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A SIMPLIFIED METHOD for MEASURING RESIDUAL STRESS DISTRIBUTIONS in WELDED STEEL PLATES

By

Raymond D. Zook

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Signed Raymond D. Zook
Raymond D. Zook

Golden, Colorado

Date: August 7, 1964

Approved: J D Lubahn
Thesis Advisor
J. D. Lubahn

AWSchlechten
Head of Department
A. W. Schlechten

Golden, Colorado

Date: August 7, 1964

ABSTRACT

The experimental work, the results of which comprise this thesis, was performed in an effort to develop a simple method of determining residual stress distributions in welded mild steel plates.

The work was performed as a part of a coordinated program for the study of the effect of residual stress levels on the fracture strength of welded ship plates. The program is supported by the U. S. Navy Marine Engineering Laboratory Contract No. N161-25683.

A dissection method was developed, utilizing the length changes of 1/4 in. strips cut parallel to the direction of the welding. The length change is converted to residual stress, using Hook's law and neglecting transverse stress; and the stress calculated is the average stress over the width, thickness and length of the strip.

The distribution of the average stress in a 1-in. thick plate was determined. The distribution was similar to distributions obtained by other experimentalists.

Certain errors in the method were investigated experimentally and others were determined by analytical methods. The largest single error

investigated was in the dial gages (7%). All other errors investigated were insignificant in comparison. Corrections can be made for the dial gage error after proper calibration of the gages. Corrections can also be made for certain other errors on the basis of the experimental work e.g. machining stresses, non-parallelism of strips.

The errors due to end effects and multiaxial stress effects were not investigated, but they were calculated as 17% for end effects and 10% if the transverse stress is 30% of the longitudinal stress.

The method is relatively simple, and certain errors, such as in non-parallelism of strips, relocating strips and temperature changes can be held to an insignificant level with relative ease.

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INTRODUCTION

Brittle fracture i.e. low strength failure of welded steel plates is a major problem in industry. Recent work by Masacuchi (Masabuchi, 1958) and Wells (Wells, 1957) indicate that there is a definite relationship between residual stresses in welds and low strength failures; however, there is a considerable amount of scatter in the experimental results thus far obtained. A research program currently sponsored by the Navy is attempting to explain and eliminate the scatter and is seeking a more quantitative relationship between residual stress distribution and fracture strength in mild steel welded plates.

It is necessary to develop a relatively simple and reliable method for determining residual stress levels in these plates as a primary step in explaining the scatter and in obtaining a more quantitative residual stress-fracture strength correlation, because the scatter may be related to the residual stress in some way. The scatter may be due to a variation in residual stress; the residual stress may have been inaccurately determined; or other factors may have a great enough effect on the scatter to mask the effect of the residual stress. It is also possible that the residual stress is not contributing to the scatter.

The Wells type test (Wells, 1957) is being used in the program to find the correlation between residual stress and fracture strength. Two 36 x 18 x 1 in. steel plates are butt-welded together with an artificial notch defect in the weld prep. The plate is tested in tension to determine the fracture strength. In this type of test the other variables which can contribute to the scatter in results are notch root radius, eccentricity of loading, welding variables, and plate properties.

If residual stresses are a factor in causing low strength failures the state of stress (that is, whether it is uniaxial, biaxial or triaxial) should be known as well as the distribution and magnitude. It is not presently known whether the contribution of the stress to low fracture strength is due to the magnitude, distribution, or state of the stress or a combination of all three, nor is the required sensitivity and accuracy known. Consequently, any method should be capable of measuring the magnitude, distribution, and stress state with known sensitivity and accuracy.

Specific questions which must be considered in light of the existing test conditions are:

1. How does the stress vary along the length of the weld? (e.g. What are the end effects)?

2. How does the stress vary through the thickness of the weld?

3. Does a state of triaxial tensile stress exist?

4. How does the stress vary across the weld and into the plate? The technique developed is primarily concerned with the determination of the distribution of the longitudinal stress across the weld and into the adjacent plate. Some consideration is given to further development of the technique to answer the other three questions.

Standard methods of measuring residual stresses in welded plates fall into two basic catagories; X-ray techniques and dissection techniques, (Sachs, 1948).

Residual stresses have been measured by the X-ray back reflection method to an accuracy of ± 2000 psi. The method is based on Bragg's law for the diffraction of X-rays ($n\lambda = 2d \sin \Theta$) and on the concept that the interplaner spacing of a metal is the gage length, the strain measured being the change in this gage length. The stress in general can only be measured to a depth of about 0.001 in. below the surface; however a technique has been developed for using X-rays to determine the approximate stress further below the surface (Fromer, 1944). A flat bottom hole is turned and etched to successive depths, and the X-ray beam is directed to the bottom of the hole. The X-ray method requires elaborate and expensive equipment, and long exposure times are necessary (Norton, 1944).

Dissection methods of residual stress measurement are based on the fact that the dimensions of an object are altered when the stresses are relieved. As an example; if a rod containing uniform residual tensile stress, fastened at both ends, was cut loose, it would shorten by some amount. The residual stress would be found by the formula: $S_1 = Ee_1$. The stress being uniaxial in this case.

The dimension change in a welded plate can be measured by various methods. The simplest method (Sachs, 1948) consists of drawing a series of lines parallel and perpendicular to the weld at a given distance apart. The plate is cut into 2-in. square blocks, and the changes in the spacings are measured by an extensometer or comparator. If the directions measured are principal directions, the stresses can be represented by the relationships:

$$S_1 = \frac{E}{1-u^2} (e_1 + ue_2)$$

$$S_2 = \frac{E}{1-u^2} (e_2 + ue_1)$$

u = Poison's ratio

e_1 and e_2 = strains

The stress is assumed to be constant through the thickness of the plate. The same basic method can be used, using wire resistance strain gages or stress-coat to determine the strain.

Rosenthal (Rosenthal, 1944) has developed a method in which the blocks cut from the plate are used to determine the residual stress through the thickness of the plate. A block is cut in half (mid-thickness of the plate); a resistance wire strain gage is placed on the cut surface; and successive layers are machined from the opposite side. The strain is measured after each layer is removed. This method was used by Nordell (Nordell, 1963) in recent work on welded mild steel plates.

Two other methods used in measuring residual stresses in welded plates are the hole method and the ring method (Soete, 1950). Both methods require the use of resistance wire strain gages arranged in a radial pattern around the hole or on the ring. In the hole method, the strains are measured when a 15/16-in. hole is drilled to a depth of approximately 5/16 in. In the ring method, a circular cut is made concentric to the hole, forming a ring. The strains in the ring are measured when the hole and circular cut are made.

All of these methods are used industrially, but their accuracy seems to be in question. With the exception of Rosenthal's method they measure only the surface stresses and are somewhat complicated by elaborate set-ups.

In 1942, Johnston determined residual stresses in railroad rails by measuring changes in length of thin, longitudinal strips (Johnston, 1942). A micrometer was used to measure the length changes.

In the present work, the length changes of strips cut parallel to the weld are measured by 0.0001-in. dial gages. This approach is used because of its simplicity and probable accuracy. The advantage of dial gages over resistance strain gages is that they are easy to handle, and their accuracy is little affected by temperature and moisture. It was believed that both the sensitivity and accuracy of the method could be increased merely by increasing the length of the strips measured. This should be true because certain errors would be independent of stress levels so that the percent error would be greater for smaller strains. An increase in gage length would decrease the error in all stress levels. The transverse stress is taken as zero in this analysis; therefore the lower the transverse stresses, the more accurate the results will be. The transverse stresses were not investigated in this work, nor were the variation of stresses through the thickness of the plate and across the width and along the length of the strips.

APPARATUS and EQUIPMENT

The following apparatus and equipment was used in the stress determination technique:

1) Modified Gorton engraving machine
2) 0.0001 in. Federal dial gages
3) Precision ground gage blocks
4) Milling machine
5) Shaper

A Gorton engraving machine was modified with dial gages mounted on each side of the horizontal moveable carriage. Figure 1 shows the modified fixture. The dial gages are mounted rigidly, so that once aligned they will not be moved when samples are positioned and removed from the fixture. The carriage upon which the plates and slices are positioned can be moved in three coordinate directions by micrometer screw mechanisms (0.001 in. graduations). The left gage mounting can be adjusted laterally to accomodate samples ranging from 6 in. to 12. in length.

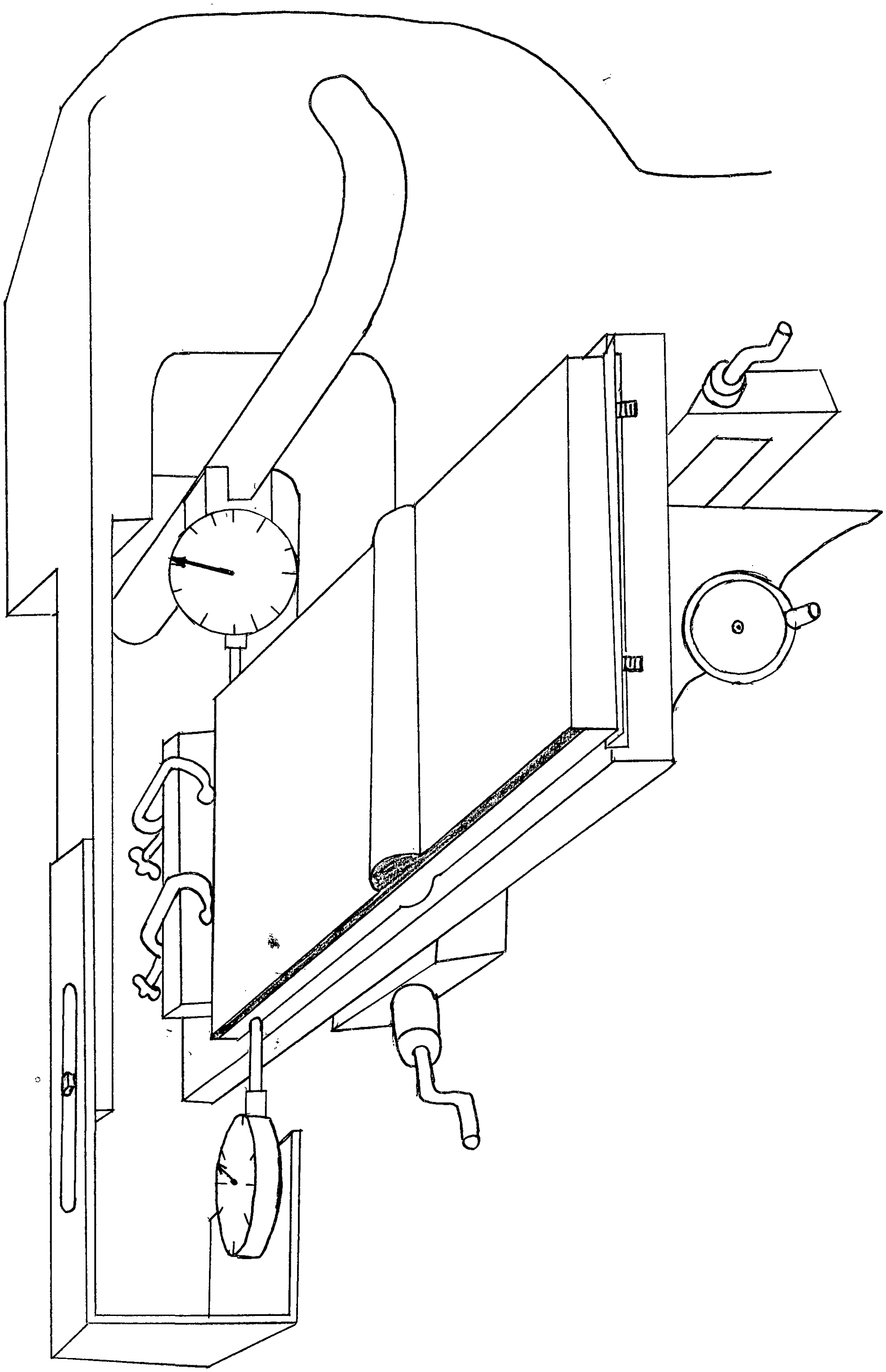


Figure 1. Measuring Fixture

EXPERIMENTAL PROCEDURE

General Procedure

The procedure in general consists of:

(a) Milling faces of welded plate smooth, flat and parallel.

