RESISTOR NETWORK MODELING IN ELECTRIC DIPOLE MAPPING

by

Augusto Cesar Bittencourt Pires

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Signed: Augusto Cesar B. Pires
Augusto Cesar B. Pires

Golden, Colorado

Date: April 11, 1975

Approved: George V. Keller
George V. Keller
Thesis Advisor

George V. Keller
George V. Keller
Head of Department

Golden, Colorado

Date: April 11, 1975

ABSTRACT

A resistor network is a powerful tool in modeling geoelectrical discontinuities on a field generated by an electric dipole source. In this paper a series of computer programs is developed to evaluate the potential field, total electric field, and apparent conductance for an earth model consisting of a thin conductive layer over a resistive basement.

Two different sets of models were studied. The first set includes a dike (1 unit wide and infinitely long), a dike (1 unit wide and 11 units long), a contact, and square discontinuities. For the first set a current of 100 amps was used. The second set of models includes faults and dikes parallel and normal to the source. For the second set a current of 1 amp was used.

From the comparison between results obtained analytically and results obtained with the resistor network we observed a good agreement between them except for the case of a conductive dike parallel to the source and a resistant dike normal to the source.

From the interpretation of field results obtained on the Black Rock Desert area of northwestern Nevada depths to the basement of 1000 m and 150 m were obtained for the Black Rock Desert and the southern portion of the Hualapai Flat, respectively. The results also lead to conclusions on

changes in depth and/or resistivity in the area.

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INTRODUCTION

Generally in direct current methods we are interested in the potential, or the electrical field in the earth due to a current driven into the ground through one or more galvanic contacts.

In the d.c. technique known as dipole mapping current is sent into the earth through an electric bipole and voltages are measured using two orthogonal dipoles. From these measurements, the total electrical field, apparent resistivity or, in some cases, apparent conductance can be obtained on the surface of the earth in all directions away from the dipole source (Keller, 1966; and Furgerson and Keller, 1974).

Analytical solutions for a layered earth and other simple geometries are well discussed in the literature (Keller, 1966; Bibby and Risk, 1973; Lee, 1973; and Furgerson and Keller, 1974).

When the geometries being modeled are not simple numerical techniques should be applied. In modeling an earth comprised of a thin conductive layer over a resistive basement we have used here a finite resistor network as an analog to the conductive layer and assumed that in our earth model all the current driven into the ground flows only in this top layer. A resistor network does not give results as precise as an analytical solution but, on the other hand, is far more powerful concept for modeling complicated geoelectrical discontinuities.

In this report, a bipole current source is simulated in the center of the resistor network and the potential field is obtained at all mesh nodes. The total electric field is obtained by calculating numerically the gradient of the potential field throughout the entire network. From the total electric field distributions we obtained apparent conductance maps using an equation presented by Furgerson and Keller (1974).

Potential field, total electric field, and apparent conductance maps were obtained for two different sets of models. The first set includes a dike one unit wide and infinitely long (infinite dike), a dike one unit wide and eleven units long (finite dike), a vertical contact, and a square discontinuity symmetric and asymmetric to the source. A second set of models was studied in order to compare apparent conductance maps obtained with the resistor network technique and maps obtained analytically by Furgerson and Keller (1974). This set of models includes a resistant dike, a conductive dike and faults.

As an example, the resistor network procedure is used in the interpretation of dipole mapping surveys carried out in the Black Rock Desert area in the State of Nevada. Here the apparent conductance maps obtained from two different sources are compared to maps obtained by the numerical procedure.

REVIEW OF THE LITERATURE

The use of resistor networks for the solution of Poisson's equation or Laplace's equation is not new in the scientific literature. The first successful use of a resistor network was made by DePack (1947) when the solution of Laplace's equation was obtained through a finite difference scheme. In 1948, Redshaw published a report on an electrical potential analyser by means of which solutions of Poisson's equation were obtained.

Liebman (1950, 1953, 1954a, 1954b, 1955a, 1955b, 1956, and Liebman and Bailey, 1954) is credited with placing the technique on a firm scientific basis and leading it into the highly accurate level that it represents today. After his first work in the solution of partial differential equations, he studied the accuracy problems on a network analog using unequal or subdivided meshes. In the solution of field problems through the use of a network, he applied the technique to the study of plane stress problems, heat conduction problems, and elastic vibration problems.

In 1958, Karplus published a book titled Analog Simulation, where the use of resistor networks is discussed in detail. His book not only deals with resistor networks but also finite-difference techniques, plate analogs, and non-passive networks, as well.

After the basic work of Liebman, papers followed dealing with field problems on well-logging (Guyod, 1955), accuracy on a network (Landau, 1957), and modeling of electrical measurements in porous media (Rink and Shopper, 1968; and Greenberg and Brace, 1969).

In more recent years Jellito and Borm (Jellito, 1969; Jellito and Borm, 1970a, 1970b; Borm and Jellito, 1972; and Borm, 1972, 1973) developed the theory of a network analog for geoelectrical measurements. They modeled a layered earth, in two and three dimensions, and obtained apparent resistivity sounding curves for the Schlumberger array for the case of two and three layers. Their work also includes the modeling of layered earth and hydrology problems using conductive plate analogs. The solution for the potential field on the analog is obtained by finding the Green's function of a current source for the potential in the conductive plate, then, the potential distribution induced by the known initial current density is obtained as a superposition by summing the current density function over the plate.

THE GENERAL RESISTOR NETWORK SOLUTION

The geoelectrical response of a field system which cannot be analysed by an analytical procedure can often be determined or at least outlined through an electric analog.

In most of the works published to this date on the use of a resistor network for the solution of field problems the authors were attempting to solve Poisson's and Laplace's equations by the numerical technique of finite difference method (DePack, 1947; Liebman, 1950, 1953; Guyod, 1955; Landau, 1957; and Karplus, 1958). In this technique the differential equation in question is replaced by its corresponding difference equation.

Consider, for example, Laplace's equation:

$$\text{div grad } U = 0 \tag{1}$$

where U is an unknown scalar function of the space cartesian coordinates x and y (assuming a two dimensional cartesian problem).

Before the difference equations can be written a discretized coordinate grid must be superimposed upon the field and the nodes on the grid given consistent numerical coordinates (Fig. 1).

Using the forward- and backward-difference approxi-

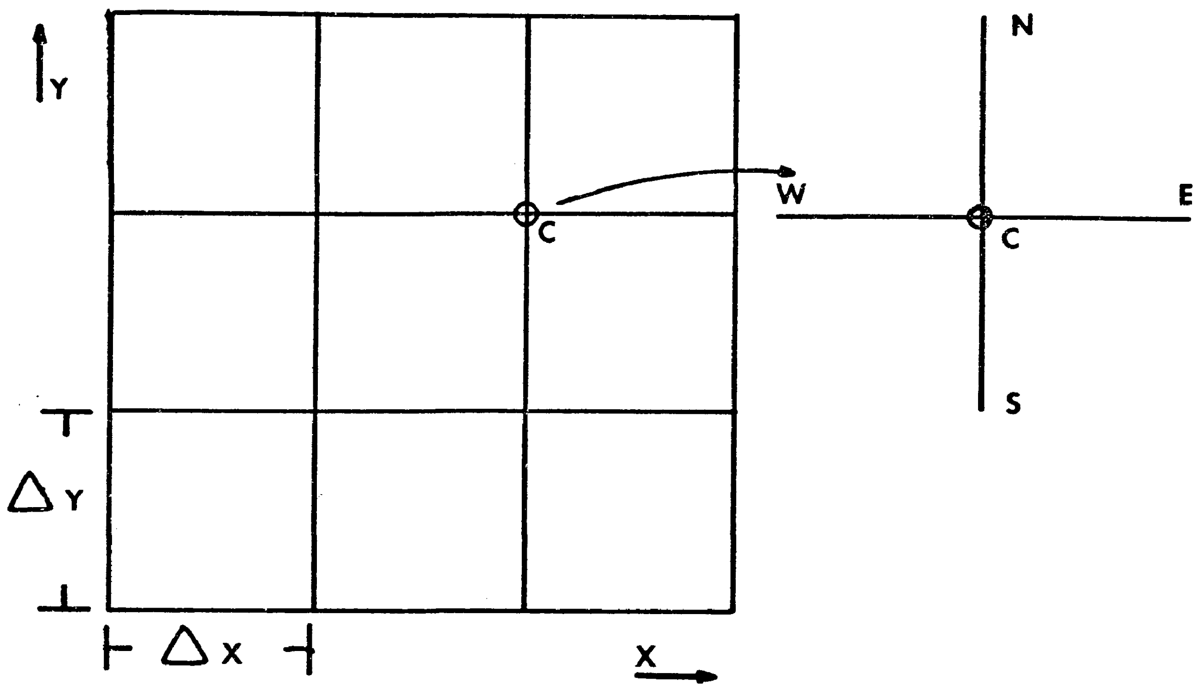


Figure 1. Two-dimensional field with finite difference grid and typical mode cross.

mations for first derivatives (Hildebrand, 1974), we can write:

$$\left(\frac{\partial U}{\partial x}\right)_{C-E} \cong \frac{U_E - U_C}{\Delta x} \tag{2}$$

$$\left(\frac{\partial U}{\partial x}\right)_{W-C} \cong \frac{U_C - U_W}{\Delta x}$$

$$\left(\frac{\partial U}{\partial y}\right)_{C-N} \cong \frac{U_N - U_C}{\Delta y}$$

$$\left(\frac{\partial U}{\partial y}\right)_{S-C} \cong \frac{U_C - U_S}{\Delta y}$$

For the second derivatives we have (Hildebrand, 1974)

$$\left(\frac{\partial^2 U}{\partial x^2}\right)_c \cong \frac{1}{\Delta x^2}\left(U_E + U_W - 2U_c\right)$$
$$\left(\frac{\partial^2 U}{\partial y^2}\right)_c \cong \frac{1}{\Delta y^2}\left(U_N + U_S - 2U_c\right) \tag{3}$$

Therefore, the two dimensional Laplace's equation will become

$$\nabla^2 U \cong \frac{1}{\Delta x^2}\left(U_E + U_W - 2U_c\right) + \frac{1}{\Delta y^2}\left(U_N + U_S - 2U_c\right) = 0 \tag{4}$$

if we make $\Delta x = \Delta y = h$ we have:

$$\nabla^2 U \cong \frac{U_N + U_S + U_E + U_W - 4U_c}{h^2} = 0$$

or

$$\Delta^2 U \cong \frac{U_N - U_c}{h^2} + \frac{U_S - U_c}{h^2} + \frac{U_E - U_c}{h^2} \quad \frac{U_W - U_c}{h^2} = 0 \tag{5}$$

Now, let us consider a node on the resistor network (Fig. 2).

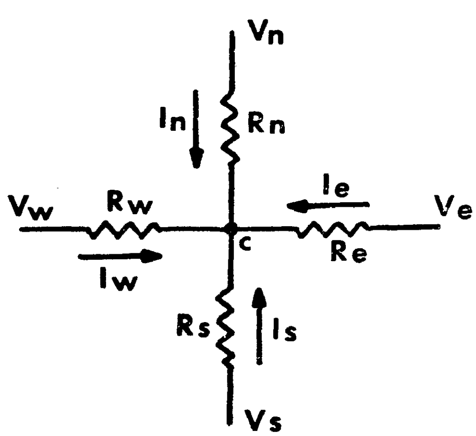


Figure 2. Typical node on the resistor network.

Using Kirchoff's current law we can write

$$i_N + i_S + i_E + i_W = 0 \tag{6}$$

or

$$\frac{V_N - V_C}{R_N} + \frac{V_S - V_C}{R_S} + \frac{V_E - V_C}{R_E} + \frac{V_W - V_C}{R_W} = 0 \tag{7}$$

By comparison of equations 5 and 7 we see that potential U and voltage V are analogous quantities provided that the resistors are proportional to h^2. This analogy allows several problems to be reduced to the finite difference algorithm.

Another technique that can be applied as a modeling tool for geoelectrical problems is the finite element method. This method is well established and has been described in detail by Zienkiewicz (1971). Applications to electrical and electromagnetic problems are presented in a paper by Coggon (1971).

The finite element method relies on some variational principle valid all over the region in consideration. For example, in electromagnetic scattering problems use is made of Hamilton's principle: Electromagnetic fields behave in such a way that total energy is minimized.

The main difference between the finite difference and the finite element techniques is that while the first approaches the solution of the differential equation directly by approximating to those in a discrete manner the finite element method deals directly with an approximate minimization of a functional.

Following an outline by Zienkiewicz (1971) let us derive the solution of two-dimensional Poisson's equation by the finite element method.

Suppose that the equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + C = 0 \tag{8}$$

is valid in a region V and subject to the condition

$$U = U_b \tag{9}$$

at the boundary C (Fig. 3a).

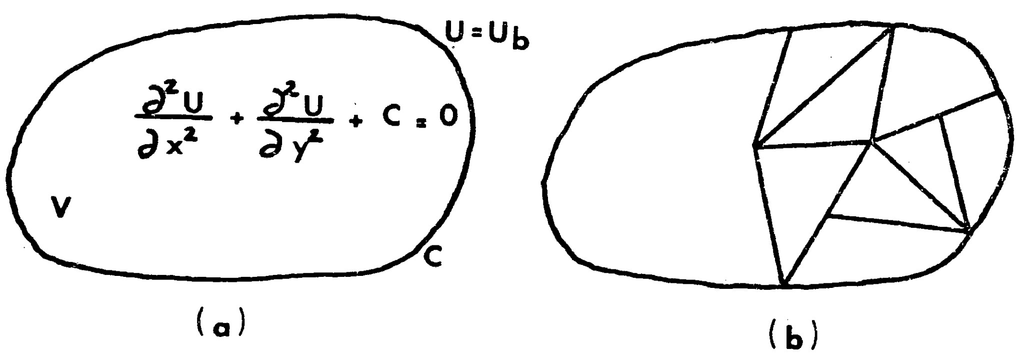


Figure 3. a) Region where Poisson's equation is valid.
b) Finite elements on region V.

Using calculus of variations we can write an equivalent expression for equation 8 (Hildebrand, 1965)

$$Z = \iint \left[\frac{1}{2}\left(\frac{\partial U}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial U}{\partial y}\right)^2 + CU \right] dx\, dy \tag{10}$$

The solution of Poisson's equation is the same function U that satisfies the boundary condition and minimizes equation 10.

If we break the region V into finite elements (Fig. 3b), then in each element we have:

$$U = \left[N_i , N_j , \cdots \right] \begin{Bmatrix} U_i \\ U_j \\ \vdots \end{Bmatrix} = \left[N \right] \left\{ U \right\}^e \qquad (11)$$

where $\{U\}^e$ represents the values of U at the element nodes and $[N]$ list the coordinates of the nodes only.

Assuming that $[N]$ is specified in such a way that U is continuous between elements, we can place our attention on a single element.

Substituting (11) into (10) and integrating over an element area we obtain:

$$\frac{\partial Z^e}{\partial U_i} = \iint_{V^e} \left[\frac{\partial U}{\partial x} \frac{\partial}{\partial U_i} \left(\frac{\partial U}{\partial x} \right) + \frac{\partial U}{\partial y} \frac{\partial}{\partial U_i} \left(\frac{\partial U}{\partial y} \right) - C \frac{\partial U}{\partial U_i} \right] dxdy$$

$$= \iint_{V^e} \left[\left(\frac{\partial N_i}{\partial x} U_i + \frac{\partial N_j}{\partial x} U_j + \cdots \right) \frac{\partial N_i}{\partial x} + \right.$$

$$\left. + \left(\frac{\partial N_i}{\partial y} U_i + \frac{\partial N_j}{\partial y} U_j + \cdots \right) \frac{\partial N_i}{\partial y} - C N_i \right] dxdy \qquad (12)$$

or

$$\frac{\partial Z^e}{\partial \{U\}^e} = \left[k \right]^e \left\{ U \right\}^e + \left\{ F \right\}^e \qquad (13)$$

where
$$k_{ij} = \iint_{V^e} \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) dx\,dy$$

$$F_i = -\iint_{V^e} C N_i \, dx\,dy$$

The minimizing set of equations will be

$$\frac{\partial z}{\partial\{U\}} = [k]\{U\} + \{F\} \qquad (14)$$

where

$$[k] = \sum [k]^e$$
$$\{F\} = \sum \{F\}^e$$

The substitution of boundary conditions and evaluation of the system of linear equations will give the solution.

If we consider the finite elements as made up of interconnected resistors we can write by making use of Ohm's law (Fig. 4):

$$I_i = \frac{1}{R^e}(V_i - V_j)$$
$$I_j = \frac{1}{R^e}(V_j - V_i) \qquad (15)$$

or in matrix notation

$$\begin{Bmatrix} I_i \\ I_j \end{Bmatrix} = \frac{1}{R^e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} V_i \\ V_j \end{Bmatrix}$$

or

$$\{I\}^e = [k]^e \{V\}^e \qquad (16)$$

In order to obtain the result for a complete network it is assumed that the potential is continuous at nodes and

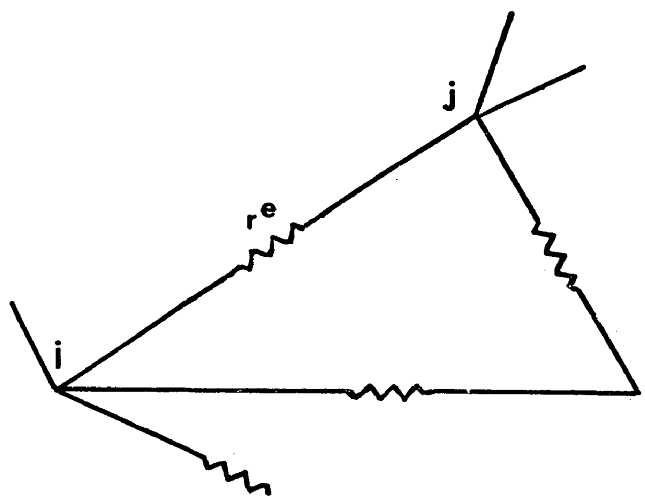


Figure 4. A finite element made up of resistors.

a current balance is imposed there. If an external input of current P_i is present we can write

$$P_i = \sum_{m=1}^{n} \sum k^e_{im} V_m \qquad (17)$$

where the second summation is over all elements and for all nodes.

Rewriting (17) in matrix form we have

$$\{P\} = [K]\{V\} \qquad (18)$$

where

$$k_{ij} = \sum k^e_{ij}$$

By comparison of equations 11, 16 and 18 we see that resistor networks used for modeling of geoelectrical problems can be solved by the finite element method.

In this report, we are concerned with the simulation of a specific field problem. This problem can be stated as such: given a bipole current source find the total electric field everywhere on the surface layer. As was mentioned before the earth model is assumed to be a thin conductive layer over a resistive basement. Assuming that current will flow only in the surface layer we can replace it by a network of resistors as in figure 5.

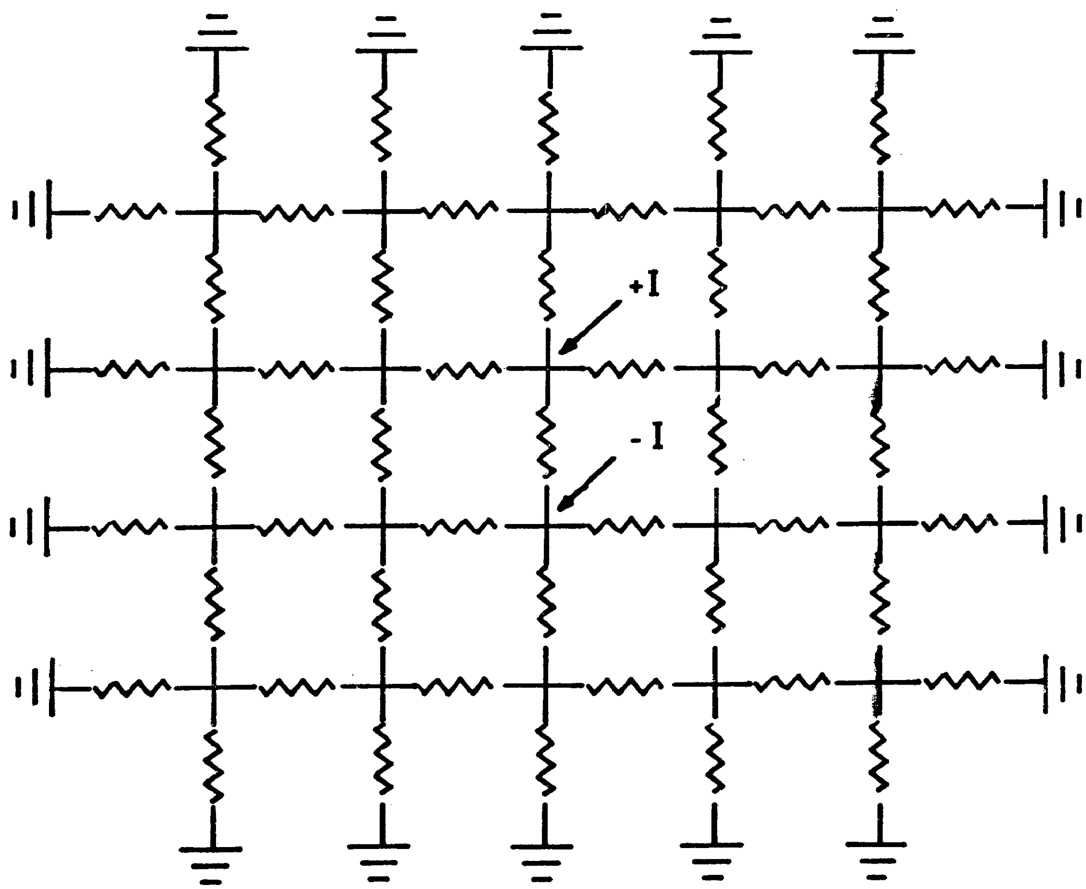


Figure 5. Resistor network analogue to a thin conductive layer.

An electrical bipole is simulated at the center of the network by feeding a current +I and a current -I into two different nodes. These nodes can be adjacent or not to each other. Square meshes are used in the network and the resistors will be in analogy with resistivity if we take the resistivity unit as ohm. unit length.

It was assumed that at great distance from the source the potential vanishes and, therefore, the most external nodes of the network were grounded.

The voltage on each node will be in analogy with the potential and can be found with respect to the reference nodes by solving a system of linear equations.

As an example let us solve the simple network presented in Figure 6.

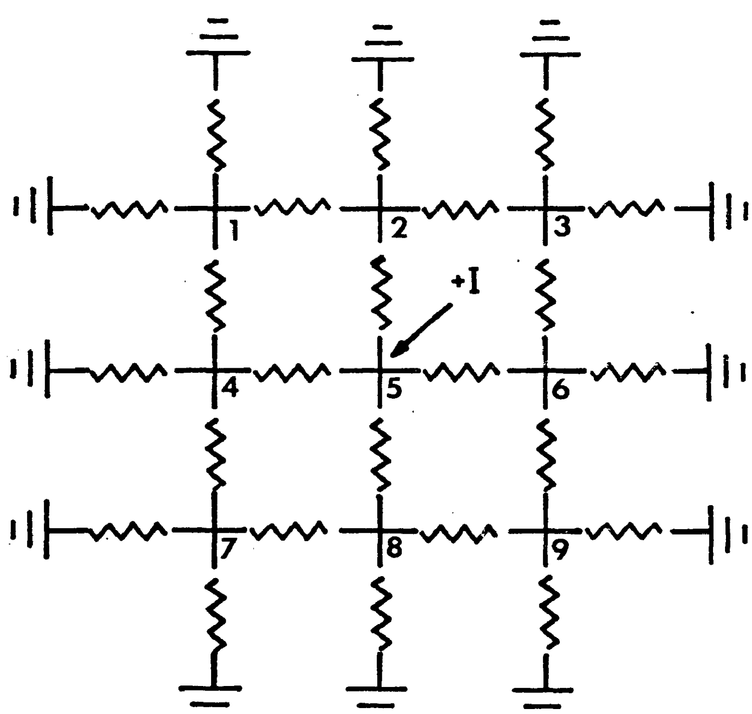


Figure 6. Resistor network with 9 nodes and square meshes.

In order to write the system of equations we apply Kirchoff's current law to the nodes at which voltages are defined. For simplicity, let us assume all resistors with the same value R and at node 5 we feed a current I.

Then we can write

$$0 = \frac{1}{R} V_1 + \frac{1}{R} V_1 + \frac{1}{R} (V_1 - V_2) + \frac{1}{R} (V_1 - V_4)$$

$$0 = \frac{1}{R} V_2 + \frac{1}{R} (V_2 - V_1) + \frac{1}{R} (V_2 - V_3) + \frac{1}{R} (V_2 - V_5)$$

$$0 = \frac{1}{R} V_3 + \frac{1}{R} (V_3 - V_2) + \frac{1}{R} (V_3 - V_6)$$

$$0 = \frac{1}{R} V_4 + \frac{1}{R} (V_4 - V_1) + \frac{1}{R} (V_4 - V_5) + \frac{1}{R} (V_4 - V_7)$$

$$I = \frac{1}{R} (V_5 - V_2) + \frac{1}{R} (V_5 - V_6) + \frac{1}{R} (V_5 - V_4) + \frac{1}{R} (V_5 - V_8) \tag{19}$$

$$0 = \frac{1}{R} V_6 + \frac{1}{R} (V_6 - V_3) + \frac{1}{R} (V_6 - V_5) + \frac{1}{R} (V_6 - V_9)$$

$$0 = \frac{1}{R} V_7 + \frac{1}{R} (V_7 - V_8) + \frac{1}{R} (V_7 - V_4) + \frac{1}{R} V_7$$

$$0 = \frac{1}{R} V_8 + \frac{1}{R} (V_8 - V_7) + \frac{1}{R} (V_8 - V_9) + \frac{1}{R} (V_8 - V_5)$$

$$0 = \frac{1}{R} V_9 + \frac{1}{R} V_9 + \frac{1}{R} (V_9 - V_8) + \frac{1}{R} (V_9 - V_6)$$

Rewriting equations 19 in matrix form we have:

$$\begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} \frac{4}{R} & -\frac{1}{R} & 0 & -\frac{1}{R} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R} & \frac{4}{R} & -\frac{1}{R} & 0 & -\frac{1}{R} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R} & \frac{4}{R} & 0 & 0 & -\frac{1}{R} & 0 & 0 & 0 \\ -\frac{1}{R} & 0 & 0 & \frac{4}{R} & -\frac{1}{R} & 0 & -\frac{1}{R} & 0 & 0 \\ 0 & -\frac{1}{R} & 0 & -\frac{1}{R} & \frac{4}{R} & -\frac{1}{R} & 0 & -\frac{1}{R} & 0 \\ 0 & 0 & -\frac{1}{R} & 0 & -\frac{1}{R} & \frac{4}{R} & 0 & 0 & -\frac{1}{R} \\ 0 & 0 & 0 & -\frac{1}{R} & 0 & 0 & \frac{4}{R} & -\frac{1}{R} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{R} & 0 & -\frac{1}{R} & \frac{4}{R} & -\frac{1}{R} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{R} & 0 & -\frac{1}{R} & \frac{4}{R} \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{vmatrix} \qquad (20)$$

The solution of the system of linear equations expressed in matrix form in 20 will give the voltage in each node of the network. It should be noted that this type of resistor network yields a coefficient matrix, symmetric and sparse in the sense that all non-zero terms are situated on a band centered on the main diagonal. These features will be of great use when a bigger network is used for modeling purposes, as can be seen in the next section.

MODELING A CONTINUOUS MEDIUM

In modeling a continuous medium a square-meshed resistor network with 23 by 24 nodes was used. Therefore, in order to obtain the voltage in each node we have to solve a system of linear equations with 552 equations.

The computing facility at the Colorado School of Mines, a model PDP-10 computer by Digital Equipment Corporation, allows 40,000 words of storage for user's usage. The matrix of coefficients generated from the above mentioned system of equations requires 244,704 storage spaces and would be impossible to solve on the PDP-10 unless use is made of properties inherent to the matrix.

The matrix of coefficients for the type of network being used is positive definite, symmetric, and band-limited. Using these features we can solve the proposed system of linear equations inside the allowable computer capacity. The procedure used in this report for the solution of system of equation is Gauss elimination with partial pivoting and it is discussed in Appendix A.

In the dipole mapping technique the source is about 3/4 to 2½ miles long and the measuring dipoles 100 to 300 feet long. The distance away from the source that measurements can be made depends upon several factors such as the length of the source, the amount of current driven into the ground, the conductivity of top layer and geology. Generally we can say that in a conductive area measurement can be made at distances up to four or five source lengths from the center of the bipole source.

Taking these facts into account we placed the source in the center of the resistor network using two adjacent nodes for the purpose (Fig. 7). This will limit our perception of changes in field properties on the region close to the source. If one is interested in studying changes close to the source one should move the two point sources farther apart.

Boundary Effect

Because we are using a finite network to represent an infinite medium errors will arise inside the region of interest. In order to minimize this effect use is made of a termination strip all around the network. The external boundary is joined by a network which rapidly becomes very coarse as shown in figure 8. The value of the resistors in the coarse network can be computed by the method of vector areas (Karplus, 1958).

As an example, let us compute the value of the resistors on the termination strip represented in figure 8.

By defining the interior resistors as

$$R \triangleq R_o \frac{\Delta x}{\Delta y}$$

we can obtain the value for resistors R_1, R_2, and R_3 by

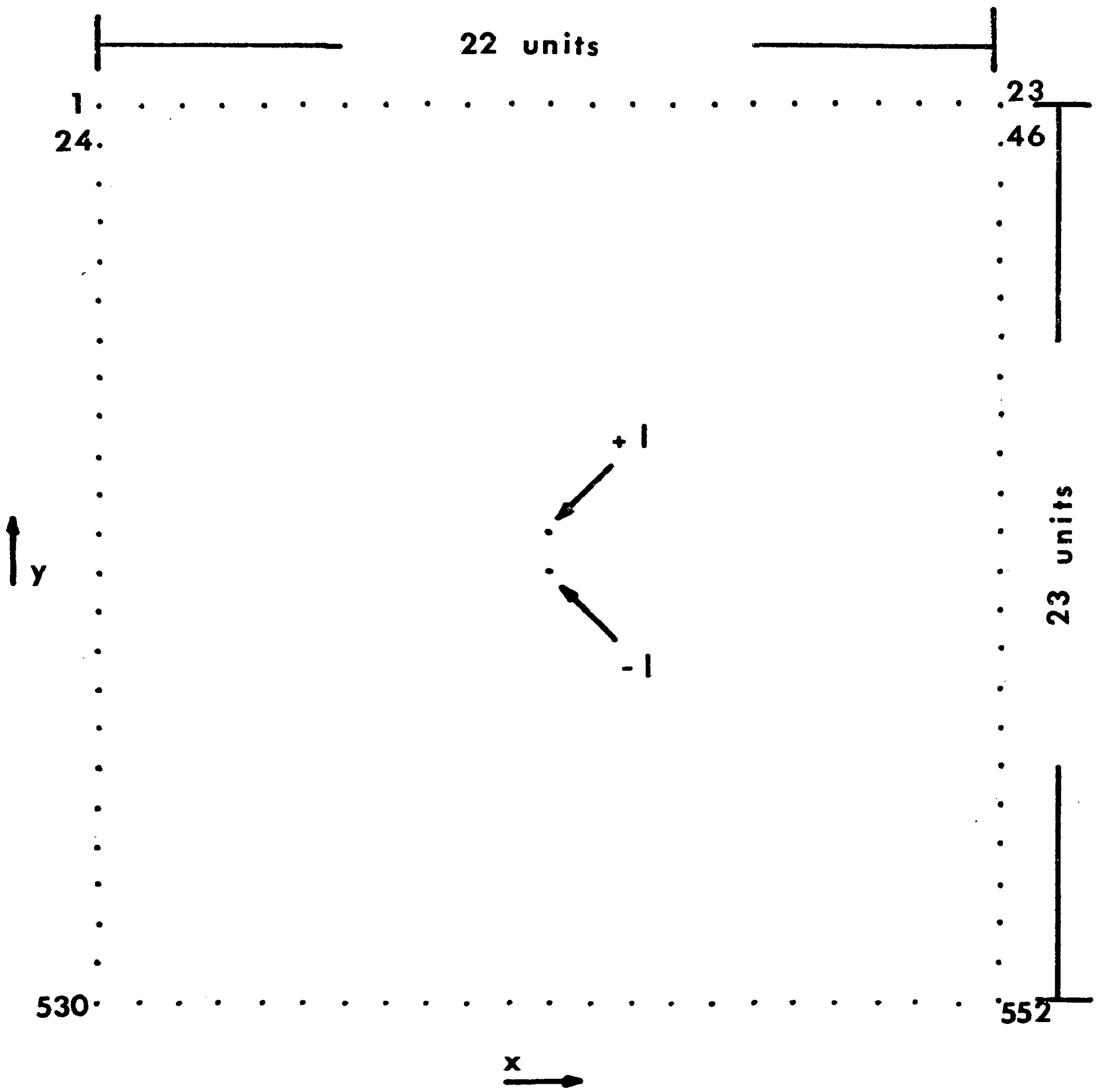


Figure 7. Resistor network designed for modeling purposes. Currents +I and -I fed into the network by two different nodes simulate a dipole.

observing the area they occupy in the termination strip.

Therefore

$$R_1 = R_o \frac{2\Delta x}{\Delta y} = 2R$$

$$R_2 = R_o \frac{\Delta x}{\Delta y} = R$$

$$R_3 = R_o \frac{4\Delta x}{\Delta y/2} = 8R$$

The procedure discussed above was used when we solved the resistor network model of a continuum. This approach yields similar results as if one assumes that the medium becomes increasingly resistive as one moves away from the current source.

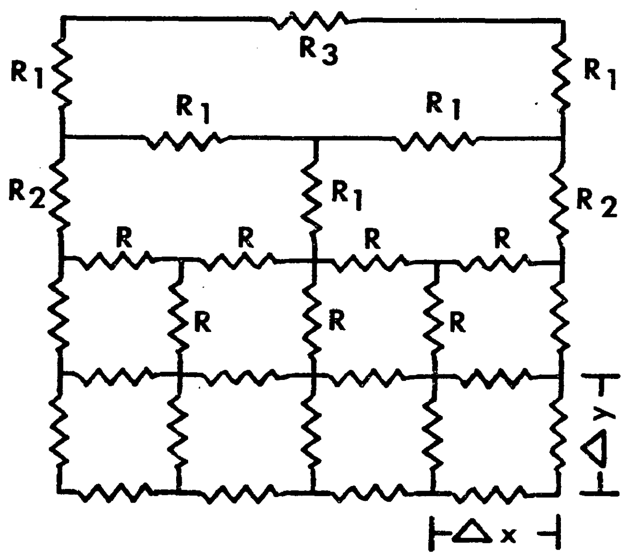


Figure 8. Termination strip for minimization of boundary effect.

In the proposed network we increased the value of the resistors in the two most external loops by two-fold and four-fold. The voltages obtained on the nodes of these loops were disregarded on the final analysis because the error was too large and these nodes are not in the area of interest. They were used with the only purpose of decreasing the error inside the network, by pushing it to the most external loops.

The improvement on the results can be observed in the results obtained for the electric field and presented on figure 14.

Source Effect

The potential gradient due to the current source placed on the center of the network is not uniform. Close to the source the gradients are steeper than on the edge of the network. In order to obtain more accurate solutions it is necessary to use a finer grid spacing around the source because there the potential undergoes sharp variations.

There are several ways, described in the literature, on how to approach the problem (Liebman, 1954a; Persico, 1952; Landau, 1957; and Karplus, 1958). In this report we used the following approach: a) the potential is obtained for each node on a coarse network; b) the mesh size of the network over an area 20 percent the size of the coarse network (including termination strip) is subdivided in half (Fig. 9a); c) the resistors on the finer network are kept with the same value as in the coarse network; d) the potential values obtained with the coarse network for the interface between the two networks are multiplied by conductor values and used as an input on the external loop of the finer

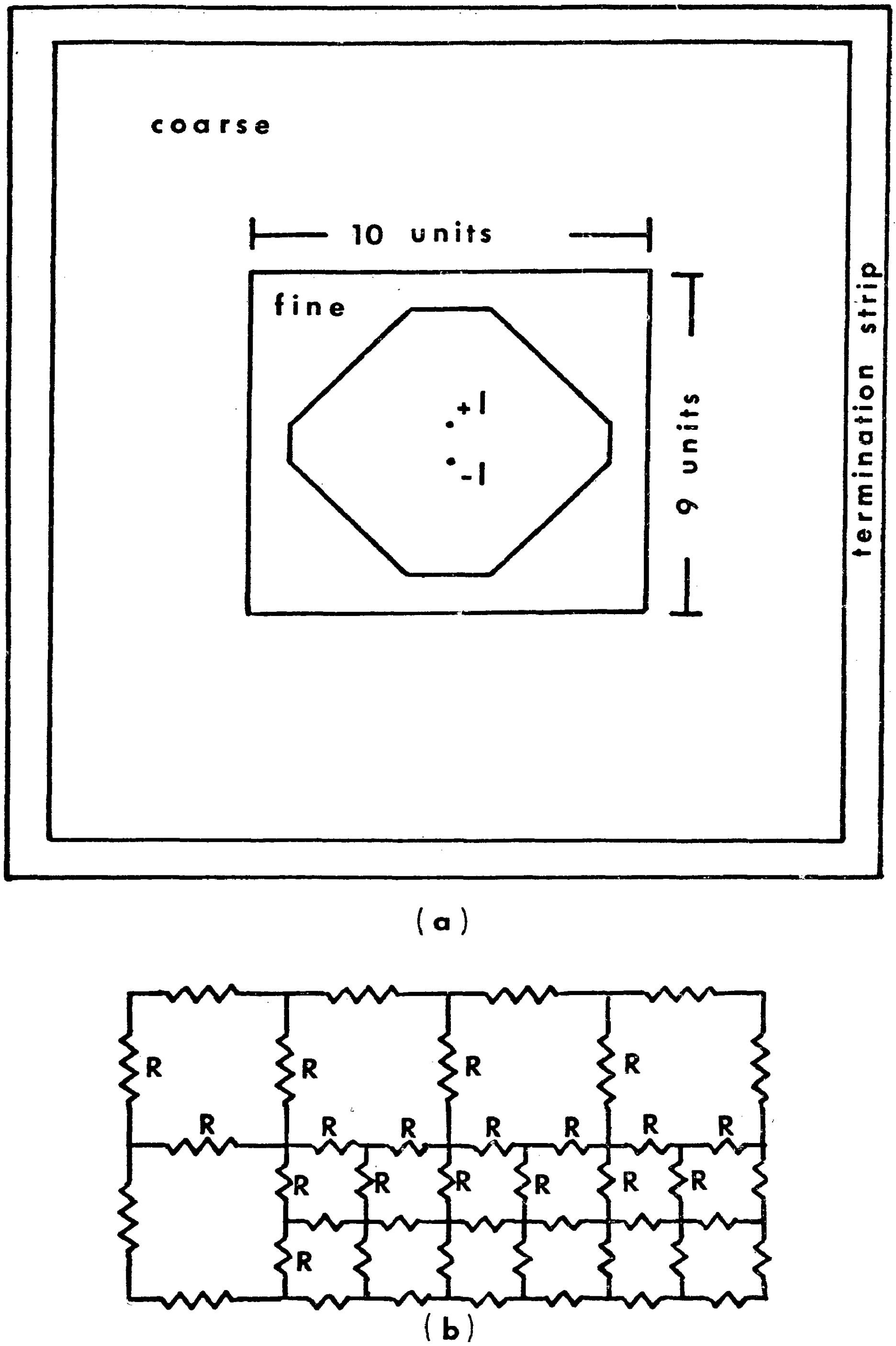


Figure 9. a) Coarse and fine networks with area (diamond-shaped) where low errors occur. b) Portion of the interface between coarse and fine networks.

network; e) the potential is obtained for each node of the finer network.

