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AN ECONOMETRIC MODEL OF COLORADO'S MINERAL INDUSTRY

By

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ABSTRACT

Out of the different econometric models which have been reviewed, the polynomial and multiple linear regression analysis techniques have been selected to portray the economic structure of Colorado's Mineral Industry. Both models have served to predict physical output and employment for a one-year period.

The error analysis has shown that one is able to predict those variables with an average actual error of 10.1%. There is no proof to favor multiple linear regression over polynomial regression as a forecasting technique.

Multiple linear regression analysis, however, has given evidence that specific minerals follow different laws of production. Lead-zinc and oil-coal production are influenced by parameters having an additive character, whereas copper-silver output has the tendency to be affected by parameters having a multiplicative impact. This has led to the conclusion that an index of technology is necessary to explain better the structural behavior of the copper-silver industry.

INTRODUCTION

The Need for a Prediction Method

That a structural metamorphosis of our society and economy is underway is obvious. The rapidly increasing national and world population entails a correspondingly higher production of all consumer goods. The supply of agricultural products and basic minerals, in particular, will have to be adjusted to meet this growing need.

Agriculture and mining are the two basic foundations on which any industrial nation grows and progressively develops. They have been the object of close scrutiny for many years. Questions which have had to be answered included how much did we produce in the past? How much can we produce in the future? And what is the price at which we can sell our products? Although most decisions concerning these questions are based on trends and statistics encountered on the domestic market, a modern state is also highly influenced by international trade and interaction with foreign markets. This factor reflects the complexity of the national economic structure in an industrial nation.

There have been many ways of analyzing business statistics, from simple graphical interpretation to such mathematical treatments as harmonic regression. Extensive data analysis on gathered statistics indicating economic phenomena require careful mathematical treatment if the results are to be meaningful. The results of such analyses are not necessarily superior to the judgments of qualified people acquainted with the field, but they can be more helpful in interpreting complex situations. However, the results are always open to discussion.

Mathematical models of economic phenomena are very useful in analysis. This paper examines certain mathematical models by which trends in the mineral industry can be recognized and evaluated. These models are applied to data pertaining to the mineral industry in an attempt to provide a means of more accurately predicting mineral production, employment, values of products, and related information. Knowledge of how these raw data are related mathematically can provide useful guides for the early recognition of trends.

Economic Systems and Their Mathematical Models

The complexity of the business economy and its technology has challenged engineers and scientists to devise better methods for its control. One of these methods is a new discipline called system analysis. The goal is to find the relationships among the variables. Once the system and its boundaries have been defined, the system in its concrete form can be transformed to an abstract image. Abstraction is not the goal for analyzing a system but a means. It describes the real world in symbolic form. The abstraction of a system is called its model. A model best represents the important criteria which contribute most significantly to the system. Mathematical symbols are commonly used to represent the systems, although other forms may be used for description. As an example, a circle represents any form of a closed system. This can be described in various ways:

a. Qualitatively one can say that the figure is round. One can add further, in evaluating the size of the circle that the figure is big, medium, or small.

b. Quantitatively, the circle can be described in either polar or cartesian coordinates.

$$r^2 - 2rr_o\cos(\varphi - \varphi_o) + r_o{}^2 = R^2 \qquad (1)$$

$$(x - x_o)^2 + (y - y_o)^2 = R^2 \qquad (2)$$

Equations (1) and (2) exactly describe the shape of the figure and its dimensions, and each is a mathematical model of the system.

Quantitative analysis still is the best way for studying the behavior of static or dynamic systems. Wherever mathematics may be used, we deal with quantitative data, data which have magnitude and dimension. Therefore, emphasis will be given to those models which can best handle the collected data quantitatively.

Models already developed for most of the operations in the mineral industry allow a precise study at any stage of operation. Although the real phenomena are applied to the model by means of certain assumptions, most of our models have proven their validity and practicability under a variety of conditions. We may mention but one discipline of mining engineering, rock mechanics; the progress made in this field recently was due largely to the recognition of similarity between rock and other materials already used for structural purposes. The model of a stress pattern around an underground opening was developed from theory already known from experience with stress in steel and concrete.

The purpose of this paper is to define the variables and constants which most influence Colorado's mining industry. Once they have been established, they can be used to forecast the future performance of the mining industry in Colorado. It is understood that Colorado's mining industry is a part of the whole American economy, which we may call the total economic system. To analyze such a total system would involve intensive and time-consuming investigations. However, we are justified in dividing the total system into its components, which we call sub-systems. The integrated sub-systems represent the total system. Sub-systems are easier to study, because they involve only manageable parameters and a limited number of variables. The Colorado mining industry will be treated as such a sub-system.

One of the most striking features of scientific research and problem solving is its ability to repeat experiments and bring nearly absolute objectivity to the interpretation of the results. The scientific approach applied to the solving of economic problems has in many instances failed because of the lack of objectivity and the influences of illogical, inconsistent, and arbitrary phenomena.

The investigation of scientific systems is facilitated through the exploitation of causality (cause and effect) and the exact definition of the system boundaries. We call these systems closed systems in contrast to open systems. The latter are influenced by random variables, which sometimes are neither useful nor valuable. Closed systems are relatively free of random input. As it will be shown later in the paper, the mining industry in Colorado is an open system; and, unfortunately, this complicates the analysis. We are fully aware of randomness in economic systems; therefore, special mathematical treatments will be applied to adjust for randomness.

In econometric terms, open systems and their representative models are called "ill-structured," and the interpretation of the result is termed "qualitative" because they contain both known and unknown variables (Optner, 1965, p. 21). ✓

Open systems offer a wide range of alternatives for interpretation. It is particularly difficult to forecast future performance. Open systems with their mixed and uncertain characteristics have to be treated most carefully.

Modeling of open-systems still has many shortcomings; therefore any attempt to bring logical order to the gathered information must be treated as experimental. With the passage of time the system will change and the old solution may not be applicable to the new situation.

Econometric Models

The econometric model is a relatively new tool for analyzing the structural behavior of the overall economy and its integrated sectors, also referred to as industries. The model has been extensively used to determine the nature of the business cycle, output of industries, employment, prices and other business indicators.

Optner states that in constructing the model, "we postulate a condition, situation or state, to describe a set of objects, attributes, and relationships in the system" (1965, p. 27). A postulated proposition is considered to be a tentative statement put forth hypothetically. The most significant step in analysis will be to test the reliability and function of the hypothetical model with actual data to determine whether the hypothesis has been valid.

The concepts for modeling consists of establishing a conceptual framework, a base on which the investigator believes the economy will work. The framework will consist of a set of equations, either to be solved simultaneously or individually, which call for interdependence among economic variables. The model builder has many options in the choice of the kind and number of variables needed to explain and work the model within reasonable limits.

In econometric models there are three kinds of variables to link the input-output relationship. The known variables, which go in the equations and then are solved for the unknowns, are referred to as predetermined. These are our empirical data collected for analysis. The unknown variables to be solved by means of the operation of the set of equations on the predetermined variables are called endogeneous.

Variables are regarded as exogeneous if they are determined essentially outside the model. In open systems, such as the economic system, there is often uncertainly in distinguishing between endogeneous and exogeneous variables. We might treat certain variables as exogeneous if they cannot adequately be expressed by means of the mathematical model, although they may have an endogeneous character (Liebenberg, 1966, p. 14).

The impact of exogeneous variables on the system is recognized. However, it is believed that in the system under investigation these have only minor influence on the magnitude and direction of the endogeneous variables.

The Input-Output Model. The model has been applied as a forecasting tool by using the national income account table as its framework. A simplified accounting matrix shows the flow of commodities between the different industries as supplier, and the households as consumer. This can be also named a transaction model: transactions between industries. Inter-industry transactions are believed to form the major part in the economy. National accounts, however, show only the final output, which equals the domestic product. The final product is at the end of the current production line and is, therefore, outside the transformation process involved in inter-industry analysis. Final products go only to the consumer, add to the stock of assets, or are exported. Hence, the product identity of the economy can be written as follows:

$$\text{Domestic product} + \text{import} = \text{consumption} + \text{assets} + \text{exports} \qquad (3)$$

In an input-output table (accounting matrix), the above identity does not show the intermediate product, which is the distinctive feature in an inter-industry analysis. The product identity for all industries is a complete input-output table and is written as follows:

$$\text{Total output} + \text{imports} = \text{intermediate product} + \text{consumption} + \text{asset production} + \text{exports} \qquad (4)$$

The intermediate product of each industry will be the principal item of discussion. We subdivide the intermediate product of each industry in a sum showing all deliveries to other industries and all purchases from all other industries. Their purchases, the intermediate input and the deliveries, the intermediate output, together with items showing the industry's purchase of primary inputs, and respectively, the industrial sales of final product to domestic buyers, and its net exports, form the system of the accounting matrix.

X in Table A denotes the output, the first and second subscript indicate the industry of origin, and the industry of destination, respectively; h, denotes the household sector, final consumer, and simultaneously the supplier of labor; t, the foreign trade balance for the output of each industry, the import treated as a negative output.

The total matrix listed in Table A will be partitioned into three sub-matrixes, showing a matrix of injections which reveal the demand placed on the producing sector by the non-producing sector (top right-hand corner in Table A).

$$\begin{bmatrix} x_{1h} & x_{1t} \\ x_{2h} & x_{2t} \end{bmatrix} \tag{5}$$

The sub-matrix in the top left-hand corner:

$$\begin{bmatrix} 0 & x_{12} \\ x_{21} & 0 \end{bmatrix} \tag{6}$$

is the matrix of transactions representing the flow of commodities between the industries.

The row

$$\begin{bmatrix} x_{h1} & x_{h2} \end{bmatrix} \tag{7}$$

is the vector of factor costs. This sum must be equal in value to the sum of the elements of the right-hand sub-matrix.

TABLE A

SIMPLIFIED ACCOUNTING MATRIX

(according to A. Ghosh)

Purchases	Industry 1	 2	Households	Rest of World
Sales				
Industry 1	0	X_{12}	X_{1h}	X_{1t}
Industry 2	X_{21}	0	X_{2h}	X_{2t}
---	--	--	---	---
Households	X_{h1}	X_{h2}	0	0

In addition, some independent estimates have to be made of the following for every industry in order to treat Table 1 mathematically as a whole matrix:

1. Total gross deliveries
2. Final demands
3. Exports
4. Imports

<u>The Multi-Sector Model of Balanced Growth.</u> This model investigates the existence of an equilibrium (the equality of supply and demand) in a total economy system. Balanced growth describes such an equilibrium in a market at all times in every time period. By employing discrete time intervals we obtain first order difference equations. The model can be prescriptive or predictive, wherever the first difference refers to the immediate future or immediate past, respectively.

Two assumptions are made to facilitate the mathematical operations. <u>Supply</u> in each market is a linear function of the existing capital stock in a sector. Capital, however, is not transferable from one industry to the other. <u>Demand</u> is a linear function of personal disposable income. The linear functions are believed to have an increasing tendency over the time period.

When the net aggregate output in the system is expressed as a function of integral time values (t), then the output can be determined at a time (t + 1) from the structural equation of the model.

The model contains three further identities (definitions): n demand equations, n supply equations, and n equilibrium conditions. The above mentioned equations are discussed in Bear (1966, p. 156-159), and only the final solution of the model will be described.

