PRINCIPLES OF HYDROLOGY

APPLIED TO LEACHING OF ORE IN PLACE

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TABLE OF CONTENTS

ABSTRACT OF THESIS

Principles of Hydrology Applied to Leaching of Ore in Place

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The method of extracting ore by means of leaching of ore in place has been introduced and defined as that method in which leaching liquids enter the orebody through injection drill holes, in which the leaching liquid dissolves the valuable minerals, and in which the pregnant solutions are recovered through extraction drill holes.

The radial flow has been studied and Muskat's equation

has been proved experimentally.

The change in rate of flow has been studied mathematically and experimentally.

The principles studied may be summarized as follows:

1. Muskat's equation is found to be mathematically and experimentally correct.
2. Different methods of measurement of permeability give different results.
3. In the laboratory the same liquid or gas should be used for the determination of the permeability of a rock that is used in practice.
4. The factor of viscosity is merely a fair approximation to find the difference in rate of flow of different liquids through the same rock with constant

permeability.

5. Formulas for permeability lack correction for the effects of contact angle, wettability, gas bubbles, and other physical properties.
6. Rate of flow decreases because of formation of gas bubbles, formation of precipitates from chemical reactions.
7. Rate of flow increases because of opening up of pores by means of removal of ore minerals; and because of vibration, which causes small gas bubbles to combine with larger bubbles, in which form the gas can escape.
8. Strong solutions tend to hasten channel forming.
9. Channel forming decreases the efficiency of leaching.
10. Mathematical formulas for the change in rate of flow have little practical value.

This abstract of _____ words is approved as to form and content.

I recommend its publication.

Date May 25, 1950

Signed E. W. Livingston

Professor in charge of thesis

INTRODUCTION

Many of the minerals present in the earth occur as minable deposits. Many other minerals cannot be recovered at a profit with the present methods of mining. These unminable deposits may be low-grade deposits, very deep-seated orebodies, or orebodies consisting of very weak ore and walls.

To recover the values from the unminable deposits, a new method of extraction of the values by means of leaching of ore in place is suggested. Leaching of ore in place can be defined as that method of extracting values from orebodies, in which the leaching liquids enter the orebody through injection drill holes, in which the leaching liquid dissolves the valuable minerals, and in which the pregnant solutions are removed from the orebody by means of extraction drill holes.

The drill holes will have to be placed in that pattern which is economically best. Drill patterns vary in the spacing of drill holes, in the diameter of drill holes, and in the shape of the pattern. Next to the pattern itself, the number of injection holes and extraction holes can be varied relatively.

Special pumps and casings for drill holes will have to be provided, as the chemicals used might be chemically corrosive.

Whether or not the leaching of ore in place is possible will depend entirely upon the characteristics of the orebody. Geologic features like fractures and shear zones might make this method of ore extraction impossible. The chemical composition of the ore minerals and of the waste minerals has to be studied. The leaching liquid chosen has to be such that most of the valuable mineral will dissolve and as little as possible of the waste mineral will be dissolved.

Physical characteristics like permeability and porosity will change during the process of leaching of ore in place. These characteristics with their change have been given much thought in this paper and are discussed next to the principles of hydrology. Assumptions have been made in this paper and are discussed in the conclusions.

CHAPTER I

BASIC PRINCIPLES OF HYDROLOGY

Turbulent and Laminar Flow

Flow of water was first studied in stream channels, in canals and ditches, and later in pipes. Observation showed that stream flow generates eddies and complex cross currents, and similar flow conditions were noticed in pipes in which water movement is rapid. For stream flow and flow in pipes it has been long considered that velocity varies approximately as the square root of the frictional resistance expressed as loss in head or hydraulic gradient.

Flow of water in capillary tubes was first studied by Hagen 1/ and Poisseuille 2/ who found that in capillary

1/ Hagen, G., Ueber die Bewegung des Wassers in engen cylindrischen Roehren, Poggendorf Annalen, vol. 46, pp. 423-442, 1839.

2/ Poiseuille, J., Recherches experimentales sur le mouvement des liquides dans les tubes de tres petit diametre, Mem. Savants Etrange, vol. 9, 1846.

tubes the relation of loss of head to flow is different from that in large pipes and that the velocity varies directly with the head consumed or hydraulic gradient. Darcy 3/ discovered

3/ Darcy, Henry, Les Fontaines publiques de la ville de Dijon (Paris: Victor Dalmont, 1856).

that this same relation of velocity to head governs the water movement in water-bearing sands.

The fundamental laws that determine the state existing for any given case of flow were determined by Osborn Reynolds

in 1883. 4/ The laws were derived from his classical

4/ Reynolds, Osborne, Trans. Royal Soc. London, 1882-1895.

experiments, wherein the relationship between the velocity of flow through a pipe and the amount of head lost in friction was a main point of investigation. The basic results are shown in Fig. 1. The ordinate in this figure is the hydraulic gradient which is designated by i and defined as the head lost per unit of distance traveled. Fig. 1 shows that at the small velocities in the Zone I the gradient is proportional to the velocity, a condition which may be considered the distinguishing characteristic of laminar flow. This rectangular motion is also designated as straight-line flow, streamline flow, or viscous flow. If the velocity is gradually increased, a point is reached at which eddies begin to form, and a range of uncertain behavior is entered. Further increase in velocity leads eventually to a relationship of velocity to gradient which is the definite curve of Zone III. If the velocity is now decreased, the smooth curve persists down to a velocity considerably lower than that at which the eddy formation previously was noted. Further decrease causes a return of laminar flow. The upper limit of Zone II, the range of uncertain behavior, is indefinite and of little interest in the present discussion. The lower

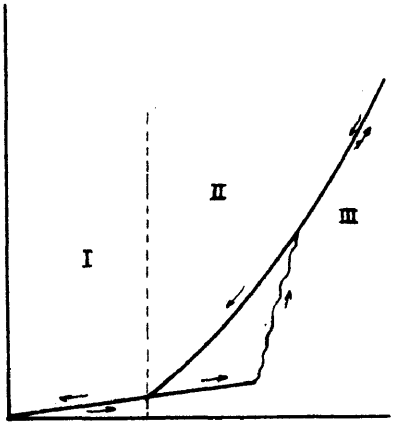


Fig.1

limit is a definite velocity V_c below which the flow in pipes is always laminar. It is known as the lower critical velocity. Reynolds found that this velocity is inversely proportional to the diameter of the pipe and gave the following general expression:

$$\frac{V_c D_w}{g} = 2000$$

where D is the pipe diameter and g the acceleration of gravity. This expression is known as the Reynolds' number for the lower critical velocity.

A special case of laminar flow is called percolation. Percolation is described by Slichter 5/ as follows. In the

5/ Slichter, C. S., The motions of underground waters, U.S.G.S. Water-Supply Paper 67, pp. 35-37, 1902.

slow motion of a viscous liquid the bounding layer adheres to the walls of the vessel containing the liquid, so that motion nearly ceases in the neighborhood of the boundary. This is not the case, however, in the movement of groundwaters, as the frictional resistance which the water meets is present in each individual pore between the soil particles, and is not transmitted from layer to layer through the water itself, as in the case of a viscous liquid. If the material is no finer near the impervious layer or boundary, the resistance to motion per unit length is no greater at the boundary of the region than in its interior. This uniform distribution of the resistance to motion throughout the mass of the groundwater, and the separation of each individual stream

in a capillary pore from every other similar stream in the neighboring pores makes the character of the motion difficult to understand.

Tolman 6/ defines percolation as slow movement of water

6/ Tolman, C. F., Groundwater (McGraw-Hill Book Co., Inc. 1937)

in interconnected pores of saturated granular material under hydraulic gradients commonly developed underground. Resistance to movement developed by friction of the moving water against the surface of innumerable grains constituting the water-bearing material determines the permeability of the material, which is commonly considered to vary roughly as the square of the average diameter of the pores. This type will always occur in materials with grain size smaller than 3 to 4 mm, even with high hydraulic gradient.

Taylor 7/ pays attention to the effect of sinuous paths

7/ Taylor, Donald, W., Soil Mechanics, p. 110, 1948.

of flow through soils. Consider the flow through the pores of a homogeneous soil mass across a plane which is normal to the general direction of flow, and compare this flow with that across another similarly chosen plane a short distance away. The pore-opening patterns at the two planes will show little resemblance, but all statistical average values relative to pore sizes and shapes will be alike for the two planes. On this basis the soil pores may be considered as a group of tubes of unvarying section. There is one essential difference between flow through a soil mass and the flow through a tube.

In the soil the flow is sinuous, winding around soil grains, whereas in the tube it follows straight lines. This condition leads to a difference between the true hydraulic gradient and the gradient used in soils. The true gradient equals the ratio of head loss to length of flow path within which this loss occurs. In soils, the gradient must be considered the ratio of head loss to the straight-line distance between initial and final points, the distance being shorter than the true length of the sinuous flow path. This ratio is a constant for a given soil and may be considered merely a shape factor item (discussed later).

Hydraulic Properties of Rocks

Hydraulic properties depend chiefly on porosity and on size, shape, arrangement, interconnection, and continuity of openings or interstices. Rock formations and the openings therein are the result of primary geologic processes that form the rocks and of secondary processes that modify them, either increasing or decreasing the porosity and the permeability.

Agencies that produce openings are processes of sedimentation; expansion of included gases forming cavities in lava and in mudflows; drying out or desiccation, especially in clays, producing mud cracks and other secondary fractures; crystallization of igneous rocks producing intercrystal interstices; fracturing or mechanical disruption; chemical actions like solution; colloidal processes and flocculation;

combination of chemical and mechanical action known as weathering; vital processes like root openings, and animal and insect borings.

Agencies that destroy openings are compaction, deflocculation of clays, slumping, cementation, metamorphism, and weathering.

Different studies have been made on the effect of grain size on permeability and ease of flow. Tolman 8/ concludes these studies as follows. 1. Percentage of pore space does not increase with the size of material; 2. Heterogeneity of size of material reduces pore space; 3. The size of the fine interstitial material controls the velocity of percolation in heterogeneous material; 4. The larger the proportion of large grains enclosed in fine material, the greater the reduction in average porosity of the formation; 5. Maximum ease of movement is always parallel to stratification, except in soils.

8/ Op. cit., p. 111.

Laws of Permeability

Darcy conducted a series of experiments on water sands in 1856, verified the relation of velocity to gradient for water-bearing sands, and suggested the application of this relation to problems of water movement through water-bearing materials.

Darcy's conclusions are expressed in the following formula.

$$V = \frac{kh}{l}$$

Where h is head
l is length of the column
V is velocity

Darcy did not attempt to determine and segregate the factors that control k, as did later experimenters, but used the method favored at the present time of experimental measurement of k for different types of materials. This factor measures the permeability of the water-transporting material, now generally indicated by ground-water hydrologists in the United States by the letter P, representing a standardized coefficient.

Between 1880 and 1892 Allen Hazen 9/ made experiments

9/ Hazen, A., Some physical properties of sands and gravels, Mass. State Board of Health 24th Annual Report for 1892, 1893.

on river sands and gravels at Lawrence, Massachusetts. He went farther than Darcy in that he differentiated the effect of grain size of water-bearing material on the coefficient of permeability, and took into consideration the effect of temperature and viscosity of the water. His experiments on homogeneous materials indicated that velocity varies as the square of the diameter of grain in the water-bearing material. Hazen's formula reads: $V=Cx\frac{h}{l}xd^2(0.70+0.03t)$

Where D is grain size
H is head
l is length
t is temperature
V is velocity

Between 1902 and 1904 Charles Slichter 10/ conducted experiments on the percolation of water through sands and gravel

10/ Slichter, C. S., Field measurements of rate of movement of underground waters, U.S.G.S. Water-Supply Paper 140, 1905.

in a horizontal tank using low gradients to approach more natural conditions. His equation is

$$q=0.2012\ \frac{pd^2s}{\mu hk}\ \text{cfm}$$

Where d is effective size of grains in mm
μ coefficient of viscosity
h is distance through which water moves in ft
K is a constant depending upon the porosity
p is pressure in ft of water
q is quantity in cu ft per min
s is area of cross section in sq ft

Terzaghi developed the following formula for the permeability of sands, in which permeability is expressed as K, 11/.

$$K=(800 \text{ to } 600)\frac{V_o}{V_t}\left[\frac{N-0.13}{\sqrt[3]{1-N}}\right]^2 d_w^2$$

Where dw is the effective rise of grain in cm
N is the void volume (voids divided by total volumn)
V_o is the coefficient of viscosity at 10° C
V_t is the coefficient of viscosity at t C

The basic assumption in this equation is that in the void channels the widest part of the channel is at least five times the diameter of the narrowest part. The loss in head in the latter would be 25 times that in the widest part of the channel.

11/ Terzaghi, C., Principles of soil mechanics, Eng. News-record, Vol. 95, p. 31, 1925.

Fair and Hatch derived a formula for the calculation of permeability: 12/ $K=1/\left\{C_H T F \left[\frac{S}{100}\ \text{sum of}\ \frac{W}{d}\right]^2\right\}$

12/ Fair, G.M., and L.P. Hatch, Fundamental factors governing streamline flow of water through sands, Jour. Amer. Water Works Assn., Vol. 25, No. 11, pp. 1551-1565, 1933.

Where C_H is a filtration constant determined as 5/gravity
d is the diameter (for sands it is taken as the geometric mean of the adjacent screen sizes
F is the porosity factor $(1-p)^2/p^3$, where p is the porosity
K is the coefficient of permeability
S is the shape factor, varying from 6 for spheres to 7.7 for angular sands
T is the temperature correction taken as viscosity / density
W is the percentage by weight of material of a given diameter d

Taylor 13/ developed a formula for the permeability in relation to grain size and void volume:

13/ Taylor, Donald W., Fundamentals of soil mechanics (New York; John Wiley & Sons, Inc. 1948) pp. 104-111.

$$K = D_s^2 \frac{\gamma_w}{\mu} \frac{e^3}{1+e} C$$

Where C is the composite shape factor
D_s is the grain size
e is the void ratio
γ_w is the specific gravity of water
K is permeability
μ is viscosity of water

As Taylor's equation will be used in later discussions, a more detailed derivation follows.

Taylor's Derivation

The equation for the average velocity of a fluid in laminar flow is

$$V_{av} = \frac{\gamma_w R^2}{8 \mu} i \qquad (1)$$

In which γ_w is the specific gravity of liquid
i is the hydraulic gradient
R is diameter of tube
μ is viscosity of liquid
V_{av} is the average velocity

If tubes of various shapes and sizes are to be studied, some characteristic dimension must be used to describe the general size of the tube. This dimension is the hydraulic radius, which is defined as the ratio of area of the tube to the wetted perimeter; it is designated herein by R_H.

For a circular tube R_H equals $\frac{\pi R^2}{2\pi R}$ or $\frac{R}{2}$.

Multiplying Equation 1 by the cross-sectional area of the tube and substituting $R=2R_H$ we find

$$Q=\frac{1}{2}\frac{wR_H^2}{\mu}ia \qquad (2)$$

In which a is the cross sectional area
Q is the quantity of fluid flowing.

For different shaped tubes it is found that this equation will change only by a constant. In general the equation is

$$Q=C_s\frac{wR_H^2}{\mu}ia \qquad (3)$$

In which C_S is the shape constant.

The shape constant can be evaluated mathematically for simple shapes only. The value of C_S is independent of size; the dimension R_H expresses the size.

Let it be assumed that Fig. 2 represents the cross-section of a tube of constant but extremely irregular shape. Let the total area of the cross-section, including the tube walls, be designated as A and the void ratio or porosity as n. The area a in equation 3 will equal nA and equation 3 becomes

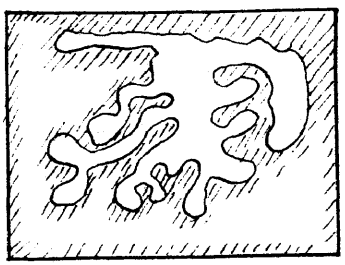


Fig.2

$$Q=\left(\frac{C_s \gamma_w R_H^2 n}{\mu}\right) iA. \qquad (4)$$

The hydraulic radius may be expressed

$$R_H=\frac{A}{P}=AL \div PL$$

In which A is the cross-sectional area
L is the length of the tube
P is the perimeter

AL is the volume of flow channel; PL is the surface of flow channel.

