A LEAST COST AIRCRAFT SCHEDULING METHOD FOR THE MILITARY

by

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ABSTRACT

The intent of this thesis is to improve the United States Army's aircraft maintenance system by offering a better method of scheduling aircraft for missions and maintenance flow, through quantitative mathematical methods. The Army's current method of scheduling aircraft is a graphing technique with visual interpretation.

The problem is to schedule aircraft for flights in such a way that the aircraft are systematically moved into a major scheduled inspection. These inspections, phase inspections, take the aircraft out of service.

Through graphing and algebraic techniques the proposed method quantifies the aspects of flowing aircraft through maintenance. Each aircraft's maintenance posture is taken into consideration in the scheduling process. The method determines, from the available pool of tail numbers, the best selection to satisfy upcoming flights. This least cost method minimizes the hours the aircraft can deviate from a planned maintenance flow. Each schedule adjusts the fleet to continue the planned maintenance flow.

This method allows interface with maintenance planners to accommodate special situations. This interface is important for military aircraft scheduling.

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To my wife, Colette, thanks for being devoted us and supporting my endeavors.

Chapter 1

INTRODUCTION AND ANNOTATED BIBLIOGRAPHY

1.1 Introduction

The United States Army's method for scheduling aircraft for daily flights and maintenance is inefficient. It is time consuming and susceptible to human error. The objectives of the maintenance system are clear: to maintain the aircraft to readiness standards and provide mission support. This can be accomplished by defining the variables in the maintenance system, and developing a method that schedules aircraft by serial number for the next day's missions in a systematic fashion. The method developed cannot violate current Army maintenance procedures or policies.

I am an aviation maintenance officer and a maintenance test pilot in the United States Army. With ten years in the Army, I have always served as either a ground or aviation maintenance officer. From December 1984 to October 1988, I personally scheduled or supervised the scheduling of as many as 62 aircraft at one time.

My experiences and the Army's growth are important to the development of this problem. The Aviation Branch is new to the Army. Established on the twelfth of April, 1983, this branch is a combat element that directly engages the enemy with an armada of helicopters. Few procedures existed on how the Aviation Branch would conduct operations. The complexity of helicopters requires special logistical planning. The maintenance practices used to manage the aircraft were the same for ground vehicles with some supporting regulations specifically drafted for aircraft management. The logistical system changed constantly in a search for better ways to provide support. Speculating that there were ways to improve the Army's aviation maintenance system, I began to research how to schedule aircraft for missions and maintenance.

1.2 Military Literature Review

The first step was to review the aircraft and the logistics system. The Colorado National Guard at Buckley Air National Guard Base, Aurora, Colorado assisted in making the current publications available for review. The Army Regulations, Technical Manuals, and Field Manuals that are important to aircraft scheduling are listed in the Bibliography.

The next step was to find any information on upcoming changes in aviation maintenance management. Mr. John J.

Griffiths at the Aviation Systems Command, AVSCOM, is the Chief of the Maintenance Management Division. AVSCOM's headquarters, located in Saint Louis, Missouri is the central facility for aviation logistics. During a visit, Mr. Griffiths displayed one of their top projects, a new automated logistical management system, Unit Level Logistics System-Aviation, ULLS-A. This new system has the capacity to do the administrative logistical work for maintenance managers. Using this software package will save hours of tedious work for maintenance personnel.

The software package is written and designed by Cobro, Earth City, Missouri. The program manager at Cobro, Mr. David Largess, pointed out during an arranged presentation that the Army originally wanted the package to do aircraft scheduling. Cobro's observation of how different maintenance officers at different Army installations scheduled aircraft produced inconsistent results. The attempt to code the results into a computer program was unsuccessful. Consequently, the scheduling portion was left out of the original version of the software package. The software package is to undergo acceptance testing in the later part of 1991.

1.3 Mathematical and Operations Research Literature Review

Once the problem was defined and the current scheduling system examined, the goal was to find possible solutions to the problem. Searching the operations research literature was necessary to locate previous work done on scheduling and capitalize on the findings. However, there were no articles found that were directly related to aircraft scheduling. Other topics like inventory and production control, military logistics, production scheduling, and transportation technology were examined from The Institute of Management Sciences, O.R./M.S Index, 1952-1987. From these articles nothing was applicable to the Army system.

The possible solution to this problem is in the formulation of a linear program or integer program. Two standard linear programming models, the assignment problem and the transportation problem, have the desired properties to solve the posed problem. A search for linear programming, optimization, and operations research techniques provided an abundance of information. These references indicated that the assignment problem is the way to formulate the aircraft scheduling problem. Assignment problems are problems that have the same number of facilities and jobs such that each facility is assigned to only one job at the cheapest cost.

Winston (1991) demonstrates the formulation of an assignment problem as a balanced transportation problem where the supply is equal to the demand. The transportation problem is a method of optimally determining how much of a commodity to distribute from each given supply location to satisfy the demand at other locations. Winston also covers the formulation of an assignment problem as a linear program. He points out another efficient algorithm for solving this type of problem - the Hungarian method.

Anton and Rorres (1987) explain the theorem that solves assignment problems using the Hungarian method. This theorem was first proved in 1931. The method is named after two Hungarian mathematicians: D. Konig and E. Evervary. Anton and Rorres provide the best description of this method.

Woolsey and Swanson (1975) show an out-of-kilter algorithm using network flows to solve the assignment problem. The out-of-kilter algorithm includes a FORTRAN program that formulates the assignment problem as a transshipment problem.

Hiller and Lieberman (1990) tie several mathematical models to the formulation of an assignment problem. These models or methods described by Hiller and Lieberman all find optimal solutions to the assignment problem. The methods discussed are the minimum cost network flow, transportation problems, transshipment problems, integer programming, and linear programming. They briefly describe some of the mathematical results and principles behind these methods.

Churchman, Ackoff, and Arnoff (1961) provide a brief description of the historical contributions by mathematicians. They point out the techniques developed by some of the pioneers in the research of assignment problems like Flood, Kuhn, and Dwyer.

Burkard and Derigs (1980) developed FORTRAN programs for several different types of assignment and matching problems. They rename what is referred to in most texts as the assignment problem, the linear sum assignment problem. They claim that the most efficient computer algorithm uses a modification to Dijkstra's shortest path algorithm. Their modification is called the shortest augmenting path. Their subroutine for the linear sum assignment problem was modified and used in the development of a program for the assignment portion of the aircraft scheduling method.

Wu and Coppins (1981) provide the best description of Dijkstra's shortest path algorithm. They point out that this algorithm was first proposed in 1959 and that it is very efficient and is simple to understand.

Luenberger (1984) in his description of minimum cost network flows and the assignment problem, points out the special mathematical structures in these problems and the benefits gained by these special structures.

1.4 Systems Design Literature Review

In order for a quantitative method to be adopted into the Army's aviation logistical system it must fit into the already established system. A systems design approach is an effective solution. The literature on this subject is often filled with specific designs in engineering. The following references provide structure to the design of a system for any type of project.

Asimow (1962) has fourteen thorough steps outlined for completing a design project. Asimow notes that these steps are not confined to a specific order or application.

Winston (1991) outlines seven steps in methodology for the operations research analysis. Contained in these seven steps are the fourteen steps described by Asimow.

Mott (1991) outlines three phases for a systems design approach to solving problems. Within the three phases are the detailed steps for structuring and solving problems as outlined in the two previous references.

Phase 1: Ascertain the functions of the system and define the problem. Collect data and establish design requirements. Set the objectives for the project.

Phase 2: Nominate alternative solutions to the system. Select the optimum solution for the system and design it in detail.

Phase 3: Make a detailed design of the system. Document the project and its results.

Chapter 2 incorporates Phase 1. Chapters 3 and 4 contain Phase 2, showing the optimal system and how it operates in detail. Chapter 5 concludes with the results of the method as described in Phase 3.

Chapter 2

PROBLEM STATEMENT AND DESIGN REQUIREMENTS

2.1 Problem Statement and System Level Requirements

The objective is to develop a quantitative method for scheduling United States Army aircraft by serial number for the next day's missions. The method will incorporate the Army's current aircraft maintenance tracking procedure, the phase flowchart. The method must pick only one aircraft per mission. Each aircraft selected must be able to do the assigned mission. The method selected must fit into the current maintenance system and be flexible enough to allow maintenance officers to make managerial decisions. The method is to be used by Army aviation organizations that are required to schedule aircraft for missions.

