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APPLICATION OF RISK ANALYSIS TO PETROLEUM INVESTMENT DECISIONS

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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science in Mineral Economics.

Golden, Colorado

Date 11/20/81

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ABSTRACT

This study is concerned with the economic evaluation and decision making regarding petroleum investment ventures.

A technique of assessing and describing the degree of uncertainty that exists in all phases of an oil and gas project, exploration, development and production, is presented. The problem involves associating probabilities with the range of profitabilities that might result from a particular project.

The variables that are used to determine the profitability of the investment, such as the variables that enter in the calculation of recoverable reserves, the variables that affect the production, and the variables used in the economic analysis, may have a probabilistic nature. This model of risk analysis combines the probabilistic distribution of those variables and generates a final distribution of physical and economic parameters used in decision making such as the recoverable reserves, life of the project, production of oil and gas, net present value, rate of return and the ratio of expected gain to total capital costs.

The model is applied to the economic evaluation of an offshore oil project in a field of the Gulf of Mexico.

A computer program to implement the simulation model is

necessary to achieve the results. This work includes a computer program constructed for that purpose.

A statistical interpretation of the resulting distributions for the solution variables is provided.

After dealing with this model of risk analysis we conclude that this model is an excellent tool in the decision making process. Similar models can add a new dimension to the communication between the engineer and management. The data assignments required for probability models are admittedly difficult, but the judgment of the individual who is most familiar with the venture represents the best information in the organization.

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DEDICATION

To my wife

Ivonne

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INTRODUCTION

The economic evaluation of oil and gas ventures is one of the classical examples of decision making under uncertainty. The decision maker should take into account the risk of recovering a certain amount of reserves needed to guarantee the profitability of the investment. He also must consider the effects of uncertain economic factors such as increasing trends and variable fluctuations in prices and costs. Conventional decision theory techniques have been used as a help for management decisions, but these techniques need precise estimates of risk and uncertainty and the success of their application depends on the accuracy of the risk analysis. Therefore, the use of these conventional techniques requires a more accurate and not subjective assessment of risk.

A risk analysis model applied to oil and gas ventures is presented. This model explicitly recognizes the probabilistic nature of each parameter affecting the success of the investment. The solution to the problem may be derived from the interrelationships among the variables or parameters. There may be only a few variables or several. Interrelationships among parameters may be explicit or implicit. Input variables are not precisely known; there is usually an uncertainty in their value. The degree of uncertainty may vary from one variable to another. Variables that are known accurately are called deterministic. The uncertainty of a parameter may result from difficulty in directly and accurately measuring the quantity. Uncertainty may also result from prediction of future parameter values. This type of uncertainty is particularly evident in investment analysis involving future costs, prices and production. The uncertainty in the solution to investment problems is often called risk, and hence the name of risk analysis.

The risk analysis model presented is Monte Carlo simulation. Simulation is a powerful tool for obtaining solutions by using random numbers. It refers to the group of numerical methods characterized by the process of making repeated solutions. Values entering into the calculations are repeatedly selected by random numbers taken from an appropriate range of values. From several hundred to several thousand trials are generally used to obtain suitable results, which makes the technique less suitable to hand calculations than to computer use. Essentially, it is a method in which the answers can be arranged in a form that gives the fractions of the total results which fall within certain ranges. Thus, it is readily adapted to solving probability problems and constructing probability distributions. The method is very flexible and the extent of its uses depends largely upon the ingenuity of the person designing the solution. A strong reason for using simulation is that it offers a means of combining the knowledge of many individuals into a single expression. It is adaptable to the team concept. Another reason for using simulation concerns the organization of knowledge, i.e. information is often easiest and best presented in segments.

The application of the Monte Carlo simulation to petroleum investment ventures is very helpful in the process of decision making because of the uncertainty that exists in oil and gas projects, specially, in the exploration phase in which the degree of uncertainty can be reduced considerably. With certain experience in geological and engineering aspects it is possible to get a satisfactory estimate of the recoverable reserves of a new oil deposit, as well as reliable financial outcomes used in the selection of economic alternatives.

The principal purpose of this work is to present the theoretical and mathematical structure of the Monte Carlo simulation, combined with its application to the economic evaluation of petroleum investment ventures in order to attain results helpful to the decision making process.

This work proceeds according to the following outline. Chapter 1 begins with a discussion of the definitions of risk and uncertainty, how they are used in decision making process and what are the basic differences between both. Chapter 2 then introduces the concept of simulation and a description of some methods of simulation. The remaining of chapter 2 will deal with the theoretical and mathematical structure of the Monte Carlo simulation and a comparison with analytical models. Chapter 3 adds more discussion about the Monte Carlo model concerning with its procedure and the use of random numbers. Chapter 4 outlines the principal aspects of the application of this model concerning petroleum investment decisions. This is the main section of this work. A procedure to evaluate the profitability of petroleum projects is presented. Chapter 5 describes the computer program constructed to achieve this model. This description presents the theoretical and mathematical procedure followed by the computer program in dealing with petroleum economic problems. Chapter 6 presents the performance of the model applied to the economic evaluation of an oil and gas project in the Gulf of Mexico. Then a summary of the principal aspects concerning this model of risk analysis is presented. An appendix has been included to provide the computer program constructed. Another appendix has been included to present a glossary of important concepts of probability theory that should be known to understand the basic structure and procedure of the Monte Carlo method.

1. RISK AND UNCERTAINTY

1.1 Definition

Risk can be expressed as the probability distribution associated with a set of possible future losses. It is used to describe the amount which will be lost if a certain event takes place. If uncertainty exists, risk cannot be reduced unless uncertainty is evaluated.

We can classify engineering and business decisions as to whether the conditions under which the decision is made are certain, risky, uncertain, or both risky and uncertain. Risk exists when each alternative will lead to one of a set of possible outcomes and there is a known probability of each outcome. Uncertainty exists when the probabilities of these outcomes are completely or partially unknown.

All economic analyses are made under condition of risk and uncertainty to varying degrees. To take account of risk explicitly, we need to apply theories of statistics, probability, probability distributions, sampling and confidence limits.

1.2 Difference between Risk and Uncertainty

The meaning of risk and uncertainty refers to the possibility that the desirability of profitability of a project will be different than it is expected at the time of evaluation. However, most of the time, management is not as disturbed if the resulting solution are more favorable than anticipated as they would if the results are less favorable.

For most economic purposes it is not necessary to differentiate between the meaning of the words risk and uncertainty and they can be used interchangeably. However, if we want to deal explicitly with risk and uncertainty, definitions of these concepts must be considered. Risk can be defined as the dispersion of the probability distribution of the event whose expected value is being predicted. Uncertainty will be defined by the degree of lack of confidence that the estimated probability distribution is correct.

Risk is the variation from the expected value which occurs randomly and unfavorable. The larger this variation, the larger the risk. The factors that cause this randomness are many and their importance is not enough to be recognized separately by any of their effects. They are combined with each other statistically and produce a distribution of values for the variable. We cannot attempt to predict those variations caused by randomness. The differences between the expected value and the values caused because of risk is, therefore, not assigned to any error in predicting any variable.

Uncertainty is caused by errors in predicting one or more factors which are significant in determining the future values of the variable. The differences between expected values and those caused by uncertainty are because of incorrect forecasting or not forecasting the factors that define the future values of the variable.

Those factors are affected by the stability of the objectives, policies and strategies of the firm which define the criteria being used to evaluate the alternatives, fluctuations of the market supply and demand of products and services, effects on the market because of consumer reaction to fluctuations in prices, effect on costs of production because of producer's research or technological progress, unlikely events that may occur and will alter a traced plan of production and operations, etc.

1.3 Probability Distribution For Risk

It is often helpful to evaluate risks by constructing a distribution showing the relative frequency of the possible values. These frequency distributions can be developed in a number of ways, with varying degrees of accuracy and uncertainty. Our purpose here is the Monte Carlo simulation which is the most effective way of handling economic problems with high degree of risk and uncertainty.

Another approach in evaluating risk and uncertainty is to estimate a range of possible values for the solution variable, rather than attempt to construct an entire frequency distribution. In addition to estimating expected value an optimistic value is estimated as well as a pessimistic value. If the project appears to be economically good even when the pessimistic values are used, it should be adopted. If the project appears noneconomical even when the optimistic values are used, it should be rejected. When the optimistic value and the expected values make the project appear as economical, but the pessimistic do not, then further study of the factors causing the risks and uncertainties must be made.

1.4 Risk and Uncertainty in Investment Decision Making

Uncertainty in the cash flows from a project could be handled by using a higher interest rate in computing net present value. Also, sensitivity analysis can help when uncertainty is present. However, these approaches consider implicitly the risks and uncertainties.

If the decision maker wants to consider risk and uncertainty explicitly or in greater detail, then he should use probability methods. A probability method can consider that the annual cash flows are only known in a probabilistic sense. For example, we may consider that the cash flow to be achieved by a project may have a certain expected value during its first year, but we are not sure. This uncertainty is expressed by explicitly assessing the chance that the cash flow might have other values. If we explicitly state information about the chance of the cash flows having different values for each year, the value we obtain for the net present value or rate of return will be uncertain but will have to be expressed in terms of probabilities. This information provides the decision maker with a more useful tool to define the possible values of the financial outcomes.

The simplest way of handling risk and uncertainty with regard to the project's financial outcomes is to perform a sensitivity analysis. It should be noted that if we desire to vary the life of the project, we must also specify how the cash flows change as the life changes. Risk considerations can also be explicitly introduced in the evaluation of investment under capital rationing when selection among competing projects must be made.

Although the explicit account of risk factors surely give an idea of how the project's financial outcomes will be, we should realize that the estimation of probability values for the input parameters may not be easy. The engineering and production aspects will create a considerable dispersion of estimates. Also in the selection of any alternative the estimation of how different projects' financial outcomes are each to other could be a problem.

2. SIMULATION

2.1 Definition

Problems with uncertainty can be quite complicated, and a solution by direct analytical methods might be difficult if not impossible. The most general and basic method is to simulate the real solution of the problem.

In this section we will present a technique to solve problems which are subject to risk and uncertainty. This technique is called simulation. Briefly stated, the concept of simulation is concerned with the description of risk and uncertainty in the form of distribution of possible values of the random variables of the problem.

These distributions are combined to construct a distribution of the possible levels of the solution variables. From these final distributions expected values of the solution variables are computed to use in the decision making process. The method is a continuous outcome model of risk and uncertainty.

Simulation as a means of risk analysis in decisions under uncertainty has a very important use in the mineral industry, specifically, in the exploration activity for oil and gas. The general logic of simulation is the repetition of computations, or passes. These repetitive computations are continued until a sufficient number of values of the solution variables are obtained and their distribution are defined. Obviously this many arithmetic solutions of equations imply a great amount of work and the only way of accomplishing it is by using the computer. Fortunately a computer can do this very quickly and for a nominal cost.

A series of repetitive solutions of the equations of interest are done each time with values of the independent variables which are selected from their respective distribution. After a sufficient number of passes have been made the values of the dependent variables are tabulated as distributions.

Some of the advantages of using simulation are as they were presented by Paul D. Newendorp[18]:

1. The risk and uncertainty are described as a range and distribution of possible values for each unknown factor. The distribution for the solution will cover all the possible values.

2. Simulation can be applied to any problem involving random variables.

3. The number of variables used in the problem can be as large as the decision maker wants.

4. It is not necessary that the final distributions for the solution variables have to be of a specific form such as lognormal, normal, etc.

5. Judgments about the distribution of input parameters and output solutions can be made according to the knowledge of the person most familiar with the parameter. Simulation models do not require that one person provide all the input variables of the problem.

6. Simulation models does not require much computer time.

7. A simulation analysis can be used to produce a sensitivity analysis just by changing the distribution of the input parameters in terms of the form or the range of values.

The use of simulation in business and economic analysis, in contrast to the relatively long history of its application to engineering and scientific studies, is a fairly recent development. The technique emerged as a major tool of analysis in business and economics, as digital computers started to become widely available.

Business and economic problems must be looked at in terms of the total system with all of the interactions between the parts, and simulation is a tool that is easily applied to this type of interactive system. The behavior of a system is a result of the combined effect of actions and reactions through all parts of the system. Thus it is necessary to consider not only the actions on one part of the problem, but also the reactions of other components and their continuing interactions.

2.2 Methods of Simulation

Decision makers involved in problems concerning risk analysis on investment decisions probably recognize some, or all, of these problems when trying to use a formal theory. Most alternatives methods of risk analysis in common use today are based on (1) subjective probability, or (2) statistical inferences from past data, or (3) mathematical models that while, valid in theory are not successful in practice.

A subjective probability approach represents an estimation of the degree of belief, or a reliability of interpretation that the investor assigns to the success of his operation. Thus, subjective probability is based on judgment and depends on the shortcoming that given the same basic data, assessments of the degree of risk made by the people involved in the operation are different. Personal bias, emotions and experience influence the subjective judgment of risk.

The subjective assessment of probability is an overall judgment usually made at the end of the evaluation. It is an estimate of how various uncertain parameters will occur. The problem of risk analysis in investment decisions must be considered largely on the basis of subjective assessments. An explicit method should consider this important fact and provide a practical use by means of subjective judgments of each individual variable affecting the solution. Therefore, the use of subjective probabilities cannot be eliminated.

The statistical inferences from past data to create inference of probabilities of the future outcomes is sometimes used. The implicit assumption that necessarily is made when using this approach is that the probability of specific outcomes do not change with time.

A few mathematical models have been constructed to assess the degree of risk in future investment programs. These models consider that the probabilities of specific solution variables change with time. The practical use of these models is difficult, however, because of the judgment or assumptions required to define the distribution probability for the solution variables those models are technically and mathematically acceptable.

Probabilities that have a binomial nature also have been proposed for use in risk analysis. A computed number of successes is used to be combined with a given probability of success. The assumption is that the probability of success is constant and known, and that one of only two possible outcomes can occur, success or failure. The use of this approach needs a subjective distinction between a stochastic variable that is considered to be successful according to what it is affecting and the range of failure of the same variable.

The methods of risk analysis described above have their assumptions and limitations. Most of the methods do not consider that probabilities of some parameter values change continuously with time. Some of these methods do not realistically give the range of possible outcomes, but rather just one outcome with the corresponding probability. Therefore, none of the theoretical approaches based on these limitations, assumptions and shortcoming can effectively handle a risk analysis problem in an investment decision. The assumptions on which they are based are not consistent with the real problem. The important difficulty is the lack of knowledge of the probabilistic nature of the variables that control the operative course after investing. Therefore the problem of risk analysis becomes one of subjective judgment. Once this is recognized the management must begin to improve their ability to analyze the probabilistic nature of the variables affecting the profitability of future investment decisions.

The purpose of this work is to present a model that recognizes explicitly the probabilistic nature of all parameters affecting profitability. The decision-maker could select the most probable value of each input variable and compute a most probable solution variable. However, it would be better to combine the range of possible values of each variable into a single distribution of profitability. A simple straightforward model of accomplishing this is the Monte Carlo simulation which will be described in the remaining sections of this work.

2.3 Monte Carlo Simulation

The Monte Carlo simulation explicitly recognizes the probabilistic nature of the parameters that determines the solution variables. This model is general and may have use in numerous investment problems. The proposed model of risk analysis basically selects at random one value of each of the random variables which have been already define with a respective probability distribution. With the combination of these values a value of the solution variable is obtained. The repetition of this process will create a final distribution for the solution variable. It is readily adaptable to the computer; indeed it probably will be more practical on the computer, particularly if there are a large number of variables to consider. The number of independent variables that can be evaluated is unlimited. If a correlation exists between two parameters one parameter can be expressed as a deterministic function of the second parameter plus a random error term independent of the second parameter. It is not necessary that the distribution be of a specific form such as normal or lognormal. The distributions can be expressed as continuous or discret functions.

This method, in effect, constitutes a shift of emphasis regarding subjective judgments. A series of judgments are made on the basis of occurrency of each individual variable. This method does not require a single judgment about how a series of variables will be combined. Also this model does not eliminate the difficult subjective probability judgment. However the model theoretically suggest that it is better to make these judgments on each individual variable than on a series of variables and how they will occur simultaneously.

Empirical solutions of complex probability models can be obtained by applying the Monte Carlo simulation. This numerical method reduces a stochastic model to a series of deterministic calculations. A conventional analytical approach to the solution of a probabilistic model requires that all elements of the model be manageable mathematically and that an analytical derivation of the solution distribution exist, but the Monte Carlo method allows either analytical or numerical model description.

The logical procedure of the Monte Carlo simulation begins with the definition and the relationship of the input variables which combined define a solution variable. It may take the form of a single equation, several equations or even a series of step-wise computations. After this, the input variables are classified into two groups. One group consists of all variables or parameters for which we know their values. The second group includes all the parameters that are considered to be uncertain and for which probabilistic distributions are assigned. At this point we have defined the simulation model. Chapter 3 will describe this procedure in greater detail. For the sampling or selection of the input variables we have to use random numbers which can be obtained from tables or from computer algorithms. A subroutine (program) that generates random numbers uniformly distributed between 0 and 1 is presented in Appendix A, and the description of this subprogram is given in chapter 7.

The random numbers do not have any effect on the resulting solutions; they are just used for the mechanical step of sampling values from each random variable distribution for each of the trials. It is a method which insures that we consider the uncertainty in the problem. We can achieve resulting solutions also without using random numbers, but the alternative procedures are much more tedious and are not amenable to computer implementation.

2.4 Monte Carlo Model Vs. Analytical Models

Monte Carlo method can be used in the solutions of problems for which

1. Methods of solving analytic models have not been developed. Many waiting-time models are in this category

2. Solvable analytic models are available, but the mathematical procedures are relatively complex, so that Monte Carlo techniques provide a simpler method of solution.

Both analytic and Monte Carlo models have some common limitations. In both cases, we must be concerned with the question of the validity of the model in terms of the behavior of the solution in the real world. However, with Monte Carlo simulations we can frequently obtain more realistic models than with analytic models.

Monte Carlo techniques have some disadvantages which are not present in analytic solution methods. In the Monte Carlo method we have the problem of statistical variations which are present in any experimental design of a real situation. We must take a large enough sample to reduce sampling variation to a sufficiently low level. In some cases, we may also have the problem of ensuring that our simulation continues for a long enough period of time to reach steady-state conditions. Thus, when an analytical model is realistic and can be manipulated to obtain the optimum solution, it possesses an important advantage over a Monte Carlo model.

Following is a comparison between sensitivity analysis and Monte Carlo model. The sensitivity analysis is made only by changing the values of the input parameters; those changes are represented in terms of percentage. The variation of change in the solution is also measured in percentage. The simulation analysis considers these changes of the inputs parameters in an overall basis and at one time, which is not made by the sensitivity analysis. The variable input is affected by a random number so that values of the input variable is obtained as many trials are made. Therefore, the Monte Carlo simulation contains a large number of sensitivity analyses.

2.4.1 <u>Other Models</u>

A variety of methods have been developed to express the anticipated outcome or value of an economic venture. Some of these methods attempt to cope with uncertainty, but most seem to be inadequate for providing a complete description.

1. Expected Values. This is a deterministic method. It makes allowances for variations, which are necessary to compensate for the unfavorable cases. This method conceals

much information.

2. More accurate information. Reduction of error in estimates might make it unnecessary to use probabilistic methods. Deterministic methods could then be used. However, the improvement of estimates takes both time and money. Opportunities may be lost while trying to refine the information. The problem faced with simulation is to estimate the uncertainty that exists in a venture due to imperfect knowledge about its several parts.

3. Empirical adjustment. Where certain estimates have consistently been too high, or too low, by about the same percentage, the use of an empirical adjustment would amount to improving the quality of the estimate.

4. Revising cutoff grades. Rejecting investment whose expected values are below a minimum reduces the risk of financial losses. Any such recommendation presumes there a large number of opportunities immediately available.

5. Three-level estimates. This procedure consist of combining first all high values, next all medium values, then all low values of the several factors that enter into an estimate of value of profitability. The problem with this approach may be that all valuable information about the relative probabilities in the center of the range would be missing.

6. Assigned probabilities. This method considers the structure of a complete probability diagram. Probability diagrams may be crude or refined. Three-level estimates have been improved by assigning a probability of one third to each of the three outcomes. Improvements in this have been made by weighting the middle estimate more heavily than the extremes. However where an outcome is the result of the interaction of several factors that can be separated and estimated more accurately individually, the best complete probability diagram of the outcome can be constructed by combining again them by simulation.

2.5 <u>The Decision-Tree Approach</u>

The decision-tree approach is a good tool in the simulation of problems involving business investment decisions because of its characteristic in showing the interaction among a present decision, chance events, competitors'moves, and possible future decisions and their consequences. It can be used in exploring a variety of problems such as new product introduction, plant modernization, research and development strategy, outlays for new facilities, and others. This approach leads to different and improved answers to investment analysis questions. It forces management to take more direct account of

1. Impact of future possible decisions
2. Impact of uncertainty
3. Values of present and future values
4. Comparative advantages of varying levels of expected profit and planned operations.

This decision-tree model differs from the Monte Carlo model in that it does not use continuous random variables. It only uses discreet random variables and the use of random numbers is not necessary for its application. However, it is considered to be a simulation model for the simple reason that it combines the possible values of the input parameters which are affected by the respective probabilities, and a probability distribution for the solution variable is obtained. The possible values for the solution variable are single values with certain probability of occurrence. Therefore, the solution will have a discreet probability distribution.

The fact that the decision-tree model can not be used when any of the input parameters has a continuous probability is a sufficient reason to consider it in disadvantage with the Monte Carlo model in terms of their applications and way of handling risk and uncertainty.

3. THE MONTE CARLO MODEL - PROCEDURE

In the previous chapter we defined theoretically the Monte Carlo simulation. In this chapter we will discuss in more mathematical detail its structure and procedure when dealing with problems involving risk and uncertainty.

Probability theory, like any other science, has its own set of descriptive terms. For the convenience of the reader, I suggest that before going into the description of the procedure of the Monte Carlo model some concepts of probability theory should be understood. For this purpose, Appendix B presents a brief description of probability terms.

3.1 General Procedure

Assume that a process has been defined by the equation

$$Y = f(X,Z,W)$$

where "X" and "Z" are random variables whose values are not exactly known, "W" is a deterministic variable, that is, its value is known. "Y" is the solution variable for the process.

The problem is to determine the solution of the process but taking into account the uncertainty in the values of "X" and "Z" and the risk in the solution "Y".

The proposed method of risk analysis to solve this problem can be described best as a sequence of steps.

1. Define the process, that is, define the variables and the relationship that ties them together.

2. Estimate the range and distribution of possible values for each of the independent random variables "X" and "Z". The value of "W" is already known.

3. From the distribution of each independent random variable randomly select one value. Then compute the value of "Y" using this combination of values. This determines one point in the final distribution of the solution variable "Y". Select at random a second value from the distribution of each independent random variable. Again compute the resulting solution variable. This is a second point in the distribution of the solution variable "Y".

4. Repeat step 3 again and again, each time with a set values selected at random from the distribution of each independent random variable and the single known value of "W". Enough combinations of variables should be considered to describe adequately the shape and range of the distribution of the solution variable "Y".

5. The distribution of values of the solution variable can be arranged to determine the probability of obtaining various ranges of the solution. Or, by rearranging on a cumulative frequency basis, the probability of obtaining at least a specific value of the solution can be determined.

Fig. 3.1 shows the graphic sequence of the steps described above.

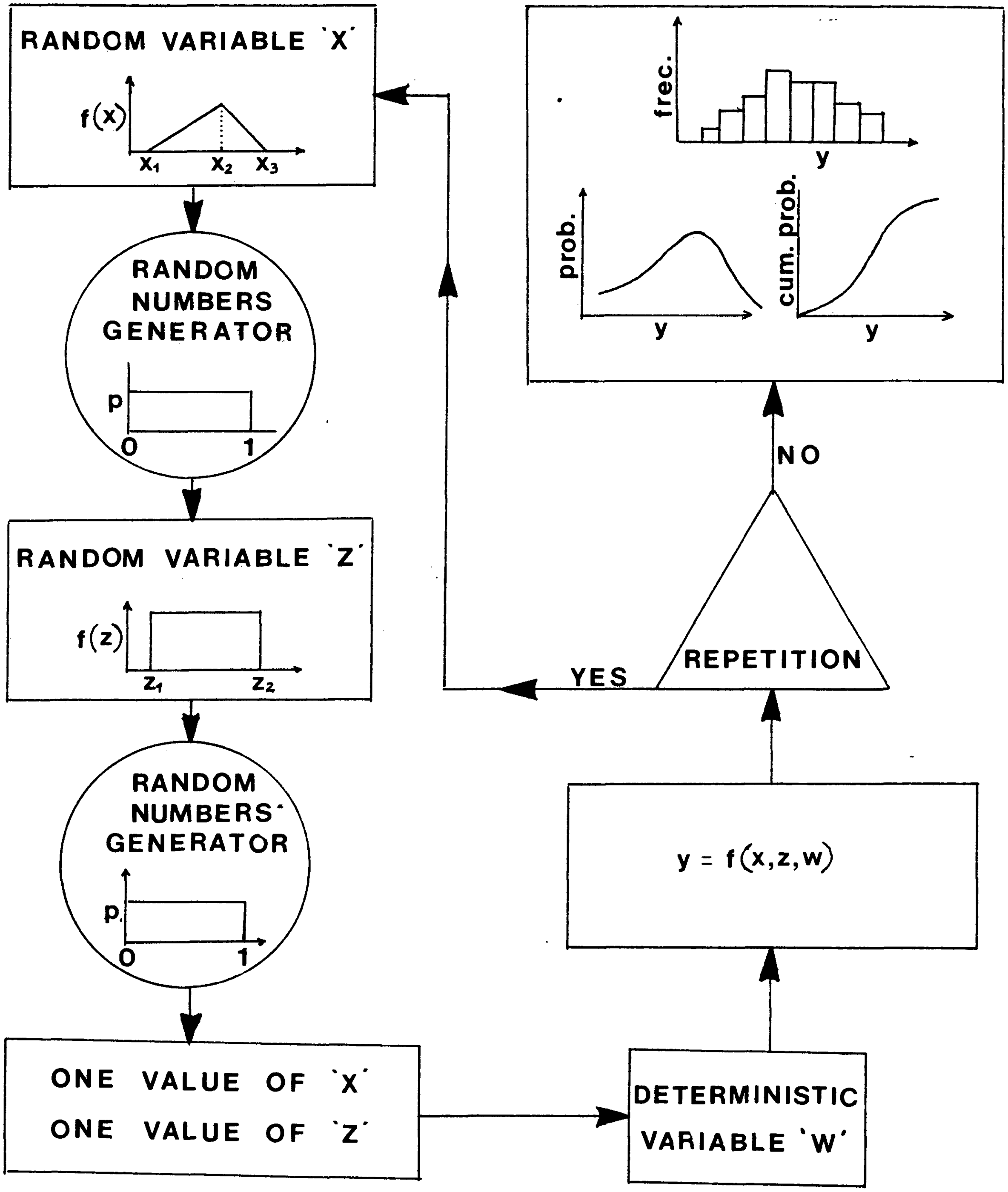

Fig. 3.1 Monte Carlo Model. General Procedure

We will now discuss each of these steps in order to make more understandable the use of the method.

