A STUDY OF LARGE ELECTRIC POTENTIAL GRADIENTS ASSOCIATED WITH GEOMAGNETIC STORMS

BY
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ABSTRACT

The earth's magnetic field is subject to perturbations caused by interaction of the field with the solar wind. The more severe of these perturbations include geomagnetic storms which are accompanied by telluric-current storms, caused by induction.

Records of orthogonal components of the horizontal component of the earth's electric field are used to estimate extreme values of the amplitude of the field. The records were taken in the interval September 1969 to June 1971 at Athol, Idaho; Long Beach, Wash.; and Boulder City, Nevada.

Estimates obtained indicate that protection of power transmission systems against potential gradients of 10 to 100 volts/kilometer should be adequate insurance against catastrophic interference.

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INTRODUCTION

Although the electric field of the earth has been the subject of considerable speculation since Oersted's discovery of the relationship between electric currents and magnetic fields, relatively few measurements of the earth's electric field have been made, and its origin has been understood for less than a century.

It is now known that the earth's crust has at most only a very small, constant electric field. It is true that some localized electric fields, which derive from electrochemical processes, exist in the crust, but the only appreciable large-scale, time-varying, electric fields are induced by fluctuations in the geomagnetic field. The geomagnetic fluctuations of interest here have durations of a few hours or less, and are caused by interaction of the geomagnetic field with the solar wind.

The largest geomagnetic variations of this type occur during geomagnetic storms and have amplitudes with magnitudes of a few per cent of the main field. Because of its dependence on the solar wind, geomagnetic activity of this type has a strong cyclical variation, with a period of about 11 years, which derives from the 11-year cycle of sunspot activity. Increased sunspot activity is accompanied by increased geomagnetic activity. However, the increase is only in the average activity, and it is not possible to say that the greatest magnetic storms occur only during periods of peak sunspot activity. In Figure 1, great magnetic storms which have occurred in the past are indicated on a graph of the Wolf index of sunspot activity, R[1]. The same storms are listed in Table 1.

Of the 21 storms listed in Table 1, only 4 or 5 (1784, 1822, 1903, 1921 and perhaps 1909) occurred at times when the Wolf index was less than half its value at the preceding or subsequent peak. Two of those (1784 and 1833) are assumed to be great storms only on the basis of Poey's report that aurora were seen in Havana on those dates (Loomis et al, 1859). Thus, although great storms do not occur exclusively during periods of high sunspot activity, they evidently have a strong tendency to occur then. One could test, statistically, the hypothesis that storms are equally likely to occur during any part of the cycle, but because of the difficulty in

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1 R is defined by
$R=K(10g + s)$ where
g is the number of sunspot groups,
s is the number of observable individual spots,
K is a correction factor depending upon the observer and his equipment.

selecting a criterion[2] for great storms, it does not seem worthwhile to do so.

Associated with a geomagnetic storm is a telluric current storm, as it has historically been called. The relatively large amplitudes which the earth's electric field may attain during a telluric current storm pose a potential threat to systems of extended, grounded conductors. Although such systems have experienced only minor difficulties in the past, there are several reasons for believing that a catastrophic outage of such systems could be caused by a large telluric current storm.

Perhaps the most obvious reason for concern is that modern systems of grounded conductors tend to be electrically continuous over distances approaching continental dimensions. Large potentials can and have occurred on such systems as a result of natural electric fields of a few volts/kilometer. A few tens of volts/kilometer might be intolerable. High-voltage direct-current power transmission systems might be particularly susceptible to disturbance by telluric current storms because they utilize grounded electrodes to maintain the current balance in the system.

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2 The criterion used in selecting the storms in Table 1 was that a Kp value of 9_0 was attained or that the range in at least one component was greater than 1000 at a mid-latitude station. The storms before 1872 were selected simply because they seemed to be exceptional.

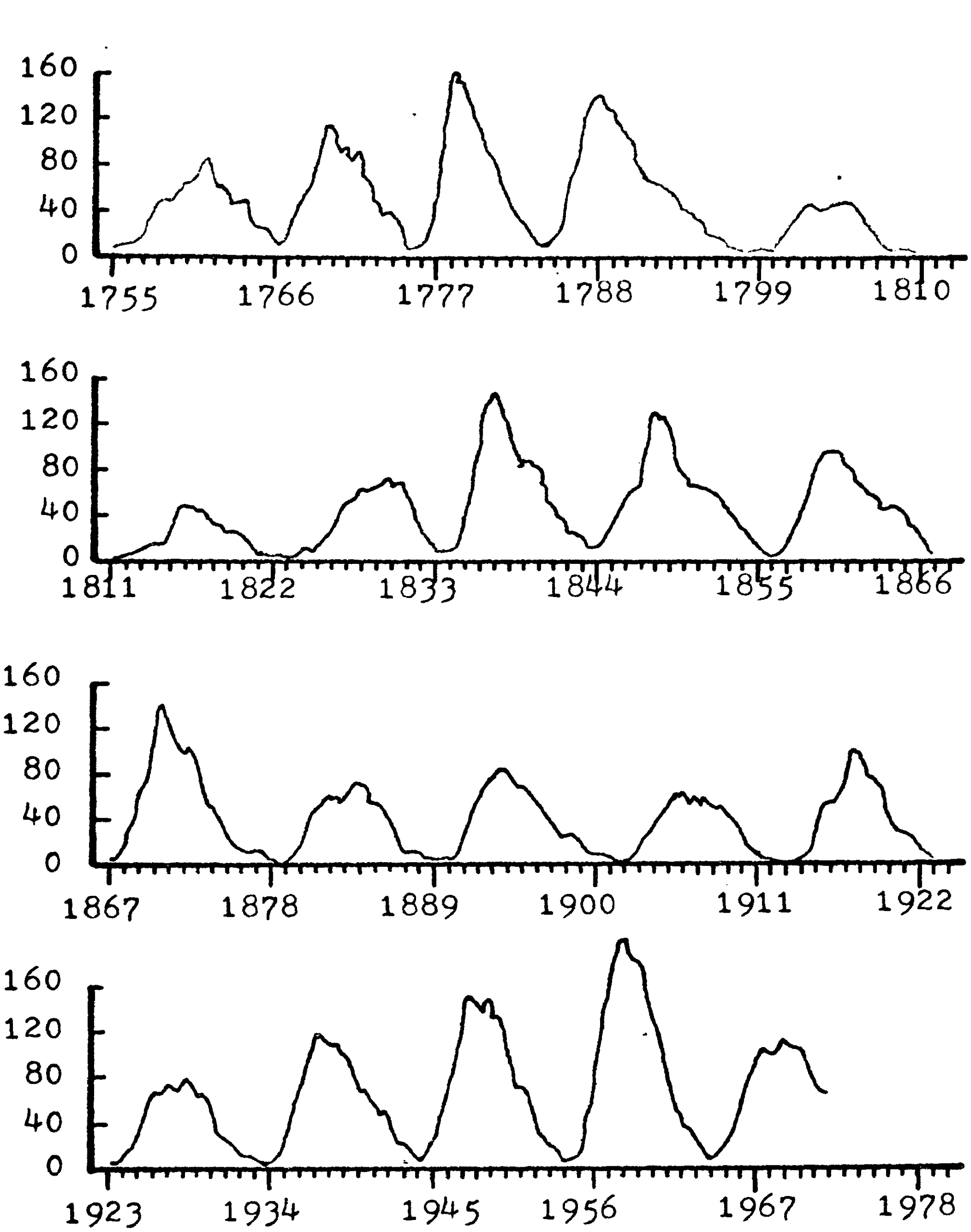


Figure 1. Smoothed sunspot numbers (after World Data Center A, 1972). Great magnetic storms are listed in Table 1.

Date	Ranges			Station	Remarks
	Declination	Horizontal intensity	Vertical intensity		
1784, Nov. 13	?	?	?	Havana	Aurora seen in Havana (Loomis et al, 1859)
1789, Nov. 14	?	?	?	Havana	Aurora seen in Havana (Ibid)
1833	?	?	?	Havana	Aurora seen in Havana (Ibid)
1848, Nov. 17	?	?	?	Havana	Aurora seen in Havana (Ibid) Telegraph disturbances in England. (Walker, 1861)
1859, Aug. 28-Sept. 7	140'	4% 700 γ (?)	1% 400 γ (?)	Kew, England	Photographically recorded (Stewart, 1861)
1872, Feb. 4	?	960 γ	?	Bombay, India	Aurora seen at Bombay (Chapman and Bartels, 1940)
1882, Nov. 17-21	115'	1090 γ	1060 γ	Greenwich England	(Ibid)
1903, Oct. 31-Nov. 1	186'	950 γ	950 γ	Potsdam Germany	(Ibid)

Table I. Great Magnetic Storms

Date	Ranges			Station	Remarks
	Declin-ation	Horizontal intensity	Vertical intensity		
1909, Sept. 25	210'	1500 γ	1100 γ	Potsdam Germany	(Ibid)
	68'	800 γ	235 γ	San Juan Puerto Rico	(McNish, 1940)
1921, May 13-16	199'	1060 γ	1100 γ	Potsdam Germany	(Chapman and Bartels, 1940)
	120'	800 γ	1000 γ	Cheltenham Maryland	(McNish, 1940)
	96'	1100 γ	453 γ	Watheroo Australia	(Ibid)
1938, Apr. 16	328'	1900 γ	600 γ	Potsdam Germany	(Chapman and Bartels, 1940)
	25'	1320 γ	118 γ	Huancayo Peru	(McNish, 1940)
	291'	1120 γ	1020 γ	Cheltenham Maryland	(Ibid)
1940, Mar. 24-25	135'	2300 γ	900 γ	Potsdam Germany	(Ibid)
	137'	850 γ	1100 γ	Cheltenham Maryland	
	21'	1390 γ	122 γ	Huancayo Peru	
1941, Mar. 1-7	?	1367 γ	761 γ	Cheltenham Maryland	3 occurrences of $K = 9_o$
					(Journal of Terrestrial Magnetism and Atmospheric Electricity, 1941)

Table I Continued

Date	Ranges			Station	Remarks
	Declination	Horizontal intensity	Vertical intensity		
1946, Feb. 7-8	?	?	?	Cheltenham Maryland	$K = 9_o$ from 0900 to 1200 Feb 7^o (Journal of Terrestrial Magnetism and Atmospheric Electricity, 1946)
1946, Mar. 23-29	?	2160 γ	1080 γ	Cheltenham Maryland	(Ibid)
1946, July 26-27	?	1500 γ	1130 γ	Cheltenham Maryland	(Ibid)
1946, Sept. 21 -24	?	1150 γ	530 γ	Cheltenham Maryland	(Ibid)
1957, Sept. 4	?	?	?		$K_p = 9_o$ from 1500 to 1800 UT (Bartels, 1961)
1958, Feb. 10-11	?	1000 γ	?	Fredricksburg, Va.	$K_p=9_o$ 0000 to 0300 UT (Bartels, 1961) Large earth-potentials recorded on East coast. (Winckler et al, 1959)
1958, July 8					$K_p=9_o$ 1500 to 1800 UT (Bartels, 1961)
1970, Mar. 8	?	500 γ 5000 γ	?	Fredricksburg, Va. College Alaska	$K_p=9_o$ 1800 to 2100 UT (Kawasaki et al, 1971)

Table I Continued

Another reason for concern is that systems of extended conductors are relatively recent developments. The first such system, the electric telegraph, has been in use for less than 150 years. In that time, there have been only about 20 great magnetic storms, so that such systems have not been exposed to large electric fields of this type very many times. Furthermore, systematic observations of the geomagnetic variation field have been made only since 1836, and continuous photographic records only since 1857, so that there is little additional information on large electric fields in the earth to be inferred from geomagnetic records of great magnetic storms.

The aim of the study described here was to arrive at plausible estimates of the largest electric field levels which might occur in the near future, i.e., up to 100 years in the future. Because this study was done during a time interval when the 11-year cycle was near its peak, the estimates obtained here are probably biased toward high values. That is, electric field values estimated to occur once every 10 years are more likely to have a return-period of 20 to 30 years and so forth.

Historical Observations and Theories of Earth-Currents

Although the phenomenon of static electricity was apparently known to the ancient Greeks, the continuous current was apparently unknown until Volta's invention of the voltaic pile. Following his report of his discovery to the Royal Society of London in 1800, many cells were constructed and used in various experiments, and in 1820 A.D. Oersted described his discovery of the relationship between electrical and magnetic phenomena.

Among the researchers who, motivated by Oersted's work, began to experiment with electrical currents was Sir Humphrey Davey. In 1821, Davey, describing his experimental results, proposed the hypothesis that currents flowing in the earth might give rise to the geomagnetic field.

The first attempts to detect earth-currents were apparently made by R. W. Fox in the Cornish mines. In his report to the Royal Society in 1830, Fox noted that he had detected currents in wires attached to mineral veins, and he attributed them to a rather vague thermo-electric mechanism which he believed to become more intense with depth. He concluded that vein structures in the earth might well be capable of carrying currents which cause the geomagnetic field.

In 1831 Barlow described a series of experiments with a scale model earth having latitudinal windings. By making current flow in the windings, he had reproduced the gross features of the geomagnetic field, and he proposed that analogous currents in the real earth were caused by thermoelectric effects.

Faraday reported on his research in electromagnetism in 1832. Judging from his results with conductors moving in the earth's field, he proposed that an electric field, directed from the equator to the poles, should be induced in the earth by virtue of its rotation in its own magnetic field. However, he believed that"...without the conductors or something equivalent to them, it is evident these currents could not exist, as they could not be discharged." He also thought that aurora could be caused by a discharge related to the electric fields which he postulated. The fact that he did not recognize that time variation in the geomagnetic field might produce currents is, at least in part, due to the fact that no systematic observations of either phenomenon had yet been made.

Becquerel demonstrated, as he related to the Academy of Science of Paris in 1844, that the chemical nature of the water in the vicinity of grounded electrodes had a very marked effect on potentials measured between those electrodes. Although his aim was apparently to refute Barlow and Fox's theory, he concluded that if electrochemical effects were mainly responsible for the currents which Fox measured, they were inadequate to account for the geomagnetic field since there was obviously no system of east-west oriented conductors to discharge the currents. Becquerel's paper is significant because it is the first recognition of a problem which is apparently unavoidable by those who wish to study earth-currents, namely potential variations due to electrochemical effects at electrodes.

In 1838 an electromagnetic telegraph was installed between the Paddington and West Drayton stations of the Great Western Railway in England. By 1862 there were 1500 miles of telegraph lines in Great Britain and in the interim the first systematic observations of earth-currents were conducted. Because early telegraph installations employed earth-return, they were very susceptible to interference by natural earth-currents. In fact, the receiving apparatus was redesigned after the large disturbances of 1848 so that operation might continue during periods when the needle would have otherwise been against its stop (Walker, 1861).

Barlow (1849) apparently conducted the first systematic observations of earth currents. He noted that simultaneous deflections were often produced in different lines, that periods of great magnetic disturbance corresponded to periods of great earth-current activity and that aurora very often coincided with periods of large earth-current activity. Barlow concluded only that there was an actual flow of current in the ground and did not propose a theory explaining its cause.

The great magnetic storm of 1859 prompted Walker (1861) to instigate a series of semi-quantitative observations of earth-current disturbances in English telegraph lines during periods of large earth-current activity. Walker notes that the 11-year cycle of earth-current activity was known at the time of this work and also that earth-currents "...go with magnetic disturbance." His observations are remarkable only because he presented the first set of data which could be reduced to quantitative electric field values---provided that the potential of his calibration cell of amalgamated zinc and platinized graphite with "...1 sulphuric acid + 10 water" could be deduced. Walker proposed rather tentatively that magnetic disturbances might be due to earth-currents.

Stewart (1861) in his report on observations at Kew Observatory of the magnetic storm of 1859, noted that a bright eruption had been observed in a sunspot group at the time of the commencement of the storm, and he attributed the storm to that disturbance on the sun. He supposed that earth-currents and aurora were both induction phenomena and that the upper atmosphere was a good conductor of electricity. Stewart's ideas, in contrast to those of many subsequent investigators, are remarkably close to contemporary theories of magnetospheric disturbances and, to the author's knowledge, he was the first proponent of the concept of earth-currents as induction phenomena.

Charles Matteucci, who first reported telegraph disturbances due to earth-currents in 1847, reported the results of a series of earth-current observations in 1864. Although he used zinc-zinc sulfate half-cells and attempted to control their environment very closely (he went so far as to have an observer maintain the water level in the hole containing the electrode), Matteucci nevertheless found a constant component of current flow. He demonstrated with an artificial current source that stray currents were attenuated too rapidly with distance to be a possible cause of the constant component which he observed. Having observed a tendency for earth-currents to flow uphill, he proposed that atmospheric electricity might be the source of earth currents. He did not advocate a theory of the relationship between earth-currents and geomagnetism, but he stated that his results were not consistent with Faraday's theory of induction.

In 1868, Sir George Airy, the Astronomer Royal of England, reported the results of an experiment, done at Greenwich Observatory, in which magnetic "tendencies" were computed from earth-current records and then compared with magnetic records. Airy stated that the curves were in general agreement with certain exceptions, namely that the current records had more "irregularities" than magnetic records, that current events preceded magnetic ones (in one case by ½ hour!) and that the sense of the computed north magnetic component was exactly opposite the sense of the observed north component. Airy's conclusion was "...that the magnetic disturbances are produced by terrestrial galvanic currents below the magnets...", but "...they do not explain the existence of the principal part of terrestrial magnetism."

Ellis (1892) confirmed Airy's observation that electric disturbances seem to precede magnetic disturbances but he found no similar relationship for the magnetic diurnal variation. For that reason, and because he believed that only the horizontal magnetic component was correlated with the electric variations during disturbances, he questioned the theory that earth-currents cause magnetic storms.