2.2 Background and Justification of Need

Technological advancements in the military have traditionally grown faster than society's ability to use the advancements to their full potential. The U. S. Military prides itself on finding better hardware to fight the battle. Modernization usually results in a lack of skilled personnel to operate and maintain new equipment. One new concept for bringing advanced equipment into the inventory is the Attack Helicopter Training Brigade at Fort Hood, Texas. Developed especially for the new attack helicopter, the AH-64, this training brigade has become an Army standard on how to field new technology. The AH-64 requires special pilot training for tactical flying and special diagnostic and support training for the maintenance crews. Conducted in a team atmosphere, this training has proven successful.

One of the largest advancements in technology has been in aviation. The military forces from all nations forced the growth of aviation into the viable combat role of air power. The Army's inventory has the latest in attack, utility, and cargo helicopters and electronically sophisticated fixed wing airplanes. Army aviation asserted itself in the combat roles as a viable force with the gain of the Aviation Branch.

In the U.S. Army, aircraft are the property of aviation flight companies. The number of personnel and aircraft that belong to a flight company depends on the type and quantity of aircraft. Approximately four flight companies, a maintenance company, and a headquarters company constitute an aviation battalion. The flight company commanders' responsibilities are to maintain their airframes and to schedule the aircraft for maintenance and missions. Poor scheduling by a single flight company could cripple the company's combat posture. These actions would place greater manpower demands on the maintenance company, thus reducing the battalion's combat power.

2.3 The Army Scheduling System

The logisticians are responsible for building systems that will sustain the equipment. Maintenance officers plan and execute maintenance operations; they inform the commander of the most effective way to use equipment under given circumstances.

The maintenance officers directly oversee the scheduling of aircraft maintenance to meet mission requirements. The Army's current scheduling method outlined in the Army Aviation Maintenance manual, FM 1-500, is the sliding scale scheduling method. This method simply involves plotting the aircraft against the hours remaining until the crafts' next phase inspection onto a chart. Maintenance officers refer to this diagram as the phase flowchart.

A typical phase flowchart is illustrated in figure 2.1. This flowchart is set up for seven utility helicopters. The UH-1 is a multipurpose transport. The paragraphs following figure 2.1 will explain all the facts in FM 1-500 on scheduling.

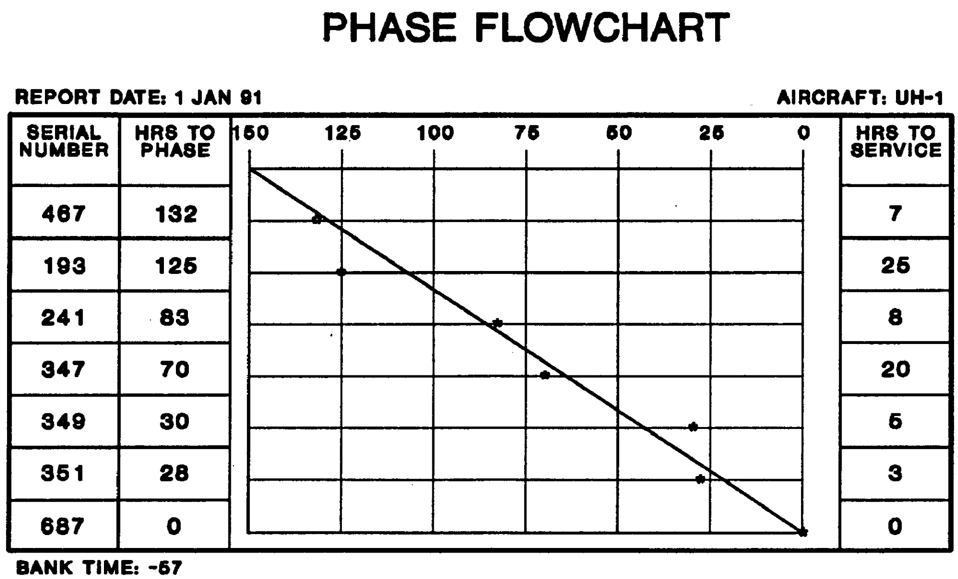


Figure 2.1 Phase Flowchart

The sliding scale scheduling method has a diagonal line that is drawn from the upper left hand corner of the chart down to the lower right hand corner. FM 1-500 defines this line as the optimal line. This line is used to maintain the hourly spacing between each aircraft. Ideally, if all of the aircraft were plotted exactly on the optimal line each aircraft would have equal spacing in hours apart from the next aircraft on the flowchart. When scheduling by this method, aircraft plotted to the right of the optimal line are considered overflown and should be restricted from flights until they return to the line. The ability to turn down a flight is more of a political issue than just ideal management. Consequently, scheduled missions are very seldom turned down unless they are generated by the unit itself for training or other purposes. Airframes that are plotted left of the line are available to fly missions until they reach the optimal line. This chart is only effective when updated daily, and common practice in the unit is to put the aircraft status somewhere on the flowchart. Aircraft that are not flyable, or are grounded, remain on the flowchart but are not available for flights.

The left side of the chart contains the aircraft's serial numbers or tail numbers. All aircraft of the same type and series are listed on the flowchart. A separate flowchart is required for each type of aircraft in the company.

The hours to phase column represents the flyable hours remaining on the aircraft until next phase inspection. Phase maintenance is a major airframe inspection that requires the disassembly of the aircraft and replacement of any questionable components during the inspection. The phase inspection also provides time to replace any of the time life components, and a thorough inspection of the airframe's forms and records. These phase intervals are established by AVSCOM for each type or mission design series of aircraft in the Army's inventory. For example, the utility helicopter, UH-1, has a 150 flight hour interval while the observation helicopter, OH-58, has a 300 flight hour interval. This inspection can take months to complete and hundreds of man-hours, or it can take less than a week. Some factors that determine phase completion time are the condition of the aircraft when it is accepted for the inspection, the number of components to be replaced and the availability of the parts, and the number of trained mechanics available to complete the inspection. When the aircraft is released from the phase inspection it must be test flown by a maintenance officer to certify the craft's airworthiness. After the maintenance officer has released the aircraft for general aviation use, the production control section places the craft's tail number at the top of the hours to phase column. This is the manner in which the aircraft cycle through the flowchart.

The right hand column shows hours until next scheduled service. The UH-1 has a scheduled service every 25 flight hours. In addition to the 25 hour service, there are other routine inspections that are required either by flight hour or calendar dates. The next upcoming inspection should be listed in that column. Figure 2.1 only shows those hours until the next 25 hour service. With the sliding scale method, the individual scheduling the aircraft must be familiar with the aircraft and the amount of hours between each required inspection.

The last management tool described in FM 1-500 is bank time, which is located in the lower left hand corner of Figure 2.1. Bank time is mathematically related to the optimum line on the phase flowchart and is recognized as the area of a triangle. Bank time is used as an indicator to determine if the scheduling is meeting the maintenance objectives. The formula for the optimum bank time from FM 1-500 is:

$$(N)(P)(1/2) = O_{bt}.$$

Where:

N = The number of aircraft by type assigned to a unit.

P = The phase interval for the particular type of aircraft.

O_{bt} = Optimum bank time.

The optimum bank time for the sample flowchart is:

[7(aircraft)] x [150(hours)] x [1/2] = 525 (aircraft hours).

Actual bank time is the sum of the time remaining on all of the aircraft until the next phase inspection. The bank time figure is required for each type of aircraft in the unit.

$$A_{bt} = \sum_{i=1}^{n} h_i .$$

Where:

$N = \{1,2,\ldots,n\}$, The number of aircraft by type.

h_i = The hours remaining to phase for each aircraft.

A_{bt} = Actual bank time.

The actual bank time for the sample flowchart:

132 + 125 + 83 + 70 + 30 + 28 + 0 = 468 (aircraft hours).

Actual bank time is compared to the optimal bank time. In practice, the optimal bank time is subtracted from the actual bank time, $A_{bt} - O_{bt} = -57$ (aircraft hours) in this example. If the difference in hours is negative the fleet is overflown. When negative bank time is greater than or equal to the established phase interval for the aircraft, it is interpreted as the number of phase inspections that maintenance needs to complete. This indicator provides a general trend to the flow of aircraft with regard to the established optimal line.

