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THE EFFECT OF THE LINDE TREATMENT ON RESIDUAL STRESSES AND BRITTLE FRACTURE IN BUTT-WELDED PIPES

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ABSTRACT

This report describes progress to date on a long-range project to develop the Linde treatment as a method for reducing the residual stresses in butt-welds in pipe, thereby avoiding brittle fracture. The equipment development, the experimental-technique-development and the analytical-technique-development have been largely completed. Pipes have been satisfactorily butt-welded and methods of measuring residual stresses in and near these butt-welds have been developed. Equipment for applying the Linde treatment and equipment for fracture-testing the butt-welded pipe have been completed and shown to function properly.

A numerical technique for determining temperature distributions has been applied to Linde treatment-type boundary conditions and Tall's method of determining residual stress in welds has been applied to the Linde treatment. A preliminary elastic analysis was conducted to show how much less restraint exists against contraction of localized hot zones in a pipe than in a plate.

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INTRODUCTION

The fact that low-strength brittle-failure is a serious problem has been the subject of many investigations. Rosenstein has an excellent summary of the literature (Rosenstein, 1966).

The designers have been interested in the effect on brittle failure of metallurgical damage and residual stresses because, of all the factors involved, these two can most easily be altered. It is hard to evaluate these factors individually because they invariably occur together; therefore the separation of these two factors has been a subject of many investigations on welded-structures (Rosenstein, 1966). Greene and Holzbaur showed that a "controlled low-temperature stress-relieving" heat treatment (the "Linde treatment") reduced welding residual stresses to zero and even compression (Greene and Holzbaur, 1946). Greene also produced low-stress brittle-failure in non-Linde-treated welded plates and not in Linde treated plates by bending them, but this method is not as straightforward as fracturing the plates in a tensile test (Greene, 1949). Kennedy showed in tensile tests on notched and welded plates of the same size that the Linde treatment did not conclusively improve brittle failure behavior; but he did not measure the residual stresses after the Linde treatment (Kennedy, 1957). If the stresses had not been reduced to zero, as in Greene's specimens, then the

effect on brittle failure behavior of the full stress reduction could not be observed.

Since large-plate tensile tests were inconvenient and expensive, it was thought that the low-stress brittle-failure caused by the loading of the weld and notch in Greene's plate specimen could be produced in an internally-pressurized butt-welded pipe of the same thickness (Furman, 1967). With the available equipment it was impossible to produce on the pipe a double-V weld like the one on Greene's plate specimen; therefore a single-V weld was used. When pressurized, the circumferential stresses at the inner surface of the 18-inch O. D. by 16-inch I. D. pipe were 13% higher than those produced at the outer surface. The pressurized welded pipe, therefore, did not simulate pure tensile loading nor did it contain the same notch geometry as in a tensile plate specimen. However the butt-welded pipe did provide a simple means of application of the Linde treatment to the weld since all that was needed was to rotate the pipe under the torch.

The determination of the effect of the Linde treatment on the brittle failure of butt-welded pipe should proceed as follows:

1) for a given torch configuration and speed find the transient temperature distribution in a mathematical model of the pipe.
2) measure the transient temperature distribution in a physical specimen and check the results with the predictions of step 1.
3) from this transient temperature distribution calculate the residual stresses produced upon cooling the mathematical model (in welding and in the Linde treatment).
4) measure the residual stress distribution before and after the

Linde treatment in a welded specimen and check the results with the predictions of step 3.

5) run a fracture-strength test on as-welded specimens and on Linde-treated specimens and show improvement in fracture strength.

6) show any effects of the Linde treatment on the metallurgical damage produced by welding and on the notch acuity, and show how these effects might be controlled.

This thesis lays the groundwork for future investigations into the effect of the Linde treatment on residual stresses for various torch configurations. A numerical method of transient temperature determination in a plate is developed and compared with an algebraic solution. Surface temperature measurements were made on a pipe heated with a Linde torch. An algebraic method of calculating the circumferential elastic stresses produced in a pipe heated non-uniformly along the axis was made to show how the pipe behaves under an axial temperature gradient. Since the residual circumferential stresses in the pipe are produced upon cooling after plastic straining has occurred, a step by step numerical method of stress calculation must be used. The shape of the circumferential residual stress distribution measured in the as-welded pipe was the same as that measured in as-welded plate specimens of comparable dimensions.

Just what affect the Linde treatment had on metallurgical damage in the notch-root vicinity, if any, is not clear. Greene showed with Charpy impact specimens what appeared to be a definite improvement in the brittle behaviour of material in the heat-affected-zone (near the notch root) after the Linde treatment. But since the results were somewhat scattered and the tests limited in number, a

definite conclusion was not justified (Greene, 1949). Jonassen claims that the amount of plastic flow caused by the Linde treatment at the notch root may be sufficient to cause a significant reduction in severity of this stress-raiser and thereby improve the low temperature performance of the specimen (Jonassen, 1949). But the beneficial effect of reduction in notch acuity may be partially counteracted by a loss of ductility resulting from the plastic flow at the notch root. These two problems might be avoided by spacing the torches and cooling the region between them so that the heat-affected-zone is not heated and the notch not affected; and by cutting the notch near the weld after the heat treatment, as did Kihara and Masabuchi after welding to avoid the same problem (Kihara and Masabuchi, 1959).

Even though the notched and as-welded pipe did not fail before it yielded when pressurized at -40C (Furman, 1967), the equipment developed for the test performed satisfactorily. If as-welded pipes could be made to exhibit low-strength fracture by increasing the severity of the conditions then this equipment could be used to show the improvement due to the Linde treatment.

EQUIPMENT DEVELOPMENT

The equipment developed and built for this project performed very well. The available shop equipment was adapted to cut the large diameter pipe into 18-inch lengths in preparation for welding and to cut the welded pipe into residual-stress-measurement rings. Notches were cut into the weld preparation as shown in fig. 1. The automatic welding machine used by Rosenstein to butt-weld plates (Rosenstein, 1966) was adapted to butt-weld pipe, fig. 2. The residual stresses in the butt-welded pipe were determined by measuring the changes in circumference of rings cut out of the pipe. A torch similar to the one Greene used in his original investigation (Greene and Holzbaur, 1946) was made and checked. Pressure testing equipment was built and tested. This consisted of the welded pipe, two end plates, the bolts to hold the end plates together, a core to reduce the volume of compressed methanol, a high pressure pump, and end seals. These seals permitted the end plates to move axially, the pipe to contract axially and the pipe to move radially without leaking when the vessel was pressurized. A refrigeration system consisting of a compressor, a heat exchanger, a pump to circulate the methanol coolant and a cooling tank to contain the pressure equipment was made and tested. For a description of the pressure equipment and cooling system see (Furman, 1967). Experiments showed that with this equipment 18-inch O.D. by 16-inch I.D. pipe could be pressurized to 40,000 psi circumferential

stress at -40C.

Welding Procedure

In the butt-welding of Rosenstein's plates the Aircomatic welding-head was mounted on a motor-driven carriage designed for carrying the head on a straight path past the work (the Radiagraph) which travelled on a smooth track. The pipe was butt-welded by keeping the welding head stationary and rotating the pipe under the welding head on ball-mill rollers, chain driven by a sprocket which replaced the drive wheel on the Radiagraph, fig. 2. The pipe was grounded through two channel beams clamped on with a bolt, which, in turn, was grounded through a friction contact to the welding cable, fig. 2. The remaining welding equipment was not changed. The conditions of welding are shown in table 1. The weld beads did not fill the notch (fig. 1) as was indicated by examination of a notched section cut out after welding.

Measurement of Residual Stresses

The circumferential residual stresses in a butt-welded pipe were found by measuring the change of the circumference of the rings into which the pipe was dissected. The change in circumference was measured by a tape as shown in fig. 3. This circumferential deflection divided by the original mid-thickness circumference is the average elastic strain released when the ring is freed from the pipe. The average residual stress that would shrink the ring back to its original diameter in the pipe is this elastic strain multiplied by the modulus of elasticity of the material. The circumferential deflection at the surface of the pipe is the difference in the mean readings of an average of six micrometer measurements across the blocks, as indicated in fig. 3, before and after each ring was

cut out. The measured difference at the block corners is the same as that at the outer surface, which is also the same through the wall thickness (see Appendix II). The pipe can be considered thin-walled so that the neutral axis remains at mid-thickness when the diameter changes. The standard deviation of the mean readings was about 0.002 inch for all measurements. The maximum error in the deflection would therefore be about $\pm$ 0.004 inch, which corresponds to a stress measurement error of $\pm$ 2100 psi.

If the pipe were cut into infinitesimally thin rings, then the circumferential residual stress in each parted off ring would be the average residual stress (averaged over the radial thickness) for a point location along the pipe axis. We shall refer to this stress as the "radially averaged stress" (averaged over the wall thickness). "Radially averaged stress" is a single-valued function of axial location. Since finite width rings are cut, a stress gradient exists across the axial width of the ring in addition to the radial variation through the thickness, fig. 4. In regions of the pipe where the slope of this stress distribution changes rapidly along the axis, narrow rings should be cut so that the distribution across the ring width is nearly linear. The measured residual stress will be the "radially averaged stress" at midwidth of the ring, fig. 4, left-hand ring. If the stress distribution across the axial width of the ring is not linear, then the measured residual stress is still the axial average of the "radially averaged stress", but its location across the ring width will be unknown, fig. 4, right-hand ring.

The dissection scheme in fig. 5 was used for the butt-welded pipe because published residual stress measurements on butt-welded plates showed that the stress gradient changes most rapidly near the weld. The shape of the measured

residual stress distribution in the butt-welded pipe was very similar to distributions measured in butt-welded plates of the same thickness.

In order to compare the residual stress distribution through the thickness of a single-V weld with that measured in a double-V weld (Nordell and Hall, 1965), a ring 1.5-inches wide containing the weld was split and the opening at midthickness (D) was measured, fig. 6.

Before the ring was split the circumference at the inner surface was $2\pi R_o$ and the circumference at the midthickness was $2\pi(R_o + t/2)$.
After splitting, the mid-thickness radius was

$$\frac{2\pi(R_o + t/2) + D}{2\pi} = R_o + D/2\pi + t/2.$$

The final radius at the inner surface was $R_o + D/2\pi$.
The separation distance at the inner surface was

$$D' = D \frac{R_o + D/2\pi}{R_o + D/2\pi + t/2.}$$

The final length of an inner fiber of the ring was

$$2\pi(R_o + D/2\pi) - D' = 2\pi R_o + D - D'.$$

The released elastic strain at the inner surface was

$$e_{I.D.} = \frac{(2\pi R_o + D - D') - 2\pi R_o}{2\pi R_o} = \frac{1}{2\pi R_o}(D - D')$$

Substituting for D' and simplifying

$$e_{I.D.} = (Dt/2)/(2\pi R_o^2 + \pi R_o t + DR_o)$$

Since $D = 1.056$ inch

$t = 0.948$ inch

$R_o = 8.062$ inch

then $e_{I.D.} = 0.495/(408 + 24 + 9)$

$e_{I.D.} = 0.00112$

The compressive stress at the inner surface and the tensile stress at the outer surface that would cause this ring to open when split (the stress distribution is symmetrical because the neutral axis remains at the mid-thickness) is calculated from this released elastic strain:

$$S' = e_{I.D.} E$$

$$S' = 0.00112(30)(10)^6$$

$$S' = 33,600 \text{ psi}$$

The residual stresses released at the outer surface, the mid-thickness and the inner surface of the ring containing the weld after it was cut from the pipe were calculated from the measured outer surface circumference change of 0.056 inches.

(This value was the same as that measured at the block corners as previously discussed):

$$S_{I.D.} = \frac{0.056E}{2\pi(8.063)} = 33,200 \text{ psi}$$

$$S_{M.T.} = \frac{0.056E}{2\pi(8.532)} = 31,300 \text{ psi}$$

$$S_{O.D.} = \frac{0.056E}{2\pi(9.001)} = 29,700 \text{ psi}$$

These stresses when added to the linear stress distribution gives a residual tensile stress in the outer fibers of the weld of 63,300 psi and a residual compressive stress at the bottom of the single-V weld of 400 psi, fig. 7.

Linde Treatment

The purpose of the Linde treatment is to relieve welding stresses by heating two regions, one on each side of the weld, so that when they expand under partial restraint the weld will flow plastically (because it is already under yield-point residual tensile stresses). When the heated regions cool and contract the weld will contract elastically (compatibility has to be satisfied) and relieve its tensile stresses (Green and Holzbaur, 1946).

If, in a butt-welded plate of finite size all of the plate excluding the weld was heated, then the total strain in the weld will be equal to the difference in the thermal strains in the plate and in the weld (compatibility requires that the weld remain attached to the plate and the weld negligibly restrains the expanding plate). If, in a butt-welded plate of infinite size, two finite regions were heated near the

weld, then the weld would not be strained at all, because the heated regions would be prevented from expanding by the complete restraint of the remaining infinite plate. The tensile strain in the weld of a finite Linde-treated plate will be equal to the difference between the expansion of the hot zones and the thermal strain of the weld. The tensile strain in the weld can be no larger than the total strain permitted in the hot zone by the surrounding plate. If, in a butt-welded plate the partially restrained hot zones are too far from the weld, then their expansion will be fully restrained by the large amount of metal between them, which includes the unstrained weld. If the hot zones are too close together, then the weld becomes very difficult to keep cool and the difference in thermal strain between the hot zone and the weld, which causes the tensile straining, will be decreased or may even cause compression.

Greene and Holzbaur studied the effect of varying the torch widths, varying the distance of the torch from the weld and varying the hot zone temperature (by changing the travel speed) on residual tensile stresses in butt-welded plates. They showed that the residual stresses in the weld could be reduced to zero by using a six-inch-wide torch with its centerline five inches from the weld centerline. A heat-treating temperature of 350F was reached in the heated region by moving the torch at a rate of sixteen inches-per-minute. The weld was cooled with a water spray between the torches. A water spray was located about six inches behind each torch to remove surplus heat, localize the application to that portion of the weld being treated, prevent any possible expansion, buckling or distortion of the plate and, most importantly, cause a contraction behind the heated zone which would reduce the restraint on this heated zone.

Apparatus was built for using the Linde treatment to relieve the welding stresses in a butt-welded pipe. An oxy-acetylene torch was made to the proper dimensions and when fired on a rotating pipe it functioned properly. The accompanying quenching-spray was made and it also functioned properly. The combination torch and quenching-spray is shown in fig. 8-B.

The torch was a 3/8-inch copper tube threaded to screw into an oxy-acetylene mixing head, fig. 8-A. The nozzles were punched with a needle to the proper depth to produce a 0.020-inch hole, fig. 8-C.

Temperature measurements were made, on the inner and outer surfaces of a pipe rotating past the Linde torches, to determine what Greene meant by "heat-treating temperature". When the pipe was rotated at ten inches-per-minute, the inside surface reached a maximum of 350F about 1.5 inches behind the torch (lines were drawn in the direction of rotation with crayons indicating temperatures of from 150 F to 400 F), fig. 9-A. When the pipe was rotated at sixteen inches-per-minute, the inside surface of the pipe never exceeded 250 F, fig. 9-B. When the pipe was rotated at ten inches-per-minute, the temperature on the outer surface of the pipe exceeded 400 F inside a five-inch long parabolic isotherm following the torch, fig. 9-A. At sixteen inches-per-minute the 400 F isotherm was less than one inch behind the torch, fig. 9-B.

From these experiments it was concluded that the pipe reached an average temperature of 350 F when rotated at sixteen inches-per-minute, while it reached a minumum of 350 F at a rate of ten inches-per-minute. Therefore, when Greene refers to a heat-treating temperature of 350 F at a torch travel rate of sixteen inches-per-minute, he means the average plate temperature.