When flow tubes place through a soil mass, the volume of the flow channels is the effective pore volume, which is expressed by eV_S, wherein e is the void ratio and V_S is the volume of solids. The surface area of flow channels is the total surface area of the soil grains H_S. Thus an expression for hydraulic radius in soils is

$$R_H=e\,\frac{V_s}{H_s} \qquad (5)$$

Let D_S be the diameter of the spherical grain which has the same ratio of volume to surface area that holds collectivity for all grains of a given soil. This ratio of volume to surface area may be expressed for the spherical grain as

$$\frac{V_s}{H_s}=\frac{1/6\ \pi D_s^3}{\pi D_s^2}=1/6\ D_s \qquad (6)$$

or

$$R_H=e\,\frac{D_s}{6} \qquad (7)$$

Substitution of equation 7 in equation 4, replacement of the porosity term by $\frac{e}{1+e}$, and introduction of a composite shape factor C to include the factor C_S, the

numerical constant, and all other shape effects, gives

$$Q=\left(D_s^2 \frac{\gamma_w}{\mu} \frac{e^3}{1+e} C \right) iA. \tag{8}$$

From comparison of this equation with Darcy's Law it follows that

$$K=D_s^2 \frac{\gamma_w}{\mu} \times \frac{e^3}{1+e} C. \tag{9}$$

The composite shape factor takes care of not only the constant C_S, but also, according to Taylor, of the effect of sinuous paths of flow through soils. The author sympathizes with this formula, because it gives a view of the facts and matters effecting the permeability, it has a safety factor (which takes care of non-measureable factors like surface tension, contact angle, ionization of liquid, enclosed air in voids, and others), and it agrees with Darcy's Law.

CHAPTER II

MATHEMATICAL APPROACH

This chapter deals with the derivations of formulas for (1) the radial flow of fluids from and into a well and (2) the leaching of ores in places.

Hydrodynamic Relations

In analyzing the fundamental principles of hydrodynamics it is found that they are essentially nothing more than reformulations of the corresponding principles of mechanics in such a way as to be appropriate to the flow of fluids. The law of conservation of matter can be stated as 1/: The net excess of mass flux, per unit of time, into

1/ Muskat, M., The flow of homogeneous fluids through porous media, 1937, p. 121.

or out of any infinitesimal volume element in the fluid system is exactly equal to the change per unit of time of the fluid density in that element multiplied by the free volume of the element. In the rectangular system shown in Fig. 3, we observe element dx, dy, dz, with as center the coordinates x, y and z. The mass flow into the side dy dz is

$$\left[\rho v_x - \frac{\partial}{\partial x}(\rho v_x)\frac{dx}{2}\right] dy.dz$$

Where ρ is the fluid density
v is the fluid velocity

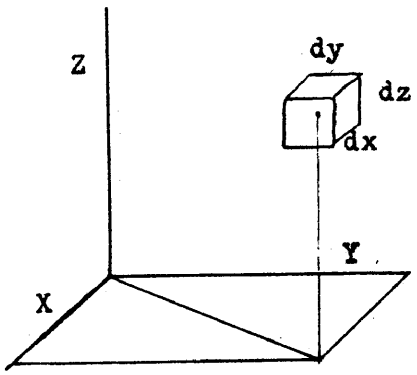


Fig.3

The flow out of the parallel side will be

$$\left[\rho V_x + \frac{\partial}{\partial x}(\rho V_x)\frac{dx}{2}\right] dy \cdot dz \ .$$

The net flux leaving the parallelepiped through these two sides will be the difference of the flux of the two sides, or

$$\frac{\partial}{\partial x}(\rho V_x)\, dx \cdot dy \cdot dz \ .$$

By breaking the other sides similarly and adding up the results, we will find the total flux

$$\left[\frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z)\right] dx \cdot dy \cdot dz \ . \qquad (1)$$

Now the mass of fluid in the volume element is evidently

$$f,(\rho\, dx\, dy\, dz) \ .$$

Hence the loss of mass in the element per unit time can also be expressed as $-f.\left(\frac{\partial \rho}{\partial t}\, dx\, dy\, dz\right)$.

So we have the following equation

$$\left[\frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z)\right] = -f\frac{\partial \rho}{\partial t} = \rho \bar{v}$$

where $\bar{v}$ is the vectorial velocity as a resultant of v_x, v_y and v_z.

This equation is called the equation of continuity.

When the rate of flow is independent of the time and when flow is steady, the term $-f\frac{\partial \rho}{\partial t}$ becomes 0.

Darcy's Law

Darcy's 2/ law is generally accepted. Darcy made a

2/ Darcy, H., Les fontaines publique de la ville de Dijon, 1856.

study of flow of fluid through porous media in 1856. His experiment gave the following results, which are now generally referred to as Darcy's law. The rate of flow Q of water through a filter bed is directly proportional to the area A of the sand and to the difference h between the fluid heads of the inlet and outlet faces of the bed, and inversely proportional to the thickness L of the bed, or

$$Q = \frac{c A \Delta h}{L}$$

where c is a constant characteristic of the sand.

Another way of stating Darcy's law is: The flow of fluid is directly proportional to the pressure gradient $\frac{dp}{dl}$ or

$$Q = -c \frac{dp}{dl}$$

in which c is the same constant.

The experiments relating to Darcy's law were restricted to columns or beds of porous materials in which the macroscopic flow is necessarily of a one-dimensional character. It is necessary to generalize these emperical results before a complete theory can be developed.

First it is supposed that in a general three-dimensional flow system the resultant velocity at any point is directly proportional to, in magnitude, and in the same direction as the resultant pressure gradient at that point. Inversely we can resolve the velocity into three components. Darcy's law takes then the form

$$V_x = -\frac{K_x}{\mu} \frac{\partial p}{\partial x}$$

$$V_y = -\frac{K_y}{\mu} \frac{\partial p}{\partial y}$$

$$V_z = -\frac{K_z}{\mu} \frac{\partial p}{\partial z} .$$

Where μ is the viscosity of the fluid
K is the permeability of the porous material.

Two-dimensional System

Now we consider a two-dimensional system. By the two-dimensional system is meant a system in which the velocity-distribution vector $\bar{v}$ varies only with two of the rectangular coordinates and is independent of the third. Physically all fluid systems necessarily extend in three dimensions. However, the significant characteristic of two-dimensional systems is that all the features of the motion may be observed in a single plane, the motion being identically the same in parallel planes. In a porous bed, being horizontal and completely filled with water and pierced by wells through the whole thickness of the bed, the bed being of the same thickness all over, two-dimensional study is possible.

From the above considerations it follows that the general equation

$$\gamma \bar{V} = \frac{\partial}{\partial x}(\gamma V_x) + \frac{\partial}{\partial y}(\gamma V_y) + \frac{\partial}{\partial z}(\gamma V_z) = 0$$

becomes

$$\gamma \bar{V} = \frac{\partial}{\partial x}(\gamma V_x) + \frac{\partial}{\partial y}(\gamma V_y) = 0 \qquad (1)$$

Furthermore we have the three general equations for Darcy's law. The last equation (for the Z direction) is eliminated. So we have left

$$V_x = \frac{-K_x}{\mu}\frac{\partial p}{\partial x} \qquad (2a) \qquad \text{and} \qquad V_y = \frac{-K_y}{\mu}\frac{\partial p}{\partial y} \qquad (2b)$$

We can further assume that $K_x = K_y = K$.

By dividing Equation 1 by γ and substituting the values of Equations 2a and 2b into Equation 1, we find

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \qquad (3)$$

The Radial Flow into a Well 3/

3/ Muskat, M., The flow of homogeneous fluids through porous media, pp. 150-156, 1937.

By radial flow is meant a two-dimensional fluid motion symmetrical about an axis, its details varying only with the distance from the axis of symmetry. Taking advantage of the symmetry of the problem we can use the cylindrical-coordinate system reduced to two dimensions. Equation 2a, 2b, and 3 then become

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} = 0 \qquad (4)$$

$$v_r = -\frac{K}{\mu}\frac{\partial p}{\partial r} \qquad (5) \text{ and } \qquad v_\theta = -\frac{K}{\mu r}\frac{\partial p}{\partial \theta} \qquad (6)$$

For the more particular case of radial flow, it follows that

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = 0 \qquad (7)$$

$$v_r = -\frac{K}{\mu}\frac{\partial p}{\partial r} \qquad (8)$$

$$v_\theta = -\frac{K}{\mu r}\frac{\partial p}{\partial \theta} \qquad (9)$$

Integrating Equation 7, there results first

$$r\frac{\partial p}{\partial r} = \text{constant} = C_1$$

and then

$$p = C_1 \ln r + C_2 \qquad (10)$$

as the general expression for the pressure distribution in a system of radial flow. Since in the present case it has already been assumed that the flow is radial, we are no longer free as to the corresponding boundaries and boundary conditions. In fact, it is not difficult to show that the only system giving radial flow is that bounded by two concentric circles over each of which the pressure is uniform. Thus the radial-flow system may be characterized by the conditions

$r = r_w \qquad p = P_w$

$r = r_e \qquad p = P_e$

(see Fig. 4)

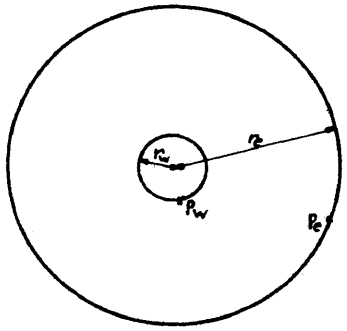


Fig. 4

Entering the conditions p_w, r_w, p_e, and r_e into Equation 10 we can solve for c_1 and c_2. Or

$$P_w = C_1 \ln r_w + C_2 \qquad P_e = C_1 \ln r_e + C_2$$

whence

$$C_1 = \frac{P_e - P_w}{\ln r_e / r_w} \qquad C_2 = \frac{P_w \ln r_e - P_e \ln r_w}{\ln r_e / r_w}$$

so that

$$P = \frac{P_e - P_w}{\ln r_e/r_w} \ln r + \frac{P_w \ln r_e - P_e \ln r_w}{\ln r_e/r_w} = \frac{P_e - P_w}{\ln r_e/r_w} \ln \frac{r}{r_w} + P_w .$$

Applying now Equation 8, v_r takes the form

$$V_r = -\frac{K}{\mu} \frac{\partial P}{\partial r} = -\frac{K}{\mu r} \frac{P_e - P_w}{\ln r_e/r_w} .$$

Finally the total outflow per unit time from the sand into the well is evidently given by

$$Q = -h \int_0^{2\pi} r V_r \, d\theta = -h \, 2\pi \, r V_r = 2\pi \frac{K}{\mu} h \frac{P_e - P_w}{\ln r_e/r_w}$$

where h is the sand thickness.

The equation $Q = \frac{2\pi K h (P_e - P_w)}{\ln r_e/r_w}$ will be referred to as Muskat's equation.

Taking Muskat's equation $Q = \frac{2\pi K h (P_e - P_w)}{\ln r_e/r_w}$ we see that if the permeability of the bed, the thickness of the bed, and the viscosity of the fluid are constant, the flow is directly proportional to the pressure drop and inversely proportional to the natural logarithm of the ratio of the diameters of the concentric circles. Or

$$Q = \frac{c (P_e - P_w)}{\ln r_e/r_w}$$

in which

$$c = \frac{2\pi K h}{\mu}$$

To show these proportional relationships the following mathematical graph is plotted in Fig. 5.

It can be seen that in the calculations the diameter of the hole will not be the most important. To increase the flow, everything else being the same, the ratio of the

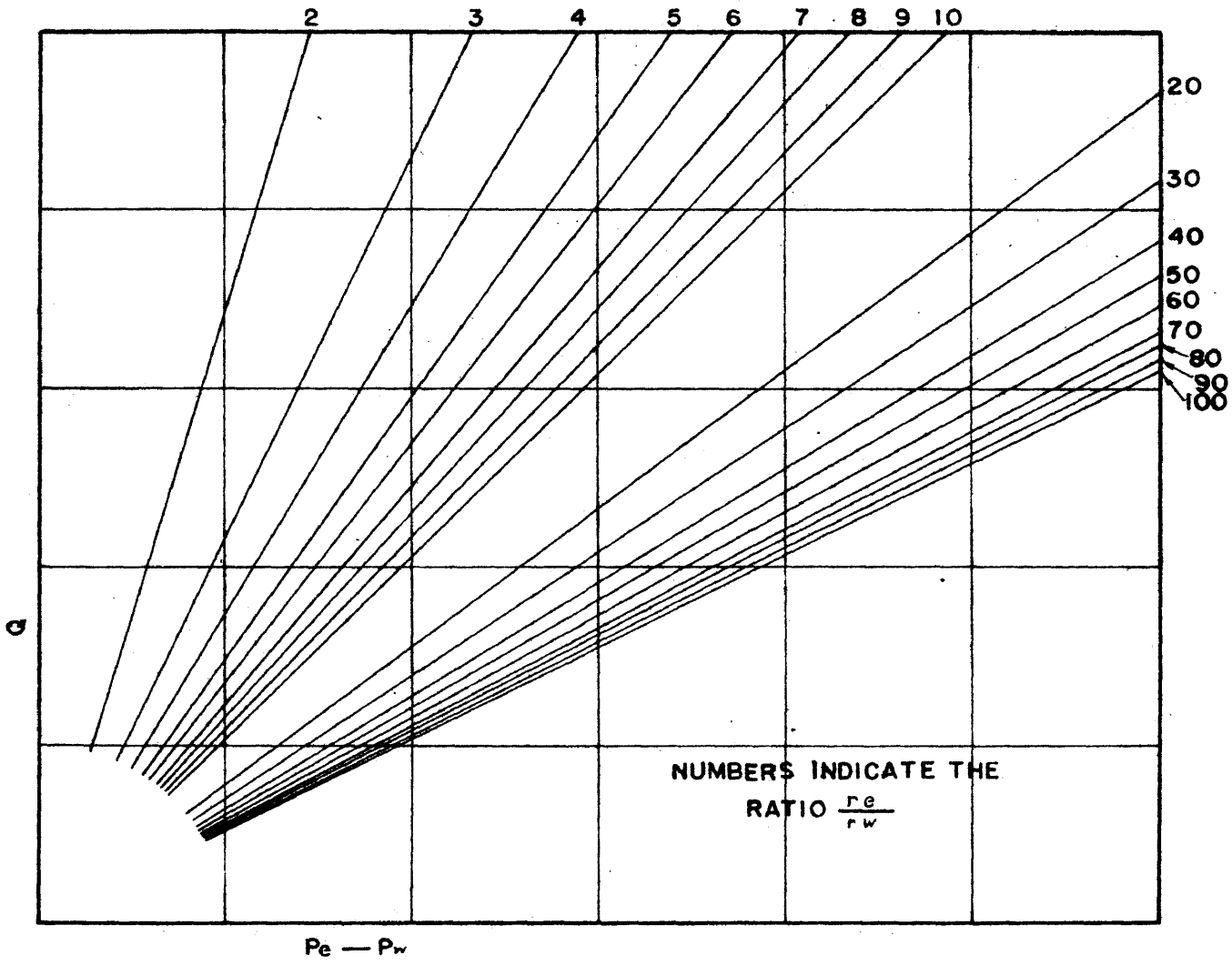

2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100
NUMBERS INDICATE THE
RATIO re/rw
Pe — Pw

FIG. 5

diameters of the concentric circles has to be increased. In a circular drainage area with a 100-ft diameter we will have to increase the hole diameter from 1 ft to 10 ft to double the amount of flow.

Leaching of Ore in Place

During any sub-surface leaching process the constants of permeability, porosity, viscosity, and many others will vary with the time. In the following an attempt is made to derive functions and formulas for the variation of some of these factors.

During leaching, the effective porosity at the end of the process will be made up by the original effective porosity, the amount of material leached out, and a part of the difference between the original total porosity and the original effective porosity. The last factor is due to the partly or entirely leaching out of the walls that formed the difference between total and effective porosity. For the flow of fluids through sands the effective porosity only is considered. In the discussions and derivations in this chapter the following assumption will be made. The effective porosity equals the total porosity. This assumption has to be made, as it is impossible to make mathematical derivations without it.

According to the Berthoud-Valeton Theory 4/, the rate

4/ Jour. Cem. Phys., 10,625 (1912). See Chemical Engineers' Handbook, McGraw-Hill Book Co., Inc., 1934, p. 1474.

of solution of a crystalline substance depends upon the rate of diffusion and a first-order surface reaction. This fact can be most simply expressed as

$$\frac{dw}{dt} = KS(C_s - C_t) \quad (1)$$

Where C_s is the concentration of solute in solution at saturation
C_t is the concentration of solute in solution at time t
K is a negative constant
S is the surface area of the undissolved solute
t is the time
W is the weight of undissolved solute

Equation 1 is for a second-order process. However, by maintaining negligible concentration of solute in the solution, the diffusional resistance drops out and a first-order process results wherein the rate of solution depends only upon the surface area of undissolved solute. Equation 1 becomes

$$\frac{dw}{dt} = K'S \quad (2)$$

Since, for geometrical solids, the surface area is proportional to the weight to the two-thirds power we may express equation[2] as

$$\frac{dw}{w^{2/3}} = K'dt \quad (3)$$

Integration of Equation 3 gives

$$3w^{1/3} = K't + C \quad (4)$$

where C is the constant of integration which may be obtained by substituting the weight of the geometrical solid at zero time.

