A TRI-DIMENSIONAL ANALYSIS
OF FLUID DISPLACEMENT
BY MEANS OF A POTENTIOMETRIC MODEL

By

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ABSTRACT

An electrolytic potentiometric model investigation of the performance of a three dimensional fluid flow program is presented.

The study developed a method whereby tri-dimensional predictions of reservoir performance can be made by combining conventional two-dimensional potential surfaces. Mapping of the top and bottom of a fluid-containing formation indicated the usefulness of such procedure to obtain closer approximation to actual flow conditions.

The effects of incomplete well penetration on fluid displacement were analytically investigated.

The results show that considerable error in performance predictions may result from analyses of potential distribution on only one two-dimensional surface when the wells are only partially penetrating.

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INTRODUCTION

An important question arising in fluid displacement operations is how the injected fluid will spread through the reservoir, how soon and in what sequence it will invade the producing wells. In answer to this question, it is necessary to map the expected progress of the interface between the injected and the displaced fluids as the interface proceeds from the injection to the producing wells.

One of the most convenient and practical methods of mapping the interface between the two fluids is to simulate the system by means of an electrical model. The theory of these models is based upon the correspondence between the hydrodynamics of steady-state fluid flow through porous media and the problems of current flow in continuous conductors. Such an analogy, whereby Darcy's law is replaced by Ohm's law, permits simulating fluid flow by means of electric currents.

A number of limiting assumptions are, however, necessary for the models to work: that the reservoir be isotropic and its permeability uniform, that the effects of relative permeability may be neglected, that the viscosity and the compressibility of driven and driving fluids be the same, and that the gravitational effects of differences in fluid densities may also be neglected. The electrical analogy may then be established, wherein electric currents proportional to the injection and extraction rates are passed through the conducting body by means of electrodes at points corresponding to the position of the wells in the field (Lee, 1948).

Previous Studies

In the early 1930's it was recognized that the flow of liquids through porous media follows Darcy's law, which states that the velocity of flow is proportional to the pressure gradient.

Wyckoff, Botset, and Muskat (1933) pointed out that this law is essentially a statement of facts obtained from experimental results. It applies when the flow is in the viscous range. However, experiments show that the actual flow of liquids in a producing horizon must be viscous except, perhaps, in the vicinity of a well producing at a high rate.

The application of potential theory to certain problems of viscous flow of dead liquids was illustrated by Muskat (1932). Wyckoff, et al. (1933) showed later that the analytical solution of the differential equations involved in problems of practical interest presents extreme difficulties because of the complex geometrical configurations of input and output wells. They further stated that a convenient and useful method of obtaining the potential distribution in steady-state flow of liquids in porous media is by means of electrical analogs.

The formal correspondence between the hydrodynamics of steady-state fluid flow through porous media and the problems of current flow in continuous conductors was clearly stated by Muskat (1937). He also pointed out that there is no direct electrical analogue to the effect of gravity in the flow of fluid through porous media. In such an analogy the potential distribution in the conductor is exactly equivalent to the pressure distribution in the field, and the current lines correspond exactly to the flow lines in the field. Streamlines, therefore, can be obtained graphically by drawing a system of curves which intersect the equipotentials at right angles (Lee, 1948).

Hurst and McCarty (1941) presented a method to determine the wave fronts swept out by dry gas in recycling operations. By this method, the potential distribution at various points was determined with a stainless-steel probe connected to the null point of a potentiometer. The procedure employed was rather laborious and lengthy. This procedure required considerable graphical work and computations in converting from the model data to the desired result.

Lee (1948) presented an improvement over previous methods, which involved replacing the single probe with a four-contact probe. In this way streamlines could be obtained directly by rotating the exploring probe until the null indicator connected to two of the contacts read zero. The other two contacts, located perpendicularly to the null contacts, were used to read the voltage drop along the fluid streamline.

Green (1948) applied a servomechanism and relaying system to a conventional electrolytic mapping instrument so that the streamline plot could be made automatically on a plotting board by means of a pantograph.

Although the method thus developed was quite expeditious and more accurate than those employing mechanical models, it was some time before this new instrument found extensive application. This fact was probably due to the prevalence of an erroneous impression that the method was not applicable to two fluids which differ in viscosity, density, or relative permeability (Wolf, 1948).

Marshall and Oliver (1947) investigated the limitations of the accuracy with which model studies can depict the course of cycling in any reservoir. They determined that the accuracy of the models is essentially a function of the accuracy with which the reservoir configuration is known.

In some cases the discrepancy between model predictions and field performance led to a reexamination of the engineering and geological data. Kelton (1943) showed that a condensate field had performed better than predicted by comparing model predictions obtained two years earlier with production data. A successive reevaluation of the Grapeland Field of Texas, subject of that investigation, revealed that the reservoir was actually larger than it had originally been estimated.

Purpose and Scope of Investigation

Most of the models encountered in the literature were two-dimensional. Such models, although of considerable help in reservoir-engineering studies, seemed to be limited as to the extent that they could not conveniently predict the tri-dimensional patterns of fluid displacement. The problem becomes one of particular interest when the wells do not equally or fully penetrate the oil-bearing sand.

