FORECASTING THE TIMELINE FOR QUANTUM ADVANTAGE AND ECONOMIC VIABILITY USING OPTION PRICING

by

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ABSTRACT

The potential for quantum computing to significantly reduce the time necessary to process and analyze data has attracted the attention of major financial institutions. These companies are continually researching ways to accelerate data processing and analysis to react faster to market changes and improve the results of their decisions. Quantum computing holds promise as a "game changer" for several different financial applications such as modeling, risk analysis, portfolio planning and automated trading. However, it is unclear if and/or when a quantum computing advantage will exist over current methods and computers. Estimates for such an advantage range anywhere from 3 to 20+ years in the future. The purpose of this project is to provide guidance on when quantum computing will be economically viable for the finance industry. To focus the research, a specific type of finance application: option pricing, was chosen for analysis. Based on this research, an economically viable quantum computer for option pricing will not be available until 2037 at the earliest, with a more likely timeframe of 2040. Further, the payoff period for investment could take another 3 to 5 years. This paper covers available data for predicted improvements in quantum computing hardware, and improvements in algorithms for option pricing, to forecast when such a computer will be available.

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CHAPTER 1 INTRODUCTION

Quantum Computing shows promise as a technology to drastically reduce the time necessary to find an optimal solution for a problem given many inputs. Quantum computers use attributes of quantum mechanics to solve complex problems such as optimization. First is superposition which allows a quantum bit (called a qubit) to exist in both a 0 and 1 state at the same time. Second is entanglement which creates a physical relationship between multiple qubits such that changing one has an impact on the others. These two attributes enable quantum computers to perform operations on all possible solutions to a problem simultaneously and significantly reduce the number of steps required to find an optimal solution. This capability is well suited to solve problems of high complexity where complexity is measured based on the growth of computations required to solve a problem as a function of the number of inputs. Using this definition, the number of computations for highly complex problems grow quadratically, exponentially, or even factorially based on the number of inputs.

Today's computers are based on binary operations where a bit can only be a 1 or 0. These "classical computers" (the term used throughout the remainder of this paper to distinguish from quantum computers) are limited in their ability to address highly complex problems. These problems quickly grow to the point where classical computers are unable to solve them in our lifetime. Quantum computers, on the other hand, will be well suited to address these types of problems [1]. Using the quantum mechanical attributes described above, quantum computers will markedly reduce the number of computations required to find a solution. A fully functioning quantum computer would reduce the number of steps to a point where solutions can be found in a time frame valuable to the researcher or user.

Today's quantum computers demonstrate the potential for solving these types of problems. However, there are numerous technical challenges that must be addressed before a quantum computer is capable of outperforming current classical computers. These challenges include reducing the time necessary to initialize the quantum computer, the error rate for qubit gates, the length of time qubits retain valuable information, the ability to identify and address error in quantum calculations and the number of qubits in a quantum system [2].

The objective for researchers and companies in the space is to address these challenges sufficiently to deliver a *fault-tolerant quantum computer*. Fault-tolerance means the computer can address any errors and reliably complete quantum algorithms. When available, these computers are expected to deliver quantum advantage over classical computers and consistently deliver results without concern of errors impacting the answers. Due to the complexity of challenges that need to be address, the timeline for delivering fault-tolerant quantum computers is currently expected to be at least 10 years.

In the meantime, researchers are investigating the potential value of quantum computers that are not yet fault tolerant. These types of computers are termed Noisy, Intermediate-Sized Quantum computers (NISQ). The question is if there are business or scientific use cases where a good enough answer provides value and where full fault tolerance is not required [3]. The finance industry offers numerous opportunities to use quantum computers to deliver better answers faster than what classical computers can provide today. Answers to finance questions are often never perfect, so a NISQ solution that can provide a better answer, or good enough answer faster, can provide business value [1].

The purpose of this research is to estimate when the different challenges of quantum computers will be addressed sufficiently to provide value as compared with existing classical computers in the finance industry. Solutions to the challenges of quantum computing are progressing at different rates, and in many cases improvement in one area negatively impacts performance in another [4]. And, while significant investment is being made in quantum computers ($2.35 billion in quantum startups in 2022 [5]), performance in classical computers continues to improve [6]. It is not sufficient for a quantum computer to beat today's classical computers; it must outperform a projection of future classical computer performance.

Once quantum computing has progressed to the point where it can outperform the best available classical computers, the next steps is to estimate the economic value of such performance. This research focuses on quantum computers reducing the time necessary to price a stock option and allow faster execution of an option trade. Determining the actual dollar value associated with faster option pricing is not captured in available literature or research studies. Therefore, estimates were necessary to compare potential value of faster pricing. Finally, an estimated cost of a quantum computer is required. The time to payback the cost of the quantum computer using the value derived from faster option pricing, is also included in the time estimate.

It is necessary to make assumptions across all these different factors. To the extent possible, these assumptions have been documented. This provides the reader the opportunity to decide as to the likelihood of the estimated timeframe.

This research is structured as follows: Chapter 2 covers the different types of finance applications that may be able to benefit from functioning NISQ solutions. To focus the study, option prediction pricing is chosen as representative application where the potential value of quantum computers can be measured. Chapter 3 reviews the different types of quantum algorithms available for performing option pricing. Chapter 4 covers the prediction on when a quantum computer with advantage will be available. It includes a detailed review of the challenges that must be overcome for NISQ computers to become viable solutions. Chapter 5 examines the payback period for a quantum computer using value from improved option pricing and trading. It includes highwater benchmark that NISQ systems would need to exceed to demonstrate value. Chapter 6 covers a few technologies that can accelerate the timeline for quantum advantage availability. Finally, Chapter 7 summarizes the study with a roadmap showing different milestones leading to quantum advantage and payback.

CHAPTER 2 FINANCIAL APPLICATION REVIEW

Many financial institutions are investing in quantum computing research. They expect solutions to yield huge competitive advantage over companies that do not use quantum computing. A few examples include Barclays worked with IBM on transaction settlement [7]. BBVA researched potential for investment-portfolio optimization [7]. J.P. Morgan partnered with the Chicago Quantum Exchange to research post-quantum cryptography [8]. Goldman Sachs worked with QC Ware on accelerating Monte Carlo analysis for risk analysis and price estimation [8]. The Commonwealth Bank of Australia has invested in quantum computing since 2014 with start-up company Silicon Quantum Computing to research risk and return optimization as well as post quantum cryptography [9]. These are a few examples where financial institutions are betting on quantum computing. There are likely investments and research not announced as a breakthrough represents competitive advantage for the first mover [7].

2.1 Terminology

Computers are used in numerous ways to execute financial applications. Table 1 describes how each of these approaches is used to perform the application. It is necessary to understand these different approaches, and the different types of financial applications where they can be applied, to determine the potential benefit of quantum computing. Following is a quick overview of different terms used.

2.1.1 Artificial Intelligence

Artificial Intelligence is process of training a computer to perform human like tasks with little to no human intervention and is used extensively to perform financial analysis. Each of the following approaches are applied to different financial applications.

Machine Learning – Earlier attempts at artificial intelligence used a series of rules intended to emulate how humans make decisions and perform activities. In machine learning, computers review large amounts of data and develop a set of patterns that can be applied to a previously unseen set of data.

Deep learning – This is a type of machine learning that emulates a human brain with a set of "neurons" where a calculation is performed on a set of inputs. Connections between neurons are weighted based on the training data and the output of one neuron is sent as data to the next.

Recurrent Neural Networks – While traditional deep learning solutions feed the data in only one direction, a recurrent neural network allows for data to move back in the network to incorporate historical information into the machine learning training.

Long short-term memory – Is a more complex neural network to address some of the issues found with Recurrent Neural Networks. It does a better job of modeling long term dependencies between data.

Regression – A classic mathematical model to find a formula that best represents a set of data. This formula can then be used on new data to determine the appropriate output. The formula can be linear, polynomial, or even more complex depending on the type of data analyzed.

Principal Component Analysis – An approach to simplify a set of data without losing the pertinent information under analysis. It takes a set of data with lots of different columns (features) and strips out any columns that are redundant or do not help distinguish between different entities.

Natural Language Processing – A machine learning approach to interpret human text or speech and determine its meaning.

Large Language Model – is a neural network based on a large set of human language to support more powerful natural language processing. It has analyzed this large set of human language data to improve understanding of previously unseen human text.

Clustering – A machine learning approach to determine how different groups of data points relate to each other. In a large data set, there are certain groups or clusters where these data points are similar in some way.

Reinforcement Learning – A process by which a computer performs an action based on a set of data and is then rewarded for the right actions and punished for wrong actions. These rewards and punishments are used to train the computer to perform better actions in the future, thus learning the data set.

2.1.2 Quantum Speed-up

Quantum computing can dramatically accelerate or speed-up the processing of information by reducing the number of steps. The following terms are used to describe the potential performance improvement.

Polynomial – this type of speed up occurs when the number of steps is reduced by the factor of an exponential. For example, if a classical computer algorithm receives 'n' inputs and requires n^3number of calculations for an answer, it is said to be a polynomial algorithm. If the same problem requires a quantum computer to perform only n^2number of steps, then the quantum computer has a polynomial speed up.

Quadratic – a quadratic formula is specific type of polynomial where the exponent is 2. Quadratic speed up occurs when a classical computer uses n^2 of steps and a quantum computer uses a linear number of steps such as 2n.

Sub-quadratic – represents a speed up where the exponent becomes lower than 2. In some cases, it would be greater than one.

Exponential – Exponential algorithms are those where the number of inputs is the exponent of a complexity calculation. This can look like 2^n number of steps. An exponential speed up occurs when the number of calculations move from exponential to polynomial. For example – an algorithm that required 2^n steps improves to n^2 number of steps. Theoretically exponential speed up could result in a linear number of calculations where the number of steps is on the order of n such as $2n$

To give a sense of scale for speed up, choosing the optimal portfolio of a set of 10 stocks from the S&P 500 would require analyzing over 245 billion trillion different combinations ($\binom{500}{10} = 2.45\text{x}10^{20}$ combinations). This assumes looking at every possible combination of 10 stocks from 500 and is considered a *brute force* approach. An example of quadratic speed up would be to analyze all combinations with only the square root number of the brute force approach. This is an impressive speed up at just over 15 billion steps to cover all combinations. ($\sqrt{2.45x10^{20}} = 15\text{x}10^9$ combinations). However, an exponential speedup would require something on the order of only 50 steps. Exponential speed up allows for calculations that today's classical computers simply cannot perform.

2.1.3 Financial Terminology

Assets – A financial asset is an investment or resource that has the potential to increase in value or provide monetary benefit in the future. A financial asset can be a security (defined below) or cash, loans, or bank deposits.

Security – A security is a specific type of financial asset that is tradeable in financial markets. Securities are investments in corporations (stocks, options), or government entities (bonds).

Derivatives – A financial asset that derives its value from an underlying asset. The value of the derivative is tied to the underlying asset. They allow for investing in anticipated change of the value of the underlying asset.

Options – An option is a specific type of derivative that provides the buyer the opportunity, but not obligation, to purchase or sell an asset at a specified price within a restricted timeframe. The cost of an option is typically a fraction of the cost of the underlying asset. This allows investors to profit from a change in the asset price without having to purchase the entire asset. Options based on company stock price are the most commonly traded.

Call Option – Investors who believe the value of an asset will increase can purchase a Call Option. This gives them the opportunity to purchase the asset at a specified price. If the price of the asset is higher than the price specified by the option, the investor will "exercise the option". The difference between the current price of the asset, and the price specified by the option is the value realized by the investor.

Put Option – Can be considered the opposite of a call option. Investors who believe an asset will decrease in value will purchase a Put Option. In similar fashion, if the asset price does decrease, the investor exercises the option, purchases the asset at the lower price and selling at the higher price, thus realizing financial gain.

Settlement – Completing the process of a transaction. This can include sale of an asset where ownership of the asset is transferred from one party to another and the payment for the asset has been fulfilled. Settlement also includes recording of the transaction. The sequence or order of settlement is a very complex process due to the vast number of transactions that happen daily.

Volatility – Represents the frequency and degree of change in the value of an asset or in the overall financial market. Volatility can be used to represent the risk and opportunity of an investment. High frequency and a large degree of change in the value of an asset leads to the potential for high losses or gains when investing.

Financial Forecasting – Prediction in the future financial performance of a company or entity. These predictions can and are performed on future company revenues, profits as well as cost of selling goods and services. Financial forecasting is based on numerous different factors including historical performance of the company, broad economic situations, and prediction of the specific market where the company competes.

Financial Regulations – define how a company needs to operate and report the performance of operations. These regulations are defined and enforced by the governments where a company conducts business. Examples of regulations include estimating and reporting fair market value of owned assets, timing of transactions for reporting periods, and disclosure of appropriate financial information for investors. Financial institutions such as banks, investment brokerages, insurance companies, are subject to specific regulations to protect the investments of consumers.

Accounting – the process of managing, tracking, recording, and reporting the financial transactions made by a company or other entity. These transactions can include sales of products or services, purchasing of the same, investments to grow the company, leases, purchases, or sales of assets. Accounting includes tracking budgets for each organization as set forth by leadership, summarizing financial data and producing analysis and reports for internal and external stakeholders.

Auditing – the process of reviewing a company or government entity's financial records to verify compliance with financial regulations and internal company policies. Auditing usually involves reviewing the documentation recorded through the process of accounting. Auditing includes identifying areas where a transaction or financial activity does not comply with regulations, or where there is a risk that such activity is not in compliance. Auditing can be performed by an organization internal to the company to identify compliance risks, or external to the company when deemed necessary.

Market Making – an approach to accelerate and simplify the trading process for investors and financial institutions. A Market Maker holds in reserve a large number of securities they wish to sell. These reserves are used to purchase and sell stocks and other securities on a continuous basis which accelerates the process. Market Makers benefit from the spread between the bid price and ask price for a security and will routinely use high frequency trading technology to perform at the appropriate speed.

Bid Ask Spread – When purchasing a security, a "bid" is the highest price an investor is willing to spend to purchase a security. The "ask" is the seller's side of the equation. This is the lowest price that a seller is willing to sell a security. The difference between the bid and ask prices is called the spread. This difference is where many financial institutions can make profit by brokering the transaction between purchasers and sellers.

2.1.4 Option Pricing Algorithms

Black Scholes – Also called the Black Scholes Merton model, is a formula that calculates the price of an option based on a set of assumptions and underlying stock information. It can be used for a very basic type of option called a European option. Thus, it is limited in its ability to predict an option price for more complex options.

Monte Carlo – is a simulation process whereby samples from different data sets that have relevance to the option are combined to a predicted option price. Examples of the data sets include changes in the stock price on which the option is based, changes in how much money is in the market (liquidity), number of trades that have been performed up to this point in the day and so on. Monte Carlo simulations are much more robust than the Black Scholes model.

2.2 Quantum Computing Benefits

Potential benefit for quantum computing exists in numerous areas of financial services companies. The following table combines the results of several different research papers into a summarized view of quantum benefits to finance industry applications. It describes a set of financial applications, how classical computers are used today to perform the application, and how quantum computers can provide an advantage.

The table includes three general sections.

- The first focuses on improving the results from purchasing and selling financial securities. This includes determining a fair price for an asset including stocks and derivatives, optimizing the value of a portfolio of stocks, and performing transactions using automation.

- The second section includes broad financial market activities including settling financial transactions, accelerating the process of security buying and selling and tracking the volatility in the market.
- The third section focuses on activities done by individual companies and financial institutions. These applications include identifying potential for fraud, determining the risk of providing credit, following regulations, and performing the tasks of forecasting and accounting.

Table 2.1 Classical and quantum approach to perform complex financial applications

Financial Application	Description	Classical computer Approach	Quantum Benefit
Asset Pricing	Predicting future financial asset prices based on historical price data	A deep learning method called Recurrent Neural Networks using Long Short-Term Memory shows value in improving prediction results as compared with commonly used approaches [10].	Training these networks are computationally intensive and quantum LSTM solutions demonstrate improvement over training using classical computers [11]
Multi-Asset Trend Following	Predict 1-day return of multi-asset class portfolio	Machine learning through regression is an approach used to predict the returns of a class of assets. Regression uses historical data to create a function that approximates the data and can be used to predict future values.	Different quantum algorithms for regression analysis have been proposed and studied to offer speedup of analysis compared with classical computers. [1]
Prediction of Binary Options / Derivatives Pricing	Predict the outcome of exotic options.	Option pricing is done using Monte Carlo methods where the probability of an outcome is based on random sampling of relevant data.	Quantum computers theoretically offer a quadratic speedup using amplitude estimation as a substitute for Monte Carlo estimation. [12]
Stock Selection and Portfolio Optimization	Identifying a set of stocks with combination of features to balance profit and loss.	Cluster stocks into groups with similar returns and identify those with lower risk, or vice versa. Can be done dynamically based on changing data in the market.	Dynamic Quantum Classification can identify subsets of data critical to accurate clustering. This allows for analysis of large sets of data and improved accuracy of the results [13].

