REDUCED-ORDER ENERGY MODELING FOR ADVANCED SETPOINT CONTROLS OF RESIDENTIAL BUILDINGS WITH TIME-OF-USE RATES

by

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ABSTRACT

In the United States, approximately 40% of energy is consumed by commercial and residential buildings, primarily attributed to a building's heating, ventilation, and air conditioning (HVAC) system. During periods of extreme temperatures, these cooling systems are pushed to their limits and components of the grid system are stressed. This can result in grid system instability, lead to potential health concerns for building occupants, and result in higher energy costs for utilities and building owners. One solution to address this problem is implementing variable electric rates with the use of energy management strategies such as incentivized demand-response (DR) programs and load shifting to control AC units or large appliances. Variable electric rate programs often address the energy demands of commercial and industrial buildings; however, there is increased interest in curtailing the electricity use in the residential sector via the same methods. Key topics of interest regarding load-shifting in residential homes include accurate models to predict costs and electricity use, and the use of advanced controls of temperature setpoints under variable electric rates.

Presented in this thesis are (i) an analysis of grey-box and black-box reduced-order models for cooling load predictions of two residential homes; and (ii) a demonstration of the impacts of time-of-use (TOU) rates applied to residential homes using an MPC. Of the two models presented, the black-box model is better suited for application to the MPC. The studies also demonstrate that the electricity rate greatly impacts the benefits of an MPC, both from a consumer cost-savings perspective and in its ability to shift AC electricity.

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LIST OF ABBREVIATIONS

Air Conditioning . . . . . AC

A Modeling Language for Mathematical Programming . . . . . AMPL

Artificial Neural Network . . . . . ANN

Application Program Interface . . . . . API

Auto Regressive with Exogenous Inputs . . . . . ARX

Auto Regressive with Moving Average Exogenous Inputs . . . . . ARMAX

American Society of Heating, Refrigerating, and Air-Conditioning Engineers . . ASHRAE

Building Control Virtual Test Bed . . . . . BCVTB

Building Energy Optimization . . . . . BEopt

Box Jenkins . . . . . BJ

Coefficient of Performance . . . . . COP

Co-variance Root Mean Squared . . . . . CV-RSME

Dry Bulb Temperature . . . . . DBT

Department of Energy . . . . . DOE

Indoor Air Temperature . . . . . IAT

International Energy Conservation Code . . . . . IECC

Heating Ventilation and Air Conditioning . . . . . HVAC

House Simulation Protocols . . . . . HSP

Matrix Laboratory . . . . . MATLAB

MATLAB-EnergyPlus Co-simulation Interface . . . . . MLE+

Model Predictive Control . . . . . MPC

Nonlinear Auto Regressive outputs with Exogenous Inputs . . . . . NLARX

Normalized Mean Biased Error . . . . . NMBE

National Renewable Energy Laboratory . . . . . NREL

Outdoor Air Temperature . . . . . OAT

Photovoltaic . . . . . PV

Thermal Resistance and Capacitance . . . . . RC

Reduced-Order Model . . . . . ROM

State Space Model . . . . . SS

Time-Of-Use . . . . . TOU

Timestep . . . . . TS

Temperature Set Point . . . . . TSP

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CHAPTER 1

INTRODUCTION

Buildings account for a significant portion of the electricity use in the United States, which is primarily driven by air-conditioning equipment [1]. One side-effect of increased AC usage during a cooling season is the large spike in electricity demand that coincides with the hottest parts of the day, and is more prominent when temperatures are elevated [2]. High electricity loads result in strain on the grid and higher operating costs for electricity utilities. These costs come from inefficient reserve technologies, excessive strain on generation and transmission equipment, and the potential need for upgrades to plant capacity. Furthermore, excessive peak demand can lead to service interruptions, and potential health risks for sensitive groups [3]. Therefore, reducing electricity use, and specifically air-conditioning use in buildings, may be beneficial to numerous stakeholders. This thesis focuses on the benefits to residential consumers and electricity providers from a cost and peak-demand perspective when variable electric rates are applied.

Electricity providers have historically applied variable electric rates to commercial and industrial customers to incentivize load-shifting; however, this technique is becoming increasingly more popular for residential customers. A common rate structure for residential buildings is a time-of-use (TOU) rate, which have higher than normal costs during on-peak hours, but cheaper rates during off-peak hours. Due to varying outdoor temperatures and occupancy profiles, there is a large amount of variability in electricity demand throughout the day, as shown in Figure 1.1. TOU rates are designed so consumers can reduce energy costs by shifting their primary load to off-peak times, which helps smooth this demand curve. Figure 1.2 gives an example of how TOU prices might change over a 24-hour period, and how the rate spike occurs during the period of highest demand. There are several methods to take advantage of variable rates including: (i) energy-efficient retrofit technologies,

(ii) demand-response programs of appliances and other electric devices, and (iii) control strategies of building systems that react to time-dependent inputs such as electricity cost, internal loads, and occupant behavior. The latter can be implemented for the HVAC system by introducing controls applied to the thermostat setpoint via rule-based controls, or more advanced controls that use optimization techniques.

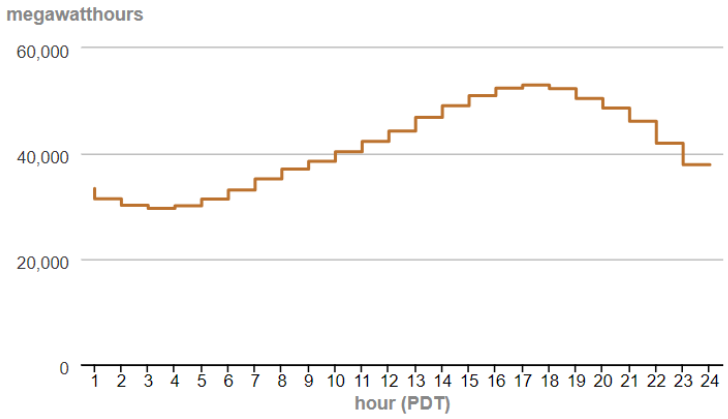

Figure 1.1: Electricity demand of California for a single day in the summer [4].

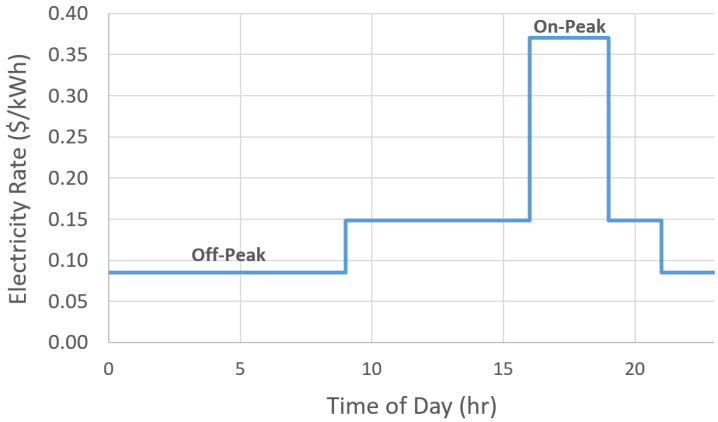

Figure 1.2: Two-tiered time-of-use pricing structure in Sacramento, CA [5].

Accurate predictive building models are essential in both the study of advanced controls and their application. Building energy models include white-box, grey-box, and black-box, and are capable of predicting the energy demands of a building using building construction data, environmental inputs, historical data, or a combination of the three.

- White-box: Introduces all relevant heat transfer mechanisms impacting the building, and tend to be the most accurate in predicting building responses and electricity use. These require very detailed input information about the building, have longer development and computation times, and are not easily generalized across different home types.

- Black-Box: Uses statistical methods to link input data to the output, creating equations that are data-driven. Black-box methods can use linear and nonlinear equations that are fit to simulated or experimental data. These models are fast and are chosen because of their simplicity and potential accuracy, but are heavily dependent on the training data set.

- Grey-Box: Considers relevant physical heat transfer phenomena for a simplified building geometry, reducing the number of variables and differential equations needed, and applies an empirical approach to calibrate model input parameters. These models often utilize the RC network approach to simplify heat transfer mechanisms with resistances (R) and capacitances (C) of building structures.

Reduced-order models are capable of predicting residential and commercial building responses for tests using both simulated and actual buildings [6, 7], while reducing computation time compared to commercially available whole-building energy simulations. Therefore, ROMs are most commonly used in advanced building controls that require a predictive model [8].

Designing controls of building systems can be complex, as they have numerous time-dependent inputs such as electricity cost, environmental conditions, and occupancy behavior. Pre-cooling is one control strategy that uses the thermal mass of the building envelope to store energy by lowering the temperature setpoint prior to peak cost times, and constitutes one of the most cost-effective ways to reduce electricity costs in existing buildings [9–14]. With the onset of smart meters, smart thermostats, and variable time-of-use rates, smart

controls of thermostats are becoming more common for residential buildings. Optimization-based model predictive controls (MPCs) are especially suitable for large input-output systems, such as buildings. MPCs operate by taking feedback signals from past system states and the environment to determine future system responses, then optimizing to meet all constraints, e.g., thermal comfort and energy usage [15–17]. Variables such as temperature setpoint, electricity rate, and thermal storage may be used to find an optimal schedule. The constraints might include comfort index, building codes (ventilation and other standards), or control schedules. Many studies have demonstrated the potential for MPCs applied to commericial and residential buildings with variable electric rates. Literature suggests that significant cost, energy, or peak demand reductions are possible when optimizing control of temperature setpoints, PV, subsystem components, or other building equipment [16, 18–22].

Time-of-use rates or other variable rates are often the driver behind MPCs for buildings. Several studies have optimized setpoints for varied electric rates or optimized variable electric rates, but have failed to focus on analysis of TOU rates for a single location that uses an MPC for temperature setpoints.

1.1 Research Objectives

The objectives of this thesis are to:

- Develop two reduced-order models capable of predicting the cooling electricity load of two different detached homes
- Compare the models in the context of prediction accuracy, development, computation speed, and ability to apply to a temperature setpoint controls strategy
- Integrate the reduced-order black-box model with a previously developed model-predictive control framework to optimize temperature setpoints in a detached home
- Analyze how TOU rates influence the response of an MPC for a residential home

1.2 Technical Approach Overview

Below is a technical outline of the steps taken to achieve the objectives:

1. Develop two grey-box models in MATLAB using simulated EnergyPlus [23] homes for training data. These models take a limited amount of physical data from the home, and estimate heat flow through the building envelope. The system of equations is then calibrated by optimizing specific building parameters with a least-squares minimization that limits the error between EnergyPlus output and the grey-box model output.

2. Identify a time-series forecasting model (black-box model) to predict cooling electricity for the same two simulated homes. These models stem from previous work [24] that identified an ARX model as sufficient for building cooling load predictions using inputs of dry-bulb temperature, temperature setpoint, and occupancy. These models are identified in MATLAB using the same training set as the grey-box model, with some analysis done in AMPL.

3. Compare the two model types by validating with the same dataset, and considering the challenges in development and future application in an MPC.

4. Using a previously developed framework [24], apply the black-box model to an MPC for optimization of temperature setpoints to minimize electricity costs and thermal discomfort for a new detached home. The identified ARX equation is implemented as a constraint in the MPC optimization formulation, which uses AMPL and CPLEX to formulate and solve. The MPC is optimized at every timestep using a finite prediction horizon for each run before stepping forward. A Simulink model is used to link the MPC output with an EnergyPlus simulation each timestep, which simulates a real-time building response. This simulation is run for a new detached home for two months in the summer, and under varied time-of-use rate.

5. Analyze the response of the MPC under multiple TOU rates from a cost-savings and a peak-demand perspective by observing the trends in response to the electricity rate.

1.3 Thesis Contributions

The primary contributions of this thesis, not previously published in the literature, include:

1. A robust comparison of grey-box and black-box models for application to a model predictive controller

2. An analysis of the impacts of variable electric rates on residential electricity customers and providers using an MPC in a single location

1.4 Thesis Organization

The thesis is organized as follows: Chapter 2 provides an in-depth comparison of two reduced-order models, a grey-box and a black-box model, and the unique challenges of applying them to an optimization of building controls; Chapter 3 details the implementation of an ARX model to a model-predictive controller, and analyzes the impact that time-of-use rates can have when applied to a residential home using smart controls; and finally, Chapter 4 summarizes the findings and impacts of the thesis.

CHAPTER 2

ANALYSIS OF GREY-BOX AND BLACK-BOX RESIDENTIAL ENERGY MODELS FOR APPLICATION TO BUILDING TEMPERATURE CONTROLS

2.1 Introduction

Residential and commercial buildings account for approximately 75% of the electric energy usage in the United States, and within the residential sector, air conditioning is the largest component [1]. Electricity use from buildings is particularly high during the cooling season, due to increased AC usage, which can lead to undesired stress on the electric grid and high energy costs during peak-demand hours.

To mitigate peak electric demand, utility companies encourage users to shift their electricity needs using peak demand programs such as: time-of-use (TOU) pricing, peak pricing, real-time pricing, and peak rebates [25]. To shift cooling or electric energy demand, there are different technologies in the market such as batteries, smart appliances, variable thermostats, learning thermostats such as NEST [26], and advanced controls. For thermostat controls, pre-cooling is often used, an approach that stores cooling energy by lowering the zone temperature when the cost of electricity is low, then relaxes the temperature setpoint to reduce energy demand at peak time [9, 10, 12, 14, 27]. The use of pre-cooling may require advanced controls to maximize cost savings. Studies using advanced controls include artificial neural networks, model predictive controls (MPC), and fuzzy logic control [28]. MPC is rapidly advancing the field of research for temperature control of buildings due to the potential of large improvements using existing equipment, and the possibility of energy savings from optimization [29]. However, advanced controls such as MPCs can have complex models of the controllable system, the reby creating a potential for long solution times. Although there are optimization studies using robust white-box models, such as EnergyPlus, TRNSYS, or Modelica [30–32], many studies use reduced-order models (ROMs), such as grey-box or

black-box models, to increase the computational speed of the problem [16, 24, 33–40]. Picard et al. (2016) [31] showed that both a linearized white-box predictive model and a grey-box model can be used to develop effective controllers as long as sufficient identification data sets are used. ROMs reduce the simulation run time, the development time, and the amount of physical building data required. For example, EnergyPlus, with a warm up period, takes approximately 20 seconds per daily run of a single zone residential building, which can become intractable in an optimization that requires hundreds to thousands of iterations per solve [33].

Among energy models used for predictions in these control strategies, there are three main categories:

- White-box: Introduces all relevant heat transfer mechanisms impacting the building, and tend to be the most accurate in predicting building responses and energy use. However, these require very detailed input information about the building, have longer development and computation times, and are not easily generalized across different home types. Solution methods depend on the software, and may include radiant time series, heat balance method, or thermal circuits [41]. Examples of white-box building energy models include EnergyPlus, ESP-r, DOE 2, and TRNSYS [23, 42].

- Black-Box: Uses statistical methods to link input data to the output, creating equations that are data-driven. Black-box methods can use linear and nonlinear equations that are fit to simulation or experimental data. These models are fast, can be applied to whole buildings or specific building technologies, and are chosen because of their simplicity and potential accuracy. Typical algebraic models previously used for buildings include box-jenkins (BJ), auto-regressive with exogenous input (ARX), auto-regressive with moving average exogenous input (ARMAX), artificial neural network (ANN), state-space models (SS), and non-linear auto-regressive with exogenous input (NLARX). Cole et al. (2014) [33] applied the ARX structure to model residential buildings for MPC. Chintala et al. (2015) [20] developed a platform to automatically

develop ARX model structures for HVAC systems. Chen (2015) [43] generated a SS model using data-driven methods to incorporate occupant feedback about thermal comfort, while reducing the modeling complexity of the system. Ansari et al. (2005) [44] created simple regression models for multiple HVAC components to estimate the total cooling load of the system. One drawback of black-box models is that they are highly dependent on the quality of historical training data used [45].

