THE SECTIONAL METHOD - A NEW APPROACH
TO SURVEY CALCULATION AND ANALYSIS FOR DIRECTIONAL DRILLING

by

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ABSTRACT

This paper presents a new derivation of a circular arc method (herein referred to as the Sectional Method of derivation) for calculation of dogleg severities and wellbore trajectories from directional surveys. Assumptions are: 1) trajectories follow a circular arc and 2) three-dimensional unit vectors[7], derived from azimuth and inclination measurements, at adjacent survey stations are coplanar. Basis for this new derivation stems from an analysis of projections of trignometric entities onto vertical and horizontal planes. A consequence of this approach is a graphical presentation, not previously published, of a derivation of directional drilling equations for a circular arc interpretation of a wellbore. This presentation provides a new level of understanding of concepts for a circular arc interpretation. Such an understanding is essential to all concerned with directional drilling for both proper application of directional drilling equations and selection of a method which best describes a particular wellbore.

The dogleg angle equation presented in the Minimum Curvature section of the API Bulletin D20[3] was obtained from Blythe[11]. His derivation is unpublished. His equation is derived from Taylor's Minimum Curvature Method[1]. A derivation of his equation is published within this thesis. A second derivation is published to demonstrate the equivalency of this equation with the equation developed from the Sectional Method. A summary of example calculations of trajectories and dogleg severities is presented for comparison of the Sectional Method and the Minimum Curvature Method. A second summary is presented which compares dogleg severities calculated with the Sectional Method equation with those calculated with equations published in the API Bulletin D20.[3]

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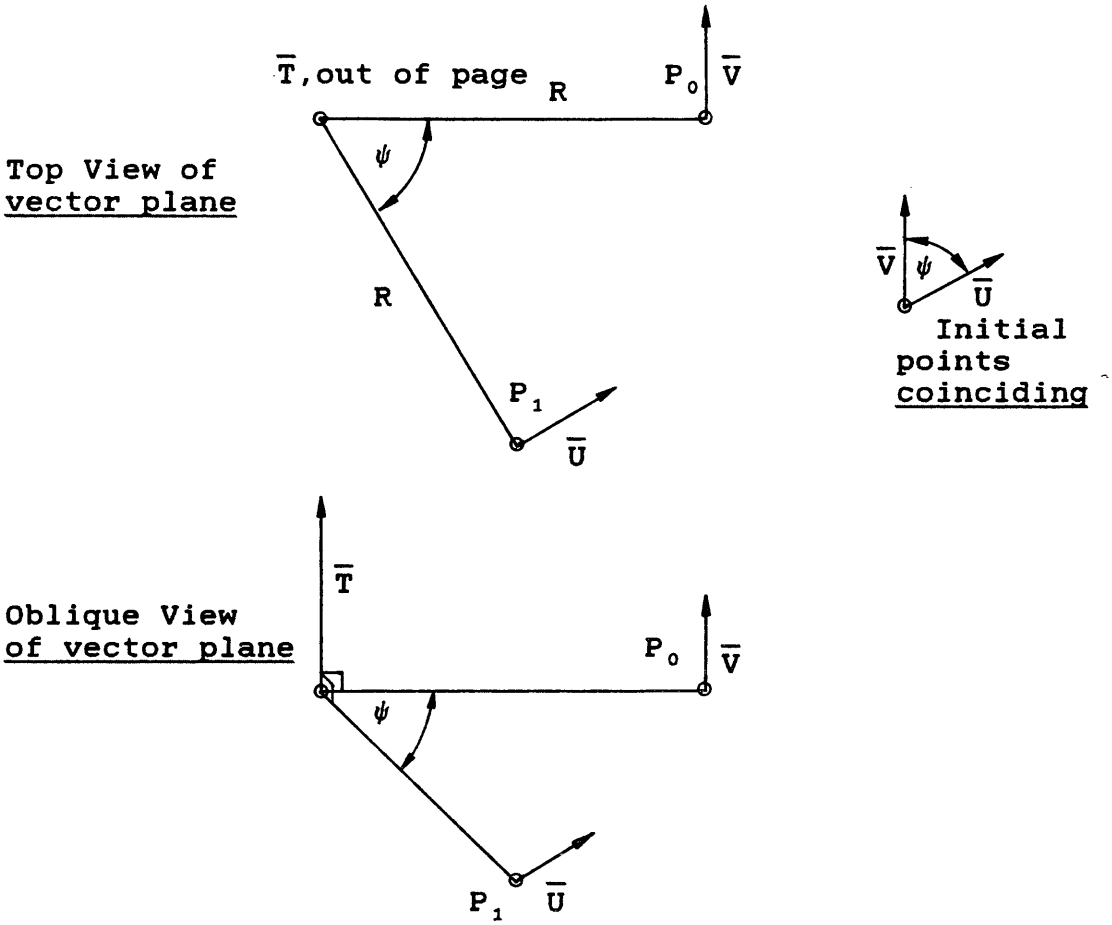
INTRODUCTION

A recognized definition of a surveying method in directional drilling is a set of transformation equations which transform an entity from spherical space to rectangular space. The Sectional Method, derived herein, is a directional surveying method. It is assumed that the wellbore follows a circular arc between survey stations. The value of this method is that its derivation is based on graphics rather than abstract equations employed by other methods. For correlation and ease of comparison, terms and symbols associated with a circular arc in Taylor's derivation of his Minimum Curvature Method[1] are observed. As described in Taylor's derivation, the plane of turn (Figure 1) is defined as "... the plane through P_0 and normal to a vector that is perpendicular to the directions U= (Ux,Uy,Uz) and V= (Vx,Vy,Vz) at P_1 and P_0, respectively." P_0 and P_1 are survey stations at each end of his circular arc.

Both the Sectional Method and Taylor's Minimum Curvature Method describe a circular arc trajectory. Like all other methods published in API Bulletin D20[3], neither of these two methods address the problem which occurs if actual wellbore curvature cannot be approximated by a circular arc. It is not intended, in this paper, to rederive all other



From reference number 12:

The scalar (or dot) product of $\overline{U}$ and $\overline{V}$ is written as $\overline{U} \cdot \overline{V}$ and is defined: $\overline{U} \cdot \overline{V} = |U|\ |V| \text{ Cos}(\psi)$. Angle ψ is the angle between $\overline{U}$ and $\overline{V}$ "computed when the vectors have their initial point coinciding". As noted in the top figure, angle ψ has the same value as an angle between the radii, R, whose tangents are $\overline{U}$ and $\overline{V}$. Thus, if one defines a dot product of two vectors in a three dimensional space, then a plane containing the two vectors is also defined. Vector $\overline{T}$ is the vector normal to the vector plane containing $\overline{U}$ and $\overline{V}$ and defined from crossproduct, $\overline{U}\text{x}\overline{V}$, of the two vectors. The direction of vector $\overline{T}$ is the direction referred to by Taylor in development of his Minimum Curvature Method. His plane through P_0 and normal to $\overline{T}$ results from mathematical definition associated with the dot product.

FIG. 1: VECTOR CONCEPTS

methods published by the API for directional calculations. However, to provide a basis for comparison of the Sectional Method with other common methods, a short description of methods listed in API Bulletin D20[3] is provided below.

As noted in the Bulletin, "There are several known methods of computing directional surveys. However, of these methods, only six appear to be distinct and commonly used. These methods are the Tangential, Average Angle, Balanced Tangential, Mercury, Minimum Curvature and Radius of Curvature." Because the Mercury Method does not provide a dogleg severity equation and is a combination of the Tangential and Balanced Tangential methods, it is not mentioned further in this thesis.

Origin of the Tangential Method is not known. The first published evalution of it was by Walstrom[6] in 1971. Bulletin D20 describes the method in this manner, "By definition, the Tangential Method uses only the inclination and direction angles measured at the lower end of the course length. The wellbore path is assumed to be tangential to these angles throughout the course." Curvature is not described with this method. Thus, an equation was developed by Wilson[2] to be employed to calculate dogleg severity when the Tangential Method was applied for interpretation of wellbore survey data. And because no curvature is described with the method, the Tangential Method "will result in greater horizontal displacement and indicate less vertical displacement for the build section and oppositely for the dropoff section of the wellbore." The Average Angle and Balanced Tangential Methods are also limited in that they, too, do not interpret any curvature between survey stations. They are simply methods wherein errors caused by "straight line" interpretation is reduced in curved sections of the wellbore as compared to the Tangential Method.

To minimize misinterpretation between two similar methods, Bulletin D20 descriptions of key characteristics of the Radius of Curvature and Minimum Curvature Methods are quoted in full. Description for Radius of Curvature is written as, "The Radius of Curvature Method uses sets of angles measured at the upper and lower ends of the course length to generate a space curve (representing the wellbore path) that has the shape of a spherical arc passing through the measured angles at both the upper and lower ends of the measured course. This method assumes that the wellbore is of a smooth curvature, which describes the segment by the three-dimensional vectors (survey points) with a known distance between the survey points." And the description for Minimum Curvature is quoted, "The Minimum Curvature Method, like the Radius of Curvature Method, uses the space vectors defined by inclination and direction measurements which are smoothed onto the wellbore. In this method, smoothing is done by the use of a ratio factor which is defined by the curvature (dogleg) of the wellbore section. The method does produce a circular arc as does the Radius of Curvature Method; this is not, however, an assumption of the method but a result of minimizing the total curvature within the physical constraints on a section of wellbore."

Comment on the above descriptions is supplied from references[14]: "Mathematical methods, and the notation which describes them, are deliberate creations designed to communicate ideas as precisely as possible. But the concepts used in mathematical procedures frequently appear paradoxical and even selfcontradictory." The following definitions are quoted from the same source to help clarify comparison of the Minimum Curvature Method with the Radius of Curvature Method: "A space curve which does not lie in a plane is called a skew or twisted curve. For example, a circle drawn on a sphere is a plane spatial curve; an ellipse projected on a sphere is a skew spatial curve."

The Radius of Curvature Method assumes a wellbore trajectory "to be curved in either or both vertical and horizontal projections". These curvatures are considered to be circular arcs (from wellbore description associated with

Figures 4 and 5 by Wilson[2]). Consequently, the method does not necessarily "produce a circular arc", as described in API Bulletin D20. The Radius of Curvature Method might be described more accurately as one which produces, 1) a plane spatial curve when data are most closely described by a horizontal or vertical circular arc, 2) a skew spatial curve when data cannot be described as in the first case. The lack of ability of this Method to describe a circular arc in an oblique plane is demonstrated by the fact[15] that there is no orientation of a circular arc, other than vertical or horizontal, which will produce a circular arc projection in the vertical and horizontal planes as required by the Method. Though proof will not be presented in this thesis, it is this author's opinion that the only skew spatial curve which will produce a circular arc in horizontal and vertical projection is a helix. As a demonstration of Radius of Curvature interpretation, one can show that bottom hole location calculated from two survey stations on a helical arc does not change regardless of how many additional survey stations, between the first two, are added to the list of survey data for the calculation. Thus, the Radius of Curvature Method might be said to "produce a helical arc" in some cases.

As noted above, both the Sectional Method and the

Minimum Curvature Method "produce a circular arc". Thus, data tables calculated from application of the two methods should be identical. Any difference in either calculated trajectory or dogleg severity between the methods, for a particular set of survey data, can only be explained by lack of ability of one of the applications to describe a circular arc consistently.

In summary, the assumption of a circular arc trajectory is made to provide a basis for development of trignometric relationships essential in the derivation of the Sectional Method. Based on this assumption, a "plane of turn" and a "center of turn" are described. In addition, tangents to the arc at each survey station, contained in the plane of turn, can be defined. Line segments described from these basic concepts provide a basis for derivation of equations for calculation of dogleg severity and trajectories between survey stations.

THE SECTIONAL METHOD

Space and Terminology

A three dimensional space is assumed in this thesis for development of the Sectional Method. This space is described by the orthogonal axes shown in Figure 2. Point A is the origin of the axes. The positive X direction is east and the positive Y direction is north. The positive Z direction is downward, in a direction of increasing True Vertical Depth, TVD.

Line segments are referenced by their endpoints as in the following example: P1-P2 is the line segment between points P1 and P2. Triangles are referenced in the same manner except that three points are used to indicate the three line segments which form the sides of the triangle. Where right triangles are mentioned, the right angle will be located at the middle point as in the following example: TI-A-P2 is the notation for a right triangle with the right angle located at point A. Angles, in general, are referenced the same as right triangles (angle PP1-A-P2 is at point A and between line segments PP1-A and P2-A).