(b) Placing welded plate between two fixed dial gages and taking initial readings at equal intervals along the width of the plate.

(c) Cutting longitudinal strips (parallel to the weld) from the plate.

(d) Measuring the change in length of the strips by placing the strips back between the dial gages.

Apparatus Alignment

The retaining block on the moveable carriage is made square with the carriage with a toolmaker's square. A 6 x 5/8 x 5/8 in. precision-ground gage block is used to align the dial gages with the retaining block. This is done by removing the spindle tips and fixing each dial gage spindle shank parallel to the gage block in both directions. The aligning was done by eye, the spindle being considered parallel to the gage block when no space could be seen at any point between the spindle shank and the block. Figure 2 shows the alignment.

Welded Plate

A mild steel plate of the size and shape shown in Figure 3 was examined.

Joint Preparation

A double-V-joint weld preparation was made as shown in Figure 4.

Welding Procedure

The two 14 x 6-in. plates were butt-welded together by an automatic gas-shielded-arc method, using 10 passes.

Machining

The sides parallel to the joint were milled parallel to the weld prep before welding. One of these sides was used as a reference face in further machining and strain measurements.

After welding, a 1-in. slice was cut from each end of the welded plate, and the cut surfaces were milled perpendicular to the reference surface. The surfaces were milled as smooth as possible, perpendicular to the top and bottom of the plate. (Figure 3.)

Plate Marking

The upper 1/4 in. of the left surface was painted red, and lines 1/8 in. apart were scribed in the paint and numbered from the reference surface (Figure 5). The scribed positions were numbered 1 through 50.

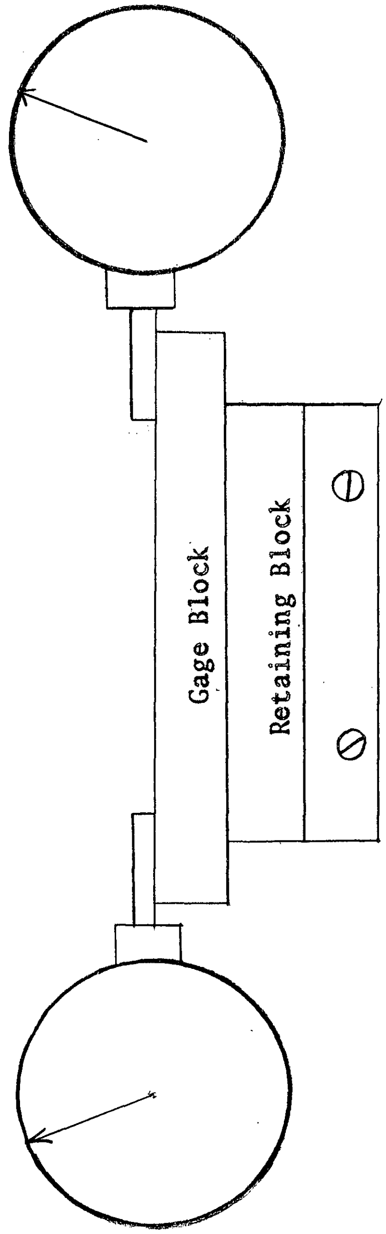


Figure 2. Dial Gage Alignment

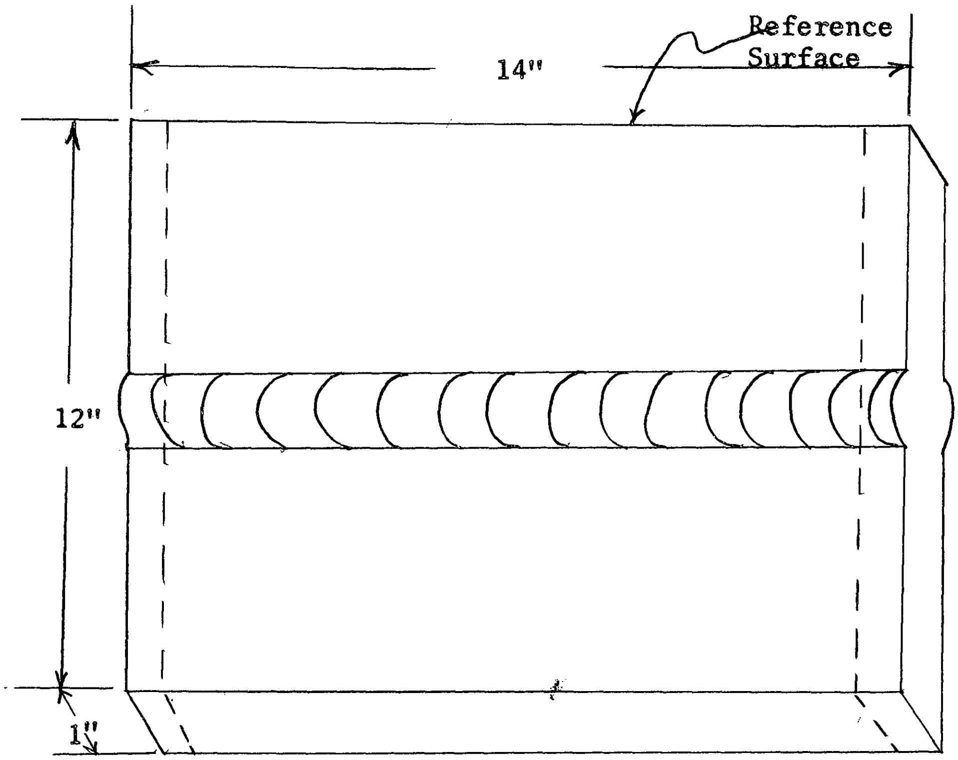


Figure 3. Welded Plate

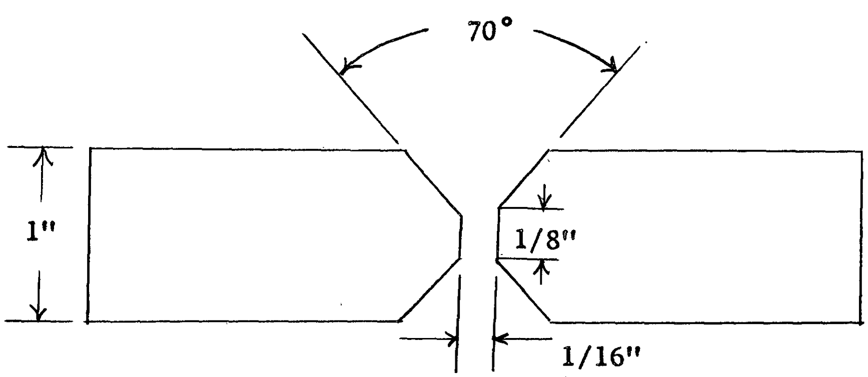


Figure 4. Weld Joint Preparation

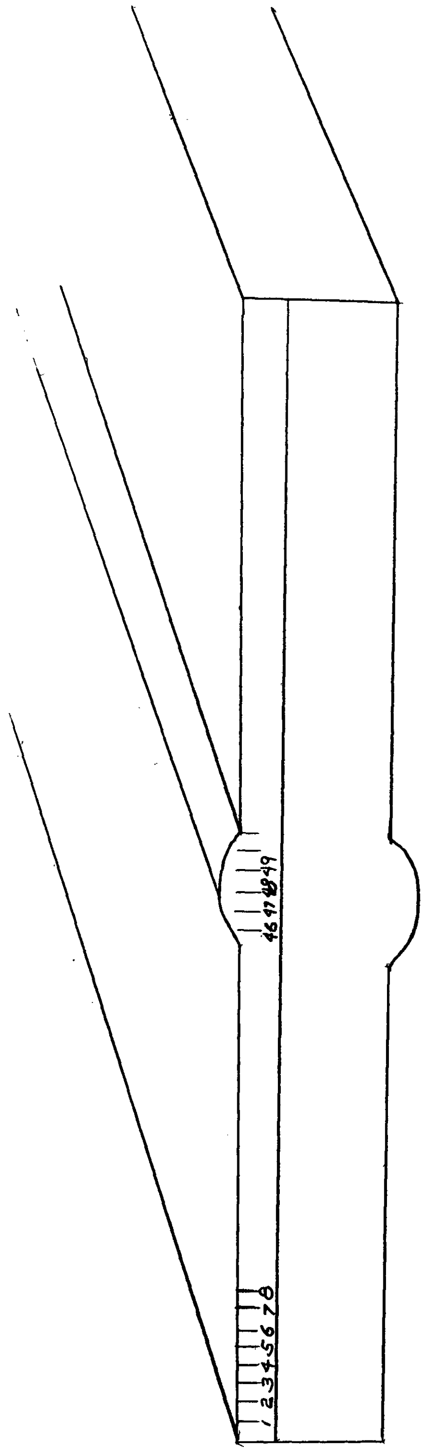


Figure 5. Plate Marking

Initial Measurements

The plate was placed in the measuring fixture with the reference surface against the retaining block. Two small clamps were placed as shown in Figure 1 to prevent movement of the plate. Starting from the edge against the block, readings of both gages were taken every 1/8 in. along the mid-thickness of the plate at the scribed positions. The actual positions were governed by the micrometer screw. Each division on the crank wheel is 0.001 in.

The plate was removed and then repositioned in the fixture. A new set of readings was taken at the same 50 positions according to the micrometer screw. A total of 5 sets of readings were taken, the plate being removed from the fixture between each set of readings.

Slicing

The center line of the weld was estimated at 6 in. from the reference surface. A line was scribed at this distance, parallel to the reference surface.

The plate was cut on a line approximately 1/4 in. past the center line with a vertical band saw.

The plate was placed in a milling machine with the reference surface made parallel with the milling machine carriage. A dial gage was used for this alignment.

A 1/4 in. thick slice was made through the weld with a 5-in dia. x 1/16 in. thick side-tooth slitting saw, the scribed centerline being in the center of the slice. Using the lowest speed possible on the milling machine, four 1/4-in. deep cuts were made to machine through the full 1-in. thickness of the plate. The plate was progressively sliced by the same method into eighteen 1/4-in strips; the final strip was slightly thicker. The micrometer screw on the horizontal carriage travel was used to govern the thickness of each slice.

The clamps had to be removed once and repositioned before making the last few slices. The edge became nonparallel to the carriage. Corrections were made to account for the error in length change caused by the nonparallelism. These corrections are described in the discussion.

Final Measurements

Each strip was placed in the measuring fixture, with the fixture at the number 1 position. The strips were clamped tight against the retaining block so that there was no noticeable curvature. Then the final lengths were measured at the mid-thickness of the strips. Each strip was measured 5 times, the strips being removed between each measurement. The points measured on the strips were not exactly at the same locations where the original measurements were made. The points at which these measurements were made were determined by adding the measured thicknesses of the slices and the thickness of the saw cuts. The details of the determination are in the discussion.

