DETECTION AND PREDICTION OF CHAOTIC BEHAVIOR IN MINERAL COMMODITY MARKETS

by

Thomas F. Brady

A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Mineral Economics).

Golden, Colorado

Date 3-25-96

Signed: Thomas F. Brady
Thomas F. Brady

Approved: Wade E. Martin
Dr. Wade E. Martin
Thesis Advisor

Golden, Colorado

Date March 22, 1996

John E. Tilton
Dr. John E. Tilton
Professor and Head,
Division of Economics and Business

ABSTRACT

Despite the global importance of mineral commodity prices, economists and financial analysts have poor records in explaining, compensating for and predicting price behavior. Forecasting models of commodity prices have traditionally been directly or indirectly derived using autoregressive methods. One assumption inherent in the use of such models is that the process generating commodity price time series is stochastic or random.

This study examines the alternative that mineral commodity prices may be the result of nonlinear deterministic or chaotic processes. Chaos is determined behavior which gives the appearance of randomness. Modifications to traditional univariate time series model specification processes are presented to detect the class of systems which possess conditions necessary for chaotic behavior, including determinism, nonlinearity and sensitive dependence upon initial conditions. The process relies upon techniques derived in the physical sciences and culminates with the specification of non-traditional modeling techniques which incorporate the characteristics of chaotic behavior.

Chaos in mineral commodity processes calls into question heretofore accepted theories of the stochastic behavior of prices. Price movements may not be the result of an indeterminant number of influences, but rather from a specific deterministic prescription. As a result, commonly utilized capital market theories based on this assumption may give misleading results as measures of expected returns and asset risk are questionable. If the economic systems of mineral commodity prices are indeed found to be chaotic, the techniques presented in this study provide avenues for a more robust and theoretically correct description of these systems.

The modified model specification process described here is applied to zinc and copper price series. While the prediction results from non-traditional models appear to be very similar to those from a conventional second-order autoregressive model, chaotic behavior is detected within the zinc price series. Reasons for the lack of superior results with the new models may include the excessive noise inherent within any economic time series as well as the limited length of this series.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGMENTS

I would like to thank all of the individuals who made it possible for me to complete this dissertation. First, I would like to recognize my advisor Wade Martin for his generous assistance in shaping, organizing and guiding this study. Thanks to my committee members who lent direction in their particular areas of expertise: Michael Walls, Carol Dahl, Erik VanVleck and Bill Astle. In particular, I appreciate the humor and jocularity provided by Michael Walls who helped to keep this project in the appropriate perspective. I also would like to thank the Division of Economics and Business and Graduate Office for the financial support received over the years.

Finally, I wish to thank my family and friends who truly made it possible to complete this degree program.

To my grandfather Bernard "Beef" Moroney who always stressed the importance of education and hard work.

Chapter 1
INTRODUCTION

Most economists agree that the ultimate goal of macroeconomics and microeconomics is to uncover truths about economic relationships. To uncover these truths, economists have utilized a myriad of methods. Many look for empirical verification as the evidence which must support a given theory. Others assert that economic truths can be discovered only through logical deduction (Holcombe, 1990). Common throughout these types of economic analyses is a reliance upon modeling. When used correctly, economic models simplify complex systems into manageable components. By understanding a model, an economist gains insight into the behavior of actual systems in the existing world.

The use of modeling has been extensive within the analysis of economic time series. This is understandable as the systems of concern are invariably complex with many influencing factors and interactions. For example, a given economic system, such as that governing the price history of a mineral commodity, may be affected by factors such as trading volume, prices of substitutes, resource availability, political and weather conditions, etc. It is not uncommon for an economic time series to be affected by so many agents that one cannot possibly hope to completely identify them let alone understand the forces that describe the interactions among these agents.

A traditional tactic around this often insurmountable task of identifying each influencing agent, has been the assumption by economists that changes in a given economic system are exogenous and occur in a random fashion; in other words, that the system is stochastic. This assumption has profound consequences to the economist in terms of the type of models to be utilized. The modeling of a stochastic economic system often entails inferring that uncertainty arrives through an additive error term which has mean zero and constant variance. This, in turn, allows for model parameters to be estimated through traditional techniques such as ordinary least squares. That these error terms are uncorrelated over time, allows for the use of linear models and/or those nonlinear models which are easily transformed to linear processes. Linear models are intuitively appealing in that traditional economic theory is upheld as these models yield only one solution or equilibrium position. Linear models, moreover, are generally relatively simple and can be solved explicitly rather than by using complicated iterative numerical procedures (Creedy and Martin, 1994).

In stochastic processes, each observation is assumed to be a random variable. In an economic series, for example, each equilibrium state is considered random as are any changes to an equilibrium state. Such a determination allows for the use of models such as Random Walks, Autoregressive (AR), Moving Average (MA), or Autoregressive Moving Average (ARMA). These models are tractable and intuitively appealing due to their relative simplicity and wide applicability. Forecasts using these models are obtainable, as model parameters may be obtained through straightforward techniques.

Over the past ten years economists have begun to question the validity of assuming that the processes which generate certain economic time series are truly stochastic. Certain classes of nonstochastic or deterministic processes may generate behavior which appears as randomness. The distinction being that the observed, apparent random behavior is not due to an indeterminant number of exogenous causes as with stochastic systems but the result of a specific deterministic equation or system of equations. A deterministic system is one in which certain results can be fully known in advance (Peters, 1991). With a deterministic system, the possibility exists for predicting the future with a deterministic model.

Of particular interest are nonlinear systems which evolve in a deterministic manner. Nonlinear deterministic systems are often referred to as chaotic systems. These systems may appear random in that they can exhibit monotonicity and even-order cycles but also cycles of odd-order (non-periodic) (Creedy and Martin, 1994). As interest in chaos is deep, crossing a vast magnitude of disciplines, much confusion exists in terms of its definition. Thus, this study will utilize the definition that chaotic systems are the class of deterministic processes that mimic random or stochastic dynamics (Brown, 1995). More detailed explanations of chaotic behavior and its implications are discussed in chapter 2 of this study.

During this century, the most sweeping revolution in the world view of science may be associated with chaotic dynamics. Part of the reason for this broad application is that chaotic dynamics is not something which is part of a specific physical model limited to one small area of science, but rather, a consequence of mathematics itself and hence appears in a broad range of physical systems (Rasband, 1990). Although the mathematical representations of these systems can be very different, they often share common properties. For this reason, it is possible to utilize the characterization techniques of nonlinear and chaotic systems originally derived for physical systems, but applied to the economic systems of mineral commodity spot price series.

This study explores the alternative that the oft-observed, random appearing fluctuations in mineral commodity price series may not be due to stochastic behavior of the underlying system but to low-dimensional, deterministic chaos.[1] Ultimately, predictive models of these series must account for this distinction as in the presence of mis-specification, any prediction will be questionable. This distinction is important because the prediction of the future behavior of a system is only possible to the extent that the system

[1] Low dimensional infers that the random appearing behavior may be due to a low (finite) number of contributing factors.

is deterministic, meaning that the information from the systems past unambiguously determines its future states.

Specifically, this study examines alternatives to traditional, temporally dependent time series modeling techniques through examination of models which incorporate spatial dependence. A modification to traditional univariate time series model specification processes is presented to detect the class of systems which possess conditions necessary for chaotic behavior. This process culminates with the specification of non-traditional models which incorporate characteristics of chaotic behavior. Prior to introduction of this modified process, a review of past modeling methods and economic paradigms is addressed.

Chapter 2 provides a descriptive, historical analysis of the paradigms through which economic models have flowed from static analysis up to the present need for nonlinear modeling. Chapter 3 presents a modified time series model specification process which includes methods to provide for the distinction between stochastic and deterministic processes. The methods are relatively new and more complex than traditional tests and models.[2] The detection techniques as well as resulting prediction methods are completely described in this chapter. Chapter 4 will present empirical examples of mineral commodity price time series applied to the modified specification process. Conclusions and suggested areas for further research are addressed in Chapter 5.

---

[2] Many nonlinear dynamic analytical techniques were derived in the early 1980s due to the findings of Packard et al. (1979) which were formalized by Takens (1981). The 1980s also saw the advent of powerful enough computers to complete these analytic processes.

Chapter 2
MODELING IN ECONOMICS

An understanding of why nonlinear and chaotic dynamics has come to the forefront of economic system modeling requires a review of past paradigms from which such modeling has flowed. For example, the prevalent use of linear modeling methods throughout this century is closely tied to past economic paradigms. Both nonlinear and chaotic dynamical modeling, as with past analytical techniques which break from traditional, more established methods, are not without criticism. However, these modeling methods attempt to satisfy a void which previous modeling techniques have failed to meet. Specifically, the failure of stochastically based models to achieve accurate prediction results.

A review of history reveals that following the occurrence of an unexpected event of significant magnitude, theorists invariably question the validity of existing models to describe real world system behaviors. As stated by Kuhn (1962):

> Because it demands large-scale paradigm destruction and major shifts in the problems and techniques of normal science, the emergence of new theories is generally proceeded by a period of pronounced professional insecurity. As one might expect, that insecurity is generated by the persistent failure of the puzzles of normal science to come out as they should. Failure of existing rules is the prelude to a search for new ones.

Shifting of paradigms has repeatedly occurred throughout economic history. Within each shift, however, a common theme has arisen; the assumption of linearity has proven crucial due to both the analytical ease of linear models and the lack of resources and knowledge necessary to solve nonlinear models.

Static Analysis

During this century, static analysis has evolved to become the fundamental method in both macroeconomic and microeconomic analyses. Static analysis deals with the question of what the equilibrium position will be under certain given conditions of a model. For example, given an output price, a static model would generate an equilibrium output quantity level. The equilibrium positions, A and B in Figure 2.1, are examples of static equilibrium points.

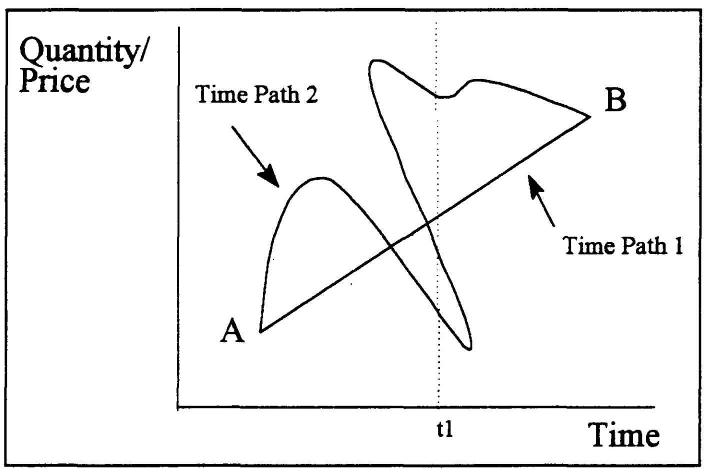


Figure 2.1 Economic System with Two Equilibrium Points A and B

Leontief's (1951) input-output models, which depicted the U.S. economy in 1919 to be made up of 46 industrial sectors, was one such use of static analysis. This model deals with the particular question, "What level should each of the 46 sectors produce in order that production will just be sufficient to satisfy total demand ?" (Chiang, 1984). For simplification, production processes were presented in linear forms to allow for a more direct conversion of such processes into empirical studies. This type of model quickly became widely used as this industry analysis had wide utilization in the World War II war effort in the United States and was later expanded by the U.S. Bureau of Labor Statistics to show the sources, amounts and destination of key materials (Oser, 1953).

There are two particular problems with static analyses. The first is that static analysis does not consider the adjustment process necessary to achieve an equilibrium position. As a result, if the adjustment process takes a long time, then the equilibrium position attained may have lost all empirical relevance. A second problem is that of attainability. The static method fails to address whether the achieved equilibrium state is attainable for given input criteria. In other words, static models may produce unstable equilibrium points. To disregard the adjustment process, economists have traditionally assumed away the problem of attainability of equilibrium (Chiang, 1984).

Comparative Statics

Comparative statics can be viewed as an extension of simple static analysis in that comparative statics is concerned with the effect that one or more exogenous variables has on the equilibrium value of the system. In the past, comparative statics has proven useful as often the goal of an analysis is to determine how a disequilibrating change in a parameter will affect an equilibrium state of the system. Within economic applications, these exogenous or shift parameters can be either changes in income, demand, supply, technology, consumer tastes and/or preferences, etc. Comparative statics does not examine these changes over time, but rather examines two different possible situations which hypothetically could occur (Holcombe, 1990). In short, analyses based upon comparative statics are devoid of time.

Figure 2.1 depicts an economic system with two equilibrium points A and B. A comparative static analysis would only be concerned with the two equilibrium positions (A

& B). The time paths between these two positions, such as time paths 1 and 2, would be of no concern. By its nature comparative statics ignores the process of adjustment from the old equilibrium (pt. A) to the new (pt. B). Furthermore, comparative statics neglects the time element involved in the adjustment process. When the time path of an economic system is considered, dynamic analysis must be utilized. If the time path of concern is nonlinear (time path 2) then nonlinear dynamic analysis or some type of linear approximation must be employed.

Comparative statics is useful because it allows economists to make predictions about the long-run direction and magnitude of a change in economic variables. Such analysis, however, lacks the ability to describe how variables react to exogenous changes. Further, it has become increasingly apparent that comparative static analysis can provide misleading views of the behavior of an economic system. Fully capturing the behavior of a system requires the use of dynamic methods of analysis (Levacic, 1976).

Comparative Static Analysis Tools

The reliance on comparative statics as the method of choice under past economic paradigms has resulted in tools which lend themselves to this type of analysis becoming popular. It has been customary in the construction of both empirical and theoretical models for economists to adopt linear specifications or those nonlinear specifications which can be easily linearized following a transformation process. This preference for linear models is based upon both intuitive and practical reasons. Within the context of comparative statics, linear models have been effective because they result in single solutions or solitary equilibria. As a result, different equilibria can be compared. In addition, linear models produce equilibrium conditions which have the property of stability. The emphasis on linear models has also been one of practical convenience. Linear models also have the desirable features of being easily understood in great detail and are straightforward to implement. This has been necessary because the computing technologies needed to solve difficult mathematical expressions have only been available to economic analysts for the last ten to twenty years (Gershenfeld and Weigned, 1994).

As economics has been termed, by and large, a science of choice, one estimation tool which has proved to be extremely useful within the realm of comparative static analysis is linear programming. For example, when an economic project such as the production of a specified level of output is to be completed, there are generally a number of ways to accomplish this task. It is the essence of linear programming to choose on the basis of the specified criteria, the best method or alternative. Consequently, linear programming occupies a solid niche in economics as under the neoclassical school of thought, both firms and consumers are portrayed as striving to optimize subject to certain constraints, such as limited income or restricted productive capacity (Vandermeulen, 1971).

The origin of linear programming occurred in about 1758 when economists first began to describe economic systems in mathematical terms. Francois Quesnay's *Tableau E'conomique* is an early example of a linear programming model attempting to interrelate the roles of the landlord, the peasant and the artisan (Pressman, 1994). In 1874, Leon Walras proposed a sophisticated mathematical model which included fixed technology constraints within its structure. Until the 1930s, however, there was little in the way of exploitation of the linear type of model. In 1937, John Von Neumman formulated a dynamic linear programming model which introduced alternative methods of producing given commodities singularly or jointly (Dantzig, 1963). While this model did not contain any explicit objective, it did provide a solution which occurred when certain market forces were maximized.

The development of the general linear programming model is completely independent of the early work of Von Neumann and others. This model arose due to the empirical programming needs of the United States Air Force (USAF) to formulate its operations during World War II (Vandermeulen, 1971). Derivation of the general linear programming model relied upon the Leontief Input-Output model, a quantitative model of the United States macroeconomy, initiated in 1936 (Leontief, 1966). This input-output model of the economy provided both the stimulus and guide to the solution of similar problems of allocation within the USAF (Vandermeulen, 1971). Essentially more of a descriptive and predictive method, the Leontief model did not contain a criterion for optimization. In 1948, George Dantzig developed the simplex algorithm, a computational method devised for finding an optimal solution.

Static model analysis requires the use of relatively simple differential calculus to obtain the necessary first and second order conditions needed to describe the minima or maxima of a given system. One weakness of this approach is that it is essentially myopic in nature in that derivatives or differentials can usually locate local extrema without difficulty; however, finding a global or absolute extrema usually requires additional information and/or further investigation. Furthermore, complications arise and complexity increases when the dynamic behavior of a system is considered. To describe the behavior of economic variables within dynamic models requires the use of difference or differential equations.

Dynamic Analysis

Samuelson (1939) states, "The stationary state is not statical . . . [rather] a solution of a dynamical process." Dynamics refers to the analysis in which the objective is to study the time paths of a system. Broadly defined, dynamics is the systematic study of change and the forces that generate change. Dynamics attempts to determine how things change, why changes occur and what kinds of change might occur in the near or distant future. One feature of dynamical analysis is the dating of system variables which inevitably introduces time into the picture. Within economics there are numerous areas which lend themselves to dynamical study. Day (1994) lists some of the following examples. Economic dynamics is the systematic study of changes in production, consumption, trade, resource allocation, prices and welfare. Market dynamics is concerned with the behavior of supply and demand when prices and/or quantities adjust to evolving conditions. Business cycle studies are concerned with fluctuations in macroeconomic data. Growth theory examines long run economic expansions whereas development theories are concerned with the evolution of economic structures, including changes in technology and in forms of economic organization.

Attempting to describe the dynamical behavior of economic systems is not new to economists and is rooted in attempts to model macroeconomic behavior. To describe the dynamic behavior of a system, economists can employ either quantitative or qualitative techniques. Quantitative techniques involve the use of differential equations, difference equations and/or Bernoulli equations. Differential equations are equations involving derivatives or differentials. These equations express rates of change of continuous functions over time. The objective in working with differentials is to find a function, without derivatives or differentials which satisfies the differential equation. Such a function is called the solution of the differential equation.

Quantitatively, a dynamical economic system may be defined as a deterministic mathematical prescription for evolving the state of a system forward in time. An example of a dynamical system in which time (t) is a continuous variable, is the following system of N first order, autonomous, ordinary differential equations:

$$dx^{(1)}/dt = F_1(x^{(1)}, x^{(2)}, \ldots, X^{(n)})$$
$$dx^{(2)}/dt = F_2(x^{(1)}, x^{(2)}, \ldots, X^{(n)})$$
$$\vdots$$
$$dx^{(n)}/dt = F_n(x^{(1)}, x^{(2)}, \ldots, X^{(n)})$$

which can be written in vector form: $dx/dt = \dot{x} = \mathbf{F}[\mathbf{x}(t)]$. This is a dynamical system because for any initial state of the system x(0), the above equations can be solved to obtain a future system state, x(t), for t>0 (Ott, 1993). With simple dynamical systems, the differential equations may be solved quantitatively. That is, in every case, a time path for $\dot{x}$, for which each value of t, tells the corresponding value of the system. The dynamic behavior of these types of systems which can be quantitatively explained by exact differential equations, separable-variable differential equations or Bernoulli differential equations.

At times, more often than not, it may be difficult or impossible to find a quantitative solution for a given differential equation or system of differential equations. All is not lost , however, as it may be possible to determine the qualitative properties of the time path or trajectory. One method to assist in this qualitative process, involves the use of phase portraits or phase diagrams, originally utilized in dynamical analysis of physical systems. Figure 2.2, is an example of a phase diagram and displays the path in phase space followed by a 3-dimensional system as it evolves in time. The specific path (the dotted line) may be referred to as an orbit or trajectory of the system. To further distinguish dynamical systems the distinction between simple and complex dynamical systems is needed.

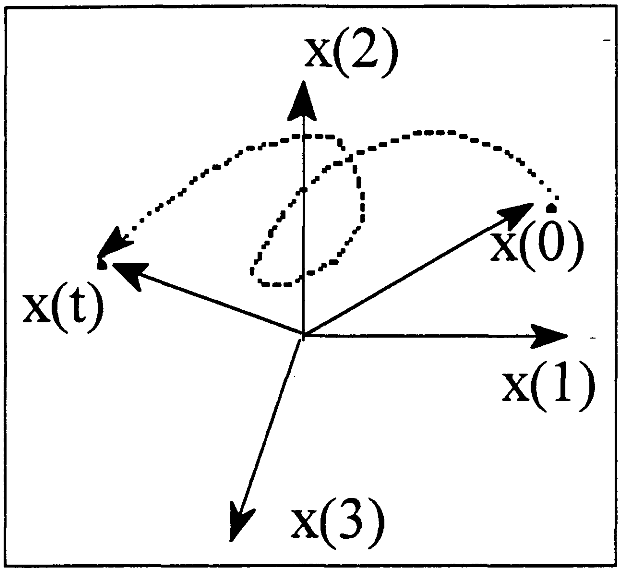


Figure 2.2 Example Phase Diagram

Simple Dynamical Systems

With simple dynamical systems, the differential equations utilized to describe system behavior over time may be solved quantitatively. That is, in every case a time path, $\dot{x}$, for which each value of t (time), tells the corresponding value of the system. In the realm of simple dynamical system, distinct patterns of change can be identified including stationary states in which a solution or position will repeat endlessly; and periodic cycles in which all aspects of a system change by the same proportion (Day, 1994). Note that a steady state can also be thought of as a cycle of period one. These are examples of simple dynamical systems because these types of patterns can be characterized in a finite or simple manner. In addition, the equations used to describe simple behavior are linear and thus can be quantitatively solved.

The roots of economists' interest in dynamics can be found in the non-mathematical literature on attempts to describe fluctuations in business cycles. Again, within economics as within the physical sciences, there has in the past been a dependence upon linear methods in dynamical analyses. The reasons for this dependence are twofold. First, prior to the 1980s, there was a lack of computer resources powerful enough to analyze nonlinear processes. Second, as previously stated, linear models give rise to single equilibrium points and thus correspond to past economic paradigms. Typically, past attempts to describe business cycle dynamics have included the determination of a large number of equations, each of which were designed to provide a set of conditions sufficient to generate oscillatory behavior in the economy. These models, however, were usually vague and difficult to verify (Baumol and Benhabib, 1989). Samuelson (1939) utilized difference equations, differential equations and mixed models to generate deterministic time paths. Samuelson demonstrated that a wide range of values chosen for parameters of such models will yield oscillatory time paths. For example, Samuelson's multiple-accelerator model is composed of the following three equations:

$$Y_t = C_t + I_t \tag{2.1}$$

$$C_t = cY_{t-1} + k \tag{2.2}$$

$$I_t = b(Y_{t-1} - Y_{t-2}) \tag{2.3}$$

where Y is national income or output, C is consumption, c is the marginal propensity to consume, I is investment and k can be seen as an error or uncertainty term. Substituting (2.2) and (2.3) into (2.1) yields

$$Y_t = (c + b)Y_{t-1} - bY_{t-2} + k \tag{2.4}$$

which is Samuelson's second-order linear difference equation. The Samuelson model proved that for wide ranges of c and b, this equation was able to generate broad oscillations.

Initially received with immense enthusiasm, interest in these models quickly dissipated (Baumol and Benhabib, 1989). One reason was that, qualitatively, linear models of this nature are only capable of generating four types of time paths (a) oscillatory and stable (that is, convergence with oscillations of decreasing amplitude toward some fixed equilibrium value); (b) oscillatory and explosive (cycles of ever-increasing amplitude); (c) nonoscillatory and stable; and (d) nonoscillatory and explosive (Baumol and Benhabib, 1989). These trajectories or time paths are displayed in Figure 2.3. It was soon recognized that linear equations even more complex (of higher order) than Samuelson's would not generate any time paths basically different from those displayed in Figure 2.3 (Baumol and Benhabib, 1989). Correspondingly, it was determined that this

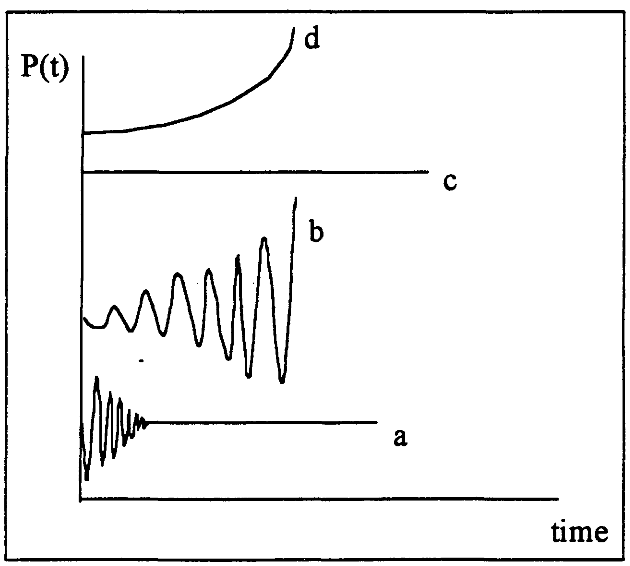


Figure 2.3 Time Paths Generated by Linear Dynamic Systems

range of possible time path configurations was simply not rich enough for economists' needs to more accurately reflect reality. Time paths are much more complicated and many oscillations do not simply explode or dampen toward disappearance.

Complex Dynamical Systems

There are, however, dynamical systems which do not converge to a steady state nor to a periodic cycle. As a result, the behavior of these systems cannot be quantitatively determined. Figure 2.4 displays daily aluminum spot prices on the London Metal Exchange (LME) during 1993. This dynamical system may well be complex as it does not appear to converge to any sort of periodic behavior. Furthermore, complex systems can involve nonperiodic fluctuations, overlapping waves and/or switches in regime or structural change (Day, 1994). Even though the variables which drive complex systems fluctuate in a nonperiodic manner, as may be the case with mineral commodity spot prices, it is still desirable to find models which explain their behavior. This, in turn, has given rise to economists' interest in nonlinear dynamics and consequently chaotic dynamics.

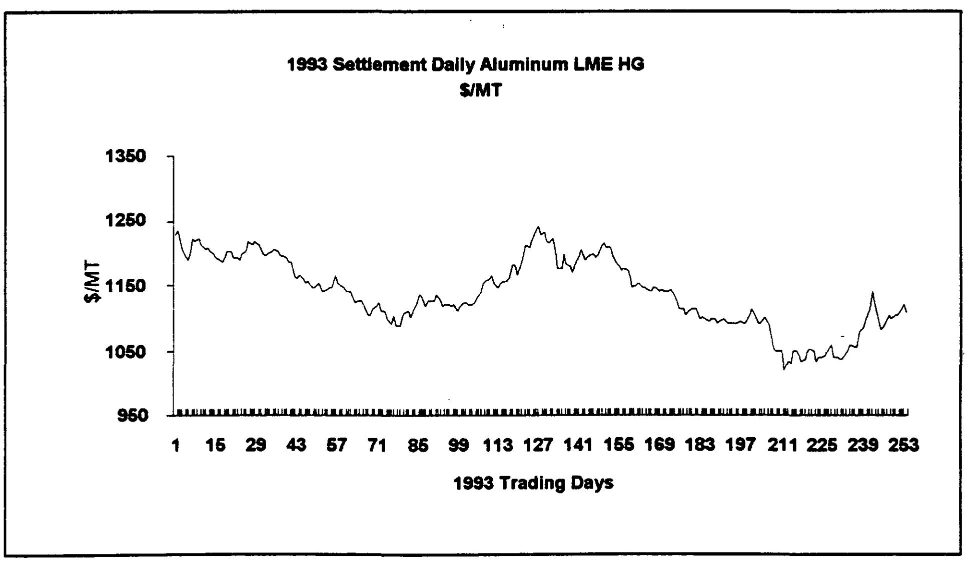


Figure 2.4 Daily Spot Prices for LME High Grade Aluminum

Chaotic Dynamics

A subset on nonlinear dynamic systems are systems which exhibit chaotic dynamic behavior. Chaotic dynamics arise when certain conditions are satisfied. These conditions include: (1) sensitive dependence upon initial conditions (SDIC); (2) existence of fractal dimensions; and (3) existence of critical levels. Each of these will be further explained later in the text.

The very word "chaos" implies some observation of a system, perhaps through some measurement, and that these observations vary unpredictably (Rasband, 1990). However, the distinction must be made that chaos does not imply randomness or random events such as flipping a coin. Chaotic dynamics refers to the deterministic development of a chaotic outcome. In other words, from moment to moment, the system of concern is evolving in a deterministic way, i.e., the current state of a system depends on the previous state of the system in a rigidly determined way (Rasband, 1990). There is some type of nonlinear dependence inherent in the system. A chaotic system differs from a random system in that with the latter, the current system state is independent, i.e., a causal relationship with previous states does not exist.

Chaos History

The philosophical implications of chaotic dynamics were first detailed by Henri Poincare in the late nineteenth century. In his work, *Science and Method*, Poincare states:

> A very slight cause, which escapes us, determines a considerable effect which we cannot help seeing, and then we say that this effect is due to chance. If we could know exactly the laws of nature and the situation of the universe at the initial instant, we should be able to predict exactly the situation of this same universe at a subsequent instant. . . . [It] may happen that slight differences in the initial conditions produce very great differences in the final phenomena; a slight error in the former would make an enormous error in the latter. Prediction becomes impossible.

This effect is presently referred to as sensitivity upon initial conditions (SDIC) and is an important characteristic of chaotic dynamic systems. Following this work by Poincare, interest into what is now referred to as chaotic systems was largely forgotten but reappeared in connection with meteorology with Lorenz's work in 1963 and in hydrodynamic turbulence by Takens in 1981. In the early 1980s chaotic dynamics intruded into physics, not just as a philosophically important idea but as something which could be empirically tested. For example, it was shown through detailed experiments that chaos is present in hydrodynamic turbulence (Ruelle and Eckmann, 1985).

Chaotic behavior has profound implications in economics both to analysts and to policy makers. Chaos warns that apparent random behavior may, in reality, be truly deterministic. Furthermore, it warns of the dangers of extrapolation and the difficulties that can beset economic forecasts (Baumol and Benhabib, 1989). However, chaotic behavior may provide for a deeper understanding of systems and the persistence of fluctuations. Traditional paradigms such as the efficient market hypothesis may be proved invalid.

