HORIZONTAL ELECTRICAL PROFILING IN GROUNDWATER EXPLORATION

by

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ABSTRACT

It is important for the Geophysicist doing ground-water exploration, to be able to map vertical geologic structures such as dikes,faults, joints, solution cavities and geologic contacts.

A method that can be used for this purpose is the horisontal profiling method, using a Wenner array of electrodes with a constant electrode separation. Considering first a single vertical plane, then two vertical planes to simulate the above mentioned geological structures the image theory were used to solve the Laplace equation.

These solutions were programmed to yield a set of master curves that can be used to interpret horizontal electrical profiling data.

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INTRODUCTION

In groundwater exploration, the Exploration Geophysicist is faced with two basic kinds of problems. In the first place he must be able to map horizontal structures with the main purpose of obtaining an accurate profile of the bedrock in alluvial-filled valleys. Vertical electrical profiling methods are extensively used for this purpose with many sets of master curves available to facilitate in the interpretation of the data.

The second kind of problem is the mapping of vertical structures such as dikes, faults, joints, solution cavities and geologic contacts. These structures can be mapped by means of horizontal profiling methods where a chosen array is moved along a horizontal profile, keeping a constant electrode separation.

Unlike the case of vertical profiling, there are hardly any master curves available for the interpretation of horizontal profiles. In this report the image theory is used to find solutions for the Laplace equation, considering three basic situations. First I considered a single vertical plane, representing geologic contacts or faults. Secondly I considered two vertical planes with the same resistivities in the first and third layers, representing dikes, joints, brecciated zones and solution cavities. In the third place I considered the more general situation where none of the resistivities of the three vertical layers are the same.

The solutions were programmed and an extensive set of master curves were calculated and plotted, using the FORTRAN 1V language and a IBM 360 computer.

CHAPTER 1

OCCURRENCE OF GROUNDWATER

Before a geophysicist can decide on the geophysical method to be used in a groundwater exploration project, it is essential to know which kind of target to look for. It is thus necessary to know where groundwater occur in different geologic environments. From a hydrological point of view, we divide the geology into the following environments.

1. Nonindurated sediments.
2. Sedimentary rocks
3. Metamorphic and igneous rocks

1.1. Groundwater in Nonindurated Sediments

Nonindurated sediments are usually the most popular terrain to do groundwater exploration in. They have relatively high permeabilities, are easy to drill and are normally in favourable locations for groundwater development. On the basis of their origin, nonindurated sediments can be divided into different categories. The more important of these are:

1.1.1: River valleys

Although river valley sediments, which normally consist of clay, silts, sand and gravel, display great lateral variability, they normally have a simple vertical succession from coarse sands or gravels near the bottom of the channel to silts and clays at the top. (Davis & De Wiest, 1966).

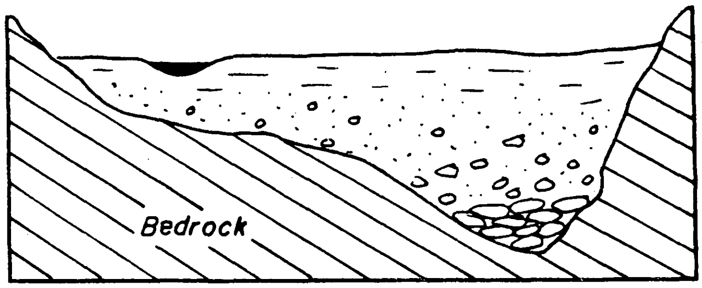


Figure 1.1: A typical cross-section of a river valley.

The main problem of the geophysicist in these regions is mapping the bedrock. Vertical electrical profiling would be inexpensive, especially where the bedrock consists of high resistivity rocks.

1.1.2: Valleys of Tectonic Origin

Large valleys often originate through tectonic movements. They are usually bounded by long fault systems and filled by debris eroded from surrounding mountains.

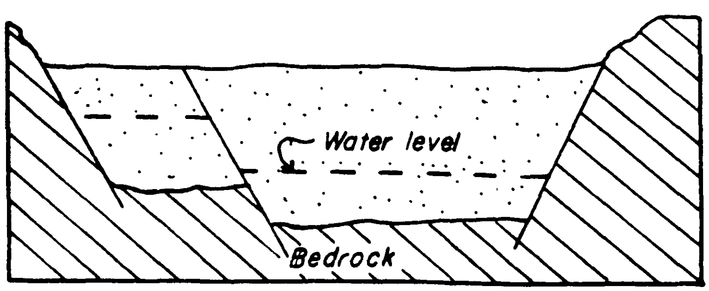


Figure 1.2: Cross-section of a tectonic valley.

Individual aquifers are usually burried channel deposits and are extremely difficult to locate. A very important target in these regions would be the mapping of faults. In unconsolidated sediments, faults tend to form impermeable boundaries, having a damming action where there is a groundwater gradient. Seismic refraction and horizontal electrical profiling are both methods that can be successfully used in mapping these faults.

1.1.3: Coastal Plains

These plains vary in size from small isolated valley deposits, to broad plains fringing hundreds of miles of coastlines. Sediments usually represent both alluvial and marine environments. As in the case of tectonic valleys, water exploration consists mainly in locating and mapping permeable horizons as well as faults that might act as barriers in these aquifers. Seismic techniques are often used to trace these aquifers.

1.1.4: Glaciated Terrain

About 20% of the presently exposed land surface of the Earth was once covered by ice. This ice cover had a profound influence on the hydrogeology of these areas. Glacial deposit, called till, is characterized by poor sorting and scattered boulders. This unsorted character, combined with the compacting weight of the glacial ice, makes till rather dense and impermeable.

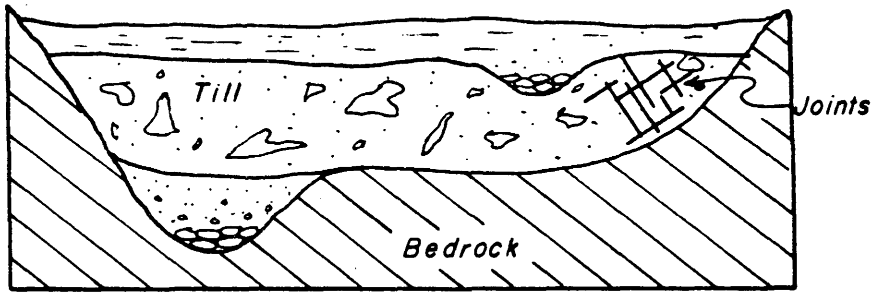


Figure 1.3: Old river valley filled with glacial till.

Though a limited amount of water can be obtained from fracture systems in the till, the groundwater explorer would concentrate more on finding gravel-filled channels that were formed during retreat of the glacial margin. Till-covered valleys should also be an important target as many of these valleys have thick layers of permeable sands and gravels underneath the till.

1.2. Groundwater in Sedimentary Rocks

About 50% of all sedimentary rocks consist of shale, claystone, siltstone and other fine grained detrital rocks. Next in abundance is sandstone, then carbonate rocks, while about 2% are made up out of conglomerate, gypsum, tillite and salt. (Davis & de Wiest, 1966).

1.2.1. Fine Grained Detrital Rocks

Although these rocks may have high porosities, they normally have very low permeabilities and consequently act as barriers to groundwater movements. These beds yield very little water to a well. They are usually more important as marker beds. They are continuous over very large areas and can be mapped by using seismic refraction methods. When a water well has to be sighted in these rocks, the main target would be the location and mapping of faults and fractures. Fractures in these rocks often exist to great depths.

1.2.2. Sandstones

Sandstones have low porosities and permeabilities.Most favourable areas for groundwater developement are along fault zones and jointed zones. Wells should be located in broad valleys or upland plateaus rather than hill crests or valley slopes.

1.2.3. Carbonate Rocks

Carbonate rocks can yield enormous amounts of water. It is ,however, extremely difficult to predict yields from carbonate rocks, which makes the sighting of successful wells very hard.

Groundwater in these rocks is mainly confined to vertical solution cavities, along faults or joints and horizontal solution cavities along bedding planes.

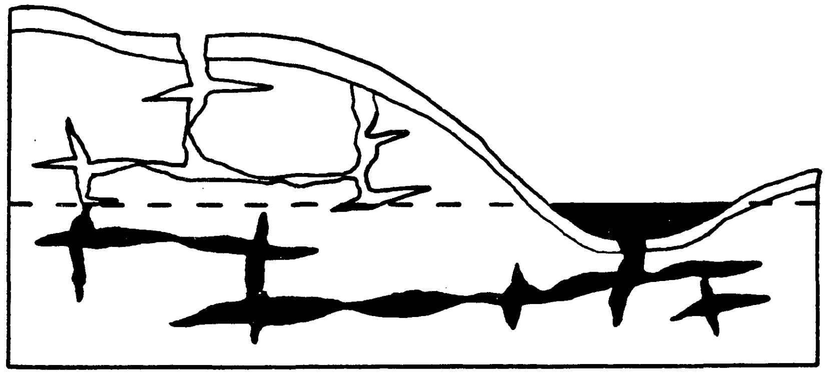


Figure 1.4: Occurence of groundwater in limestone and dolomite.

From the standpoint of water production, the horizontal cavities are more important as they tend to remain open over large distances and are much easier intersected by a well than the relatively widely spaced vertical cavities. As these horizontal cavities tend to be better developed near faults, the mapping of faults is a primary target when exploring for groundwater in carbonate rocks.

1.3. Groundwater in Igneous and Metamorphic Rocks

These form the most difficult geologic environments in which to successfully locate a well. Extreme variations in lithology, coupled with highly localized water-producing zones, make geophysical exploration very difficult.

The water bearing targets in these rocks are faults, dikes and geologic contacts, especially the fracture zones associated with faults. These faults and fractures tend

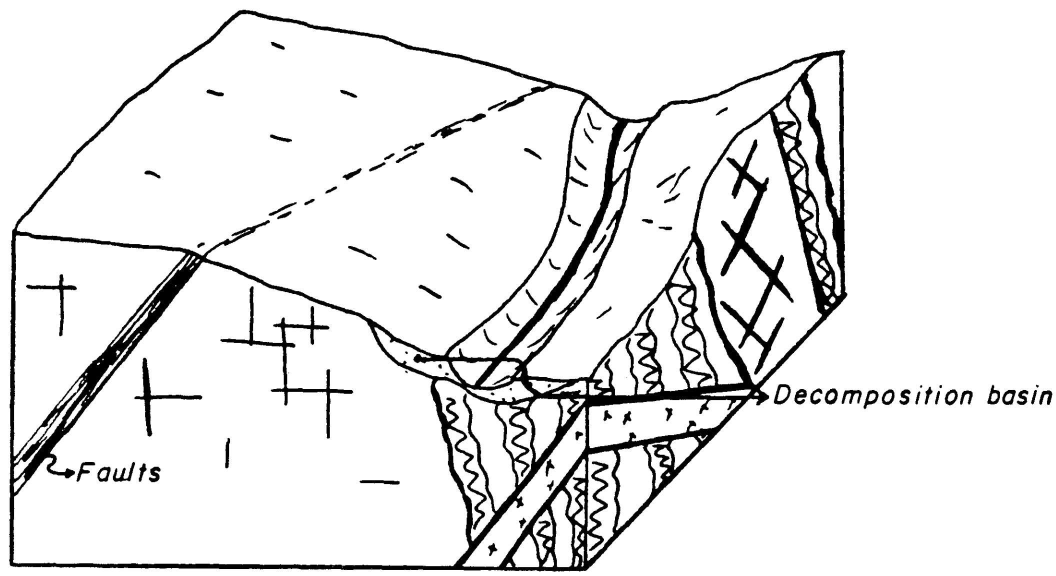


Figure 1.5: Targets in metamorphic and igneous rocks.

to be vertical and it has been found unwise to drill wells deeper than 200 feet. Another relatively easy target to look for in these regions is a decomposition basin near a source of recharge.

From the dicussions in this chapter it follows that the basic problems for a geophysicist exploring for groundwater, are the mapping of bedrock, faults, fractures, dikes and vertical geologic contacts. The mapping of bedrock can quite easily and accurately be done by the use of seismic refraction methods and vertical electrical profiling methods. More difficult from the geophysical point of view is the mapping of faults, fractures, dikes and vertical contacts. These features can be mapped by using the horizontal electrical profiling method, provided the explorer has a proper understanding of the possible shapes his field data curves can assume.

It is the main purpose of this work to generate examples of master curves that can result from different geologic situations.

CHAPTER 2

AVAILABLE METHODS IN GROUNDWATER EXPLORATION

2.1 GEOLOGIC METHODS

A short summery of geologic methods used in groundwater exploration, are:

(a) Lithological mapping in which different lithological units with similar water bearing properties are grouped together (Davis & De Wiest, 1966).

(b) Stratigraphic methods can be used in determining the position and thickness of a water bearing horizon.

(c) In conjunction with stratigraphy, structural geology is essential in locating a displaced water-bearing horizon. It is also useful in locating fracture systems and faults.

(d) In many regions, geomorphology is considered an indispensable tool. Many permeable sediments, such as glacial deposits, ancient beach ridges and terrace deposits, are reflected in landforms.

2.2 AIR-PHOTO INTERPRETATION

As the occurrence of groundwater depends to a large extent upon terrain characteristics, proper air-photo interpretation can provide valuable information in this regard (Todd,1967). Vegetation,landform and use, drainage patterns erosion, color and special features such as terraces, alluvial planes and faults, are apparent on air-photos. These are strong indications of subsurface conditions.

2.3 GEOPHYSICAL METHODS

Geophysical methods used in groundwater exploration are:

(a) Gravity methods; These methods measure lateral changes in the Earth's gravitational pull associated with near-surface changes in density (Dobrin, 1952). Small geologic changes are, however, very difficult to detect by means of the gravity method. This method is thus of limited use in groundwater exploration, although it has been used with some success in mapping large burried valleys (Hall, 1962).

(b) Magnetic methods; Magnetic methods are probably the most rapid geophysical technique. However many magnetic anomalies are related to causes which have no relation with hydrological situations, making this method of little use in groundwater exploration.

(c) Seismic methods; Seismic methods used in groundwater exploration are mainly seismic refraction methods. The depth to bedrock, as well as faults, are mapped by using the contrast in seismic velocities of different geologic layers (Musgrave, 1967). These are, however, relatively expensive methods to use.

(d) Electrical methods; Of all geophysical methods, the electrical method has been most widely used in groundwater exploration. Although electrical prospecting methods are extremely diversified, the groundwater explorer is mainly interested in methods based on the determination of the resistivity, real or apparent, of successive geologic layers.

These methods consist of introducing an electric current through two electrodes into the Earth, measuring the resulting potential difference between two other electrodes and then calculating a value for the apparent resistivity. By using existing computer programs (Crous 1971), in conjunction with master curves (Orellana and Mooney, 1966), depths to horizontal boundaries can be determined.

The electrical resistivity methods, which differ mainly with respect to the relative positions of the current and potential electrodes, can be divided into two groups; namely vertical profiling methods and horizontal profiling methods.

In the vertical profiling methods, the centre of the electrode configuration stays fixed while the electrodes are successively moved outward. This generates a curve of apparent resistivity versus depth, yielding a vertical profile of apparent resistivities.

In horizontal profiling methods the entire array, with fixed electrode separation, is moved along a straight line. This yields a horizontal profile that shows the variation of the apparent resistivity at a chosen depth along the profile.

Vertical profiling methods work excellent when used to map horizontal, or near horizontal, layers. The theory is well known and master curves for two, three, four and five layers are available.

Horizontal profiling methods should be used when mapping vertical structures such as contacts between two geologic units, faults, fractures, dikes and solution cavities. Although these methods are not unknown, few master curves exist.

In the following chapters I will find solutions for the Laplace equation, using the image theory. The equation is solved for a single vertical plane as well as two vertical planes.

CHAPTER 3

A VERTICAL PLANE

(Faults and Geologic Contacts)

3.1. GENERAL EXPRESSIONS FOR THE POTENTIAL CAUSED BY A SINGLE SOURCE

There are two ways to obtain expressions for the potential in a medium, namely (i) by direct intergration of Laplace's equation, or (ii) by the method of images. As shown by Keller (1953) the method of images can only be applied to certain geometric shapes, the vertical plane being one of them.

In figure 3.1 we have a current electrode, C, supplying a current of strength I, and we wish to determine the potential at P in medium 1, and at P' in medium 2.

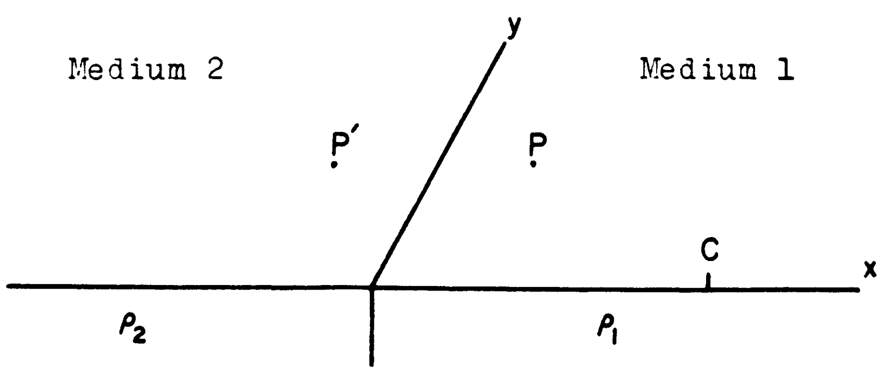


Figure 3.1: A vertical plane separating two homogeneous, isotropic media of resistivities ρ_1 and ρ_2.

The method of images imagines a mirror in the place of the separating plane, and then divides the problem into two parts, as shown in figure 3.2.

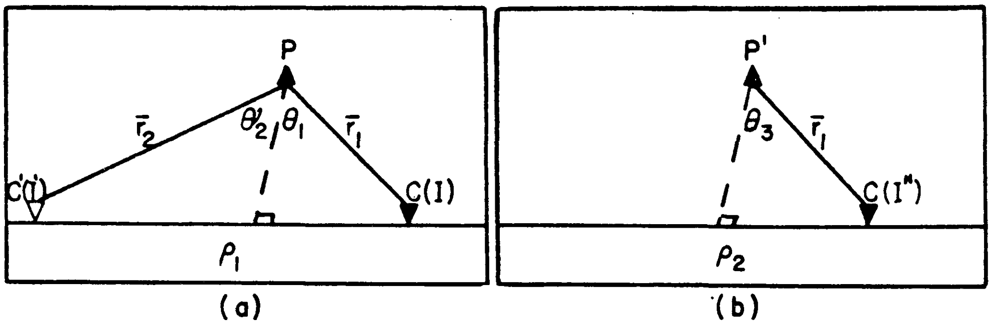


Figure 3.2: Application of the method of images to the problem depicted in figure 3.1.

First, the potential at P can be described as if we had only one medium with resistivity ρ_1 , a source I at C and an image source I' at C', as shown in figure 2(a). Applying the solution of a point source, we have

$$U_{1A} \triangleq U_{P_1} = \frac{I\rho_1}{2\pi r_1} + \frac{I'\rho_1}{2\pi r_2} \tag{1}$$

The potential at P' can be described as if we had only one medium with resistivity ρ_2 and a source I" at C, as shown in figure 2(b). This gives us

$$U_{2A} \triangleq U_{P_2} = \frac{I''\rho_2}{2\pi r_1} \tag{2}$$

We use the notation (Van Nostrand and Cook, 1966) U_{mn} where

m = 1 refers to the potential in medium 1,

m = 2 refers to the potential in medium 2,

n = A implies that the source is in medium 1 and

n = B implies that the source is in medium 2.

The potential functions have to meet the following boundary conditions: (Keller and Frischknecht, 1966).

1. The potential function U, has to be continuous across the boundary between the two media.

2. The normal component of current flow across the boundary must be continuous.

The first condition gives us

$$U_{1A} = U_{2A} \quad \text{at} \quad x = 0$$

$$\therefore \quad I\rho_1 + I'\rho_1 = I''\rho_2 \tag{3}$$

The second condition says

$$J_n = J_n'$$

now

$$\vec{J} = \frac{I\vec{r}_1}{2\pi r_1^3} + \frac{I'\vec{r}_2}{2\pi r_2^3}$$

$$\vec{J}' = \frac{I''\vec{r}_1}{2\pi r_1^3}$$

$$J_n = \frac{I\sin\theta_1}{2\pi r_1^2} + \frac{I'\sin\theta_2}{2\pi r_2^2} = \frac{I''\sin\theta_3}{2\pi r_1^2} = J_n' \tag{4}$$

At the boundary we have

$$\sin\theta_1 = -\sin\theta_2 = \sin\theta_3$$

$$r_1 = r_2$$

$$\therefore \quad I'' = I' - I \tag{5}$$

equations (3) and (4) gives us

$$I' = k_{12} I \tag{6}$$

and
$$I'' = (1 - k_{12}) I \tag{7}$$

with
$$k_{12} = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \triangleq \text{Reflection coefficient}$$

Substituting I' and I" in equations (1) and (2) gives us

$$U_{1A} = \frac{I\rho_1}{2\pi}\left[\frac{1}{r_1} + \frac{k_{12}}{r_2}\right] \quad \text{and} \quad U_{2A} = \frac{I\rho_2}{2\pi r_1}\left[1 - k_{12}\right] \tag{8}$$

In exactly the same way we can show that, when the source is in medium 2 we have

$$U_{1B} = \frac{I\rho_1}{2\pi r_3}\left[1 + k_{12}\right] \quad \text{and} \quad U_{2B} = \frac{I\rho_2}{2\pi}\left[\frac{1}{r_3} + \frac{k_{12}}{r_4}\right] \tag{9}$$

where r_3 and r_4 are defined as in figure 3.2.

3.2 APPLICATION TO THE WENNER ARRAY

3.2.1 Horizontal profiling perpendicular to the strike of the plane

When applying the method of images to the situation depicted in figure 3.3, equations (8) and (9) will generate

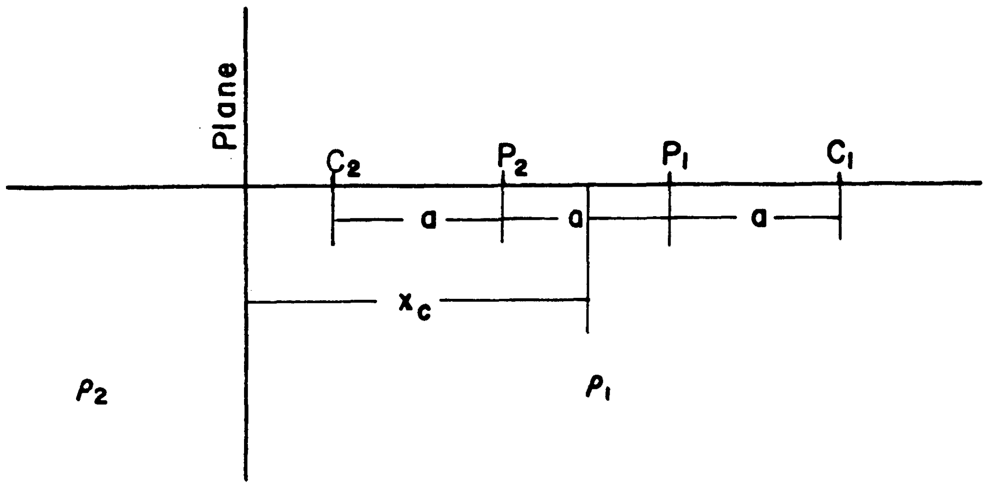


Figure 3.3: The Wenner array along a horizontal profile perpendicular to the strike of the plane.

a variety of potential functions, depending on the position of the array relative to the plane. These are divided into 5 groups, according to the 5 basic ranges of values x_c can assume. We thus have 5 equations for the apparent resistivity necessary to describe a single horizontal profile.

The procedure to derive these equations is to calculate the total potential at each potential electrode due to both the source electrodes, determine the potential difference between the two, and then substitute into the equation

$$\rho_a = 2\pi a \frac{V_{12}}{I} \tag{10}$$

which gives us the apparent resistivity for the Wenner configuration of electrodes (Dobrin, 1952).

As an example, we consider the case in figure 3.3, where we have $x_c > 3a/2$. Here we have all the electrodes in medium 1 and can thus use the expression for U_{1A}, equation 8, in all cases.

(a) The potential at P_1, due to the source at C_1, is

$$U_{P_1C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{2(x_c + a)}\right] \tag{11}$$

(b The potential at P_1, due to the source at C_2, is

$$U_{P_1C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + \frac{k_{12}}{2x_c - a}\right] \tag{12}$$

$$U_{TP_1} = U_{P_1C_1} + U_{P_1C_2}$$

$$= \frac{I\rho_1}{2\pi a}\left[\frac{1}{2} + \frac{3a^2 k_{12}}{2(x_c + a)(2x_c - a)}\right] \tag{13}$$

(c) The potential at P_2, due to the source at C_1, is

$$U_{P_2C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + \frac{k_{12}}{2x_c + a}\right] \tag{14}$$

(d) The potential at P_2, due to the source at C_2, is

$$U_{P_2C_2} = \frac{-I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{2(x_c - a)}\right] \tag{15}$$

$$U_{TP_2} = U_{P_2C_1} + U_{P_2C_2}$$

$$= \frac{I\rho_1}{2\pi a}\left[-\frac{1}{2} + \frac{3a^2k_{12}}{2(x_c - a)(2x_c + a)}\right] \tag{16}$$

Using equations (13) and (16) we get

$$U_{12} = U_{TP_2} - U_{TP_1}$$

$$= \frac{I\rho_1}{2\pi a}\left[-1 + \frac{3a^2k_{12}(x_c - a)(2x_c + a) - 3a^2k_{12}(x_c + a)(2x_c - a)}{2(x_c^2 - a^2)(4x_c^2 - a^2)}\right] \tag{17}$$

Substituting equations (17) into equation (10) gives us

$$\frac{\rho_a}{\rho_1} = -1 - \frac{3a^3x_ck_{12}}{(x_c^2 - a^2)(4x_c^2 - a^2)} \tag{18}$$

Just as it has become customary to use the ratio ρ_a/ρ_1 instead of an absolute value for ρ_a it is also customary to use the ratio x_c:a, instead of x_c. (18) thus becomes

$$\frac{\rho_a}{\rho_1} = -1 - \frac{3\eta k_{12}}{(\eta^2 - 1)(4\eta^2 - 1)} \tag{19}$$

where $\eta = \frac{x_c}{a}$

In the same way, using the appropriate expressions for U, given by equations (8) and (9), we can derive expressions for ρ_a/ρ_1 for the other 4 ranges of η. However when we are using the horizontal profiling technique to map a vertical plane, we often do not know the strike of the plane, consequently we seldom have our profile oriented perpendicular to the strike. This leads me to consider the more general situation of the horizontal profile crossing the strike at an angle.

3.2.2 The Horizontal Profile Making an Angle α with the Strike of the Plane.

As in paragraph 3.2.1, I will again demonstrate the solution for the case where we have all the electrodes in medium 1, as in figure 3.4, again only needing the expression for U_{1A}, equation 8 .

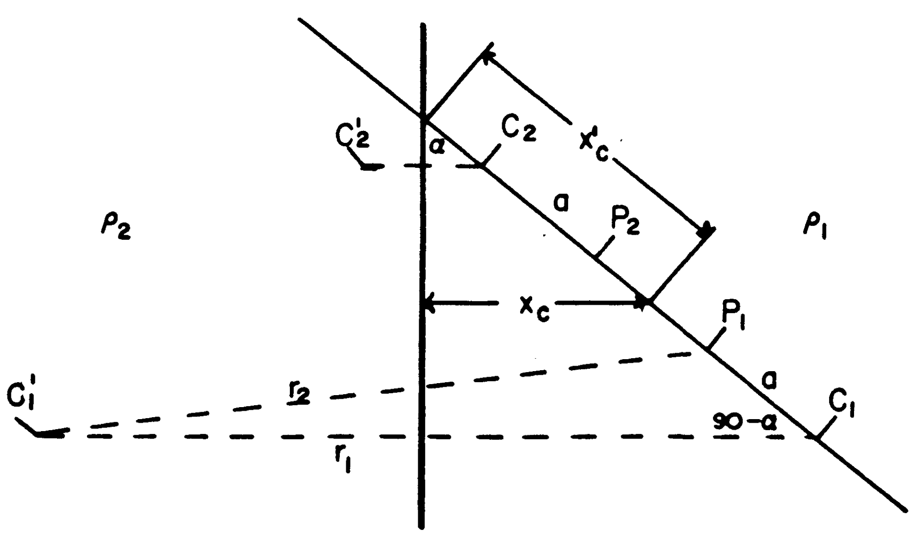


Figure 3.4: The Wenner array along a horizontal profile crossing the strike of the plane at an angle α.

(a) The potential at P_1, due to the source at C_1, is

$$U_{P_1C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{\sqrt{a^2 + (2x_c + a\sin\alpha)(2x_c + 3a\sin\alpha)}}\right] \qquad (20)$$

The r_2 substituted into the equation for U_{1A}, is the r_2 indicated in figure 3.4. The value is obtained by using the cosine rule and substituting $x_c = x_c' \sin\alpha$, and $\cos(90 \pm \alpha) = \mp \sin\alpha$. The expressions used for the corresponding r_2's in the following equations were all derived in the same way.

(b) The potential at P_1 due to the source at C_2, is

$$U_{P_1C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + \frac{k_{12}}{\sqrt{4a^2 + (2x_c - 3a\sin\alpha)(2x_c + a\sin\alpha)}}\right] \qquad (21)$$

$$U_{TP_1} = \frac{I\rho_1}{2\pi a}\left[\frac{1}{2} + \frac{ak_{12}}{\sqrt{a^2 + (2x_c + a\sin\alpha)(2x_c + 3a\sin\alpha)}} - \frac{ak_{12}}{\sqrt{4a^2 + (2x_c - 3a\sin\alpha)(2x_c + a\sin\alpha)}}\right] \qquad (22)$$

(c) The potential at P_2 due to the source at C_1, is

$$U_{P_2C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + \frac{k_{12}}{\sqrt{4a^2 + (2x_c + 3a\sin\alpha)(2x_c - a\sin\alpha)}}\right] \qquad (23)$$

(d) The potential at P_2 due to the source at C_2, is

$$U_{P_2C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{\sqrt{a^2 + (2x_c - 3a\sin\alpha)(2x_c - a\sin\alpha)}}\right] \qquad (24)$$

$$U_{TP_2} = \frac{I\rho_1}{2\pi a}\left[-\frac{1}{2} + \frac{a k_{12}}{\sqrt{4a^2 + (2x_c + 3a\sin\alpha)(2x_c - a\sin\alpha)}} - \frac{a k_{12}}{\sqrt{a^2 + (2x_c - 3a\sin\alpha)(2x_c - a\sin\alpha)}}\right] \quad (25)$$

Taking $V_{12} = U_{TP_2} - U_{TP_1}$ and substituting into equation (10) we have

$$\frac{\rho_a}{\rho_1} = -1 + \frac{k_{12}}{\sqrt{4 + (2\eta + 3\sin\alpha)(2\eta - \sin\alpha)}} - \frac{k_{12}}{\sqrt{1 + (2\eta - 3\sin\alpha)(2\eta - \sin\alpha)}} - \frac{k_{12}}{\sqrt{1 + (2\eta + \sin\alpha)(2\eta + 3\sin\alpha)}} + \frac{k_{12}}{\sqrt{4 + (2\eta - 3\sin\alpha)(2\eta + \sin\alpha)}} \quad (26)$$

By taking $\alpha = 90^\circ$, equation (26) reduces to equation (19).

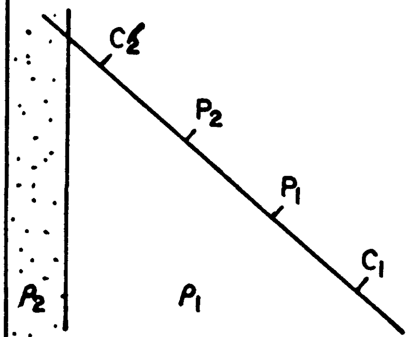
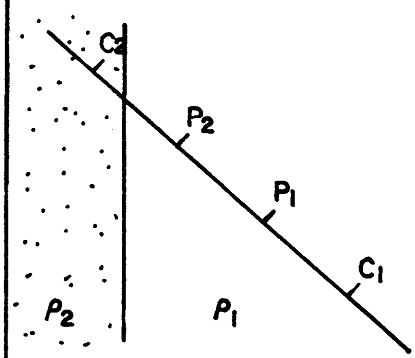
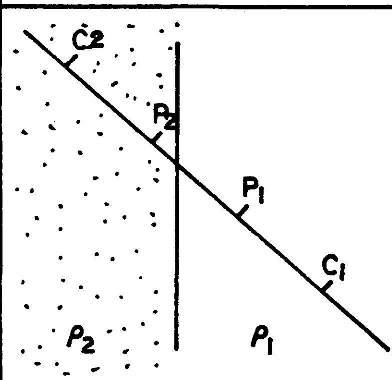
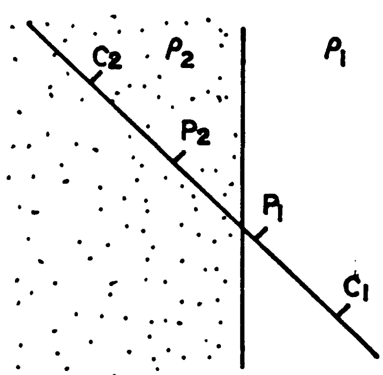
3.3 CALCULATING CURVES OF ρ_a/ρ_1 VERSUS η

The procedure described in paragraph 3.2, was used in developing program 1. The program is designed to calculate the ratio ρ_a/ρ_1 as a function of η. Basically each curve is built up out of 5 sections, each section being determined by the position of the array relative to the plane. For calculating purposes however, I have divided the curve into 6 parts. The 6 sections are calculated by using the appropriate formulae given in table 1. In all 6 cases we have

$$\eta = x_c/a$$

$$x_c = x_c' \sin\alpha$$

$$r_2 = a\sqrt{1 + (2\eta + \sin\alpha)(2\eta + 3\sin\alpha)}$$

Range of η	Position of array relative to vertical plane.	Formulae to be used.
$\eta > \frac{3}{2}\sin a$		$U_{P_1C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_2}\right]$ $U_{P_1C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + \frac{k_{12}}{r_3}\right]$ $U_{P_2C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + \frac{k_{12}}{r_4}\right]$ $U_{P_2C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_5}\right]$
$\frac{3}{2}\sin a > \eta > \frac{1}{2}\sin a$		$U_{P_1C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_2}\right]$ $U_{P_1C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1 + k_{12}}{2a}\right]$ $U_{P_2C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + \frac{k_{12}}{r_4}\right]$ $U_{P_2C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1 + k_{12}}{a}\right]$
$\frac{1}{2}\sin a > \eta > 0$		$U_{P_1C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_2}\right]$ $U_{P_1C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1 + k_{12}}{2a}\right]$ $U_{P_2C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1 + k_{12}}{2a}\right]$ $U_{P_2C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1 + k_{12}}{1 - k_{12}}\right]\left[\frac{1}{a} - \frac{k_{12}}{r_6}\right]$
$0 > \eta > -\frac{1}{2}\sin a$		$U_{P_1C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_6}\right]$ $U_{P_1C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1 + k_{12}}{2a}\right]$ $U_{P_2C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1 + k_{12}}{2a}\right]$ $U_{P_2C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1 + k_{12}}{1 - k_{12}}\right]\left[\frac{1}{a} - \frac{k_{12}}{r_2}\right]$

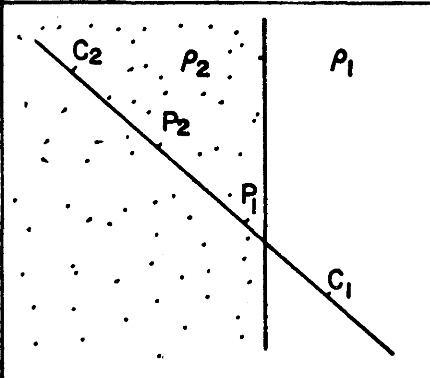
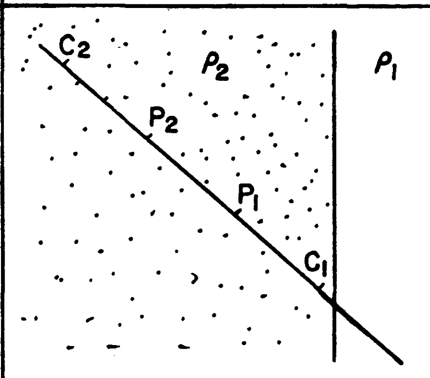
Range of η	Position of array relative to vertical plane.	Formulae to be used.
$-\frac{1}{2}\sin\alpha > \eta > -\frac{3}{2}\sin\alpha$		$U_{P_1C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1+k_{12}}{a}\right]$ $U_{P_1C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1+k_{12}}{1-k_{12}}\right]\left[\frac{1}{2a} - \frac{k_{12}}{r_4}\right]$ $U_{P_2C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1+k_{12}}{2a}\right]$ $U_{P_2C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1+k_{12}}{1-k_{12}}\right]\left[\frac{1}{a} - \frac{k_{12}}{r_2}\right]$
$-\frac{3}{2}\sin\alpha > \eta$		$U_{P_1C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1+k_{12}}{1-k_{12}}\right]\left[\frac{1}{a} - \frac{k_{12}}{r_5}\right]$ $U_{P_1C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1+k_{12}}{1-k_{12}}\right]\left[\frac{1}{2a} - \frac{k_{12}}{r_4}\right]$ $U_{P_2C_1} = \frac{I\rho_1}{2\pi}\left[\frac{1+k_{12}}{1-k_{12}}\right]\left[\frac{1}{2a} - \frac{k_{12}}{r_3}\right]$ $U_{P_2C_2} = -\frac{I\rho_1}{2\pi}\left[\frac{1+k_{12}}{1-k_{12}}\right]\left[\frac{1}{a} - \frac{k_{12}}{r_2}\right]$

Table 1: Formulae used to calculate the master curves for a single vertical plane.

$$r_3 = a\sqrt{4 + (2\eta - 3\sin\alpha)(2\eta + \sin\alpha)}$$

$$r_4 = a\sqrt{4 + (2\eta + 3\sin\alpha)(2\eta - \sin\alpha)}$$

$$r_5 = a\sqrt{1 + (2\eta - 3\sin\alpha)(2\eta - \sin\alpha)}$$

In the program I have left I/2 as well as $2\pi/I$ out of the formulae in table 1, as they finally cancel out in any case.

CHAPTER 4

TWO VERTICAL PLANES

(Dikes, open faults, fractures and solution cavities)

4.1 The Potential due to a Single Source

Before considering the Wenner array, we first find the expressions for the potentials caused by a single source located in medium 1,2 and 3 successively; figure 4.1.

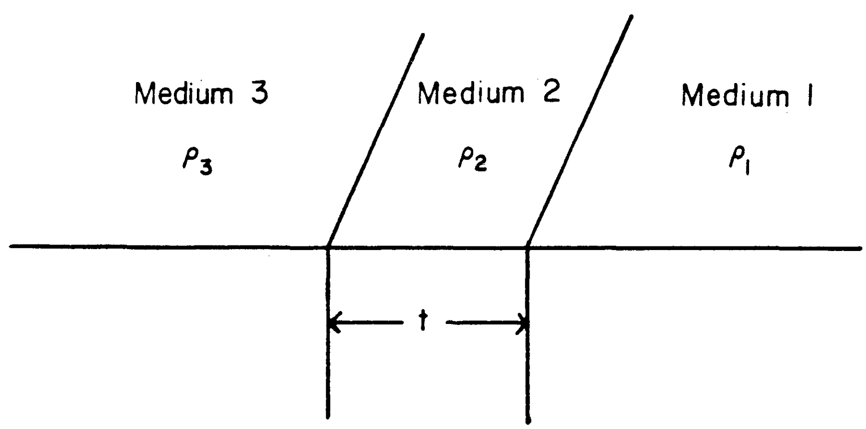


Figure 4.1: Two vertical planes a distance t units apart separating three media with resistivities ρ_1 , ρ_2 and ρ_3 respectively.

We now expand our notation of chapter 2, using U_{mn} such that

m = 1 refers to the potential in medium 1

m = 2 refers to the potential in medium 2

m = 3 refers to the potential in medium 3

n = A implies that the source is located in medium 1

n = B implies that the source is located in medium 2

n = C implies that the source is located in medium 3.

Using the image theory, we now need an infinite number of images, as shown in figures 4.2, 4.3 and 4.4, to satisfy the boundary equations at both planes (Dakhnov, 1959).

Considering that the images in a medium does not contribute to the potential in that medium, we have the following expresions:

(a) With the source in medium 1 (figure 4.2)

$$U_{1A} = \frac{I\rho_1}{2\pi}\left[\frac{1}{r_1} + \frac{k_{12}}{r_2} + (1+k_{12})(1-k_{12})\sum_n \frac{k_{21}^{n-1}\, k_{23}^{n}}{r_n^1}\right] \tag{27a}$$

$$U_{2A} = \frac{I\rho_2}{2\pi}(1-k_{12})\left[\frac{1}{r_1} + \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{r_n^1\, k_{21}} + \frac{1}{r_n^2}\right\}\right] \tag{27b}$$

$$U_{3A} = \frac{I\rho_3}{2\pi}(1-k_{12})(1-k_{23})\left[\frac{1}{r_1} + \sum_n \frac{k_{21}^n\, k_{23}^n}{r_n^1}\right] \tag{27c}$$

(b) With the source in medium 2 (figure 4.3)

$$U_{1B} = \frac{I\rho_1}{2\pi}(1+k_{12})\left[\frac{1}{r_1} + \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{k_{21} r_n^1} + \frac{1}{r_n^2}\right\}\right] \tag{28a}$$

$$U_{2B} = \frac{I\rho_2}{2\pi}\left[\frac{1}{r_1} + \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{k_{21} r_n^1} + \frac{1}{r_n^2} + \frac{1}{r_n^3} + \frac{1}{k_{23} r_n^4}\right\}\right] \tag{28b}$$

$$U_{3B} = \frac{I\rho_3}{2\pi}(1-k_{23})\left[\frac{1}{r_1} + \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{r_n^1} + \frac{1}{k_{23} r_n^2}\right\}\right] \tag{28c}$$

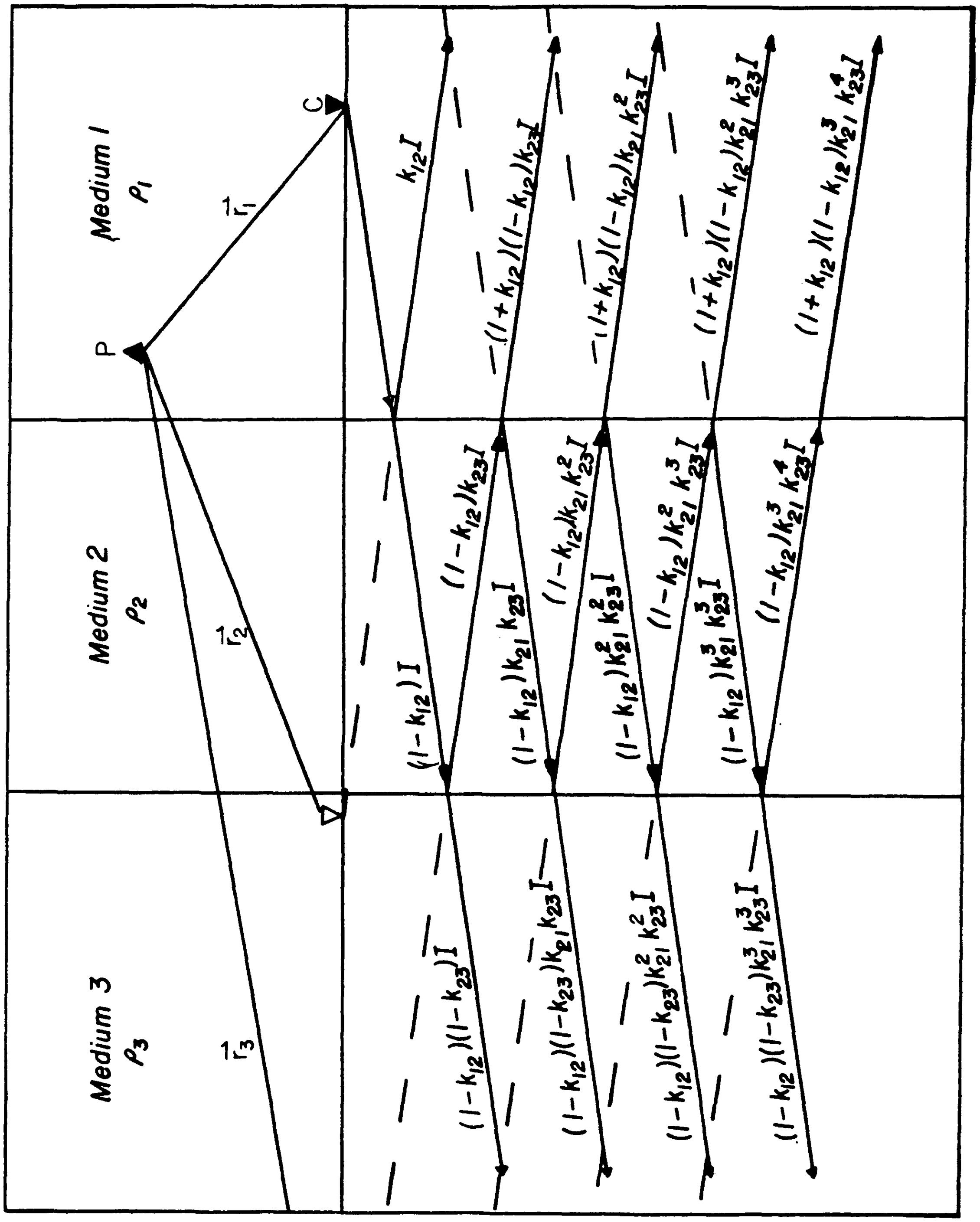


Figure 4.2 The images caused by a source in medium 1.

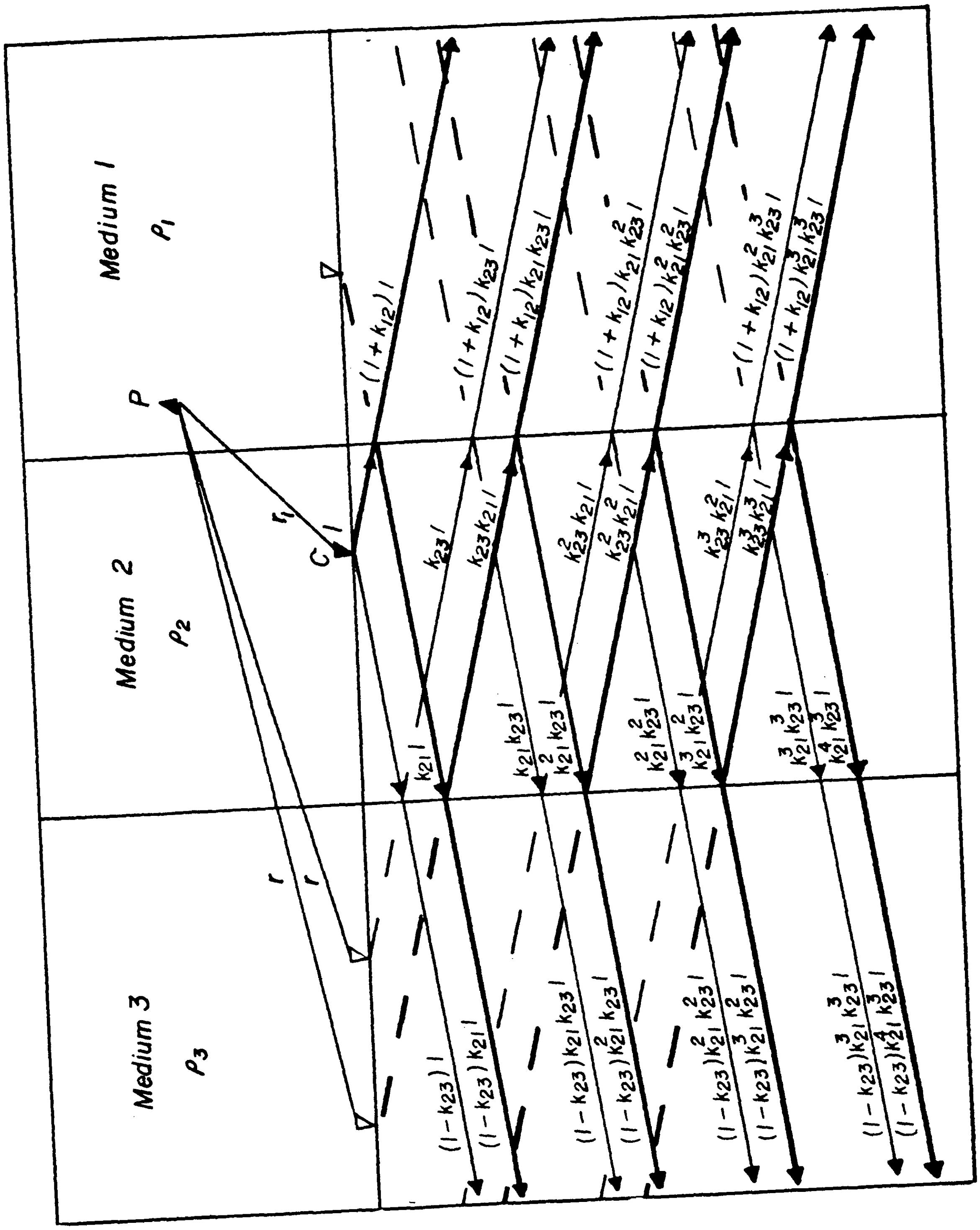


Figure 4.3 The images caused by a source in medium 2.

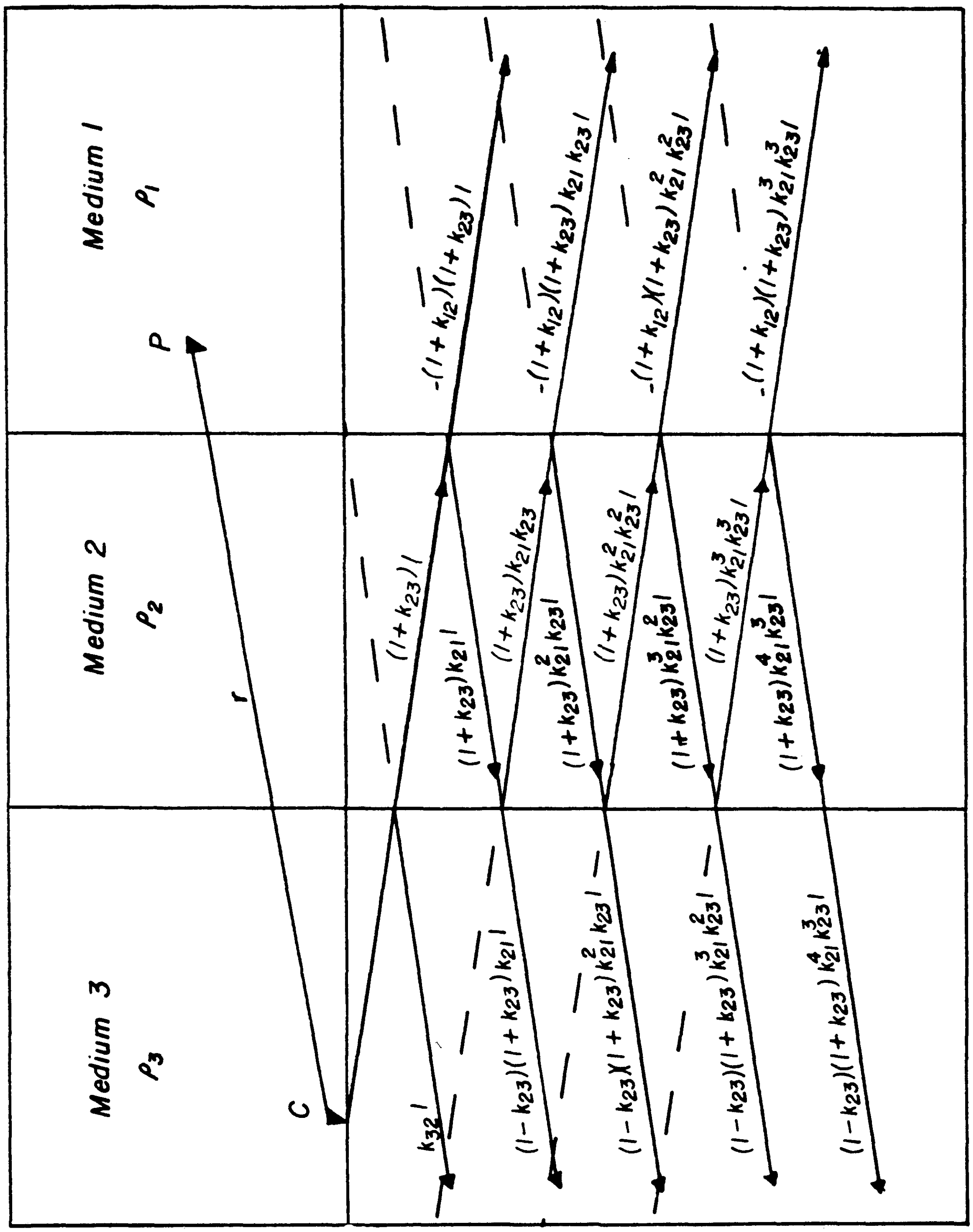


Figure 4.4 The images caused by a source in medium 3.

(c) With the source in medium 3 (figure 4.4)

$$U_{1c} = \frac{I\rho_1}{2\pi}(1+k_{12})(1+k_{23})\left[\frac{1}{r_1} + \sum_n \frac{k_{21}^n k_{23}^n}{r_n^1}\right] \quad (29)a$$

$$U_{2c} = \frac{I\rho_2}{2\pi}(1+k_{23})\left[\frac{1}{r_1} + \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{k_{23} r_n^1} + \frac{1}{r_n^2}\right\}\right] \quad (29)b$$

$$U_{3c} = \frac{I\rho_3}{2\pi}\left[\frac{1}{r_1} - \frac{k_{23}}{r_2} + (1-k_{23})(1+k_{23})\sum_n \frac{k_{21}^n k_{23}^{n-1}}{r_n^1}\right] \quad (29)c$$

In applying these equations to the Wenner array, we have to consider the following possible ranges of the thickness t, of medium 2.

4.2: $2a \sin\alpha < t < 3a \sin\alpha$

In this range of t we can have 1, 2 or 3 electrodes in the second medium at one time. Ten possible positions of the array, as shown in figure 4.5, was considered to determine the formulae used to calculate the master curves. Using the equations in paragraph 4.1, I calculated the potential at P_1 due to the source at C_1, then the potential due to the sinc at C_2 and added the expresions to find the total potential at P_1. The same with P_2. This yielded:

For position 1 figure 4.5

with $\beta \triangleq \frac{t}{a}$

$$\eta > \frac{3}{2}\sin\alpha + \frac{1}{2}\beta$$

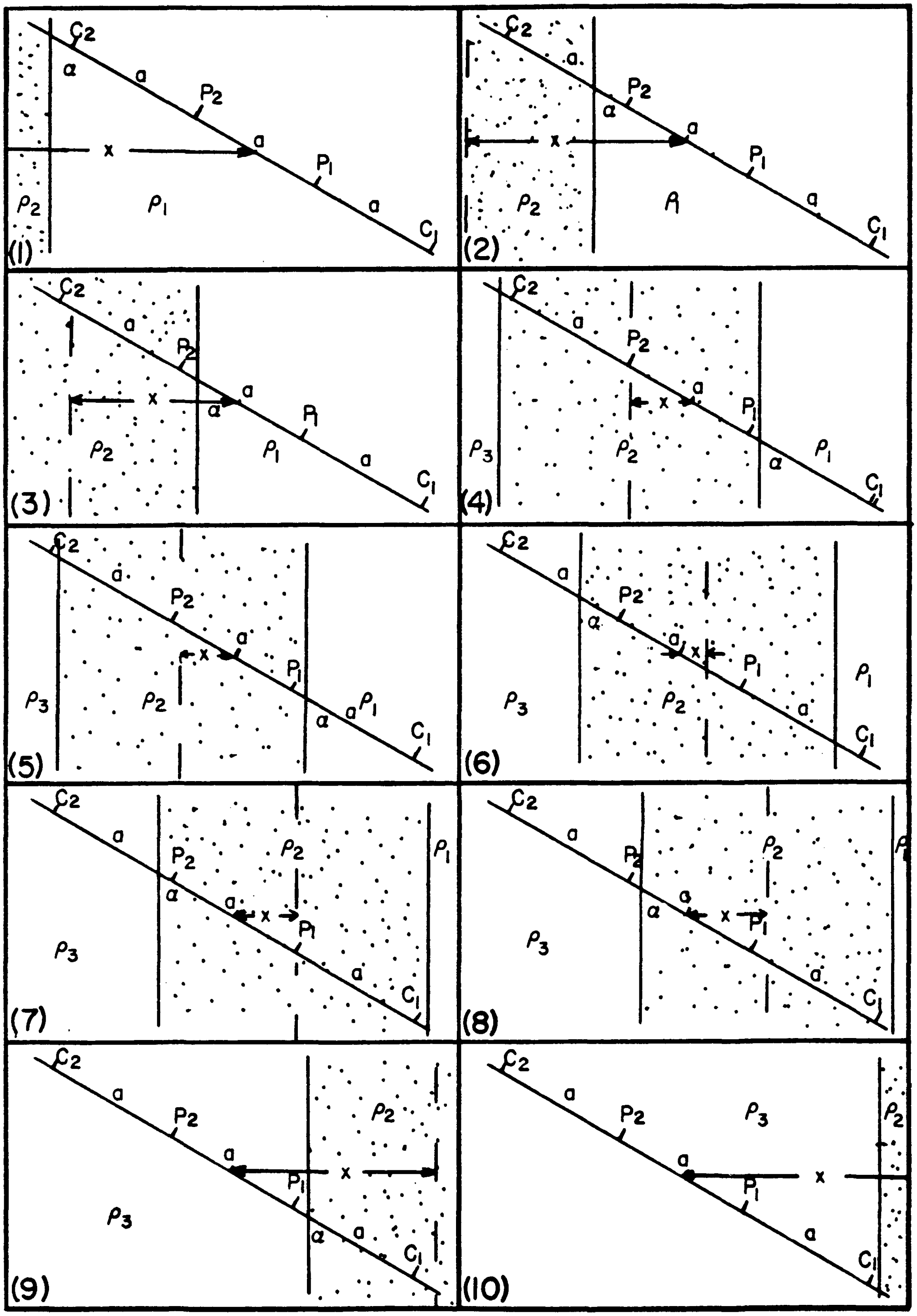


Figure 4.5 Relative positions af the array used to calculate the master curves

$$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + k_{12}\left\{\frac{1}{r_2} - \frac{1}{r_4}\right\} + (1-k_{12})(1+k_{12})\sum_n k_{21}^{n-1} k_{23}^{n}\left\{\frac{1}{r_n^1} - \frac{1}{r_n^2}\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_1}{2\pi}\left[-\frac{1}{2a} + k_{12}\left\{\frac{1}{r_3} - \frac{1}{r_5}\right\} + (1-k_{12})(1+k_{12})\sum_n k_{21}^{n-1} k_{23}^{n}\left\{\frac{1}{r_n^3} - \frac{1}{r_n^4}\right\}\right]$$

For position 2 figure 4.5:

$$\frac{3}{2}\sin\alpha + \frac{1}{2}\beta > \eta > \frac{1}{2}\sin\alpha + \frac{1}{2}\beta$$

$$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + k_{12}\left\{\frac{1}{r_2} - \frac{1}{2a}\right\} + (1+k_{12})\sum_n k_{21}^{n} k_{23}^{n}\left\{\frac{1-k_{12}}{k_{21}r_n^1} - \frac{1}{k_{21}r_n^2} - \frac{1}{r^6}\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_1}{2\pi}\left[-\frac{1}{2a} + k_{12}\left\{\frac{1}{r_3} - \frac{1}{a}\right\} + (1+k_{12})\sum_n k_{21}^{n} k_{23}^{n}\left\{\frac{1-k_{12}}{k_{21}r_n^3} - \frac{1}{k_{21}r_n^4} - \frac{1}{r_n^9}\right\}\right]$$

For position 3 figure 4.5:

$$\frac{1}{2}\sin\alpha + \frac{1}{2}\beta > \eta > -\frac{1}{2}\sin\alpha + \frac{1}{2}\beta$$

$$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + k_{12}\left\{\frac{1}{r_2} + \frac{1}{2a}\right\} + (1+k_{12})\sum_n k_{21}^{n} k_{23}^{n}\left\{\frac{1-k_{12}}{k_{21}r_n^1} - \frac{1}{k_{21}r_n^2} - \frac{1}{r_n^6}\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_2}{2\pi}\left[-\frac{1}{2a} - \frac{k_{12}}{2a} + \sum_n k_{21}^{n} k_{23}^{n}\left\{\frac{1-k_{12}}{k_{21}r_n^3} + \frac{1-k_{12}}{r_n^6} - \frac{1}{k_{21}r_n^4} - \frac{1}{r_n^7} - \frac{1}{r_n^9} - \frac{1}{k_{23}r_n^8}\right\}\right]$$

For position 4 figure 4.5:

$$-\frac{1}{2}\sin\alpha + \frac{1}{2}\beta > \eta > \frac{3}{2}\sin\alpha - \frac{1}{2}\beta$$

$$U_{TP_1}=\frac{I\rho_2}{2\pi}\left[\frac{1}{2a}-\frac{k_{12}}{a}+\sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^1}+\frac{1-k_{12}}{r_n^9}-\frac{1}{k_{21}r_n^2}-\frac{1}{r_n^{11}}-\frac{1}{r_n^6}-\frac{1}{k_{23}r_n^{10}}\right\}\right]$$

$$U_{TP_2}=\frac{I\rho_2}{2\pi}\left[-\frac{1}{2a}-\frac{k_{12}}{2a}+\sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^3}+\frac{1-k_{12}}{r_n^6}-\frac{1}{k_{21}r_n^4}-\frac{1}{r_n^7}-\frac{1}{r_n^9}-\frac{1}{k_{23}r_n^8}\right\}\right]$$

For position 5 figure 4.5:

$$\frac{3}{2}\sin\alpha-\frac{1}{2}\beta>\eta>0$$

$$U_{TP_1}=\frac{I\rho_2}{2\pi}\left[\frac{1}{2a}-\frac{2k_{12}+k_{23}}{2a}+\sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^1}+\frac{1-k_{12}}{r_n^9}-\frac{1+k_{23}}{k_{23}r_n^{10}}-\frac{1+k_{23}}{r_n^6}\right\}\right]$$

$$U_{TP_2}=\frac{I\rho_2}{2\pi}\left[-\frac{1}{2a}-\frac{k_{12}+2k_{23}}{2a}+\sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^3}+\frac{1-k_{12}}{r_n^6}-\frac{1+k_{23}}{k_{23}r_n^8}-\frac{1+k_{23}}{r_n^9}\right\}\right]$$

For position 6 figure 4.5:

$$0{\cdot}0>\eta>-\frac{3}{2}\sin\alpha+\frac{1}{2}\beta$$

$$U_{TP_1}=\frac{I\rho_2}{2\pi}\left[\frac{1}{2a}-\frac{2k_{12}+k_{23}}{2a}+\sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^8}+\frac{1-k_{12}}{r_n^9}-\frac{1+k_{23}}{k_{23}r_n^3}-\frac{1+k_{23}}{r_n^6}\right\}\right]$$

$$U_{TP_2}=\frac{I\rho_2}{2\pi}\left[-\frac{1}{2a}-\frac{k_{12}+2k_{23}}{2a}+\sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^{10}}+\frac{1-k_{12}}{r_n^6}-\frac{1+k_{23}}{k_{23}r_n^1}-\frac{1+k_{23}}{r_n^9}\right\}\right]$$

For position 7 figure 4.5:

$$-\frac{3}{2}\sin\alpha+\frac{1}{2}\beta>\eta>\frac{1}{2}\sin\alpha-\frac{1}{2}\beta$$

$$U_{TP_1} = \frac{I\rho_2}{2\pi}\left[\frac{1}{2a} - \frac{k_{23}}{2a} + \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{k_{21} r_n^8} + \frac{1}{r_n^9} + \frac{1}{r_n^7} + \frac{1}{k_{23} r_n^4} - \frac{1+k_{23}}{k_{23} r_n^3} - \frac{1+k_{23}}{r_n^6}\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_2}{2\pi}\left[-\frac{1}{2a} - \frac{k_{23}}{2a} + \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{k_{21} r_n^{10}} + \frac{1}{r_n^6} + \frac{1}{r_n^{11}} + \frac{1}{k_{23} r_n^2} - \frac{1+k_{23}}{k_{23} r_n^1} - \frac{1+k_{23}}{r_n^9}\right\}\right]$$

For position 8 figure 4.5:

$$\frac{1}{2}\sin\alpha - \frac{1}{2}\beta > \eta > -\frac{1}{2}\sin\alpha - \frac{1}{2}\beta$$

$$U_{TP_1} = \frac{I\rho_2}{2\pi}\left[\frac{1}{2a} - \frac{k_{23}}{2a} + \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{k_{21} r_n^8} + \frac{1}{r_n^9} + \frac{1}{r_n^7} + \frac{1}{k_{23} r_n^4} - \frac{1+k_{23}}{k_{23} r_n^3} - \frac{1+k_{23}}{r_n^6}\right\}\right]$$

$$U_{TP2} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{2a} - k_{23}\left\{\frac{1}{2a} - \frac{1}{r_2}\right\} + (1-k_{23}) \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{r_n^6} + \frac{1}{k_{23} r_n^2} - \frac{1+k_{23}}{k_{23} r_n^1}\right\}\right]$$

For position 9 figure 4.5:

$$-\frac{1}{2}\sin\alpha - \frac{1}{2}\beta > \eta > -\frac{3}{2}\sin\alpha - \frac{1}{2}\beta$$

$$U_{TP_1} = \frac{I\rho_3}{2\pi}\left[\frac{1}{2a} - k_{23}\left\{\frac{1}{a} - \frac{1}{r_3}\right\} + (1-k_{23}) \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{r_n^9} + \frac{1}{k_{23} r_n^4} - \frac{1+k_{23}}{k_{23} r_n^3}\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{2a} - k_{23}\left\{\frac{1}{2a} - \frac{1}{r_2}\right\} + (1-k_{23}) \sum_n k_{21}^n k_{23}^n \left\{\frac{1}{r_n^6} + \frac{1}{k_{23} r_n^2} - \frac{1+k_{23}}{k_{23} r_n^1}\right\}\right]$$

For position 10 figure 4.5:

$$-\frac{3}{2}\sin\alpha - \frac{1}{2}\beta > \eta$$

$$U_{TP_1} = \frac{I\rho_3}{2\pi}\left[\frac{1}{2a} - k_{23}\left\{\frac{1}{r_5} - \frac{1}{r_3}\right\} + (1-k_{23}^2)\sum_n k_{21}^n k_{23}^{n-1}\left\{\frac{1}{r_n^4} - \frac{1}{r_n^3}\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{2a} - k_{23}\left\{\frac{1}{r_4} - \frac{1}{r_2}\right\} + (1-k_{23}^2)\sum_n k_{21}^n k_{23}^{n-1}\left\{\frac{1}{r_n^2} - \frac{1}{r_n^1}\right\}\right]$$

In all the above and following equations, we have

$$r_2 = a\sqrt{1+(2\eta+\sin\alpha)(2\eta+3\sin\alpha)+\beta^2-4\eta\beta-4\beta\sin\alpha}$$

$$r_3 = a\sqrt{4+(2\eta-\sin\alpha)(2\eta+3\sin\alpha)+\beta^2-4\eta\beta-2\beta\sin\alpha}$$

$$r_4 = a\sqrt{4+(2\eta-3\sin\alpha)(2\eta+\sin\alpha)+\beta^2-4\eta\beta+2\beta\sin\alpha}$$

$$r_5 = a\sqrt{1+(2\eta-3\sin\alpha)(2\eta-\sin\alpha)+\beta^2-4\eta\beta+4\beta\sin\alpha}$$

$$r_6 = a\sqrt{1+(2\eta-3\sin\alpha)(2\eta-\sin\alpha)+\beta^2+4\eta\beta-4\beta\sin\alpha}$$

$$r_n^1 = a\sqrt{1+(2\eta+3\sin\alpha)(2\eta+\sin\alpha)+\beta^2(2n-1)^2+4(2n-1)\eta\beta+4(2n-1)\beta\sin\alpha}$$

$$r_n^2 = a\sqrt{4+(2\eta-3\sin\alpha)(2\eta+\sin\alpha)+\beta^2(2n-1)^2-4(2n-1)\eta\beta-2(2n-1)\beta\sin\alpha}$$

$$r_n^3 = a\sqrt{4+(2\eta+3\sin\alpha)(2\eta-\sin\alpha)+\beta^2(2n-1)^2+4(2n-1)\eta\beta+2(2n-1)\beta\sin\alpha}$$

$$r_n^4 = a\sqrt{1+(2\eta-3\sin\alpha)(2\eta-\sin\alpha)+\beta^2(2n-1)^2-4(2n-1)\eta\beta-4(2n-1)\beta\sin\alpha}$$

$$r_n^5 = a\sqrt{4+(2\eta-3\sin\alpha)(2\eta-7\sin\alpha)+\beta^2(2n-1)^2+4(2n-1)\eta\beta+10(2n-1)\beta\sin\alpha}$$

$$r_n^6 = a\sqrt{4+4n^2\beta^2+8n\beta\sin\alpha}$$

$$r_n^7 = a\sqrt{1+4n^2\beta^2-4n\beta\sin\alpha}$$

$$r_n^8 = a\sqrt{1+(2\eta-3\sin\alpha)(2\eta-\sin\alpha)+(2n-1)^2\beta^2-4(2n-1)\beta\eta+4(2n-1)\beta\sin\alpha}$$

$$r_n^9 = a\sqrt{1+4n^2\beta^2+4n\beta\sin\alpha}$$

$$r_n^{10} = a\sqrt{4+(2\eta-3\sin\alpha)(2\eta+\sin\alpha)+(2n-1)^2\beta^2-4(2n-1)\beta\eta+2(2n-1)\beta\sin\alpha}$$

$$r_n^{11} = a\sqrt{4+4n^2\beta^2-8n\beta\sin\alpha}$$

4.3 $a\sin\alpha < t < 2a\sin\alpha$

As shown in figure 4.6, we can now have a maximum of two electrodes in medium 2 at one time. To calculate the curves far this range of t, I used the following formulae:

For position 1 figure 4.6 The equations are the same as in position 1, paragraph 4.2.

