A THEORY OF THE SOLAR WIND INTERACTION WITH THE GEOMAGNETIC FIELD

BY

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ABSTRACT

The mechanism of injection of charged particles into the trapping zones (radiation belts) is not well understood. The solar wind, which is responsible for the presence of the charged particles in the radiation belts, has been represented as a uniform dielectric plasma. The energy density of the solar wind in the uniform model is insufficient to penetrate the geomagnetic field to distances less than 10 R_E. However, if the solar wind is thought of as an aggregate of clusters or "bundles" of charged particles, then it is possible for the solar wind particles to enter the trapping zones directly. This mechanism is investigated using the Störmer theory, and agreement with the morphology of storms is noticed.

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INTRODUCTION

Spherical harmonic analysis of magnetic-field measurements at sites distributed over the surface of the earth shows that less than one per cent of the field is due to external causes. The field due to internal causes is approximately that of a dipole and undergoes very slow changes, which are appreciable only over periods of time of the order of a century. These variations are called secular variations and are not of importance in this thesis.

On the other hand, the external field undergoes very rapid variations. These variations may be subdivided into nearly periodic and non-periodic variations. The former show periodicities ranging from about 0.1 second to 11 years (Chapman & Bartels, 1940; Chapman, 1964). Periodicities of a day or more have been studied extensively, and their causes have been established. In recent years, there has been increased interest in periodicities of less than a day. Variations with periods ranging from about 0.1 sec to 2-3 hours have been reported by a large number of researchers. The amplitudes of these variations range from about 0.1 γ to 500 γ, compared with the nearly static field of approximately 50,000 γ (Pritchard, 1964). In the earth are observed closely associated variations in the electrical currents, producing voltage-gradient amplitudes ranging from about 0.1 mV/km to about 10 mV/km in sedimentary basins and much higher amplitudes in more resistive rocks. These earth currents are induced by the rapidly varying magnetic field at the surface of the earth. Large, non-periodic variations in the electromagnetic field, lasting from 1-3 days, are generally termed storms. Nearly periodic variations, lasting less than 2-3 hours, are termed transient pulsations or micropulsations.

In this thesis the primary cause of storms and micropulsations is treated. Lindemann's suggestion of solar corpuscular radiation was used by Chapman and Ferraro in their theory of geomagnetic storms. It was considered that the solar corpuscular radiation, the primary cause of geomagnetic storms and micropulsations, was effectively a dielectric medium. This plasma stream would be stopped as it moved into regions of increasing density of magnetic field energy (Chapman, 1964). At the boundary of the plasma stream, electric currents flowed which shielded the plasma from the geomagnetic field. These currents were considered responsible for the initial phase of geomagnetic storms. This theory, however, was qualitative, and more refined approaches using the same idea have been developed by many researchers. A bibliography of major contributions in this field is given in review articles by Troitskaya (1964), Matsushita (1964), White (1966) and Beard (1964). The predictions obtained by the more refined methods are in agreement with observations of the initial phase of geomagnetic storms and account for sudden commencements when a shockfront is present in the solar wind. These methods, however, do not describe the morphological structure of storms and micropulsations, the trapping of charged particles, and the ring currents causing the main phase of geomagnetic storms.

The aim of this thesis is to present a solar-corpuscular-radiation model requiring a minimum of hypotheses. The thesis describes the formation of ring currents and the trapping of charged particles.

The sun emits energy in the form of electromagnetic radiations and charged particle flux, referred to as the solar wind. The solar wind has been regarded as an essentially continuous dielectric plasma stream with occasional enhancements. Detailed investigation of a plasma may be approached on either a microscopic or macroscopic basis with a continuous fluid model as an extreme case. A plasma is generally an electrically neutral, fully ionized state of matter. The solar plasma contains mostly protons and electrons. A distribution function describes the microscopic state of the plasma.

On the other hand, a macroscopic study treats average properties of the bulk of the plasma. These properties are the density, temperature, pressure, and charge-density fluctuations. The plasma may be treated as a fluid when uniformity of macroscopic variables is assumed. The bulk motion of such a fluid produces electromagnetic fields, which in turn affect the motion. Such a study is termed magneto-hydrodynamics or simply hydromagnetics. Disturbances of low frequency in a plasma propagate as hydromagnetic waves, which involve motion of the fluid without separation of charges. Plasma oscillations are disturbances of high frequency that are produced as a result of charge separations.

In the fluid model, plasma flows from the sun at a uniform speed, progressively occupying larger volumes. Two hypotheses for this process that can be considered are (1) that the plasma spreads itself uniformly and (2) that this process leads to the breaking up of the plasma into discrete regions of high density. Helmholtz' Theorem in hydrodynamics states that the flux of vorticity over any surface moving with the liquid remains constant throughout its motion. In a non-idealized fluid, frictional forces cause vortices to appear rapidly. In his investigations of the equilibrium configurations of magnetic stars, Ferraro used Hill's spherical vortex, a solution of the magnetohydrostatic equation (see references in Kendall and Plumpton, 1964). Ferraro's results lead to the supposition that vortices of Hill's type are produced in the solar wind in the process of streaming out. These vortices have a tendency to exclude penetration by external electromagnetic fields and carry a small magnetic field with them. This characteristic may facilitate the explanation of the emission of the solar wind. The mechanism proposed by Parker (1964) causes the emission of these vortices rather than of individual particles or of a continuous stream.

A microscopic theory to show the appearance of these vortices met with limited success. They cannot represent a thermodynamic and statistical equilibrium configuration (Landau and Lifshitz, 1958) and, hence, represent essentially a non-equilibrium state whose lifetime is finite and may be deduced. It may also be observed that a system of particles obeying the Bose-Einstein Statistics, bosons, has the tendency to cluster; this phenomenon has been experimentally observed in the counting rate of bosons in dense light beams (Henley and Thirring, 1962). The problems of dense, high-temperature plasmas seem to offer the possibility of being "solved" by Pair Theory.

At this stage, the problem of ascertaining the dimensions and number of particles forming a vortex of the Hill's type, or a "bundle," arises. Microscopic theory has given a means of estimating these quantities in a historically notable technique first considered by Debye in the case of electrolytic solutions. Consider a test particle in a plasma; the other charges distribute themselves around the test particle, forming an ion cloud (Landau and Lifshitz, 1958; Holt and Haskell, 1965). The radius of the effective cloud is computed by assuming that the particles obey a Maxwell-Boltzmann distribution and that the Poisson's Law of electrostatics is applicable. For plasmas having temperatures of $10^4 - 10^6$ °K and particle densities of 10^{18} to 10^{20} per cubic meter, the number of particles in an "ion cloud" having the dimensions of a Debye-Hückel radius of $10^{-2} - 10^{-3}$ cm is $10^2 - 10^5$. Bostick experimented with plasmoids in vacuum chambers containing strong magnetic fields and found that plasmoids retain their shape in a "very uncanny manner" (Bostick, 1961). The well-known pinch effect may also be interpreted as indicative of the strong tendency for plasmas to produce motions which contain the particles spatially.

It is predicted by theory that electromagnetic waves shorter than the Debye length cannot propagate in a plasma. It is argued that electrons can respond very easily to the effect of these fields and damp the disturbances. This observation may be explained on a bundle model as follows: Disturbances of wave lengths shorter than the Debye length strain a bundle in shape and lose their energy, whereas those of longer wave lengths displace bundles spatially without much strain.

Finally Oort, from his studies of the absorption lines and 21-cm radiation from interstellar gas, came to the conclusion that the observed fine structure could be adequately described on the assumption that the clouds are actually "regions of high density separated by regions of negligible density" (Oort, 1955). Examples of this sort, together with the idea that an expanding stream of interacting particles would tend to form bundles, suggest that the solar wind may be best described as consisting of bundles having negligible interactions among them. This conjecture is here raised to the status of a hypothesis called the Bundle Theory, and also introduces another hypothesis, that the bundles are destroyed in collisions with other bundles and with thermal plasma.

Stőrmer's treatment of the problem of charged-particle motion in a magnetic dipole field is frequently applied to the study of auroras and cosmic rays (see review of early work, Stőrmer, 1955). The Stőrmer theory is utilized in considering the orbits of bundles in the geomagnetic equatorial plane. Figure 1 shows typical trajectories in the equatorial plane, viewed from the galactic north pole.

In the vicinity of the earth, the bundles produce electric fields due to the separation of positive and negative bundles. The electric fields produced in this manner (shown in Fig. 2) enable the bundles reaching region A to transfer their kinetic energy to the diamagnetic plasma already present in the ionosphere-magnetosphere cavity. The bundles which thus lose their kinetic energy stagnate and diffuse into the cavity. The stagnation maintains the electric field in the region A, and diffusion replenishes the plasma in the cavity, allowing a more or less continuous pumping of plasma towards the inner part of the magnetosphere. The diamagnetic plasma drifts closer to the earth in region A because of the drift due to the interaction of the electric and induction fields, and gains energy since the magnetic field is inhomogeneous.

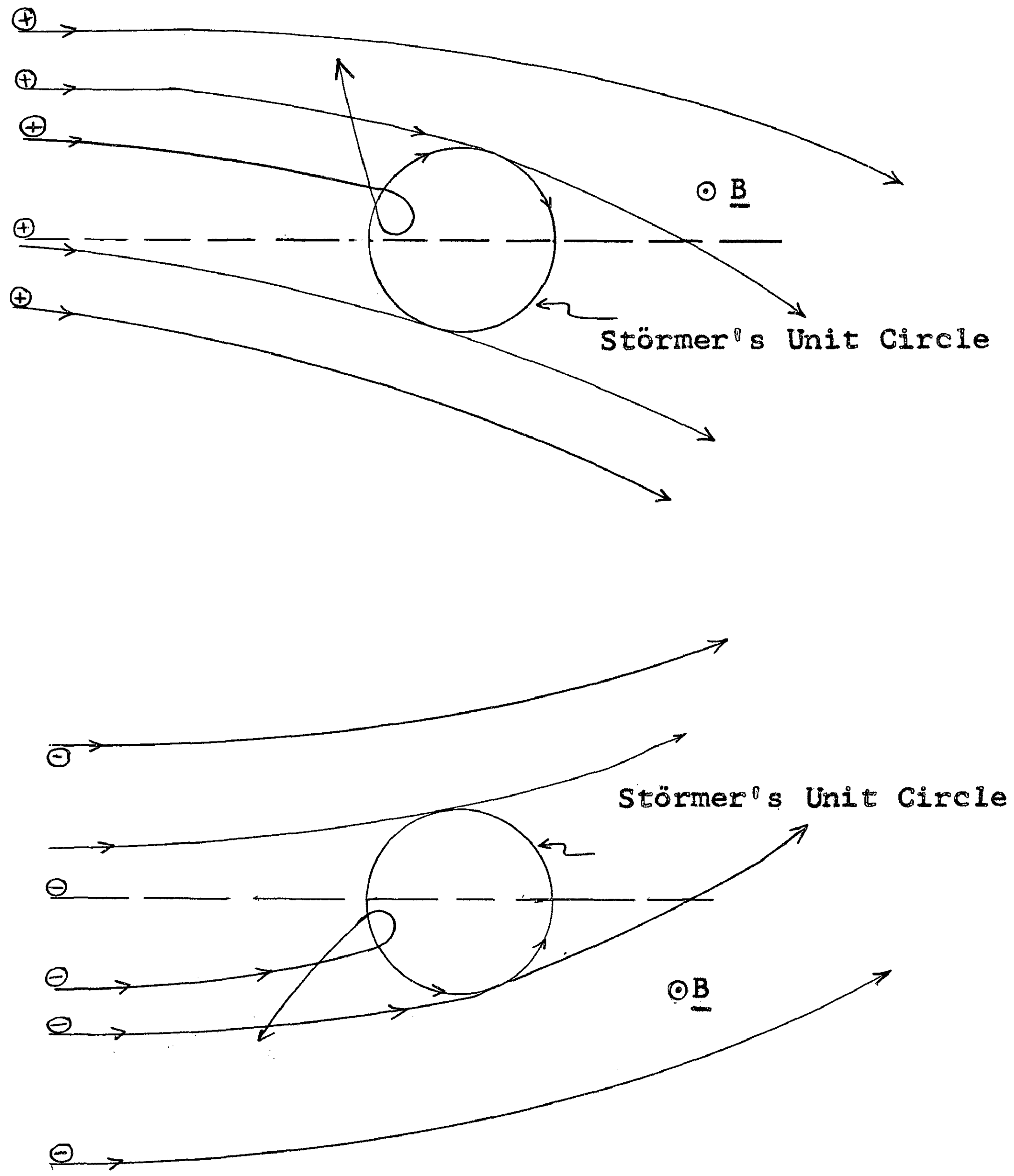


Fig. 1. Typical trajectories of bundles in the geomagnetic equatorial plane (for pitch angle zero). After Störmer (1955)

