T-927

CALCULATION OF THE GRAVIMETRIC EFFECTS OF TOPOGRAPHY AND ISOSTATIC COMPENSATION WITH AN ELECTRONIC DIGITAL COMPUTER

by

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T-927

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INTRODUCTION

The method in common use for calculating the gravimetric terrain and isostatic compensation corrections was introduced by Hayford about 1910. In principle the method is a graphical integration of the effect at a gravity station of all the irregularities in the elevation of the land and sea bottoms over the entire surface of the earth. The surface of the earth with the station as the center is divided into circular zones and the zones in turn into compartments. Hayford used fifteen zones from the station out to 166.7 kilometers, which he identified by the letters A to O; and 18 zones from 166.7 kilometers to the antipodes of the station. The latter he identified by numbers 18 to 1. The zones between the station and 166.7 kilometers are usually referred to as the lettered zones and those beyond as the numbered zones. The effect of each compartment is found by estimating the mean elevation of the land surface within its boundaries and looking up in a set of tables the corresponding effect of topography and compensation. The accuracy of the method thus depends on the skill

of the computer in correctly estimating the mean elevations of all the compartments. The process is tedious and time consuming. This difficulty is especially true for stations located in mountainous country where extra care must be taken.

In recent years, a number of investigators have used electronic digital computers to reduce the amount of work and time required in the calculations. For example, Heiskanen (1953) has discussed their use in computing the effect of isostatic compensation; and Bott (1958) has described a program he developed for computing terrain corrections.

The method described by Bott involves the substitution of a square grid for circular zones and compartments; and the calculation of the effect of the mass underlying each square as if it were concentrated along a vertical line at the center of the square. The squares are made small enough compared to their distances from the station so that sufficient accuracy is retained. This technique has been called the mass-line method by Heiskanen (1958).

A fast, reliable method of computing the effect of topography and compensation is needed particularly for gravity surveys now being done by students of the Colorado School of Mines Geophysics Department in the mountains of Colorado. Here where large changes in elevation may occur within a compartment, the accuracy of Hayford's method rests too heavily upon the judgment of the inexperienced student computer. Also, the time spent by the student in making the corrections is out of all proportion to the reward in knowledge and

experience gained.

It was, in fact, the tiresome experience with Hayford's method which stimulated an interest in the problem of devising a computing program for the LGP-30, the electronic digital computer installed in the Colorado School of Mines mathematics building. Work on the problem was begun in the summer of 1959, and writing and testing a workable program was finally completed in the late fall of 1960.

GENERAL PLAN OF ATTACK

The program was planned with a particular area in mind, namely, that part of Central Colorado for which a complete set of $7\frac{1}{2}$-minute U. S. Geological Survey quadrangle sheets were available. This area is enclosed by the parallels of latitude 38°52'30" N and 40°00'00" N, and meridians 105° W and 106° W. These maps have a scale of 1:24,000. Available also were Army Map Service maps having a scale of 1:250,000. The latter were used to obtain elevations needed for the zones reaching out to 166.71 kms beyond the area covered by the $7\frac{1}{2}$-minute quadrangle sheets.

Bott's use of square grid with the mass-line method of computing the gravimetric effects seemed to offer the simplest approach to the mathematical formulation of the problem. Formulas for the attraction of a line mass are easy to derive and are reasonably simple in form.

It was unnecessary to consider the problem of writing a program for the numbered zones; since Qureshy's interpolation map (1958) for the State of Colorado gives these corrections with

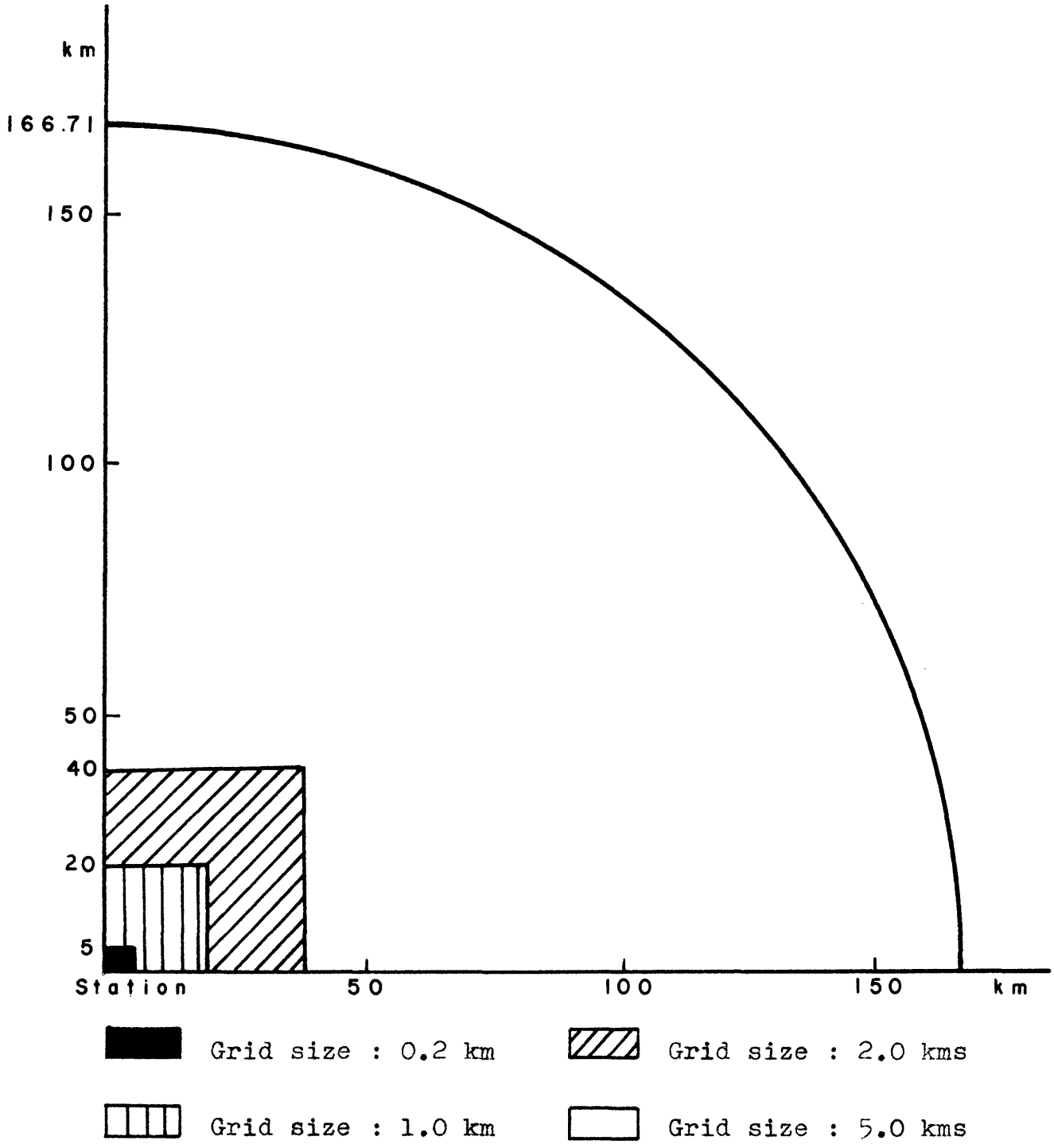


Figure 1 . Areas covered by different-size grids on one quadrangle

sufficient accuracy. The problem chosen was the lessor but still difficult one (as it turned out) of devising a program that would compute the separate corrections for topography and isostatic compensation in one series of operations for the area of the lettered zones. Hayford's division into fifteen lettered zones was not used. Instead, the circular area of Hayford's lettered zones was divided into the four areas shown in figure 1.

The outer area or zone was bounded on the outside by a circle of radius 166.71 kms, and on the inside by a square measuring 80 kms on a side. The remaining three inner zones were bounded by squares with outside dimensions of 80, 40, and 10 kms, respectively. The grid size chosen for each zone was 5, 2, 1, and 0.2 kms, respectively, beginning with the outside zone. Two programs were written, one for the innermost zone and one for the remaining three outer zones.

In deriving the formulas for the three outermost zones, the earth's curvature was taken into account; it was ignored in the derivation of the formula for the zone containing the station, since the curvature effect within 7 kilometers of the station is very small. In what follows the four zones will be identified by the Roman numerals I, II, III, IV, starting at the outer zone and counting inward toward the station.

The Airy-Heiskanen system of isostatic compensation was chosen; with the thickness of the earth's crust at sea level T=30 kms. This figure, the value used by Qureshy in his isostatic reduction

of Holmer's Colorado gravity data, was considered desirable for use in the same system in future additions to the Colorado data.

DERIVATIONS OF THE FORMULAS

Zone IV

In figure 2 the (x, y) plane is at sea level. The part of the prism above ABCD contains the mass between sea level and the surface of the ground at Q. This mass is compensated by the mass at the bottom of the prism whose thickness is t and whose depth is T. These two masses are regarded to be concentrated along the two line segments at the center of the prism. The coordinates of the station where the effects of the two line masses are to be computed are S(x , y, e).

The vertical component of the attraction A_z at the point S(x, y, e) of the line mass is given by

$$A_z = k \rho \Delta u \ \Delta v \int_a^b (e-w)dw \ / \ \left[(u-x)^2 + (y-v)^2 + (e-w)^2\right]^{\frac{3}{2}}$$

and this becomes, when the integration is carried out,

$$A_z = k \rho \Delta u \ \Delta v \ \Big| \ 1/\left[(x-u)^2 + (y-v)^2 + (e-w)^2 \right]^{\frac{1}{2}} \ \Big|_a^b .$$