Table 2.1 Continued

Financial Application	Description	Classical computer Approach	Quantum Benefit
Algorithmic Trading	Executing trades by accounting for market variables with little to no human intervention.	Algorithmic trading can be performed using Reinforcement Learning whereby the system is trained based on historical data across several data sources to determine the next best action for training [1]. The amount of relevant data is limited based on the performance of classical computers. This leads to assumptions that may negatively impact financial results.	Research shows Quantum Reinforcement Learning can offer a quadratic speed up over classical computers thus enabling more data to be analyzed for trading decisions [14].
Implied Volatility Estimation	Market's view of likelihood of change in each security's price	Tuning implied volatility modeling is necessary to adjust for market changes and improve estimate results. Tuning in real time requires reduction of the number of features in a machine learning model, typically done using Principal Component Analysis [1].	Identifying the principal components can theoretically be done exponentially faster with quantum computers [15]. However, some research suggests that this exponential speed up is due to assumptions on preparing for PCA and not the algorithm itself [16]
Market Making	Maintains a set of sell orders and buy orders at various quantities and prices. Make profit based on the spread of asks and bids.	Like algorithmic trading, market making can benefit from reinforcement learning [17].	Quantum computers have the potential to improve results and/or accelerate results through Quantum Reinforcement Learning [14].
Transaction Settlement	Which trades should be settled at what time and in what order. This is a complicated optimization problem as it covers total value, legal requirements, and appropriate levels of liquidity [18].	The complexity of this problem grows factorially with the number of payments that need to be settled.	Quantum computers can be used to accelerate the time to develop a near-optimal solution [19].

Table 2.1 Continued

Financial Application	Description	Classical computer Approach	Quantum Benefit
Credit Scoring	Credit risk of loan applications based on experience. Classification to either accept or deny credit.	Use Natural Language Processing to create risk assessment for underbanked applicants where normal financial data is not available. Determining the credit worthiness of applicants based on a set of features compared with historical results. Reducing the number of features makes the process more efficient [1].	Quantum computers have the potential to speed up the process of natural language processing [20].
Fraud Detection	Anomaly detection used to identify out of norm situations that point to fraud. Identify anomalies in financial statements to detect and prevent fraud.	Fraud detection is performed using a machine learning technique called *clustering* where data point anomalies are identified as potential fraud [21].	Quantum machine learning can include additional features as part of training and improve the precision in fraud identification [22].
Financial regulations	Basel III requirements set complicated polices that investment firms must follow and execute stress tests to verify the investments comply [23].	Monte Carlo simulations are used to perform stress tests. These simulations require significant compute resources to run and determine the risk at the required level of precision.	Quantum computing may provide a quadratic speedup and use the acceleration to improve accuracy of the models. Higher accuracy allows banks to use their capital more efficiently [23].
Financial Forecasting	Planning tool to adapt to uncertainty based on predictions. One example would be forecasting annual earnings. This analysis can include non-structured data such as macroeconomic factors, analysis of news articles and sentiment to improve results	News articles, market announcements and analysis on market trends impact future earnings for companies. Financial forecasting is improved through the use Natural Language Processing and Large Language Models to identify and categorize this type of information based on future impact [1].	Research into quantum NLP algorithms shows theoretical sub-quadratic speed up over classical computers which can improve timeliness of adjustments to financial forecasts [20].

Table 2.1 Continued

Financial Application	Description	Classical computer Approach	Quantum Benefit
Accounting and auditing	Identify anomalies in financial statements to detect and prevent fraud.	Natural Language Processing is used to review financial statements [24].	As with Financial Forecasting, quantum computers have the potential to speed up the process of natural language processing [20].

CHAPTER 3 QUANTUM ALGORITHM OVERVIEW

The financial application chosen for this study is the "prediction of binary options / derivatives pricing". An option is a contract that provides the buyer the opportunity, but not obligation, to purchase or sell an asset at a specified price within a restricted timeframe. Options are a type of financial derivative, and the terms "option pricing" and "derivative pricing" are routinely used interchangeably. Option pricing is a process to determine the fair market value of an option. This includes estimating how much it will be worth at the end of the timeframe, and then discounting that value to today's dollars. The result shows what amount should be paid for an option today. The difference between the cost of an option on the market, and the estimated fair market value, is the opportunity for profit.

As mentioned in Table 2.1, Option pricing is routinely performed using Monte Carlo methods where the probability of an outcome is based on random sampling of relevant data. Monte Carlo analysis is performed by repeatedly pulling a set of random samples for each variable with potential impact to the option price. Each set of random samples is termed a path. The option payoff for each path is computed and then the payoffs are averaged to estimate the value for the option.

The accuracy of the Monte Carlo process is based on the number of random samples taken across all the data sets. Specifically, the error of the estimate is inversely proportional to the square root of the number of samples taken. This presents a challenge as more samples requires more computational power to perform in a specified time. Faster and more accurate is better when determining what investments to make on highly volatile stock options. [25]. It is important to note that the Monte Carlo process uses an estimated price calculation for each path. This process converges to an estimate of value rather than using historical true prices for the option

Researchers have experimented with numerous quantum algorithms for option pricing. Following is a subset of these algorithms (identified based on the level of research for each on the specific problem at hand). This section describes each algorithm, how it may provide benefit over Monte Carlo running on classical computers, and perspective on running on NISQ devices. Applicability to this research is also noted.

3.1 Quantum Monte Carlo

The first algorithm covered can be used as a quantum version of Monte Carlo. Quantum Amplitude Estimation, as described below, is used to determine the highest likelihood outcome from a set of probabilities, like the Monte Carlo method [26].

3.1.1 Quantum Amplitude Estimation (QAE)

Algorithm overview: One approach for using quantum computers is to represent a specific occurrence or outcome as a quantum state. The probability of a particular state occurring is represented as an amplitude. QAE is used to estimate the likely outcome by estimating the amplitudes of each state.

Option pricing approach: One type of option, called European options, are "path-independent" where the value of the option is only dependent on the underlying stock price at maturity. For European options, estimating the price is based on prediction of what the stock price will be at a specified time. The following steps are taken to execute Quantum Amplitude Estimation to determine a fair price for the option (this information is based on IBM's Pricing European Options tutorial [27]):

Step 1 – Load the probability distribution for different option values

- The different potential stock prices are limited within a set range.
- That range is divided into the number of potential quantum states that can be represented with the number of qubits available. Assuming n qubits, there are $2^n - 1$ possible price ranges.
- The amplitude for each possible quantum state represents the probability of a particular price at maturity

Following figure shows an example for an option price between $1.25 and $3.00 mapped to 8 different potential quantum states using 3 qubits.

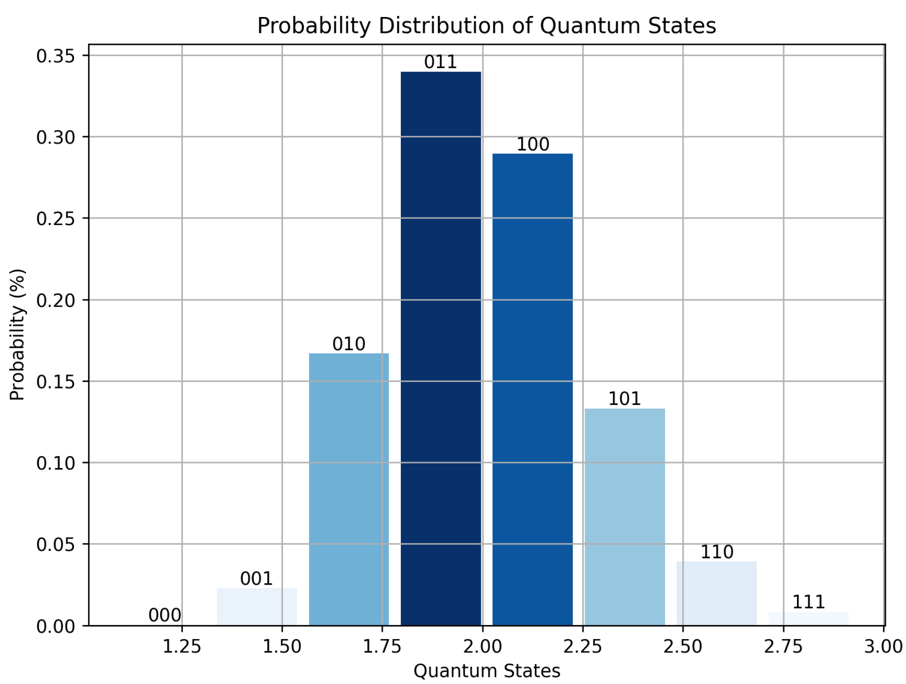

Figure 3.1 Probability distribution based on Black-Scholes-Morton log normal distribution for different option prices

The distribution is represented in a quantum computer as amplitudes for each of the quantum states. As an example, an option price of $2 is associated with the 4^{th} bin in the distribution which is captured in the $|011\rangle$ quantum state. The probability of hitting that price is 34% which is translated into an amplitude and associated with the $|011\rangle$ quantum state. The quantum circuit required for loading this entire distribution is as follows.

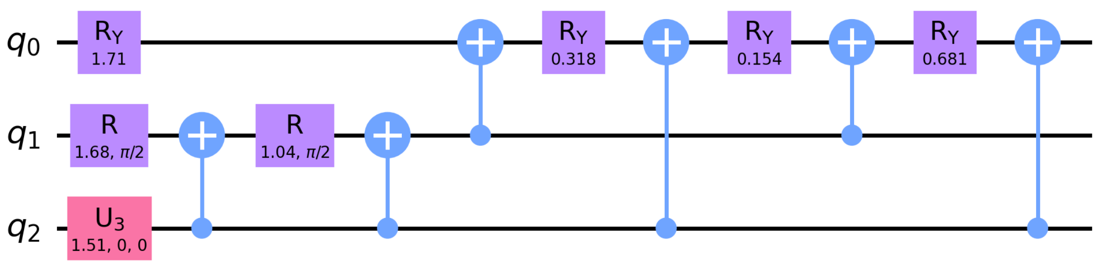

Figure 3.2 Quantum circuit to load probability distribution for eight different option prices

Step 2 – Compute Payoff

The next step uses the distribution captured in step one and calculates the likeliest option price. The result of the circuit is an expectation value, representing the predicted option price. One additional component is to verify the payoff will be above the strike price, meaning it is beneficial to execute the option. This activity is captured in the circuit as well.

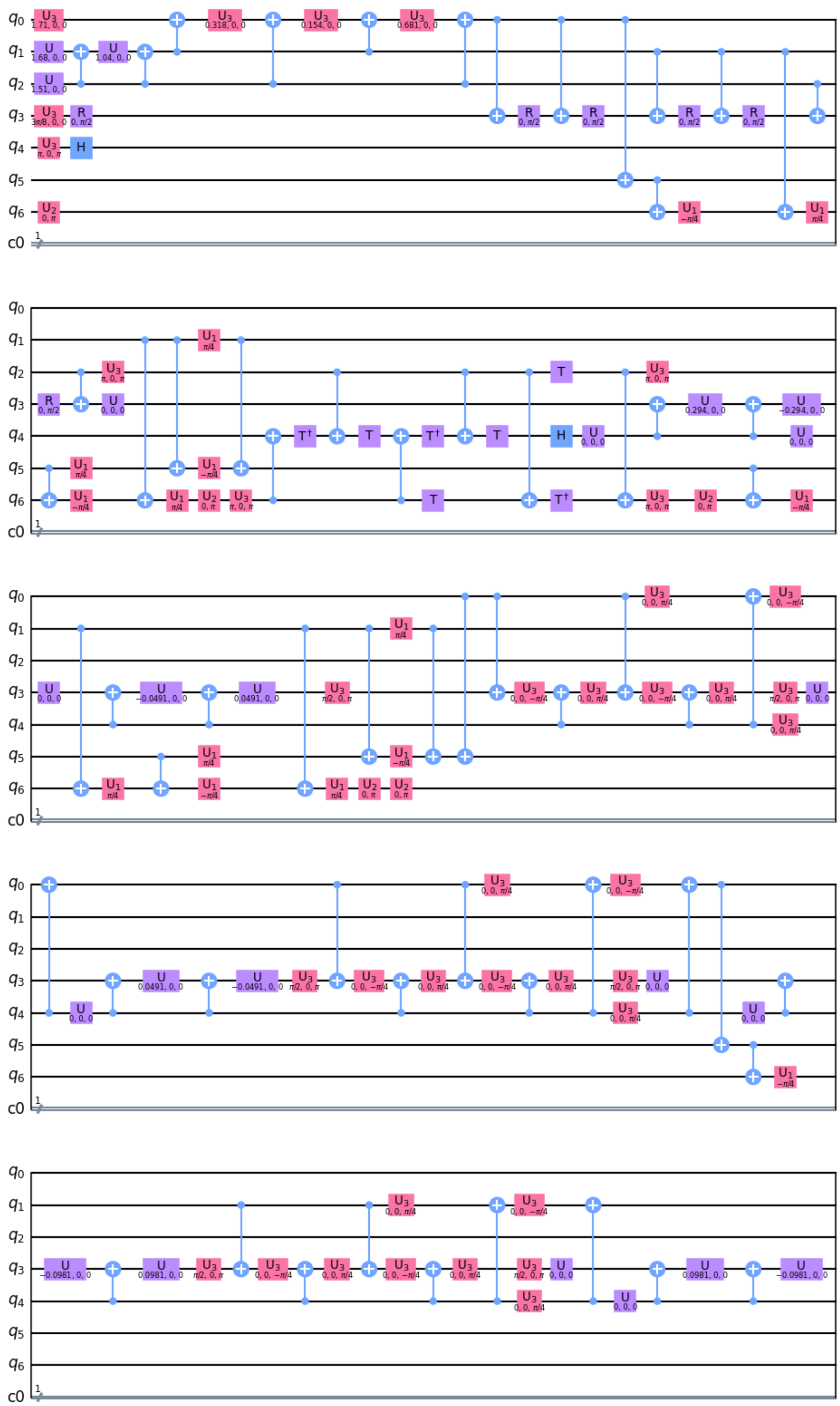

Figure 3.3 Quantum circuit to compute the payoff for an option price using 3 qubits to encode probabilities and 7 qubits to capture results

Step 3 – Evaluate the payoff

The final step is to use Quantum Amplitude Estimation to extract the expectation value from the payoff calculation. Figure 3.4 is a summarized view of the quantum circuit required for payoff evaluation. The circuit amplifies the amplitudes of specific quantum states and encodes the expectation value for each of the 3 qubits onto evaluation qubits and then performs an inverse Quantum Fourier Transform which transforms the encoded information into a measurable result.

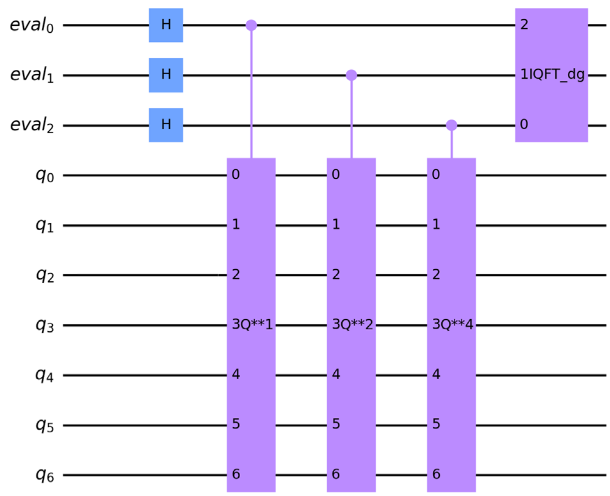

Figure 3.4 Summarized Quantum Amplitude Estimation circuit for reporting the expectation value and payoff of the option

Algorithm Benefit: The number of measurements determines the accuracy of the result with this quantum algorithm requiring quadratically fewer measurements than Monte Carlo performed on a classical computer for the same accuracy. If the desired accuracy for a Monte Carlo analysis, ϵ then the number of samples taken is $\frac{1}{\epsilon^2}$. For QAE, the same accuracy is $1/\epsilon$ [29]. This is the quadratic speedup mentioned above.

