$ and θ decreases, for some θ, $\bar{b}^*_i = 0$, this gives the minimum value of θ.

$$\text{Min } \theta = \text{Max } [-\bar{b}_i/\bar{B}^1_{ir}] \quad (\text{i such that } \bar{B}^1_{ir} > 0)$$

Therefore the stability limits of the basis are:

$$b_r + \text{Max } [-b_i/\bar{B}^1_{ir}] \leq b_r \leq b_r + \text{Min } [-b_i/\bar{B}^1_{ir}]$$

$$\bar{B}^1_{ir} > 0 \qquad\qquad \bar{B}^1_{ir} < 0$$

Applying these results to the numerical example, we get:

b_i	Lower Bound b_r + Max$[-b_i/B_{ir}]$ $\bar{B}^1_{ir} > 0$	Enter. Var.	Exit. Var.	Upper Bound b_r + Min$[-b_i/B_{ir}]$ $\bar{B}^1_{ir} < 0$	Enter. Var.	Exit. Var.
12	12 + -6/ 3/2 = 8	I_2	X_3	12 + -6/ -1/2 = 24	I_1	X_1
24	24 + -6/ 1/2 = 12	I_1	X_1	24 + -6/ -1/2 = 36	X_3	I_2

The i^{th} row of the inverse, in which the corresponding $\bar{B}^1_{ir}$ was found, indicates the exiting variable.

The entering variable is found in the following way: in the row of the exiting variable, find the minimum ratio of $\overline{C}_j/\overline{a}_{sj}$, with j so that $a_{sj} < 0$. This relation is obtained from the dual simplex algorithm.

4. <u>Uncertain Input-Output Coefficients for Nonbasic Variables</u>

Problem: What is the effect of changing a matrix entry (a_{ij}) of a nonbasic variable?

In terms of the numerical example: there is only one nonbasic variable (X_2), its matrix entries are 1 (the truck takes 1 driver) and 2 (the truck takes 2 hours of maintenance per shift). This question arises: How many hours of maintenance per shift should the second type of trucks take to be considered in the optimum-operation plan? (There is no point in doing the sensitivity analysis of the number of drivers per shift.)

Relation 2c:

$$\overline{C}_j = C_j - \pi A_j$$

One of the nonbasic coefficients of A_j will be changed by θ. The new $\overline{C}_j$ will be $\overline{C}^*_j$.

$$\overline{C}^*_j = C_j - \pi a_{1j} - \ldots\ldots \pi_r(a_{rj} + \theta) - \ldots\ldots \pi_m a_{mj}$$

$$\overline{C}^*_j = \overline{C}_j - \pi_r\theta$$

1) If $\pi_r > 0$ and θ increases, then $\overline{C}^*_j = 0$ for $\theta = \overline{C}_j/\pi_r$

2) If $\pi_r < 0$ and θ decreases, then $\overline{C}^*_j = 0$ for $\theta = \overline{C}_j/\pi_r$

Therefore the limits of stability of the basis are:

$\pi_r > 0 \qquad -\infty \leq a_{rj} \geq a_{rj} + \overline{C}_j/\pi_r$

$\pi_r < 0 \qquad a_{rj} + \overline{C}_j/\pi_r \leq a_{rj} \geq \infty$

Applying these results to the numerical example we get: the only nonbasic C_j is $C_2 = -1400$, π_1 and π_2 are < 0

$\pi_1 = -800 \qquad \pi_2 = -400$

a_{rj}	Lower Bound $a_{rj}+\overline{C}_j/\pi_r$	Enter. Var.	Exit. Var.	Upper Bound ∞	Enter. Var.	Exit. Var.
$a_{22}=2$	$2+200/-400 = 3/2$	X_2	X_3 or X_1	∞		

5. <u>Effect of Introducing a New Variable</u>

Problem: What is the effect of adding a new variable in the optimum solution?

In terms of the numerical example: Would a new truck with the following characteristics be considered in the optimum solution?

Truck type 4: takes 3/2 hr of maintenance
hauls 1500 T/shift

All that has to be done is to calculate $\overline{C}_4$, if

$\overline{C}_4 > 0$ It will not be considered.

