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ECONOMIC EVALUATION
OF LEVERAGED INVESTMENTS

by

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ABSTRACT

This work examines two critical points of contention from the current literature. These are the validity of the implicit reinvestment assumption in discounted cash flow rate of return (DCFROR) analysis and the selection of the appropriate discount rate for use in performing economic evaluation.

Basic microeconomic analysis leads to the following conclusions:

(1) There is no reinvestment assumption implied or required by DCFROR calculations.

(2) Opportunity cost, not cost of capital, is the appropriate discount rate for use in performing economic evaluations.

This work also uses basic economic evaluation theory to develop a mathematical model defining proportionality, the functional relationship between growth rate of return and the proportion of debt used in project financing. This relationship can be used to define opportunity costs appropriate for the evaluation of investments involving borrowed funds or to quantify the impact of debt financing on growth rate of return results.

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Chapter 1

INTRODUCTION

This research has two primary objectives. The first is to resolve the following questions from the economic evaluation literature:

(1) Is reinvestment of intermediate project cash flows implied in performing discounted cash flow rate of return (DCFROR) calculations?

(2) What is the appropriate discount rate for use in economic evaluation?

The second objective is to develop a mathematical model defining proportionality, or the functional relationship between project growth rate of return (GRR) and the proportion of borrowed funds used in the investment. Once defined, this proportionality relationship can be used to determine the discount rate appropriate for use in evaluating investments involving borrowed funds. This relationship can also be used to calculate project growth rates of return resulting from leveraging an equity-funded project with a specified debt proportion and balloon or mortgage-loan type repayment schedule.

The conclusions drawn in the discussions of the validity of the reinvestment assumption are important for both academic and practical reasons. First, the collection and evaluation in one work of the principal arguments in favor of the implicit reinvestment assumption should resolve any doubt about its validity. At present, arguments and counterarguments appear individually and out of context, making them difficult for students and practitioners to understand and assess. Resolution of this question has immediate practical ramifications because DCFROR is one of the most widely used economic evaluation techniques. Chapter 3 will show that use of the assumption must seriously limit the use of DCFROR as an economic evaluation tool.

The implied reinvestment question will be examined historically, logically, and mathematically using basic microeconomic analysis. A literature survey will track the evolution of microeconomic thought and analysis to determine the origin of the implied reinvestment assumption. The rationale for this review is that if it is possible to locate the first use or discussion of the assumption, then it may be easier to assess its validity.

Once an economic evaluation history has been developed, and the roots of the implicit reinvestment assumption are known, current arguments regarding the existence and validity of the assumption will be evaluated. At this point the primary arguments favoring the reinvestment assumption will be summarized. Each argument will then be critiqued both mathematically and logically. The mathematical critique is intended to determine whether an implicit reinvestment assumption is required by the mechanics of the evaluation calculations. The second critique explores the logical ramifications of the use of the implied reinvestment assumption. The mathematical analysis examines whether the implicit reinvestment assumption must be accepted to carry out the arithmetic of economic evaluation calculations. The logical analysis treats the desirability of using the implied reinvestment assumption and the logical inconsistencies resulting from its use.

Resolution of the appropriate discount-rate problem is important because use of the wrong discount rate may cause the firm to make suboptimal investments. This question also has ramifications for the performance of theoretical economic analysis.

The literature survey contained in this work will show that there are two schools of thought regarding the appropriate discount rate question. One group contends that cost of capital is the correct discount rate or economic performance standard. Opponents contend that opportunity cost of capital is a better discount and hurdle rate. This conflict will be resolved through a thorough review and analysis of existing arguments using basic microeconomic tools.

The proportionality relationship will be mathematically defined for balloon and mortgage-type loan repayment schedules. The models developed will be extensions of the basic GRR mathematics.

It has long been argued that the introduction of leverage could increase project DCFROR and GRR, and so, distort evaluation results. The models developed here will remove the distortion and allow the accurate evaluation of leveraged projects.

Chapter 2 is a literature review of the evolution of economic evaluation thought. Chapter 3 examines the appropriate discount rate question and summarizes and evaluates implied reinvestment rate arguments. Chapter 4 develops the mathematics of the proportionality model, and Chapter 5 summarizes and makes suggestions for further research.

Chapter 2

THE HISTORICAL AND ANALYTICAL DEVELOPMENT OF ECONOMIC EVALUATION THEORY

Section 2.1 of Chapter 2 contains a brief review of some of the major contributions to the theory of economic evaluation over the last one hundred years. Works considered include analyses by Hoskold, Fisher, Keynes, Alchian, Dean, Solomon, Berry, and O'Neil.

Section 2.2 examines the principal analytical techniques currently employed in economic evaluation and discusses the strengths and weaknesses of each.

Section 2.3 introduces the reader to several important points of confusion, economic evaluation questions about which there is significant difference of opinion in the current text and periodical literature. Specifically, arguments regarding the discount rate appropriate for use in economic analysis and the validity of the implicit reinvestment assumption will be briefly introduced. Chapter 3 of this work is devoted to resolving these and other fundamental questions pertaining to basic economic evaluation.

2.1 Historical Overview

Basic time value of money concepts have been incorporated in economic and financial analysis for more than one hundred years. Adaptation of these general principles to the evaluation of mining investments began with Hoskold (1877) and continued through O'Neil (1982).

2.1.1 Hoskold and Fisher

Basic time value of money concepts have been incorporated in economic and financial analysis for many years in the mining industry. Hoskold based the mine valuation techniques he presented in The Engineer's Valuing Assistant (1877). Hoskold's method incorporated a sinking fund provision which essentially assumed that the owners of the property under construction would dedicate an equal amount of the venture's profits each year to investment in safe securities like government bonds. The amount of this annual commitment would be calculated by determining what annual investment put to interest at the assumed "safe" rate would be required to grow to the original acquisition cost of the mine by project termination. The purpose of this annual commitment was to provide funds to open a new mine when the current property was exhausted. Any remaining annual profit was then discounted at a higher, speculative rate to determine the value of the property. The rate used in this discounting step was to be higher than the safe rate assumed to apply to the sinking fund commitment. This "speculative rate" was higher because it represented the return the firm must receive to be induced to make the investment, its minimum rate of return or hurdle rate.

This process became widely used in mine valuation despite some significant shortcomings. First, the analysis is based on profit rather than cash flows. Subsequent valuation and evaluation tools emphasize cash flows as inputs because cash is the asset that can be consumed, accumulated, or invested. Annual profits do not accurately reflect the costs that the firm must bear in any given year nor the revenues that it will receive as a benefit of past investment. Second, the sinking fund assumption is used in all Hoskold valuation, regardless of its validity in a particular case. If a sinking fund will not actually be used as an investment for project profits, incorporation of the assumption in mine valuation would tend to undervalue the property. This distortion is a result of the assumed commitment of some of the profits to a "safe" investment with a yield below that actually realized on other investments. Despite these shortcomings, Hoskold's work shows a working knowledge of time value of money concepts and the basic discounting process.

On the same general subject, Irving Fisher (1896, 19) wrote, "The ordinary definition of the present value of a given sum due at a future date is that sum which put to interest today will amount to the given sum of that future date." This concept is known as net present value (NPV) analysis in the current literature.

These works demonstrate that time value of money concepts have been understood for over 100 years. The correct application of these simple ideas to real problems and the development of reliable analytical tools occurred much more recently, however.

In the mid-1930s, in his The Theory of Interest, Fisher introduced a measure he called "rate of return over cost," which is equivalent to DCFROR analysis. In developing the analytical logic of the rate of return over cost measure, Fisher (1926, 161) wrote that

> the rate of return over cost is always that rate which, employed in putting the present worth of all costs and the present worth of all returns, will make these two equal. . . . [T]he rate of return over cost compared with the market interest rate is our guide as to how far to go (in a series of incremental investments). We thus reach the marginal rate of return over cost.

Fisher illustrated the application of his rate of return over cost with the case of a farmer faced with progressive incremental investments which will improve the productivity of his acreage. If, as an alternative to spending $100 on farm improvement, the farmer could deposit his money in a savings bank yielding 5 percent per annum, then the return received on the additional or incremental investment associated with each increase in capital outlay should be compared against the 5 percent standard. If the incremental DCFROR exceeds 5 percent, the additional expenditure is economically justified.

A thorough discussion of the mechanics of this process of incremental analysis is presented in section 2.1.1. Fisher's contribution in The Theory of Interest is important for several reasons. First, it is one of the first treatments of economic evaluation in which the rate of return, or unknown "i," associated with an investment is determined. This process is the basis for modern DCFROR analysis. Second, Fisher's interpretation of the interest rate derived is based on comparing that rate with the yield from investing the same funds elsewhere, or the opportunity cost associated with making any commitment of resources. The opportunity cost concept has been overlooked frequently in subsequent works, as will be discussed later in section 3.1 of Chapter 3.

Finally, in basing his rate of return over cost on the differences between costs and revenues resulting from each of two possible courses of action, Fisher has performed what is known in modern literature as incremental analysis. As will be shown in the discussion of economic evaluation techniques contained at the end of this chapter, incremental analysis must be performed in order to properly compare mutually exclusive investment opportunities using rate of return analysis. Failure to perform incremental analysis when evaluating mutually exclusive alternatives using rate of return analysis has led to persistent confusion in the literature regarding the assumptions implicit in DCFROR analysis and its utility as an evaluation tool (Solomon 1956; Van Horne 1980; Weston 1982).

2.1.2 Keynes and Alchian

In his General Theory of Employment, Interest and Money, Keynes (1936, 135) introduced an economic measure he called the "marginal efficiency of capital" which he defined as

> being equal to that rate of discount which would make the present value of the series of annuities given by the returns expected from the capital asset during its life just equal to its supply price.

Keynes argues in the same work that his "marginal efficiency of capital" is conceptually and analytically the same as Fisher's "rate of return over cost."

Alchian (1955) disputed Keynes's claim. Alchian contended that Keynes was mistaken, and careless, in his assertion that the "marginal efficiency of capital" is equivalent to Fisher's "rate of return over cost." Specifically, Alchian argued that Fisher's "rate of return over cost" was developed to rank investment alternatives by the universally correct criterion of maximum present value, and that the measure could be defined only by reference to at least two alternative investment options. Keynes's "marginal efficiency of capital," on the other hand, was designed to find the discount rate that equated costs and revenues for a single project. Alchian (1955, 39) concludes:

> If one makes the careless step of thinking of the advantages as a simple receipt stream and disadvantages as a simple cost stream for a <u>single</u> investment, he will than be able to think that IRR is the rate of return over cost. Fisher's rate of return over cost always involved a comparison of two options, not a discounting of merely one option.

Alchian went on to say that "A ranking of substitutable alternatives by the Keynesian IRR is not consistent with maximizing net present worth." In effect, Alchian is arguing that ranking projects in order of rate of return will not necessarily lead to the best economic choices at either the macro level or for an individual firm.

An examination of the arguments reveals that Alchian is at least partly correct. As will be analytically demonstrated in section 2.2.2, DCFROR calculations based on a single project cash flow cannot be used reliably to select from among two alternative or mutually exclusive projects. The project with the highest DCFROR is not always the best economic choice. Further, ranking of projects in order of descending rate of return will not always result in investment in the best combination of projects for a given capital budget. However, Alchian's contention that Fisher's "rate of return over cost" will rank investment alternatives by the universally correct criterion of maximum present value is itself in serious error. As will be shown in section 2.2.2, incremental analysis cannot be used to rank nonmutually exclusive investment alternatives in order of economic desirability.

Alchian's argument that Fisher's "rate of return over cost" must be applied to two or more projects is also in error. Fisher indicates in his discussion of his method that one possible option in most situations is to do nothing. If a firm faces a single investment option, its incremental cash flow would indeed be represented by the difference between the costs and revenues associated with the project in question and doing nothing. In this situation, DCFROR and "rate of return over cost" will give the same result. In the modern literature, this case is known as the accept/reject decision. DCFROR is a valid economic tool for evaluating a single investment on an accept/reject basis. DCFROR can also be used to compare the economic desirability of several options if only one may be accepted. In this case, an incremental cash flow is obtained and evaluated as discussed by Fisher in the development of "rate of return over cost." The mathematics of these concepts are illustrated in detail in section 2.2.3. Alchian (1955, 41) concluded his comparison of Keynes's and Fisher's measures by writing:

> In order to have the Keynesian ranking agree with Fisher's, either we must assume exactly similar time periods, or we must assume that the net receipts from the two alternatives can be immediately and perpetually reinvested at their own internal rates of return.

This statement is significant because it introduces the concept of reinvestment of project cash flows in rate of return analysis. This argument assumes that any positive revenues generated by the project considered will be reinvested elsewhere to earn interest or returns until project termination. As will be discussed in section 2.3, some authors have argued that every DCFROR and NPV analysis implicitly assumes that any intermediate project cash flows are reinvested to earn at that project's DCFROR. Chapter 3 of this work analytically examines this assumption and demonstrates that no implicit reinvestment assumption is contained in either DCFROR or NPV analysis.

Joel Dean popularized the rate of return concept in Capital Budgeting (1951, 25) in which he wrote:

> The rate of return on the investment refers in theory to the rate of interest that will make the present value of future gross earnings just equal to the cost of the machine.

This is nearly identical to what Fisher and Keynes had been arguing for nearly 20 years. Dean's contribution was to bring these evaluation concepts to the business community in an applied context. His work was often cited by authors writing in the fields of financial management and economic evaluation. (Solomon et al., The Management of Corporate Capital, 1959; Berry 1972; Weston 1975)

2.1.3 Solomon and Berry

Ezra Solomon (1956) introduced what would later come to be known as growth rate of return in an attempt to reconcile the apparent discrepancy in the economic decisions made by DCFROR and NPV analyses of the same project.

Solomon defined two investment options, shown in Examples 2.1 and 2.2. Using DCFROR analysis, he determined Option X to be the best choice because it shows the highest DCFROR. Solomon then analyzed the same two projects with NPV analysis, using a 10 percent discount rate, and found Option Y to have the highest NPV, making it the best economic choice.

EXAMPLE 2.1

	Year 0	Year 1
Option X	C=$100	R=$120

DCFROR = 20%
NPV at 10% discount rate = $109.09

EXAMPLE 2.2

	Year 0	Year 1	Year 2	Year 3	Year 4
Option Y	C=\$100	R=\$0	R=\$0	R=\$0	R=\$174.90

DCFROR = 15%
NPV at 10% discount rate = \$119.46

Solomon blamed the inconsistent results on different implicit reinvestment rates built into the DCFROR analysis. As will be shown in Chapter 3, the inconsistency in results is not due to any reinvestment assumption, but rather to failure to complete the DCFROR analysis by performing incremental rate of return analysis. Once Solomon had concluded that the inconsistent results were the fault of differing implicit reinvestment assumptions, he resolved the inconsistency by making a common, explicit reinvestment assumption for both options. In this analysis, intermediate positive project cash flows for both projects were assumed to be reinvested at 12 percent through year four, the termination date of the longest-lived alternative, or to accumulate to a single terminal value. Both options could then be represented as consisting of a single investment yielding a single lump-sum payoff at project termination year four. A DCFROR calculation was then made to determine the rate of growth of the initial investment. This analysis is illustrated in Examples 2.3 and 2.4.

EXAMPLE 2.3

	Year 0	Year 1	Year 2	Year 3	Year 4
Option X	C=$100	R=$0	R=$0	R=$0	R=$159.72

Growth rate of return = 13.9%
NPV at 10% discount rate = $9.09

EXAMPLE 2.4

	Year 0	Year 1	Year 2	Year 3	Year 4
Option Y	C=$100	R=$0	R=$0	R=$0	R=$174.90

Growth rate of return = 15%
NPV at 10% discount rate = $119.46

A thorough discussion of the mathematics of growth rate of return analysis and its application in the evaluation of mutually exclusive and nonmutually exclusive project evaluations is contained in section 2.2.3.

This analysis led Solomon (1956, 129) to argue:

> The apparent conflict between the two approaches results only from differing assumptions that each makes about the future. . . . Our conclusion is that correct and consistent ranking of the investment worth of competing proposals can be obtained only if the following factors are taken into account:
>
> 1. The valid comparison is not simply between two projects but between two alternative courses of action. The ultimate criterion is the total wealth that the investor can expect from each alternative by the terminal date of the longer-lived project. In order to make a fair comparison, an explicit and common assumption must be made regarding the rate at which funds released by either project can be reinvested up to the terminal date.
>
> 2. If the rate of return is to be used as an index of relative profitability, then the relevant

> rate is the per annum yield promised by each alternative course of action from its inception to a common terminal date in the future (usually the terminal date of the longer-lived project).

Solomon then concludes that if this analytical adjustment is made to DCFROR analysis its results will be consistent with those obtained using NPV analysis.

