RADIUS-OF-DRAINAGE EQUATION
FOR
PRESSURE BUILD-UP

by

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ABSTRACT

For two boundary conditions, radius-of-drainage relationships are derived from pressure build-up analysis for a single well.

The first boundary condition considers a single well near an infinite linear barrier in an otherwise homogeneous and infinite reservoir. An actual field analogy would be a well near an extensive linear fault. Another approximation for this idealized case would be a well in a relatively thin reservoir near a high-contrast fluid-fluid contact, or a reservoir exhibiting a sudden change in formation properties, such as thickness, porosity, or permeability.

In the second case, a single well is assumed near a linear and infinite pressure source in an otherwise infinite and homogeneous reservoir. A field approximation

would be a well near a linear pressure-maintenance system whereby the pressure at that boundary is essentially constant with time.

Theoretical pressure build-up curves were plotted, the points at which the effect of the boundary was felt were graphically determined, and then the correlation was made to arrive at equations for the radius of drainage.

A conventional plot of build-up pressure as a function of the logarithm of the ratio of total time to shut-in time gives a straight line until the effect of a discontinuity is felt by the test. Prior to the deviation from the straight line, the equations derived in this study can be used to determine the radius of drainage. This radius of drainage represents the distance within which no discontinuity exists. It may be considered the distance to which pore space has been proven. Since the points at which the effect of the boundary was felt by the well were graphically determined, accuracy is affected by the scale of the drawing.

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INTRODUCTION

When a single well is placed on production, a radius of drainage which increases with time results. Variations of pressure transients with time show that for some flow time the pressure drop is very small and it approaches zero at some distance from the well bore. Even though it appears that for a specific flow time the pressure change has a value of zero at some distance from the well bore, this is only relative. According to the diffusivity equation, pressure change occurs everywhere in the reservoir as soon as pressure changes in the well bore. It was this apparent constancy of the pressure at some point out in the reservoir that led to the concept of "radius of drainage."

The radius of drainage may be defined in many ways depending on the assumed conditions of flow and pressure. For example, the radius of drainage can be defined as that point in the reservoir or aquifer beyond which the change in pressure is only 1% of that at the well bore, or that point or boundary across which flow is only 1% of the flow into the well bore.

Depending on their assumptions, many authors in the field of transient fluid behavior derived different radius-of-drainage equations. Before using these equations, considerable attention must be paid to the assumptions on which the equations are based.

Previous Radius-of-Drainage Equations

Van Poollen (1965) discussed the published radius-of-drainage equations and presented a possible new equation. He emphasized the arbitrariness of these equations and pointed out that since some of the assumptions made are not exactly accurate, the resulting equations represent approximate relationships between radius of drainage and time. Table 1 shows these equations and the names of the authors who developed them.

TABLE I

SUMMARY OF VARIOUS RADIUS-OF-DRAINAGE EQUATIONS

(After van Poollen, Oil and Gas Jour., Sept. 14, 1964, p. 140)

Author		P. E. Jones	Tek, Grove and Poettman	Hutchinson and Kern	Hurst	van Poollen
Radius of drainage	c-g-s Units	$4\left(\frac{k\,t}{\phi\,\mu\,c}\right)^{1/2}$	$4.29\left(\frac{k\,t}{\phi\,\mu\,c}\right)^{1/2}$	$1.5\left(\frac{k\,t}{\phi\,\mu\,c}\right)^{1/2}$	$2.6408\left(\frac{k\,t}{\phi\,\mu\,c}\right)^{1/2}$	$2\left(\frac{k\,t}{\phi\,\mu\,c}\right)^{1/2}$
	Field Units	$\left(\frac{k\,t}{10\,\phi\,\mu\,c}\right)^{1/2}$	$\left(\frac{k\,t}{9\,\phi\,\mu\,c}\right)^{1/2}$	$\left(\frac{k\,t}{70\,\phi\,\mu\,c}\right)^{1/2}$	$\left(\frac{k\,t}{22.5\,\phi\,\mu\,c}\right)^{1/2}$	$\left(\frac{k\,t}{39.2\,\phi\,\mu\,c}\right)^{1/2}$

For a detailed comparison of these equations, summary of the assumptions made, and references to the original authors, van Poollen's work (1965) is recommended.

The equations presented in Table 1 relate the radius of drainage (r) to permeability (k), porosity (ϕ), viscosity (μ), compressibility (c), and time (t). These equations may be grouped in the following equation:

$$r = A \left(\frac{k\ t}{\phi\ \mu\ c}\right)^{1/2} \qquad (1)$$

where A is a constant that differs from author to author.

A conventional plot of pressure against the logarithm of producing time gives a straight line until the effect of a discontinuity is reflected by the test. Prior to the deviation from the straight line, these equations can be used to find the radius of drainage. One then has the distance within which no discontinuity occurs, and connected pore space has been proven at least to that distance. It must be noted here that the symbol t refers to flow time for a draw-down test.

Equation (1) also applies to a convention pressure build-up curve after a steady state condition in the immediately preceding production period has been reached. The reason equation (1) applies is that pressure build-up curves have the same shape as draw-down curves following steady-state production periods. In a pressure-maintenance system, for instance, an essentially steady-state condition can be reached and if the well is then shut in for a period of time, the build-up curve resembles an inverted draw-down curve and equation (1) can be used to find the radius of drainage. Then the symbol t refers to the build-up time.

Detection of a Linear Fault

Horner (1951) discussed the detection of a linear fault from pressure build-up data. He used the principle of superposition and the method of images to find the distance to an infinite linear barrier in an otherwise homogeneous and infinite reservoir. The presence of such a barrier affects the conventional pressure build-up curve whereby the pressure drop, ΔP, is plotted against the logarithm of the ratio of total time to shut-in time, $(\frac{t + \theta}{\theta})$. In this case and after a comparatively long shut-in time, the curve is characterized by two straight-line segments connected by a transition zone. Horner extended these two straight-line segments and concluded that the following equation is satisfied at the point of intersection:

$$-Ei\ \left(\frac{-a^2\ \phi\ \mu\ c}{k\ t}\right) = \ln\ \left(\frac{t + \theta}{\theta}\right) \qquad (2)$$

where Ei is the exponential integral

$$Ei\ (\chi) = -\int_{\chi}^{\infty} \left(\frac{e^{-u}}{u}\right) du$$

Knowing the reservoir and fluid properties included in equation (2) and the value of $\left(\frac{t + \theta}{\theta}\right)$ at the intersection of the two straight-line segments, the equation can be solved for the distance to the fault.

Davis and Hawkins (1963) showed that when $\left(\frac{t + \theta}{\theta}\right)$ is greater than 30, the relationship in equation (2) may be simplified with considerable accuracy to give:

$$a = 0.0122\ \left(\frac{k\ \theta}{\phi\ \mu\ c}\right)^{1/2} \qquad (3)$$

Standing (1964) extended the work of Davis and Hawkins with a simplification which is valid for $(\frac{t + \theta}{\theta})$ greater than 2. His equation is as follows:

$$a = 0.0122 \; (1 + \frac{0.4}{D}) \; (\frac{k \; \theta}{\phi \; \mu \; c})^{1/2} \qquad (4)$$

In equations (3) and (4) the units are as follows:

k = permeability in millidarcies

ϕ = porosity; a dimensionless fraction

c = compressibility in psi^{-1}

a = distance to the barrier in feet

μ = viscosity in dentipoises

$D = (\frac{t + \theta}{\theta})$

where t and θ are, respectively, flow time and shut-in time at the intersection of the two straight-line segments, in hours.

Purpose and Scope of Investigation

The equations shown in Table 1 are applicable to a pressure build-up test only after a steady-state condition has been reached. In Horner's method, a considerably long shut-in time is required to establish the second straight-line segment with reasonable accuracy. It is often desirable to determine the radius of drainage under unsteady-state conditions and with short shut-in time. A drill-stem test, where the sum of flow time and shut-in time is usually in the order of half an hour to two hours, is a good example of that situation.

The purpose of this investigation is to develop a relationship for finding the radius of drainage from pressure build-up analysis under unsteady-state conditions.

Two cases of reservoir boundary conditions are considered.

Case 1:

The presence of an infinite linear barrier that completely obstructs fluid flow in an otherwise infinite and homogeneous reservoir is assumed. A field example of this case would be the presence of an extensive linear fault in a very large field. Another actual analogy would be a well in a relatively thin reservoir near a high-contrast fluid-fluid contact, or a reservoir exhibiting a sudden change in formation properties such as thickness, porosity, or permeability.

Case 2:

The presence of an infinite linear pressure source in an otherwise infinite and homogeneous reservoir is assumed. A field example would be a linear and complete pressure-maintenance system whereby the pressure at the outer boundary is kept virtually constant with time.

THEORETICAL ANALYSIS

A brief discussion of the theoretical concepts related to this investigation will be presented. The equations used in this study are based on the application of the diffusivity equation, the principle of superposition, and the method of images to pressure build-up. Using these equations, theoretical pressure build-up curves are plotted and analysed to determine radius-of-drainage equations.

The Diffusivity Equation

The equation governing the unsteady-state flow of a single and slightly compressible fluid in a homogeneous porous medium is expressed with pressure as a function of radius and time:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\phi \mu c}{k}\frac{\partial P}{\partial t} \qquad (6)$$

The symbols and units are defined in the Appendix. Equation (6) is applicable to the fluid flow and it was derived by many authors such as Muskat (1946), and Collins (1961).

Equation (6) is commonly known as the diffusivity equation and it is based on the following assumptions:

1. The reservoir is homogeneous, i.e., permeability and porosity are constant and uniform.
2. The reservoir is horizontal with a uniform thickness throughout.
3. The fluid is present in one phase and flow is in accordance with Darcy's Law.
4. The density of the fluid obeys an exponential Law.
5. The viscosity of the fluid is constant.
6. The compressibility of the fluid is small and constant.
7. The flow is radial into a common source or sink.

For various boundary conditions, many solutions to the diffusivity equation have been presented in the literature. The solutions commonly considered in dealing with fluid flow through porous media were presented by Muskat (1946), Van Everdingen and Hurst (1949), and Horner (1951).

In a truely infinite system, which is the situation considered in this study, the solution frequently used has been referred to as the point-source solution or the Lord Kelvin solution. This solution is:

$$P_{r,t} = \frac{q\ B\ \mu}{4\ \pi\ k\ h} \text{Ei}\ (\frac{-\phi\ \mu\ c\ r^2}{4\ k\ t}) \qquad (7)$$

<u>Principle of Superposition and the Method of Images</u>

The diffusivity equation is a linear homogeneous equation and therefore, the principle of superposition applies to its solution. In this type of equation, it is possible to add two or more solutions and obtain a net effect with different boundary conditions.

The application of the principle of superposition may be accomplished by the use of "image wells". The premise of image wells can be demonstrated by considering

Case 1 (presence of an infinite linear barrier in an otherwise infinite reservoir). With the presence of an extensive linear fault that completely obstructs fluid flow, the energy which would be supplied for production is decreased. This decrease in energy must be supplied to the well from the side on which the well is drilled. This energy is equal to the energy that the well would draw from the far side of the fault. Since the principle of superposition applies, the effect of this loss of energy can be superposed on the effect of the loss of energy on the near side of the fault. This can be done by assuming an image well which is identical to the actual well and at the same distance from the fault. The net pressure response is the sum of the response of the actual well and the image well.