$\overline{C}_4 < 0$ It will be considered.

Relation 2c can be used for this purpose:

$$\bar{c}_4 = c_4 - \pi A_4$$

$$\bar{c}_4 = -1500 + (800 \quad 400) \cdot \begin{vmatrix} 1 \\ 3/2 \end{vmatrix} = -100$$

$\bar{c}_4$ is negative; therefore, if this variable goes into the basis the objective function will be increased. Consequently, this type of truck will be in operation.

As a variable is going into the basis, another variable must leave the basis. The exiting variable can be found by calculating the ratios $\bar{b}_i/\bar{a}_{i4}$, the row that gives the minimum ratio corresponds to the exiting variable:

$$\bar{b}_1/\bar{a}_{14} = 6/1 = 6$$

$$b_2/a_{24} = 6 \; 3/2 = 4$$

The second row corresponds to the variable X_1; therefore, this variable leaves the basis.

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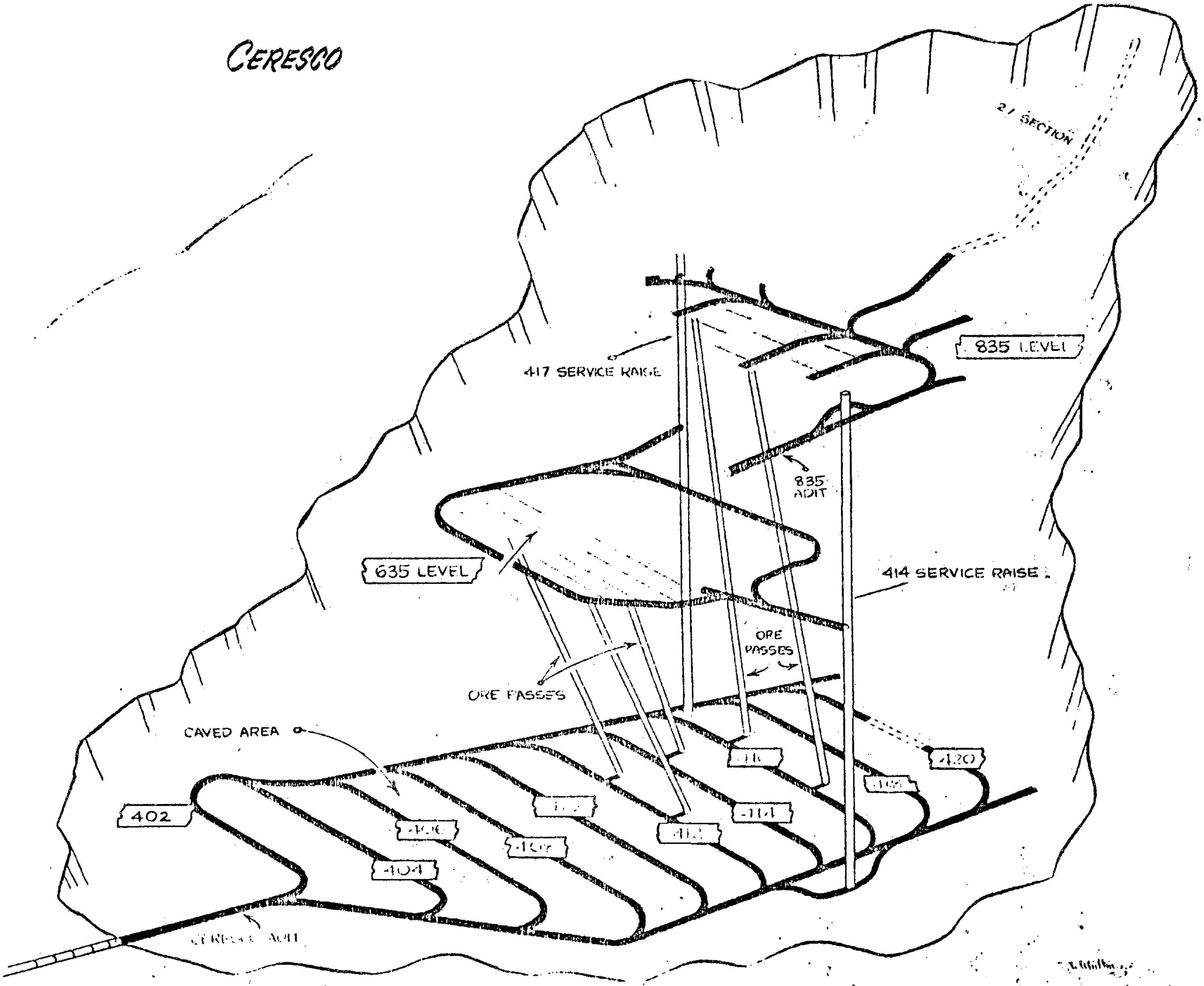