Equation 1 is applicable for the second-order process. Cs is a constant as obtained from soluability tables, but C_t

is a variable quantity and is evaluated as

$$C_t = W_o - W \tag{5}$$

Where W is the weight of the solid at any time
W_o is the weight of the unsubmerged solid at zero time

Substitution of $W_o - W$ for C_t and $W^{2/3}$ for S gives

$$\frac{dw}{dt} = K' w^{2/3} (C_s - W_o + W) \tag{6}$$

$C_s - W_o$ is a constant which can be designated by C_o. Substituting this value and rearranging for integration between limits gives

$$\int_{w_o}^{w} \frac{dw}{w^{1/3}(C_o + W)} = \int_{o}^{t} K' dt . \tag{7}$$

Equation 4 and Equation 7 were experimentally checked by Gleeson 5/. Gleeson ran experiments with salt cubes as

5/ Gleeson, George W., Some simple experiments dealing with rates of solution. Journal of Chemical Education, Vol. 15, p. 187, 1938.

the solid and water as the solute. His results agreed very well with the two mathematical solutions.

In Equation 4 the constant of integration C can be found by substituting w_o for w and O for t, being the values for w and t at zero time. We find $C = 3w_o^{1/3}$ and Equation 4 becomes

$$3w^{1/3} = K't + 3w_o^{1/3} \qquad \text{or} \qquad w = (K''t + w_o^{1/3})^3 \tag{8}$$

in which $K'' = 1/3\, K'$

Integration of Equation 7 gives as a result

$$3C_o^{-2/3}\left\{\frac{1}{6}\ln\frac{(C_o^{-1/3}w^{1/3}+1)^2}{C_o^{-2/3}w^{2/3} - C_o^{-1/3}w^{1/3}+1} + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2C_o^{-1/3}w^{1/3}-1}{\sqrt{3}}\right)\right\} = K't \Big|_{w_o}^{w} \tag{9}$$

Porosity

The porosity, at any time, of a sand containing soluble solid can be written as $e_t = \frac{e+a}{e+a+b}$. (10)

Where a is the amount of dissolved material
b is the amount of undissolved material
e is the original porosity

We can write $e+a+b=1$ by taking decimal values for a, b, and e. The amount a is a volume and equals $\frac{W_o - W}{\gamma}$, where γ is the value for the density of the material.

By substituting the value for a, and the value for $a+b+e$ in Equation 10 we find the general equation for the porosity

$$e_t = \frac{W_o - W}{\gamma} + e \qquad (11)$$

In which e_t is the porosity at any time
w is the weight of the undissolved soluble material at any time
w_o is the weight of the undissolved unsubmerged solid at time zero

Substitution of the values for W gives the relation between the porosity and the time.

Permeability

From Taylor's Equation $K = C D_s^2 \frac{\gamma w}{\mu} \frac{e^3}{1+e}$, derivations will be made for the permeability during the leaching process. We can write above equation as

$$K_t = C_L D_t^2 \frac{e_t^3}{1+e_t} \quad . \qquad (12)$$

Where C_L is a constant being equal to $C \frac{\gamma w}{\mu}$

D_t is the value for D_s at any time
e_t is the porosity at any time
K_t is the permeability at any time

The representative grain consists both of soluble grains

and insoluble grains. Therefore we write D_t as $a d_t + b d_s$. The value ad_t represents the soluble part and the value bd_s represents the insoluble part. To express ad_t in W we write

$$\left(\frac{w}{\gamma}\right)^{2/3} = n\pi d_t^2 \quad . \tag{13}$$

This equation is eligible, remembering the relationship between the surface area and the volume of a geometric body. The number of the soluble grains is given by n. So

$$a d_t = a\sqrt{\left(\frac{w}{\gamma}\right)^{2/3}\frac{1}{n\pi}} \quad . \tag{14}$$

Substituting the values from Equations 13 and 14, and the value for D_t in Equation 12 we find

$$K_t = C_L\left[a\sqrt{\left(\frac{w}{\gamma}\right)^{2/3}\frac{1}{n\pi}} + b d_s\right]^2 \frac{e_t^3}{1+e_t} \tag{15}$$

which is the general equation for the permeability during a leaching process.

Summary of the Formulas for Leaching of Ore in Place

Summarizing the general formulas for permeability and porosity for both the first and second-order process we have

$$e_t = \frac{w_o - w}{\gamma} + e \tag{11}$$

$$K_t = C_L\left[a\sqrt{\left(\frac{w}{\gamma}\right)^{2/3}\frac{1}{n\pi}} + b d_s\right]^2 \frac{e_t^3}{1+e_t} \tag{15}$$

$$w = \left(K'' t + w_o^{1/3}\right)^3 \tag{8}$$

$$3 C_o^{-2/3}\left\{\frac{1}{6}\ln\frac{\left(C_o^{-1/3} w^{1/3}+1\right)^2}{C_o^{-2/3} w^{2/3} - C_o^{-1/3} w^{1/3}+1} + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2 C_o^{-1/3} w^{1/3}-1}{\sqrt{3}}\right)\right\} = K' t \Big|_{w_o}^{w} \tag{9}$$

To find the porosity we have to substitute the values for W from Equation 8 or Equation 9 into Equation 11 for respectively the first- and the second-order process. To obtain the value for the permeability these values must be substituted into Equation 15.

It can be seen that too many factors are involved and that too complicated formulas are the result. The only thing that has been shown in this second part of this chapter is that change in porosity and permeability cannot very well be demonstrated by means of mathematics.

CHAPTER III

EXPERIMENTAL APPROACH

In this chapter two different types of tests are described: (1) Muskat's Equation and two additional permeability tests and (2) the leaching of ores in place.

Test on Muskat's Equation

To check Muskat's Equation $Q = \frac{2\pi Kh(P_e - P_w)}{\mu \ln r_e/r_w}$ a test described in the following was run. The test was started by injecting water through vertical holes in sandstone cores. Two cores were used, one 8 in. high, the other 3.5 in. high. The cores respectively had axially centered holes 1 in. and 2.25 in. in diameter. The flow and the pressure were measured and plotted to curves.

Apparatus

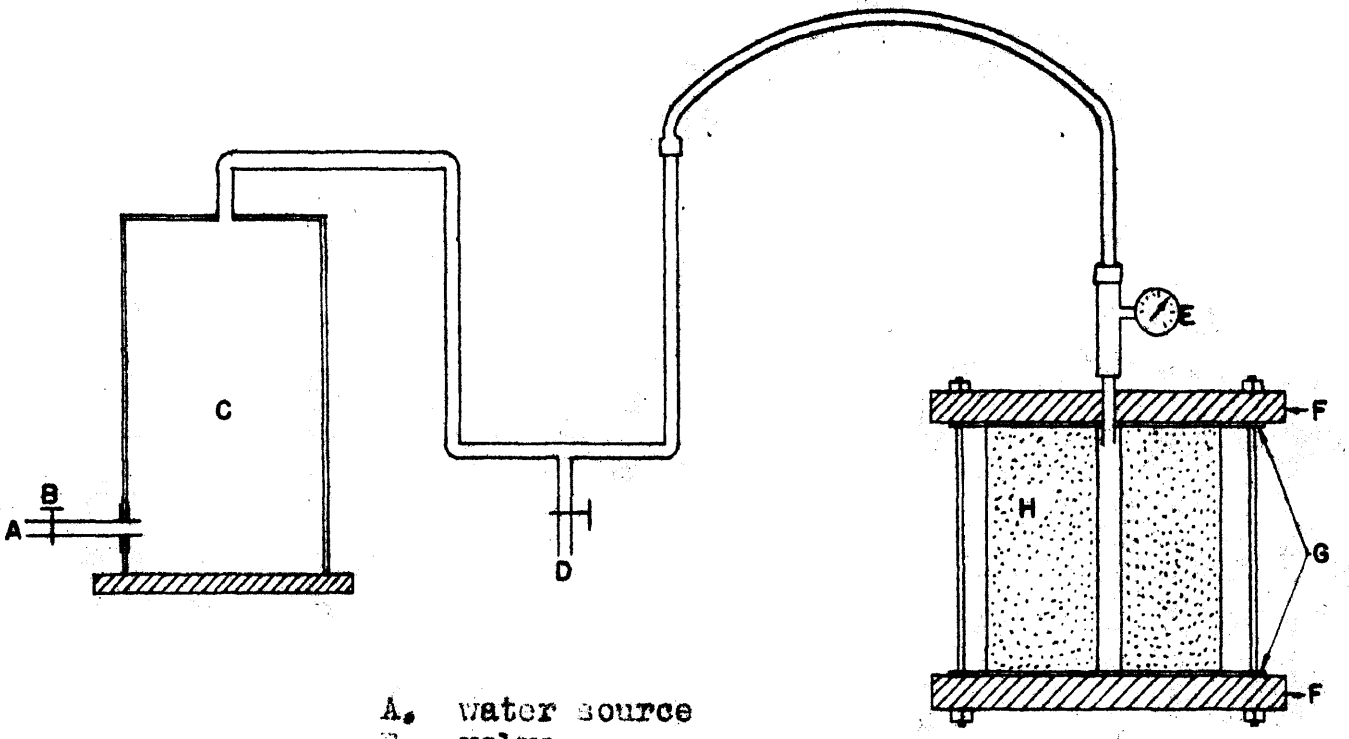
A sandstone core, 8 in. in diameter, obtained from diamond drilling in Navajo sandstone for blasting purposes, was cut by means of a diamond saw into cylinders 8 in. and 3.5 in. high. A 1 in. hole was drilled in the center of the first core in the vertical direction with a pressure drill, and a 2.25 in. hole was drilled in the second core. Standard pipe mounted in the pressure drill acted as a Calyx bit and the shot used consisted of iron filings.

The perforated core was placed in a steel clamp with gasket between the sandstone cylinder and the plates. The original clamp consisted of two 1/8-in. steel plates reinforced with angle iron. The plates, 10 in. square, were

fastened to the sandstone cylinder by means of four bolts. The 1/8-in. plates did not seem to be sufficient to withstand the original 35-psi pressure of the water injected. So these plates were replaced by 1-in. steel plates 1 ft square. A hole threaded for a 1/8-in. pipe was drilled in the top plate. The top part of the 1/8-in. pipe was connected to the pressure source, and the bottom part acted as a lead to the hole in the cylinder. Many gaskets and cements have been tried to withstand the pressure between the core and the plates. Sponge rubber, glazier's putty, rubber sheet, and rubber sheet with "aviation" cement were insufficient to withstand the original 35-psi pressure. Asbestos gasket and aviation cement was sufficient and has been used for pressures up to 110 psi.

Originally a pump with a maximum capacity of 35 psi was used as a pressure source. Later it was found that the water supply of the city of Golden gave a pressure up to 110 psi. The pressure was regulated by means of a bypass valve. The original results were not satisfactory because of rust and other kinds of dirt from the pipelines of the system. The walls of the hole in the sandstone cylinder became impervious or nearly so. For this reason a filter was connected between the water source and the core. A diagram and a picture are shown in Fig. 6 and Fig. 7.

The pressure of the water entering the pressure pot (the core-clamp combination) is measured at a pressure gauge. The water is caught in a zinc reservoir from which



A. water source
B. valve
C. filter filled with sand
D. bypass valve
E. pressure gauge
F. clamp
G. gasket and cement
H. sandstone core

Fig.6

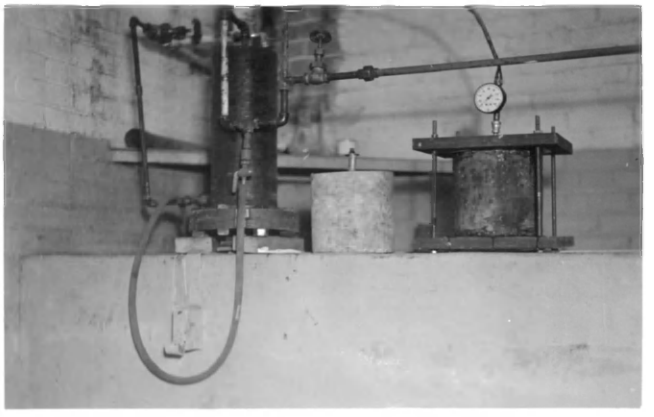


Fig.7

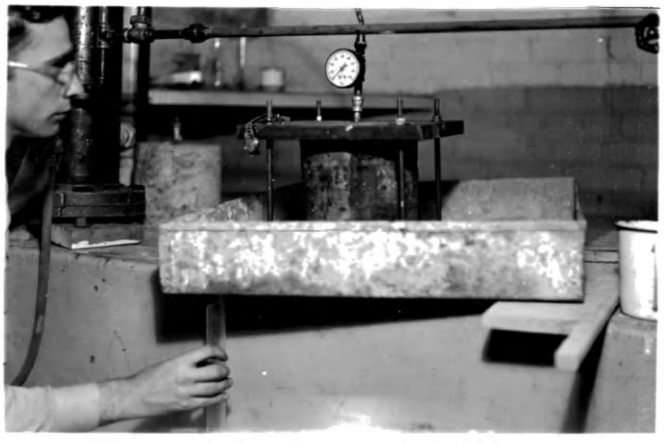


Fig.8

the water runs through a hole into a graduate shown in Fig. 8.

Results

The data obtained are listed in Table I and II, and the results are plotted in Fig. 9 so that the value for $p_e - p_w$ (gauge pressure in psi) are the abscissas and the values for Q(flow in ml per min) are the ordinantes. A straight-line relation between Q and $p_e - p_w$ seemed to exist, as was expected from Muskat's equation. The tangent of the angle which the straight line makes with the horizontal axis will be designated as b $\left(b = b' \frac{14.7}{60}\right)$. The value b corresponds with the $\frac{2\pi K h}{\mu \ln r_e/r_w}$ – part of Muskat's equation, and can be derived from the line obtained. Now the value K for the permeability can be calculated from the value b, by writing the equation for b as $K = \frac{b \mu \ln r_e/r_w}{2\pi h}$.

TABLE I, DATA FROM TEST ON MUSKAT'S EQUATION

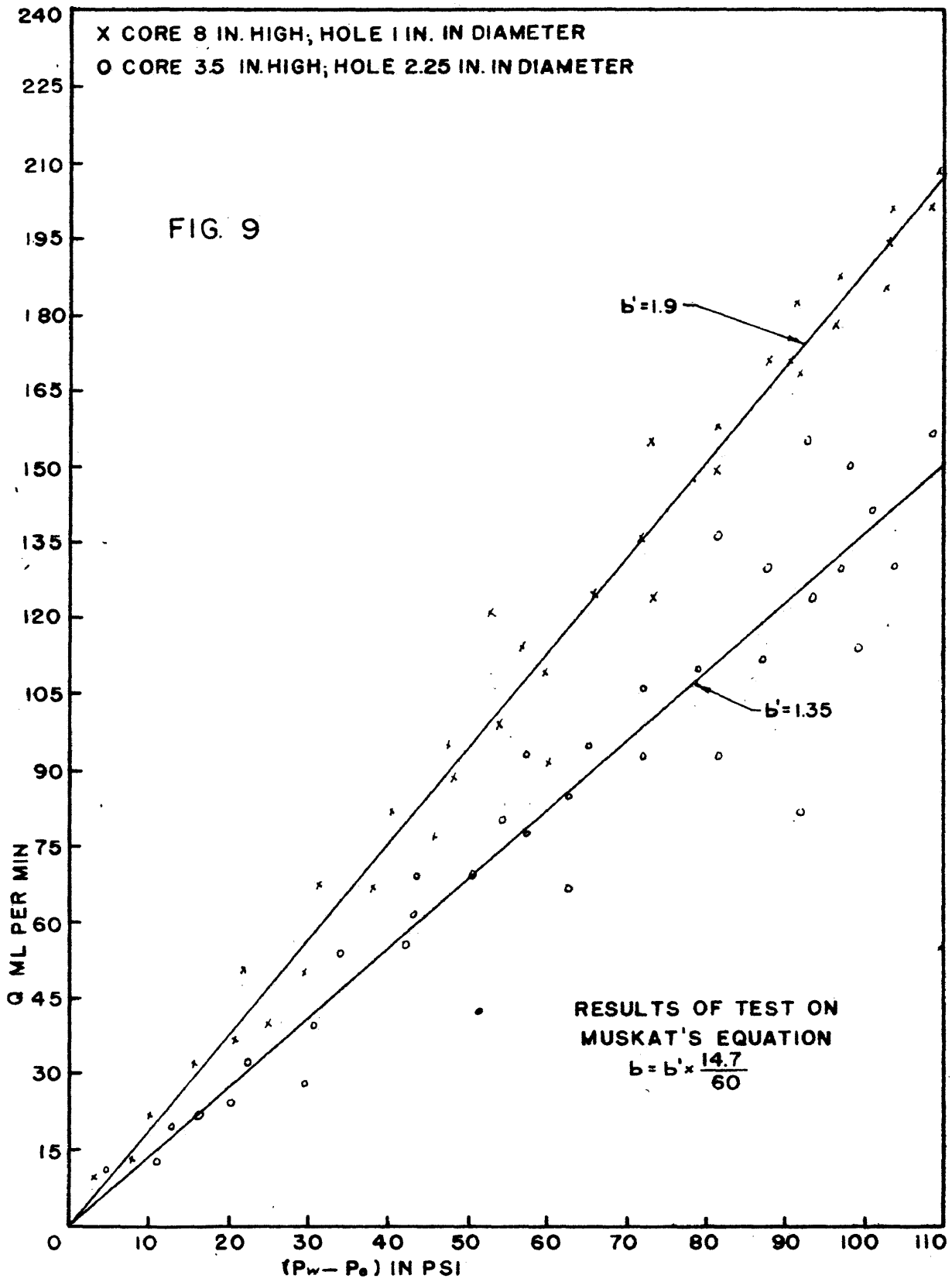
height core - 8 in.
diameter hole - 1 in.
diameter core - 8 in.