A simple example will prove Solomon's contention wrong. Consider the project cash flows defined in Examples 2.5 and 2.6. Growth rates of return and NPVs are calculated for each and the results summarized in Table 2.1. In this case a 12 percent reinvestment rate was specified for positive project cash flows through year four, the termination date of the longest-lived project. As Table 2.1 demonstrates, growth rate of return and NPV will not necessarily yield consistent results if project costs are not the same for each alternative considered.

EXAMPLE 2.5

	Year 0	Year 1	Year 2	Year 3	Year 4
Option A	C=$100	R=$0	R=$0	R=$0	R=$174.90

Growth Rate of Return = 15%
NPV at 12% Discount = $11.1521

EXAMPLE 2.6

	Year 0	Year 1	Year 2	Year 3	Year 4
Option B	C=$1000	R=$0	R=$0	R=$0	R=$1685.9136

Growth Rate of Return = 13.95%
NPV at 12% Discount = 71.4286

TABLE 2.1
Comparison of Results of GRR and NPV Analysis
of Mutually Exclusive Investments

Option	Growth Rate of Return	NPV at 10%	NPV at 12%
A	15%	$19.4591	$11.1521
B	13.95	90.9091	71.4286
Economic Choice	A	B	B

Solomon's work here is important for several reasons. First, like Alchian, he has erroneously concluded that the inconsistent results obtained by DCFROR and NPV analyses are the fault of differing implicit reinvestment assumptions contained in DCFROR calculations. However, in Alchian's analysis the reinvestment argument was not the emphasis of the work. Solomon spends a great deal of time and effort discussing the implicit reinvestment's role in DCFROR analysis.

Second, Solomon's work reached a wide audience, coming at a time when the business world was attempting to learn, understand, and use DCFROR and NPV. The lasting impact of his work on this question is illustrated by the many references to his analysis in important text and periodical literature (Weston 1975; Van Horne 1980). Finally, the analytical technique Solomon offered to resolve the discrepancy in DCFROR and NPV results has come to be known as growth rate of return analysis in the modern literature.

Berry (1972) modified Solomon's basic work by postulating that the reinvestment rate used in calculating growth rates of return may change as the firm "learns" how to invest its funds most profitably. Solomon's analysis provided for only one reinvestment rate. Berry's performance measure, called the "wealth growth rate," assumes that the reinvestment rate for positive project cash flows will increase as the firm finds better and better investment opportunities.

2.1.4 Recent Developments in Mineral Evaluation

Thomas O'Neil (1982) has discussed several factors that have recently complicated the economic analysis of mineral properties.

O'Neil cites the Lakeshore copper mine in southern Arizona as an example of how using capital costs and unstable mineral prices caused the abandonment of an important property. In 1969, original capital cost estimates for development of the Lakeshore property were in the neighborhood of $100 million. The investment appeared sound in light of the strong copper prices prevailing at the time. By the time Hecla Mining Company and El Paso Natural Gas abandoned the property in 1978, total investment exceeded $200 million, and the world copper price was depressed.

O'Neil (1982, 1670) summarizes:

> Hecla and El Paso were forced to make the painful decision to walk away from the Lakeshore venture when bankruptcy became a real possibility for Hecla. Hecla simply did not have the financial resources to withstand depressed market conditions for a prolonged period.

This increased risk has led to the development of new evaluation techniques. A weakness of traditional DCFROR or NPV analyses is their dependence upon and sensitivity to assumed commodity prices as an input. Wide fluctuations in commodity prices have weakened the credibility of any technique which requires specification of an input price. In an attempt to deal with this problem some firms have begun performing supply curve analyses. Instead of assuming a commodity price and calculating the NPV or DCFROR for an individual project, the firm derives the supply curve for all the producers of the commodity. Once the relationship between breakeven production cost, including minimum required rate of return, and annual output has been determined, the firm attempts to locate the breakeven cost for the property in question on the curve. To quote O'Neil (p. 1670),

> One major producer screens perspective projects requiring that their estimated [breakeven] operating costs be in the lower quartile of all major producers of that commodity. Then if prices sink, that operation is protected by a cushion of higher cost producers who would presumably be forced to curtail production at an earlier date. . . . Such curves are rapidly gaining in importance for

investment decision making in mining. It comes as little surprise to find that in the copper industry similar curves for early 1982 show virtually all domestic mines operating at a loss.

2.2 Review of Economic Evaluation Techniques

This section reviews the mathematics and the limitations of the economic evaluation techniques that will be used in Chapters 3 and 4. This review is important because much of the confusion surrounding the validity of the implicit reinvestment rate assumption in DCFROR analysis was caused by misunderstanding of the limits of NPV and DCFROR analyses.

2.2.1 Net Present, Annual, and Future Value Analyses

Perhaps the easiest analytical technique to use is net present value analysis (NPV), in which project revenues and costs are discounted at a specified interest rate to some common point in time. If the discounted revenues exceed discounted costs the NPV is positive and the investment is economically attractive in that undertaking it will increase the present value of the firm. If NPV is negative the investment will not increase the present value of the firm because the present value of project costs exceeds present value of project revenues. A third possible outcome exists in which NPV exactly equals zero indicating that discounted revenues exactly equal discounted costs. In this situation the present value of the firm is neither increased nor decreased by undertaking the investment, and the firm would be economically indifferent to accepting the project. A sample net present value calculation is shown below.

EXAMPLE 2.7

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$100	R=$50	R=$50	R=$50	R=$50	R=$50

$$\text{NPV @ } 10\% = -\ \$100 + \$50[1/(1.1)] + \$50[1/(1.1)]^2 + \$50[1/(1.1)]^3 + \$50[1/(1.1)]^4 + \$50[1/(1.1)]^5$$

Initial cost at year zero is $100 yielding net positive cash flows of $50 a year for the following five years. If the appropriate discount rate is 10 percent compounded annually the NPV associated with this project is approximately $89.54. The economic interpretation of this result is that if this project is undertaken, the present value of revenues exceed present value of costs by almost $90, and so the present value of the firm is increased by $90 by accepting the project.

Analyses similar to NPV calculations may be performed in which future or terminal values associated with project cash flows are determined. In this situation project costs and revenues are carried forward to some common point in time, often the project termination date. Instead of discounting backwards at a specified rate of return, revenues and costs are compounded forward to a common future date to determine a net future value, or NFV. Performing a NFV analysis of the project presented in Example 2.7 yields the following result.

EXAMPLE 2.8

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=\$100	R=\$50	R=\$50	R=\$50	R=\$50	R=\$50

$$NFV = -\ \$100(1.1)^5 + \$50(1.1)^4 + \$50(1.1)^3 + \$50(1.1)^2 + \$50(1.1) + \$50 = \$144.2$$

The economic interpretation of NFV is similar to that of NPV. If the NFV is positive, the future value of the firm is increased and the project is economically acceptable. If the NFV is negative, acceptance of the project will decrease the future value of the firm and the project is economically unacceptable.

A final variant of NPV and NFV analysis is net annual value (NAV) in which the NPV or NFV is converted to an equivalent annual value. For example, instead of calculating net value at the beginning or end of a project, the result can be prorated over the project life, incorporating the time value of money. The resulting net annual value may be interpreted as the uniform value accruing to the investor in each year of the project. NAV calculations for Example 2.9 are shown below.

EXAMPLE 2.9

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$100	R=$50	R=$50	R=$50	R=$50	R=$50

NPV @ 10% = $89.54

NAV @ 10% = $89.54 $\{.1(1.1)^5/[(1.1)^5 - 1]\}$ = $23.61

As was the case in the interpretation of NPV and NFV, a positive NAV indicates an economically acceptable investment while a negative NAV indicates that the project is economically unattractive. The principle advantage of net value analyses is their ease of calculation. A primary disadvantage is that the correct discount rate must be known in advance and that results may be very sensitive to changes in the discount rate used. If NPV calculations are made for the data in Example 2.9 using a 25 percent discount rate instead of the 10 percent rate used in the initial analysis, the NPV falls from $89.54 to $34.46. If the rate used is 50 percent, NPV becomes negative, yielding -$13.169.

In summary, individual projects should be accepted if their NPVs are positive. If two mutually exclusive investment options are being compared, the project showing the highest NPV is the best economic choice. NPV fails, however, as an accurate tool for ranking nonmutually exclusive investment alternatives in order of economic desirability.

Net value analysis may be used to select the best economic choice from among competing, mutually exclusive investment opportunities. If only one of several investments may be accepted, the investment promising the highest net present, annual, or future value is the most economically desirable.

Net value analysis cannot be used, however, to rank nonmutually exclusive investment alternatives in order of economic desirability. This is the decision firms are often called upon to perform in the context of capital budget allocation. In this situation funding projects in order of highest NPV to lowest will not necessarily lead to funding the combination of projects that maximizes the present worth of the firm.

To illustrate this concept, consider the seven investment options shown in Table 2.2. The NPVs for each option are calculated using a discount rate of 10 percent compounded annually. A ranking of projects in NPV order is shown in Table 2.3.

If a firm facing a $100 budget constraint were to allocate its capital by funding projects with the highest NPVs first, it would end up accepting projects B, E, and C, thereby using up its entire $100 capital budget. This policy would result in a cumulative NPV of $30.69 for the three investments. Although this strategy leads to funding the individual projects with the highest NPVs, it does not

TABLE 2.2
Cash Flows for Nonmutually Exclusive
Investment Alternatives

Project A

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25.00	R=$8.36	R=$8.36	R=$8.36	R=$8.36	R=$8.36

NPV @ 10% = $6.691

Project B

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25.00	R=$0	R=$0	R=$0	R=$0	R=$57.50

NPV @ 10% = $10.703

Project C

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$37.50	R=$6.375	R=$6.375	R=$6.375	R=$6.375	R=$6.375

NPV @ 10% = $9.9508

Project D

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25.00	R=$12.75	R=$12.75	R=$12.75	R=$12.75	R=$12.75

NPV @ 10% = $5.834

Project E

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$37.50	R=$12.54	R=$12.54	R=$12.54	R=$12.54	R=$12.54

NPV @ 10% = $10.036

TABLE 2.2 (continued)

Project F

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$12.50	R=$0	R=$0	R=$0	R=$0	R=$27.4056

NPV @ 10% = $4.516

Project G

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$12.50	R=$4.18	R=$4.18	R=$4.18	R=$4.18	R=$4.18

NPV @ 10% = $3.346

TABLE 2.3
NPV Project Selection for $100 Capital Budget

Option	Cost	NPV @ 10%
B	$ 25.00	$10.70
E	37.50	10.04
C	37.50	9.95
Total	$100.00	$30.69

lead to maximization of the present worth of the firm.

Instead of funding projects B, E, and C, the firm could fund A, B, F, and E, as shown in Table 2.4. This combination yields a total NPV of $31.947 for the $100 capital budget.

This example illustrates an inability of simple NPV analysis to identify the best combination of nonmutually exclusive projects to fund given a limited capital budget. As will be shown later in this section, incremental DCFROR

TABLE 2.4
Present Value Ratio Project Ranking for
$100 Capital Budget

Option	Present Worth Cost (PWC) @ 10%	NPV @ 10%	PVR = $\frac{NPV}{PWC}$
B	$ 25.00	$10.70	.428
F	12.50	4.52	.3616
E	37.50	10.04	.2677
A	25.00	6.69	.2676
Total	$100.00	$31.95	

Conclusion: Fund projects B, F, E, and A given a $100 capital budget. Resulting total NPV = $31.947.

analysis will reach conclusions consistent with those obtained using NPV analysis. Therefore, incremental rate of return analysis will also fail to consistently rank nonmutually exclusive projects in order of economic desirability.

In order to correctly rank nonmutually exclusive alternatives in order of economic desirability using NPV analysis, the NPV associated with a given project must be divided by the discounted or present worth of the net costs required to generate the NPV. The resulting ratio, called present value ratio (PVR), can be used to rank nonmutually exclusive investment options in order of economic desirability. Table 2.4 summarizes the PVR rankings for the seven investment options introduced in Table 2.1.

In performing present value ratio analysis it is important to include in the ratio denominator only those costs not covered in project revenues occurring concurrently or before incidence of cost. To illustrate, consider Example 2.10, which requires cash outlays in Years 0 and 2. The calculation of the present worth cost (PWC) of this investment is shown in Equation 2.1.

EXAMPLE 2.10

Year 0	Year 1	Year 2	Year 3	Year 4
C=$100	R=$50	R=$70	R=$100	R=$100

NPV @ 10% = 31.0361

$$PWC = \$150 + \$70[1/(1.1)]^2 - \$50[1/(1.1)] = \$112.3967 \quad (2.1)$$

The PWC is reduced by $50 to yield to net cost to the firm of undertaking this project, or the net expenditure required from the corporate treasury. The $50 in project revenue received in Year 1 provides part of the capital required for the $70 expenditure required in Year 2.

In summary, NPV calculations may be used to make accept/reject decisions for a single investment, select the best economic choice from two or more mutually exclusive alternatives, or to place a value on an investment. NPV analysis alone cannot, however, rank nonmutually exclusive investment alternatives in order of economic desirability. If an NPV-based ranking tool is required, present value ratio (PVR) analysis must be performed to yield the correct ordering.

2.2.2 Discounted Cash Flow Rate of Return Analysis

One of the most commonly used investment evaluation criteria is discounted cash flow rate of return, also known as internal rate of return. In DCFROR analyses, project cash flows are evaluated to determine what discount rate sets project costs exactly equal to project revenues. In other words, a present worth equation is set up incorporating the discount rate, "i," as an unknown. The equation is then solved for the value of "i" which yields an NPV of zero. This value of "i," the DCFROR, is then compared with some minimum economic performance standard, or hurdle rate, to determine whether the project is economically acceptable.

One advantage of DCFROR analysis is that no discount rate need be specified to perform the required calculations. The DCFROR is solved for within the context of the project cash flows; no external data is required to determine DCFROR. The DCFROR cannot be interpreted, however, without reference to an economic performance standard like minimum required rate of return. In NPV analysis, if the standard or discount rate changes, all project NPVs must be recalculated using the new discount rate. In DCFROR calculations, the DCFROR determined for any project is independent of the minimum required rate of return. Therefore, if the minimum rate of return changes, no new calculations are necessary to determine whether projects are still economically acceptable.

For example, the DCFROR associated with Example 2.10 equals 41.04 percent. If the minimum required rate of return equals 10 percent, this project is economically acceptable. If the minimum rate of return increased to 50 percent, no new calculations are required to determine that this project is no longer attractive economically.

The primary disadvantage associated with the use of DCFROR is misinterpretation of its results and misunderstanding of its assumptions and its application in some situations.

A characteristic of DCFROR analysis that continues to cause confusion is its inability to indicate which of two mutually exclusive alternative investments is the best economic choice. This evaluation context may arise if an investor has insufficient funds to undertake more than one investment, or if alternative means of providing a required service (i.e., haul trucks vs. conveyer systems for moving overburden and coal in surface coal mines) are being evaluated. Because DCFROR is a percentage calculation which does not explicitly take into account the magnitude of the investment or base to which the result applies, the DCFRORs associated with two or more individual projects may not be meaningfully compared to select the best economic choice from among several mutually exclusive alternatives.

To illustrate, consider a comparison of the project depicted in Example 2.10 with Example 2.11.

EXAMPLE 2.10

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$100	R=$50	R=$50	R=$50	R=$50	R=$50

DCFROR = 41%

EXAMPLE 2.11

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$1000	C=$500	R=$750	R=$750	R=$750	R=$750

DCFROR = 25%

The DCFROR for Example 2.10 was previously calculated to be 41 percent, while the DCFROR for Example 2.11 is determined to be 25 percent. A comparison of DCFROR results would lead to selection of Example 2.10 because its DCFROR is substantially larger than that of Example 2.11. But comparisons of DCFROR results ignore the base to which the rate of return applies. It may be economically better to receive a lower rate of return on a greater investment than a very high rate of return on a smaller investment.

In order to determine which mutually exclusive alternative is the best economic choice, a two-phased analysis must be performed. Each candidate investment must first be evaluated to see that its DCFROR meets or exceeds the minimum required rate of return.

Next, incremental analysis must be performed on each qualifying investment. The purpose of the incremental analysis is to determine whether the increased investment associated with higher cost alternatives is justified by any incremental increase in cash flow that the incremental investment may generate. In executing incremental analysis, the lowest cost option is used as a base. The cash flow associated with the lowest cost alternative is subtracted from the cash flow associated with the next lowest cost option to yield an incremental cash flow composed of the annual differences between revenue and cost fofor each alternative. This incremental cash flow is then evaluated to determine its DCFROR. This process is illustrated below using Example 2.11 and Example 2.12 cash flows.