Chaos Implications

Chaotic dynamic systems are capable of generating a rich variety of time series patterns, which may be so complex as to appear random. If such a nonlinear structure is shown to exist, the implications are far reaching for modeling of economic time series. These implications include the following:

A) Chaos generates uncertainty as to the validity of the "random walk" hypothesis. This hypothesis states that historical levels in an economic series will not influence future levels; i.e., economic levels are independent over time. Models which are based on the stochastic assumption of price changes are thus questionable.

B) If the true system exhibits nonlinear dependence, then treating the resulting time series as if it were generated by a stochastic process, as do AR, ARCH and ARMA models, then these models may lead to the exaggerated conclusion of the amount of randomness affecting the system.

C) Nonlinearity implies that long-term forecasting is very difficult, if not impossible. Short-term forecasts, however, may be feasible.

D) Chaotic systems develop in a deterministic manner yet produce results which give the appearance of random behavior. If this is the case then academic concepts such as the *efficient market hypothesis* are inappropriate in describing metal market pricing behavior. Metal markets may have some degree of inefficiencies. Consequently, this could lead to the theoretical acceptance of technical investment techniques such as trend analysis, value investing, tactical asset allocation and market timing (Peters, 1991a).

If time series are proven to be chaotic, it must be accepted that the idea of making precise, detailed quantitative predictions about the future state(s) of a system may have to be abandoned, but not the goal of predictability altogether. In fact, knowing that a chaotic model applies to some mineral commodity price series would enable precise qualitative predictions (relative to the values of the state variables) about large scale properties (attractors) to be made, particularly concerning under what conditions behavior would change in form (move between attractors, become chaotic, etc.) and when the precise qualitative predictability of behaviors must give way to probabalistic estimates. Attractors will be discussed later in the chapter.

In working with chaotic models, it must be accepted that the understanding obtained is of a different character than that obtainable with traditional stochastic models. However, this information is no less meaningful. This different type of understanding may require giving up some of the traditional goals, such as some aspects of determinism, in favor of a different conception that Kellert (1993) called, "dynamic understanding." Such understanding is, "holistic, historical and qualitative, eschewing deductive systems and causal mechanisms and laws" (Kellert, 1993).

Ingredients Necessary for Chaotic Behavior

As stated earlier, chaotic behavior may appear random but in reality be generated by a low number of contributing factors. Prior to determining how to detect this type of behavior, it is necessary to describe the factors of chaos. Chaotic systems are referred to as nonlinear deterministic systems. The first ingredient of chaos is that the system is deterministic, thus in theory it is possible to unambiguously determine future states or prices with knowledge of past states. Through examination of the attractor of a system it is possible to distinguish deterministic from stochastic systems. The second necessary condition of chaos to be possible is that a system, for example, the mapping:

$$P_t = f(P_{t-i})$$

must incorporate some form of nonlinearity. If the mapping or relationship is linear, then chaos can be ruled out. However, nonlinearity is a necessary but not sufficient condition for the emergence of chaotic behavior. Chaos also requires that a nonlinear system possess sensitivity to initial conditions. This characteristic is measured through Lyapunov exponents and places limits on the predictability of a system in the long term.

Attractors

An important concept of dynamics is that systems can be characterized by the presence of attracting sets or attractors in phase space. In other words, attractors allow a researcher to examine and classify the dynamical behavior of a system geometrically. An attractor represents the equilibrium properties of a system. An attractor can be thought of as the equilibrium or time path of a dynamical system. Roughly speaking, attractors are what the behavior of a system settles toward or is attracted to. Crutchfield (1986) details that an attractor can be classified as belonging to one of four different geometrical forms: a single point, limit cycle, torus or chaotic. In an N-dimensional system, if there is global stability, then there is a single equilibrium point and thus the attractor has zero dimension. If the equilibrium of a system is characterized by a limit cycle, the attractor is a line and thus has one dimension. In both of these instances, the equilibrium characteristics of the system are constructed to a geometric form with a dimension that is both less than the dimension of the system and is an integer.

Fixed-Point Attractors

The simplest form of attractor is a fixed point. Point attractors correspond to systems that settle into a steady state equilibrium. Point attractors are the only type of attractor associated with linear systems (Radzicki, 1990). Nonlinear systems, however, can have attractors which correspond to any of the four types previously mentioned. Probably the easiest manner to examine the dynamic behavior of a system is with the aid of a 45° line diagram with the values of P_t on the vertical axis and the values of P_{t-1} on the horizontal. Consider Figure 2.5. This diagram can be read by taking a starting value of a system at P(0). The dynamics of the system can be explored by tracing the sequence of P(t) in the figure. For example, moving vertically from P(0) to the curve determines P(1). A move horizontally from P(1) to the 45° line thus determines P(2) and so on. The system depicted in Figure 2.5 converges to an equilibrium point. Correspondingly, the attractor for this type of system is a fixed point (P*) as the system is "attracted to" this value. Figure 2.6 shows the same process but in a time domain.

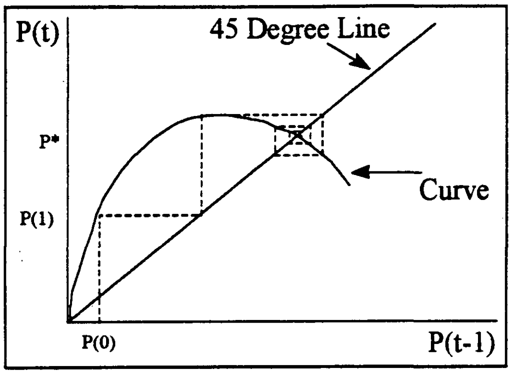


Figure 2.5 Steady State System 45° Diagram

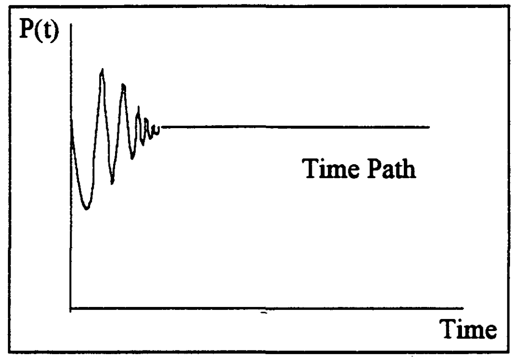


Figure 2.6 Time Path of Figure 2.5 System

Limit Cycle Attractors

Limit cycle attractors are associated with systems which fluctuate in a repetitive manner, according to some identifiable periodicity. For an economic example of this type of attractor, consider the cobweb cycle model which is one of the simplest form of an economic dynamic model. In this model, the supply of a good in year (t) is a function of the goods price in year (t-1) and where, in any period, the price is adjusted to "clear the market." Examples may include agricultural settings where the decision to grow a crop or raise a herd are based on conditions during the previous year. This type of phenomenon was observed during the 1930s in the United States with hog prices and, in principal, the cycle can repeat endlessly granted that producers do not learn from previous experiences. Figure 2.7 is an example of a cobweb cycle. In this cycle, supply in year 1 (S1), is dependent upon prices in year zero (P0). Correspondingly, as prices rise to P(0), demand falls off to D(1). With the decrease in demand in year 1, suppliers decrease their supply to S(2).

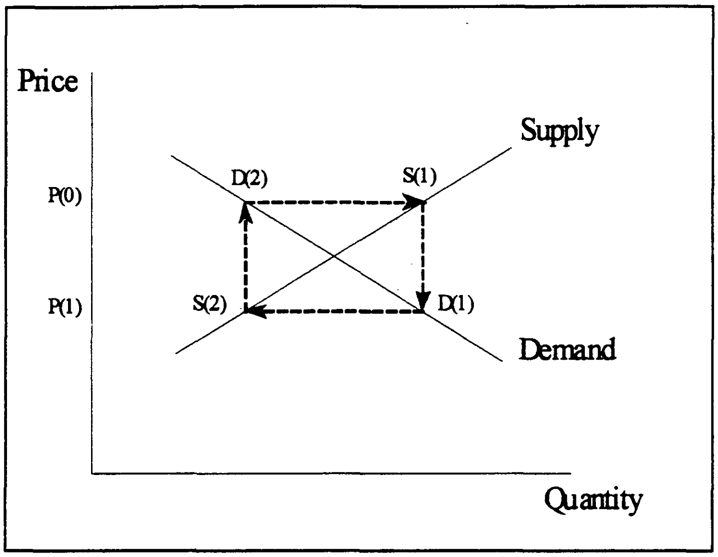


Figure 2.7 Example Cobweb Cycle

At the lower price, P(1), demand increases to D(2) which has a price level equal to the initial prices, thus forming a 2-period cycle. Figure 2.8 depicts this process in a phase-space domain whereas Figure 2.9 shows the same process but in a time domain.

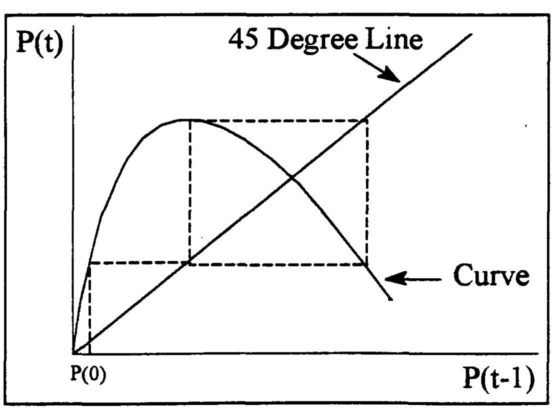


Figure 2.8 Two Cycle Period 45° Diagram

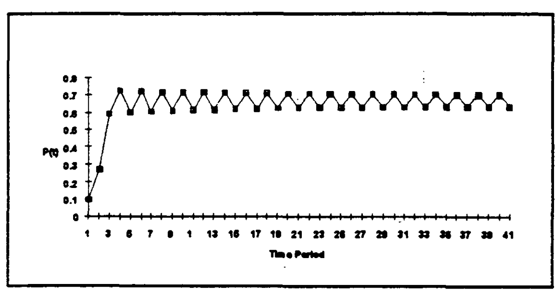


Figure 2.9 Time Path of a Two Period Cycle

The attractor for a limit cycle may be depicted as Figure 2.10. Given any initial condition, W or Z, it can be seen that the system time paths converge to the dashed ellipse. This ellipse would be considered the attractor. Other examples of two-period cycles include the beating of a human heart and/or a metronome.

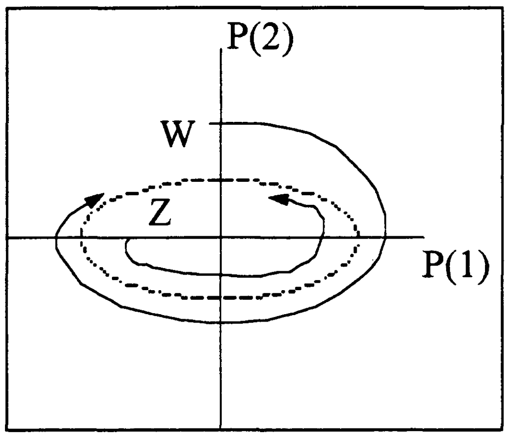


Figure 2.10 Example of a Limit Cycle Attractor

Nonperiodic Attractors

Both tori and chaotic attractors are associated with systems which fluctuate nonperiodically. In other words, they have periods of infinity. As stated by Radzicki (1990):

> The difference between tori attractors and chaotic attractors involves the predictive or information-preserving nature of their fluctuations. Systems with tori attractors fluctuate in an information-preserving manner, while systems with chaotic attractors fluctuate in an information-generating or

> unpredictable fashion. More specifically, in the case of tori attractors, two points (prices) that lie infinitesimally close on the surface of the attractor will remain infinitesimally close as the system moves through simulated time.

This lends itself to the property of ergodicity which will be explained below. With chaotic attractors, two prices infinitesimally close on the attractor will separate or diverge at an exponential rate. Interestingly, measurements of the rate of separation, called Lyapunov Exponents, are methods which have been used in the past to detect sensitive dependence on initial conditions.

Strange Attractors (Chaotic Attractors)

The relationship between strange attractors and chaos is that if a system has a strange attractor, then the system is chaotic. A special property of strange attractors is that they possess non-integer dimensions which in the literature are referred to as fractal dimensions or fractals. In particular, chaotic motion has been identified when the process or processes jump irregularly over the attractor (Creedy and Martin, 1994). According to Ruelle and Takens (1971) for an attractor to be strange, the following properties are needed:

A) All trajectories remain within a bounded region
B) Sensitive Dependence upon Initial Conditions
C) The attractor cannot be split into two or more pieces

Trajectories within a bounded region implies that the attractor which chaotic trajectories move over has a finite dimension. Sensitivity to initial conditions refers to the fact that microscopic changes in initial conditions of a chaotic system will have profound effects on the behavior of that system as it evolves through time. The third condition alludes that chaotic trajectories cannot cross one another.

A necessary quality of a chaotic system is that the system resides in a fractal dimension. A fractal dimension is a dimension which is non-integer, for example, a dimension between 2 and 3 such as 2.5 would be considered a fractal dimension. As stated by Creedy and Martin,

> For an attractor to be identified as strange, the dimension of a continuous time system needs to be at least equal to three. The reasons are as follows. In a one-dimensional continuous time model the trajectory of the state variable is smooth and thus the irregular jumps identified in the one-dimensional discrete maps are precluded in the one-dimensional continuous time maps. Two-dimensional continuous time systems also cannot exhibit chaotic behavior since the trajectory cannot intersect itself. It is only for three-dimensional or higher-order continuous time systems that a smooth trajectory can behave in a supposedly irregular pattern over the attractor without intersecting another trajectory.

Nonlinearity

The problem of modeling real-world systems with mathematical equations usually begins with a linear model. However, when finer details or more accurate results are desired, additional nonlinear terms must be added (Rasband, 1990). It is, therefore, necessary to thoroughly define the concept of nonlinearity. In the context of this study, nonlinearity will be assumed to mean that the measured properties of a system in a later state depend in some complicated manner on the measured values in an earlier state. Complicated meaning that this dependency is something other than just proportional to, differing by a constant or by a combination of these two (Rasband, 1990). As will be seen in the following sections, nonlinearity and complex dynamical systems go hand in hand. Again, as attention is turned to the nonlinear aspects of a system, it must be noted that nothing beyond the most general statements can be said about the solution(s). It is almost never possible to write down a solution in closed form as can be the case with simple dynamical systems or those easily linearized nonlinear dynamical systems. Rasband (1990), therefore, suggests that many techniques including averaging, asymptotic analysis, numerical integration, etc., should be applied in order to gain as much qualitative information about the solution as possible. With these definitions in order, it is now possible to examine how economists in the past have handled nonlinearity.

Economists Past Confrontation with Nonlinearity

Nonlinearity, by its very nature, is messy. Particularly in economics, nonlinearity disrupts the established foundations of exclusive equilibrium levels. Furthermore, nonlinear confounds traditional time series models which assume linearity. In the past, when confronted with nonlinear systems, economists have generally assumed three courses of action:

A) Assume linear approximations could adequately describe nonlinear behavior.
B) Transform nonlinear variables to linear approximations.
C) Construct crude, non-assuming nonlinear models.

The idea behind linear approximations is that a point on a nonlinear curve could be adequately described by its tangent line to that point. Figure 2.11 displays how a linear approximation(s) is used on an nonlinear curve. A single linear approximation of this nonlinear system might take the form of the solid straight line in Figure 2.11. In many cases a single linear approximation can be improved upon by dividing the domain into subregions (regions r1, r2 and r3 on Figure 2.11).

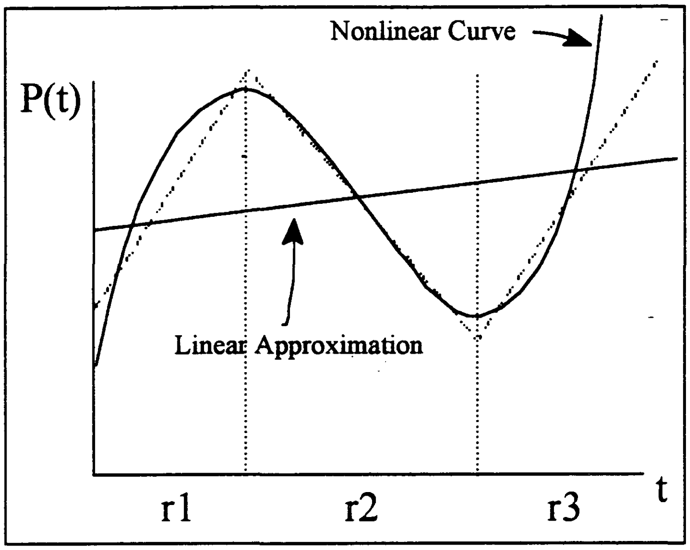


Figure 2.11 Linear Approximations to a Nonlinear Curve

In other cases, economists attempted to preserve the nonlinear relationships of their models by transforming nonlinear variables to a linear approximation and thus achieving models which they could use. For example, consider the following nonlinear equation:

$$F(y) = Ax^b \quad (2.5)$$

This relationship can easily be transformed by taking the logarithm of both sides and achieving the following equation which is linear in two variables log(F(y)) and log(x):

$$\log(F(y)) = \log(A) + b\log(x) \quad (2.6)$$

Unfortunately, there is only a limited class of nonlinear relationships of which an effective linear transformation can occur.

The work of Slade (1982) examines past nonlinear behavior as she noted that relative price series for natural resources do not appear linear, but rather U-shaped. In order to assess the validity of U-shaped price paths, Slade estimated a quadratic time trend equation for the relative price indices of eleven minerals and fuels and one mineral aggregate over the period from 1870-1973 using the following:

$$P_t = a + b_1T + b_2T^2 + e_t$$

where a and the b's are parameters, T is a time trend, P is the relative price of the resource, and t indicates that the relative price and error terms are indexed by time. Slade finds that the coefficient on the quadratic terms (b_2) to be statistically significant in 11 out of 12 energy-related commodities examined.

Time Series Modeling under the Linear Paradigm

Prior to the 1920s, forecasting of an economic time series was simply completed by extrapolating from the series a globally fit regression line. The single linear approximation line on Figure 2.11 is an example of this technique. The beginning of modern time series analysis can be traced to Yule's 1927 prediction of the number of sunspots during a year. Briefly, the autoregressive technique models the current estimate of the future value of a variable as the weighted sum of previous observations. The autoregressive technique proved to be the basis for time series analysis for the next five to six decades. Yule's model, as with all autoregressive techniques assumes the underlying system is stochastic and thus linear modeling techniques may be utilized. To obtain more complex behavior, outside intervention is needed in the form of exogenous shocks must be assumed (Gershenfeld and Weigend, 1994). Again, note the correspondence with past economic paradigms.

Unfortunately, linearity can only give rise to dynamic behavior which is periodic. The sacrifice in realism by making linear approximations of truly nonlinear systems may be too great and thus the descriptive power of the model is questionable and/or lost.

The Need for Nonlinearity

One reason to look at nonlinear approaches to modeling economic time series is based on the idea that large swings in these time series may be the result of the system jumping from one stable equilibrium to another. To encompass this notion of multiple equilibria, nonlinearity needs to be introduced as multiple equilibria cannot arise in linear models. When the time path of concern is nonlinear, analysis becomes vastly more difficult and the results much more unclear. For example, Figure 2.12 again displays an economic system with equilibrium points A and B. If the time path follows a nonlinear course such as time path 2, multiple equilibria can result. This can be seen on Figure 2.12 at time period t_1 where multiple quantity levels or prices occur. This poses significant problems to economists attempting to predict an equilibrium quantity and/or price over time.

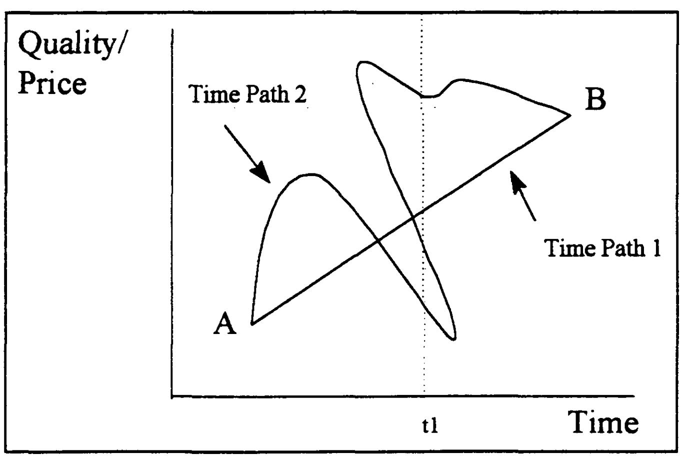


Figure 2.12 Example of a Nonlinear Time Path Resulting in Multiple Equilibria

As stated by Tong and Lim (1990):

> Just as a linear differential equation is totally inadequate as a tool to analyze more intricate phenomena such as limit cycles, time irreversibility, amplitude-frequency dependency, and jump resonance, a linear time series model should give place to a much wider class of models if we are to gain deeper understanding into the structure of the observed data.

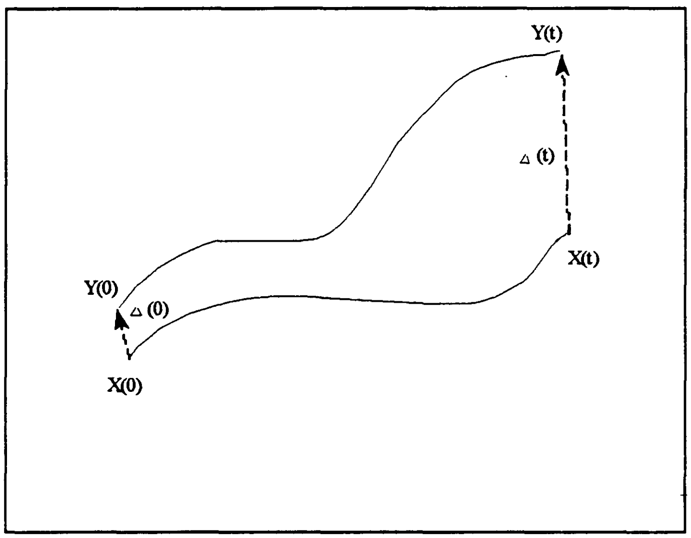
Nonlinear dynamics is not new to economists; however, recently there has been a renewed interest. Hicks (1950) and Goodwin (1951) proposed general nonlinear models which were applied to trade and business cycles. Specifically, Goodwin was concerned with business cycles which would not converge to a steady state nor explode as was the case with past modeling attempts which utilized linear dynamic methods. These authors showed that such nonlinear models can yield stable limit cycles toward which all possible time paths of the variable Y_t (national output) converge. Rather than converging to a fixed equilibrium value, with oscillations damping out toward zero amplitude, these nonlinear models proved that a stable equilibrium cycle could exist, with Y_t forever wandering from peak to trough along an equilibrium cycle path. The importance of these works was that they responded to a real economic issue and were not simply formal mathematical models. The limited acceptance of these models can be traced to both the difficulty in handling nonlinear systems and empirical success of linear stochastic models, particularly, linear difference equations. Specifically, time series analysts returned to the linear approach by utilizing the results of Slutzky (1927). In this work, Slutzky observes that stable low order stochastic difference equations could generate cyclic processes that mimicked actual business cycles (Scheinkman, 1990). As a result, the use of dynamic models which incorporated nonlinearities was limited until the past 10-15 years.

The re-emergence of nonlinear time series modeling can be traced to the development of powerful computing technologies. During the 1980s, powerful computers have made it possible to collect and analyze time series orders of magnitude longer than any time before in history. These technologies have allowed for the simulation of nonlinear processes and to show that their behavior often violates expectations borne out of traditional linear paradigms (Gershenfeld and Weigend, 1994). Further, the results of these new, nonlinear algorithms could be visualized which in return facilitated deeper understanding. An important result of nonlinear models is that they can exhibit not only monotonic and periodic cycles as with traditional linear economic models, but also nonperiodic fluctuations. With the advent of sufficient computing technologies, researchers could begin to face nonlinearity head on and the subsequent implications of nonlinearity. One such implication is chaotic dynamics.

Sensitive Dependence upon Initial Conditions (SDIC)

The exponential sensitivity of chaotic solutions means that, as the system moves through time, small errors in the solution can grow very rapidly (i.e., exponentially). Therefore, after some time period, effects such as noise, measurement errors and computer roundoff can totally change the solution from what it would be in the absence of these effects. The dynamics on an attractor are said to be chaotic if there is exponential sensitivity on initial conditions. As an example consider Figure 2.13. This figure contains two trajectories X and Y. Consider two initial conditions X(0) and Y(0) and imagine that they evolve forward in time by a dynamical system to X(t) and Y(t), at time t, the separation between orbits is $\Delta(t)$. As stated by Ott (1993), if in the limit $|\Delta(0)| \rightarrow 0$, t is large, orbits remain bounded and the difference between the solutions, $\Delta(t)$, grows exponentially for typical orientation of the vector $\Delta(0)$ (i.e., $\Delta(t)/\Delta(0) \sim \exp[(ht)$, $h>0$), then we say that the system displays sensitive dependence upon initial conditions (SDIC) and is chaotic. By possessing SDIC, long term prediction of chaotic systems becomes extremely difficult, if not impossible.

Bounded solutions imply that there is some portion of phase space (< infinite) which the solutions never leave. This implication of chaotic systems is far reaching and is one of the factors which has led to the overwhelming interest in chaos by economists and other disciplinary researchers.



(Source: Ott, 1993)

Figure 2.13 Example of SDIC system

The profound implications of SDIC can be illustrated by the use of the first order logistic equation:

$$P_{t+1} = \lambda P_t(1 - P_t)$$

which is capable of generating chaotic output when the parameter, λ, is varied. The contrast between the two time paths shown in Figures 2.14 and 2.15 are striking when initial conditions are changed by 0.01 and λ is held constant. Further, as seen in these figures, in earlier time periods this slight change in initial conditions has little effect but has profound consequences as this system passes through time.

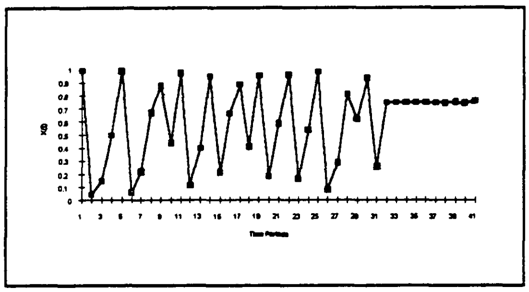


Figure 2.14 Logistic Equation in a Chaotic Zone with Initial Condition P(0) = 3.94.

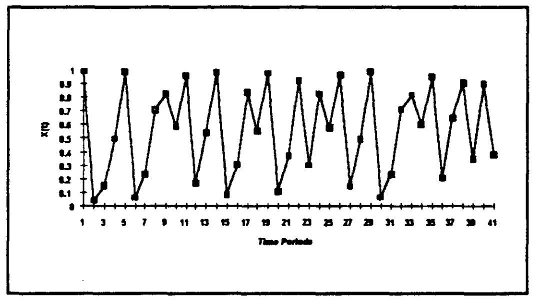


Figure 2.15 Logistic Equation in a Chaotic Zone with Initial Condition P(0) = 3.95.

Critical Levels

One other characteristic of chaotic systems is the existence of critical levels. Critical levels are specific points when the behavior of a system radically changes. Again with the first order logistic equation, when λ passes certain levels, the number of equilibrium levels increases. For small values of λ, the logistic curve converges to a steady state. This equation passes a critical level when λ is equal to 3.2. At this point the system converges to a 2-period cycle. This point is often referred to as a bifurcation point. As λ increases, this system experiences numerous bifurcation points as the cycles increase (or cascade) to a point where chaotic behavior commences. Roughly speaking as a system passes bifurcation points, behavior becomes increasingly complex and the qualitative pattern increasingly unstable (Day, 1994).

Another example of a system passing critical levels is that of cigarette smoke rising in a draftless room. In a draftless room, a column of smoke will rise from a cigarette and suddenly break into swirls and dissipate. This point can be considered a critical level of the system as once the velocity of the smoke passes this level, the smoke column breaks up as the smoke column can no longer overcome the density of air.

Time Series Modeling

With dynamical analyses, time series modeling has traditionally and continues to be a fundamental modeling method available to economists, financial and market analysts and other social scientists. A price time series typically consists of a set of observations on a variable, P, taken at equally spaced intervals over time. Time series data differ from cross-sectional data in that the former has a temporal order. When a price time series is plotted over time, interesting patterns such as trends, cycles or other irregularities may appear. At this point, an analyst has two distinct courses of action which may be taken. Ideally, one would attempt to account for a pattern by introducing explanatory variables, thus deriving a cause and effect relation. The second approach is to attempt to explain the pattern(s) of prices by examining the history of the price itself. This approach leads to time series modeling which is aimed at uncovering an internal process responsible for the price's behavior.

When faced with economic systems which have numerous influencing factors and interactions, economists have turned to the utilization of time series modeling as a method to both describe the behavior of the system as well as a device for predicting future values or states of the system. The distinguishing feature of a time series model as opposed to an econometric model is that no attempt is made to formulate a behavioral relationship between the dependent variable and the independent variables (Harvey, 1994). This is quite understandable considering that both the independent and dependent variables may not be fully known.[3] Further, this may be the case when the system of concern is a mineral commodity price series. In a univariate time series model, the movements in the dependent variable such as the current price of a mineral commodity are explained solely in terms of its own past or by its position in relation to time. Forecasts can be subsequently made by extrapolation.