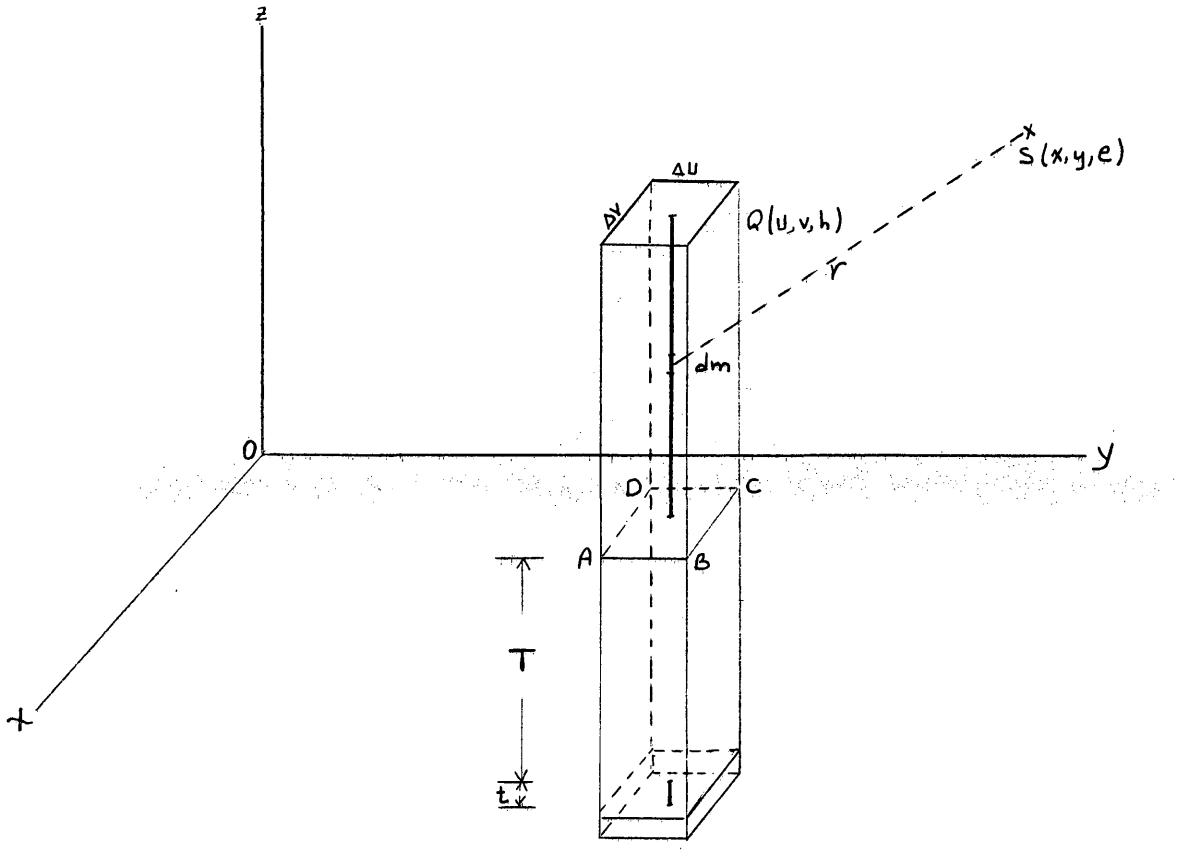


Figure 2. Mass-line method for computing the corrections without considering the effect of the earth's curvature.

Setting the upper limit b=-T and the lower limit a=-(T+t)

this gives for the effect of compensation,

$$A_c = k\rho\,\Delta u\,\Delta v\left[\,1/\left[(x-u)^2+(y-v)^2+(e+T)^2\right]^{\frac{1}{2}} - 1/\left[(x-u)^2+(y-v)^2+(e+T+t)^2\right]^{\frac{1}{2}}\right]$$

Two cases have to be considered for the topographic correction A_b.

The first is the case where the elevation of the grid point Q(u, v, h) is less than the elevation of the station S(x, y, e); that is, when $e > h$. Here we put the upper limit b=h and the lower limit a=0 to get

$$A_b = k\rho\,\Delta u\,\Delta v\left[1/\left[(x-u)^2 + (y-v)^2 + (e-h)^2\right]^{\frac{1}{2}} - 1/\left[(x-u)^2 + (y-v)^2 + e^2\right]^{\frac{1}{2}}\right]$$

The second case arises when the elevation of the grid point Q(u, v, h) is greater than the elevation of the station S(x, y, e); that is, when $e < h$. That part of the mass above the station would have an effect opposite in sign to that part of the mass below the station. For this case A_b is given by

$$A_b = k\rho\,\Delta v\,\Delta u\left[2/\left[(x-u)^2 + (y-v)^2\right]^{\frac{1}{2}} - 1/\left[(x-u)^2 + (y-v)^2 + (e-h)^2\right]^{\frac{1}{2}} - 1/\left[(x-u)^2 + (y-v)^2 + e^2\right]^{\frac{1}{2}}\right]$$

For the terrain correction only, the upper limit is b=e, and the lower limit is a=h. This yields

$$A_t = k\rho\,\Delta v\,\Delta u\left[1/\left[(x-u)^2 + (y-v)^2\right]^{\frac{1}{2}} - 1/\left[(x-u)^2 + (y-v)^2 + (e-h)^2\right]^{\frac{1}{2}}\right].$$

This formula was used to check the accuracy of Bott's approximation, which is

$$A_t = 1/2\,k\rho\Delta u\,\Delta v\left[(e-h)^2 / \left[(x-u)^2 + (y-v)^2\right]^{3/2}\right].$$

In the preceding formulas

k is the constant of gravitation,
ρ is the volume density of the mass,
$\Delta u\,\Delta v$ is the area of the grid,
T is the sea-level thickness of the earth's crust,
t is the thickness of the compensating layer,
e is the elevation of the station S(x, y, e),
h is the elevation of the grid point Q(u, v, h).

Zones I, II, and III

With reference to figure 3, the component of the attraction

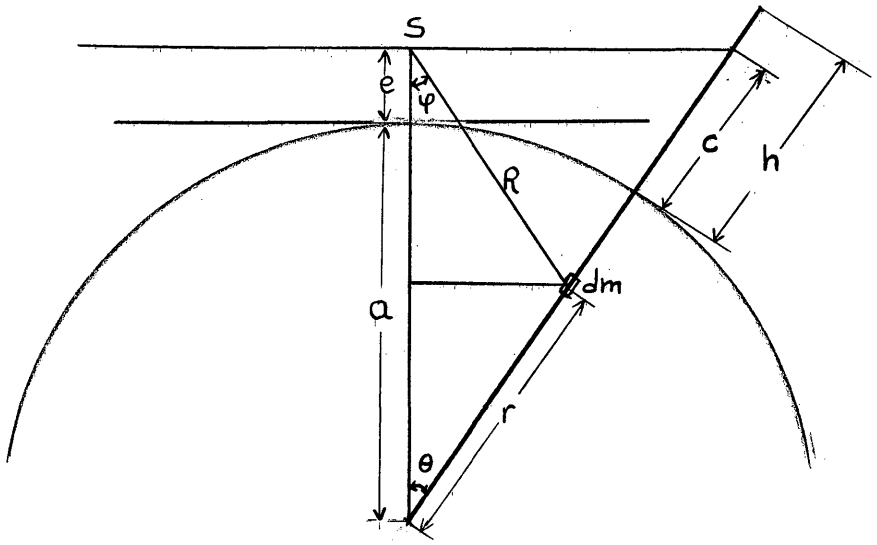


Figure 3. Mass-line method for computing the corrections with the effect of the earth's curvature.

in the radial direction A_ρ at the point S of the line mass element dm is given by

$$dA_\rho = k\lambda \cos\varphi \, dr/R^2$$
$$= k\lambda \left[(b-r\cos\Theta)\, dr/(r^2+b^2-2br\cos\Theta)^{3/2}\right],$$

where λ is the mass per unit length of the line and b=a+e. The effect A_ρ of that part of the line between r_1 and r_2 is

$$A_\rho = k\lambda \int_{r_1}^{r_2} (b-r\cos\Theta)\, dr/(r^2-2br\cos\Theta+b^2)^{3/2}.$$

Setting r=m+bcosΘ, this becomes

$$A_\rho = k\,\lambda\, b\sin^2\theta \int_{r_1}^{r_2} dm/(m^2+b^2\sin^2\theta)^{3/2} - k\,\lambda\cos\theta \int_{r_1}^{r_2} m\,dm/(m^2+b^2\sin^2\theta)^{3/2},$$

where $r_2 = m_2 + b\cos\theta$ and $r_1 = m_1 + b\cos\theta$. Upon carrying out the integration and reverting to the original symbols, the last equation may be written in the form

$$A_\rho = k\,\lambda/b \left[r_2 / (r_2^2 - 2br_2\cos\theta + b^2)^{\frac{1}{2}} - r_1 / (r_1^2 - 2br_1\cos\theta + b^2)^{\frac{1}{2}} \right].$$

To use this formula for the effect of compensation A_c, set $r_2 = a-T$ and $r_1 = a-T-t$ and the result is

$$A_c = k\lambda_2 / a+e \left[a-T / \left[(a-T)^2-2(a-T)(a+e)\cos\theta+(a+e)^2\right]^{\frac{1}{2}} - a-T-t / \left[(a-T-t)^2-2(a-T-t)(a+e)\cos\theta+(a+e)^2\right]^{\frac{1}{2}} \right].$$

Here a is the average value of the earth's radius, and λ_2 is mass per unit length of the compensating mass. The value used for a was 6,371.2 kms and for λ_2, the value was 0.6 $\Delta u\ \Delta v$.

For the topographic corrections, there are two cases: $c > h$ and $c < h$. When $c > h$, the upper limit is $r_2 = a+h$ and the lower limit $r_1 = a$. The topographic correction A_b is then given by

$$A_b = k\,\lambda_1 /(a+e) \left[a+h / \left[(a+h)^2-2(a+h)(a+e)\cos\theta+(a+e)^2\right]^{\frac{1}{2}} - a/\left[a^2-2a(a+e)\cos\theta+(a+e)^2\right]^{\frac{1}{2}} \right],$$

the value used for λ_1 was 2.67 $\Delta u\ \Delta v$.

When $c < h$, that part of the mass line which extends beyond c will produce a negative effect at the station S, so that A_b must be

written

$$A_b = k\,\lambda_1/b\,\left| r/\,(r^2-2br\cos\theta+b^2)^{\frac{1}{2}}\right|_a^{a+c} - k\,\lambda_1/b\left| r/\,(r^2-2br\cos\theta+b^2)^{\frac{1}{2}}\right|_{a+c}^{a+h}$$

Substituting these limits for r, the expression for A_b may be written

$$A_b = k\,\lambda_1/(a+e)\left\{ 2\left[a+c\,/\,[(a+c)^2+(a+e)^2-2(a+c)(a+e)\cos\theta\,]^{\frac{1}{2}} - a\,/\,[a^2+(a+e)^2-2a(a+e)\cos\theta]^{\frac{1}{2}}\right] - \left[a+h\,/\,[(a+h)^2+(a+e)^2-2(a+h)(a+e)\cos\theta\,]^{\frac{1}{2}} - a\,/\,[a^2+(a+e)^2-2a(a+e)\cos\theta\,]^{\frac{1}{2}}\right]\right\}.$$

To calculate the thickness of the compensating layer, a formula given by Heiskanen (1938) was used. It is

$$t = \lambda h\left[1 + [2T+(1+\lambda)h]/a + (2T+\lambda h)\,[2T+(1+\lambda)h\,]/a^2 - T(T+\lambda h)\,/a^2 - (\lambda^2-1)h^2\,/\,3a^2\right].$$

Setting λ=4.45, Heiskanen value, T=30 kms, and a=6371.2 kms, this formula may be put into a form better suited for computing, namely,

$$t=\left[(Ah+b)h + C\right] h,$$

where A=0.00000197, B=0.00385707, and C=4.49220336.