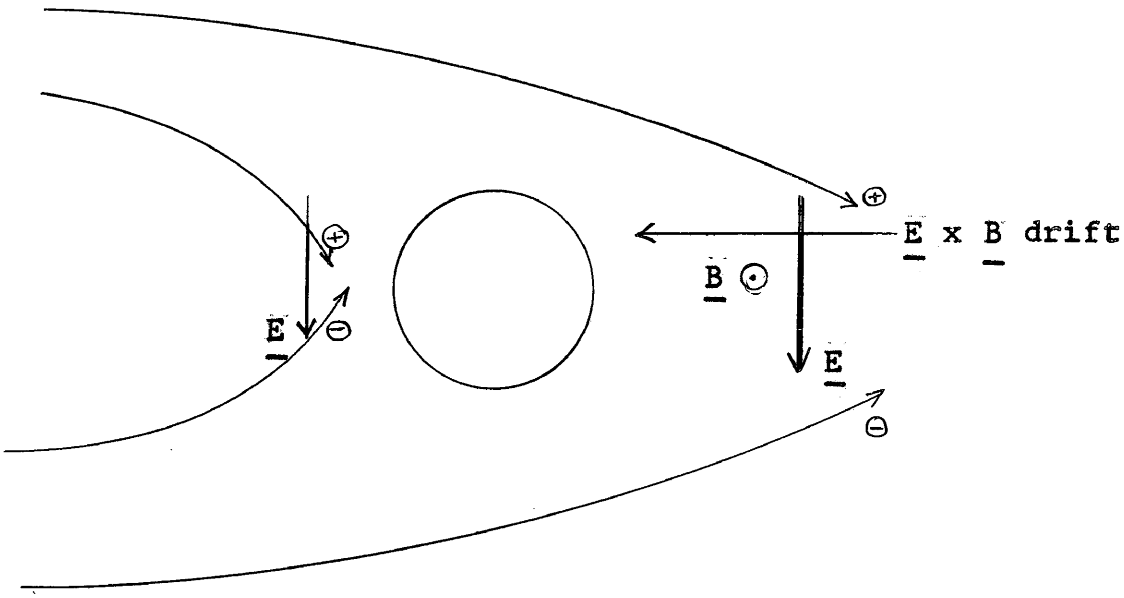


Fig. 2. Plasma Flow due to the Influx of Bundles.

In a region where diamagnetic plasma particle density is not appreciable, the bundle trajectories cross mostly along a line extending from the sun through the earth, and collisions in these regions would increase the density of diamagnetic plasma. In this process, plasma oscillations and hydromagnetic waves may be generated.

These disturbances propagate in the intervening plasma towards the lower levels of trapped radiation and cause micropulsations.

An analysis of the drift motion of diamagnetic plasma in the geomagnetic field together with the electric field leads to the conclusion (in agreement with observations) that the drifting particles approach closer to the earth on the night side than on the day side (O'Brien, 1963).

THE ELECTROMAGNETIC FIELD
OF THE EARTH AND ITS SURROUNDINGS

The electromagnetic field of the earth may be divided into time-independent and time-dependent parts. The former is termed the steady field.

The magnetic field of the earth has been approximated to be that of a centered magnetic dipole of moment 8.1 x 10^{22} amp - m^2 inclined 11.6° to the axis of rotation. Major anomalies have been found which have been represented as being due either to a centered quadrupole or to eccentric dipoles. The steady magnetic field thus represented accounts for more than 99% of the observed magnetic field. The remaining part is due to the current systems ever present in the earth's surroundings and is subject to the influence of tides, ionizing radiations, and corpuscular streams. Hence, it is not surprising that this part undergoes variations, some of which are of prime concern in this thesis.

The earth is surrounded by an atmosphere which is ionized in layers or strata. The ionization is primarily due to the sun's ionizing radiations and, hence, generally varies from a maximum at noon to a minimum at midnight.

The region beyond the ionosphere is of prime concern here. Figure 3 shows the region extending from about 1.2 earth radii from the earth's center to approximately 8 earth radii. In this region fluxes of energetic protons and electrons, termed the Van Allen Belts, are encountered. The proton belt is termed the inner Van Allen belt, and the electron belt, the outer Van Allen belt (Dessler, 1960).

The region beyond the Van Allen belt is generally believed to be a region of space where the geomagnetic field is confined by the solar wind as shown in figure 4. Gold termed this region the magnetosphere (Dessler, 1960). The boundary of the magnetosphere, termed the magnetopause, is believed to be a region of intense activity resulting in current systems. This boundary varies in distance from the earth, depending on the solar wind pressure. The disturbances at this boundary are believed to be transmitted to the earth in the form of hydromagnetic waves through the intervening plasma.

A right-handed spherical polar coordinate system is used as shown in figure 5. The polar axis is chosen to be the magnetic azimuthal axis. A point P is represented by the ordered triad (r, θ, ϕ), and a vector $\underline{A}$ is represented by either a row or a column matrix with the proper subscripts to denote the components thus:

$$\underline{A} = \begin{bmatrix} A_r & A_\theta & A_\phi \end{bmatrix}$$

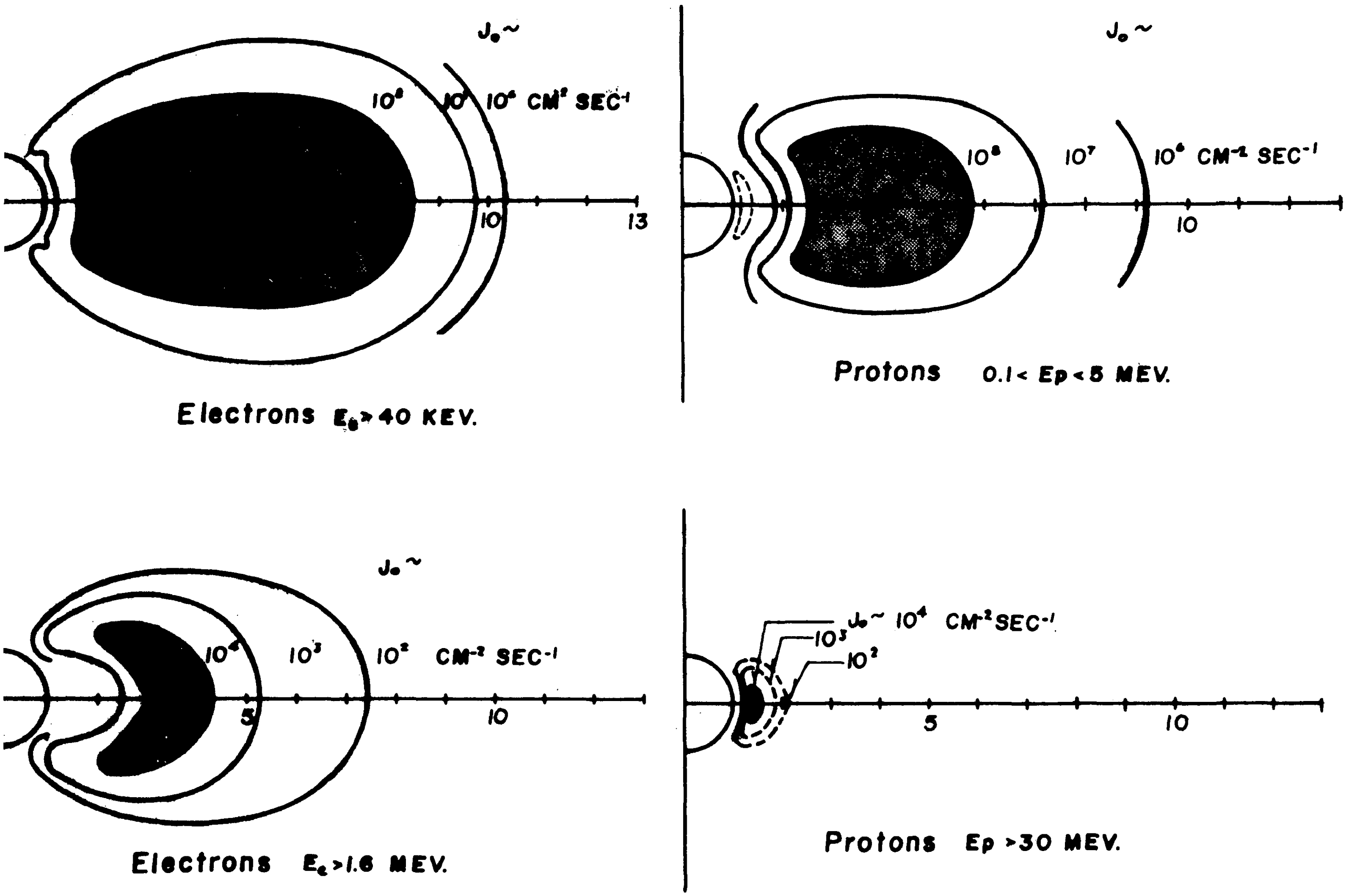
$$\text{or} \quad \underline{A} = \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

In this system of notation, the magnetic induction field of the earth at point P (r, θ, ϕ) as seen by an observer fixed to the earth is represented thus:

$$\underline{B} = \begin{bmatrix} B_r & B_\theta & B_\phi \end{bmatrix}$$

$$= - \frac{\mu_o M_E}{4\pi r^3} \begin{bmatrix} 2 \cos \theta & \sin \theta & 0 \end{bmatrix}, \quad r > R_E$$

where M_E is the moment of the earth's magnetic dipole, μ_o is the permeability of free space, and R_E is the radius of the earth. The MKS system will be used throughout this study.



Electrons $E_e > 40$ KEV.

Protons $0.1 < E_p < 5$ MEV.

Electrons $E_e > 1.6$ MEV.

Protons $E_p > 30$ MEV.

