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**MEASURING THE EFFICIENCY OF U.S. ARMY
COMBAT UNITS: AN APPLICATION OF
DATA ENVELOPMENT ANALYSIS**

by

John R. Crino

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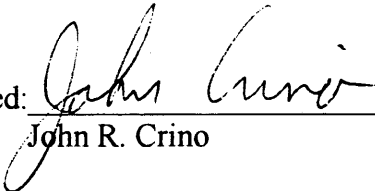
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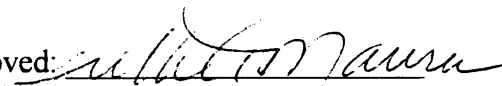
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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Master of Science (Mathematical and Computer Sciences).

Golden, Colorado

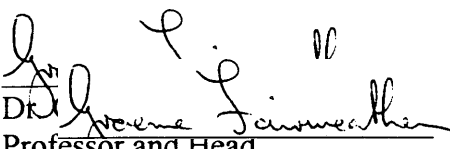
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ABSTRACT

The purpose of this study is to determine the applicability of Data Envelopment Analysis (DEA) to the efficiency evaluation of United States Army combat units in a Combat Training Center (CTC). Like most public sector organizations, the Army has difficulties measuring how well its sub-units perform. Most of the Army's measurements of efficiency are based on ratio analyses. A ratio analysis such as the Army currently uses is limited on what it can measure, and what it provides for information.

This study provides an alternative measure of efficiency that the Army may use to help evaluate its sub-units performances. The method introduced is called Data Envelopment Analysis. DEA has many more capabilities than the traditional forms of efficiency measures. DEA can measure multiple input and output factors and provide prescriptive information to the evaluated units.

The results of this study indicate that Data Envelopment Analysis is a feasible measure of efficiency. DEA performed much better than the traditional Force Exchange Ratio currently used to measure combat unit efficiency. With the resources already available to the Army, DEA can easily be incorporated into the system.

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ACKNOWLEDGMENTS

I would like to express my sincere gratitude to Dr. Ruth Maurer for her help as my thesis advisor and instructor, Dr. Gene Woolsey and Professor Bill Astle for their instruction over the past two years, and especially Dr. Frank Grange for all his time and effort in making this thesis possible. I can confidently say that the four of you provided me with a superb education over the last two years.

Special thanks go to workers at the Center for Army Lessons Learned and Combat Training Center Warrior Information Network for assisting my data collection efforts. You provided assistance beyond my expectations.

Finally, I would like to thank my wife Laura for her continuous support during the entire grueling process. You provided me uninterrupted time to complete this study, and never ending emotional support.

Chapter 1

INTRODUCTION

1.1 General Discussion

Data Envelopment Analysis is a mathematical model developed by Charnes, Cooper, and Rhodes in 1978. They initially developed this model as a linear programming model for use in evaluating the activities of not-for-profit entities in the public sector. The model provides a single measure for the efficiency indicator, as well as determining objective weights for the observational data which include multiple input and output factors (Charnes, Cooper, Rhodes, 1978). One of the main advantages in using DEA is its ability to measure the efficiency of an organization using multiple input and output factors. The model is also capable of measuring the strengths that each organization possesses against its peer group. One of the model outputs is a set of objective weights that make each organization look as good as possible against all the other evaluated organizations. The model performs this analysis with fairness and without any bias.

Throughout this study, the term “efficiency” is used quite often. Efficiency in this study is a measure of a unit’s use of resources to obtain outcomes. This is relative to

other units' ability to do the same. Charnes and Cooper define the term "efficiency": "100% relative efficiency is attained by any (unit) only when comparisons with other relevant (units) do not provide evidence of "inefficiency" in the use of any input or output"(Charnes and Cooper, 1985, 59).

In the private sector, a common measure of efficiency is a company's net present value. Generally, a company that provides a sufficient return on investment to its investors is considered efficient, while a company that does not is considered inefficient. Measures are currently in place to help determine if a private company is efficient or not, but there are few measures for public sector organizations. Public sector organizations whose performance is not profit based are more difficult to measure. Examples are public schools, universities, hospitals, city police forces, government branches, and the military.

Studies already exist of the use of DEA for public organizations. These studies include measuring the efficiency of national universities and national liberal arts colleges (Breu and Raab, 1994), forest management (Chang and Hwang, 1993), hospitals (Hogan and Wroblewshi, 1993), and military recruitment (Lewis and Morey, 1981).

An organization under evaluation is referred to as a Decision Making Unit (DMU). For example, a DMU could be a university department, city police precinct, or

hospital branch. The process of choosing DMUs is explained in Chapter 3. The data used to evaluate DMUs are referred to as input and output factors.

Data Envelopment Analysis is becoming a common approach to measuring efficiency. Other than DEA, there are two methods more commonly used to measure public sector efficiencies. These two methods are ratio analysis and multiple regression analysis. Each method has its shortcomings, and many times proves unsuitable in measuring organization efficiency (Silkman, 1986, 8). For example, ratio analysis will not accommodate multiple inputs and outputs. Numerous ratios must be used to measure all the inputs and outputs in an organization. Many times, one ratio may indicate efficiency in the unit while another shows inefficiency. There is no collective ratio to indicate an overall efficiency score. Multiple regression analysis has three principal drawbacks. First, single equation regression requires that there be only one output for all the inputs. Second, regression analysis measures the efficiency relative to the average efficiency in the group rather than the best efficiency. Finally, regression analysis requires a parametric specification of an equation detailing how inputs are combined to produce outputs (Silkman, 1986, 9.)

1.2 Measuring Army units' efficiency using DEA

A military unit's efficiency is usually not measured by any method commonly used in the private sector. A military unit is more closely measured by non profit type

characteristics. The Army standard of measurement for military units is the Unit Status Report (USR). The USR is a monthly report that each unit must submit to its higher headquarters. The USR starts at the battalion level and continues all the way to the Department of the Army. At the Department of the Army level, an analyst may look at the USR report for any battalion, brigade, division, or corps.

The USR indicates the “go to war” status of each military unit. The report generally includes four areas of a military unit. These areas are training, maintenance, logistics, and personnel. The actual performance measures for each area vary. For example, in training, units are measured more qualitatively than quantitatively. Commanders give a general rating based on needs in their training. For maintenance, measures are on a percentage basis. The number of “down” days of a unit’s equipment are compared to the number of “working” days of their equipment. In logistics, the measure is also based on percentages. The number of needed equipment components are compared to the number of components on hand. For personnel, the percentage of assigned strength to on hand strength is the main measure. Each of these four areas has numerous sub-categories. Once each area’s sub-categories are measured, a rating of C1, C2, C3, or C4 is assigned based on specific set army regulations. A rating of C1 is best, and C4 is the worst. Once all four areas are assigned a rating, an overall rating of C1, C2, C3, or C4 is assigned to each unit based on specific regulations. The regulations vary

depending on what type of unit is evaluated, and it is beyond the scope of this study to explain the specifics.

Currently, there are no Army wide standard quantitative measures for evaluating a unit in a combat/training environment. Measures currently employed are mostly based on subjective evaluations by observer controllers in a field exercise. An observer controller is a fellow soldier that observes the battle and provides feedback on the unit's performance. Divisions regularly perform External Evaluations (EXEVALS) for their brigades and battalions in order to provide feedback on how well a unit performs in a combat environment. The most common quantitative measure of a unit's performance is the number of friendly casualties and friendly equipment destroyed compared to the number of enemy casualties and equipment destroyed. The unit's subjective and quantitative performance measures usually go no higher than the division level.

The Army does have three central training facilities that military units frequently go to for realistic combat training. The training facilities are explained in detail in section 1.5. It is here that data is collected at the Army level on how well units perform in a combat environment. The main measure of unit efficiency is based on the Force Exchange Ratio (FER) which is explained in detail in section 1.6.

The Army has measures in place to evaluate military units, but these measures have some weaknesses. The USR is the Army's main measure to evaluate units, but it provides a very general evaluation score. The measure is not prescriptive and provides

little feedback on how well a unit compares to other units in specific areas. In a combat/training environment, a commonly used measure is the Force Exchange Ratio. The FER is limited in what it can measure. It can not measure multiple input and output factors of a unit.

Data Envelopment Analysis is a model that can provide the Army with unit efficiency measures. The model may be used for either an overall unit measure like the USR, or a combat scenario measure. The model allows multiple input and output factors. The efficiency score from the model indicates whether a unit compares efficiently against other units. The model's output provides prescriptive measures that can help a unit determine where it is weak compared to its peers, and by how much they need to fix their input and output factors. The model also provides the ability to rank order each unit based on its efficiency score.

The scope of this study focuses primarily on the efficiency of a unit in a combat environment. The organizations used in this study are battalion size army combat units. The battalions evaluated are Mechanized and Armor task forces and Cavalry squadrons. DEA is used as a model to determine the task forces' efficiency scores. The task forces are evaluated on their performances at the National Training Center (NTC). This is a small application of DEA that is a basis for potential use on a more grand scale in the Army.

1.3 Current Measures of Efficiency

There are some measures of efficiency (MOE) used in the military that are presented in The Military Applications of Modeling by John A. Battilega and Judith K. Grange. There were three main types of MOEs that were discussed. One MOE is a single measure determined by an established criterion of success for a mission. Examples of criteria for ground combat established in the book are: probability of success in achieving a mission, rate of advance of a unit, time to reach an objective, and attrition rate inflicted on the enemy (Battilega and Grange, 41). The MOE of a unit is determined by the above criteria. A unit's performance in combat is then based on how well it scored on a single criterion. The second form of MOE is a ratio analysis. Two examples mentioned in the book are Expected Exchange Ratio, number of enemy to friendly casualties, and Resource Exchange Rate, friendly resources to achieve the mission vs. enemy resources expended, (Battilega and Grange, 40). The third MOE mentioned in the book was developed by Tiede and Leake in 1971. This MOE provides a methodology for rank ordering units when multiple criteria of success were used. Tiede and Leake placed battle outcomes into three different classes. Class I contained units that completed the mission, class II contained units that partially completed the mission, and Class III had units that did not complete the mission. Within each class, units were ranked on how well each unit fared in strength compared to the enemy. The unit with

the highest final strength vs. lowest enemy strength remaining after the battle ranked best in that class. Each class was then ranked in this fashion.

Battilega and Grange discuss later in their book that there currently is a problem with evaluating a unit when multiple criteria are presented (Battilega and Grange, 58-61). Even single criterion MOEs easily turns into a multiple criteria analysis. For example, four units partially complete a mission at different levels of success. An analyst will then further look into the different conditions that each unit performed in or the different course of action that the enemy used. An analysis of this nature becomes more difficult to measure quantitatively. Data Envelopment Analysis may be a possible solution to this problem. This study is a preliminary look at using DEA to measure the efficiency of units with multiple input and output factors.

1.4 Mechanized and Armor Task Forces and Cavalry Squadrons

A Mechanized Infantry Battalion is a military organization consisting of approximately 822 soldiers and 109 armored vehicles (SH7-176, 1989, 5-1). These vehicles are primarily M2 Bradley Infantry Fighting Vehicles and M113 Armored Personnel Carriers. The mission of the Infantry is to close with the enemy by means of fire and maneuver, in order to destroy or capture him, or repel his assault by fire, close combat, and counterattack. An Armor Battalion is also a military organization, and it consists of approximately 562 soldiers and 91 armored vehicles (SH7-176, 1989, 8-1).

These vehicles are primarily M1 Abram Tanks. The mission of the Armor is to close with and destroy enemy forces using fire, maneuver, and shock effect. The Cavalry Squadron is a mix of infantry, armor, and field artillery assets. The actual makeup of the squadron tends to vary from unit to unit. The squadron generally has up to 1000 soldiers and 89 armored vehicles. The vehicles are a mix of M1 Abram Tanks and M3 Cavalry Fighting Vehicles.

Mechanized Infantry Battalions and Armor Battalions work closely with each other on the battlefield. Mechanized Infantry Divisions have Armor Battalions assigned within the division and Armor Divisions have Mechanized Infantry Battalions assigned within the division. In battle, Mechanized Infantry Battalions usually have Armor Companies attached to them, and Armor Battalions usually have Mechanized Infantry Companies attached to them. The cross attachment of companies makes up a task force. A task force is a Battalion size organization with a mixture of Infantry and Armor Companies. The DMUs in this study are task forces. Task forces also have other elements attached to them in the battlefield. Those other elements mainly include Engineer, Artillery, and Air Defense assets. Task forces are not always the same size, as they tend to vary due to the type mission that they have been given. Military commanders determine makeup and size of the task forces for each mission.

The primary decision maker is the task force commander, usually with 18 years experience in the military, who is supported by a staff. The staff consists of an executive

officer, personnel officer, intelligence officer, operations officer, and supply officer.

Prior to each mission, the commander and his staff go through a decision making process to develop the task force plan. The plan is then issued and executed. Success or failure on the battlefield is due partly to this plan.

1.5 The National Training Center

The National Training Center (NTC) located at Fort Irwin, California is one of three Combat Training Centers operated by the Army. Joint Readiness Training Center (JRTC) located in Fort Polk, Louisiana is primarily for the Army's Light Infantry Divisions, and the Combat Maneuver Training Center in Germany is primarily for the Army units overseas.

The National Training Center is an Army run organization that is used to help evaluate and train Army combat units. The mission of the NTC is to provide units with combat scenarios, and then evaluate them in the execution of those scenarios. The NTC is located in the California desert. A permanent party of soldiers is stationed there. These soldiers make up the opposing forces, observer controllers, and administrative personnel. Combat units sent there for evaluation usually consists of one brigade command group and a combination of Armor, Cavalry, Mechanized Infantry, and Light Infantry battalions. The NTC is usually the highlight of a unit's training calendar for

the year. It is the most stressful and realistic training that any unit will endure. Many months are spent preparing for a deployment to NTC.

The NTC is a sophisticated operation that involves numerous support personnel. The units under evaluation are given a mission and then are required to plan and execute the mission. Observer Controllers (OCs) are assigned to each unit down to the platoon level. They are experienced military officers and noncommissioned officers whose job is to collect, process, and disseminate the data from the battle. At the end of each mission, they hold an After Action Review (AAR) which provides the units feedback on their performance. The feedback consists of Battle Damage Assessment (BDA), the execution of standard Army tactics, and internal procedures. AARs are performed at platoon, company, battalion, and brigade levels for each mission. There are typically five to eight missions consisting of offensive and defensive force on force operations, and offensive and defensive live fire operations. Unit rotations through the combat training scenarios last from ten to fourteen days and there are typically ten to thirteen rotations per year.

For each force on force mission, the evaluated units goes up against opposing forces (OPFOR). These are highly trained soldiers that are familiar with Soviet tactics and the terrain at the NTC. Both the OPFOR and BLUFOR (the evaluated unit) are equipped with the Multiple Integrated Laser System (MILES). This system simulates the rounds fired by a weapon system and also detects a hit from a weapon system. For example, a friendly M1 Tank will fire a blank tank round at an OPFOR T-82 tank. The

M1 Tank MILES sends a laser signature at the T-82 Tank. If the laser hits the T-82 Tank, the receiver box on the Tank will set off flashing lights and will register a code indicating a M1 Tank hit it. The Observer Controller with that unit will assess the damage on the Tank and call it into the TAFF (Tactical Analysis Feedback Facility). The TAFF records all the battle damage for use in the AARs. After the completion of an NTC rotation, each unit is given a take home package consisting of the AARs and data collected from the rotation. Units will then use this information to help set up their future training in order to train on their weaknesses. The take home packages and rotation data are also given to the Center for Army Lessons Learned (CALL) at Fort Leavenworth, Kansas for use in research.

1.6 Current Efficiency Rating System

The AAR system at NTC focuses primarily on helping units identify their strengths and weaknesses under extreme wartime conditions. The entire process is a learning process and is not treated as a measure of a unit's performance. The focus is to help units so that when placed in real wartime conditions, they perform better as a unit. Units will see the Battle Damage Assessment (BDA) in their AARs. BDA consists of friendly and enemy casualties and equipment destruction.

The efficiency rating system is used more for the Army organizations that conduct research on trends. Such organizations include the Army Research Institute and the

Center for Army Lessons Learned. They use data collected at NTC to determine the direction the Army is heading in its units' performance. The current efficiency measure that is used is the Force Exchange Ratio. This is a simple ratio that can give a quick and general analysis on a unit's performance of a mission at NTC. The ratio is defined as:

$$FER = \frac{\left(\begin{array}{c} \textit{Initial Red Forces} \\ \textit{Initial Blue Forces} \end{array} \right)}{\left(\begin{array}{c} \textit{Final Red Forces} \\ \textit{Final Blue Forces} \end{array} \right)} \quad (\text{equation 1.1})$$

The Red Forces are the OPFOR and the Blue Forces are the evaluated units. The advantages to this ratio analysis is that it is simple and easy to use. It gives a quick indicator of the unit's performance. The common rule of thumb used by army analysts, is if a unit has a score greater than one, it performed well. There are also numerous disadvantages. The ratio does not have the ability to look at numerous inputs and outputs. It is restricted in what it can measure. For example, it cannot measure the number of aircraft destroyed, the amount of personnel casualties and the total vehicles destroyed with one ratio. It also assumes all types of vehicles are of the same importance. For example, the M1A1 Tank is heavily armored and carries a 120mm cannon as its main armament. The M3 IFV is lightly armored and carries a TOW missile system and 25mm gun as its main armament. Each type of vehicle has a different

mission and different capabilities. The Force Exchange Ratio does not account for the differences and therefore has its flaws in measuring unit efficiencies.

1.7 DEA's Ability to Measure Multiple Factors

One of DEA's main advantages is the ability to measure a number of different factors that have different characteristics. This is useful when attempting to measure efficiencies on the battlefield because there are a large number of factors with differing characteristics affecting a unit's efficiency. A unit uses soldiers, tanks, infantry fighting vehicles, artillery, aircraft, obstacles, and a myriad of other types of equipment. It is fairly obvious that each of the factors mentioned above does not have the same effect on the battlefield. A soldier and an aircraft have totally different effects on the outcome of a battle. Each is important and needs to be included in any unit's performance measure. DEA is modeled to use each of these factors in measuring the efficiency of a unit. The model is run without having to pre-assign weights to each of these factors indicating their effect on the battlefield. The weights are inherently fair because they are not subjectively assigned. The weights are assigned by optimizing the program. Any other type of ratio analysis must have weights pre-assigned to each factor in order to account for the factor effects.

1.8 Criteria for Determining DEA's applicability as a Method of Efficiency

When trying to decide if DEA is an applicable model for measuring unit efficiency, some form of criteria should be established for comparison. The criteria chosen for comparison is from The Military Applications of Modeling by Battilega and Grange. Chapter 4 discusses the outcome of the study with the established criteria.

The list of criteria follows:

1. The model should fit the customer (staff, budget, technical background, data access).
2. The cost of the model should be less than the effectiveness of the model.
3. Uncertainties in the model should be dealt with appropriately.
4. The model can be designed in great breadth or great detail, but not both.
5. The model is more rigid as the chronological time decreases between comparisons.
6. Modeling assumptions should not be biased.
7. Model rigidity should meet the analytic environment which is modeled.

1.9 Organization of the study

The Introduction of this study contained a general discussion on the use of DEA as a model to measure efficiency. The introduction further described how DEA is used as a measure of efficiency for Army combat units at the National Training Center.

Chapter 2 will explain the methodology behind the DEA model. An example is used to

discuss how DEA works. Two primal DEA programs are presented and two dual DEA programs are presented for use in the study.

Chapter 3 is the DEA application. The chapter starts with a discussion on how the DMUs and input and output factors were chosen for the study. The model is then applied and results are displayed. A discussion of the results is the last part of the chapter.

Chapter 4 contains the conclusions and recommendations of the study. Comparisons are made between the DEA results and the FER results. A list of DEA advantages and weaknesses are then explained. The criteria established in section 1.8 are also addressed. The final chapter contains the areas for further research. An explanation on how DEA may be used in other areas of the Army is presented. Also, there are different ways to run the model to fit the customers needs.

Chapter 2

METHODOLOGY

2.1 General Discussion

In order to explain the methodology of Data Envelopment Analysis, this chapter begins with a hypothetical example. This simple example will help visualize the concept of DEA. Once the concept is explained, the actual formulations of DEA is introduced and defined.

The simple example displayed consists of five decision making units (DMUs) that each have two inputs and one output. Consider each DMU as an infantry rifle company (companies A, B, C, D, E). Each DMU conducts a similar mission where the resources used are friendly soldiers and ammunition. The inputs used are input 1, number of friendly soldier casualties, and input 2, pounds of ammunition fired. The output is the number of enemy casualties. The data are illustrated in figure 1. Notice that the inputs are resources that the decision making unit would like to be as small as possible. The output is a factor the DMU would like to see become as numerous as possible.

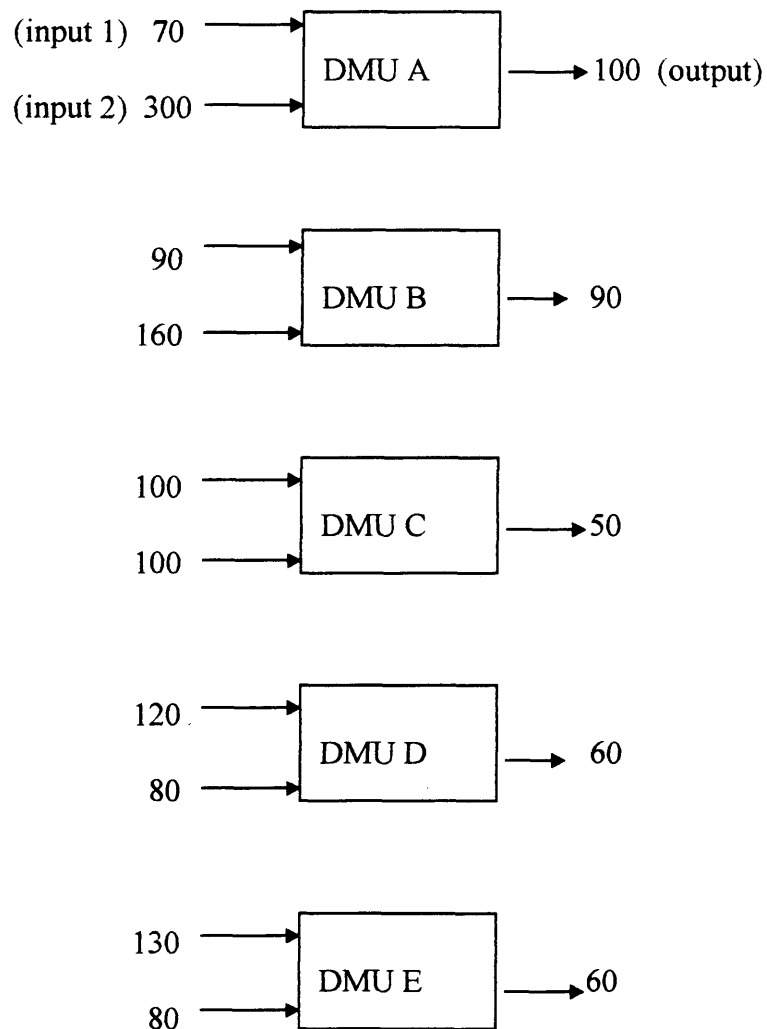


Figure 1. Input / Output Illustration

To graphically portray the example, table 1 displays the inputs divided by the output for each DMU. It is intuitively obvious that the smaller the ratio the more

efficiently each DMU used its inputs to achieve the output. The algebraic results are listed in table 1 and graphed in figure 3.

