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THE PRELIMINARY DESIGN BY GEOMETRIC PROGRAMMING OF WATER TREATMENT PLANTS FOR ENHANCED RECOVERY OF OIL

T-2479

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ABSTRACT

The purpose of this thesis is to determine a mathematical method for preliminary design of water treatment plants for stimulated recovery of oil. Two models using a geometric programming technique are presented, assessed, and analyzed to establish the validity of their application for deriving efficient and least cost water treatment plant designs. A method for simplifying the solution of multi-degree of difficulty design problems for injection water and wastewater treatment plants using geometric programming is developed. Hypothetical design examples, based on the two models, are given and solved to illustrate the simplified solution procedure for deriving optimal (least cost) designs.

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DEDICATION

To my wife

RITA

and children

EVELYN, NEIL, VICTOR, and MICHELLE

CHAPTER I

INTRODUCTION

The explosive increase in the demand for oil and the gains associated with it, gave good reason for the operators in the oil industry to extract more of the oil remaining in the ground after the primary recovery had reached its economic limit. Once oil reaches the surface, it becomes a commercial product from which revenues become immediately available to the producers. To increase their revenues, operators engaged in oil production have made deliberate attempts, employing different techniques, to stimulate oil recovery from producing reservoirs. Water injection is one of the oldest and most widely used of these techniques. At the present time, it is regarded as a very reliable and economic recovery technique.

The recovery of oil from the reservoirs is a displacement process. Oil does not have any inherent ability to expel itself from the reservoir but must be displaced from the formation to the wellbore by a displacing agent. Generally, gas or water acts as the displacing agent. Usually, one or both of these fluids occur naturally in the pore space of the reservoir rock or within its environs. The effect of these fluids, singly or in combination, provides the primary driving

force for moving the oil into the wellbore. Once at the wellbore, oil is pushed by the underlying pressure or is lifted by pumps to the surface.[1]

As oil production continues, the reservoir pressure declines unless a fluid such as water flows into the reservoir to replace the oil produced. Reservoirs that have a contiguous aquifer are replenished by water influx from the aquifer as fast as the oil is normally produced. Thus nearly all oil reservoirs experience decreasing reservoir pressures throughout their production history.[2]

Traditionally, water is injected into or near to an oil reservoir for increasing oil recovery. The injected water will (1) delay the pressure decline in a reservoir during oil production; or (2) again increase oil production from a reservoir after a decline in pressure has occured causing a decline in the oil production rate from this reservoir. It is normally difficult to separate the factors that are important to each case since the important factors to one are also important to the other case. The two cases will be defined respectively as follows.

> "Water-pressure maintenance is a process whereby water is injected into an oil-producing reservoir to supplement natural energy indigenous to the reservoir and to improve oil-producing characteristics of the field prior to the time that economic productive limits have been reached.

> Waterflooding is a secondary-recovery method in which water is injected into a reservoir to obtain additional oil recovery by movement of reservoir oil to a producing well after the reservoir has approached its economic limit by primary recovery methods."[3]

Whether it is for pressure-maintenance or waterflooding, stimulated oil recovery from a reservoir remains the major objective of operators engaged in water injection.

Confusion often arises in attempting to distinguish between waterflooding, pressure-maintenance, and other methods used for stimulating oil recovery from reservoirs. Therefore, it is simpler to use an overall term to encompass the variety of processes which are used to increase oil recovery above the limits that would have been achieved under primary recovery. Enhanced oil recovery will be used in this thesis to encompass pressure-maintenance and waterflooding processes.

Waterflooding was an accidental development that took place in western Pennsylvania over one hundred years ago when water from a shallow water-bearing formation leaked into an oil producing well. While this virtually eliminated further oil production from this well, it did result in an increase in oil producing rates from surrounding wells.

The initial success of waterflooding in this reservoir can be attributed to a number of favorable factors. The reservoir had no water encroachment and contained oil with

little gas in solution. As a result, primary recovery was limited and the oil recovery by water injection was many times that achievable by pressure depletion.[4]

Oil extracted by primary recovery amounts to about one third of the original oil in the ground. Water injection applied to oil reservoirs permits the recovery of additional volumes of oil that was considered years ago as residual or unrecoverable oil. This process aids in recovering about 40-65 percent of the original oil in place.[5]

The operators in the petroleum industry, anxious to benefit from this process, have used it since its accidental discovery to increase the recovery of oil from oil producing reservoirs. Water is injected into or near to the oil reservoir through producing wells converted to water injectors or through new wells drilled solely for this purpose. Water injection has since been developed and improved immensely through field applications and extensive research.

A predominant factor controlling water injection operations is the location of injection and production wells with respect to each other and with respect to reservoir boundaries. In the case of continuous reservoirs with relatively large areal extent, it is common practice in conducting water injection operations to locate injection

and production wells in a regular geometric pattern which forms a symmetrical and interconnective network of wells. Water injection projects that are designed according to one of these well-known geometrical arrangements are called pattern floods. Figure 1.1 illustrates some of the typical arrangements of the various flooding patterns. It is often impractical or impossible to design water injection operations which simulate one of the standard geometrical flood patterns. In this case, the operator must select a less sophisticated well network — the choice being either a peripheral or random-type injection pattern. Figures 1.2 and 1.3 are typical representations of peripheral and random-type flood networks respectively.[6]

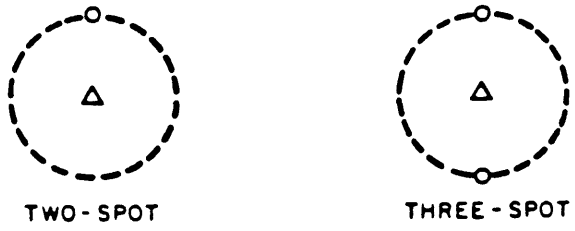
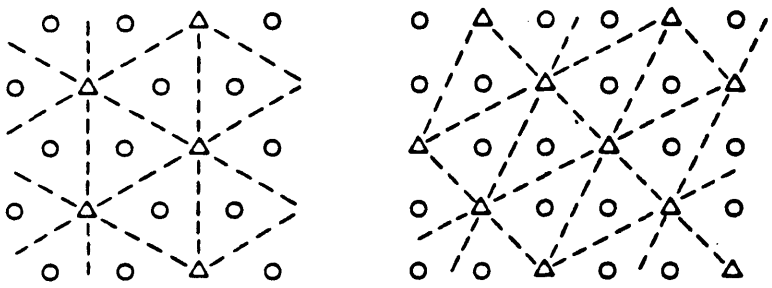
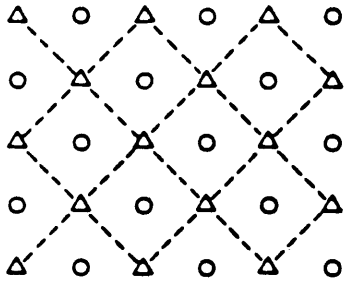


Figure 1.1 - Flooding Patterns

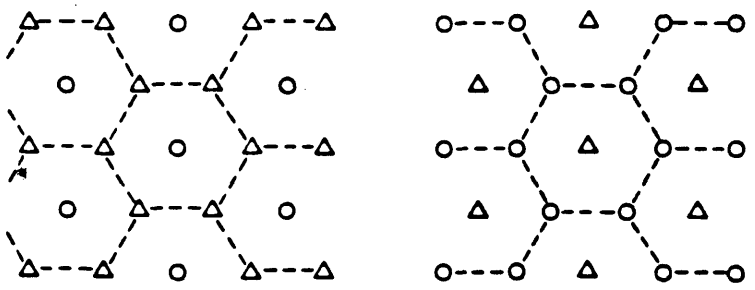


REGULAR FOUR-SPOT

SKEWED FOUR-SPOT



FIVE-SPOT



SEVEN-SPOT

INVERTED SEVEN-SPOT

Figure 1.1 - Flooding Patterns.
(Continued)

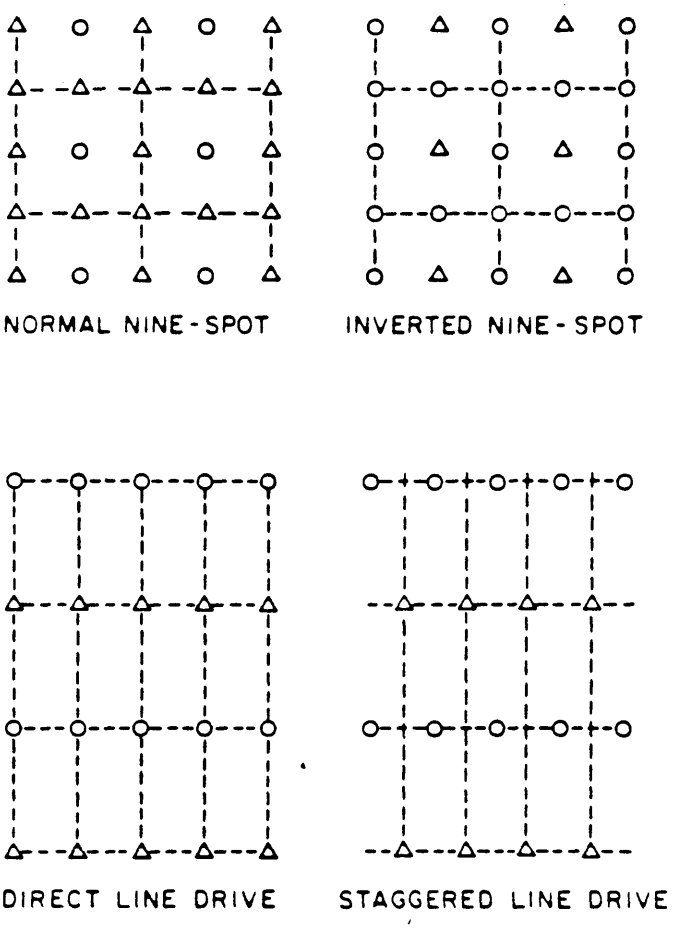


Figure 1.1 - Flooding Patterns.
(Continued)

Source: Secondary and Tertiary Oil Recovery Processes.
Interstate Oil Compact Commission.
op. cit. p. 14-16

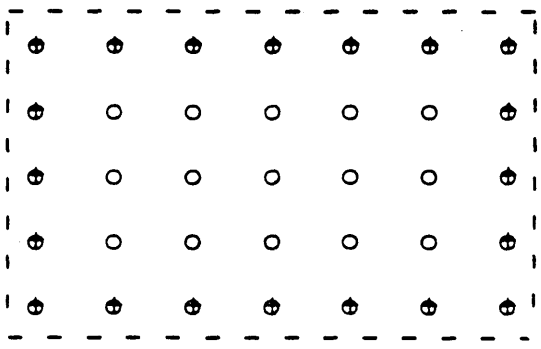


Figure 1.2 - Peripheral Flooding Pattern.

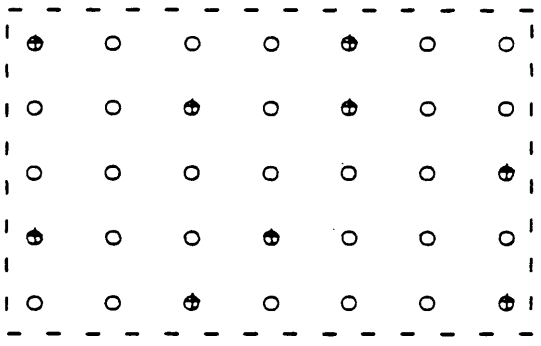


Figure 1.3 - Random-Type Flooding

⊕ Injection Well
o Production Well
- - - Arbitrary Reservoir Boundary

Notice that water is the major oil displacing agent in water injection. The operator must therefore secure extensive and inexpensive supplies of water before designing and implementing the water treatment and injection operations. Water for injection may be taken from one or several of the following common supply sources.

a. Surface water from oceans, rivers, streams, lakes, or reservoirs.
b. Wells specially drilled for this purpose which draw water from shallow aquifers which are connected to a surface water body.
c. Brine or fresh water from other subsurface zones.
d. Produced water which accompanies the crude petroleum in the producing reservoir.

