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A METHOD FOR COMPUTING THE TERRAIN CORRECTION FOR THE NEAR ZONE BY THE USE OF TRIANGULAR PRISMS

by

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ABSTRACT

A method applying triangular prisms to approximate topography for zones near the gravity station is used to obtain the terrain correction for these near zones. In order to obtain a formula for the vertical component of gravitational attraction of a triangular prism, a triple integral of the form

$$\int_{u=x_1}^{x_1+a} \int_{v=y_1}^{c-u} \int_{v=o}^{z} \frac{w\,du\,dv\,dw}{\left[u^2+v^2+w^2\right]^{3/2}}$$

was evaluated. The formula contains 30 terms, after the limits of integration have been substituted. A computer program was written to find the sum of the effects of the prisms within a circular zone of 600-m radius.

The results obtained by using this method were shown to be accurate. The computing time per station proved to be long on the CDC-8090 digital computer. However, by

making modification in the formula and using a larger computer, considerable amount of computing time can be saved. It became apparent that rectangular zones should be used instead of circular zones for defining the boundaries of near and far topography.

ACKNOWLEDGMENTS

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INTRODUCTION

Gravity measurements taken in the field are reduced by a series of corrections to a datum plane before they are used to estimate subsurface geology. One of these corrections is the terrain correction, which removes the effects of irregularities in the topography upon the gravity measurements. The total terrain correction may be divided into two parts, corrections for the near zone irregularities (for this thesis, areas within 600 m of the station) and those for the far zones. Since irregularities in the near zones may contribute large values to the correction, they must be studied in detail. This thesis sets forth a method for the accurate estimation and removal of the effects due to the near-zone irregularities.

The classical method for computing terrain corrections is to approximate the terrain by simple geometric forms. Now that high-speed electronic computers are available, more complex geometric forms may be used to get a better

approximation of the terrain, and as a result more precise terrain corrections may be obtained. The following is a brief history of the development of the methods used to obtain terrain corrections.

In 1912 Hayford and Bowie proposed a method in which they used block cylinders to approximate the terrain. This method made use of a template to divide the topography around the station into circular zones which were in turn divided into compartments. Using the average elevation of the compartments, Hayford constructed a set of standard tables to find the gravitational effects of these compartments. The sum of the effects of the individual compartments is the terrain correction. An extension of this idea was developed by Sandberg (1958), who found that, for certain areas, an inclined plane is a better approximation for the topography in the zones near the station.

The next development came when Bott (1958) succeeded in developing a program for electronic digital computers to compute the terrain correction. Although the speed of the operation was greatly increased, templates were still required to correct for zones near the station. Bott used elevations on the corners of a square grid to approximate the terrain. The mass underlying each square grid was then assumed to be concentrated along a vertical line at the

center of the square. Heiskanen (1958) called this technique the mass-line method.

Work done by Altan (1961), who used the mass-line method, showed a limitation in computing the terrain correction for the near zones, especially for stations located in rough terrain.

In this thesis triangular prisms are used to approximate the topography for zones near the stations, and a program was developed for an electronic digital computer to correct for the near-zone topography. The triangular prism method eliminates the need for graphical methods in the near zones; when this method is used in conjunction with the mass-line method, a complete program may be written so that digital computer can be used to correct for the effect of both the far and near topography.

APPROXIMATION OF THE TERRAIN WITH TRIANGULAR PRISMS

When terrain corrections are made in gravity surveys, some geometric form, whose attraction can be computed, is used as an approximation to the terrain. Obviously, the geometric form which best fits the true terrain gives the most precise results. Examples of the geometric forms used are the block cylinder, as proposed by Hayford and Bowie (1912); the inclined slope, as proposed by Sandberg (1958); and the flat-topped rectangular prism, as proposed by Bott (1958). For distant topography, which contributes little to the terrain correction, the geometric forms mentioned above give good solutions to the problem.

In order to obtain a more precise approximation to the terrain in near-station zones, the idea of using the triangular prism is proposed. The topography is divided into a square grid, as shown in figure 1. The squares are subdivided into two triangles by using corners of the squares for the vertices of the triangles. The two triangles have a common side as shown.

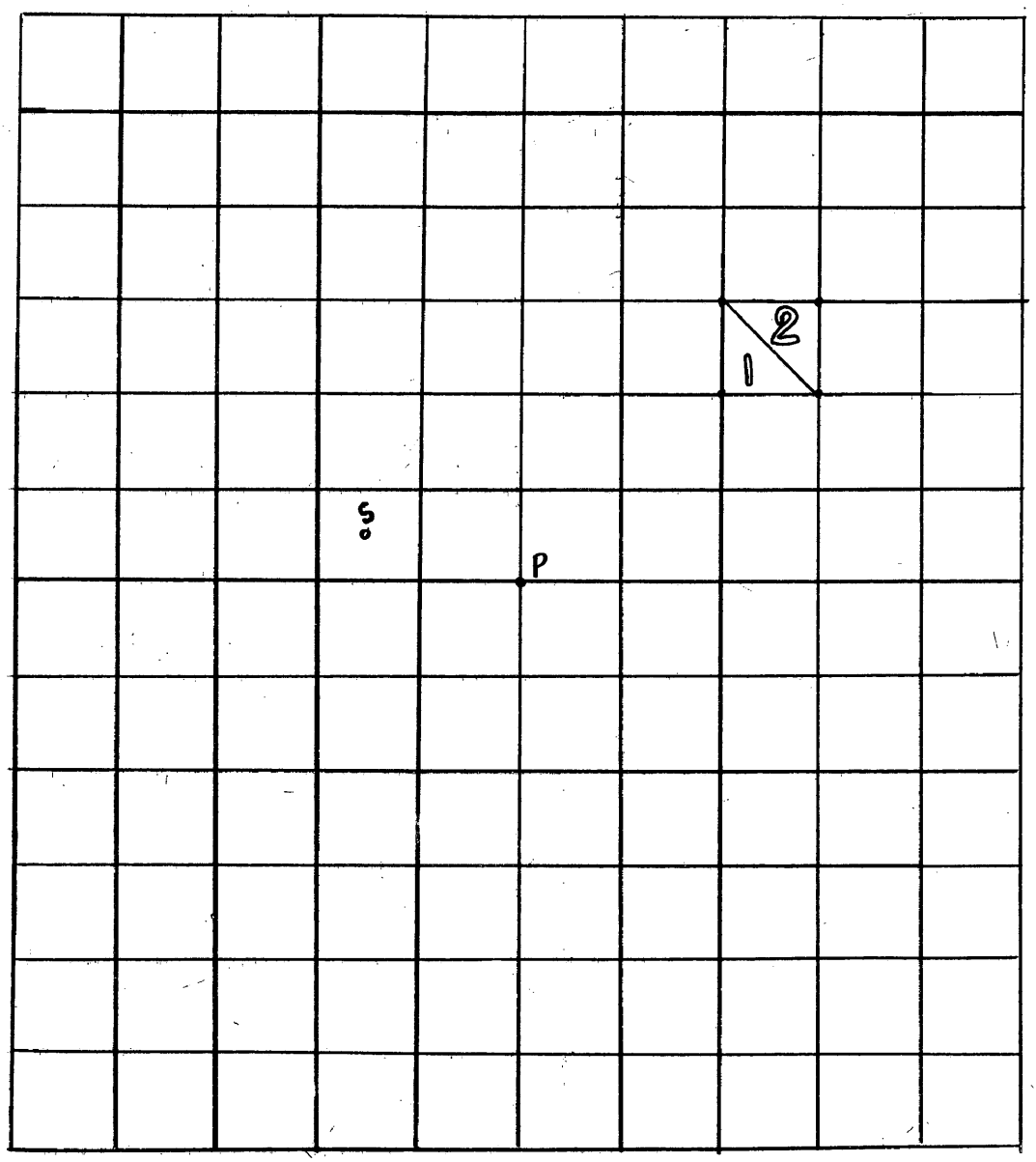

Figure 1 Topography Divided by Square Grids.

Figure 2 shows how those triangles are controlled by, and therefore approximate, the topography. The elevations and positions of the vertices define a plane surface, which is the upper plane of the triangular prism. In a uniform topography, the plane of the prism surface is identical to that of the topography surface. In a non-uniform topography, the upper surface of the prism is an approximation to the topographic surface. The degree of accuracy of the approximation is determined by the following two factors:

A. The size of the square grid: In this thesis, squares with 100-m side gave accurate determination of the terrain correction for the examples studied.

B. Placement of the triangles: This factor is controlled by changes in topography. The prisms must be placed so that their upper surfaces are an approximation to the slope of the topography.

The terrain was divided into a square grid, and the elevations and coordinates of the corners of each square were recorded. This procedure is a modification of the procedure used by Bott (1959). Bott also divided the topography into squares; however, he recorded only the average elevation within each square.