Fig. 3. Radiation Belts

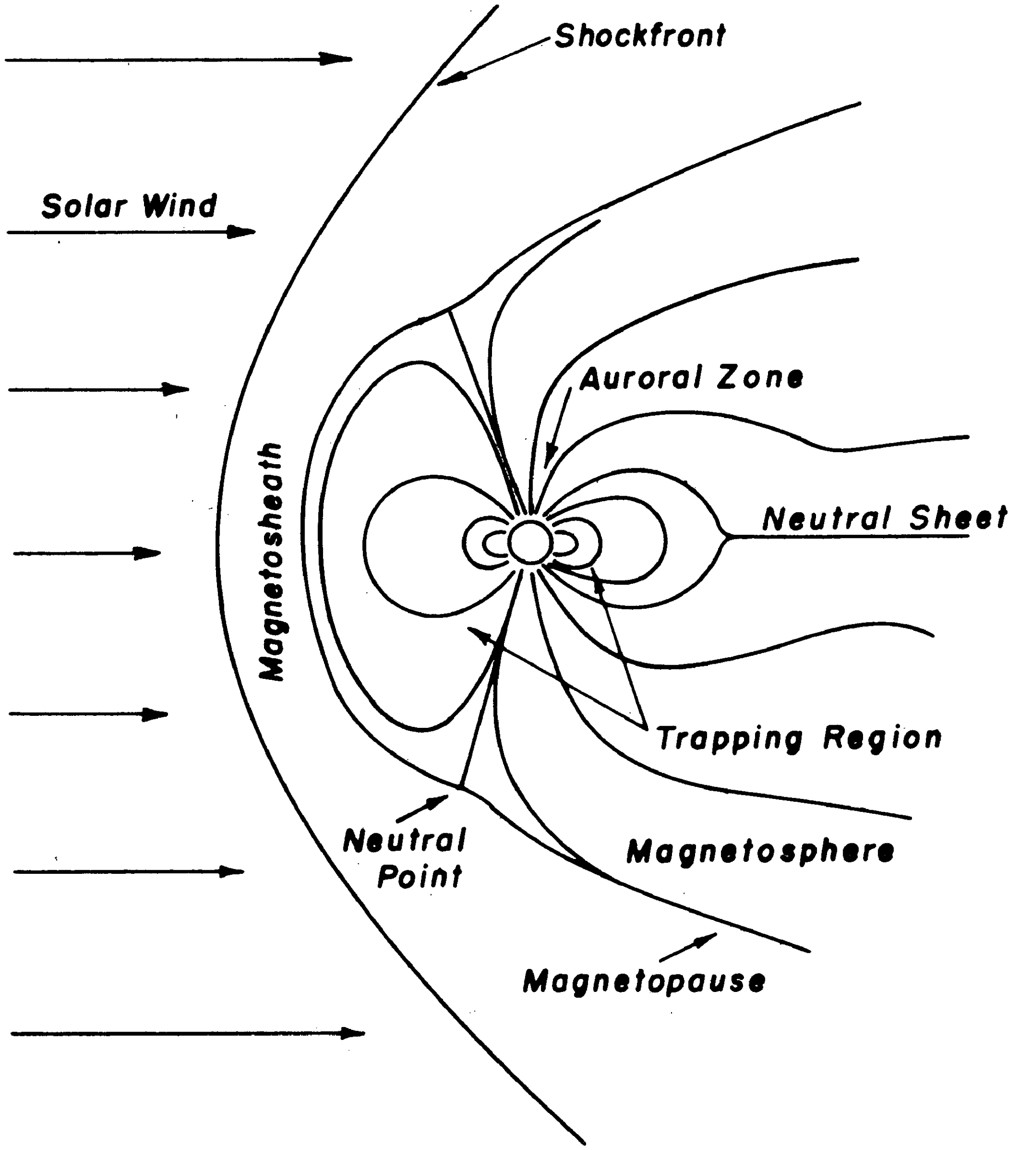


Fig. 4. The Magnetosphere

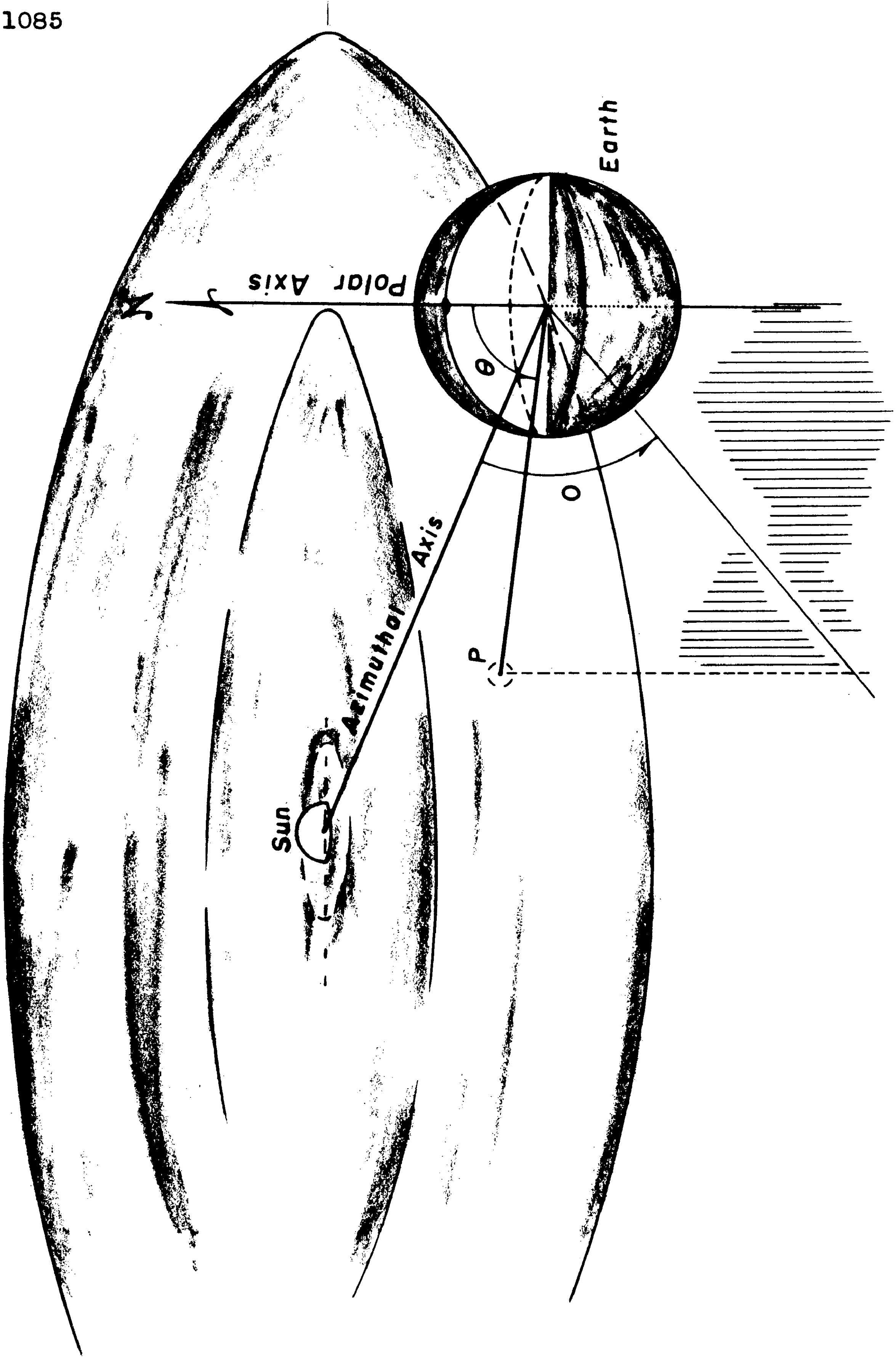


Fig. 5. The Earth-fixed Coordinate System

THE TIME DEPENDENCE
OF THE ELECTROMAGNETIC FIELD OF THE EARTH

The part of the electromagnetic field of the earth undergoing rapid changes is of external origin. Variations in this field, correlated to the 11 year solar cycle, the 27-day period of rotation of the sun, and the sunspot numbers, have been observed. Annual and seasonal variations resulting from the differing orientation of the geomagnetic dipole axis relative to the sun have also been observed. Variations in the telluric field are the direct result of the variations in the magnetic field, and hence, the terminology customarily associated only with geomagnetic field variations is used to describe both. The terminology is used without prefixes.

Daily Variations

Daily variations in the electromagnetic field of the earth have been analyzed, and a major part has been found to be dependent on the earth's position relative to the sun and the moon. The sun-dependent part is termed the S-variation or the S-field, and the moon dependent part is termed the L-variation or the L-field. The S- and L-variations are the result of tidal effects, convection currents, and the changing ionization in the ionosphere. The convection currents are due to the uneven solar heating of the atmosphere. The changing ionization depends on local time.

The sun emits electromagnetic radiations (mainly in the ultra-violet region) and a steady stream of low-velocity protons and electrons, termed the solar wind. This activity of the sun, which causes the ionization of the upper atmosphere, may be termed normal. Days when the sun is normal are called quiet days, and the S-variations on these days are termed Sq variations.

However, there are days when the sun is abnormally active, that is, when the sun emits enhanced fluxes of particles. These days are called disturbed days, and the S-variations are termed S_d-variations. The S_d variations are generally thought of as resulting from the superposition on S_q of another variation termed the disturbance daily variation, denoted by S_D (or Ds).

Periods of several days when the disturbance is very large are termed storms and are of great interest in this thesis. The S_q variations depend on latitude and local time, whereas the L-variations depend on latitude and local lunar time and have a semi-diurnal periodicity.

Storms

The abnormal activity of the sun results in the enhancement of the solar wind. The particle flux on a normal day is estimated to be 4 - 40 x 10^6 particles per cubic meter, having velocities of 3 - 7 x 10^5 meters per second. On disturbed days these figures change to 2 x 10^8 and 1 - 2 x 10^6 respectively (Parker, 1964). An idea of the duration of the abnormal activity may be obtained by noting that the lives of solar flares, coronal arcs, and solar prominences are 10^3, 10^3, 10^5 seconds, respectively (Alfven, 1963).

Chapman studied geomagnetic storms and found that the average characteristic of storms may be divided into an initial, a main, and a recovery phase. During the initial phase, the horizontal component of the geomagnetic field increases rapidly; the increase lasts 2 - 4 hours.

The end of the initial phase is the beginning of the main phase. During the main phase, the field decreases and reaches a minimum which is much more below the original undisturbed value than the maximum was above it. On reaching the minimum, the recovery phase begins. During this phase, the field returns very slowly to the normal state. This recovery lasts several days.

These phases have a dependence on the local time as well as on time measured from the beginning of the storm. The latter is termed the storm-time variation, denoted by D_{st}, and the former is termed the disturbance daily variation, denoted by D_S or S_D. Figure 6 shows the results obtained by Sugiura and Chapman (1960) from the study of many storms and shows the average features of storms.

Storms are sometimes characterized by the suddenness of the variation, referred to as a Sudden Commencement (SC), and by the nature of the SC. The SC characteristics have been employed by Akasofu and Chapman (1959) to classify storms. This classification describing the various types is shown on Table I. On the basis of existing theories of solar wind interactions with the geomagnetic field, many major phenomena remain unexplained. The unanswered questions concerning storms are briefly discussed in the following.