For position 2 figure 4.6 The equations are the same as in position 2 paragraph 4.2.

For position 3 figure 4.6 The equations for the total potentials at P_1 and P_2 are the same as in position 3 paragraph 4.2, but the range of η changes to

$$\tfrac{1}{2}\sin\alpha + \tfrac{1}{2}\beta > \eta > \tfrac{3}{2}\sin\alpha - \tfrac{1}{2}\beta$$

For position 4 figure 4.6

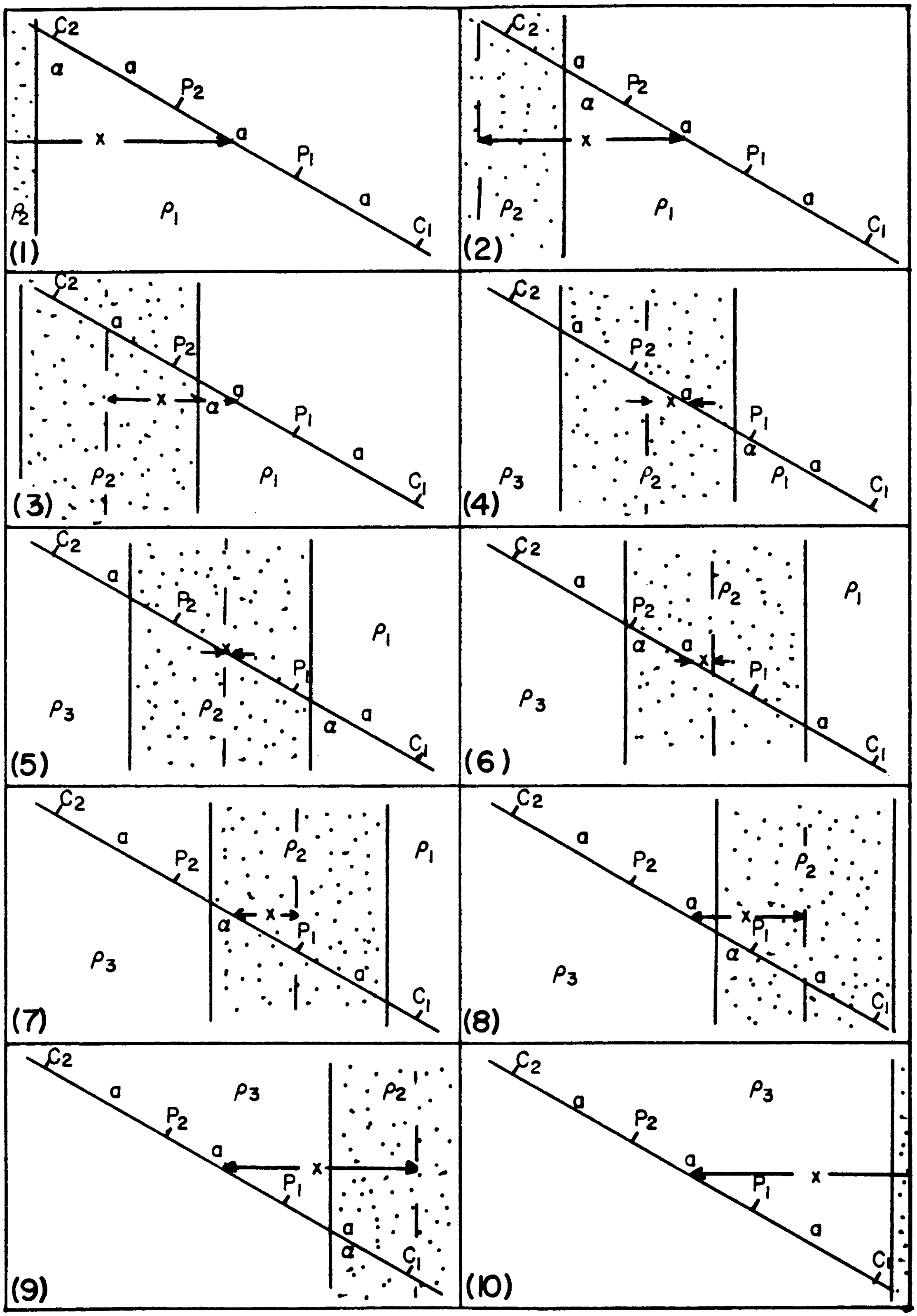


Figure 4.6 Relative positions of the array used th calculate the master curves

$$\tfrac{3}{2}\sin\alpha - \tfrac{1}{2}\beta > \eta > -\tfrac{1}{2}\sin\alpha + \tfrac{1}{2}\beta$$

$$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_2} + (1+k_{12})\left\{-\frac{1+k_{23}}{2a} + \sum_n k_{21}^n k_{23}^n\left(\frac{1-k_{12}}{k_{21}r_n^1} - \frac{1+k_{23}}{r_n^6}\right)\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_2}{2\pi}\left[-\frac{1}{2a} - \frac{k_{12}}{2a} - \frac{k_{23}}{a} + \sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^3} + \frac{1-k_{12}}{r_n^6} - \frac{1+k_{23}}{k_{23}r_n^8} - \frac{1+k_{23}}{r_n^9}\right\}\right]$$

For position 5 figure 4.6 The equations for the total potentials at P_1 and P_2 are the same as in position 5 paragraph 4.2, but the range of η changes to

$$-\tfrac{1}{2}\sin\alpha + \tfrac{1}{2}\beta > \eta > 0$$

For position 6 figure 4.6 The equations for the total potentials at P_1 and P_2 are the same as in position 6 paragraph 4.2, but the range of η changes to

$$0 > \eta > \tfrac{1}{2}\sin\alpha - \tfrac{1}{2}\beta$$

For position 7 figure 4.6

$$\tfrac{1}{2}\sin\alpha - \tfrac{1}{2}\beta > \eta > -\tfrac{3}{2}\sin\alpha + \tfrac{1}{2}\beta$$

$$U_{TP_1} = \frac{I\rho_2}{2\pi}\left[\frac{1}{2a} - \frac{k_{12}}{a} - \frac{k_{23}}{2a} + \sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^8} + \frac{1-k_{12}}{r_n^9} - \frac{1+k_{23}}{k_{23}r_n^3} - \frac{1+k_{23}}{r_n^6}\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_3}{2\pi}\left[\frac{(1-k_{12})(1-k_{23})}{2a} - \frac{1}{a} + \frac{k_{23}}{r_2} + (1-k_{23})\sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{r_n^6} - \frac{1+k_{23}}{k_{23}r_n^1}\right\}\right]$$

For position 8 figure 4.6 The equations for the total potentials at P_1 and P_2 are the same as in position 8 paragraph 4.2, but the range of η changes to

$$-\tfrac{3}{2}\sin\alpha+\tfrac{1}{2}\beta>\eta>-\tfrac{3}{2}\sin\alpha+\tfrac{1}{2}\beta$$

For position 9 figure 4.6 The equations are the same as for position 9 paragraph 4.2.

For position 10 figure 4.6 The equations are the same as for position 10 paragraph 4.2.

4.4 $t<a\sin\alpha$

For this range of t we cannot have more than one electrode at a time in medium 2. The positions used for calculating the curves are shown in figure 4.7.

For position 1 figure 4.7 The equations are the same as for position 1 paragraph 4.2.

For position 2 figure 4.7 The equations for the total potentials at P_1 and P_2 are the same as for position 2 paragraph 4.2, but the range of η changes to

$$\tfrac{3}{2}\sin\alpha+\tfrac{1}{2}\beta>\eta>\tfrac{3}{2}\sin\alpha-\tfrac{1}{2}\beta$$

For position 3 figure 4.7

$$\tfrac{3}{2}\sin\alpha-\tfrac{1}{2}\beta>\eta>\tfrac{1}{2}\sin\alpha+\tfrac{1}{2}\beta$$

$$U_{TP_1}=\frac{I\rho_1}{2\pi}\left[\frac{1}{a}+\frac{k_{12}}{r_2}-\frac{(1+k_{12})(1+k_{23})}{2a}+(1+k_{12})\sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^1}-\frac{1+k_{23}}{r_n^6}\right\}\right]$$

$$U_{TP_2}=\frac{I\rho_1}{2\pi}\left[\frac{1}{2a}+\frac{k_{12}}{r_3}-\frac{(1+k_{12})(1+k_{23})}{a}+(1+k_{12})\sum_n k_{21}^n k_{23}^n\left\{\frac{1-k_{12}}{k_{21}r_n^3}-\frac{1+k_{23}}{r_n^9}\right\}\right]$$

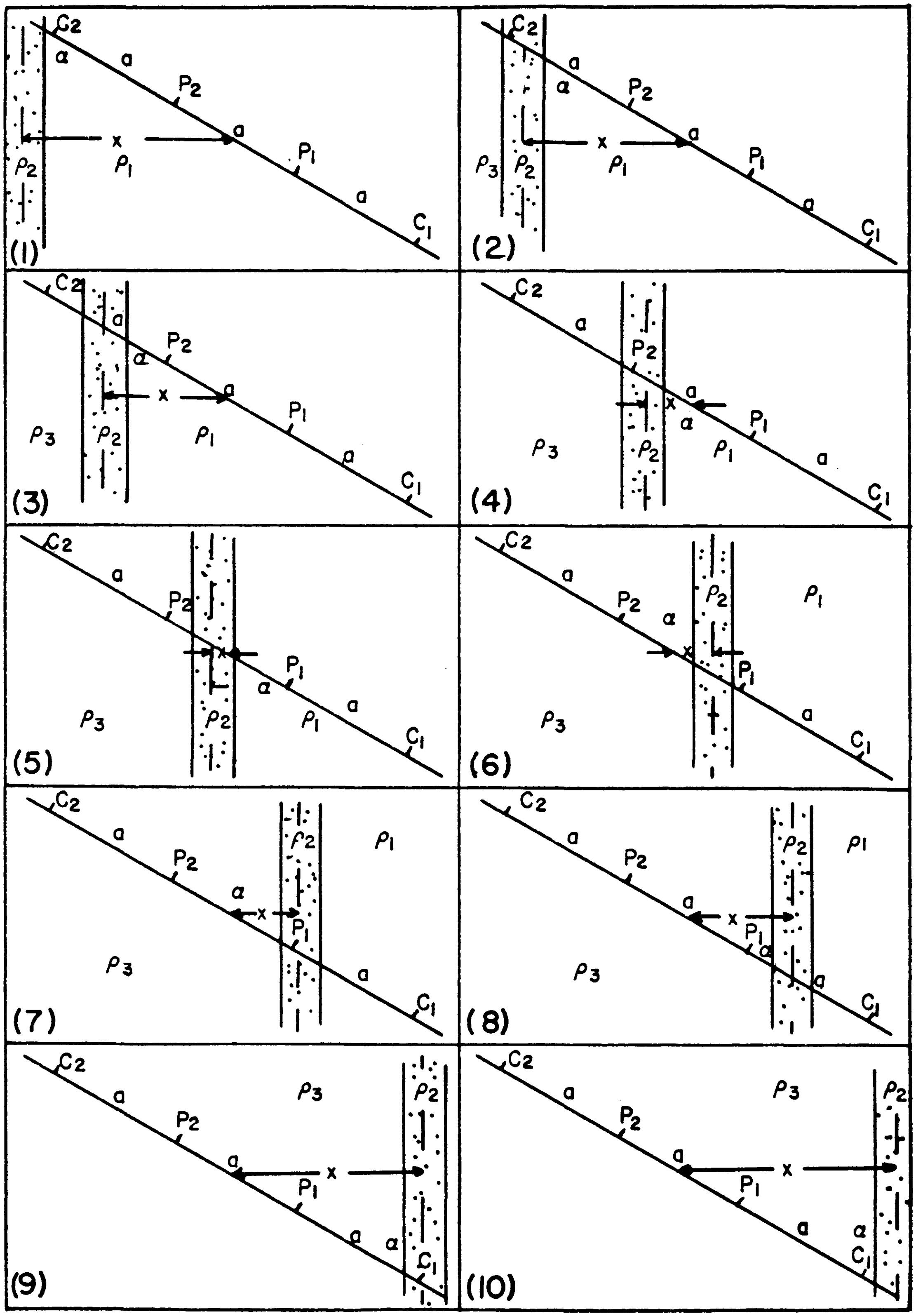


Figure 4.7 Relative positions of the array used to calculate the master curves

<u>For position 4 figure 4.7</u> The equations for the total potentials at P_1 and P_2 are the same as for position 4 paragraph 4.3, but the range of η changes to

$$\frac{1}{2}\sin\alpha + \frac{1}{2}\beta > \eta > \frac{1}{2}\sin\alpha - \frac{1}{2}\beta$$

<u>For position 5 figure 4.7</u>

$$\frac{1}{2}\sin\alpha - \frac{1}{2}\beta > \eta > 0$$

$$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_2} + (1+k_{12})\left\{-\frac{(1-k_{12})}{2a} + \sum_n k_{21}^n k_{23}^n \left(\frac{1-k_{12}}{k_{21} r_n^{\prime}} - \frac{1+k_{23}}{r_n^6}\right)\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{a} + \frac{k_{23}}{r^6} + \frac{(1-k_{12})(1-k_{23})}{2a} + (1-k_{23})\sum_n k_{21}^n k_{23}^n \left\{\frac{1-k_{12}}{r_n^6} - \frac{1+k_{23}}{k_{23} r_n^8}\right\}\right]$$

<u>For position 6 figure 4.7</u>

$$0 > \eta > -\frac{1}{2}\sin\alpha + \frac{1}{2}\beta$$

$$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r^6} - \frac{(1+k_{12})(1+k_{23})}{2a} + (1+k_{12})\sum_n k_{21}^n k_{23}^n \left\{\frac{1-k_{12}}{k_{21} r_n^8} - \frac{1+k_{23}}{r_n^6}\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{a} + \frac{(1-k_{12})(1-k_{23})}{2a} + \frac{k_{23}}{r_2} + (1-k_{23})\sum_n k_{21}^n k_{23}^n \left\{\frac{1-k_{12}}{r_n^6} - \frac{1+k_{23}}{k_{23} r_n^{\prime}}\right\}\right]$$

<u>For position 7 figure 4.7</u> The equations for the total potentials at P_1 and P_2 are the same as for position 7 paragraph 4.3, but the range of η changes to

$$-\frac{1}{2}\sin\alpha + \frac{1}{2}\beta > \eta > -\frac{1}{2}\sin\alpha - \frac{1}{2}\beta$$

For position 8 figure 4.7

$$-\frac{1}{2}\sin\alpha - \frac{1}{2}\beta > \eta > -\frac{3}{2}\sin\alpha + \frac{1}{2}\beta$$

$$U_{TP_1} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{2a} + \frac{k_{23}}{r^3} + \frac{(1-k_{12})(1-k_{23})}{a} + (1-k_{23})\sum_n k_{21}^n k_{23}^n \left\{\frac{1-k_{12}}{r_n^9} - \frac{1+k_{23}}{k_{23} r_n^3}\right\}\right]$$

$$U_{TP_2} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{a} + \frac{k_{23}}{r} + \frac{(1-k_{12})(1-k_{23})}{2a} + (1-k_{23})\sum_n k_{21}^n k_{23}^n \left\{\frac{1-k_{12}}{r_n^6} - \frac{1+k_{23}}{k_{23} r_n^1}\right\}\right]$$

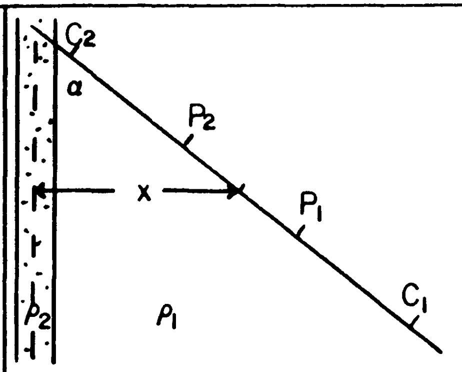
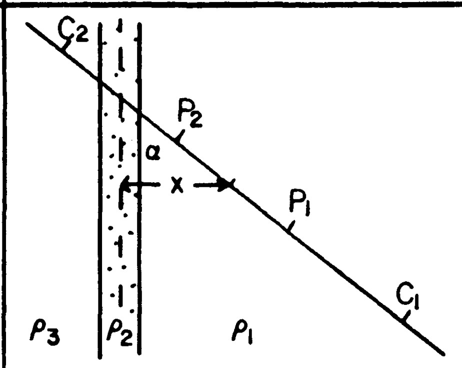
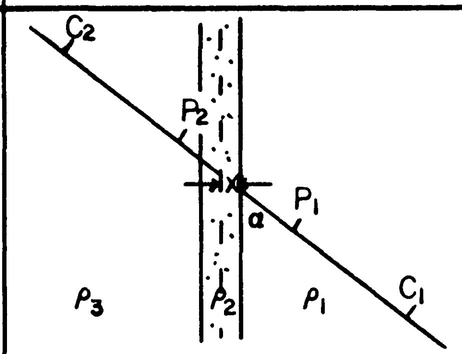
For position 9 figure 4.7 The equations for the total potentials at P_1 and P_2 are the same as for position 9 paragraph 4.2, but the range of η changes to

$$-\frac{3}{2}\sin\alpha + \frac{1}{2}\beta > \eta > -\frac{3}{2}\sin\alpha - \frac{1}{2}\beta$$

For position 10 figure 4.7 The equations are the same as for position 10 paragraph 4.2.

4.5 $t << a \sin\alpha$

I finally consider a narrow vertical structure, assuming that t is small enough that none of the electrodes will ever be in medium 2. This leads to only 6 possible positions of the array relative to the vertical structure, as shown in figure 4.8. The ranges of η as well as the formulae used in calculating the master curves, are all shown in the figure.

$\eta > \frac{3}{2}\sin\alpha$		$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + k_{12}\left\{\frac{1}{r_2} - \frac{1}{r_4}\right\} + (1-k_{12})(1+k_{12})\sum_n k_{21}^{n-1}k_{23}^{n}\left\{\frac{1}{r_n^1} - \frac{1}{r_n^2}\right\}\right]$ $U_{TP_2} = \frac{I\rho_1}{2\pi}\left[-\frac{1}{2a} + k_{12}\left\{\frac{1}{r_3} - \frac{1}{r_5}\right\} + (1-k_{12})(1+k_{12})\sum_n k_{21}^{n-1}k_{23}^{n}\left\{\frac{1}{r_n^3} - \frac{1}{r_n^4}\right\}\right]$
$\frac{3}{2}\sin\alpha > \eta > \frac{1}{2}\sin\alpha$		$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_2} - \frac{(1+k_{12})(1+k_{23})}{2a} + (1+k_{12})\sum_n k_{21}^{n}k_{23}^{n}\left\{\frac{1-k_{12}}{k_{21}r_n^1} - \frac{1+k_{23}}{r_n^6}\right\}\right]$ $U_{TP_2} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + \frac{k_{12}}{r_3} - \frac{(1+k_{12})(1+k_{23})}{a} + (1+k_{12})\sum_n k_{21}^{n}k_{23}^{n}\left\{\frac{1-k_{12}}{k_{21}r_n^3} - \frac{1+k_{23}}{r_n^9}\right\}\right]$
$\frac{1}{2}\sin\alpha > \eta > 0$		$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_2} - \frac{(1+k_{12})(1+k_{23})}{2a} + (1+k_{12})\sum_n k_{21}^{n}k_{23}^{n}\left\{\frac{1-k_{12}}{k_{21}r_n^1} - \frac{1+k_{23}}{r_n^6}\right\}\right]$ $U_{TP_2} = \frac{I\rho_1}{2\pi}\left[\frac{1}{2a} + \frac{k_{12}}{r_3} - \frac{(1+k_{12})(1+k_{23})}{a} + (1+k_{12})\sum_n k_{21}^{n}k_{23}^{n}\left\{\frac{1-k_{12}}{k_{21}r_n^3} - \frac{1+k_{23}}{r^9}\right\}\right]$

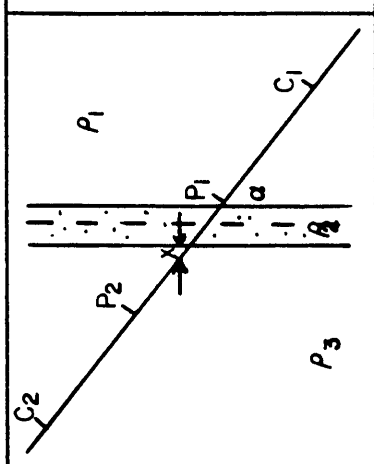
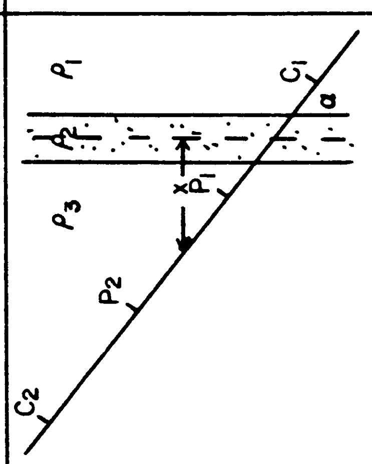
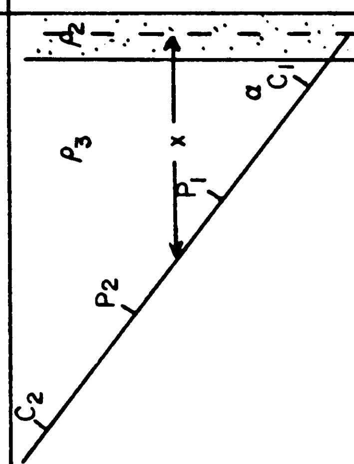
$0 > \eta > -\frac{1}{2}\sin\alpha$		$U_{TP_1} = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + \frac{k_{12}}{r_6} - \frac{(1+k_{12})(1+k_{23})}{2a} + (1+k_{12})\sum_n k_{21}^n k_{23}^n \left\{\frac{1-k_{12}}{k_{21} r_n^8} - \frac{1+k_{23}}{r_n^6}\right\}\right]$ $U_{TP_2} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{a} + \frac{k_{23}}{r_2} + \frac{(1-k_{12})(1-k_{23})}{2a} + (1-k_{23})\sum_n k_{21}^n k_{23}^n \left\{\frac{1-k_{12}}{r_n^6} - \frac{1+k_{23}}{k_{23} r_n^1}\right\}\right]$
$-\frac{1}{2}\sin\alpha > \eta > -\frac{3}{2}\sin\alpha$		$U_{TP_1} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{2a} + \frac{k_{23}}{r_3} + \frac{(1-k_{12})(1-k_{23})}{a} + (1-k_{23})\sum_n k_{21}^n k_{23}^n \left\{\frac{1-k_{12}}{r_n^9} - \frac{1+k_{23}}{k_{23} r_n^3}\right\}\right]$ $U_{TP_2} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{a} + \frac{k_{23}}{r_2} + \frac{(1-k_{12})(1-k_{23})}{2a} + (1-k_{23})\sum_n k_{21}^n k_{23}^n \left\{\frac{1-k_{12}}{r_n^6} - \frac{1+k_{23}}{k_{23} r_n^1}\right\}\right]$
$-\frac{3}{2}\sin\alpha > \eta$		$U_{TP_1} = \frac{I\rho_3}{2\pi}\left[\frac{1}{2a} - k_{23}\left\{\frac{1}{r_5} - \frac{1}{r_3}\right\} + (1-k_{23}^2)\sum_n k_{21}^n k_{23}^{n-1}\left\{\frac{1}{r_n^4} - \frac{1}{r_n^3}\right\}\right]$ $U_{TP_2} = \frac{I\rho_3}{2\pi}\left[-\frac{1}{2a} - k_{23}\left\{\frac{1}{r_4} - \frac{1}{r_2}\right\} + (1-k_{23}^2)\sum_n k_{21}^n k_{23}^{n-1}\left\{\frac{1}{r_n^2} - \frac{1}{r_n^1}\right\}\right]$

Figure 4.8 Relative positions of the array as well as the formulae used to calculate the master curves.

4.6 CALCULATING THE MASTER CURVES

The preceding formulae were used to calculate five basic sets of curves. These are given in appendix B, C D, E, and F and are for the following situations respectively:

(a) where we have the same material on both sides of medium 2, that is $\rho_1 = \rho_3$

(b) where $2a \sin \alpha < t < 3a \sin \alpha$

(c) where $a \sin \alpha < t < 2a \sin \alpha$

(d) where $t < a \sin \alpha$

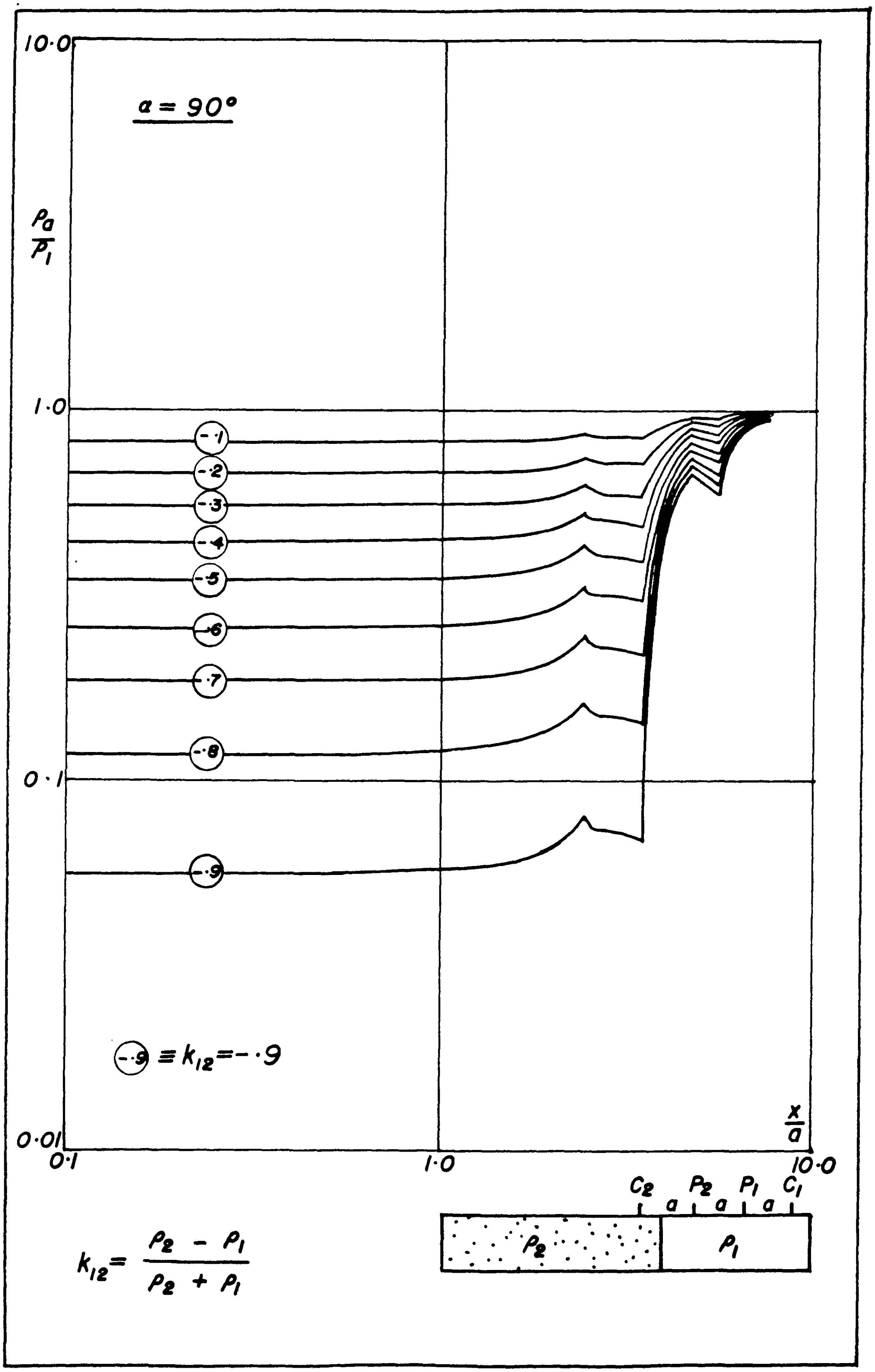
(e) where $t \ll a \sin \alpha$

APPENDIX A

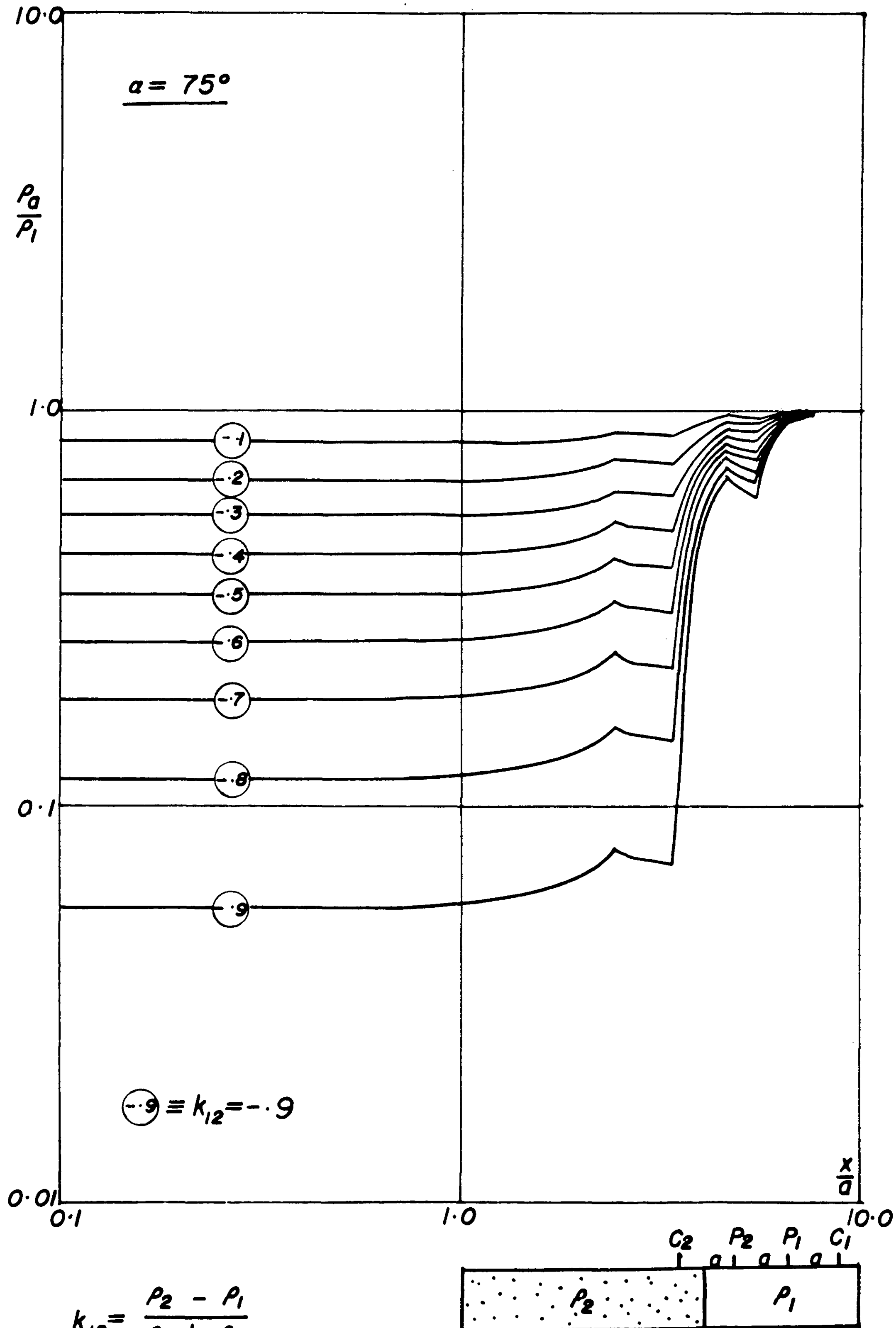
This appendix contains six sets of curves for the horizontal profile crossing the vertical plane at six different angles. Values of the reflection coefficient varies from -.9 to -.1. These correspond to the following resistivity ratios:

k_{12}	ρ_1	ρ_2
-.9	1	.053
-.8	1	.111
-.7	1	.177
-.6	1	.250
-.5	1	.333
-.4	1	.43
-.3	1	.54
-.2	1	.67
-.1	1	.82

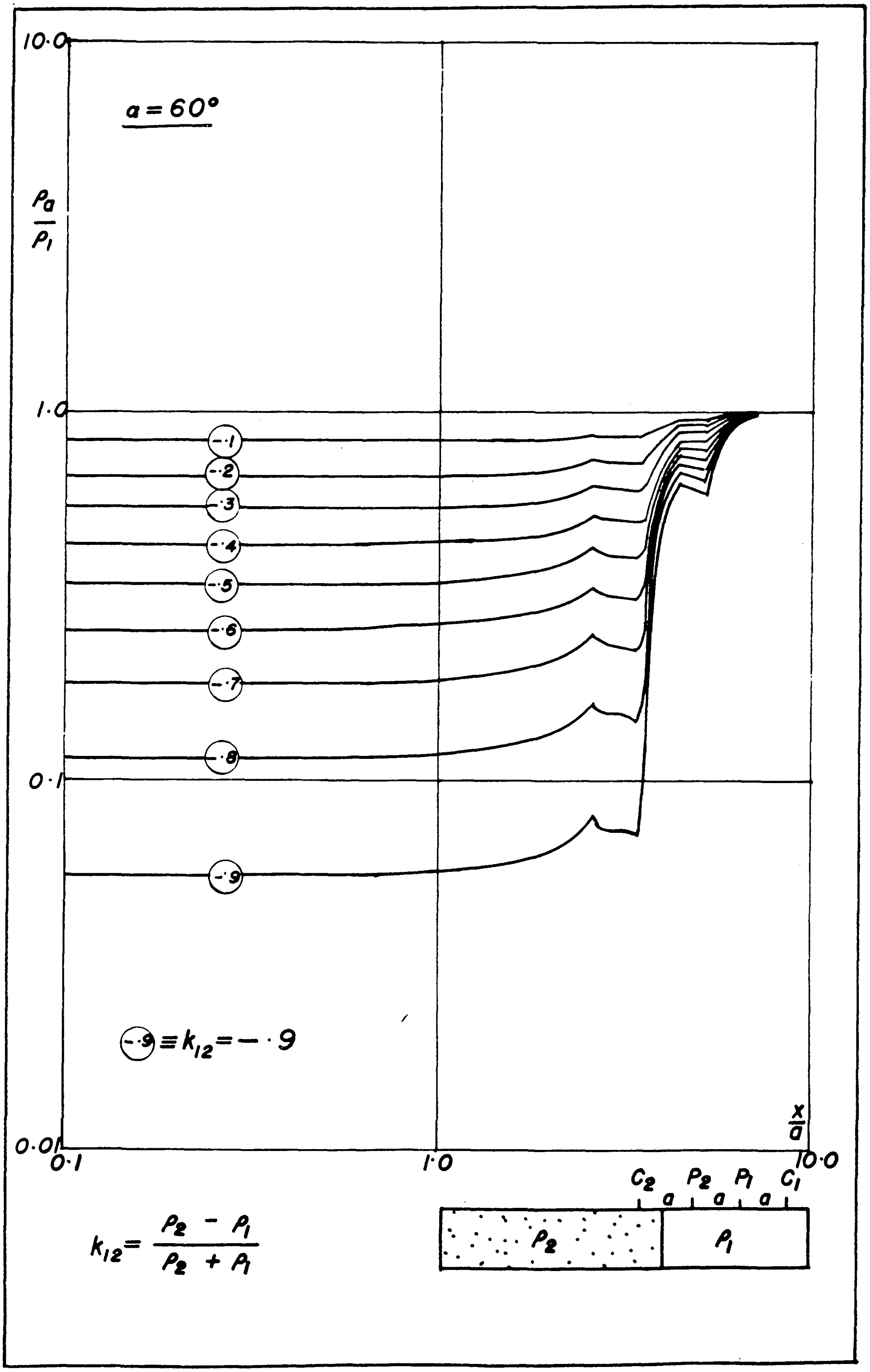
Program 1 was used to calculate these curves. All curves are plotted on 3×3 log-log paper (no. 19.1330).

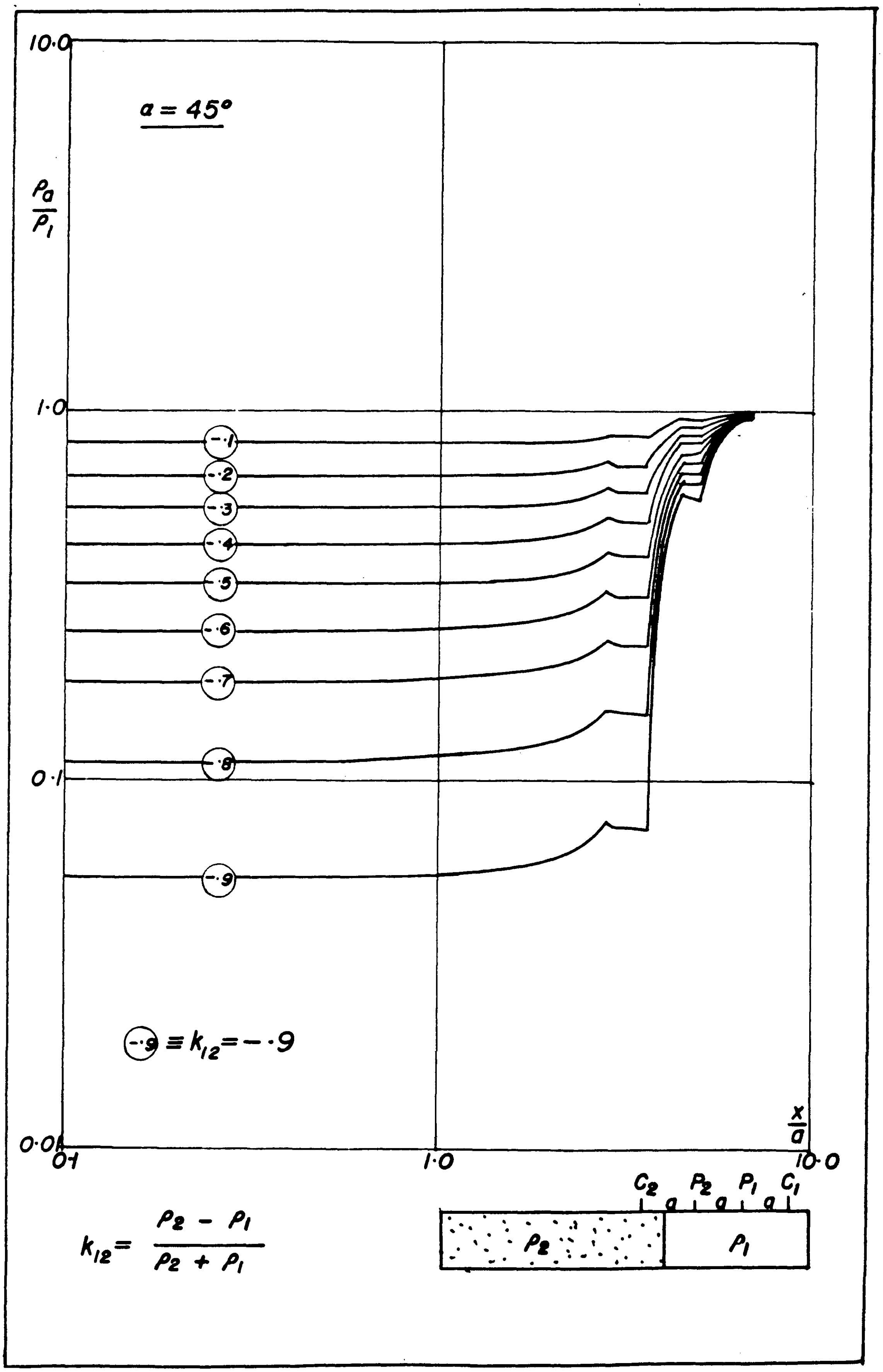


10·0
a = 90°
$\frac{\rho_a}{\rho_1}$
1·0
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
0·1
-·9
-·9 ≡ k_{12}=-·9
0·01
$\frac{x}{a}$
0·1
1·0
10·0
C_2
a
P_2
a
P_1
a
C_1
ρ_2
ρ_1
$k_{12} = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$

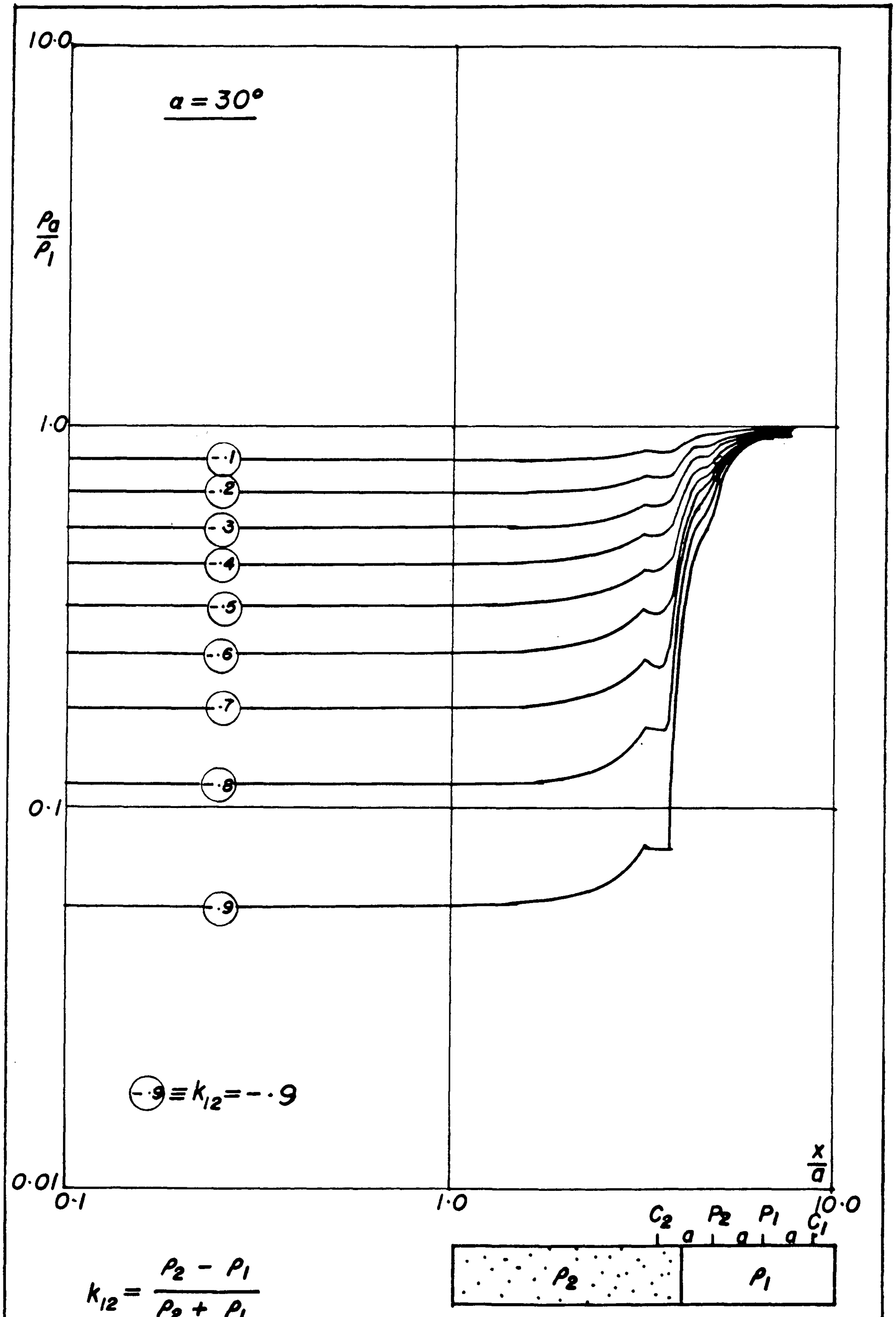


$$k_{12} = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

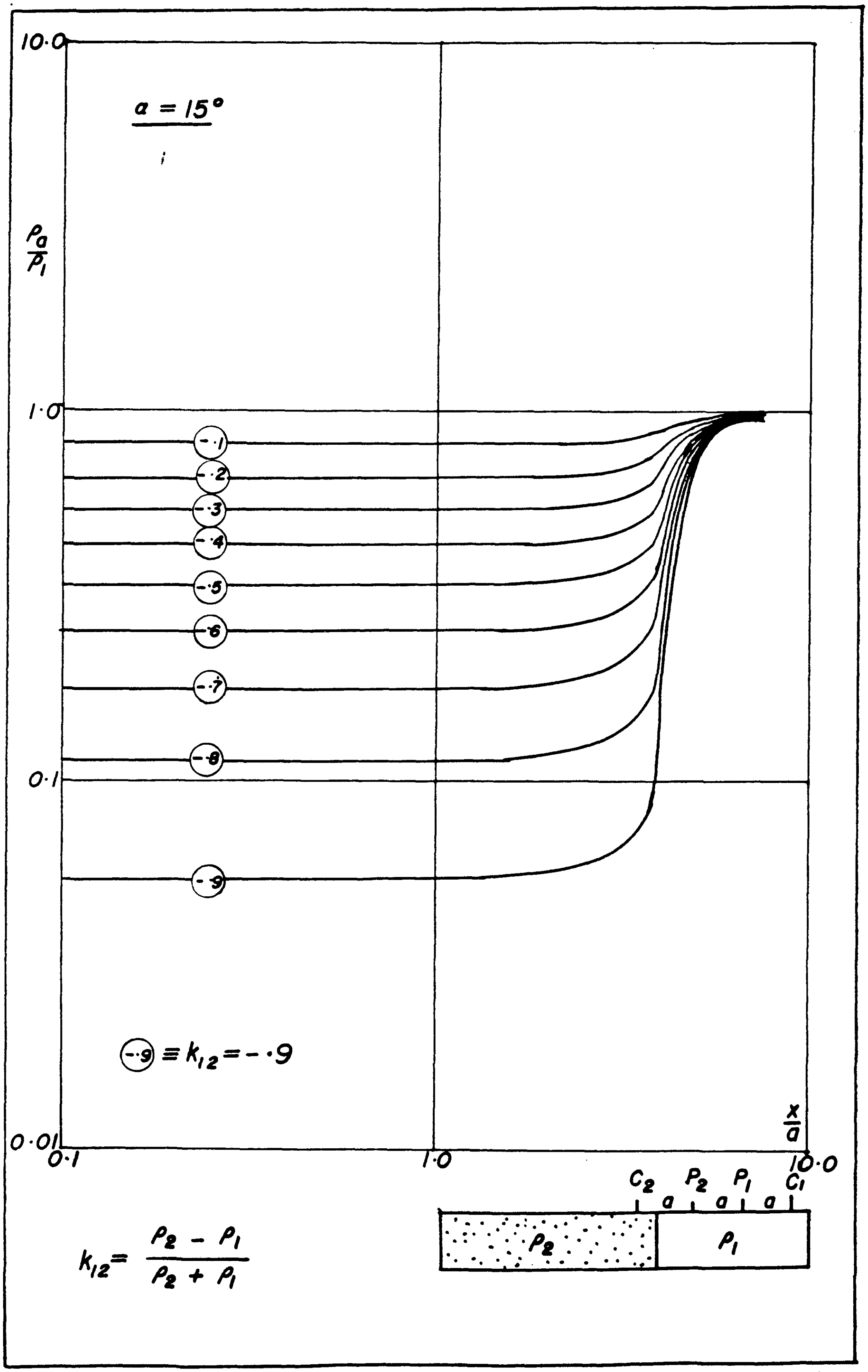

10·0
a = 60°
ρa / ρ1
1·0
–·1
–·2
–·3
–·4
–·5
–·6
–·7
–·8
0·1
–·9
–·9 ≡ k12 = – ·9
x/a
0·01
0·1
1·0
10·0
C2 a P2 a P1 a C1
ρ2
ρ1
k12 = (ρ2 – ρ1)/(ρ2 + ρ1)



a = 45°
ρa/ρ1
10·0
1·0
0·1
0·01
0·1
1·0
10·0
x/a
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
(-·9) ≡ k12 = -·9
k12 = (ρ2 - ρ1)/(ρ2 + ρ1)
C2 a P2 a P1 a C1
ρ2
ρ1

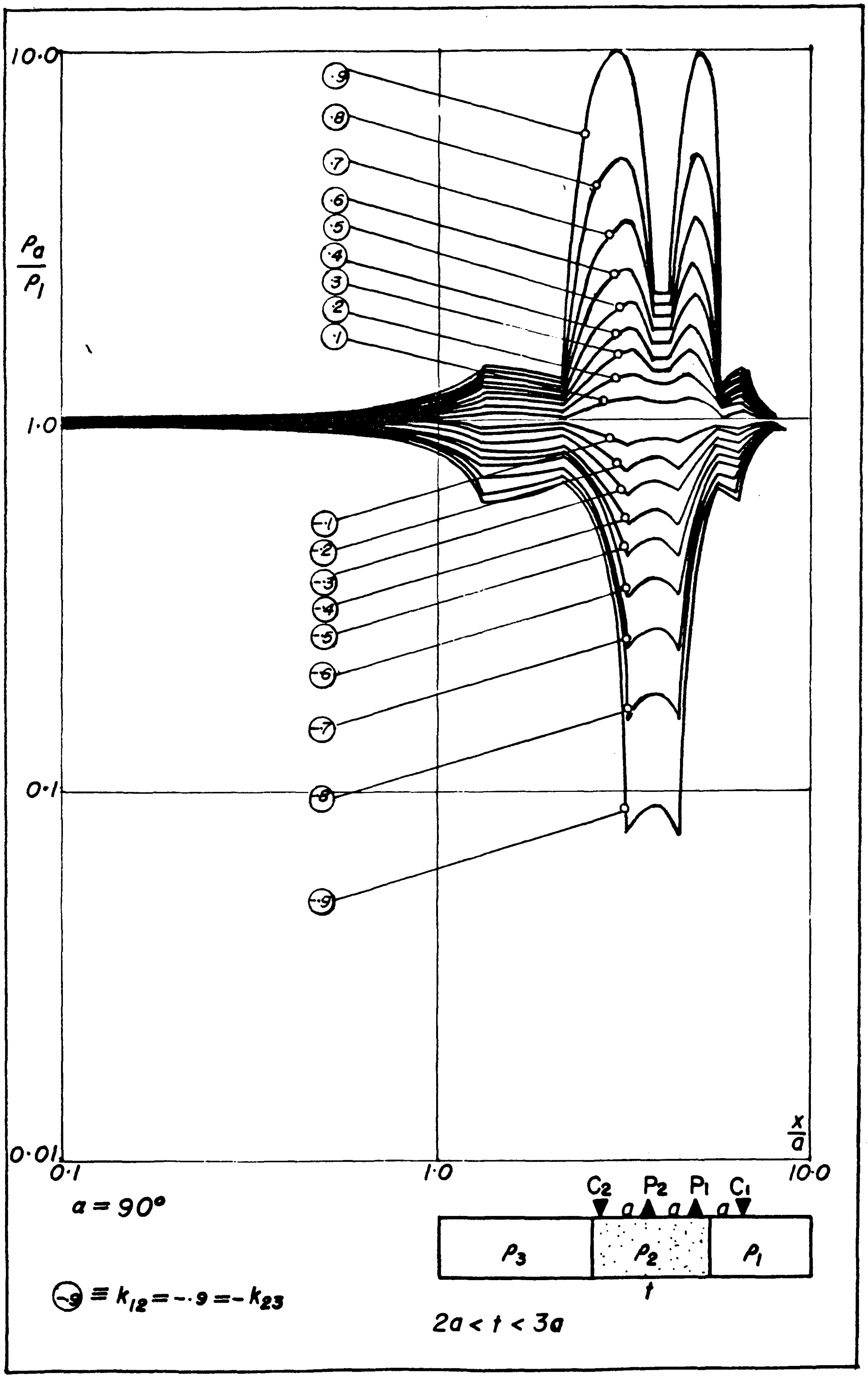


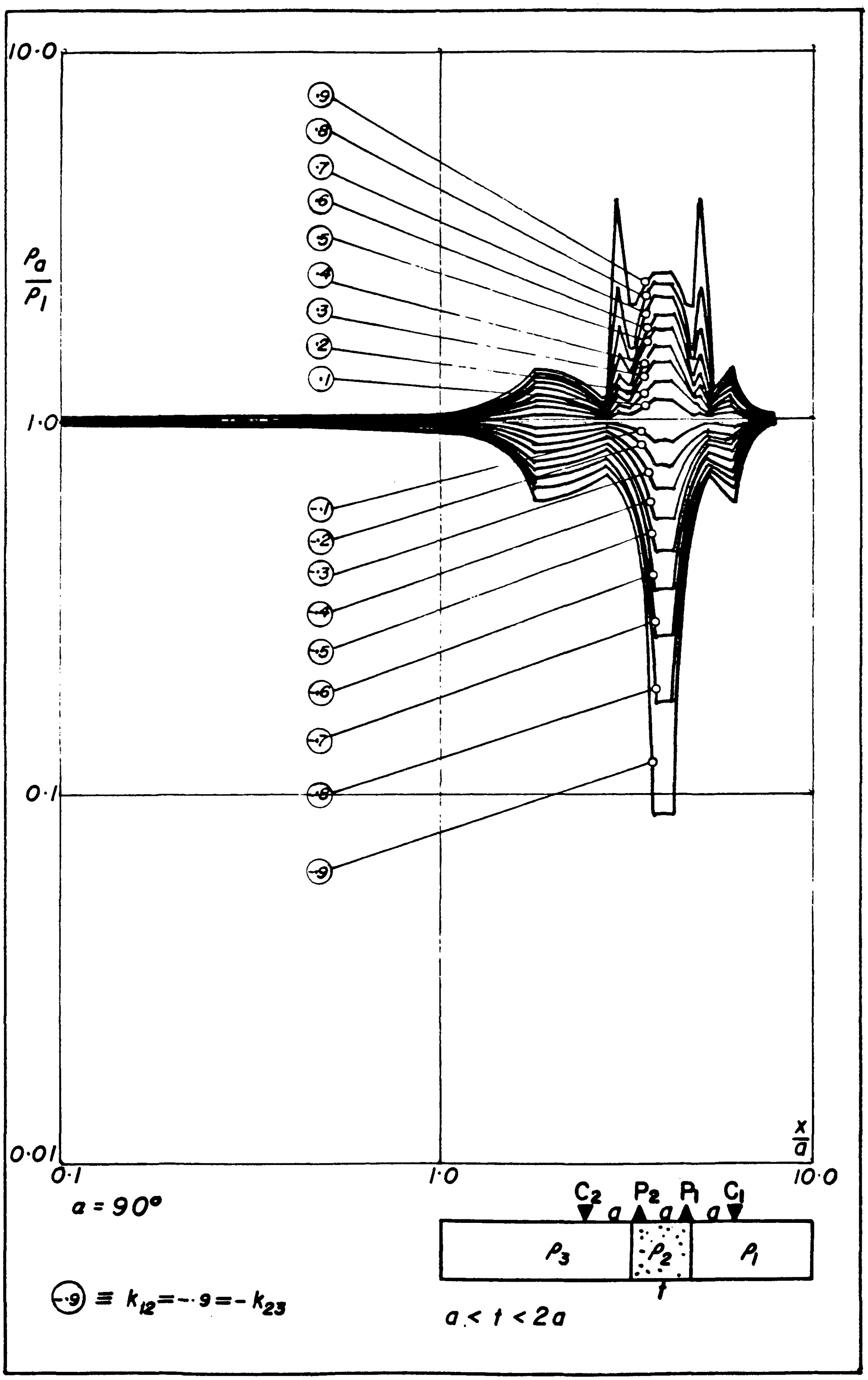
$$k_{12} = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