From figure 9b we can observe that the coarse network yields only every other potential value to be used as input on the most external loop of the finer network. If we use an interpolation technique we can obtain values between the ones yielded by the coarse network and proceed in the solution of the finer network. The numerical technique used in this report to carry out the interpolation is Lagrangian interpolation (Appendix A).

Because the values from the coarse network used for input on the fine network are in error the results obtained with the fine network will be in error in addition to the errors inherent in the numerical procedures. Therefore not all values obtained are used in the composition of the final picture of the problem being modeled. Actually only the values obtained on a diamond-shaped area around the source are used because in this area we have the lowest error in the numerical procedure (see Fig. 9a).

Potential and Electrical Fields

If we see our two current inputs as a simulation of a bipole source used in field measurements and if a resistor on a mesh branch is seen as an analog to resistivity (ohm. unit length) we can assume that the voltage distribution obtained on the network is in complete analogy with the potential distribution obtained in field measurements.

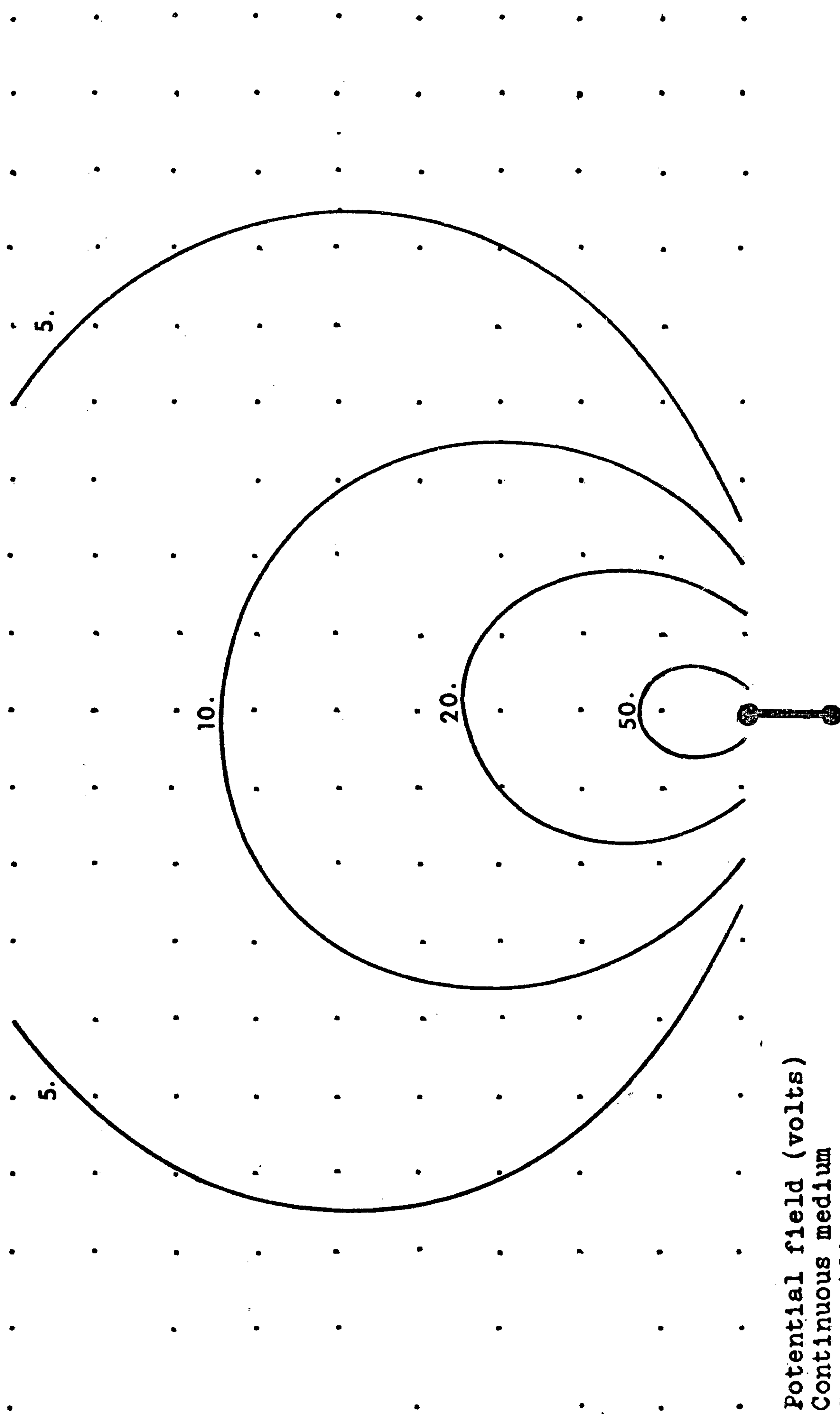
In figure 10 we present the potential field distribution obtained with a uniform network. The resistors used are 5 ohm resistors and currents of +100 and -100 amp fed through two different nodes were used to simulate the bipole. This potential field distribution was obtained by combining the results yielded by computer programs GRID and FIGRID that are presented in Appendix B. Program GRID gives the potential distribution on the coarse network and computer program FIGRID gives the results for the fine network. The distribution is corrected for boundary and source effects according to the procedures explained before.

From the result we can see that due to the fact that the bipole is represented by two adjacent point sources no information can be obtained in between the sources. Because generally, we are interested in measurements away from the bipole source this fact poses no problems.

In dipole mapping we deal with the total electric field rather than the potential field. Because the electric field is the negative gradient of the potential it may be calculated numerically from the gradient of the potential throughout the resistor network.

The gradient of the potential can be represented as:

$$E_T = -\nabla U \cong -\sqrt{(Dx)^2 + (Dy)^2} \qquad (21)$$



Potential field (volts)
Continuous medium
Current 100 amps
Resistivity 5 ohm.unit length

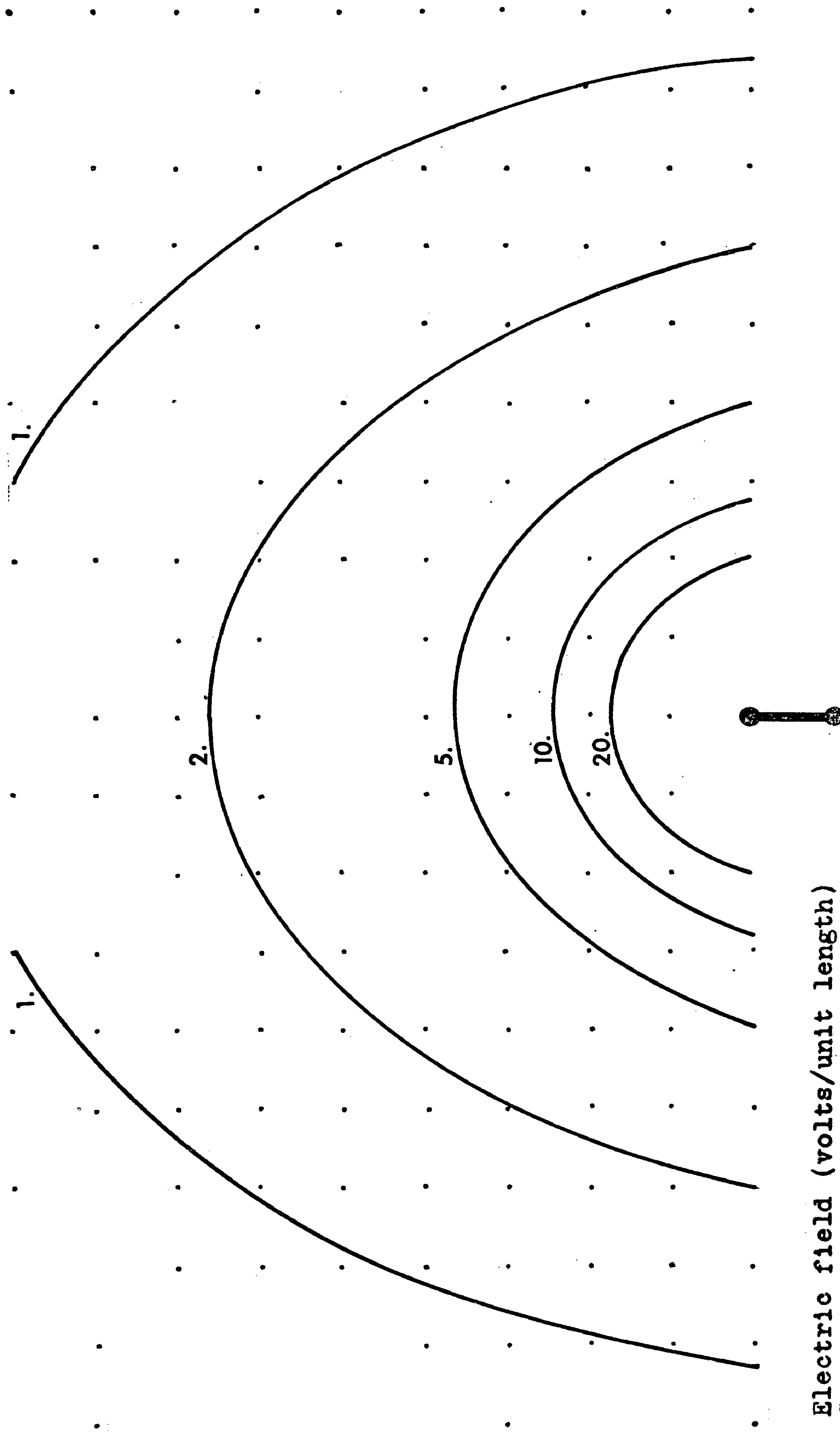
Figure 10

where Dx is the derivative of the potential in the x-direction and Dy is the derivative in the y-direction. These derivatives are obtained by differentiating a second order Lagrangian polynomial fitted over three consecutive points. This approach is explained in Appendix A and is used in computer programs EFD and EFFIGR presented on Appendix B.

The electric field distribution presented in figure 11 was obtained from the potential field distribution shown on figure 10. The distribution obtained with computer program GRID is used as input for program EFD that evaluates the gradient of the potential for the coarse grid. The potential distribution obtained from FIGRID is used as input for computer program EFFIGR that gives the gradient of the potential for the fine resistor network. The result presented in figure 11 is the composition of the results obtained with EFD and EFFIGR (for program EFFIGR only those numbers on the low-error area around the source, Figure 9a).

At this point I will show how the boundary and source corrections improve the results and also how the network behaves as a modeling tool. To achieve this goal I will compare my results for the electrical field with results obtained analytically.

Here, we should recall the earth model proposed at the beginning of this report and presented on figure 12.



Electric field (volts/unit length)
Continuous medium
Current 100 amps
Resistivity: 5 ohm.unit length

Figure 11

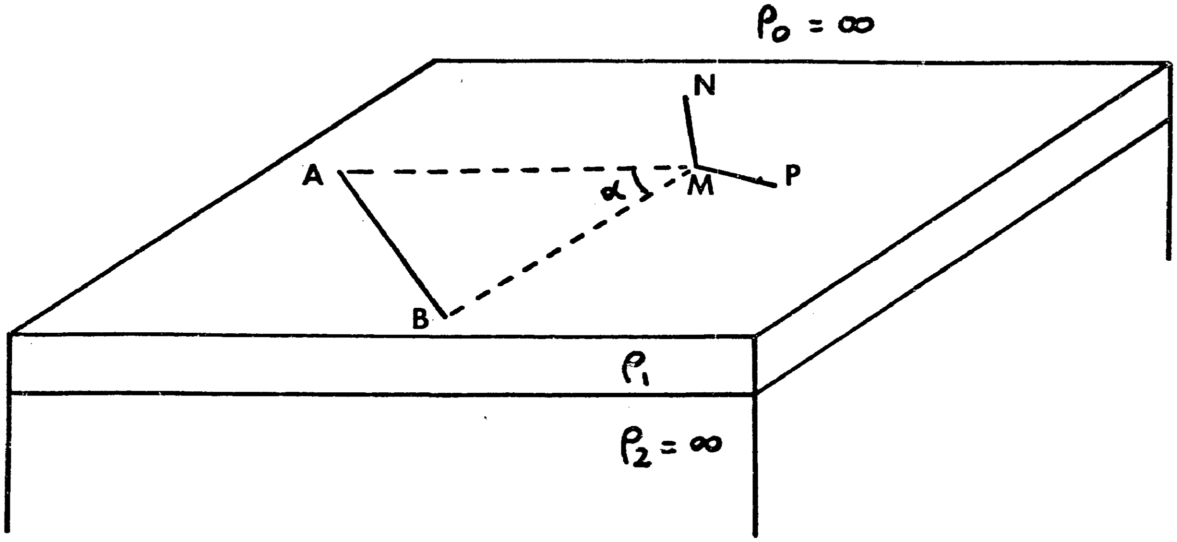


Figure 12. Earth model being modeled with dipole mapping set-up.

If we have a single source on the top layer we can say that at some distance R which is large compared to the thickness of the first layer current flows parallel to the boundaries and has a uniform density from top through the bottom of this top layer (Keller and Frischknecht, 1967).

The electric field can be obtained by using Ohm's law

$$\vec{E} = \rho_1 \vec{J} \tag{22}$$

where, ρ_1 is the resistivity of the top layer and $\vec{J}$ is the current density vector.

By integrating the current density over an equipotential surface, in this case, a cylindrical shell of radius R and height H at great distances from the source, a relationship between the electric field and current from the source can be established.

$$I = \int_S \vec{J}\, ds = 2\pi R h J \tag{23}$$

Therefore

$$E = \frac{\rho_1 I}{2\pi R h} \tag{24}$$

By defining

$$S \triangleq \frac{h}{\rho_1} \tag{25}$$

where S is the conductance of the top layer we have the electric field at the surface of a thin layer for a current I as

$$E = \frac{I}{2\pi R S} \tag{26}$$

Using the dipole mapping scheme presented in figure 12 we have that (Furgerson and Keller, 1974):

$$E_1 = \frac{I}{2\pi R_1 S} \tag{27}$$

and

$$E_2 = \frac{-I}{2\pi R_2 S} \tag{28}$$

The total electric field is obtained by adding vectorially these two electric fields:

$$E_T = \frac{I}{2\pi R_1 S}\left\{1 + \left(\frac{R_1}{R_2}\right)^2 - 2\left(\frac{R_1}{R_2}\right)\cos\alpha\right\}^{1/2} \quad (29)$$

Using in equation 29 a value of 100 amp for I, 0.2 mho for S and the coordinates of each node on the network to obtain the values of R_1, R_2, as well as the angle α we can evaluate the total electric field analytically and compare with our result obtained with a resistor network.

In figures 13, 14, and 15 electric field error maps are presented. They represent the absolute percentage deviation of the results obtained with the resistor network from the results obtained analytically. In figure 13 the electric field obtained with the network has no correction for boundary and source effects and therefore, errors higher than 60 percent can be observed. In figure 14 the correction for the boundary was applied and nowhere inside the network the error goes over 30 percent but, as we can see, the source effect is still present. Applying both boundary and source corrections to the resistor network we obtained the result of figure 15. Here the source effect is minimized and if we assume that results with errors up to 15 percent are valid, we may use more than 60 percent of the network area for interpretative and modeling purposes. This means that we can go as far as six source lengths in every direction without going over the error limit of 15

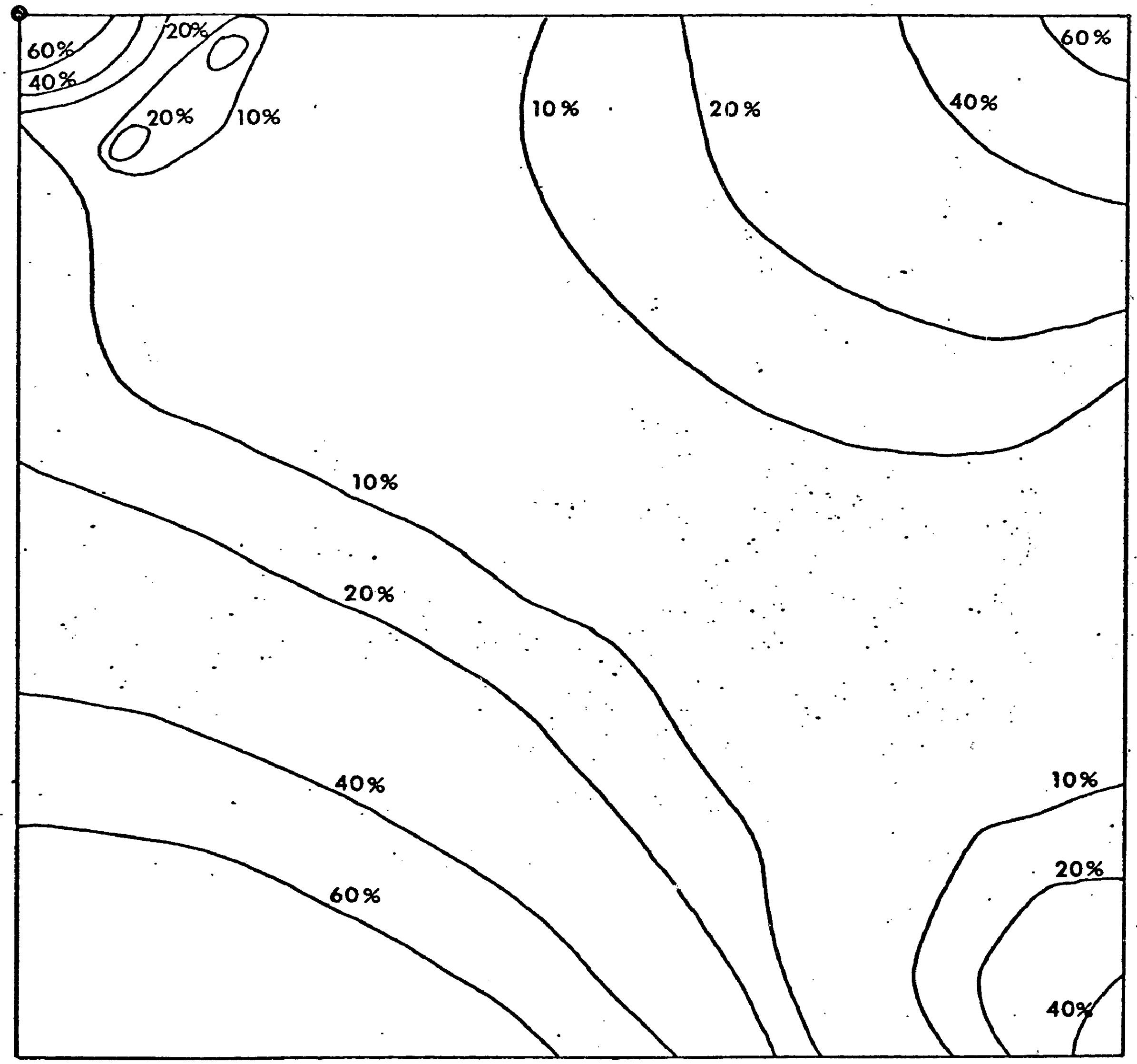


Figure 13. Absolute error map for the electric field (no corrections applied).

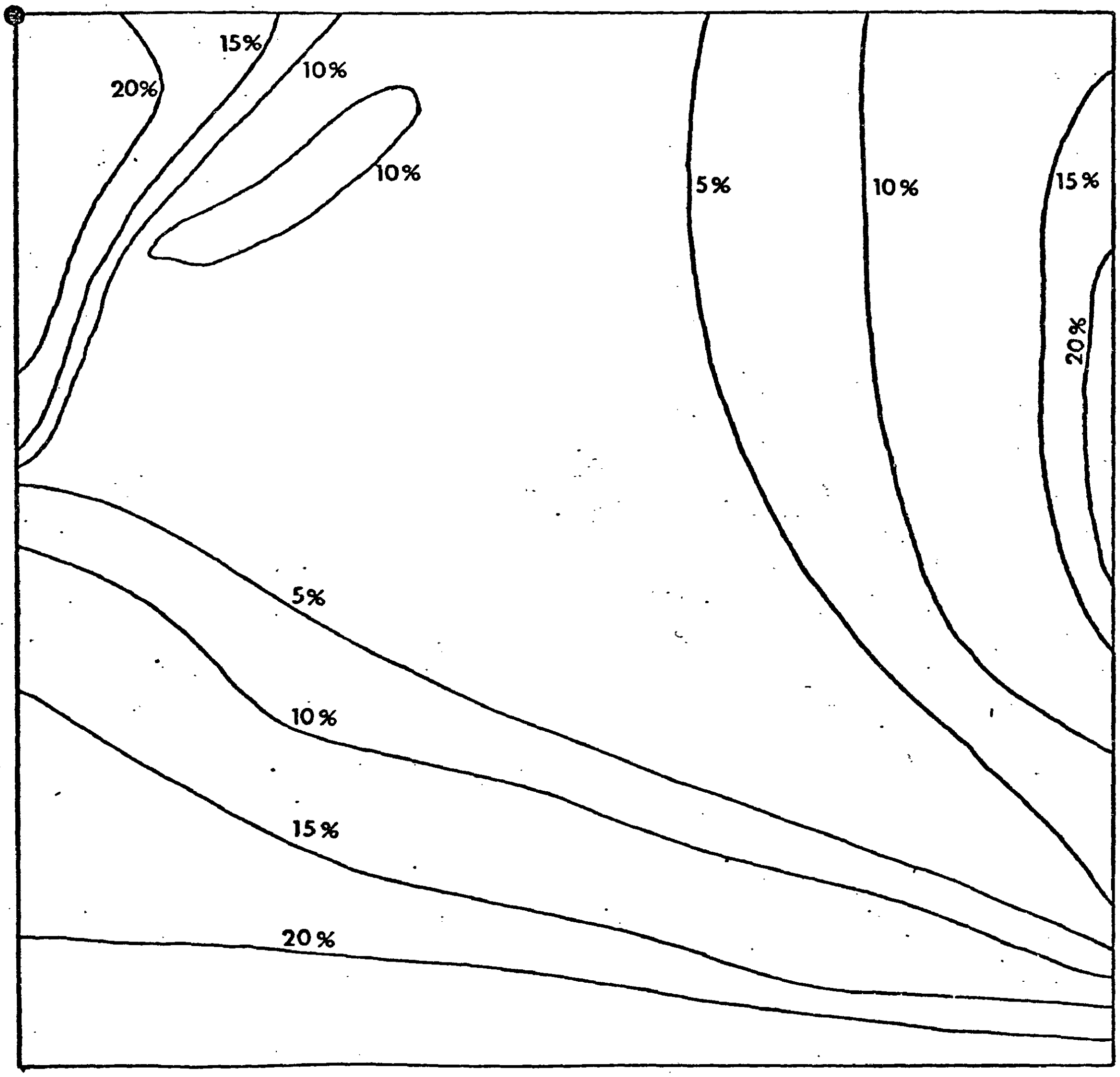


Figure 14. Absolute error map for the electric field - boundary correction applied.

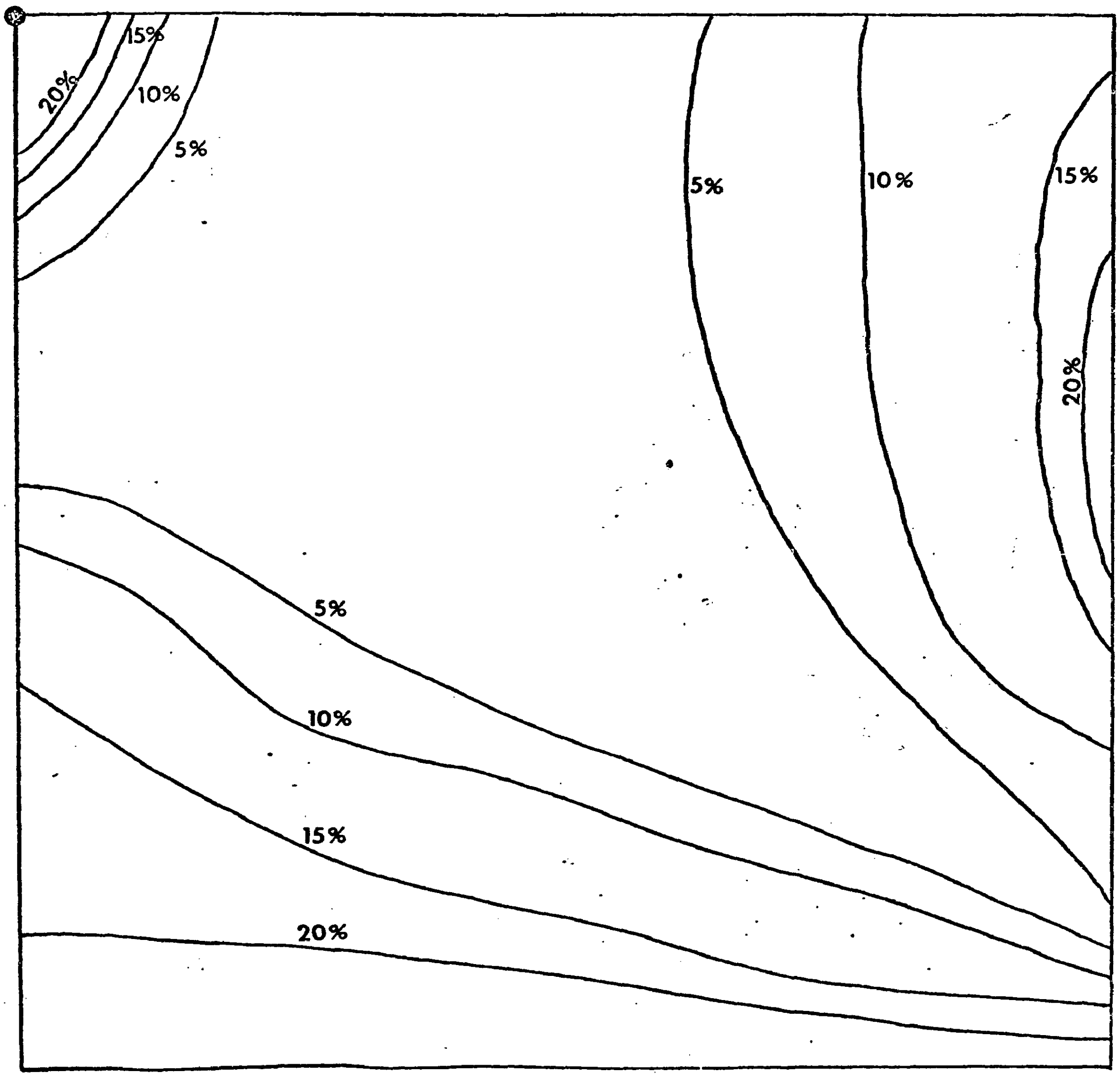


Figure 15. Absolute error map for the electric field-boundary and source corrections applied.

percent.

As was mentioned before, in a conductive area we can obtain measurements with the dipole mapping technique at about four to five source lengths from the center of the dipole source. From the results presented above we can see that the resistor network proposed in this work will be capable of modeling dipole mapping surveys if the 15 percent error margin can be accepted. It should be noted that in field surveys, an accuracy of 10 to 20 percent is standard practice.

MODELING OF BASIC GEOELECTRICAL DISCONTINUITIES

The computer programs presented in Appendix B were written with the purpose of evaluating the potential field and total electric field distributions of a continuous medium. In modeling geoelectrical discontinuities the resistors representing the discontinuity have to have their values changed. This is achieved by making appropriate changes on the subroutine COND, that is called in the program GRID. If the discontinuity is totally or partially inside the area where correction for the source effect is applied, the assignment of conductances in the computer program FIGRID should be changed accordingly.

In order to make the appropriate changes in programs GRID and FIGRID it should be remembered that the 552 nodes of the coarse network are set up in 24 rows of 23 nodes each. The numbering of the nodes is from left to right and from top to bottom (Fig. 7). For each node we have four resistors numbered from one to four starting with the resistor on the right side of a node and proceding clockwise. Here we should stress that because the two most external loops of the coarse network are used for boundary correction the values obtained on these loops are not printed out. Therefore we are left with a 380-node network with 20 rows of 19 nodes each.

The fine network has 399 nodes laid on 19 rows of

21 nodes each. The numbering of the nodes and resistors is done here in the same way as for the coarse network.

In addition to the potential field and electric field distributions, values of apparent conductance were computed. Solving equation 29 for the apparent conductance we obtain

$$S_a = \frac{1}{2\pi R_1}\left[1 + \left(\frac{R_1}{R_2}\right) - 2\left(\frac{R_1}{R_2}\right)\cos\alpha\right]^{1/2} \frac{I}{E_T}$$

Because we have the total electrical field distribution, the current and the geometry of the network for defining the values of R_1, R_2 and the angle α, we can obtain apparent conductance maps.

The computer program APCON1, presented in Appendix B, uses the total electric field distribution obtained by program EFD and evaluates the apparent conductance on the coarse network. Program APCOM2 uses the total electric field distribution obtained by program EFFIGR and evaluates the apparent conductance on the fine network.

In this report the potential field, total electric field and apparent conductance were obtained for two different sets of models.

The first set of models includes:

(a) dike (one unit wide and infinitely long) parallel to the source

(b) dike (one unit wide and eleven units long) parallel to the source

(c) contact parallel to the source

(d) square discontinuity symmetric to the source

(e) square discontinuity asymmetric to the source.

The current used in all these models was 100 amps. The resistors representing the discontinuities are twice the value of the resistors in the continuous part of the medium, respectively 10 ohm and 5 ohm. For the infinite dike I also obtained results for a 20:1 contrast where the resistors comprised by the dike were 100-ohm resistors.

These results are presented in contour maps and when symmetry is present only half of the map is shown. In the case of a square discontinuity only the half of the map showing the discontinuity is presented. Due to the analogy existent between the resistor network and the field problems being modeled, the contours in the total electric field distribution map have units of volts per unit length, the contours of the potential field are in volts, and the contours of apparent conductance are in mhos.

For the second set of models I evaluated the potential field, total electrical field, and apparent conductance maps for the following models:

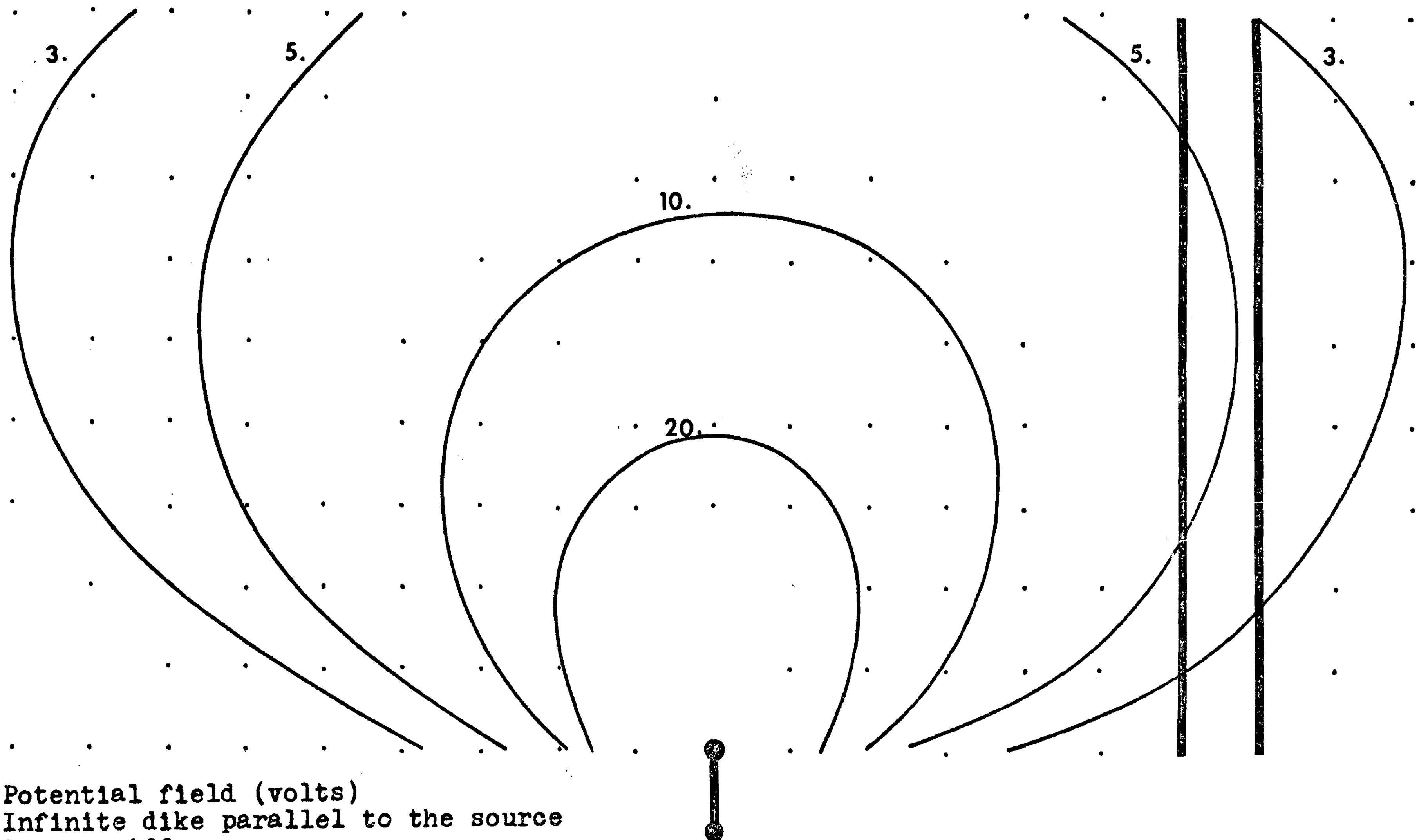
(a) vertical contact parallel to the source

(b) vertical contact normal to the source

(c) dike parallel to the source

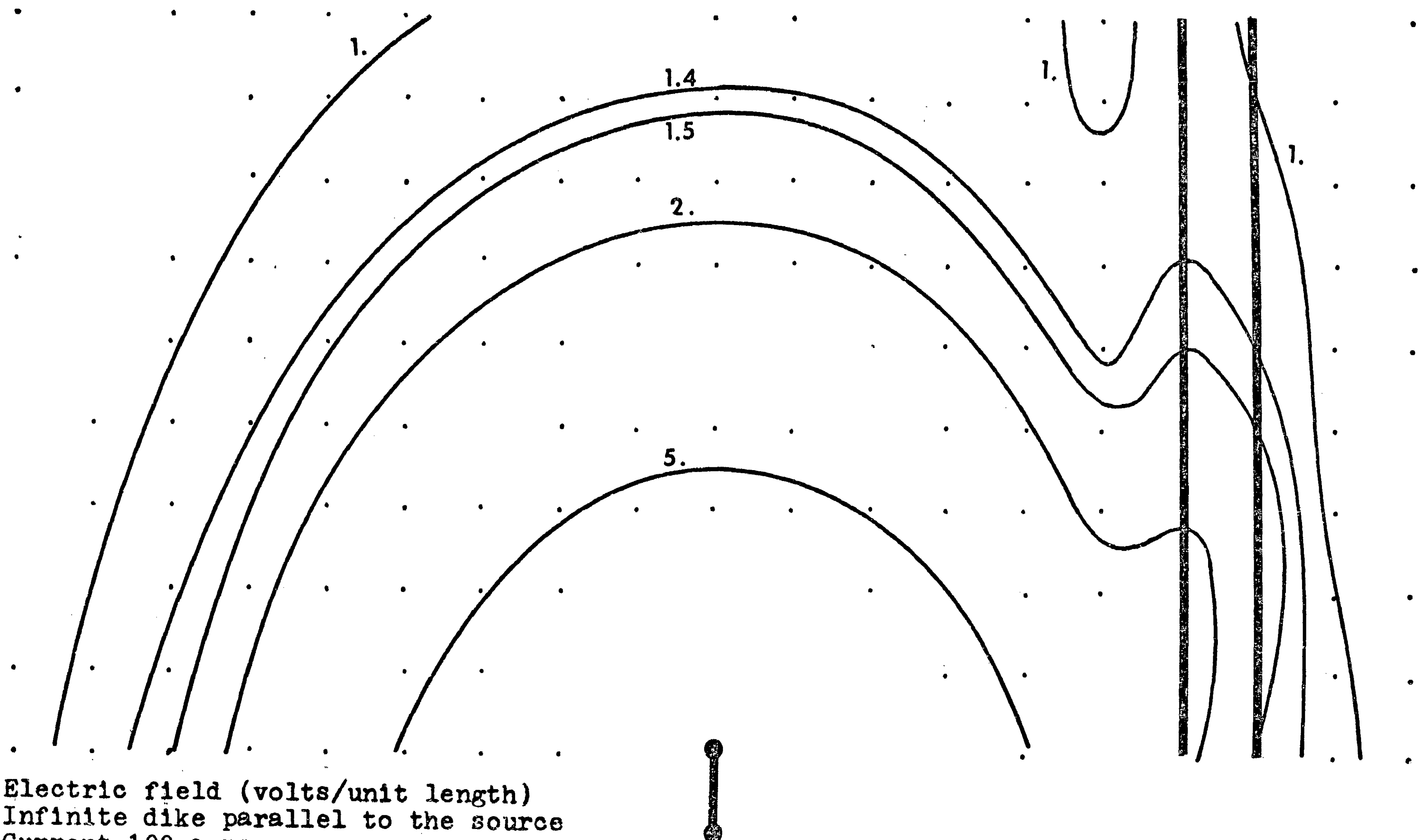
(d) dike normal to the source

(text continues on page 57)



