T-1627

INTERPRETATION OF ELECTROMAGNETIC SOUNDINGS USING A LAYERED EARTH MODEL

By

Jeffrey J. Daniels

A Thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Geophysical Engineering.

Signed: Jeffrey J. Daniels
Jeffrey J. Daniels

Golden, Colorado

Date: April 2, 1974

Approved: George V. Keller
G. V. Keller
Thesis Advisor

Ralph Holmer
Ralph Holmer
Head of Department

Golden, Colorado

Date: April 2, 1974

ABSTRACT

Theoretical expressions for the electromagnetic field components are computed fast and accurately by using linear filter theory. In this paper a computer program is developed to numerically evaluate the electromagnetic field components (E_x, E_y, E_c and H_z) for a current dipole source over a layered earth.

Scaled H_z and E_c ($E_c = \sin(z\theta)Ex - \cos(z\theta)E_y$) field component curves simplify catalog curve matching by eliminating the curve dependence on θ, I and ds. A sample catalog for the H_z component is given in Appendix A.

A least-squares curve fitting procedure is developed to aid in the interpretation of electromagnetic sounding data. This procedure is tested on two and three layer theoretical sounding curves. Results of these tests can be summarized as follows:

(1) For the two layer case convergence of the layered earth parameters (resistivity and thickness) is accurate to within ten percent.

(2) Three layer curve fitting, with four or more parameters, requires a good first guess to insure convergence. The first guess curve should be within twenty percent of the actual field component values.

CONTENTS

ILLUSTRATIONS

Figure | Page

Table

ACKNOWLEDGMENTS

I express my appreciation to George R. Pickett, David Butler, L. Trowbridge Grose, and George V. Keller for their helpful suggestions. I also wish to express my thanks to Walt Anderson and Frank Frischnecht of the U.S.G.S. and Jorge Parra of C.S.M. Support for the project was provided by the Office of Naval Research (Contract No. N00014-71-A-0430-0003). Special thanks to the Colorado School of Mines Computing Center for providing unlimited computer time.

INTRODUCTION

The objective of controlled-source electromagnetic sounding is to obtain an accurate representation of the subsurface geology. This goal can be achieved by mathematical modeling (Keller and Frischknecht, 1966, Keller, 1968, Frischknecht, 1967, Vanyan, 1965, Pritchard, 1971, Sinha and Bhattacharyya, 1967, Wait, 1966, and others).

Recent advances in numerical techniques have made possible the solution of practical mathematical models. Since these models can only approximate the real geologic situation the interpreter must make use of all auxiliary information at his disposal. This information may come from knowledge of the geology of a given area, well data, or other geophysical data.

One of the more useful models is that for a horizontally layered earth where each layer is represented by an electrical resistivity and a layer thickness. This approach is basically the same as the "apparent resistivity" approach described for electromagnetic sounding data by Vanyan (1967) and for direct-current electrical sounding data by Crous (1971). The layered earth model is applicable to many geological problems (e.g. layering in a sedimentary basement, determination of depths and thicknesses of oil shale and tar sand formations, determination of highly resistant wave guides, etc.). Digital linear filter theory (Ghosh, 1971) provides a rapid technique for computing theoretical electromagnetic sounding curves based on the layered earth model.

Model-based direct interpretation can be achieved by one of the following techniques.

1. Catalog Interpretation - the observed sounding curve is compared to a catalog of theoretical curves (geological) parameters are given by the theoretical curves which matches the observed data),

2. "Cut and Try" Interpretation - model parameters are repeatedly adjusted and new models are generated until the theoretical curve which matches the observed data is found, and

3. Automatic Interpretation - first guess parameters are iteratively adjusted according to some statistical criteria until the theoretical curve matches the observed data. This paper uses a modification of the basic least-squares criterion to develop an automated interpretation scheme.

The purpose of this paper is to develop a computer oriented interpretation technique for electromagnetic sounding data. A layered earth model is assumed to fit the geological conditions represented by the sounding data.

ELECTROMAGNETIC FIELD COMPONENTS

Figure 1 illustrates the theoretical model considered in this paper. The source is a horizontal current dipole. The model is a horizontally stratified isotropic medium, where ρ_i and h_i represent the electrical resistivity and thickness of the i-th layer. Although the following discussion considers an N-layered earth the interpretation scheme based on this model assumes a maximum of three layers. The computer programs developed in this paper can easily be

Following Wait's development (1966) the fields vary according to . The free-space wave number is assumed to be equal to zero. The vertical magnetic field component and the horizontal electric field components are expressed as

$$H_z = \frac{I\,ds}{4\pi} \sin\theta \int_0^\infty \lambda (1 + R_1(\lambda)) J_1(\lambda r)\, d\lambda \qquad (2.1)$$

$$E_x = -\frac{I\,ds}{4\pi} \gamma_1^2 \rho_1 \int_0^\infty (1 + R_1(\lambda)) J_0(\lambda r)\, d\lambda$$

$$+ \frac{\rho_1 I\,ds}{4\pi} \Big\{ -\cos^2\theta \int_0^\infty \lambda^2 \Big[-\frac{\gamma_1^2}{\lambda^2} + \frac{2 Z_1(\lambda)}{\lambda \rho_1} - \frac{(\lambda^2 + u_1^2 R_1(\lambda))}{\lambda^2}$$

$$+ (1 + R_1(\lambda)) \Big] J_0(\lambda r)\, d\lambda + \frac{\cos(2\theta)}{r} \int_0^\infty \lambda \Big[-\frac{\gamma_1^2}{\lambda^2} + \frac{2 Z_1(\lambda)}{\lambda \rho_1}$$

$$- \frac{(\lambda^2 + u_1^2 R_1(\lambda))}{\lambda^2} + (1 + R_1(\lambda)) \Big] J_1(\lambda r)\, d\lambda \qquad (2.2)$$

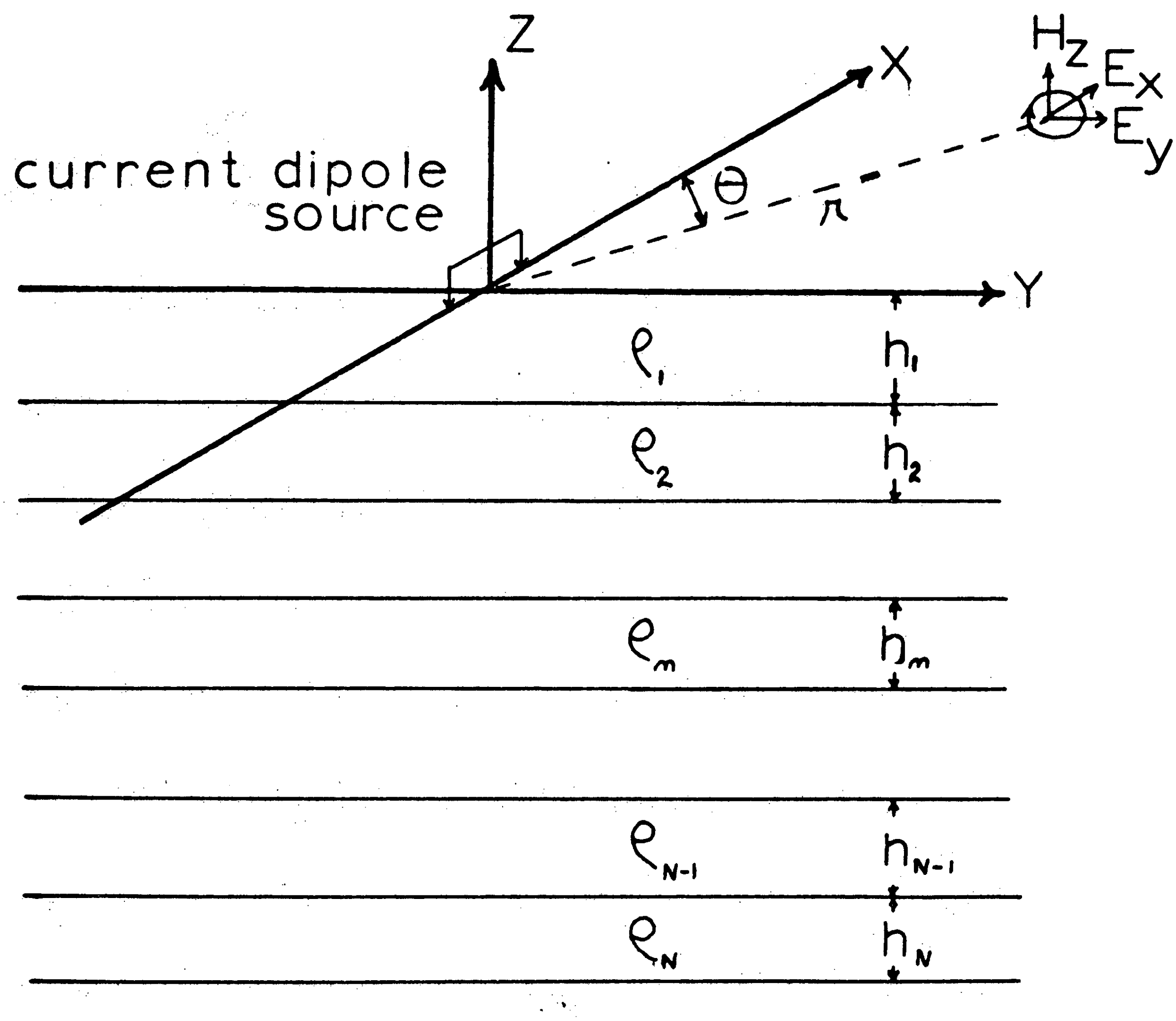


Figure 1: Theoretical model.

and $$E_y = \frac{\rho_1 I\,ds}{4\pi}\left\{-\frac{\sin(2\theta)}{2}\int_0^\infty \lambda^2\left[-\frac{\gamma_1^2}{\lambda^2} + \frac{2Z_1(\lambda)}{\lambda\rho_1}\right.\right.$$

$$\left. - \frac{(\lambda^2+u_1^2 R_\perp(\lambda))}{\lambda^2} + (1+R_\perp(\lambda))\right]J_0(\lambda r)\,d\lambda + \frac{\sin(2\theta)}{r}\int_0^\infty \lambda\left[-\frac{\gamma_1^2}{\lambda^2}\right.$$

$$\left. + \frac{2Z_1(\lambda)}{\lambda\rho_1} - \frac{(\lambda^2+u_1^2 R_\perp(\lambda))}{\lambda^2} + (1+R_\perp(\lambda))\right]J_1(\lambda r)\,d\lambda \quad (2.3)$$

Variables in these equations are defined as follows:

ρ_1 = resistivity of the first layer

J_0 = zeroth-order Bessel function,

J_1 = first-order Bessel function,

I = current into source dipole,

λ = dummy variable of integration,

ds = source dipole length,

γ_n = n^{th} layer wave number $(j\mu_0\omega\sigma_n)^{1/2}$, and

$u_n^2 = \lambda^2 + \gamma_n^2$

$R_\perp(\lambda)$ and $Z_1(\lambda)$ are correction factors which account for the effect of layering on the electromagnetic fields. The recursive expressions used for solving $R_\perp(\lambda)$ and $Z_1(\lambda)$ are written as follows:

$$R_\perp(\lambda) = \frac{N_0 - Y_1}{N_0 + Y_1} \quad (2.4) \qquad \text{where} \quad N_0 = \frac{\lambda}{j\mu_0\omega}$$

$$Y_n = N_n\,\frac{Y_{n+1} + N_n \tanh(u_n h_n)}{N_n + Y_{n+1}\tanh(u_n h_n)}, \quad N_n = \frac{u_n}{j\mu_0\omega}, \quad n = 1,2,\ldots,N$$

and $Y_N = \dfrac{u_N}{j\mu_0\omega}$

$$Z(\lambda) = K_n\,\frac{Z_{n+1} + K_n\tanh(u_n h_n)}{K_n + Z_{n+1}\tanh(u_n h_n)} \quad (2.5)$$

where $K_n = u_n\rho_n$, $n = 1,2,\ldots,N$ and $Z_N = u_N\rho_N$

The electric field components can be combined so that

$$E_c = \sin(2\theta) E_x - \cos(2\theta) E_y = \frac{I\, ds\, \rho_1 \sin(2\theta)}{4\pi} \int_0^\infty - \gamma_1^2 \left(1 + R_\perp(\lambda)\right)$$

$$+ \frac{\lambda^2}{2} \left[\frac{\gamma_1^2}{\lambda^2} + \frac{(\lambda^2 + u_1^2 R_\perp(\lambda))}{\lambda^2} - \frac{2 Z_1(\lambda)}{\lambda \rho_1} - (1 + R_\perp(\lambda)) \right] J_0(\lambda r)\, d\lambda \quad (2.6)$$

Combining E_x and E_y in this manner simplifies numerical evaluation and minimizes computational error.

To insure convergence of the integrals in equations (2.1),(2.2),(2.3), and (2.6) homogenous half-space expressions (assuming a resistivity of ρ_1) are subtracted from the integrand. The closed-form expressions for a homogenous half-space are then added to the resulting integration. The final computational form for the field components then becomes:

$$E_x = -\frac{I\, ds\, \gamma_1^2 \rho_1}{4\pi} \int_0^\infty \left[R_\perp(\lambda) - \frac{\lambda - u_1}{\lambda + u_1} \right] J_0(\lambda r)\, d\lambda$$

$$+ \frac{\rho_1 I\, ds}{4\pi} \left\{ -\cos^2\theta \int_0^\infty \lambda^2 \left[\frac{2}{\lambda} \left(\frac{Z_1(\lambda)}{\rho_1} - u_1 \right) - \frac{u_1^2}{\lambda^2} \left[R_\perp(\lambda) - \frac{\lambda - u_1}{\lambda + u_1} \right] \right. \right.$$

$$\left. + R_\perp(\lambda) - \frac{\lambda - u_1}{\lambda + u_1} \right] J_0(\lambda r)\, d\lambda + \frac{\cos(2\theta)}{r} \int_0^\infty \lambda \left[\frac{2}{\lambda} \left(\frac{Z_1(\lambda)}{\rho_1} - u_1 \right) \right.$$

$$\left. \left. + \left(1 - \frac{u_1^2}{\lambda^2}\right) \left(R_\perp(\lambda) - \frac{\lambda - u_1}{\lambda + u_1} \right) \right] J_1(\lambda r)\, d\lambda \right.$$

$$+ \frac{I\, ds\, \rho_1}{2\pi r^3} \left[(3\cos^2\theta - 2) + (1 + \gamma r) e^{-\gamma_1 r} \right] \quad (2.7)$$

$$E_y = \frac{\rho_1 I ds}{4\pi}\left\{-\sin(2\theta)\int_0^\infty \lambda^2\left[\frac{2}{\lambda}\left(\frac{Z_1(\lambda)}{\rho_1} - u_1\right)\right.\right.$$

$$\left. + \left(1 - \frac{u_1^2}{\lambda^2}\right)\left(R_\perp(\lambda) - \frac{\lambda - u_1}{\lambda + u_1}\right)\right] J_0(\lambda r)\, d\lambda + \frac{\sin(2\theta)}{r}\int_0^\infty \lambda\left[\frac{2}{\lambda}\left(\frac{Z_1(\lambda)}{\rho_1} - u_1\right)\right.$$

$$\left. - \left(1 - \frac{u_1^2}{\lambda^2}\right)\left(R_\perp(\lambda) - \frac{\lambda - u_1}{\lambda + u_1}\right)\right] J_1(\lambda r)\, d\lambda + \frac{3\rho_1 I ds}{2\pi r^3} \sin\theta\cos\theta \quad (2.8)$$

$$E_c = \frac{\rho_1 I ds \sin(2\theta)}{2\pi}\left\{\frac{1}{2}\int_0^\infty \left[\frac{\gamma_1^2}{2}\left(\frac{\lambda - u_1}{\lambda + u_1} - R_\perp(\lambda)\right) + \lambda\left(u_1 - \frac{Z_1(\lambda)}{\rho_1}\right)\right] J_0(\lambda r)\, d\lambda\right.$$

$$\left. + \frac{1}{r^3}\left[-\frac{1}{2} + (1 + \gamma_1 r)\, e^{-\gamma_1 r}\right]\right\} \quad (2.9)$$

and

$$H_z = \frac{I ds}{2\pi}\sin(\theta)\left\{\frac{1}{2}\int_0^\infty \lambda\left[R_\perp(\lambda) - \frac{\lambda - u_1}{\lambda + u_1}\right] J_1(\lambda r)\, d\lambda\right.$$

$$\left. + \frac{1}{\gamma_1^2 r^4}\left[3 - (3 + 3\gamma_1 r + \gamma_1^2 r^2)\, e^{-\gamma_1 r}\right]\right\} \quad (2.10)$$

To aid computations the following substitutions are made in equations (2.7), (2.8), (2.9), and (2.10):

$\lambda = \frac{g}{\delta}$, $K_m = \frac{\rho_1}{\rho_m}$, $D_m = \frac{2 h_m}{\delta}$, $B = \frac{r}{\delta}$

and $u_m = \frac{V_m}{\delta}$ where $\delta^2 = \frac{2\rho_1}{\mu_0 \omega}$, and $V_m^2 = g^2 + 2j K_m$

Equations (2.7), (2.8), (2.9), and(2.10) are then written as:

$$E_c = \frac{I ds}{4\pi\delta}\sin(2\theta)\int_0^\infty \left\{\gamma_1^2\left(\frac{g V_1 (F_1 - 1)}{(g + V_1)(g + V_1 F_1)}\right) + \frac{g V_1}{\delta^2}\right\} J_0(gB)\, dg$$

$$+ \frac{I ds\, \rho_1 \sin(2\theta)}{2\pi r^3}\left\{-\frac{1}{2} + \left[(1+j)B + 1\right] e^{-(1+j)B}\right\} \quad (2.11)$$

$$E_x = j\frac{\mu_0 \omega I\,ds}{4\pi}\left\{ \frac{\cos^2\theta}{\delta}\int_0^\infty g\left[i\,V_1(L_1-1) + \frac{2V_1(1-F_1)}{(g+V_1F_1)(g+V_1)} \right] J_0(gB)\,dg \right.$$

$$- \frac{1}{\delta}\int_0^\infty \frac{2gV_1(1-F_1)}{(g+V_1F_1)(g+V_1)} J_0(gB)\,dg - \frac{\cos(2\theta)}{r}\int_0^\infty \left[i\,V_1(L_1-1) \right.$$

$$\left.\left. + \frac{2V_1(1-F_1)}{(g+V_1)(g+V_1F_1)} \right] J_1(gB)\,dg \right\} + \frac{I\,ds\,\rho_1}{2\pi r^3}\left[(3\cos^2\theta - 2) + (1+(1+j)B)e^{-(1+j)B} \right] \quad (2.12)$$

$$E_y = j\frac{\mu_0 \omega I\,ds}{4\pi}\left\{ -\frac{\sin(2\theta)}{r}\int_0 \left[j\,V_1(L_1-1) + \frac{2V_1(1-F_1)}{(g+V_1)(g+V_1F_1)} \right] J_1(gB)\,dg \right.$$

$$+ \frac{\sin(2\theta)}{2\delta}\int_0^\infty g\left[j\,V_1(L_1-1) + \frac{2V_1(1-F_1)}{(g+V_1)(g+V_1F_1)} \right] J_0(gB)\,dg$$

$$+ \frac{3\rho_1 I\,ds}{2\pi r^3}\sin\theta\cos\theta \quad (2.13)$$

$$H_z = \frac{I\,ds}{2\pi\delta^2}\sin\theta \int_0^\infty \frac{g^2 V_1(1-F_1)}{(g+V_1)(g+V_1F_1)} J_1(gB)\,dg$$

$$- \frac{j\,I\,ds\,\delta^2 \sin\theta}{4\pi r^4}\left\{ 3 - (3 + 3(1+j)B + 2jB^2)e^{-(1+j)B} \right\} \quad (2.14)$$

L_1 and F_1 are solved recursively, starting with the last layer and working upward, using

$$F_m = \frac{V_{m+1}F_{m+1} + V_m\left(\frac{1-e^{-V_mD_m}}{1+e^{-V_mD_m}}\right)}{V_m + V_{m+1}F_{m+1}\left(\frac{1-e^{-V_mD_m}}{1+e^{-V_mD_m}}\right)}, \text{ where } F_N = 1$$

and $$L_m = \frac{\rho_{m+1}V_{m+1}L_{m+1} + \rho_m V_m\left(\frac{1-e^{-V_mD_m}}{1+e^{-V_mD_m}}\right)}{\rho_m V_m + \rho_{m+1}V_{m+1}L_{m+1}\left(\frac{1-e^{-V_mD_m}}{1+e^{-V_mD_m}}\right)} \text{ where } L_N = 1.$$

COMPUTATION OF THE ELECTROMAGNETIC FIELD COMPONENTS

Numerical Procedure

Computation of the integrals in equations (2.11), (2.12), (2.13), and (2.14) is carried out using the linear-digital-filter approach explained by Ghosh (1971). This technique is superior to other methods (e.g. spline integration (Vanyan, 1967), Gaussian quadrature (Anderson, 1973)) in its computational speed and ease of application. Linear filter integration yields about four significant figures of accuracy (Anderson, 1973).

The basis for linear filter integration is the sampling theorem. This theorem states that a function f(y) can be reconstructed by replacing each of the sampled values (y=a) by

$$f(y) = \frac{f(a) \sin\left(\frac{2\pi(y-a)}{2\Delta y}\right)}{2\pi\left(\frac{y-a}{2\Delta y}\right)} \qquad (3.1)$$

The function f(y) is equal to the infinite sum of the sinc-function weighted by f(a) $\left(f(y) = \lim_{n\to\infty} \sum_{i=1}^{n} f(a_i)\,\mathrm{sinc}(y)\right)$.

The error involved in using this approximation is optimized when $y = \frac{\ln(10)}{10}$ (Koeford and others, 1972). This sample interval is used throughout the remainder of this paper. Substituting $x=\ln(B)$ and $y=\ln(\frac{1}{g})$ into equations (2.11), (2.12), (2.13), and (2.14) yields

$$E_c = \frac{I\,ds\,\rho_1 \sin(2\theta) e^{-x}}{4\pi\delta} \int_0^{\infty} E(\rho_m, h_m, y) e^{x-y} J_0(e^{x-y})\,dy + \frac{I\,ds\,\rho_1 \sin(2\theta)}{2\pi r^3}\left[-\frac{1}{2} + \left((1+j)B+1\right) e^{-(1+j)B}\right]$$

$$E_x = j\frac{\mu_0 \omega I\,ds}{4\pi} e^{-x}\left\{\frac{\cos^2\theta}{\delta}\int_{-\infty}^{\infty} E_{x1}(\rho_m, h_m, y)\, e^{x-y} J_0(e^{x-y})\, dy\right.$$

$$\left. - \frac{1}{\delta}\int_{-\infty}^{\infty} E_{x2}(\rho_m, h_m, y)\, e^{x-y} J_0(e^{x-y})\, dy - \frac{\cos(2\theta)}{r}\int_{-\infty}^{\infty} E_{x3}(\rho_m, h_m, y)\, e^{x-y} J_1(e^{x-y})\, dy\right\}$$

$$+ \frac{I\,ds\,\rho_1}{2\pi r^3}\left\{\left[3\cos^2\theta - 2\right] + \left[1 + (1+j)B\right]e^{-(1+j)B}\right\}, \quad (3.3)$$

$$E_y = j\frac{\mu_0 \omega I\,ds}{4\pi} e^{-x}\left\{-\frac{\sin(2\theta)}{r}\int_0^{\infty} E_{y1}(\rho_m, h_m, y)\, e^{x-y} J_1(e^{x-y})\, dy\right.$$

$$\left. + \frac{\sin(2\theta)}{2\delta}\int_0^{\infty} E_{y2}(\rho_m, h_m, y)\, e^{x-y} J_0(e^{x-y})\, dy\right\} + \frac{3\rho_1 I\,ds}{2\pi r^3}\sin\theta\cos\theta, \quad (3.4)$$

and $$H_z = \frac{I\,ds\,\sin\theta\, e^{-x}}{2\pi\delta^2}\int_0^{\infty} H(\rho_m, h_m, y)\, e^{x-y} J_1(e^{x-y})\, dy$$

$$- j\frac{I\,ds\,\delta^2 \sin\theta}{4\pi r^4}\left\{3 - (3 + 3(1+j)B + 2jB^2)\, e^{-(1+j)B}\right\}, \quad (3.5)$$

where $$E(\rho_m, h_m, y) = \gamma_1^2\left(\frac{e^{-y} V_1(e^{-y})[F_1 - 1]}{(e^{-y} + V_1 F_1)(e^{-y} + V_1)} + \frac{e^{-y} V_1(1 - L_1)}{\delta^2}\right),$$

$$H(\rho_m, h_m, y) = \frac{e^{-2y} V_1(1 - F_1)}{(e^{-y} + V_1 F_1)(e^{-y} + V_1)}, \quad E_{x2}(\rho_m, h_m, y) = \frac{2e^{-y} V_1(1 - F_1)}{(e^{-y} + V_1 F_1)(e^{-y} + V_1)},$$

$$E_{y1}(\rho_m, h_m, y) = jV_1(L_1 - 1) + \frac{2V_1(1 - F_1)}{(e^{-y} + V_1 F_1)(e^{-y} + V_1)},$$

$$E_{x1}(\rho_m, h_m, y) = e^{-y} E_{y1}(\rho_m, h_m, y), \quad E_{y2}(\rho_m, h_m, y) = E_{x1}(\rho_m, h_m, y),$$

$$E_{x3}(\rho_m, h_m, y) = E_{y1}(\rho_m, h_m, y).$$

Equations (3.2), (3.3), (3.4), and (3.5) are in the form of convolution integrals.

If we let $E(\rho_m, h_m, y)$ = input function, $e^{(x-y)} J_0(e^{(x-y)})$ = filter function, and apply the sampling theorem so that $E(\rho_m, h_m, y) = E(a_i)\,\mathrm{sinc}(y)$, we obtain the relation

$$E(\rho_m, h_m, y) = \frac{I\, ds\, \rho_1 \sin(2\theta)\, e^{-x}}{4\pi S} \sum_{i=1}^{m} E(a_i) \int_0^{\infty} \mathrm{sinc}(y)\, e^{x-y} J_0(e^{x-y})\, dy \quad (3.6)$$

Equation (3.6) can now be put into the following form:

$$E_c = \frac{I\, ds\, \rho_1 \sin(2\theta)}{4\pi S} e^{-x} \sum_{i=1}^{m} E(a_i)\, c(y_i) \qquad (3.7)$$

where $c(y_i) = \mathrm{sinc}(x)\, e^x J_0(x)$ and $a_i = e^{-(x-y_i)}$. $c(y_i)$ is called the sinc-response of the filter function. The sinc-response is computed and stored as data in the computer program which calculates the field components.

Using the identity $$\int_0^{\infty} e^{-ag} J_0(gB)\, dg = \frac{1}{\sqrt{a^2 + B^2}} \qquad (3.8)$$

and substituting $x = \ln(B)$ and $y = \ln(\frac{1}{g})$ the following integral is obtained

$$\int_0^{\infty} e^{-ae^{-y}} \left\{ e^{x-y} J_0(e^{x-y}) \right\} dy = \frac{e^x}{\sqrt{a^2 + e^{2x}}} \qquad (3.9)$$

where "a" can be any positive constant. From equations (3.9) it follows that

$$\int_0^{\infty} \left[e^{-e^{-y}} - e^{-2e^{-y}} \right] \left\{ e^{x-y} J_0(e^{x-y}) \right\} dy = \frac{e^x}{\sqrt{1+e^{2x}}} - \frac{e^x}{\sqrt{4+e^{2x}}} \qquad (3.10)$$

The sinc-response is now found using the expression

$$\text{sinc-response} = \mathcal{F}^{-1}\left[\mathcal{F}(\text{sinc})\; \mathcal{F}\left(\frac{\text{output function}}{\text{input function}} \right) \right] \qquad (3.11)$$

where denotes the fourier transform operator and the inverse fourier transform operator, ' denotes the input function equal to $e^{-e^{-y}} - e^{-2e^{-y}}$, and the output function is equal to $\frac{e^x}{\sqrt{1+e^{2x}}} - \frac{e^x}{\sqrt{4+e^{2x}}}$.