ANALYTICAL-TECHNIQUE DEVELOPMENT

Numerical Method of Finding Temperature Distribution

Rosenthal has developed a theory of heat flow due to a moving source which gives results that agree well with experiments involving simple heat-source geometries (Rosenthal, 1946). The mathematics involved in this theory prevents an easy application to a specific heat-flow model. Therefore, a numerical method of finding the transient temperature distribution through the plate thickness and parallel to the plate surfaces due to the Linde torches was made.

Considering heat flow only in a plane perpendicular to the weld centerline, a heat balance is made on an element and its four surrounding elements of a grid superposed on the plane, fig. 10. Equating the sum of the amounts of heat conducted from the surrounding elements to the center element with that heat stored in the center element during a time interval, Δt,

$$Q_{1-0} + Q_{2-0} + Q_{3-0} + Q_{4-0} = \frac{Q_o}{\Delta t},$$

where $$Q_{1-0} = k(\text{area})\frac{T_1 - T_o}{\Delta x}$$ (Fourier's Law for finite differences)

(Q_{2-0}, Q_{3-0}, Q_{4-0} are similar)

and $$Q_o = pC_p \text{ (volume)}(T_{o, \Delta t} - T_o).$$

Substituting,

$$k(\Delta x)^2 \frac{(T_1 - T_o)}{\Delta x} + k(\Delta x)^2 \frac{(T_2 - T_o)}{\Delta x} + k(\Delta x)^2 \frac{(T_3 - T_o)}{\Delta x} +$$

$$k(\Delta x)^2 \frac{(T_4 - T_o)}{\Delta x} = pC_p(\Delta x)^3 \frac{(T_{o, \Delta t} - T_o)}{\Delta t}$$

Solving for the temperature at the end of the time interval,

$$T_{o, \Delta t} = \frac{T_1 + T_2 + T_3 + T_4 + (M - 4)T_o}{M}$$

where $$1/M = \frac{k \Delta t}{(\Delta x)^2 pC_p}$$

The quantity M can have any positive value. For a given grid size (Δx) the time intervals become very small for large values of M and the complete solution becomes more accurate, but it also becomes too long. For values of M less than 4 the solution becomes mathematically unstable. A value of M = 4 was chosen in order to maintain stability and keep the solution as short as possible. Therefore,

$$T_{o, \Delta t} = \frac{T_1 + T_2 + T_3 + T_4}{4}$$

The time interval between successive temperatures at a point in the grid is determined by the grid size (Δx) alone, because p, C_p and k are specified. For the chosen grid size ($\Delta x = 0.2$ inch) and assuming p, C_p and k constant with temperature the time interval for mild steel is constant at 0.58 seconds.

The boundry conditions assumed during heating were as indicated in fig. 10. The 2000F torch boundary in the same figure was replaced by 100 F during cooling. Since the surface temperature under the torch during heating is presently unknown, an arbitrary value was used in order to illustrate the method and emphasize the heat flow perpendicular to the weld centerline and through the thickness of the plate. Fixing the other boundary temperatures means that heat losses through these surfaces would be larger than if the surfaces were allowed to warm up (steeper temperature gradient means larger heat losses). No heat losses are allowed perpendicular to the plane of the grid; this would be true if the torch was not a line source normal to the weld, but a plane source of heat finite in both directions in the plane of the plate. The grid straddled the boundary between the heated and un-heated surfaces so that the temperature gradient perpendicular to the weld centerline could be found simultaneously with the temperature gradient through the plate thickness.

The numerically-calculated temperature gradient through the plate thickness at the middle of the torch agreed very well with an analytical gradient for the same type of boundary conditions and after the same time interval (Carslaw and Jaeger, 1959), fig. 11. This is to be expected, because the heat flow in the plane of the plate is negligible at mid-torch width; and Carslaw and Jaeger's solution is for no heat flow in the plane of the plate (one-dimensional heat flow).

The numerically-calculated temperature gradient through the plate thickness upon cooling is shown in fig. 11-B. A sample calculation of a transient temperature distribution is shown in fig. 12.

For purposes of determining more realistically what boundary conditions should be used in the above calculations, the surface temperature variation with time was determined by an experiment in which a single-nozzle oxy-acetylene torch was held stationary for 25 seconds a half-inch from the outer surface of an 18-inch diameter, one-inch thick pipe. When the torch was removed the outer surface had reached 500 F and the inner surface 250 F (measured with temperature-indicating crayons). These surface temperature variations could be determined analytically if the heat losses were calculated from the heat transfer coefficients and fluid characteristics at both surfaces. Since heating from one side only gives a steep through-thickness temperature gradient, the effect of this gradient on residual stress calculations should be shown. The through-thickness temperature distribution produced by heating both surfaces and its effect on residual stress calculations should be compared with that of the single-surface heating.

Numerical Method Of Residual Stress Calculation

The residual stresses produced by a cooled weld in a plate can be calculated in the following manner (Tall, 1964):

1) Divide the plate into small increments wherever steep stress gradients are expected and into larger increments where less steep gradients are expected, fig. 13. Because the temperature distribution is symmetrical about the weld centerline, the calculations were made on one half of the plate, line 1 (all "lines"

refer to table 2).

2) Determine the maximum yield stress, line 3, for the temperatures at the end of the time interval, line 2.

3) Calculate the thermal stresses, $E\alpha T$ (plane strain), line 5, from the temperature change, line 4. The change of the material properties E and α are then found as functions of temperature.

4) Add these thermal stresses, line 5, to the stresses present in each longitudinal element at the beginning of the temperature interval, line 6.

5) Limit the stress to yield, line 7.

6) Sum the forces over the plate (on the plane perpendicular to the weld centerline) and divide the resultant force by the area upon which all the non-yielding forces act to get an "equilibrium stress", line 8.

7) Add the negative of this "equilibrium stress" to the sum of the initial and thermal stresses on each element, line 9, unless it would cause the stress to drop below yield.

8) Check force summation again, line 10. If the sum is near zero, then the stresses from step 7 will be used for the initial stresses on the next interval. If the sum is not near zero, then repeat steps 6, 7 and 8.

9) Continue this procedure until the entire plate has cooled to room temperature.

This method was applied to typical Linde-type treatments, one of which was two six-inch strips centered five inches from the weld center-line, heated by 500 F and cooled as shown in fig. 14. For simplicity, $E\alpha$ was considered independent of temperature. This method of calculation was used to show the effect of various torch widths upon tensile residual stress reduction in a butt-welded plate (see table 3). A modification of the method could be used also to show the effect on welded pipe. Table 3 lists the torch widths and other significant features for applying Tall's method to the Linde treatment. The welding stress distributions to which the various Linde treatments were applied were also found by Tall's method, fig. 15.

The torch application that most closely represented Greene's condition in his original investigation reduced the weld tensile stresses by 50%, but the region under the torches contained tensile stresses near the yield point of the plate, fig. 15-F. Greene measured tensile residual stresses in the region heated by the torches of only ten to twenty percent of the yield stress of the plate material. This difference between analytical and experimental results may be due to the fact that the wide torches require a different compatibility condition than does a weld. In his calculation of welding residual stresses, Tall assumes that transverse planes remain plane and perpendicular to the weld centerline for each temperature interval considered (Tall, 1964). This may not be true for the Linde treatment on a finite plate.

The Determination Of The Circumferential Stress Distribution In A Pipe Caused By A Non-linear Temperature Distribution Along The Axis

This derivation begins with the determination of the radial elastic displacement

in a pipe caused by an internal pressure distribution. When the differential equation relating radial displacement to internal pressure has been found, the solution will be modified to take account of temperature distribution. The equation will then be solved for radial displacement as a function of axial distance. Knowing the radial displacement function, we can find the circumferential stress distribution along the axis for a given temperature distribution. An infinitely-long thin-walled pipe will serve as a model for determining the integration constants in the complementary solution, but the total solution will apply to a finite length of pipe because the radial displacement is damped out within a short distance along the axis, fig. 19.

For the element of fig. 16, the summation of moments-per-circumferential-unit-length about point A, the summation of forces in the axial direction and the summation of forces in the radial direction must all equal zero. The moment, M_t, is zero along the circumference because the pipe remains round during internal pressurization. The radial stresses are zero because the pipe is considered as thin-walled. Equating summation of forces to zero,

$$\Sigma F_x = 0 = S_x R_o h d\theta - (S_x + \frac{dS_x}{dx} dx) R_o h d\theta$$

$$\frac{dS_x}{dx} R_o h dx d\theta = 0 \qquad (1)$$

$$\Sigma F_r = 0 = \tau_x R_o h d\theta - (\tau_x + \frac{d\tau_x}{dx} dx) R_o h d\theta -$$

$$2 S_t h dx \sin\frac{d\theta}{2} + Z(x) R_o d\theta dx$$

$$\frac{d\gamma_x}{dx} + \frac{S_t}{R_o} = \frac{Z(x)}{h} \qquad (2)$$

$$\Sigma M_A = 0 = M_x - (M_x + \frac{dM_x dx}{dx}) + \gamma_x h dx$$

$$\frac{dM_x}{dx} = h\gamma_x \qquad (3)$$

The average axial stress, S_x, is zero because the ends of the pipe are not loaded ($S_x = 0$ at $x = \infty$) and S_x does not change. If S_x is not zero the corresponding displacement can be added to the displacement caused by the internal pressure.

We now need to find S_t and M_x in terms of the radial displacement, w. First we apply Hooke's Law to find S_t as a function of the strains, then find the strains in terms of the radial displacement. From Hooke's Law

$$Ee_x = S_x - uS_t$$

$$Ee_t = S_t - uS_x$$

Solving for S_x and S_t

$$S_x = \frac{E}{1 - u^2}(e_x + ue_t) \qquad (4)$$

$$S_t = \frac{E}{1 - u^2} (e_t + ue_x) \tag{5}$$

The circumferential strain (see fig. 17) is

$$e_t = \frac{(R_o + w)d\theta - R_o d\theta}{R_o d\theta}$$

$$e_t = \frac{w}{R_o} \tag{6}$$

Since there is uniaxial circumferential loading, the circumferential stress is equal to the circumferential strain times the modulus of elasticity

$$S_t = \frac{Ew}{R_o} \tag{7}$$

Now consider a longitudinal element one unit wide along the circumference, fig. 18. The moment in the axial direction bends this element of the pipe into an arc of radius, p, to the neutral axis. This moment arises from an axial bending stress (not the average axial stress, which is zero) which is distributed linearly and symmetrically through the pipe wall. The moment is calculated and defined by integrating over the thickness, h, the product of the axial bending stress, S_x^* , its moment arm, y, and the area upon which the stress acts, (1) dy. From Hooke's Law

$$Ee_x^* = S_x^* - uS_t^*$$

$$Ee_t^* = S_t^* - uS_x^*$$

where e_x^* and S_x^* are the axial bending strain and stress and e_t^* and S_t^* are the circumferential bending strain and stress. Because an axial bending moment distributed uniformly around the circumference will not warp a pipe out of round, the circumferential bending strain, e_t^*, is zero, therefore

$$S_t^* = uS_x^*$$

Substituting and solving for S_x^*

$$S_x^* = \frac{Ee_x^*}{1 - u^2} \tag{8}$$

The axial bending strain (fig. 18) is

$$e_x^* = \frac{(p + y)d\phi - pd\phi}{pd\phi}$$

$$e_x^* = \frac{y}{p} \tag{9}$$

For small slopes the radius of curvature of bending is

$$\frac{1}{p} = \frac{d^2w}{dx^2}$$

Substituting for 1/p in eq. 9

$$e_x^* = \frac{d^2w}{dx^2}$$

Substituting for e_x^* in eq. 8

$$S_x^* = \frac{Ey}{1 - u^2} \left[\frac{d^2w}{dx^2}\right] \qquad (10)$$

Knowing the bending stress we can find the bending moment M_x

$$M_x = \int_{h/2}^{h/2} (S_x^*)(y)(1)dy$$

Substituting for S_x^* and integrating

$$M_x = \frac{Eh^3}{12(1 - u^2)} \left[\frac{d^2w}{dx^2}\right]$$

where $\frac{Eh^3}{12(1 - u^2)}$ is defined as the flexural rigidity of a shell, D, which is

similar to the flexural rigidity of a beam, EI. Therefore

$$M_x = D \frac{d^2w}{dx^2} \tag{11}$$

We now have the circumferential stress and the axial bending moment as functions of radial displacement. Now differentiating eq. 3

$$\gamma_x = \frac{1}{h} \frac{dM_x}{dx}$$

$$\frac{d\gamma_x}{dx} = \frac{1}{h} \frac{d^2M_x}{dx^2}$$

Substituting for $d\gamma_x/dx$ in eq. 2

$$\frac{d\gamma_x}{dx} + \frac{S_t}{R_o} = \frac{Z(x)}{h}$$

$$\frac{1}{h} \frac{d^2M_x}{dx^2} + \frac{S_t}{R_o} = \frac{Z(x)}{h}$$

Substituting for M_x and S_t and multiplying through by h/D

$$\frac{d^4w}{dx^4} + \frac{hEw}{DR_o^2} = \frac{Z(x)}{D}$$

Letting $\frac{hE}{DR_o^2} = 4\beta^4$ to facilitate handling of the complementary solution (Appendix I)

$$\frac{d^4w}{dx^4} + 4\beta^4 w = \frac{Z(x)}{D} \tag{12}$$

Equation 12 describes the radial displacement behavior of a pipe under the influence of an internal pressure distribution. This displacement behavior can also be produced by an axial temperature distribution. The form of the complementary solution of eq. 12 doesn't depend on the type of displacement cause (e.g. pressure or temperature). The particular solution due to a temperature distribution can be found in the following manner:

1) Cut the pipe into thin rings by planes normal to the axis.
2) Let each ring expand in accordance with its temperature increase.
3) By means of external pressure, squeeze each ring back to its original size.
4) Attach the rings into a pipe again, keeping M_x and e_t equal to zero.
5) Remove the external pressure and find the displacements equivalent to applying internal pressure, Z(x), which is equal to the external pressure which was removed. This internal pressure can be found from equilibrium of force and the compatibility condition

$$S_t = \frac{Z(x)R_o}{h}$$

$$\frac{S_t}{E} = a\,\Delta T(x)$$

Solving for Z(x) and substituting for S_t

$$Z(x) = \frac{Eha\,\Delta T(x)}{R_o}$$

The stresses produced in the ring by this internal pressure are obtained from the displacements found by integrating eq. 12 which becomes

$$\frac{d^4w}{dx^4} + 4\beta^4 w = \frac{1}{D}\,\frac{Eha\,\Delta T(x)}{R_o} \qquad (13)$$

The temperature distribution chosen to illustrate the method of solution does not closely approximate physical reality, but it serves to show feasibility of the method because it is non-linear.

Letting

$$\Delta T(x) = 1000e^{-0.75x} \qquad (\text{for } 0 < x < \infty)$$

$$\Delta T(x) = 1000e^{+0.75x} \qquad (\text{for } -\infty < x < 0) \qquad (14)$$

The particular solution to eq. 13 becomes

$$w_p = \frac{Eha}{DR_o}\,\frac{1000e^{-0.75x}}{(-0.75)^4 + 4\beta^4} \qquad (\text{for } 0 < x < \infty)$$

Since $h = 1$ inch

$E = (30)(10)^6$ psi

$a = (6)(10)^{-6}$ / F

$R_o = 8.5$ inch

$u - 0.2$

Then $\beta = 0.446\ in^{-1}$

$D = 2.75\ (10)^6$ in-lb

and $w_p = 0.0172e^{-0.75x}$

For the general case letting

$$w_p = Ke^{-\theta x}$$

The complementary solution becomes (from Appendix I)

$$w_c = e^{-\beta x}\ (C_3 \cos\beta x + C_4\ \sin\beta x)$$

The total solution is the sum of the complementary and particular solutions

$$w = e^{-\beta x}\ (C_3 \cos\beta x + C_4 \sin\beta x) + Ke^{-\theta x}$$

The two constants C_3 and C_4 can be found from the following boundary conditions:

1) The slope of the displacement curve is zero at x = 0 due to the symmetry of the temperature distribution.