NISQ Application: Using Quantum Amplitude Estimation for Monte Carlo analysis can provide a quadratic speed up over classical computers. [28]. As described in the algorithm overview above, initial versions of QAE are performed using a method that requires a fully error corrected quantum computer. The inverse Quantum Fourier Transform circuit grows so large such that NISQ computer would be unable to perform the analysis without errors overriding any valuable results [26]. The simple example described above requires over 14,000 different gates to perform step 3.

However, more recent research has shown that Quantum Amplitude Estimation can be implemented without the need for the Fourier transform and can therefore be explored on near term quantum computers [29]. Using this approach for option pricing would entail completing several iterations of the circuit shown in Figure 3.3 based on the desired accuracy level. A comparison of the circuits required for each approach are shown in Appendix A.

3.2 Optimization Algorithms

The objective of optimization is to find the best solution to a problem out of all possible solutions. Examples of optimization algorithms would be transportation: finding the optimal route to visit multiple sites, retail: scheduling resources for the best combination of cost and customer satisfaction and finance: selecting an optimal set of stocks for a portfolio. Two approaches described below, quantum annealing and quantum walk optimization provide different approaches to solve these types of problems.

3.2.1 Quantum Annealing

Algorithm overview: Quantum annealing is an algorithm that takes its name from materials science. To reduce defects in materials, those materials are heated to the melting point and then slowly cooled to allow molecules to settle to a uniform state. This *annealing* process reduces the number of defects. For quantum annealing, a quantum computer starts in a superposition state of all possible outcomes. As mentioned above, superposition means that a single qubit can be both 0 and 1 at the same time. This concept is expanded to all qubits in the computer. The superposition state is analogous to the melting point of a material where all molecules can flow to any position. It represents a high energy point for the quantum computer.

Slowly the quantum state is adjusted using changes to the electromagnetic fields for each qubit. This process continues until further adjustments no longer reduce the total energy. The lowest energy state is called the ground state, the true solution. However, the process may not find the lowest energy state, and thus it won't find the best solution.

Option pricing approach: Quantum annealing for option pricing requires defining a function where the minimum (or lowest cost) represents the likeliest price for the option. The steps in this process are as follows:

- Determine the function based on the set of relevant data including previous stock prices, market conditions, volatility and so forth. In the case of Sheir Yarkoni et al [30], they used a Mean-Variance Analysis as the cost function for portfolio optimization. This cost function is a tradeoff between the expected return (Mean) and risk (Variance) of a particular portfolio.
- Define a matrix (called a Hamiltonian) that represents the cost function. The solution to this matrix will be represented as the lowest possible energy state in the quantum annealing system (the ground state).
- Adjust the Hamiltonian over time until the system represents the ground state of the desired Hamiltonian and the solution to the minimization function.

Finding the actual ground state (the real solution) is not guaranteed, so this process will typically be used to find the best possible solution in each number of iterations [30].

Algorithm Benefit: Quantum annealing is designed to solve optimization problems, identifying the best solution across several different possible solutions. In the case of option pricing, any price within set constraints is possible. Optimization is getting as close to the real price as possible to make the best investment decision. Empirical evidence shows that quantum annealing solutions converges faster to a non-ideal solution than classical computing [31].

NISQ Application: Quantum annealing is not based on quantum gates and so the required level of control over qubits is not as severe. This allows for current state quantum annealers with thousands of qubits whereas quantum computers using gates have only reached ~500. The challenge is there is no proof yet that the process for quantum annealing can provide a quantum advantage over the potential capabilities of classical computers [30]. Therefore, quantum annealing is not a viable solution for this study.

3.2.2 Quantum Walk Optimization Algorithm (QWOA)

Algorithm overview: A random walk is a search algorithm over a set of connected data points (a graph) to model what is presumably a random process. Each step in a random walk is subject to a specific probability, leading to an end state that is not dependent on history. A Quantum Walk uses the characteristics of quantum computers to reduce the number of steps required to reach a solution endpoint [35].

Option pricing approach: For option pricing, finding the optimal price needs to be defined as an optimization problem, like quantum annealing. However, the design of the problem is different between quantum annealing and quantum walk. With quantum annealing a cost function, that represents the difference between a predicted price and actual price, is defined that can be minimized. Unlike quantum annealing, the problem for a quantum walk is designed as a graph with each node as a potential portfolio configuration

Algorithm Benefit: Slate et al [36] used this approach to optimizing stock portfolios. If the graph in question is designed as a mapping of possible portfolio configurations, the quantum walk searches the graph (with defined constraints) to identify the optimized configuration. QWOA can reduce the number of steps by eliminating any positions that are not valid configurations given the defined constraints. Modifying the graph to model potential values of data impacting the price of an option allows for the use of QWOA for option pricing. David Orrell [37] researched this particular use case for QWOA where the model uses investor decision making as a method for pricing.

NISQ Application: Slate et al [36] mention in their abstract this model is targeted at NISQ devices. However, the paper focuses on comparison of three different quantum algorithms, rather than characteristics that make QWOA a candidate for NISQ. Kalleri et al [38] demonstrate the ability to run quantum walk algorithms on NISQ computers.

3.3 Machine Learning

The idea of using quantum computers for machine learning was first identified in 2008 and has rapidly grown as an area of research since that time. Quantum computers offer potential improvement over classical for machine learning in a couple of different ways. The first is acceleration of solving linear algebra problems. Many machine learning techniques require solving these types of equations. If the number of variables in these equations reaches a certain point, the complexity of the problem can overwhelm classical computers. Quantum computers have shown the potential for exponential speedup for these types of equations [36].

A second area of research is to use both classical and quantum computers to train neural networks on large sets of data. The quantum computer uses adjustable quantum circuits (called parameterized circuits) as the different neurons in deep learning. The classical computer is used to calculate improved settings for each neuron which are then fed back into the quantum computer circuits. Following are descriptions of different machine learning algorithms with application in finance and potentially for option pricing.

Machine learning is, in general, composed of two parts: training the model to effectively analyze unseen data, and then using the model on unseen data to perform the desired analysis and output. This separation between training and execution gives the opportunity for speedup over Monte Carlo methods for option pricing (regardless of the type of computer used, classical or quantum). Training of the machine learning system can be based on historical data related to the option for pricing. Then the execution can be executed immediately on new available data. Monte Carlo on the other hand requires random sampling from data at the moment of pricing, with the number of random samples based on the desired error rate. This results in computation time greater than that of machine learning execution [37].

3.3.1 Quantum Circuit Born Machine (QCBM)

Algorithm Overview: A Quantum Circuit Born Machine is like a neural network in classical computers. A set of adjustable quantum circuits are modified to produce a probability distribution that best represents as set of training data. Once 'trained' the QCBM uses random nature of quantum mechanics to produce samples of a probability distribution [32]. QCBM has a technical advantage over other types of generative modeling as direct sampling can be performed rather than a time-consuming measured sampling [33]. Hibat-Allah et al [46] demonstrated the potential for quantum advantage of QCBMs over classical algorithms such as Transformers, Recurrent Neural Networks, Variational Autoencoders and Wasserstein Generative Adversarial Networks. Their research shows that QCMBs are more efficient than other classical methods when training over a limited data set.

Option pricing approach: A QCBM begins with a random quantum state, and then through iterations, a set of parameterized quantum circuits (PQC) are adjusted to match the state with the probability distribution [34]. Samples are then taken from this distribution like a Monte Carlo analysis to determine the likely price for the option.

Algorithm Benefit: Demonstrations have shown that Quantum Circuit Born Machines can perform Monte Carlo analysis. In addition, Hibat-Allah et al [46] demonstrated the potential for quantum advantage of QCBMs over classical algorithms such as Transformers, Recurrent Neural Networks, Variational Autoencoders and Wasserstein Generative Adversarial Networks. Their research shows that QCMBs are more efficient than other classical methods when training over a limited data set. However, the advantage is related to quality of results and not speed. Therefore, QCBM is not a viable solution for determining timing of quantum advantage for option pricing.

NISQ Application: Researchers have designed algorithms using QCBM specifically for use on NISQ systems. Kiss et al [32] developed a circuit design that requires less connectivity than previous designs. This improves results on NISQ systems. In fact, their results suggest that noise, which is inherent in NISQ devices, is included in the training and can provide benefit assuming the error rate is below a certain threshold.

3.3.2 Quantum Gradient Algorithms (QGA)

Algorithm overview: Gradient Descent is an algorithm that defines a function using a set of adjustable parameters to predict a specific outcome. First, a starting point for each of the parameters is defined based on historical data or random values. Next a prediction is made and compared with the true outcome. The difference between predicted and true outcome is the error function. An adjustment is made to the parameters and the error is calculated again. The impact of the change on the error represents a slope or gradient, which is used to determine the direction of parameter change most likely to reduce the error. This process continues across all parameters until the error rate no longer reduces or a target number of iterations is achieved.

Computing the gradient becomes increasingly complex and computationally intensive as the number of parameters increases. For each iteration of the process, the gradient for each parameter must be calculated individually. If the number of parameters is high enough, classical computers will not be able to perform this analysis in a reasonable timeframe.

On the other hand, quantum computers have the potential to provide an exponential speedup when calculating the gradient. Quantum computers can use superposition to calculate the gradient for all parameters at the same time [39]. At a technical level, each parameter is encoded on a set of qubits which are put into superposition. A set of quantum gates are then applied to calculate the gradients. This information is captured in the phase of the quantum state. Then an Inverse Quantum Fourier Transform is used to obtain the gradient information from the quantum state phase. The research by Chen et al [43] shows that this process can happen with just a single step, significant improvement over the classical process which requires one calculation for every parameter.

Option pricing approach: For pricing options, the data in question can be historical stock price fluctuations, changes in standard ratios call "greeks" that apply to options, or even market trend data.

Algorithm Benefit: Stamatopoulos et al [44] demonstrated the ability to apply Quantum Gradient Algorithms to estimating financial derivative risks. Based on their analysis, QGA could provide a quadratic speed up for determining risk at a defined tolerance level over classical computer methods.

NISQ Application: The research performed by Stamatopoulos [44] provides an estimate of the quantum processing capabilities required to deliver quantum advantage. Specifically, being able to estimate risks associated with the options in less than 1 second. The purpose of this study is to estimate when quantum computers will achieve the defined processing capabilities.

3.3.3 Quantum Support Vector Machine (QSVM)

Algorithm overview: A Support Vector Machine is a type of supervised machine learning algorithm that classifies a set of data points into two categories [42]. Each data point can have any number of different features that describes it. The higher the number of features, the more complicated the process is for classification.

Each feature can be thought of as a different dimension that needs to be analyzed. Quantum computers are highly effective at handling multi-dimensional data. They can be used to execute a support vector machine much more efficiently than classical computers.

Option pricing approach: SVMs have been used to predict whether an 'all-or-nothing' option will pay out. These types of options must meet specific criteria for the payout. The SVM is used to predict if a particular option will meet those criteria [1].

Algorithm Benefit: Quantum computing versions of SVM can provide a quadratic speed up over classical computing SVM [45].

NISQ Application: Kariya et al [46] studied the performance of Quantum Support Vector Machines on currently available quantum hardware (NISQ). Their results show that, as of 2021, technical challenges make the use of NISQ hardware for QSVM infeasible. 62.5% accuracy will not be effective for option pricing predictions, and therefore QSVM is not a viable solution for this study.

3.3.4 Hybrid Quantum Generative Adversarial Network

Algorithm overview: A generative adversarial network (GAN) is a machine learning model that uses two machine learning systems. The first system (generator) attempts to generate output that is indistinguishable from naturally occurring output. The second system (discriminator) attempts to determine if the output was created by the generator or is natural output. GANs are widely used in machine learning on classical computers. A quantum GAN replaces one, or both, of the two systems with a quantum computer [44].

Option pricing approach: A hybrid quantum GAN is where the generator is quantum, and the discriminator is classical. This type of setup is used when input is classical data such as a range of potential prices for the option. Fuchs et al [47] use a quantum generator, and classical computer for a discriminator to generate a probability distribution across the option prices.

Algorithm Benefit: Generating a range of potential option prices is complex due to the number of different factors involved. Each factor can be considered an additional dimension which increases the complexity of calculation for classical computers. However, reviewing a market state and option price is straightforward and well suited for classical computers. Accelerating the process of generation using a quantum computer improves the speed and accuracy of the pricing process.

NISQ Application: Using this hybrid approach increases the efficiency of performing the generative process by reducing the number of quantum gates that need to be adjusted. The hybrid quantum GAN reduces the complexity of loading the distribution from an exponential number of rotation gates to a polynomial number of gates. This reduction improves the potential for using this algorithm on NISQ quantum computers [44].

3.4 Signal Processing

Signal processing is a field of engineering focused on analyzing, measuring, and manipulating both analog and digital signals. Examples include audio, video, digital imaging and so forth. These techniques have started to make their way into the finance industry [46]. Following is an application of a quantum approach to signal processing for option pricing.

3.4.1 Quantum Signal Processing

Algorithm overview: Quantum Signal Processing is a relatively new technique [40] that enables quantum computers to execute a generalized set of mathematical actions called polynomial transformations [41]. These transformations can be used to replace quantum computations and thus the number of quantum resources required for many algorithms.

Option pricing approach: In the case of option pricing, the payoffs for options are encoded directly as quantum amplitudes [41].

Algorithm Benefit: The intent of option pricing is to determine the likely payoff for the option and thus estimate the price. In quantum signal processing, the potential payoffs are encoded into the quantum amplitudes which reduces the number and depth of circuits required to execute option pricing [41].

NISQ Application: The challenge with quantum computers in the NISQ era is the unstable nature of quantum gates and information processing. Reducing the number of gates for option pricing means fewer resources required and can potentially accelerate the availability of a quantum computer with advantage over classical computers [40].

CHAPTER 4 PREDICTED AVAILABILTY

This chapter provides a forecasted time when a quantum computer, capable of demonstrating quantum advantage for option pricing, will be available. Estimated time frames for when quantum computers will provide competitive advantage vary widely. Different components of quantum computing will need to improve to reach quantum advantage for option pricing. These include:

Table 4.1 Set of technical quantum computing challenges that need to be addressed to deliver quantum advantage

Challenge Areas	Description
Initialization	The process of preparing a quantum state to begin calculations is time consuming. This is necessary when attempting to analyze a set of classical data using a quantum computer. A distribution of potential option prices is classical data, and the information must be loaded into the quantum state.
2-qubit gate fidelity	Gate fidelity is a measure of accuracy of the output of a gate in a quantum circuit. Represented as a percentage, gate fidelity shows how close the gate output is compared with the expected output (i.e., the ideal state) based on the mathematical action approximated by the gate. 1-qubit gate fidelity is generally considered to be at a necessary level of accuracy for quantum computation. However, 2-qubit gate fidelity, where the gate reads information from 2-qubits and produces an output on one or both qubits, is a considerable challenge.
Decoherence	Represents how long one or many qubits will maintain their state and the desired quantum information. External noise as well as interference between qubits leads to decoherence where the information is destroyed. Extending the life of quantum information is necessary to allow for quantum calculations to complete and provide the desired output.
Quantum Error Correction (QEC)	One way to address issues with gate fidelity and short decoherence times is to identify and correct quantum errors. Quantum error correction uses several physical qubits to operate as an error corrected logical qubit. Depending on the quantum algorithm used, hundreds, thousands or even millions of physical qubits are required to act as one logical qubit. QEC increases the scalability requirements for quantum computers and decreases the quantum advantage due to the error correction time.
Scalability	Scalability is measured simply as the number of addressable qubits on a quantum computing chip or in a quantum computer. The architecture of the chip needs to be designed so that control devices can act on qubits, and information can transfer between qubits. This is a challenge due to the complexity of controlling qubits which increases as the number of qubits grows.
Readout	Measuring and reading the results of quantum computation also introduces errors into the process. Measurement is the process of reading the final quantum state and transitioning to a classical computer to analyze the results. This process has an error rate (like gate fidelity) that must be accounted for, or improved, to have a viable result from the quantum computer.