The purpose of measuring the plate and strips 5 times, removing them from the fixture between each measurement, was to find the error in relocating samples and repeatability of the dial gages.

Calculations

The difference between the original dial gage readings on the plate and the dial gage readings on the strips is the change in length of the strip caused by the average residual stress in the regions from which the strip was taken. The change in length of a strip is used to calculate the average residual stress over the length, width and thickness of the strip, using the uniaxial stress-strain relationship $S_1 = Ee_1$. E is the modulus of elasticity, and e_1 is the longitudinal strain.

$$e_1 = \frac{\text{change in length of the strip}}{\text{original length of plate}}$$

The length changes and calculated stresses are in Table 1.

TABLE 1. Length Changes and Calculated Stresses

Strip	Point	Length Change	Stress
1	47	-0.01648	40,800
2	45	-0.01271	31,590
3	42	-0.00376	9,330
4	40	-0.00141	3,500
5	37	0.00229	-5,670
6	35	0.00529	-13,140
7	32	0.00476	-11,820
8	30	0.00410	-10,170
9	27	0.00400	-9,930
10	25	0.00343	-8,540
11	22	0.00312	-7,770
12	20	0.00253	-5,800
13	17	0.00171	-4,260
14	14	0.00101	-2,510
15	12	0.00148	-3,680
16	9	0.00152	-3,770
17	7	0.00154	-3,830
18	4	0.00053	-1,320
19	1	0.00028	-750

DISCUSSION

Theoretical Considerations

The necessity for equilibrium of forces gives one an insight as to the distribution of longitudinal residual stresses in a flat welded plate. A tensile stress would be expected in the weld, because the weld attains the highest temperature and consequently has the tendency for the most shrinkage upon cooling. The weld metal is prevented from shrinking fully by the cooler base metal. The base metal near the weld has also been heated, so on cooling it too will try to contract if it has flowed plastically on heating. The resistance to the contraction causes residual tensile stress. Compressive forces are necessary to balance the tensile forces in satisfying the equilibrium of force condition; therefore there must be a point of zero stress at some distance from the weld. Moreover, the compressive forces must be distributed from the point of zero stress to the edge of the plate in such a way that their total value equals the total tensile force.

The longitudinal stress distribution across the width of the plate

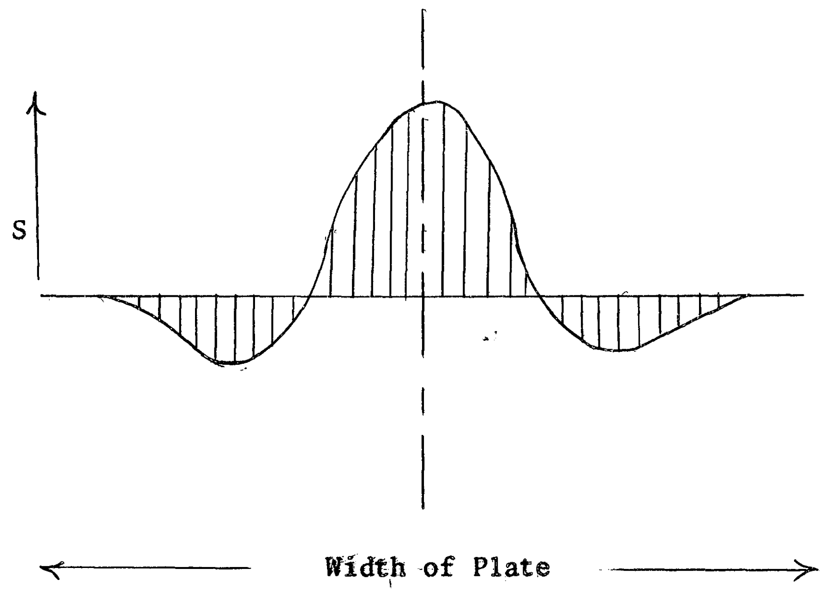


Figure (a) Expected Longitudinal Stress Distribution should be similar to that shown in (a). A similar stress distribution was found by Norton (Norton, 1944) in a locally heated 3-in. wide by 10-in. long thin steel strip. Nordell (Nordell, 1963) also found this type of distribution in a 1-in. thick welded steel plate. The longitudinal stress would not have to be zero at the edge of the plate. This would depend on the shape of the plates being joined.

The length change of a strip cut longitudinally (parallel to the weld) is the result of the average stress acting across the width and along the length of the strip, but if the stress is calculated by the relation $S_1 = Ee_1$, the actual stress might not be accurately described. The accuracy of this method will depend on the thickness of the strip in relation to the spatial extent of the distribution of the longitudinal stress and the magnitude of the normal and transverse stresses.

To visualize the effect of strip thickness in relation to the longitudinal stress distribution across the width of the plate, let us consider that the transverse and normal stresses are negligible. Consider the distribution and relative size of the strip in (b). The strip would shorten by some amount, but some residual stress would remain in the strip due to the initial variations of the stress. The residual stress distribution remaining in the strip would be as shown in (c), and the stress actually determined by $S_1 = Ee_1$ would be the difference in the peaks shown by the arrows. The same distribution is shown in (d) and (e) with a smaller strip. Because the tensile stress is not uniform, a small amount of residual stress still remains in the strip, as shown in (e). The error decreases as the strip gets smaller. The average stress approaches a constant or

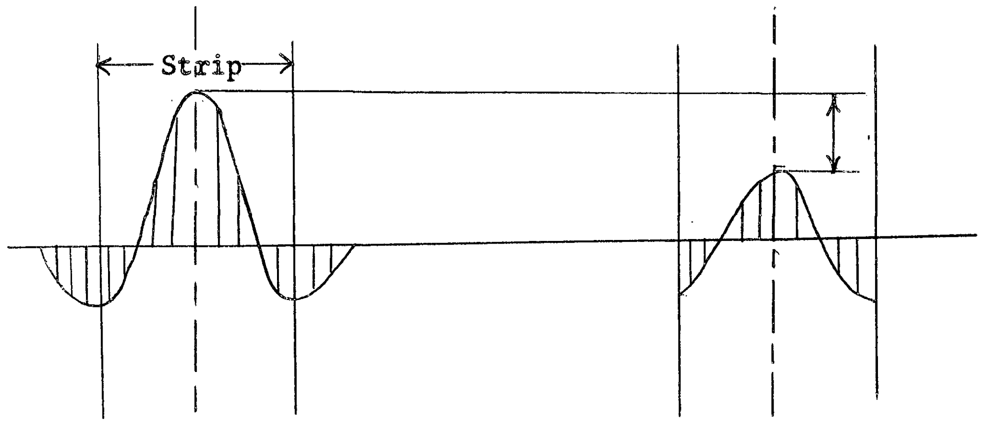


Figure (b) Strip Width to Stress Distribution Relation

Figure (c) Residual Stress in Strip

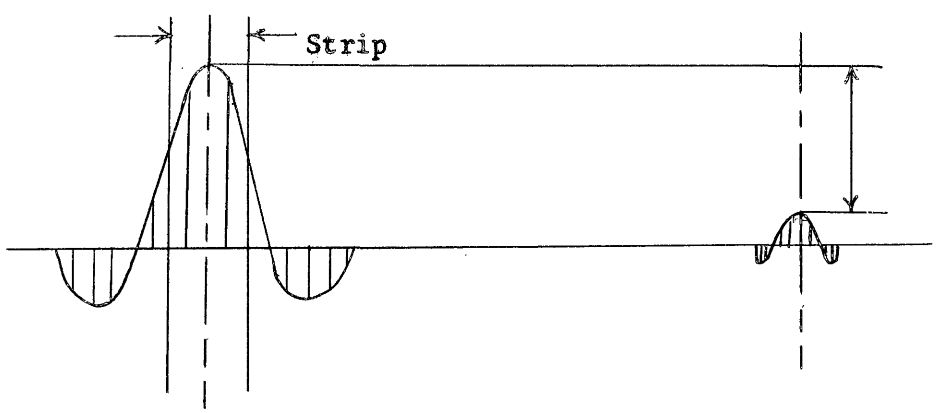


Figure (d) Strip Width to Stress Distribution Relation

Figure (e) Residual Stress in Strip

single value of stress. If a wider stress distribution existed, thicker strips could be taken with the same accuracy as with thinner strips in a narrower stress distribution, (f) and (g).

If a strip were taken as shown in (h), there would be no average length change, because the tensile and compressive stresses would balance in this region. However, there would be bending of the strip.

The greatest amount of error in any distribution would be in the region where the initial stress distribution has the greatest curvature.

In the slicing method, it follows that slices as thin as possible should be made for greatest accuracy, because the smaller the segment the smaller the curvature in the stress distribution will be. A sufficiently small segment in any curve will be a straight line. The thinness of the strip is limited by the magnitude and depth of machining stresses. For heavy cuts, stresses due to machining could have an absolute depth of 0.010 in. and therefore become more important as the strip becomes thinner.

Error Analysis

The error introduced by the variation of the longitudinal stress along the length of the strip will now be considered. In using the relationship $S_1 = Ee_1$, we are in effect considering the stress to be constant along the length of the strip; when in fact the stress must vary, because it is zero at the ends of the strip. Two possible distributions, representing extremes, are shown in (i) and (j). The nearer the distribution is to that in (j), the closer the average stress is to the maximum stress, and the smaller the error will be in calculating the stress by

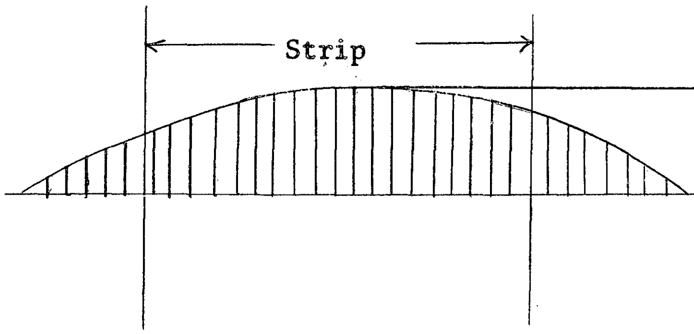


Figure (f) Strip Width to Stress Distribution Relation.

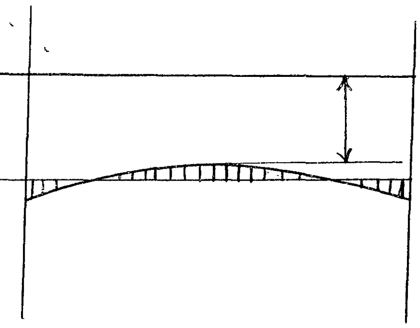


Figure (g) Residual Stress in Strip.

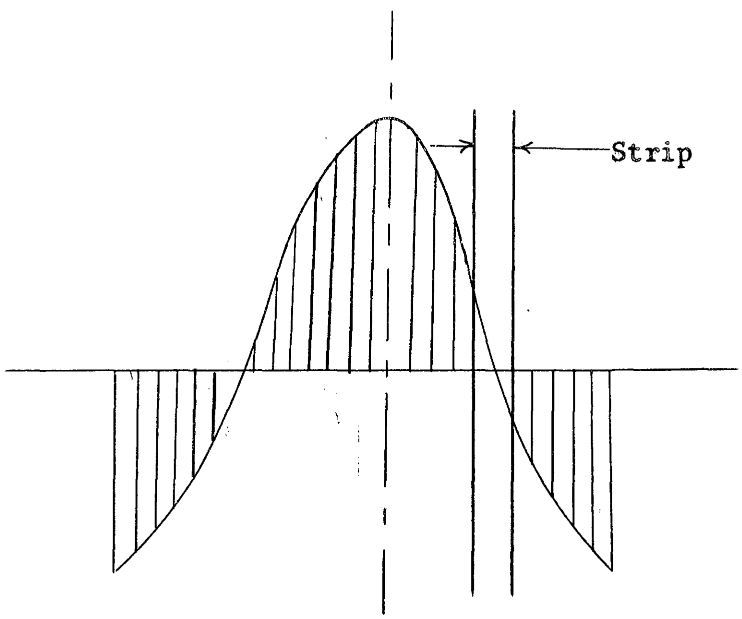


Figure (h) Strip Width to Stress Distribution Relation.