The real expertise in using the sliding scale scheduling method is in the experience of the maintenance officers and production control officers. This daily scheduling process is usually a major obstacle for the production control officer. If an aircraft is incorrectly scheduled it could easily end in a failed mission or extensive unplanned maintenance. These results quickly obtain the attention of higher commanders.

2.4 A Proposed Quantitative Method

Incorporating the phase flowchart as described in FM 1-500, the distance from the optimal line can be obtained through a calculation of the slope. Given the amount of hours forecasted for each flight, a cost table can be constructed in aircraft hours. Once the table is formed, the objective is finding the optimal assignment of aircraft to missions by minimizing the cost relative to the optimal line. The actual optimization can be done by using one of several mathematical solution methods. The aircraft scheduling method, or the least cost method, provides a systematic approach that incorporates the Army policies for unit level maintenance and solves the problem with optimization techniques.

Chapter 3

THE LEAST COST SCHEDULING METHOD

3.1 Introduction

This chapter begins with aircraft and system limitations. These limitations dictate the way in which the data is prepared and the assignment solutions are interpreted. Next is a discussion on the preparation of data. The data is placed into a cost table and then formulated as an assignment problem. A step-by step example on how to develop a cost table follows the discussion. The least cost scheduling method requires that the data is prepared to minimize the cost, in aircraft hours, relative to the phase flowchart. Once the cost table is formulated, the use of a assignment problem solution method will determine the best schedule. This chapter shows the formulation of the assignment problem as a linear program and finds a solution to the example problem with the Hungarian method.

3.2 Aircraft Restrictions

All aircraft have design features that limit the aircraft in some manner. The aircraft operators manual outlines the design and functional limitations. To guarantee the safe operation of aircraft, the Federal Aviation Administration, FAA, and the Army impose flight restrictions. Some restrictions apply to all aircraft, while others apply only to particular design limitation. Possible design limitations include the maximum safe altitude the aircraft can obtain, airspeed restrictions, load capacity, armament, available seating, and the aircraft's ability to withstand the elements of weather. Military aircraft design is based on mission description which has a large impact on aircraft capabilities and special equipment. Currently, there are 23 different aircraft listed in the Army Regulation-Logistics Readiness and Sustainability, AR 700-138. This regulation defines each type of aircraft by category and provides a brief description of each aircraft's mission. The aircraft are designated by mission description; for example, A-Attack, O-Observation, and U-Utility.

The condition of the aircraft and all of its systems determines the status of the aircraft. The maintenance managers are required to track each airframe's status in hours and report the status daily. A written report is prepared monthly by type, in fleet percentages, and by aircraft serial number, in hours. This report is sent through both logistic and command channels to the Aviation Systems Command. The Army has four categories for tracking the status of its aircraft. The status of the aircraft determines its scheduling potential.

Fully mission capable, FMC - the aircraft and all aircraft systems are fully functional. An aircraft in this status can be scheduled without restrictions.

Partial mission capable, PMC - the aircraft is flyable but one or more of the aircraft's subsystems are not operational. Aircraft with a PMC condition are restricted from performing certain missions.

Not mission capable maintenance, NMCM - the aircraft is grounded and needs repair. Aircraft in this status are being repaired, or they are awaiting repair. These aircraft are not available for flights.

Not mission capable supply, NMCS - the aircraft in this status are grounded, and remain in this status until the repair part becomes available. When the part arrives, the status is changed to NMCM. Airframes with the condition status NMCS are not available for mission scheduling.

3.3 System Limitations

The flying hour program is the basis for all logistics, mission support, and pilot training for a unit. Every year, aviation units develop a flying hour program, which is a forecast of the amount of hours the unit intends to fly during the next fiscal year. If the unit has three different types of aircraft, then there are three portions to the program. This program is developed based on maintenance capabilities, trained pilots, training requirements, and the commander's estimate. Once the unit's forecast is approved by the Department of the Army, a plan is formulated to complete the flying hour program. The number of phase inspections required to satisfy the flying hour program is calculated and distributed over a 12 month period to establish flying hour goals. For example, if maintenance plans to complete two UH-1 phase inspections per month, and the phase interval on the aircraft is 150 hours, then maintenance needs the fleet to fly 300 hours to sustain the flying hour program.

The scheduling of the aircraft is a daily event. The aircraft undergo routine maintenance every flight. When they are not scheduled for flights, other inspections are done and faults are corrected on the aircraft. When grounding faults are detected, the aircraft becomes unavailable for flight missions. If there exist more missions than available aircraft, the missions are ranked by priority. The flight operations section confers with the production control section and both maintenance and operations present options to the commander. The commander makes the final decision on which missions to fly.

3.4 Preparing the Data

The preparation of the data leads to the formulation of a cost table, which contains an entry for each possible aircraft and mission combination. To standardize for this application, c_{ij} represents the cost, in hours, to fly aircraft *i* on mission *j*. Allowing the cost matrix to be p x q, where p is the number of aircraft available and q is the number of missions.

$$\mathbf{C} = \begin{bmatrix} C11 & C12 & \cdots & C1q \\ C21 & C22 & \cdots & C2q \\ \vdots & \vdots & & \vdots \\ Cp1 & Cp2 & \cdots & Cpq \end{bmatrix} .$$

These costs are determined by the following calculation:

$$c_{ij} = d_i + m_j.$$

Where:

d_i = the distance in hours aircraft *i* is away from the optimal line on the flowchart.

m_j = the amount of flight hours estimated to do mission *j*.

The estimated flight time, m_j, to do mission *j* is a value that is determined by the flight operations section and is a variable that must be given or determined by a pilot after planning the flight. The d_i must be calculated for each aircraft.

To calculate the distance each aircraft is from the optimal line, the slope of the optimal line must be determined. The vertical axis or Y axis will represent aircraft and the horizontal axis or X axis will represent hours. The intervals on the vertical axis are one because the vertical axis represents aircraft. Therefore, the y-intercept will equal the total number of aircraft on the phase flowchart. The only predetermined x values on the line are 0, at the origin, and P the phase interval. The x values of the optimum line at each Y interval are the desired values. Figure 3.1 shows the standard formula for the calculation of the slope.

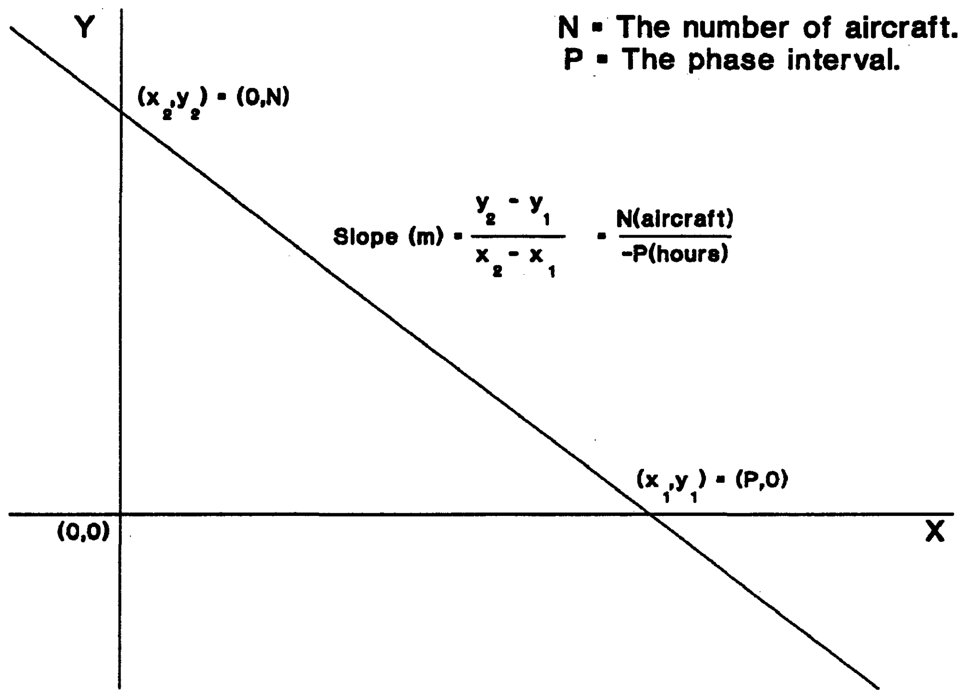


Figure 3.1 Calculation of the Slope

The x coordinates, hours at the optimal line, for each interval on the Y axis, is determined from a derivation of the point-slope formula:

$$y - y_1 = m(x - x_1).$$

Substitute the following variables for:

$y = y_i$ to represent each vertical interval.