The aggregate output Y_t for a given industry with its initial income I_o is an exponential function of time written as:

$$Y = I_o \left(1 + \frac{\alpha_n}{\sum_{i=1}^{n} \alpha_i \beta_i}\right)^t \tag{8}$$

The term $\frac{\alpha_n}{\sum_{i=1}^{n} \alpha_i \beta_i}$ is the growth rate of the industry under consideration where α_n is referred to as the marginal propensity to save; α_i, the marginal propensity to consume the ith good; and β_i, the marginal capital--output ratio in the ith sector (sub-industry).

Equation (8) contains a weighted average of the capital-output ratios of the n sectors. Because of the linearity of the supply and demand functions, the weighted average does not change with time. Therefore the growth rate in equation (8) is considered to be the share of net investment divided by a weighted average of the marginal capital-output ratio.

In the calculation of the growth rate, the output Y_t at time t, can be determined; or for prediction purposes at a time t + 1, Y_{t+1} can be estimated.

The Regression Model. The statistical approach of regression analysis is applied to economic problems when the data under consideration are believed to follow certain economic laws. Rather than derive the quantitative relationship with the final solution of a mathematical equation, as in the multi-sector model of balanced growth, the regression requires a hypothesis, that is, a hypothetical model of the manner in which the investigator believes the economy has behaved in the past or will function in the future. Regression analysis is then applied to test how well the hypothesis coincides with the real dependence among variables.

The most accepted models in the economy call for linear relationships between the variables, either one dependent (9) or multiple dependents (10) for the independent one. Two economic conditions postulated below are believed to be of linear character:

$$(C_t)\ D_t = \alpha_o + {}_1 I_t + u_{1t} \tag{9}$$

$$I_t = \alpha_o + \alpha_1 P_t + \alpha_2 K_t + u_{2t} \tag{10}$$

In equation (9) a linear relationship is hypothesized to the effect that demand D_t at a time t, or consumption (C_t) at a time t, is a function of income I_t at time t; u_{1t} is referred to as a disturbance term. Equation (10) shows income I_t as a linear function of non-wage income P_t and net capital stock K_t at time t, u_{2t} is the disturbance term, and α_o, α_1 and α_2 are the regression coefficients.

α_0, α_1 and α_2 have to be estimated; u signifies the stochastic element in the model, the most important feature of regression models.

Whenever estimates are made, we want to know the confidence limits and the magnitudes of our estimation. These limits reflect the effort of the random inputs, u, in the open system which we are unable to detect at the present time.

D_t in equation (9) is the endogeneous variable; I_t, the predetermined one. Equation (9) and (10) are structural and in the present form are useful only as descriptive devices. For usefulness as a forecasting instrument, the structural equation has to make the endogeneous variable a quantitative function of the predetermined variables alone.

Except for specific linear relations between dependent variables, non-linear dependence among variables is the rule rather than the exception. The concept of the regression model remains; only data transformations are necessary.

Polynomial regression analysis of the general form

$$Y = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \ldots . \alpha n t^n \qquad (11)$$

will be applied to all gathered data, where time t is the predetermined variable. Hence, the linear case is treated only as a first degree polynomial and included in the process of analysis.

A polynomial curve of regression does not always satisfactorily describe the association between variables.

Frequently, the relations are more complex and can be handled only by other mathematical functions, as:

a. Exponential

b. Trigonometric

c. Logarithmic

The regression model, when expressed by these functions, keeps its main characteristics; only the symbolic representation differs.

INTERPRETATION AND RESULTS

Data Source

The data used in the study have different sources. *The Colorado Yearbook 1962-1964* provided most of the general statistics and some mining data. The United States Department of Commerce publishes data continuously on general business indicators in the monthly issue of *Survey of Current Business*. Data concerning earnings and employment come from the United States Bureau of Labor Statistics. *The Annual Report* of the Colorado State Bureau of Mines is responsible for most of the information about mineral production and mineral value. Some production figures were taken from the *Mineral Industry Survey* of the United States Bureau of Mines. National mining statistics have been obtained from *Minerals Yearbook*, Part I-III. Mineral fuel data are published by the American Petroleum Institute's *Petroleum Facts and Figures*. Useful information on employment in Colorado was obtained from the School of Business, University of Colorado.

All data, in the form of yearly averages, are presented in Appendix A. For assurance of sufficient storage in the computer memory, time has been coded for calculations.

Time in years starts with an initial observation at time 0, ascending in even number to time t, where t corresponds to equals $n - 1$ observations.

ORIGINAL	CODED	OBSERVATION
1955	0	1
1956	1	2
--	--	--
1965	t	11

Data Manipulation

It was initially planned to include all variables available for the model in the analysis. The total number of variables, however, would have exceeded 100. Early computations have shown that only a few variables are really significant in the model, and that more parameters, exclusively related to the mining industry, have to be added. For the final analysis, the following mining statistics are believed to contribute most pertinently to the models in any case:

1. Employment services
2. Production and its dollar-value
3. Capital inputs
4. Metal prices
5. Capital sources
6. Consumption of metal products.

Tables I through IX, Appendix A, present data in each of these categories.

Regression analysis is most helpful in determining the variation of the data with respect to time. Several variations in technique provide useful insights.

<u>Polynomial Regression</u>. Polynomial regression analysis considers only one variable at a time. Increasing degrees of refinement are obtained by including increasingly higher powers. By this means very close fits to the given data can be obtained, and the most accurate predictions for the immediate future can be made. This technique has been applied to the available data on the yearly basis with the intent of determining the degree of correlation. This work is discussed in detail subsequently.

Once the general picture of the variables had been obtained, the next step was to determine the interdependence between the mining activity indicators. By means of multiple regression analysis, two models have been tested:

a. Employment

b. Production

<u>Multiple Regression</u>. On examining the results of these analyses involving multiple variables it became apparent that better correlations might be obtained if suitable transformations of the data were made.

In certain instances it seemed appropriate to substitute the logarithms of the numbers for the numbers themselves.

In other cases it appeared that the effects of certain changes might be delayed longer in appearing in some variables than in others. Better correlations were in fact obtained when the raw data for one or more of the variables were advanced or retarded in time, with respect to the unaltered variables. This transformation will henceforth be referred to as "lagging."

Once the multiple regression coefficients had been computed, a quantitative relationship could be established to enable us to predict future change of one indicator with respect to several others.

An attempt was made to combine polynomial regression and multiple regression to obtain improved accuracy in prediction. Polynomial regression was used on each variable to predict the values for the next year. These terms were then used with the multiple regression technique to obtain its predictions.

All of this work was done on the G.E. 225 computer at the Denver Mining Research Center of the U. S. Bureau of Mines.

Analytical Comparison of Models

The previous discussed models have had various applications in economic forecasting and business cycle descriptions. Their principle of operation relates to three key elements:

a. Data collection

b. Hypothesis postulation

c. Analysis of variables

The collected data should show the patterns (amplitudes and duration) of the business cycle. It is understood that the data are unbiased and present the average of input or output in the industry over a period of time, t. The yearly average is more a convenient presentation than a desirable one. Short term averages, on a quarterly basis, would contribute much more to the analysis. Unfortunately most of the accessible information is available only as yearly issues. Other than through the selection of the data, the investigator has no influence on the data.

How different mathematical treatments can be applied to gathered business indicators has been demonstrated by means of three models. To favor a specific one is not to indicate or prove that the selected model is superior to the others. Each model serves a different purpose, and rather than compete for attention they should be considered as mutually valuable compliments to the analysis.

The input-output model can best describe the overall national economy. The model requires a detailed tabulation of inter-industry flows of commodities in addition to import-export estimations. The number of variables used for input are in the range of 200. These variables cover most of the industries participating in the economy. As it can be seen from Equation (4), the matrixes become quite extensive and tedious to handle mathematically. Especially the inter-industry transactions, referred to as intermediate products, are hidden in the complexity of supply-delivery actions. An individual analysis of every industry has to be performed prior to mathematical execution of the national account table.

The input-output table is not suitable for state-wide economic application, like the one for the economy of Colorado, because of uncertainties in the estimation of imports and exports. These uncertainties raise the question of numerical accuracy of the figures. An input-output model for state economy analysis would require a rather drastic disaggregation of information. Disaggregation, however, has not advanced far enough to allow direct yielding of the actual physical structure of various industries.

The model in the present form has its best application as a device for use in planning the future economical structure. Due to the lack of detailed statistical information, an input-output table represents in excellent fashion the statistical reconciliation of separate data. With respect to Colorado's mining industry, the accounting matrix as presented in Table A will well serve as an excellent planning tool. For planning we are concerned with what the producer must do rather than will choose to do to maintain a given level of future final output. For prediction we wish to know what the producer will choose to do now in response to recent levels of demand for final output. For final demand, however, in an input-output table, the categories tend to be decreed rather than foreseen.

The model of balanced growth was presented in order to illustrate modeling of what is known as a production system. Only part of the economic information is used for the structural analysis. We are interested mainly in output or change in productivity whereby the main input variables in the model are labor input and capital input over a period of time. It is further assumed that the economy balances out in a kind of equilibrium. Unfortunately, because the reality faces a rather complex discontinuity, it is substantially different from the model. The functioning of the model will largely depend on how well we are able to estimate or determine the input variables in equation (8). Applied to the mining industry, the model can adequately describe the production, the productivity index, and the output in dollar value. The model, however, will definitely fail to predict such variables as employment, earnings, and mineral values.

The growth model as postulated in equation (8) requires the estimation of three mutually independent variables, αn, αi, and βi. The determination of these parameters has to be performed by means of another model. Frequently it will happen that these constants can be derived only with the use of data which are now out of date and which no longer apply to the present or future situations. Therefore, the accuracy of the output estimation by the growth model may be open to question.

Whenever data from an open system with additional random input and no specific orientation in dependence are available, statistical tests, in addition to graphic and numeric presentation, are today's best mathematical tools to gain insight into the physical structure of the system. Regression models account quantitatively for the stochastic element in the system under investigation, here also referred to as disturbance. Equation (12) simply states that production P is a linear function of labor input, L, and capital input, K.

$$P = \alpha_1 L + \alpha_2 K + u \tag{12}$$

where the disturbance term u allows the deviation from the ideal linear model. The term u can be determined to any desired accuracy, depending on the number of observations available and their accuracy. However, u does by no means explain the character of the deviation, or source of external influence.

The regression model is a mathematical checking device, in a diagnostic sense, which permits us to test the hypothesis of a certain dependence between variables.

The input-output model is rather a display of data input, well structurized, but does not account for randomness. The model of balanced growth being a more theoretical approach toward modeling the physical structure of the economy does not account for randomness either.

With the aid of high-speed electronic digital computers, mass data collections can be processed in extremely short time periods, as compared with conventional methods. This results in considerable savings in tedious and time-consuming hand calculations; hence more time can be spent on the important output analysis.

The possibility of reducing the data with already available statistical programs on high-speed digital computers, the possibility of determining stochastic elements in the model, and the possibility of manipulating comparatively limited amounts of data for a reasonably accurate descriptive model are considered to be the main reasons for favoring the regression model over the others in this specific research project.

Formulation of a Production Model

An extended classical production function has been used to prove the validity of the proposition that any physical output, Q, of an industry is essentially determined by a change of the input parameters. These key input parameters are those variables which have the most influence on the output. Although a variety of input parameters can be used, the model will restrict the key variables to capital input K, employment services E, and time t.

In the past years, however, many empirical studies have supported what many economists have suspected -

physical output and increase of productivity have to be considered as a consequence also of changes in technology rather than increased capital only. These changes extend our production function to

$$Q = A \cdot f\,(K,E)_t \tag{13}$$

Where A is an index of the level of technology and f indicates a generalized function. More will be said of this later.

