The Design of a
Long Period Horizontal Station Seismometer
Employing a Resistance Bridge Transducer

by

H. N. Opland

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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the Degree of Master of Science in Geophysics.

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INTRODUCTION

The object of this work was to design a horizontal, long period seismometer which would employ a resistance-bridge transducer and to include computations, specifications, and working drawings for the construction of such a seismometer. It was further desired to outline the possibilities for remote pen-recording.

The purpose of a seismograph is to make as faithful and as legible a record as possible of one component of the actual earth's motion. The term seismograph is usually applied to the entire system which makes such records, while the term seismometer is used with reference to the instrument which contains the "seismic" mass. Because a pendulum which possesses a large mass tends to remain at rest while its support moves, such arrangements are usually employed. To record the earth's motion we require the aforementioned "seismic" mass, a means of recording the motion in different planes or directions, a means of recording the vibrations of different frequencies, a damping device, a recording device, a timing device, and a magnification device.

A horizontal seismometer must be so constructed as to be affected only by the horizontal component of the earth's vibration. It follows, therefore, that it must be allowed to rotate only about a vertical axis. Further, because the motion produced by an earthquake (seismic waves) is, in general, oscillatory in motion, a linear oscillator will readily respond to such motion. Therefore nearly all seismometers are forced linear oscillators. The equation for a forced linear oscillator is given by $m\, d^2x/d\tau^2 + a\, dx/d\tau + kx = F(\tau)$ where the restoring force (the force resulting from Hooke's law, and pushing the body of mass (m) back to its position of equilibrium (x = 0) is -kx, the damping force is -a dx/dt, and F(t) is the forcing function, a function of time. Therefore a seismometer that is essentially a linear damped oscillator represents the best compromise we can make. Such a seismometer employs a restoring force proportional to its displacement and a damping force proportional to its velocity. Most modern seismometers are constructed according to this precept.

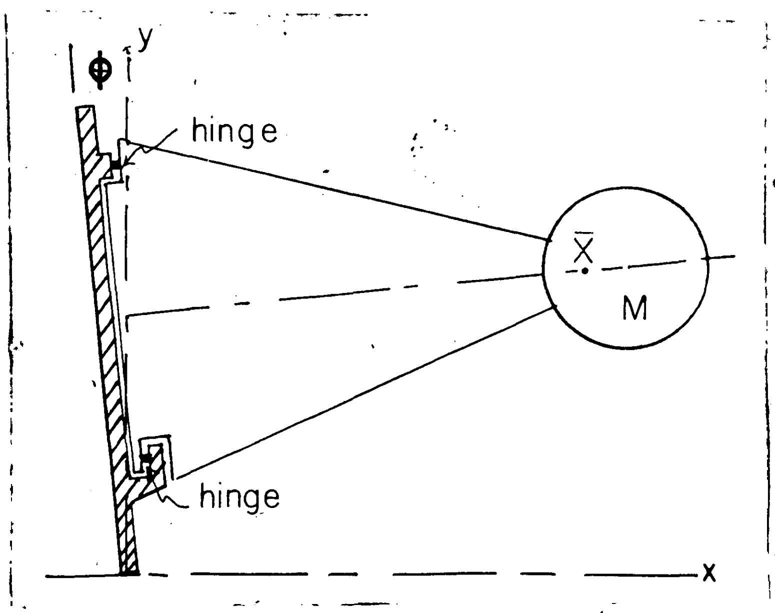

Figure 1

A common type of horizontal seismometer is shown schematically in Figure 1. The x,y plane is maintained vertical, thus insuring only horizontal motion and the mass (M) is hinged to a support so that there is equal tension on each of the hinges. The pendulum is pictured inclined from the horizontal axis by an angle ϕ to illustrate the effect of gravity on the system. If the boom were exactly horizontal, the only restoring torque acting would be the torque exerted by the hinging arrangement. As pictured, gravity tends to diminish this restoring torque by an amount proportional to the mass (M), the sine of the angle ϕ, and the length of the effective moment arm, from the hinge-line to the center of gravity ($\bar{X}$). Such a negative restoring torque is termed a labilizing torque.

Long period seismometers are those having natural periods of oscillation that range from ten to thirty seconds. One method of adjusting the period of the pendulum in Figure 1 would be to vary the inclination (ϕ). An increase in its shown direction would increase the period until instability develops and an inclination of the boom toward or below the horizontal would decrease the period. It is often desirable to have the seismometer adjustable as to period, covering a range from three or four seconds up to thirty seconds. Period adjustment has been provided for in these designs.

When a linear oscillator is contained in a frame or support, the simple expression $m\, d^2x/d\tau^2 = F$ may be applied, where the force (F) acts on the mass (m) whose acceleration is given by $d^2x/d\tau^2$ and x is the position of the mass relative to a hypothetical fixed point. If the support is displaced an amount y by the movement of the earth, the force acting on the mass will be the sum of (1) $k(y - x)$, the force proportional to the relative displacement, and (2) $a\,(dy/d\tau - dx/d\tau)$ the force proportional to the relative velocity. We then have

$$m\, d^2x/dt^2 = a\left(\frac{dy}{dt} - \frac{dx}{dt}\right) + k(y - x)$$

or

$$m\frac{d^2x}{dt} + a\left(\frac{dx}{dt} - \frac{dy}{dt}\right) + k(x - y) = 0$$

Subtracting d^2y/dt^2 from both sides will give

$$m\left(\frac{d^2x}{dt^2} - \frac{d^2y}{dt^2}\right) + a\left(\frac{dx}{dt} - \frac{dy}{dt}\right) + k(x - y) = -m\, d^2y/dt^2$$

The relative displacement $(x - y)$ of the mass (m) is the measurable variable for the seismometers herein discussed. If we let this variable equal u $(u = x - y)$, we write the equation of motion as

$$m\, d^2u/dt^2 + a\, du/dt + k\, u = -m\, d^2y/dt^2$$

and we see that the measurable variable behaves as though a force had been applied which is proportional to the mass of the linear oscillator and the ground acceleration.

THE HORIZONTAL SEISMOMETER

Period Formulae

The period of an undamped pendulum is given by:

$$T = 2\pi \sqrt{\frac{\text{Moment of inertia}}{\text{Restoring torque per unit angle}}}$$

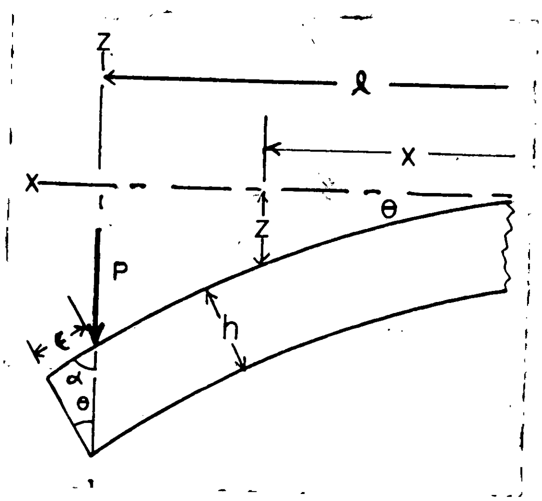

Figure 2

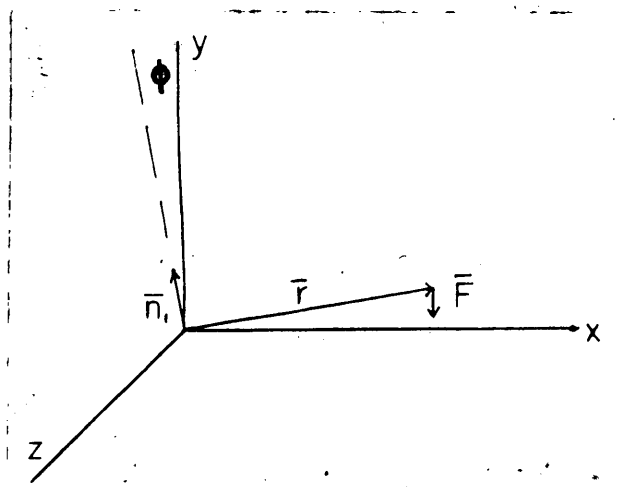

Figure 3

First considering Figure 2, we have a vector $\bar{F}$ acting on the vector $\bar{r}$ where $\bar{n}_1$ is a unit vector making an angle of ϕ with the y axis in the x,y plane. $\bar{F}$ is the force due to gravity (g) acting on the mass (m) and in vector notation may be written as $-\bar{j}\,mg$. The vector $\bar{r}$ corresponds to a pendulum arm and it is displaced an angle θ in the z direction. Then

$$\bar{r} = \bar{i}x + \bar{j}y + \bar{k}z \qquad \text{AND} \qquad \bar{n}_1 = -\sin\phi\,\bar{i} + \cos\phi\,\bar{j}$$

The torque about $\bar{n}_1$ is then:

$$L_{\bar{n}_1} = \bar{n}_1 \cdot (\bar{F} \times \bar{r}) = \begin{vmatrix} -\sin\phi & \cos\phi & 0 \\ x & y & z \\ 0 & -mg & 0 \end{vmatrix} = -Z\,mg\sin\phi$$

For small angles of displacement ($\sin\theta \simeq \theta$), $z = x\,\theta$ where x is taken as the horizontal distance from the hinge line to the center of gravity (see $\overline{X}$, Figure 1). Therefore $L_{\bar{n}_1} = -x\,\theta\, mg \sin\phi$ or the torque per unit angular displacement is $-x\, mg \sin\phi$. If the axis (represented by the unit vector $\bar{n}_1$) is inclined so that $\bar{n}_1 = +\sin\phi\,\bar{i} + \cos\phi\,\bar{j}$, the torque per unit angle is $+x\, mg \sin\phi$.