$x = x_i$ to represent each horizontal interval corresponding with each vertical interval.

$y_1 = 0$.

$x_1 = P$.

$m = -N/P$.

Where:

N = The number of aircraft by type.

P = The phase interval for the type of aircraft.

Therefore,

$$y_i = (-N/P)(x_i - P).$$

Solving for x_i;

$$\frac{P(y_i - N)}{- N} = x_i.$$

Figure 3.2 shows a way to simplify these calculations by changing the position of the triangle captured in the first quadrant.

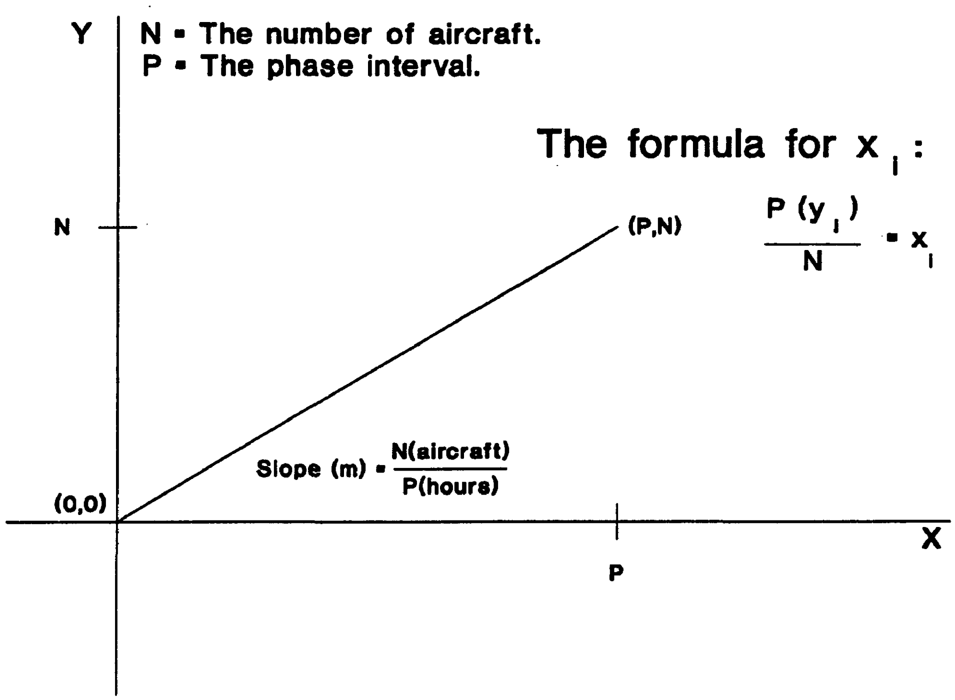


Figure 3.2 Simplified Calculations

Now that the hours at the optimal line can be determined the distance is:

$$d_i = x_i - h_i.$$

Where:

x_i = The hours at the optimal line for each aircraft.

h_i = The hours remaining to phase for each aircraft.

d_i = The distance each aircraft is from the line, in hours.

There is one other consideration in determining each cost - the penalty cost. If a particular aircraft cannot do a certain mission a penalty cost must be assigned. This penalty cost, M, replaces the corresponding c_{ij} in the cost matrix. Since the phase interval, P, is the largest conceivable number in the formulation, then M = P. The intent of the penalty cost is to eliminate the possibility of an aircraft being selected for a mission that it is incapable of performing.

Each cost, c_{ij}, can be interpreted as a future cost. If aircraft *i* flew mission *j*, the cost c_{ij} would represent the next day's distance, d_i, or the new position on the flowchart relative to the optimal line. The way in which the c_{ij}'s are calculated formulates a minimization problem. Simply, the objective is to minimize the cost, in hours, relative to the optimal line. Since all of the costs are relative to the slope of a line this method adjusts to maintain the determined slope.

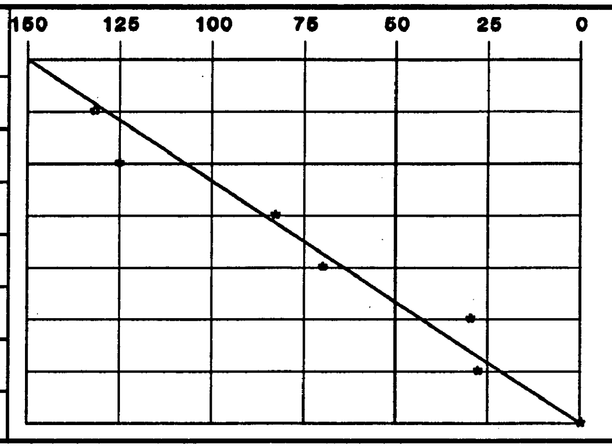
3.5 Step-By-Step Example

The following eight steps prepare the data into cost in units of hours. There will be a cost for each available aircraft to fly each possible mission. Figure 3.3 will start the example with an illustration of a flowchart.

PHASE FLOWCHART

REPORT DATE: 1 JAN 91 AIRCRAFT: UH-1

SERIAL NUMBER	HRS TO PHASE	HRS TO SERVICE
467	132	7
193	125	25
241	83	8
347	70	20
349	30	5
351	28	3
687	0	0



BANK TIME: -57

Figure 3.3 Phase Flowchart

STEP 1: Find the slope of the optimal line. In the form of a rise over run calculation, the slope is obtained by dividing the number of aircraft on the flowchart by the phase interval. This is the first step in finding the distance from each aircraft plotted on the phase flowchart to the optimal line. The slope of the flowchart in figure 3.1 is:

7(aircraft) / 150(hours) = .04666667(aircraft/hour).

Step 2: Determine the coordinates of the phase line at each increment on the vertical axis. The aircraft at the bottom of the chart is on the zero line. Figure 3.4 shows the calculations and the coordinates:

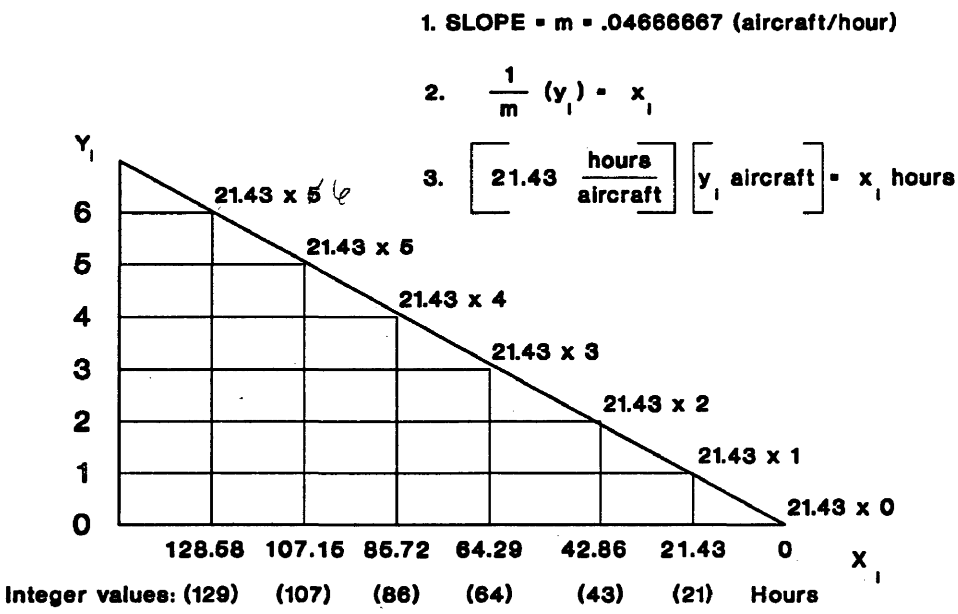


Figure 3.4 Hours at the optimal line

Step 3: Calculate the distance each aircraft is from the optimal line. Take the values found in step 2, the horizontal coordinates of the slope, and subtract the aircraft hours until the next phase inspection for each line on the flowchart, use only the integer coordinate values for this example. Table 3.1 shows step 3.