EXAMPLE 2.11

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$1000	C=$500	R=$750	R=$750	R=$750	R=$750

EXAMPLE 2.12

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$100	R=$50	R=$50	R=$50	R=$50	R=$50

Incremental Cash Flow

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$900	C=$550	R=$700	R=$700	R=$700	R=$700

Incremental DCFROR = 24.43%

In this case the incremental DCFROR, or the return associated with the incremental or marginal investment equals 24.43 percent. The incremental investment would be attractive if the minimum required rate of return were less than or equal to 24.43 percent. In this case, Example 2.12 would be a better economic choice than Example 2.11 despite the fact that Example 2.11 had the higher project rate of return. The incremental analysis revealed that the $900 incremental cost associated with funding Example 2.12 instead of Example 2.11 is more than justified by the incremental revenue generated by the project, if the minimum required rate of return for this firm is less than or equal to 24.43 percent. If a third, higher cost alternative existed, another incremental analysis would be performed using Example 2.12 as a base to see if the marginal or incremental cost required to undertake the higher cost project would be justified by the incremental revenues generated. If the incremental analysis performed using this third project with Example 2.12 as a base yielded an incremental rate of return exceeding the minimum required rate of return, the higher cost project would again be accepted over the lower cost Example 2.12. If the incremental DCFROR were determined to be less than the minimum required rate of return, the investment would be rejected and Example 2.12

would remain the base. This process would continue until all projects had been incrementally analyzed.

This is exactly the kind of analysis introduced by Fisher in 1936 in The Theory of Interest to illustrate application of "rate of return over cost" as discussed in section 2.1.1.

There remains substantial confusion in important works in the literature and in practice regarding the use of DCFROR for analyzing mutually exclusive projects. Many authors (Solomon 1956; Van Horne 1980; Weston 1982) commented on the apparent discrepancies in results obtained when evaluating the same mutually exclusive projects with NPV and DCFROR techniques, and have concluded that these discrepancies result from reinvestment assumptions implicit in both NPV and DCFROR calculations. As will be shown in Chapter 3 of this work, the primary cause of this persistent confusion is incomplete DCFROR analysis. Chapter 3 will also show that a properly executed incremental rate of return analysis will always yield results consistent with those obtained from NPV calculations.

A second weakness of DCFROR is its inability to reliably rank nonmutually exclusive alternatives in order of economic attractiveness. In the nonmutually exclusive situation, selection of any one project does not preclude undertaking any of the others. Ranking of nonmutually exclusive alternatives is often associated with the capital budgeting process in which investment opportunities are ordered according to their economic desirability. Projects are then funded according to their rank until the funds available are exhausted.

To illustrate this process, assume that a firm with a 10 percent discount rate has the opportunity to invest in as many of the seven projects displayed in Table 2.5 as its financial resources will allow. In other words, the firm must rank these seven nonmutually exclusive investment alternatives in order of economic preference. Ranking by project rate of return yields the ordering shown in Table 2.5. Project D promises the highest DCFROR of 21.43 percent, with projects A, E, and G tied for second with anticipated rates of return of 20 percent each. Projects B, C, and F are the least attractive investments, according to rate of return ranking.

If the budget restriction is $100,000, selection of projects in DCFROR order would result in funding projects D, A, E, and G, as shown in Table 2.6, exhausting the entire $100,000 budget and generating $25,909 in net present value.

Examination of these results will reveal that this investment plan will not yield the best return for the firm despite the fact that the project with the highest indi-

TABLE 2.5
Cash Flows for Nonmutually Exclusive
Investment Alternatives

Project A

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25,000	R=$8360	R=$8360	R=$8360	R=$8360	R=$8360

DCFROR = 20%
NPV @ 10% = $6691

Project B

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25,000	R=$0	R=$0	R=$0	R=$0	R=$57,500

DCFROR = 18.126%
NPV @ 10% = $10,703

Project C

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$37,500	R=$6375	R=$6375	R=$6375	R=$6375	R=$6375

DCFROR = 17%
NPV @ 10% = $9950.8

Project D

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25,000	R=$12,750	R=$12,750	R=$12,750	R=$12,750	R=$12,750

DCFROR = 21.4333%
NPV @ 10% = $5834.9374

Project E

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$37,500	R=$12,540	R=$12,540	R=$12,540	R=$12,540	R=$12,540

DCFROR = 20%
NPV @ 10% = $10,036.5

TABLE 2.5 (continued)

Project F

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$12,500	R=$0	R=$0	R=$0	R=$0	R=$27,406

DCFROR = 17%
NPV @ 10% = $4516.97

Project G

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$12,500	R=$4180	R=$4180	R=$4180	R=$4180	R=$4180

DCFROR = 20%
NPV @ 10% = $3346

TABLE 2.6
Project Ranking by DCFROR Criteria
for a $100,000 Capital Budget

Project	Cost	DCFROR	NPV @ 10%
D	$ 25,000	21.43%	$ 5,835
A	25,000	20	6,691
E	37,500	20	10,037
G	12,500	20	3,346
Total	$100,000		$25,909

vidual DCFRORs have been funded. Table 2.7 summarizes the results obtained if projects B, C, F, and A are funded. This plan involves funding the three alternative projects with the lowest rates of return.

The investment plan summarized in Table 2.7 results in an NPV of $31,862, almost $6,000 NPV greater than the results obtained from funding the projects promising the highest individual project rates of return. This example

TABLE 2.7
Alternate Project Ranking for a
$100,000 Capital Budget

Project	Cost	DCFROR	NPV @ 10%
B	$ 25,000	18%	$10,703
C	37,500	17	9,951
F	12,500	17	4,517
A	25,000	20	6,691
Total	$100,000		$31,682

clearly demonstrates that DCFROR is an unreliable means of ranking nonmutually exclusive projects for capital budgeting purposes. A discussion of the historical evolution of this kind of analysis is presented in section 2.1.2.

In summary, simple DCFROR analysis may only be used to make accept/reject decisions for individual investment opportunities. If only one of two or more projects may be undertaken, an incremental analysis must be performed if DCFROR is to be used as a decision criterion. DCFROR will not reliably rank nonmutually exclusive investment options in order of economic preference. If a rate of return ranking tool is required, growth rate of return may be used as will be illustrated in the following section.

2.2.3 Growth Rate of Return Analysis

As mentioned earlier in this chapter, the confusion surrounding the reinvestment assumption argued by some to be implicit in performance of rate of return calculations led

Solomon (1956) to develop an economic performance measure that made common, explicit assumptions regarding the reinvestment of intermediate project cash flows. This technique, now called growth rate of return, determines the rate of growth of the initial investment through some specified point in time, usually the project termination date. Cash inflows generated by the project are explicitly assumed to be reinvested at a known rate of return to yield a future value as of the specified termination date. The project cash flows are then expressed as an initial investment, or investments, which grow to a single payoff at some specified future date. The rate of return is then found which equates the single required investment with the single resulting terminal value, or the rate of growth of the project investment.

To illustrate, consider Example 2.14. This project requires initial investment of \$100 to yield future cash flows of \$50 and \$60 in the first two years, and \$70 annually for the last three years of the project life.

EXAMPLE 2.14

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=\$100	R=\$50	R=\$60	R=\$70	R=\$70	R=\$70

If positive cash flows may be reinvested to earn 15 percent by project termination at year five, the project will have accumulated $421.78, which may be interpreted as the single terminal value associated with this investment. This reinvestment process can be represented by Equation 2.2.

$$\text{Terminal Value} = \$50(1.15)^4 + \$60(1.15)^3 + \$70(1.15)^2 + \$70(1.15) + \$70 = \$421.78 \qquad (2.2)$$

This calculation yields an investment opportunity depicted by Example 2.15, in which the initial $100 investment grows to a $421.78 terminal value over the five year project life.

EXAMPLE 2.15

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$100	R=$0	R=$0	R=$0	R=$0	R=$421.78

The growth rate of return associated with this project is the discount rate which equates project costs with the single terminal value of $421.78. Solving Equation 2.3 for "i" reveals the rate of growth of the initial $100 investment to be 33.36 percent.

$$\$100 = \$421.78[1/(1.1)]^5 \qquad (2.3)$$

Alternatively, if this investment is undertaken, the firm will receive a 33.36 percent return on its entire intitial investment over the five year project life.

As discussed in section 2.1.3 of this chapter, Ezra

Solomon (1956) popularized growth rate of return to reconcile the apparent discrepancies in results obtained from NPV and DCFROR analyses of the same projects.

Proponents of growth rate of return also argue that using an explicit reinvestment assumption growth rate of return resolves the logical inconsistencies that some feel accompany simple DCFROR calculations. Specifically, the explicit reinvestment assumption used in growth rate of return calculations makes more sense than the use of different implicit reinvestment rates for different, perhaps competing projects, that could be undertaken at the same time in the same or similar economic environments. A brief discussion of the implicit reinvestment argument is introduced in section 2.3, with a mathematical analysis demonstrating the invalidity of the implicit reinvestment argument following in Chapter 3.

In Chapter 3 it will also be shown that it is never necessary to perform growth rate of return calculations to arrive at correct economic decisions. However, if a firm uses rate of return analysis, as opposed to NPV or ratio analysis, growth rate of return will be required to rank nonmutually exclusive projects in order of preference.

Table 2.8 displays growth rate of return calculations for the seven nonmutually exclusive investment alternatives

TABLE 2.8
GRR Calculations for Nonmutually Exclusive Evaluations

Option A

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25,000	R=$8360	R=$8360	R=$8360	R=$8360	R=$8360

$$\$8360(1.10)^4 + \$8360(1.10)^3 + 8360(1.10)^2$$

$$+ \$8360(1.10) + \$8360 = \$51,037.8$$

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25,000	R=$0	R=$0	R=$0	R=$0	R=$51,037.8

Growth ROR = 15.34%

Option B

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25,000	R=$0	R=$0	R=$0	R=$0	R=$57,500

Growth ROR = 18.13%

Option C

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$37,500	R=$6375	R=$6375	R=$6375	R=$6375	R=$6375

$$\$6375(1.1)^4 + \$6375(1.1)^3 + \$6375(1.1)^2$$

$$+ \$6375(1.1) + \$6375 = \$76,419.38$$

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$37,500	R=$0	R=$0	R=$0	R=$0	R=$76,419.38

Growth ROR = 15.3014%

TABLE 2.8 (continued)

Option D

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25,000	R=$12,750	R=$10,200	R=$7650	R=$5100	R=$2550

$$\$12,750(1.1)^4 + \$10,200(1.1)^3 + \$7650(1.1)^2$$

$$+ \$5100(1.1) + \$2550 = \$49,661$$

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$25,000	R=$0	R=$0	R=$0	R=$0	R=$49,661

Growth ROR = 14.71%

Option E

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$37,500	R=$12,540	R=$12,540	R=$12,540	R=$12,540	R=$12,540

$$\$12,540(1.1)^4 + \$12,540(1.1)^3 + \$12,540(1.1)^2$$

$$+\$12,540(1.1) + \$12,540 = \$76,556.7$$

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$37,500	R=$0	R=$0	R=$0	R=$0	R=$76,556.7

Growth ROR = 15.34%

Option F

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$12,500	R=$0	R=$0	R=$0	R=$0	R=$27,405

Growth ROR = 17%

TABLE 2.8 (continued)

Option G

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$12,500	R=$4180	R=$4180	R=$4180	R=$4180	R=$4180

$$\$4180(1.1)^4 + \$4180(1.1)^3 + \$4180(1.1)^2$$

$$+ \$4180(1.1) + \$4180 = \$25,519$$

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$12,500	R=$0	R=$0	R=$0	R=$0	R=$25,519

Growth ROR = 15.34%

considered in the preceding capital budgeting analysis (Tables 2.4 through 2.7). Once individual project growth rates of return have been determined, projects may be accurately ranked in order of economic desirability by their growth rates of return, as shown in Table 2.9.

TABLE 2.9
Growth Rate of Return Project Ranking
for $100,000 Capital Budget

Project	Growth ROR	Cost	NPV @ 10%
B	18.13%	$ 25,000	$10,703
F	17.00	12,500	4,517
A	15.34	25,000	6,691
E	15.34	37,500	10,036
Total		$100,000	$31,947

Funding projects in order of growth rate of return will result in the highest possible cumulative NPV. Given the data shown in Table 2.9, projects B, F, A, and E should be funded, using up the $100,000 budget and yielding a cumulative NPV of $31,947.

In summary, growth rate of return analysis can be used to make accept/reject decisions for single investments and to rank nonmutually exclusive investment alternatives in order of economic desirability. Section 2.1.3 demonstrated, however, that growth rate of return is not a reliable tool for selecting the best economic choice from among two or more mutually exclusive alternatives. If growth rate of return is to be used to make mutually exclusive investment decisions incremental analysis must be performed.

2.3 Two Problems in Economic Evaluation

A review of the managerial finance and engineering economics literature reveals that there remain at least two important issues about which there are significant differences of opinion. The first of these is the selection of the appropriate discount rate for performing economic analysis. The second is the validity of the implicit reinvestment rate assumption argued by some to exist in the performance of DCFROR and NPV analysis.

There are essentially two schools of thought regarding the appropriate discount rate. One group (Weston 1982; Van Horne 1980; Solomon 1956) emphasizes the cost of capital as the appropriate discount rate. If DCFROR analysis is to be used in making the decision, the project DCFROR must be greater than the cost of the capital expressed as a percentage, that must be devoted to it. If the firm is able to obtain financing at 6 percent, any project promising a return greater than 6 percent ought to be accepted because the benefits associated with the project exceed the costs. In reality, the analysis can become much more complex as implicit, explicit, and weighted average costs of capital are calculated (Weston 1982; Van Horne 1980), but the basic premise remains unchanged.

Another philosophy (Grant and Ireson 1982; Stermole 1984) maintains that the real measure of the cost of any investment is the rate of return that could have been earned on the investment forgone to fund the project under consideration. This is essentially an opportunity cost argument. Its proponents argue that if, in funding any given project, the firm passes up the chance to earn a return of 15 percent on another investment of approximately equal risk, then the economic performance standard that any project under consideration should have to meet is not the 6 percent of the capital, but the 15 percent return afforded by the forgone opportunity.

The implicit investment rate question is divided along similar lines. Section 2.1.2 briefly reviewed the development of the reinvestment assumption. Today some important authors (Weston 1982; Van Horne 1980; Porterfield 1961) contend that DCFROR analysis incorporates an explicit reinvestment assumption that says that all the project costs must be borrowed at an interest rate equal to the project DCFROR. Also, all positive project cash flows must be reinvested at an interest rate equal to the DCFROR until project termination. These authors contend that unless these strict conditions are met, DCFROR results cannot be meaningfully interpreted. Obviously these conditions severely limit the utility of DCFROR as a tool for practical decision making.

But there is another side to this argument. Some authors (Grant and Ireson 1982; Stermole 1984) argue that there is no implicit reinvestment assumption built into DCFROR analysis. These authors argue that the reinvestment opportunities, or lack of them, that exist for positive project cash flows are irrelevant in the calculation and interpretation of the DCFROR for any given project. Project DCFROR is independent of the borrowing rate, or cost of funds, and the reinvestment rate. In this case DCFROR becomes a much more useful tool in economic decision making. Considering the popularity of DCFROR as an evaluation tool, the appropriate discount rate and validity of the reinvestment assumption are important questions. These questions will be considered in detail and resolved in Chapter 3.

Chapter 3

THE DISCOUNT RATE AND THE IMPLICIT REINVESTMENT ASSUMPTION

Section 3.1 examines the discount-rate question and concludes that opportunity cost, or the return associated with the most attractive forgone investment opportunity, is the appropriate rate to use in performing economic analysis.

Section 3.2 explores the validity of the implicit reinvestment rate assumption and finds that there is no reinvestment assumption implied or required to perform valid DCFROR analysis for accept/reject decisions. Section 3.2.1 points out logical and analytical inconsistencies resulting from attempts to apply the assumption consistently. Section 3.2.2 determines that the assumption is not required to make correct economic decisions. Section 3.2.3 elaborates on the breakeven nature of the DCFROR calculations. Section 3.2.4 discusses various other arguments offered in support of the reinvestment assumption.