The generalization of equation (13), however, has some limitations which restrict the quantity of output, Q, if certain environment conditions change. The environment for a product is the domestic market, with its determination of price P, consumption C, capital source K_s, capital expenditures K_e, and national income I.

Including these variables in our production model causes it to take the form:

$$Q_t = A \cdot f\,(K_e, K_s, C, I, P, E)_t \tag{14}$$

After the definition of the input variables, the function f has been postulated to be either of additive or multiplicative character. This means that our treatment of equation (14) as a sum or a product leads to equation (15) and (16):

$$Q_t = \alpha_0 + \alpha_1 K_{et} + \alpha_2 K_{st} + \alpha_3 C_t + \alpha_4 I_t + \alpha_5 P_t + \alpha_6\ t \tag{15}$$

$$Q_t = \alpha_0 \cdot A . K_{et}^{\alpha 1} . K_{st}^{\alpha 2} . C_t^{\alpha 3} . I_t^{\alpha 4} . P_t^{\alpha 5} . t^{\alpha 6} \tag{16}$$

The change of technology index in equation (15) has been omitted; it is believed that the time variable t comprises A. The reason for incorporating A in equation (16) is the evidence that only the product model permits one to deduce A mathematically. It has also been proven that A has a multiplicative action on productivity. It is not the purpose of this paper to calculate A quantitatively; we might conclude, however, that industries following the model presented as equation (15) produce on a constant level of technology during the time period t, whereas industries following the model of equation (16) are believed to adjust their production immediately according to innovations in technology.

In the final form of the model, employment services E have been omitted because of a lack of detailed information. Since it is assumed that there is no resource limitation with labor, changes in output are assumed to produce corresponding change in the number of people employed, rather than fluctuations in employment causing corresponding change in output.

The Mineral Production Model

The mineral production model affords a means of predicting the production of specified metals if one knows the manner in which their production is related to that of other key metals. Thus there may be a relationship between the production of vanadium from the Colorado Plateau to the production of uranium in that area, since both elements occur in carnotite, a common source of uranium.

In using the mineral production model, however, one must make sure that deduced production is actually related to the key production. Copper and uranium are not related, as they seldom occur in the same deposit; and copper is produced for sale in an open market, whereas uranium has, until very recently, been sold in a closely controlled market. Furthermore, production figures may not be entirely accurate for uranium and vanadium, whereas copper production is not considered to be a matter of strategic information.

Normally an entrepreneur will expand and contract production in accordance with market requirements. This is true regardless of the kind of market in which he sells. Typically, when his costs of production exceed his revenue, he will reduce production drastically or close down altogether. There are, however, two major exceptions:

(1) The producer has a near monopoly on his product. In this case he finds it relatively easy to adjust his prices upward, while working to lower his costs, until his operations are again profitable.

(2) The producer is subsidized in some fashion, the subsidy representing the difference between his costs and his income. For example, a captive mine may in effect be subsidized by the industry for which it produces. Or a government may, in an attempt to encourage the production of uneconomic

metals, provide a guaranteed market or a subsidy on production as a means to induce a desired level of output.

In either case the metals involved are not suitable for use in a mineral production model. Both molybdenum and uranium-vanadium are relevant as examples of the two exceptions. In addition, most of their data being concealed or classified is unaccessible for interpretation.

Gold has also been omitted because it cannot be traded freely in this country, and the government sets the fixed gold price.

To represent the mineral fuel sector, oil and coal have been analysed in order to compare their economic behavior in contrast to metals.

The input variables in the model have been specified according to equation (14). Capital input in the mining industry is represented by new plant and equipment expenditures. The capital inputs in oil production are the expenditures to maintain, service, and develop production. It is postulated that the capital sources come from the total U. S. mineral production.

The domestic consumption is treated as demand and does not include minerals going into the stockpile.

The national income can be considered as the source for demand and additional source for capital expenditures, respectively. The mineral price is believed to be production-determined. The entrepreneur will adjust his operations according to the profits on the domestic market. Unfortunately, the above listed variables have exogenous character; they are essentially determined outside our model. The materials chosen are those marketed nationally because there is too little information about materials sold only in local markets. Coal and oil provide the two exceptions. Their local markets are large enough to be well defined. But in contrast to metal prices they simply represent the ratio of dollar value to sold quantity. This representation means that the actual mineral fuel price is calculated after production, whereas the metal price is known before production. However, there is good reason to believe that the factors which influence the local markets in these commodities are very similar to those which affect producers anywhere in the country, so it is not inconsistent to use these commodities in the model.

Practical considerations commonly prevent a producer from making immediate changes in his rate of production in response to changes in costs and market demand. The rapidity with which he can respond appears to vary somewhat from mineral to mineral, or metal to metal. Better correlations can frequently be obtained if some data are lagged. This lagging requires a slight modification in the production model equations.

After least square estimation of the α_i parameters, denoted by Roman letters a_i, our mineral production model will consist of three basic equations:

$$Q_t = a_o + a_1 K_{et} + a_2 K_{st} + a_3 C_t + a_4 I_t + a_5 P_t + a_{6t} + u_1 \tag{17}$$

$$Q_t = a_o + a_1 K_{et} + a_2 K_{s(t-1)} + a_3 C_{t-1} + a_4 I_{t-1} + a_5 P_{t-1} + a_6 t + u_2 \tag{18}$$

$$\begin{aligned} \mathrm{Log}Q_t = {} & \mathrm{Log}a_o + a_1 \mathrm{Log}K_{et} + a_2 K_{s(t-1)} + a_3 \mathrm{Log}C_{t-1} + a_4 \mathrm{Log}I_{t-1} \\ & + a_5 \mathrm{Log}P_{t-1} + a_6 \mathrm{Log}t + \mathrm{Log}u_3 \end{aligned} \tag{19}$$

$$+ a_5 \mathrm{Log}P_{t-1} + a_6 \mathrm{Log}t + \mathrm{Log}u_3$$

The Employment Model

The model which will be described in this section has been formulated by Brechling and O'Brien. The main features are as follows:

1. Employment in an industry is demand-determined. There are no supply constraints.
2. Desired labor or employment services E_s can be written as a function of three variables, namely, output, Q; capital input, K_e; and an index of technology, A.

$$E_s = A \cdot f \cdot (K_e, Q)_t \tag{20}$$

3. We assume a log-linear function and regard capital input, K_e, and index of technology, A, as concealed in the time variable, t.

Entrepreneurs seldom adjust their employment upward until they are compelled to do so by a rising demand for their services or product, which cannot be met with their existing personnel. Similarly, when demand slackens, they seldom react immediately by cutting employment, waiting until the buildup of inventory forces action. Thus their response to changes in the business cycle exhibits a lag. For this reason a more accurate form of equation (20) becomes:

$$\text{Log } E_t = \text{Log}\alpha_0 + \alpha_1 \text{Log} Q_t + \alpha_2 \text{Log } E_{t-1} \qquad (21)$$

One can easily obtain data about employment and output as a function of time. Coefficients for equation (21) can be obtained through the application of regression analysis. Letting a_1 represent $_i$, where a_i is the coefficient obtained for the least square estimate, we obtain:

$$\text{Log } E_t = \text{Log } a_0 + a_1 \text{Log } Q_t + a_2 \text{Log } E_{t-1} + u \qquad (22)$$

Equation (22) is the type-equation of the employment model.

Polynomial Regression, an Auxiliary Model

After evaluation of the multiple regression equation, revealing the multiple regression coefficients a_i, we obtained the functional relationship between the dependent variable y and its independent predetermined variables X_j.

Due to a time lag, we covered a period of observations from t-(N +1) to t, where N = number of observations.

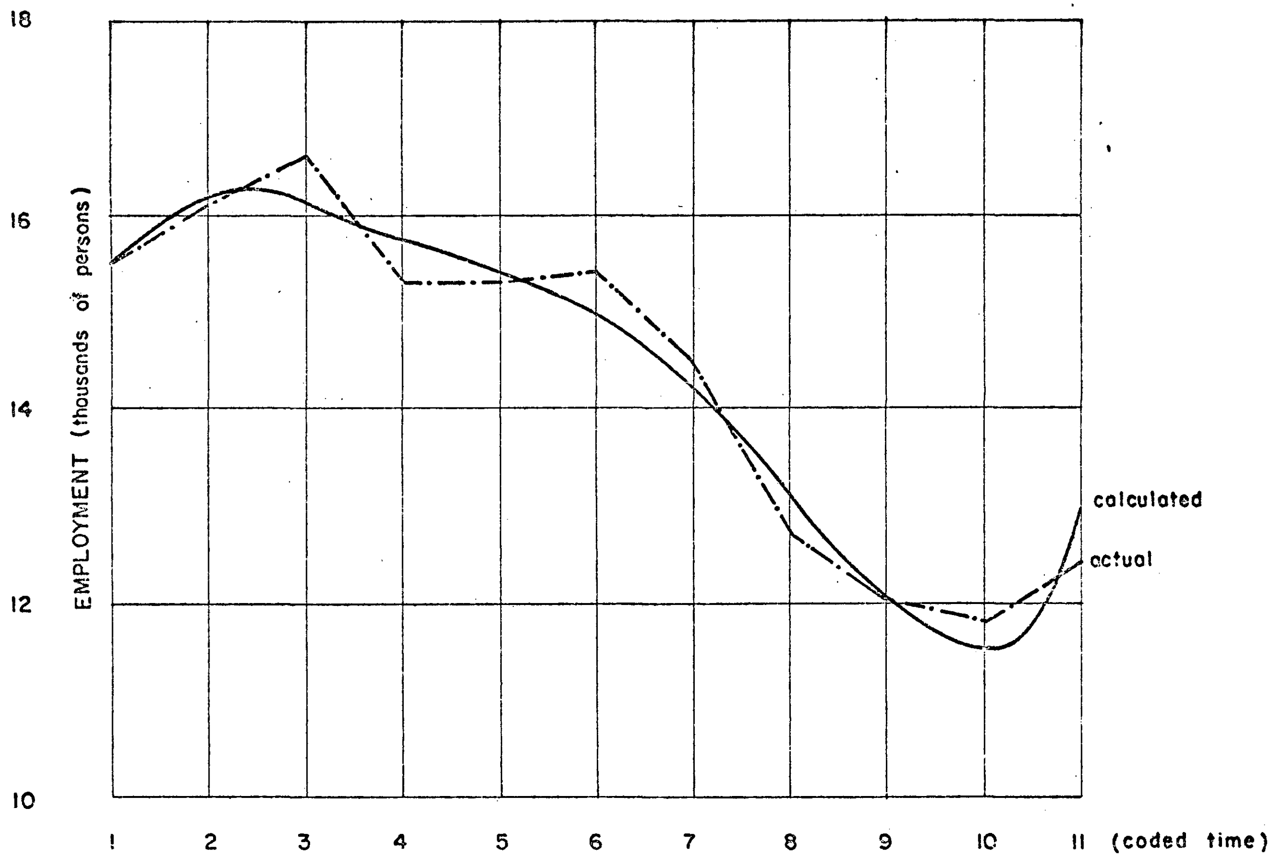
In order to get independent estimates of predetermined variables X_j, we use the polynomial regression technique to estimate X_j at the time t +1.