The object of this investigation was to determine whether two-dimensional potential maps of reservoir models could be combined practically to yield a tri-dimensional picture of fluid flow during a displacement operation. The effects of differential penetration of the output sources were also investigated.

The study was confined to a fluid displacement operation on a direct line-drive pattern in which the input well was located at the periphery of the zone of interest.

In the systems investigated the lengths of the electrodes were variably set at one-third, two-thirds, and maximum well penetration. All combinations of the above penetrations were studied, except when the penetrations of the two producers were equal but not complete. This problem was in fact the subject of previous investigations.

MODEL INVESTIGATION

The following presentation is a description of the equipment and procedure employed in this study. A discussion of the basic theory involved in potentiometric studies is not within the scope of this thesis because copious literature has been published on this subject. The interested reader is referred to the fine developments by Muskat (1949,b) and Lee (1948).

Potentiometric Field Mapper

The electrolytic-potentiometric field mapper available at the Colorado School of Mines is comprised of two main sections: the control console, and the model table. (Fig. 1).

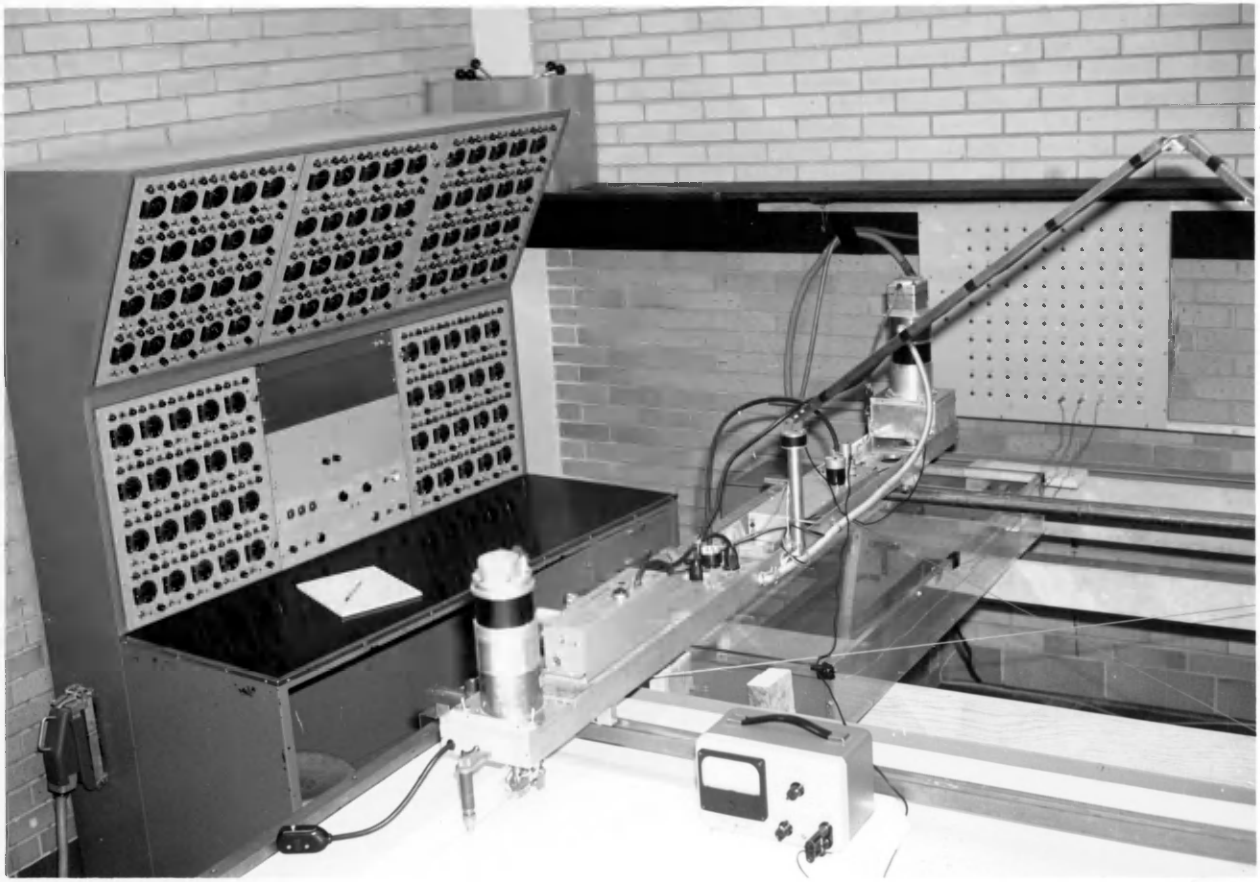

Figure 1

POTENTIOMETRIC FIELD MAPPER

The control console contains all the circuits and components used in establishing well currents, potential surface, and gradients.

The model table consists of two parts, one containing the actual model in which all potentiometric measurements are made, and the other supporting the map upon which streamlines and isopotentials are plotted. The two parts of the model table are related by means of a one-to-one pantograph used for translating the potentiometric measurements to the output map.