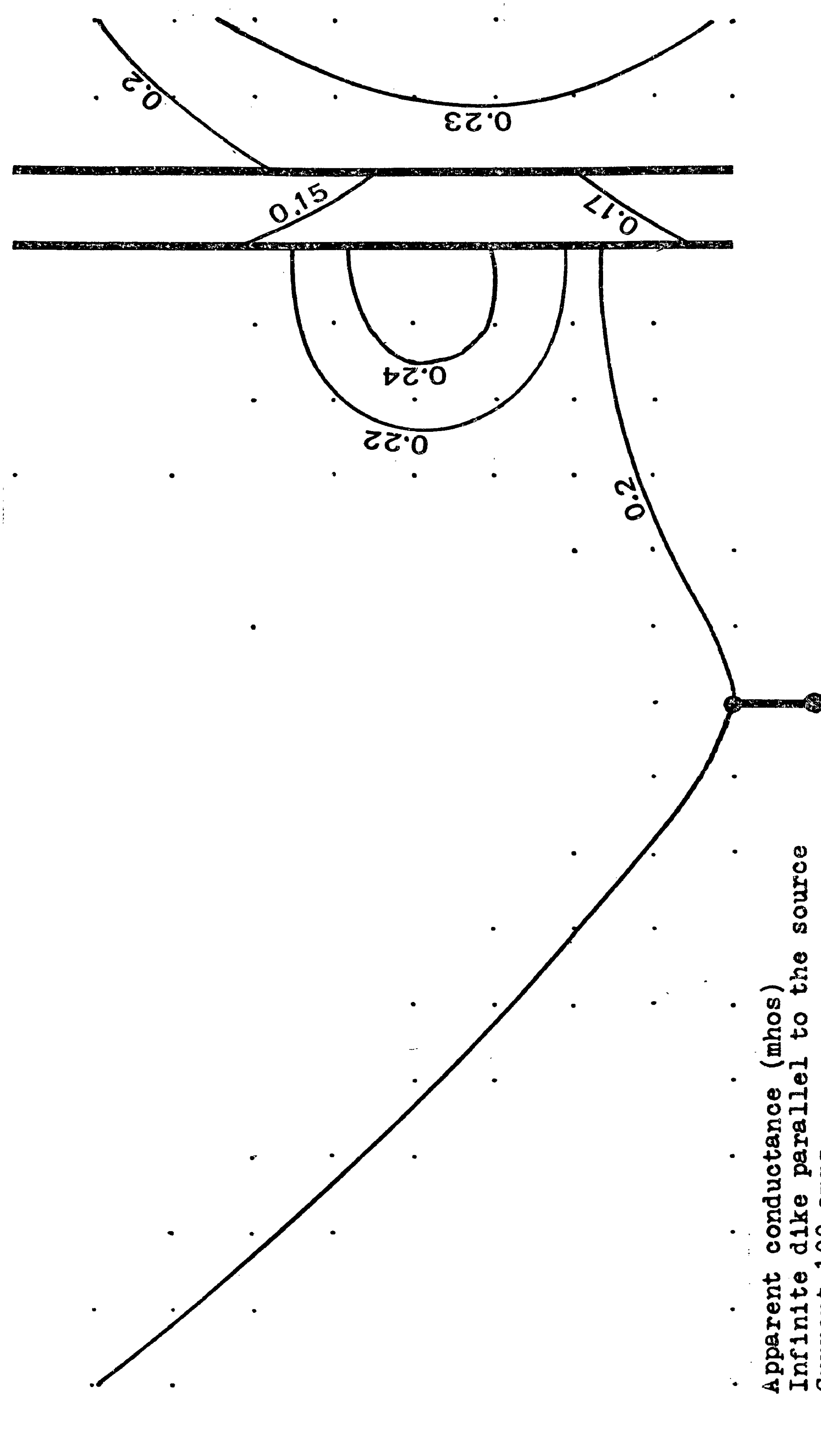
Potential field (volts)
Infinite dike parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 16



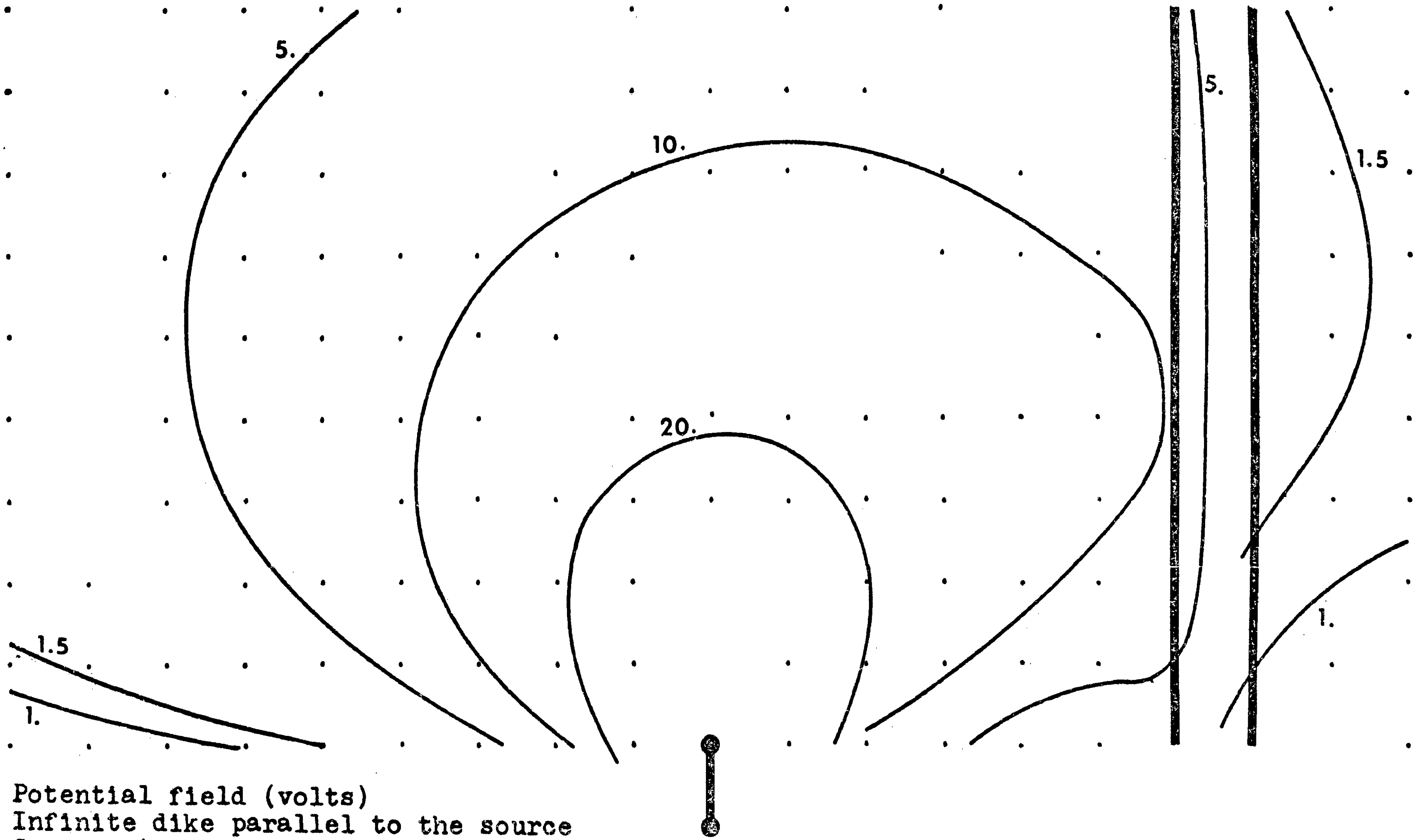
Electric field (volts/unit length)
Infinite dike parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 17



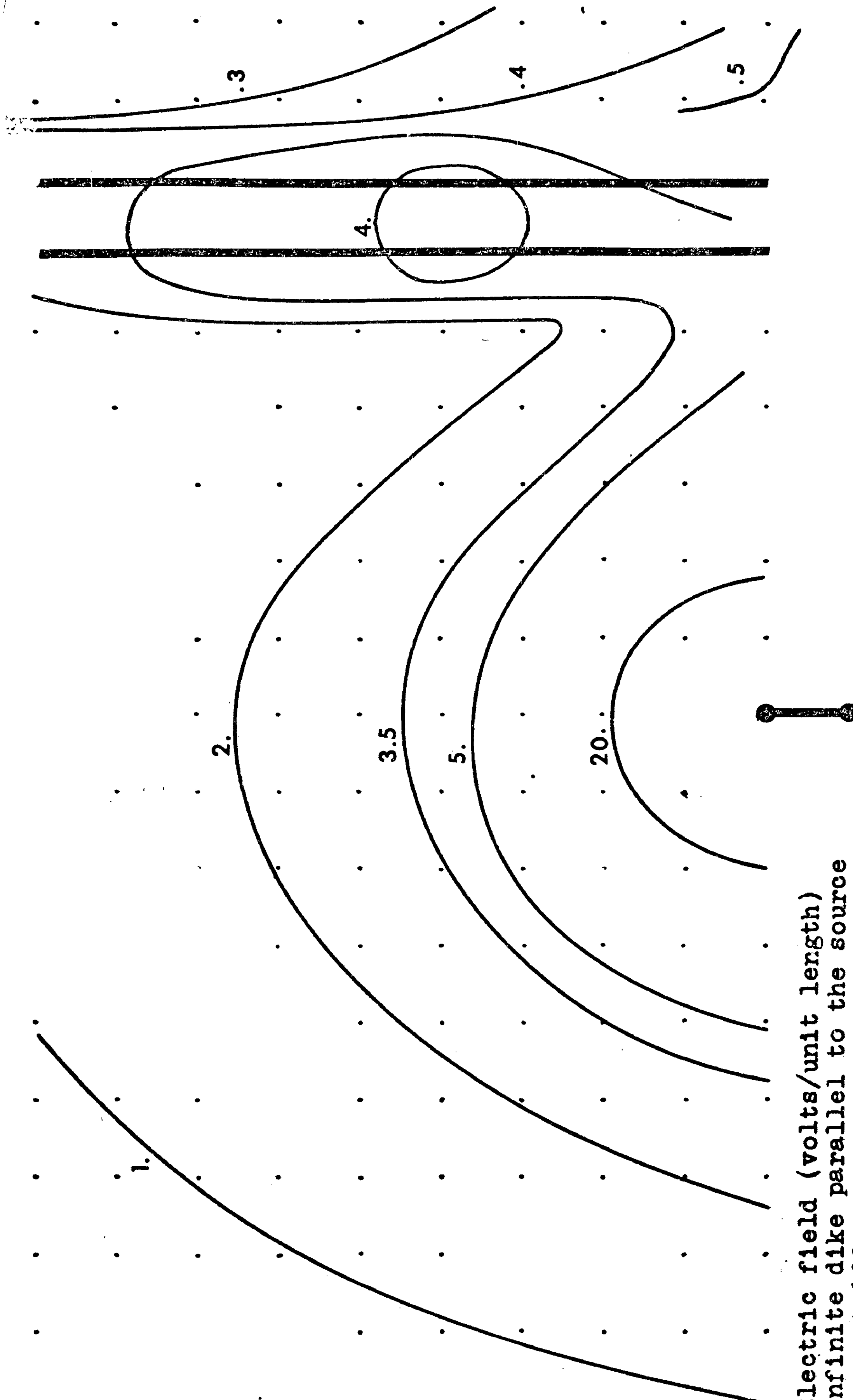
Apparent conductance (mhos)
Infinite dike parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm. unit length
discontinuity: 10 ohm. unit length

Figure 18



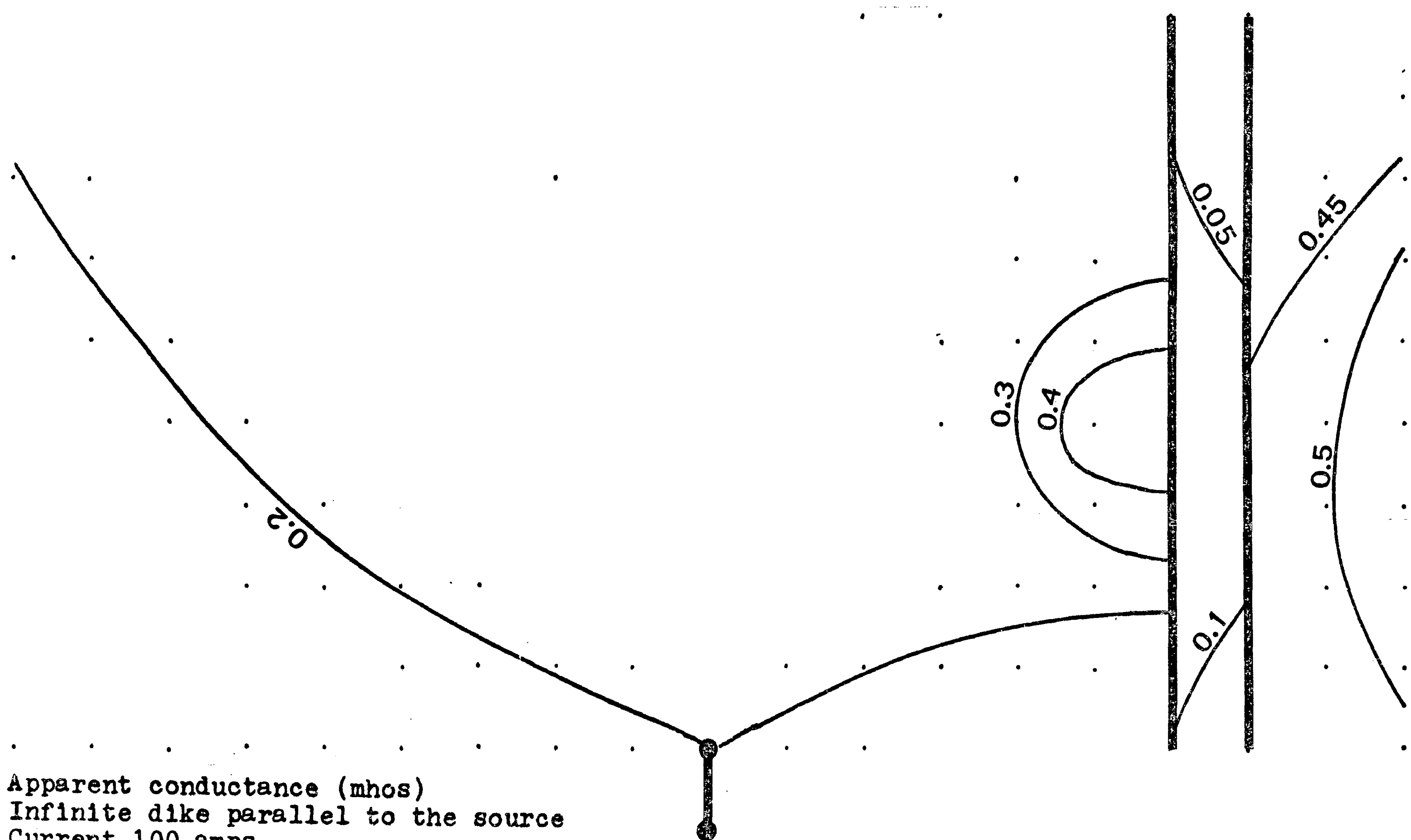
Potential field (volts)
Infinite dike parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 100 ohm.unit length

Figure 19



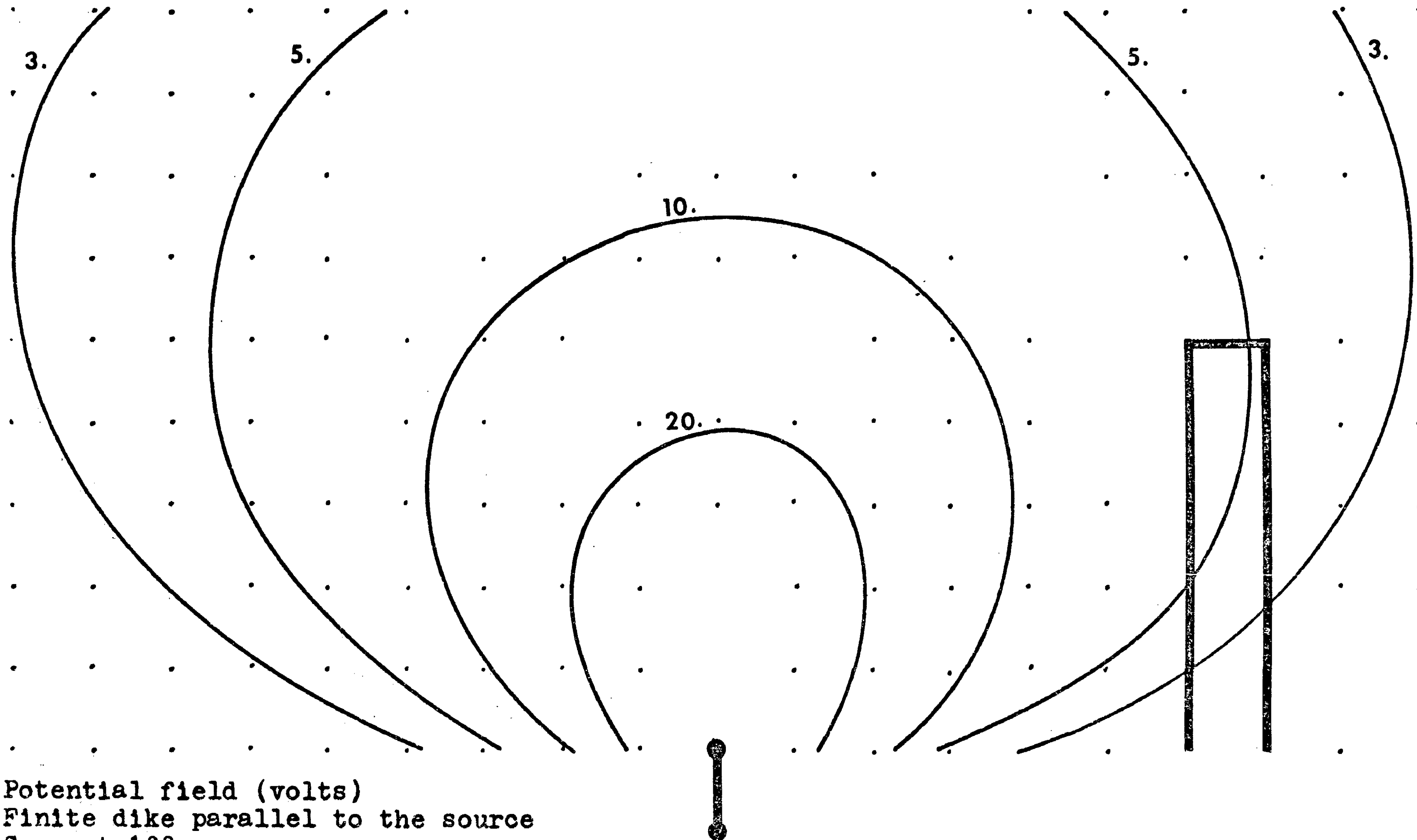
Electric field (volts/unit length)
Infinite dike parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 100 ohm.unit length

Figure 20



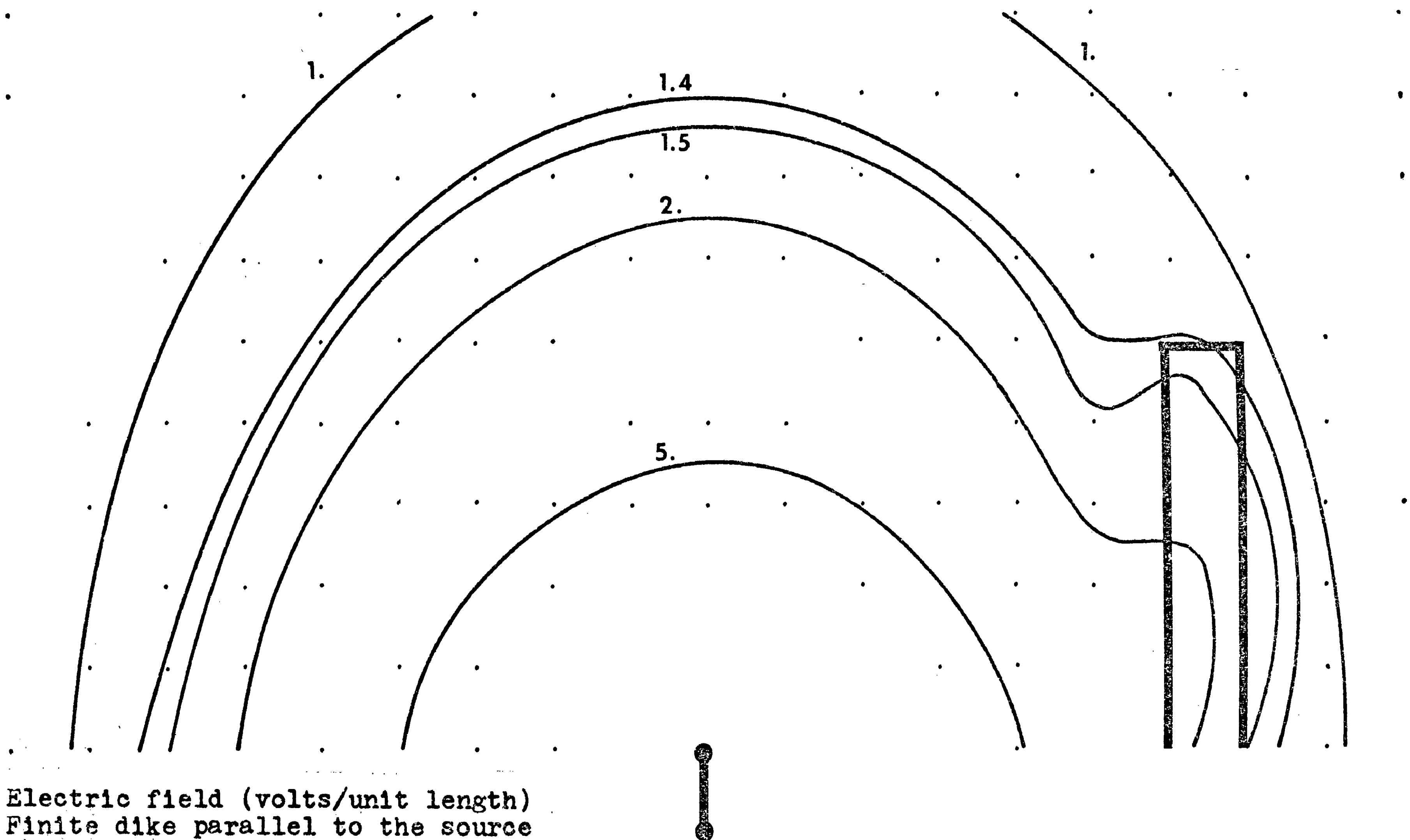
Apparent conductance (mhos)
Infinite dike parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm. unit length
discontinuity: 100 ohm. unit length

Figure 21



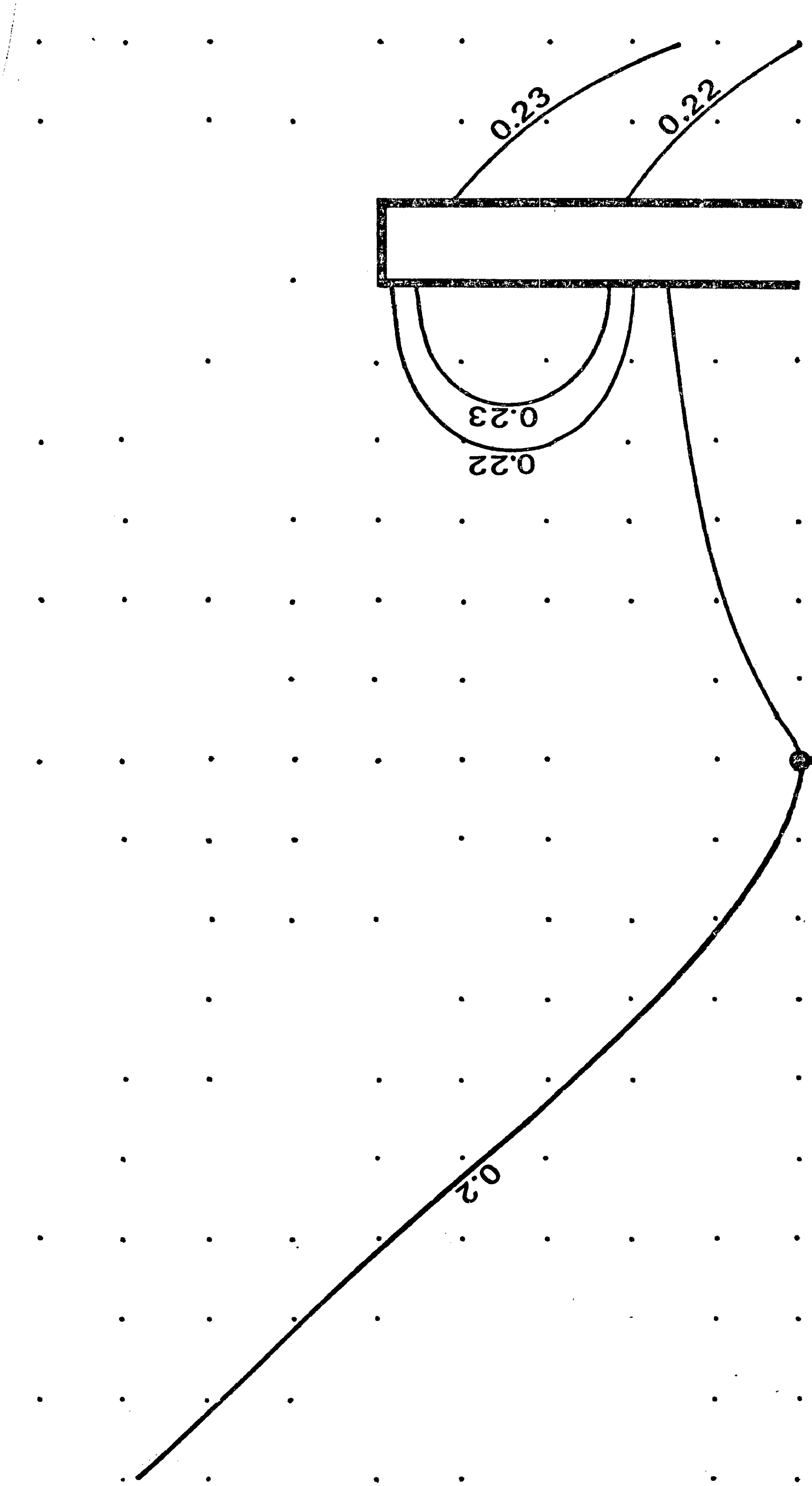
Potential field (volts)
Finite dike parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 22



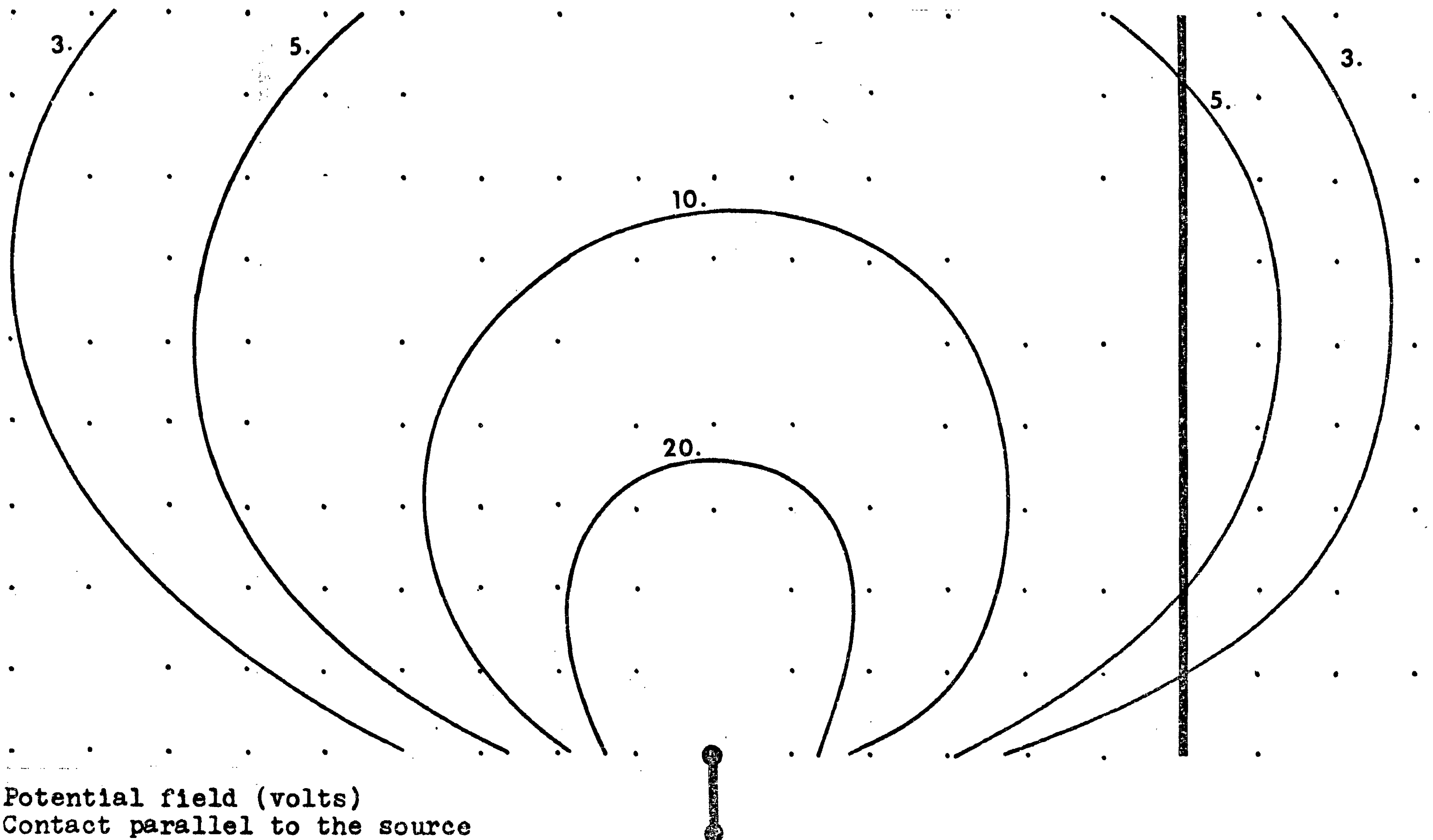
Electric field (volts/unit length)
Finite dike parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 23



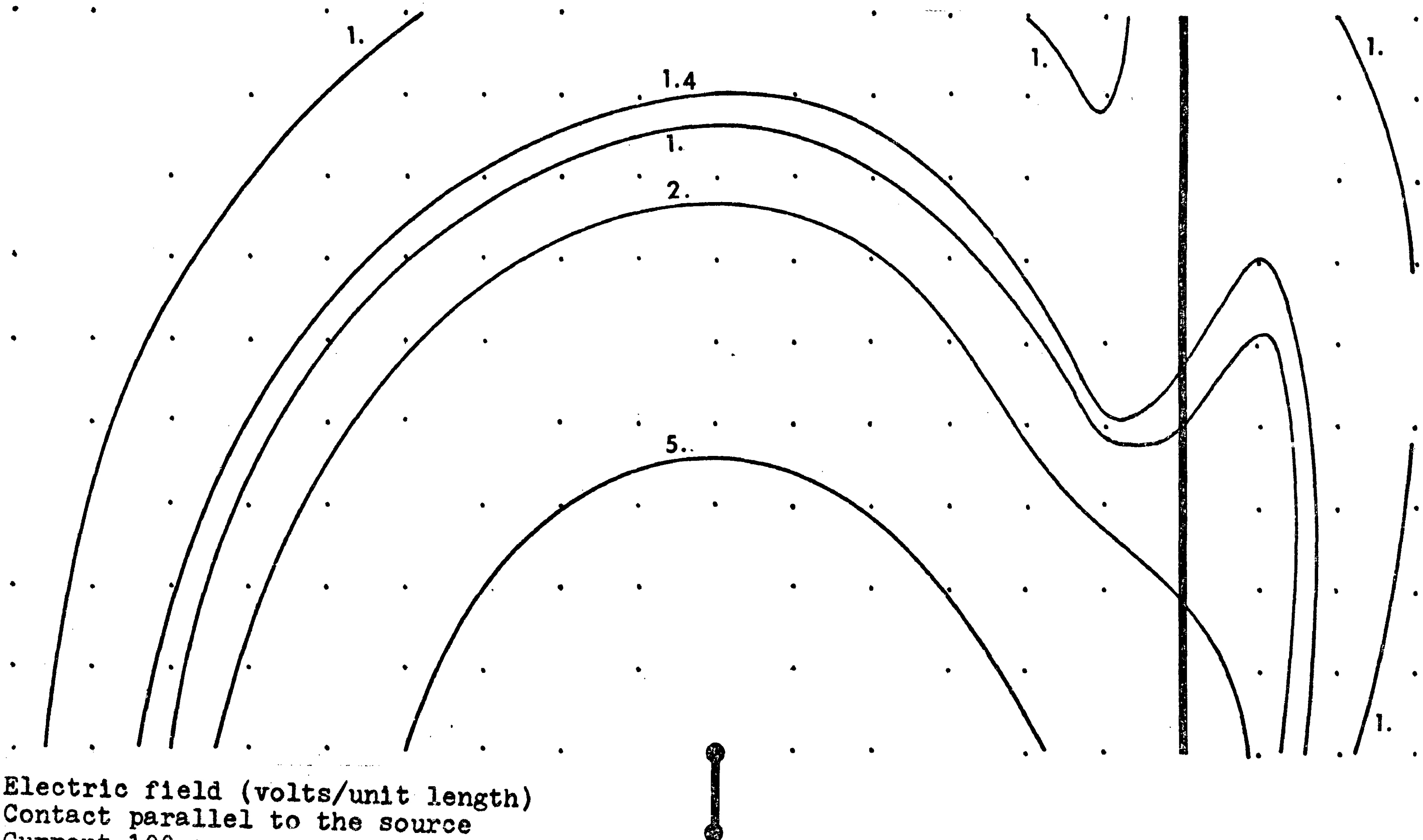
Apparent conductance (mhos)
Finite dike parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm. unit length
discontinuity 10 ohm. unit length

Figure 24



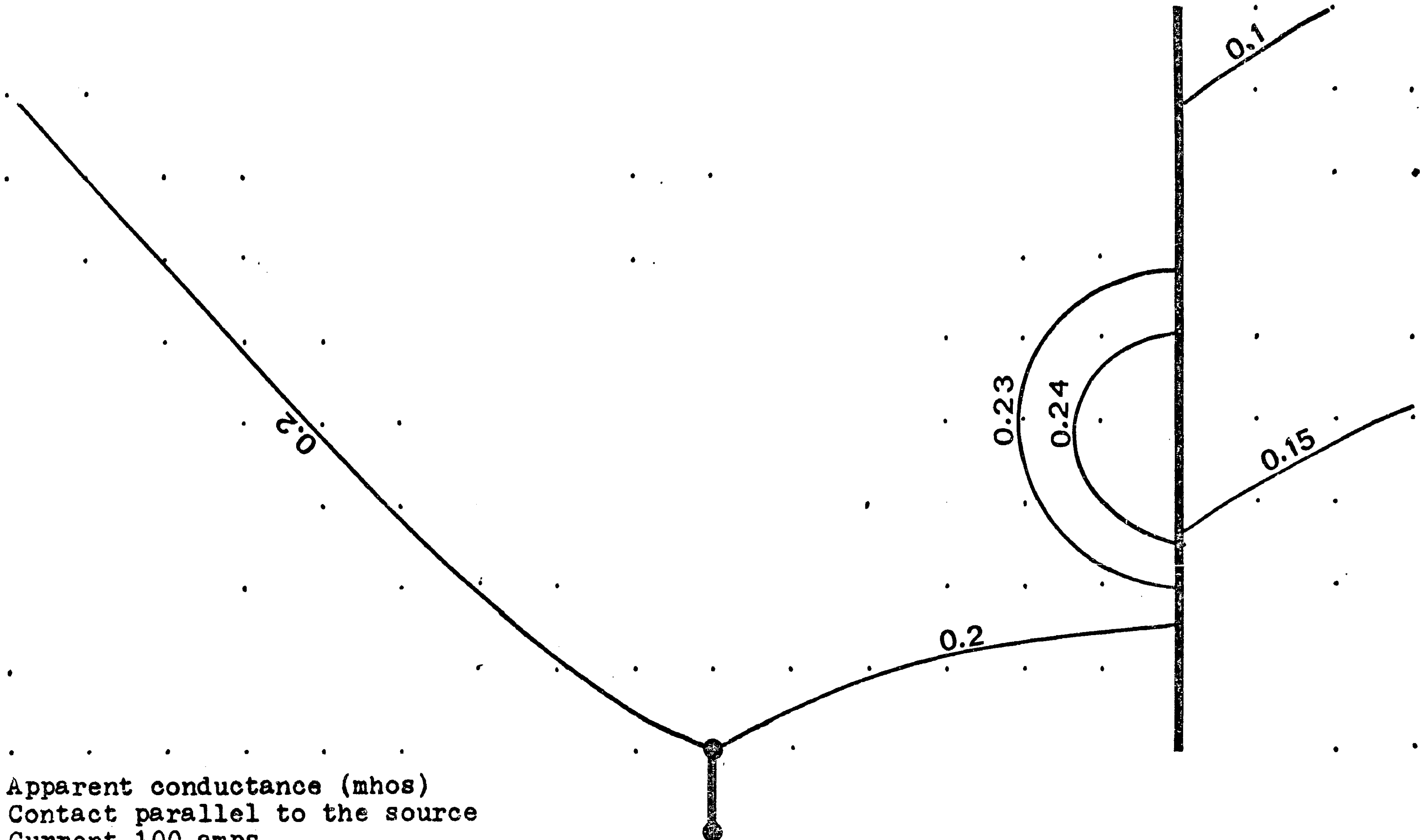
Potential field (volts)
Contact parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 25



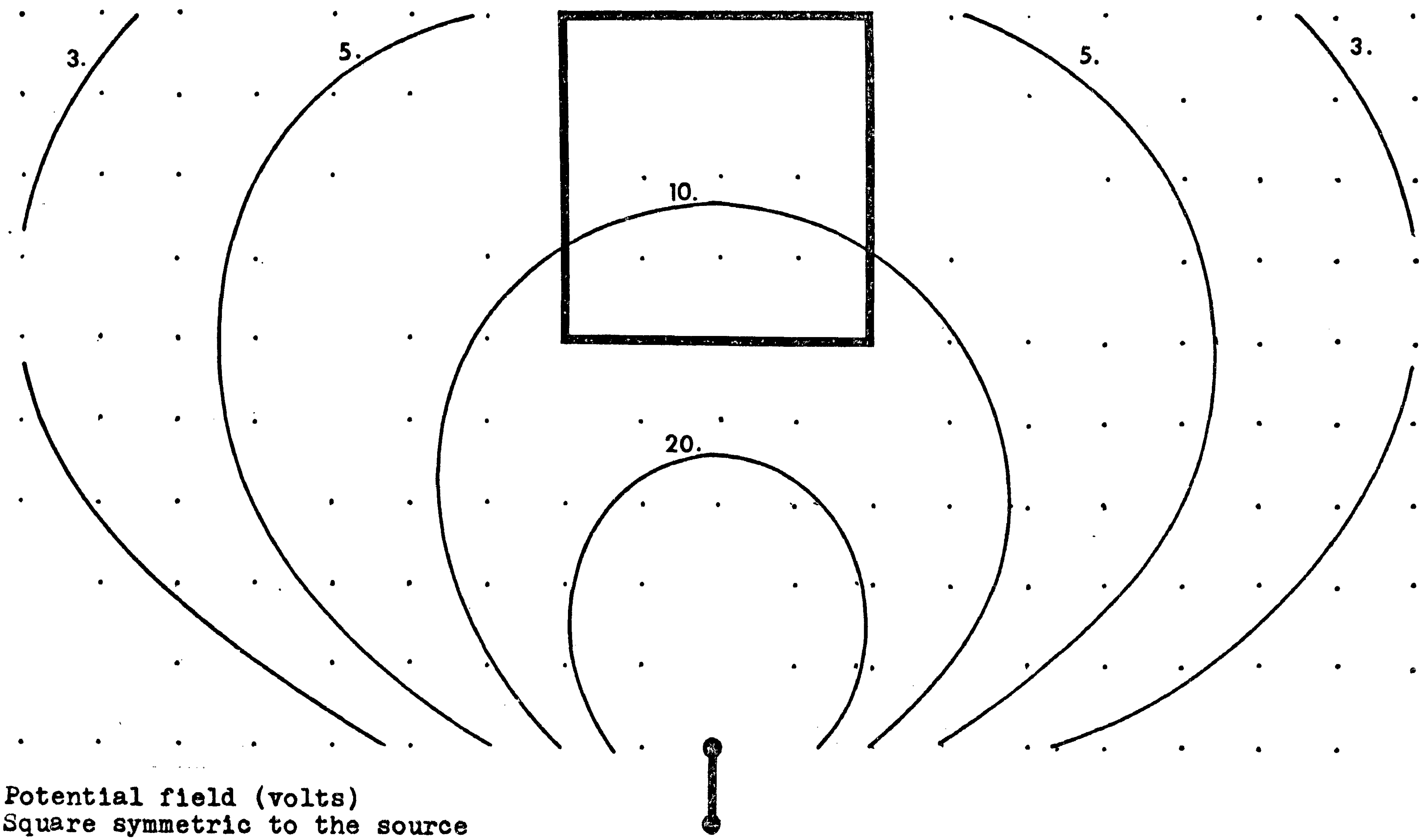
Electric field (volts/unit length)
Contact parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 26