$p_w - p_e$ psi	Time min	Amount ml	Flow ml per min
3.4	40	392	9.8
8.0	30	399	13.3
10.1	30	651	21.7
15.9	30	954	31.8
20.7	30	1092	36.4
22.0	30	1528	50.9
25.0	30	1199	40.0
29.7	30	1517	50.6
31.5	30	2016	67.2
38.2	30	1988	66.3
40.6	20	1620	81.0
47.9	20	1829	91.5
46.0	20	2230	111.5
48.2	20	1768	88.4
53.0	20	1767	88.4
53.0	20	2406	120.3
54.0	20	2020	101.0
57.0	20	2280	114.0
59.8	20	2180	109.0
60.4	20	1847	91.4
66.0	20	2590	124.5
72.0	20	2700	135.0
73.6	20	3089	154.5
73.8	20	2482	124.1
81.6	20	2971	148.6
82.0	20	3146	157.3
88.0	20	3357	167.9
91.0	20	3353	167.7
92.0	10	1816	181.6
92.0	10	1665	166.5
96.4	10	1783	178.3
96.5	10	1867	186.7
103.0	10	1961	196.1
103.5	10	1845	184.5
104.0	10	2008	200.8
109.5	10	2007	200.7
110.0	10	2092	209.2

TABLE II - DATA FROM TEST ON MUSKAT'S EQUATION

height core - 3.5 in.
diameter hole - 2.25 in.
diameter core - 8 in.

$p_w - p_e$, psi	Time, min	Amount, ml	Flow, ml per min
4.8	60	667	11.1
11.0	60	744	12.4
13.1	60	1169	19.5
16.4	60	1118	18.6
20.3	40	961	24.0
22.4	40	1291	32.3
29.8	40	1113	27.8
30.7	40	1567	39.2
39.0	30	1610	53.7
42.5	30	1667	55.6
43.6	20	1228	61.4
44.0	20	1320	66.0
50.8	30	1981	66.0
52.0	30	1260	42.0
54.6	20	1591	79.6
57.4	20	1860	93.0
57.5	20	1554	77.7
63.0	20	1322	66.1
63.0	20	1680	84.0
65.0	20	1890	94.5
72.0	20	2123	106.2
72.0	20	1853	92.7
79.0	20	2192	109.6
81.8	20	1857	92.9
81.8	20	2721	136.1
87.0	20	2224	111.2
87.6	20	2598	128.9
92.0	20	1625	81.3
93.0	20	3091	154.6
93.0	20	2464	123.2
97.0	20	2476	128.8
98.3	10	1500	150.0
99.0	10	1141	114.1
101.0	10	1414	141.4
104.0	10	1292	129.2
109.0	10	1563	156.3


X CORE 8 IN. HIGH; HOLE 1 IN. IN DIAMETER
O CORE 3.5 IN. HIGH; HOLE 2.25 IN. IN DIAMETER
FIG. 9
b'=1.9
b'=1.35
RESULTS OF TEST ON
MUSKAT'S EQUATION
b = b' x 14.7/60
Q ML PER MIN
(Pw − Pe) IN PSI
240
225
210
195
180
165
150
135
120
105
90
75
60
45
30
15
0
10
20
30
40
50
60
70
80
90
100
110

The calculations for the two tests on Muskat's equation are shown below.

Explanation	Core I	Core II
b^1 obtained from Fig. 9	1.9	1.35
$b = b^1 \frac{14.7}{60}$	0.466	0.331
height of core in cm	20.32	8.89
temperature in C	25	25
viscosity in centipoises	0.8937	0.8937
hole diameter in cm	10.16	10.16
outside diameter in cm	1.27	2.86
r_e/r_w	8.0	3.55
ln r_e/r_w	2.0794	1.267
permeability in darcys	0.0069	0.0067

By core I is meant the 8-in. core, 8-in. high with a hole diameter of 1 in.

By core II is meant the 8-in. core, 3.5 in. high with a hole diameter of 2.25 in.

The value for the viscosity is found in Handbook for Physics and Chemistry, 31st ed., 1949, page 1755.

The close values of the permeability prove that the equation derived by Muskat is correct as far as the height and the diameter ratio in the formula. These two tests do not prove whether Muskat's method is correct for the determination of the exact permeability of a rock.

To check the exact value of the permeability as derived above, two additional tests were run. The principle for

these tests is Darcy's law

$$K = \frac{\mu Q L}{A P} .$$

Where A is the cross-sectional area of the sample in cm^2
K is the permeability in darcys
l is the length of the sample in cm
μ is the absolute viscosity of the fluid or the gas used in centipoises
P is the pressure drop through the sample in atmospheres
Q is the quantity of fluid or gas passes in ml per second

Method for Additional Permeability Tests

Test plugs are cut from the sandstone by means of a pressure drill, steel tubing, and crushed steel. A drag bit of 3/4-in. tubing was used. The test plugs were thoroughly cleaned with water only. Usually the plugs have to be washed with carbon tetrachloride, as the material flowing in most cases in the field is oil. For these experiments in which water is used as the flowing liquid (here the pressure pot experiments is referred to) washing with carbon tetrachloride was not considered to be necessary.

After the length and diameter have been measured, the plugs are inserted in a rubber cork or hose. The cork or hose is placed in the testing apparatus, and measurements of flow and pressure are made, from which the permeability can be calculated. The two tests will be segregated as "Test with Air" and "Test with Water".

Test with Air

The apparatus for the permeability test in which air is

used as flowing gas is designed as shown in Fig. 10.1/

1/ American Petroleum Institute Code No. 27, Standard Procedure for Determining Permeability, 1935. To be contained from American Petroleum Institute, 50 West 50th Street, New York, N.Y.

Air cleaned from oil and other impurities enters the apparatus under a pressure of approximately 65 lbs. The pressure is reduced by means of a reducing valve. The apparatus has two pressure gauges. The first one measures the higher values, at which time the second one is closed by a valve. In case the values become low (in the neighborhood of 5 lbs) the valve to the second gauge is opened. This gauge is a common U-tube filled with mercury and has a scale in inches.

After the air passes the reducing valve, it goes through the plug inserted in the perforated rubber stopper. The rubber stopper is placed in a clamp of the apparatus as shown in Fig. 10. The pressure of the air leaving the sandstone plug has the tubes connected by a hose. To maintain a constant pressure, (atmospheric) the two levels of the liquid (colored water) in the graduate tubes are held even by moving the right tube. The time necessary for a certain amount of air to flow is measured by a stop watch.

Results and Calculations

The data obtained from the experiments are listed below.

TABLE III

DATA AND CALCULATION FOR PERMEABILITY TEST WITH AIR

	Plug 1	Plug 2	Plug 3
Temp., °C	24	24	24
Viscosity centipoises*	.0182	.0182	.0182
Bar., in. Hg	26.3	26.3	26.3
Bar., atm.	.878	.878	.878
Gauge press., in. Hg	4.5	4.5	4.5
Gauge press., atm.	.1503	.1503	.1503
Inlet pressure	1.028	1.028	1.028
Outlet pressure	.878	.878	.878
Average pressure throughout sample	.953	.953	.953
Applied air volume, cc	50	50	50
Corrected air volume, cc	52.4	52.4	52.4
Length plug, cm	2.5	2.6	2.1
Area plug cross-sectional, cm^2	1.76	2.22	1.76
Time, sec	288	274	286
Permeability, darcys	.0295	.0272	.0265

* The values for viscosity are found in Handbook for Chemistry and Physics, 31st ed., p. 1762, 1949.

If water or any other liquid is used, the formula $K = \frac{\mu Q L}{P A}$ is valid. In calculating permeabilities, where air or gas has been used instead of fluid, it is necessary to make a correction for the pressure. It is necessary to determine the average pressure throughout the length of the sample instead of using the difference between input and output pressure as may be done with a liquid. This correction is necessary because the pressure drop in a flow tube carrying gas may not be a straight line from inlet to outlet as it is with a liquid. The principle involved is the same as that in pipeline work.

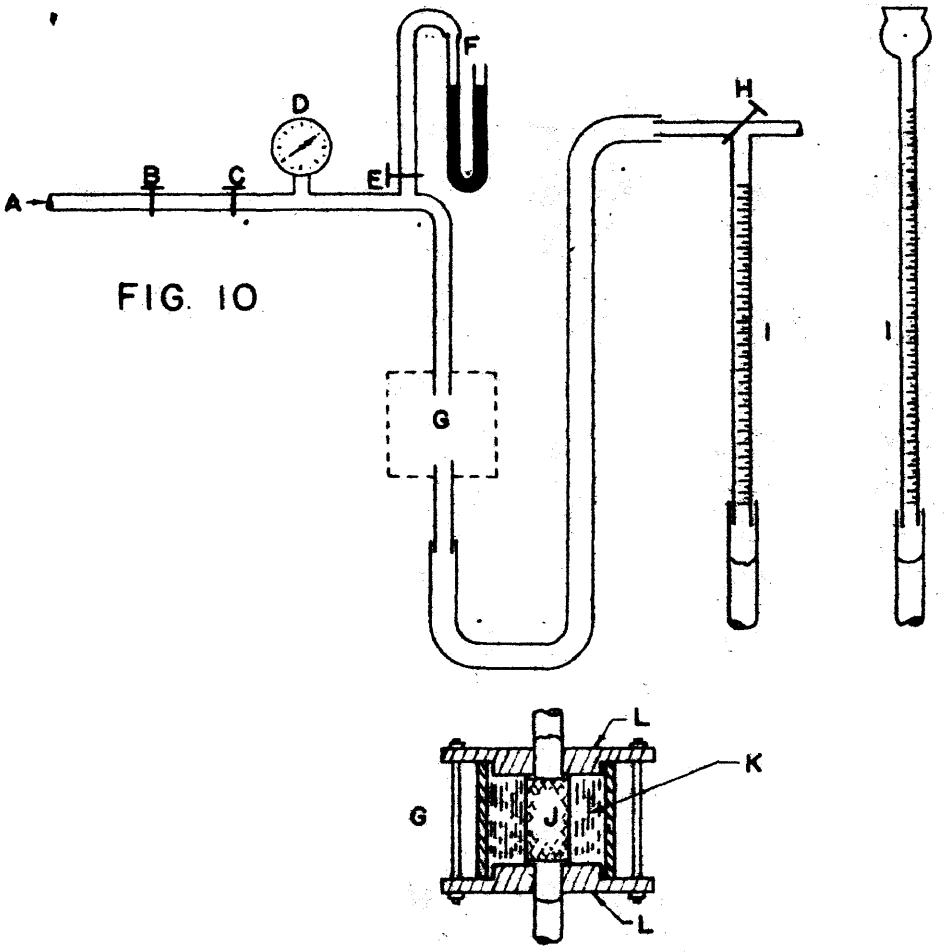


FIG. 10

A. air pressure
B. valve
C. reducing valve
D. pressure gauge
E. valve
F. mercury pressure gauge
G. specimen clamp
H. triple valve
I. graduated tubes
J. specimen
K. rubber stopper
L. clamp

Test with Water

The principle of the test is to inject a known amount of a liquid (water) with a known viscosity, during a known time and under a known pressure through a horizontal specimen of which the permeability is to be found. Darcy's law is accepted and used for the calculations.

The apparatus built is shown in Fig. 11. Low pressures in the order of a few inches of liquid used are applied. The specimens are plugs as described, and are the same as those used for the test with air. The plugs should be so inserted in the rubber hose used to contain the specimen that the cylindrical axis of the plug is horizontal during the test. This horizontal position is used so that no pressure drop or back pressure occurs in the specimen because of the force of gravity.

The data and calculations are shown below:

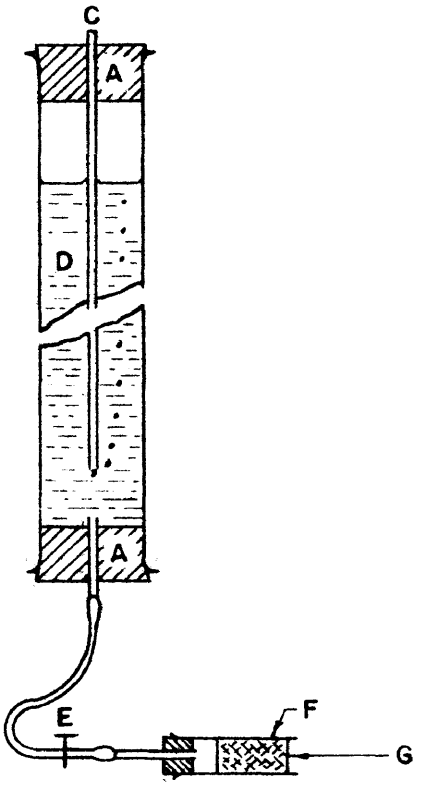


Fig.11

A. bored rubber stopper
B. reservoir
C. constant-pressure tube
D. liquid
E. valve
F. rubber hose
G. test plug

Fig.11

TABLE IV

DATA AND CALCULATIONS FOR PERMEABILITY TEST WITH WATER

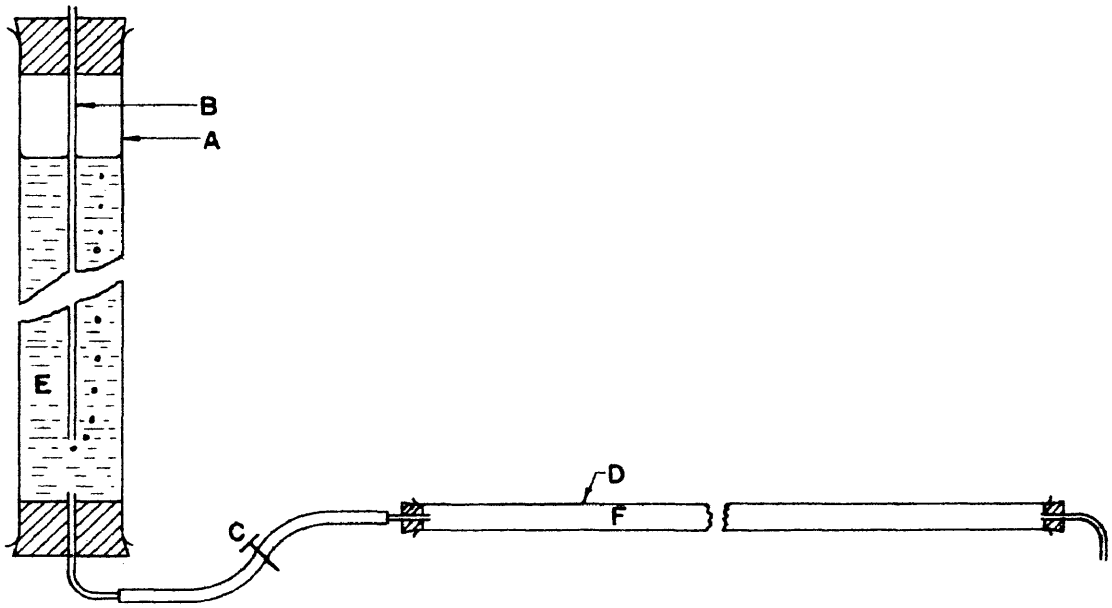
Explanation	Plug 1	Plug 2	Plug 3	Plug 4
Length, cm	2.5	2.6	2.1	6.3
Diam, cm	1.5	1.7	1.5	2.0
Area, cm	1.76	2.22	1.76	3.14
Temp water, °C	25	29	27	30
Viscosity water	.8937	.8180	.8545	.8007
Applied pressure, in. of water	27	27	27	27
Applied pressure, atm	.067	.067	.067	.067
Volume, ml	9.4	26.7	17.2	54.3
Time, min	300	600	480	720
sec	18,000	36,000	28,800	43,200
Permeability	.013	.011	.009	.010

Experiments on Leaching

To gain a better understanding of the change in rate of flow during an underground leaching process, tests as described in the following were run. Mixtures of sand and soluble materials, and crushed ore were leached under a constant head.