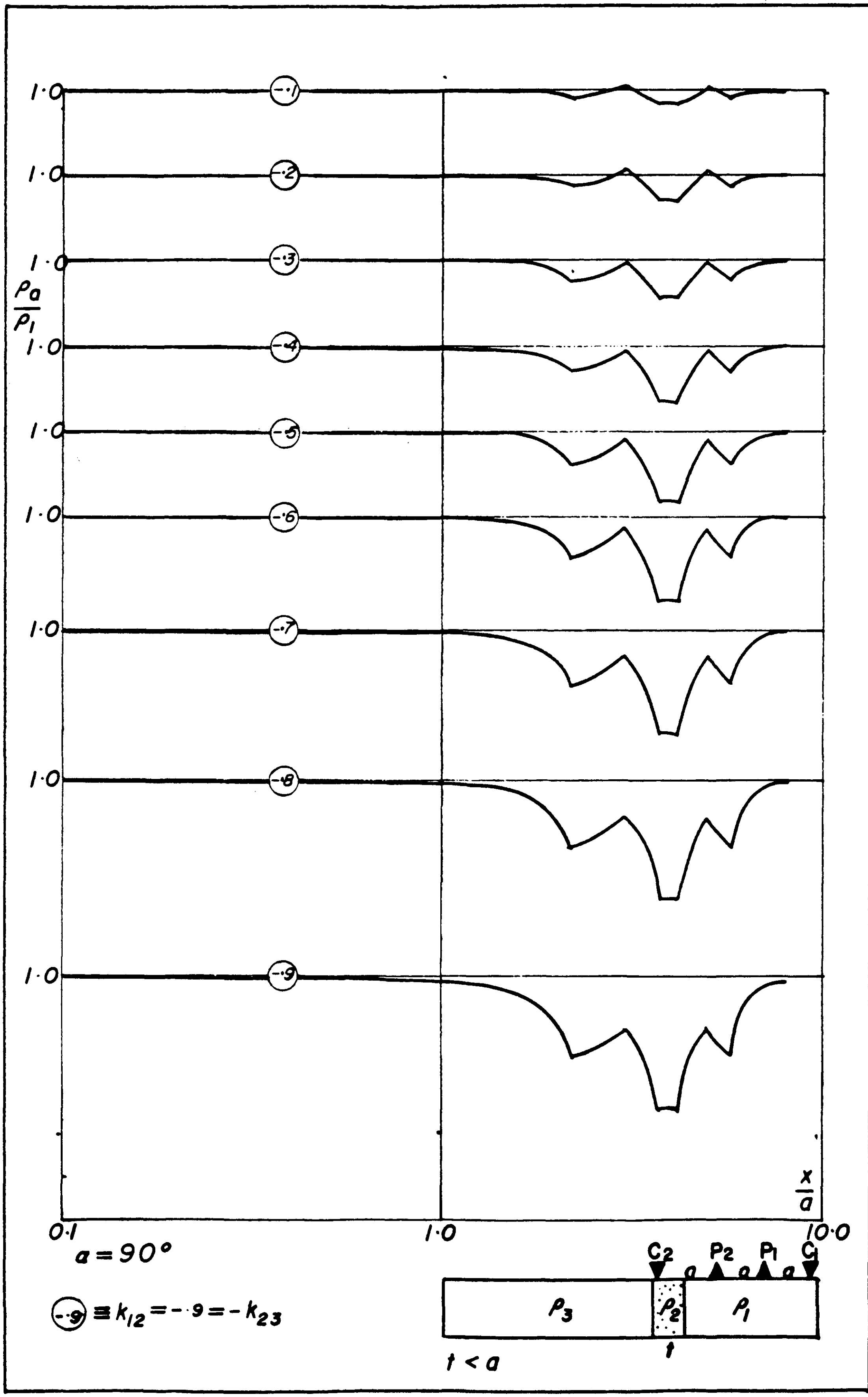

a = 15°
ρa/ρ1
10·0
1·0
0·1
0·01
0·1
1·0
10·0
x/a
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
-·9 ≡ k12 = -·9
C2 a P2 a P1 a C1
ρ2
ρ1
k12 = (ρ2 - ρ1)/(ρ2 + ρ1)

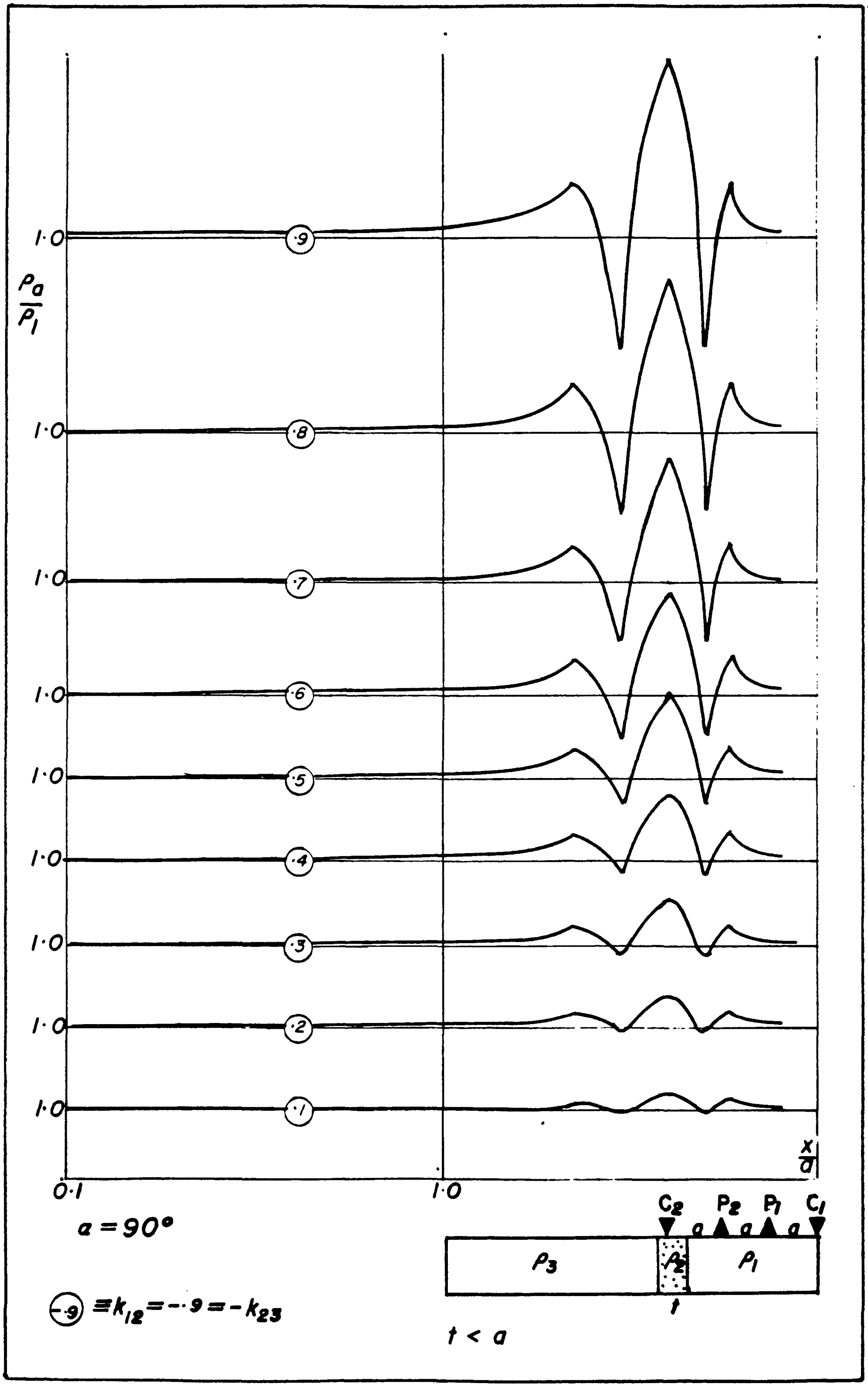
APPENDIX B

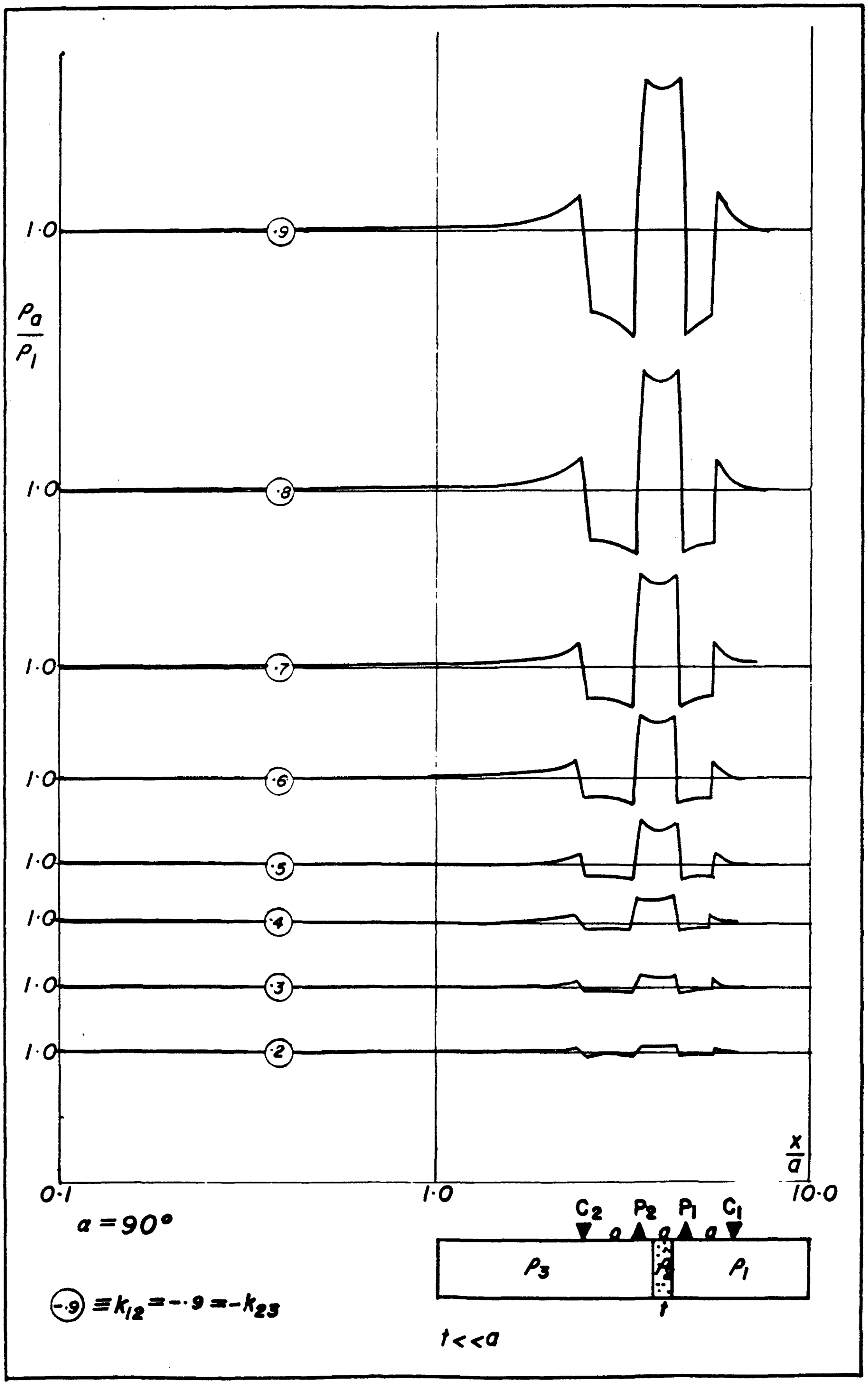
This appendix contains master curves for two vertical planes separating three media with resistivities ρ_1, ρ_2 and ρ_3, with $\rho_1 = \rho_3$. Although $\alpha = 90^\circ$ throughout, the user may specify any angle when calculating additional curves. For the cases where $t < a$ and $t << a$ the curves were separated for clarity.

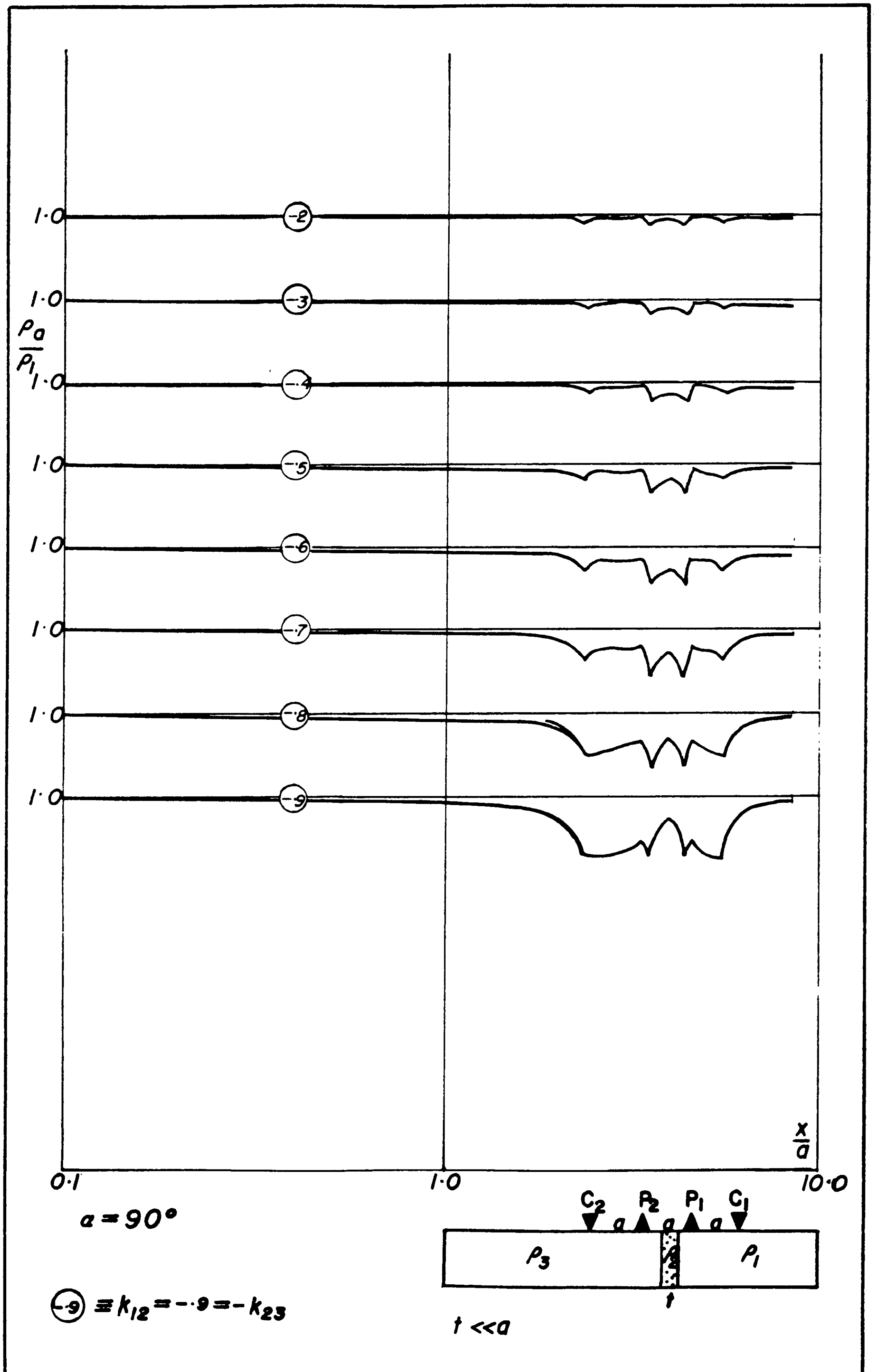

10·0
1·0
0·1
0·01
ρ_a / ρ_1
0·1
1·0
10·0
x/a
·9
·8
·7
·6
·5
·4
·3
·2
·1
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
$\alpha = 90°$
C2 P2 P1 C1
a a a
ρ_3 ρ_2 ρ_1
t
-·9 $\equiv k_{12} = -\cdot 9 = -k_{23}$
$2a < t < 3a$


10·0
1·0
0·1
0·01
0·1
1·0
10·0
ρ_a / ρ_1
x/a
·9
·8
·7
·6
·5
·4
·3
·2
·1
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
a = 90°
C2 P2 P1 C1
a a a
ρ3 ρ2 ρ1
t
a < t < 2a
(-·9) ≡ k12 = -·9 = -k23


1·0
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
$\frac{\rho_a}{\rho_1}$
$\frac{x}{a}$
0·1
1·0
10·0
$\alpha = 90°$
-·9 ≡ $k_{12} = -·9 = -k_{23}$
C2
P2
P1
C1
a
ρ_3
ρ_2
ρ_1
t
t < a


1·0
1·0
1·0
1·0
1·0
1·0
1·0
1·0
1·0
$\frac{\rho_a}{\rho_1}$
·9
·8
·7
·6
·5
·4
·3
·2
·1
$\frac{x}{a}$
0·1
1·0
$a = 90°$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
t
$t < a$
(·9) $\equiv k_{12} = -·9 = -k_{23}$


1·0
·9
$\frac{\rho_a}{\rho_1}$
1·0
·8
1·0
·7
1·0
·6
1·0
·5
1·0
·4
1·0
·3
1·0
·2
$\frac{x}{a}$
0·1
1·0
10·0
$\alpha = 90°$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
t
(-·9) $\equiv k_{12} = -·9 = -k_{23}$
$t << a$


1·0
-2
1·0
-3
$\frac{\rho_a}{\rho_1}$
1·0
-4
1·0
-5
1·0
-6
1·0
-7
1·0
-8
1·0
-9
$\frac{x}{a}$
0·1
1·0
10·0
$\alpha = 90°$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
t
$t \ll a$
(-9) $\equiv k_{12} = -.9 = -k_{23}$

APPENDIX C

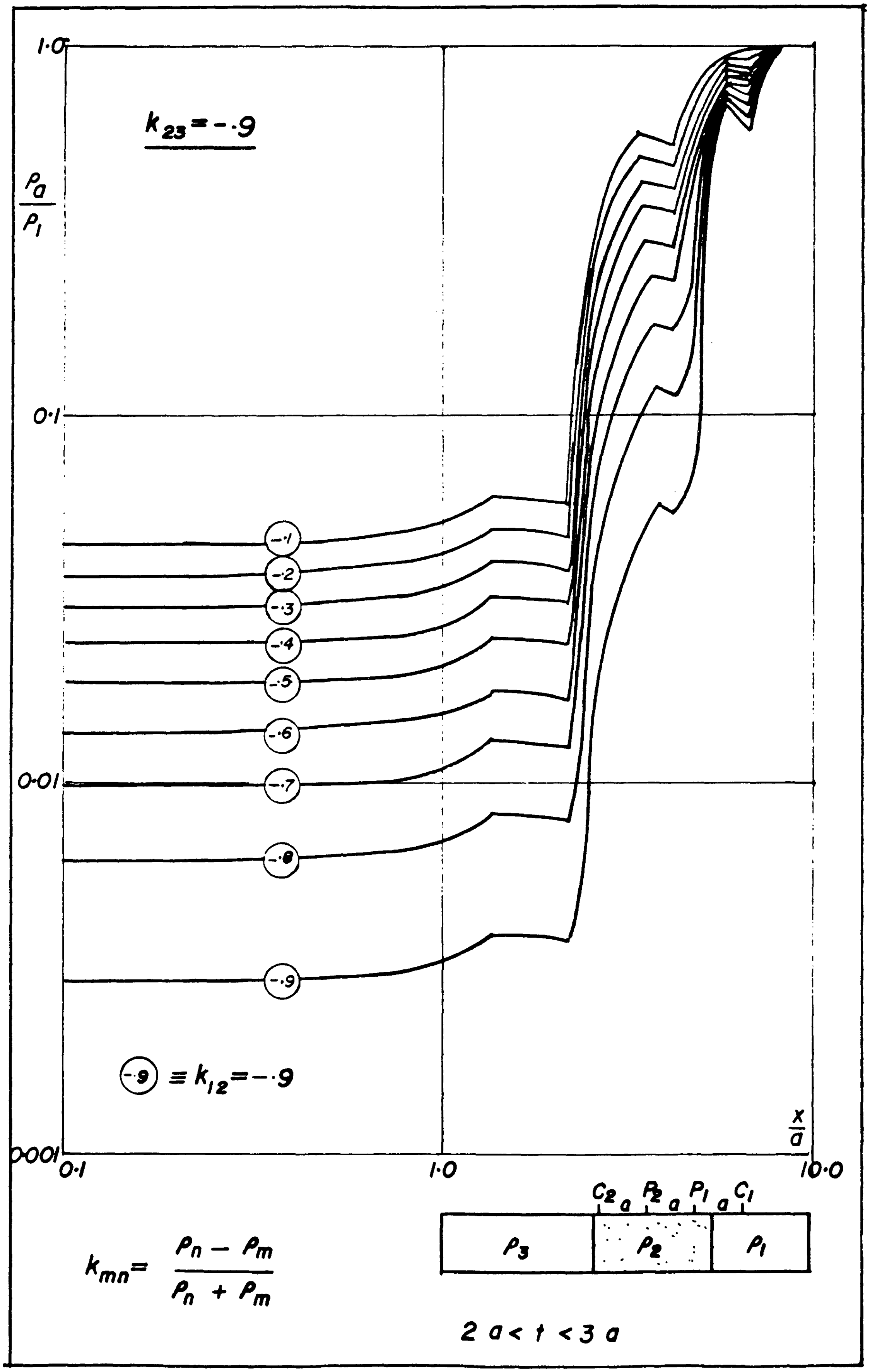
This appendix contains 180 master curves for the case where $2a < t < 3a$. Allthough all curves are for $\alpha = 90^{o}$, program 2 that was used to calculate these curves, is such that the user can specify any angle for calculating additional master curves. The curves were calculated for k_{12} varying from $-.9$ to $+.9$ in steps of .1, and k_{23} from $-.9$ to $+.9$ in steps of .2. These correspond to the following resistivity ratios:

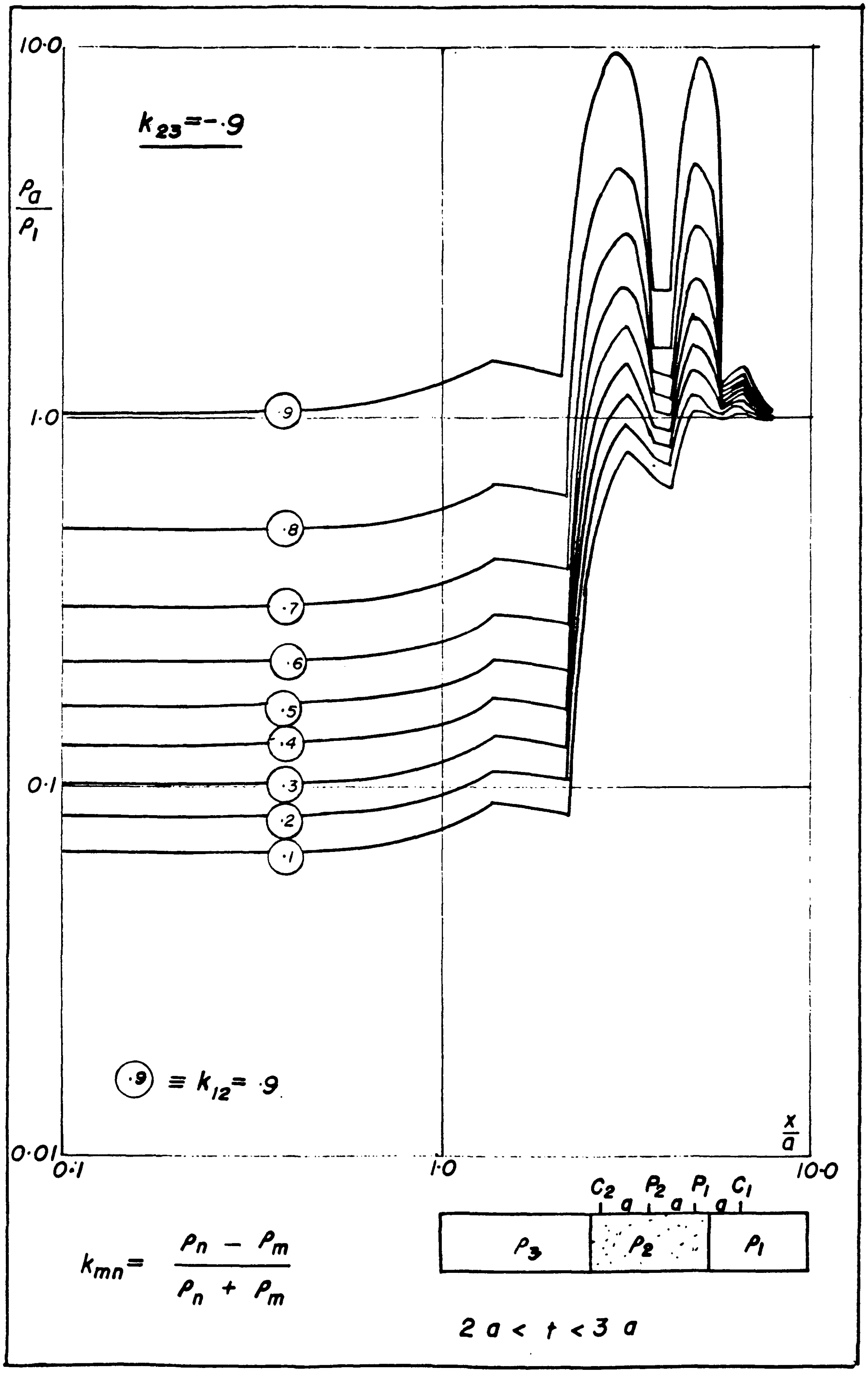
k_{12}	k_{23}	ρ_1	ρ_2	ρ_3	k_{12}	k_{23}	ρ_1	ρ_2	ρ_3	k_{12}	k_{23}	ρ_1	ρ_2	ρ_3
-.9	-.9	1.0	.05	.003	-.5			.33	.059	-.1			.82	.273
-.8			.11	.006	-.4			.43	.076	.1			1.22	.406
-.7			.18	.009	-.3			.54	.095	.2			1.50	.497
-.6			.25	.013	-.2			.67	.118	.3			1.86	.617
-.5			.33	.018	-.1			.82	.145	.4			2.33	.774
-.4			.43	.022	.1			1.22	.215	.5			3.00	1.00
-.3			.54	.028	.2			1.50	.263	.6			4.00	1.33
-.2			.67	.035	.3			1.86	.327	.7			5.66	1.88
-.1			.82	.043	.4			2.33	.410	.8			9.00	3.03
.1			1.22	.064	.5			3.00	.535	.9			19.00	6.29
.2			1.50	.079	.6			4.00	.706					
.3			1.86	.097	.7			5.66	.997	-.9	-.3	1.0	.05	.028
.4			2.33	.122	.8			9.00	1.60	-.8			.11	.060
.5			3.00	.159	.9			19.00	3.33	-.7			.18	.095
.6			4.00	.210						-.6			.25	.135
.7			5.66	.280	-.9	-.5	1.0	.05	.018	-.5			.33	.179
.8			9.00	.478	-.8			.11	.037	-.4			.43	.231
.9			19.00	.992	-.7			.18	.059	-.3			.54	.289
					-.6			.25	.083	-.2			.67	.359
-.9	-.7	1.0	.05	.009	-.5			.33	.111	-.1			.82	.441
-.8			.11	.020	-.4			.43	.143	.1			1.22	.656
-.7			.18	.031	-.3			.54	.179	.2			1.50	.803
-.6			.25	.044	-.2			.67	.222	.3			1.86	.996

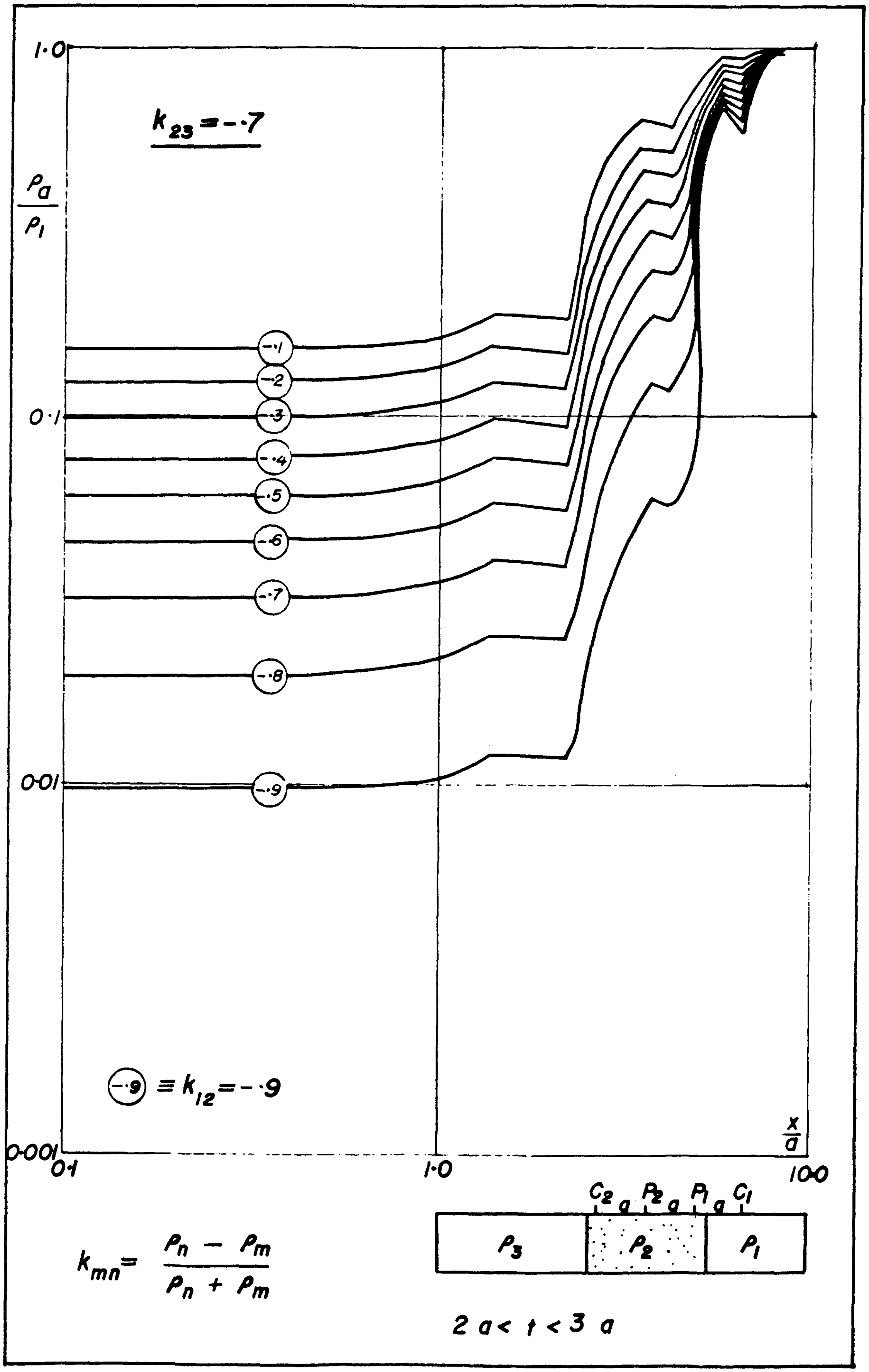
k_{12}	k_{23}	ρ_1	ρ_2	ρ_3
.4			2.33	1.25
.5			3.00	1.63
.6			4.00	2.15
.7			5.66	3.04
.8			9.00	4.89
.9			19.00	10.2
-.9	-.1	1.0	.05	.043
-.8			.11	.091
-.7			.18	.145
-.6			.25	.205
-.5			.33	.273
-.4			,43	.351
-.3			.54	.440
-.2			.67	.545
-.1			.82	.671
.1			1.22	.998
.2			1.50	1.22
.3			1.86	1.52
.4			2.33	1.90
.5			3.00	2.48
.6			4.00	3.27
.7			5.66	4.62
.8			9.00	7.44
.9			19.00	15.4
-.9	.1	1.0	.05	.064
-.8			.11	,136
-.7			.18	.216
-.6			.25	.306
-.5			.33	.407
-.4			.43	.524
-.3			.54	.657
-.2			.67	.815
-.1			.82	1.00
.1			1.22	1.49
.2			1.50	1.82
.3			1.86	2.26
.4			2.33	2.84
.5			3.00	3.67

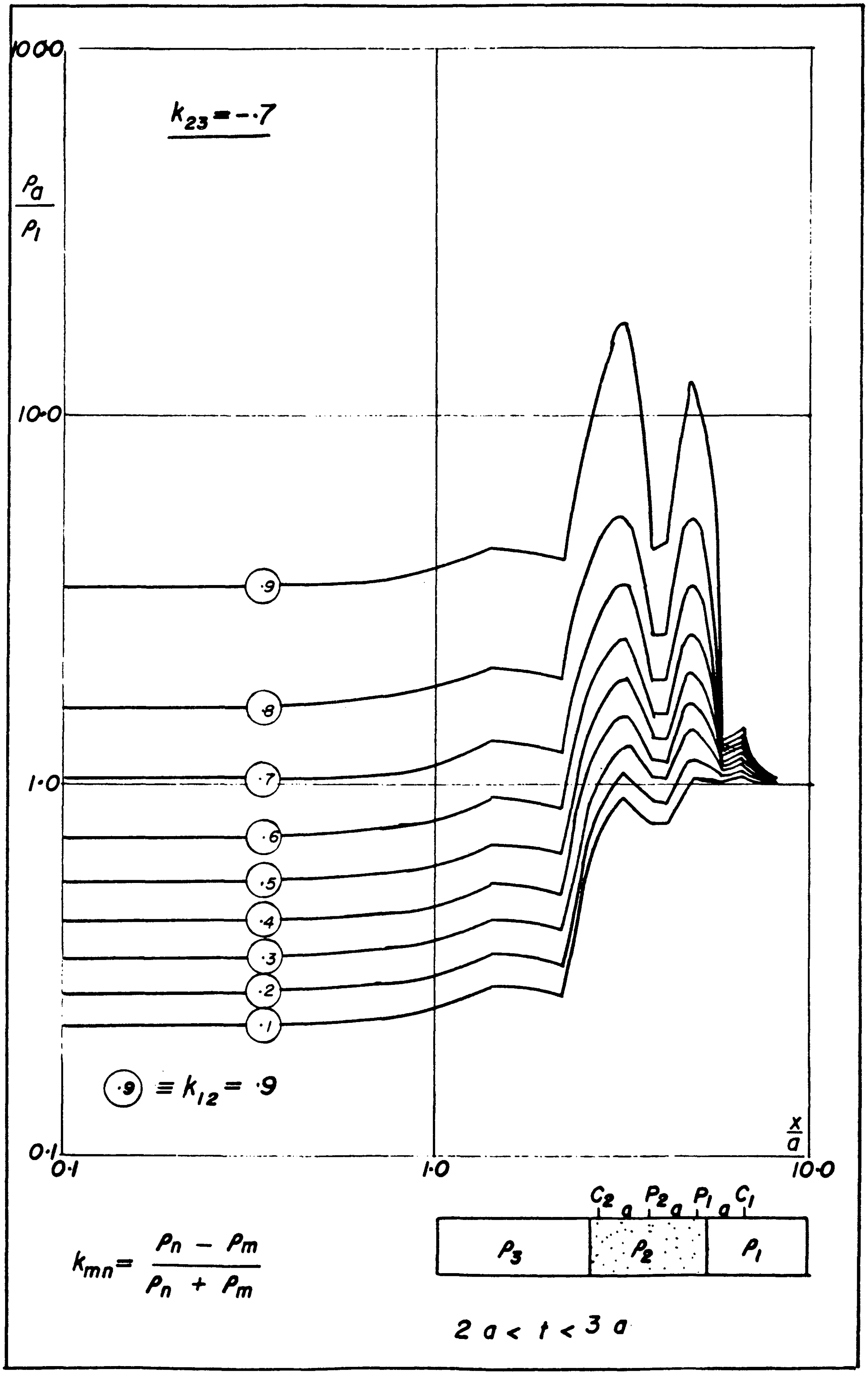
k_{12}	k_{23}	ρ_1	ρ_2	ρ_3
.6			4.00	4.89
.7			5.66	6.88
.8			9.00	11.1
.9			19.00	23.0
-.9	.3	1.0	.05	.098
-.8			.11	.206
-.7			.18	.328
-.6			.25	.464
-.5			.33	.619
-.4			.43	.795
-.3			.54	.998
-.2			.67	1.24
-.1			.82	1.52
.1			1.22	2.26
.2			1.50	2,77
.3			1.86	3.44
.4			2.33	4.32
.5			3.00	5.63
.6			4.00	7.43
.7			5.66	10.5
.8			9.00	16,9
.9			19.00	35.0
-.9	.5	1.0	.05	.158
-.8			.11	.333
-.7			.18	.530
-.6			.25	.750
-.5			.33	1.00
-.4			.43	1.29
-.3			.54	1.61
-.2			.67	2.00
-.1			.82	2.46
.1			1.22	3.66
.2			1.50	4.48
.3			1.86	5.56
.4			2.33	6.98
.5			3.00	9.09
.6			4.00	12.0
.7			5.66	16.9

k_{12}	k_{23}	ρ_1	ρ_2	ρ_3
.8			9.00	27.3
.9			19.00	56.6
-.9	.7	1.0	.05	.298
-.8			.11	.630
-.7			.18	1.00
-.6			.25	1.42
-.5			.33	1.89
-.4			.53	2.43
-.3			.54	3.05
-.2			.67	3.78
-.1			.82	4.64
.1			1.22	6.91
.2			1.50	8.46
.3			1.86	10.5
.4			2.33	13.2
.5			3.00	17.2
.6			4.00	22.7
.7			5.66	32.0
.8			9.00	51.5
.9			19.00	106
-.9	.9	1.0	.05	1.00
-.8			.11	2.11
-.7			.18	3.36
-.6			.25	4.75
-.5			.33	6.33
-.4			.43	8.14
-.3			.54	10.2
-.2			.67	12.7
-.1			.82	15.6
.1			1.22	23.2
.2			1.50	28.4
.3			1.86	35.2
.4			2.33	44.2
.5			3.00	57.6
.6			4.00	76.0
.7			5.66	107
.8			9.00	171
.9			19.00	358

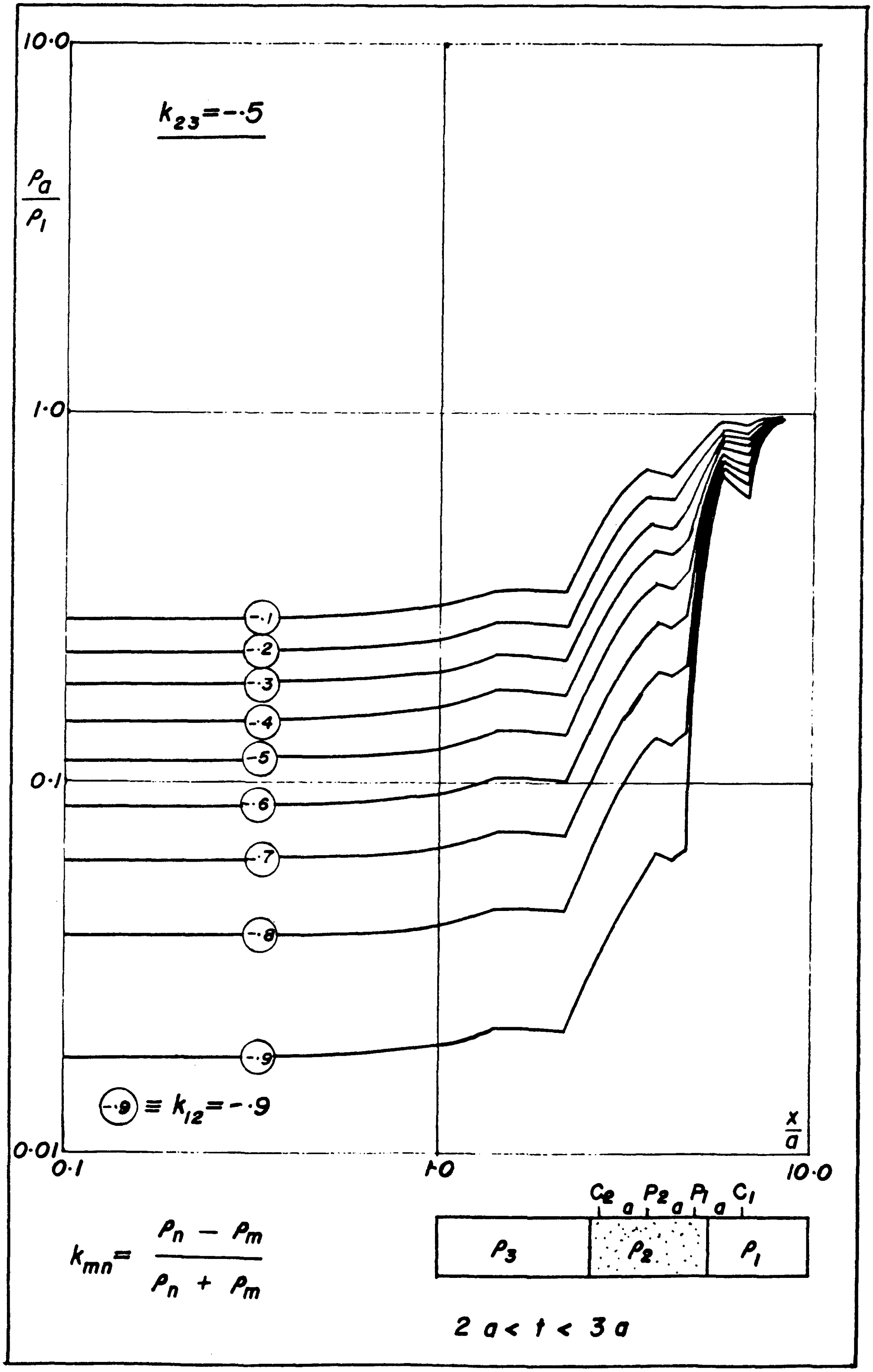

$k_{23} = -.9$
ρ_a / ρ_1
1·0
0·1
0·01
0·001
0·1
1·0
10·0
x/a
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
(-·9) ≡ $k_{12} = -·9$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
2 a < t < 3 a

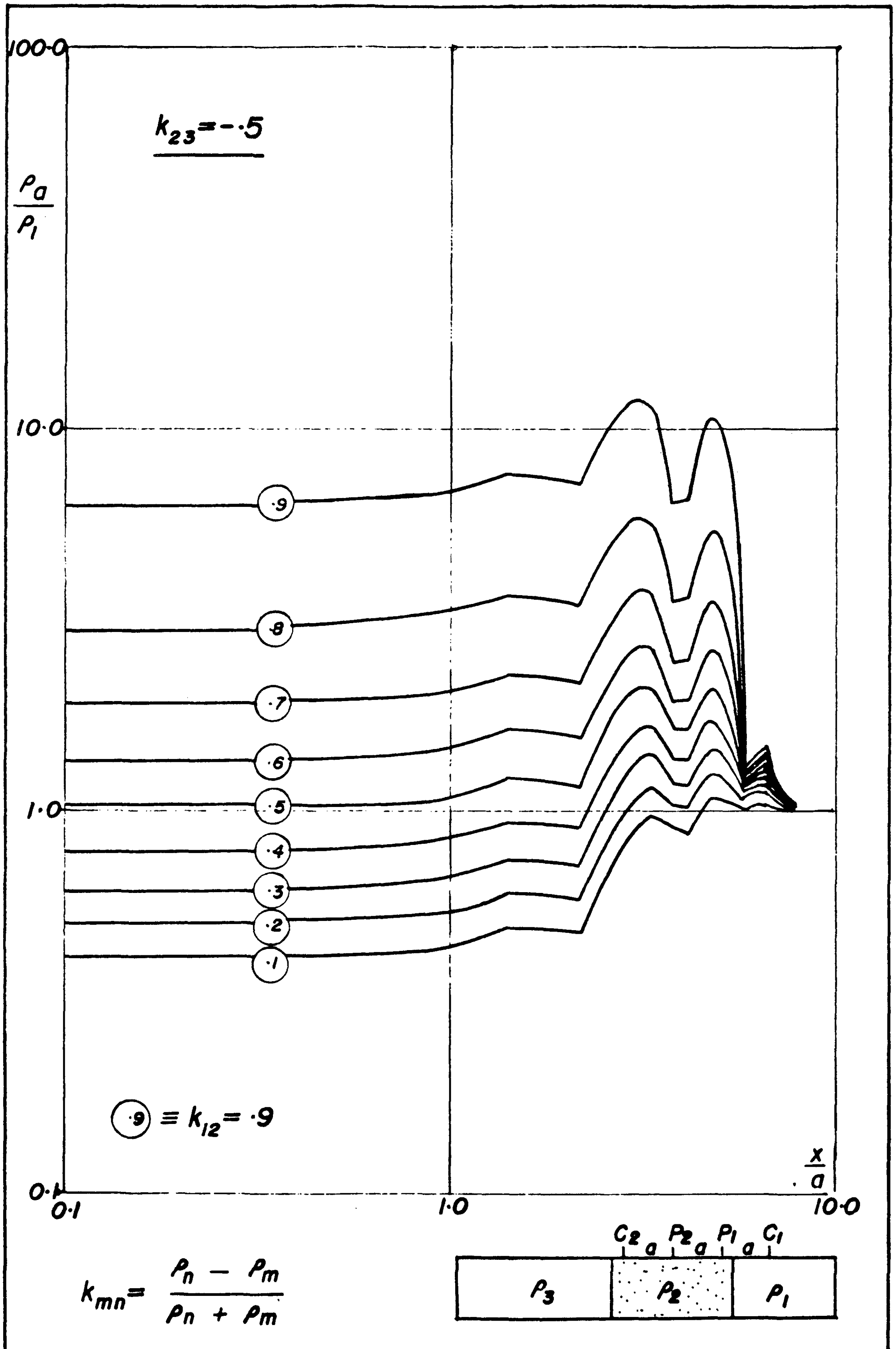

$k_{23} = -.9$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
0·1
1·0
10·0
$\frac{x}{a}$
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ $k_{12} = \cdot 9$
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$2a < t < 3a$


$k_{23} = -\cdot 7$
ρ_a / ρ_1
1·0
0·1
0·01
0·001
0·1
1·0
10·0
x/a
$-\cdot 1$
$-\cdot 2$
$-\cdot 3$
$-\cdot 4$
$-\cdot 5$
$-\cdot 6$
$-\cdot 7$
$-\cdot 8$
$-\cdot 9$
$-\cdot 9 \equiv k_{12} = -\cdot 9$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$2a < t < 3a$



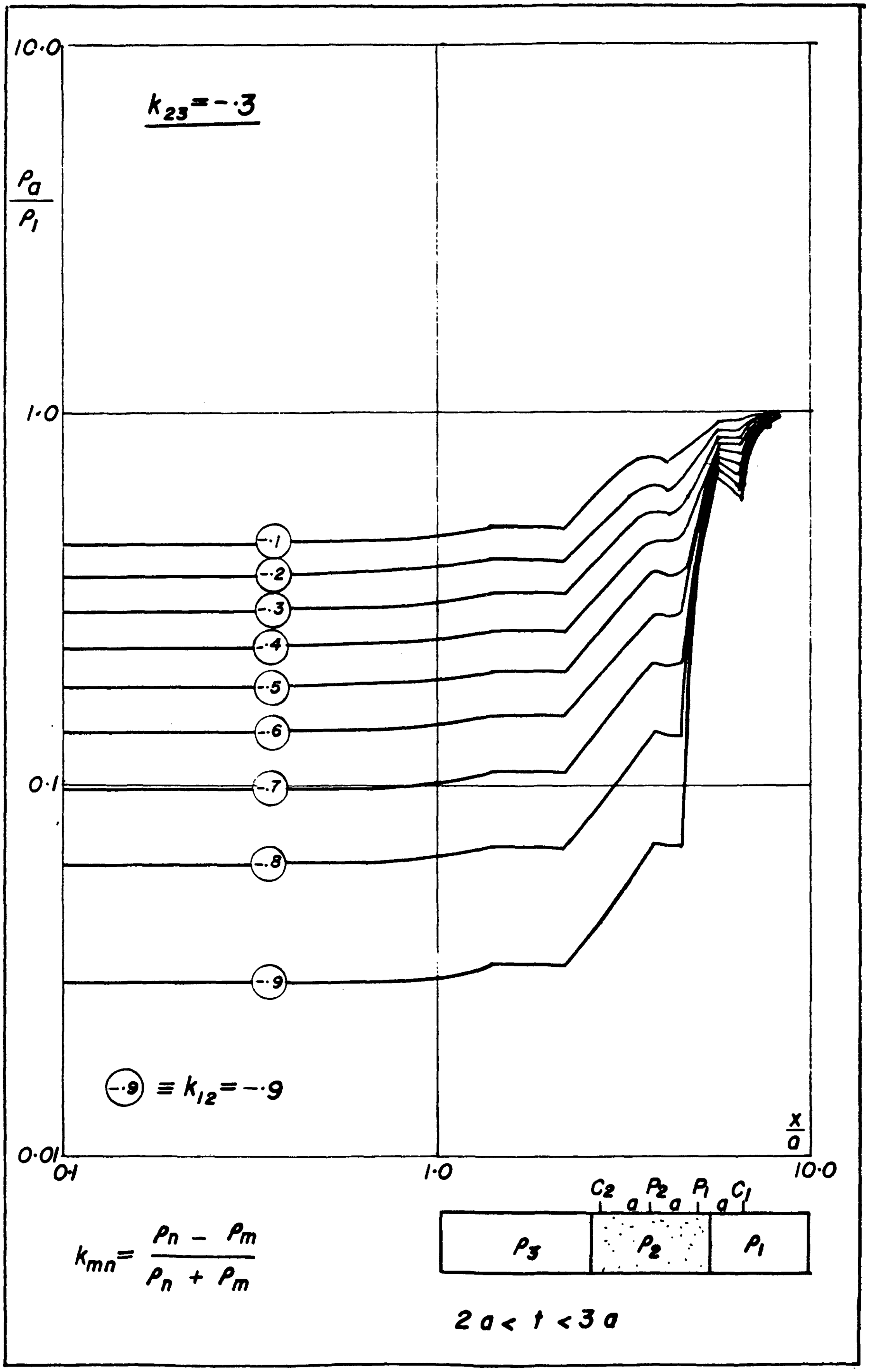
$k_{23} = -\cdot 7$
ρ_a / ρ_1
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ $k_{12} = \cdot 9$
x/a
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \dfrac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$2a < t < 3a$


$k_{23} = -.5$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
0·1
1·0
10·0
$\frac{x}{a}$
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
(-·9) ≡ $k_{12} = -·9$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$2a < t < 3a$

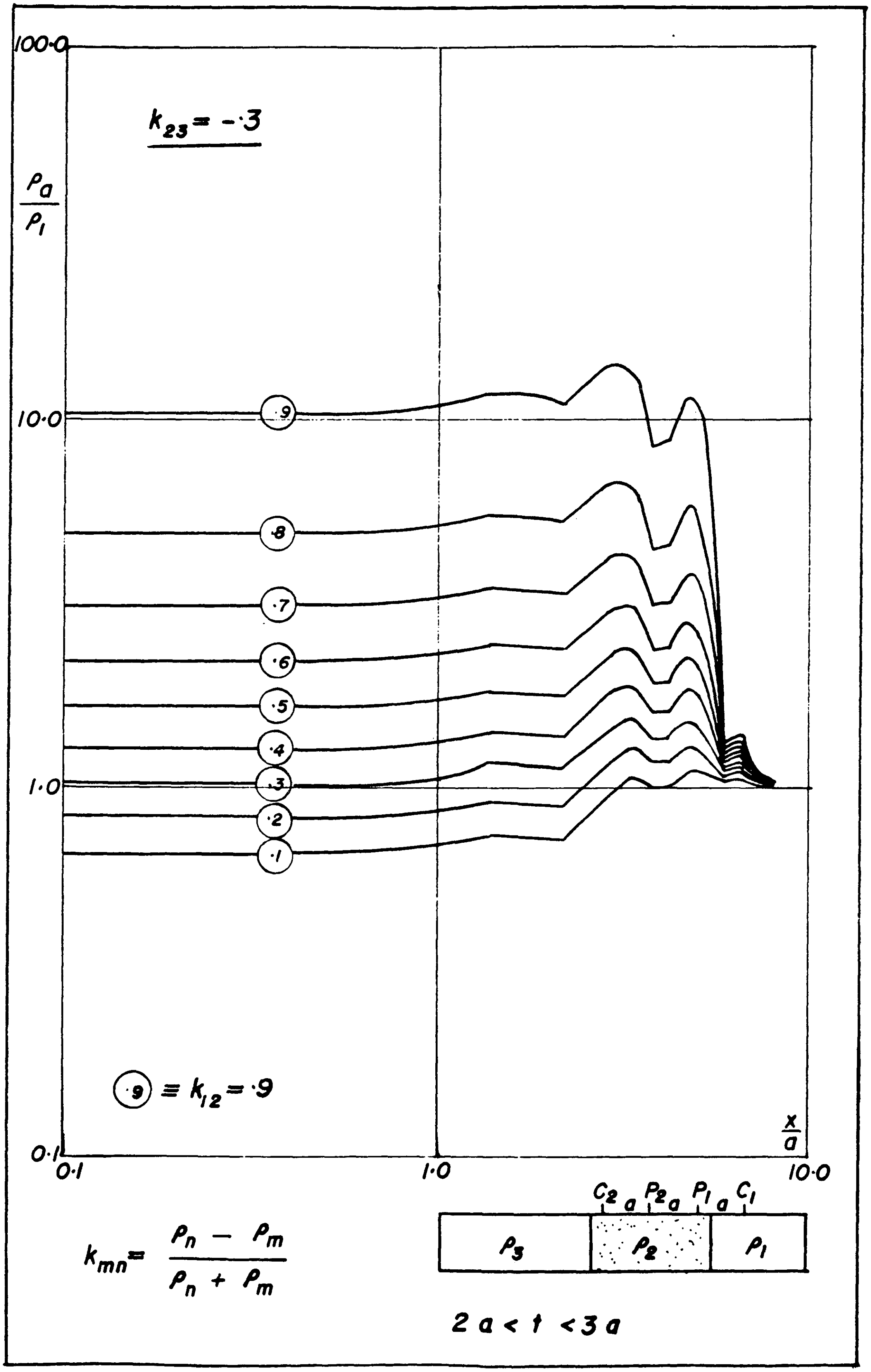


$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

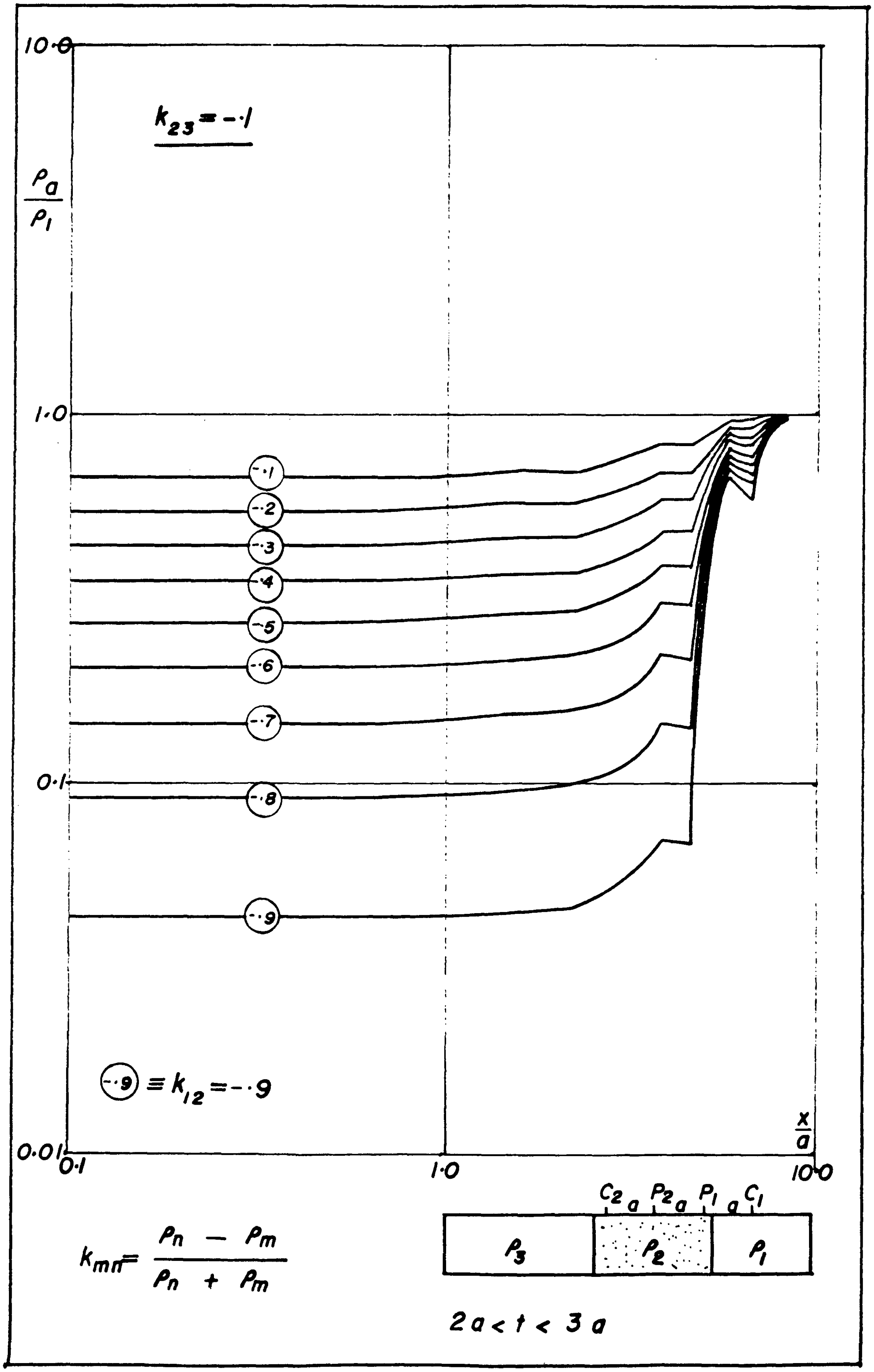
$2a < t < 3a$

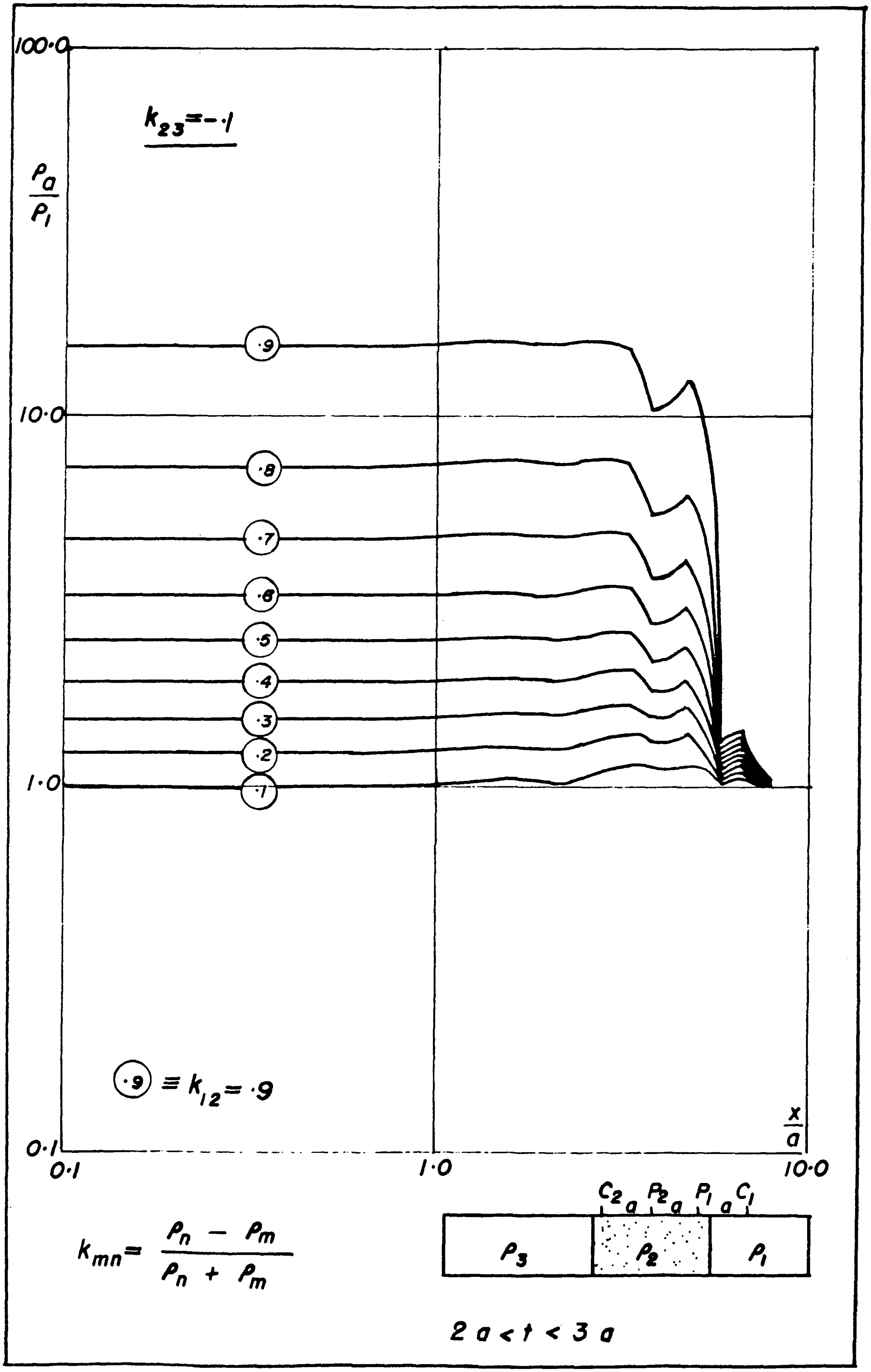


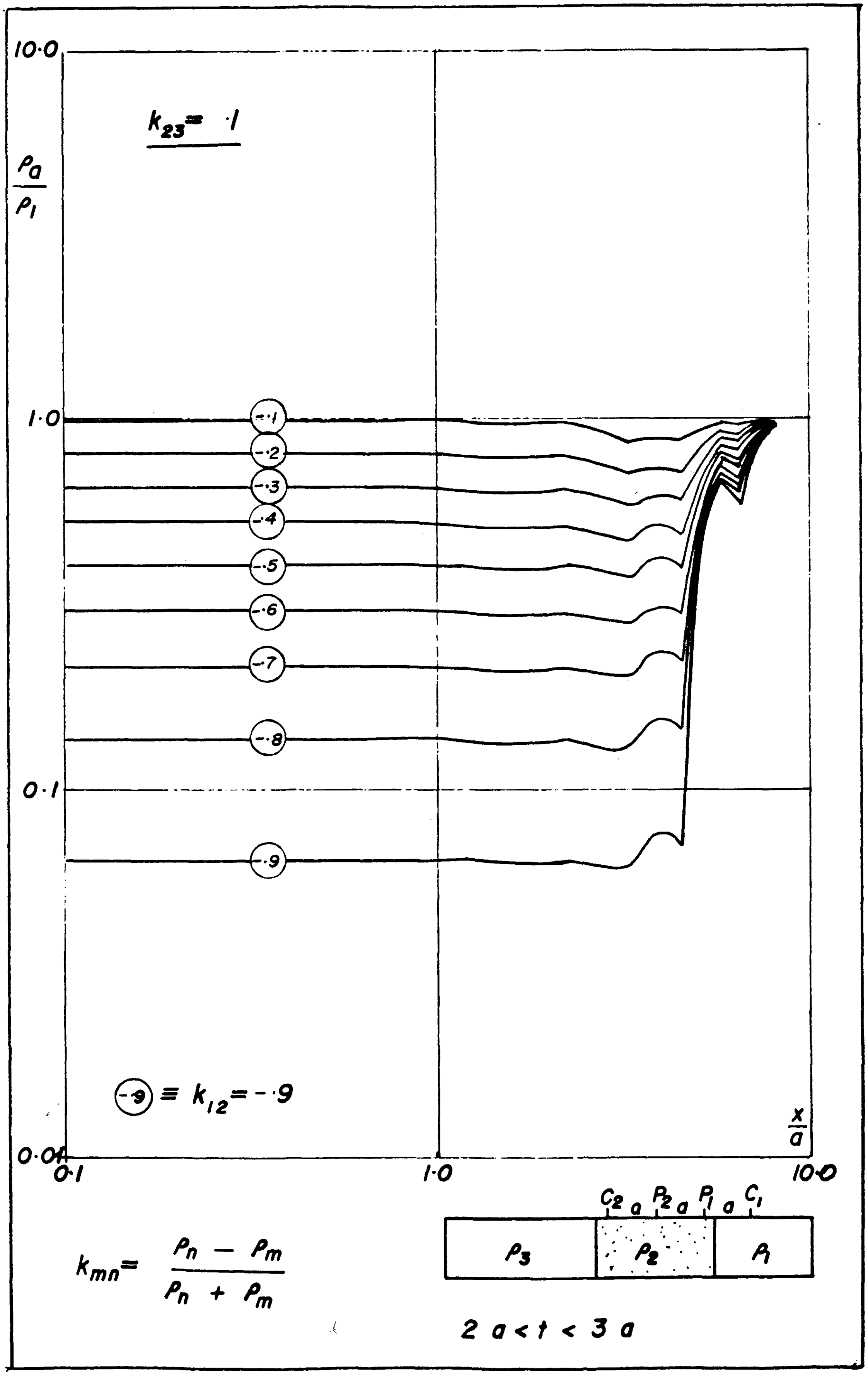
$k_{23} = -.3$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
−·1
−·2
−·3
−·4
−·5
−·6
−·7
−·8
−·9
−·9 ≡ $k_{12} = -.9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$2a < t < 3a$