To determine the restoring torque supplied by the suspension hinges we first consider Figure 3 which is drawn in the x,z plane. If flat spring hinges are used, we can assume a cantilever action and use the elastic curve equation to calculate the force (P) necessary to produce a displacement z_ℓ at x equal to the hinge length ℓ. The force (P) (see Appendix, p. 1) was found to be $P = 3EIz_\ell/\ell^3$. (E is Young's modulus of the material, I is the second moment of the cross-section of the hinge perpendicular to a neutral axis). We now proceed to calculate the strain in a filament on the sides of such a hinge and we assume that α (Figure 3) is a right angle for small angles of θ. Then $\epsilon = h \tan\theta$ where h is the perpendicular distance across the hinge and $dz/dx = \tan\theta$. The strain is therefore given by

$$\epsilon/\ell = \frac{h\tan\theta}{\ell} = \frac{dz/dx\ h}{\ell}$$

and dz/dx (see Appendix, p. 1) equals $P\ell^2/2EI$. Substituting the value given for P (above), $dz/dx = 3z_\ell/\ell$ and the strain is given by $\epsilon/\ell = 3z_\ell h/2\ell^2$

The restoring torque per unit angle created by the hinges will then be the force (P) times the effective moment arm divided by the unit angle.

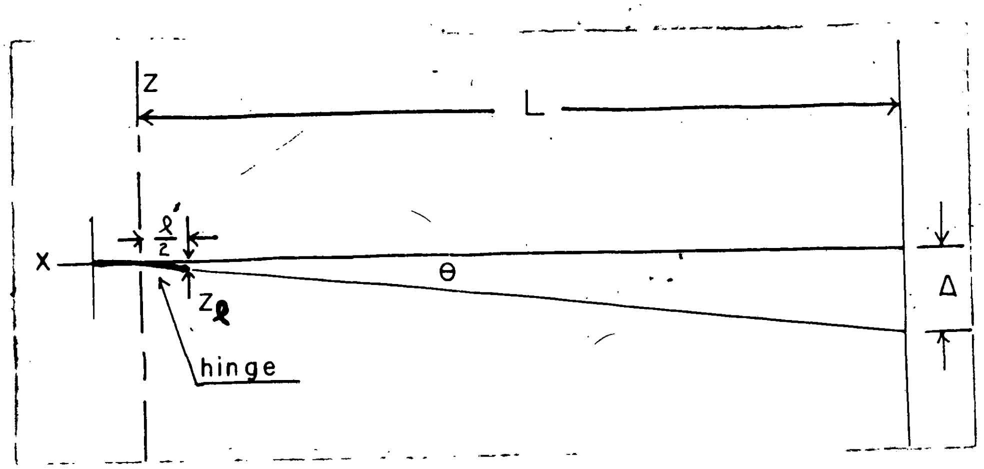

Figure 4

Figure 4 shows the hinge in relation to the boom and its angular displacement Δ . The figure also shows a hinge line passing through the center of the hinge. The equivalent axis of rotation of a hinge that is fixed at one end and displaced at the other is approximately one-third of the hinge length from the fixed end (Heiland, 1946, p. 100). Our pendulum will require two such hinges, one fixed at the opposite end with respect to the other. Therefore our actual hinge line will be slightly displaced from the vertical. However, this displacement is so small due to the small dimensions of the hinges compared to the length of the boom and the distance between the hinges that we assume a hinge line passing through the center of each hinge. For small angles of θ ($\tan\theta = \theta$) we may write z_ℓ (Figure 4) as $\theta \ell'/2$ and the second moment of such a hinge whose width is b' and whose thickness is h is given by b' h / 12. Therefore the force (P) is:

$$P = \frac{3 E I z_\ell}{(\ell')^3} = \frac{3}{12} \cdot \frac{E_h b' h^3}{(\ell')^3} \cdot \frac{\theta \ell'}{2}$$

and the torque per unit angle (T_h) will be:

$$T_h = \frac{P\ell'}{\theta 2} = \frac{3}{12} \cdot \frac{E_h b' h^3}{\theta (\ell')^3} \cdot \frac{\theta \ell'}{2} \cdot \frac{\ell'}{2} = \frac{1}{16} E_h \, b' h^3 / \ell'$$

For two hinges

$$T_h = \frac{1}{8} E_h b' h^3 / l'$$

where the subscript ($_h$) is used to designate "hinge."

The period formula now becomes

$$T = 2\pi \sqrt{\frac{I}{T_h - \bar{X}\, mg \sin\phi}}$$

where ϕ is the angle of inclination from the horizontal in an upward direction.

Moment of Inertia of the Boom

For a unit mass at a distance r from the desired axis of rotation, the amount of inertia is given by $I = \Sigma\, mr^2$.

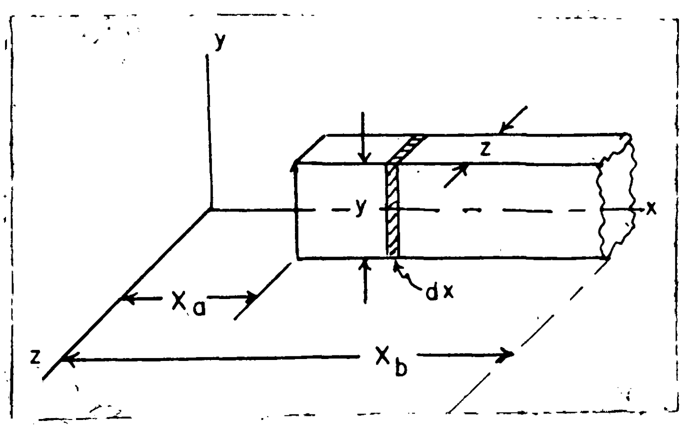

Figure 5

The moment of inertia of the mass shown in Figure 5 taken about the y axis will be $I_y = \int x^2 dm = \rho \int x^2 dv$ where ρ is the density of the material. Also from the figure $dv = yz\, dx$;

$$\therefore I_y = \rho yz \int_{x_a}^{x_b} x^2 dx = \rho yz \left[\frac{x^3}{3} \right]_{x_a}^{x_b}$$

For more than one mass the total moment of inertia (I_T) will be $I_1 + I_2 + I_3$ etc. Likewise the center of gravity ($\bar{X}$) can be calculated for any number of associated masses by

$$\bar{X} = \frac{m_1\bar{X}_1 + m_2\bar{X}_2 + m_3\bar{X}_3 \text{ (ETC.)}}{m_1 + m_2 + m_3 \text{ (ETC.)}}$$

Design A

Design A is pictured in the three-view drawing Plate 1 (enclosed). It is a simple horizontal pendulum which is supported by two flat spring steel hinges under equal tension and is damped by an aluminum vane moving in the field of permanent magnets. The support should be annealed steel, the major pendulum mass of brass, and the other parts of the boom should be cast or machined aluminum.

Note: If bonded strain gages are used to record the strain produced by a unit angular deflection of the boom, they would be attached to either side of the spring hinges. Such an arrangement would add little or nothing to the restoring torque (T_h). However, it was found that such gages would not give the proper sensitivity, and wires which would undergo strain were introduced. The mathematical justification of this design principle is given fully under the section The Electro-Mechanical Transducer which follows. It is necessary at this time, however, to introduce these wires into our discussion of Designs A and B because they add to the restoring torque acting on the boom and hence affect the period of both designs.

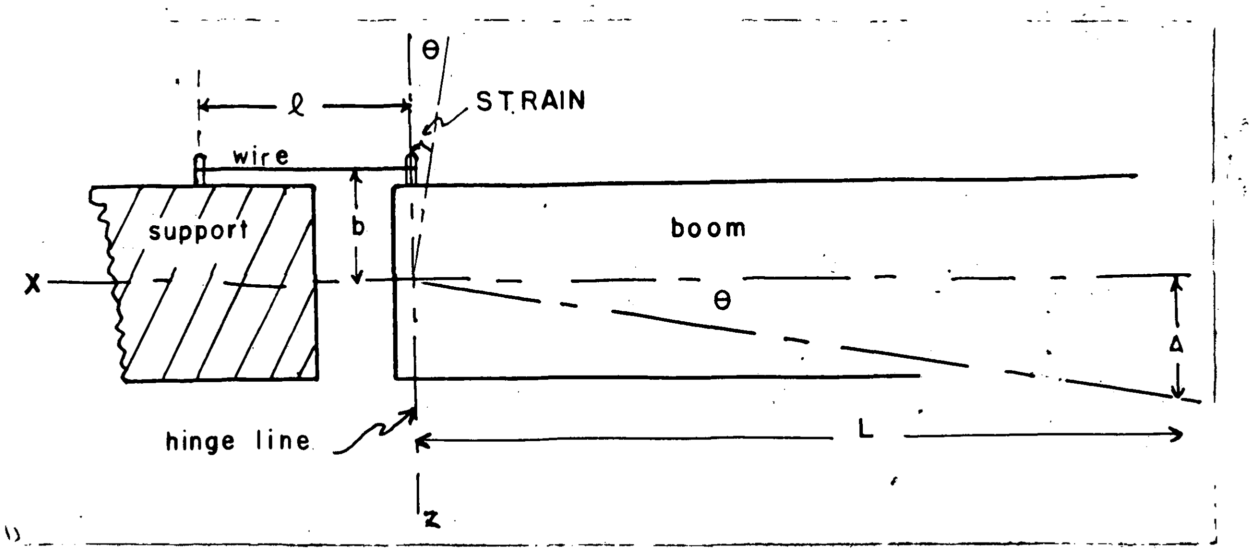

Figure 6

If wires are attached to the boom at the hinge line and then to the support as shown in Figure 6, the strain they will undergo may be written as $\theta b/\ell$

The restoring force per unit angle (θ) due to wires attached in this manner will be

$$\text{Force} = 2 \cdot N \cdot A \cdot \text{Stress} = 2 \cdot N \cdot A \cdot E_w \cdot \theta b/\ell$$

where N is the number of wires per side, A is the cross sectional area of the wire and E_w is Young's modulus of the wire material. The numeral "2" merely indicates that there are wires on both sides. The torque per unit angle (T_w) may then be written as

$$T_w = \frac{2 \cdot N \cdot A \cdot E_w}{\theta} \cdot \frac{\theta b}{\ell} \cdot b = 2 \cdot N \cdot A \cdot E_w \cdot b^2/\ell$$

where ℓ is the wire length and b is the distance out from the center of the boom or hinge point -- the effective lever arm.