Values of the constants occurring in the various formulas which were used in the calculations are re-stated as follows:

k is equal to $6.67\ 10^{-8}$ cm^3 gm^{-1} sec^{-2},
ρ is equal to 2.67 gm cm^{-3},
$\Delta u\,\Delta v$ is equal to the area of grid square in cm^2,
T is equal to 30 kms,
a is equal to 6,371.2 kms,
λ_2 is equal to $0.6\,\Delta u\,\Delta v$ gm cm^{-1},
λ_1 is equal to $2.67\,\Delta u\,\Delta v$ gm cm^{-1}.
t = Kms
h = Kms.

APPLICATION

Estimating Elevations of Grid Points

Elevations for zone I were estimated from the 1:250,000 Army Map Service maps at intervals of 5 kilometers along east-west profiles. The Transverse Mercator plane coordinate system given on these maps was used to assign plane coordinates (u, v) to each grid point. The coordinates actually given on the maps were used. The area covered by the grid is bounded on the west and east sides respectively by the lines u=250 kms and u=680 kms; and on the north and south sides respectively by the lines v=4,600 kms and v=4,100 kms. There were 101 east-west profiles, each 430 kms long. The elevations on each profile were punched on tape and identified by the coordinates of the first station on the west end of the profile.

Elevations for zone II were estimated at two-kilometer intervals from the same maps. A total of 131 profiles was put on tape, within an area bounded on the west and east respectively by the lines u=370 kms and u=630 kms; and on the north and south respectively by the lines

v=4,470 kms and v=4,210 kms.

Elevations for zones III and IV were taken from U. S. Geological Survey 7½-minute quadrangle sheets. The coordinate system was the same as that given on the Army Map Service maps. For zone III, 69 profiles on a 1-km grid were picked. These profiles covered an area bounded by the lines u=447 - 500 kms and v=4411 - 4,373 kms. A 200-meter grid was used for zone IV, and 121 profiles bounded by the lines u=463-484 kms and v=4,419 - 4,395 kms were picked for this zone.

The total number of points at which elevations were estimated was 42,500. All elevations were punched on tape, a single profile on each tape.

Data Storage

The first problem that had to be solved in writing the computing program was that of finding an efficient method of making available the elevation data. Zone I, for example, required 68 profiles with a total of 5,916 grid points. These grid points with two identifying coordinates on each profile meant a total of 6,052 data words needed for computing the gravimetric effects for this one zone alone.

The memory drum of the LGP-30 has a capacity of 4,096 words, and since 14 tracks or 896 sectors were needed for storing the program and necessary sub-routines, there were only 50 tracks or 3,200 sectors available for data storage. This limitation meant that either the data would have to be fed into memory in a piecemeal fashion when

needed or two data words would have to be stored in one memory location. For zone I the last-mentioned method was used and for zones II, III, and IV, the first.

The data on one profile in zone I needed 45 memory locations; so that with 3,200 memory locations available for data, 71 profiles could be stored. Thus enough data could be stored to compute the gravitational effects at stations lying within an east-west band 15 kms wide.

The first and second words on each profile tape were the u and v coordinates of the first grid point on the profile going from west to east. The total number of grid points on the profile depended on the zone and grid spacing. There were 87 grid points on the 5-km spaced profiles for zone I. The profile tapes for zone I contained, therefore, 89 data words. When these data words were stored on the memory drum, only 45 locations were required per profile, since two words were placed in one location, the 45th location containing, of course, only one word.

Zone	Boundary in km		Grid size in km	Number of	
	Outer	Inner		Profile	Grid Point
I	166.71	80	5	68	87x68=5,916
II	80	40	2	48	50x48=2,400
III	40	10	1	55	56x55=3,080
IV	10	0.2	0.2	121	106x121=12,826

Table 1. Showing boundaries, number of profiles, and points on each zone.

The Program

The program and a flow chart, which gives the function of the various parts of the program, appear at the end of this report. The flow chart contains enough detail to enable one to read through the program with an understanding of the purpose of each group of instructions but has rather too much detail for the main structure to stand out clearly.

The function of the program can be visualized with the help of figure 4. This figure shows a circle which represents the outer boundary of zone I, which has a radius of 166.71 km. The square inside represents the inner boundary of the zone. Its sides measure 80 km. The horizontal lines are profiles crossing the zone. The profiles are 5 km apart; at 5-km intervals on each profile elevations have been read and stored on the memory drum. Although only 48 profiles are shown crossing the zone in the figure, actually there were 68; 20 have been omitted for clarity.

A vertical line mass is located at each 5-km grid point on each profile. The program was written to calculate the gravimetric effect at the center of the circle, of the line mass at each grid point which falls inside the circle and outside the square. The effect of each line mass was added to that of all the rest, and the sum printed out.

When a data word was called up from memory, its coordinates were tested to ascertain whether the point fell within the zone or outside it. If the point fell within the circle and outside the square, the effect of the line mass was calculated; if it fell outside the circle or within the square, it was rejected and the next point tested.

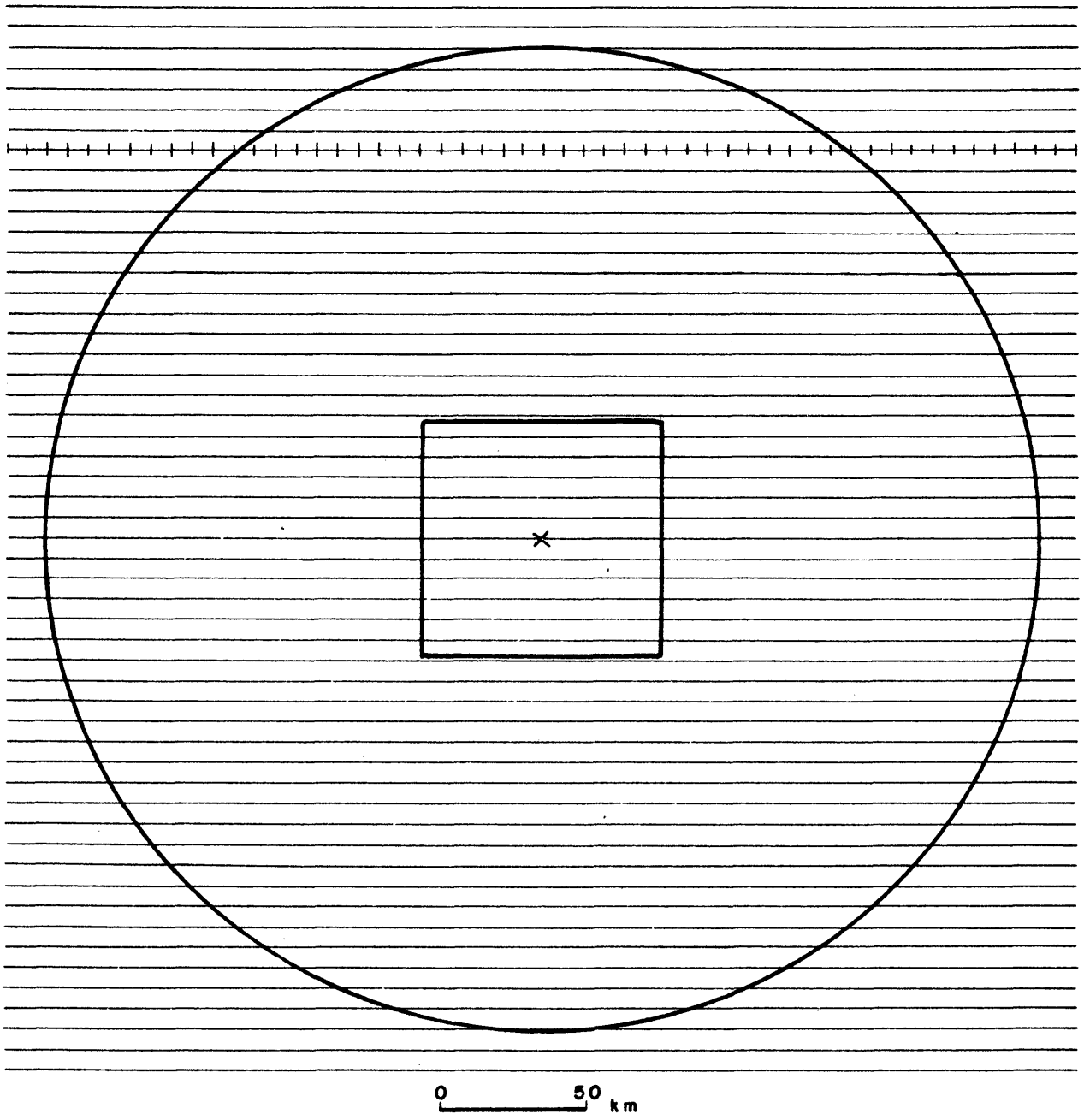


Figure 4 . Profiles on zone I

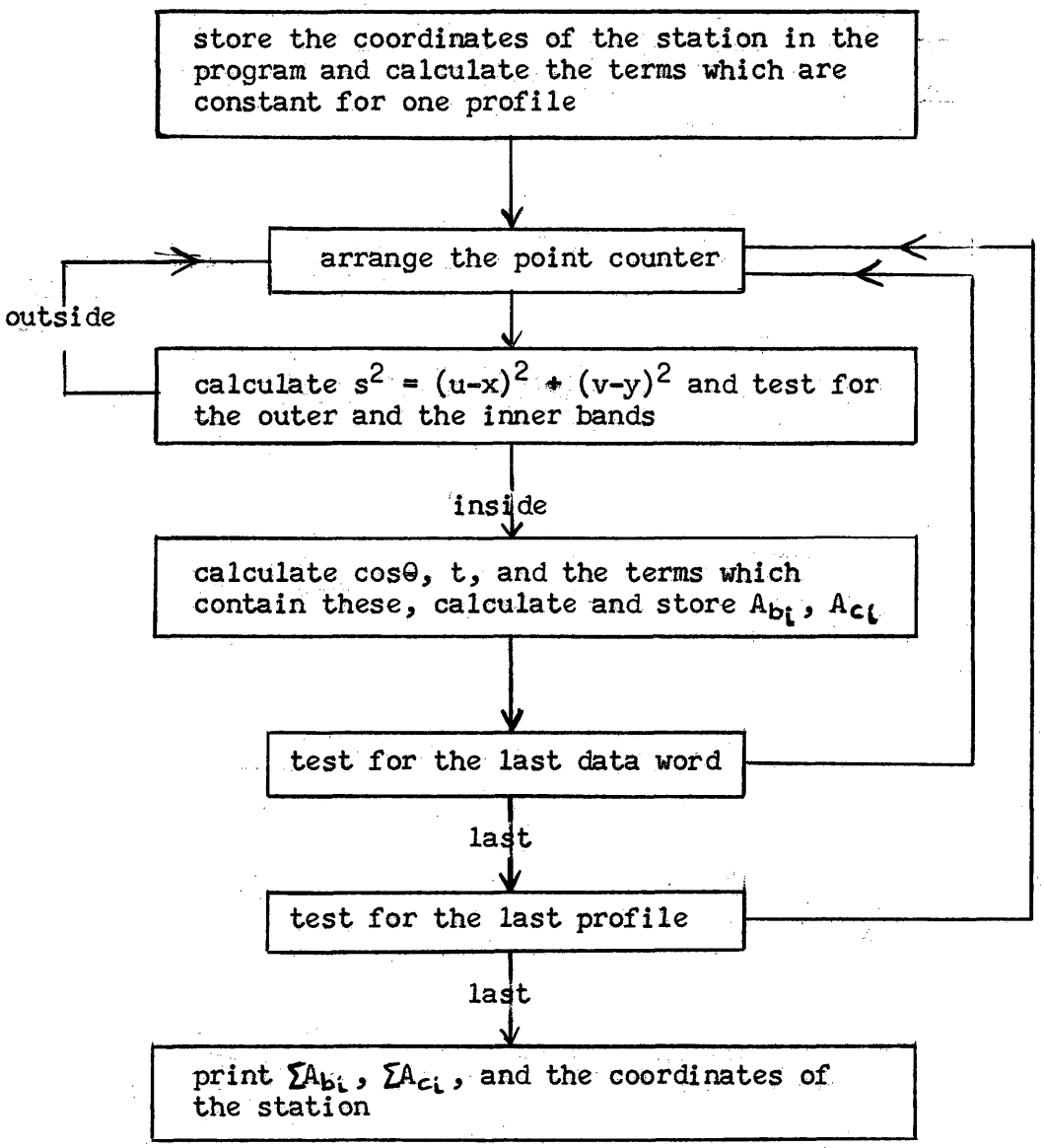


Figure 5. Flow chart to illustrate the main divisions of the program

Figure 5 is a functional chart of the program.