<u>DMU</u>	<u>INPUT1/OUTPUT</u>	<u>RATIO</u>	<u>INPUT2/OUTPUT</u>	<u>RATIO</u>
A	70/100	.70	300/100	3.00
B	90/90	1.00	160/90	1.78
C	100/50	2.00	100/50	2.00
D	120/60	2.00	80/60	1.33
E	130/60	2.17	80/60	1.33

Table 1. Input Output Ratios

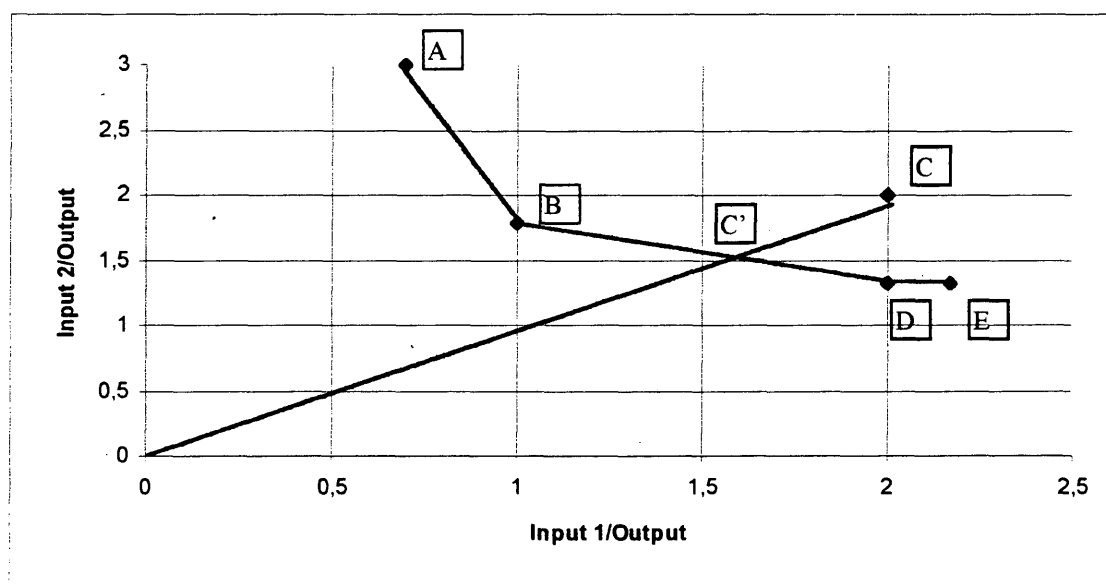


Figure 2. Graphical Representation of the Normalized Input

The piecewise linear curve on the graph is the efficient frontier. The frontier (when minimizing) is defined as the piecewise linear curve that connects the convex set of points closest to the origin. When maximizing, the frontier is defined as the piecewise linear curve that connects the concave set of points furthest from the origin. In this example, any point on the curve is considered efficient, and any point to the right of the curve is considered inefficient. Notice that DMUs A, B, D, and E are on the frontier, and DMU C is to the right. DMUs A, B, and D are efficient, but C and E are not and will be explained in detail later. In principle (when minimizing), any DMU that is lower and to the left of another DMU is more efficient because it uses less inputs to achieve the same amount of output (Silkman, 1986, 15). As DMUs become more efficient, they move closer to the origin.

The efficient frontier is also called the production function. This line is a piecewise linear curve that connects all the best practice DMUs in the set. Any point on the line between two DMUs is a linear combination of the two points and is considered a possible outcome. For example, DMUs B and D are the reference set for the hypothetical DMU C', because pseudo DMU C' is a linear combination of DMU B and D. Notice that DMU C is off the production function. If a line is drawn from the origin to DMU C, it intersects the production function. The location where DMU C intersects the production function, i.e. pseudo DMU C', is the location that DMU C needs to be in order to achieve efficiency. The efficiency rating for DMU C is the ratio of the distance

from the origin to C' , divided by the distance from the origin to C . All inefficiency ratings are between zero and one. Obviously, any DMU on the piece wise linear curve is efficient and has an efficiency rating of one.

DMU E is not efficient even though it is on the production function line. Even though DMU E is producing the same amount of output, it uses more inputs than DMU D and therefore in no way is considered as efficient as DMU D. One way to avoid this problem is to define the horizontal line of the production function as a line that connects DMU D to a point on the horizontal axis at infinity (Silkman, 1986, 16). Now the line is sloping very slightly and DMU E is no longer on the production function. This same logic also extends to the vertical axis as well if there was a DMU perfectly vertical of DMU A. Another method is to just extend the line that connects points B and D out to the horizontal axis and points B and A out to the vertical axis. The extension of the last sloped lines prior to the horizontal or vertical lines will prevent a DMU that uses more inputs to achieve a same amount of output from getting measured as efficient

Notice that each DMU's efficiency measure is based on its position to the other DMUs. If a new DMU is added and its location on the graph is lower and to the left of DMU A, B, or D, then the new DMU will move the production function. This causes some DMUs to become inefficient. If a new DMU is added and is to the right of the production function, then the new DMU will not affect the efficiencies of the other DMUs.

This simple example is just a way to visualize how DEA works. Most all applications of DEA has numerous inputs and outputs and visualizing how the frontier theory works is difficult.

2.2 Formulation of the DEA linear program

The basis of the linear equation used in DEA is the ratio analysis form of measure. An entity's performance could be measured by a single output divided by a single input. For example, in combat, a unit's performance could be measured by the number of enemy casualties (Y) versus the number of friendly casualties (X). The ratio Y/X is a simple and quick measure to indicate performance. But the situation usually entails many more factors than just one input and one output in a performance measure. Measuring more inputs and outputs introduces the total-factor productivity ratio. This ratio is a means to summarize performance by weighting the outputs and inputs in a single ratio (Ganley and Cubbin, 1992, 14). This requires the pre-assigning of weights to each input and output. The ratio is:

$$\frac{\sum_{i=1}^l V_i Y_i}{\sum_{k=1}^m W_k X_k} \quad (\text{equation 2.1})$$

The Y_i represents the outputs of the organization and the corresponding V_i represent the weights that is assigned to each respective output. The X_k represents the inputs from the organization. The W_k are the weights that are assigned to each input. The equation then produces a scalar for the total-factor productivity measure. The problem with this equation is that weights must be pre-assigned to the inputs and outputs. In the private sector, market prices may be used to weight the inputs and outputs, but in the public sector, prices on outputs and inputs are not present and a total factor view of efficiency requires an alternate source of weights (Ganley and Cubbin, 1992, 15). The DEA model does not require the pre-assigning of weights and this is where DEA becomes useful in measuring the efficiency of non-private organizations.

The DEA model was developed by Charnes, Cooper, and Rhodes based on the concept of efficiency originally proposed by Farrel (M.J. Farrel, 1957). The DEA procedure is designed to evaluate relative efficiency of public sector decision making units which perform similar missions and have measurable input and output factors (Lewin and Morey, 1981, 270). Charnes, Cooper, and Rhodes generalized the total-factor productivity measure into a fractional linear program formulation (Lewin and Morey, 1981, 270). “ The relative technical efficiency of any particular DMU is calculated by forming the ratio of a weighted sum of outputs to a weighted sum of inputs, where the weights for both outputs and inputs are to be selected in a manner that

calculates the Pareto efficiency of the unit” (Lewin and Morey, 1981,270). Pareto Efficiency states that a unit is 100% efficient when (a) none of its outputs can be increased without either increasing one or more of its inputs or decreasing some of its other outputs; (b) none of its inputs can be decreased without either decreasing some of its outputs or increasing some of its other inputs (Norman and Stoker, 1992, 15).

Technical efficiency is the standard for the model as opposed to allocative efficiency, which is the measure of the right mix of inputs to produce outputs. Charnes, Cooper, and Rhodes developed the following fractional linear equation to measure the efficiency of decision making units in a public sector organization. Their proposed measure of the efficiency of any DMU is obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to the condition that the similar ratios for every DMU be less than or equal to unity (Charnes, Cooper, Rhodes, 1978, 430).

In more precise form,

$$\text{MAX } h_0 = \frac{\sum_{r=1}^s U_r Y_{r0}}{\sum_{i=1}^m V_i X_{i0}}$$

subject to:

$$\frac{\sum_{r=1}^s U_r Y_{rj}}{\sum_{i=1}^m V_i X_{ij}} \leq 1 \quad j=1, \dots, n \quad (\text{program 2.1})$$

$$U_r, V_i \geq 0; \quad r=1, \dots, s; \quad i=1, \dots, m$$

This program maximizes the outputs to the inputs. h_0 is the efficiency measure of the evaluated DMU. The Y_{rj} and X_{ij} are the known outputs and inputs of the j th DMU respectively. There are n DMUs in the set. The number of inputs is represented by m , and the number of outputs is represented by s . The variable weights of each input and output are represented by $V_i, U_r \geq 0$, which will be determined by solving the problem. The subscript '0' indicates which DMU is getting optimized relative to the set of DMUs. The set of DMUs serve as the constraints to the DMU in the objective function. Solving this equation gives an efficiency rating to the DMU in the objective function relative to the DMUs in the constraints. The optimization gives the rated DMU the most favorable weighting that the constraints allow (Charnes, Cooper, Rhodes, 1978, 430). Because the DMU getting maximized in the objective function is also formulated as a constraint, a solution will always exist falling between zero and one. The objective function will equal one if the evaluated DMU is efficient relative to all the other DMUs in the set. As

each DMU is rotated into the objective function for an efficiency rating, the optimal U_r, V_i will vary based on how each DMU in the objective function gets maximized against the set of DMUs in the constraints. The U_r, V_i represent the relative value system that provides the highest possible rating for the particular DMU in the objective function, consistent with the notion that the solution meets the constraints for all the DMUs. No other DMU using the values of U_r, V_i can achieve an efficiency greater than one. This method therefore goes out of its way to rate each DMU as optimal as possible. When a DMU in the objective function has a rating of less than one, it is considered inefficient or inferior as compared to a subset of DMUs in the constraints that do have a rating of one. These efficient DMUs are considered the reference set of DMUs for the DMU in the objective function (Lewin and Morey, 1982, 271).

The term 'Envelopment' in Data Envelopment Analysis is when a output vector from the DMU in the objective function is 'enveloped from above' when there are combinations of other output vectors (with the some input vector as the rated DMU) whose values are equal to or greater than the DMU output vector. Similarly, the input vector from the rated DMU is 'enveloped from below' when the model finds a combination of other input vectors whose values are smaller than or equal to all the rated DMU input vector. If the input and output vectors of the rated DMU cannot be enveloped simultaneously by a combination of other DMUs, then the rated DMU is said

to be efficient. If the DMU can be enveloped by a combination of DMUs, then the rated DMU is not efficient and that subset of DMUs defines one 'facet' of the piece-wise linear curve. This linear combination of DMUs then serve as the reference point for the measure of inefficiency of the rated DMU (Golany and Roll, 1989, 248).

The next step in the DEA linear formulation is to transform the Charnes, Cooper, and Rhodes' non-linear non-convex program 2.1 to a linear program solvable with most linear programming computer software. Given Charnes, Cooper, and Rhodes original equation,

$$\text{MAX } h_0 = \frac{\sum_{r=1}^s U_r Y_{r0}}{\sum_{i=1}^m V_i X_{i0}}$$

subject to:

$$\frac{\sum_{r=1}^s U_r Y_{rj}}{\sum_{i=1}^m V_i X_{ij}} \leq 1 \quad j=1, \dots, o, \dots, n \quad (\text{program 2.1})$$

$$U_r, V_i \geq 0; \quad r=1, \dots, s; \quad i=1, \dots, m$$

a transformation will change this non linear equation into four different forms of a linear equation. The four forms will 1) Maximize outputs (primal) 2) Minimize inputs (primal)

3) Minimize quantities of inputs to meet stated levels of outputs (dual) 4) Maximize outputs to the stated level of inputs (dual).

Maximizing outputs in the primal requires the denominator of the original equation get set to unity (Charnes, Cooper, Rhodes, 1978, 431).

$$\sum_{i=1}^m V_i X_{i0} = 1 \quad (\text{equation 2.2})$$

This equation indicates that the weighted inputs are fixed at unity and the equation then maximizes the outputs. Equation 2.2 then becomes a constraint. Therefore, the objective function is

$$\text{MAX } h_0 = \sum_{r=1}^s U_r Y_{r0} \quad (\text{equation 2.3})$$

subject to:

$$\sum_{i=1}^m V_i X_{i0} = 1 \quad (\text{equation 2.2})$$

and the other constraint

$$\frac{\sum_{r=1}^s U_r Y_{rj}}{\sum_{i=1}^m V_i X_{ij}} \leq 1 \quad j=1, \dots, n \quad (\text{equation 2.4})$$

by simple algebra is transformed to

$$\sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij} \leq 0 \quad (\text{equation 2.5})$$

thus ending with the final linear equation for maximizing outputs in the primal as:

$$\text{MAX } h_0 = \sum_{r=1}^s U_r Y_{r0}$$

subject to:

$$\sum_{i=1}^m V_i X_{i0} = 1$$

$$\sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij} \leq 0 \quad (\text{program 2.2})$$

$$U_r, V_i \geq 0; \quad r=1, \dots, s; \quad i=1, \dots, m; \quad j=1, \dots, n$$

In order to minimize inputs as the next linear formulation, the original equation must be written as a reciprocal (Charnes, Cooper, Rhodes, 1978, 431).

$$\text{MIN } f_o = \frac{\sum_{i=1}^m V_i X_{i0}}{\sum_{r=1}^s U_r Y_{r0}}$$

subject to:

$$\frac{\sum_{i=1}^m V_i X_{ij}}{\sum_{r=1}^s U_r Y_{rj}} \geq 1 \quad (\text{program 2.3})$$

$$V_i, U_r \geq 0 \quad ; \quad r = 1, \dots, s \quad ; \quad i = 1, \dots, m \quad ; \quad j = 1, \dots, o, \dots, n$$

The same process of transformation used for the max equation will then be used for the min equation. This time the denominator will be set to unity. The following equation is then created as the primal for minimizing inputs.

$$\text{MIN } f_o = \sum_{i=1}^m V_i X_{i0}$$

subject to:

$$\sum_{r=1}^s U_r Y_{r0} = 1$$

$$\sum_{i=1}^m V_i X_{ij} - \sum_{r=1}^s U_r Y_{rj} \geq 0 \quad (\text{program 2.4})$$

$$V_i, U_r \geq 0 \quad ; \quad r = 1, \dots, s \quad ; \quad i = 1, \dots, m \quad ; \quad j = 1, \dots, o, \dots, n$$

The next formulation that must be performed is the dual of each of the primal equations. The dual has many useful purposes in DEA. The main use of the dual is the ability to obtain the input or output levels of the hypothetical DMU from a linear combination of the DMUs that make up the reference set. The dual will indicate what multiples of the reference set to use to make up the hypothetical DMU (Silkman, 1986, 32). Also, the dual is more tractable than the primal because the primal constraints are indexed on all the DMUs. The dual constraints are indexed on the inputs and outputs (Ganley and Cubbin, 1992, 19).

The dual formulation for the linear programs are described by Ganley and Cubbin. For the primal that maximizes outputs, program 2.2, the dual is:

$$\begin{aligned} \text{MIN } g_0 &= g_0 - \varepsilon \left(\sum_{i=1}^m S_i + \sum_{r=1}^s S_r \right) \\ \text{subject to: } X_{i0} * g_0 - S_i &= \sum_{c=1}^z X_{ic} \lambda_c \\ Y_{r0} + S_r &= \sum_{c=1}^z Y_{rc} \lambda_c && \text{(program 2.5)} \\ \lambda_c &\geq 0, \quad c = 1, \dots, z \\ S_r &\geq 0, \quad r = 1, \dots, s \\ S_i &\geq 0, \quad i = 1, \dots, m \end{aligned}$$

For the primal that minimizes inputs, program 2.4, the dual is:

$$\begin{aligned} \text{MAX } z_0 &= z_0 + \varepsilon \left(\sum_{i=1}^m S_i + \sum_{r=1}^s S_r \right) \\ \text{subject to: } Y_{r0} * z_0 + S_r &= \sum_{c=1}^z Y_{rc} \lambda_c \\ X_{i0} - S_i &= \sum_{c=1}^z X_{ic} \lambda_c && \text{(program 2.6)} \\ \lambda_c &\geq 0, \quad c = 1, \dots, z \\ S_r &\geq 0, \quad r = 1, \dots, s \\ S_i &\geq 0, \quad i = 1, \dots, m \end{aligned}$$

In program 2.5, there is a variable in the objective function noted as g_0 . The optimal value of g_0 equals the optimal value of the primal h_0 , thus making g_0 the efficiency rating of the evaluated DMU in the dual. If the DMU is efficient, the value is one. If it is not efficient, then the value is less than one. The other variables are noted by λ and are located with each DMU in the constraints. These are the weights of each DMU in the facet for the evaluated DMU. If a DMU is efficient, these weights are zero for all the DMUs except the DMU getting evaluated. If the DMU is not efficient, these weights represent the values of the linear combination of efficient DMUs in the reference set that make the evaluated DMU efficient (Silkman, 1986, 23). To ensure the strict positivity of the decision variables, a ε is introduced into the dual which gives a lower

bound constraint on each of the variables (Lewin and Morey, 1981, 272). ε is a small constant of magnitude 10^{-6} . This ensures that its presence in the equation will not affect the solution to a significant degree. It also forces the slack values to move to their maximum values. Caution must be taken when assigning the value to ε . In a study by Ali and Seiford, 1993, the value of ε must be within a certain bound for the dual to work properly. The surplus and slack values of the equation are represented by S_i and S_r , respectively. If the evaluated DMU is efficient, then the slack values will all equal zero. If the evaluated DMU is not efficient, then the slack values represent the additional marginal improvements to make the DMU efficient (Lewin and Morey, 1981, 275). The value of the slacks is simply the amount that each constraint failed to achieve equality (Norman and Stoker, 1991, 238). This means that the evaluated DMU has some constraints not equal where there are some inputs greater than the inputs of the weighted combination or some outputs less than the outputs of the weighted combination, indicating that additional improvements may be made (Norman and Stoker, 1991, 238).

The explanation of program 2.6 is similar to program 2.5 except that now the dual finds the maximum value of outputs attainable for a given set of inputs. The evaluated DMU is efficient when z_0 is equal to one and inefficient when it is less than one. The slacks represent the marginal increase that the outputs should achieve to form a DMU that is efficient given the linear combination of the DMUs in the efficient reference set.

Chapter 3

DEA APPLICATION

3.1 Definition and Selection of Decision Making Units (DMUs)

The Selection of DMUs is one of the first considerations when applying DEA. As briefly explained in Chapter 1, a DMU is a Decision Making Unit which represents a unit or organization that is getting evaluated on its performance. This section of the paper explains the technique implemented to select the DMUs used in this study.

A recognized procedure to select DMUs was outlined in “An Application Procedure for DEA” by B. Golany and Y. Roll. They state that “DEA is a technique for assessing the relative efficiency of ‘comparable’ units, with a view to improving their performance. This statement implies a basic assumption that differences in performance among ‘like’ units exist and are measurable. Even under quite similar conditions, one always finds differences in the way units are managed, if only because they are led by different decision makers. Thus on one hand we look for a ‘homogeneous’ set of units, where comparison makes sense, and on the other, we try to identify the differences between them.” When selecting DMUs they must first be a homogeneous group of units. There are three assumptions that must first be satisfied to consider a group of DMUs homogeneous 1) the units under consideration perform the same tasks, with

similar objectives 2) all the units perform under the same set of market conditions 3) the input and output factors characterizing the performance of all units in the group, are identical, except for differences in intensity or magnitude.

The next consideration by Golany and Roll is the size of the comparison group. The basic rule of thumb established is that the number of DMUs should be at least twice as large as the total number of input and output factors (Golany and Roll, 1989, 238). The advantages of a large comparison group is that the larger the population, the more factors may be used in the study, and the better the chance of finding high performance units that push the efficiency frontier. The disadvantage of a large comparison group is the chance that the homogeneity of the comparison group will decrease due to external factors that may exist.

The other DMU selection guidance mentioned by Golany and Roll is the boundaries of DMUs. One kind of boundary is the organizational, physical, or regional boundary. The other is the time period used in measuring the DMU activities. These boundaries should be considered when selecting the DMUs to ensure that the DMUs are comparable to each other for use in the study.

The final consideration outlined by Golany and Roll is the process of sifting out DMUs that might be considered outliers. This is important because DEA is a sensitive measure based on extreme conditions. One DMU may change part of an efficiency frontier. DMUs that deviate from general conditions of the group should be carefully

analyzed before entering them into the DEA application. An example of a DMU that can throw off part of the frontier is a DMU that performs exceptionally well due to an external factor not common to all the DMUs in the study. For example, a task force is given more tactical air support for the defense mission. This might be an experiment that the NTC is running. The task force, due to the extra assets, destroys the enemy before the enemy even gets within range of the ground weapons. This scenario would predictably cause the efficiency score of the task force to be one. As a result, a task force that was previously efficient might become inefficient.

A further condition mentioned by Vakkuri and Malkia when selecting DMUs is the ability to measure all the input and output factors that will determine the performance of the DMU. If a DMU does not have measurable input and output factors, then the DEA model becomes questionable. This is usually a condition when selecting the input and output factors for the DMUs, but should also be a consideration when selecting the DMUs.