Although Burbank (1905) recognized that, "In view of the uncertainty of electrochemical action at the earth-plates, any statement of potential difference will have little meaning", in 1922 Bauer published a discussion of earth-currents at Ebro in which potential values were treated exactly like geomagnetic data, that is, no allowance was made for the electrochemical reactions at the electrodes. Consequently he found values of the potential gradient as high as 1/3 volt per kilometer for <u>quiet</u> days during 1914-1918. Chapman and Whitehead (1923) demonstrated that his values for the diurnal variation were about 5 times larger than those predicted from the magnetic diurnal variation.

During the 1920's and 1930's a rather extensive investigation of earth-currents was conducted under the auspices of the Carnegie Institute of Washington. Two earth-current observatories were maintained, one at Watheroo, Western Australia, and the other at Huancayo, Peru. (Lest the reader take exception to the use of the word 'extensive' in reference to two observatories, the author wishes to point out that Gish (1931) lists only 6 observatories as having operated prior to 1931. These are listed in Table 2.)

Gish and Rooney (1928) describe the recording system used and note that a multiple-electrode, multiple-record system was used successfully at Watheroo to permit detection of electrochemical potentials. Gish (1931) notes that the diurnal variation is apparently caused by induction but alludes to the "fact" that, during storms, currents apparently cause the magnetic disturbances. He suggests that aurora may provide a source of the earth-currents. Gish's failure to recognize that storm-time earth-currents are caused by induction is at least partially excused by the fact, as Rooney (1932) says, that "So far most of the study has been restricted to the diurnal variation, partly because the intermittent type of record given by the recording potentiometers at Watheroo and Huancayo is not too well adapted to the study of storms and partly because the diurnal variation is more readily treated statistically."

The great magnetic storm of 1940 demonstrated that earth-current disturbances could disrupt power distribution systems. Davidson (1940) reported that 10 distribution systems in New York, New England, eastern Pennsylvania, Minnesota, Ontario and Quebec, reported disturbances. These disturbances consisted of

"1. Voltage dips ranging up to 10 per cent but generally of short duration (7 cases).
2. Tripping transformer banks by differential relay operation (5 cases involving 15 transformer banks).
3. Large increases or swings in reactive kva (4 cases)."

Germaine (1940) reproduces a record of earth-potential between Minneapolis and Fargo which indicates a maximum excursion of about 800 volts peak-to-peak in about 10 minutes. Winckler et al (1950) report that a peak-to-peak potential fluctuation of about 2550 volts was observed on a section of the Atlantic cable between Clarenville and Sydney Mines, Nova Scotia, during the great storm of February 10-11, 1958.

Although there is evidence that large potentials do occur on systems of grounded conductors during telluric current storms, and that they may interfere with the operation of some systems, there has apparently been no work done on estimation of the magnitude of rare electric field levels in the earth. That is the subject of this paper.

Station	Period of Operation
Greenwich Observatory, England	1865-1867
Berlin, Germany	1883-1891
Parc St. Maur, France	1892-?
Ebro, Spain	1910 thru 1931
Watheroo, Western Australia	1924 thru 1931
Huancayo, Peru	1924 thru 1931

Table II. Telluric "current" observatories in operation before 1931 (after Gish, 1931).

DATA COLLECTION

The data used in this study were collected at 3 stations in the western U.S. in the period september 1969 to June 1971. The stations were at Long Beach, Washington; Athol, Idaho; and Boulder City, Nevada. They were unattended except for monthly record changes and unscheduled trouble-shooting. They were part of an array of seven recording stations. The other 4 stations were at Centerville, Washington; Malin, Oregon; Reno, Nevada; and Phoenix, Arizona.

Data from the latter 4 were not used because too few of the records were representative of the true electric field activity. The reasons were usually either insufficient sensitivity or noise. The data used are equivalent to about 12 months of recording for Athol, 9 months for Long Beach, and 7 months for Boulder City. The number of records involved is about 30% of the total number obtained from all stations.

The impetus for this study was provided by the establishment of a high-voltage, direct-current (HVDC), power-transmission line between south-central Oregon and Los Angeles. Because large electric fields were of primary interest, no particular pains were taken to obtain data suitable for spectral estimation. Many sample spectra of the geomagnetic field have been published (Herron, 1967), and they are in good agreement with respect to the general features of the spectrum. The spectra of the total field and the horizontal component of the geomagnetic field are relatively insensitive to the conductivity of the earth, and, therefore, the published spectra are good approximations to the spectra of the field at other points with similar dipole latitudes.

The spectrum of the horizontal component of the electrical field is, however, quite sensitive to the conductivity of the earth. Therefore, the magnitudes of rare, large, electric-field events will depend upon the resistivity structure of the area of interest as well as the magnitude and spectrum of the inducing field.

Recording System

The recording systems at the three sites were very similar. The typical recording system consisted of an electrode array, a two-channel signal conditioner, and a chart recorder.

The electrode array consisted of 3 lead electrodes buried at the vertices of an isoceles right triangle. The triangle was situated so that the perpendicular legs were along the north-south and east-west directions. The equal legs were 1000 feet long at Athol and Boulder City, and 710 feet long at Long Beach. The electrode at the vertex of the right angle served as electrical common, and the potentials between it and the other two electrodes were the inputs to the signal conditioning unit.

The typical signal conditioning unit consisted of two channels, each of which was composed of a twin-T, 60-Hertz, passive, notch filter at the input, a bias circuit following the notch filter, then a high-input-impedance (10 megohms) operational amplifier, a low-pass, active filter, and finally a voltage divider at the output. The low-pass filter used initially had a cut-off frequency of 5 Hz. It was later replaced by one with a cut-off frequency of 1 Hz. Figure 3 is a schematic diagram of the unit.

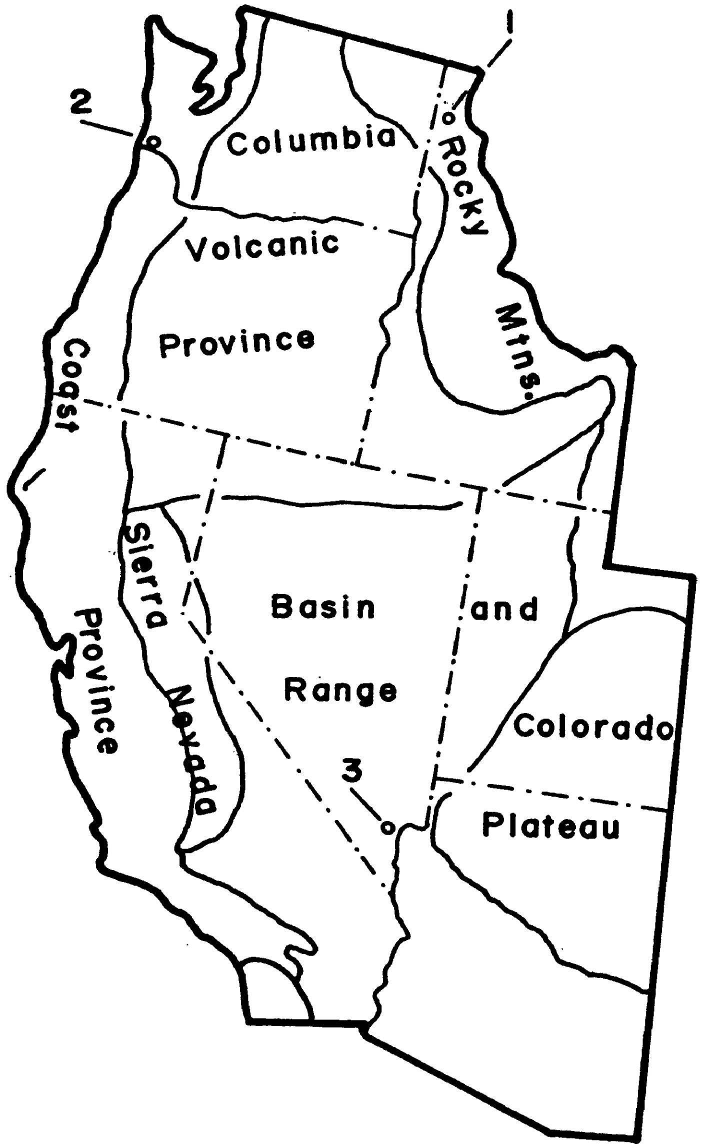
The chart recorder was an Esterline-Angus model E1124E multipoint recorder. Input voltages to this type recorder are indicated on the chart by a point stamped by an inked printhead. As used in this study, the recorder stamped one point every 30 seconds. Because the input channels were sampled sequentially, each channel was sampled once per minute. A chart speed of 2 inches per hour was used.

Geological Environment

A description of the recording system used in a study such as this must include the geological environments of the recording stations. Unfortunately, the relevant geological parameters are much less well known than the characteristics of the rest of the system, and, therefore, their effects on the recorded potentials can only be roughly estimated.

Site	Geographic Latitude	Geographic Longitude	Geomagnetic Dipole Lat.	Geomagnetic Dipole Long.
Athol, Idaho	48.0 N	116.6 W	55.2 N	36.3 W
Boulder City Nevada	36.0 N	114.8 W	43.7 N	53.3 W
Long Beach, Washington	46.3 N	124.0 W	52.0 N	66.8 W

Table III. Geographic and Geomagnetic coordinates of the stations used in this study.



1. Athol, Ida.
2. Long Beach, Wash.
3. Boulder City, Nev.

Figure 2. Physiographic map of the western U.S. with the recording stations used in this study indicated.

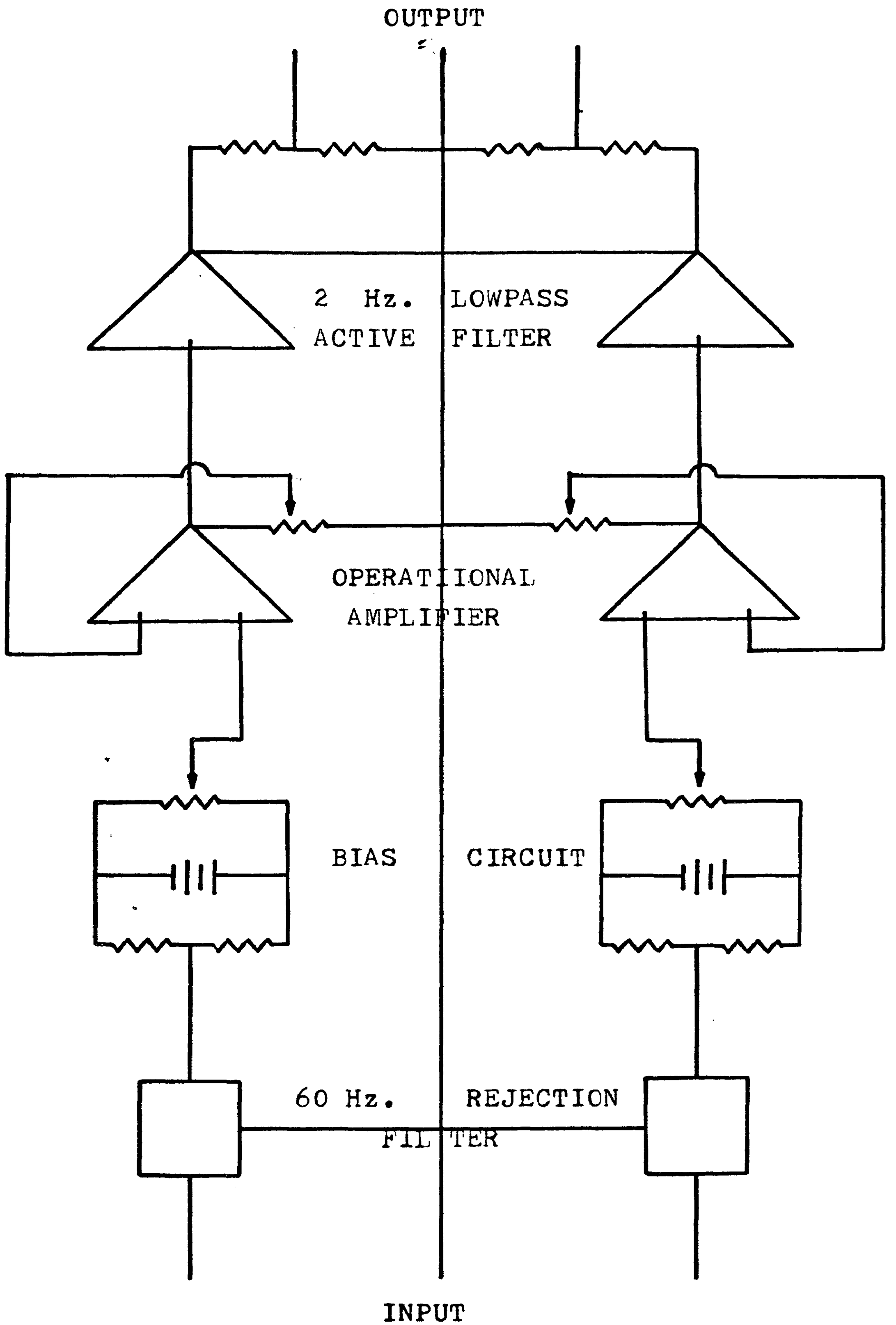


Figure 3. Schematic diagram of the signal-conditioning unit

The geological environment of a given station is conveniently described in 2 parts, namely the features which have surface expression, called here the local characteristics, and the deeper features, inferred from geophysical measurements, called here the regional characteristics. This usage is somewhat at odds with convention, but is justifiable here on the grounds that surface features of regional extent are not considered except insofar as they define physiographic regions.

Local Characteristics

Athol, Idaho. Athol is located on the Rathdrum Prairie, at the southeast end of Lake Pend Orielle. Rathdrum Prairie is bounded on the east by the Coeur d'Alene mountains and on the north, south and west by parts of the Selkirk range (Kinnison, 1955). It lies on the Purcell trench, which extends from the vicinity of Spokane to about 200 miles north of the U.S.-Canada border, and is probably related to the Rocky Mountain trench to the east. The surrounding rocks are low-grade metamorphics of the Belt series (Precambrian) and acidic intrusives, (late Cretaceous) probably associated with the Idaho batholith. The basement is overlain at Athol by about 500 feet of poorly sorted glacial outwash and/or ground moraine. This cover thins to the southwest, and is about 200 feet thick at the Washington state line.

Long Beach, Washington. The site at Long Beach lies on the southern part of the long narrow spit on the seaward side of Willapa Bay. According to Etherington (1931), the Willapa-Grays Harbor basin contains Tertiary sediments, overlying a series of basaltic and andesitic flows which form the higher portions of the Wahkiakum hills in eastern Pacific County. Etherington (1931) indicates a total thickness of sediments of about 9000 feet in the Chehalis valley, but he states that at Cape Flattery the Oligocene member of the section alone is 19,000 feet thick, According to Keller (1966), Tertiary sediments range in resistivity from several tens to several hundreds of ohm-meters. Typical basement rocks have resistivities ranging from several thousands to several tens of thousands of ohm-m.

The influence of the ocean probably is far greater than that of the sediments for the relatively high frequency events. Schmucker (1964) noted apparent ocean effects as far inland as the California-Nevada border for bay-type events, so that ocean effects are, no doubt, large for these events at Long Beach. The characteristic east-west polarization is probably an ocean effect.

Boulder City, Nevada. Boulder City lies at the north end of a relatively narrow trough-like valley which extends over 50 miles southward into California and which Callaghan (1938) calls Piute Valley. To the east, this valley is bounded by the Black mountains, which according to Longwell (1936) are mostly lava flows, intrusives and related deposits. Further south, near Searchlight, Callaghan (1938) describes them as mostly Precambrian gneiss overlain by several series of Tertiary lava flows. To the north and west, the valley is probably bounded by fault blocks. Jackson (1966) reports that the electrical basement is at a depth of about 1 km and has a resistivity no less than 1000 ohm-m.

Regional Characteristics

As indicated in Figure 2, the Boulder City site lies in the Basin and Range Province; the Long Beach site lies in the Pacific Coast Province and the Athol site lies in the Northern Rocky Mountain Province.

A characteristic which the Basin and Range and the Northern Rocky Mountain provinces have in common is relatively high terrestrial heat flow. At least part of the anomalous heat flow is caused by heat sources in the lower crust and upper mantle.

Roy et al (1968) discovered that heat flow and heat production in plutons, are related by

$$Q=a + bA$$

where Q is the heat flow in HFU (1HFU=10^{-6} cal./cm.^{2}-sec.), A is the heat production (in units of 10^{-13} cal./cm.^{3}-sec.) of surface rocks as inferred from their contents of uranium, thorium and potassium, a is the amount of heat flow contributed by rocks in the lower crust and upper mantle, and b is a scale height. Furthermore, they discovered that heat-flow provinces can be defined in terms of a and b.

The Basin and Range and the Northern Rocky Mountains both belong to that heat-flow province for which a is greater than 1.0 HFU. In particular, Blackwell (1971) and Roy et al (1971) give a value of 1.4 HFU for the Basin and Range. The generally accepted value of a for the eastern U.S. is 0.8 HFU.

The few data available for the northern Pacific Coast Province indicate that the total heat flow, and, therefore, the reduced heat flow (a), are low. Sass et al (1971) give a value of 0.83 HFU for the corrected total heat flow at Chehalis, Washington.