Water used during the early days of waterflooding was given the consideration only of quantity, not quality. As these early years passed, it was not long before it was noted that higher injection pressures were required to maintain suitable injection rates and that corrosion problems mounted. The operators of these early installations began to realize that quality of the water was just as important as quantity, and that poor water treating was proving disastrous to waterfloods that might have been successful. Water-treating practices have improved as the waterflood industry has grown.[7]

The compatibility of injection water plays a primary role in water injection. Injecting incompatible water may lead to obstructions, clogging, and deterioration of equipment caused by corrosion in the various parts of the injection, collection, and production systems. The result is very costly reconditioning of the wells, replacement of the pipes, decline in production rate, wear in pumps, and in the most serious cases, loss of the wells.[8]

Waters from these diversified sources vary in composition and present different potential problems. They vary in the nature and concentrations of impurities that they contain. The substances in suspension or in solution that may be found in the water extracted from one or a combination of the sources mentioned above are listed in Table 1.1. Water from these supply sources will require a certain degree of treatment before it is injected into the reservoir. Type of treatment depends on the quality of the source of supply and the quality desired in the finished product. The sources of water supply fall into two categories; (1) surface water which comes from the sources in (a) above; and (2) ground water extracted from the remaining sources (b,c, and d) noted above.

Surface waters normally tend to be variable in quality. These waters are often contaminated by microorganisms which

TABLE 1.1

SUBSTANCES THAT MAY BE FOUND IN INJECTION WATER

In solution	Solids	Minerals	Anions	Chlorides: (Cl^-); bromides: (Br^-); fluorides: (F^-) Sulfates: (SO_4)$^{=}$; carbonates: (CO_3)$^{=}$; bicarbonates: (CO_3H)$^-$
			Cations	Sodium: (Na^+); potassium: (K^+); calcium; (Ca^{++}); barium: (Ba^{++}); magnesium: (Mg^{++}); iron; (Fe^{++}); manganese: (Mn^{++})
		Organic	plants and animals (plankton)	
	Gases	Oxygen: (O_2); nitrogen: (N_2); hydrogen sulfide (H_2S); carbon dioxide (CO_2)		
In suspension	Minerals	Ferric hydroxide: $Fe(OH)_3$ Silica: (SiO_2) and sand Clay and mud Alkaline earth sulfates and carbonates		
	Organic	Algae, bacteria (plant and animal materials) Oil		

Source: Enhanced Oil Recovery
Secondary and Tertiary Methods (1978), op. cit. p. 25

include molds, iron and manganese bacteria, sulfur or sulfate reducing bacteria, algae, and many other forms of microbiological growths. They are also subject to extreme turbidity changes during rainy seasons or to variable organic content. Surface waters also tend to contain lower concentrations of minerals and relatively higher levels of dissolved gases.[9]

Ground waters are more uniform in quality. Microbiological growth seldom occurs in uncontaminated deep wells. As water seeps through the rock strata, it picks up impurities and dissolves some part of almost everything it contacts. These waters normally contain greater amounts of dissolved substances. The natural filtering effect of rock and sand usually keep these waters free and clear of suspended matter. Produced water may in addition contain hydrogen sulfide and small amounts of oil.[10]

Information on the quality of water source is a basic requirement for the design of treatment plants. This information may be readily available if the source has been used for water supply purposes. Records that may represent several years' experience coupled with surveys and sampling will serve as a reasonable basis for design. If information is totally lacking, then more extensive surveys and sampling will be required.

Where water from more than one source will be used, the quality of the mixture must be determined. If the mixture is to consist of fresh water and brine or produced water, then it becomes imperative that the quality of this water is established under both surface and simulated reservoir conditions.

In evaluating the chemical and bacteriological quality of a water source where turbidity removal is the matter of primary concern, it is desirable to include as much long-term data on particulate matter and microbiological parameter as possible. If any special water quality problems exist, a history should be developed for these parameters as well. General mineral quality is also an important aspect of understanding the characteristics of a water supply. This is especially true where water treatment is concerned because certain aspects of the mineral quality, especially alkalinity and pH, have an important influence on the performance of the treatment processes.[11]

Armed with this information together with the short-term and long-term amounts of injection water required, the designer selects the treatment processes that would produce the water quality desired. These processes must be capable of conditioning, in varying degrees, every barrel of water that will be injected.

Profitable financial returns in the form of greater recoveries of lower-cost oil, together with trouble-free thus lower-cost injection operations will depend on the controls exercised over the quality of injection water. The series of processes in the treatment system must be capable of removing the impurities from the water so that the finished product (plant effluent) will conform to the designated quality. The major impurities that should be controlled are (1) suspended matter; (2)chemical precipitates; and (3) algae and bacteria. In addition, the operator must also control (1) corrosion; (2) clay swelling; and (3) surface forces. The technology and information required for effectively applying these controls are presently available at a relatively advanced stage.[12]

The designer selects the treatment processes that he or she believes would produce water with the desired quality. In the selection of the equipment and the associated facilities, the designer normally incorporates his or her preferences and experience in the design. Often a number of alternative preliminary designs involving various types and combinations of treatment processes to meet the desired performance may be prepared. The information generally provided in these designs may include (1) process flow sheets with some details; (2) preliminary material balance sheet;

(3) equipment list with size and capacity; (4) utility requirement; (5) engineering data - buildings and general site requirements; and (6) preliminary plot plan. A preliminary cost estimation of the alternatives based on this information is then calculated. Ranking of the alternatives, based on costs, merits, and liabilities of each design is then determined.[13]

The least cost preliminary design obtained from using this technique is not necessarily the optimal design. Its lowest cost is only relative to the cost of each of the various alternatives under consideration. If it turns out that this design is actually an optimal design, then its derivation was an accidental occurence. Further, this design method has a number of short-comings some of which are listed below.

1. This technique is highly sensitive to the preferences of the designer.
2. It requires a large number of computations which may be time consuming and costly.
3. Major policy changes will lead to more tedious calculations.
4. Ranking of the various alternatives may be biased by the preferences of the decision makers.

5. It does not provide an integrated quantity-quality-cost-effective design.

In spite of its short-comings, this design method is still widely used by designers in the various industries. The problems associated with this design technique are normally more pronounced when the treatment plant is implemented and is in operation. Variations in the actual quality of the plant effluent from the designated quality are rather difficult to detect and control because the efficiency of performance of each process is not determined and incorporated in the design of the treatment system. The operators will have to rely on making adjustments in various parts of the plant, based on trial and error, to establish the designated water quality. This method of quality control may prove to be very costly to the operators particularly if it leads to plant shut-downs and interruption of production operations.

It is the intention of this author to apply mathematical models to obtain optimal (least cost) treatment systems. These models will provide a quick and simple method for the preliminary design of treatment systems.

The mathematical models, using a geometric programming technique, will enable calculation of the total costs of each of the preliminary design alternatives before the design variables in each process in the system are determined. In

addition, this technique will facilitate the calculation of the cost associated with each process in the individual alternatives. Further, the capability of this technique in providing the designer with a simple method for performing sensitivity analysis on the effects of designated injection water quality on total operating costs will be illustrated. The inherent capability of geometric programming that aids in determining the effects of inflation on total operating costs will also be illustrated. Finally, a method for simplifying the solution of difficult design problems by geometric programming will be developed. The method is referred to as the "Simplified Solution Method." The validity of this solution method for deriving optimal solutions for multi-degree of difficulty water treatment design problems using geometric programming will be illustrated by applying it to a number of hypothetical treatment plant design problems.

CHAPTER II

BASIC CONCEPTS OF WATER TREATMENT

The characteristics of the proposed source of injection water and the required changes in these characteristics determine the water treatment procedures and the plant design. The primary objective of the treatment plant is to condition the water so that it meets the quality and the quantity requirements for injection into the reservoir.

The water treatment plant must be designed to treat the maximum amount of water required for daily injection. The absolute maximum capacity must therefore be incorporated into the building of the plant. The maximum design capacity must not be exceeded to obtain additional volumes of treated water because these additional volumes cannot be obtained without sacrificing water quality.

The quality of the source water, with due consideration given to variations, future changes, and the quality goals form the basis for selecting the type of system and the associated processes to achieve the finished product. Generally, the treatment systems are classified as open systems or closed systems.

OPEN SYSTEMS

In this type of system, there is no attempt made to exclude the contact between water and air. The system is open to the atmosphere and large amounts of oxygen dissolve into the water. Surface water sources such as rivers, lakes, and oceans are already saturated with air and are generally handled in complex open systems. Figure 2.1 is a sketch of a typical open system.

Some advantages associated with open systems are (1) they can be exposed to the atmosphere; (2) they allow for storage of large volumes of water in open pits or retention ponds; (3) suspended solids settle out of the stored water and reduce the load on the system; (4) water has an opportunity to neutralize; (5) water is aerated and any amounts of H_2S and CO_2 occuring in the water are removed.

The major disadvantages of this type of treatment system are as follows.

1. Oxygen, which is a very corrosive element and promotes the growth of certain types of micro-organisms, remains in the system.
2. Large amounts of water are lost from the system through evaporation.

Many operators with access to adequate water sources use this type of system and depend on chemicals for corrosion control.[14]