The rest of this section is the application of Golany and Roll's DMU selection technique to this study. The first paragraph is the decision process used to determine where the DMUs were found for use in the application. The rest of the section indicates how well the DMUs fit the criteria that Golany and Roll outline.

The overall goal of this study is to assess the suitability of DEA to measure the efficiency of U S Army combat units. In searching how to accomplish this goal, a

number of options were discussed. A few options were 1) comparing the Unit Status Report (USR) of each unit 2) collecting data from a real world operation such as Operation Desert Storm or Operation Just Cause 3) collecting data from a Combat Training Center such as JRTC, NTC, or CMTC. Upon further research it became obvious that collecting data from a Combat Training Center was the easiest and most accurate approach because the CTC is purposely set up to collect data for evaluation and research purposes. It was not possible to collect USR data because the data is top secret. Real world conflict data is difficult to collect because data is not accurately collected on the battlefield, stored, or distributed. All CTC data is collected and stored at Fort Leavenworth in the Combat Training Center Warrior Information Network (CTC WIN) as well as the actual combat training center itself. The data is collected and stored for purposes of further research and to indicate trends. When comparing the Combat Training Centers, the National Training Center (NTC) by far had collected and stored the most data. Also, the "Star Wars" system used by the NTC is far superior to any system used at JRTC or CMTC in collecting data. The type of units that typically perform at NTC are Mechanized Infantry battalions, Armor battalions, Cavalry squadrons, and Light Infantry battalions. The Light Infantry battalions are composed much differently than the heavy battalions and they tend to have different type missions. Also, the Light Infantry battalions are not the main focus at NTC, and are used mainly to assist the heavy forces. Therefore, not much data is collected of the Light Infantry Battalions and will not be used

in this study. The Light Infantry will only be part of the study when a light infantry unit is attached to a heavy battalion task force.

The DMUs considered are the Mechanized, Armor, and Cavalry task forces and squadrons that have performed a rotation at NTC in the past five years. These DMUs are fairly homogeneous. The first assumption of homogeneity is the performance of similar tasks and similar objectives. A task for this application is considered a mission. Each NTC rotation has five to eight missions for each battalion. Missions include a movement to contact, deliberate attack, defense, offensive live fire, and defensive live fire. The mission which is probably the most similar for each DMU is the defense. The defense is usually conducted on the same terrain against the same size opposing force. Each DMU is given a similar mission statement which includes not letting a certain size enemy force past a specific phase line. The other missions listed are not usually as consistent from unit to unit or rotation to rotation. Therefore the task that each DMU undergoes is the defensive mission, and the objective that each DMU must achieve is the standard expressed in the mission statement. The mission and mission statement for each DMU are fairly comparable.

The next assumption of homogeneity is the DMUs performance under the same set of market conditions. Market conditions in this application are considered weather, time of day, and time to prepare. Market conditions do vary to some extent at NTC. The weather is much different in the winter than the summer. It might be raining for one unit

and not for another. Time of day tends to vary. Generally the enemy attack is performed at night, but on occasion, the enemy attack may occur during daylight. Finally, one unit may have more time to prepare for the defense than another. A unit with less time to prepare could be less likely to achieve its objective. Although the market conditions do vary from unit to unit, there is a neutralizing affect. The neutralizing effect is the fact that the enemy must also perform under the same conditions. When a unit must defend in the rain, its equipment and weapons may become less effective. The neutralizing element is that the enemy must also perform in the rain where its equipment and weapons may also become less effective. Therefore, although market conditions can be a factor in DMU homogeneity, the fact that the attacking enemy is under the same conditions lessens some of the market condition effect between DMUs.

The third assumption of homogeneity is that the input and output factors that characterize the performance of all units in the group are identical, except for differences in intensity or magnitude. The next section of this chapter explains the selection of the input and output factors. All the DMUs have the same inputs and outputs except for the magnitude of each. There are dissimilarities in the number of types of weapons and vehicles in each task force, but the input factors are general enough to categorize these dissimilarities into a common element. Therefore, this assumption is met.

The next consideration for DMU selection is the size of the comparison group. Data was collected on eighty units that performed a defensive operation from 1990 to

1994. After sifting out some DMUs that did not have some of the required data collected, the final number of DMUs selected was sixty-six. This number meets the requirement that the number of DMUs be at least twice the number of input and output factors.

The boundaries that affect the DMUs are similar to each other. The organizational boundaries are similar in the fact that each task force is composed of a battalion commander, a staff, three or four companies, and a headquarters company. The physical boundaries are similar within each type of battalion, but are dissimilar between the three types of battalions. There is a difference between the Armor, Mechanized and Cavalry battalions in the type of vehicles and weapons that are employed. This minor discrepancy is interesting because DEA can compare how each type of battalion performed against the other. Also DEA can just compare that particular type of battalion against themselves. Keep in mind that for the study, the DMUs represent a task force which is a mix of Armor and Mechanized companies. Therefore, for each DMU there is a good mix of all the types of weapons and vehicles on the battlefield. Regional boundaries make no difference in comparing these DMUs because depending where each unit is from, they are all training based on the same Army standard. Finally, time period boundaries are similar in the amount of time each unit has to perform the mission, but are dissimilar in the year that each unit rotated through NTC. The data collected ranges from 1990 to 1994. Although there is a time period difference, the effects are minimal

because there has not been much of a change in tactics, missions, or conditions. If there was a change, then DEA could be a good measure in analyzing trends. A unit performing under one condition last year can be compared to another unit performing under a different controlled condition this year.

The final DMU selection condition outlined by Golany and Roll is the sifting out of DMUs that may be considered outliers. As mentioned earlier, there were eighty DMUs eligible for consideration, but only sixty-six were selected. DMUs were sifted out because they had missing data. Other DMUs were eliminated because of unusual circumstances. For example, a couple DMUs were eliminated because the size of the enemy force that attacked was unusually small, making the results look extremely favorable for the DMU evaluated.

3.2 Selection of Input and Output Factors

The selection of input and output factors is the next step in the DEA application. Golany and Roll outline a sound procedure for this process. They suggest that the initial list of factors generated be as large as possible. From there, they outline a process to reduce the size of the list to a workable number of inputs and outputs. They suggest that the factors initially generated probably fall into the following classes: fully, partially, or uncontrollable factors by the DMUs, quantitative in nature, and qualitative in nature. Note that the more the input and output factors in the model, the more DMUs will be

located on the efficiency frontier (Golany and Roll, 1988, 239). Therefore, it is better to limit the number of inputs and outputs so that a good efficiency comparison may be made for the DMUs.

An input factor is data that the DMU would like to make smaller. For example, if data collected on a military unit was the number of friendly casualties, then it is an input factor because that is something the unit would like to make smaller. An output is data that the DMU would like to make bigger. For example, if data collected on a military unit is the number of enemy casualties, then it is an output factor because that is data that the unit would like to see increase.

The three stages for refining the list of inputs and outputs outlined by Golany and Roll are as follows 1) judgmental screening 2) non- DEA quantitative analysis 3) DEA based analysis. The first step discussed is the judgmental screening that the list of factors will endure. This process is performed by expert decision makers in the field of study. In this study, the list of factors was generated by company commanders in the Army who have been through a CTC. All the factors were put to the test outlined by Golany and Roll 1) Is the factor related to, or contributing to, one or more of the objectives set for the application? 2) Is the factor conveying pertinent information not included in the other factors? 3) Does the factor contain elements which interfere with the notion of technical efficiency? 4) Are data on the factor readily available and generally reliable?

The next step that was used in this study to reduce the number of factors was the use of non-DEA quantitative methods to assign numerical values to the factors. Golany and Roll suggest that for quantitative factors, the ‘natural’ choice would be the physical units by which the factors are normally measured. For example, if a factor was economically oriented, then the factor units would be in dollars, if the factor was a number of vehicles, then it would be measured in the quantity of vehicles. For qualitative factors that do not come with a natural measure, a numerical value must be assigned in order for the factor to be used in the model. There may be several ways to assign values to qualitative factors, but the usual practice is to find a surrogate variable which has a known relation to the varying levels of the qualitative factor (Golany and Roll, 1988, 239). A good selection example for choosing a surrogate variable is displayed in a article by D. Serussi “Applying DEA to the I.D.F. Air Force Maintenance Units.”

In this study, there was one output factor that was qualitative in nature. That output factor was mission accomplishment by the unit. Did the unit accomplish the mission set out by higher headquarters? Since a mission statement in the defense is usually straight forward, the unit either accomplishes the mission to the set standard or not. Therefore, it was determined to treat the factor as a zero or one variable with no intermediate numerical rating. It is possible that the task force can partially complete the mission. Battilega and Grange mention in their book that mission accomplishment can

be given partial values. But for the purpose of this study, the standard is full completion because the data for partial completion was not available. Further research and data collection may some day give mission accomplishment a partial quantitative value.

The next process in the non-DEA quantitative method step is to classify the factors into inputs and outputs. Golany and Roll state that “resources utilized by the units or conditions affecting their operation are typical inputs, while measurable benefits generated constitute the outputs.” Another method to classify the factors is to let the inputs be conditions or units that the DMU would prefer to decrease. The outputs are then conditions or units that the DMU would prefer to increase.

The final step in choosing inputs and outputs is to perform a DEA based analysis. This step requires trial runs of the DEA model with various combinations of the factors. This helps determine if some factors do not affect the efficiency ratings of the DMUs in general. Basically, if a factor does not help discriminate the efficiency scores between DMUs, there is no reason to keep it in the model. Factors which are consistently associated with very small multipliers relative to the other multipliers may be dropped due to the minor impact on the efficiency scores.

Once all these steps are accomplished, the list of factors are decided upon for inclusion into the study. There are eleven factors that were decided upon for use in this study, five input factors and six output factors. The input factors are:

1. The number of friendly dismounted casualties
2. The number of friendly tanks destroyed by enemy forces
3. The number of friendly armored personnel carriers or infantry fighting vehicles destroyed
4. The number of friendly combat multipliers destroyed
5. The number of fratricides.

The number of friendly dismounted casualties is defined as either light infantry soldiers attached to the unit or organic infantry soldiers killed or wounded outside of a vehicle.

The number of tanks destroyed is defined as any kind of mobility or weapon system disablement on the M1 or M60 tank. The number of armored personnel carriers (APCs) or infantry fighting vehicles (IFVs) destroyed is any kind of mobility or weapon system that is disabled on the M113, M2, or M3. The number of combat multipliers destroyed is defined as any Artillery, Engineer, Air Defense Artillery, or other attached unit in direct support of the task force, that have a major weapon system or vehicle disabled. This also includes indirect fire systems internal to the task force. Fratricides are defined as the number of friendly personnel, weapon systems, or vehicles that are killed, wounded, destroyed, or disabled by friendly assets.

These inputs all passed the elimination process. They are all considered costs to the unit and each DMU ideally would like to decrease each of these inputs down to zero. They are not levels of input that may be set by the unit prior to the operation but they

may be influenced during the mission. Also, each factor is quantitative and their natural units are used in the model.

The outputs used for the model are listed below:

1. The number of enemy casualties.
2. The number of enemy tanks destroyed.
3. The number of enemy armored personnel carriers destroyed.
4. The number of enemy combat multipliers destroyed.
5. The number of enemy aircraft destroyed.
6. Mission accomplishment by the task force.

The number of enemy casualties is the number of enemy considered killed or wounded.

The number of enemy tanks destroyed is classified as T-72 or T-80 tanks whose weapon system or mobility capability is disabled. The number of enemy armored personnel carriers destroyed is defined as the number of enemy BMPs, BRDMs, or BTRs whose weapon system or mobility capability are disabled. The number of enemy combat multipliers destroyed is the number of attached enemy elements whose weapon system or vehicle is rendered disable. The number of aircraft destroyed is the disablement of both the helicopter assets (HIND D) and the fixed wing assets. Fixed wing assets are the fighter jets. Mission accomplishment by the task force is whether or not the task force accomplished the mission given to them by higher headquarters.

These output variables meet the elimination process outlined earlier. The first five outputs are similar to the inputs in the fact that they are considered costs to the enemy. A unit's performance on the battlefield is many times measured by its ability to destroy numerous enemy assets. Outputs one through five are all measured by their natural units. Output six is different from the others because it is not immediately quantifiable. It is considered more as a goal for the DMU. This output was measured as a zero or one variable because the unit either accomplished the mission or did not accomplish the mission.

There are numerous input and output combinations that may be used to measure a military unit's performance on the battlefield. It is certain that other persons performing this study would use other combinations of inputs and outputs. Much of the decision process of which type of input and output factor to use was based on availability of data and measurability of the data. These input and output factors will help measure how well the decision makers in the task force employed their assets compared to the other task force elements performing the same mission.

The final discussion on the input and output factors involves data reliability. Under the system currently used by the NTC; data collection, storage, and presentation have the potential for extreme accuracy. If the decision makers who run NTC know exactly what type of data they want to collect, store, and present, then the data is fairly reliable. The data that is used in this study is from the years 1990 to 1994. Understandably, the

decision makers who determined what data was collected, stored and presented, did not have this study in mind. The data that is used in this study comes from the 'Take Home Packages' that NTC sent to the CTC WIN for storage. My findings on the data show some differences on the type of data collected, stored, and presented over that four year time period. For example, one year of 'Take Home Packages' has accurate data on friendly fixed wing assets destroyed, and the next year the packages do not have any of the aircraft data. The primary reasons for data variability is 1) turnover in the persons making decisions on what gets collected, stored and presented 2) preferences in what the evaluated unit leaders wanted collected on their units 3) collecting data based on what the trend analyzers wanted collected.

Data reliability for this study is also based on how it is collected. During the time period 1990 to 1994, the data was collected on the ground by the OCs and then recorded in the TAFF. Numerous factors can cause the collection of data to become inaccurate. Some of the reasons are 1) a large battlefield filled with confusion 2) MILES malfunctions 3) human error. Different rules also existed for collecting data. For example, one year the rule for collecting data on friendly infantry casualties was, if the vehicle was destroyed and they were in it, then they were a casualty. The following years, the rule was, if the vehicle was destroyed, then the infantry could dismount the vehicle and continue with the mission. They would then become casualties when their personal MILES system sounded.

As a result, some assumptions were made on the data collected for this study. The following is the list of assumptions:

1. If no input or output data was presented, then it received a zero value
2. Data sent to the TAFF by the OCs was accurate
3. All BDA was stored and presented in the Take Home Packages
4. An infantry squad has nine personnel in it during the battle
5. Infantry casualties indicated that they were dismounted
6. Enemy casualties included both mounted and dismounted

With the new Star Wars system in place at NTC, and a conscious effort to collect data that will fit a DEA model more comfortably, these assumptions will most probably be reduced. Data reliability will also increase to a much higher accuracy level.

The data used for this study is located in Appendix A. There are sixty-six DMUs and eleven input and output factors. The DMUs are labeled with a simple system so that the type of battalion may be identified and the year that the unit rotated through NTC may be identified. "A" means Armor, "M" identifies Mechanized, and "C" stands for Cavalry. The next two numbers are the year that the DMU rotated through NTC. The final two numbers are identifiers, with no meaning to them. The inputs and outputs are also labeled so that they may be simply identified. The labeling is as follows:

1. Fcas - friendly infantry dismounted casualties
2. Ftank - friendly tanks destroyed
3. FIFV - friendly infantry fighting vehicles destroyed
4. Fcom - friendly combat multiplier vehicles destroyed
5. Frat - number of fratricides
6. Ecas - number of enemy casualties
7. Etank - number of enemy tanks destroyed
8. Eapc - number of enemy APCs destroyed
9. Ecom - number of enemy combat multipliers destroyed
10. Eair - number of enemy aircraft destroyed
11. MA - mission accomplishment by friendly forces

3.3 Application of the DEA Model

The Charnes, Cooper, and Rhodes model (CCR) for DEA application is the model of choice for applying the data for this study. In Chapter two, four forms of the model were presented for use in the DEA application. All four of the model forms were derived from the CCR model. Each of the four programs optimizes the evaluated DMU differently. For this study, the primal program 2.2 is used to calculate the DMU efficiencies. The primal emphasizes input reduction, and optimizes by the virtual multipliers of the inputs and outputs (Golany and Roll, 1989).

The procedure to fit the data to the program is noted below. The program used for this example is program 2.2. The data set is from Appendix A.

$$\text{MAX } h_0 = \sum_{r=1}^s U_r Y_{r0}$$

subject to:

$$\sum_{i=1}^m V_i X_{i0} = 1$$

$$\sum_{r=1}^s U_r Y_{rj} - \sum_{i=1}^m V_i X_{ij} \leq 0 \quad (\text{program 2.2})$$

$$U_r, V_i \geq 0; \quad r = 1, \dots, s; \quad i = 1, \dots, m; \quad j = 1, \dots, 0, \dots, n$$

The initial weights U_r, V_i are the variables. For the purpose of the example, DMUs A9001, A9002, and C9305 are used. The input factors are represented by X_i and the output factors by Y_i . The DMU getting optimized is A9001.

$$\text{Maximize (A9001)} = u_1 * 82 + u_2 * 26 + u_3 * 61 + u_4 * 11 + u_5 * 5 + u_6 * 0$$

subject to:

$$\text{(A9001)} \quad v_1 * 0 + v_2 * 28 + v_3 * 16 + v_4 * 2 + v_5 * 4 = 1$$

$$\begin{aligned} \text{(A9001)} \quad & (u_1 * 82 + u_2 * 26 + u_3 * 61 + u_4 * 11 + u_5 * 5 + u_6 * 0) - \\ & (v_1 * 0 + v_2 * 28 + v_3 * 16 + v_4 * 2 + v_5 * 4) \leq 0 \end{aligned}$$

$$\begin{aligned} \text{(A9002)} \quad & (u_1 * 41 + u_2 * 24 + u_3 * 44 + u_4 * 8 + u_5 * 3 + u_6 * 0) - \\ & (v_1 * 9 + v_2 * 21 + v_3 * 35 + v_4 * 9 + v_5 * 1) \leq 0 \end{aligned}$$

continued for all the DMUs

$$\begin{aligned} \text{(C9305)} \quad & (u_1 * 26 + u_2 * 21 + u_3 * 91 + u_4 * 7 + u_5 * 8 + u_6 * 0) - \\ & (v_1 * 70 + v_2 * 30 + v_3 * 22 + v_4 * 4 + v_5 * 0) \leq 0 \end{aligned}$$

$$U_r, V_i \geq 0; \quad r = 1, \dots, 6; \quad i = 1, \dots, 5; \quad j = 1, \dots, 66$$

The program is then run on a spreadsheet with an optimizer function. The results of this specific example follows:

$$\begin{array}{cccccc}
 A9001 = 1.000 & v_1 = .0139 & v_2 = .0085 & v_3 = .0314 & v_4 = .1298 & v_5 = .0000 \\
 u_1 = .0000 & u_2 = .0174 & u_3 = .0000 & u_4 = .0090 & u_5 = .0897 & u_6 = .0000
 \end{array}$$

The tool best used to apply the data collected for the DEA model in this study is a spreadsheet with a solver function. For this specific application, the spreadsheet 'EXCEL' is utilized. Each column represents a DMU. The column is labeled with the DMU identifier, the inputs and outputs are stored beneath the label, and then the equations are below the data. The results are then stored in a column below the DMU in a matrix format. Each column and row represented a DMU. The "EXCEL" commands used for the primal model are in Appendix B. The results for the primal model are located in Appendix C.

A summary of the results is listed in Appendix D. The first five rows are displayed below to indicate what the results look like. The first column is the DMU identifier. The next column is the DMU efficiency score for the model. The following eleven columns are the virtual multipliers for the DMU inputs and outputs. These multipliers are the numerical values used to optimize the DMU against its peer group.

DMU	eff.	Input virtual multipliers					Output virtual multipliers					
	score	Fcas	Ftank	FIFV	Fcom	Frat	Ecas	Etank	Eapc	Ecom	Eair	MA
A9001	1.000	0.0139	0.0085	0.0314	0.1298	0.0000	0.0000	0.0174	0.0000	0.0090	0.0897	0.0000
A9002	0.463	0.0042	0.0648	0.0000	0.0846	0.0000	0.0000	0.0253	0.0000	0.0412	0.0208	0.0000
A9003	0.994	0.0115	0.0077	0.0000	0.0623	0.2193	0.0000	0.0000	0.0000	0.0523	0.0058	0.0000
A9004	1.000	0.0000	0.0282	0.0150	0.0056	0.0000	0.0005	0.0000	0.0000	0.0428	0.0028	0.0000
A9101	1.000	0.0008	0.0146	0.0110	0.0497	0.0145	0.0010	0.0088	0.0000	0.0033	0.0297	0.0000

Table 2. Model Results

Note that the range in values for the efficiency scores range from zero to one. Also recognize that there are twenty-three scores that equal one. The next section discusses methods to break the tie between DMUs that have efficiency scores of one.

3.4 Reference Sets

There are two areas of the model results that are analyzed. The two areas are reference sets and cross efficiencies. Each area is discussed and then data from the model is displayed.

The first result analyzed is the reference set for each evaluated DMU. Reference sets are the DMUs in the constraints that receive an efficiency score of one. These DMUs represent the ‘facet’ on the linear frontier that the evaluated DMU is compared against. The list of the reference sets for each DMU is listed in Appendix E.

The results in section 3.3 indicated that there were twenty-three DMUs that received a score of one. As stated earlier, a reference set is the efficiency frontier that a evaluated DMU is aiming for. Sometimes a DMU achieves a score of one simply because it specializes in a particular area. This is considered a ‘niche’ player (Doyle and

Green, 1991, 634). As a result, that DMU is not used in many reference sets because its efficiency is based mostly on performing extremely well in a specialized area that no other DMU performed well in, hence the term “niche.” One test to determine if a DMU specializes is to count the number of times it is used in a reference set. A DMU that is used many times in reference sets is considered a ‘broad’ player (Doyle and Green, 1991, 634). Listing the number of times a DMU is listed in a reference set helps order the DMUs based on how well they performed overall as opposed to a specialized area. The list of ordered DMUs follow.