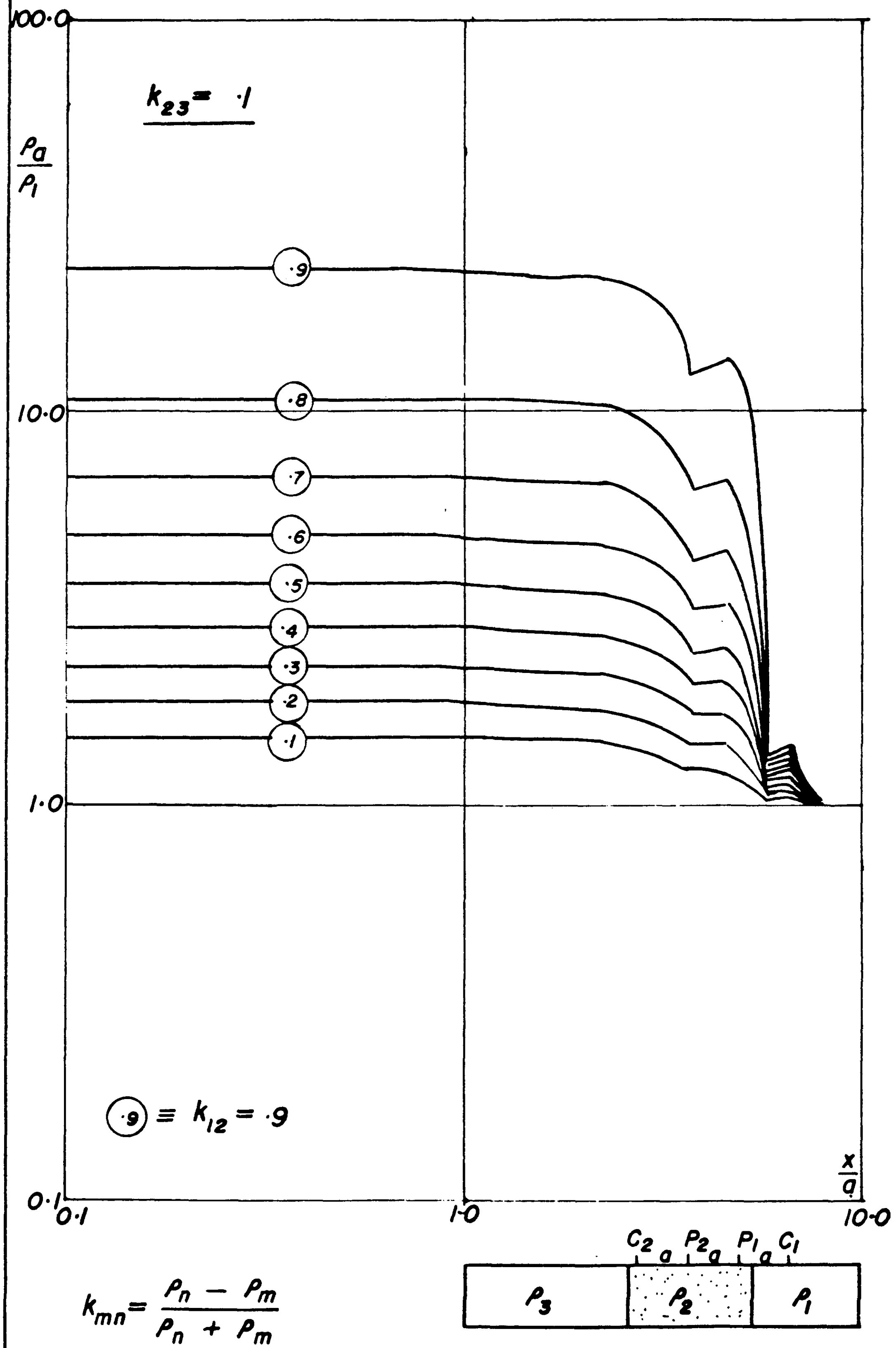
$k_{23} = -\cdot 3$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
$\cdot 9 \equiv k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$2a < t < 3a$


$k_{23} = -\cdot 1$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
-·9 ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$2a < t < 3a$


$k_{23} = -\cdot 1$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ $k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$2a < t < 3a$

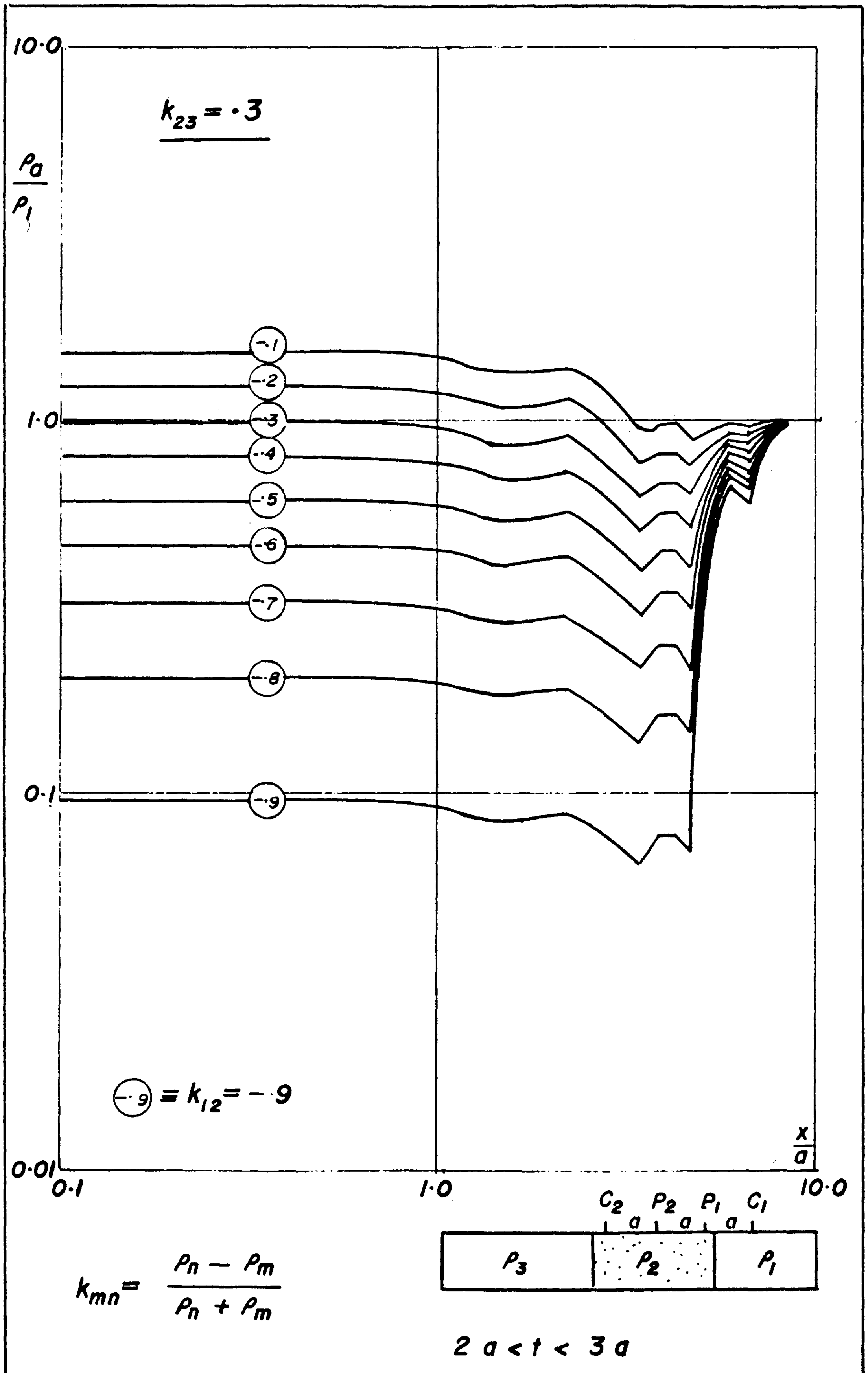


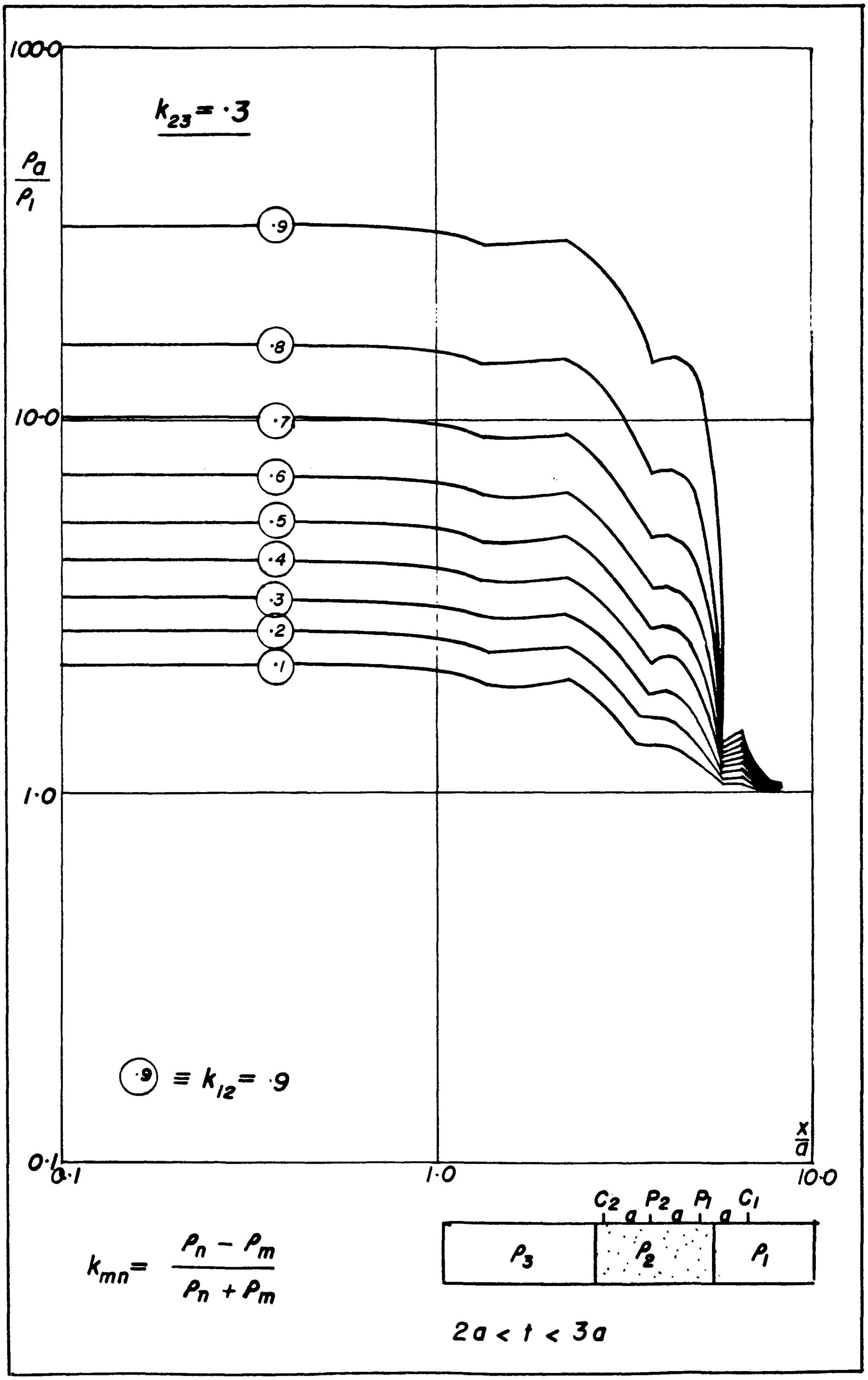
10·0
1·0
0·1
0·01
$\frac{\rho_a}{\rho_1}$
$k_{23} = \cdot 1$
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
(-·9) ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
2 a < t < 3 a

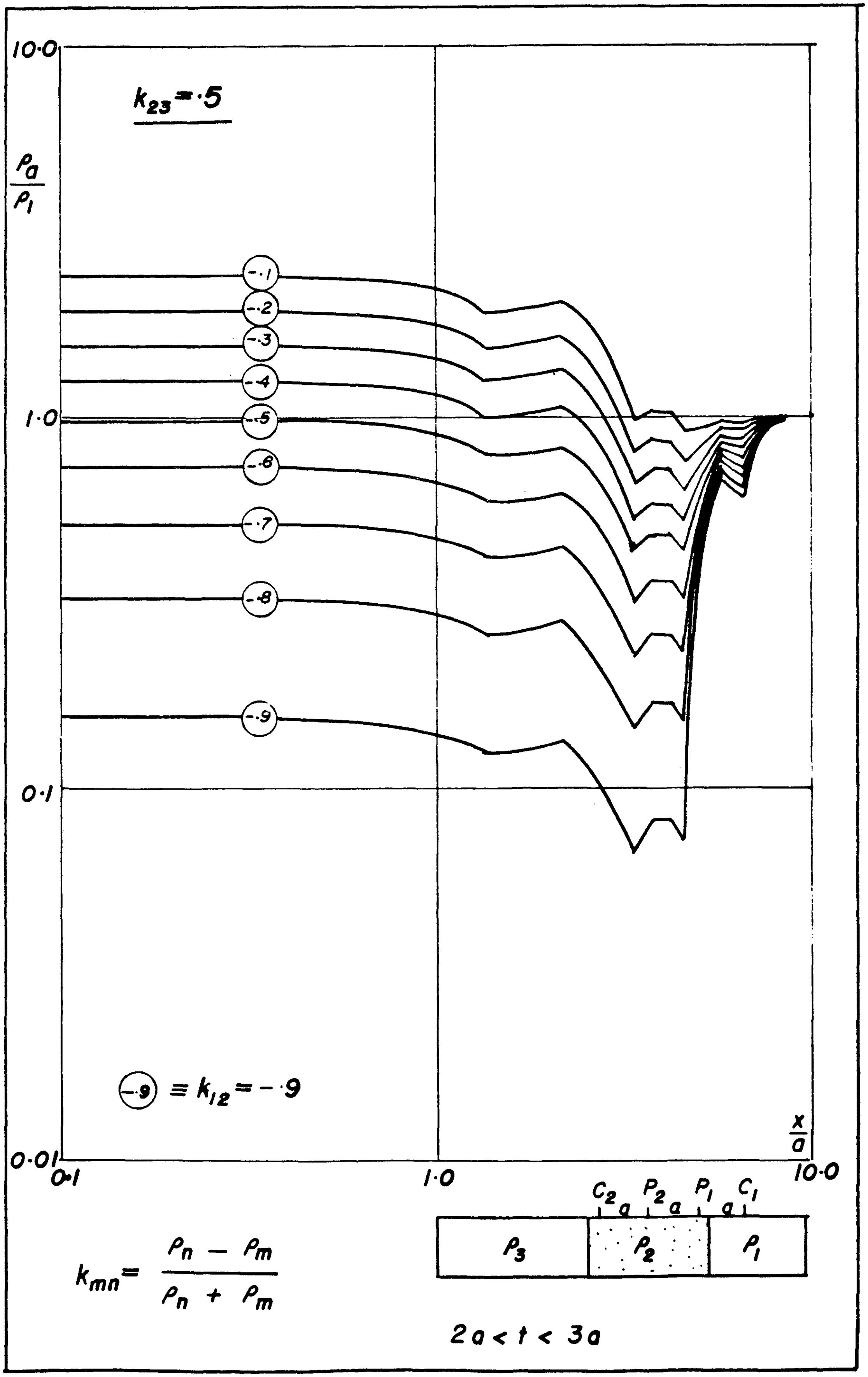


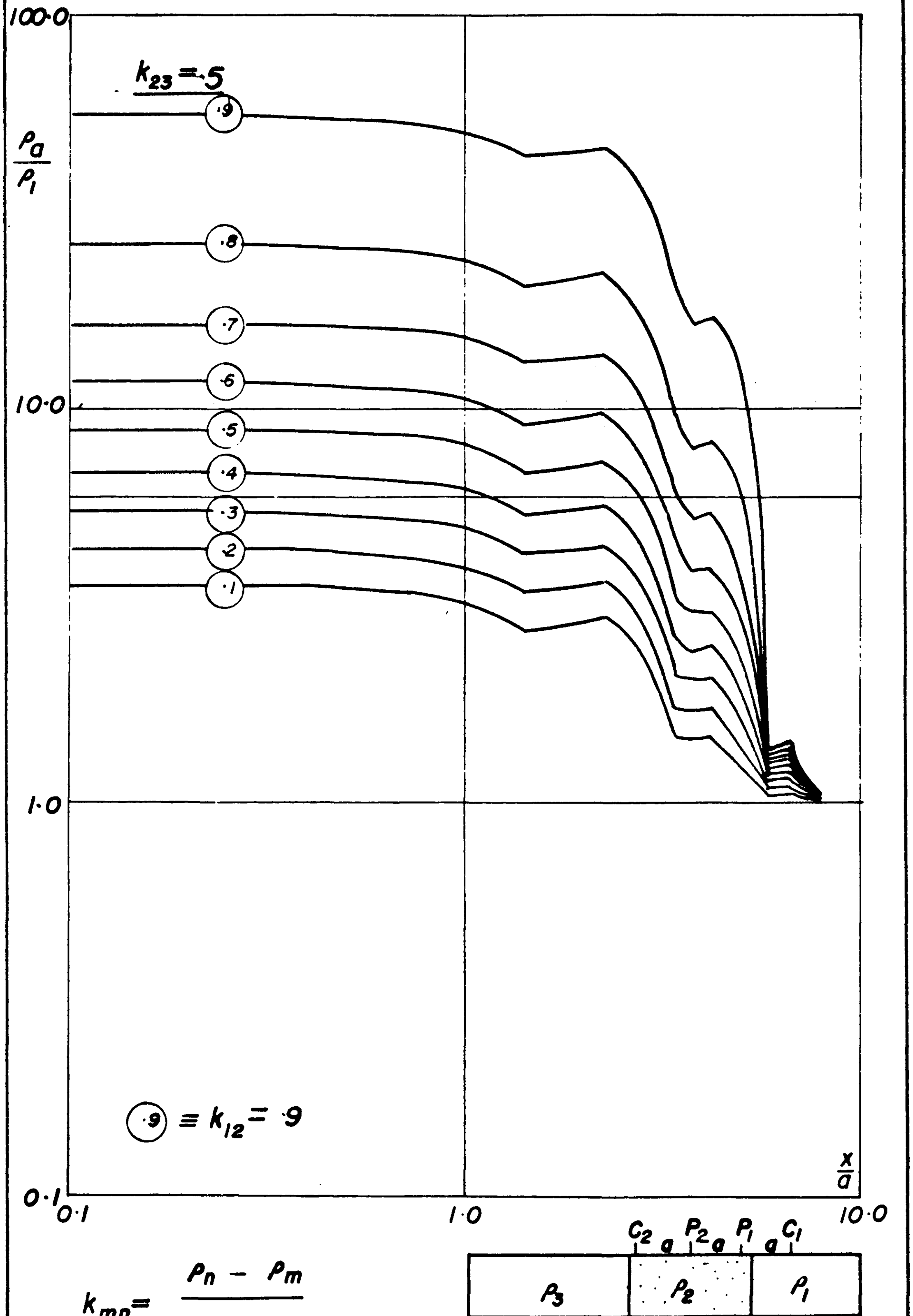
$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

$2a < t < 3a$


10·0
$k_{23} = \cdot 3$
$\frac{\rho_a}{\rho_1}$
-·1
-·2
1·0
-·3
-·4
-·5
-·6
-·7
-·8
0·1
-·9
-·9 = $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·01
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$2a < t < 3a$

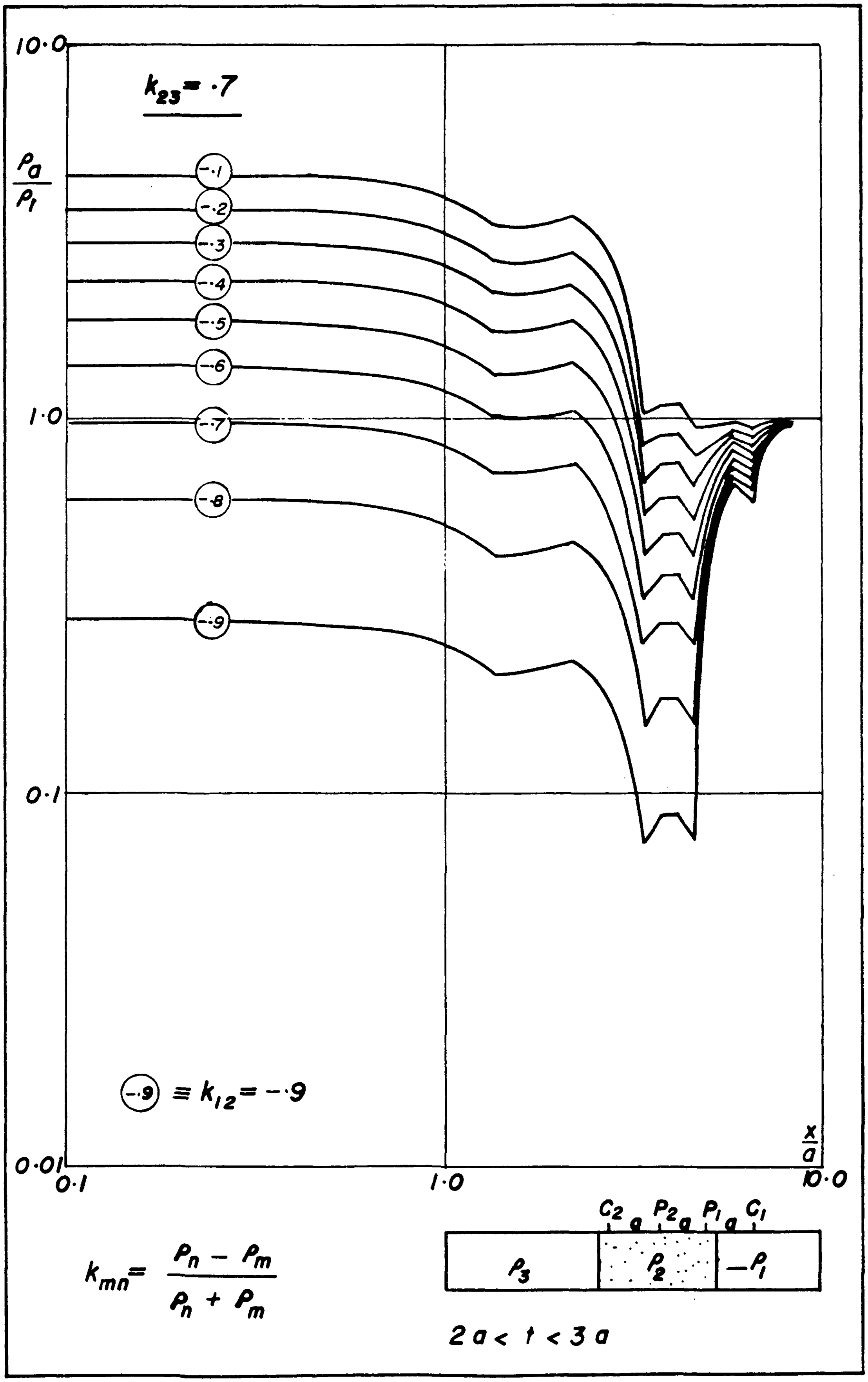

$k_{23} = \cdot 3$
ρ_a / ρ_1
100·0
10·0
1·0
0·1
0·1
1·0
10·0
x/a
·9
·8
·7
·6
·5
·4
·3
·2
·1
·9 ≡ $k_{12} = \cdot 9$
$k_{mn} = \dfrac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$2a < t < 3a$

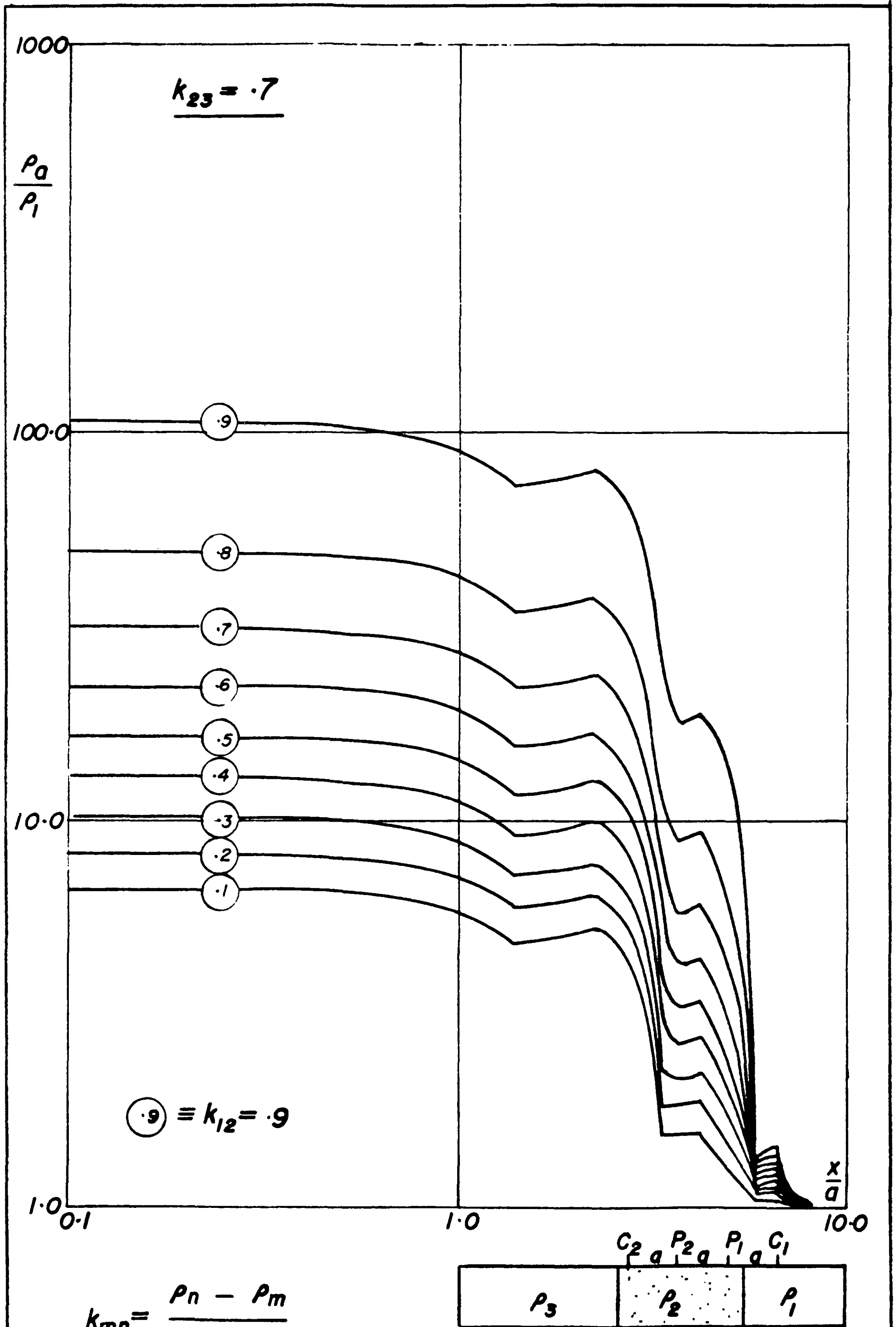

$k_{23} = \cdot 5$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
(-·9) ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$2a < t < 3a$



$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

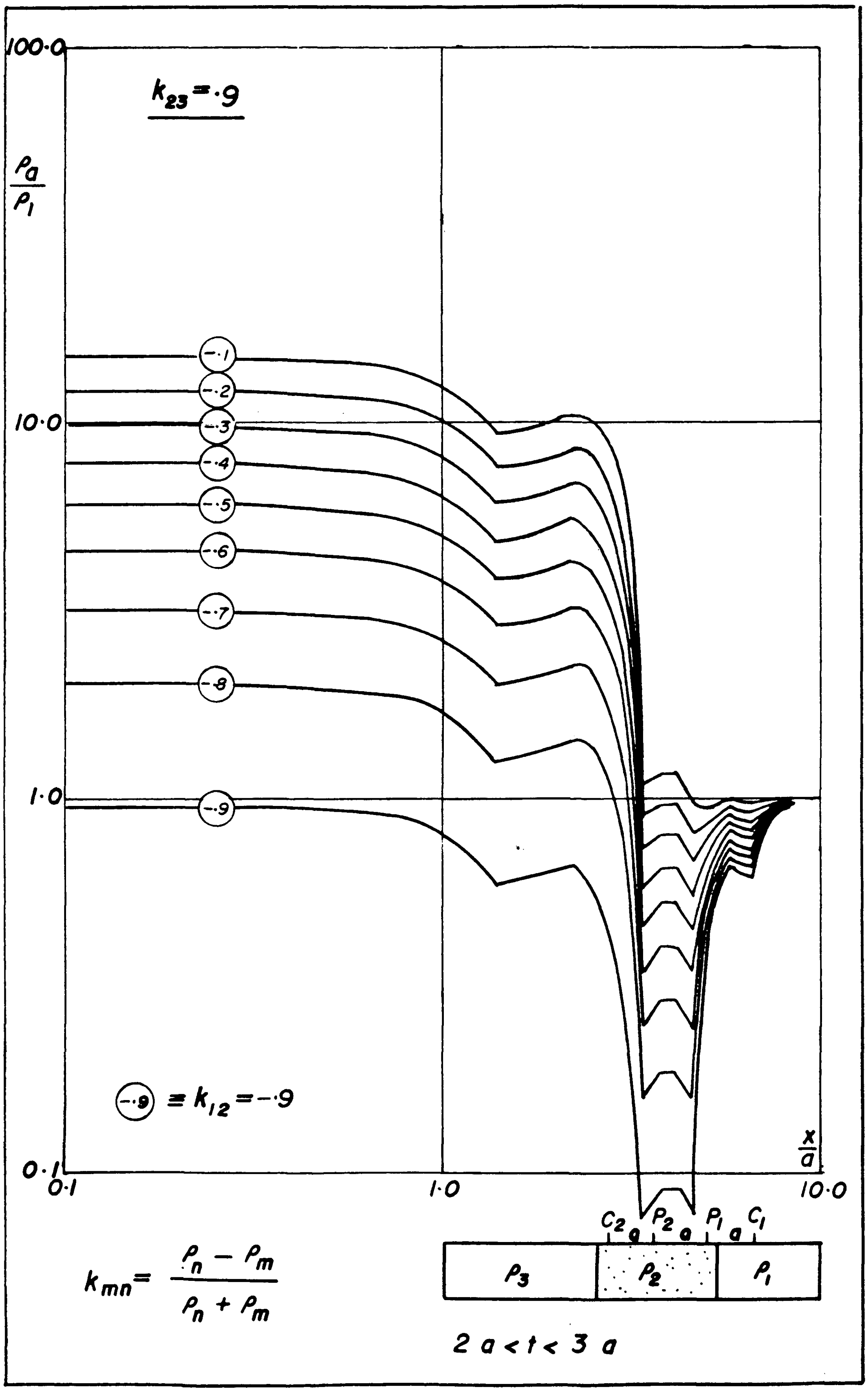
$2a < t < 3a$

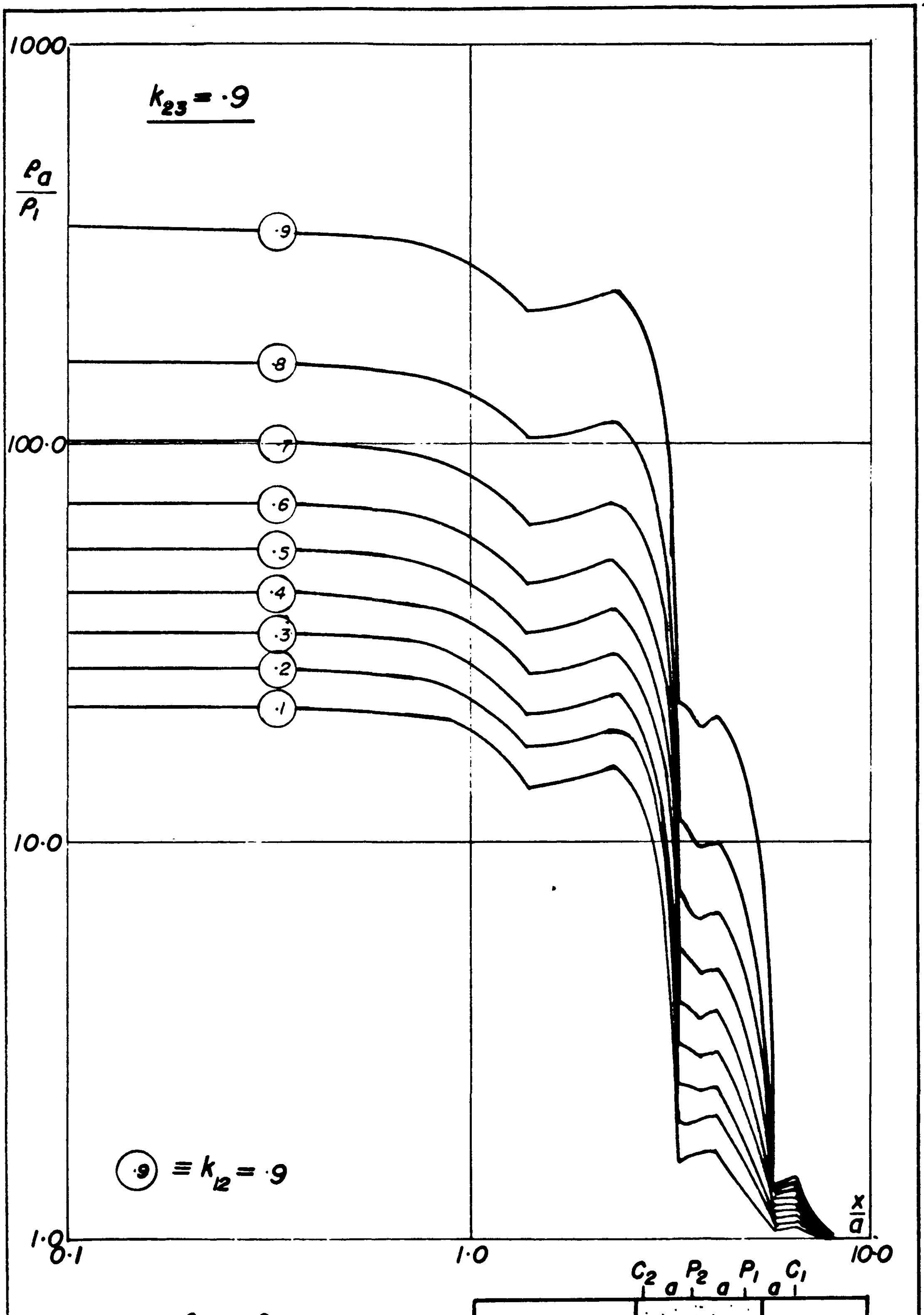

10·0
$k_{23} = \cdot 7$
ρ_a / ρ_1
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
1·0
0·1
(-·9) ≡ $k_{12} = -\cdot 9$
x/a
0·01
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
2 a < t < 3 a



$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

$2a < t < 3a$


$k_{23} = \cdot 9$
ρ_a / ρ_1
100·0
10·0
1·0
0·1
0·1
1·0
10·0
x/a
−·1
−·2
−·3
−·4
−·5
−·6
−·7
−·8
−·9
(−·9) ≡ $k_{12} = -\cdot 9$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$2a < t < 3a$

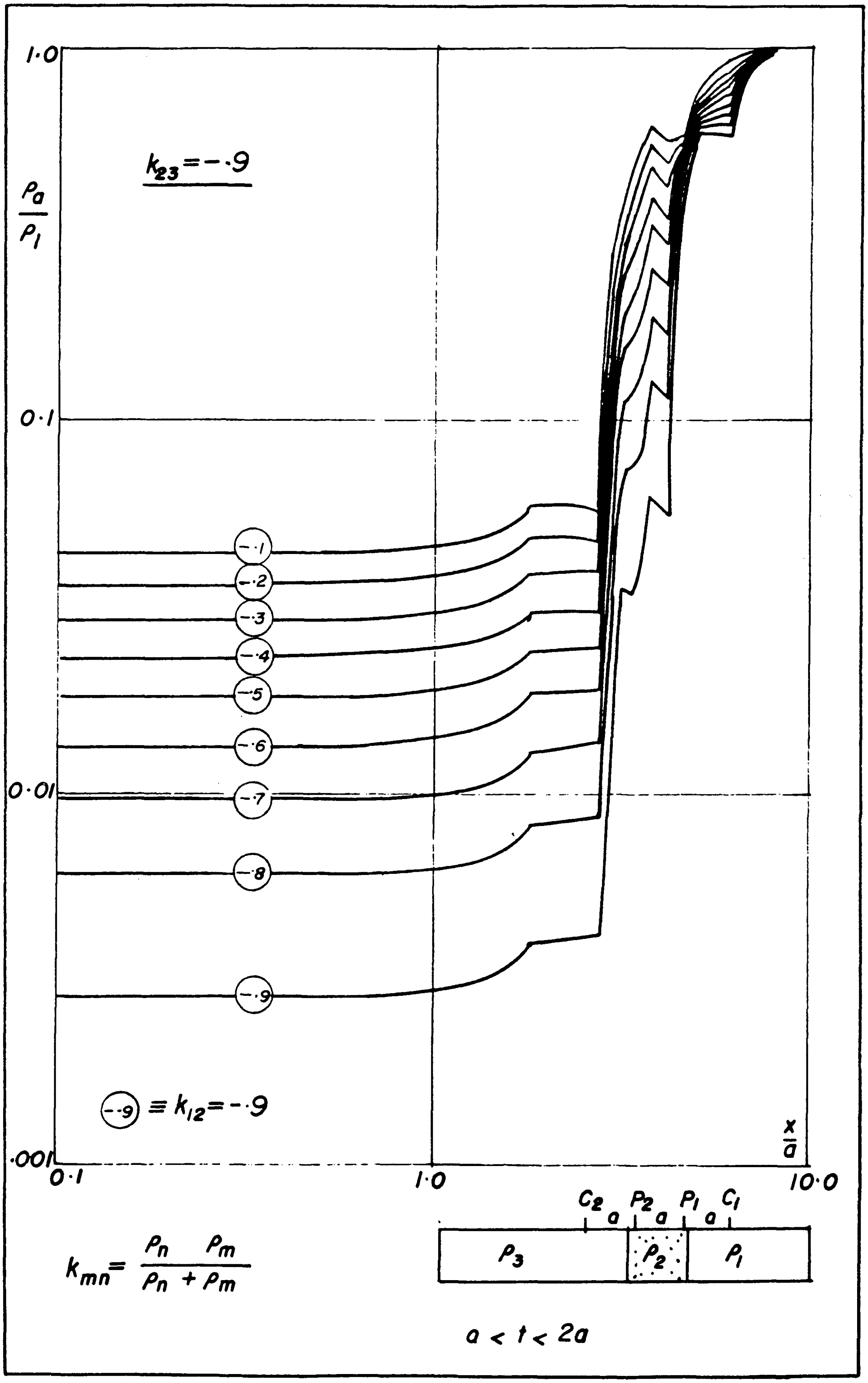


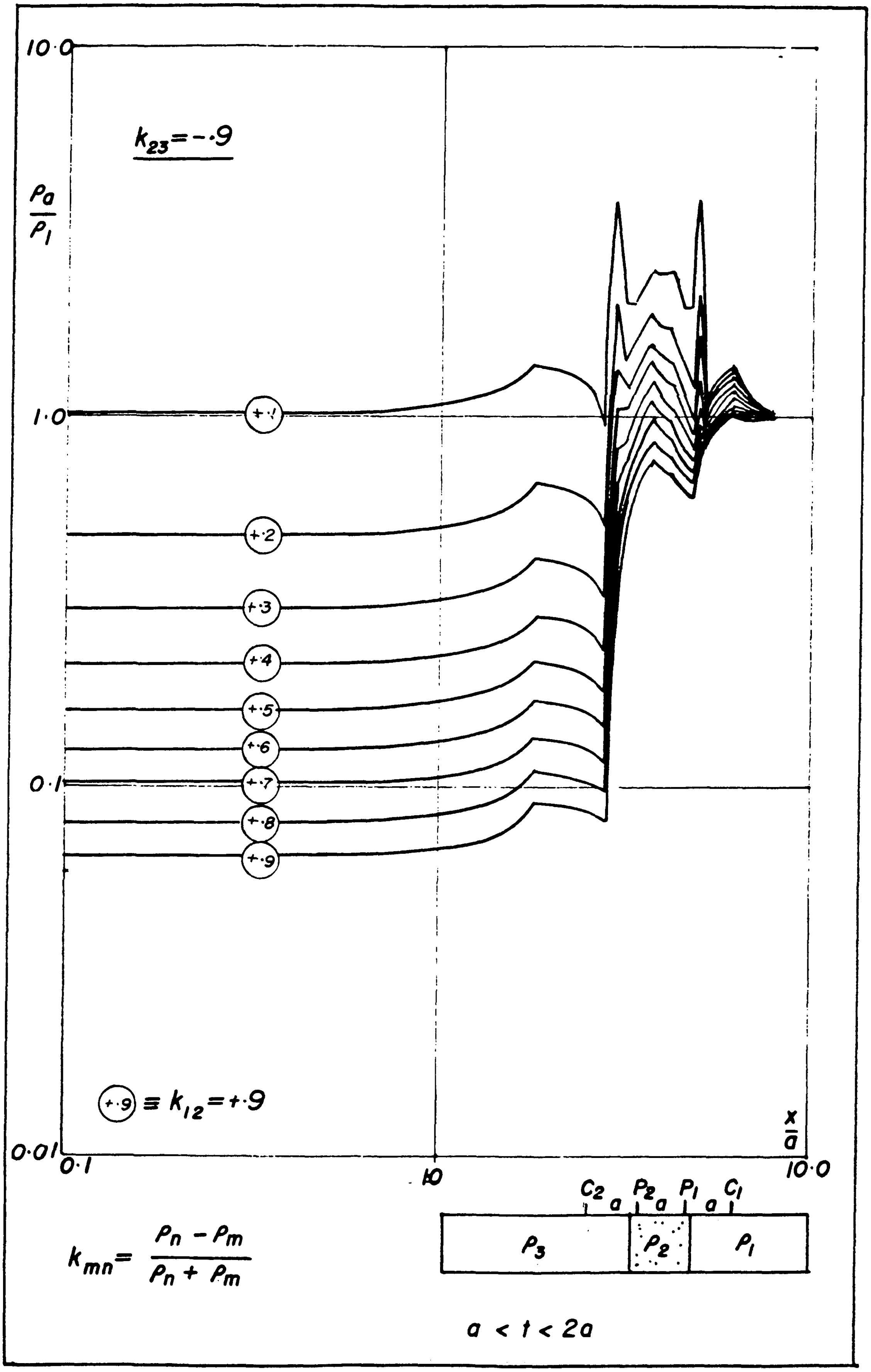
$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

$2a < t < 3a$

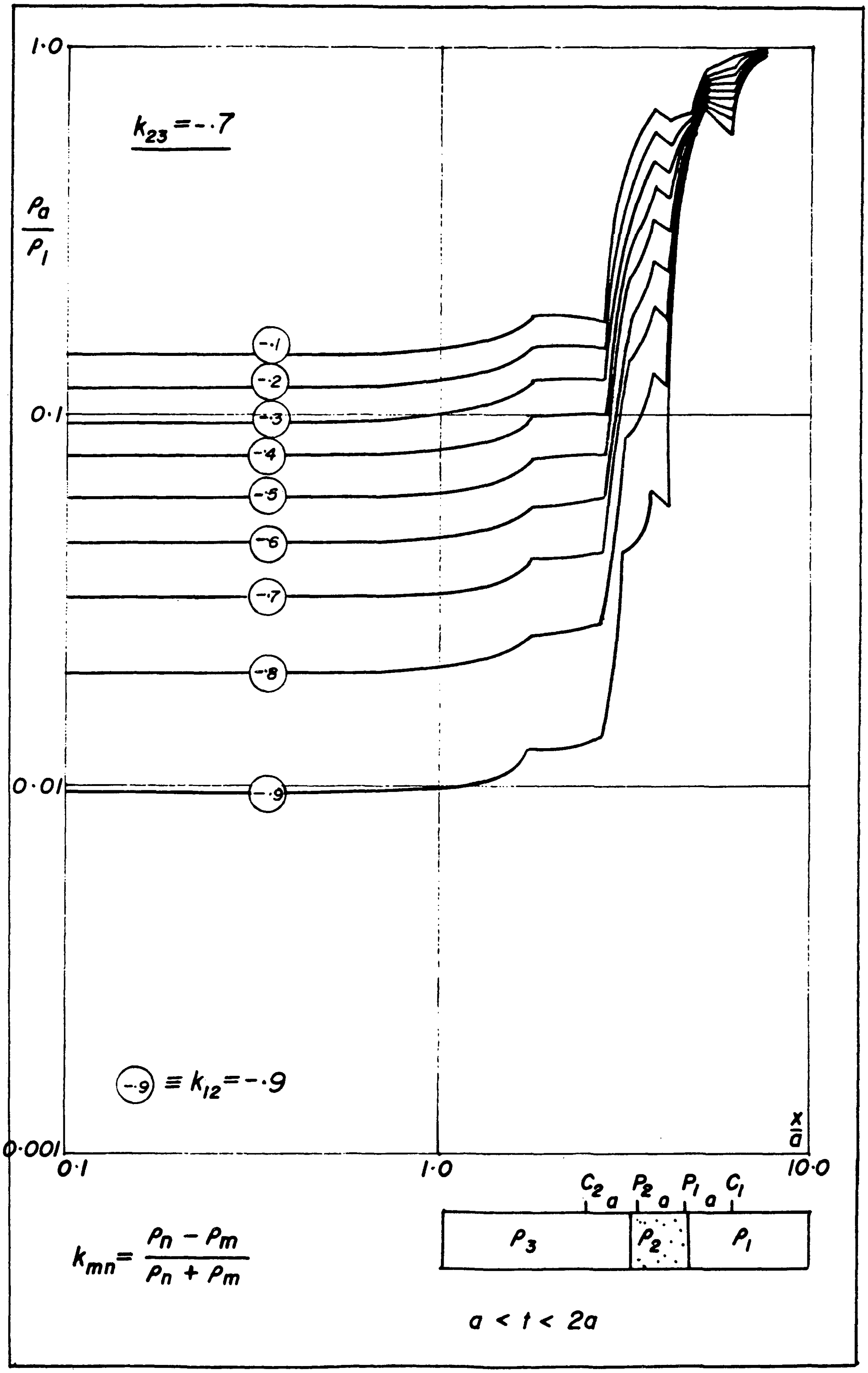
APPENDIX D

This appendix contains 180 master curves for the case where $a < t < 2a$. Although $\alpha = 90^{\circ}$ throughout, program 3 that was used to calculate these curves, is such that the user may specify any angle when calculating additional curves.

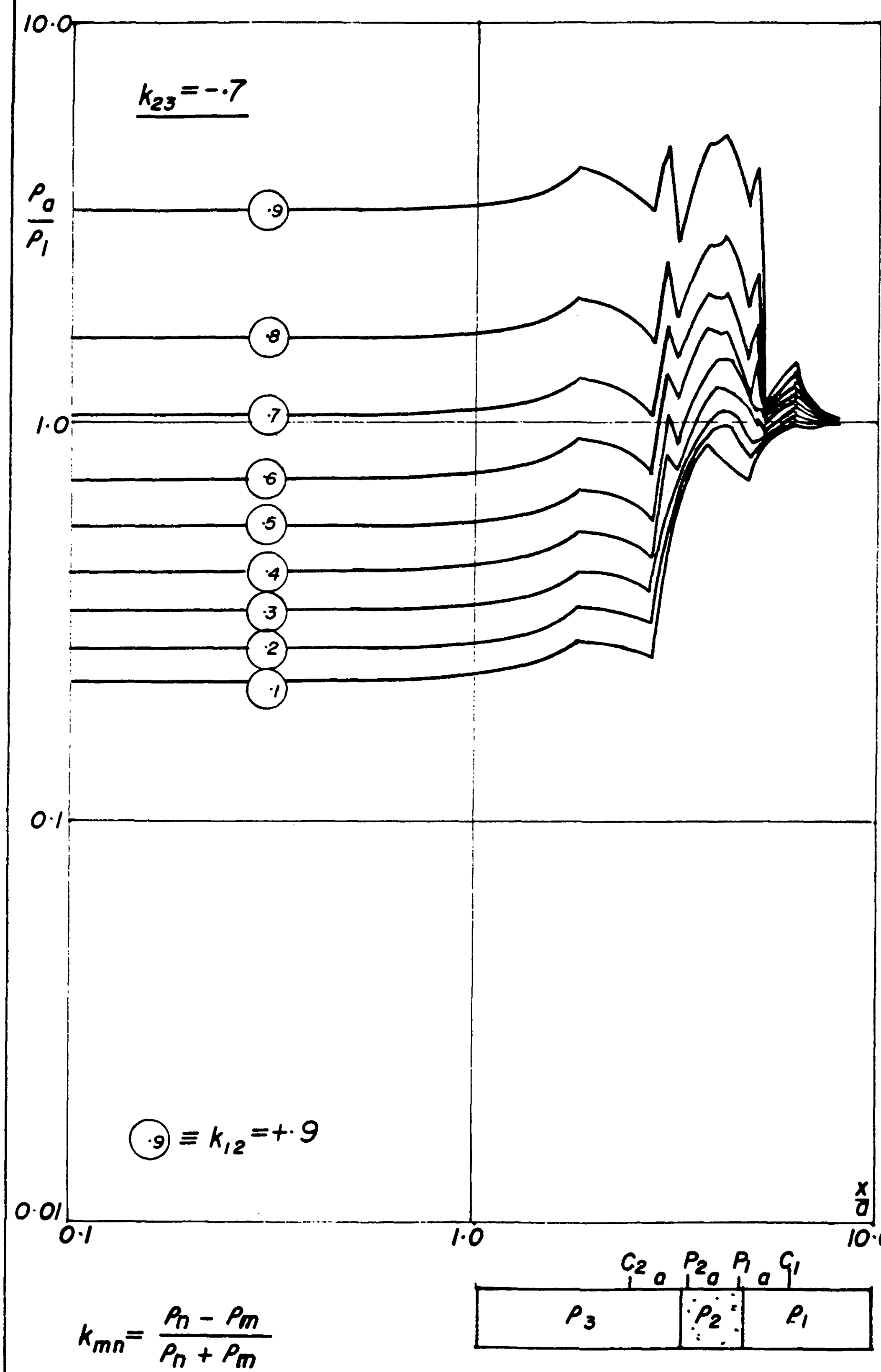

1.0
0.1
0.01
.001
$k_{23} = -.9$
ρ_a / ρ_1
-.1
-.2
-.3
-.4
-.5
-.6
-.7
-.8
-.9
$(-.9) \equiv k_{12} = -.9$
x/a
0.1
1.0
10.0
C_2
a
P_2
a
P_1
a
C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n \quad \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$



10·0
1·0
0·1
0·01
$k_{23} = -\cdot 9$
$\frac{\rho_a}{\rho_1}$
+·1
+·2
+·3
+·4
+·5
+·6
+·7
+·8
+·9
(+·9) ≡ $k_{12} = +\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C2
a
P_2
a
P_1
a
C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$

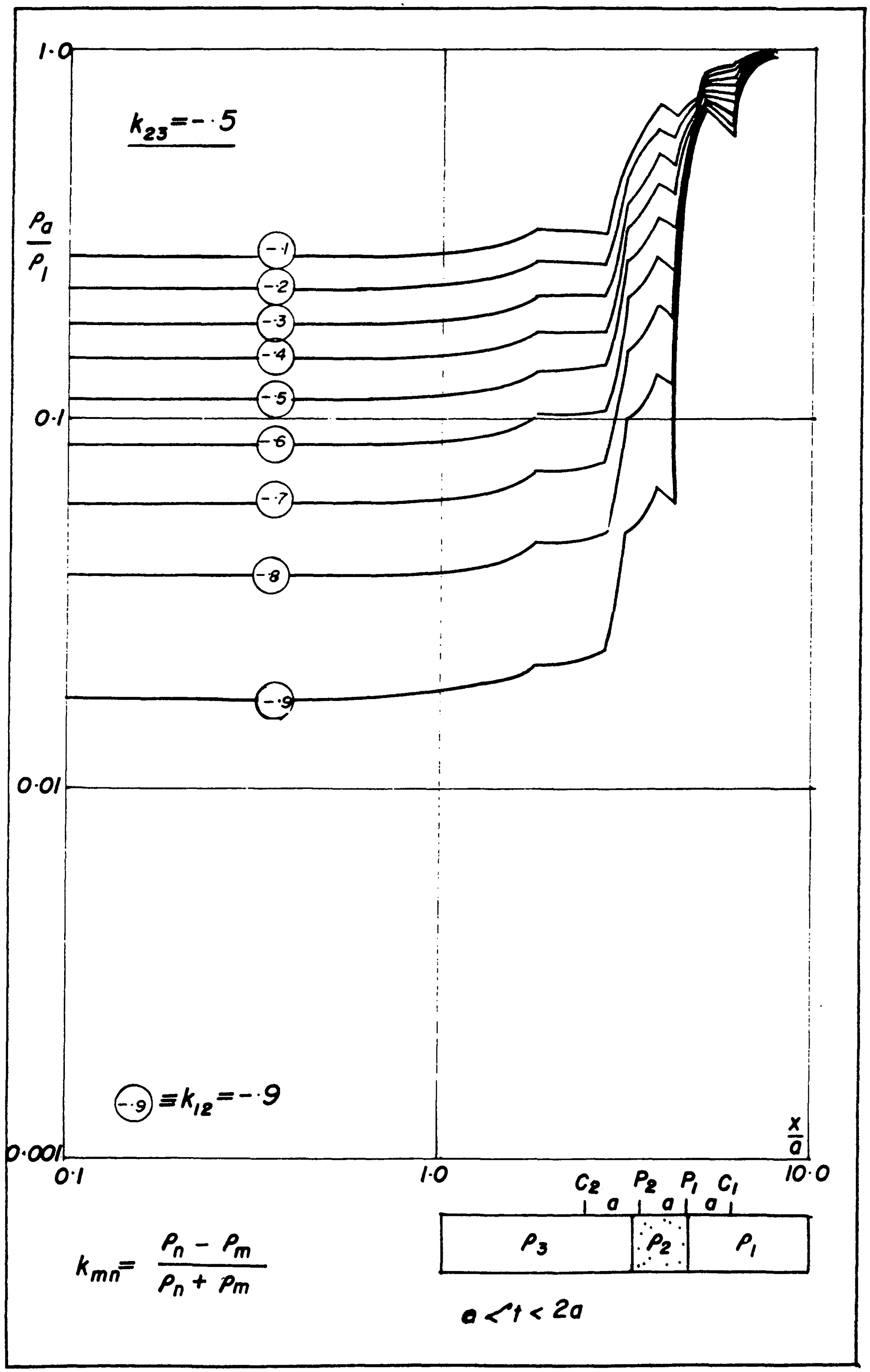


$k_{23} = -.7$
ρ_a / ρ_1
1.0
0.1
0.01
0.001
-.1
-.2
-.3
-.4
-.5
-.6
-.7
-.8
-.9
(-.9) ≡ $k_{12} = -.9$
x/a
0.1
1.0
10.0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$

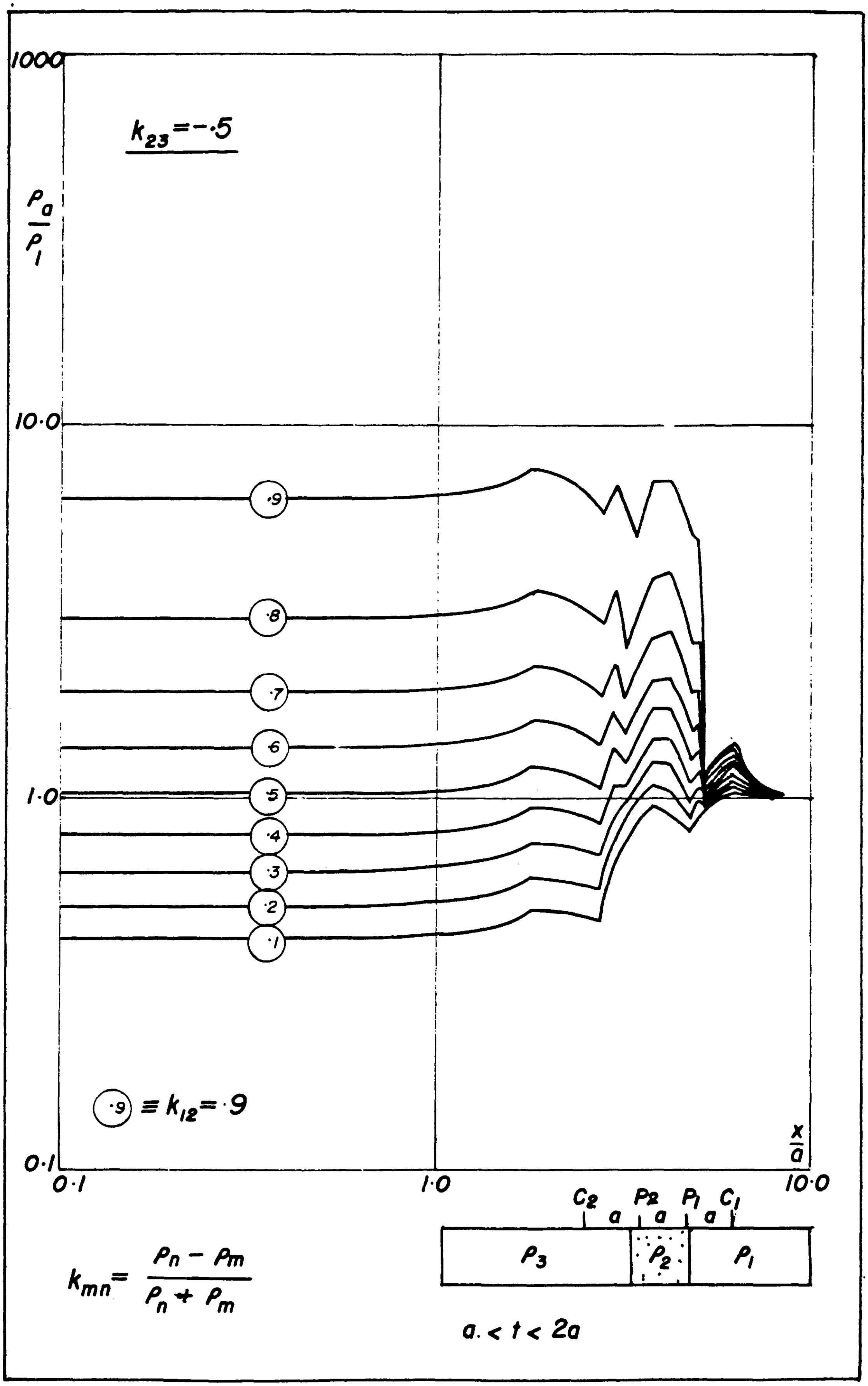


$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

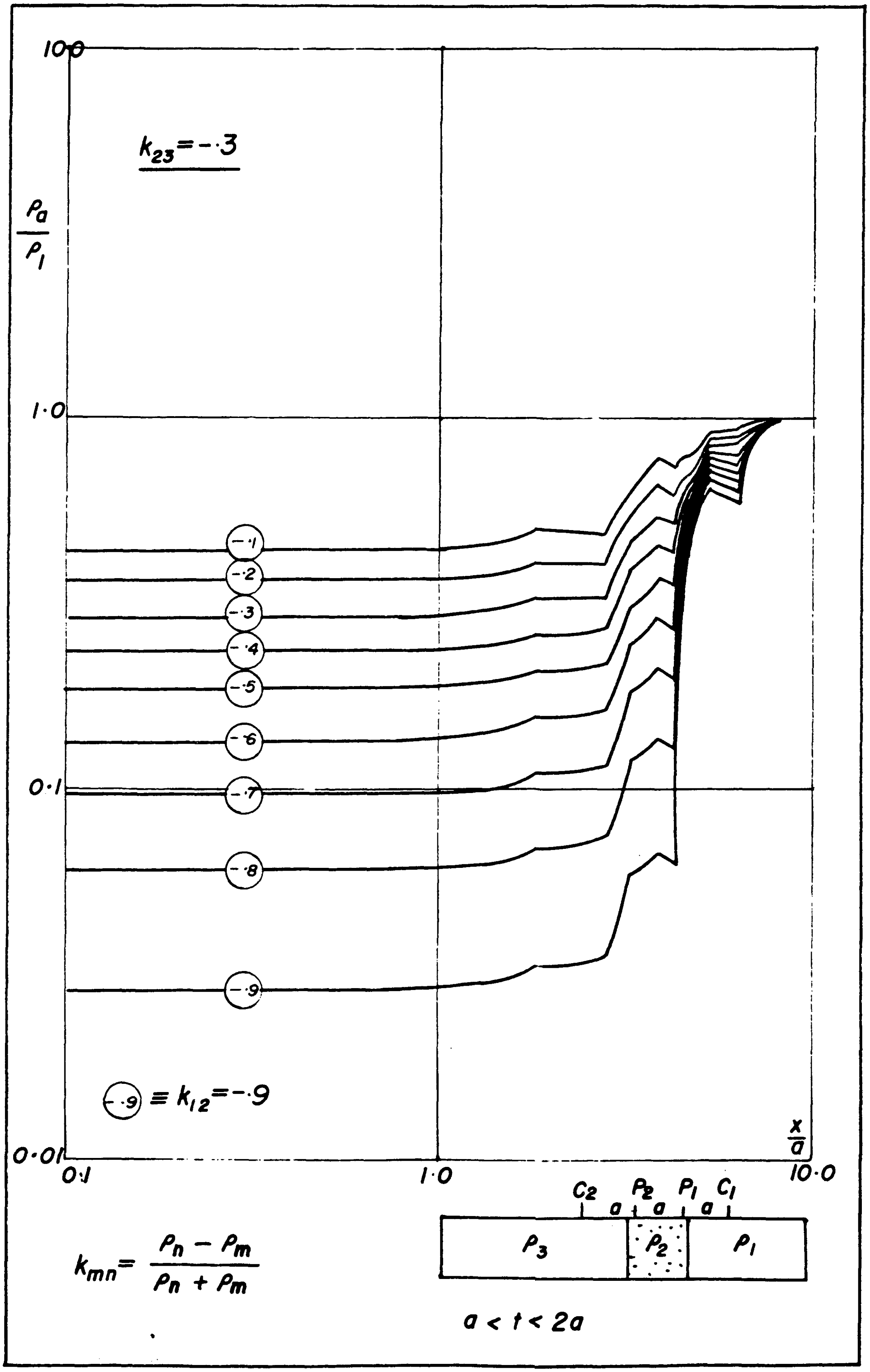
$a < t < 2a$



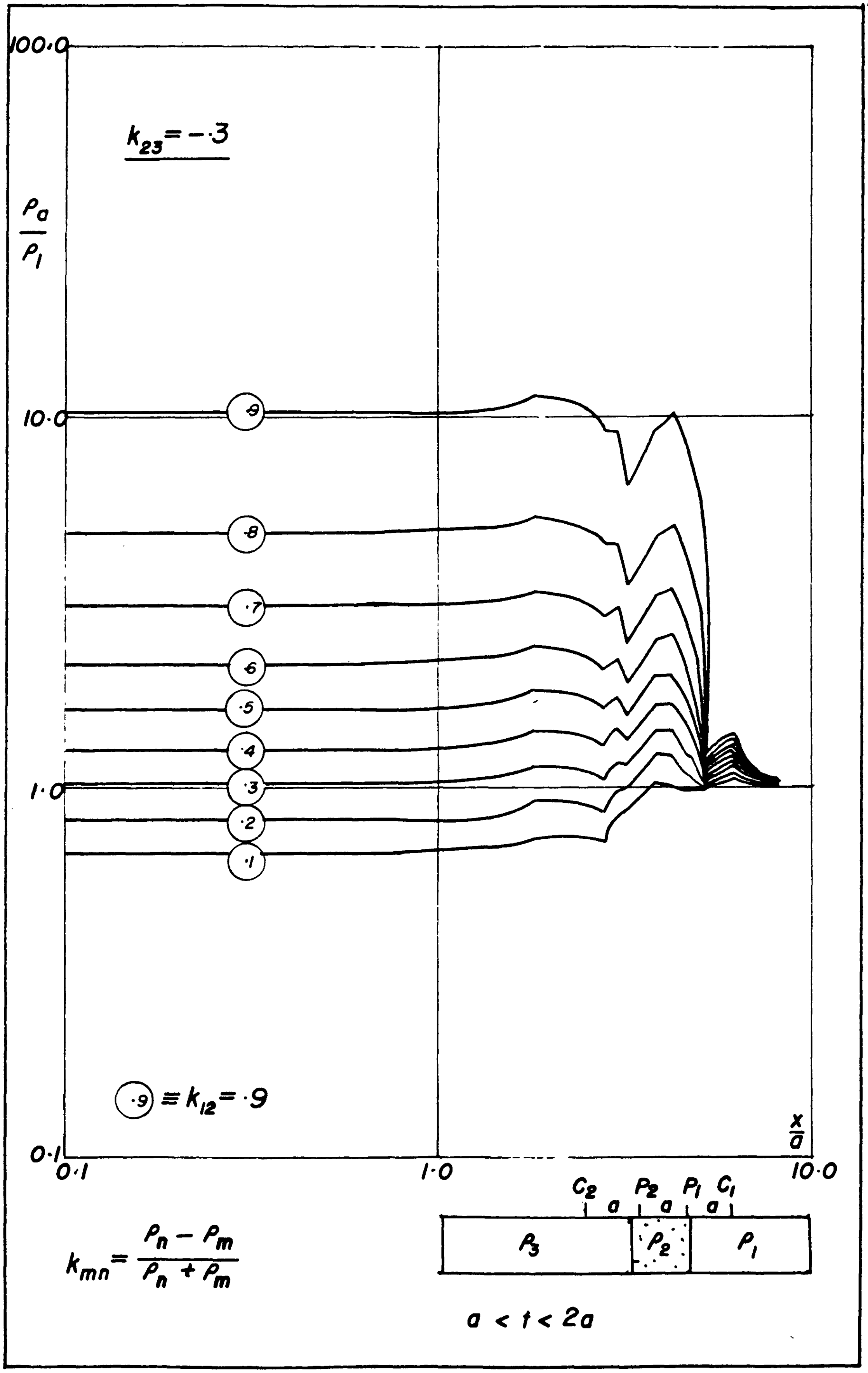
$k_{23} = -.5$
ρ_a / ρ_1
1.0
0.1
0.01
0.001
-.1
-.2
-.3
-.4
-.5
-.6
-.7
-.8
-.9
(-.9) $\equiv k_{12} = -.9$
x/a
0.1
1.0
10.0
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$