Two counter registers were established: one to count the number of profiles computed and another to count the grid points. The latter has been labeled the "point counter" in the program and flow chart. The point counter was used to augment the address of the "bring" instruction whose function was to select a data word from memory. Since there were two words in one location and since the unit digit in the sector portion of the address is at a q of 29, 1 at a q of 30 was added to the address of the "bring" instruction each time a data word was selected from memory. The notation q is adopted for the location of the binary point. Provision of course had to be made for skipping the address of the sector containing the initial grid-point coordinates at the beginning of each profile.

When the data word was brought from memory to the accumulator register, an "extract" command was used to select the half of the data word which was next in line. The point counter was used to enable the "extract" order to make the proper choices. The point counter was tested, and if it contained a bit at a q of 30, it would extract the last half of the data word; but if there were a zero at a q of 30, it would extract the other half.

Wherever possible the address part of each instruction was "optimized"; that is, each address was chosen so that access time was a minimum. This process often required inserting an unconditional transfer instruction where logically it would not be required.

Minor changes were made in the program for zones II, III, and IV; but the main structure remained the same for all zones. For zones II and III and IV, one data word instead of two was stored in each location. In zone IV the effect of the earth's curvature was ignored. At the boundary of this zone the angle θ was so small that $\cos\theta$ differed from unity in the sixth decimal place and by less at smaller distances from the station.

Accuracy of Mass-Line Method

The accuracy of the mass-line method and the correctness of the program was checked by computing the topographic effect of a uniformly thick layer in zone one based on another more exact method. The thickness of the layer h and the elevation of the station e was chosen to be 9,000 ft. The formula for the attraction of a spherical cap of density ρ is

$$A_b = 2/3\pi k\rho\left[(a+h)^3 - a^3/(a+h)^2\right] + 2\pi k\rho\left[w^2(1/3\ r^2/b^2 + 1/3\ r/b\cos\theta + \cos^2\theta - 2/3) - b\sin^2\theta\cos\theta\log(w+r-b\cos\theta)\right]\Big|_a^{a+h}$$

where $w=(r^2+b^2-2br\cos\theta)^{\frac{1}{2}}$, was applied twice to find the difference between the attraction of a spherical cap extending out to 166.71 km and one extending out to 40 km. This result exceeded the attraction of zone I by an amount equal to the attraction of four times that of a 9,000-ft.-thick layer having the shape of the shaded portion of figure 6. The attraction of the latter was found by a graphical method.

A comparison between the results of the two methods

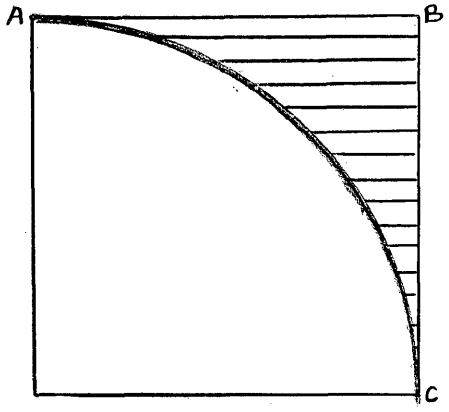


Figure 6. Graphical calculation of the attraction due to the mass of ABC.

shows a difference of less than one tenth of a milligal. The topographic effect by the mass-line method computed by machine was 10.04 milligals; whereas the same effect computed by a combination of formula and graph was 9.96 milligals, a difference of 0.08 milligal. The reason the mass-line method gives a result slightly too large is that the 5-km prism was considered to be uniform in cross-section although actually its cross-section diminishes with depth. Correspondingly the linear density of the line masses would increase with depth instead of remaining constant. The difference, however, is not great enough to be considered.

Whenever a calculation is made from the prepared elevation profiles, the gravity station may not fall at the center of the

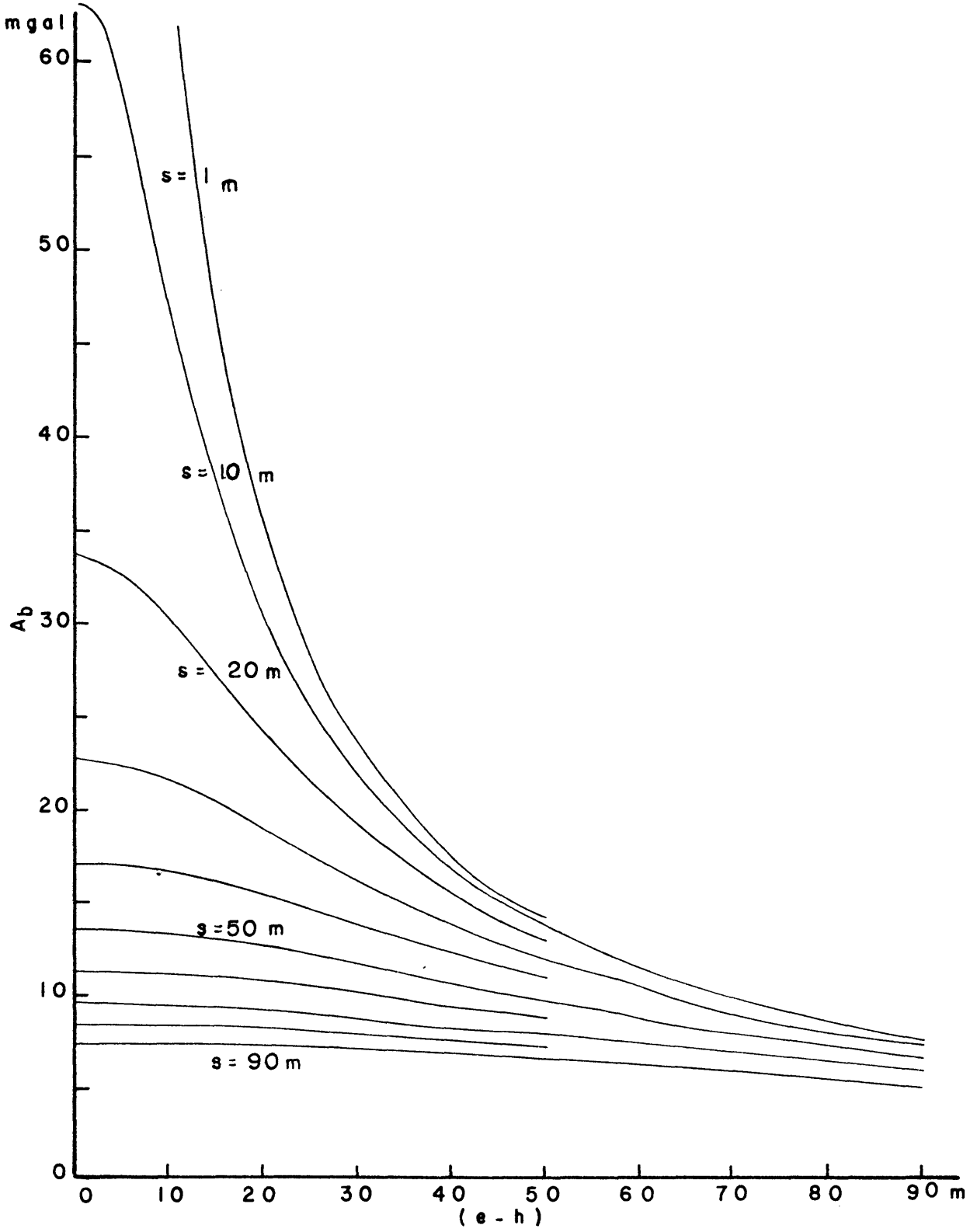


Figure 7a . A_b vs elevation differences for different distances from the station to the mass-line