Seismic evidence indicates that the Basin and Range and Northern Rocky Mountains are characterized by a relatively thin crust and low upper-mantle velocities (Hales and Herrin, 1971; Pakiser and Zietz, 1965). Hales and Herrin also state that the travel-time residual for S-waves is unusually large relative to the P travel-time residual, and they argue that a partial-melt zone in the upper mantle is indicated. The crust is, evidently, thicker in the pacific Coastal Province. According to White et al (1968), the thickness of the crust exceeds 50 kilometers at Vancouver, B.C.

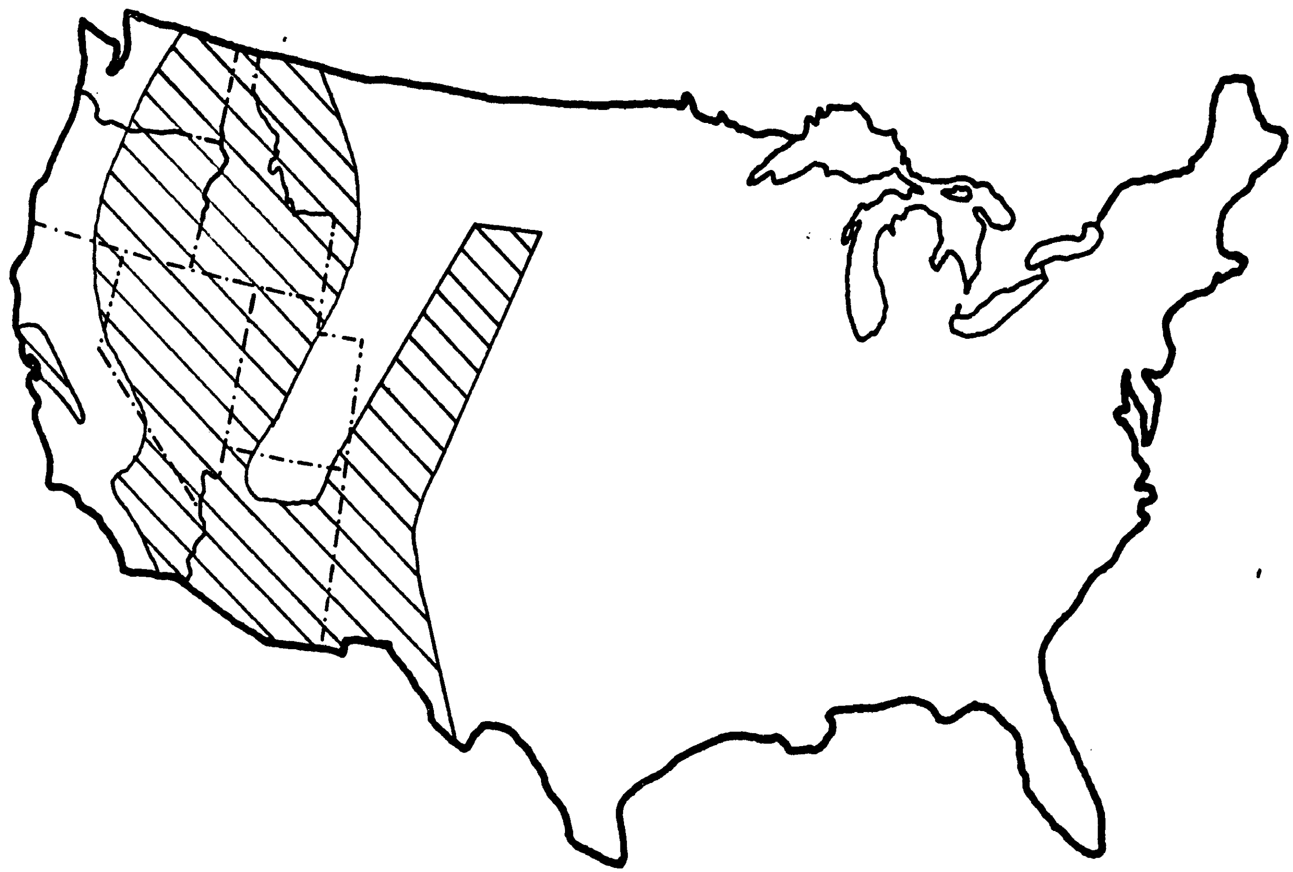


Figure 4. A map of the U.S. with the area of high reduced heat flow cross-hatched. (after Roy et al, 1972)

Heat flow and seismic data from the Basin and Range and Northern Rocky Mountains indicate that these regions may be more conductive at depth than the eastern U.S. The Pacfic Coast Province, however, seems to resemble the eastern U.S., both in crustal thickness and heat flow. The resemblance is based on rather sparse data, however, and seems to be coincidental when geologic considerations are taken into account.

Atwater (1970) has proposed that the Pacific coast of North and Central America is a zone of interaction among several crustal plates, the most important ones being the American, the Pacific and the Farallon plates. Initially a subduction zone, where the American plate was overthrusting the Farallon plate, the coastal zone evolved into a transform fault as the Farallon plate was consumed and the Pacific and American plates came together. At present, the transform fault consists of the San Andreas fault zone, and extends from the Gulf of California to north of San Francisco. The subduction zone may be still active in the Pacific Northwest.

Atwater's hypothesis is in agreement with the seismic and heat-flow evidence already discussed. The down-going material of the Farallon plate would cause heat flow at the surface to be low (Toksoz et al, 1971), and the crust would appear to be thick over the down-going slab. The fact that there is no Benioff zone associated with the proposed zone of subduction would seem to rule out the hypothesis, but Atwater suggests that the absence of the Benioff zone may be due to the youth of the Juan de Fuca plate which is being consumed, and which is a remnant of the Farallon plate. Because of its youth, the Juan de Fuca plate is relatively thin and warm, and therefore, would reheat and soften quickly. The down-going plate would also be wet, and, therefore, the conductivity at depth might be lower than in a stable crust of similar thickness.

Numerous magneto-telluric (MT) and geomagnetic depth soundings (GDS) have been conducted in the western U.S. and Canada. Schmucker (1964) did the earliest GDS work in the southwestern U.S. Although it was already known that the vertical component of magnetic disturbances is quite small at Tucson, Schmucker discovered that between Las Cruces and Cornudas, New Mexico, there is a rather abrupt change in the character of the vertical component. To the east of Cornudas the vertical component is typically about 3 times larger than it is at Tucson. Schmucker also found that the vertical component is strongly correlated with what he calls the D component, and which is apparently an east-west, horizontal component, between Las Cruces and Cornudas. A rather abrupt thickening of the resistive portion of the crust and, possibly, the upper mantle is indicated. Schmucker was using bay variations in this work, and those are the longest-duration events considered in this study.

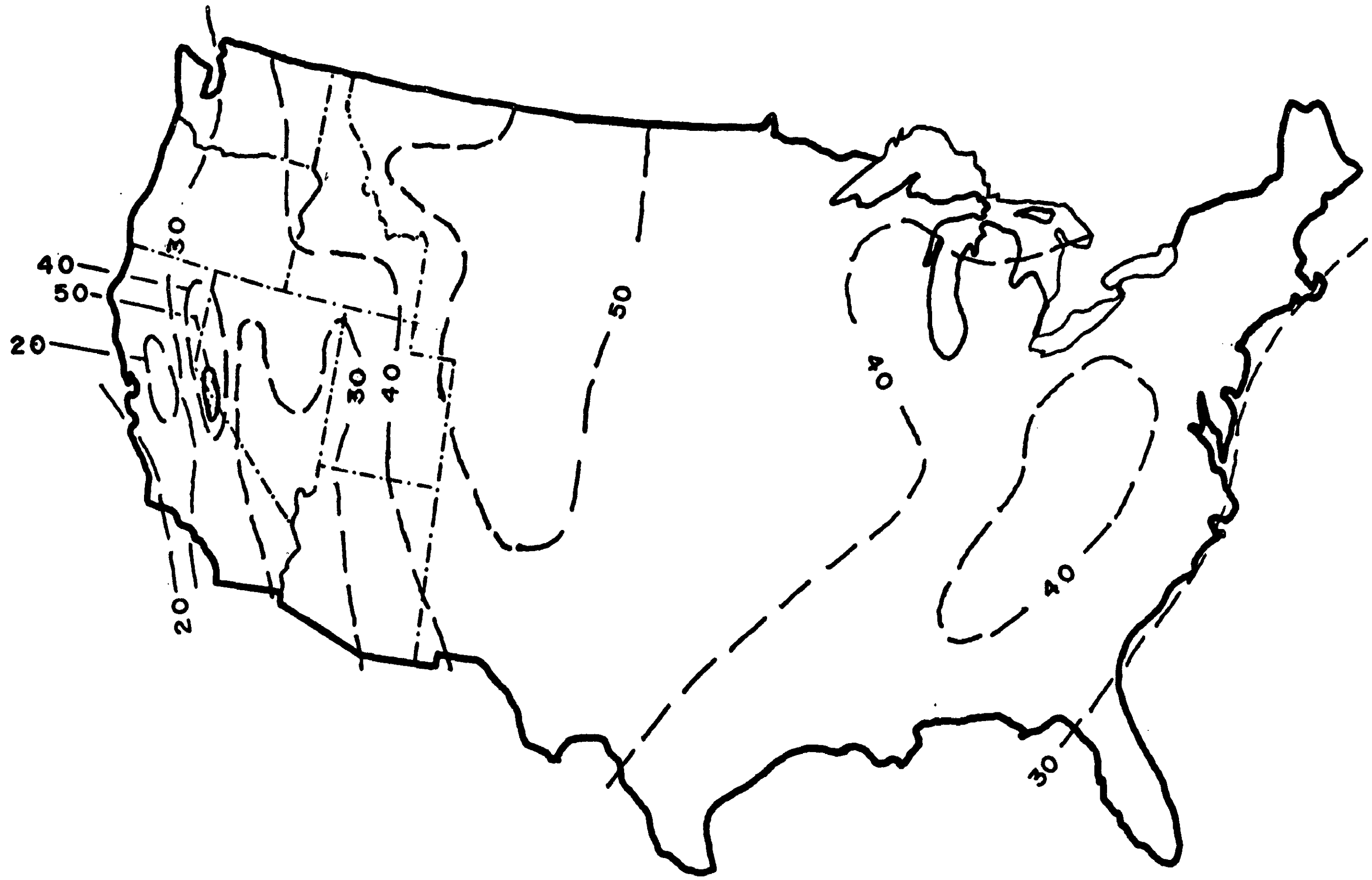


Figure 5. A map of the U.S. with contours on the M-discontinuity.
Indicated depths are in kilometers. (after Pakiser and Zietz, 1965)

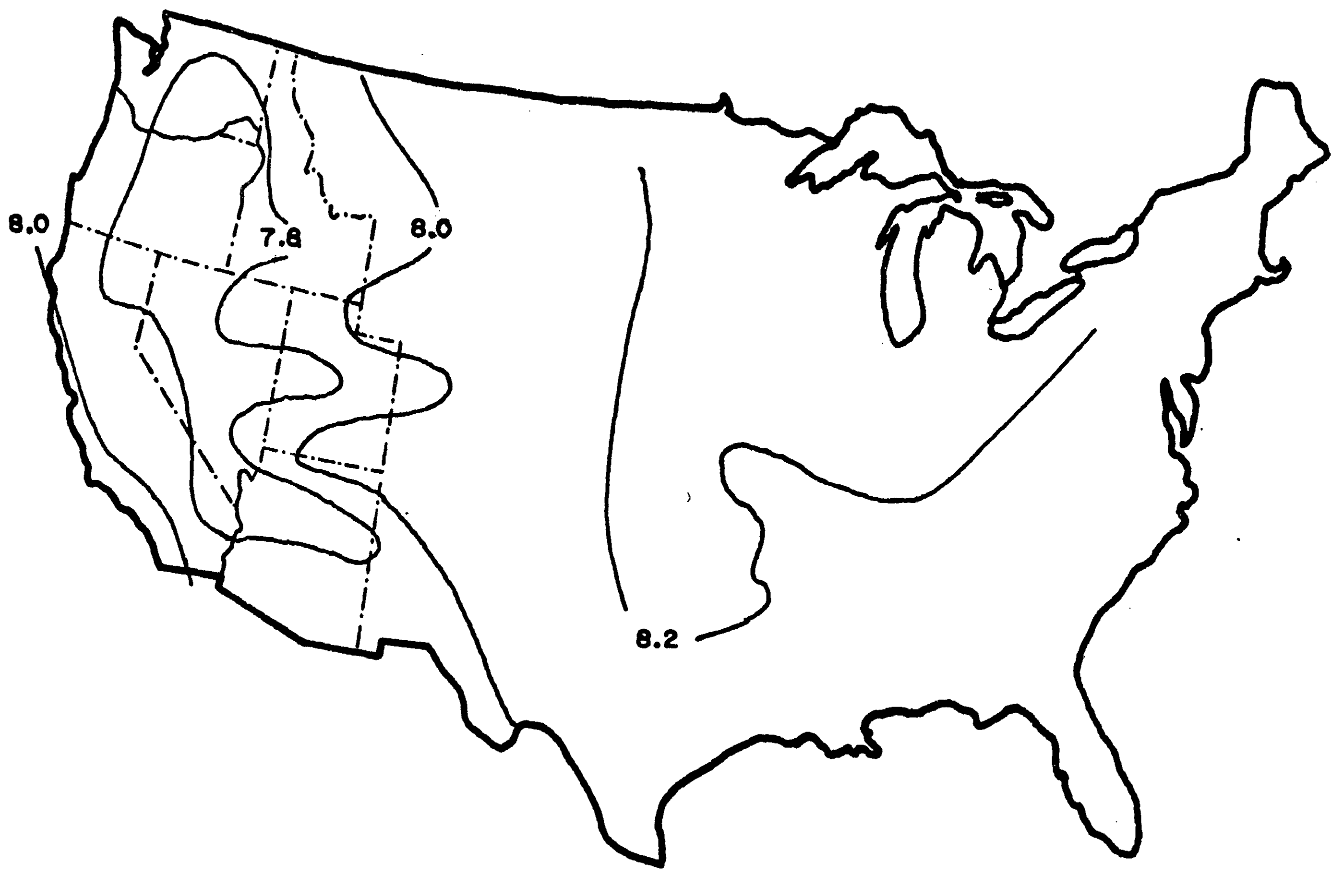


Figure 6. A map of the U.S. with contours of P_n velocity.
Velocities are in kilometers/sec. (after Hales and Herrin, 1972)

In a later paper Schmucker (Schmucker, 1970) indicates that, for 2-dimensional source fields and a homogenous earth, the electric field is related to the vertical component of the magnetic variation by

$$E(w,k) = (w\mu/k)Z(w,k)$$

where

E = the electric field magnitude
Z = the vertical magnetic field component
w = the radian frequency
k = the radian wavenumber
μ = the magnetic permeability.

The point is that an increase in Z like the one described above would be accompanied by a similar increase in E. The situation is somewhat different when k is zero, but the change in thickness of the resistive layer would still change E, as will be demonstrated later.

Caner et al (1967) described the results obtained from three GDS profiles across the western U.S. and Canada. Two of the profiles were across the transition zone, which had been discovered by Schmucker, into the Basin and Range, in the U.S., and one was across the northern extension of that zone in Canada.

In another paper, Caner et al (1969) described a MT (magneto-telluric) survey in western Canada. Their interpretation of this survey calls for different sections to the east and west of a line running roughly along the Alberta-

British Columbia border. The eastern section has a 250 ohm-meter layer, 30 to 35 kilometers thick, overlying a 30 to 50 ohm-meter layer of indefinite thickness. The site at Athol, Idaho lies very near the indicated boundary between the eastern and western sections. The results of this study seem to indicate that the site lies to the east of the boundary.

Caner et al (1969) cite results obtained at Vulcan, Alberta, by Vozoff and Ellis (1966) as support for their eastern section, and in a later paper, Caner (1971) finds support for this model in GDS data.

Porath (1971a) criticizes Caner's model, with its conductive crustal layer, noting that the observed fields are not consistent with fields computed for two-dimensional models. Conductive structures in the upper mantle, on the other hand, as modeled in 2 dimensions, are consistent with the observed fields. He also criticizes the MT interpretation for the eastern section, because he believes that the stations used were too near conductivity inhomogeneities.

Camfield et al (1970) describe the results obtained from four GDS east-west lines lying between 44° and 51° north latitude. Attenuation of the vertical variation field in the Northern Rockies is attributed to a westward rise in the conductive mantle. Spectral amplitude maps for 85 and 102 minute periods indicate that the mantle may rise to the south.

Porath (1971b) finds that a GDS profile from eastern Colorado to eastern Nevada <u>can</u> result from a two-dimensional crust-mantle structure consisting of 4 layers. The first layer is of variable thickness, 160 kilometers under the Great Plains, 45 kilometers under the Basin and Range, with a resistivity of 1000 ohm-meters. The second layer represents a variable-thickness, low-velocity zone. Its resistivity is 2 ohm-meters and its thickness, is 40 kilometers under the Great Plains, 155 kilometers under the Basin and Range. The third layer is 200 kilometers thick and has a resistivity of 100 ohm-meters. The fourth layer has a resistivity of 10 ohm-meters.