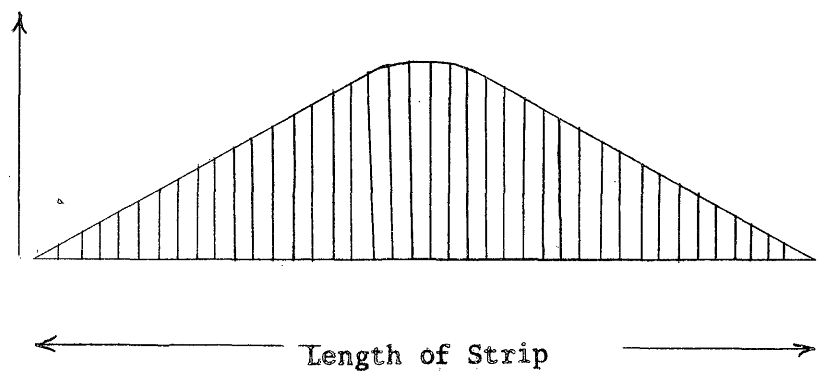


Figure (i) Possible Longitudinal Stress Distribution Along Length of Strip.

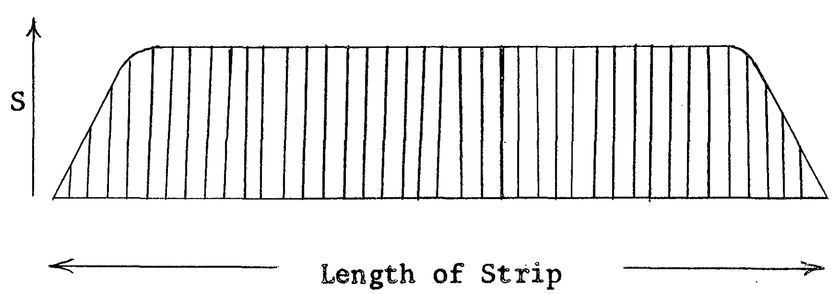


Figure (j) Possible Longitudinal Stress Distribution Along Length of Strip.

the change in length.

There is reason to believe that the distribution is nearer to that shown in (j), based on St. Venant's principle and on work done by Soete (Soete, 1950). Soete found the surface stress was nearly constant over a 4-ft. length on a 6-ft. long welded plate. St. Venant's principle indicates the stress should die out at each end in a distance equal to the critical dimension. If the width of the longitudinal tensile stress distribution (2 in. in this work) is taken as the critical dimension, the approximate error from taking S_1 as constant along the strip can be calculated. The calculated error is approximately 17 per cent.

The transverse stress has not been investigated, so the error resulting from using the uniaxial stress relationship is not known. The factors which govern the distribution of the transverse stress are the compatability condition and equilibrium of forces and moments. If we consider a case where the surfaces are insulated and a single-pass weld is made, a longitudinal distribution of transverse stress as shown in (k) would be expected. A moment is acting on the solidified metal as the weld progresses across the plate. The moment reaches such a magnitude that some of the weld metal is plastically deformed, and residual stress results on cooling. If there were no insulation and more than one pass was made, the surfaces would be allowed to cool more rapidly than the center. If plastic flow has occurred in the center, a residual stress distribution will form through the thickness of the weld during cooling. This distribution should be constant along the length of the weld, and would add to the distribution

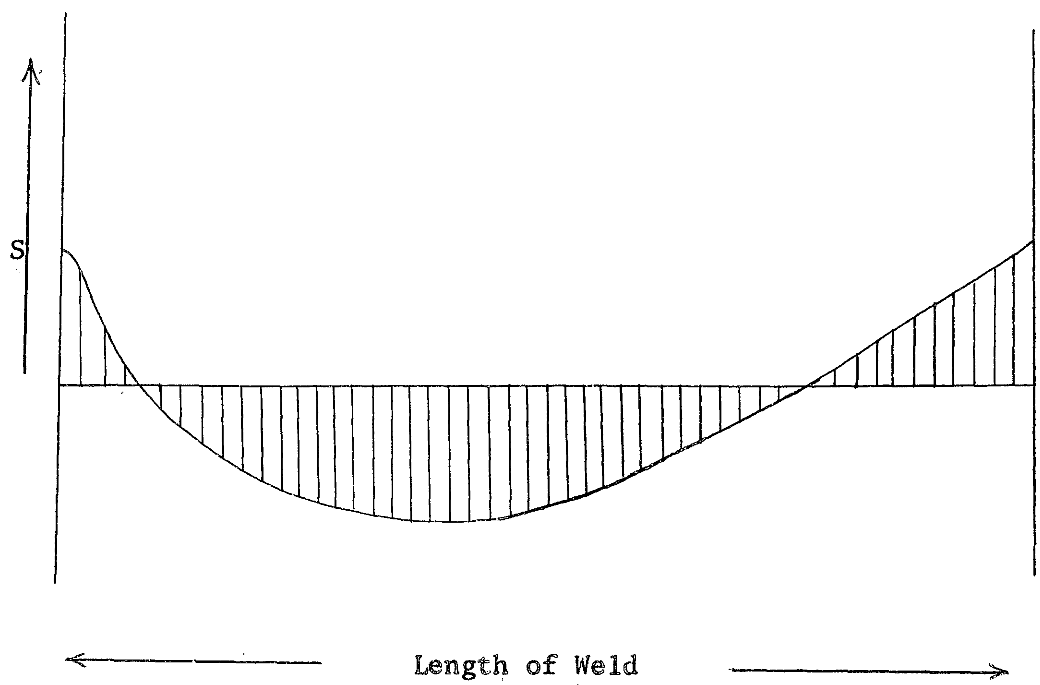


Figure (k) Longitudinal Distribution of Transverse Stress.

in (k).

If the normal stress is considered negligible, the multiaxial stress relationship is: $Ee_1 = S_1 - uS_t$. The error caused by neglecting S_t can be calculated if hypothetical values of stress are used.

If S_t is 30% of S_1

$S_1 = 40,000$ psi

$S_t = 12,000$ psi

$Ee_1 = 40,000 - 1/3\ (12,000 + 0)$

$= 36,000$ psi

The value of S_1 which would be calculated neglecting S_t would be 36,000 psi, which would result in an error of 10%.

Other errors in the measured average stresses across the thickness and width and along the length of the strips may be introduced as a result of the following factors, which have been investigated.

Temperature changes

Nonparallelism of the two faces of a slice

Relocation of the strips at the original measured points

Calibration of the gages

Repeatability in putting the plate and strips in and out of the fixture.

Possible length changes due to clamping of the strips for measuring

Initial gage length measurement (length of plate)

Repeatability of dial gages

Machining stresses

If a classical error analysis approach is used, the effects of all the errors on the total percentage error in the average stresses can be seen.

S is determined by: $S = Ee$

Relative error $= \frac{ds}{s}$

$$ds = e\,dE + E\,de$$

$$\frac{ds}{s} = \frac{edE}{eE} + \frac{Ede}{eE} \qquad \frac{ds}{s} = \frac{dE}{E} + \frac{de}{E}$$

$$\delta = \ell - \ell_o \qquad e = \frac{\delta}{\ell_o}$$

$$de = \frac{\delta d\ell_o - \ell_o\, d\delta}{\ell_o^2}$$

$$\frac{ds}{s} = \frac{dE}{E} + \frac{\delta d\ \ell_o - \ell_o\, d}{\ell_o^2} \quad \frac{\ell_o}{\delta}$$

$$\frac{ds}{s} = \frac{dE}{E} + \frac{\ell_o\, d\, \ell_o}{2} - \ell 2\frac{d\delta}{\ell_o^{\,2}\delta}$$

$$\frac{ds}{s} = \frac{dE}{E} + \frac{d\ell_o}{\ell_o} - \frac{d\delta}{\delta}$$

$$\frac{ds}{s} = \frac{dE}{E} + \frac{d\,\ell_o}{\ell_o} - \frac{de - \ell_o}{\ell - \ell_o}$$

Where $\frac{dE}{E}$ represents the relative error in the modulus of elasticity and $\frac{d\ell_o}{\ell_o}$ represents the relative error in the original length $d\ell$ is the absolute error in the final length, and $d\ell_o$ is the absolute error in the original length.

$\ell - \ell_o = \delta =$ the change in length of the strip.

The expression can be rearranged to: $\frac{dE}{E} + \frac{\ell_o}{\ell_o} = \frac{d\ell - d\ell_o}{e\ell_o}$

since $\ell - \ell_o = e\,\ell_o$.

This expression shows that the relative error should decrease with larger strains and with longer strips. The modulus of elasticity was not measured, so its relative error will not be considered in this analysis. A modulus of 30 million was arbitrarily used in the stress calculations.

<u>Error in original length</u>: The measurements of the original length of the plate were made with a micrometer with a sensitivity of approximately 0.0002 in. An average of all the measurements taken along the full width of the plate was used for all of the stress calculations, inasmuch as all deviations from the mean were less than 0.0010 in. It is apparent that the error in this measurement is insignificant.

if $d\ell_o = 0.001$ $\frac{d\ell_o}{\ell_o}$ would equal 0.00008 or 0.008%.

The original measurements are shown in Table 2.

The significant errors are in the change in length measurements.

$$= \frac{d\ell - d\ell_o}{\ell - \ell_o}$$

The remaining sources of error listed contribute to the error $d\ell_o$ or to

TABLE 2. Original Length Measurements of Welded Plate. (Measurements taken with micrometer every 1/2 in.)

	RUN 1	RUN 2	
	12.0720	12.0720	
	12.0720	12.0722	
	12.0722	12.0722	
	12.0720	12.0720	
	12.0720	12.0721	
	12.0722	12.0720	
	12.0722	12.0720	
	12.0722	12.0720	
	12.0723	12.0722	
	12.0723	12.0724	
	12.0724	12.0724	
	12.0724	12.0725	
	12.0725	12.0725	
Average	12.0722	12.0719	Overall Average 12.0720

$d\ell$; therefore each of the sources will be listed under $d\ell_o$ and $d\ell$, and their effects will be examined.

<u>Errors contributing to $d\ell_o$:</u>

Dial gages

Reproduceability

(1) Push-pull test

With a sample in the fixture, the spindle was pulled back and released 10 times. The dial gage was read each time. The error was less than the sensitivity of the gage, the sensitivity being 0.00001 in.

(2) Putting the plate in and out of the fixture.

In taking initial readings (before slicing) on the plate investigated, dial gage readings were taken at 50 points along the edge of the plate. The plate was removed and put back in the fixture. A new set of readings were taken at the same points. A total of 5 sets of readings were taken, the plate being removed between each set of readings. The errors calculated varied for different points from 0.00000 to $\pm$ 0.00004.

The data is on Table 3.