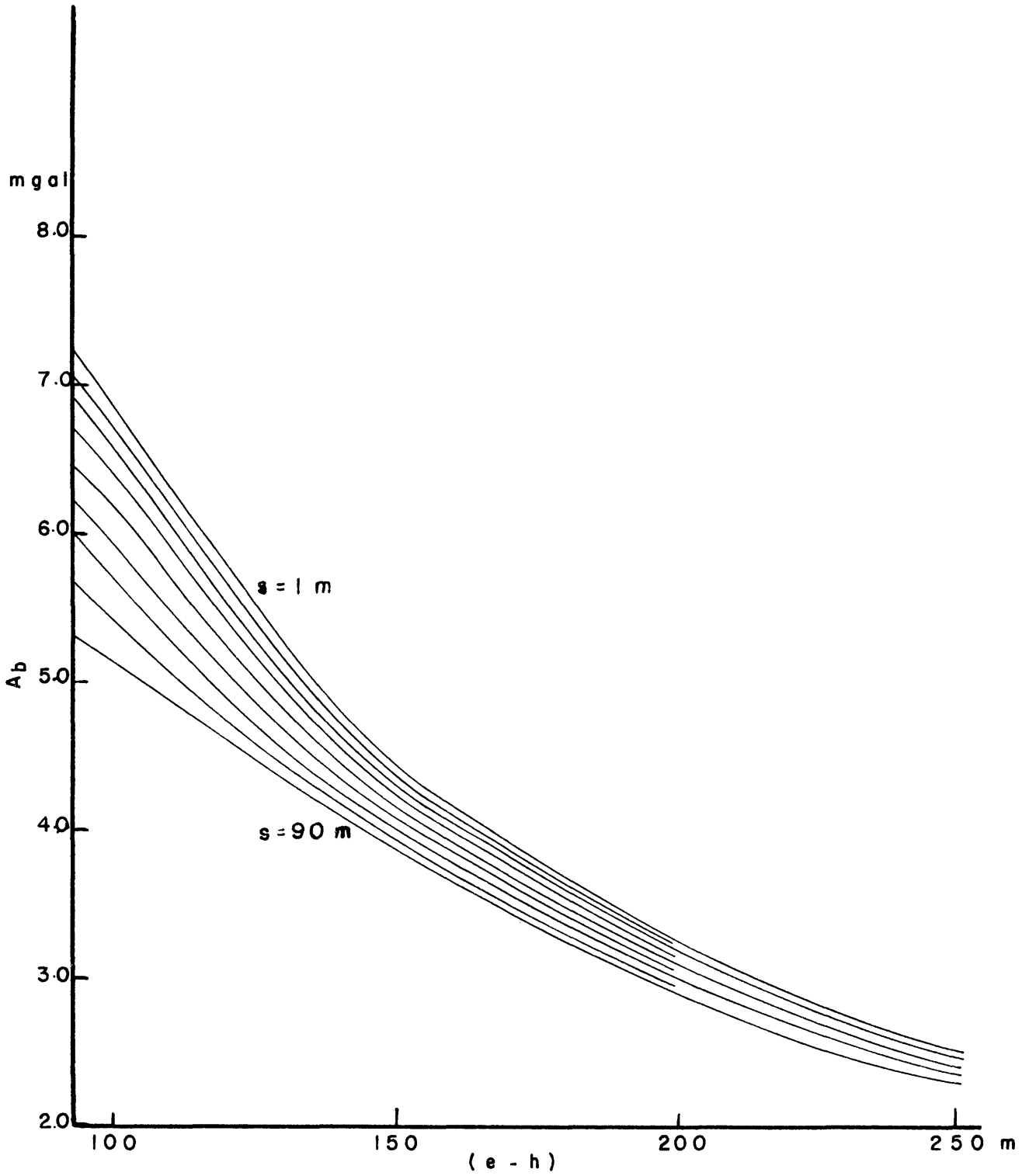


Figure 7b . A_b vs elevation differences for different distances from the station to the mass line

various zones. For zone I, the station may fall a maximum distance of 2.5 km from the center of the zone. However, no significant difference in the results was found when the station was displaced from center of zones I, II, and III. For zone IV, the results were very sensitive to the position of the station with respect to the grid points. Figures 7a, b show the variation of the effect with distance of a mass line in the 200-meter grid. This effect is so serious that the method is not practical without modification for the zone containing the station. A suggested change in method for zone IV is that of computing the effect of the entire mass lying between the station and sea level from an exact formula. The terrain effect could then be computed by the mass-line method.

Calculations with and without Corrections for Curvature

The topographic and isostatic corrections were calculated at ten stations in the Ralston Buttes quadrangle with the formulas which took the curvature of the earth's surface into account and with those that did not. The comparison of the two methods is shown in tables 2, 3. In the topographic correction there was an average of 3 milligals difference between the results of the two methods, whereas in the isostatic correction the average difference was 13 milligals. About seventy-five percent of both differences arose from zone I. Table 4 shows that there is no important difference in time between the two methods.

Elevation of the station (in m)	Topographic correction calculated with the earth curvature (1)	without the earth curvature (2)	Differences in mgal (1)-(2)
1945	202.409	199.740	2.669
2106	201.439	199.104	2.335
2247	235.943	232.690	3.253
2466	271.424	268.376	3.048
2357	256.423	253.261	3.162
2274	249.747	246.245	3.502
2297	250.606	247.213	3.393
2489	277.326	273.808	3.518
2327	250.402	247.320	3.082
2385	230.205	227.077	3.128

Table 2. The effect of the earth's curvature on the value A_b.

Elevation of the station (in m)	Isostatic correction calculated with the earth curvature (1)	without the earth curvature (2)	Differences in mgal (1)-(2)
1945	194.230	207.413	-13.183
2106	198.534	211.532	-12.998
2247	200.730	214.244	-13.514
2466	203.863	217.955	-14.092
2357	200.863	214.348	-13.485
2274	201.747	214.736	-12.989
2297	202.933	216.487	-13.554
2489	200.217	213.368	-13.151
2327	202.239	214.959	-12.720
2385	202.673	216.364	-13.691

Table 3. The effect of the earth's curvature on the value A_c.

	Computation time for zones (both corrections)				Entire area
	I	II	III	IV	
	h m	h m	h m	h m	h m
With the curvature effect	2 58	1 25	1 51	1 36	7 52
Without curvature effect	2 41	1 21	1 46	1 36	7 24
Only the terrain correct.	1 48	- 34	- 48	- 55	4 05

Table 4. Computation time spent for the calculations of A_b, A_c, and the terrain correction at one station.

Comparison Between the Mass-Line and Hayford's Method of Computing Terrain Effects

Only the terrain effect was computed out to 166.71 km for the ten test stations mentioned in the last section. Three methods were used: 1. the program developed here modified to compute only the terrain effect, 2. the same as 1 except that Bott's approximation formula was used, and 3. Hayford's method. The effect was computed with four different inner boundaries for zone IV: 10 cm, 100 meters, 200 meters, and 1000 meters. The earth's curvature was not taken into account.

As seen in table 5, the results using an inner boundary of 10 cm in zone IV were not significantly different from those using inner

Elevation of the station in meters	Terrain correction computed with chart method	Difference (chart--10 cm inner boun)	Terrain correction due to the inner boundary of							
			10 cm		100 m		200 m		1000 m	
			without approx.	with approx.	without approx.	with approx.	without approx.	with approx.	without approx.	with approx.
1945	5.56	-1.2	6.85	4.01	6.83	4.01	6.35	4.01	4.78	2.60
2106	4.34	0.6	3.72	2.80	3.72	2.80	3.56	2.80	2.98	2.37
2247	3.83	0.1	3.73	2.81	3.70	2.81	3.64	2.81	3.09	2.51
2266	6.26	-0.4	6.67	5.79	6.48	5.79	6.44	5.79	4.46	3.99
2357	4.78	0.18	4.60	3.34	4.32	3.34	4.15	3.34	3.49	2.94
2274	5.67	-0.6	6.33	4.68	5.73	4.68	5.57	4.68	4.59	4.02
2297	5.73	-0.3	6.08	4.81	5.99	4.81	5.69	4.81	4.65	4.10
2489	3.95	-0.8	4.84	3.00	4.00	3.00	3.81	3.00	3.25	2.46
2327	5.46	0.8	4.29	2.91	4.01	2.91	3.71	2.91	2.68	2.61
2385	3.82	0.9	2.92	2.37	2.92	2.37	2.89	2.37	2.75	2.32

Table 5. Terrain corrections with different inner boundary (results are in mgal)

boundaries of 100 and 200 meters. Except for the results found for an inner boundary of 1 km, there was fair agreement with values computed by Hayford's method. However, if the machine-computed values are accepted as correct, then the values computed by Hayford's method show deviations of as much as 1.2 milligals. Since Hayford's method relies on the judgment of the computer in correctly estimating elevations in fairly large compartments, it is more reasonable to suspect error here than in the machine-computed values.