- Grey-Box: Bridges the gap between white- and black-box models by considering limited physical phenomena as well as an empirical training method. Grey-box models consider relevant physical heat transfer phenomena for a simplified building geometry, reducing the number of variables and differential equations needed. Grey-box models also use actual data or simulated data from a white-box model to calibrate different input parameters. These models often utilize the RC network approach to simplify heat transfer mechanisms with resistances (R) and capacitances (C) of building structures. Braun et al. (2002) [6] used transfer functions and a nonlinear regression algorithm to optimize building parameters for an R13C8 building configuration and predict cooling demand and indoor temperature for various office buildings. Reynders et al. (2014) [46] identified a fourth-order (R6C4) model as the most accurate grey-box model for heating predictions of residential buildings, with no increased accuracy at higher orders. Harb et al. (2016) [47] determined that an R4C2 structure was best for predictions of indoor temperatures of occupied buildings, while Berthou et al. (2014) [48] used an R6C2 model to predict cooling energy and the indoor temperature of occupied buildings, and showed that two weeks of training data was sufficient for parameter estimation. Xu et al. (2008) [49] used a 3R2C configuration for both the walls and roof but optimized parameter values only for a simplified internal mass (R2C2). Deque et al. (2000) [50] predicted indoor temperature with a refined grey-box model by eliminating less significant inputs prior to a statistical optimization. Kim et al. (2015) [51] applied the unit zone model successfully to multiple zone buildings. Afram et al. (2015) [52]

modeled individual HVAC components with grey-box models for application of HVAC system controls.

Four studies have compared grey- and black-box models based on their effectiveness and accuracy in building-related energy predictions [7,45,52,53]. Reynders et al. (2013) [7] found that both model types can produce accurate day-ahead predictions, but the ARX black-box model is more robust for longer horizons when simulating heating demand for a single-family home. Li et al. (2015) [53] presented a novel black-box model which outperformed existing black-box models and an existing grey-box model for simulating a mid-sized commercial building, and showed that the accuracy of the existing models was highly dependent on training and forecasting data. The other studies looked at models of HVAC system components [52] and visually compared building simulations for indoor temperature predictions [45]. However, no study has compared grey-box and black-box models for predicting residential cooling loads. As smart thermostats and variable electric rates become more common for residential buildings, useful predictive models will be important in implementing advanced cooling setpoint controls.

The objective of this study is to provide a quantitative analysis of ARX models and robust grey-box models that predict cooling loads for two different residential homes. In addition to comparing model prediction performance, this study looks at the adaptiveness of the model types when applied to varying home types, specifically a newer home and an older home. Other metrics for comparison include the potential for integration into an advanced control model, and development and solution times.

2.2 Methodology

The general methodology of this study is as follows: 1) selection of residential buildings with available sub-hourly data from EnergyPlus, 2) identification of the black-box models using historical data and development and calibration of the grey-box models, and 3) comparison of ROMs with validation data.

2.2.1 Black-Box Model Methodology

The investigated black-box model is an autoregressive model with exogenous inputs (ARX) for prediction of cooling loads. An ARX model is chosen in this study as it has been shown to be effective in modeling space conditioning loads for residential buildings, it can be easily implemented into an advanced control strategy, and a more robust model that captures noise is not needed for the buildings modeled [54]. A previous study compared an ARX, ARMAX, and non-linear ARX, and showed no significant change in cooling prediction accuracy for an ARX model applied to a single-zone residential building [24]. Development of the black-box models rely on the following methodology: 1) simulate the data for training and validation from EnergyPlus; 2) identify coefficients for the ARX model using a least-squares optimization; and 3) simulate the response of the ROM by using various prediction horizons to represent the maximum expected forecasting response of an MPC.

The formulation for the ARX identification is shown below. The system model is identified using a least-squares objective function, $\mathcal{Z}_1$, shown in Equation (2.1). The ARX formulation, Equation (2.2), is a constraint in the model, and is characterized by two core parameter sets: the exogenous inputs, $U_{t,j}$, for time t and input j, and the output, Y'_t, for time t. The variables, $\mathbf{a}_i$, $\mathbf{b}_{i,j}$ constitute the tunable coefficients for previous timestep i and input j. The set $\mathcal{I}$ contains all previous timesteps used to predict electricity use ($\mathbf{Y}_t$) at time t. This model uses multiple current and previous inputs to predict the cooling load at time t, making it a multiple input single output (MISO) linear model. The objective function can also include an optional penalty term, P_t, which is applied when the training electricity use is zero. This term increases the importance of capturing periods when the HVAC system is off, which can be an important measure for load-shifting controls. Identification of the coefficients ($\mathbf{a}_i$, $\mathbf{b}_{i,j}$) is accomplished using the mathematical modeling language AMPL [55] and the optimization solver CPLEX v12.7.1.0 [56], with three weeks of EnergyPlus training data (Y'_t). In the formulation, bolded characters represent variables, while non-bolded characters represent parameters.

Sets

$t \in \mathcal{T}$	Timesteps encompassing the training period
$i \in \mathcal{I}$	Set of lag times {1, 2, ..., cardinality of weighting factors $\mathbf{a}_i, \mathbf{b}_{i,j}$}
$j \in \mathcal{J}_t$	Set containing the inputs outdoor drybulb temperature, building occupancy, and cooling setpoint at timestep t $\{DBT_t, T_t, Occ_t\}$

Parameters

Y'_t	EnergyPlus HVAC electricity use at timestep t (kWh)
$U_{t,j}$	Exogenous data for timestep, t, and input j (°C, °C, # of people)
DBT_t	Outdoor dry bulb air temperature at timestep t (°C)
T_t	Setpoint temperature for timestep t (°C)
Occ_t	Number of people occupying the zone at timestep t (people)

Variables

$\mathbf{Y}_t$	ARX predicted HVAC electricity use at timestep t (kWh)
$\mathbf{a}_i$	Weighting coefficient to system outputs for previous timestep i (-)
$\mathbf{b}_{i,j}$	Weighting coefficient to exogenous inputs for previous timestep i, and exogenous input j (-)

$$\mathcal{Z}_1 = \min \left[\sum_{t \in \mathcal{T}} (Y'_t - \mathbf{Y}_t)^2\right] \tag{2.1}$$

subject to:

$$\mathbf{Y}_t = \sum_{i \in \mathcal{I}} \left[-\mathbf{a}_i \cdot Y'_{t-i} + \sum_{j \in \mathcal{J}} \mathbf{b}_{i,j} \cdot U_{t-i+1,j}\right] \quad \forall t \in \mathcal{T} \tag{2.2}$$

2.2.1.1 Selection of Black-Box Input Parameters

The cardinality of the weighting coefficients, or the model order, is determined through a parametric analysis varying the number of previous timesteps used for historical data. The goal in choosing the model order is to include all previous timesteps that incite a system

response, which is proportional to the magnitude of the weighting factor for the earliest input. Twelve previous timesteps, or 6 hours, is sufficient for predicting the system response for the two homes modeled.

Model inputs, $U_{t,j}$, are essential in accurately predicting a response of a complex system such as a building. Correctly choosing the number of inputs is not an insignificant task; too few would incorrectly simulate the system, and too many might reduce the accuracy of the model by developing correlations to noise, known as overfitting [20, 57]. A previous study compared ARX models for detached residential buildings and showed that the accuracy of a three-input ARX model was comparable to a six-input model in predicting the cooling electricity load [24]. The ARX presented in the formulation above uses dry-bulb temperature, temperature setpoint, and occupancy as the exogenous inputs, chosen because they are expected to be closely correlated with heat transfer through a building envelope, while having limited correlations amongst themselves [41].

2.2.2 Grey-Box Model Methodology

Grey-box models include physical heat transfer mechanisms to predict energy use or indoor temperature of buildings. The developed grey-box model uses a resistance-capacitance (RC) approach to generate the governing heat transfer equations. The model is considered reduced because certain complexities of a typical home, such as directionally dependent radiation and multi-layered wall surfaces, are omitted; however, the model includes a calibration to adjust certain parameters and account for the discrepancies. These omissions reduce the required inputs and allow for quicker computation times than typical white-box models, while maintaining accurate predictions. The grey-box model simplifies the geometry by considering only one-dimensional heat transfer for one roof, one wall, one window, and the floor. The floor, roof, and wall use a 2R1C network, while the window uses a single resistance with no mass, as shown in Figure 2.1. Incident beam radiation to the wall is estimated as the average of beam radiation to vertical surfaces facing North, South, East, and West. Figure 2.1 shows a visual representation of how two of the building structures are

coupled with transfer mechanisms.

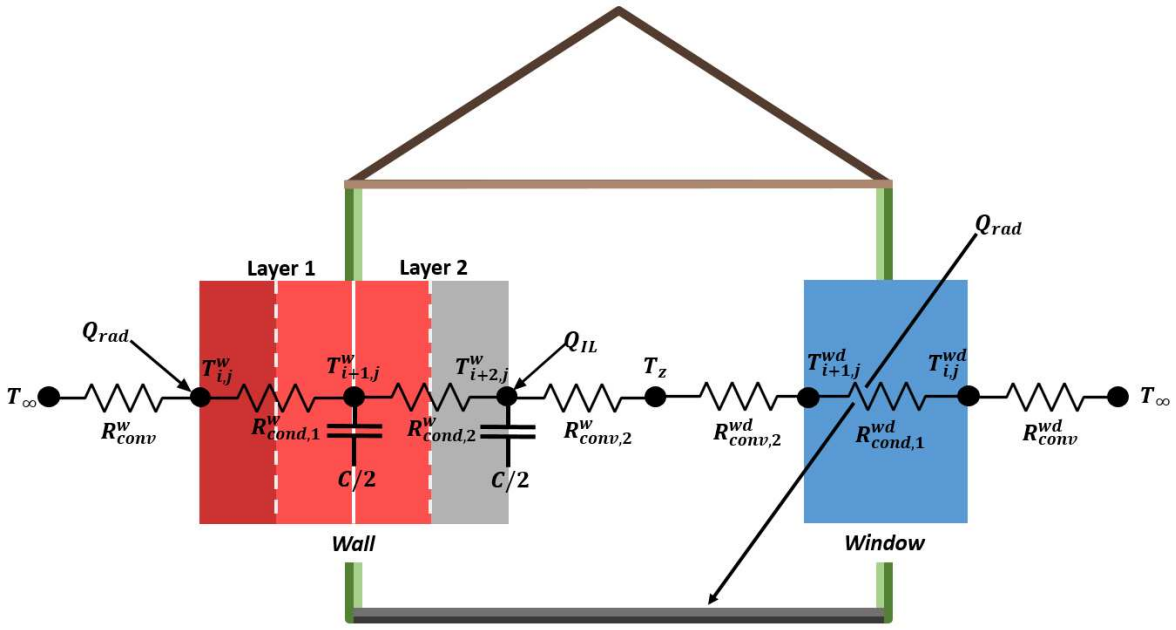

Figure 2.1: Reduced building geometry with RC networks shown for the wall and window.

In this network, the R's and C's represent the resistances and capacitances, the ∞, w, and wd represent ambient, wall, and window, respectively, and the Q's are direct heat flows, either as the net solar and sky radiation (rad) or the convective and radiative internal loads (IL). The 1, 2, and 1/2 notations indicate external, internal/external interface, and internal nodes, respectively. Finally, the first index on the temperature variables, T, indicates the node location, while the second index, j, indicates the timestep. Not shown in the figure is the heat removal from the zone via the HVAC system, infiltration gains, and the roof and floor structures, which have identical networks to the wall.

Heat flow through the building envelope is modeled as one-dimensional and transient. Energy balances for the exterior and interior wall nodes are shown in Equations (2.3) and (2.4), respectively. The interior wall node has a storage term due to the higher thermal mass, while the storage at the exterior wall is considered negligible, limiting the number of computationally expensive differential equations. Similar energy balances are applied to nodes of each building structure, as well as the zone air.

$$0 = G_{t+1}^{vert} + \frac{\mathbf{T}_{t+1}^{mid} - \mathbf{T}_{t+1}^{ext}}{R^{ext}} + \frac{T_{t+1}^{sky} - \mathbf{T}_{t+1}^{ext}}{R_t^{rad}} + \frac{T_{t+1}^{\infty} - \mathbf{T}_{t+1}^{ext}}{R^{cv,ext}} \tag{2.3}$$

$$\frac{\mathbf{dT}}{\mathbf{dt}} \frac{C^{int}}{2} = Q^{IL,rad} + \frac{\mathbf{T}_t^{mid} - \mathbf{T}_t^{int}}{R_{int}} - \frac{\mathbf{T}_t^{int} - \mathbf{T}_t^{zone}}{R^{cv,int}} \tag{2.4}$$

Where,

$\mathbf{T}$	Temperature (K)
$\mathbf{Q}^{IL,rad}$	Direct radiative heat flux from internal loads (W/m^2)
$\frac{\mathbf{dT}}{\mathbf{dt}}$	Derivative of temperature with respect to time (K/s)
R	Thermal Resistance ($K \cdot m^2/W$)
C	Thermal Capacitance ($J/K \cdot m^2$)
G^{vert}	Incident solar radiation on vertical surfaces (walls) (W/m^2)
ext	Exterior structure node
mid	Interior/exterior structure interface node
int	Interior structure node
rad	Radiation (sky)
$zone$	Zone node
cv	Convection
∞	Outdoor air
t	Current timestep

The system of heat transfer equations was coded using a finite difference method to solve for each node temperature; the equivalent finite difference form of Equation (2.4) is shown in Equation (2.5).

$$c_{p,i}\rho_i \Delta x \frac{T_i^{j+1} - T_i^j}{\Delta t} = k_i \frac{T_{i+1}^{j+1} - T_i^{j+1}}{\Delta x} + k_i \frac{T_{i-1}^{j+1} - T_i^{j+1}}{\Delta x} \tag{2.5}$$

Where,

T_i^j	Temperature of node i at timestep j
Δt	Time between each timestep (s)
Δx	Distance between nodes (m)
$c_{p,i}$	Specific heat of node i ($J/kg \cdot K$)
ρ_i	Density of node i (kg/m^3)
k_i	Thermal conductivity of node i ($W/m \cdot K$)

To solve for the node temperatures in the differential equations (e.g., Equation (2.5)), the Crank-Nicolson method is applied, and the entire system of equations is solved via the Gauss-Siedel iterative solver. Equation (2.6) shows the Crank-Nicolson formulation that calculates temperature one timestep in the future.

$$T_{i,j+1} = T_{i,j} + \left(\left.\frac{dT}{dt}\right|_{T=T_{i,j},t=t_j} + \left.\frac{dT}{dt}\right|_{T=T_{i,j+1},t=t_{j+1}}\right)\frac{\Delta t}{2} \tag{2.6}$$

The energy removed by the HVAC system at time t is assumed to be equal to the total gains to the zone, and therefore, the total convective gains from the interior structures to the zone node can be used to predict the sensible energy removed, as shown in Equation (2.7). This is possible once all temperatures of interior nodes are determined via the equations above.

$$Q_t^{HVAC} = \sum_{i \in \mathcal{I}} \frac{T_{i,t}^{int} - T_t^{zone}}{R_i^{cv,int}} \tag{2.7}$$

Where,

$i \in \mathcal{I}$	Set containing all building structures {wall, roof, floor, window}
Q_t^{HVAC}	Predicted heat energy removed at time t (kWh)

To simplify the model and reduce the required number of equations and variables, the model follows these assumptions:

- HVAC system instantaneously removes heat gains
- There are just four building structures (wall, roof, floor, and window)
- The window and wall face one direction
- Latent heat transfer is neglected
- Internal radiation between floor, walls, and roof is neglected
- All internal mass from the slab, appliances, and furniture is combined into the floor structure
- Transmitted solar radiation to the windows is absorbed entirely by the internal mass
- The convection and radiation from the internal mass is coupled
- There is no exterior shading

2.2.2.1 Grey-Box Model Calibration

Once the heat transfer model is established, elements of the grey-box model are calibrated to increase the accuracy. In this case, a statistical method optimizes weighting factors for the parameters shown in Table 2.2. The baseline for these parameters (i.e., guess values) come from Building Energy Optimization (BEopt) v2.3 building data [58], which is a program used for designing residential buildings and identifying cost-optimal efficiency packages, or a TMY3 weather file, which represents typical weather for specific locations. Each of these starting points has an associated weighting factor variable by which it is multiplied. Starting with a value of 1, and bounded by values of 0.25 and 3.0, the weighting factors are individually adjusted using a least-squares optimization routine to minimize the error between EnergyPlus and the ROM output.