Mogel and Dellinger observed that storms are accompanied by radio fadeouts, and this phenomenon is referred to as a sudden ionospheric disturbance (S I D). It was also observed that the ionization in the E-layer is enhanced while the electron density in the F-layer decreases, mainly in polar regions (Obayashi, 1964). The foregoing

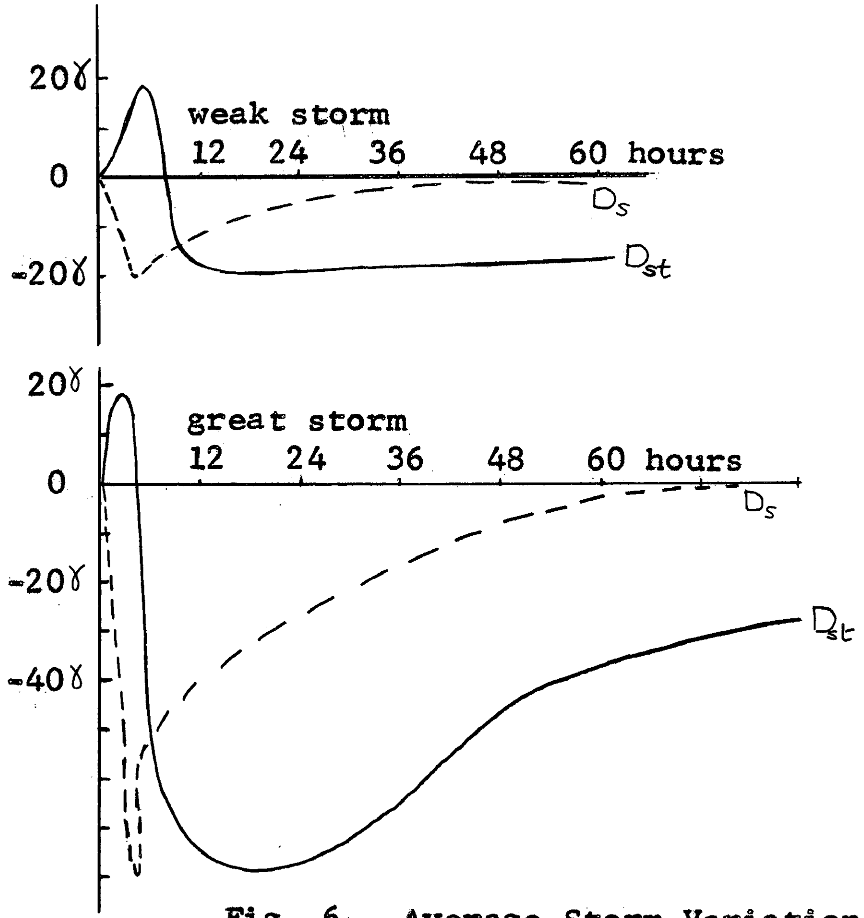


Fig. 6. Average Storm Variation

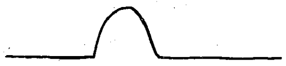
Type	Variation in the Horizontal Component	Other notations
Sc (+)		SSC
Sc (-)		rev. SSC
Sc (-+)		SSC*
Sc (+-)		rev. SSC*

Table I. Types of Sudden Commencements.

observation led to the conclusion that particles of the solar wind are directly injected into the upper ionosphere; the manner in which this happens has not yet been satisfactorily explained on the basis of current theories. The disturbance in the auroral zone and the middle latitudes is larger at night than in the daytime, whereas at the polar cap it is predominant in the daytime. The reverse impulse type of Sc (Sc^{-+}) occurs during the daytime in the magnetic equatorial zone (Matsushita, 1964). Matsushita has also come to the conclusion that the "mere distortion of the earth's field caused by incoming plasma front" cannot produce the Sc variation. Solutions to the above problems may be obtained by the Bundle Theory.

Micropulsations

Micropulsations are disturbances in the electromagnetic field of the earth having periods in the range 0.1 sec. to 10 min. and sometimes 2-3 hours. The amplitudes vary from less than 0.1 γ to tens of γ's in the geomagnetic field and from 0.1 millivolt to tens of millivolts per kilometer in the telluric field, measured in sedimentary basins.

A bay is a disturbance in the field, characterized by a very rapid increase or a decrease; it usually lasts 20-40 min. This is believed to be due to the augmentation of the

S_q field by increased ionization in the E and F regions of the ionosphere. Bays occur mostly around midnight and are accompanied by Pt pulsations and high frequency pulsations with periods of 10^{-3} to 1 seconds.

Pt pulsations consist of several series of oscillations. Each series consists of heavily damped oscillations lasting 10-20 min. or 1-1/2 - 8 cycles. The periods range from 40 sec. to a few minutes (Troitskaya, 1964), and 10 - 100 sec. (Keller, private communication). These occur mostly around midnight and often accompany or precede a bay. The Pt pulsations have a microstructure consisting of short irregular pulsations (denoted SIP) which are sometimes followed by pearl type (PP) pulsations. Pt pulsations always begin with an augmentation of the north component of the geomagnetic field. Troitskaya (1964) observes that "The properties of Pt-excitation in magnetically conjugate regions show that the mechanism of solar particle trapping, drift and periodical injection is apparently less probable than direct injection into the atmosphere of rapid charged particles trapped into dense clouds of solar plasma and released near the earth." This idea of Troitskaya is in excellent agreement with the Bundle Theory.

Pc pulsations are continuous pulsations having periods of 10-600 secs. and last many hours. They are divided into three types: Pc I, Pc II, and Pc III, having periods of 10-50, 60-150, and 150-600 seconds, respectively. Pc I and Pc II have a maximum around noon, whereas Pc III has a maximum in the morning and evening hours.

Pearl type pulsations are regular amplitude-modulated sinusoidal oscillations with periods ranging from 0.3 - 4 sec. They occur in the form of separate bursts, gradually developing into a series of pulsations lasting from tens of minutes to tens of hours. The pearl type pulsations are believed to be the result of bunches of charged particles moving along the magnetic field lines of the earth and, at the same time, drifting around the earth.

Study of the fine structure of geomagnetic storms shows that pp^s and SIP^s occur before storms. At this time an increase in energetic protons is also observed.

Table II summarizes the major types of micropulsations.

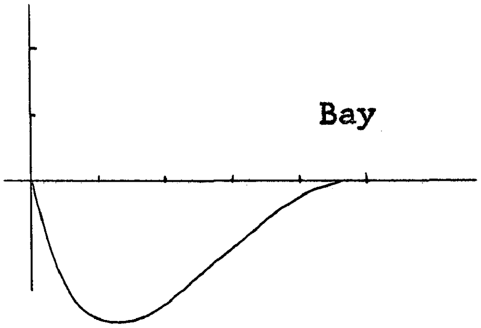
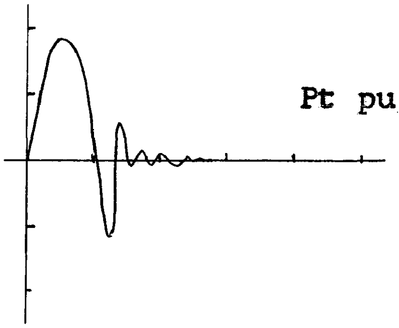
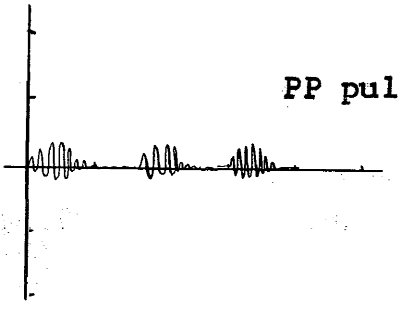
		Amplitude	Period
	Bay	20-40 γ	20-40 min.
	Pt pulsations	$\sim 10\gamma$	10-100 sec.
	PP pulsations	$\sim 10^{-2}\gamma$	.3-4 sec.

Table II. Micropulsations.

THE BUNDLE THEORY: A STUDY OF STREAMING PLASMAS

The study of plasmas may be approached in four different ways (Rose and Clark, 1960) characterized by the relative importance of interactions in quantum-mechanical, Debye, fluid, and gas-kinetic regions. The study of quantum mechanical interactions becomes important when the probability of collisions at Compton Wavelengths is appreciable. The gas-kinetic treatment of a plasma is applicable when the mean free path of a plasma particle is large compared to the dimensions of the plasma body.

The two-fluid model has been used with at least partial success in the case of the solar wind.

A remarkable feature of this description is its success because this ignores "the very nature of the plasma" (Stix, 1962).

> The plasma does manifest coherent motions, and the real physical question is to account, on the one hand, for the collisionless coupling of particle motions which give the plasma a fluid

> like behaviour and, on the other hand, to establish the correspondence between a hot plasma of charged particles and a free streaming neutral gas.
>
> The Coulomb force between any two particles is minute, but the interaction achieves strength by the coherent or collective motion of the plasma particles. The motion of different charged particles passing through a fluctuation of electric field are all modified in a coherent if not identical way, and these coherent motions in turn lead to macroscopic currents and space charge which, in turn affect the coherent particle motions, that is responsible for the propagation of motion in a hot plasma and which replaces the familiar collision process.

Consider a plasma having an average particle density of n_o protons and n_o electrons per cubic meter and characterized by a temperature T^oK.

The electrostatic potential surrounding a test particle may be evaluated by assuming that the Maxwell-Boltzmann distribution and Poisson's Law of electrostatics are applicable. The electrostatic potential, Ψ (r) in a spherically symmetrical distribution, is given by

$\Psi(r) = \frac{q}{4\pi\epsilon_o}\frac{1}{r}\exp\left(-\frac{r}{h}\right)$, where q is the charge on the test particle, ϵ_o the permittivity of free space and $h = \left(\frac{\epsilon_o kT}{n_o q^2}\right)^{\frac{1}{2}}$. h is termed the Debye-Hückel or the Debye radius (Holt & Haskell, 1965; Landau & Lifshitz, 1958).

This is the distance from the test particle at which the electrostatic potential of the plasma distribution around the test particle drops to $\frac{1}{e}$ of its value at the surface of the test particle. Thus h is a measure of the distance to which a plasma distribution influences other particles.

The number of particles of like charges in a Debye sphere is given by

$$N = 1.38 \times 10^6 \quad \frac{T^{3/2}}{(n_o)^{\frac{1}{2}}}$$

The following table (III) gives the number of particles in a Debye sphere in regions of interest to us. It may thus be reasonable to consider aggregates of this magnitude to move collectively and with negligible interactions with other aggregates. Yourgrau, van der Merwe and Raw (1966) observe that "clouds" of charged particles interacting through a screened Coulomb potential may be considered equivalent to an assembly of virtually independent "quasi-particles."