3.1.1 Definition of the variables and their relationship

We must specify the measure of value of interest to us which is "Y", and all the variables which affect its value which are "X", "Z" and "W". Then we need to define the relationship which ties all the variables together. The relationship of interest here is the equation. This relationship or equation is the model or process we try to analyze. Also in the definition of the independent variables we have to classify them into two groups, stochastic variables and deterministic variables. The stochastic variables are those whose values are unknown or there is randomness in their values. In this case "X" and "Z" are the stochastic variables. The deterministic variables are those whose values are exactly known; in this case "W" is the only deterministic variable in the process. Some of the random variables may be related to one another, in which case the dependency relationship must be defined.

3.1.2 Definition of the range and probability distribution for the independent random variables

This is the step where professional expertise and judgment is necessary. So this step is really the key or the critical point in the entire analysis.

Definition of the distributions for each of the random variables can be defined based on the following general guidelines:

(1) The distributions can be of any shape, range or form. We do not have to use a few standard statistical distributions such as the normal, lognormal, etc. The distributions can be discrete or continuous.

(2) The judgments about the distribution for each variable do not have to be made by a single person. Use of the expertise of the persons most related to the variable will lead to better judgments.

(3) The distributions can be based on histograms or frequency distributions of data. The distributions also can be based on knowledge that certain variables characteristically follow a common distribution; or they might be based on a purely subjective judgment.

(4) If opinions vary about the range and distribution of a variable try several possible distributions. This will allow us to determine how sensitive the solution is to a particular type of distribution.

3.1.3 Selection at random of values of the independent random variables

Suppose that for our process "X" has been defined by a triangular probability distribution where "X_1" is the lowest value, "X_2" is the most probable value and "X_3" is the highest value. "Z" has been defined by an uniform distribution where "Z_1" is the lowest value of "Z" and "Z_2" is the highest value of "Z".

The selection of one value of each independent random variable is made by using random numbers and introducing them into the probability distribution defined for the random variable. These random numbers have to be uniformly distributed. Uniformly distributed random numbers can be obtained by referencing a table of random digits for manual computations or by using a random number generator for computer processing. In chapter 5 a generator of random numbers uniformly distributed between 0 and 1 is presented.

The transformations used to generate a random value for the stochastic variables "X" and "Z" are illustrated in Fig. 3.2. The random number "R_s" generated is assigned as a value for $P(X < x)$ or $P(Z < z)$ and the corresponding value, x_s or z_s are determined by the stochastic variables. It is important to note that a random number is generated each time a selection of a random variable value is being made. Thus the number of random numbers generated should be equal to the number of stochastic variables in the process or model. In our process, $P(X < x)$ and $P(Z < z)$ are given by the cumulative distribution functions $F(X)$ and $F(Z)$. For computer calculations, the indicated graphical determination in Fig. 3.2 is replaced with direct calculations of x and z.

At this point we have define a value for "X" and a value for "Z", and we know the value of "W" which is a deterministic independent variable in the process "Y", so we can compute a value for the solution "Y". This computed value is one point in the distribution of the solution.

3.1.4 <u>The repetition or simulation</u>

The selection at random of values of the stochastic variables is repeated again and again, and each time this is done a new value for the solution is obtained. Necessarily this procedure is repeated a sufficient number of times in order that the shape and range of the distribution of the solution can be described adequately.

This sampling process is the key part of the analysis, and it must be done in a manner such that the sequence of values sampled will be distributed in exactly the same manner as its original distribution. This objective can only be achieved by the use of random numbers as entry points on the cumulative probability distribution of the random variables. The random number is set equal to the numerically equivalent probability and the corresponding value of the random variable is determined from the equation that defines its cumulative probability.

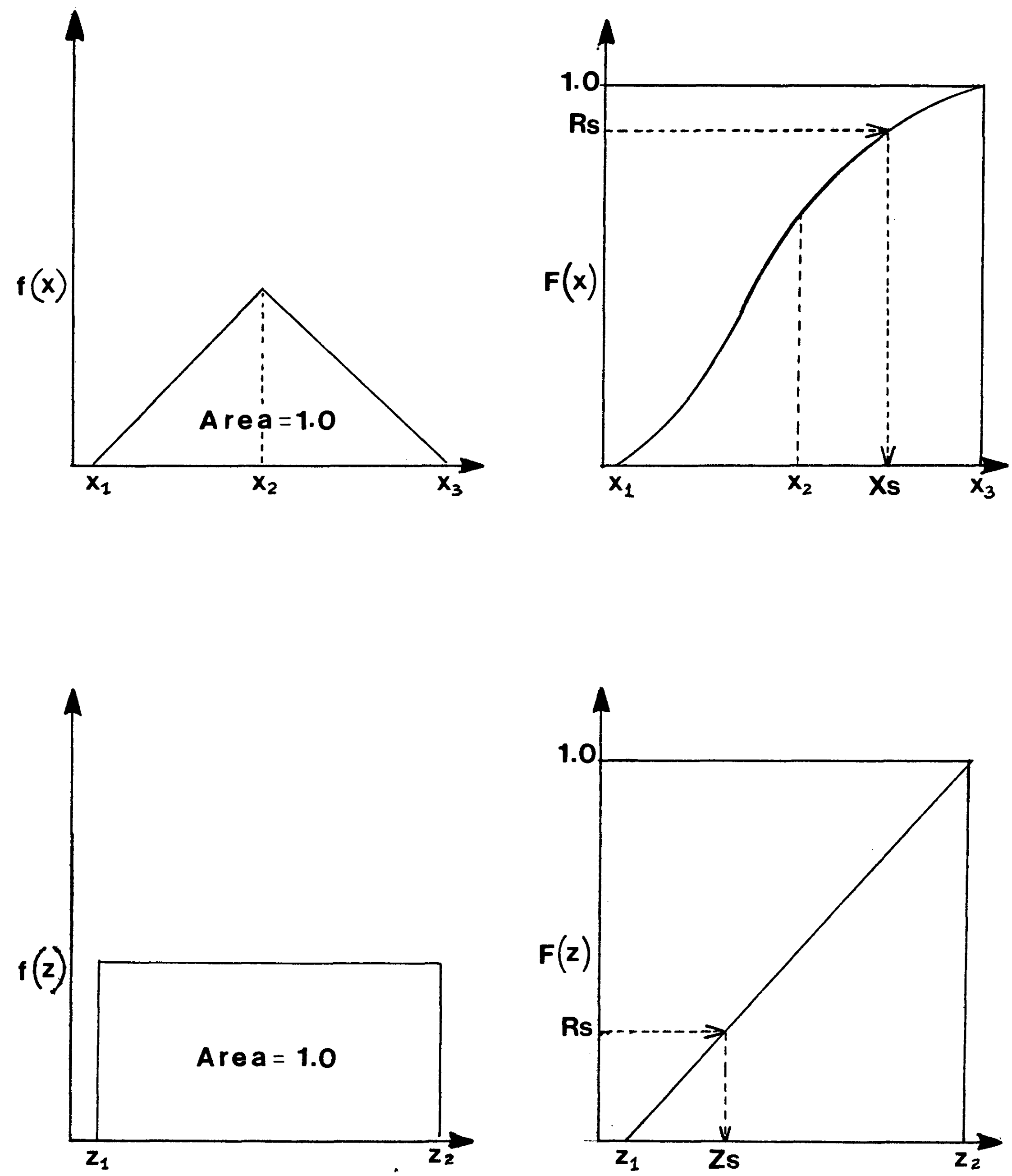

Fig. 3.2 Selection of random variable values, Xs, Zs from continuous triangular and uniform distributions, given uniformly distributed random numbers, Rs

This is the value of the stochastic variable for a particular trial. On the next trial another random number is generated and set equal to its numerically equivalent probability and another value of the random variable is determined. Over a series of such samplings the sampled values of the stochastic variable will be distributed in exactly the same form as the originally specified distribution. This sampling scheme, although just one of several possible ways, is by far the most efficient method on a computer and it is used almost exclusively in performing the simulation trials.

3.1.5 Probability distribution for the solution

Statistical analysis summarizing the model computations should include numerical distributions and averages for all random variables pertinent to the solution. A numerical distribution of "Y" can be obtained by ordering the trial values and computing the probability and the cumulative relative frequency, $F(Y_i)$, for each Y_i as shown in Fig. 3.3. Since "Y" could have a wide range of values, a confidence interval expressed as a fraction of the mean value of "Y" is convenient. Based on the assumption that the distribution of the mean of "Y" approaches a normal distribution, comparison of the variance after n trials and the allowed variance for the mean yields the calculation of the confidence interval.

Graphical presentations of solution distributions are most useful for constructing probability statements, but plotting is often impractical in computer process. A tabulation of the solution distribution on even probabilities is a workable substitute for a plot.

3.2 Interpretation of the Results

Fig. 3.3 shows the probability and cumulative relative frequency distributions for the solution "Y". In addition to these distributions the model solution would include a summary of statistical values such as the mean and standard deviation of the solution, the minimum and maximum values of the solution, and a confidence interval for the mean of the solution. The mean value of the solution is not necessarily the best single value of the solution, better solution values depend on the probability distribution. However, the mean value of the solution is usually the most probable value of the solution.

Probability statements can be formed by referring to the tabulated distribution. For example we can state the probability of obtaining a solution value less than a certain value of interest.

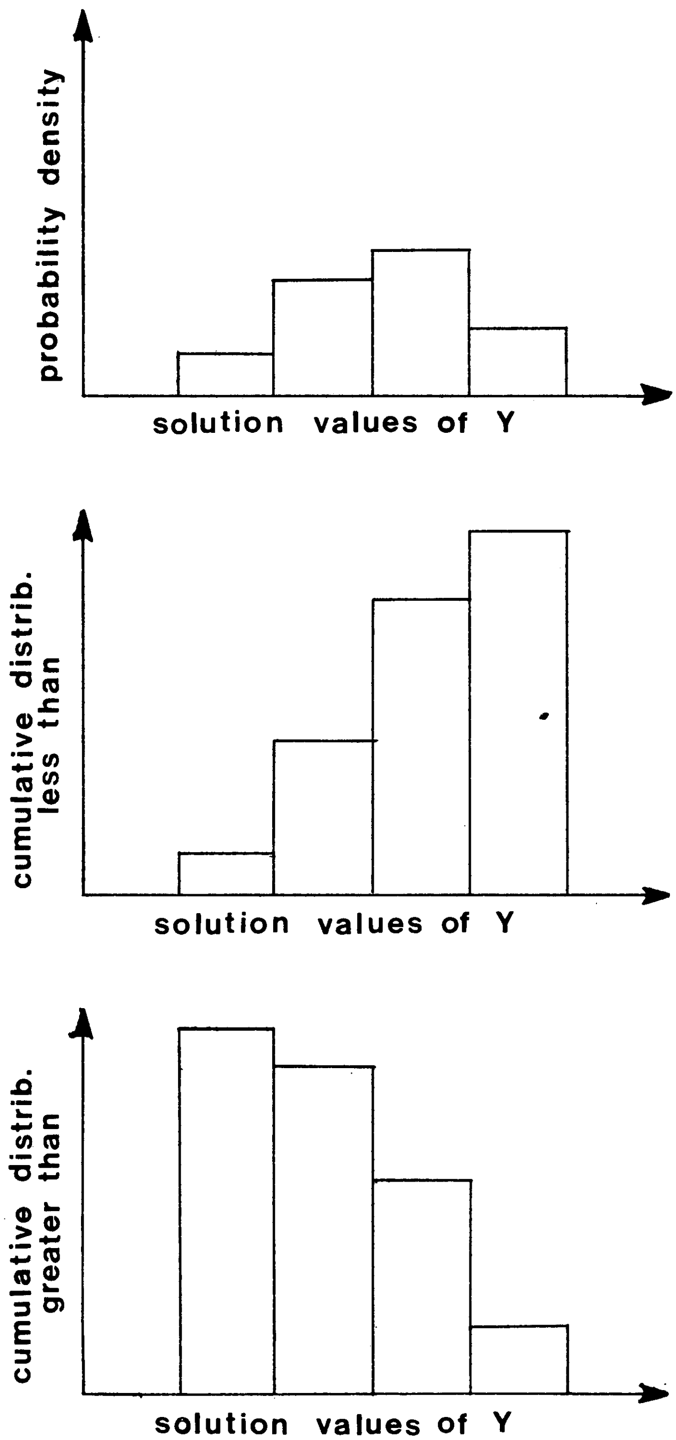

Fig. 3.3 Representation of the Solution Distribution

3.2.1 Uncertainty in the solution

With this type of analysis a degree of uncertainty is assigned to some or all of the input variables in a problem; it is clear that there will be a corresponding uncertainty in the solution. The magnitude and nature of this uncertainty is expressed quantitatively for the solution by a probability density distribution or histogram. A problem involving indeterminate variables does not have a single deterministic solution and to present a single solution to such a problem is incorrect. The maximum point in the probability distribution for the solution is the most probable value for the solution since a solution around this value has the greatest chance of occurrence. The median value is the value at which the distribution is divided into two equal parts. A solution is equally likely to fall either above or below this value and the median value of many solutions would be equal to or close to this value.

Although distributions for the input random variables may all be symmetrical, this does not necessarily imply that the distribution for the solution will also be symmetrical. The form of the solution distribution is determined by the interrelationship between the input parameters and the type of probability distributions for each of the input random variables. The median, mean and most probable values of the solution do not necessarily coincide. If the most probable values of all the input random variables are used to determine a single value of the solution, this value will not necessarily coincide with the most probable value of the solution.

3.2.2 Handling of the solution

The objective of the model is simultaneously to obtain a valid solution of the problem and to determine the degree of uncertainty associated with the solution.

The model can also be used to test the sensitivity of a solution to uncertainty in various input variables. Such sensitivity testing can often delineate areas in which more data and information should be obtained, or indicate that it is unnecessary to investigate in depth certain input random variables for which the solution is not sensitive.

In more complicated problems involving hundreds of variables, the model allows a more detailed assessment of uncertainty to the input random variables. The technique thereby integrates the skill and judgment of many persons.

The best accomplishment of the probability distribution for the solution is that this distribution presents not only the most probable value, but also the more attractive and unfavorable possibilities, together with a measure of the chance that these various outcomes will occur.

It is important to note that if three deterministic solution values are calculated using three sets of values of the input random variables corresponding to pessimistic, most probable and optimistic results, these three deterministic outcomes mean little since it is unlikely that all the pessimistic or optimistic values for the input random variables would occur simultaneously. In fact, many combinations of these pessimistic and optimistic input values can be used to obtain a wide spread of possible outcomes. Unless the probability distribution is known for such a range of possible values of the solution, these limit values have little significance. This is a very important difference between analytical or deterministic models and statistical techniques to solving engineering problems.

3.3 The necessity of Computer Program

Because of the large number of calculations required by the Monte Carlo method, most problems are programmed for computer solution, and several schemes have been developed for generating a series of uniformly distributed random numbers. It is necessary that the random number generator meet the statistical requirements, that the series be both uniformly distributed and random.

A computer program for making a simulation analysis consists of the following parts:

1. Procedures for reading into the computer the random variable probability distributions.

2. Procedures for reading into the computer the deterministic variable values.

3. Procedures for reading into the computer the equation used to compute the solution variable and any equations that determines interrelationship or dependency among input random variables and intermediate variables.

4. A random number generator.

5. The programming required to make the actual simulation trials.

6. Procedures for output of the computed data relating to probability distribution for the solution variable, the computed mean of the solution variable, as well as the standard deviation, minimum and maximum values and confidence interval for the mean. Any graphical displays generated by the computer are optional additions to the output.

There are really only two tools we need for computer sampling, an array of cumulative probability values and an array of random numbers. No matter whether the cumulative probability values are originally from continuous or discreet random variables, or whether the random numbers are generated internally in the computer or read from punched cards, the computer sampling procedure is basically the same. The important thing is to bring these tools together into a working computer program.

3.4 Comments about the Model

Although the model described here is only a tool in the solution of problems involving risk and uncertainty similar models can add a new dimension to the improvement in analyzing this kind of problem. The assignment of probability distribution to the stochastic input data used in probability models could be difficult, but the judgment of the persons who are most familiar with the stochastic variables or parameters represents the best data sources for the model. Certainly this valuable information should be processed using the most sophisticated methods possible at the current level of technology.

Several observations can be made concerning probabilistic models:

1. It is possible to represent the risk and uncertainty associated with the problem separately in several stages.

2. Probabilistic models should be data oriented and should require only that data which can be estimated at the time of the analysis.

3. A general model must include enough detail to represent adequately any consideration that might have an effect on the solution.

4. The time and effort required to develop a computer program for a general Monte Carlo model are great, but a single improved decision concerning the solution can more than pay for these costs.

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4. APPLICATION OF MONTE CARLO SIMULATION TO PETROLEUM INVESTMENT VENTURES

The evaluation of petroleum exploratory ventures is concerned with the economic consequence of success or failure of a certain investment. For an individual well, even when it is located with geologic guidance, there is quite a low probability of discovering enough oil to repay the drilling and completion costs. However, if a drilling program consists of a number of such wells, there is often a high probability that the better discoveries will repay the costs of the entire program as well as yield the necessary profits.

Several of the methods that have been proposed to evaluate petroleum investment ventures have used an expected value. The binomial distribution has been used for describing the distribution of occurrences of successful and unsuccessful trials, and the lognormal distribution has been recognized as adequate for describing the distribution sizes of oil reservoirs. The use of decision trees, a sophisticated method for obtaining expected values, has been applied to making choices among various prospective petroleum exploratory ventures. Also the incorporation of utility preference theory has been considered in the risk analysis to select among suitable investment prospects.

Among the aspects that should be considered in any petroleum investment project the most important is the distinction of the different phases of the venture. From the inception to the final period these phases can be identified as (1) exploration, (2) development, and (3) production. Early exploration decisions concerning geological and geophysical investments are investigated with the entire trend as the basic unit. After prospects have been defined, the trend phase ends and the prospect phase begins. This phase, which continues until the exploratory test is drilled, is the phase in which various land trading and drilling decisions are made. The development phase, which is contingent upon a discovery, is the time when investments for field wells are considered. Decisions relating to the sale or acquisition of producing properties characterize the production phase, which extends to abandonment.

4.1 Risk and Uncertainty in Oil and Gas Projects

The degree of uncertainty associated with a petroleum investment project decreases as the project goes from the earlier phases to the production phase. Unfortunately many of the most important decisions in petroleum exploration and production must be made during the early phases when dry-hole risk and uncertainty about economic factor are greatest. In order to quantify risk and uncertainty a stochastic model for each of the phases, should be defined.

As we have seen in each of the phases of a petroleum project there exists a certain degree of uncertainty and risk. The greater uncertainty is observed in the exploratory phase. This degree of uncertainty decreases as the project begins its development phase, and continues with the production phase until the economic limit is reached.

The exploration phase which is based on geological studies permits the detection of producible structures; however, even though the geological studies show the existence of producible structures, the exploratory wells have to prove the presence of marketable quantities of oil and gas. On the other hand, it is possible that the production could not be attractive because of small quantity of recoverable reserves. Furthermore, if the geological studies are successful and the exploratory wells prove the existence of commercial quantities of oil and gas there will still be uncertainty in the development phase.

As we observe, the results that are obtained in each of the different phases are subject to a certain degree of uncertainty which implies risk at the moment of making a decision. Thus, it is necessary to quantify the uncertainty in each of these phases in such a way that risk is reduced as much as possible in the financial outcomes and in the selection of the best decision.

Among the techniques proposed, the Monte Carlo model seems to be the most promising approach to quantify the risk and uncertainty in petroleum economic problems. A Monte Carlo model applied to petroleum investment ventures will be presented in the remainder of this chapter.

4.2 Probability Distribution for Stochastic Parameters

Although probability theory is characterized by a complex mathematical structure and theory, petroleum economic problems are usually unable to take advantage of them because of insufficient statistical data. An acceptable statistical sample of parameters related to the different phases of a oil and/or gas project is not available to the decision maker.

Fortunately, the judgment of an experienced engineer or geologist can provide acceptable definitions of the probability distributions for the physical and economic parameters instead of dealing with the relatively large samples required by methods based on probability theory.

The definition of the probability distribution for the random input parameters can be made based on three type of probability distributions, continuous, triangular, and uniform distribution, and discrete distribution. Such type of distributions provide the basic initialization to solve any petroleum economic problem, and it is not necessary to introduce in the problem sophisticated forms of probability distribution such as normal, lognormal, exponential, etc.

As stated before the judgment of experienced professionals determines the shape and range of the distribution of the random input parameters. However, the presence of disagreement in the definition of such distributions among the persons most familiar with those parameters is very possible. Furthermore, it is possible to have the situation that a certain random parameter which has been usually considered to be stochastic is defined with a single value instead of a range of values with some degree of uncertainty. This happens when the decision-maker has a strong belief in the estimation of the value of that parameter and does not consider the presence of uncertainty in this value.

One of the most difficult parts of this model in the solution of petroleum economic problems is just what was described above, i.e. the definition of the probability distribution for the random input parameters. Thus, we not only have uncertainty in the values of the stochastic parameters, but also uncertainty in the kind of probability distributions for those parameters. This difficulty is a shortcoming of the model. However, it yields to the possibility of making sensitivity analysis referring to changes in the probability distributions for some stochastic parameters which makes the model more sophisticated and effective in solving problems involving risk and uncertainty.

The probability distributions used for random input parameters for most petroleum economic problems are the continuous triangular distribution, the continuous uniform distribution and the discrete distribution. The continuous triangular distribution is defined by three values, the lowest, the most probable and the highest. The random variable can take any value between the lowest value and the highest value; however, values around the most probable value have greater chance of occurrence. This distribution is characterized by the fact that there is certain belief that the possible value of the random parameter could be the most probable value. The uncertainty in the value of the random parameter decreases as the value of the random parameter tends to approximate the extreme values. The continuous distribution defines a range of values with equal chance of occurrence between a lower limit and an upper limit. All the values within this range have the same probability of occurrence. The continuous uniform distribution differs from the continuous triangular distribution in the degree of uncertainty involving the value of the random parameter. The continuous uniform distribution presents higher degree of uncertainty in the value of the stochastic parameter because the weight of occurrence is distributed uniformly between the lower value and the higher value. The discrete distribution defines the random variable with single values and its respective probabilities of occurrence. The variable can only take these values because of the discontinuity in the distribution. The discrete distribution would have less degree of uncertainty than the continuous triangular and uniform distributions because single values are supposed to occur and not all the values within the defined range are possible.

Appendix B presents a more detailed description of these types of distribution discussed above. Fig. 4.1 presents some of the most used probabilistic distributions in economic problems.

4.3 The Model

We have shown that there is certain degree of uncertainty in each of the phases of an oil and gas project. Thus, it will be convenient to develop a model for each of the phases, and then bring them into an economic model in such a way that the method of evaluating the project becomes more understandable.