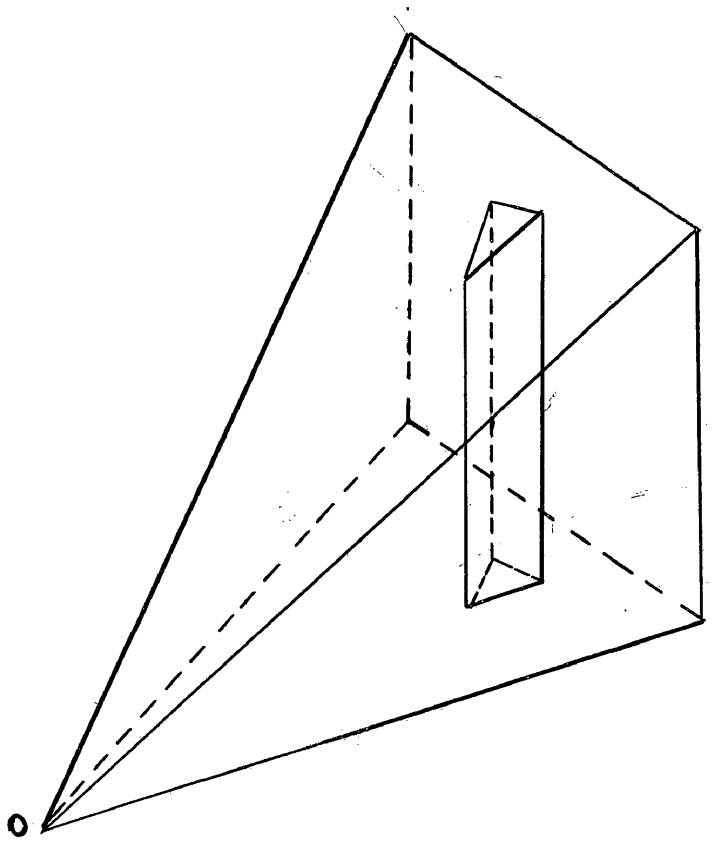

Figure 2 Approximation of a Hillside by Triangular Prisms.

DERIVATION OF THE FORMULAS

Because a square grid was used for digitizing the topography, two triangular prisms were defined by each square (figure 1). The relative positions with respect to the gravity station of triangles 1 and 2 are also shown in figure 1.

Attraction of Uniform Triangular Prism on a Particle Placed at the Origin

It is assumed that the normal cross-section of the prism is a right isosceles triangles, whose side is a (see figure 3, prism 1) and that the prism is bounded on the bottom by a plane passing through the points (x_1, y_1), (x_1+a, y_1), (x_1, y_1+a) and on the top by a plane passing through the points (x_1, y_1, z_2), (x_1+a, y_1, z_1), (x_1, y_1+a, z_3).

If R is the distance from any point (u, v, w) within the prism to the origin, then

$$R^2 = u^2 + v^2 + w^2$$

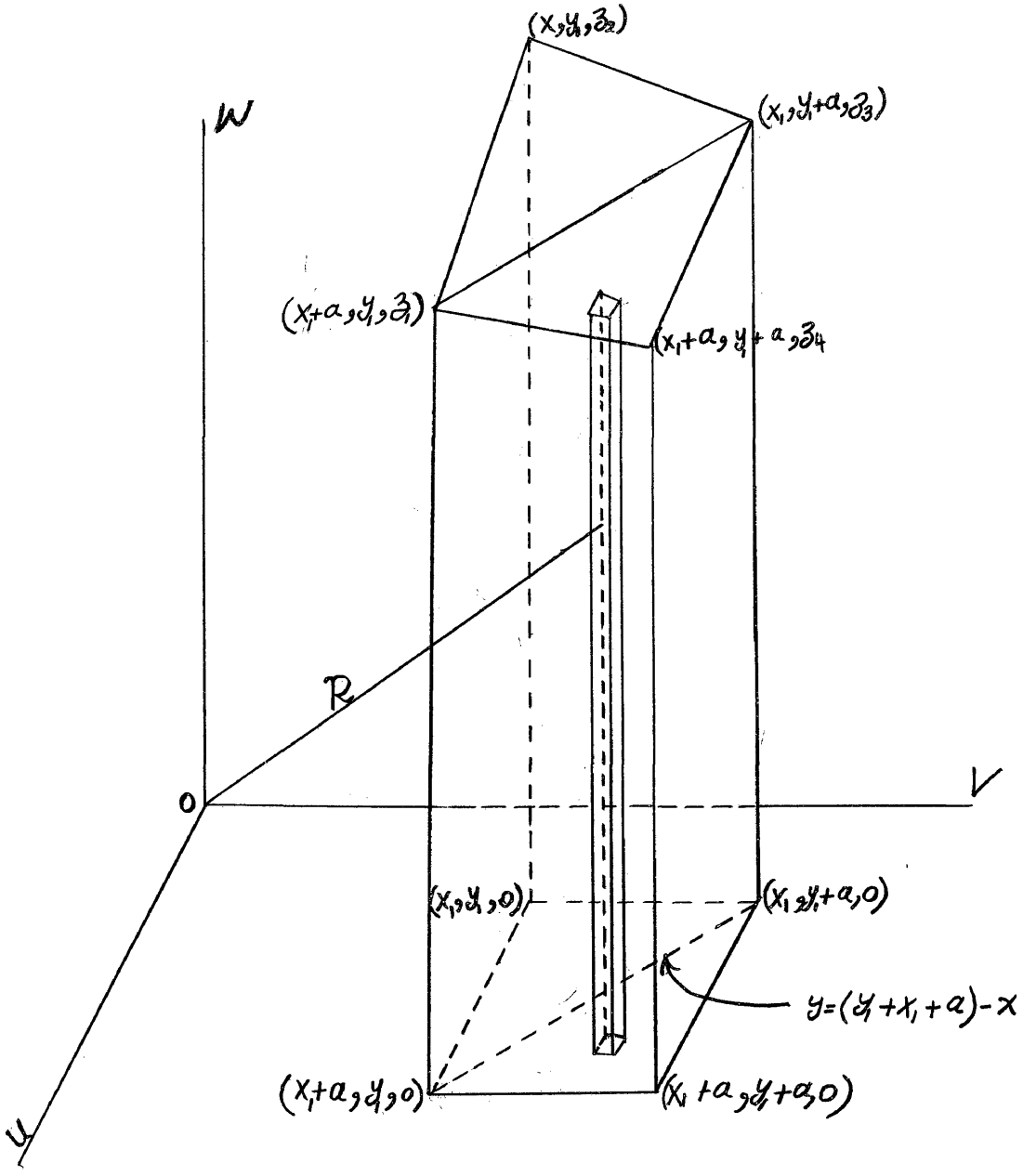

Figure 3 Position of the Prisms With Respect to the Coordinate System.

The vertical component of the attraction A_w', at the origin of the triangular prism, is given by:

$$A_w' = \frac{Aw}{k\rho} = \int_{u=x_1}^{x_1+a} \int_{v=y_1}^{c-u} \int_{w=o}^{w=z} \frac{w\,du\,dv\,dw}{\left[u^2+v^2+w^2\right]^{3/2}} \qquad (1)$$

In equation (1)

k is the gravitational constant and

ρ is the mass per unit volume of the prism.

The Limits of Integration

The attraction of the prism at the origin is found by determining the sum of the attraction due to infinitesimal rectangular parallelepipeds. The limits of integration on u, v, and w are:

1. Integrating with respect to w and holding u and v constant from

$$w = o$$

to $$w = z = \frac{1}{a}\left[(u-x_1)(z_1-z_2)+(z_3-z_2)(v-y_1) + az_2\right]. \qquad (2)$$

Equation (2) is the equation for the upper plane of the triangular prism.

2. Integrating with respect to v and holding u constant from

$v = y_1$

to $v = (y_1+x_1+a)-u = c-u$ (3)

Equation (3) is the equation of the line passing through the points (x_1+a, y_1), (x_1, y_1+a).

3. Integrating with respect to u from.

$u = x_1$

to $u = x_1+a$.

Evaluation of the Triple Integral Given by Equation (1)

$$A'_w = \frac{Aw}{\rho k} = \int\int\int_{w=o}^{w=z} \frac{w\,du\,dv\,dw}{\left[u^2+v^2+w^2\right]^{3/2}} \qquad (4)$$

Integration is performed for the variables w, v, and u, respectively. To avoid confusion, the limits of integration will be shown only for that variable with respect to which integration is to be performed. For example, in equation (4) only the limits for the variable w are shown, indicating that integration will be performed only with respect to the variable w. Reference will be made to Tables of Integrals whenever they are used.