The period formula now becomes

$$T = 2\pi \sqrt{\frac{I_T}{T_h + T_w - \bar{X}\, mg \sin\phi}}$$

Design A: Calculations

Using the dimensions indicated on Plate 1 and the following densities (Al = 2.5, Brass = 8.5) the total moment of inertia (I_T), the center of gravity ($\bar{X}$) and the mass (m) were calculated to be

$$I_T = 5{,}390{,}000 \text{ gms-cm}^2$$

$$\bar{X} = 24.8 \text{ cm.}$$

$$m = 8840 \text{ gms.}$$

From the previously derived expressions for T_h and T_w and using the following dimensions for the hinges and wires, the total restoring torque was calculated.

Hinge			Wire		
Thickness	= h =	0.02 cm.	Diameter	= 1 mil =	0.00254 cm.
Width	= b' =	1.0 cm.	Length	= ℓ =	3.0 cm
Length	= ℓ' =	1.0 cm.	Number	= N =	4.0
Modulus	= E_h =	20.5×10^{11}	Modulus	= E_w =	15.0×10^{11}
			Moment arm	= b =	0.8 cm.
(steel)			(constanten)		

$$T_h = \tfrac{1}{8} E_h \cdot b' \cdot h^3 / \ell' = \tfrac{1}{8} \times 20.5 \times 10^{11} \times 1 \times (.02)^3 / 1 = 2.05 \times 10^6 \text{ dynes-cm.}$$

$$T_w = 2 \cdot N \cdot \pi r^2 \cdot E_w \cdot b^2 / \ell = 2 \times 4 \times \pi (1.27 \times 10^{-3})^2 \times 15 \times 10^{11} \times (.8)^2 / 3$$

$$= 12.8 \times 10^6 \text{ dynes-cm.}$$

The only variable remaining in the period formula is ϕ, the angle of axis inclination. If this angle is set at $3^\circ\ 51'$ by means of the leveling screws, we shall obtain the following period:

$$T = 2\pi \sqrt{\frac{I_T}{T_h + T_w - \bar{X} mg \sin\phi}} = 2\pi \sqrt{\frac{5.39 \times 10^6}{(2.05 + 12.8) \times 10^6 - 24.8(8840 \times 980) \times .0667}}$$

$$= 2\pi \sqrt{\frac{5.39 \times 10^6}{5.39 \times 10^5}} = 2\pi \sqrt{10} = 19.9 \text{ seconds.}$$

and when the pendulum is horizontal, $(\phi = 0)$ the period will be

$$T = 2\pi \sqrt{\frac{5.39 \times 10^6}{14.85 \times 10^6}} = 2\pi \sqrt{.363} = 3.8 \text{ Seconds.}$$

Lowering the boom below the plane of the horizontal will further decrease the period.

The damping recommended to be applied to this design is the conventional type which consists of a copper or aluminum vane attached to the end of the boom which moves in the field produced by permanent magnets. The damping is changed or adjusted by moving the magnets. It is possible that the damping arrangement outlined under Design B may be applicable to this design also and its limitations are cited under the section on <u>Damping</u>.

A graduated leveling bubble is suggested as a means of roughly checking period control and the usual limit stops near the boom mass are provided to prevent large displacements which might snap the strain-gage wires. A clamp bar is provided for locking the boom and also serves to hold it in the correct position for changing or adjusting the hinges which are held in place by fillets and set screws. The method of attachment of the wires will be covered more fully under <u>Design B</u>. The base plate should be a brass casting with marginal grooves for an air-tight transparent plastic cover box. It might be desired to drill the clamping yoke for an air jet by means of which the boom may be given controlled impulses for calibration and adjustment.

Design B

Design B is pictured in the three-view drawing Plate 2 (enclosed). It is mounted on the same type base as Design A and has the same type limit stops, clamp bar, level vial and leveling screws. The hinges are the same material but of smaller dimensions due to the smaller mass which they must support. Damping is supplied by a moving coil of the loud-speaker type moving in the field of a permanent magnet of the speaker design. This arrangement will be described in detail under Damping. The main difference in this design is that the labilizing (negative restoring) torque is produced by a pair of elinvar springs which are attached to the boom to the right of the hinge line. The support contains a passage for the lower spring and the support connections of the spring are adjustable (micrometers) so that the degree of astatization may be varied -- and hence the period. If the seismometer is maintained level and T_s designates the torque per unit angle applied by each spring, the period may be calculated from

$$T = 2\pi \sqrt{\frac{I_r}{T_h + T_w - 2T_s}}$$

Another feature of Design B is the use of the damping magnet as the main mass of the boom. The magnet is welded to a small block and this in turn is screwed to the rod of the boom. The damping coil is fixed firmly to the base plate and is centered in the field of the magnet by suitable adjusting screws.

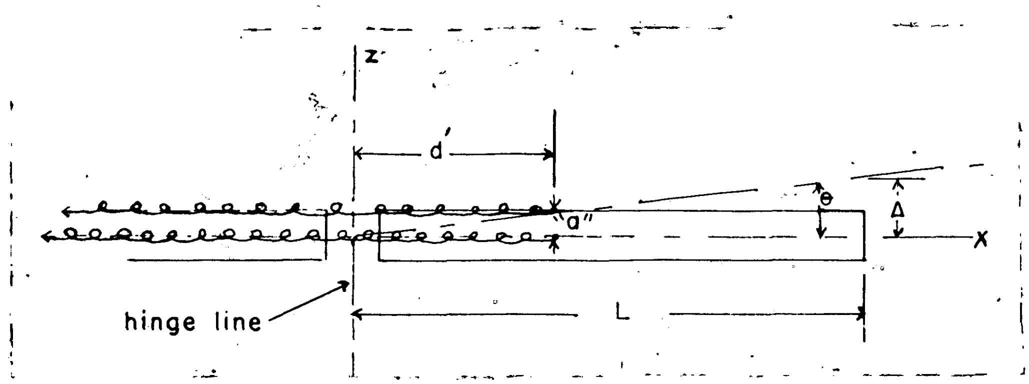

Figure 7

Assuming the stationary attachments of the springs far-distant to the left of the hinge line, the lever arm of the spring shown in Figure 7 would approximate "a" in length. Assuming $\theta = \tan\theta$ we see that "a" $= d'\theta$.

The force exerted by a cylindrical helical spring of circular cross section when under tension is given (Marks, 1941, p. 486) by:

$F = G_s d^4 f / 64 e^3 n$ where G_s is the shear modulus of the material, d is the wire diameter, f is the longitudinal displacement producing the force F, e is the coil radius and n is the number of turns in the coil. The longitudinal displacement (f) is taken as the distance between the spring connections minus the length of the closed spring.

If the labilising torque per unit angle (θ) is T_s for each spring, we have

$$T_s = \frac{F \times \text{Lever Arm}}{\theta} = \frac{G_s d^4 f}{64 e^3 n} \cdot \frac{d'\theta}{\theta} = \frac{G_s d^4 f d'}{64 e^3 n}$$

where $G_s d^4 / 64 e^3 n = C$, the spring constant, and d' is the distance from the point of attachment (spring to boom) to the pivot axis of the pendulum.

Design B: Calculations

Using elinvar for the spring material (because of the stable elastic characteristics which this material possesses) and the following dimensions of the spring, we calculate the spring constant (c).

$G_s = 4.9 \times 10^{11}$ (elinvar) $\qquad n = 57$ (turns)

$d = 0.015" = 0.04$ cm. $\qquad e = 0.2$ cm. (coil radius)

$$c = G_s d^4/64e^3n = \frac{4.9\times10^{11}\times(.04)^4}{64\times(.2)^3\times57} = 43{,}000$$

and if $d' = 5.0$ cm. (the boom distance of attachment) and $f = 30$ cm. (attached spring length minus closed spring length), the labilizing torque per unit angle becomes

$$T_s = cd'f = 43\times10^3\times5\times30 = 6.45\times10^6 \text{ dynes-cm.}$$

The moment of inertia of the boom shown in Plate 2 is calculated as follows:

I_{1a} = I of part to left of hinge line

I_{1b} = I of part to right of hinge line

I_2 = I of rod

I_3 = I of connecting assembly from boom to magnet

I_4 = I of magnet

I_T = Total I = the sum of the above.

From the previously developed moment of inertia formula,

$$I_y = \rho yz\left[\frac{x^3}{3}\right]_{x_a}^{x_b} = \rho yz\left[\frac{x_b^3}{3} - \frac{x_a^3}{3}\right]$$

where x_b is the distance from the hinge line to the outer edge of a unit and x_a is the distance from the hinge line to the inner edge of the unit under consideration, we calculate

$$I_{1a} = 2.5 \times 10 \times 1 \left[1^3/3 - 0 \right] = 8 \text{ gms-cm}^2$$
$$I_{1b} = 2.5 \times 10 \times 1.5 \left[5.5^3/3 - 0 \right] = 2060 \text{ " "}$$
$$I_2 = 2.5 \times \pi \times (.5)^2 \left[\frac{10.5^3}{3} - \frac{5.5^3}{3} \right] = 456 \text{ " "}$$
$$I_3 = 7.0 \times 3 \times 3 \left[\frac{12.5^3}{3} - \frac{10.5^3}{3} \right] = 16700 \text{ " "}$$

For a cylinder of radius r and length ℓ taken at an axis a distance a from the center of the cylinder, I is given (Hodgman, 1947, p. 2338) as: $I = m(r^2/4 + \ell^2/12) + ma^2$. The mass of the magnet is suggested as 400 gms., the radius as 2.5 cm., the length as 5.0 cm., and the distance to the hinge line from its center is 15 cm. I_4 may be written as:

$$I_4 = 400\left(\frac{2.5^2}{4} + \frac{5^2}{12}\right) + 400 \times 15^2 = 9.1 \times 10^4 \text{ gms-cm}^2.$$

The total moment of inertia (I_T) is 1.08×10^5 gms-cm^2 and the mass of the boom is approximately 750 gms.

