T-995

COMPARISON OF FIVE-SPOT AREAL-SWEEP-EFFICIENCY CALCULATION PROCEDURE TO MODEL PERFORMANCE FOR MOBILITY RATIOS OTHER THAN ONE

By

Ching Huang Wu

A Thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science in Petroleum Engineering.

Signed: Ching Huang Wu

Ching Huang Wu

Approved: D. M. Bass

D. M. Bass
Thesis Advisor

D. M. Bass

D. M. Bass
Head, Department of
Petroleum Engineering

Golden, Colorado

Date: May 20, 1964

CONTENTS

TABLES, FIGURES, AND PLATES

ABSTRACT

Higgins and Leighton's method for predicting a five-spot water-flood performance handles the saturation distribution during the water-flood operation both before and after breakthrough. Its ability to match the oil recovery data obtained from experimental floods for a range of mobility ratios has been demonstrated.

Higgins and Leighton's method calculates the areal sweep efficiency at any flood period, but the accuracy of these calculations is yet to be proved. This investigation made several experiments with a simulated quadrant of a five-spot pattern, consolidated oil-wet model, to ascertain the validity of the calculation procedure to predict areal sweep efficiency.

A comparison of experimental and calculated areal sweep efficiencies indicates agreement within six per cent for mobility ratios less than 1.5. The percentage error increased for mobility ratios greater than 1.5.

This study indicated that the Higgins and Leighton's method to yield an excellent approximation of the total oil recovery efficiency.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to Prof. D.M. Bass for suggesting the investigation, for giving consistent and helpful guidance, and for serving on the thesis committee.

Many thanks are extended to Prof. R.V. Hughes and Prof. P.F. Dickson for serving on the thesis committee and for their critical examination of this work, and to Prof. E.G. Fisher for his assistance in preparing the final manuscript.

The author is indebted to Mr. R.V. Higgins of the Bureau of Mines in San Francisco for having processed all the data through an IBM 7090 Digital Computer to obtain the calculated results.

The author also wishes to express his appreciation to Messrs. W.C. Sheldon, R.K. Harley, and H.P. Meabon of Marathon Oil Company for supplying reference materials and for their helpful suggestions.

Grateful acknowledgment is made to the Colorado School of Mines for the graduate fellowship made available.

INTRODUCTION

The theoretical considerations of fluid displacement processes, fractional flow and frontal advance, based on Darcy's law and mass-conservation were initially derived by Leverett (1941) and Buckley and Leverett (1942). The two basic relations were extended by Welge (1952). These relations were subsequently applied in two main areas; first by Johnson, Bossler, and Naumann (1959) in relative permeability calculations for laboratory displacements and second by Higgins and Leighton (1960; 1961) in water-flood-performance calculations. This study relied heavily upon the last two applications.

A computer method to predict a five-spot water-flood performance was recently proposed by Higgins and Leighton (1961; 1962; 1963). In their computation technique, stream lines are used, and relative permeability changes due to changing saturation gradients behind the water front and after breakthrough are taken into account. Nevertheless, one major assumption is made: the stream lines for a five-spot model are independent of mobility variations in both displaced and displacing phases. This means that a one-to-one mobility ratio is assumed to be representative of the stream lines that would exist for any mobility ratio.

The stream lines in a flow system are governed by the potential distribution in the system, and the potential distribution should vary with time with changing mobility ratios. Therefore, it is pertinent to consider that mobility ratios other than one will possibly have remarkable effect on the stream lines assumed in the five-spot water-flood calculations and consequently on the calculated water-flood performances.

This investigation was undertaken to determine if the Higgins and Leighton's calculation procedure can be applied to determine the areal sweep efficiency of a five-spot water flood under condition of various mobility ratios. The range of mobility ratios, if any, in which the calculation technique can satisfactorily replace the laboratory model is investigated.

In this investigation, mobility ratios of 0.83 to 4.10 were used in a consolidated-sand five-spot model. The results obtained from the calculation technique and from the experiments are compared on the bases of areal sweep efficiency at breakthrough and cumulative oil production.

THEORETICAL CONCEPTS OF FLUID DISPLACEMENT

Leverett (1941) abandoned the prevailing "capillary tube" concept of porous structure and clarified the function of capillary pressure in an immiscible two-phase flow system. In the same presentation he derived an equation for an oil-water flow system which expressed the fraction of water in the flow stream passing a point in the sand. This relation, fractional flow equation, included a capillary, a gravity, and a viscosity terms as well as an external pressure gradient, a saturation gradient, and relative permeabilities.

The derived fractional flow equation indicated that the fractional flow of any fluid phase in the flowing stream at any position was a function of the saturation existing at that particular position. Combining the fractional flow relation with mass-conservation considerations, Buckley and Leverett (1942) developed an expression relating fractional flow to space and time. This expression is commonly referred to as the frontal advance equation.

By means of the two relations mentioned previously, the saturation at the water front, the average saturation and the saturation distribution behind the water front, average saturation and saturation distribution after breakthrough for the system can be determined.

Fractional Flow

In engineering units, Darcy's law is expressed as

$$v = -1.127 \frac{k}{u} \left(\frac{dP}{dl} - 0.434 \rho \sin \theta\right) \qquad (1)$$

where v is velocity of one phase fluid flow in barrels per day per square foot,

k is the absolute permeability in darcy,

u is the viscosity of the fluid in centipoise,

$\frac{dP}{dl}$ is the driving gradient in psi per foot in the flow direction of 1,

$0.434 \rho \sin\theta$ is the hydrostatic gradient in psi per foot,

ρ is the fluid specific gravity,

θ is the angle measured between the horizontal and the direction of 1.

By analogy to equation 1, it is possible to write an equation for oil and water in two-phase flow system as follows:

$$v_o = -1.127 \frac{k_o}{u_o} \left(\frac{d\ P_o}{dl} - 0.434 \rho_o \sin\theta\right) \qquad (2)$$

$$v_w = -1.127 \frac{k_w}{u_w} \left(\frac{d\ P_w}{dl} - 0.434 \rho_w \sin\theta\right) \qquad (3)$$

two additional equations are formulated by definition:

$$v_t = v_o + v_w \qquad (4)$$

$$f_w = \frac{v_w}{v_t} \qquad (5)$$

and the capillary pressure existing at the oil water interface may be expressed as

$$\frac{d}{dl} (P_o - P_w) = \frac{d\ P_c}{dl} = \frac{d\ P_c}{d\ S_w} \cdot \frac{d\ S_w}{dl} \qquad (6)$$

where v_o, v_w, and v_t are the velocities of oil, water, and total flow in the direction of l,

k_o and k_w are the effective permeability to oil and water,

u_o and u_w are the oil and water viscosities,

$\frac{d\ P_o}{dl}$ and $\frac{d\ P_w}{dl}$ are the pressure gradients in oil and water,

ρ_o and ρ_w are oil and water specific gravities,

f_w is the fraction of water in the flow stream.

Equation 6 shows that the capillary pressure is a function of saturation which, in turn, is a function of the distance, where $\frac{d\,S_w}{dl}$ is termed the "saturation gradient".

From equations 2, 3, 4, 5, and 6 the fractional flow equation is derived and results in equation 7,

$$f_w = \frac{1 - 1.127\,\frac{k_o}{u_o}\,\frac{1}{v_t}\left[0.434\,(\rho_o - \rho_w)\,\sin\theta - \frac{d\,P_c}{d\,S_w}\cdot\frac{d\,S_w}{dl}\right]}{1 + \frac{k_o}{k_w}\cdot\frac{u_w}{u_o}} \qquad (7)$$

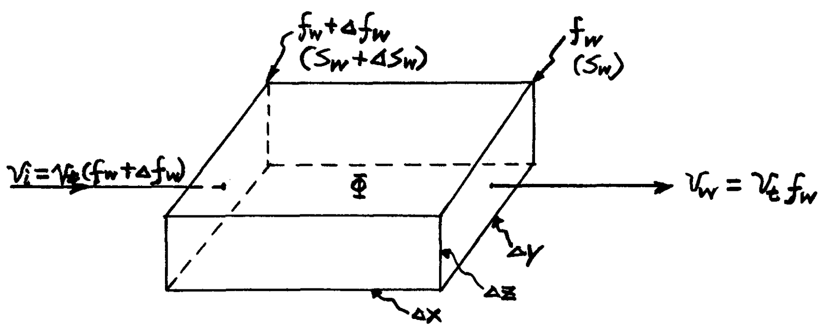
In a horizontal flow system θ equals zero reducing the gravity term to zero. Under field reservoir condition or in a laboratory experiment with high rate of displacement the capillary and gravity terms are usually neglected (Rapoport and Leas, 1953, p. 139). Therefore, equation 7 may be reduced to a much simpler form shown in equation 8,

$$f_w = \frac{1}{1 + \frac{k_o}{k_w}\cdot\frac{u_w}{u_o}} \qquad (8)$$

Frontal Advance

Figure 1 represents an incremental block within a flow system.

Figure 1 A Block of a Flow System.



where v_i is the input velocity in length per unit time,

v_w and v_t are the water and total flow velocities in length per unit time,

Δf_w is an increment of fractional flow of water,

ΔS_w is the water-saturation increment,

Δx, Δy, and Δz are the length, width and height of the block,

Φ is the porosity.

Let Δt be the period of flowing, then within this time period the amount of water left in the block will equal the volume difference of the input water and the output water; the volume conservation (incompressible fluids assumed) can be expressed as

$$\left[v_t(f_w + \Delta f_w)\Delta z\,\Delta y - v_t f_w \Delta z \Delta y\right]\Delta t = \Phi \Delta x \Delta y \Delta z \cdot \frac{\Delta S_w}{\Delta t} \cdot \Delta t \qquad (9)$$

Simplifying,

$$\frac{\Delta x}{\Delta t} = \frac{v_t}{\Phi} \cdot \frac{\Delta f_w}{\Delta S_w} \qquad (10)$$

If Δt, Δx, Δy, and Δz approach zero equation 10 reduces to

$$\lim_{\substack{\Delta t \to o \\ \Delta S_w \to o}} \frac{\Delta x}{\Delta t} = \frac{\partial x}{\partial t} = \frac{v_t}{\Phi} \cdot \frac{\partial f_w}{\partial S_w} = \frac{v_t}{\Phi} f' \qquad (11)$$

Equation 11 is commonly called "frontal advance equation".