Concepts

As shown in Figure 2, a "turn plane" is oriented at an

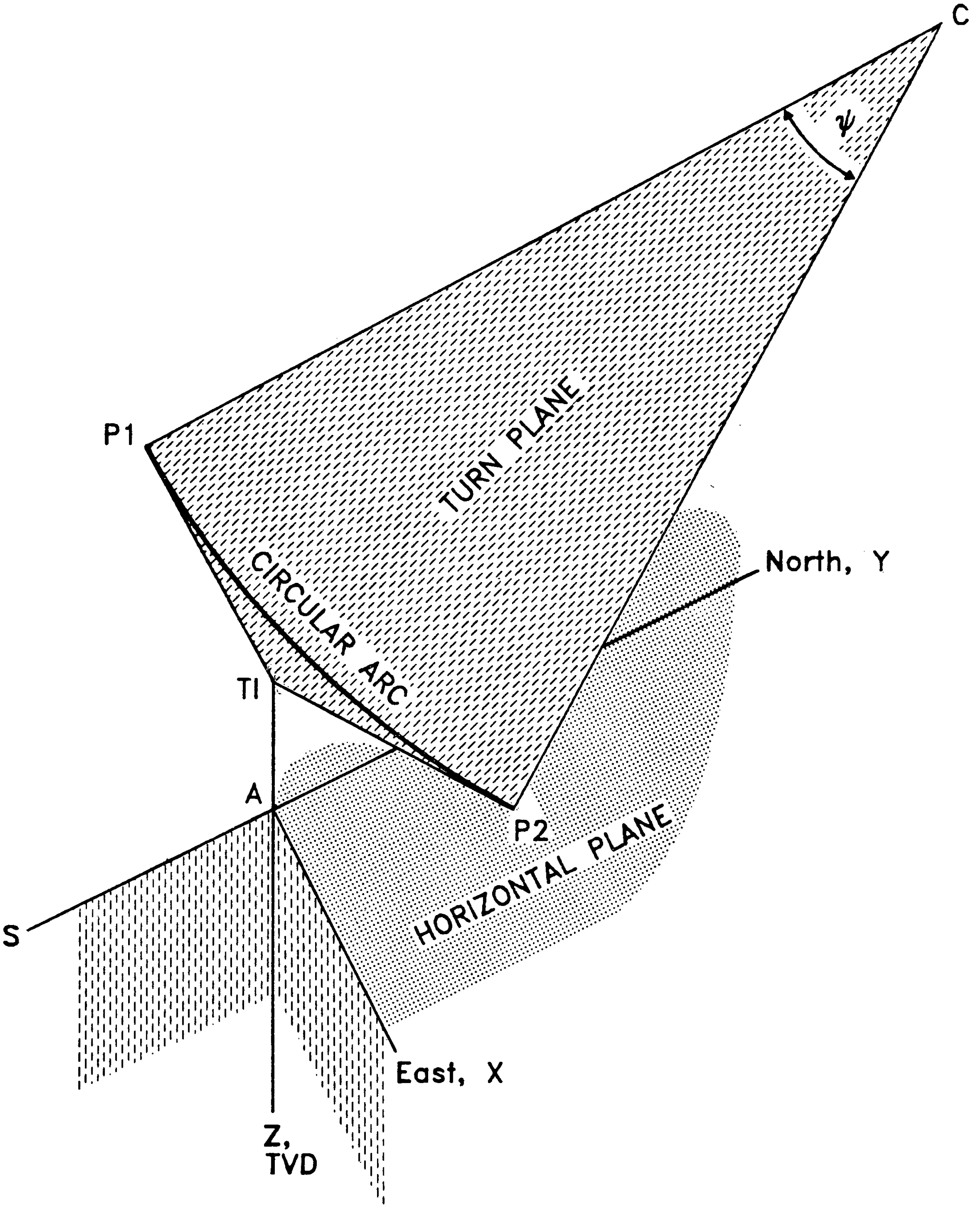


FIG. 2: TURN PLANE ORIENTATION

oblique angle above a "horizontal plane". Point P2 is a point in common with the two planes. Points, C, P1 and TI, are contained in the turn plane and are above the horizontal plane. The turn plane contains a "circular arc" which is assumed to represent a wellbore trajectory. The point, C, is the center of turn of the arc. Points, P1 and P2, represent the first and second survey stations respectively. P1 is above P2. The angle, ψ, is the angle swept by the radius of the circular arc between points, P1 and P2. Point TI is the point of intersection of tangents drawn at P1 and P2.

Two labeled vertical planes are shown in Figure 3. The first plane, labeled "1st PLANE", is defined by a vertical crossection through the tangent of the arc which contains line segment P1-TI. Intersection of this plane and the horizontal plane forms line "AZM1". In a similar manner, the second vertical plane, labeled "2nd PLANE", is defined by a vertical crossection drawn through the tangent which contains line segment P2-TI. Line AZM2 is formed by the intersection of this plane and the horizontal plane.

Angles, θ_1 and θ_2, are azimuth angles measured clockwise from north in the horizontal plane. Angle $\Delta\theta$ is the absolute value of the minimum difference angle between lines AZM1 and AZM2. For example: If $\theta_1 = 245^\circ$ and $\theta_2 = 10^\circ$, then $(\theta_2 - \theta_1) = -235^\circ$ and $\Delta\theta = 125^\circ$ $(360^\circ - (+235^\circ) = 125^\circ)$.

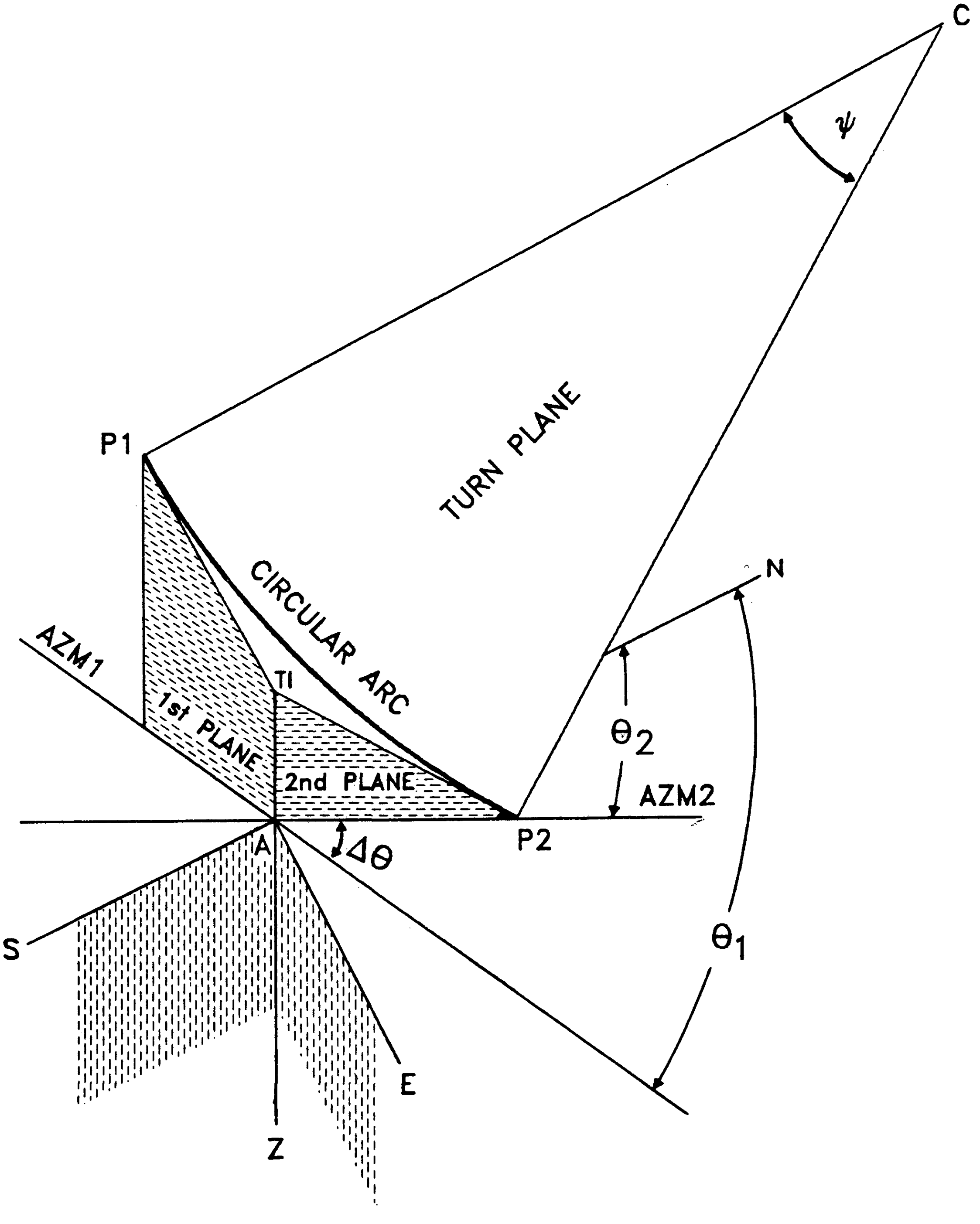


FIG. 3: VERTICAL PLANES AND AZIMUTH

Trignometric Relationships

The Turn Plane

Reference will now be to Figure 4. The "Turn Plane" contains line segments, labeled "R", which represent the radius of the circular arc. Line segments P1-TI and P2-TI are tangents to the arc at stations P1 and P2. The length of each of these tangents is equal to the value, $R \cdot \mathrm{Tan}(\psi/2)$. Observe that this value equals the length of the side opposite the angle, $\psi/2$, in both right triangles, C-P1-TI and C-P2-TI. The line segment, TI-C, labeled "Line of Bisection", is the hypotenuse for both of these triangles.

Line segment P1-B-P2 is the chord of the circular arc and the "straight line" distance between stations P1 and P2. Line segment TI-C bisects both angle ψ and segment P1-B-P2. Line segments P1-B and P2-B are created by this bisection and each is part of a right triangle wherein each adjoining radius is the hypotenuse. These triangles respectively are C-B-P1 and C-B-P2. The lengths of line segments P1-B and P2-B can each be expressed as $R \cdot \mathrm{Sin}(\psi/2)$. And, the "straight line" distance from P1 to P2 is $2R \cdot \mathrm{Sin}(\psi/2)$.

Vertical Planes

Refer now to Figure 5. Angle ϕ_1 is the inclination at the first survey station, P1. By costruction, point TI is