The hinges used on Design B need not be so large as those of Design A since they support a considerably smaller mass. E_h and ℓ remain the same (see p. 12) but the width (b') is now 0.5 cm. and the thickness (h) is 0.012 cm. T_h for Design B was computed to be $.221 \times 10^6$ dynes-cm. per unit angle.

The same arrangement of wires are used and T_w is again 12.78×10^6 dynes-cm. The natural period (from the formula given on page 14) was computed as 6.28 seconds. An increase in the spring length (f) of 0.01 cm. will increase T_s sufficiently to cause a period change of 0.17 seconds. The adjustment on the spring tension at the support ends must, therefore, be controlled to 0.01 cm. An adjustment of 0.06 cm. at both of the spring support connections will effect a one-second period change. Increasing the tension will increase the labilizing force ($2T_s$) and lengthen the period. Decreasing the tension will shorten the period. The period calculated above (6.28 sec.) is for an f of 30.00 cm. If f is increased to 31.1 the period will be 25 seconds and if f is decreased to 29.92 cm. the period will be 5.0 seconds, giving a range of twenty seconds of period change for a movement of 1.18 cm. at the support connections of the springs. The adjustable spring connection posts, their sliding tracks, and the micrometer screws should be constructed and mounted with reasonable precision, and reference marks that are equidistent from the hinge line should be scribed on each track for comparison purposes.

The constantan wires, which undergo strain when the boom is displaced, should be attached to the boom exactly at the hinge line and equidistant from the center, as shown on Plates 1 and 2. This may be accomplished by setting two small posts of fused quartz or sapphire into the cast aluminum boom at the positions indicated on the plates. Similar pins are set into the support and the wires are snubbed once around each pin and then led to the soldered connection points. It is desirable to connect these wires to the pins in such a manner that they will all be under similar tension. It is also desirable that these pins all be of the same length and that the wires are connected to them 0.8 cm. from the center line of the boom. An opening is suggested in the support directly behind the connecting posts so that the wires, which make up a Wheatstone bridge circuit, may be connected properly without excessively long lead wires. This circuit is discussed in the following section.

THE ELECTRO-MECHANICAL TRANSDUCER

It has long been known that when a conductor is placed either in tensile or compressive stress, the electrical resistance changes. When a wire is placed under tensile stress, the wire becomes longer and the cross section decreases. The change in dimensions of the wire can be expressed in terms of unit elongation and Poisson's ratio. However, the values of resistance change predicted from dimensional changes do not check closely with the changes measured experimentally (Neilsen, 1943, p. 107).

For most metals, the strain sensitivity (defined as the ratio of change of unit resistance per unit strain) would be expected to be between 1.5 and 1.8 from the consideration of only the dimensional changes of the wire. Instead, most metals and alloys have sensitivity factors considerably higher. Constanten is given as 2.0 and the iso-elastic wire used in other commercial gages is listed as high as 3.5. This has never been explained. The reason probably lies in an actual change of volume resistivity of the metal (Neilsen, 1943, p. 106).

Bonded strain gages are generally made up of a few loops of fine wire that are mounted on a sheet of thin paper. The ends of the wire terminate in stronger leads for connection ease. These gages are then cemented to the article undergoing strain and accurately transmit the strain when their resistance change is measured by a Wheatstone bridge or other amplified means. If this type of strain-recording device were to be used in these designs the gages would be attached to the sides of the spring hinges. The strain on the sides of these hinges was derived on page 7 and found to be:

$$\epsilon/\ell = 3 z_\ell h / 2\ell^2$$

where h is the thickness, ℓ is the length, and z_ℓ is the displacement at the boom end of the hinge. z_ℓ equals $\Delta\ell/L$ where Δ is the displacement of the boom mass due to ground motion and L is the effective length of the pendulum (see Figure 4).

According to Robertson (1948, p. 26) the amplitude of the vibrations caused by earthquakes is measured in microns (10^{-4} cm.) and we shall let Δ (the boom displacement) equal 5×10^{-4} cm. From Design B, h equals 0.012 cm., ℓ equals 1 cm., L equals 15 cm. and therefore $z_\ell = \Delta \cdot \ell / L = 5 \times 10^{-4} \times 1/15 = 3 \times 10^{-5}$ cm.

The strain is then given by

$$\epsilon/\ell = \frac{3 \times 3 \times 10^{-5} \times .012}{2 \times 1^2} = 8.4 \times 10^{-7}$$

As has been stated, the better type of commercial strain gages have a sensitivity factor of 3.45 which is expressed mathematically as $\Delta R/R \,/\, \epsilon/\ell$ where ϵ/ℓ is the strain. Then $\Delta R = 3.45 \times$ strain $\times$ total resistance. For an iso-elastic gage (available commercially) where total resistance is equal to 1000 Ω

$\Delta R = 3.45 \times 8.4 \times 10^{-7} \times 1000 = 0.003\ \Omega$ change in 1000 Ω.

Calculations under Magnification in a later section show that this change in resistance is not sufficient to permit the recording of any but the larger earthquakes which might occur relatively near to the seismograph station.

Referring back to Figure 6 (page 11) the strain on the wire of length ℓ at a distance b from the centerline of the boom's rest position may be written as $\Delta \ell / L$. Assuming small angles of displacement ($\theta = \tan\theta$) the strain will be

$$\epsilon/\ell = \frac{\Delta \cdot \ell}{L} = \frac{b \cdot \theta}{L} = \frac{b \cdot \Delta/\ell}{L} = \frac{b\Delta}{\ell L}$$

Using the dimensions suggested in Design B, namely, h equals 0.8 cm., ℓ equals 3 cm., L equals 15 cm. and Δ equals 5×10^{-4} cm., we see that the strain is

$$\epsilon/\ell = \frac{.8 \times 5 \times 10^{-4}}{3 \times 15} = .89 \times 10^{-5}$$

The resistance of two 3 cm. lengths of constantan wire one mil in diameter can be calculated from $R = \rho \frac{\ell}{A}$ where ρ is the resistivity (5×10^{-5} ohm-cm. for constantan), ℓ is the length (6 cm.) and A is the cross sectional area (πr^2).

$$R = \rho \frac{\ell}{A} = \frac{5 \times 10^{-5} \times 6}{\pi (1.27 \times 10^{-3})^2} = 60\,\Omega.$$

The sensitivity factor of constantan is 2 so we may write

$$\Delta R = 2 \times \epsilon/\ell \times R = 2 \times .89 \times 10^{-5} \times 60$$

$$\Delta R = 0.00107\,\Omega \text{ change in } 60\,\Omega.$$

Calculations under <u>Magnification</u> in a later section prove this resistance change to be easily recordable for ground displacement of 5×10^{-4} cm. and even less.

Recording Circuit and Formulae

As was stated earlier, four resistance arms, either bonded strain gages or wires are subjected to a strain by displacement (Δ) of the pendulum. This strain produces a change in resistance and this, in turn, unbalances a Wheatstone bridge circuit and deflects a galvanometer or some other indicating device.

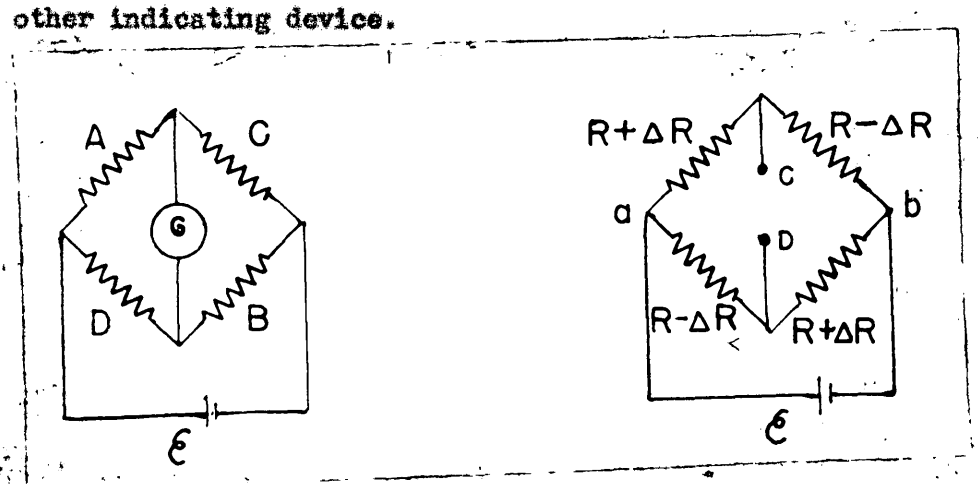

Figure 8 Figure 9

If A and B are the wire loops on one side of the pendulum and C and D are the loops on the other side, we have the circuit shown in Figure 8. If all the loops are of the same length and same material, then for any strain

$$A = B = R + \Delta R \quad \text{and} \quad C = D = R - \Delta R \quad \text{(Figure 9)}$$

The resistance between a-b (Figure 9) is 2R and the current flowing in each branch is $I = \mathcal{E}/2R$. The potential drop in $R + \Delta R$ (from b) equals $I(R + \Delta R)$ and the potential drop in $R - \Delta R$ (from b) equals $I(R - \Delta R)$. The difference in potential across c-d is then $I(R + \Delta R) - (R - \Delta R))$ or $\mathcal{E}\, \Delta R/R$.