If $\frac{\partial f_w}{\partial S_w}$ is evaluated at a constant water saturation, equation 11 can be changed to

$$dx = \frac{v_t}{\Phi} (f')_{S_w}\, dt \qquad (12)$$

Integration of equation 12 between the limits of $o \leq x \leq L$ and $o \leq t \leq T$ results in

$$L = \frac{v_t}{\Phi} (f')_{S_w}\, T \qquad (13)$$

Equation 13 implies that a plane of constant water saturation will advance a distance L which is proportional to the elapsed time, the magnitude of flowing velocity, and the value of the derivative of the fractional flow equation at that saturation.

MODEL INVESTIGATIONS

The model study is discussed in the following subheadings: Materials and Models, Rock and Fluid Property Determination, and Five-Spot Experiments.

Materials and Models

Sands used in this investigation were a mixture of 100- to 180-mesh and 180- to 240-mesh from American Graded Sand Co. The binder used in preparing consolidated sands was a mixture of X-85 Resin and Activator "E" of Armstrong Products Co. and Epon V 40 of Sherwood Solvents, Inc..

Fluids used were Texaco Crystalite kerosene, two mixtures of Texaco Crystalite kerosene and Texaco U.R.S.A. P-30, and distilled water with viscosities 2.74, 21.98, 78.27, and 0.92 centipoise, respectively.

The following steps were followed in the preparation of a five-spot model and a core sample for the rock and fluid property determinations,

1. Preparation of sand mixtures:
   a. The total sand mixtures required were determined.
   b. Sixty per cent by weight of the total sand mixture was 100- to 180-mesh and forty per cent was 180- to 240-mesh.
   c. The measured sands were mixed in a rolling mixer.
2. Preparation of the binder:
   a. One seventy-fifth by weight of the total sand mixture was the total binder required.
   b. Resin X-85 required was seven tenths by weight of the total binder.
   c. Epon V 40 required was three tenths by weight of the total binder.
   d. The measured Resin X-85 in step b and Epon V 40 in step c were mixed.
   e. The mixture of step d was added with Activator "E" one tenth by weight of the Resin X-85 used and were thoroughly mixed.
3. The sands and the binder were thoroughly mixed to make sure the sand grains were evenly coated with the binder, then the mixture was molded into a five-spot model (1/4 x 4- x 5-inch) and in a sample box. The density of the packing was prefixed

at 1.689 grams per cubic centimeter. The setting time was 2 hours at 165°F.

4. After the sands had consolidated, the sample was cored with 1 1/2-inch core drill for rock and fluid property determination. The core sample was coated with a mixture of Resin X-85 and Activator "A" and both ends were attached with fittings as illustrated on Plate 1, page 14.

5. The consolidated sands for the five-spot model were also coated with a mixture of Resin X-85 and Activator "A". After the coating had set, four fittings were attached at four corners of the model as illustrated in Plate 1, page 14.

Rock and Fluid Property Determination

Rock and fluid properties of the model were determined by laboratory displacement of a core sample and calculations. These data obtained were used in Higgins and Leighton's calculations. Equipment and procedure, relative permeability calculation, and results are presented in the following paragraphs.

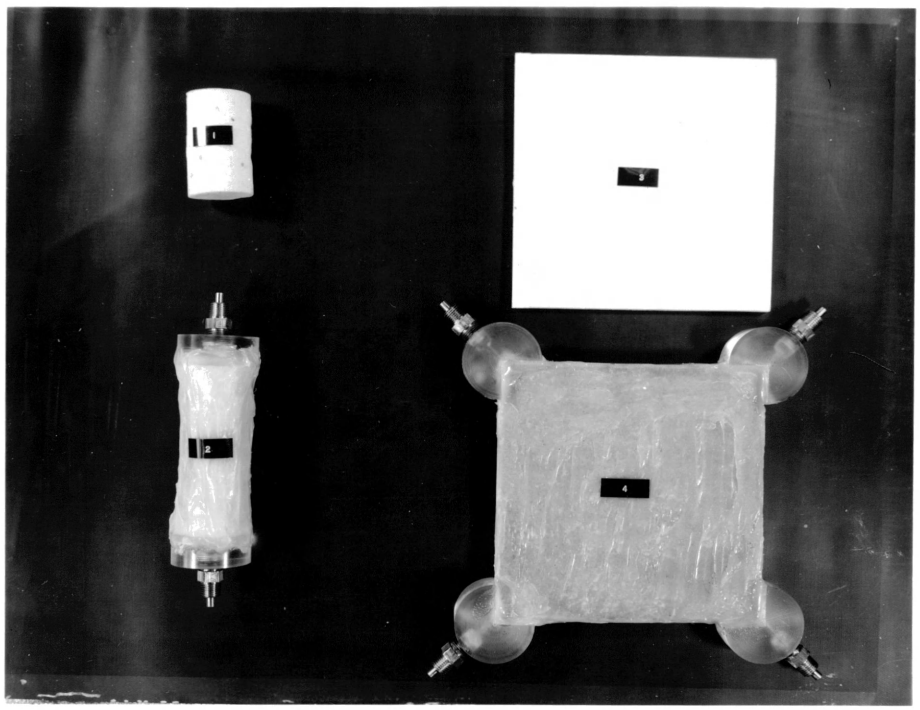


Plate 1 Core Sample and Five-spot Model.

1 Drilled core sample before coating

2 Coated core sample

3 Molded five-spot sand before coating

4 Coated five-spot model

Equipment and Procedure

A constant rate pump was used to inject water into a measuring chamber. The measuring chamber served two purposes: to displace the oil with water during the oil saturation process and to measure the injected oil volume. A pressure gage constantly checked with a manometer for accuracy was used to measure the changing input pressures. On the output end of the core sample, measuring cylinders and a stop watch were provided. A vacuum pump was used to extract the air in the sample to ensure the initial water saturation as perfect as possible, a manometer was attached to the vacuum pump to detect leakage. A schematic of the equipment is shown in Figure 2, page 19.

Determination of the rock and fluid properties was conducted according to the following steps:

1. The core sample was weighed.
2. The core sample was evacuated at 24-inch mercury column (one atmosphere pressure is equivalent to 24.5-inch mercury column at Golden.)
3. The core sample was saturated with water.
4. The core sample was weighed again to calculate the porosity.

5. The water was injected at a constant rate and the stabilized pressure drop was measured to calculate the absolute permeability to water.
6. Water in the core sample was displaced with oil and the output water and oil volumes were measured; the resulting water saturation and oil saturation were calculated.
7. The oil in the core sample was then displaced with water at a constant rate to obtain data for relative permeability calculation.

Relative Permeability Calculation

The relative permeability calculation method used was proposed by Johnson, Bossler, and Naumann (1959). The procedure used is summarized below:

1. Data obtained from a displacement experiment were: flowing time interval, total output, oil output, water output, and average input pressure.
2. The cumulative total output volume (equals water injected) and the cumulative total output volume in terms of pore volume of the core sample (W_i) were calculated for each time interval.

3. The cumulative output oil volume and then the average water saturation $(S_w)_{avg}$ were calculated at each time interval.

4. $(S_w)_{avg}$ against W_i was plotted and the derivative $\frac{d(S_w)_{avg}}{d\ W_i}$ was taken to obtain the fractional flow of oil and water at the outlet face $(f_o)_2$ and $(f_w)_2$.

5. The total output velocity per unit pressure differential was calculated for each time interval. The ratio of each calculated value with respect to that of initial time interval was calculated to obtain the reciprocal of relative injectivity $\frac{1}{I_r}$.

6. Using the reciprocal value of W_i from step 2, $\frac{1}{W_i\ I_r}$ was calculated.

7. $\frac{1}{W_i\ I_r}$ against $\frac{1}{W_i}$ was plotted and the derivative $\frac{d(\frac{1}{W_i\ I_r})}{d(\frac{1}{W_i})} = s$ was graphically (or by other means) calculated.

8. The relative permeability to oil at the outlet face is calculated as $(k_{ro})_2 = \frac{(f_o)_2}{s}$. The corresponding water saturation was calculated as $(S_w)_2 = (S_w)_{avg} - (f_o)_2\ W_i$.

9. The relative permeability to water at the outlet face was calculated as

$$(k_{rw})_2 = (k_{ro})_2 \times \frac{(f_w)_2}{(f_o)_2} \times \frac{u_w}{u_o}$$

where u_w and u_o are viscosities of water and oil.

Results

The data obtained from the fluid-displacement experiment and resulting calculation are as follows:

Porosity	Φ = 34%
Absolute Permeability	k = 1.466 darcys
Initial Water Saturation	S_{wi} = 13%
Initial Oil Saturation	S_{oi} = 87%

Relative permeability to oil and water is listed in Table 1, page 20, and plotted in Figure 3, page 21.

The sand is preferentially oil-wet.

Five-Spot Experiments

A model simulated a quadrant of a five-spot was used in this study. The dimension of the model was 1/4- x 5- x 5-inch. Three runs were made with mobility ratios ranging from 0.83 to 4.10.