Computations to obtain values for E_c are carried out in the following manner:

1. abcissa values "y_i" are read into the computer and
2. $E_c(\rho_m, h_m)$ is calculated using the relation

$$E_c(\rho_m, h_m) = \frac{I\, ds\, \rho_1 \sin(2\theta)\, e^{-x}}{2\pi S} \sum_{i=1}^{m} E(a_i)\, c_i + \frac{I\, ds\, \rho_1 \sin(2\theta)}{2\pi R^3} \left[-\frac{1}{2} + \left((1+j)B + 1 \right) e^{-(1+j)B} \right] \quad (3.12)$$

The procedure for calculating H_z, E_x, and E_y is similar to the method for calculating E_c. Putting H_z in the form of equation (3.7) we obtain

$$H_z(\rho_m, h_m) = \frac{I\, ds \sin\theta\, e^{-x}}{2\pi S^2} \sum_{k=1}^{m} H(b_k)\, d_k - j\frac{I\, ds\, S^2 \sin\theta}{4\pi R^4} \left\{ 3 - (3 + 3(1+j)B + 2j B^2) e^{-(1+j)B} \right\} \quad (3.13)$$

where $d_k = \text{sinc}(x) * e^x J_1(x)$ and $b_k = e^{-(x-y_k)}$. The sinc-response for H_z is calculated in the same manner as the E_c sinc-response except that the identity

$$\int_0^\infty g\, e^{-ag} J_1(gB)\, dg = \frac{B}{\sqrt{(a^2+B^2)}^{3/2}}$$

is used instead of equation (3.8).

Using the procedure developed for E_c and H_z (zeroth and first order Hankel transforms) the final computational form for equations (3.3) and (3.4) becomes:

$$E_x(\rho_m, h_m) = j\mu_0 \omega I\, ds\, e^{-x} \left\{ \sum_{i=1}^{m} E_{x1}(a_i)\, c(y_i) - \frac{1}{S} \sum_{i=1}^{m} E_{x2}(a_i)\, c(y_i) - \frac{\cos(2\theta)}{R} \sum_{k=1}^{m} E_{x3}(b_k)\, d(y_k) \right\}$$

$$+ \frac{I\,ds\,\rho_1}{2\pi r^3} \left\{ \left[3\cos^2\theta - 2\right] + \left[1 + (1+j)B\right] e^{-(1+j)B} \right\} \quad (3.14)$$

$$E_y(\rho_n, h_n) = \frac{j\mu_0 \omega I\,ds}{4\pi} \left\{ -\frac{\sin(2\theta)}{r} \sum_{k=1}^{m} E_{y1}(b_k)\, d(y_k) + \frac{\sin(2\theta)}{2\delta} \sum_{i=1}^{m} E_{y2}(a_i)\, C(y_i) \right\} + \frac{3\rho_1 I\,ds}{2\pi r^3} \sin\theta\cos\theta \quad (3.15)$$

<u>Computer Program for Calculating the Electromagnetic Field Components</u>

A Fortran computer program to numerically calculate the equations for the electromagnetic field components is given in Appendix B. The program is called EMFWD. It computes E_x, E_y, E_c, and H_z using the numerical technique described in the previous section.

The results of this program are in agreement with results obtained by Anderson (1973). Figure 2 is a comparison of EMFWD results and results obtained by Anderson.

parameter set		EMFWD results	Anderson's results
A	E_x	$-.8296\times10^{-13}-j.1632\times10^{-12}$	$-.8291\times10^{-13}-j.1633\times10^{-12}$
	E_y	$.1310\times10^{-12}+j.2548\times10^{-12}$	$.1312\times10^{-12}+j.2548\times10^{-12}$
	H_z	$.1075\times10^{-11}-j.5272\times10^{-12}$	$.1075\times10^{-11}-j.5266\times10^{-12}$
B	E_x	$-.9804\times10^{-10}-j.1987\times10^{-10}$	$-.9811\times10^{-10}-j.1988\times10^{-10}$
	E_y	$.1570\times10^{-10}+j.3135\times10^{-10}$	$.1570\times10^{-10}+j.3134\times10^{-10}$
	H_z	$.6397\times10^{-09}-j.3018\times10^{-09}$	$.6398\times10^{-09}-j.3019\times10^{-09}$
C	E_x	$-.3190\times10^{-12}-j.5761\times10^{-12}$	$-.3190\times10^{-12}-j.5764\times10^{-12}$
	E_y	$.4999\times10^{-12}+j.9027\times10^{-12}$	$.5002\times10^{-12}+j.9026\times10^{-12}$
	H_z	$.7590\times10^{-12}-j.4212\times10^{-12}$	$.7591\times10^{-12}-j.4210\times10^{-12}$

parameter values:
set A; r=50000, frequency=1 hertz, ρ_1=100Ω-m, ρ_2=1Ω-m, h_1= 1000
set B; r=10000, frequency=1 hertz, ρ_1=100Ω-m, ρ_2=1Ω-m, h_1= 1000
set C; r=50000, frequency=1 hertz, ρ_1=100Ω-m, ρ_2=1Ω-m, h_1= 1000

for all sets $I\,ds$=12.566371
r and h_1 are in meters

Table 1 : Comparison of EMFWD results with values obtained by Anderson (personal communication, 1973).

Figures 2, 3, and 4 are examples of computational results for the electric field components. Examples for the magnetic field component are given in Appendix A.

The theoretical sounding curves in figures 2 and 3 approach unity at high frequencies. In figure 2 the asymptotic behavior appears at a relatively low frequency. This effect is caused by the conductive first layer. Comparison of figures 3 and 4 indicates that the asymptotic behavior in figure 3 is caused by a thick first layer. Checks such as these establish the accuracy of EMFWD results and the validity of the numerical procedure which it utilizes.

Computation time for all complex electromagnetic field components is approximately 1.2 seconds per sample frequency (using the Colorado School of Mines DEC PDP-10 computer). EMFWD results are accurate to between 3 and 4 significant figures.

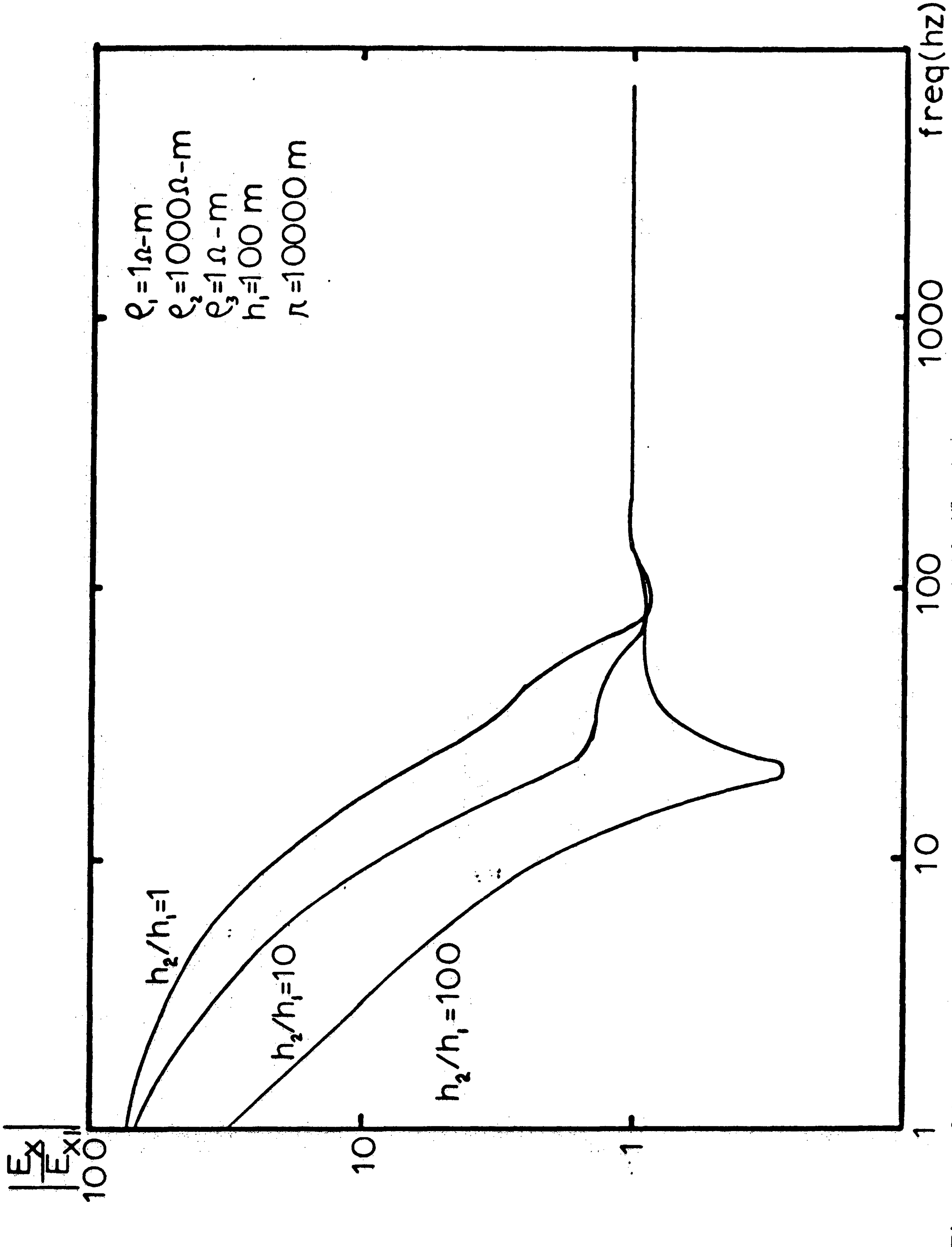


Figure 2: Example curve for the scaled amplitude of E_x

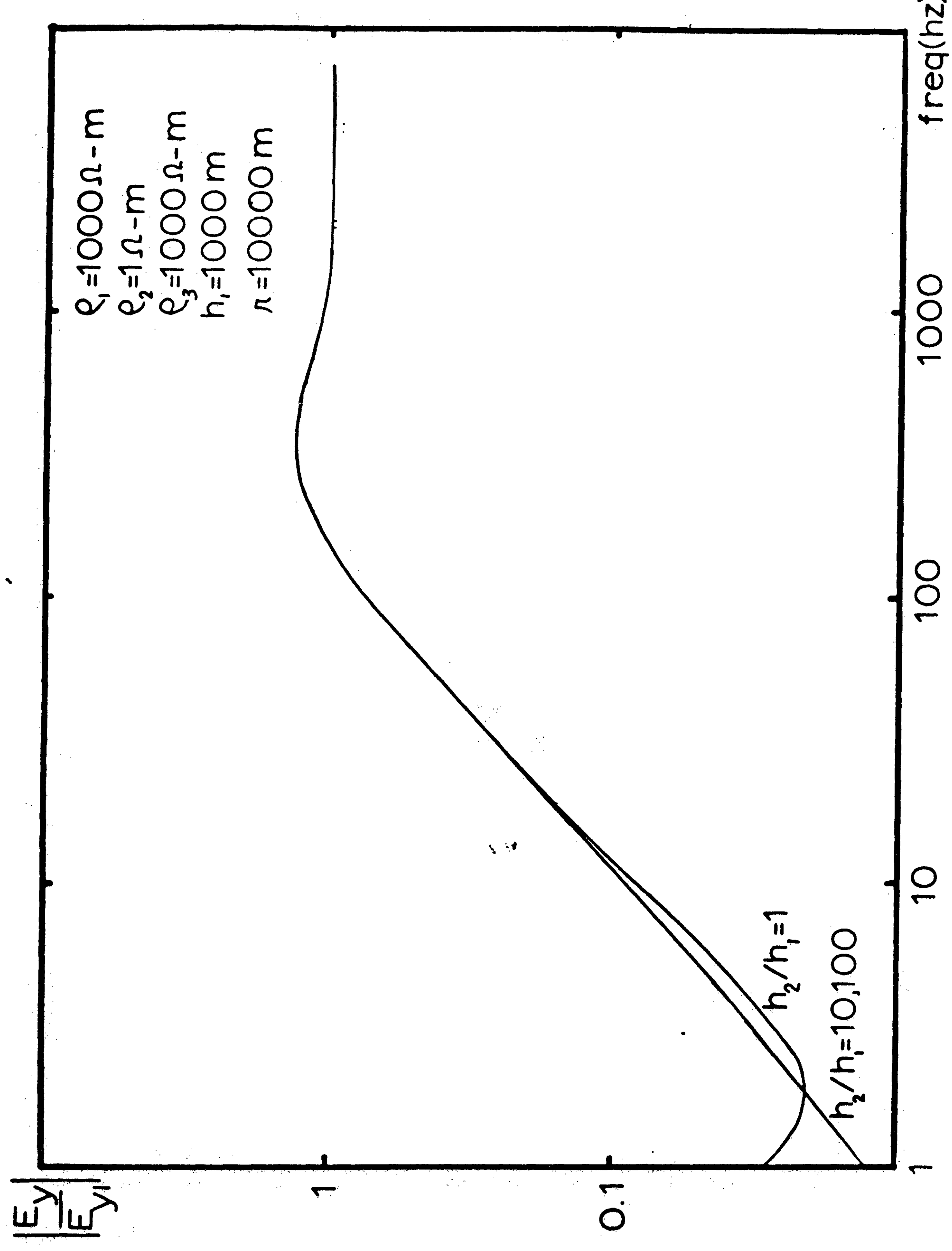


Figure 3: Example curve for the scaled amplitude of E_y (h_1=1000).

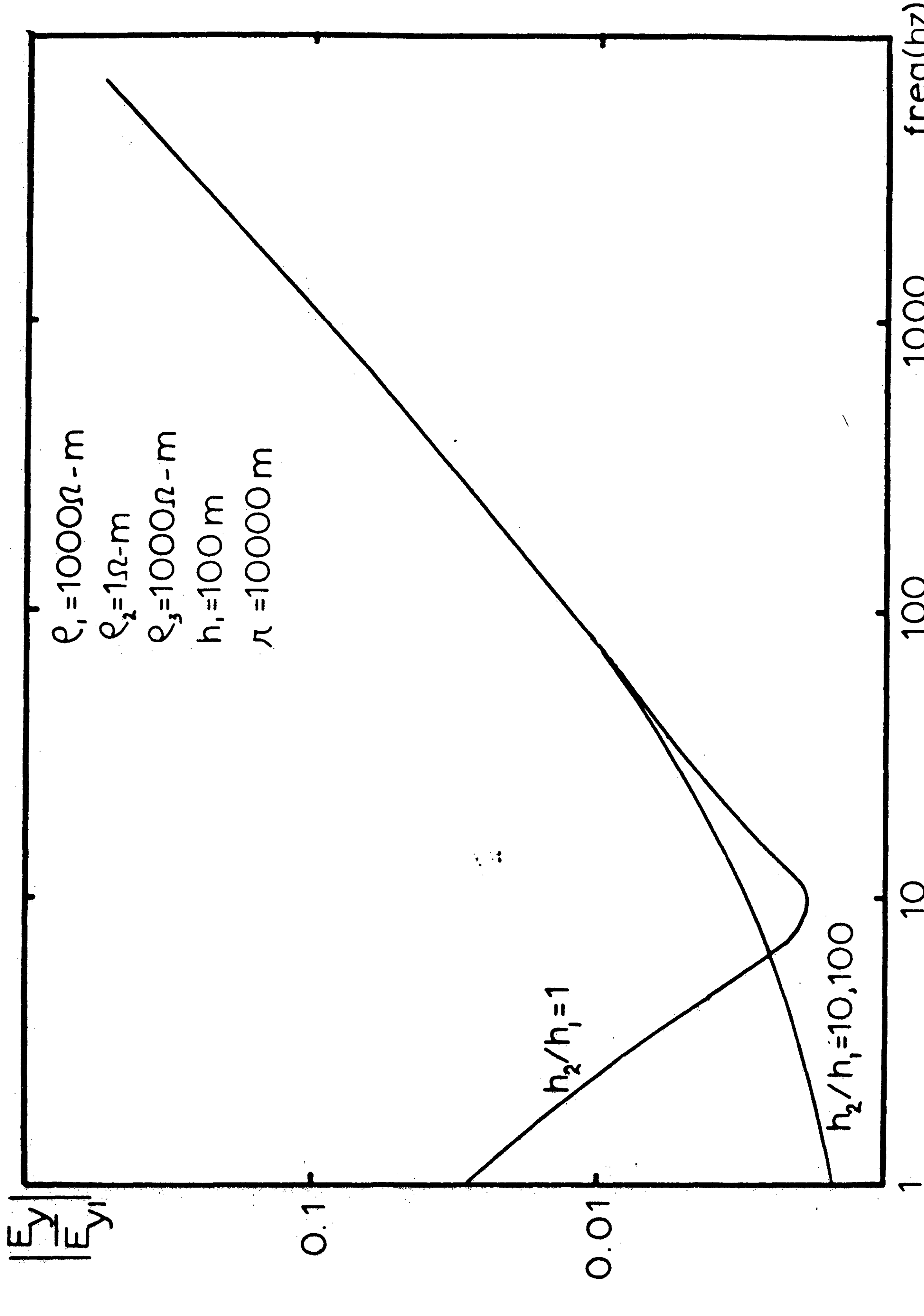


Figure 4: Example curve for the scaled amplitude of E_y (h_1=100).

LEAST-SQUARES CURVE FITTING

Theory of Least-Squares Curve Fitting

The objective of least squares curve fitting is to provide the best possible model approximation ($\hat{Y}_i$) to the observed data (Y_i). This is done by adjusting the parameters (P_j) until $\phi = \sum_{i=1}^{m} [Y_i - \hat{Y}_i]^2$ is minimized. $\hat{Y}_i$ is the independent variable which is a function of x_i (i=1,2,...,n) and P_j (j=1,2,...,k). For the electromagnetic case considered in this paper $\hat{Y}_i$ represents the theoretical field components and Y_i represents the observed field components at $x_1, x_2, \ldots, x_n$ frequencies. The parameters "P_j" represent the layer thicknesses and layer resistivities.

The Newton-Gauss least-squares method approximates the field components using a Taylor series so that

$$\hat{Y}(x_i, \hat{P} + \delta_t) = f(x_i, \hat{P}) + \sum_{j=1}^{k} \left(\frac{\partial f_i}{\partial \hat{P}_j} \right) \delta_{tj} , \qquad (4.1)$$

where $\hat{P}$ = least-squares estimate of P, $f(X_i, P) = \hat{Y}_i$ and δ_t = correction vector to $\hat{P}$.

The least-squares criterion now becomes

$$\hat{\phi} = \sum_{i=1}^{m} \left[Y_i - f(x_i, \hat{P}) - \sum_{j=1}^{k} \left(\frac{\partial f_i}{\partial P_j} \right) \delta_{tj} \right. \qquad (4.2)$$

δ_t is solved by setting $\frac{\partial \hat{\phi}}{\partial \delta_j} = 0$ for all "j" values.

In matrix form the solution to the problem becomes

$$A \vec{\delta}_t = \vec{g} \qquad (4.3)$$

where $A^{[k \times k]} = P^T P$,

$$P^{[m \times k]} = \frac{\partial f_i}{\partial P_j} \quad , \quad i = 1, 2, \ldots, m \; ; \; j = 1, 2, \ldots, k$$

and $$g^{[K \times 1]} = \sum_{i=1}^{m} \left[(Y_i - f_i) \frac{\partial f_i}{\partial P_j} \right] \qquad (4.4)$$

In order to insure convergence with the Newton-Gauss method the first guess must be linearly close to the proper solution. The first guess is linearly close to the solution when the linearized model (equation 4.1) is a good approximation to the actual model.

To circumvent this problem Marquardt (1963) uses scaled matrices in the normal equations and introduces an adjustment factor, λ^r, which controls the convergence. According to the theory which he develops there always exists a λ (at the r-th iteration) such that $\phi^{r+1} < \phi^r$.

Following Marquardt's algorithm, the least-squares normal equations at the r-th iteration become

$$(\bar{A}^{*r} + \lambda^r \bar{I}) \vec{\delta}^{*r} = \vec{g}^{*r} \qquad (4.5)$$

where $$\bar{A}^* = (a^*_{jj'}) = \frac{a_{jj'}}{\sqrt{a_{jj}}\sqrt{a_{j'j'}}},$$

$$\vec{g}^* = g^*_j = \frac{g_j}{\sqrt{a_{jj}}},$$

and $$\delta_j = \frac{\delta^*_j}{\sqrt{a_{jj}}}$$

Equation 4.5 is solved for δ^{*r} and the new parameter values become $\hat{P}^{r+1} = \hat{P}^r + \delta^r$. (4.6)

These parameter values are then used to compute a new set of theoretical points $\hat{Y}_i$ which are compared to the observed data. The iterative procedure is stopped when $\vec{\phi}$ is considered small enough for the particular problem being considered.

Marquardt suggests the following procedure be used to find the proper value of λ.

Let $\nu > 1$.
Let $\lambda^{(r-1)}$ denote the value of λ from the previous iteration. Initially let $\lambda^{(0)} = 10^{-2}$, say.
Compute $\phi(\lambda^{r+1})$ and $\phi(\lambda^{r+1}/\nu)$.
i. If $\phi(\lambda^{r+1}/\nu) \leq \phi^{r}$, let $\lambda^{r} = \lambda^{r-1}/\nu$.
ii. If $\phi(\lambda^{r-1}/\nu) > \phi^{r}$, and $\phi(\lambda^{r-1}) \leq \phi^{r}$, let $\lambda^{r} = \lambda^{r-1}$.
iii. If $\phi(\lambda^{r-1}/\nu) > \phi^{r}$, and $\phi(\lambda^{r-1}) \geq \phi^{r}$, increase λ by successive multiplication by ν until for some smallest w, $\phi(\lambda^{r-1}\nu^{w}) \leq \phi^{r}$. Let $\lambda^{r} = \lambda^{r-1}\nu^{w}$.

Application of Least-Squares Curve Fitting

Marquardt's algorithm is applied to electromagnetic sounding data as follows:

1. The appropriate theoretical model is chosen. In this paper a horizontally stratified earth is assumed to approximate the geological situation represented by the electromagnetic sounding data.

2. A first guess of the parameters is made. The parameters for a layered earth are electrical resistivity and layer thickness.

3. ϕ^{a} is computed.

4. Equation (4.5) is solved for δ^{*a}.

5. New parameters $\hat{P}^{a+1}$ are computed according to equation (4.6).

6. A new theoretical solution is calculated using the parameters $\hat{P}^{a+1}$.

7. ϕ^{a+1} is computed. At this point the computations are stopped if ϕ^{a+1} is considered small enough. If ϕ^{a+1} is not small enough a new λ is computed and steps 3 through 7 are repeated.

The computer program to apply Marquardt's algorithm to geophysical data was originally written by Jorge Parra (1972).

A modified version of this program, called EMINT, is listed in Appendix B.

First Guess

Marquardt's algorithm requires an initial "guess" of the parameters. Obtaining an accurate first guess is critical for insuring fast and accurate convergence of the algorithm. A first guess can be obtained by catalog curve matching, by empirical "rules-of-thumb", or by using available geologic information.

Using the theoretical curves generated by EMFWD the following generalizations can be made concerning two and three layer curves:

(1) For a resistive first layer both the two and three layer $\left|\frac{H_3}{H_{31}}\right|$ curves increase positively with increasing frequency. This effect becomes negligible for large first layer thicknesses. Figures (A-2) and (A-11) illustrate this behavior.

(2) For a conductive first layer $\left|\frac{H_3}{H_{31}}\right|$ is equal to one at high frequencies. Figure (A-3) is an example of this condition.

(3) The ratio $\left|\frac{H_3}{H_{31}}\right|$ for two and three layer curves is generally less than one at low frequencies for a resistive first layer and greater than one at low frequencies for a conductive first layer. See figures (A-1) and (A-12).

(4) Amplitude curves for the scaled electric field component approachs unity at high frequencies when the first layer is conductive. A thick resistive first layer will give the same effect. See figures 3 and 4 .

(4) At low frequencies the amplitude of the scaled electric field curves are less than unity for a resistive

first layer and greater than unity when the first layer is conductive. Figures (2.3), (2.4), and (2.5) illustrate this.

These rules alone are not sufficient to obtain an accurate first guess. A first guess made using these rules should be checked either with a curve catalog or with a test curve generated by EMFWD (Appendix B).

Appendix A contains a variety of two and three layer scaled H_z sounding curves. This catalog is by no means complete but it can serve to aid in obtaining a first guess. A major advantage of using scaled H_z field components

$$\frac{H_z}{H_{z1}(\rho_1)} = \frac{\frac{1}{2}\int_0^{\infty} H(\rho_m, h_m, y)\, e^{x-y} J_0(e^{x-y})\, dy + \frac{1}{\gamma^2 r^4}\left[3 - (3 + 3(1+j)B + 2jB)e^{-(1+j)B}\right]}{\frac{1}{\gamma_1^2 r^4}\left[3 - (3 + 3(1+j)B + 2jB)e^{-(1+j)B}\right]} \quad (4.7)$$

can be seen in their lack of dependence on the direction angle θ, the source current I, and the depole length ds. This simplification reduces the number of catalog curves required to make a first guess. Scaled E_c components have this same property. The scaled E_c components can be written in the form

$$\frac{E_c}{E_{c1}(\rho_1)} = \frac{\frac{1}{2\delta}\int_0^{\infty} E(\rho_m, h_m, y)\, e^{x-y} J_0(e^{x-y})\, dy + \frac{\rho_1}{r^3}\left\{-\frac{1}{2} + \left[(1+j)B + 1\right]e^{-(1+j)B}\right\}}{\frac{\rho_1}{r^3}\left\{-\frac{1}{2} + \left[(1+j)B + 1\right]e^{-(1+j)B}\right\}} \quad (4.8)$$

Least-Squares Examples

Five examples (figures 5, 6, 7, 8 and 9 are shown to illustrate the results of least-squares curve fitting. The two layer examples show a resistive first layer overlaying a conductive basement while the three layer examples illustrate a resistive bed between two conductive beds.

Figure 5 shows a two layer example for the amplitude of H_z. ρ_1 is fixed laeaving only two variable parameters. Although it is not necessary to fix any parameters convergence is enhanced if either ρ_1 or λ_1 is fixed. In this example the least-squares procedure was stopped after three iterations.

Table 2 shows the importance of chosing a good initial value for λ. From this example it appears that a smaller λ will yield a better fit. This is only true in cases where the first guess is close to the actual solution. When $\lambda = 0$ Marquardt's algorithm is equivalent to the Newton-Gauss procedure which will diverge for many cases where the parameters are non-linear.

λ	ϕ	RMS error
0.001	207.45×10^{-11}	3.38×10^{-11}
0.010	214.40×10^{-11}	3.44×10^{-11}
0.100	277.70×10^{-11}	3.92×10^{-11}
1.000	643.40×10^{-11}	5.96×10^{-11}
10.000	1071.30×10^{-11}	7.70×10^{-11}

where $\phi = \sum_{i=1}^{\#\text{ of Points}} (Y_i - \hat{Y}_i)^2$, RMS error $= \sqrt{\dfrac{\phi}{\text{number of sample points}}}$

Table 2 : Effect of variation of λ on the interpretational error.