The formulation for the model calibration is shown below. The objective function, shown in Equation (2.8), is minimized using the sequential-quadratic programming (sqp) algorithm in MATLAB by adjusting the weighting factors, $\mathbf{x}$. The sqp algorithm is sufficient because the optimization problem is small-scale and linear [59]. The decision to use cooling electricity in Equation (2.8), rather than cooling and indoor air temperature in the objective function, is based on trials that show no additional gain in accuracy of temperature predictions, but a

decrease in cooling accuracy when including both. The constraints include lower and upper bounds for the weighting matrix (Equation (2.9)), relation of the sensible cooling delivered and the electricity use (Equation (2.10)), as well as the equations described in Section 2.2.2. Like the black-box model, the least-squares error was minimized using three weeks of data, which is considered the training data.

A sensitivity analysis helped determine the appropriate parameters to weight by altering individual measures and observing the impact on the model. Parameters associated with large overall changes to the cooling load, as well as those with transient impacts, were selected for the optimization run. Additionally, parameters that have limited correlations to each other were prioritized.

Sets

$t \in \mathcal{T}$	Timesteps encompassing the training period

Parameters

${Y'}_t$	EnergyPlus HVAC electricity use at time t (kWh)
u	Vector of upper bounds of weighting factors (-)
l	Vector of lower bounds of weighting factors (-)
COP	Coefficient of performance for the air-conditioning equipment (-)

Variables

$\mathbf{Y}_t$	Grey-box predicted HVAC electricity use at time t (kWh)
$\mathbf{Q}_t^{HVAC}$	Energy removed by the HVAC system (kWh)
$\mathbf{x}$	Vector of weighting factors to building parameters in $Table$ 2.2 (-)

$$\mathcal{Z}_2 = \min [\sum_{t \in \mathcal{T}} (Y'_t - \mathbf{Y}_t(\mathbf{x})^2] \tag{2.8}$$

subject to:

$$l \leq \mathbf{x} \leq u \tag{2.9}$$

$$\mathbf{Y}_t(\mathbf{x}) = \frac{\mathbf{Q}_t^{HVAC}}{COP} \quad \forall t \in \mathcal{T} \tag{2.10}$$

Equations (2.3) - (2.7)

2.2.3 Data for Training and Validation

The process of training and validation uses data from two simulated homes in EnergyPlus, new detached and existing detached residential buildings. The simulated homes were built using BEopt v2.3 and run with EnergyPlus v8.4. Figure 2.2 shows the building archetypes, which have been used in other studies to represent a cross-section of homes [60, 61].

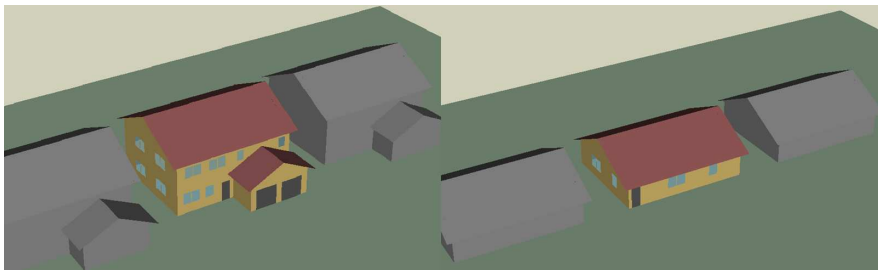

Figure 2.2: New detached (left) and existing detached (right) BEopt homes.

The simulated homes represent two different residential buildings by using current practices for homes built around 2010 and homes built before 1980. All homes have central air conditioning and heating systems. Table 2.1 summarizes the differences between new and existing homes; this table shows values prior to the grey-box parameter calibration. All new homes are similar to or more energy efficient than the B10 Benchmark from the Building America House Simulation Protocols [62]. The B10 Benchmark is consistent with: (1) the 2009 International Energy Conservation Code [63], (2) federal appliance standards of January 1, 2010, and (3) lighting and miscellaneous electric load (MEL) characteristics common in homes built in 2010. Existing homes follow the House Simulation Protocols for pre-retrofit cases of existing (old) buildings, and assumes houses were built between 1950-1979, and the

AC is from 1981-1991.

Table 2.1: New and existing detached home details.

Building Parameter	**2010 Home (new)**	**Pre-1980 Home (existing)**
Wall Insulation	R-13+R5	Uninsulated
Attic Insulation	R-30	R-13
Windows (U/SHGC)	0.37/0.53	0.84/0.63
Infiltration (ACH@50Pa)	4	20
Foundation	Uninsulated slab-on-grade	Uninsulated slab-on-grade
A/C (SEER)	13	8
A/C Size (Tons)	5	6
Plug Loads (kWh/y)	3270	2730
Finished Area (ft^2)	2496	1280

The ARX and grey-box models are trained with three weeks of data from the two simulated homes using the methods described in Sections 2.2.1 and 2.2.2, respectively. Table 2.2 displays the grey-box calibrated parameters and the resulting weighting factors from the formulation in Section 2.2.2.1.

Table 2.2: Optimized weighting factors for associated parameters.

Input Parameter	**New Detached**	**Existing Detached**
Wall External Resistance	0.45449	0.34817
Roof External Resistance	0.5994	0.41767
Floor External Resistance	0.72945	1.518
Infiltration Flow Rate	2.1332	1.3519
Wall Incident Solar Radiation	0.79022	0.95573
Roof Incident Solar Radiation	1.3272	1.4383
Floor Capacitance	1.6068	1.3319

2.2.4 Validation Methods

The grey-box and black-box models are validated using the EnergyPlus models described in Section 2.2.3. An NREL study has summarized methods for assessing the accuracy of residential energy building models [64]. Among the commonly used accuracy metrics for pre-

dictive energy models are the coefficient of variation of root mean square error (CV-RMSE) and the normalized mean biased error (NMBE) [65]. The CV-RMSE (Equation (2.11)) is used to determine the error between the ARX model and the EnergyPlus simulation data. The NMBE (Equation (2.12)) provides the overall bias of the resulting models prediction. The coefficient of determination, R^2, is also used in assessing how well the models fit the training set.

$$CVRMSE = \frac{100}{\bar{Y}'}\sqrt{\frac{\sum_{t=1}^{N}(Y'_t - Y_t)^2}{N-1}} \tag{2.11}$$

$$NMBE = \frac{100}{N \cdot \bar{Y}'}\sum_{t=1}^{N}(Y'_t - Y_t) \tag{2.12}$$

$Where,$

Y'_t	EnergyPlus cooling electricity at time t (kWh)
Y_t	ROM cooling electricity at time t (kWh)
$\bar{Y}'$	EnergyPlus mean cooling electricity (kWh)
N	Total number of timesteps (-)

2.3 Results

Cooling electricity outputs from EnergyPlus are compared to predictions from the black-box and grey-box reduced-order models for the new and existing detached homes. Figure 2.3 shows the temperature setpoint pre-cooling schedule used in the validation of each model, and the corresponding zone temperature calculated in EnergyPlus. This schedule is chosen to help analyze each models potential for implementation into an advanced control strategy. Cooling loads of the two residential homes are predicted for a 48-hour period and a 12-hour period in August. The 12-hour predictions are representative of a possible prediction horizon used in a model-predictive control strategy, and extends from 8:00 AM to 8:00 PM. The 48-hour predictions better show model capabilities over an extended period, starting at

midnight on the same day as the 12-hour run. The models are individually validated with EnergyPlus, then compared to each other.

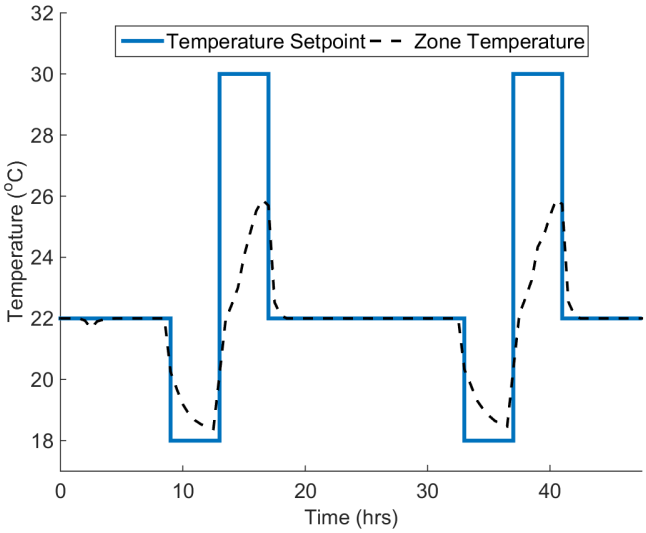

Figure 2.3: Temperature setpoint pre-cooling schedule and zone temperature used in model validation.

2.3.1 Black-Box Model Results

There are two methods in which an ARX model can be implemented, referred to in this study as the simulation and prediction methods. The simulation method assumes that all previous cooling data is known while stepping through the prediction horizon. This would be the case in a real-time control strategy that uses a smart-meter, where the electricity use is updated after every timestep. In the prediction method, the very first timestep is assumed to have known previous data; however, future computations rely on the previous ARX predictions instead of known data. Because less data is known the further out the model predicts, unreliable outputs are more likely with longer prediction horizons. The prediction method is often used in control strategies to estimate future loads that may be impacted by the current timestep. Figure 2.4 and Figure 2.5 show the cooling load outputs of both methods, as well as the EnergyPlus validation data for a 12-hour period and a 48-hour period, respectively. Both prediction horizons start on August 7, the 48-hour horizon starts

at midnight, and the 12-hour horizon starts at 8:00 AM.

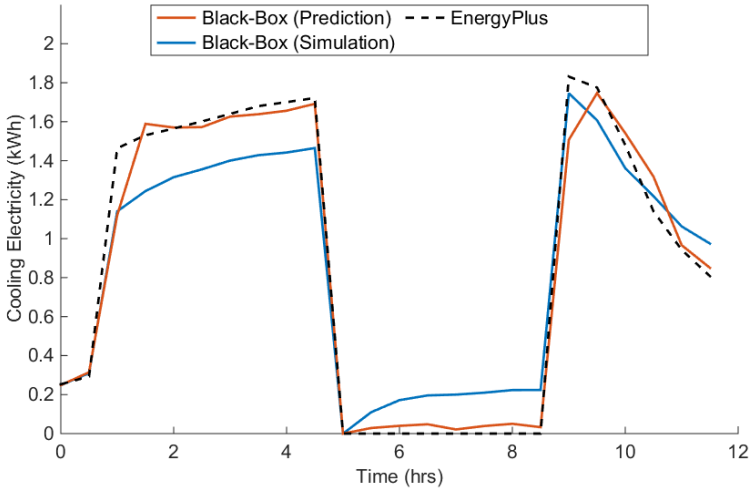

Figure 2.4: 12-hr ARX model predictions compared to EnergyPlus outputs for a new detached home.

Because the ARX is a linear model fit to a nonlinear system, the ARX deviates from EnergyPlus when the cooling load is zero, and may predict negative electricity use. This is not as apparent using the simulation method; however, it could be problematic when using the prediction method in identifying an advanced control strategy. To account for negative predicted cooling loads, the ARX output can be bound above zero after a prediction is made. Another way to reduce the error when electricity use is zero is to utilize a penalty, P_t, by which the objective function, Equation 2.1, is multiplied during system identification. The figures above are the black-box validation with no penalty term, while Figure 2.6 and Figure 2.7 show results with a penalty imposed. The ARX error during the load shifting period (zero values) is decreased when trained with a penalty; however, accuracy is impacted in other areas, such as capturing the peaks during the pre-cooling times.

2.3.2 Grey-Box Model Results

Similar to the black-box model, the grey-box model was run for 48-hour and 12-hour periods, and compared to EnergyPlus data. The grey-box model is capable of predicting

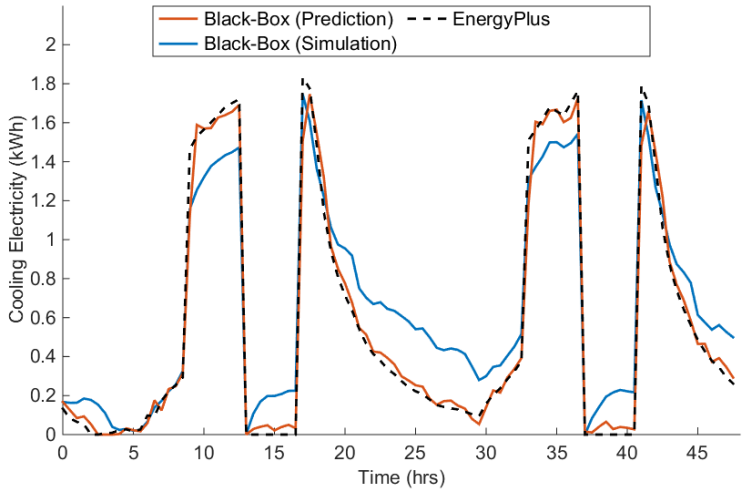

Figure 2.5: 48-hr ARX model predictions compared to EnergyPlus outputs for a new detached home.

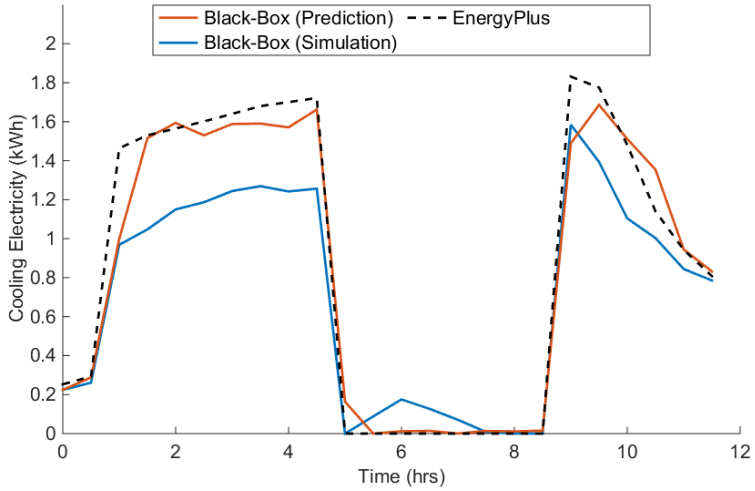

Figure 2.6: 12-hr ARX model predictions compared to EnergyPlus outputs for a new detached home. The model is trained using a penalty term applied to the objective function when $Y_t' = 0$.