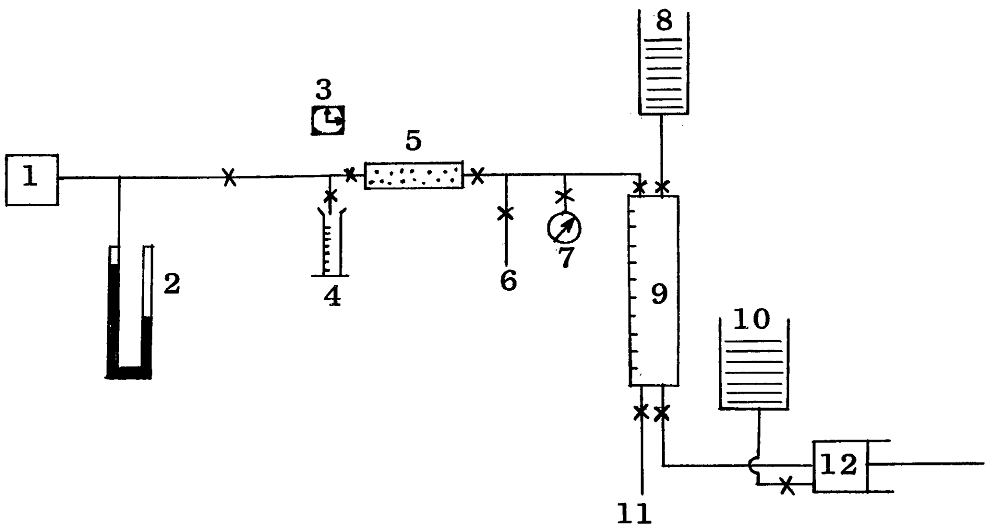


Figure 2 Schematic of Relative Permeability Measurement Apparatus.

1.	Vacuum Pump	7.	Pressure Gage
2.	Manometer	8.	Oil Supply
3.	Stop Watch	9.	Measuring Chamber
4.	Measuring Cylinder	10.	Water Supply
5.	Core Sample	11.	Drainage
6.	Bleeding	12.	Constant Rate Injection Pump

Table 1 Water and Oil Relative Permeability

Water Saturation	Relative Permeability Ratio to Water	Relative Permeability Ratio to Oil	Water Saturation	Relative Permeability Ratio to Water	Relative Permeability Ratio to Oil
0.1769	0.0000	1.0000	0.41	0.1431	0.3450
0.19	0.0079	0.9590	0.42	0.1500	0.3200
0.20	0.0138	0.9295	0.43	0.1575	0.2945
0.21	0.0198	0.9000	0.44	0.1694	0.2680
0.22	0.0259	0.8700	0.45	0.1739	0.2437
0.23	0.0315	0.8385	0.46	0.1818	0.2178
0.24	0.0368	0.8057	0.47	0.1895	0.1931
0.25	0.0432	0.7748	0.48	0.1980	0.1678
0.26	0.0496	0.7465	0.49	0.2069	0.1416
0.27	0.0557	0.7150	0.50	0.2160	0.1169
0.28	0.0619	0.6842	0.51	0.2245	0.0950
0.29	0.0732	0.6555	0.52	0.2335	0.0762
0.30	0.0744	0.6280	0.53	0.2430	0.0593
0.31	0.0799	0.6010	0.54	0.2535	0.0446
0.32	0.0865	0.5750	0.55	0.2640	0.0324
0.33	0.0928	0.5470	0.56	0.2739	0.0224
0.34	0.0989	0.5208	0.57	0.2844	0.0148
0.35	0.1054	0.4965	0.58	0.2962	0.0105
0.36	0.1160	0.4728	0.59	0.3079	0.0079
0.37	0.1177	0.4469	0.60	0.3202	0.0035
0.38	0.1238	0.4215	0.61	0.3322	0.0012
0.39	0.1298	0.3955	0.62	0.3454	0.0000
0.40	0.1362	0.3700			

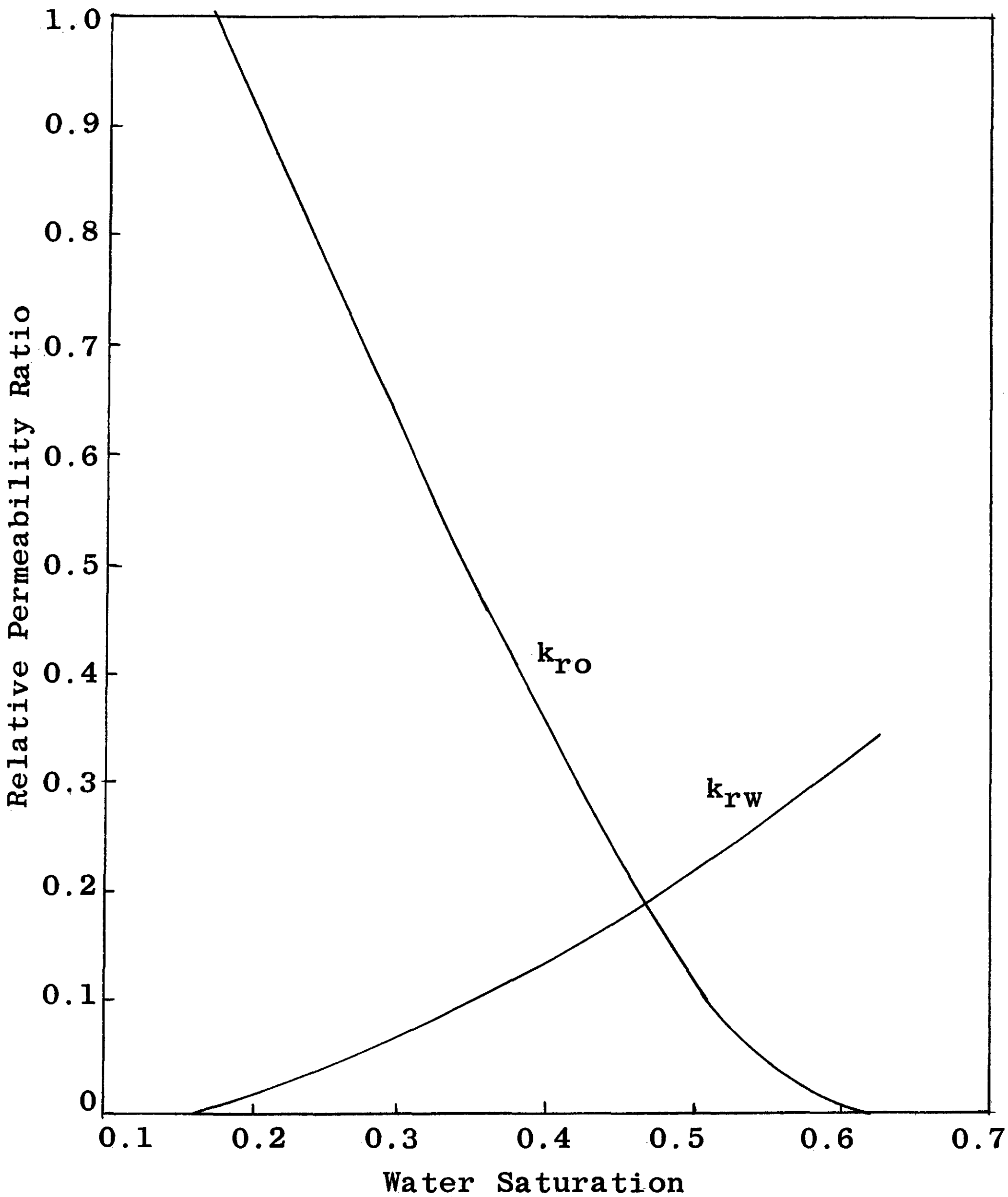


Figure 3 Water and Oil Relative Permeability.

k_{ro} Relative Permeability Ratio to Oil

k_{rw} Relative Permeability Ratio to Water

Equipment and Procedure

A centrifugal pump was used to inject water into the air-pressure chamber, and a constant water head in the chamber was maintained. Air pressure was directed through an air regulator to exert a constant air pressure in the chamber, and a manometer was installed to measure the air pressure. From the lower end of the air-pressure chamber, the water was lead to the injection well of a five-spot model, and on the diagonal corner, at the producing well, measuring cylinders and a stop watch were furnished. The experimental equipment is shown in Figure 4, page 25, and Plate 2, page 26.

The experimental procedure for the five-spot-model water-flood is as follows:

1. The model was weighed.
2. The model was then evacuated at 24-inch mercury column.
3. The model was saturated with water.
4. The saturated model was weighed again to calculate the porosity for checking purpose (around 3 per cent difference with the absolute value obtained from core sample).
5. Water in the model was displaced with oil and the output water and oil volumes were measured. The

initial water saturation and oil saturation were calculated to check with the data from core sample (about 2 per cent difference of absolute values).

6. The water was dyed with washable black ink and the colored water was injected through the injection well at a constant pressure. At the producing well, the time interval, total output, water output, and oil output were measured and recorded. The water front was traced at breakthrough for areal sweep efficiency calculation.

7. Three runs were made with a constant-viscosity displacing fluid and displaced fluid viscosities of 2.74, 21.98, and 78.27 centipoise.

Results

The areal sweep efficiencies at breakthrough were experimentally determined to be 77.92, 58.96, and 44.54 per cent, for the mobility ratios of 0.83, 2.90, and 4.10 with viscosity ratios of 2.98, 23.89, and 85.08, respectively.

The cumulative oil production and cumulative total production (equal total injection) were calculated in terms of the pore volume of the model for the convenience of comparison with the computed data discussed later. The observed results of this experiment are listed in tables

2, 3, and 4, page 27, and are plotted in figures 10, 11, and 12, pages 46, 47, and 48.

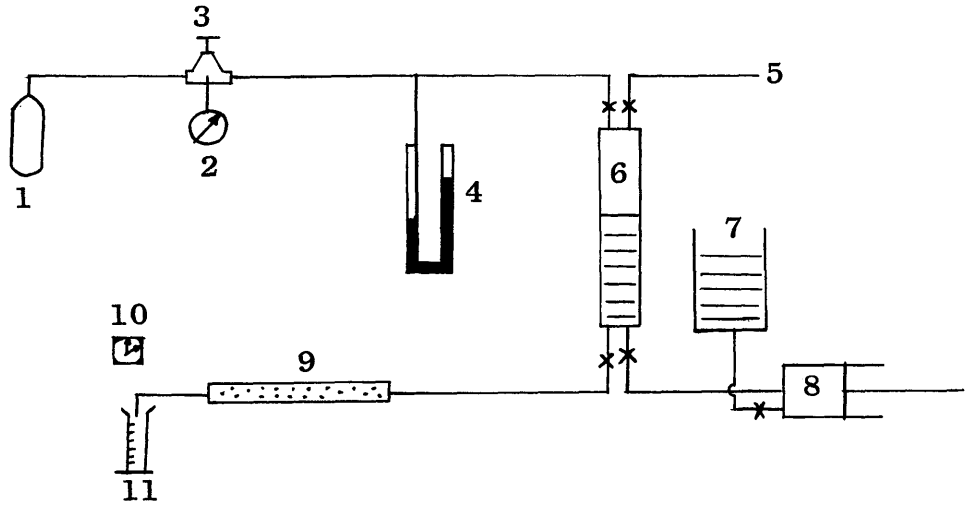


Figure 4 Flow Diagram for Five-Spot Water-Flood Experiment.