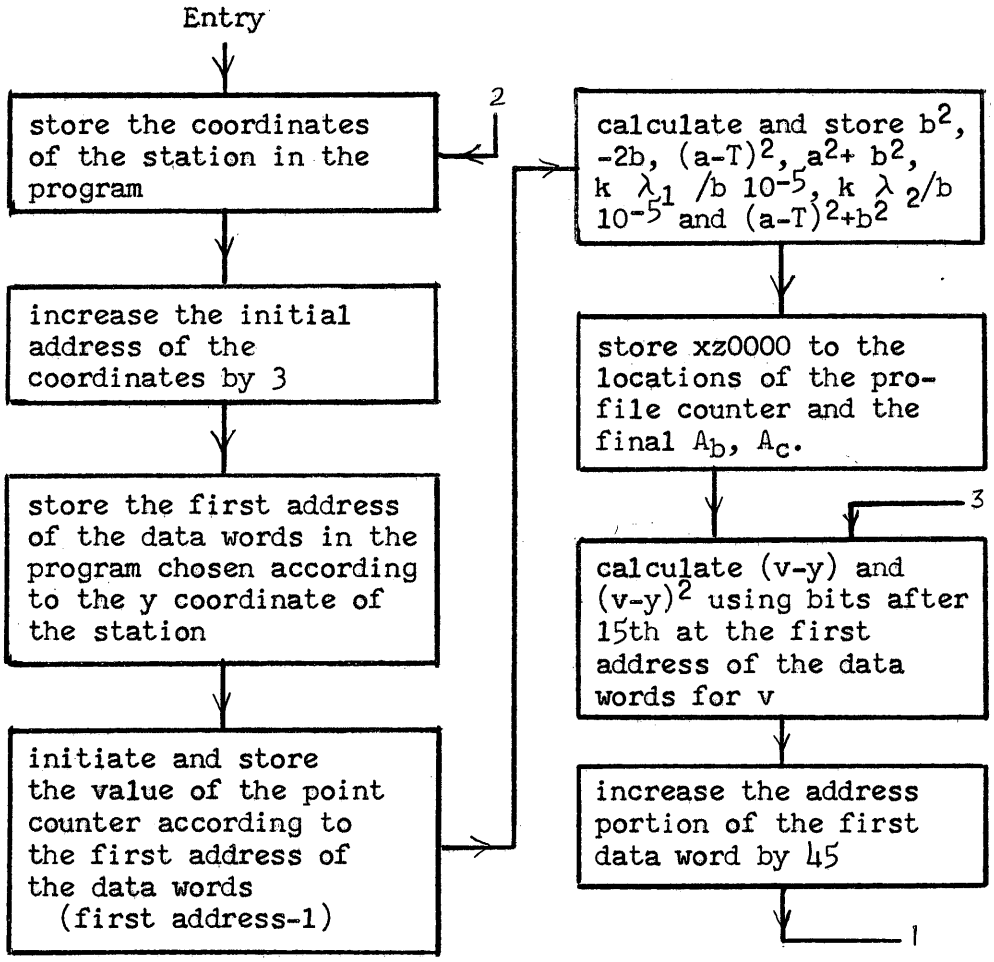
CONCLUSION

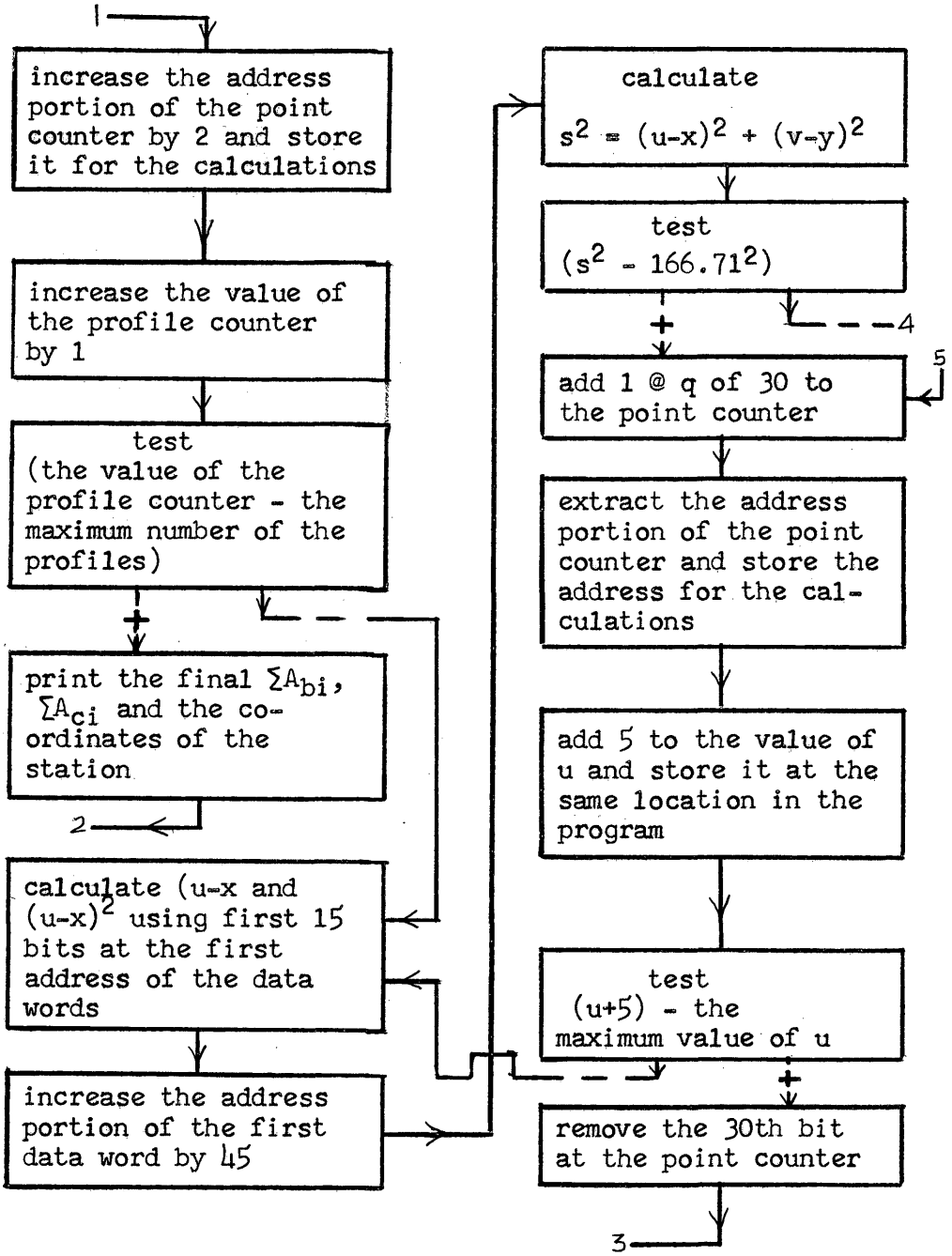
The program and methods described in this report do not offer an entirely satisfactory solution to the problem of computing topographic effects. Although the mass-line method appears to give satisfactory results in zones I, II, and III, it proved to be unreliable in zone IV. The trouble in zone IV arose from the fact that line-mass approximations to the actual mass distribution do not give a good approximation to the true effect in the neighborhood of the station. The very good results obtained in computing terrain effects suggest that a much more accurate method of computing the topographic effect would be first to compute from a formula the effect of a spherical cap of thickness equal to the elevation of the station; and then to correct this value for the terrain effect using the mass-line method.

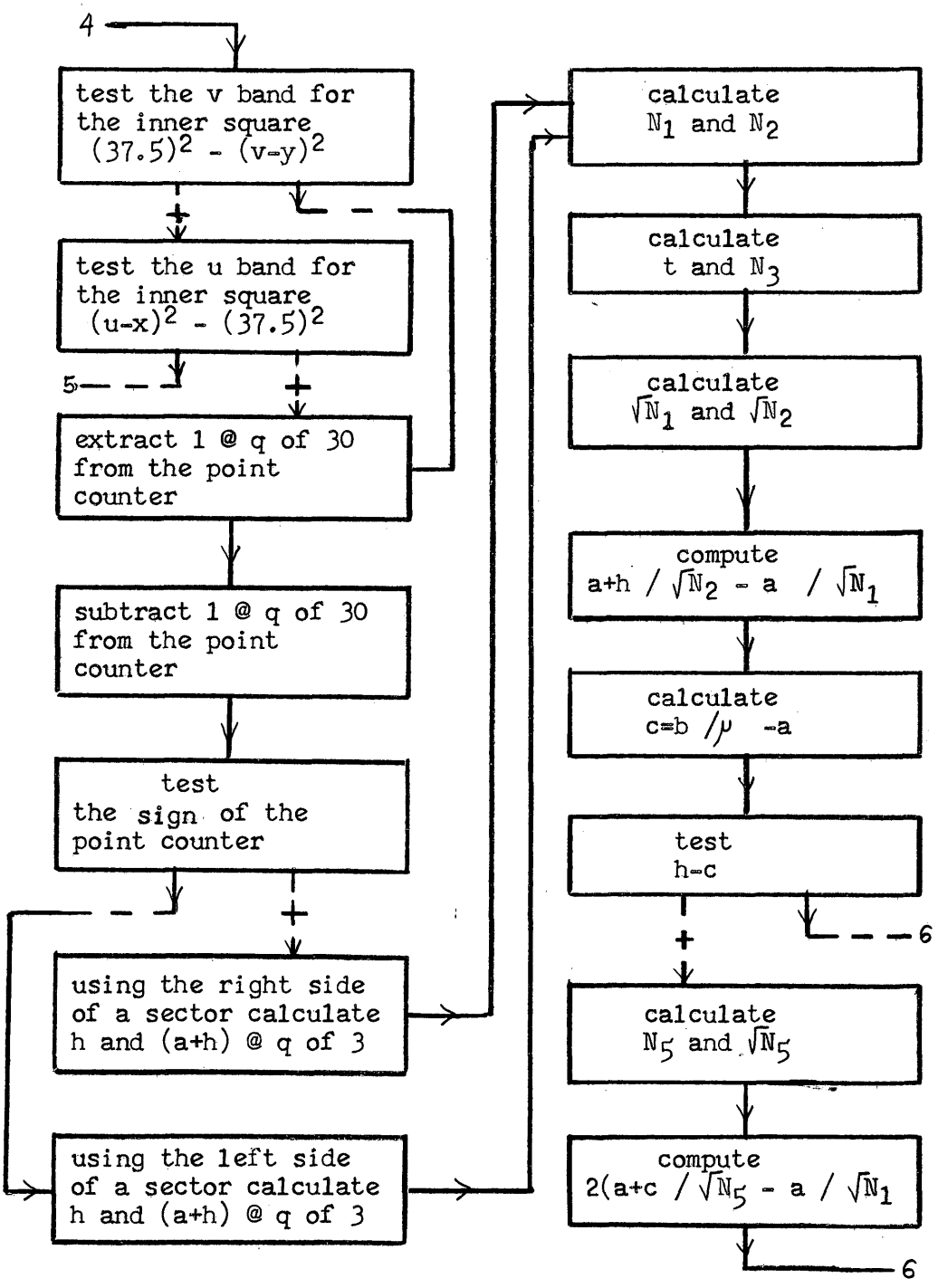
Since the masses involved in computing the isostatic effect are always more than 30 km from the station, the mass-line method gives good results in all zones.

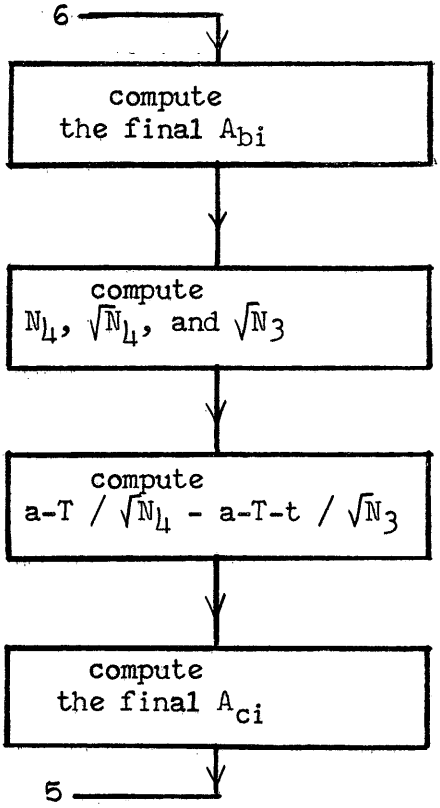
The failure to reduce computing time to a reasonable value was a great disappointment. An increase in grid size might reduce the time by one half; but even this time would be much too long. Perhaps this can only be done with the larger, more elaborate computers. Computing the topographic and isostatic effects out to 166.71 km by Hayford's method usually takes between $3\frac{1}{2}$ to 4 hours per station. As seen in table 4, the time for the program developed was nearly 8 hours per station.

FLOW CHART OF THE PROGRAM:




1
increase the address portion of the point counter by 2 and store it for the calculations
increase the value of the profile counter by 1
test (the value of the profile counter - the maximum number of the profiles)
print the final ΣA_{bi}, ΣA_{ci} and the co-ordinates of the station
2
calculate (u-x and $(u-x)^2$ using first 15 bits at the first address of the data words
increase the address portion of the first data word by 45
calculate $s^2 = (u-x)^2 + (v-y)^2$
test $(s^2 - 166.71^2)$
4
add 1 @ q of 30 to the point counter
5
extract the address portion of the point counter and store the address for the calculations
add 5 to the value of u and store it at the same location in the program
test (u+5) - the maximum value of u
remove the 30th bit at the point counter
3


4
test the v band for the inner square $(37.5)^2 - (v-y)^2$
test the u band for the inner square $(u-x)^2 - (37.5)^2$
5
extract 1 @ q of 30 from the point counter
subtract 1 @ q of 30 from the point counter
test the sign of the point counter
using the right side of a sector calculate h and (a+h) @ q of 3
using the left side of a sector calculate h and (a+h) @ q of 3
calculate N_1 and N_2
calculate t and N_3
calculate $\sqrt{N_1}$ and $\sqrt{N_2}$
compute $a+h / \sqrt{N_2} - a / \sqrt{N_1}$
calculate $c=b / \mu - a$
test h-c
6
calculate N_5 and $\sqrt{N_5}$
compute $2(a+c / \sqrt{N_5} - a / \sqrt{N_1}$
6


6
compute
the final A_{bi}
compute
N_4, $\sqrt{N_4}$, and $\sqrt{N_3}$
compute
a-T / $\sqrt{N_4}$ - a-T-t / $\sqrt{N_3}$
compute
the final A_{ci}
5

