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A GEOMETRIC PROGRAMMING APPROACH TO THE SOLUTION OF MULTISTAGE COUNTERCURRENT HEAT EXCHANGER SYSTEMS

by

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ABSTRACT

This paper developed a method for solving multistage countercurrent heat exchanger system (CHES) models. The method used geometric programming to analyze the CHES model in order to solve the model quickly. In this work, the author does not presume to be an expert in heat exchangers. What is done here is to use CHES models as an example of an optimization technique that yields a simpler, more straightforward understanding of CHES engineering design. Additionally, this paper also provides a computer program developed from the CHES algorithm.

The CHES algorithm is significant because previous heat exchanger models focused only on the economic costs for shell-and-tube heat exchanger configurations. The focus on shell-and-tube configurations came from an industry preference and the mathematical complexity of CHES models. Although CHES configurations are more efficient than shell-and-tube models, the complexity of determining a solution to CHES models impeded the development of economic solutions. The CHES algorithm removes the mathematical complexity for CHES models.

Most heat exchanger network economic models must know the temperatures of the incoming and outgoing cold and hot processed streams to determine the minimum costs of a system. The CHES algorithm quickly computes the temperatures for the cold and hot streams for CHES models. This enables the analyst to swiftly calculate

different CHES configurations based upon different starting values for the cold and hot streams.

The ability to determine the temperatures for the cold and hot streams allows the CHES algorithm's use with the previously designed economic network models based on shell-and-tube designs. This is significant because it permits industry to compare the costs of shell-and-tube and CHES models in an exclusive, or a mixed, configuration.

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To the sheep out there I have met. Go Cowboys!

Chapter 1
INTRODUCTION

1.1 Background

Operations research optimization techniques can be used in a variety of areas. One such area involves finding the optimal synthesis of chemical processes. Lekary, in his book Engineering Design, Better Results through Operations Research Methods (1988), section 6.2, he discusses methods of the manufacturing of chemical products processing of heat exchangers using networks. Since 1960 there has been continuing development in the optimizing of heat exchanger networks using economic measures. The developed methods on heat-recovery (exchanger) networks use various techniques to optimize the processing of streams (heating or cooling of the fluid) to needed specifications to minimize total annual operating costs.

Minimizing annual operating cost is an important aspect of the development of the chemical process network. To minimize the annual operating cost, most methods focus on how to heat or cool the process streams cheaply. It is this function of changing the temperatures of process streams that contributes to the high annual operating costs (Cerda and Westerberg 1983). To control the annual operating cost, plant operators must use an efficient heat-recovery (exchanger) system.

The heat exchanger system is designed to use the heat available from streams that need to be cooled to heat streams that need to be warmed.

A typical design within a heat exchanger consists of two fluids separated by some type of composite wall that imposes a minimum element of thermal resistance (Lienhard 1981). The efficiency of the heat transfer from one fluid to another then becomes a question of design. Some of the questions to be posed are how much space must be allowed between the fluids, which materials are best suited, what the flow rate of the fluid should be, and what is the best configuration for the heat exchanger system.

An enormous variety of heat exchangers use a dividing wall between two fluids or gases. There are, however, three basic categories that include most commercial heat exchangers (Lienhard 1981):

1) Simple parallel or countercurrent (counterflow) configuration
2) Shell-and-tube configuration
3) Cross-flow configuration

1.1.1 Countercurrent Configuration

The countercurrent configuration for heat exchangers is the most basic and versatile (Lienhard 1981). The two fluid streams flow in opposite directions. The heat is then exchanged by the fluids. Figure 1.1 depicts a version of this configuration. The countercurrent heat exchanger is considered to be the most efficient because it can make the best use of available temperature differences between the streams (Spaulding 1988). As the cold stream flows from left to right, it

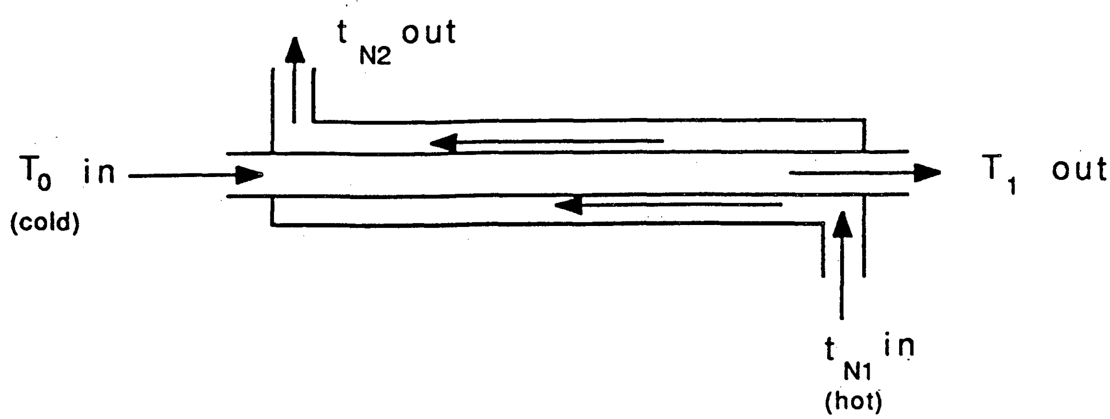

Figure 1.1 Countercurrent Configuration
(Source: D. B. Spaulding. 1988. Heat Exchanger Design Handbook. New York: Hemisphere Publication Corporporation.)

encounters hotter temperatures from the hot stream which flows right to left. These exchangers can obtain the greatest heat transfer between streams (Spaulding 1988). It is this efficiency that causes some problems for the countercurrent configuration built on a large scale. Thermal stress may develop within the tubes, shearing the tube joints and causing leaks. Furthermore, the rapid expansion and contraction of the tubes can create stress cracks in the tubes which may also lead to fluid leaks (Mueller 1954). As the repair of such leaks is usually quite expensive, industry has an aversion to the use of pure countercurrent heat exchangers. Integration of the shell-and-tube configuration with portions of the countercurrent design elements is the current preference (Cerda and Westerberg 1983). The result is an exchanger easier to maintain and cheaper to repair.

1.1.2 Shell-and-Tube Configuration

The second configuration, the shell-and-tube design, is the form most used for very large exchangers. The design reduces the thermal stress between the tubes, thus reducing the possibility of fluid leaks (Mueller 1954). Figure 1.2 illustrates a typical flow pattern of the fluids.

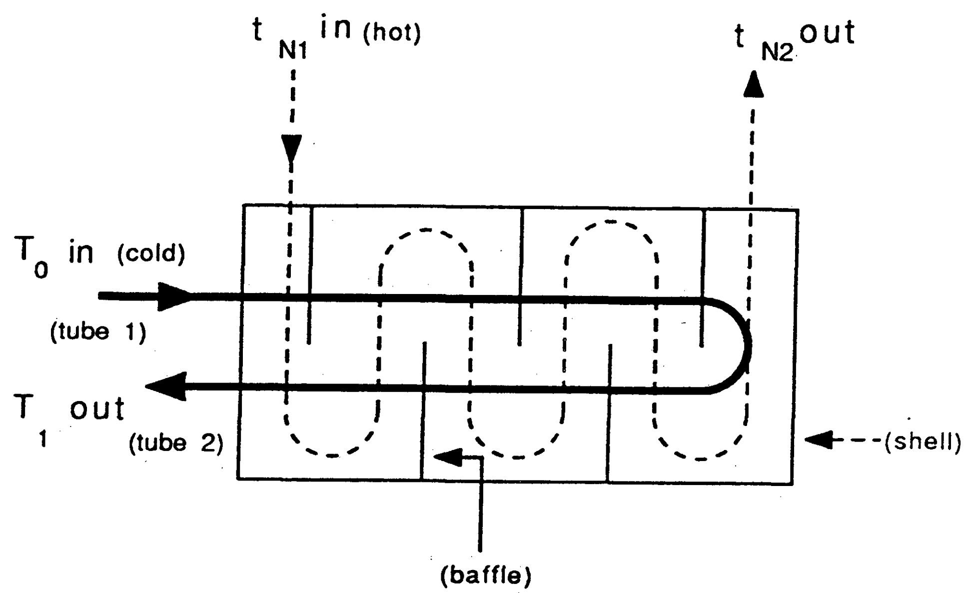

Figure 1.2 Shell-and-Tube Configuration
(Source: J. W. Lienhard. 1981. A Heat Transfer Textbook. New Jersey: Prentice Hall, Inc.)

Shell-and-tube heat exchangers are classified by the number of internal tubes and number of *passes* within the shell. The diagram above shows an industry standard 2-1, two tube passes and one shell pass, shell-and-tube configuration.

The shell-and-tube exchanger controls the hot stream's flow over the cold stream's flow using baffles which direct the hot flow over the cold stream's tubes. The baffles are designed to allow the hot flow to *mix* right or left over the cold stream's tube. This use of baffles, however, makes the shell-and-tube design more complicated. The flow patterns are complex and almost defy analysis. The flow, directed by the baffles, might leak through the baffle joints or even bypass the baffles near the wall if not properly designed, making the heat exchanger inefficient (Lienhard 1981).

The heat transfer with a shell-and-tube configuration can be just as efficient as a countercurrent exchanger if the design is good. The flow is longer and slower than in a countercurrent heat exchanger, lessening thermal stress (Lienhard 1981). Since heat is not exchanged as rapidly, the shell-and-tube configuration defuses the quick expansion and contraction within the tubes that occurs in the countercurrent design.

1.1.3 Cross-flow Configuration

The cross-flow heat exchanger configuration is similar to the shell-and-tube design. Instead of using baffles to direct the hot stream's flow over the cold stream's tubes, the cross-flow design relies on tubes. The cross-flow heat exchanger configuration avoids the *mixing* of the flows that occurs in the shell-and-tube design because the hot stream is directed by tubes, not baffles. Each flow, hot and cold, within the cross-flow design must stay in a prescribed path through the exchangeras directed by the tube design. The hot flow must follow a prescribed path over the cold flow. This does not allow the hot flow to *mix* to the right or left as is the case with the shell-and-tube design. Figure 1.3 is a typical cross-flow setup.

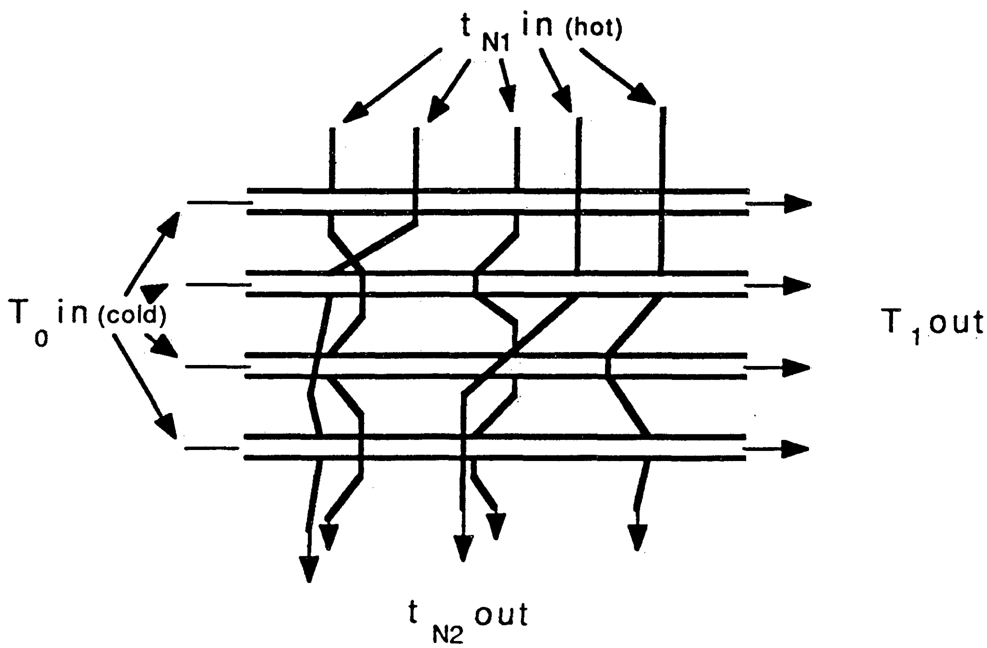

Figure 1.3 Cross-flow Configuration
(Source: J. W. Lienhard. 1981. A Heat Transfer Textbook. New Jersey: Prentice Hall, Inc.)