1. Air-Pressure Supplier
2. Pressure Gage
3. Air Regulator
4. Manometer
5. Bleeding
6. Air-Pressure Chamber
7. Water Supply
8. Injection Pump
9. Five-Spot Model
10. Stop Watch
11. Measuring Cylinder

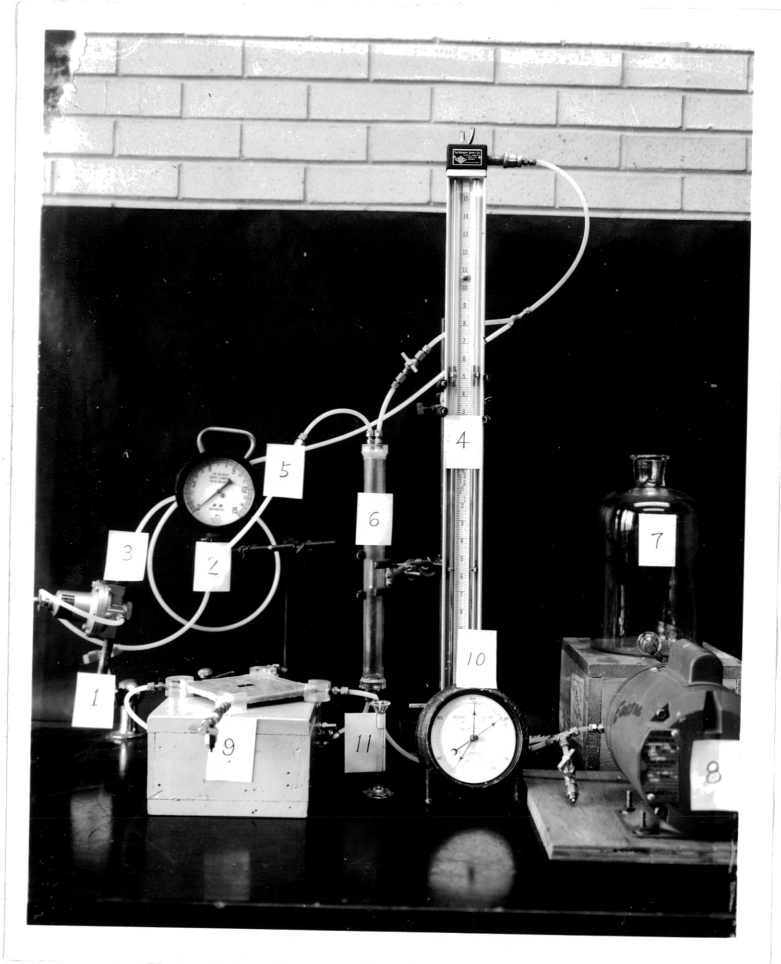


Plate 2 Experimental Equipment.

1. Air-Pressure Supplier
2. Pressure Gage
3. Air Regulator
4. Manometer
5. Bleeding
6. Air-Pressure Chamber
7. Water Supply
8. Injection Pump
9. Five-Spot Model
10. Stop Watch
11. Measuring Cylinder

Table 2

Experimental Oil Recoveries
for Mobility Ratio = 0.83

Cumulative Oil Produced in Pore Volume	Cumulative Water Injected in Pore Volume
0.000	0.000
0.047	0.047
0.096	0.096
0.146	0.146
0.198	0.198
0.252	0.252
0.306	0.306
0.352	0.361
0.387	0.416
0.420	0.473
0.440	0.532

Table 3

Experimental Oil Recoveries
for Mobility Ratio = 2.90

Cumulative Oil Produced in Pore Volume	Cumulative Water Injected in Pore Volume
0.000	0.000
0.032	0.032
0.065	0.065
0.100	0.100
0.135	0.135
0.169	0.172
0.192	0.224
0.215	0.279
0.232	0.339
0.249	0.403
0.262	0.469
0.274	0.541

Table 4

Experimental Oil Recoveries
for Mobility Ratio = 4.10

Cumulative Oil Produced in Pore Volume	Cumulative Water Injected in Pore Volume
0.000	0.000
0.032	0.032
0.065	0.065
0.105	0.105
0.130	0.137
0.145	0.165
0.155	0.192
0.162	0.220
0.169	0.252
0.176	0.284
0.183	0.317
0.189	0.354
0.195	0.390
0.200	0.427
0.206	0.464
0.211	0.504

HIGGINS AND LEIGHTON'S CALCULATION TECHNIQUES

Higgins and Leighton's calculation techniques are designed to be processed by a computer. The computing time for a run is about one minute. The idea of stream lines used in the technique, the data required, the calculation procedure, and the results calculated are discussed in the succeeding paragraphs.

Stream Lines

The stream lines used in the Higgins and Leighton's calculation are determined by a potential model wherein the mobility ratio is one. For a one-to-one mobility consideration the stream lines can be determined using a numerical method (Harley and Meabon, 1962) or a graphical method under the assumptions of a homogeneous system with one-phase steady-state flow.

Since stream lines are always perpendicular to the isopotential lines, the fluid-flow direction is necessarily parallel to the stream lines. If an octant of a five-spot is divided into four channels whose boundaries correspond to the stream lines, there would be no cross flow between the channels. If each channel is divided into a series of small cells, the flow condition for each small cell can be properly adjusted to a linear flow system except the portion close to the wells. A shape factor was devised to convert radial flow near the wells into a counterpart of a linear flow and to adjust the flow condition of each cell into a linear flow system where Darcy's equation applies.

The pressure differential between wells is assumed to be a constant in the calculation, which means a fixed pressure differential exists over the total length of each channel but the pressure gradient at any point in the channel is a function of the length and the resistivity with the channel. If the shape factor, which includes the average length, is combined with the resistivity factor for each cell and summed to form an overall resistivity for a channel, then the flow rate through a channel may be calculated for any pressure differential.

Data Requirement

The data required in the pattern-displacement calculation are as follows,

1. Rock and fluid properties:

   a. Porosity Φ

b. Absolute permeability k

c. Viscosity of displacing fluid u_w

d. Viscosity of displaced fluid u_o

e. Initial water saturation S_{wi}

f. Water-oil relative permeabilities k_{rw} and k_{ro}

2. Reservoir conditions and operation method:

a. Well spacing d

b. Pressure differential Δp

c. Thickness of the formation h

d. Wellbore radius r_w

The quantities d and r_w are required in the shape-factor calculation for the first and the last cell of each channel. The calculation time can greatly be reduced if a per-unit-thickness basis is processed.

In the calculations related to the experimental study, the rock and fluid properties were obtained from the model investigation discussed previously. The reservoir conditions and operation method were arbitrarily chosen as follows (Higgins and Leighton, 1963, p. 56):

Distance of well spacing $d = 7.37$ ft

Pressure differential $\Delta p = 100$ atmospheres

Thickness of the formation $h = 1$ ft

Wellbore radius $r_w = 0.01516$ ft

The comparison should be valid as all quantities are reduced to dimensionless terms.

Calculation Procedure

The calculations relative to the three experimental runs were computed on IBM 7090 Digital Computer by the Bureau of Mines. The details of the computer procedure are presented in the two papers by Higgins and Leighton (1962; 1963).

The following steps summarize the calculation procedures:

1. Determine the stream lines in an octant of a five-spot. Divide the octant into four channels and measure the angles subtended by each channel at each well.
2. Determine the bulk volume and pore volume (V_p) for each channel.
3. Divide each channel into 40 cells and calculate a shape factor (G_n) for each cell.
4. Using premeabilities determined, calculate the fractional flow for water with equation 8,

$$f_w = \frac{1}{1 + \frac{k_o}{k_w} \cdot \frac{u_w}{u_o}} \qquad (8)$$

5. Plot calculated f_w against the corresponding S_w to find saturation at the water front $(S_w)_f$ and the average water saturation behind the front $(S_w)_{avg}$.
6. Take the derivative, $f' = \frac{\partial f_w}{\partial S_w}$, corresponding to the water saturations from $(S_w)_f$ to S_{or}, where S_{or} is the residual oil saturation.
7. Plot $\frac{1}{k_{ro}}$ and $\frac{1}{k_{rw}}$ against f' and calculate the areas under curves $\frac{1}{k_{ro}}$ and $\frac{1}{k_{rw}}$.
8. Compute the performance of primary phase:
   a. Calculate the average oil volume to be produced during the water invasion of each cell with equation 14,

$$(V_o)_j = \frac{V_p}{40} \; (\overline{S}_{wf} - S_{wi}) \qquad (14)$$

   $(V_o)_j$ is oil to be produced.

   b. With equations 15 and 16 calculate the area-mean relative permeabilities of each cell when the water front reaches the end of the cell,

$$(k_{rw})_{mn} = \frac{f'_{f/j}}{\text{Area under } \frac{1}{k_{rw}} \text{ vs. } f' \text{ curve in } n^{th} \text{ cell}} \qquad (15)$$

   f'_f is designated corresponding to $(S_w)_f$.

$$(k_{ro})_{mn} = \frac{f'_{f/j}}{\text{Area under } \frac{1}{k_{ro}} \text{ vs. } f' \text{ curve in the nth cell}} \tag{16}$$

$(k_{rw})_{mn}$ and $(k_{ro})_{mn}$ are mean relative permeabilities at nth cell.

j is the number of cells invaded.

c. Compute initial instantaneous oil rate at the beginning of flood with equation 17,

$$(q_o)_{j=o} = \frac{k \Delta p}{\sum_{n=1}^{40} \frac{u_o}{k_{roi}} G_n} \tag{17}$$

where k_{roi} is the initial relative permeability to oil.

d. When the first cell is invaded (j = 1), calculate the instantaneous oil rate with equation 18,

$$(q_o)_j = \frac{k \Delta p}{\sum_{n=1}^{j} \frac{G_n}{\frac{(k_{rw})_{mn}}{u_w} + \frac{(k_{ro})_{mn}}{u_o}} + \sum_{j+1=n}^{40} \frac{u_o}{k_{roi}} G_n} \tag{18}$$

e. Calculate the average oil rate with equation 19,

$$(q_o)_{avg} = \frac{(q_o)_{j+1} + (q_o)_j}{2} \tag{19}$$

$(q_o)_{avg}$ is average oil rate.

f. Calculate the elapsed time between two instantaneous rates (the time to flood one cell) with equation 20,

$$t_j = \frac{(V_o)_j}{(q_o)_{avg}} \tag{20}$$

g. Repeat steps a, b, c, d, e, and f until breakthrough.