2) The shear stress, γ_x, is equal to zero at x = 0, also because of the symmetry involved.

Applying the first boundary condition

$$\left.\frac{dw}{dx}\right|_{x = 0} = 0 = \beta(C_4 - C_3) - K\theta$$

Applying the second boundary condition

$$\gamma_x\Big|_{x = 0} = \frac{D}{h}\left[\frac{d^3w}{dx^3}\right]_{x = 0} = 0 = 2\beta^3(C_4 + C_3) - K\theta^3$$

Solving for C_3 and C_4

$$C_3 = \frac{K\theta(\theta^2 - 2\beta^2)}{4\beta^3}$$

$$C_4 = \frac{K\theta(\theta^2 + 2\beta^2)}{4\beta^3}$$

Since $K = 0.0172$ in

$\theta = 0.75$ in^{-1}

$\beta = 0.446$ in^{-1}

then $C_3 = 0.005935$ in

$C_4 = 0.034650$ in

Substitution of C_3, C_4, K, θ and β into eq. 15 will give an expression for the radial displacement caused by the given axial temperature distribution, eq. 14. The circumferential stress can be found from this radial displacement, w, which is related in turn to the total circumferential strain, which is

$$\lambda_t = e_t + a\Delta T(x) = \frac{w}{R_o}$$

Since $S_x = S_r = 0$

then $e_t = \frac{S_t}{E}$

Substituting and solving for S_t

$$S_t = \frac{Ew}{R_o} - Ea\Delta T(x)$$

Substituting for w and for $\Delta T(x)$

$$S_t = \frac{E}{R_o} e^{-\beta x}(C_3 \cos\beta x + C_4 \sin\beta x) + Ke^{-\theta x} - Ea(1000e^{-\theta x})$$

Substituting for constants and combining terms

$$S_t = e^{-0.446x}(20,950\cos 0.446x + 122,300\sin 0.446x) - 119,300e^{-0.75x}$$

This stress distribution is shown in fig. 19.

Conformity to equilibrium of forces can be checked by integrating the product of the circumferential stress, S_t, and the differential area upon which it acts, (2)dx, over the length of half of the pipe, $0 \rightarrow \infty$

$$\int_0^\infty S_t(2)dx = 2\left[20,950\int_0^\infty e^{-\beta x}\cos\beta x dx + 122,300\int_0^\infty e^{-\beta x}\sin\beta x dx - 119,300\int_0^\infty e^{-0.75x}dx\right]$$

$$\int_0^\infty S_t(2)dx = 2\left[20,950\frac{\beta}{\beta^2+\beta^2} + 122,300\frac{\beta}{\beta^2+\beta^2} - \frac{119,300}{(-0.75)}(0-1)\right]$$

$$\int_0^\infty S_t(2)dx = +2,740 \text{ lbs.}$$

This force is negligible compared with $\int_{1.5}^{6.2} S_t(2)dx$, (see fig. 19),

$$\int_{1.5}^{6.2} S_t(2)dx = -112,600 \text{ lbs.}$$

CONCLUSIONS

This report describes progress, to date, on a long-range project to develop the Linde treatment as a method for reducing the residual stresses in butt welds in pipe, thereby avoiding brittle fracture.

The equipment-development and experimental-technique-development phases have been largely completed. They consist of:

1) The Airco automatic gas-shielded arc-welding machine has been converted to make butt welds in 18" pipe. These welds are entirely satisfactory.

2) Methods of measuring residual stress in and near butt welds have been developed. These methods provide a detailed axial distribution and an approximate radial distribution. The axial distribution is similar to that in plate, but the radial stress is much more non-uniform in single-V butt welds than in double-V plate welds.

3) Equipment for applying the Linde treatment has been completed and shown to function properly. Preliminary exploration of the temperature distributions has been conducted. From these measurements it can be shown that Greene's "heat

treating temperature" is mid-plate temperature and that through-thickness temperature variations are large.

4) Equipment for fracture testing butt-welded pipe has been completed. This equipment has been shown to be capable of producing 40,000 psi circumferential stress in 18-inch O.D. by 1" wall-thickness pipe at - 40 C without applying axial stress.

Using this equipment, developed jointly with F. J. Furman, it was found that the notched and butt-welded pipe (with no benefit of stress relief), when loaded at - 40 C, withstood yield point stresses. Apparently the notch in the pipe did not simulate the same degree of severity necessary for brittle failure as that in Greene's plate specimen.

The analytical aspects of the work consist mainly of analytical-technique development, as follows:

1) A two-dimensional numerical technique for determining temperature distribution has been applied for boundary conditions which roughly simulate the Linde treatment. This analysis shows that through-thickness variations are too large to be ignored in the subsequent thermal-stress analysis. This analysis applies to plate or pipe.

2) Tall's method of determining residual stress in welds has been applied to the Linde treatment. The results show a decrease in residual stress at the weld similar to what is measured, but the analysis showed an unexpectedly large tensile residual stress at the hot zones which is not observed experimentally. It is suggested that the compatibility condition which is applicable to (narrow) welds is not applicable to (wide) hot zones

produced by Linde torches. This analysis applies only to plate.

3) Since pipe provides less restraint against contraction of localized hot zones than plate does, a preliminary elastic analysis was conducted to show how much less restraint exists in the pipe. Under arbitrary conditions selected for illustrative purposes, a thermal stress of 118,000 psi was developed. This is considerably less than the 180,000 psi that would have occurred under full restraint (e.g. in a large plate).

RECOMMENDATIONS

1) In the neighborhood of the weld, the local residual stresses need to be delineated in greater detail by parting narrower rings.
2) The temperature distributions corresponding to various combinations of torch width, speed, and spacing need to be investigated more in detail, including the effects of follow-up water cooling.
3) Film coefficients and torch temperatures should be measured so that accurate temperature distributions can be determined by the method described. These distributions should be checked against the measurements.
4) The thermal stress analysis needs to be modified to take account of compatibility in a more rigorous manner. Also, the effects of through-thickness temperature variations needs to be assessed with respect to its effect on the thermal stress analysis. The thermal stress analysis may have to be extended to cover through-thickness temperature variation.
5) The analysis of the lessened restraint due to pipe flexing should be extended by numerical methods to include plastic

deformation. Before proceeding with the plastic case, the numerical methods should be applied to the elastic case investigated in the body of this report, and the results checked against the algebraic solution.

6) The conditions in fracture testing of welded pipes should be made more severe so that low strength fracture will be obtained. If it is demonstrated that the Linde treatment does not affect notch sharpness or metallurgical damage, then the Linde treatment can be used to show that reduction of residual stress is the cause of improvement in fracture strength.

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Table 1. Conditions of Welding

Welding wire	1/16-in diameter Airco A-675 (mild steel)
	Nominal composition of deposit:
	0.09% C 0.44% Cu 0.90% Mn 0.027% S (max) 0.42% Si 0.014% P (max)
	Mechanical properties of deposit:
	U.T.S. = 76,000 psi % elongation = 25 Y.S. = 60,000 psi % red. of area = 40
Shielding gas	Argon + 2% oxygen (for arc stability)
	Gas flow = 40 - 50 cubic feet per minute
Voltage	25.4 $\pm$ 0.2 volts
Current	Root pass = 380 $\pm$ 10 amps Cover passes = 360 $\pm$ 10 amps
Wirefeed	195 $\pm$ 5 inches per minute
Pipe surface speed past head	14.5 $\pm$ 1.0 inches per minute
Time to complete	1 1/2 hours (13 passes at 4 minutes per pass plus 12 interpass pauses at 3 minutes each)

Table 2. Tall's method applied to the Linde treatment (see fig. 14, 1/2 heat losses)

t_o (1)	X	0.5	1.5	5.0	8.5	9.5	11.0	13.0	15.5	18.5	Summation of Forces
	ΔX	1.0	1.0	6.0	1.0	1.0	2.0	2.0	3.0	3.0	
(2)	T_o	0	0	500	0	0	0	0	0	0	
(3)	S_{max}	50	40	26.7	40	40	40	40	40	40	(all stresses in ksi)
(4)	ΔT	0	0	+500	0	0	0	0	0	0	
(5)	$S_t = -E\alpha\Delta T$	0	0	-100	0	0	0	0	0	0	(E = 200 psi/°F)
(6)	$S_o = S_i + S_t$	+43.0	+25.0	-125	-6.5	-1.5	+3.0	+4.5	+4.5	+4.5	(S_i = welding residual stresses)
(7)	$S_o \leq S_{max}$	+43.0	+25.0	-26.7	-6.5	-1.5	+3.0	+4.5	+4.5	+4.5	
(8)	P	+43.0	+25.0	-160.2	-6.5	-1.5	+6.0	+9.0	+12.5	+12.5	+ = $\frac{-60.0}{20.0}$ = -3.0 NO!
	$S_{eq} = +3.0$										
(9)	S_o	+46.0	+28.0	-23.7	-3.5	+1.5	+6.0	+7.5	+7.5	+7.5	
(10)	P	+46.0	+28.0	-142.2	-3.5	+1.5	+6.0	+15.0	+22.5	+22.5	= $\frac{-2.3}{20.0}$ = -0.1 OK!
t_1	T_1	0	0	400	0	0	0	0	0	0	
	S_{max}	50	40	29.3	40	40	40	40	40	40	
	ΔT	0	0	400	0	0	0	0	0	0	
	$S_t = -E\alpha\Delta T$	0	0	+20.0	0	0	0	0	0	0	
	$S_1 = S_o + S_t$	+46.0	+28.0	-3.7	-3.5	+1.5	+6.0	+7.5	+7.5	+7.5	
	$S_1 \leq S_{max}$	Same as above									
	P	+46.0	+28.0	-22.2	-3.5	+1.5	+12.0	+15.0	+22.5	+22.5	= $\frac{+122.3}{20.0}$ = +6.2 NO!
	$S_{eq} = -6.2$										
	S_1	+39.8	+21.8	-9.9	-9.7	-4.7	-0.2	+1.3	+1.3	+1.3	
	P	+39.8	+21.8	-59.4	-9.7	-4.7	-0.4	+2.6	+3.9	+3.9	= $\frac{-1.7}{20.0}$ = 0.1 OK!

Table 2. Cont'd.

t_2	X	0.5	1.5	5.0	8.5	9.5	11.0	13.0	15.5	18.5	Summation of forces
	ΔX	1.0	1.0	6.0	1.0	1.0	2.0	2.0	3.0	3.0	
	T_2	0	0	300	200	0	0	0	0	0	
	S_{max}	50	40	32.0	34.7	40	40	40	40	40	
	ΔT	0	0	-100	+200	0	0	0	0	0	
	$S_t = -E\alpha\Delta T$	0	0	+20.0	-40.0	0	0	0	0	0	
	$S_2 = S_1 + S_t$	+39.8	+21.8	+10.1	-34.7	-4.7	-0.2	+1.3	+1.3	+1.3	
	$S_2 \leq S_{max}$	Adjustment made in line above									
	P	+39.8	+21.8	+60.6	-34.7	-4.7	-0.4	+2.6	+3.9	+3.9	$= \frac{+92.8}{20.0} = +4.6$ NO
	$S_{eq} = -4.6$										
	S_2	+35.2	+17.2	+5.5	-34.7	-9.3	-4.8	-3.3	-3.3	-3.3	
	P	+35.2	+17.2	+33.0	-34.7	-9.3	-9.6	-6.6	-9.9	-9.9	$= \frac{+5.4}{20.0} = +0.3$ NO
	$S_{eq} = -4.9$										
	S_2	+34.9	+16.9	+5.8	-34.7	-9.6	-5.1	-3.6	-3.6	-3.6	
	P	+34.9	+16.9	+31.2	-34.7	-9.6	-10.2	-7.2	-10.8	-10.8	$= \frac{-0.3}{20.0} = -0.015$ OK
t_3	T_3	0	0	250	200	50	0	0	0	0	
	S_{max}	50	40	33.3	34.7	39.0	40	40	40	40	
	ΔT	0	0	-50	0	+50	0	0	0	0	
	$S_t = -E\alpha\Delta T$	0	0	+10.0	0	-10.0	0	0	0	0	
	$S_3 = S_2 + S_t$	+34.9	+16.9	+15.2	-34.7	-19.6	-5.1	-3.6	-3.6	-3.6	
	$S_3 \leq S_{max}$	Adjustment made in line above									
	P	+34.9	+16.9	+91.2	-34.7	-19.6	-10.2	-7.2	-10.8	-10.8	$= \frac{+49.7}{20.0} = +2.5$ NO
	$S_{eq} = -2.5$										
	S_3	+32.4	+14.4	+12.7	-34.7	-22.1	-7.6	-6.1	-6.1	-6.1	
	P	+32.4	+14.4	+76.2	-34.7	-22.1	-15.2	-12.2	-18.3	-18.2	$= \frac{+2.2}{20.0} = +0.1$ OK

Results of calculations after another four time interval shown in fig. 15-F

Table 3. Significant features of Tall's method applied to Linde treatment

Torch	Figure	Distance from weld centerline	Weld Y.S. (ksi)	Plate Y.S. (ksi)	Plate Width (inch)	Reduction in weld stresses	Stress under torch after treatment (%Y.S.)	Temp. rise between torches	Heat losses from plate
0.8	15-A	3 inches	40	40	20	17%	45	linear	none *
2.0	15-B	3 inches	40	40	20	40%	80	linear	none
4.0	15-C	5 inches	50	40	40	33%	90	linear	none
4.0	15-D	5 inches	50	40	40	27%	87	none	none
6.0	15-E	5 inches	50	40	40	33%	85	none	none
6.0	15-F	5 inches	50	40	40	50%	87	none	50% **