The reasoning for Case 2 (presence of a linear pressure source) follows in a similar manner with one exception. Since there is a gain of energy from the far side of the boundary, the image well is an injection well instead of a production well.

It must be noted that the observation point is at the actual well and that once the image well is used, the boundary is no longer present.

Pressure Build-up

The analysis of pressure build-up curves is based on solutions to the diffusivity equation and the application of the principle of superposition (Horner, 1951).

After producing for a time t, a well can be shut-in for a time θ to obtain data for pressure build-up analysis. Mathematically, the shut-in period may be considered as a production at a constant rate qB, and a simultaneous injection at the same rate. The principle of superposition is applied by adding the effect of production for time $(t + \theta)$ to the effect of injection for a time θ. This principle is described in Figure 1. It can be seen from this graph that $\Delta P'_{t+\theta}$ is the pressure draw-down due to the constant rate of production qB since time zero. The pressure increase due to the injection at the same rate which started at time t is equal to $\Delta P'_{\theta}$.

The total effect at time $(t + \theta)$ is:

$$\Delta P_{t+\theta} = \Delta P'_{t+\theta} + P'_{\theta} \tag{8}$$

When this concept is applied to the point-source solution of the diffusivity equation, the following pressure build-up equation results:

$$P_o - P_w = \Delta P = \frac{-q\ B\ \mu}{4\ \pi\ k\ h} \left\{ Ei\ \left[\frac{-\phi\ \mu\ c\ r_w^2}{4\ k\ (t+\theta)}\right] - Ei\ \left[\frac{-\phi\ \mu\ c\ r_w^2}{4\ k\ \theta}\right] \right\} \tag{9}$$

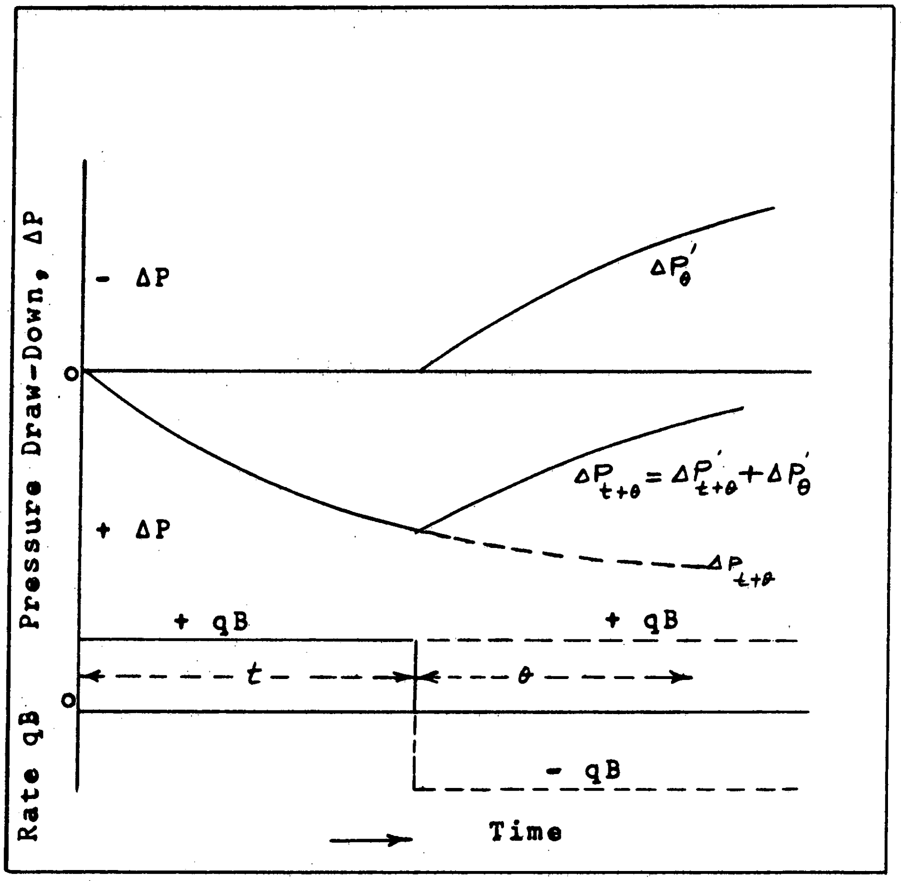


Figure 1

Shut-in Period Analogy

(After van Poollen, Oil and Gas Jour., Nov. 1, 1965, p. 118)

Equations Used

Case 1: Presence of a Linear Barrier

Figure 2a shows the method of images for one well near a no-flow boundary.

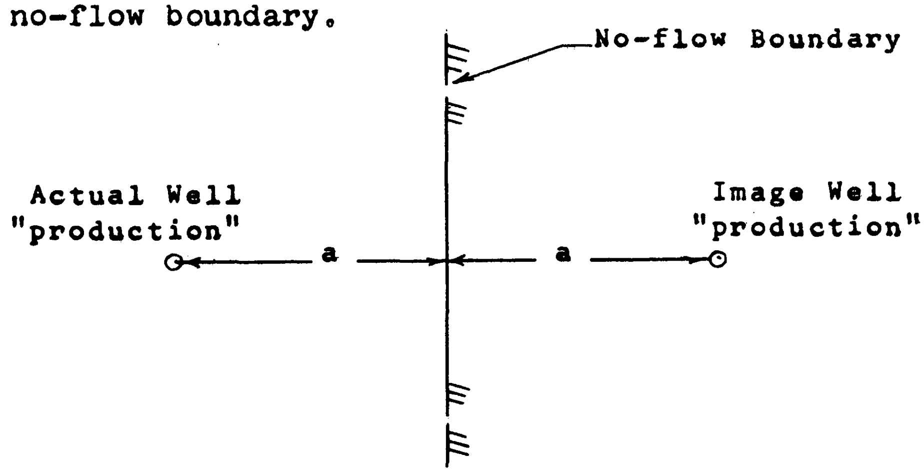


Figure 2a

Method of Images

Case 1

If the principle of superposition is applied the following build-up equation results:

$$P_o - P_w = \Delta p = \frac{-q B \mu}{4 \pi k h} \left\{ Ei \left[\frac{-\phi \mu c r_w^2}{4 k (t + \theta)}\right] - Ei \left[\frac{-\phi \mu c r_w^2}{4 k \theta}\right] + Ei \left[\frac{-\phi \mu c (2a)^2}{4 k (t + \theta)}\right] - Ei \left[\frac{-\phi \mu c (2a)^2}{4 k \theta}\right] \right\}$$

The first two Ei terms in equation (9) are due to the well itself while the second two Ei terms are due to the image well.

When dimensionless time and dimensionless pressure drop are introduced, equation (9) becomes

$$(\Delta P)_D = - \text{Ei} \left[\frac{-r_w^2}{a^2 \, (t + \theta)_D}\right] + \text{Ei} \left[\frac{-r_w^2}{a^2 \, \theta_D}\right] - \text{Ei} \left[\frac{-4}{(t + \theta)_D}\right] + \text{Ei} \left[\frac{-4}{\theta_D}\right] \qquad (11)$$

where

$$(\Delta P)_D = \frac{4 \, \pi \, k \, h \, \Delta P}{q \, B \, \mu},$$

$$\theta_D = \frac{4 \, k}{\phi \, \mu \, c \, a^2}, \text{ and}$$

$$(t + \theta)_D = \frac{4 \, k \, (t + \theta)}{\phi \, \mu \, c \, a^2}$$

Case 2: Presence of a Linear Pressure Source

Figure 2b shows the method of images for one well near a constant-pressure boundary.

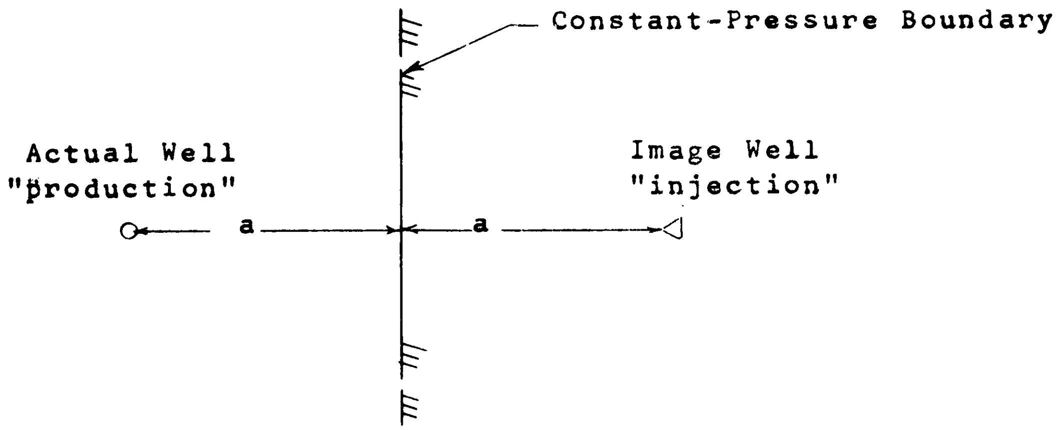


Figure 2b

Method of Images

Case 2

By use of the principle of superposition and the method of images, the buildup equation in terms of dimensionless time and pressure drop is

$$(\Delta P)_D = -\,Ei\left[\frac{-r_w^2}{a^2\,(t+\theta)_D}\right] + Ei\left[\frac{-r_w^2}{a^2\,\theta_D}\right] + Ei\left[\frac{-4}{(t+\theta)_D}\right] - Ei\left[\frac{-4}{\theta_D}\right] \qquad (12)$$

The dimensionless terms are as defined in the case of a linear barrier.

METHOD OF INVESTIGATION

The method followed in this investigation is somewhat similar to that presented by Hurst, Haynie, and Walker (1961). Using equations (11) and (12) for Case 1 and Case 2 respectively, theoretical pressure buildup curves are plotted. For different dimensionless flow times and an arbitrary distance to the boundary, a set of these pressure build-up curves is plotted whereby the ordinate is dimensionless pressure drop $(\Delta P)_D$ and the abscissa is the logarithm of the dimensionless time ratio $(\frac{t + \theta}{\theta})$. See Figures 3 through 10. A total of 26 different values is assumed for the dimensionless flow time. Flow times range from 10 to 10,000. The distance to the boundary is then varied and another set of build-up curves is obtained. The distances considered range from 50 to 2,000 units.

Since within this range the curves are essentially identical for each boundary condition, only sample curves are shown in the Appendix. Sample curves have been photographically reduced from a 15- by 20-in. grid to the present size. In all calculations the value of r_w is assumed to be 0.5 units.

The calculations were programmed in Algol for a Burroughs B5500 digital computer, and the curves were plotted by use of a Calcomp plotter. The flow diagrams and a listing of the program are shown in the Appendix.

ANALYSIS OF RESULTS

For the boundary conditions considered in this study, the shape of the curves in Figures 3 through 10 is characteristic of pressure build-up behavior. It is seen that for early shut-in times (for large values of $\frac{t + \theta}{\theta}$) the curves follow the logarithmic approximation to the point-source solution and form straight lines. As soon as the effect of the outer boundary is felt by the well, the curves start deviating from the straight-line form. When the ratio of $\frac{t + \theta}{\theta}$ approaches one, i.e., when shut-in times are very large compared to flow times, all the curves converge towards a zero pressure drop.