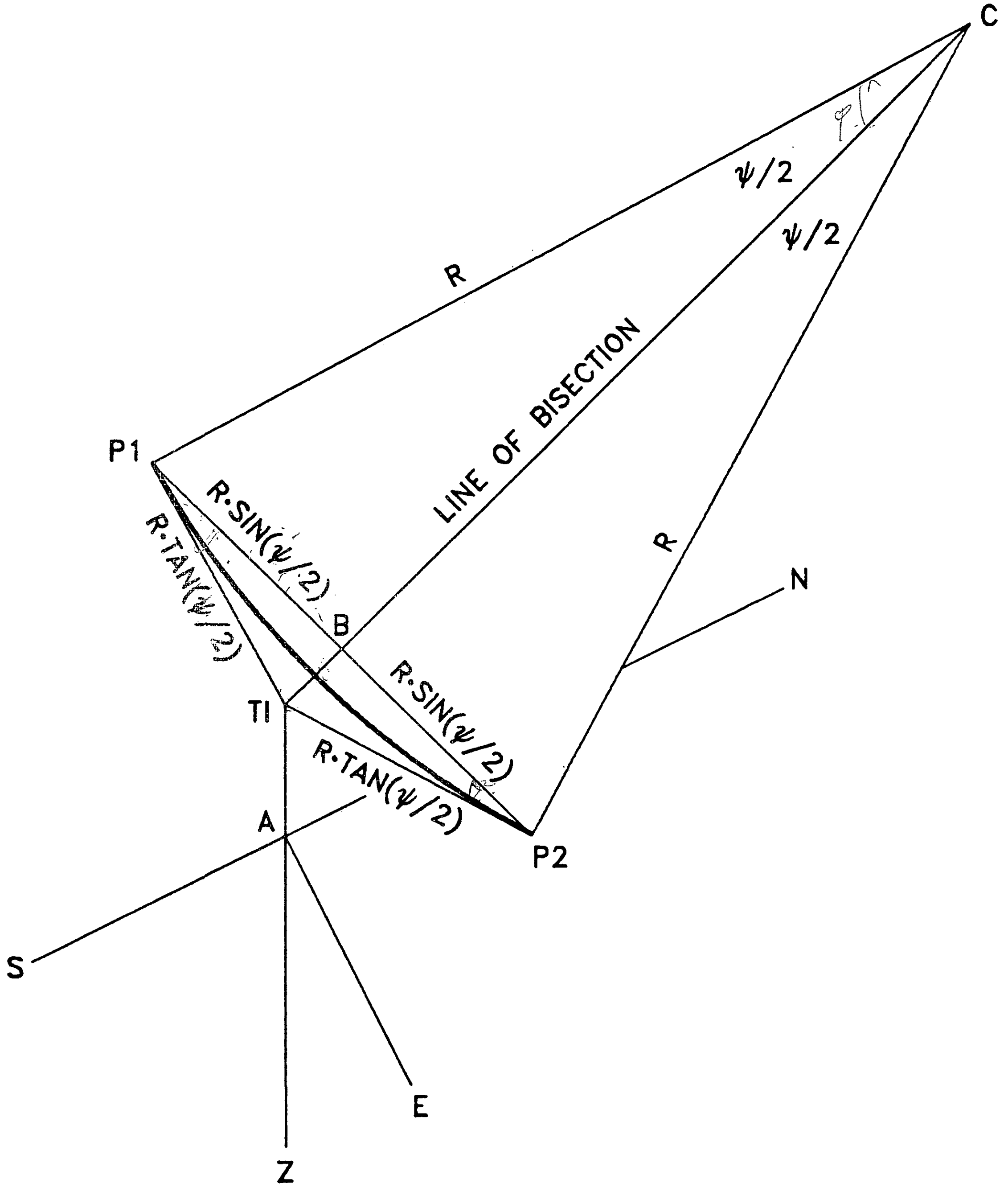


FIG. 4: TURN PLANE DETAIL

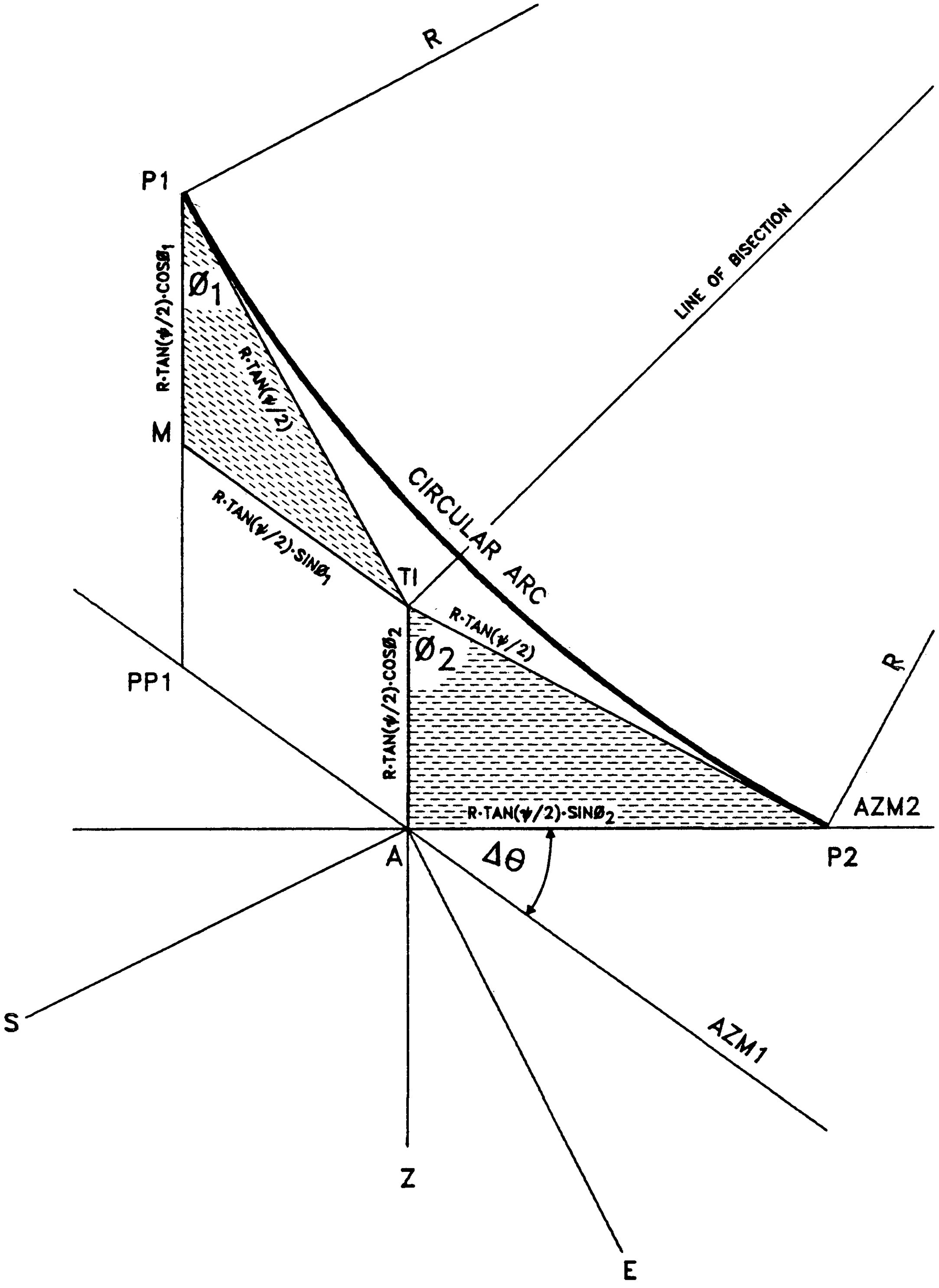


FIG. 5: VERTICAL PLANE DETAIL

directly above point A. Line segments P1-TI, M-TI and PP1-A are contained in the same vertical plane. Segment M-TI is a horizontal projection of segment P1-TI and is the side opposite angle ϕ_1 in the right triangle P1-M-TI. Thus, the length of M-TI is P1-TI$\cdot$Sinϕ_1 or, by substituting R$\cdot$Tan(ψ/2) for P1-TI, this length is R$\cdot$Tan(ψ/2)$\cdot$Sinϕ_1. Line segment P1-M is a vertical projection of P1-TI. Its length is R$\cdot$Tan(ψ/2)$\cdot$Cosϕ_1.

In a similar analysis, ϕ_2 is the inclination angle at P2, the second survey station. Line segment P2-A is the side opposite angle ϕ_2 in right triangle P2-A-TI. The length of this segment is R$\cdot$Tan(ψ/2)$\cdot$Sin(ϕ_2). The length of the vertical line segment, TI-A, is R$\cdot$Tan(ψ/2)$\cdot$Cos(ϕ_2). It should be noted that the expression for the length of P1-PP1, the total change in true vertical depth between the two survey stations, is R$\cdot$Tan(ψ/2)$\cdot$[Cos(ϕ_1) + Cos(ϕ_2)].

The Horizontal Plane

Reference is now made to Figure 6 where consideration is given to triangle PP1-A-P2. The length of line segment PP1-P2 in this triangle is given by the Law of Cosines. By substitution of -Cos$\Delta\theta$ for the COSINE of angle PP1-A-P2, the Law of Cosines for the length of PP1-P2 is written:

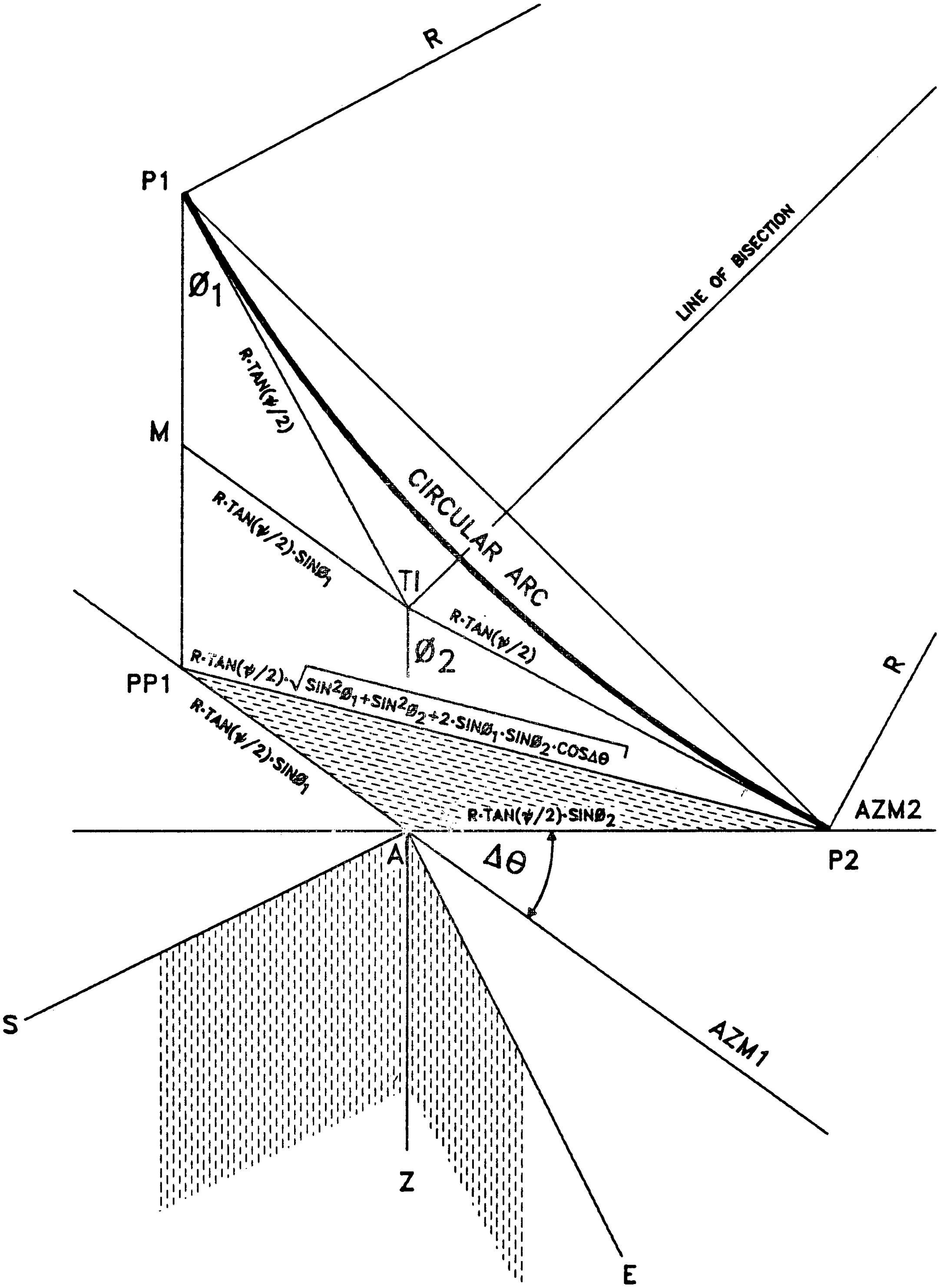


FIG. 6: HORIZONTAL PLANE DETAIL

$$\underline{PP1\text{-}P2} = R\cdot \mathrm{Tan}(\psi/2)\cdot\sqrt{\mathrm{Sin}^2\phi_1 + \mathrm{Sin}^2\phi_2 + 2\cdot\mathrm{Sin}\phi_1\cdot\mathrm{Sin}\phi_2\cdot\mathrm{Cos}\Delta\theta}$$

Dogleg Severity

Refer to Figure 7. Expressions now exist for the sides of the right triangle, P1-PP1-P2. The equation for angle ψ is developed in the following:

Substitution of expressions for sides of the right triangle P1-PP1-P2 into the Pythagorean Theorem yields.

$$(1)\quad [2R\cdot\mathrm{Sin}(\psi/2)]^2 = [R\cdot\mathrm{Tan}(\psi/2)\cdot(\mathrm{Cos}(\phi_1) + \mathrm{Cos}(\phi_2))]^2 + [R\cdot\mathrm{Tan}(\psi/2)\cdot\sqrt{\mathrm{Sin}^2\phi_1 + \mathrm{Sin}^2\phi_2 + 2\cdot\mathrm{Sin}\phi_1\cdot\mathrm{Sin}\phi_2\cdot\mathrm{Cos}\Delta\theta}]^2$$

Substitution of $\mathrm{Sin}(\psi/2)/\mathrm{Cos}(\psi/2)$ for $\mathrm{Tan}(\psi/2)$ and division of both sides by $[R\cdot\mathrm{Sin}(\psi/2)]^2$ yields:

$$(2)\quad 4 = [(\mathrm{Cos}\phi_1 + \mathrm{Cos}\phi_2) / \mathrm{Cos}(\psi/2)]^2 + [\sqrt{\mathrm{Sin}^2\phi_1 + \mathrm{Sin}^2\phi_2 + 2\cdot\mathrm{Sin}\phi_1\cdot\mathrm{Sin}\phi_2\cdot\mathrm{Cos}\Delta\theta} / \mathrm{Cos}(\psi/2)]^2$$

Simplification and solution for $\mathrm{Cos}(\psi/2)$ results in:

$$(3)\quad \mathrm{Cos}(\psi/2) = \sqrt{1/4\cdot[(\mathrm{Cos}\phi_1 + \mathrm{Cos}\phi_2)^2 + \mathrm{Sin}^2\phi_1 + \mathrm{Sin}^2\phi_2 + 2\cdot\mathrm{Sin}\phi_1\cdot\mathrm{Sin}\phi_2\cdot\mathrm{Cos}\Delta\theta]}$$