Figures 6 and 7 show a nearly perfect fit for the imaginary part of H_z and the real part of E_y. This fit was obtained after six iterations. Table 3 indicates the variation of error with respect to the number of iterations applied to this case. It is impossible to estimate the number of iterations which will yield a good fit for any given case. The following procedure will help to optimize the number of iterations necessary

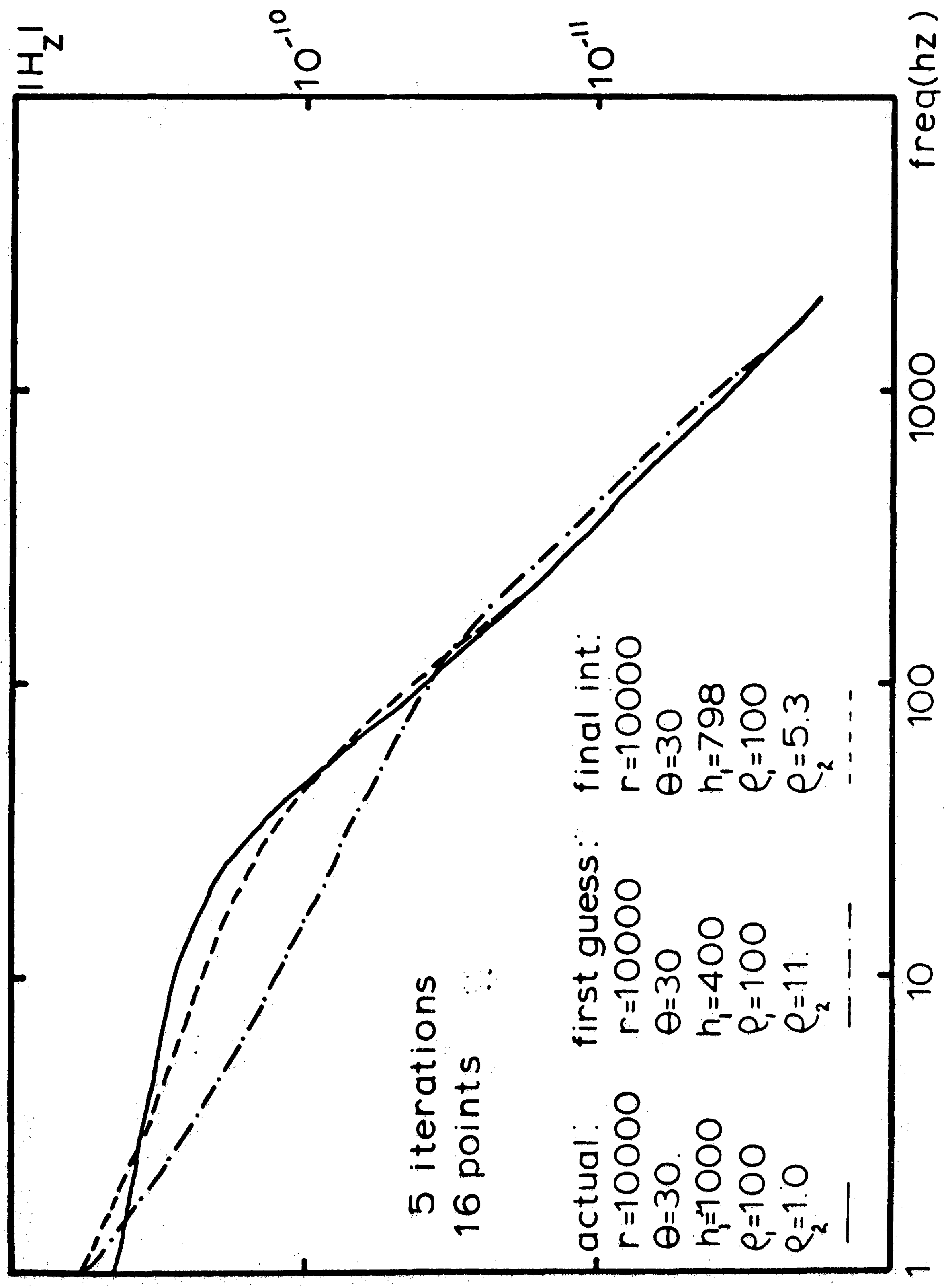


Figure 5: Two layer $|H_z|$ least squares fit.

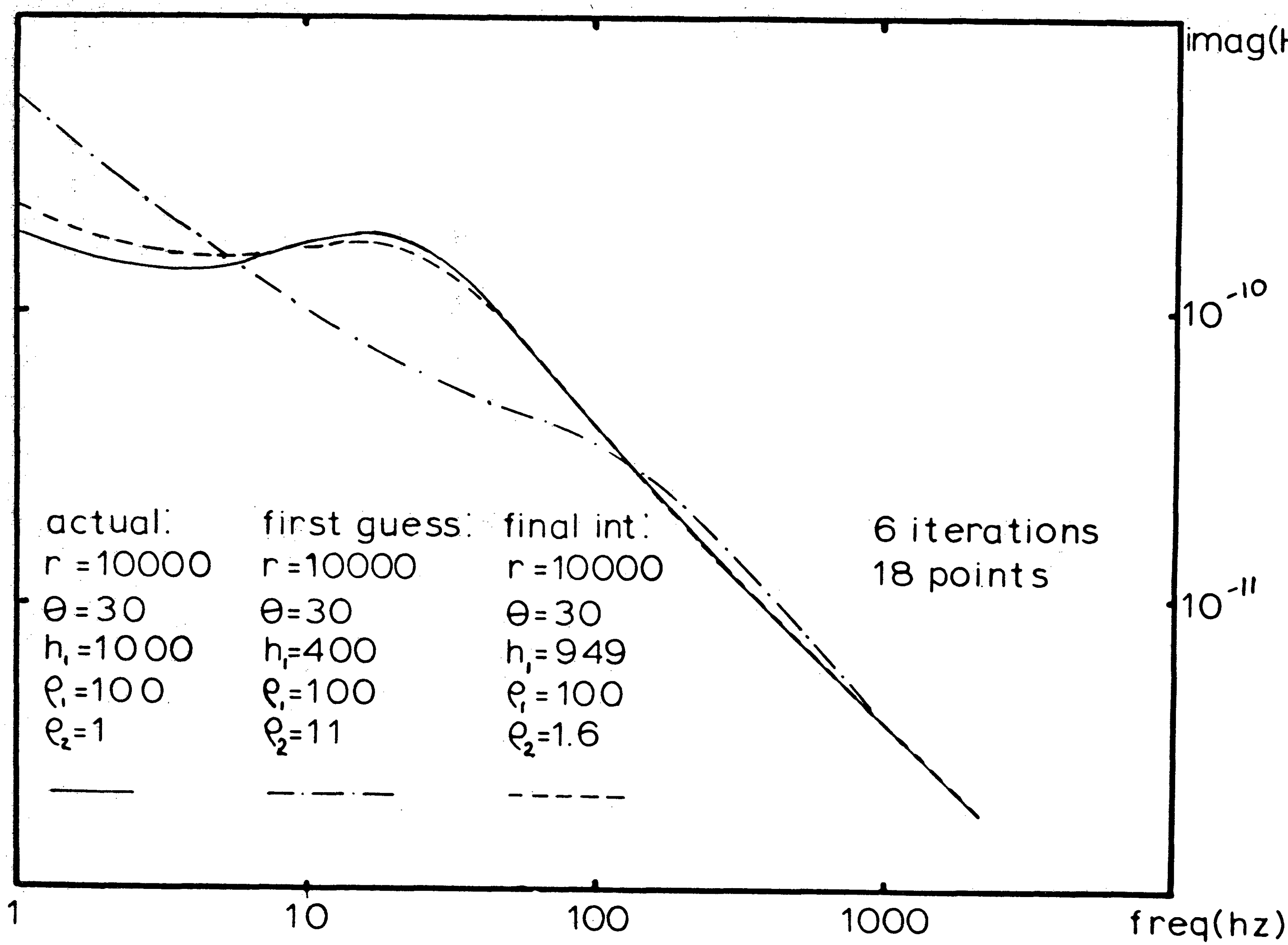


Figure 6: Two layer imaginary part of H_z least squares fit.

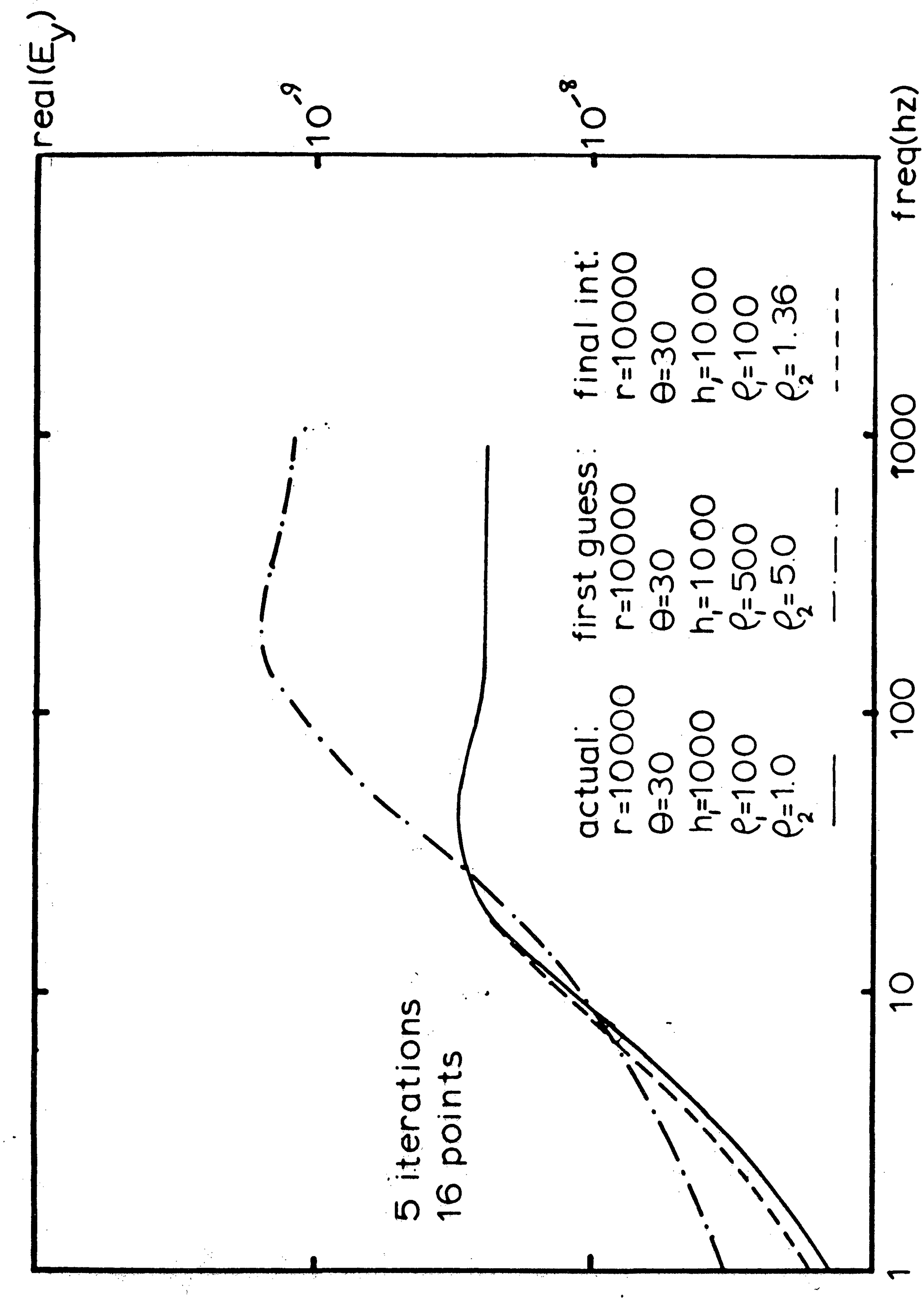


Figure 7 : Two layer real part of E_y least-squares curve fit.

to obtain a good fit.

Run EMINT through three iterations at a time.

i. If $\phi_3 < \phi_2$ by more than ten-percent, continue the iterative procedure.

ii. If $\phi_2 < \phi_1$ by more than ten-percent and $\phi_3 < \phi_2$ by less than ten-percent check the closeness of fit between the observed data and the calculated values. Stop the iterative procedure if the fit is considered adequate. Change λ or the first guess if the fit is inadequate.

iii. If $\phi_2 < \phi_1$ by less than ten-percent and $\phi_3 < \phi_2$ by less than ten-percent change λ or the first guess and continue the iterative procedure.

Iteration	ϕ	RMS error	(ohm-m)	(meters)
3	306.3×10^{-10}	10.1×10^{-10}	3.30	726.4
4	182.7×10^{-10}	2.12×10^{-10}	2.64	823.4
6	49.1×10^{-10}	2.9×10^{-11}	1.68	949.4
Actual parameter values			1.00	1000.0
First guess parameter values			11.00	400.0

Table 3 : Variation of converged parameters and error with the number of iterations.

Table 4 indicates how the accuracy of fit is affected by the number of sample points used for interpretation. Increasing the number of sample points does not necessarily improve the fit. As long as the curve is adequately defined, only about three sample points per decade of frequency are required for a good fit. This conclusion is based solely on theoretical results. More points are necessary when the data contains geologic or other non-random noise.

Two Layers : $Imag(E_y)$

Actual parameters r=10000m., θ=30 , ρ_1 =100Ω-m, h_1=100m., ρ_2=1.Ω-m

First guess r=10000m., θ =30, ρ_1 =500Ω-m, h_1=100m., ρ_2=5.Ω-m

minimum frequency= 1hertz maximum frequency= 1000hertz

all points evenly spaced

Number of points	RMS error	Interpreted parameters	
6	1.06×10^{-11}	ρ_1=100.02Ωm	ρ_2=1.142 m
8	1.13×10^{-11}	ρ_1=100.01Ωm	ρ_2=1.145 m
10	1.07×10^{-11}	ρ_1=100.01Ωm	ρ_2=1.137 m
12	1.22×10^{-11}	ρ_1=100.01Ωm	ρ_2=1.148 m

Three Layers : $|H_z|$

Actual parameters r=10000m., θ=30°, ρ_1 =1Ω-m, ρ_2 =1000Ω-m, ρ_3=1.Ω-m, h_1=100m., h_2=100m.

First guess r=10000m., θ =30°, ρ_1 =1.Ω-m, ρ_2 =1300.Ω-m, ρ_3=5.Ω-m, h_1=200m., h_2=300m.

minimum frequency=1 hertz maximum frequency=6000 hertz

all points evenly spaced

Number of points	RMS error	Interpreted parameters	
7	1.63×10^{-12}	h_1=196.2 m ρ_2=1121.5 Ωm	h_2=134.0 m ρ_3 =1.38 Ωm
13	1.86×10^{-12}	h_1=196.4 m ρ_2=1126.4 Ωm	h_2=147.9 m ρ_3 =1.53 Ωm
20	1.78×10^{-12}	h_1 =187.4 m ρ_2 =1177.4 Ωm	h_2 =156.7 m ρ_3 =1.111 Ωm

Table 4 : The effect of varying the number of sample points on the interpretation error and interpreted parameter values.

Figures 8 and 9 show test results of applying Marquardt's least-squares algorithm to three layer cases. For the three layer case proper convergence can only occur when the first guess is reasonably close to the exact solution.

Computer time and cost for the least-squares interpretation program can be summarized as follows (cost and time per sample point):

(1) For H_z and E_c two layer cases each iteration takes approximately two seconds ($1.20) while E_x and E_y takes four seconds ($2.40) of computer time.

(2) For H_z and E_c three layer cases each iteration takes approximately three seconds ($1.80) while E_x and E_y takes five seconds ($3.00) of computer time.

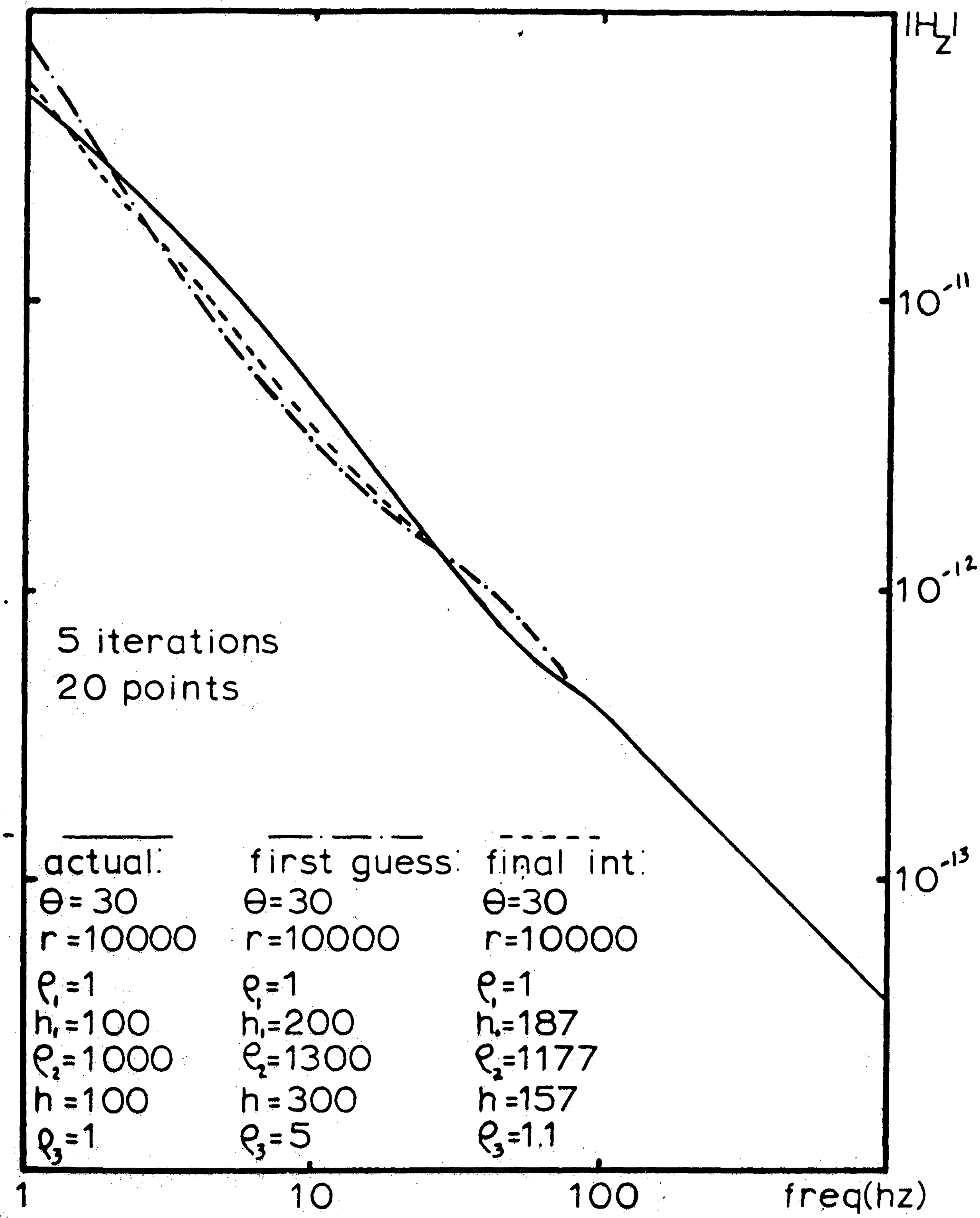


Figure 8: Three layer $|H_z|$ least squares-curve fit.

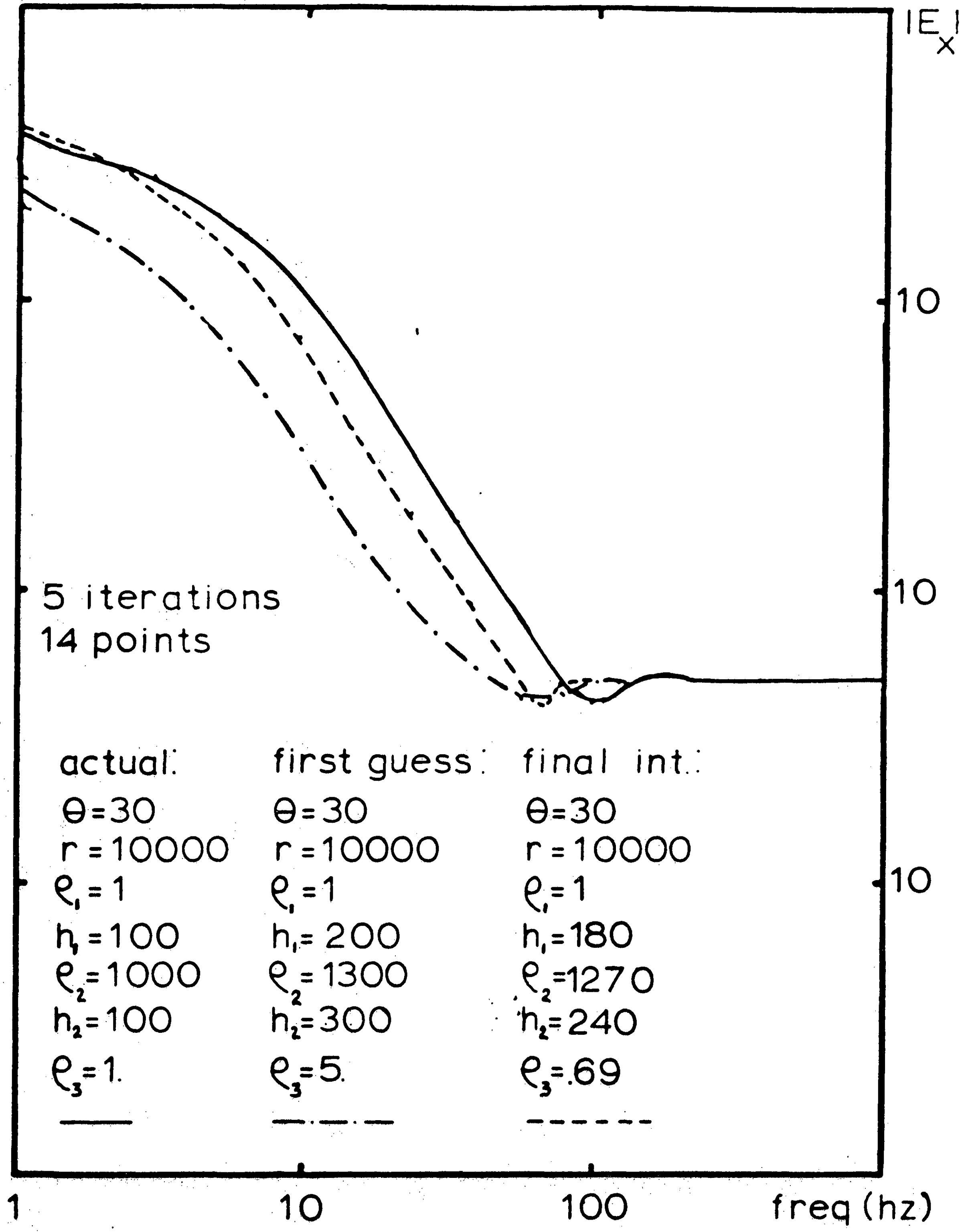


Figure 9: Three layer $|E_x|$ least-squares curve fit.

CONCLUSIONS

Marquardt's algorithm for least-squares curve fitting provides a good method for interpreting electromagnetic sounding data. The theoretical expressions for the electromagnetic field components can be evaluated numerically by making the proper substitution of variables in the integral expressions and applying principles of linear filter theory to the resulting integrals.

Test results of applying Marquardt's algorithm to electromagnetic sounding data can be summarized as follows:

(1) For the two layer case convergence of the layered earth parameters is generally fast and accurate. Convergence of the layered earth parameters (resistivity and thickness) is accurate to within ten-percent. Convergence is enhanced by fixing either the first layer resistivity or the first layer thickness.

(2) Three layer curve fitting, with four or more parameters, requires a good first guess to insure convergence. Parameter convergence is generally slower for the three layer case than for the two layer case.

Scaled H_z field component curves (Appendix A) can simplify catalog curve matching by eliminating the curves dependence on θ, I, and ω. Similarity, scaling E_c (which is a mathmatical combination of E_x and E_y) simplifies catalog interpretation for the E_x and E_y components of the electric field. These modifications of the field components can enhance the ability of the interpreter to obtain a good first guess.

The overall least-squares computer program is too expensive to operate on a commercial basis. For H_z and E_c two layer cases, each iteration takes approximately two seconds per sample point, while for E_x and E_y each iteration consumes four seconds of computer time per sample point. Computation time can be reduced by calculating the partial derivatives of the layered-earth parameters in closed form.

Further computer time could be saved by using separate computer programs to individually fit each field component.

Regardless of how "automated" an interpretation scheme may be, meaningful results still require good judgement on the part of the interpreter. The geophysicist who uses the technique developed in this paper should not lose sight of the assumptions which are inherent in the method. The final interpretation of electromagnetic sounding data must be geologically meaningful.

APPENDIX A

Two and Three Layer Scaled H_z Amplitude Curves

The graphs in this section were computed using Program EMFWD. They may serve as a guide for a first guess interpretation.

Graph symbols:

$$\frac{H}{H1} = \frac{\text{amplitude of } H_z \text{ (layered earth model)}}{\text{amplitude of } H_z \text{ (homogenous half-space)}}$$

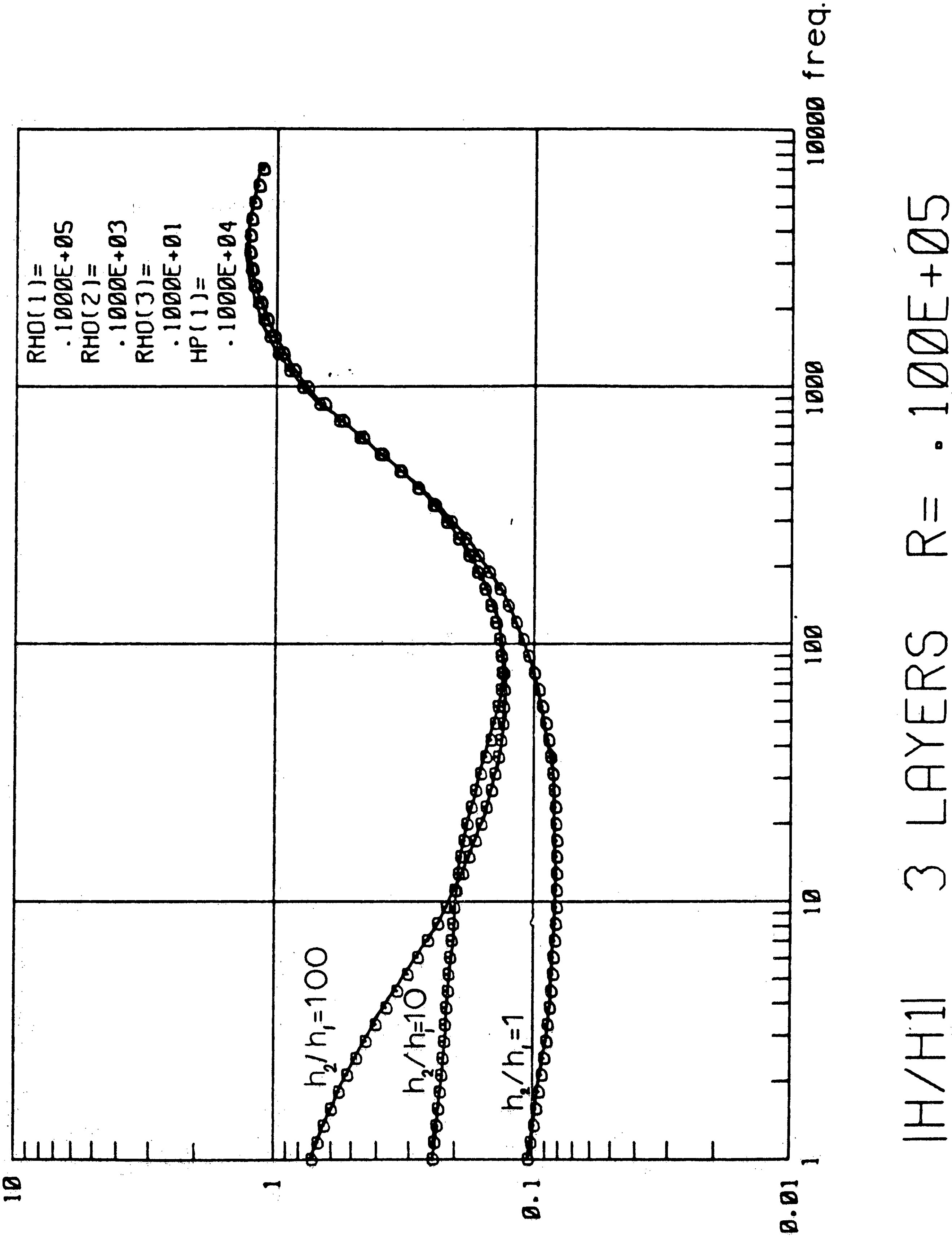
$\frac{H}{H1}$ represents the ordinate values on each graph.