By Thevenin's Theorem (Cruft, 1947, p. 110) we can construct the equivalent circuit shown in Figure 10.

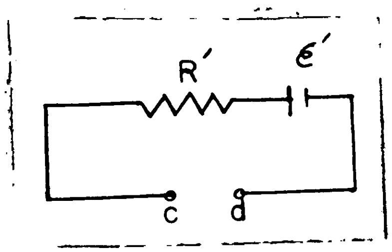

Figure 10

$\mathcal{E}'$ equals the difference in potential across c-d which is $\mathcal{E}\, \Delta R/R$ and R' is given as

$$R' = \frac{(R+\Delta R)+(R-\Delta R) \times (R-\Delta R)+(R+\Delta R)}{(R+\Delta R)+(R-\Delta R)+(R-\Delta R)+(R+\Delta R)} = \frac{2R \times 2R}{4R} = R$$

(see Figure 9)

Galvanometer statistics are given in terms of sensitivity, coil resistance and external resistance needed for critical damping. Galvanometers may have an external critical damping resistance of less than R, and in that case the galvanometer must be shunted with an additional resistance so that the equivalent series resistance is the specified value if critical damping is desired.

Our circuit may be expressed as:

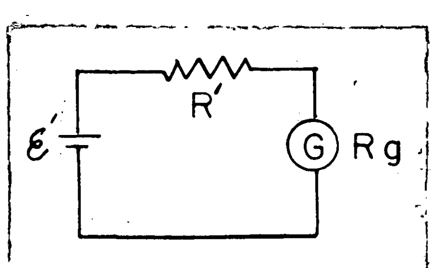

Figure 11

where R_s is the resistance desired for any given degree of damping and R_g is the resistance of the galvanometer coil. If the external critical damping resistance specified for the galvanometer is greater than R for any such bridge circuit, a simple series resistance may be added. However, if a shunt resistance is required we have the following circuit:

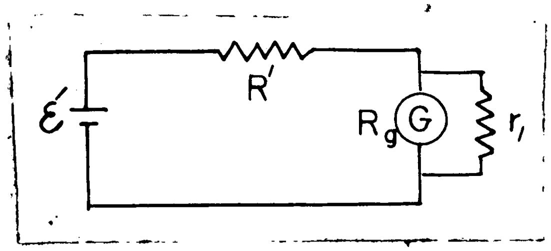

Figure 12

where $R_s = \frac{r_1 + R'}{r_1 R'}$ and therefore $r_1 = \frac{R' R_s}{R' - R_s}$ $(R' = R)$

The current flowing in the galvanometer without the shunt resistance r_1 may be expressed by

$$I = \frac{\mathcal{E}'}{(R_s + R_g)} = \frac{\mathcal{E}\,\Delta R}{R(R_s + R_g)}$$ (see Figures 9 and 11)

while the current flowing in the galvanometer with the shunt resistance r_1 in the circuit is

$$i_g = I \cdot \frac{r_1}{r_1 + R_g} = \frac{\mathcal{E}}{(R' + R_g)} \cdot \frac{r_1}{(r_1 + R_g)} = \frac{\mathcal{E}\,\Delta R \; r_1}{R(R + R_g)(r_1 + R_g)}$$

From the equation

$$\eta\,\omega_0 = \frac{(BAN)^2 \times 10^{-3}}{2K_\tau(R_s + R_g)}$$ (Merideth, 1949)

where η is the degree of damping, ω_0 is the undamped natural frequency of the galvanometer coil, B is the field strength of the galvanometer magnet in kilogauss, A is the area of the coil in square cm., N the number of turns in the coil, K_t the moment of inertia of the coil, R_s the series resistance and R_g the coil resistance, we may write the following constant (C):

$$C = \frac{(BAN)^2 \times 10^{-3}}{2 K_I \omega_0} = \eta (R_s + R_g)$$

R_g is usually given for commercial galvanometers and R_s is given for η equal to unity. Therefore the constant C can be evaluated for any galvanometer and the series resistance (R_s) necessary for other values of η can be calculated. From this R_s and a known R' (R' = R), r_1 may be obtained. Knowing the impressed voltage ($\mathcal{E}$) and the change in resistance of each arm of the bridge (ΔR), i_g may be readily evaluated.

DAMPING

The principle of a coil with annular magnetic field has been employed to effectively damp Design B and possibly Design A. From the equation of motion

$$I \frac{d^2\theta}{d\tau^2} + a\frac{d\theta}{d\tau} + k\theta = F(\tau)$$

where k is the restoring force per unit angle and a is the damping torque per unit angular velocity and I is the moment of inertia of the boom.

For a coil of length ℓ moving in a field of B gauss with a current of i cgsm units flowing a equals B x ℓ x i (cgsm units). Conventionally, $i = \frac{I}{10}$ amperes and $a = B\ell I/10$. The electromotive force (cgsm units) generated by a moving coil is given by: $e = B\ell\, dx/d\tau$ where $\frac{dx}{d\tau}$ is the unit linear velocity ($r\frac{d\theta}{d\tau} = \frac{dx}{d\tau}$ where r is the distance from the hinge line to the damping coil center).

The Voltage (E) may then be written as:

$$E = \frac{B\ell r}{10^8} \cdot \frac{d\theta}{d\tau} \text{ VOLTS}$$

and the current (I) as:

$$I = \frac{E}{R} = \frac{B\ell r}{R\, 10^8} \cdot \frac{d\theta}{d\tau} \text{ AMPERES}$$

It follows that:

$$a\frac{d\theta}{d\tau} = \frac{B\ell}{10}\left[\frac{B\ell r}{R\, 10^8} \cdot \frac{d\theta}{d\tau}\right] = \frac{B^2\ell^2 r}{R\, 10^9} \cdot \frac{d\theta}{d\tau} .$$

and $a = B^2\ell^2 r / R\, 10^9 =$ force/unit angular velocity.

The damping ratio (1 for critical damping) is expressed by the symbol η and equals ϵ/ω_0 where $\epsilon = a/2I$, the damping constant and $\omega_0 = \sqrt{k/I}$ the undamped natural frequency. Then $\eta = \frac{\epsilon}{\omega_0} = \frac{a/2I}{\sqrt{k/I}}$ and $a = \eta\sqrt{k/I} \times 2I$.

If the damping applied to an oscillating system is proportional to the velocity, the closest approach to a correct reproduction of the forcing motion is obtained. (For a seismometer this forcing motion is the ground motion). From a study of the dynamic response of such an oscillating system it is found that resonance occurs when the frequency of the ground motion equals the natural frequency of the seismometer. Since resonance is highly undesirable, damping is applied.

Critical damping ($\eta = 1$) adds no particular advantage — it cuts amplitudes, reduces the range of frequencies for which the amplitude is essentially constant, and introduces an additional phase shift — but with η equal to 0.7, resonance is eliminated (Heiland, 1946, p. 588).

Returning to our formula for a and letting η equal 0.7, we have

$$a = 1.4\, I \sqrt{k/I}$$

Substituting $2\pi/T$ for $\sqrt{k/I}$, where T is the natural period, we have

$$a = 2.\dot{8}\,\pi\, I/T$$

the damping torque required for 0.7 critical damping of a system whose moment of inertia is I and whose natural period is T.

The dimensions and characteristics of a three cm. coil and annular magnet are given below.

Field strength $= B = 10{,}000$ gauss

Wire length $= \ell = n\pi d$ where $n =$ no. of turns $= 12$
$d =$ diameter of coil $= 3$ cm.

then $\ell = 113$ cm.

Distance, hinge line to coil center $= r = 15$ cm.

$R = R_c + R_s$ coil resistance + shunt resistance

where $R_c = \rho \ell / A$

Resistivity of copper $= \rho = 1.7 \times 10^{-6}$ ohms-cm.

Area (cross sectional) of wire $= A = \pi r^2 = 1.96 \times 10^{-3}$ cm.^{2}

where $r = 0.0025$ cm. wire radius

$R_c = 0.098$ ohms.

If we short circuit the coil ($R_s = 0$), we find that

$$a = B^2 \ell^2 r / R_c 10^9 = 10^8 \times (113)^2 \times 15 / .098 \times 10^9 = 1.92 \times 10^5 \text{ dynes-cm.}$$

This, then, is the damping torque available from a coil and magnet commonly found as parts of a three cm. loud-speaker.

Design B

If the seismometer of Design B is set for a period of twenty seconds, the damping torque necessary for 0.7 critical damping is given by (a) when I equals 10^5 gm-cm^2 as:

$$a = 2.8 \pi I / T = 2.8 \pi \times 10^5 / 20 = .44 \times 10^5 \text{ dynes-cm.}$$

If R_s is included as a shunt across the coil terminals, the damping torque produced can be made equal to the damping torque desired, namely, $.44 \times 10^5$ dynes-cm.

From the expression for the damping torque we solve for a constant C and evaluate

$$a = \frac{B^2 \ell^2 r}{(R_c + R_s)10^9} \qquad a R_c + a R_s = \frac{B^2 \ell^2 r}{10^9} = C$$

$$R_s = \frac{C - a R_c}{a} \qquad C = \frac{10^8 (113)^2 15}{10^9} = 1.92 \times 10^4$$

where R_s is the resistance to be varied as the period is varied to maintain 0.7 critical damping ($R_c = .098 \cong 0.1\,\Omega$).

With the period set at twenty seconds we have calculated the desired damping torque (a) to be $.44 \times 10^5$ dynes-cm. Solving for R_s we obtain

$$R_s = \frac{1.92 \times 10^4 - .44 \times 10^5 \times .1}{.44 \times 10^5} = 0.336\,\Omega$$

With the period set at five seconds the damping torque required for 0.7 critical damping is:

$$a = 2.8\pi \times 10^5 / 5 = 1.76 \times 10^5 \text{ dynes-cm.}$$

and R_s is given by

$$R_s = \frac{1.92 \times 10^4 - 1.76 \times 10^5 \times .1}{1.76 \times 10^5} = 0.01\,\Omega$$

Therefore we see that the damping may be accurately controlled with some arrangement of variable resistor connected across the damping coil. The range of this variable resistor should be from 0.01 to 0.45 ohms for periods of from five to twenty-five seconds, respectively.