Integrating equation (4) with respect to z, we get

$$A'_w = -\int\int \frac{du\,dv}{\left[u^2+v^2+w^2\right]^{\frac{1}{2}}} \Bigg]_{w=o}^{w=z}$$

Dwight 201.03

Substituting the limits for w, we get

$$A'_w = \iint \frac{dudv}{\left[u^2+v^2\right]^{\frac{1}{2}}} - \iint \frac{dudv}{\left[u^2+v^2+z^2\right]^{\frac{1}{2}}} \qquad (5)$$

The first term in the right-hand member of (5) is designated α, and the second term γ, so that

$$A'_w = \alpha + \gamma \ .$$

Using Dwight (1955) equation 201.01 we see that

$$\alpha = \int_{v=y_1}^{c-u}\int \frac{dudv}{\left[u^2+v^2\right]^{\frac{1}{2}}} = \int_u \log\left[v+\left[v^2+u^2\right]^{\frac{1}{2}}\right]du \Bigg]_{v=y_1}^{c-u} .$$

Substituting the limits for v, we get

$$\alpha = \int_u \log\left[(c-u) + \left[(c-u)^2+u^2\right]^{\frac{1}{2}}\right]du - \int_u \log\left[y_1+\left[y_1^2+u^2\right]^{\frac{1}{2}}\right]du \ . \qquad (6)$$

The first term in the right-hand member of equation (6) will be called β and the second term ζ, so that

$$\alpha = \beta + \zeta$$

We have

$$\zeta = -\int_{u=x_1}^{x_1+a} \log\left[y_1+\sqrt{y_1^2+u^2}\right]du,$$

which upon integration becomes

$$\zeta = -u\log\left[y_1+\left[y_1^2+u^2\right]^{\frac{1}{2}}\right]+ y_1\log\left[u+\left[y_1^2+u^2\right]^{\frac{1}{2}}\right]- u\Bigg]_{u=x_1}^{x_1+a}.$$

Now consider the term β,

$$\beta = \int_{u=x_1}^{x_1+a} \log\left[(c-u)+\left[(c-u)^2+u^2\right]^{\frac{1}{2}}\right]du .$$

Factoring out log(u), we get

$$\beta = \int_{u=x_1}^{x_1+a} \log(u)du + \int_{u=x_1}^{x_1+a} \log\left[\left|\frac{c}{u} - 1\right| + \left[\left|\frac{c}{u} - 1\right|^2 + 1\right]^{\frac{1}{2}}\right]du.$$

Now let

$$Y = \left|\frac{c}{u} -1\right| + \left[\left|\frac{c}{u} -1\right|^2 + 1\right]^{\frac{1}{2}}$$

$$Y - \left|\frac{c}{u} -1\right| = \left[\left|\frac{c}{u} -1\right|^2 + 1\right]^{\frac{1}{2}}.$$

Then

$$u = \frac{2Yc}{Y^2+2Y-1} \text{ , and } du = \frac{-2c(Y^2+1)}{(Y^2+2Y-1)^2} dY, \text{ and}$$

$$\beta = \int_{Y=Y_1}^{Y_2} \frac{-2c(Y^2+1)}{(Y^2+2Y-1)^2} \log Y dy + \left[u\log u - u \right]_{u=x_1}^{x_1+a} . \qquad (7)$$

In equation (7), let

$n = \log Y$ and

$$dm = \frac{-2c(Y^2+1)}{(Y^2+2Y-1)^2} dY,$$

Using the formula for integration by parts,

$\int ndm = nm - \int mdn$, we obtain

$$\int_{Y=Y_1} \frac{-2c(Y^2+1)}{(Y^2+2Y-1)^2} \log Y dY = \left[u\log Y \right]_{u=x_1}^{x_1+a} - \int \frac{2c}{(Y^2+2Y-1)} dY$$

and hence

$$\beta = [u\log u - u] + \left[u\log Y - \frac{c}{\sqrt{2}} \log \left[\frac{Y-\sqrt{2}+1}{Y+\sqrt{2}+1} \right] \right]_{Y=Y_1}^{Y_2}$$, Dwight, 1955, 160.01

where

$$Y_1 = \left[\left(\frac{c}{x_1} - 1 \right)^2 + 1 \right]^{\frac{1}{2}} + \left(\frac{c}{x_1} - 1 \right)$$

$$Y_2 = \left[\left(\frac{c}{x_1+a} - 1\right)^2 + 1\right]^{\frac{1}{2}} + \left(\frac{c}{x_1+a} - 1\right)$$

The total result for the term α is given by

$$\alpha = \int_{u=x_1}^{x_1+a}\int_{v=y_1}^{c-u} \frac{dudv}{u^2+v^2\ ^{\frac{1}{2}}} = \Bigg\{ -\left[u\log\left[y_1+\left[y_1^2+u^2\right]^{\frac{1}{2}}\right]+\right.$$

$$\left. y_1\log\left[u+\left[y_1^2+u^2\right]^{\frac{1}{2}}\right] - u\log|u|\right]_{u=x_1}^{x_1+a} + \left[u\log Y - \frac{c}{\sqrt{2}}\cdot\right.$$

$$\left.\log\left[\frac{Y+1-\sqrt{2}}{Y+1+\sqrt{2}}\right]\right]_{Y=Y_1}^{Y_2} \Bigg\}$$

The symbol "log" is used here for natural or Napierian logarithms. We have

$$A_w' = \alpha + \gamma \quad .$$

In the previous pages the term α was evaluated; now consider the term γ, where

$$\gamma = -\int_{v=y_1}\int^{c-u} \frac{dudv}{\left[u^2+v^2+z^2\right]^{\frac{1}{2}}} \tag{8}$$

and

$$z = \frac{1}{a}\left[(u-x_1)(z_1-z_2)+(z_3-z_2)(v-y_1) + az_2\right]$$

In the equation for z, let

$$s = \frac{1}{a}\left[(u-x_1)(z_1-z_2)\right], \text{ and}$$

$$p = (z_3-z_2)/a, \text{ so that}$$

$$z^2 = \left[s+p(v-y_1)+z_2\right]^2 \quad \text{or}$$

$$z^2 = p^2v^2+v\left[2sp-2p^2y_1+2z_2p\right]+\left[s^2+p^2y_1^2 - 2spy +z_2^2 + 2z_2s-2z_2py_1\right]$$

$$\left[u^2+v^2+z^2\right]^{\frac{1}{2}} = \left[Av^2+Bv+D\right]^{\frac{1}{2}} = A^{\frac{1}{2}}\left[v^2 + \frac{B}{A}\,v+ \frac{D}{A}\right]^{\frac{1}{2}} . \qquad (9)$$

where

$$A = 1 + p^2$$

$$B = 2sp-2p^2y_1+2z_2p$$

$$D = u^2+s^2+p^2y_1^2 - 2spy_1+z_2^2+2z_2s-2z_2py_1$$

Substituting equation (9) into equation (8) we get

$$\gamma = \frac{-1}{\sqrt{A}} \int\int_{v=y_1}^{c-u} \frac{dudv}{\left[v^2 + \frac{B}{A} v + \frac{D}{A}\right]^{\frac{1}{2}}} = \frac{-1}{\sqrt{A}} \int \left[\log \left[2 \left(v^2 + \frac{B}{A} v + \frac{D}{A} \right)^{\frac{1}{2}} + 2v + \frac{B}{A} \right] \right]_{v=y_1}^{c-u} du.$$

If we substitute the limits for v, we get

$$\gamma = \frac{1}{\sqrt{A}} \int_u \log \left[2 \left(y_1^2 + \frac{B}{A} y_1 + \frac{D}{A} \right)^{\frac{1}{2}} + 2y_1 + \frac{B}{A} \right] du - \frac{1}{\sqrt{A}} \int_u \log \left[2 \left[(c-u)^2 + \frac{B}{A} (c-u) + \frac{D}{A} \right]^{\frac{1}{2}} + 2(c-u) + \frac{B}{A} \right] du.$$

In the expression for γ, the first term is designated G and the second term E, so that

$$\gamma = G + E$$

where

$$G = \frac{1}{\sqrt{A}} \int_{u=x_1}^{x_1+u} \log \left[2 \left(y_1^2 + \frac{B}{A} y_1 + \frac{D}{A} \right)^{\frac{1}{2}} + 2y_1 + \frac{B}{A} \right] du.$$

B and D are functions of the variable u defined by

$$B = 2sP-2P^2y_1+2z_2P$$

$$D = s^2+P^2y_1^2 - 2sPy_1+z_2^2+2z_2s-2z_2Py_1+u^2$$

In the expressions for B and D

$$s = \frac{1}{a}(u-x_1)(z_1-z_2) = n(u-x_1) \text{ where } n = \frac{1}{a}(z_1-z_2).$$

If we substitute the expression for s, in B and D, then insert B and D in the expression for G, we get

$$G = \frac{1}{\sqrt{A}}\int_{u=x_1}^{x_1+a} \log\left[2\left[\frac{u^2}{A}(1+n^2)+\frac{u}{A}\left(2z_2n-2n^2x_1\right) + \frac{1}{A}\left(y_1^2A-p^2y_1^2+z_2^2-\right.\right.\right.$$

$$\left.\left.\left.2z_2nx_1+n^2x_1^2\right)\right]^{\frac{1}{2}} + \frac{u}{A}np + \frac{1}{A}\left(z_2p-nx_1p-p^2y_1+Ay_1\right)\right] du .$$

Factoring out the term $\log\left[\frac{2(1+n^2)^{\frac{1}{2}}}{\sqrt{A}}\right]$, we get

$$G = \frac{1}{\sqrt{A}}\int_{u=x_1}^{x_1+a} \log\left[\frac{2(1+n^2)^{\frac{1}{2}}}{\sqrt{A}}\right]du + \frac{1}{\sqrt{A}}\int_{u=x_1}^{x_1+a} \log\left[(u^2+Ru+F)^{\frac{1}{2}} + \right.$$

$$\left.Ku+N\right] du$$

where

$$R = \frac{1}{1+n^2}\left[2z_2 n - 2n^2 x_1\right] ,$$

$$F = \frac{1}{1+n^2}\left[Ay_1^2 - P^2 y_1^2 + z_2^2 - 2z_2 n x_1 + n^2 x_1^2\right] ,$$

$$k = \frac{nP}{\sqrt{A(1+n^2)}} , \text{ and}$$

$$N = \frac{1}{\sqrt{A(1+n^2)}}\left[z_2 p - n x_1 p - p^2 y_1 + A y_1\right] .$$