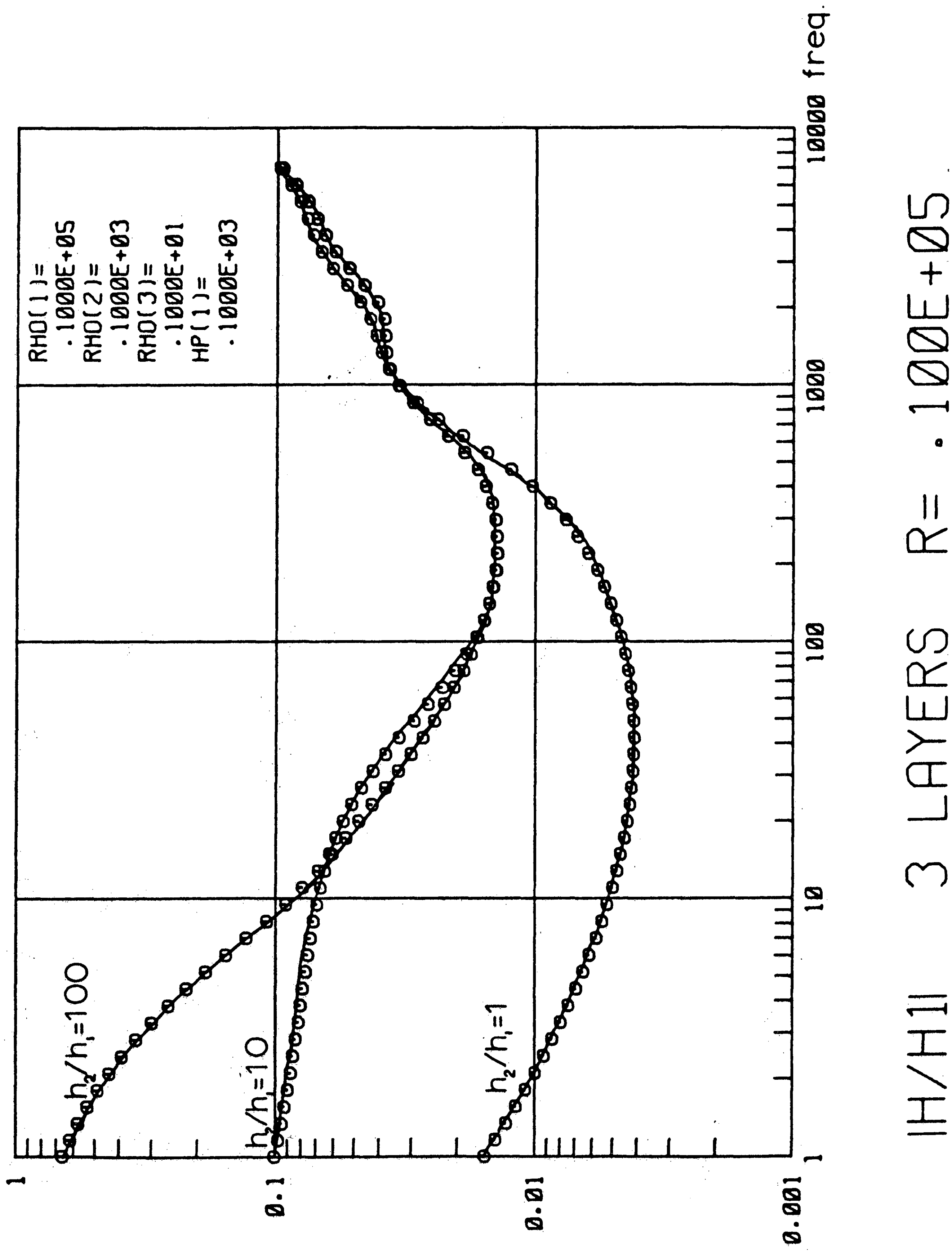
R = source receiver seperation

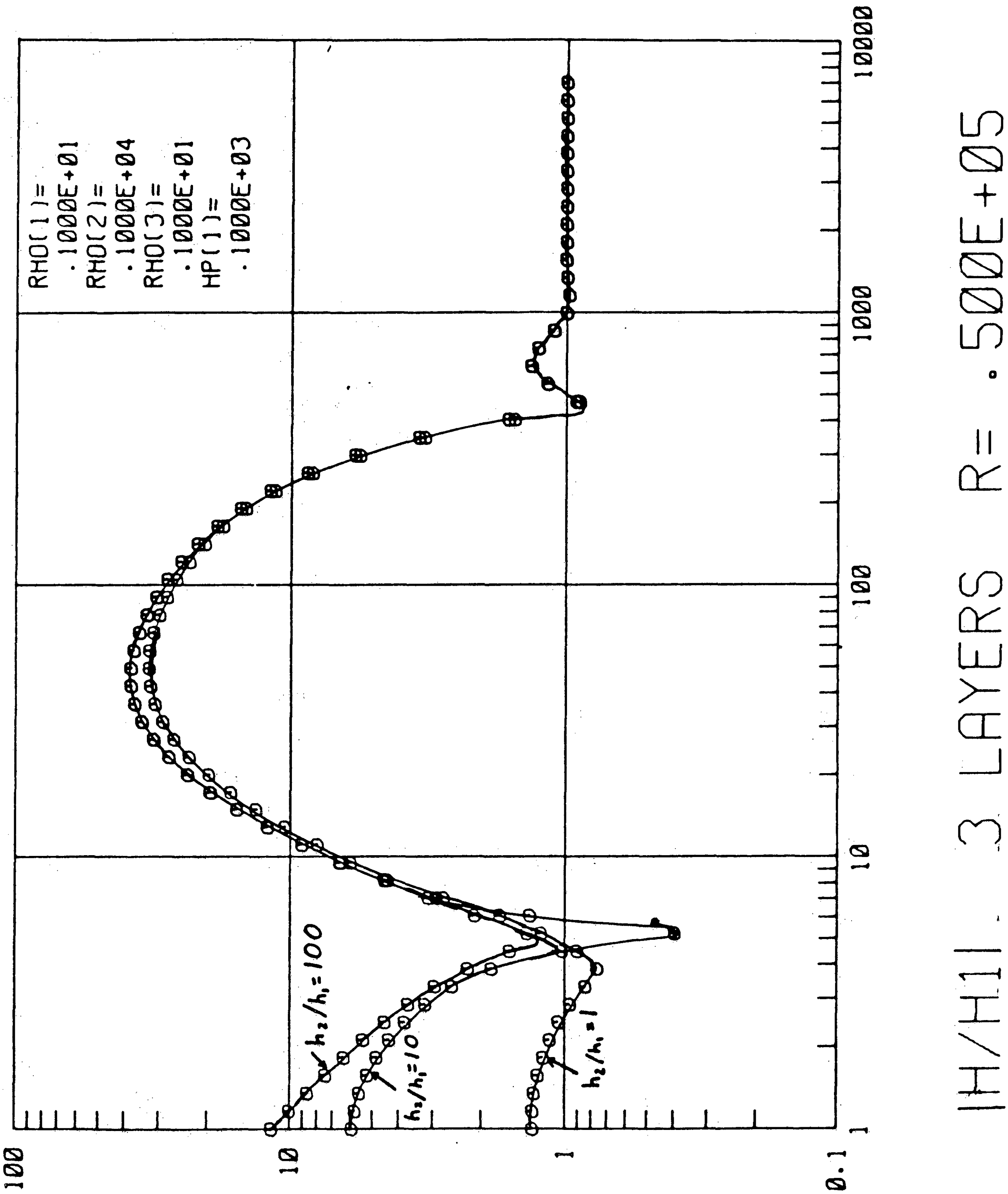
RHO(i) = resistivity of the i-th layer

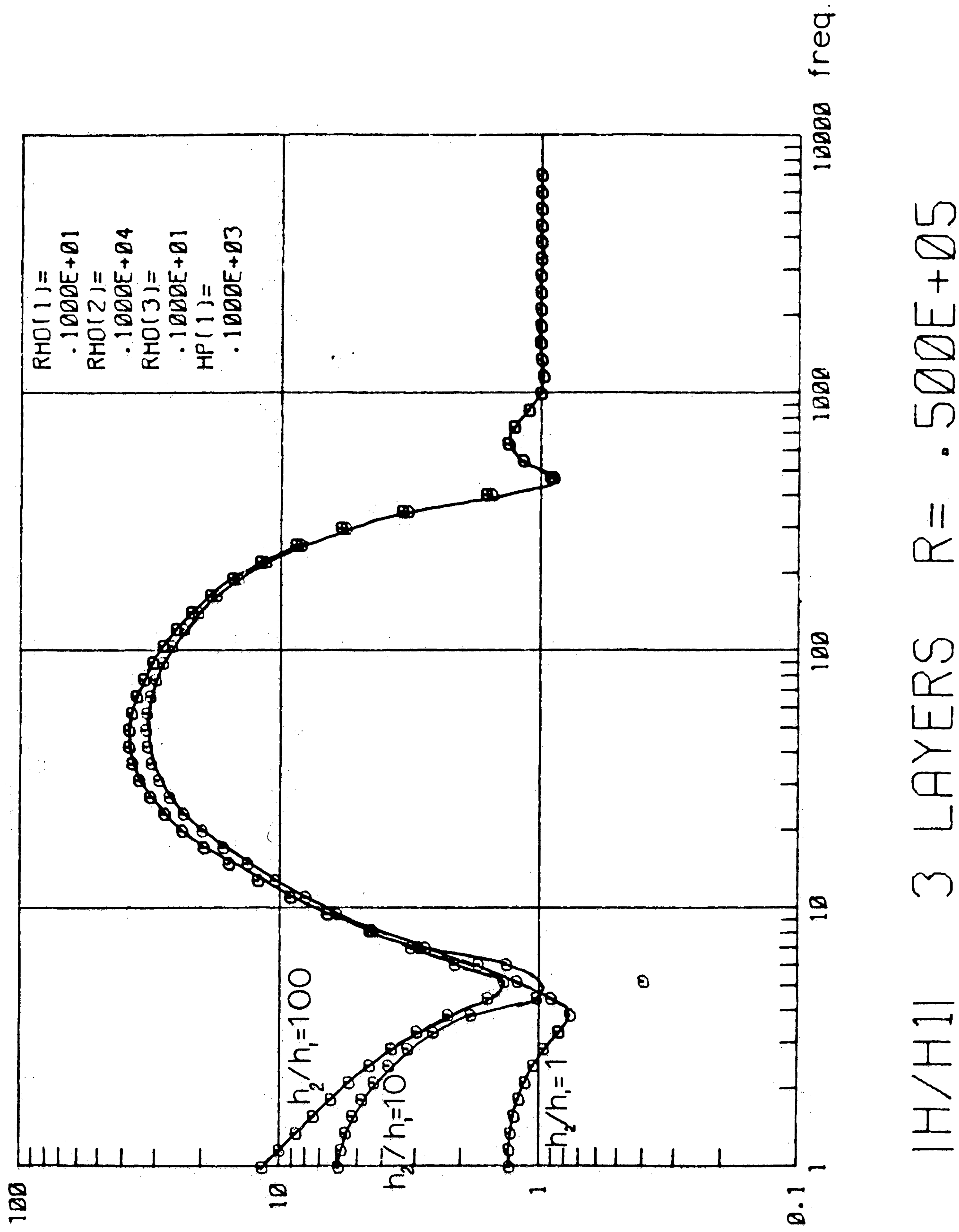
HP(i) = thickness of the i-th layer

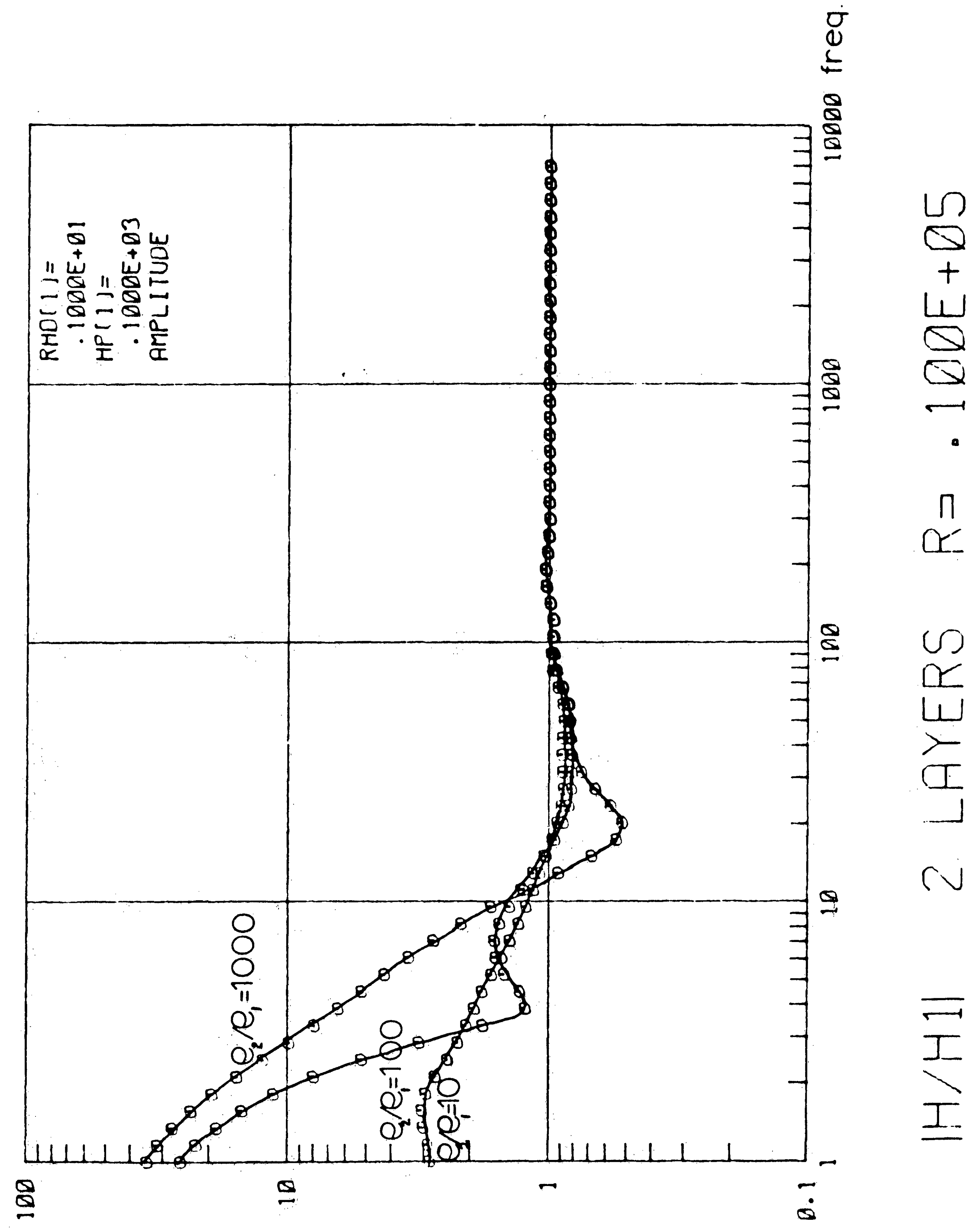
abscissa values= frequency (in hertz)

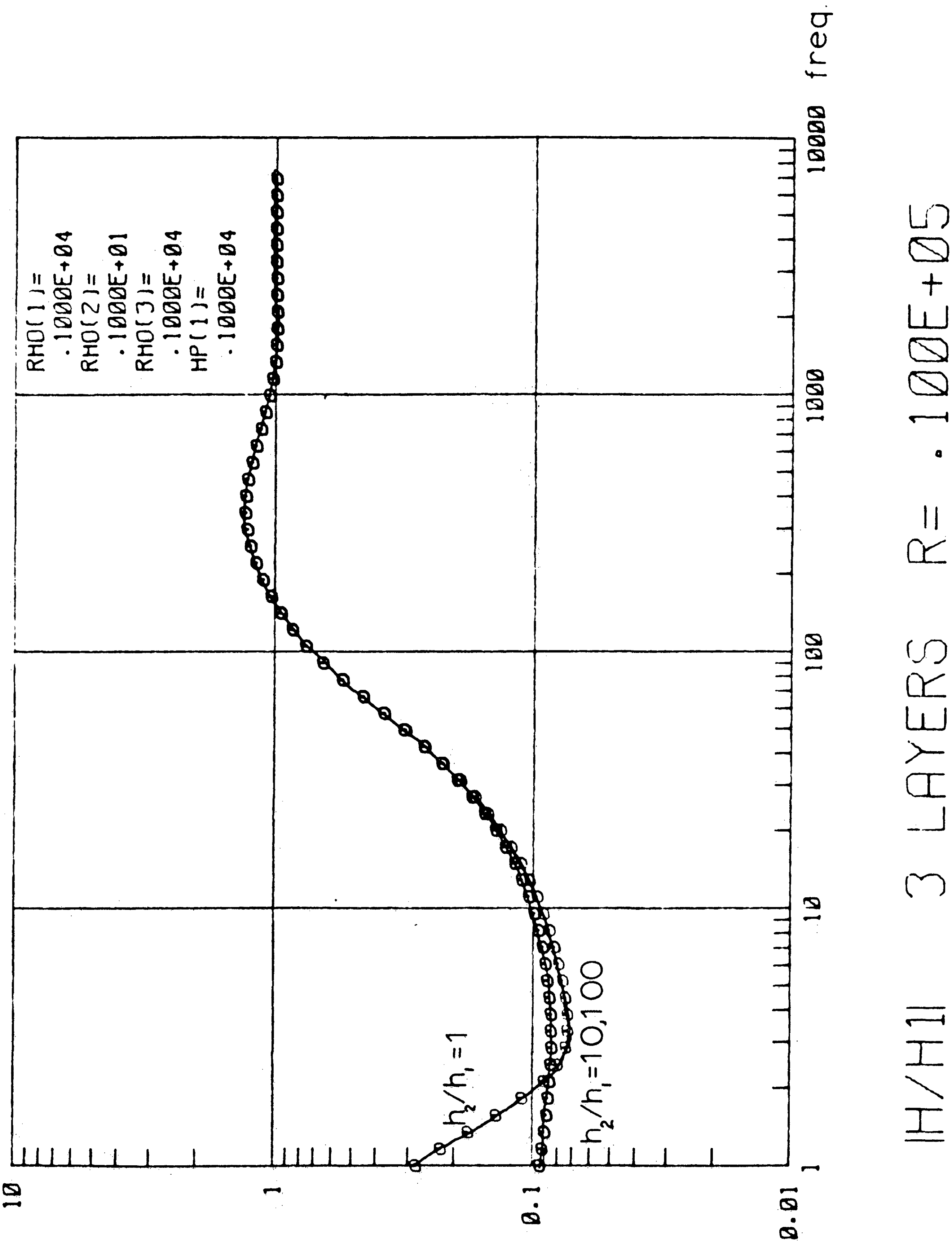

RHO(1)=
.1000E+05
RHO(2)=
.1000E+03
RHO(3)=
.1000E+01
HP(1)=
.1000E+04
h2/h1=100
h2/h1=10
h2/h1=1
10
1
0.1
0.01
1
10
100
1000
10000
freq.
|H/H1| 3 LAYERS R= .100E+05

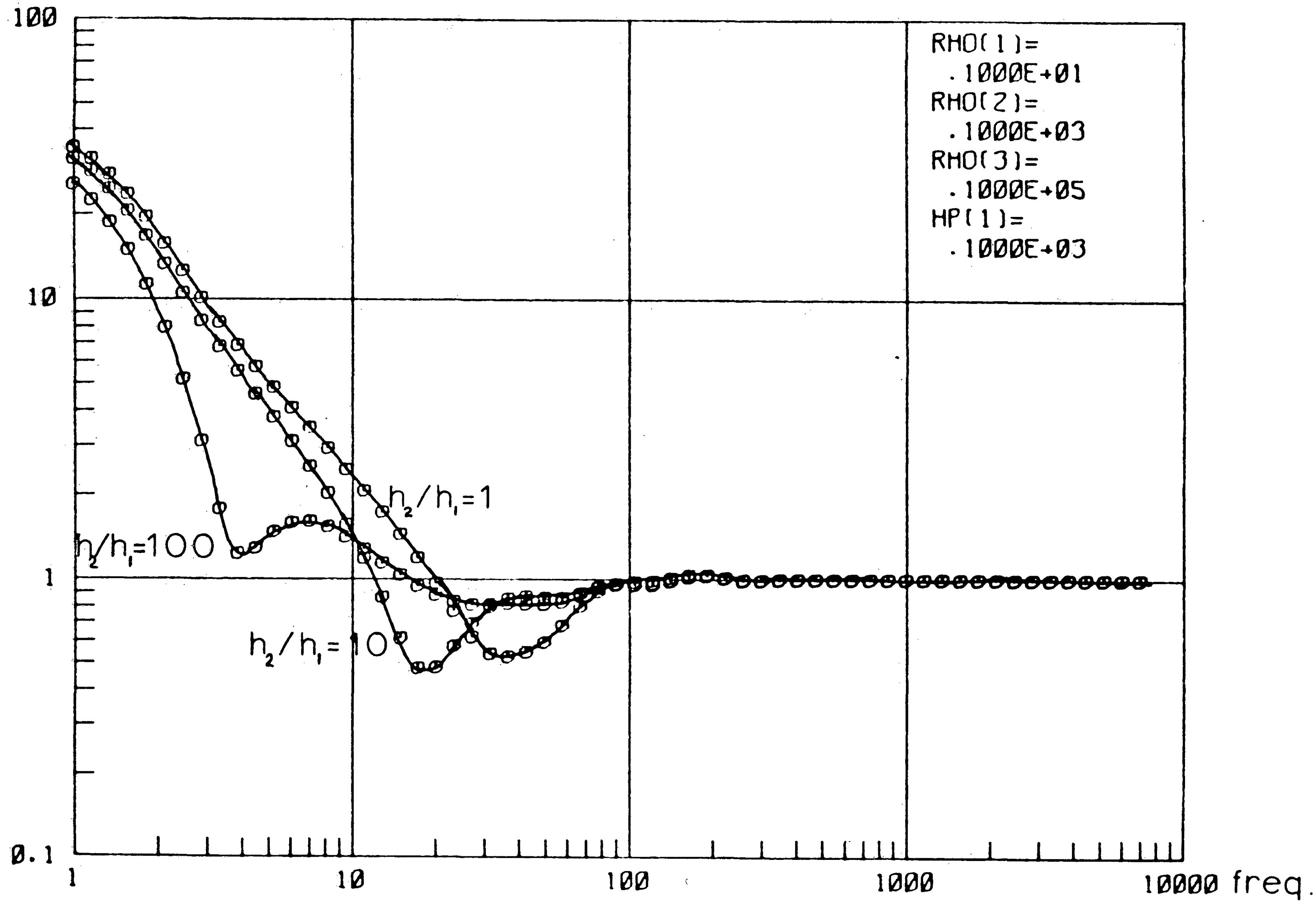

|H/H1| 3 LAYERS R= .100E+05
RHO(1)= .1000E+05
RHO(2)= .1000E+03
RHO(3)= .1000E+01
HP(1)= .1000E+03
h2/h1=100
h2/h1=10
h2/h1=1
1
0.1
0.01
0.001
1
10
100
1000
10000 freq.