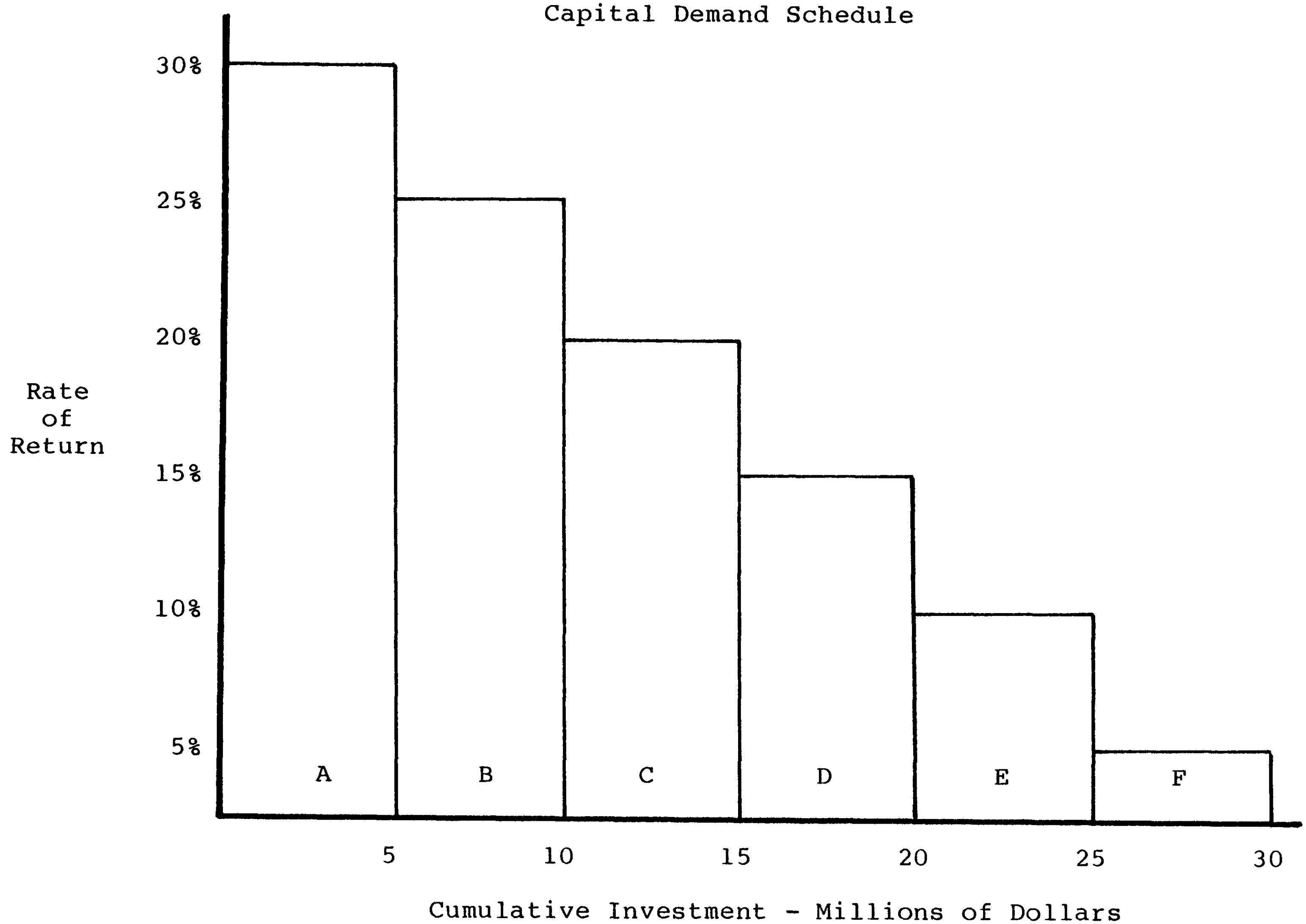
3.1 The Discount Rate

Once the mechanics of the various economic performance indicators are understood, questions remain regarding the appropriate interest rate to be used as an NPV discount rate or as a hurdle rate for DCFROR calculations. Many authors (Van Horne 1980; Weston 1982) emphasize that NPV analysis should be performed using the firm's cost of capital as the relevant discount rate. Similarly, all projects evaluated using DCFROR or growth rate of return should be accepted if their anticipated rates of return exceed the firm's cost of capital. It can be demonstrated, however, that use of cost of capital as the relevant discount or hurdle rate can lead to suboptimal decisions (or decisions that do not maximize the present value of the firm) in all but the most ideal situations. In all cases, however, the opportunity cost defined by the highest-return project not undertaken can be used as a discount rate for NPV analysis or as a hurdle rate for DCFROR analysis to arrive at optimal decisions.

As is the case with many financial or economic evaluation questions, basic microeconomic theory provides a framework within which the questions regarding appropriate discount rates can be analyzed and resolved. The firm's demand for capital may be depicted graphically by ranking investments in order of decreasing rates of return, as illustrated in Figure 3.1, in which projects are aggregated according to promised rates of return. Category A is composed of investments promising a 30 percent return. Implementation of all of these projects requires expenditure of $5 million. Category B is composed of all opportunities yielding 25 percent. Pursuit of all Category B investments requires an additional

FIGURE 3.1

Capital Demand Schedule



$5 million investment beyond the $5 million already committed to Category A, and so on. The histogram, then, represents the marginal return (or marginal revenue) associated with any level of cumulative investment. In order to maximize profit or to maximize its present value, the firm continues to invest, or to demand capital, so long as the yield (rate of return) or marginal revenue exceeds the cost of investing or using each additional or marginal dollar.

Acceptance of the above conditions makes it possible to determine the investment level that a rational firm will undertake if the cost of investing is known. As is the case with all inputs, the firm will continue to demand capital so long as the value of the product of the use of capital exceeds the input cost of capital.

This is the line of reasoning pursued by Weston (1982) in arguing that the intersection of the marginal cost of capital curve and the marginal revenue curve defines the discount rate that should be used in making NPV or DCFROR analysis. Using this approach in an accept/reject context, all nonmutually exclusive projects offering rates of return exceeding the rate defined by the intersection of the MR and MC curves are acceptable. This is equivalent to continuing investment until the marginal return on the last dollar invested exactly equals the cost of the funds devoted to the investment. Similarly, using the rate of return defined by the intersection of the MR and MC curves as a discount rate in NPV analysis results in positive NPVs for all projects falling above and to the left of the point of intersection of the two curves. Acceptance of these projects constitutes an optimal decision.

Two general types of costs are associated with investment in a given project:

1. Costs associated with procurement and use of the funds invested, including interest, dividends, flotation, and financing.

2. Opportunity costs associated with investing in a given project instead of taking advantage of other available investment alternatives. The relevant opportunity costs are the returns the firm could have realized on the projects forgone to fund the projects undertaken.

An illustration of the difference in these costs would be a case in which a firm has two investment opportunities, A and B, each costing $100,000 and promising returns of 15 and 20 percent, respectively. If the firm borrows the capital at 12 percent and invests it in project B which offers a 20 percent rate of return, the firm's cost of borrowing, or cost of capital, is 12 percent, but it also incurs an opportunity cost of 15 percent, because of having passed up the chance to earn a 15 percent return on project

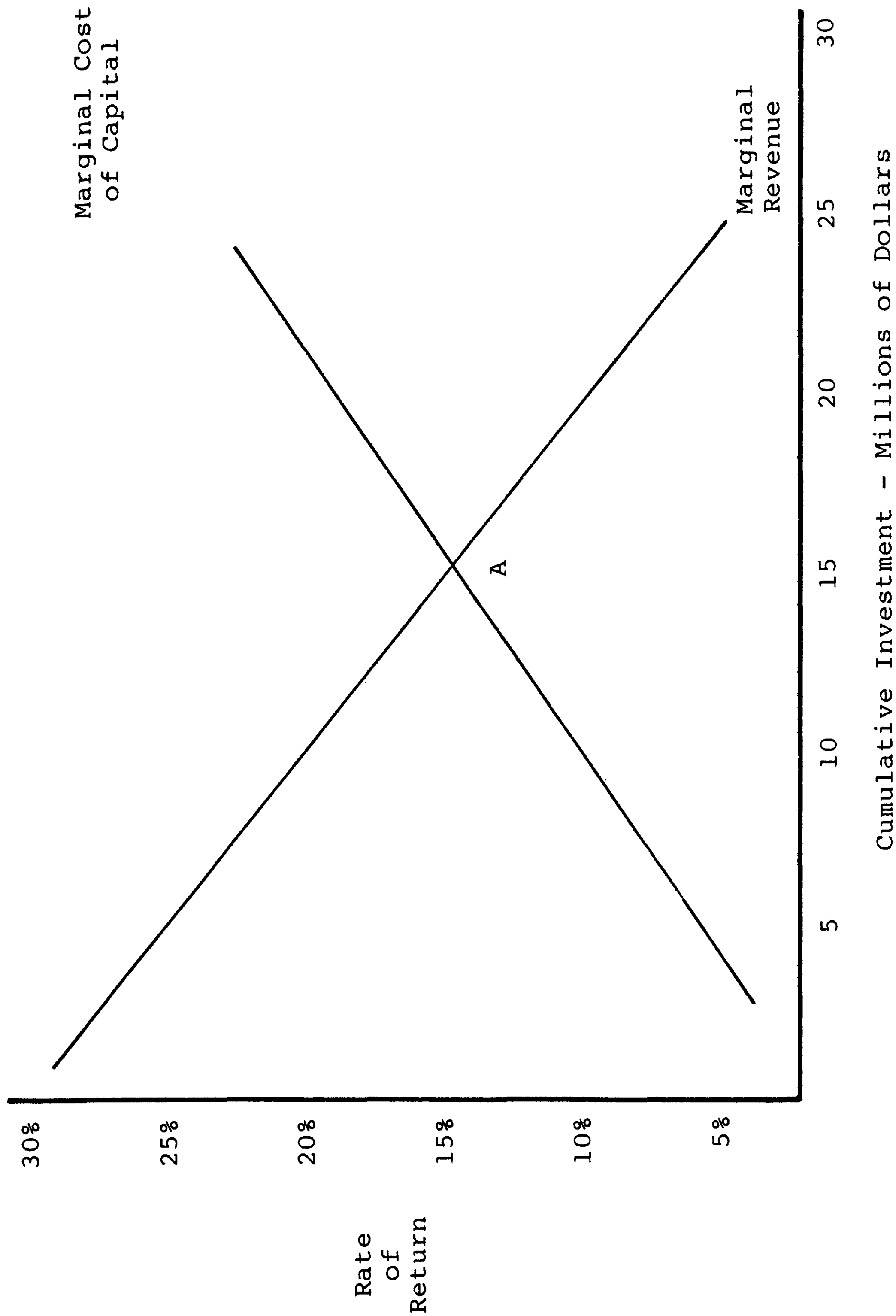
A by investing in project B.

The analysis depicted in Figure 3.1, in which the relevant discount rate is the marginal cost defined by the intersection of the two curves, explicitly considers only the first of the costs discussed above.

In Figure 3.2, the capital demand schedule, or marginal revenue curve, is depicted as a smooth curve, indicating a continuous return distribution over the 5 percent to 30 percent range considered. In this case, the difference between the marginal cost of capital, defined by the intersection of the MR and MC curves, and the opportunity cost, defined by the highest-return project not undertaken, is slight; so slight that the two rates can be considered the same for all practical purposes. If the marginal revenue curve is depicted as a histogram, as illustrated in Figure 3.1, the marginal and opportunity costs, defined by the intersections of the MR and MC curves, will be identical. In the ideal situation of unrestricted borrowing, or no capital rationing, marginal cost of capital and opportunity cost will be the same, and either can be used to reach an optimal decision. Use of the discount or hurdle rate to evaluate investment opportunities shown in Figure 3.2 would result in correctly accepting those projects lying above point A and rejecting those lying below, the result required by microeconomic theory.

FIGURE 3.2

Optimal Investment Level in a Noncapital-Rationing Environment

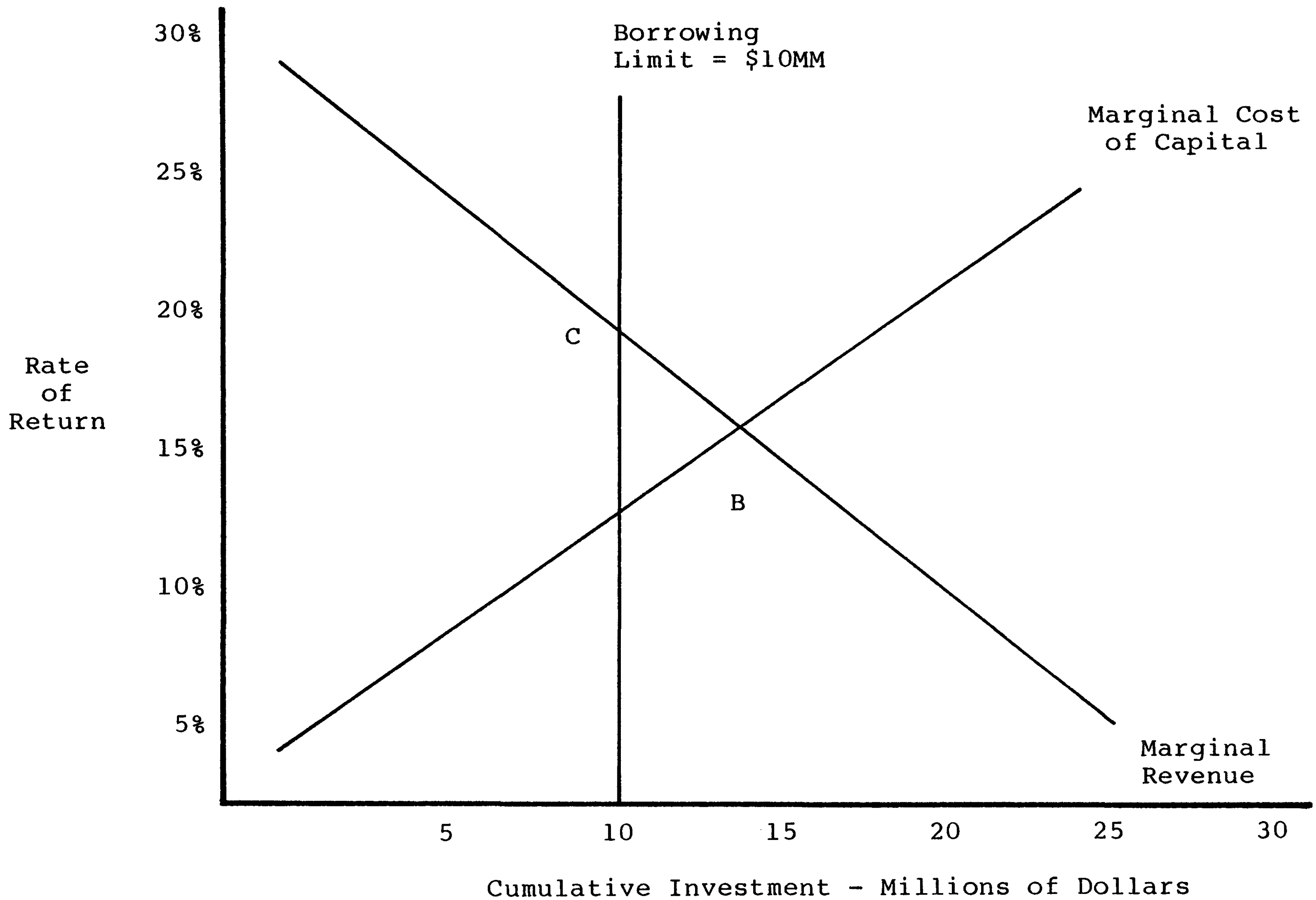


Van Horne (1980), however, points out that, in practice, firms often face borrowing restrictions or capital rationing which limits the amount that the firm, or some part of the firm (an individual department, for instance), can use. A capital rationing situation is shown in Figure 3.3. Although the firm should continue to borrow until point B is reached (where the marginal return associated with the next dollar borrowed exactly equals that dollar's cost), the firm's imposition of a $10 million borrowing ceiling would result in the situation shown in Figure 3.3. At the $10 million ceiling, the firm's cost of capital is no longer determined by the intersection of the MR and MC curves, but instead by the intersection of the MC curve and the vertical line defining the borrowing limit. This change lowers the cost of capital from 15 to 10 percent. Obviously, use of a 10 percent discount or hurdle rate for accept/reject decisions will not result in optimal decisions because the lower discount rate leads to acceptance of lower rate-of-return projects than was the case when borrowing was unrestricted and the marginal and opportunity costs were both 15 percent.