For each type of boundary, and within the range of distances considered in this study, it is seen that the curves virtually coincide for corresponding flow times regardless of the distance to the boundary. A very slight deviation occurs at very large values of $\frac{t + \theta}{\theta}$. This deviation is noticeable only when curves for small distances are superposed on curves for very large distances. Such small deviation can be seen, for instance, when Figures 3 and 6 on the original 15- by 20-in. grid are superposed and placed on a light table.

Since in each type of boundary all curves for corresponding dimensionless flow times essentially coincide, it seems reasonable to conclude that, within the range of distances investigated the points of deviation from a straight line occur at the same values of $(\frac{t + \theta}{\theta})$. Due to the inherent inaccuracy in determining such points graphically, readings were taken from all the sample curves shown in the Appendix (Figures 3 through 10). For each case of boundary condition, the readings were averaged numerically and tabulated in Tables 2 and 3. For the sake of simplicity, the values of $(\frac{t + \theta}{\theta})$ at which the curves deviate from being straight lines were referred to as the "points of deviation" or simply $(\frac{t + \theta}{\theta})_{dev}$.

Especially in the case of a linear barrier, some difficulty was encountered in determining the points of deviation for large dimensionless flow times. This difficulty is mainly due to the scale of the drawing. Therefore, these curves were ignored.

CORRELATION OF RESULTS

In trying to arrive at a radius-of-drainage relationship for each type of boundary condition, the average values of $(\frac{t + \theta}{\theta})$ at the deviation points were plotted against corresponding dimensionless flow times on a logarithmic scale. See Figures 11 and 12. It can be seen that the points fall approximately on a straight line. An average straight line is drawn through these points. In drawing such a straight line emphasis is placed on the points corresponding to small values of dimensionless time. As mentioned earlier, the points of deviation on these curves can be determined more accurately than on those for large dimensionless flow time.

In each type of boundary conditions, an equation is fitted to the straight line as follows:

Case 1:

From Figure 11, the slope = 0.890 and the intercept to the vertical axis = 14.9.

Therefore

$$t_D = 14.9 \left(\frac{t + \theta}{\theta}\right)^{0.890} \qquad (13)$$

but

$$t_D = \frac{4\,k\,t}{\phi\,\mu\,c\,a^2} \text{ by definition} \qquad (14)$$

Equating (13) and (14) and solving for the distance a, it is found that

$$a = 0.518 \left[\frac{k\,t}{\phi\,\mu\,c\left(\frac{t + \theta}{\theta}\right)^{0.890}}\right]^{1/2} \cong$$

$$0.5 \left[\frac{k\,t}{\phi\,\mu\,c\,(\frac{t + \theta}{\theta})}\right]^{1/2} \qquad (15)$$

Case 2:

From Figure 12, the slope of the line = 1.015 and its intercept to the vertical axis = 12.

Therefore,

$$t_D = 12 \left(\frac{t + \theta}{\theta}\right)^{1.015} \qquad (16)$$

but

$$t_D = \frac{4\,k\,t}{\phi\,\mu\,c\,a^2} \quad \text{by definition} \qquad (17)$$

Equating (16) and (17) and solving for the distance a, it is found that

$$a = 0.577 \left[\frac{k\,t}{\phi\,\mu\,c \left(\frac{t+\theta}{\theta} \right)^{1.015}} \right]^{1/2} \cong$$

$$0.6 \left[\frac{k\,t}{\phi\,\mu\,c \left(\frac{t+\theta}{\theta} \right)} \right]^{1/2} \qquad (18)$$

CONCLUSION

For two cases of boundary conditions, approximate radius-of-drainage equations have been derived from pressure build-up.

In Case 1, where a well was assumed to be near a linear and infinite barrier in an otherwise infinite and homogeneous reservoir, the following equation was derived:

$$a \cong 0.5 \left[\frac{k\, t}{\phi\, \mu\, c\, \left(\frac{t + \theta}{\theta}\right)} \right]^{1/2} \qquad (15)$$

In Case 2, where a well was assumed to be near a linear and infinite pressure source in an otherwise infinite and homogeneous reservoir, the following equation was derived:

$$a \cong 0.6 \left[\frac{k\ t}{\phi\ \mu\ c\ (\frac{t + \theta}{\theta})} \right]^{1/2} \qquad (18)$$

A plot of build-up pressure vs. the logarithm of $(\frac{t + \theta}{\theta})$ gives a straight line until the effect of a discontinuity is felt. Prior to the time when the curve deviates from being a straight line, equations (15) and (18) can be used to find the radius of drainage which is the distance within which no discontinuity exists.

The radius-of-drainage equations presented in Table 1 apply to pressure build-up curves only after a steady-state condition has been reached. Equations (15) and (18), however, apply to pressure build-up under unsteady-state conditions.

Equation (4), which is a simplification by Standing (1964) to an equation originally presented by Horner (1951), can be used to find the distance to a linear barrier from pressure build-up curves. This equation has the same variables as equation (15) which deals with the same boundary conditions. To use equation (4), it is necessary to find the point of intersection of two straight-line segments. Long shut-in time may be required to establish the second straight-line segment accurately. Equation (15), however, can be used to find the radius of drainage at any point on the first straight-line segment before reaching the transition zone. The radius of drainage then represents the distance to which connected pore space has been proven.

APPENDIX

SECTION I

NOMENCLATURE

a = radius of drainage in centimeters

t = flow time in seconds, which is production time prior to the build-up test

k = permeability in darcies

ϕ = porosity; a dimensionless fraction

μ = viscosity in centipoises

c = compressibility in atmospheres^{-1}

q = production or injection rate in cubic centimeters at standard surface conditions

B = formation volume factor in reservoir units per standard surface units

h = formation thickness in centimeters

$P_{r, t}$ = pressure at time t and radius r, in atmospheres

P_o = initial reservoir pressure, or reservoir pressure at moment of completion of well, in atmospheres

P_w = pressure in the well during build-up, in atmospheres

ΔP = pressure drop = $P_o - P_w$, in atmospheres

ΔP_D = dimensionless pressure drop

t_D = dimensionless flow time

r_w = well radius in centimeters

In discussing previous studies, it was necessary to use the units introduced by the original authors. In this study, however, the symbols and their units are as specified in the above nomenclature. If a different set of consistent units is desired, proper conversion factors should be introduced.

TABLE 2

Case 1

"Points of deviation," from Figures 3 through 6

t_D	$(\frac{t + \theta}{\theta})$dev.				
	A=100 Fig. 3	A=500 Fig. 4	A=1000 Fig. 5	A=2000 Fig. 6	Numerical Average
20	17	18	16	16	16.8
30	24	26	24	26	25.0
40	29	35	27	30	30.3
50	35	37	40	40	38.0
60	47	49	47	47	47.5
70	50	58	60	52	55.0
80	70	73	71	70	71.0
90	95	80	85	95	88.8
200	160	155	165	155	158.8
300	260	270	280	260	267.5
400	380	370	400	320	367.5
500	580	430	470	450	482.5
600	700	570	700	600	642.5
700	950	680	950	700	820.0
900	1000	900	1000	950	962.5

TABLE 3

Case 2

"Points of Deviation," from Figures 7 through 10

t_D	$(\frac{t + \theta}{\theta})$dev.				
	A=100 Fig. 7	A=500 Fig. 8	A=1000 Fig. 9	A=2000 Fig. 10	Numerical Average
20	15	15	15	15	15.0
30	24	20	24	25	23.3
40	28	30	35	35	32.0
50	37	38	37	40	38.0
60	52	55	46	45	49.5
70	60	66	60	60	61.5
80	75	78	73	65	72.8
90	83	85	84	85	84.3
200	160	190	160	150	165
300	310	300	270	280	290
400	400	380	320	320	355
500	520	430	370	430	437.5
600	590	500	500	470	515
700	650	600	600	550	600
800	670	800	670	720	715
900	920	910	850	800	870
2000	1700	1850	1700	1700	1737.5
3000	2700	2600	2300	2300	2475
4000	3800	3800	3500	3500	3650
5000	4400	4600	4500	4600	4525

SECTION II

Figures 3 through 6 - Build-up Curves for Case 1

Figures 7 through 10 - Build-up Curves for Case 2

T - dimensionless flow time

A - distance to a discontinuity

$(\frac{t + \theta}{\theta})$ - dimensionless time ratio

ΔP_D - dimensionless pressure drop

Figure 11 - Radius of Drainage for Case 1

Figure 12 - Radius of Drainage for Case 2

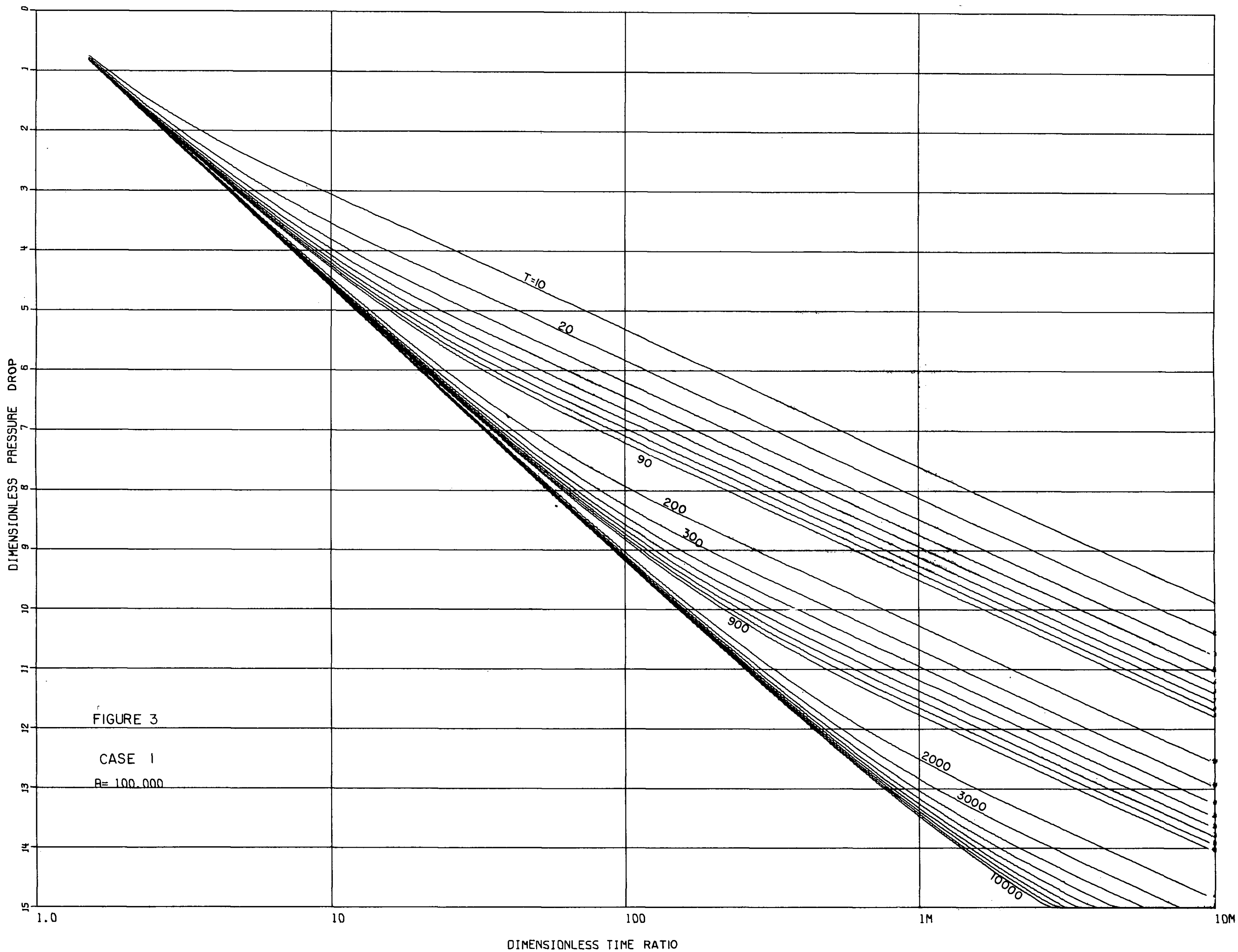


FIGURE 3
CASE I
A= 100,000
T=10
20
90
200
300
900
2000
3000
10000
DIMENSIONLESS PRESSURE DROP
DIMENSIONLESS TIME RATIO
1.0
10
100
1M
10M
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