9. Compute the performance of subordinate phase:

a. Reduce the abscissa of f' by a predetermined decrement to yield f'_{brm}.

b. Once f'_{brm} is determined, the corresponding value of S_{wm} can be determined with equation 21,

$$\overline{S}_{wm} = S_{wm} + \frac{1 - f_{wm}}{f'_{brm}} \tag{21}$$

$\overline{S}_{wm}$ is average saturation at a particular moment corresponding to f'_{brm} after breakthrough.

S_{wm} is outlet face water saturation corresponding to f'_{brm}.

f_{wm} is the fractional flow of water corresponding to f'_{brm}.

c. Calculate the oil produced between each decrement of f'_{brm} with equation 22,

$$V_{ojm} = V_p \, (\overline{S}_{wm} - \overline{S}_{wm-1}) \tag{22}$$

$m = 1, 2, 3, \ldots\ldots.$

d. Calculate the water-oil ratio corresponding to f'_{brm} with equation 23,

$$WOR = \frac{k_{rw}}{k_{ro}} \cdot \frac{u_o}{u_w} \qquad (23)$$

WOR is water-oil ratio.

e. Calculate the area-mean relative permeabilities for each cell with equations 24 and 25,

$$(k_{rw})_{m\ nm} = \frac{f'_{brm}/40}{\text{Area under } \frac{1}{k_{rw}} \text{ vs. } f' \text{ curves in nth cell}} \qquad (24)$$

$$(k_{ro})_{m\ nm} = \frac{f'_{brm}/40}{\text{Area under } \frac{1}{k_{ro}} \text{ vs. } f' \text{ curves in nth cell}} \qquad (25)$$

f. Calculate the total instantaneous rate for each f'_{brm} decrement with equation 26,

$$(q_o + q_w)_m = \frac{k\,\Delta p}{\sum_{n=1}^{40} \dfrac{G_n}{\left[\dfrac{(k_{rw})_{m\ nm}}{u_w} + \dfrac{(k_{ro})_{m\ nm}}{u_o}\right]}} \qquad (26)$$

q_w is water rate.

g. Calculate the instantaneous oil rates with equation 26 and 27,

$$(q_o)_{j\ =\ m} = \frac{q_{wm}}{WOR} \qquad (27)$$

h. Calculate the average oil rate for each f'_{brm} decrement with equation 28,

$$(q_o)_{j=m\ avg} = \frac{(q_o)_{j=m} + (q_o)_{j=m-1}}{2} \tag{28}$$

m = 1, 2, 3,.....

i. Calculate the time elapsed between each f'_{brm} decrement with equation 29,

$$t_{j=m} = \frac{V_{ojm}}{(q_o)_{j=m\ avg}} \tag{29}$$

j. Repeat from step a again until the predetermined water-oil ratio is reached.

10. Calculate the cumulative time, oil recovery, water injected, and record water-oil ratio.
11. Convert the calculated result to basis for comparison.
12. From the results of step 10, calculate the water fronts in each channel at breakthrough. Plot the water fronts and calculate the areal sweep efficiency.
13. Calculate the mobility ratio at breakthrough using equation 30,

$$M_o = \frac{\left(\frac{k_{rw}}{u_w} + \frac{k_{ro}}{u_o}\right) \text{swept at breakthrough}}{\left(\frac{k_{rw}}{u_w} + \frac{k_{ro}}{u_o}\right) \text{unswept}} \tag{30}$$

M_o is a mobility ratio.

Calculated Results

The areal sweep efficiencies at water breakthrough calculated by the Higgins and Leighton's method were 78.00, 74.72, and 72.34 per cent for mobility ratios of 0.83, 2.90, and 4.10 and viscosity ratios of 2.98, 23.89, and 85.08, respectively.

The cumulative oil production and cumulative water injection are listed in tables 5, 6, and 7, page 38, and are plotted in figures 10, 11, and 12, on pages 46, 47, and 48.

Table 5

Calculated Oil Recoveries for Mobility Ratio = 0.83

Cumulative Oil Produced in Pore Volume	Cumulative Water Injected in Pore Volume
0.000	0.000
0.052	0.052
0.087	0.087
0.140	0.140
0.193	0.193
0.236	0.246
0.317	0.317
0.346	0.352
0.391	0.435
0.415	0.517
0.432	0.600

Table 6

Calculated Oil Recoveries for Mobility Ratio = 2.90

Cumulative Oil Produced in Pore Volume	Cumulative Water Injected in Pore Volume
0.000	0.000
0.036	0.036
0.064	0.064
0.108	0.108
0.138	0.138
0.168	0.173
0.192	0.218
0.215	0.269
0.234	0.326
0.243	0.356
0.252	0.386
0.293	0.552

Table 7

Calculated Oil Recoveries for Mobility Ratio = 4.10

Cumulative Oil Produced in Pore Volume	Cumulative Water Injected in Pore Volume
0.000	0.000
0.029	0.029
0.062	0.062
0.086	0.086
0.120	0.147
0.140	0.205
0.149	0.240
0.158	0.278
0.167	0.321
0.176	0.366
0.185	0.414
0.192	0.488
0.199	0.524

COMPARISON OF EXPERIMENTAL AND CALCULATED RESULTS

The experimental and calculated results were compared on the bases of areal sweep efficiency as a function of mobility ratio and of the pore-volume oil produced as a function of pore-volume water injected. Both comparisons used a dimensionless unit.

Areal Sweep Efficiencies

The areal sweep patterns and the areal sweep efficiencies of both experimental and calculated results are shown in figures 5, 6, and 7, on pages 41, 42, and 43 . The numerical data are listed in Table 8.

Table 8

Experimental and Calculated
Areal Sweep Efficiency

Mobility Ratio	Viscosity Ratio	Areal Sweep Efficiency	
		Experimental	Calculated
0.83	2.98	77.92%	78.00%
2.90	23.89	58.96%	74.72%
4.10	85.08	44.54%	72.34%

The relations between mobility ratio and areal sweep efficiency are presented in Figure 8, page 44, and the relations between viscosity ratio and areal sweep efficiencies are presented in Figure 9, page 45.

Figures 8 and 9 show that the experimental and calculated areal sweep efficiencies diverge quickly as the mobility ratio increases. This comparison suggests that the Higgins and Leighton's calculation techniques will be unsatisfactory when used to predict areal sweep efficiency. Figure 8 also shows that if the range of mobility ratio is within 1.5, the error will be within six per cent.

Oil Recoveries

The plots of oil recoveries and cumulative water injected for different mobility ratios are presented in figures 10, 11, and 12, on pages 46, 47, and 48. These figures reveal that the experimental and calculated data are in an excellent agreement. The maximum difference in the two methods is 1.5 per cent for total water injection between 0 and 0.5 pore volumes. The two methods also indicate good agreement at time of water breakthrough.

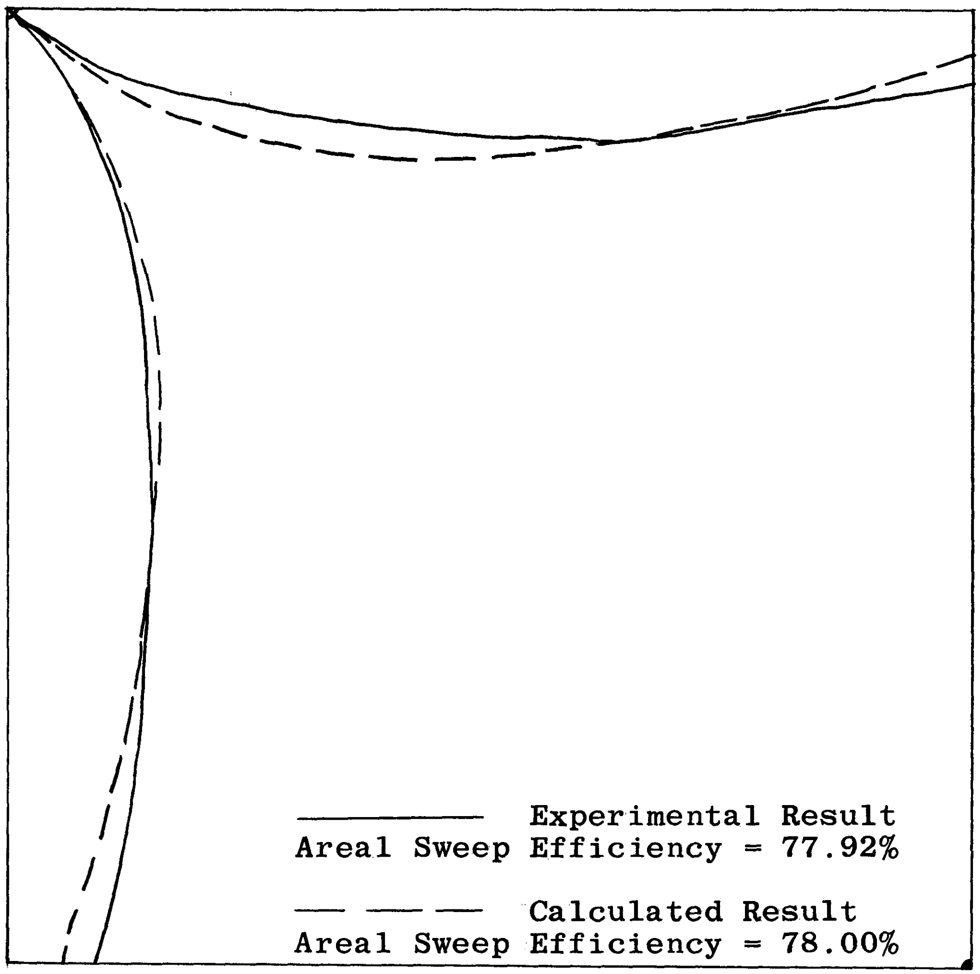


Figure 5 Experimental and Calculated Areal Sweep Patterns at Breakthrough for Mobility Ratio = 0.83.