Use of the formerly defined 15 percent rate of return in the new environment would also lead to suboptimization in the capital rationing case because projects offering returns lying on the MR curve between points C and B could be

FIGURE 3.3

Investment in a Capital-Rationing Environment



accepted instead of those lying above point C. In the capital rationing situation, the relevant rate of return to be used for economic analysis must be defined by the intersection of the MR curve and the $10 million dollar vertical capital limit, which falls at point C. The rate defined at point C is the opportunity cost, or rate associated with investments not undertaken because of the limited capital availability. In this case, the 20 percent opportunity cost defined becomes the relevant discount rate to be used in NPV or DCFROR analyses, while the cost of capital becomes irrelevant. This conclusion stems from the assumption that the managers of the firm will attempt to maximize the present value of the firm, given the economic environment and constraints they face. In the situation depicted in Figure 3.3, use of a 15 percent rate of return will not necessarily lead to maximization of the present value of the firm. Use of a 15 percent standard will lead to acceptance of projects with rates of return between 15 percent and 20 percent at the expense of projects promising returns of 20 percent and higher. Therefore, although use of a cost of capital standard in a capital rationing environment will lead to decisions that increase the wealth of the firm, the cost of capital standard will not lead to decisions that maximize the wealth of the firm. The cost of capital standard fails to satisfy the requirements of a decision rule if maximization of the present value of the firm is management's goal.

This discussion has illustrated that marginal cost of capital can be used to make proper investment decisions only in the most ideal conditions of unlimited access to capital. In the ideal case, it has been demonstrated that marginal cost of capital equals opportunity cost, which means that opportunity cost also can be used in evaluating investments under ideal conditions of unlimited capital access. This discussion has also shown that in capital rationing situations, use of the firm's marginal cost of capital can lead to significant errors in evaluating projects for acceptance or rejection, while opportunity cost continues to be the appropriate rate for economic evaluation. Thus, marginal cost of capital works only sometimes, while opportunity cost works in all situations.

The general conclusion that opportunity cost is the appropriate discount rate to be used in NPV and DCFROR analyses is supported by the work of Van Horne (1980) and Grant and Ireson (1983). According to Grant and Ireson,

> [if] there is a limit on total funds available for investment in capital assets from all sources including borrowing, and if there are many proposals for investments in assets that seem likely to yield high returns, the controlling element in determining the minimum attractive rate of return (or discount rate) would still have been the 15 percent opportunity cost. . . . In principle opportunity costs within the enterprise normally

should determine the choice of i^* when the capital budget is limited.

Conclusions developed to this point regarding the proper discount rate can be applied to the practical situation in which the firm faces the task of evaluating a stream of investment opportunities and must select the appropriate discount rate to account for the time value of money. In practice, the firm will probably not construct a demand for capital curve, a marginal cost of capital curve, determine their point of intersection and use that point to define the investment quantity, marginal cost of capital and opportunity cost. Nor will the firm represent capital rationing with a vertical line on a graph to identify opportunity cost defined as the relevant discount rate. The firm's management, however, will probably have some idea of the cost of capital as well as some feel for the returns promised by all the projects that it would like to develop but cannot for lack of capital.

The point of this analysis is to illustrate that if opportunity cost exceeds cost of capital, use of the latter in evaluating projects will lead to a suboptimal investment program. Opportunity cost will always be the appropriate discount rate because it is an accurate representation of what the firm has given up, or the "cost" that it has incurred to fund an individual project. Also, as shown by the graphical analysis, opportunity cost can never fall significantly below the cost of capital.

3.2 The Reinvestment Assumption

Early in the discussion of rate of return, a distinction must be made between DCFRORs based on unamortized balances and those based on the total initial investment required. Only the latter require use of a reinvestment assumption.

3.2.1 Logical and Analytical Inconsistencies

In using a reinvestment assumption, whether implicit or explicit, the rate of growth of the total initial investment is determined. The interpretation of rate-of-return results using a reinvestment assumption is a specialized subset of growth rate of return, which uses intermediate project cash flows and a reinvestment assumption to determine a terminal value at some point in the project life. Once this terminal value is calculated, the interest rate is found which equates the terminal value with the investment required to generate it. The rate that sets initial cost equal to terminal value is the rate at which the initial investment has grown. Including an implicit reinvestment assumption in the interpretation of ordinary DCFROR calculations turns DCFROR into growth rate of return.

Given the cash flow shown in below, an ordinary

DCFROR of 15.24 percent is calculated. If an implicit reinvestment assumption is used, the intermediate cash flows must be reinvested at 15.24 percent.

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
C=$1000	R=$300	R=$300	R=$300	R=$300	R=$300

$$\$300(1.1524)^4 + \$300(1.1524)^3 + \$300(1.1524)^2 + \$300(1.1524) + \$300 = \$2032.43 \qquad (3.1)$$

Reinvestment at the assumed 15.24 percent would yield a terminal value of $2032.43, leaving calculation of growth rate of return of 15.24 percent as shown in Equation (3.2).

$$\$1000(1.1524)^5 = \$2032.43 \qquad (3.2)$$

Thus, the only difference in growth rate of return and interpretation of ordinary DCFROR using a reinvestment assumption is that in making GRR analysis, the reinvestment assumption is made explicitly, while in the interpretation of DCFROR the reinvestment assumption is made implicitly, once the rate of return has been calculated. Use of an implicit reinvestment rate assumption, however, can lead to logical and analytical inconsistencies when evaluating projects on an accept/reject or comparative basis.

Assume that an investor with a required rate of return of 10 percent is faced with two alternatives, A and B.

Project A

Year 0	Year 1	Year 2	Year 3	Year 4
C=$23,616	R=$10,000	R=$10,000	R=$10,000	R=$10,000

DCFROR = 25%

Project B

Year 0	Year 1	Year 2	Year 3	Year 4
C=$30,000	R=$0	R=$5000	R=$10,000	R=$45,000

DCFROR = 21.06%

In the examples offered above, the DCFRORs calculated are 25 percent and 21 percent, respectively, for projects A and B. If the implicit reinvestment assumption is used, we must be prepared to assume that the firm simultaneously faces reinvestment opportunities yielding different rates of return, and that the reinvestment rate is a function of the initial investment selected.

Specifically, in the situation displayed above, advocates of the implicit reinvestment assumption argue that for the DCFROR to equal 25 percent in case A, the firm must face a borrowing rate of 25 percent and be able to reinvest intermediate positive project cash flows at 25 percent. At the same time, in evaluating case B, the firm must believe that it can borrow and reinvest at 21 percent in order to embrace the implicit reinvestment assumption. In calculating rates of return for various projects, the firm must be prepared to assume that each project has a different borrowing and reinvestment rate, notwithstanding the fact that the projects would occur over the same time periods and in similar financial and economic environments. The simultaneous existence of different borrowing and reinvestment rates that depend exclusively on which projects are being examined or accepted is difficult to defend.

If the borrowing and reinvestment rate for the investments evaluated above equals 25 percent, as is suggested by project A, analysis shows A to be preferred. If the borrowing and reinvestment rate equals 21 percent, as would be suggested by the assumptions seemingly contained in the project B analysis, project A would still be preferred, as shown by the net present value analyses in Example 3.1. Using either of the borrowing and reinvestment rates implied by the use of the reinvestment assumption in this case indicates selection of project A. The actual interpretation of borrowing and reinvestment rates, based on the implicit assumptions made in the evaluation of the projects considered above, remains unclear. In practice, if simultaneous opportunities existed to borrow at 25 percent or 21 percent, the firm would elect to borrow at 21 percent. Similarly, if concurrent reinvestment opportunities of 25 percent and 21 percent existed, the firm would opt to reinvest at 25 percent.

EXAMPLE 3.1
Mutually Exclusive Analysis of Alternative Investments

Project A	Year 0	Year 1	Year 2	Year 3	Year 4
	C=$23,616	R=$10,000	R=$10,000	R=$10,000	R=$10,000

DCFROR = 25%
NPV @ 25% = 0
NPV @ 21% = +$1788

Project B	Year 0	Year 1	Year 2	Year 3	Year 4
	C=$30,000	R=$0	R=$5000	R=$10,000	R=$45,000

DCFROR = 21%
NPV @ 25% = $3248
NPV @ 21% = 0

The question becomes no clearer if incremental analysis is performed. The alternatives discussed above yield the incremental cash flow shown in Table 3.2. Incremental DCFROR analysis yields a result of 17.19 percent.

EXAMPLE 3.2
Incremental Analysis

(B-A)	Year 0	Year 1	Year 2	Year 3	Year 4
	C=$6384	C=$10,000	C=$5000	R=$0	R=$35,000

DCFROR = 17.19%
NPV @ 25% = $3248
NPV @ 21% = $1736
NPV @ 17.19% = 0

The question remains as to which borrowing and reinvesting rate assumption to use: that of original project A or original project B, or a third set implied by the incremental analysis. Investigation demonstrates that use of either 25 percent or 21 percent shows the incremental investment yielding 17.19 percent to be unsatisfactory, and again indicates selection of project A. These results are summarized in Example 3.3.

If the 17.19 percent rates are used, the firm would be indifferent to investment B as shown by the calculations contained in Example 3.4. But, recall that the 17.19 percent rates were implied by an incremental analysis and are not associated with either of the original candidate proposals. In fact, the 17.19 percent rate derived from the incremental analysis actually pertains to incremental project B cash flows, which were initially assumed to have 21 percent borrowing and reinvesting rates associated with them.

If the firm actually has a 10 percent return requirement, as was stipulated at the beginning of this discussion, it becomes clear that the decisions made on the basis of the borrowing and reinvestment rates implicitly assumed in these project calculations have led to the wrong choice. Use of any of the three rate sets implied in the preceding analysis shows project B to be a breakeven proposition at best (using the 17.19 percent rates determined by the incremental analysis).

EXAMPLE 3.3
Growth Rate of Return Calculations
Using Implicit Reinvestment Rates

Assuming Reinvestment @ 25%

Project A

Year 0	Year 1	Year 2	Year 3	Year 4
C=$23,616	R=$10,000	R=$10,000	R=$10,000	R=$10,000

Terminal Value at 25% Reinvestment = $57,656.25

Year 0	Year 1	Year 2	Year 3	Year 4
C=$23,616	R=$0	R=$0	R=$0	R=$57,656.25

$\$23,616 = \$57,656.25[1/(1 + i)]^4$
i = 25% = Growth Rate of Return

Project B

Year 0	Year 1	Year 2	Year 3	Year 4
C=$30,000	R=$0	R=$5000	R=$10,000	R=$45,000

Terminal Value at 25% Reinvestment = $65,312.50

Year 0	Year 1	Year 2	Year 3	Year 4
C=$30,000	R=$0	R=$0	R=$0	R=$65,312.50

$\$30,000 = \$65,312.50[1/(1 + i)]^4$
i = 21.47% = Growth Rate of Return

Therefore, Project A Preferred at a 25% reinvestment rate

Growth Rate of Return @ 21%

Terminal Value at 21% Reinvestment = $54,456.61

Project A

Year 0	Year 1	Year 2	Year 3	Year 4
C=$23,616	R=$0	R=$0	R=$0	R=$54,456.61

$\$23,616 = \$65,312.50[1/(1 + i)]^4$
i = 23.23% = Growth Rate of Return

EXAMPLE 3.3 (continued)

Terminal Value at 21% Reinvestment = $64,307.66

	Year 0	Year 1	Year 2	Year 3	Year 4
Project B	C=$30,000	R=$0	R=$0	R=$0	R=$64,307.66

$$\$30,000 = \$64,307.66[1/(1 + i)]^4$$
$$i = 21\% = \text{Growth Rate of Return}$$

Therefore, Project A preferred at 21% reinvestment rate

EXAMPLE 3.4
Growth Rate of Return Using Correct Explicit
Reinvestment Rate of 10%

Growth Rate of Return @ 10%

Terminal Value at 10% Reinvestment = $46,400

	Year 0	Year 1	Year 2	Year 3	Year 4
Project A	C=$23,616	R=$0	R=$0	R=$0	R=$46,410

$$\$23,616 = \$46,410[1/(1 + i)]^4$$
$$i = 18.40\% = \text{Growth Rate of Return}$$

Terminal Value at 10% Reinvestment = $62,050

	Year 0	Year 1	Year 2	Year 3	Year 4
Project B	C=$30,000	R=$0	R=$0	R=$0	R=$62,050

$$\$30,000 = \$62,050[1/(1 + i)]^4$$
$$i = 19.92\% = \text{Growth Rate of Return}$$

Therefore, Project B preferred at 10% reinvestment rate

If a net present value analysis is performed using a minimum rate of return of 10 percent, a different picture emerges. As shown below, project B shows a clear advantage at a 10 percent minimum rate of return.

Net Present Value at 10%

Project A	$8,083
Project B	$12,381

This result is verified by the results of the incremental analysis performed earlier, which show a return on the incremental investment of 17.19 percent, substantially above the minimum requirement of 10 percent. Both net present value and properly executed and interpreted rate of return analysis (i.e., incremental rate of return analysis) show project B to be far superior to project A, a result that could not have been derived using the multiplicity of simultaneous implicit assumptions that must be made if the implied borrowing and reinvestment arguments are accepted. Example 3.4 confirms the superiority of Project B using GRR analysis with a 10 percent reinvestment and hurdle rate.

Unrealistic and confusing interpretations resulting from the use of the implicit borrowing and reinvestment assumptions are not limited to examination of mutually exclusive alternatives. They may also affect the analysis of individual projects. For example, if an investment opportunity shows a rate of return of 2 percent, the implicit reinvestment argument requires that the firm borrow and reinvest at that rate. Similarly, if an individual project shows a 50 percent rate of return, the implicit assumption interpretation would show borrowing at 50 percent and reinvestment at 50 percent, regardless of the actual availability of other projects offering 50 percent returns as reinvestment candidates and of the actual cost (marginal or opportunity) of investment dollars. The interpretation of the implicit assumptions of both high and low rate-of-return results has led to the conclusion that rate of return overstates the economic desirability of high rate-of-return investments (because of the high reinvestment rate assumption) and understates the desirability of low rate-of-return projects in a similar fashion (Berry 1972). The validity of the assertion, or the degree of distortion actually involved, must be offset, at least in part, by the borrowing rate assumption which is implicitly low for low rate-of-return projects and higher for those projects showing a higher rate of return.

This discussion is not designed to point out common errors in practice resulting from use of the implicit borrowing and reinvesting rate assumption, but rather to demonstrate the analytical and logical ramifications resulting from attempts at their consistent application. In general, authors writing in this field have not attempted to reconcile those inconsistencies. The most common approach to discussion of rate-of-return analysis has been to note the existence of the implicit borrowing and reinvestment rate assumptions. The inconsistency in the results of DCFROR and NPV analysis of mutually exclusive projects is then blamed on the different borrowing and reinvestment rates argued to be implicit in each calculation. This apparent discrepancy is used to argue that DCFROR is a poor tool for the evaluation of mutually exclusive alternatives, and because of the required implicit borrowing and reinvestment assumptions which may not be met, a suspect tool for analyzing individual investments. Nevertheless, DCFROR analysis continues to be one of the most widely taught and used economic evaluation techniques. The implicit borrowing and reinvestment assumptions also continue to be taught and acknowledged in practice, despite the logical inconsistencies their use breeds.

3.2.2 Validity of Returns Based on Unamortized Investment Balances

Rate of return calculations incorporating implicit or explicit reinvestment assumptions are in effect determining rates of return based on the total investment required to generate that return. The preceding discussion demonstrated the logical inconsistencies involved with the use of implicit reinvestment rates. The preceding discussion also showed that the use of explicit reinvestment rates in growth rate-of-return calculations involves extra assumptions, calculations, and yields results which may be very sensitive to changes in the reinvestment rate used. Some authors argue that either implicit or explicit reinvestment rates must be accepted, however, because calculation of a rate of return based on total initial investment is necessary to arrive at correct economic decisions. They point out that if a reinvestment assumption is not accepted, the rate of return calculated does not apply to the total initial investment, but only to the unamortized investment balance, or to the part of the initial investment that has not been recovered by positive project cash flows. Because some doubt may exist as to the validity of rates of return based on unamortized balances as economic decision criteria, this question will be explored in detail here.

In calculating a return based on unamortized balance, the firm determines the returns accruing to funds tied up in the project considered which are unavailable for use elsewhere in the corporation. The fact that different projects will have different unamortized balances at a given time, and that these balances will constantly change, simply reflects the reality of what will actually occur during the given project life. Calculating rate of return based on the actual unrecovered balance accurately reflects the return that the firm receives on the investment remaining in the project throughout the project life.

The appropriateness of calculating rate of return based on the unamortized investment balance may be further clarified by an argument involving the base used to determine the capital, interest, or opportunity costs incurred by the project. Assume, for example, that the firm borrows money at 12 percent to invest in a project, and project cash flows are to be used to make loan principal and interest payments. The interest liability for a given year depends on the amount the project owes the lender at that time. The amount owed will depend on previous payments and on previous interest assessed.

Suppose the project in question is funded from the corporate treasury and incurs the marginal or weighted average cost of capital or opportunity cost on funds invested. This cost is incurred only on those funds that have not been paid back to the corporate treasury through intermediate project cash flows.