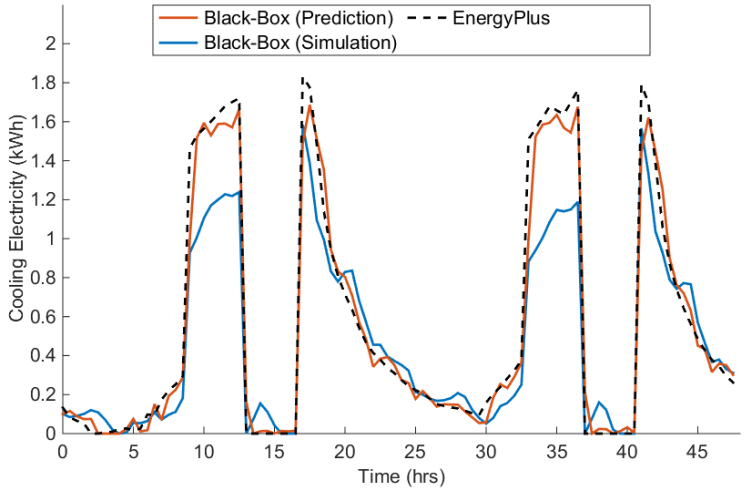

Figure 2.7: 48-hr ARX model predictions compared to EnergyPlus outputs for a new detached home. The model is trained using a penalty term applied to the objective function when $Y_t' = 0$.

indoor air temperature in addition to the cooling electricity, which can be useful to better account for user comfort in certain applications. Figure 2.8 shows the cooling electricity rate and indoor air temperature predictions for a 48-hour period. Unlike the black-box prediction method, which is influenced by the last known data point, the grey-box predictions are not impacted by the length of the prediction horizon, as no previously known data is used as an input - therefore, the results of the 12-hour simulation are encompassed within the 48-hour simulation.

2.3.3 Black-Box and Grey-Box Comparison

The ARX model and the grey-box model are compared using the validation cooling predictions for 48- and 12-hour time periods. Figure 2.9 and Figure 2.10 show the two models compared with EnergyPlus outputs for both simulation periods. The simulation method is used for the comparison, as that is the typical implementation of the ARX.

A summary of the error metrics for the reduced-order models applied to the new and existing homes can be found in Table 2.3 and Table 2.4, respectively. The results include the predictions from the training period, and two validation periods compared to EnergyPlus.

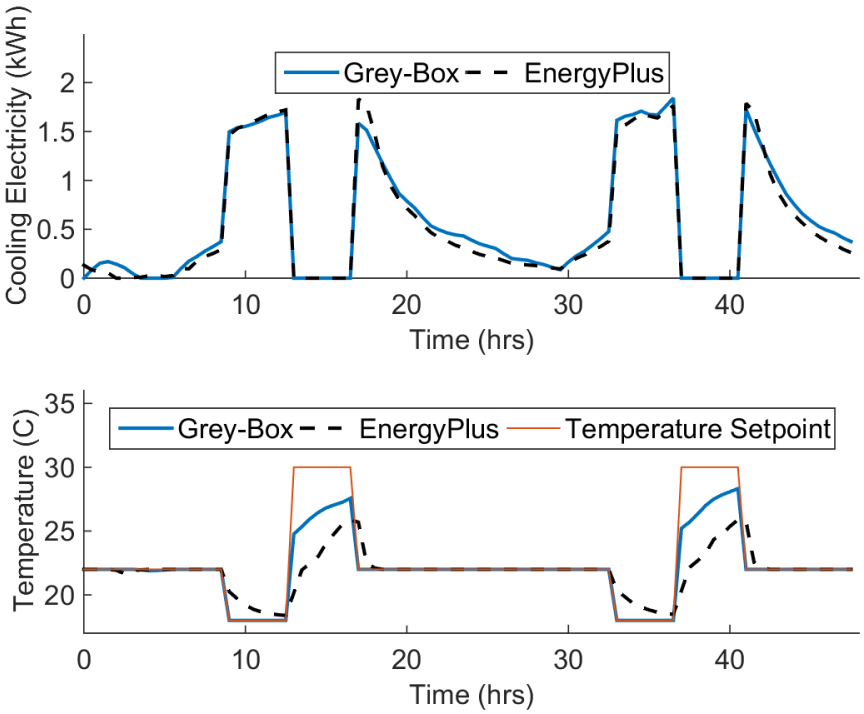

Figure 2.8: Grey-box predicted cooling loads and indoor temperature compared to EnergyPlus.

Only the simulation method includes training data for the black-box results, as this is method used in training the model.

2.4 Discussion

The value of building energy management strategies is marked by significant reductions in building energy costs. A barrier to achieving advanced control strategies for residential homes is identifying predictive models that can be generalized across weather inputs, building constructions, and occupant behavior. Presented in this study is a comparison of two reduced-order grey-box and a black-box models capable of being implemented into a model-predictive control strategy.

Table 2.3, Table 2.4, Figure 2.9, and Figure 2.10 demonstrate that both model types are able to accurately predict cooling electricity to a similar degree. The models perform better when simulating the new detached home based on the error metrics chosen; however,

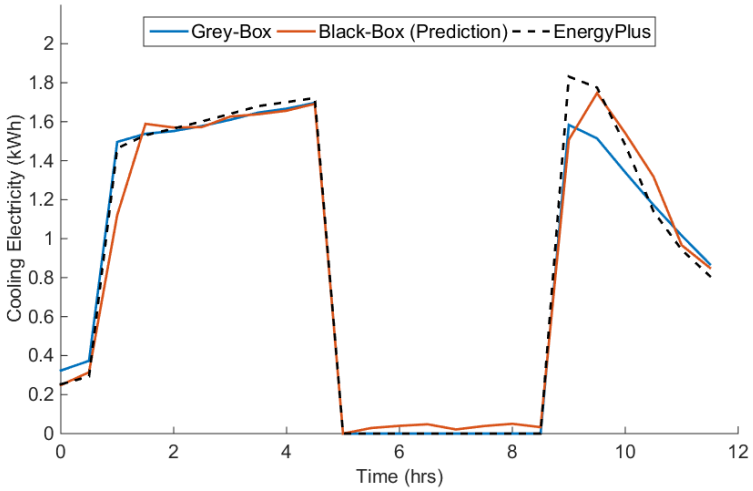

Figure 2.9: 12-hr predictions using the ARX model and grey-box model, compared to EnergyPlus outputs for new a detached home.

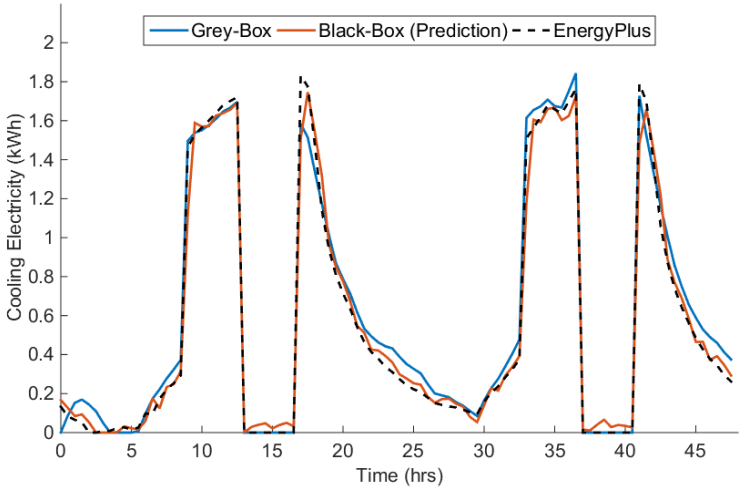

Figure 2.10: 48-hr predictions using the ARX model and grey-box model, compared to EnergyPlus outputs for new a detached home.

Table 2.3: Cooling electricity prediction error metrics for the grey-box and black-box models, compared to EnergyPlus for a new detached home.

		Black-Box (Simulation)	**Black-Box (Prediction)**	**Grey-Box**
Training	CVRMSE	-	19.74%	26.37%
	NMBE	-	0.14%	2.10%
	R^2	-	0.973	0.955
Validation (48 hrs)	CVRMSE	34.97%	13.96%	14.21%
	NMBE	-14.74%	-0.72%	-5.71%
	R^2	0.950	0.986	0.987
Validation (12 hrs)	CVRMSE	21.99%	12.11%	9.59%
	NMBE	3.61%	0.97%	2.10%
	R^2	0.972	0.981	0.989

Table 2.4: Cooling electricity prediction error metrics for the grey-box and black-box models, compared to EnergyPlus for an existing detached home.

		Black-Box (Simulation)	**Black-Box (Prediction)**	**Grey-Box**
Training	CVRMSE	-	23.94%	34.32%
	NMBE	-	0.16%	0.95%
	R^2	-	0.959	0.923
Validation (48 hrs)	CVRMSE	33.38%	24.78%	27.59%
	NMBE	-11.44%	-1.12%	2.68%
	R^2	0.945	0.956	0.950
Validation (12 hrs)	CVRMSE	30.72%	23.31%	24.60%
	NMBE	-9.12%	-0.59%	3.72%
	R^2	0.950	0.950	0.954

this is likely due to the lower electric load values associated with the smaller, existing home. The overall absolute errors for the existing detached is lower than the new detached. While the grey-box and black-box models perform similarly, they maintain key differences beyond the error metrics. The black-box model is shown to be accurate across extended prediction horizons when real-time data is available as an input; however, without updated previous data, the CVRMSE for the 48-hour validation increases from 13.96% to 34.97%, and the NMBE from -0.72% to -14.74% for the new detached home. The 12-hour prediction horizon reduces these gaps, with the CVRMSE ranging from 12.11% to 21.99% and NMBE ranging from 0.97% to 3.61%. An ARX model in a setpoint control optimization must rely on the prediction method to control the current timestep, and therefore, a prediction horizon beyond 12 hours would likely be unhelpful and could misinform the model. Additionally, as shown in Figure 2.5 and Figure 2.7, the ARX method may provide inaccurate solutions when the cooling load is zero, even with a penalty applied in the objective function. This is particularly true when using the prediction method, which can lead to unnecessarily high pre-cooling demands to reduce incorrect non-negative value.

The grey-box model has similar prediction errors to the black-box, but can better predict zero values. Additionally, the grey-box model can predict indoor temperature and has the benefit of not needing any real-time data for predictions, but instead relies on knowledge of the construction of the building, historical input data for training, and real-time inputs for simulation. The grey-box model, however, requires more computing power than the ARX. Prior to parameter calibration, the grey-box is able to provide a decent representation of the home through a system of ordinary differential equations, which require numerous iterations to converge for each timestep. This not only increases the complexity of introducing this model into an optimization problem, but increases the computation time. The ARX is the fastest model, solving in approximately 7ms, the grey-box solves in approximately 600ms, while EnergyPlus solves in approximately 30 seconds. The ROMs both show significant improvements in computation time compared to a white-box model; however, the grey-

box model solve times are still substantially longer than the black-box, which could be an important factor, depending on the application.

2.5 Conclusion

This study details the development of two commonly used reduced-order models for building energy analysis. An ARX black-box model is compared to a simplified RC grey-box model for cooling electricity predictions. Both models show potential for accurate solutions with significantly reduced computation times compared to a robust white-box model. However, they each have unique advantages and disadvantages when considered in the context of an advanced controls optimization. The results indicate that the ARX model is highly dependent on available data as predictions are made, contrasting with the grey-box, which does not require any real-time data for predictions. However, the black-box model introduces the benefits of quicker computations and ease of implementation, both important factors in application to a model-predictive controller.

CHAPTER 3

MODEL-PREDICTIVE CONTROL OF TEMPERATURE SETPOINTS FOR HOMES SUBJECT TO TIME-OF-USE RATES

3.1 Introduction

As populations grow, total and peak electricity demand is likely to follow. Periods of peak electricity demand strain the grid, particularly in the summer when air conditioning use is highest, increasing the potential for service interruptions and shortening the lifetime of certain infrastructure. Additionally, higher peak demands may lead to increased costs for electric utilities due to less efficient reserve technologies and the potential need for upgrades to plant capacity [66]. Variable electricity rates, such as time-of-use (TOU), are becoming an increasingly popular method to address peak electricity of residential homes. Historically applied to commercial and industrial buildings, these rate structures increase electric rates when demand is highest, but have lower rates than normal during off-peak, encouraging consumers to shift or eliminate their peak electricity to save money. Strategies to reduce peak demand and take advantage of variable rates include: (i) energy-efficient retrofit technologies, (ii) demand-response controllers that delay electrical devices until off-peak times, and (iii) advanced-building or other thermostat control strategies such as rule-based, pre-cooling, or use of learning and programmable thermostats (e.g., NEST [26]). Pre-cooling uses the thermal mass of the building envelope to store energy by lowering the temperature setpoint prior to peak times, and constitutes one of the most cost-effective ways to reduce electricity costs in existing buildings [9–14]. Advanced building optimization control algorithms include: neural networks, fuzzy logic, as well as feedback and feed-forward equations [28].

Optimization-based model predictive controls (MPCs) are especially suitable for large input-output systems, such as buildings. MPCs operate by taking feedback signals from past system states and the environment to determine future system responses, then optimizing

to meet all constraints, e.g., thermal comfort and energy usage [15–17]. Variables such as temperature setpoint, electricity rate, and thermal storage may be used to find an optimal schedule. The constraints might include comfort index, building codes (ventilation and other standards), or control schedules. MPC controllers operate over two horizons (Figure 3.1): (i) a *finite receding prediction horizon* over which a model forecasts the system output, and (ii) an *operation horizon*, called the control horizon, over which the controllable variable values are determined using optimization. To increase the tractability of large problem instances, the prediction horizon is often larger than the control horizon, which decreases the total number of optimization variables [67]. An MPC uses system predictions over a prediction horizon to determine optimal control variables, and fixes these values across the control horizon. The prediction horizon is as long as or longer than the control horizon [67].

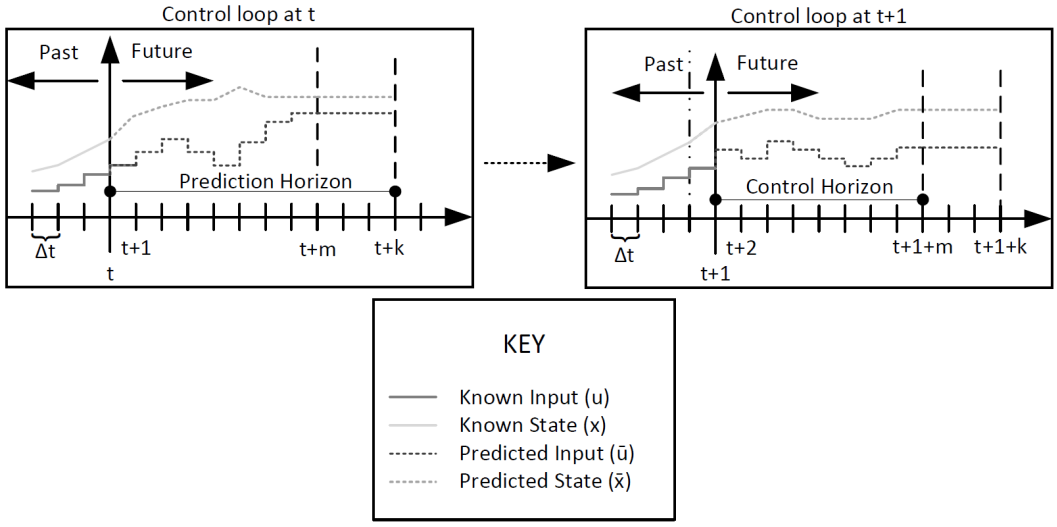

Figure 3.1: The operational basis for the receding (t), control (m), and prediction (k) horizons for an MPC over two time steps $(t, t+k)$.