* solid lines in fig. 14

** dotted lines in fig. 14

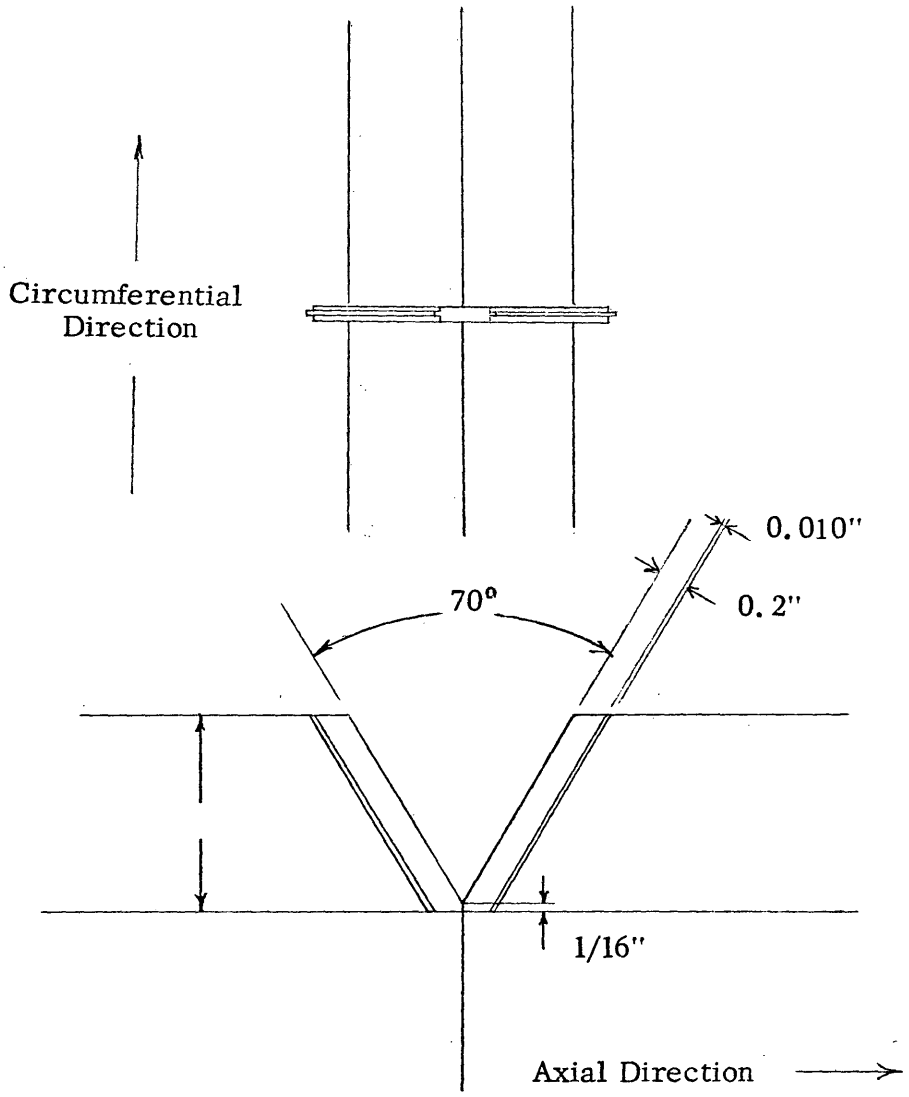


Fig. 1. Single-V notch in pipe

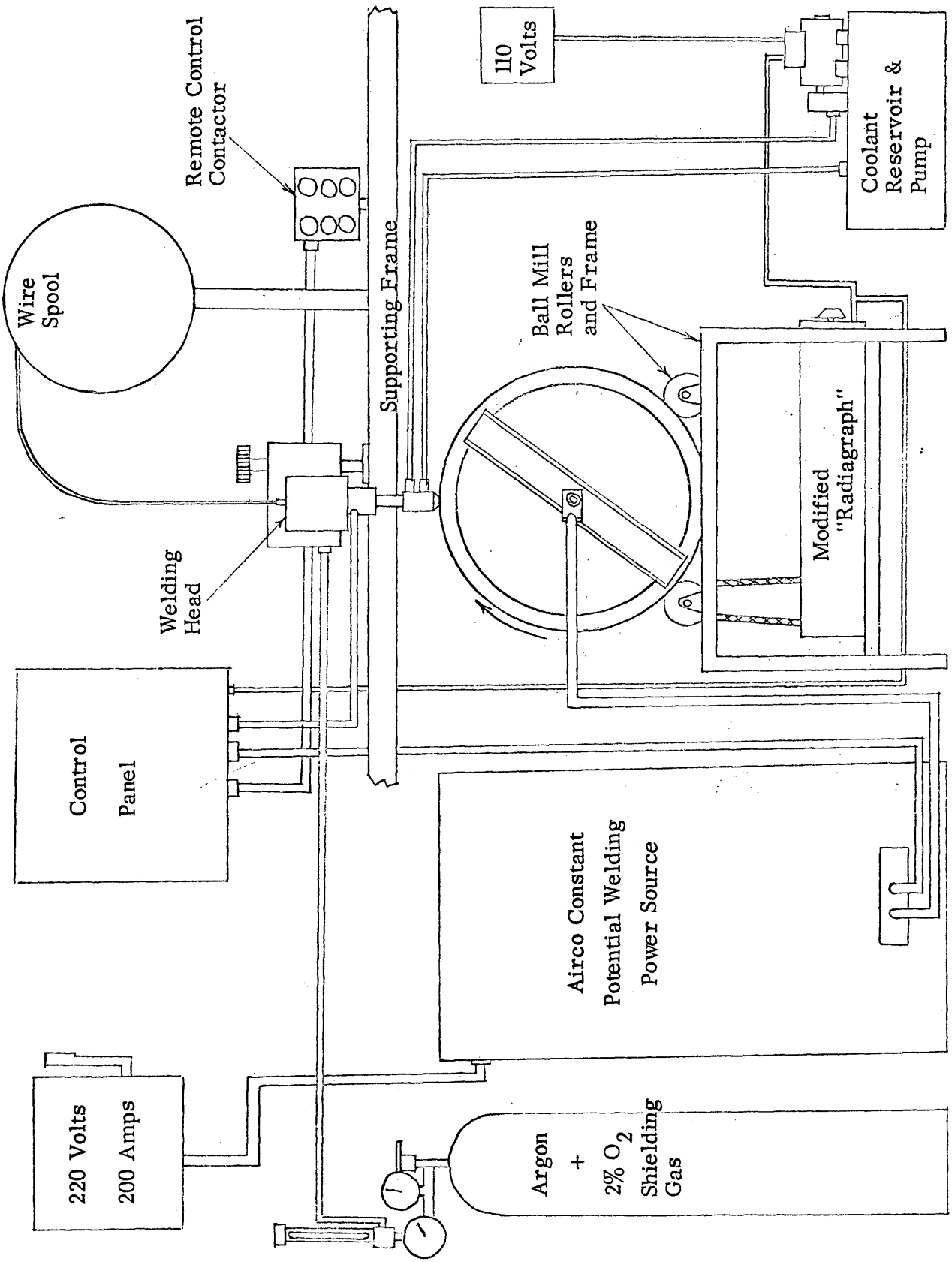


Fig. 2-A. Welding equipment modified for butt-welding pipe

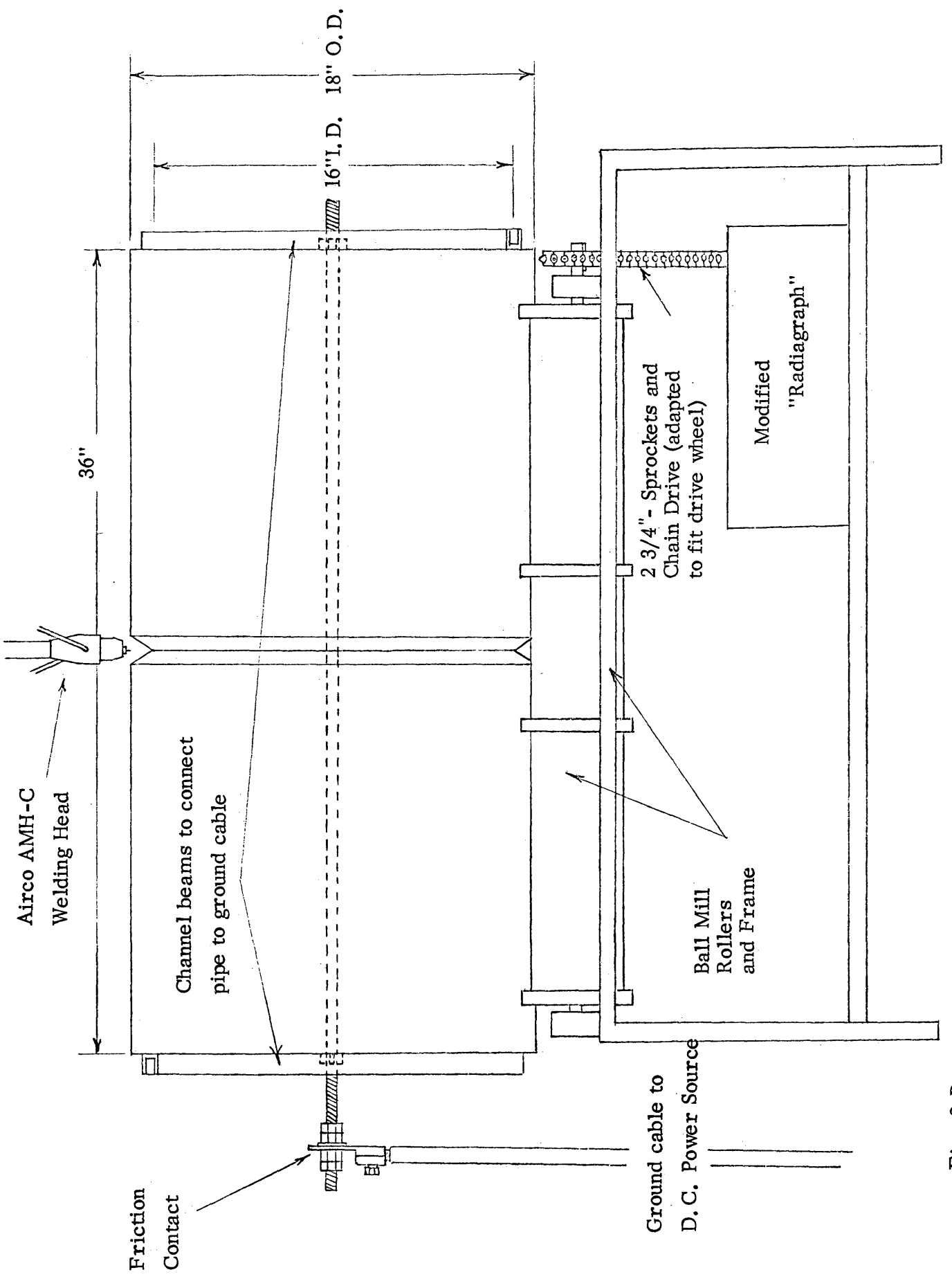

18" O.D.
16" I.D.
36"
Airco AMH-C
Welding Head
Channel beams to connect
pipe to ground cable
2 3/4" - Sprockets and
Chain Drive (adapted
to fit drive wheel)
Modified
"Radiagraph"
Ball Mill
Rollers
and Frame
Friction
Contact
Ground cable to
D.C. Power Source

Fig. 2-B.