Figure 2 is a schematic representation of the entire system used in this study.

The probing device, consisting of a five-electrode array, permits the measurements of two basic types of potentiometric data -- point potentials and gradients.

With reference to Figure 3, point potentials can be measured by observing on a voltmeter the potential at the central electrode, P, of the probe assembly with respect to an electrical reference point within the well current generating circuit. The gradients can be determined by means of the S electrodes of the probe assembly, which

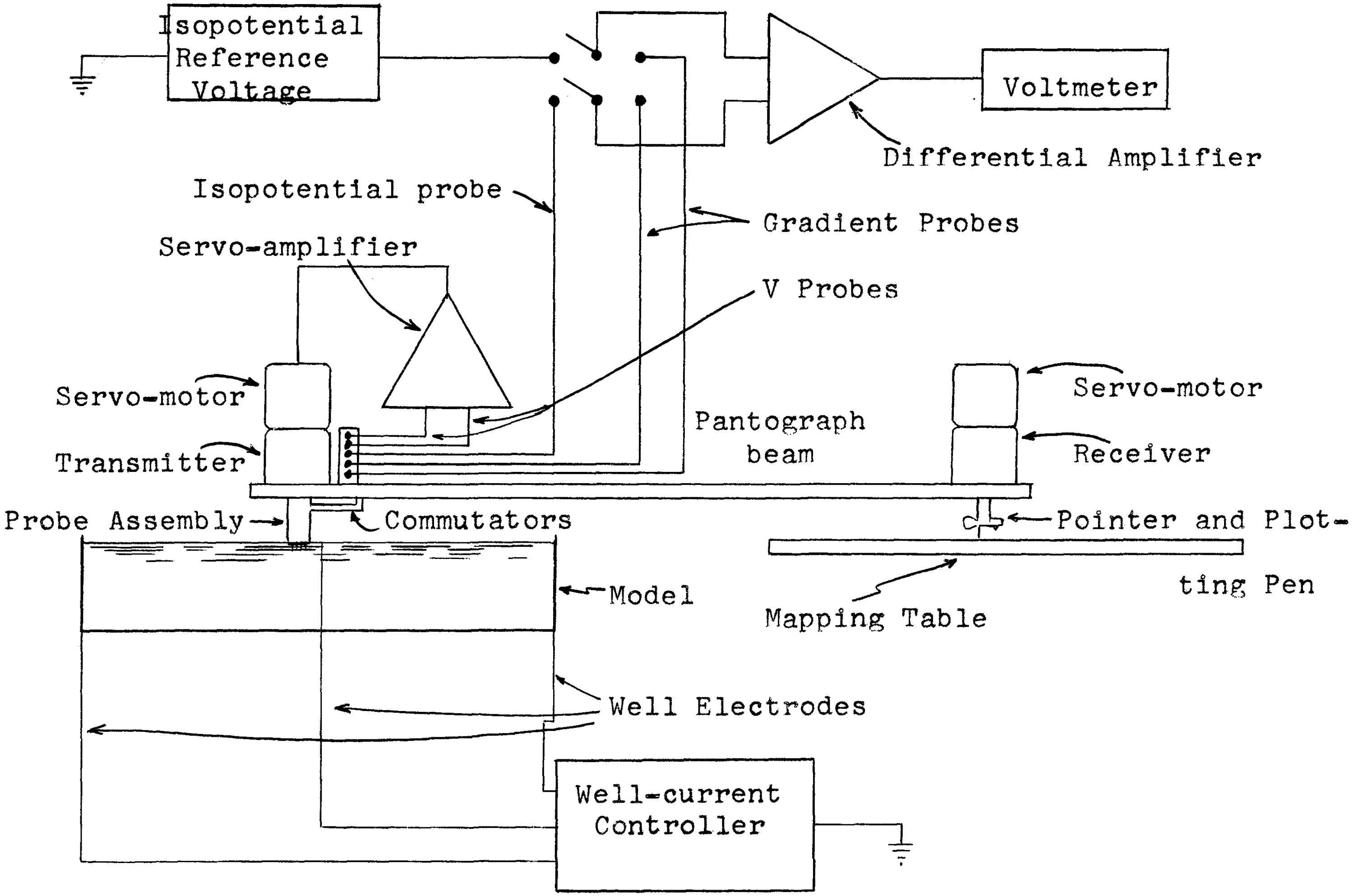

Figure 2

SCHEMATIC DIAGRAM OF POTENTIOMETRIC FIELD MAPPER

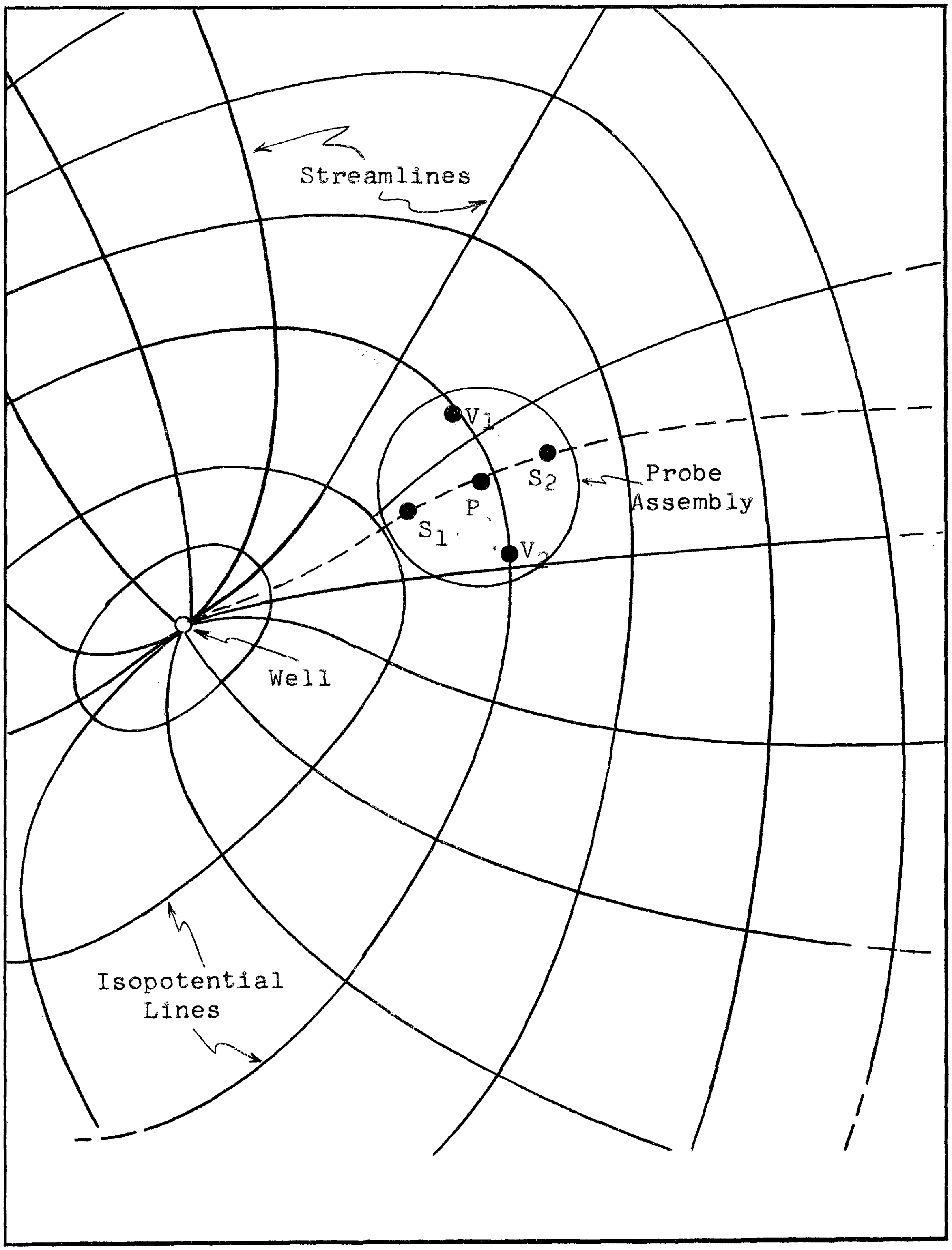

Figure 3
ORIENTATION OF FIVE-PROBE ASSEMBLY IN POTENTIAL FIELD

are on a line perpendicular to the line of the V electrodes (Miller, 1961).

The actual method of gradient measurement relies upon establishing an orientation of the probe assembly so that the V electrodes observe the same potential. Streamlines are obtained by plotting a series of associated gradient segments sequentially on the map.

Reservoir Model

The study model consisted of two 48 x 12 x 4 in. lucite tanks. One tank was used to map streamlines along a perpendicular plane through the wells. In this tank the electrodes representing the wells were introduced across the surface of the medium and barely covered with electrolyte in order to avoid the formation of a meniscus around the wires. The second tank was used to map streamlines on both the top and the bottom of the formation. Here, the electrodes were introduced from the top on one side of the tank.

Partial penetration of the wells was simulated by coating sections of the wire-electrodes with insulating material.

Tap water, previously filtered to eliminate the suspended solid particles, was used as the electrolyte. The medium had a specific conductance of 136 micromhos-cm, and was maintained at 70°F throughout all the runs.

The well-electrodes consisted of 0.040 in. copper wires.

Procedure

Electrodes were installed in the model in accordance with their geometric location in a line-drive pattern. A well-current controller was connected to each well electrode. By means of rheostats, the current at each electrode was adjusted to correspond to the fluid-flow rate at each well and also to represent fluid production or injection. Whenever possible, the output rate was set equally for the two producing wells.

The probe system, located at one end of the pantograph arm of the model table, was brought in contact with the electrolyte surface. The potentiometric data were measured by means of a vacuum-tube voltmeter and recorded on the map by means of the plotting pen at the other end of the pantograph arm. Streamlines were graphically obtained by drawing a smooth curve through the plotted points.

A total of seven sets of three runs each was made. Each set comprised the streamline mapping on the top and the bottom of the tank, as well as on the vertical plane along the line of the wells.