Table 4.1 Continued

Challenge Areas	Description
Cosmic Ray Event	Quantum computers are currently very fragile, and calculations are negatively impacted by noise between qubits as well as external variability such as vibrations, heat, and electromagnetic forces. These sources of noise may be overcome using quantum error correction (mentioned previously). However, there are situations where a catastrophic event (at a quantum level) happens, and the calculation cannot be recovered. A Cosmic Ray Event occurs when particles from the sun reach the quantum computer during computation and destroy the quantum information. There are proposed error correction schemes to address these challenges which may reduce the number of catastrophic events [45].
Quantum algorithm	The challenges above drive the need for shorter quantum circuits with lower depth and fewer qubits. Heavy focus is on optimizing existing quantum algorithms or identifying new algorithms with fewer hardware requirements. This reduces the impact of challenges above and moves closer to consistent quantum advantage compared with classical computers.

Superconducting Qubits and Trapped Ions are the two most researched quantum computing hardware platforms. Based on the research available, estimates will be based on performance of superconducting qubits. Companies are researching additional hardware approaches for quantum computing [46]. However, unforeseen technology advances will be required for these platforms to “catch-up” to superconducting qubits and trapped ions. These advances cannot be reasonably predicted; therefore, these platforms are outside the scope of this study.

4.1 Performance Projection Methodology

Moore’s Law is an oft mentioned performance prediction for the increase in the number of transistors on a classical computer chip. The prediction was that this number would double every 24 months, and this improvement has held relatively accurate since 1965 [46].

Unfortunately, such historical information, and simple definition of performance improvement, doesn’t exist yet for quantum computers. As mentioned above, numerous components of a quantum computer must improve to achieve the necessary level of performance for quantum advantage. These measures of improvement are also not consistently captured in the literature.

The estimated time to achieve a target state for each component, and ultimately a quantum computer that can compete with today’s classical computer solutions, is based on:

- A review of literature since 2000 for the component in question
- Identification of performance level for that point in time
- Trajectory of improvement based on available data
- Target date based on the assumption that performance will continue to improve at the previously seen rate.

4.2 Superconducting Qubits

Researchers have studied superconducting qubits as viable solution for quantum computers for well over 20 years. This section of the document will cover several challenges for superconducting qubits including decoherence, 2-qubit gate fidelity, scalability and quantum error correction through logical qubits.

4.2.1 Superconducting Qubit Decoherence

William Oliver and Paul Welander [47] researched the growth in coherence times over the 15 years prior to their paper publication. They found near doubling of coherence time on an annual basis, leading to roughly 100 microseconds before decoherence occurs. Figure 1 shows this growth with a subset of data points identified by [47] up until 2013.

Since that time, there has been no measurable improvement in the decoherence times. Mergenthaler et al [48] state that "many groups around the world now routinely report qubit lifetimes in the 10-100 microsecond range, with average dephasing times that depend on the chosen qubit design." Only a few research articles were found for this study that referenced any specific numbers after 2013. The additional data points shown on Figure 1 represent the current state of superconducting qubit decoherence.

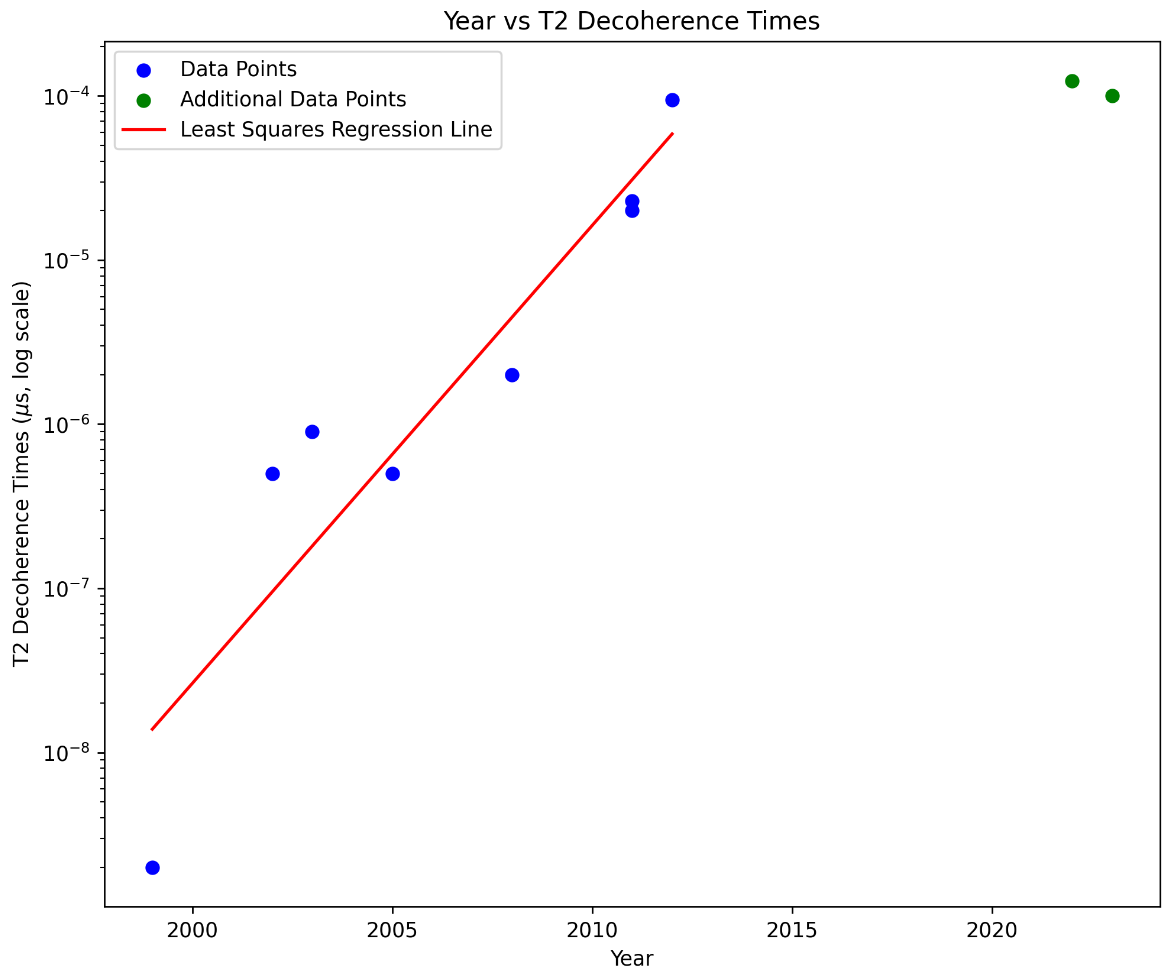

Figure 4.1 The increase in T2 coherence time for superconducting qubits since 1999

Based on this information, it is unlikely to see decoherence times for superconducting qubits improve beyond the 100-microsecond mark.

4.2.2 Fidelity for Two-qubit Gates

Quantum computers require several types of logic gates to perform computational tasks. Each gate requires a different physical implementation. This leads to variations in the gate fidelity. In fact, research papers will typically focus on only one type of gate in the analysis. Table 2 shows the varying degrees of two-qubit gate fidelity across different gate types.

Table 4.2 Highest reported superconducting two-qubit gate fidelities since 2006

Year	CZ	$\sqrt{iSWAP}$	CR	$\sqrt{bSWAP}$	MAP	RIP	CNOT
2006							87%[1]
2007							
2008							
2009	94%[2]						
2010		78%[3]					
2011			81%[3]				
2012		90%[3]		86%[3]			
2013	87%[3]				87.20%[6]		93.47%[3]
2014	99.40%[4]	90%[6]					
2015							
2016	98.53%[3]	98.23%[3]	99.10%[5]			98.50%[6]	
2017	93.60%[3]						
2018		94.00%[3]					99%[6]
2019	99.54%[3]	99.66%[3]					
2020	99.70%[6]						
2021	99.76%[7]	99.87%					99.50%[8]
2022							

The consensus is that two-qubit gate fidelity must be greater than 99.9% to support surface code error correction [4]. Mapping the improvement rate is done using gate error (calculated as $1 - gate\ fidelity$) instead of gate fidelity.

[1] Fidelity information provided by [110]
[2] Fidelity information provided by [108]
[3] [109]
[4] [111]
[5] [112]
[6] [3]
[7] [113]
[8] [114]

One consideration for predicting a target for gate error is the physical limit. While the underlying mechanisms for these limits at the quantum level are not well understood, it is reasonable to assume there is a physical limit to the amount of control that can be exercised over materials used in superconducting qubits. This would set a floor at which point no improvement in gate error is achievable. The following graph assumes physical limits of either 10^{-5} or 10^{-4} for CZ gate error.

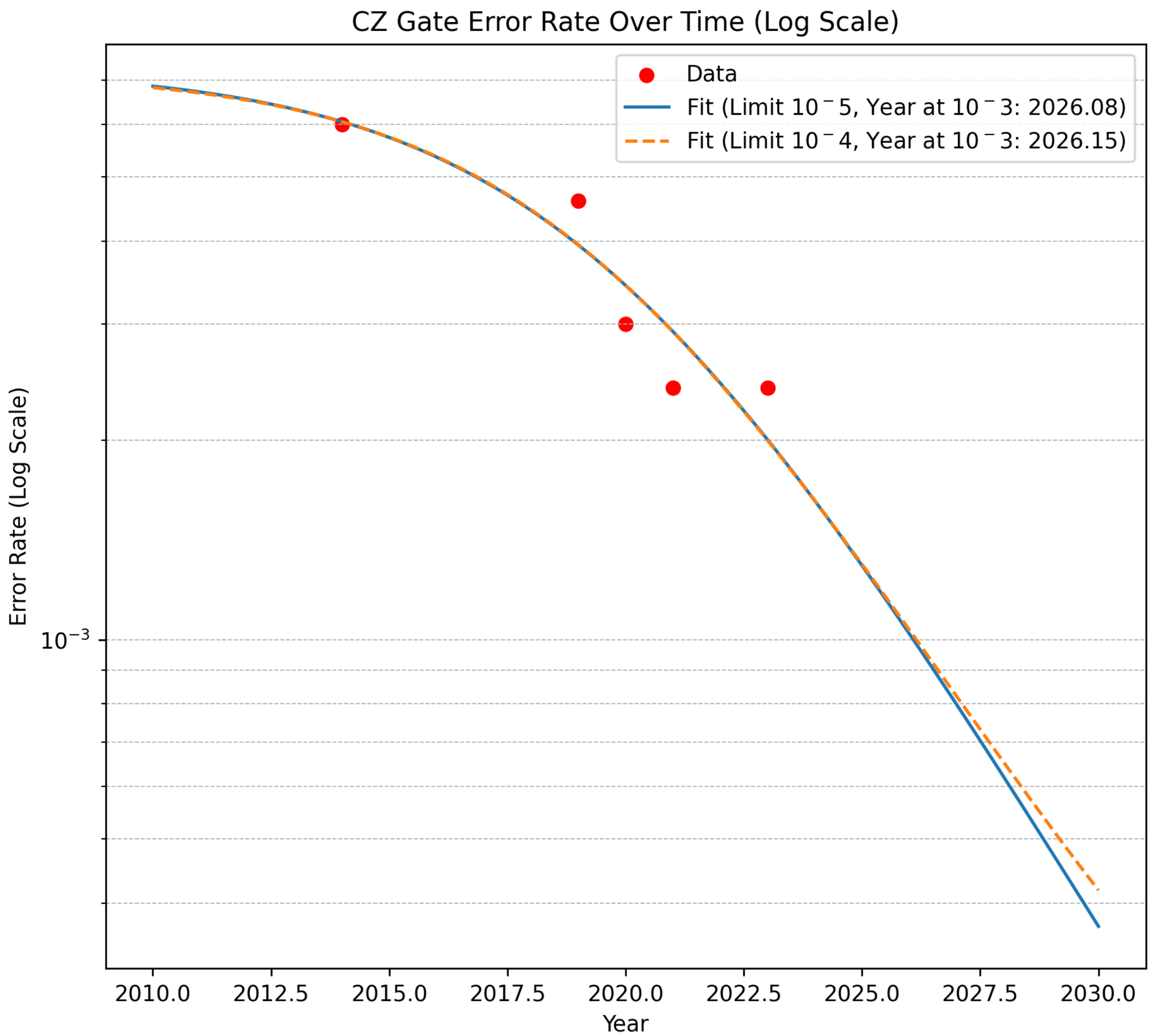

Figure 4.2 Gate error rate improvement for superconducting CZ gate[9] assuming physical limit of either 10^{-5} or 10^{-4}

[9] Note that the recent rapid improvement in gate fidelity would indicate 99.9% should have already been achieved. Since there is no published research showing that achievement, the data is manipulated to assume a constant 'best rate' at 99.76%.

In either case, achieving the target of 10^{-3}(or 99.9%) would occur in 2026. This shows that two qubit gate fidelity is likely not an impediment to quantum advantage for option pricing.

4.2.3 Scalability

Scalability is the number of addressable qubits in a quantum computing device. Achievements in "qubit count" are generally reported as the main measure of increased performance. As mentioned, previous, other improvements in other factors are necessary to achieve quantum advantage, however, it is important to track the increase in qubits.

The growth rate of transistors in classical computing chips have doubled every 1 ½ to 2 years since 1965 [49]. This growth rate has also held consistent across many companies. Superconducting qubit counts have demonstrated similar exponential growth across companies such as D-Wave, IBM, Google and Rigetti as seen in Figure 3. While previous growth patterns show exponential growth, recent data shows a slowing (and in some cases stalling) in the growth of qubit counts [4].

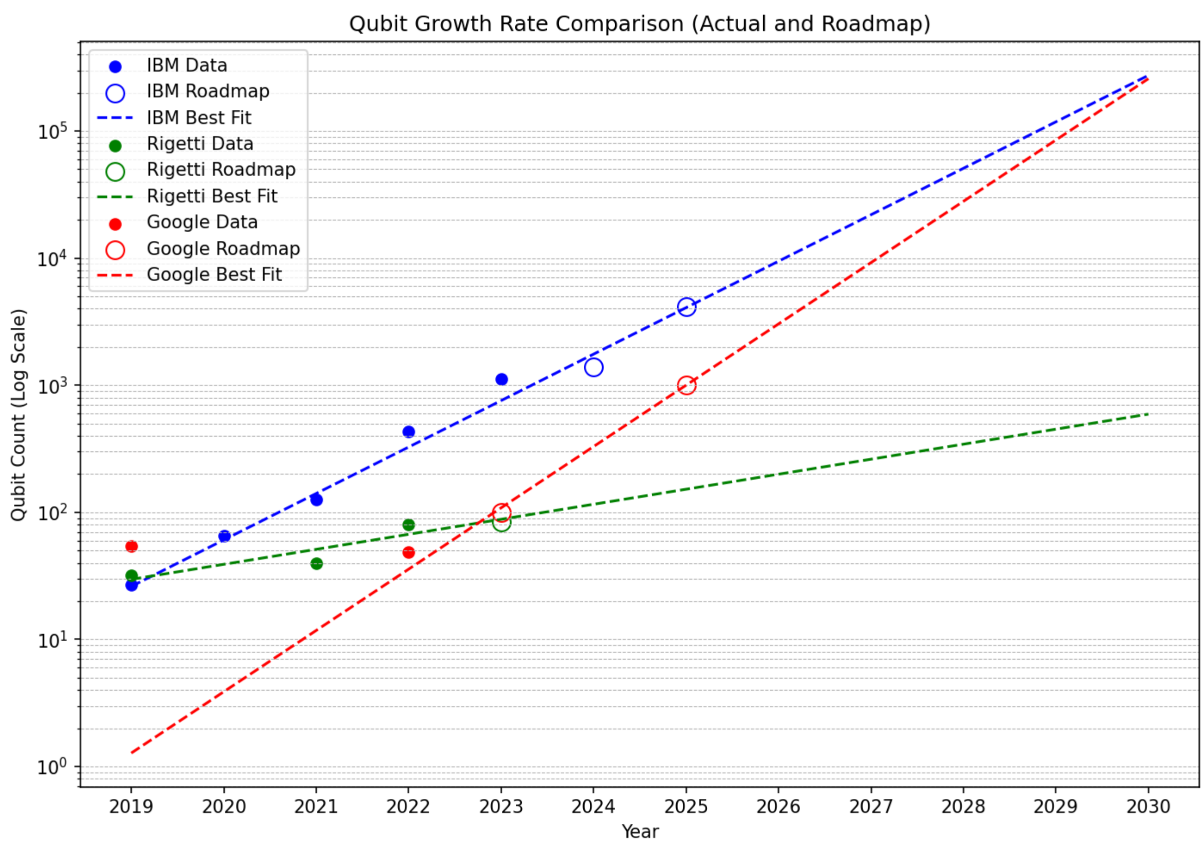

Figure 4.3 Actual and planned growth in qubit counts for IBM, Rigetti and Google starting in 2019

IBM to this point has matched their 2022 roadmap [50] on an annual basis having just announced their 1,121 qubit quantum processor called Condor. One caveat for the future stages of the 2022 roadmap is their hardware architecture transitions from single QPU (Quantum Processing Unit) to multiple QPUs with quantum communication links. According to IBM, the path to superconducting scalability is creating multiple qubit "modules" each with a set of addressable qubits (see roadmap example from IBM [51]). Inter-module connections are required to scale beyond the limit of a single chip. These connections introduce errors into the quantum computation that will need to be limited [46]. It is unclear if that technology will negatively impact the performance and error rate for those computers.