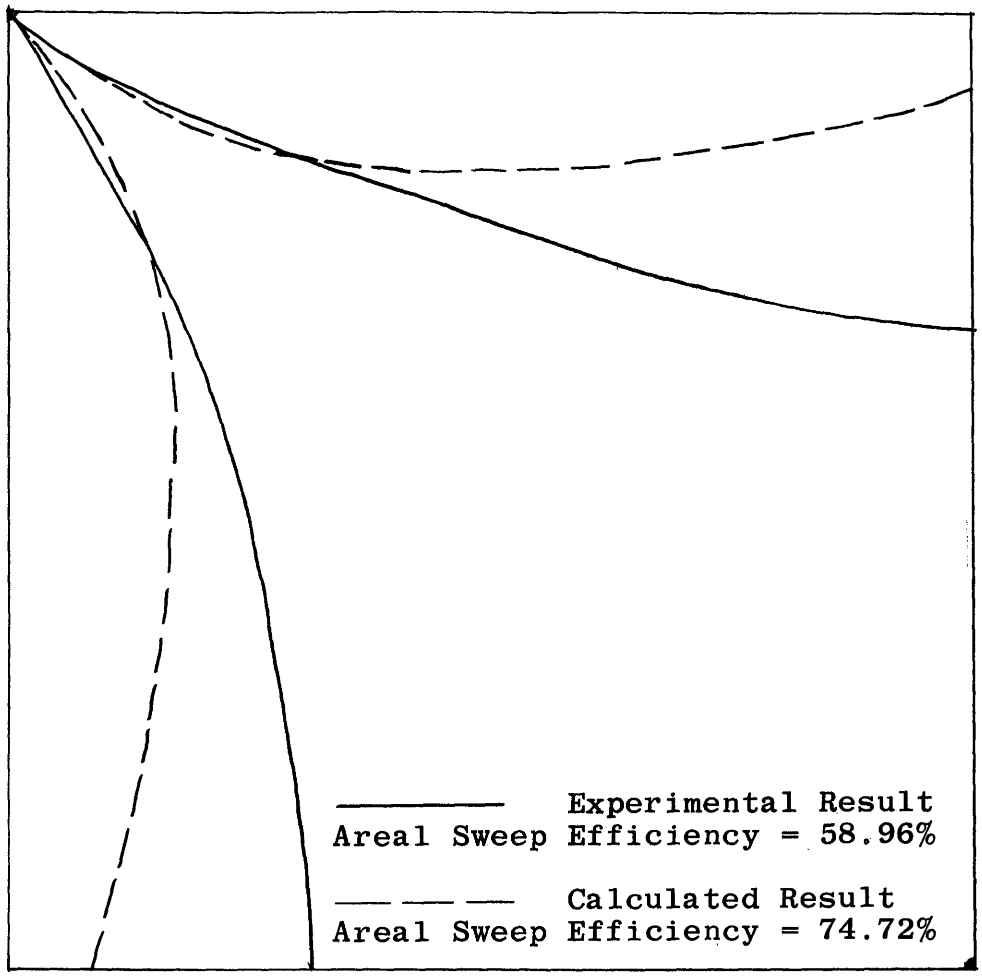


Figure 6 Experimental and Calculated Areal Sweep Patterns at Breakthrough for Mobility Ratio = 2.90.

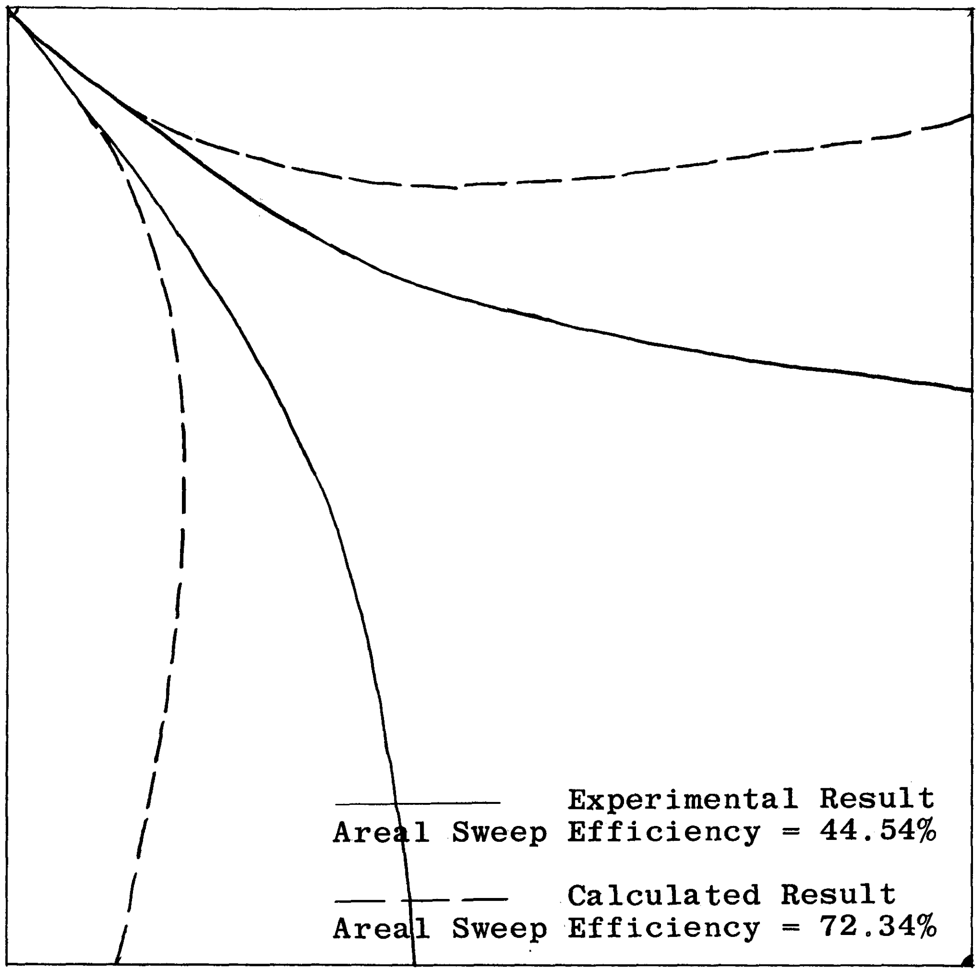


Figure 7 Experimental and Calculated Areal Sweep Patterns at Breakthrough, Mobility Ratio = 4.10 .

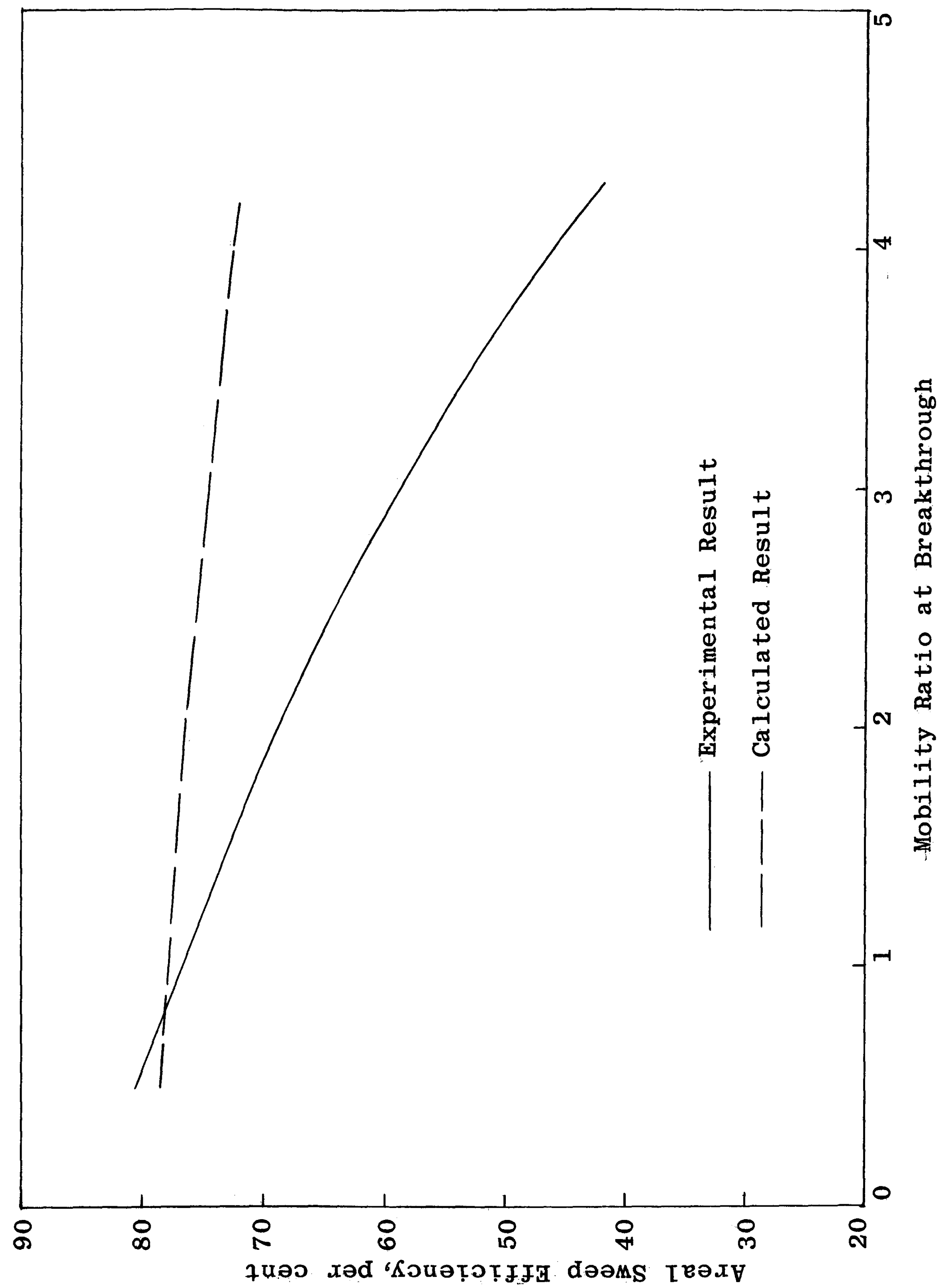


Figure 8 Effect of Mobility Ratio on Areal Sweep Efficiency.