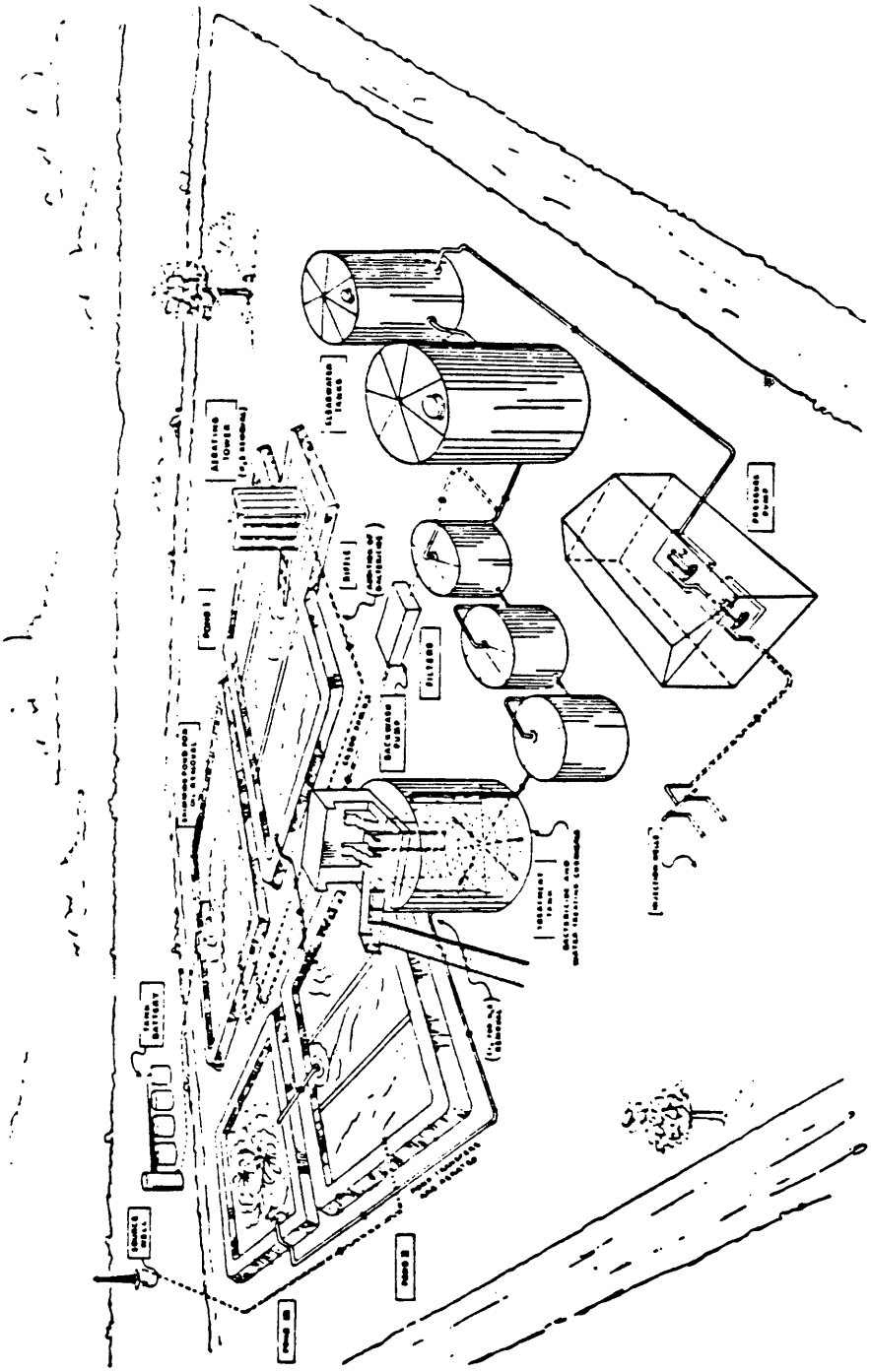


Figure 2.1. Sketch of a typical open waterflood system

Source: Sharpley, J.M., Applied Petroleum Microbiology. op. cit. p.150

CLOSED SYSTEMS

The objective in the design of a closed system is to exclude air-water contact. In this type of system, the water is prevented from contacting air and dissolving oxygen. This system is traditionally used for the treatment of water which contains very small amounts of oxygen or is initially free of air. Figure 2.2 is a sketch of a typical closed system.

The main advantages of this type of installation are (1) the elimination of the cost of auxiliary treating equipment; (2) the elimination of the cost for extensive chemical treatment; (3) the attendant cost of settling basins, pumps, and filter equipment; (4) most dissolved elements are maintained in solution as a means of preventing the formation of excessive quantities of suspended or precipitated solids.

Some of the disadvantages of this system are listed below.

1. This type of system cannot be conveniently adapted to handling waters that require complex treatment.
2. Air leakages may be noticed only after corrosion and scaling have set in the system. The detection of leakage points in the system is usually very difficult.

This type is used extensively in the oil industry.[15]

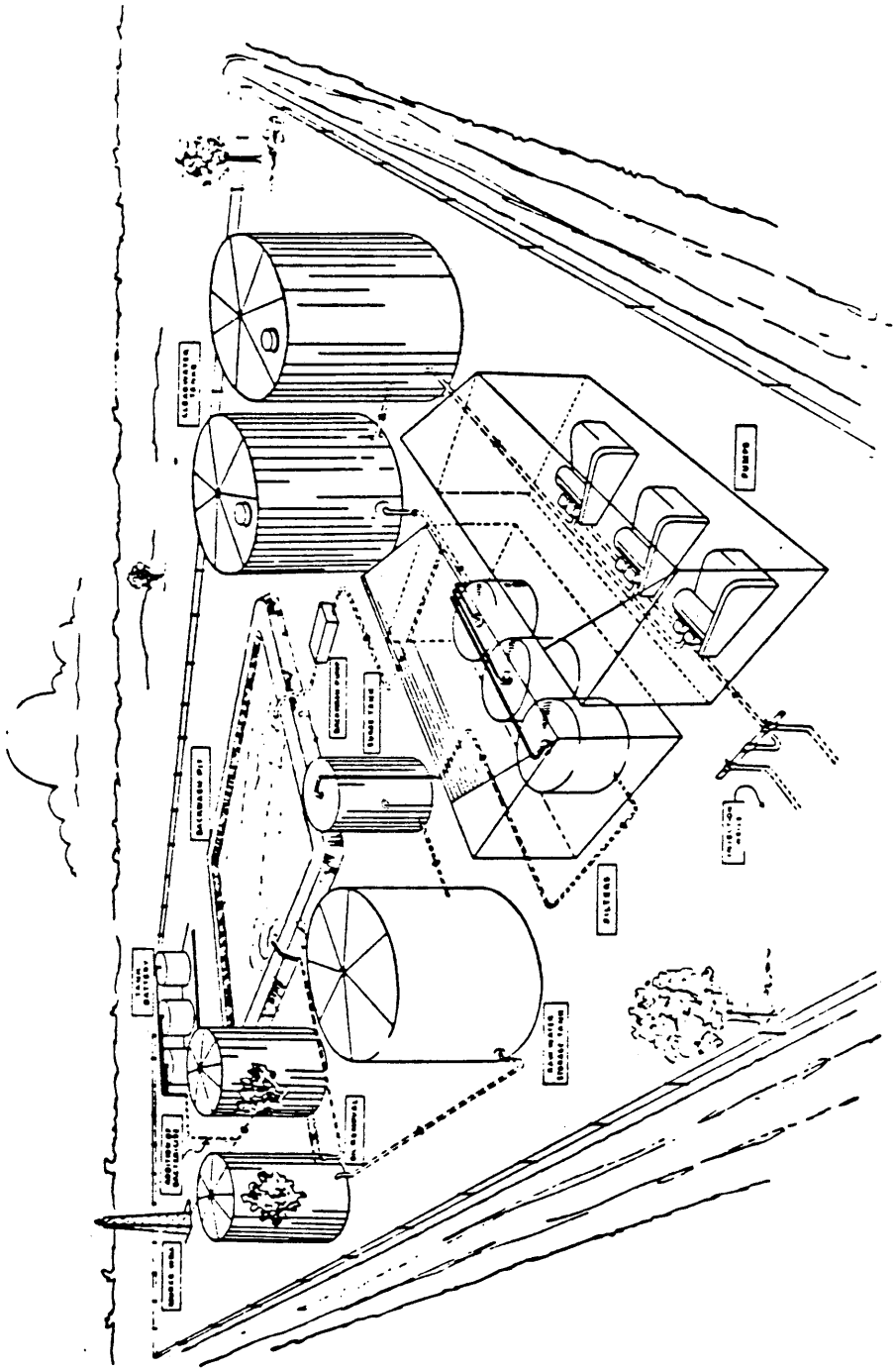


Figure 2.2. Sketch of a typical closed waterflood system

Source: Sharpley, J.M., Applied Petroleum Microbiology. op. cit. p.151

In the cases where the supply source is inadequate to meet the maximum daily requirements and the disposal of the water produced with the oil (produced water) is less economical, it may be necessary to consider produced water as the logical injection fluid. In such cases, the treatment plant may be designed as a semi-closed system comprising a combination of an open system and closed system. In a semi-closed system, water will remain in the open system until it is treated to remove the dissolved oxygen and then will be flowed into a closed system from there on to the injection well.

SELECTION OF TREATMENT SYSTEM

The selection of the treatment system must comply and be consistent with the individual characteristics of each project. The designer is obligated to direct his or her efforts and determination toward developing a design that is (1) feasible to produce and construct; (2) capable of conditioning the required quantity of the water to the desired quality; (3) to operate with the least amount of problems; and (4) to fulfill the performance objectives with the least cost possible.

The type of system selected will consist of unit processes or combinations of unit processes. These processes must be capable to effect physical and chemical changes in

the water and produce an effluent consistent with the designated quality.

In many water treatment plants for enhanced recovery of oil, sedimentation may be all that is necessary to remove suspended solids from the water. Other waters may require coagulation, sedimentation, filtration, disinfection, as well as additional auxiliary processes such as chemical treatments for scale inhibition and corrosion control.

Turbid waters are normally coagulated and clarified to remove finely divided colloids that always present problems in water treatment. These colloids are often negatively charged particles kept in suspension by mutual repulsion. The addition of a chemical, called a coagulant, to the water causes the aggregation of some fine particles and the absorbtion of others to produce a larger particle called floc. The process is speedy and occurs within seconds of the application of the coagulating reagent to the water. Rapid mixing at the point of application of the chemical is necessary to insure uniform chemical distribution and exposure to the fine particles in the water to the coagulating agent before the coagulation reaction is completed. Coagulation and flocculation aid in attaining complete envelopement of the suspended particles within the floc particles, and to condition the floc particles so that they

will readily be removable in the subsequent processes of sedimentation, or filtration, or both.[16,17]

The solids that have been rendered more settleable by coagulation and flocculation are then allowed to separate from the water by gravity. The separation process, called sedimentation, takes place in sedimentation basins. These basins may also be identified as settling basins, settling tanks, sedimentation tanks, or clarifiers.[18]

Sedimentation is one of the most widely used processes in the treatment of water, second perhaps to chlorination. This process is usually applied in two principal ways in water treatment — plain sedimentation or sedimentation after coagulation and flocculation. Plain sedimentation is used to remove settleable solids that occur naturally in surface waters. The solids settle out of the water without any previous treatment. This process is usually used as a preliminary process to reduce heavy sediment loads prior to subsequent treatment processes such as coagulation.

The selection and sizing of the settling basins depends on the amount of information available on the characteristics of the solids in the water and the experience of the engineer. There are no simple mathematical formulations available that would permit "plugging" some pertinent values into algebraic formulas and arrive at a good settling tank. Therefore, the

engineer should attempt to select or design settling units with hydraulic properties best suited to the settling properties involved. Because there are many different raw waters, most of the published design criteria and the manufactured basins available from different suppliers are quite broad in nature, thus further complicating the selection or design procedure to the engineer.[19]

Filtration normally follows sedimentation in the treatment system. Water filtration is a physical and chemical process of separating suspended and colloidal impurities from water by passage through a porous medium, usually a bed of sand or other granular material. The impurities in the water are left behind in the filter material or are adsorbed to the surface of the filter grains as the water passes through the filter. Water filters may be classified in various ways. Hydraulically they may be identified as rapid or slow filters, depending on the rate of flow per unit of surface area. Filters are also classed according to the kind or type of filter media employed, such as sand, coal (or anthracite), coal-sand, multilayered, mixed bed, or diatomaceous earth. They may also be described according to the direction of flow through the bed, that is, downflow, upflow, fine to coarse, or coarse to fine. Another distinction is between pressure and gravity filters.

Selection of the size and type of filter is of primary importance to insure efficiencies and effect the final clarification in the injection waters. Filters are back-washed with clear water whenever the quality of the water is impaired or filters become choked. Clear water is usually supplied from clear water tanks or from separate tanks specially incorporated in the design of the treatment plant to serve this purpose.[20]

The occurence of microorganisms in injection waters causes many of the problems and difficulties experienced in the recovery of oil by water injection. The problems associated with the microorganisms may be expected to occur at certain points in the system, depending on the type and characteristics of the microorganisms in the water. Most of the kinds of organisms that live in water and cause problems are known. However, the greatest damage in water injection operations is caused by only a few genera. Algae and bacteria are the two major groups that consist of the many classes of microorganisms that create the most serious problems. The problems associated with algae may be filter clogging, contraction of pipes and wellbores, scaling and corrosion of production and injection facilities, and the scaling of the producing formations at the bottom of the wells. They also grow in clear water tanks and reservoirs

thus corroding them and contaminating the water. Since bacteria are in many characteristics very similar to the algae, the problems caused by these two groups are in general very similar.[21,22]

Disinfection of water eliminates many of the problems caused by the microorganisms in the injection water. This process, as now ordinarily considered, involves using chemicals for the destruction or inactivating the problem causing organisms. The microorganisms occuring in the water usually dictate the type of chemicals used for disinfection. Chlorine is the most widely used chemical because it is the most economical and easy to use disinfectant. There are many other types of chemicals used for bacterial control which are broadly classed in several ways. The interested reader is referred to the AWWA book, "Water Quality and Treatment", and many more technical publications on the subject of chemical disinfectants.[23]

Scaling and corrosion are among the common problems in water injection. Where such problems occur, chemicals are used to either eliminate their source or control it. A large number of scales are corrosion products and the use of proper corrosion inhibitors eliminates such scales. Other types of scale may require special chemicals to inhibit their formation.