The objective is to evaluate the investment prospect by using four models:

The Exploration model

The Development model

The Production model

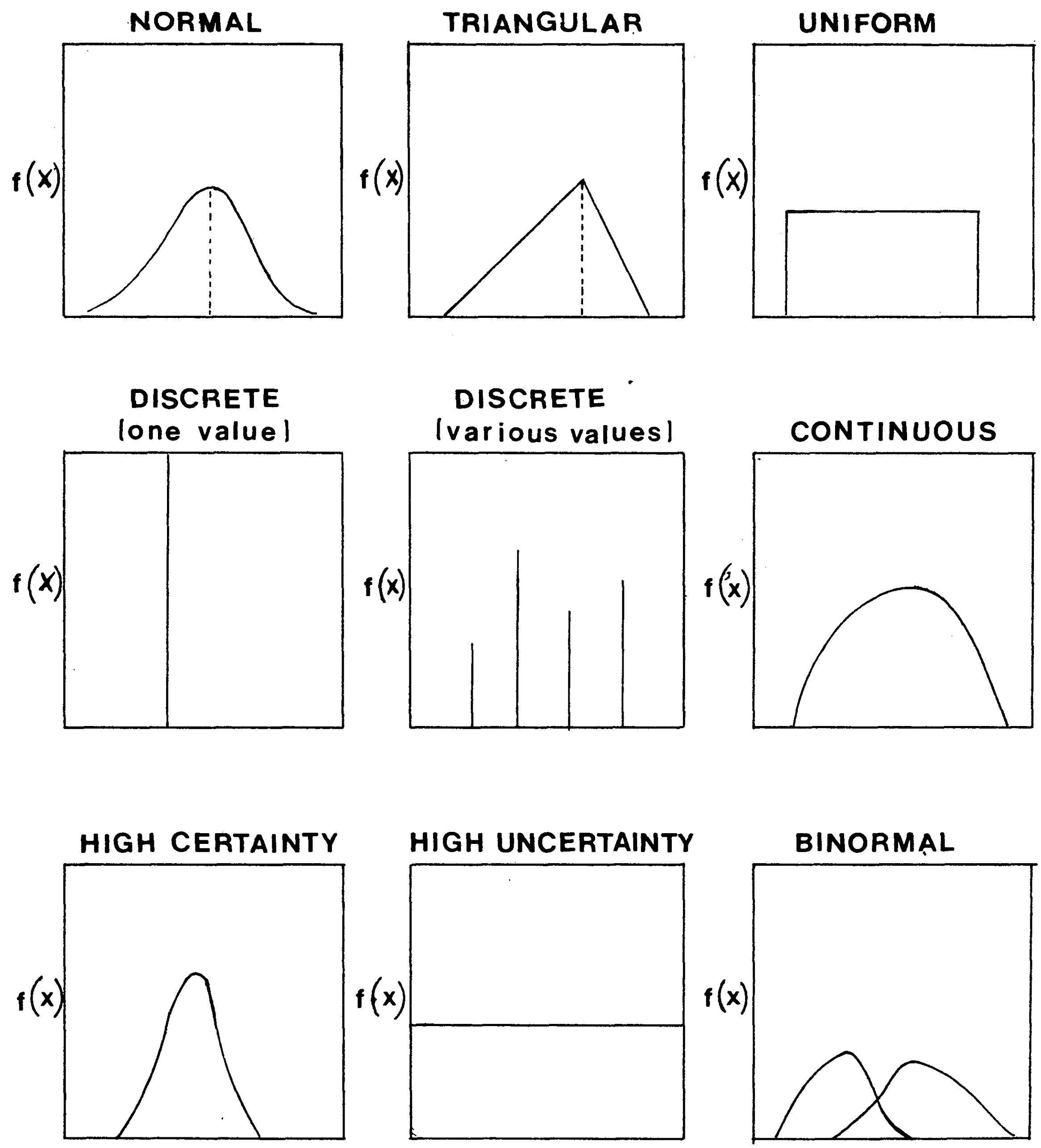

Fig. 4.1 Types of Probability Distribution used in Economic Problems

The Economic analysis model.

Note that these four models conform a general model which is the principal objective. This general model is the articulation of those four models in order that the project can be evaluated from the exploration phase to the production phase. In other words, the solution parameters determined in the exploration model are used to perform the development and production models which in their turns provide the economic analysis model of the necessary outcomes to carry out the economic analysis. There is a strong dependency among the models. Because of this dependency among the models, in a computer simulation program these four models are not analytically separated as they will be in this theoretical analysis. For computer procedures it is impractical to consider the four models separately. We will now discuss each of these models.

4.3.1 The Exploration model

The exploration model is concerned with the estimation of recoverable reserves. This model involves greater uncertainty than the other models. Once geological studies have been made, judgments about the value of physical parameters that define the amount of recoverable reserves are made. From these subjective judgments the parameters that affect the quantity of recoverable reserves can be divided into two groups: stochastic parameters and deterministic parameters.

After defining which parameters are stochastic and which are deterministic, then a type of probability distribution, such as continuous triangular, continuous uniform or discrete, is assigned to each of the stochastic parameters. Using a generator of random numbers which are uniformly distributed between 0.0 and 1.0 hundreds of combinations of the input parameters are obtained and the recoverable reserves are computed as many times as the number of trials. The number of solutions will determine a probability distribution for the recoverable reserves. Note that this procedure is the same as that explained in section 3.1. Fig. 4.2 is a graphical representation of the exploration simulation model.

The exploration simulation model requires the following input basic data:

Productive area

net pay thickness

porosity of the productive zone

water saturation

gas saturation

oil volume factor and

recovery factor

The calculation of the recoverable reserves is made by using the equation

$$RR = \frac{7758 \times A \times h \times P \times (1-SW-SG) \times RF}{BO}$$

where

7758 is a conversion factor

A is the productive area in acres

h is the net pay thickness in feet

SW is the water saturation in percentile

SG is the gas saturation in percentile

RF is the recovery factor in percentile

P is the porosity in percentile

BO is the oil volume factor to convert bls of oil at the deposit conditions to bls of oil at normal conditions

RR is the quantity of recoverable reserves in bls.

The most important thing concerning the exploration model is that the results obtained will determine if the amount of recoverable reserves are economically exploitable or not. The risk involved in this decision is quantified by the probability distribution of the solution. If the amount of recoverable reserves is considerable, there is still uncertainty in the economic value of the deposit. Thus, the exploration simulation model cannot determine by itself the range of profitability of the investment, we need to move into the development, production and economic analysis models to have a better idea of the success of the investment.

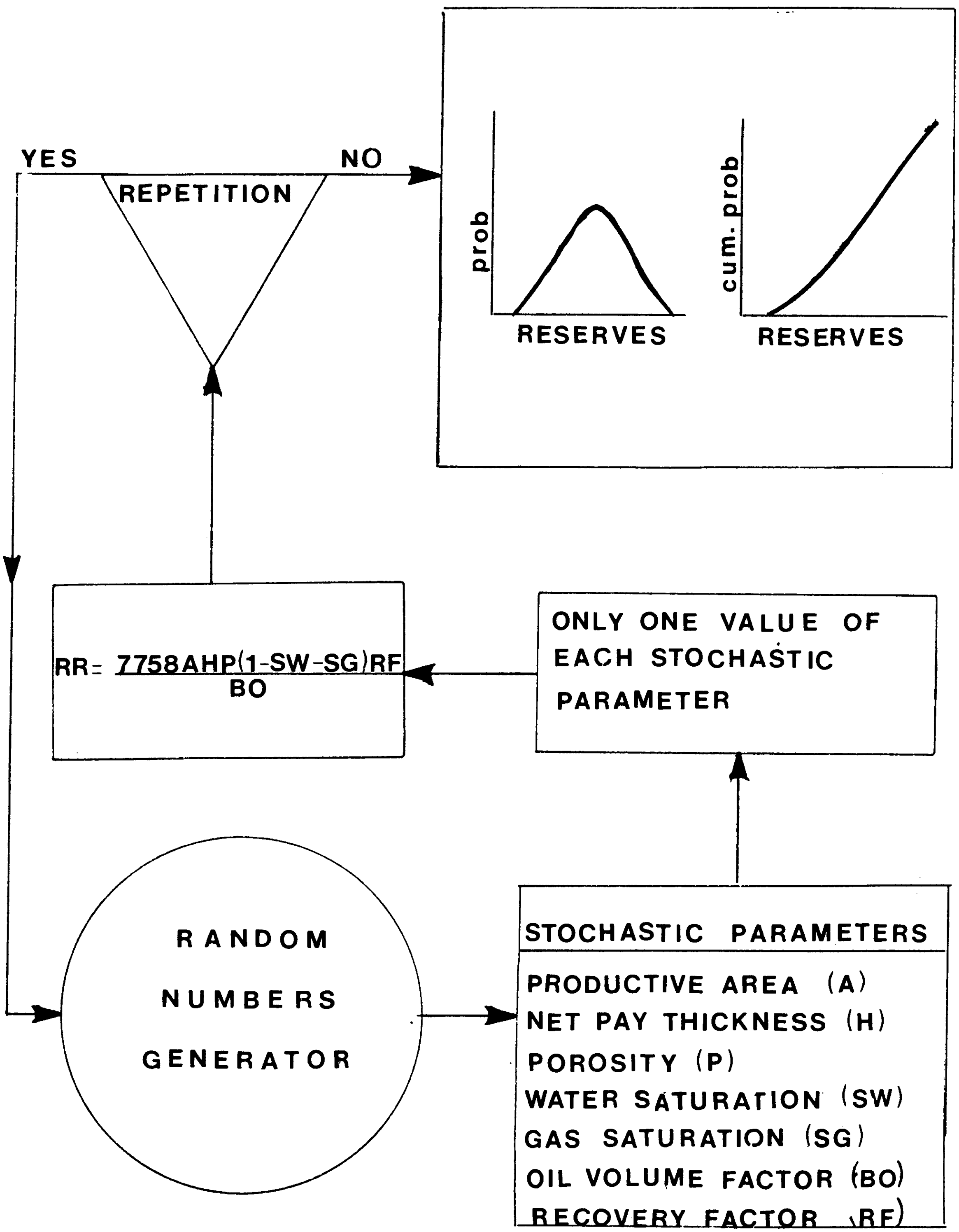

Fig. 4.2 Exploration Model

The computer simulation program constructed to achieve this method of risk analysis considers all the input parameters in the exploration model to be stochastic. However, the program was constructed in such a way that it gives to the user flexibility to assign a single value to any or all of the input random parameters. What this means, is that, if the decision maker considers a stochastic parameter to be deterministic, the computer program will permit him to do that. Chapter 5, will explain the procedure of the program and how to use it.

While discussing the other three models we will indicate what input parameters should be deterministic and the reason for why they should be.

4.3.2 The Development model

When in the exploration phase the geological studies have defined prospective structures and the exploratory wells have proved the possibility of commercial amounts of recoverables reserves, then we move into the development of the oil and gas deposit. In the development phase the company begins its drilling operations. The number of producing wells the company drills is based on the area of the deposit. The amount of recoverable reserves does not have any relation to the number of wells to drill. What determines the number of producing wells is the relation between the productive area and the drainage area per well. This is a very empirical assumption but of great use in the industry.

The fact that the productive area is a stochastic parameter implies that the number of wells to be drilled is also a stochastic parameter and then the following questions must be answered:

(a) How many wells per year we have to be drill to develop the entire deposit?

(b) How many years we need to complete these wells?

(c) How many of these wells will result in dry holes?

The answer to question (a) is concerned with our previous discussion about the drainage area. The answer to question (b) embodies a considerable degree of uncertainty. The time necessary to develop a deposit depends on many things such as the success of the operations, the availability of necessary equipment, the availability of capital and human resources, political circunstances, etc. Thus this parameter should be stochastic. However, it does not mean that the management cannot estimate the time to drill a certain number of wells. Here we must rely on the judgments of experienced professionals in the organization. The answer to question (c) is based on past experience, however, it is usually treated as a stochastic parameter and given as a percentage of the number of development wells drilled.

The dedication of capital in the development phase is relatively bigger than in the exploration phase. The capital invested in the exploration phase includes the acquisition of mineral rights or land acquisition, cost of drilling equipment, cost of seismic exploration and cost of exploratory test holes. In the development phase the capital invest is the cost per foot drilled, the cost of recompleted wells, and the operating costs.

The cost of a drilled well is a function of the depth of the productive zone. The depth of the productive zone could be a stochastic parameter because of the uncertainty in the thickness of the productive zone. The exploratory wells could detect the depth and thickness of the productive zone in some areas of the deposit. However, we need as many exploratory wells as changes in the shape of the oil structure which is economically impossible.

Because of changes in the hardness of the drilled formation, the rate of drilling also changes which implies a wide range in the cost per foot drilled. Thus, the cost per foot drilled could be considered a stochastic parameter.

The operating costs are costs that depend on the rate of production of oil. The cost per produced barrel of oil or operating cost could also be considered a stochastic parameter. The operational problems present in a producing well are the factors that cause uncertainty in the value of the operating costs.

The cost of recompleting a well is usually unknown. It depends on the depth of the well, the operation time, the kind of completion equipment in the well, etc. Usually in the oil industry wells are on average recompleted each four years after production.

The rate of production of oil and gas and the declination of the production are undoubtedly stochastic parameters. These parameters are affected by properties of the reservoir such as permeability, pressure, temperature, amount of reserves, viscocity of the oil, etc. The rate of production of gas is a function of the quantity of oil produced if the deposit is not a gas reservoir. The rate of gas produced is equal to the rate of oil produced times the gas oil ratio. The gas oil ratio is a measure of the quantity of gas associated with each barrel of oil produced.

We have made a brief description of each of the input parameters that enter into the development model. In summary the development model is defined by the following input parameters:

(1) Depth of productive zone

(2) number of development wells per year

(3) number of years necessary to drill the total number of development wells or development time

(4) number of dry holes

(5) rate of production per well

(6) gas oil ratio

(7) declination of production

(8) cost of development well

(9) cost of recompletion

(10) cost of dry hole

(11) operating costs.

All of these parameters could have a stochastic nature. The nature of the probability distribution for these parameters is defined by the judgment of the professional individuals who are most familiar with them. It is possible that some of these parameters are defined with single values in instead of a range of values. It also depends on the experience of people that make the subjective judgments.

The most important thing in the development phase is that management begins to see some return on the capital invested because a certain production will have been achieved. However, we need to determine the economic value of the project to quantify the risk and uncertainty involved in the financial outcomes.

4.3.3 The Production model

Once the development wells have been drilled the maximum point in the rate of production is reached. At this point the production phase starts. This phase is also called the declination phase of the production. This phase continues until the rate of production has reached its economic limit which marks the end of the life of the project.

The input data required by the simulation production model has been defined in the development model, and they are:

(1) the rate of production per well

(2) declination of production

(3) operating costs

(4) cost of recompletion wells.

The rate of production of a well is affected by the annual declination factor of production immediately after its first year of production. In fact this factor of declination will continue affecting the production until a non-economical level of production is reached.

Three important results obtained from the simulation production model are:

(a) Average rate of production of oil

(b) Average rate of production of gas

(c) Life of the project

This results are presented in the form of probability distributions and cumulative probability distributions. These distributions will indicate to management a range of possible values of production and project timing which can give them an idea about the economic value of the project. The uncertainty indicated in the solutions obtained in this simulation production model reflects the risk involved in the project. Fortunately this uncertainty has been quantified in such a way that the risk is reduced.

For convenience, the development model and production model have been presented graphically together in Fig. 4.3. In this graph those parameters related to disbursement or return of money have been excluded. The objective of doing this is to present a more practical understanding of the determination of the solution parameters, life of the project and average production of oil and gas and avoid any interaction with the economic analysis model.

4.3.4 The Economic Analysis model

We have discussed the exploration model which helps in the determination of the probability distribution for the recoverable reserves. Then we described the development and production model in which we discussed how the amount of recoverable reserves will be produced or at most exhausted. We have seen that the life of the project is the time of reaching the economic limit of production or producing the total amount of recoverable reserves.

In the three previously discussed models we have defined the possible annual production schedule, the annual operating costs, the disbursement of costs for drilling operations and

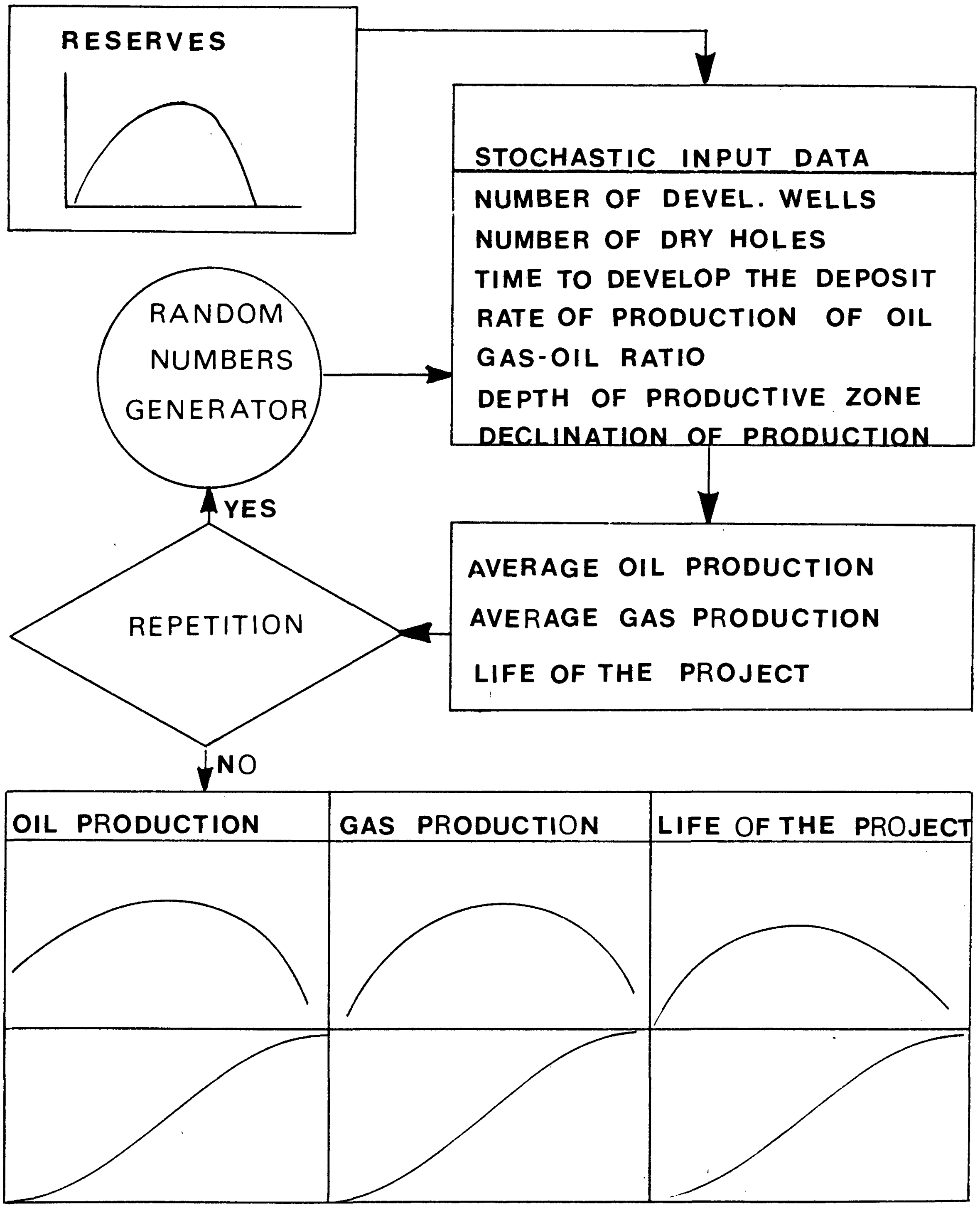

Fig 4.3 Development and Production Models

recompletion of wells. Thus, all the necessary information to construct an economic analysis simulation model is complete.

The objective of the economic analysis model is to determine a range of possible annual cash flows which will be used to determine probability distributions for the present value, discounted cash flow rate of return and the ratio of present value to present total capital costs (see Fig. 4.4).

We are not going to present in detail the procedure to calculate the annual cash flow and determine the net present value, discounted cash flow rate of return and net present value-present capital cost ratio. If the reader is not familiar with the procedure of investment decision methods and economic analysis to evaluate economic alternatives we recommend him consult Barish and Kaplan [2] and Stermole[24].

The important thing here is to indicate the input data required by the economic analysis model. The two types of input data are listed below:

<u>Deterministic data</u>

(1) Cost of land acquisition or mineral rights

(2) Cost of drilling and recompletion equipment

(3) Number of drilling and recompletion equipment

(4) Kind of depreciation:

4.a straight line

4.b double declining balance

4.c sum of year's digits

4.d units of production

4.e double decline switching to straight line

4.f double declining balance switching to sum of year's digits

(5) depreciation life

(6) tangible costs

(7) salvage value of the equipment

(8) percentage depletion (if applicable)

(9) income tax rate

(10) royalties

(11) investment tax credit (if desirable)

(12) financial loan on the year zero investment (if desirable), if yes, percentage of the capital costs invested in year zero that is borrowed and the interest rate at which the money is borrowed

(13) actual oil price

(14) actual gas price

(15) minimum rate of return

<u>Stochastic data</u>

(1) annual rate of production of oil and gas

(2) annual operating costs

(3) annual drilling and recompletion costs

(4) cost of seismic exploration

(5) number of test holes

(6) cost of test holes

(7) cost escalation factor

(8) price escalation factor

(9) life of the project

Note that the annual rate of production of oil and gas and the life of the project have been determined in the development and production models from previous input data that have been considered as stochastic so their solution values also have a probabilistic nature.

The price and escalation factors are perhaps the most important parameters in the economic analysis model. In today's inflationary economic scene it is important to reconsider carefully investment measures that worked well in the past, because they may not work as well today. Explicit inclusion of inflation effects can change the real expectancy of an investment. If we consider properly the inflation effects then the objectives of our investment decisions can be changed. But the proper consideration of inflation is a matter of uncertainty. We do not know exactly at what rate prices and costs will escalate in the future. Therefore, we have to consider in our model the annual price and cost escalation factors as stochastic parameters.

Finally, when the probability distribution for the financial outcomes have been generated, then the decision maker has very useful tools to evaluate the uncertainty in the solution for these financial outcomes to make the best decision concerning the project or referring to the selection of other investment alternative.

4.4 The computing procedure

We have previously discussed the theoretical structure of the proposed model of risk analysis applied to petroleum economic problems. In that discussion we separated the model into four submodels: exploration, development, production and economic analysis models. In the discussion of those four models a description of each of the principal compounding elements was presented, that is, a description of the input parameters and solution parameters was provided. Now we move into the mathematical framework of the procedure followed by the model. The computing procedure of the model is based principally on the generation of random numbers to select the values of the stochastic parameters. This is the main characteristic that make Monte Carlo simulation differ from analytical models. Hundreds of values of each parameter are generated which makes possible a large number of combinations of the variables that affect the functional relationships determining the solution variables.

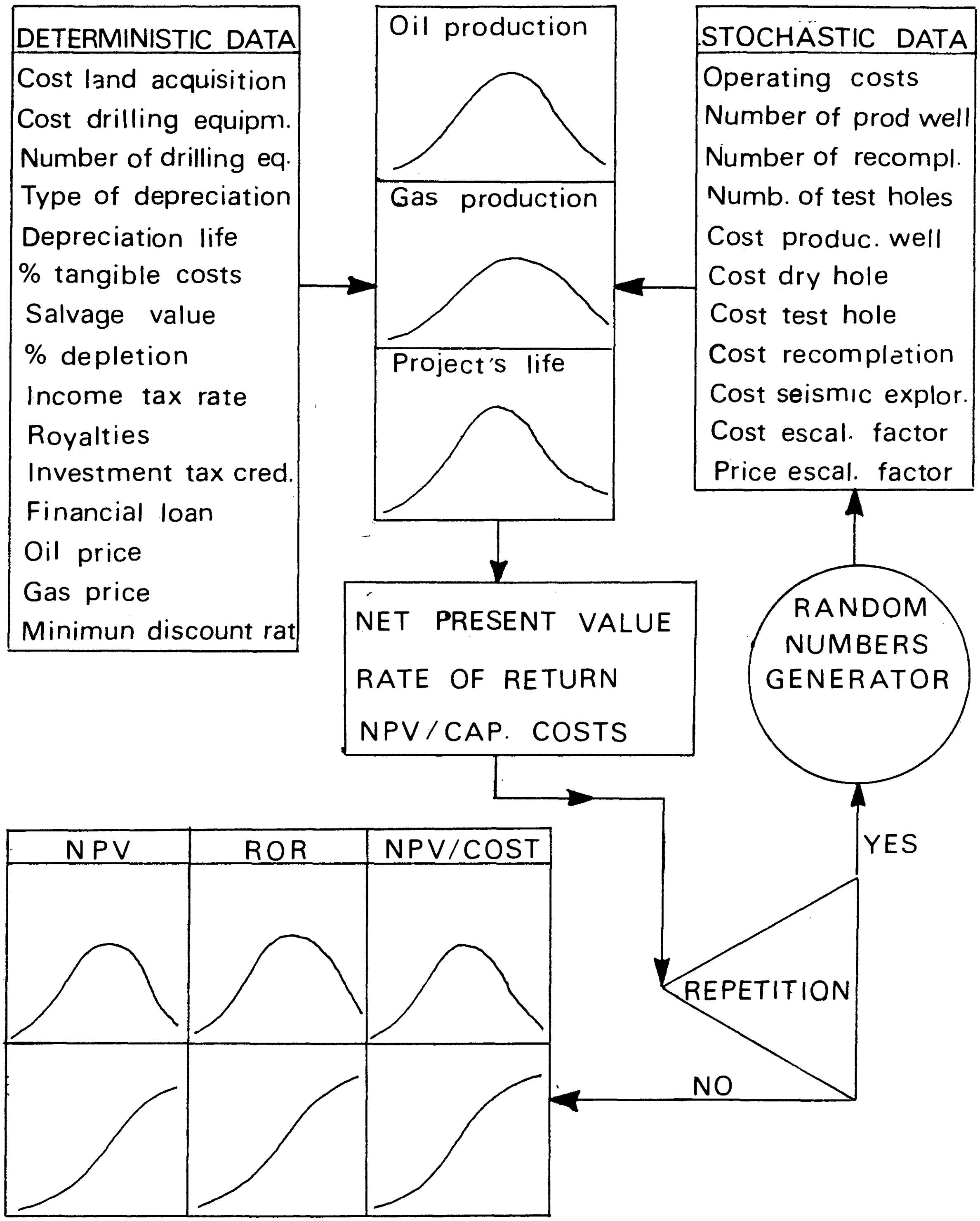

Fig. 4.4 Economic Analysis Model

The generation of random numbers will be explained in chapter 5 together with the random selection of the stochastic parameters. In this section we describe how the model proceeds from the calculation of recoverable reserves in the exploration model, to the determination of the financial outcomes, in the economic analysis model.