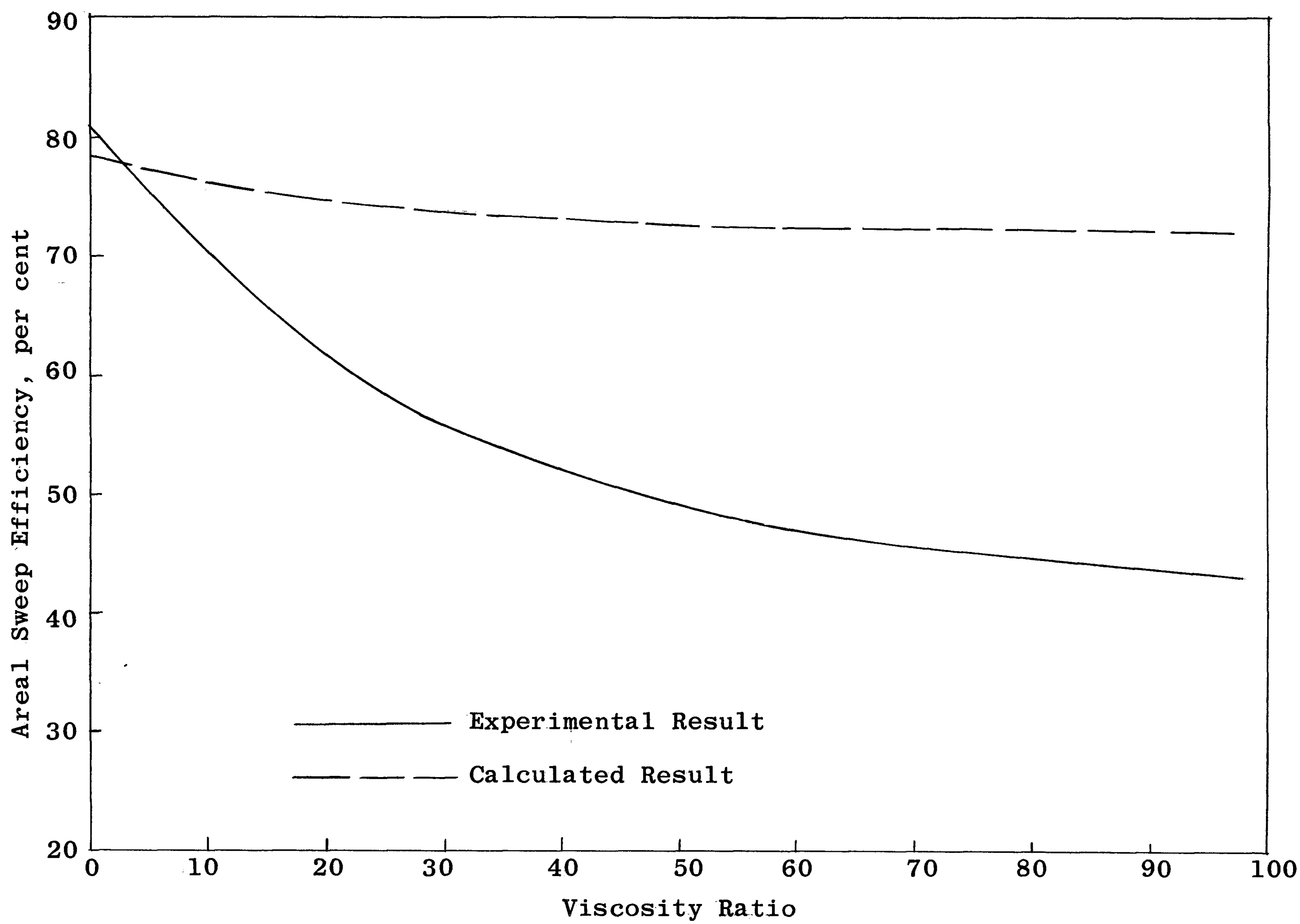


Figure 9 Effect of Viscosity Ratio on Areal Sweep Efficiency.

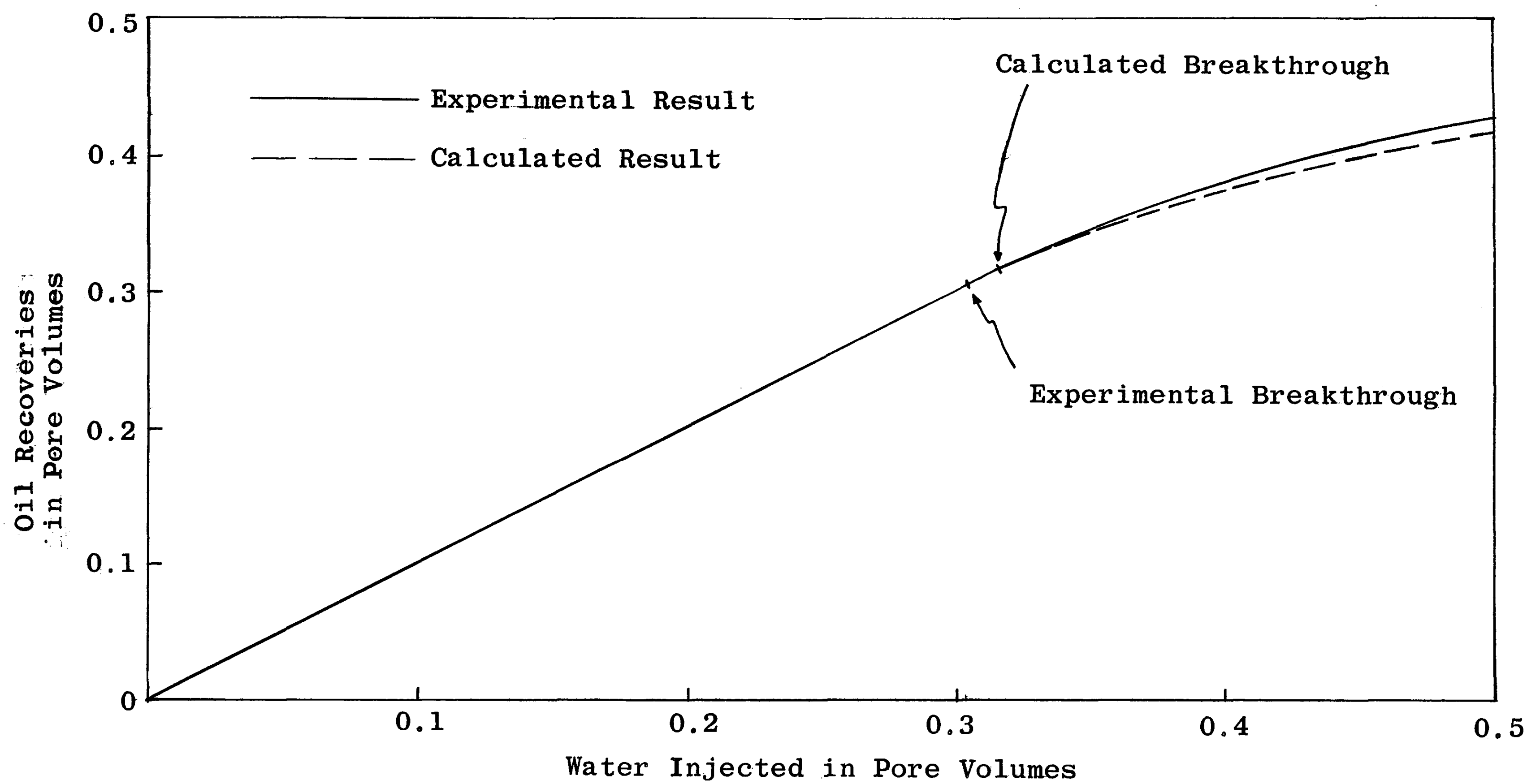


Figure 10 Comparison of Experimental and Calculated Oil Recoveries for Mobility Ratio = 0.83 at Breakthrough.