(3) Parallax

The error due to parallax in reading the gages was less than the sensitivity of the gages.

TABLE 3. Initial Dial Gage Differences $(\times 10^{-3})$

Point	Run 1	Run 2	Run 3	Run 4	Run 5	Average	Av. Deviation
1	0	.15	.10	.10	.03	.078	.03
2	.22	.16	.15	.16	.08	.15	.01
3	.20	.29	.22	.22	.14	.21	.02
4	.22	.28	.25	.24	.17	.23	.01
5	.18	.30	.28	.26	.05	.21	.04
6	.25	.25	.17	.22	.17	.21	.01
7	.14	.28	.12	.31	.12	.19	.04
8	.15	.30	.28	.26	.12	.22	.03
9	.22	.29	.14	.20	.10	.19	.02
10	.20	.25	.23	.19	.06	.19	.02
11	.15	.20	.18	.12	.0	.15	.02
12	.08	.22	.12	.14	.10	.13	.02
13	.20	.25	.30	.28	.25	.25	.01
14	.18	.23	.25	.20	.09	.19	.02
15	.12	.20	.20	.16	.02	.14	.02
16	.20	.33	.30	.25	.12	.24	.03
17	.10	.19	.13	.12	.04	.12	.01
18	-.03	.10	.13	.09	.02	.06	.02
19	.24	.25	.23	.20	.10	.20	.02
20	-.12	.22	.21	.15	.10	.11	.02
21	.05	.09	.10	.10	.10	.09	.01
22	-.10	.09	.09	.0	-.02	.01	.03
23	-.08	.0	.04	-.06	-.14	-.05	.02
24	-.15	.24	-.04	-.07	-.08	-.07	.02
25	.01	.01	-.03	-.04	-.08	-.03	.01
26	-.20	-.08	-.10	-.18	-.23	-.16	.02
27	-.14	-.14	-.16	-.22	-.33	-.20	.03
28	-.19	-.10	-.14	-.18	-.27	-.18	.02
29	-.22	-.18	-.20	-.21	-.32	-.22	.02
30	-.19	-.19	-.15	-.21	-.37	-.22	.03
31	-.32	-.32	-.27	-.31	-.42	-.33	.02
32	-.30	-.28	-.25	-.35	-.48	-.35	.03
33	-.40	-.46	-.42	-.52	-.54	-.47	.02
34	-.52	-.58	-.52	-.60	-.60	-.56	.02
35	-.70	-.65	-.56	-.70	-.67	-.66	.02
36	-.57	-.52	-.52	-.55	-.61	-.55	.02
37	-.62	-.64	-.66	-.70	-.78	-.68	.02
38	-.70	-.68	-.66	-.80	-.80	-.73	.02
39	-.78	-.84	-.86	-.86	-.94	-.86	.02
40	-1.15	-.93	-.94	-.95	-.90	-.97	.03
41	-.98	-.98	-1.00	-.99	-1.00	-.99	.00
42	-1.18	-1.23	-1.27	-1.25	-1.27	-1.25	.01
43	-1.16	-1.18	-1.20	-1.18	-1.17	-1.18	.00
44	-1.42	-1.35	-1.45	-1.36	-1.40	-1.40	.01
45	-1.38	1.40	1.50	-1.37	-1.39	-1.40	.01
46	-1.40	-1.43	-1.45	-1.40	-1.41	-1.41	.01
47	-1.30	-1.42	-1.45	-1.45	-1.52	-1.43	.02
48	1.46	-1.54	-1.52	-1.55	-1.56	-1.53	.01
49	1.72	-1.75	-1.72	-1.77	-1.78	-1.75	.01
50	1.88	-1.90	-1.90	-1.88	-1.90	-1.90	.00

Errors contributing to D_ℓ:

Dial gages

Calibration

(1) A 12-in. long plate was placed in the fixture and moved to the right 0.0010 in. at a time through the measuring range used for the stress measurements. Readings were taken on both gages after each 0.0010-in. movement. The calibration curve is shown on Graph 1. A maximum error of 7% was found in the movement. If greater accuracy were needed, the gages could be calibrated against a primary standard, and the measured length changes could be corrected.

Repeatability

(1) Push-pull test

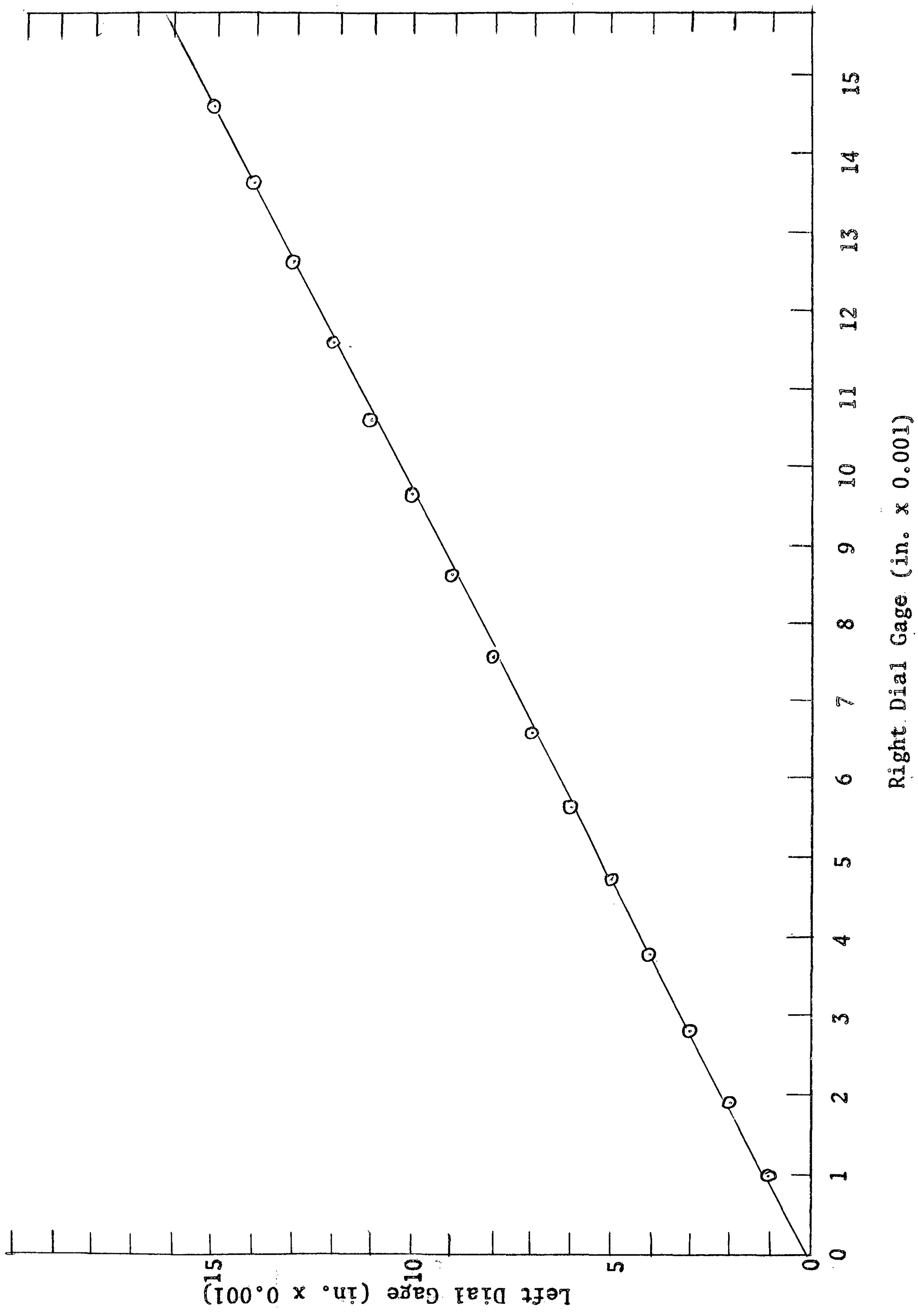
The result was the same as in $d\ell_o$ as expected.

(2) Putting strips in and out of the fixture.

Each strip was put in the fixture at the number one position, and dial gage readings were taken. A total of 5 readings were taken on each strip, the strip being removed from the fixture between readings. The calculated error varied for each strip and ranged from 0.00000 to ± 0.00004. The results are in Table 4.

(3) Parallax

The error due to parallax is the same as in $d\ell_o$, as expected.



GRAPH 1. Gage Calibration Curve

TABLE 4. Final Dial Gage Differences on Strips ($x\ 10^{-3}$)

Strip	Point	Run 1	Run 2	Run 3	Run 4	Run 5	Average	Av. Deviation
1	47	2.05	2.00	2.12	2.20	2.12	2.09	.03
2	45	5.90	5.82	5.85	5.92	5.95	5.89	.02
3	42	14.90	15.00	15.00	15.00	15.02	14.99	.02
4	40	17.88	17.62	17.60	17.52	17.50	17.62	.04
5	37	1.62	1.59	1.63	1.60	1.63	1.61	.01
6	35	4.60	4.60	4.70	4.65	4.60	4.63	.02
7	32	4.30	4.40	4.45	4.43	4.40	4.41	.02
8	30	3.94	3.85	3.95	3.90	3.88	3.88	.02
9	27	3.81	3.80	3.80	3.80	3.79	3.80	.00
10	25	3.38	3.42	3.42	3.42	3.38	3.40	.01
11	22	3.16	3.13	3.15	3.10	3.12	3.13	.01
12	20	2.65	2.60	2.68	2.67	2.63	2.64	.01
13	17	1.89	1.78	1.88	1.83	1.75	1.83	.02
14	14	1.42	1.35	1.50	1.38	1.35	1.40	.02
15	12	1.86	1.80	1.78	1.85	1.85	1.83	.01
16	9	1.90	1.91	2.00	2.00	1.85	1.93	.02
17	7	1.94	1.88	1.95	1.92	1.98	1.93	.01
18	4	1.00	1.02	1.10	1.08	1.15	1.07	.02
19	1	17.80	17.80	17.80	17.78	17.80	17.80	.00

Relocation of strips to original measured points.

An error would be expected, resulting from measuring the strips at different locations than those measured on the plate if the original measurements on the plate are used to calculate the strains. We can determine the maximum possible error using Graph 2, a plot of initial plate readings against distance from the reference edge of the plate. The maximum change on the curve is 0.0007 per in. The greatest distance any point, measured on the strips, is from an original measured point on the plate is 0.067 in. This point is actually located where the slope is much smaller.