Throughout scientific research, measured time series are the basis for characterizing an observed system and for predicting its future behavior. Specifically, time series analysis has three goals: forecasting, modeling and characterization (Gershenfeld and Weigend, 1994). The intention of forecasting of a time series is to accurately predict the short-term evolution of the system. The goal of modeling is to accurately capture the features of long-term behavior of the system. As stated by Gershenfeld, (1994), "These are not necessarily identical: finding governing equations with proper long-term properties may not be the most reliable way to determine parameters for good short-term forecasts and a model useful for short-term forecasts may have incorrect long-term properties." The final objective of time series analysis, characterization, attempts to determine the fundamental properties of the system. These properties may include determining the dimension or degrees of freedom of the system and/or the amount and sources of randomness.

Traditionally, time series modeling has not meshed with economic paradigms and empirical practices in that the causes which determine a time series were not obtained. To an extent, this may be true as many view the only goal of time series modeling is for prediction of future values. However, this study views the two approaches to the examination of fluctuations in a time series as complementary rather than competitive. As

---

[3] Complete knowledge of the factors affecting system behavior and the interactions between those factors are not known.

commodity price series have numerous influencing factors and even more interactions between these factors, utilizing time series methods may lead to deeper understandings whereby explanatory relationships may be modeled in future research. At present, the process which underlies or generates mineral commodity price series is unknown. Because the only evidence available of this underlying process is the price series itself, this study examines the question, "What can be deduced about the underlying system dynamics through examination of the time series?" Specifically, this study will utilize and comment upon univariate time series models and techniques applied to mineral commodity price series. Univariate time series are of the form:

$$P_t = f(P_{t-i}) + e_t$$

Current and future prices are only a function of past prices and current and/or past uncertainty terms, e_t, representing random disturbances and traditionally assumed to have mean zero and constant variance.

The beginning of time series analysis lies with the work of Yule (1927) in his work to predict the annual number of sunspots. Specifically, Yule invented the autoregressive technique:

$$Y_t = \theta_1 Y_{t-1} +, \ldots, + \theta_p Y_{t-p} + \varepsilon_t$$

This model predicts that the next value in a time series will be a weighted sum of previous observations in a series. This is a linear model because observations are linearly correlated across time with neighboring observations. The error term is assumed to have mean zero and constant variance.[4] This methodology has given rise to other stochastic methods such

[4] This term captures all external shocks to the system. Such a system in engineering terms is said to thus be driven by noise (a stochastic system).

as moving-average processes in which each observation is generated by a weighted average of random disturbances:

$$Y_t = \mu + +\varepsilon_t + \theta_1 \varepsilon_{t-1} +, \ldots, + \theta_q \varepsilon_{t-q}$$

where the parameters $\theta_1, \ldots, \theta_q$ can be either positive or negative and the error terms are random.[5] Extensions, of these methods include combinations of autoregressive and moving-average (ARMA) techniques.

Many financial time series such as stock returns can exhibit changes in variance over time. These changes tend to be serially correlated, with groups of highly volatile observations occurring together[6] (Harvey, 1994). Relaxing the assumption that the variance of the error terms must be constant leads to models such as autoregressive conditional heteroskedasticity (ARCH) and the generalized autoregressive conditional heteroskedasticity (GARCH).[7]

ARCH and GARCH provide refinements to linear stochastic methods and have found significant application and success in the financial arenas. However, these techniques are still stochastic. Two occurrences in the early 1980s provide for the ability to question if the processes are truly stochastic or may be deterministic in some fashion. The first development is state (or phase) space reconstruction by time delay embedding which draws on ideas from differential topology and dynamical systems to provide a technique for recognizing when a time series has been generated by deterministic governing equations (Gershenfeld and Weigend, 1994). The second is the advent of powerful computing technologies which have enabled research into nonlinear systems.

---

[5] Note that the mean of the process is independent of time, since $E(Y_t) = \mu$.

[6] This can be thought of as large changes or adjustments following large changes (high volatile) and small changes following small changes (low volatility).

[7] These methods are referred to in the literature as stochastic variance models.

Over the past decade, many studies have found applications within economic systems. Barnett and Chen (1987) examine monetary aggregates with the correlation integral and find low correlation dimensions. Correlation integrals have also been used in the analysis of macroeconomic time series by Brock and Sayers (1988). Scheinkman and LeBaron (1989) uncover evidence of nonlinear dependencies on weekly returns for the Center for Research on Security Price's (CRSP) value-weighted index. Decoster et al. (1992) utilize the correlation dimension to search for chaotic structure in daily futures prices for four commodities: sugar, coffee, silver and copper. This study found that there was strong evidence of nonlinear structure in all of the four price series which could not be explained by his alternative hypothesis of ARCH models. Unfortunately, evidence for the presence of chaos was provided, but, as the author states, "Further research is needed before we can confirm or reject the discovery of chaos."[8] Before examining specific, relatively new, techniques, a review of economists past confrontations with dynamical behavior is in order.

Stochastic and Deterministic

A major focus of this study lies in differentiating stochastic from deterministic systems. The word "stochastic" has a Greek origin meaning random or choice. The antonym of stochastic is "sure" or "deterministic" or "certain." A deterministic model attempts to predict a single outcome from a given set of circumstances whereas a stochastic model predicts a set of possible outcomes weighed by their likelihoods or probabilities. For example, if tomorrow's price for a given commodity is uncertain, it must therefore be described by a probability distribution if the time series is generated by a

[8] Chapter 4 of this study will examine the behavior of the systems which generate copper spot price series.

stochastic process.[9] If the process is truly deterministic, then the possibility exists to predict tomorrow's outcome with certainty. Consider Figure 2.16 which details degrees of freedom for the two type of processes. A deterministic system may contain low enough degrees of freedom to allow for precise predictions whereas as descriptions of stochastic systems are limited to descriptions based upon traditional statistical and probability theories.

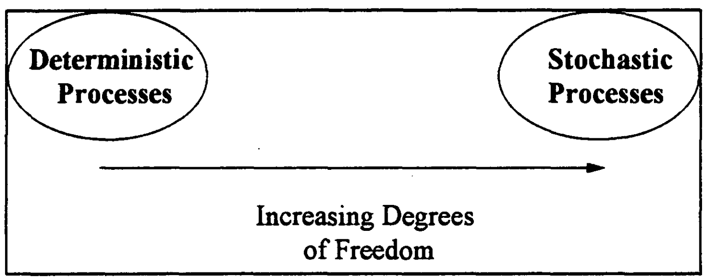


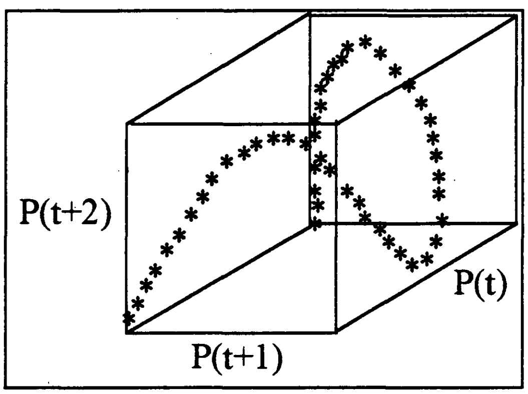
Figure 2.16 Stochastic and Deterministic Processes

The distinction between stochastic and deterministic systems is critical and has far reaching implications. If the underlying system which generates mineral commodity prices is truly deterministic, then treating the time series as if it were generated by a linear stochastic difference equation will lead to an exaggerated view of the amount of randomness which is affecting the system (Scheinkman, 1990). For example, this could occur if mineral commodity prices are represented by an ARMA model but the underlying system which produces these prices is truly deterministic and not stochastic.

---

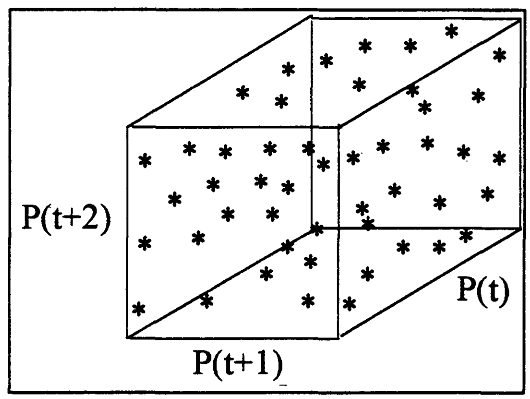
[9] A stochastic process is a family of random variables P_t, where t is a parameter running over a suitable index set T. In most instances, t corresponds to discrete units of time and the index set is $T = \{1, 2, 3, \ldots\}$. A stochastic process is a description of a random phenomenon changing with time.

The use of stochastic models for truly chaotic systems in the past is not unfounded as chaotic behavior gives the appearance of being random but in fact is not random. In other words, random appearing behavior may simply be a veil for behavior which is truly chaotic. For clarity between these two types of systems, consider Figures 2.17 and 2.18. An important property of nonlinear models is that they can generate random looking results even though the model is deterministic. Even though both deterministic and stochastic systems can generate random behavior, the dimensions of these models are completely different. Processes arising from deterministic systems have finite dimensions whereas processes arising from stochastic systems have infinite dimension.



(Source: Creedy (1994))

Figure 2.17 Deterministic Process in Three Dimensions



(Source: Creedy (1994))

Figure 2.18 Stochastic Process in Three Dimensions

Figure 2.17 shows a deterministic system which is at least a two-dimensional process while Figure 2.18 displays a stochastic system which is at least a three dimensional process. This result could be continued to higher and higher dimensional systems with the result that the stochastic series will always fill the entire space (Creedy and Martin, 1994). In other words, a stochastic system has a dimension equal to whatever dimension it is placed (also called the embedding dimension) because its components are uncorrelated. A deterministic system is held together by its correlations and thus will retain its dimension regardless of the dimension in which it is embedded. Therefore, a stochastic series is said to have infinite dimension while chaotic systems possess finite dimensions. This result is important as it forms a basis of determining if a resulting time series is generated from a deterministic or stochastic process.

Stochastic/Deterministic and Independence/Dependence

It is necessary to differentiate between the issues of independence and dependence in a time series and the stochastic or deterministic (nonstochastic) nature of the process which underlies or generates the time series. For example, consider the generalized relation in Figure 2.19.

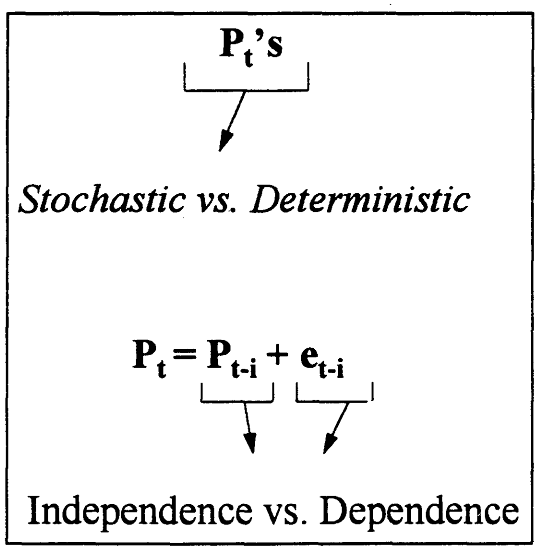


Figure 2.19 Origins of Dependence and Nonstochasticity

Stochastic behavior assumes that the price time series is a collection of random variables with each price, P_t, assumed to be drawn randomly from a probability distribution. Determinism implies that a given price is unambiguously determined by the past. Dependence in a time series is by the relationships between present prices and past prices as well as current and past error terms. A deterministic process is inherently dependent as each price in a series is merely the result of a precise relation with past prices. A stochastic process can be either independent in which case prices follow a random walk or dependent where prices are either linearly or nonlinearly correlated with other prices.

Over the past decades, dynamic analysis of economic and financial systems has relied upon methods derived in the rigidly disciplined and controlled physical sciences. Application of techniques such as differential and difference equations has generally required severe restrictions and/or overwhelming assumptions. Questioning the validity of such assumptions, this study applies the physics-based methods of differential topology and dynamic systems theory to economic systems. Chapter 3 presents a modified time series model specification process which aims at uncovering the characteristics necessary for chaotic behavior from an observed time series. This process culminates with the specification of non-traditional models.

Chapter 3
DETECTION OF CHAOTIC BEHAVIOR: A MODIFIED TIME SERIES MODEL SPECIFICATION PROCESS

Commodity prices are one of the most important determinants of world economic performance. Their influence is far reaching from the stability of developing countries' incomes, industrial countries' inflation and world investment and growth to their prominent role in the private sector's business decisions for production, consumption and expansion. Explaining and analyzing the behavior of commodity prices has therefore been an extensive area of study in economics. Much can be gained by fully understanding commodity markets; economists and policy makers have studied commodity prices attempting to uncover future patterns of general price levels and overall economic stability of an economy whereas financial analysts have examined commodity prices attempting to unlock potential arbitrage opportunities. However, despite the importance of commodities, economists traditionally have poor records in explaining and forecasting price behaviors.

A financial time series model for prices or returns is a detailed description of how successive prices are determined. [10] Critical to the successful application of models is that the model incorporate the properties of the underlying economic system in question or the model is properly specified. For a model to be empirically helpful, forecasts from the model should be statistically optimal and the model should enhance the rational decision making process (Taylor, 1986). Traditionally, it has been assumed that most economic

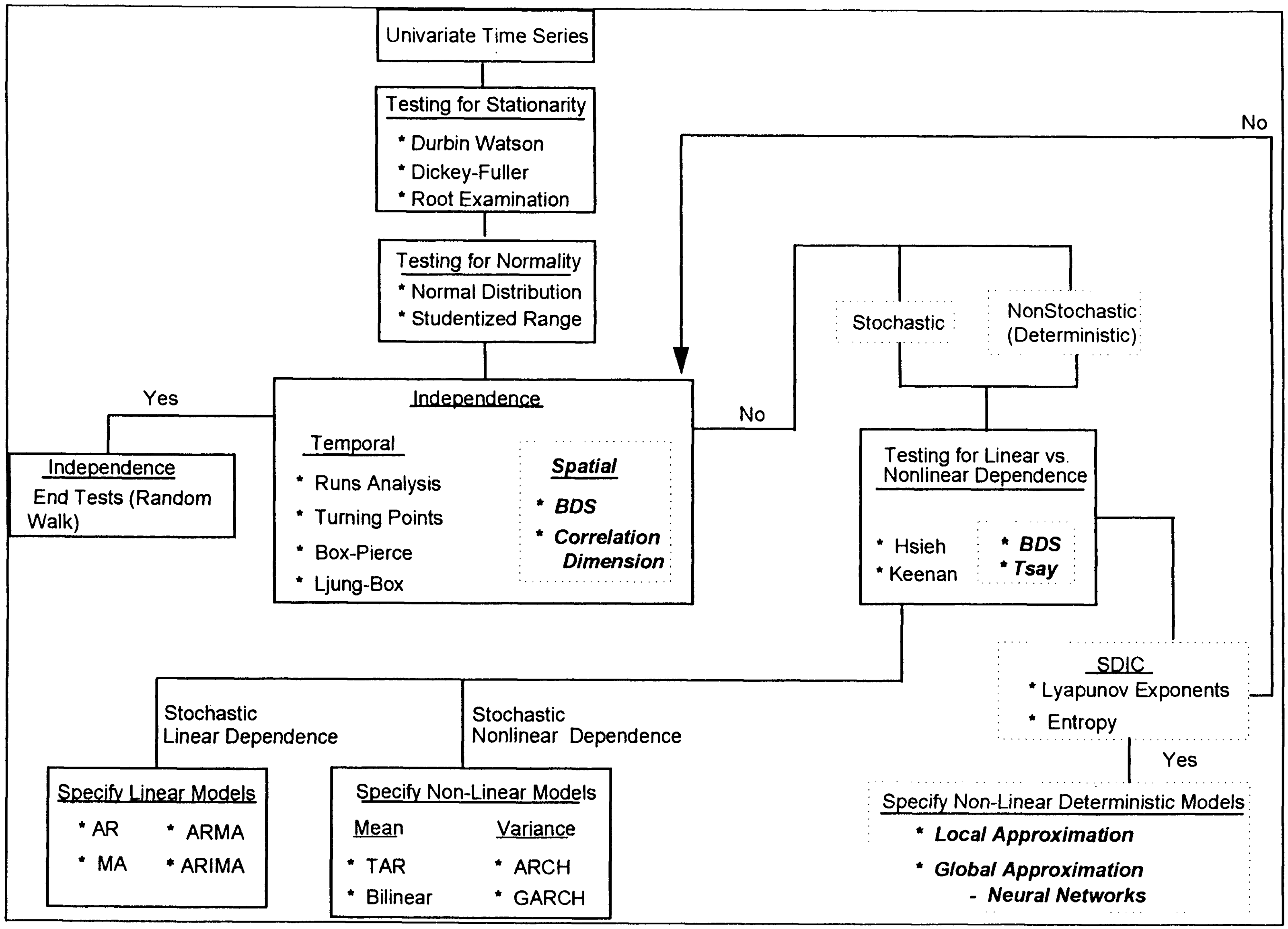
[10] One method of obtaining returns is to take the first difference of successive prices (i.e., $R_t = P_t - P_{t-1}$). Other methods include taking the logarithmic difference.

systems, particularly those associated with the financial markets, are stochastic. Subsequent models of how prices behave in these markets have typically assumed prices to change in a stochastic manner. However, it is well documented that a class of low-dimensional nonlinear deterministic (or chaotic) systems may give the appearance of randomness both visually and to many elementary statistical tests. Reliance upon these traditional tests may result in the incorrect specification of stochastic models which may over estimate the randomness effecting the system (Scheinkman, 1990).

Proper specification of any model is necessary as in the presence of mispecification, resulting estimates are unreliable thus making forecasts questionable. To assist in the determination of the underlying behavior, rather than simply assuming it to be stochastic, and the proper specification of subsequent univariate time series models, Figure 3.1 is provided. Specifically, this process aims at uncovering characteristics necessary for chaotic behavior. The dotted elements on this figure detail these steps. If these characteristics are present, alternative modeling methods are presented. For each step in the modified specification process, the following chapter thoroughly describes the tests utilized and their relevance toward the goal of a properly specified model.

Verification of Stationarity

As seen in Figure 3.1, the first step toward proper model specification is to test a price time series for stationarity. It is necessary to test for stationarity in a time series because most statistical models require that the underlying generating process be stationary. When a series of observations is generated by a stationary process, they fluctuate around a constant level and there is no tendency for their spread to increase or decrease over time (Harvey, 1994). Figure 3.2 displays two sample time series, one of



(Derived from Cromwell et al., 1994)

Figure 3.1 Modification of Univariate Testing and Model Specification Process which is stationary.[11]

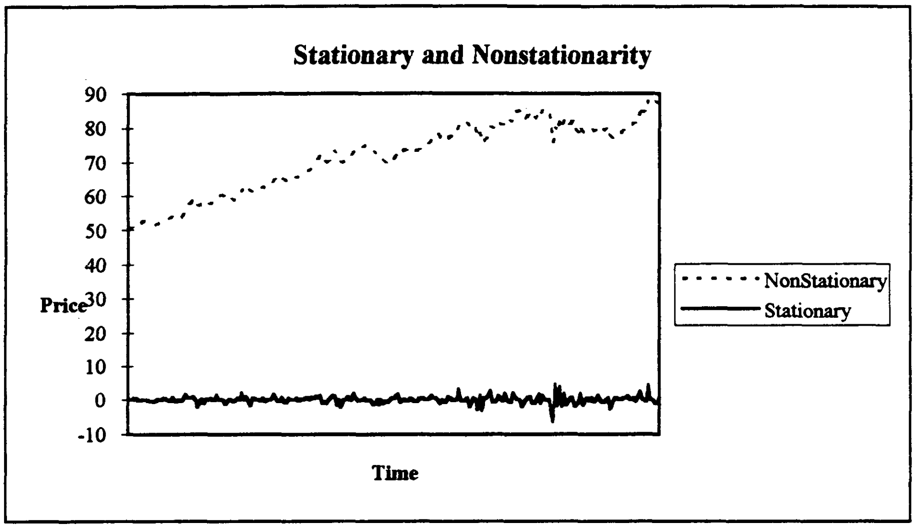


Figure 3.2 Stationary and Nonstationary Series

Defined formally, for a process to be stationary, the following conditions must be satisfied for all values of time (t) (Harvey, 1994):

A)	$E[P_t] = \mu$		3.1
B)	$E[P_t - \mu]^2 = \sigma_P^2$		3.2
C)	$Cov(P_t, P_{t+k}) = E[(P_t - \mu) = E[(P_{t+k} - \mu)]$	for k = 1, 2, 3, ...	3.3

Equations 3.1 and 3.2 define that the mean and variance of the generating process is constant. Equation 3.3 ensures that each autocovariance is independent of time and only dependent on the lag value (k). These above three equations provide a definition for weak stationarity.

[11] The stationary series is achieved through differencing the original price series.

Occasionally, the condition of strict stationarity may be imposed. This is a stronger definition in that the joint probability distribution of a set of T observations at times $t_1, t_2, ..., t_T$ is the same as the joint probability of the observations at times $t_{1+\tau}, t_{2+\tau}, ..., t_{T+\tau}$. The literature states that the distinction between weak and strong stationarity is not important (Harvey, 1994). Therefore, in this study the term stationarity will always imply weak stationarity.

Stationarity is necessary for analyzing a univariate time series because during a specific time interval, only a single measurement of observations is available. As a result, attention must shift from the aggregation of observations at particular points in time, as is done with cross-sectional data, to averaging the observations over time. This is only possible if the data generating the process is such that the mean, variance and covariance are independent of time (Harvey, 1994). Stationarity is a necessary restriction on the generating process so that meaningful inferences can be made. If a stochastic process is stationary, the probability distribution Prob(P_t) is the same for all t and it is possible to use the properties of discrete random variables to characterize the observable time series as well as the underlying process itself.

Most economic time series are not stationary, however, many of them can at least be approximated by a stationary process if they are differenced (Harvey, 1993). Nonstationarity of a time series is reflected by the presence of a unit root. Thus, one method which can be utilized to verify stationarity is to examine a time series for the existence of unit roots. Consider the autoregressive model:

$$P_t = \rho P_{t-1} + e_t \qquad t = 1, 2, ... \qquad (3.4)$$

In the explanation of unit roots, it is helpful to utilize the notation of a lag or backshift operator (B). Equation 3.4 is thus defined:

$$(1 - \rho B)P_t = e_t$$

where $BP_t = P_{t-1}$ and $B^2P_t = P_{t-2}$, etc.

This notation can be extended to higher order models. For example, an AR(2) model,

$$P_t = \rho_1 P_{t-1} + \rho_2 P_{t-2} + e_t \qquad t = 1, 2, \ldots$$

using the lag notation would appear as:

$$(1 - \rho_1 B - \rho_2 B^2)P_t = e_t$$

Through the process of solving the above polynomials for B, roots of a time series are determined. A stationary series will have all roots outside the unit circle ($B > 1$). A non-stationary time series will have one or more roots on the unit circle with the rest outside.[12] If a process is found to be nonstationary, transformations of the price series can be performed in such a manner as to render the process stationary. For example, the simplest nonstationary process (or process which has a unit root) is the random walk:

$$P_t = P_{t-1} + e_t$$

The first difference of this model, $\Delta P_t = P_t - P_{t-1}$, is a transformation which yields stationarity. Transformations, either through differencing, logarithmic, etc. act like filters to remove trend components.

---

[12] Unit roots inside the unit circle ($B < 1$) indicate another type of nonstationarity and are associated with explosive time series (i.e., approach $\pm \infty$). Since explosive behavior is not realistic for financial series it should not be considered (Dickey et al., 1986).

One of the simplest and most widely used tests for unit roots was developed by Dickey and Fuller (1979). This test examines the condition where the process has a unit root and where differencing helps to remove that root. The DF test will allow for the determination that a mineral commodity price series follows a random walk. If so, the series will have to be first-differenced in order to remove the unit roots thus yielding a stationary time series. This is the most common test for stationarity in the literature and will be used in the empirical studies described in chapter 4.

The DF test requires running three regression models:

A) $\Delta P_t = \alpha_1(P_{t-1}) + \varepsilon_t$
B) $\Delta P_t = \alpha_0 + \alpha_1 (P_{t-1}) + \varepsilon_t$
C) $\Delta P_t = \alpha_0 + \alpha_1 (P_{t-1}) + \beta t + \varepsilon_t$

The null hypothesis in each of these equations is that P_t is a random walk (i.e., $\alpha_1 = 0$). Three regression models are run to test this null hypothesis of what type of trend (or where the nonstationarity arrives from).

Specifically, the null hypothesis for each of the above regression models:

A) H_0: P_t is a random walk; $\alpha_1 = 0$

B) H_0: P_t is a random walk plus a drift term (α_0); again $\alpha_1 = 0$

C) H_0: P_t is a random walk plus a drift term (α_0) around a stochastic time trend (β); again α_1=0

The calculated statistics do not have a Student's-t distribution, even asymptotically, and are usually referred to as a τ-statistic because of this reason. As a result, these statistics are compared to critical values which have be computed by Fuller (1976).[13]

Confirmation of Normality

Following the process outlined in Figure 3.1, the next step is to test for normality. Many of the estimation and testing procedures are based on the assumption that the observations from a given process are normally distributed. This is especially true if the generating process is later deemed stochastic through completion of subsequent steps in the modified specification process. Once a price time series has been transformed to achieve stationarity, if it passes a test of normality, it may be possible to specify a linear model to describe the behavior of the prices (Cromwell et al., 1994). If, on the other hand, a series is not approximately normal then using standard estimation tools such as ordinary least squares is questionable. Testing for normality provides evidence of the type of dependence in a stationary process (Cromwell et al., 1994).

One method to assess whether a given time series is normally distributed is by a normal probability plot. Alternatively, normal distribution tests provide for more traditional statistical measures in which a series is analyzed to see if it follows a normal distribution. These tests generally examine the skewness and kurtosis properties of the distribution of prices. These values are thus compared to the same measures from a normal distribution. It is common to refer to a normal distribution in terms of its moments in that odd moments of this distribution greater than 2 are statistically equivalent to zero (i.e., the first moment is the mean, the second moment the variance and skewness is the third, and kurtosis the fourth, etc.). Bera and Jarque, 1981 provide a test for normality also and is simply an extension of the normal distribution test with a joint statistic being computed for skewness and kurtosis.

[13] The specific programming code for this procedure is included as an appendix.

Normal Distribution Tests

A well known property of normal distributions is that all odd moments greater than two are statistically equivalent to zero. This test evaluates the condition in which a given distribution is normal by examining the condition that if a random variable is normal, its skewness is statistically equivalent to zero and its kurtosis is approximately three. A distribution with kurtosis greater than three has thicker tails than a normally distributed random variable and is typically called leptokurtic. This test calculates the following statistics:

$$\text{Skewness: } \frac{T^{-1}\sum_{i=1}^{T}(P_i-\overline{P})^3}{T^{-1}\sum_{i=1}^{T}(P_i-\overline{P})^{3/2}}=(\beta_1)^{1/2}$$

$$\text{Kurtosis: } \frac{T^{-1}\sum_{i=1}^{T}(P_i-\overline{P})^4}{T^{-1}\sum_{i=1}^{T}(P_i-\overline{P})^{2}}=(\beta_2)$$

With these statistics the following values are calculated:

$v_1 = (\beta_1)^{1/2}/SE(\beta_1)^{1/2}$ where $SE(\beta_1)^{1/2} = (6/T)^{1/2}$

$v_2 = (\beta_2 - 3)/SE(\beta_2)$ where $SE(\beta_2) = (24/T)^{1/2}$

The null hypothesis of this test is that prices (P_i) are normally distributed. The null hypothesis is rejected when v_1 and v_2 are greater than a selected value from a standard normal distribution.

Jarque-Bera Normality Test

The Jarque-Bera test for normality is similar to the normal distribution tests, above, with the exception that the following joint statistic is computed:

$$S = (T/6)\beta_1 + (T/24)(\beta_2 - 3)^2$$

Under the null hypothesis of normality, S is distributed as a chi-square with two degrees of freedom.

There are a number of normality tests available. The skewness and kurtosis measures along with the Jarque-Bera statistic are common in the reviews of the literature and thus are computed for each time series utilized in the empirical studies detailed in chapter 4.

Examination(s) of Independence

Step three involves testing a price time series for independence. When a time series is said to be independent, this infers that prices are stochastically independent. In other words, prices are uncorrelated. If a price series is independent, then the process is completely random and no useful model can be constructed. Under the *efficient market hypothesis*, a time series of commodity prices is assumed to be independent and thus any form of technical analysis and/or modeling is futile (i.e., prices follow a random walk). To verify or dispute this assumption, a myriad of independence tests are available.

The modified specification process allows for more robust estimation of the dependencies which may exist within a time series. As seen in Figure 3.1 in the testing for independence step, enhancement is made to standard specification processes to provide for a distinction between temporal and spatial dependence. In particular, this distinction will assist in the identification of the class of processes which appear independent to the traditional temporal tests but possess forms of spatial dependencies. Cromwell et al. (1994), for example, analyzes a time series of voter partisanship and finds while many temporal tests conclude independence, spatial dependencies may exist. This section will present and discuss in detail measures such as the correlation dimension and BDS test which examine spatial dependencies. Through the use of these spatial dependence tests, the process of determining whether a system is stochastic or deterministic can be initiated.

The more traditional tests of temporal independence include those such as the Turning Point and Runs tests. Temporal based tests examine dependencies through the comparison of subsequent points (i.e., P_t is compared to P_{t-1} and P_{t+1}, etc.). The Turning Point tests examine the number of turning points in a price series to determine independence or dependence. Other temporal tests are available including the Runs test which examines the number of sign changes, either from (+) to (-) or from (-) to (+) to calculate a statistic from which independence can be determined.[14]

Many of the temporal tests available are quite popular in that they are intuitively straightforward and easy to complete. However, in light of this computational ease, spatial dependencies may go undetected. Thus, the need for more vigorous dependence detection techniques. The temporal tests of independence will be discussed in the first part of this section while the latter portions detail the more complex spatial dependence tests.