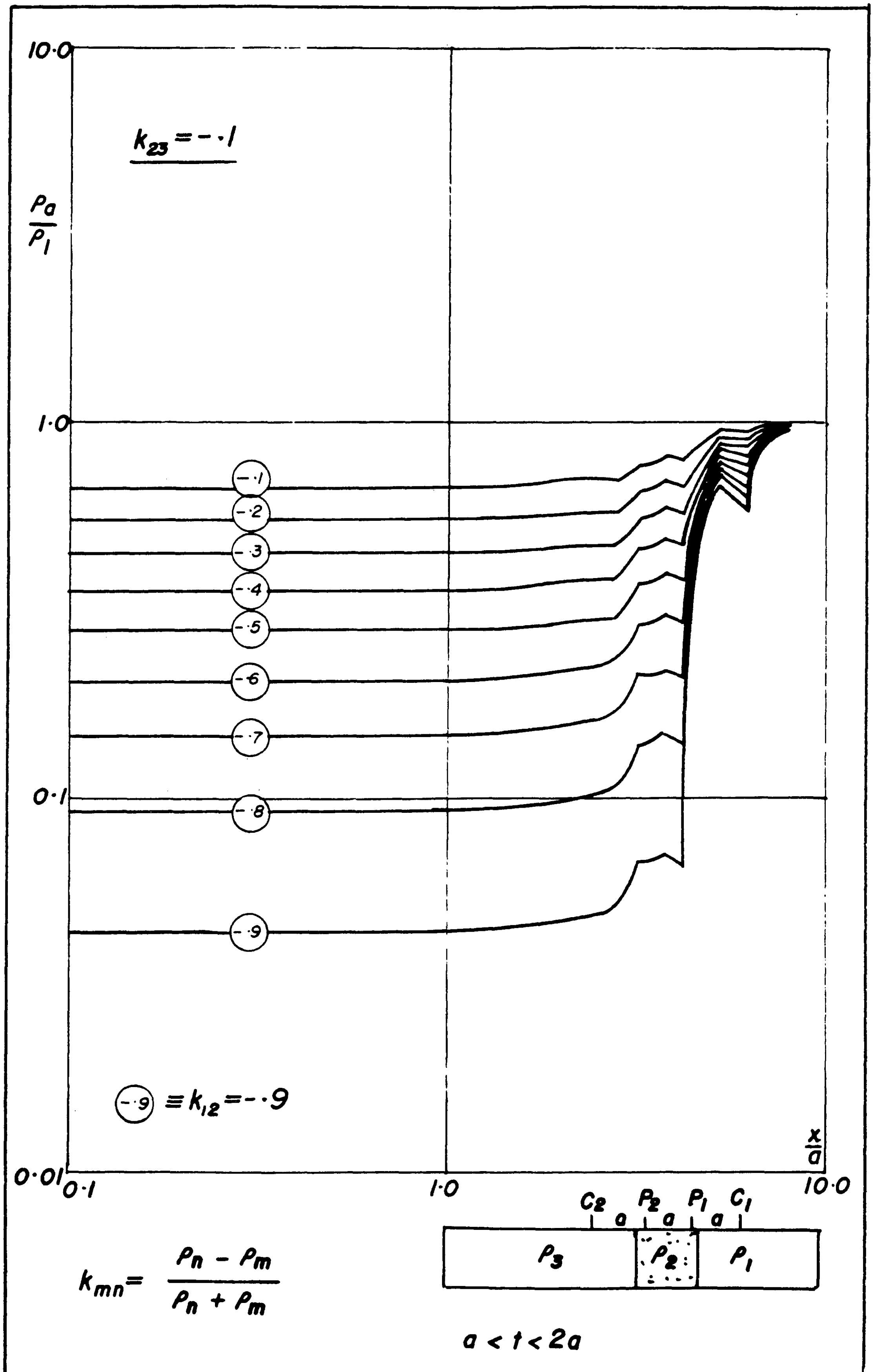
$k_{23} = -.5$
$\frac{\rho_a}{\rho_1}$
1000
10·0
1·0
0·1
0·1
1·0
10·0
$\frac{x}{a}$
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ $k_{12} = \cdot 9$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
a. < t < 2a



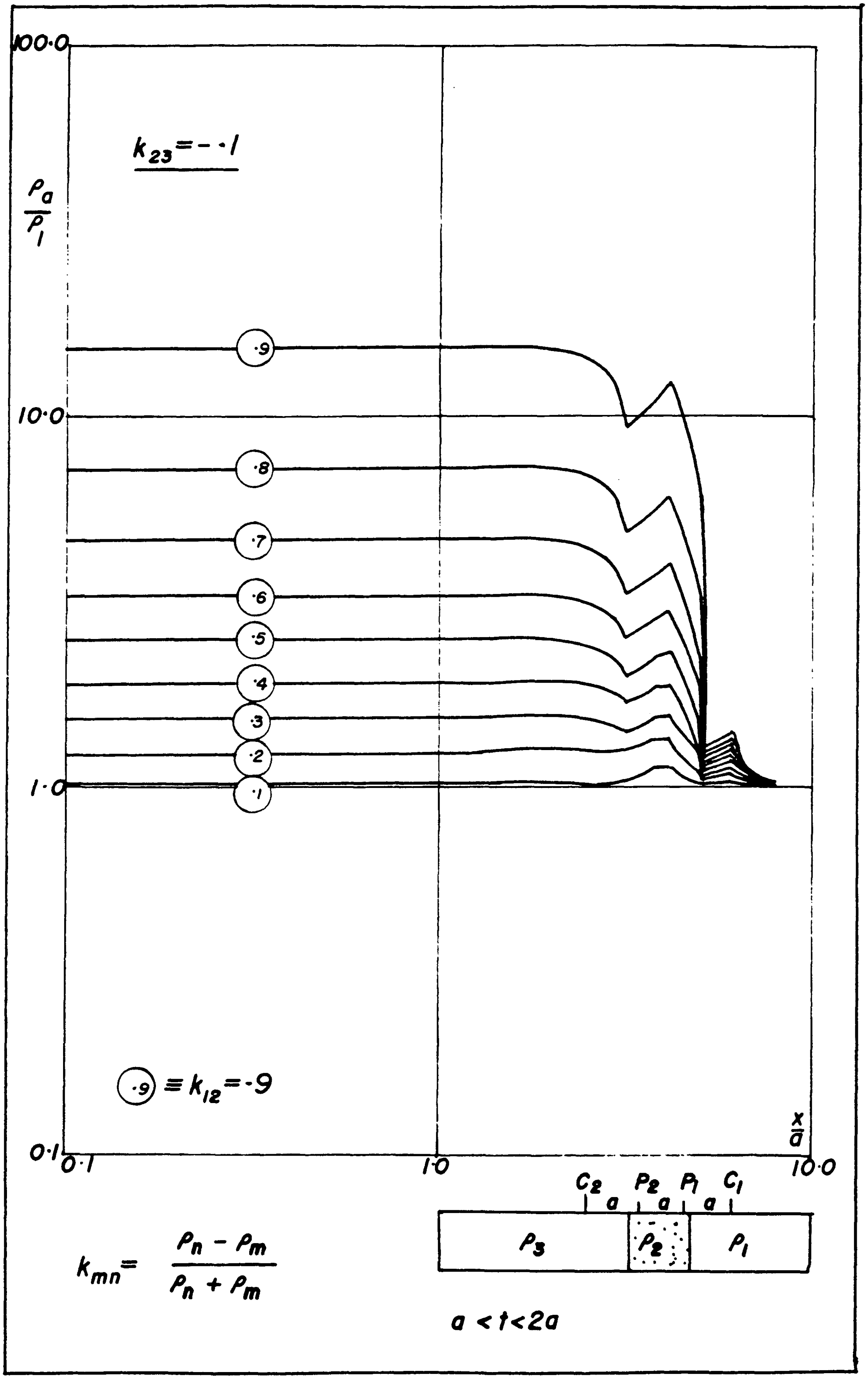
$k_{23} = -\cdot3$
ρ_a / ρ_1
10·0
1·0
0·1
0·01
0·1
1·0
10·0
x/a
−·1
−·2
−·3
−·4
−·5
−·6
−·7
−·8
−·9
(−·9) ≡ $k_{12} = -\cdot9$
C2
P2
P1
C1
a
a
a
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
a < t < 2a



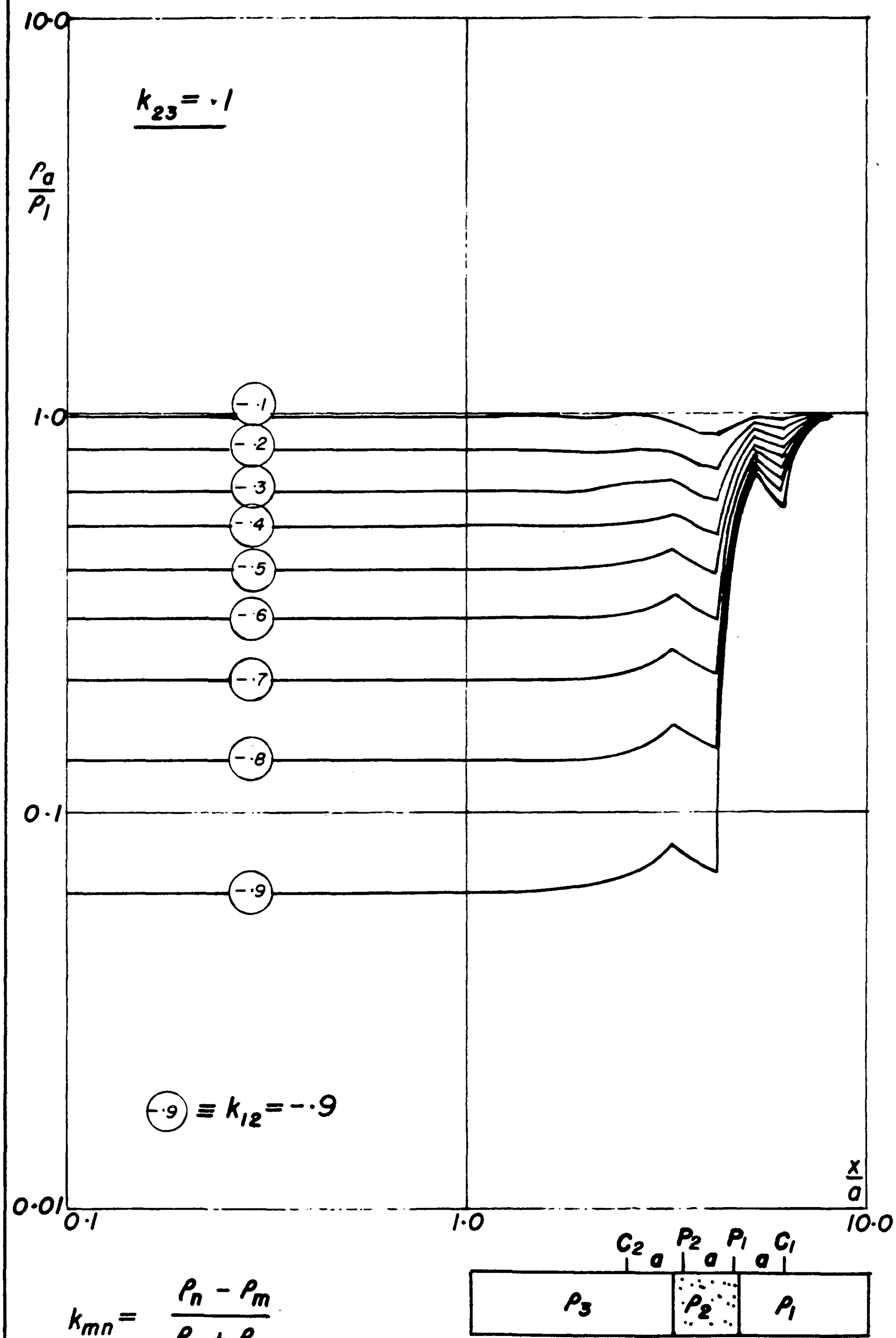
$k_{23} = -\cdot 3$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
$\frac{x}{a}$
0·1
1·0
10·0
(·9) ≡ $k_{12} = \cdot 9$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$a < t < 2a$



10·0
$k_{23} = -\cdot 1$
$\frac{\rho_a}{\rho_1}$
1·0
-·1
-·2
-·3
-·4
-·5
-·6
-·7
0·1
-·8
-·9
-·9 ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·01
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$

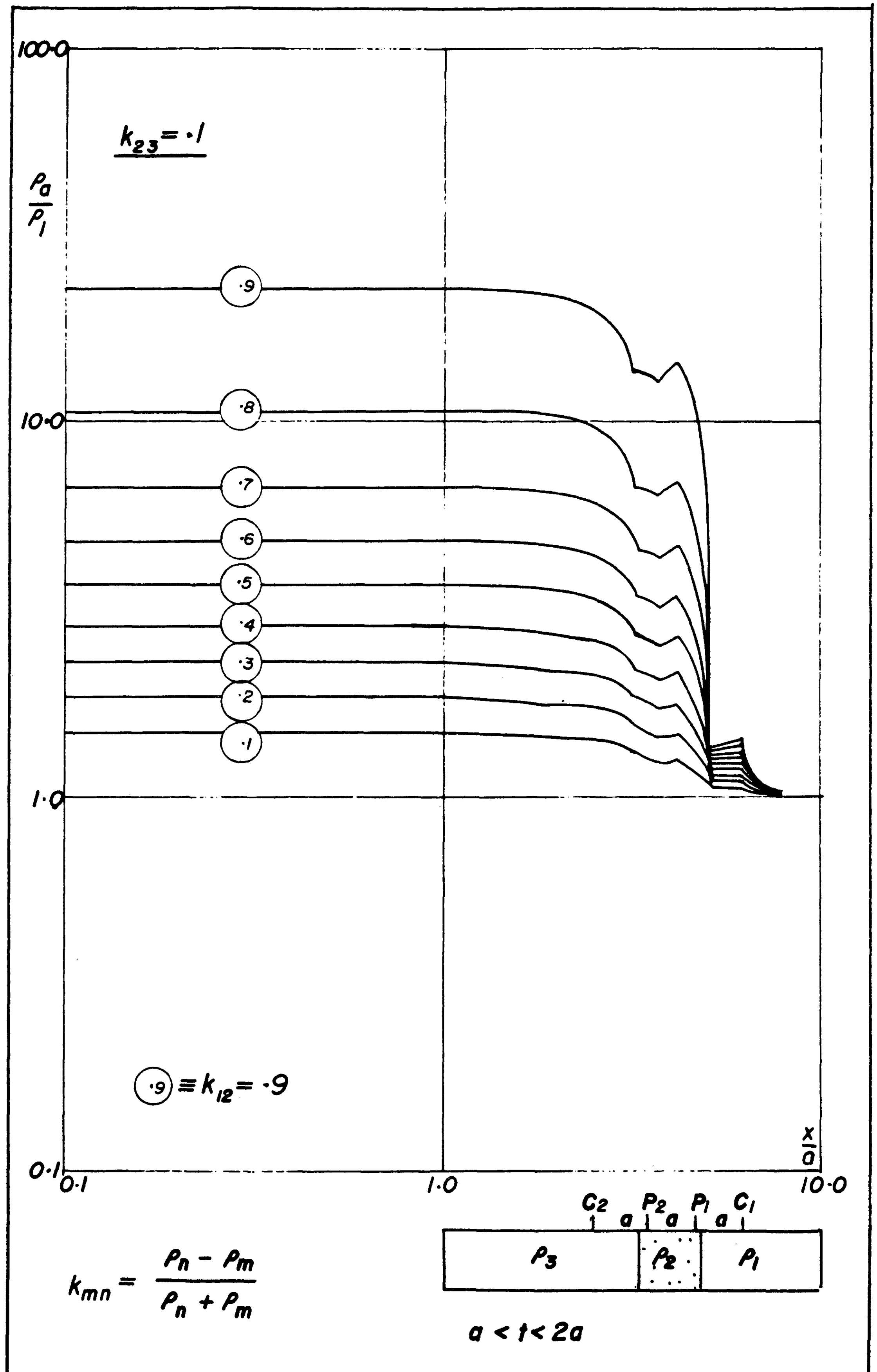


$k_{23} = -.1$
ρ_a / ρ_1
100·0
10·0
1·0
0·1
0·1
1·0
10·0
x/a
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) $\equiv k_{12} = ·9$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
$k_{mn} = \dfrac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$

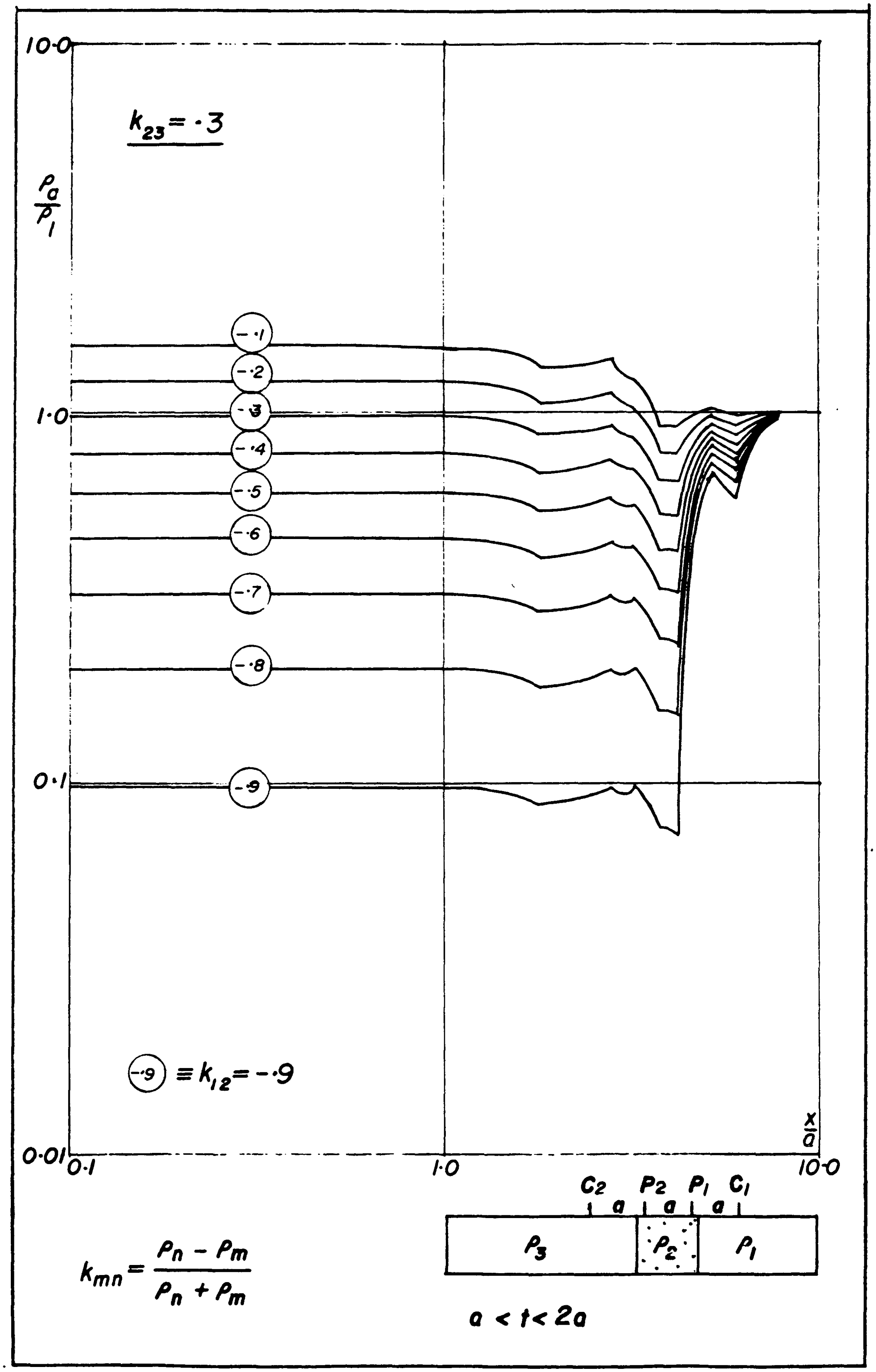


$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

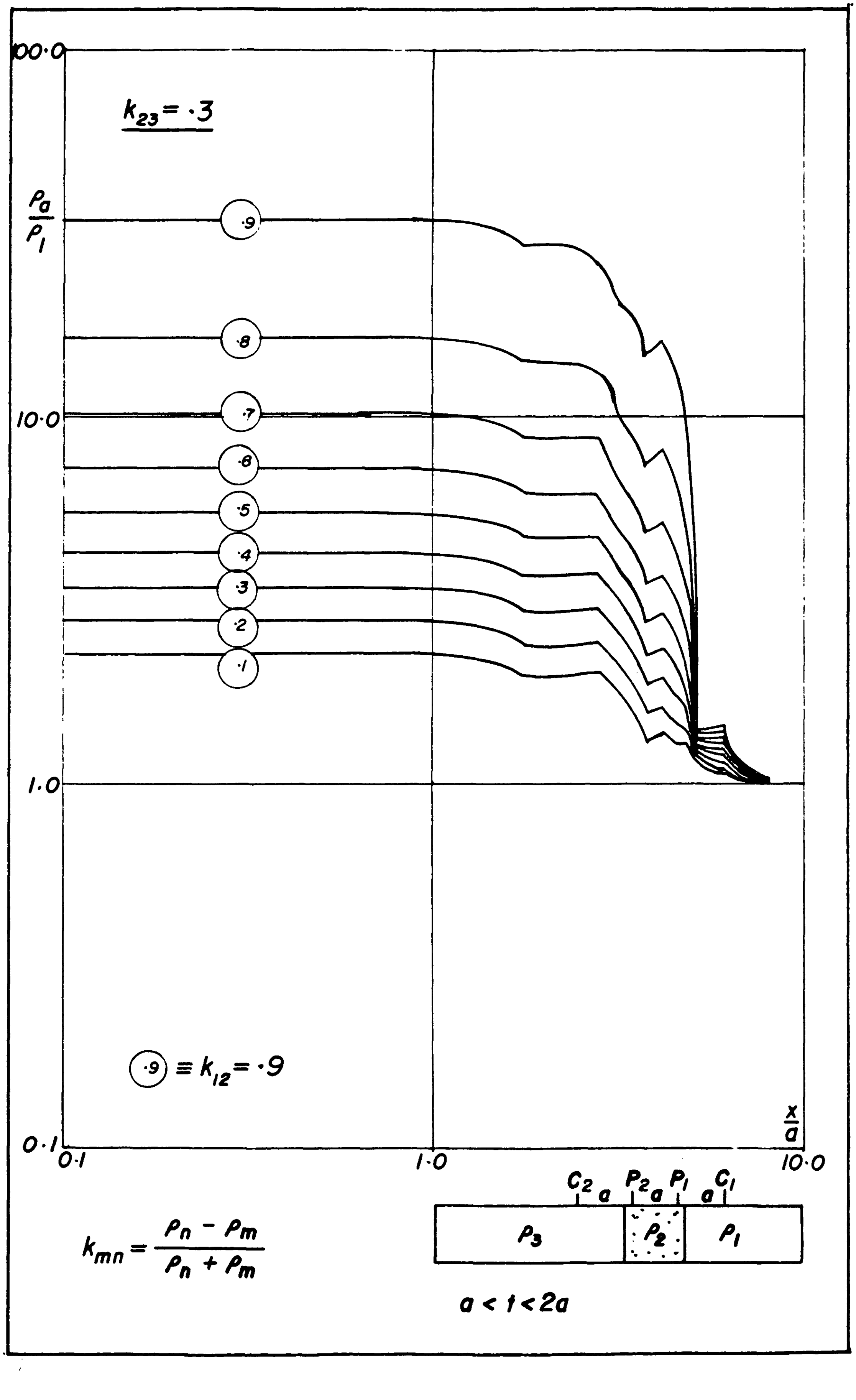
$a < t < 2a$



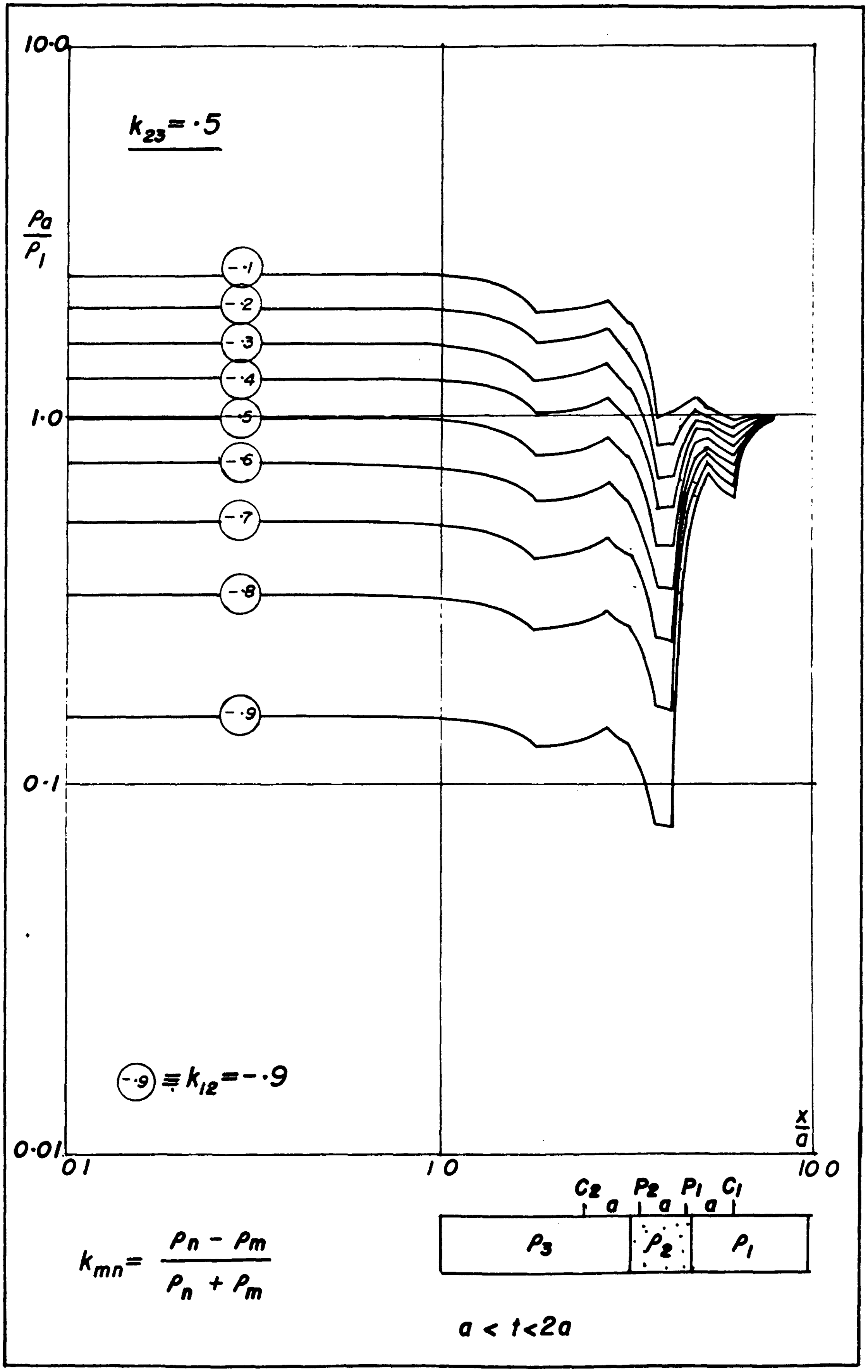
$k_{23} = \cdot 1$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ $k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C2 a P2 a P1 a C1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$

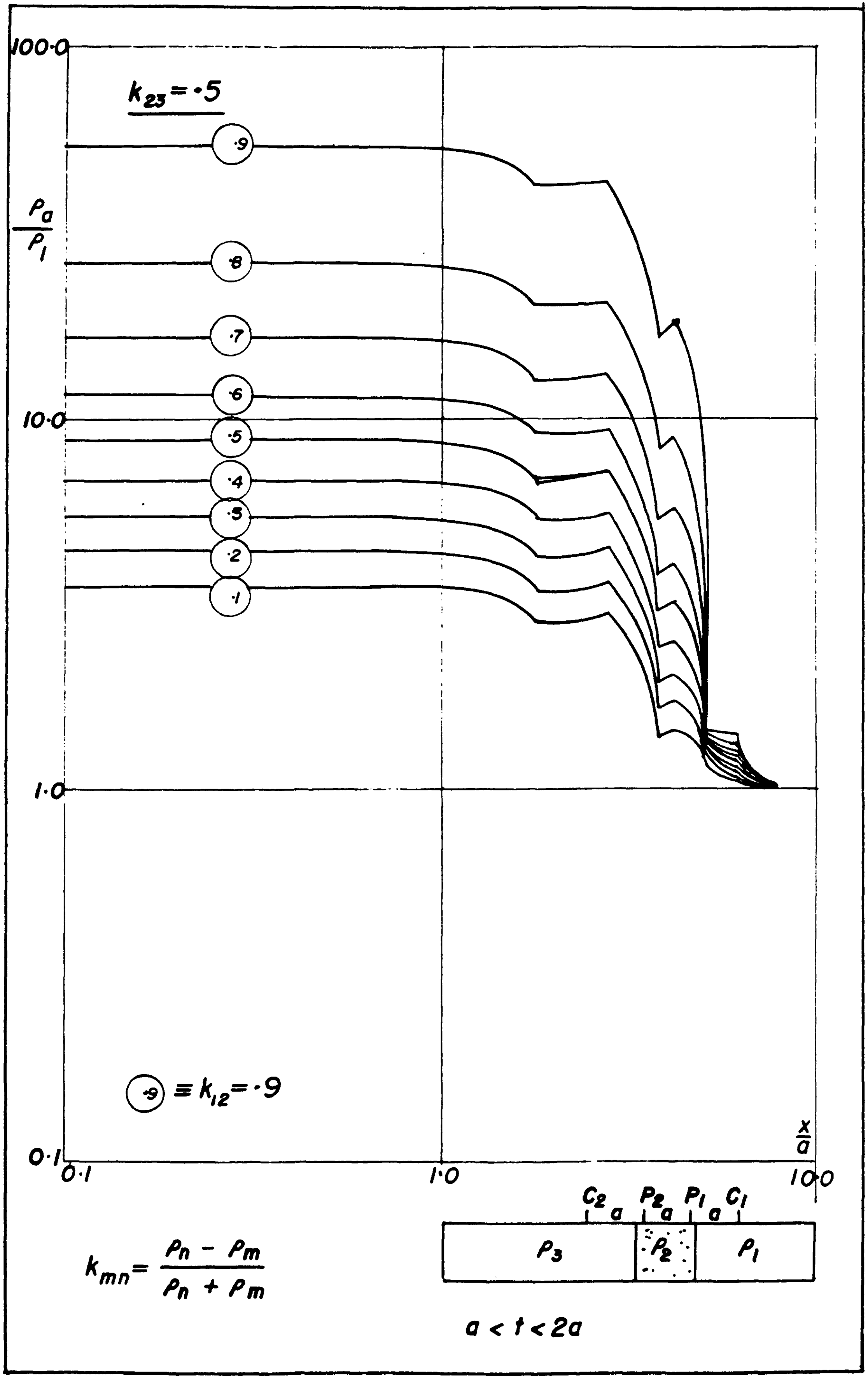


$k_{23} = \cdot 3$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
−·1
−·2
−·3
−·4
−·5
−·6
−·7
−·8
−·9
$(-\cdot 9) \equiv k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C2
P2
P1
C1
a
a
a
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$

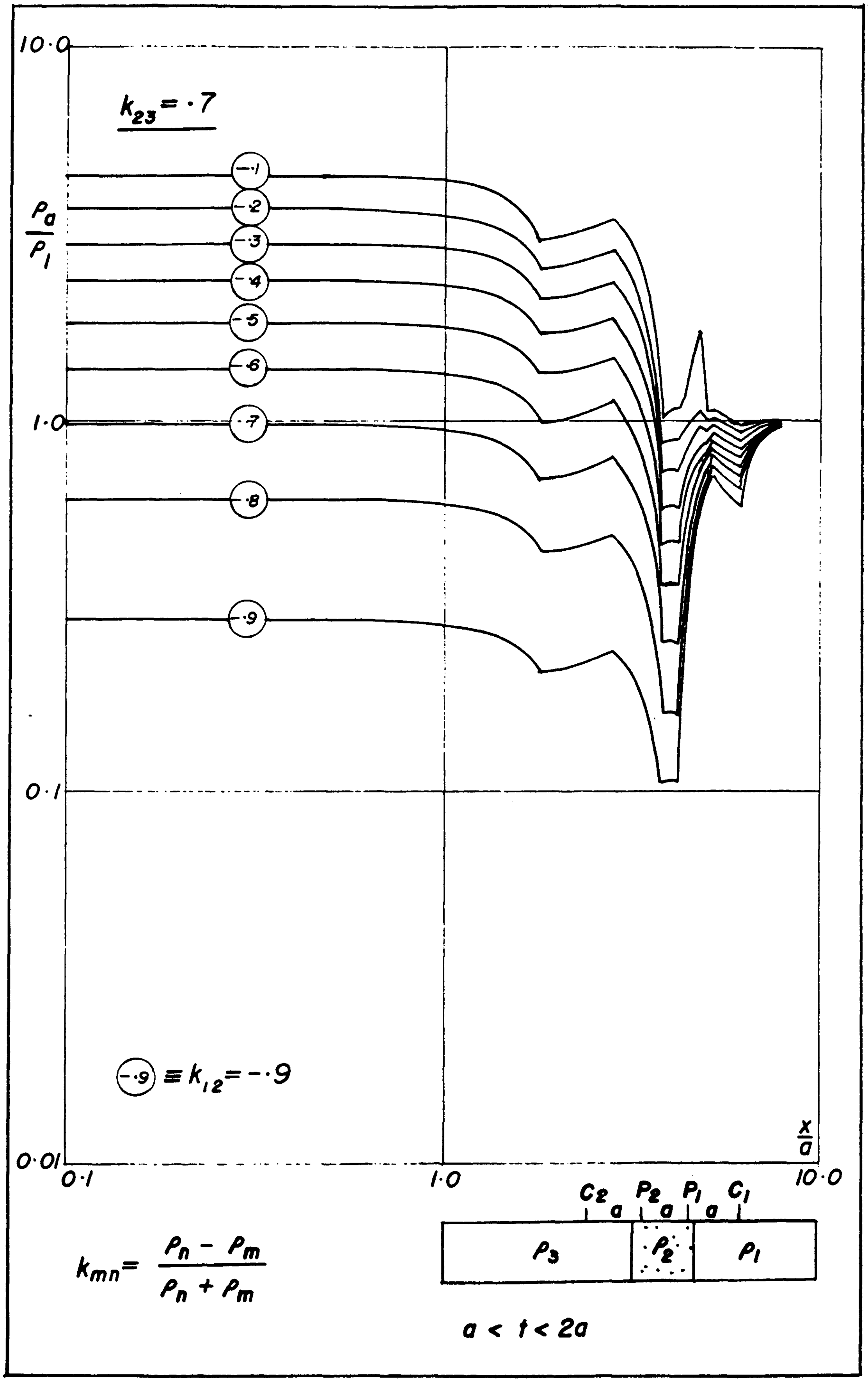


$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

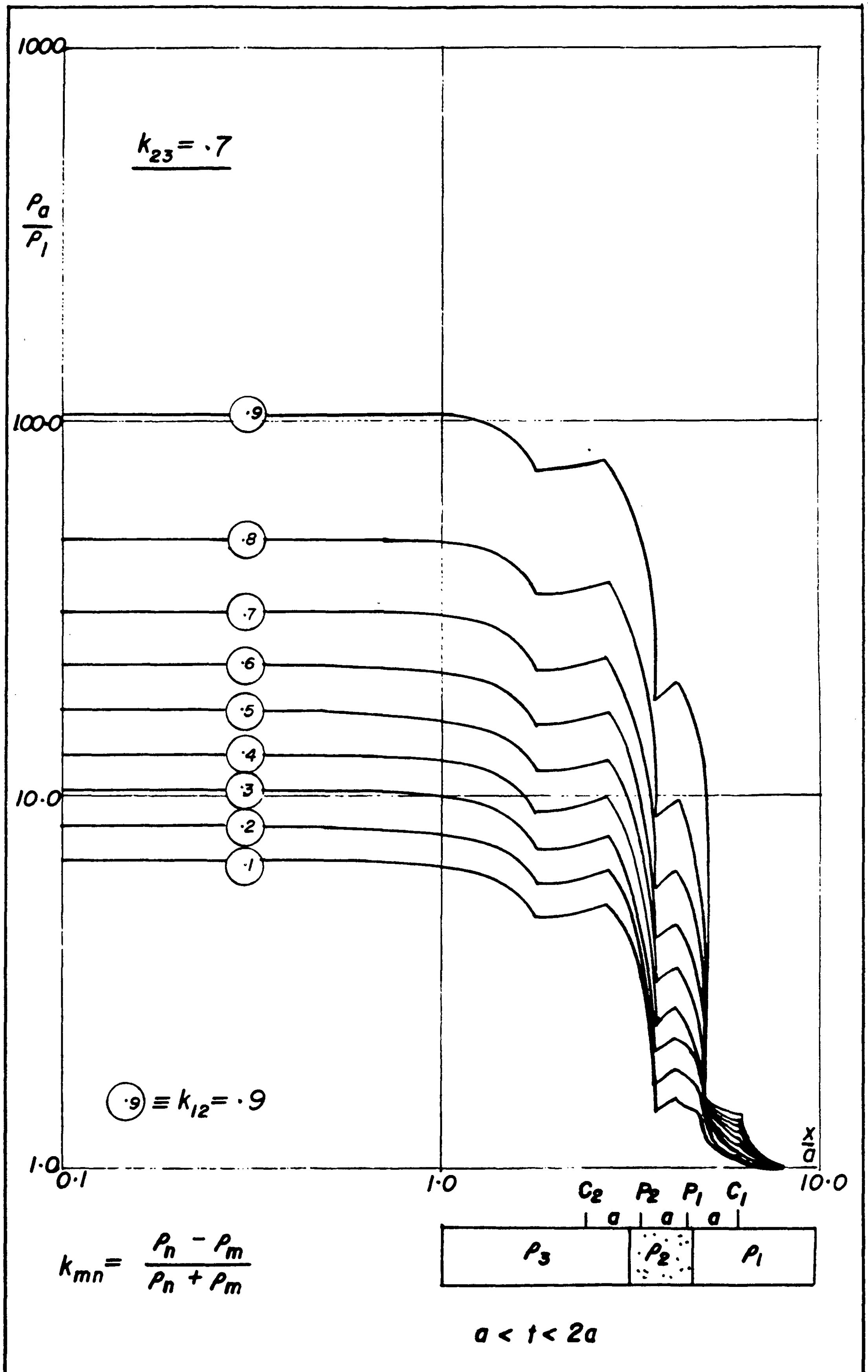

10·0
$k_{23} = \cdot 5$
$\frac{\rho_a}{\rho_1}$
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
1·0
0·1
-·9 ≡ $k_{12} = -\cdot 9$
0·01
$\frac{x}{a}$
0 1
1 0
10 0
C2 a P2 a P1 a C1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$



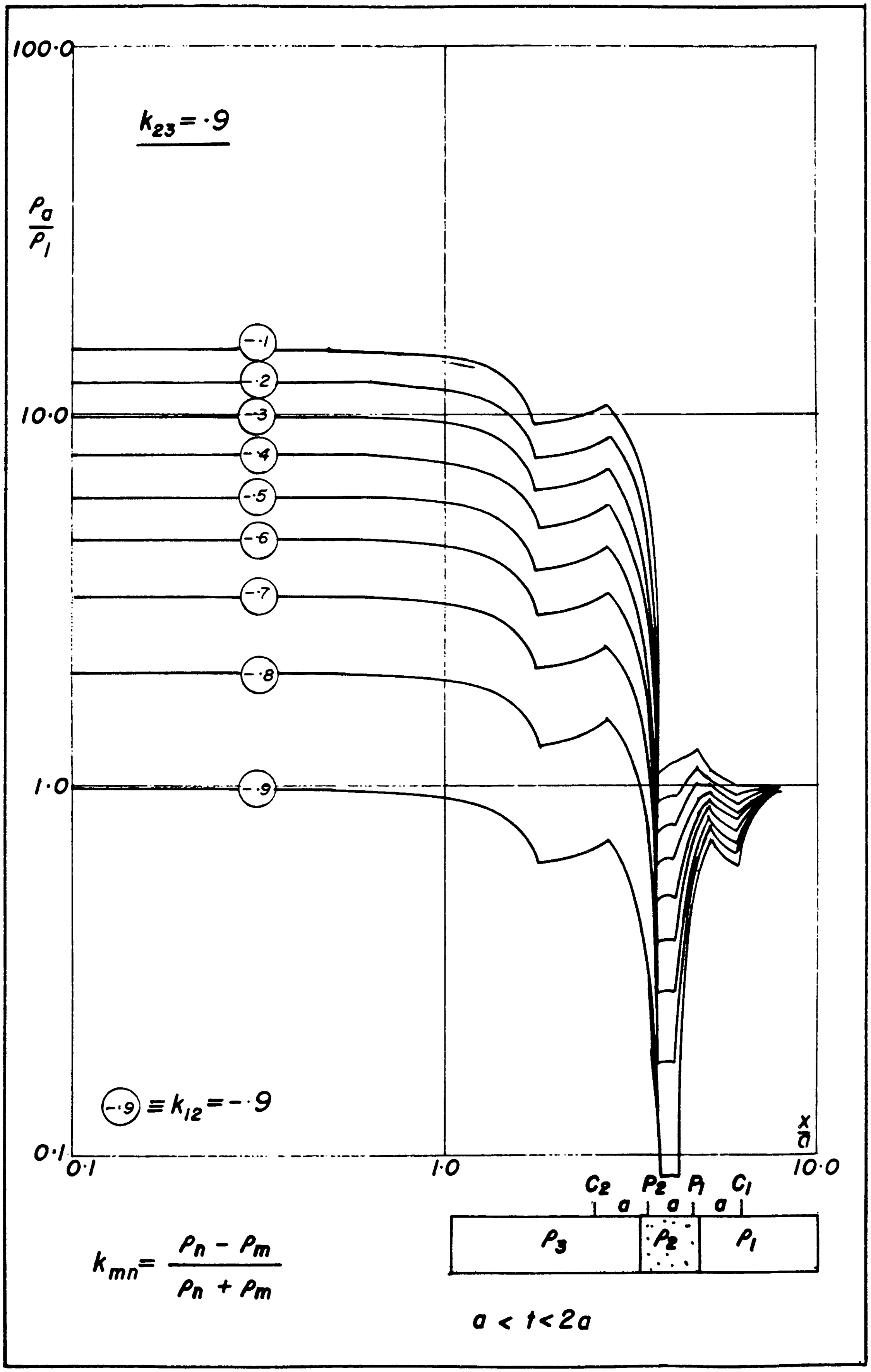
$k_{23} = \cdot 5$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
·9 $\equiv k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$



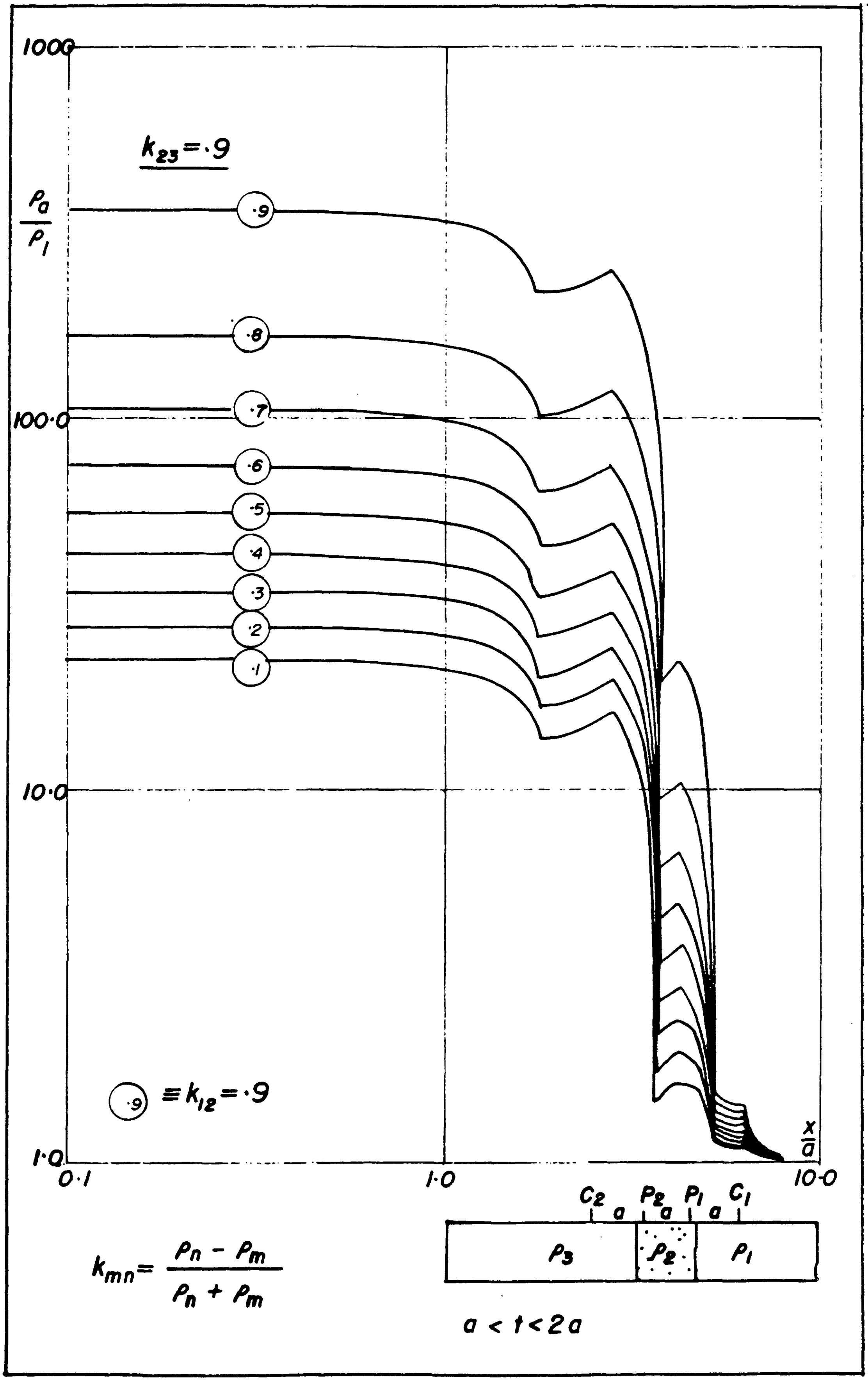
$k_{23} = \cdot 7$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
−·1
−·2
−·3
−·4
−·5
−·6
·7
·8
·9
−·9 ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$



$k_{23} = \cdot 7$
ρ_a / ρ_1
1000
100·0
10·0
1·0
0·1
1·0
10·0
x/a
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ $k_{12} = \cdot 9$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
$a < t < 2a$

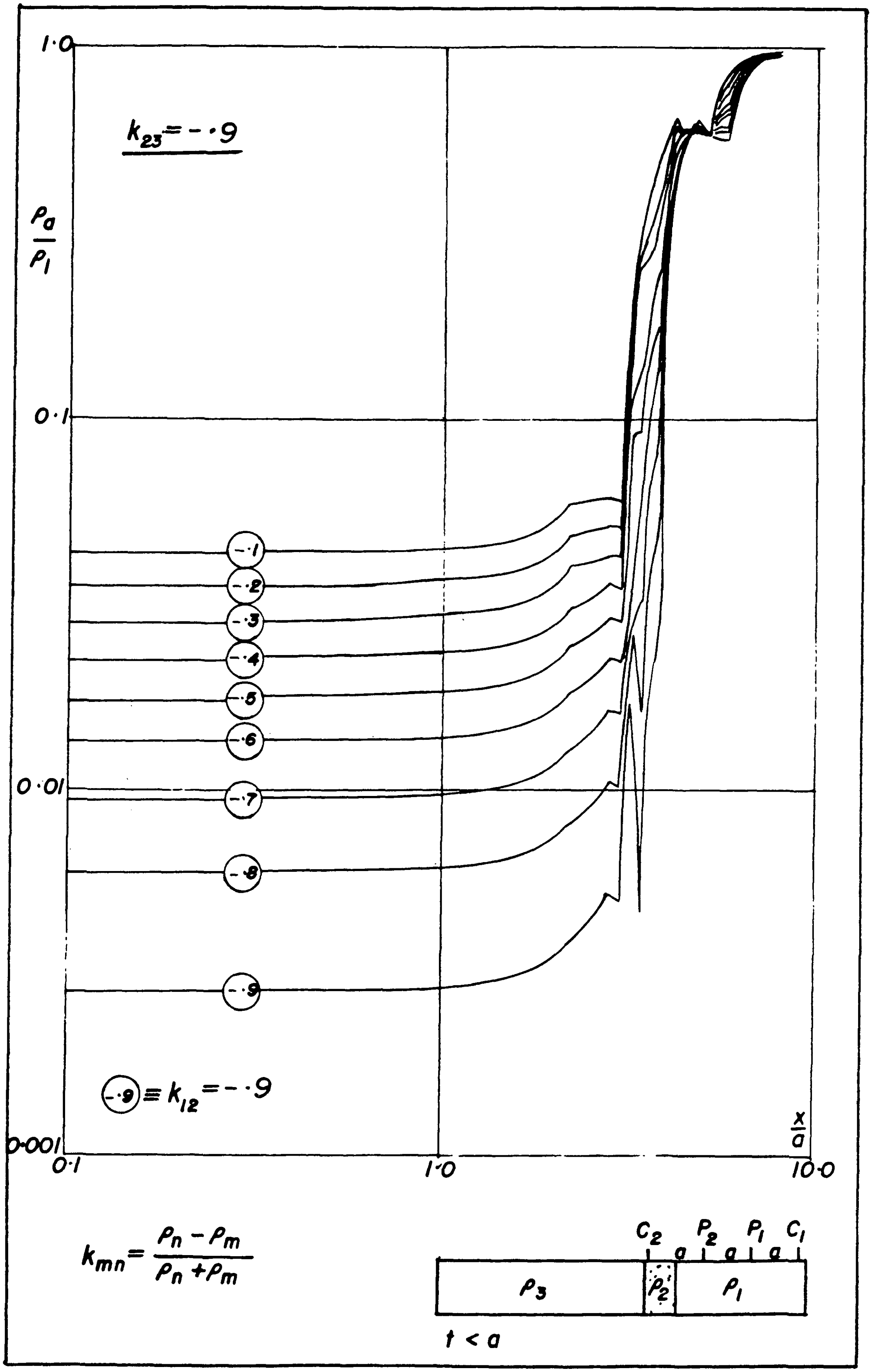


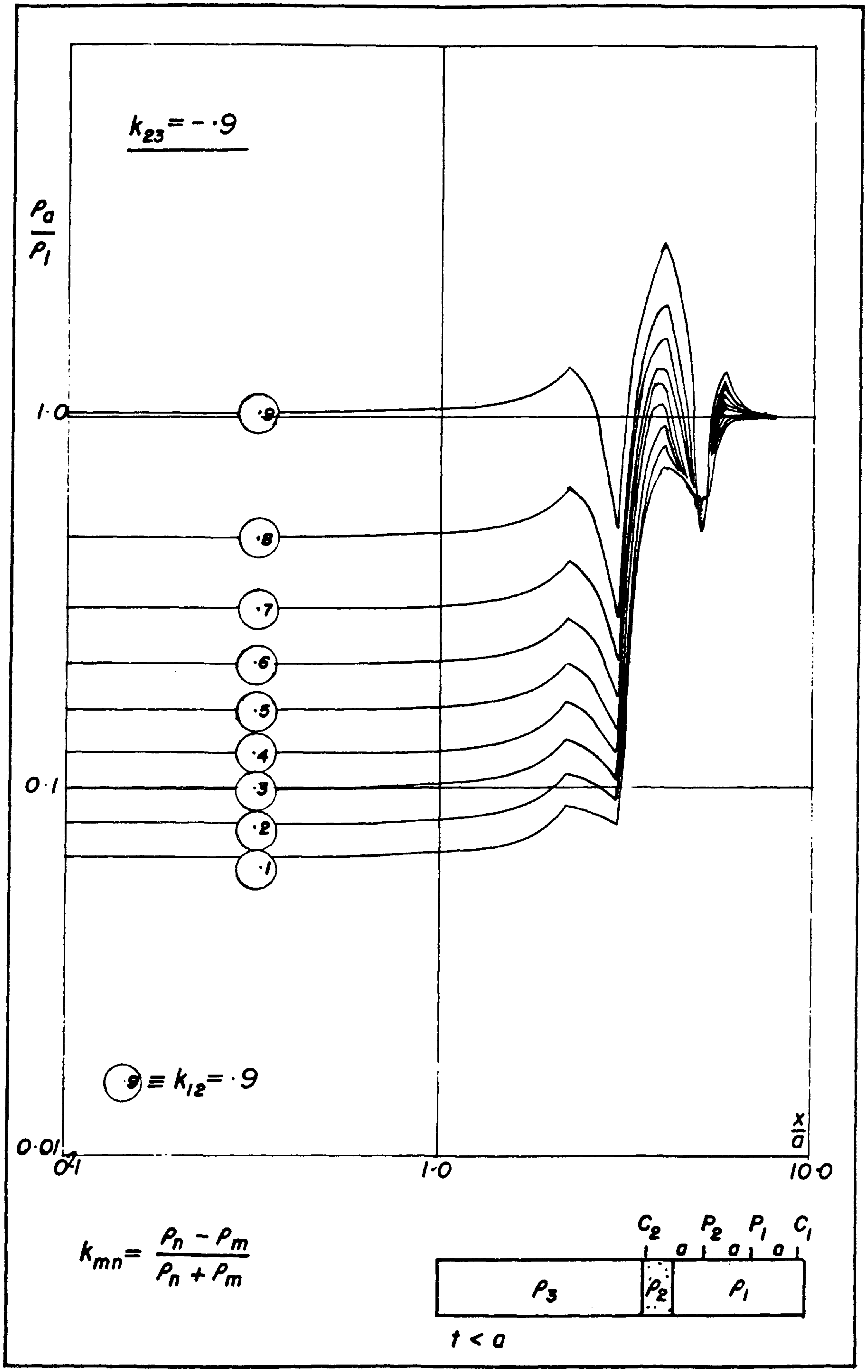
$k_{23} = \cdot 9$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
0·1
1·0
10·0
$\frac{x}{a}$
−·1
·2
·3
·4
·5
·6
·7
·8
·9
$(-\cdot 9) \equiv k_{12} = -\cdot 9$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$


1000
100·0
10·0
1·0
0·1
1·0
10·0
$k_{23} = \cdot 9$
$\frac{\rho_a}{\rho_1}$
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) $\equiv k_{12} = \cdot 9$
$\frac{x}{a}$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$a < t < 2a$

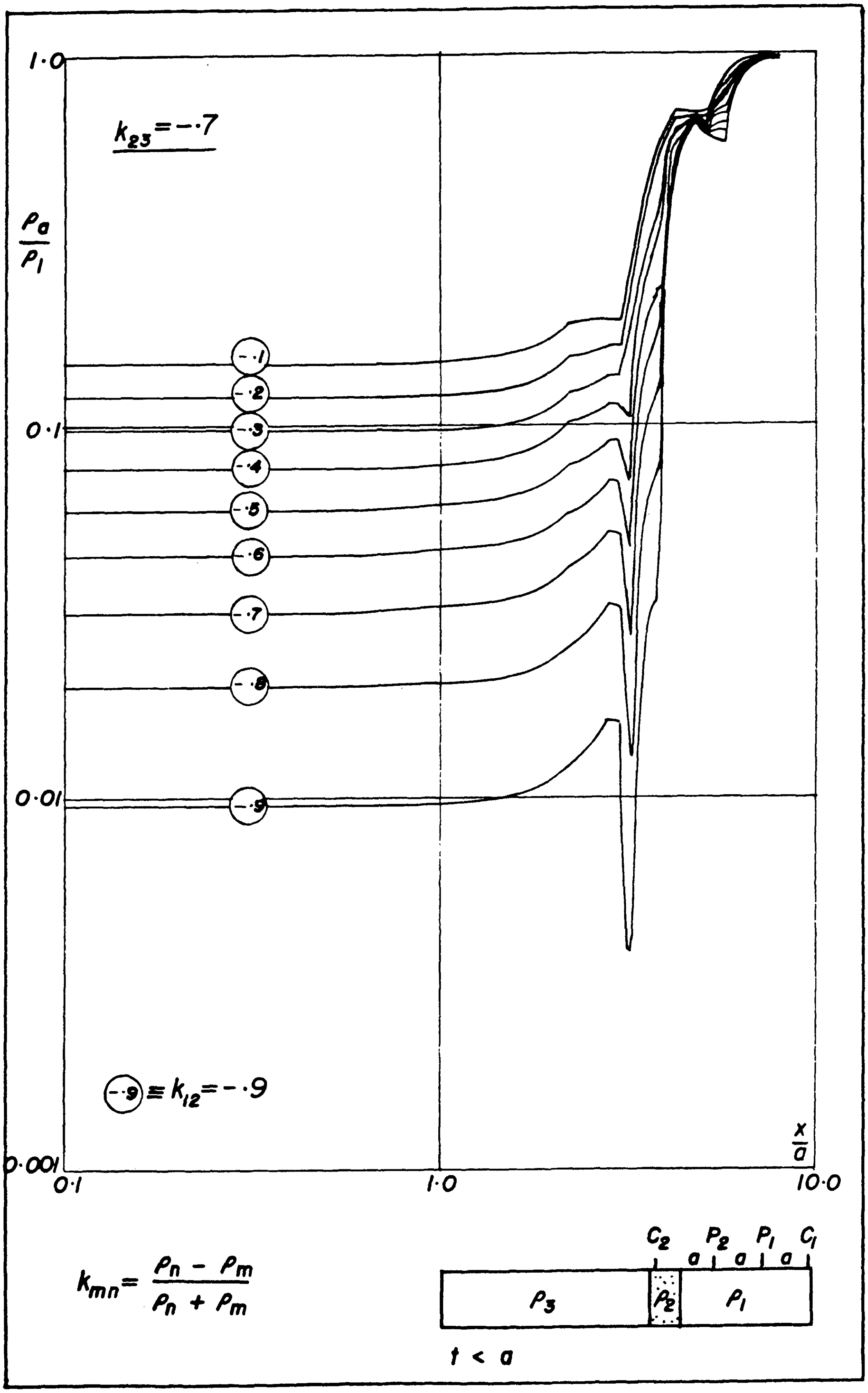
APPENDIX E

This appendix contains 180 master curves for the case where $t < a$. Although $\alpha = 90^{\circ}$ throughout, program 4 that was used to calculate these curves, is such that the user may specify any angle when calculating additional master curves.