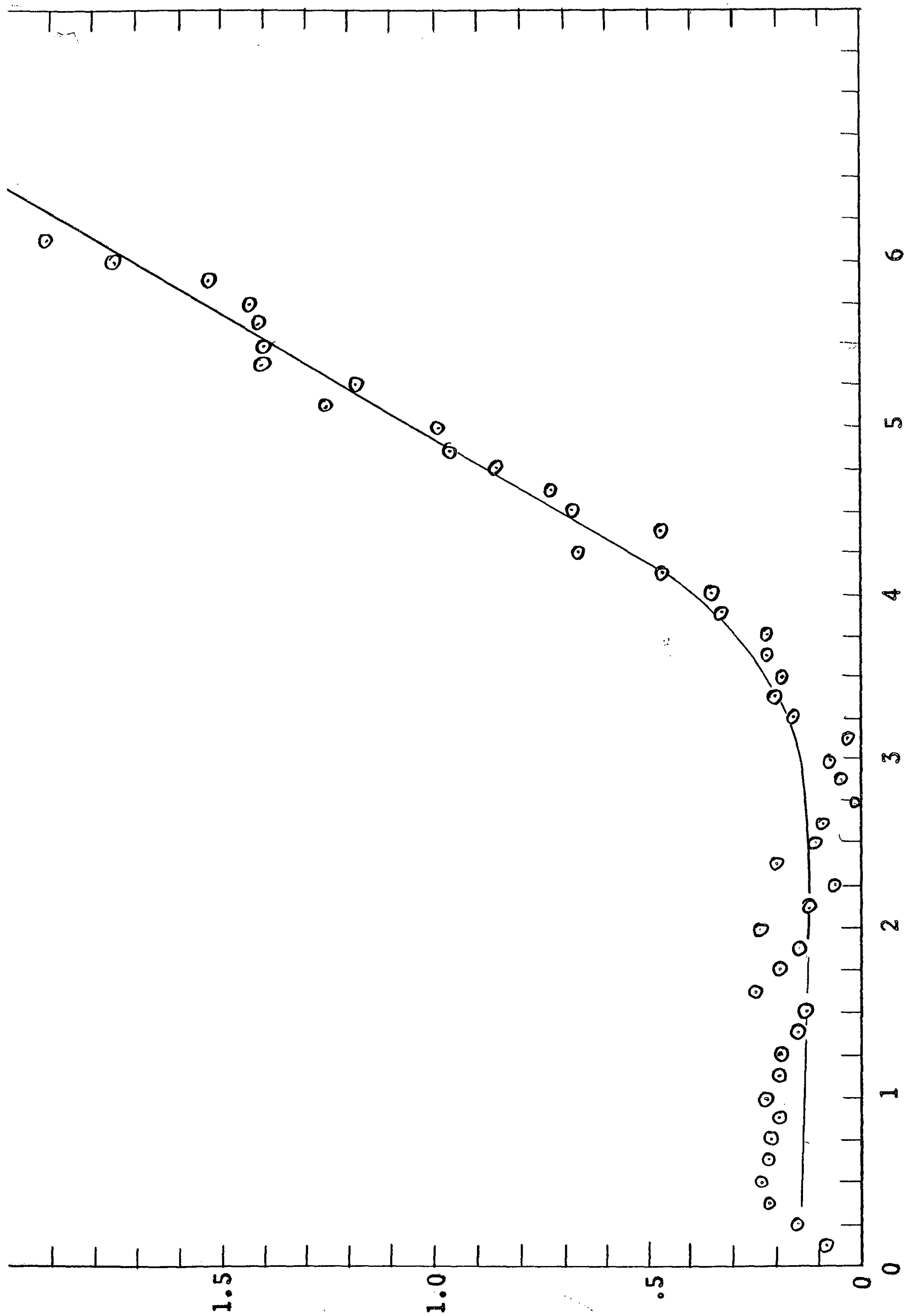
$$(.067) \quad (0.0007) \quad = \quad 0.00005 \text{ max.}$$

This value is less than the maximum deviation from the mean in the initial readings. The error would be greater if there was more taper in the plate. When other plates are tested, the taper should be held at a minimum, or correction curves should be used.

The actual locations of the measured points on the strips are shown in the Appendix 8.

Clamping of strips

Some of the strips exhibited a slight curvature. The strips were clamped in the fixture for ease of measurement and to insure that arc length rather than chord length is measured. In clamping, all of the noticeable curvature was removed from the strips. The maximum arc-height in any of the strips was no more than 0.010 in. There is a possible error that can be introduced by clamping, namely the change in length caused by the clamping action. A



GRAPH 2. Curve of Gage Readings vs Distance From Edge of Plate

calculation using equilibrium of forces and moments indicates the error would be no more than 0.00001.

Temperature changes

An error can be introduced in the calculated length change if the strip or plate is not at the same temperature as the fixture during measuring. Since the fixture is more massive than the strips, it will take longer for it to reach the same temperature as the room, that is, the lag time will be greater than that of the strips. If the temperature of the room changes rapidly, there could be an error.

A plate was put in the fixture, and the temperature and length changes were checked over a three day period. The temperature of the room varied less than one degree in 24 hours, and the error observed was 0.00003 in. over that period. The data is shown in Table 5.

The temperature was not checked during the measurements of the plate and strips, but both the plate and strips were allowed to sit in the room for several hours before the measurements were made. It is probable that the maximum possible error would be 0.00003.

The error is so small because the temperature of the room changes so slowly over a long period of time. Conditions might change at different times of the year, and changes could be expected if the door to the room is left open. More rapid changes of the temperature would affect the strips, and much greater errors could be expected. A one degree temperature change can cause a length change

TABLE 5. Temperature Check

Date	Temp.	Time	Gage Readings x 10^{-3} Left	Right
June 27	78.8	12:25 P.M.	5.95	9.68
	78.8	1:50	5.95	9.68
	78.6	2:50	5.95	9.68
	78.5	3:15	5.95	9.68
	78.5	6:25	5.95	9.68
	78.5	7:40	5.94	9.66
	78.6	8:50	5.99	9.62
	78.6	10:50	5.98	9.62
June 28	78.5	9:00 A.M.	5.99	9.62
	78.5	10:00	6.00	9.61
	78.5	11:30	6.00	9.61
	78.5	12:40 P.M.	6.00	9.61
June 29	78.0	7:30 A.M.	6.00	9.60
	78.0	12:20 P.M.	6.00	9.60
	78.0	1:20	6.00	9.60
	78.2	2:30	6.00	9.60
	78.2	3:30	6.00	9.60
	78.2	5:00	6.00	9.60
	78.2	6:00	6.00	9.60
	78.2	8:00	5.99	9.60
	78.0	9:00	5.99	9.60
	78.0	10:00	5.99	9.60

in the strip of 0.0008 in.

Non-parallelism of strips

A certain amount of error will be introduced in the length change if the sides of the strips are not parallel because the sides of the strips were used for positioning in the fixture. An error can be computed by geometry, which however does not take into consideration the curvature of the spindle tip of the dial gage. A calculation was made to determine the change in length due to the cocking of the piece which would affect the dial gages. To find the actual change in length, a 10-in. long precision ground gage block was placed in the fixture and cocked at various angles. Dial gage readings were taken at the various cocking angles. Details of the method, calculations, and corrections made are shown in Appendices 3,4,5,6 and 7. The data shows that any strip can be out of parallel by as much as 0.010 in. before a significant error is introduced. If the thicknesses of the strips are measured, a correction can be made to make the error negligible. Because of mechanical difficulties in slicing, the last 5 strips were out of parallel to the reference surface far enough to introduce significant errors. Following are the errors introduced in the lengths of the strips:

Strip	Error
18	0.00022
17	0.00020
16	0.00022
15	0.00022
14	0.00020

Corrections were made which are shown in Appendix 7.

Machining stresses

To study the effect of machining stresses on the length changes of the strips, a plate was annealed and sliced. A 12 x 2 1/2 by 1-in. plate was heated at 1600 F for 1 hr. and furnace-cooled for 20 hr.

Two 1/16-in. strips and two 1/4-in. strips were cut on the milling machine, using the same procedure as was used on slicing the test plate. An increase in length averaging 0.0008 in. was measured on the two 1/16-in. strips which were cut on both sides. This would represent an error in stress of 1,980 psi, all other things being equal.

An increase in length averaging 0.0001 in. was measured on the two 1/4-in. strips. This would represent an error in stress of 248 psi, other things being equal. The change in length actually falls within the scatter in the repeatability in putting the strips in and out of the fixture; therefore one may not draw a conclusion on the basis of these two 1/4-in. strips alone. The combined results from the 1/4-in. and the 1/16-in. strips indicate that the error is real and predictable. A correction could be made in each stress measured to allow for the error. Furthermore the change in length due to machining stresses, can be predicted for strips of other thicknesses by using the following formula:

$$\frac{\delta}{\delta_o} = \frac{t_o - 2X}{t - 2X}$$

δ = change in length of strip of desired thickness

δ_o = change in length of 1/4 in. strip

t = desired thickness

t_o = 1/4 in.

X = depth of machining stress (0.002 in. calculated)

The length change data are in Appendix 1.

Following is a summary of the discussed maximum possible errors in the method. The absolute errors, such as those in repeatability and sensitivity of the dial gages, are calculated for an arbitrary stress of 30,000 psi for comparison.

Relative errors

Longitudinal distribution (end effects)	+17%
Neglecting transverse stress	$\pm$10%
Dial gage calibration	$\pm$ 7%
Original length	$\pm$ 0.01%

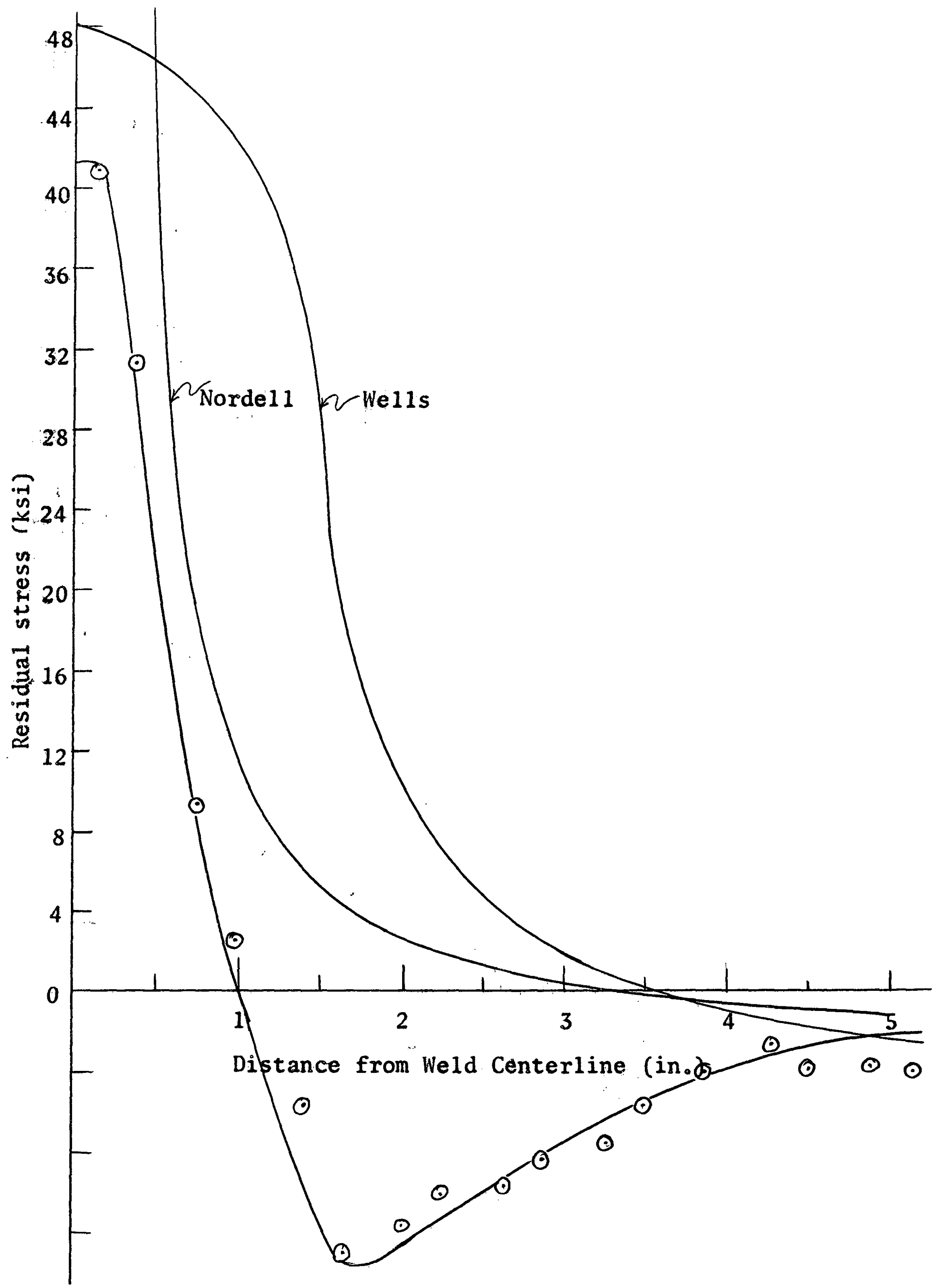
Calculated for 30,000 psi

Dial gage repeatability	$\pm$0.1%
Dial gage sensitivity	$\pm$0.3%
Putting strips in and out of fixture	$\pm$0.3%
Parallax	$\pm$0.1%
Relocation of strips (uncorrected)	$\pm$ .4%
Clamping	$\pm$0.1%
Temperature changes	$\pm$0.2%
Non-parallelism of strips (corrected	-0.4%
Machining stresses	-0.8%

The + sign indicates that the stresses determined would be consistantly below the actual value; the - sign indicates the converse. All of the errors in the average stresses are negligible compared to the 7% error in dial gage calibration (the first two errors listed do not pertain to the average stress). The calibration error can be 7% for any value of stress; whereas the other errors in the average stress will increase at lower stresses. At 10,000 psi the errors can triple, but they would still be insignificant compared to the calibration error.