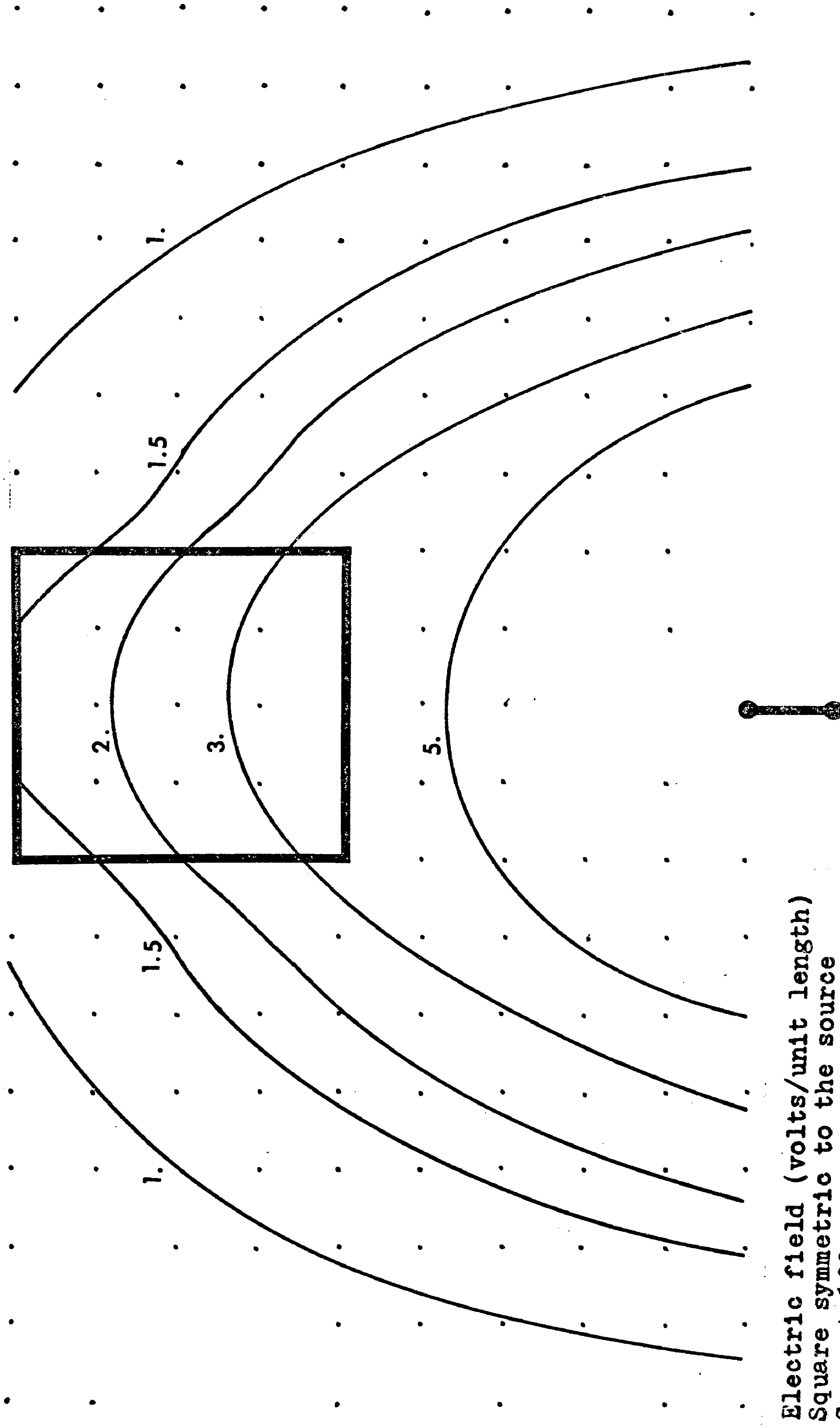
Apparent conductance (mhos)
Contact parallel to the source
Current 100 amps
Resistivities: medium: 5 ohm. unit length
discontinuity: 10 ohm. unit length

Figure 27



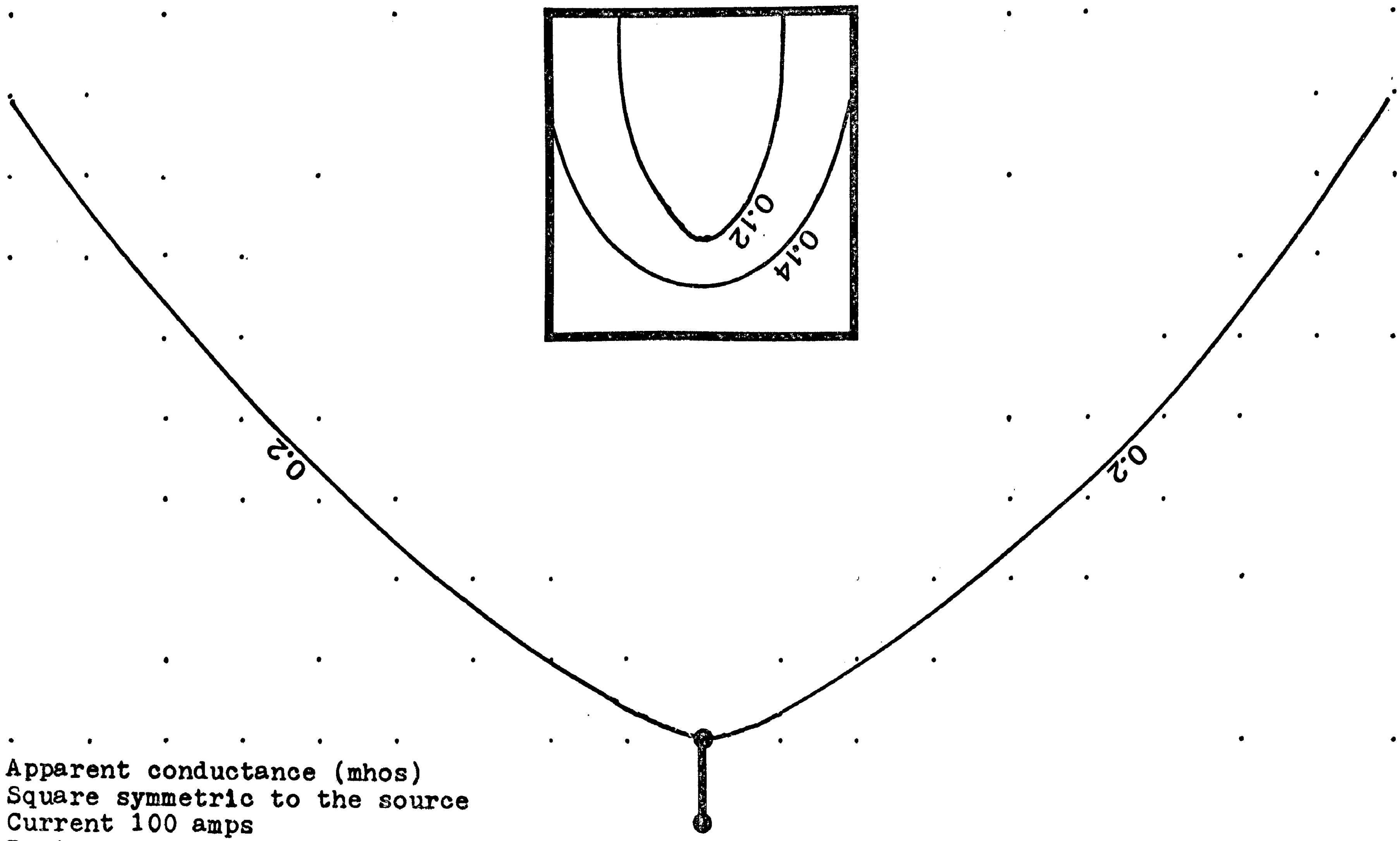
Potential field (volts)
Square symmetric to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 28



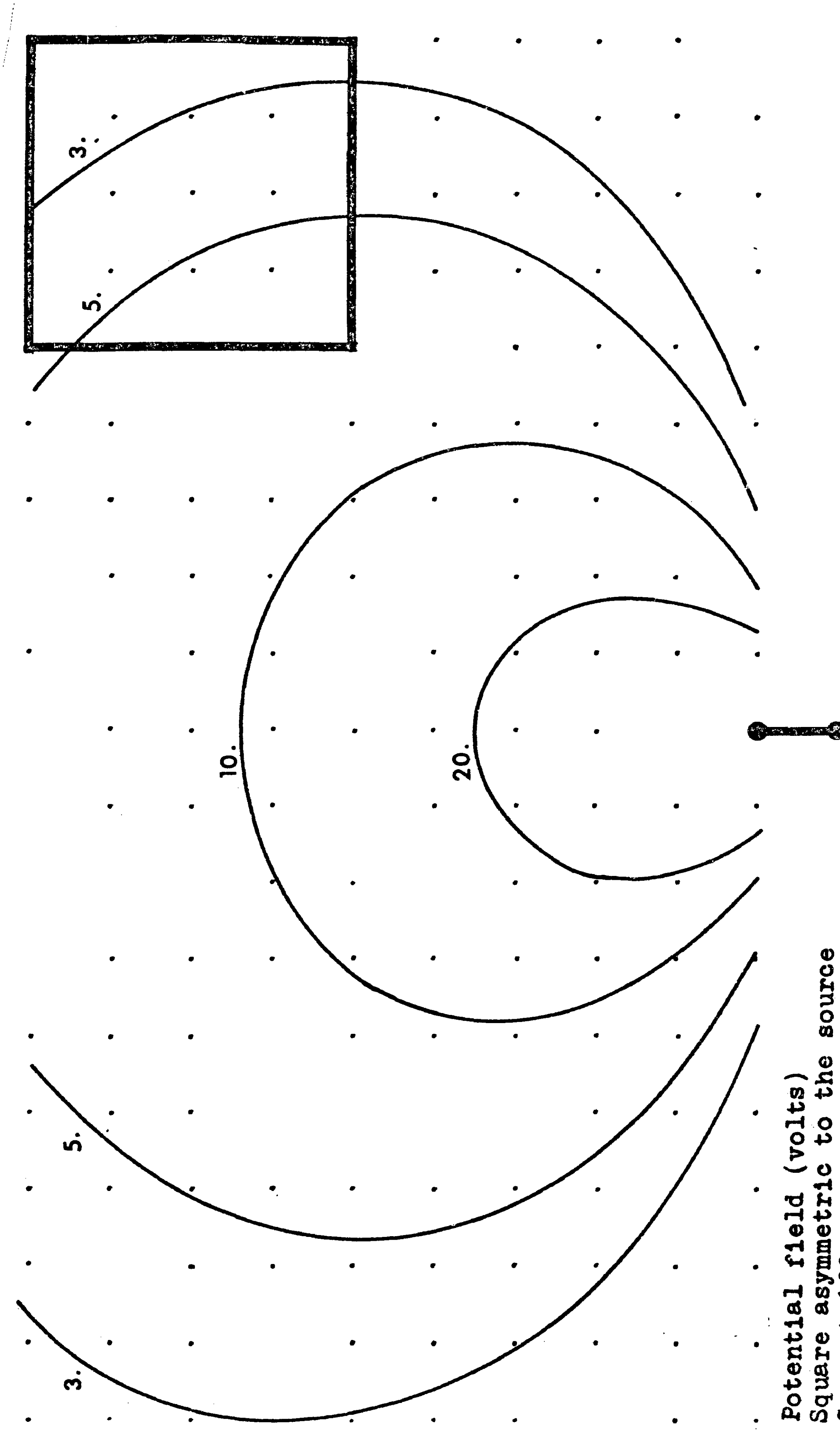
Electric field (volts/unit length)
Square symmetric to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 29



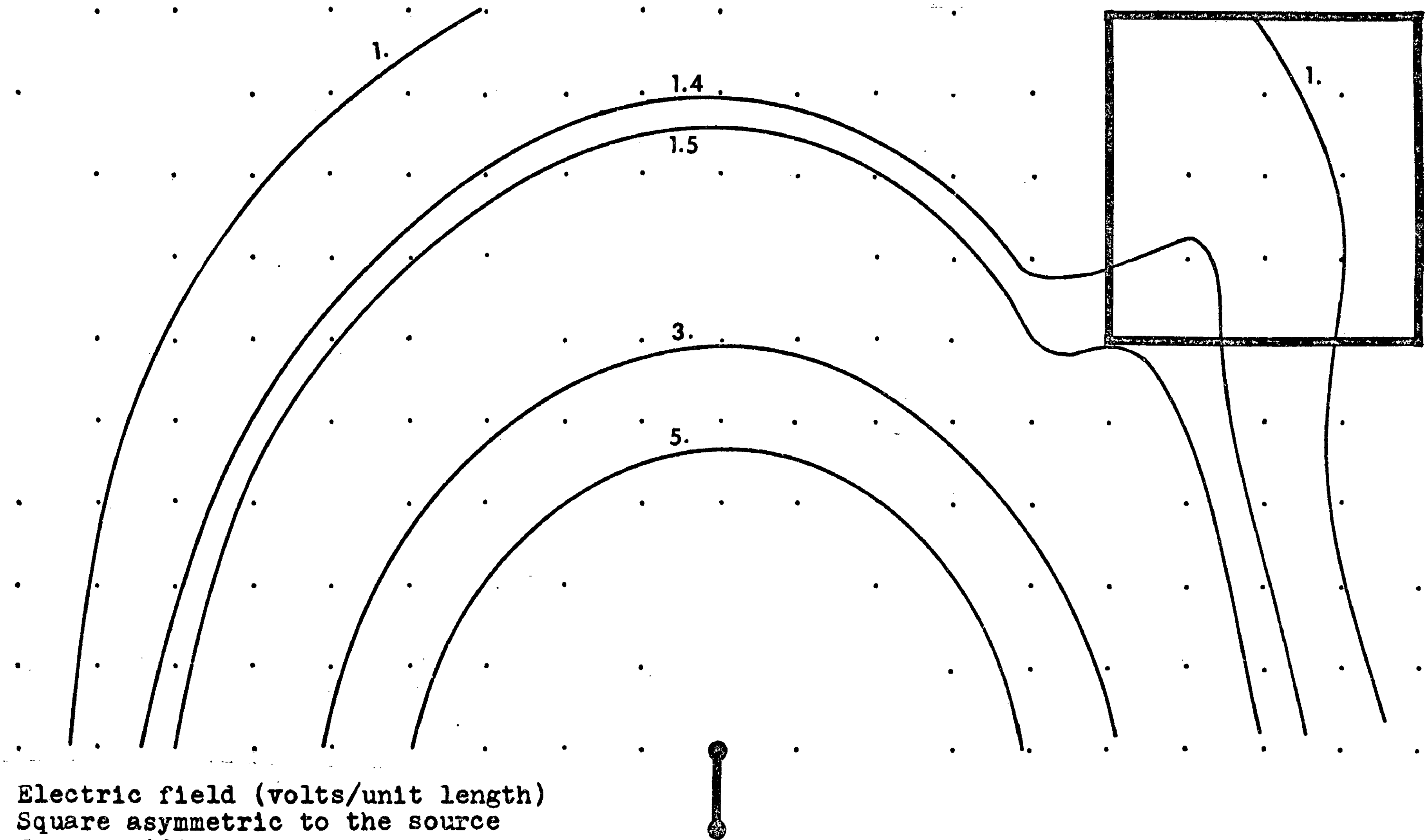
Apparent conductance (mhos)
Square symmetric to the source
Current 100 amps
Resistivities: medium: 5 ohm. unit length
discontinuity: 10 ohm. unit length

Figure 30



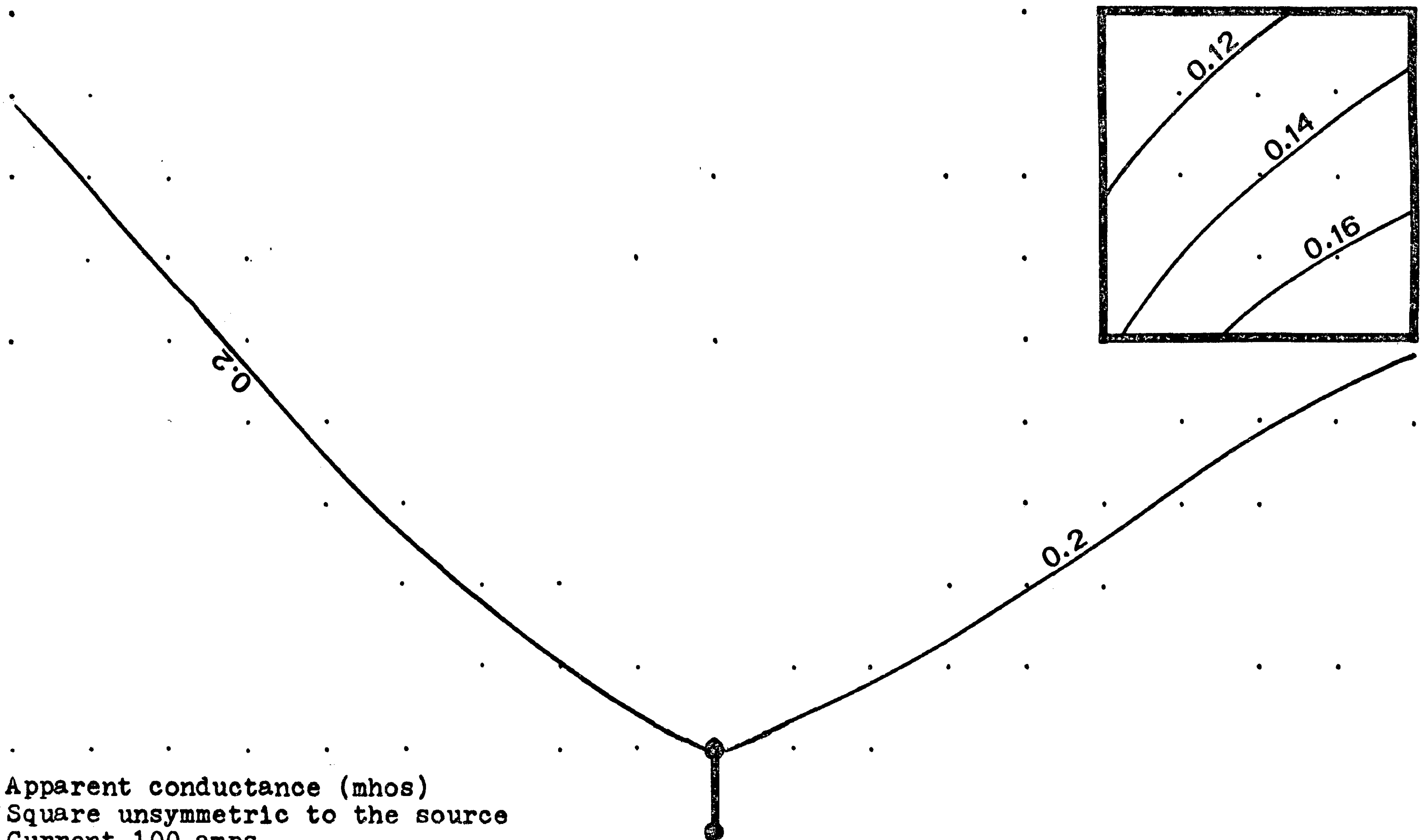
Potential field (volts)
Square asymmetric to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 31



Electric field (volts/unit length)
Square asymmetric to the source
Current 100 amps
Resistivities: medium: 5 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 32



Apparent conductance (mhos)
Square unsymmetric to the source
Current 100 amps
Resistivities: medium: 5 ohm. unit length
discontinuity: 10 ohm. unit length

Figure 33

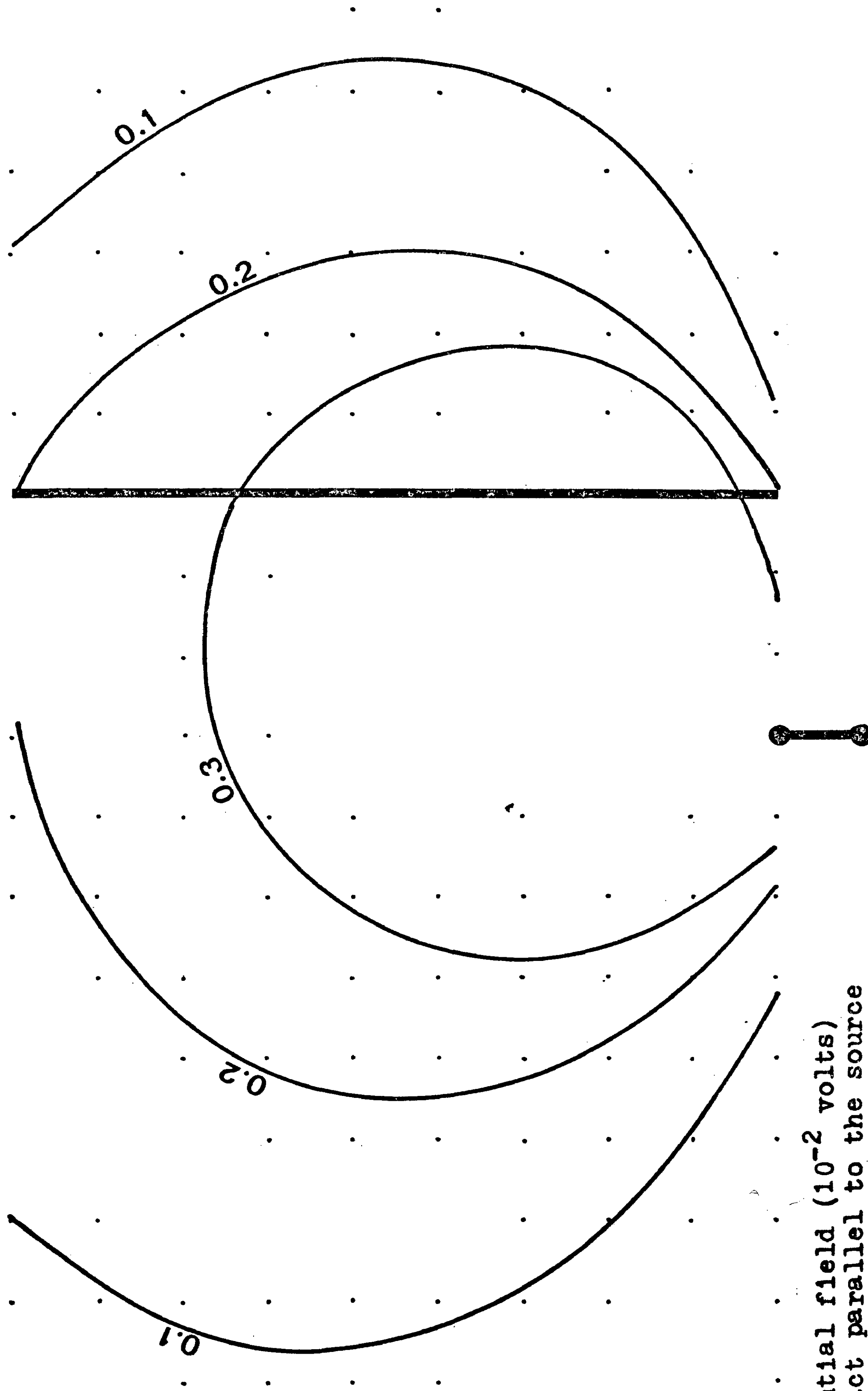
The reason for running these models is that the apparent conductance maps have been obtained analytically by Furgerson and Keller (1974). This fact will enable us to compare the analytical results with the results obtained with the resistor network and reach some conclusions about the capabilities and limitations of the numerical procedure.

For each model I obtained two sets of results. First I considered the discontinuity more resistant than the host medium and second I assume the discontinuity to be more conductive than the continuous part of the medium. The current used in obtaining the results is 1 amp and the resistivity contrast between the medium and the discontinuity being 10:1 or 1:10, depending on the problem.

Here, we should point out that for the case when the source is normal to the discontinuity, Furgerson and Keller (1974) placed the discontinuity 3 units of length away from the center of the dipole. Due to the geometry of the resistor network we had to place the discontinuity at 3 units of length from the closest end of the source. As can be seen from the results, this does not alter the picture in such a way that would prevent a comparative analysis of the results.

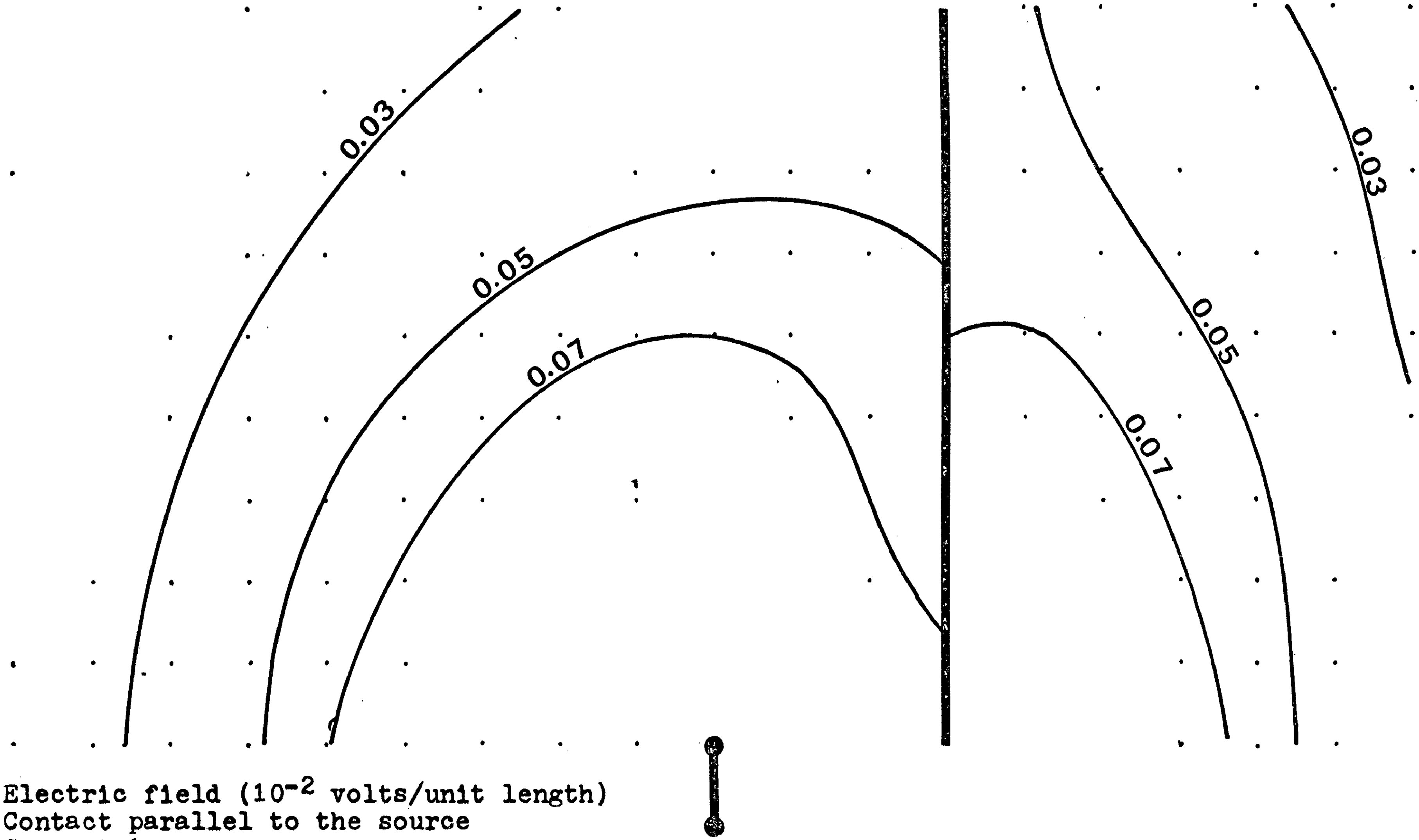
Inasmuch as Furgerson and Keller presented no contour maps of potentials or electric field values, we presented them in a different scale in order to permit a broader picture of the results. For the apparent conductance map I present only the area that is covered by the results of Furgerson and Keller.

The units on these maps are the same as the ones in the previous set of results.



Potential field (10^{-2} volts)
Contact parallel to the source
Current 1 amp
Resistivities: medium: 0.1 ohm. unit length
discontinuity: 1 ohm. unit length

Figure 34



Electric field (10^{-2} volts/unit length)
Contact parallel to the source
Current 1 amp
Resistivities: medium: 0.1 ohm.unit length
discontinuity: 1 ohm.unit length

Figure 35

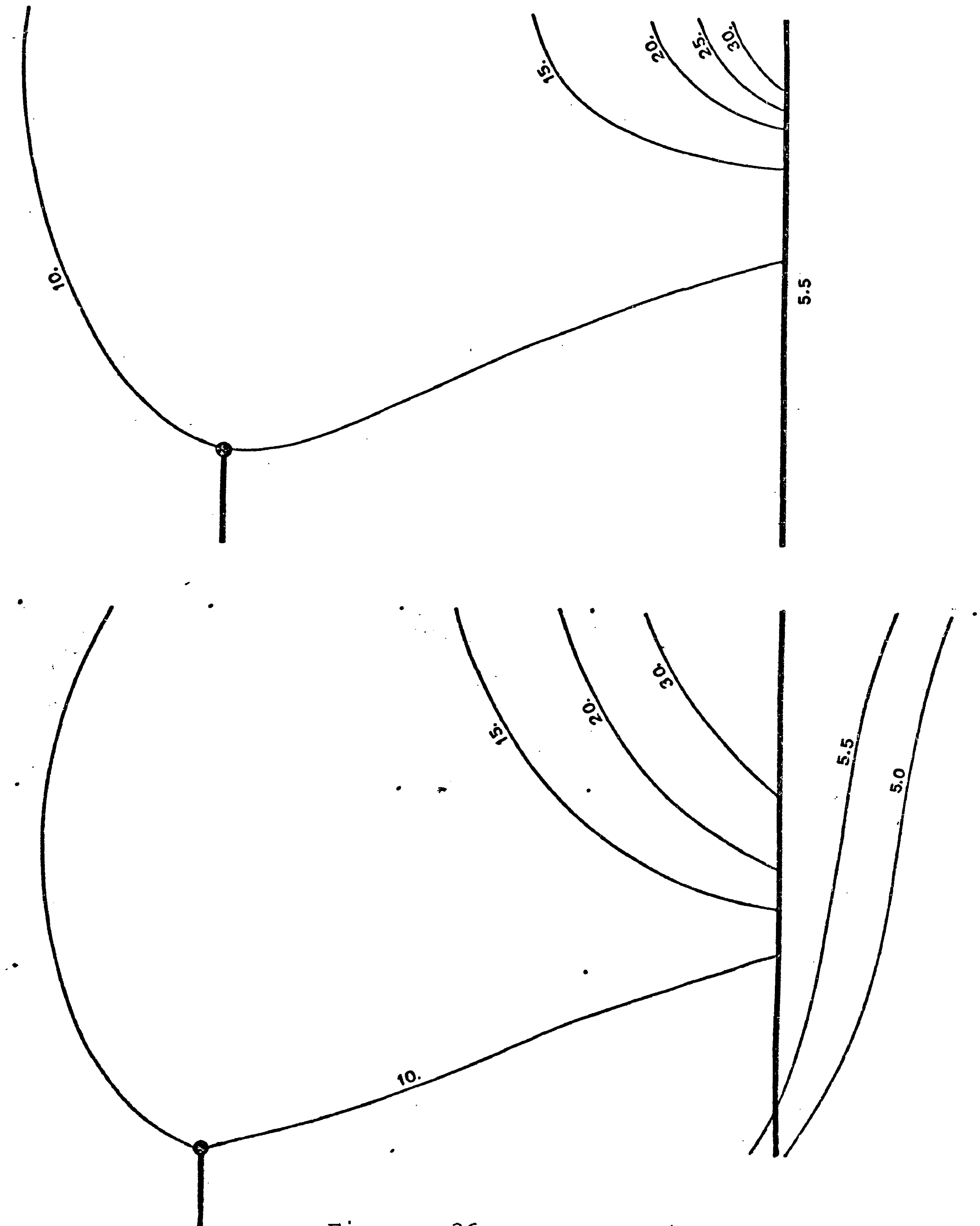
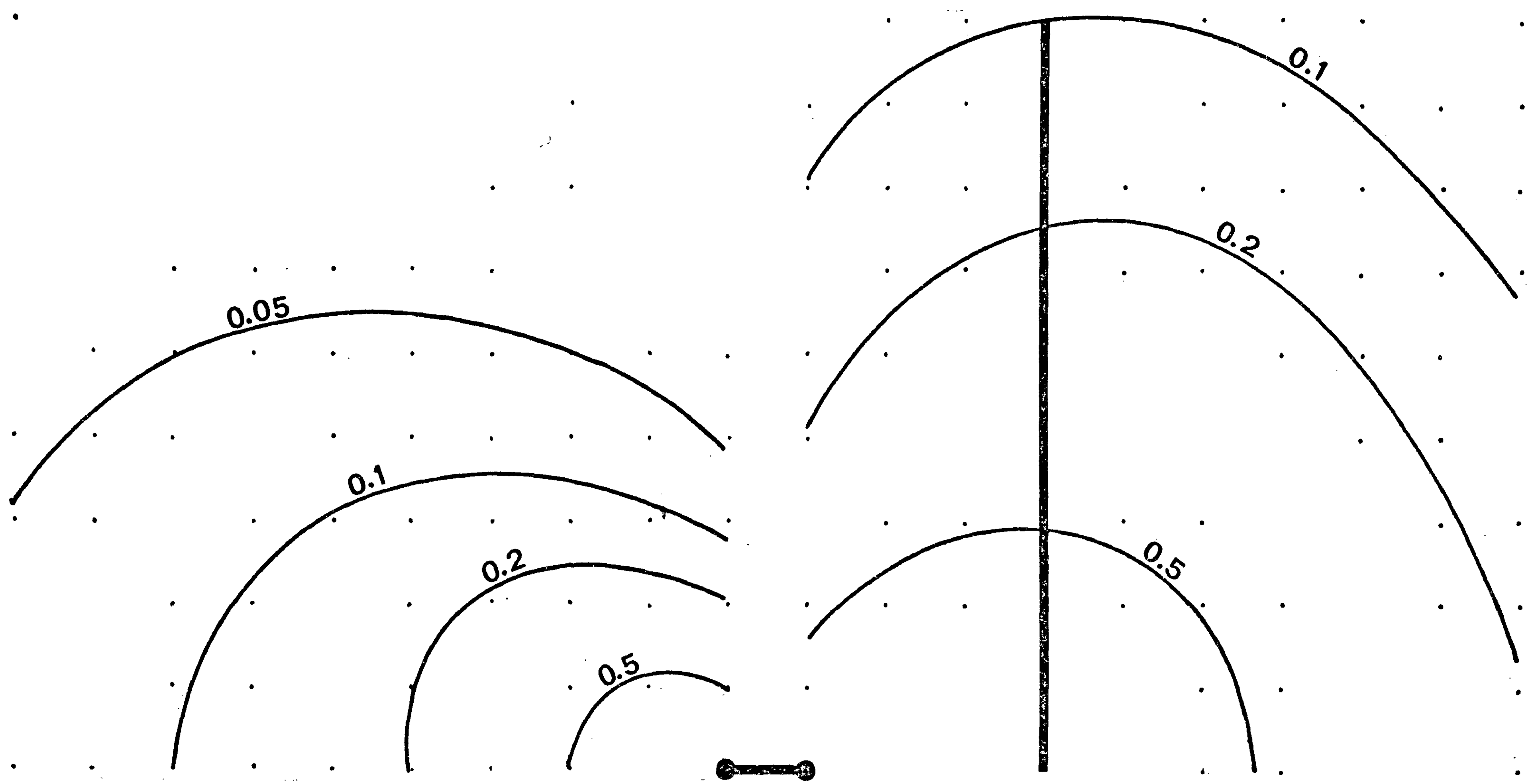


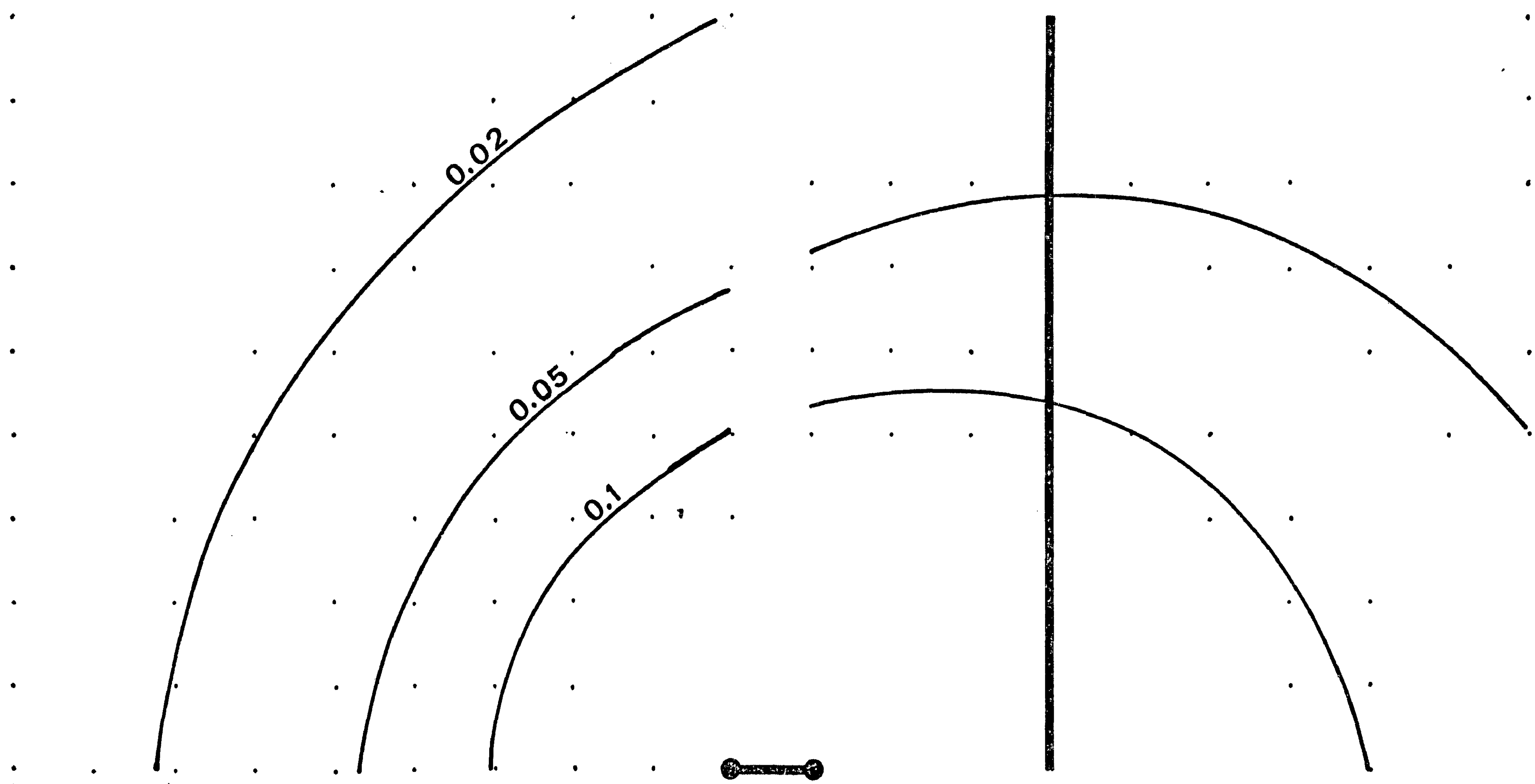
Figure 36

Apparent conductance (mhos) for a contact parallel to the source. A current of 1 amp and resistivities of 0.1 ohm. unit length (medium) and 1 ohm. unit length (discontinuity) are used. Results on top after Furgerson and Keller (1974).