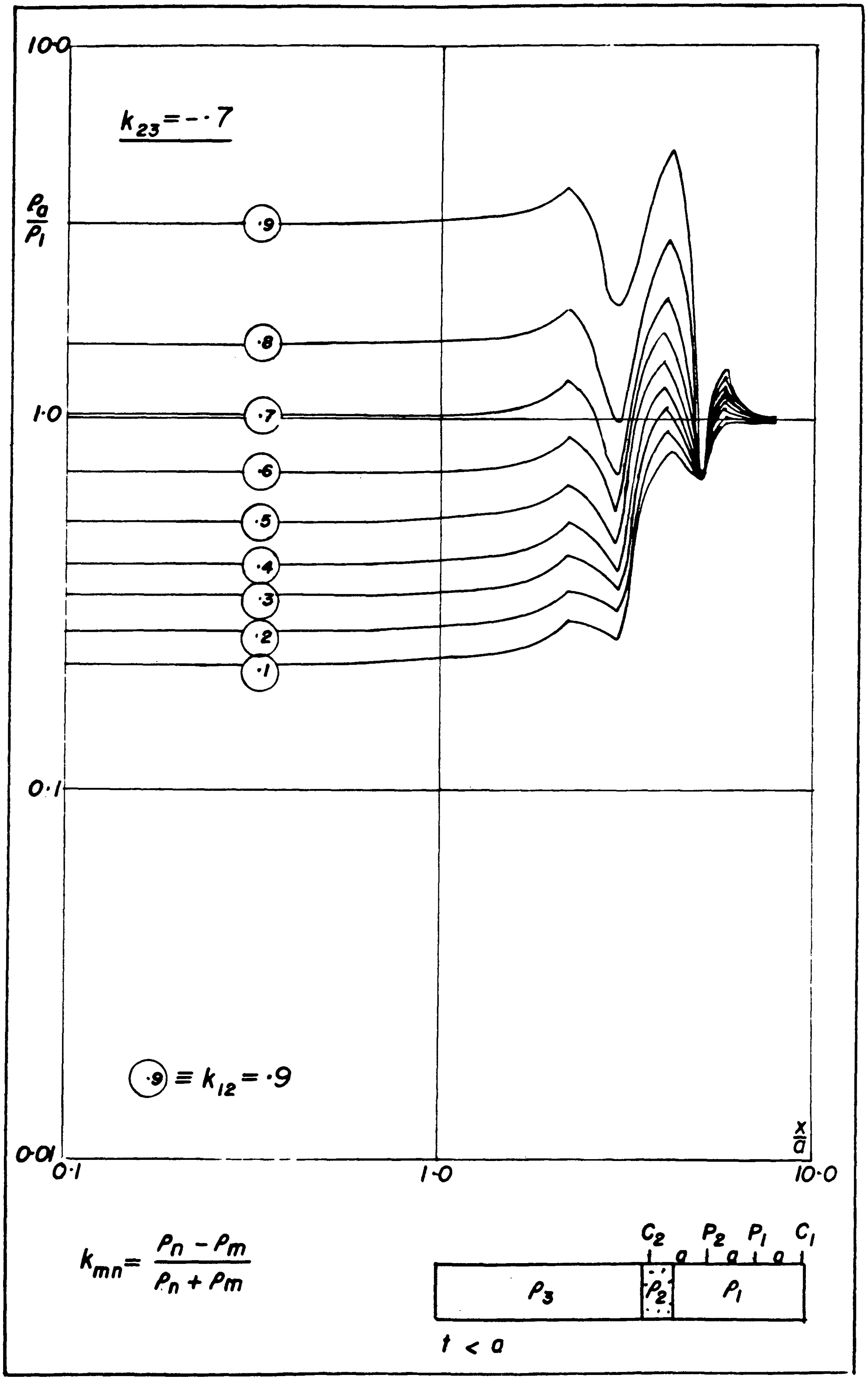

$k_{23} = -.9$
$\frac{\rho_a}{\rho_1}$
1·0
0·1
0·01
0·001
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
$(-·9) \equiv k_{12} = -·9$
$\frac{x}{a}$
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
$t < a$



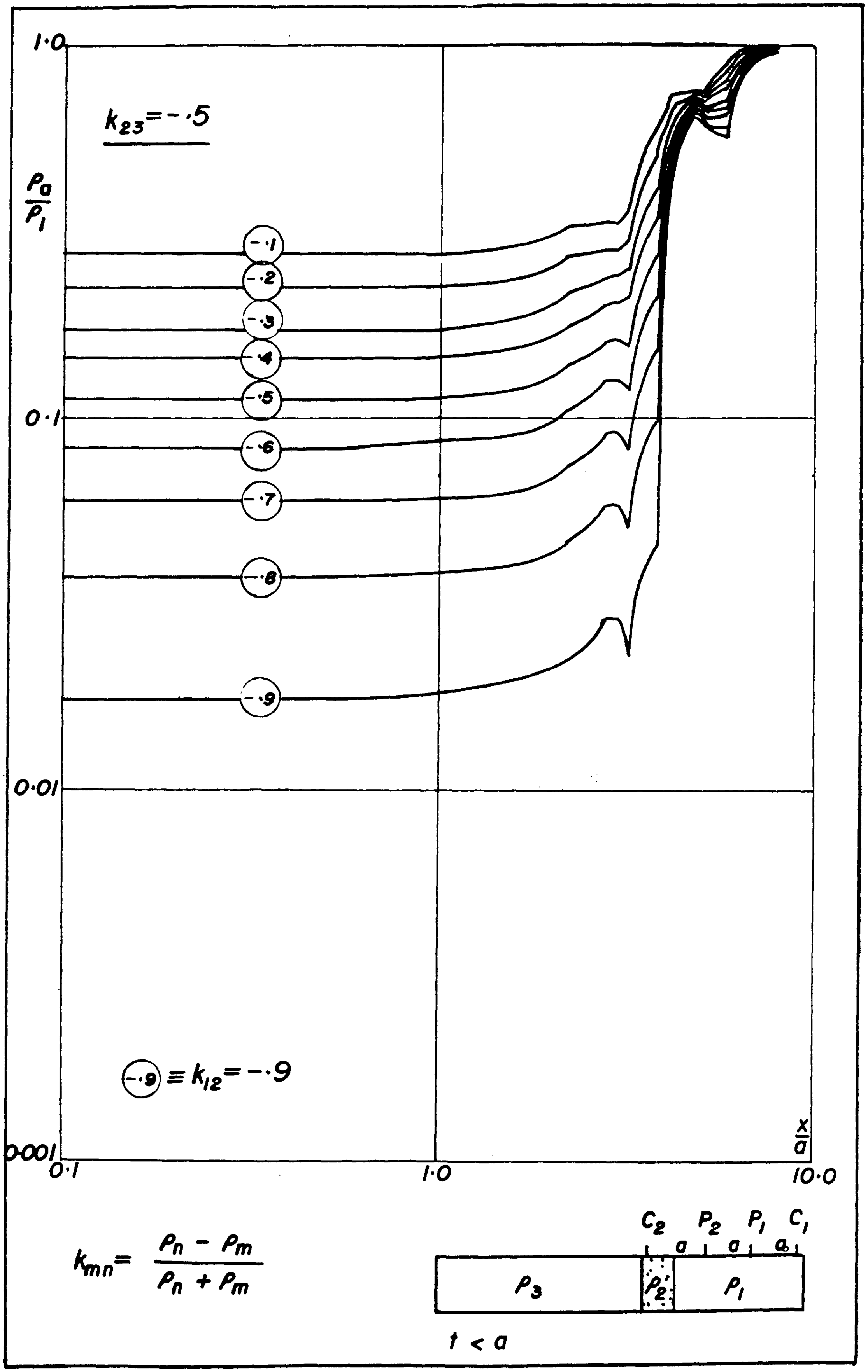
$k_{23} = -\cdot 9$
$\frac{\rho_a}{\rho_1}$
1·0
0·1
0·01
·9
·8
·7
·6
·5
·4
·3
·2
·1
$\cdot 9 \equiv k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
$t < a$



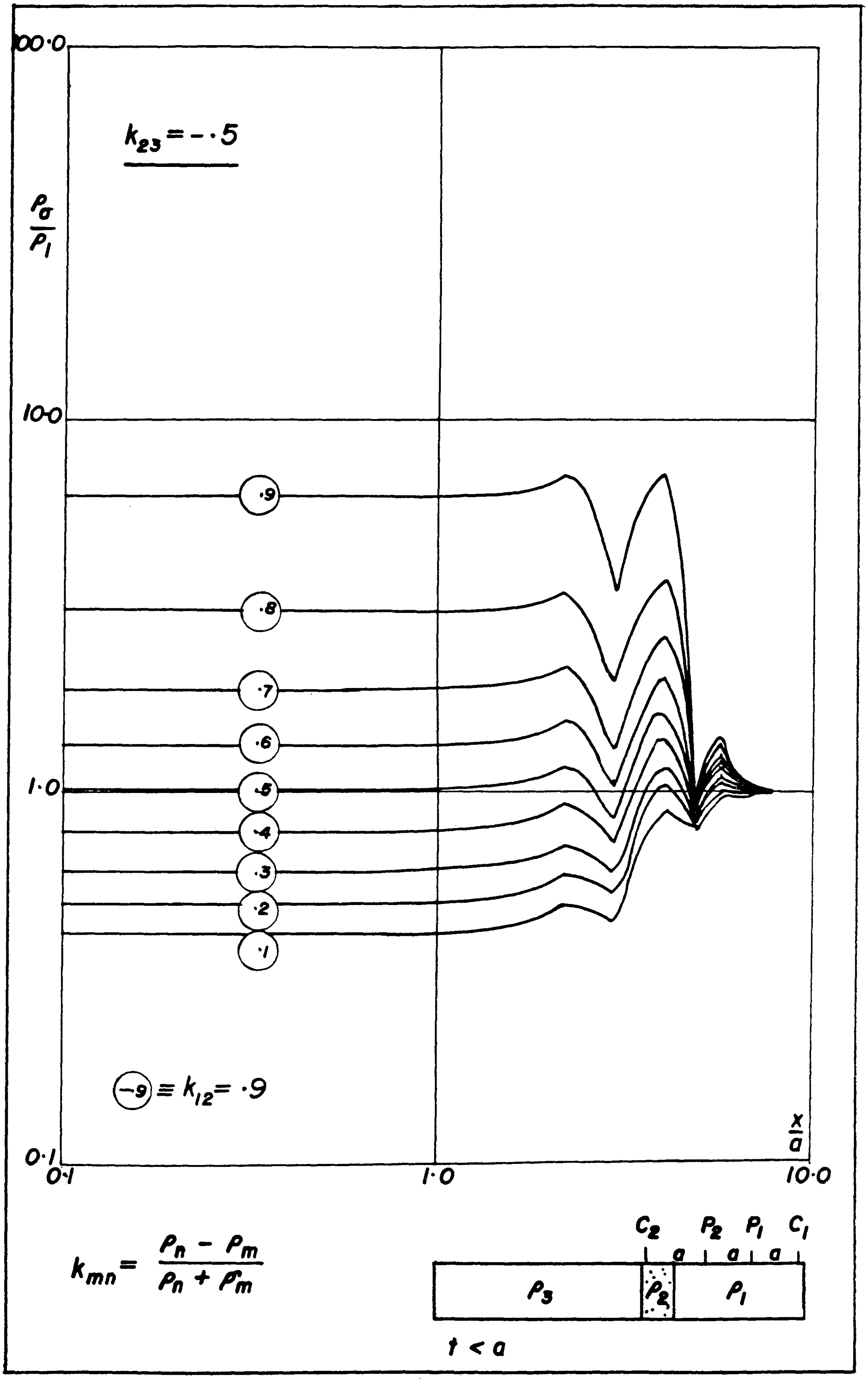
$k_{23} = -\cdot 7$
$\frac{\rho_a}{\rho_1}$
1·0
0·1
0·01
0·001
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
(-·9) ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
t < a



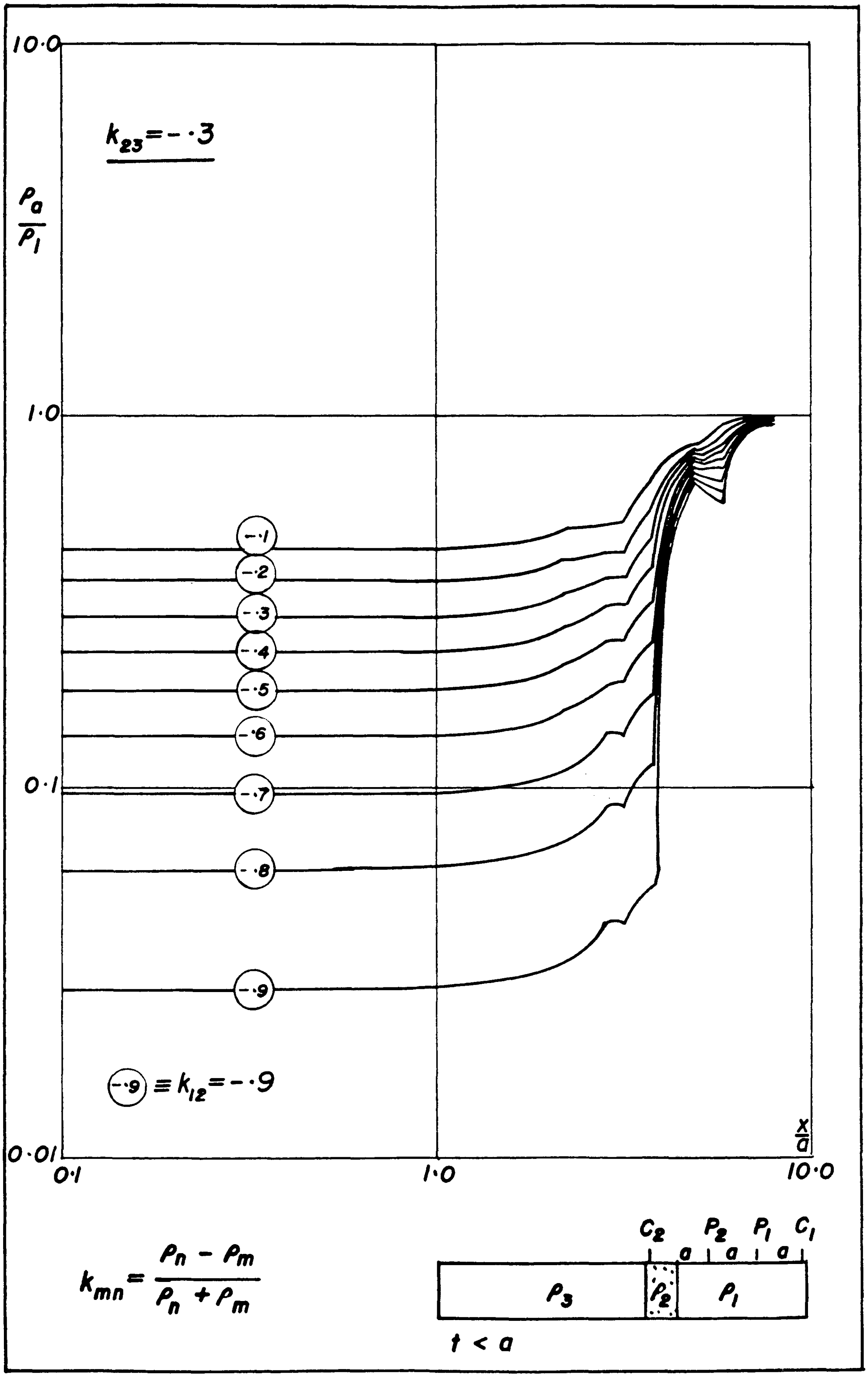
10·0
$k_{23} = -\cdot 7$
$\frac{\rho_a}{\rho_1}$
·9
·8
1·0
·7
·6
·5
·4
·3
·2
·1
0·1
(·9) ≡ $k_{12} = \cdot 9$
0·01
$\frac{x}{a}$
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2 P_2 P_1 C_1
a a a
ρ_3
ρ_2
ρ_1
t < a



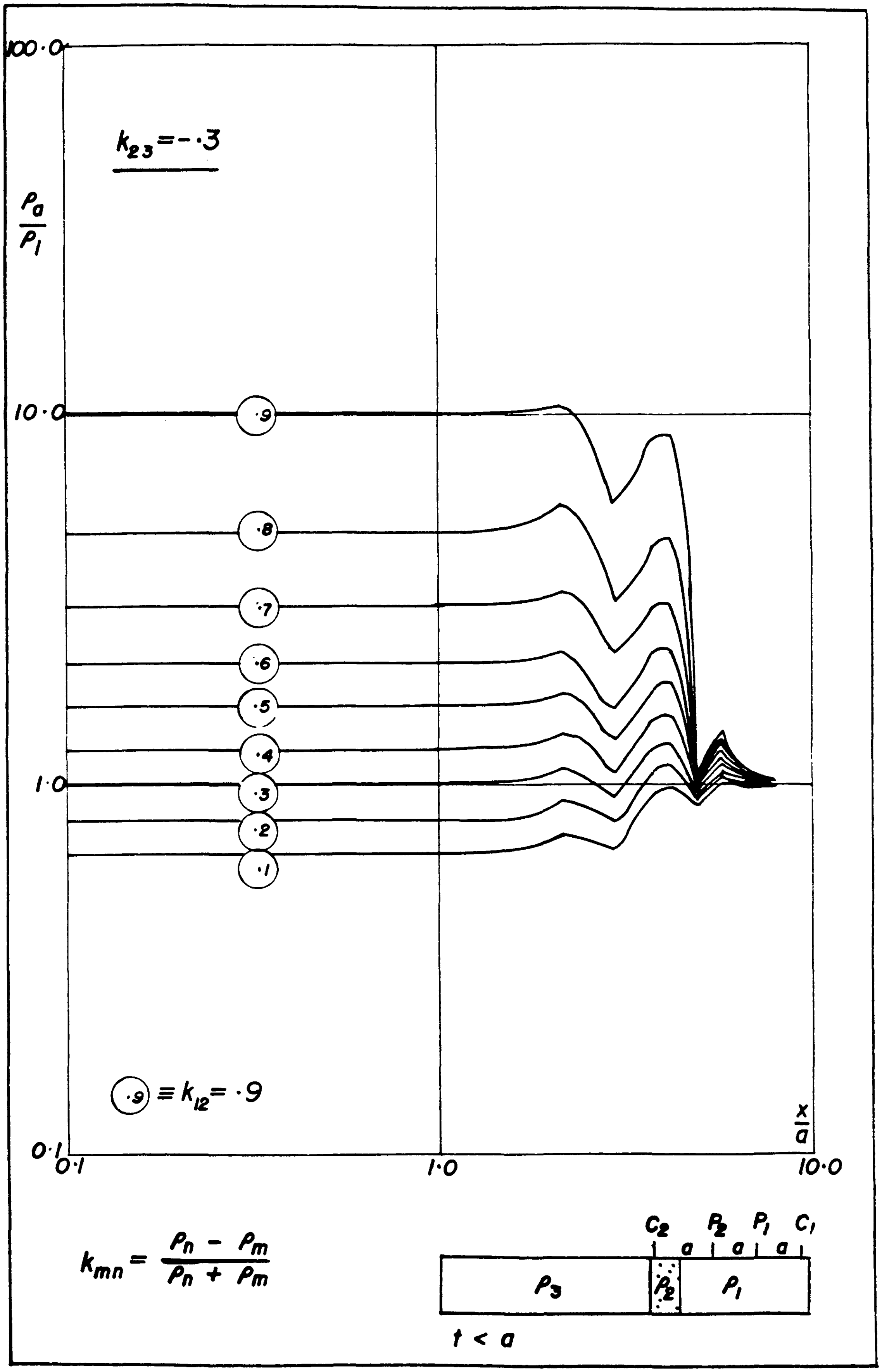
$k_{23} = -\cdot 5$
$\frac{\rho_a}{\rho_1}$
1·0
0·1
0·01
0·001
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
(-·9) ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2'
ρ_1
$t < a$



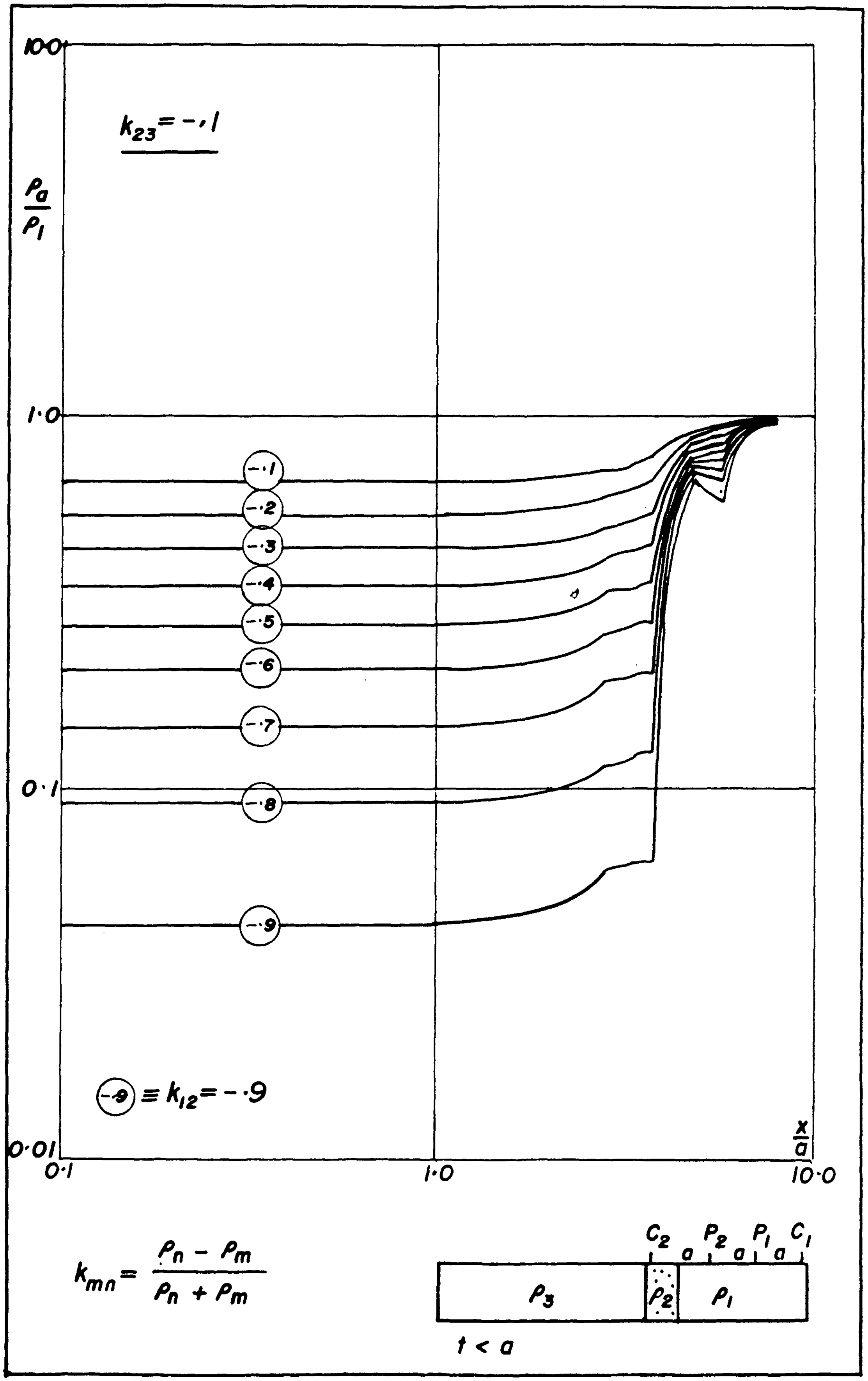
$k_{23} = -\cdot 5$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ $k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
$t < a$

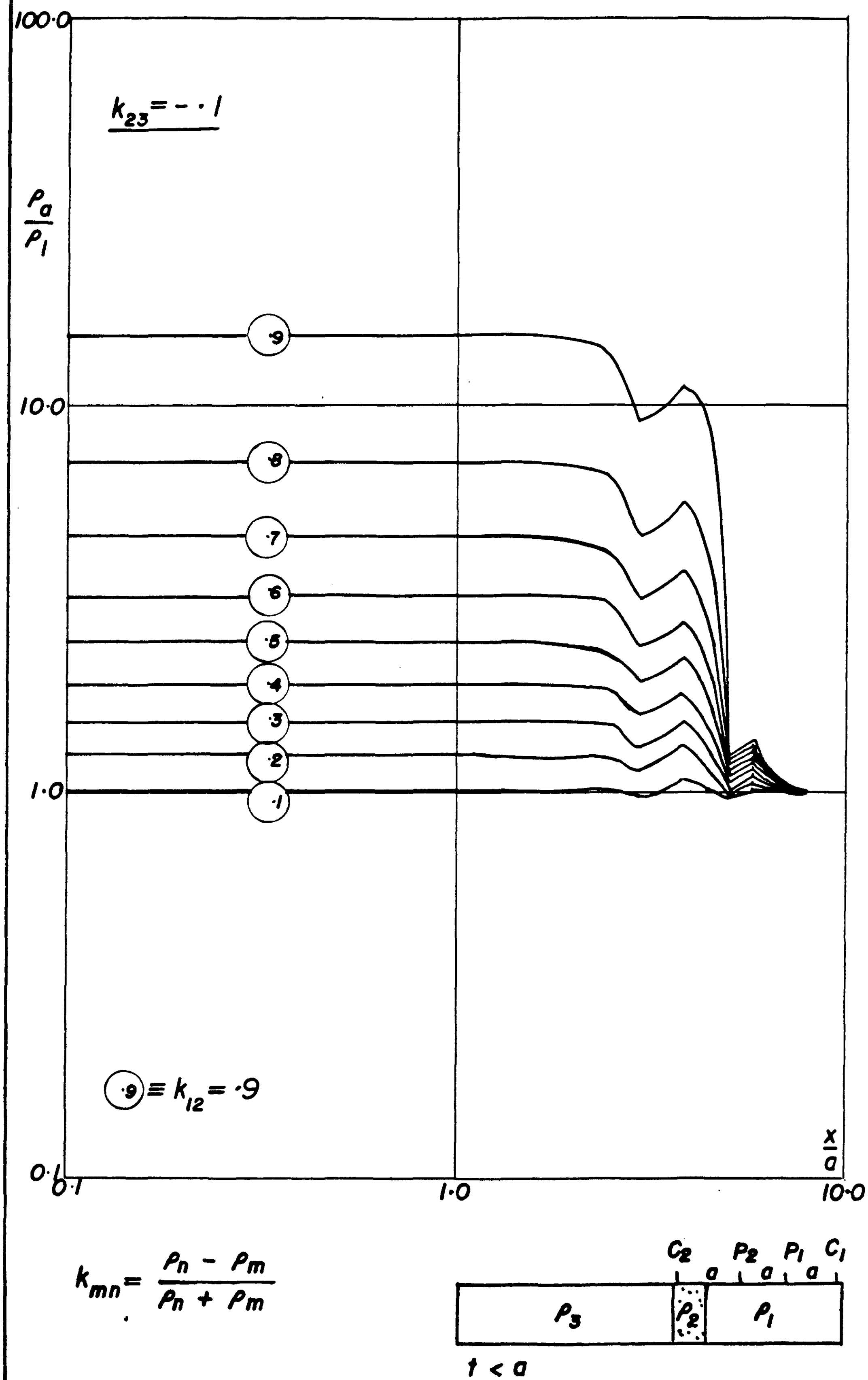


10·0
$k_{23} = -\cdot 3$
$\frac{\rho_a}{\rho_1}$
1·0
-·1
-·2
-·3
-·4
-·5
-·6
0·1
-·7
-·8
-·9
$(-\cdot 9) \equiv k_{12} = -\cdot 9$
$\frac{x}{a}$
0·01
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
$t < a$

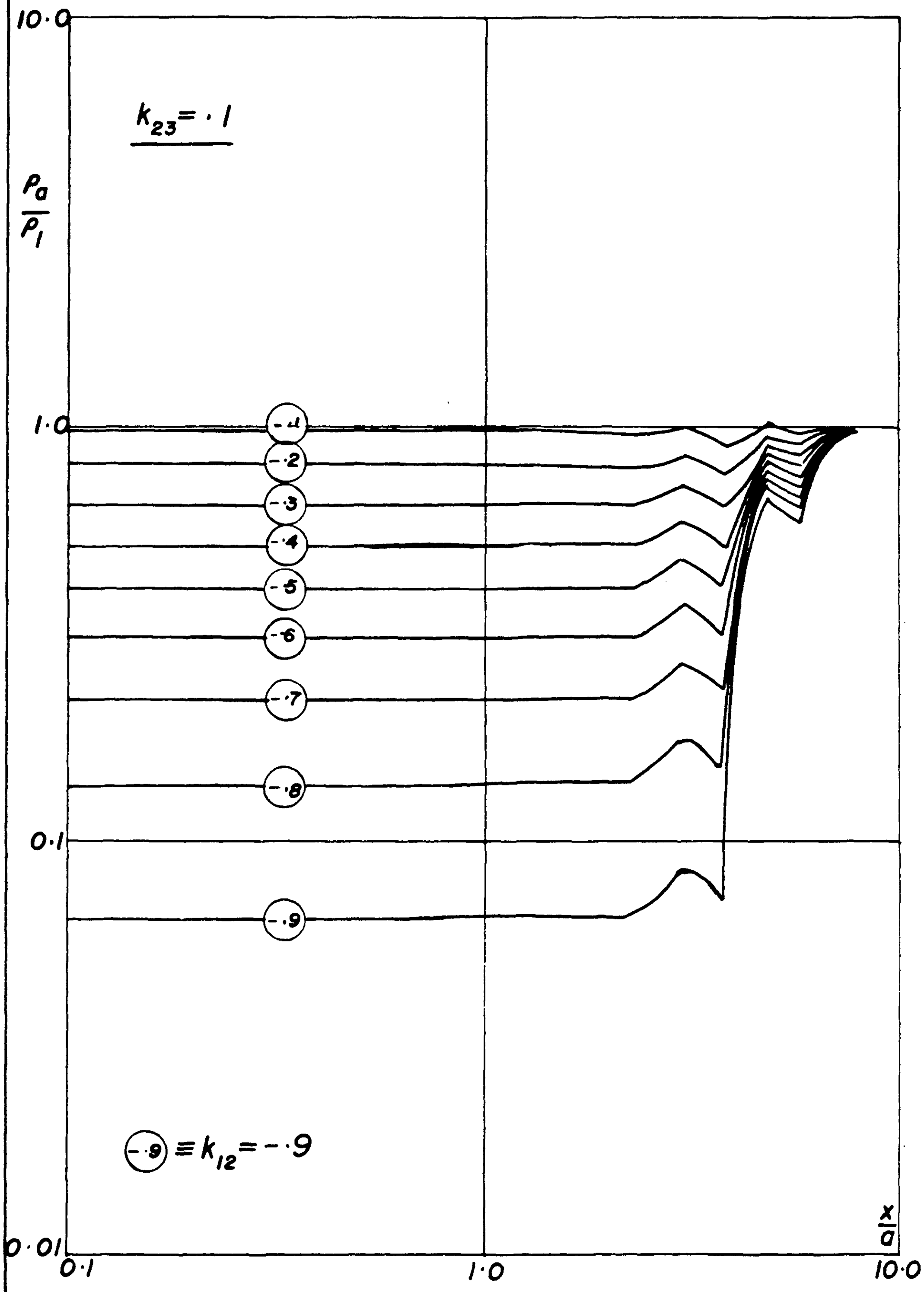


$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

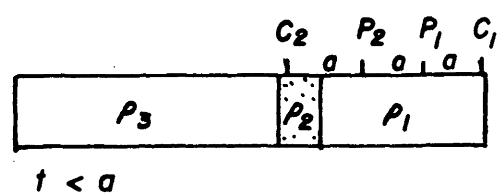

$k_{23} = -.1$
$\frac{\rho_a}{\rho_1}$
100
1·0
0·1
0·01
0·1
1·0
10·0
$\frac{x}{a}$
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
(-·9) ≡ $k_{12} = -·9$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
t < a

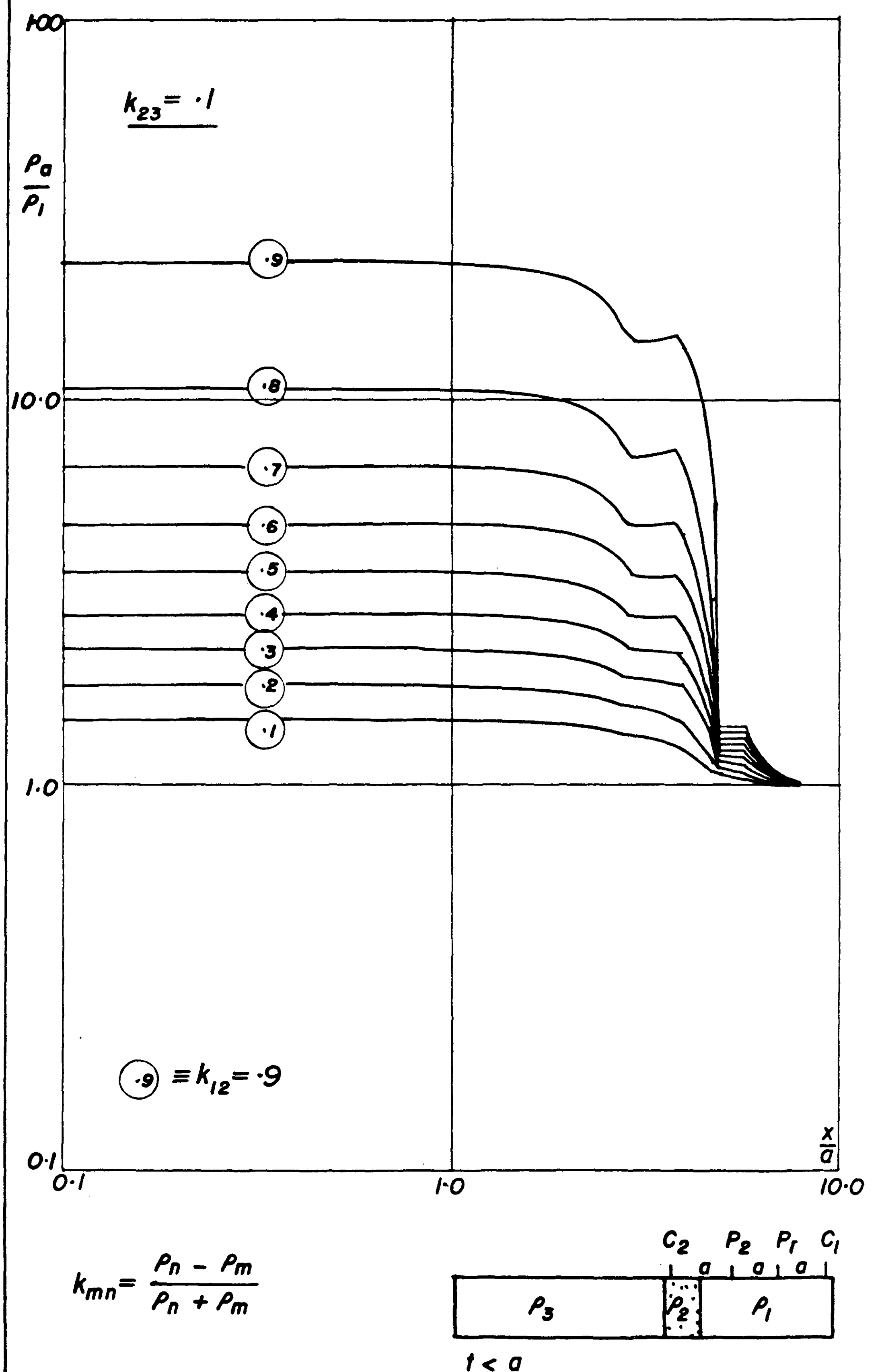


$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

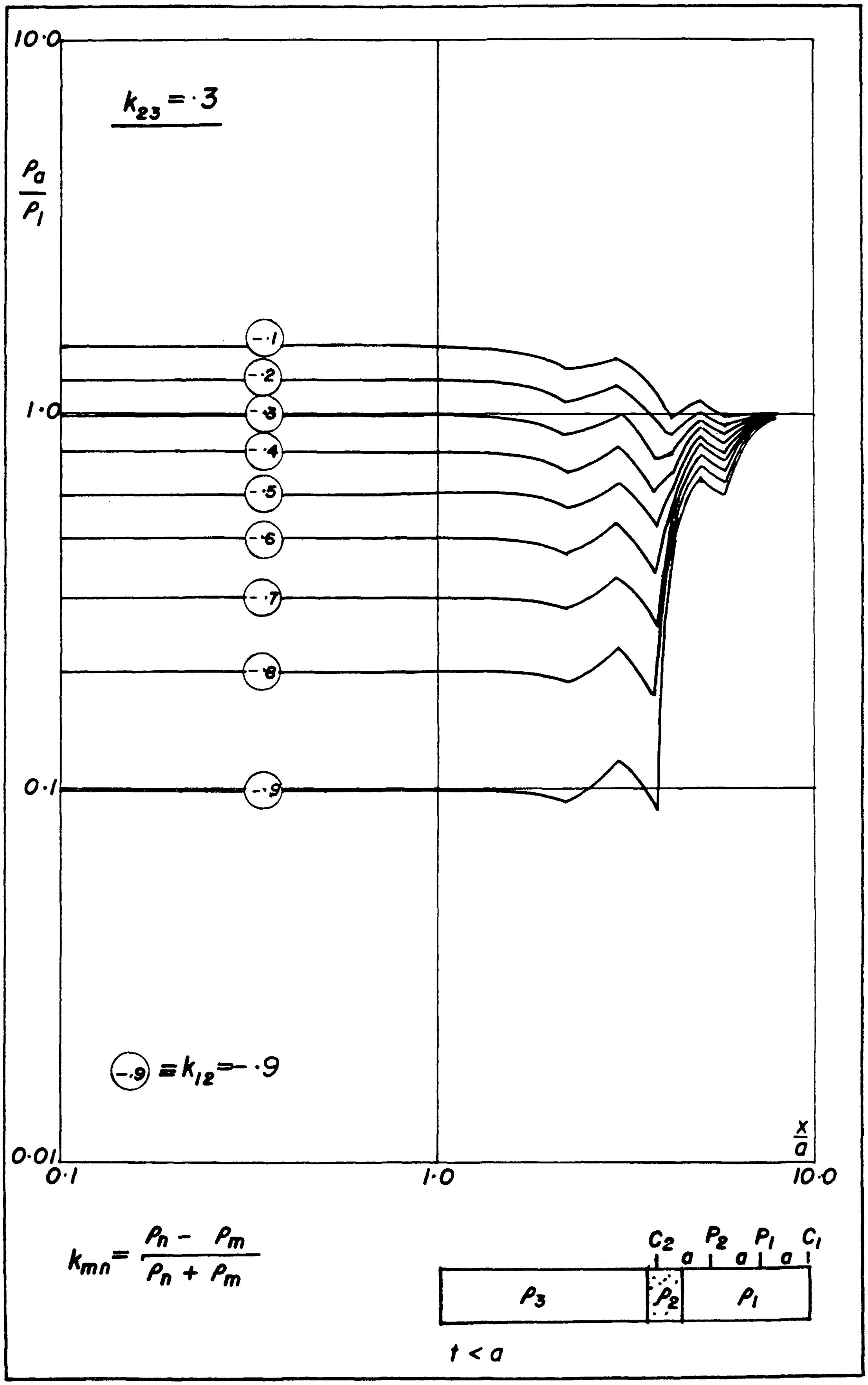


$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

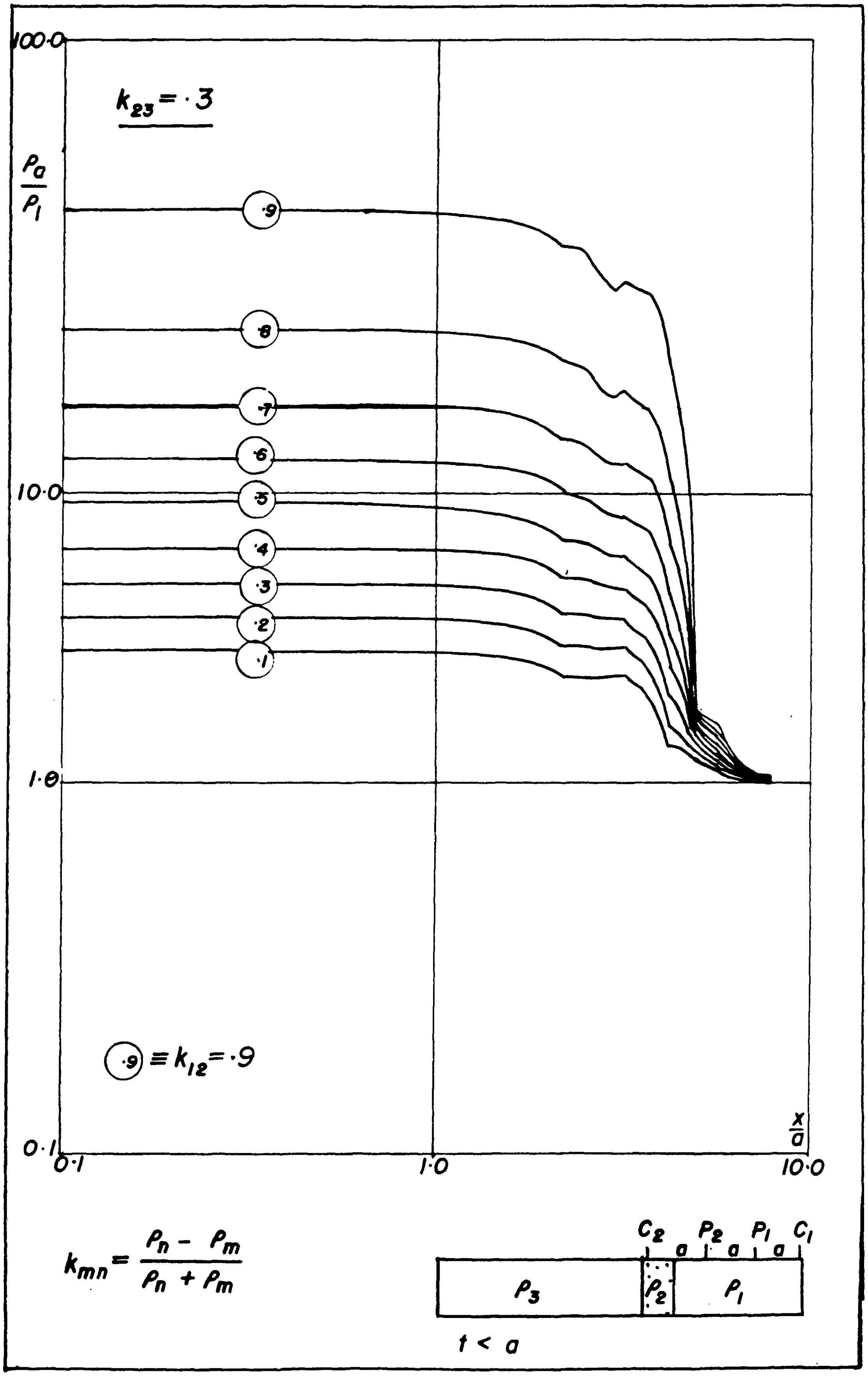


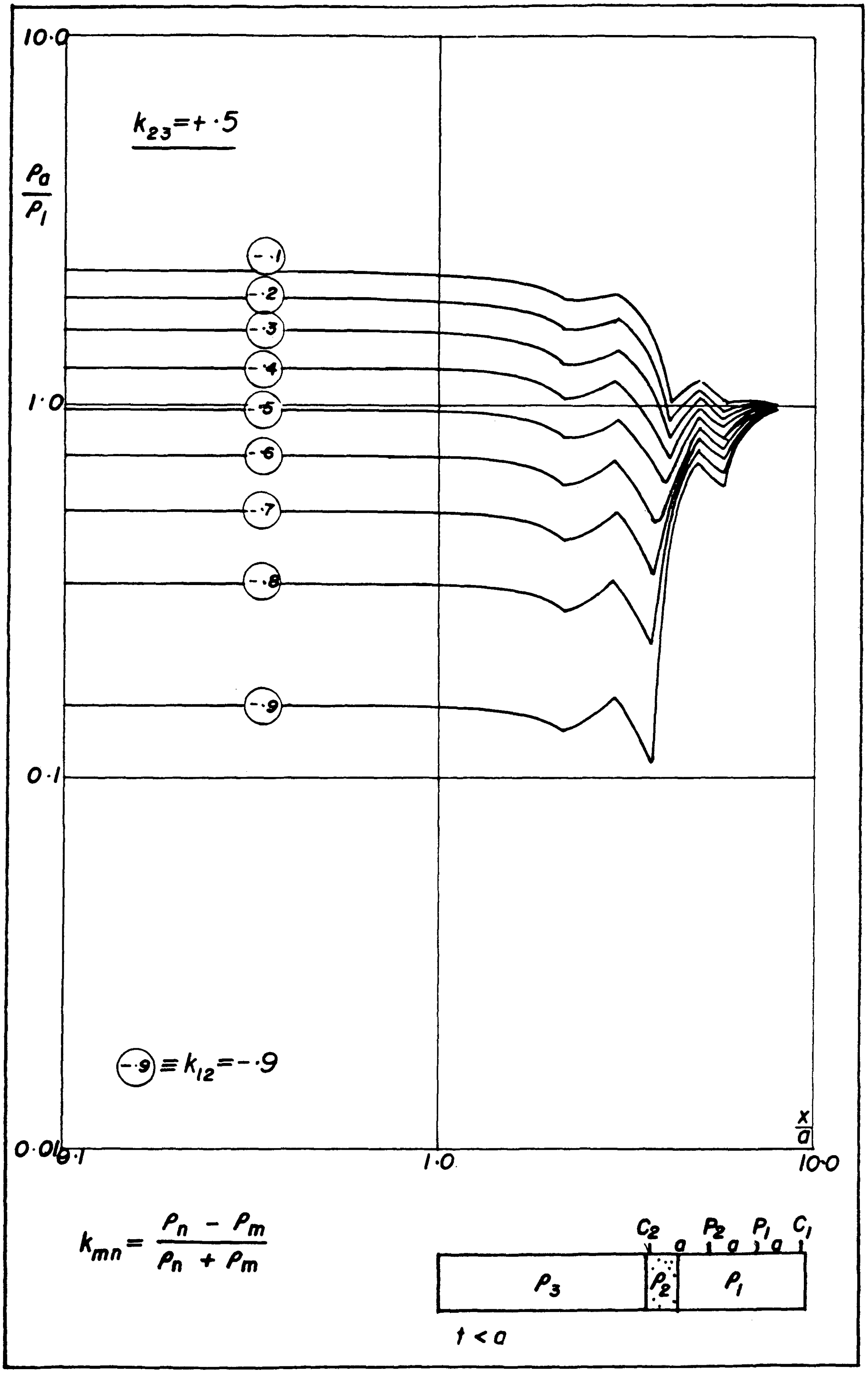


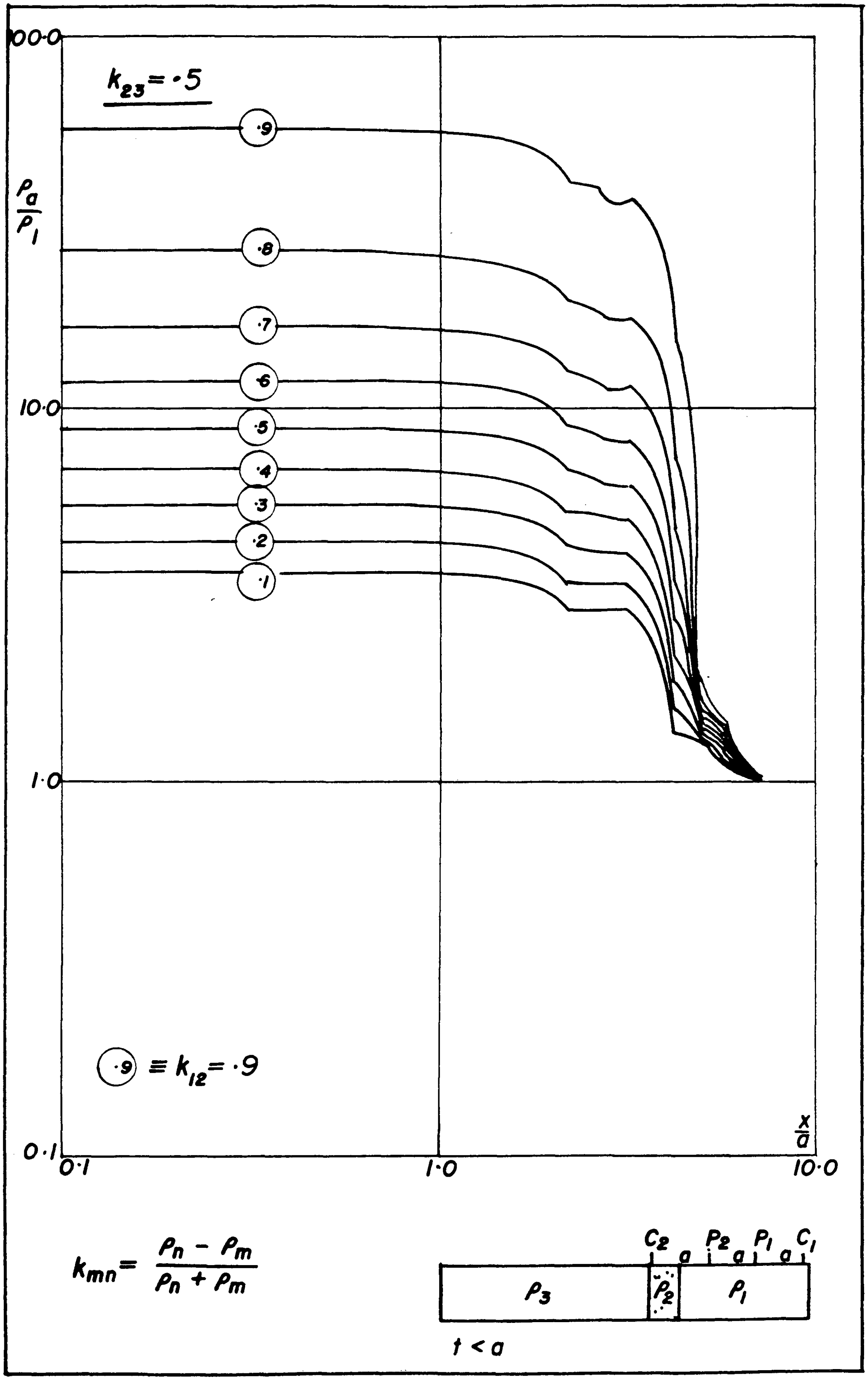
$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$



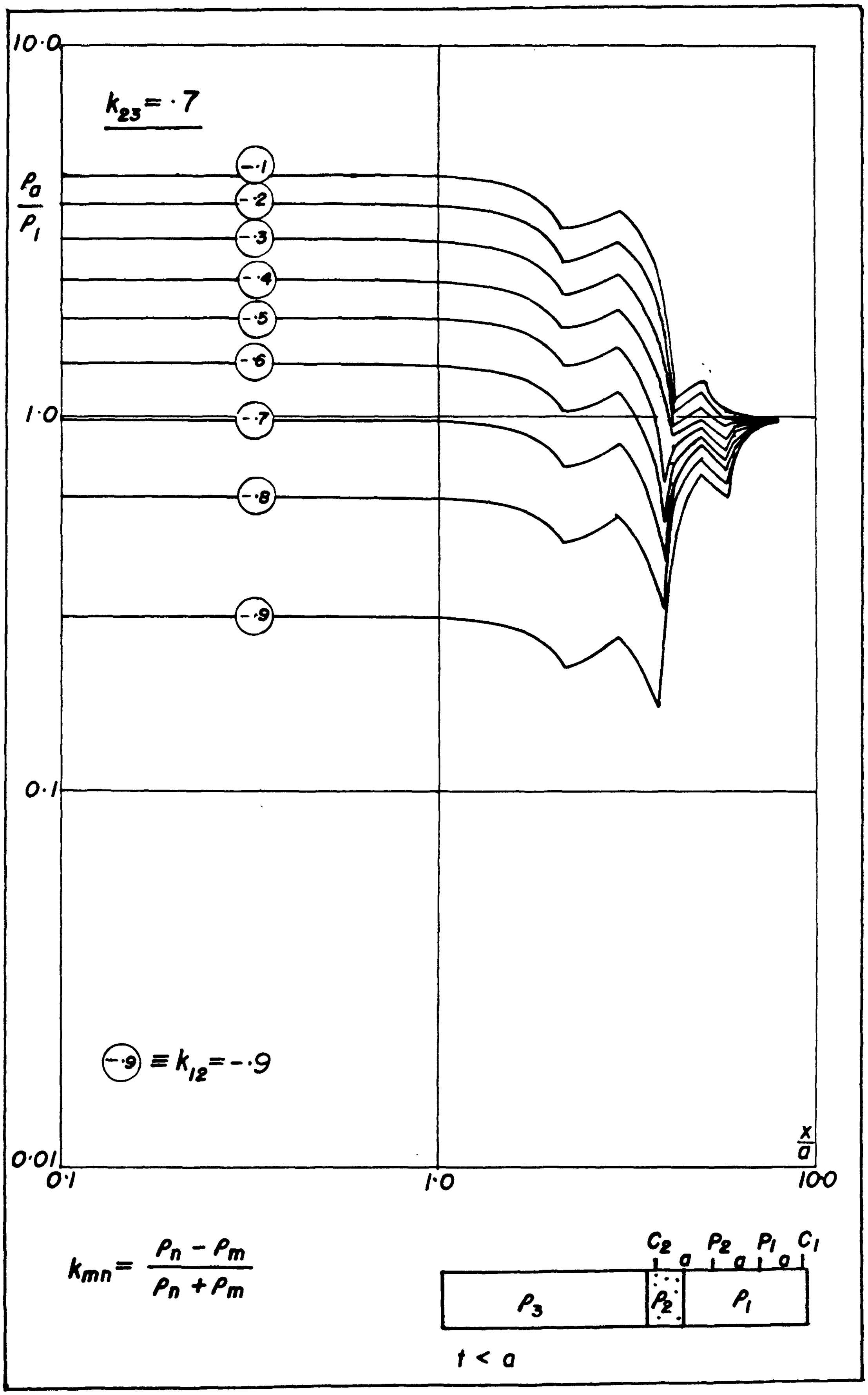
$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

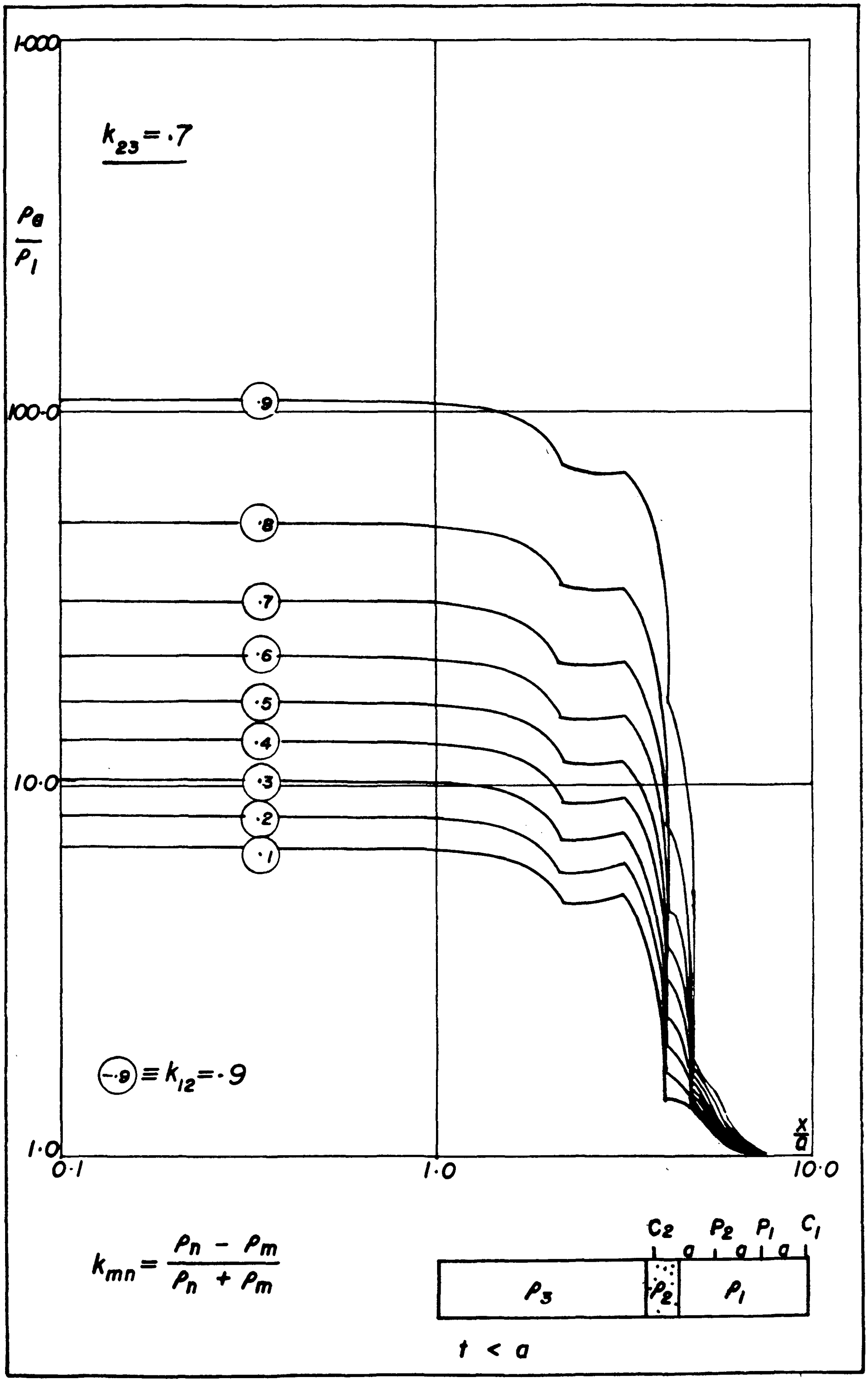

$k_{23} = \cdot 3$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
$\cdot 9 \equiv k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$t < a$


10·0
$k_{23} = +·5$
$\frac{\rho_a}{\rho_1}$
-·1
-·2
-·3
-·4
1·0
-·5
-·6
-·7
-·8
-·9
0·1
$(-·9) \equiv k_{12} = -·9$
0·01
0·1
1·0
10·0
$\frac{x}{a}$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C2
a
P2
a
P1
a
C1
ρ_3
ρ_2
ρ_1
t < a

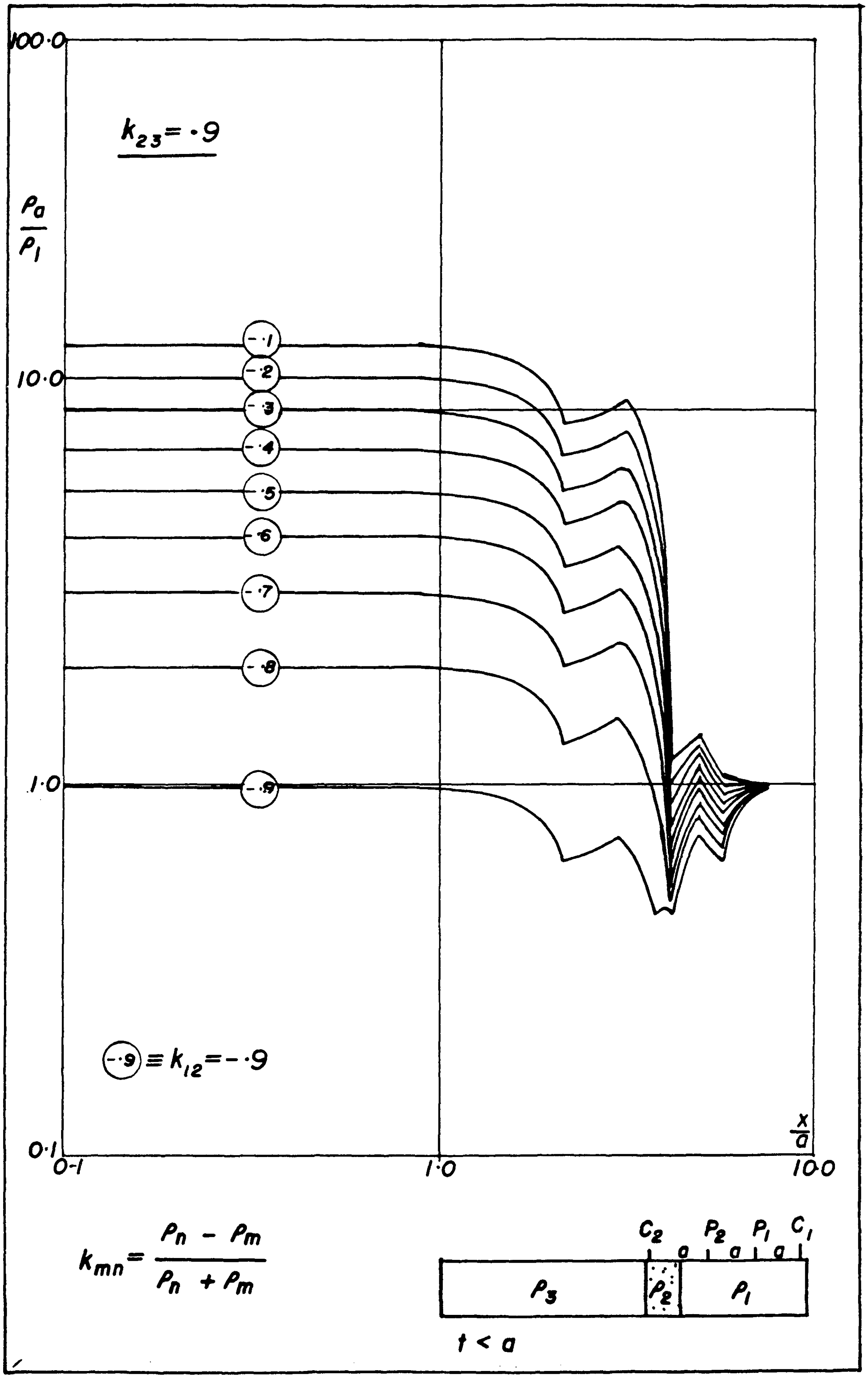


$k_{23} = \cdot 5$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
$\frac{x}{a}$
0·1
1·0
10·0
(·9) $\equiv k_{12} = \cdot 9$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C2
a
P2
a
P1
a
C1
ρ_3
ρ_2
ρ_1
t < a

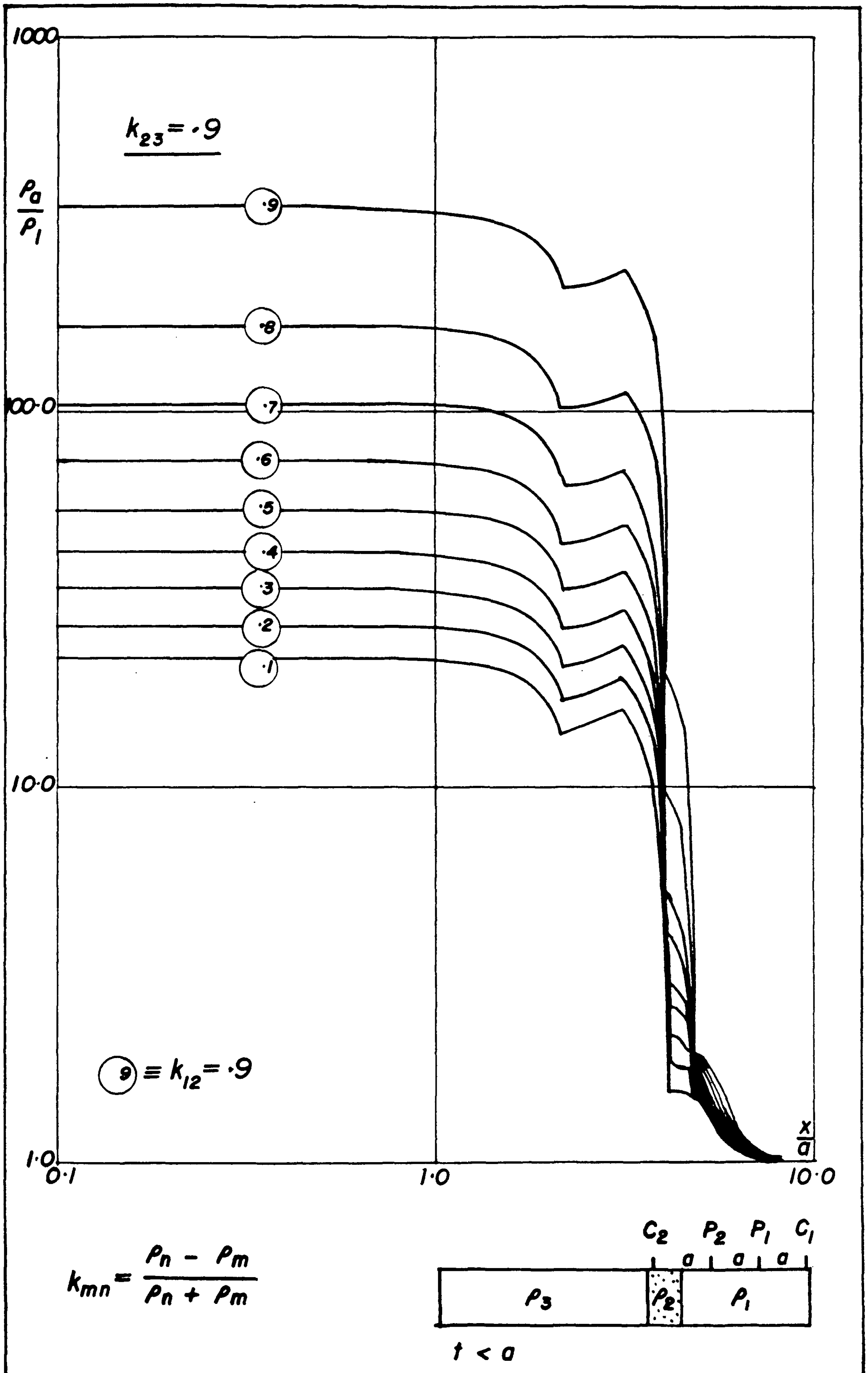

10·0
$k_{23} = \cdot 7$
ρ_a / ρ_1
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
1·0
0·1
0·01
-·9 ≡ $k_{12} = -\cdot 9$
x/a
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2
a
P_2
a
P_1
a
C_1
ρ_3
ρ_2
ρ_1
t < a