The first step is the configuration of the problem in terms of the risk involved. Management realizes that there will be uncertainty in the solution variables and a certain degree of risk in the investment decision, and that this risk is caused by the uncertainty in the input parameters that affect the profitability of the investment. Therefore management and the engineering, geology sections should provide subjectives estimation of the input parameters as well as the type of probability distribution in the case of the stochastic parameters.

When all the input variables have been defined, that is, classified into two groups, deterministic and stochastic, and then for those that possess a probabilistic nature a kind of probability distribution has been assigned the model is set and the procedure followed is:

(1) Specify the number of simulation trials. This is the number of times the solution variables will be determined. It is best to use more than three hundred simulation trials

in order to obtain normalized distributions for the solution parameters.

(2) Specify the number of class boundaries for the solution probability distributions. The number of class boundaries is arbitrarily chosen by the decision maker. If there is a considerable spread of value within the range of the solution the number of class boundaries should be small. If there is a concentration of values around certain point in the distribution a large number could be used.

(3) Select at random a value of each parameter that has a probabilistic nature. A random number is generated each time a value of a stochastic variable is being selected.

(4) Calculate the recoverable reserves.

(5) Calculate the annual production of oil and gas.

(6) Determine the life of the project. This is done by using the amount of recoverable reserves and the annual production of oil or an estimated economic level of production of oil.

(7) Determine the total annual revenues from the annual production of oil and gas.

(8) Compute the annual operating costs. This is a direct function of the annual production of oil.

(9) Compute the annual intangible capital costs. The annual capital costs are represented by the costs of drilling

and recompletion equipment, land acquisition and exploration in year zero and in the following year by the cost of drilling the annual total of development wells and recompleting them.

(10) Calculate the annual depreciation by using any of the methods mentioned in section 4.3.4. The depreciation is based on the annual tangible capital costs.

(11) Obtain the annual income before depletion by subtracting from the total revenues, calculated in step 7, the annual royalties, intangible costs, operating costs and depreciation.

(12) Determine the annual allowed depletion. The major oil and gas producers are not elegible for percent depletion and should use cost depletion calculations. Small oil and gas companies use both percentage depletion and cost depletion and deduct the one that allows less taxation costs.

(13) Calculate a reduction of tax liabilities by means of investment tax credit if it is applicable.

(14) Obtain the annual taxable income by subtracting from the annual income before depletion the allowed depletion and adding the investment tax credit.

(15) Obtain the net annual profit by applying the corporate income tax rate.

(16) Determine the salvage value after tax. To determine this, the final book value of the depreciable assets should be determined. The after tax salvage value is

calculated by subtracting from the salvage value the book value of the depreciable assets and then deduct the tax from the capital gain.

(17) Determine the net annual cash flow. This is done by subtracting from the net annual profit the annual capital costs and adding back depreciation, depletion and intangible costs or development costs. The after tax salvage value of the equipment should be added to the last year computed cash flow.

(18) If a loan has been used to pay part or all of the year zero investment costs, adjust the net annual cash flow calculated in step (17) taking into account that interest costs on the loan are tax deductible and capital costs up to the amount of the loan are to be amortized through its life. The amount of money borrowed to finance the year zero investment must be subtracted from the year zero net cash flow calculated in step (17).

(19) Determine the net present value of the investment at the minimum discount rate.

(20) Determine the discounted cash flow rate of return.

(21) Calculate the ratio of net present value to total present capital costs.

(22) Repeat the procedure from step (3) to step (21) as many times as indicated by the number of simulation trials given in step (1).

(23) Arrange the number of values determined for each of the solution parameters. The arrangement consists on determining the number of values that fall in any of the class boundaries defined in step (2).

(24) Determine the probability of occurrence and the cumulative probability of each of the class boundaries for each of the solution parameters.

When step (24) is done, probability distributions for the solution parameters have been constructed and the complete execution of the model has been done. The next step is to determine an effective way of interpreting the results which have been described in section 3.2.

5. THE COMPUTER SIMULATION PROGRAM

In the previous chapter we described the procedure followed by the model to determine probability distributions for the recoverable reserves, average rate of production of oil and gas, life of the project, net present value, rate of return and net present value-present capital cost ratio. We have seen that to obtain useful probability distribution for those parameters it is necessary to calculate each of them hundred of times. Because of the need of a generator of random numbers uniformly distributed between zero and one, and because of the large number of calculations required by the model a computer program is extremely necessary.

The program developed for this study is written in FORTRAN IV language. The random number generator used in the sampling process is a multiplicative pseudorandom type generator and is contained in the program as a subroutine. The calculation time required for executing a 1000 simulation trials is about 3 minutes in a DEC system-1091 701v1 and the cost incurred is about $5.00. The computer program and a list of symbols used within the program are presented in appendix A. Program input data and computer output will be provided in chapter 6 when we apply the model to a case study.

5.1 Structure of the program

The program is divided in 8 parts, they are: the main program, and the subroutines LECT, WRISD, SELECT, RAND, RAIL and DIST. Each of these parts has a specific objective.

The main program performs the calculation of the solution parameters according to the procedure described in section 4.4. It necessarily uses and articulates the subroutines whose description will follow immediately afterwards.

The subroutine LECT is a subprogram that allows the user to enter the stochastic input data. This subroutine gives the flexibility to the user of assigning to each of the stochastic parameters any of the three types of distribution discussed in section 4.2, i.e. continuous triangular, continuous uniform and discrete. Furthermore, this subprogram is so flexible that if the user considers a stochastic parameter to be defined by a single value, the subroutine transforms the stochastic parameter to a deterministic parameter. The subroutine LECT is so powerful that if the user enters incorrect data not in accordance with the type of distribution that he has selected a prompt is given by the program to make the corresponding correction.

The subroutine WRISD provides a computer output of the stochastic input data entered and the corresponding probability distributions assigned to each of the stochastic parameters.

The subroutine SELECT selects the value of the stochastic parameter using the random number generated and the type of probability distribution assigned to the parameter. The reader should consult Appendix B to see the mathematical approach used by the subroutine SELECT to combine the random number with the type of distribution assigned to the stochastic parameter.

The subroutine RAND is the generator of random numbers uniformly distributed between zero and one. This subroutine internally uses four other subroutines: TIME, SETRAN, RAN, and SAVRAN. These four subroutines provide a very complex and sophisticated mathematical procedure to generate a random number. When the main program calls the subroutine RAND, this subroutine call the subroutine TIME which gives the time of the day based a 24-hour day in hours, minutes and seconds. Then, the subroutine SETRAN is called. SETRAN has one argument, which must be a non-negative integer, in this case the argument is the time given by TIME. The starting value of the function RAN is set to the value of this argument, unless the argument is zero. In this case RAN uses its normal starting value. An entry at RAN is provided to return a floating-point number, which is the conversion of a generated random integer. An entry at SAVRAN is provided to return the last random integer generated by a call to RAN, and an entry at SETRAN provides the random number generator with a nonstandard starting point. The result of a call to SAVRAN used as the argument in a call to SETRAN continues the sequence of random number generation rather than restarting the same sequence from the standard starting point. The algorithm used by the RAN function is taken from an article by W.H. Payne, J.R. Rabring and T.P. Bogyo; it is presented in appendix B. The important feature of this random numbers generator is that it provides a different set of random numbers each time the program is run.

The subroutine RAIL calculates the unbiased form of the standard deviation and the mean of the sampled solutions.

The subroutine DIST provides the results of the model. This subprogram finds the minimum and maximum values of each of the solutions parameters. Then the length of the interval of the class boundaries is computed by dividing the maximum value found by the number of class boundaries provided by the user. The calculated values for each solution parameter are arranged and classified in order to put them into the respective class boundary. The probability of each class is obtained by dividing the number of values that fall in the class by the number of simulation trials. The cumulative probability is determined by accumulating the probability density values. Finally, the subroutine DIST calculates a 95% confidence interval for the mean.

5.2 Effectiveness and Flexibility of the Computer Program

If this computer program is run twice with the same input data, the results obtained will be slightly different, because the random numbers generated are different each time the program is run. However the results obtained are statistically equal. This is a clear indication of the effectiveness of the program.

The program was constructed in a conversational format for the convenience of the user.

Another important characteristic of this computer program is its flexibility. Its structure is so general that it allows the user to operate in the way he considers most convenient. For example, the user can apply any of the current methods of depreciation. If he wants, he can reduce the tax liability by using the investment tax credit on the depreciable assets. If the decision maker wants, he can make the economic analysis of the project under conditions of financial loan. Finally the decision maker can assume a stochastic parameter to be deterministic if he wants. If he considers the parameter to have a probabilistic nature, then any type of probability distribution of the three presented in section 4.2 can be assigned to the parameter.

The diversification of opinions about the probabilistic distribution of a certain stochastic parameter could create a difficulty of assessing this probabilistic distribution. In those cases, because of the relative low cost incurred in running the program sensitivity analysis can be made by changing the distribution of stochastic parameters.

6. CASE STUDY

In this chapter we present an economic evaluation of an oil and gas project by using the model previously discussed. The investment-project selected for this study is a dedication of capital on a small reservoir in the Gulf of Mexico. The oil field is located 100 miles offshore Louisiana. The development of the field was planned to begin in 1980 with the setting of the platform and immediate drilling.

This chapter follows with the presentation of the economical and physical data of the oil and gas project as well as the probability distributions of the solution parameters and the interpretation of the results. The manner of introducing the input data into the computer simulation program is described in Appendix A.

INPUT DATA
****

INPUT DATA FOR THE MODEL
****

NUMBER OF TRIALS= 1000
NUMBER OF CLASS BOUNDARIES FOR THE SOLUTIONS= 14

INPUT DATA FOR DETERMINISTIC PARAMETERS
****

COST OF LAND ADQUISITION ($)= 15000000.
COST OF DRILLING AND RECOMP. EQUIPMENT ($)= 6500000.
NUMBER OF DRILLING AND RECOMP. EQUIPMENTS= 2
TANGIBLE COSTS (%)= 70.0
ACTUAL OIL PRICE ($/BL)= 30.0
ACTUAL PRICE GAS ($/MCF)= 2.0
TAX INCOME RATE (%)= 50.0
ROYALTIES (%)= 16.7
MINIMUM DISCOUNT RATE (%)= 15.
PERCENTAGE DEPLETION (%)= 0.0

COMPANY IS NOT ELEGIBLE FOR PERCENT DEPLETION

DRILLING EQUIPMENT AND WELL COSTS WILL BE
DEPRECIATED BY USING THE METHOD

STRAIGHT LINE

DEPRECIATION LIFE= 15 YEARS

SALVAGE VALUE= 0.

INVESTMENT TAX CREDIT IS APPLIED TO 10% OF THE FULL TANGIBLE EQUIPMENT AND WELL COSTS AND WILL BE USED AGAINST TAX OBLIGATIONS AND OTHER TAXABLE INCOME IN THE YEARS WHEN COSTS ARE INCURRED.

100. % OF THE YEAR ZERO CAPITAL COSTS (DRILLING AND RECOMPLETION EQUIPMENT, MINERAL RIGHTS, EXPLORATION TEST HOLES AND SEISMIC EXPLORATION) IS BORROWED AT 12. % INTEREST RATE PER YEAR WITH THE LOAN PAID OFF OVER THE LIFE OF THE PROJECT

INPUT DATA FOR STOCHASTIC PARAMETERS
****

PRODUCTIVE DRAINAGE AREA (ACRES)

ONLY ONE VALUE IS ESTIMATED

5000.0

NET PAY THICKNESS (FT)

TRIANGULAR DISTRIBUTION

LOWEST VALUE	MOST LIKELY VALUE	HIGHEST VALUE
30.0	36.0	38.0

WATER SATURATION (%)

UNIFORM DISTRIBUTION

LOWEST VALUE	HIGHEST VALUE
8.0	12.0

GAS SATURATION (%)

UNIFORM DISTRIBUTION

LOWEST VALUE	HIGHEST VALUE
6.0	10.0

POROSITY (%)

TRIANGULAR DISTRIBUTION

LOWEST VALUE	MOST LIKELY VALUE	HIGHEST VALUE
20.0	25.0	28.0

OIL FORMATION VOLUME FACTOR

UNIFORM DISTRIBUTION

LOWEST VALUE	HIGHEST VALUE
1.0	1.2

RECOVERY FACTOR (%)

TRIANGULAR DISTRIBUTION

LOWEST VALUE	MOST LIKELY VALUE	HIGHEST VALUE
45.0	50.0	56.0

GAS OIL RATIO (CF/BL)

ONLY ONE VALUE IS ESTIMATED

700.0

DEPTH OF PRODUCTIVE ZONE (FT)

UNIFORM DISTRIBUTION

LOWEST VALUE	HIGHEST VALUE
9000.0	10000.0

DECLINATION OF PRODUCTION (%/YEAR/WELL)

ONLY ONE VALUE IS ESTIMATED

4.0

PRODUCTION PER WELL (BLS/DAY)

TRIANGULAR DISTRIBUTION

LOWEST VALUE	MOST LIKELY VALUE	HIGHEST VALUE
630.0	750.0	900.0

NUMBER OF TEST HOLES

ONLY ONE VALUE IS ESTIMATED

25.0

NUMBER OF DEVELOPMENT WELLS PER YEAR

ONLY ONE VALUE IS ESTIMATED

6.0

NUMBER OF YEARS NECESSARY TO DRILL THE
TOTAL OF DEVELOPMENT WELLS TO DEVELOP THE DEPOSIT

UNIFORM DISTRIBUTION

LOWEST VALUE HIGHEST VALUE

6.0	9.0

NUMBER OF DRY HOLES AS A PERCENTAGE OF DEV.WELLS

TRIANGULAR DISTRIBUTION

LOWEST VALUE	MOST LIKELY VALUE	HIGHEST VALUE
17.0	20.0	24.0

COST OF AVERAGE EXPLORATION TEST HOLE ($/HOLE)

ONLY ONE VALUE IS ESTIMATED

300000.0

COST OF DEVELOPMENT WELL ($/FOOT)

DISCRETE DISTRIBUTION

VALUE	PROBABILITY
180.0	22.0
185.0	20.0
190.0	35.0
193.0	23.0

COST OF DRY HOLE ($/FOOT)

DISCRETE DISTRIBUTION

VALUE	PROBABILITY
210.0	20.0
217.0	23.0
225.0	30.0
230.0	27.0

COST OF RECOMPLETED WELL ($/WELL)

UNIFORM DISTRIBUTION

LOWEST VALUE	HIGHEST VALUE
600000.0	850000.0

COST OF SEISMIC EXPLORATION ($)

ONLY ONE VALUE IS ESTIMATED

70000.0

OPERATING COSTS ($/BL)

UNIFORM DISTRIBUTION

LOWEST VALUE	HIGHEST VALUE
1.5	3.8

COST ESCALATION FACTOR ($/YEAR)

TRIANGULAR DISTRIBUTION

LOWEST VALUE	MOST LIKELY VALUE	HIGHEST VALUE
6.0	9.0	12.0

PRICE ESCALATION FACTOR (%/YEAR)

TRIANGULAR DISTRIBUTION

LOWEST VALUE	MOST LIKELY VALUE	HIGHEST VALUE
8.0	10.0	15.0

THE FOLLOWING ASUMPTIONS ARE CONSIDERED
IN THIS PROGRAM:

1.- ALL TEST HOLES WILL BE DRILLED DURING THE FIRST YEAR OF THE PROJECT

2.- WELLS WILL PRODUCE IN THE FOLLOWING YEAR AFTER THEY ARE COMPLETED

3.- WELLS WILL BE RECOMPLETED EACH FOUR YEARS AFTER PRODUCTION

RESULTS OBTAINED BY USING MONTE CARLO SIMULATION
****

THE RESULTS OBTAINED BY USING THE MONTECARLO SIMULATION ARE PRESENTED IN THE FORM OF PROBABILISTIC DISTRIBUTION OF THE SOLUTION VARIABLES. THE FOLLOWING TABLES SHOW THE DISTRIBUTION OF RECOVERABLE RESERVES,RATE OF PRODUCTION OF OIL AND GAS, LIFE OF THE PROJECT, AND ECONOMIC MEASURES USED IN DECISION MAKING SUCH AS THE NET PRESENT VALUE, RATE OF RETURN AND THE RATIO OF EXPECTED GAIN TO CAPITAL COSTS

THE OBJETIVE OF THE METHOD IS SIMULTANEOUSLY TO OBTAIN A VALID SOLUTION OF THE PROBLEM AND TO DETERMINE THE DEGREE OF UNCERTAINTY ASSOCIATED WITH THE SOLUTION, THE NARROWER THE DISTRIBUTION FOR THE SOLUTION, THE GREATER THE CERTAINTY OF THE SOLUTION; A FLAT DISTRIBUTION WITH CONSIDERABLE SPREAD BETWEEN THE UPER AND LOWER VALUE LIMITS INDICATES A GREATER UNCERTAINTY IN THE SOLUTION.

THERE WILL ALSO BE FOUND THE MEAN AND STANDAR DEVIATION OF THE SOLUTION VARIABLES. A TABULATION OF COMPUTED CUMULATIVE PROBABILITY FOR EACH SOLUTION VARIABLE IS GIVEN. ALSO A 95% CONFIDENCE INTERVAL FOR THE MEAN IS GIVEN FOR EACH OF THE SOLUTION VARIABLES.

LOAN WILL BE PAID OFF OVER THE LIFE OF THE PROJECT WITH ANNUAL END OF YEAR AVERAGE MORTGAGE PAYMENTS OF 1.654 MILLION DOLLARS

RECOVERABLE RESERVES (MILLION OF BLS)
****

MEAN= 120.88
STANDARD DEVIATION= 13.292
MINIMUM VALUE= 80.382
MAXIMUM VALUE= 169.084

CLASS *****		FREQUENCY *********	PROBABILITY ***********	CUMULATIVE PROBABILITY ***********
0.00	12.08	0.	.0000	.0000
12.08	24.15	0.	.0000	.0000
24.15	36.23	0.	.0000	.0000
36.23	48.31	0.	.0000	.0000
48.31	60.39	0.	.0000	.0000
60.39	72.46	0.	.0000	.0000
72.46	84.54	1.	.0010	.0010
84.54	96.62	22.	.0220	.0230
96.62	108.70	158.	.1580	.1810
108.70	120.77	320.	.3200	.5010
120.77	132.85	323.	.3230	.8240
132.85	144.93	147.	.1470	.9710
144.93	157.01	27.	.0270	.9980
157.01	169.08	2.	.0020	1.0000

95 OUT OF 100 TIMES THE MEAN WILL BE
BETWEEN 94.825 AND 146.929

AVERAGE RATE OF PRODUCTION OF OIL (THOU. OF BLS/DAY)
****

MEAN= 15.48
STANDARD DEVIATION= 2.645
MINIMUM VALUE= 7.340
MAXIMUM VALUE= 21.274

CLASS *****		FREQUENCY *********	PROBABILITY ***********	CUMULATIVE PROBABILITY ***********
0.00	1.52	0.	.0000	.0000
1.52	3.04	0.	.0000	.0000
3.04	4.56	0.	.0000	.0000
4.56	6.08	0.	.0000	.0000
6.08	7.60	1.	.0010	.0010
7.60	9.12	3.	.0030	.0040
9.12	10.64	25.	.0250	.0290
10.64	12.16	87.	.0870	.1160
12.16	13.68	144.	.1440	.2600
13.68	15.20	188.	.1880	.4480
15.20	16.72	196.	.1960	.6440
16.72	18.24	204.	.2040	.8480
18.24	19.75	119.	.1190	.9670
19.75	21.27	33.	.0330	1.0000

95 OUT OF 100 TIMES THE MEAN WILL BE
BETWEEN 10.295 AND 20.664

AVERAGE RATE OF PRODUCTION OF GAS (MILLI ON OF CF/D)
****

MEAN= 10.84
STANDARD DEVIATION= 1.852
MINIMUM VALUE= 5.138
MAXIMUM VALUE= 14.892

CLASS *****		FREQUENCY *********	PROBABILITY ***********	CUMULATIVE PROBABILITY ***********
0.00	1.06	0.	.0000	.0000
1.06	2.13	0.	.0000	.0000
2.13	3.19	0.	.0000	.0000
3.19	4.25	0.	.0000	.0000
4.25	5.32	1.	.0010	.0010
5.32	6.38	3.	.0030	.0040
6.38	7.45	25.	.0250	.0290
7.45	8.51	87.	.0870	.1160
8.51	9.57	144.	.1440	.2600
9.57	10.64	188.	.1880	.4480
10.64	11.70	196.	.1960	.6440
11.70	12.76	204.	.2040	.8480
12.76	13.83	119.	.1190	.9670
13.83	14.89	33.	.0330	1.0000

95 OUT OF 100 TIMES THE MEAN WILL BE
BETWEEN 7.206 AND 14.465

LIFE OF THE PROJECT (YEARS)
****

MEAN= 22.55
STANDARD DEVIATION= 5.373
MINIMUM VALUE= 13.000
MAXIMUM VALUE= 56.000

CLASS *****		FREQUENCY *********	PROBABILITY ***********	CUMULATIVE PROBABILITY ***********
0.00	4.00	0.	.0000	.0000
4.00	8.00	0.	.0000	.0000
8.00	12.00	0.	.0000	.0000
12.00	16.00	66.	.0660	.0660
16.00	20.00	343.	.3430	.4090
20.00	24.00	313.	.3130	.7220
24.00	28.00	150.	.1500	.8720
28.00	32.00	77.	.0770	.9490
32.00	36.00	33.	.0330	.9820
36.00	40.00	9.	.0090	.9910
40.00	44.00	6.	.0060	.9970
44.00	48.00	1.	.0010	.9980
48.00	52.00	1.	.0010	.9990
52.00	56.00	1.	.0010	1.0000

95 OUT OF 100 TIMES THE MEAN WILL BE
BETWEEN 12.023 AND 33.085

NET PRESENT VALUE (MILLION OF DOLLARS)
****

MEAN= 768.67
STANDARD DEVIATION= 154.850
MINIMUM VALUE= 421.931
MAXIMUM VALUE= 1409.864

CLASS		FREQUENCY	PROBABILITY	CUMULATIVE PROBABILITY
0.00	100.70	0.	.0000	.0000
100.70	201.41	0.	.0000	.0000
201.41	302.11	0.	.0000	.0000
302.11	402.82	0.	.0000	.0000
402.82	503.52	12.	.0120	.0120
503.52	604.23	118.	.1180	.1300
604.23	704.93	254.	.2540	.3840
704.93	805.64	262.	.2620	.6460
805.64	906.34	169.	.1690	.8150
906.34	1007.05	104.	.1040	.9190
1007.05	1107.75	54.	.0540	.9730
1107.75	1208.45	21.	.0210	.9940
1208.45	1309.16	5.	.0050	.9990
1309.16	1409.86	1.	.0010	1.0000

95 OUT OF 100 TIMES THE MEAN WILL BE
BETWEEN 465.163 AND 1072.176

NPV-CAPITAL COSTS RATIO
****

MEAN= 6.63
STANDARD DEVIATION= 1.352
MINIMUM VALUE= 3.586
MAXIMUM VALUE= 12.185

CLASS *****		FREQUENCY *********	PROBABILITY ***********	CUMULATIVE PROBABILITY ***********
0.00	0.87	0.	.0000	.0000
0.87	1.74	0.	.0000	.0000
1.74	2.61	0.	.0000	.0000
2.61	3.48	0.	.0000	.0000
3.48	4.35	17.	.0170	.0170
4.35	5.22	115.	.1150	.1320
5.22	6.09	256.	.2560	.3880
6.09	6.96	259.	.2590	.6470
6.96	7.83	162.	.1620	.8090
7.83	8.70	108.	.1080	.9170
8.70	9.57	56.	.0560	.9730
9.57	10.44	17.	.0170	.9900
10.44	11.31	9.	.0090	.9990
11.31	12.19	1.	.0010	1.0000

95 OUT OF 100 TIMES THE MEAN WILL BE
BETWEEN 3.982 AND 9.280

DISCOUNTED CASH FLOW RATE OF RETURN (%)
****

MEAN= 56.15
STANDARD DEVIATION= 3.984
MINIMUM VALUE= 45.668
MAXIMUM VALUE= 67.660

CLASS *****		FREQUENCY *********	PROBABILITY ***********	CUMULATIVE PROBABILITY ***********
0.00	4.83	0.	.0000	.0000
4.83	9.67	0.	.0000	.0000
9.67	14.50	0.	.0000	.0000
14.50	19.33	0.	.0000	.0000
19.33	24.16	0.	.0000	.0000
24.16	29.00	0.	.0000	.0000
29.00	33.83	0.	.0000	.0000
33.83	38.66	0.	.0000	.0000
38.66	43.50	0.	.0000	.0000
43.50	48.33	15.	.0150	.0150
48.33	53.16	187.	.1870	.2020
53.16	57.99	505.	.5050	.7070
57.99	62.83	257.	.2570	.9640
62.83	67.66	36.	.0360	1.0000

95 OUT OF 100 TIMES THE MEAN WILL BE
BETWEEN 48.346 AND 63.963

6.3 Interpretation of the Solution

The computer outputs previously presented provide the probability distributions for the solution parameters, that is, for the recoverable reserves, average rate of production of oil and gas, life of the project, net present value, ratio of net present value to present capital cost, and discounted cash flow rate of return.