IBM released an updated roadmap [52] at the end of 2023. This roadmap extends to 2033 but moves from showing physical qubit growth to increases in the number of quantum gates and logical qubits. The physical qubits count no longer doubles each year. Instead, the roadmap describes improving quantum circuit quality to increase the number of gates on a quantum chip with 1092 qubits.

Google's roadmap [53] shows 54 qubits in 2019, 10^2 in 2023 and 10^3 in 2025+. Beyond 2025, they have no timelines for future qubit counts. Based on a report in February [54], Google has also transitioned its strategy from growing the number of qubits to improving the stability of their existing systems. The 54-qubit high water mark in 2019 (which was used to demonstrate quantum supremacy in a highly debated paper [55]) was replaced with a 49 qubit computer with improved 2-qubit gate error rates.

Rigetti announced last year they were pushing back availability of their greater than 1000 qubit computers due to rising inflation leading to reduced investment capital and a challenging job market [56]. In their latest investor presentation (August 2023) they show their focus over the past 2 years has been to improve the gate fidelity of their processors rather than growing qubit counts. This is similar to both IBM's and Google's approach mentioned above. Rigetti's largest computer currently has 84 qubits, with an unspecified timeline to get to 336 qubits [57].

4.2.4 Logical Qubits

A logical qubit is a set of physical qubits that work together to perform quantum error correction and improve the fidelity of any computation. The number of physical qubits required to produce a logical qubit is based on the gate fidelity specifically two-qubit gate fidelity as described in SUBSECTION 4.2.2 above.

Based on the previous analysis two-qubit gate fidelity is not an impediment to a NISQ computer capable of demonstrating quantum advantage for option pricing. However, the documents above do not list the number of qubits in the system tested. According to numerical research by Kubo et al [58], a small-scale quantum computer with fewer than 10 superconducting qubits may be able to demonstrate 99.99% success rate. But, to achieve quantum advantage, this fidelity rate must be delivered consistently with computers that have thousands or hundreds of thousands of qubits.

Sevilla et al [59] collected data from 2007 until 2019 that combines both gate fidelity and physical qubit counts. The Quantum Computing report [60] includes data from 2020 through 2022. Following are a few data points with computers that have more than 20 qubits:

Table 4.3 Best achieved gate fidelity for superconducting quantum computers with 20+ qubits since 2018

Company	Designation	Number of Qubits	2-qubit Gate Fidelity	Date
Google	Bristlecone [61]	72	99.4000%	3/5/2018
Google	Sycamore [62]	53	99.6400%	10/23/2019
Rigetti	Aspen-7 [63]	32	95.2000%	10/24/2019
IBM	IBM Q System One [64]	20	98.3100%	1/8/2019
IBM	Brooklyn [65]	65	98.9100%	9/15/2020
IBM	Sherbrooke [65]	127	99.1600%	12/21/2021
IBM	Seattle [65]	433	99.7860%	11/9/2022

Returning to the research by Sevilla [59], the authors developed a model of estimating the number of logical qubits for a given quantum computer based on the number of physical qubits and the average two-qubit gate fidelity. Using this model and the historical growth of qubits and gate fidelity, they have also created a forecast for when quantum computers will likely be able to factor 2048-bit RSA integers. When this occurs, quantum computers could be used to break today's cryptographic secure protection of data (further details on this milestone are in SUBSECTION 4.3.1).

Using their model provides a view to potential timing for quantum advantage. However, there are several assumptions used to develop the model and forecast which need to be considered:

- The authors assume continued exponential growth in the number of qubits and gate fidelity. While exponential growth for qubit count has been demonstrated to this point by IBM [4], other companies have slowed their growth. This makes it a risky assumption for the robustness of the model.
- Further, the authors assume exponential growth in gate fidelity. Ezratty's research [4] shows questionable evidence that gate fidelity has or will continue with exponential improvement.
- Qubit count and 2-qubit gate fidelities are the two variables used to calculate the findings. A third variable, qubit decoherence time, is mentioned as a crucial part of the equation. However, it is not included as part of the model.

Sevilla [59] recognizes these assumptions and state their forecast is to be considered an optimistic lower bound for quantum computing performance growth. Based on the available information, the forecasted years for seeing computers with quantum advantage would be the earliest possible.

Their formula for the number of generalized logical qubits, based on the number of physical qubits and the average 2 qubit gate fidelity is

$$GLQ \approx N_p * \left[4 \frac{\log\left(\frac{\sqrt{10}p_P}{p_l}\right)}{\log\left(\frac{p_{th}}{p_P}\right)} + 1 \right]^{-2} \tag{4.1}$$

This formula is the relationship between the number of physical qubits, two-qubit gate error rate, and required logical qubit error rate. N_p is the number of physical qubits in the system. p_P is the two-qubit error rate. $p_{th}$is the approximate two-qubit gate error rate threshold for error correction ($\approx 10^{-2}$). Finally, $p_l$is the acceptable logical qubit error rate ($= 10^{-18}$).

Using this formula, the authors generate a contour plot showing the combination of physical qubits and two-qubit gate error and the resulting logical qubit count. Then based on this formula, and historical information on qubit counts and error rates since 2007 for superconducting qubits, they generate a prediction on when two goals will be realized. The first is when to expect a single logical qubit that will perform fault-tolerant computation. This prediction is that it will happen no sooner than 2026. The second goal is when we can use a quantum computer to factor RSA 2048 integers. The authors use 4100 as the estimated number of logical qubits required. Based on this information, they predict such a computer no earlier than 2039.

The authors collected data on qubit counts and average two-qubit gate error rates from several different sources. This information covered 2007 to 2019, and as such did not include qubit counts and gate fidelity for more recent quantum computers, specifically IBMs releases of 65, 127 and 433 qubit computers. When adding those numbers into the prediction model, the estimated time to achieve 4100 qubits increases to 2040, indicating the growth in physical qubit counts and fidelities are less than the assumptions made by the authors.

A bootstrapping method is used to determine confidence level in extrapolating the available data to a future date. The variability is in the growth estimates for physical qubit counts and two-qubit error rates using log-linear minimum square error. They calculate the results for 5%, 50% and 95% quartiles showing a distribution of target dates for 1 and 4100 logical qubits. The following chart shows this information across 5 different percentiles.

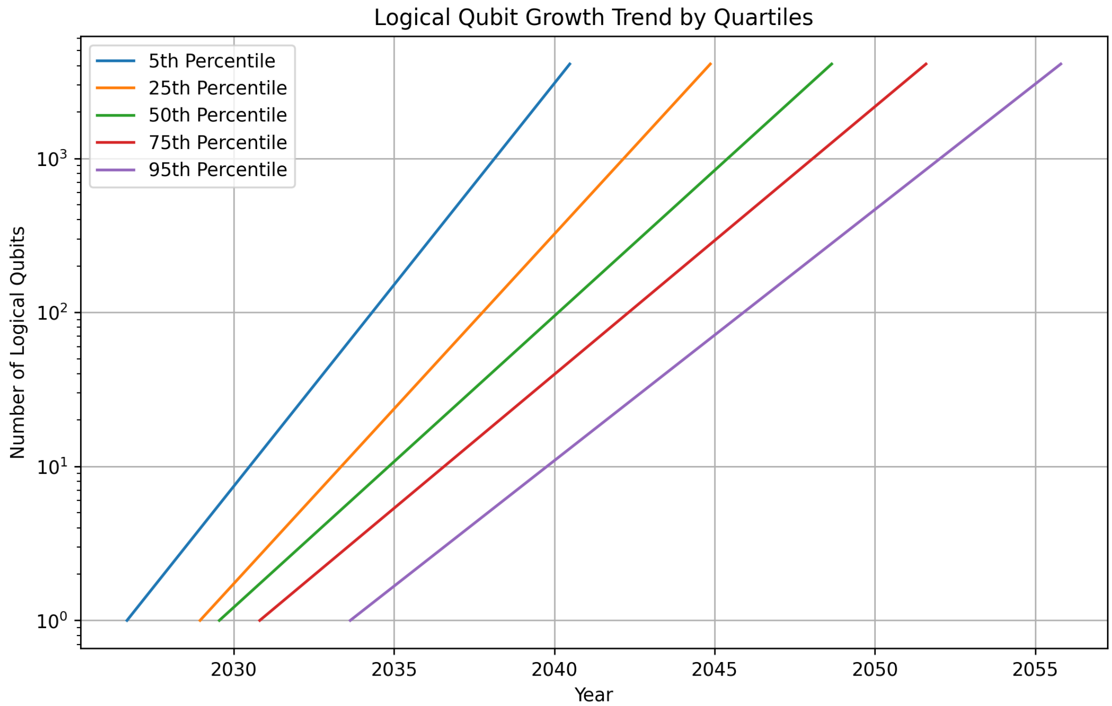

Figure 4.4 Range of estimated increase in the number of logical qubits over time based on the analysis in [59]

4.2.5 Currently Available Logical Qubits

As noted previously, the authors of [59] set the earliest year of availability for a single logical qubit in 2026. Logical qubits have been physically demonstrated, however the error rate when performing two-qubit gates has been greater than 10^{-3} [66]. In December of 2023, a programmable quantum processor was able to demonstrate 48 logical qubits using neutral atoms [67]. The scale of this result is an impressive step forward in demonstrating logical qubits. Again, however, the error rate is still higher than 10^{-3}. Based on the formulation above, a logical qubit will need to have an error rate at 10^{-18} to operate effectively at scale. Current examples of logical qubits demonstrate they can work, but continued, substantial improvement in error rate is necessary.

4.3 Quantum Algorithm Optimization

The next step in forecasting when to expect a quantum advantage for option pricing is to look at the quantum algorithm. The previous section covers how quickly quantum computing physical attributes will improve. Optimizing the quantum algorithm will reduce the physical requirements for option pricing and thus accelerate availability.

Chapter 3 describes several different tested algorithms for option pricing. Chakrabarti et al [68] used Quantum Amplitude Estimation as an algorithm for pricing options. The authors developed what they termed an upper bound for the compute resources required to demonstrate quantum advantage for option pricing. A quantum computer would have to have 8000 logical qubits. The authors also estimate a specific measure of the type and sequence of quantum gates called T-Depth. T-Depth indicates the number of quantum gates that cannot be simulated on classical computers. In this case, the requirement is a T-Depth of 54 million.

In 2022, new research was conducted using Quantum Gradient algorithms for options. Stamatopoulos et al [39] estimated that a quantum computer would require 6000 logical qubits and a T-Depth of 9 million. Further decrease in the number of logical qubits was demonstrated by Stamatopoulos et al [69] using Quantum Signal Processing. In this case, the authors claim a requirement of 4700 logical qubits for quantum advantage.

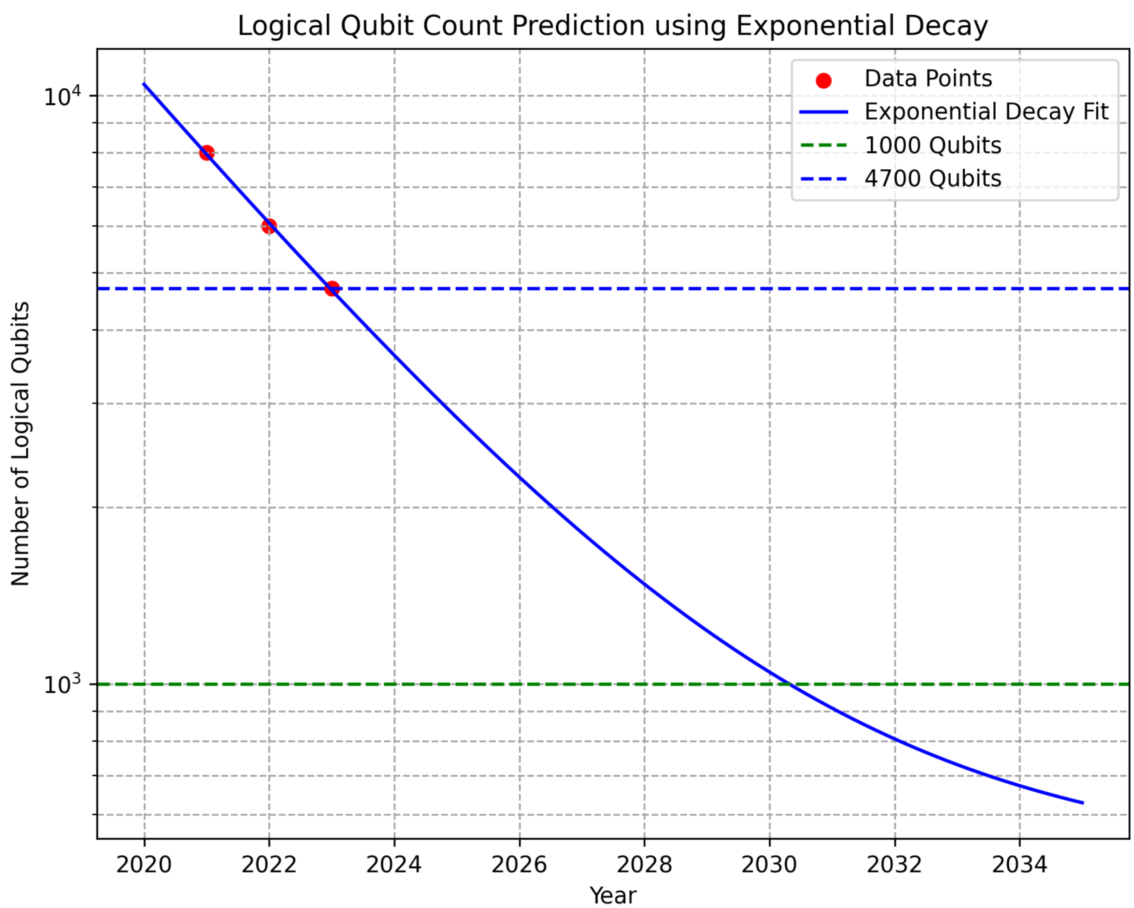

Figure 4.5 Potential reduction in qubit count required for option pricing quantum advantage assuming exponential decay

A prediction on availability of a quantum computer for option pricing could be made using the 4700 logical qubits (represented by the blue dashed line in Figure 4). However, it is not unreasonable to assume continued optimization of the algorithms leading to a lower requirement for the number of logical qubits. Figure 4 shows an exponential decay view of the required logical qubit count (with an assumed floor of 500 logical qubits). Predictions based on three data points have low confidence, however it does provide a view of lower bound when the requirement lines up with the available qubits.

4.3.1 RSA Factorization as an Analogy

As mentioned above, assuming exponential decay for the requirement for qubits based on only three data points provides low confidence. It is possible though to look at requirement improvement in other algorithms to determine if such assumptions can be made. It turns out this rapid and significant reduction in quantum resources for an algorithm would not be unprecedented.

Data encryption is used to protect stored data and secure communications for computerized transactions. Data encryption solutions are based on the challenge classical computers have factoring large numbers. The current standard for encryption is called RSA-2048 which uses numbers large enough that for all practical purposes, classical computers would be unable to factor.

In 1994 Peter Shor developed an algorithm that would allow a sufficiently powerful quantum computer to factor large numbers at a significantly faster rate than classical computers [70]. While classical computers (based on today's technology) would take millions of years to factor large numbers, a quantum computer using Shor's algorithm could take hours or even less. If this capability existed, encryption methods such as RSA-2048 could be broken, exposing business, financial, government and personal data and communications.