Analysis of Results

The measured average longitudinal stress as a function of the distance from the centerline of the weld is shown in Graph 3. This is the curve passing through the experimental points. Nordell (Nordell, 1963) and Wells (Wells, 1957) found similar distributions. A comparison is made of their distributions on Graph 3. These results were obtained from 1-in. thick plates, Nordell's plate being 24-in. wide and Well's plate being 36-in. wide. The compressive stresses of these two distributions were more uniform and did not approach zero at the edges of the plate. This would be expected, because the plates were twice and three times the size of the plate used in my work; the spatial distribution of the compressive stresses should be greater. The tensile stress region is considerably wider which is a result of different welding conditions. Because of the different plate sizes and welding conditions, the stress values cannot be compared; however the general shapes of the curves are the same. This fact, along with the theoretical considerations discussed earlier in the report, indicates that the curve obtained



GRAPH 3. Longitudinal Residual Stress Distribution

in this work is what should be expected.

In order for the equilibrium of force conditions to be satisfied, the areas under the curve above and below the zero stress line must be equal. The compression area is actually approximately 18% greater than the tension area. A difference in areas would be expected in light of the errors in the method. The errors which would cause the most discrepancy are those due to neglecting the transverse stress and those inherent in the dial gages. Some of the error could be due to the fact that the maximum stress measured is not in the center of the weld. The actual weld centerline might be further to the left, in which case the tensile area would be greater.

It is not known what accuracy is necessary nor to what degree the stresses must be described for the Navy program. It is possible that a correlation between the maximum average stress in the weld and fracture strength may be sufficient. It is probable that the triaxial stress condition existing at the root of the notch, caused by the severity of the notch itself, is severe enough to dwarf the effect of the multiaxial residual stress state. The distribution of the longitudinal stress may be the only important consideration.

The work done in this thesis constitutes the groundwork from which a more refined and accurate method can be developed for describing the magnitude, distribution and state of stress. By expanding on the slicing method which has been described here, one could determine:

1) Distribution of the longitudinal stress along the length of the weld.

2) Transverse stresses.

3) Distribution of the longitudinal stresses through the thickness of the plate.

The distribution of the longitudinal stresses along the length of the weld could be determined by placing wire resistance strain gages in a line on both the top and bottom of the plate, then cutting a strip out of the plate. The strain gages will record the strains along the length of the strip. The average stress over the gage length of the strain gage can be computed for each gage location. X-ray measurements can also be made along the length of the plate before a strip is removed, for determining surface stresses.

The average transverse stresses can be determined by cutting strips perpendicular to the weld, and measuring the strain changes.

The longitudinal stress distribution through the thickness of the plate can be determined by progressively machining one side of the strip and measuring the length and curvature changes.

CONCLUSIONS

Conclusions which can be made in light of the results obtained in this investigation and some theoretical considerations are as follows:

1) The method is relatively simple, requiring less apparatus than most methods.

2) The stress distribution obtained on the welded plate is as would be expected and conforms in its general characteristics to the distributions found by Nordell and Wells.

3) The error due to machining stresses in 1/4-in. strips is negligible compared to other errors in the method; however if other errors were reduced to such an extent that the machining stress error became significant, one could correct for it. Furthermore, the errors for strips of other thicknesses could be predicted on the basis of the results of tests performed in this investigation, and corrections could be made. The error is a relative error which will always represent an increase in length.

4) The error caused by the non-parallelism of the sides of the

strip is a relative error, experimentally determined. This error could be quite large if care is not taken in slicing; however a correction can be readily made for this error. The maximum possible error after correction is 0.4% at 30,000 psi. This is due to the scatter in the experimental points in the method of determining the error. The corrected error is negligible compared to other errors in the method. The corrected error is absolute.

5) The error in relocating the strips, that is measuring the strips in different locations than those where the original measurements were made on the plate, can be significant only if the sides of the plate are considerably out of parallel. Even in that case the error would be insignificant compared to other errors in the method. A correction can be easily made, but in the plate examined the corrected values would have uncertainties as high as their uncorrected values. The uncertainty would always be as high as that in the original dial gage differences. This is an absolute error.

6) The error in putting strips and plates in and out of the fixture is an absolute error, insignificant compared to the large errors in the method. This is an error for which no correction can be made, and is the type of error which would always be in the method. However it is an error easily determined experimentally.

7) The following absolute errors can be neglected, because they are so small. They will always be insignificant compared to other errors.

dial gage sensitivity

dial gage repeatability

parallax

clamping

8) The largest error investigated was that in the dial gage, i.e. 7%. This is only an estimate of what the error could be. Each gage could have an error of 3 1/2 per cent or either gage can have an error to 7 per cent. The real error in each gage can be determined experimentally by calibrating the gages against a standard. A correction can be made using a calibration curve.

9) The error in the original length of the plate is the most insignificant of all the errors.

10) The largest possible error in the method appears to be that due to end effects. The longitudinal distribution along the length of the strip can be determined by experimental methods. The error could be eliminated or reduced to a negligible value if the distribution were known. This error is a relative error.

11) The error in neglecting transverse stress effects can be higher or lower than 10% depending on the ratio of the transverse stress to the longitudinal stress and whether it is compressive or tensile stress. The transverse stress could also be determined experimentally, in which case the error could be eliminated or reduced.

12) The error which might be introduced through temperature changes could be quite large if room temperature changed rapidly during measuring. The error can be determined experimentally, but the error can easily be held to an insignificant value if the room temperature is maintained constant and the pieces to be measured are allowed to reach room

temperature before the measurement. The temperature error will always be an absolute error.

APPENDIX 1. Error Due to Machining Stresses. (Plate was heated at 1600°F for 1 hour and furnace cooled for 20 hours.)

Initial Dial Gage Readings on Plate x 10^{-3}

(Left gage read; readings areamounts less than zero on right gage).

Strip	Thickness (in.)	Run 1	Run 2	Run 3	Run 4	Run 5	Average
1	1/16	-.90	-.80	-.82	-.80	-.78	-.82
2	1/16	-1.10	-1.08	-1.10	-1.13	-1.08	-1.09
3	1/4	-1.80	-1.70	-1.76	-1.72	-1.75	-1.75
4	1/4	-2.20	-2.15	-2.20	-2.17	-2.17	-2.18
		Final Dial Gage Readings on Strips					
1	1/16	.10	.12	.0	.0	.1	+.06
2	1/16	-.30	-.30	-.28	-.30	-.30	-.30
3	1/4	-1.62	-1.75	-1.69	-1.60	-1.62	-1.66
4	1/4	-2.00	-2.02	-2.00	-2.15	-2.10	-2.05

Change in Length of Strips

Strip	Length Change
1	0.00088
2	0.00079
3	0.00009
4	0.00013

APPENDIX 2. Thickness Measurements of Strips.

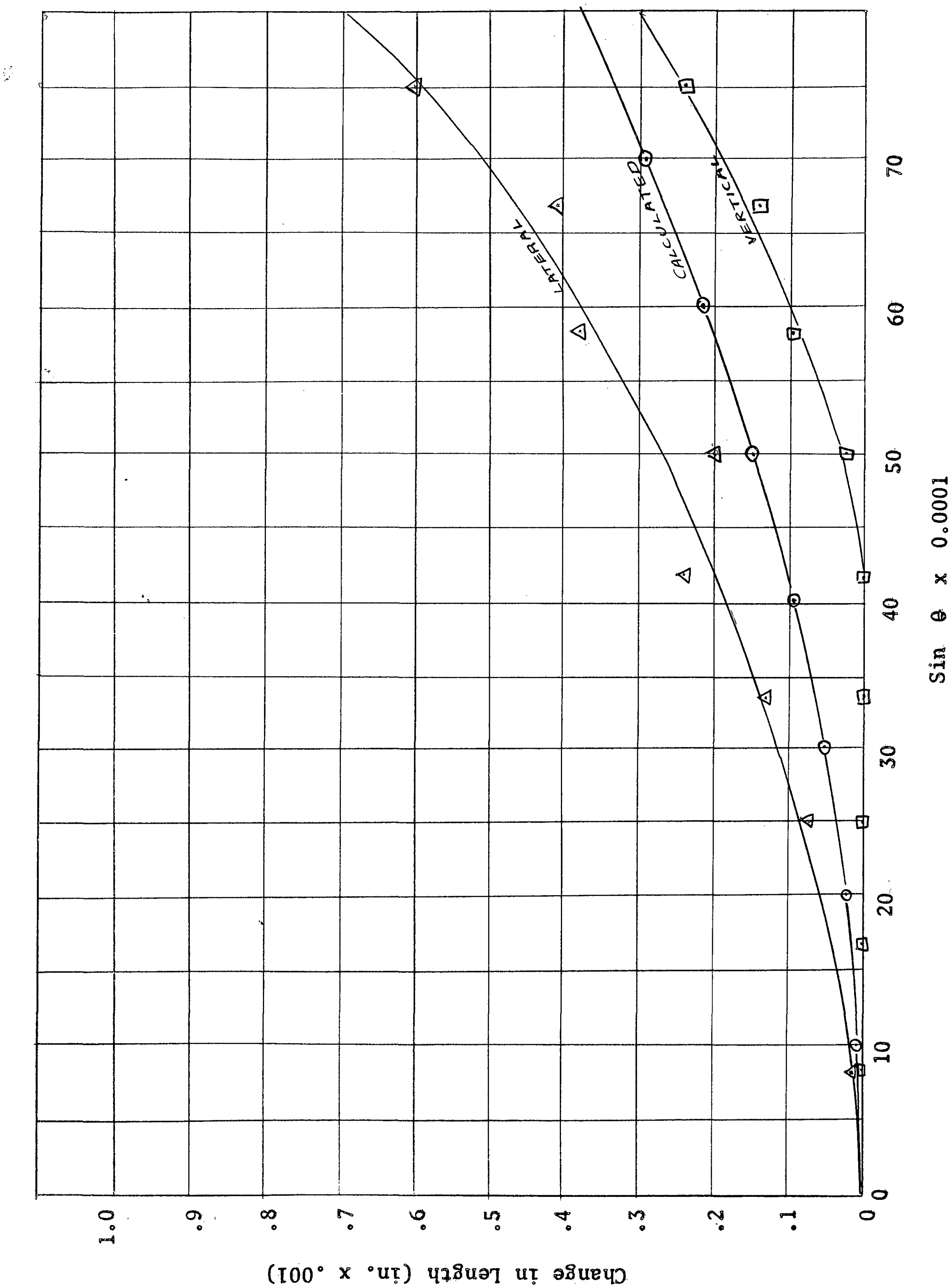
Strip	Reference End	Other end	Difference
1	.2490	.2456	.0034
2	.2494	.2476	.0018
3	.2475	.2475	.0000
4	.2432	.2420	.0012
5	.2492	2489	.0003
6	.2430	.2425	.0005
7	.2467	.2476	.0009
8	.2472	.2472	.0030
9	.2480	.2482	.0002
10	.2453	.2458	.0003
11	.2456	.2478	.0022
12	.2473	.2465	.0008
13	.2480	.2490	.0010
14	.2448	.2890	.0442
15	.2459	.2476	.0017
16	.2451	.2455	.0004
17	.2448	.2472	.0024
18	.2430	.2446	.0016
19	.3846	.3283	.0563

APPENDIX 3. Vertical and Lateral Cocking of 12" Plate.