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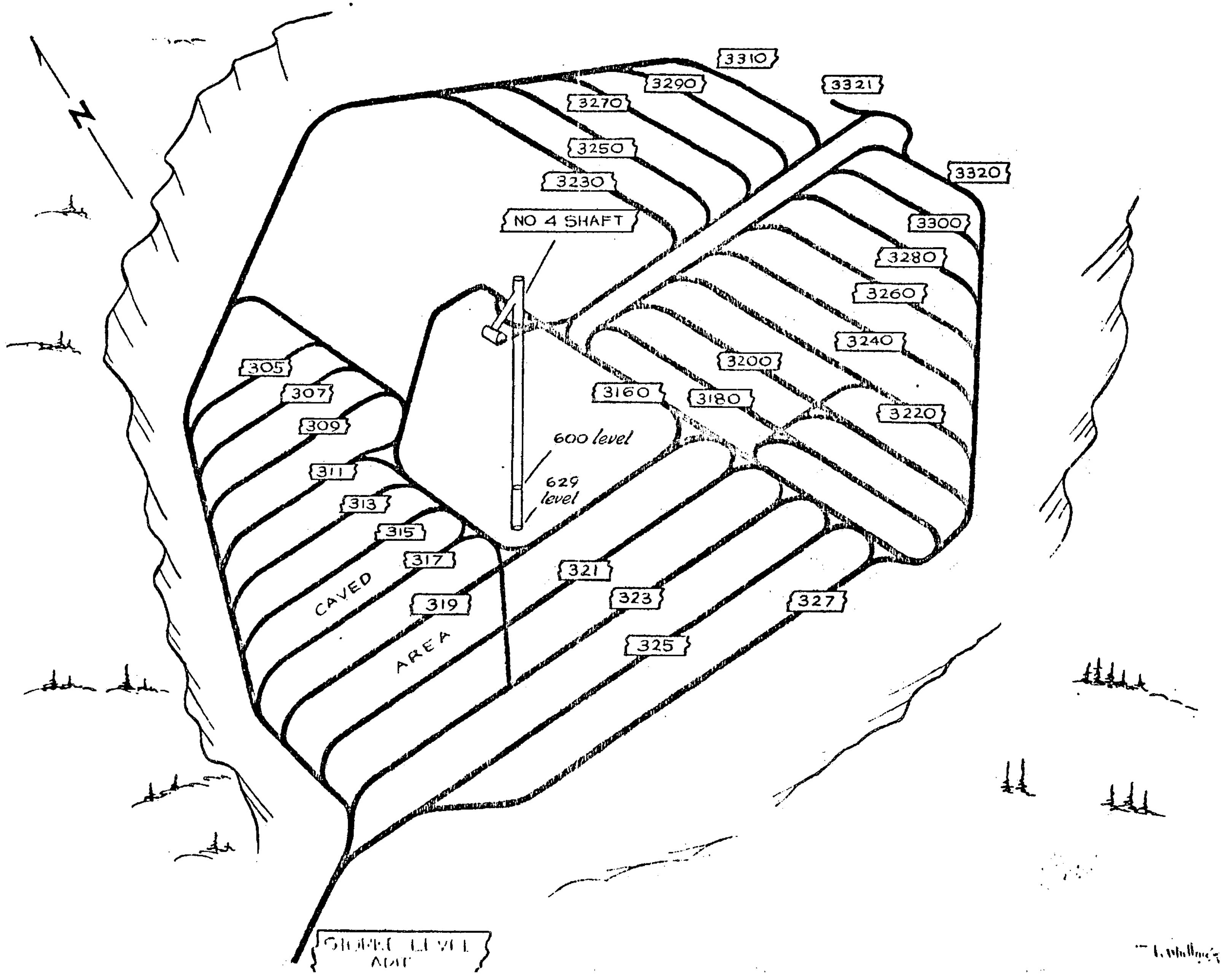
plate #2
T 1424