THE PROGRAM

Location	Instruction	Contents	Notes
6214	xb1300	u @ q of 8	v @ q of 28
	xe0334	0000wwww	v @ q of 28
	xd0401	1 @ q of 15	v @ q of 13
	xs0434	y @ q of 13	v-y @ q of 13
	xh0440	v-y @ q of 13	
	xm0440	v-y @ q of 13	$(v-y)^2$ @ q of 26
6220	xc0448	$(v-y)^2$ @ q of 26	
	b0014	xb1300	
	xa0820	xz0045	
	y0014		
	xb0331	point counter add.	initially 1263
	xa0447	2 @ q of 29	
	xy0349		
	xy0450		
	xc0331	point counter add.	

```
                xb0539    profile counter    initially xz0000
6230            xa0401    1 @ q of 15
                xh0539    profile counter
                xs0545    68 @ q of 15       number of profiles
                xt0300                       to calculations
                xb1059    ∑A_bi @ q of 0
                xR0144
                xU0132    print ∑A_bi
                xz0000
                xb1060    ∑A_ci @ q of 0
                xR0144
6240            xU0132    print ∑A_ci
                xz0000
                xb1061    x @ q of 13
                xd0436    1 @ q of 3         x @ q of 10
                xR0144
                xU0132    print x
                xz0003
                xb1062    y @ q of 13
                xm0437    1 @ q of 1         y @ q of 14
                xR0144
6250            xU0132    print y
                xz0004
                xb1063    e @ q of 13
```

```
                xd0544    1 @ q of 9        e @ q of 4
                xR0144
                xU0132    print e
                xz0001
start program   xb1200    x @ q of 13
                xh0444    x @ q of 13
                xc1061    x @ q of 13
      6260      xb1201    y @ q of 13
                xh0434    y @ q of 13
                xc1062    y @ q of 13
                xb1202    e @ q of 13
      6300      xc1063    e @ q of 13
                 b0057    xb1200
                xa0805    xz0003
                 y0057
                 b0060    xb1201
                xa0805    xz0003
                 y0060
                 b0063    xb1202
                xa0805    xz0003
                 y0063
      6310      xb1038    xz1300 or xz1345 or xz1426
                xy0300
                 y0014
```

	xb1053	point counter initial value	
	xc0331	point counter add.	
	xb0339	a @ q of 13	
	xa1063	e @ q of 13	b=a+e @ q of 13
	xh0744	b=a+e @ q of 13	
	xm0744	b @ q of 13	b^2 @ q of 26
	xm0445	1 @ q of 1	b^2 @ q of 27
6320	xh0742	b^2 @ q of 27	
	xh0446	b^2 @ q of 27	
	xm0445	1 @ q of 1	b^2 @ q of 28
	xc0811	b^2 @ q of 28	
	xb0819	a-T @ q of 13	
	xm0430	a-T @ q of 14	$(a-T)^2$ @ q of 27
	xa0742	b^2 @ q of 27	$(a-T)^2+b^2$ @ q of 27
	xc0449	$(a-T)^2+b^2$ @ q of 27	
	xb0339	a @ q of 13	
	xm0339	a @ q of 13	a^2 @ q of 26
6330	xm0445	1 @ q of 1	a^2 @ q of 27
	xa0446	b^2 @ q of 27	a^2+b^2 @ q of 27
	xc0505	a^2+b^2 @ q of 27	
	xb0744	b @ q of 13	
	xm1056	-1 @ q of 1	-b @ q of 14
	xm0437	2 @ q of 2	-2b @ q of 16
	xd1044	1 @ q of 2	-2b @ q of 14

```
                xc0839    -2b @ q of 14
                xb1041    k λ1 10^-5 @ q of 5
                xd0744    b @ q of 13      k λ1 / b 10^-5 @ q of -8
6340            xh1047    k λ1 / b 10^-5 @ q of -8
                xc1057    k λ1 / b 10^-5 @ q of -8
                xb1042    k λ2 10^-5 @ q of 5
                xd0744    b @ q of 13      k λ2 / b 10^-5 @ q of -8
                 u0145
                xc1043    k λ2 / b 10^-5 @ q of -8
                xb1058    xz0000
                xh0539    profile counter
                xh1059    ∑Ab_i
                xc1060    ∑Ac_i
6350             u0014


0300            xb1300    u @ q of 8       v @ q of 28
                 e0037    wwww0000         u @ q of 8
                 m0038    1 @ q of 5       u @ q of 13
                 c0457    u @ q of 13
                 b0000    xb[1300]
                 a0520    z0045
                 y0000
                 b0457    u @ q of 13
```

```
                s0144    x @ q of 13        u-x @ q of 13
                h0531    u-x @ q of 13
0310            m0531    u-x @ q of 13      (u-x)^2 @ q of 26
                u0012
                a0148    (v-y)^2 @ q of 26
                h0528    (u-x)^2+(v-y)^2 =s^2 @ q of 26
                s0129    166.71^2 @ q of 26  s^2-166.71^2 @ q of 26
                t0332
                b0031    point counter
                a0403    1 @ q of 30
                h0031    point counter
                e0462    3wwj               extract add. portion
0320            y0049
                y0150
                b0457    u @ q of 13
                a0102    5 @ q of 13        u+5 @ q of 13
                h0457    new u or u+5 @ q of 13
                s0404    681 @ q of 13      (u+5)-681 @ q of 13
                t0007
                b0031    point counter
                e0414    3wwj               remove bit @ q of 30
                c0031    point counter
0330            xu6214
                xz1263   initial value of point counter
```

```
,0000013          2        1 @ q of 30
                  2        1 @ q of 30
                  wwww
                  00000000 √N1 @ q of 9
                  wwww
                  wwww0000
                  4000000  1 @ q of 5
                  638jjjjj a @ q of 13
          0340    00000000
                  00000000
                  00000000
                  00000000
                  00000000
                  b0031    point counter
                  e0032    1 @ q of 30
                  s0033    1 @ q of 30
                  t0150
                  xb[0000]
          0350    e0036    wwww    right or odd h @ q of 18
                  d0423    1 @ q of 5      h @ q of 13
                  h0438    h @ q of 13     for Ab routine
                  d0439    1 @ q of 10     h @ q of 3
                  h0233    h @ q of 3      for Ac routine
                  u0056
```

	c0135	h @ q of 3	
	u0059		
	xz0000		
	b0438	h @ q of 13	
0360	a0039	a @ q of 13	a+h @ q of 13
	h0440	a+h @ q of 13	
	c0441	a+h @ q of 13	
	b0528	s^2 @ q of 28	begin cosine routine
0400	u0201		
,0000002	10000	1 @ q of 15	
	140000	5 @ q of 13	
	b0118	B @ q of 0	begin t routine
	m0233	h @ q of 3	Bh @ q of 3
	a0234	C @ q of 3	Bh+C @ q of 3
	m0135	h @ q of 3	t=(Bh+C)h @ q of 6
	m0529	-1 @ q of 8	-t @ q of 14
	a0130	a-T @ q of 14	a-T-t @ q of 14
	h0459	a-T-t @ q of 14	
0410	h0460	a-T-t @ q of 14	
	m0461	-2bμ @ q of 14	-2b(a-T-t)μ @ q of 28
	c0562	-2b(a-T-t)μ @ q of 28	
	u0116		
	xz0000	N_2 @ q of 18	
,0000001	200000	1 @ q of 10	

	b0502	N_2 @ q of 18
	u0119	
,0000001	7q636f	B @ q of 0
	h0248	N_2 @ q of 18
0420	u0256	
,0000001	638jjjjj	a @ q of 13
	u0258	
	b0459	a-T-t @ q of 14
	m0460	a-T-t @ q of 14 $(a-T-t)^2$ @ q of 28
	a0511	b^2 @ q of 28 $b^2+(a-T-t)^2$ @ q of 28
	a0562	$-2b(a-T-t)\mu$ @ q of 28
	c0642	$(a-T-t)^2-2b(a-T-t)\mu+b^2=N_3$ @ q of 28
	u0300	
,0000021	k91j0	166.71^2 @ q of 26
0430	318f6666	a-T @ q of 14
	00000000	
	00000000	
	00000000	
	00000000	y @ q of 13
	00000000	h @ q of 3
	10000000	1 @ q of 3
	40000000	1 @ q of 1
	00000000	
	00000000	$\sqrt{N_2}$ @ q of 9

```
0440    00000000    v-y @ q of 13
        00000000    (a+c)^2+b^2 @ q of 27
        00000000
        00000000    N3 @ q of 18
        00000000    x @ q of 13
         8000000    1 @ q of 4
        00000000    b^2 @ q of 27
               8    2 @ q of 29
        00000000    (v-y)^2 @ q of 26
        00000000    (a-T)^2+b^2 @ q of 27
0450      xb0000    wwww0000      left or even h @ q of 3
        u0154
        xz0000
        h0233       h @ q of 3
        u0156
        h0135       h @ q of 3
        m0115       1 @ q of 10       h @ q of 13
        u0159
        h0438       h @ q of 13
0460    a0518       a @ q of 13       a+h @ q of 13
        h0440       a+h @ q of 13
        c0441       a+h @ q of 13
        b0528       s^2 @ q of 26     begin cosine routine
0500    u0201
```

	m0430	$1/24a^2$ @ q of -26 $1/24(s/a)^2$ @ q of 0
	s0431	1/2 @ q of 0
	m0232	$1/a^2$ @ q of -25
	u0206	
	xz0000	
	m0528	s^2 @ q of 26
	a0422	1 @ q of 1 $\cos\theta=\mu$ @ q of 1
	d0530	1 @ q of 1 μ @ q of 0
	h0638	μ @ q of 0
0510	m0539	-2b @ q of 14 $-2b\mu$ @ q of 14
	h0461	$-2b\mu$ @ q of 14
	m0441	a+h @ q of 13 $-2b(a+h)\mu$ @ q of 27
	a0442	b^2 @ q of 27 $b^2-2b(a+h)\mu$ @ q of 27
	c0443	$b^2-2b(a+h)\mu$ @ q of 27
	b0444	b @ q of 13
	d0638	μ @ q of 0 b/μ @ q of 13
	c0246	b/μ @ q of 13
	b0440	a+h @ q of 13
	m0441	a+h @ q of 13 $(a+h)^2$ @ q of 26
0520	m0556	1 @ q of 1 $(a+h)^2$ @ q of 27
	a0443	$b^2-2b(a+h)\mu$ @ q of 27
	d0244	1 @ q of 9
	c0502	$(a+h)^2-2b(a+h)\mu+b^2=N_2$ @ q of 18
	b0503	a @ q of 13