A given set of binary data can be described by a variety of polynomial equations; and if sufficient care is taken, an equation can be obtained which exactly fits the data. However, this is not necessarily the most useful equation for extrapolation. In practice it seldom pays to go beyond a 5th-power equation. Within this range the best fit is indicated by the "F-statistics," the ratio of the variance removed by the correlation to the residual variance of the estimate.

It is not entirely satisfactory to rely on the F-estimate to indicate the best equation for predicting future values of the dependent variable, since the best-fitting curve has the most curvature. It has been demonstrated that a straight line can cut a plane curve of x-power only x times. Thus there is no possibility of the curve recrossing the line of the real data beyond the limits of the data used. This is illustrated by the data of Table 10 (Appendix B), plotted in Figure 1. It is unlikely that actual employment will increase in the next year or two as rapidly as the calculated curve indicates.

A better criterion is to select that equation which indicates values for the dependent variable closest to those for the last year or two. This technique introduces a personal bias; however, it has considerably improved the forecasting technique.

Predetermined input variables for the multiple regression model are listed in Table 11 and Table 12. They are called <u>endogenous</u> and <u>exogenous</u> due to their character of determination: either directly related to Colorado's mineral industries or determined outside the domestic industry.



REDUCED COMPUTER PLOT FOR EMPLOYMENT

FIGURE 1

Table 11 and Table 12 also list the standard error, STD, of the estimation and actual deviation from the true value for 1966, called actual and percentage error (%).

Polynomial regression has also been applied to production and employment statistics, Table 16 and Table 17. The results from this study will serve as a comparison to the analysis of multiple regression.

The coefficients of the equations by means of which Tables 11, 12, 16 and 17 have been derived are listed in Tables 13, 14, 15, and 15A, respectively (Appendix B). The regression coefficients are denoted by a_1.

Comparison of Predicted Variables With Their Actual Value

The multiple linear regression model, represented in mathematical form by equations (15), (16), and (22), has been used to forecast mineral production and employment in Colorado. Selection of the appropriate model for each individual mineral was based on the F-statistic. The F-statistic is the overall correlation indicater enabling us to determine the significance of multiple correlation. The nalysis revealing the highest F-statistic has been chosen as the representative model.

The summary of the total analysis for the different minerals is tabulated in Tables 20 to 25 (Appendix C). Testing equation (15) reveals statistics listed under Case I, testing equations (16) and (22) produces the information listed under Case II and III, respectively. As already mentioned, Case I treated the variables as synchronous with respect to time; Case II had certain independent variables lagged one year. Whereas in Case III a logarithmic data transformation was applied, using Case II structure.

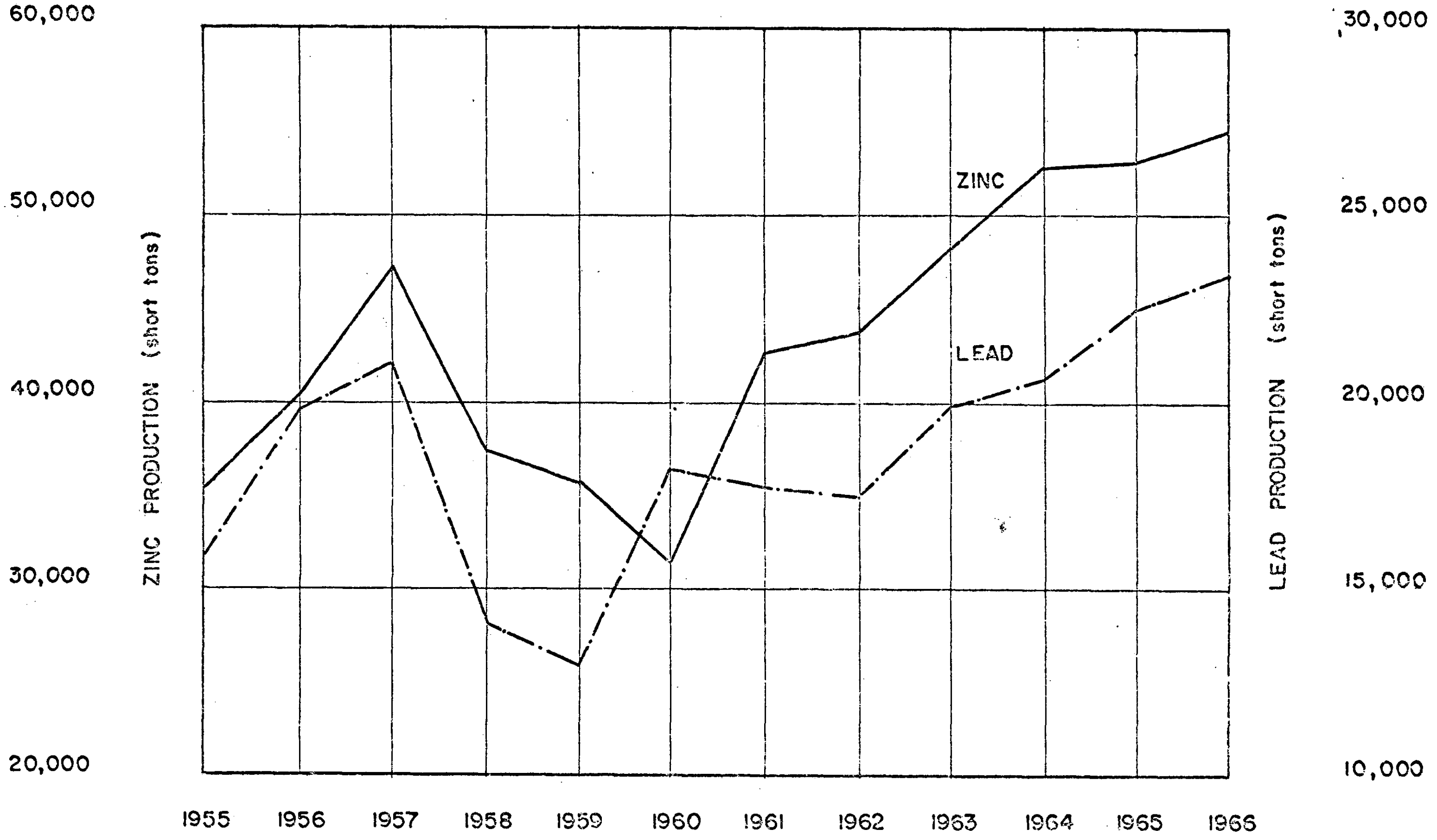
Tables 18 and 19 list the regression coefficients with their standard error for the production and employment function (Appendix C). By means of these equations the estimated or predicted variable is calculated with the use of input estimations from the polynomial regression model.

An error analysis has also been performed on the input variables in order to demonstrate the quality of mathematical forecasting.

Whenever a certain forecasting technique is applied, one is interested in how well the technique is able to approach the actual value. For a quantitative understanding of the deviations, it is convenient to select actual values which are known one time period beyond that of data used.

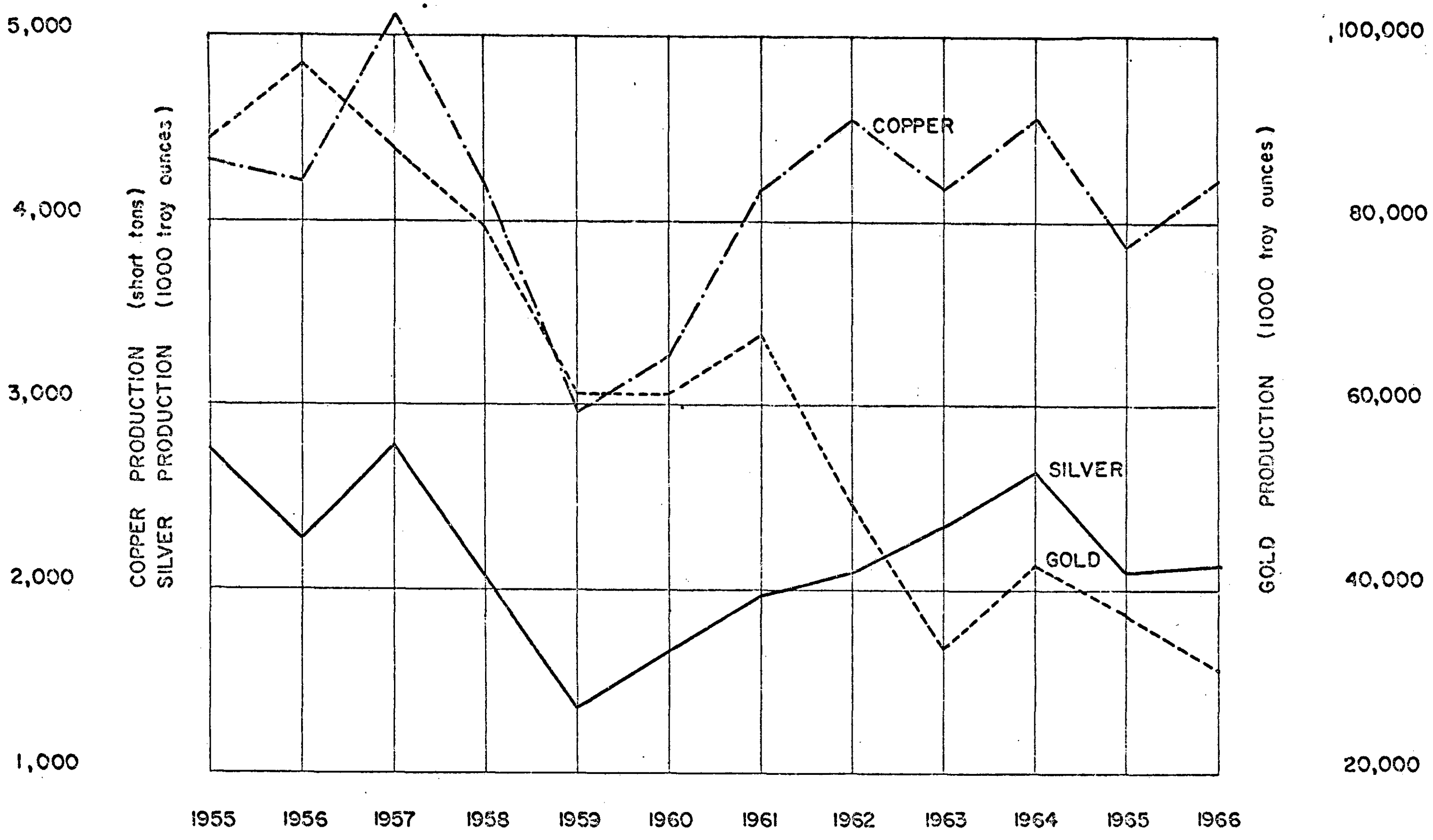
In our example, statistics from 1954 to 1965 have been used to construct the models. These models have been used to predict values for 1966, which are compared with true actual values obtained later (Table 26, Appendix C).

Mineral Production. Forecasting selected mineral production in Colorado with the polynomial and multiple linear regression model has revealed that specific mineral industries show different deviations in magnitude when the predicted value is compared with the actual one.