RHO(1)=
.1000E+01
RHO(2)=
.1000E+04
RHO(3)=
.1000E+01
HP(1)=
.1000E+03
$h_2/h_1=100$
$h_2/h_1=10$
$h_2/h_1=1$
100
10
1
0.1
1
10
100
1000
10000
|H/H1| 3 LAYERS R= .500E+05

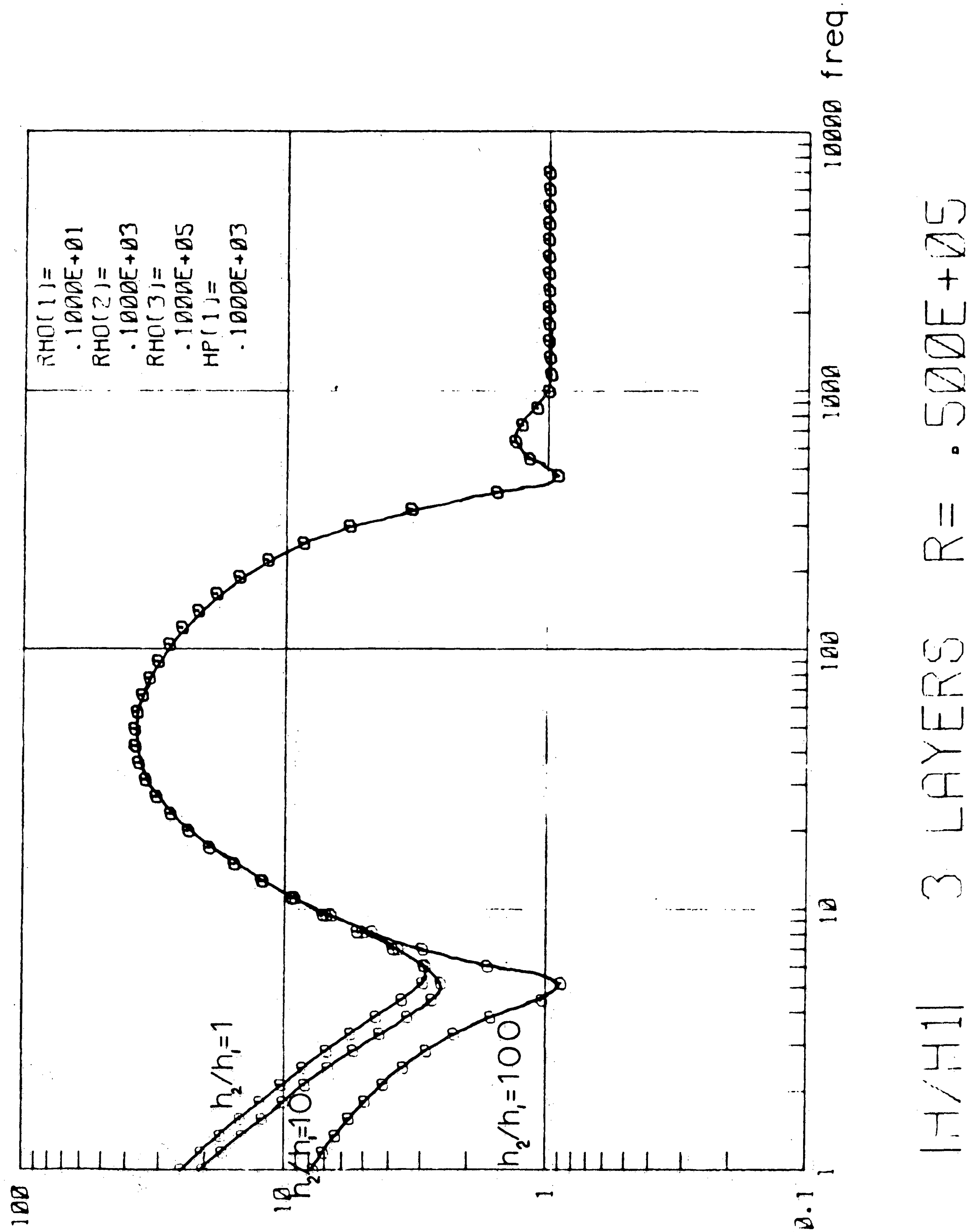

RHO(1)=
.1000E+01
RHO(2)=
.1000E+04
RHO(3)=
.1000E+01
HP(1)=
.1000E+03
h_2/h_1=100
h_2/h_1=10
h_2/h_1=1
100
10
1
0.1
1
10
100
1000
10000
freq.
|H/H1| 3 LAYERS R= .500E+05

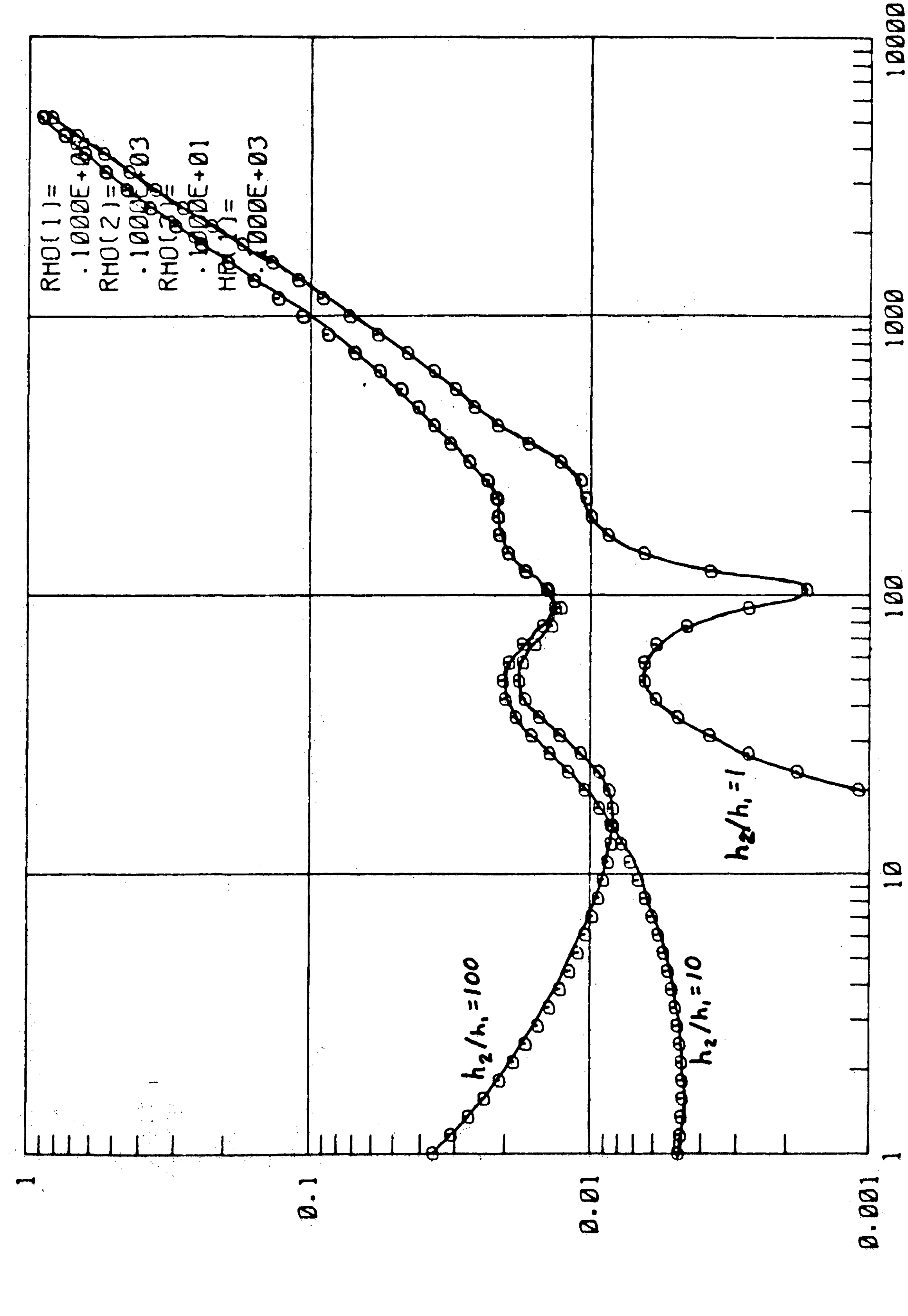

RHO(1)=
.1000E+01
HP(1)=
.1000E+03
AMPLITUDE
ρ2/ρ1=1000
ρ2/ρ1=100
ρ2/ρ1=10
100
10
1
0.1
1
10
100
1000
10000 freq.
|H/H1| 2 LAYERS R= .100E+05


|H/H1| 3 LAYERS R= .100E+05
RHO(1)=
.1000E+04
RHO(2)=
.1000E+01
RHO(3)=
.1000E+04
HP(1)=
.1000E+04
h2/h1=1
h2/h1=10,100
freq
1
10
100
1000
10000
10
1
0.1
0.01

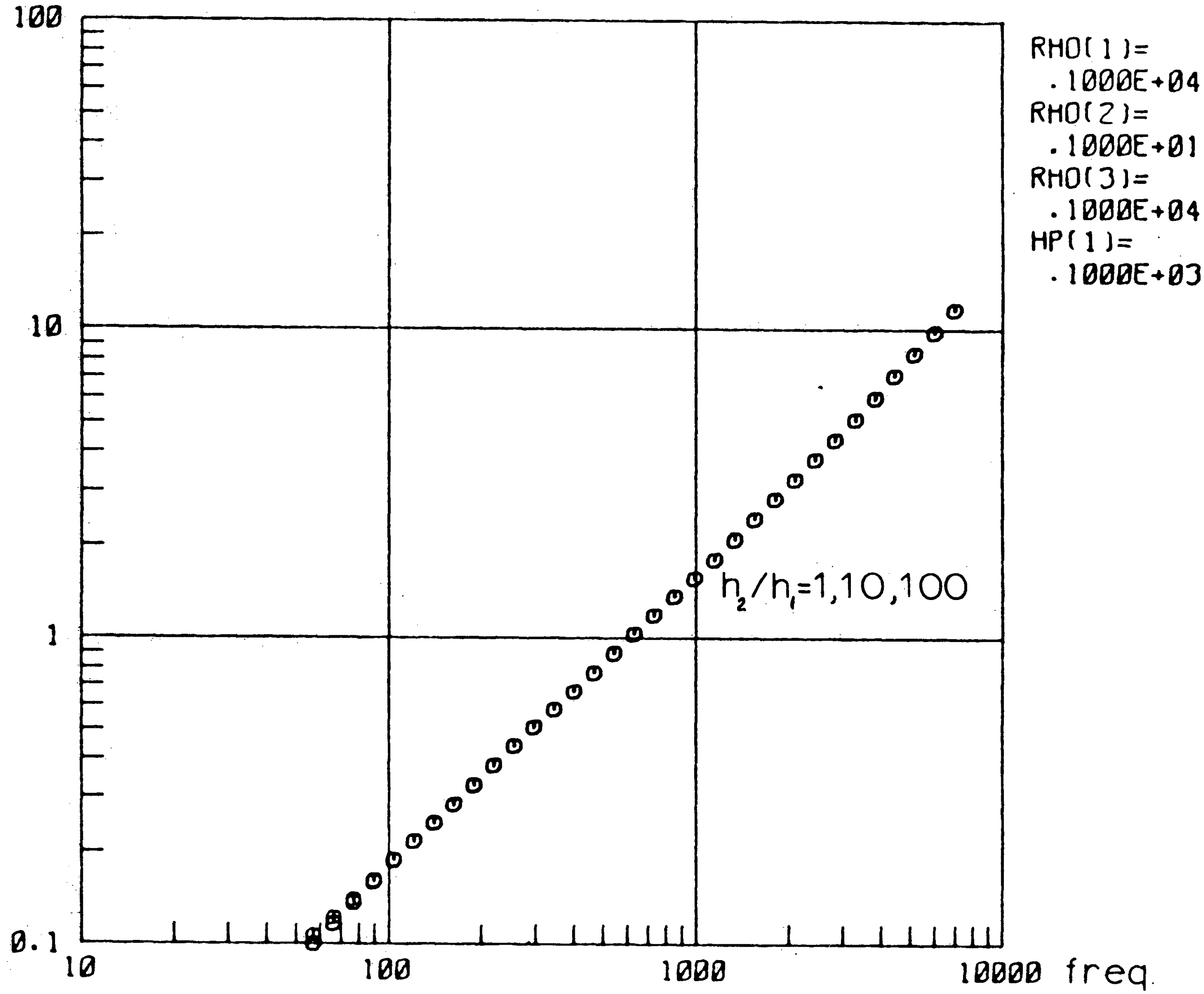


|H/H1| 3 LAYERS   R= .100E+05

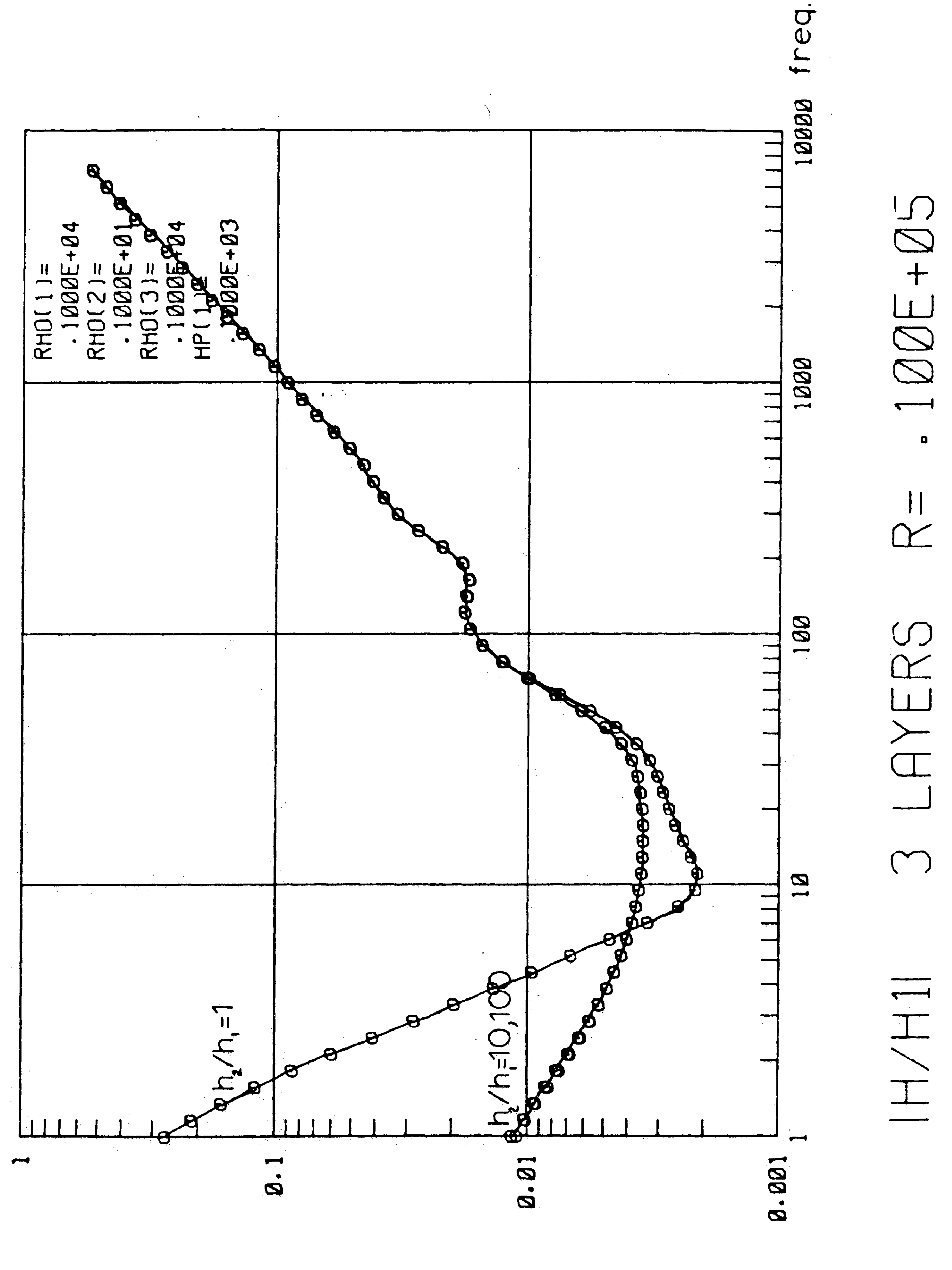

|H/H1| 3 LAYERS R= .500E+05
RHO(1)=
.1000E+01
RHO(2)=
.1000E+03
RHO(3)=
.1000E+05
HP(1)=
.1000E+03
h_2/h_1=1
h_2/h_1=10
h_2/h_1=100
100
10
1
0.1
1
10
100
1000
10000 freq.

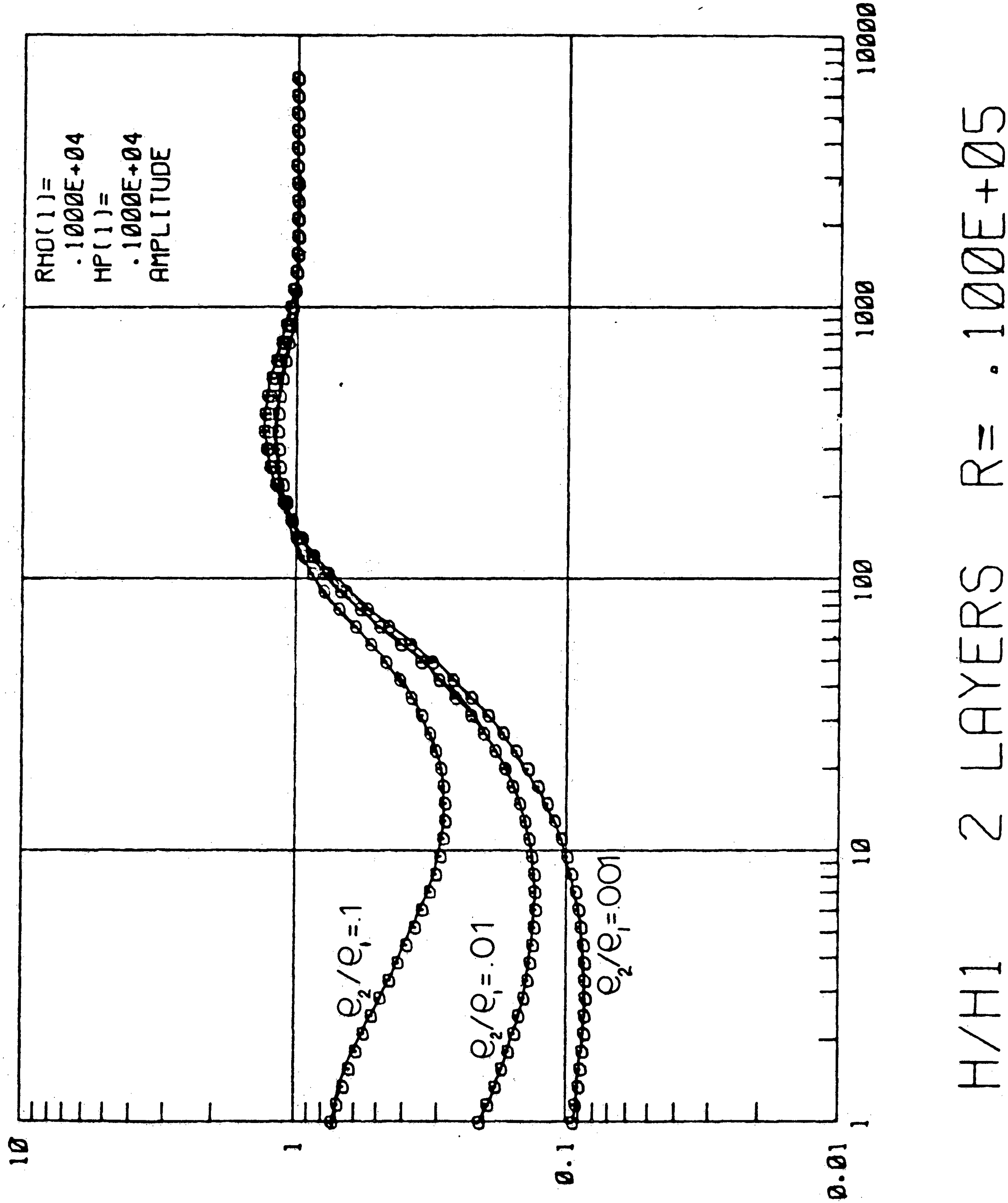


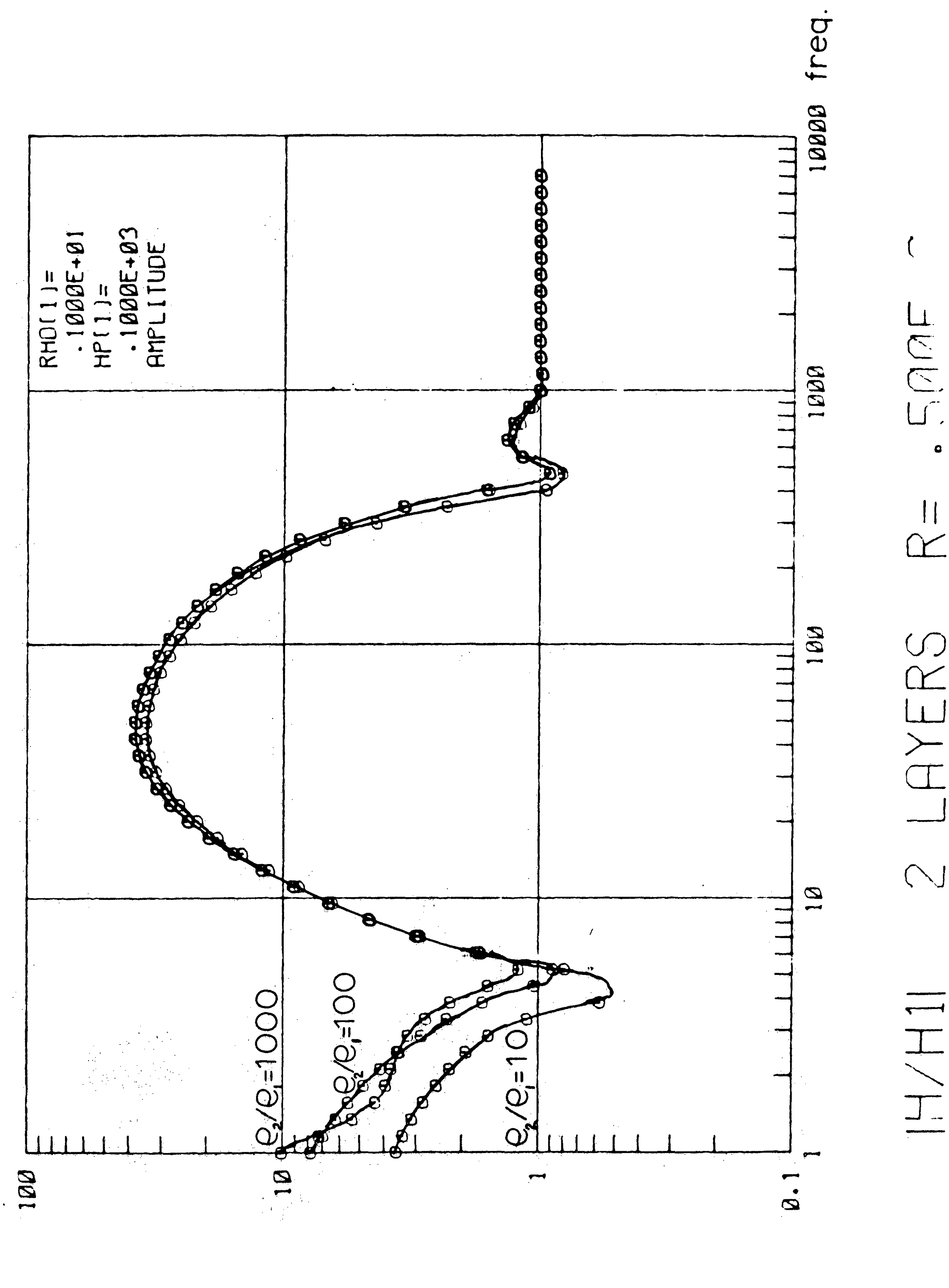
|H/H1| 3 LAYERS R= .500E+05

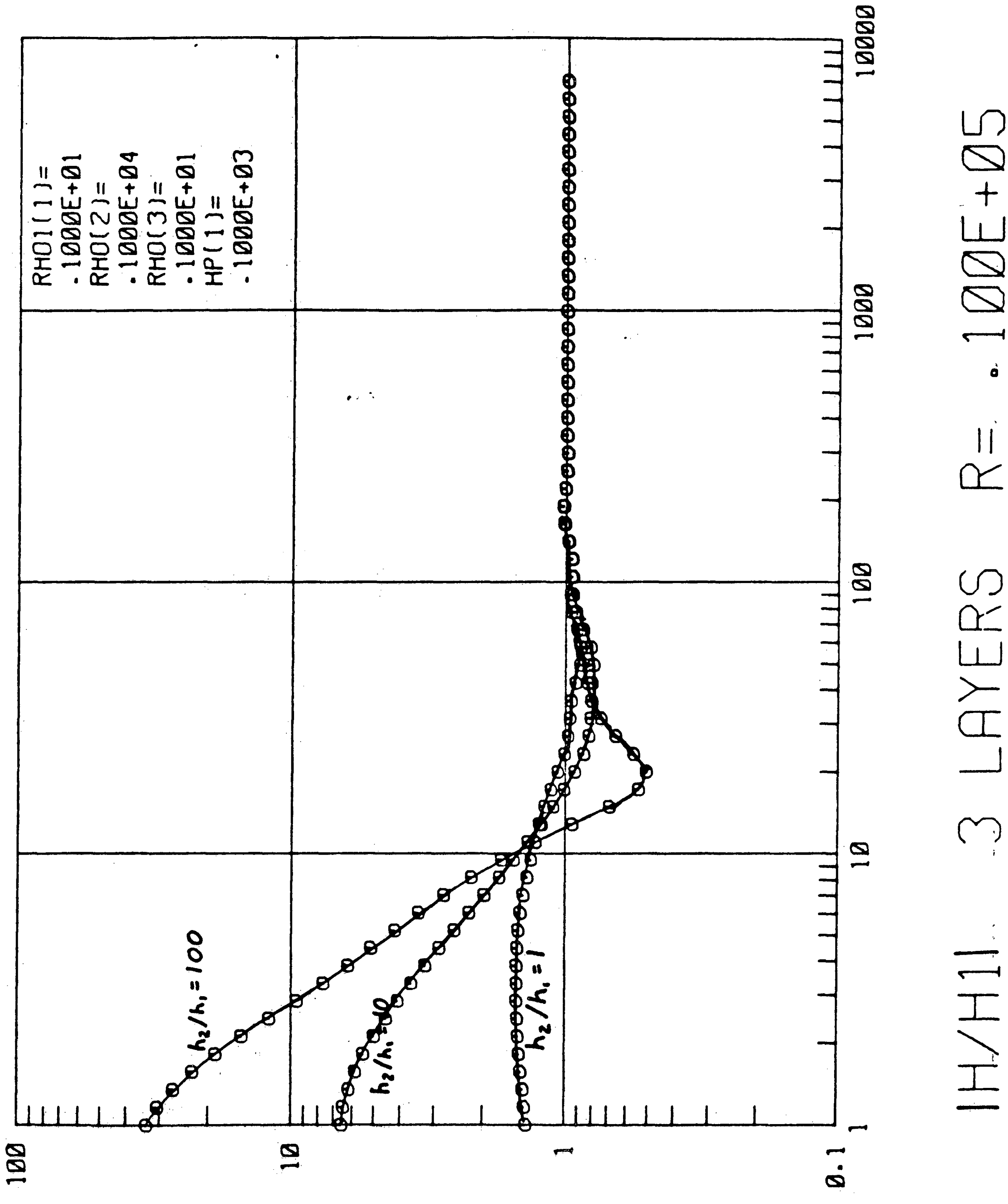


|H/H1| 3 LAYERS R= .500E+05

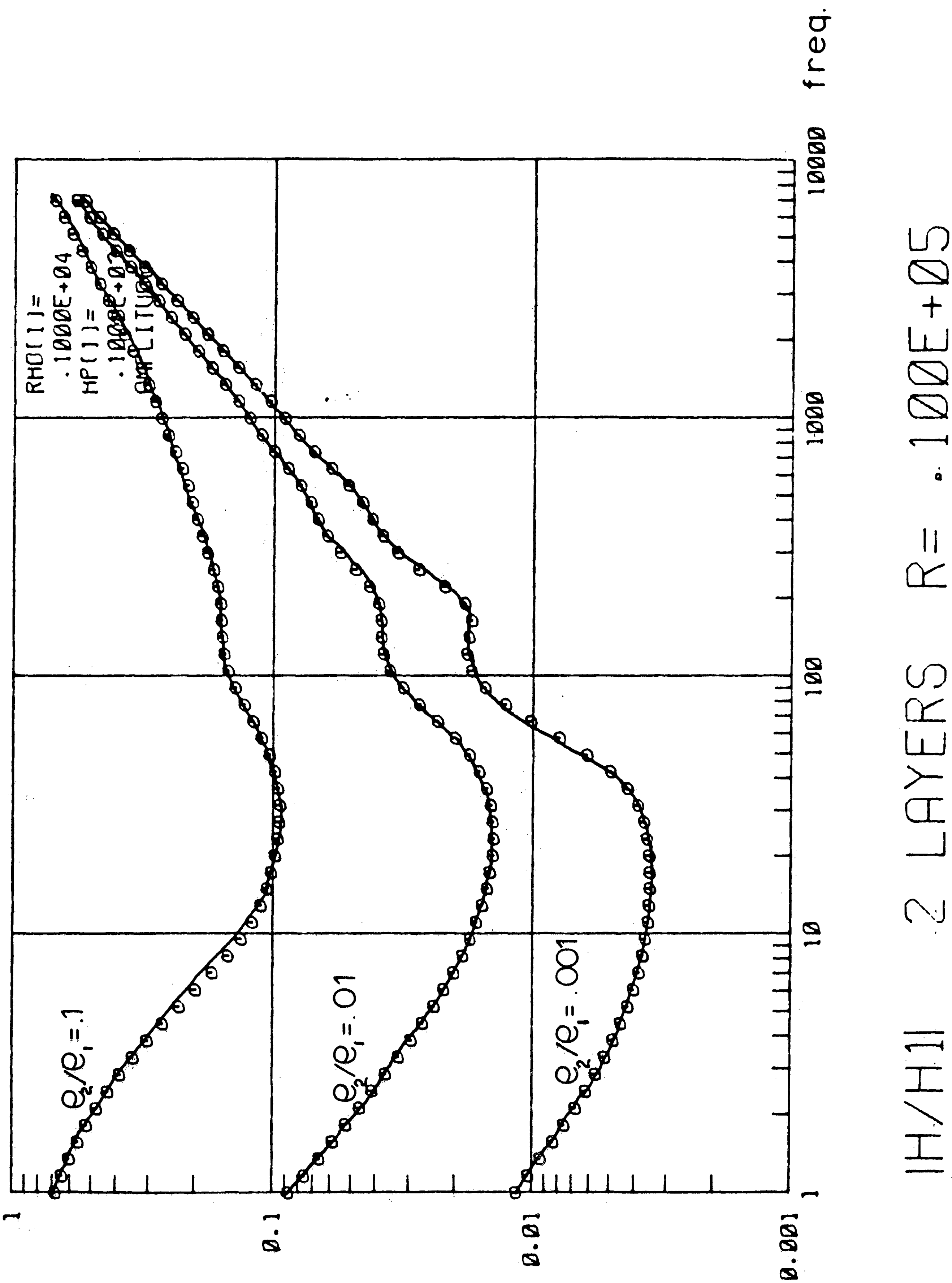

|H/H1| 3 LAYERS R= .100E+05
freq.
RHO(1)= .1000E+04
RHO(2)= .1000E+01
RHO(3)= .1000E+04
HP(1)= .1000E+03
h_2/h_1=1
h_2/h_1=10,100
1
10
100
1000
10000
1
0.1
0.01
0.001