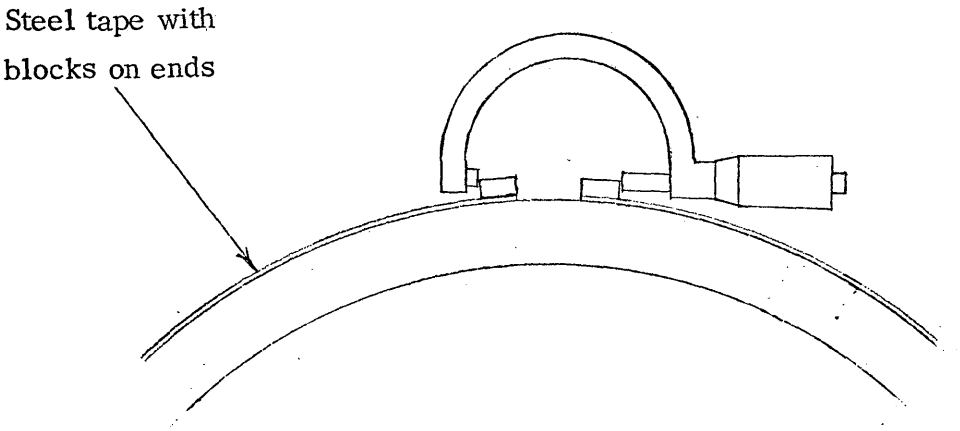


Fig. 3. Residual stress measuring technique

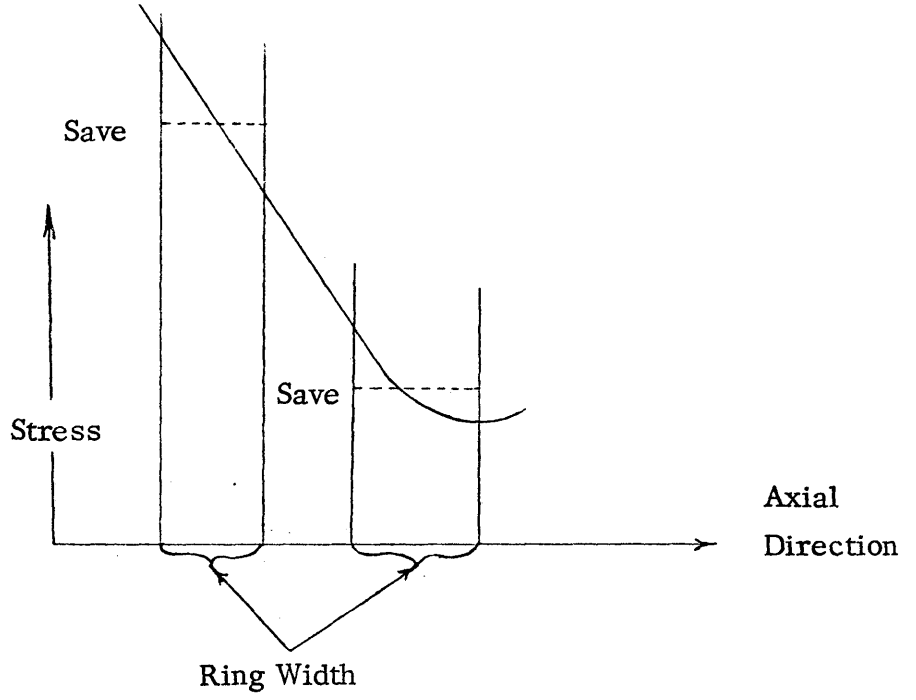


Fig. 4. Error in average stress location

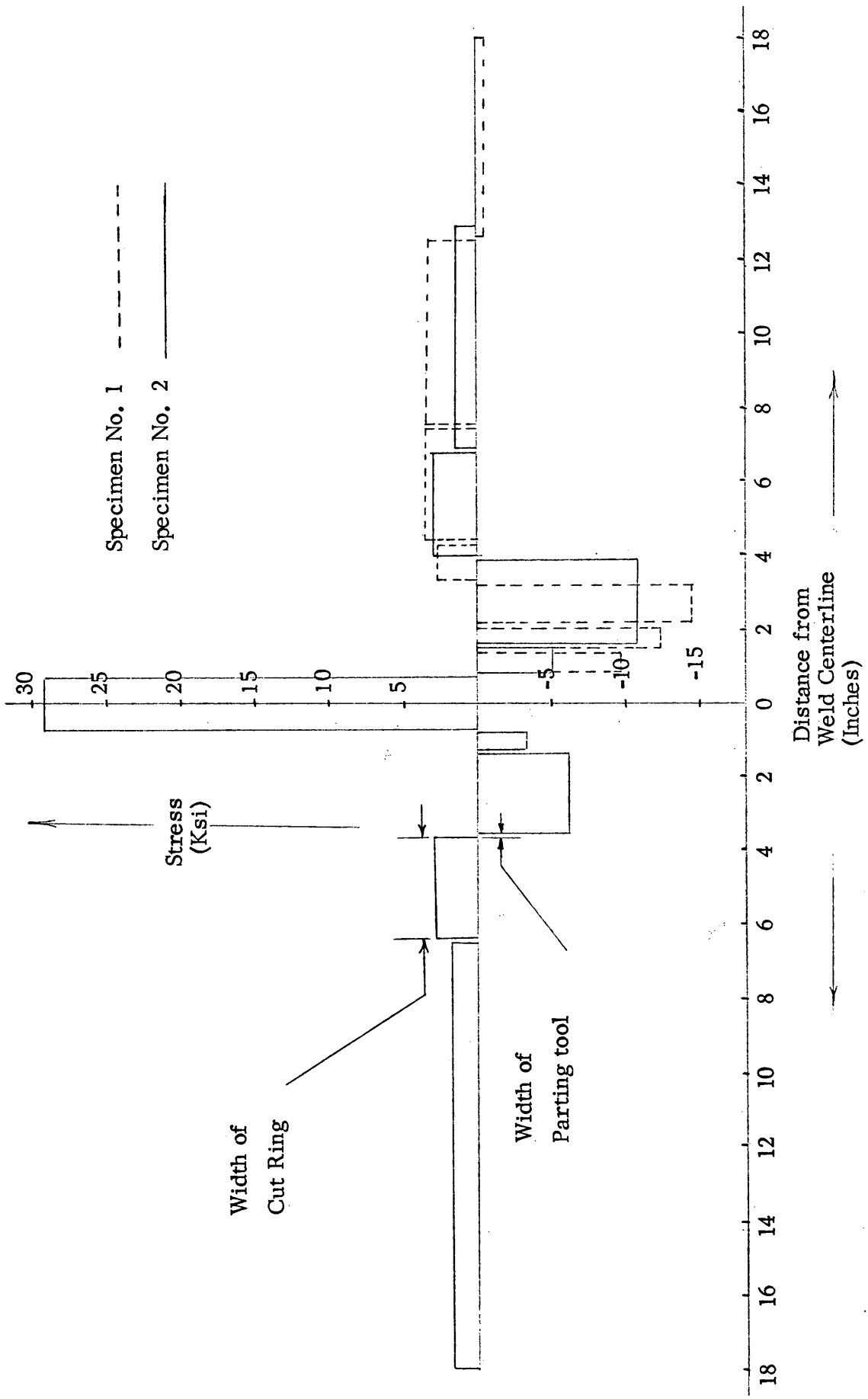


Fig. 5. Residual Stresses in butt-welded pipe

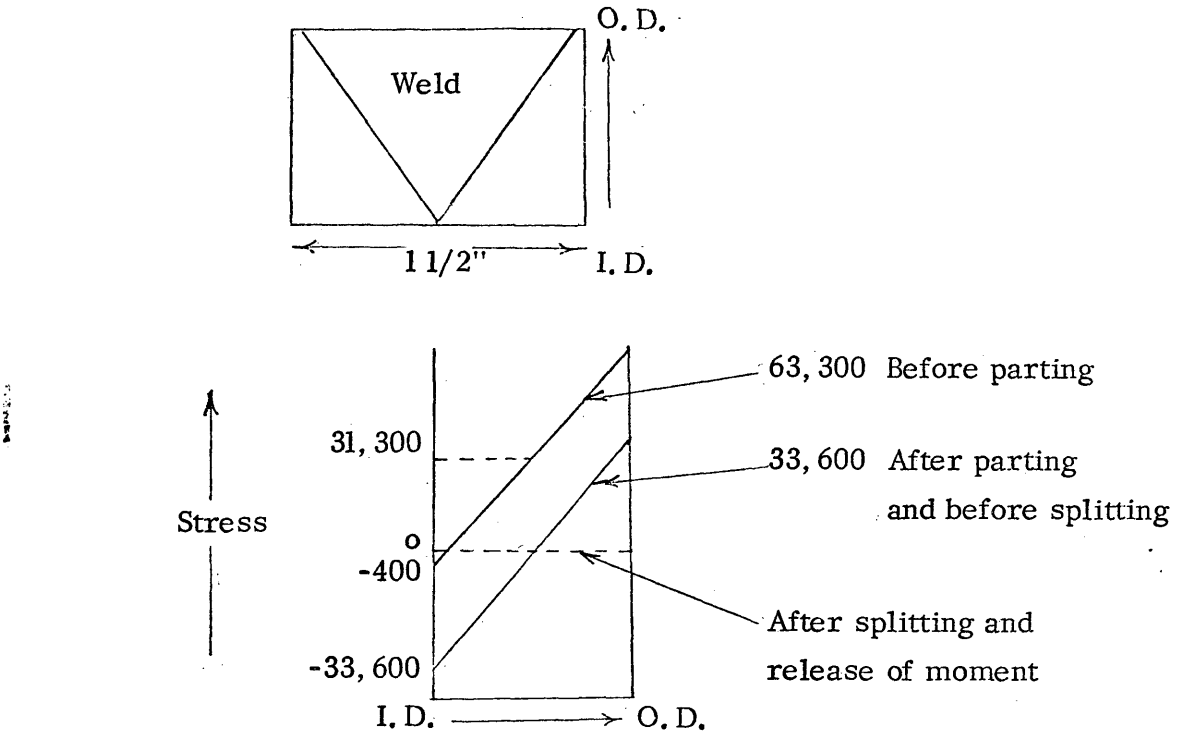


Fig. 7. Distribution of circumferential residual stresses in weld

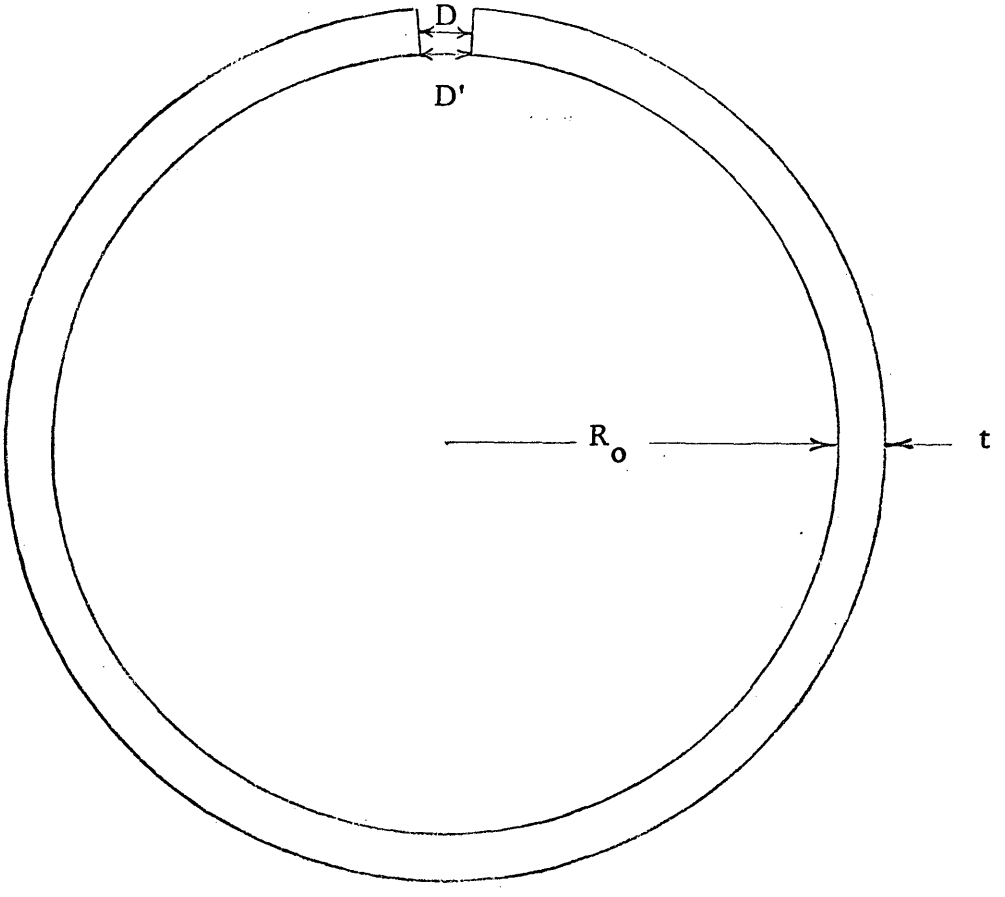


Fig. 6-A. Split ring containing weld

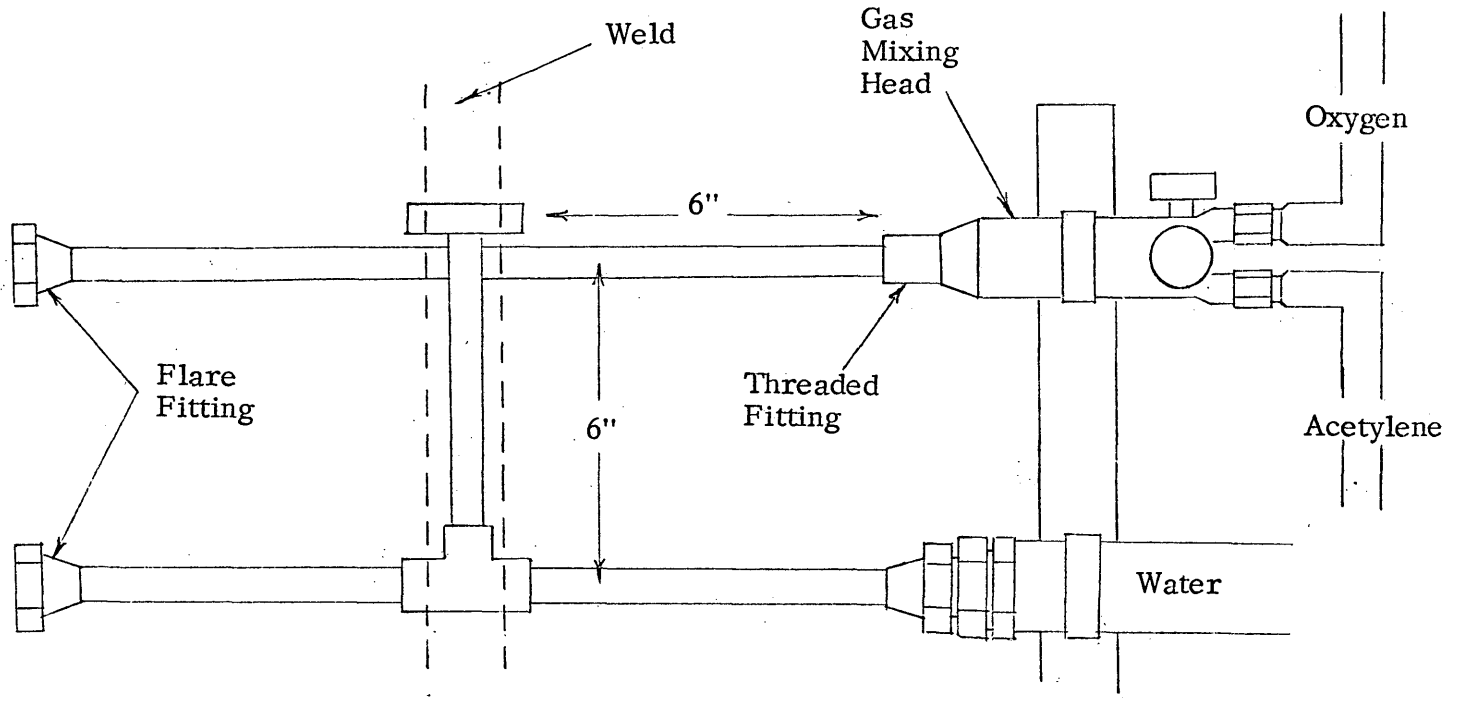


Fig. 8-A. Modified Linde torch

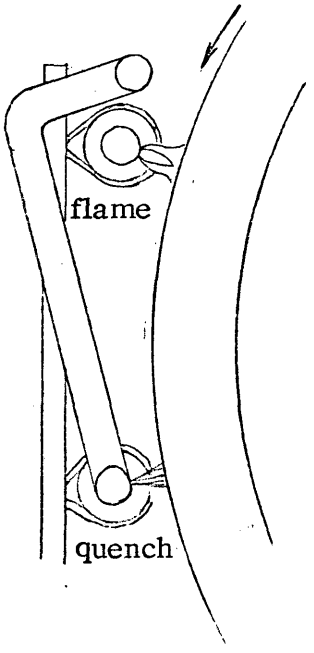


Fig. 8-B.

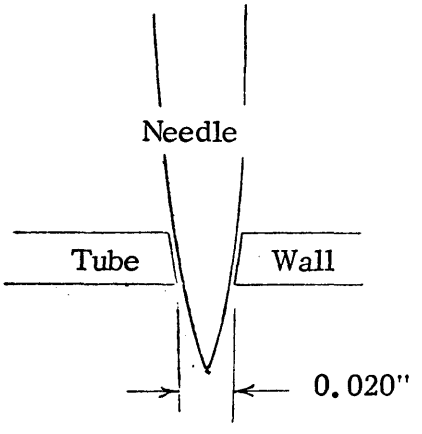


Fig. 8-C.

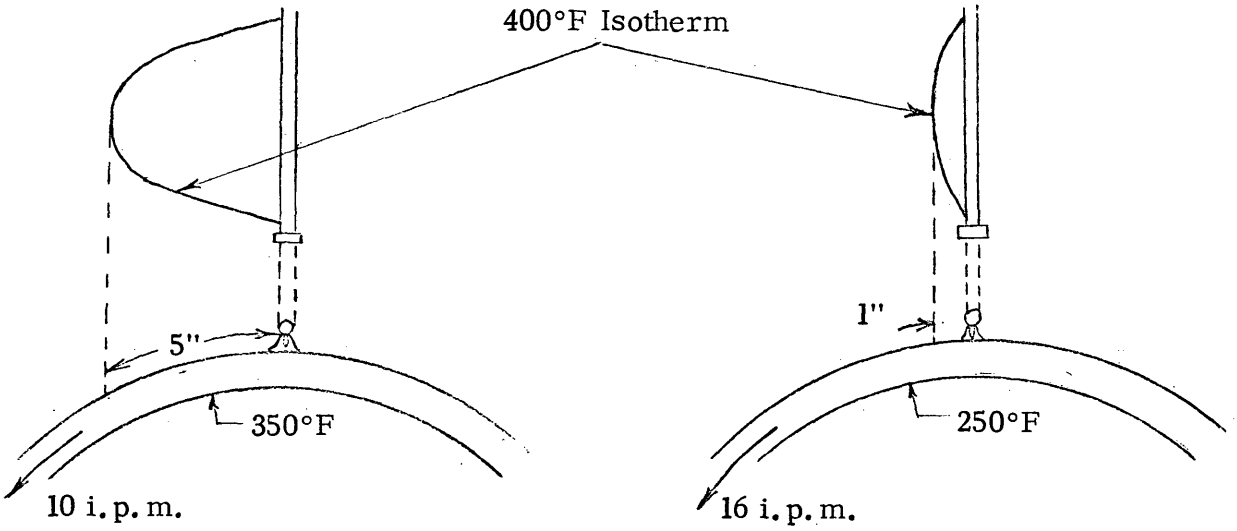


Fig. 9-A. Surface temperature measurements Fig. 9-B.