Data for 12-in. Plate

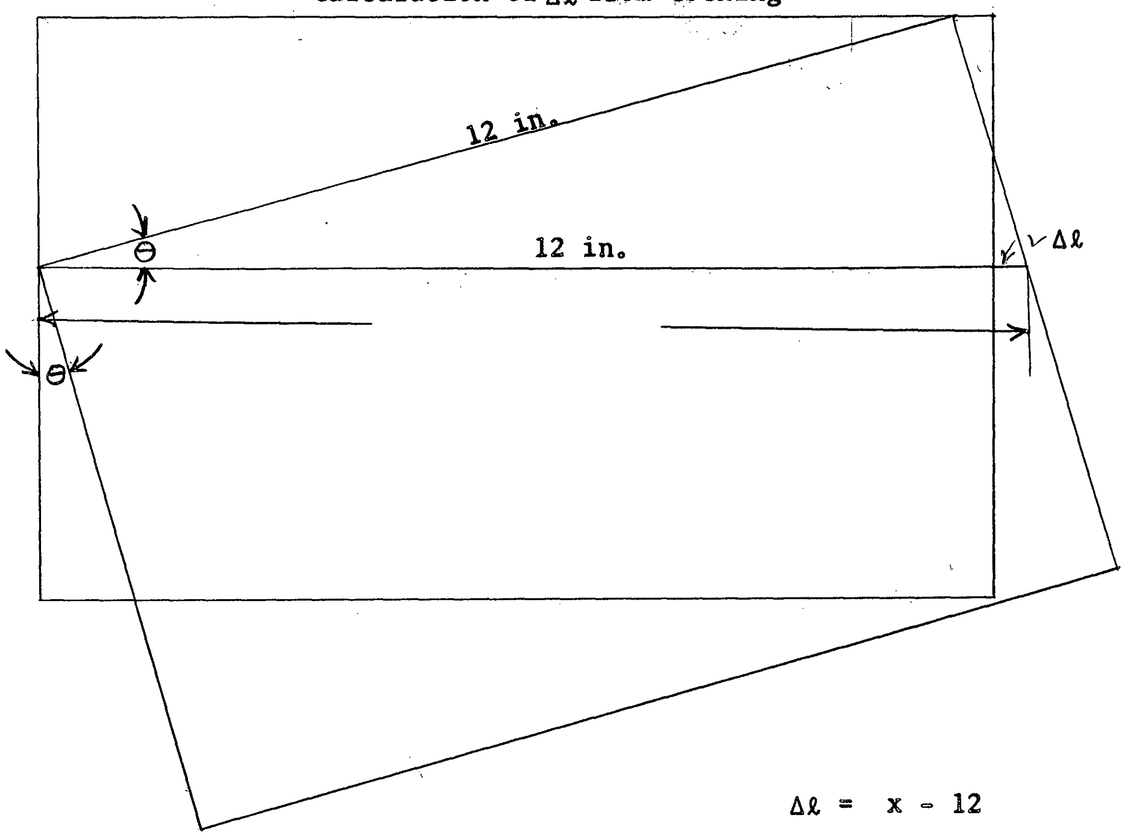
	Measured $\times 10^{-3}$			Calculated $\times 10^{-3}$
Sin θ	Vertical	Lateral	Sin θ	Vertical & Horizontal
.000833	0.00	0.01	0.001	0.006
.00167	0.00	0.00	0.002	0.024
.00250	0.00	-0.02	0.003	0.060
.00333	0.00	0.07	0.004	0.092
.00416	0.00	0.13	0.005	0.150
.00500	0.02	0.24	0.006	0.217
.00583	0.10	0.20	0.007	0.294
.00667	0.14	0.38	0.008	0.380
.00750	0.24	0.41		
.00833	0.34	0.60		
.00917	0.42	0.91		
.01000	0.48	1.64		

APPENDIX 4. Curve of Length Change vs Sine of Cocking Angle



APPENDIX 5.

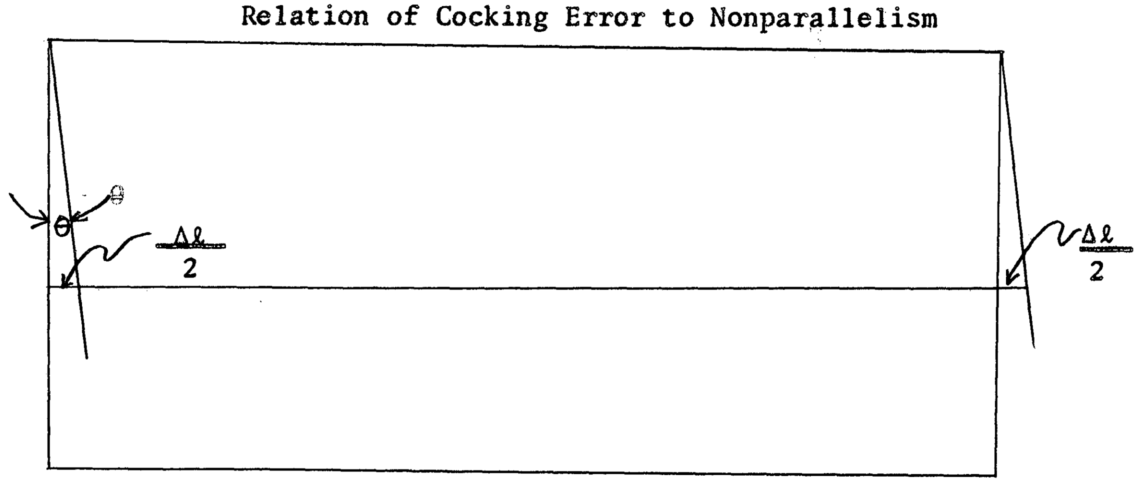
Calculation of $\Delta \ell$ from Cocking



$$\Delta \ell = x - 12$$

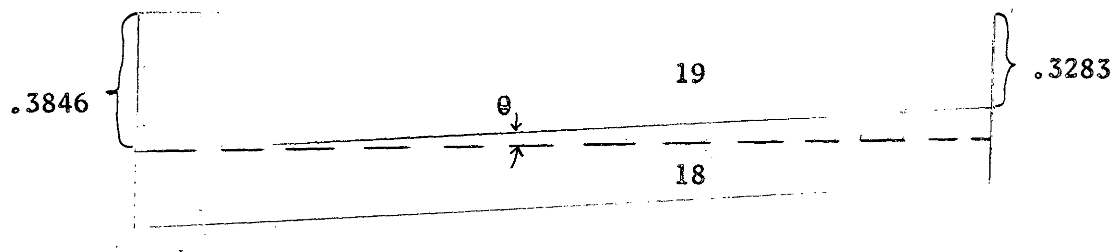
$$x = 12 (\sec \theta)$$

Relation of Cocking Error to Nonparallelism

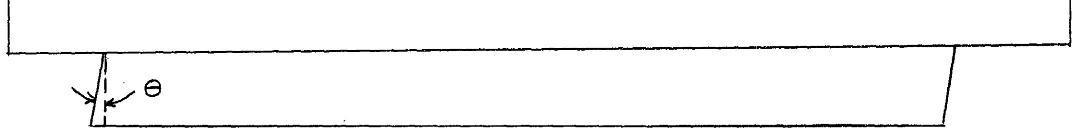


APPENDIX 6

Sample Calculation for Correction for Non-Parallelism of Strips



$$\sin\theta = \frac{0.057}{12.072} = 0.0048$$



From Graph = .00022

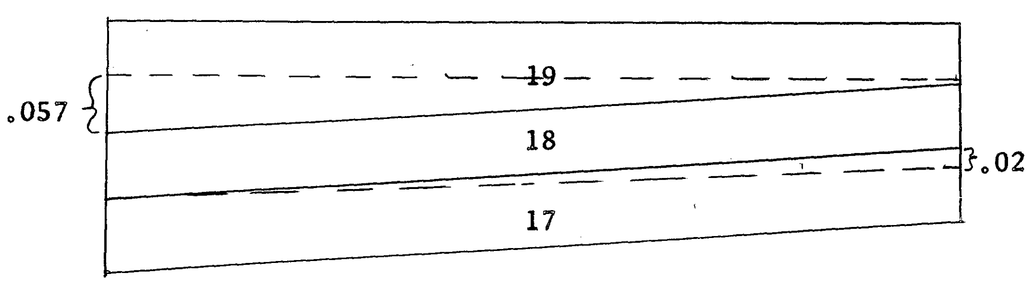
Correction in Stress Calculation:

Initial gage difference	0.00023
Final gage difference	0.00107

$$0.00107 - 0.00022 = 0.00075$$

$$0.00075 - 0.00023 = 0.00053$$

$$S = \frac{0.00053}{12.072} \times 30{,}000{,}000 = 1{,}320 \text{ psi}$$



APPENDIX 7. Corrections for Non-Parallelism of Strips

Strip	Sin θ	Error	Corrected Stress
18	0.00480	0.00022	1,320
17	0.0458	0.00020	3,830
16	0.00480	0.00022	3,770
15	0.00480	0.00022	3,680
14	0.00458	0.00020	2,510
13	0.0010	0.00001	No correction

Note: 0.0001 is a negligible error; the errors in all other strips are negligible.

APPENDIX 8. Location of Points Measured on Strips.

Distance of Initial Measured Point and
Points Measured on Strips from Reference Edge. (inches)

Strip	Initial Point	Point Measured on Strip	Difference
19	0.125	0.125	.0
18	0.500	0.515	0.0156
17	0.875	0.820	0.055
16	1.125	1.123	0.058
15	1.500	1.490	0.010
14	1.750	1.796	0.046
13	2.000	2.067	0.067
12	2.375	2.377	0.002
11	2.625	2.686	0.062
10	3.000	2.994	0.006
9	3.250	3.301	0.051
8	3.625	3.601	0.024
7	3.875	3.911	0.036
6	4.250	4.219	0.031
5	4.500	4.520	0.020
4	4.875	4.835	0.040
3	5.125	5.130	0.005
2	5.500	5.499	0.001
1	5.750	5.761	0.011

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