Scale inhibitors aid in delaying, reducing, or preventing the scale formation when added in small amounts to a normally scaling water. Most of the scale inhibitors used in the oilfield function by one or both of the following mechanisms.

1. When scales first begin to form, very tiny crystals precipitate from the water. At this point, the scale inhibitor adsorbs onto the surface of the crystals while they are still very tiny and prevents further growth. This is thought to be the primary mechanism by which most oilfield scale inhibitors work.
2. In some cases scale inhibitors prevent the precipitated scale crystals from adhering to the solid surfaces such as piping and vessels.

The scale inhibitors must be applied into the system where scaling tendencies of the water are most likely to occur. They must be present in the water on a continuous basis so that they are available to inhibit the growth of each scale crystal as it precipitates from the water. Therefore, either continuous chemical injection or a "batch treatment" method, which provides a continuous supply of inhibitor to the system between treatments, must be used.[24]

Salts and gases dissolved in the water have a marked influence on the corrosivity of water. At extremely low

concentrations of dissolved salts, different anions and cations show varying degrees of influence on the corrosivity of the water. Gases such as oxygen, carbon dioxide, and hydrogen sulfide are very corrosive when they occur in water. Deaeration is frequently used to remove oxygen from the water. Degasification on the other hand is a general term applied to the removal of dissolved gases such as CO_2 and H_2S.

Corrosion is usually reduced or eliminated by the addition of chemical substances, called corrosion inhibitors, to the water. By the addition of corrosion inhibitors to the water, a simple and inexpensive solution to the corrosion problem is often found. There is a wide variety of chemical substances used as corrosion inhibitors. The conditions of the environment and type of corrosion govern the choice of inhibitor. Usually, the choice of the inhibitor is based on the experience of the corrosion engineer and some trial and error testing. Often, the method costing the least in money and manpower, yet giving adequate protection is chosen.[25]

Corrosion should always be considered in the design of a water treatment system. A little thought and a minimum of effort during the various stages of design may save a significant amount of money and extend the life of the system. Poor planning can often lead to the use of more costly corrosion

mitigation methods than one that could have been used had the system been adequately designed. The reader, if interested, is referred to the numerous technical publications on the subject of scaling and corrosion in water handling systems.

The above discussion presents a brief explanation of the principal process elements of a water treatment system. Ancillary process elements include chemical storage, feeding of chemicals, and sludge disposal. It is of primary importance in the design of a treatment system to understand that the individual process elements and the flow processes affect each other. Although the designer may be aware of these effects, the exact position and function of the separate elements in the flow diagram must be kept in mind. The proper selection of the process elements and their position in the flow diagram will determine the quality of the finished product.

There are several approaches available to the designer for choosing the various processes and the components of each process to produce a number of alternative designs that would produce water of the designated quality. Among these approaches are a few presented in a number of reports published by the Environmental Protection Agency.

In 1978, the Environmental Protection Agency published Estimating Costs for Water Treatment as a Function of Size

and Treatment Efficiency. This publication presents different cost curves for flows ranging between 2,500 gpd and 1 mgpd. In 1979, the same agency published Estimating Water Treatment Costs, and in September of the same year, a two volume report Managing Small Water Systems: A Cost Study, was published. The first of these reports presents cost curves for flows ranging between 1 to 200 mgpd treatment plants. These publications provide the designer with a simple methodical procedure to design treatment plants of various capacities and produce water of designated qualities. Total cost curves for construction, operation, and maintenance of the plant are presented. Detailed costs of each process and the costs of the associated components including energy, maintenance materials, and labor are also presented. Contingency allowance, interest during construction, contractors overhead profits, and many more cost factors are also provided. The designer can select a number of alternative combinations of processes for a particular treatment plant and determine the total costs by simply reading each cost from the appropriate curve.[26]

In spite of the simplicity of this design and costing method, using it does not necessarily yield the least cost design. The selection of the flow diagram and the unit processes are chosen by the designer and are influenced by

his or her experience, past performance of similar designs, and personal preferences. This method also requires a large number of computations for each design alternative. The calculations will have to be repeated whenever major policy changes are initiated, a time consuming and costly process to the operator.

A mathematical model for the optimum design of a treatment plant using precoat filtration has been developed by E. Robert Baumann.[27] The design procedure uses a computer program to determine the optimum design. Prior to applying the computer program, a large number of data must be determined first. The optimum water turbidity, specific resistance of filter cake, body feed concentration, suspended solid concentration, and a large number of additional variables must be predetermined. The calculation of these parameters, in some cases, will require additional computer programs specifically formulated for each specific case, water source, and designated effluent quality. In addition, where turbidity is high in the source, the conventional treatment processes, namely, coagulation, flocculation, and sedimentation must be used to render the influent to the filter consistent with the filter handling capacity. Further, the majority of the costs involved in using precoat filters are in the variable operating cost area which are subject to

significant inflationary costs. This method may yield an optimum design under specific static assumptions, but once the filter is put into operation, the optimality will diminish rapidly because there is no method provided in the design to control the numerous variables that determine the operation of the filter.

The mathematical models discussed in the next chapter of this thesis will be used to determine optimal designs for water treatment plants for enhanced recovery of oil. These models have the capability of providing a preliminary design more quickly than the methods discussed above.

CHAPTER III

GEOMETRIC PROGRAMMING MODELS

A geometric programming technique is used in this thesis for the preliminary design of water treatment plants for the enhanced recovery of oil. The geometric programming technique of Duffin, Peterson, and Zener[28] often yields the total cost of a design before the design variables are established. This technique permits the calculation of the total cost of a number of alternative designs from which the policy makers may choose the one alternative that best suits their corporate policies. With the design alternative chosen, the optimal dimensions of the design variables may then be determined.

The percentage contribution of each process or combination of processes to the total cost can be determined from the total cost of the selected design. The benefit associated with this step is that if costs need to be cut due to policy changes, the cuts will be directed toward those processes that contribute the most to the total cost to achieve the most saving.[29]

The benefits derived from using geometric programming for design purposes are summed up in the following statements:

> "Geometric programming has the property that you may find the optimal solution before knowing the optimal values of the design parameters.

> Put another way, with geometric programming one can determine the cost of a design before the design is final."[30]

The theory and techniques of geometric programming will not be presented in this thesis. The interested reader is referred to the authors mentioned above and to Beightler, Charles, and Phillips (1976). It is sufficient to note that by using geometric programming, an optimal design may often be determined by solving a number of simultaneous linear equations to evaluate a number of unknowns. This process involves simple algebra as opposed to the usual analytical techniques that require calculus or some other relatively more difficult mathematical technique.

Two geometric programming models will be discussed to determine the possibility and extent of their application to the preliminary design of water treatment plants for the enhanced recovery of oil.

Ecker and McNamara, in 1971, presented a geometric programming model for the preliminary design of an industrial waste treatment plant.[31] The basic characteristics of this model are:

1. It considers only one pollutant;
2. It does not consider plant capacity as a variable parameter;
3. It does not include waste disposal.

This model may not represent a realistic situation in waste treatment. In real situations, most wastes contain more than one pollutant and the sludge accumulations from the various treatment processes have to be disposed of eventually. Further, the plant capacity will affect the total investments incurred by the operators which are of primary importance in corporate policy decisions.[32]

The model, however, is consistent with injection water treatment plants. It considers the treatment plant as a series of processes each of which removes a certain fraction of the pollutant from the influent to that process. In their discussions, Ecker and McNamara indicated that each process will remove a fraction of the BOD from the influent to that process. The fraction of BOD remaining in the effluent from the last process will be in accordance with the specified amount. This is in a way similar to the functions of the process or combination of processes through which water flows in the injection water treatment plant to emerge from the last process with the designated quality. In this respect, the objective of the waste treatment plant and that of the injection water treatment plant are consistent in that in each case the effluent from the last process must conform to some predetermined quality.

The fact that this model considers only one pollutant

does not render it an unrealistic model for injection water treatment plants. The situation where only one pollutant needs to be removed from the injection water arises quite frequently. Where water is produced from a deep water producing sand, the water, quite often, is practically free from microbiological contamination and dissolved or precipitated minerals. It may contain, though, some sand particles which have to be removed before this water is injected into the reservoir. In such a case, there will be only one pollutant that will need to be removed from the water. Another case where only one pollutant needs to be removed from the injection water may be represented by a water injection project where only produced brine is used as the injection fluid. In such a case, the brine, which may be carrying some oil, will be skimmed to remove all traces of oil and then allowed to flow by gravity or under pressure to the input well.

In the two cases mentioned above, or any other cases similar to these, the operator will not be burdened with the problem of waste disposal. The sand from the former case may be stockpiled or spread into layers in the environs of the plant with no apparent difficulty or significant costs. Oil from the latter case, may, on the other hand, be pumped into the oil storage tanks or oil pipelines.

The model as presented by Ecker and McNamara, may still be used to derive an optimal solution although it does not consider the plant capacity as a parameter. The optimal solution may be based on the cost of each unit of flow through each process or combination of processes in the treatment plant. The total cost per unit flow will be the sum of the unit cost in each term in the cost function. Since the various alternative designs will be required to handle the same capacity, the optimal total unit cost will yield the optimal design. In other words, the total cost for each alternative design will be the total unit cost for that design multiplied by a common constant for all the alternative designs, namely, the maximum plant capacity.

It can be concluded from the above analysis that the model presented by Ecker and McNamara may be effectively applied to design optimal water treatment plants for the enhanced recovery of oil.

According to Ecker and McNamara, the costs may be written in the form

$$y_i = c_i x_i^{a_i} \qquad (1)$$

Where;

y_i = total annual cost of the i-th process

c_i = a positive constant

x_i = fraction of the BOD (Biochemical Oxygen Demand) remaining after each process.

a_i = a negative constant.

The authors also determined that the maximum fraction (K) of BOD remaining in the effluent from the last process is equal to the product of the x_i's. Further, the exponent a_i is negative because as the percent of BOD increases in each process, the costs incurred for that process decrease.

It has been noted earlier that the treatment plant consists of a number of processes in series, therefore the total annual operating cost is the sum of the annual costs incurred in each process. These costs are represented in the model presented by Ecker and McNamara in a mathematical formulation as follows:

$$\text{Minimize ATC} = C_1 x_1^{a_1} + C_2 x_2^{a_2} + \ldots, + C_n x_n^{a_n} \qquad (2)$$

$$\text{Subject to:} \quad x_1 x_2 \ldots\ldots x_n \leq K \qquad (3)$$

$$x_i > 0 \qquad (4)$$

$$K > 0 \qquad (5)$$

The formulation of this model as applied to a water treatment plant for enhanced recovery of oil may be written as follows:

$$\text{Minimize ATC} = C_1 x_1^{a_1} + C_2 x_2^{a_2} + C_3 x_3^{a_3} \quad (6)$$

$$\text{Subject to: } \frac{1}{K} x_1 x_2 x_3 \leq 1 \quad (7)$$

(Equation 7 is written in the proper form required for geometric programming).

$$x_1, x_2, x_3 > 0 \quad (8)$$

Assuming:

1. The plant removes only one pollutant from the water.
2. The final designated water quality may be achieved by three processes in series.
3. K is the maximum designated fraction of pollutant remaining in the water after the final process.