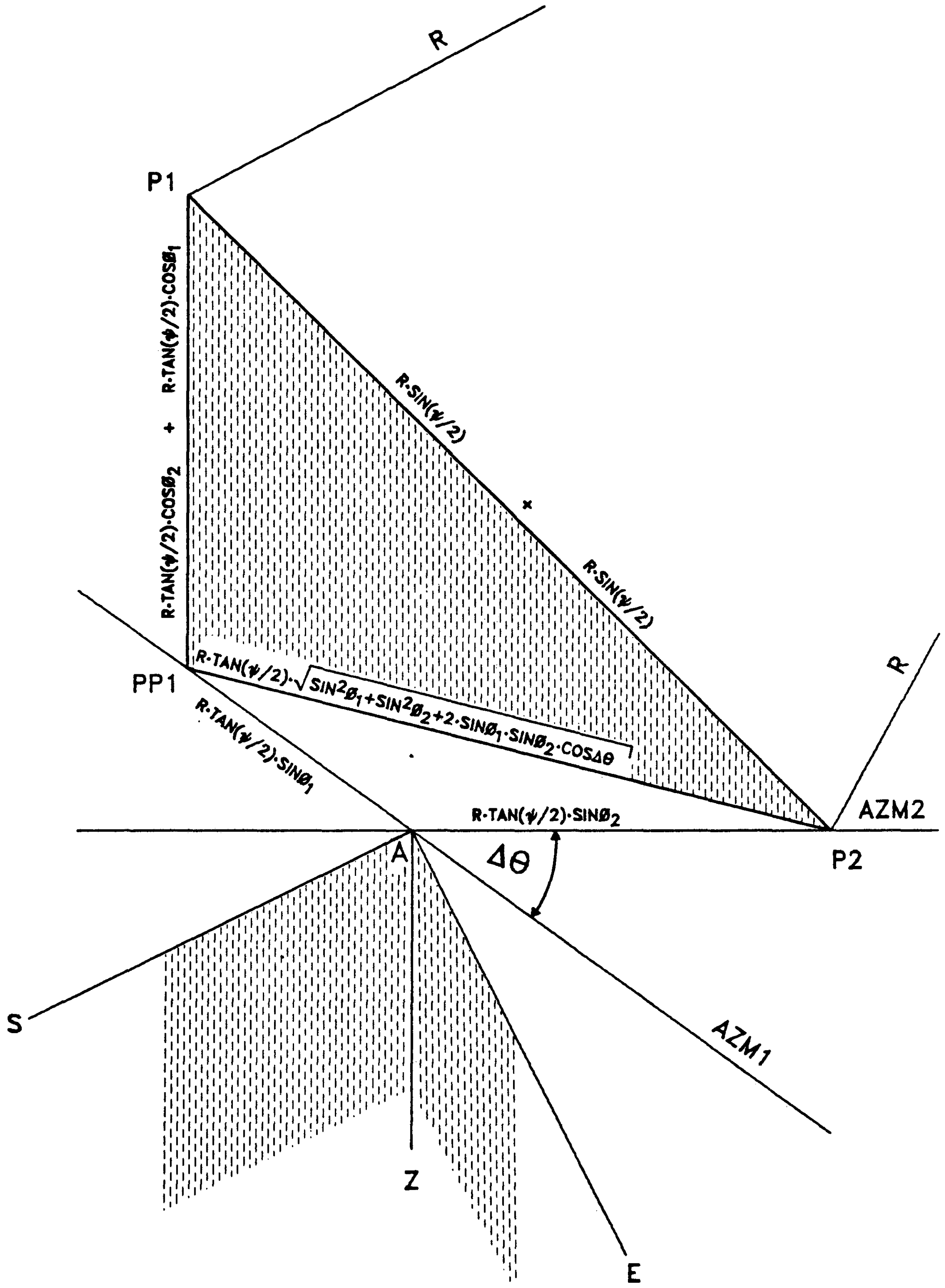


FIG. 7: DOGLEG SEVERITY DETAIL

Expansion of the term, $(\mathrm{Cos}\phi_1 + \mathrm{Cos}\ \phi_2)^2$, substitution of the equality, $\mathrm{Sin}^2\phi + \mathrm{Cos}^2\phi = 1$, and collection of terms gives:

$$(4)\qquad \mathrm{Cos}(\psi/2) = \sqrt{1/2\cdot(1 + \mathrm{Cos}\phi_1\cdot\mathrm{Cos}\phi_2 + \mathrm{Sin}\phi_1\cdot\mathrm{Sin}\phi_2\cdot\mathrm{Cos}\Delta\theta)}$$

And solving for ψ results in:

$$(5)\qquad \psi = 2\cdot\mathrm{Cos}^{-1}\sqrt{1/2\cdot(1 + \mathrm{Cos}\phi_1\cdot\mathrm{Cos}\phi_2 + \mathrm{Sin}\phi_1\cdot\mathrm{Sin}\phi_2\cdot\mathrm{Cos}\Delta\theta)}$$

Dogleg severity equals the dogleg angle, ψ, divided by arc length expressed in hundreds of feet. Arc length is assumed here to be equal to the difference in measured depth, ΔMD, between survey stations. In oilfield units, the equation is:

$$(6)\qquad \mathrm{DLS} = (200/\Delta\mathrm{MD})\cdot \mathrm{Cos}^{-1}\sqrt{1/2\cdot(1 + \mathrm{Cos}\phi_1\cdot\mathrm{Cos}\phi_2 + \mathrm{Sin}\phi_1\cdot\mathrm{Sin}\phi_2\cdot\mathrm{Cos}\Delta\theta)}$$

<u>Displacements</u>

Refer to Figure 8. From this figure, it should be observed that the north/south displacement of survey station P2 from survey station P1 is expressed:

$$(7)\qquad \Delta N/S = \underline{PP1\text{-}A}\cdot\mathrm{Cos}\theta_1 + \underline{A\text{-}P2}\cdot\mathrm{Cos}\theta_2$$

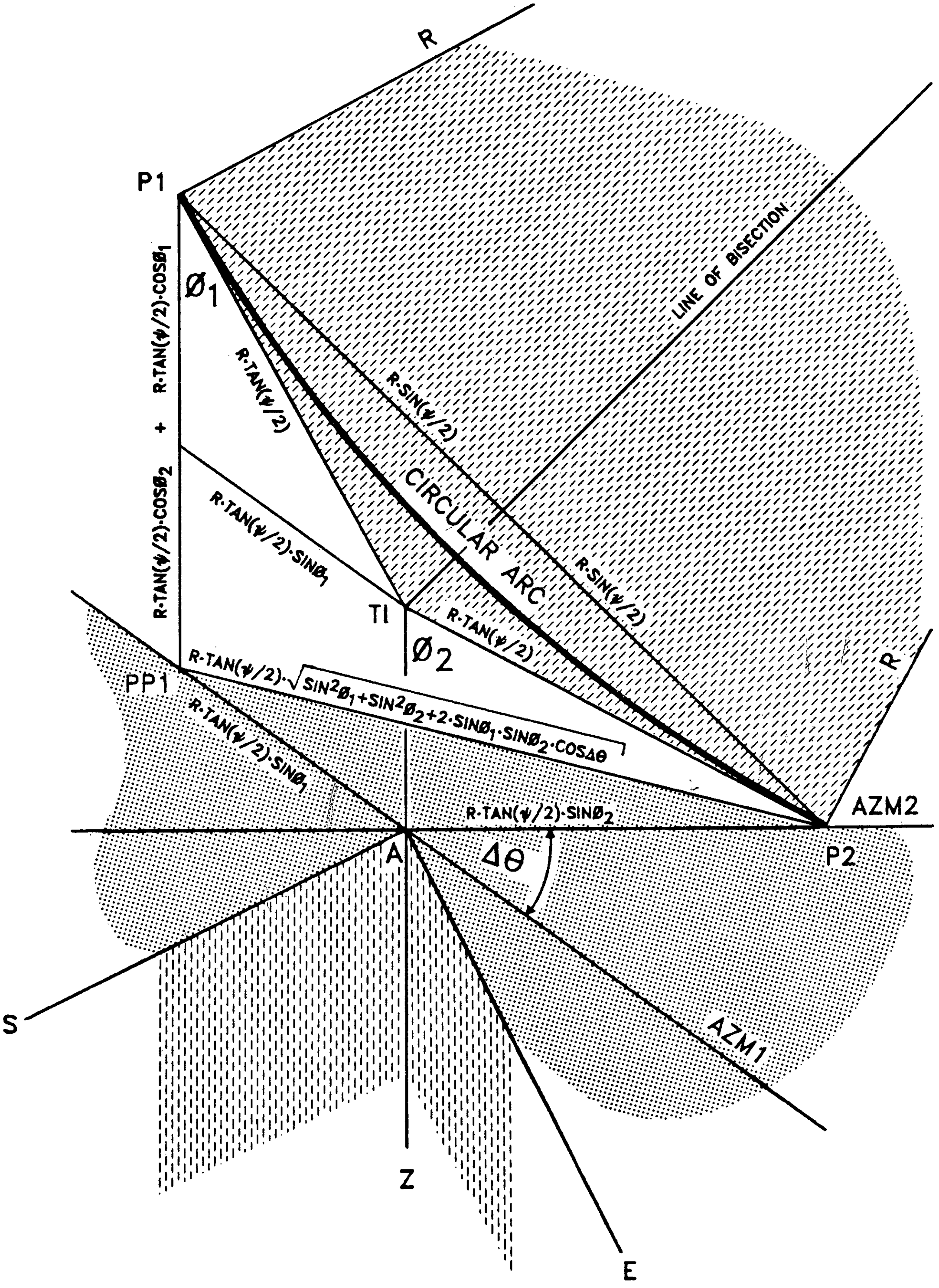


FIG. 8: DISPLACEMENT DETAIL

Substitution of expressions for the line segments in equation 7 yields:

$$(8)\quad \Delta N/S =$$

$$[R\cdot\mathrm{Tan}(\psi/2)\cdot\mathrm{Sin}\phi_1]\cdot\mathrm{Cos}\theta_1 + [R\cdot\mathrm{Tan}(\psi/2)\cdot\mathrm{Sin}\phi_2]\cdot\mathrm{Cos}\theta_2$$

$$= R\cdot\mathrm{Tan}(\psi/2)\cdot(\mathrm{Sin}\phi_1\cdot\mathrm{Cos}\theta_1 + \mathrm{Sin}\phi_2\cdot\mathrm{Cos}\theta_2)$$

In terms of known variables and in oilfield units, the radius of a circular arc is $(180\cdot\Delta MD)/(\pi\cdot\psi)$. Substitution of this expression for R in equation 8 gives:

$$(9)\quad \Delta N/S = [(180\cdot\Delta MD)/(\pi\cdot\psi)] \cdot \mathrm{Tan}(\psi/2) \cdot (\mathrm{Sin}\phi_1\cdot\mathrm{Cos}\theta_1 + \mathrm{Sin}\phi_2\cdot\mathrm{Cos}\theta_2)$$

And in a similar manner, the equation for the east/west displacement can be expressed:

$$(10)\quad \Delta E/W = [(180\cdot\Delta MD)/(\pi\cdot\psi)] \cdot \mathrm{Tan}(\psi/2) \cdot (\mathrm{Sin}\phi_1\cdot\mathrm{Sin}\theta_1 + \mathrm{Sin}\phi_2\cdot\mathrm{Sin}\theta_2)$$

It was demonstrated in the "Vertical Planes" section that total change in True Vertical Depth, TVD, can be expressed:

$$(11)\quad \Delta TVD = R\cdot\mathrm{Tan}(\psi/2)\cdot[\mathrm{Cos}(\phi_1) + \mathrm{Cos}(\phi_2)]$$

Again substituting known variables for R, the change in True Vertical Depth is:

(12) $\Delta TVD = [(180 \cdot \Delta MD)/(\pi \cdot \psi)] \cdot \mathrm{Tan}(\psi/2) \cdot (\mathrm{Cos}\phi_1 + \mathrm{Cos}\phi_2)$

It should be noted that if ϕ_1 is equal in value to ϕ_2 and θ_1 is equal in value to θ_2, then the dogleg angle, ψ, is zero. Division by zero in equations 9, 10 and 12 for this case makes them invalid. It is recognized, however, that the familiar Tangential Method[3] provides the following equations which produce no error for this case:

$$\Delta N/S = \Delta MD \cdot \mathrm{Sin}\phi_2 \cdot \mathrm{Cos}\theta_2$$

$$\Delta E/W = \Delta MD \cdot \mathrm{Sin}\phi_2 \cdot \mathrm{Sin}\theta_2$$

$$\Delta TVD = \Delta MD \cdot \mathrm{Cos}\phi_2$$

COMPARISON OF METHODS

Mathematical Comparison

The following mathematical analyses are performed to demonstrate equivalency of the API equation for the Minimum Curvature dogleg angle and the Sectional Method equation: 1) An estimation of Blythe's derivation of the API equation is made; 2) This equation is modified by substitution of trignometric identities to give the Sectional Method form of the dogleg angle equation. These analyses follow:

From API Bulletin D20, the Minimum Curvature equation for dogleg angle is:

$$(13)\ \mathrm{Cos}(\psi) = \mathrm{Cos}(\phi_1 - \phi_2) - \mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\phi_2)\cdot[1 - \mathrm{Cos}(\theta_2 - \theta_1)]$$

From SPE paper #3362 by Mason and Taylor, the dogleg angle, ψ, is defined and solved for by finding the dot product of the two "unit tangent" vectors, $\bar{U}$ and $\bar{V}$, located at adjacent survey stations. The direction cosines for these unit vectors are expressed as:

$\bar{U}x = \mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\theta_1)$ $\bar{V}x = \mathrm{Sin}(\phi_2)\cdot\mathrm{Sin}(\theta_2)$

$\bar{U}y = \mathrm{Sin}(\phi_1)\cdot\mathrm{Cos}(\theta_1)$ $\bar{V}y = \mathrm{Sin}(\phi_2)\cdot\mathrm{Cos}(\theta_2)$

$\bar{U}z = \mathrm{Cos}(\phi_1)$ $\bar{V}z = \mathrm{Cos}(\phi_2)$

Definition of the dot product gives:

(14) $\overline{U}\cdot\overline{V} = |\overline{U}||\overline{V}|\mathrm{Cos}(\psi)$