STORKE LEVEL



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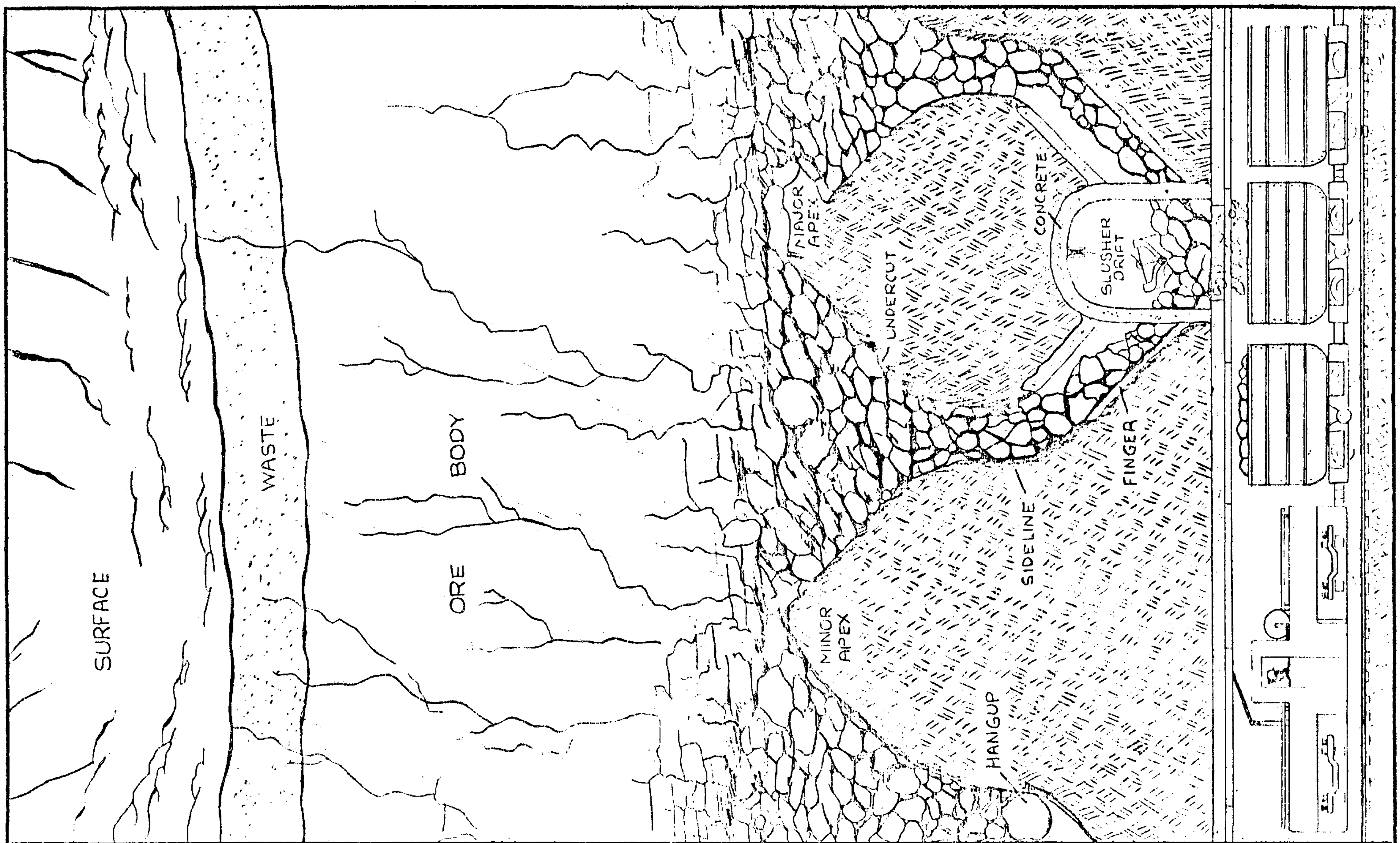