Although they differ in detail, the various models proposed for the transition from the Great Plains to the Basin and Range are characterized by a resistive surface layer, thinner in the west, underlain by a conductive layer. If these models are valid, the relative electric field strengths for magnetic bay-type variations can be estimated from the relation

(Wait, 1962)

$$\frac{E_x}{H_y} = \left(\frac{i\omega\mu_0}{\sigma_1}\right)^{1/2} Q$$

(Wait, 1962)

where

E_x = the electric field of a plane-polarized electromagnetic wave at the surface of a 2-layered earth

H_y = the magnetic field associated with E_x

σ_1 = first layer conductivity
$\mu_0 = 4\pi \times 10^{-7}$ Henries/m
$\omega = 2\pi f$ = 2π (linear frequency)
Q = 2-layer surface impedance correction factor

$$= \frac{\gamma_1 + \gamma_2 \tanh(\gamma_1 h_1)}{\gamma_2 + \gamma_1 \tanh(\gamma_1 h_1)}$$

where

$\gamma_1 = (i\omega\mu_0\sigma_1)^{1/2}$
h_1 = first layer thickness
σ_2 = second layer conductivity
$i = \sqrt{-1}$
$\gamma_2 = (i\omega\mu_0\sigma_2)^{1/2}$

For $\sigma_1 \approx 0.01$ mhos/m, $\sigma_2 \approx 1$ mho/m, $f \approx 1/1800$, the following result is obtained for $|\gamma_1|$, $|\gamma_1| = 2\pi \times 10^{-6}$. Thus

$$Q = \frac{(\gamma_1/\gamma_2) + \tanh(\gamma_1 h_1)}{1 + (\gamma_1/\gamma_2)\tanh(\gamma_1 h_1)}$$ becomes

$$Q \cong \frac{(\sigma_1/\sigma_2)^{1/2} + \gamma_1 h_1}{1 + (\sigma_1/\sigma_2)^{1/2}\gamma_1 h_1}$$ for

$h_1 \lessdot 10^5$ m . Now $(\sigma_1/\sigma_2)^{1/2} \doteq 0.1$ so that Q becomes

$Q \cong (\sigma_1/\sigma_2)^{1/2} + \gamma_1 h_1$.

Thus for a resistive and relatively thin first layer over a conductive second layer, the electric field associated with a bay-type magnetic variation will, to first approximation, increase linearly with the thickness of the first layer.

The implication of this result is that the electric fields, associated with bay-type variations, recorded in the present study may be considerably reduced in amplitude compared to otherwise similar events occuring in regions where the lithosphere is thicker.

DATA REDUCTION AND ANALYSIS

Data Reduction

The method of data reduction used in this study was intended to facilitate handling of large amounts of data. It was therefore designed to provide a quick grouping of data on the basis of amplitude classes or, as they will be called henceforth, "amplitude indices".

An amplitude index was assigned to each three hour interval on a given record. The amplitude index (AI) given to an interval is the least integer such that

$$AI \geq \log_{\sqrt{2}} (A) + 1$$

where A is the amplitude in tenths of inches, of the largest event in the interval. The amplitude index, the duration and the character of the largest event were recorded for each 3-hour interval on a record so that month-long records provided samples of between 200 and 240 members. Other samples could be derived from the basic samples by grouping AI's from events of like character, at least for periods in which system gain was unchanged.

The character of an event, as used here, refers to both the frequency content and duration of the event. Because the recording system was not capable of distinguishing frequencies higher than a few tenths of a cycle per minute,

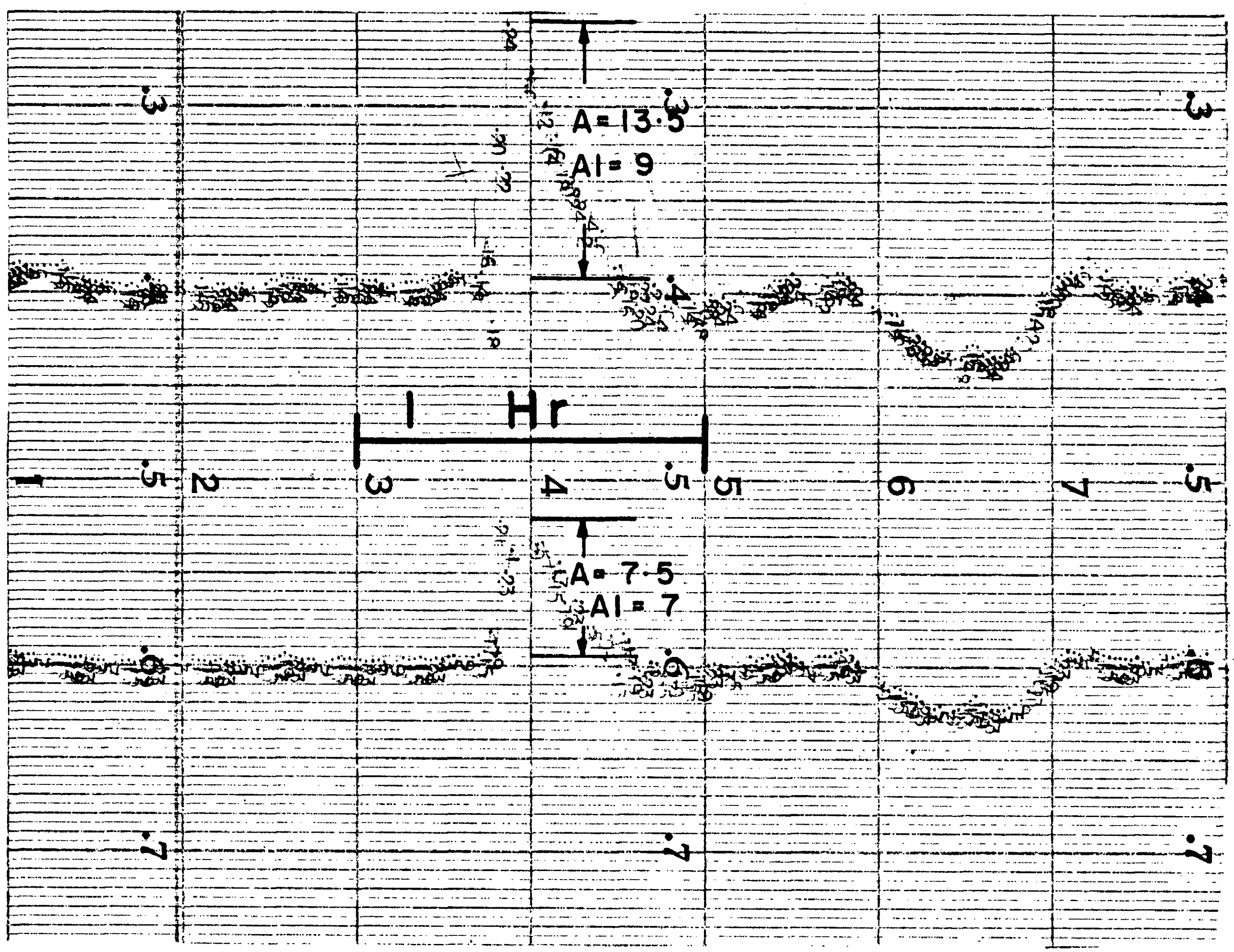


Figure 7a. A segment of an electric field record from
Boulder City, Nevada, 18 Oct. 1969,
Upper: E-W, max. excursion 26 mv/km
Lower: N-S, max. excursion 22 millivolts/kilometer

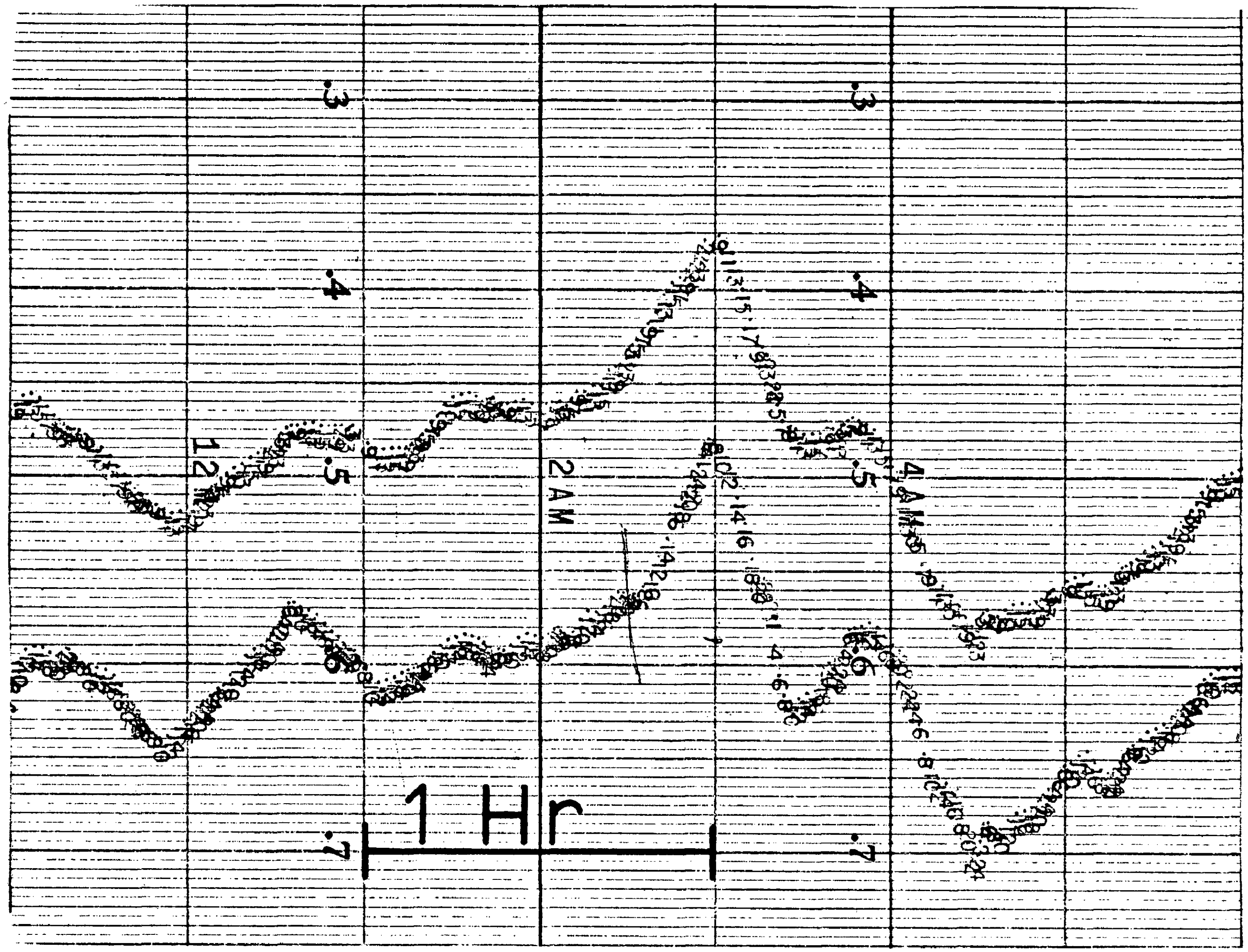


Figure 7b. Electric field record from Boulder City, Nevada, 9 Nov. 1969.
Upper: N-S, max. excursion 57 mv/km
Lower: E-W, max. excursion 57 mv/km

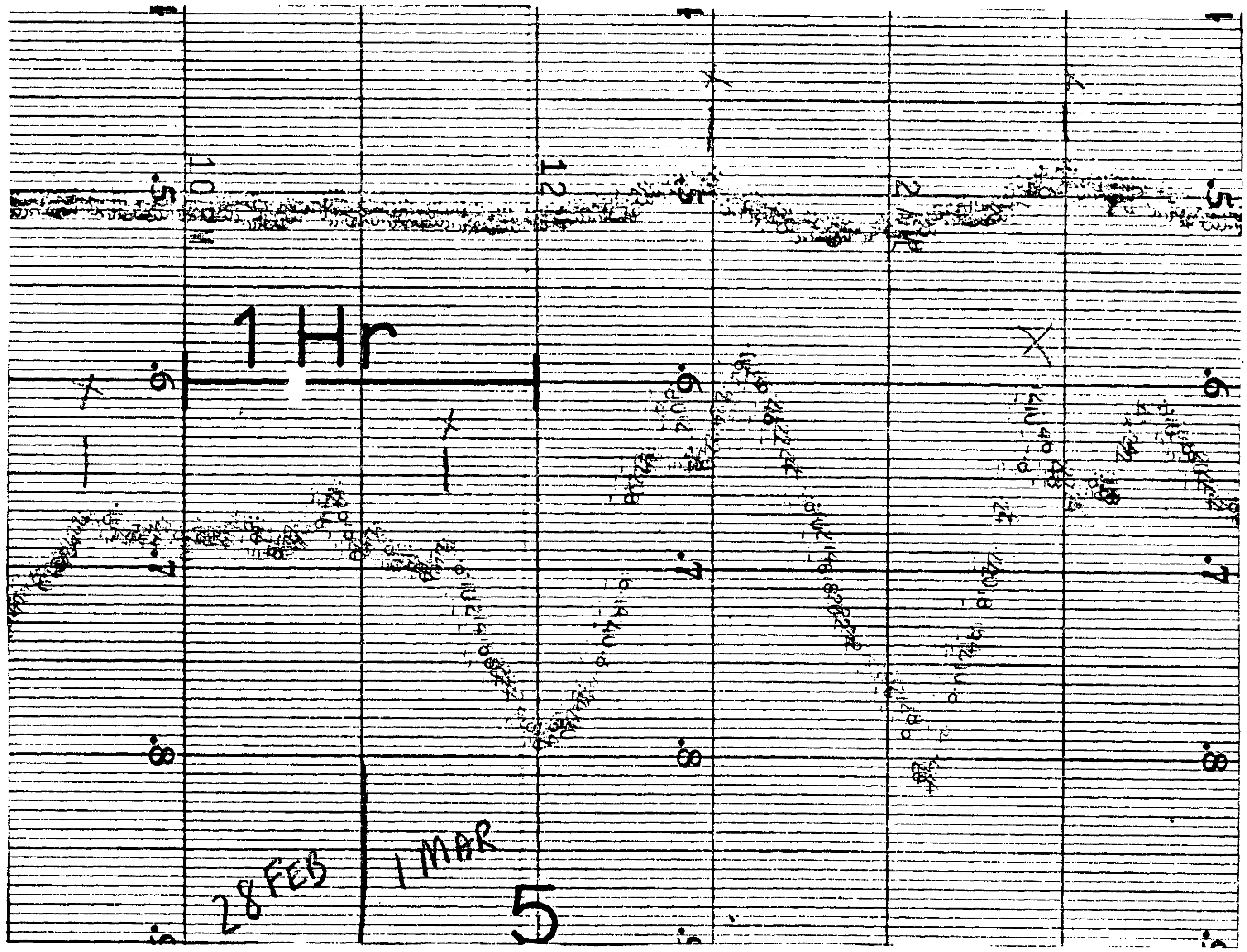


Figure 7c. Electric field record from Long Beach, Wash., 1 March 1970.
Upper: N-S, max. excursion 15 mv/km
Lower: E-W, max. excursion 90 mv/km

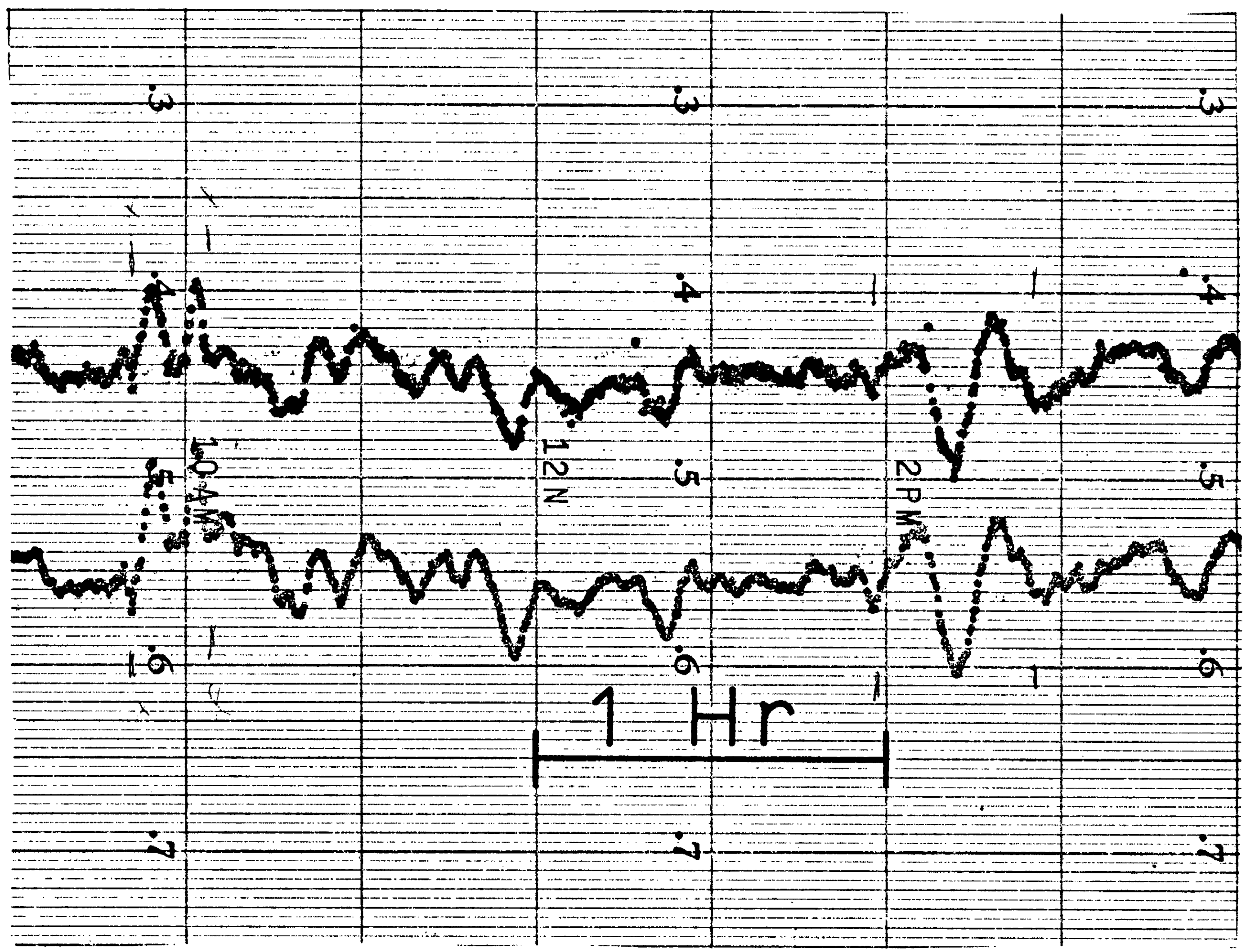


Figure 7d. Electric field record from Athol, Idaho, 18 Nov. 1970.
Upper: N-S max. excursion 100 mv/km
Lower: E-W max. excursion 100 mv/km