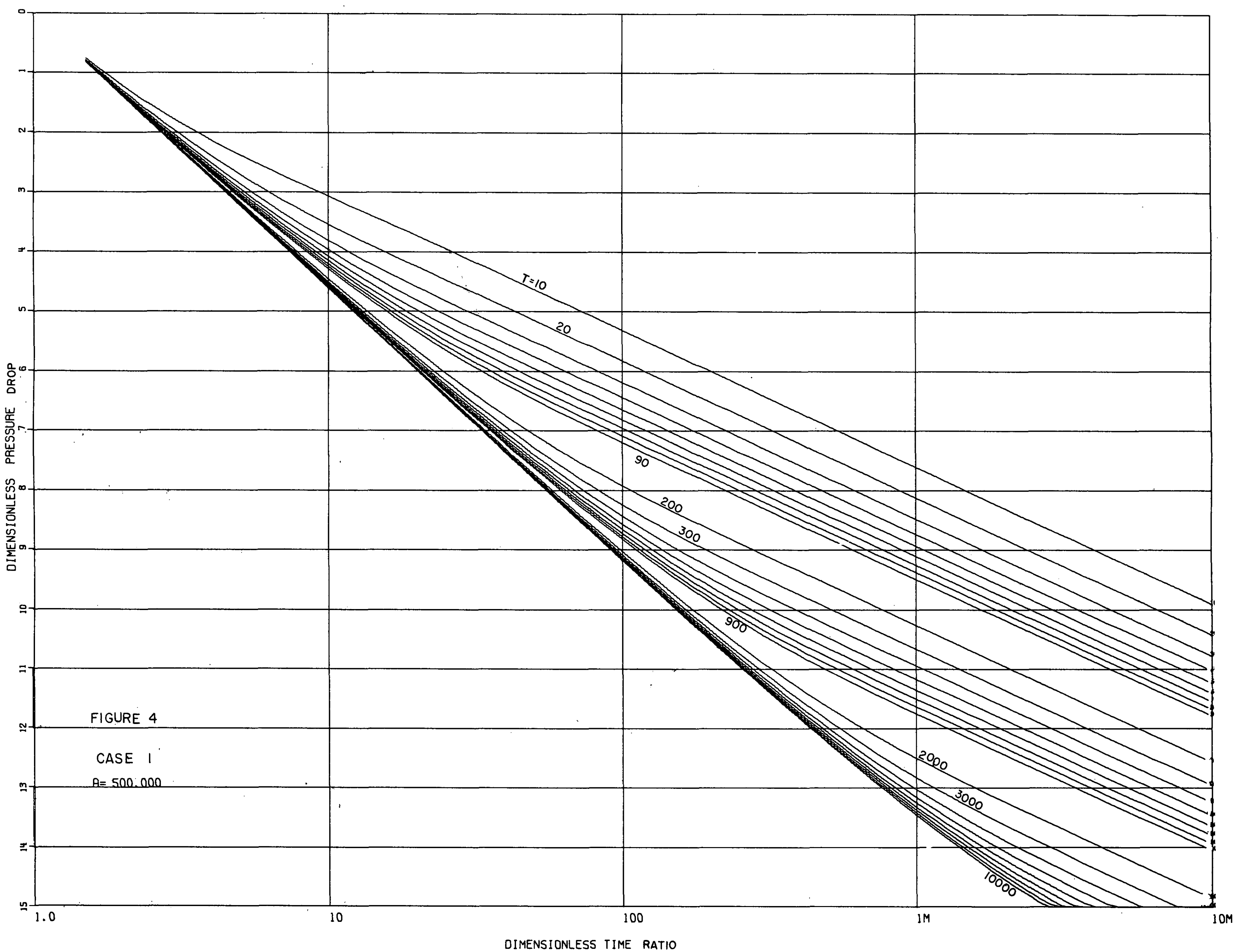


FIGURE 4
CASE I
B= 500,000
T=10
20
90
200
300
900
2000
3000
10000
DIMENSIONLESS PRESSURE DROP
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
DIMENSIONLESS TIME RATIO
1.0
10
100
1M
10M

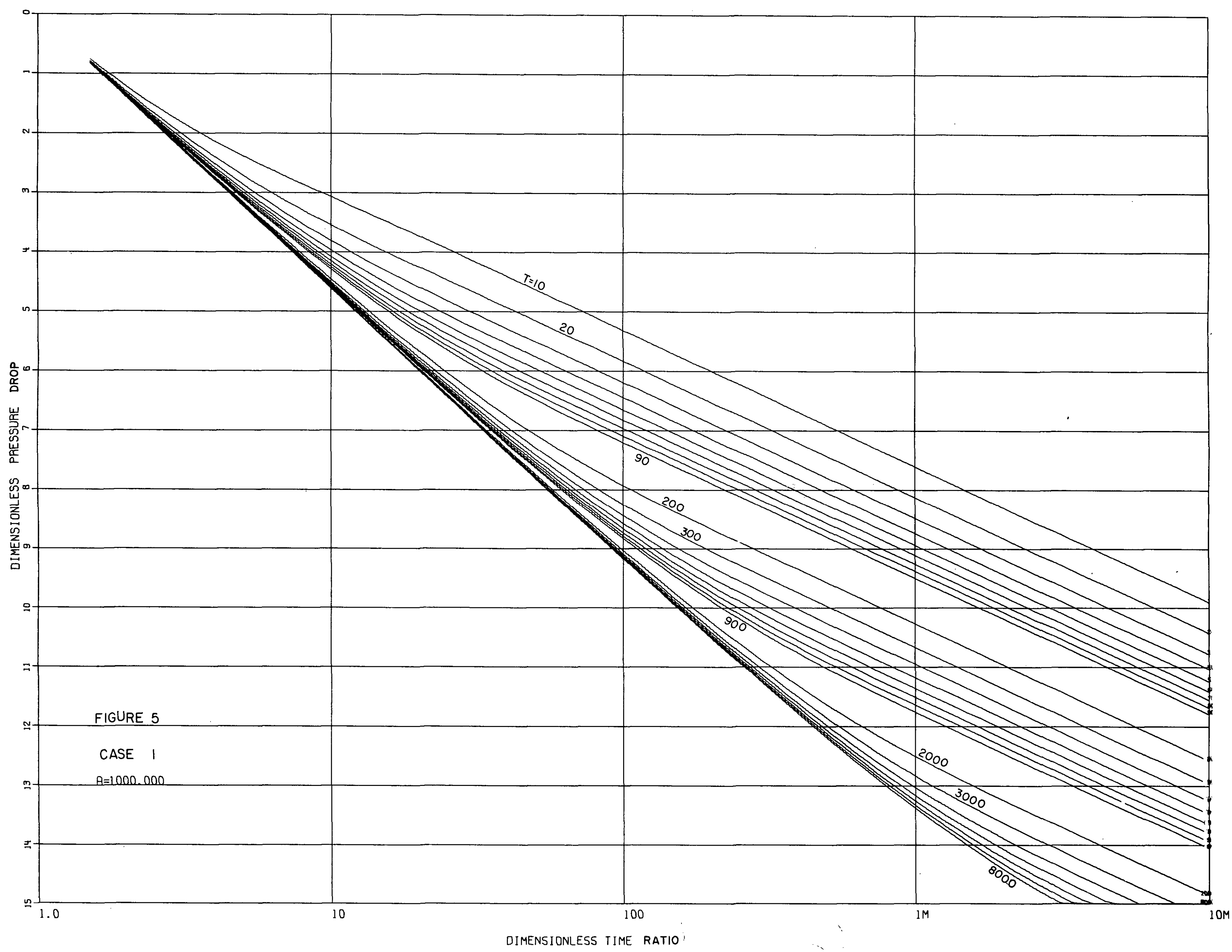
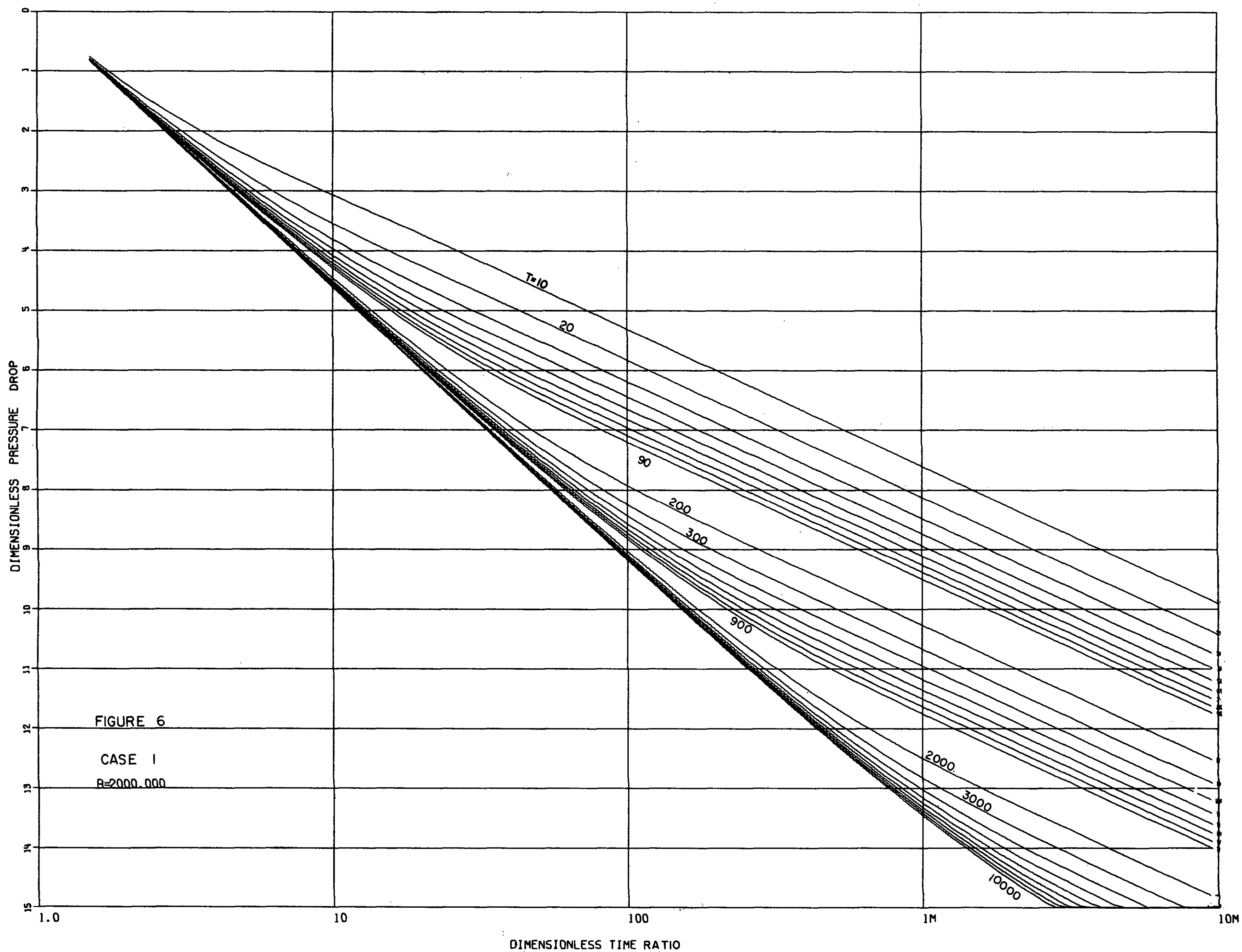