The cross-flow exchanger is seldom used in industry for chemical processing because this design is not as efficient as the first two discussed. The cross-flow exchanger is typically used in car radiators.

1.1.4 Study's Focus

This study focuses on the countercurrent configuration of heat exchanger design. Although the countercurrent configuration is not typically the sole exchanger design element in large-scale industrial applications, it is still used as a part to the overall system because of its efficiency. This paper will outline the steps needed to rapidly find the optimal heat transfer area in terms of the total area, as measured in square feet, needed for the construction of a multistage countercurrent heat exchanger system (CHES). The steps outlined in this solution require the foreknowledge of inlet temperatures of both the hot and cold streams, number of stages the heat exchanger needs, heat transfer coefficients, flow rate, and specific heat. This information will be discussed in the next section in more detail. It is because of the known design variables and the relationship of the unknown variables to the known variables that the solution to the CHES problem becomes one of finding the optimal area (Lienhard 1981).

1.2 The Design Variables

In designing a heat exchanger, one must be aware of the different optimization methods that can be used. An optimal design of the system in terms of heat exchange may not reflect the optimal design in cost. A smaller heat exchanger design may require higher temperatures for the initial inlet hot and cold streams to achieve the process objective. Preheating the two streams may raise operating costs over the projected budget. Therefore, a larger heat exchanger design with lower initial inlet temperatures may be desired.

This paper's main focus is the optimal design as defined by the minimum area needed for the heat exchange instead of minimum annual operating costs of the system. There are three reasons for this focus.

First, the mathematical problem statement is reduced when cost is not a design factor. This enables quick numerical solutions. This advantage is that the CHES algorithm limits the number of unknown variables and uses an efficient search technique. Secondly, the CHES algorithm is flexible. The construction of the algorithm allows the results to be analyzed independently or integrated into a network that does use cost. Finally, once the decision is made to build a heat exchanger, both capital and operating costs become a function of the design. Both costs influence the known controllable variables that affect the size of the area needed for the heat exchange. Cost, as a design input, is usually discussed in terms of capital and annual operating costs. Cost is discussed briefly in this and the following section, providing insight for conducting a comparative operating cost analysis. For now, the focus is on the design variables.

The design variables used in the CHES algorithm are the initial inlet temperature (T_0), the target ending temperature (T_N), the hot stream inlet temperature for each stage (t_{N1}), the heat transfer coefficients for each stage (U_i), the flow rate (w), and the specific heat (C_p). The optimal area of each stage of the CHES is determined by the above variables (Boas 1963). There exists a strong relationship between the amount of operating costs incurred and how hot the initial inlet temperatures of the exchanger must be.

The initial inlet temperature costs are influenced mostly by the type of heating system used, such as petroleum, natural gas, electricity, or solar energy. The tradeoffs from the optimal solution, using the CHES algorithm, are dependent on whether a lower initial inlet temperature and larger heat transfer area per stage could be used rather than a higher initial inlet temperature and smaller heat transfer area.

By using a larger area, the initial inlet streams to each stage do not need to be as hot as in a smaller heat exchanger system. In heating the initial streams (operating cost), the larger area would be cheaper than the smaller area. To examine which design is better suited for the user's application, both capital and operating costs must be considered.

If the focus of the heat exchanger system is on cost design, then the real annualized investment and annual operating costs must be considered. Cost functions should be used for the objective function when the goal is to minimize cost with respect to efficiency of the heat exchanger system. The use of these particular cost functions in the objective function creates a difficulty for the mathematical description of the problem (Cerda and Westerberg 1983). The use of the cost functions leads to a mixed integer nonlinear programming formulation that causes the objective function to be nonconvex within the feasible space. Additionally, this formulation also requires the inclusion of many variables in the model. (Grossmann and Sargent 1977). Optimizing the problem becomes difficult and a solution may not be possible. Techniques have been developed in the past few years to avoid the difficulty of nonconvexity in the feasible space.

1.3 Approaches Using Cost as an Input

Researchers have developed methods that *optimize* a heat exchanger system with respect to cost. The approaches are varied and some are built upon other models. The approaches used in the past include dynamic programming (Boas 1963), mixed integer linear programming (MILP), transshipment models (Lekary 1988, Cerda and Westerberg 1983) and network formulations (Linnhoff and Flower 1978). All of these models, except for the dynamic programming model, integrate cost into the formulation of the problem.

The desired goal of these models is to minimize the total expected cost of the heat exchanger. The total expected cost is the sum of the capital cost of the design and the expected annual operating cost. The approaches that used cost in the objective function only targeted the annual operating cost. There are three reasons for targeting annual operating cost alone.

First, the cost of the material does not represent a significant capital cost factor. Capital cost of the material used in the exchanger is controlled by the number of transfer units used in the design (Flower and Linnhoff 1980). Second, the variation of the size of the area of individual heat transfer units to account for different lower or higher initial inlet temperatures in the system, in terms of material used, is not significant for capital cost. Third, the expected rising costs of energy do indeed affect the design of the exchanger system significantly (Cerda and Westerberg 1983). The annual operating cost represents the cost to heat or cool the initial inlet temperatures to the design starting values. The economic network design of a heat exchanger system focuses only on minimizing the heating and cooling costs between the two streams (Grossmann and Sargent 1977, Flower and Linnhoff 1980).

1.3.1 Capital Cost

As mentioned, capital cost can be controlled by the number of transfer units needed in the design. Linnhoff and Hindmarsh (1983), have noted that the minimum number of heat transfer units needed in a system is a function of the number of processed streams minus one. They also found that this generality is violated when a *pinch* point is discovered in the heat exchanger system. This *pinch* point is analogous to a bottleneck in a production system. What happens when a *pinch* point exists is that the needed energy from the heat transfer area is not sufficient to complete a given task. An additional heat transfer unit is needed.

Grimes, Rychener, and Westerberg (1982) point out that one way to control the capital cost of the heat exchanger system is to minimize the total area required. They also point out that if the number of heat transfer units can be minimized, capital costs are reduced even more.

1.3.2 Annual Operating Cost

Anderson (1976) developed a method for annualizing the operating cost of heat exchanger systems for shell-and-tube designs using geometric programming. Anderson's method relies on knowing all the design variables. His method focuses only on shell-and-tube designs, but he provides insight on modifying the program to account for countercurrent heat exchanger systems. This information from the modification can be used to determine the annual cost for countercurrent heat exchanger systems.

Anderson's method does not focus its approach on the overall mechanical design process of the heat exchanger, nor does this paper. The strength in Anderson's approach is that given the design data, such as listed in section 1.2, from which the expected annual operating cost can be determined.

1.4 Objective

The objective of this study is to develop a method for finding the minimum heat transfer area needed in a design of a countercurrent heat exchanger system. The recent documented methods for determining the number of transfer units needed (capital cost) and the various methods for determining the operating cost will not be addressed in this dissertation. The author hopes to fill an intermediate bridge between these two cost methods by establishing an easier and faster method for determining the amount of heat transfer between the heat transfer stages by accurately computing the total area, as measured in square feet, needed for each stage.

The method employed here does not use cost for optimizing the total area needed for the heat exchanger. Instead, the solution to the problem focuses on finding the unknown temperatures of the cold stream being passed to the next transfer unit. The knowledge of the target ending temperature, the heat coefficients, flow rate, and specific heat influences the design.

Because the method addressed in chapter 2, when used with the computer program addressed in chapter 3, is extremely fast, the solution can be applied to a modified method developed by Anderson (1976) to determine the operating cost. Anderson's method also needed to find the temperatures of the processed streams. Since the CHES algorithm finds these temperatures, the now known temperatures can be used in a modified version of Anderson's method focusing on cost. The information gained allows the comparison of various heat exchanger systems in the more significant cost factor (operating cost). This study's method will also allow heat exchanger designers to vary the known variables to conduct sensitivity of the design size based on operating cost.

Chapter 2
RECURSION MADE SIMPLE

2.1 Introduction

The approach used in this study for developing a method capable of solving the heat exchange area, as measured in square feet for multistage countercurrent heat exchanger systems, has its roots in geometric programming (GP). Appendix A contains a brief synopsis of the history and use of GP (Knowles 1990).

The algorithm developed here uses GP to analyze the problem in its basic format. GP provided the author with insight into the structure of the problem that enabled the discovery of a recursion, or pattern, in the formulations for multistage countercurrent heat exchanger systems (CHES). To use the CHES algorithm, it is not really necessary to use GP. GP was initially used to evaluate the structure of the problem. This evaluation of the problem led to the discovery of a recursion which could be solved using known GP techniques. The GP techniques used to solve the CHES problem were built into the computer program. One should be able to understand the variable substitutions and the pattern of the recursion. Furthermore, it is important to be aware of the relationships the variable transformations have with the basic problem in order to develop a solution that the design engineer can interpret.

2.2 The Basic Problem Formulation

The diagram above represents the interaction between successive stages in a multistage CHES (Boas 1963). The initial cold stream inlet temperature, T_0, and the ending cold stream outlet temperature, T_N, are known. Additionally, the initial hot stream inlet temperatures, t_{11}, $t_{21}, \ldots,$ t_{N1}, into each of the stages are known. The unknown variables $T_1, \ldots, T_{N-1}$, represent the cold stream outlet temperatures from the preceding heat transfer area (A_i). The subscript i represents the number of stages (i equals 1 to N). The additional unknown variables $t_{12}, \ldots, t_{N2}$, represent the outlet hot stream temperatures from the heat transfer area (A_i). The solution of hot and cold stream unknown variables is used to determine the heat exchange area for each stage in square feet as well as the total area (A_T). The method developed in this chapter determines the values of all unknown variables.

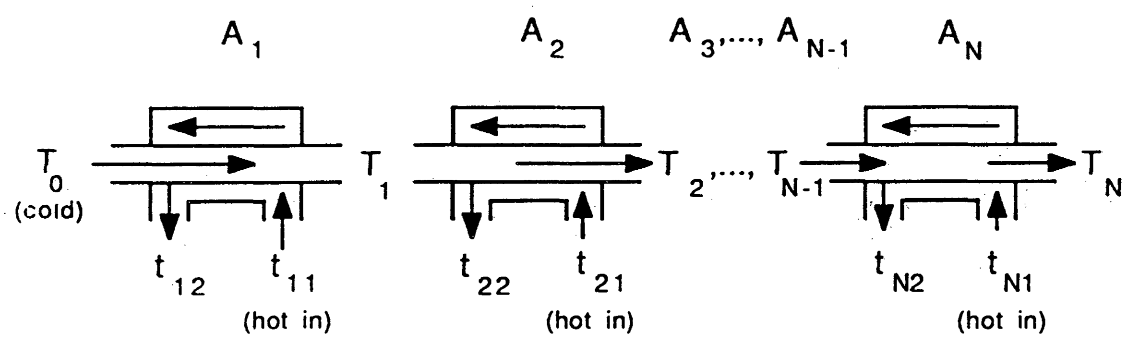

Figure 2.1 Multistage Countercurrent Heat Exchanger (Source: A. H. Boas. 1963. "Optimization Via Linear and Dynamic Programming." Chemical Engineering, vol. 70: 85-88)

There are three other variables to this problem that must also be considered. These variables are the flow rate (w), the specific heat (C_p) and the heat transfer coefficients (U_i). Both streams passing through the heat transfer areas, for this problem, have the same flow rate and specific heat in order to maintain a constant heat transfer coefficient in the given design. Controlling the flow rate and specific heat allows the overall heat transfer coefficients to be known constants (Avriel 1973). Since these terms are all known, their interaction with one another forms the heat transfer coefficient scaling factor ($\hat{U}$). The scaling factor $\hat{U}$ is determined by dividing the product of the flow rate and specific heat into the heat transfer coefficient.

The objective of the countercurrent heat exchanger, and other types of heat exchangers, is to heat the cold steam in each stage to a certain temperature in order to conduct some type of chemical process. This is accomplished by gradually raising the cold stream inlet temperatures, T_i, through a heat transference from the hot stream, t_{i1}, in each of the various stages. To optimize this problem, the goal becomes to find the smallest heat exchange area, as measured in square feet, needed for all the individual process stages (Lienhard 1981).