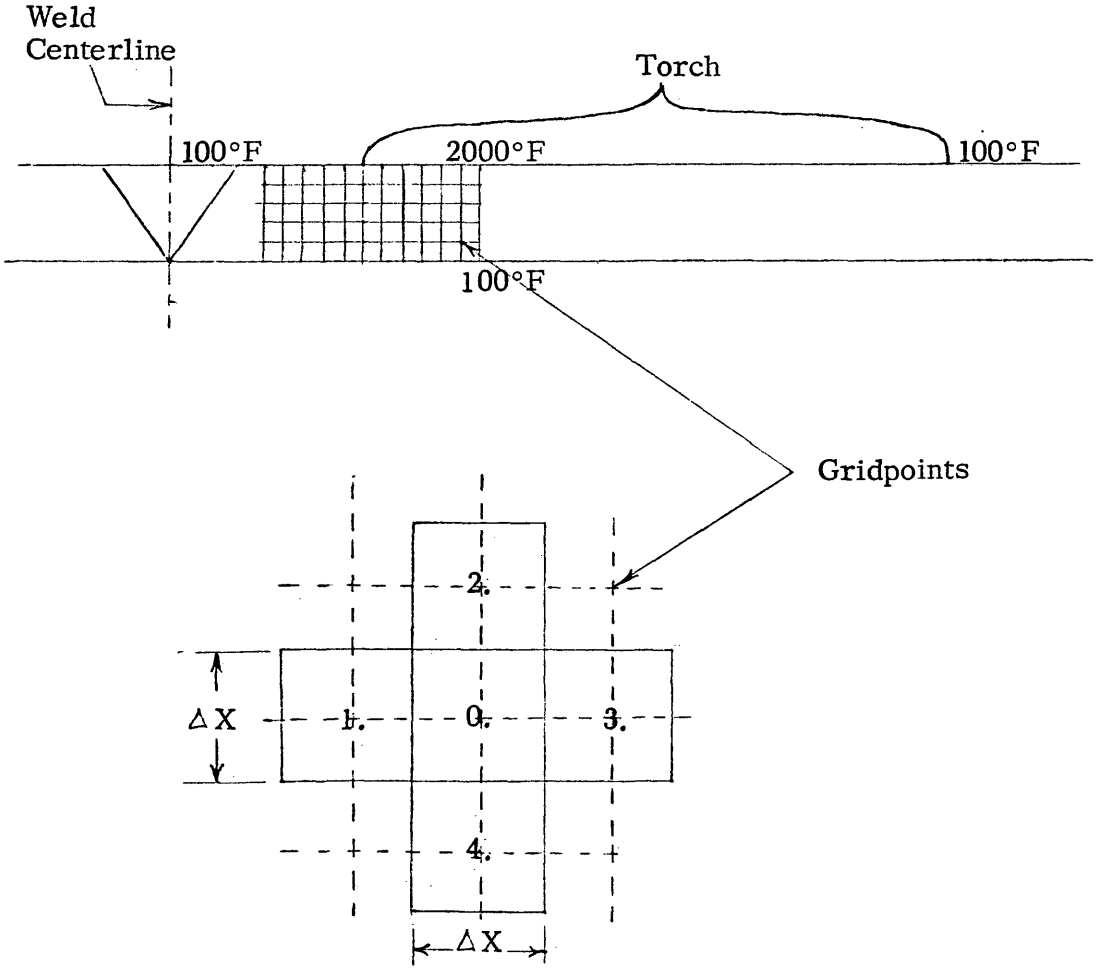
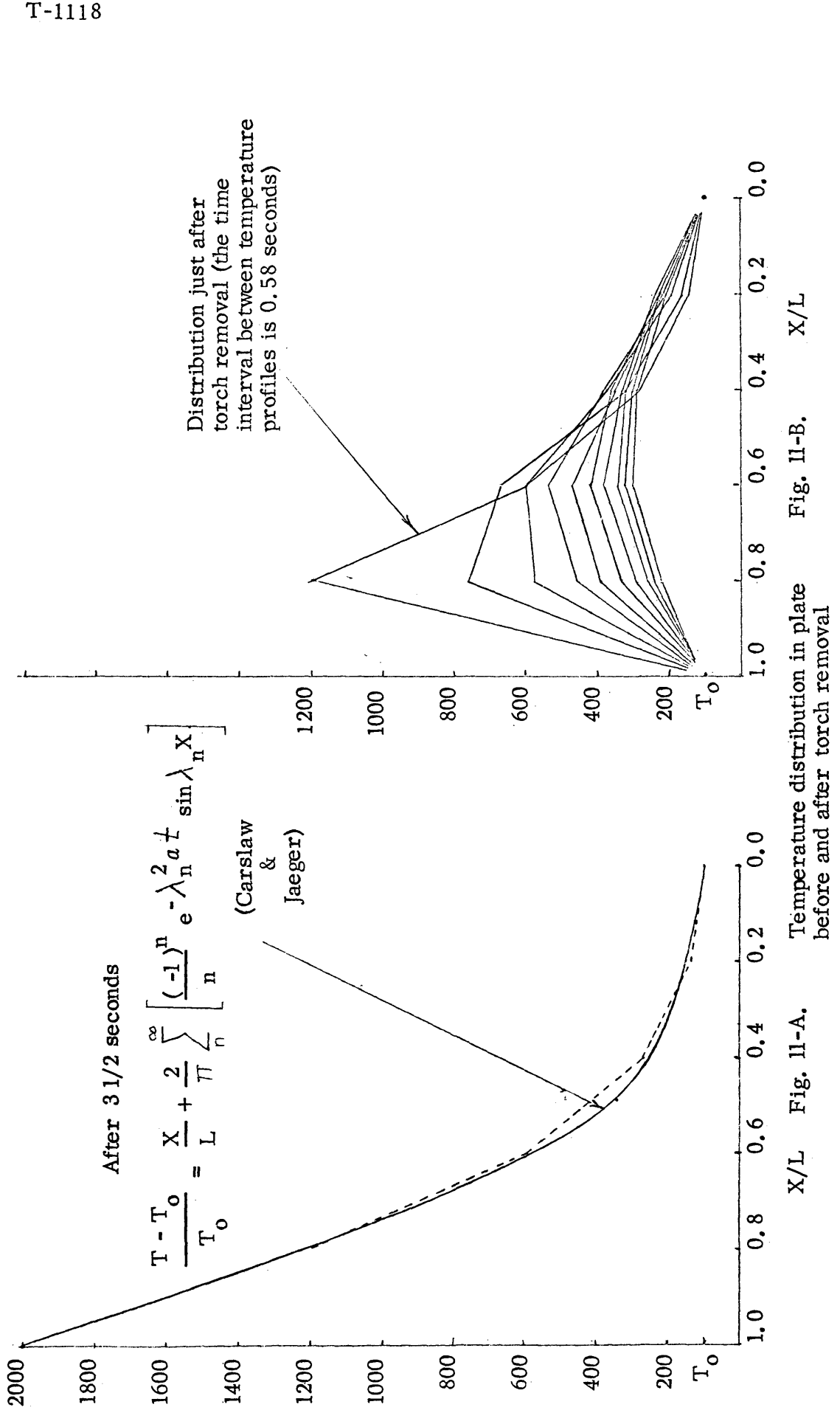


Fig. 10. Transient temperature distribution grid



X/L Fig. 11-A. Temperature distribution in plate before and after torch removal Fig. 11-B. X/L

++100		+100		+100		+100		+2000		+2000		+2000		+2000		+2000			
+100	100	+100	107	+100	137	+100	301	+100	865	+100	1028	+100	1058	+100	1065	+100	1065		
100	102	100	110	100	158	100	324	575	926	575	1086	575	1134	575	1141	575	1143		
100	103	100	117	100	167	219	354	694	955	813	1141	813	1188	813	1202	813	1203		
100	105	100	121	130	181	249	368	813	990	932	1175	961	1236	961	1249	961	1253		
+100	100	+100	102	+100	124	+100	195	+100	373	+100	420	+100	442	+100	442	+100	442		
100	101	100	108	100	134	100	230	100	408	100	505	100	524	100	531	100	531		
100	102	100	112	100	152	100	251	219	464	219	558	219	599	219	605	219	608		
100	104	100	120	100	162	160	283	278	496	338	617	338	656	338	671	338	672		
+100	100	+100	100	+100	100	+100	123	+100	152	+100	175	+100	175	+100	175	+100	175		
100	100	100	101	100	112	100	137	100	195	100	214	100	225	100	225	100	225		
100	101	100	106	100	118	100	161	100	219	100	262	100	271	100	176	100	276		
100	103	100	108	100	131	100	175	130	255	130	297	130	320	130	324	130	326		
+100	100	+100	100	+100	100	+100	100	+100	107	+100	107	+100	107	+100	107	+100	107		
100	100	100	100	100	100	100	106	100	115	100	122	100	122	100	122	100	122		
100	100	100	100	100	103	100	113	100	131	100	138	100	142	100	142	100	142		
100	100	100	102	100	108	100	124	100	143	100	159	100	163	100	165	100	165		
+100		+100		+100		+100		+100		+100		+100		+100		+100			

Grid spacing = 0.2 inches
Time interval between successive temperature intervals = 0.58 seconds
+ = grid point

$$T_{o,\,\Delta t} = \frac{T(1) + T(2) + T(3) + T(4)}{4}$$

Fig. 12. Calculation of transient temperature distribution upon heating as indicated

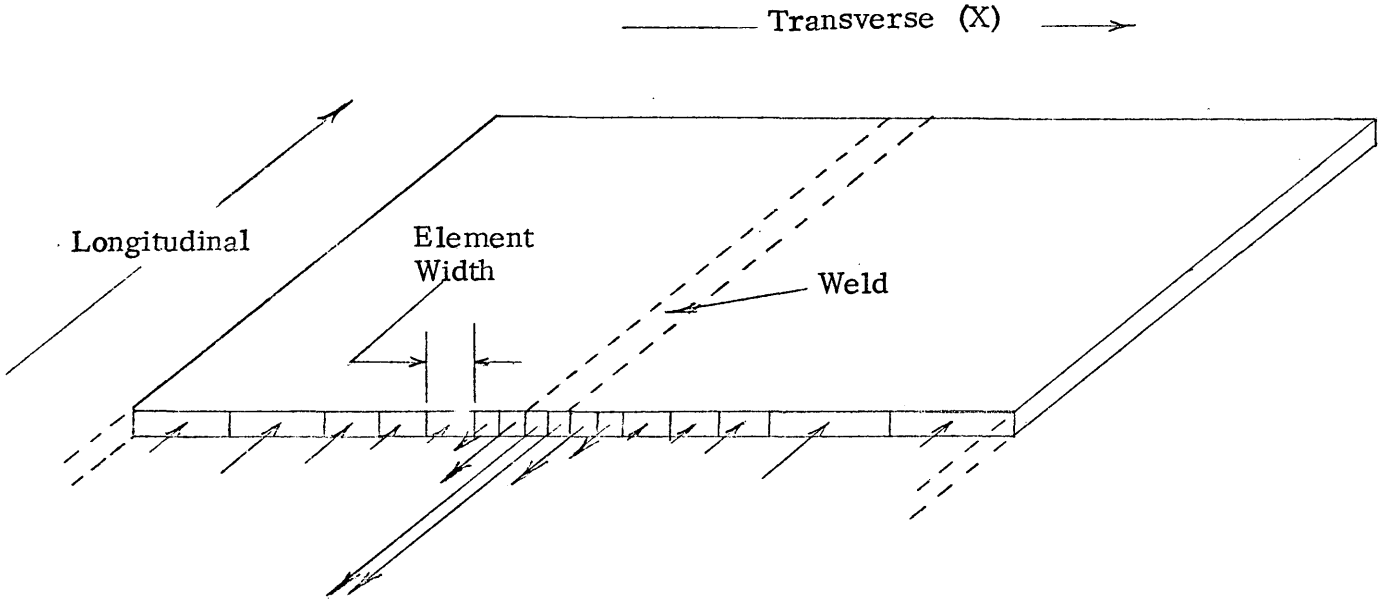


Fig. 13. Forces acting on transverse plane at the weld midlength

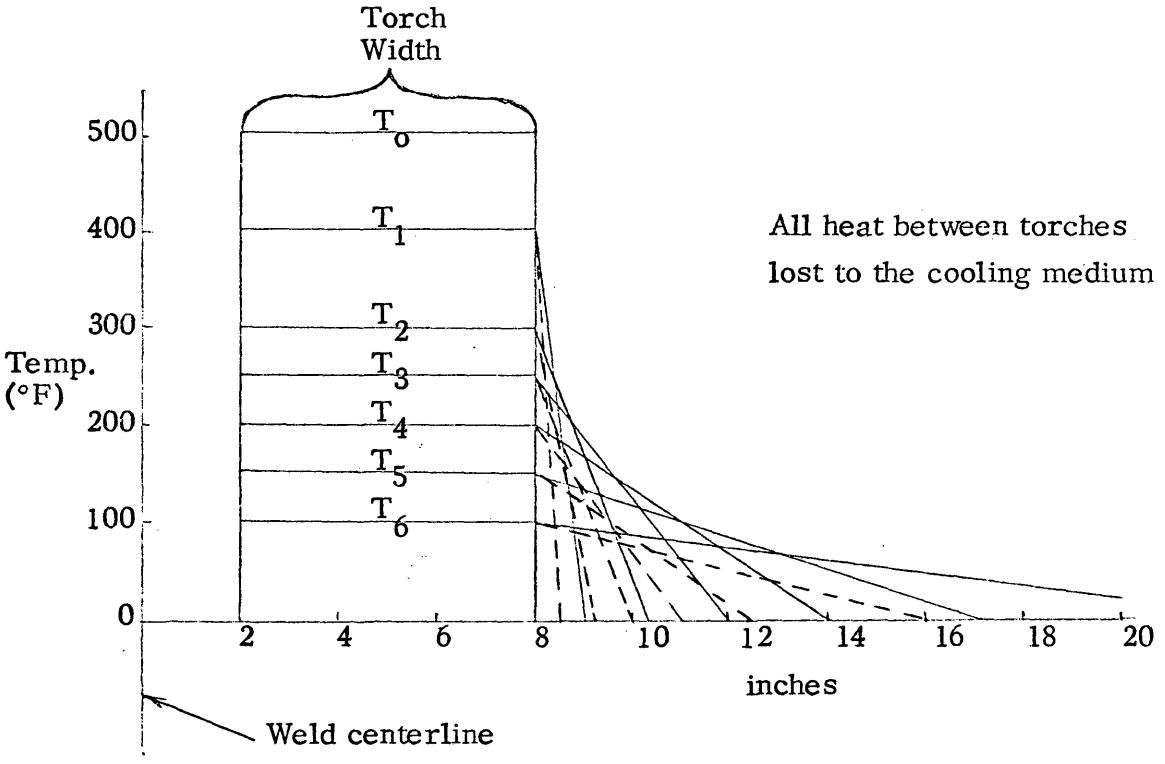


Fig. 14 Transient temperature distribution due to Linde treatment

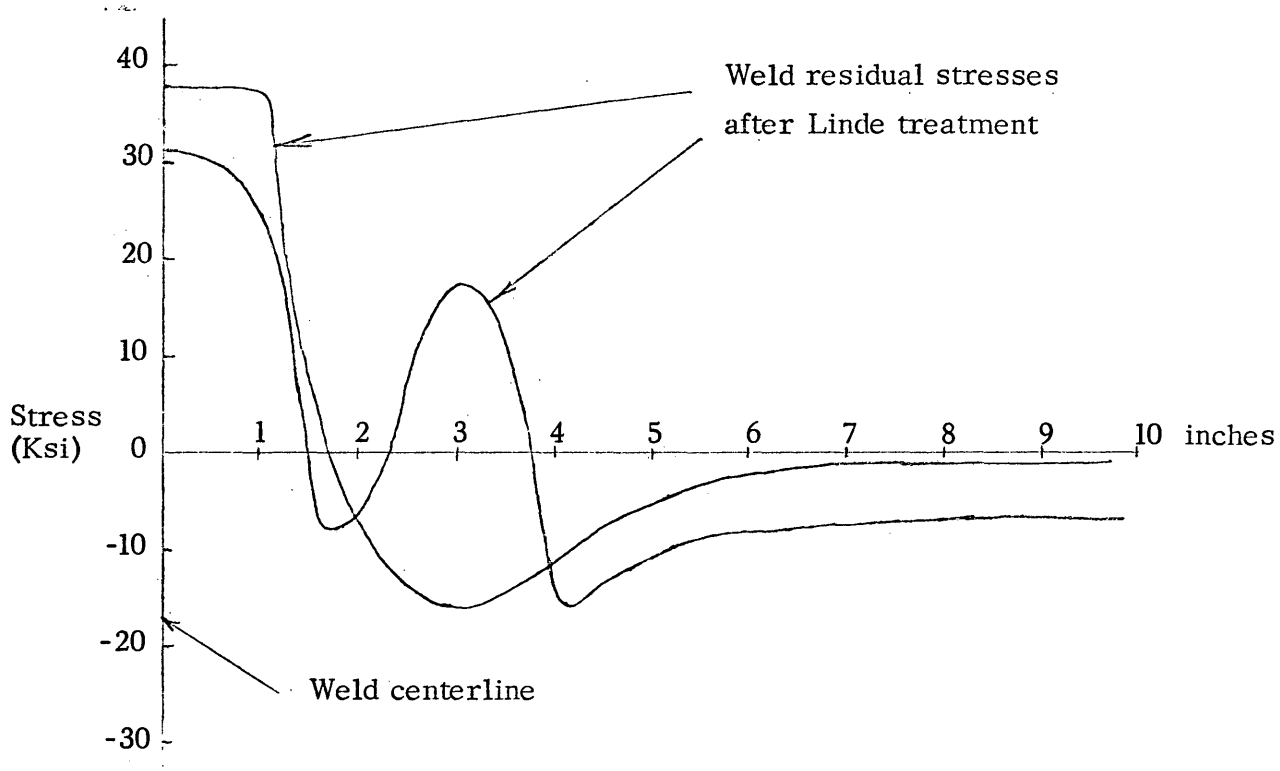


Fig. 15-A. Stress distribution after Linde-type treatment (refer to Table 3)