Potential field (10^{-2} volts)
Contact normal to the source
Current 1 amp
Resistivities: medium: 0.1 ohm. unit length
discontinuity: 1 ohm. unit length

Figure 37



Electric field (10^{-2} volts/unit length)
Contact normal to the source
Current 1 amp
Resistivities: medium: 0.1 ohm.unit length
discontinuity: 1 ohm.unit length

Figure 38

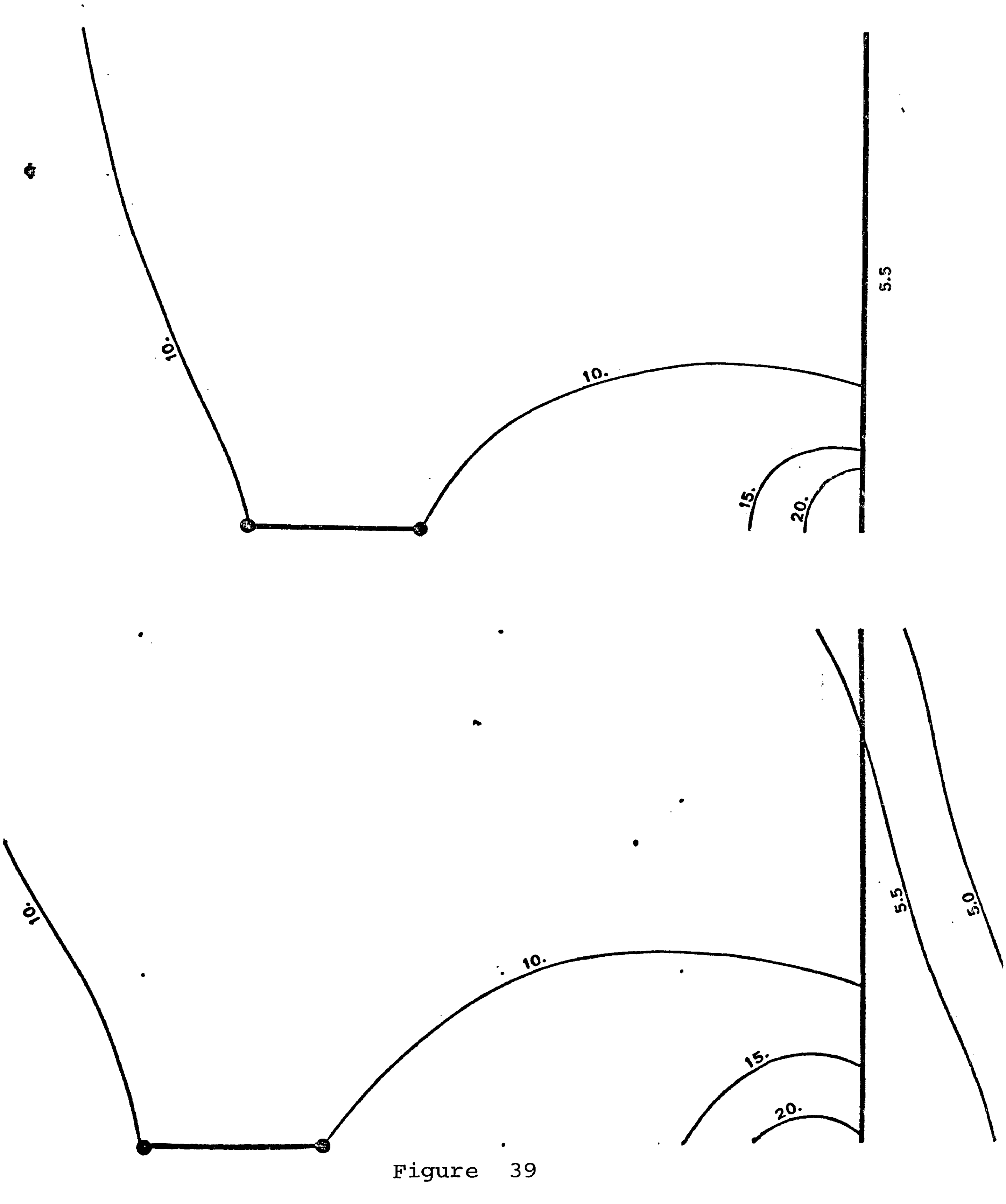
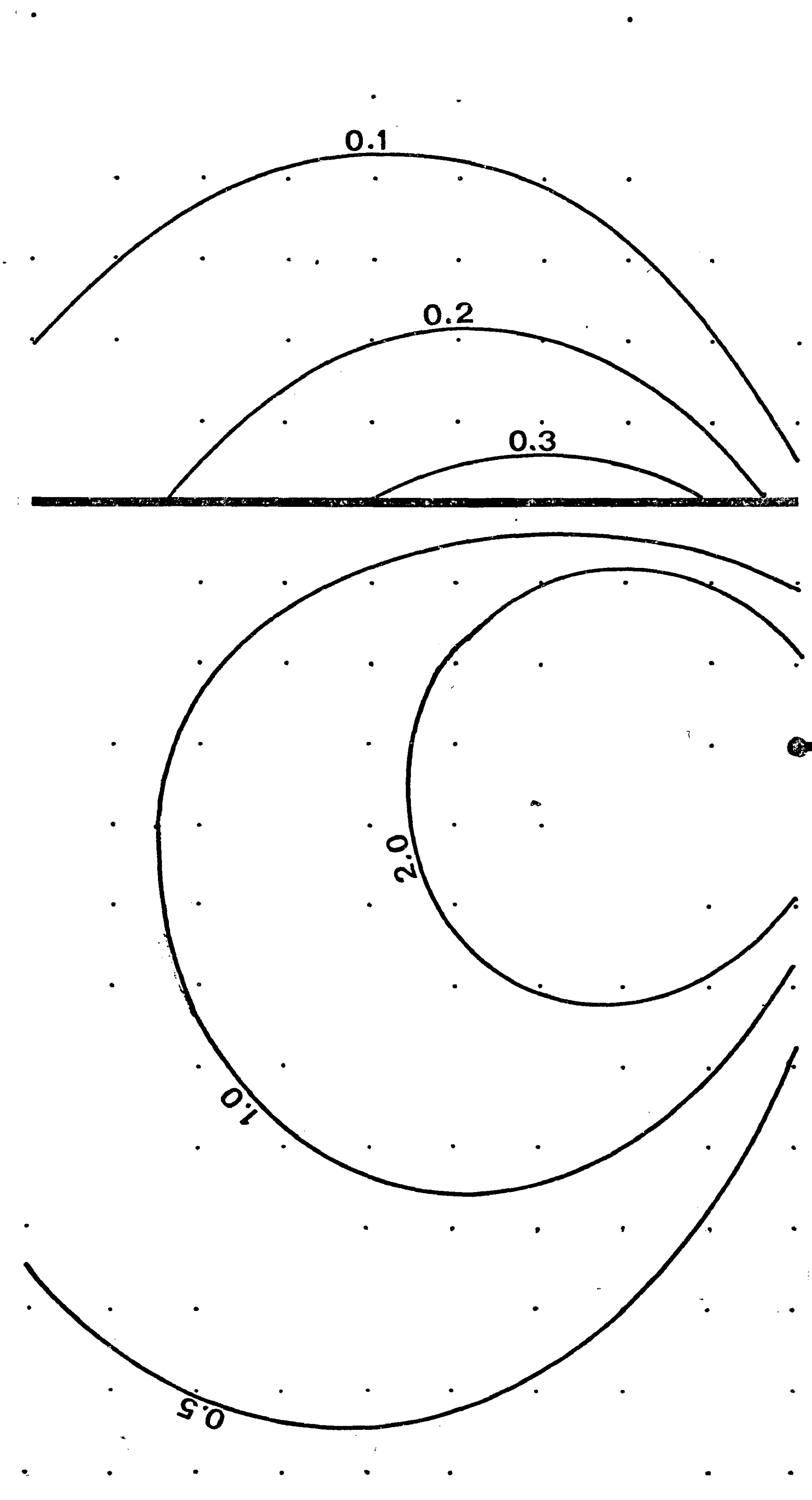


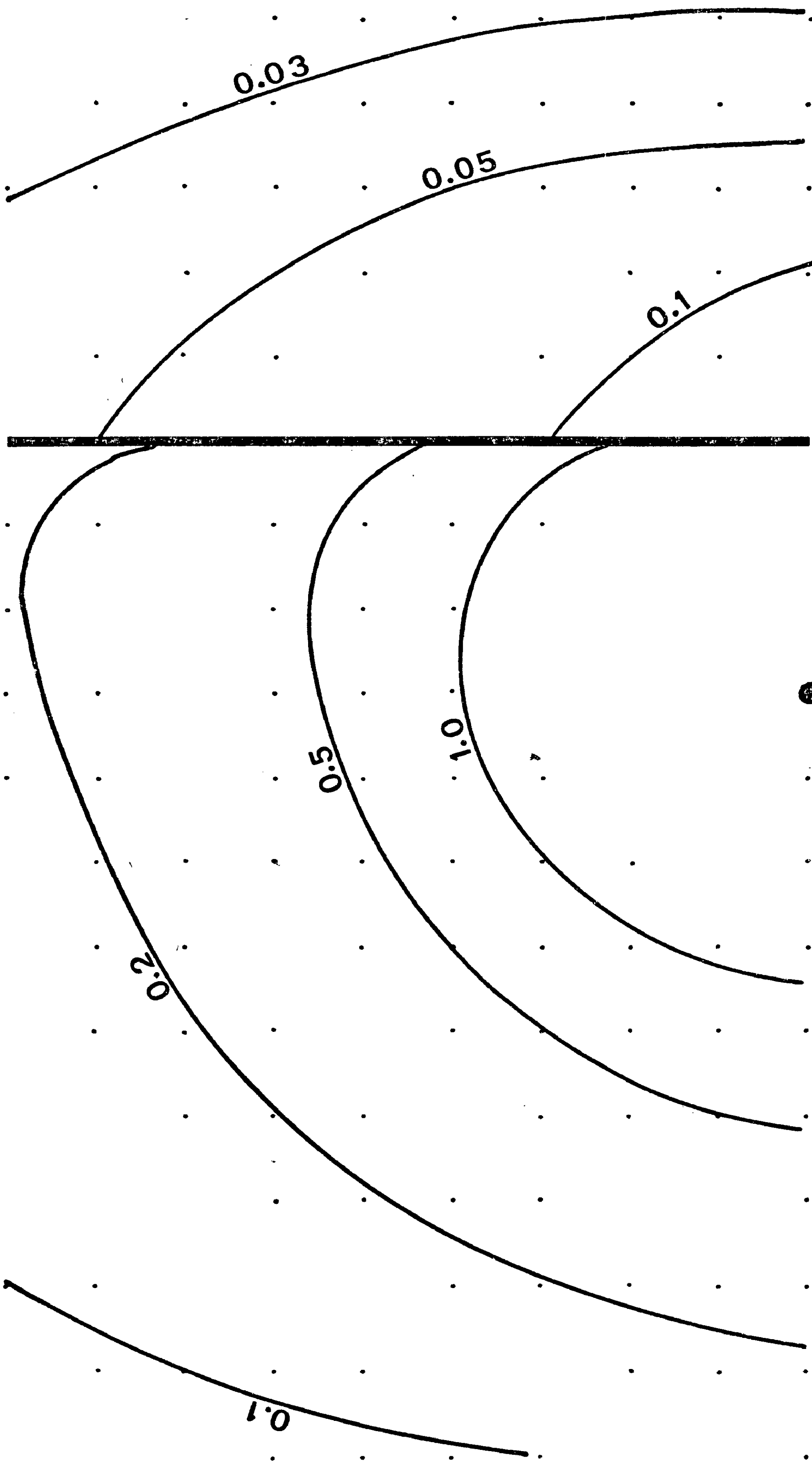
Figure 39

Apparent conductance (mhos) for a contact normal to the source. A current of 1 amp and resistivities of 0.1 ohm. unit length (medium) and 1 ohm. unit length (discontinuity) are used. Results on top after Furgerson and Keller (1974).



Potential field (10^{-2} volts)
Contact parallel to the source
Current 1 amp
Resistivities: medium: 1 ohm. unit length
discontinuity: 0.1 ohm. unit length

Figure 40



Electric field (10^{-2} volts/unit length)
Contact parallel to the source
Current 1 amp
Resistivities: medium: 1 ohm.unit length
discontinuity: 0.1 ohm.unit length

Figure 41

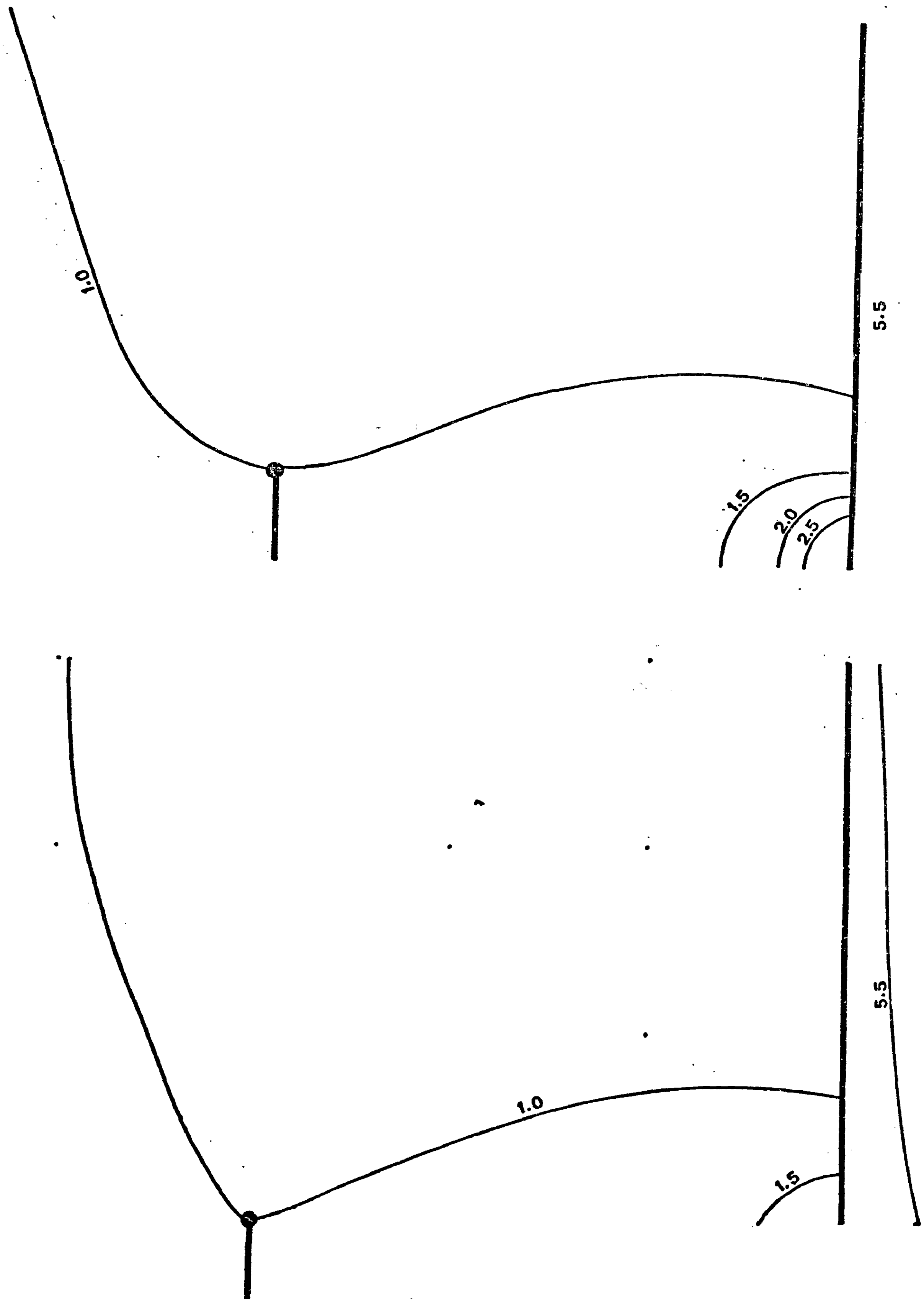
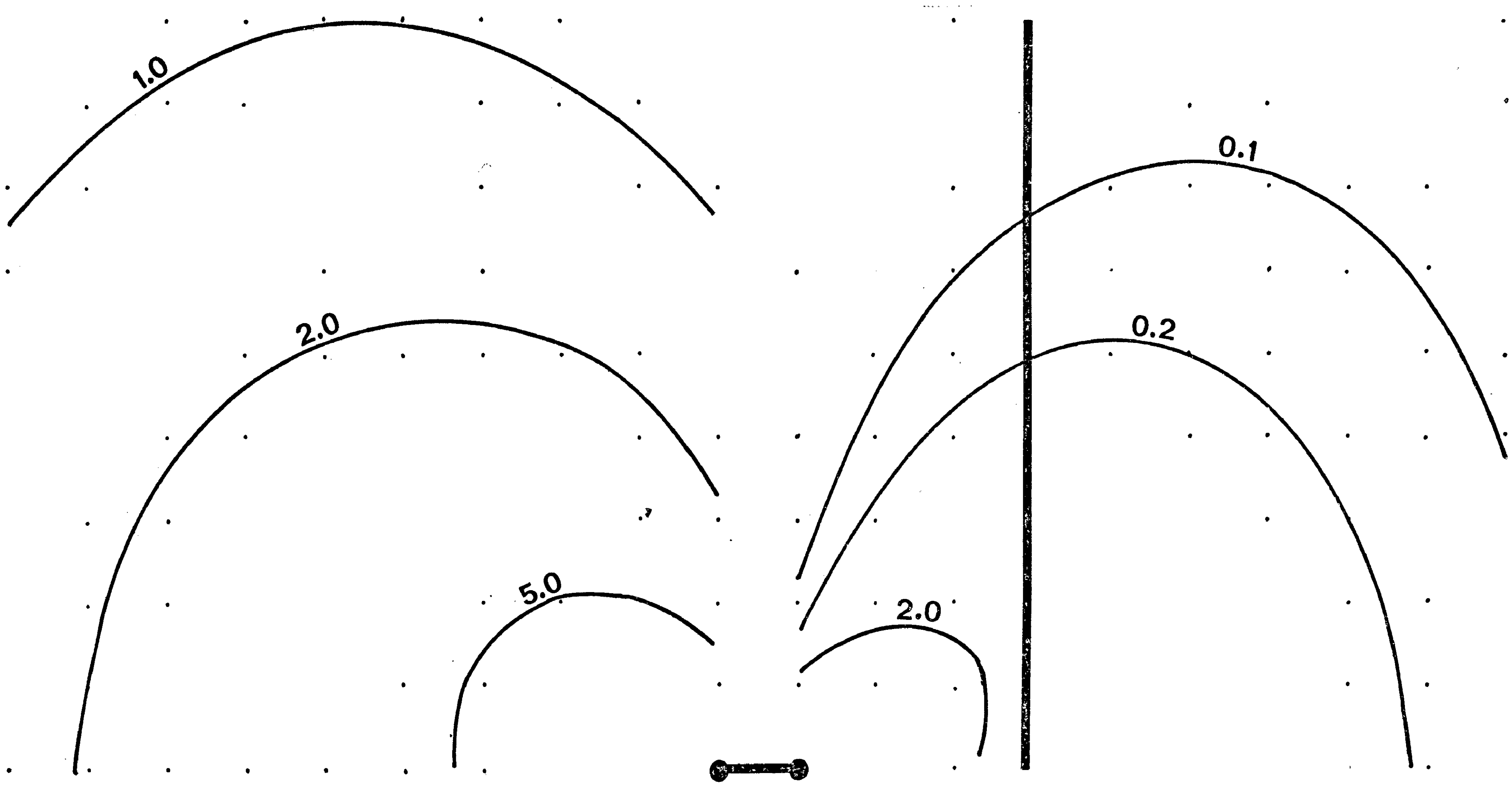


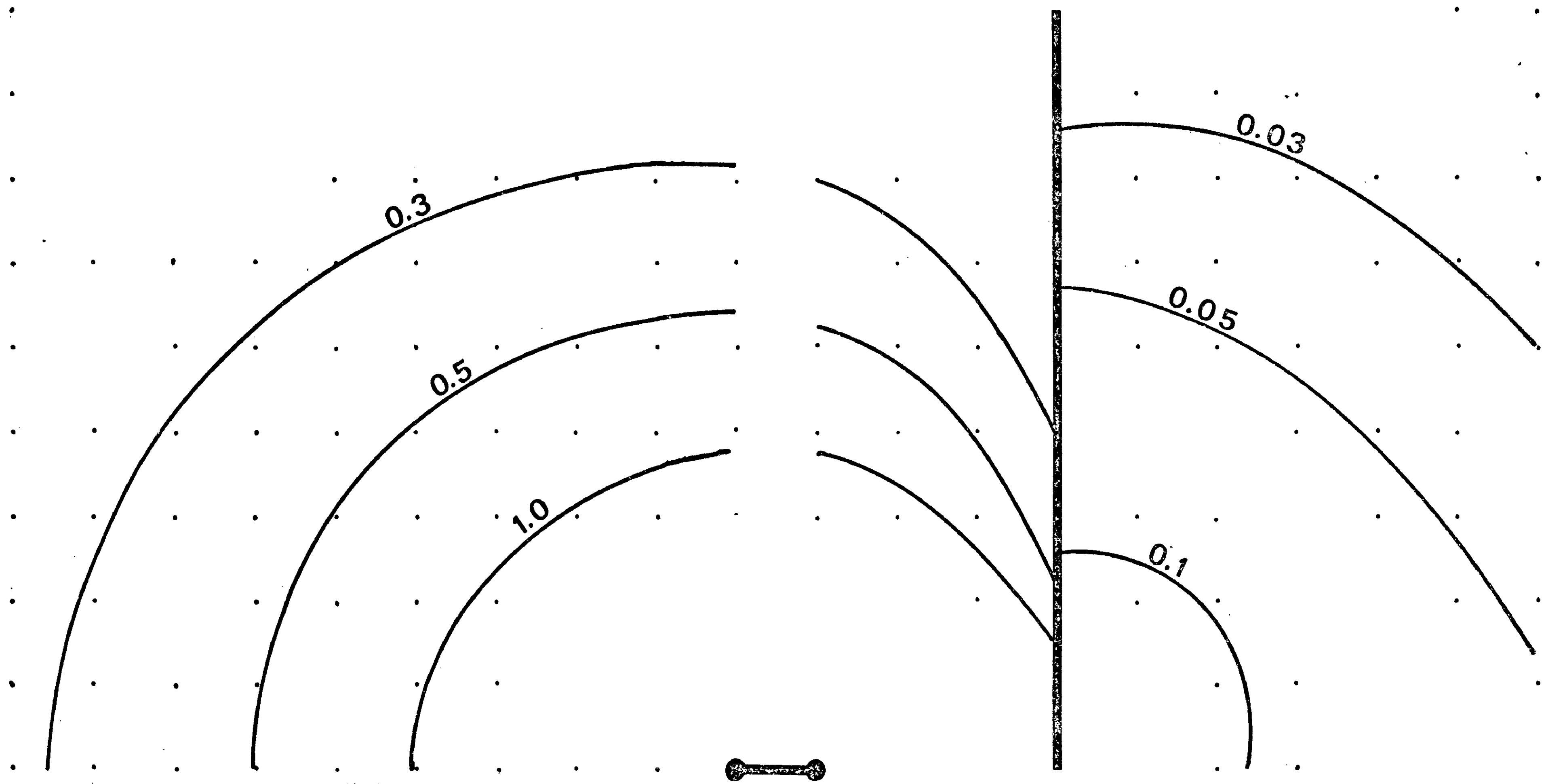
Figure 42

Apparent conductance (mhos) for a contact parallel to the source. A current of 1 amp and resistivities of 1 ohm. unit length (medium) and 0.1 ohm. unit length (discontinuity) are used. Results on top after Furgerson and Keller (1974).



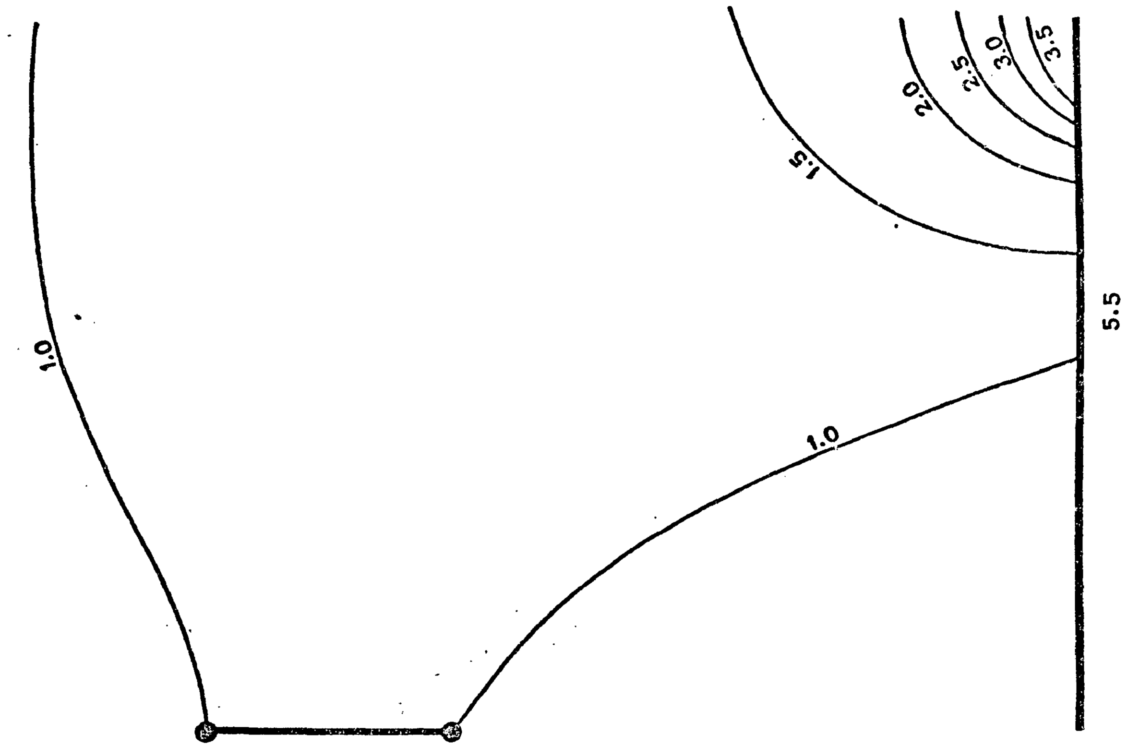
Potential field (10^{-2} volts)
Contact normal to the source
Current 1 amp
Resistivities: medium: 1 ohm. unit length
discontinuity: 0.1 ohm. unit length

Figure 43



Electric field (10^{-2} volts/unit length)
Contact normal to the source
Current 1 amp
Resistivities: medium: 1 ohm.unit length
discontinuity: 0.1 ohm.unit length

Figure 44



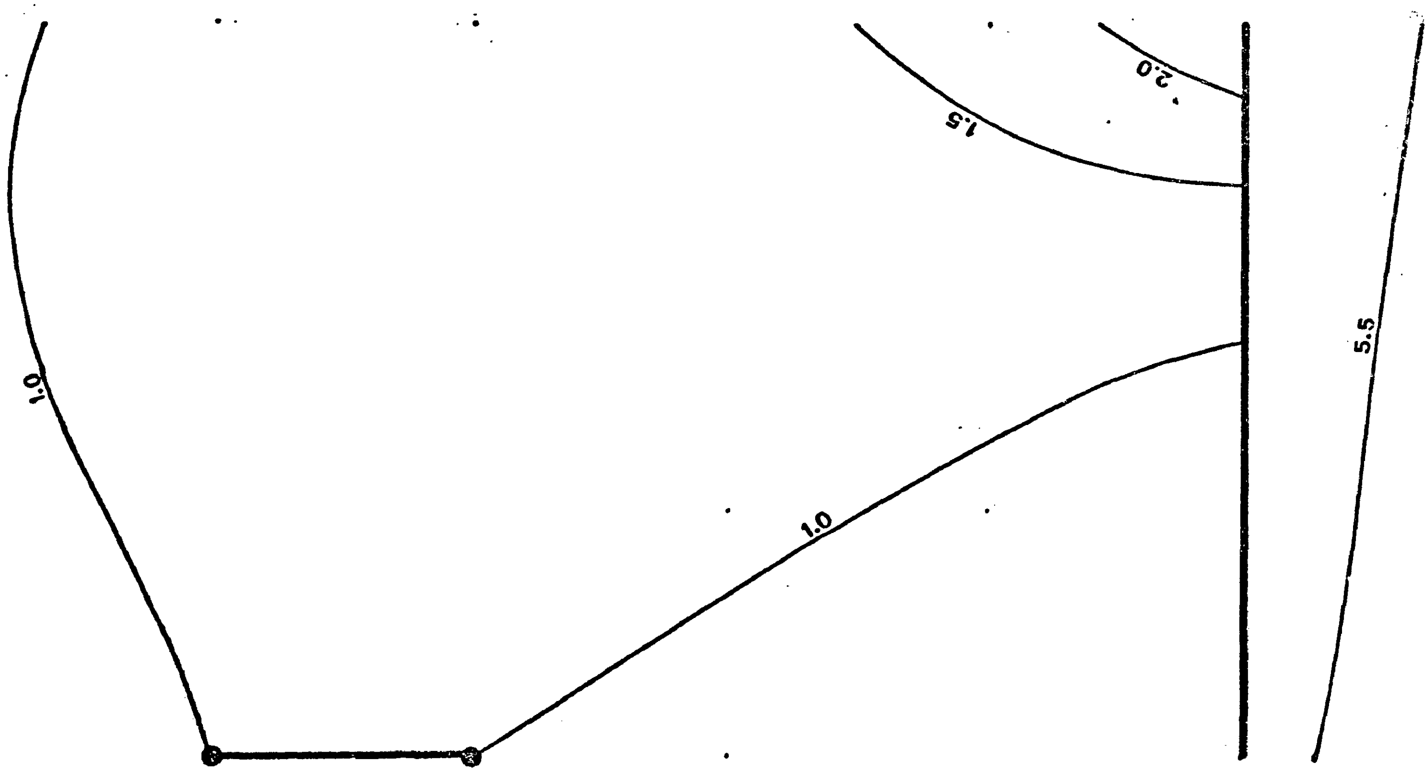
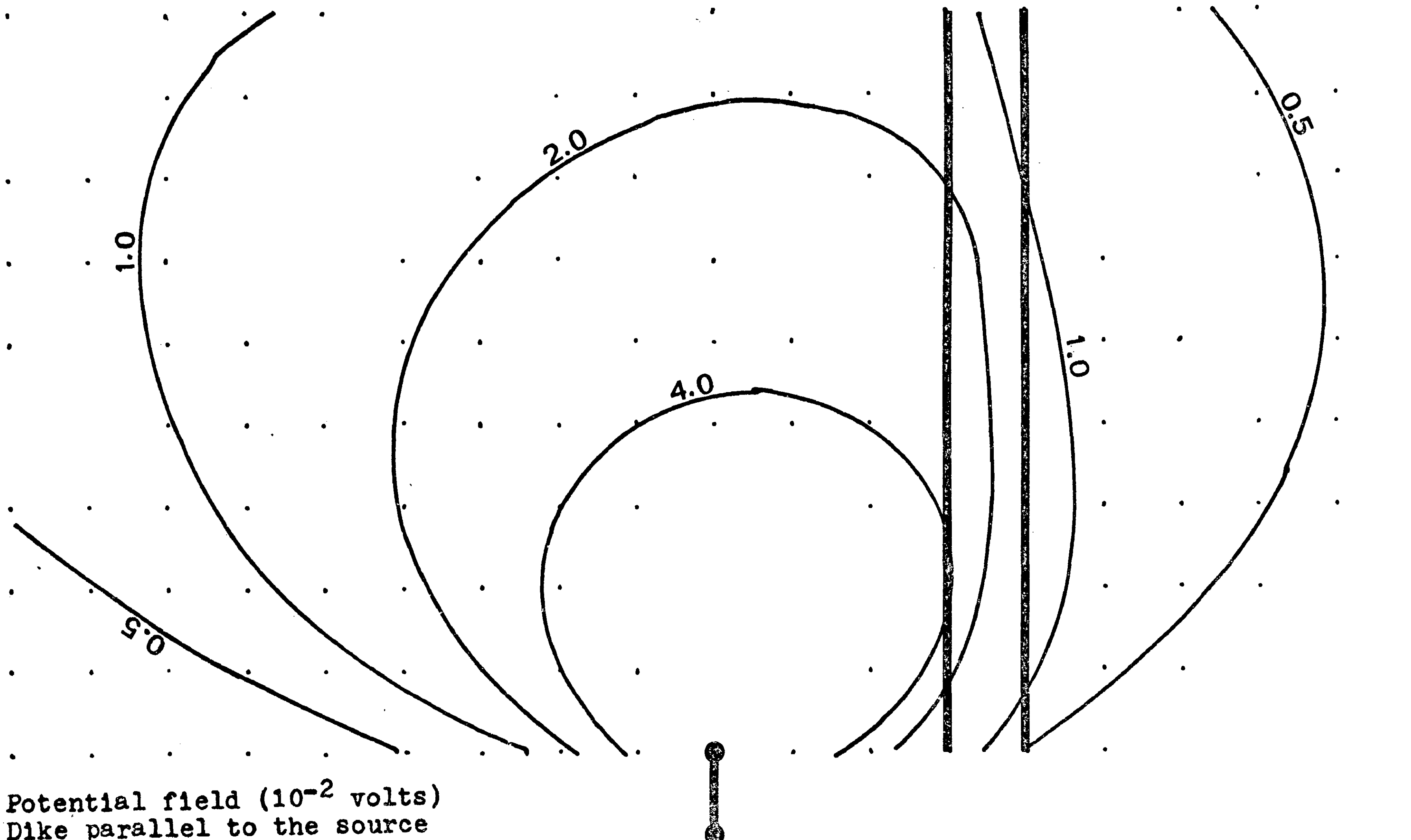


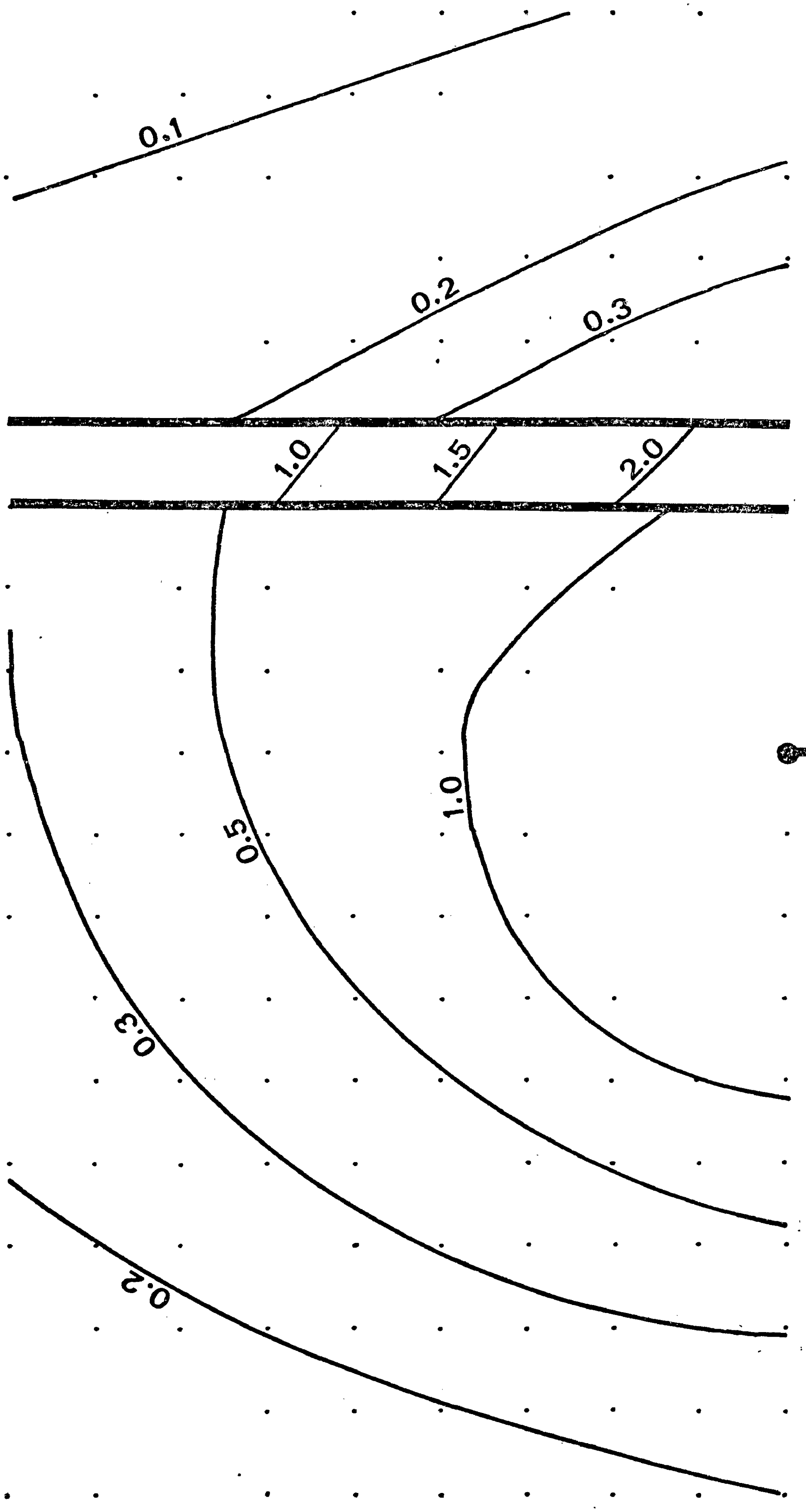
Figure 45

Apparent conductance (mhos) for a contact normal to the source. A current of 1 amp and resistivities of 1 ohm. unit length (medium) and 0.1 ohm. unit length (discontinuity) are used. Results on top after Furgerson and Keller (1974).