DMU	Efficiency Score	# times Referenced
A9201	1	31
M9201	1	27
C9202	1	24
M9308	1	23
A9209	1	19
A9306	1	15
M9307	1	13
A9101	1	12
M9305	1	12
C9302	1	6
A9204	1	5
C9201	1	3
A9004	1	2
A9001	1	2
M9101	1	2
A9403	1	2
M9301	1	1
M9007	1	1

Table 3. The Number of Times DMUs are Referenced

Notice that DMUs A9201 and M9201 are referenced 31 and 27 times respectively. They would definitely be considered as organizations that performed well in most areas. Also notice that DMUs M9301 and M9007 were only referenced once each, indicating they were not in any other DMU reference set. These would definitely be considered DMUs that only performed well in a specialized area. There is no defined cutoff point as to the number of times a DMU must be referenced to be considered a 'broad' or 'niche' player. In this study, there were nine DMUs that were referenced twelve times or more, and fourteen DMUs that were referenced under six times each. DMUs referenced twelve times or more are considered a "broad" performer and DMUs referenced under six times are considered "niche" performers for the purpose of this study.

Reference sets are also used to determine how an inefficient DMU may become efficient. Reference sets are used as the prescriptive device for the inefficient DMUs. In order to calculate how a DMU can become efficient, the dual calculation must be performed or a sensitivity analysis of the primal must be performed. On most spreadsheet optimizers, a sensitivity analysis may be performed with a touch of a button. The primal formulation with a sensitivity analysis is much less time consuming than setting up and performing the dual formulation.

Once the sensitivity analysis is performed, the marginal values are extracted. The marginal values come from the reference DMUs of the evaluated DMU. The sum of the marginal values multiplied by the input and output factors of the reference DMUs will

produce a hypothetical DMU for the inefficient DMU. In theory, the evaluated DMU, with the new input and output factors is now on the efficient frontier. An example of the calculation is given below using DMU A9301 as an example. The efficiency score of A9301 is .706. The reference set for DMU A9301 is A9201, A9209, A9306, M9201, and M9308.

DMU	mar val	Input factors					Output Factors					
A9201	0.0086	0	16	13	7	1	143	35	118	14	8	1
A9209	0.4290	18	23	13	0	0	115	30	135	7	3	0
A9306	0.5850	0	15	19	8	3	225	27	120	4	13	0
M9201	0.0370	131	0	18	7	0	175	32	97	5	6	0
M9308	0.1800	30	18	42	0	0	139	29	121	8	11	1
A9301'		18.0	22.0	25.0	5.0	1.8	213.7	35.4	154.5	7.1	11.2	0.2

Table 4. Calculation of Hypothetical DMU

Notice that the reference set of DMUs are in the first column with their marginal values in the second column. The marginal value is then multiplied by each input and output factor and then the sum of each result is given in the bottom row. The values in the bottom row are the new values that will make DMU A9301 become efficient. The new input and output factors for DMU A9301 multiplied by the calculated weights give a efficiency score of one.

$$\text{EfficiencyScore} = \frac{\text{Weighted Outputs}}{\text{weighted Inputs}}$$

$$E = \frac{0 * 213 + 0 * 35 + .0091 * 154 + .0014 * 7 + 0 * 11 + 0 * .2}{.0044 * 18 + .0492 * 22 + .0023 * 25 + .0396 * 5 + 0 * 1.7} = 1$$

Therefore, if a unit wants to know how to become efficient, it calculates its new input and output factors as described above. The results give the unit a goal to strive for to become efficient.

3.5 Cross Efficiencies

Another method used to break the ties between efficient DMUs is to determine their cross efficiencies. A cross efficiency is the score of a DMU given the optimal weights of another DMU. Every row in Appendix C represents how a DMU scored with the optimal multipliers of the evaluated DMU. On average, a DMU that is an overall performer should score closer to one compared to a DMU that specializes in performance. This would also be another method to order DMUs that have efficiency scores of one. The following is a list of DMUs with their average cross efficiency score, efficiency score, and number of times referenced by another DMU. They are in order by their cross efficiency score. Only the first twelve rows are displayed, the rest of the results are given in Appendix F.

DMU	Efficiency score	# times referenced	Cross efficiency average
A9201	1	31	0.9639
C9202	1	24	0.8623
M9308	1	23	0.8388
A9101	1	12	0.7904
M9201	1	27	0.7904
A9306	1	15	0.7740
A9003	0.9938	0	0.7614
A9209	1	19	0.7605
M9305	1	12	0.7498
A9204	1	5	0.7249
M9307	1	13	0.7083
M9301	1	1	0.6493

Table 5. Cross Efficiency Scores

The first eleven ordered cross efficiency scores had all the “broad” performers with one exception. The DMUs that were more specialized were located in the twelfth through forty-fourth position. The following is a chart that will help visualize the correlation between the cross efficiency average and the efficiency score ordering.

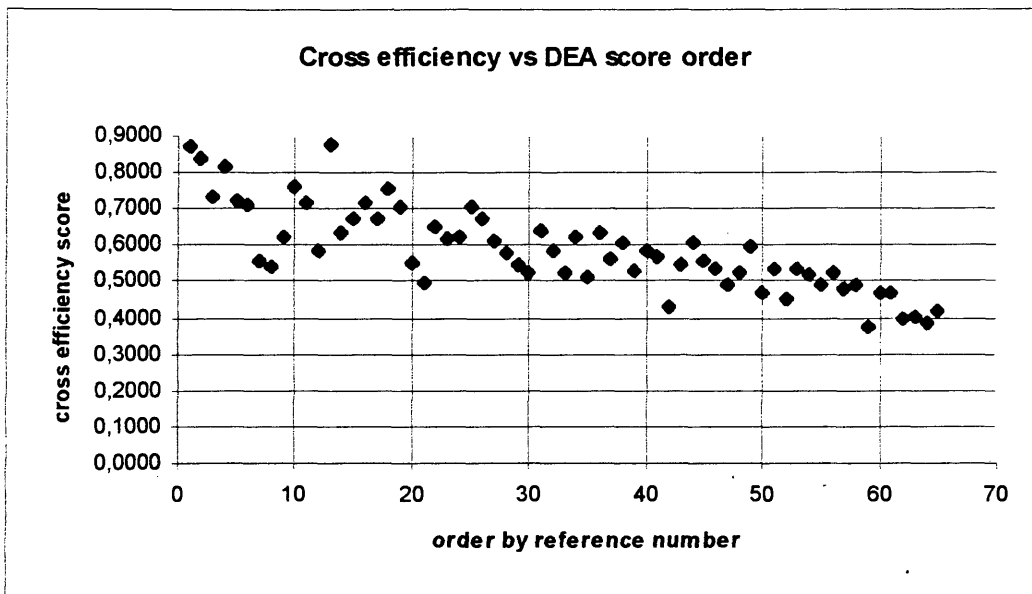


Figure 3. Cross Efficiency Average versus DEA Score Order

The scatter plot indicates that there is a some form of correlation between the cross efficiency average and the ordering of DMUs by efficiency scores.

The next comparison is the rank ordering of the cross efficiency score versus the rank ordering of the number of times referenced.

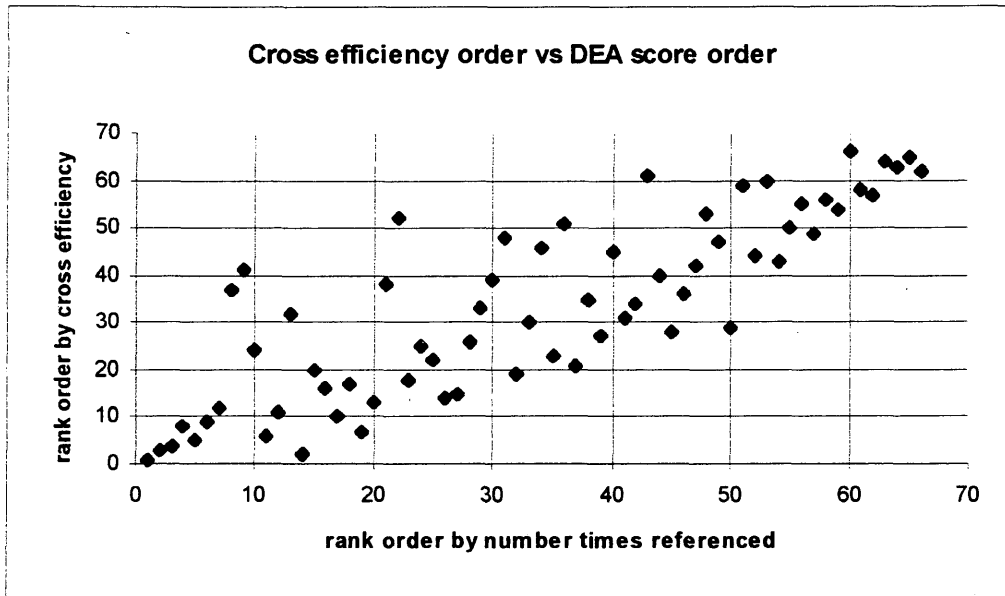


Figure 4. Cross Efficiency Score Order versus DEA Score Order

The rank ordering by the cross efficiency and number times referenced tend to indicate some correlation. In order to check the correlation, a Spearman rank order test was conducted. The test is taken out of Practical Nonparametric Statistics by W.J. Conover. The measure of correlation is designated by ρ (rho) and the equation is given below.

$$\rho = 1 - \frac{6 \sum_{i=1}^n [R(X_i) - R(Y_i)]^2}{n(n^2 - 1)} \quad (\text{equation 3.1})$$

Let $R(X_i)$ be the rank of X_i when compared to the other values, and the same for $R(Y_i)$. The variable n is the number of points of comparison. The result of the correlation test gave a value of .840 for Spearman's Rho. The closer the value to 1, the more positively correlated the results. A hypothesis test outlined on page 254 of Conover's book tests the independence between the two random variables. The one tailed test for positive correlation is stated below.

H_0 : The X_i and Y_i are mutually independent

H_1 : There is a tendency for the larger values of X and Y to be paired together.

Spearman's Rho is used as a test statistic for the preceding hypothesis. The value of ρ is checked against the quantiles of ρ in table A10. H_0 is rejected if the calculated value of ρ is greater than the table value. The table indicates that with a $\alpha = .05$, $\rho = .204$. Therefore, we reject H_0 , indicating that there is positive correlation.

Chapter 4

CONCLUSIONS AND RECOMMENDATIONS

4.1 DEA Vs FER Comparison

The objective of this paper is to explore the utility of DEA as a potential method to help evaluate the efficiencies of combat units. The current system that the Army uses is the Force Exchange Ratio, explained earlier in Chapter 1. The following section is a comparison of the DEA results against the FER results for each DMU. The first comparison uses a limited amount of data points. The data used will concentrate only on friendly and enemy tanks and APCs. The second comparison uses all the data in the study.

For the first comparison, the FER model was run using the number of friendly and enemy tanks and APCs. The model used for comparison follows:

$$FER = \frac{\left(\frac{\text{Initial Enemy Tanks} + \text{Initial Enemy APCs}}{\text{Initial Friendly Tanks} + \text{Initial Friendly APCs}} \right)}{\left(\frac{\text{Final Enemy Tanks} + \text{Final Enemy APCs}}{\text{Final Friendly Tanks} + \text{Final Friendly APCs}} \right)} \quad (\text{equation 1.1})$$

The model was solved on a spreadsheet and the results are listed in Appendix G. The next computation involved placing the data into the DEA model. The model was solved similarly to the original DEA model explained in Appendix B. The difference is the input and output factors used. The following inputs and outputs were used:

INPUTS

1. Initial friendly tanks
2. Initial friendly APCs
3. Final enemy tanks
4. Final enemy APCs

OUTPUTS

1. Initial enemy tanks
2. Initial enemy APCs
3. Final friendly tanks
4. Final friendly APCs

The inputs represent the numbers that the DMUs would like to be as small as possible and the outputs represent the numbers that the DMUs would like to be large. Understand that on the battlefield, the decision makers do not want to be outnumbered, but if the friendly units can perform well using limited resources, allowing extra resources to get allocated elsewhere, then that is more preferred.

The inputs and outputs were then run in the DEA model. The results are displayed by DMU and placed in order by efficiency score and the number of times each DMU was referenced. The DMU cross efficiency average is located in the fourth column and the FER model results are displayed in the last column. The results of the first ten DMUs are displayed. The complete table of results are listed in Appendix H.

DMU	Efficiency score	# Times Referenced	Cross Efficiency Average	FER
M9201	1.0000	41	0.9427	0.2660
M9307	1.0000	35	0.8747	1.1281
A9209	1.0000	29	0.7325	1.4862
A9201	1.0000	23	0.8807	4.0046
C9202	1.0000	23	0.8374	2.3382
M9302	1.0000	21	0.8187	0.8307
M9007	1.0000	8	0.7256	1.7647
A9003	1.0000	6	0.7150	2.5630
M9003	1.0000	4	0.6221	0.6340
M9005	1.0000	4	0.5586	0.2655

Table 6. Condensed Model Results

Note that there is some resemblance between the two model scores. The FER model scores are fairly high in the top end and low in the bottom end. Also note there are a few DMUs that do not correspond between the two models. For example, DMU M9201 is rated by the DEA model as the most efficient DMU, but is in the bottom 25% in the FER model. The FER model is inaccurate for this DMU. Looking at the raw data for DMU M9201 and then placing it into FER model, it is obvious where the model becomes inaccurate.

<u>type vehicle</u>	<u>initial # vehicles</u>	<u>final number of vehicles</u>	<u># vehicles destroyed</u>
friendly tank	0	0	0
friendly APC	18	1	17
enemy tank	39	7	32
enemy APC	124	27	97

Table 7. FER Model Data for DMU M9201

$$FER = \frac{\frac{39 + 124}{7 + 27}}{\frac{0 + 18}{0 + 1}} = \frac{9.055}{34} = .2663 \quad (\text{equation 1.1})$$

Notice that the friendly forces lost only 17 total vehicles and the enemy forces lost 129 total vehicles. The FER model gave a rating of .2663 as its efficiency score. The rule of thumb that the military uses is, if a unit's FER is greater than one, then it performed well. The above example would not fit very well into this rule, because most all military decision makers would consider this a successful battle. Had four friendly vehicles remained instead of one, then the FER model score would be 1.065, and would have been considered a successful battle by the model. Therefore, the difference of 14 total friendly

vehicles lost instead of 17 to the enemy's total of 129 would have made a big difference in FER model score.

The following two figures help illustrate the results between the two model methods. The figures are comparisons between the DEA efficiency scores (by cross efficiency) versus the FER ordering of the DMUs and the FER ordering versus the DEA ordering of the DMUs.

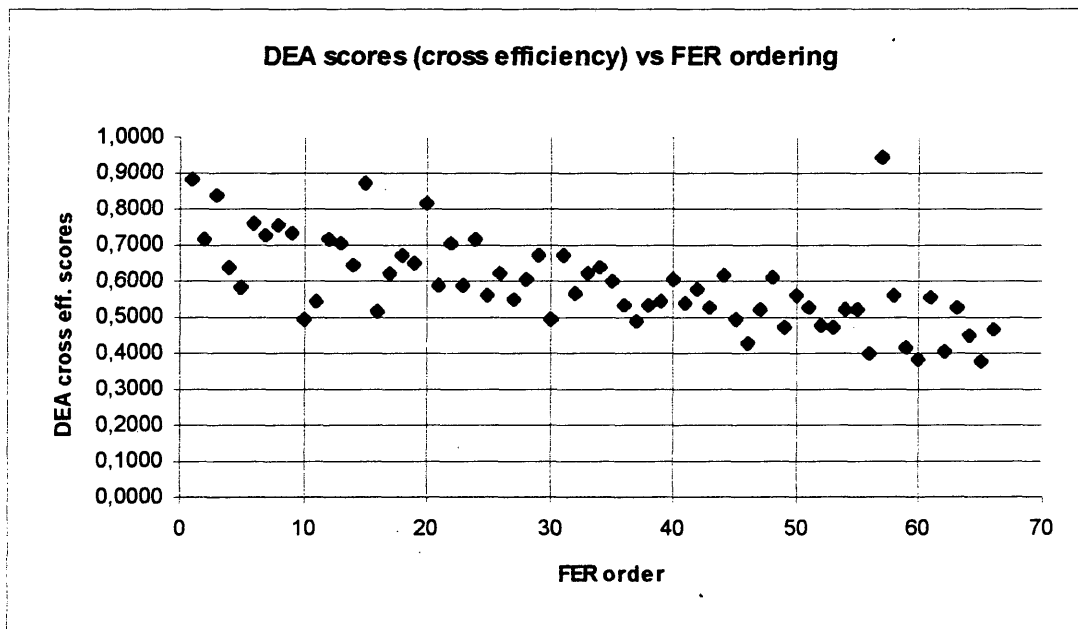


Figure 5. DEA Cross Efficiency Score versus FER Order

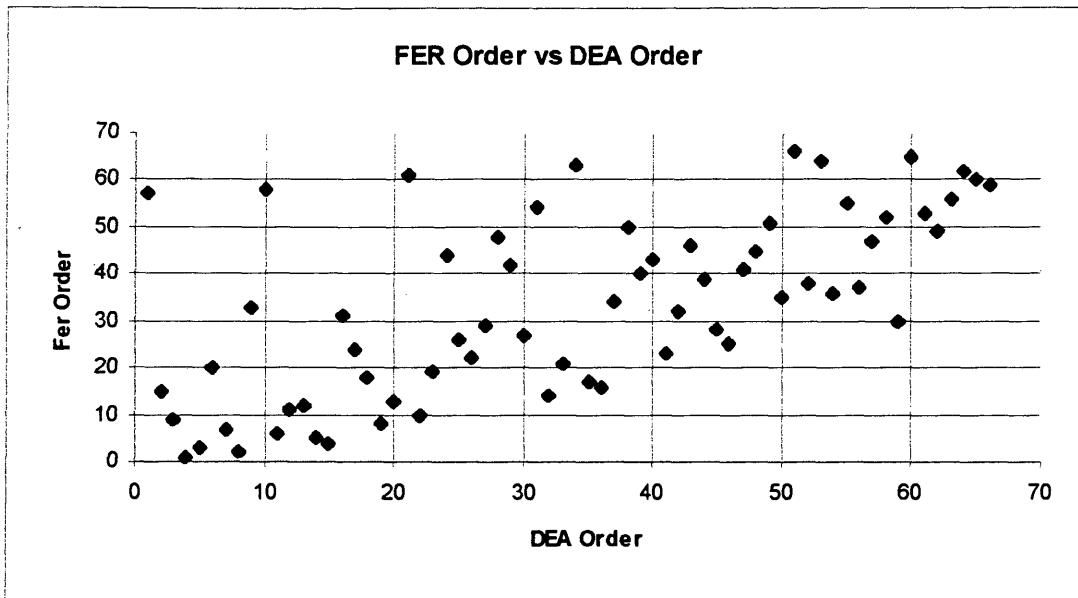


Figure 6. FER Order versus DEA Order

Figure 5 indicates some correlation between the two models. Notice that the higher the DEA efficiency score, the better the rank order placement by the FER model. There is one outlier that does not seem to fit. This outlier is the one explained earlier on page 65.

Figure 6 displays the DMU ordering of the two models against each other. The figure indicates that the two models are generally correlated in the ordering of DMUs. A Spearman rank order test was conducted to check the correlation. The same procedure in Chapter 3 was used for the Spearman test. A value of .704 was calculated for ρ , therefore we reject the hypothesis that the two measures are independent.

The next part of this comparison between the two models is the comparison of the models using all the inputs and outputs in the study. The results for the DEA model were already shown in Chapter 3. The FER scores displayed in Appendix I are taken directly from the computer at the CTC WIN office at Ft. Leavenworth. There are only fifty-three DMUs with a FER value because the computer program did not have a value for all the DMUs.

Using the FER model results shown in Appendix I and the DEA model results from Chapter 3, a graph of the FER ordering results versus the DEA ordering results follows:

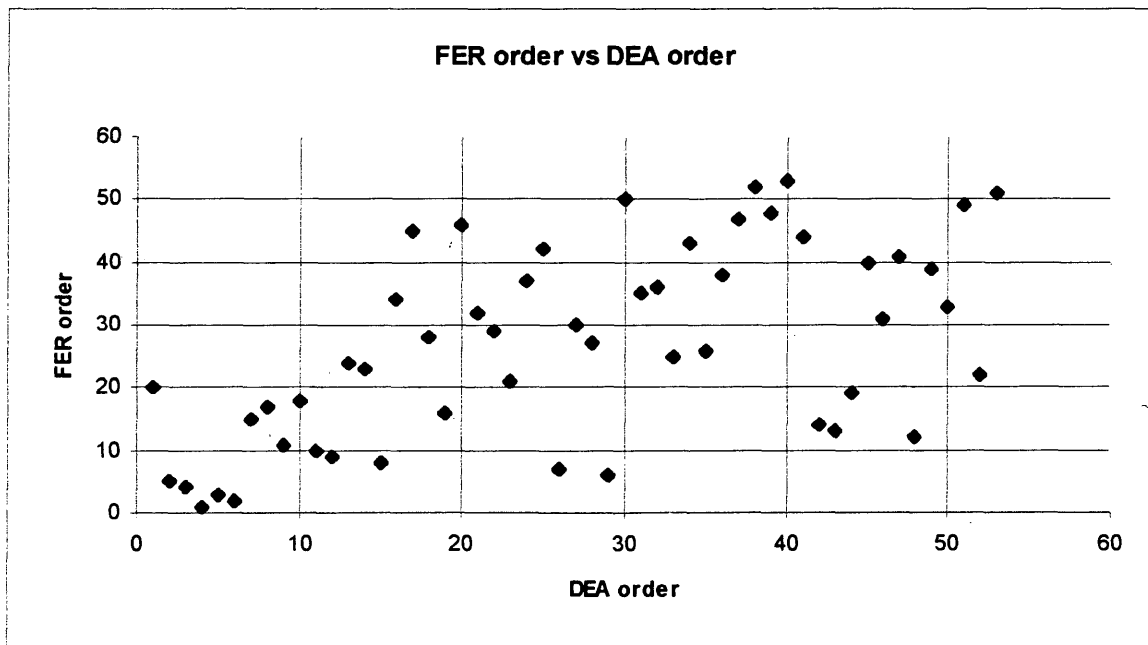


Figure 7. FER Order versus DEA Order, Full Model

There appears to be some indication of correlation. A Spearman's rank order test similar to Chapter 3 was conducted. A value of .569 was calculated for ρ . Therefore, when compared to the table value of .230, we reject the hypothesis that the two random variables are independent.

4.2 DEA Model Advantages

Chapter 3 of this study displayed the DEA model applied to measuring US Army combat units' performance efficiency in a CTC environment. Chapter 4 then displayed the DEA model results versus the current FER model. This section will explain the advantages that the DEA model has over the FER model.