There is a probability of 0.320 that the amount of recoverable reserves is between 108.70 and 120.77 million of bls. The range between 120.77 and 132.85 also has the same probability of occurrence. This implies that the most probable value of the distribution is around 120 million of bls. Note that this value is very close to the mean value, 120.88. An important measurement provided by this probability distribution is that, there is a 82.4% chance that the amount of recoverable reserves is less than 132.85 million of bls and that the minimum possible value is 80.38 million of bls. This makes one conclude that there is a concentration of values around the mean which is reflected by the limits defined by the 95% confidence interval for the mean.

The most probable value of the average rate of production of oil lies between 15,200 and 18,240 bls/day. Note that the mean value is not the same as the most probable value. The probability distribution for the average rate of production of gas is exactly the same as for the average rate of production of oil. The reason of this, is that the production of gas is a direct function of the production of oil, and that the gas-oil ratio was defined with a single value of 700 cubic feet per one barrel of oil produced. From both distributions we can conclude that there is a 11.6% chance that the production of oil and gas are less than 12,160 bls/day and 8.5 million of c.f./day respectively.

The probability distribution for the life of the project shows a greater chance for the interval between 16 and 24 years. In accordance with the statistical measures of this distribution, i.e. standard deviation, mean, confidence interval for the mean, etc., the life of the project would be approximately 20 years.

The net present value has a very important role in the decision making process. The uncertainty about this financial outcome is quantified by the resulting probability distribution which indicates a major chance between the values of 604 and 806 million of dollars. This probability distribution involves a greater degree of uncertainty than the ones we have previously discussed. However, the fact that the net present value is above 400 million dollars is a clear indication of the profitability of the investment. The spread of values between 400 and 1200 million of dollars is reflected by the high standard deviation calculated, 155 million of dollars.

The probability distribution for the ratio of net present value to total present capital costs would indicate a better understanding of the risk involved in the profitability of the project. Evidently the most probable value of this parameter lies around 6. This value indicates that for each dollar invested there will be a gain of 6 dollars. Note that the minimum value found is 3.6 which implies a good reason to assure the profitability of the project.

The probability distribution for the rate of return indicates that the value of this parameter would be between 43.5% and 67.7% and that the most probable range of value is between 53% and 58% with a chance of 50%. Note that the distribution is very narrow which reflects a low degree of uncertainty involving the value of the parameter. This is also indicated by the low standard deviation calculated.

CONCLUSIONS

1. The use of probabilistic petroleum economic models makes possible the quantification of the degree of uncertainty present in the exploration development and production phases. This will reduce the risk associated with the profitability of the project.

2. The Monte Carlo technique adapted to petroleum economic problems seems to be the most promising approach for evaluating the risk and uncertainty associated with most petroleum projects.

3. A comparable set of financial outcomes can be obtained for a group of petroleum projects that the best alternative can be selected.

4. The Monte Carlo simulation provides the decision maker the opportunity of studying the variation of engineering solutions in accordance with the range of values of any of the input parameters. Therefore, improvements in the subjective judgments of these parameters can be made in order to narrow the uncertainty in the solution.

5. Subjective probability judgments about the stochastic parameters are made individually for each parameter rather than collectively. The judgments should be made by those persons in the organization who are most familiar with each parameter.

6. The probability distribution for the solutions are independent of the particular form of probability distributions used for expressing the probability of the stochastic input parameters.

7. The method developed permits expressing the economic character of an investment venture in terms of confidence intervals for the expected values of the financial outcomes or conversely as a risk.

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APPENDIX A

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C     APPLICATION OF RISK ANALYSIS TO
C     PETROLEUM INVESTMENT VENTURES
C     MONTECARLO SIMULATION
C
      DIMENSION DNA(10),PDA(10),DNH(10),PDH(10),DNYDW(10)
      DIMENSION DNSW(10),PDSW(10),DNSG(10),PDSG(10),PDYDW(10)
      DIMENSION PDP(10),DNBO(10),PDBO(10),DNRF(10),PDRF(10)
      DIMENSION PDGR(10),DNRC(10),PDRC(10),DNDC(10),PDDC(10)
      DIMENSION DNOPC(10),PDOPC(10),DNCE(10),PDCE(10),DNDP(10)
      DIMENSION PDDP(10),DNPE(10),PDPE(10)
      DIMENSION DNPG(10),PDPG(10),DNCS(10),PDCS(10),DNPW(10)
      DIMENSION DNTH(10),PDTH(10),DNDW(10),PDDW(10),DNDH(10)
      DIMENSION PDDH(10),DNCDH(10),PDCDH(10),DNCR(10),PDCR(10)
      DIMENSION DNP(10),DNGR(10),PDPW(10),TZI(200)
        DIMENSION AVP(1000),AVG(1000),ATIC(0:200),PLP(1000)
        DIMENSION PPW(300),PGW(300),RR(1000),ATSV(0:200)
        DIMENSION CPB(200),PPB(200),PPG(200),CDP(200),CSP(200),CREP(200)
        DIMENSION CEH(200),RIC(200),YVO(200),CHAR(0:200),ASM(0:200)
        DIMENSION RUT(0:200),RON(0:200),YSA(0:200),CIR(0:200),DAV(0:200)
        DIMENSION RAU(200),DAG(200),CIG(200),FIDE(0:200),TAXI(0:200)
        DIMENSION EMI(0:200),ABU(0:200),GAN(0:200),CFLOW(0:200)
        DIMENSION PVAL(1000),DCF(0:200),DCFROR(1000),TCOST(0:200)
        DIMENSION DCFW(0:200),RNPVC(1000),ALB(0:200),FED(0:200)
        DIMENSION GLOR(200),CAR(200),BOOK(200),VAS(200),SAU(0:200)
        REAL NRPV(50),ICPV,MEDPV,ICRNP,NRRNP(50),MEDRA,ICROR,NRROR(50)
        REAL MEDRO,ICRR,IFR(50),MEDP,ICP,NRP(50),MEDQ,ICG,NRG(50),MROR
        REAL MEDG,ICL,NRL(50),MEDL,LEON(0:200),LI(50),LS(50),LIFE(1000)
        DOUBLE PRECISION FCPV(50),FCRNP(50),FCROR(50),PROB(50)
        DOUBLE PRECISION FCRR(50),FCP(50),FCG(50),FCL(50)
C
C     READING DATA FOR STATISTICAL ANALYSIS
C
        INC=4
        INR=4
        OPEN(UNIT=INC,FILE='VEN.DAT')
        OPEN(UNIT=INR,FILE='ZUL.DAT')
        WRITE(INR,410)
410     FORMAT(/,15X,'APPLICATION OF RISK ANALYSIS TO',/,15X,
     1  'PETROLEUM INVESTMENT DECISIONS',////,15X,
     1  'MONTE CARLO SIMULATION',/////,3X,
     1  'THIS PROGRAM IS CONCERNED WITH THE ECONOMIC EVALUATION',/,1X,
     1  'AND DECISION MAKING REGARDING PETROLEUM INVESTMENT',/,1X,
     1  'VENTURES. IT INVOLVES ASSOCIATING PROBABILITIES WITH',/,1X,
     1  'THE RANGE OF PROFITABILITIES THAT MIGHT RESULT FROM A',/,1X,
     1  'PARTICULAR PROJECT.',//,3X,
     1  'THE VARIABLES USED TO DETERMINE THE PROFITABILITY',/,1X,
     1  'OF THE INVESTMENT CAN HAVE A PROBABILISTIC NATURE',/,1X,
     1  'DEPENDING ON YOUR JUDGMENT. THE PROGRAM GIVES YOU THE',/,1X,
     1  'OPPORTUNITY TO DECIDE WHICH VARIABLES ARE DETERMINISTIC',/,1X,
     1  'AND WHICH ARE STOCHASTIC; AND ALSO TO ASSIGN THE KIND',/,1X,
     1  'OF DISTRIBUTION FOR THE STOCHASTIC VARIABLES.',//,3X,
     1  'THIS MODEL OF RISK ANALYSIS COMBINES THE PROBABILISTIC',/,1X,
     1  'DISTRIBUTION OF THOSE STOCHASTIC VARIABLES AS WELL AS',/,1X,
     1  'THE VALUES OF THE DETERMINISTIC ONES AND GENERATES A',/,1X,
     1  'FINAL DISTRIBUTION OF ECONOMIC PARAMETERS USED IN',/,1X,
     1  'DECISION MAKING SUCH AS THE NET PRESENT VALUE AND THE',/,1X,
     1  'RATE OF RETURN ON THE INVESTMENT.')
700       WRITE (INR,1)
1     FORMAT(/,1X,'ENTER NUMBER OF TRIALS')
      READ(INC,*)NT
```

```
        IF(NT.GT.1000)GO TO 695
        IF(NT.LT.1000)GO TO 466
        IF(NT.EQ.1000)GO TO 466
695     WRITE(INR,696)
696     FORMAT(/,1X,'DO NOT ENTER MORE THAN 1000 TRIALS')
        GO TO 700
466      WRITE(INR,4)
4      FORMAT(/,1X,'ENTER NUMBER OF CLASS BOUNDARIES',/,1X,
     1  'DO NOT USE MORE THAN 50 CLASS BOUNDARIES')
       READ(INC,*)NC
        IF(NC-50)467,467,461
461     WRITE(INR,462)
462     FORMAT(/,1X,'DO NOT ENTER MORE THAN 50 CLASS BOUNDARIES')
        GO TO 465
C
C      READING DETERMINISTIC DATA
C
467     WRITE(INR,3)
3       FORMAT(///,3X,'INPUT DATA FOR DETERMINISTIC PARAMETERS',//)
       WRITE(INR,5)
5      FORMAT(/,1X,'ENTER COST OF LAND ACQUISITION',/,1X,
     1  '(COST OF MINERAL RIGHTS)')
       READ(INC,*)CLA
463     WRITE(INR,81)
81      FORMAT(/,1X,'WHICH METHOD OF DEPRECIATION WILL YOU USE',/,1X,
     1  'TYPE 1 IF STRAIGHT LINE ',/,1X,
     1  'TYPE 2 IF DOUBLE DECLINING BALANCE',/,1X,
     1  'TYPE 3 IF UNITS OF PRODUCTION ',/,1X,
     1  'TYPE 4 IF SUM OF YEAR DIGITS',/,1X,
     1  'TYPE 5 IF DOUBLE DECLINING SWITCHING TO STRAIGHT LINE',/,1X,
     1  'TYPE 6 IF DOUBLE DECLINING SWITCHING TO SUM OF YEAR DIGITS',/)
        READ(INC,*)KD
        IF(KD-6)468,468,691
691     WRITE(INR,464)
464     FORMAT(/,1X,'TYPE ANY INTEGER NUMBER BETWEEN 1 1ND 6',/)
468     IF(KD.EQ.3)GO TO 773
        WRITE(INR,82)
82      FORMAT(/,1X,'HOW MANY YEARS OF DEPRECIATION LIFE WILL YOU USE?')
        READ(INC,*)NYD
773     WRITE(INR,87)
87      FORMAT(/,1X,'WHAT PERCENT. OF YOUR EQUIP. AND WELL COSTS',/,1X,
     1  'ARE CONSIDERED TO BE TANGIBLE COSTS (%)?',/)
        READ(INC,*)PTC
        WRITE(INR,2104)
2104    FORMAT(/,1X,'IS THE COMPANY ELEGIBLE FOR PERCENT DEPLETION?',/,
     1  1X,'TYPE "1" IF YES OR "2" IF NO')
2108    READ(INC,*)EPD
        IF(EPD.GT.2.0)GO TO 2105
        IF(EPD.LT.1.0)GO TO 2105
        IF(EPD.EQ.2.0)GO TO 2106
        IF(EPD.EQ.1.0)GO TO 2107
2105    WRITE(INR,664)
        GO TO 2108
2107    WRITE(INR,91)
91      FORMAT(/,1X,'WHAT PERCENTAGE FOR DEPLETION PURPOSES WILL',/,1X,
     1  'YOU USE (%)?',/)
        READ(INC,*)PDE
2106    PDE=0.0
       WRITE(INR,6)
6      FORMAT(/,1X,'ENTER TAX INCOME RATE  (%)')
```

```
      READ(INC,*)TAX
      WRITE(INR,7)
7     FORMAT(/,1X,'ENTER ROYALTIES  (%)')
      READ(INC,*)ROY
      WRITE(INR,8)
8     FORMAT(/,1X,'ENTER MINIMUM RATE OF RETURN  (%)')
      READ(INC,*)MROR
      WRITE(INR,9)
9     FORMAT(/,1X,'ENTER COST OF  DRILLING AND RECOMPLETION EQUIPMENT')
      READ(INC,*)EQC
        WRITE(INR,92)
92      FORMAT(/,1X,'HOW MANY OF THOSE EQUIPMENTS WILL YOU USE?',/)
        READ(INC,*)NEQC
        WRITE(INR,2121)
2121    FORMAT(/,1X,'WHAT IS THE SALVAGE VALUE OF YOUR EQUIPMENT?')
        READ(INC,*)SALVA
        WRITE(INR,96)
96      FORMAT(/,1X,'ENTER ACTUAL OIL PRICE ($/BL)')
        READ(INC,*)PRO
        WRITE(INR,97)
97      FORMAT(/,1X,'ENTER ACTUAL GAS PRICE ($/MCF)')
        READ(INC,*)PRG
        WRITE(INR,495)
495     FORMAT(/,1X,'WOULD YOU LIKE INVESTMENT TAX CREDIT TO BE',//,1X,
     1  'APPLIED ON DEPRECIABLE EQUIPMENT COSTS?',//,1X,
     1  'TYPE 1 IF YES, TYPE 2 IF NO.')
465     READ(INC,*)ITC
        IF(ITC-2)469,469,663
663     WRITE(INR,664)
664     FORMAT(/,1X,'TYPE 1 FOR YES OR 2 FOR NO')
        GO TO 465
469     IF(ITC.EQ.1)GO TO 496
        IF(ITC.EQ.2)GO TO 497
496     WRITE(INR,498)
498     FORMAT(//,4X,'INVESTMENT TAX CREDIT IS APPLIED TO 10%',//,1X,
     1  'OF THE FULL TANGIBLE EQUIPMENT AND WELL COSTS AND WILL',//,1X,
     1  'BE USED AGAINST TAX OBLIGATIONS AND OTHER TAXABLE',//,1X,
     1  'INCOME IN THE YEARS WHEN COSTS ARE INCURRED.')
497     WRITE(INR,857)
857     FORMAT(/,1X,'DO YOU PLAN TO BORROW THE TIME ZERO CAPITAL COSTS?'
     1  ,//,1X,'TYPE "1" IF YES, TYPE "2" IF NO')
860     READ(INC,*)LBM
        IF(LBM.LT.1)GO TO 859
        IF(LBM-2)858,858,859
859     WRITE(INR,664)
        GO TO 860
858     IF(LBM.EQ.1)GO TO 856
        IF(LBM.EQ.2)GO TO 861
856     WRITE(INR,830)
830     FORMAT(/,1X,'WHAT PERCENTAGE OF THE TIME ZERO INVESTMENT WILL',/
     1  /,1X,'YOU BORROW?',/)
        READ(INC,*)PLI
        WRITE(INR,1407)
1407    FORMAT(/,1X,'WHAT IS THE INTEREST RATE (%) AT WHICH THE MONEY',/
     1  ,1X,'IS BORROWED?',/)
        READ(INC,*)PIR
C
C     READING STOCHASTIC DATA
C
861     WRITE(INR,2)
```

```
2       FORMAT(///,5X,'NOW, YOU WILL ENTER THE INFORMATION FOR THE',/,
     1  1X,'STOCHASTIC PARAMETERS. BE SURE YOU FOLLOW CORRECTLY',/,1X,
     1  'THE DIRECTIONS',//)
        WRITE(INR,10)
10     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'PRODUCTIVE DRAINAGE AREA  (ACRES)')
        CALL LECT(IDA,A1,A2,A3,A,ND,PDA,DNA)
       WRITE(INR,18)
18     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'NET PAY THICKNESS (FEET)')
        CALL LECT(IDA1,H1,H2,H3,H,ND1,PDH,DNH)
       WRITE(INR,20)
20     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'WATER SATURATION  (%)')
        CALL LECT(IDA3,SW1,SW2,SW3,SW,ND3,PDSW,DNSW)
       WRITE(INR,21)
21     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'GAS SATURATION  (%)')
        CALL LECT(IDA4,SG1,SG2,SG3,SG,ND4,PDSG,DNSG)
       WRITE(INR,22)
22     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'POROSITY  (%)')
        CALL LECT(IDA5,P1,P2,P3,P,ND5,PDP,DNP)
       WRITE(INR,23)
23     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'OIL FORMATION VOLUME FACTOR')
        CALL LECT(IDA6,BO1,BO2,BO3,BO,ND6,PDBO,DNBO)
       WRITE(INR,24)
24     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'RECOVERY FACTOR  (%)')
        CALL LECT(IDA7,RF1,RF2,RF3,RF,ND7,PDRF,DNRF)
       WRITE(INR,25)
25     FORMAT(/,1X,'TO ENTER VALUE OF GAS OIL RATIO  (CF/BL)')
        CALL LECT(IDA8,GR1,GR2,GR3,GR,ND8,PDGR,DNGR)
       WRITE(INR,26)
26     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'COST OF AVERAGE EXPLORATION TEST HOLE ($/WELL)')
        CALL LECT(IDA9,RC1,RC2,RC3,RC,ND9,PDRC,DNRC)
       WRITE(INR,27)
27     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'WELL DEVELOPMENT COST  ($/FOOT)')
        CALL LECT(IDA10,DC1,DC2,DC3,DC,ND10,PDDC,DNDC)
        WRITE(INR,83)
83      FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1  'COST OF DRY HOLE ($/FT)')
        CALL LECT(IDA11,CDH1,CDH2,CDH3,CDH,ND11,PDCDH,DNCDH)
        WRITE(INR,28)
28      FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1  'COST OF RECOMPLETION ($/WELL)')
        CALL LECT(IDA12,CR1,CR2,CR3,WCR,ND12,PDCR,DNCR)
        WRITE(INR,85)
85     FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1'OPERATING COSTS  ($/BL)')
        CALL LECT(IDA13,OPC1,OPC2,OPC3,OPC,ND13,PDOPC,DNOPC)
        WRITE(INR,29)
29      FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1  'COST ESCALATION FACTOR (%)')
        CALL LECT(IDA14,CE1,CE2,CE3,CE,ND14,PDCE,DNCE)
        WRITE(INR,30)
30      FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
```

```
     1  'DECLINATION OF PRODUCTION, (%/YEAR)')
        CALL LECT(IDA15,DP1,DP2,DP3,DP,ND15,PDDP,DNDP)
        WRITE(INR,31)
31      FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1  'PRICE ESCALATION FACTOR (%)')
        CALL LECT(IDA16,PE1,PE2,PE3,PE,ND16,PDPE,DNPE)
        WRITE(INR,33)
33      FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1  'DEPTH OF PRODUCTIVE ZONE (FT)')
        CALL LECT(IDA18,PG1,PG2,PG3,PG,ND18,PDPG,DNPG)
        WRITE(INR,34)
34      FORMAT(/,1X,'TO ENTER VALUE OF',/,1X,
     1  'COST OF SEISMIC EXPLORATION ($)')
        CALL LECT(IDA19,CS1,CS2,CS3,CS,ND19,PDCS,DNCS)
        WRITE(INR,35)
35      FORMAT(/,1X,'WHAT PRODUCTION PER WELL PER DAY DO YOU EXPECT?',/,
     1  1X,'TO ENTER THIS PARAMETER',/)
        CALL LECT(IDA20,PW1,PW2,PW3,PW,ND20,PDPW,DNPW)
        WRITE(INR,36)
36      FORMAT(/,1X,'HOW MANY TEST HOLES ARE NECESSARY TO SET UP',/,
     1  1X,'THE PRODUCTION PROGRAM?',/,1X,
     1  'TO ENTER THIS PARAMETER',/)
        CALL LECT(IDA21,TH1,TH2,TH3,TH,ND21,PDTH,DNTH)
        WRITE(INR,37)
37      FORMAT(/,1X,'TO ENTER THE NUMBER OF DEVELOPMENT WELLS',/,1X,
     1  'DRILLED PER YEAR TO DEVELOP THE DEPOSIT')
        CALL LECT(IDA22,DW1,DW2,DW3,DW,ND22,PDDW,DNDW)
        WRITE(INR,508)
508     FORMAT(/,1X,'HOW MANY YEARS ARE NECESSARY TO DEVELOP',/,1X,
     1  'THE DEPOSIT?, THAT IS, HOW MANY YEARS YOU NEED TO',/,1X,
     1  'DRILL THE TOTAL OF DEVELOPMENT WELLS?',/,1X,
     1  'TO ENTER THIS PARAMETER')
        CALL LECT(IDA2,YDW1,YDW2,YDW3,YDW,ND2,PDYDW,DNYDW)
        WRITE(INR,38)
38      FORMAT(/,1X,'WHAT PERCENTAGE OF THE DEVELOPMENT WELLS',/,1X,
     1  'DO YOU EXPECT TO BE DRY HOLES?',/,1X,
     1  'TO ENTER THIS PARAMETER',/)
        CALL LECT(IDA23,DH1,DH2,DH3,DH,ND23,PDDH,DNDH)
C
C       WRITING INPUT DATA
C
        WRITE(INR,93)
93      FORMAT(///,5X,'THE FOLLOWING ASUMPTIONS ARE CONSIDERED',
     1  /,5X,'IN THIS PROGRAM:',///,1X,
     1  '1.- ALL TEST HOLES WILL BE DRILLED DURING THE FIRST',/,5X,
     1  'YEAR OF THE PROJECT',//,1X,
     1  '2.- WELLS WILL PRODUCE IN THE FOLLOWING YEAR AFTER THEY',/,5X,
     1  'ARE COMPLETED'
     1  ,//,1X,'3.- WELLS WILL BE RECOMPLETED EACH FOUR YEARS',/,5X,
     1  'AFTER PRODUCTION',//)
        WRITE(8,190)
190     FORMAT(///,20X,'INPUT DATA',/,20X,'**********')
        WRITE(8,191)
191     FORMAT(///,15X,'INPUT DATA FOR THE MODEL',/,15X,
     1  '************************')
        WRITE(8,200)NT,NC
200     FORMAT(//,1X,'NUMBER OF TRIALS=',2X,I4,/,1X,'NUMBER
     1   OF CLASS BOUNDARIES FOR THE SOLUTIONS=',2X,I2,/)
        WRITE(8,204)CLA,EQC,NEQC,PTC,PRO,PRG,TAX,ROY,MROR,PDE
204     FORMAT(///,15X,'INPUT DATA FOR DETERMINISTIC PARAMETERS',/,15X,
```