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CLIMAX MOLYBDENUM COMPANY
CLIMAX, COLORADO

BROKEN ORE BODY

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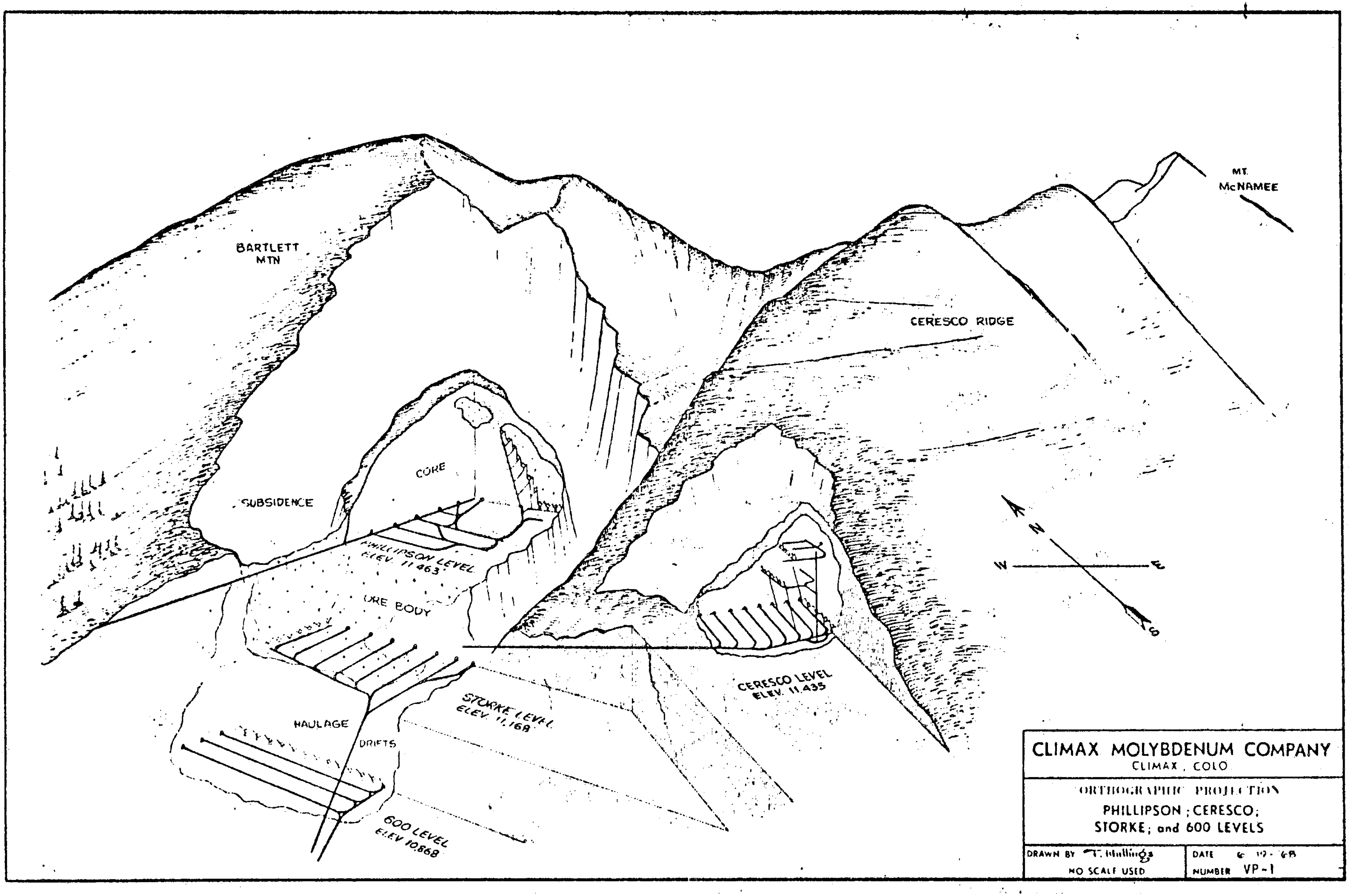
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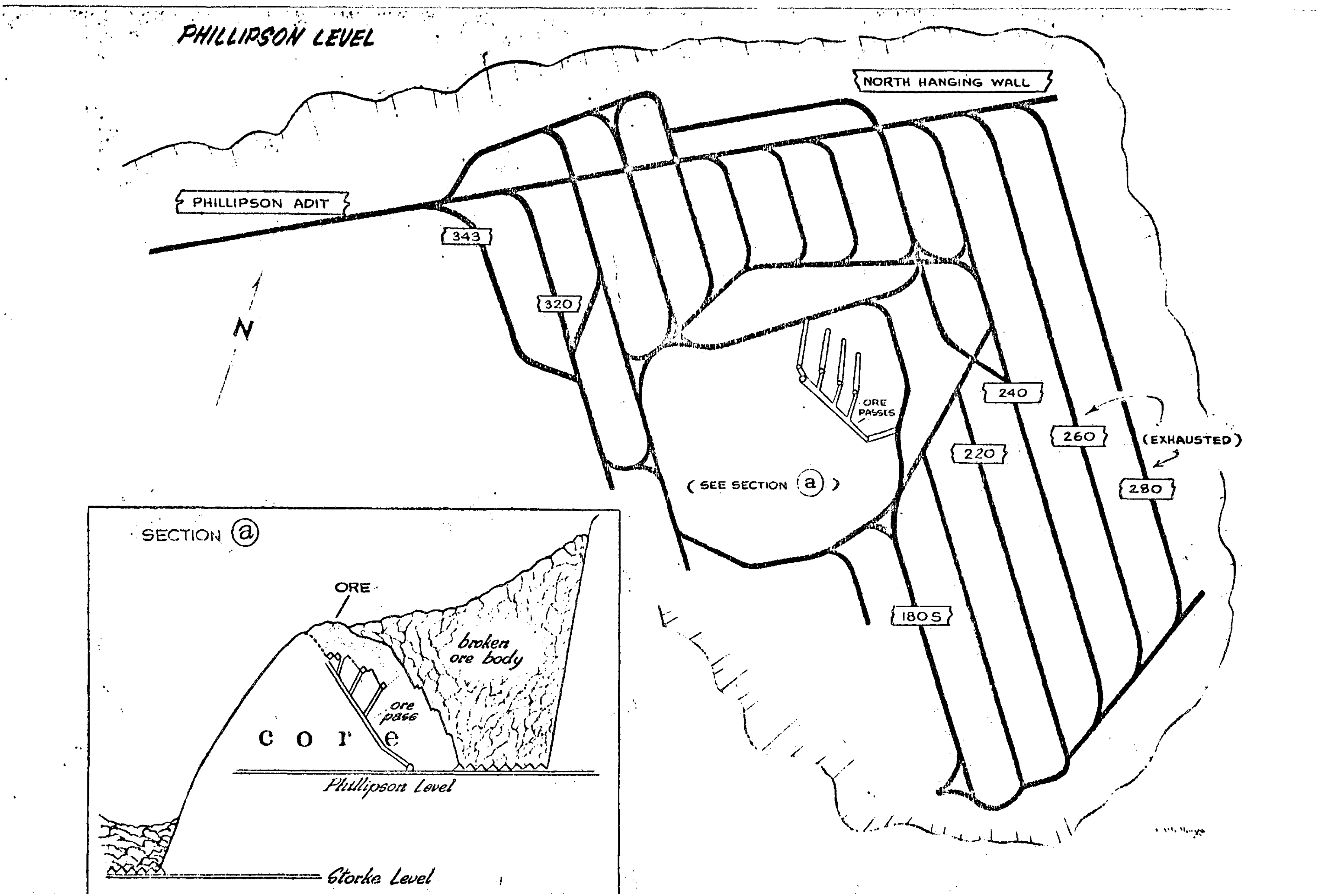