The risk posed by quantum computers capable of executing this algorithm has led governments and businesses to develop quantum resistant encryption methods. The question becomes, how long before such a capability exists? The ability to execute Shor's algorithm to factor RSA-2048 integers has become a measuring stick for when true quantum advantage occurs. As such, significant research has gone into determining the quantum computing resources required.

According to Gidney and Ecker [71], the first estimate of resource volume required to factor 2048-bit RSA integers was conducted in 2009 by Meter et al [72]. They estimated 6.5 billion physical qubits would require 410 days to factor. Since that time, the estimate for physical qubits required has dropped significantly. Jones et al [79] estimated 620 million in 2010, Gorman and Campbell [80] estimated 230 million in 2017. Gdiney and Eker's paper in 2019 estimated 20 million qubits were required.

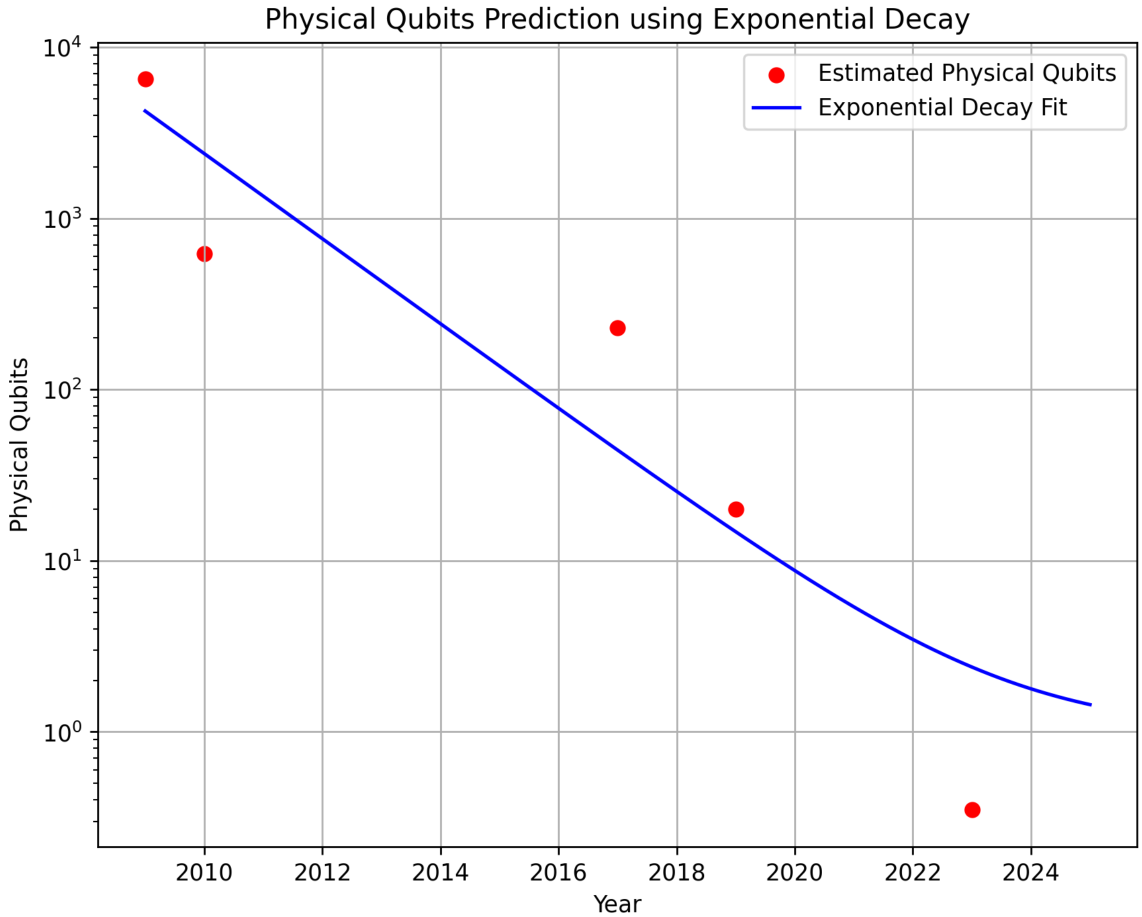

Figure 4.6 Decreasing physical number of qubits to factor RSA2048 integers

The previous set of figures is based on adjustments to the quantum algorithm. Changes to the algorithm reduce the computational power required and thus, the physical hardware. Other researchers propose different topological approaches, using the same superconducting qubits, but operating in a different physical form. One such approach called "Cat-Qubits" demonstrates the potential for even fewer physical qubits required for RSA-2048 factorization. According to Gouzien et al [73] using cat-qubits it is possible to factor such numbers using just under 350,000 in 4 days.

The RSA-2048 factorization quantum volume shows improvement like the predicted reduction in logical qubits required for option-pricing. While not a verification of the potential improvement, this analogy shows empirical evidence that such improvement is possible.

4.4 Timeline Prediction

Combining the information from the superconducting qubit progress and challenges, with the improvements in quantum algorithms for option pricing, allows for a prediction on the earliest to expect a computer with quantum advantage. Overlaying the reduction in logical qubits required from Figure 4 with the prediction of number of logical qubits available over time from Figure 3 shows the following:

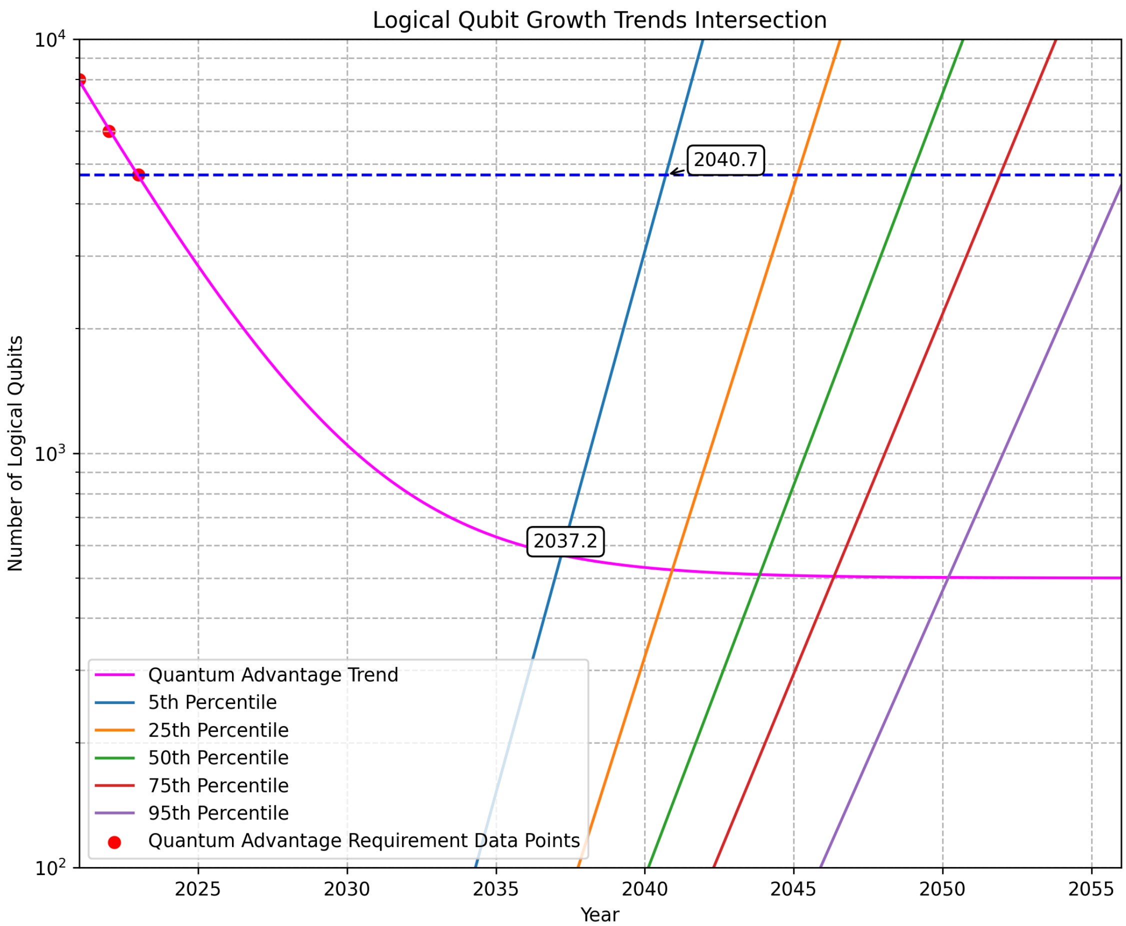

Figure 4.7 Combining predicted reduction in logical qubits count with growth in number possible to estimate year for quantum advantage for option pricing

Based on this information, using several very optimistic assumptions, the earliest to expect a quantum computer capable of outperforming classical computers for pricing options, is 2037. Assuming no improvement in the algorithm for option trading, such a quantum computer wouldn't be available until late 2040.

CHAPTER 5 ECONOMIC VIABILITY OF QUANTUM COMPUTERS

Determining the economic viability of a quantum computer goes beyond estimating when quantum advantage occurs. It is also necessary to determine when the value derived from faster option pricing can cover the investment in purchasing a quantum computer. Calculating when quantum computers will be economically viable for option pricing requires:

- Comparison with classical solutions
- Value associated with the enhanced capability
- Cost of classical and quantum computers

This information can be used to determine the increased revenue from quantum computer option pricing, and how long it will take before the added cost can be recouped.

5.1 Classical Computing Benchmark

The first step, comparison with classical solutions, requires determining how fast a classical computer can perform option pricing. Then estimating how much faster classical computers will be when quantum computers with advantage are available. Three research papers were used for their specific estimate of the necessary performance for quantum advantage in option pricing:

- A Threshold for Quantum Advantage in Derivative Pricing [68]
- Towards Quantum Advantage in Financial Market Risk using Quantum Gradient Algorithms [39]
- Derivative Pricing using Quantum Signal Processing [69]

Each paper refers to a target of pricing options within one second. Chakrabarti [68] mentions that Monte Carlo methods using classical computers can take between five to ten seconds. Stamatopoulos et al[39] claim that calculating Greeks (measurements of risk for option investment) for a basket option can be estimated in 8 seconds. Unfortunately, none of these papers provide reference to a source that validates this performance target.

To determine if the estimates provided above are reasonable, another source is required. A chapter of <u>High-Performance Computing in Finance: Problems, Methods, and Solutions</u> [75] covers the use of classical supercomputers for option pricing. Peter Schober [74] conducts performance testing for option pricing using S&P 500 data. The testing was performed on a small-scale computer with just over 13,000 cores. Time comparisons were performed with increasing number of the cores used.

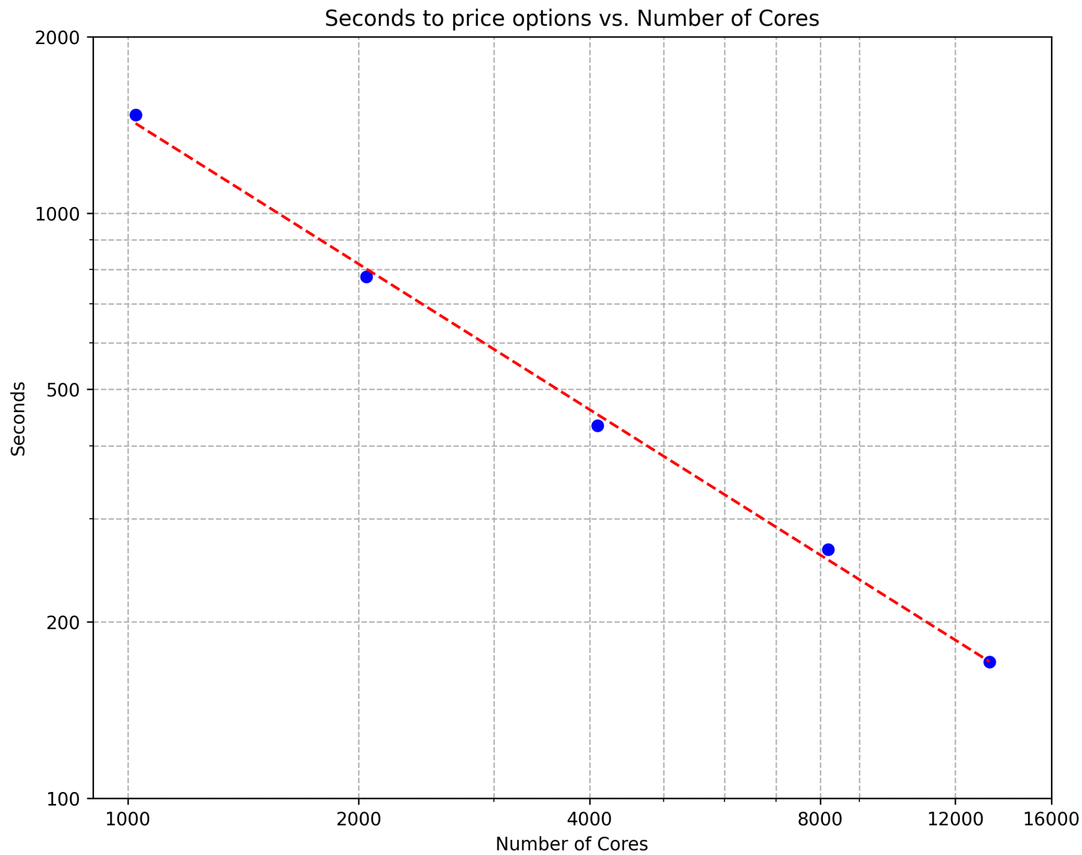

Figure 5.1 Number of seconds required to price options with S&P 500 data based on number of CPU cores used

The testing shows strong scaling correlation between the number of cores and reduction in time to perform the pricing. The performance improvement is due to massive parallelization of the option pricing process. The algorithm priced 493 stocks from the S&P 500 and parallelization maxed out at 13,000 CPU cores, meaning any additional cores would provide no additional speed up. This configuration resulted in pricing taking 170 seconds. Significantly longer than the five to ten second estimate used by Chakrabarti [68]. Because this analysis is fully parallelized, the use of GPUs will likely not provide any additional speedup. GPU performance gains are based on the internal ability to parallelize computations, something that is done using multiple CPUs in this testing.

The testing published by Dempster et al [75] was performed in 2017. The authors used a supercomputer named JUQUEEN which was originally installed in 2012 per the Jülich Supercomputing Centre (JSC) [76]. CPU performance has improved over the past 11 years. Using newer CPUs for the same calculation should result in less time for calculation. According to Hobbhahn and Besiroglu [77] performance of CPUs has doubled every 2.3 years between 2011 and 2021. With this rate of doubling, we would expect the option pricing (assuming a direct correlation between CPU performance and reduction in calculation time) to take 6.17 seconds in 2023. This is right in line with the 5-10 second range mentioned in [68]. If this doubling performance were to continue, the option pricing calculation time would take just over 1/10th of a second, 10 times *faster* than the target of 1 second used in [68], [39] and [69].

However, other sources show the rate of performance improvement of CPUs is dropping. Data from PassMark Software benchmarking [6] is based on a performance test composite called CPU Mark applied to reader submitted CPUs. Their data shows that from 2012 until 2023 the performance improvement has averaged 9% annually for server CPUs and 17% annually for desktop CPUs. A doubling rate of every 7 years and every 4 years respectively. Assuming the doubling rate is very 4 years, the option pricing time in 2037 would be 2.52 seconds.

Some researchers believe the ability to improve CPU hardware performance will reach a stopping point. This is when the number of transistors on a CPU can no longer be increased due in part to the physical limitations of fabricating transistors at such small scale. Shalf [78] provides a graph that shows all components of CPU performance flattening by 2025. This represents the end of performance improvement using semiconductor lithography.

For purposes of value estimation, this paper will use 2 seconds for classical computing processing time for option pricing. This is a very optimistic view of performance scaling for classical computers.

5.2 Value Estimation

Determining the value of quantum advantage requires estimating the financial gain from faster pricing of options. There are numerous factors that impact potential financial gain, making it difficult to find research that can help answer the question. For example: changing the dollar amounts traded using quantum option pricing would drastically impact the value realized. Another example is the probability that the result from option pricing is correct and a profit is realized from the trade.