```
     1    '*****************************************',///,1X,
     1    'COST OF LAND ADQUISITION ($)=',2X,F10.0,/,1X,
     1    'COST OF DRILLING AND RECOMP. EQUIPMENT ($)=',2X,F10.0,/,1X,
     1    'NUMBER OF DRILLING AND RECOMP. EQUIPMENTS=',2X,I2,/,1X,
     1    'TANGIBLE COSTS (%)=',2X,F6.1,/,1X,
     1    'ACTUAL OIL PRICE ($/BL)=',2X,F6.1,/,1X,
     1    'ACTUAL PRICE GAS ($/MCF)=',2X,F6.1,/,1X,
     1    'TAX INCOME RATE (%)=',2X,F5.1,/,1X,
     1    'ROYALTIES (%)=',2X,F5.1,/,1X,
     1    'MINIMUM DISCOUNT RATE (%)=',2X,F4.0,/,1X,
     1    'PERCENTAGE DEPLETION (%)=',2X,F5.1,/)
          IF(EPD.EQ.1.0)GO TO 2114
          IF(EPD.EQ.2.0)GO TO 2115
2115      WRITE(8,2113)
2113      FORMAT(//,2X,'COMPANY IS NOT ELEGIBLE FOR PERCENT DEPLETION')
2114      WRITE(8,205)
205       FORMAT(/,1X,'DRILLING EQUIPMENT AND WELL COSTS WILL BE',/,1X,
     1    'DEPRECIATED BY USING THE METHOD')
          IF(KD.EQ.1)GO TO 206
          IF(KD.EQ.2)GO TO 207
          IF(KD.EQ.3)GO TO 208
          IF(KD.EQ.4)GO TO 209
          IF(KD.EQ.5)GO TO 210
          IF(KD.EQ.6)GO TO 211
206       WRITE(8,212)
212       FORMAT(/,1X,'STRAIGHT LINE ')
          GO TO 199
207       WRITE(8,214)
214       FORMAT(/,1X,'DOUBLE DECLINING BALANCE')
          GO TO 199
208       WRITE(8,215)
215       FORMAT(/,1X,'UNITS OF PRODUCTION ')
          GO TO 426
209       WRITE(8,216)
216       FORMAT(/,1X,'SUM OF YEAR DIGITS')
          GO TO 199
210       WRITE(8,217)
217       FORMAT(/,1X,'DOUBLE DECLINING SWITCHING TO STRAIGHT LINE')
          GO TO 199
211       WRITE(8,218)
218       FORMAT(/,1X,'DOUBLE DECLINING SWITCHING TO SUM OF YR DIGITS')
199       WRITE(8,213)NYD
213       FORMAT(/,1X,'DEPRECIATION LIFE=',2X,I2,2X,'YEARS')
          WRITE(8,2123)SALVA
2123      FORMAT(/,1X,'SALVAGE VALUE=',2X,F15.0)
426       IF(ITC.EQ.1)GO TO 513
          IF(ITC.EQ.2)GO TO 514
513       WRITE(8,498)
          GO TO 202
514       WRITE(8,515)
515       FORMAT(//,4X,'INVESMENT TAX CREDIT IS NOT APPLIED TO',/,1X,
     1    'THE TANGIBLE EQUIPMENT AND WELL COSTS')
202       IF(LBM.EQ.1)GO TO 827
          IF(LBM.EQ.2)GO TO 829
827       WRITE(8,1301)PLI,PIR
1301      FORMAT(//,5X,F5.0,1X,'% OF THE YEAR ZERO CAPITAL COSTS',/,5X,
     1    '(DRILLING AND RECOMPLETION EQUIPMENT, MINERAL RIGHTS,',/,5X,
     1    'EXPLORATION TEST HOLES AND SEISMIC EXPLORATION) IS',/,5X,
     1    'BORROWED AT',1X,F4.0,1X,'% INTEREST RATE  PER YEAR WITH THE',/,
     1    5X,'LOAN PAID OFF OVER THE LIFE OF THE PROJECT',/)
```

```
829     WRITE(8,224)
224     FORMAT(///,15X,'INPUT DATA FOR STOCHASTIC PARAMETERS',/,15X,
     1  '*************************************')
        WRITE(8,225)
225     FORMAT(//,1X,'PRODUCTIVE DRAINAGE AREA (ACRES)')
        CALL WRISD(IDA,A1,A2,A3,A,ND,PDA,DNA)
        WRITE(8,235)
235     FORMAT(//,1X,'NET PAY THICKNESS (FT)')
        CALL WRISD(IDA1,H1,H2,H3,H,ND1,PDH,DNH)
        WRITE(8,247)
247     FORMAT(//,1X,'WATER SATURATION (%)')
        CALL WRISD(IDA3,SW1,SW2,SW3,SW,ND3,PDSW,DNSW)
        WRITE(8,253)
253     FORMAT(//,1X,'GAS SATURATION (%)')
        CALL WRISD(IDA4,SG1,SG2,SG3,SG,ND4,PDSG,DNSG)
        WRITE(8,259)
259     FORMAT(//,1X,'POROSITY (%)')
        CALL WRISD(IDA5,P1,P2,P3,P,ND5,PDP,DNP)
        WRITE(8,265)
265     FORMAT(//,1X,'OIL FORMATION VOLUME FACTOR')
        CALL WRISD(IDA6,BO1,BO2,BO3,BO,ND6,PDBO,DNBO)
        WRITE(8,271)
271     FORMAT(//,1X,'RECOVERY FACTOR (%)')
        CALL WRISD(IDA7,RF1,RF2,RF3,RF,ND7,PDRF,DNRF)
        WRITE(8,277)
277     FORMAT(//,1X,'GAS OIL RATIO (CF/BL)')
        CALL WRISD(IDA8,GR1,GR2,GR3,GR,ND8,PDGR,DNGR)
        WRITE(8,283)
283     FORMAT(//,1X,'DEPTH OF PRODUCTIVE ZONE (FT)')
        CALL WRISD(IDA18,PG1,PG2,PG3,PG,ND18,PDPG,DNPG)
        WRITE(8,289)
289     FORMAT(//,1X,'DECLINATION OF PRODUCTION (%/YEAR/WELL)')
        CALL WRISD(IDA15,DP1,DP2,DP3,DP,ND15,PDDP,DNDP)
        WRITE(8,295)
295     FORMAT(//,1X,'PRODUCTION PER WELL (BLS/DAY)')
        CALL WRISD(IDA20,PW1,PW2,PW3,PW,ND20,PDPW,DNPW)
        WRITE(8,301)
301     FORMAT(//,1X,'NUMBER OF TEST HOLES')
        CALL WRISD(IDA21,TH1,TH2,TH3,TH,ND21,PDTH,DNTH)
        WRITE(8,307)
307     FORMAT(//,1X,'NUMBER OF DEVELOPMENT WELLS PER YEAR')
        CALL WRISD(IDA22,DW1,DW2,DW3,DW,ND22,PDDW,DNDW)
        WRITE(8,517)
517     FORMAT(//,1X,'NUMBER OF YEARS NECESSARY TO DRILL THE',/,1X,
     1  'TOTAL OF DEVELOPMENT WELLS TO DEVELOP THE DEPOSIT')
        CALL WRISD(IDA2,YDW1,YDW2,YDW3,YDW,ND2,PDYDW,DNYDW)
        WRITE(8,313)
313     FORMAT(//,1X,'NUMBER OF DRY HOLES AS A PERCENTAGE OF DEV.WELLS')
        CALL WRISD(IDA23,DH1,DH2,DH3,DH,ND23,PDDH,DNDH)
        WRITE(8,319)
319     FORMAT(//,1X,'COST OF AVERAGE EXPLORATION TEST HOLE ($/HOLE)')
        CALL WRISD(IDA9,RC1,RC2,RC3,RC,ND9,PDRC,DNRC)
        WRITE(8,325)
325     FORMAT(//,1X,'COST OF DEVELOPMENT WELL ($/FOOT)')
        CALL WRISD(IDA10,DC1,DC2,DC3,DC,ND10,PDDC,DNDC)
        WRITE(8,349)
349     FORMAT(//,1X,'COST OF DRY HOLE ($/FOOT)')
        CALL WRISD(IDA11,CDH1,CDH2,CDH3,CDH,ND11,PDCDH,DNCDH)
        WRITE(8,356)
356     FORMAT(//,1X,'COST OF RECOMPLETED WELL ($/WELL)')
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```
      CALL WRISD(IDA12,CR1,CR2,CR3,WCR,ND12,PDCR,DNCR)
      WRITE(8,363)
363   FORMAT(//,1X,'COST OF SEISMIC EXPLORATION ($)')
      CALL WRISD(IDA19,CS1,CS2,CS3,CS,ND19,PDCS,DNCS)
      WRITE(8,370)
370   FORMAT(//,1X,'OPERATING COSTS ($/BL)')
      CALL WRISD(IDA13,OPC1,OPC2,OPC3,OPC,ND13,PDOPC,DNOPC)
      WRITE(8,387)
387   FORMAT(//,1X,'COST ESCALATION FACTOR ($/YEAR)')
      CALL WRISD(IDA14,CE1,CE2,CE3,CE,ND14,PDCE,DNCE)
      WRITE(8,394)
394   FORMAT(//,1X,'PRICE ESCALATION FACTOR (%/YEAR)')
      CALL WRISD(IDA16,PE1,PE2,PE3,PE,ND16,PDPE,DNPE)
      WRITE(8,93)
C
C     INITIALIZING CALCULATIONS
      CALL RAND(YFL)
      NCALC=0
      K=0
      JJ=0
      LL=0
      II=0
      NN=0
      MM=0
      NM=0
      LK=0
411   NCALC=NCALC+1
      K=K+1
      JJ=JJ+1
      LL=LL+1
      II=II+1
      NN=NN+1
      MM=MM+1
      NM=NM+1
      LK=LK+1
C
C     PRODUCTIVE DREINAGE AREA (ACRES)
C
      YFL=RAN(YFL)
      CALL SELECT(IDA,A1,A2,A3,A,YFL,ND,PDA,DNA,AREA)
C
C     NET PAY THICKNESS (FT)
C
      YFL=RAN(YFL)
      CALL SELECT(IDA1,H1,H2,H3,H,YFL,ND1,PDH,DNH,ESP)
C
C     WATER SATURATION (%)
C
      YFL=RAN(YFL)
      CALL SELECT(IDA3,SW1,SW2,SW3,SW,YFL,ND3,PDSW,DNSW,SAW)
      SAW=SAW/100
C
C     GAS SATURATION
C
      YFL=RAN(YFL)
      CALL SELECT(IDA4,SG1,SG2,SG3,SG,YFL,ND4,PDSG,DNSG,SAG)
      SAG=SAG/100
C
C     POROSITY
C
```

```
      YFL=RAN(YFL)
      CALL SELECT(IDA5,P1,P2,P3,P,YFL,ND5,PDP,DNP,POR)
      POR=POR/100
C
C     OIL FORMATION VOLUME FACTOR
C
      YFL=RAN(YFL)
      CALL SELECT(IDA6,BO1,BO2,BO3,BO,YFL,ND6,PDBO,DNBO,BOF)
C
C     RECOVERY FACTOR
C
      YFL=RAN(YFL)
      CALL SELECT(IDA7,RF1,RF2,RF3,RF,YFL,ND7,PDRF,DNRF,REF)
      REF=REF/100
C
C     DETERMINATION OF RECOVERABLE RESERVES (THOU. OF BLS)
C
      RR(K)=(7758.0*AREA*ESP*POR*(1.0-SAW-SAG)*REF/BOF)/1000000
C
C     DETERMINATION OF AVERAGE PRODUCTION RATE AND
C     LIFE OF THE PROJECT
C
      YFL=RAN(YFL)
      CALL SELECT(IDA15,DP1,DP2,DP3,DP,YFL,ND15,PDDP,DNDP,DPR)
C
C     DECLINATION OF PRODUCTION
C
      DPR=DPR/100
C
C     RATE OF PRODUCTION (BLS/DAY)
C
      YFL=RAN(YFL)
      CALL SELECT(IDA20,PW1,PW2,PW3,PW,YFL,ND20,PDPW,DNPW,PROD)
C
C     NUMBER OF DEVELOPMENT WELLS PER YEAR
C
      YFL=RAN(YFL)
      CALL SELECT(IDA22,DW1,DW2,DW3,DW,YFL,ND22,PDDW,DNDW,DWN)
C
C     NUMBER OF YEARS TO DEVELOP THE DEPOSIT
C
      YFL=RAN(YFL)
      CALL SELECT(IDA2,YDW1,YDW2,YDW3,YDW,YFL,ND2,PDYDW,DNYDW,YDL)
C
C     NUMBER OF DRY HOLES
C
      YFL=RAN(YFL)
      CALL SELECT(IDA23,DH1,DH2,DH3,DH,YFL,ND23,PDDH,DNDH,DHN)
      DHN=DHN/100
C
C     GAS OIL RATIO
C
      YFL=RAN(YFL)
      CALL SELECT(IDA8,GR1,GR2,GR3,GR,YFL,ND8,PDGR,DNGR,GOR)
C
C     DETERMINATION OF OIL AND GAS PRODUCTION
C
      SP=0.0
      J=1
      PPW(J)=(PROD*(DWN*(1-DHN))*365)/1000000
```

```
571     SP=SP+PPW(J)
        IF(SP-RR(K))634,635,635
634     J=J+1
        Z=J
        IF(Z-YDL)636,637,637
636     PPW(J)=PPW(J-1)*(1-DPR)+(PROD*(DWN*(1-DHN))*365)/1000000
        GO TO 571
637     PPW(J)=PPW(J-1)*(1-DPR)
        IF(PPW(J).LT.0.01)GO TO 635
        SP=SP+PPW(J)
        IF(SP-RR(K))638,635,635
638     J=J+1
        GO TO 637
635     LIFE(LL)=J
575     SS=0
        IL=LIFE(LL)
        DO 576 I=1,IL,1
576     SS=SS+PPW(I)
        AVP(JJ)=(SS*1000/IL)/365
        QQ=0
        DO 577 I=1,IL,1
        PGW(I)=(PPW(I)*GOR)
577     QQ=QQ+PGW(I)
        AVG(II)=(QQ/IL)/365
C
C       DETERMINATION OF TOTAL REVENUES BEFORE DEPRECIATION
C
        YFL=RAN(YFL)
        CALL SELECT(IDA16,PE1,PE2,PE3,PE,YFL,ND16,PDPE,DNPE,PES)
C
C       PRICE ESCALATION FACTOR
C
        PES=PES/100
C
C       COST ESCALATION FACTOR
C
        YFL=RAN(YFL)
        CALL SELECT(IDA14,CE1,CE2,CE3,CE,YFL,ND14,PDCE,DNCE,CES)
        CES=CES/100
        YFL=RAN(YFL)
        CALL SELECT(IDA13,OPC1,OPC2,OPC3,OPC,YFL,ND13,PDOPC,DNOPC,OC)
        KL=1
        CPB(KL)=OC
        PPB(KL)=PRO
        PPG(KL)=PRG
        DO 657 KL=2,IL,1
        CPB(KL)=CPB(KL-1)*(1+CES)
        PPB(KL)=PPB(KL-1)*(1+PES)
        PPG(KL)=PPG(KL-1)*(1+PES)
657     CONTINUE
C
C       COST OF DEVELOPMENT WELL
C
        YFL=RAN(YFL)
        CALL SELECT(IDA10,DC1,DC2,DC3,DC,YFL,ND10,PDDC,DNDC,CDW)
C
C       COST OF DRY HOLE
C
        YFL=RAN(YFL)
        CALL SELECT(IDA11,CDH1,CDH2,CDH3,CDH,YFL,ND11,PDCDH,DNCDH,CPS)
```

```
C
C       COST OF RECOMPLETED WELL
C
        YFL=RAN(YFL)
        CALL SELECT(IDA12,CR1,CR2,CR3,WCR,YFL,ND12,PDCR,DNCR,CRW)
C
C       COST OF EXPLORATION TEST HOLE
C
        YFL=RAN(YFL)
        CALL SELECT(IDA9,RC1,RC2,RC3,RC,YFL,ND9,PDRC,DNRC,CTH)
C
C       COST OF SEISMIC EXPLORATION
C
        YFL=RAN(YFL)
        CALL SELECT(IDA19,CS1,CS2,CS3,CS,YFL,ND19,PDCS,DNCS,CSE)
C
C       DEPTH OF PRODUCTIVE ZONE
C
        YFL=RAN(YFL)
        CALL SELECT(IDA18,PG1,PG2,PG3,PG,YFL,ND18,PDPG,DNPG,DEPTH)
C
C       NUMBER OF TEST HOLES
C
        YFL=RAN(YFL)
        CALL SELECT(IDA21,TH1,TH2,TH3,TH,YFL,ND21,PDTH,DNTH,THN)
        KL=1
        CDP(KL)=CDW
        CSP(KL)=CPS
        CREP(KL)=CRW
        CEH(KL)=CTH
        DO 722 KL=2,IL,1
        CDP(KL)=CDP(KL-1)*(1+CES)
        CSP(KL)=CSP(KL-1)*(1+CES)
        CREP(KL)=CREP(KL-1)*(1+CES)
        CEH(KL)=CEH(KL-1)*(1+CES)
722     CONTINUE
        JI=IL+1
        DO 723 I=1,IL,1
        RIC(I)=PPW(I)*PPB(I)+PGW(I)*PPG(I)/1000
        YVO(I)=RIC(I)*(1-ROY/100)
        LEON(I)=YVO(I)-PPW(I)*CPB(I)
723     CONTINUE
        I=1
        IP=I-1
        CHAR(IP)=(DWN*(1-DHN)*DEPTH*CDW+DHN*DWN*DEPTH*CPS)/1000000
        ASM(IP)=(CLA+CSE)/1000000
        RON(IP)=(THN*CEH(I))/1000000
        RUT(IP)=EQC*NEQC/1000000
        I=I+1
        X=I
        IF(X-YDL)724,724,725
724     IP=I-1
        CHAR(IP)=(DWN*(1-DHN)*DEPTH*CDP(IP)+DHN*DWN*DEPTH*CSP(IP))/
     1  1000000
        I=I+1
        V=I
        IF(V-YDL)724,724,725
725     I=1
728     IP=I+3
        G=IP
```

```
      IF(G-YDL)726,726,727
726   YSA(IP)=(DWN*(1-DHN)*CREP(IP))/1000000
      I=I+1
      GO TO 728
727   DO 729 I=1,JI,1
      IP=I-1
      CIR(IP)=(CHAR(IP)+RON(IP)+RUT(IP)+YSA(IP))*(1-PTC/100)
729   DAV(IP)=(CHAR(IP)+RON(IP)+RUT(IP)+YSA(IP))*PTC/100
C
C     DEPRECIATION
C
      IF(KD.EQ.1)GO TO 730
      IF(KD.EQ.2)GO TO 731
      IF(KD.EQ.3)GO TO 732
      IF(KD.EQ.4)GO TO 733
      IF(KD.EQ.5)GO TO 734
      IF(KD.EQ.6)GO TO 735
730   FBOO=0.0
      DO 743 I=1,IL,1
743   ALB(I)=0.0
      DO 736 I=1,IL,1
      IP=I-1
      GLOR(I)=DAV(IP)/NYD
      KI=I
      IK=1
739   CAR(KI)=GLOR(I)
      ALB(KI)=ALB(KI)+CAR(KI)
      KI=KI+1
      IK=IK+1
      IF(IK-NYD)737,737,736
737   IF(KI-IL)739,739,740
740   FBOO=FBOO+GLOR(I)
      IK=IK+1
      IF(IK-NYD)740,740,736
736   CONTINUE
      FBOO=FBOO+DAV(IL)
      GO TO 744
731   FBOO=0.0
      DO 745 I=1,IL,1
745   ALB(I)=0.0
      DO 746 I=1,IL,1
      IP=I-1
      GLOR(I)=DAV(IP)*2/NYD
      KI=I
      IK=1
      CAR(KI)=GLOR(I)
      ROB=DAV(IP)-CAR(KI)
749   ALB(KI)=ALB(KI)+CAR(KI)
      KI=KI+1
      IK=IK+1
      IF(IK-NYD)747,747,750
747   IF(KI-IL)748,748,750
748   CAR(KI)=ROB-CAR(KI)
      GO TO 749
750   BOOK(I)=ROB
      FBOO=FBOO+BOOK(I)
746   CONTINUE
      FBOO=FBOO+DAV(IL)
      GO TO 744
732   RESE=RR(K)
```

```
       FBOO=0.0
       DO 751 I=1,IL,1
751    ALB(I)=0.0
       DO 752 I=1,IL,1
       IP=I-1
       GLOR(I)=DAV(IP)*PPW(I)/RESE
       KI=I
       CAR(KI)=GLOR(I)
       ROB=DAV(IP)-CAR(KI)
756    ALB(KI)=ALB(KI)+CAR(KI)
       KI=KI+1
       IF(KI-IL)753,753,754
753    CAR(KI)=ROB*PPW(KI)/RESE
       ROB=ROB-CAR(KI)
       GO TO 756
754    BOOK(I)=ROB
       FBOO=FBOO+BOOK(I)
752    CONTINUE
       FBOO=FBOO+DAV(IL)
       GO TO 744
733    FBOO=0.0
       DO 757 I=1,IL,1
757    ALB(I)=0.0
       DO 758 I=1,IL,1
       IP=I-1
       NSD=NYD*(NYD+1)/2
       KI=I
       IK=0
761    CAR(KI)=DAV(IP)*(NYD-IK)/NSD
       ALB(KI)=ALB(KI)+CAR(KI)
       KI=KI+1
       IK=IK+1
       IF(KI-IL)759,759,760
759    IF(IK-NYD)761,758,758
760    BOOK(I)=0.0
762    IF(IK-NYD)763,763,758
763    CAR(KI)=DAV(IP)*(NYD-IK)/NSD
       BOOK(I)=BOOK(I)+CAR(KI)
       FBOO=FBOO+BOOK(I)
       IK=IK+1
       GO TO 762
758    CONTINUE
       FBOO=FBOO+DAV(IL)
       GO TO 744
734    FBOO=0.0
       DO 764 I=1,IL,1
764    ALB(I)=0.0
       DO 765 I=1,IL,1
       IP=I-1
       KI=I
       IK=1
       CAR(KI)=DAV(IP)*2/NYD
       ROB=DAV(IP)-CAR(KI)
771    ALB(KI)=ALB(KI)+CAR(KI)
       KI=KI+1
       IK=IK+1
       NS=NYD-IK+1
       IF(IK-NYD)766,766,767
766    IF(KI-IL)768,768,767
768    GLOR(KI)=ROB/NS
```

```
      VAS(KI)=ROB*2/NYD
      IF(GLOR(KI)-VAS(KI))769,770,770
769   CAR(KI)=VAS(KI)
      ROB=ROB-CAR(KI)
      GO TO 771
770   CAR(KI)=GLOR(KI)
776   ALB(KI)=ALB(KI)+CAR(KI)
      KI=KI+1
      IK=IK+1
      IF(IK-NYD)772,772,765
772   IF(KI-IL)774,774,775
774   CAR(KI)=ROB/NS
      GO TO 776
767   BOOK(I)=ROB
      GO TO 779
775   BOOK(I)=ROB/NS
897   IK=IK+1
      IF(IK-NYD)778,778,765
778   BOOK(I)=BOOK(I)+ROB/NS
      GO TO 897
779   FBOO=FBOO+BOOK(I)
765   CONTINUE
      GO TO 744
      FBOO=FBOO+DAV(IL)
735   FBOO=0.0
      DO 780 I=1,IL,1
780   ALB(I)=0.0
      DO 781 I=1,IL,1
      IP=I-1
      KI=I
      IK=1
      CAR(KI)=DAV(IP)*2/NYD
      ROB=DAV(IP)-CAR(KI)
787   ALB(KI)=ALB(KI)+CAR(KI)
      KI=KI+1
      IK=IK+1
      NSD=NYD-IK+1
      NS=NSD*(NSD+1)/2
      IF(IK-NYD)782,782,783
782   IF(KI-IL)784,784,783
784   GLOR(KI)=ROB*NSD/NS
      VAS(KI)=ROB*2/NYD
      IF(GLOR(KI)-VAS(KI))785,786,786
785   CAR(KI)=VAS(KI)
      ROB=ROB-CAR(KI)
      GO TO 787
786   CAR(KI)=GLOR(KI)
      ROB=ROB-CAR(KI)
792   ALB(KI)=ALB(KI)+CAR(KI)
      KI=KI+1
      IK=IK+1
      NSD=NYD-IK+1
      IF(IK-NYD)788,788,781
788   IF(KI-IL)790,790,783
790   CAR(KI)=ROB*NSD/NS
      ROB=ROB-CAR(KI)
      GO TO 792
783   BOOK(I)=ROB
      FBOO=FBOO+BOOK(I)
781   CONTINUE
```

```
      FBOO=FBOO+DAV(IL)
C
C     INCOME BEFORE DEPLETION
C
744   JI=IL+1
      DO 793 I=1,JI,1
      IP=I-1
793   SAU(IP)=LEON(IP)-CIR(IP)-ALB(IP)
C
C     DEPLETION
C
      IF(EPD.EQ.1.0)GO TO 2109
      IF(EPD.EQ.2.0)GO TO 2110
2109  RESE=RR(K)
      DO 794 I=1,IL,1
      RAU(I)=YVO(I)*PDE/100
794   DAG(I)=SAU(I)*0.5
      I=1
      TCLA=CLA/1000000
      CIG(I)=TCLA*PPW(I)/RESE
796   IF(RAU(I)-DAG(I))797,797,798
797   IF(RAU(I)-CIG(I))799,799,800
799   FIDE(I)=CIG(I)
      GO TO 2101
800   FIDE(I)=RAU(I)
      GO TO 2101
798   IF(DAG(I)-CIG(I))801,801,802
801   FIDE(I)=CIG(I)
      GO TO 2101
802   FIDE(I)=DAG(I)
2101  I=I+1
      IF(I-IL)2102,2102,2103
2102  RESE=RESE-PPW(I-1)
      TCLA=TCLA-FIDE(I-1)
      CIG(I)=TCLA*PPW(I)/RESE
      GO TO 796
2110  RESE=RR(K)
      TCLA=CLA/1000000
      I=1
      FIDE(I)=TCLA*PPW(I)/RESE
      DO 795 I=2,IL,1
      TCLA=TCLA-FIDE(I-1)
      RESE=RESE-PPW(I-1)
795   FIDE(I)=TCLA*PPW(I)/RESE
C
C     INVESTMENT TAX CREDIT
C
2103  IF(ITC.EQ.1)GO TO 804
      IF(ITC.EQ.2)GO TO 805
805   OLG=0.0
      GO TO 811
804   IF(KD.EQ.3)GO TO 810
      IF(NYD-4)806,806,807
806   OLG=0.333333
      GO TO 811
807   IF(NYD-6)809,809,810
809   OLG=0.66667
      GO TO 811
810   OLG=1.0
C
```