$k_{23} = \cdot 7$
$\frac{\rho_a}{\rho_1}$
1·000
100·0
10·0
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ $k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C2
P2
P1
C1
a
a
a
ρ_3
ρ_2
ρ_1
$t < a$



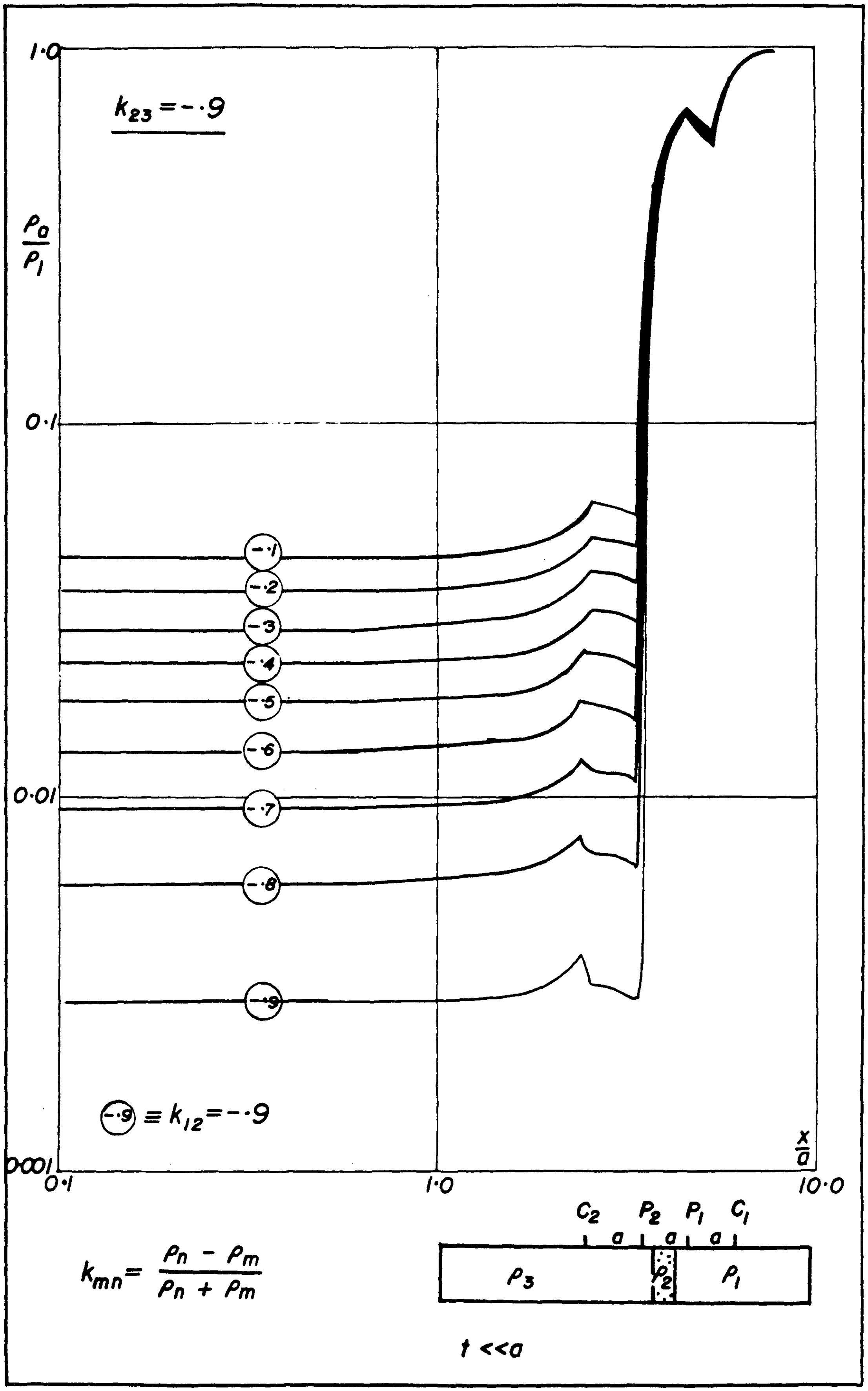
$k_{23} = \cdot 9$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
-·9 ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
$t < a$

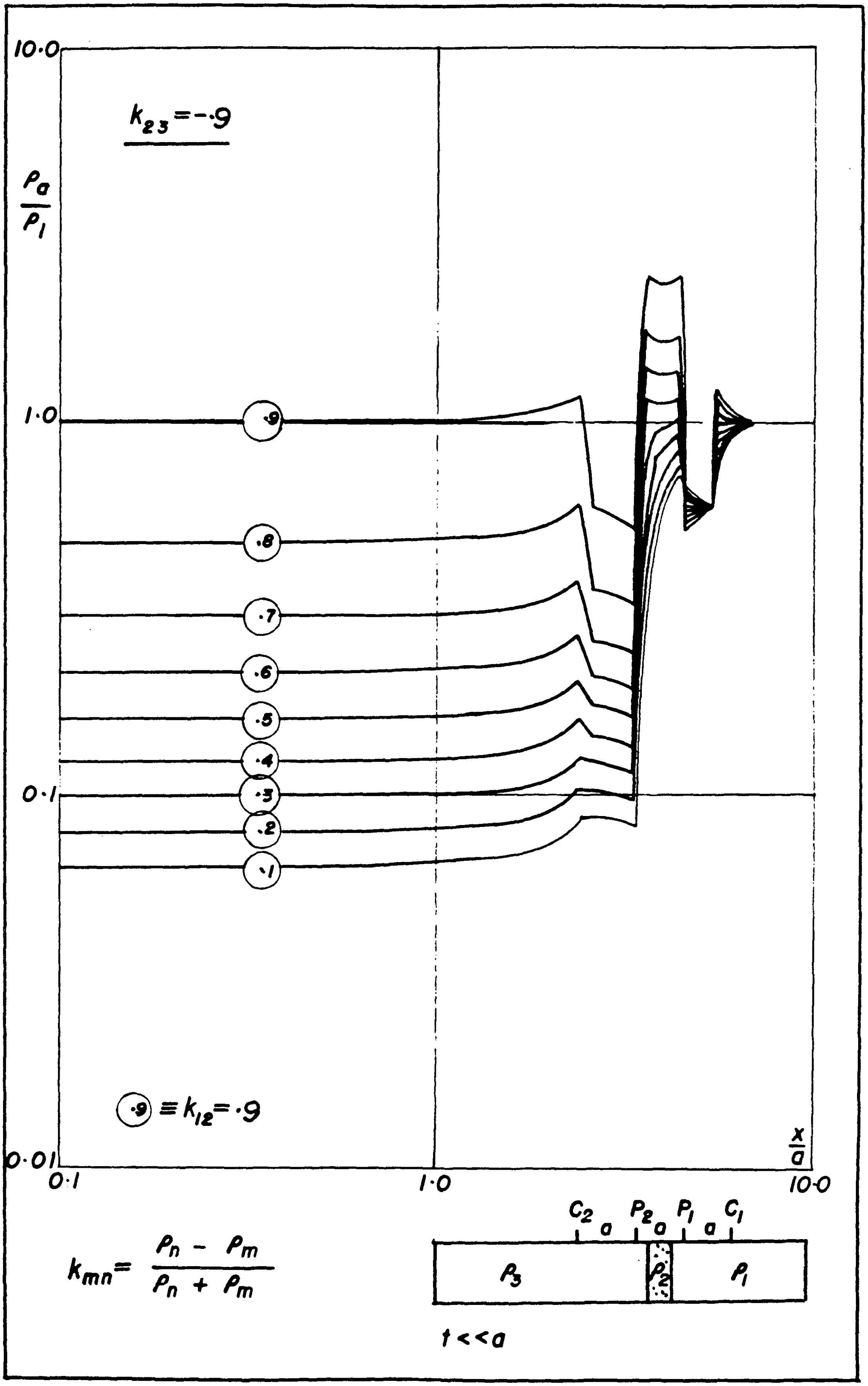


$k_{23} = \cdot 9$
$\frac{\rho_a}{\rho_1}$
1000
100·0
10·0
1·0
0·1
1·0
10·0
$\frac{x}{a}$
·9
·8
·7
·6
·5
·4
·3
·2
·1
9 $\equiv k_{12} = \cdot 9$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
$t < a$

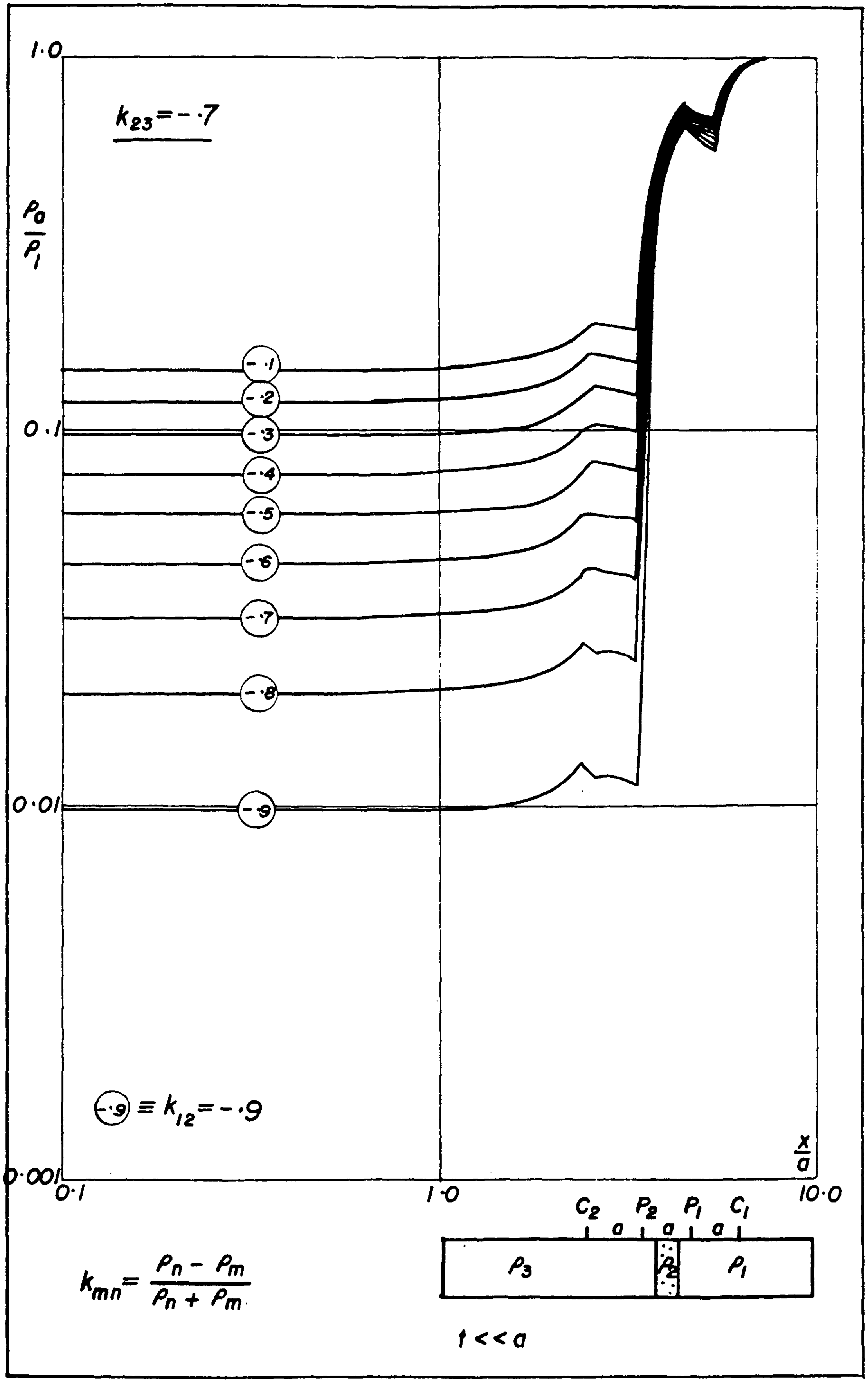
APPENDIX F

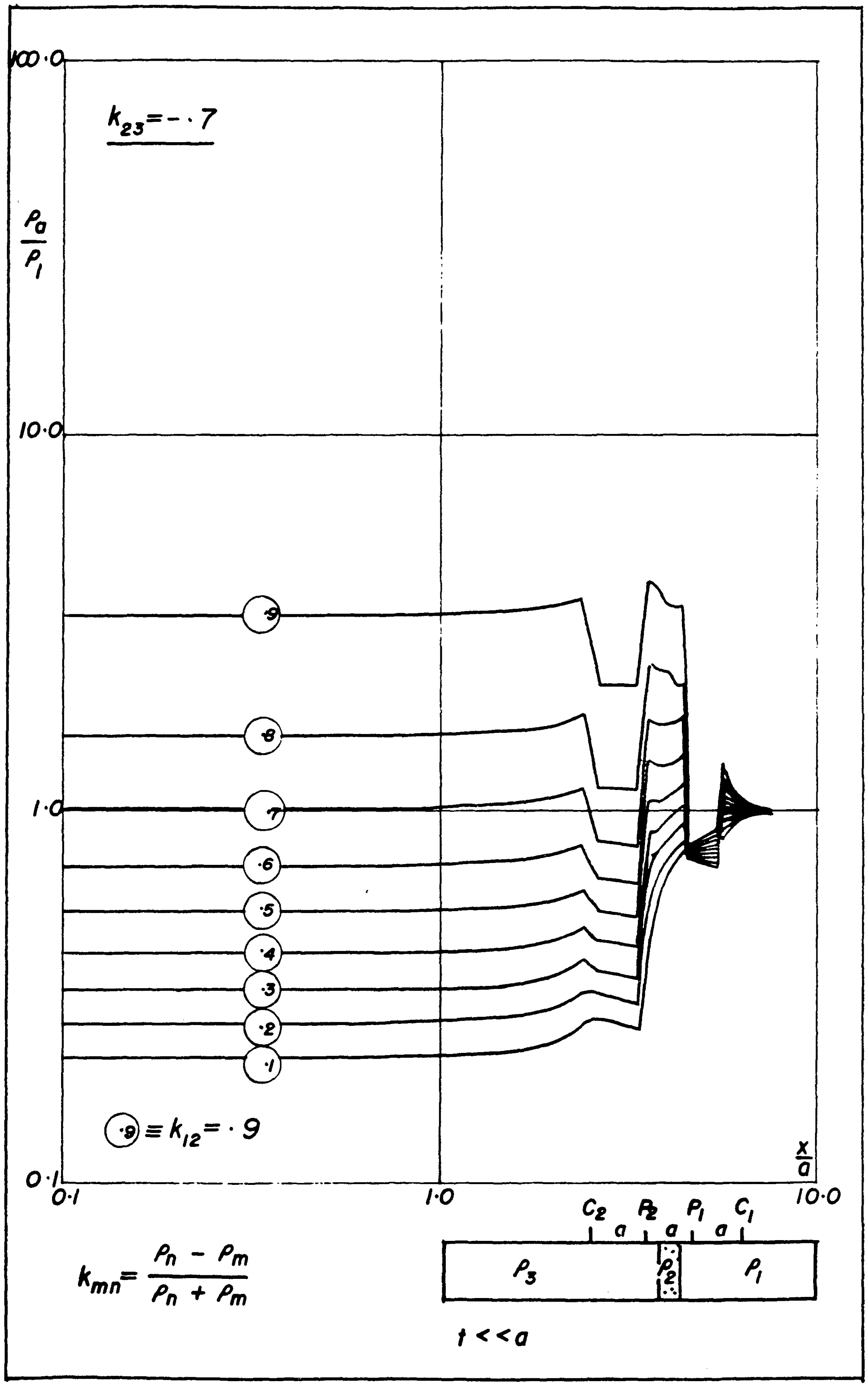
This appendix contains 180 master curves for the case where $t \ll a$. Although $\alpha = 90^{\circ}$ throughout, program 5 that was used to calculate these curves, is such that the user may specify any angle when calculating additional curves.

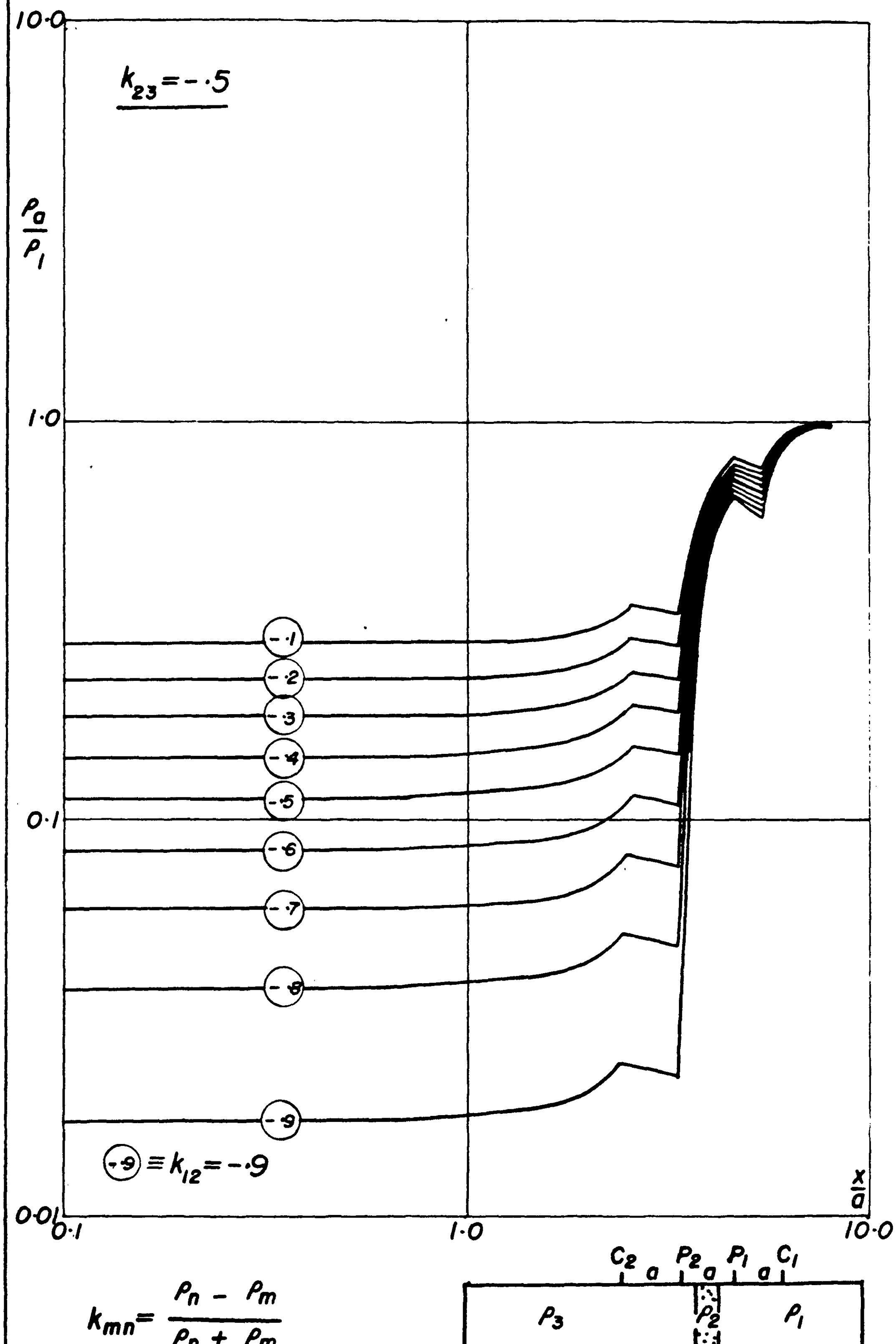

$k_{23} = -\cdot 9$
$\frac{\rho_a}{\rho_1}$
1·0
0·1
0·01
0·001
−·1
−·2
−·3
−·4
−·5
−·6
−·7
−·8
−·9
−·9 ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t \ll a$



$k_{23} = -.9$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ k_{12} = ·9
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
t << a

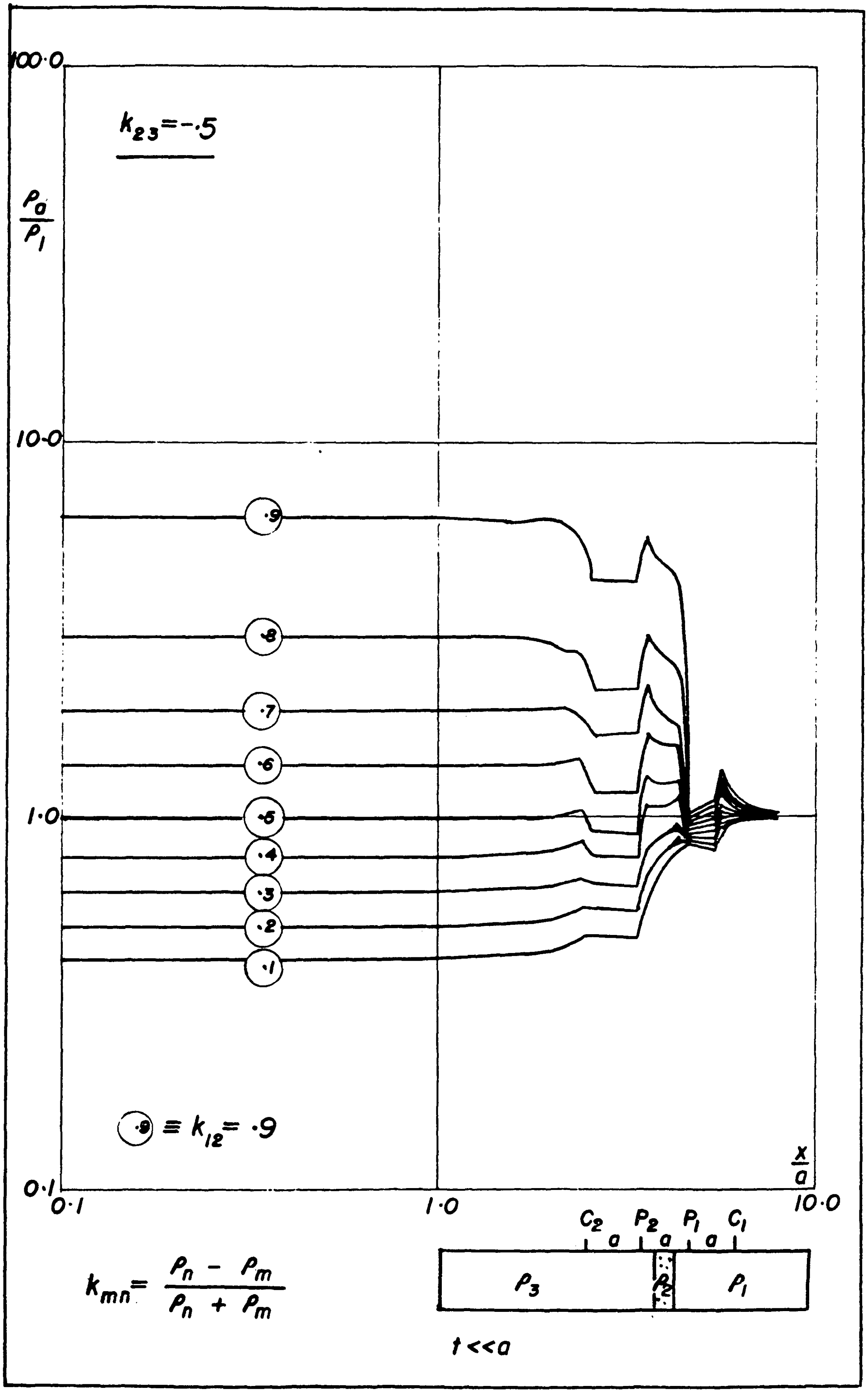

$k_{23} = -\cdot 7$
$\frac{\rho_a}{\rho_1}$
1.0
0.1
0.01
0.001
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
-·9 ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3 ρ_2 ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$


$k_{23} = -\cdot 7$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
(·9) ≡ $k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$

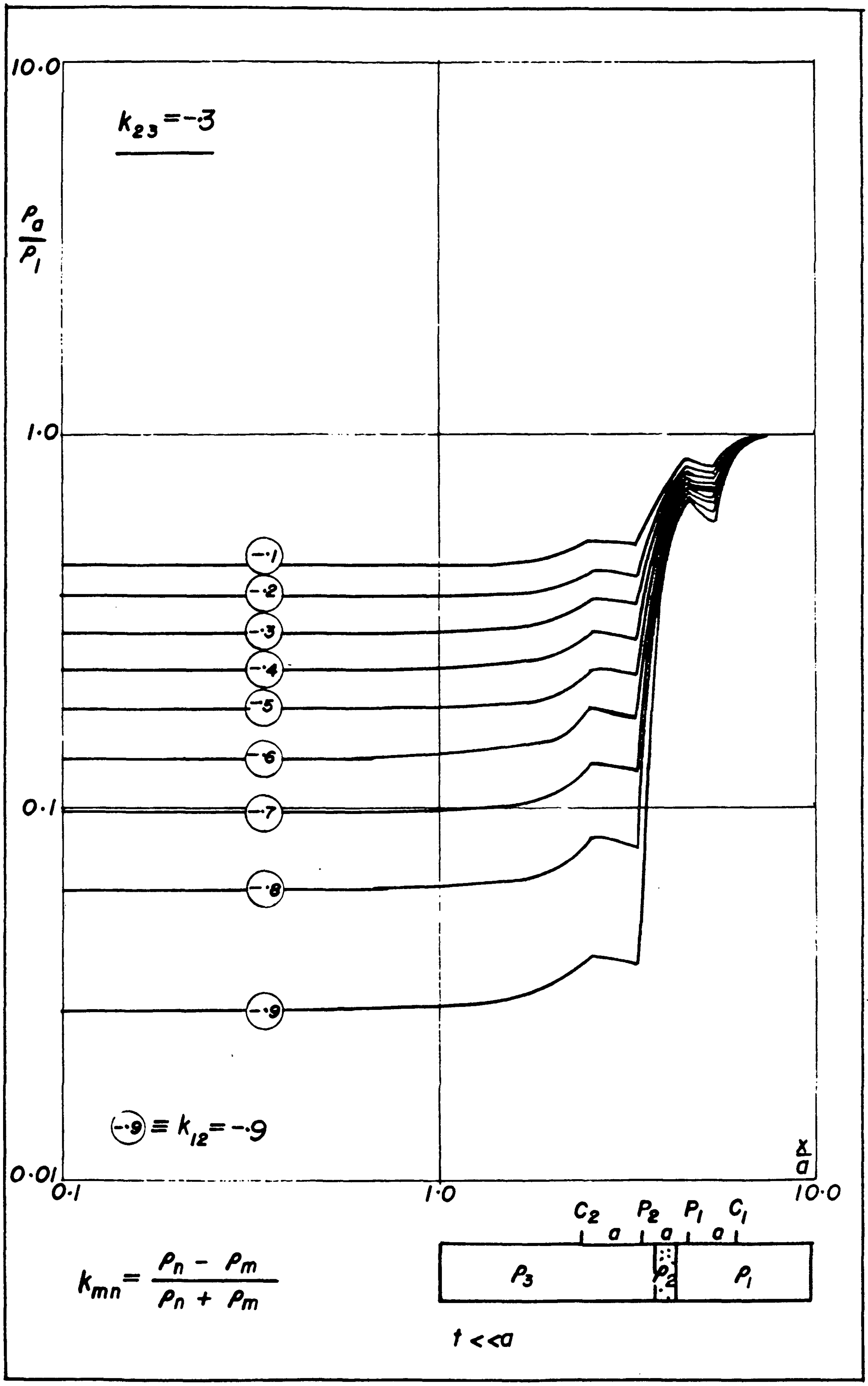


$$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$$

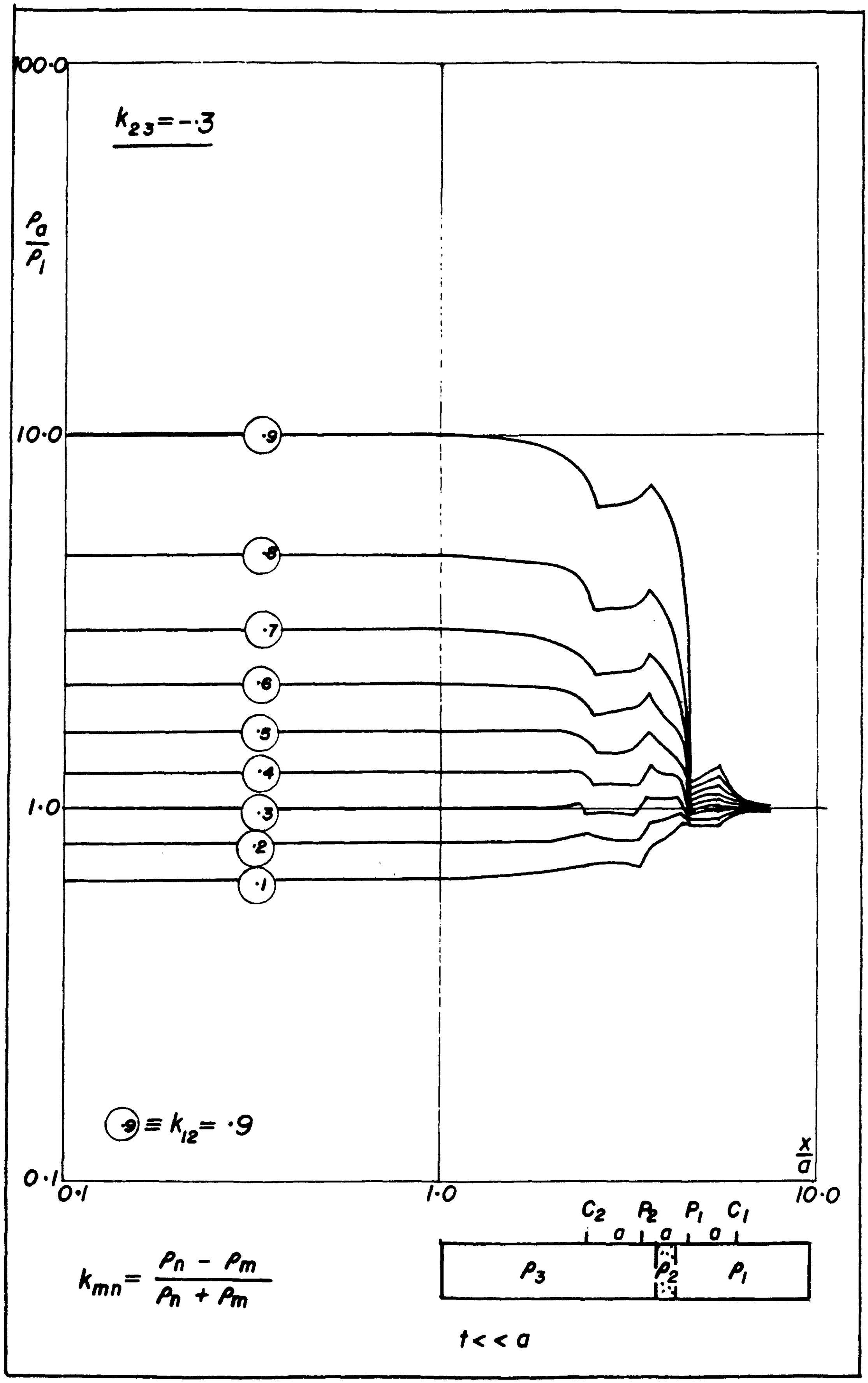
$t << a$



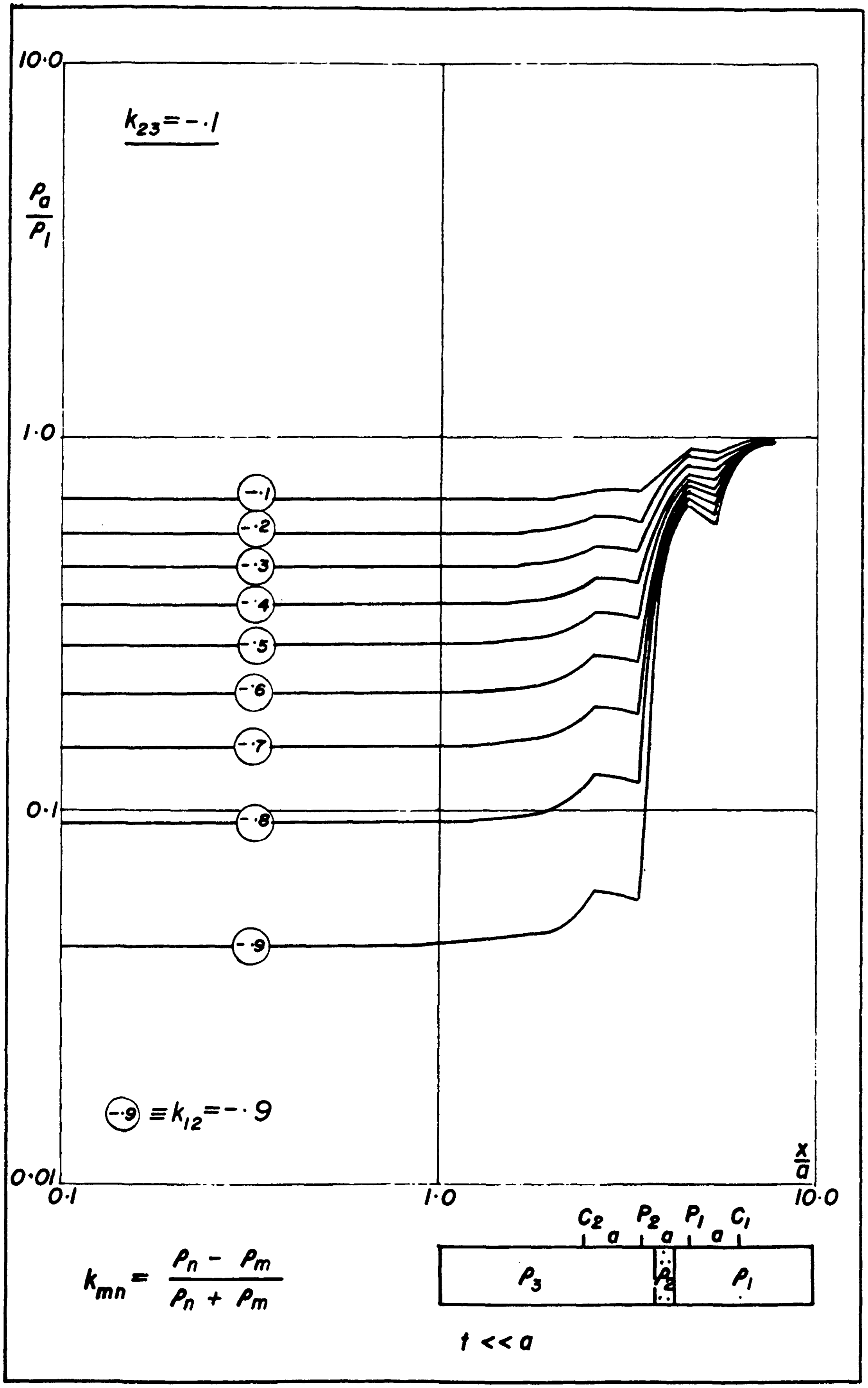
$k_{23} = -.5$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
·9 ≡ $k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$



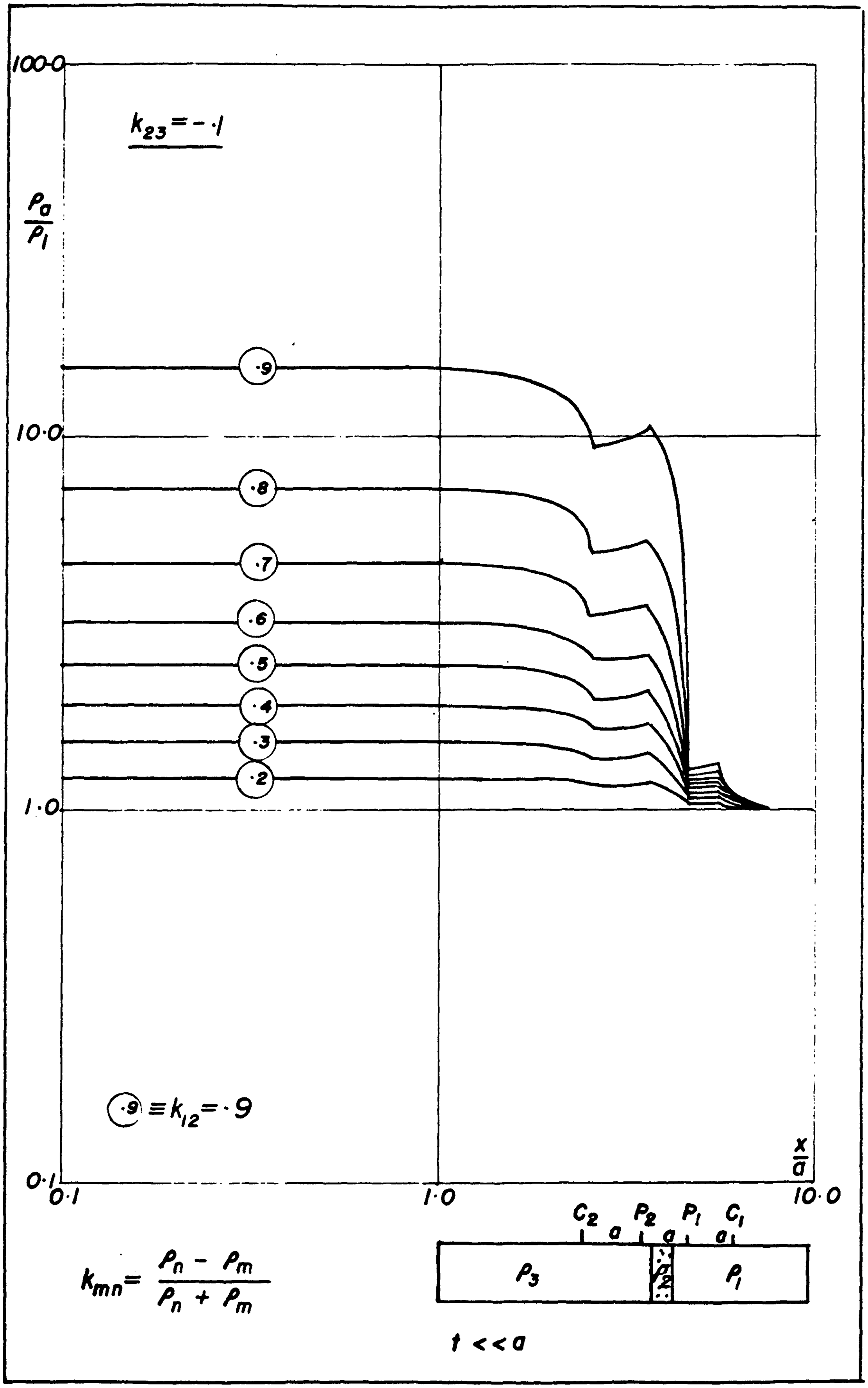
$k_{23} = -\cdot 3$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
0·1
1·0
10·0
$\frac{x}{a}$
−·1
−·2
−·3
−·4
−·5
−·6
−·7
−·8
−·9
−·9 ≡ $k_{12} = -\cdot 9$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$



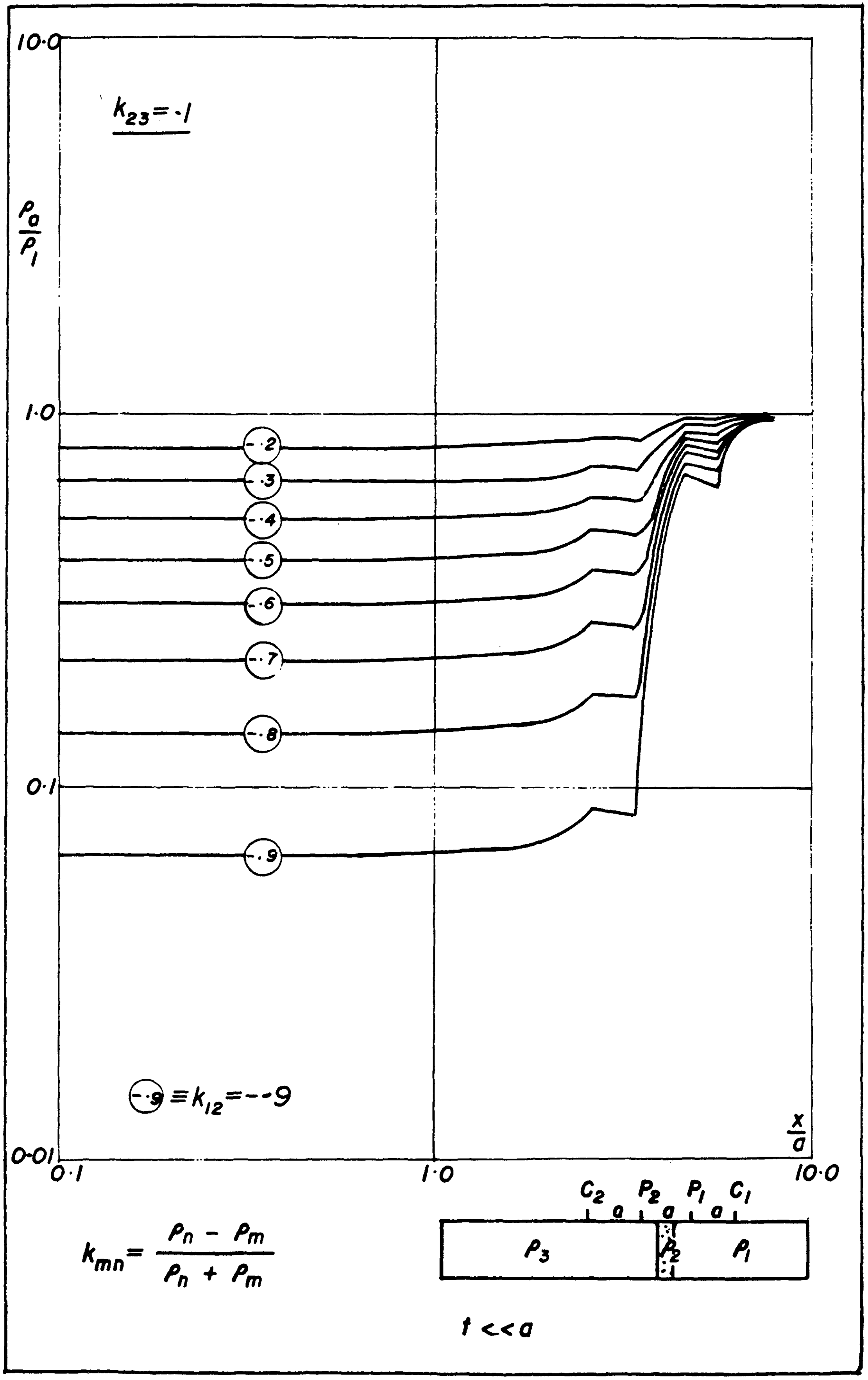
$k_{23} = -\cdot 3$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
$\cdot 9 \equiv k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2
P_2
P_1
C_1
a
a
a
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$

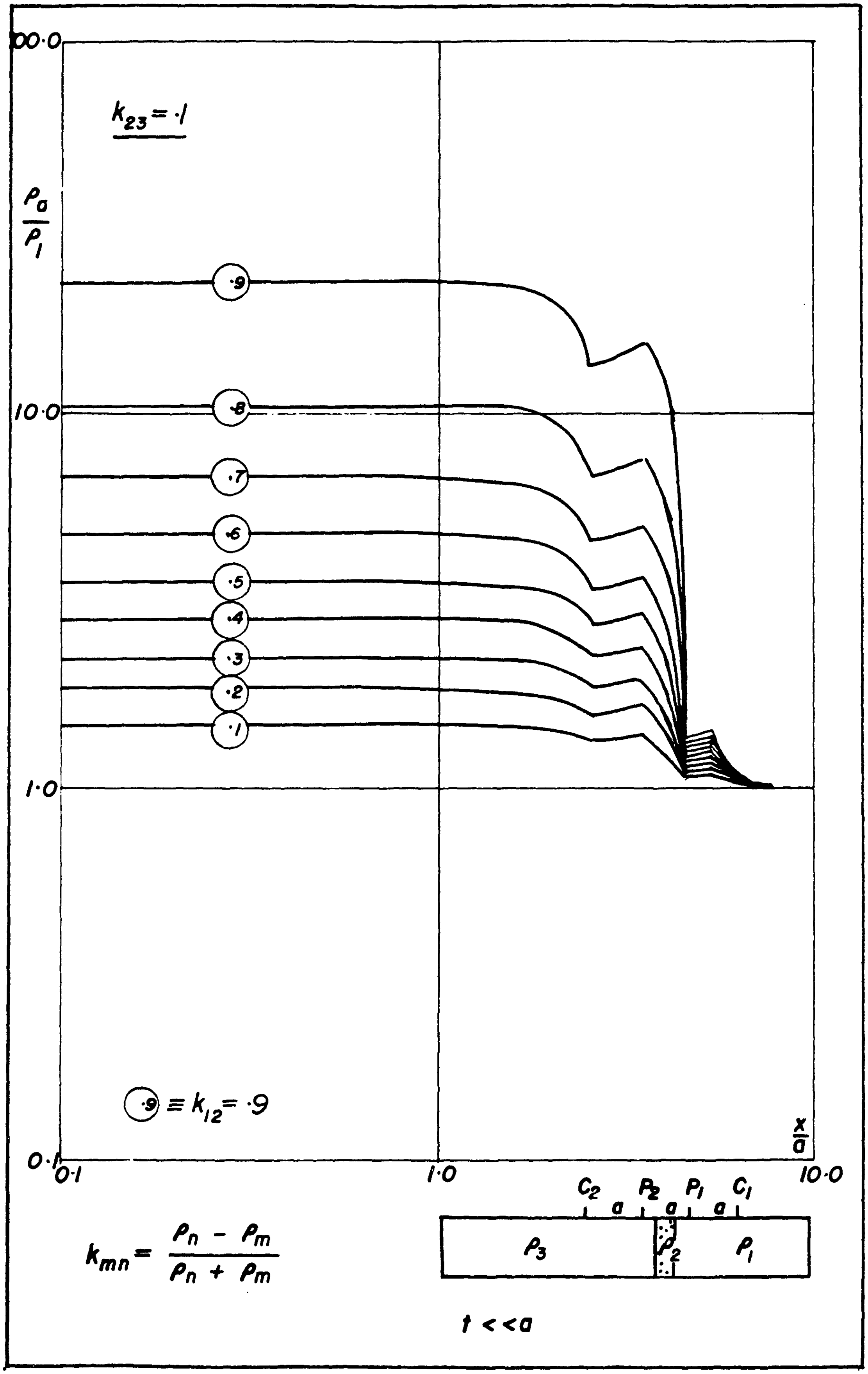


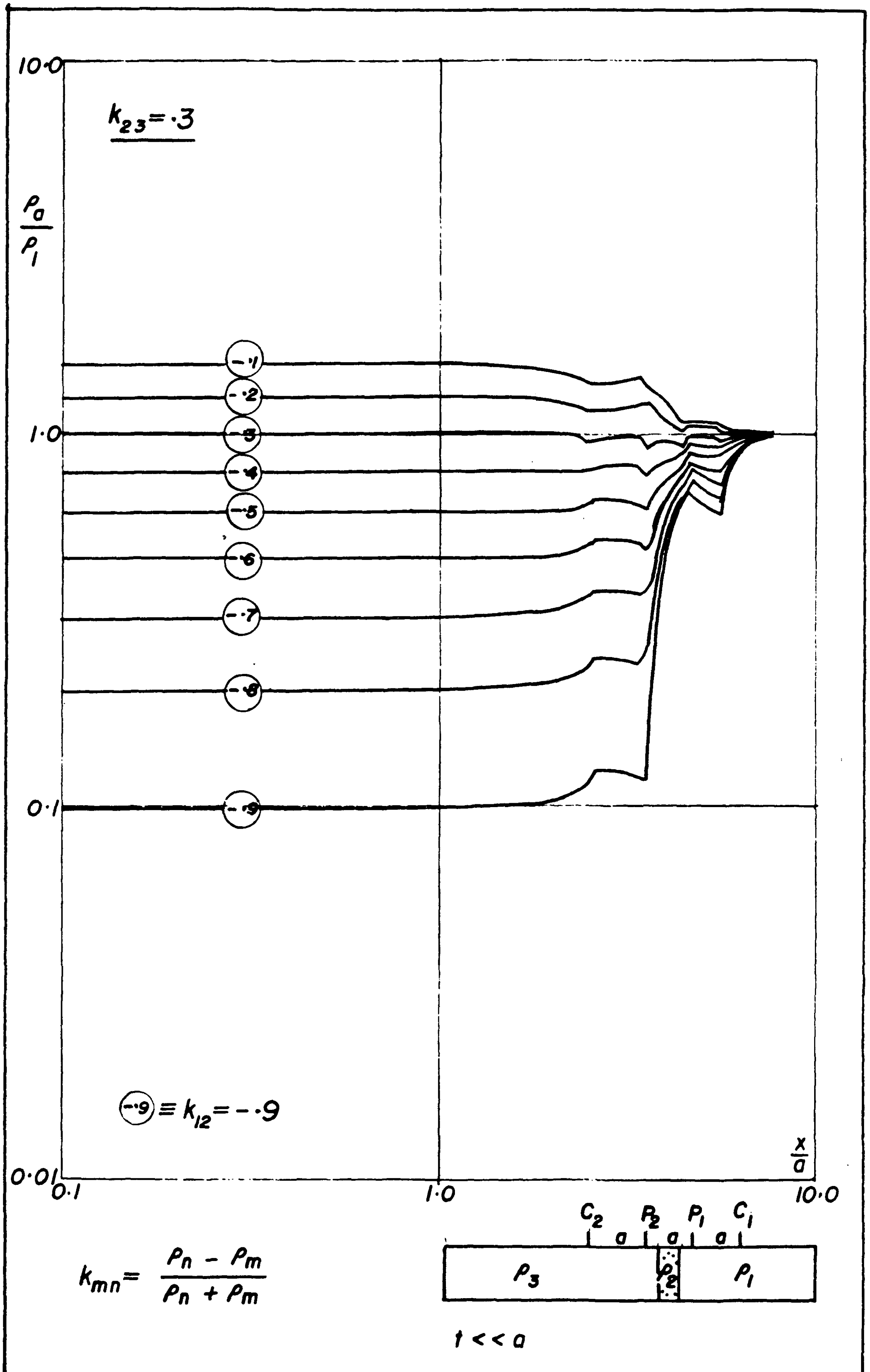
10·0
$k_{23} = -·1$
$\frac{\rho_a}{\rho_1}$
1·0
-·1
-·2
-·3
-·4
-·5
-·6
-·7
0·1
-·8
-·9
-·9 $\equiv k_{12} = -·9$
0·01
0·1
1·0
10·0
$\frac{x}{a}$
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$t << a$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$



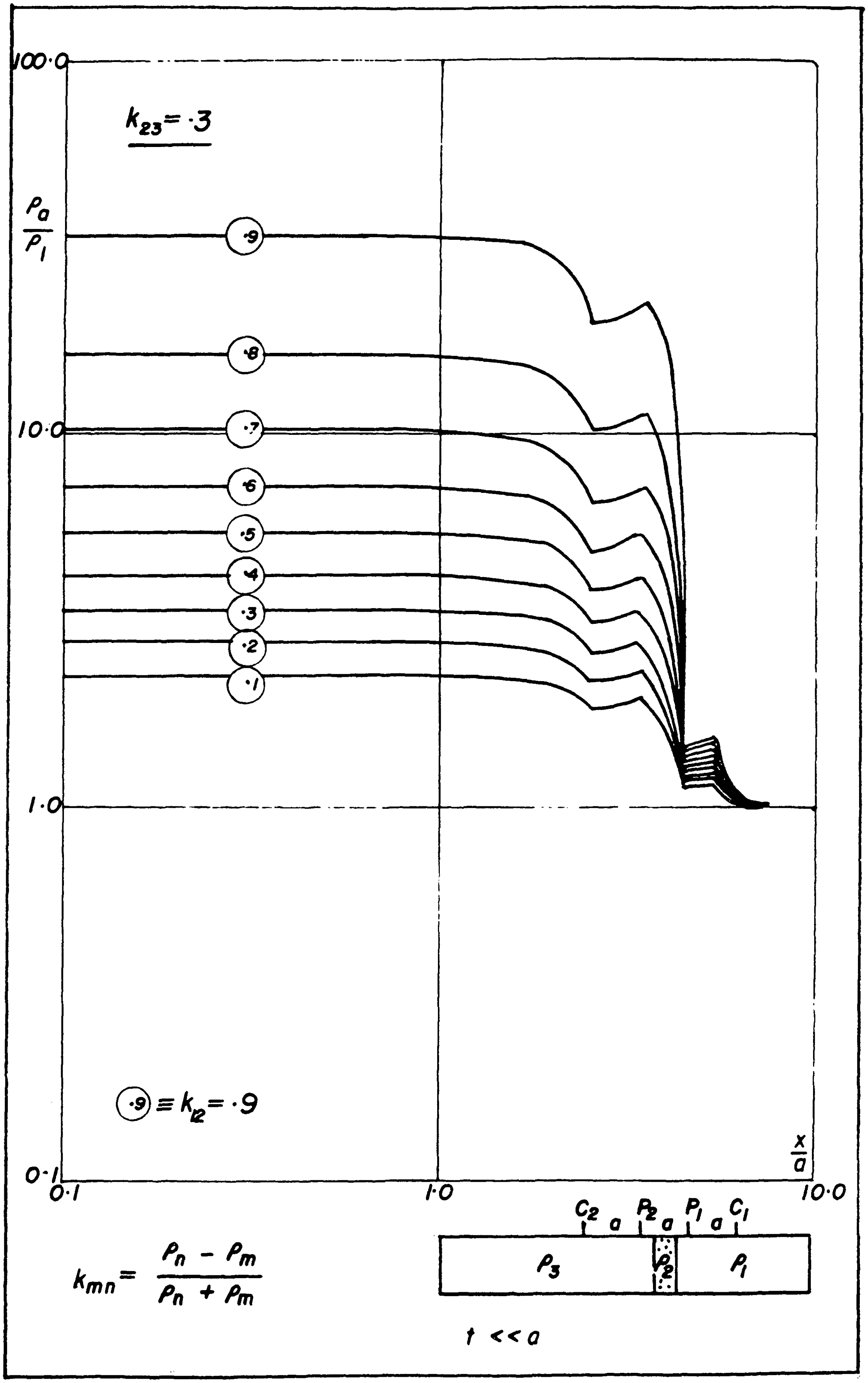
$k_{23} = -\cdot 1$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·9 $\equiv k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2
a
P_2
a
P_1
C_1
a
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$

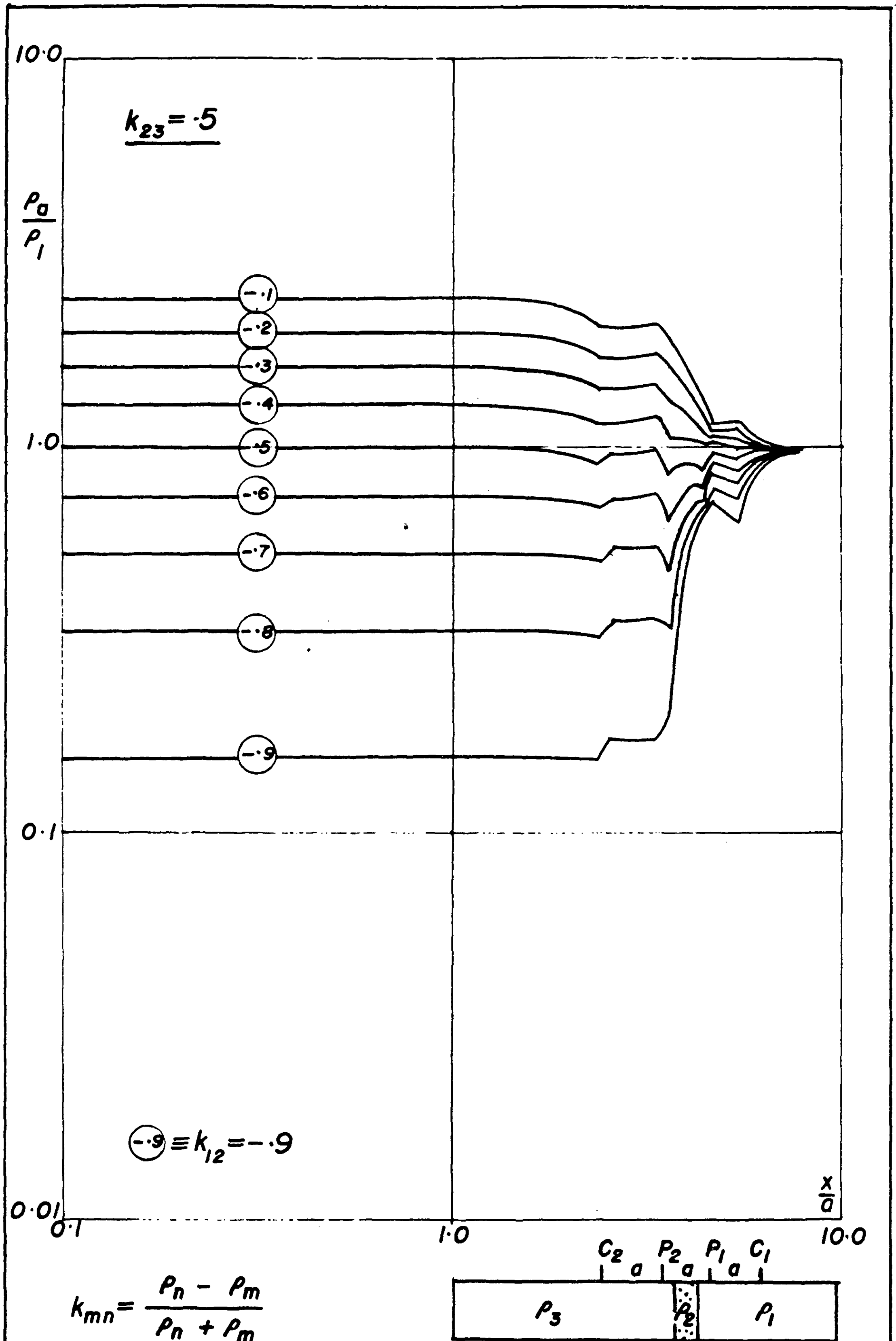

$k_{23} = \cdot 1$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
$\circled{-\cdot 9} \equiv k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$

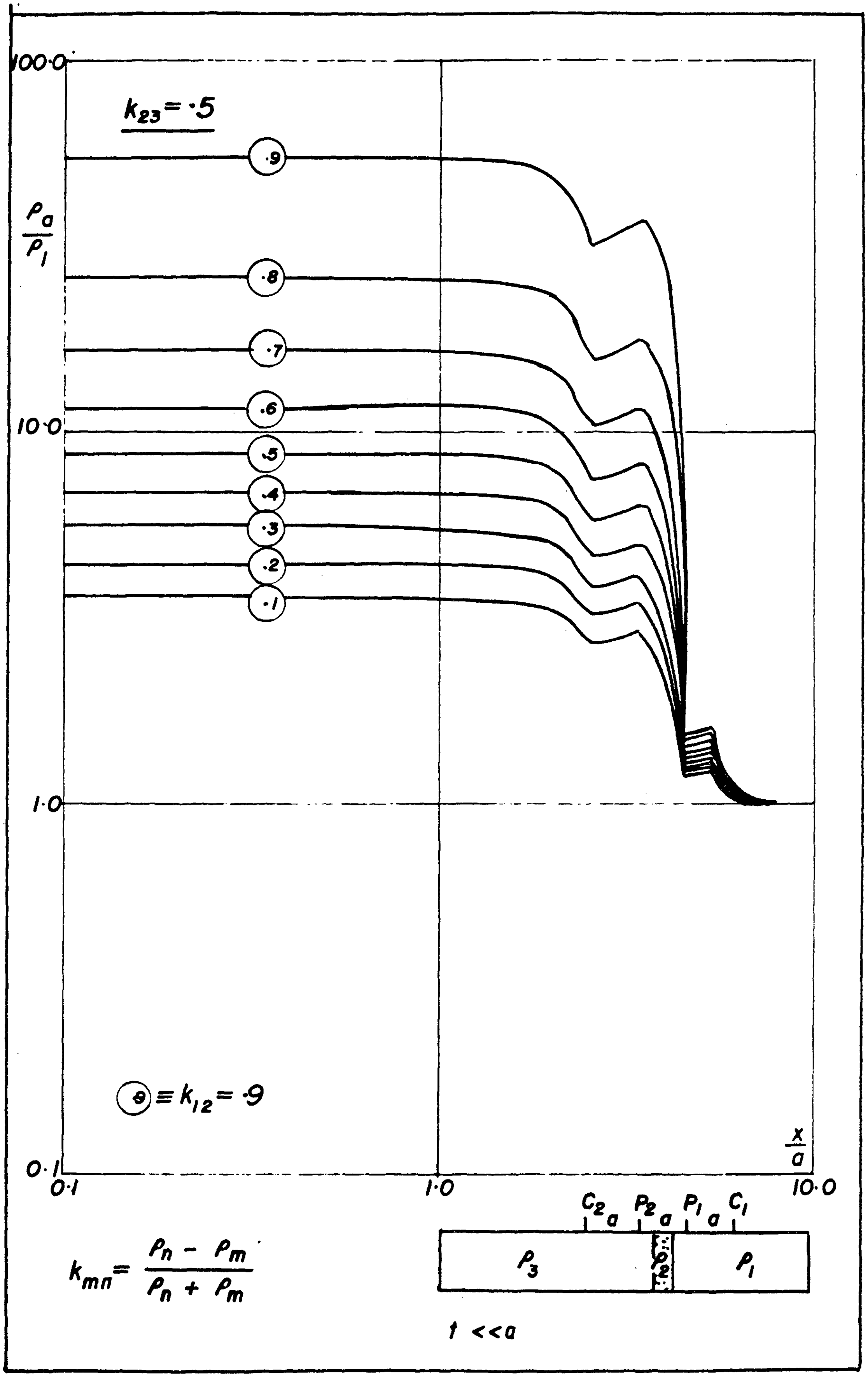

$k_{23} = \cdot 1$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
·9 ≡ $k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2
a
P_2
a
P_1
a
C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$