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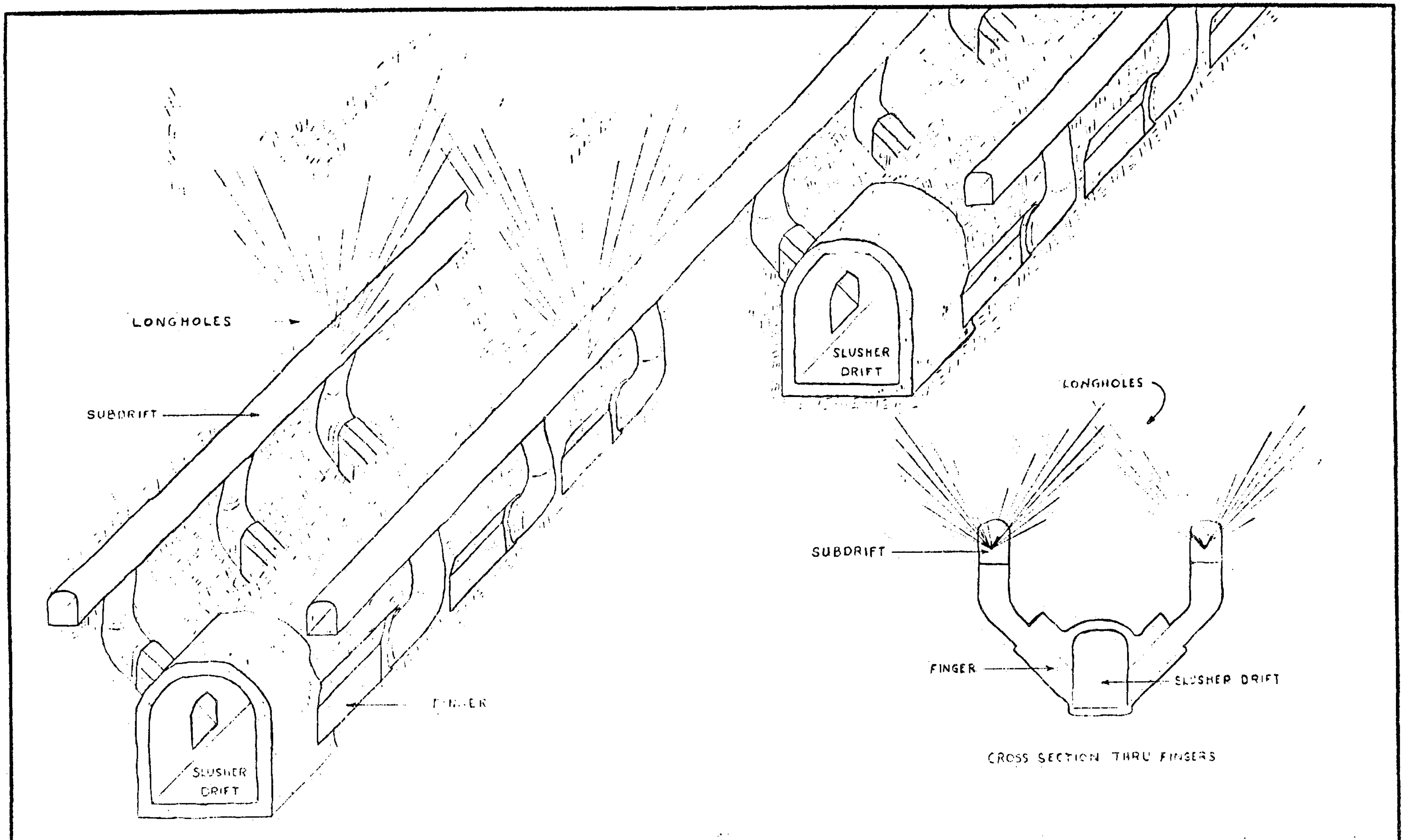

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NEW METHOD STOPES
SUBDRIFT TYPE

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Plate #7

600 AND 629 LEVELS

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