If the project initially borrows \$721 from the corporate capital pool at a 12 percent cost, and if at the end of one year no payments have been made, the unamortized loan balance will be as shown in Equation 3.3.

$$\$721 \times 1.12 = \$807.47 \qquad (3.3)$$

Equation 3.3 represents principal and interest accrued through year one, less any payments made to diminish, or amortize, the total owed. If at the end of year one the project pays back \$200, the unamortized balance becomes

$$\$807.47 - \$200 = \$607.47 \qquad (3.4)$$

If another \$200 payment is made at the end of year two, the balance becomes

$$\$607.47 \times 1.12 - \$200 = \$480.37 \qquad (3.5)$$

Thus, the cost of the financial resources devoted to the project is a function of their unamortized balances. If costs are based on unamortized balances, the revenue measure must also be based on unamortized balances in order to make consistent comparisons of project revenues and costs.

In the literature this type of analysis is known as cumulative cash position analysis (Stermole 1984). The calculations shown in Table 3.1 illustrate the amortization of loan principal and interest for a \$721 investment paid off with five annual payments of \$200 each. The example demonstrates the assessment of interest based on unamortized balances, or the unrecovered principal and accrued interest, less payments through a given point in time. This analysis also indicate that these project cash flows are able to

TABLE 3.1
Loan Amortization

Year	Initial Balance$_t$	x	Interest Assessed	=	Unamortized Balance	-	Payment	=	Initial Balance$_{t+1}$
0	$721.00								
1	721.00	x	1.12	=	$807.47	-	$200	=	$607.47
2	607.47	x	1.12	=	680.42	-	200	=	480.42
3	480.42	x	1.12	=	538.07	-	200	=	338.07
4	338.07	x	1.12	=	378.64	-	200	=	178.64
5	178.64	x	1.12	=	200.00	-	200	=	0

amortize the initial loan at 12 percent interest, or meet the payments required to provide the lender (or the corporate treasury) with 12 percent return on the unamortized loan balance. The ability of these project cash flows to meet 12 percent interest assessments on the outstanding loan balance indicates that the project is earning exactly 12 percent on the unamortized balance of the loan.

The same basic analytical approach can be used to interpret rate-of-return calculations in general. Since the definition of DCFROR is that rate which sets the present worth of project revenues equal to the present worth of project costs, the DCFROR determined by trial and error calculation is that rate which, when applied to unamortized loan balances as illustrated above, returns the cumulative cash position to zero at project end. This calculation can be performed without reference to the actual borrowing interest rate, and without reference to a reinvestment rate. The project earning rate determined by DCFROR calculations is independent of parameters existing outside of the project itself. Investment and cash flows alone determine the DCFROR. Of course, the DCFROR result cannot be interpreted without reference to some external criteria, such as a borrowing rate or opportunity cost, which serve as a minimum economic performance standard. In this case, the project has been shown to earn 12 percent on the unamortized balance. If the project incurred a 12 percent cost of capital or opportunity cost, it would exactly break even, and the firm would be indifferent to this investment. If the borrowing or opportunity cost rate was less than 12 percent, the firm would benefit, i.e., do better than break even. If the cost exceeded 12 percent, the firm would not break even on this investment. As this discussion points out, and Table 3.1 emphasizes, rate-of-return analysis is essentially a breakeven calculation.

Those arguing in favor of the implicit reinvestment assumption and rates of return based on total initial investment would interpret the same input data as follows. The total initial investment is $721 devoted to a project with a five-year life. For the project to "return" 12 percent on the initial $721 investment, that investment must grow to $1,270.65 by the end of year five. That growth target is determined using Equation 3.4. The specific calculations for this case are shown in Equation 3.5.

$$\text{Cost} \times (1 + r)^n = \text{Required Accumulation} \quad (3.4)$$
$$r = \text{Annual Interest Rate}$$

$$721 \times (1.12)^5 = \$1{,}270.65 \quad (3.5)$$

In order for this project to have accumulated $1,270.65 in cash flow by year five, the intermediate $200 cash flows must be reinvested at the DCFROR 12 percent through the end of the project life as shown in Table 3.2.

TABLE 3.2
Loan Amortization Using Implicit
Reinvestment Assumption

Year	Balance_{n-1}	x 1.12 +	Receipt n	=	Balance_n
0	$0	0	$0		$0
1	0	0	200	=	200.00
2	200.00	x 1.12 +	200	=	424.00
3	424.00	x 1.12 +	200	=	674.88
4	674.88	x 1.12 +	200	=	955.87
5	955.87	x 1.12 +	200	=	1270.57

Using the calculated DCFROR as the project reinvestment rate yields the accumulated balance required at project end. The accumulated $1,270.57 is then interpreted as being available to repay the principal and interest due the corporate treasury.

The preceding pages have examined the validity of the implicit reinvestment assumption in the basic rate-of-return calculation, which does not mathematically incorporate a reinvestment assumption. Further, it has been shown that DCFROR can be used to arrive at correct economic project acceptance decisions without incorporation of implicit or explicit reinvestment rates. Finally, simple DCFROR results may not be meaningfully interpreted in a growth rate-of-return context. Growth rate-of-return calculations require explicit consideration and statement of possible reinvestment rates.

3.2.3 DCFROR as a Breakeven Calculation

The breakeven interpretation of DCFROR introduced in the previous section merits further discussion. Interpretation of DCFROR in a breakeven context may clarify some of the ambiguity surrounding its meaning.

A simple DCFROR calculation can do no more than provide an accept/reject decision for single investments. DCFROR provides these decisions by comparing the DCFROR calculated with some standard or minimum rate-of-return requirement. Some authors have defined the DCFROR as the maximum cost of funds (marginal or opportunity) that the project can bear and still break even. They search for the interest rate which will set NPV equal to zero. If the project can bear the interest cost determined, it must be earning at that rate. Because NPV equals zero at the DCFROR determined, it is then argued that the borrowing and reinvesting rates must also equal the DCFROR. If they do not, NPV will not equal zero.

It must be borne in mind that in calculating DCFROR, the firm is looking for the maximum marginal or opportunity cost the project can bear and still break even. Determination of the maximum cost the project can bear says nothing about the actual costs that it must bear, just as knowing that a project breaks even at a given cost of capital is insufficient information to use in making a decision. In neither case can a decision be made without reference to some outside criteria.

Assume that project cash flows indicate a 20 percent rate of return for a given investment. A valid interpretation of that result is that at a 20 percent cost of capital, the firm exactly breaks even, achieves a zero NPV and will be indifferent to undertaking the investment generating these cash flows. An accept or reject decision cannot be made, however, until the project breakeven of 20 percent is compared to some external economic performance standard. For example, if the actual cost of capital is 15 percent, the project should be accepted because this project could support up to a 20 percent cost of capital and still break even. At 15 percent, it does better than break even. How much better? We do not need to know. The only decision that can be made with DCFROR is the accept/reject decision, and to make that decision we need only know whether the project breakeven cost of capital exceeds the costs it would actually have to bear. On the other hand, if the actual cost of capital were 30 percent, the cost of capital burdens that would have to be carried by the project would exceed what it could incur and still break even, so the project would be rejected.

Throughout the discussion above, however, changing of the actual cost of capital from 15 percent to 30 percent did not change the DCFROR or the project's breakeven cost, which is, in turn, its rate of return. Through it all, the project breakeven, or rate of return, remained constant at 20 percent. The breakeven nature of the rate of return calculation is partly illustrated by DCFROR's inability to rank nonmutually exclusive alternatives in order of economic preference. DCFROR can only indicate the breakeven interest rate or opportunity cost that each project could support and thereby make accept/reject decisions for individual projects. Section 2.2.2 demonstrated that funding nonmutually exclusive projects in DCFROR order will not necessarily maximize the present worth of the firm.

A similar line of reasoning applies to the analysis of mutually exclusive alternatives. In order to evaluate mutually exclusive alternatives with rate of return, or any other indicator of economic performance, an incremental analysis must be performed, which will yield an incremental rate of return on a single incremental investment. The analysis will then determine if the incremental investment is justified on a rate-of-return basis, and an accept/reject decision will be made for the single incremental investment. The analysis performed in the context of the increment will yield the DCFROR, or maximum cost of capital that the incremental investment could bear and still break even. Again, if the maximum costs that could be borne exceed those that must be borne (the cost of capital), the incremental investment is accepted. The maximum capital costs that a project could bear and still break even say nothing about the actual costs that the investment would have to bear.

If other, higher-cost mutually exclusive alternatives existed, subsequent incremental analyses would be performed, and incremental investments would be judged by comparing their DCFROR (maximum or breakeven costs of capital) with the minimum return requirement (actual marginal or opportunity costs of capital).

The breakeven interpretation of internal rate of return should help answer arguments offered by Porterfield (1965). Porterfield defined rate of return as (p. 24)

> that discount rate (interest rate) that equates the present value of its stream of cash inflows with the present value of its stream of cash outflows.

and continued by saying that (p. 26)

> [l]imitations in the rate of return method . . . result from its assumptions as to the treatment of intermediate cash flows. . . . In certain circumstances the rate of return method assumes that intermediate cash inflows will be reinvested at a rate of return equal to that calculated for the project itself.

In support of his argument, Porterfield offers the following example in which a firm invests $1,952 in a project generating three annual $1,000 cash flows.

Year 0	Year 1	Year 2	Year 3
C=$1,952	R=$1,000	R=$1,000	R=$1,000

The DCFROR determined for the investment is 25 percent. Because the project is earning 25 percent, it should be able to just pay off a loan at 25 percent for the required initial investment. Cash flows for an investment loan at 25 percent, assuming interest is payable in each period, are shown below.

Year 0	Year 1	Year 2	Year 3
R=$1,952	C=$488	C=$488	C=$2,440

The combination of investment and loan cash flows yields the final net cash flow depicted below.

Year 0	Year 1	Year 2	Year 3
C=$0	R=$512	R=$512	C=$1,440

Finally, Porterfield contends that (p. 27)

> if net intermediate cash inflows of $512 at times t_1 and t_2 were reinvested at 25%, they would cumulate to $1440 at time 3. If they were reinvested at any other rate of return, the firm would not break even. Of course, if prepayment of the loan were permitted, this prepayment itself would constitute an opportunity for reinvestment at a rate of return of 25%.

The argument is easy to believe. Once an assumption of cost of capital is made, the firm can always earn at least the cost of capital by paying back funds borrowed at that marginal or opportunity cost and freeing itself of that cost. The lower limit on the reinvestment rate, then, is the initial cost of capital. Therefore, the reinvesting rate is a function of the borrowing rate.

And so the trap is sprung. DCFROR is that rate which sets project NPV equal to zero. For NPV to equal zero, the firm must be earning at the cost of capital, and the reinvestment rate must always be at least the cost of capital. Therefore, the reinvesting rate must equal the rate of return.

The key point mentioned in this discussion, but not properly emphasized, is the breakeven concept. As previously mentioned, the DCFROR calculated is that rate which makes NPV zero. At this rate the firm breaks even, or is indifferent to undertaking the investment. It is not necessary to know any actual borrowing or reinvesting rates to know the rate at which the firm breaks even. The breakeven is a function solely of the parameters contained in the project cash flow. Once the breakeven rate is calculated, it is compared with the actual cost of capital or opportunity cost. If the breakeven rate or maximum cost of capital the project can bear exceeds the cost the project will have to bear, the project is better than a breakeven project, and a decision is made to undertake it. As discussed previously, the go, no-go decision for a single increment of investment is the only decision that can be made with a simple rate-of-return calculation.

Application of this line of reasoning to Porterfield's example clears up uncertainty regarding the validity of his arguments. Calculation of DCFROR yields the maximum cost of capital the project could incur to break even and, as Porterfield mentions, the project does not break even at any other cost. Comparison of the breakeven with these other costs yields a go decision if the actual costs are less than the breakeven and a no-go decision if the actual costs exceed the breakeven.

Chapter 4

ECONOMIC EVALUATION OF LEVERAGED INVESTMENTS

Chapters 2 and 3 of this work dealt primarily with the economic evaluation of all-equity investments. Debt financing was largely ignored in performing the analyses. Chapter 4 will extend this all-equity discussion to examine the effect on evaluation results of the introduction of borrowed funds into the analyses.

Specifically, this chapter will mathematically define proportionality, or the functional relationship between the proportion of project costs which are funded with debt and the resulting change in project GRR.

Two forms of the proportionality relationship will be examined and quantified through the development of mathematical models. The first model describes the effects of leverage on GRR results for projects funded with balloon financing. The second form of the model will define the effect of leverage on projects where borrowing is repaid with equal annual mortgage-type payments.

These proportionality models may be used to determine appropriate, consistent minimum GRR requirements for leveraged investments. Once determined, the minimum leveraged GRR can be used as a discount or reinvestment rate in performing NPV, GRR and PVR analyses. These models can also be used to determine the leveraged GRR that would result from financing any given project with any debt proportion.

These developments allow simultaneous economic and financial evaluation of leveraged projects. As will be shown in the following section, leveraged project evaluations have traditionally been performed in two phases to avoid distorting economic results with the effects of financing.

4.1 Distortion of Economic Analysis Results Owing to Leverage

Authors (Vancil 1963; Elgers 1980; Pritchard 1980; Bierman 1982; Stermole 1984) writing in the field of economic evaluation have often cautioned against attempting to combine evaluation of the economic attractiveness of a project with financial considerations, such as methods of project funding. Financing options considered might include retained earnings, debt, leases, as well as other sources of capital. Combination of economic and financial evaluation may lead to erroneous conclusions and investment in unacceptable projects that appear attractive because of the effects of leverage. Introduction of leverage into economic analysis may make a poor project look good.

The distortion of evaluation results by leverage is illustrated by the following example. Example 4.1 contains a cash flow for a project showing a 10 percent GRR.

EXAMPLE 4.1

```
          Year   Year   Year   Year   Year   Year
            0      1      2      3      4      5
            |______|______|______|______|______|
        C=$100  R=$0   R=$0   R=$0   R=$0  R=$322.20
                                                 .5 Tax Rate
      GRR = 10%                             $161.10
```

This is an all-equity investment. The project is funded with retained earnings. No borrowed funds are included in this evaluation. The project would be economically unacceptable if a 15 percent GRR is required.

Example 4.2 considers the same project funded with 50 percent leverage. In this case, 50 percent of the initial $100 investment is borrowed at 5 percent interest to be repaid at project termination in year five. The project cash flow shown here combines project economics and financing to yield a leveraged GRR of 15.8 percent. If this leveraged GRR is compared with the 15 percent minimum required rate of return, the project could be erroneously accepted. This acceptance would be in error because the high GRR shown is primarily the result of financing. Project economics have not changed. The use of leverage in the analysis makes the project appear attractive.

Project GRR will generally increase dramatically as a function of the proportion of borrowed money included in

EXAMPLE 4.2

Year 0	Year 1	Year 2	Year 3	Year 4	Year 5		
C=$50	R=$0	R=$0	R=$0	R=$0	R=	$322.200	
						13.814	Interest
						308.386	
						.5	Tax Rate
						154.193	
						-50.000	Principal
						$104.193	

GRR = 15.817%

project financing. As long as funds can be borrowed at an interest rate which is less than the all-equity GRR earned by the project without leverage, borrowing will increase the project GRR. Table 4.1 summarizes for Example 4.1 cash flow data the leveraged GRR results of evaluations including leverage proportions from 25 percent to 90 percent.

TABLE 4.1
Project GRR as a Function of Leverage Proportion

Leverage Proportion	Leveraged GRR
0%	10.000%
25	12.080
40	14.010
50	15.817
75	24.818
90	42.455

It is these kinds of distortions that have led authors to conclude that project evaluations should be performed in two phases. The first phase evaluates only the economic attractiveness of the project. In order to avoid distortions introduced by financing considerations, this phase of the analysis is performed assuming that no borrowed monies will be used in project funding. The first phase is an all-equity analysis. If the project meets the minimum all-equity DCFROR requirements, then the second phase of the analysis is begun. In this second phase, the firm searches for the best available financing for use on the project it already knows to be economically attractive. As long as the financing used carries an interest cost which is less than the all-equity GRR shown by the economic evaluation of the investment, leverage will work to the advantage of the project.

4.2 Leveraged Opportunity Cost

Section 4.1 pointed out the distortions introduced by the incorporation of leverage in economic analysis. It was shown that comparison of a leveraged project GRR with an all-equity opportunity cost could result in incorrect economic choices. Comparison of leveraged GRR results with leveraged opportunity cost standards would, however, yield correct economic choices.

In order to define a leveraged opportunity cost standard against which to compare leveraged project GRR, the projects defining the all-equity GRR must be leveraged in the same proportion at the same terms as are to be used in financing the project in question. The leveraged GRR resulting from leveraged analysis of the opportunity cost-defining project becomes the minimum leveraged GRR standard that all projects financed with the same leverage proportion and terms must meet.