	m0461	$-2b\mu$ @ q of 14 $-2ab\mu$ @ q of 27
	a0205	a^2+b^2 @ q of 27
	d0713	1 @ q of 9
	h0450	$a^2-2ab\mu+b^2=N_1$ @ q of 18
	u0230	
0530	u0259	
,0000004	40000000	1 @ q of 1
	69jqj72j	$1/a^2$ @ q of -25
	00000000	h @ q of 3
	47q010f1	C @ q of 3
	a0121	a @ q of 13 a+c @ q of 13
	h0522	a+c @ q of 13
	c0523	a+c @ q of 13
	u0103	
	xz0000	profile counter initially xz0000
0540	h0255	N_5 @ q of 18
	u0242	
	c0521	N_5 @ q of 18
	u0405	
,0000005	400000	1 @ q of 9
	440000	68 @ q of 15
	00000000	b/μ @ q of 13
	638jjjjj	a @ q of 13
	00000000	N_2 @ q of 18

	d0721	1 @ q of 9
0550	u0240	
,0000002	00000000	
	10000000	1 @ q of 3
	a0759	$\sum A_{b_i}$ @ q of 0
	u0420	
	xz0000	N_5 @ q of 18
	c0114	N_2 @ q of 18
	u0122	
	u0123	
	c0724	N_1 @ q of 18
0560	b0246	b/μ @ q of 13
	s0247	a @ q of 13 c=b/μ -a @ q of 13
	h0541	c @ q of 13
	u0235	
0600	b0450	N_1 @ q of 18
	xR1150	
	xU1100	
	c0035	$\sqrt{N_1}$ @ q of 9
	u0305	
	b0248	N_2 @ q of 18
	xR1150	
	xU1100	
	c0139	$\sqrt{N_2}$ @ q of 9

	u0705	transfer to compute $a+h/\sqrt{N}_2-a/\sqrt{N}_1$
,0000022	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
0620	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
0630	00000000	
	00000000	
	b0347	$(37.5)^2$ @ q of 26

```
                s0148    (v-y)^2 @ q of 26  37.5^2-(v-y)^2 @ q of 26
                t0045
                b0531    u-x @ q of 13
                m0531    u-x @ q of 13    (u-x)^2 @ q of 26
                s0352    (37.5)^2 @ q of 26 (u-x)^2-37.5^2 @ q of 26
                t0016
                u0045
,0000016        00000000
                00000000
                00000000
                00000000
                00000000
                40000000 1 @ q of 1
                00000000
                  fwj8   (37.5)^2 @ q of 26
                00000000
                00000000
     0650       00000000
                00000000
                  fwj8   (37.5)^2 @ q of 26
                00000000
                00000000
                00000000
                  t0728
```

```
                b0522    a+c @ q of 13
                m0523    a+c @ q of 13    (a+c)^2 @ q of 26
                m0231    1 @ q of 1       (a+c)^2 @ q of 27
     0660       a0146    b^2 @ q of 27    b^2+(a+c)^2 @ q of 27
                u0362
                c0141    b^2+(a+c)^2 @ q of 27
                u0400
     0700       b0522    a+c @ q of 13
                u0411
,0000003        00000000
                      2  1 @ q of 30
                ff40000  681 @ q of 13
                b0255    N5 @ q of 18
                xR1150
                xU1100
                c0532    √N5 @ q of 9
                u0557
     0710       xz0000
                m0461    -2bμ @ q of 14   -2b(a+c)μ @ q of 27
                a0141    (a+c)^2+b^2 @ q of 27
                u0249
,0000006           3wwj
                00000000
                00000000
```

	00000000	
	00000000	
	00000000	
0720	c0759	ΣA_{b_i} @ q of 0
	u0432	
,0000010	40000000	1 @ q of 1
	4000000	1 @ q of 5
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
	00000000	
0730	8k13g43	$1/2a^2$ @ q of -26
	40000000	1/2 @ q of 0
	b0461	$-2b\mu$ @ q of 14
	m0519	a-T @ q of 13 $-2b(a-T)\mu$ @ q of 27
	a0149	$(a-T)^2+b^2$ @ q of 27
	d0650	1 @ q of 9
	h0458	$(a-T)^2-2b(a-T)\mu+b^2=N_4$ @ q of 18
	u0526	
,0000025	00000000	h @ q of 13
	200000	1 @ q of 10
0740	00000000	a+h @ q of 13

	00000000	a+h @ q of 13
	00000000	b^2 @ q of 27
	00000000	$b^2-2b(a+h)\mu$ @ q of 27
	00000000	b @ q of 13
	00000000	
	00000000	
	00000000	
	00000000	N_4 @ q of 18
	00000000	
0750	00000000	N_1 @ q of 18
	00000000	
	00000000	
	00000000	
	00000000	
	638jjjj	a @ q of 17
	00000000	
	00000000	u @ q of 13
	00000000	N_4 @ q of 18
	00000000	a-T-t @ q of 14
0760	00000000	a-T-t @ q of 14
	00000000	$-2b\mu$ @ q of 14
	3wwj	
	xz1300	
0800	c0143	N_3 @ q of 18

```
                u0600
,0000003        00000000    N2 @ q of 18
                638jjjjj    a @ q of 13
                00000000
                xz0003
                b0642       N3 @ q of 28
                d0736       1 @ q of 10      N3 @ q of 18
                u0524
,0000004        wwww0000
  0810          00000000    N3 @ q of 18
                00000000    b^2 @ q of 28
                00000000
                s0735       a-T-t/√N3 @ q of 8
                m0743       k λ2/ b 10^-5 @ q of -8   Aci @ q of 0
                a0760       ΣAci @ q of 0
                c0760       ΣAci @ q of 0
                u0016
,0000002        638jjjjj    a @ q of 13
                6314jjjj    a-T @ q of 13
  0820          xz0045
                xz0000      N5 @ q of 18
                xz0000      a+c @ q of 13
                xz0000      a+c @ q of 13
                h0510       N3 @ q of 18
```

```
                u0500
                c0448     N4 @ q of 18
                u0506
,0000011        00000000  s^2 @ q of 26
                ww800000  -1 @ q of 8
0830            40000000  1 @ q of 1
                00000000  u-x @ q of 13
                00000000  √N5 @ q of 9
                00000000
                00000000
                00000000
                00000000
                 8000000  1 @ q of 4
                00000000
                  xz0000  -2b @ q of 14
0840              xz0000
                  xz0000  c @ q of 13
,0000005        00000000
                00000000
                00000000
                00000000
                00000000
                  xz0000
                  xz0000
```

```
                c0759    ΣA_bi @ q of 0
0850            u0432
,0000006        00000000
                00000000
                00000000
                00000000
                00000000
                40000000    1 @ q of 1
                b0522       a+c @ q of 13
                m0537       1 @ q of 4          a+c @ q of 17
                u0560
0860            d0532       √N_5 @ q of 9       a+c/√N_5 @ q of 8
                u0700
                xz0000      -2b(a-T-t)μ @ q of 28
                xz0000
0900            b0143       N_3 @ q of 18
                xR1150
                xU1100
                c0734       √N_3 @ q of 9
                u0605
                b0448       N_4 @ q of 18
                xR1150
                xU1100
                c0732       √N_4 @ q of 9
```

```
                    u0644
,0000014            00000000
                    00000000
                    00000000
                    00000000
                    00000000
                    00000000
                    00000000
                    00000000
                    00000000
                    00000000
                    00000000
0920                00000000
                    00000000
                    00000000
                    00000000
                    00000000
                      d0703     1 @ q of 2
                      s0752     a+h/√N2-a/√N1 @ q of 8
                      u0725
,0000017            00000000
                    00000000
                    00000000
0930                10000000    1 @ q of 3
                    10000000    1 @ q of 3
                    00000000
```

```
                00000000
                00000000
                00000000
                00000000
                40000000    2 @ q of 2
                00000000    μ @ q of 0
                00000000
          0940  00000000
                00000000
                00000000    N3 @ q of 28
                00000000
                  b0459     a-T-t @ q of 14
                  m0631     1 @ q of 3        a-T-t @ q of 17
                  u0655
,0000008        00000000
                00000000
                00000000
          0950    400000    1 @ q of 9
                00000000
                00000000
                00000000
                00000000
                  d0734     √N3 @ q of 9      a-T-t/√N3 @ q of 8
                  c0735     a-T-t/√N3 @ q of 8
```

```
                b0722    a-T @ q of 14
                m0630    1 @ q of 3          a-T @ q of 17
                u0660
0960            d0732    √N4 @ q of 9        a-T/√N4 @ q of 8
                u0513
                xz0000
                xz0000
1000            s0750    a/√N1 @ q of 8      a+c/√N5-a/√N1 @ q of 8
                m0737    2 @ q of 2          2[a+c/√N5-a/√N1] @ q of 10
                u0624
,0000001        20000000 1 @ q of 2
                xz0000
                b0455    a @ q of 17
                d0035    √N1 @ q of 9        a/√N1 @ q of 8
                c0750    a/√N1 @ q of 8
                b0440    a+h @ q of 13
                m0145    1 @ q of 4          a+h @ q of 17
1010            d0139    √N2 @ q of 9.       a+h/√N2 @ q of 8
                u0714
,0000002        00000000
                400000   1 @ q of 9
                s0750    a/√N1 @ q of 8      a+h/√N2-a/√N1 @ q of 8
                u0716
                h0752    a+h/√N2-a/√N1 @ q of 8
```

```
                u0718
                b0438   h @ q of 13
                s0541   c @ q of 13      h-c @ q of 13
1020            u0356
,0000004        400000  1 @ q of 9
              318f6666  a-T @ q of 14
              00000000
              00000000  N1 @ q of 18
                m0747   k λ1/b 10^-5 @ q of -8    Abi @ q of 0
                a0759   ΣAbi @ q of 0
                u0549
                b0757   k λ1/b 10^-5 @ q of -8
                u0730
1030            m0752   a+h/√N2-a/√N1 @ q of 0    Abi @ q of 0
                u0253
,0000006        00000000 √N4 @ q of 9
              00000000
              00000000  √N3 @ q of 9
              00000000  a-T-t/√N3 @ q of 8
                200000  1 @ q of 10
              20000000  2 @ q of 2
                xz1300
                xz1345
1040            xz1426
```

,0000012	1j7q868	$k\lambda_1\ 10^{-5}$ @ q of 5
	667381	$k\lambda_2\ 10^{-5}$ @ q of 5
	00000000	$k\lambda_2/b\ 10^{-5}$ @ q of -8
	20000000	1 @ q of 2
	00000000	
	00000000	
	00000000	$k\lambda_1/b\ 10^{-5}$ @ q of -8
	00000000	
	00000000	
1050	00000000	$a/\sqrt{N_1}$ @ q of 8
	00000000	
	00000000	$a*h/\sqrt{N_2}-a/\sqrt{N_1}$ @ q of 8
	xz1263	initial values of the point counter
	xz1344	chosen according to the y coordinate
	xz1425	of the station
,0000002	j0000000	-1 @ q of 1
	00000000	$k\lambda_1/b\ 10^{-5}$ @ q of -8
	xz0000	
	xz0000	$\sum A_{b_i}$ for print out
	xz0000	$\sum A_{c_i}$ for print out
	xz0000	x for print out
	xz0000	y for print out
	xz0000	e for print out
.0000000		

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