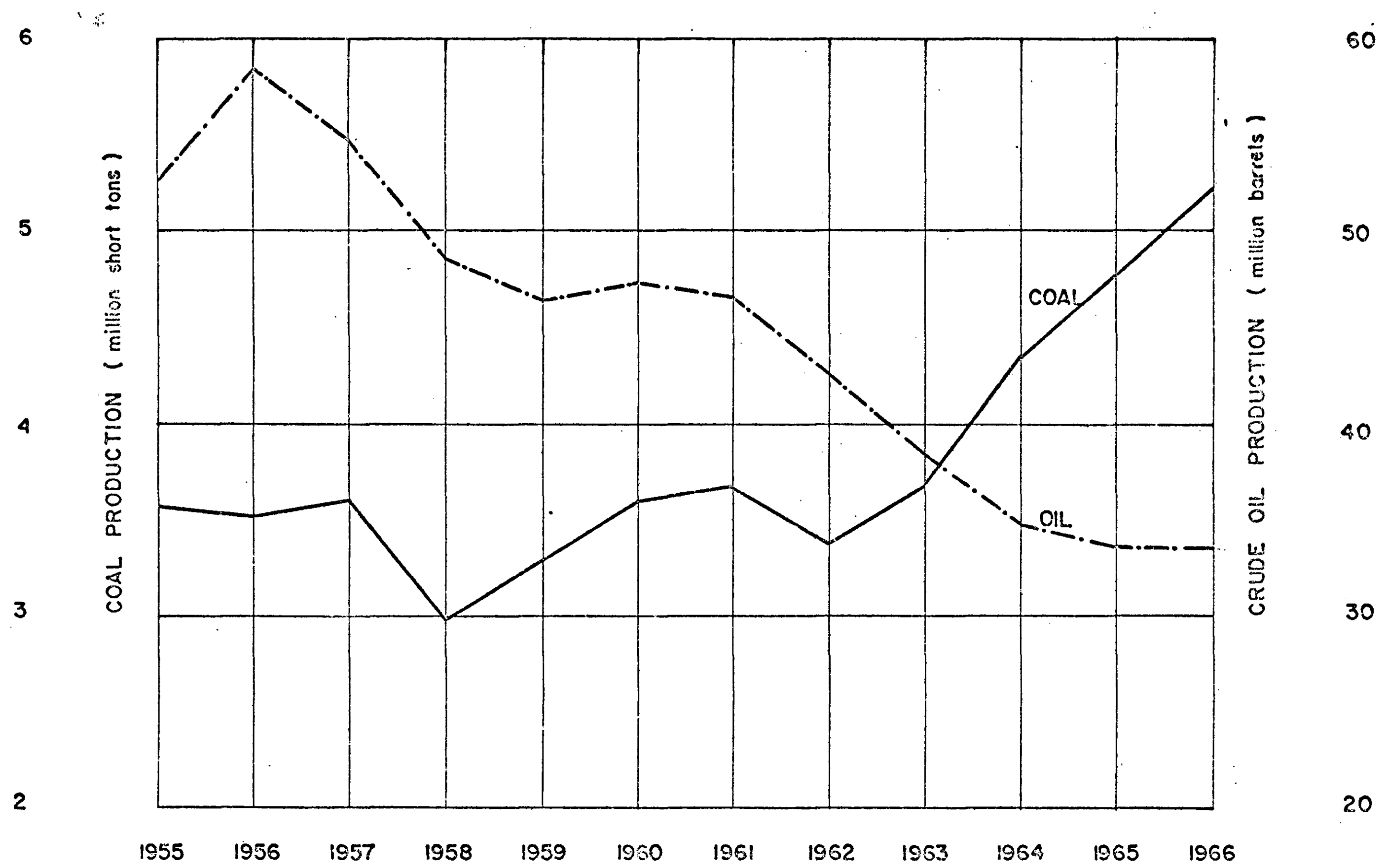
COLORADO METAL PRODUCTION 1955 - 1966

FIGURE 2



COLORADO METAL PRODUCTION 1955 - 1966

FIGURE 3



COLORADO MINERAL FUEL PRODUCTION 1955 - 1966

FIGURE 4

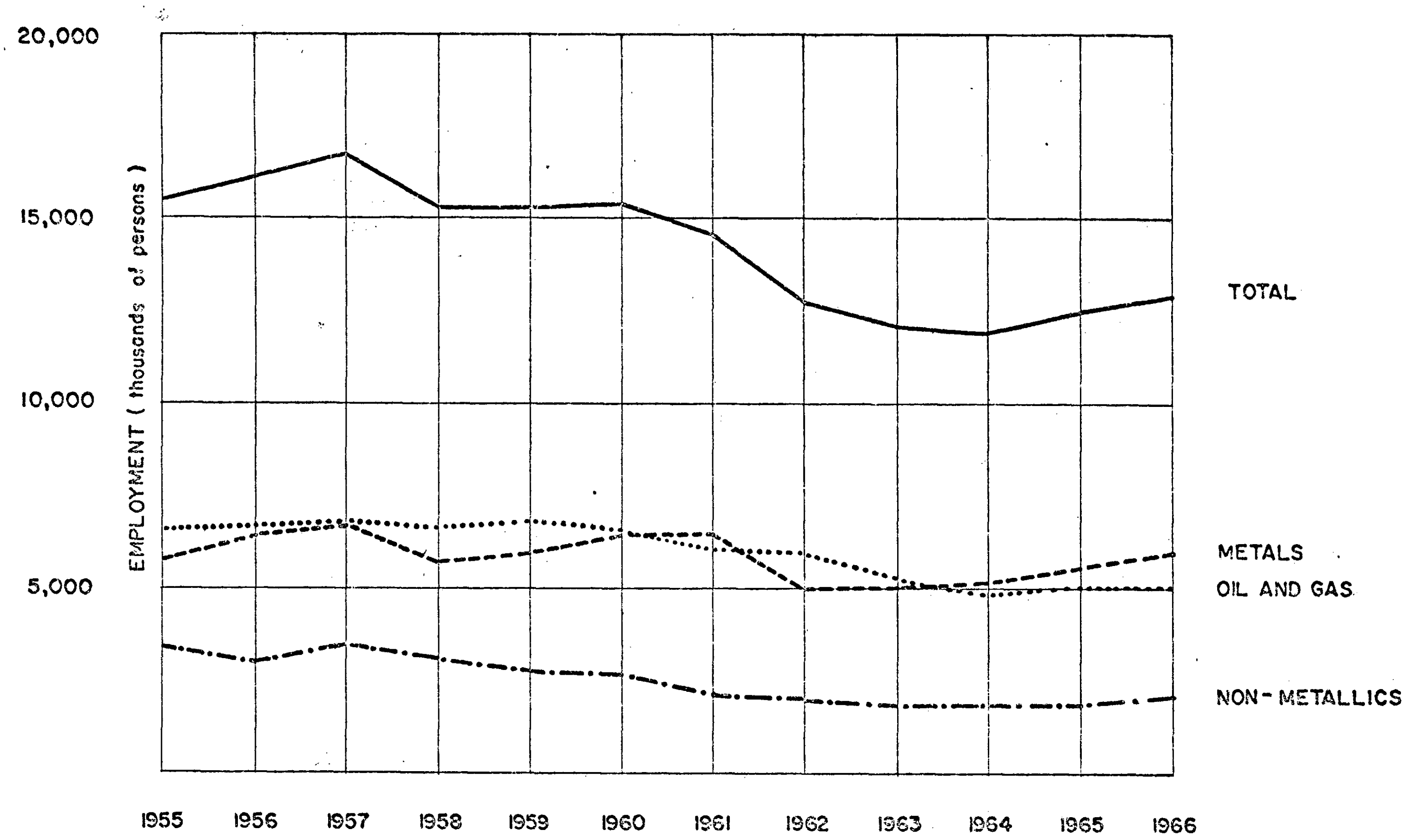
The average error with the polynomial regression (P.R.) technique is 5.20%, whereas the multiple linear regression (M.L.R.) techniques indicate an average error of 17.28%. With respect to the actual or average error in per cent it can be stated that polynomial regression is superior to multiple linear regression.

It is striking that the data from the copper-silver industry does not fit the M.L.R. model formulation, whereas lead-zinc is relatively easy to analyze. One might conclude from the statistics in Table 26 that the expected forecasting error in an industry is closely related to the volume of mineral production. The observation is at least valid in a follow-up comparison of silver and lead-zinc.

It should be mentioned that out of twelve estimates, nine showed a strong tendency to be underestimated. In the M.L.R. model five estimates out of six are underestimated. It is believed that this is personal error induced by the estimator. It can be removed by incorporating an adjustment factor.

The statistics on metal and mineral fuel production have been displayed graphically on Figures 2, 3, and 4. These figures familiarize the reader with the production cycle over the past eleven years and serves as a supplement to Table 26.

Employment. Forecasting employment for three different mineral industries can easily be performed with the polynomial



EMPLOYMENT IN THE MINERAL INDUSTRY OF COLORADO
1955 – 1966

FIGURE 5

regression model. Deviations of the estimations, as the error analysis shows, are extremely small; the average error is 1.88%.

The average error in the M.L.R. model is calculated to be 15.74%. One is inclined to reject the M.L.R. model as a forecasting tool for employment. It is also noted that all predictions are underestimated (Table 26).

The graphical display of the employment statistics is shown in Figure 5.

Error Analysis in Forecasting Models

Forecasting errors, as has already been pointed out, will occur in any model. First the data collection may contain errors; some data may have to be adjusted after compilation. Second, the estimation of the predetermined variables may differ greatly from the actual value. These two error sources should be distinguished from the different error which stems from the formulation of the model. As the analysis has indicated, it is not possible to take full account of all causal factors that determine the dependent variable in its magnitude and direction.

For example, the stochastic character of some of the variables has been noted. Even if the model includes all the pertinent factors, the predicted value will be likely to differ from the actual one because of the elements of uncertainty in the various forms.

Needless to say, a personal bias toward variable selection and estimation of the predetermined variables will influence the forecasting technique. However, experience may indicate the natural and direction of the bias.

The Econometric Structure of Colorado's Mineral Industry

Upon decision on the appropriate model for the structural behavior for a specific mineral industry, it becomes necessary to define the key variables which are believed to have the most impact on a change in physical output. It is of importance also to eliminate the variables which do not contribute considerable to the overall change.

Selection of the key variables will be based on their individual correlation with the dependent. Tables 20-25 in Appendix C present variable correlation for a number of important mineral products.

Lead-Zinc Industry. The metal industries similar in behavior are the lead-zinc and the silver-copper. For the lead-zinc, the metal prices do not have the expected effect on production. In both instances, the metal price reveals the lowest correlation to metal production, (Lead $r^2 = .418$, zinc $r^2 = .430$). The magnitude of metal production will therefore not be determined by the metal price. The time variable containing the index of technology has the second lowest rank (lead $r^2 = .449$, zinc $r^2 = .677$).

It might be concluded that the lead-zinc industry is rather stable technologically. This is true for the mining methods as well as for mineral-processing techniques.

Variation in output will be caused by consumption and capital expenditures in new plant and equipment.

Zinc mining is able to change production immediately. Lead requires a lag period with respect to demand. Lead production developed its highest correlation to demand with a one year time-lag, $r^2 = .699$, whereas zinc production adjusted synchronous with demand, $r^2 = .723$. The high correlation of Colorado's lead-zinc production to U. S. mineral value is partly due to the fact that production itself is contained in the total mineral value. (Mineral value = physical output . price). The major conclusion is that Colorado's lead-zinc industry follows the national trend in production magnitude. The analysis also indicates the arithmetic structure of the production model. It can be anticipated that variables, having an additive character, as in the lead-zinc industry model, are all significant with respect to physical output. We miss extreme overall correlation deviations (Tables 21, 22).