Apparatus

The apparatus used is shown in Fig. 12. A 2.5-in. glass reservoir is connected by means of rubber stoppers and glass tubing to a horizontal leaching tube in which the material to be leached is placed. A constant head for the leaching liquid is provided by means of a glass tube (b in Fig. 12) in the reservoir connected to the atmosphere. The amount of liquid flowing is measured at the reservoir as an outage. Outage can be defined as the drop in height of the liquid in the reservoir. A test run determined that 1.5 in. of outage equaled 100 ml of leaching liquid. Paper strips pasted to



A. reservoir
B. constant-pressure tube
C. valve
E. leaching liquid
F. material to be leached

Fig. 12

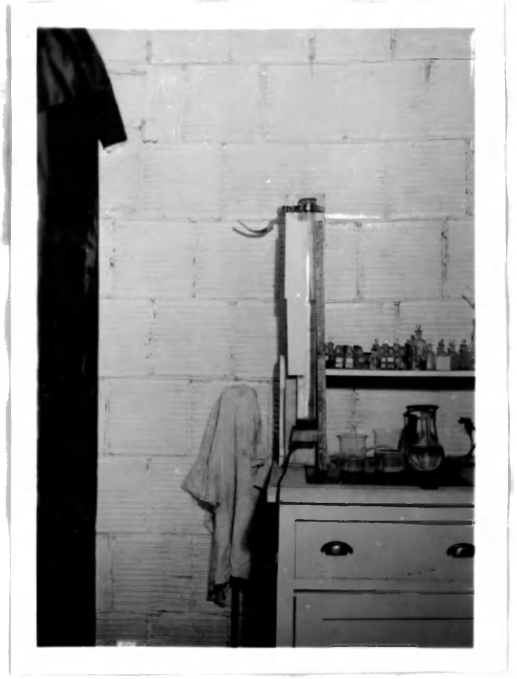


Fig.12a

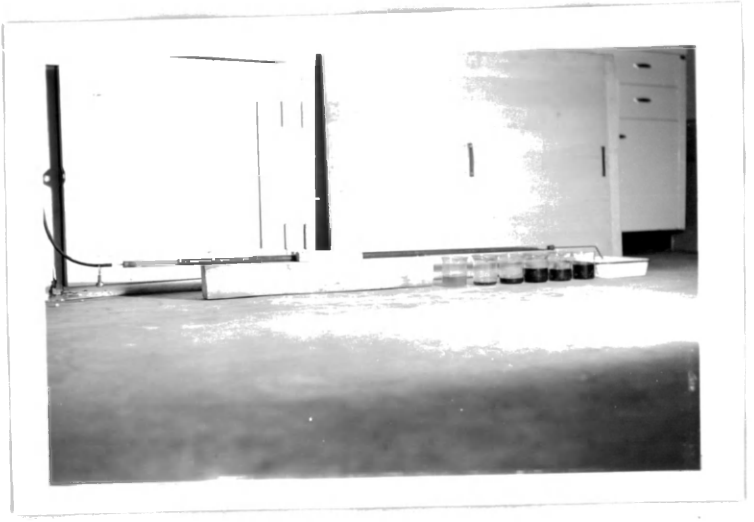


Fig.12b

the glass reservoir were used to record the outage. At different time intervals the position of the surface of the liquid was marked on these strips. The marks were later measured to give the data recorded.

Tests Run

The following tests were run.

Test	Material to be Leached	Leaching Liquid	Head of Liquid, in.
1	Salt, 5% in Ottawa Sand	water	7.5
2	Salt, 10% in Ottawa Sand	water	7.5
3	Salt, 20% in Ottawa Sand	water	7.5
4	Salt, 50% in Ottawa Sand	water	7.5
5	Salt, 100%	water	7.5
6	Sugar, 20% in Ottawa Sand	water	7.5
7	Baking Soda, 10% in Ottawa Sand	water	7.5
8a	Cal Seal, 10% in Ottawa Sand	water	7.5
8b	Cal Seal, 10% in Ottawa Sand		7.5
9a	Copper ore crushed to -14 mesh	water	27
9b	"	ammonia	27
9c	"	H_2SO_4, 1%	27

The sand used in the different mixtures is the so-called Ottawa sand. The pure silica sand, 20 to 30 mesh, had well-rounded grains. The percentages given are by volume.

The data obtained from the different tests are given in Table V through Table XIV. Curves are plotted for the accumulated value of the quantity of solvent against the time in Fig. 13 through Fig. 21. The left-side scale of ordinates show the accumulative quantity in milliliters and the abscissas

show the time in minutes. The rate of flow is calculated from curves mentioned by means of graphical differentiation. The velocity obtained is plotted with the time in minutes as abscissas and the velocity in milliliters per minute as ordinates referred to the right-side scale.

Salt, 5% in Ottawa Sand--The total amount of leaching liquid (water) used for the leaching of the salt was 800 ml. The time necessary for this flow was 180 minutes. After a period of 60 minutes the rate of flow started to decrease. The velocity curve shows a constant velocity with a break at 60 minutes, however, inaccuracy in the readings may have influenced this apparent decrease in rate of flow, especially because only initial and final readings were taken for the period from 60 to 180 minutes.

Salt, 10% in Ottawa Sand-- The total amount of leaching liquid (water) passed through the sand-salt mixture is 1400 milliliters over a period of 105 minutes. An increase in rate of flow could be observed after the saturation point of the sand-salt mixture was reached. The saturation of the mixture with water required 8 minutes. During the period from 8 minutes to 70 minutes the gradual increase in rate of flow was not outstanding, while from 70 minutes until 150 minutes no increase in rate of flow could be seen. The steady flow between 70 and 150 minutes was due to the fact that most of the salt was leached out.

Salt, 20% on Ottawa Sand--The total amount of leaching

TABLE V - SALT, 5 % IN OTTAWA SAND

Leaching Liquid - Water
Temperature - 29 C
Head - 7.5 in. of water

Length Tube - 47 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, in.	Volume, ml	Remarks
0	0	0	Start
3	0.7	45.7	
5	0.97	64.6	
7.5	1.16	77.4	Saturated
10	1.36	90.7	
15	1.70	113.3	
20	2.06	137.4	
25	2.40	160.0	
30	2.78	185.4	
35	3.12	208.0	
40	3.50	233.4	
45	3.82	254.8	
52	4.30	286.8	
55	4.50	300.0	
60	4.82	321.3	
180	12.00	800.0	Stop

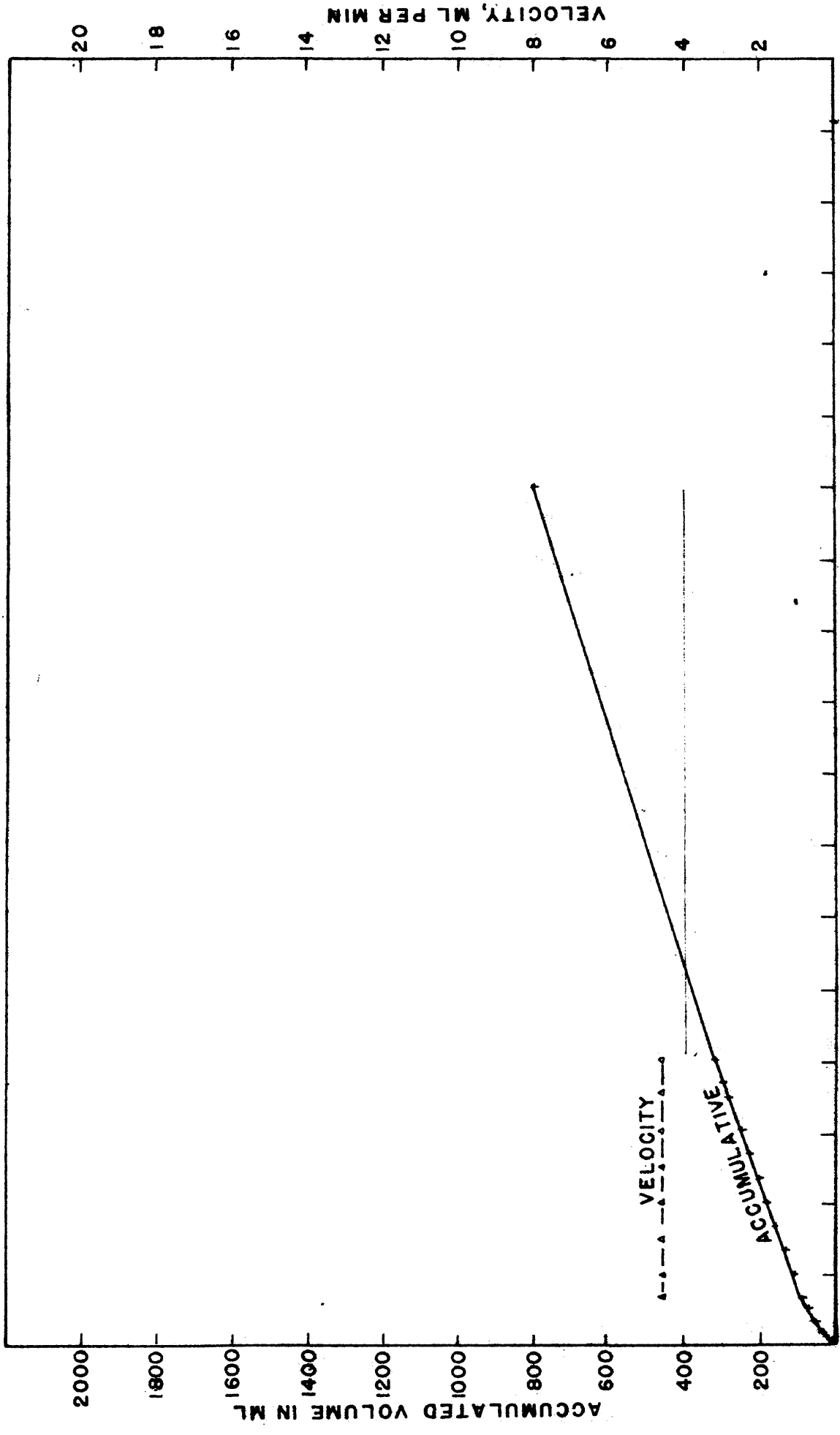


Fig.13 Salt, 5% in Ottawa Sand

TABLE VI - SALT, 10 % IN OTTAWA SAND

Leaching Liquid - Water
Temperature - 29 C
Head - 7.5 in. of water

Length Tube - 47 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, in.	Volume, ml	Remarks
0	0	0	Start
1	0.28	18.67	
2	0.4	26.67	
3	0.53	35.32	
4	0.68	45.33	
5	0.72	48.00	
6	0.98	65.34	
7	1.1	73.32	Saturation
8	1.2	80.00	
10	1.47	97.98	
15	2.18	145.3	
20	2.92	194.7	
25	3.73	248.7	
30	4.64	309.3	
36	5.92	394.5	
40	6.77	453.0	
45	7.82	521.6	
50	8.98	598.4	
55	10.13	675.0	
60	11.41	760.0	
65	12.6	839	
70	13.84	922	
75	15.07	1005	
80	16.38	1092	
85	17.5	1167	
90	18.9	1260	
95	20.02	1335	
100	21.2	1413	
105	22.3	1486	Stop

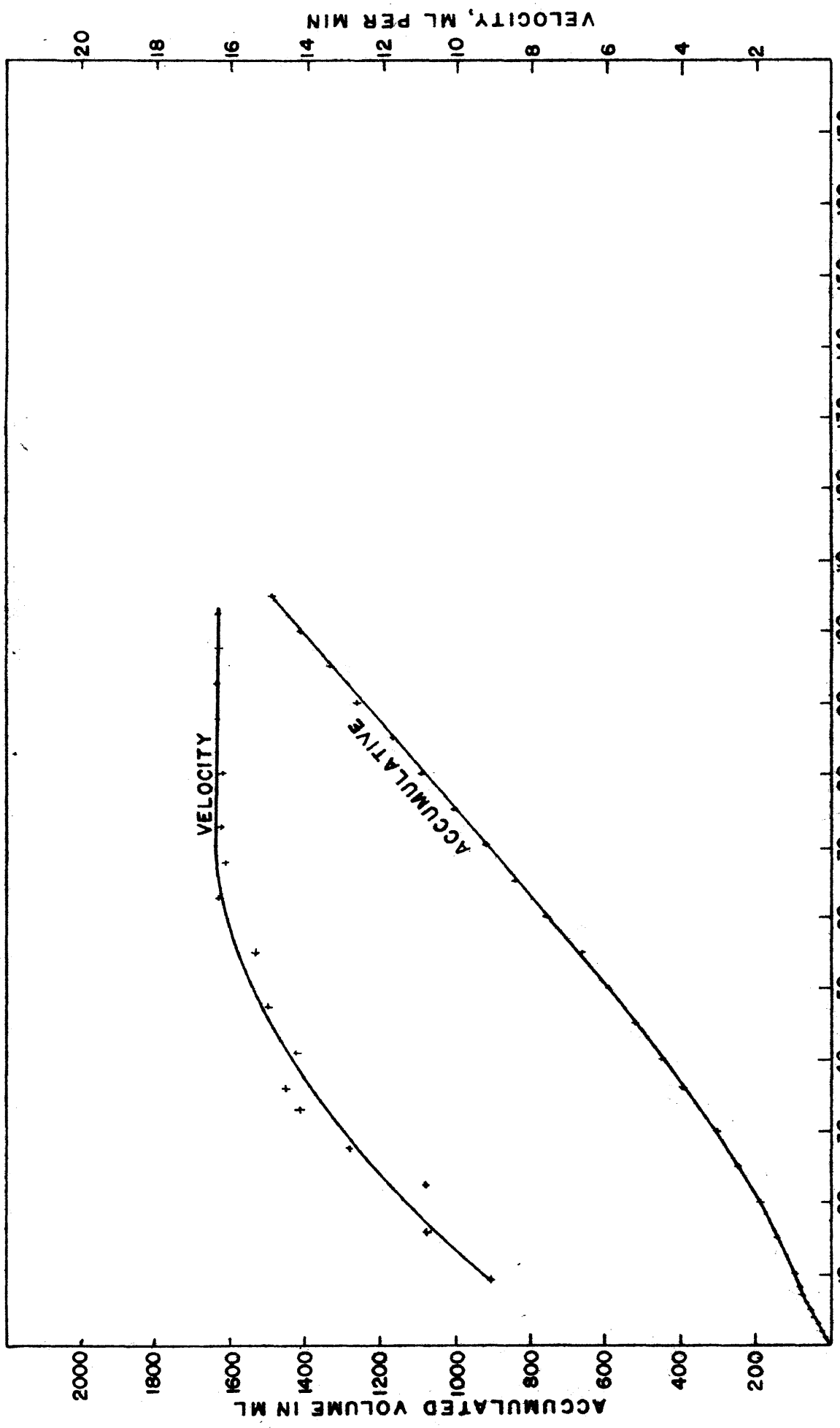


Fig. 14 Salt, 10% in Ottawa Sand

TABLE VII - SALT, 20 % IN OTTAWA SAND

Leaching Liquid- Water
Temperature - 29 C
Head - 7.5 in. of water

Length Tube - 47 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, in.	Volume, ml	Remarks
0	0	0	Start
1	0.3	20	
2	0.48	32	
3	0.67	44.7	
4	0.86	57.4	
5	1.02	68.0	
6	1.22	81.3	Saturated
10	2.13	142.0	
12	2.68	178.7	
13	2.97	198.1	
14	3.23	214.4	
15	3.52	234.8	
16	3.85	256.8	
17	4.17	278.1	
18	4.45	296.7	
19	4.80	320.0	
20	5.21	347.5	
21	5.44	362.8	
22	5.80	386.8	
23	6.10	406.8	Tapped glass,
24	6.43	427.7	air bubbles
25	6.78	452.2	coagulated
26	7.39	493.0	
27	7.89	526.2	
28	8.42	561.6	
29	8.92	594.4	
30	9.43	628.7	
31	9.93	662.0	
32	10.42	694.4	
33	10.96	730.4	
34	11.48	765.0	
35	11.94	796.0	
36	12.45	830.0	
37	12.95	863.0	
38	13.42	894.4	
39	13.91	927.2	
40	14.42	961.6	
41	14.92	994.0	