Four equally spaced traverses or streamlines were plotted on the vertical plane of the model, whereas on the two horizontal surfaces eight streamlines were traced. All maps started from a common distance of one inch from the input electrode and radiating from it 10 degrees apart from each other.

The injection well was kept at full penetration throughout the experiment; the penetration of the two producing wells was changed by one-third of total at a time. All combinations of partial penetrations of the producers were investigated, except where such penetrations were equal but not full.

RESULTS AND DISCUSSION

The experimental data gathered in this investigation, using one input well and two producing wells in the same plane, are expressed graphically in Figures 4 through 10. Each illustration represents one case studied and summarizes the streamline plotting on three different planes of the model. The upper, middle, and lower parts of each figure show streamlines on the surface of the model, on the vertical plane passing through the wells, and on the bottom of the model respectively. The plots are reproduced to scale.

The conditions under which each investigation was carried out are given on the page preceding each illustration. Producing flow-rates are expressed in percent of input rate. When the penetration of the farthest removed producing well was one-third of total, the relative rates could not be set equally for the two producing wells.

In Case 1 only two runs were made. The wells were all fully penetrating, and displacement patterns on top and bottom of the sand were, therefore, identical.

Discussion of Results

In previous investigations, involving only two-dimensional mapping on the surface of the reservoir model, recovery determinations were based on areal calculations of sweep efficiency. This procedure is acceptable when the wells are known to fully penetrate the reservoir, as the potential distribution in the vertical direction is uniform. However, when the penetration of one or more producing wells is not complete, it results in correlating errors. Streamlines tend to follow paths closer to the line of the wells or break into the output sources, but fluid displacement greatly varies with depth.

Predictions based on surface mapping alone would, therefore, be overestimated. The error is larger when shallower wells are present and is maximum when the middle well has deeper partial penetration than the farthest well.

Qualitatively the effects of incomplete well penetration were amply proved in this study. From a quantitative standpoint, however, the data available may be insufficient to predict displacement performance to a satisfactory degree of accuracy. A tentative method would be that of averaging the areal sweep efficiencies of the two vertical boundaries of the reservoir. The error thus resulting would be less than that deriving from a single two-dimensional map, whether differential penetration is considered or neglected.

Furthermore, this work indicates an approach for further study. By mapping streamlines at different levels in the reservoir model and by calculating the sweep efficiency at each depth, a volumetric sweep could be more closely approximated. A comparison with the results obtained by averaging between the two vertical boundaries of the reservoir may indicate whether or not the error is of sufficient magnitude to justify the longer approach.

Comparison of Mapped Data

The general appearance of the surface streamlines indicates that deflection of the streamlines around or into the first well occurs whenever one of the two producing wells is not fully penetrating the reservoir. The deflection is greater when the decrease in penetration occurs at the farthest removed well. The difference is more noticeable in the middle part of the mapped surface, and is particularly significant when the penetration of the farthest removed well is low and that of the middle well is intermediate. The greatest divergence from a situation in which all three wells are fully penetrating is represented by the system investigated in Case 5. Figure 11 shows a comparison of performances displayed by the two systems.

On the vertical plane through the wells the streamlines are horizontal throughout five-sixths of the distance between the input well and the first producer. Streamlines curve upward in the neighbourhood of a producing well whenever such well is not fully penetrating the sand.

The coning effect is greater the shallower is the well penetration. In all cases, the direction of flow reverses itself in a region past the first producer for a distance which ranges between one-sixth and one-fourth of the separation between the two producing wells.

The pattern of the bottom streamlines is very sensitive to variations in penetration of both producing wells, but to a larger degree whenever the variation occurs in the farthest removed well.

The bottom maps of the model do not show streamlines in the regions neighbouring the producing wells. Streamlines do actually radiate from the injection well between 0 and 10°, but for comparative purposes they were not shown on the maps. The first streamline mapped in all cases was, in fact, the one originating at 10° from the input well.

Figure 4

Case 1: Streamline maps

A - surface and bottom of model

B - vertical plane through the wells

	First producing well W_{p_1}	Second producing well W_{p_2}
Well penetration	Full	Full
Relative output rate (percent of input rate)	50	50