The main advantage of the DEA model is its ability to measure a unit's efficiency using multiple input and output factors. The factors do not have to be aggregated in any way in order to run the model. The data may be entered in its original numeric form. One factor may be in units of dollars and another may be in units of soldiers or vehicles. The FER model has no way to separate the differences in the input and output factors. All the data entered is assumed in the same type of unit. All the data is lumped together, whether it is a tank, aircraft, or soldier. FER may be accurate with only one or two similar factors, but loses accuracy when multiple factors with different weights are measured.

A second advantage that the DEA model has is its ability to measure the best that each DMU has to offer against its peer group. DEA uses an optimization model where its factors are multiplied by generated weights. These weights are generated by running the DMU under evaluation against all the other DMUs. The outcome is a set of weights that make the evaluated DMU look as good as possible against its peer group. For example, if a task force performed better than any other task force in shooting down enemy aircraft, then it was rewarded in the model for doing so. If a task force performed worse than any other unit in killing enemy tanks, then that factor was probably weighted on the low end of the spectrum. Therefore, the DEA model finds the best that each unit has to offer and then rates it against its peers. The weights can then be used to identify where each unit performed well or poorly. Units may be picked to perform certain type missions based on their strengths. For example, if a unit performed better than its peers in air defense, then that unit might be selected to defend in an area with more enemy air traffic. On the other hand, the FER model does not perform this function. All the evaluated units in the FER model have their factors placed into the model with the same weight.

The third advantage that the DEA model has is its ability to identify a reference set for each unit under evaluation. If a unit is not efficient, then there is a reference set that was evaluated using the same weighted factors as the evaluated unit. The reference set then can be used to identify where the unit may make improvements. Improvements

in a specific input or output may make this unit an efficient unit. An example on the prescriptive method to make a DMU efficient is given in Section 3.4.

4.3 DEA Model Weaknesses

With any model, there are weaknesses or some kind of limitations. DEA is not any different. There are a few weaknesses worth discussing that are applicable to this study. The main weakness is the sensitivity of the model. The second weakness is the generation of a large number of efficient units.

The sensitivity of the model is the main weakness of the model. Because this model is based on an efficiency frontier, and each unit is measured against that frontier, any error causing the frontier to change will change the efficiency rating of some DMUs. For example, if a unit has an efficiency rating of .955, and an error in the data was found, where the unit actually killed 25 enemy tanks instead of 20 enemy tanks, then this could shift the curve. This unit would become efficient and the unit that once was in the reference set would become inefficient. As another result, other DMUs in the reference set would then become less efficient. The point of this example is that an error in the data, from the collection on the battlefield to the input into the model, could change the model results for other unit performance ratings.

The second weakness is the generation of a large number of efficient units with a rating of one. In this study, one third of the evaluated units received an efficiency score

of one. This is mainly due to a number of 'niche' players. Some units performed well in a 'broad' spectrum of factors, while the 'niche' players performed well in specialty areas. There were thirteen 'niche' players found in this study. Units with an efficiency rating of one become more frequent with the number of factors added to the study. With eleven factors in the study, there were twenty-three efficiency ratings of one and thirteen 'niche' players. When the study was performed with only eight factors, the number of efficient units were reduced to fifteen. The number of 'niche' players were reduced to eight. Therefore, with more factors added to the study (keeping the same number of DMUs), more units will rate efficient and more 'niche' players will be present. Although DEA models can handle numerous factors, the model better discriminates the DMUs with fewer factors in the study. There were two methods used in this study to correct this weakness. One method was ranking the efficient DMUs by the number of times it was referenced and the second method was ranking the DMUs by cross efficiency scores. There is another method that may be explored to try to break the efficient DMU ties. The method is presented by Per Anderson and Neils Peterson in A Procedure for Ranking Efficient Units in DEA. They reveal that efficiency ties may be broken by letting the evaluated DMU efficiency scores get greater than one. To accomplish this, they loosen the constraint on the evaluated DMU. This method is not yet proven, but is mentioned as a possible way to break ties.

4.4 Model Criteria Results

The model criteria presented in Chapter one is discussed in this section. Some criteria can be answered as a result of this study, while other criteria can not. The criteria that is not answered can be addressed with further research, but for now, it is beyond the scope of this study.

The first two model criteria address the issue of cost and if the customer has the adequate personnel and equipment to run the model. The answer is “yes”. In general, the Army has a vast amount of personnel and equipment resources that can easily run this model. The agencies that would use this model have trained personnel in Operations Research that are familiar with linear programming. The equipment required to run the model is a computer with a spreadsheet. Most agencies in the Army use EXCEL as a spreadsheet. The most resource intensive element of this model is the collection and storage of the raw data. This is also not a problem because the resources are already in place performing the collection and storage of the raw data.

The third criterion mentioned is if the uncertainties in the model are properly addressed. Model uncertainties are still unknown at this point in the study. Further research is required to determine if there exist uncertainties.

The fourth criterion requires that the model be designed in great breadth or great detail, but not both. The model passes this criteria. It is reasonable to design the model in great detail because the model requires known quantitative numbers. This is apparent

throughout the study. If the model was to calculate in great breadth, the results would not be accurate due to quantitative uncertainty which usually accompanies a model with great breadth. When numbers are uncertain, the efficiency frontier is uncertain and efficiency scores are not accurate.

The fifth criterion requires that the model becomes more rigid as chronological time decreases. Chapter three addresses this issue with the selection of DMUs. DMUs should be fairly homogenous, and time is an element in the determination of DMU homogeneity. If two DMUs are compared, where one DMU from ten years ago is compared against a DMU from the present time, homogeneity between DMUs is lost due to technological and training differences. When homogeneity is lost, model rigidity is less.

The sixth criterion requires that the model is not biased. One of the model strengths is the fact that the model is fair and efficiency scores are calculated with no bias. Chapter two addresses the methodology of the model. The fact that weights are assigned to raw data by the model and not by subjective analysis is the underlying reason for model fairness.

The final criterion requires that the model rigidity meet the analytic environment for which it is modeled. The environment that this model is based is a learning environment. Units go to NTC to learn how to become better units. The data from NTC is collected and stored for the purposes of trend analysis. The fact that a unit was ranked

20th out of 100 units instead of 15th does not make a big difference in the purpose of combat unit training. The fact that a unit ranked 90th out of 100 would make a difference. The model rigidity does not need to be exact in how it rank orders units, but the ability to rank order units based on multiple input and output factors give a unit general guidance on where it stands overall compared to the other units. Therefore, the model does provide the appropriate rigidity for the analytic environment.

Chapter 5

AREAS FOR FURTHER RESEARCH

This model has considerable potential for uses in the military. The model has already been used to determine efficiencies in Naval recruiting, Air Force maintenance programs, and now Army combat units. This Chapter discusses the use of DEA as a model specifically for evaluating Army combat units. Combat units may be evaluated in a variety of settings to include a CTC rotation (already explored in this study) or during normal garrison operations.

This study evaluated units in the CTC environment. The study included eleven factors that were determined based on expert opinion and data readily available. There are numerous areas that may be further researched in this study. One area is to perform the study again with factors determined by the high level decision makers at a CTC or in the Center for Army Lessons Learned. Once decided upon, collect the specific data to support those factors, and run the model again. Some factors that may be of interest to collect is the amount of money that each unit spent to get ready for the CTC rotation. Another factor of interest is the number of training days spent by each unit preparing to deploy to the CTC. Other factors that could be explored are the cost of ammunition spent for each battle, number of obstacles employed, or the task force leadership. Factors

can also be given utility functions and then placed into the model with their new utility values. For example, if the first tank destroyed is worth more than the twentieth tank destroyed, then the utility factor will reflect this in the input data.

This model may also be used to evaluate the performance of combat units based on the USR report as mentioned in Chapter 1. The model may also be used to evaluate combat units in garrison. This data includes factors such as unit's budget for the year, number of training days, number of days off, equipment readiness, reenlistment retention scores, number of AWOLs, MOS skill tests, number of disciplinary actions, and physical training scores. This would then give the unit commanders a model to help them determine which subunits are performing well compared to the others. Units that are not performing efficiently can be identified and helped.

This model may be applied to many various studies that a military decision maker would like to see performed. Following the procedures outlined in this study should give a head start. The base requirements to perform a study similar to this study are a spreadsheet with an optimizer function, some knowledge of linear programming, and data for the study.

REFERENCES CITED

- Ali, A. and Seiford, L. 1993. Computational Accuracy and Infinitesimals in Data Envelopment Analysis.
- Anderson, P. and Peterson, N. 1993. A Procedure for Ranking Efficient Units in Data Envelopment Analysis. Management Science. 39: 1261-1264.
- Battilega, John A. and Grange, Judith K. The Military Applications of Modeling. Wright-Patterson Air Force Base, Ohio: AFIT Press.
- Breu, T. and Raab, R. 1994. Efficiency and Perceived Quality of the Nation's "Top 25" National Liberal Arts Colleges: An Application of Data Envelopment Analysis to Higher Education. Socio-Econ. Plann. Sci. 28: 33-45.
- Chang, P. and Hwang, S. 1993. Data Envelopment Analysis in Measuring the Efficiency of Forset Management. Journal of Environmental Management. 38: 73.
- Charnes, A. and Cooper, W. 1985. Preface to Topics in Data Envelopment Analysis. Annals of Operations Research. 2: 59-64
- Charnes, A. , Cooper, W. and Rhodes, E. 1978. Measuring the Efficiency of Decision Making Units. European Journal of Operations Research. 2: 429-444.
- Conover, W. J. 1982. Practical Nonparametric Statistics. 2nd ed. New York: John Wiley and Sons Inc.
- Doyle, J. and Green, R. 1991. Comparing Products Using Data Envelopment Analysis. OMEGA International Journal of Management Science. 19: 631-638.
- Farrell, M.J. 1957. The Measurement of Productive Efficiency. Journal of the Royal Statistical Society. 120: 253-290.
- Ganley, J.A. and Cubbin, J.S. 1992. Public Sector Efficiency Measurement, Applications of Data Envelopment Analysis. Amsterdam, Netherlands: North Holland.

- Golany, B. and Roll, Y. 1989. An Application Procedure for DEA. Omega International Journal of Management Science. 17: 237-250.
- Hogan, A. and Wroblewski, R. 1993. The Determination Of Hosiptal Efficiency: A Data Envelopement Analysis. Applications of Management Science. 7: 115.
- Lewin, A. and Morey, R. 1981. Measuring the Relative Efficiency and Output Potential of Public Sector Organizations: An Application of Data Envelopement Analysis. International Journal of Policy Analysis and Information Systems. 5: 267-285.
- Norman, M. and Stoker, B. 1991. Data Envelopement Analysis, An Assessment of Performance. Chichester, England: John Wiley and sons Inc.
- Serrussi, D. 1987. Applying DEA to the I.D.F. Air Force Maintainence Units. MSc Thesis, Technion -Isreal Institute of Technology.
- Silkman, R. 1986. Measuring Efficiency: An Assessment of Data Envelopement Analysis. San Francisco, CA: Jossey-Bass Inc.
- Tiede, Roland V. and Leake, Lewis A. 1971. A Method for Evaluating the Combat Effectiveness of a Tactical Information System in a Field Army. Operations Research. 19: 587-604.
- Vakkuri, J. and Malkia, M. 1995. The Applicability of DEA Method in Efficiency Evaluation: The Case of Finnish University Departments. Prepared for INFORMS Conference, fall 1995.
- _____. 1989. Infantry Reference Data Book. Fort Benning, GA: United States Army Infantry School.

SELECTED BIBLIOGRAPHY

- Ali, A., Cook, W. and Seiford, L. 1991. Strict versus Weak Ordinal Relations For Multipliers in Data Envelopment analysis. Management Science. 37: 733-738.
- Chang, K. and Guh, Y. 1989. Linear Production Functions and the Data Envelopment Analysis. European Journal of Operations Research. 52: 215-223.
- Charnes, A. and Neralic L. 1992. Sensitivity Analysis in Data Envelopment Analysis. Glasnik Matematicki. 27: 191-201.
- Golany, B., Phillips, F., and Rousseau, J. 1993. Models For Improved Effectiveness Based On DEA Efficiency Results. IIE Transactions. 25:2-10.
- Jenkins, L. 1991. Weight Restricted Data Envelopment Analysis and Regression as Approaches to Measuring Relative Efficiency. Msc. RMC of Canada.
- Kao, C. 1994. Efficiency Improvement in Data Envelopment analysis. European Journal of Operations Research. 73: 487-494.
- Roll, Y., Cook, W., and Golany, B. 1991. Controlling Factor Weights in Data Envelopment Analysis. IIE Transactions. 23: 2-9.

APPENDIX A
CCR MODEL DATA

DMU	Input Factors					Output Factors					
	Fcas	Ftank	FIFV	Fcom	Frat	Ecas	Etank	Eapc	Ecom	Eair	MA
A9001	0	28	16	2	4	82	26	61	11	5	0
A9002	9	21	35	9	1	41	24	44	8	3	0
A9003	9	24	27	8	1	170	41	105	18	10	1
A9004	0	23	20	9	2	212	30	71	21	2	0
A9101	18	32	28	3	4	260	42	113	28	9	1
A9102	18	20	26	14	0	26	13	56	4	5	0
A9103	27	18	13	4	0	50	16	45	4	3	0
A9201	0	16	13	7	1	143	35	118	14	8	1
A9202	36	30	21	5	1	101	18	63	9	2	0
A9203	18	23	41	11	2	164	28	96	10	8	1
A9204	36	19	23	2	4	75	31	101	13	11	1
A9205	18	27	30	8	2	72	15	64	2	10	0
A9206	0	24	18	4	1	105	30	83	10	5	0
A9207	36	27	29	8	3	4	13	59	2	5	0
A9208	54	21	14	5	1	106	9	56	5	6	0
A9209	18	23	13	0	0	115	30	135	7	3	0
A9301	18	22	25	5	3	117	21	109	5	7	0
A9302	36	21	24	5	6	126	12	67	4	0	0
A9303	45	22	18	4	4	118	16	52	10	10	0
A9304	72	20	11	13	2	110	26	110	9	6	1
A9305	90	25	31	17	1	108	19	84	9	9	0
A9306	0	15	19	8	3	225	27	120	4	13	0
A9307	36	21	23	19	4	50	15	36	0	7	0
A9308	36	25	21	5	2	113	21	87	3	12	0
A9309	0	22	19	10	0	90	21	80	4	5	1
A9310	0	13	16	20	5	70	18	73	8	6	0
A9401	18	23	18	7	0	40	5	16	6	6	0
A9402	39	20	16	5	0	83	15	72	4	6	0
A9403	18	11	11	1	0	0	9	34	0	3	1
A9404	0	37	14	5	4	53	28	72	11	3	0
M9001	0	20	36	19	5	190	17	42	8	4	0
M9002	0	20	20	3	3	100	10	37	4	1	0
M9003	54	18	24	19	2	72	17	38	3	5	0
M9004	130	18	34	14	0	156	24	62	6	3	0

DMU	Fcas	Ftank	FIFV	Fcom	Frat	Ecas	Etank	Eapc	Ecom	Eair	MA
M9005	45	21	21	6	2	47	34	81	11	8	0
M9006	18	20	21	13	1	142	33	95	14	6	0
M9007	36	18	23	10	3	45	28	78	20	4	0
M9008	9	23	19	11	0	72	32	36	12	5	0
M9101	0	23	32	20	0	145	24	65	3	14	1
M9102	86	12	19	9	0	99	24	47	5	7	0
M9201	131	0	18	7	0	175	32	97	5	6	0
M9202	76	23	18	30	0	60	28	95	7	5	0
M9203	79	18	9	7	0	65	17	66	4	4	0
M9204	92	23	19	10	1	50	4	46	3	2	0
M9301	9	20	31	10	0	189	26	123	13	7	1
M9302	28	16	7	6	0	97	19	85	0	7	0
M9303	24	20	17	10	6	74	15	66	5	1	0
M9304	22	18	19	8	4	111	17	94	8	5	1
M9305	23	18	3	4	0	105	15	64	2	9	0
M9306	22	21	15	9	0	90	13	56	2	5	0
M9307	7	11	4	6	0	71	14	60	3	6	0
M9308	30	18	42	0	0	139	29	121	8	11	1
M9401	96	15	31	1	0	90	31	111	4	4	0
M9402	125	17	30	5	0	115	18	50	5	4	0
M9403	9	15	16	11	1	90	24	93	7	2	0
M9404	230	23	27	6	2	62	26	85	6	8	0
C9001	0	30	46	3	2	40	13	27	3	1	0
C9002	0	33	44	7	6	133	27	44	9	4	0
C9003	9	33	22	10	6	65	25	55	6	6	0
C9201	0	29	31	1	7	89	38	95	10	5	1
C9202	0	12	9	7	0	182	37	82	10	1	1
C9301	0	36	37	6	5	97	26	94	2	1	0
C9302	0	19	17	1	0	165	18	92	3	2	0
C9303	0	26	13	3	5	18	29	109	9	3	0
C9304	0	25	24	1	4	0	6	21	9	6	0
C9305	70	30	22	4	0	26	21	91	7	8	0

APPENDIX B

EXCEL FORMULATIONS FOR THE CCR MODEL

The following table is an example of the spreadsheet used to optimize the data with the CCR model. In order to keep this on one page, the example table represents all columns A through BP (all 66 DMUs). Notice rows 14 through 18 are repeated twice, this represents what the values in those cells were before the model was solved, and after the model was solved.

	A	B	C	D	E	F	G	H	I	J	...	BP
1	factors	mult.	A9001	A9002	A9003	A9004	A9101	A9102	A9103	A9201	...	C9305
2	Fcas	1	0	9	9	0	18	18	27	0	...	70
3	Ftank	1	28	21	24	23	32	20	18	16	...	30
4	FIFV	1	16	35	27	20	28	26	13	13	...	22
5	Fcom	1	2	9	8	9	3	14	4	7	...	4
6	Frat	1	4	1	1	2	4	0	0	1	...	0
7	Ecas	1	82	41	170	212	260	26	50	143	...	26
8	Etank	1	26	24	41	30	42	13	16	35	...	21
9	Eapc	1	61	44	105	71	113	56	45	118	...	91
10	Ecom	1	11	8	18	21	28	4	4	14	...	7
11	Eair	1	5	3	10	2	9	5	3	8	...	8
12	MA	1	0	0	1	0	1	0	0	1	...	0
13												
14	Before Optimization (multipliers =1)											
15	Objective func.		44	65	60	43	78	64	58	29	...	126
16	constraint #1		186	127	343	345	450	113	119	318	...	153
17	constraint #2		-142	-62	-283	-302	-372	-49	-61	-289	...	-27
18	ratio		3.7	1.6	5	6.222	5.329	1.333	1.903	8.622	...	1.214
14	After Optimization (multipliers will change values)											
15	Objective func.		1	2.57	2.215	1.992	1.791	3.054	1.456	1.453	...	2.438
16	constraint #1		1	0.759	1.773	0.891	1.791	0.711	0.583	1.453	...	1.146
17	constraint #2		3E-10	1.811	0.442	1.101	3E-10	2.343	0.872	0	...	1.292
18	ratio		1	0.295	0.8	0.447	1	0.233	0.401	1	...	0.47

optimized weights for column B (transposed)

	Fcas	Ftank	FIFV	Fcom	Frat	Ecas	Etank	Eapc	Ecom	Eair	MA
new weights	.014	.009	.031	.130	.000	.000	.017	.000	.009	.090	.000

Row one represents the DMUs in the model, rows two through twelve in columns C through BP is the input and output factors, and column B is the initial multiplier values.

Rows 15 through 18 represent the objective function and the constraints, the formulations used for each follows:

ROW 15

For cell C15, $C15 = \text{SUMPRODUCT}(C2:C6; \$B2:\$B6)$

For cell D15, $D15 = \text{SUMPRODUCT}(D2:D6; \$B2:\$B6)$

...

For cell BP15, $BP15 = \text{SUMPRODUCT}(BP2:BP6; \$B2:\$B6)$

ROW 16

For cell C16, $C16 = \text{SUMPRODUCT}(C7:C12; \$B7:\$B12)$

For cell D16, $D16 = \text{SUMPRODUCT}(D7:D12; \$B7:\$B12)$

...

For cell BP16, $BP16 = \text{SUMPRODUCT}(BP7:BP12; \$B7:\$B12)$

ROW 17

cell C17 = $\text{SUMPRODUCT}(C2:C6; \$B2:\$B6) - \text{SUMPRODUCT}(C7:C12; \$B7:\$B12)$

cell D17 = $\text{SUMPRODUCT}(D2:D6; \$B2:\$B6) - \text{SUMPRODUCT}(D7:D12; \$B7:\$B12)$

...

BP17 = $\text{SUMPRODUCT}(BP2:BP6; \$B2:\$B6) - \text{SUMPRODUCT}(BP7:BP12; \$B7:\$B12)$

ROW 18 cell C18= C16/C15

cell D18=D16/D15

...

cell BP18=BP16/BP15

To solve for a specific DMU (for this example, solve for DMU A9001), the following must be performed in the program solver:

1. Identify the cell that will get optimized: \$C\$15
2. Maximize or Minimize: Minimize
3. Identify the variable cells: \$B\$2:\$B\$12
4. Give constraints: \$C\$16=1
 \$C\$17>=0
 \$D\$17>=0
 ...
 \$BP\$17>=0

The results are given in the identified cells of the solver program (located in the example table above, cell C15, cell C16, cells C17 through BP17, and cells C18 through BP18) .

The weights for each factor are located in column B. The results are then copied and transferred to another worksheet. Once the transfer is performed, the weights are reset to one, and the solver is set up to solve for the next DMU. To solve for the next DMU, just change the cell getting optimized (from \$C\$15 to \$D\$15) and change the first constraint (from \$C\$16=1 to \$D\$16=1). Repeat this process until all the DMUs are solved.

The values in row 15 are the efficiency scores of the DMUs given the weights of the optimized DMU. The values in row 16 are the constraint values of each DMU given the weights of the optimized DMU. The values in row 17 is the efficiency score of each DMU when compared to the optimized DMU. The values in row 18 is a ratio of the DMU objective function divided into the value for constraint one. The ratio analysis in row 18 makes all the DMU efficiency scores between zero and one.