FIGURE 5
CASE I
A=1000,000
T=10
20
90
200
300
900
2000
3000
8000
DIMENSIONLESS PRESSURE DROP
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
DIMENSIONLESS TIME RATIO
1.0
10
100
1M
10M



DIMENSIONLESS PRESSURE DROP
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
1.0
10
100
1M
10M
DIMENSIONLESS TIME RATIO
T=10
20
90
200
300
900
2000
3000
10000
FIGURE 6
CASE I
B=2000,000

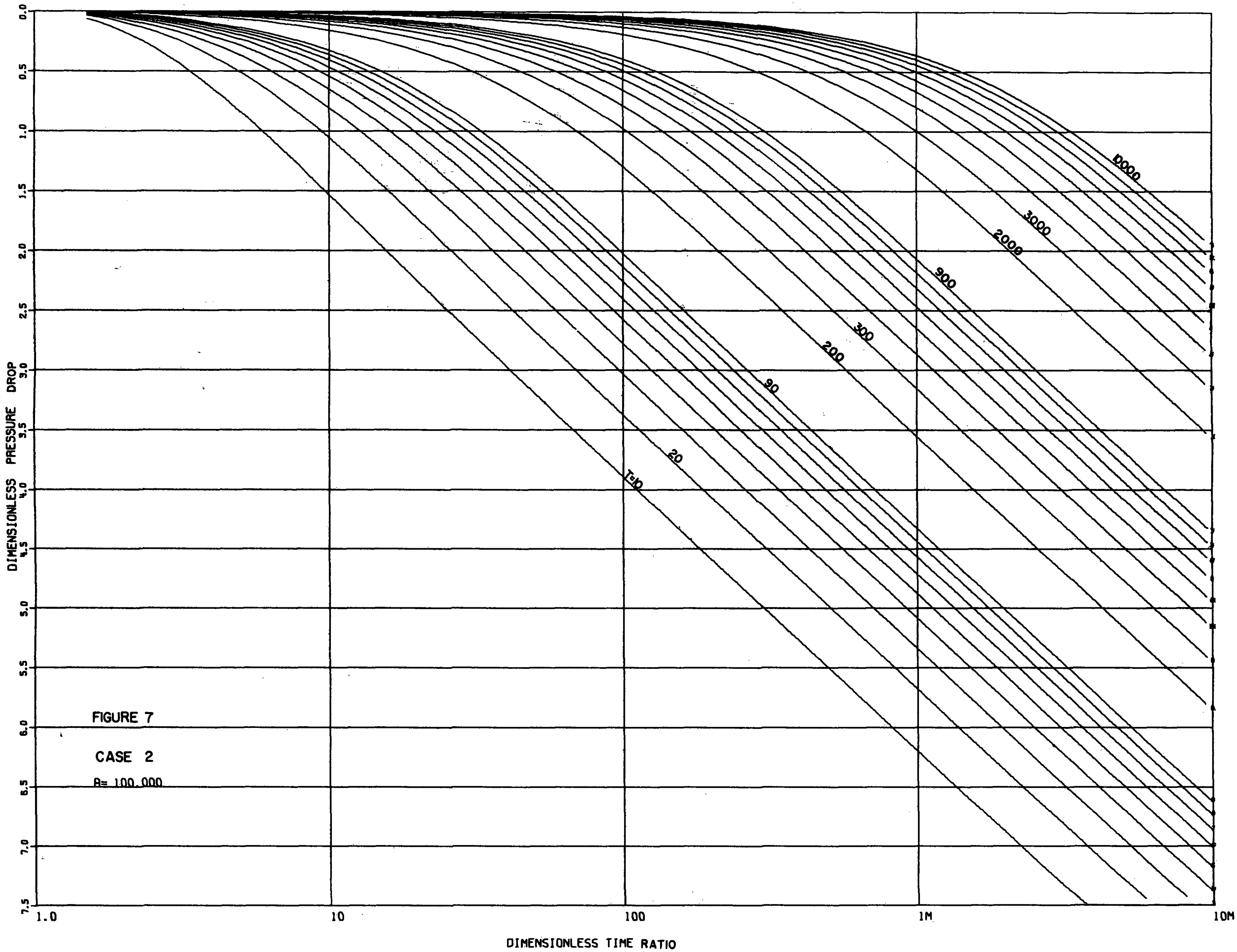


FIGURE 7
CASE 2
B= 100,000
DIMENSIONLESS PRESSURE DROP
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
DIMENSIONLESS TIME RATIO
1.0
10
100
1M
10M
10000
3000
2000
900
300
200
90
20
T=10

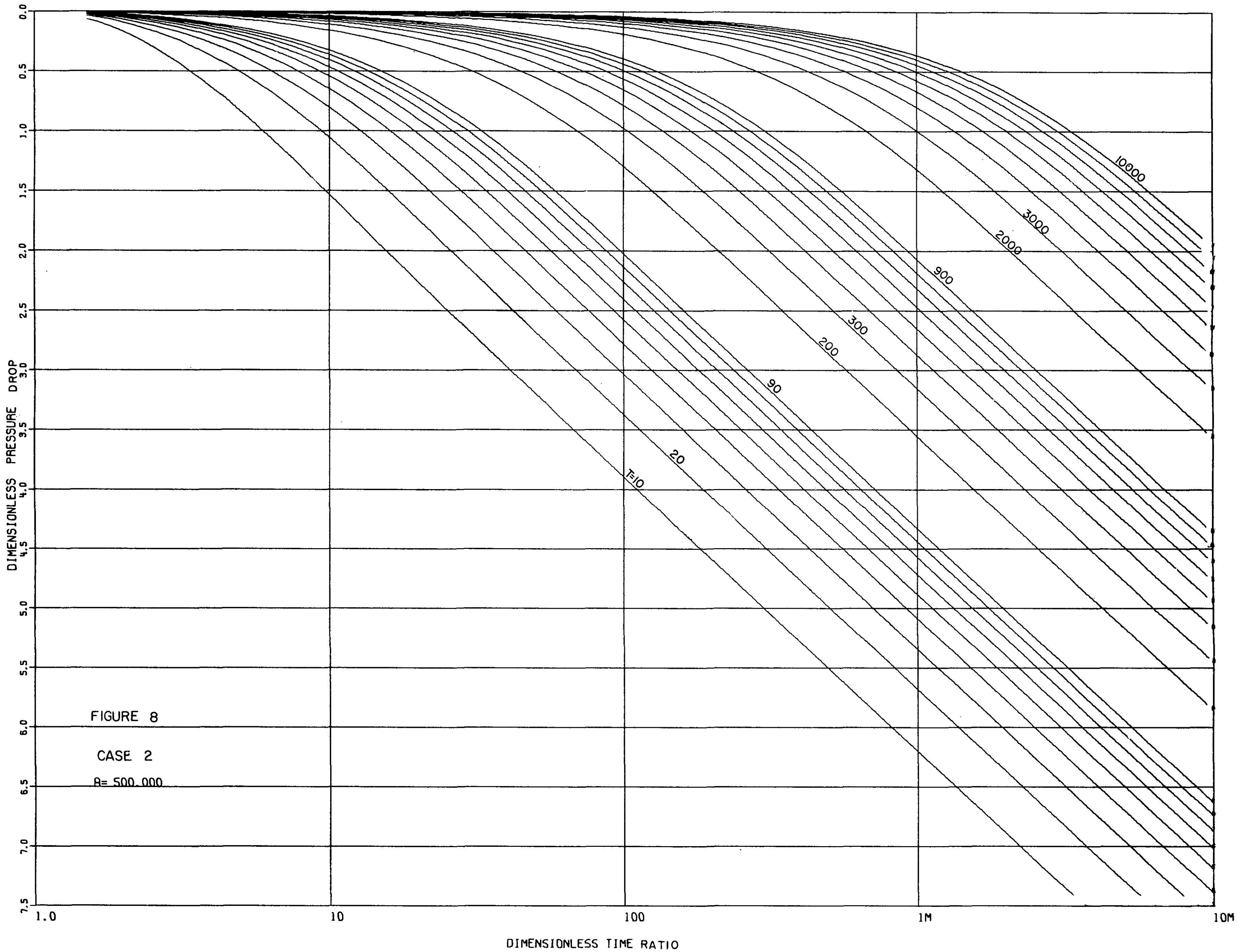


FIGURE 8
CASE 2
B= 500,000
10000
3000
2000
900
300
200
90
20
T=10
DIMENSIONLESS PRESSURE DROP
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
1.0
10
100
1M
10M
DIMENSIONLESS TIME RATIO