Nonetheless some estimate of value to help answer the question is necessary for this research. Following is an extremely simple example to provide a basis for discussion. This example

- Is based on TSLA option trading data for one day
- Assumes the binary option pricing is completed in 2 seconds and quantum pricing is completed in 1 second. According to [68] Monte Carlo analysis using classical computers for option pricing can take 5 to 10 seconds. In [39] the authors claim that calculating Greeks (measurements of risk for option investment) for a basket option can be estimated in 8 seconds.
- Uses a range of probabilities the trade is made in the right direction (between 70% and 50%)
- Doesn't assume any transaction fees, which would have an impact if the number of trades is roughly doubled

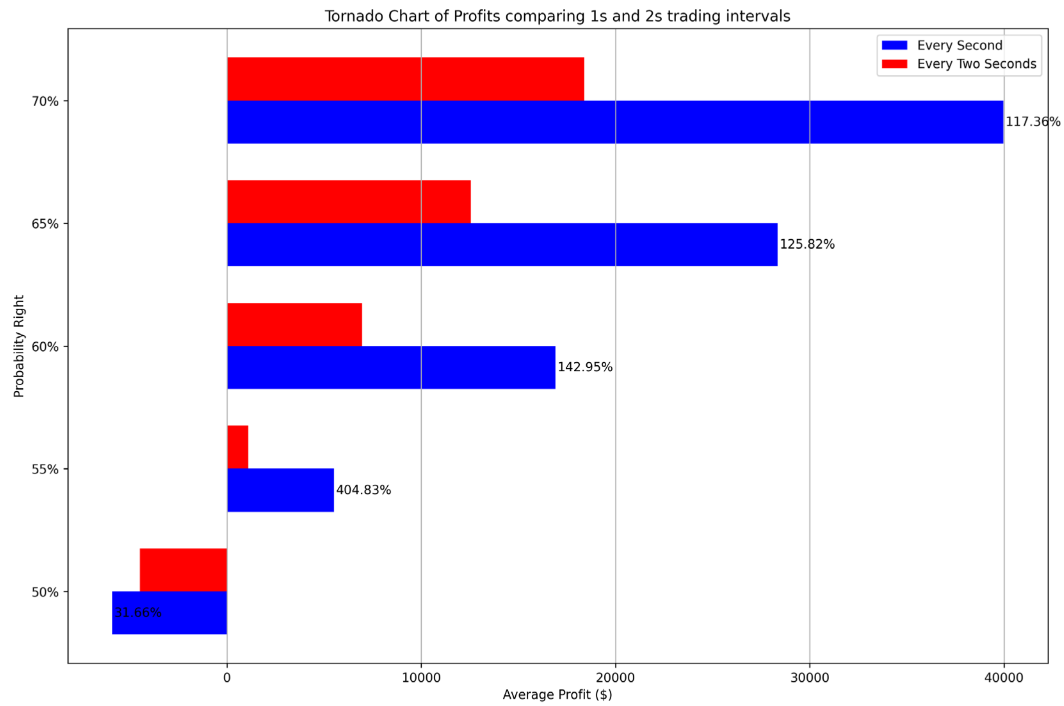

Figure 5.2 Average profit of trading at 1 second vs. 2 second intervals[10] using small sample of option data

The ability to complete option pricing faster and act on the information certainly shows a financial gain. In the case of this simple example, the average total profit for option pricing every second is 145% of the profit for every two seconds.

5.3 Classical Computer Cost Estimate

As mentioned at the outset of this chapter, a value estimation would entail comparison of costs between classical computer capable of option pricing in 2 seconds with a quantum computer capable of the same calculation in 1 second.

A simpler, but no less effective approach would be to assume a financial institution already has a classical computer with the 2 second pricing capability. The benefit of pricing options as fast or faster than competitors will justify the investment to maintain top performing classical computers. Likely, when a quantum computer capable of pricing options at 1 second is available, the classical computer performing this task is in place and a sunk cost.

With this assumption, the payoff period would be for the entire cost of the quantum computer. This avoids the need to estimate the growth in supercomputing costs.

[10] Data obtained from https://www.cboe.com/, The Chicago Board Options Exchange

5.4 Quantum Computer Cost Estimate

Estimating the costs of a quantum computer in the late 2030s is certainly not easy either. Quantum computers of the size required do not exist, and pricing for quantum computers that do exist are rarely published. One example where the cost was published was in mid 2023. Rigetti is a quantum computing company developing superconducting qubits. They sold a 9-qubit quantum computer [79]. The cost was less than $1 million in part because the purchasing company already had the necessary cooling system for the QPU. Different approaches to estimate potential costs for quantum computers are necessary.

5.4.1 Estimate Based on Existing Quantum Technology

One article from a website called “quantumzeitgeist” [80] does provide estimated costs for different elements of superconducting quantum computers based on referenced information from several quantum journals[11]:

- Single superconducting qubit: $1000 to $2000
- Dilution refrigerator[12]: $500,000
- Quantum Software and Algorithm Development: $200,000 to $500,000
- Maintenance and Operational Costs: $1 to $2 million

These estimates are based on today’s numbers, where the number of physical qubits is small (433 at the top end). Based on the analysis in Chapter 4, in 2037, a quantum computer required for option pricing at one second could require anywhere from 70,000 to 380,000 physical qubits. Applying the price information from above to the physical qubit counts results in a quantum computer costing between $71 million and $760 million.

However, there are problems with these estimates. The first, the costs don’t include any information on microwave pulse-based control and readout systems. These systems are constructed using many different off the shelf products to mix and filter desired wave frequencies, amplify the signal and the ability to switch signals off when necessary. The combined cost can exceed ten thousand per qubit. In 2022, Fermilab created a purpose-built control and readout system for superconducting qubits. They claim this solution is a tenth of the cost of current systems [81].

The second problem with the estimate, calling into question the veracity of the report, are the references used. The report claims the cost estimates were based on reports from several different quantum journals. However, this author was unable to find any of the provided references. Searches on the web sites for Journal of Quantum Computing, and Journal of Quantum Information Science did not

[11] The report references The Journal of Quantum Computing, The Journal of Quantum Information Science, The Journal of Quantum Software, and The Journal of Quantum Mechanics

[12]

result in the stated reports for cost information. And based on general web searches, the Journal of Quantum Software, and The Journal of Quantum Mechanics do not exist.

5.4.2 Estimate Based on Existing Super Computers

Today's quantum computers are much like today's supercomputers. A small number of very expensive solutions are purchased and installed at companies, universities, or governments around the world. For example, according to Forbes, IBM has 20 installations of their IBM Quantum System One throughout the world [82]. For supercomputers, the Top500 list which lists the top performing supercomputers in the world, lists the individual location for each system on the list [83]. These are not mass-produced computers, but instead individually built and installed for specific customers.

The best approach to estimate the cost of a quantum computer may be to use costs for classical supercomputers. Current estimated cost for today's supercomputers range between $100 million and $300 million [84]. Meta purchased a supercomputer in early 2022 with an estimated cost of over $400 million. They are now in the process of expanding the computer which could bring the total cost to over $900 million [85]. The Forbes article mentioned above [82] is an announcement from IBM and the Cleveland Clinic to install an IBM Quantum System One on the clinic's main campus. According to the article, this effort was funded in part by the State of Ohio at $500 million. Assuming a significant portion of the funding went to the computer, this is in line with existing supercomputer costs. So, for purposes of the payback analysis, the range of $100 to $300 million will be used.

5.4.3 Energy Costs

These estimates do not include a major component of today's supercomputer costs: energy. An opinion piece by the CMO at QuEra claims that the largest supercomputer cost over $23 million in energy costs each year [86]. Assuming a 10-year lifespan of such a computer, the energy costs are approaching the original cost to purchase the hardware. This is based on consumption of 21.1 megawatts. In contrast, existing superconducting quantum computers use only 10-25 kilowatts. A significant reduction in the energy costs required to operate. It remains to be seen how much energy a scaled quantum computer will consume, but there does exist the potential to reduce energy costs using quantum.

5.5 Payback Period

The payback period is based on the incremental value provided by a quantum computer performing option pricing twice as fast as a classical computer. This allows a financial company to perform twice as many transactions and take advantage of market changes faster.

Estimation of return value based on option pricing depends on numerous factors. Depending on the size of the trade, payback period could theoretically be performed in one day. A more sensible approach is setting a reasonable range for a set of variables that impact the value of a trade and perform Monte Carlo analysis using those ranges. The following chart is a simple model that uses the following value ranges:

- Bid ask spread is .03 with a standard deviation of .01
  - Bid-ask spread is the difference between the highest dollar figure an entity is willing to pay for an option (bid) and the amount an entity is willing to sell it for (ask). This spread represents the potential profit when conducting high speed trading such as what is proposed here.
- Liquidity factor is 1 with a standard deviation of .05
  - Liquidity factor is the amount of available money being traded and will impact the speed at which options will be bought or sold.
- Transaction cost is .005 with a standard deviation of .002
  - Transaction cost is the amount of money required to perform a trade. This impacts the profit of each trade as a required cost of executing the trade.
- Number of shares bought or sold for classical computer is 5000 with standard deviation of 1000
  - For each trade, this is the number of shares an entity will buy or sell, impacting the overall profit or loss. This is the number of shares used for classical computers as part of the calculation
- Number of shares bought or sold for quantum computer is 2500 with standard deviation of 500
  - Because the number of transactions is double for a quantum computer, it is assumed that each transaction must be half of the classical computer trade. Otherwise, the quantum computer would have a profit advantage if it is selling or buying more shares

This model is performed using three different success rates – 70%, 60% and 50%. These values represent the percentage of times a particular trade is profitable. It assumes the ability to trade 365 days per year over 3 years. The model performs 1000 simulations to estimate the profit returns for classical computer performing 10,000 trades per day. For each simulation, the model assumes the quantum computer will perform the same trade as a classical computer, and then interleaves an additional trade by the quantum computer for a total of 20,000 trades per day.

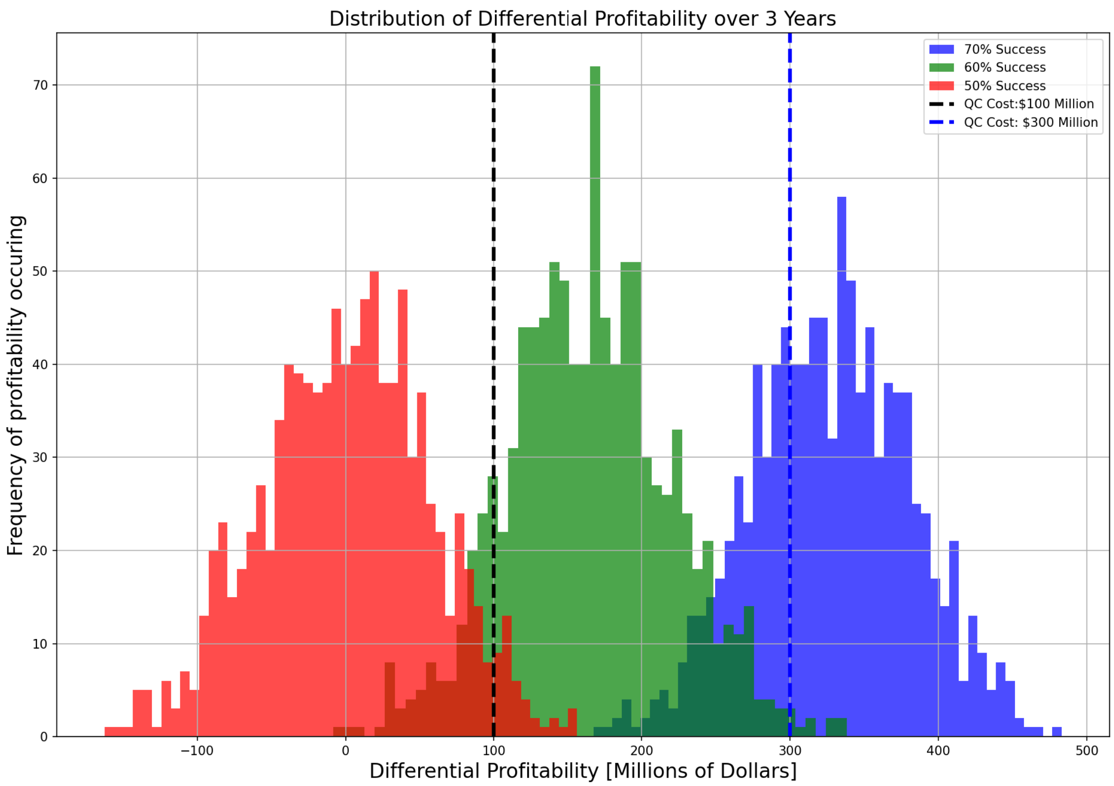

Figure 5.3 Quantum computing profitability over classical computers based on success rate of option pricing results

The groupings show the increasing value of faster trading as the success rate increases. This result is not surprising, as each additional trade provides the opportunity for more profit. Estimating the payback period depends greatly on the cost of the quantum computer. Assuming a $100 million price tag, a 3-year payback period is highly likely for 55% success rate or above. On the other hand, at $300 million cost will likely not be paid back in 3 years. Further analysis shows that a 70% success rate for 5 years would be required for the $700 million payback.

5.6 Exponential Speedup

A consideration for the payback period is the potential for discovering an algorithm that provides exponential speedup using quantum computers for option pricing. Research does lead to the discovery of new algorithms with exponential speedup [87] [88] Recall that Table 1 describes the quantum algorithms for option pricing provide a quadratic speedup over classical algorithms. This speedup allows for the faster completion of pricing an option leading to a faster decision on trades and potential profit.

As described in section 2.1.2, an exponential speedup on high complexity problems is a dramatic increase as compared with quadratic speedup. Application of such an algorithm could be used to further accelerate the option pricing process. This could radically reduce the payback period. Another approach would be to increase the number of datapoints used to perform analysis. Increasing the accuracy of pricing could have just as much impact on reducing the payback period as accelerating option pricing.

The physical requirements for a quantum computer running an exponential speedup algorithm would likely remain the same. The earliest availability of such a computer around 2037. However, the value realized and time to payoff could be significantly faster.

CHAPTER 6 DISCUSSION

The above research is based on extrapolation of ongoing trends in technology. It uses a heavily explored quantum computing technology in superconducting qubits as the basis for prediction of quantum advantage. This document also uses a de-facto standard in classical computing processing speed in Moore's law. Which is based on increasing the number of transistors on a CPU every two+ years. The low watermark for classical computers performing option pricing is based on the slowdown in Moore's law.

This research does not take into consideration the potential for transformational technology. A situation where an unforeseen or an unlikely technology change occurs that delivers faster performance before it was predicted. The potential for transformational technology exists for both quantum computing as well as classical computing. At least seven different qubit technologies are in various stages of research [89]. Each has their own strengths and weaknesses in addressing the different challenges in quantum computing mentioned previously. At the same time, the semiconductor industry has been researching alternatives to the standard general purpose classical computer chips to address the waning year over year performance improvements.

This discussion covers a couple of potential qubit technologies that may accelerate availability of quantum advantage for option pricing. It also covers a couple of approaches to accelerate classical computers whereby the performance target of one second may not be fast enough to demonstrate quantum advantage.

6.1 Alternative Quantum Computing Technologies

This document uses superconducting quantum computing as the hardware basis for analysis. Investment and research in superconducting qubits over the past 30+ years provides a wealth of documentation on how the technology is progressing and forecasted timelines for achieving quantum advantage. This research information is necessary to provide a reasonable estimate for when option pricing will be a viable solution for quantum computers. However, there are numerous other hardware solutions for quantum computers that are in various stages of research.

6.1.1 Photonic Qubits

One such technology is quantum photonic chips which use a single photon of light as a qubit. Photons offer several advantages over superconducting qubits. They are not impacted by electromagnetic interference or heat and can use fiber optic networks to transition between devices. Also, photons can hold their quantum state and do not suffer from decoherence.

PsiQuantum is a quantum computing company working solely on the use of photons as qubits. They are taking advantage of semiconductor fabrication technologies to create chips with all the necessary components for creating photon-based quantum qubits. While challenging, the process for fabricating these chips has proven successful. The challenge now is to use fiber optic connections to join chips together to grow the number of qubits [90].

PsiQuantum recently announced they target delivery of a commercial quantum computer in less than six years [91]. Such a computer would require at least 1 million qubits to be viable for commercial use. If this target holds true, the availability of a computer that provides quantum advantage for option pricing may come much earlier than predicted based on the analysis in this paper.