APPENDIX C
CCR MODEL RESULTS

DMU	A9001	A9002	A9003	A9004	A9101	A9102	A9103	A9201	A9202	A9203	A9204	A9205	A9206	A9207
score	1	0.463	0.994	1	1	0.481	0.557	1	0.512	0.685	1	0.604	1	0.314
A9001	1	0.613	0.497	0.501	0.67	0.281	0.363	0.67	0.438	0.225	0.67	0.485	1	0.475
A9002	0.295	0.463	0.417	0.316	0.317	0.366	0.358	0.317	0.403	0.15	0.317	0.222	0.366	0.216
A9003	0.8	0.875	0.994	0.776	0.836	0.909	0.959	0.836	0.941	0.758	0.836	0.824	0.795	0.645
A9004	0.447	0.74	0.943	1	0.593	0.541	0.711	0.593	0.917	0.33	0.593	0.168	0.74	0.293
A9101	1	1	1	1	1	0.561	0.764	1	1	0.658	1	0.542	1	0.66
A9102	0.233	0.234	0.193	0.19	0.235	0.481	0.331	0.235	0.204	0.186	0.235	0.298	0.16	0.287
A9103	0.401	0.391	0.325	0.279	0.469	0.373	0.557	0.469	0.433	0.187	0.469	0.277	0.303	0.397
A9201	1	1	1	1	1	1	1	1	0.929	1	1	1	1	0.904
A9202	0.278	0.345	0.411	0.367	0.365	0.254	0.441	0.365	0.512	0.142	0.365	0.122	0.292	0.28
A9203	0.41	0.516	0.378	0.395	0.498	0.479	0.403	0.498	0.38	0.685	0.498	0.408	0.347	0.439
A9204	1	0.998	0.477	0.695	1	0.483	0.521	1	0.557	0.818	1	0.548	0.563	1
A9205	0.478	0.268	0.121	0.116	0.437	0.359	0.26	0.437	0.084	0.326	0.437	0.604	0.215	0.478
A9206	0.823	0.673	0.845	0.504	0.726	0.506	0.656	0.726	0.672	0.286	0.726	0.803	1	0.558
A9207	0.259	0.2	0.075	0.082	0.23	0.161	0.134	0.23	0.073	0.124	0.23	0.218	0.136	0.314
A9208	0.367	0.278	0.226	0.335	0.499	0.367	0.47	0.499	0.341	0.308	0.499	0.294	0.152	0.52
A9209	1	0.709	1	0.426	1	0.426	1	1	1	0.247	1	0.533	1	1
A9301	0.555	0.459	0.225	0.28	0.586	0.298	0.271	0.586	0.215	0.37	0.586	0.44	0.358	0.663
A9302	0.118	0.242	0.095	0.232	0.266	0.046	0.071	0.266	0.118	0.166	0.266	0	0.143	0.24
A9303	0.667	0.525	0.321	0.557	0.728	0.386	0.418	0.728	0.392	0.438	0.728	0.427	0.293	0.649
A9304	0.334	0.428	0.227	0.563	0.479	0.366	0.419	0.479	0.361	0.545	0.479	0.185	0.209	0.533
A9305	0.263	0.302	0.209	0.362	0.351	0.463	0.446	0.351	0.326	0.292	0.351	0.227	0.138	0.382
A9306	0.949	0.678	0.224	0.41	1	0.568	0.369	1	0.17	1	1	1	0.532	1
A9307	0.23	0.168	0.016	0.04	0.246	0.175	0.092	0.246	0	0.244	0.246	0.216	0.095	0.243
A9308	0.727	0.412	0.167	0.203	0.738	0.442	0.388	0.738	0.15	0.443	0.738	0.657	0.291	0.775
A9309	0.408	0.352	0.301	0.235	0.427	0.502	0.413	0.427	0.249	0.563	0.427	0.571	0.371	0.425
A9310	0.288	0.359	0.186	0.543	0.304	0.29	0.214	0.304	0.196	0.374	0.304	0.238	0.215	0.339
A9401	0.354	0.231	0.425	0.304	0.316	0.564	0.644	0.316	0.442	0.224	0.316	0.541	0.189	0.29
A9402	0.448	0.355	0.268	0.271	0.545	0.496	0.629	0.545	0.368	0.303	0.545	0.404	0.233	0.595
A9403	0.52	0.333	0.05	0.017	0.484	0.332	0.326	0.484	0	0.758	0.484	0.485	0.253	0.602
A9404	0.609	0.434	0.402	0.392	0.425	0.204	0.298	0.425	0.367	0.104	0.425	0.243	0.712	0.323
M9001	0.193	0.29	0.181	0.363	0.288	0.207	0.171	0.288	0.185	0.368	0.288	0.153	0.188	0.171

DMU	A9001	A9002	A9003	A9004	A9101	A9102	A9103	A9201	A9202	A9203	A9204	A9205	A9206	A9207
M9002	0.252	0.283	0.215	0.249	0.333	0.111	0.142	0.333	0.195	0.192	0.333	0.106	0.341	0.214
M9003	0.187	0.219	0.078	0.179	0.247	0.238	0.166	0.247	0.098	0.231	0.247	0.15	0.11	0.209
M9004	0.153	0.317	0.132	0.306	0.334	0.266	0.334	0.334	0.304	0.244	0.334	0.065	0.114	0.233
M9005	0.628	0.72	0.417	0.546	0.687	0.492	0.572	0.687	0.555	0.296	0.687	0.376	0.422	0.616
M9006	0.448	0.622	0.552	0.714	0.552	0.687	0.666	0.552	0.636	0.372	0.552	0.341	0.438	0.449
M9007	0.384	0.747	0.584	0.975	0.439	0.487	0.589	0.439	0.757	0.195	0.439	0.167	0.405	0.381
M9008	0.475	0.573	0.68	0.562	0.494	0.749	0.809	0.494	0.699	0.231	0.494	0.425	0.521	0.269
M9101	0.448	0.321	0.168	0.188	0.467	1	0.431	0.467	0.115	0.795	0.467	0.823	0.242	0.433
M9102	0.356	0.505	0.184	0.412	0.594	0.614	0.662	0.594	0.398	0.427	0.594	0.234	0.181	0.483
M9201	0.346	1	0.153	1	1	0.707	0.9	1	0.658	1	1	0.15	0.176	1
M9202	0.175	0.253	0.135	0.313	0.229	0.411	0.314	0.229	0.207	0.16	0.229	0.118	0.132	0.253
M9203	0.283	0.325	0.157	0.31	0.449	0.335	0.558	0.449	0.346	0.206	0.449	0.154	0.149	0.486
M9204	0.082	0.098	0.081	0.158	0.138	0.128	0.174	0.138	0.15	0.099	0.138	0.059	0.043	0.206
M9301	0.467	0.614	0.818	0.609	0.593	1	0.857	0.593	0.79	0.801	0.593	0.583	0.476	0.574
M9302	0.629	0.377	0.05	0.108	0.751	0.54	0.469	0.751	0	0.435	0.751	0.581	0.271	0.841
M9303	0.169	0.27	0.113	0.286	0.235	0.08	0.098	0.235	0.133	0.131	0.235	0.038	0.172	0.243
M9304	0.39	0.446	0.253	0.485	0.464	0.265	0.268	0.464	0.285	0.676	0.464	0.244	0.279	0.528
M9305	1	0.405	0.241	0.275	1	0.746	1	1	0.237	0.496	1	1	0.356	1
M9306	0.326	0.233	0.137	0.162	0.387	0.389	0.341	0.387	0.137	0.27	0.387	0.375	0.183	0.386
M9307	0.738	0.482	0.356	0.439	0.756	1	0.859	0.756	0.355	0.524	0.756	0.931	0.415	0.815
M9308	0.828	1	1	0.382	1	1	1	1	1	1	1	1	0.596	1
M9401	0.364	0.708	0.182	0.249	0.718	0.327	0.533	0.718	0.553	0.277	0.718	0.144	0.231	0.758
M9402	0.206	0.364	0.152	0.289	0.442	0.298	0.486	0.442	0.446	0.256	0.442	0.106	0.116	0.35
M9403	0.302	0.483	0.336	0.477	0.401	0.37	0.362	0.401	0.377	0.24	0.401	0.149	0.359	0.403
M9404	0.244	0.362	0.099	0.282	0.476	0.241	0.372	0.476	0.288	0.204	0.476	0.116	0.098	0.548
C9001	0.164	0.215	0.19	0.096	0.174	0.087	0.104	0.174	0.14	0.066	0.174	0.104	0.273	0.105
C9002	0.354	0.417	0.246	0.28	0.373	0.165	0.178	0.373	0.222	0.21	0.373	0.2	0.412	0.2
C9003	0.429	0.333	0.152	0.23	0.368	0.169	0.163	0.368	0.145	0.183	0.368	0.265	0.315	0.297
C9201	0.889	0.753	0.303	0.374	0.662	0.185	0.211	0.662	0.272	0.471	0.662	0.303	0.942	0.508
C9202	0.637	1	1	1	0.926	0.867	1	0.926	1	1	0.926	0.173	1	0.494
C9301	0.249	0.268	0.063	0.082	0.28	0.042	0.047	0.28	0.055	0.104	0.28	0.06	0.337	0.255
C9302	0.63	0.472	0.807	0.262	0.774	0.297	0.441	0.774	0.419	0.355	0.774	0.962	1	0.661
C9303	0.839	0.602	0.329	0.425	0.526	0.184	0.24	0.526	0.311	0.106	0.526	0.236	0.858	0.596
C9304	0.661	0.379	0.447	0.375	0.353	0.288	0.325	0.353	0.373	0.179	0.353	0.586	0.502	0.357
C9305	0.47	0.383	0.322	0.278	0.502	0.462	0.772	0.502	0.536	0.193	0.502	0.356	0.237	0.608

DMU	A9208	A9209	A9301	A9302	A9303	A9304	A9305	A9306	A9307	A9308	A9309	A9310	A9401	A9402
score	0.593	1	0.706	0.499	0.873	1	0.517	1	0.412	0.831	0.532	0.81	0.686	0.645
A9001	0.505	0.705	0.382	0.43	0.584	0.161	0.336	0.67	0.281	0.38	0.471	0.404	0.314	0.318
A9002	0.164	0.311	0.273	0.21	0.255	0.052	0.311	0.317	0.143	0.174	0.125	0.352	0.331	0.324
A9003	0.664	0.809	0.614	0.673	0.809	0.548	0.849	0.836	0.684	0.601	0.704	0.825	0.945	0.861
A9004	0.576	0.604	0.441	0.783	0.64	0.056	0.387	0.593	0.118	0.123	0.418	0.62	0.576	0.574
A9101	1	1	0.582	1	1	0.49	0.589	1	0.528	0.542	1	0.759	0.601	0.624
A9102	0.148	0.224	0.307	0.119	0.253	0.114	0.44	0.235	0.285	0.263	0.136	0.34	0.35	0.379
A9103	0.349	0.466	0.349	0.283	0.368	0.125	0.334	0.469	0.243	0.294	0.307	0.266	0.444	0.513
A9201	0.761	1	1	0.792	1	1	1	1	1	0.686	1	1	1	1
A9202	0.351	0.368	0.312	0.361	0.325	0.051	0.193	0.365	0.098	0.125	0.3	0.223	0.317	0.373
A9203	0.454	0.479	0.51	0.522	0.473	0.349	0.548	0.498	0.479	0.398	0.471	0.564	0.375	0.419
A9204	0.756	1	0.766	0.575	1	0.675	0.724	1	0.897	1	0.971	0.856	0.402	0.499
A9205	0.384	0.414	0.327	0.234	0.43	0.193	0.523	0.437	0.455	0.551	0.29	0.387	0.311	0.308
A9206	0.55	0.724	0.558	0.525	0.572	0.152	0.467	0.726	0.3	0.386	0.474	0.447	0.716	0.608
A9207	0.123	0.226	0.289	0.061	0.21	0.1	0.244	0.23	0.231	0.275	0.139	0.205	0.13	0.171
A9208	0.593	0.485	0.345	0.416	0.572	0.229	0.442	0.499	0.429	0.495	0.477	0.356	0.325	0.447
A9209	0.894	1	1	0.735	0.566	0.119	0.331	1	0.206	0.326	1	0.29	1	1
A9301	0.551	0.582	0.706	0.522	0.515	0.163	0.436	0.586	0.387	0.498	0.468	0.42	0.257	0.34
A9302	0.359	0.295	0.427	0.499	0.199	0	0.036	0.266	0	0	0.306	0.1	0.042	0.095
A9303	0.75	0.73	0.33	0.515	0.873	0.308	0.58	0.728	0.634	0.786	0.561	0.624	0.32	0.374
A9304	0.377	0.5	0.552	0.356	0.539	1	0.436	0.479	0.752	0.356	0.619	0.448	0.259	0.455
A9305	0.31	0.338	0.328	0.264	0.432	0.171	0.517	0.351	0.42	0.385	0.241	0.453	0.3	0.445
A9306	1	1	1	0.967	0.993	0.403	1	1	1	1	0.758	1	0.415	0.484
A9307	0.18	0.249	0.164	0.109	0.252	0.176	0.334	0.246	0.412	0.306	0.142	0.291	0.105	0.124
A9308	0.698	0.718	0.488	0.415	0.699	0.318	0.66	0.738	0.663	0.831	0.558	0.49	0.39	0.446
A9309	0.337	0.422	0.482	0.325	0.357	0.603	0.45	0.427	0.524	0.309	0.532	0.328	0.579	0.507
A9310	0.213	0.314	0.46	0.244	0.411	0.22	0.469	0.304	0.54	0.297	0.197	0.81	0.183	0.237
A9401	0.296	0.286	0.101	0.175	0.43	0.183	0.514	0.316	0.37	0.416	0.198	0.387	0.686	0.506
A9402	0.517	0.525	0.476	0.374	0.514	0.207	0.515	0.545	0.422	0.505	0.437	0.362	0.501	0.645
A9403	0.252	0.458	0.452	0.068	0.314	0.961	0.419	0.484	0.753	0.532	1	0.239	0.371	0.445
A9404	0.233	0.459	0.327	0.237	0.353	0.099	0.2	0.425	0.142	0.167	0.288	0.253	0.252	0.267
M9001	0.323	0.292	0.216	0.438	0.315	0.068	0.272	0.288	0.191	0.165	0.208	0.429	0.144	0.164
M9002	0.416	0.344	0.298	0.502	0.261	0.029	0.113	0.333	0.064	0.087	0.305	0.166	0.116	0.144
M9003	0.191	0.25	0.182	0.171	0.252	0.123	0.329	0.247	0.313	0.23	0.143	0.307	0.137	0.187
M9004	0.338	0.335	0.275	0.376	0.286	0.054	0.242	0.334	0.155	0.156	0.227	0.241	0.149	0.33
M9005	0.393	0.688	0.497	0.298	0.644	0.218	0.591	0.687	0.49	0.581	0.347	0.593	0.416	0.53
M9006	0.421	0.553	0.544	0.49	0.573	0.164	0.615	0.552	0.378	0.335	0.34	0.663	0.556	0.643
M9007	0.221	0.442	0.495	0.334	0.573	0.103	0.475	0.439	0.255	0.253	0.208	0.766	0.386	0.499
M9008	0.286	0.493	0.209	0.281	0.462	0.146	0.539	0.494	0.302	0.291	0.207	0.504	0.835	0.654

DMU	A9208	A9209	A9301	A9302	A9303	A9304	A9305	A9306	A9307	A9308	A9309	A9310	A9401	A9402
M9101	0.402	0.444	0.298	0.308	0.498	0.549	1	0.467	0.835	0.567	0.408	0.669	0.643	0.519
M9102	0.494	0.585	0.319	0.381	0.592	0.222	0.699	0.594	0.6	0.574	0.334	0.549	0.33	0.622
M9201	1	1	1	1	1	0.222	1	1	1	0.887	0.633	1	0.241	1
M9202	0.127	0.237	0.325	0.135	0.237	0.153	0.36	0.229	0.308	0.169	0.137	0.319	0.212	0.379
M9203	0.368	0.459	0.397	0.27	0.43	0.222	0.35	0.449	0.364	0.342	0.368	0.268	0.255	0.553
M9204	0.165	0.135	0.215	0.155	0.176	0.058	0.137	0.138	0.121	0.12	0.155	0.127	0.096	0.191
M9301	0.578	0.562	0.764	0.708	0.6	0.437	0.757	0.593	0.543	0.413	0.614	0.701	0.914	0.88
M9302	0.669	0.757	0.666	0.437	0.638	0.483	0.676	0.751	0.741	0.691	0.651	0.381	0.494	0.714
M9303	0.196	0.262	0.399	0.281	0.208	0.033	0.092	0.235	0.069	0.062	0.209	0.181	0.066	0.118
M9304	0.437	0.477	0.646	0.492	0.517	0.621	0.372	0.464	0.589	0.353	0.66	0.491	0.209	0.297
M9305	1	1	0.509	0.499	1	1	0.864	1	1	1	1	0.508	1	1
M9306	0.374	0.38	0.337	0.301	0.36	0.18	0.41	0.387	0.349	0.339	0.31	0.269	0.377	0.415
M9307	0.624	0.753	0.673	0.453	0.811	0.686	1	0.756	0.964	0.74	0.59	0.663	1	1
M9308	1	0.869	1	1	0.75	0.393	1	1	0.657	1	0.816	0.77	1	1
M9401	0.6	0.688	0.802	0.567	0.408	0.08	0.355	0.718	0.235	0.428	0.442	0.291	0.227	0.618
M9402	0.49	0.427	0.28	0.42	0.386	0.081	0.312	0.442	0.228	0.319	0.301	0.269	0.19	0.417
M9403	0.279	0.416	0.686	0.403	0.333	0.072	0.311	0.401	0.167	0.14	0.282	0.388	0.296	0.437
M9404	0.39	0.481	0.321	0.207	0.487	0.173	0.342	0.476	0.416	0.528	0.327	0.294	0.149	0.352
C9001	0.124	0.17	0.147	0.165	0.097	0.013	0.077	0.174	0.035	0.056	0.086	0.091	0.102	0.101
C9002	0.329	0.38	0.206	0.393	0.298	0.054	0.203	0.373	0.136	0.184	0.213	0.278	0.153	0.159
C9003	0.246	0.389	0.241	0.198	0.331	0.145	0.254	0.368	0.273	0.284	0.222	0.279	0.149	0.166
C9201	0.436	0.707	0.572	0.486	0.451	0.381	0.258	0.662	0.364	0.332	0.684	0.37	0.176	0.216
C9202	0.747	0.969	0.856	1	0.657	1	0.437	0.926	0.533	0.103	1	0.57	1	1
C9301	0.216	0.3	0.41	0.288	0.114	0.016	0.05	0.28	0.035	0.047	0.218	0.06	0.042	0.1
C9302	1	0.752	0.831	1	0.437	0.066	0.243	0.774	0.142	0.222	0.77	0.195	0.768	0.6
C9303	0.19	0.589	0.704	0.244	0.395	0.115	0.22	0.526	0.189	0.232	0.383	0.315	0.191	0.265
C9304	0.249	0.324	0.154	0.122	0.485	0.142	0.385	0.353	0.314	0.482	0.181	0.445	0.297	0.262
C9305	0.336	0.479	0.421	0.173	0.479	0.198	0.469	0.502	0.387	0.524	0.343	0.343	0.506	0.678

DMU	A9403	A9404	M9001	M9002	M9003	M9004	M9005	M9006	M9007	M9008	M9101	M9102	M9201	M9202
score	1	0.879	0.628	0.555	0.357	0.386	0.792	0.772	1	0.835	1	0.729	1	0.553
A9001	0.481	1	0.195	0.667	0.448	0.308	0.713	0.401	0.456	0.314	0.272	0.321	0.67	0.365
A9002	0.175	0.279	0.126	0.246	0.293	0.166	0.344	0.429	0.441	0.331	0.192	0.356	0.317	0.197
A9003	0.789	0.685	0.457	0.699	0.69	0.635	0.773	0.865	0.899	0.945	0.784	0.944	0.836	0.529
A9004	0.439	0.89	0.608	1	0.4	0.656	0.49	0.971	1	0.576	0.129	0.641	0.593	0.492
A9101	1	1	0.513	1	0.555	0.759	0.828	0.883	1	0.601	0.517	0.667	1	0.523
A9102	0.157	0.133	0.083	0.094	0.304	0.123	0.246	0.252	0.226	0.35	0.359	0.393	0.235	0.275
A9103	0.347	0.255	0.164	0.165	0.371	0.271	0.501	0.262	0.296	0.444	0.291	0.515	0.469	0.357
A9201	1	1	0.596	0.907	1	0.726	1	1	1	1	1	1	1	1
A9202	0.289	0.304	0.201	0.249	0.215	0.301	0.331	0.328	0.378	0.317	0.113	0.372	0.365	0.322
A9203	0.514	0.269	0.444	0.393	0.411	0.514	0.418	0.474	0.468	0.375	0.553	0.42	0.498	0.335
A9204	1	0.501	0.23	0.349	0.734	0.407	1	0.622	0.689	0.402	0.826	0.492	1	0.549
A9205	0.35	0.117	0.171	0.143	0.399	0.287	0.391	0.11	0.097	0.311	0.534	0.297	0.437	0.236
A9206	0.517	0.777	0.291	0.67	0.52	0.46	0.727	0.487	0.548	0.716	0.358	0.599	0.726	0.495
A9207	0.15	0.092	0.011	0.03	0.243	0.053	0.267	0.087	0.089	0.13	0.249	0.144	0.23	0.22
A9208	0.483	0.155	0.274	0.174	0.353	0.424	0.39	0.26	0.276	0.325	0.444	0.449	0.499	0.388
A9209	0.827	0.703	0.31	0.469	0.495	0.6	1	0.361	0.467	1	0.257	0.748	1	0.899
A9301	0.46	0.26	0.331	0.323	0.441	0.459	0.524	0.244	0.261	0.257	0.42	0.276	0.586	0.487
A9302	0.235	0.148	0.343	0.234	0.12	0.349	0.199	0.151	0.175	0.042	0	0.059	0.266	0.303
A9303	0.623	0.315	0.304	0.283	0.543	0.456	0.634	0.429	0.453	0.32	0.577	0.4	0.728	0.333
A9304	0.561	0.235	0.278	0.157	0.596	0.313	0.541	0.426	0.425	0.259	0.549	0.416	0.479	0.843
A9305	0.275	0.139	0.22	0.123	0.382	0.274	0.321	0.375	0.354	0.3	0.457	0.479	0.351	0.352
A9306	0.818	0.358	1	0.777	1	1	0.847	0.313	0.277	0.415	1	0.421	1	0.706
A9307	0.183	0.053	0.14	0.046	0.396	0.148	0.272	0.025	0.012	0.105	0.361	0.119	0.246	0.158
A9308	0.63	0.183	0.27	0.192	0.597	0.464	0.68	0.161	0.155	0.39	0.717	0.417	0.738	0.409
A9309	0.527	0.279	0.273	0.291	0.434	0.351	0.426	0.239	0.24	0.579	0.569	0.463	0.427	0.446
A9310	0.205	0.272	0.361	0.246	0.631	0.211	0.328	0.498	0.385	0.183	0.384	0.228	0.304	0.529
A9401	0.264	0.203	0.111	0.179	0.28	0.207	0.248	0.324	0.323	0.686	0.453	0.653	0.316	0.132
A9402	0.463	0.186	0.237	0.17	0.428	0.396	0.499	0.244	0.263	0.501	0.503	0.625	0.545	0.465
A9403	1	0.108	0.003	0	0.39	0.086	0.553	0.025	0.025	0.371	0.827	0.356	0.484	0.323
A9404	0.27	0.879	0.095	0.376	0.337	0.146	0.536	0.312	0.355	0.252	0.134	0.256	0.425	0.377
M9001	0.235	0.197	0.628	0.409	0.266	0.428	0.211	0.363	0.315	0.144	0.192	0.174	0.288	0.179
M9002	0.287	0.287	0.33	0.555	0.158	0.411	0.238	0.199	0.226	0.116	0.069	0.126	0.333	0.213
M9003	0.181	0.092	0.211	0.083	0.357	0.195	0.262	0.172	0.147	0.137	0.303	0.193	0.247	0.198
M9004	0.267	0.102	0.349	0.112	0.274	0.386	0.304	0.304	0.315	0.149	0.182	0.342	0.334	0.269
M9005	0.44	0.386	0.128	0.198	0.642	0.234	0.792	0.531	0.574	0.416	0.52	0.546	0.687	0.469
M9006	0.382	0.461	0.437	0.41	0.598	0.457	0.551	0.772	0.734	0.556	0.42	0.684	0.552	0.58
M9007	0.233	0.501	0.143	0.321	0.458	0.168	0.491	1	1	0.386	0.253	0.544	0.439	0.511
M9008	0.308	0.514	0.202	0.343	0.537	0.267	0.556	0.63	0.627	0.835	0.364	0.829	0.494	0.277