The optimal preliminary design may be determined if values, corresponding to a number of alternative designs, are assigned to the constant factors and exponents in the above formulation. This model will be illustrated by applying it to a hypothetical design in the next chapter of this thesis.

In 1978, Maurer presented a general geometric programming model in a Doctoral Dissertation titled "A Geometric Programming Approach to the Preliminary Design of Wastewater Treatment Plants." This model is highly flexible and can be expanded to include complex treatment processes which are often required to produce finished products conforming to the

standards set by various regulatory agencies. The model is so formulated that (1) it can be expanded to include any pollutants which the regulators decide to control; (2) it includes plant flow as a variable parameter; (3) it includes the cost term for maintaining a certain minimum level of dissolved oxygen which is not considered a pollutant in wastewater treatment; and (4) it includes waste disposal.[33]

It was noted earlier in this thesis that the quality of the source and the quality goals of the finished product form the basis for selecting a treatment process or combination of processes of which the treatment facilities consist. In the wastewater treatment, the complexity of the treatment facilities is also determined by the quality of the wastewater supply to the plant. If the most adverse conditions exist where the wastewater requires a complex treatment plant to achieve the designated quality in the finished plant effluent, the design process becomes rather complicated. The design process becomes even more complicated where a number of alternative designs will be required to facilitate the selection of the most suitable one that meets with the corporate policies. The general model presented by Maurer, (see Appendix A), provides for including all the cost terms associated with a complex design by simply adding the additional cost terms to the objective function. If, on the

other hand, a rather simple treatment facility is dictated by the quality of the influent to the treatment plant, the objective function may be formulated to include only those cost terms that are appropriate to the design. In its simplest form, the model will reduce to the form presented by Ecker and McNamara while incorporating all the advantages associated with this model.

Similar situations may face the designer of injection water treatment plants. The need for a complex design may arise if the water supply source or combination of sources contain any number of pollutants that have to be removed from the water to render it compatible for injection. On the other extreme, the water may simply require treatment to remove only one pollutant as in the cases mentioned earlier. In either situation, the geometric programming model developed by Maurer may be effectively used for the design of the injection water treatment plants. The objective function for the injection water treatment plant will include only those cost terms that are associated with the process or processes comprising the treatment system.

An additional advantage that may be derived from this model comes from the cost term of adding dissolved oxygen to the final effluent from the treatment plant. According to Maurer:

> "The expense of adding dissolved oxygen can be treated in the same way in the objective function as the case of removing pollutants."[34]

The costs associated with adding dissolved oxygen to the effluent may be treated in a different aspect in the design of injection water treatment plants. The operators engaged in water injection often add chemicals such as scale inhibitors, corrosion inhibitors, disinfectants, and chemical aids to the injection water as the needs may arise. The expenses of adding these chemicals contribute to the total costs incurred in the treatment operations. If one chemical is added to the water, then a cost term will need to be added to the objective function to include the cost of adding this chemical. Additional such terms may be added to the objective function to accommodate for each chemical added to the water. The objective function for an injection water treatment plant may therefore include all the cost terms associated with the process or processes that remove the pollutants from the water in addition to the cost terms of the chemicals added to the water in the treatment system.

Waste disposal is a serious and costly problem faced by both the operators in the oil industry and state agencies; they have spent large sums of money for the determination of methods to dispose of wastes in general and produced brines in particular. Freshwater waste is relatively easier to

handle and dispose of than the produced brines. Many disposal methods are used to avoid contaminating surface and subsurface waters. Some of the more common means of salt-water disposal include (1) disposal into seepage sumps; (2) controlled disposal into streams; (3) collection and evaporation in surface pits; (4) injection into salt-water-bearing formations; and (5) piping to ocean or other salt-water body.[35]

The costs associated with waste disposal may be significant; therefore, an optimal disposal method must be determined. Where waste requires treatment before disposal, alternative plans must be considered and the optimal plan adopted. This means that the waste treatment system must be designed in a manner similar to that used for the optimal design of wastewater or injection water treatment plants. According to Maurer:

> " The sludge treatment and handling processes are essentially independent of the rest of the treatment train. Hence, the sludge processes can be considered as a separate treatment train, ..., which can be optimized subject to the parameters of the ultimate disposal choice."[36]

This author is in total agreement with the opinion presented by Maurer in this respect. Whether the waste is from wastewater or injection water treatment plants, an optimal design for the disposal method must be determined.

The geometric programming model presented by Maurer can be used for this purpose.

The primary objective of this study is to use geometric programming to determine optimal designs for injection water treatment plants. The subject of waste disposal will therefore not be considered since it falls beyond the scope of this study. It is the opinion of this author that waste disposal methods must be optimized independently and should be treated as a significant factor in corporate policies. It is sufficient to note that waste disposal must be considered prior to initiating water injection. If this issue is not properly and adequately planned in advance, the operators will, at a point in time, find themselves face to face with a serious problem which may jeopardize the whole project.

Because waste disposal is not considered here, the cost terms associated with it are therefore deleted from the mathematical formulation of the expanded model thus reducing it to a simpler form. The reduced form will include only those cost terms associated with each pollutant removed in each process.

The generalized model considers (n) processes in series used to remove (m) pollutants from the plant influent. The objective function for the model may be written as follows:

$$g_o(x) = \sum_{i=1}^{n} \sum_{j=1}^{m} y^{K_i} c_{ij} x_{ij}^{a_{ij}} \quad (9)$$

Subject to: $x_{1j} x_{2j} \ldots\ldots x_{nj} \leq K_j \quad (10)$

$$x_{ij} > 0 \quad (11)$$

$$K_j > 0 \quad (12)$$

Where:

$c_{ij} x_{ij}^{a_{ij}}$ = The total annual cost to remove pollutant j in process i.

x_{ij} = Fraction of pollutant j remaining after process i.

K_j = Maximum allowable fraction of pollutant j remaining in the final plant effluent.

This formulation of the model will be used in the design of injection water treatment plants. The number of terms in the objective function will represent the processes that will be used in removing the pollutants from the plant influent. Where chemicals are added to the water for scale or corrosion inhibition, terms representing these expenses will be added to the objective function subject to appropriate constraints.

The advantages of this model will be illustrated by applying it (without sludge disposal) to two hypothetical designs of injection water treatment plants.

In conclusion, it can be said that the model presented by Maurer may be easily and effectively applied to the preliminary design of injection water treatment plants for enhanced recovery of oil. The model is flexible and can be easily adapted to suit different situations. This characteristic permits the use of this model for the design of complex as well as simpler water treatment plants.

CHAPTER IV

SOLUTION PROCEDURE

This section illustrates the use of geometric programming in the solution of three hypothetical plant designs for injection water treatment for enhanced recovery of oil. These examples present detailed analysis of water treatment systems incorporating several treatment processes in series and are typical of current practice. The objective is to determine, for each design, the combination of processes and the operating level of each chosen process that will yield the final designated water quality at a minimum total operating cost.

DESIGN 1S/A

The first design example, referred to here as DESIGN 1S/A, involves a treatment system consisting of three processes in series used to remove one pollutant from the water. The design is planned for an influent design capacity of ten million gallons per day (mgd). In this example, it is assumed that 98 percent of the original pollutant load in the influent to the treatment plant must be removed to attain the final designated water quality. Thus, each process must operate at an appropriate level that will remove a portion of the pollutant from the influent to that process. The percent of

pollutant remaining in the effluent from the third process must therefore not exceed 2 percent of the original pollutant load in the influent to the treatment plant.

The mathematical formulation of the total daily operating cost (TDOC) and the constraint for this design are derived from the model presented by Ecker and McNamara. By allocating numerical values to the coefficients and exponents of the terms in the objective function and substituting these values in equations (6), (7), and (8), the minimization problem can be written as follows:

$$\text{Minimize } TDOC_A = 36x_1^{-1.1} + 14x_2^{-1.2} + 10x_3^{-1.3} \qquad (13)$$

$$\text{Subject to:} \quad 50x_1x_2x_3 \leq 1 \qquad (14)$$

$$x_1, x_2, x_3 > 0 \qquad (15)$$

Where K = Percent of pollutant remaining in the plant effluent = .02 which gives $1/K = 50$

The positivity constraints in (15) will be satisfied at a minimizing point because of the inverse relationship between process costs and the variables x_1, x_2, and x_3. In other words, as each of these variables approaches zero, the corresponding cost terms $36x_1^{-1.1}$, $14x_2^{-1.2}$, and $10x_3^{-1.3}$ will, respectively, approach infinity. By providing the calculations for DESIGN 1S/A, the solution procedure will be illustrated.

The above equations (13,14, and 15) are formulated as the primal program for the design under consideration. The dual program associated with this design will be formed and solved for the respective dual variables (w_i's) in accordance with the geometric programming technique of Duffin, Peterson, and Zener referred to in Chapter III of this thesis. The optimal value for the dual program will determine the optimal design.

This procedure may be used for selecting the optimal design from a number of alternative designs. Dual programs associated with each design may be formed and solved for the respective dual variables. Then, the optimal value for each dual program is calculated. The dual program with the largest optimal value will be chosen as the optimal design.

The dual program for DESIGN 1S/A may be formulated using an equation of the form

$$\text{Maximize } P(w) = \left(\frac{C_1}{w_1}\right)^{w_1} \left(\frac{C_2}{w_2}\right)^{w_2} \left(\frac{C_3}{w_3}\right)^{w_3} \left(\frac{1/K}{w_4}\right)^{w_4} (w_4)^{w_4}$$

[This equation is referred to as (16)].

$$\text{Subject to: } w_1 + w_2 + w_3 = 1 \quad \text{(normality)}$$

$$\left.\begin{aligned} x_1: \quad -1.1w_1 + w_4 &= 0 \\ x_2: \quad -1.2w_2 + w_4 &= 0 \\ x_3: \quad -1.3w_3 + w_4 &= 0 \end{aligned}\right\} \text{(orthogonality)}$$

$$w_1, w_2, w_3, w_4 > 0 \quad \text{(positivity)}$$

To solve the dual program it is necessary to find the values of the dual variables (w_i's) which satisfy the above constraints. Linear algebra may be used for solving the above simultaneous equations to determine the values of the w_i's. The values of the dual variables may then be substituted into the dual objective function (16) to determine the optimal total daily operating cost (TDOC) associated with the design. The values of the dual variables calculated by solving the above linear equations are

$$w_1 = .362$$

$$w_2 = .332$$

$$w_3 = .306$$

$$w_4 = .398$$

Substituting these values into (16) gives

$$\text{Maximize } P(w) = \left(\frac{36}{.362}\right)^{.362} \left(\frac{14}{.332}\right)^{.332} \left(\frac{10}{.306}\right)^{.306} x \left(\frac{50}{.398}\right)^{.398} (.398)^{.398} \qquad (17)$$

$$= 252.456 \text{ (thousand dollars)}$$

$$= \text{Min. } TDOC_A$$

Solution of (17) with 1/K = 50 indicates that the least TDOC for meeting this water quality requirement is $252,000 per day.

Comparing the two dual programs for this example reveals

an interesting feature of this technique. Notice that the primal coefficients are not used in the formulation of the dual constraints, that is, the two dual programs remain essentially unchanged due to changes in the primal coefficients. This means that the relative weight carried by each term representing a respective process in the primal objective function does not change if the primal coefficients change.