RHO(1)=
.1000E+04
HP(1)=
.1000E+04
AMPLITUDE
e2/e1=.1
e2/e1=.01
e2/e1=.001
10
1
0.1
0.01
1
10
100
1000
10000
H/H1 2 LAYERS R= .100E+05


RHO(1)=
.1000E+01
HP(1)=
.1000E+03
AMPLITUDE
100
10
1
0.1
1
10
100
1000
10000
freq.
e_2/e_1=1000
e_2/e_1=100
e_2/e_1=10
|H/H1| 2 LAYERS R= .5


RHO1(1)=
.1000E+01
RHO(2)=
.1000E+04
RHO(3)=
.1000E+01
HP(1)=
.1000E+03
h2/h1=100
h2/h1=10
h2/h1=1
100
10
1
0.1
1
10
100
1000
10000
|H/H1| 3 LAYERS R= .100E+05

T-1627


|H/H1| 2 LAYERS R= .100E+05
RHO(1)=
.1000E+04
HP(1)=
1
0.1
0.01
0.001
1
10
100
1000
10000
freq.
e_2/e_1=.1
e_2/e_1=.01
e_2/e_1=.001

APPENDIX B

COMPUTER PROGRAMS

```
C****************** PRØGRAM EMFWD ************************
C
C       THIS PRØGRAM CALCULATES THE VERTICAL H FIELD, THE X-CØMPØNENT
C  ØF THE ELECTRIC FIELD, THE Y-CØMPØNENT ØF THE ELECTRIC FIELD,
C  AND THE CØMBINED HØRIZØNTAL ELECTRIC FIELD CØMPØNENTS (EC=
C  SIN(2*ANG)EX-CØS(2*ANG)EY) FØR A 1,2,ØR 3 LAYERED EARTH.  THE SØURCE
C  IS A HØRIZØNTAL CURRENT DIPØLE
C
C
C      FIELDS FØR A HØMØGENØUS HALF-SPACE (1 LAYER) MAY BE CØMPUTED
C  BY USING A TWØ LAYER MØDEL WITH RHØ(2)=RHØ(1).  MØDELS
C  FØR MØRE THAN THREE LAYERS MAY BE CØMPUTED SIMPLY BY INCREASING
C  THE DIMENSIØNS ØF THE ARRAYS.
C
C
C       PRØGRAM EMFWD WAS DEVELØPED ØN A DEC PDP-10 CØMPUTER AT THE
C  CØLØRADØ SCHØØL ØF MINES.  FØRTRAN PRØGRAMING LANGUAGE IS USED
C THRØUGHØUT
C
C
C                            JEFF DANIELS
C                        CØLØRADØ SCHØØL ØF MINES
C                            FEBRUARY 1974
C
C
C************************ VARIABLES ******************
C
C NFW= THE NUMBER ØF FREQUENCIES TØ BE CALCULATED
C R= SØURCE-RECEIVER SEPERATIØN
C ANG=ANGLE,IN DEGREES, DEFINING SØURCE-RECEIVER ØRIENTATIØN
C FF=FREQUENCY IN HERTZ
C F=ANGULAR FREQUENCY
C N= THE NUMBER ØF LAYERS
C HP(I)= LAYER THICKNESS
C RH(I)= LAYER RESISTIVITY
C CH(I)= CØEFFICIENTS FØR CALCULATING  THE J1 HANKEL TRANSFØRM
C YH(I)= ABSCISSA VALUES FØR CH(I)
C CE(I)= CØEFFICIENTS FØR CALCULATING THE JØ HANKEL TRANSFØRM
C YE(I)= ABSSCISSA VALUES FØR CALCULATING CE(I)
C CI = SØURCE-DIPØLE CURRENT
C DS= SØURCE-DIPØLE LENGTH
C D = NØRMATIZED THICKNESSES
C RK= NØRMALIZED RESISTIVITIES
C DEL= MØDIFIED WAVE NUMBER
C IIN= INPUT AREA
C IØUT= ØUTPUT AREA
C
C
C*************************************************************
      CØMPLEX H,E,EX,EY
      DIMENSIØN HPP(3),HP(3)
```

```
      COMMON /QC/D(3),RK(3),RH(3),N,DEL,R
      COMMON /CV/CI,DS, ANG,F,X,YH(48),CH(48),YE(61),CE(61),TM
      DATA
     &YH/-4.5307316E 0,-4.3004731E 0,-4.0702146E 0,-3.8399561E 0,
     &   -3.6096976E 0,-3.3794391E 0,-3.1491806E 0,-2.9189221E 0,
     &   -2.6886636E 0,-2.4584051E 0,-2.2281466E 0,-1.9978881E 0,
     &   -1.7676296E 0,-1.5373711E 0,-1.3071126E 0,-1.0768541E 0,
     &   -8.4659563E-1,-6.1633713E-1,-3.8607863E-1,-1.5582013E-1,
     &    7.4438369E-2, 3.0469687E-1, 5.3495537E-1, 7.6521387E-1,
     &    9.9547237E-1, 1.2257309E 0, 1.4559894E 0, 1.6862479E 0,
     &    1.9165064E 0, 2.1467649E 0, 2.3770234E 0, 2.6072819E 0,
     &    2.8375404E 0, 3.0677989E 0, 3.2980574E 0, 3.5283159E 0,
     &    3.7585744E 0, 3.9888329E 0, 4.2190914E 0, 4.4493499E 0,
     &    4.6796084E 0, 4.9098669E 0, 5.1401254E 0, 5.3703839E 0,
     &    5.6006424E 0, 5.8309009E 0, 6.0611594E 0, 6.2914179E 0/
      DATA
     &CH/3.1010561E-6, 1.8802098E-5, 5.4819540E-5, 9.2891602E-6,
     &   1.5523239E-4, 3.0344652E-5, 3.5338744E-4, 1.4793002E-4,
     &   7.7342377E-4, 5.3570357E-4, 1.7170605E-3, 1.6387239E-3,
     &   3.9247683E-3, 4.5796508E-3, 9.2111468E-3, 1.2130467E-2,
     &   2.1938415E-2, 3.0853660E-2, 5.1973594E-2, 7.4661566E-2,
     &   1.1775455E-1, 1.6353574E-1, 2.3127545E-1, 2.7368461E-1,
     &   2.8059285E-1, 1.2875840E-1,-1.5380437E-1,-4.5659951E-1,
     &  -3.6077766E-2, 4.2985683E-1,-2.1506075E-1,-2.3624312E-2,
     &   8.9316746E-2,-7.4344203E-2, 4.8572965E-2,-3.0088872E-2,
     &   1.8846544E-2,-1.2158687E-2, 8.0708759E-3,-5.4706275E-3,
     &   3.7554604E-3,-2.5929707E-3, 1.7909426E-3,-1.2320277E-3,
     &   8.4095286E-4,-5.6749747E-4, 3.7718405E-4,-1.5891835E-4/
      DATA
     &YE/-6.8348046E 0,-6.6045461E 0,-6.3742876E 0,-6.1440291E 0,
     &   -5.9137706E 0,-5.6835121E 0,-5.4532536E 0,-5.2229951E 0,
     &   -4.9927366E 0,-4.7624781E 0,-4.5322196E 0,-4.3019611E 0,
     &   -4.0717026E 0,-3.8414441E 0,-3.6111856E 0,-3.3809271E 0,
     &   -3.1506686E 0,-2.9204101E 0,-2.6901516E 0,-2.4598931E 0,
     &   -2.2296346E 0,-1.9993761E 0,-1.7691176E 0,-1.5388591E 0,
     &   -1.3086006E 0,-1.0783421E 0,-8.4808358E-1,-6.1782508E-1,
     &   -3.8756658E-1,-1.5730808E-1, 7.2950416E-2, 3.0320892E-1,
     &    5.3346742E-1, 7.6372592E-1, 9.9398442E-1, 1.2242429E 0,
     &    1.4545014E 0, 1.6847599E 0, 1.9150184E 0, 2.1452769E 0,
     &    2.3755354E 0, 2.6057939E 0, 2.8360524E 0, 3.0663109E 0,
     &    3.2965694E 0, 3.5268279E 0, 3.7570864E 0, 3.9873449E 0,
     &    4.2176034E 0, 4.4478619E 0, 4.6781204E 0, 4.9083789E 0,
     &    5.1386374E 0, 5.3688959E 0, 5.5991544E 0, 5.8294129E 0,
     &    6.0596714E 0, 6.2899299E 0, 6.5201884E 0, 6.7504469E 0,
     &    6.9807054E 0/
      DATA
     &CE/7.3260937E-4, 5.6326423E-4, 1.3727237E-4, 7.5331222E-4,
     &   3.5918326E-4, 1.0500608E-3, 7.1530982E-4, 1.5160070E-3,
     &   1.2841617E-3, 2.2497985E-3, 2.1906186E-3, 3.4076782E-3,
     &   3.6321245E-3, 5.2376028E-3, 5.9212519E-3, 8.1315877E-3,
     &   9.5527062E-3, 1.2708615E-2, 1.5305589E-2, 1.9941086E-2,
```

```
      &    2.4396626E-2,  3.1333652E-2,  3.8683065E-2,  4.9127993E-2,
      &    6.0824806E-2,  7.6314344E-2,  9.3928346E-2,  1.1545027E-1,
      &    1.3868663E-1,  1.6243847E-1,  1.8114332E-1,  1.8424433E-1,
      &    1.5556741E-1,  6.8592481E-2,-8.8339029E-2,-2.8819226E-1,
      &   -3.5565260E-1,-5.6288677E-2,  4.8186942E-1,-5.1516453E-2,
      &   -2.6102989E-1,  2.1416490E-1,-9.4490687E-2,  2.6196370E-2,
      &   -5.1097828E-4,-6.6032943E-3,  7.5193619E-3,-6.7854344E-3,
      &    5.8044372E-3,-4.9354894E-3,  4.2323106E-3,-3.6733648E-3,
      &    3.2266260E-3,-2.8649137E-3,  2.5677680E-3,-2.3202655E-3,
      &    2.1115187E-3,-1.9334662E-3,  1.7800248E-3,-1.6465436E-3,
      &    1.3468317E-3/

14     FØRMAT(1X,I2,1X,'LAYERS',2X,'SØURCE CURRENT=',
      &F8.3,2X,'SØURCE LENGTH=',F8.2,/,'ANGLE=',F8.2,2X,
      &'SØURCE-RECEIVER SEPERATIØN=',E12.4,/)
15     FØRMAT(3X,'LAYER',3X,'RESISTIVITY',3X,'THICKNESS')
16     FØRMAT(1X,I5,4X,E11.5,3X,E10.4)
10     FØRMAT(//,3X,'FREQUENCY',4X,'H(REAL)',6X,'H(IMAG)',6X,'EC
      &(REAL)',5X,'EC(IMAG)',/,3X,'FREQUENCY',4X,'EX(REAL)',
      &5X,'EX(IMAG)',5X,'EY(REAL)',5X,'EY(IMAG)',//)
12     FØRMAT(5(1X,E12.6))
 5     FØRMAT(1F)
 1     FØRMAT(2F,2I)
28     FØRMAT(2F)
 3     FØRMAT(2F)
       IIN=3
       IØUT=8
       READ(IIN,1) ANG,R,N,NFW
       DØ 11 LT=1,N
11     READ(IIN,28) HP(LT),RH(LT)
       READ(IIN,3) CI,DS
       WRITE(IØUT,14) N,CI,DS,ANG,R
       WRITE(IØUT,15)
       WRITE(IØUT,16) (LT,RH(LT),HP(LT),LT=1,N)
       NB=2*N
       TM=12.566371E-7
       ANG=(3.1415927/180.)*ANG
       DØ 2 JJ=1,N
 2     RK(JJ)=RH(1)/RH(JJ)
       WRITE(IØUT,10)
       DØ 20 I=1,NFW
       READ(IIN,5) FF
       F=6.2831853*FF
       DEL=SQRT(2.*RH(1)/(TM*F))
       DØ 6 JJ=1,N
 6     D(JJ)=2.*HP(JJ)/DEL
       X=ALØG(R/DEL)
       CALL FV(E,H,EX,EY)
       WRITE(IØUT,12) FF,H,E
       WRITE(IØUT,12) FF,EX,EY
20     CØNTINUE
       END
```

```
      SUBRØUTINE FV(EC,H,EX,EY)
C
C***************SUBRØUTINE FVALUE ***************************
C   THIS SUBRØUTINE CALCULATES THE CØNVØLUTIØN SUMS FØR EX,
C   EY, EC, AND HZ ELECTRØMAGNETIC FIELD CØMPØNENTS
C
C*********************** VARIABLES ****************************
C
C   SUM= CØNVØLUTIØN SUM FØR HZ
C   SUEY1,SUEYØ, SUEX1, SUEXØ AS DEFINED ØN PAGE 10 ØF THESIS
C   SUEY1= CØNVØLUTIØN SUM FØR EY1
C   SUEYØ= CØNVØLUTIØN SUM FØR EX1
C   SUEX1= CØNVØLUTIØN SUM FØR EX3
C   SUEXØ= CØNVØLUTIØN SUM FØR EX2
C   H1= CLØSED-FØRM HØMØGENØUS HALF-SPACE EXPRESSIØN FØR HZ
C   EX= X-CØMPØNENT ØF THE ELECTRIC FIELD FØR A LAYERED EARTH
C   EY= Y-CØMPØNENT ØF THE ELECTRIC FIELD FØR A LAYERED EARTH
C   EX1= CLØSED-FØRM EXPRESSIØN FØR A HØMØGENØUS HALF-SPACE
C         (X-CØMPØNENT)
C   EY1= CLØSED-FØRM EXPRESSIØN FØR A HØMØGENØUS HALF-SPACE
C        (Y-CØMPØNENT)
C   E= CLØSED-FØRM EXPRESSIØN FØR A HØMØGENØUS HALF-SPACE
C        ( CØMBINED ELECTRIC FIELD)
C   EC= CØMBINED ELECTRIC FIELD FØR A LAYERED EARTH
C   F1= LAYERED EARTH CØRRECTIØN FACTØR
C   Z1= LAYERED EARTH CØRRECTIØN FACTØR
C   V1= NØRMALIZED FIRST LAYER PSEUDØ WAVE NUMBER
C
C
C************************************************************

      CØMMØN /CV/ CI,DS,ANG,F,X,YH(48),CH(48),YE(61),CE(61),TM
      CØMMØN /QC/ D(3),RK(3),RH(3),N,DEL,R
      CØMPLEX H,V1,F1,SUM,H1,Z1,E1,EC,C1,C2,
     &EX1,EX0,EY1,EY0,SUEX1,SUEY1,SUEX0,SUEY0,EX,EY
      SUEX1=CMPLX(0.0,0.0)
      SUEY1=CMPLX(0.0,0.0)
      SUEX0=CMPLX(0.0,0.0)
      SUEY0=CMPLX(0.0,0.0)
      SUM=CMPLX(0.0,0.0)
      BB=EXP(X)
      C1=CMPLX(0.0,1.0)
      C2=CMPLX(1.0,1.0)
C***********************************
CALCULATE THE J1 HANKEL TRANSFØRMS
C************************************
      DØ 8 J=1,48
      Y=EXP(-(X-YH(J)))
      CALL CALC(F1,Z1,V1,Y)
      EX1=C1*V1*(Z1-1)+2.*V1*(1-F1)/((Y+V1*F1)*(Y+V1))
      SUEX1=SUEX1+CH(J)*EX1
      H=Y*Y*V1*(1.-F1)
```

```
      H=H/((Y+V1*F1)*(Y+V1))
      SUM=SUM+CH(J)*H
 8    CØNTINUE
      SUEY1=SUEX1
      H=(1/BB)* CI*DS*SIN(ANG)*SUM/(6.2831853*DEL*DEL)
      R2=R*R
      H1=3.-(3.+3.0*C2*BB+2.*C1*BB**2)*CEXP(-C2*BB)
      H1=(-C1*CI*DS*DEL*DEL*SIN(ANG)*H1)/(12.566371*R2*R2)
12    FØRMAT(2(1X,E14.8))
      H=H+H1
      SUM=CMPLX(0.0,0.0)
C************************************
CALCULATE THE J0 HANKEL TRANSFØRMS
C*************************************
      DØ 4 I=1,61
      Y=EXP(-(X-YE(I)))
      CALL CALC(F1,Z1,V1,Y)
      EY0=Y*(C1*V1*(Z1-1.)+2.*V1*(1.-F1)/((Y+V1)*(Y+V1*F1)))
      EX0=(CØS(ANG)**2/DEL)*EY0-(1/DEL)*2.*Y*V1*(1.-F1)/
     &((Y+V1*F1)*(Y+V1))
      SUEX0=SUEX0+CE(I)*EX0
      SUEY0=SUEY0+CE(I)*EY0
      EC=C1*V1*Y*(Z1-1.0)/2.0
      EC=EC-Y*V1*(1.0-F1)/((Y+V1*F1)*(Y+V1))
 4    SUM=SUM+CE(I)*EC
      EC=C1*TM*F*CI*DS*SIN(2.*ANG)*SUM/(12.566371*DEL*BB)
      E1=-.500+(1+C2*BB)*CEXP(-C2*BB)
      E1=CI*DS*RH(1)*SIN(2.*ANG)*E1/(6.2831853*R*R2)
      EC=E1+EC
      EX=(C1*TM*F*CI*DS/(12.566371*BB))*(SUEX0-CØS(2.*ANG)
     &*SUEX1/R)
      EY=(C1*TM*F*CI*DS/(12.566371*BB))*SIN(2.*ANG)*(-SUEY1/R
     &+SUEY0/(2.*DEL))
      EX1=CI*DS*RH(1)*((3*CØS(ANG)**2-2)+(1.+(1+C1)*BB)*CEXP(-C2*BB)
     &)/(6.2831853*R*R2)
      EY1=3.*RH(1)*CI*DS*SIN(ANG)*CØS(ANG)/(6.2831853*R*R2)
      EX=EX+EX1
      EY=EY+EY1
      RETURN
      END
```

```
      SUBRØUTINE CALC(F1,Z1,V1,Y)
C*************************************************
C SUBRØUTINE CALC CALCULATES THE LAYERED EARTH CØRRECTIØN FACTØRS
C (F1 AND Z1) FØR UP TØ 3 LAYERS*************************
      CØMMØN /QC/ D(3),RK(3),RH(3),N,DEL,R
      CØMPLEX V1,F1,V2,AEX,Z1
      Y2=Y*Y
      T=2.*RK(N)
      V2=CSQRT(CMPLX(Y2,T))
      DØ 9 LL=2,N
      IF(LL.GT.2) GØ TØ 4
      F1=CMPLX(1.0,0.0)
      Z1=CMPLX(1.0,0.0)
 4    I=N-LL+1
      DD=D(I)
      T=2.*RK(I)
      V1=CSQRT(CMPLX(Y2,T))
      AEX=(1.-CEXP(-V1*DD))/(1.+CEXP(-V1*DD))
      F1=(V2*F1+V1*AEX)/(V1+V2*F1*AEX)
      Z1=(V2*RH(I+1)*Z1+V1*RH(I)*AEX)/(V1*RH(I)+V2*RH(I+1)*Z1*AEX)
 9    V2=V1
      RETURN
      END
```

```
C***************** PRØGRAM EMINT ****************************
C       THIS PRØGRAM USES MARQUARDT'S ALGØRITHM TØ ADJUST FIRST
C  GUESS PARAMETERS TØ FIT THE ØBSERVED DATA.  THE SØURCE IS
C  ASSUMED TØ BE A CURRENT DIPØLE.  RECEIVER CØMPØNENTS WHICH THIS
C PRØGRAM WILL FIT ARE CHØSEN BY THE VARIABLE NCØMP LISTED BELØW.
C
C        PRØGRAM EMINT WAS DEVELØPED ØN A DEC PDP-10 CØMPUTER
C  AT THE CØLØRADØ SCHØØL ØF MINES.  FØRTRAN PRØGRAMMING LANGUAGE IS
C  USED THRØUGHØUT THE PRØGRAM.
C
C
C
C                             JEFF DANIELS
C                        CØLØRADØ SCHØØL ØF MINES
C                            FEBRUARY 1974
CC***************************** VARIABLES  *******************
C NCØMP=10 - ABSØLUTE VALUE ØF EC CALCULATED
C NCØMP=20 - REAL PART ØF EC CALCULATED
C NCØMP=30 - IMAGINARY PART ØF EC CALCULATED
C NCØMP=11 - ABSØLUTE VALUE ØF HZ CALCULATED
C NCØMP=21 - REAL PART ØF HZ CALCULATED
C NCØMP=31 - IMAGINARY PART ØF HZ CALCULATED
C NCØMP=40 - ABSØLUTE VALUE ØF EX CALCULATED
C NCØMP=50 - REAL PART ØF EX CALCULATED
C NCØMP=60 - IMAGINARY PART ØF EX CALCULATED
C NCØMP=70 - ABSØLUTE VALUE ØF EY CALCULATED
C NCØMP=80 - REAL PART ØF EY CALCULATED
C NCØMP=90 - IMAGINARY PART ØF EY CALCULATED
C ANG= ANGLE (IN DEGREES) DEFINING THE SØURCE-RECEIVER ØRIENTATIØN
C R= SØURCE-RECEIVER SEPERATIØN
C CI= SØURCE CURRENT
C DS= SØURCE LENGTH (IN METERS)
C YNØRM= NØRMALIZATIØN FACTØR FØR THE INPUT Y VALUES ( E.G. IF
C       THE AVERAGE VALUE FØR 20 EX VALUES (AT 20 DIFFERENT FREQUENCIES)
C       IS .1000E-10, YNØRM SHØULD BE .10000E-10)
C IE= THE NUMBER ØF X-Y PAIRS ØF VALUES TØ BE INPUT
C X(I)= THE ITH FREQUENCY
C Y(I)= THE ITH VALUE ØF THE ELECTRØMAGNETIC FIELD CØMPØNENT
C        CØRRESPØNDING TØ THE ITH FREQUENCY
C N= THE NUMBER ØF LAYERS IN THE FIRST-GUESS MØDEL
C HP(I)= ITH LAYER THICKNESS
C RH(I)= ITH LAYER RESISTIVITY
C ** CHØICE ØF PARAMETERS TØ BE VARIED **
C IFXE(J)=0 THE JTH PARAMETER IS NØT VARIED
C IFXE(J)=1 THE JTH PARAMETER IS VARIED
C TWØ LAYER ØRDER:
C   J=1 - R; J=2 - RH(1); J=3 - HP(1); J=4 - RH(2)
C THREE LAYER ØRDER:
C  J=1 - R; J=2 - RH(1); J=3 - HP(1); J=4 - RH(2);J=5 - HP(2);
C  J=6 - RH(3)
C*****************************************************************
```

```
      CØMMØN /CV/ CI,DS,ANG,FX,XX,TM
      CØMMØN /QC/ DD(3),RKK(3),RHH(3),N,DEL,RR
      DIMENSIØN X(25),Y(25),B(7),WTS(25),
     &BDB(7),ATA(7,7),IFXE(6),HP(3),RH(3)
      REAL LAM
      IN=1
C****************************************
C READ IN THE DATA UNDER THE LØGICAL UNIT "IN"
C****************************************
      READ(IN,1)ANG,R,CI,DS,IE,N,NCØMP
 7    FØRMAT(E)
 1    FØRMAT(4F,3I)
      READ(IN,9)(X(I),Y(I),WTS(I),I=1,NFW)
 9    FØRMAT(3E12.6)
 2    FØRMAT(2F)
      ANG=(3.1415927/180.)*ANG
      NB=2*N
      READ(IN,8) (IFXE(J),J=1,NB)
 8    FØRMAT(I6)
      READ(IN,2) (HP(LT),RH(LT),LT=1,N)
      READ(IN,7) YNØRM
      DØ 4 JJ=1,NFW
 4    Y(JJ)=Y(JJ)/YNØRM
      NPAR=NB
      B(1)=R
      DØ 3 JCT=1,N
      NCT=2*JCT
      B(NCT)=RH(JCT)
      B(NCT+1)=HP(JCT)
 3    CØNTINUE
      CALL NØNLS2(NPAR,IFXE,IE,X,Y,WTS,B,NCØMP,YNØRM)
      END
```

```
      SUBRØUTINE NØNLS2 (NPAR,IFXE,IE,X,Y,WTS,B,NCØMP,YNØRM)
C****************** SUBRØUTINE NØNLS2 *************************
C  THIS SUBRØUTINE CØNTRØLS THE LEAST-SQUARES CURVE FITTING
C ALGØRITHM AS ØUTLINED BY MARQUARDT (1963)
C
C
C   THIS IS A MØDIFIED VERSIØN ØF THE ØRIGINAL
C GEØPHYSICS LEAST-SQUARES PRØGRAM WRITTEN BY JØRGE
C PARRA.
C
C
C**********LEAST SQUARES SUBRØUTINE**************************
C  NVAR=# ØF UNFIXED PARAMETERS
C  NPAR=# ØF UNKNØWN PARAMETERS ØF F > LINØUT
C  NPTS=#ØF ØBSERVATIØN PØINTS
C  IW=ØUTPUT (FLAG:-1=NØ ØUTPUT,0,1,2=ØUTPUT)
C  FAIL=ØUTPUT(0,1,2=CØNVERGENCE:3,4,5=NØ CØNVERGENCE)
C  MAXITR=THE MAXIMUM # ØF ITERATIØNS ALLØWED
C  IFXE=ACCURACY(IFXE(K)=1,THE KTH PARAMETER IS ALLØWED TØ VARY:
C       IFXE(K)=0, THE KTH PARAMETER IS FIXED AT ITS INPUT VALUE)
C  NEXT=0
C  LINMIN=0
C  X(IE,2)=DØUBLY DIMENSIØNED ARRAY HØLDING THE ØBSERVATIØNS
C          (INDEPENDENT VARIABLES) AS RØWS ØF X
C  Y(IE)= ARRAY HØLDING THE DEPENDENT VARIABLE ØBSERVATIØNS
C  WTS=ARRAY HØLDING THE RELIABILITY WEIGHTS FØR EACH PØINT
C      /IF WTS=0 THE WEIGHT =1.0/
C  ATA=A* MATRIXFRØM MARQUARDT'S FØRMULATIØN
C  B=HØLDS THE INITIAL ESTIMATES ØF THE PARAMETERS. IF IFAIL=0
C    ØR 1, B HØLDS THE CØNVERGED PARAMETER SET
C  BDB= THE FINAL CØRRECTIØN VECTØR TØ THE CØNVERGED PARAMETERS
C  IP=10**-5
C  LAM=MARQUARDT'S DAMPING CØNSTANT
C  PHI=(Y(ØBS)-Y(MØDEL))**2
C  DAC=.05 / PARABLØIC MINIMUMIZATIØN
C  FVALUE=SUBRØUTINE THAT CALCULATES THE MØDEL
C  PARVAL=SUBRØUTINE THAT CALCULATES THE FIRST DERIVATIVE ØF THE
C         MØDEL
C  ANS=CØRRECTIØN VECTØR AT THE PØINTS ØF LINEAR MINIMUMIZATIØN
C      ***************************************************************

      DIMENSIØN ATF(7),ANS(7),ANS1(7),EX(7)
      DIMENSIØN X(25),Y(25),B(7),BDB(7),WTS(25),ATA(7,7)
      REAL LAM,LDØ,L10,NØRM
      DIMENSIØN INX(7),IFXE(7)
        INTEGER FAIL
1003  FØRMAT(4(2X,E14.8))
C*********************************
C  SET THE CØNSTANT VALUES
C***********************************
      LAM= 0.01
      NPTS=IE
```

```
      ID=NPAR
      MAXITR=6
      EP=1.E+0
      NEXT=0
      GAMA0=45.
        IW=3
      ITER=0
      TØLR=1.E-8
      IØUT=2
      IR=3
C     DETERMINE THE PARAMETERS TØ BE VARIED.
C
      NVAR=0
      DØ 10 I=1,NPAR
      BDB(I)=B(I)
      ANS(I)=0.E0
      ANS1(I)=0.E0
      IF(IFXE(I).LE.0) GØ TØ 10
      NVAR=NVAR+1
      INX(NVAR)=I
10    CØNTINUE
3001  FØRMAT(I)
      IF(IW.GE.0) WRITE(IØUT,11700) NPAR,NVAR,NPTS,MAXITR,IW,LINMIN,EP,
     *DAC
C
C      EXAMINE THE WTS ARRAY
C
      NØRM=0.E0
      DØ 40 I=1,NPTS
      IF(WTS(I)) 20,30,40
20    WTS(I)=1.E0/WTS(I)**2
      GØ TØ 40
30    WTS(I)=1.E0
40    NØRM=NØRM+WTS(I)
        NØRM=FLØAT(NPTS)/NØRM
      IF(NØRM.EQ.1.E0) GØ TØ 60
      DØ 50 I=1,NPTS
50    WTS(I)=WTS(I)*NØRM
C
C     CALCULATE THE INITAL SUM ØF SQUARES
C
60    PHI=0.E0
      IF(IW.GT.0) WRITE(IØUT,10200)
      DØ 70 I=1,NPTS
      NPVFC=1
      CALL EMCALC (X,B,IE,I,EX,FV,IFXE,NPVFC,NCØMP,YNØRM)
      SS=Y(I)-FV
      IF(IW.GT.0) WRITE(IØUT,10000) X(I),Y(I),FV,SS
 70   PHI=PHI+SS**2*WTS(I)
C********************************
C         START THE DAMPED GAUSSIN PRØCEDURE
```

```
C**************************************
C
80     ITER=ITER+1
       IF(IW.GT.0) WRITE(IØUT,11800)
       IF(ITER.GT.MAXITR) GØ TØ 380
       DØ 90 I=1,NPAR
90     BDB(I)=B(I)
       DØ 100 I=1,NVAR
       ATF(I)=0.E0
       DØ 100 J=I,NVAR
100    ATA(I,J)=0.E0
C**********************************
C          GENERATE THE ATA AND ATF ARRAYS
C************************************
       DØ 110 I=1,NPTS
       NPVFC=1
       CALL EMCALC(X,B,IE,I,EX,FV,IFXE,NPVFC,NCØMP,YNØRM)
       NØRM=WTS(I)*(Y(I)-FV)
       NPVFC=0
       CALL EMCALC(X,B,IE,I,EX,FV,IFXE,NPVFC,NCØMP,YNØRM)
       DØ 110 L=1,NVAR
       J=INX(L)
       ATF(L)=ATF(L)+NØRM*EX(J)
       DØ 110 M=L,NVAR
       K=INX(M)
       ATA(L,M)=ATA(L,M)+EX(J)*EX(K)*WTS(I)
110    CØNTINUE
       IF(IW.GT.0) WRITE(IØUT,11500) ITER,PHI,(B(I),I=1,NPAR)
C******************************************
C      PERFØRM A LØCAL SCALING ØN THE ATA MATRIX TØ AID CALCULATIØNS.
C******************************************
       DØ 120 I=1,NVAR
       IF(ATA(I,I).EQ.0.D0) GØ TØ 410
120    EX(I)=SQRT(ATA(I,I))
       DØ 140 I=1,NVAR
       ATF(I)=ATF(I)/EX(I)
       DØ 140 J=I,NVAR
       IF(I.EQ.J) GØ TØ 130
       ATA(I,J)=ATA(I,J)/(EX(I)*EX(J))
       GØ TØ 140
130    ATA(I,I)=1.E0
140    CØNTINUE
C*****************************************************
C      DETERMINE A VALID LAMDA FØR THE SCALED PARTIAL MATRIX.
C  GAMA=DIRECTIØN VECTØR ANGLE
C  PHI1=SS ERRØR FØR NEW LAMBDA
C************************************************************

       FAC=1.E+0
       CALL NEWLAM(ATA,LAM,BDB,ATF,ANS,EX,GAMA,FVALUE,PHI1,X,Y,WTS,B,
      *ID,IE,NVAR,NPTS,NPAR,IFAL,INX,IFXE,IR,NCØMP,YNØRM)
       IF(IW.GT.0) WRITE(IØUT,10600) LAM,PHI1,GAMA
```

```
C*****************************************************************
C  EP= RELATED TØ THE FIRST GUESS (CØNTRØLS LAMBDA SØ THAT
C      LAMBDA DØESN'T GET TØØ LARGE) EXAMPLE:EP=.5 THEN THE
C      GREATEST CHANGE THAT THE PARAMETERS CAN HAVE IS 50% ØF
C      THE ØRIGINAL FIRST GUESS
C*****************************************************************
      IF(IFAL-1) 150,390,400
150   DØ 160 I=1,NVAR
      IF(ABS(ANS(I)).GT.(TØLR+EP*ABS(B(INX(I))))) GØ TØ 180
160   CØNTINUE
      IF(PHI.LT.PHI1) GØ TØ 440
      DØ 170 I=1,NVAR
170   B(INX(I))=B(INX(I))+ANS(I)
      PHI =PHI1
      GØ TØ 440
180   IF(PHI1.GE.PHI) GØ TØ 220
      IF(LAM.LE.TØLR) GØ TØ 320
      DØ 1213 JP1=1,NVAR
1213  B(JP1)=BDB(JP1)
      PHI=PHI1
      LDØ=LAM/10.0E0
      CALL NEWLAM(ATA,LDØ,BDB,ATF,ANS1,EX,GAMA,FVALUE,PHI2,X,Y,WTS,B,
     *ID,IE,NVAR,NPTS,NPAR,IFAL,INX,IFXE,IR,NCØMP,YNØRM)
      IF(IW.GT.0) WRITE(IØUT,10700) LDØ,PHI2,GAMA
      IF(IFAL-1) 190,390,400
190   IF(PHI2.GE.PHI1) GØ TØ 320
200   LAM=LDØ
      DØ 1207 JP1=1,NVAR
1207  B(JP1)=BDB(JP1)
      PHI=PHI2
      DØ 210 I=1,NVAR
210   ANS(I)=ANS1(I)
      PHI1=PHI2
      GØ TØ 320
220   LDØ=LAM/10.0E0
      CALL NEWLAM(ATA,LDØ,BDB,ATF,ANS1,EX,GAMA,FVALUE,PHI2,X,Y,WTS,B,
     *ID,IE,NVAR,NPTS,NPAR,IFAL,INX,IFXE,IR,NCØMP,YNØRM)
      IF(IW.GT.0) WRITE(IØUT,10700) LDØ,PHI2,GAMA
      IF(IFAL-1) 230,390,400
230   IF(PHI2.LT.PHI) GØ TØ 200
      L10=LAM
240   L10=L10*10.E0
      CALL NEWLAM(ATA,L10,BDB,ATF,ANS,EX,GAMA,FVALUE,PHI3,X,Y,WTS,B,
     *ID,IE,NVAR,NPTS,NPAR,IFAL,INX,IFXE,IR,NCØMP,YNØRM)
      IF(L10.GT.1.E+6) STØP
      IF(IW.GT.0) WRITE(IØUT,10900) L10,PHI3,GAMA
      IF(IFAL-1) 250,390,400
250   IF(PHI3.GE.PHI.AND.L10.GT.1.E+3) GØ TØ 360
      IF(PHI3.GE.PHI) GØ TØ 260
      PHI1=PHI3
      PHI=PHI3
```

```
      DØ 1208 JP1=1,NVAR
1208  B(JP1)=BDB(JP1)
      LAM=L10
      GØ TØ 320
260   IF(GAMA.GE.GAMA0) GØ TØ 240
      FAC=FAC/2.E0
      DØ 270 I=1,NVAR
270   ANS(I)=ANS(I)/2.E0
      DØ 280 I=1,NVAR
      IF(ABS(ANS(I)).GT.(TØLR+EP*ABS(B(INX(I))))) GØ TØ 290
280   CØNTINUE
      GØ TØ 430
290   DØ 300 I=1,NVAR
300   BDB(INX(I))=B(INX(I))+ANS(I)
      PHI3=0.D0
      DØ 310 I=1,NPTS
      NPVFC=1
      CALL EMCALC(X,BDB,IE,I,EX,FV,IFXE,NPVFC,NCØMP,YNØRM)
310   PHI3=PHI3+(Y(I)-FV)**2*WTS(I)
      IF(IW.GT.0) WRITE(IØUT,10300) FAC,PHI3
      GØ TØ 250
320   CØNTINUE
      IF(IW.LE.1) GØ TØ 360
      IF(NVAR.EQ.NPAR) GØ TØ 340
      DØ 330 I=1,NPAR
330   EX(I)=0.E0
340   DØ 350 I=1,NVAR
350   EX(INX(I))=ANS(I)
      WRITE(IØUT,11600) (EX(I),I=1,NPAR)
360   CØNTINUE
      GØ TØ 440
380   IF(IW.GE.0) WRITE(IØUT,12300)
      FAIL=2
      GØ TØ 450
390   IF(IW.GE.0) WRITE(IØUT,12000)
      FAIL=3
      RETURN
400   IF(IW.GE.0) WRITE(IØUT,11900) GAMA,LAM
      FAIL=4
      RETURN
410   IF(IW.GE.0) WRITE(IØUT,11400) I,(B(K),K=1,NPAR)
      FAIL=6
      RETURN
420   IF(IW.GE.0) WRITE(IØUT,10500)
      FAIL=5
      RETURN
430   IF(IW.GT.0) WRITE(IØUT,12400)
      FAIL=1
      GØ TØ 450
440   IF(ITER.LE.4) GØ TØ 80
C**********************************
```

```
C     THE PRØCEDURE HAS CØNVERGED
C******************************
      FAIL=0
      IF(IW.GE.0) WRITE(IØUT,11800)
      IF(IW.GE.0) WRITE(IØUT,11800)
      IF(IW.GE.0) WRITE(IØUT,12200) ITER
 450  IF(IW.GE.0) WRITE(IØUT,12100) PHI,(B(I),I=1,NPAR)
C**********************************************
C     CALCULATE THE STANDARD ERRØR. USE NPTS-NVAR-NEXT AS THE DF.
C****************************************
        NØRM=SQRT(PHI/FLØAT(NPTS-NVAR-NEXT))
C********************************************
C     REMAKE THE PARTIAL MATRIX INSTEAD ØF USING THE RE-SCALED MATRIX.
C     THIS CØULD EASILY BE CHANGED.
C********************************************
      DØ  460 I=1,NPAR
460   BDB(I)=0.E0
      DØ 470 I=1,NVAR
      DØ 470 J=I,NVAR
470   ATA(I,J)=0.E0
      DØ 480 I=1,NPTS
      NPVFC=0
      CALL EMCALC(X,B,IE,I,EX,FV,IFXE,NPVFC,NCØMP,YNØRM)
      DØ 480 L=1,NVAR
      J=INX(L)
      DØ 480 M=L,NVAR
      K=INX(M)
480   ATA(L,M)=ATA(L,M)+EX(J)*EX(K)*WTS(I)
      CALL INVRT (NVAR,ATA,ID,IFU)
      IF(IFU.EQ.1) GØ TØ 420
      DØ 490 I=1,NVAR
      EX(I)=SQRT(ATA(I+1,I))
 490  BDB(INX(I))=NØRM*EX(I)
C*******************************
C     CALCULATE THE CØRRELATIØN MATRIX
C*********************************
      DØ 500 I=1,NVAR
      DØ 500 J=I,NVAR
500   ATA(I,J)=ATA(J+1,I)/(EX(I)*EX(J))
      IF(IW.LT.0) RETURN
C************************************************
C       PRINT THE PARAMETERS AND THE STANDARD ERRØRS ASSØCATED TØ THEM
C*************************************************
      WRITE(IØUT,11300) (INX(I),B(INX(I)),BDB(INX(I)),I=1,NVAR)
      IF(NVAR.EQ.1) GØ TØ 520
      IF(FAIL.EQ.2) GØ TØ 540
C***************************
C     PRINT THE CØRRELATIØN MATRIX
C*******************************
      WRITE(IØUT,11200)