For example, assume that a firm has a minimum rate of return requirement of 15 percent for an all-equity investment. If it borrows at 5 percent interest with balloon financing terms, the minimum required rate of return for five different leverage proportions are shown in Table 4.2.

TABLE 4.2
Minimum Required GRR as a Function of Leverage Proportion

Leverage Proportion	Leveraged GRR
0%	15.000%
25	18.146
40	20.992
50	23.593
75	35.865
90	58.065

These leveraged results define the minimum performance standard or hurdle rate for all other projects financed with equivalent leverage proportions and terms. For example, leveraging the project defined in Example 4.1 by borrowing 50 percent of the initial cost required at 5 percent interest with a five-year balloon loan resulted in a leverage GRR of 15.82 percent. This leveraged GRR was attractive when compared with the 15 percent all-equity minimum rate of return. The 15.817 percent leverage project GRR is not attractive when compared with the 23.59 percent leveraged minimum GRR required of projects financed with 50 percent leverage borrowed at the indicated terms. Here comparison of leveraged project GRR results with leveraged minimum rate of return requirements yields the correct choice and the investment is rejected.

Table 4.3 compares the leveraged GRR's obtained by leveraging Example 4.1 with the leveraged minimum rates of return shown in Table 4.3. Comparison of the GRR results from leveraging Example 4.1 with the appropriate leveraged opportunity costs yields the correct economic choice for all proportionality levels. The project defined by Example 4.1 cash flows is rejected at all proportionality levels.

TABLE 4.3
Evaluation of Leveraged Projects Using
Leveraged Minimum Required GRR

Leverage Proportion	Leverage GRR From Example 4.1	Leveraged Minimum Required GRR
0%	10.000%	15.000%
25	12.080	18.146
40	14.010	20.992
50	15.817	23.593
75	24.818	35.865
90	42.455	58.065

This analysis demonstrates that correct economic decisions for projects involving borrowed funds can be made if leveraged evaluation results are compared against appropriate leveraged minimum rate of return requirements. These minimum rate of return requirements must be defined by leveraging the all-equity opportunity cost-defining investment in the same proportion and terms as will be used in financing the project under evaluation.

4.3 A Mathematical Model Defining Proportionality

Section 4.2 demonstrated that correct economic evaluations of leveraged projects can be made if leveraged project GRRs are compared with leveraged minimum GRR requirements for the equivalent leverage terms and proportions. Section 4.3 will develop a mathematical model defining the functional relationship between leverage terms and proportions included in an economic analysis and leveraged GRR.

Equation 4.1 illustrates the formula defining all-equity GRR (i) for projects consisting of a single investment (C), growing to a single termination value (F) at project end (L) years after initial investment.

$$C = [1/(1 + i)]^{L} (F) \qquad (4.1)$$

Equation 4.1 can be mathematically rearranged to yield Equation 4.2.

$$1 + i = [1/(C/F)]^{1/L} \qquad (4.2)$$

At this point leverage may be introduced to determine the effect on "i," the leveraged DCFROR, of "P," the proportion of borrowed funds under consideration. Equation 4.3 quantifies this relationship for a single cost, single termination value case in which the borrowed funds are repaid in a single balloon payment at project termination.

$$1 + i = 1/[\{C(1 - P)\}/\{\sum_{t=0}^{L} F_t(1 + m)^{L-t} - (CP(1 + r)^L - CP)(1 - T) - CP\}]^{1/L} \qquad (4.3)$$

C = Negative net cash flow in all-equity project evaluation

F = Positive net cash flow in all-equity project evaluation

L = Project life (and loan life in this case)

P = Leverage proportion expressed as a decimal

m = Minimum leveraged DCFROR - used as a discount and reinvestment rate as appropriate

r = Borrowing rate or interest rate on borrowed funds

T = Tax rate

i = Leveraged GRR

If the values of variables C, F, L, m, r, and T in Equation 4.3 are known for a given analysis, Equation 4.3 defines the functional relationship between the two remaining variables, leverage proportion "P" and leveraged GRR, "i." Example 4.3 illustrates the application of this model. The calculations contained in Example 4.3 are validated in Appendix A using basic cash-flow analysis.

A slight modification of this basic model allows it to be used in defining leveraged opportunity cost standards against which to compare leveraged project economics for acceptability. The modified model is shown in Equation 4.4.

k = minimum all-equity opportunity cost

$$1 + i = 1/[\{C(1 - P)\}/\{C(1 + k)^L - (CP(1 + r)^L - CP)(1 - T) - CP\}]^{1/L} \quad (4.4)$$

In this model the term $C(1 + k)^L$ has replaced the net receipt, or "F" term, in Equation 4.3. This term defines the future receipt as the net value that an investment of "C" would grow to over an "L" year project if invested in a project earning at "k," the all-equity opportunity cost.

Thus, the future receipt is defined by the all-equity investment cost and minimum rate of return. Subtracted from that future receipt are the interest and principal costs associated with leveraging at any proportion, "P." This form of the equation accommodates balloon financing only.

Equation 4.5 is designed for use with projects incorporating a single mortgage-type loan which is repaid with a series of equal annual payments. Section 4.5 will use Equation 4.5 to evaluate a hypothetical leveraged mineral investment.

EXAMPLE 4.3
All-equity Cash Flow

	Year 0	Year 1	Year 2	Year 3
C =	$400	R=$0	R=$0	R=$1216.08
				.5 Tax Rate
				$608.04

Application of Equation 4.3

C = 400
F = 608.04
r = 10%
T = 50%
L = 3

GRR Calculation Using 0% Leverage

$$1 + i = 1/[\{400(1 - 0)\}/\{608.4 - (400(0)(1.1)^3 - 400(0))(1 - .5) - 400(0)\}]^{1/3}$$

GRR = 15%

GRR Calculation Using 25% Leverage

$$1 + i = 1/[\{400(1 - .25)\}/\{608.4 - (400(.25)(1.1)^3 - 400(.25)(1 - .5) - 400(.25)\}]^{1/3}$$

GRR = 17.9%

GRR Calculation Using 50% Leverage

$$1 + i = 1/[\{400(1 - .50)\}/\{608.4 - (400(.50)(1.1)^3 - 400(.5))(1 - .5) - 400(.5)\}]^{1/3}$$

GRR = 23.3%

EXAMPLE 4.3 (continued)

GRR Calculation Using 75% Leverage

$$1 + i = 1/[\{400(1 - .75)\}/\{608.04 - (400(.75)(1.1)^3 - 400(.75))(1 - .75) - 400(.75)\}]^{1/3}$$

GRR = 37.22%

$$1 + i = 1/[\{C(1 - P) + CP\{(r(1 + r)^n/<(1 + r)^n - 1>\} \sum_{t=1}^{n} X_t(1/[1 + m])^t - CP\{r(1 + r)^n/<(1 + r)^n - 1>\} \sum_{t=1}^{n} X_t\{1 - (1 - r/[1 + r]^{n+1-t}\}T(1/[1 + m])^t\}/ \{\sum_{t=0}^{L} F_t(1 + m)^{L-t} - CP\{r(1 + r)^n/<(1 + r)^n - 1>\} \sum_{t=1}^{n} Z_t(1 + m)^{L-t} + CP\{r(1 + r)^n/<(1 + r)^n - 1>\} \sum_{t=1}^{n} Z_t\{1 - (1 - r/[1 + r])^{n+1-t}\}T(1 + m)^{L-t}\}]^{1/L} \quad (4.5)$$

4.4 Sensitivity of Evaluation Results

The equations contained in this chapter indicate that proportionality is a complex function of many variables including interest rate, loan life, project life, and repayment schedule, and Table 4.4 illustrates the sensitivity of growth rate-of-return results to changes in interest rate,

TABLE 4.4
Sensitivity of Evaluation Results

20-Year Project Life

Loan Life (Years)	Interest Rate	Leverage Proportion					
		0%	25%	40%	50%	75%	90%
				GRR			
3	5%	15.0%	15.4476%	15.7490%	15.9667%	16.5814%	17.0203%
	10	15.0	15.3577	15.5959	15.7665	16.2433	16.5725
	12	15.0	15.3197	15.5316	15.6829	16.1030	16.3903
5	5	15.0	15.6898	16.1820	16.5542	17.7088	18.6460
	10	15.0	15.5542	15.9437	16.2351	17.1167	17.8018
	12	15.0	15.4929	15.8379	16.0943	16.8620	17.4479
10	5	15.0	16.1206	16.9908	17.6988	20.3152	23.3880
	10	15.0	15.9212	16.7023	17.2080	19.2857	21.5649
	12	15.0	15.8157	16.4439	16.9498	18.7539	20.6622
20	5	15.0	16.5013	17.7062	18.7143	22.7166	28.3512
	10	15.0	16.3117	17.3916	18.3103	22.0592	27.5006
	12	15.0	16.1726	17.1560	18.0038	21.5429	26.8172

TABLE 4.4 (continued)

10-Year Project Life

Loan Life (Years)	Interest Rate	Leverage Proportion					
		0%	25%	40%	50%	75%	90%
		GRR					
3	5%	15%	15.9249%	16.5827%	17.0782%	18.5961%	19.7938%
	10	15	15.7406	16.2626	16.6528	17.8290	18.7320
	12	15	15.6624	16.1276	16.4742	17.5119	18.2998
5	5	15	16.4422	17.5554	18.4523	21.6392	24.9432
	10	15	16.1648	17.0574	17.7729	20.2656	22.7246
	12	15	16.0385	16.8336	17.4684	19.6604	21.7758
10	5	15	17.3593	19.3553	21.0897	28.4637	39.8563
	10	15	16.9728	18.6842	20.1986	26.8569	37.5743
	12	15	16.7630	18.3138	19.7008	25.9243	36.2103

TABLE 4.4 (continued)

5-Year Project Life

Loan Life (Years)	Interest Rate	Leverage Proportion 0%	25%	40%	50%	75%	90%
		GRR					
3	5%	15.0%	16.9669%	18.5440%	19.8468%	24.6801%	29.9705%
	10	15.0	16.5795	17.8436	18.884	22.6939	26.7610
	12	15.0	16.4144	17.5455	18.4751	21.8592	25.3774
5	5	15.0	18.15	21.00	23.6	35.87	58.08
	10	15.0	17.5678	19.9325	22.1243	32.7815	52.9137
	12	15.0	17.31	19.45	21.44	31.3	50.35

3-Year Project Life

Loan Life (Years)	Interest Rate	Leverage Proportion 0%	25%	40%	50%	75%	90%
		GRR					
3	5%	15.0%	18.6041%	21.9987%	25.2143%	41.7396%	76.5243%
	10	15.0	17.9154	20.6905	23.3435	37.2862	67.7501
	12	15.0	17.6193	20.1239	22.5282	35.2962	63.7130

loan life, and project life. The data displayed in Table 4.4 supports the following general conclusions regarding the sensitivity of evaluation results:

(1) For a given loan life, project life, repayment schedule, and all-equity GRR, variation in interest rate has a relatively insignificant impact on evaluation results.

(2) For a given project life, interest rate, repayment schedule, and all-equity GRR, variation in loan life has a relatively significant impact on evaluation results. GRR results are much more responsive to 3-year loans than to 10-year loans for all project lives and all interest rates examined.

(3) For a given loan life, interest rate, repayment schedule, and all-equity GRR, variation in project life has the most significant impact on evaluation results of any variable examined. GRR results are much more responsive for 3-year project lives than for 20-year project lives for all loan lives and interest rates examined.

Appendix B contains calculations based on Equation 4.6 verifying some of the sensitivity analysis conclusions drawn from Table 4.4.

Equation 4.6 is similar to Equations 4.3 and 4.4 in that it is designed for use with a single cost, single revenue project leveraged with one balloon-type loan.

$$1 + i = 1/[\{C(1 - P) + w([CP(1 + r)^n - CP](1 - T) + CP)(1/1 + m)^n\}/\{F - V([CP(1 + r)^n - CP](1 - T) + CP)\}]^{1/L} \quad (4.6)$$

w = Binary (0,1) variable. w equals 0 if loan life equals project life. w equals 1 if loan life is less than project life.

V = Binary (0,1) variable. V equals 0 if loan life is less than project life. V equals 1 if loan life equals project life.

However, Equation 4.6 can accommodate a loan life which is shorter than the project life for the investment under consideration. Both Equations 4.3 and 4.4 require that loan life equal project life.

4.5 Mineral Industry Case Study

The following case study illustrates the application of leverage to a mineral investment. The hypothetical investment described here is taken from Stermole (1984, 249).

> An individual investor is considering acquiring and developing a mineral property believed to contain 500,000 units of developable mineral reserves (mineral units could be barrels of oil, tons of coal, ounces of gold, etc.). The mineral rights acquisition cost for the property would be $900,000 at year zero. Mineral development (or petroleum drilling) cost of $1,200,000, tangible equipment costs (mining equipment or oil and gas producing equipment and pipelines) of $1,000,000 and working capital costs of $300,000 are projected to be incurred at year 0. 10% investment tax credit will be taken on the tangible equipment costs and used as soon as possible against project tax. ACRS depreciation will be based on a 5 year depreciation life starting in year 1 when assets are placed into service. Mineral production is projected to be uniform for each year of the 5

> year project life at 100,000 mineral units per year with mineral reserves depleted at the end of year 5. Product selling price is estimated to be $30 per mineral unit produced in year 1 and escalating 10% per year in succeeding years. Operating expenses are estimated to be $1,000,000 in year 1, escalating 8% per year in succeeding years. Royalties are 15% of revenues each year. The property and equipment are expected to have no net salvage value although recovery of the $300,000 working capital investment is expected from inventory liquidation at the end of year 5. Assume the mineral produced is a 15% depletion rate mineral and that the investor is either an "individual" mineral producer (so expense 100% of mineral development cost in year 0) or an "independent" petroleum producer (so expense 100% of development drilling cost in year 0 and be eligible for percent depletion). All mineral producers are eligible for percent depletion.

These projected costs and revenues yield the cash flows shown in Table 4.5. Reinvestment of intermediate project cash flows at a 15 percent minimum rate of return yields a GRR of 24 percent as shown in Example 4.4. Example 4.4 also shows the results of leveraging this project at 25, 50, and 75 percent leverage. In each case a single loan is paid off with a series of equal annual payments using a 12 percent interest rate. Because mortgage loan-type financing is used in this case, Equation 4.5 is used to calculate leveraged GRRs. The results obtained by Equation 4.5 are validated in Appendix C using basic cash-flow calculations.