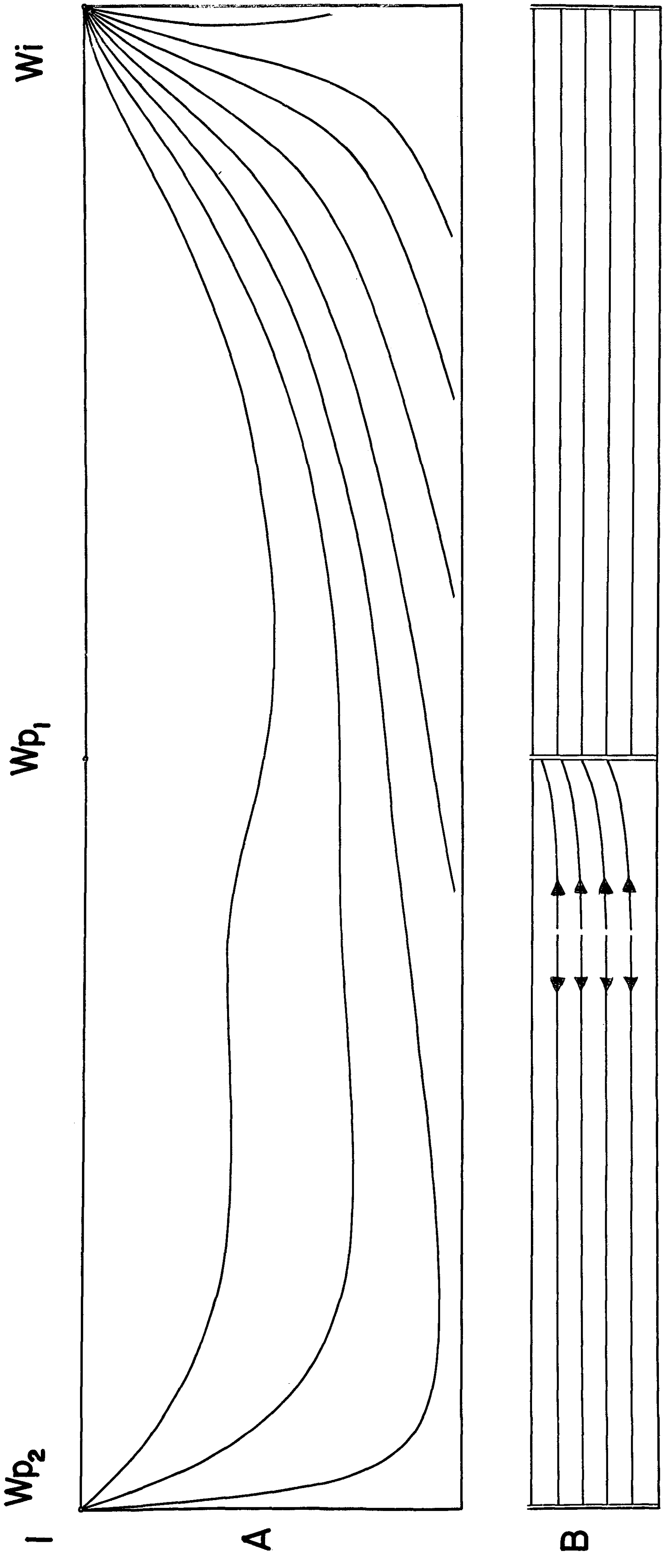
Wi
Wp$_1$
Wp$_2$
I
A
B

Figure 5

Case 2: **Streamline maps**

A – surface of model

B – vertical plane through the wells

C – bottom of model

	First producing well W_{p_1}	Second producing well W_{p_2}
Well penetration	Full	2/3
Relative output rate (percent of input rate)	50	50