DMU	A9403	A9404	M9001	M9002	M9003	M9004	M9005	M9006	M9007	M9008	M9101	M9102	M9201	M9202
M9101	0.487	0.147	0.424	0.242	0.625	0.444	0.419	0.217	0.154	0.643	1	0.594	0.467	0.255
M9102	0.456	0.148	0.337	0.118	0.627	0.436	0.61	0.408	0.401	0.33	0.668	0.729	0.594	0.352
M9201	0.795	0.14	1	0.139	1	1	1	1	1	0.241	1	1	1	1
M9202	0.146	0.138	0.135	0.074	0.456	0.125	0.292	0.33	0.272	0.212	0.283	0.379	0.229	0.553
M9203	0.37	0.141	0.173	0.089	0.435	0.26	0.515	0.235	0.259	0.255	0.352	0.526	0.449	0.601
M9204	0.129	0.052	0.107	0.055	0.105	0.139	0.104	0.13	0.137	0.096	0.121	0.167	0.138	0.252
M9301	0.634	0.407	0.603	0.611	0.466	0.718	0.462	0.791	0.777	0.914	0.68	0.917	0.593	0.56
M9302	0.659	0.156	0.351	0.157	0.781	0.525	0.793	0.041	0.034	0.494	0.775	0.565	0.751	0.842
M9303	0.165	0.204	0.221	0.181	0.223	0.192	0.257	0.202	0.212	0.066	0.053	0.088	0.235	0.374
M9304	0.582	0.303	0.37	0.322	0.418	0.389	0.421	0.401	0.407	0.209	0.467	0.259	0.464	0.549
M9305	1	0.279	0.351	0.241	0.891	0.62	1	0.172	0.163	1	1	1	1	0.788
M9306	0.332	0.135	0.261	0.166	0.371	0.353	0.352	0.131	0.127	0.377	0.403	0.39	0.387	0.355
M9307	0.62	0.39	0.41	0.31	1	0.536	0.782	0.368	0.338	1	0.983	1	0.756	1
M9308	1	0.29	0.451	0.455	0.527	1	0.721	0.545	0.649	1	1	1	1	0.427
M9401	0.509	0.152	0.255	0.108	0.415	0.472	0.707	0.261	0.354	0.227	0.324	0.477	0.718	0.495
M9402	0.374	0.098	0.272	0.101	0.277	0.416	0.379	0.273	0.32	0.19	0.281	0.449	0.442	0.242
M9403	0.262	0.358	0.377	0.318	0.461	0.343	0.44	0.504	0.484	0.296	0.181	0.366	0.401	0.677
M9404	0.387	0.085	0.097	0.044	0.387	0.182	0.56	0.212	0.248	0.149	0.382	0.341	0.476	0.375
C9001	0.108	0.135	0.088	0.209	0.104	0.136	0.166	0.111	0.133	0.102	0.048	0.094	0.174	0.082
C9002	0.262	0.311	0.267	0.482	0.252	0.322	0.338	0.266	0.289	0.153	0.15	0.166	0.373	0.144
C9003	0.254	0.305	0.129	0.198	0.377	0.171	0.428	0.183	0.191	0.149	0.243	0.161	0.368	0.233
C9201	0.644	0.661	0.205	0.693	0.434	0.29	0.722	0.316	0.368	0.176	0.311	0.191	0.662	0.349
C9202	1	1	1	1	0.893	1	0.963	1	1	1	0.475	1	0.926	1
C9301	0.191	0.196	0.178	0.246	0.19	0.221	0.304	0.057	0.078	0.042	0.039	0.043	0.28	0.262
C9302	0.716	0.441	0.573	1	0.323	1	0.547	0.199	0.254	0.768	0.196	0.388	0.774	0.567
C9303	0.282	0.988	0.047	0.375	0.457	0.085	0.745	0.33	0.381	0.191	0.163	0.211	0.526	0.638
C9304	0.239	0.46	0.003	0.396	0.253	0.072	0.304	0.367	0.4	0.297	0.334	0.299	0.353	0.14
C9305	0.376	0.202	0.05	0.089	0.395	0.149	0.543	0.273	0.311	0.506	0.46	0.686	0.502	0.423

DMU	M9203	M9204	M9301	M9302	M9303	M9304	M9305	M9306	M9307	M9308	M9401	M9402	M9403	M9404
score	0.622	0.263	1	0.96	0.43	0.749	1	0.504	1	1	1	0.524	0.791	0.583
A9001	0.365	0.383	0.285	0.411	0.368	0.169	0.67	0.274	0.351	0.272	0.673	0.419	0.281	0.609
A9002	0.2	0.186	0.332	0.189	0.185	0.138	0.317	0.213	0.244	0.192	0.403	0.248	0.275	0.322
A9003	0.539	0.508	0.884	0.544	0.501	0.671	0.836	0.766	0.691	0.784	0.754	0.744	0.579	0.76
A9004	0.477	0.433	0.575	0.369	0.407	0.177	0.593	0.5	0.321	0.129	0.629	0.81	0.412	0.389
A9101	0.524	0.506	0.589	0.531	0.479	0.54	1	0.574	0.526	0.517	1	1	0.437	0.781
A9102	0.279	0.275	0.423	0.286	0.282	0.183	0.235	0.297	0.363	0.359	0.174	0.122	0.361	0.248
A9103	0.379	0.368	0.361	0.382	0.365	0.167	0.469	0.352	0.425	0.291	0.422	0.357	0.305	0.483
A9201	1	1	1	0.985	1	1	1	0.904	1	1	0.843	0.784	1	0.895
A9202	0.336	0.324	0.263	0.307	0.315	0.125	0.365	0.254	0.285	0.113	0.368	0.452	0.258	0.298
A9203	0.341	0.343	0.531	0.368	0.35	0.625	0.498	0.535	0.467	0.553	0.408	0.421	0.517	0.424
A9204	0.55	0.596	0.49	0.701	0.591	0.859	1	0.461	0.691	0.826	1	0.497	0.562	1
A9205	0.247	0.264	0.38	0.34	0.274	0.213	0.437	0.439	0.454	0.534	0.21	0.164	0.299	0.438
A9206	0.517	0.51	0.499	0.523	0.506	0.241	0.726	0.488	0.568	0.358	0.695	0.586	0.475	0.681
A9207	0.227	0.249	0.154	0.276	0.256	0.154	0.23	0.143	0.287	0.249	0.184	0.042	0.255	0.269
A9208	0.402	0.408	0.406	0.459	0.4	0.197	0.499	0.48	0.471	0.444	0.238	0.443	0.289	0.425
A9209	1	1	0.445	1	1	0.357	1	0.536	1	0.257	1	1	0.765	1
A9301	0.501	0.552	0.347	0.592	0.569	0.335	0.586	0.435	0.583	0.42	0.429	0.347	0.596	0.521
A9302	0.297	0.356	0.089	0.322	0.362	0.158	0.266	0.16	0.191	0	0.265	0.268	0.33	0.14
A9303	0.325	0.34	0.418	0.445	0.325	0.23	0.728	0.463	0.45	0.577	0.454	0.455	0.251	0.641
A9304	0.8	0.857	0.395	0.769	0.835	0.641	0.479	0.381	0.545	0.549	0.383	0.342	0.539	0.445
A9305	0.351	0.345	0.473	0.368	0.344	0.216	0.351	0.413	0.416	0.457	0.228	0.276	0.338	0.328
A9306	0.698	0.8	0.728	0.903	0.838	0.598	1	1	0.921	1	0.508	0.525	1	0.819
A9307	0.151	0.177	0.204	0.222	0.186	0.142	0.246	0.245	0.235	0.361	0.129	0.055	0.186	0.249
A9308	0.428	0.464	0.484	0.58	0.473	0.297	0.738	0.607	0.669	0.717	0.378	0.319	0.408	0.735
A9309	0.463	0.464	0.506	0.464	0.477	0.605	0.427	0.515	0.535	0.569	0.313	0.288	0.513	0.4
A9310	0.452	0.53	0.318	0.475	0.543	0.304	0.304	0.28	0.349	0.384	0.23	0.174	0.635	0.265
A9401	0.133	0.108	0.494	0.169	0.097	0.1	0.316	0.406	0.297	0.453	0.15	0.272	0.091	0.301
A9402	0.495	0.494	0.505	0.538	0.496	0.25	0.545	0.549	0.627	0.503	0.341	0.408	0.422	0.535
A9403	0.357	0.373	0.295	0.44	0.386	1	0.484	0.304	0.591	0.827	0.398	0	0.372	0.642
A9404	0.377	0.398	0.191	0.38	0.382	0.127	0.425	0.144	0.275	0.134	0.481	0.28	0.257	0.414
M9001	0.167	0.17	0.283	0.171	0.169	0.135	0.288	0.345	0.174	0.192	0.201	0.284	0.255	0.183
M9002	0.216	0.231	0.16	0.223	0.232	0.113	0.333	0.24	0.188	0.069	0.29	0.346	0.238	0.203
M9003	0.189	0.197	0.273	0.213	0.2	0.137	0.247	0.277	0.229	0.303	0.173	0.133	0.215	0.234
M9004	0.273	0.264	0.346	0.253	0.264	0.159	0.334	0.372	0.267	0.182	0.306	0.382	0.277	0.279
M9005	0.474	0.478	0.465	0.524	0.469	0.278	0.687	0.37	0.551	0.52	0.708	0.369	0.423	0.748
M9006	0.57	0.549	0.692	0.518	0.545	0.286	0.552	0.562	0.533	0.42	0.504	0.505	0.595	0.481
M9007	0.487	0.468	0.44	0.426	0.445	0.246	0.439	0.207	0.371	0.253	0.602	0.446	0.464	0.412
M9008	0.273	0.226	0.643	0.238	0.209	0.129	0.494	0.427	0.317	0.364	0.502	0.409	0.212	0.492

DMU	M9203	M9204	M9301	M9302	M9303	M9304	M9305	M9306	M9307	M9308	M9401	M9402	M9403	M9404
M9101	0.258	0.259	1	0.34	0.271	0.579	0.467	0.886	0.545	1	0.213	0.231	0.399	0.439
M9102	0.356	0.341	0.665	0.4	0.337	0.256	0.594	0.623	0.508	0.668	0.462	0.417	0.316	0.609
M9201	1	1	1	1	1	1	1	1	1	1	1	1	1	1
M9202	0.509	0.499	0.388	0.412	0.5	0.193	0.229	0.252	0.349	0.283	0.205	0.137	0.433	0.236
M9203	0.622	0.617	0.348	0.599	0.596	0.209	0.449	0.356	0.505	0.352	0.347	0.326	0.355	0.47
M9204	0.257	0.263	0.143	0.251	0.261	0.107	0.138	0.151	0.212	0.121	0.08	0.159	0.195	0.107
M9301	0.579	0.556	1	0.555	0.567	0.775	0.593	0.857	0.724	0.68	0.473	0.691	0.828	0.472
M9302	0.885	0.924	0.614	0.96	0.933	0.358	0.751	0.815	0.938	0.775	0.38	0.336	0.633	0.757
M9303	0.349	0.426	0.102	0.371	0.43	0.167	0.235	0.12	0.208	0.053	0.253	0.174	0.353	0.178
M9304	0.532	0.602	0.306	0.591	0.609	0.749	0.464	0.338	0.461	0.467	0.366	0.342	0.58	0.37
M9305	0.842	0.855	0.775	1	0.831	0.305	1	1	1	1	0.384	0.53	0.442	1
M9306	0.37	0.375	0.428	0.397	0.382	0.186	0.387	0.504	0.452	0.403	0.205	0.265	0.339	0.349
M9307	1	1	0.984	1	1	0.387	0.756	1	1	0.983	0.393	0.422	0.721	0.724
M9308	0.466	0.461	1	0.552	0.479	1	1	1	1	1	1	1	0.807	1
M9401	0.541	0.549	0.373	0.579	0.562	0.389	0.718	0.417	0.71	0.324	1	0.65	0.626	0.823
M9402	0.256	0.247	0.353	0.271	0.244	0.169	0.442	0.391	0.332	0.281	0.407	0.524	0.235	0.417
M9403	0.666	0.684	0.403	0.592	0.7	0.31	0.401	0.356	0.51	0.181	0.416	0.344	0.791	0.349
M9404	0.388	0.406	0.252	0.458	0.391	0.209	0.476	0.248	0.405	0.382	0.452	0.264	0.248	0.583
C9001	0.086	0.085	0.094	0.087	0.087	0.058	0.174	0.098	0.105	0.048	0.235	0.146	0.122	0.163
C9002	0.142	0.142	0.194	0.154	0.14	0.103	0.373	0.224	0.165	0.15	0.396	0.283	0.168	0.303
C9003	0.227	0.254	0.181	0.281	0.253	0.132	0.368	0.194	0.24	0.243	0.316	0.155	0.205	0.355
C9201	0.35	0.398	0.199	0.422	0.401	0.546	0.662	0.207	0.345	0.311	0.87	0.307	0.4	0.614
C9202	1	0.947	1	0.784	0.946	1	0.926	1	0.698	0.475	0.932	1	0.964	0.731
C9301	0.272	0.309	0.074	0.295	0.322	0.147	0.28	0.133	0.238	0.039	0.324	0.159	0.338	0.246
C9302	0.625	0.635	0.454	0.636	0.656	0.298	0.774	0.777	0.737	0.196	0.586	0.987	0.683	0.558
C9303	0.63	0.748	0.169	0.72	0.744	0.26	0.526	0.108	0.444	0.163	0.688	0.217	0.526	0.549
C9304	0.137	0.122	0.259	0.179	0.109	0.114	0.353	0.181	0.242	0.334	0.295	0.217	0.107	0.356
C9305	0.456	0.447	0.402	0.501	0.442	0.218	0.502	0.334	0.585	0.46	0.408	0.279	0.344	0.605

DMU	C9001	C9002	C9003	C9201	C9202	C9301	C9302	C9303	C9304	C9305
score	0.351	0.533	0.441	1	1	0.504	1	1	1	0.772
A9001	0.931	0.872	1	0.786	0.567	0.637	0.513	0.95	0.906	0.363
A9002	0.397	0.302	0.297	0.427	0.232	0.365	0.244	0.257	0.225	0.358
A9003	0.807	0.745	0.791	0.848	0.793	0.703	0.82	0.719	0.776	0.959
A9004	0.864	1	0.398	0.618	0.518	0.538	0.725	0.584	0.338	0.711
A9101	1	1	0.93	1	1	0.724	1	1	1	0.764
A9102	0.141	0.113	0.236	0.22	0.189	0.217	0.218	0.201	0.236	0.331
A9103	0.235	0.173	0.399	0.434	0.412	0.323	0.417	0.27	0.313	0.557
A9201	1	1	1	1	1	1	1	1	0.903	1
A9202	0.26	0.232	0.257	0.339	0.342	0.278	0.411	0.276	0.208	0.441
A9203	0.347	0.372	0.412	0.466	0.511	0.445	0.523	0.384	0.406	0.403
A9204	0.47	0.38	0.967	0.995	1	0.653	0.738	0.712	1	0.521
A9205	0.149	0.154	0.497	0.332	0.381	0.283	0.389	0.335	0.558	0.26
A9206	0.981	0.815	0.837	0.808	0.617	0.767	0.66	0.82	0.712	0.656
A9207	0.099	0.058	0.265	0.231	0.188	0.215	0.16	0.196	0.252	0.134
A9208	0.113	0.14	0.347	0.265	0.507	0.19	0.578	0.251	0.399	0.47
A9209	0.729	0.493	1	1	1	1	1	1	0.668	1
A9301	0.299	0.313	0.567	0.521	0.546	0.568	0.568	0.537	0.548	0.271
A9302	0.132	0.18	0.113	0.228	0.298	0.281	0.322	0.197	0.035	0.071
A9303	0.235	0.255	0.631	0.499	0.667	0.26	0.656	0.406	0.727	0.418
A9304	0.165	0.147	0.321	0.373	0.549	0.299	0.478	0.254	0.262	0.419
A9305	0.113	0.112	0.252	0.245	0.312	0.177	0.376	0.189	0.269	0.446
A9306	0.442	0.716	1	0.771	0.923	0.855	1	0.819	1	0.369
A9307	0.058	0.053	0.242	0.192	0.208	0.138	0.185	0.113	0.217	0.092
A9308	0.186	0.186	0.738	0.518	0.682	0.385	0.653	0.439	0.778	0.388
A9309	0.333	0.316	0.428	0.402	0.487	0.455	0.434	0.384	0.356	0.413
A9310	0.215	0.258	0.286	0.288	0.251	0.302	0.291	0.278	0.268	0.214
A9401	0.172	0.179	0.338	0.204	0.261	0.092	0.344	0.249	0.467	0.644
A9402	0.168	0.156	0.443	0.385	0.516	0.317	0.572	0.32	0.442	0.629
A9403	0.149	0.052	0.542	0.472	0.747	0.383	0.296	0.311	0.49	0.326
A9404	0.632	0.51	0.603	0.54	0.348	0.476	0.315	0.591	0.425	0.298
M9001	0.21	0.349	0.19	0.231	0.266	0.21	0.346	0.184	0.176	0.171
M9002	0.378	0.54	0.251	0.313	0.333	0.363	0.386	0.359	0.189	0.142
M9003	0.084	0.079	0.19	0.202	0.211	0.141	0.22	0.104	0.159	0.166
M9004	0.091	0.089	0.149	0.246	0.319	0.16	0.37	0.097	0.096	0.334
M9005	0.343	0.236	0.613	0.701	0.545	0.45	0.492	0.397	0.532	0.572
M9006	0.424	0.406	0.436	0.539	0.463	0.467	0.572	0.403	0.368	0.666
M9007	0.414	0.343	0.341	0.535	0.318	0.385	0.399	0.388	0.322	0.589
M9008	0.503	0.4	0.47	0.553	0.37	0.369	0.405	0.319	0.374	0.809

DMU	C9001	C9002	C9003	C9201	C9202	C9301	C9302	C9303	C9304	C9305
M9101	0.19	0.241	0.472	0.346	0.461	0.285	0.465	0.29	0.495	0.431
M9102	0.13	0.109	0.35	0.422	0.525	0.218	0.531	0.164	0.299	0.662
M9201	0.123	0.112	0.336	0.582	1	0.315	1	0.18	0.232	0.9
M9202	0.108	0.082	0.174	0.218	0.19	0.191	0.207	0.133	0.124	0.314
M9203	0.105	0.084	0.273	0.312	0.436	0.217	0.43	0.173	0.214	0.558
M9204	0.036	0.043	0.075	0.074	0.149	0.084	0.195	0.089	0.085	0.174
M9301	0.517	0.571	0.457	0.526	0.628	0.596	0.749	0.559	0.475	0.857
M9302	0.153	0.146	0.652	0.503	0.734	0.449	0.729	0.378	0.548	0.469
M9303	0.159	0.169	0.166	0.252	0.223	0.282	0.231	0.206	0.089	0.098
M9304	0.257	0.294	0.378	0.4	0.555	0.424	0.491	0.414	0.364	0.268
M9305	0.199	0.218	1	0.559	1	0.394	1	0.512	1	1
M9306	0.131	0.147	0.335	0.27	0.367	0.252	0.422	0.236	0.313	0.341
M9307	0.305	0.311	0.754	0.546	0.686	0.507	0.78	0.532	0.712	0.859
M9308	0.55	0.44	0.839	1	1	0.879	1	0.758	1	1
M9401	0.164	0.108	0.362	0.631	0.676	0.422	0.605	0.24	0.224	0.533
M9402	0.087	0.082	0.197	0.288	0.433	0.156	0.46	0.115	0.164	0.486
M9403	0.348	0.324	0.304	0.443	0.351	0.519	0.419	0.352	0.175	0.362
M9404	0.066	0.046	0.23	0.293	0.465	0.145	0.359	0.116	0.198	0.372
C9001	0.351	0.269	0.173	0.254	0.14	0.267	0.14	0.18	0.11	0.104
C9002	0.466	0.533	0.361	0.445	0.314	0.38	0.321	0.321	0.293	0.178
C9003	0.268	0.242	0.441	0.392	0.305	0.313	0.261	0.298	0.358	0.163
C9201	1	1	0.93	1	0.67	1	0.423	1	0.699	0.211
C9202	1	1	0.647	1	1	0.998	1	0.612	0.247	1
C9301	0.319	0.289	0.278	0.372	0.262	0.504	0.232	0.317	0.081	0.047
C9302	1	1	0.676	0.684	0.816	1	1	1	0.481	0.441
C9303	0.773	0.609	0.853	0.789	0.426	0.872	0.326	1	0.541	0.24
C9304	0.598	0.586	0.632	0.377	0.252	0.211	0.282	0.746	1	0.325
C9305	0.171	0.11	0.453	0.41	0.437	0.286	0.419	0.306	0.459	0.772