Now, let us consider the integral E

$$E = \frac{-1}{\sqrt{A}} \int_{u=x_1}^{x_1+a} \log\left[2\left[(c-u)^2 + \frac{B}{A}(c-u) + \frac{d}{A}\right]^{\frac{1}{2}} + 2(c-u) + \frac{B}{A}\right] du .$$

If we follow the same procedure which was followed with the term G, we get

$$E = \frac{-1}{\sqrt{A}} \int_{u=x_1}^{x_1+a} \log\left[\frac{2(n^2+A-2np+1)^{\frac{1}{2}}}{\sqrt{A}}\right] du - \frac{1}{\sqrt{A}} \int_{u=x_1}^{x_1+a} \log\left[\left[u^2 + R_1 u + F_1\right]^{\frac{1}{2}} + k_1 u + N_1\right] du ,$$

where

$$R_1 = \frac{1}{n^2+A-2np+1}\left[2npc-2cA+2nx_1p+2p^2y_1-2z_2p-2n^2x_1-2py_1n+2z_2n\right],$$

$$F_1 = \frac{1}{n^2+A-2np+1}\left[c^2A-2npcx_1-2p^2cy_1+2z_2pc+n^2x_1^2+p^2y_1^2+2pny_1x_1+z_2^2-\right.$$

$$\left.2z_2nx_1-2z_2py_1\right],$$

$$k_1 = \frac{1}{\left[A(n^2+A-2np+1)\right]^{\frac{1}{2}}}\left[np-A\right], \text{ and}$$

$$N_1 = \frac{1}{\left[A(n^2+A-2np+1)\right]^{\frac{1}{2}}}\left[cA-npx_1-p^2y_1+z_2p\right].$$

The $\int \log\left[\left[u^2+Ru+F\right]^{\frac{1}{2}} + ku+N\right]du$ is common to both E and G.

Evaluation of the Integral $\left[\int \log\left[\left[u^2+Ru+F\right]^{\frac{1}{2}} + ku+N\right]du\right]$

In this integral let

$$T-u = \left[u^2+Ru+F\right]^{\frac{1}{2}}$$

Then $u = \frac{T^2-F}{R+2T}$, $du = \frac{2(T^2+RT+F)}{(R+2T)^2}\,dT$. Hence

$$\int \log\left[\left[u^2+Ru+F\right]^{\frac{1}{2}} + ku+N\right]du = \int \frac{2(T^2+RT+F)}{(R+2T)^2} \log\left[\right.$$

$$\frac{T^2(k+1)+T(R+2N)+NR+F-kF)}{(R+2T)}\Bigg]dT = -\int\frac{2(T^2+RT+F)}{(R+2T)^2}\log\ (2T+R)\ dT +$$

$$\int\frac{2(T^2+RT+F)}{(R+2T)^2}\log\left[T^2(k+1)+T(R+2N)+(NR+F-kF)\right]dT.$$

Integrating the firms term by parts, setting

$$n = \log\ (R+2T)$$

$$dm = \frac{2(TR+T^2+F)}{(R+2T)^2},\quad m = \frac{T^2-F}{R+2T}\ ,$$

and using

$$\int ndm = nm - \int mdn,\ \text{we get}$$

$$-\int\left[\log\ (R+2T)\right]\left[\frac{2(T^2+RT+F)}{(2T+R)^2}\right]dT = -u\log(R+2T)+\int\frac{2(T^2-F)}{(R+2T)^2}\ dT$$

$$= -u\log(2T+R)+\tfrac{1}{4}\left[2T+R-2R\log(2T+R)\ -\frac{R^2}{2T+R}\right]+\frac{F}{2T+R}$$

$$= \frac{T^2+RT+F}{2T+R}\ -\left(u\ +\frac{R}{2}\right)\log(2T+R)$$

Consider the term defined by

$$\int\left[\log\left[T^2(k+1)+T(R+2N)+(NR+F-kF)\right]\right]\ \left[\frac{2(T^2+RT+F)}{(2T+R)^2}\right]dT \triangleq H$$

$$H = \int \frac{2(T^2+RT+F)}{(2T+R)^2} \log[k+1]\, dT + \int \frac{2(T^2+RT+F)}{(2T+R)^2} \log[T^2+TJ+Q]\, dT$$

where

$$J = \frac{R+2N}{k+1},$$

$$Q = \frac{NR+F-kF}{k+1}.$$

Integrating by parts, setting

$$n = \log[T^2+TJ+Q] \text{ and } dm = \frac{2(T^2+RT+F)}{(R+2T)^2}\, dT,\ m = \frac{T^2-F}{2T+R}$$

we get

$$H = \frac{T^2-F}{2T+R} \log\left[(k+1)(T^2+TJ+Q)\right] - \int \frac{(T^2-F)(2T+J)}{(2T+R)(T^2+TJ+Q)}\, dT.$$

Let

$$I \triangleq -\int \frac{(T^2-F)(2T+J)}{(2T+R)(T^2+TJ+Q)}\, dT.$$

In this integral the integrand is an improper fraction (the degree of the numerator is equal to the degree of the denominator).

By division

$$I = -\int \left[1 - \frac{(R+J)T^2+(2F+2Q+RJ)T+(FJ+RQ)}{(2T+R)(T^2+TJ+Q)}\right] dT.$$

A proper fraction may be written as the sum of partial fractions.

$$I = -T + \int \frac{B}{2T+R}\,dT + \int \frac{AT+C}{T^2+TJ+Q}\,dT.$$

$$I = -T + \int \frac{B}{R+2T}\,dT + \int \frac{AT}{T^2+TJ+Q}\,dT + \int \frac{C}{T^2+TJ+Q}\,dT$$

On this page only A and C are used for the expressions given in the next equation,

$$I = -T + \frac{B}{2}\log[2T+R] + \int \frac{\frac{1}{2}(J+R-B)T}{T^2+TJ+Q}\,dT + \int \frac{\frac{1}{R}(FJ+RQ-BQ)}{(T^2+TJ+Q)}\,dT$$

$$I = -T + \frac{B}{2}\log[2T+R] + \tfrac{1}{2}(J+R-B)\left[\tfrac{1}{2}\log\left[T^2+TJ+Q\right]\right.$$

$$\left. - \frac{J}{(4Q-J^2)^{\frac{1}{2}}}\tan^{-1}\left[\frac{2T+J}{(4Q-J^2)^{\frac{1}{2}}}\right]\right] + \frac{(FJ+RQ-BQ)}{R(4Q-J^2)^{\frac{1}{2}}}\tan^{-1}\left[\frac{2T+J}{(4Q-J^2)^{\frac{1}{2}}}\right].$$

Now

$$\int \log\left[(u^2+Ru+F)^{\frac{1}{2}} + ku+N\right]du = \frac{T^2+RT+F}{R+2T} - \frac{2T^2-2F+R^2+2TR}{2(R+2T)}\log(R+2T)$$

$$+ u\log\left[(k+1)(T^2+JT+Q)\right] - T + \frac{B}{2}\log(2T+R) + \tfrac{1}{4}(R+J-B)\log(T^2+TJ+Q)$$

$$+ \left[\frac{\frac{1}{R}(FJ+RQ-BQ) - \frac{J}{2}(R+J-B)}{(4Q-J^2)^{\frac{1}{2}}}\right]\tan^{-1}\left[\frac{2T+J}{(4Q-J^2)^{\frac{1}{2}}}\right],$$

where

$$B = \frac{R^2J-R^3-4FJ+4FR}{2RJ-R^2-4Q}$$

$$J = \frac{R+2N}{k+1}$$

$$Q = \frac{NR+F-kF}{k+1}$$

In the previous pages, the triple integral which gives the vertical component of the attraction A_w of a triangular prism was evaluated. The following is the total result for A_w.

$$A_w = k\rho\int_{u=x_1}^{X_1+a}\int_{v=y_1}^{c-u}\int_{w=o}^{z}\frac{wdudvdw}{\left[u^2+v^2+w^2\right]3/2} = \left\{-\left[u\log\left[y_1+\left[y_1^2+u^2\right]^{\frac{1}{2}}\right]+\right.\right.$$

$$y_1\log\left[u+\left[y_1^2+u^2\right]^{\frac{1}{2}}\right]- u\log|u|\Big]_{u=x_1}^{x_1+a}+\left[u\log|Y| - \frac{c}{\sqrt{2}}\log\left[\frac{Y+1-\sqrt{2}}{Y+1+\sqrt{2}}\right]\right]_{Y=Y_1}^{Y_2}$$

$$\left.+\left\{\frac{1}{\sqrt{A}}\log\left[\left[\frac{n^2-2np+A+1}{1+n^2}\right]^{\frac{1}{2}}\right]u\right\}_{u=x_1}^{x_1+a}+\frac{1}{\sqrt{A}}\left\{\frac{T^2+RT+F}{R+2T}+\left[\frac{B}{2}\quad\frac{2T^2-2F+R^2+2R}{2(R+2T)}\right]\log(R+2T)\right.\right.$$

$$+u\log\left[(k+1)(T^2+TJ+Q)\right] - T + \tfrac{1}{4}(R+J-B)\ \log\left[T^2+TJ+Q\right] +$$

$$\left[\frac{\frac{1}{R}(FJ+RQ-BQ) - \frac{J}{2}(R+J-B)}{(4Q-J^2)^{\frac{1}{2}}}\right]\tan^{-1}\left(\frac{2T+J}{4Q-J^2}\right)\Bigg\}_{T=T_1}^{T_2} - \frac{1}{\sqrt{A}}\left\{\frac{T^2+R_1T+F_1}{R_1+2T} +\right.$$

$$\left[\frac{B_1}{2} - \frac{2T^2-2F_1+R_1^2+2R_1}{2(R_1+2T)}\right]\log\ (R_1+2T) + u\log\left[(k_1+1)(T^2+TJ_1+Q_1)\right] -$$

$$T + \tfrac{1}{4}(R_1+J_1-B_1)\ \log\left[T^2+TJ_1+Q_1\right]$$

$$+\left[\frac{\frac{1}{R_1}(F_1J_1+R_1Q_1-B_1Q_1) - \frac{J_1}{2}(R_1+J_1-B_1)}{(4Q_1-J_1^2)^{\frac{1}{2}}}\right]\tan^{-1}\left(\frac{2T+J_1}{4Q_1-J_1^2}\right)\Bigg\}_{T=T_1^1}^{T_2^1}$$

In order to avoid writing lengthy terms, the limits of integration for the variable u will not be substituted into the above formula.