Region	Particles per Cubic Meter	Temp °K	Debye Radius in Meters	No. of Particles in Debye Sphere
Ionosphere	10^{10}-10^{13}	10^{2}-10^{3}	$7\text{x}10^{-3}$-$7\text{x}10^{-4}$	10^{4}
F_1 Layer	10^{16}	10^{2}-10^{3}	$7\text{x}10^{-6}$-$2\text{x}10^{-5}$	10-400
Exosphere	10^{7}-10^{11}	---	---	---
Interplanetary Space	10^{6}-10^{7}	10^{4}-10^{5}	7	10^{9}-10^{10}
Interstellar Space	10^{6}	10^{2}-10^{4}	1-7	10^{6}-10^{9}
Solar Corona	10^{12}-10^{14}	10^{6}-10^{7}	.07-.02	10^{9}
Solar Chromosphere	10^{17}-10^{20}	10^{3}-10^{4}	10^{-5}-10^{-6}	10^{2}

Table III. Debye Sphere and related parameters describing plasmas (Adapted from Alfven & Falthammar, 1963 and MacDonald, 1964)

MOTION OF CHARGED BUNDLES IN THE EARTH'S DIPOLE FIELD

The right-handed spherical coordinate system illustrated in Fig. 5 is used. The basic assumptions used in considering the motion of a bundle are summarized as follows:

1. The bundles obey the laws of motion of point charges of mass m, charge q, and initial speed V.

2. The dipole magnetic field is not appreciably altered by the charged bundles and hence is constant in time.

3. The dipole field is the only field interacting with the bundle. (The bundles are assumed to be non-interacting among them.)

4. The magnetic and electric dipole moments of bundles due to internal motions and polarization are assumed to be negligible.

The essence of the above consideration is that the energy density of a bundle is considered infinite. However, a bundle whose energy density is comparable to that of the dipole field will not follow the trajectories of the bundles treated here. This problem is best treated by the Chapman-Ferraro method.

The following equation of motion applies to a bundle at position $\underline{r}$ (r, θ, ϕ).

$$m \frac{d^2}{dt^2} \underline{r} = q \frac{d\underline{r}}{dt} \times \underline{B}$$

This may be written thus:

$$\begin{bmatrix} \ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2 \sin^2 \theta \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2 \sin\theta \cos\theta \\ r \sin\theta\, \ddot{\phi} + 2\dot{r}\dot{\phi} \sin\theta + 2r\dot{\theta}\dot{\phi} \cos\theta \end{bmatrix} = \frac{\mu_0 M_E}{4\pi r^3} \frac{q}{m} \begin{bmatrix} r\dot{\phi} \sin^2\theta \\ -2r\dot{\phi} \sin\theta\cos\theta \\ 2r\dot{\theta}\cos\theta - \dot{r} \sin\theta \end{bmatrix}$$

where $\dot{r} = \frac{d}{dt} r$, $\ddot{r} = \frac{d^2}{dt^2} r$ and so on.

Two integrals of the motion are readily obtained. Firstly the energy of a bundle is constant since

$$q \frac{d\underline{r}}{dt} \times \underline{B} \cdot \frac{d\underline{r}}{dt} = 0$$

Hence $V^2 = r^2 + (r\dot{\theta})^2 + (r\dot{\phi} \sin \theta)^2$

Secondly, the ϕ-equation yields another integral of motion as follows:

$$r^2 \sin^2 \theta\, \dot{\phi} - \frac{K \sin^2\theta}{r} = L \text{ , where } K = \frac{\mu_0 M_E}{4\pi} \frac{q}{m}$$

and L is a constant of integration.

If $(\frac{K}{V})^{\frac{1}{2}}$, the radius of Störmer unit circle, is used as the measure of distances, the equations may be written in the familiar forms (Störmer, 1955) as follows:

$$\dot{r}_s^2 + (r_s\dot{\theta})^2 + (r_s\dot{\phi}\sin\theta)^2 = 1$$

$$r_s^2 \sin^2\theta\,\dot{\phi} - \frac{\sin^2\theta}{r} = 2\gamma$$

where the differentiation is with respect to arc length along a trajectory and r_s is the distance measured in units of the Störmer radius.

The trajectories of the bundles can be computed only by numerical methods since a third integral of motion is not available.

However, the analysis of the angular momentum integral yields some knowledge of the trajectories.

From the energy integral it follows that

$$-1 \leq r_s\dot{\phi}\sin\theta \leq +1$$

Since $r_s\dot{\phi}\sin\theta = \dfrac{2\gamma}{r_s\sin\theta} + \dfrac{\sin\theta}{r_s^2}$,

$$-1 \leq \frac{2\gamma}{r_s\sin\theta} + \frac{\sin\theta}{r_s^2} \leq +1$$

Thus for a given value of γ, this relation imposes a restriction on the values of r and θ that are possible or in other words divides the space surrounding the dipole into allowed and forbidden regions as shown in Figs. 7 and 8.

An analytical expression for bundle trajectories can be obtained for motion in the equatorial plane as given below.

$$\int d\phi = -\tfrac{1}{2}\left[\sin^{-1}\frac{1}{v}\left(\frac{L_o}{r}+\frac{K}{r^2}\right)\right]_{r=\infty}^{r} + \frac{L_o}{2}\,\frac{1}{\sqrt{2Kv}}\left[F\left(\sqrt{\frac{v+Ka^2}{2v}}\,,\right.\right.$$

$$\left.\cos^{-1}\sqrt{\frac{K}{v+Ka^2}}\left(\frac{1}{r}+\frac{L_o}{2K}\right)\right]_{r=\infty}^{r}$$

where v is the speed, L_o, the initial angular momentum, $a = \frac{L_o}{2K}$ and F, the Legendre's form of the elliptic integral of the first kind.

Typical trajectories are shown in Fig. 1.

From the above considerations it is clear that the distance of closest approach of bundles to the dipole depends on the radius of Störmer's unit circle, "C_{st}".

The following table gives the values of the radius for bundles of various sizes.

No. of protons in bundle of unit charge	Speed in m/sec	Radius in units of the earth's radius
10^2	10^6	15
10^2	10^5	50
10^3	10^6	5
10^3	10^5	15
10^4	10^6	1.5
10^4	10^5	5

Table IV. Radius of Störmer's unit circle for various bundles.

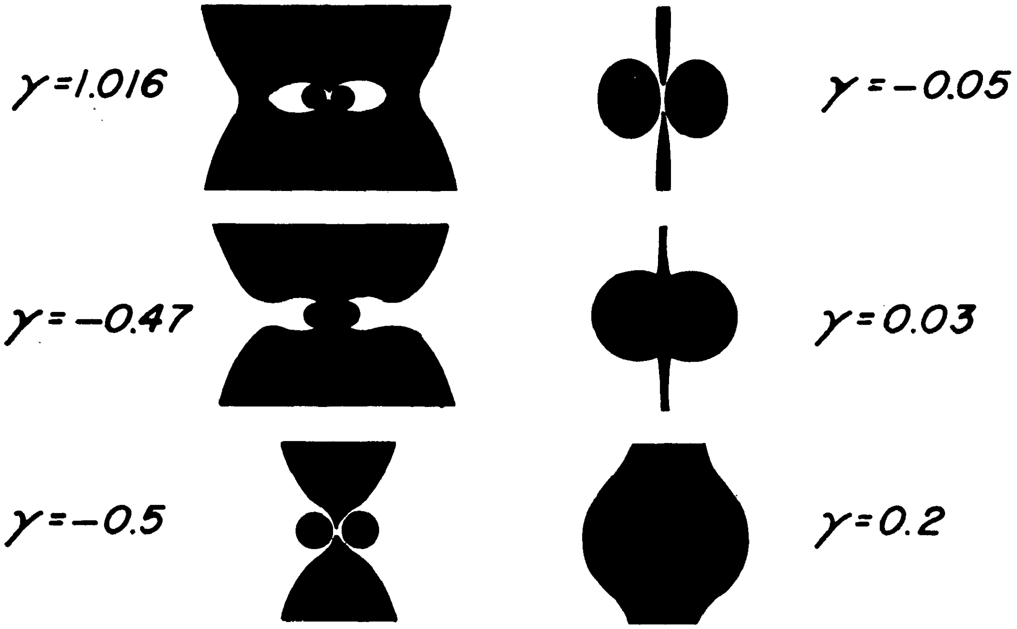


Fig. 7. Sketch of the Allowed and Forbidden Regions for Various Values of γ

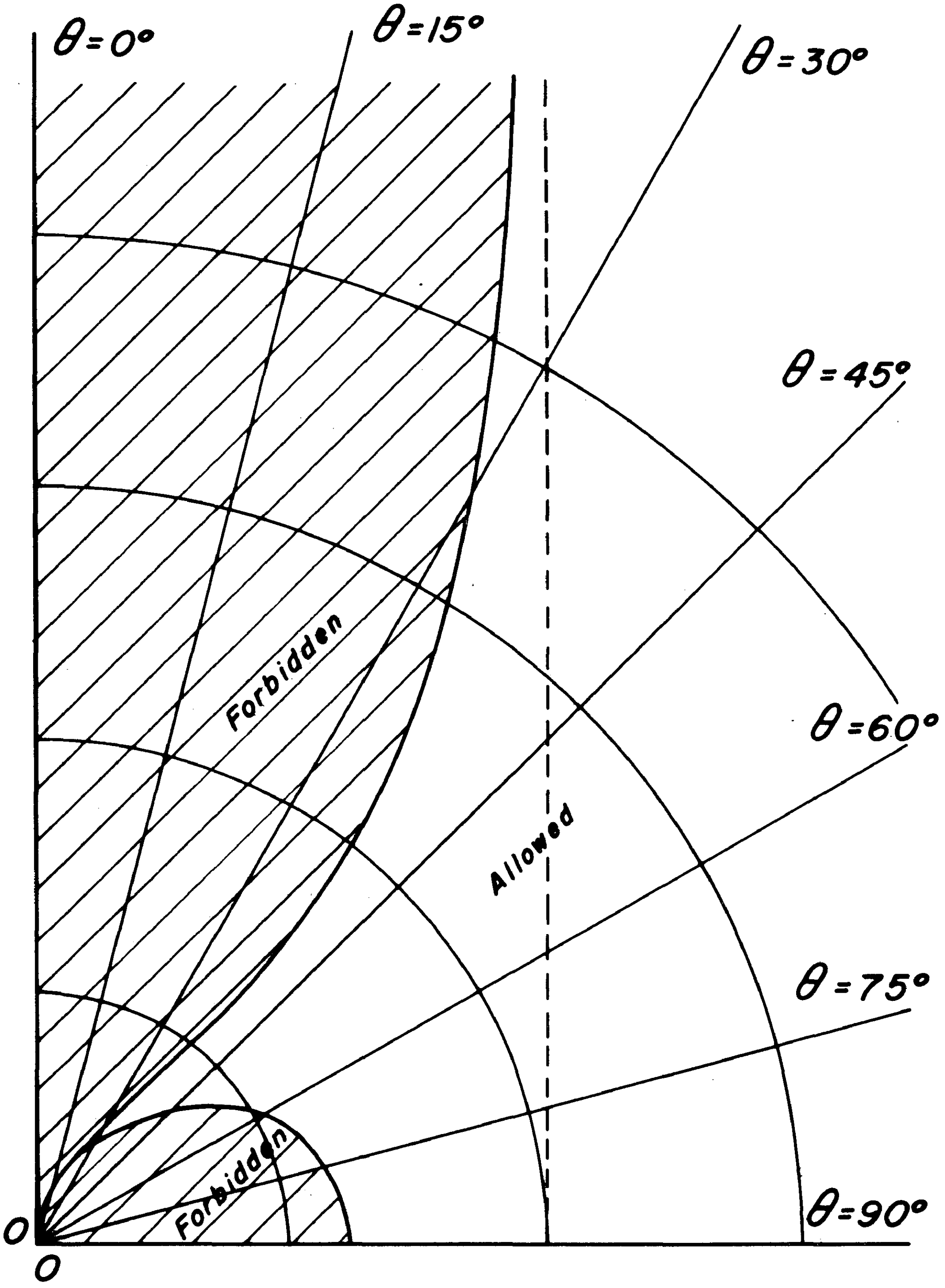


Fig. 8. Sketch of the Allowed and Forbidden Regions for a Fixed γ

MOTION OF CHARGED PARTICLES IN AN ELECTROMAGNETIC FIELD

The motion of a charged particle in an electromagnetic field may be described by the following equation.