Temporal Tests: The Turning Point Test

The Turning Point test is a very simple and common method to declare dependence or lack thereof in a sequence of observations. This determination is made by computing the number of turning points present in a given sequence of observations. If a

---

[14] The data used in the Runs test is first differenced.

time series is independent then the probability of a turning point at time t is 2/3 (Cromwell et al., 1994). Specifically, turning points are defined as:

A) $P_{t-1} < P_t > P_{t+1}$

B) $P_{t-1} > P_t < P_{t+1}$

Under this test, the number of turning points (V) is calculated. Based on the following definitions of the mean and variance, a test statistic is computed:

$$E\{V\} = 2(T-2)/3$$
$$(\sigma_V)^2 = (16T) - 29)/90$$

where T is the number of observations.

The following test statistic is computed:

$$Z = |V - E\{V\}| / \sigma_V$$

This value is compared to a critical value based on the standard normal distribution, N(0, 1).

Spatial Tests

As seen in Figure 3.1, to verify the existence of spatial dependence, two relatively new methods can be utilized: (1) computing the correlation dimension, which is a graphically based technique and (2) the BDS test which provides a statistical measure. These tests are structurally much more complex than the aforementioned temporal tests but will hopefully capture dependencies which elude these tests. Both the correlation dimension and the BDS tests examine the null hypothesis of independence in a price time series, P_t, by using the concept of spatial correlation. Through examination of spatial correlations, the dynamical behavior underlying a price series is investigated. Specifically, a time series can be embedded into a reconstructed phase space to determine the dynamical properties of the system which governs the price series. Prior to discussion of the spatial correlation techniques, an understanding of phase space and the process to reconstruct a phase space are necessary.

Phase Space

A necessary step toward the determination of spatial dependence or any measurement of a dynamical system is to embed the price series into phase space. A phase space can be used as a preliminary tool for identifying the dynamics of a process. Briefly, a phase space is a graphical representation which shows all possible states of a system. In a phase space, the value of a variable is plotted against possible values of the other system variables *at the same instant in time*. For example, if a dynamical system has three descriptive variables, the phase space is plotted in three dimensions, with each variable taking one dimension. To construct a true phase space, it is necessary to know all of the relevant variables of the system. If there are five relevant variables in a system then the phase space should utilize five dimensions. Further, it is necessary to also know the equations of motion (i.e., the explicit structural form of the system of equations for the model).

Once a system has been incorporated into or embedded into a phase space, measures can then be taken to characterize the system as stochastic or deterministic. One such measure which assists in this distinction is the presence of attracting sets or attractors within a phase space. In other words, attractors allow a researcher to examine and classify the dynamical behavior of a system geometrically. It is the dimension of a system's attractor which allow distinction between deterministic and stochastic systems. Deterministic systems have finite dimensions whereas processes arising from stochastic systems have infinite dimension.

As examples of phase spaces consider the aforementioned, Figures 2.17 and 2.18. The system in Figure 2.17 is spatially correlated as trajectories of that system will converge (or are attracted toward) a subset of the phase space. In this case the dimension of the system is said to be finite and the system is deterministic. This attraction can be seen as the trajectories in Figure 2.17 cover only a portion of the three-dimensional phase space. In contrast, the system trajectories in Figure 2.18 are not spatially correlated and fill the entire three-dimensional phase space which it is embedded into.[15] The system in Figure 2.18 is thus said to possess infinite dimension. Thus, dimension forms the distinction between deterministic and stochastic processes. In particular, the correlation dimension as well as the BDS test look for finite dimensions.

Reconstructing Phase Spaces

In practice, all of the relevant variables in a dynamic system are not known. Reasons for this may include that it is simply just to expensive to obtain data on all of the variables or, as in the case of mineral commodities, that complete knowledge of the systems variables and interactions between these variables is unknown. As a consequence, knowledge of the equations of motion of the system is not possible. This scenario is exemplified by the following example. Consider a five dimensional system of an open economy consisting of daily exchange rates, domestic interest and inflation rates and foreign interest and inflation rates (Creedy and Martin, 1994). [Note that even this simple example may be unrealistic because in practice the number of dimensions (contributing

---

[15] This result could be continued to higher and higher dimensions with the stochastic system always filling the entire space.

factors) is not known]. Suppose further that the only information available on this system is a single univariate time series of historical exchange rates with no information on the remaining four system variables. With this one piece of information available, how is it possible to obtain a phase space of the system to begin the understanding of the dynamical behavior of the system?

In the absence of a complete system of differential equations it is possible to obtain a representation of the true attractor of a dynamical system using techniques based on delay coordinates.[16] The term delay coordinates refers to delayed observations of the one observable variable. Packard, et al. (1979) first outlined using delay coordinates which enables reconstruction of attractors (if they exist) with only one observable dynamical variable of an *n*-dimensional system. With this method, it is not necessary to obtain information on the remaining *n*-1 variables of an n-dimensional system if information on one variable is known. Properly used, delay coordinates will provide a one-to-one correspondence between a reconstructed attractor and the true attractor of the system (one-to-one correspondence or mappings refers to a diffeomorphic relationship). Takens later independently provided theorems which formalized this finding in his 1981 work on turbulent flows. Further, because of this finding, for an *n*-dimensional system, it is not even necessary to know the explicit structural form of the system of equations which determine the dynamics of the *n* processes (Creedy and Martin, 1994).

The technique of using delay vectors to define coordinates in a reconstructed phase space aims at reproducing the set of dynamical states of an unknown underlying system. The central result of this method is that the behavior of the system in the reconstructed phase space and the true underlying system will differ only by a smooth local inevitable change of coordinates. In other words, it is desired for the mapping between the reconstructed system by use of delay vectors, and the true underlying system

[16] Alternatively termed delay vectors.

to be related by a diffeomorphic mapping (i.e., the mapping is one to one). As examples of one-to-one and non-one-to-one mappings consider Figures 3.3 and 3.4. Figure 3.3 displays a mapping which would be considered to be diffeomorphic, for every coordinate in the original space, Y, there exists a corresponding point in X.[17] Further, if the mapping, Φ_1, is diffeomorphic, the inverse exists (i.e., the mapping Φ_1^{-1} exists) and thus for every point in X (which is obtainable) there exists a corresponding point in Y. In Figure 3.4, Φ_2 is not a diffeomorphism as it is not a one-to-one inevitable mapping.

The process of incorporating delay vectors into a reconstructed phase space is referred to as the embedding process. The choice of embedding dimension (m) or the dimension of the reconstructed phase space is crucial because it is desirable for the mapping between the reconstructed system and the true one to be diffeomorphic. If this is the case, then an event occurring in the observable time series, will also occur in the underlying system. In the analysis of

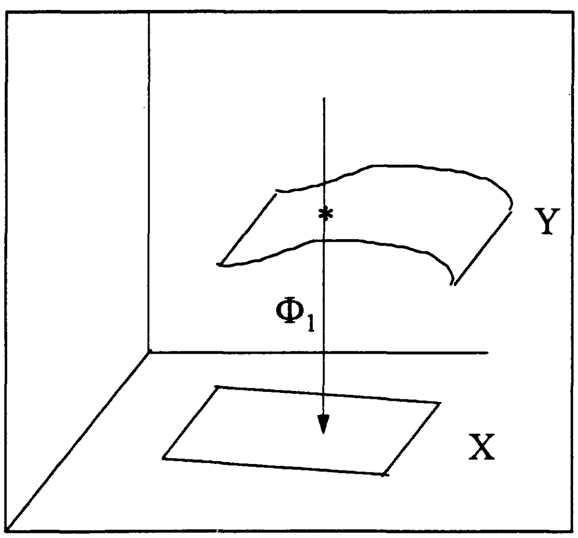


Figure 3.3 One-to-One Mapping

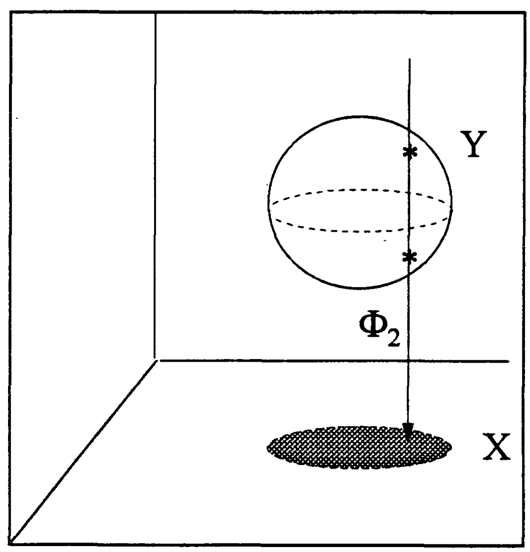


Figure 3.4 Non-One-to-One Mapping

[17] The original space, Y, can be thought of as the true system underlying a time series and space X would be the reconstructed phase space determined through the use of delay vectors of the time series.

deterministic dynamical systems, there is power in finding one-to-one mappings (Casdagli and Weigend, 1994).

Casdagli (1991) provides insight into the process of phase space reconstruction. Consider Figure 3.5 which depicts the true behavior of a dynamical system, *f*, and the states of this system, the initial state, s(0), and a later state, s(τ), which is τ time periods later. This system can be thought of as the system which underlies or generates a mineral commodity price series. Each state of this system is represented by the following mapping:

$$S(t) = f^{\tau}(s(0));$$

where S(t) is the state at time t.

As is the case with this study, the only view of the underlying system is through a time series, P(t), generated by the system (Note: Figure 3.6). In the absence of noise, the observable time series and the unobservable true system are related by the following:

$$P(t) = h(s(t))$$

where *h* is the measurement function and is also unobservable (i.e., it is not known how the price series is generated). Because both *f* and *h* are unknown, it is not possible to reconstruct states (i.e., s(t)'s) to their original form. However, it may be possible to construct a phase space that is in some sense equivalent to the original (Casdagli, 1991). Thus, the term phase space reconstruction.

The reconstruction process begins with the construction of delay vectors of the observable time series (Figure 3.7). The length of each delay vector is a function of two parameters, the embedding dimension, m, and the time delay, τ. Each delay vector defines

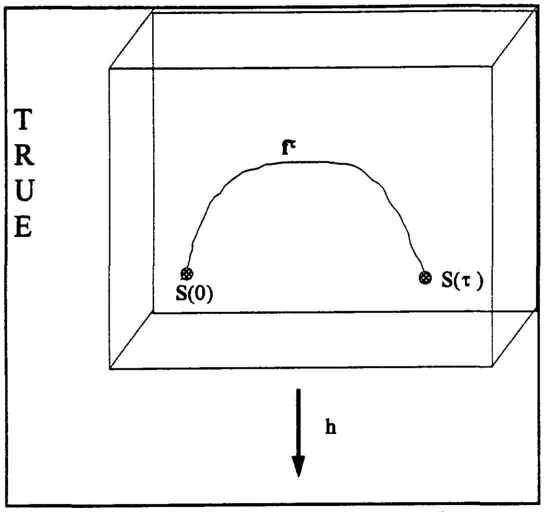


Figure 3.6 True Representation of a Dynamical System

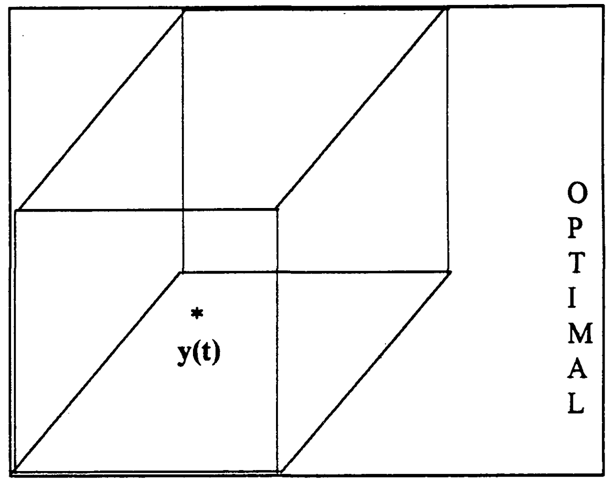


Figure 3.5 Corresponding Point in Reconstructed Phase Space

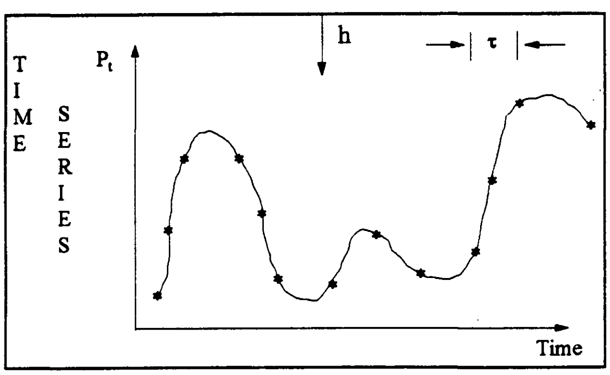


Figure 3.7 Observable Time Series

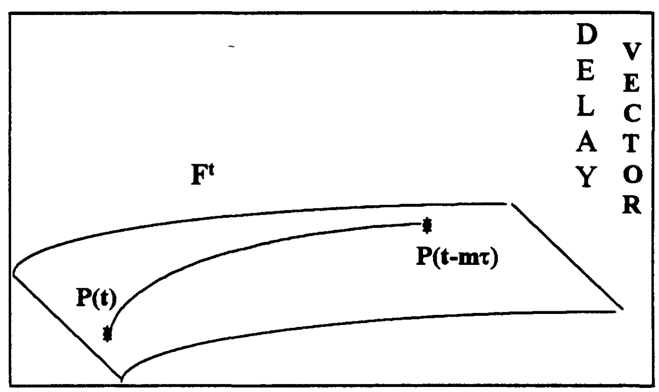


Figure 3.8 Delay Reconstruction Map

a point in the reconstructed phase space (Figure 3.8). With multiple delay vectors it is possible to obtain multiple points in the reconstructed phase space. From these reconstructed points, dynamical properties can be determined. If the delay vectors have been properly constructed, the mapping (or relation) between the reconstructed space (Figure 3.8) and the true, unknown, underlying process (Figure 3.5) will be diffeomorphic. Thus it is possible to verify/predict the behavior in the true system through observation and testing of the reconstructed system.

As a simple explanatory example, consider an annual copper prices series in cents/Lb. for 10 years (T=10). Suppose we want to understand the dynamic behavior of an economic system which has *n*-variables, of which, one variable is copper prices. The question is can we uncover the dynamic behavior of this unknown n-dimensional system which governs the behavior of copper prices with just the 10 observation time series of copper prices? The reconstruction process requires embedding delay vectors of the copper price series into an m-dimensional space called the reconstructed phase space. In terms of this study, the embedding dimension (m) will be formally estimated but for this specific example, suppose the embedding dimension is four. Another input which is required is the delay time (τ) between observations. This parameter refers to which observations should be included in the delay vectors.[18] Table 3.1 shows the m-dimensional delay vectors of the copper price series when $\tau = 1$ is selected. The columns, P^4_{T-1}, P^4_{T-2}, ..., P^4_4 are the delay vectors.

These delay vectors are used to reconstruct the phase space from which it is possible to measure the dynamical properties. Note that only those observations above the shaded cells in Table 3.1 are used in the reconstruction. The four observations below

---

[18] τ refers to how prices will be used to construct delay vectors. τ=1 implies neighboring points will be used (i.e., P_t, P_{t-1}, P_{t-2}, ...). $\tau = 2$ would imply every other observation is included (i.e., P_t, P_{t-2}, P_{t-4}, ...).

this line are lost in the process of reconstructing the space (when m=4). This is one reason that this process requires significant data.

Table 3.1 Example Delay Vectors

Observation	Delay Vectors				
	Original Series (P^4_T)	P^4_{T-1}	P^4_{T-2}	...	P^4_4
T	40	38	41	...	37
T-1	38	41	39		41
T-2	41	39	38		44
T-3	39	38	32		45
T-4	38	32	37		.
T-5	32	37	41.		.
T-6	37	41	44		
T-7	41	44	45		
T-8	44	45	.		
T-9	45	.			.

Input Parameters Needed for Spatial Dependence Techniques

The correlation dimension as well as the BDS test rely upon reconstructing attractors in phase space from a set of delay vectors. The existence of these reconstructed attractors form the basis of whether or not a given system is spatially dependent or correlated. Critical in this process is the appropriate selection of two input parameters: (1) the embedding dimension (m) and (2) the time delay (τ).

In the absence of noise, a reconstructed attractor would be obtainable for any sufficiently large m (Takens, 1981). However, when dealing with financial or any economic system, to obtain an accurate representation, it is critical to select the appropriate dimension of the reconstructed phase. If an attractor reconstruction is performed in a space whose dimension is too low, then the resulting mapping will not be smooth (i.e., not 1 to 1) and any measure of the dynamics would be questionable. If m is chosen to be too large, noise (originating from measurement, dynamic or estimation error) will tend to decrease the density of points defining the attractor (Wolf et al., 1985). Further, this noise, because it is assumed to be inherently random, will have an infinite dimension and thus fill an entire region into which it is embedded. Increasing m past what is minimally required will, in effect, unnecessarily increase the contamination present in the data (Wolf et al., 1985). This problem could eventually lead to a conclusion that a system is stochastic when in reality it is deterministic.

Several methods have been suggested for selecting an appropriate dimension, m. Takens (1981) states that an embedding will be obtained if m is chosen such that m is greater than twice the dimension of the underlying attractor (Specifically, if $m > 2d+1$).[19] For example, if the dimension of the underlying system is three, then the dimension of the reconstructed phase space should be at least equal to seven. However, it has been found that attractors reconstructed using smaller values of m often yield reliable measures of the dynamics such as the Lyapunov exponents (Wolf et al., 1985). Ding et al. (1993) state that the correlation dimension will be preserved under a delay coordinate mapping as long as $m > d$. Therefore, it is assumed that the requirement, $m > 2d+1$, by Takens will provide for an upper bound for m.

Initially, it might appear that the embedding dimension might be the most crucial element in determining spatial dependence. However, all that is required is that the reconstructed attractor is embedded into a phase space which is greater than the dimension of the underlying (true) system. As will be demonstrated, the techniques utilized to uncover spatial dependence actually use various values of the embedding dimension. Therefore, it becomes necessary to obtain robust estimates of the other input parameter, the time delay. In his original work, Takens (1981) states that, in principal,

[19] Literature notation provides the following definition: d = dimension of the true, underlying, system.

when using delay vectors to reconstruct an attractor, an embedding of the original attractor is obtained for any sufficiently large m and almost any choice of time delay, τ. Unfortunately, the literature is extremely vague on how to specifically choose an appropriate time delay. However, Wolf et al. (1985) defines the following relation:

$$\tau = Q/m$$

where: τ = time delay
m = embedding dimension
Q = the mean orbital period or average cycle length

The above relation states that the time delay (τ) is the ratio of the mean orbital period and the embedding dimension or percent of an orbit within each dimension. This ratio ensures that the orbital period does not change as the dimension is increased (Peters, 1991a).

This modified specification process utilizes the above relation to obtain sound estimates of the time delay which will be used in the construction of the delay vectors. Figure 3.9 details the process which will be used to obtain robust estimates of the embedding dimension, m, and the time delay, τ. The process for obtaining time delay estimates involves finding the mean orbital period inherent in the time series. The mean orbital period will be found through the use of rescaled range (R/S) analysis.

With estimates of m and τ, it will be possible to obtain a reconstructed system in phase space. Estimates of these will allow for the subsequent robust determination of the correlation dimension and the BDS test statistic of the dynamical systems governing behavior of mineral commodity prices. However, as seen by Figure 3.9, to obtain τ and m, the determination of the mean orbital period (Q) for a given price series must first be completed.

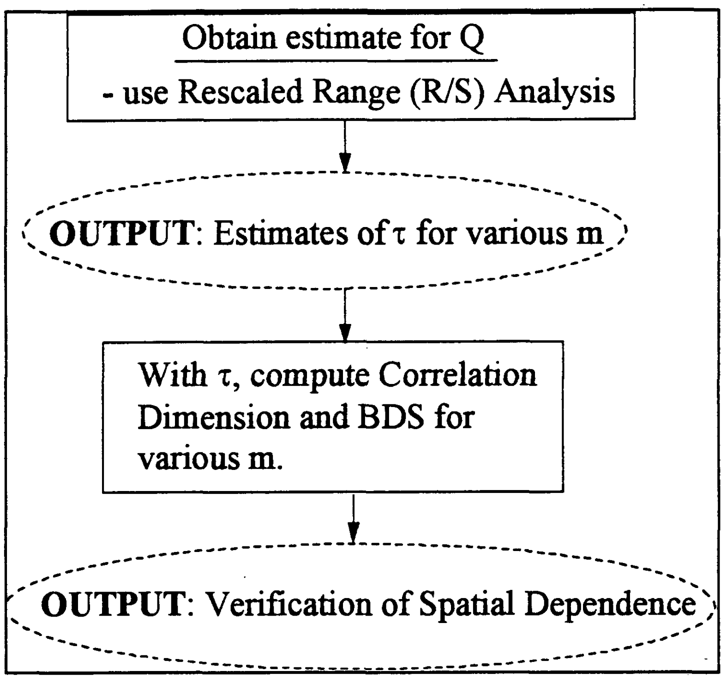


Figure 3.9 Methods Used to Determine Spatial Dependence

Calculating the Mean Orbital Period. The mean orbital period may be thought of as the point in a time series at which the effect of an initial condition is no longer felt. For example, if the orbital period is equal to five, then events occurring in period one would be fully accounted for by the market by the sixth time period. In other words, the mean orbital period refers to how long a particular event takes to dissipate through a system. If investors obtain information, the mean orbital period refers to how long it takes for the impact of that information to be fully realized. It provides an indication of the memory process inherent in the system. Important to note is that under the hypothesis that a series follows a random walk, the mean orbital period would be equal to one as prices do not contain memory of prior events (i.e., prices are independent and uncorrelated). Further, under a random walk, people react immediately to new information.

To estimate the mean orbital period within a mineral commodity price series, an analytical technique developed by Hurst in the 1920s called Rescaled Range (R/S) analysis will be utilized. Hurst first used R/S analysis in his examination of rainfall levels in the ecological regions of the Nile river basin. Specifically, Hurst utilized R/S analysis to test the then accepted belief that rainfall levels from year to year followed a random walk. Through the use of R/S analysis, Hurst discovered that many of the time series encountered in nature, such as rainfall levels in the Nile region do not follow random walks (Hurst, 1920). In other words, the Nile had a memory in that rainfall levels from year to year were related in some manner. Hurst also applied this technique to other natural events such as the number of sunspots and the annual growth of rings of American trees finding similar results.

R/S is a structurally straightforward analytical method and has been applied in a great variety of applications, particularly in the analysis of dissipative systems. Bunkov and Volovik (1993) used R/S analysis to verify the existence of long-lived signals introduced to magnetic fluids. Yang's (1962) work concerns the long-range correlation of particle order in liquid helium and superconductors. Nunes et al. (1992) analyzes the diffusion rates in systems whose spin dynamics are initially perturbed. In the financial literature, Peters (1991b) uses R/S analysis to examine long-run memory in various stock prices and in the S&P 500.

Rescaled Range (R/S) Analysis. R/S analysis will be applied to mineral commodity price series. The benefits of using R/S are twofold. First, an estimate of the mean orbital period for each price series will be obtained. In addition, R/S will provide another statistic called the Hurst exponent associated with each series. The R/S process involves two phases.

Phase one involves the calculation of the actual R/S estimates for each time lag τ. With either the logarithmic returns (i.e., $S_t = \ln(P_t/P_{t-1})$) or the raw closing prices, themselves, the following procedure is executed:

$$X(t, \tau) = X(t,\tau) = \int_0^t \left(P_s - \overline{P}_\tau\right) ds$$

where $\overline{P}_\tau = \frac{1}{\tau}\int_0^\tau P_s ds$ is the average price of P over period τ.

$X(t, \tau)$ is the cumulative deviation of t prices about a mean price over τ periods. The standard deviation of prices over the period τ is given by

$$S(\tau) = \left[\tau^{-1}\int_0^\tau \left(P_s - \overline{P}_\tau\right)^2\right]^{\frac{1}{2}}$$

For each period τ, obtain the maximum deviation (range) of $X(t, \tau)$:

$$\text{Range} = R(\tau) = \underset{0<t<\tau}{Max}\, X(t,\tau) - \underset{0<t<\tau}{Min}\, X(t,\tau)$$

By dividing the Range by the standard deviation (S) of the τ observations, it is possible to obtain a dimensionless measurement R/S for the lag period (τ) in question. In other words, for a time series with 12 observations, this procedure will give six R/S estimates for N = 2, four R/S estimates for N=3, etc. The final R/S estimate for each lag period is the average R/S for the N calculated estimates. The R/S values should increase as the time lag (N) increases. The reason for this is that the standard deviation decreases by this procedure while the range itself is unaffected (Giaever and Keese, 1989).

Hurst Exponents. Aside from estimating the mean orbital period of a dynamical system, the use of R/S analysis provides another useful measure, the Hurst exponent (H). Hurst exponents contain information about each time series in their own right. Hurst exponents provide estimates of the likelihood that two consecutive events are likely to occur. For example, if $H = .75$, then if the move from P_{t-2} to P_{t-1} was positive then there is a 75% probability that the move from P_{t-1} to P_t will also be positive.[20] The Hurst exponent can be separated into three distinct classifications (Table 3.2).

Table 3.2 Hurst Exponent Classifications

Hurst Exponent	Series Characteristics
$H = .5$	Denotes a random series. Events are random and uncorrelated.
$0 \leq H < .5$	Refers to a system which is mean reverting. If the move from P_{t-1} to P_t was positive, then most likely the move from P_t to P_{t+1} will be negative. There is trend antipersistent behavior.
$.5 < H \leq 1.0$	The time series is trend reinforcing. If the move from P_{t-1} to P_t was positive then most likely the move from P_t to P_{t+1} will also be positive. Closer values are to 1.0 indicates the strength of this trend reinforcing behavior.

[20] However, the Hurst exponent states nothing about the magnitude of the change.

The Hurst exponent measures the impact of information on the mineral commodity price series. Under the efficient market hypothesis, yesterday's events do not impact today's price. The events are uncorrelated and thus $H \approx .50$. Old information has already been discounted by the market. However, if $H > .50$, this implies that yesterday's events do impact future prices. Information received today continues to be discounted by the market after it has been initially received. As stated by Peters (1991a), "This is not simply serial correlation, where the impact of information quickly decays. It is a long memory function; the information can impact the future for long periods of time." Therefore, by using R/S analysis it is possible to obtain an estimate of how long information impacts future prices or the mean orbital period (Q).

The Hurst exponent is formally defined via the previously defined Rescaled Range procedure:

$$\frac{R(\tau)}{S(\tau)} \underset{\tau \to \infty}{\approx} \tau^{H}$$

Taking the log of both sides yields the simple equation.

$$\mathrm{Log}(R/S) = H(\mathrm{Log}(\tau)) + \mathrm{Log}(A)$$

where A is considered to be a constant. H, can therefore be estimated through the use of linear regression techniques.

Correlation Dimension

With robust estimates of the delay time and the embedding dimension, it is possible to calculate the dimension of the dynamical systems which underlie mineral commodity prices. Roughly speaking, the dimension measures the number of degrees of freedom that are relevant to the dynamics of the system. Most of the early work on dimension estimation has concerned maps such as the logistic and Henon mappings. Other work has concerned systems of coupled differential equations such as the Lorenz and Rossler models. Again, these are systems in which the complete system of differential equations are known.

The correlation dimension has become the most widely used measure of chaotic behavior (Albano et al., 1988). One of the reasons for the popularity of the correlation dimension is the relative ease which it can be calculated from a univariate or scalar time series. The calculation typically starts by reconstructing the system's trajectories in an "embedding" or phase space using the method of time delays or delay vectors. The dimension of the reconstructed trajectory (or attractor) is found by using the Grassberger-Procaccia (GP) algorithm (1983).

There are unfortunately many potential problems that can hinder the successful application of dimension estimation techniques such as the GP algorithm. Generally, these problems result from noise which is inherent in the system and from a lack of data. Ding et al. (1993) test the correlation dimension to determine the effects which occur from a lack of data. In this analysis, results from a data set consisting of 50,000 points are compared to those from a set consisting of 2,000 observations. They find that a lack of data will not only delay plateau onset, from which the correlation dimension is determined, but also make deviations from the plateau behavior occur at smaller values of m, thus shortening the lengths from both sides. Noise clouds the dimension estimate in that estimates of the correlation dimension will be biased upwards which can lead to the spurious result that a system is stochastic when it may truly be deterministic.

Another problem with the correlation dimension is that when a deterministic process has a very high dimension, the relationship between the correlation dimension and m, for moderate sizes of m, become similar for deterministic and stochastic processes. This is why the literature has in the past and currently focuses on finding low-dimensional deterministic processes. This study, therefore, attempts to uncover low dimensional deterministic behavior within mineral commodity prices.

One problem with computing the correlation dimension using the GP algorithm is that it is a graphical approach and does not represent a formal statistical test. Dimension using the GP method relies on determining the linear portion of the log of the correlation integral versus the log of the Euclidean distance curve. Brock, Dechert and Scheinkman (1987) overcome this problem with their derivation of the BDS test statistic.