In the above formula the constants are as follows:

$$c = x_1+y_1+a$$

$$Y_1 = \left[\left(\frac{c}{x_1} - 1\right)^2 + 1\right]^{\frac{1}{2}} + \left(\frac{c}{x_1} - 1\right)$$

$$Y_2 = \left[\left(\frac{c}{x_1+a} - 1\right)^2 + 1\right]^{\frac{1}{2}} + \left(\frac{c}{x_1+a} - 1\right)$$

$$p = (z_3-z_2)/a$$

$$n = (z_1-z_2)/a$$

$$H = 1 + p^2$$

$$s = \frac{1}{a}\left[(u-x_1)(z_1-z_2)\right]$$

$$R_1 = \frac{1}{n^2+A-2np+1}\left[2npc-2cA+2nx_1p+2p^2y_1-2z_2p-2n^2x_1-2py_1n+2z_2n\right]$$

$$F_1 = \frac{1}{n^2+A-2np+1}\left[c^2A-2npcx_1-2p^2cy_1+2z_2pc+n^2x_1^2+p^2y_1^2+2pny_1x_1+\right.$$

$$\left.z_2^2-2z_2nx_1-2z_2py_1\right]$$

$$k_1 = \frac{1}{\left[A(n^2+A-2np+1)\right]^{\frac{1}{2}}}\left[np-A\right]$$

$$N_1 = \frac{1}{\left[A(n^2+A-2np+1)\right]^{\frac{1}{2}}}\left[cA-npx_1-p^2y_1+z_2p\right]$$

$$B = \frac{R^2J-R^3-4FJ+4FR}{2RJ-R^2-4Q}$$

$$J = \frac{R+2N}{k+1}$$

$$Q = \frac{NR+F-kF}{k+1}$$

and

$$R = \frac{1}{1+n^2}\left[2nz_2-2n^2x_1\right]$$

$$F = \frac{1}{1+n^2}\left[Ay_1^2-p^2y_1^2+z_2^2-2z_2nx_1+n^2x_1^2\right]$$

$$K = \frac{np}{\sqrt{A(1+n^2)}}$$

$$N = \frac{1}{\sqrt{A(1+n^2)}}\left[z_2p-nx_1p-p^2y_1+Ay_1\right]$$

$$B_1 = \frac{R_1^2J_1-R_1^3-4F_1J_1+4F_1R_1}{2R_1J_1-R_1^2-4Q_1}$$

$$J_1 = \frac{R_1+2N_1}{k_1+1}$$

$$Q_1 = \frac{N_1R_1+F_1-kF_1}{k_1+1}$$

$$T_1 = \left[x_1^2+R^2x_1+F\right]^{\frac{1}{2}} + x_1$$

$$T_2 = \left[(x_1+a)^2 + R^2(x_1+a)+F\right]^{\frac{1}{2}} + (x_1+a)$$

$$T_1^1 = \left[x_1^2 + R_1^2 x_1 + F_1 \right]^{\frac{1}{2}} + x_1$$

$$T_1^2 = \left[(x_1+a)^2 + R_1^2 (x_1+a) + F \right]^{\frac{1}{2}} + (x_1+a)$$

The Attraction of Triangular Prism 2 at the Origin

The prism is bounded on the top by a plane passing through the points $(x_1+a,\ y_1,\ z_1)$, $(x_1,\ y_1+a,\ z_3)$, $(x_1+a,\ y_1+a,\ z_4)$, and on the bottom by a plane passing through the points $(x_1+a,\ y_1)$, $(x_1,\ y_1+a)$, $(x_1+a,\ y_1+a)$.

The vertical component of attraction Aw_1 at the origin of the triangular prism is given by:

$$Aw_1 = kp \int_{u=x_1}^{x_1+a} \int_{c-u}^{y_1+a} \int_{w=o}^{z'} \frac{dudvdw}{\left[u^2+v^2+w^2\right]^{3/2}} \qquad (1')$$

where

$$z' = \frac{1}{a}\left[(v-y_1)(z_4-z_1) + (u-x_1)(z_4-z_3) + a(z_1-z_4+z_3) \right] . \qquad (2')$$

Since the only difference between equation (1), which gives the vertical component of attraction of triangular prism 1, and formula (1'), which gives the vertical component of attraction of triangular prism 2, is the limits of integration, the answer for equation (1') will be given

without going through the evaluation of that equation.

$$Aw_1 = k\rho \int_{u=x_1}^{x_1+a} \int_{c-u}^{y_1+a} \int_{w=o}^{z'} \frac{dudvdw}{u^2+v^2+w^2\ ^{3/2}} = k\rho \Bigg\{ \Bigg[u\log \Bigg[$$

$$\frac{u}{(y_1+a)+\left[(y_1+a)^2+u^2\right]^{\frac{1}{2}}}\Bigg] - (y_1+a)\ \log\left[u+\left[(y_1+a)^2+u^2\right]^{\frac{1}{2}}\right]\Bigg] +$$

$$\left[u\log Y - \frac{c}{\sqrt{2}}\ \log\left[\frac{Y+1-\sqrt{2}}{Y+1+\sqrt{2}}\right]\right]_{Y=Y_1}^{Y_2} + \left\{\frac{1}{\sqrt{A}}\ \log\left[\left[\frac{n^2-2np+A+1}{1+n^2}\right]^{\frac{1}{2}}\right]u\right\}_{u=x_1}^{x_1+a} -$$

$$\frac{1}{\sqrt{A}}\Bigg\{ \frac{T^2+R_2T+F_2}{R_2+2T} + \left[\frac{B_2}{2} - \frac{2T^2-2F_2+R_2^2+2R_2}{2(R_2+2T)}\right]\log(R_2+2T) + u\log\Big[$$

$$(K_2+1)(T^2+TJ_2+Q_2)\Big] - T + \tfrac{1}{4}(R_2+J_2-B_2)\ \log\ (T^2+TJ_2+Q_2) +$$

$$\left[\frac{\frac{1}{R}(F_2J_2+R_2Q_2-B_2Q_2) - \frac{J_2}{2}(R_2+J_2-B_2)}{(4Q_2-J_2^2)^{\frac{1}{2}}}\right]\tan^{-1}\left(\frac{2T+J_2}{4Q_2-J_2^2}\right)\Bigg\}_{T=T_1}^{T_2} +$$

$$\frac{1}{\sqrt{A}}\left\{\frac{T^2+R_3T+F_3}{R_3+2T}+\left[\frac{B_3}{2}-\frac{2T^2-2F_3+R_3^2+2R_3}{2(R_3+2T)}\right]\log(R_3+2T)+u\log\left[\right.\right.$$

$$\left.(K_3+1)(T^2+TJ_3+Q_3)\right]-T+\tfrac{1}{4}(R_3+J_3-B_3)\log(T^2+TJ_3+Q_3)+$$

$$\left[\frac{\frac{1}{R}(F_3J_3+R_3Q_3-B_3Q_3)-\frac{J_3}{2}(R_3+J_3-B_3)}{(4Q_3-J_3^2)^{\frac{1}{2}}}\right]\tan^{-1}\left(\frac{2T+J_3}{4Q_3-J_3^2}\right)\Bigg\}_{T=T_1'}^{T_2'}$$

where

$$c = x_1+y_1+a$$

$$Y_1 = \left[\left(\frac{c}{x_1}-1\right)^2+1\right]^{\frac{1}{2}}+\left(\frac{c}{x_1}-1\right)$$

$$Y_2 = \left[\left(\frac{c}{x_1+a}-1\right)^2+1\right]^{\frac{1}{2}}+\left(\frac{c}{x_1+a}-1\right)$$

$$p = \frac{1}{a}(z_4-z_1)$$

$$n = \frac{1}{a}(z_4-z_3)$$

$$A = 1+p^2$$

$$s = z_1+z_3-z_4$$

$$F_2 = \frac{1}{1+n^2}\left[s^2-2npx_1-p^2y_1^2+2spa+n^2x_1^2-2snx_1+A(y_1+a)^2\right]$$

$$R_2 = \frac{1}{1+n^2}\left[2np-2n^2x_1+2sn\right]$$

$$K_2 = \frac{np}{\left[A(1+n^2)\right]^{\frac{1}{2}}}$$

$$N_2 = \frac{1}{\left[A(1+n^2)\right]^{\frac{1}{2}}}\left(Ay_1+aA-npx_1-p^2y_1+sp\right)$$