      DØ 510 I=1,NVAR
```

```
510    WRITE(IØUT,11000) I,(ATA(J,I),J=1,I)
C***************************
C      PRINT THE INVERSE MATRIX
C*******************************
       WRITE(IØUT,11800)
       WRITE(IØUT,11800)
520    WRITE(IØUT,11100)
       M=NVAR+1
       DØ 530 I=2,M
       K=I-1
530    WRITE(IØUT,10400) I,(ATA(I,J),J=1,K)
540    CØNTINUE
       WRITE(IØUT,11800)
       WRITE(IØUT,11800)
       WRITE(IØUT,10100)
       DØ 550 I=1,NPTS
       NPVFC=1
       CALL EMCALC(X,B,IE,I,EX,FV,IFXE,NPVFC,NCØMP,YNØRM)
       SS=Y(I)-FV
550    WRITE(IØUT,10000) X(I),Y(I),FV,SS
       WRITE(IØUT,12500) YNØRM
       RETURN
10000 FØRMAT(4E14.8)
10100 FØRMAT(1HO,'FINAL DEVIATIØNS'/1H ,1X,'FREQUENCY',3X
     &,'ØBS',13X,'CAL',12X,'Ø-C')
10200 FØRMAT(1HO,'INITIAL DEVIATIØNS'/1H ,13X,'ØBS',13X,'CALC',12X,
     *'Ø-C')
10300 FØRMAT(1HO,'FAC=',E14.8,6X,'  PHI(FAC) = ',E14.8)
10400 FØRMAT(1H ,'RØW ',I2/(1H ,8E15.6))
10500 FØRMAT(1HO,'FINAL A TRANSPØSE A IS NØT PØSITIVE DEFINE')
10600 FØRMAT(1HO,' IN-L = ',E14.8,'     PHI(IN-L) = ',E14.8,
     *'        GAMA = ',F10.4)
10700 FØRMAT(1HO,' L/10 = ',E14.8,'     PHI(L/10) = ',E14.8,
     *'      GAMA = ',F10.4)
10800 FØRMAT(3HOL ,7E14.8/(3H    ,16X,6E14.8))
10900 FØRMAT(1HO,' L*10 = ',E14.8,'       PHI(L*10) V ',E14.8,
     *'    GAMA = ',F10.4)
11000 FØRMAT(1H ,'RØW',I2/(1H ,8E15.6))
11100 FØRMAT(1HO,   ' INVERSE MATRIX - LØWER TRIANGULAR PØRTIØN')
11200 FØRMAT(1HO,'CØRRELATIØN MATRIX LØWER TRIANGULAR PØRTIØN RØW BY RØW
     *PRINT')
11300 FØRMAT(1HO,'  VARIABLE',6X,'PARAMETER VALUE',5X,'STANDARD ERRØR'/
     * (1HO,4X,I2,9X,E15.6,7X,E10.4))
11400 FØRMAT(1HO,'THE DIAGØNAL ELEMENT RESULTING FRØM THE PARTIAL WRT
     *B(',I2,') IS = 0.D'/1H ,'THE PØINT AT WHICH THE FAILURE ØCCURED
     *IS'/(1H ,7E14.8))
11500 FØRMAT(1HO,'ITERATIØN ',I3/1H ,7E14.8/(1H ,16X,6E14.8))
11600 FØRMAT(1HO,'DIR-VEC',5X,6E14.8/(1HO,12X,6E14.8))
11700 FØRMAT(1H1,T3,'NPAR',T11,'NVAR',T19,'NPTS',T27,'MAXITR',T37,'IW',
     *T42,'LINMIN',T56,'EP',T71,'DAC'/1H ,T4,I2,T12,I2,T20,I2,T29,
     *I2,T37,I2,T44,I2,T53,E10.4,T68,E10.4)
```

```
11800 FØRMAT(1HO/)
11900 FØRMAT(1HO,'GAMA = ',E14.8,' WHEN LAM = ',E14.8/
     *1H ,'THERE PRØBABLY EXISTS EXCESSIVELY HIGH CØRRELATIØNS BETWEEN
     * THE PARAMETERS')
12000 FØRMAT(1HO,'THE (ATA +LAM*I) MATRIX FAILED TØ BE PØS.DEF.')
12100 FØRMAT(1HO,7E14.8/(1H ,16X,6E14.8))
12200 FØRMAT(1HO,'ØPTIMAL PØINT REACHED IN ',I5, ' ITERATIØNS')
12300 FØRMAT(1HO,'MAXIMUM NUMBER ØF ITERATIØNS REACHED-BEST PØINT PRIN
     *TED'/)
12400 FØRMAT(1HO,'THE DELTA-B VECTØR REDUCED TØ CØNVERGENCE LEVEL WHILE
     *GAMA LESS THAN GAMAO.'/1H ,'THE PØINT IS PRØBABLY ØPTIMAL WITHIN
     *RØUNDING ERRØRS.')
12500 FØRMAT(1X,'NØRMALIZATIØN FACTØR = ',E14.8)
      END
```

```
      SUBROUTINE NEWLAM(ATA,LAM,B1,ATF,ANS,EX,GAMA,FVALUE,PHI,
     &X,Y,WTS,B,ID,IE,NVAR,NPTS,NPAR,IFAL,INX,IFXE,IR,NCOMP,YNORM)
C******** SUBROUTINE NEWLAM ******************************
C       SUBROUTINE NEWLAM CALCULATES THE NEW LAMBDA FOR THE LEAST
C  SQUARES PROCEDURE CONVERGENCE ACCORDING TO THE ALGORITHM
C  STATED BY MARQUARDT (1963).
C
C
C                  ORIGINAL PROGRAMER   JORGE PARRA
C                  MODIFIED BY          JEFF DANIELS
C
C*************************************************************
        DIMENSION ATA(7,7),B1(7),ATF(7),ANS(7),EX(7)
        DIMENSION Y(IE),X(IE),WTS(IE),B(7)
        DIMENSION INX(20),IFXE(7)
      REAL LAM
      CON=57.295779E0
      IFAL=0
      DO 10 I=1,NVAR
      ATA(I,I)=1.E0
10    ATA(I,I)=ATA(I,I)+LAM
      ERRORX=.001E+0
      CALL DSOL (ATA,ATF,ANS,25,NVAR,ERRORX,10,IDET,1,IFAIL)
1005   FORMAT(I5)
1006   FORMAT(3(4X,E14.8))
      IF(IFAIL.EQ.1) GO TO 20
      IFAL=1
      RETURN
 20   DO 30 I=1,NVAR
30    ANS(I)=ANS(I)/EX(I)
      IFAIL=0
      IF(NVAR.NE.1) GO TO 40
      GAMA=0.E0
      GO TO 70
40    SUM1=0.E0
      SUM2=0.E0
      SUM3=0.E0
      DO 50 I=1,NVAR
      SUM1=SUM1+ANS(I)*ATF(I)
      SUM2=SUM2+ATF(I)**2
 50   SUM3=SUM3+ANS(I)**2
      COSGAM=SUM1/SQRT(SUM2*SUM3)
        UV=SQRT(1.E0-COSGAM*COSGAM)
        GAMA=ATAN(UV/COSGAM)*CON
      IF(COSGAM.GT.0.E0) GO TO 60
      GAMA=180.E0-GAMA
      IF(LAM.LT.1.E0) GO TO 60
      IFAIL=1
60    IF(IFAIL.EQ.0) GO TO 70
      IFAL=2
      RETURN
70    DO 80 I=1,NVAR
      K=INX(I)
80    B1(K)=B(K)+ANS(I)
      PHI=0.E0
      DO 90 I=1,NPTS
      NPVFC=1
      CALL EMCALC(X,B1,IE,I,EX,FV,IFXE,NPVFC,NCOMP,YNORM)
90    PHI=PHI+(Y(I)-FV)**2*WTS(I)
      RETURN
      END
```

```
      SUBROUTINE INVRT(N,A,IE,IFL)
C******************SUBROUTINE INVRT**************************
C    THIS PROGRAM PERFORMS A MATRIX INVERSION ON THE
C TWO DIMENSIONAL ARRAY A.
C************************************************************


      DIMENSION A(7,7)
      IFL=0
      DO 40 I=1,N
      I1=I+1
      DO 40 J=I,N
      J1=J+1
      X=A(I,J)
      K=I-1
10    IF(K.LT.1) GO TO 20
      X=X-A(J1,K)*A(I1,K)
      K=K-1
      GO TO 10
20    IF(J.NE.I) GO TO 30
      IF(X.LE.0.E0) GO TO 90
      Y=1.E0/SQRT(X)
      A(I1,I)=Y
      GO TO 40
30    A(J1,I)=X*Y
40    CONTINUE
      NL=N-1
      DO 60 I=1,NL
      KL=I+1
      DO 60 J=KL,N
      Z=0.E0
      J1=J+1
      K=J-1
50    IF(K.LT.I) GO TO 60
      Z=Z-A(J1,K)*A(K+1,I)
      K=K-1
      GO TO 50
60    A(J1,I)=Z*A(J1,J)
      J1=N+1
      DO 80 I=1,N
      DO 80 J=I,N
      Z=0.D0
      KL=J+1
      DO 70 K=KL,J1
70    Z=Z+A(K,J)*A(K,I)
80    A(KL,I)=Z
      RETURN
90    IFL=1
      RETURN
      END
```

```
      SUBROUTINE DSOL (A,B,X,NAMAX,NA,ERRORX,MAXIT,K,IENTRY,ISING)
C
C     THIS SUBROUTINE SOLVES A SET OF SIMULTANEOUS LINEAR EQUATIONS
C     USING CROUTS FACTORIZATION WITH PARTIAL PIVOTING, EQUILIBRATION,
C     ITERATIVE IMPROVEMENT, AND DOUBLE PRECISION OF PRODUCTS TO REDUCE
C     ROUND-OFF ERROR.
C
C     SUBROUTINE PARAMETERS .........................................
C     A       = THE COEFFICIENT MATRIX
C     B       = THE CONSTANT VECTOR
C     X       = THE SOLUTION VECTOR (INFORMATION RETURNED)
C     NAMAX   = THE NUMBER OF ROWS IN THE DIMENSION OF A IN
C               THE CALLING PROGRAM.
C     NA      = THE NUMBER OF ROWS IN A
C     ERRORX  = THE MAXIMUM ALLOWABLE ERROR BETWEEN SUCCESSIVE ITERATIONS
C     MAXIT   = THE MAXIMUM ALLOWABLE NUMBER OF ITERATIONS
C     K       = THE ACTUAL NUMBER OF ITERATIONS PERFORMED (INFORMATION RE
C     IENTRY  = 1 (FIRST CALL TO SUBROUTINE)
C             = 2 (SUBSEQUENT CALLS WITH UNCHANGED A)
C     ISING   = 1  THE MATRIX IS NON-SINGULAR (INFORMATION RETURNED)
C             = 2  THE MATRIX IS SINGULAR. PROCEDURE DISCONTINUED.
C     ...............................................................
C     IF A SINGULAR MATRIX IS ENCOUNTERED THE PROCEDURE IS DISCONTINUED
C     AND ERROR MESSAGES ARE PRINTED OUT.
C     IF A SOLUTION IS REQUIRED FOR THE SAME A-MATRIX, BUT FOR DIFFERENT
C     B-VECTORS, THE SUBROUTINE MAY BE SUBSEQUENTLY DIRECTED TO ENTRY
C     POINT -- TWO.
C     ...............................................................
C                           PROGRAMED BY
C                        DONALD SNYDER 1968
C                      COLORADO SCHOOL OF MINES
C
C
      DOUBLE PRECISION S
      DIMENSION A(7,7),B(7),X(7),BIGB(7,7),Q(7),R(7)
      DIMENSION IPIV(10)
      IOUT=2
C
      GO TO (100,200),IENTRY
C
C     ENTRY -- ONE ..................................................
  100 CONTINUE
C
C     STORE MATRIX A IN ARRAY BIGB
      DO 101 I=1,NA
      DO 101 J=1,NA
  101 BIGB(I,J)=A(I,J)
C
C     DETERMINE ELEMENT WITH MAXIMUM MODULUS IN EACH ROW. STORE IN Q.
      DO 103 I=1,NA
      PLAINQ=0.0
```

```
      DØ 102 J=1,NA
      TEST=ABS(A(I,J))
      IF (TEST.GT.PLAINQ) PLAINQ=TEST
  102 CONTINUE
      Q(I)=PLAINQ
C     BØX 13 -- CHECK FØR SINGULARITY ..................................
.
      IF (PLAINQ.LT.0.1E-30) GØ TØ 301
C
  103 CONTINUE
C
C
C     DETERMINE THE ELEMENTS ØF THE LØWER TRIANGULAR FACTØR ØF A
C     STØRE ØVER BIGB
      DØ 110 IR=1,NA
      AMAX=0.0
      IRMAX=IR
      LIM=IR-1
      DØ 106 I=IR,NA
      S=DBLE(BIGB(I,IR))
      IF (IR.EQ.1) GØ TØ 105
      DØ 104 J=1,LIM
  104 S=S-BIGB(I,J)*BIGB(J,IR)
  105 BIGB(I,IR)=SNGL(S)
      TEST=ABS(BIGB(I,IR)/Q(I))
      IF (TEST.LE.AMAX) GØ TØ 106
      AMAX=TEST
      IRMAX=I
  106 CONTINUE
C
C     BØX 37 - CHECK FØR SINGULARITY ..................................
      IF (AMAX.LT.0.1E-30) GØ TØ 302
C
      Q(IRMAX)=Q(IR)
      IPIV(IR)=IRMAX
C
C     INTERCHANGE THE IR AND IRMAX RØWS ØF BIGB
      DØ 107 I=1,NA
      PLAINQ=BIGB(IR,I)
      BIGB(IR,I)=BIGB(IRMAX,I)
  107 BIGB(IRMAX,I)=PLAINQ
C
C     DETERMINE THE ELEMENTS ØF THE UPPER TRIANGULAR FACTØR ØF A
C     STØRE ØVER BIGB
      IF (IR.EQ.NA) GØ TØ 110
      LØW=IR+1
      DØ 109 I=LØW,NA
      S=DBLE(BIGB(IR,I))
      IF (IR.EQ.1) GØ TØ 109
      DØ 108 J=1,LIM
  108 S=S-BIGB(IR,J)*BIGB(J,I)
  109 BIGB(IR,I)=SNGL(S/DBLE(BIGB(IR,IR)))
```

```
  110 CØNTINUE
C
C     FACTØRIZATIØN IS CØMPLETE. NØW FIND SØLUTIØN
C
C     ENTRY -- TWØ  ...................................................
  200 CØNTINUE
C
C     PRESET ITERATIØN PARAMETERS, AND INITIALIZE SØLUTIØN AND ERRØR VEC
      K=0
      IFIN=0
      ISING=1
      DØ 201 I=1,NA
      X(I)=0.0
  201 R(I)=B(I)
  202 CØNTINUE
C
C     PERFØRM FØRWARDS SUBSTITUTIØN
      DØ 204 I=1,NA
      L=IPIV(I)
      S=DBLE(R(L))
      R(L)=R(I)
      IF (I.EQ.1) GØ TØ 204
      LIM =I-1
      DØ 203 J=1,LIM
  203 S=S-BIGB(I,J)*R(J)
  204 R(I)=SNGL(S/DBLE(BIGB(I,I)))
C
C     PERFØRM BACKWARDS SUBSTITUTIØN
      I=NA+1
      DØ 206 N=1,NA
      I=I-1
      S=DBLE(R(I))
      IF (I.EQ.NA) GØ TØ 206
      LØW=I+1
      DØ 205 J=LØW,NA
  205 S=S-BIGB(I,J)*R(J)
  206 R(I)=S
C
C     CØMPUTE NØRMS
      ANØRMX=0.0
      ANØRME=0.0
      IF (K.EQ.0) GØ TØ 210
      DØ 207 I=1,NA
      TEST=ABS(X(I))
      IF (TEST.GT.ANØRMX) ANØRMX=TEST
      TEST=ABS(R(I))
      IF (TEST.GT.ANØRME) ANØRME=TEST
  207 CØNTINUE
C
      IF (K.NE.1) GØ TØ 208
C
```

```
C       BØX 94 - CHECK FØR SINGULARITY ...................................
        IF ((ANØRME/ANØRMX).GT.0.5) GØ TØ 303
C
  209   IF ((ANØRME/ANØRMX).LT.ERRØRX) IFIN=1
  210   CØNTINUE
C
C       CØMPUTE CURRENT APPRØXIMATE SØLUTIØN
        DØ 211 I=1,NA
211     X(I)=X(I)+R(I)
C
C       CØMPUTE ERRØR AND STØRE BACK IN R
        DØ 213 I=1,NA
        S=DBLE(B(I))
        DØ 212 J=1,NA
  212   S=S-A(I,J)*X(J)
213     R(I)=SNGL(S)
        K=K+1
        IF (IFIN.EQ.1) RETURN
        IF (K.LT.MAXIT) GØ TØ 202
C
C       PRINT ERRØR MESSAGE AND RETURN
        WRITE(IØUT,11) MAXITR
        RETURN
  301   IBØX=13
        GØ TØ 304
  302   IBØX=37
        GØ TØ 304
  303   IBØX=94
304     WRITE(IØUT,12) IBØX
        ISING=2
C
   11   FØRMAT(1H0,53HSØLUTIØN HAS NØT CØNVERGED TØ DESIRED ACCURACY AFTER
     1,I5,11H ITERATIØNS)
   12   FØRMAT(1H0,31HSINGULAR MATRIX DETECTED AT BØX,I3)
C
        RETURN
        END
```

```
      SUBROUTINE EMCALC(FF,B,IE,I,DERIV,FV,IFXE,NPVFC,NCOMP,YNORM)
C******************* SUBROUTINE EMCALC ******************
C THIS SUBROUTINE HANDLES BOTH THE CALCULATION OF THE FORWARD
C SOLUTION AND THE PARTIAL DERIVATIVES OF THE FORWARD
C SOLUTION WITH RESPECT TO THE LAYERED EARTH PARAMETERS
C AND THE SOURCE RECEIVER SEPERATION.
C
C********************VARIABLES*************************
C  FF= FREQUENCY IN HERTZ
C  F= ANGULAR FREQUENCY
C  D(I)= NORMALIZED THICKNESS FOR THE I-TH LAYER
C  RK(I)= NORMALIZED RESISTIVITIES FOR THE I-TH LAYER
C  DEL= MODIFIED WAVE NUMBER
C
C
C*****************************************************

      COMPLEX  H,E
      COMMON /PD/ EXX(6),HX(6),DINX(6),KFXE(6),EC,HZ,ECR,ECI,HZR,HZI
     &,EXR(6),EXI(6),HXR(6),HXI(6)
      COMMON /QC/DD(3),RKK(3),RHH(3),N,DEL,RR
      COMMON /CV/CI,DS, ANG,F,XX,TM
      COMMON /PART/ RK(3),RH(3),HP(3),HPP(3),D(3),
     &X,R,NB
      DIMENSION B(7),FF(25),DERIV(7),IFXE(6)

      R=B(1)
      NB=2*N
      DO 1 JCT=1,N
      NCT=2*JCT
      RH(JCT)=B(NCT)
      HP(JCT)=B(NCT+1)
 1    CONTINUE
      DO 3 JC=1,NB
 3    KFXE(JC)=IFXE(JC)
      TM=12.566371E-7
      F=FF(I)*6.2831853
      DEL=SQRT(2.*RH(1)/(TM*F))
      DO 2 JJ=1,N
      D(JJ)=2.*HP(JJ)/DEL
 2    RK(JJ)=RH(1)/RH(JJ)
      X=ALOG(R/DEL)
      P=1.001
      RR=R
      XX=X
      DO 9 J=1,N
      RHH(J)=RH(J)
      RKK(J)=RK(J)
 9    DD(J)=D(J)
      CALL FVAL(E,H,NCOMP)
      EC=CABS(E)
      HZ=CABS(H)
      ECR=REAL(E)
```

```
      ECI=AIMAG(E)
      HZR=REAL(H)
      HZI=AIMAG(H)
      IF(NPVFC.EQ.0) GØ TØ 17
      IF(NCØMP.EQ.10) FV=EC/YNØRM
      IF(NCØMP.EQ.40) FV=EC/YNØRM
      IF(NCØMP.EQ.70) FV=EC/YNØRM
      IF(NCØMP.EQ.20) FV=ECR/YNØRM
      IF(NCØMP.EQ.50) FV=ECR/YNØRM
      IF(NCØMP.EQ.80) FV=ECR/YNØRM
      IF(NCØMP.EQ.30) FV=ECI/YNØRM
      IF(NCØMP.EQ.60) FV=ECI/YNØRM
      IF(NCØMP.EQ.90) FV=ECI/YNØRM
      IF(NCØMP.EQ.11) FV=HZ/YNØRM
      IF(NCØMP.EQ.21) FV=HZR/YNØRM
      IF(NCØMP.EQ.31) FV=HZI/YNØRM
17    CØNTINUE
      IF(NPVFC.EQ.1) GØ TØ 4
      CALL PARDER (IFXE,DERIV,NCØMP,YNØRM)
 4    CØNTINUE
      RETURN
      END
```

```
      SUBROUTINE PARDER(IFXE,DERIV,NCOMP,YNORM)
C*******SUBROUTINE PARDER*****************************
C THIS SUBROUTINE CONTROLS THE PARTIAL DIFFERENTATION WRT THE
C LAYERING PARAMETERS (RESISTIVITY AND THICKNESS)
C************************************************************
      COMMON /PD/ EXX(6),HX(6),DINX(6),KFXE(6),EC,HZ,ECR,ECI,
     &HZR,HZI,EXR(6),EXI(6),HXR(6),HXI(6)
      COMMON /OC/ DD(3),RKK(3),RHH(3),N,DEL,RR
      COMMON /CV/CI,DS,ANG,F,XX,TM
      COMMON /PART/ RK(3),RH(3),HP(3),HPP(3),
     &D(3),X,R,NB
      DIMENSION DERIV(7),IFXE(6)
      COMPLEX H,E
      JP=1
      IF(IFXE(JP).EQ.0) GO TO 6
      P=1.001
      RR=P*RR
      XX=ALOG(RR/DEL)
      DINX(JP)=RR-R
      CALL FVAL(E,H,NCOMP)
      EXX(JP)=CABS(E)
      HX(JP)=CABS(H)
      EXR(JP)=REAL(E)
      EXI(JP)=AIMAG(E)
      HXR(JP)=REAL(H)
      HXI(JP)=AIMAG(H)
      XX=X
      GO TO 7
 6    EXX(JP)=EC
      HX(JP)=HZ
      EXR(JP)=ECR
      EXI(JP)=ECI
      HXR(JP)=HZR
      HXI(JP)=HZI
 7    JP=JP+1
      DO 12 J=1,N
      IF(IFXE(JP).EQ.0) GO TO 8
      RHH(J)=P*RH(J)
      RKK(J)=RHH(1)/RHH(J)
      DINX(JP)=RHH(J)-RH(J)
      CALL FVAL(E,H,NCOMP)
      EXX(JP)=CABS(E)
      HX(JP)=CABS(H)
      EXR(JP)=REAL(E)
      EXI(JP)=AIMAG(E)
      HXR(JP)=REAL(H)
      HXI(JP)=AIMAG(H)
      RHH(J)=RH(J)
      RKK(J)=RK(J)
      GO TO 13
 8    EXX(JP)=EC
```

```
      HX(JP)=HZ
      EXR(JP)=ECR
      EXI(JP)=ECI
      HXR(JP)=HZR
      HXI(JP)=HZI
13    JP=JP+1
      IF(J.EQ.N) GØ TØ 12
      IF(IFXE(JP).EQ.0) GØ TØ 10
      HPP(J)=P*HP(J)
      DD(J)=2.*HPP(J)/DEL
      DINX(JP)=HPP(J)-HP(J)
      CALL FVAL(E,H,NCØMP)
      EXX(JP)=CABS(E)
      HX(JP)=CABS(H)
      EXR(JP)=REAL(E)
      EXI(JP)=AIMAG(E)
      HXR(JP)=REAL(H)
      HXI(JP)=AIMAG(H)
      HPP(J)=HP(J)
      DD(J)=2.*HP(J)/DEL
      GØ TØ 12
10    EXX(JP)=EC
      HX(JP)=HZ
      EXR(JP)=ECR
      EXI(JP)=ECI
      HXR(JP)=HZR
      HXI(JP)=HZI
12    JP=JP+1
5     CØNTINUE
      CALL FINDIF (NB,NCØMP,DERIV)
      DØ 19 JPZ=1,NB
19    DERIV(JPZ)=DERIV(JPZ)/YNØRM
4     CØNTINUE
      RETURN
      END
```

```
      SUBRØUTINE FINDIF (NB,NCØMP,DERIV)
C************SUBRØUTINE FINDIF****************************
C THIS SUBRØUTINE CALCULATES THE PARTIAL DERIVATIVES ØF THE
C PARAMETERS USING FINITE DIFFERENCES
C*********************************************************

      CØMMØN /PD/ EX(6),HX(6),DINX(6),INX(6),EC,HZ,ECR,ECI,HZR,HZI,
     &EXR(6),EXI(6),HXR(6),HXI(6)
      DIMENSIØN DERIV(7)
      DØ 1 JP=1,NB
      IF(INX(JP).EQ.0) GØ TØ 2
      IF(NCØMP.EQ.10) GØ TØ 10
      IF(NCØMP.EQ.40) GØ TØ 10
      IF(NCØMP.EQ.70) GØ TØ 10
      GØ TØ 3
10    DERIV(JP)=(EX(JP)-EC)/DINX(JP)
1005  FØRMAT(3(2X,E14.8))
      GØ TØ 1
 3    IF(NCØMP.NE.11) GØ TØ 4
      DERIV(JP)=(HX(JP)-HZ)/DINX(JP)
      GØ TØ 1
 4    IF(NCØMP.EQ.20) GØ TØ 11
      IF(NCØMP.EQ.50) GØ TØ 11
      IF(NCØMP.EQ.80) GØ TØ 11
      GØ TØ 5
11    DERIV(JP)=(EXR(JP)-ECR)/DINX(JP)
      GØ TØ 1
    5 IF(NCØMP.EQ.30) GØ TØ 12
      IF(NCØMP.EQ.60) GØ TØ 12
      IF(NCØMP.EQ.90) GØ TØ 12
      GØ TØ 6
12    DERIV(JP)=(EXI(JP)-ECI)/DINX(JP)
      GØ TØ 1
 6    IF(NCØMP.NE.21) GØ TØ 7
      DERIV(JP)=(HXR(JP)-HZR)/DINX(JP)
      GØ TØ 1
 7    IF(NCØMP.NE.31) GØ TØ 1
      DERIV(JP)=(HXI(JP)-HZI)/DINX(JP)
      GØ TØ 1
 2    DERIV(JP)=0.0
 1    CØNTINUE
      RETURN
      END
```

```
      SUBRØUTINE FVAL (EC,H,NCØMP)
C***************SUBRØUTINE FVAL *****************************
C THIS SUBRØUTINE CALCULATES THE FØRWARD SØLUTIØN ØN CØMMAND FRØM
C EMCALC USING LINEAR DIGITAL FILTER THEØRY
C
C***********************VARIABLES*************************
C  CH= CØEFFICIENTS FØR CALCULATING THE J1 HANKEL TRANSFØRM
C  YH= ABCISSA VALUES FØR CH
C  CE= CØEFFICIENT FØR CALCULATING THE J0 HANKEL TRANSFØRM
C  YE= ABCISSA VALUES FØR CALCULATING CE
C  SUM= CØNVØLUTIØN SUM FØR HZ
C  SUEY1,SUEX1,SUEY0,SUEX0, ARE AS DEFINED ØN PAGE 10 ØF THESIS
C  SUEY1= CØNVØLUTIØN SUM FØR EY1
C  SUEX1= CØNVLØUTIØN SUM FØR EX3
C  SUEY0= CØNVØLUTIØN SUM FØR EX1
C  SUEXØ= CØNVØLUTIØN SUM FØR EX2
C  H1= CLØSED FØRM HØMØGENØUS HALF-SPACE EXPRESSIØN FØR HZ
C  EX1= CLØSED FØRM EXXPRESSIØN FØR A HØMØGENØUS HALF-SPACE
C      ( X-CØMPØNENT)
C  EY1= CLØSED FØRM EXPRESSIØN FØR A HØMØGENØUS HALF-SPACE
C      (Y-CØMPØNENT)
C  EX= X-CØMPØNENT ØF THE ELECTRIC FIELD FØR A LAYERED EARTH
C  EY= Y-CØMPØNENT ØF THE ELECTRIC FIELD FØR A LAYERED EARTH
C  EC= CØMBINED ELECTRIC FIELD FØR A LAYERED EARTH
C  F1= LAYERED EARTH CØRRECTIØN FACTØR
C  Z1= LAYERED EARTH CØRRECTIØN FACTØR
C  V1= NØRMALIZED FIRST LAYER PSEUDØ WAVE NUMBER
C
C
C*************************************************************

      CØMMØN /CV/ CI,DS,ANG,F,X,TM
      CØMMØN /QC/ D(3),RK(3),RH(3),N,DEL,R
      CØMPLEX EX1,EX0,EY1,EY0,SUEX1,SUEX0,SUEY1,SUEY0,EX,EY
      CØMPLEX H,V1,F1,SUM,Z1,E1,EC,H1,C1,C2
      DIMENSIØN YH(48),CH(48),YE(61),CE(61)
      DATA
     &YH/-4.5307316E 0,-4.3004731E 0,-4.0702146E 0,-3.8399561E 0,
     &   -3.6096976E 0,-3.3794391E 0,-3.1491806E 0,-2.9189221E 0,
     &   -2.6886636E 0,-2.4584051E 0,-2.2281466E 0,-1.9978881E 0,
     &   -1.7676296E 0,-1.5373711E 0,-1.3071126E 0,-1.0768541E 0,
     &   -8.4659563E-1,-6.1633713E-1,-3.8607863E-1,-1.5582013E-1,
     &    7.4438369E-2, 3.0469687E-1, 5.3495537E-1, 7.6521387E-1,
     &    9.9547237E-1, 1.2257309E 0, 1.4559894E 0, 1.6862479E 0,
     &    1.9165064E 0, 2.1467649E 0, 2.3770234E 0, 2.6072819E 0,
     &    2.8375404E 0, 3.0677989E 0, 3.2980574E 0, 3.5283159E 0,
     &    3.7585744E 0, 3.9888329E 0, 4.2190914E 0, 4.4493499E 0,
     &    4.6796084E 0, 4.9098669E 0, 5.1401254E 0, 5.3703839E 0,
     &    5.6006424E 0, 5.8309009E 0, 6.0611594E 0, 6.2914179E 0/
      DATA
     &CH/3.1010561E-6, 1.8802098E-5, 5.4819540E-5, 9.2891602E-6,
     &   1.5523239E-4, 3.0344652E-5, 3.5338744E-4, 1.4798002E-4,
```

```
&    7.7342377E-4, 5.3570857E-4, 1.7170605E-3, 1.6387239E-3,
&    3.9247683E-3, 4.5796508E-3, 9.2111468E-3, 1.2130467E-2,
&    2.1938415E-2, 3.0853660E-2, 5.1973594E-2, 7.4661566E-2,
&    1.1775455E-1, 1.6353574E-1, 2.3127545E-1, 2.7368461E-1,
&    2.8059285E-1, 1.2875840E-1,-1.5380437E-1,-4.5659951E-1,
&   -3.6077766E-2, 4.2985683E-1,-2.1506075E-1,-2.3624312E-2,
&    8.9316746E-2,-7.4344203E-2, 4.8572965E-2,-3.0088372E-2,
&    1.8846544E-2,-1.2158687E-2, 8.0708759E-3,-5.4706275E-3,
&    3.7554604E-3,-2.5929707E-3, 1.7909426E-3,-1.2320277E-3,
&    8.4095286E-4,-5.6749747E-4, 3.7718405E-4,-1.5891835E-4/
 DATA
&YE/-6.8348046E 0,-6.6045461E 0,-6.3742876E 0,-6.1440291E 0,
&   -5.9137706E 0,-5.6835121E 0,-5.4532536E 0,-5.2229951E 0,
&   -4.9927366E 0,-4.7624781E 0,-4.5322196E 0,-4.3019611E 0,
&   -4.0717026E 0,-3.8414441E 0,-3.6111856E 0,-3.3809271E 0,
&   -3.1506686E 0,-2.9204101E 0,-2.6901516E 0,-2.4598931E 0,
&   -2.2296346E 0,-1.9993761E 0,-1.7691176E 0,-1.5388591E 0,
&   -1.3086006E 0,-1.0783421E 0,-8.4808358E-1,-6.1782508E-1,
&   -3.8756658E-1,-1.5730808E-1, 7.2950416E-2, 3.0320892E-1,
&    5.3346742E-1, 7.6372592E-1, 9.9398442E-1, 1.2242429E 0,
&    1.4545014E 0, 1.6847599E 0, 1.9150184E 0, 2.1452769E 0,
&    2.3755354E 0, 2.6057939E 0, 2.8360524E 0, 3.0663109E 0,
&    3.2965694E 0, 3.5268279E 0, 3.7570864E 0, 3.9873449E 0,
&    4.2176034E 0, 4.4478619E 0, 4.6781204E 0, 4.9083789E 0,
&    5.1386374E 0, 5.3688959E 0, 5.5991544E 0, 5.8294129E 0,
&    6.0596714E 0, 6.2899299E 0, 6.5201884E 0, 6.7504469E 0,
&    6.9807054E 0/
 DATA
&CE/7.3260937E-4, 5.6326423E-4, 1.3727237E-4, 7.5331222E-4,
&    3.5918326E-4, 1.0500608E-3, 7.1530982E-4, 1.5160070E-3,
&    1.2841617E-3, 2.2497985E-3, 2.1906186E-3, 3.4076782E-3,
&    3.6321245E-3, 5.2376028E-3, 5.9212519E-3, 8.1315877E-3,
&    9.5527062E-3, 1.2708615E-2, 1.5305589E-2, 1.9941086E-2,
&    2.4396626E-2, 3.1333652E-2, 3.8683065E-2, 4.9127993E-2,
&    6.0824806E-2, 7.6314344E-2, 9.3928346E-2, 1.1545027E-1,
&    1.3868663E-1, 1.6248847E-1, 1.8114332E-1, 1.8424433E-1,
&    1.5556741E-1, 6.8592481E-2,-8.8339029E-2,-2.8819226E-1,
&   -3.5565260E-1,-5.6288677E-2, 4.8186942E-1,-5.1516453E-2,
&   -2.6102989E-1, 2.1416490E-1,-9.4490687E-2, 2.6196370E-2,
&   -5.1097828E-4,-6.6032948E-3, 7.5193619E-3,-6.7854344E-3,
&    5.8044372E-3,-4.9354894E-3, 4.2323106E-3,-3.6733648E-3,
&    3.2266260E-3,-2.8649137E-3, 2.5677680E-3,-2.3202655E-3,
&    2.1115187E-3,-1.9334662E-3, 1.7800248E-3,-1.6465436E-3,
&    1.3468317E-3/
 SUEX1=CMPLX(0.0,0.0)
 SUEY1=CMPLX(0.0,0.0)
 SUEX0=CMPLX(0.0,0.0)
 SUEY0=CMPLX(0.0,0.0)
 SUM=CMPLX(0.0,0.0)
 BB=EXP(X)
 C1=CMPLX(0.0,1.0)
```

```
      C2=CMPLX(1.0,1.0)
C****************************************
CALCULATE THE FIRST ØRDER HANKEL TRANSFØRMS
C*****************************************
      DØ 8 J=1,48
      Y=EXP(-(X-YH(J)))
      CALL CALC(F1,Z1,V1,Y)
      EX1=C1*V1*(Z1-1)+2.*V1*(1-F1)/((Y+V1*F1)*(Y+V1))
      SUEX1=SUEX1+CH(J)*EX1
      H=Y*Y*V1*(1.-F1)
      H=H/((Y+V1*F1)*(Y+V1))
      SUM=SUM+CH(J)*H
 8    CØNTINUE
      SUEY1=SUEX1
      H=(1/BB)* CI*DS*SIN(ANG)*SUM/(6.2831853*DEL*DEL)
      R2=R*R
      H1=3.-(3.+3.0*C2*BB+2.*C1*BB**2)*CEXP(-C2*BB)
      H1=(-C1*CI*DS*DEL*DEL*SIN(ANG)*H1)/(12.566371*R2*R2)
12    FØRMAT(2(1X,E14.8))
      H=H+H1
      IF(MØD(NCØMP,10).EQ.1) GØ TØ 6
      SUM=CMPLX(0.0,0.0)
C*****************************************
CALCULATE THE ZERØTH ØRDER HANKEL TRANSFØRMS
C*******************************************
      DØ 4 I=1,61
      Y=EXP(-(X-YE(I)))
      CALL CALC(F1,Z1,V1,Y)
      EY0=Y*(C1*V1*(Z1-1.)+2.*V1*(1.-F1)/((Y+V1)*(Y+V1*F1)))
      EX0=(CØS(ANG)**2/DEL)*EY0-(1/DEL)*2.*Y*V1*(1.-F1)/
     &((Y+V1*F1)*(Y+V1))
      SUEX0=SUEX0+CE(I)*EX0
      SUEY0=SUEY0+CE(I)*EY0
      EC=C1*V1*Y*(Z1-1.0)/2.0
      EC=EC-Y*V1*(1.0-F1)/((Y+V1*F1)*(Y+V1))
 4    SUM=SUM+CE(I)*EC
      EC=C1*TM*F*CI*DS*SIN(2.*ANG)*SUM/(12.566371*DEL*BB)
      E1=-.500+(1+C2*BB)*CEXP(-C2*BB)
      E1=CI*DS*RH(1)*SIN(2.*ANG)*E1/(6.2831853*R*R2)
      EC=E1+EC
      EX=(C1*TM*F*CI*DS/(12.566371*BB))*(SUEX0-CØS(2.*ANG)
     &*SUEX1/R)
      EY=(C1*TM*F*CI*DS/(12.566371*BB))*SIN(2.*ANG)*(-SUEY1/R
     &+SUEY0/(2.*DEL))
      EX1=CI*DS*RH(1)*((3*CØS(ANG)**2-2)+(1.+(1+C1)*BB)*CEXP(-C2*BB)
     &)/(6.2831853*R*R2)
      EY1=3.*RH(1)*CI*DS*SIN(ANG)*CØS(ANG)/(6.2831853*R*R2)
      EX=EX+EX1
      EY=EY+EY1
      IF(NCØMP.GE.10.AND.NCØMP.LE.30) GØ TØ 7
      IF(NCØMP.GE.40.AND.NCØMP.LE.60) GØ TØ 11
```

```
      IF(NCOMP.GE.70.AND.NCOMP.LE.90) GO TO 9
 7    EC=EC
      GO TO 6
11    EC=EX
      GO TO 6
 9    EC=EY
 6    CONTINUE
      RETURN
      END
```

```
      SUBRØUTINE CALC(F1,Z1,V1,Y)
C************************************************
C SUBRØUTINE CALC CALCULATES THE LAYERED EARTH CØRRECTIØN FACTØRS
C (F1 AND Z1) FØR UP TØ 3 LAYERS************************
      CØMMØN /QC/ D(3),RK(3),RH(3),N,DEL,R
      CØMPLEX V1,F1,V2,AEX,Z1
      Y2=Y*Y
      T=2.*RK(N)
      V2=CSQRT(CMPLX(Y2,T))
      DØ 9 LL=2,N
      IF(LL.GT.2) GØ TØ 4
      F1=CMPLX(1.0,0.0)
      Z1=CMPLX(1.0,0.0)
 4    I=N-LL+1
      DD=D(I)
      T=2.*RK(I)
      V1=CSQRT(CMPLX(Y2,T))
      AEX=(1.-CEXP(-V1*DD))/(1.+CEXP(-V1*DD))
      F1=(V2*F1+V1*AEX)/(V1+V2*F1*AEX)
      Z1=(V2*RH(I+1)*Z1+V1*RH(I)*AEX)/(V1*RH(I)+V2*RH(I+1)*Z1*AEX)
 9    V2=V1
      RETURN
      END
```