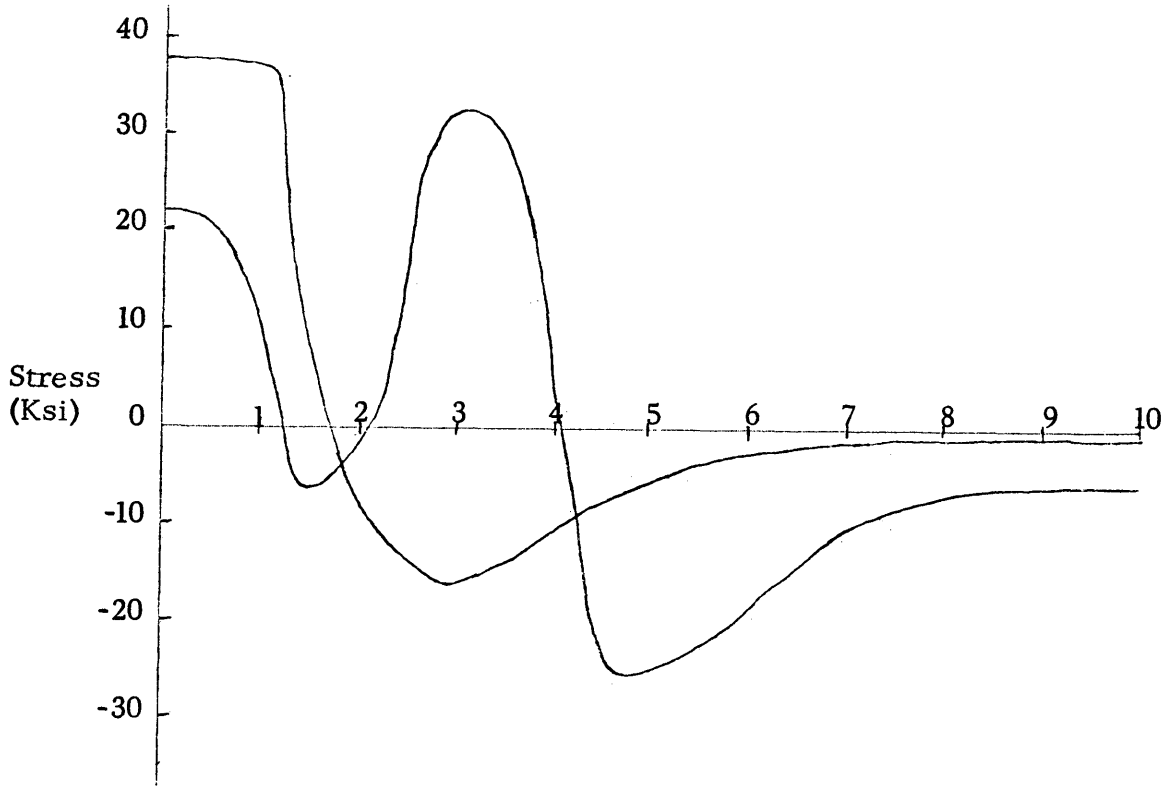


Fig. 15-B.

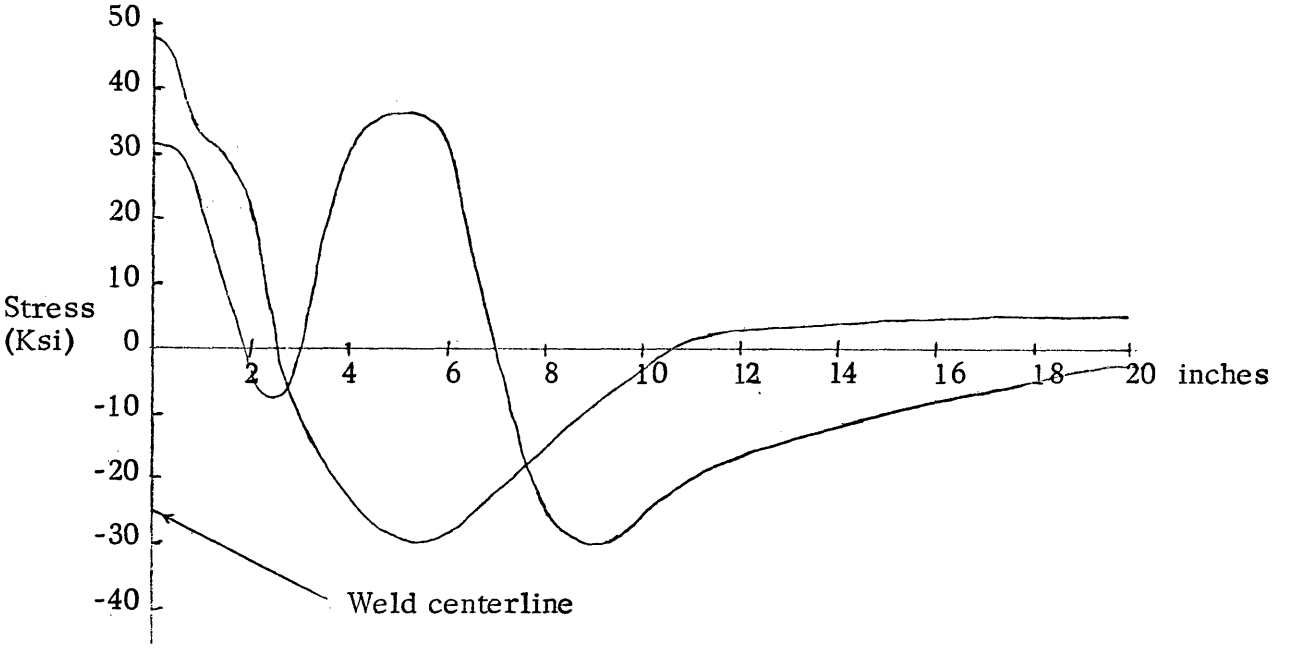


Fig. 15-C.

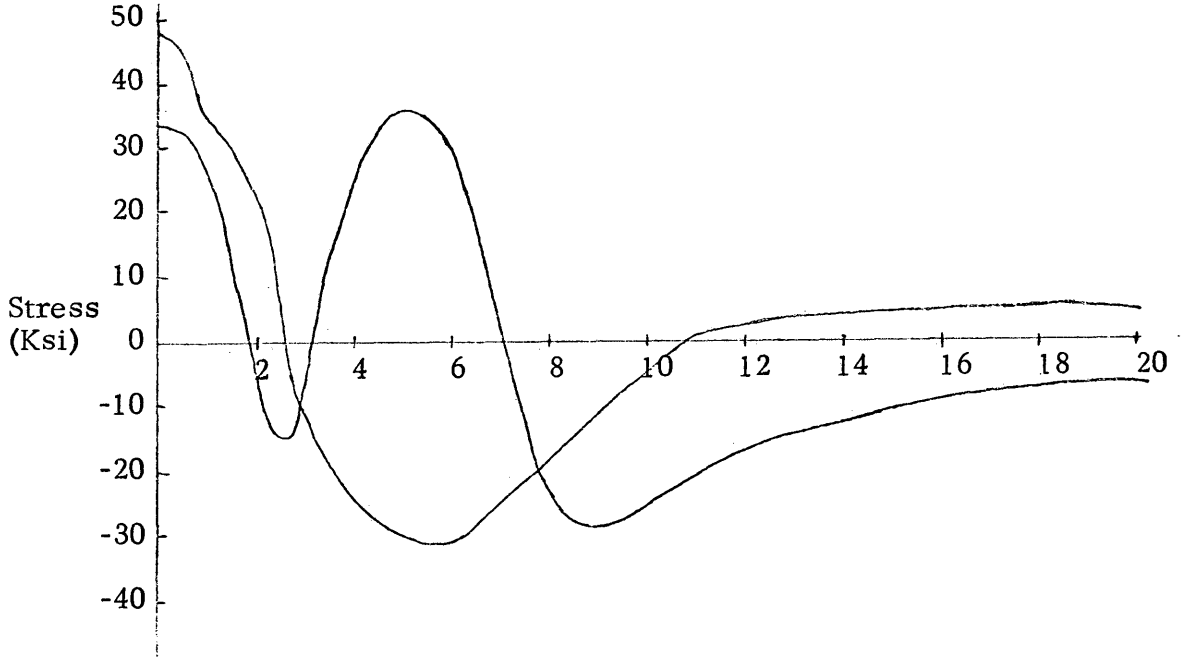


Fig. 15-D.

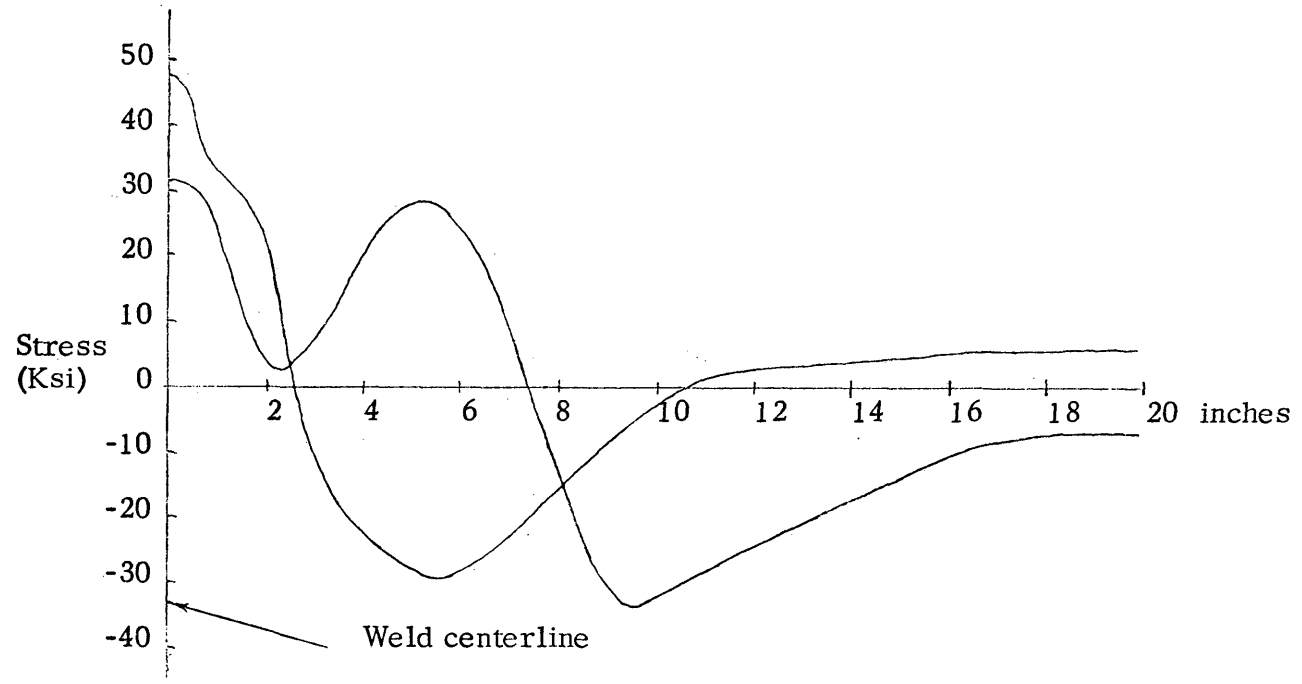


Fig. 15-E.

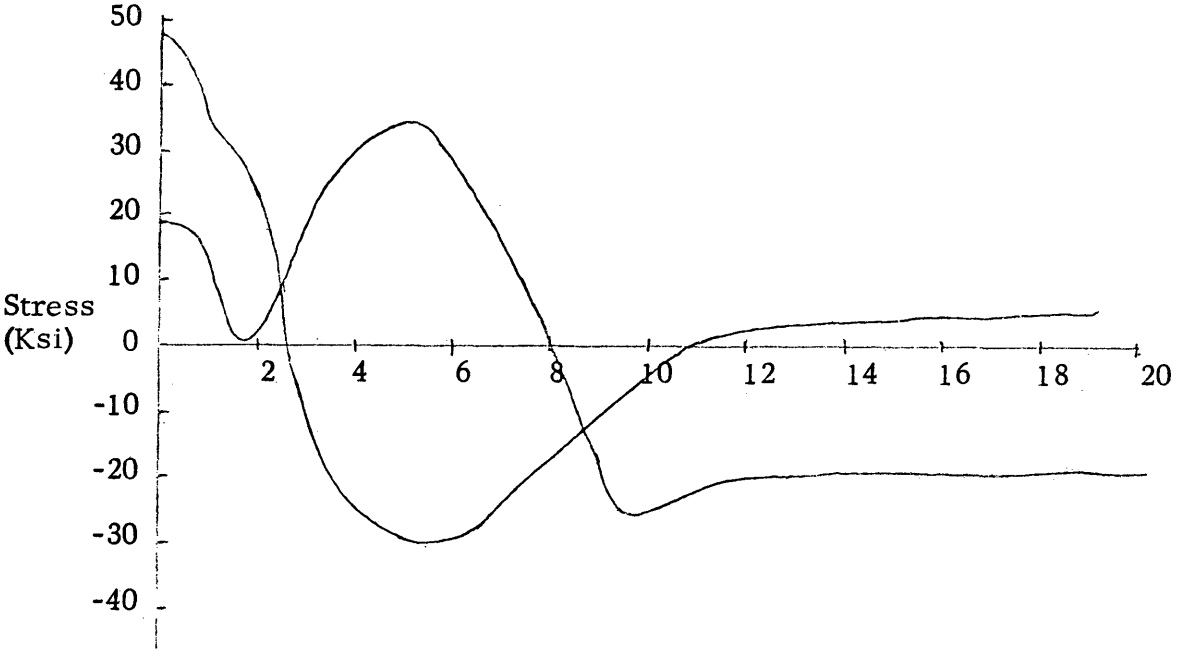


Fig. 15-F.