From these known and unknown values, the following geometric programming problem can be written as indicated in equation 2.1 (Avriel 1973). The variables have been discussed in the preceding paragraphs and the constraints will be discussed in detail in section 2.2.2.

$$\begin{aligned}
&Minimize \quad A_T \\
&subject \quad to \\
&\frac{\sum_{i=1}^{N} A_i}{A_T} \leq 1 \\
&\frac{T_i + t_{i2}}{t_{i1} + T_{i-1}} \leq 1 \\
&\frac{T_i + T_{i-1}\hat{U}_i A_i}{T_{i-1} + \hat{U}_i A_i t_{i2}} \leq 1
\end{aligned} \tag{2.1}$$

where

All variables are positive

$i = 1 \quad to \quad N$

$\hat{U}_i = \frac{U_i}{wC_p}$

A_T *is the total heat exchange area in* ft^2

$T_i, \; t_{i1}, \; t_{i2}$ *units are* $°F$

wC_p *units are* $(BTU/(hr(°F))$

U_i *units are* $(BTU/(hr(sq.ft.)(°F))$

As the problem is formulated, there will be cancellation of the units of measure so that the final problem is to find the minimum area of the heat exchanger in terms of square feet (Boas 1963). There are many terms and three unknown variables per stage, except for the first and last stage. The first and last stages have only two unknown variables because either the starting or ending temperature is known. Therefore, this model has a multiple degree of difficulty. To begin to reduce the number of terms and variables, the objective function must first be examined.

2.2.1 The Alternative Objective Function Form

To solve this problem using GP, it is necessary to reduce the degrees of difficulty of the problem. One method is to reduce the number of terms and constraints. To accomplish this task, examination of the objective function is in order.

The objective function in equation 2.1 is a single-term expression consisting of the variable A_T. In examining the constraints, A_T appears only in the first constraint. By focusing only on the objective function and the first constraint, the following GP sign table as developed by Woolsey (1985) is used. Term 1 (T1) refers to the objective function, terms 2,...,N (T2,...,TN) represent the variables $A_1,\ldots,A_N$, and term N+1 (TN+1) is the A_T term in the constraint.

VARIABLE	T1	T2	...	T4	TN+1
A_T	+				-
A_1		+			
. . .			+		
A_N				+	

Analysis of this table indicates that A_T is balanced in the constraint, indicating that the constraint is not an inequality but an equality. Therefore, the objective function and the first constraint are

$$Minimize \quad A_T$$

$$subject \quad to$$

$$\frac{\sum_{i=1}^{N} A_i}{A_T} = 1 \tag{2.2}$$

This is an important point. Knowing equation 2.2, the model can be rewritten as

$$Minimize \quad \sum_{i=1}^{N} A_i$$

$$subject \quad to$$

$$\frac{T_i + t_{i2}}{t_{i1} + T_{i-1}} \leq 1$$

$$\frac{T_i + T_{i-1}\hat{U}_i A_i}{T_{i-1} + \hat{U}_i A_i t_{i2}} \leq 1 \tag{2.3}$$

$$where$$

$$All \quad variables \quad are \quad positive$$

$$i = 1 \quad to \quad N$$

$$\hat{U}_i = \frac{U_i}{wC_p}$$

To reduce the model further, the relationships among the constraints need to be thoroughly examined.

2.2.2 Sign Table Analysis of the Constraints

The model has two constraints for each stage of the CHES. As the constraints are written in equation 2.3, the use of GP sign table analysis does not help. Each constraint first needs to be examined to determine the functional relationship with the objective function and secondly, reviewed mathematically.

The first constraint for each stage of the model, designated by

$$\frac{T_i + t_{i2}}{t_{i1} + T_{i-1}} \leq 1 \tag{2.4}$$

refers to the heat balances in each stage of the exchanger system. This constraint expresses the relationship that the amount of heat exchanged to raise the temperature of the cold stream, T_{i-1} to T_i, is less than or equal to the amount of heat loss to lower the temperature of the hot stream, t_{i1} to t_{i2}.

Examining and rewriting the constraint provides an alternative form.

$$\frac{T_i + t_{i2}}{t_{i1} + T_{i-1}} \leq 1$$

$$T_i + t_{i2} \leq t_{i1} + T_{i-1}$$

$$T_i - T_{i-1} \leq t_{i1} - t_{i2}$$

$$T_i - T_{i-1} + t_{i2} \leq t_{i1} \tag{2.5}$$

The second series of constraints designated by

$$\frac{T_i + T_{i-1}\hat{U}_i A_i}{T_{i-1} + \hat{U}_i A_i t_{i2}} \leq 1 \tag{2.6}$$

reflects the minimum areas needed in which the desired heat exchange will take place given the influence of the flow and the pressure rates in a system. Basically, the constraint informs the user that the difference in the beginning and ending temperatures of the cold stream, (T_{i-1} and T_i) is a function of the product between the heat transfer coefficient scaling factor ($\hat{U}$), the size of the area (A_i), and the difference between the hot outlet stream (t_{i2}) and the cold inlet stream (T_{i-1}). This important relationship will be discussed in more detail in section 2.3.

As with the first constraint for each stage, the second constraint can be rewritten.

$$\frac{T_i + T_{i-1}\hat{U}_i A_i}{T_{i-1} + \hat{U}_i A_i t_{i2}} \leq 1$$

$$T_{i-1}\hat{U}_i A_i - \hat{U}_i A_i t_{i2} \leq T_{i-1} - T_i \tag{2.7}$$

Using the objective function from equation 2.3 and equations 2.5 and 2.7 for the constraints, the following GP problem is formulated.

$$\begin{aligned}
&\textit{Minimize} \quad \sum_{i=1}^{N} A_i \\
&\textit{subject to} \\
&T_i - T_{i-1} + t_{i2} \leq t_{i1} \\
&T_{i-1}\hat{U}_i A_i - \hat{U}_i A_i t_{i2} \leq T_{i-1} - T_i \qquad (2.8) \\
&\textit{where} \\
&\textit{All variables are positive} \\
&i = 1 \ \textit{to} \ N \\
&\hat{U}_i = \frac{U_i}{wC_p}
\end{aligned}$$

Using equation 2.8, sign table analysis is used to examine the generic structure of the problem. The sign table used for this analysis was developed by Woolsey (1985). Term 1 represents the objective function, terms 2-5 represent the first constraint, and terms 6-9 represent the second constraint. Because each stage is constructed in the same manner, a generic sign table for a multistage CHES can be used.

VARIABLE	T-1	T-2	T-3	T-4	T-5	T-6	T-7	T-8	T-9
A_i	+					+	-		
T_i		+							-
T_{i-1}			-			+		+	
t_{i2}					+		-		+

Analysis of this table indicates that A_i is balanced in term 1 and term 7. The variable T_i is balanced in term 2 and term 9. The variable T_{i-1} is balanced with terms 3 and 6 or 8 while the variable t_{i2} is balanced in terms 5 and 7. Using this inference from the sign table, it can be concluded that the constraints are in fact equalities instead of inequalities. This is confirmed by other documentation in the theory of CHES (Holland 1970, Schenck 1959).

Because of this knowledge, the GP problem is now

$$Minimize \quad \sum_{i=1}^{N} A_i$$

subject to

$$T_i - T_{i-1} + t_{i2} = t_{i1}$$

subject to

$T_i.$ (2.9)

$$T_i - T_{i-1} + t_{i2} = t_{i1}$$

wl

$$T_{i-1}\hat{U}_i A_i - \hat{U}_i A_i t_{i2} = T_{i-1} - T_i$$

All variables are positive

$$i = 1 \quad to \quad N$$

$$\hat{U}_i = \frac{U_i}{wC_p}$$

2.3 A Relationship Exploited

Since it has now been determined that all the constraints are equalities, substitutions of the variables can be used to reduce the model's degree of difficulty. From equation 2.5, there exists the following relationship.

$$T_i + t_{i2} = t_{i1} + T_{i-1}$$

$$t_{i2} = t_{i1} + T_{i-1} - T_i \quad (2.10)$$

From equation 2.7, a relationship can be determined for A_i measured in square feet.

$$T_i + T_{i-1}\hat{U}_i A_i = \hat{U}_i A_i t_{i2} + T_{i-1}$$

$$T_i - T_{i-1} = A_i(\hat{U}_i t_{i2} - T_{i-1}\hat{U}_i)$$

$$A_i = \frac{T_i - T_{i-1}}{\hat{U}_i(t_{i2}) - T_{i-1}(\hat{U}_i)} \quad (2.11)$$

Substituting from equation 2.10 into 2.11, the final relationship for A_i can be determined.

$$A_i = \frac{T_i - T_{i-1}}{\hat{U}_i(t_{i1} + T_{i-1} \quad - \quad T_i) - T_{i-1}(\hat{U}_i)}$$

$$A_i = \frac{T_i - T_{i-1}}{\hat{U}_i(t_{i1} - T_i)}$$

$$A_i = \frac{T_i - T_{i-1}}{\hat{U}_i(t_{i1}) - \hat{U}_i(T_i)} \tag{2.12}$$

Exploiting the relationship of A_i from equation 2.15, the model can now be reduced to the following expression.

$$\textit{Minimize} \quad \sum_{i=1}^{N} \frac{T_i - T_{i-1}}{\hat{U}_i(t_{i1}) - \hat{U}_i(T_i)} \tag{2.13}$$

measured in square feet

2.4 The Recursion Discovered

From equation 2.13, the overall multistage CHES model has been reduced significantly. However, in its current form it still is difficult to solve because of the expression in the denominator. The next step is to do single variable substitution for the expressions in the denominator to assist in the geometric programming solution. This sets up a small recursion as indicated.

$$Let \quad Z_1 = (\hat{U}_1(t_{11}) - \hat{U}_1(T_1)) \Rightarrow T_1 = t_{11} - \frac{Z_1}{\hat{U}_1}$$

$$Let \quad Z_2 = (\hat{U}_2(t_{21}) - \hat{U}_2(T_2)) \Rightarrow T_2 = t_{21} - \frac{Z_2}{\hat{U}_2}$$

$$Let \quad Z_3 = (\hat{U}_3(t_{31}) - \hat{U}_3(T_3)) \Rightarrow T_3 = t_{31} - \frac{Z_3}{\hat{U}_3}$$

$$Let \quad Z_{N-1} = (\hat{U}_{N-1}(t_{(N-1)1}) - \hat{U}_{N-1}(T_{N-1})) \Rightarrow T_{N-1} = t_{(N-1)1} - \frac{Z_{N-1}}{\hat{U}_{N-1}}$$

The substitution recursion stops at Z_{N-1} because of the known values of the final stage.

Now the individual Z_i relationships can be substituted into equation 2.13.

$$Minimize \quad \frac{t_{11} - \frac{Z_1}{\hat{U}_1} - T_0}{Z_1} + \frac{t_{21} - \frac{Z_2}{\hat{U}_2} - t_{11} + Z_1}{Z_2}$$
$$+ \frac{t_{31} - \frac{Z_3}{\hat{U}_3} - t_{21} + Z_2}{Z_3} + \ldots + \frac{T_N - t_{N1} + \frac{Z_{N-1}}{\hat{U}_{N-1}}}{\hat{U}_N(t_{N1} - T_N)}$$

Expanding and collecting terms yields the following final equation.

$$Minimize \quad \frac{t_{11} - T_0}{Z_1} + \frac{t_{21} - t_{11}}{Z_2} + \frac{Z_1}{\hat{U}_1(Z_2)} + \frac{t_{31} - t_{21}}{Z_3} + \frac{Z_2}{\hat{U}_2(Z_3)}$$
$$+ \ldots + \frac{Z_{N-1}}{\hat{U}_{N-1}\hat{U}_N(t_{N1} - T_N)} + \frac{T_N - t_{(N-1)1}}{\hat{U}_N(t_{N1} - T_N)} - \left(\frac{1}{\hat{U}_1} + \frac{1}{\hat{U}_2} + \frac{1}{\hat{U}_3} + \ldots + \frac{1}{\hat{U}_{N-1}}\right)$$

Let C_1 equal the sum of the inverse $\hat{U}_i$ since the $\hat{U}_i$ are known constants, then the final form can be determined.

$$Minimize \quad \frac{t_{11}-T_0}{Z_1}+\frac{t_{21}-t_{11}}{Z_2}+\frac{Z_1}{\hat{U}_1(Z_2)}+\frac{t_{31}-t_{21}}{Z_3}+\frac{Z_2}{\hat{U}_2(Z_3)}$$

$$+\ldots+\frac{Z_{N-1}}{\hat{U}_{N-1}\hat{U}_N(t_{N1}-T_N)}+\frac{T_N-t_{(N-1)1}}{\hat{U}_N(t_{N1}-T_N)}-C_1 \qquad (2.14)$$

This form is the final formulation of the multistage CHES algorithm. What this formulation demonstrates is that for every increase in the number of stages, the problem increases by one degree of difficulty. In other words, a two-stage problem is a zero degree of difficulty problem, a three-stage is a 1 degree of difficulty problem, a four-stage is a 2 degree of difficulty problem and a multistage is a N-2 degree of difficulty problem. To demonstrate, the following sections examine a two, three and four-stage CHES.