Because the magnitude of all unit vectors is one, equation 14 can be written as:

(15) $\overline{U}\cdot\overline{V} = \mathrm{Cos}(\psi)$

Expansion of the above dot product gives:

(16) $$\begin{aligned}\overline{U}\cdot\overline{V} &= \overline{U}x\cdot\overline{V}x + \overline{U}y\cdot\overline{V}y + \overline{U}z\cdot\overline{V}z \\ &= \mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\theta_1)\cdot\mathrm{Sin}(\phi_2)\cdot\mathrm{Sin}(\theta_2) + \\ &\qquad \mathrm{Sin}(\phi_1)\cdot\mathrm{Cos}(\theta_1)\cdot\mathrm{Sin}(\phi_2)\cdot\mathrm{Cos}(\theta_2) + \\ &\qquad\qquad \mathrm{Cos}(\phi_1)\cdot\mathrm{Cos}(\phi_2)\end{aligned}$$

Factorization of $\mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\phi_2)$ yields:

(17) $$\begin{aligned}\overline{U}\cdot\overline{V} &= \mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\phi_2)\cdot \\ &\quad [\mathrm{Sin}(\theta_1)\cdot\mathrm{Sin}(\theta_2) + \mathrm{Cos}(\theta_1)\cdot\mathrm{Cos}(\theta_2)] + \mathrm{Cos}(\phi_1)\cdot\mathrm{Cos}(\phi_2)\end{aligned}$$

Further derivation requires the following identities:

$\mathrm{Cos}(\theta_1)\cdot\mathrm{Cos}(\theta_2) = 1/2\cdot[\mathrm{Cos}(\theta_2+\theta_1) + \mathrm{Cos}(\theta_2-\theta_1)]$ and

$\mathrm{Sin}(\theta_1)\cdot\mathrm{Sin}(\theta_2) = -1/2\cdot[\mathrm{Cos}(\theta_2+\theta_1) - \mathrm{Cos}(\theta_2-\theta_1)]$

Replacement of the SINE and COSINE products in equation 17 with the right hand side of the above identities gives:

$$(18)\ \overline{U}\cdot\overline{V} = \mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\phi_2)\cdot\mathrm{Cos}(\theta_2-\theta_1) + \mathrm{Cos}(\phi_1)\cdot\mathrm{Cos}(\phi_2)$$

Application of another identity,

$$\mathrm{Cos}(\phi_1)\cdot\mathrm{Cos}(\phi_2) = \mathrm{Cos}(\theta_2-\theta_1) - \mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\phi_2),$$

and substitution of $\mathrm{Cos}(\psi)$ for $\overline{U}\cdot\overline{V}$ in equation 20 yields:

$$(19)\ \mathrm{Cos}(\psi) = \mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\phi_2)\cdot\mathrm{Cos}(\theta_2-\theta_1) + \mathrm{Cos}(\phi_2-\phi_1) - \mathrm{Sin}(\phi_2)\cdot\mathrm{Sin}(\phi_1)$$

Factorization of $\mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\phi_2)$ yields an equation identical to equation 13, the dogleg equation published in the API Bulletin D20:

$$(20)\ \mathrm{Cos}(\psi) = \mathrm{Cos}(\phi_2-\phi_1) - \mathrm{Sin}(\phi_1)\cdot\mathrm{Sin}(\phi_2)\cdot[1-\mathrm{Cos}(\theta_2-\theta_1)]$$

The above API form of the dogleg angle equation is altered to the Sectional Method equation by the substitution of the right hand side of equation 20 into the following trignometric identity for half angles:

(21) $\text{Cos}(\psi/2) = \sqrt{[1 + \text{Cos}(\psi)]/2}$

Substitution of the right hand side of equation 20 for the term, $\text{Cos}(\psi)$, in equation 21 yields:

(22) $\text{Cos}(\psi/2) =$

$$\sqrt{[1+\text{Cos}(\phi_2-\phi_1)-\text{Sin}(\phi_1)\cdot\text{Sin}(\phi_2)\cdot(1-\text{Cos}(\theta_2-\theta_1)]/2}$$

This expression can be simplified following application of the following identity:

$$\text{Cos}(\phi_1-\phi_2) = \text{Cos}(\phi_1)\cdot\text{Cos}(\phi_2) + \text{Sin}(\phi_1)\cdot\text{Sin}(\phi_2)$$

Substitution of the right hand side of the above identity for the term, $\text{Cos}(\phi_2-\phi_1)$, in equation 22 gives:

(23) $\text{Cos}(\psi/2) =$

$$\sqrt{[1 + \text{Cos}(\phi_1)\cdot\text{Cos}(\phi_2) + \text{Sin}(\phi_1)\cdot\text{Sin}(\phi_2) - \text{Sin}(\phi_1)\cdot\text{Sin}(\phi_2)\cdot(1-\text{Cos}(\theta_2-\theta_1)]/2}$$

Factorization of $\text{Sin}(\phi_1)\cdot\text{Sin}(\phi_2)$ yields the following equality with the Sectional Method equation expressed in equation 4:

$$\text{Cos}(\psi/2) = \sqrt{1/2\cdot[1 + \text{Cos}\phi_1\cdot\text{Cos}\phi_2 + \text{Sin}\phi_1\cdot\text{Sin}\phi_2\cdot\text{Cos}(\theta_2-\theta_1)]}$$

For comparison with the Sectional Method equations, the displacement equations listed in the API Bulletin's Minimum Curvature section are as follows:

$$\Delta N/S =$$

$$[\Delta MD/2] \cdot [SIN(\phi_1)\cdot COS(\theta_1) + SIN(\phi_2)\cdot COS(\theta_2)] \cdot RF$$

$$\Delta E/W =$$

$$[\Delta MD/2] \cdot [SIN(\phi_1)\cdot SIN(\theta_1) + SIN(\phi_2)\cdot SIN(\theta_2)] \cdot RF$$

$$\Delta TVD =$$

$$[\Delta MD/2] \cdot [COS(\phi_1) + COS(\phi_2)] \cdot RF$$

The above term, RF, is explained as follows: "Since the measured distance (ΔMD) is measured along a curve and the inclination and direction angles (I and A) define straight line directions in space, it is necessary to smooth the straight line segments onto the curve. This is done by using the ratio factor RF." Where $RF = (2/\psi)\cdot Tan(\psi/2)$ and for small angles ($\psi<.25^{\circ}$), it is usual to set RF = 1.

For clarity, only the north/south displacements of the API and Sectional Method equations will be discussed from this point in the comparison. The known variables in the definition for the ratio factor are substituted for RF in the API north/south displacement equation to give:

(24) $\Delta N/S =$

$$[\Delta MD/\psi \cdot TAN(\psi/2)] \cdot [SIN(\phi_1) \cdot COS(\theta_1) + SIN(\phi_2) \cdot COS(\theta_2)]$$

The Sectional Method north/south displacement equation is:

(25) $\Delta N/S =$

$$[\Delta MD/\psi \cdot (180/\pi) \cdot Tan(\psi/2)] \cdot [Sin(\phi_1) \cdot Cos(\theta_1) + Sin(\phi_2) \cdot Cos(\theta_2)]$$

The only difference between equations 24 and 25 occurs in the first bracketed terms. The difference is that the Sectional Method equation has a term, $180/\pi$, not included in the API equation. This term occurred in the Sectional Method derivation where substitution was made for the radius of a circular arc ($R = 180 \cdot \Delta MD/(\pi \cdot \psi)$). The term, $180/\pi$, was required for consistency of oilfield units and is recognized as a radian-to-degree conversion factor. Thus, if this conversion is not used in the API equation for ratio factor, the dogleg angle, ψ, should be measured in radians. Though units are not specified in the API Bulletin D20, this is believed to be the only place where radians are required.

Degrees are cited in all other investigated references in the Bulletin. The Sectional Method displacement equations differ with the API equations only in this radian-to-degree conversion factor.

Application Comparison

For further comparison, the Sectional Method was applied to data published with Taylor's original paper[1], wherein the Minimum Curvature Method was derived. Table 1 is a summary for comparison. Data for the Minimum Curvature Method were listed as published instead of being calculated with the standard equations published in API Bulletin D20. It should be noted from Table 1 that data calculated from the two methods are equal. All dogleg severities agree to four decimal place accuracy. Bottomhole location agrees to within 0.04 feet. This minor difference is due to the fact that the application in the original paper assumed a straight line for low values of dogleg severity.

Thus, the Sectional Method has been demonstrated to be in agreement with the Minimum Curvature Method. The only differences occur in approach taken in derivation and the fact that the Minimum Curvature Method solves for the full angle, ψ, while the Sectional Method actually solves for the bisection angle, $\psi/2$, and doubles this to arrive at the

TABLE 1

Comparison of Survey Methods

Base Survey Data			Minimum Curvature Solution				Sectional Solution			
INC	AZM	Measured Depth	Vertical Depth	RelativeCoords.		Dogleg Severity Deg. per 100 Feet	Vertical Depth	RelativeCoords.		Dogleg Severity Deg. per 100 Feet
				North= + South= -	East= + West= -			North= + South= -	East= + West= -	
0.00	360.00	0.00	0.00	2.56	-45.00	-	0.00	2.56	-45.00	-
0.00	360.00	50.00	50.00	2.56	-45.00	0.0000	50.00	2.56	-45.00	.0000
0.25	16.00	100.00	100.00	2.66	-44.97	0.5000	100.00	2.66	-44.97	0.5000
0.25	350.00	150.00	150.00	2.88	-44.96	0.2250	150.00	2.88	-44.96	0.2250
0.50	44.00	200.00	200.00	3.14	-44.83	0.8138	200.00	3.14	-44.83	0.8138
0.25	57.00	250.00	250.00	3.36	-44.58	0.5250	250.00	3.36	-44.58	0.5250
1.75	282.00	300.00	299.99	3.58	-45.24	3.8697	299.99	3.58	-45.24	3.8697
1.50	285.00	350.00	349.96	3.90	-46.62	0.5280	349.97	3.90	-46.62	0.5280
1.75	276.00	400.00	399.94	4.15	-48.01	0.7131	399.95	4.15	-48.01	0.7131
2.25	276.00	450.00	449.91	4.34	-49.74	1.0000	449.92	4.34	-49.74	1.0000
2.25	275.00	500.00	499.88	4.52	-51.70	0.0785	499.88	4.52	-51.70	0.0785
2.50	269.00	581.00	580.81	4.63	-55.05	0.4349	580.81	4.63	-55.05	0.4349
1.75	250.50	600.00	599.79	4.53	-55.74	5.3011	599.80	4.53	-55.74	5.3011
0.75	142.50	650.00	649.78	4.01	-56.26	4.2124	649.79	4.01	-56.26	4.2124
1.00	151.00	700.00	699.77	3.37	-55.85	0.5620	699.78	3.37	-55.85	0.5620
2.50	201.50	750.00	749.74	1.98	-56.03	4.0345	749.76	1.98	-56.03	4.0345
9.25	213.00	850.00	849.16	-6.80	-61.22	6.8183	849.18	-6.80	-61.22	6.8183
10.75	225.00	900.00	898.40	-13.47	-66.70	5.1196	898.43	-13.47	-66.70	5.1196
13.50	231.00	950.00	947.28	-20.44	-74.54	6.0429	947.31	-20.45	-74.54	6.0429
19.75	234.50	1050.00	1043.06	-37.62	-97.39	6.3270	1043.08	-37.62	-97.39	6.3270
22.25	239.00	1100.00	1089.74	-47.40	-112.38	5.9467	1089.76	-47.40	-112.38	5.9467
24.25	239.50	1150.00	1135.67	-57.49	-129.34	4.0194	1135.70	-57.49	-129.35	4.0194
26.50	234.50	1200.00	1180.85	-69.18	-147.28	6.2106	1180.88	-69.18	-147.28	6.2106
28.75	234.50	1300.00	1269.43	-96.10	-185.02	2.2500	1269.47	-96.11	-185.03	2.2500
29.00	235.00	1350.00	1313.21	-110.04	-204.74	0.6951	1313.25	-110.04	-204.74	0.6951
29.50	236.50	1400.00	1356.84	-123.78	-224.93	1.7744	1356.88	-123.79	-224.94	1.7744
29.00	237.00	1450.00	1400.46	-137.18	-245.36	1.1130	1400.50	-137.18	-245.37	1.1130
27.25	238.00	1550.00	1488.64	-162.51	-285.11	1.8123	1488.69	-162.52	-285.12	1.8123
26.25	241.00	1617.00	1548.47	-177.82	-311.07	2.5075	1548.53	-177.83	-311.09	2.5075
25.25	243.00	1663.00	1589.90	-187.21	-328.71	2.8796	1589.96	-187.22	-328.73	2.8796
21.50	243.00	1832.00	1745.00	-217.64	-388.44	2.2189	1745.06	-217.65	-388.46	2.2189
19.00	243.00	1925.00	1832.23	-232.25	-417.12	2.6882	1832.30	-232.27	-417.14	2.6882
16.75	242.00	2019.00	1921.68	-245.56	-442.71	2.4157	1921.76	-245.57	-442.73	2.4157
14.75	242.00	2114.00	2013.10	-257.67	-465.47	2.1053	2013.19	-257.68	-465.50	2.1053
13.50	243.00	2209.00	2105.22	-268.38	-486.03	1.3406	2105.32	-268.39	-486.06	1.3406
12.75	245.00	2302.00	2195.79	-277.64	-505.01	0.9427	2195.89	-277.66	-505.03	0.9427
12.00	246.00	2395.00	2286.63	-285.91	-523.14	0.8387	2286.73	-285.93	-523.16	0.8387
10.50	243.00	2430.00	2320.95	-288.84	-529.30	4.5990	2321.05	-288.86	-529.33	4.5990
12.00	241.00	2523.00	2412.16	-297.37	-545.31	1.6663	2412.26	-297.39	-545.34	1.6663
11.75	238.00	2685.00	2570.69	-314.28	-574.03	0.4111	2570.80	-314.30	-574.06	0.4111
11.75	237.00	2727.00	2611.81	-318.87	-581.24	0.4849	2611.92	-318.89	-581.27	0.4849
10.75	233.00	2791.00	2674.58	-326.02	-591.47	1.9811	2674.69	-326.03	-591.50	1.9811
11.25	233.00	2852.00	2734.46	-333.02	-600.77	0.8197	2734.57	-333.04	-600.80	0.8197
10.75	233.00	2914.00	2795.32	-340.14	-610.22	0.8065	2795.43	-340.16	-610.25	0.8065
10.25	232.00	2983.00	2863.16	-347.79	-620.19	0.7712	2863.27	-347.81	-620.22	0.7712
8.75	232.00	3068.00	2946.99	-356.43	-631.25	1.7647	2947.10	-356.45	-631.28	1.7647
8.00	228.00	3159.00	3037.02	-364.93	-641.41	1.0432	3037.13	-364.94	-641.44	1.0432
7.50	230.00	3230.00	3107.37	-371.21	-648.63	0.8000	3107.48	-371.23	-648.66	0.8000
7.00	228.00	3319.00	3195.65	-378.57	-657.11	0.6292	3195.77	-378.59	-657.14	0.6292
7.00	230.00	3410.00	3285.98	-385.85	-665.48	0.2678	3286.10	-385.87	-665.51	0.2678
7.50	228.00	3503.00	3378.23	-393.55	-674.33	0.6022	3378.35	-393.57	-674.36	0.6022
7.75	229.00	3592.00	3466.44	-401.38	-683.17	0.3180	3466.56	-401.39	-683.20	0.3180
8.00	230.00	3663.00	3536.77	-407.69	-690.57	0.4015	3536.89	-407.71	-690.60	0.4015