Grassberger-Procaccia Algorithm

To measure the spatial correlation of scattered points or discrete prices in an m-dimensional phase space, the GP algorithm utilizes a measure called the correlation integral ($C_{m,T}(\varepsilon)$). The correlation integral is defined as the probability that a pair of points chosen randomly on the attractor are separated by a Euclidean distance less than (ε). The correlation integral can be interpreted as the fraction of the first m, T-histories that are within ε of each other.

$$C_{m,T}(\varepsilon) = \frac{1}{T(T-1)} \sum_{i=1}^{T} \sum_{j=1}^{T} H(\varepsilon - \|P_i^m - P_j^m\|), \text{ for all } i \neq j.$$

where: $\| \; \|$ is the Euclidean distance[21] and H(y) is referred to as a Heavyside unit step function defined as:

[21] The original GP algorithm uses the Euclidean norm, however, many studies utilize the maximum norm which provides similar results with greatly reduced computational time.

$$H(y) = \begin{cases} 1 & \text{if} \quad y>0 \\ 0 & \text{otherwise} \end{cases}$$

The correlation dimension (D^m) is defined as

$$D^m = \lim_{\varepsilon \to 0} \left\{ \frac{\log C_{m,T}(\varepsilon)}{\log(\varepsilon)} \right\}$$

In practice, one searches to see if the dimension of a system, D^m, stabilizes (or plateaus) at some value D, as the dimension, m, is increased (i.e., D^m is independent of m as m increases).[22] If this occurs, then the value of D^m at the point of plateau onset is the correlation dimension estimate. If, however, D^m increases at the same rate as which m is increased toward infinity then the system is taken to be high dimensional or "stochastic." At present, there is no theoretical difference between high dimensional chaos and stochastic behavior. If a low value of D^m is found then the system is said to be deterministic.

The GP procedure essentially consists of plotting log ($C_{m,T}$ (ε)) against log(ε) and looking for a portion over which the plot is approximately linear; the slope over that portion is the estimator of dimension. The embedding dimension, m, is then increased until the values of D appear stable. Peters (1991a) states that the initial value of the distance should be approximately 10% of the difference between the maximum and minimum values in the original time series. The program which calculates correlation integrals is included in the appendix.

Figure 3.10 displays an example Log Log plot of the correlation integrals over increasing Euclidean distances for embedding dimensions, m = 2, 4, 6, 8, 11. For each

[22] The correlation integrals are said to be invariant over this plateau region.

curve, dimension estimates using the GP method relies on determining the linear portion of the log of the correlation integral versus the log of the Euclidean distance curve.[23]

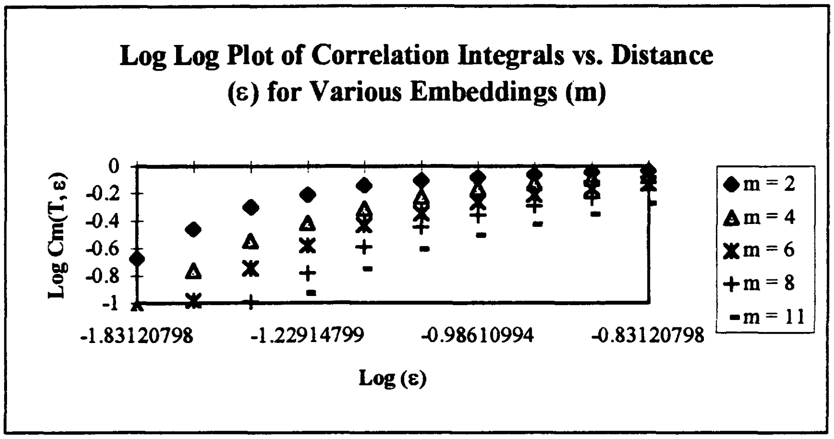


Figure 3.10 Example Correlation Integrals

Figure 3.11 displays correlation dimension estimates for both a deterministic and a stochastic example time series. Notice that the correlation dimension estimates appear to plateau for the deterministic series. This provides an indication for spatial dependence. In contrast, the stochastic series does not plateau at any portion of the figure but rather increases as the embedding dimension is increased.

The GP algorithm has been used in the analyses of many financial and economic applications. Frank and Stengos (1989) use the correlation integral to estimate dimension for gold and silver rates of return. Barnett and Chen (1987) examine monetary aggregates with the correlation integral to find low correlation dimensions. Correlation integrals have also been used in the analysis of macroeconomic time series by Brock and Sayers (1988).

[23] The linear portion can be determined visually (Peters, 1991a) or by more refined statistical methods which obtain the linear portion of the curve through minimum variance of best fit estimation curves (Krantz et al., 1995).

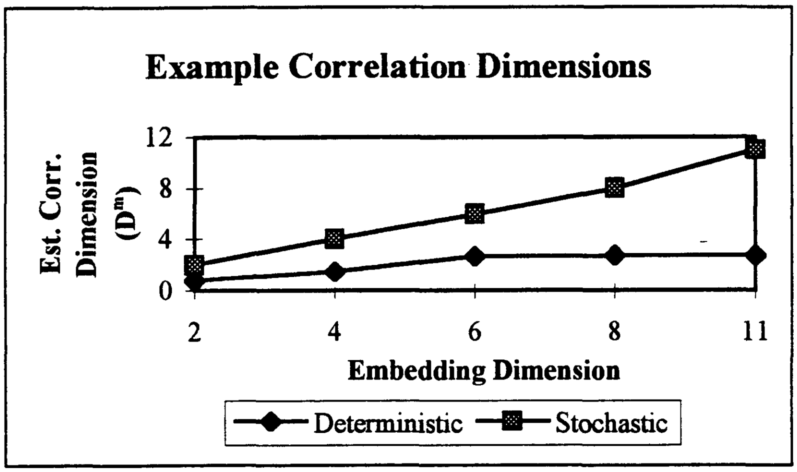


Figure 3.11 Dimension Estimates for Increasing Embedding Dimensions

BDS Test Statistic

To overcome the problems of computing a correlation dimension of finite length time series using the Grassberger-Procaccia method, Brock, Dechert and Scheinkman (1987) have developed a BDS test statistic. The BDS test examines spatial dependence in a price series through the use of the correlation integral. The BDS test for spatial independence provides a test statistic which under the null hypothesis is asymptotically normally distributed. As is the case in the determination of the correlation dimension, the BDS test relies upon the two input parameters, m and τ. The benefit of utilizing the BDS test is that the determination of spatial correlations is based on a statistical test rather than on graphical analyses in which the analyst has vast discretion in determining slopes and plateau regions.

The BDS test also employs correlation integrals to obtain a statistical test of spatial dependence. Recall that $C_{m,T}(\varepsilon)$ can be interpreted as the fraction of the first m, T-histories that are within ε of each other. Under the null hypothesis of this test, if a series

(P_t, P_{t-1}, P_{t-2}, ..., P_{t-T-1}) is independent and identically distributed (IID), BDS have shown $C_{\varepsilon, m} = (C_{\varepsilon, 1})^m$. The BDS test statistic is given by:

$$W(\varepsilon,m,T) = \frac{T^{1/2}[C_{m,T}(\varepsilon) - C_{m,T}(\varepsilon)^m]}{V^{1/2}}$$

where the estimated variance, V, is defined by:

$$V = 4\{K(\varepsilon)^m + 2[\sum_i K(\varepsilon)^{m-i} C(\varepsilon)^{2i} + (m-1)^2 C(\varepsilon)^{2m} - m^2 K(\varepsilon) C(\varepsilon)^{2m-2}]\}$$

for i=1 to m-1

The variance term is a function of two indicator function C(ε) and K(ε) which are defined as:

$C(\varepsilon) = E\{I[p_i, p_j; \varepsilon]\}$ and

$K(\varepsilon) = E\{I[p_i, p_j; \varepsilon]\ I[p_i, p_j, p_k; \varepsilon]\}$

The BDS statistic is dependent upon two unknown parameters, m and ε, which much be provided by the user. For a given m, ε cannot be too small because C(ε, T) will capture too few points. In addition ε cannot be too large or C(ε, T) will capture too many points (Cromwell et al., 1994). Brock et al. (1993) suggests selecting ε over a range based on the standard deviation of the time series (i.e., from 25% to 200% of the std. dev.). The embedding dimension is also chosen over a range. Brock et al. (1993) suggest $m \leq 5$ where as Cromwell et al. (1994) states the range should be from 1 to 15. From the range of the two input parameters, the BDS statistic is computed over a lattice.

The benefit of determining spatial dependence either through the correlation dimension or with the BDS test, is that a system can be classified as deterministic or stochastic. Both of these tests will be utilized in the empirical studies detailed in chapter 4. The next step in the modified specification process is to test for nonlinear dependence which is one of the requirements necessary for a deterministic system to be chaotic.

Testing for the Type of Dependence

As seen in Figure 3.1, through the use of the temporal and spatial dependence techniques, it is possible to determine if a given series possesses some type of dependencies or is independent (following a random walk). Aside from providing this distinction, the spatial techniques state whether a series is stochastically or deterministically dependent. Regardless, it is necessary to determine the nature of the dependence present as either linear or nonlinear. With dependency in a price series, the task of the analyst is to examine the intrinsic nature of the type of dependence between prices at different points and attempt to construct models that can reproduce this dependency. In the simplest case this dependence may reflect a linear trend; more complex processes may require a nonlinear representation (Cromwell et al., 1994).

Normalized BDS Test Statistic

The benefit of utilizing the BDS test is twofold in that it can be used to test for dependence within a price series and also test what form this dependence expresses itself, linear or nonlinear. It has been shown that to ensure that the BDS test is not reflecting linear dependence in the data, the data can be prefiltered. Thus any type of linear dependence which exists in the original data can be removed before applying the BDS test. As a result, the BDS test can test for nonlinear dependence within a price series.

To allow for the BDS statistic to test for nonlinear dependence, the time series is first transformed to the $[0,1]$ interval with the following:

$$P_{norm} = \frac{P - P_{\min}}{P_{\max} - P_{\min}}$$

where $P = P_{min}$, then $P_{norm} = 0$ and where $P = P_{max}$, then $P_{norm} = 1$.

With the normalized series, test statistics are computed over a grid of values.

Tsay Test for Nonlinearity

Tsay (1986) provides an alternative to the BDS test statistic for nonlinearity. This test is selected over the others such as the Keenan as the powers of these tests could be very low (Tsay, 1986). However, this test does retain the simplicity of the Keenan test. The Tsay test procedure is as follows:

A) An AR(2) model is fit to the returns series (x_t) and the residuals $\{e_t\}$ are retained (LeBaron, 1994).[24]

B) The vector (x^2_{t-1}, $x_{t-2}x_{t-1}$, x^2_{t-2}) is regressed on the vector (1, x_{t-1}, x_{t-2}) and the residuals $\{u_t\}$ are retained.

C) The model, $e_t = \beta u_t + \varepsilon_t$ is estimated through linear regression and an F-test is obtained testing the significance of the regression.

$$\hat{F} = \frac{\left\{\left(\sum \hat{u}_t \hat{e}_t\right)\left(\sum \hat{u}_t^T \hat{u}_t\right)^{-1}\left(\sum \hat{u}_t^T \hat{e}_t\right)/m\right\}}{\left\{\sum \hat{\varepsilon}_t^2/(n - M - m - 1)\right\}}$$

where summations are from $t = 1 + M$ to n and T is the matrix transpose.[25]

[24] The returns series is the raw price series which has been logarithmically differenced to achieve stationarity.

Tsay shows that the test statistic, $\hat{F}$, is asymptotically distributed following an F-distribution with 3 and n-4 degrees of freedom.

As seen above, Step 2 of the Tsay test, requires the use of vector autoregression (VAR). The use of VAR modeling has been advocated, most notably, by Sims (1980) as a method to estimate dynamic relationships between jointly endogenous variables without imposing strong a priori constraints. One convenience of using VARs is that the analyst does not have to decide which variables are endogenous and which are exogenous. In other words, VARs allow for the data rather than the analysts to specify the dynamic structure of a model. Normally, allowing the data rather than economic structure to specify model structure is very questionable. However, in the case of the Tsay test, only the determination of nonlinear structure is desired and not the actual specification of that nonlinear structure. Further, all of the problems associated with simultaneous equations models are avoided because VARs do not include any current variables among the regressors (Davidson and MacKinnon, 1993). One problem of VARs is the method requires the estimation of a great number of parameters resulting in the estimated parameters are quite imprecise.

VARs provide an alternative to multiple equation models and are useful when there is a question concerning the economic theory used to determine the correct specification of a model. A VAR makes minimal theoretical demands on the structure of a model. Thus, VAR analysis can assist in the determination of whether a given time series is linear or nonlinear. With a VAR, the analyst needs to specify only two things: (1) the set of exogenous and endogenous variables that is believed to interact and (2) the largest number of lags needed to capture most of the effects that the variables have on each other (Pindyck and Rubinfeld, 1991). The format of a VAR model is

---

[25] M in Tsay statistic calculation is the order of the autoregressive process which is this case is equal to 2 in accordance with LeBaron (1994).

$$x_{1,t} = a_{10} + \sum_{j=1}^{p} a_{11j} x_{1,t-j} + \sum_{j=1}^{p} a_{12j} x_{2,t-j} + \ldots + \sum_{j=1}^{p} a_{1nj} x_{n,t-j} + \sum_{j=1}^{r} b_{11j} z_{1,t-j} + \ldots + \sum_{j=1}^{r} b_{1mj} z_{m,t-j} + \varepsilon_{it}$$

$$\vdots$$

$$x_{n,t} = a_{n0} + \sum_{j=1}^{p} a_{n1j} x_{1,t-j} + \sum_{j=1}^{p} a_{n2j} x_{2,t-j} + \ldots + \sum_{j=1}^{p} a_{nnj} x_{n,t-j} + \sum_{j=1}^{r} b_{n1j} z_{1,t-j} + \ldots + \sum_{j=1}^{r} b_{nmj} z_{m,t-j} + \varepsilon_{nt}$$

with $x_1, x_2, \ldots, x_n$ being the endogenous variables, $z_1, z_2, \ldots, z_m$ the exogenous, p is the number of lags for the endogenous variables, and r is the number lags for the exogenous variables. The model can be estimated by ordinary least squares. When choosing the number of lags there is a tradeoff between fully capturing the dynamics of the system and having a sufficient number of free degrees of freedom (Pindyck and Rubinfeld, 1991).[26] For this analysis, the Tsay nonlinearity test will use two or less constraints.

If the process has been designated as stochastic, the next step is to specify a linear or nonlinear model based on the tests for nonlinear dependence. With a deterministic system, if nonlinear dependence is uncovered, the next step in the specification procedure requires determining if the underlying process is sensitive to initial conditions (another characteristic of chaotic behavior).

Indication of SDIC

The correlation dimension and BDS test furnish methods for determining if a system is deterministic or stochastic through the presence or lack of an attractor with a finite dimension. The normalized BDS test and Tsay tests provide verification of nonlinear dependence. However, these measures provide neither necessary nor sufficient conditions for a process to be chaotic. For example, if as a result of obtaining the

[26] In the equations given, there are a total of n(1+np+nr) parameters. Thus the number of parameters to estimate can increase very quickly as the number of lags increase.

correlation dimension, a system is found to have a fractal dimension, this does not lead to the conclusion that the system is chaotic. There exist strange attractors which have integer dimensions as well as attractors with fractal dimensions which are not strange (Creedy and Martin, 1994). If nonlinear spatial dependencies are found, the sixth step of the specification process calls for determining if a given process is sensitive to initial conditions. To test for SDIC in a deterministic system, Lyapunov exponents can be utilized. Asymptotic properties of a dynamical system can be characterized by the attractor's spectrum of Lyapunov exponents. Specifically, these exponents provide a qualitative and quantitative characterization of dynamical behavior.

For each dimension in the phase space of a dynamical system, there exists a Lyapunov exponent. Lyapunov exponents measure the average rate of exponential convergence of trajectories onto the attractor when negative, and the average rate of exponential divergence of nearby trajectories within the attractor when positive. The magnitude of an attractor's positive exponents, if any, provide a measure of its "degree of chaos" (Froehling et al., 1981). For example, in a three dimensional space, a system with all three exponents negative (-, -, -) would be a fixed point. A three dimensional limit cycle attractor would have (0, -, -) as its Lyapunov exponent spectrum. Chaotic attractors in three dimensions have a spectrum of (+, 0, -). With this type of attractor, the positive exponent indicates exponential spreading across the attractor and the negative exponent provides indication of exponential contraction onto the attractor (Froehling et al., 1981). The "0" Lyapunov exponent provides evidence that the dynamical behavior of the systems is acting like a periodic orbit. Figure 3.12 displays an example attractor for clarification. Negative exponents provide evidence that volumes (of the systems trajectories) are decreasing whereas positive exponents indicate expansions in certain directions but overall volumes are decreasing. Lyapunov exponents are related to how fast divergence or convergence of nearby orbits occur in phase space.

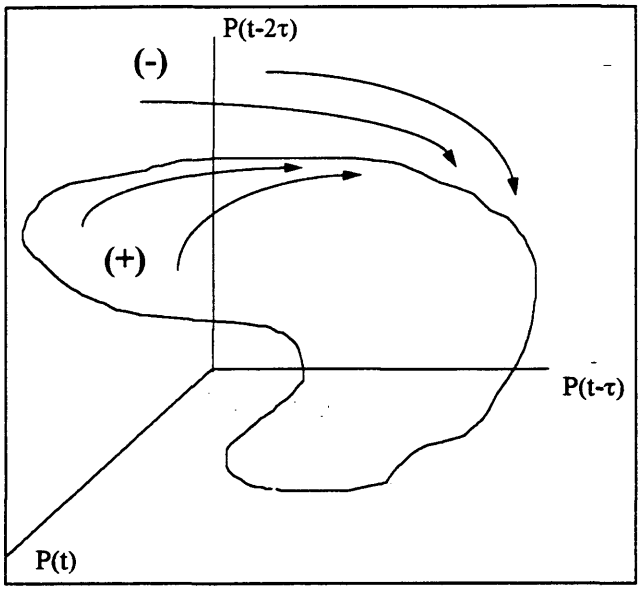


Figure 3.12 Example Chaotic Attractor

Without complete information concerning the dynamics of a system, obtaining the complete Lyapunov spectrum is impossible.[27] Wolf et al. (1985) provides a straightforward method for estimating the largest Lyapunov exponent directly from the rate of separation of neighboring points in a phase space. Aside from the embedding dimension and the time delay, this algorithm requires the analyst to input the estimated evolution time. The evolution time is the amount of time elapsed after which the distance is calculated between two previously neighboring points. The evolution time should be chosen such that it measures the stretching of the attractor in phase space but not the

[27] Using experimental data, a full spectrum of Lyapunov exponents cannot be calculated because the equations of motion (i.e., the difference or differential equations) are not known.

folding of the attractor.[28] Therefore, Wolf et al. (1985) states that the evolution length should not be greater than 10% of the attractors length in phase space.[29]

A positive Lyapunov exponent indicates the exponential spreading with the reconstructed attractor. Completion of the Wolf et al. (1985) algorithm should result in convergence to a stable value of the largest Lyapunov exponent. If stable convergence does not occur, either the parameters have not been chosen well (i.e., the embedding dimension, time delay or evolution time) or there are not sufficient data or the system is not truly nonlinear (Peters, 1991a). The programming code for this algorithm is included in the appendix.

Model Specification

Traditionally, time series analysis and prediction has fallen into two categories, the time domain or the frequency domain. In the time domain attention is centered upon the relationship between observations at different points in time. Time domain models are quite familiar including autoregressive, moving average, etc. In the frequency domain, interest is focused upon the influences of various periodic components of a time series.[30] One way to identify periodic components is to utilize spectral analysis methods (i.e., Fourier analysis). However, when a process is chaotic or may possess non-periodic cyclical behavior, spectral analysis fails as this type of analysis results in the spectrum being flat (Creedy and Martin, 1994). One alternative to analyzing a time series in either the time or frequency domain is to utilize the space domain. Use of space domain techniques provides the advantage that forecasting non-linear time series is possible.

---

[28] Creedy and Martin (1994) provides an introduction of the folding and stretching of an attractor. For more quantitative descriptions see Ott (1993).

[29] In accordance with Wolf et al. (1985), the evolution time will be chosen such that it equals 10% of the standard deviation of the data.

[30] Regular cyclic behavior is unusual in economic series, thus, when talking of periodic components, what is inferred that tendencies toward cyclical movements are centered about particular frequencies.

The final step in the model specification process is to formulate models which consider spatial correlations of the data. Resulting predictions from these models can be compared to traditional predictive models which assume only linear temporal correlations in the data such as Autoregressive (AR) techniques.[31] For time series prediction and modeling, the existence of low dimensional chaotic behavior suggests that nonlinear deterministic models should be constructed. The characteristics of low-dimensional chaos imply that precise short term prediction models are possible but predictions are impossible over longer time periods. This is due to the exponential increase of uncertainty over time (Casdagli and Weigend, 1994).

Stochastic Modeling Methods

As seen by Figure 3.1, if a price series is deemed stochastic, rather than assumed, then an appropriate stochastic model should be specified (either linear or nonlinear). AR models as all linear stochastic models assume the future value of a variable to be a linear function of past values. An AR model is estimated so as to evaluate predictions assuming the generating process of the mineral commodity price series is completely stochastic. In choosing the length of the autoregressive process to use in the model, Creedy and Martin (1994) suggests using the embedding dimension (m) which is in turn determined by the dimension of the system which generates the time series. Typically, the order of an AR model is determined through examination of the partial autocorrelation function. However, for this study, *m*-order models are used in comparison with *m*-dimensional deterministic models. Thus, a *m*-order autoregressive model will be utilized:

$$p_{t+1} = \delta + \theta_1 p_t + \theta_2 p_{t-1} + \ldots + \theta_m p_{t-m} + \varepsilon_{t+1}$$

[31] Note, this step in the specification process includes testing for lag order (i.e., how many past prices and error terms to include in the model) and subsequent testing of the residuals.

This autoregressive model is structurally straightforward and easy to compute.[32]

There are numerous extensions of the AR model including those which allow conditional variances (autoregressive conditional heteroskedastic (ARCH) and the generalized ARCH (GARCH)) and those which also allow conditional means (Bilinear and Threshold Autoregressive (TAR)). For further discussions of stochastic modeling methods, refer to Harvey, 1994.

Specifying Phase Space Modeling Methods

Methods developed by Farmer and Sidorowich (1987) and later extended by Creedy and Martin (1994) can be utilized which make use of the spatial correlation of observations in a price series. Specifically, Farmer and Sidorowich (1987) introduces *local linear models* for phase space forecasting. This method bases next-period predictions on nearest neighbors in the reconstructed phase space to the current price.[33] Local linear models recognize that any manifold (attractor) is locally linear (i.e., locally a hyperplane) (Gershenfeld and Weigend, 1994). In other words, regardless of the topological characteristics of a systems attractor region in a reconstructed phase space, within small (local) areas, this region is linear. Therefore, the Farmer methods as with its extension by Creedy, formulate predictions with familiar linear regression techniques which utilize the information from *n* neighboring points.[34]

Figure 3.13 identifies an example three dimensional attractor region in a reconstructed phase space. As this is a three-dimensional system, three points are utilized to define the reconstructed phase space, P_t, $P_{t-\tau}$, and $P_{t-2\tau}$.[35] For example, if the current position of the time series is P_{100}, local approximation methods would identify the closest

---

[32] Most commercial statistical software has the ability to solve linear stochastic models.
[33] For this reason, these techniques are termed local approximation methods, as only a subset of the dataset (i.e., those prices closest to the price of concern) are used to estimate model parameters.
[34] Casdagli (1989) suggests choosing $n = 2(m+1)$, where m is the embedding dimension.
[35] Note that τ is the time delay which is found through the rescaled range analysis.

nearest neighbors around P_{100}, to forecast $P_{(101)}$. The Farmer and Creedy methods assume that the prediction model f(t) in Figure 3.13 is linear, however, this function can also be nonlinear.

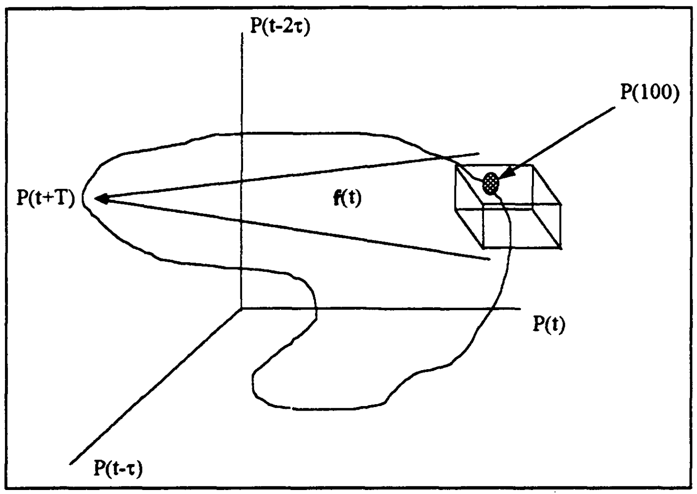


Figure 3.13 Local Approximation

Tong and Lim (1990) attempts to formulate a nonlinear class of models for natural cyclic processes based on threshold autoregression.[36] Further, simulation of nonlinear systems is the basis for neural network modeling. Nonlinear models with many parameters can be extremely flexible which may allow for them to extract features that are genuine to the underlying process (Casdagli and Weigend, 1994). However, this flexibility also gives this class of models the potential to overfit a time series. Therefore, in addition to fitting the dynamic behavior specific to a system, these models also fit the noise present in the time series. This mis-specification may result in questionable predictions.

[36] These include data from Canadian lynx , sunspots, mink-muskrat, and Kanna riverflow and rainfall.

Dynamical systems theory and the analytical measurement tools therein, originate in the physical sciences. In the realm of the physical sciences, data are usually derived from more controlled environments. For example, in the heating of a liquid, precise measurements can be used to verify the boiling point. In other words, it is easy to see when the liquid goes from a stable to an unstable state. From this point, analysis could be completed to see if the unstable or turbulent state was indeed chaotic. In the analysis of mineral commodity prices as with all economic time series, stable and unstable states are all mixed together. At this point in time, the luxury of knowing whether the data collected is from a stable or unstable state of the system is not known. As a result, economic data, especially in the financial markets, as well as other real world systems are said to have noise present. Black (1986) and DeLong (1990) discuss how the effects of noise traders in financial markets can cloud prices as well as make markets inefficient.[37] Noise complicates this type of analysis in that the presence of noise will make a truly deterministic system appear stochastic.

Because the nonlinearities of the dynamical behavior of the systems which generate mineral price series are presently not known and thus the impact of noise can not be quantified, utilizing local linear models appears to be a good first step in modeling a low-dimensional nonlinear deterministic system. The phase space prediction methods of Farmer and Creedy construct non-parametric forecasting models using only information that is nearest to the point being forecast. The local approximation methods impose a metric, the Euclidean distance ($\| \; \|$), on the reconstructed phase space to find the k nearest neighbors. Specifically, the metric defines points based on the Euclidean distance from a desired point.

---

[37] Examples of noise trading include investors not acting on actual information but on intuition or not acting rationally (i.e., the Bank of France purchasing francs, not for monetary gain but to artificially prop up the value of the franc in international markets).

For clarification consider: Suppose it is desired to predict P_{101} in Figure 3.13 from a 100 observation series of past prices:

$$\hat{P}_{101} = f(P_{100}, .., P_1)$$

As stated above, linear stochastic models such as AR methods would utilize the entire past data set (P_{100}, ..., P_1) to estimate model parameters. Local approximation methods would be more selective in utilizing only nearest neighboring points to P_{100} to estimate model parameters. The local approximation prediction methods would begin by finding the nearest points to P_{100} by minimizing the following Euclidean distances:[38]

$$\text{Distance} = || P_{100} - Z_i ||$$

where Z_i are points in the reconstructed phase space.

Note, the 2(m+1) nearest neighbors are not chosen based on a specified Euclidean distance rather on the basis which minimizes this distance. With Euclidean distances defined, algorithms can be computed to find the points with the smallest distances.

Continuing with the above example, suppose the following prices are found to be the closest (minimal distance) to P_{100}:

[38] Also called phase space prediction methods.

$$\begin{bmatrix} P_5 \\ P_{32} \\ P_{17} \\ P_{98} \\ P_{77} \\ P_{48} \end{bmatrix}$$

with P_5 being the closest in terms of Euclidean distance.

The local approximation methods then determine the next period values of the nearest neighbors which for the above example would be:

$$\begin{bmatrix} P_6 \\ P_{33} \\ P_{18} \\ P_{99} \\ P_{78} \\ P_{49} \end{bmatrix}$$

Farmer and Sidorowich (1987) present two local linear methods for which predictions can be made. The first is the *zero-order approximation* technique which simply states that the predicted value will be the next period value of the nearest point. In terms of this example:

$$P_{101} = P_6$$

as P_5 is the closest point to P_{100}.

Farmer and Sidorowich (1987) also state that a superior method to the zero-order approximation is to take the following linear approximation:

$$\begin{bmatrix} P_6 \\ P_{33} \\ P_{18} \\ P_{99} \\ P_{78} \\ P_{49} \end{bmatrix} = \begin{bmatrix} 1 & P_5 \\ 1 & P_{32} \\ 1 & P_{17} \\ 1 & P_{98} \\ 1 & P_{77} \\ 1 & P_{48} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$

Through fitting this model the following prediction is obtained:

$$\hat{P}_{101} = \alpha_1 + \beta_1(P_{100}) + \varepsilon$$

The difference between this method and a normal Autoregressive model is that the parameters α_1 and β_1 are estimated with only the nearest neighbors and not the entire data set. The implication of the local approximation models is that can better predictions occur utilizing smaller amounts of data? Further, by incorporating information from the entire price series, do stochastic methods dilute valuable information contained in the nearest neighbors?

Creedy and Martin (1994) provide an extension of the Farmer linear approximation technique. Creedy also finds the nearest neighbors of the present price in a time series, but estimates the following model:

$$\begin{bmatrix} P_6 \\ P_{33} \\ P_{18} \\ P_{99} \\ P_{78} \\ P_{49} \end{bmatrix} = \begin{bmatrix} 1 & P_5 & P_4 \\ 1 & P_{32} & P_{31} \\ 1 & P_{17} & P_{16} \\ 1 & P_{98} & P_{97} \\ 1 & P_{77} & P_{76} \\ 1 & P_{48} & P_{47} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$

This method attempts to add more information to the prediction process through the inclusion of lagged one-period prices of the nearest neighbors. Through fitting this model the following prediction is obtained:

$$\hat{P}_{101} = \alpha_1 + \beta_1(P_{100}) + \beta_2(P_{99}) + \varepsilon$$

Again, the difference between this method and a normal AR(2) model is that the parameters α_1, β_1 and β_2 are estimated with only the nearest neighbors and not the entire data set.