2.4.1 Two-Stage CHES

The initial objective function formulation of the two-stage CHES from the developed algorithm is

$$Minimize \quad \frac{T_1-T_0}{\hat{U}_1(t_{11}-T_1)}+\frac{T_2-T_1}{\hat{U}_2(t_{21}-T_2)} \qquad (2.15)$$

Graphically, this formulation is depicted in figure 2.2.

Substitute the following variables into the expresssion,

$$Z_1=\hat{U}_1(t_{11}-T_1) \Rightarrow T_1=t_{11}-\frac{Z_1}{\hat{U}_1}$$

$$K_1=\hat{U}_2(t_{21}-T_2)$$

where

K_1 *is a constant*

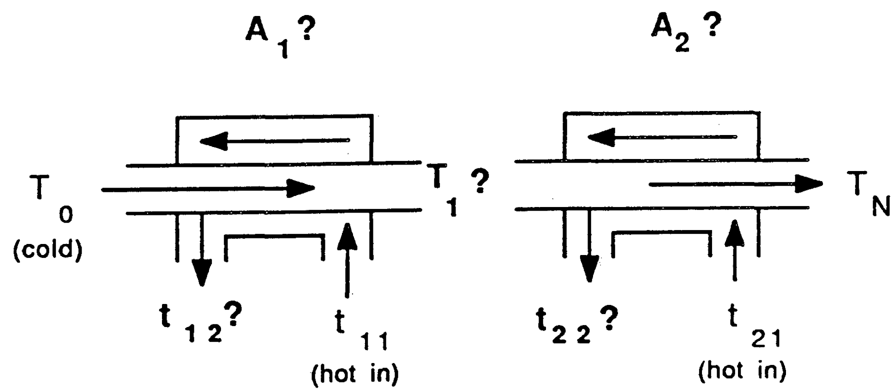

Figure 2.2 Two-stage CHES
(Adapted from: A. H. Boas. 1963. "Optimization Via Linear and Dynamic Programming." Chemical Engineering, vol. 70: 85-88)

and the expression becomes

$$Minimize \quad \frac{t_{11}-\frac{Z_1}{\hat{U}_1}-T_0}{Z_1}+\frac{T_2-t_{11}+\frac{Z_1}{\hat{U}_1}}{K_1} \tag{2.16}$$

Expand equation 2.16 and the expression becomes

$$Minimize \quad \frac{t_{11}-T_0}{Z_1}-\frac{1}{\hat{U}_1}+\frac{T_2-t_{11}}{K_1}+\frac{Z_1}{K_1\hat{U}_1} \tag{2.17}$$

Substitute in the following variables,

$$K_2=t_{11}-T_0$$

$$K_3=\frac{T_2-t_{11}}{K_1}-\frac{1}{\hat{U}_1}$$

and the expression becomes

$$Minimize \quad \frac{K_2}{Z_1}+\frac{Z_1}{K_1\hat{U}_1}+K_3 \tag{2.18}$$

There are two terms and a variable (Z_1) in equation 2.18. The third term (K_3) is a constant and does not influence the calculation for the degree of difficulty. Since two terms minus one variable minus one equals zero, the degree of difficulty for equation 2.18 is zero.

2.4.2 Three-Stage CHES

The initial formulation of the three-stage CHES is

$$Minimize \quad \frac{T_1-T_0}{\hat{U}_1(t_{11}-T_1)}+\frac{T_2-T_1}{\hat{U}_2(t_{21}-T_2)}+\frac{T_3-T_2}{\hat{U}_3(t_{31}-T_3)} \tag{2.19}$$

Graphically, this formulation is represented in figure 2.3.
Substitute in the following variables,

$$Z_1=\hat{U}_1(t_{11}-T_1) \Rightarrow T_1=t_{11}-\frac{Z_1}{\hat{U}_1}$$

$$Z_2=\hat{U}_2(t_{21}-T_2) \Rightarrow T_2=t_{21}-\frac{Z_2}{\hat{U}_2}$$

$$K_1=\hat{U}_3(t_{31}-T_3)$$

where

K_1 *is a constant*

and the expression becomes

$$Minimize \quad \frac{t_{11}-\frac{Z_1}{\hat{U}_1}-T_0}{Z_1}+\frac{t_{21}-\frac{Z_2}{\hat{U}_2}-t_{11}+\frac{Z_1}{\hat{U}_1}}{Z_2}+\frac{T_3-t_{21}+\frac{Z_2}{\hat{U}_2}}{K_1} \tag{2.20}$$

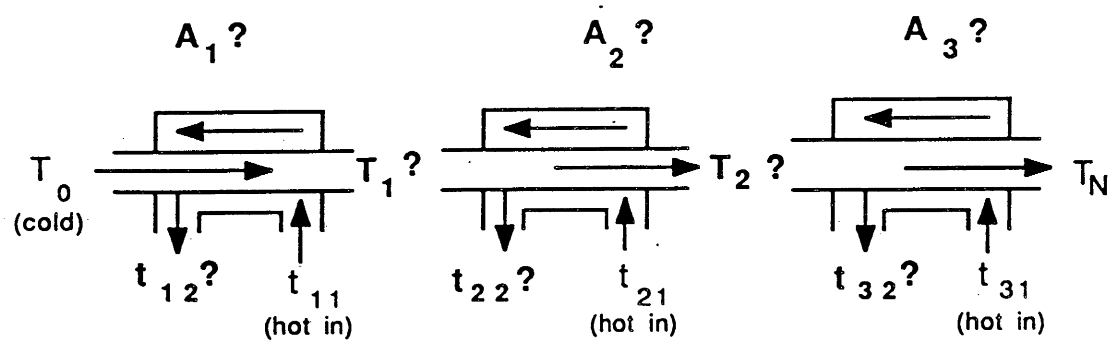

Figure 2.3 Three-stage CHES
(Adapted from: A. H. Boas. 1963. "Optimization Via Linear and Dynamic Programming." Chemical Engineering, vol. 70: 85-88)

Expanding equation 2.20, the expression becomes

$$\textit{Minimize} \quad \frac{t_{11}-T_0}{Z_1}-\frac{1}{\hat{U}_1}+\frac{t_{21}-t_{11}}{Z_2}+\frac{Z_1}{\hat{U}_1 Z_2}-\frac{1}{\hat{U}_2}+\frac{T_3-t_{21}}{K_1}+\frac{Z_2}{K_1\hat{U}_2} \qquad (2.23)$$

Substitute in the following variables,

$$K_2=t_{11}-T_0$$

$$K_3=t_{21}-t_{11}$$

$$K_4=\hat{U}_2 K_1$$

$$K_5=\frac{T_3-t_{21}}{K_1}-\frac{1}{\hat{U}_1}-\frac{1}{\hat{U}_2}$$

and the expression becomes

$$\textit{Minimize} \quad \frac{K_2}{Z_1}+\frac{K_3}{Z_2}+\frac{Z_1}{\hat{U}_1 Z_2}+\frac{Z_2}{K_4}+K_5 \qquad (2.22)$$

There are only two variables (Z_1 and Z_2) and four terms in equation 2.22. The last term (K_5) is a constant and does not influence the calculation for the degree of difficulty. Since four terms minus two variables minus one equals one, the degree of difficulty for equation 2.22 is one.

2.4.3 Four-Stage CHES

The initial formulation of the four-stage CHES is

$$\text{Minimize} \quad \frac{T_1 - T_0}{\hat{U}_1(t_{11} - T_1)} + \frac{T_2 - T_1}{\hat{U}_2(t_{21} - T_2)} + \frac{T_3 - T_2}{\hat{U}_3(t_{31} - T_3)} + \frac{T_4 - T_3}{\hat{U}_4(t_{41} - T_4)} \tag{2.23}$$

This is represented graphically in figure 2.4.

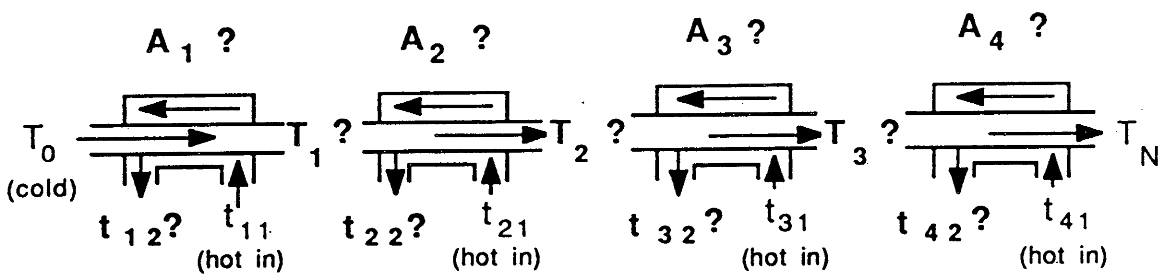

Figure 2.4 Four-stage CHES
(Adapted from: A. H. Boas. 1963. "Optimization Via Linear and Dynamic Programming." Chemical Engineering, vol. 70: 85-88)

Substitute in the following variables,

$$Z_1 = \hat{U}_1(t_{11} - T_1) \Rightarrow T_1 = t_{11} - \frac{Z_1}{\hat{U}_1}$$

$$Z_2 = \hat{U}_2(t_{21} - T_2) \Rightarrow T_2 = t_{21} - \frac{Z_2}{\hat{U}_2}$$

$$Z_3 = \hat{U}_3(t_{31} - T_3) \Rightarrow T_3 = t_{31} - \frac{Z_3}{\hat{U}_3}$$

$$K_1 = \hat{U}_4(t_{41} - T_4)$$

where

K_1 *is a constant*

and the expression becomes

$$\text{Minimize} \quad \frac{t_{11} - \frac{Z_1}{\hat{U}_1} - T_0}{Z_1} + \frac{t_{21} - \frac{Z_2}{\hat{U}_2} - t_{11} + \frac{Z_1}{\hat{U}_1}}{Z_2} + \frac{t_{31} - \frac{Z_3}{\hat{U}_3} - t_{21} + \frac{Z_2}{\hat{U}_2}}{Z_3} + \frac{T_4 - t_{31} + \frac{Z_3}{\hat{U}_3}}{K_1} \tag{2.24}$$

Expanding equation 2.24, the expression becomes

$$\text{Minimize} \quad \frac{t_{11} - T_0}{Z_1} - \frac{1}{\hat{U}_1} + \frac{t_{21} - t_{11}}{Z_2} + \frac{Z_1}{\hat{U}_1 Z_2} - \frac{1}{\hat{U}_2} + \frac{t_{31} - t_{21}}{Z_3} + \frac{Z_2}{\hat{U}_2 Z_3} - \frac{1}{\hat{U}_3} + \frac{T_4 - t_{31}}{K_1} + \frac{Z_3}{K_1 \hat{U}_3} \tag{2.25}$$

Substitute in the following variables,

$$K_2 = t_{11} - T_0$$

$$K_3 = t_{21} - t_{11}$$

$$K_4 = t_{31} - t_{21}$$

$$K_5 = \hat{U}_3 K_1$$

$$K_5 = \frac{T_4 - t_{31}}{K_1} - \frac{1}{\hat{U}_1} - \frac{1}{\hat{U}_2} - \frac{1}{\hat{U}_3}$$

and the expression becomes

$$Minimize \quad \frac{K_2}{Z_1} + \frac{K_3}{Z_2} + \frac{Z_1}{\hat{U}_1 Z_2} + \frac{K_4}{Z_3} + \frac{Z_2}{\hat{U}_2 Z_3} + \frac{Z_3}{K_5} + K_6 \qquad (2.26)$$

There are only three variables (Z_1, Z_2 and Z_2) and six terms in equation 2.26. The last term (K_6) is a constant and does not influence the calculation for the degree of difficulty. Since six terms minus three variables minus one equals two, the degree of difficulty for equation 2.26 is two.