<u>Design A</u>

For a five second period of the seismometer described as Design A the damping torque required for critical damping will be given by

$a = 2.8\pi I/5$ dynes-cm. The moment of inertia (I) of Design A is 5.4×10^6 gm-cm^2 and the required damping torque is 9.5×10^6 dynes-cm. The available damping torque from the three cm. coil outlined above is $.45 \times 10^6$ (at a distance from the hinge line of 35 cm.) and the coil constant ($C = B^2 l^2 r / 10^9$) must be at least $.95 \times 10^6$. To provide this the coil diameter and the number of turns must be increased considerably.

If a magnet and coil arrangement which satisfies the above requirements can be obtained, the magnet should be included as part of the boom mass and the coil mounted on the base plate as in Design B. Because of the rigid requirements for such a damping coil and magnet the conventional vane-type damping is recommended for Design A.

Vane-type damping is provided by the physical fact that a closed conductor moving in a non-homogeneous magnetic field sets up eddy currents within itself and these, in turn, set up an induced magnetic field which opposes the generating field. This is the same principle used in the coil and magnet design but the coil-magnet method can be controlled with more precision through a change in the coil resistance. The vane-type damping torque is dependent mainly on the vane material, the cross sectional area, the speed with which the vane moves relative to the generating field, and the strength of this field. Of these, the strength of the field is generally varied by movement of the magnets, thus controlling the damping (see Plate 1).

MAGNIFICATION

The Galvanometer

The specifications for most commercial galvanometers usually include (1) the sensitivity (in microamperes per mm. or mms. per milliampere), (2) the coil resistance (R_c), (3) the external series resistance for critical damping (R_s when $\eta = 1$), and the natural period (T_g).

Leeds and Northrup (1946, p. 10) list the following characteristics for their type HS, D-C, moving coil, reflecting galvanometers:

	Sensitivity	Period,	Resistance, ohms	
List No.	per mm. at 1 meter	seconds	R_s	R_c
2284-a	0.008 microamps	1.5	40	21
2284-e	0.005 microamps	1.5	1200	300

The Heiland Research Company (1948) manufactures a galvanometer (Type A) which has a listed sensitivity of 800 mm/milliamp at 30 cms., a coil resistance of 35 ohms, a period of 0.025 seconds, and an external damping resistance of 275 ohms at $\eta = 0.65$.

From the formula developed on page 26 we can obtain the constants (C) for the three galvanometers outlined above. The values of the respective constants were calculated to be

Leeds and Northrup #2284-a	C =	61 ohms
Leeds and Northrup #2284-e	C =	1500 "
Heiland Research, Type A	C =	462 "

and R_s at 0.65 critical damping for the Heiland Type A galvanometer is, as previously stated, 275 ohms.

It is suggested that a galvanometer which has a natural undamped period of 1.5 seconds will adequately follow the motion of a seismometer whose period is twenty seconds. Galvanometer damping is taken as seven-tenths critical ($\eta = 0.7$) and the following magnification calculations are obtained from formulae previously developed on page 25. The source of D. C. voltage ($\mathcal{E}$) is suggested as three volts (Neilsen, 1943, p. 107).

Bonded Strain Gages

The total resistance (R) of one type of commercially available strain gage is 1000 ohms (Neilsen, 1943, p. 106) and the resistance change (Δ R) which such a gage undergoes when attached to the hinges of Design A is given on page 21 as 0.003 ohms for a seismometer boom deflection of 5×10^{-4} cm. Using the Leeds and Northrup #2284-a galvanometer, ($C = 61$, $R_g = 21$) we obtain the series resistance necessary for seven-tenths critical damping ($R_{s(.7)}$) from

$$R_{s(.7)} = \frac{C}{0.7} - R_g = \frac{61}{0.7} - 21 = 66\ \Omega$$

Since the equivalent series resistance ($R' = R$) is 1000 ohms we must use a shunting resistance (r_1) as shown in Figure 12. From the formula for this resistance given on page 25 we have

$$r_1 = \frac{R\ R_{s(.7)}}{R - R_{s(.7)}} = \frac{1000 \times 66}{1000 - 66} = 71\ \Omega$$

Then the current flowing in the galvanometer coil is given by

$$i_g = \frac{\mathcal{E}\ \Delta R\ r_1}{R(R+R_g)(r_1 + R_g)} = \frac{3 \times 3 \times 10^{-3} \times 71}{1000(1000+21)(71+21)} = 6.8 \times 10^{-9} \text{ AMPS.}$$

The sensitivity of the Leeds and Northrup #2284-a galvanometer is given as one mm. (at one meter) for a current of 8×10^{-9} amperes. Therefore the galvanometer deflection obtained will be $6.8 \times 10^{-9}/8 \times 10^{-9}$ = 0.085 cm. for a seismometer deflection of 0.0005 cm., or a magnification of 170.

The Strain-Wire Transducer

The total resistance (R) of one six centimeter length of one mil diameter constantan wire was calculated to be 60 ohms and the resistance change (Δ R) for a boom deflection of 5×10^{-4} cm. was found to be 0.00107 ohms (page 22).

The galvanometer constant (C) for the Leeds and Northrup #2284-a galvanometer is 61 ohms and the series resistance necessary for seven-tenths critical damping ($R_{s(.7)}$) will again be 66 ohms. Since the resistance of one arm of the bridge network is 60 ohms (when wires are used), we need only to add 6 ohms in series with the galvanometer to produce seven-tenths critical damping. The current (I) will be given by

$$I = \frac{\mathcal{E}\,\Delta R}{R(R_{s(.7)} + R_g)} = \frac{3 \times 1.07 \times 10^{-3}}{60\,(66+21)} = .61 \times 10^{-6} \text{ Amps.}$$

This will produce a galvanometer deflection of 7.6 cm. for a boom deflection of 5×10^{-4} cm., or a magnification of 15,300.

Using the Leeds and Northrup galvanometer #2284-e (C = 1500) we find $R_{s(.7)}$ to be 1500/0.7 - 300 = 1840 ohms, and this series resistance may be obtained by adding 1780 ohms (1840-60) in series with the galvanometer. The current (I) will be

$$I = \frac{\mathcal{E}\,\Delta R}{R(R_{s(.7)} + R_g)} = \frac{3 \times 1.07 \times 10^{-3}}{60(1840+300)} = .025 \times 10^{-6} \text{ Amps.}$$

Such a current will produce a galvanometer deflection of 0.5 cm. for a boom deflection of 5×10^{-4} cm. (the sensitivity of #2284-e is 0.005 microamps per mm. at one meter), or a magnification of 1000.

Similar calculations for the Heiland Research Type A galvanometer arrive at a calculated magnification of 27 when the galvanometer is 0.65 critically damped ($R_{s(.65)} = 275\,\Omega$) and 215 ohms are added in series with the galvanometer. This magnification is based on a 30 cm. distance from the galvanometer mirror and would be approximately three times this amount at one meter, or approximately 85.

It is obvious that the Heiland Research Type A galvanometer will not be applicable to our case and the Leeds and Northrup #2284-e does not provide a magnification of more than 1000. It is therefore recommended that the Leeds and Northrup #2284-a galvanometer be used. By including two variable resistors in the bridge circuit, (1) in series with the galvanometer and (2) shunted across the galvanometer, the magnification may be varied from 15,000 to 1000. The larger value is obtained when the series resistance (1, above) is 6 ohms and the shunt (2, above) is zero. The 1000 magnification value is obtained when the series resistance is approximately 85 ohms and the shunt resistance is 187 ohms. Therefore the magnification is adjustable to an accuracy dependent on the characteristics of the variable resistors used.

<u>Note</u>: The above calculations hold for both Design A and Design B within a very close approximation since the wires are of the same dimension and are mounted in an identical manner. The strain (ϵ/ℓ) (see page 22) will be slightly less in Design A due to the increased effective length of the boom ($L = 24.8$ cm. instead of 15 cm. as in Design B) and hence the magnification from Design A for any galvanometer will be slightly less than that from Design B.

Temperature Effects

Since the four sets of strain-wires are connected in a bridge circuit, changes in temperature will affect all the arms in a similar manner and will not cause deflections of the galvanometer. When the wires are mounted on their fused quartz posts, the original tensions applied by the manufacturer will, in all probability, not be the same. However, when the boom is centered, the galvanometer may be adjusted to "zero" and subsequent deflections of the boom should be faithfully reproduced by the galvanometer. Another method of correcting for these variations in tension would be to include small variable "trimmer" resistances in each arm of the bridge in series with the strain-wires. These could then be adjusted to obtain a balanced bridge.

The spring tension of Design B will not be affected by normal temperature changes. Elinvar has a temperature coefficient of expansion of 4×10^{-6} cm/deg. C. and a ten deg. C. decrease in temperature would cause the springs (whose approximate straight-wire lengths are $30 \times \pi \times D$ where D is the coil diameter, 0.4 cms.) to shorten an amount
$4 \times 10^{-6} \times 10 \times 30 \times 0.4 = 0.0015$ cms. This change in spring length (f, page 16) would produce an increase in the period of Design B of approximately 0.02 seconds. Temperature can easily be controlled in a vault within $\pm$ 10 deg. C.

THE NORMAL RECORDING SYSTEM

Most seismograph stations employ photographic film or paper which is carried on a drum whose speed is governed very accurately. A light beam is caused to fall on the galvanometer mirror and reflected through a horizontal plano-cylindrical lens which focuses the beam on the recording film or paper on the drum. For long period seismometer recording a speed of 30 mm. per minute has been found satisfactory (Robertson, 1948, p. 31). The drum, very carefully balanced on its axis, is usually mounted on a shaft with helical thread and moves in translation at a rate depending on the pitch of the screw-shaft. Most of these drums are electrically driven and governed and provision must be made for a constant 60 cycle, 110 volt power supply.