There are two different types of optimization-based controllers: (i) deterministic model-predictive controllers and (ii) stochastic model-predictive controllers. Deterministic MPCs assume little-to-no model error or uncertainty inputs [68] and are often used due to their simplicity, despite an over-estimation of cost savings, while stochastic MPCs use a predictive model that estimates uncertainty of signals or variables. Deterministic systems are often

used to minimize HVAC electricity or cost by controlling temperature setpoints, and have shown potential for substantial cost and peak-demand reductions using prediction horizons of 12-72 hours [16, 17, 34, 36, 38–40, 69]. In addition, previous deterministic and stochastic building MPC studies have found that the following inputs have the greatest effect on MPC performance:

- Weather: Oldewurtel et al. (2010, 2012) examined how weather predictions can improve an optimization-based control model and, in turn, increase efficiency [37, 70]. The weather data selected for use within this study include outside air temperature, wetbulb temperature, and solar radiation. The results showed that the inclusion of this environmental information helps to increase prediction consistency.

- Occupancy: Oldewurtel (2013) also examined the effect of occupancy on the ROM for MPC building climate control [15]. Occupancy is a measure of how many people are in the different zones within a building. Ongoing research on predicting occupancy schedules currently uses Markov chains to decipher the location and probability of people within a building [71, 72].

Previous MPC studies have found similar energy cost savings to those presented in this chapter; however, these studies cannot be directly compared to each other due to differences in location, electricity rates, home types, and building systems. Other MPCs have also shown potential for significant cost, energy, or peak demand reductions with predictive building controls of PV, HVAC subsystem components, chillers, and whole-building energy management systems [18–22].

This chapter presents an analysis of a model-predictive controller of temperature setpoints applied to a residential home with variable electric rates. Several studies have analyzed variable electric rates using an MPC or other optimization tool. Li et al. (2016) developed an MPC for control of temperature setpoints applied to locations with differing TOU rates and weather [32]. Cole et al. (2014) presented a method to optimize variable electric rates

using fixed temperature setpoints [73]. Other studies have optimized time-of-use rates using demand-side responses from historical data [74] and with respect to plant generation costs [75].

Many academic building control systems use co-simulation environments that access multiple pieces of software for modeling single components, such as a building controller, the building envelope, and HVAC equipment of both the control and response of the virtual building [17, 21, 76]. To design MPCs and other advanced controls for buildings, researchers have developed co-simulation environments linking computer programs to allow for accurate simulation of the control and response of the virtual building. Wetter [77, 78] created a co-simulation environment based on the Ptolemy II framework [79] that links EnergyPlus to other energy simulation programs via a Java interface called Building Control Virtual Test Bed (BCVTB). Sagerschnig et al. [80] used BCVTB to couple TRNSYS [42] with MATLAB for building control design and modeling. Bernal [81] modified BCVTB to create a MATLAB-EnergyPlus interface (MLE+) that directly uses Simulink, a graphical programming environment for model-based design and simulating dynamic systems [82]. Another study developed a multi-agent, building energy management system using a co-simulation environment coupling AMPL and EnergyPlus [83]. These aforementioned co-simulation programs — BCVTB, MLE+, and text-based integration — can operate and integrate whole-building energy simulation programs such EnergyPlus with advanced controllers that allow for accurate energy predictions.

Other studies have used GenOpt [84] as an optimization solver that externally runs EnergyPlus or other simulation software to optimize building construction variables or schedules for control [32, 85, 86]. BEopt and OpenStudio are tools that also use EnergyPlus and can find optimal construction properties or retrofits to minimize costs. Both use optimization routines similar to GenOpt to determine the best design criteria (such as envelope design), but are not intended for optimizing controls. Instead, many frameworks use robust models like EnergyPlus to simulate the building response after optimization of the building control,

using Simulink Optimization and MPC toolboxes that may increase run time [21, 33, 87].

Robust white-box energy modeling programs such as EnergyPlus [23] or TRNSYS [42] are commonly used to predict the response of a building HVAC system to internal heat sources, the outdoor environment, and controllable inputs such as thermostat setpoint. These models incorporate linear and nonlinear equations based on physical laws, allowing them to accurately predict energy use at each time step over a discrete horizon. However, due to long computation times, white-box models are rarely used in optimized controllers. Reduced-order black- and grey-box models offer much faster computation times, making them ideal for advanced controls optimization. The RC method is a common grey-box model that has been applied to MPCs for building controls [16, 34–37]. Likewise, black-box models, such as auto-regressive with exogenous inputs (ARX), artifical neural network, support-vector machine, and state-space models have all been used in model-predictive controllers of building systems [38–40, 88]. These controllers have demonstrated potential for significant cost savings for consumers subject to a variable electric rate when compared to less advanced control systems. Commercial solvers and optimization-specific programming languages such as TOMLAB Optimization, APMonitor, AMPL, GAMS, and MIDACO [89,90], can be used with ROMs and linked to MATLAB and Simulink. Because AMPL and GAMS allow for the formulations to be input in a high-level algebraic form, the formulations become easier to code; additionally, presolvers both within the algebraic modeling languages (in the case of AMPL) and within the solvers, e.g., CPLEX, help expedite solutions.

No study has employed MPC for control of temperature setpoints of residential buildings paired with analysis of time-of-use rates for a single location. The contributions of this paper are: (i) a robust model predictive control platform that combines a co-simulation environment with an advanced whole-building energy simulation program and an algebraic modeling language, allowing for rapid development and deployment of optimization models; and, (ii) an analysis of the impacts of variable electric rates in an MPC that considers electricity costs and thermal comfort of residential homes, providing perspective for electricity providers and

customers alike.

3.2 Model Predictive Control Development

The development of the optimal control system is accomplished by: (i) identifying a system model for predictions of cooling energy demand, (ii) establishing the framework in which the MPC operates, and (iii) formulating the MPC optimization model to identify an optimal setpoint control schedule for different variable electric rates.

3.2.1 System Identification

A reliable model to predict AC electricity is necessary to determine an optimal operational policy of temperature setpoints. These predictions are made across a finite horizon within a formulation to find optimal setpoint variable values. The optimization model uses a linear programming algorithm which generally requires 100-1,000 iterations for an MPC applied to buildings. Therefore, detailed whole-building white-box models such as EnergyPlus might not be computationally tractable due to their long solution times. Reduced-order models (ROMs) increase computational speed by using statistical methods to reduce the number of equations that describe the system, and therefore are commonly used in MPCs.

The MPC presented in this chapter uses an ARX time-series forecasting model to predict cooling electricity. A black-box model is chosen based on a previous study that compared the capacity of a grey-box and a black-box model to predict electricity use for single-zone residential buildings [91]. The ARX is chosen based on a previous study that compared an ARX, ARMAX, and non-linear ARX, and showed no significant change in cooling prediction accuracy for an ARX applied to a single-zone residential building [24]. An ARX model with three exogenous inputs is appropriate for this application based on its ability to predict short-term energy usage and the ease with which it can be integrated in an MPC. The objective function and ARX formula are shown in Equations (3.1) and (3.2), respectively. A linear-least squares optimization procedure is performed using cooling electricity output from EnergyPlus as training data to determine the weighting coefficients $\mathbf{a}_i$ and $\mathbf{b}_{ij}$. The ARX

predicts cooling electricity at time t by using previous cooling electricity and previous and current exogenous, or external, inputs. These exogenous inputs include dry-bulb temperature (DBT), temperature setpoint (TSP), and occupancy. The indices for the exogenous inputs in the ARX model (Equation (3.2)) include the time step that is being predicted (indicated by the +1), because there is no delay between these inputs and their impact on the system. For example, if a setpoint is changed, it is expected to impact the cooling load within that same time step (at time t).

Sets

$t \in \mathcal{T}$	Time steps encompassing the training period
$i \in \mathcal{I}$	Set of lag times {1, 2, ..., cardinality of weighting factors $\mathbf{a}_i, \mathbf{b}_{i,j}$}
$j \in \mathcal{J}_t$	Set containing the inputs outdoor drybulb temperature, building occupancy, and cooling setpoint at time t $\{DBT_t, T_t, Occ_t\}$

Parameters

${Y'}_t$	EnergyPlus HVAC electricity use at time t (kWh)
$U_{t,j}$	Exogenous data for time, t, and input j (°C, °C, # of people)
DBT_t	Outdoor dry bulb air temperature at time step t (°C)
T_t	Setpoint temperature for time step t (°C)
Occ_t	Number of people occupying the zone at time step t (people)

Variables

$\mathbf{Y}_t$	ARX predicted HVAC electricity use at time t (kWh)
$\mathbf{a}_i$	Weighting coefficient to system outputs for previous time step i (-)
$\mathbf{b}_{i,j}$	Weighting coefficient to exogenous inputs for previous time step i, and exogenous input j (-)

$$\mathcal{Z} = \min [\sum_{t \in \mathcal{T}} (Y'_t - \mathbf{Y}_t)^2] \tag{3.1}$$

subject to:

$$\mathbf{Y}_t = \sum_{i \in \mathcal{I}} [-\mathbf{a}_i \cdot {Y'}_{t-i} + \sum_{j \in \mathcal{J}} \mathbf{b}_{i,j} \cdot U_{t-i+1,j}] \quad \forall t \in \mathcal{T} \tag{3.2}$$

The cardinality of the weighting coefficients, or the model order, is determined through a parametric analysis varying the number of previous time steps used for historical data. The goal in choosing the model order is to include all previous time steps that incite a system response, or in the case of this study, impact the cooling electricity prediction. Three hours of input data with four time steps per hour is able to appropriately capture the system dynamics for the residential home, with no significant improvement with longer times. A demonstration of how the ARX model predicts cooling loads from time $t = 1$ to $t = 4$, using an order of four is shown in Figure 3.2. The yellow line indicates the boundary between known and unknown cooling energy outputs, with values to the right of the line unknown, and values at or to the left of the line known, while exogenous inputs are assumed to be known across all time steps. The cooling electricity at $t = 1$ is predicted using historical data for prior time steps, the exogenous inputs at $t = 1$, and the weighting coefficients. To estimate $t = 2$, the previously predicted output from $t = 1$ becomes the input at $i = 1$. Finally, in calculating the cooling load at $t = 4$, there are three ARX-predicted values and one known value, and thus the model can become more inaccurate with longer prediction horizons. When the ARX model is applied to the MPC, the exogenous inputs are assumed to be known at every time step through statistically generated historical data, with the exception of temperature setpoint. Setpoint is a parameter prior to the prediction horizon, but is a variable within the prediction horizon.

Given the input data, a three-week training period is sufficient for accurate cooling electricity predictions. The dry-bulb temperature training set encompasses a large range of high and low temperatures, including the hottest days of the year. This allows the model to be generalized to make predictions in varied weather conditions. The occupancy input is

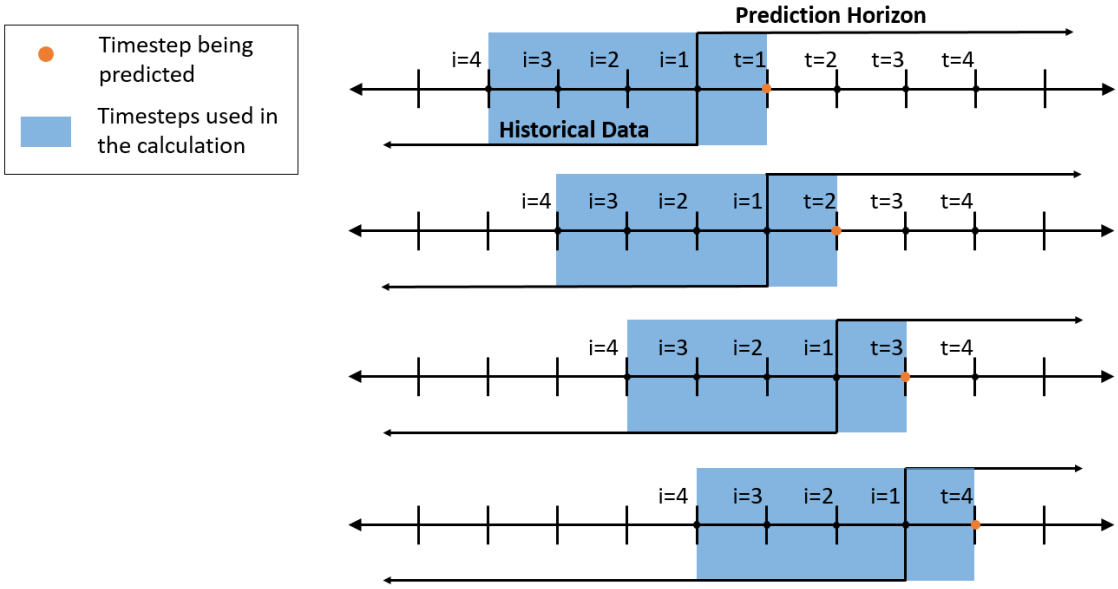

Figure 3.2: Demonstration of ARX predictions for prediction horizon of four, and model order of four

determined by EnergyPlus using the Building America House Simulation Protocols to estimate hourly occupant count [92]. Occupancy and internal loads are closely related, so the occupancy input is used to capture the building response to all internal loads, including heat gains from occupants, appliances, plug loads, and lights. Finally, the temperature setpoint training data follows a schedule that includes days with a typical pre-cooling profile, and days of a constant setpoint. Further details regarding the black-box model are discussed in [91].

Figure 3.3 and Figure 3.4 show the validation of the ARX for cooling electricity predictions across 12- and 48-hour periods compared to an EnergyPlus simulation. Two methods, *simulation* and *prediction*, are used to predict the cooling loads. In the *simulation* method, the first timestep is assumed to have known previous data; however, future computations rely on the previous ARX predictions instead of known data, as described above and shown in Figure 3.2. Because less data is known the further out the model predicts, unreliable outputs can result from longer prediction horizons. The *simulation* method is often used in

control strategies to estimate future loads that can be impacted by the current control. The *prediction* method assumes that all previous cooling data is known while stepping through the prediction horizon. This would be the case in a real-time control strategy that uses a smart-meter, where the electricity use could be updated after every timestep. The validation of 12 hours for the *prediction* method resulted in a coefficient of variation for the root mean square error (CVRMSE) of 12.11% and normalized mean bias error (NMBE) 0.97%, while the *simulation* method had a CVRMSE of 21.99% and a NMBE of 3.61%.