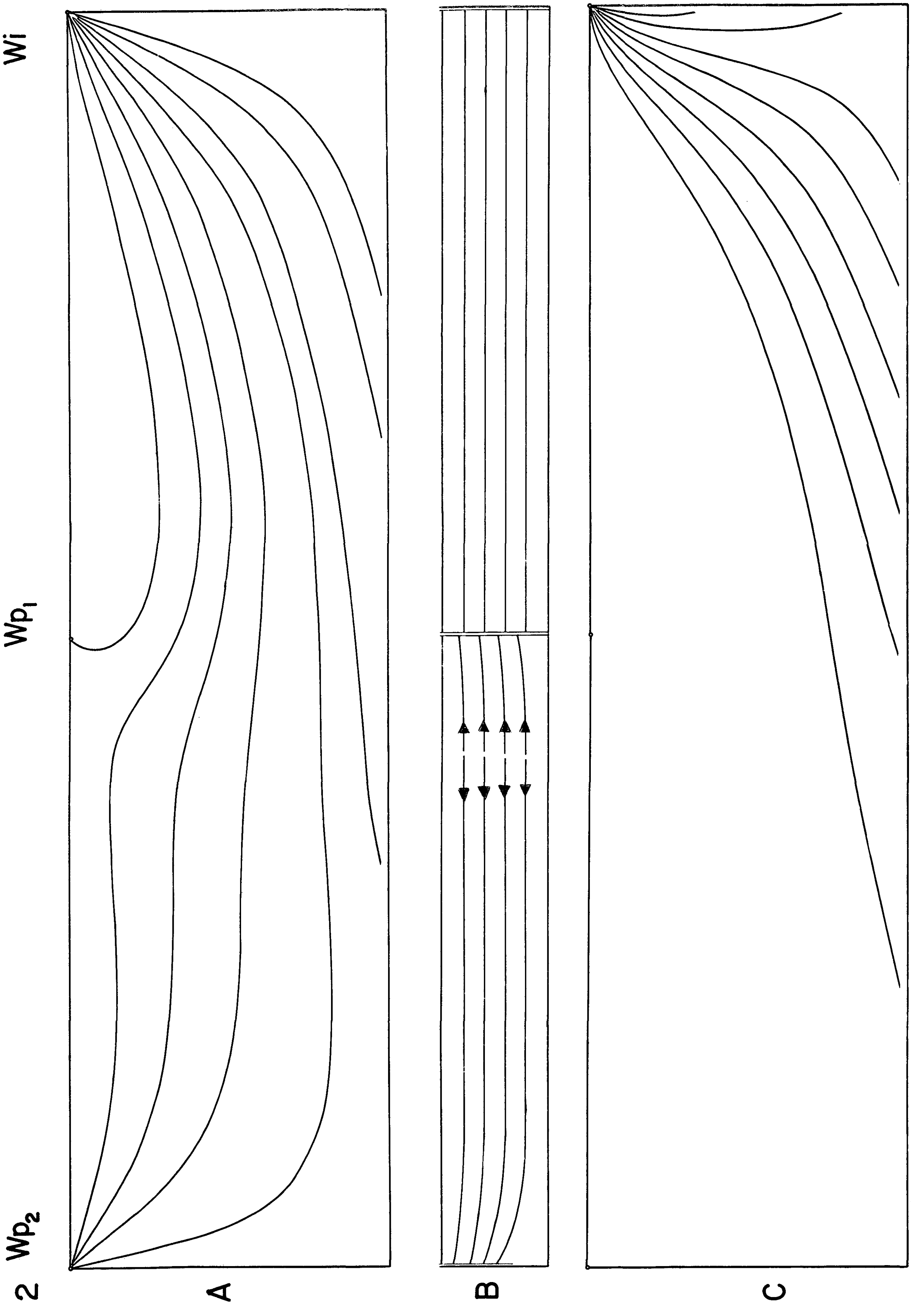

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Figure 6

Case 3: Streamline maps

A - surface of model

B - vertical plane through the wells

C - bottom of model

	First producing well W_{p_1}	Second producing well W_{p_2}
Well penetration	Full	1/3
Relative output rate (percent of input rate)	57	40