```
C       AFTER TAX SALVAGE VALUE
C
811     ATSV(IL)=(SALVA/1000000-FBOO)*TAX/100
        IF(ATSV(IL)-0.0)2141,2141,2142
2141    ATSV(IL)=0.0
C
C       TAXABLE INCOME
C
2142    DO 803 I=1,JI,1
        IP=I-1
        TAXI(IP)=SAU(IP)-FIDE(IP)
C
C       TAX
C
        EMI(IP)=TAXI(IP)*TAX/100
C
C       INVESTMENT TAX CREDIT
C
        ABU(IP)=0.1*OLG*DAV(IP)
C
C       NET PROFIT
C
        GAN(IP)=TAXI(IP)-EMI(IP)+ABU(IP)
C
C       NET CASH FLOW
C
803     CFLOW(IP)=GAN(IP)+ALB(IP)+FIDE(IP)+CIR(IP)-CHAR(IP)-RON(IP)-
     1  RUT(IP)-YSA(IP)-ASM(IP)+ATSV(IP)
        IF(LBM.EQ.1)GO TO 822
        IF(LBM.EQ.2)GO TO 823
822     I=1
        TZI(I)=(ASM(0)+RON(0)+RUT(0))*PLI/100
        PLP(LK)=TZI(I)/LIFE(LL)
        DO 824 I=2,IL,1
824     TZI(I)=TZI(I-1)-PLP(LK)
        DO 825 I=1,IL,1
825     ATIC(I)=TZI(I)*(PIR/100)*(TAX/100)
        DO 826 I=1,JI,1
        IP=I-1
826     CFLOW(IP)=CFLOW(IP)-ATIC(IP)-PLP(LK)-(RON(IP)+ASM(IP)+RUT(IP))*
     1  PLI/100
C
C       NET PRESENT VALUE
C
823     PVAL(MM)=0.0
        DO 812 I=1,JI,1
        IP=I-1
        FED(IP)=CFLOW(IP)/((1.0+MROR/100.0)**IP)
812     PVAL(MM)=PVAL(MM)+FED(IP)
C
C       DISCOUNTED CASH FLOW RATE OF RETURN
C
        T2=5.0
        T1=1.0
        T=0.0
590     SUM=0.0
        DO 813 I=1,JI,1
        IP=I-1
        DCF(IP)=CFLOW(IP)/((1.0+T/100)**IP)
813     SUM=SUM+DCF(IP)
```

```
      IF(ABS(SUM).LT.0.001)GO TO 814
      IF(SUM-0.0)815,814,816
816   T=T+T2
      GO TO 590
815   FPVA=ABS(SUM)
      FT=T
588   T=T-T1
      IF(T-0.0)819,814,485
819   T=0.0
      GO TO 814
485   SUM=0.0
      DO 818 I=1,JI,1
      IP=I-1
      DCF(IP)=CFLOW(IP)/((1.0+T/100)**IP)
818   SUM=SUM+DCF(IP)
      IF(ABS(SUM).LT.0.001)GO TO 814
      IF(SUM-0.0)588,814,1607
1607  SPVA=SUM
      ST=T
      GO TO 817
814   DCFROR(NN)=T
817   DCFROR(NN)=ST+(FT-ST)*SPVA/(SPVA+FPVA)
C
C     NET PRESENT VALUE-COSTS RATIO
C
      CAPC=0.0
      DO 820 I=1,JI,1
      IP=I-1
      TCOST(IP)=CHAR(IP)+RON(IP)+RUT(IP)+YSA(IP)+ASM(IP)
      DCFW(IP)=TCOST(IP)/((1.0+MROR/100.0)**IP)
820   CAPC=CAPC+DCFW(IP)
      RNPVC(NM)=PVAL(NM)/CAPC
      IF(NCALC-NT)411,476,476
476   WRITE(8,487)
487   FORMAT(1H1,////////,10X,'RESULTS OBTAINED BY USING MONTE CARLO
     1 SIMULATION',/,10X,50('*'),///,6X,
     1'THE RESULTS OBTAINED BY USING THE MONTECARLO SIMULATION ARE',/,
     13X,'PRESENTED IN THE FORM OF PROBABILISTIC DISTRIBUTION OF THE',
     1/,3X,'SOLUTION VARIABLES. THE FOLLOWING TABLES SHOW THE',/,3X,
     1'DISTRIBUTION OF RECOVERABLE RESERVES,RATE OF PRODUCTION OF',/,3
     1X,'OIL AND GAS, LIFE OF THE PROJECT, AND ECONOMIC MEASURES',/,3X
     1,'USED IN DECISION MAKING SUCH AS THE NET PRESENT VALUE, RATE',/
     1,3X,'OF RETURN AND THE RATIO OF EXPECTED GAIN TO CAPITAL COSTS',
     1//,6X,'THE OBJETIVE OF THE METHOD IS SIMULTANEOUSLY TO OBTAIN',/
     1,3X,'A VALID SOLUTION OF THE PROBLEM AND TO DETERMINE THE',/,3X,
     1'DEGREE OF UNCERTAINTY ASSOCIATED WITH THE SOLUTION, THE',/,3X,
     1'NARROWER THE DISTRIBUTION FOR THE SOLUTION, THE GREATER',/,3X,
     1'THE CERTAINTY OF THE SOLUTION; A FLAT DISTRIBUTION WITH',/,3X,
     1'CONSIDERABLE SPREAD BETWEEN THE UPER AND LOWER VALUE LIMITS',/,
     13X'INDICATES A GREATER UNCERTAINTY IN THE SOLUTION.',//,6X,
     1'THERE WILL ALSO BE FOUND THE MEAN AND STANDAR DEVIATION OF',/,3
     1X,'THE SOLUTION VARIABLES. A TABULATION OF COMPUTED CUMULATIVE',
     1/,3X,'PROBABILITY FOR EACH SOLUTION VARIABLE IS GIVEN. ALSO',/,3
     1X,'A 95% CONFIDENCE INTERVAL FOR THE MEAN IS GIVEN FOR EACH',/,3
     1X,'OF THE SOLUTION VARIABLES.')
      IF(LBM.EQ.1)GO TO 1302
      IF(LBM.EQ.2)GO TO 1303
1302  AEYMP=0.0
      DO 1304 LK=1,NT,1
1304  AEYMP=AEYMP+PLP(LK)
```

```
      AVMP=AEYMP/NT
      WRITE(8,1305)AVMP
1305  FORMAT(////,6X,'LOAN WILL BE PAID OFF OVER THE LIFE OF THE',/,3X
     1 ,'PROJECT WITH ANNUAL END OF YEAR AVERAGE MORTGAGE PAYMENTS',/,3
     1 X,'OF ',1X,F10.3,3X,'MILLION DOLLARS')
C
C     PROBABILISTIC DISTRIBUTION FOR RECOVERABLE RESERVES
C
      CALL RAIL(RR,NT,MEDP,DESVP)
1303  WRITE(8,488)
488   FORMAT(1H1,25X,'RECOVERABLE RESERVES (MILLION OF BLS)',/
     1 ,25X,37('*'),//)
      CALL DIST(RR,NT,NC,MEDP,DESVP,IFR,FCRR,PROB,LI,LS,ICRR)
C
C     PROBABILISTIC DISTRIBUTION FOR OIL PRODUCTION
C
      CALL RAIL(AVP,NT,MEDQ,DESVQ)
      WRITE(8,598)
598   FORMAT(1H1,15X,'AVERAGE RATE OF PRODUCTION OF OIL (THOU.
     1 OF BLS/DAY)',/,15X,53('*'),//)
      CALL DIST(AVP,NT,NC,MEDQ,DESVQ,NRP,FCP,PROB,LI,LS,ICP)
C
C     PROBABILISTIC DISTRIBUTION FOR AVERAGE GAS PRODUCTION (MCF/DAY)
C
      CALL RAIL(AVG,NT,MEDG,DESVG)
      WRITE(8,610)
610   FORMAT(1H1,15X,'AVERAGE RATE OF PRODUCTION OF GAS (MILLI
     1 ON OF CF/D)',/,15X,53('*'),//)
      CALL DIST(AVG,NT,NC,MEDG,DESVG,NRG,FCG,PROB,LI,LS,ICG)
C
C     PROBABILISTIC DISTRIBUTION FOR LIFE OF THE PROJECT
C
      CALL RAIL(LIFE,NT,MEDL,DESVL)
      WRITE(8,622)
622   FORMAT(1H1,30X,'LIFE OF THE PROJECT (YEARS)',/,30X,
     1 27('*'),//)
      CALL DIST(LIFE,NT,NC,MEDL,DESVL,NRL,FCL,PROB,LI,LS,ICL)
C
C     PROBABILISTIC DISTRIBUTION FOR NET PRESENT VALUE
C
      CALL RAIL(PVAL,NT,MEDPV,DESPV)
      WRITE(8,831)
831   FORMAT(1H1,25X,'NET PRESENT VALUE (MILLION OF DOLLARS)',
     1 /,25X,38('*'),//)
      CALL DIST(PVAL,NT,NC,MEDPV,DESPV,NRPV,FCPV,PROB,LI,LS,ICPV)
C
C     PROBABILISTIC DISTRIBUTION FOR RATIO NPV TO CAPITAL COSTS
C
      CALL RAIL(RNPVC,NT,MEDRA,DESRA)
      WRITE(8,843)
843   FORMAT(1H1,30X,'NPV-CAPITAL COSTS RATIO',/,30X,23('*'),//)
      CALL DIST(RNPVC,NT,NC,MEDRA,DESRA,NRRNP,FCRNP,PROB,LI,LS,ICRNP)
C
C     PROBABLISTIC DISTRIBUTION FOR THE DISCOUNTED
C     CASH FLOW RATE OF RETURN
C
      CALL RAIL(DCFROR,NT,MEDRO,DESRO)
      WRITE(8,863)
863   FORMAT(1H1,25X,'DISCOUNTED CASH FLOW RATE OF RETURN (%)'
     1 ,/,25X,39('*'),//)
```

```
      CALL DIST(DCFROR,NT,NC,MEDRO,DESRO,NRROR,FCROR,PROB,LI,LS,ICROR)
      END
C
C     SUBROUTINE TO DETERMINE THE MEAN AND STANDAR DEVIATION
C
      SUBROUTINE RAIL(X,N,MED,DESV)
      DIMENSION X(1000)
      REAL MED,MED1,MED2,MED3,MED4,MED5
      INTEGER N1,N2,N3,N4,N5,N6,N7,N8,N9
      N1=N/5
      N2=2*N1
      N3=N2+N1
      N4=N3+N1
      N5=N4+N1
      N6=N1+1
      N7=N2+1
      N8=N3+1
      N9=N4+1
      B=0.0
      DO 493 IK=1,N1,1
493   B=B+X(IK)
      MED1=B/N1
      B=0.0
      DO 4931 IK=N6,N2,1
4931  B=B+X(IK)
      MED2=B/(N2-N1)
      B=0.0
      DO 4932 IK=N7,N3,1
4932  B=B+X(IK)
      MED3=B/(N3-N2)
      B=0.0
      DO 4933 IK=N8,N4,1
4933  B=B+X(IK)
      MED4=B/(N4-N3)
      B=0.0
      DO 4934 IK=N9,N5,1
4934  B=B+X(IK)
      MED5=B/(N5-N4)
      MED=(MED1+MED2+MED3+MED4+MED5)/5
      BB=0.0
      DO 494 IK=1,N,1
494   BB=BB+(X(IK)**2)/(N-1)
      DESV=SQRT(BB-(N/(N-1))*MED**2)
      RETURN
      END
C
C     GENERATOR OF RANDOM NUMBERS
C
      SUBROUTINE RAND(X)
      CALL TIME(I,X)
      CALL SETRAN(X)
      X=RAN(X)
      RETURN
      END
C
C     SUBROUTINE TO SELECT AT RANDOM THE VALUE OF STOCHASTIC VARIABLES
C     ACCORDING TO TYPE OF DISTRIBUTION
C
      SUBROUTINE SELECT(ID,VAR1,VAR2,VAR3,VAR,Y,NY,PD,DN,VAL)
      DIMENSION PD(10),DN(10)
```

```
      IF(ID.EQ.1)GO TO 412
      IF(ID.EQ.2)GO TO 413
      IF(ID.EQ.3)GO TO 414
      IF(ID.EQ.4)GO TO 415
412   VAL=VAR1+Y*(VAR2-VAR1)
      GO TO 420
413   RN=(VAR2-VAR1)/(VAR3-VAR1)
      IF(Y-RN)416,417,417
416   VAL=VAR1+SQRT((VAR2-VAR1)*(VAR3-VAR1)*Y)
      GO TO 420
417   VAL=VAR3-SQRT((VAR3-VAR2)*(VAR3-VAR1)*(1-Y))
      GO TO 420
414   PCUM=0.0
      DO 418 I=1,NY,1
      PCUM=PCUM+PD(I)/100
      IF(PCUM-Y)418,419,419
418   CONTINUE
419   VAL=DN(I)
      GO TO 420
415   VAL=VAR
420   RETURN
      END
C
C     SUBROUTINE TO READ STOCHASTIC DATA
C
      SUBROUTINE LECT(ID,VAR1,VAR2,VAR3,VAR,NY,PD,DN)
      DIMENSION PD(10),DN(10)
      INR=3
      INC=1
497   WRITE(INR,2222)
2222  FORMAT(/,1X,'TYPE "1" IF DISTRIBUTION IS UNIFORM',/,1X,
     1'TYPE "2" IF DISTRIBUTION IS TRIANGULAR',/,1X,
     1'TYPE "3" IF DISTRIBUTION IS DISCRETE',/,1X,
     1'TYPE "4" IF ONLY ONE VALUE IS ESTIMATED')
      READ(INC,*)ID
      IF(ID.EQ.1)GO TO 990
      IF(ID.EQ.2)GO TO 991
      IF(ID.EQ.3)GO TO 992
      IF(ID.EQ.4)GO TO 993
      IF(ID.GT.4)GO TO 417
417   WRITE(INR,416)
416   FORMAT(/,1X,'ENTER AGAIN',/)
      GO TO 497
990   WRITE(INR,11)
11    FORMAT(/,1X,'ENTER THE TWO EXTREME VALUES, "A,B"',/,1X,
     1'THE LOWEST FIRST AND THEN THE HIGHEST,SEPARATED BY COMMAS')
      READ(INC,*)VAR1,VAR2
      GO TO 666
991   WRITE(INR,12)
12    FORMAT(/,1X,'ENTER THE THREE VALUES "A,B,C"',/,1X,
     1'FIRST ENTER THE LOWEST, SECOND THE MOST LIKELY',/,1X,
     1'AND THIRD ENTER THE HIGHEST, SEPARATED BY COMMAS')
      READ(INC,*)VAR1,VAR2,VAR3
      GO TO 666
992   WRITE(INR,15)
15    FORMAT(/,1X,'ENTER NUMBER OF DISCRET VALUES "N"',/,1X,
     1'MAXIMUN NUMBER OF DISCRETE VALUES=10')
      READ(INC,*)NY
      IF(NY-10)523,523,414
414   WRITE(INR,416)
```

```
      GO TO 992
523   WRITE(INR,13)
13    FORMAT(/,1X,'YOU WILL ENTER VALUE AND PROBABILITY PER LINE',/,
     1 1X,'REMEMBER THAT THE SUM OF PROBABILITIES SHOULD BE "100%"',
     1 /,1X,'ENTER THE PROBABILITY IN PERCENTAGE',/,1X,
     1 'PLEASE, ENTER THE VALUES OF THE VARIABLE IN INCREASING ORDER')
      SPDA=0.0
      DO 2000 I=1,NY,1
      WRITE(INR,16)
16    FORMAT(/,1X,'ENTER VALUE AND RESPECTIVE PROBABILITY')
      READ(INC,*)DN(I),PD(I)
      SPDA=SPDA+PD(I)
2000  CONTINUE
      IF(SPDA-100)70,71,70
71    GO TO 666
70    WRITE(INR,72)
72    FORMAT(/,1X,'SUM OF PROBABILITIES IS NOT "100%"',/,1X,
     1 'PLEASE ENTER AGAIN THE DISCRETE VALUES AND THE RESPECTIVE',/,1X
     1 ,'PROBABILITIES')
      GO TO 992
993   WRITE(INR,17)
17    FORMAT(/,1X,'ENTER THE ESTIMATED VALUE')
      READ(INC,*)VAR
666   RETURN
      END
C
C     SUBROUTINE TO WRITE INPUT DATA OF STOCHASTIC VARIABLES
C
      SUBROUTINE WRISD(ID,VAR1,VAR2,VAR3,VAR,NY,PD,DN)
      DIMENSION PD(10),DN(10)
      IF(ID.EQ.1)GO TO 219
      IF(ID.EQ.2)GO TO 220
      IF(ID.EQ.3)GO TO 221
      IF(ID.EQ.4)GO TO 223
219   WRITE(8,226)
226   FORMAT(/,5X,'UNIFORM DISTRIBUTION',//,1X,'LOWEST VALUE',5X,
     1 'HIGHEST VALUE')
      WRITE(8,227)VAR1,VAR2
227   FORMAT(/,F10.1,7X,F10.1)
      GO TO 332
220   WRITE(8,228)
228   FORMAT(/,5X,'TRIANGULAR DISTRIBUTION',//,1X,'LOWEST VALUE',5X,
     1 'MOST LIKELY VALUE',5X,'HIGHEST VALUE')
      WRITE(8,229)VAR1,VAR2,VAR3
229   FORMAT(/,F10.1,7X,F10.1,9X,F10.1)
      GO TO 332
221   WRITE(8,230)
230   FORMAT(/,5X,'DISCRETE DISTRIBUTION',//,6X,'VALUE',8X,
     1 'PROBABILITY')
      DO 231 I=1,NY,1
231   WRITE(8,232)DN(I),PD(I)
232   FORMAT(/,1X,F10.1,5X,F10.1)
      GO TO 332
223   WRITE(8,233)
233   FORMAT(/,5X,'ONLY ONE VALUE IS ESTIMATED')
      WRITE(8,234)VAR
234   FORMAT(/,F10.1)
332   RETURN
      END
C     SUBROUTINE TO CONSTRUCT THE PROBABILISTIC DISTRIBUTION FOR
```