A fact that may be derived from the two dual programs is that as the final percentage of pollutant remaining in the plant effluent (K) is varied, and hence 1/K, the corresponding optimal total daily operating cost of meeting the requirement will vary. Since only the constant 1/K in (14) and (17) changes and the optimal value of the dual is given as a function of 1/K in (17), the optimal total daily operating cost may be easily calculated for different values of K (designated water quality). The optimal TDOC for DESIGN 1S/B, C,D,E for respective percentages of pollutant removal 96,94, 92, and 90 are

$$TDOC_B = \left(\frac{36}{.362}\right)^{.362} \left(\frac{14}{.332}\right)^{.332} \left(\frac{10}{.306}\right)^{.306} (25)^{.398}$$

$$= 53.210\ (3.601)$$

$$= 191.609 \quad \text{(thousand dollars)}$$

$$TDOC_C = 53.210\ (16.67)^{.398} = 53.210\ (3.064)$$

$$= 163.035 \quad \text{(thousand dollars)}$$

$$TDOC_D = 53.210\ (12.5)^{.398} = 53.210\ (2.733)$$

$$= 145.423 \quad \text{(thousand dollars)}$$

$$TDOC_E = 53.210\ (10)^{.398} = 53.210\ (2.500)$$

$$= 133.025 \quad \text{(thousand dollars)}$$

The optimal cost of achieving any feasible plant effluent quality may be determined by using the above procedure. Note also that the values of the individual process variables are not yet calculated. The costs are expressed as functions of the effluent quality. This relationship aids in selecting the operating level of each process based on the quality of the source and the anticipated changes in the source quality. If for example, 90 percent of the time the source quality remains relatively stable, the operator may decide on the optimal process operating levels over this period. For the remaining 10 percent of the time, if the source quality deteriorates, the operator has the choice of either shutting-down the operations or operating at a different level, possibly not optimal, at additional operating costs. In either case, the costs corresponding to either case may be easily determined by using the above procedure. Once an optimal level of source quality is chosen, the optimal

operating level for each process in the treatment system that would yield the designated water quality may be determined.

SENSITIVITY ANALYSIS

Calculating the optimal TDOC corresponding to each effluent quality has demonstrated an important inherent characteristic of this technique. Notice that by performing these calculations, the sensitivity of TDOC to variations in the plant effluent quality is determined. Sensitivity analysis is thus performed without the need for additional calculations. Tabulating these results as shown in Table 4.1, the sensitivity of TDOC to (K) can be detected at a glance.

TABLE 4.1

SENSITIVITY ANALYSIS

DESIGN	1-K* (Percent)	TDOC ($1000)
1S/A	98	252
1S/B	96	192
1S/C	94	163
1S/D	92	145
1S/E	90	133

* $(1-K)_{percent}$ = Percent of original load of pollutant in in the plant influent removed by the treatment system.

INFLATION

The above solution obtained by geometric programming can be used to derive a lower bound on the effect of inflation on the total daily operating cost. According to Woolsey,[37] a lower bound may be found by simple substitution in the following formula:

$$\text{New (inflated) cost} = \text{Old Cost} \times \left(\frac{\text{first component's new price}}{\text{first component's old price}}\right)^{\text{first component's old weight}} \times \left(C_{2\,\text{new}} / C_{2\,\text{old}}\right)^{w_{2\,\text{old}}} \times \ldots\ldots\ldots\ldots \quad (18)$$

In other words, the lower bound on the inflated cost is equal to the old cost times the ratio of the new to old price for each component raised to the component's weight. The weights are the dual variables found by geometric programming (w_i's) at the optimum solution as shown previously. The new (inflated) total daily operating cost for the above example may now be calculated using (18) and assuming new coefficients in the primal objective function as shown below.

$C_{1,\,old} = 36$ $\qquad$ $C_{1,\,new} = 40$

$C_{2,\,old} = 14$ $\qquad$ $C_{2,\,new} = 16$

$C_{3,\,old} = 10$ $\qquad$ $C_{3,\,new} = 12$

$$\text{New (inflated) } TDOC_A = 252 \left(\frac{40}{36}\right)^{.362} \left(\frac{16}{14}\right)^{.332} \left(\frac{12}{10}\right)^{.306}$$

$$= 289.900 \quad \text{(thousand dollars)}$$

The increase in TDOC due to inflation (approximately thirty seven thousand dollars) was calculated without repeating the solution to the problem. This is an additional simple weapon provided by geometric programming.

It is interesting to point out that the optimal solution is determined, sensitivity analysis performed, and the effect of inflation on the TDOC calculated before knowing the values of the optimal design parameters. This means that the firm can have the optimal design determined long before it is implemented. The information can be stored for any period of time and then easily adjusted for inflation when needed in the simple manner described above. This feature is of extreme importance to the operators in the oil industry in that it provides them with a simple technique to plan their water treatment and injection policies well in advance before the reservoir pressure has declined.

OPTIMAL VARIABLES

The optimal values of the design variables may now be readily calculated. The individual process variables appearing in the objective function (13) are calculated from the optimal TDOC by using an equation of the form

$$TDOC = \frac{C_i x_i^{a_i}}{w_i} \qquad (19)$$

Solution for evaluating each variable can be performed individually since each variable (x_i) appears in one term of the objective function only.

The optimal value of the variable appearing in the cost term corresponding to the first process of DESIGN 1S/A is calculated by substituting the coefficient and the exponent of the first term in (13) and the w_1 from the dual program into (19) which gives

$$252 = \frac{36x_1^{-1.1}}{.362}$$

$$x_1 = .429$$

Using the same procedure, the calculated optimal values of the variables in the remaining terms of the objective function are $x_2 = .225$, and $x_3 = .208$. These values indicate that the first process in the treatment system should be used to remove 57.1 percent of the pollutant from the influent to the system. The effluent from this process is then treated where 77.5 percent of the remaining fraction of the pollutant is removed in the second process. Finally, the third process removes 79.2 percent of the fraction of pollutant remaining in the influent to this process. A total of 98 percent of

the original pollutant load is thus removed from the plant influent by the combined effect of the three processes.

Selection of the components of each process in the treatment system may proceed from this stage. The components chosen must be capable of removing the optimal fraction of pollutant from the water as determined by the optimal solution. The effectiveness of each process in the treatment system is easily determined when the plant is in operation by monitoring the quality of the influent to and effluent from each process. If the quality of water at any process in the treatment system is significantly different from that determined by the optimal solution, appropriate adjustments may be made in those parts of the treatment system where such variations are detected.

Design problems using geometric programming may be solved by following four simple rules if they are of zero degree of difficulty. These rules direct the designer through a step-by-step procedure that would lead to the calculation of the optimal total operating cost, the contribution of each process to the total cost, and the operating level of each process in the system. These rules correspond to the equations used in the preceding calculation for determining the respective variables. These four rules are referred to as Geometric Programming Rules for Posynimials[38]

and are included in Appendix B of this thesis.

MULTI-POLLUTANT TREATMENT SYSTEMS

Treatment systems may often consist of a series of processes used to remove more than one pollutant from the water influent to the system. Two designs referred to as DESIGN 2S and DESIGN 3S are presented and solved to demonstrate the procedure for solving water treatment design problems involving more than one pollutant. Technical data corresponding to each of these two designs is presented in Table 4.2 and Table 4.3 respectively.

TABLE 4.2

DESIGN 2S

Plant design capacity	25 mgd
Number of processes	3
Number of pollutants in the plant influent	2

The objective function and constraints for this design are

$$\text{Minimize TDOC} = 50x_1^{-.9} + 40x_2^{-1.0} + 35x_3^{-.5} + 30x_4^{-.7} + 60x_5^{-.3} + 45x_6^{-.6}$$

$$\text{Subject to:} \quad 50x_1x_3x_5 \leq 1$$

$$25x_2x_4x_6 \leq 1$$

$$x_i > 0 \quad \text{(for i=1,2,3,4,5, and 6)}$$

TABLE 4.3

DESIGN 3S

This design involves a treatment system consisting of three processes in series used to remove two pollutants from the water influent to the plant. The planned design capacity is for treating 50 million gallons of water per day. A chemical, for corrosion inhibition, is added to the effluent from the last process at an additional cost of 180 thousand dollars. The objective function and the constraints for this design are

$$\text{Minimize TDOC} = 105x_1^{-.9} + 80x_2^{-1.0} + 95x_3^{-.5} + 60x_4^{-.7} + 100x_5^{-.3} + 75x_6^{-.6} + 180$$

Subject to:

$$50x_1x_3x_5 \leq 1$$

$$25x_2x_4x_6 \leq 1$$

$$x_i > 0 \quad \text{(For } i=1,2,3,4,5, \text{ and } 6)$$

Determination of optimal solutions for designs involving a series of processes used to remove multiple pollutants from the plant influent are relatively more difficult than those involving a single pollutant. The degree of difficulty may br determined by an euqation devised by Duffin et al of the form

degree of difficulty = number of terms - number of variables - 1 (20)

(This equation is referred to as "Rule 4" in Appendix B). The degree of difficulty for DESIGN 1S is, according to (20), equal to zero. That for each of DESIGN 2S and DESIGN 3S is equal to 1. The degree of difficulty for each of the two design examples presented by Maurer, referred to as DESIGN 2A and DESIGN 3A, is equal to 2.

Problems with zero degree of difficulty are easily solved by hand as shown in the solution of DESIGN 1S. As the degree of difficulty for design problems increases, solutions become more difficult and optimal solutions become practically impossible to derive through hand calculation. The number of computations increases tremendously thus increasing the potential for errors. In such cases, hand calculations may be replaced by programmed computer solutions such as those presented by Maurer.

Designers, quite often, do not have access to computers

for solving problems. In such cases they are faced with either (1) indulging in numerous computations with high potential for errors or (2) contracting the work to outside sources that have access to computers. Both alternatives tend to be time consuming and costly. Contracting the work to outside sources involves administrative costs in addition to technical costs associated with the solution of the design alternatives. These facts are familiar to this author through his long experience in dealing with similar situations. Consequently, this author was inspired to look into the possibility of reducing or even eliminating the difficulties arising from such complicated situations. Cultivated efforts of this author toward achieving this goal proved fruitful. A method for simplifying the problems and deriving optimal solutions for multi-degree of difficulty water treatment design problems is developed. The method is explained in the next chapter of this thesis. This method is illustrated by applying it to the solution of DESIGN 2S and DESIGN 3S respectively. Detailed solutions for these two designs are included in Appendix C. The validity of the simplified solution method is illustrated by applying it to derive an optimal solution for DESIGN 2A presented by Maurer. The results achieved from using this method for deriving an optimal solution for DESIGN 2A agree very closely with the

computer derived results determined by Maurer. The minimum cost determined by this author using the simplified method is 758.869 thousand dollars as compared to Maurer's 757.307 thousand dollars. The variation may be attributed to the rounding of the decimal points. Detailed solution procedure for DESIGN 2A is included in Appendix C. Close agreement of primal and dual variables for this design is reflected in the results obtained by the two authors each using a different approach to deriving an optimal solution to the same problem. This fact illustrates the validity of this author's simplified solution method.