For comparative purposes, this study utilizes the same nearest neighbor methodologies but bases prediction on a simple average of the next period values of the nearest neighbors. Under this scenario, the following model is estimated:

$$\hat{P}_{101} = (P_6 + P_{33} + P_{18} + P_{99} + P_{78} + P_{49}) / N$$

where N is the number of nearest neighbors selected.

Under this process the best prediction for P_{101} is the average of the next period values of the chosen nearest neighbors to P_{100}.[39]

In summary, with a time series of mineral commodity prices, it is possible to utilize a myriad of prediction techniques for forecasting. These include linear stochastic methods such as autoregressive which assume that prices are linearly correlated with neighboring temporal prices. Zero-order and linear approximations by Farmer and Sidorowich (1987) and extensions by Creedy and Martin (1994) utilize spatial correlations between prices for prediction.

Evaluating Predictions

After obtaining predicted values from the various methods, the next step is to evaluate the predictions to identify superior techniques. Obviously only crude comparisons can be made of prediction techniques through visual observation of the results. One popular method to measure the predictive error of modeling methods in the root mean square error (RMS). The RMS has been popular for some time as a measure of predictive error in the physical sciences. The RMS can be normalized to assist in comparison.

The mean square prediction error for T pairs of prediction-observation is

$$MS = (1/T)\sum(F_i - A_i)^2$$

The error is (F_i-A_i), the difference between the numerical value of the prediction (F) and the actual numerical value (A) and T is the number of predictions. Thus, $\sum(F_i - A_i)^2$

[39] This could be extended to incorporate a weighted average prediction with weights determined Euclidean distance.

yields a measure of aggregate error in the T predictions, and MS is an average "squared error per prediction."

In the same way that the standard deviation permits us to work with the same units as the mean, so the root mean square (RMS):

$$\text{RMS}=\sqrt{(1/T)\sum(F_i - A_i)^2}$$

permits us to discuss "error" in the same units as the predicted variable. If information concerning the model generating the predictions is limited, the RMS can be used as a rough estimate of the "standard error of estimate" (Mills, 1977).

Farmer and Sidorowich (1987) provides the following definitions:

To evaluate the accuracy of predictions, compute the root mean square error:

$$\sigma_\Delta(\text{T}) = \left\langle \sum \left(p_{pred}(t,T) - p(t+T)\right)^2 \right\rangle^{1/2}$$

For convenience, this is normalized by the *rms* deviation of the actual data

$$\sigma_p = \left\langle \sum (p - \bar{p})^2 \right\rangle^{1/2},$$

formalizing the normalized error:

$$\text{E} = \sigma_\Delta(\text{T})/\sigma_\text{p}$$

If E = 0, the predictions are perfect; E = 1 indicates that the performance is no better than a constant predictor:

$$p_{pred} = \bar{p}$$

To estimate E, as many predictions as possible for reasonable convergence, typically on the order of *1000.*[40]

The aim of this chapter is to introduce and define each step involved in a modified time series model specification process. This process provides a more robust method in which the dynamical behavior of the system generating a time series can be determined rather than being simply assumed. Insight into the behavior of mineral commodity markets is gained in chapter 4 of this study as empirical testing of this process is applied to zinc and copper price time series.

---

[40] Note: The Farmer and Sidorowich (1987) article is from the *Physics Review Letters* journal. It is recognized that with financial prices series, the 1000 prediction limit may not be possible.

Chapter 4
APPLICATION OF THE MODIFIED SPECIFICATION PROCESS

The modified univariate time series specification process detailed in chapter 3 will be applied to both zinc and copper spot price time series. The zinc series consists of 564 monthly prices from the U.S. Department of Commerce Bureau of Economic Analysis' Current Business Statistics database ranging from January 1948 through June 1995. Two copper price series are analyzed and discussed later in the chapter. The zinc series represents monthly average prices from daily closing prices obtained from the *Wall Street Journal*. These prices are deflated using the producers price index (PPI).[41] The inflation deflated prices are shown in Figure 4.1. This time series appears to be interesting to this

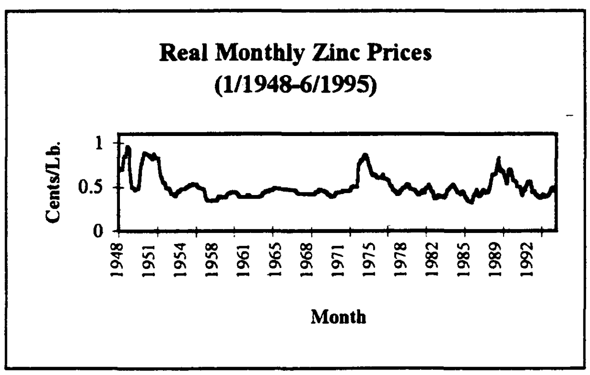


Figure 4.1 Deflated Monthly Zinc Prices

[41]The PPI is used rather than the consumer's price index (CPI) as mineral commodities are generally factors of production.

analysis as zinc prices have experienced significant influences from the tight markets which existed in the early 1950s, 1970s and 1990s to the interspersed surplus periods.

Checking for Stationarity

Following the sequence outlined in Figure 3.1, the time series is tested for stationarity. The Dickey-Fuller (DF) test is utilized in which three regression models are estimated to test whether the price series is a random walk depending on the choice of an intercept term and/or a trend term. Table 4.1 refers to the calculated τ-statistics for the zinc price series for the three regression models used in the DF test. These values are compared with critical τ values (see Davidson and MacKinnon, 1993). The results are ambiguous as the no constant model states that the real price series is nonstationary or possesses a unit root and is a random walk. However, for the constant and the constant with a trend models, the null hypothesis of nonstationarity can be rejected at the 95% confidence level. Due to these results, the real zinc price series is logarithmic first differenced.[42]

Table 4.1 D-F Results for Zinc

τ-Statistics[a]	Zinc Series
τ_{nc}	-.994
τ_{c}	-3.435
τ_{ct}	-3.474

[a] where: nc = no constant; c = constant and ct = constant with a trend.

[42] The logarithmic differenced series, $R_t = \ln(P_t/P_{t-1})$ is referred to as the zinc returns series.

Checking for Normality

The zinc price series is tested for normality in accordance with Figure 3.1. For the zinc returns series, the following test statistics are computed for skewness (v_1), kurtosis (v_2) and the Jarque-Bera joint statistic (S). These results are displayed in Table 4.2.

Table 4.2 Normality Test Results for Zinc

Test-Statistics	Zinc Series
v_1	1772.65
v_2	-14.32
S	3142

The values of v_1 and v_2 are compared to a standard normal critical value of 1.96 while the computed S values are compared to a chi-square with two degrees of freedom critical value of 5.99. These results are interesting in that the zinc series can be rejected for normality based on both skewness and kurtosis. Kurtosis in excess of 3 is an indicator of a distribution with heavy tails (i.e., more observations in the tails of the distribution than would be the case with a normal distribution). Distributions with this property are called leptokurtic.

One reason for the observed leptokurtic behavior is the number of zeros present in the returns series. This can be thought of as when the quoted price does not change from month to month. The series is found to contain ≈ 14.5% zeros. The fact that this series contains so many zeros may cause problems for traditional time series analysis and model formulation (LeBaron, 1994). Further, if the zinc price series is not normally distributed, then common statistical measures such as t-statistics, correlation coefficients, etc. are questionable and may give spurious results. Further, the case for the random walk is weakened (Peters, 1991a).

Checking Independence

The next phase of the study involves testing the price series for both temporal and spatial dependence. To test for temporal independence this study will use the Turning Point test which simply counts the number of turning points in a series. The deflated zinc price series is found to contain 303 turning points. This value along with the computed mean and variance of turning points are used to compute test statistics Z , which are compared to a critical value from a standard normal distribution. The computed test statistic for the series is

$$\hat{Z} = 7.107$$

Because this value is greater than the critical value at the 95% confidence level (1.96), the null hypothesis of independence can be rejected. This result states that the zinc returns series may possess some form of temporal dependence and it may be possible to construct a predictive model based on the observation of past prices. In contrast, if temporal independence is found, the series is assumed to follow a random walk and thus no predictive model can be constructed which would provide better results than stating tomorrow's price will be the same as today's.

Given that the turning point test states that the zinc price series is independent, the next phase of the study is to check for spatial independence. Through embedding the returns series into a reconstructed phase space, both the correlation dimension and the BDS test will be completed to provide verification of spatial correlation.

Correlation Dimension

The first step in this procedure is to obtain an accurate estimate of the time delay (τ) parameter through the measurement of the average cycle length of the price series. The average cycle length can be thought as of the length of time is takes new information to fully dissipate through the zinc market. To obtain the mean orbital periods, R/S analysis and estimates of the Hurst exponents are used. For clarification, consider Figures 4.2 through 4.4 for the zinc returns series.

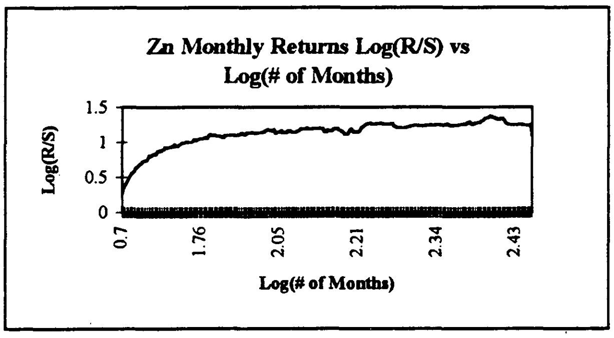


Figure 4.2 R/S Analysis for Monthly Zn Prices

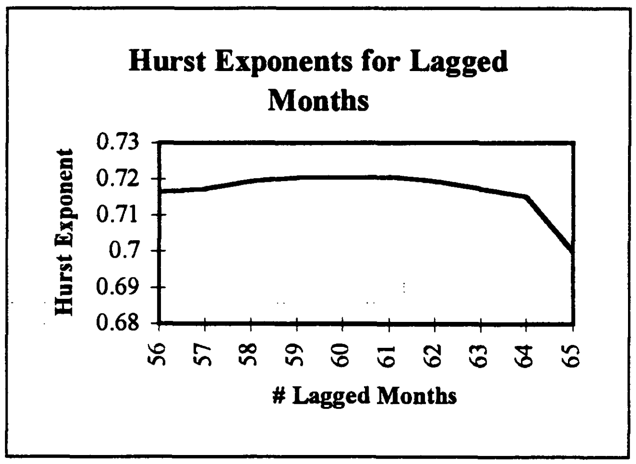


Figure 4.3 R/S Analysis to Estimate Cycle Length

Figure 4.2 displays the R/S results for various numbers of months. The slope of the R/S graph is an estimate of the Hurst exponent (H). Taking the slope of the R/S curve for various numbers of past months it is possible to identify the average cycle length as this occurs at the maximum value of H. Figure 4.3 shows estimates of the Hurst exponent for various numbers of lagged months. As seen in this figure, the maximum value occurs when the number of months is equal to approximately 61. This states that for an initial condition, such as the arrival of new information (i.e., the impact of a new environmental regulations in the Alaska mining territories on existing zinc production), on average it takes 61 months for this information to be accounted for by the market. Further, once the information has been fully accounted for, it is expected that H ≈ .50 in which case the series subsequently follows a random walk. Figure 4.4 again shows the R/S curve for the zinc series with the estimates of the Hurst exponents for the first 40 months (H = .770) and for the remaining months (H ≈ .55).

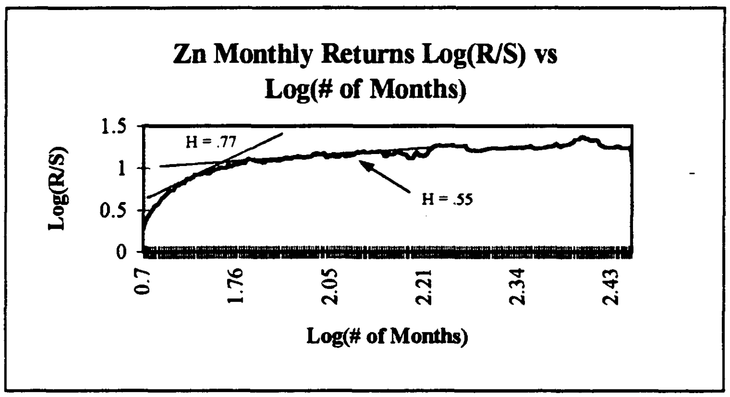


Figure 4.4 Estimated Hurst Exponents

With estimates of the mean orbital period, it is possible to obtain values of the time delay (τ) for various embedding dimensions (i.e., τ = (Avg. Cycle Length)/m). For each series, phase spaces are reconstructed based on these two input values. Correlation integrals are subsequently calculated from these inputs for increasing values of the distance within the phase space. Recall that a correlation integral calculates the probability that any two points chosen at random will be within a certain Euclidean distance from each other.

The initial Euclidean distance and the step increasing in the distance used in the calculations is equal to 25% of the standard deviation of the original price series' (Brock, and Baek, 1991). The correlation integrals, $C_{m,T}(\varepsilon)$, are computed for various embedding dimensions as the Euclidean distance (ε) is increased. The logarithms are taken of both the correlation integrals and the Euclidean distances. From the resulting Log Log plot of the $C_{m,T}(\varepsilon)$ versus (ε) the location of the linear portion is identified and the slope of the curve over that region is estimated. This slope, denoted, D^m, is taken as an estimate of the correlation dimension of the projection of the attractor to the m-dimensional reconstructed phase space (Ding et al., 1993). An estimate of the correlation dimension is obtained for increasing values of the embedding dimension. In accordance with Ding et al. (1993), correlation dimension estimates are obtained for each series for embeddings or m = 2-8 and 11.

The estimates of the correlation dimension are plotted as a function of the embedding dimension. If the estimates of the correlation dimension appear to reach a plateau for a range of large enough m values, the plateau value of the correlation dimension is taken to be an estimate of the true correlation dimension for the system underlying the zinc price series. If, however, the correlation dimension increases with increases in the embedding dimension, then this is an indication of stochastic behavior as the price trajectories are not settling down to an attracting region and are thus filling the entire region into which they are embedded.

Figures 4.5 displays a log log plot of the correlation integrals versus Euclidean distances for various embedding dimensions for the zinc returns series. Through visual inspection of the curves in this figure, linear sections of each curve are identified. Slopes of the curves over the linear sections are estimates of the correlation dimension.

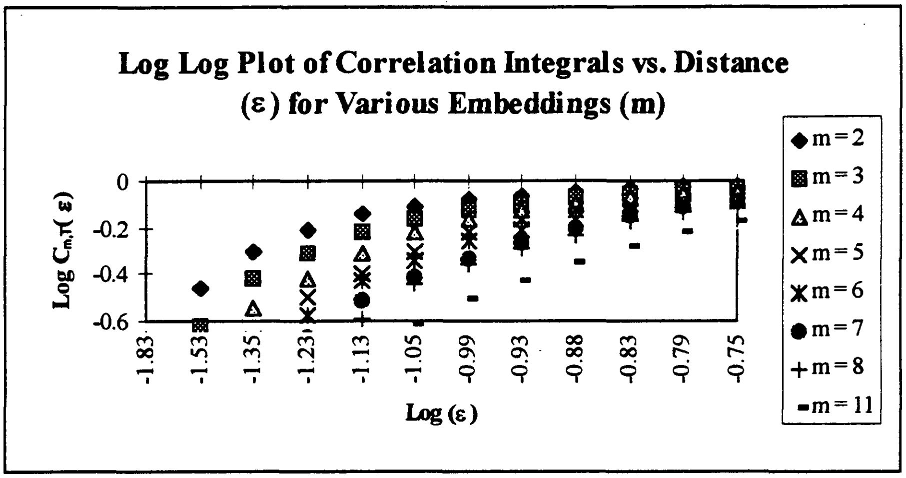


Figure 4.5 Correlation Integrals for the Zinc returns series

Figure 4.6 displays the correlation dimension estimates for the zinc returns series for increasing embedding dimensions. Notice that the correlation dimension estimates appear to plateau at approximately 1.27 as the embedding dimension is increased. This provides an indication of a fractal dimension or spatial dependence. Fractal dimension is one ingredient necessary for chaotic behavior.

BDS Test Statistics

As earlier stated to overcome the problems associated with the graphically based correlation dimension, the BDS test is used to test spatial dependence through the computation of a statistical measure. To ensure sound conclusions, the BDS test is computed over a grid of embedding dimensions and Euclidean distances. Cromwell et al., (1994) suggests choosing m over a range from 2 to 15 whereas Brock et al., (1993) state that m should be less than or equal to 5. As the zinc series is small in length, embeddings of five or less will be used. For the selection of the Euclidean distances (ε), Brock et al.,

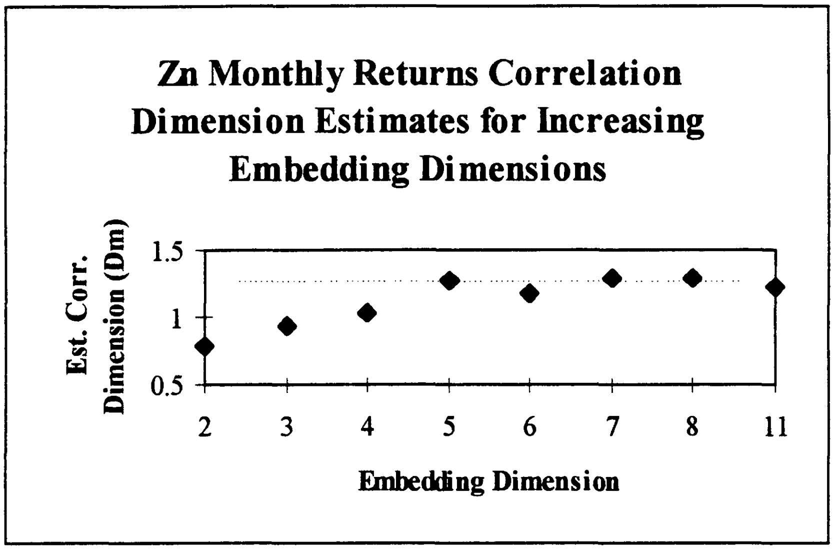


Figure 4.6 Dimension estimates for the zinc returns series.

(1993) state a range from .25 to 2 times the standard deviation of the original time series should be selected. Table 4.3 displays the grid of BDS test statistics for the zinc returns series (Note: σ_1 is the standard deviation, $\sigma_{.25}$ is 25% of the standard deviation, etc.). The first number in each cell corresponds to the raw zinc returns series whereas the second numbers are for normalized returns series (used later to test for nonlinearity). The computed statistics in Table 4.3 are compared to critical values (see Brock et al., 1993). The shaded cells in the table indicate test statistics which are over the critical value and thus signify spatial dependence in the series.

Brock et al. (1993) have shown that with series of 500 or more observations, the BDS distribution asymptotically converges to a standard normal, N(0, 1), distribution. Therefore the estimated BDS values for the zinc series are compared to standard normal critical values. As can be seen in Table 4.3, only a limited number of values exceed critical values and thus give very weak indications for spatial dependence in small dimensions (i.e., m≤5).

Table 4.3 BDS Results for Zinc

m	$\sigma_{.25}$	$\sigma_{.5}$	σ_{1}	$\sigma_{1.5}$	σ_{2}
2	1.30 1.40	1.27 1.28	.54 .80	.27 .49	.14 .32
3	1.07 1.68	1.93 1.71	1.08 1.46	.56 .97	.30 .63
4	1.50 1.53	2.34 1.87	1.60 2.02	.91 1.47	.52 .97
5	1.34 1.32	2.52 1.83	2.05 2.47	1.21 1.87	.73 1.27

Checking for Nonlinear Dependence

To ensure that the BDS test statistics are not merely reflecting linear dependencies in the data, the zinc returns series is transformed to the unit interval [0, 1] which removes linear dependencies. Through this transformation, the benefit of the BDS test is twofold, as seen in Figure 3.1, as it can be used to detect spatial dependence and verify if that dependence is nonlinear. Table 4.3 also displays the results of the BDS test for the normalized zinc returns series (bottom number in each cell). Again the BDS results are rather ambiguous for evidence of nonlinearity in the series. As a result, the Tsay test can be performed.

The Tsay test utilizes vector autoregressions (VARs) to test the statistical significance of nonlinear or cross terms (i.e., $P_{t-1}*P_{t-2}$) after the linear terms in a series have

The Tsay test utilizes vector autoregressions (VARs) to test the statistical significance of nonlinear or cross terms (i.e., $P_{t-1}*P_{t-2}$) after the linear terms in a series have be eliminated.[43] Tsay demonstrates that the test statistic is asymptotically distributed as an F-distribution with 3 and T-4 degrees of freedom, where T is the number of observations in the series. For the zinc returns series:

$$\hat{F} = 70.4$$

The critical value where T ≈ 550 at the 95% confidence level is 2.60. The Tsay test shows strong evidence of nonlinearity within the zinc returns series.

Checking the Largest Lyapunov Exponent

Through determination of the correlation dimension, it is shown that the zinc returns series shows evidence of spatial dependence or has a fractal dimension. Through use of the Tsay test, nonlinearity has been expressed in the series. However, before chaotic behavior can be concluded in the zinc returns series, the series must be analyzed for sensitive dependence on initial conditions. One method to verify the existence of this condition is to compute Lyapunov exponents. Convergence of Lyapunov exponents to a positive value provides this verification. If converge does not occur then the calculation needs to be redone or the system is not chaotic. For the returns series the set of largest Lyapunov exponents are calculated utilizing the algorithm provided by Wolf et al. (1985). As seen in Figure 4.7, the largest Lyapunov exponent for the zinc returns series appears to converge to a small positive number (approximately 0.0222). This result is interesting in that it provides slight evidence for sensitive dependence upon initial conditions. As stated

[43] VARs can be used instead of multiple equation models; first introduced by Sims (1980).

by Froehling et al. (1981),"The magnitude of an attractor's positive exponents, if any, provide a measure of its 'degree of chaos.'"

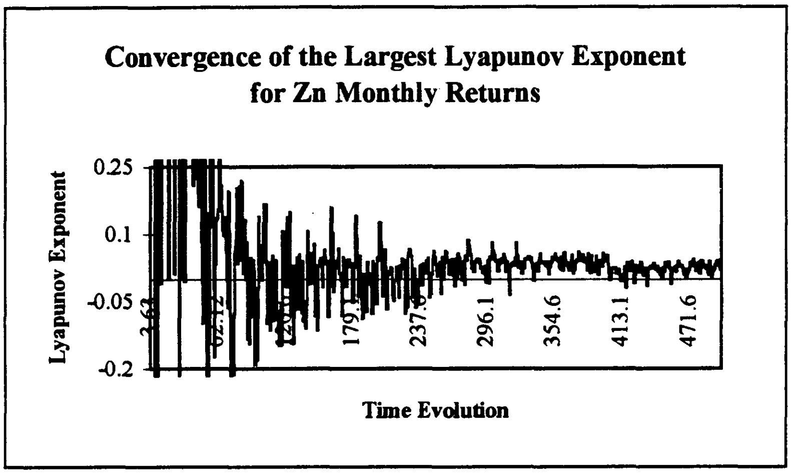


Figure 4.7 Largest Lyapunov Exponent for Zn Monthly Returns

Predictive Models

To test the predictive power of the aforementioned modeling techniques, 100 points are identified from the 564 observation zinc price series through the use of a random number generator.[44] Table 4.4 identifies the first 10 of the 100 randomly selected time periods for which predictions are completed for each of the prediction techniques introduced in chapter 3. Note, for the phase space prediction methods (i.e., Creedy, Farmer-Linear, Farmer-Zero and NN-Average), the embedding dimension for the zinc price series is found to be equal to 2, therefore, according to Casdagli (1989), the number of nearest neighbors for this series will be six.

---

[44] The random number generator is from EXCEL and based on a uniform distribution.

Table 4.4 Predictions of Random Time Periods for Various Techniques (cents/lb.)

Time Period	Actual	Creedy	Farmer-Linear	Farmer-Zero	NN-Average	AR(2)
5	0.4795	0.4845	0.4866	0.4893	0.4969	0.4874
506	0.6095	0.5945	0.6671	0.6689	0.6427	0.5858
420	0.4911	0.5043	0.5140	0.5335	0.5131	0.5195
310	0.4286	0.4298	0.4250	0.4291	0.4316	0.4312
511	0.6837	0.5684	0.7009	0.5883	0.7019	0.7132
342	0.7359	0.6999	0.7156	0.5883	0.7019	0.7283
255	0.3959	0.3964	0.3968	0.3959	0.3984	0.4011
458	0.4142	0.3977	0.3948	0.3741	0.4012	0.4167
188	0.3808	0.3836	0.3872	0.3793	0.3884	0.3842
319	0.4645	0.4890	0.4619	0.4733	0.4782	0.4652

Note: Farmer-Linear denotes the linear approximation method,
Farmer-Zero is the zero approximation technique and
NN-Average is simply the average of the nearest neighbors next period forecast

As seen on this table, the first point prediction is for the 5th observation in the zinc series which has an actual value, P_5 = 0.4795 \$/lb. For this particular observation, the Creedy method appears closest to the actual price with a forecast of 0.4845 \$/lb. The linear approximation method derived by Farmer is the next closest forecasting 0.4866 \$/lb. Point forecasts are completed for each of the 100 selected points for each modeling technique.

Based on the evaluation methods described in chapter 3 of the root mean square error (RMS) and the normalized root mean square error (E), Table 4.5 lists these values for each of the various prediction techniques over the 100 randomly selected predictions. As can be seen by Table 4.5 the second order autoregressive technique and the Farmer linear approximation method offer the best prediction results as evidenced by the lowest RMS and E values. The zero-approximation appears to be the least accurate of the chosen methods. Extending the linear approximation to incorporate lagged values of nearest neighbors (Creedy) does not appear to enhance predictive capability.

Table 4.5 Prediction Results for Various Modeling Techniques

Modeling Method	RMS	E
AR(2)	2.178368	0.220048
Farmer (Zero Approx.)	4.360455	0.440472
Farmer (Linear Approx.)	2.224386	0.224697
Creedy	3.130381	0.316216
Near Neighbor Average	2.423913	0.244852

Application to Copper Spot Price Series

To test the specification process on other mineral commodity price series as well as on series measured over different intervals, monthly and weekly copper spot price series are obtained. For this phase of the study, 204 monthly and 886 weekly copper closing price quotes were obtained ranging from January 1972 through December 1989 for the COMEX standard cathode contract. As this study is concerned with the behavior of real prices and not with the effects of inflation, prices are deflated to 1987 constant dollars using the producer price index.

To test for stationarity, the Dickey-Fuller test is utilized in which three regression models are estimated to test whether the price series is a random walk depending on the choice of an intercept term and/or a trend term. Table 4.6 refers to the calculated τ-statistics for both the monthly and weekly price series for the three regression models used in the Dickey-Fuller test.

Table 4.6 DF Results for Copper Series

τ-Statistics	**Monthly**	**Weekly**
τ_{nc}	-.966	-.893
τ_{c}	-2.013	-1.908
τ_{ct}	-2.634	-2.481

Because the computed τ statistics are less than the critical τ values in each of the three regression models for each series, the null hypothesis is accepted that the monthly and weekly copper prices are nonstationary or possess a unit root and therefore follow a random walk. It is important to note that if the model specification process were to end at this point, the conclusion would be that these series simply follow a random walk and the best prediction for tomorrow's price would simply be today's value.

Table 4.7 summarizes the results of the normality tests performed on both copper price series.

Table 4.7 Normality Test Results for Copper Series

Test Statistic	**Monthly**	**Weekly**
v_1	3.01E-17	2,740
v_2	-8.8464	8,323.8
S	76.5	76,798

These results are interesting in that the monthly series can be rejected for normality based on kurtosis. The weekly price series can reject normality based on skewness and kurtosis.

If these copper price series are not normally distributed, then common statistical measures such as t-statistics, correlation coefficients, etc. are questionable and may give spurious results. Again, the case for the random walk and the *efficient market hypothesis* is weakened.

The Turning point test is used as a test for temporal dependence in the copper price series'. The number of turning points is obtained for each price series. The monthly and weekly copper price series are found to contain 136 and 575 turning points, respectively. These values along with the computed means and variances of turning points are used to compute test statistics, Z, which are compared to a critical value from a standard normal distribution. The computed test statistics for the series are

Monthly: $Z = .3344$
Weekly: $Z = 1.12$

Because these values are lower than the critical value at the 5% significance level (1.96), the null hypothesis of independence cannot be rejected. This result states that both monthly and daily copper prices follow a random walk and thus no predictive model can be constructed which would provide better results than stating tomorrow's price will be the same as today's. Given that the turning point tests state that both copper price series are independent, the next phase of the study is to check for spatial independence. To do this, both the correlation dimension and the BDS test will be completed.

The time delay for each series is calculated so that the correlation dimension and BDS tests can be initiated. In the process of determining the maximum Hurst exponents, time delays are found to equal 40 months for the monthly series and 180 weeks for the weekly series. Again, it is important to note that these values represent on average how long it takes information to fully dissipate through these markets. This information is not, for example, similar to the arrival of an earnings report or a dividend report which is quickly/immediately accounted for by financial markets but rather information such as the impact of new technological advancements.[45] Market participants must first become aware of this technology, industry must design and establish this technology and finally investors must accept this technology. Thus, this type of information requires longer periods of time to be fully assimilated by financial markets.