Since seismograms are in reality graphs of earthquake motion versus time, accurate timing devices must be provided for. In most photographic recording methods, a prism is actuated by a relay connected to the timing clock and this movement deflects the light beam by a small amount, thus making time marks on the record. When drums rotate with a speed of 30 mm. per minute the marks are placed at intervals of approximately one minute.

The timing clock is of great importance and must have a rather small but constant rate. This clock is checked in most seismograph installations by the U. S. Naval Observatory time broadcasts. Various arrangements are used to transmit these broadcasts directly to the record.

The type of earthquake phenomenon that is to be studied has as much to do with the precision and characteristics of the recording devices as it does with the seismometer design. It is not within the scope of this work to discuss all these various phenomena, such as, microseismic recording, local shocks, and the like. The generalisations given above apply to a long period horizontal seismometer such as has been designed and to the types of earth tremors it will register best, namely, horizontal motion caused by earthquakes which originate at a distance of more than 750 miles.

A seismograph station will generally contain at least three seismometers: two long period horizontal for EW and NS motion and one short period vertical. (Periods of less than ten seconds are classed as short periods). A single drum recorder which will record the motion of all three simultaneously has recently been developed (Sprengnether, 1946, p. 85).

THE REMOTE RECORDING SYSTEM

Due to the geologic characteristics of the area in the immediate vicinity of the Colorado School of Mines, a seismograph station would have to be placed west of the school on the igneous formations above the crystalline contact, perhaps 3/4 mile from the Geophysics Laboratories. For this reason it is desirable to investigate methods of remote recording so that the seismometer or seismometers could be placed in a proper vault on piers sunk into the igneous formation and the actual recording could be done in the laboratories or offices on the campus. Pen and ink recorders would also be convenient so that the records could be observed as they were made. Photographic recorders of greater magnification could also be installed in the seismic vault and started whenever more accurate recording was desired. If two such recording channels were used on these designs it would necessitate certain changes in the strain-wire arrangements which could be worked out at that time. For the present we are interested merely in outlining some general descriptions of circuits that would facilitate remote recording from the resistance changes of our unbalanced bridge circuit.

If a carrier wave were introduced by a standard oscillator across the bridge that contains the strain-sensitive wires, and the galvanometer were replaced by a suitable amplifier, the carrier would be modulated by the unbalanced bridge (when ground motion occurs) and the amplifier would impress the signal over a set of wires to any desired remote recorder. A schematic diagram of this circuit is shown in Figure 13.

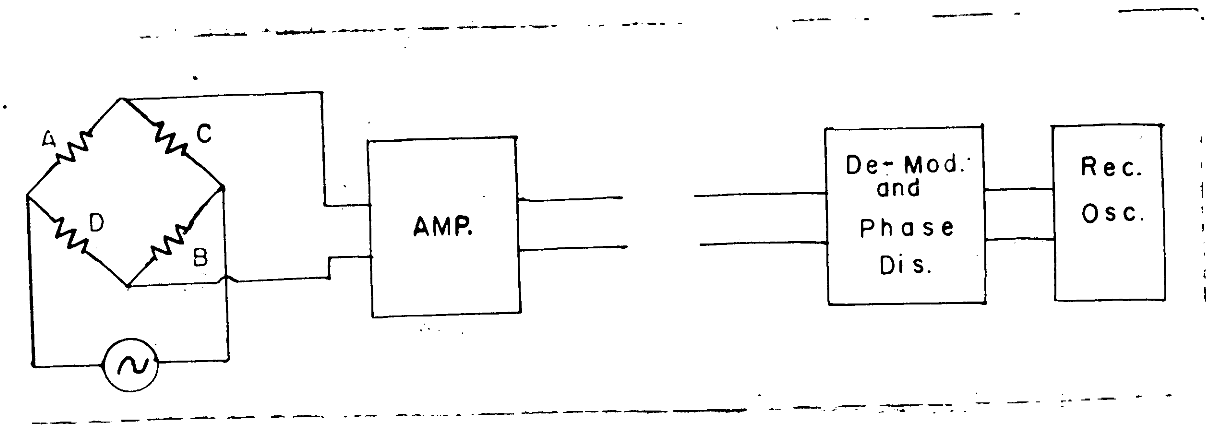

Figure 13

At the recording end of the circuit a demodulator circuit is introduced which operates on the signal so that the original modulating wave is reproduced. This part of the circuit should also contain a phase discriminator network which would assist in faithfully reproducing the modulating wave set up at the seismometer by assuring the recording of the negative deflections as such. This signal is then fed to a recording oscillograph of perhaps the magnetic direct writing type.

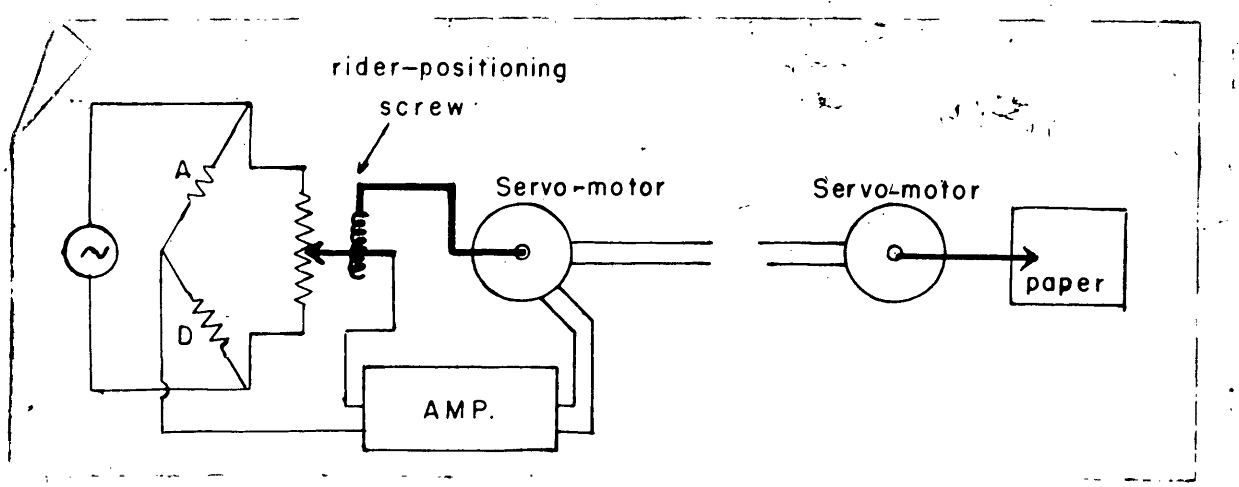

Figure 14

(Mechanical coupling is shown in heavy lines)

Figure 14 illustrates another method of remote recording. All the strain wires on each side of the boom are connected in series and placed in the circuit at A and D, respectively. The remainder of the bridge circuit consists of a fine slide wire resistor. An oscillator is connected across one corner of the bridge and the other bridge connection is made to an amplifier through the variable resistor. The amplifier is connected to a servo-motor and any unbalance in the bridge will cause amplified current to flow in the servo-motor. The rotor of this motor is connected mechanically to a screw or a gear train which moves the adjustable contact on the slide wire until the bridge is balanced. If a slave servo-motor at the remote recording location is connected to the first (thus producing a so-called Selsyn system) its rotor will follow the action of the first. A system of levers can be applied so that this motion moves a pen on a moving sheet of paper. One disadvantage of this method is the fact that the stators of the two servo-motors are composed of three-phase windings and three additional leads must be carried along with the current leads, making a five element cable necessary from vault to recorder.

The Brush Development Company of Cleveland, Ohio, recently advertised a "Strain Analyzer" (Instruments, Feb. 1949) which consists of a strain amplifier and a direct inking oscillograph. It has external connections for two strain gages, one active and one "dummy." The "dummy" gage is used in strain recording to compensate for temperature and does not undergo strain. However, it is believed (since detailed specifications were not available) that with some adjustment the two strain-wire arms employed in the Selsyn circuit could be used, as most strain amplifiers work on the principle of an unbalanced bridge. The gages that are recommended for use with this analyzer are of the Baldwin Southwark SR-4 type. These each have a total resistance of 120 ohms. The constantan wires of our design, if all four on each side of the boom were connected in series, will also have a total resistance of approximately 120 ohms. The sensitivity is given as 10^{-5} cm/cm. strain per chart division pen deflection. Due to the fact that the dimensions of the chart divisions are not known, and that the feasibility of using the direct inking oscillograph at distance from the strain amplifier is also in doubt, we may only postulate the possible use of this instrument for remote recording. It is interesting to note, however, that the strain experienced by four, three-cm. lengths of constantan wire (as mounted in Design B) will be 0.9×10^{-5} cm/cm. for a boom deflection of 5 microns. If the chart divisions mentioned above were 1 mm. each, we should obtain a magnification of 200; 5 mm. chart divisions would give 1000 magnification.

CONCLUSIONS

As is evident from the magnification calculations contained in the previous section, bonded strain gages attached to the hinges will give a magnification of only 170 times. This will not be sufficient to adequately record small earth tremors ($170 \times 5 \times 10^{-4}$ cm. = 0.85 mm.) for visual study. Therefore the strain-wire transducer principle is recommended. With this type transducer, a five micron displacement of the support due to earth motion can be magnified up to 15,000 times if desired. (This magnification is possible using the Leeds and Northrup galvanometer #2284-a when it is seven-tenths critically damped).

Design B has advantages over Design A. The mass of the boom in Design B is only about one-twentieth of the mass of the boom required in Design A. The period and the damping of Design B may be controlled with greater finesse. Boom displacement for calibration purposes may be provided with greater ease in Design B. Further, the shorter boom length of Design B produces more strain in the transducwr and hence more magnification, although this is of minor importance.