2.5 Multivariable Condensation as a Solution Technique

Normally, the larger degree of difficulty and the presence of multiple variables would still represent problems in solving larger stage CHES. This is not the case with this formulation. Research by Ratliff in 1986 developed a multivariable GP condensation method which can be applied to posynomial expressions without constraints. Sections 2.4.1, 2.4.2, and 2.4.3 clearly indicated that the CHES algorithm formulates the problem as a posynomial without constraints.

The knowledge of Ratliff's multivariable condensation method, discussed in detail in chapter 3, and the CHES algorithm allowed the development of a computer program capable of solving multistage CHES. Chapter 3 details the construction of a modified multivariable condensation program and explains how it is used. The program uses the principles as outlined by Ratliff and a computer program developed by Clayton (1990) and modified by this author for use in this paper's research.

Chapter 3
COMPUTER SOLUTION

3.1 Introduction

The increased use of computers in recent years, and the difficulty of finding the solutions to heat exchangers conventionally, have stimulated the growth of computer-aided solutions. From 1960 through 1990, different formulations of the heat exchanger problem have led to the development of computer techniques to find solutions. Some of these models are discused in the following subsections.

3.1.1 Mixed Integer Linear Program Models

Andrecovich and Westerberg (1985) developed a mathematical formulation of a heat-integrated distillation sequence synthesis that included the need to find the solution to a heat exchanger network based on a shell-and-tube design. The method formulated the problem as a mixed integer linear program (MILP). The algorithm they developed was computationally fast and focused on the economically preferred solution. The MILP model is designed to be used both as a stand-alone model or as a part in a larger model.

One of the strategies behind the use of the MILP model consisted of converting other functions formulated as mixed integer program (MIP) problems to linear forms (Lekary 1988). It was noted by Lekary that the MILP formulation took an advantage from the MIP by reducing the number of binary variables. The reduction of the binary variables was able to speed up the search for the remaining variables by reducing the bottleneck in the search area, making the MILP model computationally fast.

3.1.2 Transshipment Models

The transshipment model was used to minimize the utility cost associated with a heat exchanger problem. Lekary describes the setup of the transshipment model by regarding heat as a commodity that is shipped from hot streams to the cold streams. The exchange takes place through the temperature differences of the two streams. The constraints in the original CHES formulation represent the thermodynamics of the transfer of heat. Lekary, for example, cites from the second law of thermodynamics that heat flows from the higher to lower temperatures has to be modeled as a constraint. Cost is modeled as the associated value for the *shipping* of the heat between the hot and cold streams.

A second transshipment model was also used to discover the minimum number of heat exchanger units needed. This second model was based upon the knowledge from previous work that indicated that the number of units used will normally correspond to the optimal or near optimal solution based upon cost (Lekary). Before this second transshipment model can be used, however, the first transshipment model must be used to find the minimum utility cost.

3.1.3 Dynamic Programming Models

Boas (1963) applied dynamic programming to the solution of the countercurrent heat exchanger model. The principle behind Boas's solution is the interaction between each of the stages of the countercurrent heat exchanger system. Unlike the previously discussed models, economics is not used in finding the solution. Instead, this model was used to find the minimum area needed for heat transfer to take place.

Boas points out that the use of constraints in the basic countercurrent heat exchanger model is helpful when applying dynamic programming techniques because the constraints define the search area for each variable. The disadvantage, pointed out by Boas, is that this method may require a search for a large number of unknown variables. The optimization process can become quite complex because each stage in the heat exchanger is dependent on previous values of unknown variables.

3.1.4 Previous Geometric Programming Models

Avriel, Rijckaert, and Wilde (1973) applied geometric programming to the countercurrent heat exchanger model used by Boas. The formulation of this model was discussed in detail in chapter 2. Using an experimental code developed by Rusin and Crane, they were able to duplicate the results from Boas in a more efficient manner. This author unsuccessfully searched the literature for the computer code. Since Rusin and Crane developed the code for the Mobil Research and Development Corporation, the author suspects the code may be proprietary.

3.2 Geometric Programming Multivariable Condensation Model

The computer solution developed for this dissertation was constructed using the program MULTICON developed by Ratliff in 1982. MULTICON searches for the solution to unknown variables in an unconstrained nonlinear programming objective function format. The program is further restricted to the use of posynomial objective functions. That is, only objective functions with all positive coefficients are able to use this code.

An unpublished paper by Ratliff and Woolsey (1989) describes MULTICON's search method. Briefly, the method has its roots in geometric programming's single variable condensation search method (Ratliff 1982). As with the single variable condensation technique, each variable is *pushed* to its minimum value with a verification against the minimum objective function value. As one variable is *pushed* to its minimum value, the next variable enters the basis and is also *pushed* to its minimum value. The process repeats itself with each variable, always checking the new minimum variable values against the minimum objective function value. The search rechecks the variables and objective function in a cyclic manner until the convergence condition is met. The program developed for this paper is divided into three sections: input, MULTICON condensation, and output modules.

3.2.1 Input Module

The input module is currently programmed to handle up to twenty stages. The user needs the following information to input the problem.

1) The flow's specific heat (wC_p) is the product of the flow rate (w) and specific heat (C_p). The prompt is wCp.

2) The initial starting values of the cold stream inlet temperature (T_0) is prompted by T0.

3) The target ending temperature of the cold stream (T_N) is prompted by TN.

4) The number of stages (N) is prompted by N.

5) The heat transfer coefficient (U) is prompted by U(N).

6) The initial hot stream inlet temperatures (t_{N1}) for each stage are prompted by t(N1).

In the module's current form, the flow rate (w) and the specific heat (C_p) are assumed to be constant in each stage. Modification, if necessary, will be discussed in the section 3.3. Inputs are converted into the form discussed in detail in chapter 2. The conversion of the input data allows MULTICON to solve for all Z_i variables.

3.2.2 MULTICON Condensation Module

The MULTICON condensation module was written by Clayton (1990) for use in his thesis. It was modified and streamlined by this author to fit the countercurrent heat exchanger model. The condensation module still conducts the search for the unknown variables as described in section 3.2.

The convergence check is preset. The current setting is a 0.0001 level of accuracy. Both the variables and objective values must meet this criterion. Once the accuracy level is satisfied, the program passes the values for the Z_i variables to the output module.

3.2.3 Output Module

The output module builds upon the known relationships to the unknown values as discussed in chapter 2. It processes the Z_i variables to the previously unknown inlet cold stream temperature values passed to each successive stage. It also calculates the Z_i variables for the total heat exchange area, as measured in square feet, for each stage as well as for the total heat exchange area, also measured in square feet, of the countercurrent heat exchanger system.

3.3 Modifications to the Program

Appendix B contains a copy of the program. The program was written in Microsoft's Quick Basic, version 4.5. As this author does not claim to be a computer programmer, the code may not be as efficient as it could be.

Currently, the program will accept up to a twenty-stage countercurrent heat exchanger system. If more stages are desired, the user can modify the dimension statements appropriately.

As mentioned in section 3.2.1, the flow rate and specific heat are assumed to be constant in each stage. If this is not the case, the program can be modified to accept different flow rates and specific heats for each individual stage.

3.4 Computational Examples

To provide the user with input, computation and output examples, the two-stage, three-stage, and four-stage CHES discussed in chapter 2 will be used. Each example will demonstrate the known values and their relationship to the unknown values.

3.4.1 Two-Stage CHES Example

The computer program will process the known information to solve the CHES problem. Recalling section 3.2.1, the user must have information on the heat transfer coefficients (U_i), the product of the flow rate and the specific heat (wC_p), the initial inlet cold stream temperature (T_0), the ending temperature of the cold stream (T_N), the incoming hot stream temperatures for each stage (t_{i1}), and the number of stages (N). For this example problem, these are

$$wC_p = 100000 \quad (Btu./(Hr.)(°F))$$

$$U_1 = 120 \quad (Btu./(Hr.)(Sq.Ft.)(°F))$$

$$U_2 = 80 \quad (Btu./(Hr.)(Sq.Ft.)(°F))$$

$$T_0 = 100 \quad (°F)$$

$$T_N = 295 \quad (°F)$$

$t_{11}=300 \quad (°F)$

$t_{21}=400 \quad (°F)$

$N=2$

Located in appendix C is the computer run of the input and output of this problem. This example uses the equations listed in section 2.4.1 to give the reader an idea of how the computation is occurring in the computer program.

After the data is processed by the user, the program first computes the specific heat transfer coefficient scaling factor ($\hat{U}_i$) for each stage. For this problem, these computations are

$$\hat{U}_1=\frac{120}{100000} \Rightarrow \hat{U}_1=0.0012$$

$$\hat{U}_2=\frac{80}{100000} \Rightarrow \hat{U}_2=0.0008$$

From equation 2.15, the initial formulation of the two-stage CHES problem is

$$Minimize \quad \frac{T_1-T_0}{\hat{U}_1(t_{11}-T_1)}+\frac{T_2-T_1}{\hat{U}_2(t_{21}-T_2)}$$

Using the input numbers, the expression is

$$Minimize \quad \frac{T_1-100}{0.0012(300-T_1)}+\frac{295-T_1}{0.0008(400-295)}$$

The computer program processes the known information to make the substitution variables. These substitution variables are

$$Z_1 = 0.0012(300 - T_1) \Rightarrow T_1 = 300 - \frac{Z_1}{0.0012}$$

$$K_1 = 0.0008(400 - 295) \Rightarrow K_1 = 0.084$$

From equation 2.17 and the substitution variables, the expression becomes

$$Minimize \quad \frac{300-100}{Z_1} - \frac{1}{0.0012} + \frac{295-300}{0.084} + \frac{Z_1}{0.084(0.0012)}$$

Processing a second set of constants

$$K_2 = 300 - 100 \Rightarrow K_2 = 200$$

$$K_3 = \frac{295-300}{0.084} - \frac{1}{0.0012} \Rightarrow K_3 = -892.86$$

and the expression is now

$$Minimize \quad \frac{200}{Z_1} + \frac{Z_1}{0.084(0.0012)} - 892.86 \tag{3.1}$$

Equation 3.1 is processed through the MULTICON module as described in section 3.2.2. For this example, Z_1 was found to be 0.141984. The value Z_1 is passed to the output module to determine the unknown temperature of the cold stream (T_1). The values for Z_1 and T_1 are used to determine the areas for the two stages (A_1 and A_2). For this example, the calculations are as follows:

Recalling $Z_1 = \hat{U}_1(t_{11} - T_1)$, *then*

$$T_1 = t_{11} - \frac{Z_1}{\hat{U}_1} = 181.68 \quad (°F)$$

$$A_1 = \frac{T_1 - T_0}{Z_1} = 575.28 \quad (Sq.Ft.)$$

$$A_2 = \frac{T_2 - T_1}{\hat{U}_2(t_{21} - T_2)} = 1349.07 \quad (Sq.Ft.)$$

$$A_T = A_1 + A_2 = 1924.33 \quad (Sq.Ft.)$$

3.4.2 Three-Stage CHES Example

For the three-stage example problem, the following information was used. Appendix D is the completed computer run.