APPENDIX D

CCR MODEL RESULTS (MULTIPLIERS AND EFFICIENCY SCORES)

DMU	eff.	Input virtual multipliers					Output virtual multipliers					
	score	Fcas	Ftank	FIFV	Fcom	Frat	Ecas	Etank	Eapc	Ecom	Eair	MA
A9001	1.000	0.0139	0.0085	0.0314	0.1298	0.0000	0.0000	0.0174	0.0000	0.0090	0.0897	0.0000
A9002	0.463	0.0042	0.0648	0.0000	0.0846	0.0000	0.0000	0.0253	0.0000	0.0412	0.0208	0.0000
A9003	0.994	0.0115	0.0077	0.0000	0.0623	0.2193	0.0000	0.0000	0.0000	0.0523	0.0058	0.0000
A9004	1.000	0.0000	0.0282	0.0150	0.0056	0.0000	0.0005	0.0000	0.0000	0.0428	0.0028	0.0000
A9101	1.000	0.0008	0.0146	0.0110	0.0497	0.0145	0.0010	0.0088	0.0000	0.0033	0.0297	0.0000
A9102	0.481	0.0124	0.0863	0.0050	0.0000	0.7630	0.0000	0.0000	0.0000	0.0800	0.1360	0.0000
A9103	0.557	0.0000	0.0400	0.0441	0.1259	1.1203	0.0000	0.0000	0.0000	0.1644	0.1141	0.0000
A9201	1.000	0.0011	0.0198	0.0149	0.0672	0.0196	0.0014	0.0118	0.0000	0.0045	0.0401	0.0000
A9202	0.512	0.0000	0.0288	0.0088	0.0980	0.4120	0.0000	0.0000	0.0000	0.1111	0.0000	0.0000
A9203	0.685	0.0035	0.0541	0.0000	0.0139	0.0000	0.0018	0.0000	0.0000	0.0000	0.0407	0.3873
A9204	1.000	0.0012	0.0203	0.0153	0.0690	0.0202	0.0014	0.0122	0.0000	0.0046	0.0412	0.0000
A9205	0.604	0.0255	0.0000	0.0080	0.0724	0.1899	0.0000	0.0000	0.0000	0.0000	0.1000	0.0000
A9206	1.000	0.0306	0.0063	0.0150	0.1280	0.0675	0.0000	0.0218	0.0000	0.0288	0.0116	0.0000
A9207	0.314	0.0005	0.0472	0.0299	0.1186	0.0260	0.0000	0.0000	0.0089	0.0000	0.0952	0.0000
A9208	0.593	0.0000	0.0296	0.0232	0.1410	0.0351	0.0059	0.0000	0.0000	0.0033	0.0605	0.0000
A9209	1.000	0.0013	0.0273	0.0269	0.0934	0.0000	0.0021	0.0193	0.0000	0.0043	0.0510	0.0000
A9301	0.706	0.0044	0.0492	0.0023	0.0396	0.0000	0.0000	0.0000	0.0091	0.0014	0.0000	0.0000
A9302	0.499	0.0048	0.0615	0.0000	0.1082	0.0000	0.0064	0.0000	0.0017	0.0182	0.0000	0.0000
A9303	0.873	0.0000	0.0263	0.0215	0.0448	0.0000	0.0012	0.0000	0.0000	0.0259	0.0597	0.0000
A9304	1.000	0.0000	0.0116	0.0698	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0465	0.7209
A9305	0.517	0.0041	0.0485	0.0040	0.0050	0.1423	0.0000	0.0000	0.0000	0.0194	0.0917	0.0000
A9306	1.000	0.0010	0.0168	0.0127	0.0571	0.0167	0.0012	0.0101	0.0000	0.0038	0.0341	0.0000
A9307	0.412	0.0000	0.0635	0.0476	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1429	0.4921
A9308	0.831	0.0000	0.0278	0.0099	0.0550	0.0126	0.0000	0.0000	0.0000	0.0000	0.0833	0.0000
A9309	0.532	0.0000	0.0098	0.0364	0.0972	0.0000	0.0025	0.0000	0.0024	0.0000	0.0283	0.4394
A9310	0.810	0.0064	0.0949	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0557	0.0924	0.0000
A9401	0.686	0.0223	0.0054	0.0203	0.0812	0.8478	0.0000	0.0000	0.0000	0.0721	0.0946	0.0000
A9402	0.645	0.0024	0.0381	0.0256	0.0574	0.6664	0.0000	0.0000	0.0042	0.0676	0.0715	0.0000
A9403	1.000	0.0000	0.0297	0.0460	0.1674	0.0604	0.0045	0.0101	0.0000	0.0000	0.0798	0.6695
A9404	0.879	0.0273	0.0000	0.0498	0.0881	0.0000	0.0000	0.0135	0.0000	0.0565	0.0000	0.0000
M9001	0.628	0.0071	0.0796	0.0000	0.0000	0.0000	0.0052	0.0000	0.0000	0.0000	0.0011	0.0000
M9002	0.555	0.0779	0.0627	0.0000	0.1825	0.0000	0.0069	0.0000	0.0000	0.0770	0.0000	0.0000
M9003	0.357	0.0041	0.0691	0.0556	0.0000	0.0000	0.0000	0.0294	0.0000	0.0000	0.1000	0.0000
M9004	0.386	0.0051	0.0506	0.0000	0.0726	0.0942	0.0060	0.0000	0.0000	0.0000	0.0207	0.0000
M9005	0.792	0.0002	0.0177	0.0253	0.0584	0.0000	0.0000	0.0212	0.0000	0.0000	0.0351	0.0000
M9006	0.772	0.0029	0.0579	0.0000	0.0000	0.0850	0.0000	0.0000	0.0000	0.0689	0.0059	0.0000
M9007	1.000	0.0013	0.0341	0.0000	0.0161	0.0600	0.0000	0.0008	0.0001	0.0485	0.0000	0.0000

DMU	score	Fcas	Ftank	FIFV	Fcom	Frat	Ecas	Etank	Eapc	Ecom	Eair	MA
M9008	0.835	0.0167	0.0040	0.0152	0.0607	0.6336	0.0000	0.0000	0.0000	0.0539	0.0707	0.0000
M9101	1.000	0.0013	0.0257	0.0075	0.0084	0.0659	0.0000	0.0000	0.0000	0.0000	0.0610	0.1456
M9102	0.729	0.0017	0.0352	0.0214	0.0440	0.6895	0.0000	0.0000	0.0000	0.0840	0.0828	0.0000
M9201	1.000	0.0013	0.0222	0.0168	0.0756	0.0221	0.0016	0.0133	0.0000	0.0051	0.0451	0.0000
M9202	0.553	0.0000	0.0355	0.0550	0.0000	0.0367	0.0000	0.0000	0.0094	0.0147	0.0000	0.0000
M9203	0.622	0.0000	0.0458	0.0746	0.0158	0.0962	0.0000	0.0000	0.0141	0.0179	0.0000	0.0000
M9204	0.263	0.0000	0.0662	0.1075	0.0231	0.0000	0.0000	0.0000	0.0212	0.0084	0.0000	0.0000
M9301	1.000	0.0051	0.0445	0.0021	0.0000	0.2925	0.0012	0.0000	0.0000	0.0274	0.0595	0.0000
M9302	0.960	0.0000	0.0323	0.0486	0.0305	0.0000	0.0000	0.0000	0.0096	0.0000	0.0261	0.0000
M9303	0.430	0.0009	0.0489	0.0696	0.0144	0.0000	0.0000	0.0000	0.0152	0.0000	0.0000	0.0000
M9304	0.749	0.0024	0.0627	0.0000	0.0191	0.0000	0.0000	0.0000	0.0035	0.0000	0.0177	0.5794
M9305	1.000	0.0016	0.0284	0.0214	0.0966	0.0282	0.0020	0.0170	0.0000	0.0065	0.0577	0.0000
M9306	0.504	0.0085	0.0623	0.0120	0.0343	0.3716	0.0055	0.0000	0.0000	0.0000	0.1017	0.0000
M9307	1.000	0.0040	0.0480	0.0226	0.0589	0.2098	0.0000	0.0000	0.0092	0.0000	0.0742	0.0000
M9308	1.000	0.0016	0.0315	0.0092	0.0103	0.0807	0.0000	0.0000	0.0000	0.0000	0.0747	0.1782
M9401	1.000	0.0005	0.0411	0.0067	0.1230	0.0038	0.0000	0.0292	0.0000	0.0240	0.0000	0.0000
M9402	0.524	0.0000	0.0411	0.0142	0.1567	0.2904	0.0059	0.0000	0.0000	0.0649	0.0000	0.0000
M9403	0.791	0.0080	0.0762	0.0000	0.0000	0.0489	0.0000	0.0000	0.0108	0.0000	0.0000	0.0000
M9404	0.583	0.0000	0.0228	0.0197	0.0922	0.0519	0.0000	0.0201	0.0000	0.0000	0.0598	0.0000
C9001	0.351	0.1164	0.0471	0.0000	0.3509	0.1934	0.0000	0.0492	0.0000	0.1201	0.0000	0.0000
C9002	0.533	0.0584	0.0309	0.0000	0.1226	0.0000	0.0026	0.0069	0.0000	0.0501	0.0053	0.0000
C9003	0.441	0.0152	0.0081	0.0287	0.1233	0.0000	0.0000	0.0189	0.0000	0.0000	0.0877	0.0000
C9201	1.000	0.0060	0.0247	0.0066	0.0780	0.0000	0.0000	0.0215	0.0000	0.0087	0.0191	0.0000
C9202	1.000	0.0000	0.0194	0.0211	0.0825	0.0194	0.0022	0.0087	0.0007	0.0000	0.0373	0.1807
C9301	0.504	0.0222	0.0391	0.0000	0.0963	0.0000	0.0000	0.0187	0.0055	0.0000	0.0000	0.0000
C9302	1.000	0.0024	0.0302	0.0200	0.0858	0.1008	0.0041	0.0002	0.0018	0.0190	0.0457	0.0000
C9303	1.000	0.0304	0.0143	0.0156	0.1412	0.0000	0.0000	0.0000	0.0054	0.0307	0.0438	0.0000
C9304	1.000	0.0189	0.0084	0.0224	0.1531	0.0249	0.0000	0.0000	0.0000	0.0204	0.1361	0.0000
C9305	0.772	0.0000	0.0194	0.0213	0.0610	0.5428	0.0000	0.0000	0.0000	0.0796	0.0553	0.0000

APPENDIX E
DMU REFERENCE SETS

DMU	Reference set							
A9002	A9101	A9201	M9201	M9308	C9202			
A9003	A9101	A9201	A9209	M9308	C9202			
A9102	A9201	M9101	M9301	M9307	M9308			
A9103	A9201	A9209	M9305	M9308	C9202			
A9202	C9202	A9101	A9209	M9308				
A9203	C9202	A9201	M9201	A9306	M9308			
A9205	A9201	A9306	M9305	M9308				
A9207	A9204	A9209	A9306	M9201	M9305	M9308		
A9208	A9101	A9306	M9201	M9305	M9308	C9302		
A9301	A9201	A9209	A9306	M9201	M9308			
A9302	A9101	M9201	M9308	C9202	C9302			
A9303	A9101	A9201	A9204	M9201	M9305			
A9305	A9201	A9306	M9101	M9201	M9307	M9308		
A9307	A9201	A9306	M9201	M9305				
A9308	A9204	A9306	M9305	M9308				
A9309	A9101	A9201	A9209	A9403	M9305	C9202		
A9310	A9201	A9306	M9201					
A9401	A9201	A9209	M9305	M9307	M9308	C9202		
A9402	A9201	A9209	M9201	M9305	M9307	M9308	C9202	
A9404	A9001	A9101	A9201	C9202				
M9001	A9306	M9201	C9202					
M9002	A9004	A9101	C9202	C9302				
M9003	A9201	A9306	M9201	M9307				
M9004	A9306	M9201	M9308	C9202	C9302			
M9005	A9201	A9204	A9209	M9201	M9305			
M9006	A9201	M9007	M9201	C9202				
M9008	A9201	A9209	M9305	M9307	M9308	C9202		
M9102	A9201	M9201	M9305	M9307	M9308	C9202		
M9202	A9201	M9201	M9307	C9202				
M9203	A9201	A9209	M9201	M9307	C9202			
M9204	A9201	A9209	M9201	M9307				
M9302	A9209	M9201	M9305	M9307				

M9303	A9201	A9209	M9201	M9307					
M9304	A9201	A9403	M9201	M9308	C9202				
M9306	A9306	M9201	M9305	M9307	M9308	C9202			
M9402	A9101	A9209	M9201	M9308	C9202				
M9403	A9201	A9306	M9201						
M9404	A9204	A9209	M9201	M9305	M9308				
C9001	A9101	A9201	C9201	C9202	C9302				
C9002	A9004	A9101	A9201	C9201	C9202	C9302			
C9003	A9001	A9201	A9209	A9306	M9305				
C9301	A9201	A9209	C9201	C9302					
C9305	A9201	A9209	M9305	M9308	C9202				

APPENDIX F
CROSS EFFICIENCY SCORES

DMU	Efficiency Score	# times Referenced	Cross Efficiency Average
A9201	1	31	0.9639
C9202	1	24	0.8623
M9308	1	23	0.8388
A9101	1	12	0.7904
M9201	1	27	0.7904
A9306	1	15	0.7740
A9003	0.9938	0	0.7614
A9209	1	19	0.7605
M9305	1	12	0.7498
A9204	1	5	0.7249
M9307	1	13	0.7083
M9301	1	1	0.6493
C9302	1	6	0.6194
A9206	1	0	0.6008
M9302	0.9604	0	0.5613
A9004	1	2	0.5534
M9006	0.7719	0	0.5160
A9001	1	2	0.5075
C9201	1	3	0.5042
A9308	0.8310	0	0.5035
M9005	0.7925	0	0.5027
A9303	0.8728	0	0.4943
M9101	1	2	0.4645
M9401	1	0	0.4643
A9304	1	0	0.4552
A9203	0.6850	0	0.4485
M9008	0.8347	0	0.4465
M9102	0.7287	0	0.4440
A9301	0.7059	0	0.4424
C9303	1	0	0.4424

A9402	0.6450	0	0.4353
M9007	1	1	0.4337
M9304	0.7489	0	0.4291
A9309	0.5323	0	0.4233
C9305	0.7720	0	0.4051
A9403	1	2	0.4040
M9403	0.7910	0	0.3854
A9208	0.5933	0	0.3785
A9103	0.5566	0	0.3561
M9203	0.6221	0	0.3540
A9404	0.8789	0	0.3458
A9205	0.6035	0	0.3299
A9310	0.8103	0	0.3289
C9304	1	0	0.3282
A9305	0.5168	0	0.3213
M9306	0.5044	0	0.3164
A9401	0.6857	0	0.3141
M9404	0.5831	0	0.3121
M9402	0.5241	0	0.3046
A9202	0.5123	0	0.2953
A9002	0.4628	0	0.2773
C9002	0.5327	0	0.2680
C9003	0.4412	0	0.2601
M9004	0.3862	0	0.2509
M9202	0.5535	0	0.2496
M9002	0.5554	0	0.2479
A9102	0.4810	0	0.2471
M9001	0.6284	0	0.2459
M9003	0.3569	0	0.1973
M9303	0.4298	0	0.1942
C9301	0.5037	0	0.1937
A9302	0.4988	0	0.1906
A9307	0.4118	0	0.1802
A9207	0.3138	0	0.1798
M9204	0.263	0	0.1343
C9001	0.3505	0	0.1329

APPENDIX G
FER MODEL RESULTS

DMU	FER		DMU	FER		DMU	FER
A9201	4.0046		M9202	0.8066		C9304	0.5110
A9003	2.5630		A9310	0.7797		A9203	0.4398
C9202	2.3382		M9004	0.7461		A9307	0.4342
M9301	2.2733		A9404	0.6796		M9008	0.4069
A9101	2.2485		M9304	0.6665		A9205	0.3834
A9304	2.2257		A9208	0.6518		A9402	0.3813
M9007	1.7647		M9403	0.6449		A9309	0.3596
A9306	1.7182		C9301	0.6445		A9302	0.3314
A9209	1.4862		C9303	0.6414		A9207	0.3274
M9308	1.4845		A9102	0.6357		M9002	0.3162
A9403	1.4043		M9003	0.6340		C9305	0.2939
A9004	1.3820		M9305	0.6327		C9001	0.2761
A9206	1.2049		A9308	0.6210		M9201	0.2660
A9204	1.1964		M9401	0.6012		M9005	0.2655
M9307	1.1281		A9202	0.5607		A9002	0.2431
C9201	1.1172		A9301	0.5476		C9002	0.2287
C9302	0.9326		A9001	0.5443		M9306	0.2211
M9404	0.8938		M9402	0.5366		A9305	0.2099
M9006	0.8811		C9003	0.5300		M9303	0.1790
M9302	0.8307		M9102	0.5277		M9204	0.1742
M9101	0.8289		M9001	0.5258		A9401	0.1291
M9203	0.8150		A9103	0.5246		A9303	0.0810

APPENDIX H
RESULTS OF THE CONDENCED MODEL

DMU	Efficiency score	# Times Referenced	Cross Efficiency Average	FER
M9201	1.0000	41	0.9427	0.2660
M9307	1.0000	35	0.8747	1.1281
A9209	1.0000	29	0.7325	1.4862
A9201	1.0000	23	0.8807	4.0046
C9202	1.0000	23	0.8374	2.3382
M9302	1.0000	21	0.8187	0.8307
M9007	1.0000	8	0.7256	1.7647
A9003	1.0000	6	0.7150	2.5630
M9003	1.0000	4	0.6221	0.6340
M9005	1.0000	4	0.5586	0.2655
A9304	1.0000	3	0.7598	2.2257
A9403	1.0000	3	0.5423	1.4043
A9004	1.0000	2	0.7170	1.3820
A9101	1.0000	1	0.5860	2.2485
M9301	1.0000	0	0.6371	2.2733
C9303	0.9820	0	0.6720	0.6414
A9310	0.9659	0	0.7188	0.7797
M9404	0.9598	0	0.6717	0.8938
A9306	0.9532	0	0.7551	1.7182
A9206	0.9344	0	0.7075	1.2049
M9306	0.9301	0	0.5531	0.2211
M9308	0.9172	0	0.4947	1.4845
M9006	0.9045	0	0.6488	0.8811
A9103	0.8847	0	0.6190	0.5246
A9404	0.8822	0	0.6248	0.6796
M9203	0.8820	0	0.7041	0.8150
M9403	0.8760	0	0.6743	0.6449
M9008	0.8733	0	0.6104	0.4069
M9102	0.8727	0	0.5787	0.5277
M9304	0.8426	0	0.5473	0.6665
M9002	0.8413	0	0.5238	0.3162

A9204	0.8340	0	0.6428	1.1964
M9101	0.8234	0	0.5875	0.8289
M9303	0.8164	0	0.5257	0.1790
C9302	0.8090	0	0.6247	0.9326
C9201	0.8022	0	0.5160	1.1172
M9305	0.7948	0	0.6363	0.6327
A9402	0.7824	0	0.5630	0.3813
M9402	0.7794	0	0.6078	0.5366
M9001	0.7774	0	0.5288	0.5258
M9202	0.7615	0	0.5873	0.8066
A9102	0.7580	0	0.5662	0.6357
A9203	0.7531	0	0.4302	0.4398
A9001	0.7524	0	0.5461	0.5443
A9208	0.7505	0	0.6071	0.6518
M9004	0.7498	0	0.5595	0.7461
C9003	0.7432	0	0.5374	0.5300
C9304	0.7420	0	0.4935	0.5110
A9309	0.7380	0	0.5256	0.3596
A9308	0.7339	0	0.5974	0.6210
A9303	0.7312	0	0.4694	0.0810
A9301	0.7131	0	0.5333	0.5476
M9204	0.7044	0	0.4503	0.1742
M9401	0.6919	0	0.5349	0.6012
C9305	0.6895	0	0.5217	0.2939
A9202	0.6283	0	0.4902	0.5607
A9307	0.6252	0	0.5218	0.4342
A9302	0.6154	0	0.4781	0.3314
C9301	0.6142	0	0.4919	0.6445
A9401	0.6108	0	0.3759	0.1291
A9207	0.6103	0	0.4698	0.3274
A9205	0.6046	0	0.4701	0.3834
C9001	0.5733	0	0.3982	0.2761
A9305	0.5378	0	0.4051	0.2099
C9002	0.5346	0	0.3855	0.2287
A9002	0.5268	0	0.4182	0.2431

APPENDIX I

FER SCORES FROM CTC WIN

DMU	FER
A9001	.4419
A9002	.2632
A9003	1.3943
A9004	1.6073
A9101	7.0324
A9102	.5369
A9103	.6532
A9201	1.1395
A9202	.4229
A9203	.6527
A9204	1.3037
A9205	.4747
A9206	.8969
A9207	.3682
A9208	.6630
A9209	1.3314
A9301	.7815
A9302	.5948

DMU	FER
A9303	.6921
A9304	1.0135
A9305	.6473
A9306	3.5759
A9307	.6758
A9308	1.3374
A9309	.3364
A9310	.8405
A9401	.3173
M9001	.7081
M9002	1.1955
M9003	.5315
M9004	1.4000
M9005	.4409
M9006	.6736
M9007	.8340
M9008	.4989
M9101	.7845

DMU	FER
M9102	1.6242
M9201	2.9217
M9202	1.4025
M9203	.8562
M9204	1.0073
M9301	1.6056
M9302	.9888
M9303	1.4431
M9304	1.6896
M9305	1.5199
M9306	.4304
M9307	1.5680
M9308	2.5325
C9001	.3312
C9002	.4440
C9201	.8083
C9202	1.7940