Table 3.1 Distance from the optimal line

DISTANCE FROM OPTIMAL LINE

COORDINATE VALUE - HOURS TO PHASE = DISTANCE

SERIAL NUMBER	COORDINATE VALUE	HOURS TO PHASE	DISTANCE
467	129	132	-3
193	107	125	-18
241	86	83	+3
347	64	70	-6
349	43	30	+13
351	21	28	-7
687	0	0	0

If the aircraft represented on the phase flowchart is to the left of the optimal line, it is considered underflown. If the aircraft is to the right of the optimal line, it is considered overflown. Overflown aircraft will have a positive value, (+), and underflown aircraft will have negative value, (-).

Step 4: Determine the next required service or inspection for each aircraft. These can be listed either in flight hours or as a date. If the next inspection is listed as a calendar inspection, then the date must be posted. Calendar inspections are only an issue if an inspection is due on the date of a possible assignment.

Step 5: Determine the number of flyable aircraft. The aircraft status will show the number of airframes that are FMC and PMC. This is the total available airframes.

Step 6: Examine the mission sheet. The mission sheet has the total number of flights, and an estimate of the hours to do each flight. The sheet contains the aircraft's departure time and a brief description of the mission. The type of aircraft equipment or configuration required is determined from the description of the mission. Figure 3.5 shows an example of three proposed missions.

MISSION SHEET

DATE: 1 JAN 90

MISSION	ESTIMATED FLIGHT TIME	DESCRIPTION	TAIL NUMBER ASSIGNED
	DEPARTURE TIME		
1	6 HRS	Cross country mission, requested by the Chief of Staff.	
	0800 HRS		
2	4 HRS	Air Assault training, 12 personnel.	
	0600 HRS		
3	2 HRS	Instrument check ride.	
	1400 HRS		

Figure 3.5 Mission sheet for the next day's flights

Step 7: Make a cost table. Place the missions in a row across the top and flyable tail numbers down the left hand column. If a tail number is not flyable, or is grounded, then it is left out of the table. This table will be filled with numerical values representing cost, in aircraft hours. These costs are calculated by taking the forecasted hours to complete the mission, and adding it to the distance, in hours, the aircraft is away from the optimal line. For example, mission (1) requires six aircraft hours to complete, and tail number 467 is underflown by three hours, (-3). The cost calculation for tail number 467 is:

-3 hrs. distance + 6 hrs. mission time = 3 hrs. overflown.

This is interpreted as the cost; if tail number 467 flew mission (1) for six hours it would be three hours overflown on the flowchart.

Step 8: Adjust the cost table for aircraft inspections and restrictions. If an assignment of an aircraft is not possible for a particular mission, a large number, the phase interval, is placed in the table in the corresponding mission and tail number's location. Figure 3.6 is the adjusted cost table.

COST TABLE

AIRCRAFT	MISSION #1	MISSION #2	MISSION #3
467	3 (-3 + 6)	1 (-3 + 4)	-1 (-3 + 2)
193	-12 (-18 + 6)	-14 (-18 + 4)	-16
241	9 (3 + 6)	7	5
347	0	-2	-4
349	150 (SERVICE)	17	150 (IFR RESTRICTION)
351	150 (SERVICE)	150 (SERVICE)	-5

SERVICE RESTRICTION 349 , 5 HOURS TILL SERVICE
351, 3 HOURS TILL SERVICE

Figure 3.6 Adjusted cost table

Mission one is a six hour mission and tail number 349 has only five hours available until the next service. A value of 150 is placed in mission (1)'s column, aircraft 349's row. Suppose 349 is PMC and restricted from instrument flights. The value 150 is placed in mission (3)'s column and 349's row. Tail number 351 is 3 hours until next service. So, place in mission (1) and mission (2)'s column, 351's row, a value of 150. These large values, or high costs, will preclude these aircraft from being selected for those flights.

3.6 The Optimal Solution

The assignment problem requires certain tasks to be assigned to facilities on a one-to-one basis optimally. In general mathematical terms, *n* tasks are assigned to *n* facilities, and only one task can be assigned to each facility. When the number of tasks equals the number of facilities, it is denoted as a balanced problem.

The total possible number of assignments is *n* factorial. The objective is to find the optimal assignment, or least cost, from the *n* factorial possible choices. This can be achieved through proven techniques for solving assignment problems. The linear programming formulation is how most texts mathematically define the assignment problem. There are several software packages available to find the solution and therefore the simplex algorithm for linear programming will not be discussed. The Hungarian method is suited for manual calculations and will be demonstrated in the example problem.

3.6.1 Linear Programming.

The assignment problem is defined in mathematical notation as a linear program, where the objective is to minimize the cost of assigning facilities to tasks. The standard formulation is for a balanced problem where *n* number of facilities equals *n* number of tasks.

Minimize

$$\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \text{for } i = 1,2,.....,n.$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad \text{for } j = 1,2,.....,n.$$

$$x_{ij} >= 0 \qquad \text{for all } i \text{ and } j.$$

This formulation has a special property in that it does not allow fractional assignments. Intuitively, the assignment of aircraft 1 is made to mission 1 or it is not. This type of integer requirement makes this formulation a special case in 0-1 integer problems.

The example presented earlier is not a balanced problem and the formulation of unbalanced problems using linear programming does not require the addition of dummy missions or dummy aircraft in order to guarantee a n x n formulation. Dummy points will be explained in detail in the description of the Hungarian method. For orientation, tail number 465 is labeled aircraft 1 with the following tail numbers labeled from the top of the cost table to the bottom sequentially, e.g. 467 implies x_{1j}, 193 implies x_{2j}.

Minimize

$$3x_{11}+1x_{12}-1x_{13}-12x_{21}-14x_{22}-16x_{23}+9x_{31}+7x_{32}+5x_{33}+0x_{41}$$
$$-2x_{42}-4x_{43}+150x_{51}+17x_{52}+150x_{53}+150x_{61}+150x_{62}-5x_{63}$$

Subject to

$x_{11}+x_{12}+x_{13} <= 1$ (aircraft supply constraints)
$x_{21}+x_{22}+x_{23} <= 1$
$x_{31}+x_{32}+x_{33} <= 1$
$x_{41}+x_{42}+x_{43} <= 1$
$x_{51}+x_{52}+x_{53} <= 1$
$x_{61}+x_{62}+x_{63} <= 1$
$x_{11}+x_{21}+x_{31}+x_{41}+x_{51}+x_{61} = 1$ (mission demand constraints)
$x_{12}+x_{22}+x_{32}+x_{42}+x_{52}+x_{62} = 1$
$x_{13}+x_{23}+x_{33}+x_{43}+x_{53}+x_{63} = 1$

This problem is unbalanced because it can supply six aircraft for a demand of only three missions. When supply is greater than the demand, the right hand sides of the supply constraints are less than or equal to one, and the demand constraints are equal to one. Another unbalanced condition exists if the demand for missions is greater than the available supply. The supply constraints have right hand sides that are equal to one and the demand constraints are less than or equal to one.

3.6.2 The Hungarian Method.

The Hungarian Method solves balanced one-to-one assignment problems. It has guidelines that must be applied for finding an optimal solution. The guidelines are listed below:

Guideline 1: The number of columns must equal the number of rows in the cost matrix. Simply, the cost matrix must be square, n x n, to insure a correct solution. If the matrix is not balanced, rows or columns of zeroes are added until the matrix becomes square. These additional rows or columns are labeled dummy points. In the scheduling of aircraft it will be common to add additional dummy missions, or columns, and quite possibly under strained conditions dummy aircraft, or rows.

Guideline 2: Make the cost matrix integer. If a fractional value exists in the cost matrix, multiply the matrix by the appropriate power of 10 to make the matrix integer. Aircraft are tracked to the tenth of the hour, where one tenth of an hour is equal to 6 minutes. Therefore the cost tables are not always completely integer, and in such cases all entries are multiplied by 10. This is to reduce the arithmetic for manual computations.

Guideline 3: The Hungarian method solves minimization problems. For the Army aircraft scheduling problem, the data has been prepared to guarantee that the formulation minimizes the cost.

The cost matrix for the example problem requires that guideline 1, additional dummy missions, be added. Figure 3.7 is the new cost table.