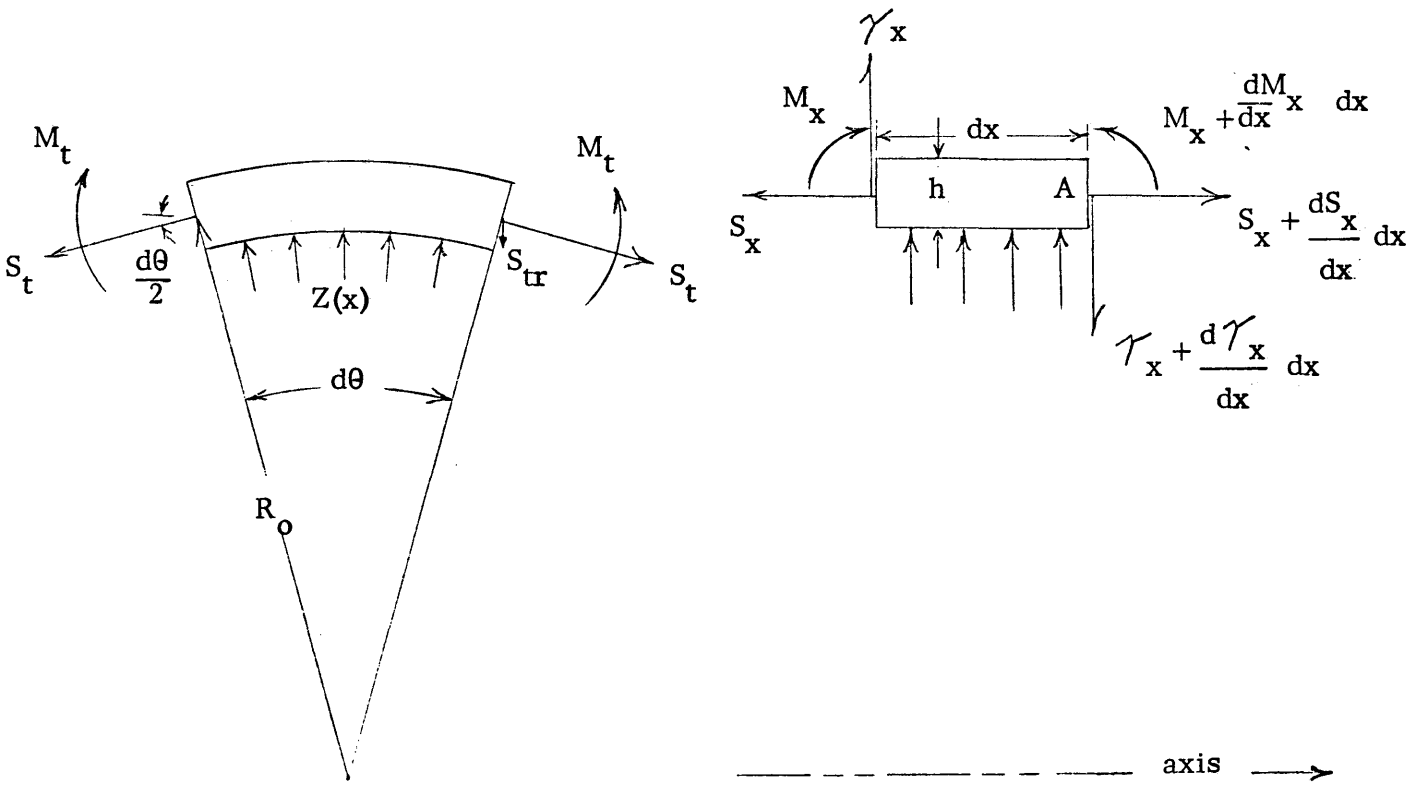


Fig. 16. Cylindrical element of a pipe under internal pressure Z(X)

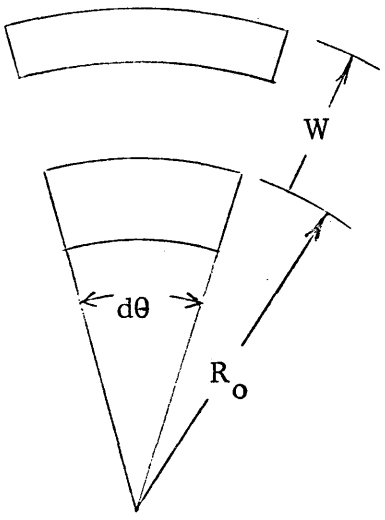


Fig. 17. Cylindrical element used to define circumferential strain

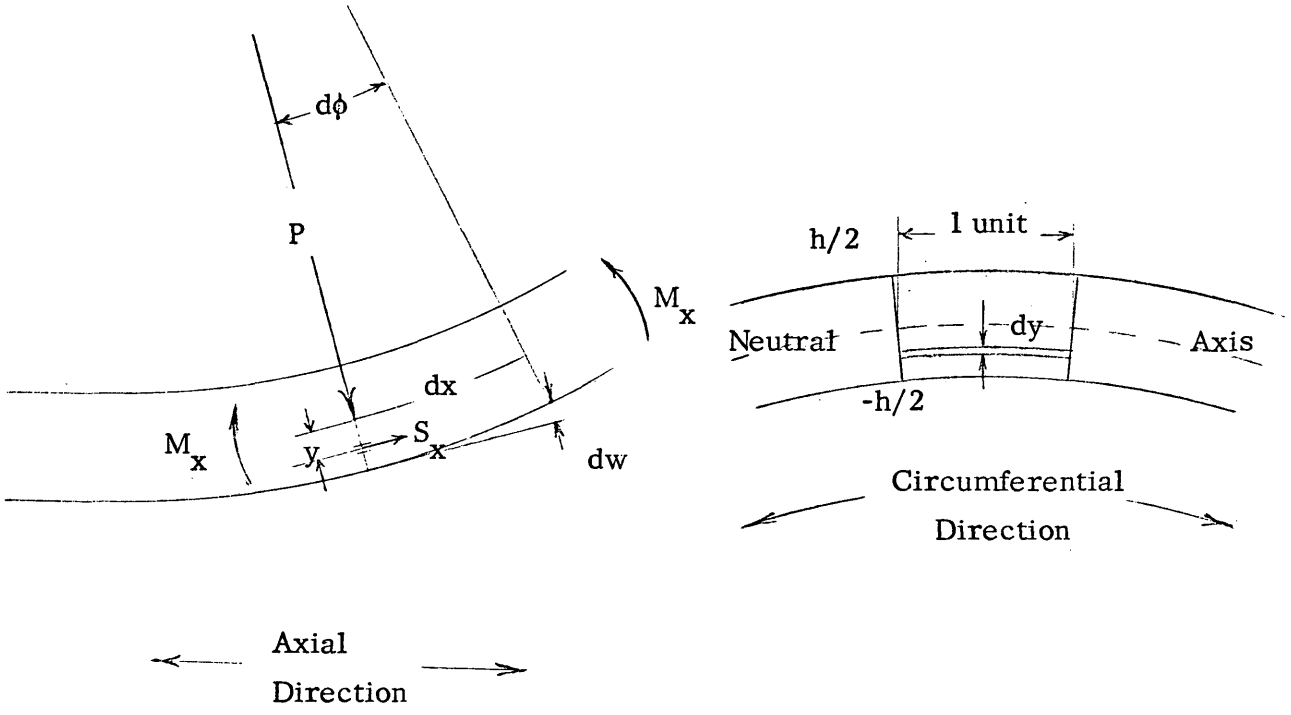


Fig. 18. Cylindrical element bent along the axis by the moment M_x

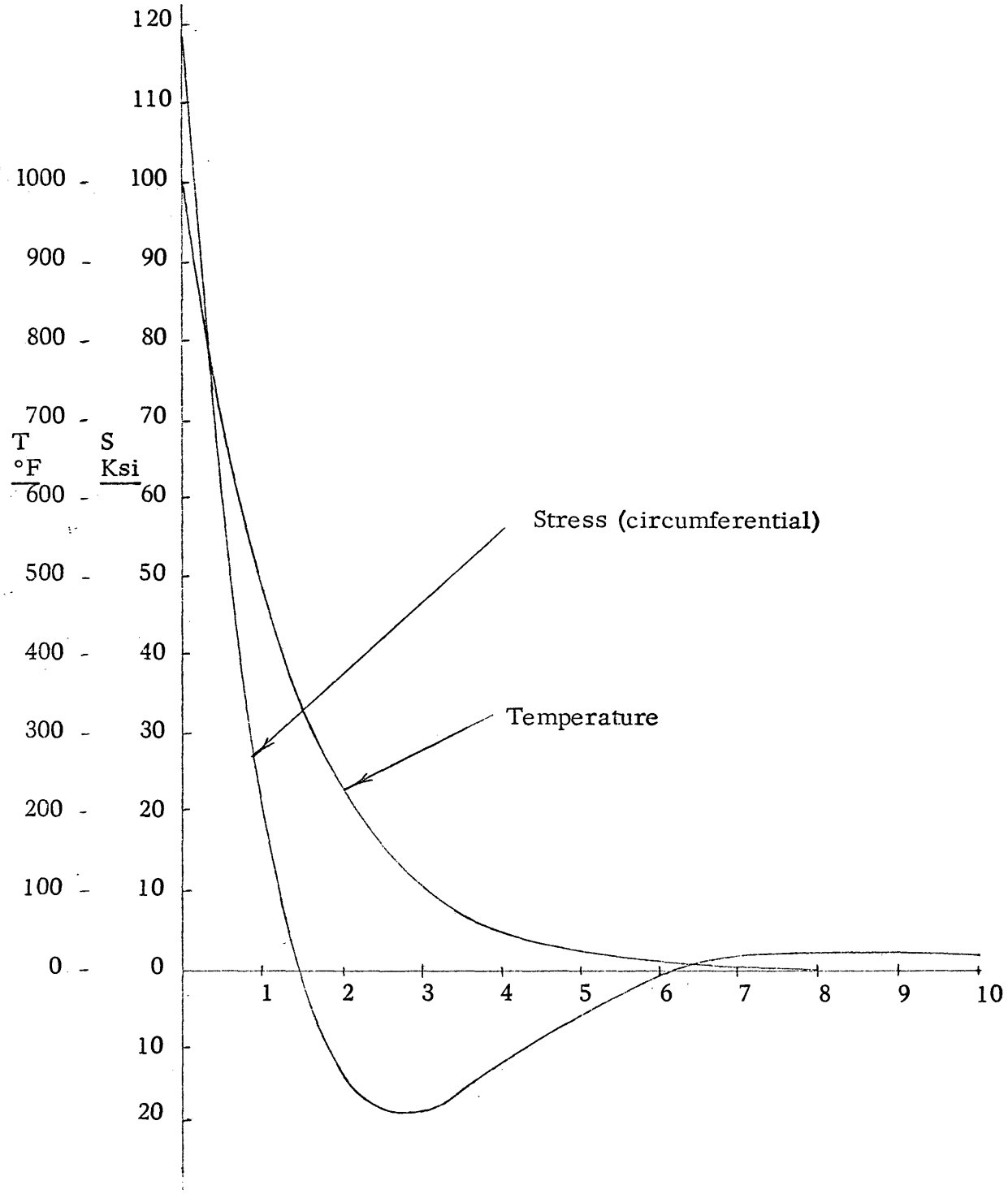


Fig. 19. Distribution of circumferential stress caused by the indicated non-linear temperature distribution

Appendix I. Complementary solution

To solve $$\frac{d^4w}{dx^4} + 4\beta^4 w = 0$$

guess a solution of the form

$$w_c = e^{mx}$$

Substituting

$$m^4 e^{mx} = -4\beta^4 e^{mx}$$

This characteristic equation has four roots

$$-m_1 = m_3 = \sqrt[4]{\frac{4\beta^4}{4}}(1+i) = \beta(1+i) = \beta + \beta i$$

$$-m_2 = m_4 = \sqrt[4]{\frac{4\beta^4}{4}}(-1+i) = \beta(-1+i) = -\beta + i$$

The general solution takes the form

$$w_c = A_1 e^{m_1 x} + A_2 e^{m_2 x} + A_3 e^{m_3 x} + A_4 e^{m_4 x}$$

If we substitute for m_1, m_2, m_3, and m_4 and let

$$e^{i\beta x} = \cos\beta x + i\sin\beta x$$

$$e^{-i\beta x} = \cos\beta x - i\sin\beta x$$

then $$w_c = e^{\beta x}(C_1\cos\beta x + C_2\sin\beta x) + e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x)$$

where $$C_1 = A_1 + A_4$$

where $C_2 = i(A_1 - A_4)$

$$C_3 = A_2 + A_3$$

$$C_4 = i(A_2 - A_3)$$

Since w_c must approach zero as $x \to \infty$, we must conclude that

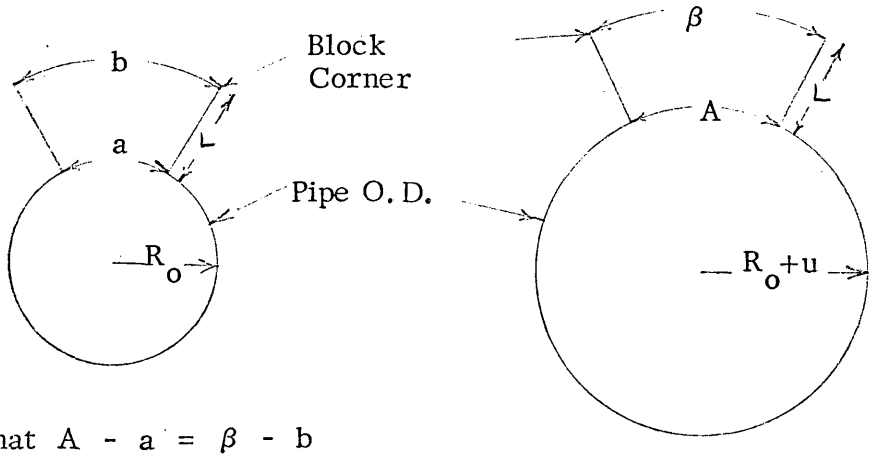
$$C_1 = C_2 = 0$$

Because as $x \to \infty$, $e^{\beta x} \to \infty$

Therefore

$$w_c = e^{-\beta x}(C_3 \cos\beta x + C_4 \sin\beta x)$$

Appendix II. Proof that block corner deflection is the same as surface deflection



Prove that $A - a = \beta - b$

Sectors of the same arc are proportional to the sector radii

$$\frac{b}{a} = \frac{R_o + L}{R_o}, \quad b = a\left(1 + \frac{L}{R_o}\right)$$

$$\frac{\beta}{A} = \frac{R_o + u + L}{R_o + u}, \quad \beta = A\left(1 + \frac{L}{R_o + u}\right)$$

Compatibility

$$\frac{A}{a} = \frac{R_o + u}{R_o}, \quad A = a\left(1 + \frac{u}{R_o}\right)$$

Solving for $A - a$ and $\beta - b$

$$A - a + a\left(\frac{u}{R_o}\right)$$

$$\beta - b = a\left(1 + \frac{u}{R_o}\right)\left(1 + \frac{L}{R_o + u}\right) - a\left(1 + \frac{L}{R_o}\right)$$

Appendix II. Cont'd.

$$\beta - b = a\left(\frac{u}{R_o} + \frac{L(R_o + u)}{R_o(R_o + u)} - \frac{L}{R_o}\frac{(R_o + u)}{(R_o + u)}\right)$$

$$\beta - b = a\left(\frac{u}{R_o}\right)$$

Therefore

$$A - a = \beta - b$$

The constancy of circumferential deflection with radius of lines perpendicular to apipe's outer surface also holds true for circumferential deflections in the pipe wall itself.