TABLE VII (continued)- SALT, 20 % IN OTTAWA SAND

Time, min	Outage, in.	Volume, ml	Remarks
42	15.40	1026.5	
43	15.91	1060.0	
44	16.40	1093.0	
45	16.91	1127.5	
46	17.39	1159.0	
47	17.88	1181.5	
48	18.32	1222.0	
49	18.85	1256	
50	19.34	1289	
51	19.84	1323	
52	20.33	1355	
53	20.81	1387	
54	21.30	1420	
55	21.78	1456	
56	22.30	1486	
57	22.76	1516	
58	23.24	1550	
59	23.71	1581	
60	24.20	1613	Stop

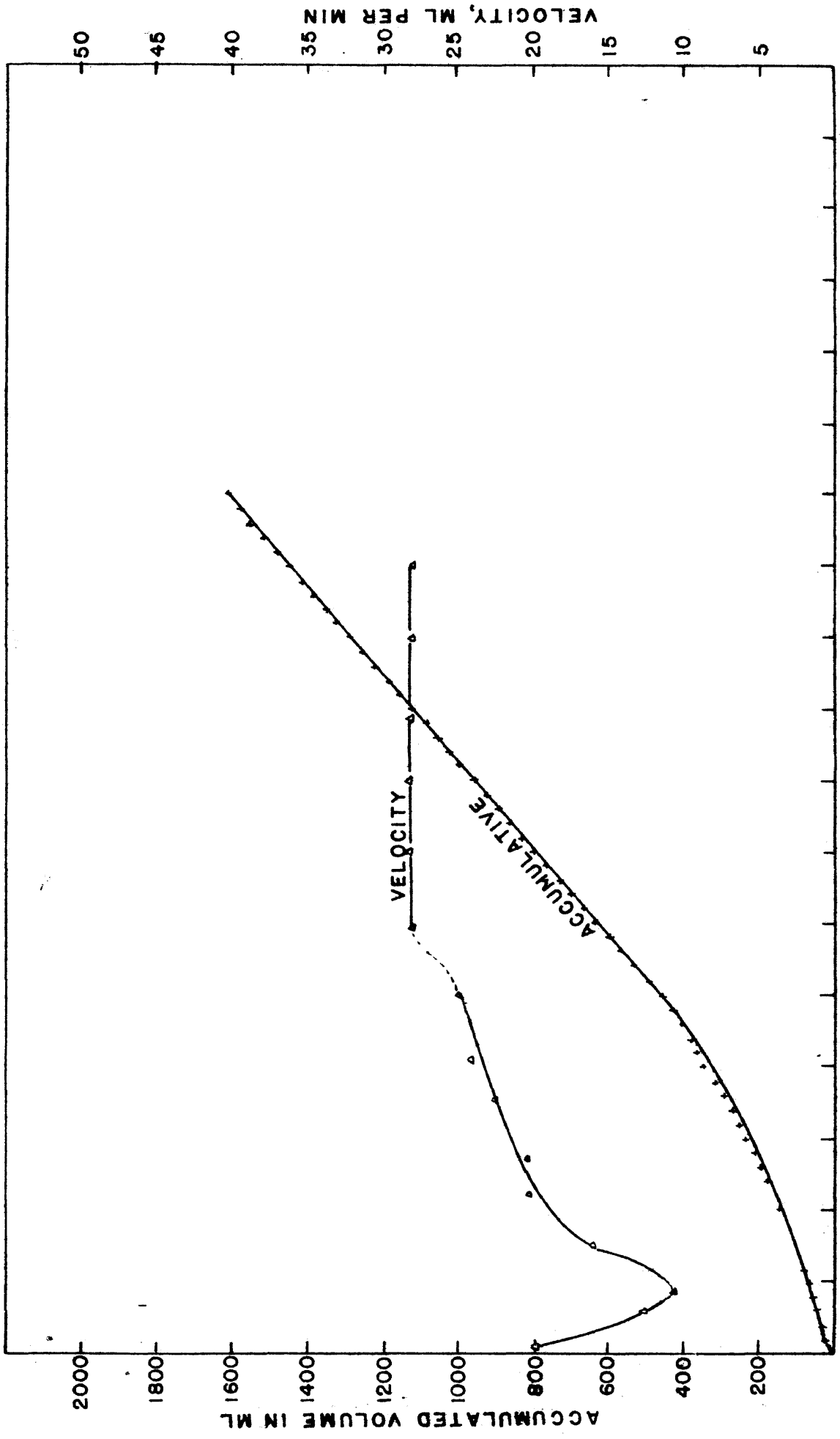


Fig. 15 Salt, 20% in Ottawa Sand

TABLE VIII - SALT, 50 % IN OTTAWA SAND

Leaching Liquid - Water
Temperature - 25° C
Head - 7.5 in. of water

Length Tube - 47 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, in.	Volume, ml	Remarks
0	0	0	Start
1	0.33	22	
2	0.60	40	
3	0.85	56.7	
4	1.08	72	Saturated
5	1.49	99.4	
6	2.07	138.0	
7	2.7	180	
8	3.3	220	
9	3.9	260	
10	4.5	300	
11	5.12	341.3	
12	5.7	380	
13	6.3	420	
14	6.88	459	
15	7.5	500	
16	8.1	540	
17	8.67	578	
18	9.25	616	
19	9.85	656	
20	10.4	694	
21	10.99	733	
22	11.5	767	
23	12.12	804	
24	12.67	844	
25	13.2	880	
26	13.73	916	
27	14.3	954	
28	14.82	988	
29	15.3	1020	
30	15.83	1056	
31	16.37	1092	
32	16.87	1126	
33	17.4	1160	
34	17.94	1196	

TABLE VIII (continued)- SALT, 50 % IN OTTAWA SAND

Time, min	Outage, in.	Volume, ml	Remark
35	18.45	1230	
36	18.93	1262	
37	19.42	1295	
38	19.95	1330	
39	20.45	1364	
40	20.95	1396	
41	21.47	1431	
42	21.98	1465	
43	22.5	1500	
44	23.04	1536	
45	23.58	1572	
46	24.1	1606	
47	24.63	1642	Stop

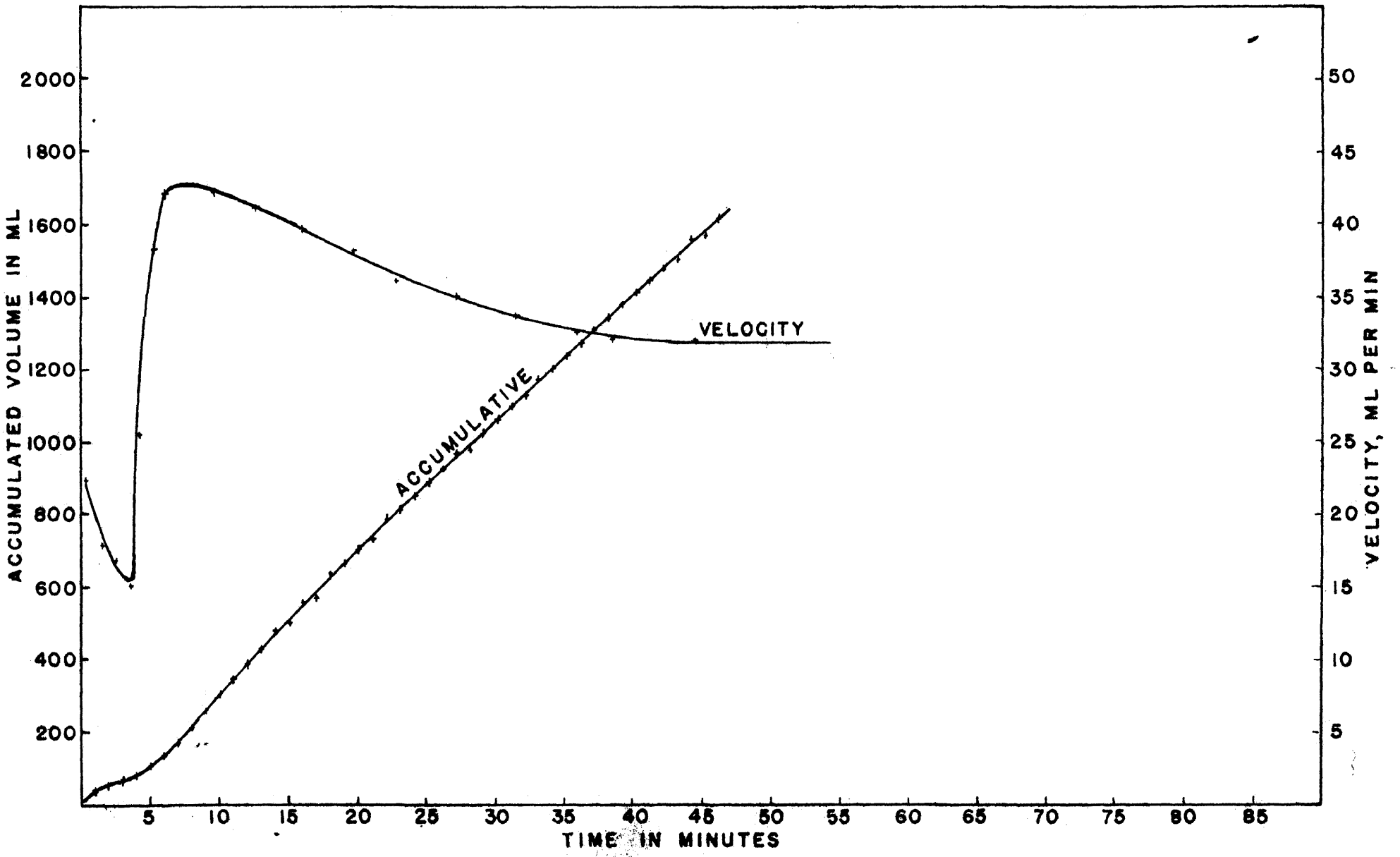


Fig. 16 Salt, 50% in Ottawa Sand

TABLE IX - SALT, 100 %

Leaching Liquid - Water
Temperature - 29 C
Head - 7.5 in. of water

Length Tube - 47 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, in.	Volume, ml	Remarks
0	0	0	Start
1	0.23	15.34	
2	0.46	30.68	
3	0.68	45.7	
4	0.92	61.33	
5	1.12	74.6	
6	1.34	89.3	
7	1.5	100.0	Saturated
8	1.6	106.7	Flow stopped,
9	1.6	106.7	helped by suction
11	1.6	106.7	
13	2.5	166.7	
14	5.25	350.0	
14.25	6.1	406.8	
14.5	6.8	457.0	
14.75	7.55	503.7	
15	8.31	554	
15.25	9.03	602	
15.5	9.78	652	
15.75	10.5	700	
16	11.3	753	
16.25	12.1	806	
16.5	12.9	860	
16.75	13.7	913	
17	14.57	972	
17.25	15.35	1027	
17.5	16.22	1081	
18	17.9	1184	
18.5	19.53	1302	
19	21.14	1410	
19.5	22.7	1514	
20	24.2	1614	Stop
20	24.2	1614	Start
20.5	26.65	1777	

TABLE IX (continued)- SALT, 100 %

Time, min	Outage, in.	Volume, ml	Remarks
21	28.65	1910	
22	32.3	2153	
22.5	34.2	2280	
23	36.0	2400	
23.5	37.75	2517	
24	39.65	2643	
24.5	41.4	2760	
25	43.2	2870	
25.5	45.05	3004	
26	46.7	3112	
26.5	48.5	3233	Stop

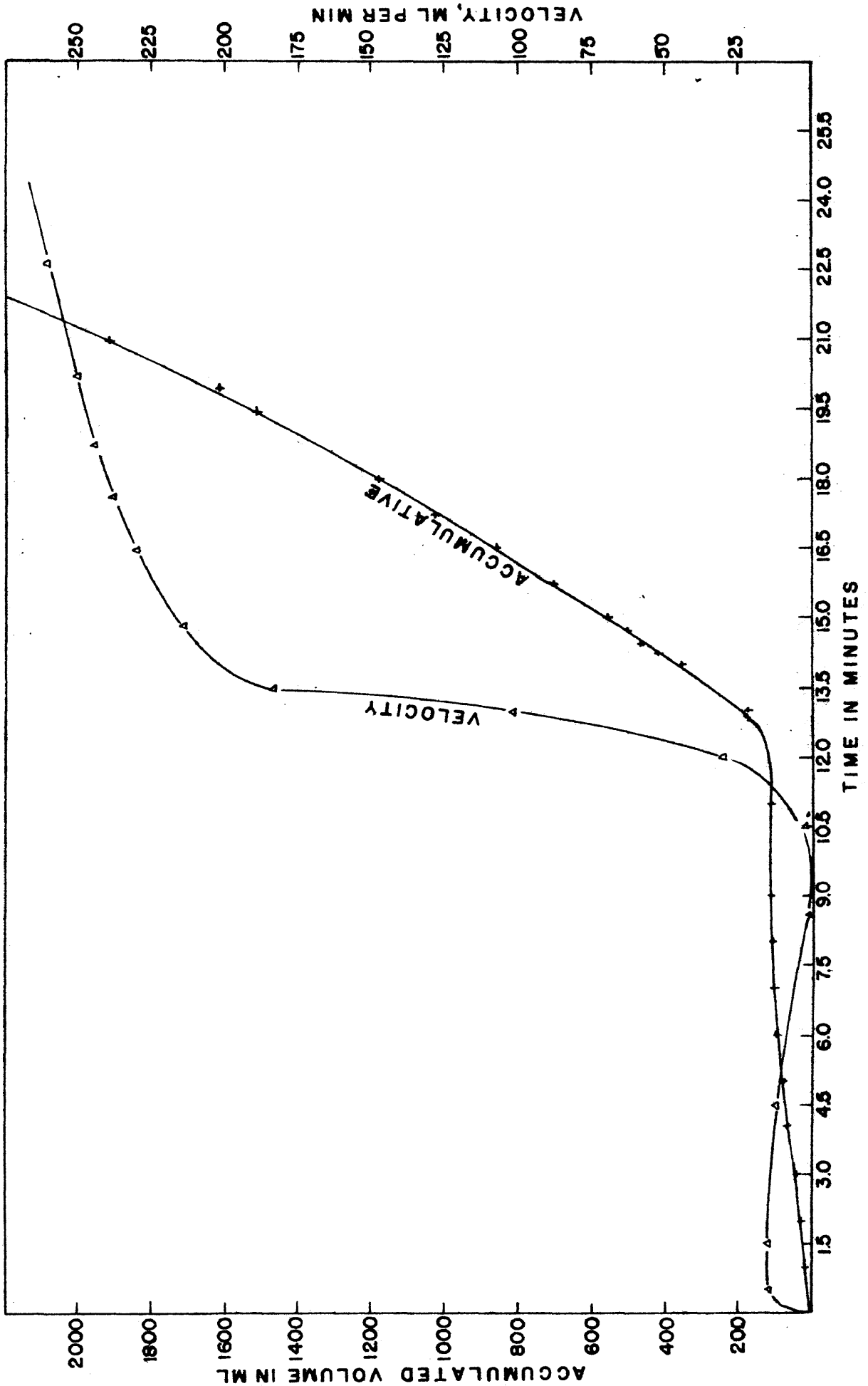


Fig. 17 Salt, 100%.