Potential field (10^{-2} volts)
Dike parallel to the source
Current 1 amp
Resistivities: medium: 1 ohm. unit length
discontinuity: 10 ohm. unit length

Figure 46



Electric field (10^{-2} volts/unit length)
Dike parallel to the source
Current 1 amp
Resistivities: medium: 1 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 47

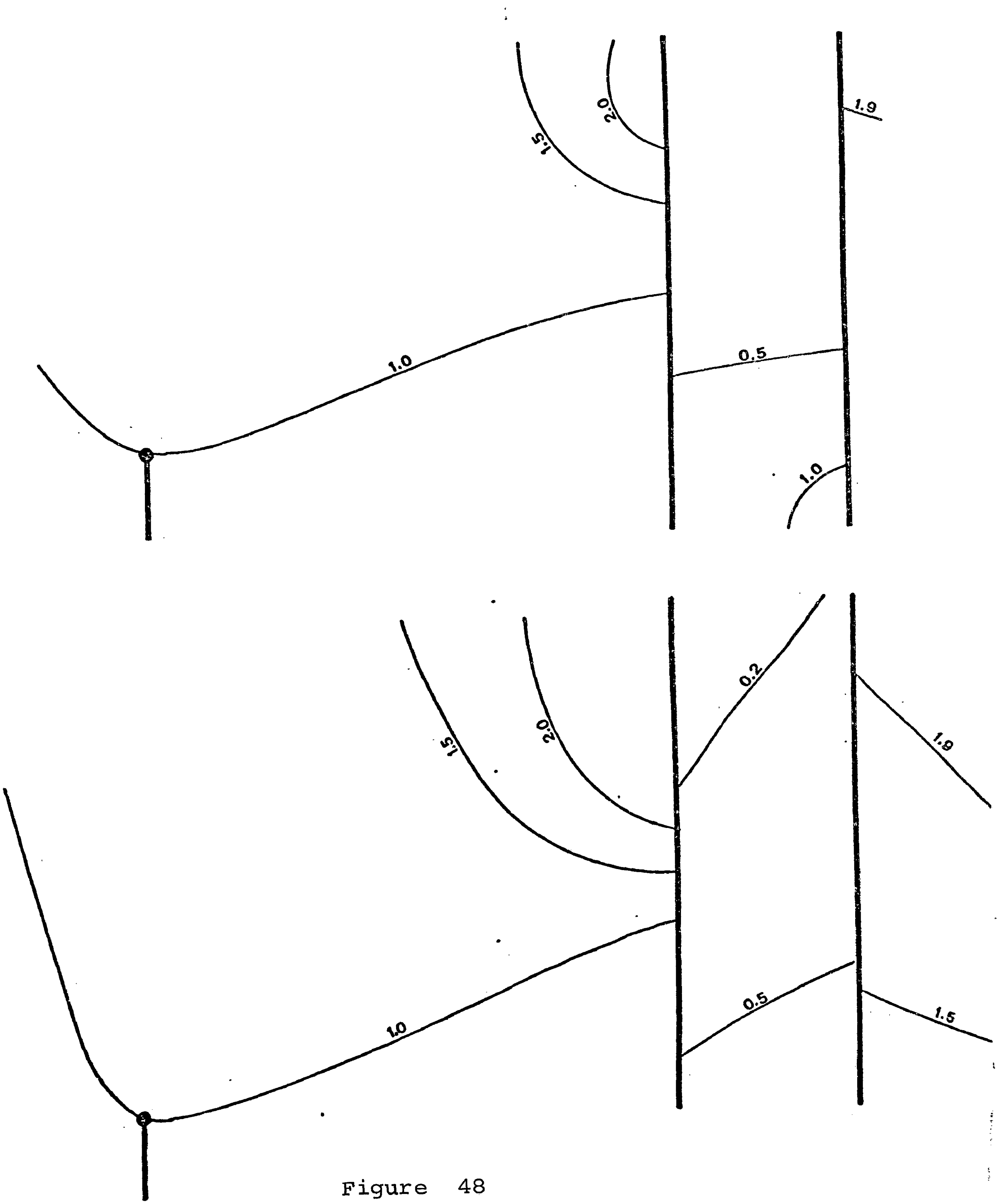
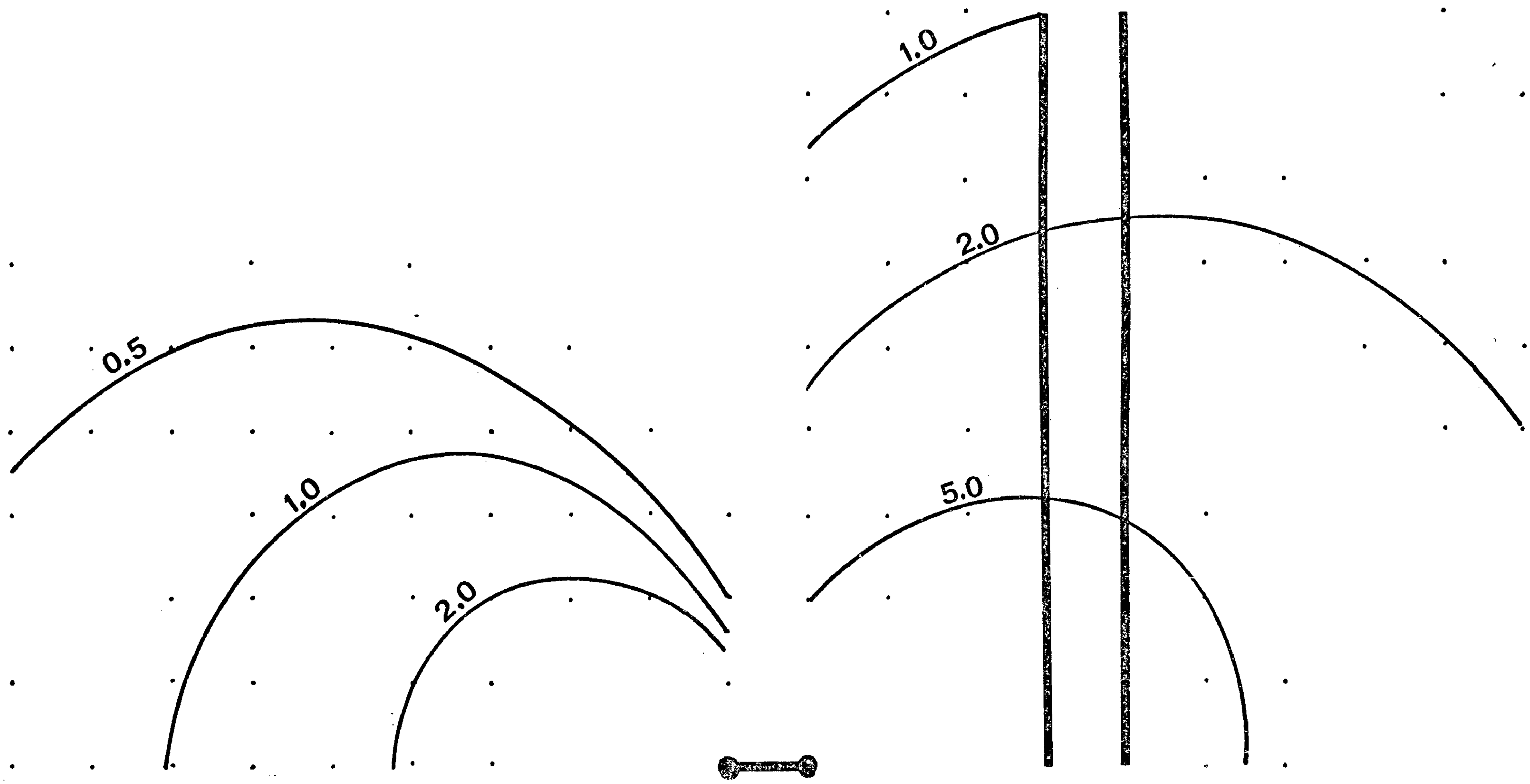


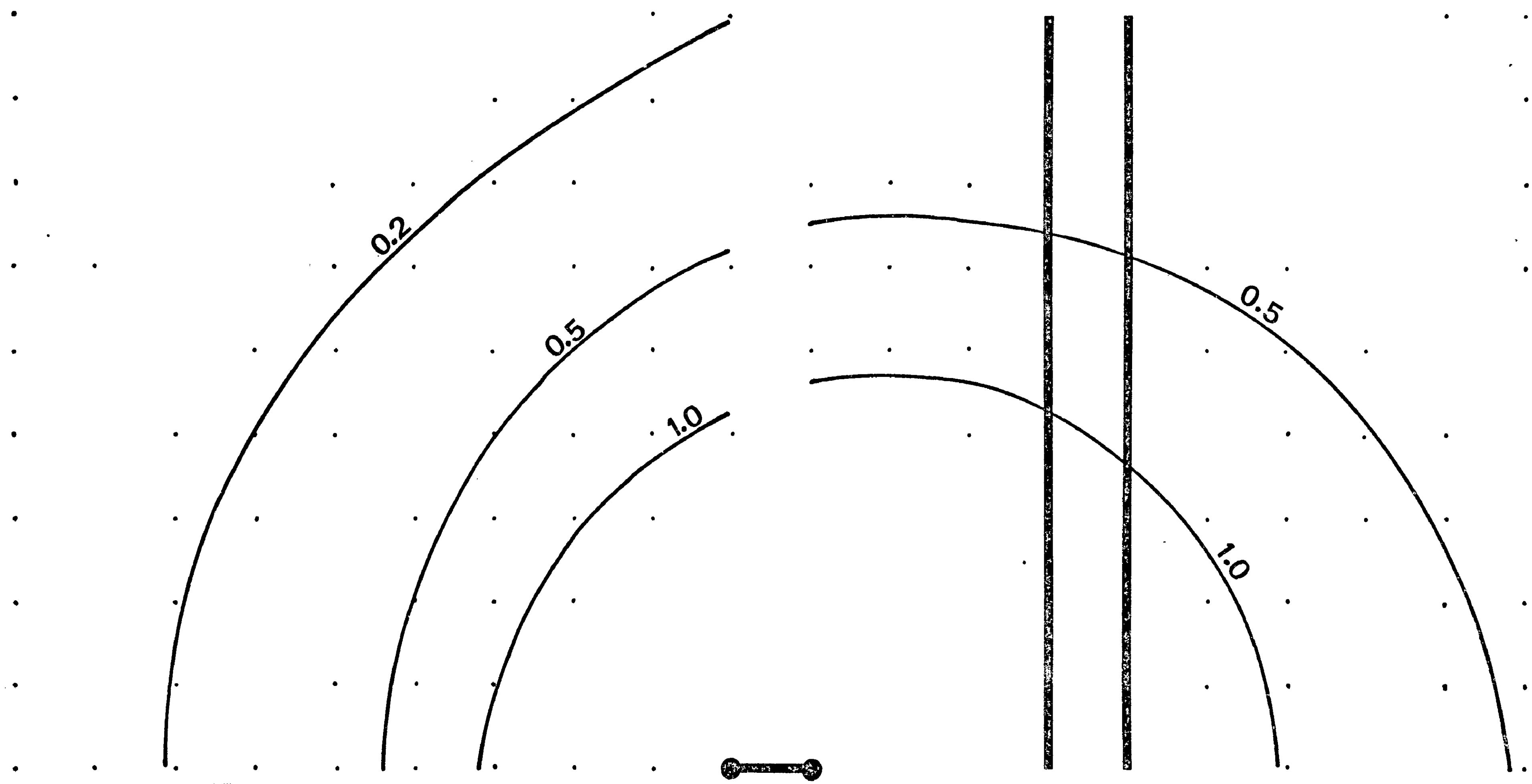
Figure 48

Apparent conductance (mhos) for a dike parallel to the source. A current of 1 amp and resistivities of 1 ohm. unit length (medium) and 10 ohm. unit length (discontinuity) are used. Results on top after Furgerson and Keller (1974).



Potential field (10^{-2} volts)
Dike normal to the source
Current 1 amp
Resistivities: medium: 1 ohm. unit length
discontinuity: 10 ohm. unit length

Figure 49



Electric field (10^{-2} volts/unit length)
Dike normal to the source
Current 1 amp
Resistivities: medium: 1 ohm.unit length
discontinuity: 10 ohm.unit length

Figure 50

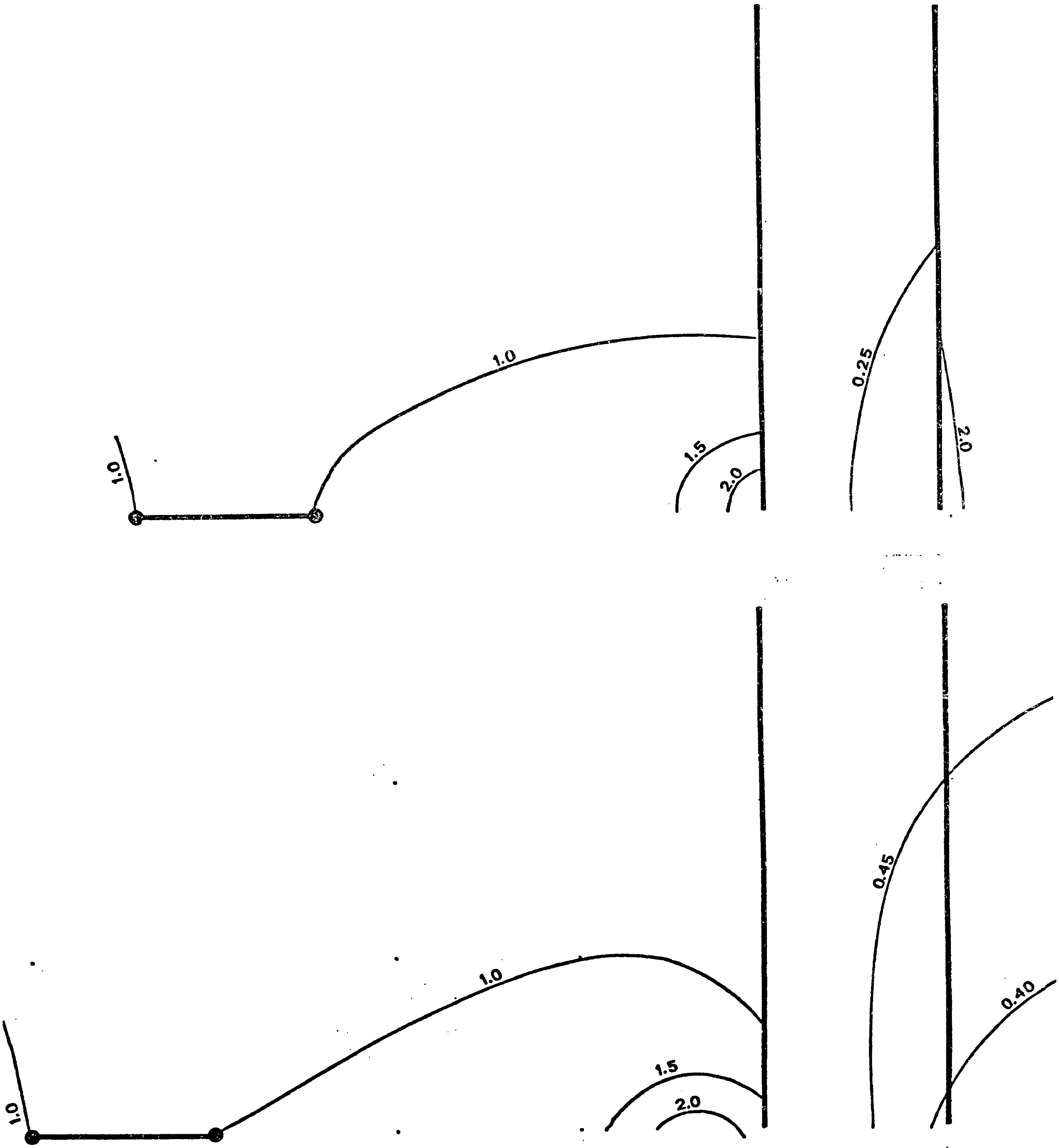
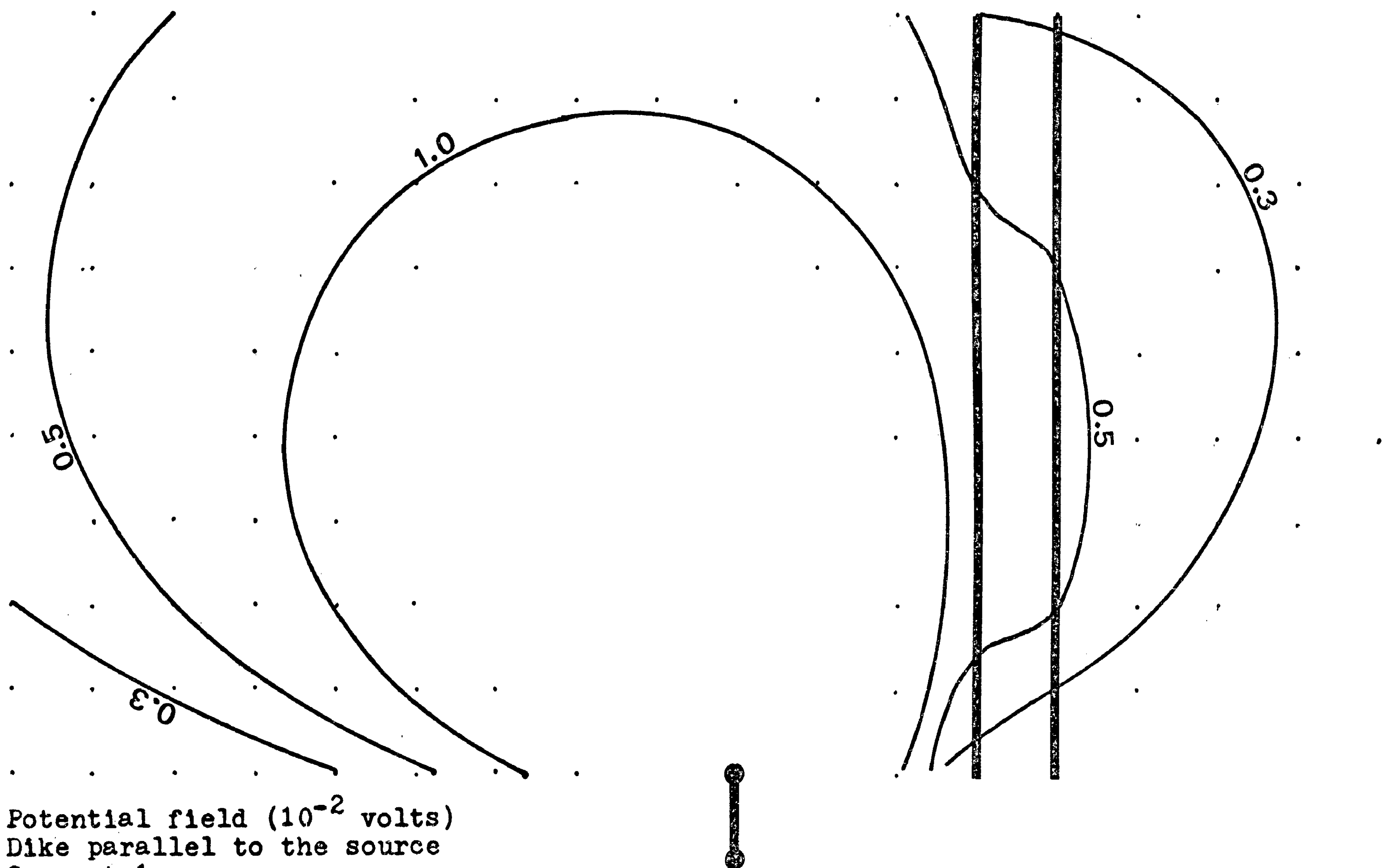


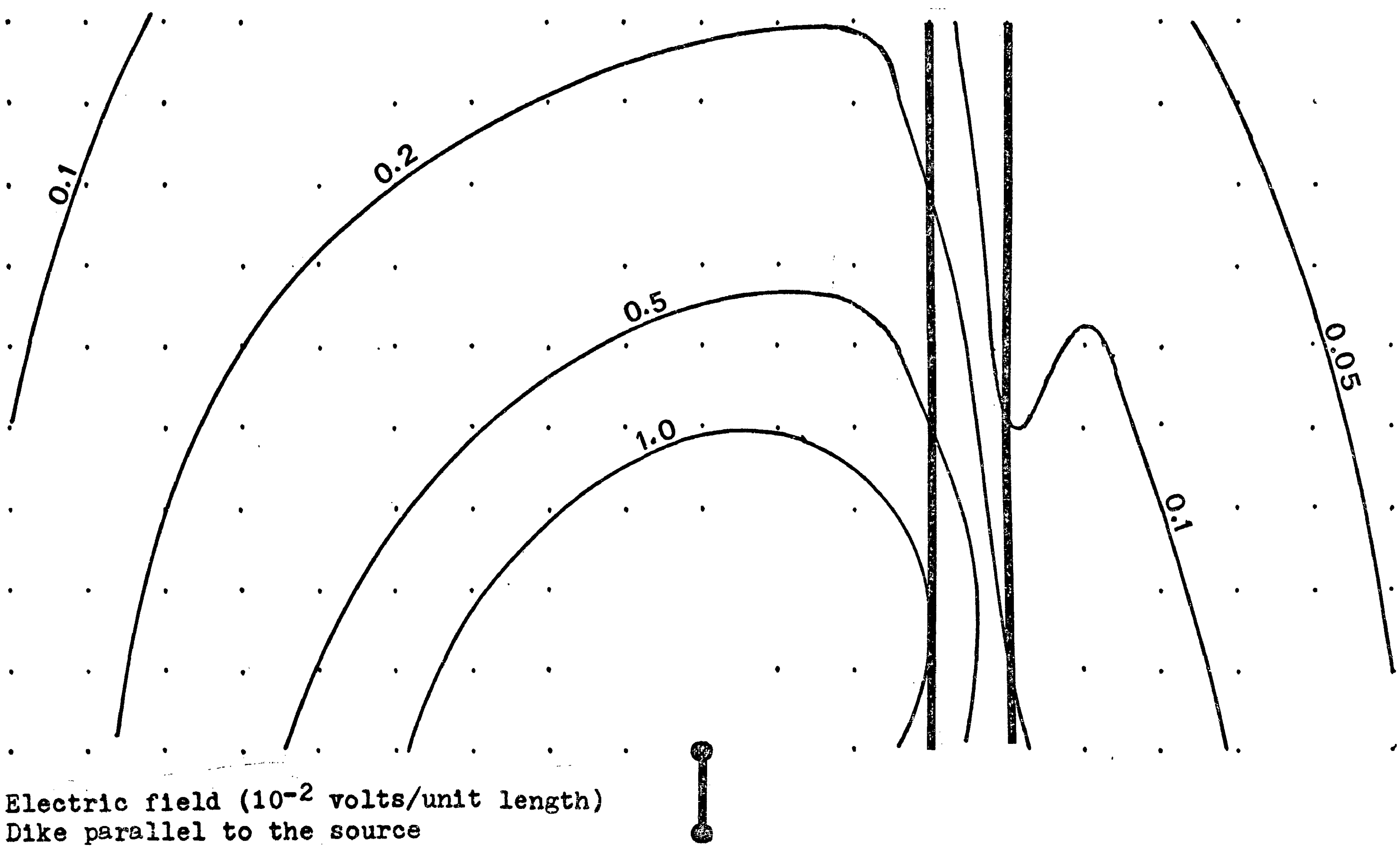
Figure 51

Apparent conductance (mhos) for a dike normal to the source. A current of 1 amp and resistivities of 1 ohm. unit length (medium) and 10 ohm. unit length (discontinuity) are used. Results on top after Furgerson and Keller (1974).



Potential field (10^{-2} volts)
Dike parallel to the source
Current 1 amp
Resistivities: medium: 1 ohm. unit length
discontinuity: 0.1 ohm. unit length

Figure 52



Electric field (10^{-2} volts/unit length)
Dike parallel to the source
Current 1 amp
Resistivities: medium: 1 ohm.unit length
discontinuity: 0.1 ohm.unit length

Figure 53

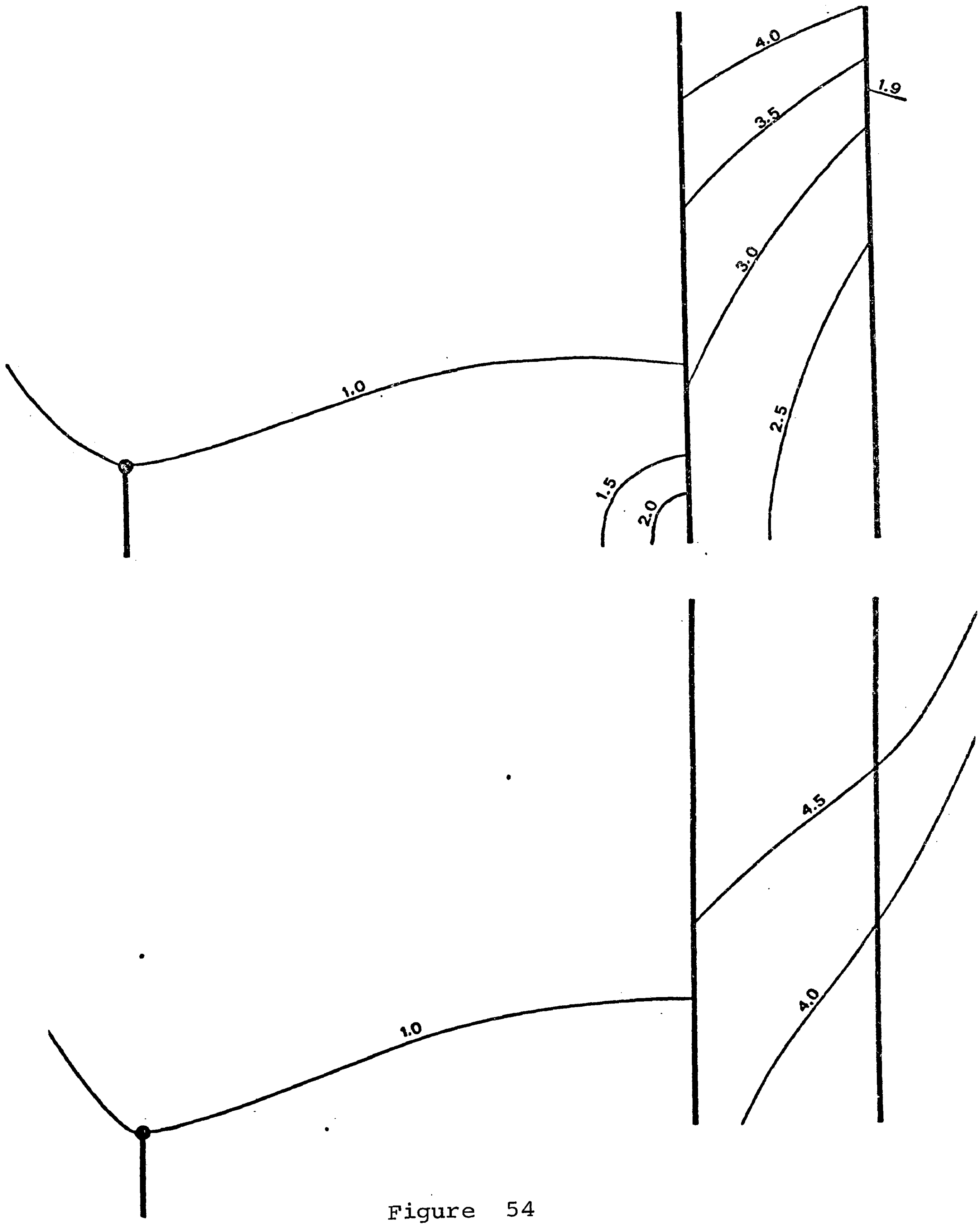
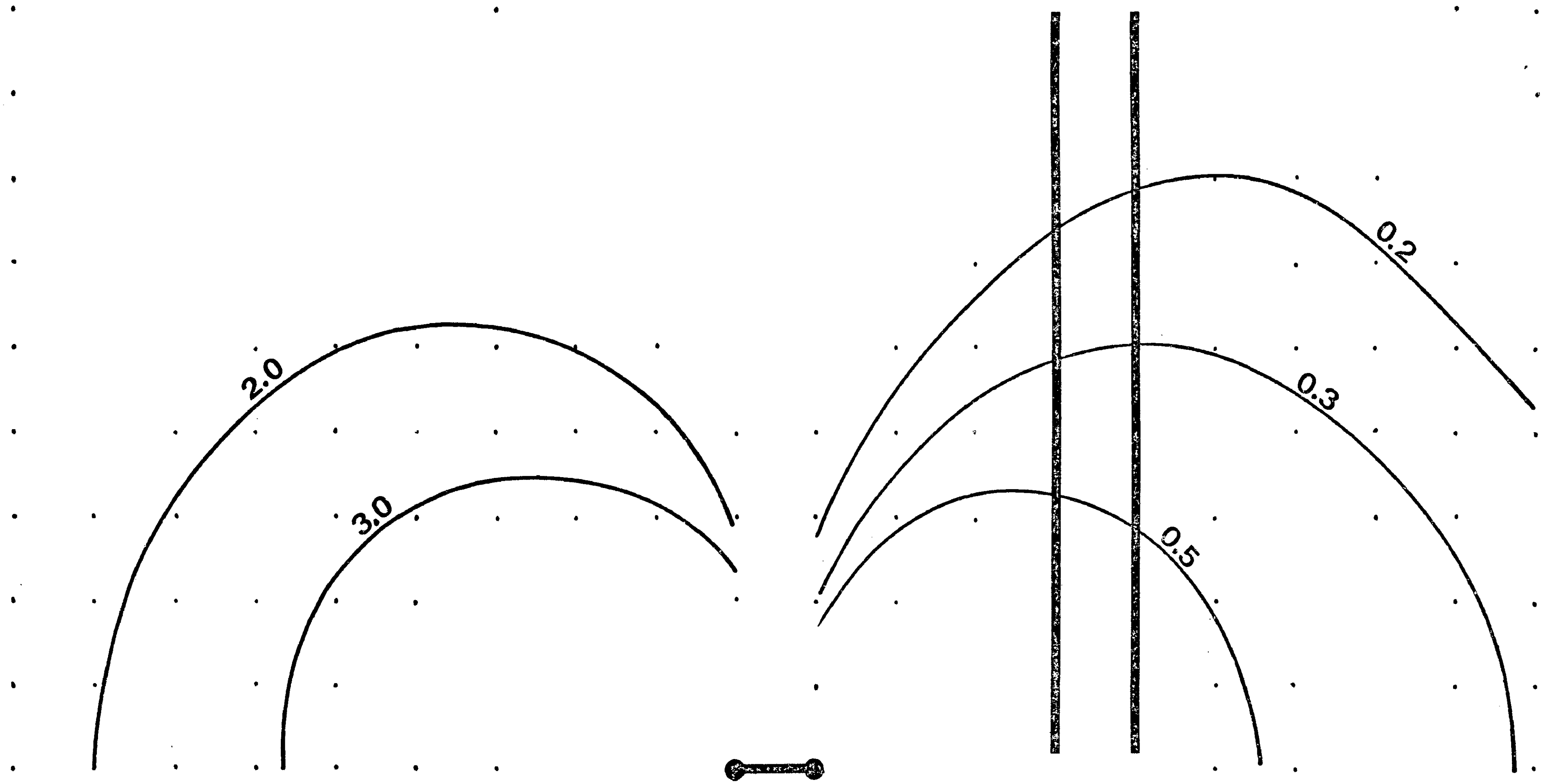


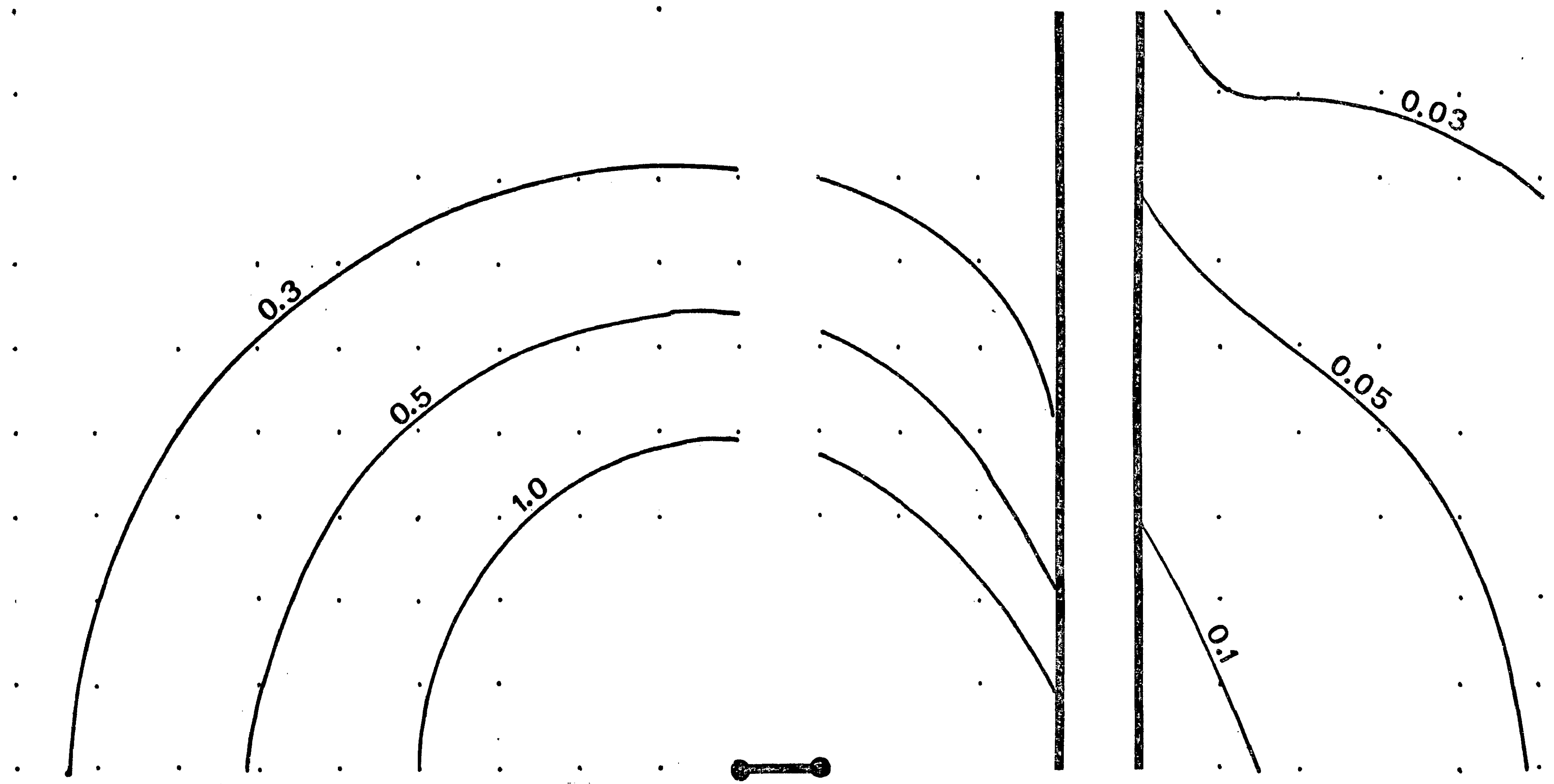
Figure 54

Apparent conductance (mhos) for a dike parallel to the source. A current of 1 amp and resistivities of 1 ohm. unit length (medium) and 0.1 ohm. unit length (discontinuity) are used. Results on top after Furgerson and Keller (1974).



Potential field (10^{-2}) volts)
Dike normal to the source
Current 1 amp
Resistivities: medium: 1 ohm. unit length
discontinuity: 0.1 ohm. unit length

Figure 55



Electric field (10^{-2} volts/unit length)
Dike normal to the source
Current 1 amp
Resistivities: medium: 1 ohm.unit length
discontinuity: 0.1 ohm.unit length

Figure 56

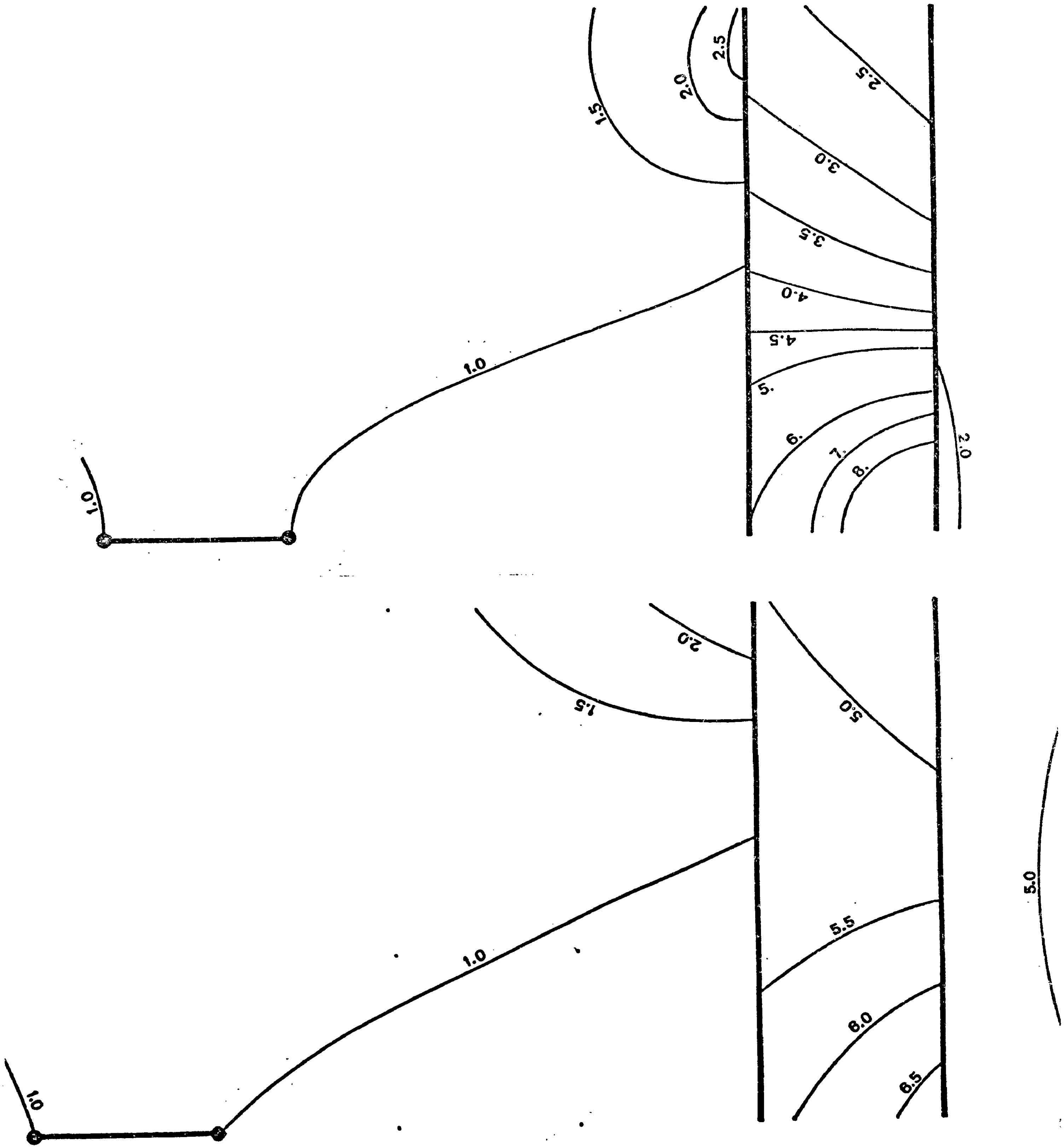


Figure 57

Apparent conductance (mhos) for a dike normal to the source. A current of 1 amp and resistivities of 1 ohm. unit length (medium) and 0.1 ohm. unit length (discontinuity) are used. Results on top after Furgerson and Keller (1974).

INTERPRETATION OF FIELD PROBLEMS

During the summer of 1974 the Geophysics Department of the Colorado School of Mines made a geothermal investigation in the state of Nevada. A dipole mapping survey was carried out on the Black Rock Desert area located about 100 miles north of Reno.

Here I have tried to model the apparent conductance maps obtained from two different source locations out of five source locations occupied during the survey. Figure 16 is a geologic map of the area in question with the location of the two sources referred to.

Source 1a

This source was located on the playa sediments of the Black Rock Desert. The western electrode was placed on the edge of the desert and I considered it as placed on volcanics.