$wC_p = 100000 \quad (Btu./(Hr.)(°F))$

$U_1 = 120 \quad (Btu./(Hr.)(Sq.Ft.)(°F)$

$U_2 = 80 \quad (Btu./(Hr.)(Sq.Ft.)(°F)$

$U_3 = 40 \quad (Btu./(Hr.)(Sq.Ft.)(°F)$

$T_0 = 100 \quad (°F)$

$T_N = 500 \quad (°F)$

$t_{11} = 300 \quad (°F)$

$t_{21} = 400 \quad (°F)$

$t_{31} = 600 \quad (°F)$

$N = 3$

The specific heat transfer coefficient scaling factors ($\hat{U}_i$) for each stage are

$$\hat{U}_1 = \frac{120}{100000} \Rightarrow \hat{U}_1 = 0.0012$$

$$\hat{U}_2 = \frac{80}{100000} \Rightarrow \hat{U}_2 = 0.0008$$

$$\hat{U}_3 = \frac{40}{100000} \Rightarrow \hat{U}_3 = 0.0004$$

The initial formulation of the three-stage CHES problem from equation 2.19 is

$$\text{Minimize} \quad \frac{T_1 - T_0}{\hat{U}_1(t_{11} - T_1)} + \frac{T_2 - T_1}{\hat{U}_2(t_{21} - T_2)} + \frac{T_3 - T_2}{\hat{U}_3(t_{31} - T_3)}$$

Using the input numbers, the expression becomes

$$\text{Minimize} \quad \frac{T_1 - 100}{0.0012(300 - T_1)} + \frac{T_2 - T_1}{0.0008(400 - T_2)} + \frac{500 - T_2}{0.0004(600 - 500)}$$

The computer processing of the known information to provide substitute variables is

$$Z_1 = 0.0012(300 - T_1) \Rightarrow T_1 = 300 - \frac{Z_1}{0.0012}$$

$$Z_2 = 0.0008(400 - T_2) \Rightarrow T_2 = 400 - \frac{Z_2}{0.0008}$$

$$K_1 = 0.0004(600 - 500) \Rightarrow K_1 = 0.04$$

From the above relationships and equation 2.21, the expression is now

$$\text{Minimize} \quad \frac{300 - 100}{Z_1} - \frac{1}{0.0012} + \frac{400 - 300}{Z_2} + \frac{Z_1}{0.0012(Z_2)} - \frac{1}{0.0008} + \frac{500 - 400}{0.04} + \frac{Z_2}{0.04(0.0008)}$$

and consolidating the expression yields

$$\text{Minimize} \quad \frac{200}{Z_1}+\frac{100}{Z_2}+\frac{Z_1}{0.0012(Z_2)}+\frac{Z_2}{0.000032}+416.666667 \tag{3.2}$$

Equation 3.2 is processed through the MULTICON module as described in section 3.2.2. For this example, Z_1 was found to be 0.14166 and Z_2 was 0.083528. These values are passed to the output module. The values for Z_1 and Z_2 are used to calculate the temperatures for T_1 and T_2. The found values for Z_i's and T_i's are used to compute the area for the three stages (A_1, A_2, and A_3). For this example, the calculations are as follows:

Recalling $Z_1=\hat{U}_1(t_{11}-T_1)$ *and* $Z_2=\hat{U}_2(t_{21}-T_2)$, *then*

$$T_1=t_{11}-\frac{Z_1}{\hat{U}_1}=181.95 \quad (°F)$$

$$A_1=\frac{T_1-T_0}{Z_1}=578.52 \quad (Sq.Ft.)$$

$$T_2=t_{21}-\frac{Z_2}{\hat{U}_2}=295.59 \quad (°F)$$

$$A_2=\frac{T_2-T_1}{Z_2}=1360.36 \quad (Sq.Ft.)$$

$$A_3=\frac{T_3-T_2}{\hat{U}_3(t_{31}-T_3)}=5110.36 \quad (Sq.Ft.)$$

$$A_T=A_1+A_2+A_3=7049.24 \quad (Sq.Ft.)$$

The results for the three-stage CHES example problem correspond to other work by Boas (1963) and Avriel, Rijckaert, and Wilde (1973), based upon the same input values using different optimization techniques.

3.4.3 Four-Stage CHES Example

For the four-stage example problem, the following information was used. Appendix E is the completed computer run.

$wC_p = 100000 \quad (Btu./(Hr.)(°F)$

$U_1 = 120 \quad (Btu./(Hr.)(Sq.Ft.)(°F)$

$U_2 = 80 \quad (Btu./(Hr.)(Sq.Ft.)(°F)$

$U_3 = 40 \quad (Btu./(Hr.)(Sq.Ft.)(°F)$

$U_4 = 20 \quad (Btu./(Hr.)(Sq.Ft.)(°F)$

$T_0 = 100 \quad (°F)$

$T_N = 500 \quad (°F)$

$t_{11} = \quad (°F)300$

$t_{21} = \quad (°F)400$

$t_{31} = \quad (°F)600$

$t_{41} = \quad (°F)700$

$N = 4$

The specific heat transfer coefficient scaling factors ($\hat{U}_i$) for each stage are

$$\hat{U}_1 = \frac{120}{100000} \Rightarrow \hat{U}_1 = 0.0012$$

$$\hat{U}_2 = \frac{80}{100000} \Rightarrow \hat{U}_2 = 0.0008$$

$$\hat{U}_3 = \frac{40}{100000} \Rightarrow \hat{U}_3 = 0.0004$$

$$\hat{U}_4 = \frac{20}{100000} \Rightarrow \hat{U}_4 = 0.0002$$

The initial formulation of the four-stage CHES problem from equation 2.23 is

$$\text{Minimize}\quad \frac{T_1-T_0}{\hat{U}_1(t_{11}-T_1)}+\frac{T_2-T_1}{\hat{U}_2(t_{21}-T_2)}+\frac{T_3-T_2}{\hat{U}_3(t_{31}-T_3)}+\frac{T_4-T_3}{\hat{U}_4(t_{41}-T_4)}$$

and using the input numbers, the expression becomes

$$\text{Minimize}\quad \frac{T_1-100}{0.0012(300-T_1)}+\frac{T_2-T_1}{0.0008(400-T_2)}+\frac{T_3-T_2}{0.0004(600-T_3)}+\frac{500-T_3}{0.0002(700-500)}$$

The computer processes known information to provide substitute variables:

$$Z_1=0.0012(300-T_1)\Rightarrow T_1=300-\frac{Z_1}{0.0012}$$

$$Z_2=0.0008(400-T_2)\Rightarrow T_2=400-\frac{Z_2}{0.0008}$$

$$Z_3=0.0004(600-T_3)\Rightarrow T_3=600-\frac{Z_3}{0.0004}$$

$$K_1=0.0002(700-500)=K_1=0.04$$

From the above relationships and equation 2.25, the expression becomes

$$\text{Minimize}\quad \frac{300-100}{Z_1}-\frac{1}{0.0012}+\frac{400-300}{Z_2}+\frac{Z_1}{0.0012(Z_2)}-\frac{1}{0.0008}$$

$$+\frac{600-400}{Z_3}+\frac{Z_2}{0.0008(Z_3)}-\frac{1}{0.0004}+\frac{500-600}{0.04}+\frac{Z_3}{0.04(0.0004)}$$

and consolidating the expression yields

$$\text{Minimize} \quad \frac{200}{Z_1}+\frac{100}{Z_2}+\frac{Z_1}{0.0012(Z_2)}+\frac{200}{Z_3}$$

$$+\frac{Z_2}{0.0008(Z_3)}+\frac{Z_3}{0.000016}-7083.33333 \qquad (3.3)$$

Equation 3.3 is processed through the MULTICON module as described in section 3.2.2. For this example, Z_1 was 0.163548, Z_2 was 0.166464, and Z_3 was 0.113708. These values are passed to the output module. The values for Z_1, Z_2, and Z_3 are used to calculate the temperatures for T_1, T_2, and T_3. The determined values for Z_i's and T_i's are used to compute the area for the four stages (A_1, A_2, A_3, and A_4). For this example, the calculations are as follows:

Recalling $Z_1=\hat{U}_1(t_{11}-T_1)$, $Z_2=\hat{U}_2(t_{21}-T_2)$, *and* $Z_3=\hat{U}_3(t_{31}-T_3)$, *then*

$$T_1=t_{11}-\frac{Z_1}{\hat{U}_1}=163.71 \quad (°F)$$

$$A_1=\frac{T_1-T_0}{Z_1}=389.52 \quad (Sq.Ft.)$$

$$T_2=t_{21}-\frac{Z_2}{\hat{U}_2}=191.92 \quad (°F)$$

$$A_2=\frac{T_2-T_1}{Z_2}=169.52 \quad (Sq.Ft.)$$

$$T_3=t_{31}-\frac{Z_3}{\hat{U}_3}=315.73 \quad (°F)$$

$$A_3=\frac{T_3-T_2}{Z_3}=1088.84 \quad (Sq.Ft.)$$

$$A_4=\frac{T_4-T_3}{\hat{U}_4(t_{41}-T_4)}=4606.67 \quad (Sq.Ft.)$$

$$A_T=A_1+A_2+A_3+A_4=6254.55 \quad (Sq.Ft.)$$

Chapter 4

CONCLUSION AND AREAS FOR FURTHER RESEARCH

4.1 Conclusion

The objectives of this dissertation were

1) to develop a method for finding the minimum heat transfer area needed for multistage countercurrent heat exchangers,

2) to develop a computer program capable of finding the solution to the method developed, and

3) to develop a method that can be used, or be modified, into existing economic models.

These objectives have been met. Through the use of advanced geometric programming principles, a generalized method was discovered for finding the solution to the multistage countercurrent heat exchangers. A computer program based on geometric programming was written to find the solution to the problem. The model is sufficiently flexible to be integrated into existing economic models.

The first of these three objectives was met by converting the countercurrent heat exchanger model into a single posynomial objective function. The second objective was met by exploiting Ratliff's MULTICON program. The final form developed in the methodology and examples sections outlined in chapter 2 aptly fitted the needed posynomial conditions for Ratliff's MULTICON program. The author was therefore able to construct an input module that transformed the countercurrent heat exchanger model into a useable form. The values of the transformed variables found by MULTICON were later transcribed in the output module of the program, enabling the information to be written in terms that a design engineer can interpret. Finally, the model allows the quick determination of the unknown values of the cold stream outlet temperatures. Since most economic models rely on these values to determine cost, the CHES algorithm is flexible enough to be integrated into previously developed economic models.

The method and computer program developed presents a significant breakthrough in the design process for countercurrent heat exchangers. Previous work found that the countercurrent heat exchanger problem was difficult to formulate in a manner which would provide insight to the construction of a computer code. As a result, the old formulation forced the solution to the countercurrent heat exchanger to be found with trial and error methods (Lienhard 1981). Additionally, previous work in the heat exchanger optimization area focused only on shell-and-tube heat exchanger configurations. The advent of this document now allows mixing of the heat exchanger configuration modules within a design system. This greatly enhances the existing economic models which previously only focused on the shell-and-tube configurations.

4.2 Areas for Further Research

The method developed solves only for the total area as measured in square feet for heat exchange based upon the optimization the heat transfer area. Although Anderson (1976) developed a method for computing annual operating cost for a heat exchanger, his method focused only on shell-and-tube designs. As mentioned in chapter 2, Anderson did provide some insight on how to modify his method to account for countercurrent heat exchangers, but to date this has not been accomplished.

A further area of research would be to integrate the method described in this document with the method of cost estimation by Anderson. This would provide a single source for computing cost and total squared area of countercurrent heat exchangers. In addition to this integration of computer codes and methods, further study could be conducted on how to integrate this paper's research into the economic methods discussed in chapter 2. This will allow the CHES algorithm to be integrated into currently used economic models.

The principal contribution of this work is the insight gained in CHES models in terms of economic modeling. Before this work, the designing of countercurrent heat exchangers was attempted only by trial-and-error methodology in terms of engineering requirements alone. This work establishes a feasible basis for constructing a cost-driven design of CHES models.

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Appendix A

Geometric Program Background

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Geometric Program Background

Background

Clarence Zener developed GP in the early 1960s. Zener, then director of science at Westinghouse, was joined by Richard Duffin, a mathematics professor at Carnegie Mellon University, and Elmor Peterson, a Ph.D. student of Duffin's. Their comprehensive study published in 1967 (Duffin, Peterson, and Zener) was instrumental in exploiting the relationship between the geometric and arithmetic means based upon Cauchy's arithmetic mean-geometric mean inequality.

Thome (1987) provided a simple explanation for the arithmetic mean-geometric mean inequality. Simply stated, the arithmetic mean of a group of terms is always greater than or equal to the geometric mean of the same group of terms. It is this concept that provides the basic building block for GP. The mathematical proof to this statement is found within the reference by Duffin, Peterson, and Zener.