However, Design B requires two springs whose constants must be as close to equal as possible and also requires a coil mounting which will facilitate the proper placement of the coil relative to the magnetic field which contains the coil. The clearance between coil and magnet in a standard three-cm. annular magnet is fairly small.

It is therefore concluded that while Design A is relatively simple to build as compared with Design B, Design B is suggested as the more precise and convenient to operate. The factors of precision and convenience of operation should make it worth while to undertake the precision of manufacture demanded by Design B.

<u>Note</u>: Construction suggestions and explanations of minor details regarding both designs as shown on Plates 1 and 2 are included as pages 2 to 4 in the Appendix.

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APPENDIX

Force Produced by a Spring Hinge

The elastic curve equation in rectangular coordinates is $M = E\,I\,d^2z/dx^2$ where M is the bending moment of the section whose distance from the coordinates is x, z is the deflection of the elastic curve (the hinge) at the same section, I is the second moment of the cross section of the hinge perpendicular to a neutral axis, and E is Young's modulus (ℓ is the length of the hinge and P is the applied force). M is also equal to $P(\ell - x)$ and therefore:

$$E I \, d^2z/dx^2 = P(\ell - x)$$

$$E I \, dz/dx = P\ell x - Px^2/2 + C_1$$

$$\left.\begin{matrix} \frac{dz}{dx} = 0 \\ x = 0 \end{matrix}\right\| \quad \therefore \quad C_1 = 0$$

$$E I \, z = P\ell x^2/2 - Px^3/6 + C_2$$

$$\left.\begin{matrix} z = 0 \\ x = 0 \end{matrix}\right\| \quad \therefore \quad C_2 = 0$$

$$\left.\begin{matrix} \text{AT } x = \ell \\ z = z_\ell \end{matrix}\right\| \therefore \quad E I z_\ell = P\ell^3/3 \quad ; \quad P = 3 E I z_\ell/\ell^3$$

$$\text{ALSO:} \quad \frac{dz}{dx} = \frac{dz_\ell}{d\ell} = P\ell^2/2EI \quad \text{AT } x = \ell ;\ z = z_\ell$$

The final value of $dz/d\ell$ is substituted on page 7 for dz/dx because the strain is being calculated at $x = \ell$.

Explanations of Design A: Plate 1

The upright support shown at the left may have to be enlarged to effectively support a boom of the size and weight suggested in this design. The dimensions of this member as shown on Plate 1 were arrived at mainly from the limiting factor of the space available on the original drafting plate.

The recessed area of both boom and support in the vicinity of the strain-wires should be milled to one cm. in thickness at the most. Both the boom and support are 2 cm. thick near the hinge line but the total distance from the boom-center to the attached wires should be 0.8 cm.; therefore the smaller dimensions for the thickness of these sections.

The magnets which surround the damping vane are welded to their movable and their stationary supports, respectively. The supporting frame might also be grooved to accommodate more movement of the damping vane to the right (see end view, Plate 1). Displacements of the magnitude allowed for in the present drawing will seldom occur from ground motion but the limit stops are designed to restrict the desired maximum motion and the damping vane should not be used for this purpose; therefore the suggested slot in the magnet-supporting frame.

The leveling screw taps in the base plate should be fitted with internal collars which contain the threads and these, in turn, might be slotted and set screws provided for, properly clamping the leveling screw adjustments at any desired position.

The boom can be centered in the x,z plane by the use of an anti-parallax optical system placed on the centerline of the boom (not shown).

Design A would best be calibrated by drilling the limit stops for an air jet and producing the displacement desired for calibration by means of a small, controlled blast of air.

The boom is adjusted to the proper period by means of the bubble level and subsequent calibration. The damping is controlled by movement of the damping magnet.

Explanations of Design B: Plate 2

The boom support shown on Plate 2 need not be solid as shown. The only requirements are that the spring adjustments be equidistant from the hinge line and that a passage for the lower spring is provided through the boom supports. The main boom support can be flanged and bolted to the upper base plate for ease in construction (in the same manner as the small support section which holds the lower hinge).

Additional holes are provided in the main boom support for moving the spring adjustment mechanisms closer to the hinge line. This might be necessary if springs cannot be obtained which have the comparatively low spring constant recommended on page 16. The support attachment of each spring consists of a post and block which slides on a track and is adjusted by a micrometer threaded shaft.

Due to the difference in thickness between the support and the boom, the clamping bar (suggested to facilitate the moving of the entire seismometer) is shown twice as thick where it comes in contact with the boom.

The stationary damping coil should be wound on either a thin cylinder of bakelite or on an anodised cylinder of aluminum and this held to a stationary support with a clamping screw which is free to move in an oversized hole when loosened. Four set screws are provided for lateral and rotational adjustment of the damping coil. Binding posts for the coil leads are shown and calibration can be accomplished by giving the boom a displacement by the simple procedure of discharging a condenser of appropriate size through the coil windings.

The area of the boom and main support in the neighborhood of the strain-wires is again recessed to a thickness of one cm. The same suggestions as given under Design A again apply to the leveling screws (with collars) and the leveling and centering arrangements. The damping magnet is welded to the connecting block which is screwed to the rest of the boom.

The boom is adjusted to the proper period by first leveling the base plate and then adjusting the tension of the springs. The damping is controlled by a series variable resistor (not shown) and both period and damping are checked by subsequent calibration.

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PLATE I

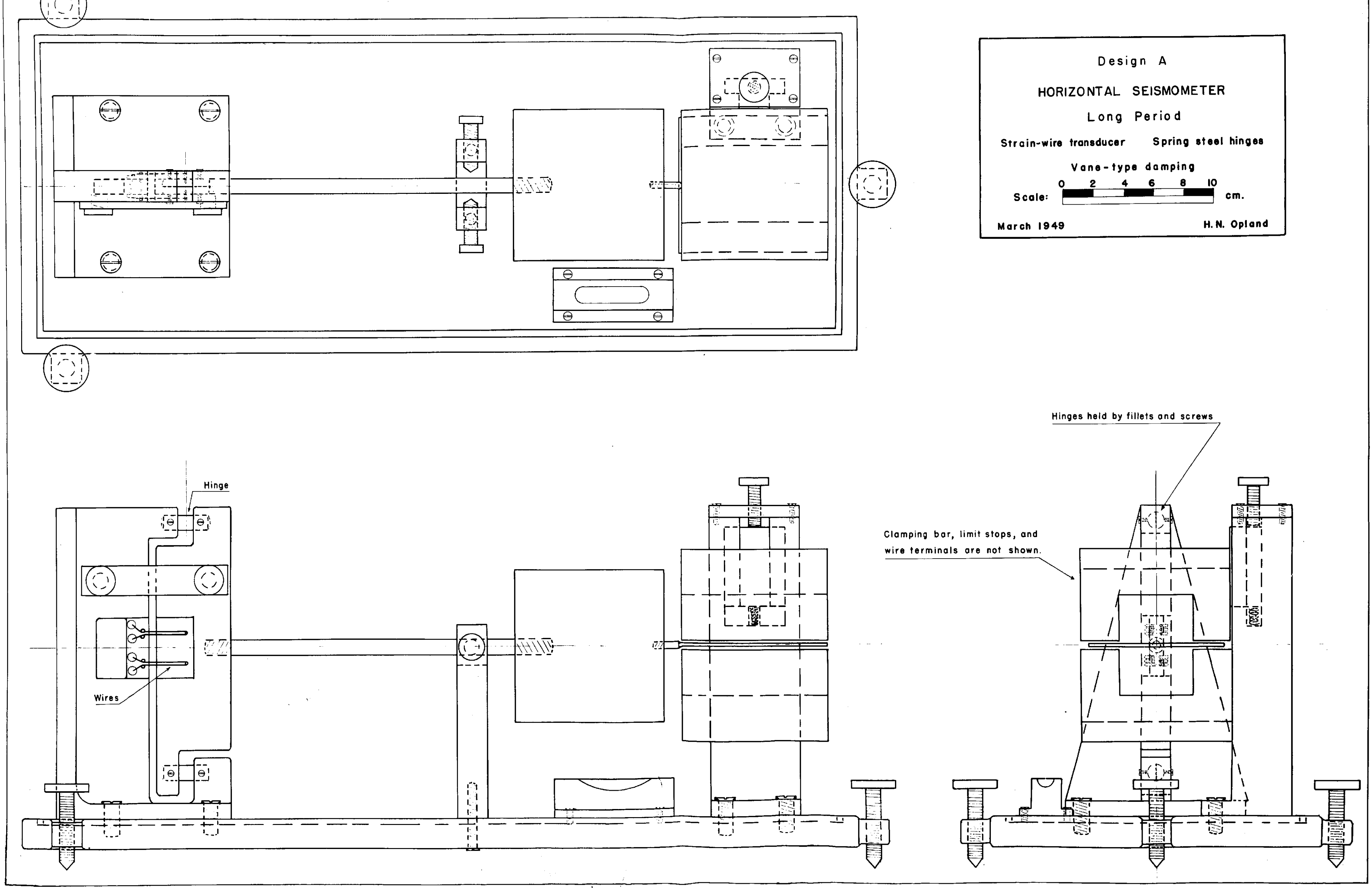
Design A
HORIZONTAL SEISMOMETER
Long Period
Strain-wire transducer
Spring steel hinges
Vane-type damping
Scale:
0 2 4 6 8 10
cm.
March 1949
H. N. Opland
Hinges held by fillets and screws
Clamping bar, limit stops, and wire terminals are not shown.
Hinge
Wires

PLATE 2

Design B

HORIZONTAL SEISMOMETER

Long Period

Strain-wire transducer Spring period control

Coil and magnet damping

Scale: 0 2 4 6 8 10 cm.

March 1949 H. N. Opland