dogleg angle.

The second computational comparison made to demonstrate application of the Sectional Method concernes calculation of dogleg severities. All equations for dogleg severity listed in API Bulletin D20 were included as a basis of comparison. The survey data base from Table 1 was used for typical well surveys. Table 2 provides a summary of the calculated data. From this analysis, it is apparent that no difference exists from dogleg severities calculated with the Sectional Method and those presently in use. The differences noted under Radius of Curvature are due to utilization of the modified Radius of Curvature dogleg severity equation listed as equation 3.5 on page 7 of Bulletin D20. As explained by API, this form of the Radius of Curvature dogleg severity equation "gives no significant difference in dogleg severity from the prior equations, unless changes in inclination and direction are excessive." Noted from the data base in this example and mentioned elsewhere[10], however, "in near vertical wells, small changes in hole direction can produce large changes in azimuth."

TABLE 2

Comparison of Common Equations for Dogleg Severity

Survey Data Base			Dogleg Severity Solutions From API Bulletin D20 (Min. Curvature DLS Values From TABLE 1 - Base For % Diff. Calculation)							
INC	AZM	Measured Depth	Sectional Method		Radius of Curvature Method		Lubinski's DLS Solution		Wilson's DLS for Tangential Method	
			DLS	% Diff.	DLS	% Diff.	DLS	% Diff.	DLS	% Diff.
0.00	360.00	0.00	-	-	-	-	-	-	-	-
0.00	360.00	50.00	.0000	0.0	0.0000	0.0	0.0000	0.0	.0000	0.0
0.25	16.00	100.00	0.5000	.0	0.5191	3.8	0.5000	.0	0.5000	.0
0.25	350.00	150.00	0.2250	.0	0.2269	0.8	0.2250	.0	0.2250	.0
0.50	44.00	200.00	0.8138	.0	1.0669	31.1	0.8138	.0	0.8138	.0
0.25	57.00	250.00	0.5250	.0	0.5127	2.3	0.5250	.0	0.5250	.0
1.75	282.00	300.00	3.8697	.0	8.7742	126.7	3.8697	.0	3.8697	.0
1.50	285.00	350.00	0.5280	.0	0.5241	0.7	0.5280	.0	0.5280	.0
1.75	276.00	400.00	0.7131	.0	0.7431	4.2	0.7131	.0	0.7131	.0
2.25	276.00	450.00	1.0000	.0	1.0000	.0	1.0000	.0	1.0000	.0
2.25	275.00	500.00	0.0785	.0	0.0785	.0	0.0785	.0	0.0785	.0
2.50	269.00	581.00	0.4349	.0	0.4468	2.7	0.4349	.0	0.4349	.0
1.75	250.50	600.00	5.3011	.0	4.9420	6.8	5.3011	.0	5.3011	.0
0.75	142.50	650.00	4.2124	.0	3.4632	17.8	4.2124	.0	4.2124	.0
1.00	151.00	700.00	0.5620	.0	0.5814	3.5	0.5620	.0	0.5620	.0
2.50	201.50	750.00	4.0345	.0	5.3300	32.1	4.0345	.0	4.0345	.0
9.25	213.00	850.00	6.8183	.0	6.9985	2.6	6.8183	.0	6.8183	.0
10.75	225.00	900.00	5.1196	.0	5.3889	5.3	5.1196	.0	5.1196	.0
13.50	231.00	950.00	6.0429	.0	6.1723	2.1	6.0429	.0	6.0429	.0
19.75	234.50	1050.00	6.3270	.0	6.3609	0.5	6.3270	.0	6.3270	.0
22.25	239.00	1100.00	5.9467	.0	6.0509	1.8	5.9467	.0	5.9467	.0
24.25	239.50	1150.00	4.0194	.0	4.0210	.0	4.0194	.0	4.0194	.0
26.50	234.50	1200.00	6.2106	.0	6.3371	2.0	6.2106	.0	6.2106	.0
28.75	234.50	1300.00	2.2500	.0	2.2500	0.0	2.2500	.0	2.2500	.0
29.00	235.00	1350.00	0.6951	.0	0.6964	0.2	0.6951	.0	0.6951	.0
29.50	236.50	1400.00	1.7744	.0	1.7839	0.5	1.7744	.0	1.7744	.0
29.00	237.00	1450.00	1.1130	.0	1.1113	0.2	1.1130	.0	1.1130	.0
27.25	238.00	1550.00	1.8123	.0	1.8089	0.2	1.8123	.0	1.8123	.0
26.25	241.00	1617.00	2.5075	.0	2.4798	1.1	2.5075	.0	2.5075	.0
25.25	243.00	1663.00	2.8796	.0	2.8576	0.8	2.8796	.0	2.8796	.0
21.50	243.00	1832.00	2.2189	.0	2.2189	.0	2.2189	.0	2.2189	.0
19.00	243.00	1925.00	2.6882	.0	2.6882	.0	2.6882	.0	2.6882	.0
16.75	242.00	2019.00	2.4157	.0	2.4132	0.1	2.4157	.0	2.4157	.0
14.75	242.00	2114.00	2.1053	.0	2.1053	.0	2.1053	.0	2.1053	.0
13.50	243.00	2209.00	1.3406	.0	1.3385	0.2	1.3406	.0	1.3406	.0
12.75	245.00	2302.00	0.9427	.0	0.9357	0.7	0.9427	.0	0.9427	.0
12.00	246.00	2395.00	0.8387	.0	0.8369	0.2	0.8387	.0	0.8387	.0
10.50	243.00	2430.00	4.5990	.0	4.5615	0.8	4.5990	.0	4.5990	.0
12.00	241.00	2523.00	1.6663	.0	1.6737	0.4	1.6663	.0	1.6663	.0
11.75	238.00	2685.00	0.4111	.0	0.4075	0.9	0.4111	.0	0.4111	.0
11.75	237.00	2727.00	0.4849	.0	0.4849	.0	0.4849	.0	0.4849	.0
10.75	233.00	2791.00	1.9811	.0	1.9495	1.6	1.9811	.0	1.9811	.0
11.25	233.00	2852.00	0.8197	.0	0.8197	.0	0.8197	.0	0.8197	.0
10.75	233.00	2914.00	0.8065	.0	0.8065	.0	0.8065	.0	0.8065	.0
10.25	232.00	2983.00	0.7712	.0	0.7692	0.3	0.7712	.0	0.7712	.0
8.75	232.00	3068.00	1.7647	.0	1.7647	.0	1.7647	.0	1.7647	.0
8.00	228.00	3159.00	1.0432	.0	1.0264	1.6	1.0432	.0	1.0432	.0
7.50	230.00	3230.00	0.8000	.0	0.7944	0.7	0.8000	.0	0.8000	.0
7.00	228.00	3319.00	0.6292	.0	0.6250	0.7	0.6292	.0	0.6292	.0
7.00	230.00	3410.00	0.2678	.0	0.2678	.0	0.2678	.0	0.2678	.0
7.50	228.00	3503.00	0.6022	.0	0.6065	0.7	0.6022	.0	0.6022	.0
7.75	229.00	3592.00	0.3180	.0	0.3192	0.4	0.3180	.0	0.3180	.0
8.00	230.00	3663.00	0.4015	.0	0.4030	0.4	0.4015	.0	0.4015	.0

EXAMPLE CALCULATION

Typical application of the Sectional Method is shown in the example below. Calculations for displacements and dogleg severity listed in table 1 for the measured depth interval, 900-950 Ft., are detailed as follows:

The following data are given as input data for calculation:
Given at 900 Ft. MD:
Azimuth, θ_1,= 225°; Inclination, ϕ_1,= 10.75°; Measured depth, MD_1,= 900 Ft.; True Vertical Depth, TVD_1,= 898.40 Ft.; North/South Displacement, N/S_1,= -13.47 Ft.; East/West Displacement, E/W_1, = -66.70 Ft.;
Given at 950 Ft. MD:
Azimuth, θ_2,= 231°; Inclination, ϕ_2,= 13.50°; Measured depth, MD_2,= 950 Ft.;

Find: 1) Dogleg severity between survey stations; 2) True vertical depth at the second station, TVD_2; 3) North/South displacement at second station, N/S_2; 4) East/West displacement at the second station, E/W_2;

Dogleg Severity: From equation 6, the dogleg severity equation is:

$$DLS = (200/\Delta MD) \cdot \cos^{-1}\sqrt{1/2\cdot(1 + \cos\phi_1\cdot\cos\phi_2 + \sin\phi_1\cdot\sin\phi_2\cdot\cos\Delta\theta)}$$

Substituting input data for the variables gives:

$$DLS = (200/50) \cdot \cos^{-1}\sqrt{1/2\cdot[1 + \cos(10.75)\cdot\cos(13.5) + \sin(10.75)\cdot\sin(13.5)\cdot\cos(6)]}$$

DLS = 6.0429 Degrees per 100 Ft.

True Vertical Depth: From above, the dogleg angle, ψ, is

$\psi = DLS\cdot\Delta MD/100 = 3.0215$; And from equation 12,

$$\Delta TVD = [(180\cdot\Delta MD)/(\pi\cdot\psi)]\cdot\tan(\psi/2)\cdot(\cos\phi_1 + \cos\phi_2)$$

Substituting for the variables gives:

$$\Delta TVD = [(180\cdot 50)/(\pi\cdot 3.0215)]\cdot Tan(3.0215/2)\cdot[Cos(10.75)+Cos(13.5)]$$

$\Delta TVD = 48.88$ Ft., And because $TVD_2 = TVD_1 + \Delta TVD$, then

$TVD_2 = 898.40 + 48.88 = 947.28$ Ft.

<u>North/South displacement</u>: From equation 9,

$$\Delta N/S = [(180\cdot\Delta MD)/(\pi\cdot\psi)]\cdot Tan(\psi/2)\cdot(Sin\phi_1\cdot Cos\theta_1+Sin\phi_2\cdot Cos\theta_2)$$

Substituting for variables:

$$\Delta N/S = [(180\cdot 50)/(\pi\cdot 3.0215)]\cdot Tan(3.0215/2)\cdot [Sin(10.75)\cdot Cos(225)+Sin(13.5)\cdot Cos(231)]$$

$\Delta N/S = -6.97$ Ft., And because $N/S_2 = N/S_1 + \Delta N/S$, then

$N/S_2 = -13.47 + (-6.97) = -20.44$ Ft.

<u>East/West displacement</u>: From equation 10,

$$\Delta E/W = [(180\cdot\Delta MD)/(\pi\cdot\psi)]\cdot Tan(\psi/2)\cdot(Sin\phi_1\cdot Sin\theta_1+Sin\phi_2\cdot Sin\theta_2)$$

Substituting for variables:

$$\Delta E/W = [(180\cdot 50)/(\pi\cdot 3.0215)]\cdot Tan(3.0215/2)\cdot [Sin(10.75)\cdot Sin(225)+Sin(13.5)\cdot Sin(231)]$$

$\Delta E/W = -7.84$ Ft., And because $E/W_2 = E/W_1 + \Delta E/W$, then

$\Delta E/W = -66.7 + (-7.84) = -74.54$ Ft.