COST TABLE

AIRCRAFT	MISSIONS #1	MISSIONS #2	MISSIONS #3	DUMMY MISSIONS #1	DUMMY MISSIONS #2	DUMMY MISSIONS #3
467	3	1	-1	0	0	0
193	-12	-14	-16	0	0	0
241	9	7	5	0	0	0
347	0	-2	-4	0	0	0
349	150	17	150	0	0	0
351	150	150	-5	0	0	0

Figure 3.7 Cost table for the Hungarian method

The following is the sequence of steps required to solve the assignment problem by the Hungarian method. This will minimize the cost, in hours, relative to the optimal line on the flowchart.

Step 1: Select the smallest entry in each row, with, for example, -2 being less than -1. Subtract the smallest entry from all the other entries in its row. At least one zero should appear in each row and all of the entries in the matrix should be nonnegative, (positive).

Step 2: Select the smallest entry in each column and subtract it from all the other entries in its column. At least one zero should appear in each row and column and all of the entries in the matrix should be nonnegative. This is the reduced cost matrix.

The Hungarian Method

STEP 1:

4	2	0	1	1	1
4	2	0	16	16	16
9	7	5	0	0	0
4	2	0	4	4	4
150	17	150	0	0	0
155	155	0	5	5	5

STEP 2:

0	0	0	1	1	1
0	0	0	16	16	16
5	5	5	0	0	0
0	0	0	4	4	4
146	15	150	0	0	0
151	153	0	5	5	5

Figure 3.8 Steps 1 and 2 of the Hungarian method

Step 3: Cover all of the zero entries by drawing lines through the appropriate rows or columns. Use the minimum amount of lines to cover the zeros. There may be many different combinations of lines through the rows and/or columns, but it is imperative that the minimum number of lines is selected.

Step 4: Determine whether an assignment can be made. One of two conditions will exist, an optimal assignment is available, or one is not available. If optimal, the number of lines will equal *n*, the size of the sides of the matrix. Select *n* zeros, such that only one zero is selected for each row and each column. The position of the selected zeros represent the assignments made by the algorithm. If an assignment is made this would complete the method.

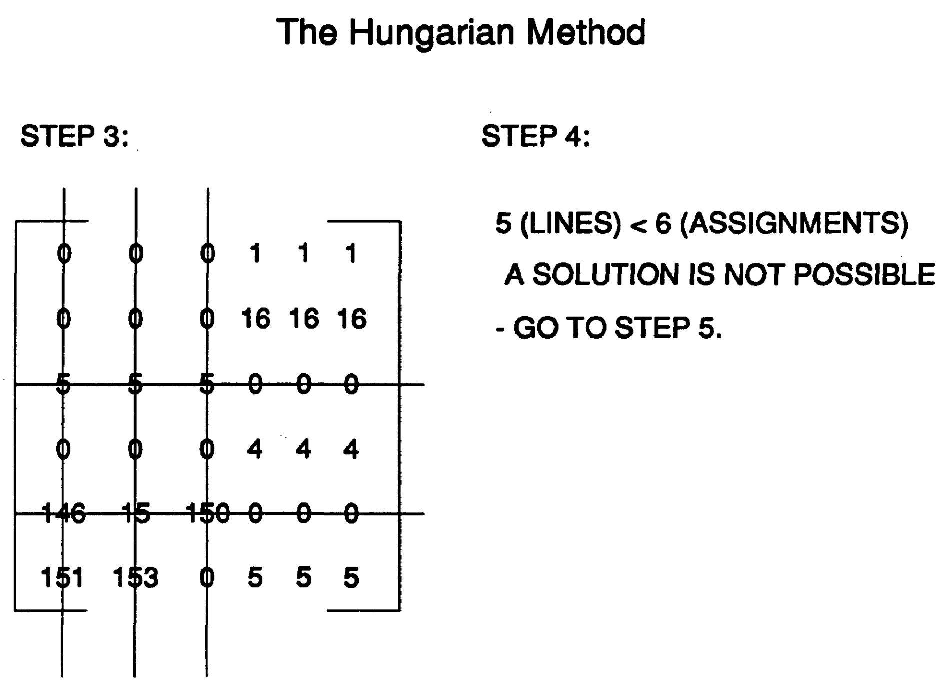


Figure 3.9 Steps 3 and 4 of the Hungarian method

If the number of lines is less than *n* then proceed to step five. In Chapter 4 there is a labeling algorithm that in effect determines the minimal number of lines. The labeling algorithm modifies the Hungarian method described in this chapter.

Step 5: Select the smallest entry in the reduced cost matrix not covered by any line, and call this entry *k*. This entry must be subtracted from all of the uncovered entries in the present reduced cost matrix. If a matrix entry is covered by both a vertical line and a horizontal line then the value for entry, *k*, is added to the all values covered by both lines. If a matrix entry is only covered by a single line, then the covered values are carried over, unchanged, in the new reduced cost matrix. When this matrix is completed, proceed to step 3. The next figure shows the completed step 5. The minimum number of lines that can be drawn to cover the zero's is six. The boxes around the zero's represent the assignment selected.

The Hungarian Method

Step 5 and the Solution:

	#1	#2	#3	D1	D2	D3
467	0	0	0	[0]	0	0
193	[0]	0	0	15	15	15
241	6	6	6	0	[0]	0
347	0	[0]	0	3	3	3
349	145	16	151	0	0	[0]
351	151	153	[0]	4	4	4

FROM STEP 5:
k = 1.

Figure 3.10 Solution to the Hungarian method

The solution is read off of the table by the position of the zero selected. The total cost is the sum of the original cost for the chosen aircraft and mission combinations. The assignments for the three proposed missions and the cost are represented in table 3.2.

Table 3.2 The optimal assignment

Fly	Mission	Cost
351	3	-5
347	2	-2
193	1	-12

Total -19

Every time step five is completed, the sum of all the entries in the new cost matrix is less than the sum of all the entries in the previous cost matrix. Step 5 is the step that guarantees convergence to an optimal solution.

3.7 Some Sensitivity

This method is flexible enough to react to the needs of an aviation maintenance officer. There are three important results that may be available from the method discussed. First there may be other assignments at no additional cost. Next, from a maintenance perspective, when there are more missions than available aircraft, this method will select the most economical missions. Last, the ability to dedicate an aircraft for a specific mission.

Every maintenance officer would like to have options, especially if the options are at no additional cost. Through close scrutiny of the final cost table there may be another assignment. In the above example there does exist another selection of zeros at the same cost.

Table 3.3 The alternative optimal assignment

Fly	Mission	Cost
351	3	-5
347	1	0
193	2	-14
		Total -19

In this selection, the same three aircraft were chosen except for different missions. This may be significant. If tail number 347 requires a routine inspection, the 0800 hour takeoff versus the 0600 hour takeoff time may prevent maintenance personnel from arriving earlier or staying late to do the scheduled maintenance.

In the situation where there are more missions than flyable aircraft, it could be useful information for the commander to know which missions least affect the maintenance posture. This also has the benefit of optional mission choices at the same cost, as described for optional aircraft choices.

Sometimes certain aircraft must be flown for maintenance scheduling purposes, or in the case of a dedicated aircraft for a General Officer. Prepare the cost matrix as outlined previously, and then circle the cost in the table for the designated aircraft and mission. Eliminate the row and the column of the selected mission and aircraft. Form a new cost matrix less the designated row and column, and proceed with the Hungarian method.

The next chapter will cover a computer program that was developed for the least cost method. In the description of the computer program some of the finer details of the assignment problem will be elaborated.

Chapter 4

THE COMPUTER PROGRAM SOLUTION METHOD

4.1 Introduction

The computer algorithm is adapted from a FORTRAN subroutine authored by Derigs (1980). The routine in its entirety follows the steps of the Hungarian method with an application of Dijkstra's shortest path algorithm of labeling rows and columns. This chapter will start with a general description of network flows and then take a more detailed look at the assignment problem. The detail outlines the theory behind the assignment problem. The methodology of the computer algorithm finishes this chapter.

4.2 Networks

Networks are a system of nodes connected with a set of arcs. The arcs have arrows to indicate the direction of flow and associated with each arc is a distance, or cost. Figure 4.1 is a network with six nodes with the following identities. The first node represents the super source, nodes 2 and 3 will represent supply points, and nodes 4 and 5 will represent demand points, while the last node in the direction of flow will be the super sink. Connecting the six nodes are eight arcs. The two arcs departing the super source and the two arcs entering the super sink having a zero cost, while the arcs between the supply nodes and the demand nodes have distance equal to the cost. In the scheduling of aircraft, the supply nodes would be the aircraft tail numbers and the demand nodes would be the daily missions. The arcs between the aircraft nodes and the mission nodes would be the cost, in hours, to fly an aircraft on a mission.