$$J_2 = \frac{R_2+2N_2}{K_2+1}, \quad Q_2 = \frac{N_2R_2+F_2-K_2F_2}{K_2+1}$$

$$T_1 = \left[x_1^2+R_2x_1+F_2\right]^{\frac{1}{2}} + x_1$$

$$T_2 = \left[(x_1+a)^2 + R_2(x_1+a) + F_2\right]^{\frac{1}{2}} + (x_1+a)$$

$$B_2 = \frac{R_2J_2-R_2^3-4F_2J_2+4F_2R_2}{2R_2J_2-R_2^2-4Q_2}$$

$$R_3 = \frac{1}{A+n^2-2np+1}\left[2npc-2cA-2n^2x_1+2ns+2npx_1-2sp-2npy_1+2p^2y_1\right]$$

$$F_3 = \frac{1}{A+n^2-2np+1}\left[Ac^2-2pncx_1-2p^2cy_1+2spc+n^2x_1^2+s^2+p^2y_1^2-2psy + 2nx_1y_1-2snx_1\right]$$

$$K_3 = \frac{[np-A]}{\left[A(1+n^2-2np+a)\right]}$$

$$N_3 = \frac{1}{\left[A(1+n^2-2np+A)\right]^{\frac{1}{2}}}\left[Ac-npx_1-p^2y_1+sp\right]$$

$$T_1' = \left[x_1^2+R_3x_1+F_2\right]^{\frac{1}{2}} + x_1$$

$$T_2' = \left[(x_1+a)^2 + R_3(x_1+a) +F_3\right]^{\frac{1}{2}} + (x_1+a)$$

$$J_3 = \frac{R_3+2N_3}{K_3+1}$$

$$Q_3 = \frac{N_3R_3+F_3-K_3F_3}{K_3+1}$$

$$B_3 = \frac{R_3^2J_3-R_3^3-4F_3J_3+4F_3R_3}{2R_3J_3-R_3^2-4Q_3}$$

TESTING THE FORMULA FOR THE ATTRACTION OF A TRIANGULAR PRISM

The derivation of the formula for the vertical component of the gravitational attraction of a triangular prism is given in the previous pages. The formula contains 30 terms, after the limits of integration have been substituted. Two examples were used to test the validity of the formula, and the results are shown in table below. Both examples use a hemisphere of uniform density, $\rho = 2.6$ g cm^{-3}, and of radius, $r = 100$ m. The two examples are:

Example A. The vertical component of attraction of a hemisphere of uniform density upon the center of its plane base. Figure 4A illustrates this example. The exact formula for the attraction at point p is given by

$A_Z = \pi k \rho r$, (see Ramsey, 1961, p. 38)

which upon substituting the given values for π, k, ρ, and r, yields $A_Z = 4 \times 1.3602$ mgal.

Triangular prisms with sides of 10 m were then used to approximate the hemisphere. The sum of the attractions of

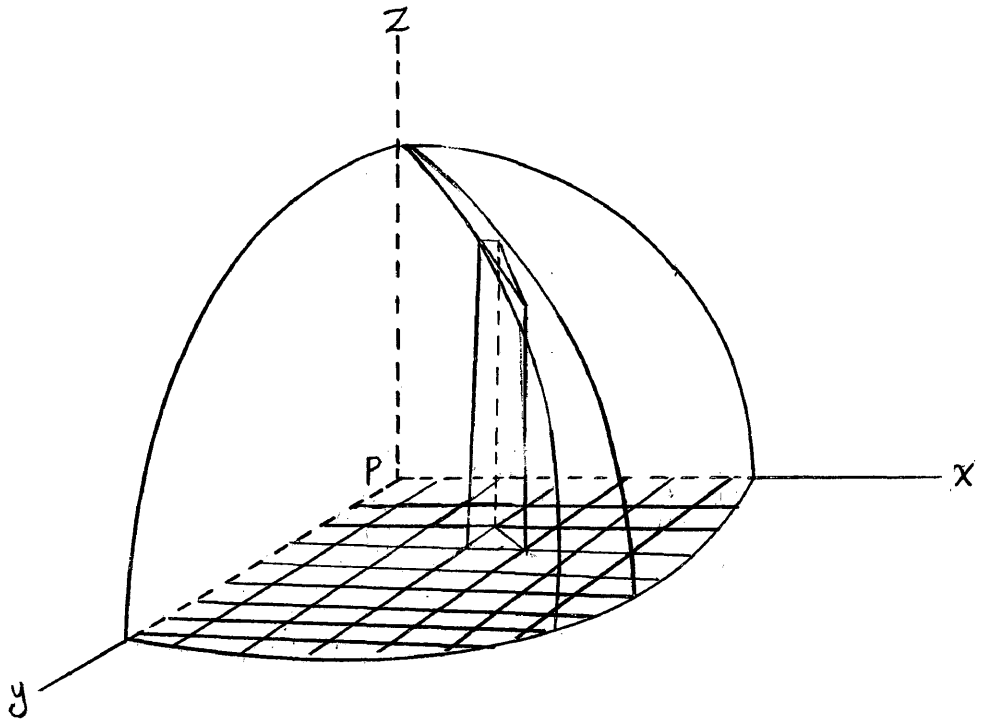

Figure 4A Approximation of a Hemisphere by Triangular Prisms.

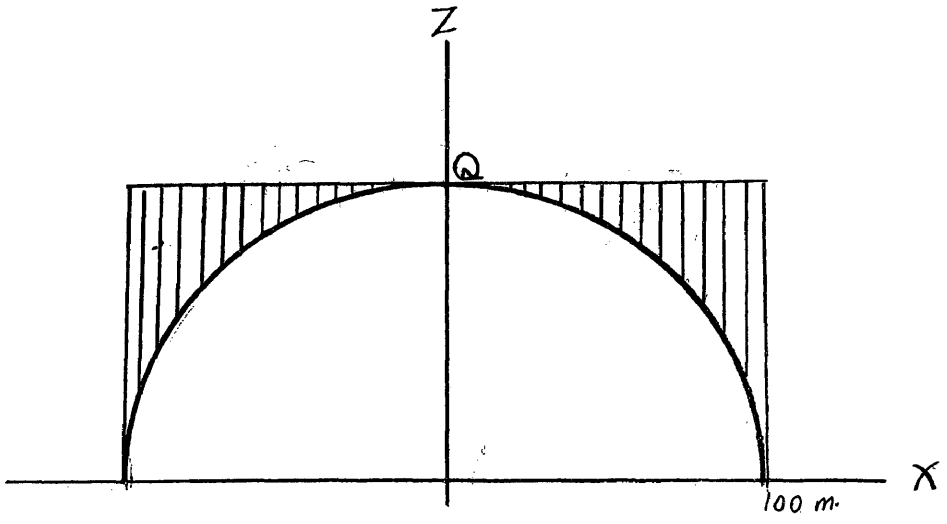

Figure 4B Mass Distribution After a Hemisphere is Subtracted From a Cylinder.

these prisms upon point p was found by means of the CDC-8090 digital computer. Because of the symmetry of the hemisphere, the attraction of only one quadrant was calculated. The result obtained for the attraction of the hemisphere was

A_z = 4 x 1.355499 mgal

Example B. The terrain correction for a gravity station located at the top of a hemisphere of uniform density. Figure 4B illustrates this example.

The attraction of a hemisphere of radius r at its top, point Q in figure 4B, is given by

$$A_h = 2\pi\rho r\left[1 - \frac{\sqrt{2}}{3}\right]$$ (see Ramsey, 1961, p. 43)

and the attraction of a cylinder of radius and height equal to r at the center of its upper plane, (point Q in figure 4B) is given by

$$A_c = 2\pi\rho r\ [2 - \sqrt{2}]$$ (see Ramsey, 1961, p. 36)

The terrain correction at Q, for a circular zone of radius r, is given by

$T_c = A_c - A_h$

= 4 x 0.1550628 mgal

Triangular prisms were again used to approximate the hemisphere, and the terrain correction was obtained. The result was

T_c = 4 x 0.1299803 mgal

Listed below is a tabulation of the results and the errors obtained for these examples.

Example A

Attraction at the center of the base plane of the hemisphere	Attraction using triangular prism method in mgal	Attraction using exact formulas in mgal	Error in mgal
	1.355499	1.3602	0.004701

Example B

Terrain correction at the top of the hemisphere	Terrain correction using triangular prism method in mgal	Terrain correction using exact formulas in mgal	Error in mgal
	0.1299803	0.1550628	0.0250825

By means of the CDC-8090 digital computer, the attraction of 150 triangular prisms was determined in 25 minutes.

The previous examples show a small error in using the triangular-prism method. The main errors arise from the following two factors:

1. Approximation of the curved surface of the hemisphere with the triangular planes,

2. Approximation of the circumference of the base of the hemisphere with triangles. Obviously this factor contributes more error to Example B than Example A.

THE PROGRAM

Below is a description of the program developed to calculate the terrain correction by the use of the formula of attraction of a triangular prism.

Modification of the Formula

In order to save computing time, the formula for the attraction of a triangular prism was rearranged to minimize the number of operations. Illegal operations; for example division by zero, were eliminated mostly through legal modifications in the formula.