CONCLUSIONS

1) A graphical derivation of a circular arc method has been developed for calculation of wellbore trajectories and dogleg severities.

2) The graphic approach demonstrated in the derivation of the Sectional Method requires only an understanding of basic trignometric concepts.

3) Evidence has been presented which indicates that trajectories and dogleg severities calculated with this method are equivalent to those calculated with the Minimum Curvature Method equations published in the API Bulletin D20.

4) Evidence has been presented which indicates that the Sectional Method equation for dogleg severity is comparable with those published in API Bulletin D20.

5) The graphical techniques presented here should help in the analysis of more complex geometric problems which occur in directional drilling.

NOMENCLATURE

DLS = Dogleg Severity.

$\overline{U}$, $\overline{V}$ = Unit tangent vectors associated with the survey stations from which direction cosines ($\overline{U}x$, $\overline{U}y$, $\overline{U}z$; $\overline{V}x$, $\overline{V}y$, $\overline{V}z$) are obtained. Unitless. From Taylor[1].

ΔMD = Change in measured depth (course length).

R = Turn radius for the circular arc interpreted between survey stations.

TVD = True vertical depth.

ϕ = Inclination angle.

θ = Azimuth angle.

ψ = Angle associated with the circular arc interpreted between survey stations. Also the Dogleg Angle.

π = Mathematical constant. $\pi = 3.14159265\ldots$

Note: Oilfield units understood unless specified otherwise.

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APPENDIX A (API TERMS)

For standardization, the following terms are taken from API Bulletin D20 and have specific application to description of survey calculational methods, procedures, and concepts mentioned herein. Refer to "Nomenclature" for terms not listed in the following:

Angle of Buildup. Rate of change (degrees/100 ft) of the inclination angle in the section of the hole where the inclination from the vertical is increasing (refer to "Buildup").

Angle of Dropoff. Rate of change (degrees/100 ft) of the inclination angle in the section of the wellbore that is decreasing toward vertical (refer to "Drop Off").

Angle of Inclination. That angle in degrees, taken at one or at several points of variation, from the vertical as revealed by a deviation survey; sometimes called the inclination or angle of deviation.

Average Angle. The arithmetic average of the two angles, one at each end of the course length.

Azimuth. Direction of a course measured in a clockwise direction from 0-360 degrees with zero degrees taken as North.

Bearing. Refer to azimuth.

Bent Sub. Sub used on top of a downhole motor to give a non-straight bottom assembly. One of the connecting threads is machined at an angle to the axis of the body of the sub.

Blind Sidetrack. Refer to "Uncontrolled Sidetrack."

Borehole. The wellbore; the hole made by drilling or boring a well.

Borehole Axis. Refer to "Hole Axis."

Borehole Direction. Refers to the azimuth in which the borehole is heading.

Borehole Directional Survey. Refers to the measurements of the inclinations, azimuths, and specified depths of the stations through a section of borehole.

Borehole Survey Calculation Methods. Refer to "Wellbore Survey Calculation Methods."

Bottom-hole. Refers to the lowest or deepest part of a borehole.

Bottom-hole Location. Position of the bottom of the hole given with respect to some known surface location.

Build-and-hold Wellbore. A wellbore configuration where the inclination is increased to some terminal angle of inclination and maintained at that angle to the specified target.

Build Angle. The act of increasing the inclination of the drilled hole; the rate of change (degrees/100 ft) of the increasing angle in the hole.

Buildup. That portion of the hole in which the inclination angle is increased; rate of buildup is usually expressed as the angular increase per 100 feet of measured depth.

Calculation Methods. Refer to "Wellbore Survey Calculation Methods."

Circular-arc Method. Refer to "Wellbore Survey Calculation Methods."

Closed Traverse. Term used to indicate the closeness of two surveys, one survey going in the hole and the second survey coming out of the hole (generally applied to Gyroscope Surveys as a check on Drift).

Controlled Directional Drilling. The art and science involving the intentional deflection of a wellbore in a specific direction in order to reach a predetermined objective below the surface of the earth.

Control Cylinders. Hypothetical limits in the form of a cylinder around the planned trajectory of the wellbore and in which the borehole is to be maintained.

Course. The axis of the borehole over an interval length.

Course Bearing. The azimuth of the course.

Course Deviation. The length of a line made by projecting a course length onto a horizontal plane. In practice, the horizontal displacement between two stations regardless of direction.

Course Length. The difference in measured depth or actual hole length from one station to another.

Crooked Hole. Wellbore which has been inadvertently deviated from a straight hole.

Crooked-hole Tendency. A characteristic of the formation, bottom-hole assembly or drilling practices to cause a crooked hole to be drilled.

Curvature in the Horizontal Plane. Projection of the curvature of the hole onto a horizontal plane.

Curvature in the Vertical Plane. Projection of the curvature of the hole onto a vertical plane.

Cylinder (in directional drilling). Refer to "Control Cylinders."

Cylinder Drilling. Refers to drilling in which the course of the borehole is held within previously determined limits set by the circumference of imaginary cylinders extending from the surface location to the desired objective of the hole.

Deflected Hole. Wellbore which is intentionally deviated.

Departure. Horizontal displacement of one station from another in an east/west or north/south direction.

Deviation. Refer to "Inclination."

Deviation Type.

> **Abnormal Deviation**. Usually associated with highly faulted formations having fracture planes on either side of a fault.
>
> **Abrupt Deviation**. Usually associated with interbedded, anisotropic, or laminar formations.

Drift Deviation. Usually associated with a gradual hole angle change.

Induced Deviation. Man-made, either intentional or unintentional.

Rotational Deviation. Usually results from the bit moving in a slow spiral due to drill stem rotation.

Direction. Refers to the azimuth; direction of vertical projection of the hole onto a horizontal plane.

Direction Control. Refers to "Controlled Directional Drilling."

Directional Drilling. Refer to "Controlled Directional Drilling.

Directonal Surveillance. Refer to "Controlled Directional Drilling.

Directional Survey. Refer to "Borehole Directional Survey."

Direction of Closure. The direction of the line of closure.

Direction of Inclination. The direction of the course.

Displacement. The lateral distance from the surface location to the primary target.

Dogleg. Total curvature in the wellbore consisting of a change of inclination and/or direction between two points.

Dogleg Angle. Refer to "Dogleg Severity."

Dogleg Control Program. Program designed specifically to decrease the severity of or eliminate doglegs in a drilled hole.

Dogleg Severity. A measure of the amount of change in the inclination and/or direction of a borehole, usually expressed in degrees per 100 feet of course length.

Dogleg Types (Qualitative).

Abrupt Dogleg. A dogleg caused by a sudden change in inclination and/or direction over a short interval.

Decreasing Dogleg. A dogleg in the borehole with the change in inclination returning the borehole toward vertical.

Excessive Dogleg (Severe Dogleg). Doglegs larger than "Permissible Doglegs" (see below).

Increasing Dogleg. A dogleg in the borehole with the change in inclination increasing the angle away from vertical.

Long Dogleg. A dogleg with a gradual change in inclination and/or direction over a long interval.

Permissible Dogleg. A dogleg through which equipment and/or tubulars can be operated without sticking tools or providing excessive stresses or wear.

Double Doglegging. Refer to "S-type Well."

Downhole Motor. A power source located just above the bit to rotate the bit; usually refers to motors where bit torque is produced by utilizing either turbine or moineau principals as drilling mud is pumped through the motor.

Drift (1). Horizontal component of the distance from the surface to any certain point in the wellbore (usually the bottom of the wellbore).

Drift (2). Normally random procession in a displacement gyro caused by stray torques from bearings, pickoffs, imperfect balance, mass shift, etc..

Drift Angle (1). The angle between the axis of the wellbore and the gravity vertical (refer to "Inclination").

Drift Angle (2). More or less constant angle at which the wellbore is carried after sufficient angle has been obtained in the buildup (refer to "Maximum Angle").

Drift Survey. Refer to "Inclination Survey."

Drop Off. That portion of the wellbore in which the inclination is reduced (refer to "Angle of Drop Off").

Drop-off Interval. The interval in the wellbore where the inclination angle is purposely decreased and returned toward the vertical.

Drop-off Rate. The rate of change of the inclination in the part of the wellbore where the inclination angle is purposely returned toward the vertical, usually expressed in degrees per 100 feet of course length.

Fatigue Failure. Failure of equipment due to cumulative effect of repeated change of stress.

Fatigue Life. Number of cycles a metal can endure at a given stress level before failure will occur.

Goodman Diagram. A plot of reversed bending stress versus the average tensile stress

Grid North. The direction from any geographical location within a grid system paralleling the Universal True Meridian as determined by observation of Polaris.

Gyro Precession. Refer to "Precession" and "Drift".

Gyro Rigidity. First property of a gyroscope. Tendency of a spinning gyroscope to maintain the original axis of rotation.

Gyroscope Survey. A directional survey conducted using a gyroscope for directional control. Usually used where magnetic directional control cannot be obtained.

Hangdown. The weight of drill stem suspended below a dogleg.

Helical Buckling. Buckling in which the pipe forms a helix or spiral shape.

High-angle Holes. Generally conceded to be holes for which the inclination angle from vertical exceeds 50 degrees.

High Side of Hole. Opposite side of the hole from the low side. The low side being determined by the force of gravity, and on which side a free length of pipe would rest.

Hold Angle. The borehole inclination and direction are maintained constant.

Hole Axis. A line through the center of the hole, generally considered to be the centralized position that would be taken by a stiff tubular member inserted through that section of the hole.

Hole Azimuth Angle. The angle between north and the projection of the hole axis onto a horizontal plane. Angle is referred to either true north, magnetic north, or grid north.

Hole curvature. Refers to the changes in inclination and direction of the borehole.

Horizontal Displacement. The distance between two points that are projected onto a horizontal plane.

Inclination (Inclination Angle). The angle of the wellbore from the vertical.

Inclination Survey. A survey to obtain the angle through which the bit was deflected from the vertical during drilling operations. Usually implies a survey where no azimuth readings are taken.

Interference (1). Occurs when drilling boreholes that are deemed too close to the borehole of another well (refer to "Intersection").

Interference (2). Refer to "Magnetic Interference".

Intersection. Occurs when two wellbores meet at a common point. Usually happens when a drilling bit or drill stem contacts the casing of a previously drilled well.

Isogonic Chart. A chart showing lines of equal magnetic declination.

Keyseat. A condition wherein the borehole is abraded and extended sideways and with a diameter smaller than the drill collars and the bit; usually caused by the tool joints on the drill pipe.

Kickoff Point (Kickoff Depth). The position in the wellbore where the inclination of the hole is first purposely increased.

Kinked Double. A bent double of drill string. Forerunner to the fabricated bent sub.

Knuckle Joint. A deflection tool employing a ball-type universal joint permitting 360 degree flexure of the lower part of the tool.

KOP. Refer to "kickoff Point."

Lambert Coordinate System. A system of coordinates on a conical projection based on two standard parallels.

Latitude. The horizontal displacement of one station from another in a north or south direction.

Lead Angle. A method of setting the direction of the wellbore in anticipation of the bit walking.

Ledge. A projecting ridge or edge in the side of the wellbore. A ledge may be created with the bit by rotating off bottom. Other ledges occur when consolidated formations are exposed by wall cavings in the softer unconsolidated formations.

Line of Closure. A straight line, in a horizontal plane containing the last station of the survey, drawn from the projected surface location to the last station of the survey.

Lined Out. Refers to being on a satisfactory trajectory with the proper angle of inclination and direction.

Locked-in. Refers to the condition where the bottom-hole assembly is held relatively fixed within the borehole by the outer diameter of the assembly being nearly the same diameter as the drill bit. The inclination and direction of the borehole are maintained.

Low-angle Holes. Generally conceded to be holes for which the inclination from vertical is less than 20 degrees.

Magnetic Declination. Angular difference, east or west, at any geographical location, between true north or grid north and magnetic north.

Magnetic Interference. That condition which occurs when extraneous magnetic forces cause a magnetic compass to read incorrectly.

Magnetic North. The direction from any geographical location on the earth's surface to the north magnetic pole.

Magnetic Pole. The area on a magnet at which the magnetic field enters and leaves the magnet. Magnets have two poles, the north pole and the south pole.

Magnetic Survey. A directional survey in which the direction is detrermined by a magnetic compass detecting the earth's magnetic field.

Magnetized Drilling Assemblies. A drilling assembly may retain residual magnetism. This magnetism affects the magnetic compass.

Master Well Course Maps. Plots showing the locations of the wellbores of several wells in an area.

Maximum Angle (Maximum Average Angle). Refers to the angle of inclination to which the wellbore is held in the "locked-in" straight section.

Maximum Permissible Dogleg. Refer to "Dogleg Types, Permissible."

Measured Course. Refer to "Course Length".

Measured Depth. Actual length of the wellbore from its surface location to any specified station (refer to "Well Depth").

Mechanical Orienting Tool. A device to orient deflecting tools without the use of subsurface surveying instruments.