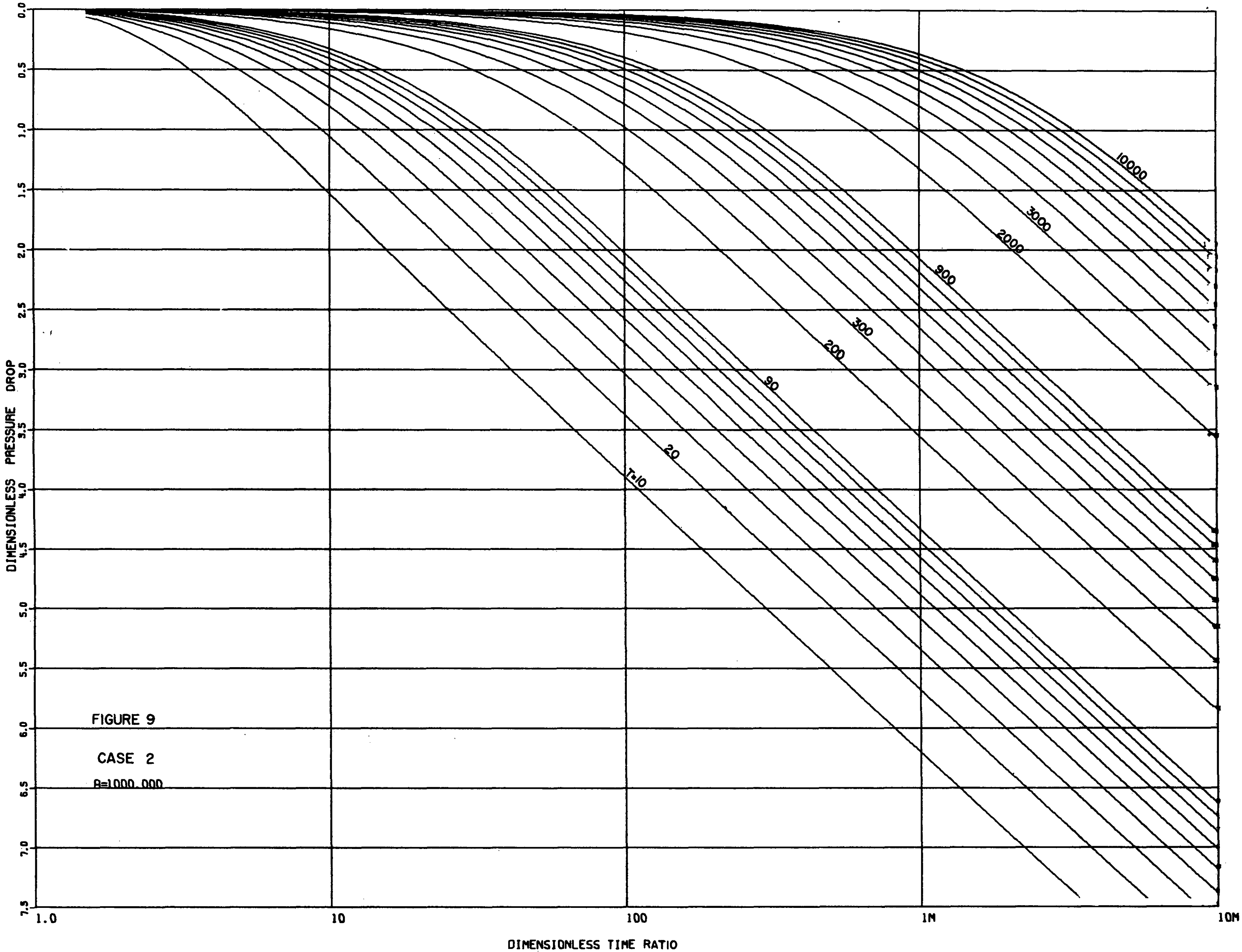
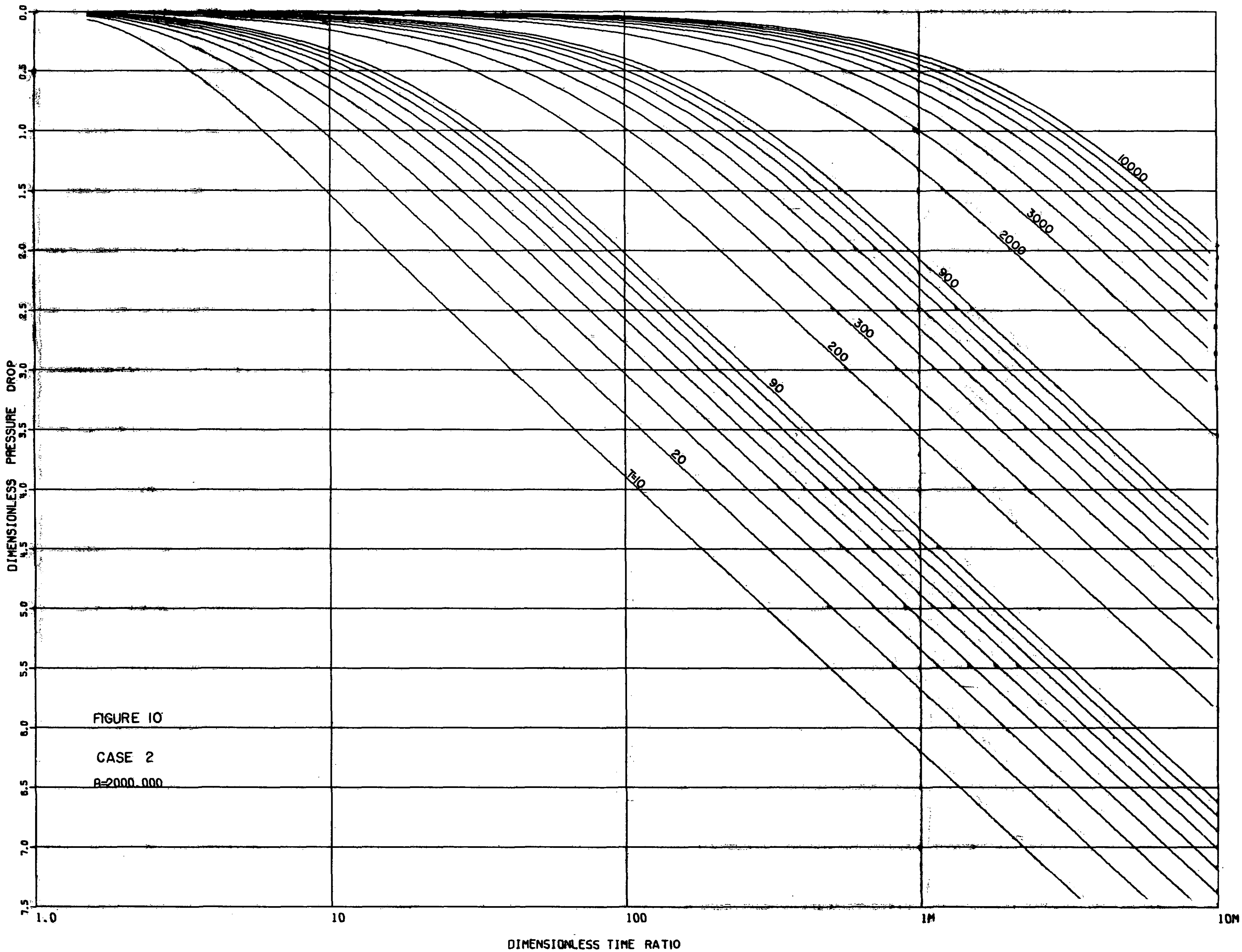


FIGURE 9
CASE 2
B=1000,000
DIMENSIONLESS PRESSURE DROP
DIMENSIONLESS TIME RATIO
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
1.0
10
100
1M
10M
10000
3000
2000
900
300
200
90
20
T=10



DIMENSIONLESS PRESSURE DROP
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
1.0
10
100
1M
10M
DIMENSIONLESS TIME RATIO
10000
3000
2000
900
300
200
90
20
T=10
FIGURE 10
CASE 2
B=2000.000

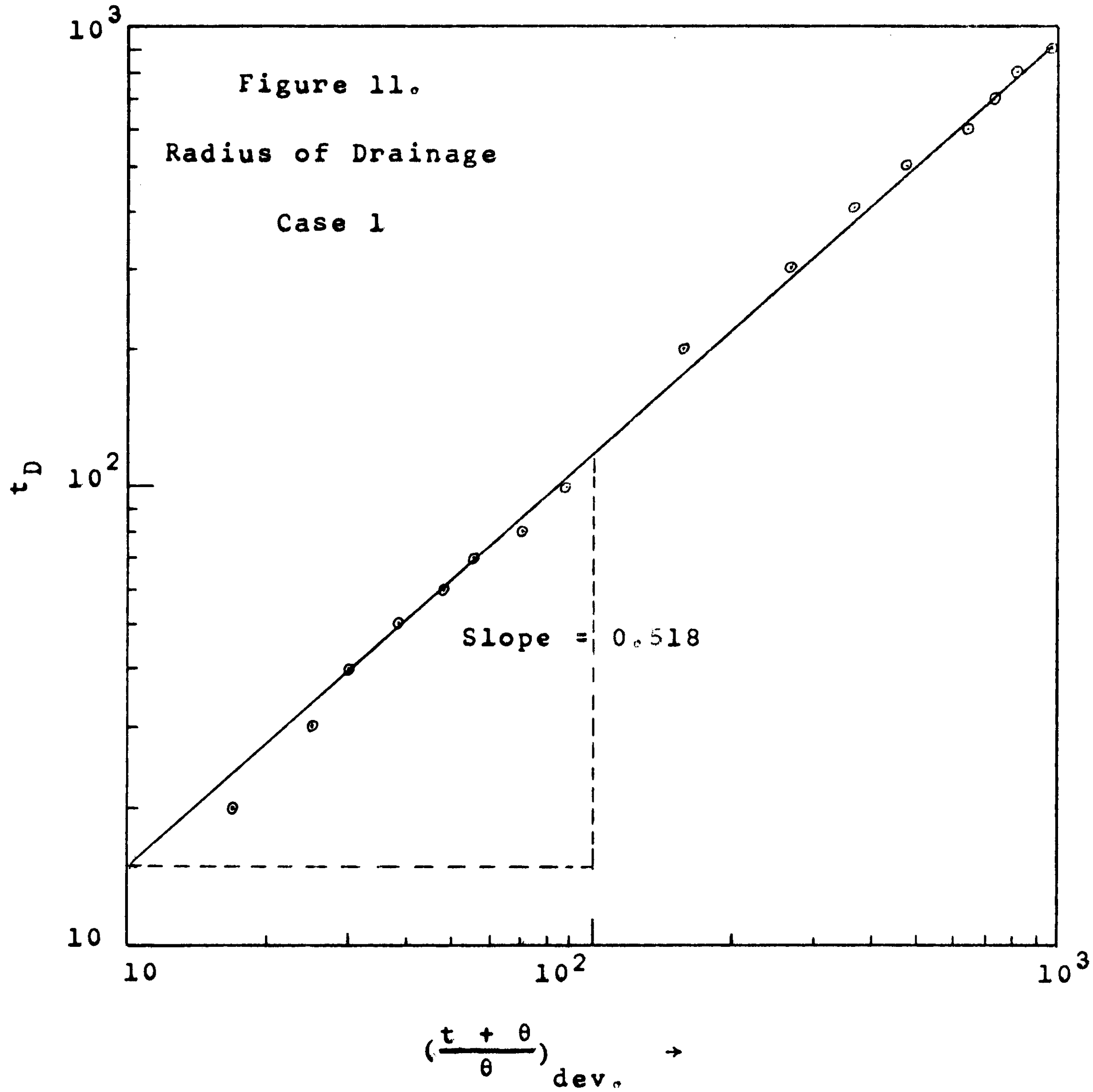


Figure 11.
Radius of Drainage
Case 1
Slope = 0.518
10^3
10^2
10
t_D
10
10^2
10^3
$\left(\frac{t + \theta}{\theta}\right)_{dev.}$ →