A program was developed to calculate the terrain correction by the use of the formula of attraction of a triangular prism. The program in Fortran language is given in appendix I. The program is provided with comments and statements to enable one to read through the program with an understanding of the purpose of each group of instructions. The program may be divided into two main parts.

1. Subroutines

Three subroutines were developed. The function of each subroutine is to evaluate part of the formula of attraction of a triangular prism. Substitution of the limits of integration in the formula was carried out through each subroutine.

2. The Main Program

The function of the program can be visualized with the aid of figure 5. This figure shows a circle which represents the outer boundary of the corrected zone, which has a radius of 600 m. The square grids have 100-m sides; the elevations at the corners of each square grid have been recorded.

The program was written to calculate the gravimetric effect at the center of the circle of the mass underlying each square grid which falls inside the circle. The effect of each rectangular prism was added to all the rest, and the sum printed out.

The first executable statement in the program is to store the number of gravity stations with their x and y coordinates. The elevations given on the first two profiles (A and B in figure 5) with the x and y coordinates of the first point on each profile were read in the computer and stored.

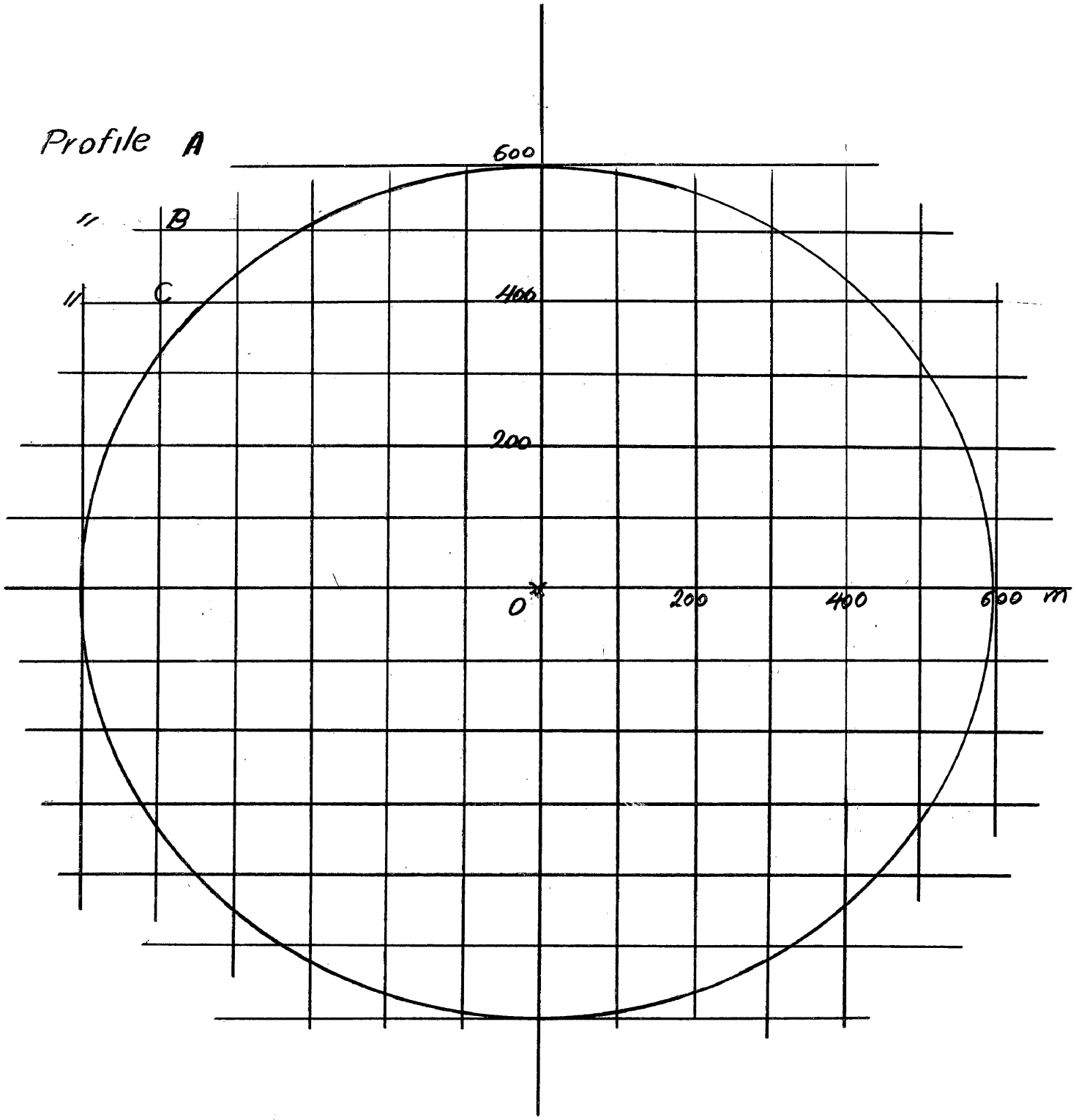

Figure 5 Terrain Correction For Station at the Center of Circular Zone.

When a data word was called up from memory, its coordinates were tested to ascertain whether the point fell within or outside the zone for each gravity station. If the point fell within the circle, the elevation of this point and the other three points, which form together a square grid inside the circle, were called up from memory, and the effect of the mass underlying that square was calculated. If the point fell outside the circle, it was rejected, and the next point was tested.

A counter register was established to count the number of rectangular prisms computed for each gravity station separately.

Testing of the Program

The calculation of terrain correction for a gravity station located at the base of a hill with a constant slope was used to test the program. For this case, square grids of 100-m sides were used in digitizing the contour map of the hill. The Sandberg graphical method was used to calculate the terrain correction for the same gravity station. The following table compares the results obtained by using the two methods mentioned above.

Terrain correction using triangular prism method in mgal	Terrain correction using Sandberg graphical method in mgal	Error in mgal
3.21147	3.12	0.09147

By the use of the CDC-8090, the attraction of 200 triangular prisms was calculated in 30 minutes.

CONCLUSIONS

The program and methods described in this thesis offer a satisfactory solution to the problem of computing terrain correction for the near zones.

The examples given in this study showed that approximation by triangles at the perimeter of the circular zone contributes errors to the results of terrain correction. The computing time per station, given in this report, should be reduced considerably in order to have an economically feasible method.

The triangular-prism method may be improved by considering the following points:

1. More precise results can be obtained by calculating the terrain effect within a rectangular zone instead of a circular zone.

2. Computing time can be saved by a simple modification in the formula of attraction of the triangular prism. Since the first part of this formula is a function of the

area only, it can be evaluated over the entire zone of correction rather than for each prism separately.