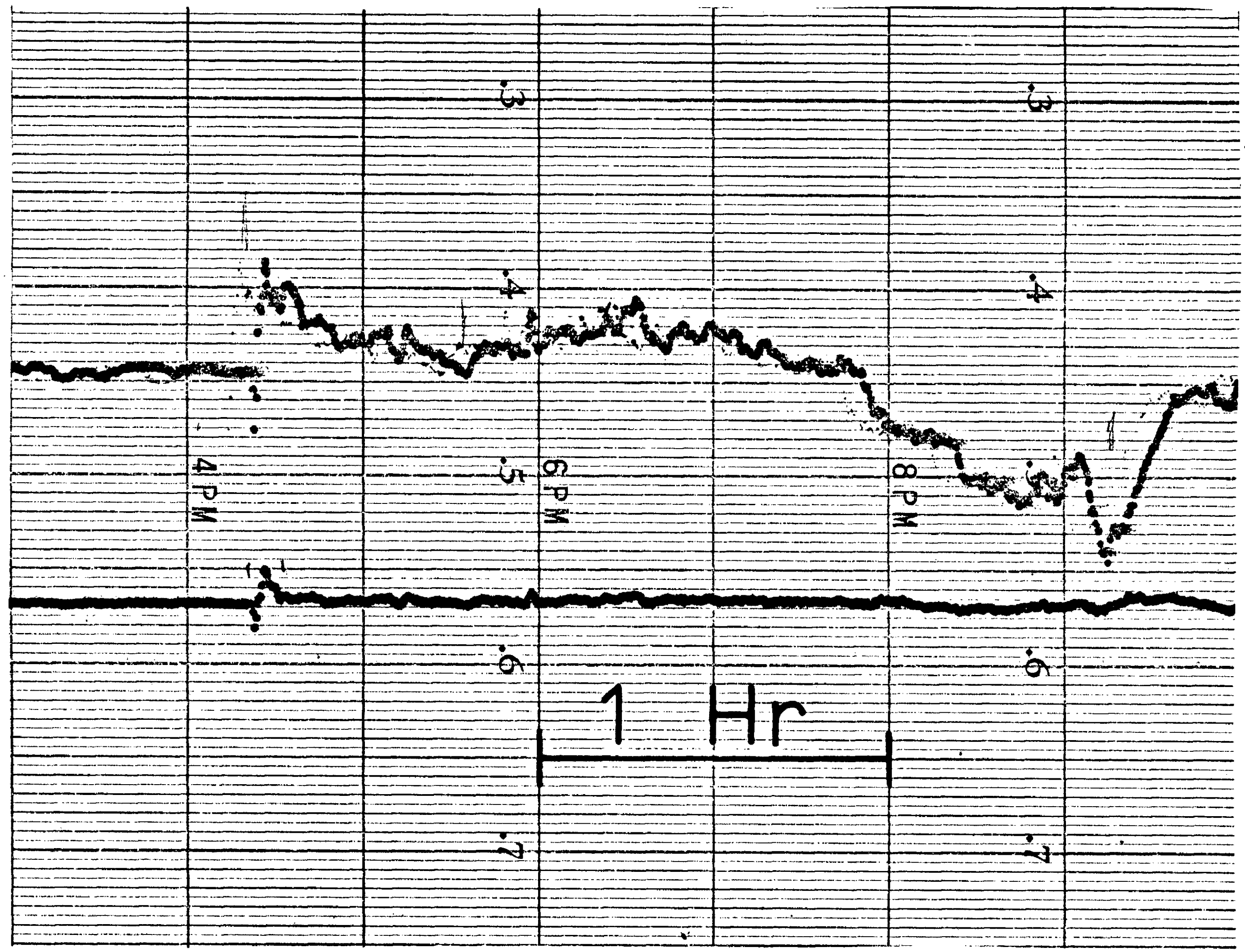


Figure 7e. Electric field record from Long Beach, Wash., 26 Jan. 1971.
Upper: E-W, max. excursion 100 mv/km
Lower: N-S, max. excursion 25 mv/km

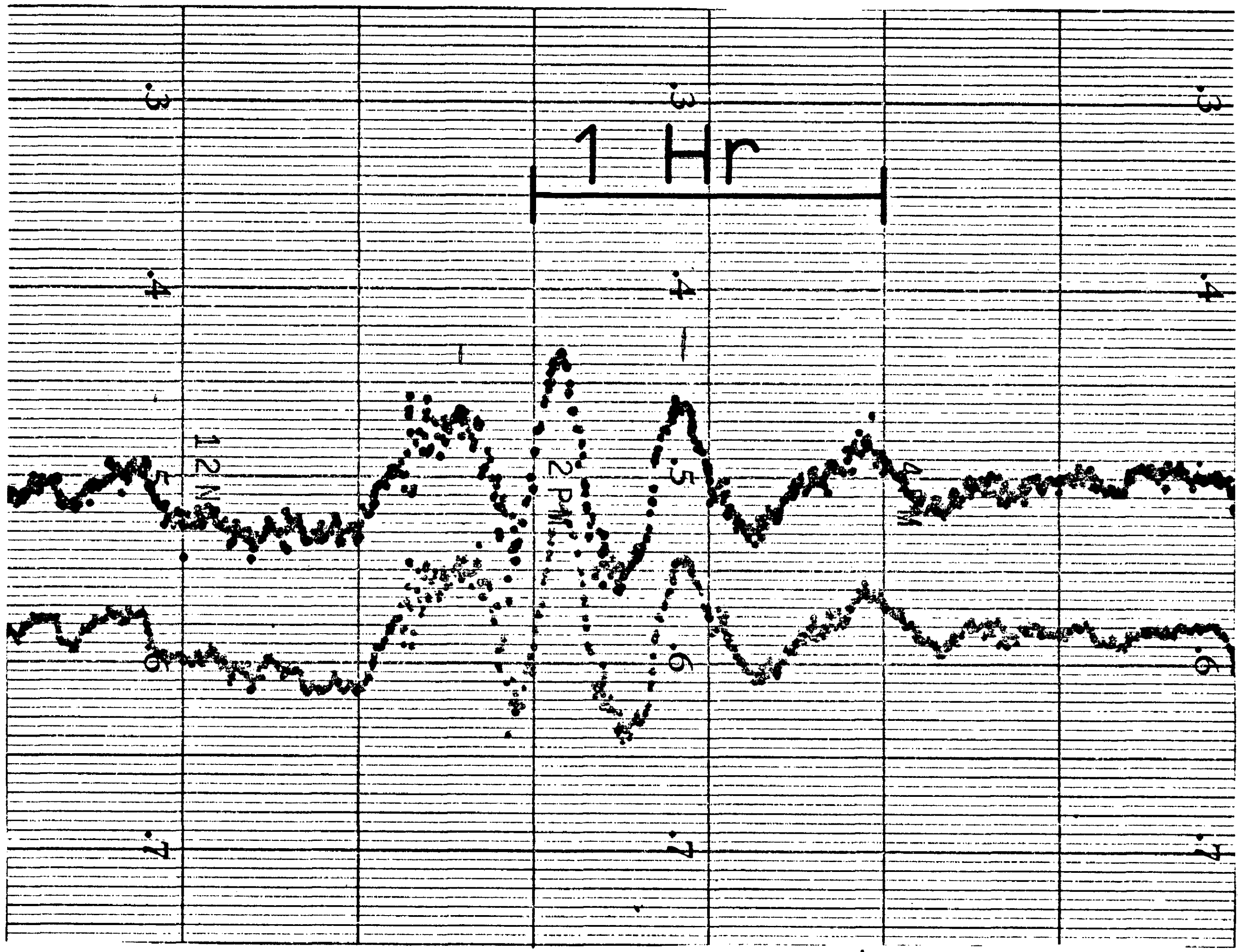


Figure 7f. Electric field record from Athol, Idaho, 27 Jan. 1971.
Upper: N-S max. excursion 170 mv/km
Lower: E-W max. excursion 160 mv/km

rapid excursions were lumped together under the character designation "PC" (see chapter V). Slower excursions were unlabeled or labeled "W" or "QP" (for quasi periodic), depending upon their shape. The slower variations could also be grouped on the basis of their durations or apparent periods.

In retrospect, the grouping of the amplitudes during data reduction seems less desirable than simply recording the amplitudes of the largest events. It did, however, make the data reduction process considerably faster.

Data Analysis

The aim of this study was, as stated previously, to establish plausible estimates of the amplitudes of large infrequent electric fields within the earth. One approach to this problem is to attempt to fit known probability functions to samples of electric field data. That is the approach used here and which is described in more detail below.

Sample Cumulative Relative Frequency Function

The data reduction process, as described previously, yielded a group of amplitude indices for each component of the electric field. The sample relative frequency function was obtained from each group simply by counting the number of occurrences of each amplitude index in a group and dividing that number by the total number of amplitude indices in the group. The sample cumulative relative frequency function was then obtained from

$$F_N(AI) = \sum_{AI' \leq AI} f_N(AI')$$

where

N = number of amplitude indices in the group

Fn(AI) = the sample cumulative relative frequency function

fn(AI) = the sample relative frequency function

The sample cumulative relative frequency will henceforth be called the sample probability function.

Although the group of AI's obtained from a given month was the basic sample, other samples could be formed by grouping AI's associated with the same event-character classification or by grouping several months' data. Grouping of data could only be done, however, for periods during which the recording system gains were unchanged.

Gain entered only as an additive constant, because of the logarithmic nature of the AI, but samples obtained at different gains could not be mixed unless some assumptions were made about the distribution of amplitudes within the interval characterized by a given AI.

Probability Functions

Two probability functions were used for comparison with the sample probability function, namely, the gaussian and the double exponential probability functions. The gaussian function was used initially because it was known that many natural phenomena can be adequately described by the log-normal probability function. Although the gaussian function gave satisfactory results for some samples, the double exponential probability function gave better results for the larger part of the data.

The double-exponential distribution is given by

$$F(x) = \exp\left[-\exp\left\{-\alpha_n (x - u_n)\right\}\right]$$

where the characteristic largest values, Un, is defined (for a sample of size n) by

$$\mathfrak{F}(u_n) = 1 - 1/n \tag{1}$$

and the extremal intensity function, α_n, is defined by

$$\alpha_n = n\, f(u_n) \doteq f(u_n) / (1 - \mathfrak{F}(u_n)) \tag{2}$$

where $\mathfrak{F}(x)$ and $f(x)$ are, respectively, an exponential-type distribution and its probability density function. An exponential-type distribution is one which has a density function such that the following condition is fulfilled for very large values of x

$$\frac{f(x)}{(1 - \mathfrak{F}(x))} \approx -\frac{f'(x)}{f(x)} \tag{3}$$

(Gumbel, 1958, p. 119)

Following Gumbel (1958, p. 166) it will be shown that the double-exponential distribution is the asymptotic ($n \longrightarrow \infty$) distribution law for largest values of samples from populations with exponential-type initial distributions.

Let F(x) be the initial distribution, and Um be the <u>characteristic mth value</u> from the top in an ordered sample of size n. Define

(4) $F(u_m) = 1 - m/n$ and expand F(x) about U_m, i.e.

$$(5) \quad F(x) = F(u_m) + \sum_{r=1}^{\infty} \frac{(x-u_m)^r}{r!} f^{[r-1]}(u_m)$$

$$= 1 - m/n + \sum_{r=1}^{\infty} m/n \frac{(x-u_m)^r}{r!} \frac{n}{m} f^{[r-1]}(u_m)$$

By analogy with (2), let

$$\alpha_m = f(u_m)/(1 - F(u_m)) = \frac{n}{m} f(u_m)$$

and then from (3) $-\frac{n}{m} f'(u_m) = \alpha_m^2$

Then by induction it can be shown that

$$\frac{n}{m} f^{[r]}(u_m) = (-1)^r \alpha_m^{r+1}$$

Thus

$$F(x) = 1 - \frac{m}{n} + \sum_{r=1}^{\infty} \frac{m}{n} \frac{(x-u_m)^r}{r!} (-1)^{r-1} \alpha_m^r$$

or

$$F(x) = 1 - \frac{m}{n} \exp[-\alpha_m (x - u_m)]$$

It should be noted that this is an asymptotic (large X) form of the original distribution.

Now for m=1 F(x) becomes

$$F(x) = 1 - \frac{1}{n} \exp[-\alpha_1 (x - u_1)]$$

It is a well known result from order statistics that for a set of an observations ordered from smallest to largest, the distribution of the largest value is given by

$$P(x_n < X_0) = (F(X_0))^n$$

F is the distribution function of the individual xi's. Thus for the case of interest here

$$P(x < X_0) = \left(1 - \frac{1}{n} \exp[-\alpha (X_0 - u)]\right)^n$$

or for $n \rightarrow \infty$

$$P(x < X_0) = \exp(-\exp[-\alpha (X_0 - u)])$$

As indicated previously, the data reduction scheme made use of the largest excursion in every three-hour interval. Since each 3-hour interval consisted of 180 samples, the appropriate value for n above is 180. Because of the rapid convergence of the series, the asymptotic form was used.

<u>Selection of the probability function</u>. Because it was not clear, a priori, which probability function most resembled a given sample probability function, the parameters of the best fitting (in the minimum-square-error sense) function were found for each type of probability function.

The computer programs used read and counted the AI's from the specified period and computed the sample probability function, which was usually defined over a range of 10 to 15 AI's, the former during periods of low activity and the latter during periods of high activity.

The sample probability function was then inverted using the desired probability function. The inversion was trivial when the double exponential function was used. When the Gaussian probability function was used, inversion was accomplished by interpolation in a table.

After inversion, the parameters of the minimum-square error line, relating the AI-inverted probability pairs, were computed from

$$a = \sum_{i=1}^{N} (x_i - \bar{x})(Y_i - \bar{Y}) / (x_i - \bar{x})^2 \quad ; \quad b = \bar{Y} - a\bar{X}$$

a = slope of the line
b = intercept
xi = linearized probability
yi = amplitude index with value i
$\bar{x}$ = average value of xi's
$\bar{y}$ = average value of yi's

The values obtained for a and b were estimates of the parameters of the probability function used for inversion. For the gaussian function,

a = σ = square root of the variance
b = μ = mean

For the double exponential function,

$a = {}^{1}/_{\alpha}$ = inverse of the extremal intensity function

$b = u$ = characteristic extreme

When these estimates were established, criteria for selection of the better fitting pf could be applied. Three parameters often used in goodness-of-fit testing were computed and were used to evaluate the adequacy of the given pf for representing the data. These were estimates of the chi-squared, the Kolmogorov and the von Mises statistics.

The chi-squared test is the most commonly used goodness-of-fit test. The quantity used in the test is

$$\chi^2 = \sum_i (O_i - E_i)^2 / E_i$$

where Oi is the observed number of events in the i^{th} interval, and Ei is the expected number of events in the i^{th} interval. The selection of intervals is somewhat arbitrary with the proviso that Ei should not be less than 2 (Johnson and Leone, 1964.). Ei is computed from the proposed distribution and the total number of events. As indicated by the name of the test, χ^2 is distributed approximately as chi-squared with the number of degrees of freedom given by the number of intervals less the number of parameters estimated from the data.

The Kolmogorov test is based upon the value of

$$Dn = \sup_x | F(x) - F_n(x) |$$

where F(x) is the assumed probability function, and Fn(X) is the sample probability function, i.e.,

$$Fn(X) = \frac{n(X)}{N}$$ where n is the number of events (in a sample of size N) less than X. The distribution of Dn is not known for finite N, but it is known that

$$L(Z) = 1 - 2\sum_{n=1}^{\infty} (-1)^{n-1} \exp(-2n^2z^2)$$ is the limiting distribution of $N^{\frac{1}{2}}D_n$ as $N \to \infty$. A table of L(Z) is given by Smirnov (1948). Birnbaum (1951) gives tables of $\Pr(D_n < c/n)$ for c from 1 to 15 and N from 1 to 100.

The von Mises test utilizes the value of

$$W_N^2 = N\int_{-\infty}^{\infty} [F_N(x) - F(x)]^2 \, dF$$ where F(x) and Fn(x) are as previously defined. Anderson and Darling (1952) give a table of

$$\lim_{N \to \infty} \Pr\{N W_N^2 < z\} .$$

The expression

$$N W_N^2 = \frac{1}{12N} + \sum_{j=1}^{N} \left[F(x)\Big|_{x=x_j} - \frac{2j-1}{2N} \right]^2$$

is given by Johnson and Leone (1964). Because the data reduction process used here did not yield individual amplitude values, the expression

$$NW_N^2 = \sum_{j=1}^{K} n_j \left[F(x)\Big|_{x=x_j} - \frac{2\sum_{i=1}^{j} n_i - 1}{2N} \right]^2 + \frac{1}{12N}$$

was used, where ni is the number of members of the group with AI equal to i.