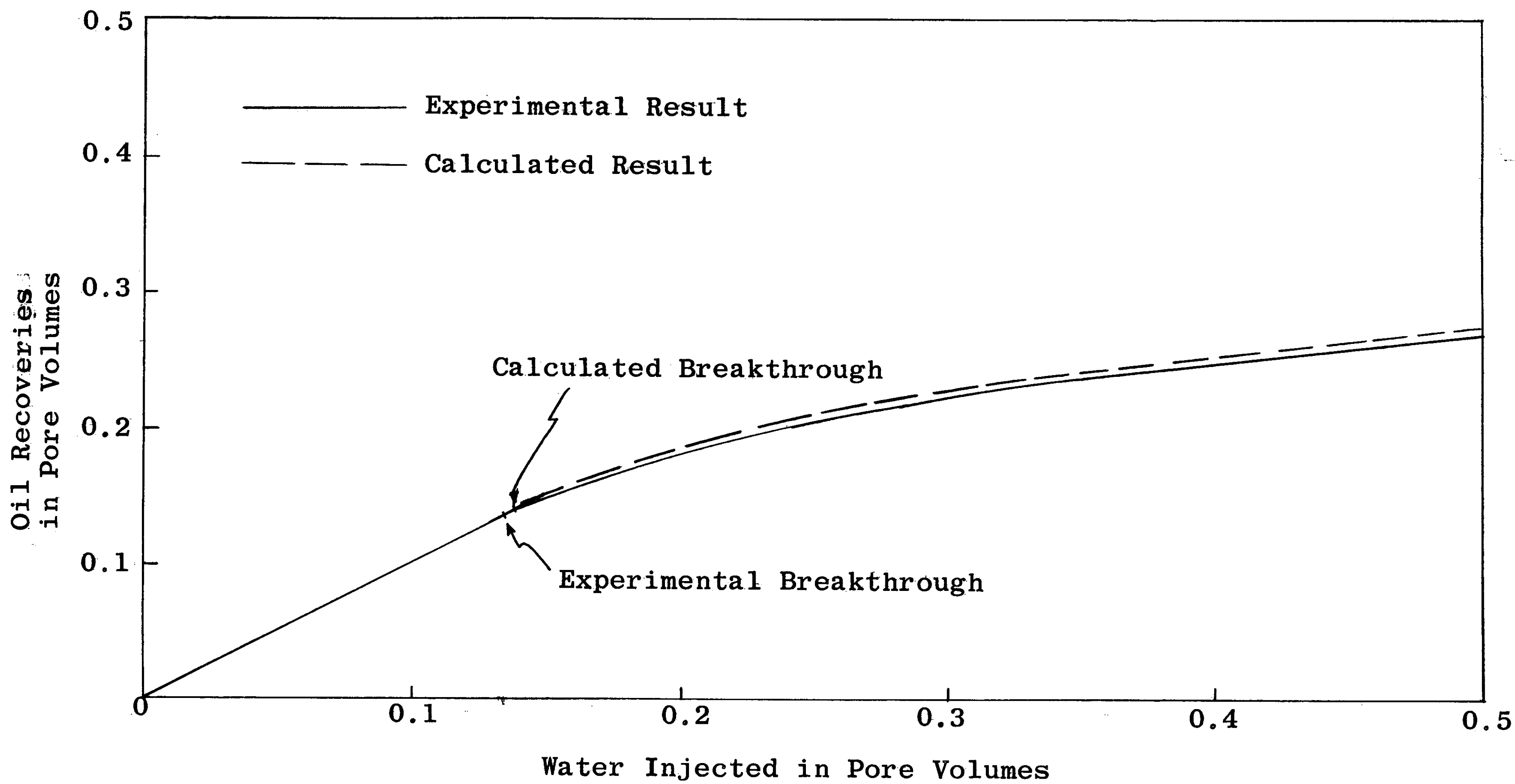


Figure 11 Comparison of Experimental and Calculated Oil Recoveries for Mobility Ratio = 2.90 at Breakthrough.

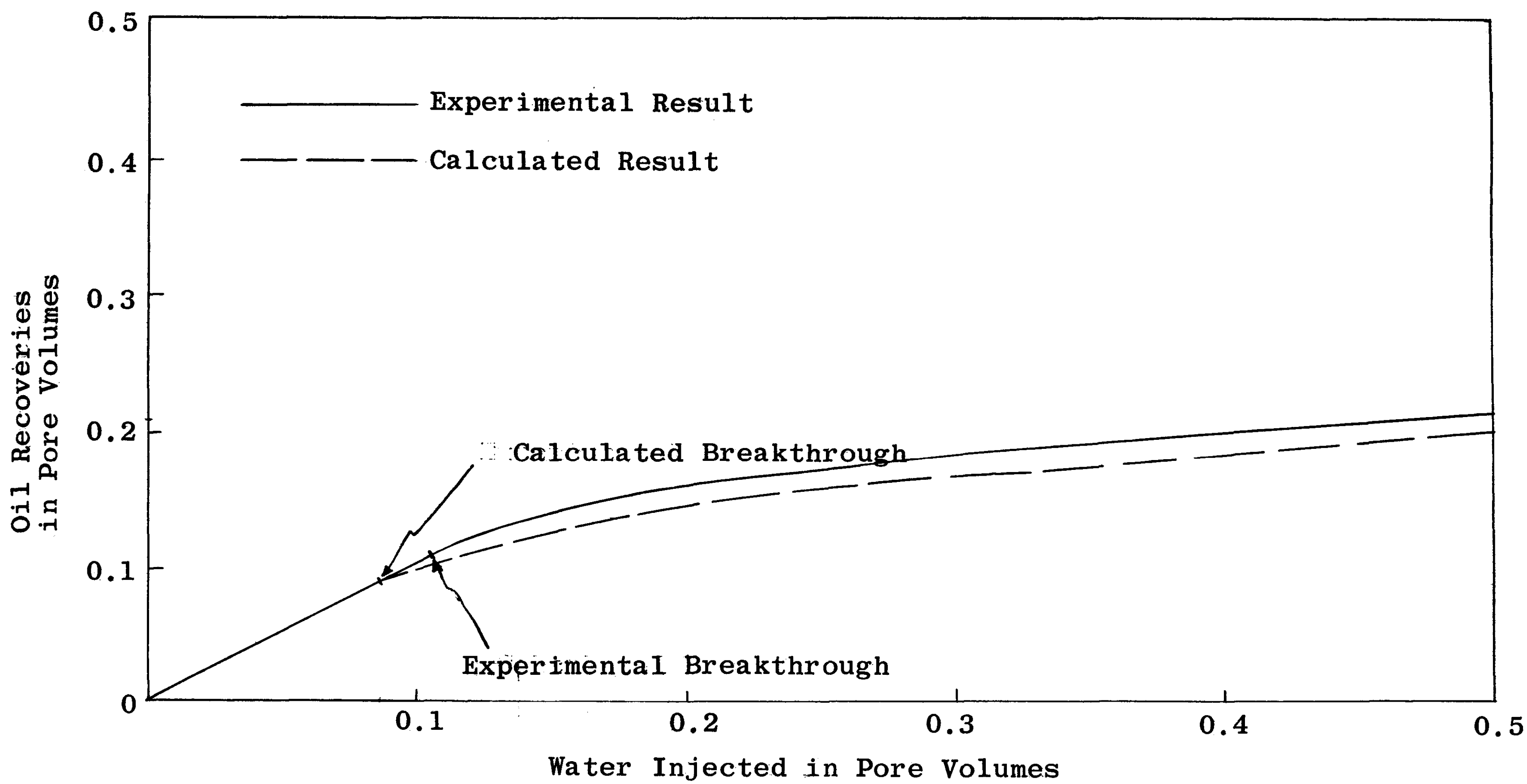


Figure 12 Comparison of Experimental and Calculated Oil Recoveries for Mobility Ratio = 4.10 at Breakthrough.

SUMMARY AND CONCLUSIONS

1. A model which represented a quadrant of a five-spot pattern was used to study the areal sweep efficiency with mobility ratios from 0.83 to 4.10.
2. In the experimental data, errors might exist due to end effects caused in preferentially oil-wet model sands.
3. In this study, Higgins and Leighton's method failed to predict the areal sweep efficiency within six-per cent error for mobility ratios beyond 1.5.
4. Higgins and Leighton's method is an excellent means to approximate an oil recovery efficiency.
5. The unit displacement efficiency suggested by frontal advance equation is lower than that of actual unit displacement efficiency in a five-spot high-mobility-ratio water-flood. Further study on the frontal advance equation and on stream lines as affected by mobility change is recommended.

BIBLIOGRAPHY

Aronofsky, J.S., and Ramey, H.J., Jr., 1956, Mobility ratio -- its influence on injection or production histories in five-spot water-flood: Am. Inst. Mining Metall. Petroleum Engineers Trans., v. 207, p. 205-210.

Buckley, S.E., and Leverett, M.C., 1942, Mechanism of fluid displacement in sands: Am. Inst. Mining Metall. Petroleum Engineers Trans., v. 146, p. 107-116.

Douglas, J., Jr., Peaceman, D.W., and Rachford, H.H., 1959, A method for calculating multi-dimensional immiscible displacement: Am. Inst. Mining Metall. Petroleum Engineers Trans., v. 216, p. 297-308.

Higgins, R.V., and Leighton, A.J., 1960, Waterflood performance in stratified reservoirs -- recovery as influenced by relative permeability curves and by continuously changing saturation: U.S. Bur. Mines Rept. Inv. 5618.

__________ 1961, Performance of five-spot water floods in stratified reservoirs using stream lines: Rocky Mountain Regional Meeting of SPE in Farmington, New Mexico, May 25 to May 26, 1961, Paper SPE-57.

__________ 1962, A computer method to calculate two-phase flow in any irregularly bounded porous medium: Jour. Petroleum Technology, June, p. 679-683.

__________ 1963, Principles and computer techniques for calculating performance of a five-spot waterflood -- two-phase flow: U.S. Bur. Mines Rept. Inv. 6305.

Harley, R.K., and Meabon, H.P., 1962, Digital computer program to predict water-flood performance: Marathon Oil Co., Denver Research Center. (Private communication).

Johnson, E.F., Bossler, D.P., and Naumann, V.O., 1959, Calculation of relative permeability from displacement experiments: Am. Inst. Mining Metall. Petroleum Engineers Trans., v. 216, p. 370-372.

Leverett, M.C., 1941, Capillary behavior in porous solids: Am. Inst., Mining Metall. Petroleum Engineers Trans., v. 142, p. 152-169.

Rapoport, L.A., and Leas, W.J., 1953, Properties of linear water floods: Am. Inst. Mining Metall. Petroleum Engineers Trans., v. 198, p. 139-148.

Welge, H.J., 1952, Simplified method for computing recoveries by gas or water drive: Am. Inst. Mining Metall. Petroleum Engineers Trans., v. 195, p. 91-98.