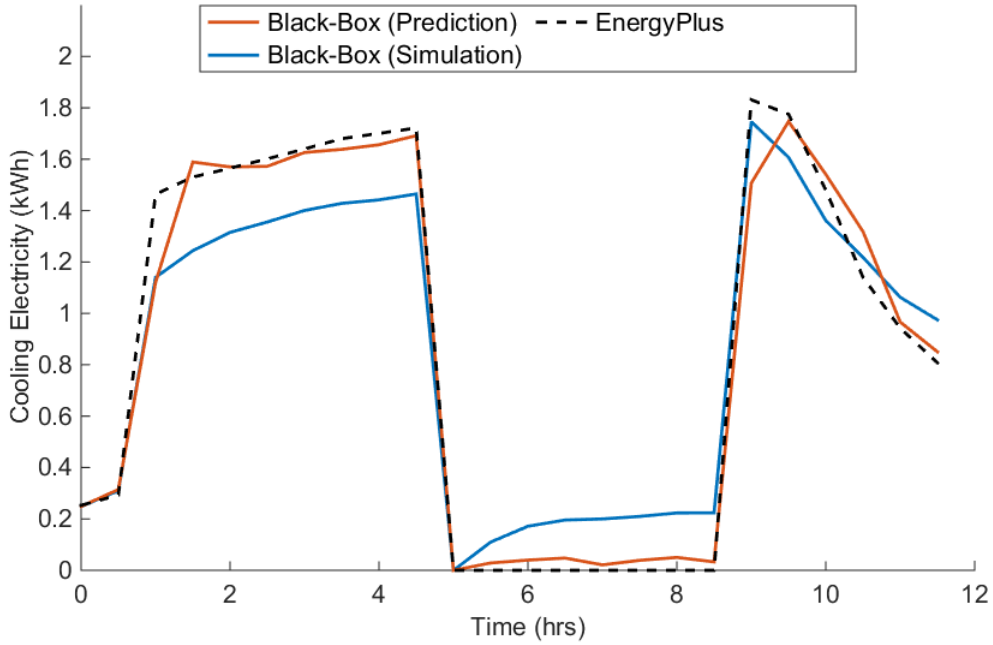

Figure 3.3: 12-hr predictions using the ARX model, compared to EnergyPlus outputs for a new detached home.

3.2.2 Prototyping Platform Description

EnergyPlus and AMPL are integrated using MATLAB and Simulink for the MPC platform. AMPL [55] is an algebraic modeling language that allows for rapid development and deployment of optimization models, and can call different solvers depending on the mathematical problem structure. The solver used in this study, CPLEX v12.7.1 [56], is particularly suitable for linear and mixed integer problems, making it appropriate for the proposed linear

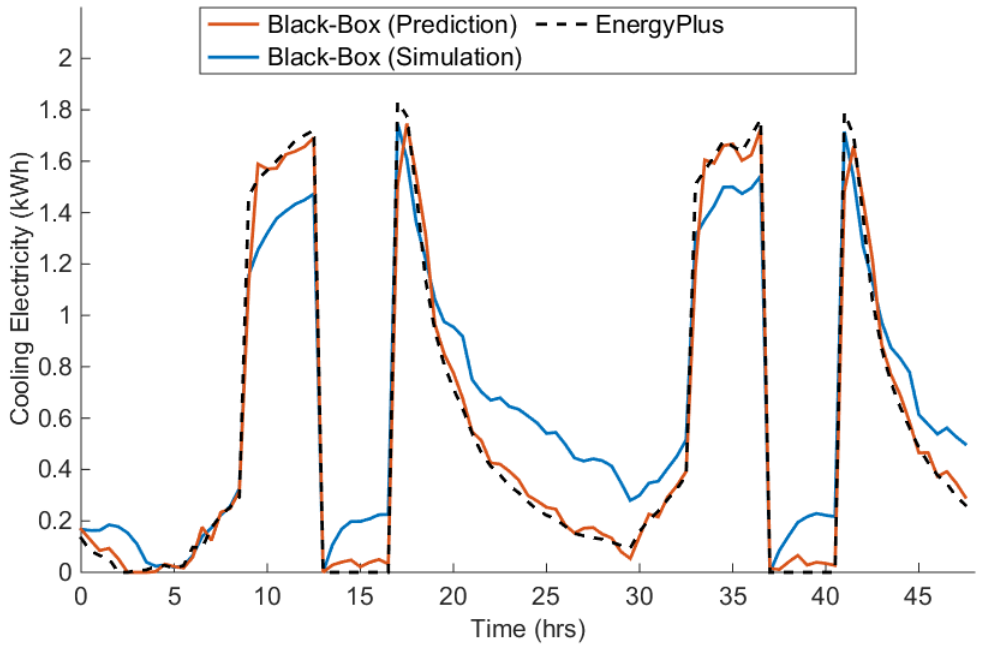

Figure 3.4: 48-hr predictions using the ARX model, compared to EnergyPlus outputs for a new detached home.

MPC formulation. This study uses an AMPL API that allows AMPL to interact with other applications such as C++, Java, Python, and MATLAB. The Simulink interface uses an S-function to call the AMPL API and perform the following operations:

1. Instantiate an AMPL object
2. Load the latest parameter values in the AMPL model (i.e., DBT, TSP, occupancy prior to t)
3. Solve the MPC optimization problem
4. Output setpoint to EnergyPlus to simulate the response for the next time step
5. Close the AMPL instance

Following the optimization run in step 3), EnergyPlus is run with the optimal temperature setpoint at global time, t, via MLE+. The resulting cooling output, Y_t, and setpoint, TSP_t,

become fixed as historical data in the next optimization. These steps are accomplished using Simulink to communicate between AMPL, MLE+, and MATLAB, which is used for initializing and processing data. Figure 3.5 shows the prototyping platform's control loop, with numbers that correspond to the steps listed above.

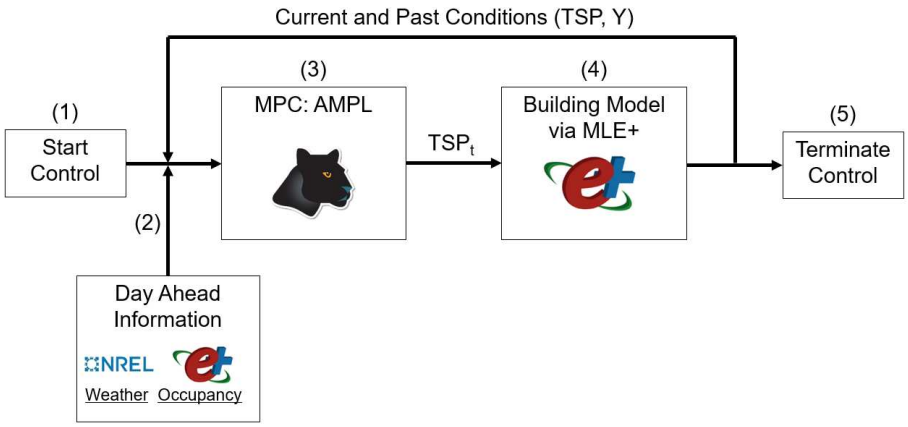

Figure 3.5: MPC modeling environment control loop, constructed with Simulink; the numbers correspond with the steps listed in the preceding paragraph.

3.2.3 MPC Optimization Model

The goal of advanced building controls is to maintain thermal comfort while reducing the cost or energy consumption of the building. The MPC presented minimizes the electricity costs due to cooling with variable time-of-use (TOU) rates by determining an optimal setpoint schedule that considers both electricity cost and the zone temperature setpoint while occupied. The ARX (Equation (3.2)) described previously is a constraint in the MPC, but the temperature setpoint inputs here are variables, while the weighting coefficients are parameters. Other important parameters include outdoor air temperature, electric rate, prediction horizon, occupancy, temperature bounds, and desired temperature. Due to the information available in the MPC, the temperature setpoint is considered to be the same as the indoor temperature when determining thermal comfort. The ARX model still captures thermal mass effects when predicting cooling loads; however, it is unable to predict indoor

temperature, unlike more robust models.

The MPC operates on a rolling horizon by solving across a fixed-length prediction horizon for each time within the simulation period (t, $t+1$,...$t+n$). The setpoint at t and the associated electricity use, as determined by EnergyPlus, becomes fixed for the next optimization run upon solving. The problem is solved similarly at the global time step $t+1$, with historical data and known future data shifted one step forward. This method allows for future predictions to influence the selection of the current temperature setpoint, while not fixing setpoints from predictions beyond time t, as those are typically be less accurate.

3.2.4 Mathematical Formulation of the MPC

The objective function, ($\mathcal{Z}_t$), and corresponding constraints of the model-predictive control are shown in Equations (3.3) - (3.10). The formulation uses unbolded lowercase and uppercase letters to represent the deterministic parameters, bold upper case letters to indicate variables, and script font to denote sets. Subscripts are used to identify indices, and should be read such that $\mathbf{Y}_{t+k}$ is the predicted energy usage at time $t+k$ predicted from time step t. The variables, which include setpoint temperature, energy usage, and one auxiliary variable, are continuous. We present the formulation for a fixed t, and denote the corresponding objective function ($\mathcal{Z}_t$).

Sets

$t \in \mathcal{T}$	Global time steps encompassing simulation period
$k \in \mathcal{K}$	Time steps encompassing the prediction horizon {0, 1, ..., prediction horizon}
$i \in \mathcal{I}$	Set of lag times {1, 2, ..., cardinality of the weighting factors a, b}
$j \in \mathcal{J}_t$	Set containing the inputs outdoor dry-bulb temperature, building occupancy, and cooling setpoint at time t $\{DBT_t, \mathbf{T}_t, Occ_t\}$
$\mathcal{T}^O$	Subset of timesteps $t \in \mathcal{T}$ for which the building is occupied
$\mathcal{T}^U$	Subset of timesteps $t \in \mathcal{T}$ for which the building is unoccupied

Parameters

a_i	Weighting coefficient to system outputs for previous time step i (-)
$b_{i,j}$	Weighting coefficient to exogenous inputs for previous time step i and exogenous input j (-)
$\underline{s}^u$	Lower bound of temperature setpoint when unoccupied (°C)
$\underline{s}^o$	Lower bound of temperature setpoint when occupied (°C)
$\bar{s}^u$	Upper bound of temperature setpoint when unoccupied (°C)
$\bar{s}^o$	Upper bound of temperature setpoint when occupied (°C)
p_t	Preprocessed parameter of 1 if zone is occupied, 0 otherwise for time t (binary)
p^+	Penalty value for positive temperature setpoint deviation ($/°C)
r_t	Charge rate for electricity at time t ($/kWh)
T'	Desired indoor air temperature (°C)
DBT_t	Outdoor dry bulb air temperature at time t (°C)
Occ_t	Occupancy of the zone at time t (people)
Y_t^E	Known cooling energy from EnergyPlus at time t (kWh)
T_t^F	Fixed temperature setpoint at time t (°C)

Variables

$\mathbf{T}_t$	Temperature setpoint for time t (°C)
$\mathbf{Y}_t$	ARX predicted HVAC electricity use at time t (kWh)
$\mathbf{W}_t$	Auxiliary variable for deviation from desired temperature at time t (°C)
$\mathbf{U}_{t,j}$	Exogenous input data for time t, and input j (°C, °C, # of people)

$$(\mathcal{Z}_t) : \min \sum_{k \in \mathcal{K}} (r_{t+k} \cdot \mathbf{Y}_{t+k} + p^+ \cdot p_{t+k} \cdot \mathbf{W}_{t+k}) \tag{3.3}$$

subject to:

$$\mathbf{Y}_{t+k} \geq \sum_{i \in \mathcal{I}} [-a_i \cdot \mathbf{Y}_{t+k-i} + \sum_{j \in \mathcal{J}} b_{i,j} \cdot \mathbf{U}_{t+k-i+1,j}] \quad \forall k \in \mathcal{K} \tag{3.4}$$

$$\mathbf{W}_{t+k} \geq \mathbf{T}_{t+k} - T' \quad \forall k \in \mathcal{K} \tag{3.5}$$

$$Y^E_{t-i} \leq \mathbf{Y}_{t-i} \leq Y^E_{t-i} \quad \forall i \in \mathcal{I} \tag{3.6}$$

$$T^F_{t-i} \leq \mathbf{T}_{t-i} \leq T^F_{t-i} \quad \forall i \in \mathcal{I} \tag{3.7}$$

$$\underline{s}^{\,o} \leq \mathbf{T}_{t+k} \leq \bar{s}^{\,o} \quad \forall k \in \mathcal{K} : t+k \in \mathcal{T}^O \tag{3.8}$$

$$\underline{s}^{\,u} \leq \mathbf{T}_{t+k} \leq \bar{s}^{\,u} \quad \forall k \in \mathcal{K} : t+k \in \mathcal{T}^U \tag{3.9}$$

$$\mathbf{Y}_{t+k}, \mathbf{W}_{t+k} \geq 0 \quad \forall k \in \mathcal{K} \tag{3.10}$$

The objective function minimizes the cost of cooling electricity, as well as the setpoint deviation from a desired temperature, T'. Constraint (3.4) estimates the cooling electricity demand, where zone temperature setpoint and electricity are variables for timesteps greater than or equal to t, and parameters for previous timesteps, or timesteps less than t. Constraints (3.8) and (3.9) provide setpoint bounds while the building is occupied and unoccupied, respectively, to ensure thermal comfort of occupants. If the building is occupied, the temperature setpoint can range from 21°C to 26°C, with a penalty p^+ applied above 24°C, and can range from 18°C to 30°C with no penalty when unoccupied, as shown in Figure 3.6.

3.3 Application of Platform For Case Study

The case study presented explores a detached home in Sacramento, CA, which is shown in Figure 3.7. It is a $2,500 ft^2$ newer build with construction properties similar to a home built in 2009. Construction specifications follow the National Renewable Energy Lab House Simulation Protocols (HSP) based on the 2009 International Energy Conservation Code (IECC) and federal appliance standards [62]. The weather data is taken from a typical meteorological year (TMY) file for for the months of July and August, which encompass the warmest months in Sacramento and the highest electricity usage due to HVAC systems.

The TOU rate schedule included in the MPC objective function, Equation (3.3), is characterized by the length and height of the rate spike. The MPC is applied to the residential building models for nine time-of-use rate scenarios, which all have off-peak rates of

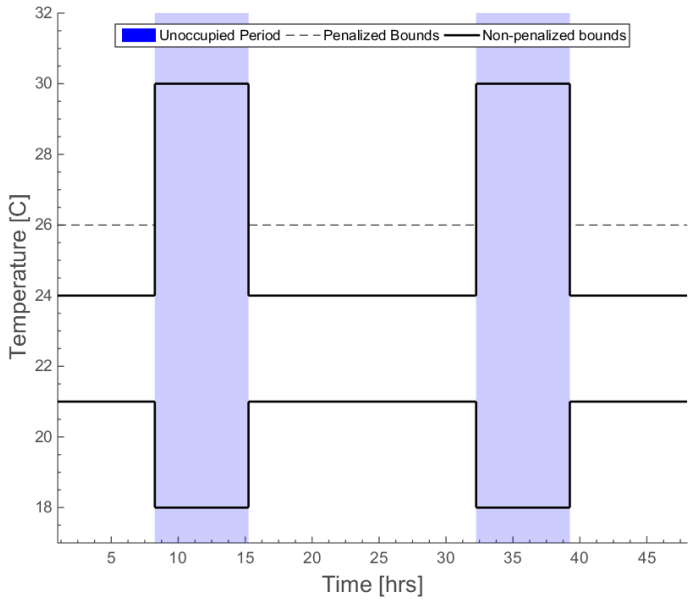

Figure 3.6: Occupant-dependent temperature setpoint bounds for the MPC

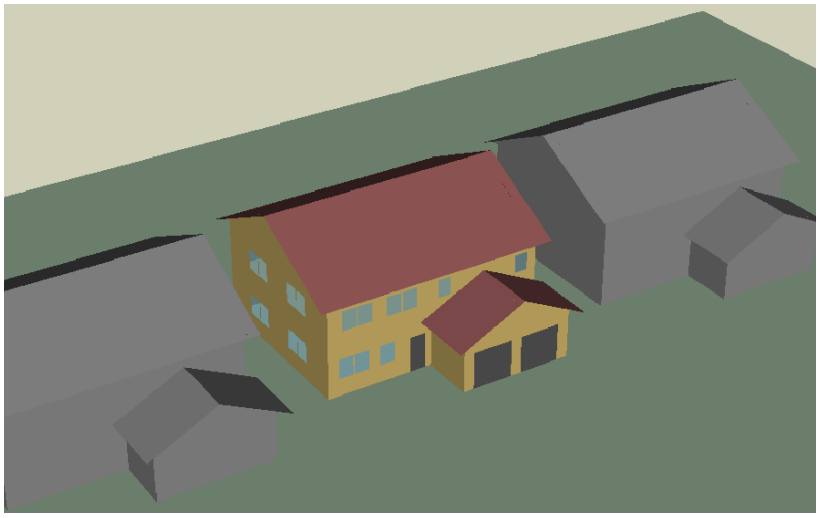

Figure 3.7: Single-zone new detached residential building model.