Geometric Programming

Geometric programming takes advantage of the arithmetic mean-geometric mean inequality to optimize a certain class of nonlinear minimization models. However, the perceived limitations of GP, that it can be applied only to posynomials or that GP is not effective with signomials, are reasons that GP is not within the mainstream use of current optimization techniques (Peterson 1976; Dembo 1978; Ecker 1980).

To solve a problem with GP, the problem must first be placed into proper form. The conventional GP form consists of an objective function to be minimized. If the objective function is constrained, then the constraints must be written in the form of less than or equal to one. The following is a simple example of GP form.

$$\text{MIN } Z = 5X^{-2} + 4Y^{-3}$$

$$\textit{subject to}$$

$$X + Y \leq 1 \tag{1.1}$$

The example also demonstrates another important characteristic of the GP form. The objective function must be a posynomial. In other words, all the coefficients are positive and all exponents are real numbers. This is in contrast to a signomial equation. The term signomial refers to problems in which the coefficients are negative and all exponents are real numbers. A signomial objective function represents a difficulty to conventional GP techniques (Ecker 1980; Woolsey 1988). Corrections for signomial objective functions are discussed in section for special techniques in GP.

The phrase "degrees of difficulty" is another way of classifying GP problems. The degree of difficulty is defined by counting the number of terms in the objective function and constraints, subtracting the total number of variables from the total amount of terms, and then subtracting one. Referring to equation 1.1, two terms are in the objective function and two terms are in the constraint, making the total number of terms four. There are only two variables (X,Y). Therefore, the degree of difficulty of this problem computes to four terms, minus two variables, minus one, equals one. This simple problem equates to a one degree of difficulty problem.

Conventional GP, as defined by Duffin, Peterson, and Zener, requires zero degree of difficulty problems. Wessels (1989) lists three further assumptions that limit the usefulness of conventional geometric programming:

(1) All variables must be positive.

(2) The objective function value must be strictly positive.

(3) All the constraints are actually equalities, i.e., all the constraints are active and binding.

As documented by Wessels, GP experienced rapid innovations during its early years. Since 1976, GP principles have been applied to numerous other models for optimization; see, for example, Smeers and Tyteca (1984); Wall, Greening, and Woolsey (1986); and Dinkel and Tretter (1987). However, despite these applications and advances, standard operations research (OR) manuals still do not emphasize the importance of GP. Current OR textbooks such as Walsh (1975), Dennis and Schnable (1983), and Luenberger (1984), ignore GP as a subject.

Special Techniques in Geometric Programming

Wessels (1989) noted that difficulties in applied conventional GP occur under any of the following conditions:

(1) Multiple degrees of difficulty exist.

(2) The objective function is a signomial.

(3) There are numerous constraints to the model.

(4) Some of the constraints are not binding; i.e., there is slack in some of the right-hand side values.

(5) The objective function has a negative value at optimality.

(6) Some variables have negative values at optimality.

To correct for some of the difficulties noted, various techniques have been developed. The approaches vary, but each technique corrects for a difficulty and allows the model to be optimized by GP. Beightler and Phillips (1976) devoted a textbook to the subject.

One technique, condensation, based upon the arithmetic mean-geometric mean inequality, has proved to be extremely effective. Beighter and Phillips explain this process in detail. A simplified overview provided by Wessels (1989) states that condensation is the process by which two or more terms in a problem are combined into a single multiplicative term. Condensation involves choosing weights for the weighted arithmetic and geometric means in order to solve the equality condition of the arithmetic mean-geometric mean inequality satisfactorily. Ratliff (1986) further documented a generalized algorithm to solve unconstrained balanced posynomials with condensation. Condensation is a proven technique for finding optimality (Greening 1982).

Signomials can be solved by condensation after transformation into posynomials. Through the process known as the Greening Transformation and Condensation Technique (GTCT), Thome (1987) was successful in solving a large class of nonlinear, unconstrained, signomial economic models of multiple degrees of difficulty.

Kirk (1988) documented an algorithm for preprocessing problems into a form capable of being solved by GP. By rewriting the objective function and the constraints, the Kirk algorithm enables the problem to be solved through conventional GP or by use of one of the developed special applications for solving problems with GP. The Kirk algorithm reduces large problems to smaller problems to assist the solver in recognizing the class of the problem.

Wessels (1989) tackled the problem of optimizing models with multiple binding and non-binding constraints. By enhancing the sign table, identifying the binding constraints, and transforming the problem into what Wessels refers to as the all constraint form, he was able to reduce the degrees of difficulty of the problem. The problem is then solved by finding the dual feasible solution and backing out the values of the primal variables. Using the feasible values of the primal variables as an initial solution, the Wessel algorithm then employs condensation to find the final optimum value.

Wilde's Sign Table Analysis

When there is an optimization model with numerous constraints, determining which constraints are active, not really there, or binding can be extremely difficult. Wilde (1978) created a diagnostic tool, referred to as the sign table, based on monotonicity analysis to help determine which constraints are active or binding. The Monotonicity Theorem allows the user to determine where a differentiable function increases or decreases (Purcell 1987). This allows the user to determine the maximum or minimum value of a function. As an addendum to this analysis, Kirk (1988) documented a system for eliminating unnecessary constraints thereby reducing the scope of Wilde's Sign Table and allowing the user an easier tool for analysis.

Wilde's Sign Table is constructed by placing the value of the exponent, by term, into a tabular format. As Wessels (1989) noted, Wilde's Sign Table is constructed from the orthogonality constraints on the dual variables, that is, the exponent value of the variable in each individual term. This is best demonstrated by an example.

Consider the following example (Woolsey 1988):

$$\text{MIN} \quad Z = 2x10^5 LWHY^{-1} + 4x10^6 LH + 1.6x10^3 LHW + 1.8x10^5 X^{-1}$$

subject to

$$(14x10^3)^{-1} XH^{-1} + \left(\frac{7x10^2}{14x10^3}\right) H^{-1} \leq 1$$

$$17HW^{-1} \leq 1$$

$$27HY \leq 1 \qquad (2.1)$$

Wilde's Sign Table for monotonicity would be defined as follows:

	T-1	T-2	T-3	T-4	T-5	T-6	T-7	T-8
L	1	1	1					
W	1		1				-1	
H	1	1	1		-1	-1	1	1
Y	-1							1
X				-1	1			

To understand this table, an explanation of the column and row headings is required. The column headings refer to the terms in the equation. Example, T-1 refers to the objective function's first term and T-6 refers to the first constraint's second term. The row headings are the variables that appear throughout the model. The numbers in the table are the exponent value of that particular variable in that stated term. Blank spaces indicate that the variable was not present in the indicated term.

The interpretation of Wilde's Sign Table indicates that in order to bound equation 2.1 for the variable *Y*, it would be necessary to use the objective function and the third constraint. The logic behind this interpretation stems from the fact that the variable *Y* in the objective function has a negative exponent term. The variable *Y* achieves a balance, that is, a positive exponent term, in the third constraint. Similar logic applies to bound the other variables in equation 2.1.

Woolsey's Sign Table Analysis

Woolsey (1985) simplified the Wilde's Sign Table by dropping the numerical value of the exponent term. Furthermore, Woolsey accounts for the coefficient's sign value by multiplying the coefficients' sign with the exponent's sign. The column and row headings on the Woolsey's Sign Table are the same as with Wilde's Sign Table. The column headings refer to the terms in the objective function and constraints. T-1 starts the headings and represents the first term in the objective function. T-8 ends the headings and represents the last term in the last constraint. Woolsey's Sign Table for equation 2.1 is as follows:

	T-1	T-2	T-3	T-4	T-5	T-6	T-7	T-8
L	+	+	+					
W	+		+				-	
H	+	+	+		-	-	+	+
Y	-							+
X				-	+			

The advantage to Woolsey's Sign Table is that it is quick to interpret. A person using GP as an analytic tool can quickly see where terms balance. An additional strength of this table is that it can account for signomials because the table takes into account the coefficient's sign before placing the value into the table.