CHAPTER V

THE SIMPLIFIED SOLUTION METHOD

A close look at the formulation of the three designs considered in Chapter IV, namely, DESIGN 2S, DESIGN 3S, and DESIGN 2A, reveals an interesting relationship that is utilized for establishing the simplified solution method. Notice that for each pollutant there is a cost term for the processes involved in removing that pollutant. The variables in these cost terms are restricted by a constraint involving only those variables associated with that pollutant. For example, in DESIGN 2A, the first cost term for each process is associated with the first pollutant and the variables in these four terms are restricted by the first constraint that includes only those variables appearing in the four terms. The same discussion applies to the remaining two pollutants. This relationship means that changes in the variables associated with the first pollutant will not affect the variables in the cost terms associated with the remaining two pollutants. Therefore, it can be concluded that an optimal solution for removing each pollutant by the series of processes in the system may be derived independently and without affecting the solution related to the remaining two pollutants. In other words, for removing three pollutants

by a series of four processes each removing a fraction of each of the pollutants from the influent to that process, will require solving three independent problems, for each of which an optimal solution will be derived. The mathematical formulation for each of the problems will consist of an objective function comprising four terms and restricted by one constraint with one term. Each of the cost terms in the objective function contains one variable. Therefore the total number of terms for each of these problems including the term in their respective constraint will be five terms. Since there are only four variables in the objective function and the constraint includes only these four variables, the degree of difficulty for each of these problems is equal to zero as calculated from (20). The optimal solution for the system will thus involve solving three problems each of zero degree of difficulty, which, as shown previously, is rather simple.

The number of zero degree of difficulty problems for any treatment system using geometric programming and formulated in a manner similar to the designs considered is determined by the number of pollutants to be removed from the plant influent. For a system involved in removing 3 pollutants from the plant influent, the number of zero degree of difficulty problems will be three problems. For DESIGN 2S

and DESIGN 3S each of which is involved in removing 2 pollutants from the plant influent, the number of zero degree of difficulty problems is equal to two for each design.

The values of primal and dual variables for each problem are easily determined by solving the zero degree of difficulty problems associated with each pollutant. The values of (x_i's) calculated represent the optimal values for the process and the system. The optimal values of the (w_i's) calculated for each process are the optimal values for that particular process and not for the system. The optimal values of the dual variables (w_i's) for the system may be calculated by an equation formulated by this author of the form

$$w_i^* = w_i R$$

where

w_i^* = the optimal value of the dual variable for the system

w_i = the optimal value of the dual variable for the process

R = the ratio of the optimal operating cost of the processes involved in removing the respective pollutant to the optimal total operating cost of the system.

The optimal total operating cost for the system is the sum of the optimal total operating cost for each of the zero degree of difficulty problems in the system. Evidence for these conclusions and relationships is provided in the

detailed solutions included in Appendix C of this thesis.

The validity of the "Simplified Solution Method" has been illustrated by applying it for deriving optimal solutions for a number of designs with degree of difficulty greater than zero. It has been shown effective in solving designs for injection water treatment as well as designs for wastewater treatment systems. The applications of this solution method may be extended to include deriving optimal solutions for designs of plants used for the treatment of domestic water supplies and industrial effluents. The method may also be used to derive optimal solutions to designs of plants used for the extraction of various minerals from brines. The mathematical formulations for such plant designs must be similar to those of the geometric programming design models presented in this thesis. This solution method provides the designer with an effective weapon to reduce the multi-degree of difficulty problems to zero degree of difficulty thus rendering them easy to solve and derive optimal solutions.

The results obtained from solving the two designs, DESIGN 2S and DESIGN 3S are shown in Table 5.1 and Table 5.2 respectively. The detailed solution procedure to each of these two designs and DESIGN 2A (presented by Maurer) is included in Appendix C .

TABLE 5.1

DESIGN 2S

SUMMARY OF RESULTS

Planned design capacity	25 mgd
Number of processes	3
Number of pollutants	2
Total daily operating cost (optimal)	$503,000

i	w_i^*	x_i^*
1	.089	.725
2	.119	.326
3	.159	.050
4	.170	.080
5	.266	.008
6	.198	.080

TABLE 5.2

DESIGN 3S

SUMMARY OF RESULTS

Planned design capacity	50 mgd
Number of processes	3
Number of pollutants	2
Total daily operating cost (optimal)	$1133,000

i	w_i^*	x_i^*
1	.076	.801
2	.098	.721
3	.137	.377
4	.140	.250
5	.228	.042
6	.163	.222

Percent contribution of corrosion inhibitor to the optimal total daily operating cost = 15.9

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Geometric programming is a simple and viable technique for obtaining optimal solutions for water treatment systems for enhanced recovery of oil. Optimal solutions can be obtained for designs where one or more pollutants need to be removed from the plant influent.

Sensitivity analysis and the effect of inflation on optimal total operating costs can be performed without repeating the problem solution. This inherent capability of geometric programming aids in deriving an optimal design well in advance of the implementation of water injection processes. Further, it provides corporations in the oil industry with a simple weapon to plan their corporate investment and water injection policies before the reservoirs show signs of depletion.

The mathematical design models selected can be effectively applied to injection water treatment systems to derive optimal system designs. The models are applied to hypothetical design problems to establish their effectiveness in meeting the desired theoretical goals. Lack of appropriate data prevented the application of the solution technique to actual designs. The difference between hypothetical and

actual designs rests in the value of the coefficients (C_i's) and exponents (a_i's) in the cost terms of the objective function and the values of the pollutant(s) removal efficiency(ies) in the constraint(s) represented by the values of the (K_i's). In real situations, one simply assigns appropriate values to these constants and follows the same solution technique, presented in this thesis, to derive the optimal design.

The "Simplified Solution Method" developed by this author is valid for reducing multi-degree of difficulty design problems, formulated in accordance with the models presented in this thesis, to zero degree of difficulty problems. The effectiveness of this method for deriving optimal solutions to injection water and wastewater treatment designs using geometric programming is illustrated through the solutions of the hypothetical examples presented in this thesis.

The major recommendations of this author are as follows.

1. Research is needed to develop statistics linking the individual costs of each process in the treatment systems to the efficiency of removal of each pollutant from the influent to the system.
2. Pilot plants may also be used to develop the statistics mentioned in (1) above to construct the

coefficients and exponents to represent actual designs.

3. Research is also needed to determine values of K_i's suitable for each type of producing formation. Various formations react differently to the quality of the injected water. The operators engaged in water injection will use these data to determine the water quality that best suits their operations.
4. Geometric programming models need to be developed for water supply systems from source to treatment plant as well as from treatment plant to injection wells. This will aid in deriving optimal solutions to totally integrated designs.
5. Finally, this author hopes that the information and recommendations provided in this thesis will form a solid base for further research in the field of enhanced oil recovery by water injection.

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LITERATURE CITED

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APPENDIX A

GENERALIZED MULTIPLE POLLUTANT MODEL

APPENDIX A

Table VI

Generalized Multiple Pollutant Model

$$\text{Minimize: } g_o(x) = y^{k_{DOA}} c_{DOA} x_{DOA}^{a_{DOA}} + \sum_{i=1}^{n} \sum_{j=1}^{m} y^{k_i} c_{ij} x_{ij}^{a_{ij}}$$

$$\text{Subject to: } g_j(x) = \frac{1}{K_j} \prod_{i=1}^{n} x_{ij} \leq 1, \quad j=1, 2, \ldots, m$$

$$g_{DOA}(x) = K_{DOA} x_{DOA}^{-1} + x_{DOA}^{-1} \prod_{i=1}^{n} x_{iDO} \leq 1$$

$$x_{DOA} > 0$$

$$x_{iDO} > 0 \text{ for each } i=1, 2, \ldots, n$$

$$x_{ij} > 0 \text{ for each } \begin{cases} i=1, 2, \ldots, n \\ j=1, 2, \ldots, m \end{cases}$$

APPENDIX A, Continued

Table VI, Continued

where

x_{ij} = percent of pollutant j influent to process i which remains after process i

x_{DOA} = percent of dissolved oxygen to be added

x_{iDO} = percent of dissolved oxygen influent to process i which is removed by process i

y = flow in mgd

K_j = maximum allowable fraction of pollutant j influent to process 1 remaining in final plant effluent

K_{DOA} = minimum allowable percent dissolved oxygen in final plant effluent

$y^{k_i} c_{ij} x_{ij}^{a_{ij}}$ = total annual cost to remove pollutant j in process i with a flow of y mgd

$y^{k_{DOA}} c_{DOA} x_{DOA}^{a_{DOA}}$ = total annual cost to add dissolved oxygen

Source: Maurer, Ruth A., A Geometric Programming Approach to the Preliminary Design of Wastewater Treatment Plants: Ph.D. Thesis, Colorado School of Mines, Golden, Colorado, (1978). p.64-65.

APPENDIX B

GEOMETRIC PROGRAMMING RULES

FOR

POSYNIMIALS

APPENDIX B

Quick & Dirty GEOMETRIC PROGRAMMING RULES FOR POSYNOMIALS

1. The optimum value of the objective function is always of the form:

   $g_0(x)^* = (\text{coefficient of 1st term}/w_1)w_1 * (\text{coefficient of 2nd term}/w_2)w_2 * \cdots * (\text{coefficient of last term}/w_{last})w_{last} * (\Sigma w\text{'s in 1st constraint})^{\Sigma w\text{'s in 1st constraint}} * \cdots * (\Sigma w\text{'s in last constraint})^{\Sigma w\text{'s in last constraint}}$.

2. Equations generated for a geometric program are:

   Σw's in objective function $= 1$
   and for each primal variable x_j, with n variables and m terms,

   $$\sum_{i=1}^{i=m} [(w_i \text{ for each term}) * (\text{exponent on } x_j \text{ in that term})] = 0$$

   $$j = 1, 2, \ldots, n$$

3. Primal variables may be found by:

   $g_0(x)^* = (\text{1st term in objective function}/w_1) = (\text{2nd term in objective function}/w_2) = \cdots = (\text{last term in objective function}/w_{last})$
   or for each constraint term,
   $w_{ith\ term} = (i\text{th term}) * (\text{sum of all } w\text{'s in that constraint})$

4. The degree of difficulty is given by:

   D.D. = (no. of terms) − (no. of variables) − 1

Source: Woolsey, R.E.D., and Swanson, H.S., Operations Research for Immediate Application - a quick and dirty manual: Harper and Row Publishers, New York, (1975). p.92.

APPENDIX C

CALCULATIONS

USING THE SIMPLIFIED SOLUTION METHOD

APPENDIX C

DESIGN 2S

SIMPLIFIED SOLUTION METHOD

Planned design capacity	25	mgpd
Number of processes	3	
Number of pollutants	2	

The problem is solved as 2 problems of zero D.D.

$$\text{Minimize TDOC} = 65\,x_1^{-.9} + 40\,x_2^{-1.0} + 35\,x_3^{-.5} + 30\,x_4^{-.7} + 60\,x_5^{-.3} + 45\,x_6^{-.6}$$

$$\text{Subject to} \quad 50\,x_1\,x_3\,x_5 \leq 1$$

$$25\,x_2\,x_4\,x_6 \leq 1$$

$$x_i > 0 \quad (\text{for } i = 1,2,3,4,5,\text{and } 6)$$

(a) Problem 1

$$\text{Min} \quad TC_a = 65\,x_1^{-.9} + 35\,x_3^{-.5} + 60\,x_5^{-.3}$$

$$\text{S.T.} \quad 50\,x_1\,x_3\,x_5 \leq 1$$

$$x_i > 0 \quad \text{for} \quad i = 1, 3, 5$$

APPENDIX C (Continued)

$$w_1 + w_3 + w_5 = 1$$

x_1 : $-.9w_1 + w_7 = 0$

x_3 : $-.5w_3 + w_7 = 0$

x_5 : $-.3w_5 + w_7 = 0$

$$.9w_1 = .5w_3 = .3w_5 = w_7$$

$$w_1 = .1724138$$

$$w_3 = .3103448$$

$$w_5 = .5172414$$

$$w_7 = .1551724$$

$$TC_a = \left(\frac{65}{.1724138}\right)^{.1724138} \left(\frac{35}{.3103448}\right)^{.3103448} \left(\frac{60}{.5172414}\right)^{.5172414} \times \left(\frac{50}{1}\right)^{.1551724}$$

$$= 258.5533075 \text{ (thousand dollars)}$$

(b) Problem 2

$$\text{Min. } TC_b = 40\, x_2^{-1.0} + 30\, x_4^{-.7} + 45\, x_6^{-.6}$$

$$\text{S.T.} \quad 25\, x_2\, x_4\, x_6 \leq 1$$

$$x_i > 0 \quad \text{for } i = 2, 4, 6$$

APPENDIX C (Continued)

$$w_2 + w_4 + w_6 = 1$$

x_2 : $-1w_2 + w_8 = 0$

x_4 : $-.7w_4 + w_8 = 0$

x_6 : $-.6w_6 + w_8 = 0$

$w_2 = .2441860$

$w_4 = .3488371$

$w_6 = .4069767$

$w_8 = .2441860$

$$TC_b = \left(\frac{40}{.2441860}\right)^{.2441860}\left(\frac{30}{.3488371}\right)^{.3488371}\left(\frac{45}{.4069767}\right)^{.4069767} \times \left(\frac{25}{1}\right)^{.2441860}$$