In modeling this problem I placed a vertical dike, at a 30 degree angle with respect to the source, in order to represent the volcanics that lay on a strip west of the source.

I used a resistivity value of 1 ohm.m for the Black Rock Desert, a resistivity of 35 ohm.m for the volcanics, and a resistivity of 10 ohm.m for the Hualapai Flat situated on the west side of the volcanics.

The apparent conductance field map and the modeling result are presented in figure 17.

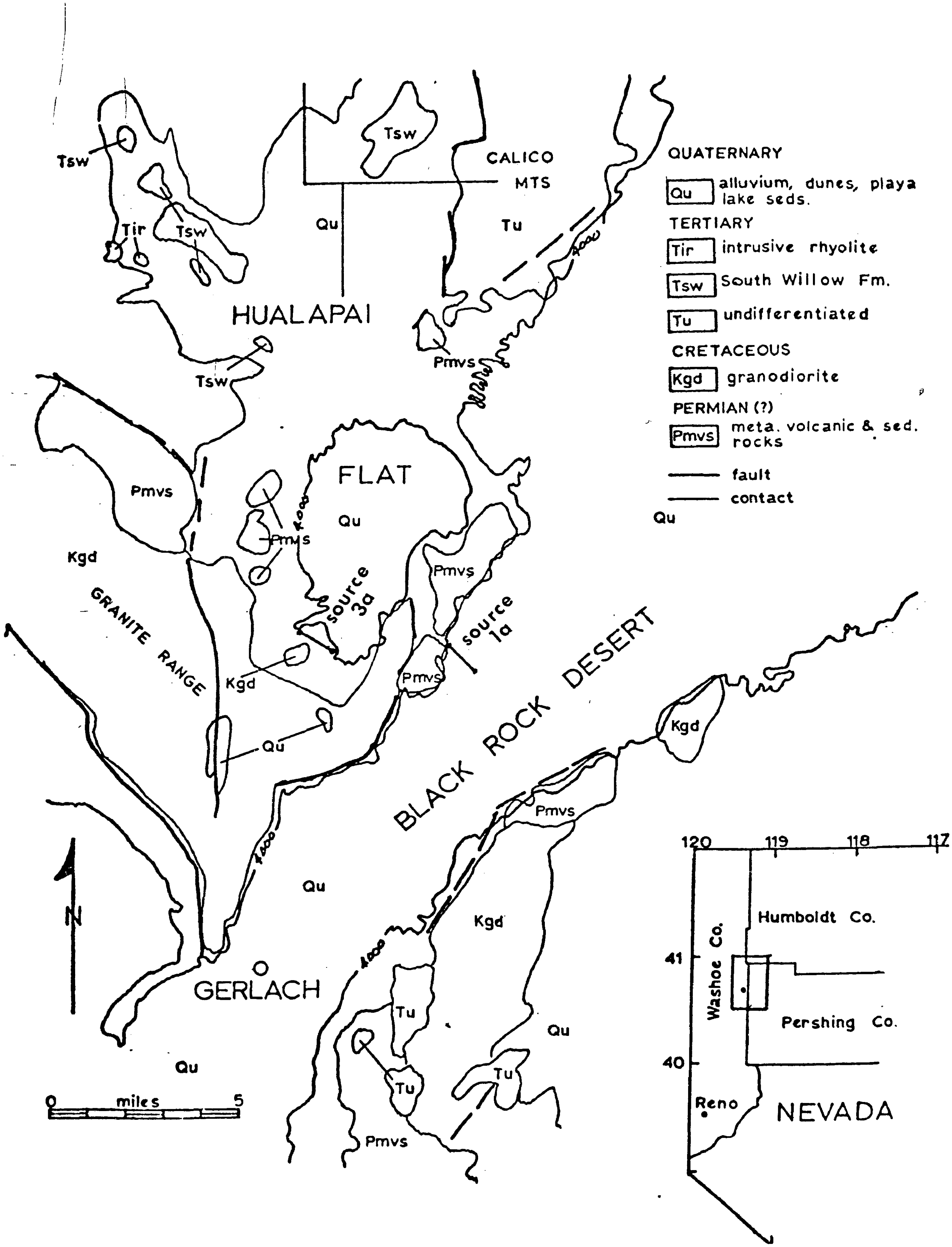


Figure 58. Reconnaissance geological map of the Black Rock Desert area of northwestern Nevada with dipole mapping source locations. (after Keller, et al, 1974)

It can be readily seen that the two maps have several features in common. It should be pointed out that if we assume that the resistivities used in the modeling procedure are correct the values obtained for apparent conductance should be multiplied by the depth in order to obtain the field values.

The steep gradients observed on the eastern portion of the apparent conductance field map are not present in the modeling results. This can be explained by the fact that while in the modeling it is assumed a horizontal top layer, the Black Rock Desert is a graben with a sharp change in thickness of the sediments as we move away from the edges toward its center. A sharp decrease in resistivity would give the same effect but this is unlikely to occur in this area due to the homogeneity of the sediments.

Apparent conductance is defined as one unit of depth per unit of resistivity and in order to obtain the field value at two source lengths from the edge of the Black Rock Desert we would have to multiply the modeling value by a depth factor of 1000 m.

In the Hualapai Flat area the depth factor we obtained is around 100 m at a distance 2.5 source lengths from the dipole source. Changes in resistivity are more likely to occur here because the sediments are not homogeneous as in the Black Rock Desert. From the modeling results we observe that there is a decrease in conductance values in the Hualapai Flat as we proceed to the north. Because this

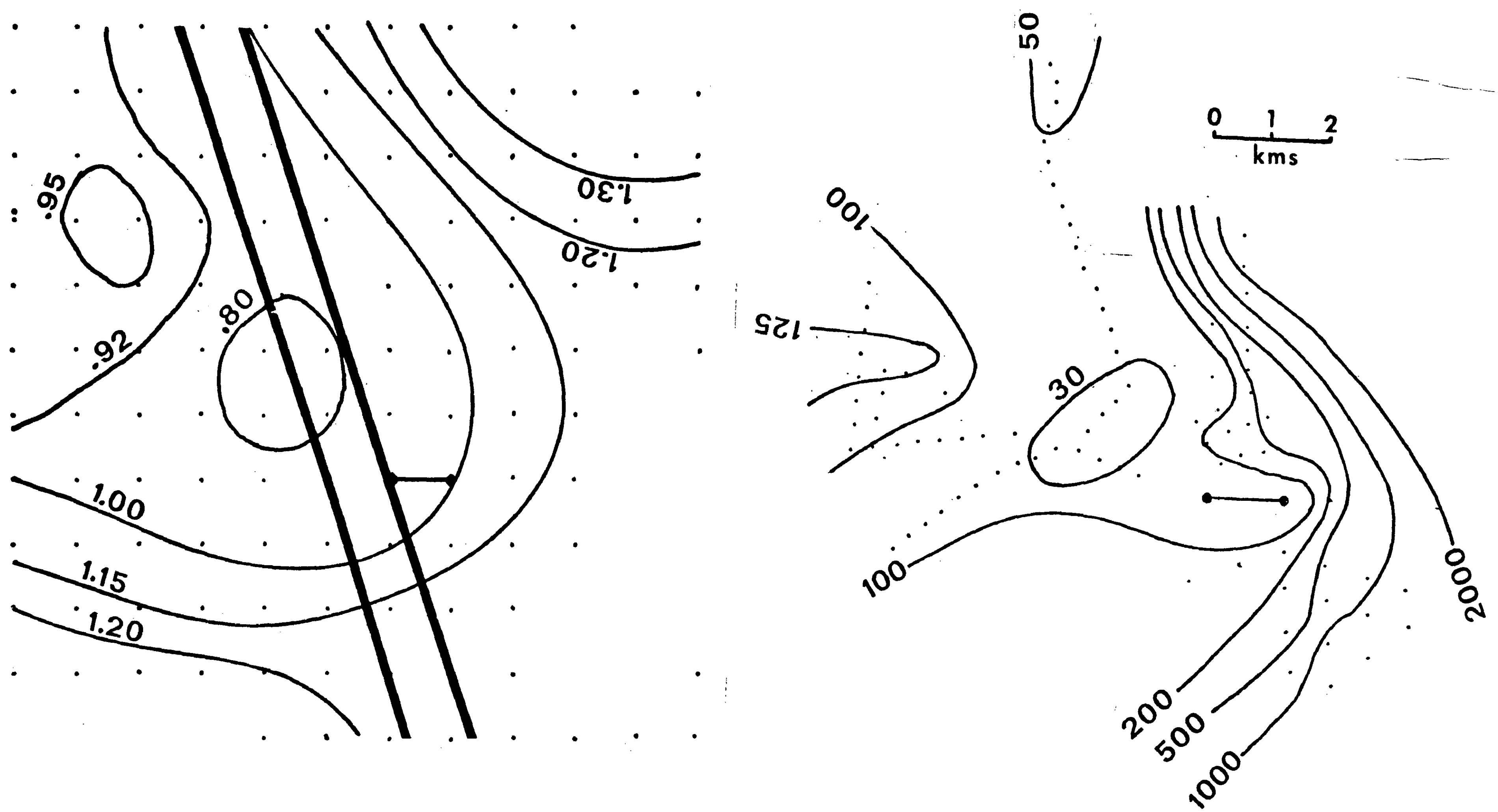


Figure 59. Apparent conductance maps for the source 1a with contours in mhos. On the right is the field map and on the left the modeling results.

fact is not present in the field results we can argue that resistivity decreases as we proceed north or depth increases toward the north.

Source 3a

This source is located on the southern portion of the Hualapai Flat and is oriented approximately in an east-west direction.

Here we have a more complicated problem that can be handled by the resistor network and give the interpreter an idea on the cause of the conductance anomalies.

I used an L-shaped discontinuity with 100-ohm.m resistivity to represent the Granite Range that is located at west and south of the source. The volcanics to the east were replaced by a dike with a resistivity of 35 ohm.m with its southern end with a resistivity of 50 ohm.m. The volcanics to the west were replaced by a square block with resistivity of 35 ohm.m. The resistivity over the Hualapai Flat area was assumed to be 10 ohm.m.

The apparent conductance field map and the modeling results are presented in figure 18. We can see from the results that most of the features in the field map are present on the map obtained with the resistor network. Here, again, it should be mentioned that if the resistivities are correct a depth factor has to be used to multiply the apparent conductance from the modeling results in order to

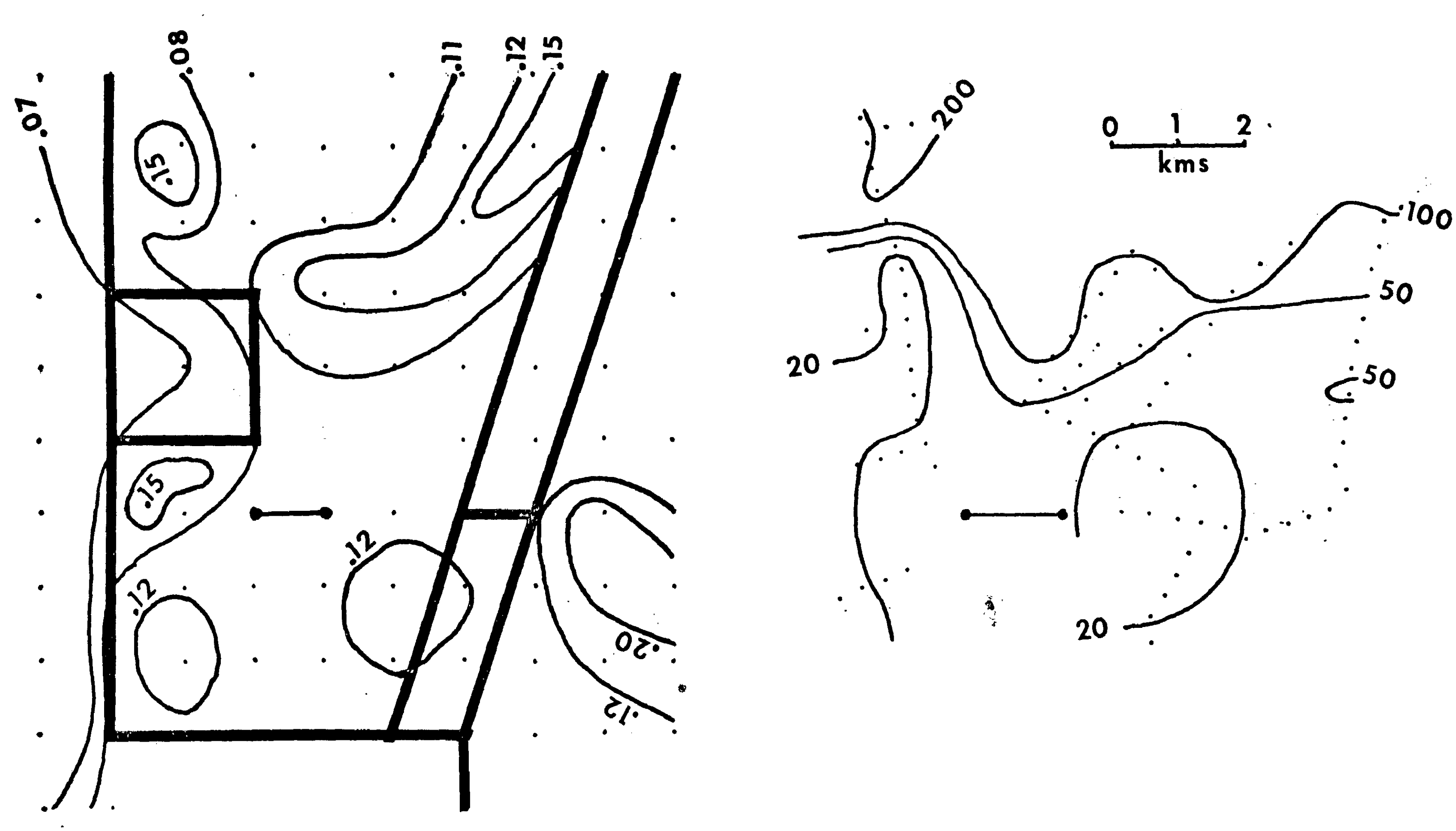


Figure 60. Apparent conductance maps for the source 3a with contours in mhos. On the right is the field map and on the left the modeling results.

obtain the field values.

For the area close to the source (1 source length) we obtain depths ranging from 150 m to 250 m. By comparison of the field results with the modeling results we conclude that a decrease on resistivity and/or an increase in depth toward north must occur in the area.

The 200 mho contour appearing on the northern portion of the field map cannot be completely explained in terms of a thickening of the sediments. In that area we have hot springs and this would account for a decrease on resistivity values. Therefore a change in resistivity and depth to the basement are likely to be responsible for that high conductance value.

DISCUSSION OF THE RESULTS

The results presented in the two previous sections make possible an analysis of the capabilities and limitations of the resistor network as a modeling tool for dipole mapping surveys.

The contour maps presented in this report show that the resistor network indicates the presence of discontinuities in the field generated by an electric bipole source.

The potential field is indicative of discontinuities only in the case of high resistivity contrasts. However, the indications on the shape of the discontinuity are so slight that we cannot surely ascertain the position of a given geometrical shape. The electric field maps and apparent conductance map show the discontinuities for low resistivity contrasts as well as for high resistivity contrasts. These maps also allow a conclusion about the kind of geoelectrical discontinuity being studied.

By comparing our apparent conductance maps with the ones obtained analytically by Furgerson and Keller (1974) we observe that for a fault problem the agreement between the results is good, despite small distortions on the contours. This conclusion applies to all models studied, whether the source is normal or parallel to the contact or located on the resistant or conductive side of the fault.

The results for the dikes have to be discussed in more detail because in some cases the modeling results do not agree completely with the ones obtained analytically.

For the case of a conductive dike we observe that the general picture is obtained with the resistor network. However, when the source is placed normal to the dike the conductance values obtained inside the dike differ from the ones obtained analytically, although they have the same order of magnitude and the contours have the same pattern. When the source is placed parallel to the dike we observe that the modeling procedure used does not show the end of the geoelectrical discontinuity, giving a higher value of conductance for the medium on the back side of the dike.

In the case of a resistant dike we observe the opposite behavior. When the source is normal to the dike we are not able to observe the far side of the discontinuity because the resistor network yields lower conductance values for the medium behind the dike. When the source is placed parallel to the contact we obtained good results with only a slight variation on the magnitude of the apparent conductance values inside the geoelectrical discontinuity.

On the modeling of field problems it should be mentioned that the source 1a can be solved analytically but for source 3a no analytical solution can be obtained due to the complicated geometries present. In both sources the resistor network was capable of showing the main features present in the field data and allowed the drawing of conclusions about changes in depth and resistivity inside the areas of interest.

CONCLUSIONS

The results presented in this report indicate that a resistor network can be a powerful tool in helping a geophysicist to understand the anomalies caused by geoelectrical discontinuities on a field generated by an electrical dipole source.

Even in the presence of a small contrast in resistivity the changes in the field pattern caused by the discontinuities can be easily observed. Here, we should point out that the potential field distribution is much less indicative of anomalies than the electric field. For higher resistivity contrasts the anomalies in the electric field are striking on their appearance.

By comparison of apparent conductance maps obtained with the resistor network with the ones obtained analytically we observe that in modeling fault problems the network behaves very well in yielding good results. In the case of dikes we observe that when we have a conductive dike parallel to the source or a resistant dike normal to the source the modeling procedure used in this report is not sensitive enough to show the far end of the geoelectrical discontinuities.

In using the resistor network for modeling field results we observed that the results obtained allow the interpreter to draw conclusions about the geology, resistivities and depths in the area being modeled.

The results presented contain errors due to the several numerical approaches used in obtaining them. However, the errors are expected to be less than 20 percent and they do not inhibit the use of the resistor network for modeling and interpretation purposes. The size and speed of the computing facility being used also influences the size of the errors in the results because it limits the dimensions of the network used as the modeling tool.

Taking into account that sometimes it is impossible to obtain analytical results for some complicated field problems, the resistor network can provide a picture of the problem at a relatively low cost. For each model studied an average of 3 minutes and 5 seconds of central processor unit time was spent.

Beside the models studied in this report a variety of geoelectrical discontinuities can be studied with the resistor network, but it is beyond the scope of this report to present results for odd-shaped geometries that seldom occur in nature.

Finally I wish to emphasize that the resistor network simulation gives a useful check on field results and enables the investigator to draw conclusions on the geology of the area, with confidence.

APPENDIX A

In this Appendix I will discuss the numerical procedures used throughout this report in obtaining the results presented.

Solution of Linear Systems of Equations by Gauss Elimination

In computer programs GRID and FIGRID, I solve a system of linear equations using Gauss elimination.

The basis of Gauss elimination is the following (Forsythe and Moler, 1967): given a square matrix A of order n, let A_k denote the principal minor matrix made from the first k rows and columns. Assuming that det $(A_k) \neq 0$ for k = 1, 2...., n^{-1} then there exists a unique lower triangular matrix L = $(m_{i,j})$, with $m_{1,1} = m_{2,2} = \ldots\ldots = m_{n,n} = 1$, and a unique upper triangular matrix U = $(U_{i,j})$ so that LU = A.

Here, a lower square triangular matrix is defined as a square matrix C = $(c_{i,j})$ such that $c_{i,j} = 0$ for $i < j$ and C is an upper triangular matrix if $c_{i,j} = 0$ for $i > j$.

Because the matrix we are concerned with in our modeling problem is a positive definite matrix we can reduce it to a lower (or upper) triangular matrix (Wilkinson, 1961; Householder, 1964). Therefore we have

$$L L^T = A$$

This is achieved by multiplying the matrix by elementary triangular and permutation matrices (Householder, 1964).

The elementary permutation matrices essentially interchange rows of the square matrix A. By interchanging rows after determining that a given element of A is an element of maximal absolute value in the column we have the pivoting. Taking the pivot to be an element of largest absolute value in a column is called Gauss elimination with partial pivoting and that is the strategy used in subroutine BANSOL used in programs GRID and FIGRID.

The interchanging of rows in Gaussian elimination improves the numerical stability of the scheme, as shown by Forsythe and Moler (1967). An extensive analysis of the roundoff errors in the Gauss elimination procedure can be found in Wilkinson (1961) and Forsythe and Moler (1967).

To obtain the solution of the system of equations a back substitution scheme is employed. This amounts to solving the n^{th} equation of the equivalent triangular system, then the n-1 equation and so forth.

Several algorithms have been presented in the literature dealing with this procedure (Thurnau, 1963; Martin and Wilkinson, 1965, 1967). Subroutine BANSOL is a translation from ALGOL of the algorithm presented by Thurnau (1963) published by Wesley et al (1972).

Lagrangian Interpolation

In order to have a suitable set of values of potential for input in program FIGRID interpolation has to be carried out on the potential values obtained from program GRID for the interface between the coarse and the fine networks. The procedure used in this report was Lagrangian interpolation.

In Lagrangian interpolation we look for a polynomial y(x) of degree n, which takes on the same values of a given function f(x) for n + 1 distinct abcissas x_0, x_1, ..., x_n (Hildebrand, 1974).

The polynomial y(x) can be written as

$$y(x) = m_0(x)f(x_0) + m_1(x)f(x_1) + \cdots + m_n(x)f(x_n) \quad \text{(A-1)}$$

or

$$y(x) = \sum_{j=0}^{n} m_j(x)f(x_j)$$

where $m_0(x)$, ..., $m_n(x)$ are polynomials of degree n or less, to be determined.

The result of replacing y(x) by f(x) has to be an identity when f(x) is an arbitrary polynomial of degree n or less. This will arise if and only if the result of replacing y(x) by f(x) is an identity when f(x) = 1, x, x^2,...., and x^n.

We see that equation A-1 will take on the value $f(x_i)$ when $x = x_i$ if $m_i(x_i) = 1$ and if $m_i(x_k) = 0$ when $k \neq i$. Using the Kronecker delta notation

$$\delta_{ik} = \begin{cases} 0 & \text{for } i \neq k \\ 1 & \text{for } i = k \end{cases} \quad \text{(A-2)}$$

this requirement becomes

$$m_i(x_k) = \delta_{ik} \quad (i = 0, \ldots, n;\ k = 0, \ldots, n) \qquad \text{(A-3)}$$

Because $m_i(x)$ is a polynomial of degree n which vanishes when $x = x_0, x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n$ we can write

$$m_i(x) = A_i\left[(x - x_0)\cdots(x - x_{i-1})(x - x_{i+1})\cdots(x - x_n)\right] \qquad \text{(A-4)}$$

where A_i is a constant that can be readily derived by taking into account that $m_i(x_i) = 1$, therefore

$$A_i = \frac{1}{(x_i - x_0)\cdots(x_i - x_{i-1})(x_i - x_{i+1})\cdots(x_i - x_n)} \qquad \text{(A-5)}$$

By substituting (A-5) and (A-4) into (A-1) we obtain the Lagrangian interpolation polynomial of degree n as:

$$y(x) = \prod_{i=0}^{n}(x - x_i)\sum_{k=0}^{n}\frac{y_k}{(x - x_k)\prod\limits_{\substack{i=0 \\ i \neq k}}^{n}(x_k - x_i)} \qquad \text{(A-5)}$$

where

$$\prod_{i=0}^{n}(x - x_i) = (x - x_0)(x - x_1)\cdots(x - x_n)$$

$$\prod_{i=0}^{n}(x_k - x_i) = (x_k - x_0)(x_k - x_1)\cdots(x_k - x_n)$$

Let us now make an error analysis of the process.

If f(x) is approximated by a polynomial $y(x) = y_{0,\ldots,n}(x)$

of maximum degree n, which coincides with it at n + 1 distinct points $x_0, \ldots, x_n$ then the error

$$E(x) = f(x) - y(x) \qquad (A\text{-}7)$$

is given by

$$E(x) = (x - x_0)(x - x_1)\ldots(x - x_n) f[x_0, \ldots, x_n, x] \qquad (A\text{-}8)$$

Considering the fact that f(x) - y(x) and $(x - x_0)(x - x_1)\ldots(x - x_n)$ vanish at the n + 1 points $x = x_0, x_1, \ldots x_n$ we can write a linear combination of these functions

$$F(x) = f(x) - y(x) - k\left\{(x - x_0)(x - x_1)\ldots(x - x_n)\right\} \qquad (A\text{-}9)$$

or

$$F(x) = f(x) - y(x) - K\,\Pi(x) \qquad (A\text{-}10)$$

and determine the constant K in such a way that F(x) vanishes at an arbitrarily chosen point $\bar{x}$.

If the smallest and the largest of the n + 2 values $x_0, x_1, \ldots, x_n, \bar{x}$ establish a closed interval $\bar{J}$, then F(x) vanishes at least n + 2 times in the interval, F'(x) vanishes at least n + 1 times, F"(x) at least n times...., and finally $F^{(n+1)}(x)$ vanishes at least once inside the interval $\bar{J}$. If we denote such point as $\bar{z}$ and use (A - 10) we can write:

$$0 = f^{(n+1)}(\bar{z}) - y^{(n+1)}(\bar{z}) - K\,\Pi^{(n+1)}(\bar{z}) \qquad (A\text{-}11)$$

Because y(x) is a polynomial of maximum degree n its (n + 1)th derivative vanishes and since

$$\pi^{(n+1)}(x) = (n+1)!$$

we can write

$$K = \frac{1}{(n+1)!} f^{(n+1)}(\bar{z}) \tag{A-12}$$

and the relation $F(\bar{x}) = 0$ becomes

$$f(\bar{x}) - y(\bar{x}) = \frac{1}{(n+1)!} f^{(n+1)}(\bar{z}) \pi(\bar{x}) \tag{A-13}$$

for some point $\bar{z}$ in the closed interval $\bar{J}$. Since $\bar{x}$ is arbitrary we can finally write

$$E(x) = \frac{1}{(n+1)!} f^{(n+1)}(z) \pi(x) \tag{A-14}$$

for some point z in the interval limited by the largest and smallest values of the numbers x_0, x_1, ..., x_n, x.

Because the error is dependent upon the value of $f^{(n+1)}(z)$, the method is satisfactory only when this derivative is suitably small. In our modeling problem it can be readily seen that the region close to the source yields larger error due to high derivative values.

Numerical Differentiation

In order to obtain the gradient of the potential numerical differentiation has to be used on the results yielded by programs GRID and FIGRID.

Here we discuss the method used to compute a vector of derivative values given the vectors of argument values and corresponding function values.

If $Z = (z_1, \ldots, z_n)$ are the derivative values, $X = (x_1, \ldots, x_n)$ are the argument values, and $Y = (y_1, \ldots, y_n)$ are the corresponding function values we can find z_i as the derivative at the point x_i of the Lagrangian interpolation polynomial of degree two relevant to these successive points (x_{i-1}, y_{i-1}), (x_i, y_i) and (x_{i+1}, y_{i+1}).

For $i = 1, \ldots, n-2$ we have to find a_i, b_i, and c_i such that

$$\bar{y}_i(x) = a_i x^2 + b_i x + c_i$$

passes through (x_i, y_i), (x_{i+1}, y_{i+1}), and (x_{i+2}, y_{i+2}).

Due to the features of the problem being solved in this report we have equidistantly spaced argument values x_i with $x_i - x_{i-1} = h$ for $i = 2, \ldots, n$.

The expressions for the derivatives z_i are (Hildebrand, 1974):

$$z_i = \begin{cases} \frac{1}{2h}(-3y_1 + 4y_2 - y_3) & \text{if } i = 1 \\ \frac{1}{2h}(y_{i+1} - y_{i-1}) & \text{if } i = 0, \ldots, n-1 \\ \frac{1}{2h}(y_{n-2} - 4y_{n-1} + 3y_n) & \text{if } i = n \end{cases}$$

Let us now see the errors that can be originated with this procedure.

The truncation errors in the expressions above are

$$T_i \doteq \begin{cases} \frac{h^2}{3} y'''(\alpha_1) & \text{where } \alpha_1 \in [x_1, x_3] \text{ if } i = 1 \\ -\frac{h^2}{6} y'''(\alpha_i) & \text{where } \alpha_i \in [x_{i-1}, x_{i+1}] \text{ if } i = 2, \ldots, n-1 \\ \frac{h^2}{3} y'''(\alpha_n) & \text{where } \alpha_n \in [x_{n-2}, x_n] \text{ if } i = n \end{cases}$$

In addition to these truncation errors, roundoff errors can be of considerable magnitude. Suppose that each of the ordinates involved could be in error by $\pm \epsilon$. Then the magnitude $|R_i|$ of the corresponding error Ri in the calculation of z_i can be as large as

$$|R_i| = \begin{cases} \frac{4\epsilon}{|h|} & \text{for } i = 1, n \\ \frac{\epsilon}{|h|} & \text{for } i = 2, \ldots, n-1 \end{cases}$$

APPENDIX B

In this appendix we present the listing of the computer programs used in obtaining the results shown in this report.