With time delay measurements, correlation integrals are subsequently calculated from these inputs for increasing values of the distance within the phase space. The correlation integrals $C_{m,T}(\varepsilon)$ are computed for various embedding dimensions (m) as the Euclidean distance (ε) is increased. The logarithms are taken of both the correlation integrals and the Euclidean distances. From the resulting Log Log plot of the $C_{m,T}(\varepsilon)$ versus (ε) the location of the linear portion is identified and the slope of the curve over that region is estimated. This slope, denoted, D^m, is taken as an estimate of the correlation dimension of the projection of the attractor to the m-dimensional reconstructed phase space. An estimate of the correlation dimension is obtained for increasing values of the embedding dimension. Figures 4.8 and 4.9 display the correlation dimension estimates for the two copper price series for increasing embedding dimensions. In Figure 4.8, notice that the correlation dimension estimates appear to plateau at approximately 2.27. However, in Figure 4.9, the dimension estimates for the weekly series do not appear to plateau. Further, in this figure the rate of increase of the estimates of the correlation dimension increase and the embedding dimension increases. Because of this result, evidence of deterministic behavior cannot be concluded for either the weekly series.

[45] An example with copper markets may be the full impact of solvent extraction and electrowinning (SX-EW) techniques on existing copper production.

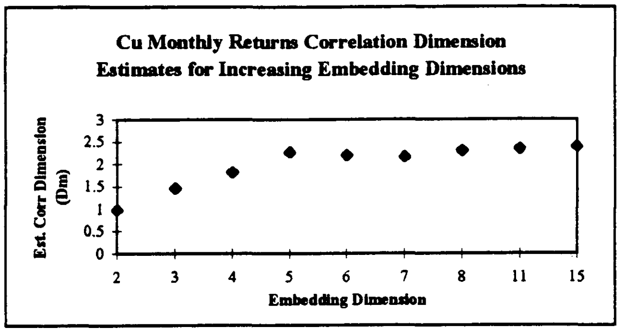


Figure 4.8 Dimension Estimates for Cu Monthly

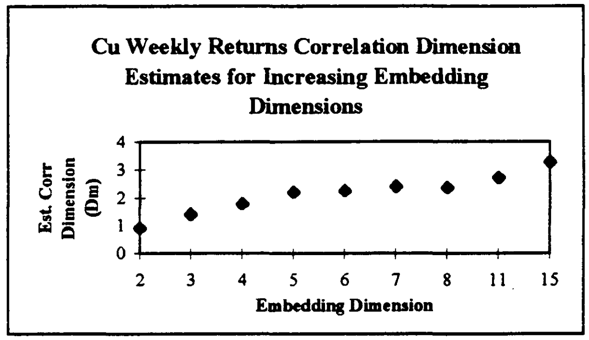


Figure 4.9 Dimension Estimates for Cu Weekly

As with the zinc price series, BDS statistics are estimated over a grid of embedding dimensions and Euclidean distances for each copper price series. Table 4.8 displays the grid of BDS test statistics for each of the copper price series (Note: σ_1 is the standard deviation, $\sigma_{.25}$ is 25% of the standard deviation, etc.). The first number in each cell corresponds to the monthly series whereas the second numbers are for the weekly series.

Table 4.8 BDS Results for Copper Series'

m	$\sigma_{.25}$	$\sigma_{.5}$	σ_1	$\sigma_{1.5}$	σ_2
2	.04045	.10521	.22845	.27514	.25854
	.00205	.00469	.01672	.04092	.06118
3	.01080	.09400	.28275	.46871	.49608
	1.3×10^{-5}	-5.2×10^{-4}	5.4×10^{-4}	.02436	.05281
4	5.8×10^{-4}	.05495	.27367	.58230	.70970
	-1.9×10^{-5}	-1.5×10^{-4}	8.7×10^{-4}	.01074	.03014
5	2.9×10^{-3}	.02844	.26956	.71899	.92240
	-5.4×10^{-7}	-1.7×10^{-5}	2.9×10^{-4}	3.7×10^{-3}	.01238

These estimated BDS values are compared to critical values. As the monthly series only has 204 observations, critical values are obtained from Brock, et al. (1993). The estimated BDS values for the weekly series are compared to standard normal critical values. As can be seen in Table 4.8, all of the computed BDS statistics over the grid are small and thus spatial dependence can be rejected for small dimensions (i.e., $m \leq 5$).

To test for nonlinear dependencies in the copper prices series', the normalized BDS and Tsay test are utilized. Under the normalized BDS test, both series are transformed to the [0, 1] interval to remove any linear dependence which may be present. Table 4.9 displays the results of the normalized BDS test for the normalized monthly (top number in each cell) and weekly series (bottom number).

Table 4.9 Normalized BDS Results for Copper Series

m	$\sigma_{.25}$	$\sigma_{.5}$	σ_1	$\sigma_{1.5}$	σ_2
2	.05781 .03692	.16178 .12068	.32261 .27324	.34261 .29555	.30758 .2196
3	.02587 .02218	.15264 .13623	.48260 .56049	.61653 .73508	.59415 .58167
4	.01327 .01067	.09848 .09956	.54440 .71556	.86894 1.196	.86068 1.0227
5	.00588 .00345	.03885 .05939	.58880 .7129	1.076 1.544	1.095 1.512

The computed BDS statistics for the normalized series' are also less than the corresponding critical values and thus low dimensional nonlinear dependence cannot be confirmed. However, notice the values in the shaded areas of Table 4.9 appear to be increasing. One reason for this is that as the standard deviation of the data is increased, the correlation integrals used in calculating the BDS statistics are capturing more and more observations. However, to explore the possibility that higher dimensional dependencies may be present, further BDS statistics are computed for higher dimensions (m = 6-11) and are displayed on Table 4.10. The shaded cells in Table 4.10 indicate BDS statistics which are larger than corresponding critical values thus infer that higher dimensional nonlinear dependence may be present in both series.

Table 4.10 Higher Dimension BDS Results

m	$\sigma_{.25}$	$\sigma_{.5}$	σ_1	$\sigma_{1.5}$	σ_2
6	.00182	.03474	.59504	1.2754	1.3459
	.00126	.00311	.63055	1.7667	1.9579
9	-1.5×10^{-6}	.00664	.44175	1.4886	1.9023
	-2.5×10^{-6}	.00326	.35986	2.0099	3.0737
11	-4.3×10^{-8}	-7.6×10^{-8}	.32606	1.5216	2.2019
	-6.8×10^{-8}	5.9×10^{-4}	.21795	1.8722	3.5356

In addition to the normalized BDS test for nonlinearity, the Tsay test is used to test for the significance of higher order terms in each copper price series. The test statistic is asymptotically distributed as an F-distribution with 3 and T-4 degrees of freedom, where T is the number of observations in the series. For the two copper price series

$$\hat{F}_{monthly} = 4.018$$

$$\hat{F}_{weekly} = 2.756$$

Given the number of observations in each copper price series, the critical value at the 95% confidence level is 2.60. The Tsay test shows evidence of nonlinearity within both copper returns series. However, at the 99% confidence level ($F_c = 3.78$), linearity in the weekly returns series cannot be rejected.[46]

The largest Lyapunov exponents are calculated for the two copper series using the Wolf et al. (1985) algorithm. Figure 4.10 displays the convergence properties for the largest Lyapunov exponent for the monthly copper price series.[47] Notice that over distinct ranges, the series oscillates between positive and negative values.[48] Similar results are found with the weekly series. Therefore, SDIC cannot be concluded for either of the copper price series as converge to a positive value does not appear to occur. These results may be due to the extremely limited number of data points in each series.

Peters (1991a) states that data requirements necessary to determine the largest Lyapunov exponent vary with the complexity of the system. A minimum of 10^m data points are needed. For example in the dimensionality of the attractor of a system is found to be two, 100 points are needed. Three-dimensional attractors, as found with both copper price series, thus require 1000 data points. Unfortunately, the monthly and weekly series only have 204 and 886 data points respectively.

---

[46] Noise in the series could be a major contributing factor to this result. This implication will be discussed in chapter 5.

[47] Graphical limitations only allow 4000 iterations to be displayed.

[48] This is in contrast to the monthly zinc series (Figure 4.7) which appears to converge to a stable, positive exponent.

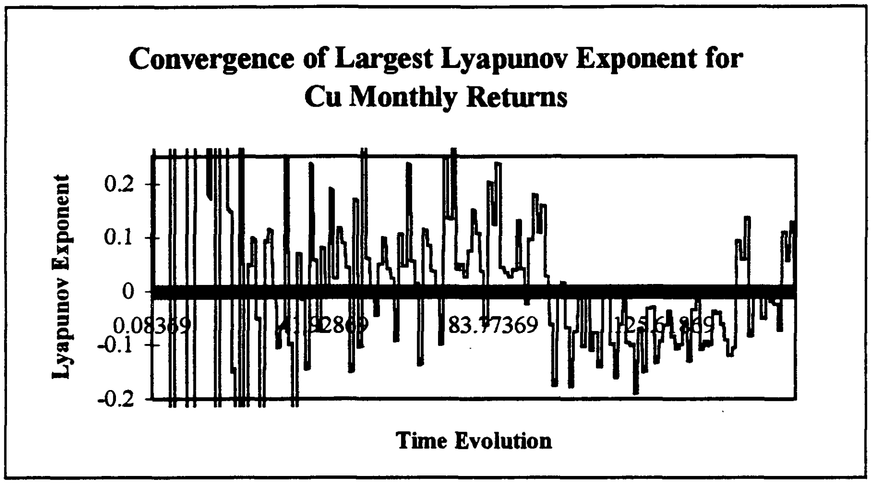


Figure 4.8 Largest Lyapunov exponent for Cu Monthly

Applications of a modified univariate time series model specification process are presented to capture the class of systems which are nonlinear deterministic or chaotic. The systems which govern monthly zinc and copper spot prices are tested. To test the specification process against more frequently measured time series, a weekly copper price series is also utilized.[49] Empirical results show that the process generating the monthly zinc series is indeed chaotic as the system displays determinism, nonlinearity and sensitivity to initial conditions (SDIC). Non-traditional predictive models which incorporate inherent spatial dependencies are thus applied to the zinc process. Chaotic behavior in the two copper spot price processes could not be concluded. While the monthly copper process is found to be deterministic and slight evidence of nonlinear dependence is uncovered, SDIC could not be detected. Stochasticity could not be rejected for the process underlying the weekly copper series.

[49] Higher frequency financial time series generally have more noise present than those measured on a less frequent basis (Peters, 1991a).

Chapter 5
CONCLUSIONS, IMPLICATIONS, AND MODELING EXTENSIONS

This study examines alternatives to traditional, temporal dependent time series modeling which assume stochastic generating processes. Techniques derived in the physical sciences aimed at uncovering the necessary characteristics of chaotic behavior of determinism, nonlinearity and sensitive dependence upon initial conditions (SDIC) are applied to the economic systems of mineral commodity spot prices. A modification to traditional univariate time series model specification processes is presented to detect the class of systems which possess these conditions necessary for chaotic behavior. This process culminates with the specification of non-traditional time series models. This chapter recaps the findings and provides implications for the modeling of commodity markets in the future.

Chapter 4 applies the modified model specification process to two monthly mineral commodity price series (zinc and copper) and one weekly copper price series. Table 5.1 summarizes the results of the tests from the modified specification process applied to each of these series. The modified specification process begins with each series being examined for stationarity. The commonly utilized and intuitively straightforward Dickey-Fuller test is applied to each series. Each of the three series is found to be nonstationary and thus are logarithmic first differenced to achieve stationarity. To assess whether each series of observations is normally distributed, simple tests of skewness and kurtosis are applied. Each series is found to lack normality either from excess kurtosis or skewness or both. As a result, if conventional statistical measures and tests are put to use, the results will be questionable.

Table 5.1 Model Specification Testing Results

Test	Test Name	Zn Monthly	Cu Monthly	Cu Weekly
Stationarity	Dickey Fuller	No	No	No
Normality	Jarque-Bera	No[a]	No[b]	No[a]
Temporal Dependence	Turning Point	Yes	No	No
Spatial Dependence	Correlation Dimension	Yes	Yes	No
	BDS	Yes -Slight	No	No
Nonlinearity	Normalized BDS	Yes - Slight	Yes - Slight	Yes-Slight
	Tsay	Yes - Strong	Yes	Yes
SDIC	Lyapunov Exponents	Yes	No	No

a Normality rejected based on both excessive kurtosis and skewness
b Normality rejected based only on excessive kurtosis

To assess dependencies within each series, tests based on two differing methodologies are exercised. The traditional Turning Point test which assesses temporal dependencies is applied to the three series. The zinc series is found to be temporally dependent while both copper price series are found to be independent using this test. Following traditional model specification processes, one might conclude that both copper price series follow a random walk and thus no type of model would assist in prediction. Again it is important to note that low dimensional nonlinear deterministic behavior is characteristically known to appear random to both the eye as well as many traditional tests such as the and Turning Point. Attempting to pick up these types of dependencies, the spatial dependence measures of the correlation dimension and BDS tests are applied to each series.

Spatial dependencies based on uncovering the existence of system attractors with fractal dimensions are found for the two monthly price series through the correlation dimension. This outcome provides evidence that the two governing processes may be deterministic. This same result could not be concluded for the weekly price series. The BDS test is also applied to each series to unveil determinstic or stochastic behavior. Low-dimensional spatial dependence could only be concluded for the zinc price series as both copper series could not reject spatial independence via the BDS test. Reasons for these results may be lack of data in the monthly copper price series and increased noise in the weekly copper price series.

To test for nonlinearity in each price series both the normalized BDS and the Tsay test are performed. Results from the normalized BDS test suggest slight evidence of nonlinearity in each series.[50] Nonlinearity is more strongly concluded in each series via the Tsay nonlinearity test. The final characteristic necessary for chaotic behavior, sensitive dependence upon initial conditions (SDIC), is verified for governing process of each series. Through convergence of the largest Lyapunov exponent in each series, SDIC is only concluded for the zinc process.

Through the specification tests performed, there is evidence of nonlinear deterministic behavior in the monthly zinc process. Attempting to model this evident deterministic behavior, simple non-traditional deterministic models which incorporate information from spatial correlations within the system are applied to the zinc price series. Local linear phase space models proposed by Farmer and Sidorowich (1987) and Creedy and Martin (1994) are utilized based on the assumption that any manifold or attractor is locally linear. The advantage of using local models is their ability to adhere to the local

[50] Computed BDS statistics on the two normalized copper price series' infer the existence of higher dimensional (dimensions of 6 or greater) nonlinear spatial dependence.

shape of an arbitrary surface; the corresponding disadvantage is that they do not lead to a compact description of the system (Gershenfeld and Weigend, 1994). Prediction results from these models are compared to a second-order autoregressive (AR(2)) model which assumes traditional stochastic behavior underlying the zinc price series.

The local linear phase space predictive model (i.e., Farmer and Sidorowich, 1987) appears to offer the same predictive errors as the traditional second-order autoregressive (AR(2)) model. This is interesting in that while the two modeling types differ vastly on the assumptions made concerning the behavior of the underlying system, the results from the predictions are very similar. Stochastic models assume that the generating process possesses many/infinite degrees of freedom while deterministic models infer a low number (quantifiable) of degrees of freedom. Reasons for these similar results may include lack of sufficient data and/or noise present within any financial price series.

Implications

One benefit of the modified time series model specification process is the provision for uncovering deterministic behavior from a single series of prices. With nonlinear deterministic behavior, the possibility exists to formulate exact predictions given knowledge of the model and initial system conditions. With a low-dimensional chaotic system, behavior in the price series could be driven by relatively few (finite) numbers of contributing factors which could be modeled. This is in direct contrast to traditional stochastic models which assume that an infinite, irreducible number of contributing factors are responsible for system behavior and thus predictions must be based upon classical probabilities.

Once deterministic behavior is confirmed, the next task lies in identifying how many and which independent variables to include in the predictive process. For example, a predictive model for zinc prices may only include, the previous price(s) and past and/or current investor sentiment, etc. as the contributing independent variables. To enhance the predictive capabilities of the phase space modeling techniques for particular systems, nonlinearity will most likely have to be introduced.

Constraining Factors of Financial and Economic Systems

In the analysis of monthly zinc prices, it was demonstrated that predictions from phase space models were similar to those from a traditional second-order autoregressive model. In many instances, in the realm of the physical sciences, data are usually derived from controlled environments. The dynamic systems techniques presented and utilized in this study are derived in such environments. As an example, consider a system in which it is desired to understand the dynamical behavior of water molecules as the temperature is increased beyond the boiling point. Under an experiment such as this, scientists may seek to verify if the molecular behavior of water in the boiling, turbulent stages is chaotic or alternatively that these molecules simply moving randomly. One benefit of applying detection techniques as described in this study to this type of system is that precise information concerning the system is already known. For example, it is known that water molecules go from stable states to turbulent states once the boiling point is achieved (212°F at sea level). In other words, it is easy to see when the system goes for a stable to a potentially chaotic state.

In the analysis of mineral commodity prices as with all economic time series, stable and unstable states are all mixed together. At this point in time, the luxury of knowing whether the data collected is from a stable or unstable state of the system is not known. For example, when is the copper market stable or is it ever? As a result, economic data, especially in the financial markets, as well as other real world systems are said to have noise present. Noise complicates this type of analysis in that the presence of noise will make a truly deterministic system appear stochastic. This may explain the lack of superior predictive results with the local approximation models applied to the zinc series or the lack of conclusive evidence in the empirical studies of copper prices for lower dimensional chaotic behavior.

Noise

Noise will exaggerate the amount of randomness which is affecting a system and will thus make a truly deterministic process appear random. Any financial time series is said to have noise present. Prices thus not only reflect the true value of the underlying asset but also the affecting noise. This noise can arrive through trading in which financial market participants act in ways which contradict investor rational behavior of maximizing profits or returns. Examples may include investors not acting upon actual information but from other parties such as a mother-in-law or an international government purchasing its own currency, not for monetary gain but to artificially prop up the value.

Much research has been completed on methods to limit the affects of noise so that true dynamical behavior can be uncovered. To alleviate the problems excessive noise has upon analyses of dynamical behavior Sauer (1994) utilizes filtered delay coordinates which incorporates discrete Fourier transformations to filter out noise. It can be shown that as with unfiltered delay coordinates, those which are filtered also provide for a diffeomorphic mapping to the true underlying dynamical behavior. Filtering may provide a means in which dynamical systems theory can be effectively applied to economic time series. “Cleaner” time series may enhance the ability of the techniques described in this study to uncover chaotic behavior as well as enhance deterministic model prediction capabilities.

Data Requirements

Another important feature necessary in testing for deterministic or stochastic behavior are data requirements. In general, enough data are needed so as to appropriately cover the attractor of a system with enough points if the system is deterministic. For example, from a recent time series prediction competition in which many of the techniques used in this study were applied, the data lengths ranged from 1000 to 34,000 observations.[51] Again, the mineral commodity price series analyzed in this study are much shorter in length.[52] Further, Peters (1991a) states that in the analysis of empirical data, data from 10 cycles of the system are needed. For example if metal prices follow the U.S. business cycle, i.e., approximately four years, this requirement states that the analysis would require 10*4 or 40 years of data.

On first examination, it appears that for the study of mineral commodity prices as with any financial time series analyses that one would have access to vast amounts of data. For example, if the data at hand is not sufficient, traditionally an analyst could choose one of two paths. First, it is possible to obtain data sampled at a more frequent rate such as tick by tick prices. However, as stated by Brock et al. (1993), "Such high frequency return dynamics are contaminated by the dynamics of the bid/ask spread, which may be of interest to the market micro-structure theorist but not to an analyst testing the stock price itself for nonlinearity." The second route available to analysts is that as an alternative to sampling more frequently, one could keep the same sampling rate and simply obtain more data (i.e., longer and longer datasets). Unfortunately, this presents a problem also as nonstationarity becomes a problem. This nonstationarity problem stands in sharp contrast to those series in the physical sciences where the majority of the nonlinear dynamical tests and statistics have been derived. For example, fluid dynamics do not depend on who is chairman of the Federal Reserve (Brock et al., 1993). As will be explained, differencing in

[51] Time Series Competition of the NATO Advanced Research Workshop on Comparative Time Series. Held in Santa Fe, New Mexico, May (1992). The datasets included: A) Fluctuations of an Infrared Laser (1,000 observations); B) Physiologic data on Heart Rates (34,000); C) High Frequency Exchange Rate data (30,000) and D) Astrophysical data (27,000).

[52] Zn monthly: 564 observations, Cu monthly: 204 observations and Cu weekly: 886 observations.

some situations can alleviate this problem, however, differencing creates other problems when dealing with financial series.

According to Takens (1981) and Sauer (1994), in most cases the original time series, P_t, and a differenced series should provide a reconstruction of the underlying dynamics.[53] Wolf et al. (1985) states that it is advisable to ensure stationarity in the underlying series so that robust estimates of the embedding dimension (m) and the time lag (τ) can be obtained. This will ensure that subsequent measures on the dynamics such as Lyapunov exponents are, in turn, robust. Specifically, differencing can be used to remove the linear effects of autocorrelation on dimension estimates and other nonlinear statistics. In cases where there is drift in the data (i.e., the series may follow a random walk), differencing provides stationarity. Differencing, however, does have drawbacks. Theiler and Eubank (1992) shows that differencing (or pre-whitening) chaotic data sets makes it more difficult to detect nonlinearity. In addition, noise tends to be amplified by differencing. This is of serious concern when applied to financial data series which inherently possess significant amounts of noise. Because of the lack of agreement in the literature between employing a stationary time series achieved through differencing and the subsequent amplification of stochastic noise because of differencing, analyses may need to be completed both the raw and differenced time series in accordance with Provenzale et al. (1993).

Modeling Extensions

This study found that mineral commodity prices may be driven by a few significant factors (i.e., the systems are low-dimensional deterministic). If mineral commodity markets are truly deterministic then models which incorporate this deterministic behavior should be utilized. The benefit of using such models is two-fold: (1) increased

[53] i.e., a diffeomorphic mapping between the reconstructed behavior and the true underlying dynamic behavior.

understanding of the dynamical behavior of mineral commodities can be gained, and (2) theoretically, improved price forecasts can be obtained as prediction models are properly specified to the underlying structure. The local-linear phase space models may be seen as a first-step in approximating the dynamical behavior in a mineral commodity market. Nonlinear techniques will most probably have to be incorporated into deterministic predictive models so that the characteristics specific to the dynamics of the mineral commodity processes can be more accurately and fully described.

Another important extension of this work would be the use of global approximation techniques such as neural networks. A neural network is a computer program that recognizes patterns. A neural network is designed to take a pattern of data, generalize from this pattern and then to make the best predictions possible. One benefit of neural networks is that it is a global approach in that all information is presented simultaneously.[54] If systems are shown to be deterministic with low-dimensionality, neural networks may present an avenue in which the significant variables contributing to market behavior can be identified. The modified time series model specification process is still valid for neural network construction. Testing for deterministic behavior, system dimension, reconstruction of phase spaces and nonlinearity are necessary inputs for neural networks.

A recent article in the *Economist* states, "Many [fund managers] have now concluded that formal chaos theory has nothing practical to offer." (*Economist*, 1996). Further, due to the noise which is inherent to any financial time series, it has been questioned whether chaos is the correct paradigm to be applied (Weigend, 1996). However, it may be that application of chaos theory to financial markets may be premature. At this point in time, the dynamical properties of markets are poorly understood. In addition, the differing regimes through which these systems flow is yet to

[54] In contrast, traditional time series analysis involves the processing of patterns over time.

be discovered. For example, when are these systems stable or more turbulent? If differing regimes can be determined, it may be possible to characterize behavior within each. Once more information concerning system behavior is known as well as better information concerning the influences input factors have on financial prices, it may be possible to apply the techniques of this study with enhanced results.

A basic truism in financial markets is that, for one participant to make money, another must lose it. Indeed, financial market forecasting has been and will remain a difficult and frustrating endeavor. For a market to remain "healthy and alive," it must be efficient and essentially unpredictable. If predictable behavior is uncovered, it must remain hidden for as long as possible. Therefore, if you do not hear from me in the future, I hope you will understand!

REFERENCES CITED

Albano, A.M., Muench, J., Schwartz, C., Mees, A.I., Rapp, P.E. 1988. "Singular-Value Decomposition and the Grassberger-Procaccia Algorithm," Physics Review A. 38, pp. 3017.

Barnett, W.A., and Chen, P. 1987. "The Aggregation-Theoretic Monetary Aggregates Are Chaotic and Have Strange Attractors," Dynamic Econometric Modeling: Proceedings of the 3rd International Symposium in Economic Theory and Econometrics. Cambridge, Mass.: Cambridge University Press.

Baumol, William J. and Benhabib, Jess. 1989. "Chaos: Significance, Mechanism, and Economic Applications." Journal of Economic Perspectives. 3 (1), (Winter), pp. 77-105.

Bera, A.K. and Jarque, C.M. 1981. "An efficient large sample test for normality of observations and regression residuals," Working Paper in Econometrics No. 40, Australian National University, Canberra.

Black, F. 1986. "Noise," Journal of Finance. 41, pp. 529-543.

Brock, W.A. and Sayers, C.L. 1988. "Is the Business Cycle Characterized by Deterministic Chaos," Journal of Monetary Economics. 22, pp. 71-90.

Brock, W.A., Dechert, W. and Scheinkman, J. 1987. "A Test for Independence Based on the Correlation Dimension," Madison, Wisconsin: University of Wisconsin Press.

Brock, W.A. and E.G. Baek. 1991. "Some Theory of Statistical Inference for Nonlinear Science," Review of Economic Studies. 58, pp. 697-716.

Brock, W.A. and Hsieh, D.A., LaBaron, B. 1993. Nonlinear Dynamics, Chaos, and Instability - Statistical Theory and Economic Evidence. Massachusetts Institute of Technology.

Brown, C. 1995. Chaos and Catastrophe Theories. Sage Publications- Series: Quantitative Applications in the Social Sciences. Thousand Oaks, Calif.

Bunkov, Y. and Volovik, G. 1993. "Homogeneous Pressing Domains in ^{3}He-B," Journal Experimental and Theoretical Physics. 76 (54), pp. 794-801.

Casdagli, M. 1989. "Nonlinear Prediction of Chaotic Time Series," Physica D.35, pp. 335-356.

Casdagli, M. 1991. "State Space Reconstruction in the Presence of Noise," Physica D. 51, pp. 52-98.

Casdagli, M. and Weigend, A. 1994. "Exploring the Continuum Between Deterministic and Stochastic Modeling," Time Series Prediction: Forecasting the Future and Understanding the Past. Proceedings Volume XV. Reading, Mass: Addison-Wesley Publishing.

Chiang, Alpha C. 1984. Fundamental Methods of Mathematical Economics. New York: McGraw-Hill Inc.

Creedy, John and Vance Martin. 1994. Chaos and Non-linear Models in Economics: Theory and Application. Brookfield, Vermont: Edward Elgar Publishing.

Cromwell, J., Labys, W.C., Terraza, M. 1994. Univariate Tests for Time Series Models. Sage Publications- Series: Quantitative Applications in the Social Sciences. Thousand Oaks, Calif.

Crutchfield, J. 1986. "Chaos," Scientific American. 255 (6), (Dec.), pp. 46-57.

Dantzig, George B. 1949. "Programming in a Linear Structure," Econometrica. V.17, No. 3,4.

Dantzig, George B., 1963. Linear Programming and Extensions. Princeton, N.J.: Princeton University Press.

Davidson, R. and MacKinnon, J.G. 1993. Estimation and Inference in Econometrics. New York: Oxford University Press,

Day, Richard. 1994. Complex Economic Dynamics, Volume 1: An Introduction to Dynamical Systems and Market Mechanisms. Cambridge: The MIT Press..

Decoster, G.P., Labys, W.C., Mitchell, D.W. 1992. "Evidence of Chaos in Commodity Futures Prices," The Journal of Futures Markets. 12 (3), pp. 291-305.

DeLong, B. 1990. "Noise Trader Risk in Financial Markets," Journal of the Political Economy. 98 (4), pp. 703- 739.

Dickey, D.A. and Fuller, W.A. 1979. "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," Journal of the American Statistical Association. 74, pp. 427-431.

Dickey, D.A., Bell, W.R., and Miller, R.B. 1986. "Unit Roots in Time Series Models: Tests and Implications," The American Statistician. 40 (1), pp. 12-26.

Ding, M. and Ott, E. 1993. "Plateau Onset for Correlation Dimension: When does it Occur?," Physics Review Letters. 70, pp. 3872.

Economist, 1996. "Chaos Under a Cloud," Economist. Jan. 13. pp. 69-70.

Frank, M. and Stengos, T. 1989. "Measuring the Strangeness of Gold and Silver Rates of Return," Review of Economic Studies. 56, pp. 553-567.

Farmer, J.D. and Sidorowich, J.J. 1987, "Predicting Chaotic Time Series," Physical Review Letters. 59 (8), pp. 845-848.

Froehling, H. Crutchfield, J.P., Farmer, D., Packard, N.H. and Shaw, R. 1981. "On Determining the Dimension of Chaotic Flows," Physica D. pp. 605-617.

Fuller, W.A. 1976. Introduction to Statistical Time Series. John Wiley and Sons, New York.

Gershenfeld, N.A. and Weigend, A.S. 1994. Time Series Prediction: Forecasting the Future and Understanding the Past. Proceedings Volume XV. Reading Mass.: Addison-Wesley Publishing.

Giaever, I., Keese, C. 1989. "Fractal Motion of Mammalian Cells," Physica D. pp. 128-133.

Goodwin, R.M. 1951. "The Nonlinear Accelerator and the Persistence of Business Cycles," Econometrica. 19 (1), (Jan.), pp. 1-17.

Grassberger, P., Procaccia, I. 1983. "Characterization of Strange Attractors," Physical Review Letters. 50 (5), (Jan.), pp. 346-349.

Grassberger, P., Procaccia, I. 1983. "Measuring the Strangeness of Strange Attractors," Physica, 189-208.

Harvey, A.C. 1993. The Econometric Analysis of Time Series: Second Edition. Cambridge Mass.: The MIT Press.

Harvey, A.C. 1994. Time Series Models: Second Edition. Cambridge Mass.: The MIT Press.