Copper-Silver Industry. To analyze the copper-silver industry seems rather complex, at least on the first glance. There is no significant correlation among the variables (time, consumption, mineral value, new plant and equipment expenditure and, national income) which might lead to a definite conclusion.

When the single correlation coefficients are ranked it appears that silver production is most affected by capital expenditures, whereas copper production shows a tendency to be metal-price oriented (Tables 20, 23). An increase in copper production can be attributed to an increase in consumption and in capital expenditures. The above statements, however, must be considered to be tentative.

The only positive conclusion we can draw is that key variables used in the input are not really pertinent. As a matter of fact, most of Colorado's copper comes from one mine. Consequently, the stochastic element is much stronger and results in misleading conclusions. For a proper portrayal of the copper-silver industry, different and new variables have to be considered. Perhaps intra-state information on capital expenditures and capital source would clarify the picture. If this were to fail, it might even raise doubts about the concept of the model formulation.

The answer lies in the peculiar structure of the silver-copper industry. Both metal industries show a strong, even a striking, tendency towards a product model. As already indicated in the previous chapter, the index of technology, defined here as an increase in productivity if labor and capital input remain constant, becomes the important feature of the industry.

Due to a raise of over 83% in the silver price (from \$1.29/ounce to \$2.37/ounce) over the past two years, a new Colorado silver boom was anticipated, or at least an intensified mining activity for silver. This was slow in developing. At the time of this writing, some months after the major price change, there is just now some indication of increased activity. This may indicate, as the model suggests, that the boom could not develop until problems in exploration and mining technology had been solved.

Mineral Fuel Industry. Figure 3 displayed graphically the mineral-fuel production in Colorado. Since 1962 coal has been produced at a rate increasing by 460,000 tons per annum. The correlation values are shown in Table 25. Consumption is the dominating factor for coal production (r^2 = .770). Despite a steadily decreasing coal price (1958 = 6.49 \$/s.t., 1966 = 4.99 \$/s.t.), the coal industry in Colorado has shown an activity to meet competition and to adjust production accordingly to the market condition. The negative price correlation (r^2 = -.859) indicates that although the tonnage produced has increased appreciably, the value of the production has remained rather stable. This indicates a trend towards low cost methods of coal mining on a larger scale. This is related to the fact that more coal production is used in the generation of power. Since the demand for power is increasing, the need for coal as a fuel is also increasing. The low price for power necessitates highly efficient coal production and a low price for the coal.

Capital expenditures with the lowest rank correlation (r^2 = .637) had the expected minor impact on production. The coal industry of Colorado produces in a closed market, where all production is sold domestically. Therefore we cannot assume an impact on production from external variables, as national capital expenditures. In addition, most of the coal is mined under long term contracts (15 years). This assures a stable production, independent from national and domestic fluctuation on the mineral-fuel market.

The future of Colorado's oil industry, according to Table 24, will depend greatly on capital expenditures and exploration activity. The correlation between production and the number of new oil wells drilled (r^2 = .746), which incorporates the capital expenditures, is significant. Drilling activity was more than two times higher in 1956 than it was in 1966 (1956 = 1222; 1966 = 548). Corresponding to a 1956 production of 58.6 millions bbls., as compared with 33.4 millions bbls. for 1966. If Colorado's oil production is to increase, more new oil fields must be found.

The national as well as the domestic oil price is negligible in its determination of production. The high negative correlation of consumption (r^2 = -.935), national income (r^2 = -.961), and U. S. mineral value (r^2 = -.827) with production in Colorado indicate that oil is too expensive to find in the state, as compared with the cost of prospecting in other areas, foreign and domestic. Surprisingly, the model shows good response in both the arithmetic and geometric versions. This suggests that the present petroleum production technology is not changing rapidly, but that an appreciable change would be accompanied by significant improvements in output level. The largest opportunity for technological change lies in oil shale.

One might conclude, therefore, that the present trend is unlikely to change until the oil shale can be developed in significant fashion.

SUMMARY

Applying polynomial and multiple-linear-regression techniques to predict future production and employment in the mineral industry of Colorado has been shown to be a useful forecasting tool. Projections for the mineral industries with an alternative model has been described by A. Ghosh (1964, p. 46). He used the input-output model in Great Britain's total economy for forecasting purposes. This is the only reference in the literature. Where an attempt has been made to predict mineral activity, the average error found with the input-output model was 8.85%. This compares with the 10.12% error with the author's regression technique.

Due to the stochastic element in econometric models an error will always accompany our prediction, irrespective of how well we are able to determine a functional relationship of business statistics.

There seems to be no reason to favor the multiple regression analysis over polynomial regression, although there is a trend that a single variable input in the model approaches the actual value closer.

The multiple regression technique, however, gives clear indication as to what parameters to observe to obtain an actual picture of the mineral industry.

Capital expenditures as a synchronous variable and consumption, adjusted for time, have the most impact on mineral production, except for copper production, which is price-oriented. Mineral price has surprisingly no effect on physical output. As in coal and oil, it even has a reverse tendency: a price declination stimulates mineral fuel production.

There is definite evidence that individual mineral industries follow a specific law of production, which mathematically can be presented as a multi-variable production function. Where the function is specified as a sum, this gives the variables an additive character (Case A).

By contrast the production function is represented as a product. These variables have a multiplicative character, and it has been postulated that the index of technology is the important parameters in the industry (Case B).

Minerals, which in production follow Case A are lead-zinc and coal-crude oil. Silver and copper are related to Case B. Oil production can also be expressed by model B.

The econometric model, as discussed in this paper, has two shortcomings, which have to be pointed out. It is most desirable to have predetermined variables with endogenous character in the model. Only Colorado mineral production value, and oil and coal price have met this requirement. There are certainly more than six independent variables as input. However, we tried to stay with the classical production function formulation.

The model has served several purposes. It has been possible to characterize the past behavior of the mineral industry's structure. From this picture, we predicted the future behavior. We have also pointed out what the important parameters are which will most influence production.

Employment has the expected trend of being production-oriented, at least for non-metallic production. The adjustment is synchronous. The employment in the metal industry does not show a significant correlation to output. A different model might be introduced to show the expected correlation.

CONCLUSIONS

The system analysis approach to gathered mining statistics on mineral production, employment, consumption, capital expenditures, and mineral prices may successfully be applied in prediction studies by means of a regression model. The regression techniques would seem to be a preferred analysis because of the absence of subjective elements. Mineral production for individual minerals follows certain laws, which can be formulated in two models.

Model "A" treats the dependent variables as arithmetic terms in the production function. The arithmetic character of the model seems to indicate the equivalence of importance of all independent variables involved in the function. The reflection of the equivalence is indicated by the low variance of the individual correlation coefficient r^2.

Model "B" describes the functional relationship between variables in form of a product of exponential factors. It appears in this model that an index of technology has to be defined in order to explain the insignificant correlations of the variables. I conclude that mineral industries, following

Model "B" in production, will greatly depend on innovations in engineering technology.

Both models, however, have to be considered as structurally incomplete. Despite some hopeful results with the limited data, the studies can point only to further research on this subject.

RECOMMENDATIONS

This paper has been the first attempt to describe some mineral industries in Colorado by means of an econometric model. Time and resources restricted the model to a small size, and eight variable relationship.

Reviewing polynomial analysis, the author concludes that the majority of the variables show a striking cycling effect with respect to time. This cycling effect is superimposed on the trend. The author suggests that harmonic regression analysis should be applied to those variables in order to reveal the frequencies and amplitudes. A disintegration of the data to quarterly-yearly information would help to distinguish between seasonal variance and the real business cycle.

Some other functions, as trigonometric and exponential, should be tested on the variables in order to improve the correlation.

Combining variables of arithmetic and geometric character can be the next step to improve the overall correlation in the multiple-regression model. This will also effect the forecasting technique positively.

The variable input then must be limited to parameters with endogenous character.

The lag-structure in all models was set to be for a one-year period. In future work this time period must be determined much more precisely. It has been suggested that an index of technology is an important factor in some mineral industries. The quantitative determination of that index for the mineral industry will enable us to improve predictions for small mineral markets.

A model describing the industrial growth (Massel, 1967, p. 330) has to be formulated in order to calculate the desired index.

To extend the model to general applications in other mineral industries, the author recommends an analysis on data taken from other domestic markets. The author sees the goal in developing an international mineral-production function, because of world-wide interactions on the mineral market.

Appendix A

Collected Data 1954-1966

TABLE 1

COLORADO METAL PRODUCTION 1955 - 1966

Year	Colo. Copper Production (Thousand Short Tons)	Colo. Gold Production (Thousand Troy Ounces)	Colo. Lead Production (Thousand Short Tons)	Colo. Silver Production (Thousand Troy Ounces)	Colo. Zinc Production (Thousand Short Tons)
1955	4.323	88.577	15.805	2.772	35.350
1956	4.228	97.668	19.856	2.285	40.246
1957	5.115	87.928	21.003	2.788	47.
1958	4.193	79.539	14.112	2.056	37.132
1959	2.940	61.097	12.907	1.341	35.388
1960	3.247	61.269	18.080	1.659	31.300
1961	4.141	67.515	17.755	1.965	42.647
1962	4.534	48.882	17.411	2.088	43.351
1963	4.169	33.605	19.918	2.307	48.109
1964	4.653	42.122	20.563	2.626	53.682
1965	3.828	37.226	22.495	2.051	53.870
1966	4.237	31.300	23.450	2.118	54.655

Source: Minerals Yearbook, Part III

TABLE 2

COLORADO MINERAL PRODUCTION VALUE 1957 - 1966

(MILLIONS U. S. $)

Year	Metals	Non-Metallics	Oil and Gas	Total
1957	134.78	38.125	182.39	355.31
1958	109.08	42.707	162.17	313.96
1959	130.69	45.635	155.53	331.86
1960	152.62	39.206	161.44	376.33
1961	170.17	43.497	159.49	396.70
1962	126.84	46.671	153.12	337.53
1963	139.46	47.905	129.49	340.87
1964	150.08	47.646	117.94	338.01
1965	154.32	47.821	116.33	345.37
1966	172.00	49.200	115.30	366.30

Source: Annual Report, <u>Colorado State Bureau of Mines</u>

TABLE 3

COLORADO MINERAL PRODUCTION VALUE 1957 - 1966 (MILLIONS U.S.$)

Year	Molybdenum	Uranium	Vanadium	Zinc	Lead	Gold	Copper	Silver
1957	50.109	40.425	12.495	11.851	6.145	3.093	3.605	2.523
1958	30.472	44.769	12.019	9.948	3.568	2.584	1.673	1.860
1959	45.696	49.404	12.793	8.269	3.060	1.921	1.646	1.213
1960	62.038	51.682	15.603	8.172	4.420	1.992	2.254	1.502
1961	65.256	60.957	17.038	9.915	4.062	2.394	2.459	1.817
1962	46.695	45.551	9.964	10.346	3.270	1.719	2.530	2.265
1963	66.393	33.479	10.117	13.102	4.578	1.355	2.553	2.951
1964	72.889	28.076	14.735	14.838	6.108	1.922	3.840	3.396
1965	78.025	21.026	15.695	15.208	7.502	1.396	3.385	2.652
1966	87.212	22.223	20.081	17.333	7.586	1.212	8.562	3.346