The application of these tests in this study did not really constitute hypothesis testing in the normal sense. Goodness-of-fit testing normally consists of selecting a level of significance and then determining the number of data necessary for an adequate test. In this study the number of data was determined in advance so that only the best level of significance could be determined. Furthermore, the grouping of the data by the data reduction technique made it impossible to determine

$$\sup_x |F_n(x) - F(x)|$$

with certainty so that the value used for D_n may have been too small in some cases. When a good fit is indicated by the value used, however, it is clear that there can be no systematic departure of Fn(x) from F(x).

Results

The aim of this study was to establish a prediction capability for large, rare electric field strength levels in the earth's surface. Ideally, such a capability would take the form of a probability function for the amplitudes of infrequent electric field levels. Such a probability function would, presumably, be consistent with any sample of data, regardless of its size, although it might not be satisfactory for data taken during other parts of the solar cycle than those used in this study.

Unfortunately, the probability functions which were used here were not consistent with the entire set of data from any station. However, one or the other of the probability functions was satisfactory for describing many subsets of data from all the stations. One contributing factor to the failure to obtain a generally satisfactory pf was that, because of the data reduction technique, samples taken at different gains could not be unambiguously reduced to a common gain. Another factor was that some samples, which contained events related to the storm of March 8, 1970, were biased by the occurrence of a rather rare event in a relatively small sample.

Table 4 contains predicted electric field bounds for events with return periods of 1, 10 and 100 years. These predictions were obtained from samples taken over approximately month-long intervals. They include only samples for which the chi-squared test indicated a best level of significance greater than 5%, for the goodness-of-fit test. Perusal of this table reveals several interesting facts.

Site	Component	Predicted Levels (volts/km) annual	decadal	centurial	K_p max	Distribution	Level of Significance (chi-sqd)
Athol, Idaho	north-south					double-exp.	
11-6-69 to 12-5-69		0.36	1.4	5.4	6_-		50%
		0.25	0.98	3.8			
1-25-71 to 2-22-71	"	0.92	2.6	7.3	7_-	"	25%
		0.65	1.8	5.2			
2-23-71 to 3-24-71	"	1.8	5.9	19.3	6_-	"	10%
		1.3	4.1	13.7			
5-1-71 to 5-21-71	"	3.3	12.8	49.5	7_-	"	25%
		2.3	9.1	35.1			
	east-west						
2-24-70 to 3-25-70		4.2	22.8	121.0	9_o	"	10%
		3.0	16.1	85.5			
4-3-70 to 4-30-70	"	1.4	4.9	17.5	8_+	"	50%
		1.0	3.5	12.3			
2-23-71 to 3-24-71	"	2.3	9.1	35.0	6_+	"	10%
		1.7	6.4	24.8			
5-1-71 to 5-21-71	"	2.5	9.4	35.0	7_-	"	5%
		1.8	6.6	24.8			
Long Beach, Wash.	east-west						
9-22-69 to 10-8-69	"	0.41	0.73	1.2	8_o	gaussian	10%
		0.29	0.51	0.85			
1-24-70 to 2-19-70	"	0.15	0.25	0.38	5_-	"	97% (!)
		0.11	0.18	0.27			
3-13-70 to 4-11-70	"	0.36	0.65	1.1	6_+	"	50%
		0.25	0.46	0.78			

Table IV. Predicted electric field levels from month-long samples

Site	Component	Predicted Levels (volts/km) annual	decadal	centur-ial	K_p max	Distribution	Level of Significance (chi-sqd)
Boulder City, Nev.	north-south						
11-6-69 to 12-6-69	"	0.29	0.54	0.92	6_-	gaussian	25%
		0.21	0.38	0.65			
1-6-70 to 2-4-70	"	0.035	0.052	0.073	5_-	"	25%
		0.025	0.037	0.051			
2-5-70 to 3-6-70	"	0.068	0.11	0.18	6_-	"	10%
		0.048	0.081	0.13			
6-10-70 to 7-4-70	"	0.24	0.44	0.74	6_+	"	25%
		0.17	0.31	0.52			
5-8-70 to 6-6-70	"	0.46	2.0	8.8	7_o	double-exp.	25%
		0.33	1.4	6.2			
4-8-70 to 5-6-70	east-west	0.79	3.3	13.9	8_+	"	50%
		0.56	2.4	9.9			
Long Beach, Wash.	east-west						
10-26-70 to 11-19-70		1.4	6.0	25.1	7_-	"	10%
		1.0	4.3	17.7			
2-19-70 to 3-13-70	"	5.3	33.2	207.0	9_o	"	10%
		3.7	23.5	146.3		"	

Table IV Continued

In only 2 instances did the gaussian function give a better fit than the double-exponential pf for samples from Athol. In those 2 the chi-squared test did not indicate a satisfactory fit, and, therefore, they are not included in Table 4. On the other hand, the double-exponential pf fit only those samples taken during periods of high electrical field activity from Long Beach and Boulder City. The apparent reason for this difference is that Athol is located in an area where the crust is thicker and/or more resistive than it is at the other 2 stations.

If the prediction obtained from the period during which the storm of March 8, 1970, is ignored, one observes that the predictions obtained from the Athol data are generally in the order of a few volts/km. for the decadal return period and in the order of a few volts/km. to a few 10's of volts/km. for the centurial return period.

The predictions obtained from the Long Beach data are less consistent than those for Athol, mostly because both pf's must be used. The predicted levels are generally lower than at Athol, however. Of the 2 sets of data which yielded high predictions, one was taken during a period in which the storm, mentioned above, occurred. The other is apparently unexceptional.

The predictions obtained from the Long Beach data are similar to those obtained for the Long Beach data in that they are, for the most part, low. The relatively high predictions evidently correspond to relatively high geomagnetic activity, as indicated by the Kp index.

Table 5 contains predictions obtained from samples composed of like types of events only. The rapid variations are those which cannot be resolved at the recording speed used. They are believed to correspond to micropulsations in the geomagnetic field. The bay-like variations are divided into 2 classes, those with durations less than 25 minutes and those with longer durations. The predictions are not greatly different than those of Table 4 except that the predicted values for rapid variations are very small for Boulder City. Rapid variations were so small at Long Beach that very few were recorded, and no prediction was attempted. The small amplitudes of variations of this type at these 2 locations is probably due to low conductivity near the surface of the ground.

The data used in this study were obtained during a period of relatively high solar activity, and therefore the predictions presented here are applicable only to like periods. Thus the nominal annual, decadal and centurial return periods may be too short by a factor of 2 or 3.

The predicted decadal levels for Athol appear to be in fair agreement with the values reported by Winckler et al (1959) and Germaine (1940) for the eastern U.S. The values were about 4.5 and about 2.5 volts/km., respectively. The author is unaware of any historical record of appreciably greater electric field levels since 1850. Hence, if the approximate levels predicted here are valid, a telluric current storm with electric field amplitudes in the order of 10 volts/km. will occur with high probability within the next 100 years.

Period	Component	Predicted Levels (volts/km)			Distribution	Level of Significance (chi-sqd)
		annual	decadal	centurial		
		Short-duration bay-like variations				
4-8-70 to 6-6-70	north-south	0.16 0.12	0.56 0.40	1.9 1.4	double-exp.	25%
		Long Beach, Wash. Bay variations				
9-22-69 to 4-11-70	east-west	0.17 0.12	0.27 0.19	0.40 0.28	gaussian	25%
		Short-duration bay-like variations				
1-20-71 to 4-18-71	east-west	0.30 0.21	0.93 0.66	2.9 2.0	double-exp.	5%

Table V Continued

Period	Component	Predicted Levels (volts/km) annual	decadal	centur-ial	Distribution	Level of Significance (chi-sqd)
		Athol, Ida. Rapid variations				
11-6-69 to 1-9-70	north-south	0.32	1.04	3.4	double-exp.	25%
		0.23	.74	2.4		
2-24-70 to 4-30-70	east-west	3.2	24.5	189.1	"	5%
		2.2	17.3	133.7		
		Bay variations				
11-6-69 to 1-9-70	north-south	0.49	2.4	11.5	"	10%
		0.35	1.7	8.2		
		Short-duration bay-like variations				
11-6-69 to 1-9-70	north-south	0.20	0.70	2.5	"	10%
		0.14	0.50	1.7		
	east-west	0.43	1.9	8.7	"	10%
		0.31	1.4	6.2		
		Boulder City, Nev. Rapid variations				
4-8-70 to 6-6-70	north-south	0.017	0.028	0.042	gaussian	25%
		0.012	0.020	0.030		
	east-west	0.030	0.051	0.077	"	10%
		0.022	0.036	0.055		
		Bay variations				
4-8-70 to 6-6-70	north-south	0.44	2.0	9.1	double-exp.	10%
		0.31	1.4	6.4		
	east-west	0.58	2.9	14.7	"	10%
		0.41	2.1	10.4		

Table V. Predicted electric field levels for various types of variations

SUMMARY AND CONCLUSIONS

Electric fields induced in the earth by perturbations in the earth's magnetic field may occasionally be large enough to interfere with the operation of various systems of grounded conductors. Such electric fields may be especially intense during geomagnetic storms.

In order to assess the seriousness of this potential hazard, one needs a capability for prediction of amplitudes of rare events. An attempt was made in this study to achieve such a capability by analysis of recordings of potentials between grounded electrodes. Data from three recording stations were used. One station was in the Basin and Range Province, one was in the Pacific Coast Province, and one was in the Northern Rocky Mountains.

Although no general prediction law was discovered, the orders of magnitude of the amplitudes which would occur every 20 to 30 years and every 200 to 300 years are estimated. The estimated values are considerably larger for the station in the Northern Rockies than for the other 2. Since that province is evidently rather like the eastern U.S. in its gross electrical properties, the values estimated for that station are believed to be adequate estimations for the eastern U.S.

APPENDIX

Geomagnetic Transient Variations

GEOMAGNETIC VARIATIONS

Classification

The predictions of the preceding section are based on statistical considerations. It is a rather obvious necessity to inquire whether there may be physical limitations of magnetospheric perturbations. Or, more correctly, since there are obvious limitations, whether there may be less obvious limitations which would become operative at low enough levels to invalidate the prediction technique of the previous section.

The remainder of this section will be devoted to (1) a brief review of the magnetospheric storm and micropulsation phenomena, (2) contemporary theories of the origin of these phenomena, and (3) whether these theories may indicate limitations of the magnitude of perturbations.

The so-called transient variations in the earth's magnetic field can be divided into several classes; S or solar variations, L or lunar variations, D or disturbance variations, bays and micropulsations. The solar variation is a regular fluctuation in the field with a period equal to one solar day. Amplitudes are typically several tens of gammas to a few hundred gammas (1 gamma = 10^{-5}gauss = 4 x 10^{-3}A/M), increasing from the auroral zone toward the geomagnetic equator. The amplitude at a given location increases from a minimum in years of solar minimum to a maximum in years of solar maximum sunspot activity. Although the solar variation is fairly large in amplitude, the very low frequency makes it relatively inefficient at inducing currents and, thus, of little interest in a study of telluric currents.

The L or lunar variation is similar to the solar variation except that its period is equal to the lunar day, and also it is much smaller in magnitude. It is of even less interest in a study of telluric currents than is the S variation.

The D or disturbance variation is caused mainly by geomagnetic storms and has two components, the Dst or storm time variation and the DS or disturbance local time variation. As the names would indicate, the Dst has classically been regarded as varying uniformly about the earth or as being a function of Universal Time only, while the DS has been regarded as being a function of local time only. Recent work indicates that this distinction is not entirely valid but the classical terminology is still used.

The separation of Dst and DS for a given storm is accomplished by averaging magnetograms from several observatories; averaging with respect to Universal or storm time gives the average Dst and averaging with respect to local time gives the average DS. These average fields can be used to determine a potential function which satisfies Laplace's equation and whose gradient gives the average fields above the earth. The tangential components of the field functions can then be used to find current systems which could cause the observed average fields at the surface. The current systems so determined are not unique, but can still provide helpful insight into the nature of the sources of the disturbances.

The typical geomagnetic storm is composed of a SC, sudden commencement, followed by the initial phase which is in turn followed by the main phase. The sudden commencement, SC, is characterized by an increase of 10 to 30 gammas in the horizontal component of the field in a matter of 2 or 3 minutes. The initial phase is characterized by a horizontal component 10 to 30 gammas greater than the quiet day level and lasts for 1 to 10 hours. The main phase is characterized by a horizontal component which is less than the quiet day level by change in the order of 100 gammas. Each phase also has a characteristic signature in the vertical component and the declination also. A particular storm may be lacking any one of these features but most storms exhibit all. A typical storm is shown in figure 8.

Often there are variations superimposed upon the main features of the storm. The superimposed variations resemble bays, being of several hours duration, but are much larger

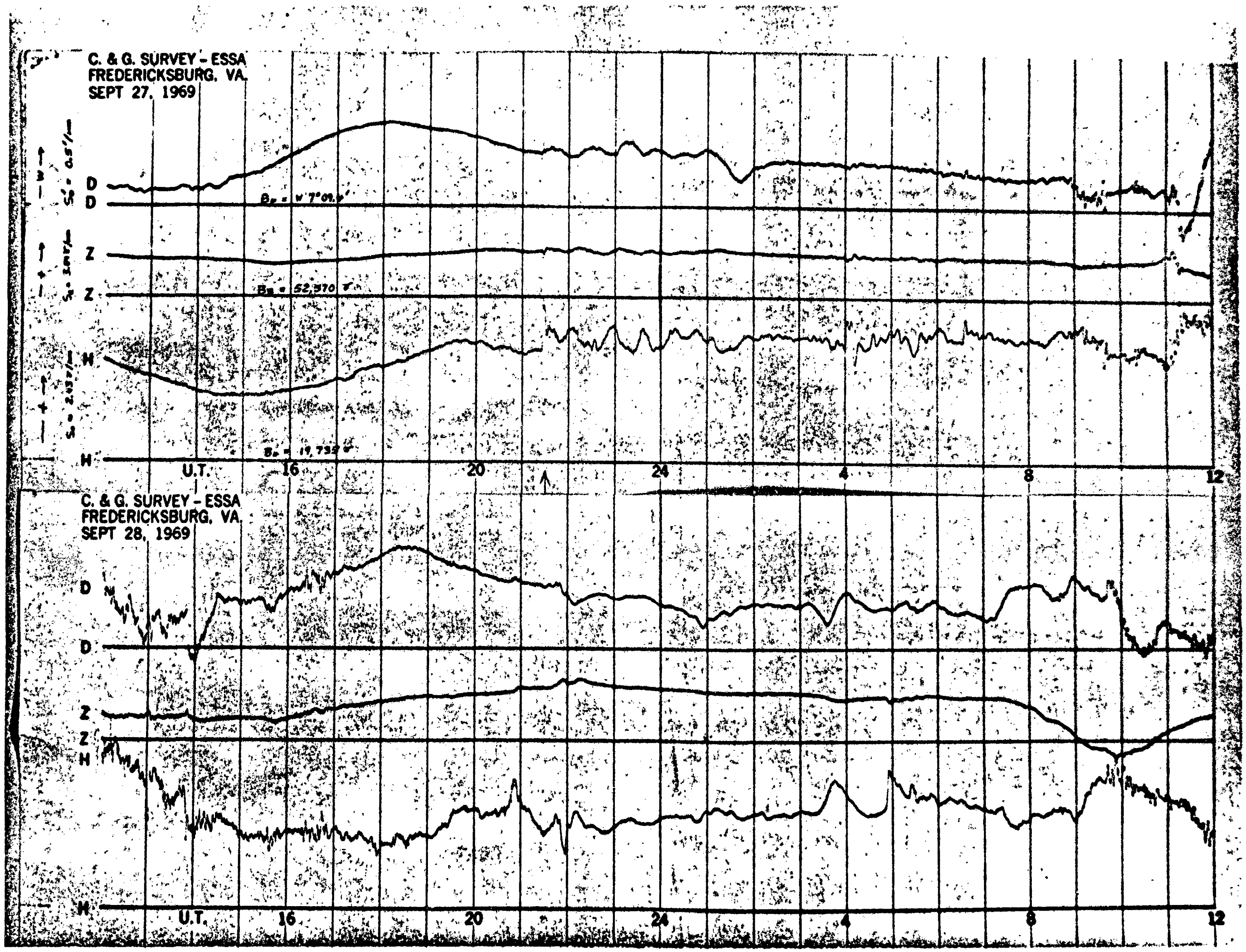


Figure 8. Magnetograms recorded on 27 and 28 September, 1969 at Fredricksburg, Virginia

than typical quiet day bays, particularly near the auroral zone. In fact, these variations are often the largest events of a given storm and for some great storms have amplitudes exceeding 1000 gammas, at near-auroral zone observatories.

The SC, initial phase and main phase are all composed of superimposed Dst and DS variations, but the main phase is dominated by the Dst variation, i.e., it is relatively uniform with respect to longitude.

An axially symmetric ring current can cause a longitudinally uniform decrease in the horizontal component of the field, and such a current has been postulated for many years. Although discovery of the trapped radiation belts seemed to provide a ring current mechanism, there is a good deal of controversy about whether or not the rather stable belts discovered thus far can account for the relatively fast main phase variation.

Possible current systems which can account for the Ds variations are less obvious. However, radio absorption studies, and the spatial distribution of aurora together with possible current systems found by solution of Laplace's equation indicate that the Ds current system is concentrated along the auroral zone with some current leakage to lower latitudes. It has been suggested that the current system is a superposition of current systems associated with what

Akasofu and Chapman (1961) have called DP substorms and Akasofu (1968) calls a polar magnetic substorm. The DP substorm current system causes geomagnetic bays, so that the bay-like disturbances associated with magnetic storms render this suggestion plausible.