Appendix B

Computer Program Listing

Appendix B

Computer Program Listing

```
REM INPUT MODULE 17 MAY 90
10 CLS : CLEAR ALL
DEFDBL A-Z
DIM TLN(20, 1), T(20), U(20), UHAT(20)
DIM POWER(20, 20), PLSPOW(20), PSCOF(20), COEFF(20), XNW(20)
DIM NGCOEF(20), DLTA(20), CNDPOW(2), CDCOF(20), WTPLS(20)
DIM PWNEG(20), WGTNEG(20), VAR$(20)
REM INIT OF VARIABLES
VH = 0: ITER = 0
REM INFO PARAGRAPH
PRINT "THIS IS THE INPUT MODULE FOR CHES."
PRINT "YOU WILL NEED TO KNOW THE HEAT TRANSFER
COEFFICIENTS (U'S),"
PRINT "INPUT TEMPERATURE (T0), ENDING TEMPERATURE (TN) AND
THE "
PRINT "SPECIFIC HEAT AND FLOW RATE (w*Cp)."
REM INPUTS
INPUT "WHAT IS THE SPECIFIC HEAT (w*Cp)"; Cp
INPUT "WHAT IS INCOMING TEMPERATURE OF THE COLD STREAM
(T0)"; T0
```

```
INPUT "WHAT IS THE OUTGOING TEMPERATURE OF THE FINAL STAGE
(TN)"; TN
INPUT "WHAT ARE THE NUMBER OF STAGES"; N
REM POWER INITIALIZATION
NTERMS = N + N - 1
FOR I = 1 TO NTERMS
   FOR J = 1 TO NTERMS
      POWER(I, J) = 0
   NEXT J
NEXT I
FOR I = 1 TO N
   PRINT : PRINT
   PRINT "WHAT IS THE HEAT COEFFICIENT OF U("; I; ")": INPUT ; U(I)
   PRINT
   PRINT "WHAT IS THE INCOMING TEMPERATURE OF THE HOT
STREAM [t("; I; ",1)]"
   : INPUT ; TLN(I, 1)
NEXT I
FOR I = 1 TO N
   UHAT(I) = U(I) / Cp
NEXT I
REM VARIABLE CONVERSION TO BOB'S CODE
REM INIT VAR ESTIMATE
NVARS = N - 1
```

```
FOR I = 1 TO NVARS
   VAR$(I) = "Z" + STR$(I) + ""
   XOLD(I) = 1
   XNW(I) = XOLD(I)
NEXT I
POWER(1, 1) = -1
POWER(2, 2) = -1
FOR I = 3 TO NTERMS - 2
   IF I MOD 2 > 0 THEN
     POWER(I, (I - 1) / 2) = 1
     POWER(I, (I + 1) / 2) = -1
   ELSE
     POWER(I, (I + 2) / 2) = -1
     END IF
NEXT I
COEFF(1) = TLN(1, 1) - T0
COEFF(2) = TLN(2, 1) - TLN(1, 1)
IF I > 2 THEN
  FOR I = 3 TO NTERMS - 2
     IF I MOD 2 = 0 THEN
        COEFF(I) = TLN(I - 1, 1) - TLN(I - 2, 1)
     ELSE
        COEFF(I) = 1 / UHAT(N - 2)
```

```
      END IF
   NEXT I
END IF

REM DEFN FOR LAST VAR TERM
POWER(NTERMS - 1, N - 1) = 1
COEFF(NTERMS - 1) = 1 / (UHAT(N - 1) * UHAT(N) * (TLN(N, 1) - TN))
REM DEFN FOR CONSTANT
CONSTANT = (TN - TLN(N - 1, 1)) / (UHAT(N) * (TLN(N, 1) - TN))
SUM = 0
FOR I = 1 TO N - 1
   SUM = SUM + 1 / UHAT(I)
NEXT I
COEFF(NTERMS) = CONSTANT - SUM
155 TL = .0001
175 CLS
'PRINT "CURRENT STARTING VECTOR": PRINT
'PRINT "VAR NUM", "VAR", "VALUE"
'FOR I = 1 TO NVARS
      'PRINT I, VAR$(I), XOLD(I)
'NEXT I
'INPUT "IF CORRECT, HIT <ENTER>, OTHERWISE ENTER NUMBER"; AA
'IF AA <= NVARS AND AA > 0 THEN
      'PRINT "ENTER NEW VALUE FOR "; VAR$(AA);
```

```
    'INPUT " :", XOLD(AA)
    'GOTO 175
'END IF
GOSUB 640
REM FINAL RESULTS
REM CALCULATION FOR OUTLET TEMPERATURES
FOR I = 1 TO N
  IF I < N THEN
    T(I) = TLN(I, 1) - XOLD(I) / UHAT(I)
  ELSE
    T(I) = TN
  END IF
NEXT I
REM CALCULATION FOR INDIVIDUAL STAGES
FOR I = 1 TO N
  STAGE$(I) = "STAGE" + STR$(I) + ""
  IF I = 1 THEN
    T(I - 1) = T0
  END IF
  AREA(I) = (T(I) - T(I - 1)) / (UHAT(I) * (TLN(I, 1) - T(I)))
NEXT I
REM CALCULATION FOR TOTAL AREA SQUARED
TOTAL = O
```

```
FOR I = 1 TO N
   TOTAL = AREA(I) + TOTAL
NEXT I
REM PRINT FINAL RESULTS
PRINT : PRINT : PRINT "    INLET TEMPS   STAGE #      AREA
SQUARED"
A$ = "  ##########.##    \          \    #########.##"
PRINT USING A$; T0
FOR I = 1 TO N
   PRINT USING A$; T(I);
   PRINT STAGE$(I);
   PRINT USING A$; AREA(I)
NEXT I
PRINT : PRINT USING "TOTAL AREA SQUARED OF THE SYSTEM IS " +
A$; TOTAL
REM NEW PROBLEM CYCLE
PRINT : INPUT "DO ANOTHER PROBLEM"; A$
IF A$ = "Y" OR A$ = "y" THEN
  GOTO 10
END IF


END
```

```
REM *** CONDENSE POS1+ ***
180 CTEMP = 1
PTEMP = 0
SUM = 0
WSUM = 0
FOR I = 1 TO NPLS
    SUM = SUM + PSCOF(I) * XOLD(VAR) ^ PLSPOW(I)
NEXT I
FOR I = 2 TO NPLS
    WTPLS(I) = PSCOF(I) / SUM * (XOLD(VAR) ^ PLSPOW(I))
    WSUM = WSUM + WTPLS(I)
    PTEMP = PTEMP + WTPLS(I) * PLSPOW(I)
    CTEMP = CTEMP * (PSCOF(I) / WTPLS(I)) ^ WTPLS(I)
NEXT I
WTPLS(1) = 1 - WSUM
CDCOF(1) = CTEMP * (PSCOF(1) / WTPLS(1)) ^ WTPLS(1)
CNDPOW(1) = PTEMP + PLSPOW(1) * WTPLS(1)
RETURN
REM *** CONDENSE NEG1 ***
360 CTEMP = 1
PTEMP = 0
SUM = 0
WSUM = 0
FOR I = 1 TO NNEG
    SUM = SUM + NGCOEF(I) * XOLD(VAR) ^ PWNEG(I)
```

```
NEXT I
FOR I = 2 TO NNEG
    WGTNEG(I) = NGCOEF(I) / SUM * (XOLD(VAR) ^ PWNEG(I))
    WSUM = WSUM + WGTNEG(I)
    PTEMP = PTEMP + WGTNEG(I) * PWNEG(I)
    CTEMP = CTEMP * (NGCOEF(I) / WGTNEG(I)) ^ WGTNEG(I)
NEXT I
WGTNEG(1) = 1 - WSUM
CDCOF(2) = CTEMP * (NGCOEF(1) / WGTNEG(1)) ^ WGTNEG(1)
CNDPOW(2) = PTEMP + PWNEG(1) * WGTNEG(1)
RETURN
REM  *** POS1 ***
540 CNDPOW(1) = PLSPOW(1)
CDCOF(1) = PSCOF(1)
WTPLS(1) = 1
RETURN
REM *** NEG 1 ***
590 CNDPOW(2) = PWNEG(1)
CDCOF(2) = NGCOEF(1)
WGTNEG(1) = 1
RETURN
REM  *** MAIN LOOP ***
640 CLS
VLUE = 0
645 PRINT " CALCULATING TOTAL AREA "
```

```
650 FOR VAR = 1 TO NVARS
NPLS = 0
NNEG = 0
FOR I = 1 TO NTERMS
    IF (POWER(I, VAR) <= 0) THEN 750
    NPLS = NPLS + 1
    PLSPOW(NPLS) = POWER(I, VAR)
    PSCOF(NPLS) = COEFF(I)
    FOR K = 1 TO NVARS
        IF (VAR <> K) THEN
           PSCOF(NPLS) = XOLD(K) ^ POWER(I, K) * PSCOF(NPLS)
        END IF
    NEXT K
750   IF (POWER(I, VAR) >= 0) THEN 815
    NNEG = NNEG + 1
    PWNEG(NNEG) = POWER(I, VAR)
    NGCOEF(NNEG) = COEFF(I)
    FOR K = 1 TO NVARS
        IF (VAR <> K) THEN
           NGCOEF(NNEG) = XOLD(K) ^ POWER(I, K) *
NGCOEF(NNEG)
        END IF
    NEXT K
815 NEXT I
REM  *** CONDENSE POSITIVE TERMS ***
```

```
830 IF (NPLS > 1) THEN GOSUB 180
IF (NPLS = 1) THEN GOSUB 540
REM   *** CONDENSE NEGATIVE TERMS ***
IF (NNEG > 1) THEN GOSUB 360
IF (NNEG = 1) THEN GOSUB 590
REM *** CALCULATE NEW DELTAS ***
DLTA(2) = 1 / (1 - CNDPOW(2) / CNDPOW(1))
DLTA(1) = 1 - DLTA(2)
REM *** CALC NEW VALUE OF VAR ***
920 FUNVL = (CDCOF(1) / DLTA(1)) ^ DLTA(1) * (CDCOF(2) / DLTA(2)) ^
DLTA(2)
XNW(VAR) = (FUNVL * DLTA(1) / CDCOF(1)) ^ (1 / CNDPOW(1))
REM *** CHK FOR VAR CONVERGENCE ***
955 XH = ABS(XNW(VAR) - XOLD(VAR)) / XOLD(VAR)
IF (ABS(XNW(VAR) - XOLD(VAR)) / XOLD(VAR) <= TL) THEN 1000
XOLD(VAR) = XNW(VAR)
ITER = ITER + 1
GOSUB 1269 'CALC CURRENT OFV
GOTO 830
1000 VLUE = 0
FOR I = 1 TO NTERMS
    TEMP = 1
    FOR J = 1 TO NVARS
        TEMP = XOLD(J) ^ POWER(I, J) * TEMP
    NEXT J
```

```
    VLUE = COEFF(I) * TEMP + VLUE
NEXT I
NEXT VAR
REM *** CHK FOR PROB CONVERG ***
IF ((ABS(VH - VLUE) / VLUE) <= TL) THEN RETURN
VH = VLUE
GOTO 650
REM *** CALC CURRENT FUNCTION VALUE ***
1269 VLUE = 0
FOR I = 1 TO NTERMS
    TEMP = 1
    FOR J = 1 TO NVARS
        TEMP = XOLD(J) ^ POWER(I, J) * TEMP
    NEXT J
    VLUE = COEFF(I) * TEMP + VLUE
NEXT I
RETURN
```

Appendix C

Two-Stage Example Problem

Appendix C

Two-Stage Example Problem

```
THIS IS THE INPUT MODULE FOR CHES.
YOU WILL NEED TO KNOW THE HEAT TRANSFER COEFFICIENTS (U'S),
INPUT TEMPERATURE (T0), ENDING TEMPERATURE (TN) AND THE
SPECIFIC HEAT AND FLOW RATE (w*Cp).
WHAT IS THE SPECIFIC HEAT (w*Cp)?  100000
WHAT IS THE INCOMING TEMPERATURE OF THE COLD STREAM (T0)?
100
WHAT IS THE OUTGOING TEMPERATURE OF THE FINAL STAGE (TN)?
295
WHAT ARE THE NUMBER OF STAGES?  2

WHAT IS THE HEAT COEFFICIENT OF U( 1 )
?  120
WHAT IS THE INCOMING TEMPERATURE OF THE HOT STREAM [t( 1 ,1)]
?  300

WHAT IS THE HEAT COEFFICIENT OF U( 2 )
?  80
WHAT IS THE INCOMING TEMPERATURE OF THE HOT STREAM [t( 2 ,1)]
?  400

CALCULATING TOTAL AREA

INLET TEMPS   STAGE #   AREA SQUARED

   100.00

   181.68     STAGE 1   575.28

   295.00     STAGE 2  1349.07


TOTAL AREA SQUARED OF THE SYSTEM IS        1924.33


DO ANOTHER PROBLEM?
```

Appendix D

Three-Stage Example Problem

Appendix D

Three-Stage Example Problem

```
THIS IS THE INPUT MODULE FOR CHES.
YOU WILL NEED TO KNOW THE HEAT TRANSFER COEFFICIENTS (U'S),
INPUT TEMPERATURE (T0), ENDING TEMPERATURE (TN) AND THE
SPECIFIC HEAT AND FLOW RATE (w*Cp).
WHAT IS THE SPECIFIC HEAT (w*Cp)? 100000
WHAT IS THE INCOMING TEMPERATURE OF THE COLD STREAM (T0)?
100
WHAT IS THE OUTGOING TEMPERATURE OF THE FINAL STAGE (TN)?
500
WHAT ARE THE NUMBER OF STAGES? 3

WHAT IS THE HEAT COEFFICIENT OF U( 1 )
? 120
WHAT IS THE INCOMING TEMPERATURE OF THE HOT STREAM [t( 1 ,1)]
? 300

WHAT IS THE HEAT COEFFICIENT OF U( 2 )
? 80
WHAT IS THE INCOMING TEMPERATURE OF THE HOT STREAM [t( 2 ,1)]
? 400

WHAT IS THE HEAT COEFFICIENT OF U( 3 )
? 40
WHAT IS THE INCOMING TEMPERATURE OF THE HOT STREAM [t( 3 ,1)]
? 600


CALCULATING TOTAL AREA


INLET TEMPS   STAGE #   AREA SQUARED

   100.00

   181.95     STAGE 1    578.52

   295.59     STAGE 2   1360.36

   500.00     STAGE 3   5110.36
```

TOTAL AREA SQUARED OF THE SYSTEM IS 7049.24

DO ANOTHER PROBLEM?

Appendix E

Four-Stage Example Problem

Appendix E

Four-Stage Example Problem

```
THIS IS THE INPUT MODULE FOR CHES.
YOU WILL NEED TO KNOW THE HEAT TRANSFER COEFFICIENTS (U'S),
INPUT TEMPERATURE (T0), ENDING TEMPERATURE (TN) AND THE
SPECIFIC HEAT AND FLOW RATE (w*Cp).
WHAT IS THE SPECIFIC HEAT (w*Cp)?  100000
WHAT IS THE INCOMING TEMPERATURE OF THE COLD STREAM (T0)?
100
WHAT IS THE OUTGOING TEMPERATURE OF THE FINAL STAGE (TN)?
500
WHAT ARE THE NUMBER OF STAGES?  4

WHAT IS THE HEAT COEFFICIENT OF U( 1 )
? 120
WHAT IS THE INCOMING TEMPERATURE OF THE HOT STREAM [t( 1 ,1)]
? 300

WHAT IS THE HEAT COEFFICIENT OF U( 2 )
? 80
WHAT IS THE INCOMING TEMPERATURE OF THE HOT STREAM [t( 2 ,1)]
? 400

WHAT IS THE HEAT COEFFICIENT OF U( 3 )
? 40
WHAT IS THE INCOMING TEMPERATURE OF THE HOT STREAM [t( 3 ,1)]
? 600

WHAT IS THE HEAT COEFFICIENT OF U( 4 )
? 20
WHAT IS THE INCOMING TEMPERATURE OF THE HOT STREAM [t( 4 ,1)]
? 700


CALCULATING TOTAL AREA


INLET TEMPS   STAGE #   AREA SQUARED

    100.00

    163.71      STAGE 1   389.52
```

191.92	STAGE 2	169.52
315.73	STAGE 3	1088.84
500.00	STAGE 4	4606.67

TOTAL AREA SQUARED OF THE SYSTEM IS 6254.55

DO ANOTHER PROBLEM?