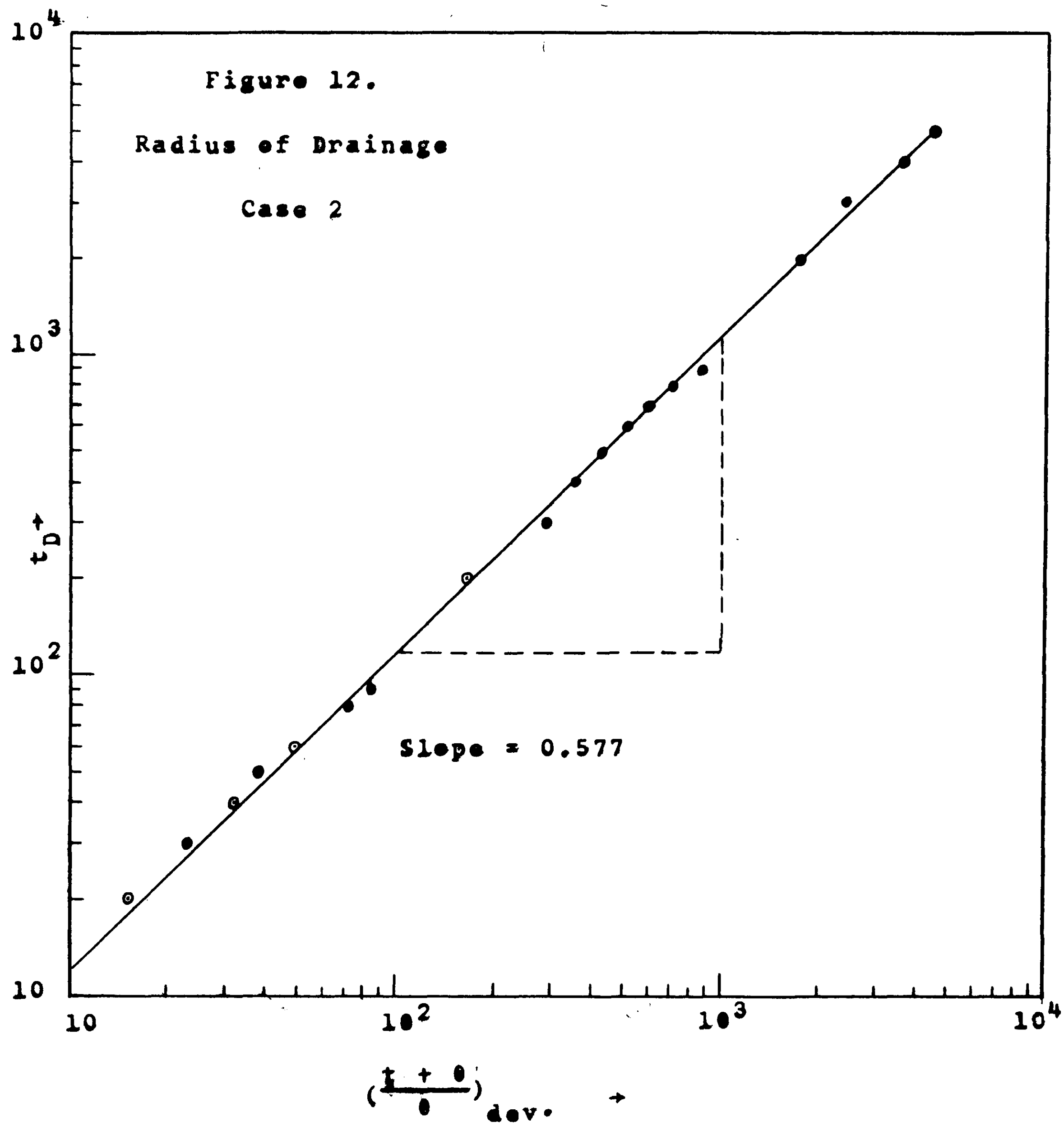


Figure 12.
Radius of Drainage
Case 2
Slope = 0.577
10
10^2
10^3
10^4
t_D →
$(\frac{t + \theta}{\theta})_{dev.}$ →

SECTION III

Flow Diagrams and Listing of Program

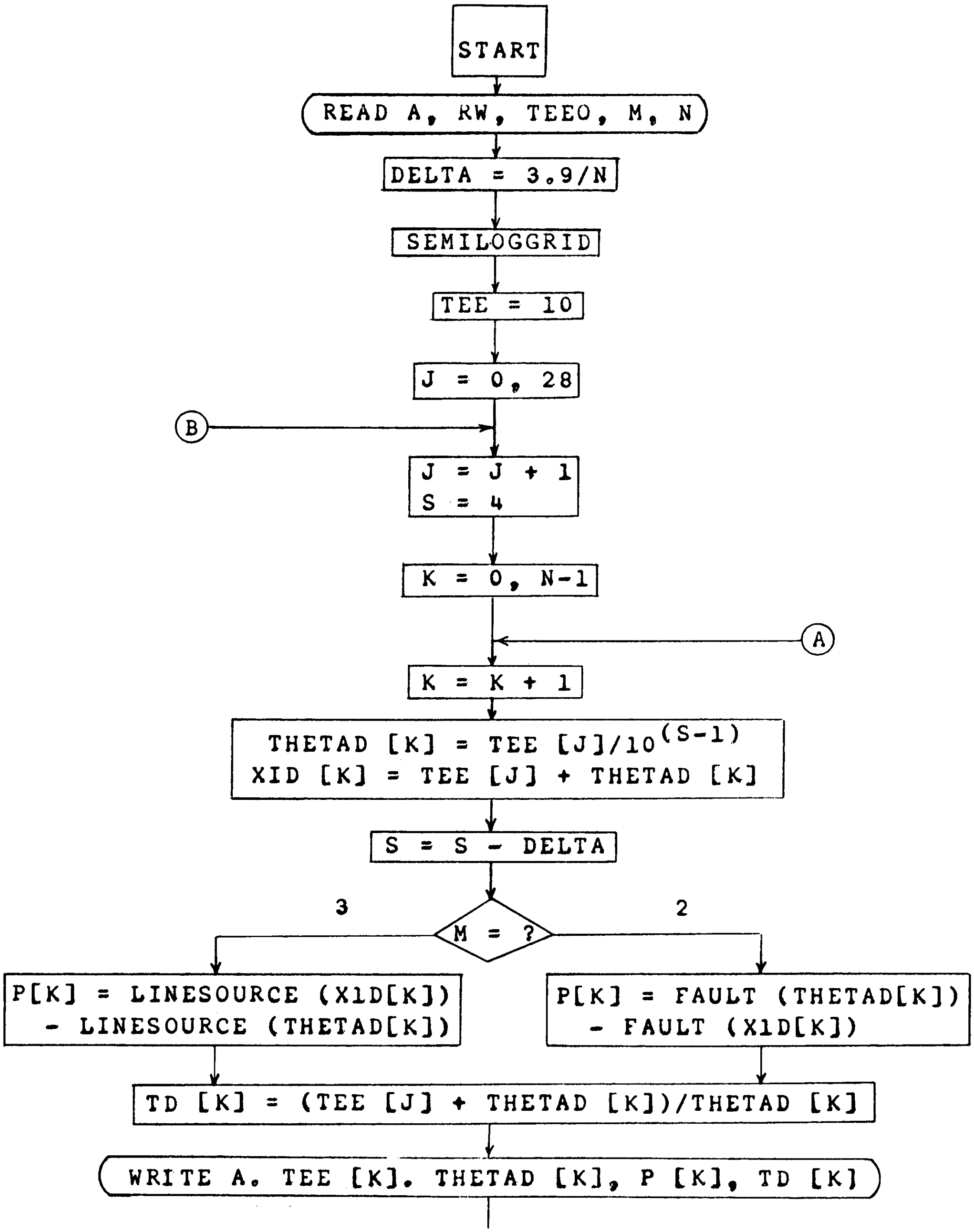


Figure 13

Flow Diagram for Main Program

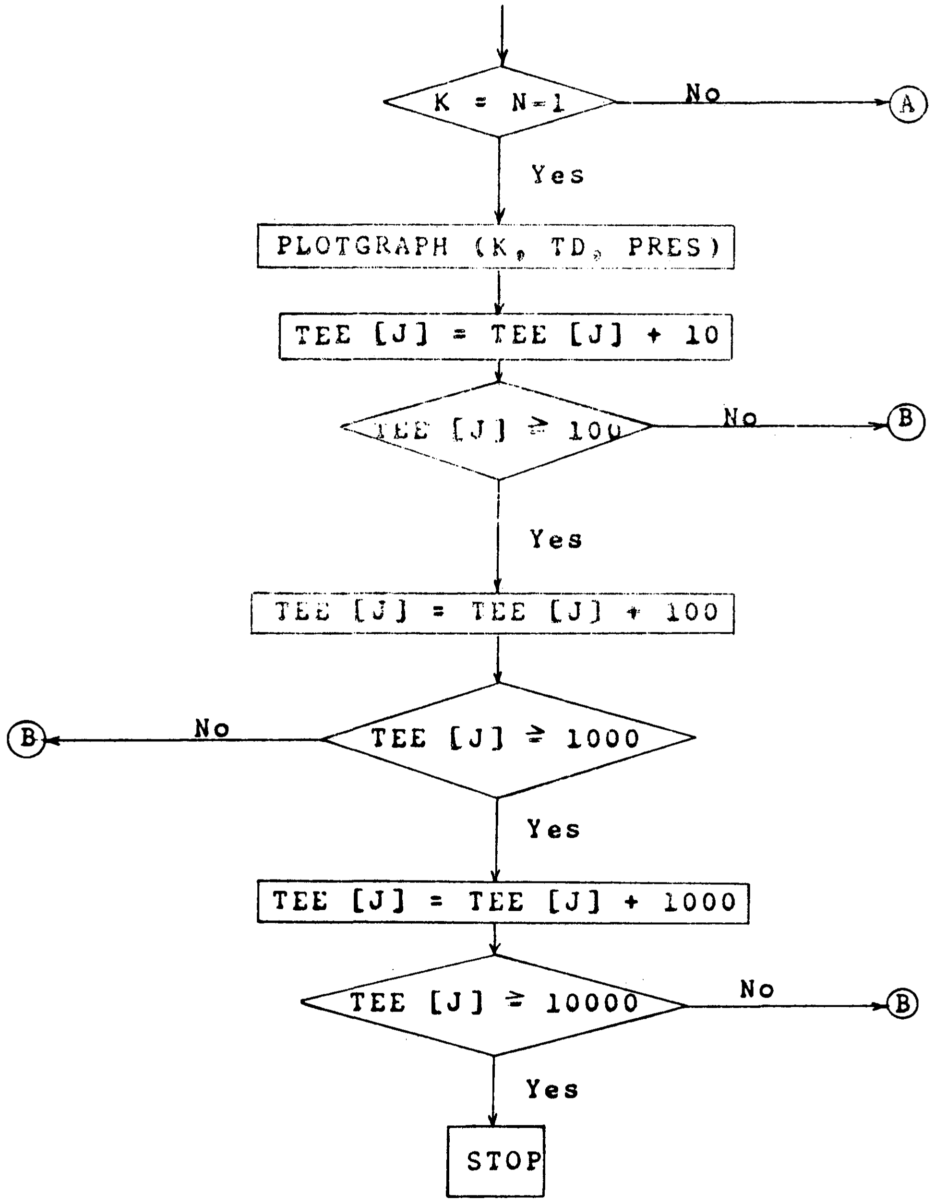


Figure 13 (Continued)