$= 244.6853493$ (thousand dollars)

$TDOC = TC_a + TC_b = 258.5533075 + 244.6853493$

$TDOC = 503.2386568$ (thousand dollars)

$$w_i^* = w_i \frac{TC_a}{TDOC} \qquad w_i^* = w_i \frac{TC_b}{TDOC}$$

$w_1^* = .0885825$ $w_2^* = .1187284$

$w_3^* = .1594486$ $w_4^* = .1696120$

$w_5^* = .2657476$ $w_6^* = .1978807$

$w_7^* = .0797243$ $w_8^* = .1187284$

$\sum_{i=1}^{6} w_i^* = .9999998 \approx 1$

$$TDOC = \frac{C_i x_i^{*a_i}}{w_i^*}$$

$x_1^* = .7254882$

$x_2^* = .3255107$

$x_3^* = .0502226$

$x_4^* = .0801479$

$x_5^* = .0075095$

$x_6^* = .0800046$

APPENDIX C (Continued)

DESIGN 3S

SIMPLIFIED SOLUTION METHOD

Planned design capacity	50	mgpd
Number of processes	3	
Number of pollutants	2	

The problem is solved as 2 problems of zero D.D.

$$\text{Minimize TDOC} = 105\, x_1^{-.9} + 80\, x_2^{-.1.0} + 95\, x_3^{-.5} + 60\, x_4^{-.7} + 100\, x_5^{-.3} + 75\, x_6^{-.6} + 180$$

$$\text{Subject to} \quad 50\, x_1\, x_3\, x_5 \leqslant 1$$

$$25\, x_2\, x_4\, x_6 \leqslant 1$$

$$x_i > 0 \quad (\text{for } i = 1, 2, 3, 4, 5, \text{ and } 6)$$

(a) Problem 1

$$\text{Min. } TC_a = 105\, x_1^{-.9} + 95\, x_3^{-.5} + 100\, x_5^{-.3}$$

$$\text{S.T.} \quad 50\, x_1\, x_3\, x_5 \leqslant 1$$

$$x_i > 0 \quad \text{for} \quad i = 1, 3, 5$$

(b) Problem 2

$$\text{Min. } TC_b = 80\, x_2^{-1.0} + 60\, x_4^{-.7} + 75\, x_6^{-.6}$$

$$\text{S.T.} \quad 25\, x_2\, x_4\, x_6 \leqslant 1$$

$$x_i > 0 \quad \text{for} \quad i = 2, 4, 6$$

APPENDIX C (Continued)

$$w_i^* = w_i \frac{TC_a}{TDOC} \qquad w_i^* = w_i \frac{TC_b}{TDOC}$$

$w_1^* = .0758870$ $w_2^* = .0979120$

$w_3^* = .1365965$ $w_4^* = .1398743$

$w_5^* = .2276609$ $w_6^* = .1631867$

$w_7^* = .0682983$ $w_8^* = .0979120$

$$\% \text{ Contribution of corrosion inhibitor to TDOC} = \frac{180}{TDOC}$$

$$= \frac{180}{1132.913298} = .1588824$$

$$\sum_{i=1}^{6} w_i^* s + .1588824 = .9999998 \simeq 1$$

$$TC_a = \frac{C_i x_i^{a_i}}{w_i} \qquad TC_b = \frac{C_i x_i^{a_i}}{w_i}$$

(for i = 1, 3, 5) (for i = 2, 4, 6)

$x_1^* = .8008060$ $x_2^* = .7212025$

$x_3^* = .3768552$ $x_4^* = .2497194$

$x_5^* = .0424994$ $x_6^* = .2223137$

APPENDIX C (Continued)

w_i's for Design 2S and Design 3S are equal because the exponents of the respective cost terms are equal.

$w_1 = .1724138$ $w_2 = .2441860$

$w_3 = .3103448$ $w_4 = .3488371$

$w_5 = .5172414$ $w_6 = .4069767$

$w_7 = .1551724$ $w_8 = .2441860$

$$TC_a = \left(\frac{105}{.1724138}\right)^{.1724138} \left(\frac{95}{.3103448}\right)^{.3103448} \left(\frac{100}{.5172414}\right)^{.5172414} \times \left(\frac{50}{}\right)^{.1551724}$$

$$= 498.6454386 \text{ (thousand dollars)}$$

$$TC_b = \left(\frac{80}{.2441860}\right)^{.2441860} \left(\frac{60}{.3488371}\right)^{.3488371} \left(\frac{75}{.4069767}\right)^{.4069767} \times \left(\frac{25}{}\right)^{.2441860}$$

$$= 454.2678599 \text{ (thousand dollars)}$$

$$TDOC = TC_a + TC_b + 180$$

$$= 1132.913298 \text{ (thousand dollars)}$$

APPENDIX C (Continued)

DESIGN 2A
PRESENTED BY MAURER

SOLUTION METHOD PROPOSED BY THIS AUTHOR

(SOLUTION AS 3 PROBLEMS OF ZERO DEGREE OF DIFFICULTY)

$$\text{Minimize } g_o(x) = 12.25x_1^{-1.38} + 24.5x_2^{-1.08} + 24.5x_3^{-1.05} + 22.65x_4^{-.68} + 17.65x_5^{-.83} + 8.83x_6^{-1.34} + 41.38x_7^{-.85} + 62.3x_8^{-.78} + 40.92x_9^{-.78} + 91.38x_{10}^{-.4} + 91.38x_{11}^{-.4} + 45.69x_{12}^{-.8}$$

$$\text{Subject to } 8.75\, x_1\, x_4\, x_7\, x_{10} \leq 1$$

$$15.70\, x_2\, x_5\, x_8\, x_{11} \leq 1$$

$$26.67\, x_3\, x_6\, x_9\, x_{12} \leq 1$$

(a) Process 1

$$\text{O.F. Min. } TC_a = 12.25x_1^{-1.38} + 22.65x_4^{-.68} + 41.38x_7^{-.85} + 91.38x_{10}^{-.4}$$

$$\text{S.T. } 8.75\, x_1\, x_4\, x_7\, x_{10} \leq 1$$

APPENDIX C (Continued)

$$w_1 + w_4 + w_7 + w_{10} = 1$$

$$x_1: \quad -1.38w_1 + w_{13} = 0$$

$$x_4: \quad -.68w_4 + w_{13} = 0$$

$$x_7: \quad -.85w_7 + w_{13} = 0$$

$$x_{10}: \quad -.4w_{10} + w_{13} = 0$$

$$1.38\, w_1 = .68\, w_4 = .85\, w_7 = .4\, w_{10} = w_{13}$$

$$w_1 + \frac{1.38w_1}{.68} + \frac{1.38w_1}{.85} + \frac{1.38w_1}{.4} + 1$$

$$.2312\, w_1 + .4692\, w_1 + .37536\, w_1 + .79764\, w_1 = .2312$$

$$1.8734\, w_1 = .2312$$

$$w_1 = .1234119782$$

$$w_4 = .2504537205$$

$$w_7 = .2003629764$$

$$w_{10} = .4257713248$$

$$w_{13} = .1703085299$$

$$TP_1 = \left(\frac{12.25}{.1234119782}\right)^{.1234119782} \left(\frac{22.65}{.2504537205}\right)^{.2504537205}$$

$$\times \left(\frac{41.38}{.2003629764}\right)^{.2003629764} \left(\frac{91.38}{.4257713248}\right)^{.4257713248}$$

$$\times \left(8.75\right)^{.1703085299}$$

$$TP_1 = 225.645338$$

APPENDIX C (Continued)

(b) Process 2 Following the same procedure gives

$$1.08\, w_2 = .83\, w_5 = .78\, w_8 = .4\, w_{11} = w_{14}$$

$$w_2 = .1565969551$$

$$w_5 = .2037647127$$

$$w_8 = .2168265533$$

$$w_{11} = .4228117789$$

$$w_{14} = .1691247116$$

$$TP_2 = \left(\frac{24.5}{.1565969551}\right)^{.1565969551} \left(\frac{17.65}{.2037647127}\right)^{.2037647127}$$

$$\times \left(\frac{62.3}{.2168265533}\right)^{.2168265533} \left(\frac{91.38}{.4228117789}\right)^{.4228117789}$$

$$\times \left(15.70\right)^{.1691247116}$$

$$TP_2 = 288.9941455$$

(c) Process 3

$$1.05\, w_3 = 1.34\, w_6 = .78\, w_9 = .8\, w_{12} = w_{15}$$

$$w_3 = .2251118613$$

$$w_6 = .1763936227$$

$$w_9 = .303035198$$

$$w_{12} = .295459318$$

$$w_{15} = .2363674544$$

APPENDIX C (Continued)

$$TP_3 = \left(\frac{24.5}{.2251118613}\right)^{.225118613} \left(\frac{8.83}{.1763936227}\right)^{.1763936227}$$

$$x \left(\frac{40.92}{.303035198}\right)^{.303035198} \left(\frac{45.69}{.295459318}\right)^{.295459318}$$

$$x \left(26.67\right)^{.2363674544}$$

$$TP_3 = 244.2295153$$

$$TOC = TP_1 + TP_2 + TP_3$$

$$TOC = 758.8689988 \text{ (thousand dollars)}$$

$$w_i^* = w_i \left(\frac{TP_i}{TOC}\right)$$

	Jabri's Results	Maurer's Results
w_1^*	.0366958429	.0367715
w_4^*	.0744709752	.0746
w_7^*	.0595767802	.0597
w_{10}^*	.1266006579	.1269
w_2^*	.0596355936	.0597586
w_5^*	.0775981218	.0776
w_8^*	.0825723604	.0827
w_{11}^*	.1610161028	.1614
w_3^*	.0724495529	.0718436
w_6^*	.0567693884	.0575846

APPENDIX C (Continued)

w_9^*	=	.0975268981	.0967125
w_{12}^*	=	.0950876359	.0942947
$\sum_{i=1}^{12} w_i^*$ s	=	.9999989101	.9998655

$$TOC = \frac{C_i x_i^{a_i}}{w_i}$$

		Jabri's Results	Maurer's Results
x_1^*	=	.5515180327	.55151802
x_2^*	=	.5665456794	.56654566
x_3^*	=	.4631112094	.46774132
x_4^*	=	.2606456735	.26064568
x_5^*	=	.2341802295	.23418023
x_6^*	=	.3064277674	.29547525
x_7^*	=	.901074627	.90107463
x_8^*	=	.9926076317	.99260764
x_9^*	=	.4677964544	.47410323
x_{10}^*	=	.8823094875	.88230946
x_{11}^*	=	.4836571122	.48365711
x_{12}^*	=	.5648236472	.57223869