The Dst, DS, and DP variations all tend to be smaller and less frequent in years of solar minimum than in years of solar maximum. In addition to these transient variations in magnetic field, there are also small fluctuations which take place almost continuously, and which are called "micropulsations." Geomagnetic micropulsations are small, relatively rapid fluctuations in the earth's magnetic field. Their amplitudes are typically between 10^{-3} gammas and several tens of gammas with slower variations having larger amplitudes. They are classified on the basis of their morphology. The two main categories are: Pc or continuous pulsations, and Pi or irregular pulsations. Pc's are approximately sinusoidal variations which generally have a long duration so that a given event contains many periods. Pi's are roughtly sinusoidal, but are much more "jagged" in appearance than Pc's.

Pc's are divided into 5 sub-categories on the basis of period. These sub-categories are, in order of increasing period: Pc1, Pc2, Pc3, Pc4, and Pc5. Pi's are subdivided into Pi1 and Pi2, also on the basis of period.

Pc1's have periods in the range 0.3 to 4 seconds. They commonly occur in sequences tens of minutes to hours in duration, which appear to be modulated. This apparent modulation prompted the use of the term "pearls" because of the resemblance of a record of such events to pearls on a string. With Pc1 oscillations, if T is the period of the modulating frequency and t is the carrier frequency, the product T (1/t) remains approximately constant.

The maximum number of Pc1's seems to occur in the years between solar maximum and solar minimum. The seasonal dependence of Pc1's is not known although there is some evidence that for mid-latitudes there is a maximum frequency of occurence in the fall and winter months, while for the auroral regions the maximum is at the equinoxes. The diurnal variation is characterized by broad maxima peaked around noon and early afternoon in high latitudes and narrower maxima peaked around midnight and early morning in low latitudes. Long series of Pc1's occur only during periods when the magnetosphere is quiet. It has been shown that more than 50% of all Pc1's occur when Kp 2. However, Pc1's are sometimes observed together with Pi1's and Pi1's in the fine structure of geomagnetic bays.

Pc2's and Pc3's have period ranges of 5 to 10 seconds and 10 to 45 seconds respectively and usually have amplitudes less than $\frac{1}{2}$ gamma. The frequency of occurence of Pc2's decreases drastically during years of solar minimum, but it remains about the same as in years of solar maximum for Pc3's. Pc3's have a strong seasonal dependence, minimum activity being in the winter and maximum activity varying from summer to equinoxes. The diurnal variation of Pc2's and Pc3's is similar, both having a maximum frequency of occurrence around local noon. The amplitudes of these events increase with increasing Kp* and attain their maximum value in the auroral zone. Pc3's are known to be excited synchronously at conjugate point stations. (Two points, one in the northern hemisphere and one in the southern hemisphere, are conjugate points if one line of force passes through both.)

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* The Kp index of geomagnetic activity The Kp index is a three-hour index of magnetic disturbance, which is intended to measure planet-wide variations. Kp is determined for 8 three-hour intervals per day, beginning at 0000GMT. It is computed from the K indices of 12 observatories located between 48 and 63 degrees geomagnetic latitude, after local effects have been removed.

The K index is in turn based upon the range, R. R, for a given component, is the difference between the highest and lowest deviations of that component from its regular daily variation. K is based on the largest range among the three components recorded.

Each observatory has its own scale by which K is determined. These scales have been determined empirically so that K indices determined for the same time interval, by different observatories, are consistent.

K is quasi-logarithmic, taking on integral values from 0 to 9. The upper range limit for K = 0 is about 100 times smaller than the lower range limit for K = 9.

Kp is also quasi-logarithmic, taking on values 0_o, 0_+, 1_-, 1_0, 1_+ etc. to 9_o, a total of 28 values.

Pc4's have amplitudes ranging from 5 to 10 gammas in high latitudes and periods ranging from 45 to 150 seconds. They occur more frequently in years of sunspot minimum than in years of sunspot maximum, but their amplitudes show no dependence on sunspot activity. They are most frequent in equinoxial months in the auroral and equatorial zones, and least frequent in winter. The diurnal maximum frequency of occurrence is around 0300 to 0600 local time in the auroral zone and shifts to around noon in lower latitudes. The amplitudes of these events are generally larger in the auroral zone. Like events often occur simultaneously at conjugate stations.

The Pc5 pulsations have periods in the range 150 to 600 seconds and amplitudes which in the extreme reach several hundreds of gammas. In the auroral zone, they occur most frequently around 0600 and 1800 local time, but there are conflicting reports regarding maxima at lower latitudes. Their amplitudes are larger in the auroral and equatorial zones than in mid-latitudes. In general, Pc5's occur neither on strongly disturbed days nor on very quiet days, but most often are in evidence on days following moderate disturbances. Records of radio absorption often show a strong correlation with magnetic records during Pc5 occurrences.

Pi1's have periods in the range 1 to 40 seconds and amplitudes in the order of 1 gamma. They typically form the fine structure of slow variations during magnetic storms and also occur with bays. They may also occur when the magnetic field is quiet. Their amplitudes attain maxima in auroral latitudes and decrease greatly at lower latitudes. They occur most frequently in the early morning and late at night. They occur most frequently in years of solar maximum and are strongly correlated with Kp, increasing with increasing Kp.

Pi2's have periods in the range 50 to 150 seconds and amplitudes in the order of 1 gamma. They are characteristically accompanied by Pi1's and sometimes followed by Pc1's. They can be divided into 2 groups; those that occur before a negative bay, and those that occur before a positive bay or during quiet times. The first group has maximum amplitudes in the auroral zone and the second group has maximum amplitudes around 50 degrees. Both have a diurnal maximum around local midnight. The yearly occurrence frequency shows an inverse relationship to solar activity. There are seasonal maxima during equinoctial months. The dominant periodicity in a Pi2 decreases with increasing Kp.

Magnetospheric Substorms

As noted previously, a characteristic feature of all magnetic storm is an irregular sequence of large bay-like variations each of which as a duration of 1 to 3 hours.

These events are generally greatest at high latitudes in the night sector, and during great storms may attain values of several thousand gammas (1 gamma = 10^{-5} gauss). According to Akasofu (1968), these events, which he calls polar magnetic substorms, are only one aspect of a phenomenon which he calls the Magnetospheric Substorm.

According to Akasofu's model, a geomagnetic storm is one aspect of a magnetospheric storm. The magnetospheric storm itself normally consists of a compression of the magnetosphere, which is caused by a gust of solar wind, and the superimposed effects of a sequence of magnetospheric substorms. The compression phase causes the initial phase or sudden commencement of the geomagnetic storm and the magnetospheric substorms include the polar magnetic substorms and the enchancement of the ring current. The greater the number and intensity of the substorms, the more severe is the magnetic storm.

A study of magnetospheric storms thus reduces to a study of magnetospheric substorms. The substorm is of interest in itself, however, in a study of earth-currents, because it is the most energetic perturbation of the magnetosphere and thus is the most probable source of extremely large earth-current events.

There are other aspects of a magnetospheric substorm in addition to the polar magnetic substorm. A brief description of each is included in order to provide a better picture of the mechanism of the polar magnetic substorm, in so far as it is known.

Auroral Substorm

The aurora was undoubtedly the first manifestation of the geomagnetic field to be observed, at least in high latitudes. The locus of auroral activity is called the auroral oval, and it is an asymmetric annular region about the magnetic pole. The auroral oval descends to the lowest latitude in the night sector (70 dip. lat. during periods of moderate activity) and is quite distinct from the auroral zone which corresponds to 67 dipole latitude.

The auroral oval is generally believed to be that region of earth where the magnetic field lines lying on the poleward boundary of the inner magnetosphere, or trapping region, intersect the earth (Fig. 9). Thus a particle approaching the earth along a field line through the auroral oval will be precipitated rather than reflected.

An auroral substorm is one aspect of a magnetospheric substorm and generally lasts for 1 to 3 hours. Although the commencement of the substorm depends on universal time, the characteristics depend strongly on local time. Onset of a substorm is indicated by increased activity of the previously quiet arcs on the equatorward side of the oval in the midnight

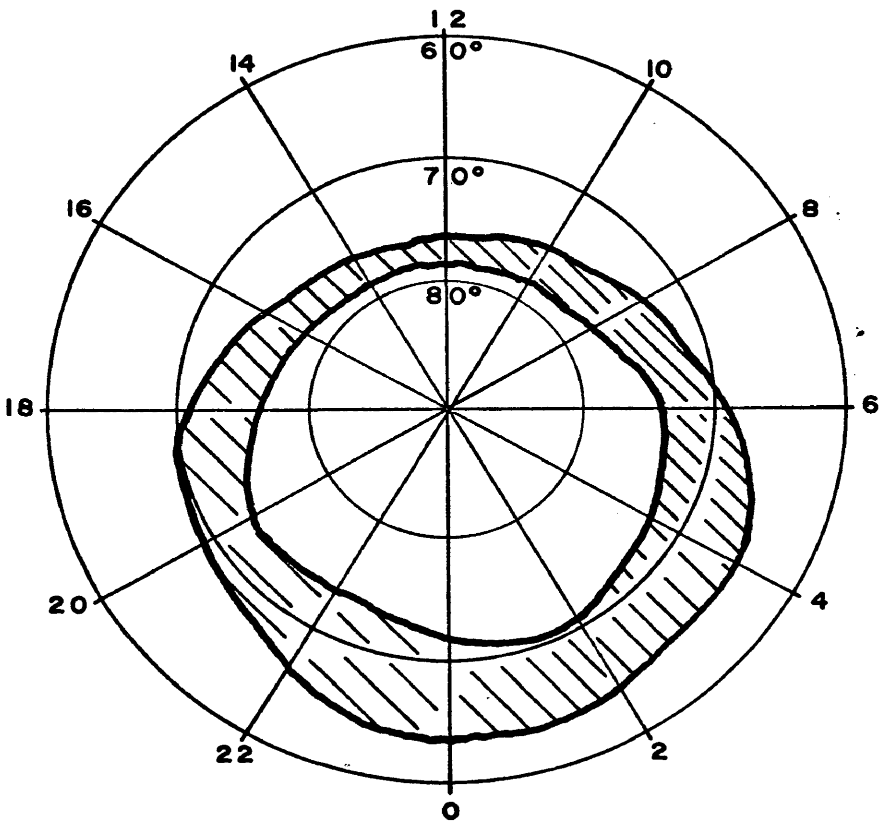

Figure 9. A map of the auroral oval in universal time and geomagnetic dipole latitude coordinates. (after Akasofu, 1968)

sector. The onset is followed by rapid spreading in all directions until the whole sky may appear to be covered in the midnight sector. One particularly noteworthy feature of the spreading is the westward traveling surge, which appears to an observer on the ground to be a fold traveling west at about 1 km/sec. The eastward spreading is manifest in the morning sector as pulsating luminous patches which have an apparent eastward speed of about 300 m/sec.

Ionospheric Substorm

The ionospheric substorm is another aspect of a magnetospheric substorm, and is essentially a state of anomalous ionization of the lower ionosphere. This condition is evidently due to precipitation of electrons more energetic than those which cause aurora. One technique for observing the state of ionization in the ionosphere is monitoring of absorption of cosmic radio noise, using a riometer.

Like aurora, the ionospheric substorm's characteristics depend strongly on local time. There are three types of events which are called E, N, and M events and which characterize the evening, night and morning sectors respectively. The E type event is a brief impulsive episode of absorption, the N type has an impulsive onset, but the absorption is greater and lasts longer than is the case for E type events. Finally the M type event commences relatively slowly, but is otherwise similar to the N type event.

As might be expected, the N type is similar in spatial and temporal occurrence to the poleward expansion phase of an auroral substorm. The E type event is associated with the westward traveling surge and occurs as the surge passes the poleward position of the station. The relatively small magnitude of the absorption may indicate that the westward traveling surge is caused by precipitation of the ''soft'' electrons. M type events are associated with the patchy aurora of the morning sector, and may be caused by relatively energetic electrons which are not efficient at exciting aurora. M type events also occur in the late morning sector, and these are associated with N type events and negative bays in the midnight sector.

X-Ray Substorm

Still another aspect of the magnetospheric substorm is the x-ray substorm. Like the other substorm phenomena, it is characterized by a burst like episode with a duration of 1 to 3 hours. It is associated with absorption events and negative bays in the midnight sector (little correlation for events in the afternoon and early evening sector). It is cause by Bremsstrahlung x-rays, from energetic electrons impinging on the atmosphere.

Proton Aurora Substorm

The proton aurora is a diffuse band, exhibitng Ha, Hb, and H emissions, which appears equatorward of quiet arcs in the evening sector. Its location in the morning sector is uncertain, but there may be two bands, one of which is caused by leakage from the ring current belt.

During a substorm, proton precipitation occurs over the entire auroral bulge. Proton precipitation appears to be fairly extensive in area in the morning sector, and the proton aurora shifts equatorward in the evening sector.

Micropulsation Substorm

It is well known that Pi2 pulsations are associated with the early phase of a magnetic bay in mid-latitudes. The situation is more complicated in high latitudes however.

During the explosive phase of the auroral substorm, a combination of Pi1 and Pc1 (IPDP) occurs equatorward of the auroral oval in the region where the positive bay is observed. Pi bursts occur in a narrow band a little poleward of the expanding bulge, and Pi1 pulsations occur within the bulge and on auroral zone in the morning sector. Pi2 pulsations are observed in mid-latitudes in the midnight sector. IPDP are seen equatorward of the surge and Pc may be associated with the surge. Later in the storm (1 hour substorm time) Pi's apread into the noon sector and Pc1's occur in the afternoon sector.

Polar Magnetic Substorm

The magnetic disturbances associated with a magnetospheric substorm are caused by currents flowing in the magnetosphere. Although the true current distribution is unknown, equivalent current systems have been found by assuming the current to be confined to the ionosphere. The equivalent current system has as its main features, two electrojets, one eastward and one westward, flowing along the auroral zone. Unfortunately, there are not enough observatories to map accurately the magnetic variation over the vicinity of the auroral zone, and since the westward surge can produce both positive and negative bays, (Akasofu and Meng, 1967) it is not possible to find a unique distribution of current for a given disturbance.

An additional problem in studying polar magnetic storms is that an ionospheric current distribution is not able to account for the world-wide distribution. Satellite observations indicate that the low-latitude disturbance is nearly the same at orbital altitudes as at ground level, indicating that extra-ionospheric currents are the source.

It is, however, clear that current <u>does</u> flow along the auroral oval, probably along active auroras. That fact coupled with the knowledge that the auroral oval descends to lower latitudes during great magnetic storms suggests that proximity to the auroral zone is a factor which has considerable importance in the estimation of rare earth-current events.

The importance of proximity to the auroral zone is suggested by several observations of large negative bays at College, Alaska. Brown et al (1968) describe a large negative bay which occurred on April 18, 1965. The bay began at 0622 UT and appears to have attained a magnitude of 2200 γ sometime before 0630 UT. Thus the lower limit of $\frac{\partial \vec{B}}{\partial t}$ is about 4.6 gamma/sec. Kawasaki et al (1971) reported a negative bay at College which attained a maximum of 4900 γ in about 30 minutes during the great storm of March 8, 1970. This bay had a magnitude of about 500 γ at Newport, Idaho, and was apparently associated with an electric field strength in excess of 1 volt/kilometer at Athol. The value of $\frac{\partial \vec{B}}{\partial t}$ for the bay at College was about 2.7 gamma/sec.

According to Akasofu (1968), the auroral oval may descend to dipole latitude 50° during great magnetic storms. Hence the possibility exists that regions in this latitude could experience electric field strengths of 10 volts/kilometer or more, solely by reason of their proximity to an auroral electrojet of intensity similar to that responsible for the magnetic variations reported above.

Theories of Magnetospheric Substorms

At the present time there is no complete theory of magnetospheric substorms. Existing theories account for some aspects of the phenomenon but not for others. The most serious flaw in most theories is their inability to explain how the energy involved in the substorm is captured from the solar wind. There is, however, one group of theories which apparently can explain the energy transfer involved.

Dungey (1961) proposed that the magnetic field of the solar wind might occasionally have a southward component. It so, then during these periods when the southward component was present, the magnetic field lines in the solar wind could become connected to the outmost field lines of the magnetosphere. The result would be that the field lines near the front of the magnetosphere would be transferred to the rear or tail of the magnetosphere. Reconnection of field lines in the tail causes contraction of the field lines which in turn accelerates charged particles because of the constancy of the first adiabatic invariant

$$\mu = \frac{m v_{\perp}^{2}}{2|\vec{B}|}$$

where m is the particle's mass, $v_{\perp}$ is the component of its velocity perpendicular to the magnetic field, $\vec{B}$. Furthermore, flow of plasma through the magnetosphere from tail to front drives a current system in the ionosphere.

Dungey's theory has received impetus from the discovery that the magnetic field of the solar wind does sometimes contain a southward component and that periods of substorm activity are correlated with periods of southward interplanetary field but that large substorms may occur with southward changes of 2 hours or so.

Another impetus for Dungey's theory comes from the fact that the tail of the magnetosphere can serve as a reservoir of magnetic energy. Since publication of Dungey's 1961 paper, it has been discovered that the magnetospheric tail extends to considerable distance (Piddington (1967) estimates 10^3 earth radii) in the anti-solar direction (Ness, 1965). In cross-section it is composed of nearly anti-parallel field lines separated by a neutral sheet of plasma. The field lines are kept from instantly reconnecting by current flow in the plasma.

Piddington (1967) believes that the tail flux may increase by 10^{17} gauss cm^2 in a few hours. The subsequent auroral substorm is then caused by growth of a plasma instability, similar to the pinch effect, which results in rapid reconnection of the anomalous tail field and resulting particle acceleration and injection of tail plasma into the trapping region.