Meridian-seeking Compass. A gyroscopic compass that has the capability to return itself to the meridian if moved away by some disturbing force.

Methods of Orientation.

> **Direct Method.** Magnets imbedded in the non-magnetic drill collar are used to indicate the position of the tool face with respect to magnetic north. A picture of

a needle compass pointing to the magnets is superimposed on the picture of a compass pointing to magnetic north. By knowing the position of the magnets in the tool, the tool can be positioned with respect to north.

Indirect Method. A method of orienting deflecting tools in which two survey runs are needed. One showing the direction of the hole and the other showing the position of the tool.

Surface Readout. A device on the rig floor to indicate the subsurface position of the tool.

Stoking. Method to orient a tool using two pipe clamps, a telescope with a hair line, and an aligning bar to determine the orientation at each section of pipe run in the hole.

Minimum Angle. The lowest angle for easy control of azimuth in a directional well, almost universally agreed to be about 18 degrees, not less than 14 degrees, and preferably 20 degrees.

Model Error. That portion of the error that is due to the difference between the position of the real well and the position derived from the model calculation under the assumption that the survey data contain no errors.

Moderate Angle Wellbores. Generally conceded to be wellbores which have an inclination from vertical between 20 and 50 degrees.

Monel (K Monel). A permanently non-magnetic alloy used in making downhole tools.

Motion Sensor. A device used in directional surveys that senses motion and will not permit the measurements of the survey until after motion ceases.

Mud Motor. Refer to "Downhole Motor".

Mule Shoe. A shaped form used on the bottom of orienting tools to position the tool. The shape resembles a mule shoe or that of the end of a pipe cut both diagonally and concave. The shaped end forms a wedge to rotate the tool when lowered into a mating seat for the mule shoe.

Multi-shot Survey. A directional survey in which multiple data points are recorded with one trip into the wellbore. Data are usually recorded on rolls of film.

Naturally Deviated Hole. A hole which has deviated from vertical without use of deflection tools. For example, many holes will drill updip.

Near-bit Stabilizer. A stabilizer placed in the bottom-hole assembly just above the bit.

Neutral Point. This term has been defined variously as (1) the point where tension is zero; or (2) where stresses are zero.

Non-magnetic Drill Collar. A drill collar fabricated with non-magnetic material such as monel steel.

Nudge. Refers to the practice where very small deflection angles are induced to displace conductor or shallow surface pipe a short distance away from an area of well congestion.

Open Hole. Wellbore in which casing has not been set.

Open-hole Survey. A survey made in the uncased section of the borehole and not within the drill string.

Orienting Techniques. Techniques used in positioning the tools that change the inclination and the direction of the wellbore (see "Methods of Orientation").

Overswing. Term denoted to excessive walk of the bit; walk of the bit greater than expected.

Packed Bottom-hole Assembly. A configuration of tools with a certain degree of rigidity and wall-bearing surfaces; used to minimize deflection of the hole from a straight course while drilling.

Pendulum Effect. Refers to the pull of gravity on a body; tendency as a pendulum to return to a vertical position.

Pendulum Hookup (Pendulum Assembly). A bit and drill collars with a stabilizer placed to attain the maximum pendulum effect.

Picture. A survey chart or film on which a survey recording has been made.

Plane of Closure. Vertical plane that contains both the surface location of the wellbore and the last station of the survey.

Plug Back. To fill part of the wellbore with cement; sometimes used for side-tracking.

Precession. Motion about the vertical and/or horizontal axis of a gyro due to imbalance, friction, earth's rotation, or externally applied forces.

Primary Deflecting Tools. Historically, the whipstock, knuckle joint, and spudding bit, and more recently, the bent sub and downhole motor.

Quadrant Bearing. An azimuth angle measured from north or south in the direction of east or west.

Reactive Torque. Based on the physical property that action equals reaction, the torque reacting on the drill stem is that torque being generated at the point in question, such as at the bit.

Reference Magnets. Magnets inset in the wall of the non-magnetic drill collar. Used to indicate a position of the deflecting tool with respect to magnetic north. A picture of a magnetic needle compass at the magnets is imposed on the picture of the magnetic north compass.

Rigidity. Usually refers to the stiffness or flexibility characteristics of a bottom-hole assembly or an element thereof.

Roll Off. Correction in the facing of the deflection tool, usually determined by experience, and which must be taken into consideration in order to give the proper facing to the tool.

Setting Off Course. A method of setting the direction of the wellbore in anticipation of the bit walking (refer to "Lead Angle").

Shot. The measurement taken or the survey reading taken as a picture or as a punched hole on a chart (refer to "Picture").

Side Track. An operation performed to redirect the wellbore by starting a new hole at a position above the bottom of the original hole.

Side-tracking Pocket. An enlargement of one side of the wellbore made to facilitate changing the direction of the wellbore. The wellbore enlargement is usually accomplished by use of jetting action.

Single-shot Survey. A measurement of the inclination and direction of a wellbore at one position with one recording.

Skew (Directional Drilling). The angular difference between the wellbore direction and the formation dip direction.

Slant Hole. A non-vertical hole; usually refers to a wellbore purposely inclined in a specific direction; also used to define a wellbore which is non-vertical at the surface.

Slant Portion of a Well. The straight portion of the wellbore that is not vertical; the "locked-in" angled portion of the wellbore.

Slant-type Directional Hole. Usually refers to a wellbore which has a vertical section, an angle-build section, and an angled-but-straight section to total depth (refer to "Straight-in Directional Hole"). Also used to define a wellbore which is non-vertical at the surface (refer to "Slant Hole").

Spiraled Wellbore. A wellbore which has attained a changing configuration as of a spiral or helical form.

Spud Bit. In directional drilling, a special bit used to change the direction and inclination of the wellbore.

Stabilizer. A tool placed in the drilling assembly to: 1) change or maintain the inclination angle in a wellbore by controlling the location of the contact point between the hole and drill collars, 2) center the drill collars near the bit to improve drilling performance, and/or 3) prevent wear and differential sticking of the drill collars.

Station Interval. The length of the course with one end at the depth described as the station course length.

Steering Readout. Directional instrument indication of the drilling tool alignment taken while drilling.

Stiff Hookup (Stiff Assembly). A well-stabilized, rigid bottom-hole assembly to maintain inclination and direction of the hole; opposite to limber hookup.

Stiffness. Quality or state of being rigid. Resistance to bending under stresses within the elastic limit.

Stoking. Refer to "Methods of Orientation."

S-type Well (S-shaped Well). Well drilled with a vertical portion, a deviated portion, and a return toward the vertical.

Straight Wellbore. Wellbore drilled with the intention to proceed in a non-changing direction.

Straight-hole Downhole Motor. A downhole motor designed to drill straight ahead; usually a straight-hole motor is longer, larger and provides more torque than does a "directional" downhole motor.

Straight-in Directional Hole. A wellbore with a build and a straight locked-in section. There is no drop-off section.

Stress Reversal. Change in stress from tension to compression, or vice versa.

String Stabilizer. Stabilizer placed anywhere in the drill stem assembly above the near-bit stabilizer.

Survey Calculation Methods. Refer to "Wellbore Survey Calculation Methods."

Survey Data Sheet. Commonly called the calculation sheet. A paper form on which to tabulate the data and results of calculations of a wellbore survey.

Surveying Frequency. Refers to the number of feet between survey records.

Survey Instrument. An instrument used to measure inclination of the wellbore and the direction of the inclination from a position within the wellbore.

Target Area. A defined area at a prescribed vertical depth, which is planned to be intersected by the wellbore.

Target Point. The coordinates in space considered to be the preferred point within the target area for the wellbore intersection. The planned point, within the target area, for the wellbore to intersect. (Also simply target).

Terminal Angle. The inclination and direction angles of the lower end of the course.

Theta Angle. The angle that will correct grid north to true north.

Tool Azimuth Angle. The angle between north and the projection of the tool reference axis onto a horizontal plane.

Tool High-side Angle. The angle between the tool reference axis and a line perpendicular to the hole axis and lying in the vertical plane.

Total Curvature. Implies three-dimensional curvature (refer to "Dogleg Severity").

Traveling Cylinder Views. A plat of the well profile within the control cylinder.

Traverse Tables. Tables of numerical values used in calculating wellbore survey results.

True North. The direction from any geographical location on the earth's surface to the north geometric pole.

True Vertical Depth (TVD). The actual vertical depth of an inclined wellbore (refer to "Vertical Depth").

Turbodrill. A downhole motor which utilizes a turbine for power to rotate the bit.

Turn. Change in bearing of the hole. Usually spoken of as the right or left turn with orientation to that of an observer who views the well course from the surface site.

Uncontrolled Sidetrack (Blind Sidetrack). The side tracking of a wellbore where direction is unimportant and not controlled.

Vertical Depth. Vertical component of the measured well depth.

Vertical Hole. A hole in which the wellbore is nearly maintained in a position vertically below the surface location.

Vertical Profile (Vertical Section). A projection of the borehole into a vertical plane parallel to the course bearing and scaled with vertical depth.

Walk (of bit). The action of the bit to change the direction of the wellbore by its tendency to turn into the side of the wellbore while rotating.

Walk (of hole). The tendency of a wellbore to deviate in the horizontal plane; generally thought to be caused by the bit rotating preferentially into the side of the hole and the anisotropic nature of the formation.

Wellbore. The hole made by drilling or boring a well.

Wellbore Survey Calculation Methods. Refers to the mathematical methods and assumptions used in reconstructing the path of the wellbore and in generating the space curve path of the wellbore from inclination and direction angle measurements taken along the wellbore. These measurements are obtained from gyroscopic or magnetic instruments of either the single shot or multishot type. The following summarized methods are recognized by the API:

> **Acceleration Method.** Utilizes the angles at the top and bottom of the course length and from these generates a curve on the assumption that the measured angles change smoothly from top to bottom of the measured course as though under the influence of a constant force of an acceleration. The results obtained are the same as "Balanced Tangential," "Trapezoidal," and "Vector Averaging" Methods.
>
> **Average Angle Method.** Uses the angles measured at both the top and bottom of the course length in such a fashion that the average of the two sets of measured

angles is the assumed inclination and direction. The wellbore survey is then calculated tangentially using these averaged angles over the course length.

Angle Averaging Method. Refer to "Average Angle Method."

Backward Station Method. Refer to "Tangential Method."

Balanced Tangential Method. Uses the inclination and direction angles at the top and bottom of the course length in a manner so as to tangentially balance the two sets of measured angles over the course length. Results obtained are the same as the "Acceleration," "Trapezoidal," and "Vector Averaging" Methods.

Circular Arc Method. Uses both sets of measured angles associated with each course length to recreate the wellbore path as a sequence of small circular arcs constrained by the measured angles to pass through the end points with inclination and direction angles as measured.

Compensated Acceleration Method. Refer to "Mercury Method."

Combined Method. Refer to "Mercury Method."

Mercury Method. A combination of the "Tangential" and "Balanced Tangential" Methods so as to treat that portion of the measured course defined by the length of the measuring tool as a straight line (tangentially) and the remainder of the measured course trapezoidally. Refer to "Compensated Acceleration Method" and "Combined Method."

Minimum Curvature Method. Uses the sets of angles measured at the top and bottom of the course length to establish coordinate velocities through which a space curve (which represents the calculated path of the wellbore) passes in a manner that minimizes its total curvature.

Quadratic Method. A method in math modeling considering the wellbore as a curve; the projections into three orthogonal planes are quadratic functions.

Radius of Curvature Method. Uses the sets of angles measured at the top and bottom of the course length to generate a space curve (representing the wellbore path) that has the shape of a spherical arc passing through the measured angles at both the upper and lower ends of the measured course.

Secant Method. This name has been applied with two different meanings: 1) meaning the "Trapezoidal Method," and 2) meaning the "Average Angle Meghod."

Simpson's Rule Method. Uses as many measured angle values as are available (a minimum of three sets) to recreate the wellbore path through Simpson's Rule for numeric integration, which approximates by passing a parabola through three points.

Tangential Method. Uses only the inclination and direction angles measured at the lower end of the course length. The wellbore path is assumed to be tangent to these angles through the course.

Terminal Angle Method. Refer to "Tangential Method."

Trapezoidal Method. Uses the measured inclination and direction angles at both ends of the measured course in a fashion that recreates the wellbore path. This is done by a sequence of trapezoidal integration segments using the measured angles as constraints on the integral over the measured course. Results obtained are essentially the same as the "Acceleration," "Balanced Tangential," and "Trapezoidal" Methods.

Vector Averaging Method. Uses inclination and direction measurements at both ends of the measured course to establish vector space direction. It is then assumed that each of these two vectors is projected for one-half the course length in creating the wellbore path. Each "half course length" segment can be treated tangentially. Results obtained are essentially the same as the "Acceleration," "Balanced Tangential," and "Trapezoidal" Methods.

Well Depth. Measured depth in the wellbore. Usually measured from the kelly bushing, derrick floor, or foundation as a datum. Refer to "Measured Depth."

Well Profile. The projection of the wellbore onto a plane.

Whipstock. A long wedge and channel-shaped piece of steel with a collar at its top through which the subs and drill stem can pass. The face of the whipstock sets an angle to deflect the bit.

Window. A section of casing milled out to provide an opening to sidetrack or kick off.