Source: Annual Report, Colorado State Bureau of Mines

TABLE 4

U. S. METAL PRICES 1954 - 1966

Year	Copper Price New York (Cents/pound)	Lead Price New York (Cents/pound)	Silver Price New York (Cents/5 ounces)	Zinc Price New York (Cents/pound)
1954	29.94	14.05	85.25	11.19
1955	37.39	15.14	89.10	12.80
1956	41.88	16.01	90.83	13.99
1957	29.99	14.66	90.82	11.90
1958	26.13	12.11	89.04	10.81
1959	30.82	12.21	91.20	11.96
1960	32.16	11.95	91.37	13.45
1961	30.14	10.87	92.45	12.05
1962	30.82	9.63	108.37	12.13
1963	30.82	11.14	127.91	12.51
1964	32.17	13.62	129.30	14.07
1965	35.19	16.00	129.30	15.00
1966	35.82	15.12	129.30	15.00

Source: Metal Statistics

TABLE 5

U. S. METAL CONSUMPTION 1954 - 1966

Year	U. S. Copper Consumption (Millions Short Tons)	U. S. Lead Consumption (Million Short Tons)	U. S. Silver Consumption, Arts & Industry (Millions Troy Ounces)	U. S. Slab Zinc Consumption (Millions Short Tons)
1954	1.235	1.094	85.	.884
1955	1.336	1.212	100.	1.119
1956	1.367	1.209	100.	1.008
1957	1.239	1.138	95.	.935
1958	1.157	.986	85.	.868
1959	1.183	1.091	103.	.956
1960	1.148	1.021	100.	.877
1961	1.237	1.027	105.	.931
1962	1.352	1.109	110.	1.031
1963	1.425	1.163	110.	1.105
1964	1.493	1.202	123.	1.207
1965	1.526	1.241	137.	1.354
1966	1.580	1.276	150.	1.289

TABLE 6

U. S. MINING AND GENERAL STATISTICS 1954 - 1966

Year	U. S. Mineral Production Value (Billions U.S.$)	U. S. New Plant & Equipment Expenditures (Billions U.S.$)	U. S. National Income (Billions U. S. $)
1954	14.032	1.021	299.673
1955	15.792	.957	330.2
1956	17.365	1.241	349.2
1957	18.113	1.243	364.0
1958	16.529	.940	400.5
1959	17.241	.990	415.5
1960	17.892	.990	427.8
1961	18.230	.980	455.6
1962	18.838	1.110	478.5
1963	19.615	1.040	517.3
1964	20.507	1.190	559.0
1965	21.433	1.300	610.0
1966	22.906	1.470	616.7

Source: Metal Statistics, Survey of Current Business

TABLE 7

MINERAL FUEL STATISTICS 1954 - 1966

Year	Coal Price Colorado ($ per Short Ton)	U. S. Coal Consumption (Millions of Short Tons)	Colo. Coal Production (Millions of Short Tons)
1954	5.54	363.0	2.899
1955	5.63	423.4	3.568
1956	5.66	432.8	3.502
1957	6.08	413.6	3.594
1958	6.49	366.7	2.974
1959	6.39	366.2	3.294
1960	5.85	380.4	3.607
1961	6.20	374.4	3.678
1962	5.92	387.7	3.379
1963	5.93	409.2	3.690
1964	5.38	431.1	4.355
1965	5.10	459.1	4.790
1966	4.99	486.2	5.222

Source: Minerals Yearbook, Volume II, Mineral Fuels

TABLE 8

MINERAL FUEL STATISTICS 1954 - 1966

Year	National Price Index (Wholesale) (1957-59=100)	Crude Oil Price Colo. ($ Per Barrel)	U. S. Oil Consumption (Billion Barrels)	Colorado Crude Oil Prod. (Million Barrels)	Capital, Expenditures in Production (Billion U. S. $)	Total New Wells Drilled in Colorado
1954	91.0	2.77	2.962	46.206	3.700	1358
1955	91.1	2.75	3.221	52.653	4.050	1509
1956	91.6	2.78	3.370	58.516	4.375	1222
1957	101.0	3.02	3.425	54.982	4.400	855
1958	101.2	2.99	3.415	48.736	3.575	836
1959	97.8	2.90	3.526	46.440	3.700	809
1960	97.2	2.90	3.609	47.469	3.575	706
1961	97.5	2.88	3.642	46.759	3.400	662
1962	97.7	2.88	3.796	42.477	3.850	758
1963	97.3	2.88	3.927	38.283	3.525	600
1964	96.9	2.88	4.032	34.755	3.800	565
1965	96.8	2.88	4.193	33.511	3.600	577
1966	97.5	2.91	4.397	33.492	3.600	548

Source: <u>American Petroleum Institute,</u> Petroleum Facts and Figures

TABLE 9

EMPLOYMENT IN THE MINERAL INDUSTRY OF COLORADO 1955 - 1966

(THOUSANDS OF PERSONS)

Year	Metals	Bituminous Coal and Lignite	Mining and Quarrying of Non-Met. Minerals	Non-Metals Total	Crude Petroleum and Natural Gas	Mining Total
1955	5.7	2.6	.7	3.3	6.5	15.5
1956	6.3	2.1	.8	2.9	6.6	16.1
1957	6.6	2.5	.8	3.3	6.7	16.6
1958	5.7	2.1	.9	3.0	6.6	15.3
1959	5.9	2.0	.7	2.7	6.7	15.3
1960	6.3	2.1	.5	2.6	6.5	15.4
1961	6.4	1.5	.5	2.0	6.1	14.5
1962	5.0	1.4	.5	1.9	5.8	12.7
1963	5.0	1.3	.5	1.8	5.2	12.0
1964	5.1	1.4	.5	1.9	4.8	11.8
1965	5.5	1.4	.5	1.9	5.0	12.4
1966	5.8	1.5	.5	2.0	5.0	12.8

Source: U. S. Bureau of Labor Statistics

Appendix B

Intermediate Results from the Regression Analysis

TABLE 10

EMPLOYMENT FUNCTIONS FOR TOTAL EMPLOYMENT

IN COLORADO'S MINERAL INDUSTRY

Degree of Polynomial	Regression Coefficients						F-Value	Prediction for 1966
	a_0	a_1	a_2	a_3	a_4	a_5		
1	17.190	-.477					38.450	11.463
2	16.193	-.017	-.038				22.868	10.466
3	13.966	1.822	-.405	.020			33.147	12.693
4	15.603	-.149	.262	-.063	.0034		28.831	14.330
5	13.103	1.375	-.016	.331	-.032	.0012	25.457	13.103

Table of Residuals for 5th degree polynomial

	Actual	Estimated	Residual
1	15.5	15.511	-.011
2	16.1	16.189	-.089
3	16.6	16.132	.467
4	15.3	15.840	-.540
5	15.3	15.467	-.167
6	15.4	14.962	.437
7	14.5	14.212	.287
8	12.7	13.190	-.490
9	12.0	12.095	-.095
10	11.8	11.500	.299
11	12.4	12.495	-.095

TABLE 11

PREDICTED AND ACTUAL ENDOGENOUS AND EXOGENOUS VARIABLES

MULTIPLE LINEAR REGRESSION MODEL (MINERAL PRODUCTION)

		Predicted 1966		Error	
	Actual 1966	Value	STD. Error	Actual	%
U. S. Lead Consumption (Millions of short tons)	1. 276*	1. 363	. 027	+ . 087	6. 82
Lead Price New York (cents/ounce)	15. 12	16. 71	1. 267	+1. 59	10. 45
U. S. Slab Zinc Consumption (millions of short tons)	1. 289	1. 526	. 021	+ . 237	18. 39
Zinc Price New York (cents/pound)	15. 00	15. 85	. 895	+ . 85	5. 67
U. S. Copper Consumption (millions of short tons)	2. 239	1. 724	. 053	- . 515	23. 02
Copper Price New York (cents/pound)	35. 82	38. 28	3. 310	+2. 37	6. 62
U. S. Mineral Production Value (Billions U. S. $)	22. 906	22. 235	1. 982	- . 671	2. 93
U. S. New Plant and Equipment Expenditures (Billions U. S. $)	1. 470	1. 345	. 120	- . 125	8. 57
U. S. National Income (Billions U. S. $)	616. 7	604. 01	16. 9	-12. 69	2. 06

*Preliminary

TABLE 11

CONTINUATION

		Predicted 1966		Error	
	Actual 1966	Value	STD. Error	Actual	%
U. S. Coal Consumption (millions of short tons)	486. 2	494. 2	12. 87	+8. 0	1. 65
Colo. Coal Price (dollars per short ton)	4. 99	4. 63	. 207	- . 36	7. 22
U. S. Silver Consumption Art and Industry (millions of troy ounces)	150.	146. 0	4. 95	-4. 0	2. 67
Silver Price (cents/ounce)	129. 3	130. 3	9. 27	+1. 0	. 77
U. S. Oil Consumption (domestic and foreign) (billions of barrels)	4. 397	4. 323	. 0123	- . 074	. 016
National Wholesale Price Index (1957-1959=100)	97. 5	99. 8	2. 20	+2. 30	2. 36
Oil Price Colorado	2. 91	2. 90	. 08	- . 01	. 035
U. S. Capital Expenditures in Crude Oil Production (billion U. S. $)	3. 600	3. 426	- . 282	. 174	4. 83
Total New Wells Drilled in Colorado	548	603	+65	55	10. 05

TABLE 12

PREDICTED AND ACTUAL ENDOGENOUS VARIABLES

MULTIPLE LINEAR REGRESSION MODEL (EMPLOYMENT)

			Predicted 1966		Error	
		Actual 1966	Value	Std. Error	Actual	%
5.0	Value of Mineral Production (millions of dollars)					
5.1	Metals	172.0	157.0	16.8	-15.0	8.72
5.2	Crude Petr. and Natural Gas	115.3	110.5	8.7	- 4.8	4.16
5.3	Non-metallics	49.2	49.8	2.1	+ .6	1.21
5.4	Total	366.3	349.4	26.5	-16.9	4.65

TABLE 13

THE REGRESSION EQUATIONS

U. S. AND COLORADO GENERAL MINING STATISTICS

Regression Coefficients	a_0	a_1	a_2	a_3	a_4
<u>U. S. Mineral Consumption</u>					
Lead	1.322	-.0961	.00829		
Zinc	1.223	-.131	.0130		
Copper	1.456	-.105	.0106		
Silver	105.357	-5.635	.752		
Coal	471.528	-34.499	3.033		
Oil	3.244	.0283	.00513		
<u>Metal and Mineral Fuel Price</u>					
Lead	19.128	-2.458	.188		
Zinc	14.125	-.872	.0847		
Copper	42.495	-3.991	.303		
Silver	75.095	4.600			
Coal	5.236	.384	-.0362		
Oil (U. S.)	83.525	8.038	-1.267	.0593	
Oil (Colorado)	2.864	.00354			
<u>General Statistics</u>					
U.S. Mineral Production Value	16.684	-.0621	/0437		
U.S. National Income	288.280	26.311			

TABLE 13

CONTINUATION

Regression Coefficients	a_0	a_1	a_2	a_3	a_4
General Statistics (Cont.)					
U.S. New Plant and Equipment Expenditures	1.210	-.0759	-.0759	.00725	
U. S. Capital Expenditures in Crude Oil Production	4.182	-.0629			
Total New Wells drilled in Colorado	2132.595	-741.840	151.84	-13.945	.463

TABLE 14

THE REGRESSION EQUATIONS

VALUE OF MINERAL PRODUCTION

	Regression Coefficients	
	a_0	a_1
Metals	124.816	3.215
Crude Petr. and Natural Gas	186.766	-7.622
Non-Metallics	38.889	1.093
Total	347.470	.193