TABLE X - SUGAR, 20 % IN OTTAWA SAND

Leaching Liquid - Water
Temperature - 31 C
Head - 7.5 in. of water

Length Tube - 47 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, cm	Volume, ml	Remarks
0	0	0	Start
0.5	0.67	17.58	
1	1.2	31.49	
1.5	1.7	44.61	
2	2.1	56.10	
2.5	2.6	68.22	
3	3.1	81.34	
5	4.2	110.21	
6	5.6	146.94	
7	6.65	174.50	
8	7.8	204.67	
9	8.9	233.54	
10	10.2	267.65	
14	14.4	377.86	
15	15.4	404.10	
17	17.4	456.58	
20	20.9	548.42	
25	22.6	593.02	
28	27.0	708.48	
29	27.9	732.10	
32	30.5	800.32	
36	34.3	900.03	
45	41.7	1094.21	
53	48.3	1267.39	
55	49.9	1309.38	Stop
55	49.9	1309.38	Start
56	52.5	1377.60	
58	56.7	1556.03	
59	59.2	1727.44	
60	64.3	2180.54	
61	72.0	2760.44	
62	79.7	3542.39	
63	87.6	4537.64	
64	95.7	5733.43	Stop

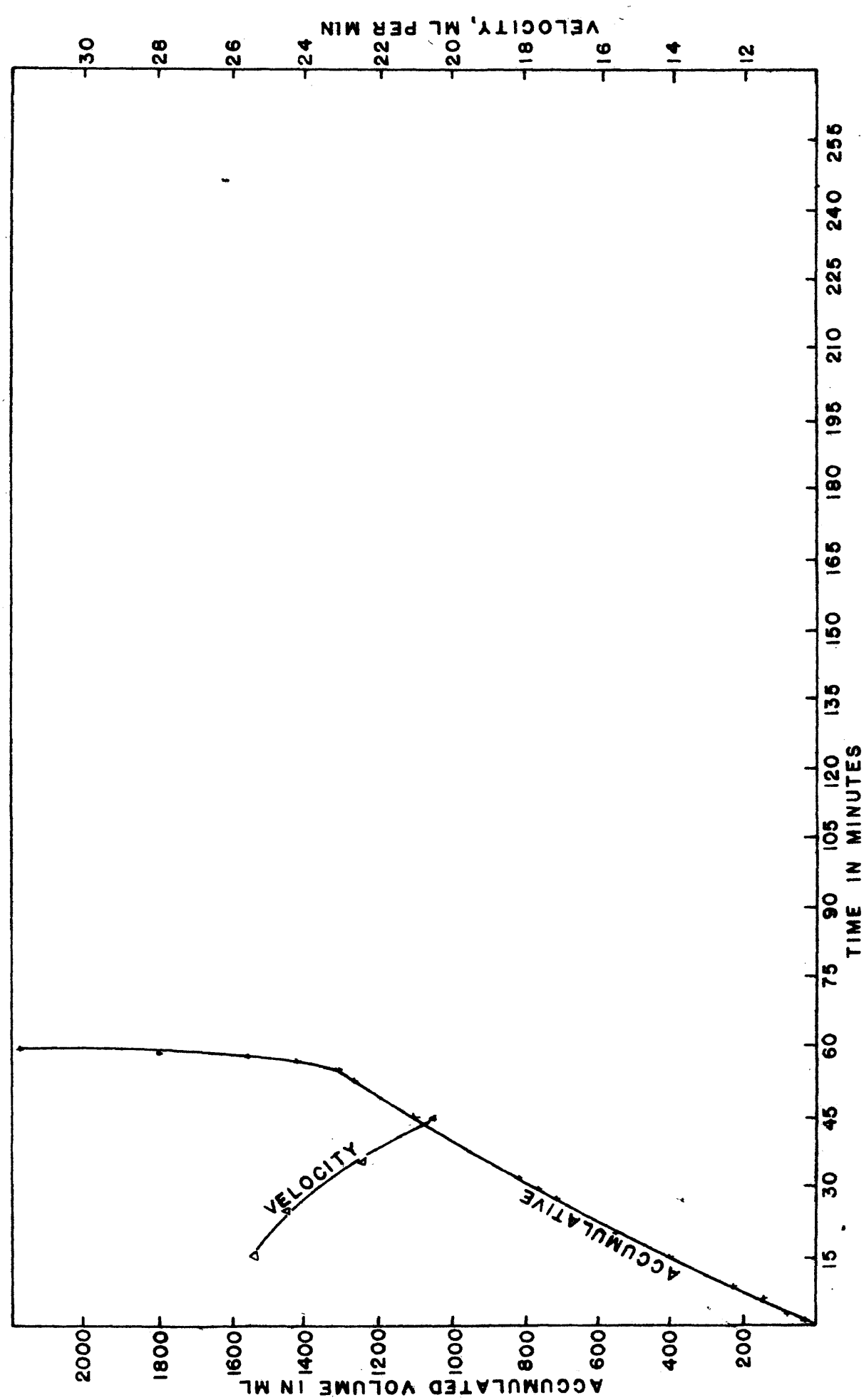


Fig. 18 Sugar, 20% in Ottawa Sand

TABLE XI - $NaHCO_3$, 10 % IN OTTAWA SAND

Leaching Liquid - Water
Temperature - 25° C
Head - 7.5 in. of water

Length Tube - 47 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, cm	Volume, ml	Remarks
0.5	0	0	Start
1	0.4	10.50	
1.5	0.8	20.99	
2	1.1	28.86	
3	1.5	39.36	
4	2.0	52.48	
5	2.4	62.98	
6	2.65	69.54	Saturated
7	3.0	78.72	
9	3.2	83.97	
11	4.1	107.58	
15	5.3	139.07	
17	5.8	152.19	
20	6.6	173.18	
23	7.6	199.42	Gas bubbles
28	9.2	241.41	are formed,
32	10.9	286.02	regular in
39	12.7	333.09	size and spacing
45	14.3	375.07	
54	16.8	440.83	
63	19.4	509.06	
67	20.6	540.54	
73	22.4	587.78	
82	25.4	666.50	
87	26.8	703.23	
95	29.1	763.58	
106	32.8	860.67	
115	35.3	926.13	
120	37.2	976.14	
130	40.8	1070.59	
136	43.1	1130.94	
144	46.2	1212.29	
150	48.5	1272.64	

TABLE XI (continued)- $NaHCO_3$, 10 % IN OTTAWA SAND

Time, min	Outage, cm	Volume, ml	Remarks
156	50.9	1335.62	
160	52.6	1380.22	
169	56.1	1472.06	
176	58.7	1540.29	
183	61.1	1603.26	Stop
183	61.1	1603.26	Start
184	62.6	1642.62	
195	66.1	1773.82	
186	72.8	2080.83	
187	60.2	2582.01	Stop

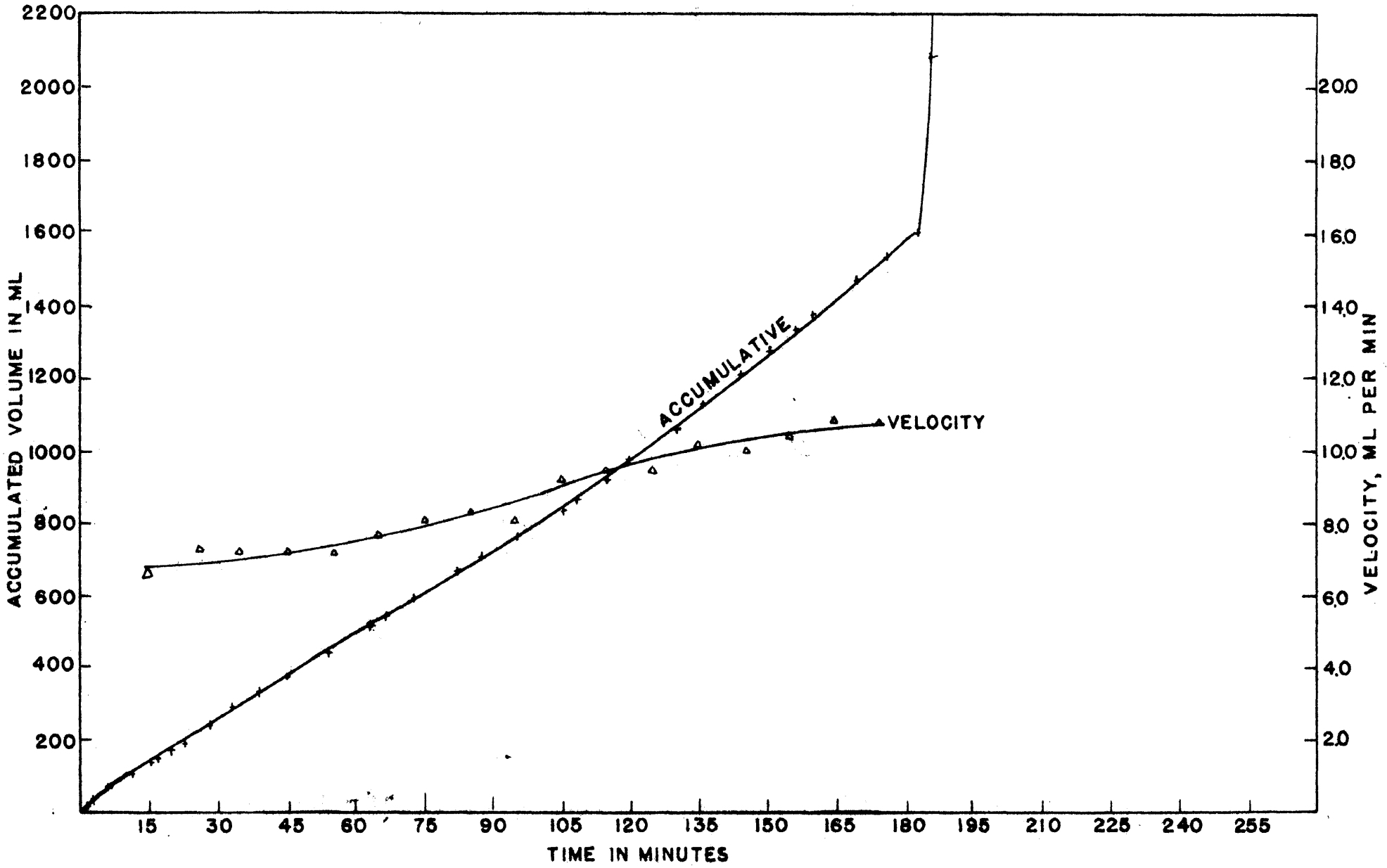


Fig. 19 Baking Soda, 10% in Ottawa Sand

TABLE XII - OAL SEAL, 10 % IN OTTAWA SAND

Leaching Liquid - water
Temperature - 30 C
Head - 7.5 in. of water

Length Tube - 47 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, cm	Volume, ml	Remarks
0	0	0	Start
0.5	0.7	18.37	
1	1.1	28.86	
2	1.6	41.98	
3	2.0	52.48	
5	2.6	68.22	
6	2.9	76.10	Saturated
10	4.0	104.96	
12	4.5	118.08	
17	5.7	149.57	
27	8.3	217.79	
31	9.3	244.03	
60	16.3	427.71	
77	20.2	530.05	
99	24.9	653.38	
112	27.7	726.85	
123	30.1	789.82	
178	42.5	1115.20	
186	44.3	1162.43	
247	58.2	1527.17	
262	61.8	1621.63	
263	62.0	1626.88	Stop
263	62.0	1626.88	Start
274	64.8	1700.35	
281	66.6	1747.58	
301	86.4	2267.14	
303	87.0	2282.88	Stop

TABLE XIII - CAL SEAL, 10 % IN OTTAWA SAND

Leaching Liquid - 1 % H_2SO_4
Temperature - 30 C
Head - 7.5 in. of water
Length Tube - 47 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, cm	Volume, ml	Remarks
0	0	0	Start
2	0.5	13.12	
5	1.1	28.86	
7	1.4	36.74	
27	3.3	86.59	
41	5.1	133.82	
58	7.8	204.67	
71	10.1	265.02	
77	11.1	291.26	
111	16.9	443.46	Stop

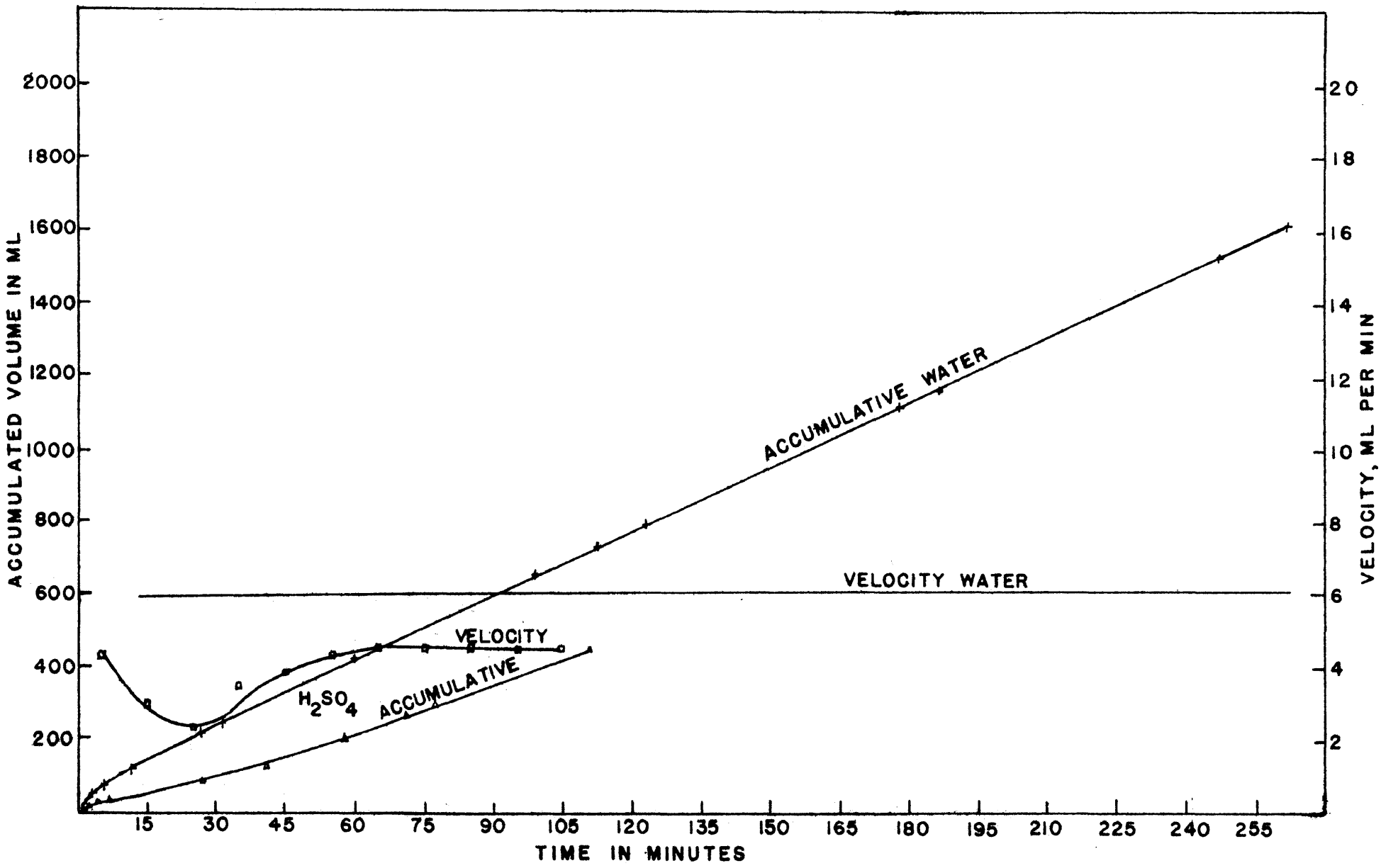


Fig. 20 Cal Seal, 10% in Ottawa Sand

TABLE [illegible] - COPPER ORE, CRUSHED TO -14 MESH

Leaching Liquid- see under Remarks
Temperature - 31 C
Head- 27 in. of leaching liquid
Length Tube - 47 in.
Length Ore in Tube - 36 in.
Length Sand in Tube - 11 in.
Diameter Tube - 0.5 in.
Diameter Reservoir - 2.5 in.