Hicks, J.R. 1950. A Contribution to the Theory of the Trade Cycle. Oxford: Oxford University Press.

Holcombe, Randall G. 1990. Economic Models and Methodology. New York: Greenwood Press.

Hurst, H.E. 1920. Long Range Planning: An Experimental Study. London: Constable Press.

Kellert, Stephen H. 1993. In the Wake of Chaos: Unpredictable Order in Dynamical Systems. Chicago: University of Chicago Press.

Krantz, R.L., Coughlin, J.P., and Billinton, S. 1995. "Studies of Stope-Scale Seismicity in a Hard-Rock Mine," U.S. Dept. of Interior, Bureau of Mines - RI 9525.

Kuhn, Thomas S. 1962. The Structure of Scientific Revolutions. University of Chicago Press, Chicago.

LeBaron, B. 1994. "Nonlinear Diagnostics and Simple Trading Rules for High-Frequency Foreign Exchange Rates," in Time Series Prediction. Santa Fe Institute, Proceedings Volume XV. Reading Mass.: Addison-Wesley Publishing.

Leontief, Wassily. 1951. Structure of American Economy 1919-1939: An Empirical Application of Equilibrium Analysis. New York: Oxford University Press.

Leontief, Wassily. 1966. Input-Output Economics. New York: Oxford University Press.

Levacic, Rosalind. 1976. The Static and Dynamic Analysis of a Monetary Economy. London: McMillan Press.

Mills, R.L. 1977. Statistics for Applied Economics and Business. McGraw-Hill Book Co., New York.

Nunes, G., Jin, C., Hawthorne, D.L., Putnam, A.M., Lee, D.M. 1992. “Spin-Polarized ^{3}He-^{4}He Solutions: Logitudinal Spin Diffusion and Nonlinear Spin Dynamics,” Physics Review B. 46(14), pp. 9082-9095.

Oser, Jacob. 1953. Economic History of Modern Europe. New York: Prentice-Hall Economic Series.

Ott, Edward. 1993. Chose in Dynamical Systems. New York: Cambridge University Press.

Packard, N.H. Crutchfield, J.P., Farmer, J.D., Shaw, R.S. 1980. “Geometry from a Time Series.” Physics Review Letters. 45 (9), pp. 712-716.

Peters, Edgar E. 1991a. Chaos and Order in the Capital Markets. New York: John Wiley and Sons.

Peters, Edgar E.. 1991b. "A Chaotic Attractor for the S&P 500," Financial Analysts Journal. (Mar. -Apr.), pp. 55-81.

Pindyck, R.S. and Rubinfeld, D.L. 1991. Econometric Models and Economic Forecasts. New York: McGraw-Hill.

Poincare, Henri. 1913. The Foundations of Science: Science and Method. Washington, D.C.: The Science Press.

Pressman, Steven. 1994. Quesney’s Tableau Economique: A Critique and Reassessment. Fairfield, New Jersey: A.M. Kelly Publishers.

Provenzale, A. Osborne, A.R., Soj, A.R. and Murante, G. 1993. “Distinguishing between low dimensional dynamics and randomness in measured time series,” Physica D. 58 (31), pp. 203.

Radzicki, Michael J. 1990. "Institutional Dynamics, Deterministic Chaos, and Self-Organizing Systems," Journal of Economic Issues. 24(1), (Mar.), pp. 57-102.

Rasband, S.N. 1990. The Chaotic Dynamics of Nonlinear Systems. New York: John Wiley and Sons.

Ruelle, D. and Takens, F. 1971. “On the Nature of Turbulence,” Communications in Mathematical Physics. 20, pp. 167

Ruelle, D. and Eckmann, J. 1985. "Ergodic Theory and Chaos and Strange Attractors," Review of Modern Physics. 57, pp. 617-656.

Sauer, T. 1994. "Time Series Prediction by Using Delay Coordinate Embedding," in Time Series Prediction. Santa Fe Institute, Proceedings Volume XV. Reading Mass.: Addison-Wesley Publishing.

Samuelson, Paul A. 1939. "Interaction Between the Multiplier Analysis and the Principle of Acceleration," Review of Economics and Statistics. 21, pp. 75-78.

Scheinkman, Jose, A. 1990. "Nonlinearities in Economic Dynamics," The Economic Journal. 100, pp. 33-48.

Scheinkman, Jose A. and LeBaron, B. 1989. "Nonlinear Dynamics and Stock Returns," Journal of Business. 62 (3), 311-337.

Sims, C.A. 1980. "Macroeconomics and Reality," Econometrica. 48, pp. 1-48.

Slade, M.E. 1982. "Trends in Natural-Resource Commodity Prices: An Analysis of the Frequency Domain," Journal of Environmental Economics and Management. 9, pp. 122-137.

Slutzky, E. 1927. "The Summation of Random Causes as the Source of Cyclic Processes," Econometrica. 5, pp. 105-146.

Takens, F. 1981. "Detecting Strange Attractors in Turbulence."- In Dynamical Systems and Turbulence, edited by D.A. Rand and L.S. Young. Lecture Notes in Mathematics. Berlin: Springer-Verlag Press, pp. 336-381.

Taylor, S. 1986. Modeling Financial Time Series. New York: John Wiley and Sons.

Theiler, J., Eubank, E. 1992 "Don't Bleach Chaotic Data," Technical Report LA-UR-92-1575, Los Alamos National Laboratory, Los Alamos, NM.

Tong, H and Lim, K.S. 1990. "Threshold Autoregression, Limit Cycles and Cyclical Data," Journal of the Royal Statistical Society B. 42, pp. 245-292.

Tsay, R.S. 1986. "Nonlinearity Tests For Time Series," Biometrica. 73 (2), pp. 461-466.

Vandermeulen, D.C. 1971. Linear Economic Theory. Englewood Cliffs, New Jersey: Prentice-Hall.

Weigend, A.S., Huberman, B.A. and Rumelhart, D.E. 1990. "Predicting the Future: A Connectionist Approach," International Journal of Neural Systems. 1: pp. 193-209.

Weigend, A.S. 1991. "Connectionist Architecture for Time Series Prediction of Dynamic Systems," Ph.D. Thesis, Stanford University.

Weigend, A.S. 1996. "Neural Networks in Financial Applications," University of Colorado School of Business Lecture Series. January 19.

Wolf, A., Swift, J.B., Swinney, H., Vastano, J.A. 1985. "Determining Lyapunov Exponents from a Time Series," Physica D. 16, pp. 285.

Yang, C.N. 1962. "Concept of Off-Diagonal Long-Range Order and the Quantum Phases of Liquid He and of Superconductors," Reviews of Modern Physics. 34(4), PP. 694-704.

Yule, G.U. 1927. "On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer's Sunspot Numbers," Philosophical Transactions: Royal Society of London. 226, pp. 267-298.

SELECTED BIBLIOGRAPHY

Brock, W.A. 1989. "Distinguishing Random and Deterministic Systems: Abridged Version," Journal of Economic Theory. 40, pp. 168-195.

Fama, Eugene F. 1965. "Behavior of Stock-Market Prices," The Journal of Business. 38, pp. 34-101.

Grandmont, Jean-Michel, Pierre Malgrange. 1986. "Nonlinear Economic Dynamics: Introduction," Journal of Economic Theory. 40 (3), pp. 3-12.

Hsieh, David A. 1991. "Chaos and Nonlinear Dynamics: Application to Financial Markets.: The Journal of Finance. 19 (5), (Dec.), pp. 1839-1877.

LeBaron, B. 1994. "Chaos and nonlinear forecastability in economics and finance," Philosophical Transactions Royal Society of London. 348, 397-404.

Labys, W., et. al. 1989. "Monetary and Economic Influences in Econometric Models of International Commodity Price Behavior," NATO ASI Series. 54, pp. 635-655.

Layton, David F. 1994. "Trends? In Natural Resource Commodity Prices," Department of Economics, University of Washington, Seattle, Washington.

Theiler, J. (1990). "Estimating Fractal Dimension," J. Opt. Soc. Am. A. 7 (6), pp. 1055-1073.

Theiler, J., Galdrikian, B., Longitin, A. and Farmer, J. 1992. "Using Surrogate Data to Detect Nonlinearity in Time Series." In Nonlinear Modeling and Forecasting, edited by M. Casdagli and S. Eubank. Santa Fe Institute Studies in the Sciences of Complexity, Proceeding Volume XII, pp. 163-188. Reading, Mass.: Addison-Wesley.

Winter, A.L. and Sapsford, D. 1989. Primary Commodity Prices: Economic Models and Policy. Cambridge University Press, New York.

APPENDIX

COMPUTER ALGORITHMS

This Macro is designed to Calculate the Correlation Dimension for a Specified Time Series. The code is initially from Peters Book Appendix. 2. 'The inputs required for this program are the time series itself, the dimension m, the time lag (tau) 'the step size and the initial distance r. These inputs are obtained through a Input Box.

```
Option Explicit

Sub CorrDim()

Dim Origts As Range
Dim NPT As Range
Dim numobs
Dim CurrRow As Integer
Dim n1
Dim c
Dim I
Dim J
Dim m
Dim tau
Dim r
Dim Lag
Dim K
Dim Theta
Dim Theta2
Dim Sum
Dim ITS
Dim CR
Dim Ind
Dim D
Dim DT
Dim L As Integer
Dim CDim As Range
Dim InitD As Range

Dim X(1 To 3000) 'Array which should contain the time series
Dim Z(2000, 19)  'Embedding Dimensions of up to 11 are allowed

'Initialize Variables
Lag = 1
K = 1
```

```
Sum = 0
ITS = 0
Theta = 0
Theta2 = 0
CR = 0
Ind = 0
L = 1

'This subroutine obtains number of observations, embedding dimension, time lag for
reconstructed
'phase space, increments to distance, and  initial distance.

DialogSheets("GetInfo").Show
m = Val(DialogSheets("GetInfo").EditBoxes("m").Text)
r = Val(DialogSheets("GetInfo").EditBoxes("r").Text)
tau = Val(DialogSheets("GetInfo").EditBoxes("tau").Text)
DT = Val(DialogSheets("GetInfo").EditBoxes("DT").Text)


numobs = CInt(InputBox("How Many Observations For This Analysis? "))

'Loop Puts the Original Time Series into the Output Sheet
CurrRow = 3
For CurrRow = 3 To numobs

   X(CurrRow) = Worksheets("Input").Cells(CurrRow, 1).Value
   Sheets("Output").Cells(CurrRow, 1).Value = X(CurrRow)
Next CurrRow


CurrRow = 3
For I = CurrRow To numobs - (m - 1) * tau
   For J = 1 To m
      Z(I, J) = X(I + (J - 1) * tau) 'Reconstruct the Phase Space
'  Sheets("Output").Cells(CurrRow, 4).Value = Z(I, J)
   Next J
Next I
'Next CurrRow

'numobs = numobs - m * tau 'Maximum Length of Phase Space
```

```
'r = r * (2 - r) 'DVORAK (1991) Modification for 6<m<11

Line1:

For K = 1 To numobs
   For I = 1 To numobs
      D = 0
      For J = 1 To m
         D = D + (Z(Lag, J) - Z(I, J)) ^ 2 'Calc. Square of the Distance
      Next J

      D = Sqr(D) 'Calculation of Distance


'Check to see if the Distance is greater than r
      If D > r Then Theta2 = 0 Else Theta2 = 1

      Theta = Theta + Theta2 'Counting Points

   Next I

   Lag = Lag + 1

Next K

CR = (1 / ((numobs) ^ 2 - numobs) * Theta) 'Calculation of the Correlation Integral
                                    'Modification from Huber, 1993)



'Set CDim = Sheets("output").Range("CR")
Sheets("output").Cells(L + 1, 2) = CR
'Set InitD = Sheets("output").Range("InitDist")
Sheets("output").Cells(L + 1, 3) = r

L = L + 1

If L > 12 Then GoTo Line2
r = r + DT
```

```
CR = 0
Theta = 0
Theta2 = 0
Lag = 1

GoTo Line1

Line2:
Sheets("output").Cells(1, 2) = "CR"
Sheets("output").Cells(1, 3) = "r"

End Sub
```

This program is used to complete Rescaled-Range analysis for a given price series. The steps involved are outlined in Peter's 1991 book "Chaos and Order in the Capital Markets" on pages 82 and 83. The program is written in FoxBase. From the output, it is possible to determine the mean orbital period and the estimated Hurst exponent for a given series.

```
*       To run this application, type DO RS.PRG
*
*
CLOSE DATA
SET TALK OFF
SET STATUS OFF

SELECT 3

USE RS3.DBF
SET SAFETY OFF
ZAP
SET SAFETY ON


SELECT 2

USE RS2.DBF
SET SAFETY OFF
ZAP SET SAFETY ON


SELECT 1

USE RS1.DBF
SKIP 1


DO WHILE .NOT. EOF()

CLOSE2 = CLOSE
SKIP -1
CLOSE1 = CLOSE
SKIP 1
```

```
DIFF1 = CLOSE2/CLOSE1
REPLACE DIFF WITH DIFF1
LOGDIFF1 = LOG(DIFF1)
REPLACE LOGDIFF WITH LOGDIFF1
SKIP

ENDDO

N=2

DO WHILE N < 282

@ 5, 5 SAY N

SELECT 2
SET SAFETY OFF
ZAP
SET SAFETY ON

SELECT 1

GO TOP
SKIP 1

DO WHILE .NOT. EOF()

REC1 = RECNO()
        IF RECNO()>= 564 - N
                EXIT
        ENDIF
SKIP N
REC2 = RECNO()
GO REC1
AVERAGE LOGDIFF FOR RECNO()>(REC1-1) .AND. RECNO()<(REC2) TO AVG1

*CALCULATE CUMULATIVE DEVIATIONS FOR EACH OF THE N PERIODS.
*(SEE PETERS BOOK PG. #63)

GO REC1
CUMDEV=0
```

```
MINDEV = LOGDIFF-AVG1
MAXDEV=LOGDIFF-AVG1
CNTR = 0
STDNUM=0

DO WHILE CNTR<N

**OBTAIN RANGES

        DEV = LOGDIFF-AVG1
CUMDEV = CUMDEV+DEV

MINDEV=MIN(CUMDEV,MINDEV)
MAXDEV=MAX(CUMDEV,MAXDEV)

**CALCULATE STANDARD DEVIATIONS

STDX=LOGDIFF-AVG1
        IF STDX<0
                STDX = -STDX
        ENDIF
        IF STDX = 0
                STDX = (STDX-1)
        ENDIF

STDXSQ = STDX^2
STDNUM=STDNUM+STDXSQ

SKIP
CNTR=CNTR+1
@ 5,25 SAY CNTR

ENDDO

RANGE=MAXDEV-MINDEV
VARIANCE = STDNUM/(CNTR-1)
STD=SQRT(VARIANCE)
RANGE_RS = RANGE/STD

SELECT 2
```

```
USE RS2
APPEND BLANK
REPLACE CNTR2 WITH CNTR
REPLACE RANGE2 WITH RANGE
REPLACE STD2 WITH STD
REPLACE RANGE_RS2 WITH RANGE_RS

RANGE=0
VARIANCE=0
STD=0
RANGE_RS=0

SELECT 1

GO REC2

ENDDO

SELECT 2

**OBTAIN AVERAGE OF R/S CALCULATED FOR EACH GROUP OF N SUBSETS

GO TOP
REC3=RECNO()
GO BOTT
REC4=RECNO()
AVERAGE RANGE_RS2 FOR RECNO()>=REC3 .AND. RECNO()<=REC4 TO AVG2

SELECT 3
APPEND BLANK
REPLACE PERIOD WITH N
REPLACE RS_OBS WITH AVG2

N=N+1

ENDDO
```

This program is used to perform the Dickey-Fuller test for stationarity in a time series. It is written in SAS 6.0. For a good review of this procedure, see Cromwell, 1994.

```
options nodate pagesize = 60 linesize=80;
data diss;
infile 'c:\thesis\analysis\orbital2\znmonth\real.txt';
input close;
run;

data tom1;
set diss;
keep close;
if close = . then delete;
run;

data tom2;
set tom1;
lagc = lag(close);
diff1 = close - lagc;
lagdiff1 = lag(diff1);
diff2 = diff1 -lagdiff1;

obs = _n_;

proc reg;
zn1: model diff1 = lagc/noint;
zn2: model diff1 = lagc;
zn3: model diff1 = lagc obs;
run;
```

This program is used to perform the Tsay (1986) test for nonlinearity in a time series. Vector Auto Regression (VAR) is utilized. Pindyck (1991) has a good introduction of VAR and why it should be used.

```
options nodate pagesize = 60 linesize=80;
data diss;
infile 'c:\thesis\analysis\orbital2\znmonth\ldiff.txt';
input x;

run;

data tom;
set diss;
xlag1 = lag(x);
xlag2 = lag(xlag1);
time = _n_;
proc autoreg;
model x = time/ nlag = (1 2);
output out = data1 r = ehat;

data tom1;
set tom;
set data1;
xlag1sq = (xlag1)**2;
xlag2sq = (xlag2)**2;
cross  = xlag1*xlag2;
run;

data tom2;
proc model data = tom1;
var one_int two_int three_int a b c d e f;
parms one_int two_int three_int a b c d e f;
label one_int = 'First intercept'
two_int = 'Second intercept'
three_int = 'Third intercept'
a = 'xlag1 coeff for xlag1sq model'
b = 'xlag2 coeff for xlag1sq model'
c = 'xlag1 coeff for cross model'
d = 'xlag2 coeff for cross model'
e = 'xlag1 coeff for xlag2sq model'
```

```
f = 'xlag2 coeff for xlag2sq model'
xlag1 = 'first lag'
xlag2 = 'second lag'
xlag1sq = 'first lag squared'
xlag2sq = 'second lag squared'
cross = 'xlag1 * xlag2';


/* Define the model equations */
xlag1sq = one_int+a*xlag1 + b*xlag2;
cross = two_int+c*xlag1 + d*xlag2;
xlag2sq = three_int+e*xlag1 + f*xlag2;

/* Specify the autoregressive process */
%ar(ar, 1, xlag1sq cross xlag2sq)

/* Estimate the model parameters */
fit xlag1sq cross xlag2sq/out=data2;

data test;
set data1;
set data2;
keep ehat xlag1sq cross xlag2sq;
proc print;
proc reg;
model ehat = xlag1sq cross xlag2sq/noint;
run;
```

This is a very simple program written in FoxPro to determine the number of turning points in a given time series. This test is used in the dissertation to test for temporal dependence in a time series.

```
*    To run this application, type DO TURNING.PRG
*
*
CLOSE DATA
SET TALK OFF
SET STATUS OFF



SELECT 1

USE rs1.DBF
SKIP 1
TURNS = 0

DO WHILE .NOT. EOF()
TP = 0
C = logdiff
SKIP -1
CLAST = logdiff
SKIP 2
CNEXT = logdiff


IF C > CLAST AND C > CNEXT

TP = 1
ENDIF

IF C < CLAST AND C < CNEXT

TP = 1
ENDIF
```

```
TURNS = TURNS + TP
@ 5, 25 SAY TURNS
* SKIP
ENDDO
```

```
'This Macro is designed to estimate the Lyapunov exponent spectrum for a given time
series. This original algorithm is from Wolf (1985) which is written in FORTRAN. An
Input Box requests the user to input the number of observations, embedding dimension,
time delay, distance size step, evolution time, maximum and minimum divergence or
trajectories and the minimum time between pairs.

Option Explicit

Sub Lyapunov()

Dim X(1 To 3000)  'Array which should contain the time series.
Dim PT1(1 To 12)
Dim PT2(1 To 12)
Dim Z(1000, 10)    'Embedding Dimensions of up to 10 are allowed.  Z(I,J) =
                   'Jth component of the Ith reconstructed attractor point

Dim NPT
Dim numobs
Dim CurrRow As Integer

Dim m
Dim tau
Dim r
Dim DT
Dim evolv
Dim scalmx
Dim scalmn
Dim lag
Dim I
Dim J
Dim SUM    'SUM holds running exponent estimate sans 1/time
Dim ITS    'ITS is the total number of propagation steps
Dim D
Dim DI
Dim DF
Dim IND    'points to a fudicial trajectory
Dim IND2   'points to second trajectory
Dim L As Integer
Dim III
```

```
Dim ZLYAP
Dim INDOLD
Dim ZMULT
Dim ANGLMX
Dim THMIN
Dim DNEW
Dim DOT
Dim CTH
Dim TH
Dim DII


'This subroutine obtains number of observations, embedding dimension, time lag for
reconstructed
'phase space, increments to distance, and  initial distance.

DialogSheets("GetInfo").Show

NPT = Val(DialogSheets("GetInfo").EditBoxes("NPT").Text)
m = Val(DialogSheets("GetInfo").EditBoxes("m").Text)
tau = Val(DialogSheets("GetInfo").EditBoxes("tau").Text)
DT = Val(DialogSheets("GetInfo").EditBoxes("DT").Text)
evolv = Val(DialogSheets("GetInfo").EditBoxes("evolv").Text)
scalmx = Val(DialogSheets("GetInfo").EditBoxes("scalmx").Text)
scalmn = Val(DialogSheets("GetInfo").EditBoxes("scalmn").Text)
lag = Val(DialogSheets("GetInfo").EditBoxes("lag").Text)

'initialize variables
SUM = 0
ITS = 0
IND = 3
L = 1

'numobs = CInt(InputBox("How Many Observations For This Analysis? "))

'Loop Puts the Original Time Series into the Output Sheet
CurrRow = 3
 For CurrRow = 3 To NPT
```

```
   X(CurrRow) = Worksheets("Input").Cells(CurrRow, 1).Value
   Sheets("Output").Cells(CurrRow, 1).Value = X(CurrRow)
  Next CurrRow

 'Nested loop to reconstruct the phase space
  For I = 3 To NPT - (m - 1) * tau
   For J = 1 To m
     Z(I, J) = X(I + (J - 1) * tau)
   Next J
  Next I

  NPT = NPT - m * tau - evolv   'Maximum length of phase space

MsgBox "Phase Space Reconstructed"


  DI = 1000000000
  For I = (lag + 1) To NPT    'Find initial pair
   D = 0
   For J = 1 To m
     D = D + (Z(IND, J) - Z(I, J)) ^ 2 'Calculate Distance
   Next J
   D = Sqr(D)
   If (D > DI) Or (D < scalmn) Then GoTo Line1     'Store Best Point

   DI = D

   IND2 = I

Line1:
  Next I
Line6:

  'Loop to Find Coodinates of Evolved Points

  For J = 1 To m
    PT1(J) = Z(IND + evolv, J)
    PT2(J) = Z(IND2 + evolv, J)
  Next J
```

```
 DF = 0

 'Loop to Compute the Final Divergence (Distance)
 For J = 1 To m
    DF = DF + (PT2(J) - PT1(J)) ^ 2
 Next J

 DF = Sqr(DF)
 ITS = ITS + 1

 SUM = SUM + (Log(DF / DI) / (evolv * DT * Log(2)))
 ZLYAP = SUM / ITS

'Output Lyapunov exponent, evolution time, and the current divergence between nearby
points.

Sheets("output").Cells(L + 1, 2) = ZLYAP
Sheets("output").Cells(L + 1, 3) = evolv * ITS
Sheets("output").Cells(L + 1, 4) = DI
Sheets("output").Cells(L + 1, 5) = DF

L = L + 1

 INDOLD = IND2
 ZMULT = 1
 ANGLMX = 0.3

Line4:

 THMIN = 3.14

 'Loop to Look For Replacement Points
 For I = CurrRow To NPT
     III = Abs(Int(I - (IND + evolv)))
     If III < lag Then GoTo Line2
     DNEW = 0

     For J = 1 To m
        DNEW = DNEW + (PT1(J) - Z(I, J)) ^ 2
     Next J
```

```
    DNEW = Sqr(DNEW)

    If (DNEW > ZMULT * scalmx) Or (DNEW < scalmn) Then GoTo Line2
    DOT = 0

    For J = 1 To m
      DOT = DOT + (PT1(J) - Z(I, J)) * (PT1(J) - PT2(J))
    Next J

    CTH = Abs(DOT / (DNEW * DF))

    If (CTH > 1) Then CTH = 1

    TH = Cos(CTH)

    If (TH > THMIN) Then GoTo Line2

    THMIN = TH
    DII = DNEW
    IND2 = I
Line2:
  Next I

  If (THMIN < ANGLMX) Then GoTo Line3

   'Cannot find replacement at 5 scale, double search angle, reset distance

  ZMULT = ZMULT + 1
  If (ZMULT < 5) Then GoTo Line4
  ZMULT = 1
  ANGLMX = 2 * ANGLMX
  If (ANGLMX < 3.14) Then GoTo Line4
  IND2 = INDOLD + evolv
  DII = DF
Line3:
  IND = IND + evolv

  'Leave Program when Fiducial Trajectory Hits End of File
  If (IND >= NPT) Then GoTo Line5
  DI = DII
```

```
  GoTo Line6

Line5:

Sheets("output").Cells(1, 2) = "LYAP"
Sheets("output").Cells(1, 3) = "Evolv*ITS"
Sheets("Output").Cells(1, 4) = "DI"
Sheets("Output").Cells(1, 5) = "DF"


End Sub
```

This Macro is designed to to find nearest neighbors through minimizing the the Euclidean Distance in a reconstructed phase space. From the nearest points, locally linear models are fit based on Farmer (1987) and Creedy (1990).

```
Option Explicit

Sub LWR()

Dim X(1 To 3000)  'Array which should contain the time series.
Dim Z(1000, 10)   'Embedding Dimensions of up to 10 are allowed.  Z(I,J) =
                  'Jth component of the Ith reconstructed attractor point

Dim NPT
Dim numobs
Dim CurrRow As Integer

Dim m
Dim tau
Dim r
Dim DT
Dim I
Dim J
Dim D
Dim ppoint
Dim L As Integer


'This subroutine obtains number of observations, embedding dimension, time lag for
reconstructed
'phase space, increments to distance, and  initial distance.

DialogSheets("GetInfo").Show

NPT = Val(DialogSheets("GetInfo").EditBoxes("NPT").Text)
m = Val(DialogSheets("GetInfo").EditBoxes("m").Text)
tau = Val(DialogSheets("GetInfo").EditBoxes("tau").Text)
DT = Val(DialogSheets("GetInfo").EditBoxes("DT").Text)
ppoint = Val(DialogSheets("GetInfo").EditBoxes("ppoint").Text)
L = 1
```

```
'numobs = CInt(InputBox("How Many Observations For This Analysis? "))

'Loop Puts the Original Time Series into the Output Sheet
CurrRow = 3
 For CurrRow = 3 To NPT

   X(CurrRow) = Worksheets("Input").Cells(CurrRow, 1).Value
   Sheets("Output").Cells(CurrRow, 1).Value = X(CurrRow)
   Sheets("Output").Cells(CurrRow, 2).Value = CurrRow
   Sheets("Output").Cells(CurrRow, 6).Value = X(CurrRow)
   Sheets("Output").Cells(CurrRow, 7).Value = X(CurrRow - 1)
   Sheets("Output").Cells(CurrRow, 8).Value = X(CurrRow - 2)

 Next CurrRow

 For CurrRow = 3 To NPT
 Sheets("Output").Cells(CurrRow, 9).Value = X(CurrRow + 1)
 Next CurrRow

 'Nested loop to reconstruct the phase space
 For I = 3 To NPT - (m - 1) * tau
   For J = 1 To m
     Z(I, J) = X(I + (J - 1) * tau)
     Sheets("Output").Cells(I, J + 2) = Z(I, J)
   Next J
 Next I

'  NPT = NPT - m * tau - evolv   'Maximum length of phase space

'MsgBox "Phase Space Reconstructed"


'Loop to calculate Euclidean Distances from specified point

 For I = 3 To NPT    'Find initial pair
   D = 0
   For J = 1 To m
     D = D + (X(ppoint) - Z(I, J)) ^ 2 'Calculate Distance
       Sheets("Output").Cells(L + 2, 5) = D
```

```
   Next J
'  D = Sqr(D)


   L = L + 1

  Next I


'Loop to find Nearest Neighbors by sorting by distance
Sheets("Output").Select
ActiveCell.Range("a3:i535").Select
   Selection.Sort Key1:=ActiveCell.Offset(0, 4).Range("A1"), Order1:=xlAscending,
Header:= _
      xlGuess, OrderCustom:=1, MatchCase:=False, Orientation:= _
      xlTopToBottom



Sheets("Output").Cells(1, 1) = "P(t)"
Sheets("Output").Cells(1, 2) = "Period"
Sheets("Output").Cells(1, 3) = "Z(1)"
Sheets("Output").Cells(1, 4) = "Z(2)"
Sheets("Output").Cells(1, 5) = "Distance"
Sheets("Output").Cells(1, 6) = "P(t)"
Sheets("Output").Cells(1, 7) = "P(t)-1"
Sheets("Output").Cells(1, 8) = "P(t)-2"
Sheets("Output").Cells(1, 9) = "P(t)+1"

'This next section performs a regression on the next period of
'the six nearest neighbors on the nearest neighbors and the lagged
'nearest neighbor points.  This is the method detailed in Creedy (1994).

   Sheets("Output").Select
   Application.ExecuteExcel4Macro String:= _
      "REGRESS([LWR.XLS]Output!R3C9:R8C9,  [LWR.XLS]Output!R3C6:R8C7,
FALSE, FALSE, , ""creedy"", FALSE, FALSE, FALSE, FALSE, , FALSE,
)"
```

```
'This next section performs a simple regression on the next period of
'the six nearest neighbors.
'This is the method detailed in Farmer (1987).

   Sheets("Output").Select
   Application.ExecuteExcel4Macro String:= _
      "REGRESS([LWR.XLS]Output!R3C9:R8C9, [LWR.XLS]Output!R3C6:R8C6,
FALSE, FALSE, , ""farmer"", FALSE, FALSE, FALSE, FALSE, , FALSE,
)"

End Sub
```