```
C       THE SOLUTION VARIABLES AND DETERMINE A 95% CONFIDENCE
C       INTERVAL FOR THE MEAN
C
        SUBROUTINE DIST(W,N,NB,MEDW,DESVW,IFW,FCW,PROB,LI,LS,ICW)
        DIMENSION W(1000)
        REAL LI(50),LS(50),MEDW,ICW,IFW(50)
        DOUBLE PRECISION FCW(50),PROB(50)
        LM=1
        U1=W(LM)
        DO 477 LM=2,N,1
        IF(U1-W(LM))478,477,477
478     U1=W(LM)
477     CONTINUE
        LM=1
        U2=W(LM)
        DO 1701 LM=2,N,1
        IF(W(LM).LT.U2)GO TO 1702
        IF(W(LM).GT.U2)GO TO 1701
        IF(W(LM).EQ.U2)GO TO 1702
1702    U2=W(LM)
1701    CONTINUE
        ICW=U1/NB
        DO 479 I=1,NB,1
479     IFW(I)=0.0
        DO 480 LM=1,N,1
        I=1
481     IF(W(LM)-ICW*I)482,482,483
482     IFW(I)=IFW(I)+1
        GO TO 480
483     IF(I-NB)484,480,480
484     I=I+1
        GO TO 481
480     CONTINUE
        DO 485 I=1,NB,1
485     PROB(I)=IFW(I)/N
        I=1
        FCW(I)=PROB(I)
        DO 486 I=2,NB,1
486     FCW(I)=FCW(I-1)+PROB(I)
        WRITE(8,489)MEDW,DESVW,U2,U1
489     FORMAT(30X,'MEAN=',F15.2,/,30X,'STANDARD DEVIATION=',F12.3,
     1  /,30X,'MINIMUM VALUE=',F12.3,/,30X,'MAXIMUM VALUE=',F12.3)
        WRITE(8,490)
490     FORMAT(//,12X,'CLASS',13X,'FREQUENCY',12X,'PROBABILITY',12X,
     1  'CUMULATIVE',/,74X,'PROBABILITY',/,12X,5('*'),13X,9('*'),12X,
     1  11('*'),12X,11('*'))
        DO 491 I=1,NB,1
        LI(I)=ICW*(I-1)
        LS(I)=ICW*I
491     WRITE(8,492)LI(I),LS(I),IFW(I),PROB(I),FCW(I)
492     FORMAT(//,F10.2,4X,F10.2,6X,F6.0,16X,F6.4,16X,F6.4)
        TALFA=1.96
        DOWL=MEDW-DESVW*TALFA/(N**(1/2))
        UPPL=MEDW+DESVW*TALFA/(N**(1/2))
        WRITE(8,1401)DOWL,UPPL
1401    FORMAT(///,30X,'95 OUT OF 100 TIMES THE MEAN WILL BE',/,30X,
     1  'BETWEEN',1X,F12.3,1X,'AND',1X,F12.3)
        RETURN
        END
```

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[LNKXCT THES execution]
              APPLICATION OF RISK ANALYSIS TO
              PETROLEUM INVESTMENT DECISIONS


              MONTE CARLO SIMULATION



  THIS PROGRAM IS CONCERNED WITH THE ECONOMIC EVALUATION
AND DECISION MAKING REGARDING PETROLEUM INVESTMENT
VENTURES. IT INVOLVES ASSOCIATING PROBABILITIES WITH
THE RANGE OF PROFITABILITIES THAT MIGHT RESULT FROM A
PARTICULAR PROJECT.

  THE VARIABLES USED TO DETERMINE THE PROFITABILITY
OF THE INVESTMENT CAN HAVE A PROBABILISTIC NATURE
DEPENDING ON YOUR JUDGMENT. THE PROGRAM GIVES YOU THE
OPPORTUNITY TO DECIDE WHICH VARIABLES ARE DETERMINISTIC
AND WHICH ARE STOCHASTIC; AND ALSO TO ASSIGN THE KIND
OF DISTRIBUTION FOR THE STOCHASTIC VARIABLES.

  THIS MODEL OF RISK ANALYSIS COMBINES THE PROBABILISTIC
DISTRIBUTION OF THOSE STOCHASTIC VARIABLES AS WELL AS
THE VALUES OF THE DETERMINISTIC ONES AND GENERATES A
FINAL DISTRIBUTION OF ECONOMIC PARAMETERS USED IN
DECISION MAKING SUCH AS THE NET PRESENT VALUE AND THE
RATE OF RETURN ON THE INVESTMENT.

ENTER NUMBER OF TRIALS
1000

ENTER NUMBER OF CLASS BOUNDARIES
DO NOT USE MORE THAN 50 CLASS BOUNDARIES
14


  INPUT DATA FOR DETERMINISTIC PARAMETERS


ENTER COST OF LAND ACQUISITION
(COST OF MINERAL RIGHTS)
15000000

WHICH METHOD OF DEPRECIATION WILL YOU USE
TYPE 1 IF STRAIGHT LINE
TYPE 2 IF DOUBLE DECLINING BALANCE
TYPE 3 IF UNITS OF PRODUCTION
TYPE 4 IF SUM OF YEAR DIGITS
TYPE 5 IF DOUBLE DECLINING SWITCHING TO STRAIGHT LINE
TYPE 6 IF DOUBLE DECLINING SWITCHING TO SUM OF YEAR DIGITS

1

HOW MANY YEARS OF DEPRECIATION LIFE WILL YOU USE?
15

WHAT PERCENT. OF YOUR EQUIP. AND WELL COSTS
ARE CONSIDERED TO BE TANGIBLE COSTS (%)?
```

```
70

IS THE COMPANY ELEGIBLE FOR PERCENT DEPLETION?
TYPE "1" IF YES OR "2" IF NO
2

ENTER TAX INCOME RATE  (%)
50

ENTER ROYALTIES  (%)
16.67

ENTER MINIMUM RATE OF RETURN  (%)
15

ENTER COST OF  DRILLING AND RECOMPLETION EQUIPMENT
6500000

HOW MANY OF THOSE EQUIPMENTS WILL YOU USE?

2

WHAT IS THE SALVAGE VALUE OF YOUR EQUIPMENT?
0

ENTER ACTUAL OIL PRICE ($/BL)
30

ENTER ACTUAL GAS PRICE ($/MCF)
2

WOULD YOU LIKE INVESTMENT TAX CREDIT TO BE
APPLIED ON DEPRECIABLE EQUIPMENT COSTS?
TYPE 1 IF YES, TYPE 2 IF NO.
1


   INVESTMENT TAX CREDIT IS APPLIED TO 10%
OF THE FULL TANGIBLE EQUIPMENT AND WELL COSTS AND WILL
BE USED AGAINST TAX OBLIGATIONS AND OTHER TAXABLE
INCOME IN THE YEARS WHEN COSTS ARE INCURRED.

DO YOU PLAN TO BORROW THE TIME ZERO CAPITAL COSTS?
TYPE "1" IF YES, TYPE "2" IF NO
1

WHAT PERCENTAGE OF THE TIME ZERO INVESTMENT WILL

YOU BORROW?

100

WHAT IS THE INTEREST RATE (%) AT WHICH THE MONEY
IS BORROWED?

12


    NOW, YOU WILL ENTER THE INFORMATION FOR THE
STOCHASTIC PARAMETERS. BE SURE YOU FOLLOW CORRECTLY
THE DIRECTIONS
```

```
TO ENTER VALUE OF
PRODUCTIVE DRAINAGE AREA  (ACRES)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
4

ENTER THE ESTIMATED VALUE
5000

TO ENTER VALUE OF
NET PAY THICKNESS (FEET)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
2

ENTER THE THREE VALUES 'A,B,C'
FIRST ENTER THE LOWEST, SECOND THE MOST LIKELY
AND THIRD ENTER THE HIGHEST, SEPARATED BY COMMAS
30,36,38

TO ENTER VALUE OF
WATER SATURATION  (%)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
1

ENTER THE TWO EXTREME VALUES, 'A,B'
THE LOWEST FIRST AND THEN THE HIGHEST,SEPARATED BY COMMAS
8,12

TO ENTER VALUE OF
GAS SATURATION  (%)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
1

ENTER THE TWO EXTREME VALUES, 'A,B'
THE LOWEST FIRST AND THEN THE HIGHEST,SEPARATED BY COMMAS
6,10

TO ENTER VALUE OF
POROSITY  (%)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
2

ENTER THE THREE VALUES 'A,B,C'
FIRST ENTER THE LOWEST, SECOND THE MOST LIKELY
AND THIRD ENTER THE HIGHEST, SEPARATED BY COMMAS
20,25,28
```

```
TO ENTER VALUE OF
OIL FORMATION VOLUME FACTOR

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
1

ENTER THE TWO EXTREME VALUES, 'A,B'
THE LOWEST FIRST AND THEN THE HIGHEST,SEPARATED BY COMMAS
1.05,1.2

TO ENTER VALUE OF
RECOVERY FACTOR  (%)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
2

ENTER THE THREE VALUES 'A,B,C'
FIRST ENTER THE LOWEST, SECOND THE MOST LIKELY
AND THIRD ENTER THE HIGHEST, SEPARATED BY COMMAS
45,50,56

TO ENTER VALUE OF GAS OIL RATIO  (CF/BL)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
4

ENTER THE ESTIMATED VALUE
700

TO ENTER VALUE OF
COST OF AVERAGE EXPLORATION TEST HOLE ($/WELL)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
4

ENTER THE ESTIMATED VALUE
300000

TO ENTER VALUE OF
WELL DEVELOPMENT COST  ($/FOOT)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
3

ENTER NUMBER OF DISCRET VALUES 'N'
MAXIMUN NUMBER OF DISCRETE VALUES=10
4

YOU WILL ENTER VALUE AND PROBABILITY PER LINE
REMEMBER THAT THE SUM OF PROBABILITIES SHOULD BE '100%'
ENTER THE PROBABILITY IN PERCENTAGE
```

```
PLEASE, ENTER THE VALUES OF THE VARIABLE IN INCREASING ORDER

ENTER VALUE AND RESPECTIVE PROBABILITY
180,22

ENTER VALUE AND RESPECTIVE PROBABILITY
185,20

ENTER VALUE AND RESPECTIVE PROBABILITY
190,35

ENTER VALUE AND RESPECTIVE PROBABILITY
193,23

TO ENTER VALUE OF
COST OF DRY HOLE ($/FT)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
3

ENTER NUMBER OF DISCRET VALUES 'N'
MAXIMUN NUMBER OF DISCRETE VALUES=10
4

YOU WILL ENTER VALUE AND PROBABILITY PER LINE
REMEMBER THAT THE SUM OF PROBABILITIES SHOULD BE '100%'
ENTER THE PROBABILITY IN PERCENTAGE
PLEASE, ENTER THE VALUES OF THE VARIABLE IN INCREASING ORDER

ENTER VALUE AND RESPECTIVE PROBABILITY
210,20

ENTER VALUE AND RESPECTIVE PROBABILITY
217,23

ENTER VALUE AND RESPECTIVE PROBABILITY
225,30

ENTER VALUE AND RESPECTIVE PROBABILITY
230,27

TO ENTER VALUE OF
COST OF RECOMPLETION ($/WELL)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
1

ENTER THE TWO EXTREME VALUES, 'A,B'
THE LOWEST FIRST AND THEN THE HIGHEST,SEPARATED BY COMMAS
600000,850000

TO ENTER VALUE OF
OPERATING COSTS  ($/BL)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
1
```

```
ENTER THE TWO EXTREME VALUES, 'A,B'
THE LOWEST FIRST AND THEN THE HIGHEST,SEPARATED BY COMMAS
1.5,3.8

TO ENTER VALUE OF
COST ESCALATION FACTOR (%)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
2

ENTER THE THREE VALUES 'A,B,C'
FIRST ENTER THE LOWEST, SECOND THE MOST LIKELY
AND THIRD ENTER THE HIGHEST, SEPARATED BY COMMAS
6,9,12

TO ENTER VALUE OF
DECLINATION OF PRODUCTION, (%/YEAR)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
4

ENTER THE ESTIMATED VALUE
4

TO ENTER VALUE OF
PRICE ESCALATION FACTOR (%)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
2

ENTER THE THREE VALUES 'A,B,C'
FIRST ENTER THE LOWEST, SECOND THE MOST LIKELY
AND THIRD ENTER THE HIGHEST, SEPARATED BY COMMAS
8,10,15

TO ENTER VALUE OF
DEPTH OF PRODUCTIVE ZONE (FT)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
1

ENTER THE TWO EXTREME VALUES, 'A,B'
THE LOWEST FIRST AND THEN THE HIGHEST,SEPARATED BY COMMAS
9000,10000

TO ENTER VALUE OF
COST OF SEISMIC EXPLORATION ($)

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
4
```

```
ENTER THE ESTIMATED VALUE
70000

WHAT PRODUCTION PER WELL PER DAY DO YOU EXPECT?
TO ENTER THIS PARAMETER


TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
2

ENTER THE THREE VALUES 'A,B,C'
FIRST ENTER THE LOWEST, SECOND THE MOST LIKELY
AND THIRD ENTER THE HIGHEST, SEPARATED BY COMMAS
630,750,900

HOW MANY TEST HOLES ARE NECESSARY TO SET UP
THE PRODUCTION PROGRAM?
TO ENTER THIS PARAMETER


TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
4

ENTER THE ESTIMATED VALUE
25

TO ENTER THE NUMBER OF DEVELOPMENT WELLS
DRILLED PER YEAR TO DEVELOP THE DEPOSIT

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
4

ENTER THE ESTIMATED VALUE
6

HOW MANY YEARS ARE NECESSARY TO DEVELOP
THE DEPOSIT?, THAT IS, HOW MANY YEARS YOU NEED TO
DRILL THE TOTAL OF DEVELOPMENT WELLS?
TO ENTER THIS PARAMETER

TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
1

ENTER THE TWO EXTREME VALUES, 'A,B'
THE LOWEST FIRST AND THEN THE HIGHEST,SEPARATED BY COMMAS
6,9

WHAT PERCENTAGE OF THE DEVELOPMENT WELLS
DO YOU EXPECT TO BE DRY HOLES?
TO ENTER THIS PARAMETER


TYPE '1' IF DISTRIBUTION IS UNIFORM
TYPE '2' IF DISTRIBUTION IS TRIANGULAR
```

```
TYPE '3' IF DISTRIBUTION IS DISCRETE
TYPE '4' IF ONLY ONE VALUE IS ESTIMATED
2

ENTER THE THREE VALUES 'A,B,C'
FIRST ENTER THE LOWEST, SECOND THE MOST LIKELY
AND THIRD ENTER THE HIGHEST, SEPARATED BY COMMAS
17,20,24


    THE FOLLOWING ASUMPTIONS ARE CONSIDERED
    IN THIS PROGRAM:

1.- ALL TEST HOLES WILL BE DRILLED DURING THE FIRST
    YEAR OF THE PROJECT

2.- WELLS WILL PRODUCE IN THE FOLLOWING YEAR AFTER THEY
    ARE COMPLETED

3.- WELLS WILL BE RECOMPLETED EACH FOUR YEARS
    AFTER PRODUCTION
```

A.2 List of symbols used within the computer program

The list given below does not include all the variables used and generated in the computer program. However the procedure followed by the program can be understood with the symbols that will be given. They are the key variables for the model.

Var.	Meaning
ABU(J)	Investment tax credit in year J (if applicable).
ALB(J)	Depreciation in year J.
AREA	Productive area in acres
ATIC(J)	After tax interest cost in year J (if applicable).
ATSV(J)	After tax salvage value in year J.
AVP(JJ)	Average rate of production of oil calculated in trial JJ in bls/day.
AVG(II)	Average rate of production of gas calculated in trial II in C.F./day.
BOF	Oil volume factor in bls at reservoir conditions per bls at normal conditions.
CDW	Cost of development well in \$/Foot.
CES	Cost escalation factor per year.
CFLOW(J)	Net cash flow in year J.
CHAR(J)	Drilling costs in year J.
CIR(J)	Intangible costs in year J

CLA	Cost of land acquisition of mineral rights.
CPS	Cost of dry-hole in \$/Foot.
CRW	Cost of recompleted well in \$/well.
CSE	Cost of seismic exploration.
CTH	Cost of exploration test hole in \$/hole.
DCFROR(NN)	Discounted cash flow rate of return calculated in trial NN.
DEPTH	Depth of productive zone in feet.
DESV	Standard deviation.
DHN	Number of dry holes as % of development wells.
DIST	Subroutine to determine the probability distribution for the solution variables.
DN	Values of a stochastic parameter with discreet distribution.
DOWL	Lower limit of the confidence interval for the mean of the solutions
DPR	Declination of production per year.
DWN	Number of development wells drilled per year.
EPD	Indicator of elegibility for percent depletion.
EQC	Cost of drilling and recompletion equipment.
ESP	Net pay thickness of the productive zone in feet.
FBOO	Final book value of depreciable assets.
FCW(I)	Cumulative probability of class boundary I.
FIDE(J)	Depletion allowed in year J.

GAN(J)	Net profit after tax in year J.
GOR	Gas oil ratio in CF/bl.
ICW	Length of the class boundaries.
IDA	Indicator for the type of probability distribution assigned to a stochastic parameter.
IFW(I)	Frequency of the class boundary I.
ITC	Indicator for the application of investment tax credit.
KD	Indicator for the kind of depreciation used.
LBM	Indicator for financial loan on the year zero capital costs.
LECT	Subroutine used to read stochastic data.
LEON(J)	Revenues after royalties and operating costs.
LI(I)	Lower limit of the class boundary I.
LIFE(LL)	Life of the project calculated in trial LL.
LS(I)	Upper limit of the class boundary I.
MED	Mean of the solution variable.
MROR	Minimum discount rate of return.
NC	Number of class boundaries for the solution distributions.
NEQC	Number of drilling and recompletion equipment.
NT	Number of simulation trials.
NY	Number of values that a stochastic parameter with a discreet distribution can take.

NYD	Depreciation life of equipment.
OC	Operating costs in $/bl.
PD	Single probability of a value of a stochastic parameter with discreet distribution.
PDE	Percentage depletion.
PES	Price escalation factor per year.
PGW(J)	Production of gas in year J in CF/year.
PIR	Annual interest rate.
PLI	Percentage of year zero investment financed on loan.
PLP(LK)	Average of principal annual mortgage payments calculated in trial LK.
POR	Porosity of the productive zone.
PPW(J)	Production of oil in year J.
PRG	Price of gas in $/CF.
PRO	Price of oil in $/bl.
PROD	Rate of production of oil in barrels per well per day.
PROB(I)	Probability of occurrence of class boundary I.
PTC	Percentage of tangible costs.
PVAL(MM)	Net present value calculated in trial MM.
RAIL	Subroutine used to calculate the standard deviation and mean of the solution parameters.
RAN	Function used in the generation of random numbers.

RAND	Generator of random numbers.
REF	Recovery factor of oil.
RNPVC(NM)	Net present value-present capital costs ratio calculated in trial NM.
ROY	Royalties.
RR(K)	Recoverable reserves calculated in trial K.
SAG	Gas saturation.
SALVA	Salvage value of the equipment.
SAW	Water saturation.
SAV	Income before depletion in year J.
SELECT	Subroutine used to make selection at random of the values of the stochastic parameters.
SETRAN	Subroutine used in the generation of random numbers.
TAX	Income tax rate.
TAXI(J)	Taxable income in year J.
TIME	Subroutine used to start the generation of random numbers by using the time of the day at which the program is run.
THN	Number of exploration test holes.
U1	Minimum value of the solution parameter.
U2	Maximum value of the solution parameter.
UPPL	Upper limit of the confidence interval for the mean of the solution.
VAL	Value selected at random of the stochastic parameters.

VAR	A single value given for a stochastic parameter.
VAR1	Lower limit value of a stochastic parameter with a continuous uniform or triangular distribution.
VAR2	Upper limit of a uniform probabilistic distribution or most probable value of a triangular probabilistic distribution.
VAR3	Upper limit of a triangular probabilistic distribution.
WRIST	Subroutine used to write stochastic input data.
YDL	Number of years to drill the development wells.
YFL	Random number generated by RAN.
YSA(J)	Cost of recompleting wells in the year J.

APPENDIX B

PROBABILITY THEORY OF THE MONTE CARLO SIMULATION

B.1 Glossary of Probability Concepts

Continuous triangular probability distribution. A distribution used when the upper and lower limit as well as a most likely value of a stochastic parameter can be specified.

Continuous uniform probability distribution. A distribution used when the upper and lower limits of the range of a variable can be specified and when any of the values between these limits is as likely to occur as any other value.

Cumulative probability distribution function. A mathematical expression that defines the probability that a random variable has a value less than or equal to a specified value. The mathematical expression is obtained by integrating the probability density function or by summing the probability function of the random variable.

Deterministic variable. Variable for which its exact value is known.

Discrete Probability distribution. A kind of distribution that is not continuous; rather only particular values can occur.

Expected value. The sum of the products of the values and respective probabilities for all possible values of a discrete random variable or the integral of the product of the variable and its probability density function over the range of values for a continuous random variable. The expected value is also known as the mean of the random variable.

Mode. The single most likely value of the stochastic variable.

Probability density function. A mathematical expression defining an area of 1.0 over the range of a continuous stochastic variable. Probabilities for intervals of a continuous stochastic variable can be obtained by integrating the probability density function.

Probability function. A mathematical expression defining a probability for each value of a discrete stochastic variable.

Probability model. A mathematical representation of the relationships between the random variables associated with a set of events.

Random variable. A variable whose value is a function of chance. Also known as stochastic variable of probabilistic variable.

Standard deviation. A measure of dispersion. It gives a numerical value, in the units of the variable itself, to the clustering tendency of the data.

95% Confidence interval. A probability statement that specifies that out of 100 calculations of a statistical parameter of a sampled data 95 indicate that the säme statistical parameter of the population will be within a specified range.

B.2 Random numbers Generation

The RAN function is a widely used generator of random numbers.

There are three entry points to the RAN function. An entry at RAN, an entry at SAVRAN which is a function that sets its argument to the last random number that has been generated by the function RAN, and an entry at SETRAN which is a function that provides the random number generator with a non-standard starting point.

RAN is 32 locations long, of which 23 are executable instructions. The argument given a call to RAN is not used by RAN. SETRAN expects an integer argument in the range $0 \leq arg < 2^{31}-1$. The result of a call to RAN is returned in accumulator Ø as a normalized floating-point number in the range $0 < X < 1$. The result of a call to SAVRAN is returned in accumulator Ø and the specified location as an integer in the range $0 < X < 2^{31}$.

The algorithm used by the RAN program is as follows

$$X_{n+1} \equiv KX_n \pmod m \qquad (1)$$

The values for K and m suggested by D.H. Lehmer and used in this program are

$m = 2^{31} - 1$

$K = 14^{29}$

$14^{29} \pmod{2^{31} - 1} = 630360016_{10}$

to avoid the division by $2^{31} - 1$ the following algorithm is used with $p = 31$

$$X_{n+1} \equiv KX_n \pmod{2^p} \qquad (2)$$

or, by definition

$$X_{n+1} = KX_n - r2^p \qquad (3)$$

for some r. Adding r to both sides of equation (3) yields

$$X^1_{n+1} = X_{n+1}{}^{+r} = KX_n \pmod{2^p - 1} \qquad (4)$$

provided that

$$X_{n+1}{}^{+r} < 2^p - 1 \quad \text{(no overflow)}$$

if

$$X_{n+1}{}^{+r} > 2^p - 1 \quad \text{(overflow)}$$

then

$$X^1{}_{n+1} = X_{n+1}{}^{+r+1} \pmod{2^p} \qquad (5)$$

which leds directly to

$$X^1{}_{n+1} = KX_n \pmod{2^p - 1} \qquad (6)$$

If the generated random number is greater than 27 bits, it is truncated to 27 bits for calls to RAN.

B.3 <u>Selection at random of the values of the stochastic variable</u>.

The selection at random of a value from the probability distribution of a stochastic variable is made by using a random number between zero and one, and the cumulative probability distribution function of the stochastic parameter, as indicated in Fig. B.1.

Three types of distributions are the most used in solving economic problems, continuous triangular, continuous uniform and discret.

The probability density function for the continuous triangular distribution is given by

$$f(X) = \begin{cases} \dfrac{2(X-X_L)}{(X_H - X_L)(X_M - X_L)} & ; \; X_L \leqslant X \leqslant X_M \\[2ex] \dfrac{2(X-X\ \ )}{(X_H - X_L)(X_M - X_H)} & ; \; X_M \leqslant X \leqslant X_H \end{cases} \qquad (1)$$

where

X_L is the lower limit of the range of X

X_M is the most probable value of X

X_H is the upper limit of the range of X

when $X_L \leqslant X \leqslant X_M$ the cumulative probability of X is given by

$$F(X) = \left(\frac{X-X_L}{X_M - X_L}\right)^2 \left(\frac{X_M - X_L}{X_H - X_L}\right) \qquad (2)$$

when $X_M \leqslant X \leqslant X_H$ the cumulative probability of X is given by

$$F(X) = 1 - \left(\frac{X_H - X}{X_H - X_M}\right)^2 \left(\frac{X_H - X_M}{X_H - X_L}\right) \qquad (3)$$

The random number generated is set equal to F(X); replacing F(X) with R_S we have:

if $\quad R_S \leqslant (X_M - X_L)/(X_H - X_L)$

$$X_S = X_L + \left[(X_M - X_L)(X_H - X_L)R_S\right]^{\frac{1}{2}} \qquad (4)$$

if $\quad R_S > (X_M - X_L)/(X_H - X_L)$

$$X_S = X_H - \left[(X_H - X_M)(X_H - X_L)(1 - R_S)\right]^{\frac{1}{2}} \qquad (5)$$

where R_S is the random number generated and

X_S is the value of X selected at random.

The graphical representation of this procedure is given in Fig. B.1.a.

The probability density function of a continuous uniform distribution is given by

$$f(X) = \frac{1}{(X_H - X_L)} \; ; \; X_L \leq X \leq X_H \qquad (6)$$

where X_H is the upper limit of the range of X

X_L is the lower limit of the range of X

The cumulative probability of X is given by

$$F(X) = \frac{X - X_L}{X_H - X_L} \qquad (7)$$

The random number generated is set equal to F(X); replacing F(X) with R_S, the uniformly distributed random number and solving for X gives

$$X_S = X_L + R_S\,(X_H - X_L) \qquad (8)$$

where

X_S is the value selected at random

Fig. B.1.b presents graphically the selection at random of a value of a stochastic variable uniformly distributed.

For a discret distributed random variable X can take single particular values. This is illustrated in Fig. B.1.c. In this case X can take values corresponding to X_1, X_2, X_3,.. ...X_n, but it can have no other values beyond or between these specific numerical values.

The manner of selecting a value of the variable X for its discret distribution is indicated in Fig. B.1.c. R_s is of course a uniformly distributed random number with a range zero to one. Also it is understood that the sum of the individual probabilities is one. To select the value of the stochastic variable proceed as follows:

make $F(X) = P(X_1)$

if $F(X) = R_s$ then $X_s = X_1$, if not

make $F(X) = P(X_1) + P(X_2)$

if $F(X) = R_s$ then $X_s = X_2$, if not

make $F(X) = P(X_1) + P(X_2) + \ldots + P(X_n)$

until $F(X)$ is equal to the random number generated.

Note that $F(X)$ is the cumulative discret probability distribution.

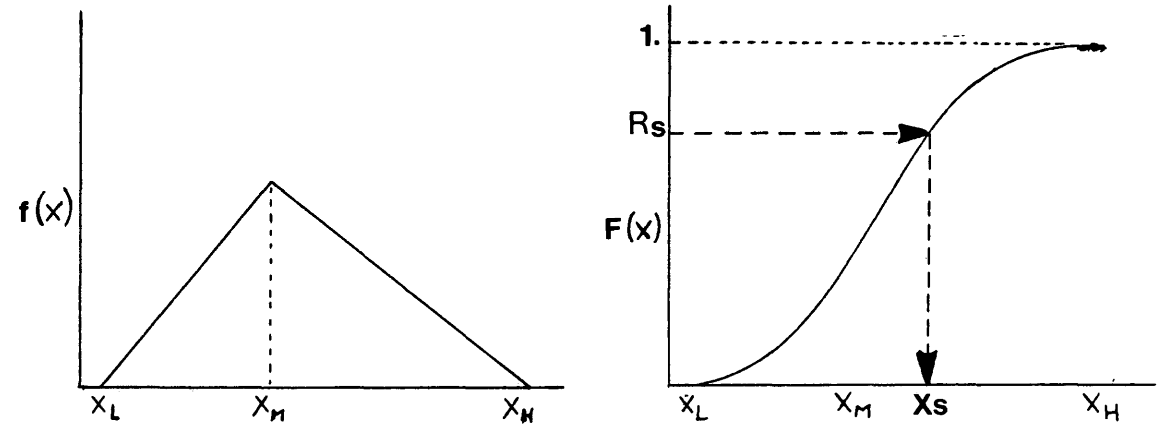

Fig. B.1.a Random Selection from Triangular Distribution

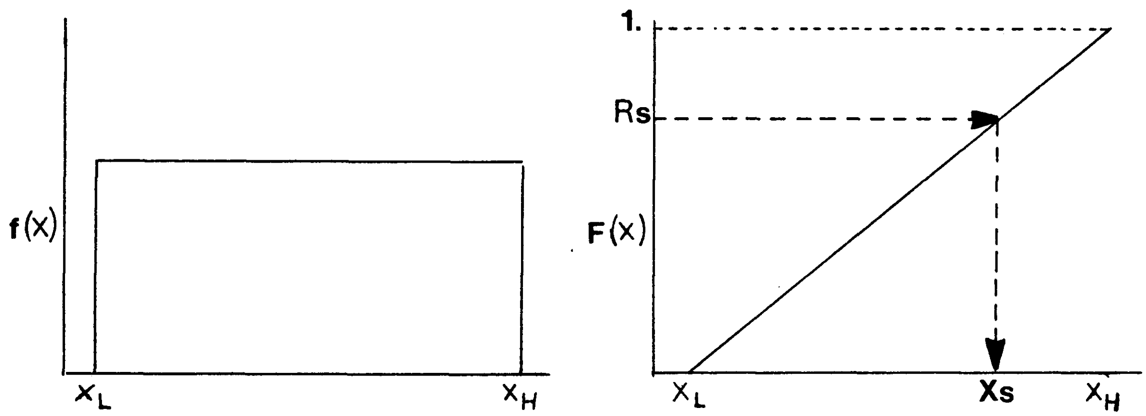

Fig. B.1.b Random Selection from Uniform Distribution

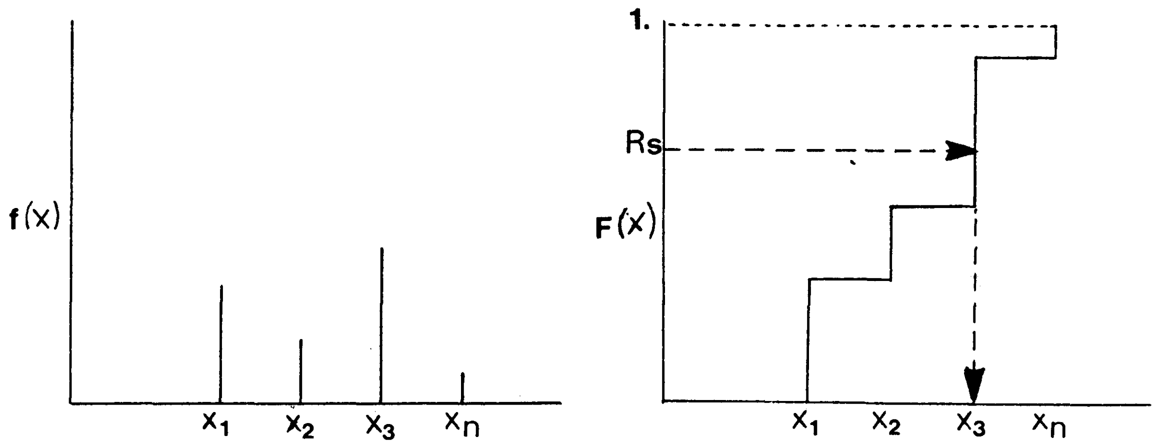

Fig. B.1.c Random. Selection from Discreet Distribution

Fig. B.1 Random Selection from Probability Distributions