$m \frac{d^2}{dt^2} \underline{r} = q \underline{E} + q \frac{d\underline{r}}{dt} \times \underline{B}$ where the symbols have the usual meaning.

Uniform Fields

When a uniform electric field acts on a charged particle, the particle continuously accelerates in the direction of the electric field, and any initial component of velocity at right angles to the electric field remains unchanged. The trajectory is in general a parabola.

In a uniform magnetic field the component of velocity parallel to the field remains unchanged, and at right angles to the field the particle gyrates with the Larmor frequency ω_c given by:

$$m\rho\omega_c^2 = q\rho\omega_c B$$

$\omega_c = \frac{q\,B}{m}$ and the energy ξ of the particle associated with the gyration = $1/2\ mV_{\perp}^2$ and is usually denoted by $\xi_{\perp}$.

Thus the trajectory of the particle is in general a helix of uniform pitch. When $\underline{E}$ and $\underline{B}$ are parallel and uniform the trajectory is a helix of increasing pitch.

When an electric field is present at right angles to the magnetic field, a drift of the center of gyration of the particle perpendicular to the $\underline{E}$ and $\underline{B}$ fields results. This may be understood by transforming the equation of motion to a frame of reference moving with a velocity $\underline{V}_D = \frac{\underline{E} \times \underline{B}}{B^2}$

In the moving frame the equation of motion reduces to $m \frac{d^2}{dt^2} \underline{r}' = q\,\underline{E}' + q\,\frac{dr'}{dt} \times \underline{B}'$ where the primed coordinates are as measured in the moving frame.

$$\underline{E}' = \underline{E} + \left(\frac{\underline{E} \times \underline{B}}{B^2}\right) \times \underline{B} = 0$$

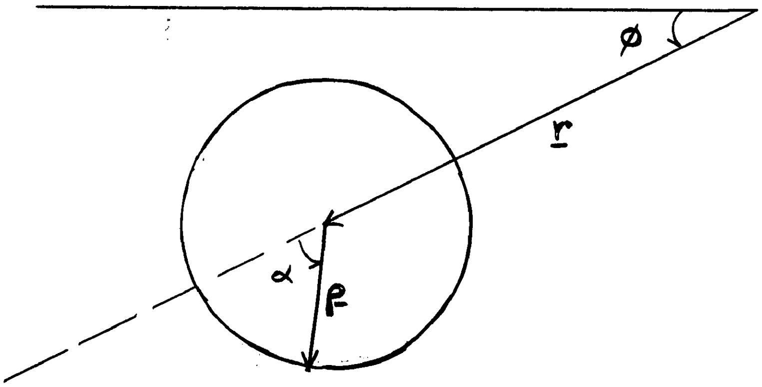
$$\underline{B}' = \underline{B} - \frac{1}{c^2}\left(\frac{\underline{E} \times \underline{B}}{B^2}\right) \times \underline{E} \cong \underline{B}$$

Thus the particle gyrates at the Larmor frequency $\frac{qB}{m}$ and drifts with a velocity $\frac{\underline{E} \times \underline{B}}{B^2}$.

Non-uniform Fields

In this case we will consider only inhomogeneous magnetic fields. This problem is best treated by a method due to Alfven (Alfven and Falthammer, 1963) termed the guiding center approximation. The guiding center is the center of gyration of a particle.

Let $\underline{r}$ denote the instantaneous location of the guiding center and ρ denote the location of the particle with reference to the guiding center as shown in the following two-dimensional sketch.



The equation of motion of the particle may be written thus

$$m \frac{d^2}{dt^2} (\underline{r} + \rho) = q \frac{d}{dt} (\underline{r} + \rho) \times \underline{B} (\underline{r} + \rho)$$

$$= q \frac{d}{dt} (\underline{r} + \rho) \times \left[\underline{B} (\underline{r}) + (\rho \cdot \nabla) \underline{B} (\underline{r}) \right]$$

to the first order of approximation.

In the absence of the inhomogeneity of the field, the equation reads $m \frac{d^2}{dt^2} \rho = q \frac{d\rho}{dt} \times \underline{B}\ (\underline{r})$.

The former equation may thus be treated as a perturbation of the latter equation, and the difference between the two yields the following equation relating the perturbations.

$$m \frac{d^2}{dt^2} \underline{r} = q \frac{dr}{dt} \times \underline{B}\ (\underline{r}) + q \frac{d}{dt}\rho \times (\underline{\rho}\cdot\nabla)\ B(\underline{r})$$

neglecting the terms of second order.

In the plane polar coordinate shown in the sketch, the perturbation equation may be written thus:

$$m \begin{bmatrix} \ddot{r} - r\dot{\phi}^2 \\ \\ r\ddot{\phi} + 2\dot{r}\dot{\phi} \end{bmatrix} = q \begin{bmatrix} r\dot{\phi}B - \rho^2 \dot{\alpha} \cos^2\alpha \ \frac{3B}{r} \\ \\ -\dot{r}B \end{bmatrix}$$

Averaging over one period of gyration and neglecting the acceleration towards the center of the coordinate system,

$$\begin{aligned} \left| r\dot{\phi} \right| &= \left| \frac{3}{2} \rho^2 \omega_c \frac{1}{r} \right| \\ &= \left| \frac{3\,\xi_\perp}{qBr} \right| \end{aligned}$$

For a positive particle $\dot{\phi} < 0$ and for a negative particle $\dot{\phi} > 0$. This is in agreement with the formula derived by Alfven $\underline{V}_D = \frac{\xi_\perp}{qB^3} \underline{B} \times \nabla B$.

When a particle is confined to move along a curved magnetic field line with a velocity V_{11} the drift velocity may be derived in a similar manner and leads to $\underline{V}_c = mV_{11}{}^2 \frac{\underline{B} \times \nabla B}{qB^3} \cdot \quad = 2 \, \xi_{11} \frac{\underline{B} \times \nabla B}{qB^3}$

The Field Due to a Diamagnetic Particle in a Dipole Magnetic Field

Let us consider a proton of energy ξ at a radial distance r in the equatorial plane.

The magnetic moment of the proton

$$\underline{\mu} = -\frac{\omega_c}{2\pi} q \pi \rho^2 \quad \underline{e}_\theta$$

$$= -\frac{\xi_1}{B} \quad \underline{e}_\theta$$

Total drift velocity due to curvature and gradient drifts $\underline{V}_D = -\frac{3}{qBr} (\xi_1 + 2\xi_{11})$. Hence the magnetic field produced at the dipole $\underline{dB} = -\frac{\mu_0}{4\pi r^3} \left[q \, (\underline{V}_D \times \underline{r}) + \frac{\xi_\perp}{B} \right]$, where $\underline{r}$ is the radius vector pointing from the origin to the charge.

$$dB = \frac{\mu_0}{4\pi r^3 B} \left[2 \, \xi_\perp + 6 \xi_{11} \right]$$

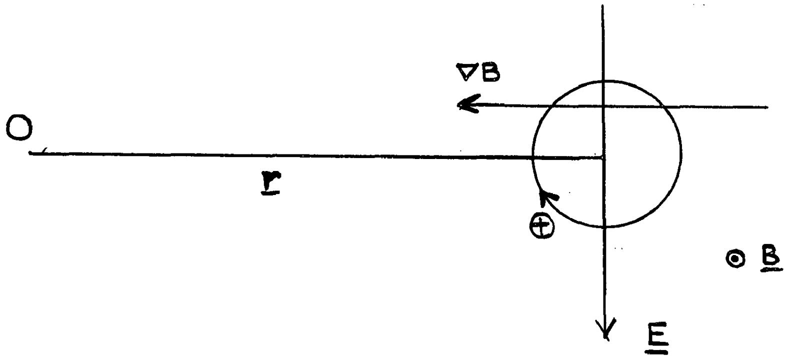
$$= \frac{2 \, \xi_\perp + 6 \xi_{11}}{M_E}$$

Thus for a given amount of energy available for injection into the ring-current system to produce a decrease in the magnetic field, an effect three times larger may be obtained if all the energy is supplied to motion parallel to the magnetic field than if the energy is supplied to gyration transverse to the magnetic field.

An Adiabatic Invariant of Motion

Consider a charged particle gyrating in a inhomogeneous magnetic field. The velocity of drift of the particle due to the gradient in the field is $\underline{V}_g = \frac{\xi}{qB^3} \underline{B} \times \nabla B$.

Consider an electric field $\underline{E}$ parallel to the direction of drift as in the sketch below. The rate of change of energy of the particle is then given by $\frac{d\xi}{dt} = q\underline{E} \cdot \underline{V}_g$.



The particle drifts toward the origin O with a velocity $\frac{\underline{E} \times \underline{B}}{B^2}$.

$$|\nabla B| = \frac{dB}{dr} \quad \text{also} \quad \frac{dr}{dt} = \frac{E}{B}$$

$$\frac{d\xi}{dt} = q\underline{E} \cdot \frac{\xi}{qB^3} \underline{B} \times \nabla B$$

Thus $\frac{d\xi}{dt} = \frac{E}{B} \cdot \frac{\nabla B}{B} \cdot \xi$ which reduces to $\frac{d\xi}{dB} = \frac{\xi}{B}$. This implies that $\frac{\xi}{B}$ = constant, and this is the dipole moment due to gyration. The above operation is valid only if the displacement of the guiding center during one period of gyration is small compared to the Larmor radius. The constancy of $\frac{\xi}{B}$ is often referred to as the First Adiabatic Invariant.

The following table gives the energies of particles of the solar wind trapped at arbitrary locations and then energized by an electric field:

Table V. Energy at $2R_E$ of Diamagnetic Plasma Particles Initially Trapped at Variable Distances

Initial Trapping Distance	Initial Energy	Energy at 2 R_E
50 R_E	10 eV	160 keV
	100 eV	1.6 MeV
	1 keV	16 MeV
40 R_E	10 eV	80 keV
	100 eV	800 keV
	1 keV	8 MeV
30 R_E	10 eV	30 keV
	100 eV	300 keV
	1 keV	3 MeV

Table V. (continued)

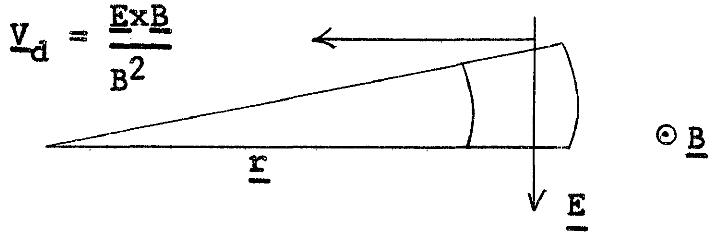
Initial Trapping Distance	Initial Energy	Energy at 2 R_E
20 R_E	10 eV	10 keV
	100 eV	100 keV
	1 keV	1 MeV
15 R_E	10 eV	4 keV
	100 eV	40 keV
	1 keV	400 keV
10 R_E	10 eV	1 keV
	100 eV	10 keV
	1 keV	100 keV
5 R_E	10 eV	100 eV
	100 eV	1 keV
	1 keV	10 keV

DEPENDENCE OF PARTICLE DENSITY DISTRIBUTION ON ELECTROMAGNETIC FIELD CONFIGURATION

Consider collisionless plasma particles in the equatorial plane of the earth's dipole field. Particles of the plasma gyrate around the magnetic field lines and drift due to the gradient and curvature of the dipole field, as considered previously. Since the plasma is collisionless, particle density distribution will remain the same as when the particles were first injected.