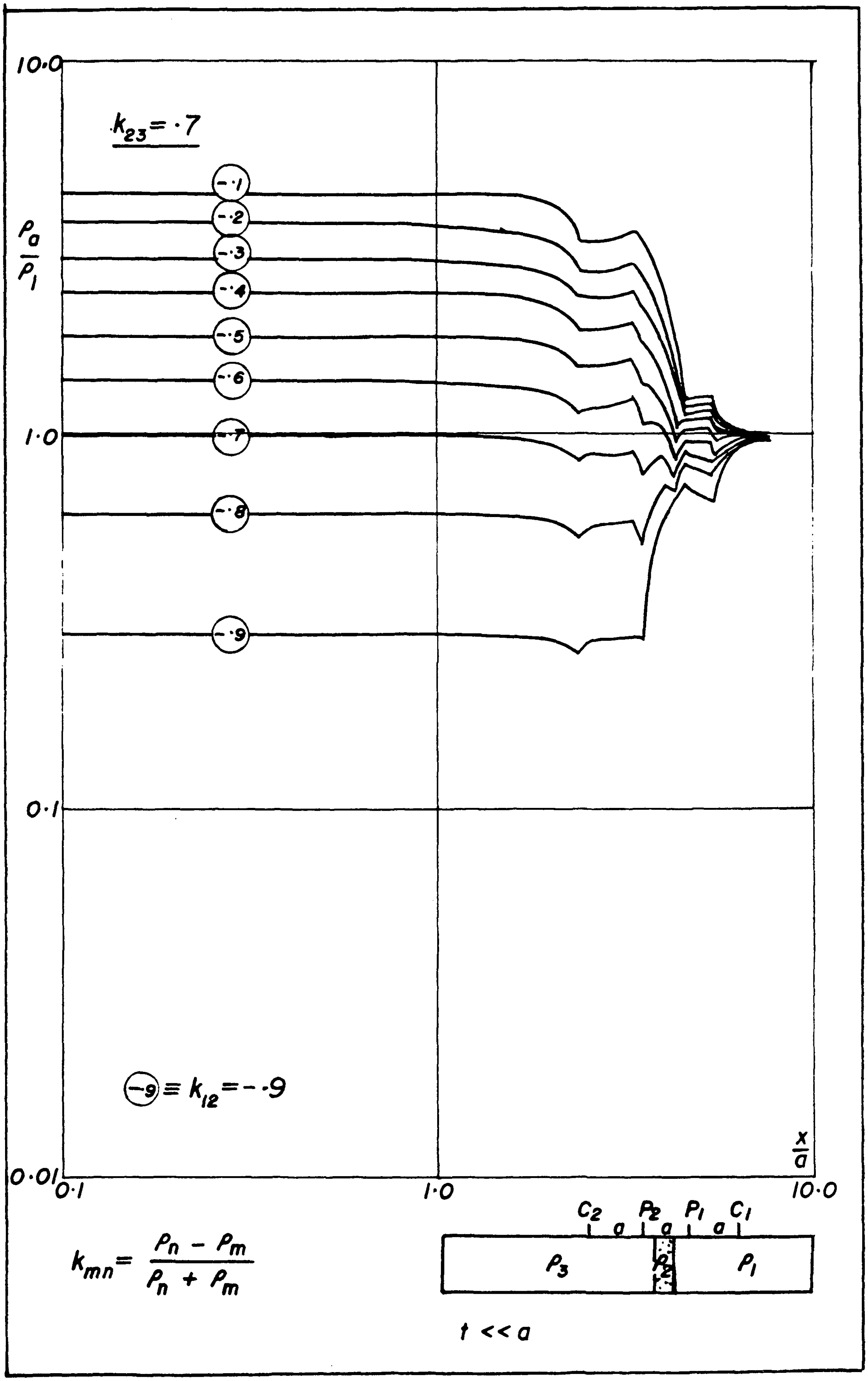
$k_{23} = \cdot 3$
$\frac{\rho_a}{\rho_1}$
10·0
1·0
0·1
0·01
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
(-·9) ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$

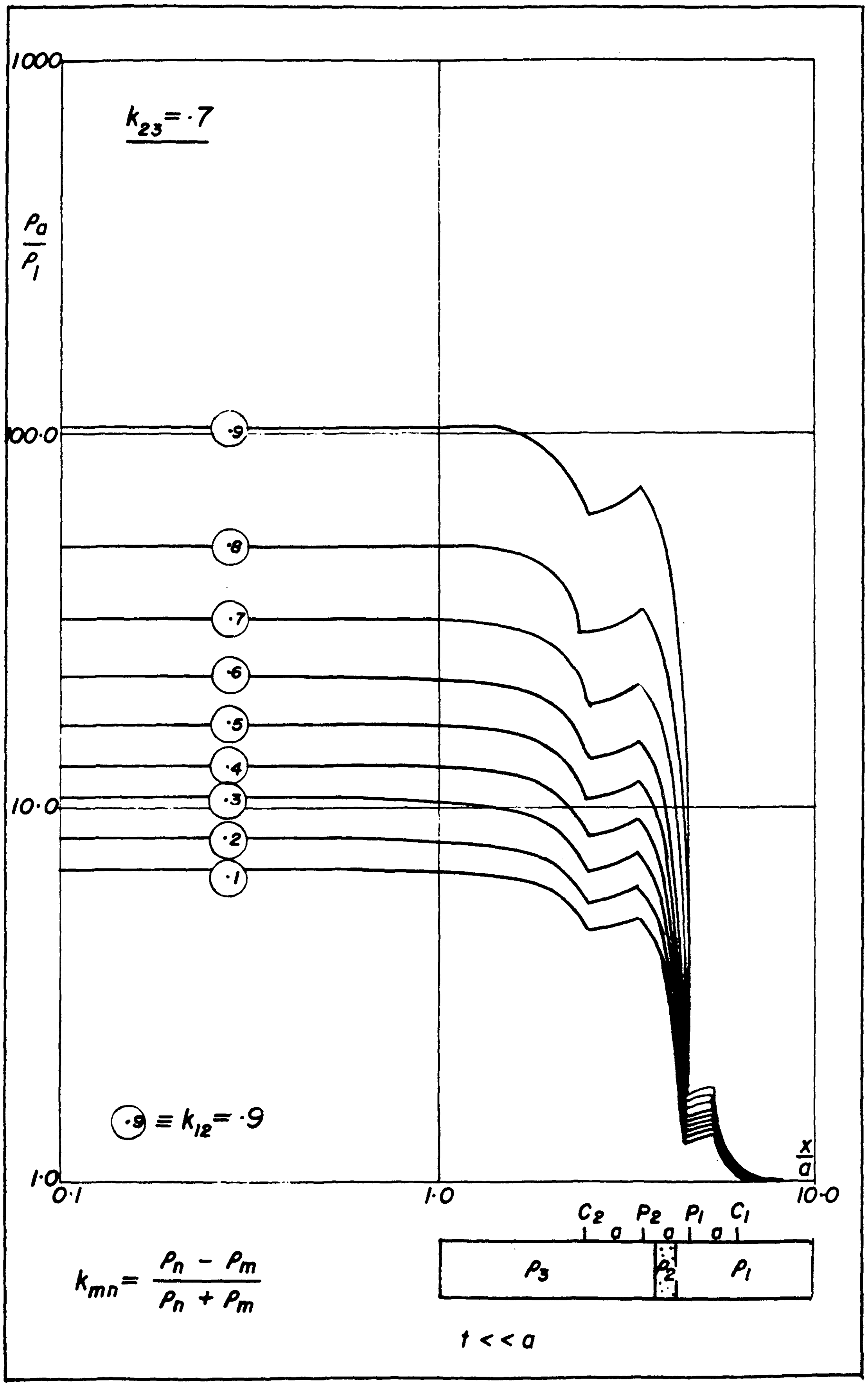

$k_{23} = \cdot 3$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
·9 ≡ $k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$


10·0
$k_{23} = \cdot 5$
$\frac{\rho_a}{\rho_1}$
−·1
−·2
−·3
−·4
−·5
−·6
−·7
−·8
−·9
1·0
0·1
−·9 ≡ $k_{12} = -\cdot 9$
0·01
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$t << a$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$

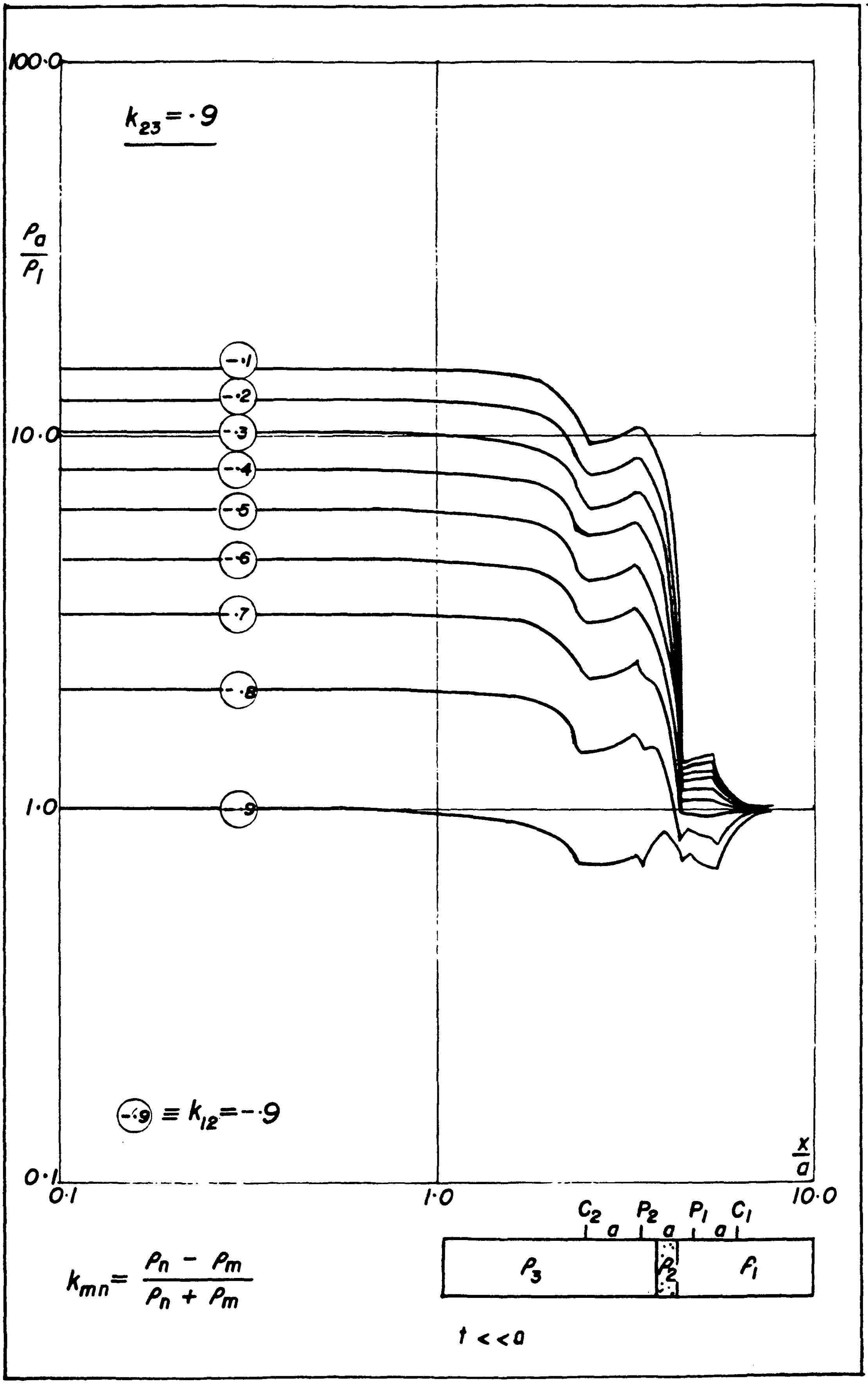


$k_{23} = \cdot 5$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
·9
·8
·7
·6
·5
·4
·3
·2
·1
$\cdot 9 \equiv k_{12} = \cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_{2a} P_{2a} P_{1a} C_1
ρ_3
ρ_2
ρ_1
$t << a$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$

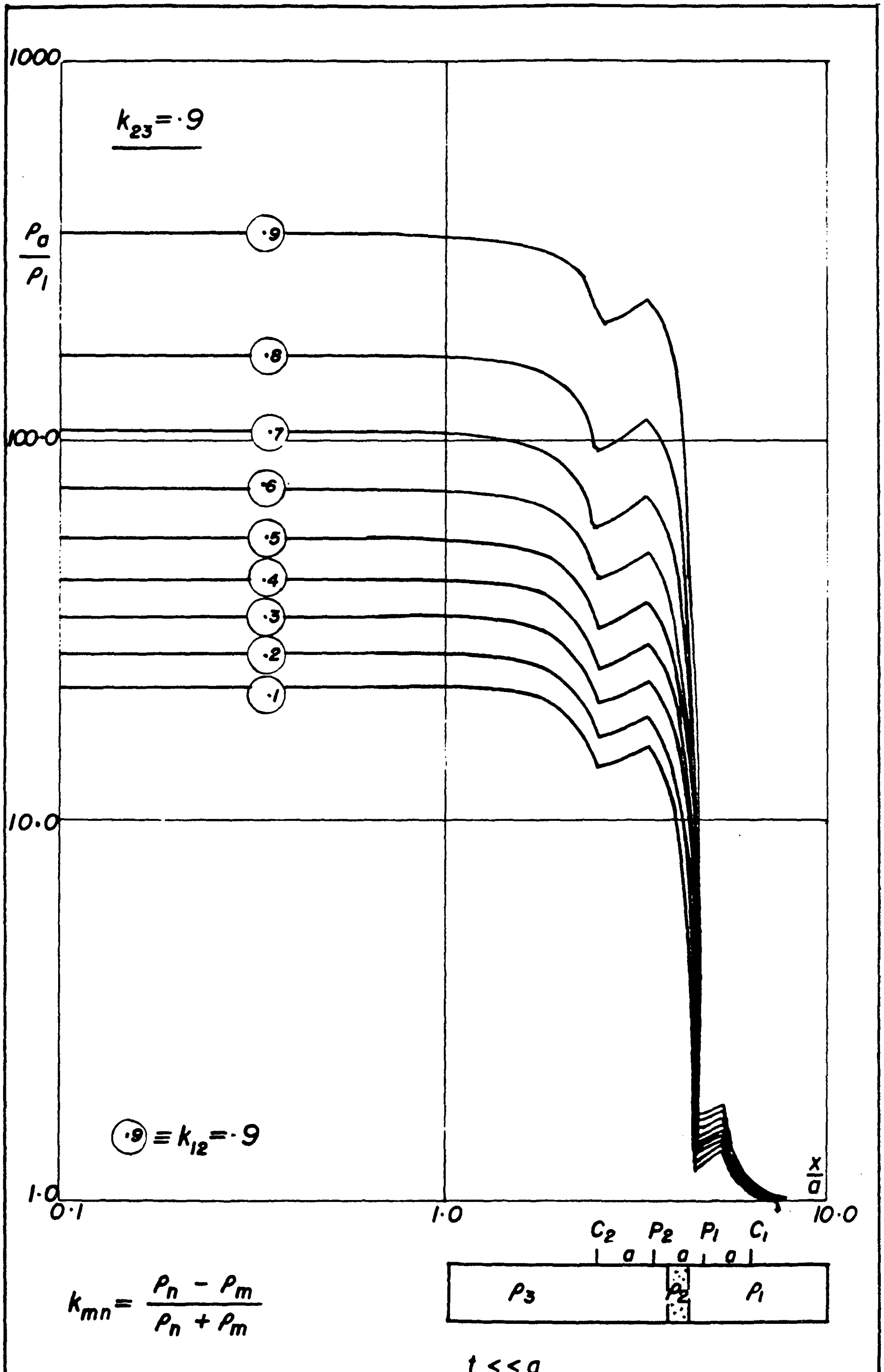

10·0
$k_{23} = \cdot 7$
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
ρ_a / ρ_1
1·0
0·1
0·01
-·9 ≡ $k_{12} = -\cdot 9$
x/a
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3 ρ_2 ρ_1
$t << a$
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$



$k_{23} = \cdot 7$
$\frac{\rho_a}{\rho_1}$
1000
100·0
10·0
1·0
0·1
1·0
10·0
$\frac{x}{a}$
·9
·8
·7
·6
·5
·4
·3
·2
·1
·9 ≡ $k_{12} = \cdot 9$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$



$k_{23} = \cdot 9$
$\frac{\rho_a}{\rho_1}$
100·0
10·0
1·0
0·1
-·1
-·2
-·3
-·4
-·5
-·6
-·7
-·8
-·9
-·9 ≡ $k_{12} = -\cdot 9$
$\frac{x}{a}$
0·1
1·0
10·0
C_2 a P_2 a P_1 a C_1
ρ_3
ρ_2
ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
t << a


$k_{23} = \cdot 9$
$\frac{\rho_a}{\rho_1}$
1000
100·0
10·0
1·0
0·1
1·0
10·0
$\frac{x}{a}$
·9
·8
·7
·6
·5
·4
·3
·2
·1
$(\cdot 9) \equiv k_{12} = \cdot 9$
C_2 P_2 P_1 C_1
a a a
ρ_3 ρ_2 ρ_1
$k_{mn} = \frac{\rho_n - \rho_m}{\rho_n + \rho_m}$
$t << a$

PROGRAM 1

```
C     A SINGLE VERTICAL PLANE
C     THE PROGRAM WILL CALCULATE A SET OF CURVES OF RHOA/RHO1 VERSUS
C     X/A FOR A GIVEN ANGLE AND A GIVEN VALUE OF A AND THE REFLECTION
C     COEFFICIENT VARYING FROM -0.9 TO 0.0 IN STEPS OF 0.1
C     ETA = X/A
C     X = THE DISTANCE FROM THE CENTRE OF THE ARRAY TO THE PLANE
C     A = THE ELECTRODE SPACING
C     RUP(I)C(I) = THE POTENTIAL AT P(I) DUE TO THE SOURCE AT C(I)
C     DIVIDED BY RHO1
C     ALPHA = THE ANGLE IN RADIANS AT WHICH THE PROFILE CROSSES THE
C     STRIKE OF THE PLANE
C     RRHO = RHOA/RHO1
      COMMON A,RK12,R2,R3,R4,R5,RUP1C1,RUP1C2,RUP2C1,RUP2C2
      READ(1,100)A,ALPHA
100   FORMAT(F5.1,F5.3)
      WRITE(3,200) ALPHA
200   FORMAT(1X,'ALPHA=',F5.3,/,1X,'___________')
      RK12=-1.0
      DO 10 I=1,10
      RK12=RK12+0.1
      WRITE(3,300)RK12
300   FORMAT(1X,'K12=',F4.1)
      ETA=-4.0
      DO 10 N=1,40
      ETA=ETA+0.2
      R2=A*SQRT(1.0+( 2.0*ABS(ETA)+SIN(ALPHA))*(2.0*ABS(ETA)+3.0*
     1SIN(ALPHA)))
      R3=A*SQRT(4.0+(2.0*ABS(ETA)-3.0*SIN(ALPHA))*(2.0*ABS(ETA)+
     1SIN(ALPHA)))
      R4=A*SQRT(4.0+(2.0*ABS(ETA)+3.0*SIN(ALPHA))*(2.0*ABS(ETA)-
     1SIN(ALPHA)))
      R5=A*SQRT(1.0+(2.0*ABS(ETA)-3.0*SIN(ALPHA))*(2.0*ABS(ETA)-
     1SIN(ALPHA)))
      IF(ETA.GT.(3.0/2.0)*SIN(ALPHA))CALL ETA1
```

```
      IF((3.0/2.0)*SIN(ALPHA).GT.ETA.AND.ETA.GT.(1.0/2.0)
     1SIN(ALPHA))CALL ETA2
      IF((1.0/2.0)*SIN(ALPHA).GT.ETA.AND.ETA.GT.0.0)CALL ETA3
      IF(0.0.GT.ETA.AND.ETA.GT.(-1.0/2.0)*SIN(ALPHA))CALL ETA4
      IF((-1.0/2.0)*SIN(ALPHA).GT.ETA.AND.ETA.GT.(-3.0/2.0)*SIN
     1(ALPHA))CALL ETA5
      IF((-3.0/2.0)*SIN(ALPHA).GT.ETA)CALL ETA6
      RV12=(RUP2C1+RUP2C2)-(RUP1C1+RUP1C2)
      RRHO=A*ABS(RV12)
      WRITE(3,400)ETA,RRHO
400   FORMAT(1X,F4.1,F10.5)
10    CONTINUE
      STOP
      END
      SUBROUTINE ETA1
      COMMON A,RK12,R2,R3,R4,R5,RUP1C1,RUP1C2,RUP2C1,RUP2C2
      RUP1C1=1.0/A+RK12/R2
      RUP1C2=-(1.0/(2.0*A)+RK12/R3)
      RUP2C1=1.0/(2.0*A)+RK12/R4
      RUP2C2=-(1.0/A+RK12/R5)
      RETURN
      END
      SUBROUTINE ETA2
      COMMON A,RK12,R2,R3,R4,R5,RUP1C1,RUP1C2,RUP2C1,RUP2C2
      RUP1C1=1.0/A+RK12/R2
      RUP1C2=-(1.0+RK12)/(2.0*A)
      RUP2C1=1/(2.0*A)+RK12/R4
      RUP2C2=-(1.0+RK12)/A
      RETURN
      END
      SUBROUTINE ETA3
      COMMON A,RK12,R2,R3,R4,R5,RUP1C1,RUP1C2,RUP2C1,RUP2C2
      RUP1C1=1.0/A+RK12/R2
      RUP1C2=-(1.0+RK12)/(2.0*A)
      RUP2C1=(1.0+RK12)/(2.0*A)
      RUP2C2=-((1.0+RK12)/(1.0-RK12))*(1.0/A-RK12/R5)
      RETURN
      END
```

```
SUBROUTINE ETA4
COMMON A,RK12,R2,R3,R4,R5,RUP1C1,RUP1C2,RUP2C1,RUP2C2
RUP1C1=1.0/A+RK12/R5
RUP1C2=-(1.0+RK12)/(2.0*A)
RUP2C1=(1.0+RK12)/(2.0*A)
RUP2C2=-((1.0+RK12)/(1.0-RK12))*(1.0/A-RK12/R2)
RETURN
END
SUBROUTINE ETA5
COMMON A,RK12,R2,R3,R4,R5,RUP1C1,RUP1C2,RUP2C1,RUP2C2
RUP1C1=(1.0+RK12)/A
RUP1C2=-((1.0+RK12)/(1.0-RK12))*(1.0/(2.0*A)-RK12/R4)
RUP2C1=(1.0+RK12)/(2.0*A)
RUP2C2=-((1.0+RK12)/(1.0-RK12))*(1.0/A-RK12/R2)
RETURN
END
SUBROUTINE ETA6
COMMON A,RK12,R2,R3,R4,R5,RUP1C1,RUP1C2,RUP2C1,RUP2C2
RUP1C1=((1.0+RK12)/(1.0-RK12))*(1.0/A-RK12/R5)
RUP1C2=-((1.0+RK12)/(1.0-RK12))*(1.0/(2.0*A)-RK12/R4)
RUP2C1=((1.0+RK12)/(1.0-RK12))*(1.0/(2.0*A)-RK12/R2)
RUP2C2=-((1.0+RK12)/(1.0-RK12))*(1.0/A-RK12/R2)
RETURN
END
```

PROGRAM 2

```
C     THE PROGRAM WILL CALCULATE A SET OF CURVES OF RHOA/RHO1
C     VERSUS X/A FOR A GIVEN ANGLE, ELECTRODE SEPPARATION AND
C     THIKNESS, T, OF THE VERTICAL STRUCTURE.
C     K12 VARIES FROM -0.9 TO +0.9 IN STEPS OF 0.1
C     K23 VARIES FROM -0.9 TO +0.9 IN STEPS OF 0.2
C     ETA=X/A
C     BETA=T/A
C     ALPHA=ANGLE IN RADIANS AT WHICH THE PROFILE CROSSES THE
C     STRIKE OF THE PLANES
C     RUT(I)=THE TOTAL POTENTIAL AT ELECTRODE I, DIVIDED BY RHO1
      COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
     1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
     1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
      READ(1,100)A,T,ALPHA
  100 FORMAT(2F5.1,F4.2)
      WRITE(3,200)ALPHA
  200 FORMAT(1X,'ALPHA=',F5.1,/,'__________')
      BETA=T/A
      RK23=-1.1
      DO 10 L=1,10
      RK23=RK23+0.2
      RK12=-1.0
      DO 10 J=1,19
      RK12=RK12+0.1
      RK21=-RK12
      WRITE(3,300)RK23
  300 FORMAT(1X,'RK23=',F4.1)
      WRITE(3,400)RK12
  400 FORMAT(1X,'RK12=',F4.1)
      ETA=-4.0
      DO 10 M=1,40
      ETA=ETA+0.2
      R2=A*SQRT(1.0+(2.0*ABS(ETA)+3.0*SIN(ALPHA))*(2.0*ABS(ETA)
     1+SIN(ALPHA))+BETA**2.0-4.0*ABS(ETA)*BETA-4.0*SIN(ALPHA)*
     1BETA)
      R3=A*SQRT(4.0+(2.0*ABS(ETA)+3.0*SIN(ALPHA))*(2.0*ABS(ETA)
```

```
      1-SIN(ALPHA))+BETA**2.0-4.0*ABS(ETA)*BETA-2.0*SIN(ALPHA)*
      1BETA)
0023   R4=A*SQRT(4.0+(2.0*ABS(ETA)+SIN(ALPHA))*(2.0*ABS(ETA)-3.0*
      1SIN(ALPHA))+BETA**2.0-4.0*ABS(ETA)*BETA+2.0*SIN(ALPHA)*
      1BETA)
0024   R5=A*SQRT(1.0+(2.0*ABS(ETA)-3.0*SIN(ALPHA))*(2.0*ABS(ETA)
      1-SIN(ALPHA))+BETA**2.0-4.0*ABS(ETA)*BETA+4.0*SIN(ALPHA)*
      1BETA)
0025   R6=A*SQRT(1.0+(2.0*ABS(ETA)-SIN(ALPHA))*(2.0*ABS(ETA)-3.0*
      1SIN(ALPHA))+BETA**2.0+4.0*ABS(ETA)*BETA-4.0*SIN(ALPHA)*
      1BETA)
0026  DO 20 K=1,10
0027  C=K
0028   RN1(K)=A*SQRT(1.0+(2.0*ABS(ETA)+3.0*SIN(ALPHA))*(2.0*ABS(
      1ETA)+SIN(ALPHA))+((2.0 C-1.0)*BETA)**2.0+4.0*ABS(ETA)*(2.0*
      1C-1.0)*BETA+4.0*SIN(ALPHA)*(2.0 C-1.0)*BETA)
0029   RN2(K)=A*SQRT(4.0+(2.0*ABS(ETA)-3.0*SIN(ALPHA))*(2.0*ABS(ETA
      1)+SIN(ALPHA))+((2.0 C-1.0)*BETA)**2.0+4.0*ABS(ETA)*(2.0*C-1.0
      1)*BETA-2.0*SIN(ALPHA)*(2.0*C-1.0)*BETA)
0030   RN3(K)=A*SQRT(4.0+(2.0*ABS(ETA)+3.0*SIN(ALPHA))*(2.0*ABS(ETA)
      1-SIN(ALPHA))+((2.0*C-1.0)*BETA)**2.0+4.0*ABS(ETA)*(2.0*C-1.0
      1)*BETA+2.0*SIN(ALPHA)*(2.0*C-1.0)*BETA)
0031   RN4(K)=A*SQRT(1.0+(2.0*ABS(ETA)-3.0*SIN(ALPHA))*(2.0*ABS(E
      1TA)-SIN(ALPHA))+((2.0*C-1.0)*BETA)**2.0+4.0*ABS(ETA)*(2.0*
      1C-1.0)*BETA-4.0*SIN(ALPHA)*(2.0*C-1.0)*BETA)
0032   RN5(K)=A*SQRT(4.0+(2.0*C*BETA)**2.0+4.0*SIN(ALPHA)*2.0*C*
      1BETA)
0033   RN7(K)=A*SQRT(1.0+(2.0*C*BETA)**2.0-2.0*SIN(ALPHA)*C*2.0*
      1BETA)
0034   RN8(K)=A*SQRT(1.0+(2.0*ABS(ETA)-3.0*SIN(ALPHA))*(2.0*ABS(E
      1TA)-SIN(ALPHA))+((2.0*C-1.0)*BETA)**2.0-4.0*ABS(ETA)*(2.0
      1*C-1.0)*BETA+4.0*SIN(ALPHA)*(2.0*C-1.0)*BETA)
0035   RN9(K)=A*SQRT(1.0+(2.0*C*BETA)**2.0+4.0*C*BETA*SIN(ALPHA))
0036   RN10(K)=A*SQRT(4.0+(2.0*ABS(ETA)-3.0*SIN(ALPHA))*(2.0*ABS(E
      1TA)+SIN(ALPHA))+((2.0*C-1.0)*BETA)**2.0-4.0*ABS(ETA)*(2.0*
      1C-1.0)*BETA+2.0*SIN(ALPHA)*(2.0*C-1.0)*BETA)
```

```
      RN11(K)=A*SQRT(4.0+(2.0*C*BETA)**2.0-8.0*C*BETA*SIN(ALPHA))
   20 CONTINUE
      IF(ETA.GT.((3.0/2.0)*SIN(ALPHA)+(1.0/2.0)*BETA))CALL POT1
      IF((3.0/2.0)*SIN(ALPHA)+(1.0/2.0)*BETA).GT.ETA.AND.ETA.GT.
     1(SIN(ALPHA)/2.0+BETA/2.0))CALL POT2
      IF((SIN(ALPHA)/2.0+BETA/2.0)GT.ETA.AND.ETA.GT.(BETA/2.0-
     1SIN(ALPHA)/2.0))CALL POT3
      IF((BETA/2.0-SIN(ALPHA)/2.0)GT.ETA.AND.ETA.GT.((3.0/2.0)
     1*SIN(ALPHA)-BETA/2.0))CALL POT4
      IF(((3.0/2.0)*SIN(ALPHA)-BETA/2.0)GT.ETA.AND.ETA.GT.0.0)
     1CALL POT5
      IF(0.0.GT.ETA.AND.ETA.GT.(BETA/2.0-(3.0/2.0)*SIN(ALPHA)))
     1CALL POT6
      IF((BETA/2.0-(3.0/2.0)*SIN(ALPHA)).GT.ETA.AND.ETA.GT.(S
     1IN(ALPHA)/2.0-BETA/2.0))CALL POT7
      IF((SIN(ALPHA)/2.0-BETA/2.0).GT.ETA.AND.ETA.GT.(-SIN(ALP
     1HA)/2.0-BETA/2.0))CALL POT8
      IF((-SIN(ALPHA)/2.0-BETA/2.0).GT.ETA.AND.ETA.GT.((-3.0/2
     1.0)*SIN(ALPHA)-BETA/2.0))CALL POT9
      IF(((-3.0/2.0)*SIN(ALPHA)-BETA/2.0).GT.ETA)CALL POT10
      RV12=RUTP2-RUTP1
      RRHO=A*ABS(RV12)
      WRITE(3,500)ETA,RRHO
  500 FORMAT(1X,F4.1,F10.5)
   10 CONTINUE
      STOP
      END
      SUBROUTINE POT1
      COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
     1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
     1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
      DO 1 N=1,10
      TERM=(1.0+RK12)*(1.0-RK12)*(((RK21**(N-1))*(RK23**N))
     1*(1.0/RN1(N)-1.0/RN2(N)))
      IF(ABS(TERM).LE.0.05.OR.N.EQ.10)GO TO 2
    1 CONTINUE
    2 NMAX=N
      I=NMAX
```

```
0064        RUTP1=1.0/(2.0*A)+RK12*(1.0/R2-1.0/R4)
0065 4      XTERM=(1.0+RK12)*(1.0-RK12)*(((RK21**(I-1))*(RK23**I))*
           1(1.0/RN1(I)-1.0/RN2(I)))
0070        RUTP1=RUTP1+XTERM
0071        I=I-1
0072        IF(I.LE.0)GO TO 3
0073        GO TO 4
0074 3      DO 5 M=1,10
0075        TERM2=(1.0+RK12)*(1.0-RK12)*(((RK21**(M-1))*(RK23**M))*(
           11.0/RN3(M)-1.0/RN4(M)))
0076        IF(ABS(TERM2).LE.0.05.OR.M.EQ.10)GO TO 6
0077 5      CONTINUE
0078 6      NMAX2=M
0079        J=NMAX2
0080        RUTP2=-1.0/(2.0*A)+RK12*(1.0/R3-1.0/R5)
0081 8      XTERM2=(1.0+RK12)*(1.0-RK12)*(((RK21**(J-1))*(RK23**J))*(
           11.0/RN3(J)-1.0/RN4(J)))
0082        RUTP2=RUTP2+XTERM2
0083        J=J-1
0084        IF(J.LE.0)GO TO 7
0085        GO TO 8
0086 7      RETURN
0087        END
0088        SUBROUTINE POT2
0089        COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
           1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
           1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
0090        DO 1 N=1,10
0091        TERM=(1.0+RK12)*((RK21**N)*(RK23**N)*((1.0-RK12)/(RK21*RN1
           1(N))-1.0/(RK21*RN2(N))-1.0/RN6(N)))
0092        IF(ABS(TERM).LE.0.05.OR.N.EQ.10) GO TO 2
0093 1      CONTINUE
0094 2      NMAX=N
0095        I=NMAX
0096        RUTP1=1.0/(2.0*A)+RK12*(1.0/R2-1.0/(2.0*A))
0097 4      XTERM=(1.0+RK12)*((RK21**I)*(RK23**I)*((1.0-RK12)/(RK21*RN1
           1(I))-1.0/(RK21*RN2(I))-1.0/RN6(I)))
0098        RUTP1=RUTP1+XTERM
```

```
0099        I=I-1
0100        IF(I.LE.0.0)GO TO 3
0101        GO TO 4
0102 3      DO 5 M=1,10
0103        TERM2=(1.0+RK12)*((RK21**M)*(RK23**M)*((1.0-RK12)/(RK21*
           1RN3(M))-1.0/(RK21*RN4(M))-1.0/RN9(M)))
0104        IF(ABS(TERM2).LE.0.05.OR.M.EQ.10)GO TO 6
0105 5      CONTINUE
0106 6      NMAX2=M
0107        J=NMAX2
0108        RUTP2=-1.0/(2.0*A)+RK12*(1.0/R3-1.0/A)
0109 8      XTERM2=(1.0+RK12)*((RK21**J)*(RK23**J)*((1.0-RK12)/(RK21*
           1RN3(J))-1.0/(RK21*RN4(J))-1.0/RN9(J)))
0110        RUTP2=RUTP2+XTERM2
0111        J=J-1
0112        IF(J.LE.0)GO TO 7
0113        GO TO 8
0114 7      RETURN
0115        END
0116        SUBROUTINE POT3
0117        COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
           1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
           1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
0118        DO 1 N=1,10
0119        TERM= (1.0+RK12)*((RK21** N )*(RK23** N )*((1.0-RK12)/(
           1RK21*RN1(N))-1.0/(RK21*RN2(N))-1.0/RN6(N)))
0120        IF(ABS(TERM).LE.0.05.OR.N.EQ.10)GO TO 2
0121 1      CONTINUE
0122 2      NMAX=N
0123        I=NMAX
0124        RUTP1=1.0/(2.0*A)+RK12*(1.0/R2-1.0/(2.0*A))
0125 4      XTERM=(1.0+RK12)*((RK21** I )*(RK23** I )*((1.0-RK12)/(
           1RK21*RN1(I))-1.0/(RK21*RN2(I))-1.0/RN6(I)))
0126        RUTP1=RUTP1+XTERM
0127        I=I-1
0128        IF(I.LE.0)GO TO 3
0129        GO TO 4
0130 3      DO 5 M=1,10
0131        TERM2=((1.0+RK12)/(1.0-RK12))*RK21**M*RK23**M*((1.0-RK12)
```

```
     1/(RK21*RN3(M))+(1.0-RK12)/RN6(M)-1.0/(RK21*RN4(M))-
     11.0/RN7(M)-1.0/RN9(M)-1.0/(RK23*RN8(M)))
      IF(ABS(TERM).LE.0.05.OR.M.EQ.10)GO TO 6
 5    CONTINUE
 6    NMAX2=M
      J=NMAX2
      RUTP2=((1.0+RK12)/(1.0-RK12))*(-1.0/A+(1.0-RK12)/(2.0*A))
 8    XTERM2=((1.0+RK12)/(1.0-RK12))*RK21**J*RK23**J*((1.0-
     1RK12)/(RK21*RN3(J))+(1.0-RK12)/RN6(J)-1.0/(RK21*RN4(J))
     1-1.0/RN7(J)-1.0/RN9(J)-1.0/(RK23*RN8(J)))
      RUTP2=RUTP2+XTERM2
      J=J-1
      IF(J.LE.0)GO TO 7
      GO TO 8
 7    RETURN
      END
      SUBROUTINE POT4
      COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
     1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
     1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
      DO 1 N=1,10
      TERM=((1.0+RK12)/(1.0-RK12))*RK21**N*RK23**N*((1.0-RK12)
     1/(RK21*RN1(N))+(1.0-RK12)/RN9(N)-1.0/(RK21*RN2(N))-1.0/RN11(N)
     1-1.0/RN6(N)-1.0/(RK23*RN10(N)))
      IF(ABS(TERM).LE.0.05.OR.N.EQ.10)GO TO 2
 1    CONTINUE
 2    NMAX=N
      I=NMAX
      RUTP1=((1.0+RK12)/(1.0-RK12))*(-1.0/(2.0*A)+1.0-RK12)/A)
 4    XTERM= ((1.0+RK12)/(1.0-RK12))*RK21**I*RK23**I*((1.0-RK12)
     1/(RK21*RN1(I))+(1.0-RK12)/RN9(I)-1.0/(RK21*RN2(I))-1.0/RN
     111(I)-1.0/RN6(I)-1.0/(RK23*RN10(I)))
      RUTP1=RUTP1+XTERM
      I=I-1
      IF(I.LE.0)GO TO 3
      GO TO 4
 3    DO 5 M=1,10
      TERM2= ((1.0+RK12)/(1.0-RK12))*RK21**M*RK23**M*((1.0-RK12)/
     1(RK21*RN3(M))+(1.0-RK12)/RN6(M)-1.0/(RK21*RN4(M))-1.0/RN7(M)
```

```
     1-1.0/RN9(M)-1.0/(RK23*RN8(M)))
0160   IF(ABS(TERM).LE.0.05.OR.M.EQ.10)GO TO 6
0161 5 CONTINUE
0162 6 NMAX2=M
0163   J=NMAX2
0164   RUTP2=((1.0+RK12)/(1.0-RK12))*(-1.0/A+(1.0-RK12)/(2.0*A))
0165 8 XTERM2=((1.0+RK12)/(1.0-RK12))*RK21**J*RK23**J*((1.0-RK12
     1)/(RK21*RN3(J))+(1.0-RK12)/RN6(J)-1.0/(RK21*RN4(J))-1.0/
     1RN7(J)-1.0/RN9(J)-1.0/(RK23*RN8(J)))
0166   RUTP2=RUTP2+XTERM2
0167   J=J-1
0168   IF(J.LE.0)GO TO 7
0169   GO TO 8
0170   RETURN
0171   END
0172   SUBROUTINE POT5
0173   COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
     1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
     1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
0174   DO 1 N=1,10
0175   TERM=((1.0+RK12)/(1.0-RK12))*RK21**N*RK23**N*((1.0-RK12)/
     1(RK21*RN1(N))+(1.0-RK12)/RN9(N)-(1.0+RK23)/(RK23*RN10(N))
     1-(1.0+RK23)/RN6(N))
0176   IF(ABS(TERM).LE.0.05.OR.N.EQ.10)GO TO 2
0177 1 CONTINUE
0178 2 NMAX=N
0179   I=NMAX
0180   RUTP1=((1.0+RK12)/(1.0-RK12))*1.0/(2.0*A)-(2.0*RK12+RK23)
     1/(2.0*A))
0181 4 XTERM= ((1.0+RK12)/(1.0-RK12))*RK21**I*RK23**I*((1.0-RK12
     1)/(RK21*RN1(I))+(1.0-RK12)/RN9(I)-(1.0+RK23)/(RK23*RN10(I))
     1-(1.0+RK23)/RN6(I))
0182   RUTP1=RUTP1+XTERM
0183   I=I-1
0184   IF(I.LE.0)GO TO 3
0185   GO TO 4
```

```
3     DO 5 M=1,10
      TERM2=((1.0+RK12)/(1.0-RK12))*RK21**M*RK23**M*((1.0-RK12)
     1/(RK21*RN3(M))+(1.0-RK12)/RN6(M)-(1.0+RK23)/(RK23*RN8(M))
     1-(1.0+RK23)/RN9(M))
      IF(ABS(TERM).LE.0.05.OR.M.EQ.1.0)GO TO 6
5     CONTINUE
6     NMAX2=M
      J=NMAX2
      RUTP2=((1.0+RK12)/(1.0-RK12))*(-1.0/(2.0*A)-(RK12+2.0*RK
     123)/(2.0*A))
8     XTERM2=((1.0+RK12)/(1.0-RK12))*RK21**J*RK23**J*((1.0-RK12)
     1/(RK21*RN3(J))+(1.0-RK12)/RN6(J)-(1.0+RK23)/(RK23*RN8(J))
     1-(1.0+RK23)/RN9(J))
      RUTP2=RUTP2+XTERM2
      J=J-1
      IF(J.LE.0)GO TO 7
      GO TO 8
7     RETURN
      END
      SUBROUTINE POT6
      COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
     1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
     1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
      DO 1 N=1,10
      TERM=((1.0+RK12)/(1.0-RK12))*RK21**N*RK23**N*((1.0-RK12)
     1/(RK21*RN8(N))+(1.0-RK12)/RN9(N)-(1.0+RK23)/(RK23*RN3(N))
     1-(1.0+RK23)/RN6(N))
      IF(ABS(TERM).LE.0.05.OR.N.EQ.10)GO TO 2
1     CONTINUE
2     NMAX=N
      I=NMAX
      RUTP1=((1.0+RK12)/(1.0-RK12))*(1.0/(2.0*A)-(2.0*RK12+RK23
     1)/(2.0*A))
4     XTERM=((1.0+RK12)/(1.0-RK12))*RK21**I*RK23**I*((1.0-RK12)/
     1(RK21*RN8(I))+(1.0-RK12)/RN9(I)-(1.0+RK23)/(RK23*RN3(I))
     1-(1.0+RK23)/RN6(I))
      RUTP1=RUTP1+XTERM
```

```
0211        I=I-1
0212        IF(I.LE.0)GO TO 3
0213        GO TO 4
0214 3      DO 5 M=1,10
0215        TERM2=((1.0+RK12)/(1.0-RK12))*RK21**M*RK23**M*((1.0-RK12
           1)/(RK21*RN10(M))+(1.0-RK12)/RN6(M)-(1.0+RK23)/(RK23*RN11(M))
           1-(1.0+RK23)/RN9(M))
0216        IF(ABS(TERM).LE.0.05.OR.M.EQ.10)GO TO 6
0217 5      CONTINUE
0218 6      NMAX2=M
0219        J=NMAX2
0220        RUTP2=((1.0+RK12)/(1.0-RK12))*(-1.0/(2.0*A)-(RK12+2.0*RK23
           1)/(2.0*A))
0221 8      XTERM2=((1.0+RK12)/(1.0-RK12))*RK21**J*RK23**J*((1.0-RK12
           1)/(RK21*RN10(J))+(1.0-RK12)/RN6(J)-(1.0+RK23)/(RK23*RN11(J))
           1-(1.0+RK23)/RN9(J))
0222        RUTP2=RUTP2+XTERM2
0223        J=J-1
0224        IF(J.LE.0)GO TO 7
0225        GO TO 8
0226 7      RETURN
0227        END
0228        SUBROUTINE POT7
0229        COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
           1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
           1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
0230        DO 1 N=1,10
0231        TERM=((1.0+RK12)/(1.0-RK12))*RK21**N*RK23**N*(1.0/(RK21
           1*RN8(N))+1.0/RN9(N)+1.0/RN7(N)+1.0/(RK23*RN4(N))-(1.0+
           1RK23)/(RK23*RN3(N))-(1.0+RK23)/RN6(N))
0232        IF(ABS(TERM).LE.0.05.OR.N.EQ.10)GO TO 2
0233 1      CONTINUE
0234 2      NMAX=N
0235        I=NMAX
0236        RUTP1=((1.0+RK12)/(1.0-RK12))*(1.0/A-(1.0+RK23)/(2.0*A))
0237 4      XTERM=((1.0+RK12)/(1.0-RK12))*RK21**I*RK23**I*(1.0/(RK21*
```

```
          1RN8(I))+1.0/RN9(I)+1.0/RN7(I)+1.0/(RK23*RN4(I))-(1.0+RK23)
          1/(RK23*RN3(I))-(1.0+RK23)/RN6(I))
0238       RUTP1=RUTP1+XTERM
0239       I=I-1
0240       IF(I.LE.0)GO TO 3
0241       GO TO 4
0242 3     DO 5 M=1,10
0243       TERM2=((1.0+RK12)/(1.0-RK12))*RK21**M*RK23**M*(1.0/(RK21
          1*RN10(M))+1.0/RN6(M)+1.0/RN11(M)+1.0/(RK23*RN2(M))-(1.0+
          1RK23)/(RK23*RN1(M))-(1.0+RK23)/RN9(M))
0244       IF(ABS(TERM2).LE.0.05.OR.M.EQ.10)GO TO 6
0245 5     CONTINUE
0246 6     NMAX2=M
0247       J=NMAX2
0248       RUTP2=((1.0+RK12)/(1.0-RK12))*(1.0/(2.0*A)-(1.0+RK23)/A)
0249 8     XTERM2=((1.0+RK12)/(1.0-RK12))*RK21**J*RK23**J*(1.0/(RK21
          1*RN10(J))+1.0/RN6(J)+1.0/RN11(J)+1.0/(RK23*RN2(J))-(1.0+RK
          123)/(RK23*RN1(J))-(1.0+RK23)/RN9(J))
0250       RUTP2=RUTP2+XTERM2
0251       J=J-1
0252       IF(J.LE.0)GO TO 7
0253       GO TO 8
0254 7     RETURN
0255       END
0256       SUBROUTINE POT8
0257       COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
          1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
          1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
0258       DO 1 N=1,10
0259       TERM=((1.0+RK12)/(1.0-RK12))*RK21**N*RK23**N*(1.0/(RK21*
          1RN8(N))+1.0/RN9(N)+1.0/RN7(N)+1.0/(RK23*RN4(N))-(1.0+RK
          123)/(RK23*RN3(N))-(1.0+RK23)/RN6(N))
0260       IF(ABS(TERM).LE.0.05.OR.N.EQ.10)GO TO 2
0261 1     CONTINUE
0262 2     NMAX=N
0263       I=NMAX
```

```
0264       RUTP1=((1.0+RK12)/(1.0-RK12))*(1.0/A-(1.0+RK23)/(2.0*A))
0265 4     XTERM=((1.0+RK12)/(1.0-RK12))*RK21**I*RK23**I*(1.0/(RK21
          1*RN8(I))+1.0/RN9(I)+1.0/RN7(I)+1.0/(RK23*RN4(I))-(1.0+RK23
          1)/(RK23*RN3(I))-(1.0+RK23)/RN6(I))
0266       RUTP1=RUTP1+XTERM
0267       I=I-1
0268       IF(I.LE.0)GO TO 3
0269       GO TO 4
0270 3     DO 5 M=1,10
0271       TERM2=(((1.0+RK12)*(1.0+RK23))/(1.0-RK12))*RK21**M*RK23
          1**M*(1.0/(RK23*RN2(M))+1.0/RN6(M)-(1.0+RK23)/(RK23*RN1(M)))
0272       IF(ABS(TERM2).LE.0.05.OR.M.EQ.10)GO TO 6
0273 5     CONTINUE
0274 6     NMAX2=M
0275       J=NMAX2
0276       RUTP2=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*(-1.0
          1/A+RK23/R2+(1.0-RK23)/(2.0*A))
0277 8     XTERM2=(((1.0+RK12)*(1.0+RK23))/(1.0-RK12))*RK21**J*RK23
          1**J*(1.0/(RK23*RN2(J))+1.0/RN6(J)-(1.0+RK23)/(RK23*RN1(J)))
0278       RUTP2=RUTP2+XTERM2
0279       J=J-1
0280       IF(J.LE.0)GO TO 7
0281       GO TO 8
0282 7     RETURN
0283       END
0284       SUBROUTINE POT9
0285       COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
          1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
          1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
0286       DO 1 N=1,10
0287       TERM=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**N*RK23**N
          1*(1.0/RN9(N)+1.0/(RK23*RN4(N))-(1.0+RK23)/(RK23*RN3(N)))
0288       IF(ABS(TERM).LE.0.05.OR.N.EQ.10)GO TO 2
0289 1     CONTINUE
0290 2     NMAX=N
0291       I=NMAX
0292       RUTP1=(((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23)))*
          1(1.0/(2.0*A)+RK23/R3+(1.0-RK23)/A)
0293 4     XTERM=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**I*RK23**I
```

```
     1*(1.0/RN9(I)+1.0/(RK23*RN4(I))-(1.0+RK23)/(RK23*RN3(I)))
      RUTP1=RUTP1+XTERM
      I=I-1
      IF(I.LE.0)GO TO 3
      GO TO 4
    3 DO 5 M=1,10
      TERM2=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**M*RK23**M*
     1(1.0/RN6(M)+1.0/(RK23*RN2(M))-(1.0+RK23)/(RK23*RN1(M)))
      IF(ABS(TERM2).LE.0.05.OR.M.EQ.10)GO TO 6
    5 CONTINUE
    6 NMAX2=M
      J=NMAX2
      RUTP2=(((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23)))*(
     11-1.0/A+RK23/R2+(1.0-RK23)/(2.0*A))
    8 XTERM2=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**J*RK23**J*
     1(1.0/RN6(J)+1.0/(RK23*RN2(J))-(1.0+RK23)/(RK23*RN1(J)))
      RUTP2=RUTP2+XTERM2
      J=J-1
      IF(J.LE.0)GO TO 7
    7 RETURN
      END
      SUBROUTINE POT10
      COMMON A,RK12,RK21,RK23,R2,R3,R4,R5,R6,RN1(10),RN2(10),
     1RN3(10),RN4(10),RN6(10),RN7(10),RN8(10),RN9(10),RN10(10),
     1RN11(10),NMAX,NMAX2,RUTP1,RUTP2
      DO 1 N=1.10
      TERM=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*RK21**N
     1*RK23**(N-1)*(1.0/RN4(N)-1.0/RN3(N))*(1.0-RK23**2)
      IF(ABS(TERM).LE.0.05.OR.N.EQ.10)GO TO 2
    1 CONTINUE
    2 NMAX=N
      I=NMAX
      RUTP1=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*(1.0/
     1(2.0*A)-RK23*(1.0/R5-1.0/R3))
    4 XTERM=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*RK21
     1**I*RK23**(I-1)*(1.0/RN4(I)-1.0/RN3(I))*(1.0-RK23**2)
```

```
0321       RUTP1=RUTP1+XTERM
0322       I=I-1
0323       IF(I.LE.0)GO TO 3
0324       GO TO 4
0325 3     DO 5 M=1,10
0326       TERM2=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*RK21
          1**M*RK23**(M-1)*(1.0/RN2(M)-1.0/RN1(M))*(1.0-RK23**2)
0327       IF(ABS(TERM2).LE.0.05.OR.M.EQ.10)GO TO 6
0328 5     CONTINUE
0329 6     NMAX2=M
0330       J=NMAX2
0331       RUTP2=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*(-1.0
          1/(2.0*A)-RK23*(1.0/R4-1.0/R2))
0332 8     XTERM2=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*RK21
          1**J*RK23**(J-1)*(1.0/RN2(J)-1.0/RN1(J))*(1.0-RK23**2)
0333       RUTP2=RUTP2+XTERM2
0334       J=J-1
0335       IF(J.LE.0)GO TO 7
0336       GO TO 8
0337 7     RETURN
0338       END
```

PROGRAM 3

This program is the same as PROGRAM 2 except for the following statements that substitute the corresponding statements in PROGRAM 2.

```
0041       IF((SIN(ALPHA)/2.0+BETA/2.0).GT.ETA.AND.ETA.GT.((3.0/2.0)
          1*SIN(ALPHA)-BETA/2.0))CALL POT3
0042       IF(((3.0/2.0)*SIN(ALPHA)-BETA/2.0).GT.ETA.AND.ETA.GT.(-SIN
          1(ALPHA)/2.0+BETA/2.0))CALL POT4
0043       IF((-SIN(ALPHA)/2.0+BETA/2.0).GT.ETA.AND.ETA.GT.0.0)CALL
          1POT5
0044       IF(0.0.GT.ETA.AND.ETA.GT.(SIN(ALPHA)/2.0-BETA/2.0))CALL
          1POT6
0045       IF((SIN(ALPHA)/2.0-BETA/2.0).GT.ETA.AND.ETA.GT.((-3.0/2.0)
          1*SIN(ALPHA)+BETA/2.0))CALL POT7
0046       IF((-3.0/2.0)*SIN(ALPHA)+BETA/2.0).GT.ETA.AND.ETA.GT.((-3.0
          1/2.0)*SIN(ALPHA)+BETA/2.0))CALL POT8
0147       TERM=(1.0+RK12)*RK21**N*RK23**N*((1.0-RK12)/(RK21*RN1(N))
          1-(1.0+RK23)/RN6(N))
0152       RUTP1=1.0/A+RK12/R2-((1.0+RK12)*(1.0+RK23))/(2.0*A)
0153 4     XTERM=(1.0+RK12)*RK21**I*RK23**J*((1.0-RK12)/(RK21*RN1(I))
          1-(1.0+RK23)/RN6(I))
0159       TERM2=((1.0+RK12)/(1.0-RK12))*RK21**M*RK23**M*((1.0-RK12)
          1/(RK21*RN3(M))+(1.0-RK12)/RN6(M)-(1.0+RK23)/(RK23*RN8(M))
          1-(1.0+RK23)/RN9(M))
0164       RUTP2=((1.0+RK12)/(1.0-RK12))*(-1.0/(2.0*A)-RK12/(2.0*A)
          1-RK23/A)
0165 8     XTERM2=((1.0+RK12)/(1.0-RK12))*RK21**J*RK23**J*((1.0-RK12)
          1/(RK21*RN3(J))+(1.0-RK12)/RN6(J)-(1.0+RK23)/(RK23*RN8(J))
          1-(1.0+RK23)/RN9(J))
0231       TERM=((1.0+RK12)/(1.0-RK12))*RK21**N*RK23**N*((1.0-RK12)/
          1(RK21*RN8(N))+(1.0-RK12)/RN9(N)-(1.0+RK23)/(RK23*RN3(N))
          1-(1.0+RK23)/RN6(N))
0236       RUTP1=((1.0+RK12)/(1.0-RK12))*(1.0/(2.0*A)-RK12/A-RK23/(2.
          10*A))
0237 4     XTERM=((1.0+RK12)/(1.0-RK12))*RK12**I*RK23**I*((1.0-RK12)/
          1(RK21*RN8(I))+(1.0-RK12)/RN9(I)-(1.0+RK23)/(RK23*RN3(I))
```

```
      1-(1.0+RK23)/RN6(I))
0243   TERM2=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**M*RK23**
      1M*((1.0-RK12)/RN6(M)-(1.0+RK23)/(RK23*RN1(M)))
0248   RUTP2=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*(((
      11.0-RK12)*(1.0-RK23))/(2.0*A)-1.0/A+RK23/R2)
0249 8 XTERM2=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**J*RK23**
      1J*((1.0-RK12)/RN6(J)-(1.0+RK23)/(RK23*RN1(J)))
```

PROGRAM 4

This program is the same as PROGRAM 3 except for the following statements that substitute the corresponding statements in PROGRAM 3.

```
0040        IF(((3.0/2.0)*SIN(ALPHA)+BETA/2.0).GT.ETA.AND.ETA.GT.(
            1(3.0/2.0)*SIN(ALPHA)-BETA/2.0))CALL POT2
0041        IF(((3.0/2.0)*SIN(ALPHA)-BETA/2.0).GT.ETA.AND.ETA.GT.(
            1SIN(ALPHA)/2.0+BETA/2.0))CALL POT3
0042        IF((SIN(ALPHA)/2.0+BETA/2.0).GT.ETA.AND.ETA.GT.(SIN(ALPHA)
            1/2.0-BETA/2.0)CALL POT4
0043        IF((SIN(ALPHA)/2.0-BETA/2.0).GT.ETA.AND.ETA.GT.0.0)CALL
            1POT5
0044        IF(0.0.GT.ETA.AND.ETA.GT.(-SIN(ALPHA)/2.0+BETA/2.0)CALL
            1POT6
0045        IF(-SIN(ALPHA)/2.0+BETA/2.0).GT.ETA.AND.ETA.GT.(-SIN(ALP
            1HA)/2.0-BETA/2.0))CALL POT7
0046        IF((-SIN(ALPHA)/2.0-BETA/2.0).GT.ETA.AND.ETA.GT.((-3.0/2.0
            1)*SIN(ALPHA)+BETA/2.0))CALL POT8
0047        IF(((-3.0/2.0)*SIN(ALPHA)+BETA/2.0).GT.ETA.AND.ETA.GT.((-
            13.0/2.0)*SIN(ALPHA)-BETA/2.0))CALL POT9
0119        TERM=(1.0+RK12)*RK21**N*RK23**N*((1.0-RK12)/(RK21*RN1(N))
            1-(1.0+RK23)/RN6(N))
0124        RUTP1=1.0/A+RK12/R2-((1.0+RK12)*(1.0+RK23))/(2.0*A)
0125 4      XTERM=(1.0+RK12)*RK21**I*RK23**I*((1.0-RK12)/(RK21*RN1(I))
            1-(1.0+RK23)/RN6(I))
0131        TERM2=(1.0+RK12)*RK21**M*RK23**M*((1.0-RK12)/(RK21*RN3(M))
            1-(1.0+RK23)/RN9(M))
0136        RUTP2=1.0/(2.0*A)+RK12/R3-((1.0+RK12)*(1.0+RK23))/A
0137  8     XTERM2=(1.0+RK12)*RK21**J*RK23**J*((1.0-RK12)/(RK21*RN3(J))
            1-(1.0+RK23)/RN9(J))
0175        TERM=(1.0+RK12)*RK21**N*RK23**N*((1.0-RK12)/(RK21*RN1(N))
            1-(1.0+RK23)/RN6(N))
0180        RUTP1=1.0/A+RK12/R2-((1.0+RK12)*(1.0+RK23))/(2.0*A)
0181 4      XTERM=(1.0+RK12)*RK21**I*RK23**I*((1.0-RK12)/(RK21*RN1(I))
            1-(1.0+RK23)/RN6(I))
0187        TERM2=((1.0+RK12)*(1.0+RK23))/ (1.0-RK12)*RK21**M*RK23**M
            1*((1.0-RK12)/RN6(M)-(1.0+RK23)/(RK23*RN (M)))
0192        RUTP2=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*
```

```
          1(-1.0/A+((1.0-RK12)*(1.0-RK23))/(2.0*A)+RK23/R6)
0193 8     XTERM2=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**J*RK23
          1**J*((1.0-RK12)/RN6(J)-(1.0+RK23)/(RK23*RN8(J)))
0203       TERM=(1.0+RK12)*RK21**N*RK23**N*((1.0-RK12)/(RK21*RN8(N))
          1(1.0+RK23)/RN6(N))
0208       RUTP1=1.0/A+RK12/R6-((1.0+RK12)*(1.0+RK23))/(2.0*A)
0209 4     XTERM=(1.0+RK12)*RK21**I*RK23**I*((1.0-RK12)/(RK21*RN8(I))
          1-(1.0+RK23)/RN6(I))
0215       TERM2=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**M*RK23**M
          1*((1.0-RK12)/RN6(M)-(1.0+RK23)/(RK23*RN1(M))
0220       RUTP2=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*(-
          11.0/A+((1.0-RK12)*(1.0-RK23))/(2.0*A)+RK23/R2)
0221 8     XTERM2=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**J*RK23**J
          1*((1.0-RK12)/RN6(J)-(1.0+RK23)/(RK23*RN1(J))
0259       TERM=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**N*RK23**N*
          1((1.0-RK12)/RN9(N)-(1.0+RK23)/(RK23*RN3(N)))
0264       RUTP1=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*(-1.0
          1(2.0*A)+RK23/R3+((1.0-RK12)*(1.0-RK23))/A)
0265 4     XTERM=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**I*RK23**I*
          1(1.0-RK12)/RN9(I)-(1.0+RK23)/(RK23*RN3(I)))
0271       TERM2=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**M*RK23**M
          1*((1.0-RK12)/RN6(M)-(1.0+RK23)/(RK23*RN1(M))
0276       RUTP2=((1.0+RK12)*(1.0+RK23))/((1.0-RK12)*(1.0-RK23))*(-1.0
          1/A+RK23/R +((1.0-RK12)*(1.0-RK23))/(2.0*A))
0277 8     XTERM2=((1.0+RK12)*(1.0+RK23))/(1.0-RK12)*RK21**J*RK23**J
          1*((1.0-RK12)/RN6(J)-(1.0+RK23)/(RK23*RN1(J)))
```

PROGRAM 5

This program is the same as PROGRAM 4 except for the following changes: (a) all IF-statements are substituted by the following

```
IF(ETA.GT.((3.0/2.0)*SIN(ALPHA))CALL POT1
IF(((3.0/2.0)*SIN(ALPHA)).GT.ETA.AND.ETA.GT.(1.0/2.0)*SIN(ALPHA
1))CALL POT2
IF((1.0/2.0)*SIN(ALPHA).GT.ETA.AND.ETA.GT.0.0)CALL POT3
IF(0.0.GT.ETA.AND.ETA.GT.(-SIN(ALPHA)/2.0))CALL POT4
IF(-SIN(ALPHA)/2.0.GT.ETA.AND.ETA.GT.-(3.0/2.0)*SIN(ALPHA))CALL
1POT 5
IF((-3.0/2.0)*SIN(ALPHA).GT.ETA)CALL POT6
```

(b) SUBROUTINE POT3 becomes SUBROUTINE POT2
SUBROUTINE POT5 becomes SUBROUTINE POT3
SUBROUTINE POT6 becomes SUBROUTINE POT4
SUBROUTINE POT8 becomes SUBROUTINE POT5
SUBROUTINE POT10 becomes SUBROUTINE POT6

The other SUBROUTINES of PROGRAM 4 are not used in PROGRAM 5.

REFERENCES

Crous, C. M.,1971, Computer-assisted interpretation of electrical sounding: C.S.M. MSc. Thesis.

Dakhnov, V.N., 1959, The Application of Geophysical Methods; Electrical Well Logging. Translated into English by Dr. G.V. Keller, Quarterly of the Colorado School of Mines, Vol.57, number2, p. 127-162

Davis, S.N., and R.J.M. de Wiest, 1966, Hydrogeology; John Wiley & Sons, Inc., New York, p. 318-417

Dobrin, M.B., 1952, Introduction to Geophysical Prospecting; Mc Graw-Hill Book Company, Inc., New York.

Hall, D.H., and Z. Hajual, 1962, The gravimeter in studies of burried valleys, Geophysics, v 27, no. 6 Part 2, p.939-951.

Keller, G.V., and F.C. Frischknecht, 1966, Electrical methods in geophysical prospecting: Oxford, Pergamon Press.

Keller, J.B., 1953,The scope of the image method: Communications on pure and applied mathematics, Vol. vi, p. 505-512.

Musgrave, A.W., 1967, Seismic refraction prospecting: The Society of Exploration Geophysicists, Tulsa, Oklahoma.

Orellana, E., and H.M. Mooney, 1966, Tables and Master Curves for vertical electrical sounding. Interciencia, Madrid, 160p.

Todd, D.K., 1967, Ground Water Hydrology: John Wiley & Sons, Inc., p. 219-234.

Van Nostrand, R.G., and K.L. Cook, 1966, Interpretation of resistivity data: U.S. Geol. Survey Prof. Paper 499, 310 p.