TABLE 4.5
Mineral Investment Case Study Cash Flows

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Revenue	--	$3,000	$3,300	$3,630	$3,993	$4,392
-Royalties	--	- 450	- 495	- 544	- 599	- 659
Net Revenue	--	+2,550	+2,805	+3,086	+3,394	+3,733
-Op. Cost	--	-1,000	-1,080	-1,166	-1,260	-1,360
-Depreciation	--	- 142	- 209	- 200	- 200	- 199
-Development	-1,200	--	--	--	--	--
Income Before Depletion	-1,200	+1,408	+1,516	+1,720	+1,934	+2,174
Cost Depletion	--	180	129	32	--	--
50% Limit	--	704	758	860	967	1,087
Percent Depl.	--	- 382	- 421	- 463	- 509	- 560
-Loss Forward	--	-1,200	- 174	--	--	--
Taxable	-1,200	- 174	+ 921	+1,257	+1,425	+1,614
-Tax @ 50%	--	--	- 460	- 628	- 712	- 807
Invest. Tax Cr.	--	--	+ 100	--	--	--
Net Income	-1,200	- 174	+ 561	+ 629	+ 713	+ 807
+Depreciation	--	+ 142	+ 209	+ 200	+ 200	+ 199
+Depletion	--	+ 382	+ 421	+ 463	+ 509	+ 560
+Loss Forward	--	+1,200	+ 174	--	--	--
+Working Cap. Ret.	--	--	--	--	--	+ 300
-Capital Cost	-2,200	--	--	--	--	--
Cash Flow	-$3,400	$1,550	$1,365	$1,292	$1,422	$1,866

EXAMPLE 4.4

GRR Calculation Using 0% Leverage

$$1 + i = 1/[3400/\{\sum_{t=0}^{5} F_t(1.15)^t\}]^{1/5}$$

GRR = 24.07%

GRR Calculation Using 25% Leverage

$$1 + i = 1/[\{3400(.75) + 3400(.25)\{(.12(1.12)^5/<(1.12)^5 - 1>\}$$

$$\sum_{t=1}^{5}(0)(1/1.1641)^t - 3400(.25)\{.12(1.12)^5/<(1.12)^5 - 1>\}$$

$$\sum_{t=1}^{5}(0)\{1 - (1 - .12/1.12)^{5-t}\}.5(1/1.1641)^t\}/$$

$$\{\sum_{t=0}^{5}F_t(1.1641)^{5-t} - 3400(.25)\{.12(1.12)^5/<(1.12)^5 - 1>\}$$

$$\sum_{t=1}^{5}(1)(1.1641)^{5-t} + 3400(.25)\{.12(1.12)^5/<(1.12)^5 - 1>\}$$

$$\sum_{t=1}^{5}(1)\{1 - (1 - .12/(1.12)^{6-t}\}.5(1.1641)^{5-t}\}]^{1/5}$$

GRR = 28.4%

EXAMPLE 4.4 (continued)

GRR Calculation Using 50% Leverage

$$1 + i = 1/[\{3400(.50) + 3400(.50)\{.12(1.12)^5/\langle(1.12)^5 - 1\rangle\}$$

$$\sum_{t=1}^{5}(0)(1/1.1841)^t - 3400(.50)\{.12(1.12)^5/\langle(1.12)^5 - 1\rangle\}$$

$$\sum_{t=1}^{5}(0)\{1 - (1 - .12/1.12)^{6-t}\}.5(1/1.1841)^t\}/$$

$$\{\sum_{t=0}^{5}F_t(1.1841)^{5-t} - 3400(.50)\{.12(1.12)^5/\langle 1.12)^5 - 1\rangle\}$$

$$\sum_{t=1}^{5}(1)(1.1841)^{5-t} + 3400(.50)\{.12(1.12)^5/\langle(1.12)^5 - 1\rangle\}$$

$$\sum_{t=1}^{5}(1)\{1 - (1 - .12/1.12)^{6-t}\}.5(1.1841)^{5-t}\}]^{1/5}$$

GRR = 35.6%

EXAMPLE 4.4 (continued)

GRR Calculation Using 75% Leverage

$$1 + i = 1/[\{3400(.25) + 3400(.75)\{.12(1.12)^5/<(1.12)^5 - 1>\}$$

$$\sum_{t=1}^{5}(0)(1.2353)^t - 3400(.75)\{.12(1.12)^5/<(1.12)^5 - 1>\}$$

$$\sum_{t=1}^{5}(0)\{1 - (1 - .12/1.12)^{6-t}.5(1/1.2353^t\}/$$

$$\{\sum_{t=0}^{5}F_t(1.2353)^{5-t} - 3400(.75)\{.12(1.12)^5/<(1.12)^5 - 1>\}$$

$$\sum_{t=1}^{5}(1)(1.2353)^{5-t} + 3400(.75)\{.12(1.12)^5/<(1.12)^5 - 1>\}$$

$$\sum_{t=1}^{5}(1)\{1 - .12/(1.12)^{6-t}\}.5(1.2353)^{5-t}\}]^{1/5}$$

GRR = 52.5%

Chapter 5

CONCLUSIONS

The first objective of this research was to draw conclusions regarding 1) the validity of the implicit reinvestment rate assumption, and 2) the discount rate appropriate for use in performing economic analysis calculations. The second research objective was to develop a mathematical model describing the proportionality relationship for two common financing options.

The analysis contained in Chapter 3 shows that there is no reinvestment assumption implied or required by the mathematics or logic of the DCFROR calculation, that incremental DCFROR analysis may be used to make correct evaluation decisions without using implicit or explicit reinvestment assumptions, and that the use of an implicit reinvestment assumption leads to logical inconsistencies which seriously limit the use of DCFROR as an evaluation tool. It was shown that opportunity cost is the appropriate discount rate for evaluation of unleveraged investments. Use of cost of capital as a discount rate in economic evaluation may lead to suboptimal investments.

The analysis contained in Chapter 4 laid out a methodology for determining opportunity costs appropriate for use in evaluating leveraged investments and for assessing the effect of leverage on any project financed with balloon or mortgage-type loans.

The concepts and methodologies developed in Chapter 4 for the evaluation of leveraged investments should be of special interest to mining and petroleum companies. These firms often undertake very capital-intensive investments involving borrowed funds and so must make precisely the kinds of evaluations discussed in Chapter 4. At present no reliable techniques are available for evaluating leveraged investments. Investment candidates are first examined on an all-equity basis to evaluate project economics. Once project economics have been shown to be acceptable, the search for the most attractive financing begins. The techniques developed in Chapter 4 allow investors to simultaneously evaluate project economics and financing. They can also be used to define the appropriate leveraged discount rate to be used as a hurdle or discount rate in DCFROR or NPV analysis.

In practice a firm's opportunity cost is defined by its best alternative use of funds, or by the DCFROR it could have received on the best investment forgone to fund the project under evaluation. The same principle applies in both leveraged and unleveraged analysis. In defining leveraged opportunity cost, the firm must apply the same leverage proportion and terms to its opportunity cost-defining investment as it is considering for the project being evaluated. The resulting leveraged DCFROR is the hurdle rate that all projects leveraged at the specified proportion and terms must meet. If a project shows a leveraged DCFROR which is less than the leveraged opportunity cost, then the firm is better off to invest the borrowed funds in the opportunity cost-defining investment.

The same line of reasoning extends to the economic evaluation of equipment leasing. Leasing is often examined as an alternative to purchasing assets. A firm may compare leasing to purchasing new equipment or evaluate the desirability of leasing to replace existing capacity. Regardless of the evaluation context, it is important to bear in mind that leasing is a financing alternative that is essentially equivalent to debt. Investments involving leased assets must, therefore, be evaluated on a leveraged opportunity cost basis.

Some firms use the cost of borrowed money as the discount rate in leasing evaluations. It was shown in Chapter 3 that use of the cost of capital instead of opportunity cost in economic evaluation could lead to suboptimal investment decisions. Further, because leasing is essentially borrowing, the appropriate discount rate for evaluating leasing projects is the leveraged opportunity cost. Use of the all-equity opportunity cost in evaluating leasing investments would lead to the kinds of distortions shown in Chapter 4 in which poor projects were made to look acceptable through the use of leverage. Use of the leveraged opportunity cost will increase the hurdle rate that the investment must meet and eliminate the distortions caused by combining economic and financial evaluations.

The models developed in Chapter 4 suggest several areas for possible further research. The two GRR models cover balloon and mortgage-type financing. It may be possible to develop a single general model which will handle all possible combinations of loans and repayment schedules. Using a general proportionality model, it may also be possible to prove mathematically that the evaluation results from leveraged analysis will always be consistent with those from unleveraged analysis. The work contained in this research points toward consistency of results but does not rigorously prove consistency. It may also be possible to develop DCFROR models similar to the GRR models offered here. DCFROR proportionality models would be useful because DCFROR analysis is one of the most widely used economic evaluation techniques.

References Cited

Alchian, A. A. 1955. "The Rate of Interest, Fisher's Rate of Return Over Cost, and Keynes' Internal Rate of Return." The American Economic Review, December 1955, pp. 39-41.

Berry, Charles E. 1972. "A Wealth Growth Rate Measurement for Capital Investment Planning." Ph.D. dissertation, The Pennsylvania State University.

Bierman, Harold and Hass, Jerome. 1982. An Introduction to Managerial Finance. New York: Norton.

Dean, Joel. 1951. Capital Budgeting. New York: Columbia University Press.

Elgers, Pieter T. and Clark, John J. 1980. The Lease/Buy Decision. New York: Macmillan.

Fisher, Irving. [1896] 1974. Appreciation and Interest. Reprint Edition. New York: A. M. Kelly.

Fisher, Irving. [1936] 1974. The Theory of Interest. Reprint Edition. New York: A. M. Kelly.

Grant, Eugene L. and Ireson, W. Grant. 1982. Principles of Engineering Economy. 7th. New York: Wiley.

Hoskold, Henry D. 1877. The Engineer's Valuing Assistant. Hyde Park, England: Longmans, Green and Company.

Keynes, John Maynard. 1936. The General Theory of Employment, Interest and Money. New York: Harcourt, Brace and World.

O'Neil, Thomas J. 1982. "Mine Evaluation in a Changing Investment Climate." Mining Engineering, December 1982, pp. 1669-1672.

Porterfield, James T. S. 1965. Investment Decisions and Capital Costs. Englewood Cliffs, New Jersey: Prentice-Hall.

Pritchard, Robert E. and Hindelberg, Thomas. 1980. The Lease/Buy Decision. New York: Amacom.

References Cited (continued)

Solomon, Ezra. 1956. "The Arithmetic of Capital-Budgeting Decisions." Journal of Business, 29 (April 1956), pp. 124-29.

Stermole, Franklin J. 1984. Economic Evaluation and Investment Decision Methods. 5th. Golden, Colorado: Investment Evaluation Corporation.

Vancil, Richard; Braken, Jerome and Christenson, Charles. 1963. Leasing of Industrial Equipment. New York: McGraw Hill.

Van Horne, James C. 1980. Fundamentals of Financial Management. 4th. Englewood Cliffs, New Jersey: Prentice-Hall.

Weston, Fred J. and Brigham, Eugene F. 1975. Managerial Finance. 5th. Hinsdale, Illinois: Dryden Press.

APPENDIX A

Validation of Example 4.3 Calculations

This appendix validates the leveraged GRR results determined in Example 4.3 using Equation 4.4. In order to validate the Example 4.3 results, leveraged GRRs will be calculated using basic cash-flow analysis.

GRR Calculation Using 0% Leverage

Year 0	Year 1	Year 2	Year 3	
C=$400	R=$0	R=$0	R=$1216.08	
			.5	Tax Rate
			$ 608.04	

GRR = 15%

GRR Calculation Using 25% Leverage

Year 0	Year 1	Year 2	Year 3	
C=$300	R=$0	R=$0	R=$1216.08	
			-33.1	Interest
			1182.98	
			.5	Tax Rate
			591.49	
			-100.00	Principal
			$ 491.49	

GRR = 17.88%

GRR Calculation Using 50% Leverage

Year 0	Year 1	Year 2	Year 3	
C=$200	R=$0	R=$0	R=$1216.08	
			-66.20	Interest
			1149.88	
			.5	Tax Rate
			574.94	
			-200.00	Principal
			$ 374.94	

GRR = 23.3%

GRR Calculation Using 75% Leverage

Year 0	Year 1	Year 2	Year 3	
C=$100	R=$0	R=$0	R=$1216.08	
			-99.30	Interest
			1116.78	
			.5	Tax Rate
			558.39	
			-300.00	Principal
			$ 258.39	

GRR = 37.22%

APPENDIX B

Validation of Selected Sensitivity Analysis Results from Table 4.4

Table 4.4 summarizes the changes in leveraged GRR results caused by variation in leverage proportion, interest rate, loan life, and project life. This appendix presents some of the calculations used to derive Table 4.4 results. These calculations are offered as examples of the use of Equation 4.6.

Project Life = 5 Years
Loan Life = 5 Years
Interest Rate = 5%
Tax Rate = 50%

GRR Calculation Using 25% Leverage

$$1 + i = 1/[300/\{400(1.15)^5 - ([100(1.05)^5 - 100].5 + 100)\}]^{1/5}$$

GRR = 18.15%

GRR Calculation Using 50% Leverage

$$1 + i = 1/[240/\{400(1.15)^5 - ([160(1.05)^5 - 160].5 + 160)\}]^{1/5}$$

GRR = 21.00%

GRR Calculation Using 75% Leverage

$$1 + i = 1/[100/\{400(1.15)^5 - ([300(1.05)^5 - 300].5 + 300)\}]^{1/5}$$

GRR = 35.87%

Project Life = 5 Years
Loan Life = 5 Years
Interest Rate = 10%
Tax Rate = 50%

GRR Calculation Using 25% Leverage

$$1 + i = 1/[300/\{400(1.15)^5 - ([100(1.1)^5 - 100].5 + 100)\}]^{1/5}$$

GRR = 17.57%

GRR Calculation Using 50% Leverage

$$1 + i = 1/[200/\{400(1.15)^5 - ([200(1.1)^5 - 200].5 + 200)\}]^{1/5}$$

GRR = 22.13%

GRR Calculation Using 75% Leverage

$$1 + i = 1/[100/\{400(1.15)^5 - ([300(1.1)^5 - 300].5 + 300)\}]^{1/5}$$

GRR = 32.80%

Project Life = 5 Years
Loan Life = 5 Years
Interest Rate = 12%
Tax Rate = 50%

GRR Calculation Using 25% Leverage

$$1 + i = 1/[300/\{400(1.15)^5 - ([100(1.12)^5 - 100].5 + 100)\}]^{1/5}$$

GRR = 17.31%

GRR Calculation Using 50% Leverage

$$1 + i = 1/[200/\{400(1.15)^5 - ([200(1.12)^5 - 200].5 + 200)\}]^{1/5}$$

GRR = 21.44%

GRR Calculation Using 75% Leverage

$$1 + i = 1/[100/\{400(1.15)^5 - ([300(1.12)^5 - 300].5 + 300)\}]^{1/5}$$

GRR = 31.30%

Project Life = 20 Years
Loan Life = 20 Years
Interest Rate = 10%
Tax Rate = 50%

GRR Calculation Using 25% Leverage

$$1 + i = 1/[300/\{400(1.15)^{20} - ([100(1.1)^{20} - 100].5 + 100)\}]^{1/20}$$

GRR = 16.31%

GRR Calculation Using 50% Leverage

$$1 + i = 1/[200/\{400(1.15)^{20} - ([200(1.1)^{20} - 100].5 + 100)\}]^{1/20}$$

GRR = 18.30%

GRR Calculation Using 75% Leverage

$$1 + i = 1/[100/\{400(1.15)^{20} - ([300(1.1)^{20} - 300].5 + 300)\}]^{1/20}$$

GRR = 22.06%

APPENDIX C

Validation of Case Study Results

This appendix contains calculations validating the leveraged GRR results obtained in section 4.5. In that section Equation 4.5 was used to evaluate a mineral project at various leverage proportions. In order to validate those results the basic all-equity mineral project cash flow will be adjusted and leveraged GRR calculations performed. This adjustment will consist of subtracting the after-tax interest and principal expenses from all-equity cash flows in each year. The GRR calculations made in this appendix may then be compared with results obtained in section 4.5.

GRR Calculation Using 25% Leverage

	Year 0	Year 1	Year 2
C=$3400(1 - .25)=	$2550	R=$1550	R=$1365
Interest Adjustment	0	-.5(102)	-.5(85.94)
Principal Adjustment	0	-133.8	-149.86
Leveraged Cash Flow	C=$2550	R=$1365.20	R=$1172.17

	Year 3	Year 4	Year 5
	R=$1292	R=$1422	R=$1866
Interest Adjustment	-.5(67.96)	-.5(47.82)	-.5(25.26)
Principal Adjustment	-167.84	-187.98	-210.54
Leveraged Cash Flow	R=$1090.18	R=$1210.11	R=$1642.83

Terminal Value @ 16.4% leveraged reinvestment rate = $8884.98 at Year 5

GRR Calculation: $\$2550 = \$8884.98(1/[1 + i])^5$

GRR = 28.4%

GRR Calculation Using 50% Leverage

	Year 0	Year 1	Year 2
C=$3400(1 - .5)=	$1700	R=$1550	R=$1365
Interest Adjustment	0	-.5(204)	-.5(171.89)
Principal Adjustment	0	-267.60	-299.71
Leveraged Cash Flow	C=$1700	R=$1180.4	R=$979.34

	Year 3	Year 4	Year 5
	R=$1292	R=$1422	R=$1866
Interest Adjustment	-.5(135.88)	-.5(95.59)	-.5(50.47)
Principal Adjustment	-335.72	-376.00	-421.13
Leveraged Cash Flow	R=$889.74	R=$998.20	R=$1419.63

Terminal value @ 18.41% leveraged reinvestment rate = $7795.54 at Year 5

GRR Calculation: $\$1700 = \$7795.54(1/[1 + i])^5$

GRR = 35.6%

GRR Calculation Using 75% Leverage

	Year 0	Year 1	Year 2
	C=$3400(1 - .75)=$850	R=$1550	R=$1365
Interest Adjustment	0	-.5(306)	-.5(257.83)
Principal Adjustment	0	-401.40	-449.56
Leveraged Cash Flow	C=$850	R=$995.60	R=$786.52

	Year 3	Year 4	Year 5
	R=$1292	R=$1422	R=$1866
Interest Adjustment	-.5(203.88)	-.5(143.46)	-.5(75.79)
Principal Adjustment	-503.51	-563.94	-631.60
Leveraged Cash Flow	R=$686.52	R=$786.33	R=$1196.50

Terminal value @ 23.53% leveraged reinvestment rate = $7016.44 at Year 5

GRR Calculation: $\$1700 = \$7016.44(1/[1 + i])^5$

GRR = 52.5%