However, an electric field superimposed on the dipole field would alter the particle density distribution due to the $\underline{E} \times \underline{B}$ drift.

For simplicity let us consider plasma particles in the equatorial plane of the earth's dipole field. Let the number density of particles be n(r), assuming cylindrical symmetry. Consider a volume element dv enclosed by the surface S. Then $\frac{\partial}{\partial t} \left[n\,(r)\,dv \right] = - \oint\!\!\oint n\,(r)\,\underline{V}_d \cdot \underline{ds}$ where $\underline{V}_d$ is the velocity of drift due to the crossed $\underline{E}$ and $\underline{B}$ fields as shown in the two-dimensional sketch below.



Using Green's theorem, we have

$$\frac{\partial}{\partial t} \left[n\,(r)\, dV \right] = - \int_{dV} \nabla \cdot (n \underline{V}_d)\, dV$$

Hence in the steady state $\nabla \cdot \left[n\,(r)\, \underline{V}_d \right] = 0$

Assuming that $\underline{E} = \frac{E_o}{r^m}\, \underline{e}_\phi$, and since $\underline{B} = -\frac{\mu_o}{4\pi} \frac{M_E}{r^3}\, \underline{e}_\theta$

$$\underline{V}_d = - \frac{4\pi E_o}{\mu_o M_E}\, r^{3-m}\, \underline{e}_r$$

Hence, in the steady state

$$\frac{1}{r^2} \frac{\partial}{\partial} \left[r^2\, n(r)\, \frac{4\pi E_o}{\mu_o M_E}\, r^{3-m} \right] = 0$$

$$\therefore \quad n(r) = n_o \frac{1}{r^{5-m}} .$$

Thus, if $\underline{E}$ has a $\frac{1}{r}$ dependence $N\,(r) = \frac{n_o}{r^4}$.

However, it may be more appropriate to use cylindrical symmetry rather than spherical symmetry in the equatorial plane of the dipole. In which case,

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r\, n(r)\, \frac{4\pi E_o}{\mu_o M_E}\, r^{\,3-m} \right] = 0 ,$$

and $n(r) = \frac{n_o}{r^{4-m}}$.

Thus if the electric field is not strongly dependent on r, the plasma density is given $n(r) = \frac{n_o}{r^4}$, or when the electric field falls off as $\frac{1}{r}$, the density is given by $n(r) = \frac{n_o}{r^3}$.

Carpenter and Smith (1964) report that in the range from 2 - 5 R_E, the plasma density falls in the range $n(r) = r^{-3}$ to r^{-4}. They also observe that the diffusive equilibrium distribution should be $n(r) = r^{-1.5}$, whereas in fact their measurements reveal a distribution $n(r) = r^{-3.5}$, at least in the range 2 - 4 R_E.

Thus this analysis may be used in support of the field configuration sketched in Fig. 2.

Drift Time and its Relation to the Initial Phase of Storms

The drift velocity for the field configuration assumed in the previous section is

$$\underline{V}_d = - \frac{4\ E_o}{M_E}\ r^{3-m}\ \underline{e}_r$$

Since the experimental evidence indicates that the outward expansion of the solar corona (the solar wind) confines the geomagnetic field in a cavity having a cross-section normal to the earth-sun line of radius $10R_E$, it is reasonable to assume that the bundles for which the radius of the stationary orbit is $10R_E$ or less yield their energy to the magnetosphere. Thus a diamagnetic particle initially at a geocentric distance of $10R_E$ will drift inward and gain energy as long as the influx of bundles continues.

The time taken for a plasma particle to drift inwards from $10R_E$ to $2R_E$, T, is given by the following

$$T = -\int_{10R_E}^{2R_E} \frac{\mu_0 M_E dr}{4\pi E_0 \, r^{3-m}} = \frac{\mu_0 M_E}{4\pi E_0} \left[\frac{1}{(2-m)} \, \frac{1}{r^{2-m}} \right]_{10R_E}^{2R_E}$$

Using $E = 10^{-2}$ V/m (Cole, 1964)
$R = 6.4 \times 10^6$m,
for $m = 1$, $T = 10^{11}$ sec.
for $m = 0$, $T = 3 \times 10^3$ sec.

Thus the existence of an average field of 10^{-2} V/m due to the penetration of charged bundles is capable of providing a process for energizing trapped radiation, as well as of redistributing the particles from a r^{-3} to a r^{-4} dependence.

The time of 3×10^3 sec. is approximately the same as the average duration of a storm. Thus this mechanism is in agreement with the observed delay between the beginning of the initial phase and that of the main phase. The main phase decrease would then be expected to continue as long as energy is supplied in excess of loss due to various removal mechanisms such as the aurora. The time of duration of solar events varies from 10^3 to 10^5 sec. Thus the development of the main phase of a storm could depend on the duration of the solar event responsible for the storm. The author is not aware of any correlations made between the duration of storms and the duration of the corresponding solar event. Speculations on the length of duration of a storm has been centered on the possibility that the solar plasma emitted at a flare has a spread of velocity, but this possibility has no foundation. The solar-wind energy has been found to have a very narrow spread of energy (Hess, Mead and Nakada, 1965). The proposed mechanism will receive more support if some method of investigation reveals that the activity at the base of the solar corona or the chromosphere continues even after the flare subsides.

The microscopic nature of the electric field assumed to be present in the magnetosphere is discussed in the following section.

Microscopic Structure of the Electric Field

The bundles penetrating into the region of trapped radiation transfer their energy to the ambient plasma through Coulomb interactions. This transfer decreases the magnetic rigidity of the bundles and traps the bundles. As shown in fig. 2 charge build-up takes place. This build-up is neutralized by the ambient plasma, if the plasma particles are free to move. However, in the presence of a magnetic field normal to the electric field the ambient plasma drifts in the direction given by $\underline{E} \times \underline{B}$.

If the potential difference maintained across the region A in fig. 2 is electrostatic then the particles cannot gain energy in excess of the potential difference. However, in this instance it is convenient to separate the mechanisms into two parts. The average electric field is thought to be responsible for the pumping of the plasma, and the individual collisions to be responsible for the energizing of the plasma. This electric field may be the same as the field inferred from observations of the time variation of magnetic field at geocentric distances of 3-4 earth radii and more. Thus the energies of solar wind particles shown in Table 5 are reasonable.

APPLICATION TO THE GEOMAGNETIC FIELD VARIATIONS

The proposed mechanism illustrates the manner in which energy of the solar wind is injected into the trapping region via the medium of bundles. This mechanism is examined in some detail in the following.

Energy Requirements

As shown earlier, the change in magnetic field is related to the change in energy of the trapped diamagnetic plasma thus: $\Delta\xi = -fM_E\Delta B$ where $1/f$ has a value of 2 to 6. Assuming thermal equilibrium of the trapped plasma, it may be shown that $2\xi_{\parallel} = \xi_{\perp}$ as the transverse motion has two degrees of freedom. Hence a reasonable average value for f is 3/10.

For a typical storm where the main phase decrease is $100\gamma\ (10^{-7}\ W/m^2)$

$$\Delta\xi = -\frac{3}{10} \times 8.1 \times 10^{22}\ (-10^{-7})\ \text{joules}$$
$$\cong 2.4 \times 10^{15}\ \text{joules}$$

Most of the kinetic energy of the solar wind is carried by protons because of their relatively larger mass, and hence the number of protons needed to carry the required amount of energy into the trapping region $\simeq \frac{2.4 \times 10^{15}}{\frac{1}{2} \times 1.67 \times 10^{-27} \times 10^{12}} \simeq 3 \times 10^{30}$, where the solar wind velocity is assumed to be 10^6 meters/sec. Cole (1964) has estimated that there are 10^{31} to 10^{32} proton-electron pairs within 4 earth radii of the earth's center. Thus the number of particles need to cause a main phase decrease of 100 γ ranges from 1/3 to 1/30 of the number of particles in the radiation belts within a geocentric distance of 4 earth radii. Thus in addition to supplying the energy, this mechanism is also capable of supplying particles to replenish the loss due to aurora.

If N is the number of bundles per cubic meter and if a bundle has n protons, the energy supplied per square meter cross-section is $\frac{1}{2} m_p N n v^2$ joules/sec, where m_p is the mass of a proton and v the velocity of a bundle.

If we assume that the bundles in a section of radius $10R_E$ transfer their energy effectively to the diamagnetic plasma effectively then the energy supplied per second to the trapped radiation is $\frac{1}{2} m_p V^2 N n \pi (10R_E)^2 v$. This is approximately equal to $Nn10^7$ joules/sec. where $v = 10^6$ m/sec.

If the energy is supplied in a period of say 5 x 10 seconds, the average duration of the main phase, then,

$$5Nn10^{11} = 2.4 \times 10^{15}$$

or

$$Nn = 5 \times 10^{3}$$

Thus the number of proton-electron pairs that need to exhibit bundle characteristics is 5×10^3 pairs/cubic meter. Satellite measurements indicate an average particle density of $10 - 50 \times 10^6$ (Bernstein, 1964). Thus if as little as 1 in every 1000 particles exhibits bundle characteristics, the required energy may be supplied to the trapping region.

The Debye Hückel radius in the chromosphere and corona is of the order of 10^{-6}m. If the size of a bundle does not alter very much (not more than a factor of 10^3) the electrostatic analyzers that are used presently would detect them as a single particle of very high energy. The energy of a proton moving with a velocity of 10^6 m/sec is approximately 5 keV. A bundle of 10^3 particles with an excess of 1 electronic charge would have an energy-to-charge ratio of 5 MeV/electronic charge. A unit of this type would not have been detected by the electrostatic analyzers used in satellites. For example, in the electrostatic analyzer used in Imp 1 (Wolfe, Silva and Meyers, 1966) the range was 0.025 - 16 keV/electronic charge. The slit opening was 0.25 cm. The range of flux measurable with the instrument was 3×10^5 - 1×10^{10} ions/cm^2/sec, and the flux of bundles would not be detected for the reason that the flux is low.

Thus the facts (1) that the bulk of the solar wind is stopped at the magnetosphere-solar wind surface by the balance in wind pressure and the magnetic field pressure; and (2) that the plasma probes did not detect plasma flowing into the trapping region across the magnetosphere interface do not contradict the theory.

The Ring Current

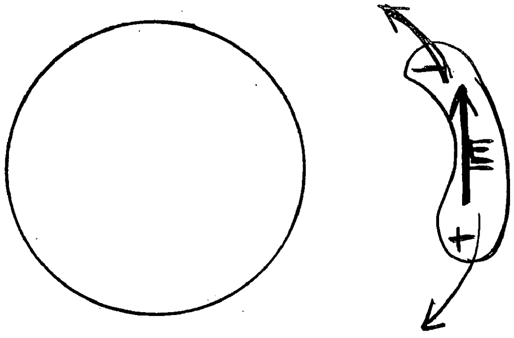
Singer proposed a model of a diamagnetic ring current to account for the main phase decrease of a storm (Apel, Singer and Wentworth, 1962). He also put forth a proposal, based on the observations of Winckler and Kellogg that the cosmic ray cut-off energies are affected at the time of the sudden commencement, that the ring current starts building up just after the sudden commencement.