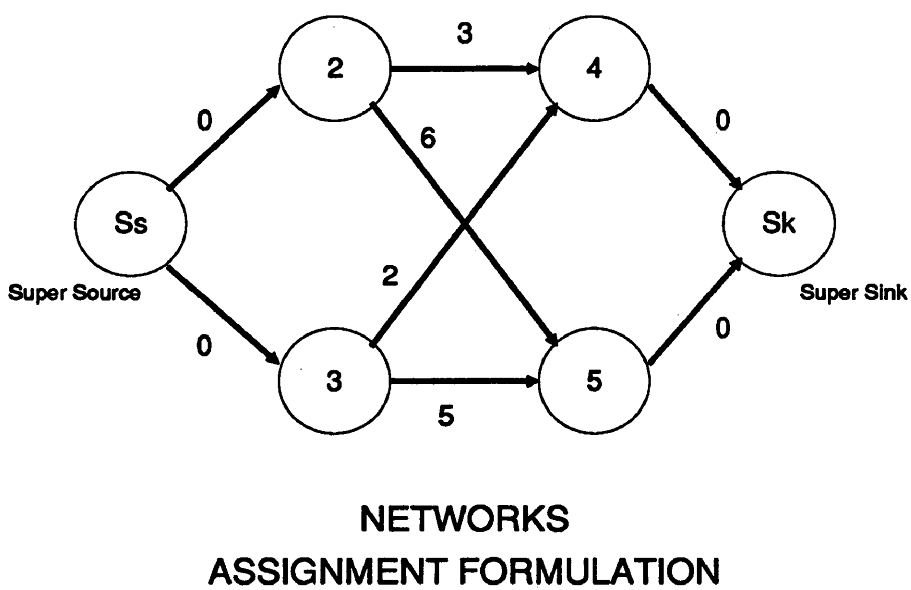


Figure 4.1 Network of an assignment problem

Dijkstra's algorithm's objective is to find the shortest route from the source to the sink. The solution to the assignment problem is finding the combination of paths from the source to the sink at the least cost as the flow passes through every supply node and demand node. This is the concept of adding the paths, or "augmenting," until the minimal total cost of all admissible paths in the network is obtained. A solution is only possible when the total flow out of the supply nodes is consumed at the demand nodes. The standard formulation of a network flows problem is:

Minimize

$$\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} x_{ij} = b_i \qquad \text{for } i = 1, 2, \ldots\ldots\ldots n.$$

and

$$0 <= x_{ij} <= u_{ij} \quad \text{for each arc i going to j.}$$

Where:

c_{ij} = cost to go from node *i* to node *j*.

b_i = Net flow at node i = (Flow out) - (Flow in).

u_{ij} = capacity on the arc between node *i* to node *j*.

Since the assignment problem has a one-to-one correspondence, the u_{ij}'s are equal to one. An incidence matrix of the two-by-two example problem in this section may clarify the constraint set:

	(Ss,2)	(Ss,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,Sk)	(5,Sk)
Ss	1	1						
2	-1		1	1				
3		-1			1	1		
4			-1		-1		1	
5				-1		-1		1
Sk							-1	-1

The incidence matrix above has the rank of at most n-1. If all rows in the matrix were added together the resulting vector would be the zero vector. With this particular matrix structure and using Gaussian elimination to find the rank, one row would become the zero vector. Therefore, any one row is redundant and can be eliminated without changing the rank of assignment constraints. The redundant constraint is a linear combination of the other constraints and in algebraic terms is referred to as linearly dependent.

The redundant constraint selected for elimination will represent a nonbasic variable. With only linearly independent rows remaining, the objective is to find a nonsingular submatrix that will solve the system of linear equations from the remaining constraint set. Any set of linearly independent row and column vectors that solves the set of equations is known as a basic feasible solution. The vectors that identify a basic feasible solution form a basis. The variables that provide the solution to the basis are called the basis variables. Therefore the solution to

$$\mathbf{Ax} = \mathbf{b}$$

Where:

- $\mathbf{A}$ = The system of equations for the original constraint set.
- $\mathbf{x}$ = The variables for the set of equation.
- $\mathbf{b}$ = The right hand sides, RHS, of the constraint set.

is

$$\mathbf{Bx} = \mathbf{b}.$$

Where:

- $\mathbf{B}$ = A nonsingular submatrix of $\mathbf{A}$.
- $\mathbf{x}$ = The basic variables, or solution set.
- $\mathbf{b}$ = The RHS.

Reform the incidence matrix into a linear program formulation by removing the columns for the two source nodes and changing the negative signs to positive. Also, picking the constraint for node 5 as the redundant row and eliminating it from the other constraints gives:

$$\begin{matrix} c_{11} & + & c_{12} & + & c_{21} & + & c_{22} \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & & \\ & & 1 & 1 \\ 1 & & 1 & \end{bmatrix} \begin{bmatrix} X11 \\ X12 \\ X21 \\ X22 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

From this matrix select a nonsingular matrix that provides a basic feasible solution. Column three is selected for elimination. However, any of the four columns may have been chosen. The following matrix represents a basic feasible solution:

$$\begin{matrix} c_{11} & + & c_{12} & + & c_{22} \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & \\ & & 1 \\ 1 & & \end{bmatrix} \begin{bmatrix} X11 \\ X12 \\ X22 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

A permutation of this matrix will result in a lower triangular matrix in which the values of the basic variables can be found through foreword substitution. Let row three become row one, row one become row two, and row two become row three, resulting in the final solution.

$$\begin{matrix} c_{11} + c_{12} + c_{22} \\ \begin{bmatrix} 1 & & \\ 1 & 1 & \\ & & 1 \end{bmatrix} \end{matrix} \begin{bmatrix} X11 \\ X12 \\ X22 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} .$$

Therefore, the solution is:

$x_{11} = 1,\ x_{12} = 0,\ x_{22} = 1.$

The total cost, TC, of this solution is:

$TC = c_{11} + c_{22}.$

The result is that all basic feasible solutions for assignment problems are lower triangular. If the cost coefficients are integer, the solution will be integer because of the triangular property. The costs coefficients are multiplied by either zero or one, and then the costs are summed together to give a total cost for the solution set.

Degeneracy occurs when a variable in the basis has a value of zero. The example basis has a degenerate basic feasible solution. Assignment problems always have degenerate basic feasible solutions, and the level of degeneracy is measurable. Let k be the number of basic variables with values equal to one. The result is that there are k-1 basic variables equal to zero. From the two-by-two sample problem k is equal to 2, therefore $2 - 1 = 1$, producing one basic variable equal to zero in the basis. This is a measure of the level of degeneracy in assignment problems, and as the problems grow in size so does the level of degeneracy. For a more in-depth explanation of triangularization and degeneracy see Luenberger (1984). In an attempt to find additional aircraft assignments to missions at the same optimal cost the computer program developed will try to capitalize on the fact that assignment problems are degenerate.

4.3 The Computer Subroutine

The program follows the steps of the Hungarian method described in Chapter 3, except that step 3 has been changed. Step 3 goes through a method of labeling rows and columns, which is a derivation of Dijkstra's shortest path algorithm. Appendix A has information on how to execute the provided diskette.

The program is loaded as a cost table and is read into the routine as a single array. The program is set up to accept a 50-by-50 cost matrix. The mission numbers are across the top and lined up as columns. The aircraft are listed down the side and arranged in rows. The total sum of all cost coefficients must be less than the supremum. Presently the supremum is set at the value of 100,000,000. From the example problem in Chapter 2 the following table would be entered into the program either through a data file, or row by row.