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APPENDIX I

```
      COMMON DX,DY,C,H,V,P,R,F,E,W,A,DH,AM1,SUM1,SUM2,DZ4 ,S
     1UM4,AM,SH
      DIMENSION W(11,2),Y(2),X(2),Z(2),TC(2),M(2)
    1 FORMAT(I5)
    2 FORMAT( 2F8.2)
    3 FORMAT(10F8.0)
    4 FORMAT(I4,2F9.3)
    5 FORMAT(10F6.0)
   10 FORMAT(F6.3)
   59 FORMAT(2E20.8,2F6.0)
   43 FORMAT(1H0,E20.8)
C     A IS THE SIDE OF THESQUARE GRID
C     K IS THE NO.OF GRAVITY STATIONS
C     RAD IS THE RADIUS OF THE CIRCULAR ZONE
C     X,Y,Z ARE THE COORDUNATES OF THE GRAVITY STATION
C     N IS THE NO) OF POINTS ON THE PROFILE
C     U1,V1 ARE THE COORDINATES OF T)E FIRST POINT ON THE
CROFILE
      W ARE  THE ELEVATIONS OF THE POINTS GIVEN ON THE
C     PROFILE
      READ 10,RAD
      READ 10,A
      READ 1 ,K
      READ 2,(X(I),I = 1,K)
      READ 2,(Y(I),I = 1,K)
      READ 2,(Z(I),I = 1,K)
      READ 4,N,U1,V1
C     THIS DO LOOP IS TO CALCULATE THE EFFECT OF EACH PRISM
C     FOR ALL STATIONS
      DO 71 I = 1,K
CIS  THECOUNTER FOR SQUARE GRIDS +OMPUUED
      M(I) = 0.
C     TC IS THE TERRAIN CORRECTION
   71 TC(I) = 0.
      READ 5,((W(I,J),I = 1,N),J = 1,2)
```

```
   9 DO 16I = 1,K
     DY = ABSF(Y(I)-V1)
C    THIS TEST IS TO ASCERTAIN IF THE POINT INSIDE OR
C    OUTSIDE THE CIRCLE
     IF(DY-RAD) 11,17,16
  17 NFU1 =  (X(I)-U1)
     NEU2 = ((X(I)-U1)  )+2
     GO TO 19
  11 HCOR = SQRTF(RAD*RAD-DY*DY)
     NFU1 = X(I)-U1-HCOR+1.5
     NEU2 = X(I)-U1+HCOR+.5
  19 A = A*100000
     DY = ABSF((DY-1)*A)
     DX = ABSF((X(I)-U1-(NFU1-1)-1)*A)
     DU = DX
C    THIS DO LOOP IS TO COMPUTE PRISMS GIVEN BY 2 PROFILES
     DO 6 J = NFU1,NEU2
  57 ATT1 = 0.
     JSUB = J+1
     IF(DU) 41,44,44
  44 DZ1 = (ABSF(W(J,2)-Z(I)))*100
     DZ3 = (ABSF(W(JSUB,1)-Z(I)))*100
     DZ4 = (ABSF(W(J,1)-Z(I)))*100
     DZ2 = (ABSF(W(JSUB,2)-Z(I)))*100
     GO TO 89
  41 DZ1 = (ABSF(W(JSUB,2)-Z(I)))*100
     DZ4 = (ABSF(W(JSUB,1)-Z(I)))*100
     DZ3 = (ABSF(W(J,1)-Z(I)))*100
     DZ2 = (ABSF(W(J,2)-Z(I)))*100
  89 C = DX+DY+A
     DX = ABSF(DX)
     CALL GALAW
     ATT1 = SUM1
     P = (DZ3-DZ2)/A
     V = (DZ1-DZ2)/A
     H = 1.+P*P
     AM = 1./SQRTF(1.+V*V)
     SH = 1./SQRTF(H)
```

```
      AA = AM*AM
      AH = SH*AM
      ZZ = DZ2*DZ2
      X2 = DX*V
      X22 = X2*X2
      Y1 = DY*P
      Y12 = Y1*Y1
      HY = H*DY
      HC = H*C
      VP = V*P
      YXZ = P*(DZ2-X2-Y1)
      R = AA*2.*V*(DZ2-X2)
      F = AA*(HY*DY+X22+ZZ-Y12-2.*DZ2*X2)
      E = VP*AH
      W = AH*(HY+YXZ)
      CALL SAMG
      SUM5 = SUM2-SUM1
      ATT1 =ATT1+SUM5
      AM1= 1./SQRTF(ABSF(H+V*V-2.*P*V+1.))
      CALL HAL
      ATT1 =ATT1+SUM2
      AH = SH*AM1
      AA = AM1*AM1
      R =AA*2.*(-YXZ+C*(VP-H)+V*(-X2-Y1+DZ2))
      F = AA*(C*(HC+2.*YXZ)+X22+ZZ+Y12+2.*(-Y1*DZ2+X2*(Y1-
     1DZ2)))
      E = AH*(VP-H)
      W = AH*(HC+YXZ)
      CALL SAMG
      SUM6 = SUM1-SUM2
      ATT1 =ATT1+SUM6
C     ATTRACTION OF THE SECOND PRISM
   21 DY = DY+A
      CALL GALAW
      ATT1 =ATT1-SUM1
      DY = DY-A
      P = (DZ4-DZ1)/A
      V = (DZ4-DZ3)/A
      H = 1.+P*P
      S = DZ1+DZ3-DZ4
```

```
      AM =1./SQRTF(1.+V*V)
      SH = 1./SQRTF(H)
      AH = SH*AM
      AA = AM*AM
      X2 = DX*V
      X22 = X2*X2
      Y1 = DY*P
      Y12 = Y1*Y1
      HY = H*DY
      HC = H*C
      VP = V*P
      ZZ = S*S
      YXZ = P*(S-X2-Y1)
      R = AA*2.*(VP*A+V*(-X2+S))
      F = AA*(2.*A*YXZ-Y12+ZZ+X22-2.*S*X2+H*(DY+A)*(DY+A))
      E = VP*AH
      W = AH*(HY+A*H+YXZ)
      CALL SAMG
      SUM5 = SUM1-SUM2
      ATT1 =ATT1+SUM5
      AM1= 1./SQRTF(ABSF(H +V *V -2.*P *V +1.))
      CALL HAL
      ATT1 =ATT1-SUM2
      AH = SH*AM1
      AA = AM1*AM1
      R = AA*2.*(C*( VP-H)+V*(-X2+S-Y1)-YXZ)
      F = AA*(C*(HC+2.*YXZ)+X22+ZZ+Y12+2.*(-Y1*S+X2*(Y1-S)))
      E = AH*(VP-H)
      W =AH*(HC+YXZ)
      CALL SAMG
      SUM6 = SUM2-SUM1
      ATT1 =ATT1+SUM6
      M(I) = M(I)+1
   20 ATT1 = -ATT1
      TC(I) = TC(I)+ATT1
      DX = ABSF(DX-A)
    6 DU = DU-A
   16 CONTINUE
      V1 = ABSF(V1-1)
      DO 18 I = 1,N
   18 W(I,1) = W(I,2)
      READ 5,(W(I,2),I = 1,N)
      IF(W(1,2) 9,85,9
```

```
      IF(W(1,2) 9,85,9
   85 DO 56 I = 1,K
   56 TC(I) = (TC(I)*2.6*.0667)/1000
      PRINT 26,(X(I),Y(I),Z(I),TC(I),I = 1,K)
   26 FORMAT(2X,2HX=F8.2,2X,2HY=F8.2,2X,2HZ=F7.0,2X,3HTC=F9.
     119HNOOF PTS COMPUTED=I7,/)
      STOP
      END
C     THIS SUBROUTINE IS TO EVALUATE THE PART OF THE FORMULA
C     DEPEND ON THE AREA ONLY
      SUBROUTINE GALAW
      COMMON DX,DY,C,H,V,P,R,F,E,W,A,DH,AM1,SUM1,SUM2,DZ4 ,S
     1UM4,AM,SH
      SUM1 = 0.
      U2 = 9999
      DX = DX-A
      DO 401 I = 1,2
      DX = DX+A
      HC = C-DX
      X22= DX*DX
      Y12= SQRTF(X22 +DY*DY)
      IF(ABSF(DX)+ABSF(DY)) 1,2,1
    2 AA = 0.
      AH = 0.
      GO TO 402
    1 IF(ABSF(DX)) 3,4,3
    4 AA = DY*LOGF(DY)
      AH = 0.
      GO TO 402
    3 AA = DX*LOGF((DY+Y12)/DX)+DY*LOGF(DX+Y12)
      IF(U2-DZ4) 21,20,21
   21 AH = 0.
      GO TO 402
   20 Y1= SQRTF(HC*HC+X22)+HC
      VP = Y1+DX
      X2 =1.414214*DX
      AH = (C/1.414214)*LOGF((VP-X2)/(VP+X2))-DX
     1*LOGF(Y1/DX)
  402 SUM1 = AA+AH -SUM1
  401 CONTINUE
      DX = DX-A
      RETURN
      END
CHIS SUBROUTINE IS TO EVALUATE PART OF THE FORMULA
      SUBROUTINE HAL
```

```
      COMMON DX,DY,C,H,V,P,R,F,E,W,A,DH,AM1,SUM1,SUM2,DZ4 ,S
     1UM4,AM,SH
      SUM2 = 0.
      AP1 = LOGF(AM/AM1)
      DX = DX-A
      DO 403 I = 1,2
      DX = DX+A
      VP  = DX*SH*AP1
      SUM2 = VP  - SUM2
  403 CONTINUE
      DX = DX-A
      RETURN
      END
C     THIS SUBROUTINE CONTAIN THE ANSWER OF THE  INTEGRAL
C     LOG((SQRTF(U*U+R*U+F)+K*U+N)
      SUBROUTINE SAMG
      COMMON DX,DY,C,H,V,P,R,F,E,W,A,DH,AM1,SUM1,SUM2,DZ4 ,S
     1UM4,AM,SH
      DH = DX+A
      X2 = R*R
      Y1 = E+1.
      Q = (W*R+F-E*F)/Y1
      HC = 4.*Q
      U=(2.*W+R)/Y1
   51 B = ((U-R)*(X2-4.*F))/(2.*R*U-X2-HC)
      P1= U*U
      A1= HC - P1
      H1= R*U
      Y12 = SQRTF(A1)
      X22 = (A1-H1+U*B)/4.+(F*(2.*P1-H1-HC)+Q*(X2-H1))/(2.*
     1H1-X2-HC)
      X22 = X22*SH
   53 DX = DX-A
      DO 405 I=1,2
      DX = DX+A
      T = SQRTF(ABSF(DX*DX+R*DX+F))+DX
      IF(ABSF(R)+ABSF(E)+ABSF(F)+ABSF(W)+ABSF(DX)) 40,41,40
   41 VP  = 0.
      GO TO 28
   40 IF(ABSF(R)+ABSF(E)+ABSF(W)) 26,17,26
   17 Y1 = SQRTF(F)
   30 VP  = (1./2.)*DX*LOGF(ABSF(DX*DX+Y1*Y1))-DX+Y1*ATANF(
     1DX/Y1)
```

```
      VP = VP*SH
      GO TO 28
   26 HY = 2.*T
      AP = R+HY
   31 AP1 = LOGF(AP)
      C1= T*R
      V1= T*T
      E1= T*U
      AA = ((V1+C1+F)/AP)+(-((2.*(V1+C1-F)+X2)/(2.*AP))+(B/2
     1))*AP1+DX*LOGF(Y1*(V1+E1+Q)-1
     1DX             *LOGF(Y1*(V1+E1+Q))
      AH = .25*(R+U-B)*LOGF(V1+E1+Q)-T
   15 SUM1 = (2.*X22/Y12)*ATANF(ABSF((HY+U)/Y12))
      VP = (AA+AH)*SH+SUM1
   28 IF(1-I) 407,408,409
  407 SUM1 = VP
      GO TO 405
  408 SUM2 = VP
      GO TO 405
  409 DX = 0.
  405 CONTINUE
      DX = DX-A
      RETURN
      END
```