main program: GRID
subprograms: COND
BANSOL

main program: EFD
subprogram: DET3

main program: FIGRID
subprograms: YLAG
BANSOL

main program: EFFIGR
subprogram: DET3

main program: APCON1

main program: APCON2

```
C-----------------------PROGRAM GRID-----------------------,--------
C
C
C       PROGRAM GRID EVALUATES THE POTENTIAL DISTRIBUTION ON A
C RESISTOR NETWORK OF 23X24 NODES. THE TWO MOST EXTERNAL LOOPS
C ARE USED FOR BOUNDARY EFFECT CORRECTION AND, THEREFORE, THE
C POTENTIAL FOR THE NODES PERTAINING TO THESE LOOPS ARE  NOT
C USED WHEN THE GRADIENT OF THE POTENTIAL IS OBTAINED (SEE PROG. EFD
C       THE PROGRAM IS WRITTEN IN FORTRAN-4 LANGUAGE AND WAS
C DEVELOPED ON A DIGITAL COMPUTER CORPORATION MODEL PDP-10
C COMPUTER, A TIME-SHARING SYSTEM. THE PROGRAM REQUIRES A
C TOTAL OF 37K WORDS OF STORAGE.
C
C
C       THE PROGRAM INPUT IS IN THE FOLLOWING FORM AND SEQUENCE:
C
C       1) VALUE OF RESISTORS BEING USED
C
C       2) CURRENT BEING USED
C
C
C       THE PROGRAM OUTPUT GIVES THE POTENTIAL VALUE ON EACH
C NODE OF THE NETWORK. THIS OUTPUT IS USED AS INPUT FOR
C PROGRAMS EFD AND FIGRID.
C
C
C       VARIABLES NAME:
C
C     N - TOTAL NUMBER OF NODES ON THE NETWORK
C     LX - NUMBER OF NODES ON X-DIRECTION
C     LY - NUMBER OF NODES ON Y-DIRECTION
C     M - WIDTH OF THE BAND OF THE MATRIX OF COEFFICIENTS
C        (IT HAS TO BE AN ODD NUMBER)
C     V - CURRENT VECTOR. AFTE IS RETURNED FROM BANSOL V
C        CONTAINS THE POTENTIAL VALUE FOR EACH NODE
C     C - IS THE DOUBLY DIMENSIONED NXM ARRAY WHICH CONTAINS
C         THE PERTINENT ELEMENTS OF THE COEFFICIENT MATRIX
C         A. THE BAND ELEMENTS OF A GIVEN ROW OF MATRIX A
C         APPEAR IN THE SAME ROW OF C BUT SHIFTED SUCH THAT
C         ELEMENT A(I,J) BECOMES C(I,J-I+(M+1)/2)
C     CDR - DOUBLY DIMENSIONED ARRAY WHICH CONTAINS THE
C           CONDUCTANCES RELATED TO EACH NODE
C     RESIST - VALUE OF THE RESISTORS BEING USED IN THE
C              NETWORK
C     CURREN - INPUT CURRENT USED IN THE NETWORK
C
C                       A.C. PIRES    OCTOBER 1974
C                       COLORADO SCHOOL OF MINES
C
C---------------------------------------------------------------------
      DIMENSION CDR(552,4),V(552),C(552,47)
```

```
      COMMON /ONE/ N
      COMMON /TWO/ CDR,RESIST
      COMMON /THREE/ M
      N=552
      LX=23
      LY=24
      ID=N
      M=2*LX+1
C
C
C-----------------------------------------------------------------
C        INPUT  DATA
C-----------------------------------------------------------------
      READ(1,3)RESIST,CURREN
3     FORMAT(2F)
C-----------------------------------------------------------------
C        PREPARE TERMINATION STRIP AND ASSIGN CONDUCTANCES
C        TO EACH NODE
C-----------------------------------------------------------------
      CALL COND
C-----------------------------------------------------------------
C        SET CURRENT VECTOR
C-----------------------------------------------------------------
      DO 20 K=1,N
20    V(K)=0.
      INP1=LY/2*LX-(LY-2)/2
      INP2=INP1+LX
      V(INP1)=CURREN
      V(INP2)=- CURREN
C-----------------------------------------------------------------
C        GENERATE MATRIX OF COEFFICIENTS
C-----------------------------------------------------------------
      DO 30 LA=1,N
30    C(LA,LY)=CDR(LA,1)+CDR(LA,2)+CDR(LA,3)+CDR(LA,4)
      DO 40 LB=1,N-1
40    C(LB,LY+1)=-CDR(LB,1)
      DO 50 LC=LX,N,LX
50    C(LC,LY+1)=0.
      DO 60 LD=1,N-LX
60    C(LD,M)=-CDR(LD,2)
      DO 70 LE=2,N
70    C(LE,LX)=-CDR(LE,3)
      DO 80 LF=LY,N,LX
80    C(LF,LX)=0.
      DO 90 LG=LY,N
90    C(LG,1)=-CDR(LG,4)
      DO 100 LH=1,LX-2
      DO 100 LI=LH+2,N-(LH+1)
100   C(LI,LX-LH)=0.
C-----------------------------------------------------------------
C        SOLVE SYSTEM OF LINEAR EQUATIONS
```

```
C---------------------------------------------------------------
      CALL BANSOL(C,V,ID)
C-------------,-------------------------------------------------
C      OUTPUT RESULTS
C-----------------------------------------------------------,----
      WRITE(9,110)(V(LJ),LJ=1,N)
110   FORMAT(8E16.4)
      STOP
      END
```

```
C--------,-----------SUBROUTINE COND-------------------------------
C
C       THIS SUBROUTINE COMPUTES THE VALLES OF THE CONDUCTANCES
C ON THD TERMINATION STRIP AND ASSIGN CONDUCTANCES TO  EACH
C NODE ON THE NETWORK.
C
C-------------------------------------------------------------------
      SUBROUTINE COND
      DIMENSION CDR(552,4)
      COMMON /ONE/ N
      COMMON /TWO/ CDR,RESIST
      DO 10 I=1,N
      DO 10 J=1,4
10    CDR(I,J)=1./RESIST
      DO 20 I=1,23
20    CDR(I,1)=1./(4.*RESIST)
      DO 30 I=1,23
      DO 30 J=3,4
30    CDR(I,J)=1./(4.*RESIST)
      DO 40 I=1,530,23
      DO 40 J=2,4
40    CDR(I,J)=1./(4.*RESIST)
      DO 50 I=530,552
      DO 50 J=1,3
50    CDR(I,J)=1./(4.*RESIST)
      DO 60 I=23,552,23
      DO 60 J=1,2
60    CDR(I,J)=1./(4.*RESIST)
      DO 70 I=23,552,23
70    CDR(I,4)=1./(4.*RESIST)
      DO 80 I=24,507,23
80    CDR(I,1)=1./(2.*RESIST)
      DO 90 I=46,529,23
90    CDR(I,3)=1./(2.*RESIST)
      DO 100 I=2,22
100   CDR(I,2)=1./(2.*RESIST)
      DO 110 I=531,551
110   CDR(I,4)=1./(2.*RESIST)
      DO 120 I=25,45
120   CDR(I,1)=1./(2.*RESIST)
      DO 130 I=25,45
      DO 130 J=3,4
130   CDR(I,J)=1./(2.*RESIST)
      DO 140 I=25,508,23
      DO 140 J=2,4
140   CDR(I,J)=1./(2.*RESIST)
      DO 150 I=508,528
      DO 150 J=1,3
150   CDR(I,J)=1./(2.*RESIST)
```

```
      DO 160 I=45,528,23
      DO 160 J=1,2
160   CDR(I,J)=1./(2.*RESIST)
      DO 170 I=45,528,23
170   CDR(I,4)=1./(2.*RESIST)
      RETURN
      END
```

```
C---------------------SUBROUTINE BANSOL-----------------------
C
C
C      THIS SUBROUTINE SOLVES A SYSTEM OF LINEAR EQUATIONS
C AX=B WHEN THE MATRIX A IS OF VERY LARGE ORDER AND SPARSE
C IN THE SENSE THAT THE ONLY NONZERO ELEMENTS ARE IN A BAND
C CENTERED ON THE MAIN DIAGONAL.
C
C      VARIABLE NAME:
C      ID - INTEGER FIRST DIMENSION OF C
C      C - (DEFINED IN MAIN PROGRAM)
C      V - (DEFINED IN MAIN PROGRAM)
C
C      RESTRICTIONS:
C      THE MATRIX A MUST BE NONSINGULAR, THE NUMBER OF
C SUPERDIAGONALS AND SUBDIAGONALS MUST BE THE SAME.
C
C      REFFRENCE:(MODIFIED AFTER)
C      WESTLEY G.W. AND WATTS J.A.,EDS.,1970,THE COMPUTING
C      TECHNOLOGY CENTER NUMERICAL ANALYSIS LIBRARY, NTIS,
C      SPRINGFIELD,VA.
C
C-------------------------------------------------------------
      SUBROUTINE BANSOL(C,V,ID)
      DIMENSION C(ID,1),V(ID)
      COMMON /ONE/ N
      COMMON /THREE/ M
      L=(M+1)/2
      L1=L-1
      DO 20 IR=1,L1
      LR=L-IR
      DO 20 I=1,LR
      DO 10 J=2,M
10    C(IR,J-1)=C(IR,J)
      NP1=N+1-IR
      MP1=M+1-I
      C(NP1,MP1)=0.
20    C(IR,M)=C(NP1,MP1)
      N1=N-1
      DO 90 I=1,N1
      IPIV=I
      IRE=I+1
      DO 30 IR=IRE,L
      IF(ABS(C(IR,1)).LE.ABS(C(IPIV,1)))GO TO 30
      IPIV=IR
30    CONTINUE
      IF(IPIV.EQ.I)GO TO 50
      T=V(I)
      V(I)=V(IPIV)
      V(IPIV)=T
      DO 40 J=1,M
      T=C(I,J)
```

```
      C(I,J)=C(IPIV,J)
40    C(IPIV,J)=T
50    V(I)=V(I)/C(I,1)
      DO 60 J=2,M
60    C(I,J)=C(I,J)/C(I,1)
      DO 80 IR=IRE,L
      T=C(IR,1)
      V(IR)=V(IR)-T*V(I)
      DO 70 J=2,M
70    C(IR,J-1)=C(IR,J)-T*C(I,J)
80    C(IR,M)=0.
      IF(L.EQ.N)GO TO 90
      L=L+1
90    CONTINUE
      V(N)=V(N)/C(N,1)
      JM=2
      DO 110 ICE=1,N1
      IR=N-ICE
      DO 100 J=2,JM
      IRM1=IR-1+J
100   V(IR)=V(IR)-C(IR,J)*V(IRM1)
      IF(JM.EQ.M)GO TO 110
      JM=JM+1
110   CONTINUE
      RETURN
      END
```

```
C---------------------------PROGRAM EFD---------------------------------
C
C
C      THIS PROGRAM EVALUATES THE GRADIENT OF THE POTENTIAL
C OBTAINED WITH THE PROGRAM GRID. SINCE THE TWO MOST EXTERNAL
C LOOPS WERE USED FOR BOUNDARY EFFECT CORRECTION THE POTENTIAL
C VALUES ON THE NODES PERTAINING TO THESE LOOPS ARE NOT TAKEN
C INTO ACCOUNT.
C      THE PROGRAM IS WRITTEN IN FORTRAN-4 LANGUAGE AND WAS
C DEVELOPED ON DIGITAL COMPUTER CORPORATION MODEL PDP-10 COMPUTER
C ATIME SHARING SYSTEM.
C
C      PROGRAM INPUT:
C
C      DATA FILE GENERATED BY PROGRAM GRID
C
C      PROGRAM OUTPUT:
C
C     1- POTENTIAL FIELD DISTRIBUTION USED AS INPUT
C     2- GRADIENT OF THE POTENTIAL FIELD
C     3- IN CASE OF ERROR IN OBTAINING THE DERIVATIVES THE
C          PROGRAM STOPS AND AN ERROR MESSAGE IS GIVEN AS OUTPUT
C
C
C      VARIABLES NAME:
C
C      LX - NUMBER OF NODES IN X-DIRECTION
C      LY - NUMBER OF NODES IN Y-DIRECTION
C      H - STEP SIZE FOR THE DERIVATIVES
C      POTEN - SINGLY DIMENSIONED ARRAY WHICH CONTAINS THE
C                POTENTIAL DISTRIBUTION ON THE NETWORK
C      POT - SINGLY DIMENSIONED ARRAY WHICH CONTAINS THE
C              POTENTIAL DISTRIBUTION ON THE NETWORK WHITOUT THE
C              TERMINATION STRIP
C      DX - DERIVATIVE ON X-DIRECTION
C      DY - DERIVATIVE ON Y-DIRECTION
C      EFIELD - GRADIENT OF POTENTIAL DISTRIBUTION
C
C
C                           A. C. PIRES  NOVEMBER,1974
C                        COLORADO SCHOOL OF MINES
C
C----------------------------------------------------------------------
      DIMENSION POTEN(600),Y(50),DX(600),DY(600),EFIELD(600),
     1  Z(50),POT(600)
      KCT=0
      LX=23
      LY=24
      H=1.
      JTOT=LX*LY
C----------------------------------------------------------------------
```

```
C       INPUT DATA: POTENTIAL DISTRIBUTION
C---------------------------------------------------------------------
        READ(9,5)(POTEN(JE),JE=1,JTOT)
5       FORMAT(8E16.4)
C---------------------------------------------------------------------
C       DELETE VALUES ON THE TERMINATION STRIP
C---------------------------------------------------------------------
        DO 6 N=1,LY-4
        DO 6 I=1,LX-4
        K=K+1
6       POT(K)=POTEN(LX+2+(N*LX)+I)
C---------------------------------------------------------------------
C       OUTPUT POTENTIAL DISTRIBUTION WHITOUT TERMINATION STRIP
C---------------------------------------------------------------------
        WRITE(11,9)
9       FORMAT(20X,'POTENTIAL FIELD(COARSE NETWORK)',///)
        WRITE(11,11)
11      FORMAT(10X,'NODE',20X,'POTENTIAL FIELD(VOLTS)',//)
        WRITE(11,12)(JB,POT(JB),JB=1,K)
12      FORMAT(10X,I4,27X,E16.4)
C---------------------------------------------------------------------
C       SET NEW PARAMETERS FOR NETWORK
C---------------------------------------------------------------------
        LLX=19
        LLY=20
        ITOT=LLX*LLY
C---------------------------------------------------------------------
C       PREPARE POTENTIAL VALUES FOR DERIVATIVE ON X-DIRECTION
C---------------------------------------------------------------------
        NDIM=LLX
        DO 30 KB=1,ITOT,LLX
        DO 10 K=1,LLX
10      Y(K)=POT(K+(KB-1))
C---------------------------------------------------------------------
C       EVALUATE DERIVATIVE ON X-DIRECTION
C---------------------------------------------------------------------
        CALL DET3(H,Y,Z,NDIM,IER)
        IF(IER.NE.0)GO TO 100
        DO 20 KP=1,LLX
20      DX(KP+(KB-1))=Z(KP)
30      CONTINUE
C---------------------------------------------------------------------
C       PREPARE POTENTIAL VALUES FOR DERIVATIVE ON Y-DIRECTION
C---------------------------------------------------------------------
        NDIM=LLY
        JD=0
        DO 60 JA=1,LLX
        DO 40 JB=JA,ITOT-(LLX-JA),LLX
        JD=JD+1
40      Y(JD)=POT(JB)
C---------------------------------------------------------------------
```

```
C       EVALUATE DERIVATIVE ON Y-DIRECTION
C-------------------------------------------------------------------------
        CALL DET3(H,Y,Z,NDIM,IER)
        IF(IER.NE.Ø)GO TO 1ØØ
        DO 5Ø JC=1,LLY
5Ø      DY(JA+LLX*(JC-1))=Z(JC)
        JC=Ø
6Ø      CONTINUE
C-------------------------------------------------------------------------
C       COMPUTE GRADIENT OF POTENTIAL DISTRIBUTION
C-------------------------------------------------------------------------
        DO 7Ø MN=1,ITOT
7Ø      EFIELD(MN)=SQRT(DX(MN)**2+DY(MN)**2)
C-------------------------------------------------------------------------
C       OUTPUT GRADIENT OF POTENTIAL
C-------------------------------------------------------------------------
        WRITE(2,8Ø)(EFIELD(N7),N7=1,ITOT)
8Ø      FORMAT(8E16.4)
        WRITE(11,91)
91      FORMAT('1',2ØX,'ELECTRICAL FIELD(COARSE NETWORK)',///)
        WRITE(11,92)
92      FORMAT(1ØX,'NODE',2ØX,'ELECTRICAL FIELD(VOLTS/UNITLENGTH)',//)
        WRITE(11,93)(JM,EFIELD(JM),JM=1,ITOT)
93      FORMAT(1ØX,I4,3ØX,E16.4)
        GO TO 12Ø
1ØØ     WRITE(15,11Ø)
11Ø     FORMAT(5X,'ERROR IN DERIVATIVE !!!!!!!')
12Ø     STOP
        END
```

```
C---------------------SUBROUTINE DET3---------------------------
C
C
C     THIS SUBROUTINE COMPUTES A VECTOR OF DERIVATIVE
C VALUES GIVEN A VECTOR OF FUNCTION VALUES WHOSE ENTRIES
C CORRESPOND TOEQUIDISTANTLY SPACED ARGUMENT VALUES. EXCEPT AT
C THE END POINTS THE DERIVATIVE OBTAINED IS THE DERIVATIVE OF
C THE LAGRANGIAN POLYNOMIAL OF DEGREE 2 RELEVANT TO THREE
C SUCCESSIVE POINTS.
C
C
C     VARIABLES NAME:
C
C     H - STEP SIZE VALUE
C     Y - GIVEN VECTOR OF FUNCTION VALUES
C     Z - VECTOR OF DERIVATIVE VALUES
C     NDIM - DIMENSION OF VECTORS Y AND Z
C     IER - ERROR PARAMETER
C           IER = -1     - NDIM IS LESS THAN THREE
C           IER =  0     - NO ERROR
C           IER = 1      - H IS ZERO
C
C     REFERENCE: IBM,1968,SYSTEM/360 SCIENTIFIC SUBROUTINE
C                PACKAGE, VERSION III, IBM,WHITE PLAINS,NY
C
C---------------------------------------------------------------
      SUBROUTINE DET3(H,Y,Z,NDIM,IER)
      DIMENSION Y(1),Z(1)
C
C     TEST OF DIMENSION
C
      IF(NDIM-3)4,1,1
C
C     TEST OF STEP SIZE
C
1     IF(H)2,5,2
C
C     PREPARE DIFFERENTIATION LOOP
C
2     HH=.5/H
      YY=Y(NDIM-2)
      B=Y(2)+Y(2)
      B=HH*(B+B-Y(3)-Y(1)-Y(1)-Y(1))
C
C     START DIFFERENTIATION LOOP
C
      DO 3 I=3,NDIM
      A=B
      B=HH*(Y(I)-Y(I-2))
3     Z(I-2)=A
```

```
C
C       END OF DIFFERENTIATION LOOP
C       NORMAL EXIT
C
        IER=0
        A=Y(NDIM-1)+Y(NDIM-1)
        Z(NDIM)=HH*(Y(NDIM)+Y(NDIM)+Y(NDIM)-A-A+YY)
        Z(NDIM-1)=B
        RETURN
C
C       ERROR EXIT IN CASE NDIM LESS THAN 3
C
4       IER=-1
        RETURN
C
C       ERROR EXIT IN CASE OF ZERO STEP SIZE
C
5       IER=1
        RETURN
        END
```

```
C---------------------PROGRAM FIGRID----------------------------
C
C
C     THIS PROGRAM EVALUATES THE POTENTIAL DISTRIBUTION ON A
C FINE NETWORK PLACED AROUND THE SOURCE. RESULTS OBTAINED FROM
C THE COARSE NETWORK, FOR THE INTERFACE BETWEEN THE TWO NETWORKS,
C ARE MULTIPLIED BY CONDUCTANCE VALUES AND USED AS INPUT CURRENTS
C ON THE MOST EXTERNAL LOOP OF THE FINE NETWORK.
C     THE PROGRAM IS WRITTEN IN FORTRN-4 LANGUAGE AND WAS
C DEVELOPED ON A DIGITAL COMPUTER CORPORATION MODEL PDP-10
C COMPUTER, A TIME-SHARING SYSTEM. THE PROGRAM REQUIRES A TOTAL
C OF 35K WORDS OF STORAGE.
C
C
C      PROGRAM INPUT:
C
C     1) DATA FILE GENERATED BY PROGRAM GRID
C     2) VALUE OF RESISTORS AND CURRENT USED ON THE NETWORK
C
C      PROGRAM OUTPUT:
C
C     1) POTENTIAL DISTRIBUTION FOR FINE NETWORK
C     2) IN CASE OF ERROR IN THE INTERPOLATIONS THE PROGRAM
C         STOPS AND AN ERROR MESSAGE IS GIVEN AS OUTPUT
C
C
C      VARIABLES NAME:
C
C      N - TOTAL NUMMBER OF NODES ON THE NETWORK
C      LX - NUMBER OF NODES IN X-DIRECTION
C      LY - NUMBER OF NODES IN Y-DIRECTION
C      M - WIDTH OF THE BAND OF THE MATRIX OF COEFFICIENTS
C          (HAS TO BE AN ODD NUMBER)
C      V - CURRENT VECTOR. AFTER IS RETURNED FROM BANSOL V
C          CONTAINS THE POTENTIAL VALUE FOR EACH NODE
C      C - IS THE DOUBLY DIMENSIONED NXM ARRAY WHICH CONTAINS
C          THE PERTINENT ELEMENTS OF THE MATRIX OF COEFFICIENTS
C          A. THE BAND ELEMENTS OF A GIVEN ROW OF MATRIX A
C          APPEAR IN THE SAME ROW OF C BUT SHIFTED SUCH THAT
C          ELEMENT A(I,J) BECOMES C(I,J-I+(M+1)/2)
C      COR - DOUBLY DIMENSIONED ARRAY WHICH CONTAINS THE
C            CONDUCTANCES RELATED TO EACH NODE
C      POTEN - SINGLY DIMENSIONED ARRAY WHICH CONTAINS THE
C              POTENTIAL DISTRIBUTION ON THE COARSE NETWORK
C      PFRAME - POTENTIAL VALUES FROM COARSE NETWORK BEING
C               USED FOR INPUT ON EXTERNAL LOOP OF FINE NETWORK
C      RESIST - VALUE OF THE RESISTORS BEING USED IN THE
C               NETWORK
C      CURREN - INPUT CURRENT USED IN THE NETWORK
C
C
```

```
C                     A.C. PIRES    DECEMBER 1974
C                     COLORADO SCHOOL OF MINES
C
C-----------------------------------------------------------------
      DIMENSION COR(399,4),V(399),C(399,43),POTEN(552),
     1  PFRAME(400),X(100),Y(100)
      COMMON /ONE/ N
      COMMON /THREE/ M
C-----------------------------------------------------------------
C     INITIALIZE CONSTANTS
C-----------------------------------------------------------------
      N=399
      LY=19
      LX=21
      ID=N
      M=2*LX+1
      IND1=0
C-----------------------------------------------------------------
C     INPUT POTENTIAL DATA FROM COARSE GRID
C-----------------------------------------------------------------
      READ(9,10)(POTEN(I),I=1,552)
10    FORMAT(8E16.4)
C-----------------------------------------------------------------
C     INPUT RESISTORS AND CURRENT VALUES
C-----------------------------------------------------------------
      READ(1,15)RESIST,CURREN
15    FORMAT(2F)
C-----------------------------------------------------------------
C     PREPARE A FRAME OF POTENTIAL VALUES FROM COARSE GRID
C-----------------------------------------------------------------
      DO 20 IA=1,LX,2
      KX=KX+1
20    PFRAME(IA)=POTEN(167+KX)
      DO 30 KA=379,399,2
      JA=JA+1
30    PFRAME(KA)=POTEN(374+JA)
      DO 40 LA=1,379,42
      LI=LI+23
40    PFRAME(LA)=POTEN(145+LI)
      DO 50 MA=LX,399,42
      MI=MI+23
50    PFRAME(MA)=POTEN(155+MI)
C-----------------------------------------------------------------
C     INTERPOLATE ON THE NORTH SIDE OF THE FRAME
C-----------------------------------------------------------------
      N1=11
      IMAX=11
      DO 70 KB=2,20,2
      DO 60 IB=1,11
      X(IB)=2*FLOAT(IB)-1
60    Y(IB)=PFRAME(2*IB-1)
```

```
      XI=FLOAT(KB)
      YX=YLAG(XI,X,Y,IND1,N1,IMAX,IEX)
      IF(IEX.NE.0)GO TO 1000
70    PFRAME(KB)=YX
C-----------------------------------------------------------------------
C     INTERPOLATE ON THE SOUTH SIDE OF THE FRAME
C-----------------------------------------------------------------------
      DO 90 KC=2,20,2
      DO 80 IC=1,11
      X(IC)=2*FLOAT(IC)-1
80    Y(IC)=PFRAME(377+2*IC)
      XI=FLOAT(KC)
      YX=YLAG(XI,X,Y,IND1,N1,IMAX,IEX)
      IF(IEX.NE.0)GO TO 1002
90    PFRAME(378+KC)=YX
C-----------------------------------------------------------------------
C     INTERPOLATE ON THE EAST SIDE OF THE FRAME
C-----------------------------------------------------------------------
      N1=10
      IMAX=10
      DO 120 KD=2,18,2
      LD=LD+42
      DO 100 IT=1,10
100   X(IT)=2*FLOAT(IT)-1
      DO 110 JD=1,379,42
      LE=LE+1
110   Y(LE)=PFRAME(JD)
      XI=FLOAT(KD)
      YX=YLAG(XI,X,Y,IND1,N1,IMAX,IEX)
      IF(IEX.NE.0)GO TO 1004
120   PFRAME(LD-21+1)=YX
C-----------------------------------------------------------------------
C     INTERPOLATE ON THE WEST SIDE OF THE FRAME
C-----------------------------------------------------------------------
      DO 150 KE=2,18,2
      LF=LF+42
      DO 130 IE=1,10
130   X(IE)=2*FLOAT(IE)-1
      DO 140 JE=LX,399,42
      LG=LG+1
140   Y(LG)=PFRAME(JE)
      XI=FLOAT(KE)
      YX=YLAG(XI,X,Y,IND1,N1,IMAX,IEX)
      IF(IEX.NE.0)GO TO 1006
150   PFRAME(LF)=YX
      PFRAME(190)=0.
      PFRAME(210)=0.
C-----------------------------------------------------------------------
C     SET CONDUCTANCE VALUES FOR EACH NODE
C-----------------------------------------------------------------------
      DO 160 I=1,N
```

```
      DO 160 J=1,4
160   CDR(I,J)=1./RESIST
C-----------------------------------------------------------------------
C     SET CURRENT VECTOR
C-----------------------------------------------------------------------
      DO 170 K=1,N
170   V(K)=0.
      V(179)=CURREN
      V(221)=-CURREN
      DO 180 KZ=1,LX
180   V(KZ)=PFRAME(KZ)*1./RESIST
      DO 190 KY=1,379,LX
190   V(KY)=PFRAME(KY)*1./RESIST
      DO 200 KW=LX,N,LX
200   V(KW)=PFRAME(KW)*1./RESIST
      DO 210 KV=379,N
210   V(KV)=PFRAME(KV)*1./RESIST
C-----------------------------------------------------------------------
C     GENERATE MATRIX OF COEFFICIENTS
C-----------------------------------------------------------------------
      DO 220 LA=1,N
220   C(LA,22)=CDR(LA,1)+CDR(LA,2)+CDR(LA,3)+CDR(LA,4)
      DO 230 LB=1,N-1
230   C(LB,23)=-CDR(LB,1)
      DO 240 LC1=LX,N,LX
240   C(LC1,23)=0.
      DO 250 LD1=1,N-LX
250   C(LD1,M)=-CDR(LD1,2)
      DO 260 LE1=2,N
260   C(LE1,21)=-CDR(LE1,3)
      DO 270 LF1=22,N,LX
270   C(LF1,21)=0.
      DO 280 LG1=22,N
280   C(LG1,1)=-CDR(LG1,4)
      DO 281 IP=2,20
      DO 281 IK=23-IP,N
281   C(IK,IP)=0.
      DO 282 IV=24,42
      DO 282 IQ=1,N-(IV-22)
282   C(IQ,IV)=0.
C-----------------------------------------------------------------------
C     SOLVE SYSTEM OF LINEAR EQUATIONS
C-----------------------------------------------------------------------
      CALL BANSOL(C,V,ID)
C-----------------------------------------------------------------------
C     OUTPUT POTENTIAL DISTRIBUTION FOR FINE NETWORK
C-----------------------------------------------------------------------
      WRITE(10,300)(V(LJ),LJ=1,N)
300   FORMAT(8E16.4)
      GO TO 310
1000  WRITE(10,1010)
```

```
      GO TO 310
1002  WRITE(10,1020)
      GO TO 310
1004  WRITE(10,1030)
      GO TO 310
1006  WRITE(10,1040)
1010  FORMAT(10X,'ERROR IN INTERPOLATION 1 !!!!!!')
1020  FORMAT(10X,'ERROR IN INTERPOLATION 2 !!!!!!')
1030  FORMAT(10X,'ERROR IN INTERPOLATION 3 !!!!!!')
1040  FORMAT(10X,'ERROR IN INTERPOLATION 4 !!!!!!')
310   CONTINUE
      STOP
      END
C
```

```
C-------------------------------FUNCTION YLAG-----------------------------
C
C
C      THIS FUNCTION CRRIES OUT LAGRANGIAN INTERPOLATION OF A
C FUNCTION OF ONE VARIABLE. GIVEN A SET OF POINTS, THIS FUNCTION
C SUBPROGRAM CALCULATES THE POLYNOMIAL FITTING THROUGH THEM
C BY THE LAGRANGIAN METHOD AND THEN INTERPOLATES AT ANY
C DESIRED LOCATION ALONG THE CURVE.
C
C
C      VARIABLES NAME:
C
C      YLAG - FUNCTION SUBPROGRAM WHICH RETURNS THE FUNCTION
C             APPROXIMATION AT XI
C      XI - ABCISSA OF THE DESIRED INTERPOLATED POINT
C      X - SINGLY DIMENSIONED ARRAY OF DISTINCT MONOTONE
C          INCREASING ABCISSAS
C      Y - SINGLY DIMENSIONED ARRAY OF CORRESPONDING ORDINATES
C      IND1 - IS THE INTEGER INDEX OF THE CENTRAL POINT OF THE
C             INTERPOLATION FORMULA. IF IND1=0, A TABLE SEARCH
C               OF THE Y-ARRAY WILL BE PERFORMED
C      N1 - NUMBER OF POINTS FOR THE INTERPOLATION POLYNOMIAL
C           (ONE MORE THAN THE DEGREE OF THE POLYNOMIAL
C           BEING USED FOR THE INTERPOLATION)
C      IMAX - NUMBER OF POINTS OF THE X-ARRAY OR Y-ARRAY
C      IEX - ERROR PARAMETER
C            IEX = -1      EXTRAPOLATION OCCURED BELOW THE TABLE
C            IEX =  0      INTERPOLATION PERFORMED
C            IEX =  1      EXTRAPOLATION OCCURED ABOVE THE TABLE
C      YX - INTERPOLATED Y-VALUE
C
C
C      REFERENCE:(MODIFIED AFTER)
C             WESTLEY G.W. AND WATTS J.A.,EDS.,1970,THE
C               COMPUTING TECHNOLOGY CENTER NUMERICAL ANALYSIS
C             LIBRARY, NTIS, SPRINGFIELD, VA.
C
C-------------------------------------------------------------------------
      FUNCTION YLAG(XI,X,Y,IND1,N1,IMAX,IEX)
      DIMENSION X(1),Y(1)
      IND=IND1
      N=N1
      IEX=0
      IF(N.LE.IMAX)GO TO 10
      N=IMAX
      IEX=N
10    IF(IND.GT.0)GO TO 40
      DO 20 J=1,IMAX
      IF(XI-X(J))30,130,20
20    CONTINUE
      IEX=1
```

```
      GO TO 70
30    IND=J
40    IF(IND.GT.1)GO TO 50
      IEX=-1
50    INL=IND-(N+1)/2
      IF(INL.GT.0)GO TO 60
      INL=1
60    INU=INL+N-1
      IF(INU.LE.IMAX)GO TO 80
70    INL=IMAX-N+1
      INU=IMAX
80    S=0.
      P=1.
      DO 110 J=INL,INU
      P=P*(XI-X(J))
      D=1.
      DO 100 I=INL,INU
      IF(I.NE.J)GO TO 90
      XD=XI
      GO TO 100
90    XD=X(J)
100   D=D*(XD-X(I))
110   S=S+Y(J)/D
      YLAG=S*P
120   RETURN
130   YLAG=Y(J)
      GO TO 120
      END
C
C
```

```
C-----------------------------PROGRAM EFFIGR-----------------------------
C
C
C      THIS PROGRAM EVALUATES THE GRADIENT OF THE POTENTIAL
C DISTRIBUTION OBTAINED WITH THE FINE NETWORK. ONLY THE VALUES
C IN A DIAMOND-SHAPED AREA AROUND THE SOURCE ARE TO BE
C USED BECAUSE THEY ARE LOW ERROR VALUES.
C      THE PROGRAM IS WRITTEN IN FORTRAN-4 LANGUAGE AND WAS
C DEVELOPED ON A DIGITAL COMPUTER CORPORATION MODEL PDP-10
C COMPUTER, A TIME-SHARING SYSTEM.
C
C
C      PROGRAM INPUT:
C
C      DATA FILE GENERATED BY PROGRAM FIGRID
C
C      PROGRAM OUTPUT:
C
C     1) POTENTIAL FIELD DISTRIBUTION USED AS INPUT
C     2) GRADIENT OF POTENTIAL FIELD
C     3) IN CASE OF ERROR IN OBTAINING THE DERIVATIVES THE
C        PROGRAM STOPS AND AN ERROR MESSAGE IS GIVEN AS OUTPUT
C
C
C      VARIABLES NAME:
C
C      (SEE PROGRAM EFD)
C
C
C                              A.C. PIRES    DECEMBER 1974
C                              COLORADO SCHOOL OF MINES
C
C-----------------------------------------------------------------------
      DIMENSION POTEN(399),Y(50),DX(399),DY(399),EFIELD(399),
     1  Z(50)
      LX=21
      LY=19
      H=.5
      DLIM=1.E-4
      ITOT=LX*LY
C-----------------------------------------------------------------------
C      INPUT DATA: POTENTIAL DISTRIBUTION FROM PROG. FIGRID
C-----------------------------------------------------------------------
      READ(10,3)(POTEN(JE),JE=1,ITOT)
C-----------------------------------------------------------------------
C      OUTPUT POTENTIAL FIELD DISTRIBUTION
C-----------------------------------------------------------------------
3     FORMAT(8E16.4)
      WRITE(8,9)
9     FORMAT(20X,'POTENTIAL FIELD(FINE NETWORK)',///)
      WRITE(8,11)
```

```
11    FORMAT(10X,'NODE',20X,'POTENTIAL FIELD(VOLTS)',//)
      WRITE(6,12)(K,POTEN(K),K=1,ITOT)
12    FORMAT(10X,I4,27X,E16.4)
C-----------------------------------------------------------------------
C     SET TO ZERO ABSOLUTE VALUES OF POTENTIAL LESS THAN .0001
C-----------------------------------------------------------------------
      DO 5 J=1,ITOT
      POTEN(J)=ABS(POTEN(J))
      IF(POTEN(J).LE.DLIM)POTEN(J)=0.
5     CONTINUE
C-----------------------------------------------------------------------
C     PREPARE POTENTIAL VALUES FOR DERIVATIVE ON X-DIRECTION
C-----------------------------------------------------------------------
      NDIM=LX
      DO 30 KB=1,ITOT,LX
      DO 10 K=1,LX
10    Y(K)=POTEN(K+(KB-1))
C-----------------------------------------------------------------------
C     EVALUATE DERIVATIVE ON X-DIRECTION
C-----------------------------------------------------------------------
      CALL DET3(H,Y,Z,NDIM,IER)
      IF(IER.NE.0)GO TO 100
      DO 20 KP=1,LX
20    DX(KP+(KB-1))=Z(KP)
30    CONTINUE
C-----------------------------------------------------------------------
C     PREPARE POTENTIAL VALUES FOR DERIVATIVE ON Y-DIRECTION
C-----------------------------------------------------------------------
      NDIM=LY
      JD=0
      DO 60 JA=1,LX
      DO 40 JB=JA,ITOT-(LX-JA),LX
      JD=JD+1
40    Y(JD)=POTEN(JB)
C-----------------------------------------------------------------------
C     EVALUATE DERIVATIVE ON Y-DIRECTION
C-----------------------------------------------------------------------
      CALL DET3(H,Y,Z,NDIM,IER)
      IF(IER.NE.0)GO TO 100
      DO 50 JC=1,LY
50    DY(JA+LX*(JC-1))=Z(JC)
      JD=0
60    CONTINUE
C-----------------------------------------------------------------------
C     COMPUTE GRADIENT OF POTENTIAL DISTRIBUTION
C-----------------------------------------------------------------------
      DO 70 MN=1,ITOT
70    EFIELD(MN)=SQRT(DX(MN)**2+DY(MN)**2)
C-----------------------------------------------------------------------
C     OUTPUT GRADIENT OF POTENTIAL DISTRIBUTION
C-----------------------------------------------------------------------
```

```
      WRITE(3,75)(EFIELD(N7),N7=1,ITOT)
75    FORMAT(8E16.4)
      WRITE(8,91)
91    FORMAT('1',20X,'ELECTRICAL FIELD(FINE NETWORK)',///)
      WRITE(8,92)
92    FORMAT(10X,'NODE',20X,'ELECTRICAL FIELD(VOLTS/UNIT LENGTH)',//)
      WRITE(8,93)(JM,EFIELD(JM),JM=1,ITOT)
93    FORMAT(10X,I4,30X,E16.4)
      GO TO 120
100   WRITE(14,110)
110   FORMAT(5X,'ERROR IN DERIVATIVE !!!!')
120   STOP
      END
```

```
C--------------------PROGRAM APCON1---------------------------
C
C      THIS PROGRAM EVALUATES THE APPARENT CONDUCTANCE
C DISTRIBUTION ON THE COARSE NETWORK. THE TOTAL ELECTRICAL
C FIELD DISTRIBUTION OBTAINED WITH THE PROGRAM EFD IS USED AS
C INPUT.
C      THE PROGRAM IS WRITTEN IN FORTRAN-4 LANGUAGE AND
C WAS DEVELOPED ON A DIGITAL COMPUTER CORPORATION MODEL
C PDP-10 COMPUTER, A TIME-SHARING SYSTEM.
C
C
C      THE PROGRAM INPUT IS:
C
C      THE TOTAL ELECTRICAL FIELD DISTRIBUTION OBTAINED
C BY PROGRAM EFD.
C
C
C      THE PROGRAM OUTPUT IS:
C
C      THE APPARENT CONDUCTANCE DISTRIBUTION ON THE RESISTOR
C NETWORK.
C
C
C      VARIABLES NAME:
C
C      LX - NUMBER OF NODES ON X-DIRECTION
C      LY - NUMBER OF NODES ON Y-DIRECTION
C      DIPOLE - LENGTH OF THE DIPOLE SOURCE
C      ETOTAL - TOTAL ELECTRICAL FIELD DISTRIBUTION
C      R1 - DISTANCE FROM ONE END OF THE DIPOLE SOURCE TO THE
C                INTERSECTION OF THE MEASURING DIPOLES.
C      R2 - DISTANCE FROM OTHER END OF THE DIPOLE SOURCE TO
C             THE INTERSECTION OF THE MEASURING DIPOLES
C      ADELTA - ANGLE FORMED BY R1 AND R2
C      APPCD - APPARENT CONDUCTANCE ON THE RESISTOR NETWORK
C
C
C
C                   A. C. PIRES        FEBRUARY 1975
C                  COLORADO  SCHOOL  OF  MINES
C------------------------------------------------------------
C
      DIMENSION FIX(20),ETOTAL(420),R1(400),R2(400),S(400),
     1  R(400),THFD(400),ADELTA(400),PART1(400),PART2(400),
     2  APPCD(400)
C-------------------------------------------------------------
C      INITIALIZE CONSTANTS
C-------------------------------------------------------------
      LX=19
      LY=20
      ITOT=LX*LY
```

```
      PI=3.14159265
      DIPOLE=1.
      CURREN=1.
      DATA (FIX(K),K=1,19)/9.,8.,7.,6.,5.,4.,3.,2.,1.,0.,
     1  1.,2.,3.,4.,5.,6.,7.,8.,9./
C-----------------------------------------------------------------
C     INPUT DATA: FILE CREATED BY PROGRAM EFD
C-----------------------------------------------------------------
      READ(2,10)(ETOTAL(IN),IN=1,ITOT)
10    FORMAT(8E16.4)
C-----------------------------------------------------------------
C     EVALUATE R1 AND R2 FOR ALL NODES
C-----------------------------------------------------------------
      N=1
      DO 22 I=1,190
      K=K+1
      J=N
      R1(I)=SQRT(FIX(K)**2+FIX(J)**2)
      R2(I)=SQRT(FIX(K)**2+(FIX(J)+1)**2)
      IF(K.LT.LX)GO TO 22
      K=0
      N=N+1
22    CONTINUE
      DO 30 I2=1,190
      R1(ITOT+1-I2)=R1(I2)
      R2(ITOT+1-I2)=R2(I2)
30    CONTINUE
C-----------------------------------------------------------------
C     EVALUATE ANGLE BETWEEN R1 AND R2 FOR ALL NODES
C-----------------------------------------------------------------
      DO 40 N3=1,180
      S(N3)=.5*(DIPOLE+R1(N3)+R2(N3))
      R(N3)=SQRT(((S(N3)-DIPOLE)*(S(N3)-R1(N3))*(S(N3)-
     1  R2(N3)))/S(N3))
      THFD(N3)=R(N3)/(S(N3)-DIPOLE)
      ADELTA(N3)=2.*ATAN(THFD(N3))
40    CONTINUE
      DO 50 N4=182,190
      S(N4)=.5*(DIPOLE+R1(N4)+R2(N4))
      R(N4)=SQRT(((S(N4)-DIPOLE)*(S(N4)-R1(N4))*(S(N4)-
     1  R2(N4)))/S(N4))
      THFD(N4)=R(N4)/(S(N4)-DIPOLE)
      ADELTA(N4)=2.*ATAN(THFD(N4))
50    CONTINUE
      DO 60 N5=1,190
      ADELTA(ITOT+1-N5)=ADELTA(N5)
60    CONTINUE
C-----------------------------------------------------------------
C     EVALUATE APPARENT CONDUCTANCE FOR ALL NODES
C-----------------------------------------------------------------
      DO 70 N6=1,180
```

```
      PART1(N6)=CURREN/(2.*PI*R1(N6)*ETCTAL(N6))
      PART2(N6)=SQRT(1.+(R1(N6)/R2(N6))**2-2.*(R1(N6)/R2(N6))*
     1  COS(ADELTA(N6)))
      APPCD(N6)=PART1(N6)*PART2(N6)
 70   CONTINUE
      DO 80 N7=182,199
      PART1(N7)=CURREN/(2.*PI*R1(N7)*ETCTAL(N7))
      PART2(N7)=SQRT(1.+(R1(N7)/R2(N7))**2-2.*(R1(N7)/R2(N7))*
     1  COS(ADELTA(N7)))
      APPCD(N7)=PART1(N7)*PART2(N7)
 80   CONTINUE
      DO 90 N8=201,ITOT
      PART1(N8)=CURREN/(2.*PI*R1(N8)*ETCTAL(N8))
      PART2(N8)=SQRT(1.+(R1(N8)/R2(N8))**2-2.*(R1(N8)/R2(N8))*
     1  COS(ADELTA(N8)))
      APPCD(N8)=PART1(N8)*PART2(N8)
 90   CONTINUE
C---------------------------------------------------------------------
C     OUTPUT RESULTS
C---------------------------------------------------------------------
      WRITE(17,121)
121   FORMAT(20X,'APPARENT CONDUCTANCE(COARSE NETWORK)',///)
      WRITE(17,122)
122   FORMAT(10X,'NODE',20X,'APPARENT CONDUCTANCE(MHO)',//)
      WRITE(17,123)(JA,APPCD(JA),JA=1,ITOT)
123   FORMAT(10X,I4,28X,E16.4)
      STOP
      END
```

```
C----------------------PROGRAM APCON2-------------------------------
C
C
C      THIS PROGRAM EVALUATES THE APPARENT CONDUCTANCE
C DISTRIBUTION ON THE FINE RESISTOR NETWORK. THE TOTAL
C ELECTRICAL FIELD DISTRIBUTION OBTAINED WITH THE PROGRAM
C EFFIGR IS USED AS INPUT.
C      THE PROGRAM IS WRITTEN IN FORTRAN-4 LANGUAGE AND WAS
C DEVELOPED ON A DIGITAL COMPUTER CORPORATION MODEL PDP-10
C COMPUTER, A TIME-SHARING SYSTEM.
C
C
C      THE PROGRAM INPUT IS:
C
C      THE TOTAL ELECTRICAL FIELD DISTRIBUTION OBTAINED
C BY PROGRAM EFFIGR
C
C
C      THE PROGRAM OUTPUT IS:
C
C      THE APPARENT CONDUCTANCE DISTRIBUTICN ON THE NETWORK
C
C
C      VARIABLES NAME:
C
C      (SEE COMPUTER PROGRAM APCON1)
C
C
C
C                          A. C. PIRES   FEBRUARY 1975
C                          COLORADO  SCHOOL  OF  MINES
C-------------------------------------------------------------------
C
      DIMENSION FIX(21),ETOTAL(400),R1(400),R2(400),S(400),
     1 R(400),THFD(400),ADELTA(400),PART1(400),PART2(400),
     2 APPCD(400)
C-------------------------------------------------------------------
C     INITIALIZE CONSTANTS
C-------------------------------------------------------------------
      LX=21
      LY=19
      ITOT=LX*LY
      PI=3.14159265
      DIPOLE=1.
      CURREN=1.
      DATA (FIX(K),K=1,21)/5.,4.5,4.,3.5,3.,2.5,2.,1.5,1.,.5,
     1 0.,.5,1.,1.5,2.,2.5,3.,3.5,4.,4.5,5./
C-------------------------------------------------------------------
C     INPUT DATA: FILE CREATED BY PROGRAM EFFIGR
C-------------------------------------------------------------------
      READ(3,10)(ETOTAL(IN),IN=1,ITOT)
```

```
10    FORMAT(8E16.4)
C-----------------------------------------------------------------------
C     EVALUATE R1 AND R2 FOR ALL NODES
C-----------------------------------------------------------------------
      N=1
      DO 20 I=1,189
      K=K+1
      J=N
      R1(I)=SQRT(FIX(K)**2+(FIX(J)-1)**2)
      R2(I)=SQRT(FIX(K)**2+FIX(J)**2)
      IF(K.LT.21)GO TO 20
      K=0
      N=N+1
20    CONTINUE
      DO 30 I2=190,210
      K2=K2+1
      R1(I2)=SQRT(FIX(K2)**2+.5**2)
      R2(I2)=R1(I2)
30    CONTINUE
      DO 40 I3=1,189
      R1(ITOT+1-I3)=R1(I3)
      R2(ITOT+1-I3)=R2(I3)
40    CONTINUE
C-----------------------------------------------------------------------
C     EVALUATE ANGLE BETWEEN R1 AND R2 FOR ALL NODES
C-----------------------------------------------------------------------
      DO 50 N4=1,178
      S(N4)=.5*(DIPOLE+R1(N4)+R2(N4))
      R(N4)=SQRT(((S(N4)-DIPOLE)*(S(N4)-R1(N4))*
     1  (S(N4)-R2(N4)))/S(N4))
      THFD(N4)=R(N4)/(S(N4)-DIPOLE)
      ADELTA(N4)=2.*ATAN(THFD(N4))
50    CONTINUE
      DO 60 N5=180,210
      S(N5)=.5*(DIPOLE+R1(N5)+R2(N5))
      R(N5)=SQRT(((S(N5)-DIPOLE)*(S(N5)-R1(N5))*
     1  (S(N5)-R2(N5)))/S(N5))
      THFD(N5)=R(N5)/(S(N5)-DIPOLE)
      ADELTA(N5)=2.*ATAN(THFD(N5))
60    CONTINUE
      DO 70 N6=1,189
      ADELTA(ITOT+1-N6)=ADELTA(N6)
70    CONTINUE
C-----------------------------------------------------------------------
C     EVALUATE APPARENT CONDUCTANCE FOR ALL NODES
C-----------------------------------------------------------------------
      DO 80 N7=1,178
      PART1(N7)=CURREN/(2.*PI*R1(N7)*ETCTAL(N7))
      PART2(N7)=SQRT(1.+(R1(N7)/R2(N7))**2-2.*(R1(N7)/R2(N7))*
     1  COS(ADELTA(N7)))
      APPCD(N7)=PART1(N7)*PART2(N7)
```

```
 80     CONTINUE
        DO 90 N8=180,220
        PART1(N8)=CURREN/(2.*PI*R1(N8)*ETCTAL(N8))
        PART2(N8)=SQRT(1.+(R1(N8)/R2(N8))**2-2.*(R1(N8)/R2(N8))*
       1 COS(ADELTA(N8)))
        APPCD(N8)=PART1(N8)*PART2(N8)
 90     CONTINUE
        DO 100 N9=222,ITOT
        PART1(N9)=CURREN/(2.*PI*R1(N9)*ETCTAL(N9))
        PART2(N9)=SQRT(1.+(R1(N9)/R2(N9))**2-2.*(R1(N9)/R2(N9))*
       1 COS(ADELTA(N9)))
        APPCD(N9)=PART1(N9)*PART2(N9)
 100    CONTINUE
C-----------------------------------------------------------------
C       OUTPUT RESULTS
C-----------------------------------------------------------------
        WRITE(16,121)
 121    FORMAT(20X,'APPARENT CONDUCTANCE(FINE NETWORK)',///)
        WRITE(16,122)
 122    FORMAT(10X,'NODE',20X,'APPARENT CONDUCTANCE(MHO)',//)
        WRITE(16,123)(JA,APPCD(JA),JA=1,ITOT)
 123    FORMAT(10X,I4,28X,E16.4)
        STOP
        END
```

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