Although there are grounds for disputing the theories which assign to the tail, the energy storage process associated with a magnetospheric substorm, these theories at least serve to point out that the energy is stored in the earth's field. It is, therefore, of interest to compare the energy of a typical substorm with the total energy of the earth's field. Akasofu (1968) estimates the energy necessary for a typical substorm at 2 x 10^{22} ergs.

The earth's field is assumed to be that of a dipole with moment 8 x 10^{25} emu. (Stacey, 1969)

Then the magnetic energy is given by

$$T = \frac{1}{2}\int_V \mu_0 |\vec{H}|^2 dV \qquad \text{(Stratton, 1941)}$$

For a dipole with moment M,

$$\vec{H} = \frac{1}{4\pi}\left(\frac{\vec{M}}{r^3} - \frac{3(\vec{M}\cdot\vec{r})\vec{r}}{r^5}\right)$$

so that

$$|\vec{H}|^2 = \vec{H}\cdot\vec{H} = \left(\frac{1}{4\pi}\right)^2\left[\frac{M^2}{r^6} + \frac{9(\vec{M}\cdot\vec{r})^2\vec{r}\cdot\vec{r}}{r^{10}} - \frac{6(\vec{M}\cdot\vec{r})^2}{r^8}\right]$$

$$= \left(\frac{1}{4\pi}\right)^2\left[\frac{M^2}{r^6}(1+3\cos^2\theta)\right].$$

Thus the energy contained in the field <u>above</u> the earth is approximately

$$T = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_a^{\infty}(4\pi\times10^{-7})\left(\frac{1}{4\pi}\right)^2\left[\frac{M^2}{r^6}(1+3\cos^2\theta)\right] \times r^2\sin\theta\, d\phi\, d\theta\, dr,$$

where

a = earth's radius $\approx$ 6.4 x 10^6 m, or

$$T = 10^{-7}\left(\frac{M^2}{3a^3}\right)$$

$$\approx 8.2 \times 10^{24} \text{ ergs.}$$

Thus the earth's field above the ground contains only about two order of magnitude more energy than the amount of energy associated with a moderate geomagnetic substorm. This suggests that processes which make use of distortion of the earth's field may be within less than two orders of magnitude of intrinsic physical limitations.

Theories of the Origin of Micropulsations

Micropulsations are generally believed to be caused by hydromagnetic waves generated on the boundary of and within the magnetosphere. They are dependent to a very great extent upon the transmission characteristics of the magnetosphere and ionosphere. Both generation mechanisms and transmission characteristics of the magnetosphere have been studied in considerable detail. Unfortunately existing theories are of little use for investigation of large amplitude phenomena.

The hydromagnetic equations are

$$\vec{\iota} = \sigma \left\{ \vec{E} + (\vec{v}/c) \times \vec{B} \right\}$$

$$\rho \frac{d\vec{v}}{dt} = \rho \vec{G} + (\vec{\iota}/c) \times \vec{B} - \text{grad } p$$

$$-\frac{\partial \rho}{\partial t} = \text{div} (\rho \vec{v})$$

and Maxwell's equations (Alfven and Falthammer, 1963), where

$\vec{v}$ = the velocity vector
σ = conductivity
ρ = density
$\vec{G}$ = non-electromagnetic force/unit mass,
$\vec{\iota}$ = current density.

Neglecting displacement currents, $\vec{G}$, and assuming, for simplicity's sake, the fluid is incompressible, one obtains

$$\rho \frac{d\vec{v}}{dt} = \frac{1}{4\pi} \operatorname{curl} (\vec{B}/\mu) \times \vec{B} - \operatorname{grad} p$$

$$= -\operatorname{grad} \left(|\vec{B}|^2 / 8\pi\mu \right) + \frac{1}{4\pi\mu} (\vec{B} \cdot \operatorname{grad}) \vec{B}$$

$$- \operatorname{grad} p$$

If the further assumptions are made that σ is very large, then

$\vec{\iota} \approx 0$

and $$\frac{\partial \vec{B}}{\partial t} = \operatorname{curl} (\vec{v}/c \times \vec{B})$$

It is evidently not known, in general, how to combine these equations in order to obtain an equation with only one variable. In lieu of a general approach, the usual stratagem is to linearize by assuming that the variation in $\vec{B}$ is a small perturbation of the static field.

It is therefore clear that a theoretical treatment of large amplitude rapid variations is not presently possible. As Jacobs (1970) says, "The conditions of generation and growth of small disturbances can be investigated in detail by a linear approximation. However, the determination of the amplitudes of stationary oscillations requires a non-linear theory."

Although a non-linear theory is lacking, there is evidence that micropulsations may be as likely to induce large earth potential gradients as larger events. Malich (1947) reports a $\frac{\partial \vec{B}}{\partial t}$ of -9.6 gammas/sec at College, Alaska in October 17, 1943. That value is considerably larger than the 2.7 gammas/sec. average $\frac{\partial \bar{B}}{\partial t}$ of the 5000 gamma bay recorded at College during the great storm of March 8, 1970 (Kawasaki et al, 1970).

REFERENCES

Airy, G.B., 1868, Comparison of magnetic disturbances recorded by the self-registering magnetometers at the Royal Observatory, Greenwich, with magnetic disturbances deduced from the corresponding terrestrial Galvanic currents recorded by the self-registering galvanometers of the Royal Observatory: Phil. Trans. Royal Soc. London, v. 158, p. 465-473

Akasofu, S.-I., and Chapman, Sydney, 1961, The ring current, geomagnetic disturbance, and the Van Allen radiation belts: J. Geophys. Research, v. 66, no. 5, p. 1321

Akasofu, S.-I., and Meng, C.-I., 1968, Low latitude negative bays: J. Atmosph. Terr. Phys., v. 30, p. 227-241

Akasofu, S.-I., 1968, Polar and magnetospheric substorms: Dordrecht-Holland, D. Reidel Pub. Co., 280 p.

Alfven, Hannes, and Falthammar, C.-G., 1963, Cosmical electrodynamics: Oxford, Oxford University Press, 228 p.

Anderson, T.W., and Darling, D.A., 1952, Asymptotic theory of certain "Goodness of Fit" criteria based on stochastic processes: Annals Math. Stat., v. 23, p. 193-211

Atwater, Tanya, 1970, Implications of plate tectonics for the Cenozoic evolution of western North America: Bull. Geol. Soc. of America, v. 81, p. 3512-3536

Barlow, Peter, 1831, On the probable electric origin of all the phenomena of terrestrial magnetism; with an illustrative experiment: Phil. Trans. Royal Soc. London, v. 121, p. 99-109

Barlow, W.H., 1849, On the spontaneous electrical current observed in the wires of the electric telegraph: Phil. Trans. Royal Soc. London, v. 139, p. 61-72

Bauer, L.A., 1922, Some results of recent earth-current observations and relations with solar activity, terrestrial magnetism, and atmospheric electricity: Terr. Magn. Atmosph. Electricity, v. 27, p. 1-30

Becquerel, A.C. , 1844, Des tentatives faites pour demontrer que le magnetisme terrestre a une origine electrique: Comptes Rendus Acad. Sci. Paris, v.19, p. 1052

Birnbaum, Z.W., 1952, Numerical tabulation of the distribution of Kolmogorov's statistic for finite sample size: Am. Stat. Assoc. J., p. 425-441

Blackwell, D.D., 1971, The thermal structure of the continental crust: in Heacock, J.G., (Ed.), The structure and physical properties of the earth's crust; Am. Geophys. Union Monograph 14, Washington, p. 169-184

Brown, W.L., Cahill, L.J., Davis, L.R., McIlwain, C.E., and Roberts, C.S., 1968, Acceleration of trapped particles during a magnetic storm on April 18, 1965: J. Geophys. Research, v. 73, no. 1, p. 153-161

Burbank, J.E., 1905, Earth-currents: and a proposed method for their investigation: Terr. Magn. Atmosph. Electricity, v. 10, p. 23-49

Calaghan, Eugene, 1938, Geology of the Searchlight District, Clark County, Nevada: U.S. Geol. Survey Bull. 906-D, p. 135-188

Camfield, P.A., Gough, D.I., and Porath, H., 1970, Magnetometer array studies in the northwestern United States and southwest Canada: Geophys. J. Roy. Astron. Soc., v. 22, p. 201-221

Caner, B., 1971, Quantitative interpretation of geomagnetic depth-sounding data in western Canada: J. G phys. Res., v. 76, no. 29, p. 7202-7216

Caner, B., Camfield, P.A., Andersen, F., and Niblett, E.R., 1969, A large-scale magnetotelluric survey in western Canada: Can. J. Earth Sci., v. 6, p. 1245-1261

Caner, B., Cannon, W.H., and Livingstone, C.E., 1967, Geomagnetic depth sounding and upper mantle structure in the Cordillera region of western North America: J. Geophys. Res., v. 72, no. 24, p. 6335-6351

Chapman, Sydney, and Whitehead, T.T., 1923, On the observations of earth-potential gradient at Ebro: Terr. Mag. and Atmos. Electricity, v. 28, p. 125-128

Davey, Humphrey, 1821, On magnetic phenomena produced by electricity: Phil. Trans. Roy. Soc. London, p. 7-19

Davidson, W.F., 1940, The magnetic storm of March 24, 1940-effects in the power system: Edison Elec. Institute Bull., July, p. 365

Davies, Kenneth, 1966, Ionospheric radio propagation: New York, Dover, 461 p.

Dungey, J.W., 1961, Interplanetary magnetic field and the auroral zones: Phys. Rev. Letters, v. 6, no. 2, p. 47-48

Ellis, William, 1892, On the simultaneity of magnetic variations at different places on occasion of magnetic disturbance, and on the relation between magnetic and earth-current phenomena: Proc. Roy. Soc. London, v.52, p. 191-208

Etherington, T.J., 1931, Stratigraphy and fauna of the Astoria Miocene of southwest Washington: Univ. Calif. Dept. Geol. Sci. Bull., v. 20, no. 5, p. 31-142

Faraday, Michael, 1832, Experimental researches in electricity - second series: Phil. Trans. Roy. Soc. London, v. 122, p. 163-194

Fleming, D.B., and Keller, G.V., 1972, Research on the probability of occurrence of unusually large earth-current storms: Rept. submitted to Bonneville Power Admin. and U.S. Bureau of Reclamation (unpub)

Fox, R.W., 1830, On the electromagnetic properties of metalliferous veins in the mines of Cornwall: Phil. Trans. Roy. Soc. London, v. 120, p. 399

Germaine, L.W., 1940, The magnetic storm of March 24, 1940-effects in the communications system: Edison Elec. Institute Bull., July, P. 367

Gish, O.H., and Rooney, W.J., 1928, On earth-current observations at Watheroo Magnetic Observatory, 1924-1927: Terr. Mag. and Atmos. Electricity, v. 33, no. 2, p. 79-90

Gish, O.H., 1931, The natural electric currents in the earth's crust: Scient. Monthly, v. 32, p. 5-21

Gumbel, E.J., 1958, Statistics of extremes: New York, Columbia University Press, 371 p.

Hales, A.L., and Herrin, Eugene, 1972, Travel times of Seismic waves: in Robertson, E.C., (Ed.), The nature of the solid earth; New York, McGraw-Hill Inc., p. 172-215

Herron, T.J., 1967, An average geomagnetic power spectrum for the period range 4.5 to 12,900 seconds: J. Geophys. Res., v. 72, no. 2, p. 759-761

Jackson, D.B., 1966, Deep resistivity probes in the southwestern United States: Geophysics, v. 31, no. 6, p. 1137-1140

Jacobs, J.A., 1970, Geomagnetic micropulsations: Heidelberg, Springer-Verlag, 179 p.

Johnson, N.L., and Leone, F.C., 1964, Statistics and Experimental design: New York, John Wiley and Sons Inc., 523 p.

Kawasaki, K., Akasofu, S.-I., Yasuhara, F., and Perreault, P.D., 1971, The geomagnetic storm of March 8, 1970: in Lincoln, J.V., and Buckman,D.B., Data on solar-geophysical activity associated with the major geomagnetic storm of March 8, 1970, report UAG-12, part 3; Ashville, N. Carolina, U.S. Dept. Commerce, National Oceanic and Atmosph. Admin., Environmental Data Service

Keller, G.V., 1966, Electrical properties of rocks and minerals: in Clark, S.P., Jr., (Ed.), Handbook of physical constants; New York, Geol. Soc. America, Inc., Memoir 97, p. 553-577

Kinnison, P.T., 1955, A survey of the groundwater of the state of Idaho: Moscow, Idaho, Idaho Bur. Mines Geol., pamphlet 103, p. 8-10

Loomis, Elias, (Ed.), Kingston, G.P., Lyman, C.S., Twining, A.C., Kirkwood, Daniel, Cornette, A., Poey, M.A., and Trask, J.B., 1859, The great auroral exhibition of August 28 to September 4, 1859: Am. J. Sci., v. 78, p. 385

Longwell, C.R., 1936, Geology of the Boulder Reservoir floor: Geol. Soc. Am. Bull., v. 47, no. 9, p. 1393-1476

Matteucci, Charles, 1864, Sur les courants electriques de la terre: Comptes Rendus Acad. Sci. Paris, v. 58, p. 942-950

Malich, C.W., 1947, Summary of largest positive and negative fluctuations, durations less than 150 seconds, horizontal intensity, H-fluxmeter, College Alaska, November 1, 1943 to January 31, 1944: in Vestine, E.H., Laporte, Lucille, Lange, Isabelle, and Scott, W.E., The geomagnetic field, its description and analysis; Washington, D.C., Carnegie Institute of Washington, Pub. 580, 390 p.

Ness, N.F., 1965, The earth's magnetic tail: J. Geophys. Res., v. 70, no. 13, p. 2989-3005

Pakiser, L.C., and Zietz, Isidore, 1965, Transcontinental crustal and upper-mantle structure: Rev. Geophys., v. 3, no. 4, p. 505-519

Piddington, J.H., 1967, A theory of auroras and the ring current: J. Atmosph. Terr. Phys., v. 29, p. 87

Porath, H., 1971a, A review of the evidence on low-resistivity layers in the earth's crust: in Heacock, J.G., The structure and physical properties of the earth's crust; Washington, D.C., Am. Geophys. Union Monograph 14, p. 127

Porath, H., 1971b, Magnetic variation anomalies and seismic low-velocity zone in the western United States: J. Geophys. Res., v. 76, no. 11, p. 2643-2648

Rooney, W.J., 1932, The significance and accuracy of measurements of earth-current potentials: Terr. Mag. Atmosph. Electricity, v. 37, p. 363

Roy, R.F., Blackwell, D.D., and Birch, Francis, 1968, Heat generation of plutonic rocks and continental heat flow provinces: Earth and Planetary Sci. Letters, v. 5, p. 1-12

Roy, R.F., Blackwell, D.D., and Decker, E.R., 1972, Continental heat flow: in Robertson, E.C., (Ed.), The nature of the solid earth; New York, McGraw-Hill, Inc., p. 506

Sass, J.H., Lachenbruch, A.H., Monroe, R.J., Greene, G.W., and Moses, T.H., Jr., 1971, Heat flow in the western United States: J. Geophys. Res., v. 76, no. 26, p. 6376-6413

Schmucker, Ulrich, 1970, An introduction to induction anomalies: J. Geomag. Geoelect., v. 22, no. 1-2, p. 9-32

Schmucker, Ulrich, 1964, Anomalies of geomagnetic variations in the southwestern United States: J. Geomag. Geoelect., v. 15, p. 193-221

Smirnov, N., 1948, Table for estimating the goodness of fit of empirical distributions: Annals Math. Stat., v. 19, p. 279-281

Stacey, F.D., 1969, Physics of the earth: New York, John Wiley and Sons, inc., 324 p.

Stewart, Balfour, 1861, On the great magnetic disturbance which extended from August 28 to September 7, 1859, as recorded by photography at the Kew Observatory: Phil. Trans. Roy. Soc. London, v. 151, p. 423-430

Stratton, J.A., 1941, Electromagnetic theory: New York, McGraw-Hill, Inc., 615 p.

Toksoz, M.N., Minear, J.W., and Julian, B.R., 1971, Temperature field and geophysical effects of a downgoing slab: J. Geophys. Res., v. 76, no. 5, p. 1113-1138

Vozoff, K., and Ellis, R.M., 1966, Magnetotelluric measurements in southern Alberta: Geophysics, v. 31, no. 6, p. 1153-1157

Walker, C.V., 1861, On magnetic storms and earth-currents: Phil. Trans. Roy. Soc. London, v. 151, p. 89-131

Wait, J.R., 1962, Theory of magnetotelluric fields: J. Res. Natl. Bur. Standards - D. Radio Propagation, v. 66D, no.5, p. 509-540

White, W.R.H., Bone M.N., and Milne W.G., 1968, Seismic refraction surveys in British Columbia, 1941-1966, a preliminary interpretation: <u>in</u> Knopoff, L., Drake, C.L., and Hart, P.J., (Eds.), <u>The crust and upper mantle of the Pacific area</u>; Washington, D.C., Am. Geophys. Union, Monograph 12, p. 81-93

Winckler, J.R., Peterson, L., Hoffman, R., and Arnoldy, R., 1959, Auroral X-rays, cosmic rays, and related phenomena during the storm of February 10-11, 1958: J. Geophys. Res., v. 64, no. 6, p. 597

World Data Center A, 1972, Catalogue of data on solar-terrestrial physics in World Data Center A subcenters; Report UAG-20: Asheville, N.C., U.S. Dept. Commerce, National Oceanic and Atmospheric Admin., Environmental Data Service, p. 38