Time, min	Outage, cm	Volume, ml	Remarks
0	0	0	Start Water
26	1.7	44.61	
48	3.1	81.34	Stop Water
0	0	0	Start NH_4OH
4	0.2	5.25	(1 to 8)
26	1.75	45.92	
33	2.13	55.89	
46	2.9	76.10	
58	3.6	94.46	
68	4.15	108.90	
74	4.45	116.77	
91	5.35	140.38	
107	6.1	160.06	
215	11.5	301.76	
265	13.6	356.86	
327	15.8	414.59	
356	16.75	439.52	
440	19.25	505.12	
497	20.85	547.10	
545	22.1	579.90	
995	33.33	874.58	
1281	39.4	1033.86	
1542	45.25	1187.36	
1546	45.35	1189.98	Stop
1546	45.35	1189.98	Start
1550	45.7	1199.17	
1556	46.04	1208.09	
1618	49.6	1301.50	
1836	61.45	1612.45	
1858	62.15	1630.82	
1992	69.85	1832.86	
2186	82.7	2170.05	Stop
2186	82.7	2170.05	Start
2189	82.8	2172.68	
2191	82.9	2175.30	
2505	103.9	2726.34	
2613	111.3	2920.51	Stop

TABLE XIV (continued)- COPPER ORE, CRUSHED TO -14 MESH

Time, min	Outage, cm	Volume, ml	Remarks
2613	111.3	2920.51	Start
2614	111.45	2924.45	
2616	111.64	2929.43	
2757	125.0	3280.00	
2820	129.8	3405.95	
2851	132.25	3470.24	
2914	137.5	3608.00	
2989	143.8	3773.31	
3054	149.5	3922.38	Stop
3054	149.5	3922.38	Start
3055	150.05	3937.31	
3060	150.7	3954.37	
3255	172.7	4531.65	Stop
3255	172.7	4531.65	Start
3885	227.65	5973.54	
3915	230.2	6040.45	Stop
3915	230.2	6040.45	Start
4004	236.3	6200.51	
4049	239.25	6277.92	
4130	243.7	6394.69	
4185	246.5	6468.16	Stop
4185	246.5	6468.16	Start
4193	247.3	6489.15	
4220	249.8	6554.75	
4257	253.4	6649.22	
4279	257.4	6754.18	
4326	259.8	6817.15	
4371	263.7	6919.49	
4479	272.6	7153.02	
4542	277.6	7284.22	
4568	279.6	7336.70	
4601	282.1	7402.30	
4635	284.7	7470.53	
4667	287.1	7533.50	
4754	293.0	7688.32	
4878	306.8	8050.43	Stop, NH_4OH
0	0	0	Start, Water
645	63.5	1666.67	Stop, Water
0	0	0	Start, 1 %
3	0.35	9.18	H_2SO_4

TABLE XIV (continued)- COPPER, ORE, CRUSHED TO -14 MESH

Time, min	Outage, cm	Volume, ml	Remarks
21	2.4	62.98	
48	5.25	137.76	
80	9.6	251.90	
158	27.7	726.85	Stop
158	27.7	726.85	Start
160	29.0	760.96	
224	30.4	797.70	
231	35.6	934.15	Stop
231	35.6	934.15	Start
234	38.3	1004.99	
359	101.8	2671.66	Stop, H_2SO_4

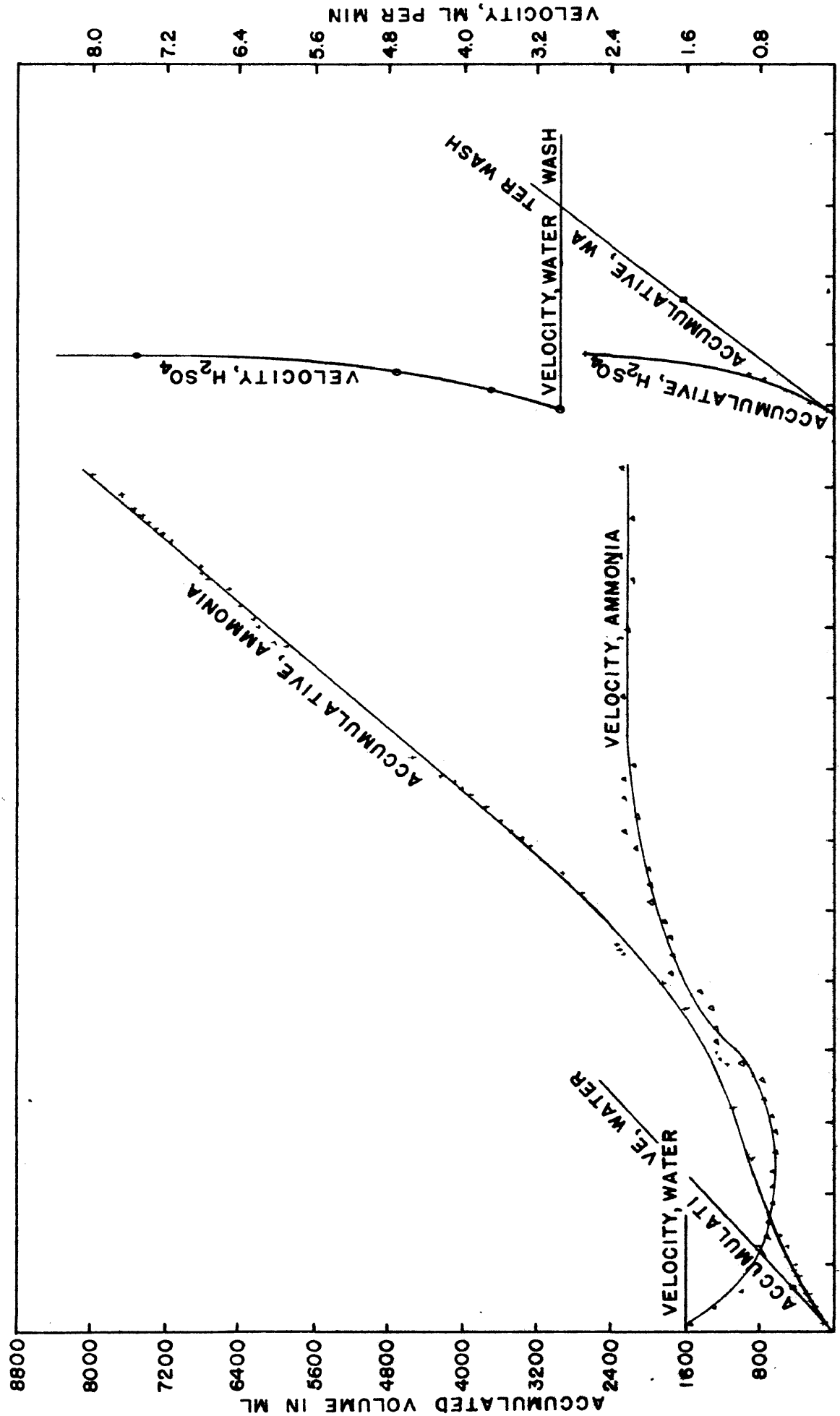


Fig. 21 Copper Ore Crushed to -14 mesh

liquid (water) passed through the sand-salt mixture amounted to 1600 milliliters. The time required for passing this quantity was 60 minutes. The sand-salt mixture was saturated with the water after 6 minutes. From 6 minutes until 25 minutes an increase in rate of flow was to be observed. During the process air bubbles of approximately 2-mm diameter formed in the tube. After 20 minutes, 36 in. from the left end of the leaching tube, these air bubbles combined into a bubble approximately 1.5 cm long and 0.7 cm wide. It was observed that this big bubble necessarily decreased the permeability. The water flowed underneath the bubble while the bubble did not move. As little bubbles were formed, they concentrated in the direction against the flow of the water next to the big bubble as is shown in Fig. 22. At a time 25 minutes after the beginning of the process the author tried to remove the bubble by tapping the leaching tube. The result was that all air bubbles combined, and that the air escaped out of the tube. Because of the removal of the air, the rate of flow increased. From this time on, the rate of flow remained constant.

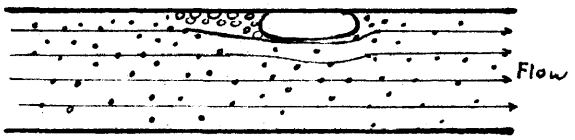


Fig. 22

Salt, 50% in Ottawa Sand--The total amount of leaching liquid (water) used during the test was 1640 ml over a period of 47 minutes. The saturation of the salt-sand mixture with water took place after 4 minutes. From 4 minutes until 47

minutes the rate of flow decreased continuously. Gas bubbles, as were described for the former test, were formed, being the cause of the decreasing rate of flow. The author was careful this time not to cause any unnecessary vibration in the leaching tube, so that the air bubbles would not combine because of this vibration. This experiment was taken over a short period of time, for the reservoir was empty at the end of the period. As will be shown in the discussion for the sugar-sand mixture, and baking soda-sand mixture, the gas bubbles formed will combine readily during a period of no flow.

Salt, 100% in Ottawa Sand--The rubber stoppers at the beginning and the end of the leaching tube were covered on the inside by a single piece of Kleenex, which was to prevent salt from flowing out of the tube in solid form. The total amount of leaching liquid (water) used during the test was 3015 ml over a period of 26.5 minutes. Saturation took place after 5 minutes. After this time, little or no flow took place. Apparently the Kleenex caused the flow to cease. The flow was restarted by suction at the exit of the leaching tube. The liquid coming out of the tube was entirely clear, no salt particles of macroscopic size were present, and the Kleenex remained intact. After the suction, the rate of flow increased tremendously up to 400 ml per minute. During the test it was noted that the high velocity caused the moving of particles. The beginning and the end of the leaching as seen during this test are shown in Fig. 23. No gas bubbles formed

because the air had a chance to escape through a channel formed at the top of the leaching tube.

Fig. 23.

Sugar, 20% in Ottawa Sand--The total amount of leaching liquid (water) used over a period of 64 minutes was 5730 ml. The sugar-sand mixture was saturated after 2.5 minutes. The rate of flow decreasedover the period from 2.5 minutes to 55 minutes. This decrease in the rate of flow was due to the formation of air bubbles as were described before, and due to the increase of viscosity of the leaching liquid. During this period of decrease in rate of flow, no attempts were made to remove the bubbles formed. The reservoir had to be refilled with water after 55 minutes. Refilling the reservoir took approximately 2 minutes, during which time the valve between the reservoir and the leaching tube was closed. The air bubbles formed all through the mixture combined, and after the intermission the rate of flow increased tremendously. The flow was so rapid that the sand particles in the leaching tube were moved.

Baking Soda, 10% in Ottawa Sand--The total amount of water used during the test was 2500 ml over a period of 187 minutes. The soda-sand mixture was saturated after 6 minutes. From 6 minutes to 183 minutes the rate of flow increased constantly. Hundreds of gas bubbles were formed, being regular in size and diameter. The gas probably consisted of air and carbon

dioxide. After the period of 183 minutes, the reservoir had to be refilled, and after the intermission a tremendous increase in rate of flow was observed. The cause for this high rate of flow was the same as that for the sugar-sand mixture described before.

Cal Seal, 10% in Ottawa Sand--Two tests were run on a Cal Seal-sand mixture. The first test, in which water was used as leaching liquid, required 260 minutes during which time 1625 ml of water were used. The mixture was saturated with water after 6 minutes. After the mixture was once saturated, no change in rate of flow took place.

The second test was to inject 5% sulphuric acid through the Cal Seal-sand mixture. Over a period of 111 minutes 440 ml of acid were used. The rate of flow decreased during the first 30 minutes, after which period the rate increased to become constant after the period of 60 minutes from the start. The rate offlow never became as high as it was when water was used. Here we see that a chemical and physical change must have taken place. During the period of decrease in rate of flow sulphates precipitated. After this precipitation the precipitates coagulated, and the rate of flow increases to become a steady flow later.

Copper Ore, Crushed to -14 mesh--Copper ore consisting of partly weathered copper sulphide mineral was crushed to minus 14 mesh. The occuring minerals were covelite, bornite, garnet, malachite, and quartz. The original rock is shown in Fig. 24. To prohibit disturbance of flow at the beginning and the end

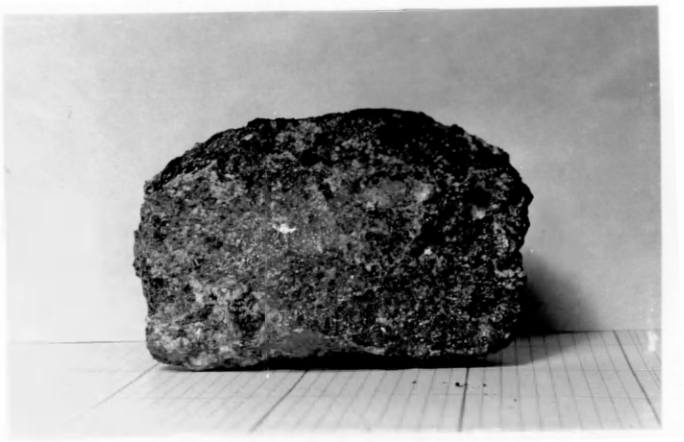


Fig.24

of the leaching tube, Ottawa sand was placed at the ends seven inches on either side. First water was injected to flood the leaching tube and to determine the original permeability of the crushed ore. After the water injection, ammonia (1 to 8) was used as leaching liquid. This leach was followed by a water wash, which again was followed by a sulphuric acid (1%) leach.

The rate of flow for the flooding water was steady over the period of 48 minutes. The rate decreased considerably during the first 1,000 minutes of the ammonia leach, to increase later. The rate of flow increased until the end of the period of 3200 minutes. After this period the rate of flow remained constant and was higher than during the water flood. The water wash gave a steady rate of flow; however, a higher rate than was observed during the last 1700 minutes of the ammonia leach. The sulphuric acid leach showed a large increase in rate of flow. During this last leach it was observed that channels formed. Material at the top of the leaching tube were affected, but no leaching took place in lower layers.

In this last test it is seen again that both change in chemical and physical properties of the material to be leached affect the rate of flow. The decrease in rate of flow during the ammonia leach was due to the precipitation of iron hydroxides which were removed later and were found in the discharge. However, the increase in rate of flow was not only due to the removal of these hydroxides but also due to the removal of copper from the ore.

CONCLUSIONS

Two types of flow of fluids are possible, turbulent flow and laminar flow. For the process of leaching of ore in place, only laminar flow is important.

Different laws of permeability have been discussed with a favorable conclusion for Taylor's equation.

Muskat's equation for the radial flow of fluids has been mathematically derived and experimentally tested. From experiments it has been found that variations in r_e, r_w, and h check with Muskat's equation. The values for the permeability K found from the experiments on the radial flow do not entirely agree with values found from other experiments with the same sandstone. From the test on radial flow, a value of 0.0068 darcys was determined for the permeability; from the test with air a value of 0.0277; and from the test with water a value of 0.011. It can be seen that different methods of determination of the permeability give different values. Comparison of the values for the permeability obtained from the pressure-pot method and obtained from the test with water shows a difference of about 30 percent. The result of the test with air differs about 400 percent from the values obtained from the pressure-pot method. It is suggested that for permeability measurements, the same liquid or gas should be used that is used in practice. The factor of viscosity should be used only to give a fair approximation in case other fluids have to be used. However, the factor of

viscosity does not take in account resistance against flow in rocks due to contact angle, wettability, and physical changes of material. A simple example will show the last: A dry clay will have a high permeability when it is tested with air. If one tries to make water flow through the same clay, little or no flow will be observed. The water makes the clay particles swell up, and the swollen particles will close the pores which provide the passways for air.

From tests of leaching of ore in place it can be seen that many variables are involved. The most important rock characteristic from the standpoint of flow of fluids, the permeability, cannot be determined by means of mathematical formulas. The permeability can be approximated from laboratory tests. Principles observed during tests on leaching of ore in place are:

1. Minerals leached out will help to increase the rate of flow.
2. Precipitates formed will decrease the permeability.
3. Precipitates formed might coagulate to larger particles or might be removed mechanically.
4. Gas bubbles formed decrease the rate of flow. (The so-called Jamin effect refers to the resistance to fluid flow through capillary openings when gas bubbles are in the path of liquid flow. This resistance is due to the energy required to deform a bubble and move it through a series of smaller openings

into larger ones. Each compression and enlargement of a bubble causes a certain loss in energy, which may become appreciable in movements over a distance.1/

1/ Buell, Arthur, W., Porosity and permeability analyses, Subsurface Geologic Methods, Colorado School of Mines, Golden, p. 168, 1949.

5. Vibration results in combining small gas bubbles to larger bubbles and will increase the rate of flow; however, channel forming might be the alternative result.
6. If a strongly dissolving leaching liquid is used the chance for channel forming is large.
7. Channel forming decreases the efficiency of leaching of ore in place.
8. As the difference in viscosity for solutions of an inorganic compound for different concentrations is small, the change in viscosity is rather unimportant.
9. The porosity varies with the amount of material leached out, but is only important in determining the reservoir volume in which pregnant solution is present.

In the tests for leaching of ore in place, different assumptions were made:

1. No formation pressure was present. If a high formation pressure is present, the particles will be able to maintain

this pressure, up to a certain point. After this point the "bridges" formed by the particles will collapse, and the permeability and the porosity will decrease suddenly. To test this effect it is suggested that a model be built in which a pressure can be applied on the material to be leached during the test.

It can be seen however, that to imitate the effects of formation pressure in the laboratory, the formation pressure in the field has to be known. Formation pressure, so difficult to determine in the field, will give only an approximation of what will happen later during the actual application.

2. Unconsolidated materials were used. An orebody will never consist of purely unconsolidated material. Consolidated materials will be able to stand formation pressure better than unconsolidated materials. Grains in consolidated material, the grains decreasing in size during the leaching process, will fall by their own weight between larger grains, and thus give fluids a better chance to pass. This falling will probably not happen in unconsolidated material.

3. Temperature was held constant. In an approximately impermeable ore increased temperature might help to start fluid flow.

4. No fractures were present. Often fractures may occur in deposits to be leached. If the fractures are very large, the method of leaching of ore in place might not be possible. Smaller fractures, however, can be grouted to prevent loss of

leaching liquid. The cement used should have approximately the same strength and permeability as the ore to form a homogeneous mass with the ore.

In practice, for experimental work only a small amount of ore to be leached in place will be available from cores. For that reason experiments must be carefully run on this material. It is suggested that in such case an extensive study be made of the core available. Physical, mineralogical, petrographical, and chemical tests must be made in such a way that the ore can be reproduced for laboratory purposes. The physical characteristics of the ore are porosity, permeability, strength necessary for withstanding the formation pressure, and particle size. Not only the mineral grains but also the mineral form of the cementing material have to be studied. The chemical characteristics are necessary to determine the type of leaching liquid to be used. After all characteristics are determined, the rock is reproduced, the formation pressure is approximated, laboratory tests can be run.

For further work it is suggested that tests as described in above paragraph be made. The effect of temperature and the effect of variation in pressure have to be studied.

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