However, Singer and his associates did not consider a mechanism by which the ring current particles are injected into the trapping regions. Chapman has pointed out that there is no necessary feeding of kinetic energy into the magnetosphere caused by the compression of the magnetic field in continuing and unsteady plasma flow (Petschek, 1964). It is observed that some storms are characterized by the absence of a main phase even when an appreciable initial phase is detected. Some storms, however, have a very negligible initial phase, although the main phase is large. The duration of the initial phase varies from about 8 min. to almost 10 hours. Thus it is reasonable to deduce that the formation of the ring current is not due to the compression of the magnetosphere caused by increased solar wind pressure but is due to an independent mechanism that supplies energy to the magnetospheric cavity. The mechanism that supplies the energy should also be capable of injecting particles to replenish the loss due to aurora. It has been estimated that the radiation belts would be drained away in a matter of seconds by a strong aurora (Cole, 1965). The bundles are capable of providing the necessary particles as well as the energy for the storm and aurora.

On this model, particle injection takes place through the entire trapping region, and the increase in particle flux is approximately 3%. This figure is deduced by noting that the number of particles injected is 1/30 of the number of particles in the radiation belts within a geocentric distance of about 4 earth radii. Here the upper value of the estimate is used. Thus it is reasonable to assume that the ring current resulting from this model is not a localized ring current visualized by Singer, but one that is merely an increase of the total particle flux in the trapping region.

This conclusion is in agreement with the unsuccessful attempt to locate the ring current by satellite experiments (Hess, Mead and Nakada, 1965).

The trapped plasma particles drift due to the curvature of the field as well as due to the gradient. The positive particles drift westward and the negative particles eastward. This drift motion will be unhindered if the drift does not produce a charge imbalance as in a homogeneous diamagnetic toroid considered by Singer. However, the formation of the ring current on the bundle model does not produce the necessary homogeneity as shown in the sketch below.



The electric field produced by the charge separation will interfere with the drift motion. If we assume that the drift is mainly controlled by the protons and that the protons drag the lighter electrons with them, the resultant motion will not contribute to a current. But the field will be increased due to the diamagnetic effect of the particles. When uniformity of particle density is established then the drift of particles can take place and cause the main-phase decrease. This may be the reason why the main phase decrease is delayed although the particles forming the ring current start their build-up soon after the sudden commencement. The time of drift of a proton of 100 keV energy at a geocentric distance of 3 earth radii is approximately 2 hours. If the particle has less energy, then the drift time will be more. Thus the effect of the ring current would not be manifest until the protons had sufficient time to create a uniform population of plasma in a ring around the earth. The time involved is again in rough agreement with the average duration of the initial phase. Thus one may conclude that the motion of the magnetospheric boundary, although contributing to the initial phase, need not be the only mechanism capable of producing the initial phase.

The inhomogeneity introduced in the trapping region by the influx of plasma in the initial stages of the formation of the ring current will also produce electromagnetic disturbances which would propagate as hydromagnetic waves to the lower levels of trapped radiation, where their effect would be observed as rapid variations of the geomagnetic field. The following section deals with the frequencies that are most likely to be observed assuming stationary wave configurations.

Hydromagnetic Waves and Plasma Oscillations in the Trapped Diamagnetic Plasma

The results derived by Chandrasekhar (1962) for the propagation of electromagnetic disturbances in plasmas are summarized as follows:

In a neutral plasma uninfluenced by external fields, transverse oscillations propagate with a group velocity given by $c/\left[1+\left(\frac{\omega_p}{kc}\right)^2\right]^{\frac{1}{2}}$, and longitudinal oscillations do not propagate but have a frequency given by $\omega_p=\left(\frac{nq^2}{m}\right)^{\frac{1}{2}}$ called the plasma frequency. k is the wave number, c the velocity of light in vacuum, n the number of density of particles of one kind, q the electronic charge, and m the mass of an electron. The disturbances of frequencies less than the plasma frequency for the transverse mode are totally reflected and do not propagate in the plasma.

In the presence of an external magnetic field B_0 the disturbances propagate with a velocity given by $B_0/(\mu_o\rho)^{\frac{1}{2}}$. This velocity is called the Alfven velocity. The frequencies of these waves are much smaller than the cyclotron frequencies of the proton and electron in the external magnetic field.

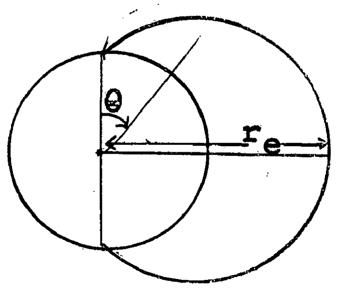
If the external magnetic field has a gradient as well as curvature, as is the case with the dipole field of the earth, the velocity of propagation transverse to the magnetic field is $(2P/\rho)^{\frac{1}{2}}$ and that along the magnetic field is $(P'/\rho)^{\frac{1}{2}}$, where ρ is the matter density of plasma

$$P = \frac{B_o^2}{2\mu} + \xi_\perp$$

$$P' = \frac{B_o^2}{\mu_o} + \frac{\xi_\perp}{2} - \xi_\parallel$$

Thus in the regions where equilibrium conditions are disturbed, hydromagnetic waves would be generated. The following section gives the various frequencies that are most likely to be the strongest.

We will consider a typical case and illustrate the problem. Consider the magnetic-field line which cuts the equatorial plane at a geocentric distance of $2R_E$ as shown in the accompanying sketch.



Using the notation in the sketch, the length of the field line between the magnetically conjugate points on the earth i[illegible] follows:

$$S = 2\int_{\sin^{-1}\left(\frac{R_E}{r_e}\right)^{\frac{1}{2}}}^{\pi/2} {}^{s'2}\sin\theta\,(1+3\cos^2\theta)^{\frac{1}{2}}\,d\theta$$

$$= r_e\left[\sqrt{1-\frac{R_E}{r_e}}\sqrt{4-\frac{3R_E}{r_e}} + \frac{1}{\sqrt{3}}\ln\left\{\sqrt{3}\sqrt{1-\frac{R_E}{r_e}} + \sqrt{4-\frac{3R_E}{r_e}}\right\}\right]$$

for $r_e = 2\ R_E$, $S = 2.8\ R_E$

$$S \equiv \underline{2 \times 10^7}\ \mathrm{m}$$

The disturbances produced in the plasma surrounding this field line will propagate along and transverse to the field line. Here we consider only the mode propagating along the field line. The velocity of propagation, assuming that the magnetic field is that due to the earth's dipole and the plasma energy density is small in comparison to the field energy density, is 10^6 m/sec.

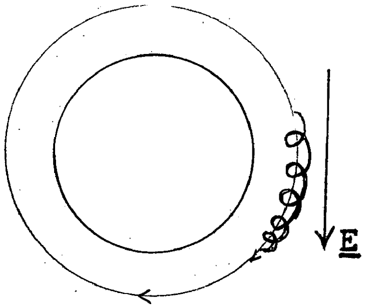
For the standing waves the frequencies are then given by the equation $f \frac{4 \times 10^7}{n} = 10^6$. These frequencies form a harmonic series. However, the actual problem is more complicated than this. The rapid fluctuations of the geomagnetic field would be given by the vector sum of the disturbances originating all over the trapping region and beyond. The above considerations are an attempt to correlate the frequencies of micropulsations with the frequencies of standing waves, to point out the possibility that the micropulsations may have an origin in the trapping zone instead of at the magnetospheric boundary as generally believed. The period for the fundamental mode considered above is 40 sec.

The Presence of Low-Energy Particles in the Radiation Belts

The mechanism by which the large number of particles found in the magnetosphere is generated is still an unsolved problem. The existing theories based on the Chapman-Ferraro model of the solar wind interaction require that the solar wind plasma be specularly reflected at the magnetospheric boundary. The speculation centers around the possibility that the plasma may at certain times be injected along the magnetic field lines which intersect the magnetosphere boundary at the neutral points shown in Fig. 4.

The particles of high energy (protons of energy greater than about 50 MeV and electrons of energy greater than about 100 keV) are adequately explained on the basis of the neutron-albedo theory. However, the energetic components of the trapped radiation form a minor part of the total number of trapped particles. These low-energy constituents, namely the low-energy protons of the inner zone and practically all the electrons of the outer zone, have not been associated with any known mechanism. The presence of low-energy protons in the inner zone may be due to injection by a bundle mechanism. This possibility is seen from Table 5, where the energies of particles at $2R_E$ are given as a function of the distance at which the bundles were initially trapped. This mechanism is similar to the one that Dungey postulated whereby particles of the solar wind may be energized by a breakdown of the third adiabatic invariant but not by the first two. However, Dungey did not consider a process by which the energy may be transferred (Hess, Mead and Nakada, 1965). The bundles provide a physical process by which the energy may be transferred, and Dungey's calculations which are in agreement with the proton spectrum measured by Explorer 12 would be applicable. The following section deals with the reason why the particles of thermal energies shift closer to the earth at night than during the day.

Consider the sketch shown below where the path of the center of gyration of a particle in the equatorial plane of an undistorted dipole is shown.



In the absence of perturbing fields, the drift path would be symmetric to the dipole axis. However, the bundle theory leads to the presence of a quasi-stationary electric field as shown in the sketch. The electric field thus represented is the time average of the fields due to the bundles and any other causes such as the charge accumulation at the boundary of the magnetosphere due to viscous interaction. This electric field would cause an $\underline{E} \times \underline{B}$ drift of the center of gyration. The drift of the guiding center in the undistorted dipole field may be represented as the gyration of a charged particle in a fictitous magnetic field of strength $B* = \frac{mv}{qr}$. The electric drift velocity would then be equal to $\frac{Eqr}{mv}$. Thus particles of energy much greater than $3/4 \ \frac{qB^2}{B\nabla B} \left(\frac{qEr}{mv}\right)^{\frac{1}{2}}$ would not be appreciably affected by the presence of the electric field. Particles of energy equal to or less than this amount would move further out during the day, and some may even be lost from the trapping region. Such outward flow of trapped particles has been detected (Dungey, 1966).

CONCLUSIONS

The problem of accounting for the particles of the radiation belts, aurora, and the main phase of magnetic storms has not been solved satisfactorily. The present theory leads to the following conclusions:

1. Energy of the solar plasma in sufficient amounts may be injected into the trapping region directly. This would account for the main phase of magnetic storms and also for the duration of the initial phase. The possibility of accounting for the initial phase is by no means slight.

2. Particles in sufficient numbers to account for the replenishment of the radiation belts are also injected into the trapping region. The auroral mechanisms are known to be capable of draining all the particles in the radiation belts in a short time. The proposed mechanism provides a continuous source of particles and energy to provide a source for auroras.

3. The theory presented here satisfactorily accounts

for the presence of low-energy particles in the inner regions of the radiation belts.

4. The possibility of micropulsations originating in the radiation belts is shown.

5. The drift of thermal particles closer to the earth on the night side (in agreement with observations) is shown to result from the presence of an electric field in the magnetosphere.

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