6.1.2 Topological Qubits

Topological qubits use a very mysterious type of particle called anyons. These are quasiparticles that exist only in two dimensions which provides potential benefits as a qubit. When used to encode quantum information, these particles provide resiliency against noise and control errors. In this way, topological qubits could provide fault tolerant quantum computing, jumping over the NISQ phase altogether [92].

The challenge with this approach is the theoretical nature of anyons. While predicted to exist, their actual existence was not demonstrated until 2020 [93]. This leaves companies working on topological qubits at a disadvantage because they have not had the lead time to experiment with actual anyons. Microsoft has been exploring topological quantum computing since 2005, and have recently demonstrated the ability to generate these type of qubits [94].

Microsoft's approach, similar with PsiQuantum, is to build a fault tolerant quantum computer using what is perceived to be a more stable type of qubit. They have not yet published a roadmap for creating a quantum computer, but if their approach proves successful, the time to a quantum computer may be reduced drastically. This would provide a platform for using quantum computers for options pricing.

6.2 Classical Computation Acceleration

Moore's law states that the number of transistors on a computer chip will double every two years. This observation has been relatively accurate since 1965. However, there is a physical limit to this growth which is fast approaching. The performance improvements based on increasing the number of transistors has slowed recently and shows signs of stopping soon.

Semiconductor companies are looking for alternatives to address this issue and maintain a high rate of performance improvement. Continued improvement is critical to support the growing computation needs of artificial intelligence solutions that have permeated the industry.

6.2.1 Specialized Chips

CPUs and more recently GPUs are designed as general-purpose computer chips. As mentioned before, Moore's law empirically has shown consistent and rapid performance improvement in general purpose chips since 1965. Because of this performance improvement it has not been economically valuable to create specialized hardware chip [78]. Generalized chips were sufficient for most computational requirements.

Now with the slowdown in performance improvement, a shift to purpose-built chip architecture is a viable solution. A specialized chip is one that is designed at the hardware level to perform a computational algorithm. This avoids overhead when using a chip designed for general use and can significantly accelerate operations.

Specialized chips for artificial intelligence and machine learning are already mainstream with solutions from Google, NVidia and Facebook. Nvidia continues to redesign it's AI chips to improve capacity to handle AI workloads [95]. Cerebras is a company that designs large scale chips purpose built for artificial intelligence solutions. Each of these chips has over 2.5 trillion transistors at a cost of $1.5m [96].

Google's Tensor Processing Unit (TPU) is a chip designed at the hardware level to operate machine learning algorithms. Experimentation shows this type of hardware can accelerate Monte Carlos simulations by as much as 250% [97]. As mentioned previously, Monte Carlo is a standard method for option pricing using classical computers. Applying purpose-built hardware for Monte Carlo analysis could decrease the time required to price options faster than forecasted previously. This would raise the stakes for a quantum computer to demonstrate advantage.

6.2.2 New Materials

Today's computer chips are based on CMOS technology which stands for complementary metal oxide semiconductor. The semiconductor industry has reached a point where shrinking CMOS based transistors further reduces the ability to carry electrical current. New materials will be necessary to continue increasing physical compute power of classical computers. Two such materials are graphene, and carbon.

Graphene is a form of graphite (just like pencil lead) that is significantly stronger than steal with electrical current density greater than copper. It has been researched since its discovery in 2004 as a solution for interconnection between transistors on classical computer chips. Graphene's properties help to address issues that occur when attempting to create nanoscale CMOS circuits. The challenge with graphene is the ability to add it to silicon chips after the CMOS transistors have been printed. New methods have been created that may enable mass production and allow for continued shrinking of transistors and increase speed of classical computers [98].

Whereas graphene is designed to connect CMOS transistors, carbon nanotubes can be used as a full replacement for CMOS. Carbon nanotubes are between one and two nanometers in diameter, within the range necessary for continued performance improvement. Research into carbon nanotubes shows the potential to create three dimensional chips. 3D allows for combining computing and memory on the same chip with 1000 times performance improvement [99].

6.3 Discussion Summary

These are just a few potential areas of transformational technology that may disrupt the current trajectory for quantum advantage with option pricing. Additional research, beyond the scope of this paper, could include a broader view of the quantum platforms to determine which has the potential to overtake superconducting qubits. Research is also warranted in the different ways classical computers may improve over time through improvements to chip architecture and materials used for integrated circuits. It may be the target for quantum computers should be much higher than previously stated above.

CHAPTER 7 CONCLUSION

Forecasting the performance growth of quantum computing is challenging due to the many different technological advances required to deliver quantum advantage. Just a few examples include increasing the number of quantum bits (qubits), reducing errors in calculation, combining physical qubits into logical qubits, increasing the lifetime of quantum computation and accelerating the process of loading data into a quantum computer. Improvements in each of these areas occur at different rates, in some cases improving one at the expense of another. The possibility exists that these challenges cannot be overcome and a quantum computer that is faster than classical computers is infeasible.

Nonetheless, many industries believe the benefits outweigh the risks and are investing heavily in quantum computing technology. The finance industry is interested in quantum computing to provide a competitive advantage for several different applications. Examples include risk analysis, fraud detection, optimized investment strategies, volatility estimation, credit scoring and transaction settlement. These applications are performed using high end classical computers, and the potential for quantum computers to outperform existing solutions creates an arms race to deliver such computers.

Options pricing is one application that can benefit from performance improvement offered by quantum computers. Faster option pricing, where a financial company determines the value of an option and performs trades based on that information, results in increased profits. If quantum computers can perform option pricing faster than classical computers, financial institutions using such computers will have a profit advantage. Options pricing has been heavily researched by financial institutions. Leveraging this research provides an opportunity for estimating when a quantum computer with this capability will be available.

Today's research shows a quantum computer would need to have 4700 logical qubits to price options faster than today's classical computers. This is a dramatic improvement over the past two years where 8000 in 2021 and 6000 in 2022 were required, and that improvement may continue. Logical qubits are combinations of physical qubits to address errors in calculation. Forecasting calculations for the availability of such a computer, using a specific technology called superconducting qubits, show when such a computer could not be available. At the earliest, a computer with sufficient power to outperform classical computers would not occur until 2037.

The cost for a quantum computer that can perform option trading and the necessary speed could be extremely high. Making an assumption that quantum computers will cost in the same range as today's supercomputers gives a range of $100 million to $300 million. Payback period for purchasing a quantum computer is highly dependent on the types and size of option trading conducted. Simple modeling shows that a 3-year payback period is likely for the low end of the cost range and 5-years is required for the high end.

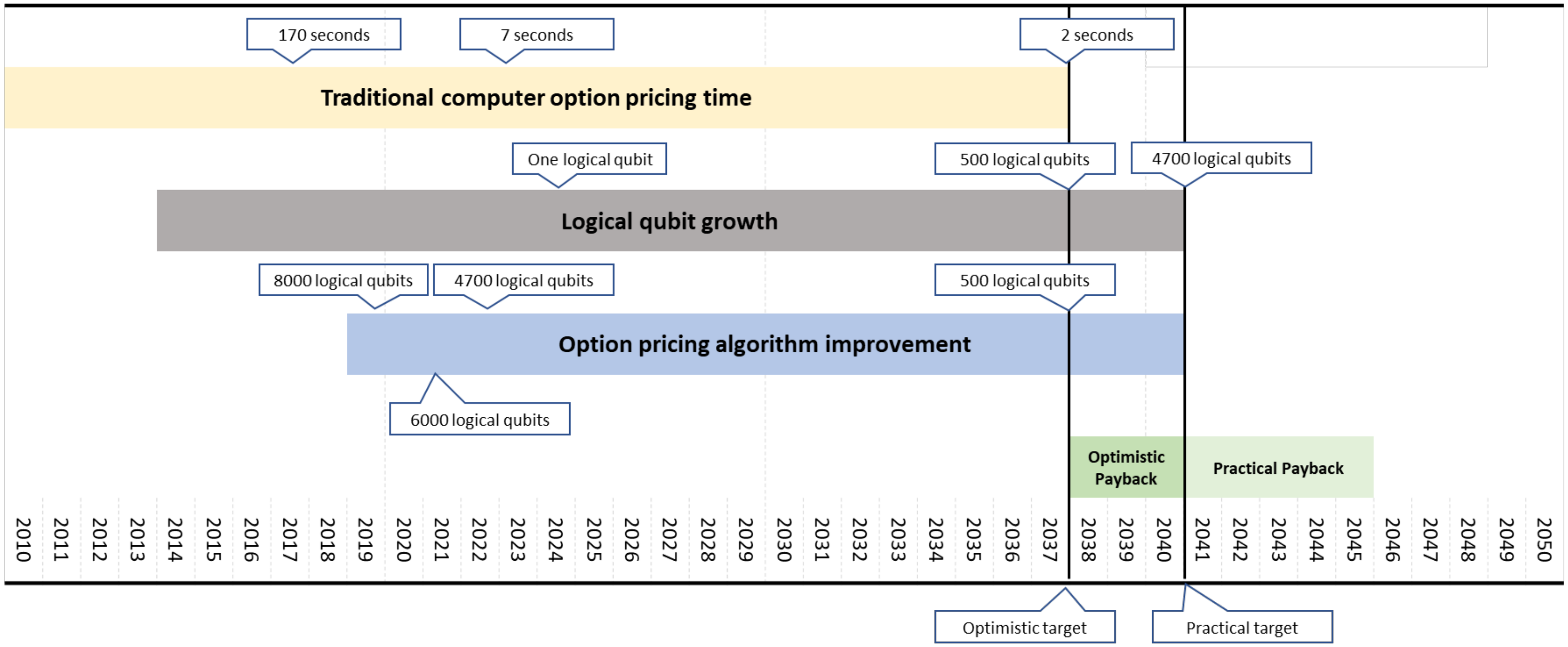

Figure 7.1 Roadmap for Quantum Computing advantage for option pricing

This forecast does not account for any breakthroughs in technology, either with superconducting qubits or otherwise. Prediction of such events is problematic and offer no reliable view into the future. However, these breakthroughs may occur, shrinking the time for availability of a quantum computer that provides advantage. This possibility, as well as a long-term view on the part of financial institutions, will likely lead to continued investment in this exciting technology.

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APPENDIX A QUANTUM CIRCUIT EXAMPLES

A.1 Iterative Quantum Amplitude Estimation

This approach to perform Quantum Amplitude Estimation was proposed by Grinko et al [106]. For option pricing, Iterative Quantum Amplitude Estimation is used to run the payoff calculation circuit multiple times to achieve the desired accuracy.

Following is the payoff calculation circuit for an option pricing that uses only one qubit. The reason for this simple model is to be able to show the circuit required for the original Quantum Amplitude Estimation:

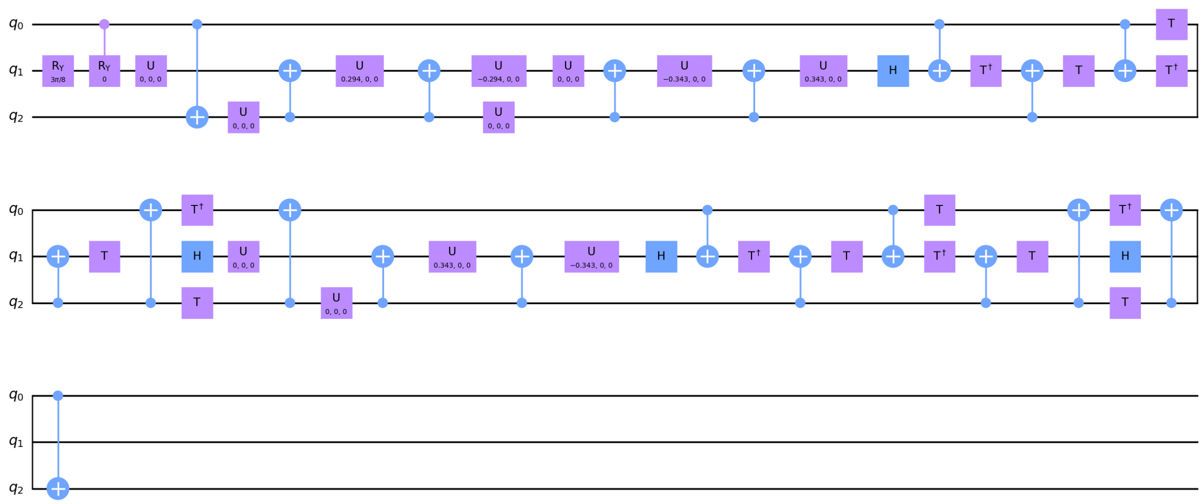

Figure A.1 Iterative Quantum Amplitude Estimation circuit for option pricing using only a single qubit for holding the probability distribution

This circuit requires 52 gates to execute but must execute multiple times to achieve the proper accuracy. The benefit is a reduced set of gates which reduces likelihood of error. This improvement is not sufficient for running on a NISQ computer but does demonstrate the potential to reduce computational requirements [12].

A.2 Original Quantum Amplitude Estimation

In contrast to the IQAE approach, the originally proposed Quantum Amplitude Estimation approach requires a complex set of quantum gates to obtain quadratic speedup over classical approaches [106]. The following circuit uses the same configuration as the IQAE circuit with only one qubit representing the probability distribution:

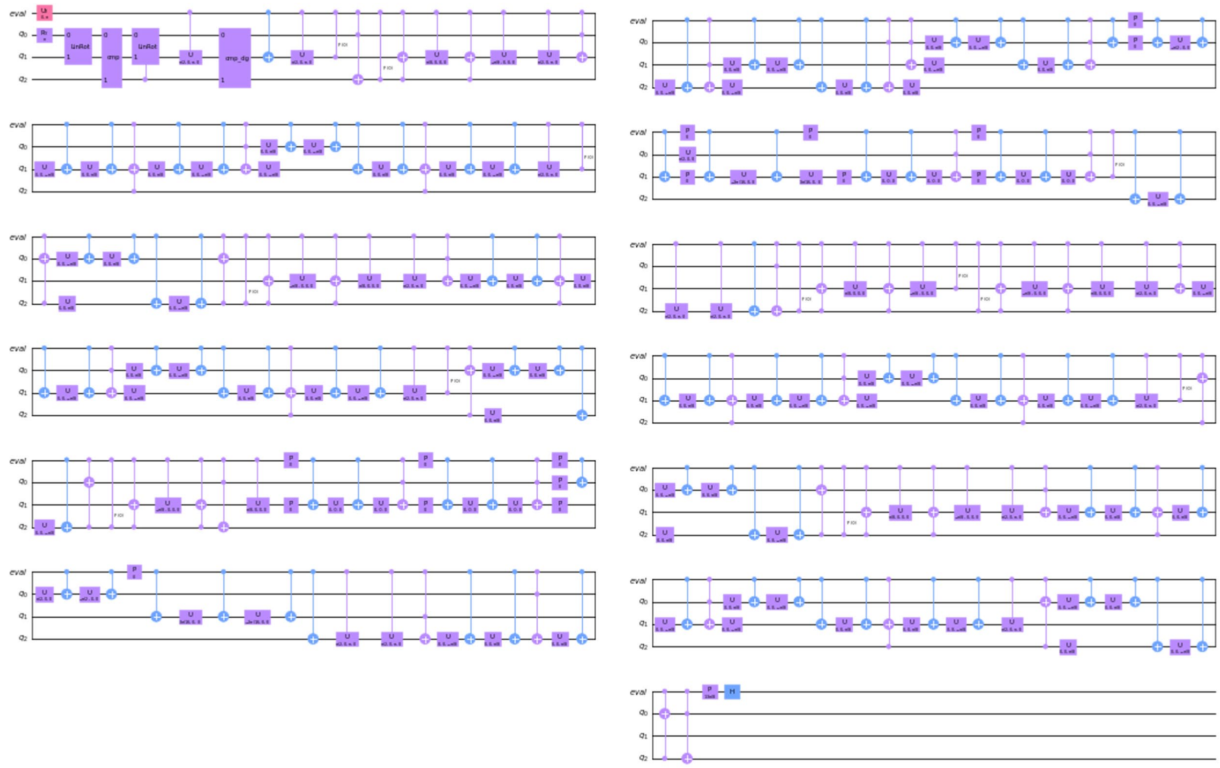

Figure A.2 Quantum Amplitude Estimation circuit for option pricing using only a single qubit for holding the probability distribution

In this case 291 gates are required as compared to 52 for IQAE. Using three qubits for capturing the probability distribution requires 146 gates for IQAE and 14,900 for QAE. This growth rate for QAE drives the need for alternative algorithms until fault tolerant quantum computers are available.