$0.0741/kWh, and varied on-peak rates. The on-peak rates for each TOU case is that of off-peak multiplied by a factor of 2, 3, or 5, which is maintained for 3, 5, or 8 hours; three of these cases are represented in Figure 3.8. For example, the (*3x, 5hr*) TOU schedule is $0.0741/kWh during off-peak, then shift to $0.2226/kWh for five hours, from 2:00 - 7:00, until reverting to the off-peak rate. The baseline comparison for all cases is a constant rate of $0.1171/kWh applied to the same building model with a setback controller. TOU rates are only applied to weekdays, while weekend days keep a constant rate equal to the off-peak rate.

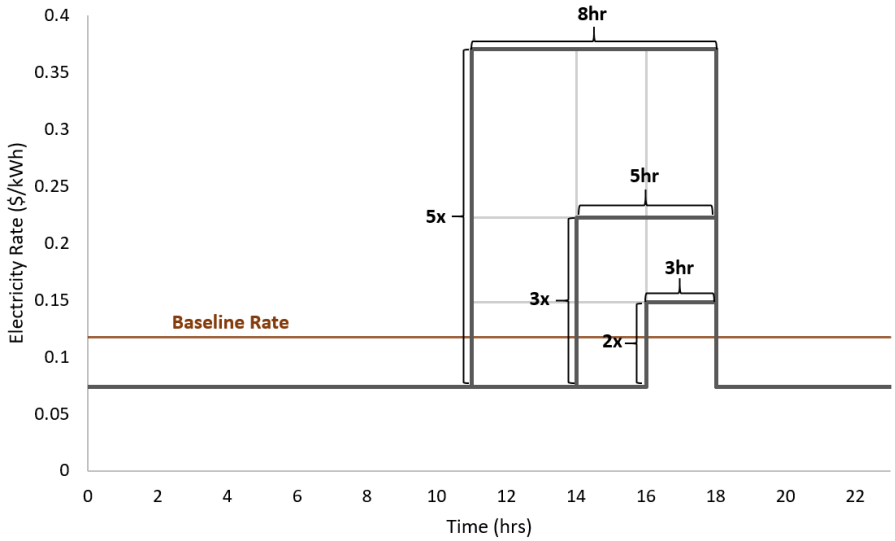

Figure 3.8: Three time-of-use rates used in the case study, and the baseline rate applied to a typical setpoint schedule

3.4 Preoptimizaton Analysis

In a model-predictive control strategy, a prediction horizon allows decisions to be made with consideration to future system impacts, and therefore, it is important that the MPC can predict sufficiently far into the future. This study performs a sensitivity analysis using the (*8-hr, 3x*) time-of-use rate to find an appropriate prediction horizon for all global timesteps. The MPC is solved to find optimal temperature setpoints using horizons of 2 hours to 24 hours. Figure 3.9 shows the sum of each objective function calculated using only the current

timestep across all $\mathcal{T}$. A 12-hour prediction horizon is chosen because the objective function shows no improvement beyond this point, and the black-box *prediction* method is capable of making useful predictions for this horizon, as discussed in Section 3.2.2.

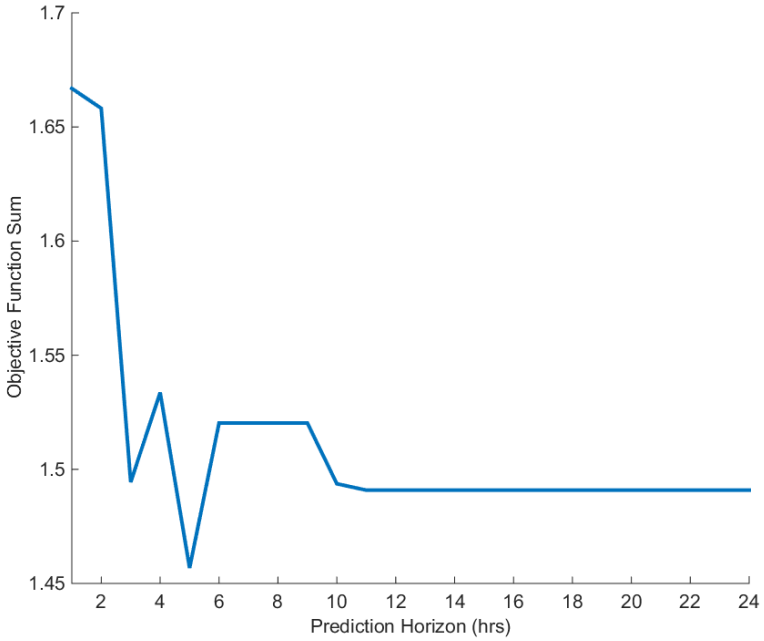

Figure 3.9: Sensitivity analysis using the sum of the objective function over five days and an increasing prediction horizon ($\mathcal{K}$).

3.5 Optimization Results

The MPC is simulated for nine TOU rate schedules over a period of two months in the summer. The results of these simulations are compared to a baseline case that uses a set-back schedule applied under a constant electricity rate of \$0.1171/kWh. Figure 3.10 shows a two-day snapshot of the optimal temperature set-point schedule, indoor air temperature, and imposed setpoint bounds for the (*8-hr, 5x*) case, as solved by the MPC. The setpoint schedule utilizes a pre-cooling strategy to store cooling energy in the home, then relaxes the temperature setpoint to reduce peak loads. This schedule also deviates above the desired temperature ($24°C$) at hours 15 and 39, which reduces user comfort but decreases electricity

use during peak-demand times. This same setpoint schedule compared to the baseline set-back schedule is shown in Figure 3.11 with the resulting electricity use shown in Figure 3.12. The electricity use is plotted with the time-of-use electricity rate, showing the effectiveness of the pre-cooling strategy in shifting the peak load. The two days shown in Figure 3.11 and Figure 3.12 are representative results of the entire two-month simulation period.

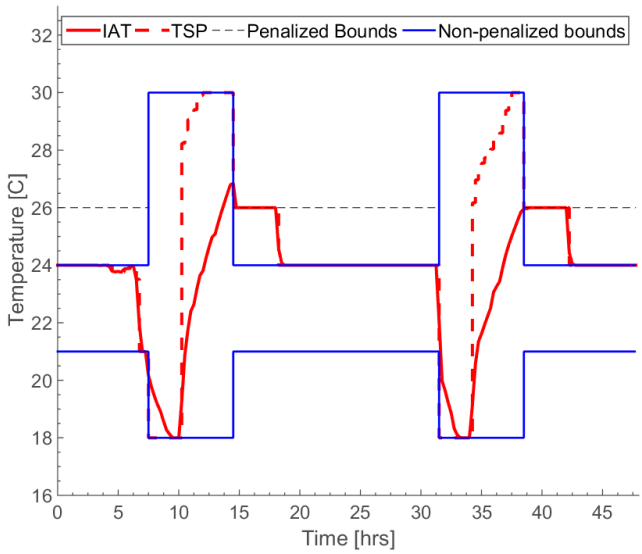

Figure 3.10: Optimal MPC setpoint schedule and the allowable setpoint bounds. IAT: indoor air temperature; TSP: temperature setpoint.

Figure 3.13 provides a view of how the MPC reacts when the TOU rate multiplier is changed. This figure displays the setpoint schedule and corresponding electricity use for the three multipliers of the *3-hr* case (*2x, 3x,* and *5x*). These plots show that as the TOU multiplier is increased (e.g., *3x* to *5x*), more pre-cooling is required to prevent peak electricity use, from zero hours for *2x* to approximately five hours for *5x*.

3.5.1 Simulation Trends

The MPC simulation spans two months, allowing for insights to the impact of TOU rates over an extended period within a cooling season. The trends shown in Figure 3.14 - Figure 3.19 compare each of the TOU cases for the full 62-day simulation. An objective

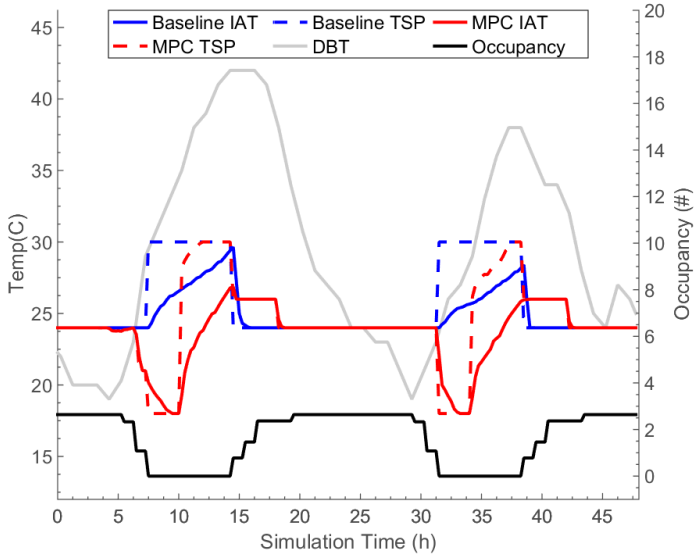

Figure 3.11: Temperature setpoint results from MPC compared to a baseline schedule for the (*8-hr, 5x*) case. IAT: indoor air temperature; TSP: temperature setpoint; DBT: dry-bulb temperature

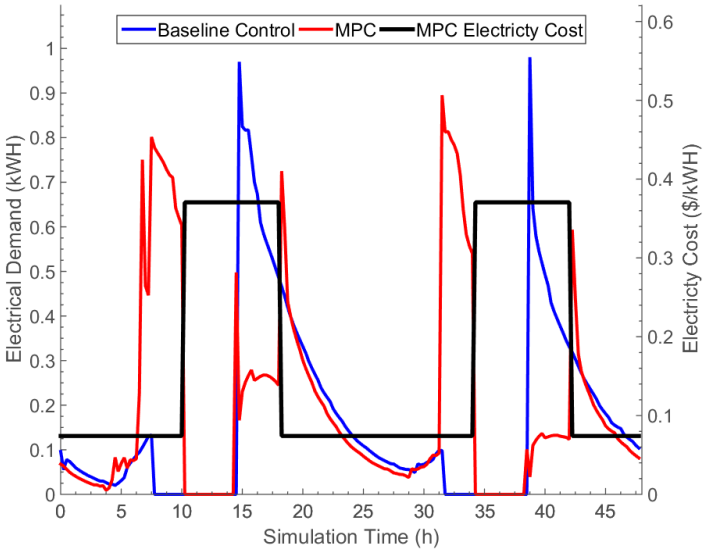

Figure 3.12: Resulting cooling electricity from the optimal MPC schedule and TOU rate compared to a baseline schedule for the (*8-hr, 5x*) case

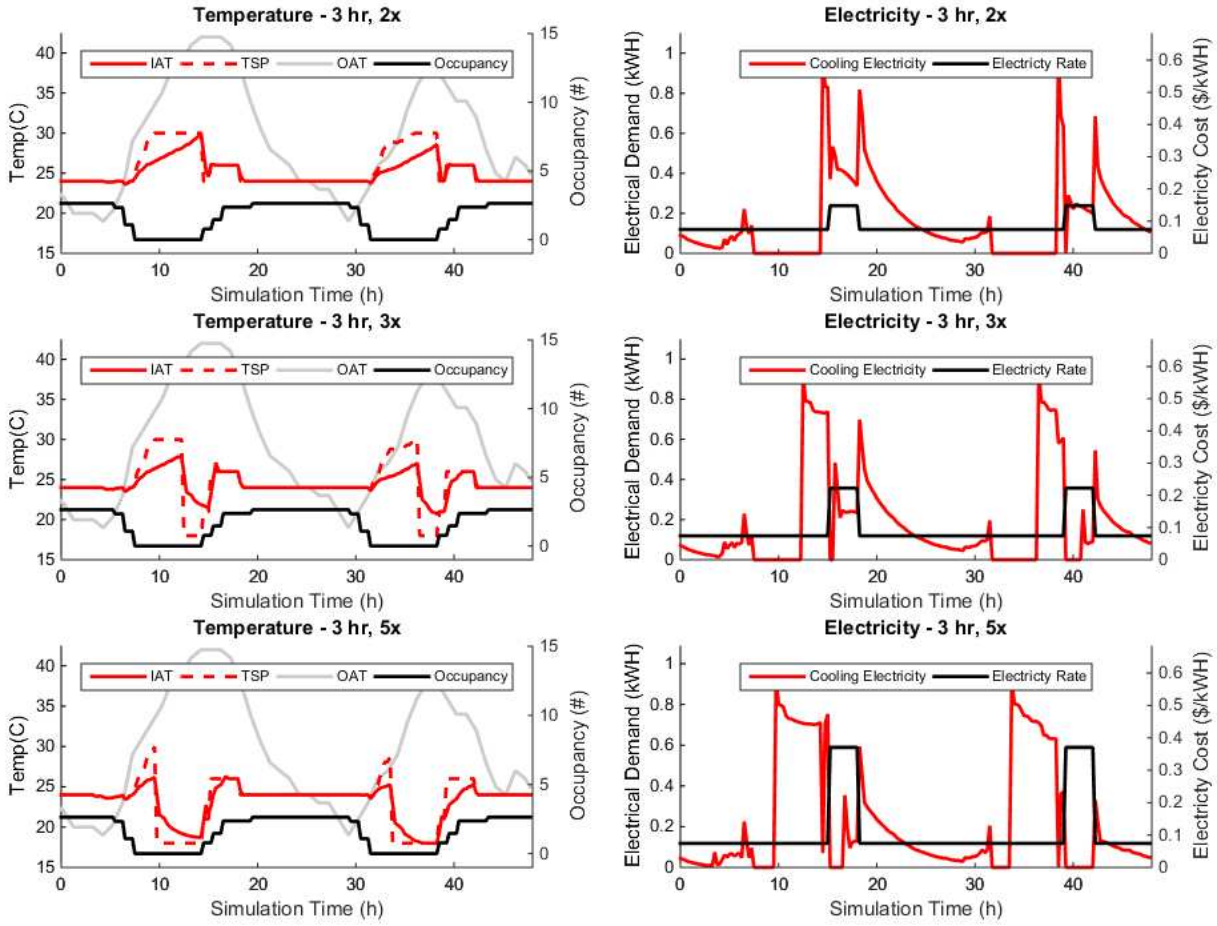

Figure 3.13: Optimal temperature setpoint schedules and corresponding electricity use for the 3-hour TOU cases. IAT: indoor air temperature; TSP: temperature setpoint; DBT: dry-bulb temperature.

function sum is calculated using Equation (3.3) for all $t \in \mathcal{T}$ after each full simulation. This objective function sum increases with more expensive and longer peak electricity rates, as shown in Figure 3.14. Likewise, consumer costs increase with higher peak electricity prices, displayed in Figure 3.15. Figure 3.16 and Figure 3.17 show the trends for the amount of pre-cooling and positive temperature deviation, respectively. Pre-cooling is defined as time when the zone temperature is set below the desired, while deviation is when the zone temperature is set above the desired. The units for both pre-cooling and temperature deviation are degree-hours, calculated as the the difference between the setpoint and the desired temperature, averaged over each hour and summed. The deviations above the desired setpoint coincide with the penalized region shown in Figure 3.6, while there is no penalty for pre-cooling below the desired. The TOU multiplier only minimally influences the total temperature deviation, as shown in Figure 3.17. The *3-hr* cases deviate the least, as the peak can be avoided with less of a temperature relaxation than the *5-hr* and *8-hr* cases. The negative trend seen between *3x* and *5x* in each of the lines does not exceed a 2% change, and is potentially due to error in the predictive ARX model.