Flow Diagram for Main Program

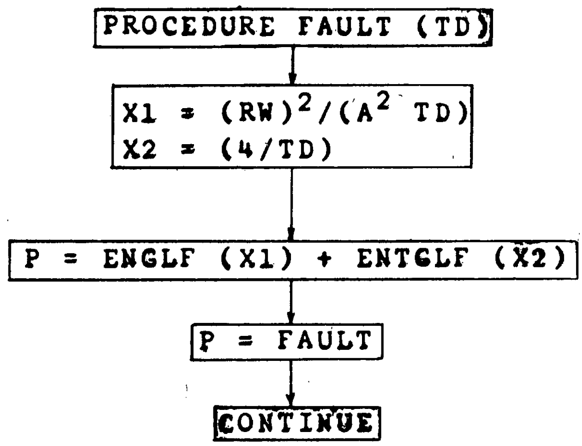


Figure 14

Flow Diagram for Procedure Fault

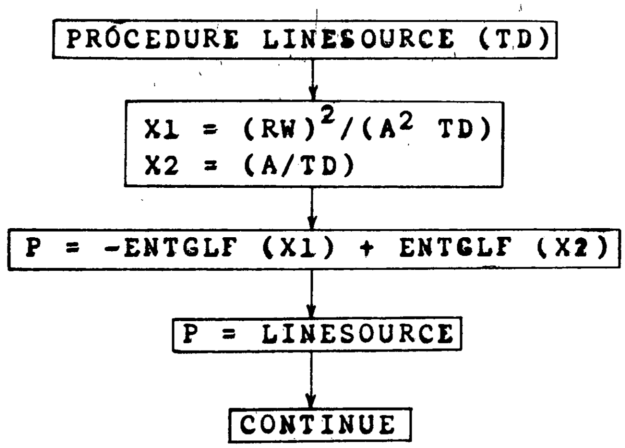


Figure 15

Flow Diagram for PROCEDURE LINESOURCE

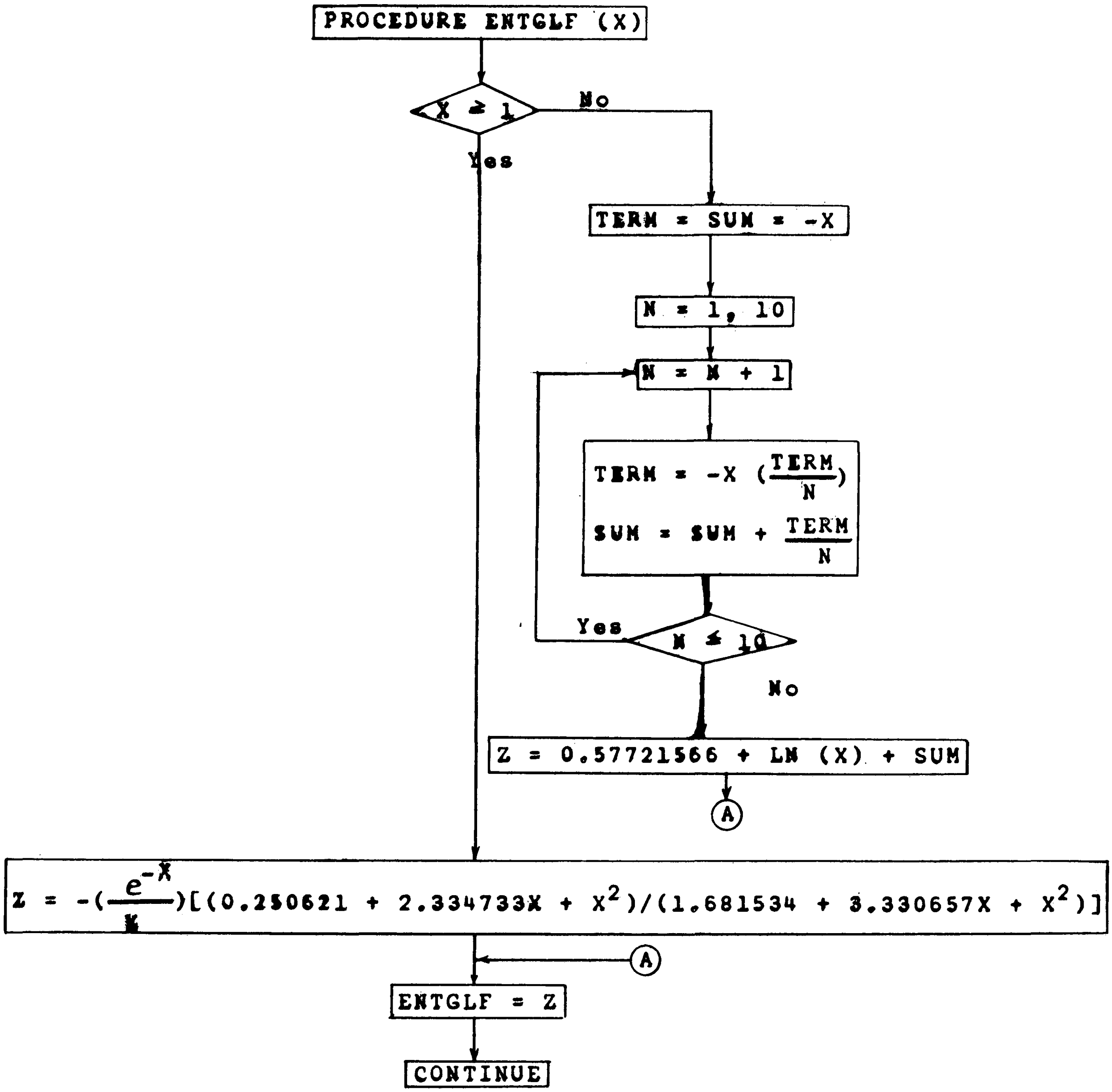


Figure 16

Flow Diagram for PROCEDURE ENTGLF

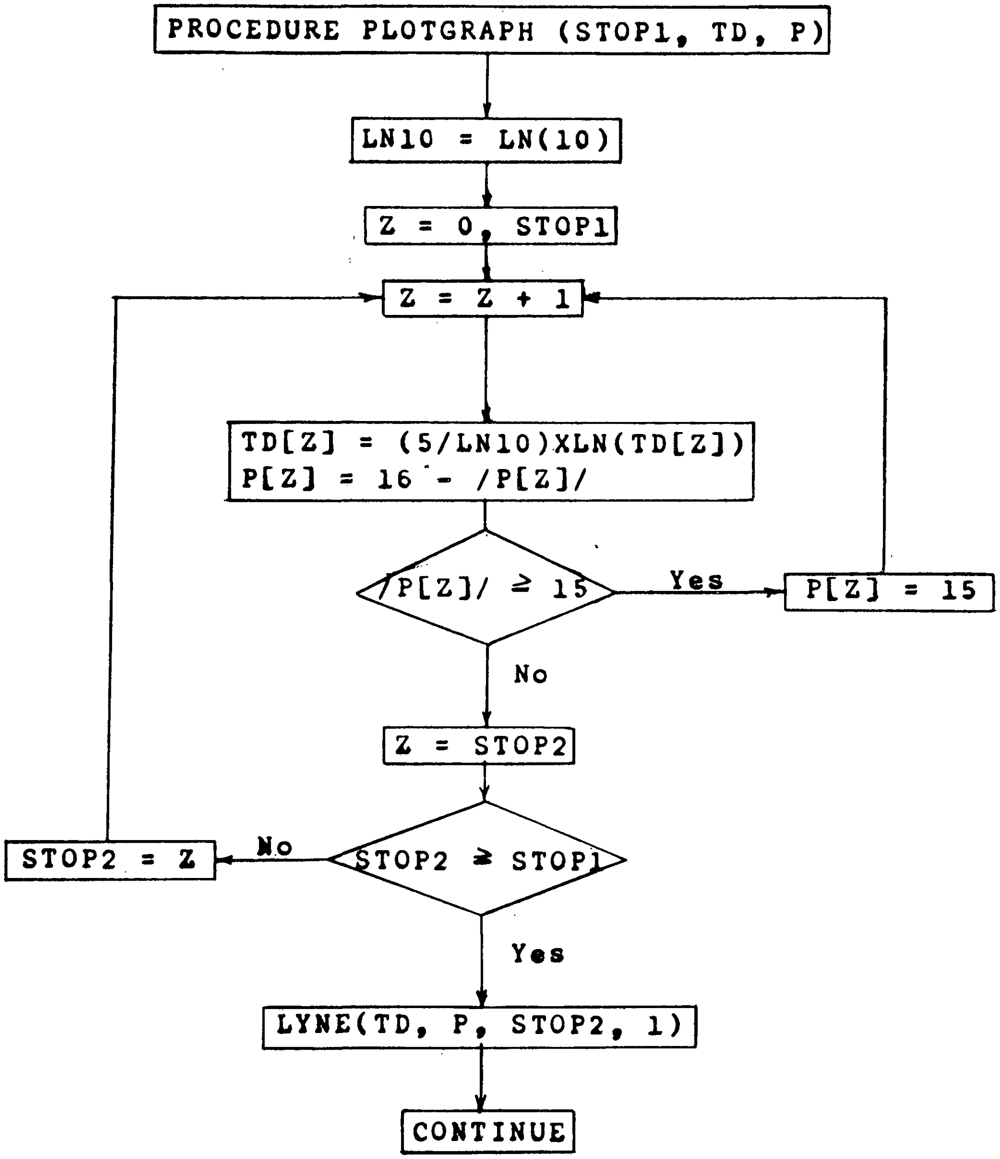


Figure 17

Flow Diagram for PROCEDURE PLOTGRAPH

BIBLIOGRAPHY

Bixel, H. C., Larkin, B. K., and van Poollen, H. K., 1963, Effects of linear discontinuities on pressure buildup and drawdown behavior: Jour. Petroleum Technology, v. 15, Aug., p. 885-895.

Collins, R. E., 1961, Flow of fluid through porous materials: New York, Reinhold Chemical Engineering Series.

Davis, E. G., and Hawkins, M. F., 1963, Linear fluid barrier detection by well pressure measurements: Jour. Petroleum Technology, v. 15, Oct., p. 1077-79.

Horner, D. R., 1951, Pressure buildup in wells: Proc. Third World Petroleum Congress, The Hage, p. 503-21.

Hovanessian, S. A., 1961, Pressure studies in bounded reservoirs: Soc. Petroleum Engineers Jour., Dec., p. 223-28.

Hurst, Leland L., and Guerro, E. T., 1958, Calculting static bottom-hole pressures in a finite reservoir: Oil and Gas Jour., v. 56, Sept. 29, p. 60-68.

Hurst, William, Haynie, O. K., and Walker, R. N., 1961, Some problems of pressure buildup: Paper SPE-145 presented at 36th Annual Fall Meeting of Soc. Petroleum Engineers in Dallas, Texas (October 8-11, 1961).

Mathews, C. S., 1954, A method for determination of average pressure in bounded reservoirs: Am. Inst. Mining Metall. Petroleum Engineers Trans., v. 201, p. 182-91.

__________, 1961, Analysis of pressure buildup in flow tests: Jour. Petroleum Technology, v. 13, Sept., p. 862-70.

Miller, C. C., Dyes, A. B., and Hutchinson, C. A., 1950, The estimation of permeability and reservoir pressure from bottom hole pressure buildup characteristics: Am. Inst. Mining Metall. Petroleum Engineers Trans., v. 189, p. 91-104.

Muskat, M., 1946, Flow of homogeneous fluids through porous media: New York, McGraw-Hill Book Company, Inc.

Perrine, R. L., 1956, Analysis of pressure buildup curves: Paper No. 801-32I presented at Spring Meeting of the Pacific District, Division of Production, Am. Petroleum Institute in Los Angeles, Calif., (may 10-11, 1956), p. 6-35.

Standing, M. B., 1964, Discussion of linear fluid-barrier detection by well pressure measurements: Jour. Petroleum Technology, v. 16, March, p. 259-62.

Van Everdingen, A. F., and Hurst, William, 1949, The Application of the Laplace transformation to flow problems in reservoirs: Am. Inst. Mining Metall. Petroleum Engineers Trans., v. 186, p. 305-24.

van Poollen, H. K., 1964, A hard look at radius-of-drainage and stabilization time equations: Oil and Gas Jour., v. 62, Sept. 14, p. 138-43.

__________, 1965, Find permeability with drawdown and buildup curves: Oil and Gas Jour., v. 63, Nov. 1, p. 118-26.