	Mission									
	1	2	3	4	5	6	7	8	9	10
467	3.0	1.0	-1.0							
193	-12.0	-14.0	-16.0							
241	9.0	7.0	5.0							
347	0.0	-2.0	-4.0							
349	150.0	17.0	150.0							
351	150.0	150.0	-5.0							
7										
8										
9										
10										
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										

1, 20 Blank Type / for Commands

/ Return, Quit, Load, Save, Clear, Export, Help, modify Tail numbers

Figure 4.2 Computer input for the example problem

The output displays the original cost table, with the optimal assignment at the optimal cost. The assignment will be listed in two columns. The first column contains the aircraft and the second column contains the missions and is displayed in the following manner:

```
          Mission Assignment Problem

Do you wish to see the cost matrix? [y/n]:n

 Optimal Assignments:

 Fly:

   Aircraft 467 ---------- Not Selected
   Aircraft 193 on Mission   1
   Aircraft 241 ---------- Not Selected
   Aircraft 347 on Mission   2
   Aircraft 349 ---------- Not Selected
   Aircraft 351 on Mission   3

     Cost of the Optimal Assignment =        -19.0
```

Figure 4.3 Computer output for the example problem

4.3.1 The Initial Solution.

The initial solution does the first two steps of the Hungarian method. In the first step, the program scans each row and selects the smallest cost coefficients.

u_i = The minimum column entry for every row.

The program then makes all possible assignments that do not have two or more zeros in the same column. If an assignment is not made, the program does step 2 of the Hungarian method:

v_j = The minimum row entry from the reduced cost coefficients determined by the following calculation:

$$v_j = c_{ij} - u_i.$$

If the modified cost coefficients $c_{ij} - u_i - v_j = 0$, then the program makes new assignments to the corresponding i^{th} row and j^{th} column. These assignments to zero cost coefficients are referred to as admissible. If the program has an optimal assignment, it enters the subroutine for the shortest augmented path and is quickly pushed to the output portion of the program after an initial check of optimality.

4.3.2. The Shortest Augmented Path.

The shortest augmented path algorithm takes the assignment determined by the initial solution and goes through a procedure of labeling rows and columns. A label is some letter or symbol that is placed outside of the matrix to identify the row of column. The labeling procedure follows the following steps:

Step 1: First locate any row that does not have an assignment and label that row. The program uses an array to maintain the row labels.

Step 2: If there are zeros in the labeled row, then label each column with a zero in the labeled row. An array is used to keep track of the labeled columns.

Step 3: If the labeled columns have assignments, then each row having an assignment in labeled columns becomes labeled. The labeling continues until all rows that do not have assignments are labeled.

The following figure shows the labeling steps applied to the example in Chapter 2. Steps 1 and 2 of the Hungarian method have already been applied, (l) will note the labeled rows and columns. A box will be drawn around the labeled rows. A circle will be placed around the smallest cost coefficient to highlight the next procedure.

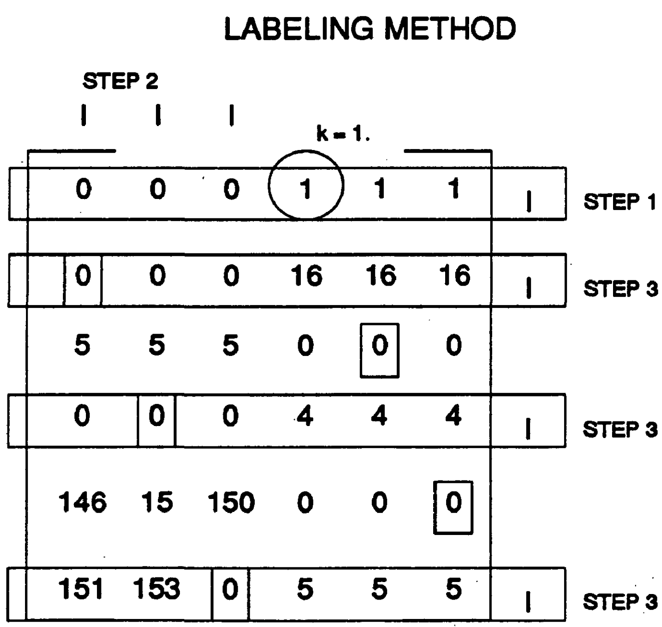


Figure 4.4 The labeling of rows and columns

The algorithm uses a modified version of step 4 in the Hungarian method shown in Chapter 3, by finding the smallest modified cost coefficient, *k*, located in a labeled row and unlabeled columns. Then *k* is subtracted from all of the labeled rows and added to all labeled columns. If the assignments do not equal *n*, then the labeling process repeats. Every time a new assignment is made it is called a breakthrough. This procedure does not draw lines through rows and columns with assignments like the method in Chapter 3, making this labeling method more suitable for a computer program.

Chapter 5

CONCLUSIONS

5.1 Introduction

This chapter will start with the implementation of the least cost method for the U. S. Army. Next, are topics for future research including further implementations and additional enhancements of the method. Ending this chapter is a brief summary of the proposed method.

5.2 Implementation

A personal goal is to see the adaptation of this method into the Army's new logistical software package for aviation, ULLS-A. It is feasible to add a scheduling module to ULLS-A because the data base for the software package has the information to formulate the mathematical problem. If ULLS-A were equipped to do aircraft scheduling, then a user could access the scheduling module, make a few managerial decisions and have a ready solution. This type of acceptance into the Army would have to be approved by the Aviation Systems Command.

Another approach is to send this method to the Aviation Branch located at Fort Rucker, Alabama which is the proponent for the Army aviation. The proponent has the power to change a Branch's methods of operations.

5.3 Topics for Further Research

The United States Air Force could possibly use this method for scheduling aircraft. It has a decentralized structure, similar to the Army, for their fighter based squadrons. The Air Force has a logistical aviation software package with a scheduling module, but this module is unreliable. Consequently, most of the scheduling is done based on the experience of the Aircraft Plans and Scheduling Supervisor. With a better understanding of the Air Force system the least cost method could be made adaptable.

There are many commercial helicopter companies and most of the larger outfits specialize in offshore transport. It is uncertain how these companies presently schedule their aircraft for flights and maintenance. It is speculated that these companies use the same concepts in scheduling as the Army, and in many cases, own the same type of helicopters. This method may be easier to adapt to the civilian sector in that the FAA requires an inspection every 100 hours for all commercial carriers. This would allow uniformity in their flowcharts.

The scheduling method proposed could be enhanced with an additional performance indicator. The bank time figure is an indication of bias. When the bank time is negative it is implied that the fleet as a whole is overflown and when the bank time is positive, the fleet is underflown. Bank time provides no indication of how far the given aircraft are dispersed from the optimal line. A calculation of mean square error would give numerical representation of dispersion. A table could be developed based on the number of aircraft as a guide for interpreting the mean square error.

The optimal line defined in FM 1-500, Aviation Maintenance manual, is nothing more than an aggregate planning line. The Air Force system uses the same optimum line as the Army. Other standard aggregate planning lines could be developed to obtain maintenance goals of an aviation unit. It is common for units to try to get positive bank time before supporting large military exercises. A standard aggregate planning line could be developed to obtain this positive bank time. The line could be switched to a line that would support heavy flying beyond the optimum line in FM 1-500 during the exercise. This line also could be standardized for heavy flying periods, or it could be customized to meet the commander's estimate of how the unit will fly in a given period. As the unit approaches this newly established line, the commander would know how many phases, or calendar days, it would take to reach the optimum line described in FM 1-500. These lines could have predetermined names like pre-exercise, mission support, and garrison.

In the area of sensitivity analysis, it may be possible to select a replacement aircraft if an aircraft becomes grounded. The aircraft selected would be at the cheapest in cost with regard to the previously selected aircraft. Most units in the army schedule a standby for this purpose. The standby should not be scheduled with the aircraft that have missions where flight time will be accrued.

5.4 Summary

The proposed method takes the phase flowchart, outlined in FM 1-500, and builds quantitative measures that support operations research optimization techniques to select a flight schedule at the least cost. The least cost method provides an efficient and systematic method for scheduling aircraft.

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Appendix A

PROGRAM DESCRIPTION

The diskette, Appendix B, is a 3.5 inch, double density disk formatted for an IBM. The program is in a directory named "AIR" and consists of two files. Once the directory has been accessed, type the file name "lcost" for least cost. The program was designed to be used in conjunction with this thesis, and is intended as a demonstrator program. This program would have to be adapted for specific commercial applications. Refer to Chapter 3 and Chapter 4 for information about the method.

Once the program is executed, there is a HELP menu that will assist the user with general information required to use the program. Refer to the HELP menu for the program's internal and external abilities.

It is only necessary to enter the actual cost. The program will add dummy rows and columns as necessary. Decimal values to the tenth may be entered. The program will also accept positive or negative values up to, and including, 999.9. It is possible to build a cost table based on information about a fleet of aircraft or by entering the cost directly into the spreadsheet.