Figure 3.18 shows the relationship between the rate schedules and the total cooling electricity. Pre-cooling strategies generally result in an increase in electricity use but a decrease in costs compared to a baseline, which is true for all nine TOU cases. The primary incentive to utilities who offer TOU rates, however, is the potential decrease in peak demand, but not necessarily a decrease in off-peak use. Figure 3.19 shows the total shifted peak electricity use changes for the rate structures. The peak demand period in Figure 3.19 is defined as the hours when the electricity rate is the highest. For each TOU length, the shifted peak electricity is greater when costs are highest, as there is a greater penalty for using electricity in this time. The case that shows the greatest potential for reducing peak demand with this MPC is the (*5-hr, 5x*) rate, which is shown in Figure 3.20.

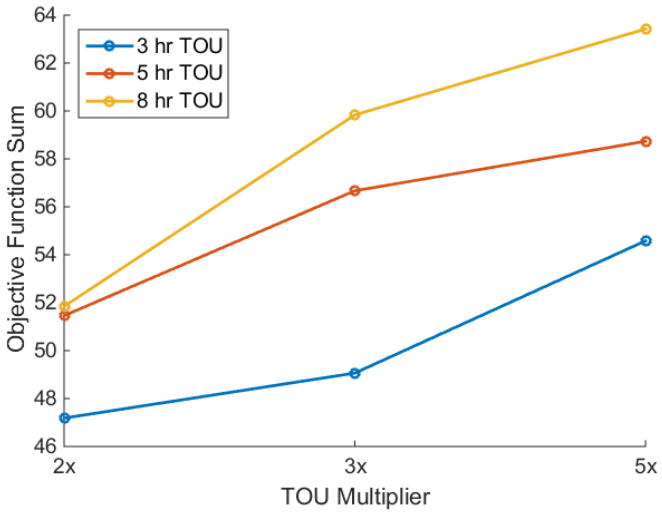

Figure 3.14: Objective function across the global time set for each TOU case

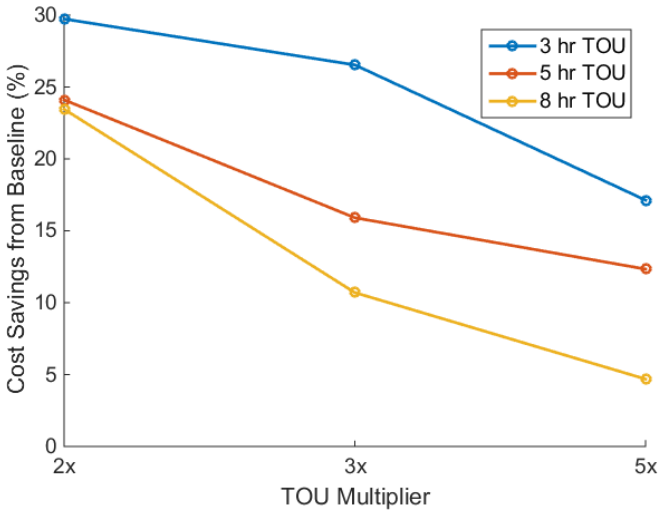

Figure 3.15: Electricity cost reduction as a percentage of the total baseline cost for each TOU case

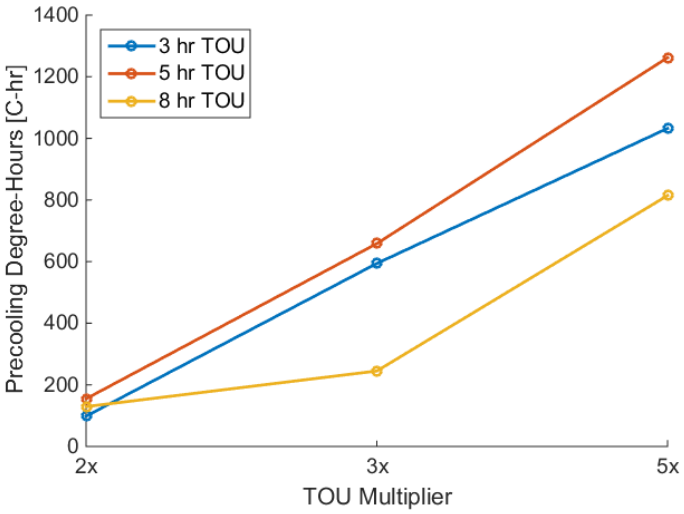

Figure 3.16: Sum of average hourly pre-cooling temperature for each TOU case

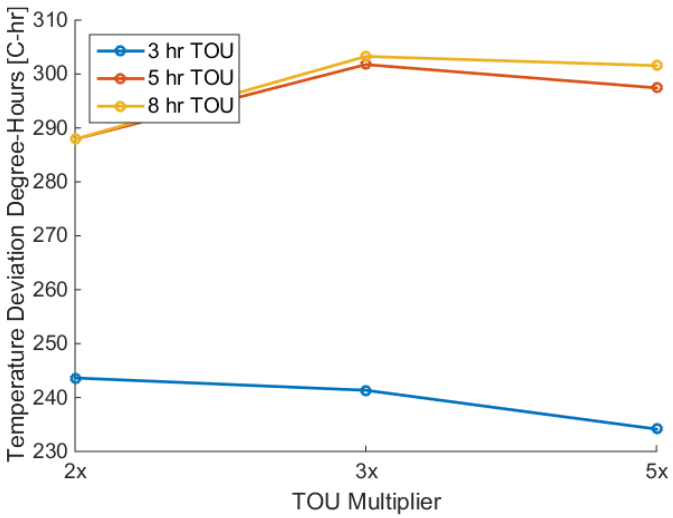

Figure 3.17: Sum of the average hourly positive deviation from the desired setpoint temperature for each TOU case

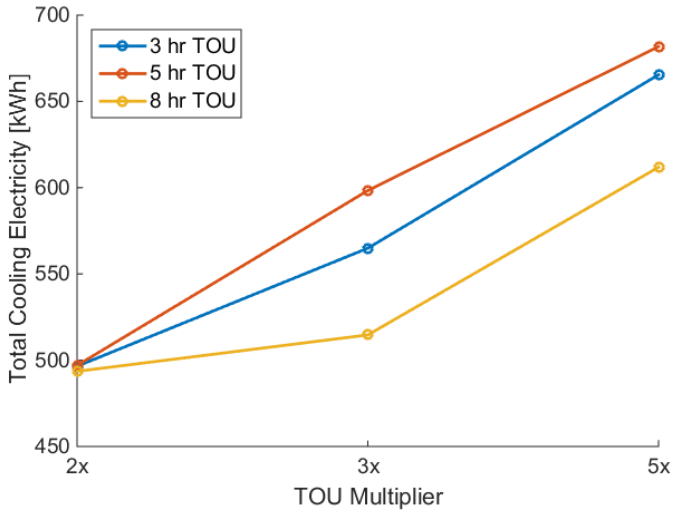

Figure 3.18: Total cooling electricity use for each TOU case

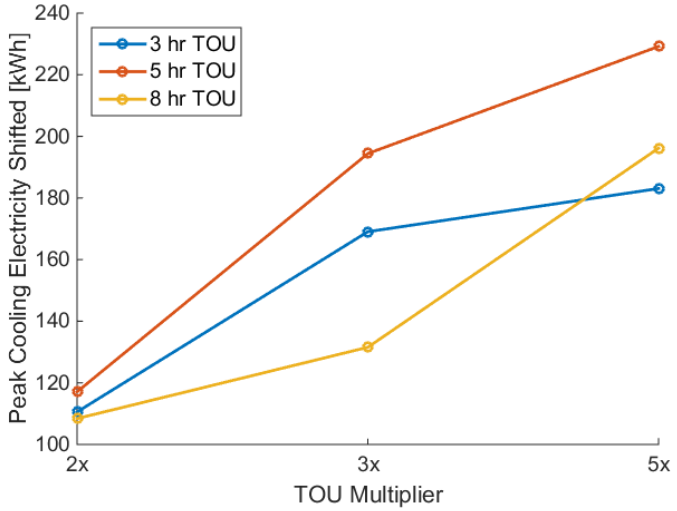

Figure 3.19: Sum of cooling electricity shifted from the peak demand times compared to a baseline setback schedule

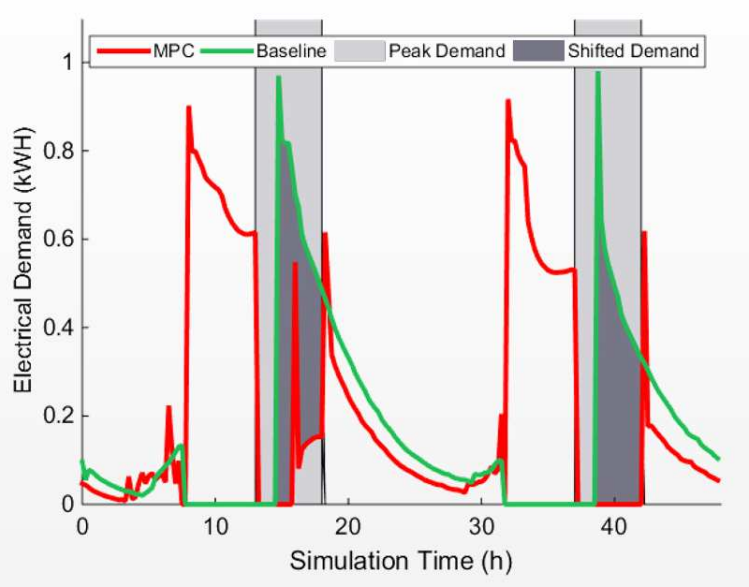

Figure 3.20: Electricity use for a (*5-hr, 5x*) MPC simulation and baseline, and shifted peak demand.

3.6 Discussion

The MPC detailed in this chapter is capable of providing significant cost savings for a residential home with variable time-of-use electricity rates compared to using a setback controller with a constant electricity rate. These cost savings are shown to be highly dependent on the TOU rate for which the MPC is operated. Of the cases presented in this study, cost savings range from approximately 5% to 30%. The (*3-hr, 2x*) TOU rate results in the highest percentage cost reduction from the baseline. However, a rate such as this is unlikely to be used when considering the motivations of an electricity provider. Due to the lack of pre-cooling, none of the *2x* rates shift much of the peak electricity demand in comparison to other cases, as seen in Figure 3.19. The load shifting is also highly dependent on the electric rate, with the (*5-hr, 5x*) case providing nearly twice as much reduction as the *2x* cases. Although the incentives are clear for consumer and provider, further market analysis and details regarding grid costs during peak demand would be needed to establish an ideal electricity rate.

3.7 Conclusions

This study documents the development of a model predictive control rapid prototyping platform for a single-zone building; the platform uses commercially available software for optimization (AMPL, CPLEX), and building energy simulations (EnergyPlus); the software is integrated within a commonly used coding language (Simulink). The prototyping platform is able to incorporate an optimization setpoint controller to determine optimal cost savings, subject to user comfort constraints. Nine TOU rates are considered, and in general, the optimal temperature setpoint follows a pre-cooling strategy to avoid electricity use during on-peak demand times. The MPC is able to maintain a balance between user comfort and reduced electricity costs for all cases, showcasing the potential for application to systems with diverse inputs. The MPC modeling platform can potentially simulate optimization of setpoint controls that include nonlinear, mixed integer, and linear programs. The ability to solve complex optimization models allows for the development of advanced controls for complex passive components, such as phase change materials (PCMs). To better understand the capability of the modeling platform, future work will include experimental tests using a thermal test chamber, as well as a multiple-zone building simulation.

CHAPTER 4

SUMMARY AND CONCLUSIONS

4.1 Summary of Efforts

This thesis describes a new methodology to analyze residential reduced-order models that predict cooling electricity and TOU rates in an MPC of temperature setpoints. Chapter 2 details the development of two reduced-order models, a grey-box and a black-box, to predict cooling electricity use in residential homes. The models were compared based on the accuracy of the predictions, but also with respect to computation time, development time, and potential for further applications. Chapter 3 described the application of the black-box model, or ARX, to a model-predictive control of temperature setpoints for a home subject to time-of-use rates. An MPC framework was used to find optimal setpoint schedules under varied rate structures for a two-month period. Two-tiered TOU rates were used that varied in length of peak pricing (from *3-hr* to *8-hr*) and the peak price (from *2x* to *5x*). The final analysis of the TOU rates included benefits to the consumer via cost savings, as well as benefits to the electric utilities through peak-demand shifting.

4.2 Key Findings

The grey-box RC model and the black-box ARX model are both capable of accurately predicting cooling electricity use of detached homes; however, the two are not equally suited for all applications. The ARX model is more simplistic and less computationally expensive than the grey-box, and therefore is more appropriate for application to an advanced controls optimization which can require many iterations of the predictive model. The ARX is only capable of accurate predictions if sufficient real-time data is available, as the formulation relies on previous and current inputs to predict future output. Furthermore, the linear ARX model has limitations in capturing some responses of the non-linear system. Specifically, the model tends to over-predict the response during a setpoint relaxation, when the electricity

use is zero. Ultimately, this limitation likely under-predicts the cost-savings potential of an MPC, because the controller unnecessarily increases the amount of pre-cooling to reduce predicted use during the relaxation. This analysis provides researchers and engineers insight into choosing a reduced-order predictive model for an MPC based on benefits and potential pitfalls.

The study of the model-predictive controller for a residential home estimates cost savings ranging from 5% - 30% compared to homes that use a setback controller and constant electric rate, and 110 kWh - 225 kWh of peak demand shedding over two months for a single home. These results indicate that (i) benefits to both homeowners and electricity providers are likely when a setpoint MPC is employed alongside TOU rates, and (ii) both cost savings and peak-load shifting are highly dependent on the length and cost of the on-peak rate. This can help inform electricity providers to determine appropriate rates, as well as customers to see where enrollment in a variable electric rate program might be financially beneficial.

4.3 Future Work

Expanding the work of the reduced-order models and the MPC can provide additional insights into residential building energy models and temperature setpoint controls. Both ROMs will be trained using data from an actual home, in addition to the two simulated homes presented. This will introduce variability in the inputs and electricity use data that is not necessarily present when using a white-box model. This could potentially expose shortcomings that accompany deterministic models such as the grey-box and ARX models. The existing detached home, discussed in Section 2.2.3, will be introduced in the MPC, further testing the model's capabilities. The same analysis of nine TOU rates will be performed, but give insight into how thermal mass might play a role, as the existing home is less massive.

4.4 Outlook

In closing, model-predictive controls in residential buildings demonstrate potential for decreasing peak-electricity use and saving consumers money when TOU rates are applied.

As variable electric rates, specifically time-of-use rates, become more prevalent for residential electricity customers, these advanced controllers are likely to be implemented in smart thermostats of homes. Furthermore, with the data that accompanies smart thermostats and smart meters, and other technology that anticipates occupancy behavior, similar reduced-order models can be identified and executed in real-time.

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