SELECTED REFERENCES

Anderson, W.L., 1973, Fortran IV programs for the determination of the transcient tangential electric field and vertical magnetic dipole for an M-layered stratified earth by numerical integration and digital linear filtering: U.S.G.S. publication PB-226 240/5, Denver, Colo.

Crous, C.M., 1971, Computer-assisted interpretation of electrical soundings: Colorado School of Mines Thesis T-1363, 108p.

Frischnecht, F.C., 1967, Fields about an oscillating magnetic dipole over a two-layer earth, and application to ground and airborne electromagnetic surveys: Colorado School of Mines Quart., v.62, no.1, 370 p.

Ghosh, D.P., 1971, The application of linear filter theory to the direct interpretation of geoelectrical resistivity sounding neasurments: Geophys. Prosp., v. 19, no. 2, p. 192-217.

Jackson, D., and Keller, G.V., 1972, An electromagnetic sounding survey of the summit of Kilauea Volcano, Hawaii: Jcur. Geophys. Res., v.77, no. 26, p.4957-4965.

Keller, G.V., 1968, Electrical prospecting for oil: Colorado School of Mines Quart., v. 63, n0. 2, 268 p.

Keller, G.V., and Frischnecht, F.C., 1966, Electrical methods in geophysical prospecting: Oxford, Pergamon Press, 527 p.

Marquardt, D.W., 1963, An algorithm for least-squares estimation of nonlinear parameters: Jour. Soc. Indus. Appl. Math., v. 11, no. 2, p. 431-441.

Meinardus, H.A., 1967, The kernal function in direct-resistivity sounding: Colorado School of Mines Thesis, no. T-1103, 151p.

Parra, J.O., 1972, A least-squares fitting method for the interpretation of magnetic anomalies caused by two-dimensional structures: Colorado School of Mines Thesis, T-1479.

Pritchard, J.I., 1971, Theory of electromagnetic sounding in the frequency domain: Colorado School of Mines Thesis, no. T-1364, 256 p.

Sinha, A.K., and Bhattacharya, P.K., 1967, Electric dipole over an anisotropic and inhomogenous earth: Geophysics, v. 32, no. 4, p.652-668.

Snyder, D.D., 1968, A gravity survey of South Park, Colorado: Colorado School of Mines Thesis, T-1174

Vanyan, L.L., 1967, Electromagnetic depth soundings: New York, Consultants Bureau.

Wait, J.R., 1966a, Electromagnetic fields of a dipole over an anisotropic half-space: Can. Jour. Phys., v. 44, p. 2387- 2401.

Wait, J.R., 1966b, Fields of a horizontal dipole over a stratified anisotropic half-space: IEEE Trans. Ant. and Prop., v. 14, no. 6 , p. 790-792.