TABLE 15

THE REGRESSION EQUATIONS

POLYNOMIAL REGRESSION ANALYSIS (POTENTIAL)

Mineral Output in Colorado	Regression Coefficients					
	a_0	a_1	a_2	a_3	a_4	a_5
Crude Oil	59.673	- .230				
Natural Gas	148.157	- .789				
Coal	3.932	- .295	.0329			
Gold	83.159	13.118	- 5.344	.533	-.0173	
Silver	3.591	- .779	.108	- .00438		
Copper	5.206	- .635	.0949	- .00412		
Lead	19.692	- 1.416	.151			
Zinc	4.588	44.751	-17.405	2.752	-.185	.00444

TABLE 15A

THE REGRESSION EQUATION

POLYNOMIAL REGRESSION ANALYSIS (POTENTIAL)

Employment and Services	Regression		Coefficients	
	a_0	a_1	a_2	a_3
Metals	5.037	.875	-.174	.00883
Bituminous Gal.	2.317	.128	.0575	.00343
Mining and Quarrying of Non-Met. Minerals	.86	-.0414	$.349*10^{-3}$	
Non-Metals (Total)	2.92	.339	-.107	.00623
Crude Petroleum and Natural Gas	7.21	-.194		
Mining (Total)	13.966	1.822	-.405	.0203
Av. Weekly Earnings	93.181	.796	.149	
Av. Weekly Hours	40.55	-.0827		

TABLE 16

PREDICTED AND ACTUAL MINING ACTIVITY FOR COLORADO 1966

POLYNOMIAL REGRESSION MODEL (POTENTIAL)

			Predicted 1966		Error	
		Actual 1966	Value	Std. Error	Actual	%
1.0	Employment (thousands of people)					
1.1	Metals	5.8	5.71	.461	-.09	1.55
1.2	Bituminous Coal and Lignite	1.5	1.56	.199	-.06	4.00
1.3	Mining and Quarrying of non-met. minerals	.5	.41	.105	-.09	18.00
1.4	Non-metals (total)	2.0	1.97	.202	-.03	1.50
1.5	Crude Petroleum and Natural Gas	5.0	4.87	.356	-.13	2.60
1.6	Mining (total)	12.8	12.69	.538	-.11	.86
2.0	Av. Weekly Earnings	124.89	124.28	2.79	-.61	.47
3.0	Av. Weekly Hours	39.9	39.55	.922	-.45	1.13

TABLE 17

PREDICTED AND ACTUAL MINING ACTIVITY FOR COLORADO 1966

POLYNOMIAL REGRESSION MODEL (POTENTIAL)

			Predicted 1966		Error	
		Actual 1966	Value	STD. Error	Actual	%
6.0	Physical Output					
6.1	Crude Oil (thousands of barrels)	33,489	32,068	2,665	-1,421	4.24
6.2	Natural Gas (billions of cubic feet)	142.6	141.8	3.89	- .8	.56
6.3	Coal (thousands of short tons)	5,296	5,133	237	143	2.70
6.4	Gold (thousands of troy ounces	31.3	32.7	8.28	1.4	4.47
6.5	Silver (thousands of troy ounces)	2,118	2,307	387	189	8.93
6.6	Copper (short tons)	4,237	4,116	681	- 114	2.69
6.7	Lead (short tons)	23,450	24,544	2,528	+1,094	4.67
6.8	Zinc (short tons)	54,655	50,299	3,692	4,356	7.97

TABLE 18

THE REGRESSION EQUATIONS

METAL PRODUCTION IN THE MINERAL INDUSTRY OF COLORADO

Regression Coefficients	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Copper Production	-61.163	28.028 (5.832)	.136 (.498)	-.354 (.427)	1.237 (.637)	1.426 (.606)	-9.074 (1.892)		
Lead Production	$-12.288*10^3$	6.312 (2.153)	27.111 (8.327)	1.001 (.660)	-2.532 (.994)	11.503 (4.913)	-.181 (.065)		
Silver Production	-74.808	32.641 (5.263)	-.111 (.939)	1.431 (.439)	.080 (1.311)	1.206 .463	-10.699 (1.514)		
Zinc Production	$5.272*10^3$	2.735 (3.257)	20.772 (15.211)	-3.480 (1.241)	8.001 (4.631)	-1.913 (24.387)	.021 (.103)		
Coal Production	$1.262*10^3$	-.645 (.346)	-1.065 (.511)	-.014 (.011)	.883 .463	-3.049 (1.922	.013 (.007)		
Oil Production	$6.228*10^3$	-3.109 (5.966)	1.807 (4.555)	2.760 (9.754)	5.250 (4.902)	-.071 (.131)	-12.041 (62.208)	-.012 (.021)	-91.021 (188.497)

Note: Number in parentheses is the standard error of the regression coefficient.

TABLE 19

THE REGRESSION EQUATIONS

EMPLOYMENT IN THE MINERAL INDUSTRY OF COLORADO

Regression Coefficients	a_0	a_1	a_2	a_3
Total Employment	704.620	-92.914 (34.432)	-.073 (.374)	.444 (.244)
Metals	599.238	-72.235 (17.737)	.057 (.209)	.627 (.135)
Non-Metallics	377.088	-49.441 (91.659)	.464 (.498)	-.486 (.687)
Oil and Gas	308.087	-40.637 (28.704)	3.444 (.339)	.224 (.317)

Appendix C

Final Results from the Multiple Linear Regression Techniques

TABLE 20

MULTIPLE REGRESSION ANALYSIS

Dependent Variable, Y

(8) Colorado Copper Production

Independent Variables, X_j

(1) Time
(5) U. S. Copper Consumption
(6) Copper Price
(7) U. S. Total Mineral Value
(9) New Plant and Equipment Expenditures
(10) U. S. National Income

Variables X_j	Correlation X_j vs. Y			Multiple Correlation			F - Value		
	I	II	III	I	II	III	I	II	III
1	-.085	-.085	-.072						
* 5	.341	.443	.470						
* 6	-.017	.632	.630						
* 7	.119	.092	.096	.904	.890	.963	2.985	2.555	8,562
9	.408	.408	.401						
* 10	-.069	.106	-.114						

* Lagged variables in Case II and III (Lagged period, 1 year)

TABLE 21

MULTIPLE REGRESSION ANALYSIS

Dependent Variable, Y

(2) Colorado Lead Production

Independent Variables, X_j

(1) Time
(2) U. S. Lead Consumption
(4) Lead Price New York
(8) U. S. Mineral Value
(9) New Plant and Equipment Expenditures
(10) U. S. National Income

Variables X_j	Correlation X_j vs. Y			Multiple Correlation			F - Value		
	I	II	III	I	II	III	I	II	III
1	.449	.449	.432						
* 3	.586	.699	.680						
* 4	.418	.043	-.005						
* 8	.744	.510	.446	.953	.971	.920	6.678	11.37	3.717
9	.819	.819	.793						
* 10	.491	.487	.429						

* Lagged variables in Case II and III (Lagged period, 1 Year)

TABLE 22

MULTIPLE REGRESSION ANALYSIS

Dependent Variable, Y	Independent Variables, X_j
(5) Colorado Zinc Production	(1) Time
	(6) U. S. Slab Zinc Consumption
	(7) Zinc Price
	(8) U. S. Mineral Value
	(9) New Plant and Equipment Expenditure
	(10) U. S. National Income

Variables X_j	Correlation X_j vs. Y			Multiple Correlation			F - Value		
	I	II	III	I	II	III	I	II	III
1	.677	.677	.647						
* 6	.732	.711	.684						
* 7	.430	.659	.664						
* 8	.858	.744	.700	.968	.890	.931	10.181	2.559	4.355
9	.717	.717	.715						
* 10	.737	.703	.637						

* Lagged variables in Case II and III (Lagged period, 1 year)

TABLE 23

MULTIPLE REGRESSION ANALYSIS

Dependent Variable, Y

(2) Colorado Silver Production

Independent Variables, X_j

(1) Time
(3) Silver Price
(4) U. S. Silver Consumption
(7) U. S. Mineral Value
(9) New Plant and Equipment Expenditures
(10) U. S. National Income

Variables X_j	Correlation X_j vs. Y			Multiple Correlation			F - Value		
	I	II	III	I	II	III	I	II	III
1	-.201	-.201	-.165						
* 3	.177	.164	.187						
* 4	.021	.021	.039	.792	.946	.978	1.127	5.682	14.926
* 7	.944	-.106	-.075						
9	.366	.366	.380						
* 10	-.137	-.195	-.201						

* Lagged variables in Case II and III (Latted period, 1 year)

TABLE 24

MULTIPLE REGRESSION ANALYSIS

Dependent Variable, Y

(3) Colorado Oil Production

Independent Variables, X_j

(1) Time
(2) National Price Index
(3) Capital Expenditures
(5) U. S. Mineral Value
(6) U. S. National Income
(7) Consumption
(8) Colorado New Oil Wells
(9) Colorado Oil Price

	Variables X_j	Correlation X_j vs. Y			Multiple Correlation			F - Value		
		I	II	III	I	II	III	I	II	III
	1	-.949	-.949	-.947	.987	.975	.988	9.74	4.81	9.99
*	2	-.235	-.571	-.532						
	4	.640	.640	.578						
*	5	-.827	-.823	-.805						
*	6	-.961	-.949	-.939						
*	7	-.935	-.901	.892						
*	8	.746	.883	.890						
*	9	-.082	-.430	-.369						

TABLE 25

MULTIPLE REGRESSION ANALYSIS

Dependent Variable, Y

(7) Colorado Coal Production

Independent Variables, X_j

(1) Time
(2) Colorado Coal Price
(3) U. S. Coal Consumption
(4) U. S. Mineral Value
(5) New Plant and Equipment Expenditures
(6) U. S. National Income

	Variables X_j	Correlation X_j vs. Y			Multiple Correlation			F - Value		
		I	II	III	I	II	III	I	II	III
	1	.665	.665	.651	.971	.956	.943	11.005	7.233	5.375
*	2	-859	-.528	-.533						
*	3	.770	.375	.343						
*	4	.832	.584	.513						
	5	.637	.637	.631						
*	6	.750	.730	.660						

* Lagged Variables in case II and III (Lagged period 1 year)

TABLE 26

COMPARISON OF FORECASTING MODELS

POLYNOMIAL REGRESSION (P. R.) VS. MULTIPLE LINEAR REGRESSION (M. L. R.)

	P. R.			M. L. R.			
	Predicted	Error		Predicted	Error		Actual
Mineral Production	Value	Actual	%	Value	Acutal	%	1966
Copper	4,116	-114	2.69	2,968	-1,269	29.95	4,237
Lead	24,544	+1,094	4.67	21,486	-1,968	8.39	23,450
Silver	2,307	+189	8.93	1,496	-622	29.38	2,118
Zinc	50,299	-4,356	7.97	50,968	-3,687	6.75	54,655
Coal	5,133	-143	2.70	4,250	-1,046	19.77	5,296
Oil	32,068	-1,421	4.24	36,900	+3,032	9.46	33,492
Employment							
Mining - Total	12.69	-.11	.86	11.12	-1.68	13.12	12.8
Metals	5.71	-.09	1.55	5.01	- .79	13.62	5.8
Non-Metallics	1.97	-.03	1.50	1.71	- .29	14.41	2.0
Oil and Gas	4.87	-.13	2.60	4.04	- .96	19.20	5.0

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