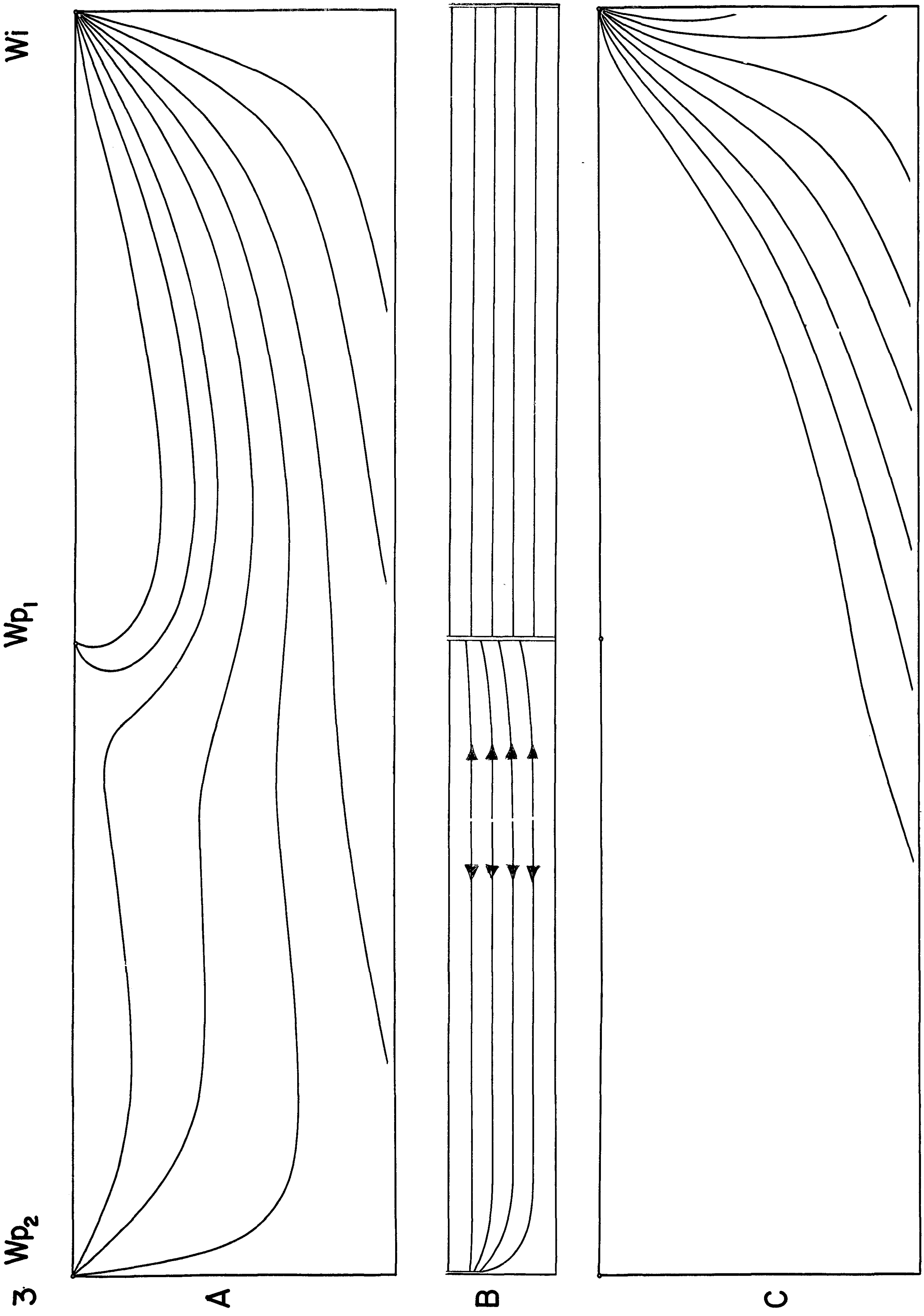

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Figure 7

Case 4: Streamline maps

A - surface of model

B - vertical plane through the wells

C - bottom of model

	First producing well W_{p_1}	Second producing well W_{p_2}
Well penetration	2/3	Full
Relative output rate (percent of input rate)	50	50

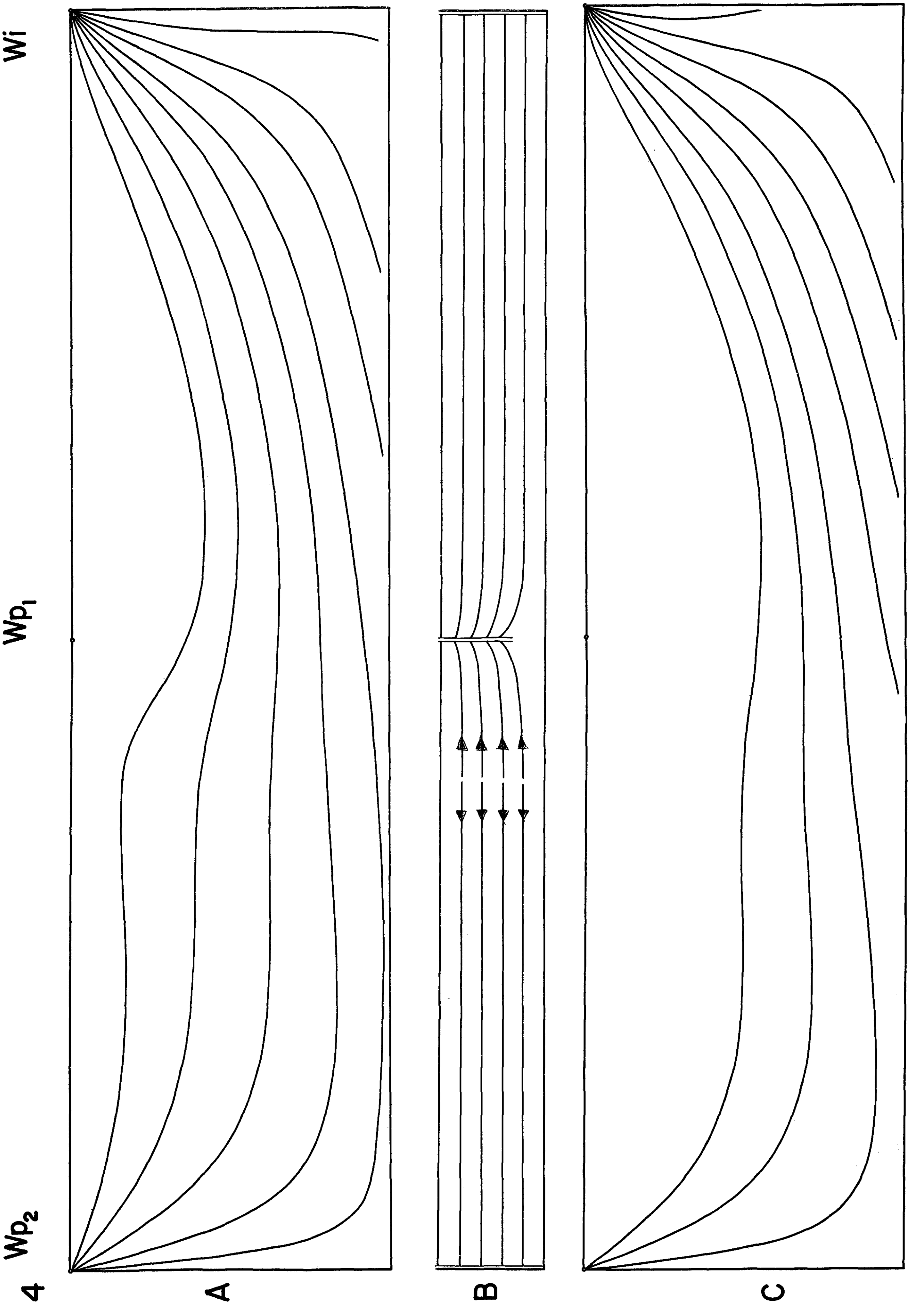

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Figure 8

Case 5: Streamline maps

A - surface of model

B - vertical plane through the wells

C - bottom of model

	First producing well W_{p_1}	Second producing well W_{p_2}
Well penetration	2/3	1/3
Relative output rate (percent of input rate)	55	43

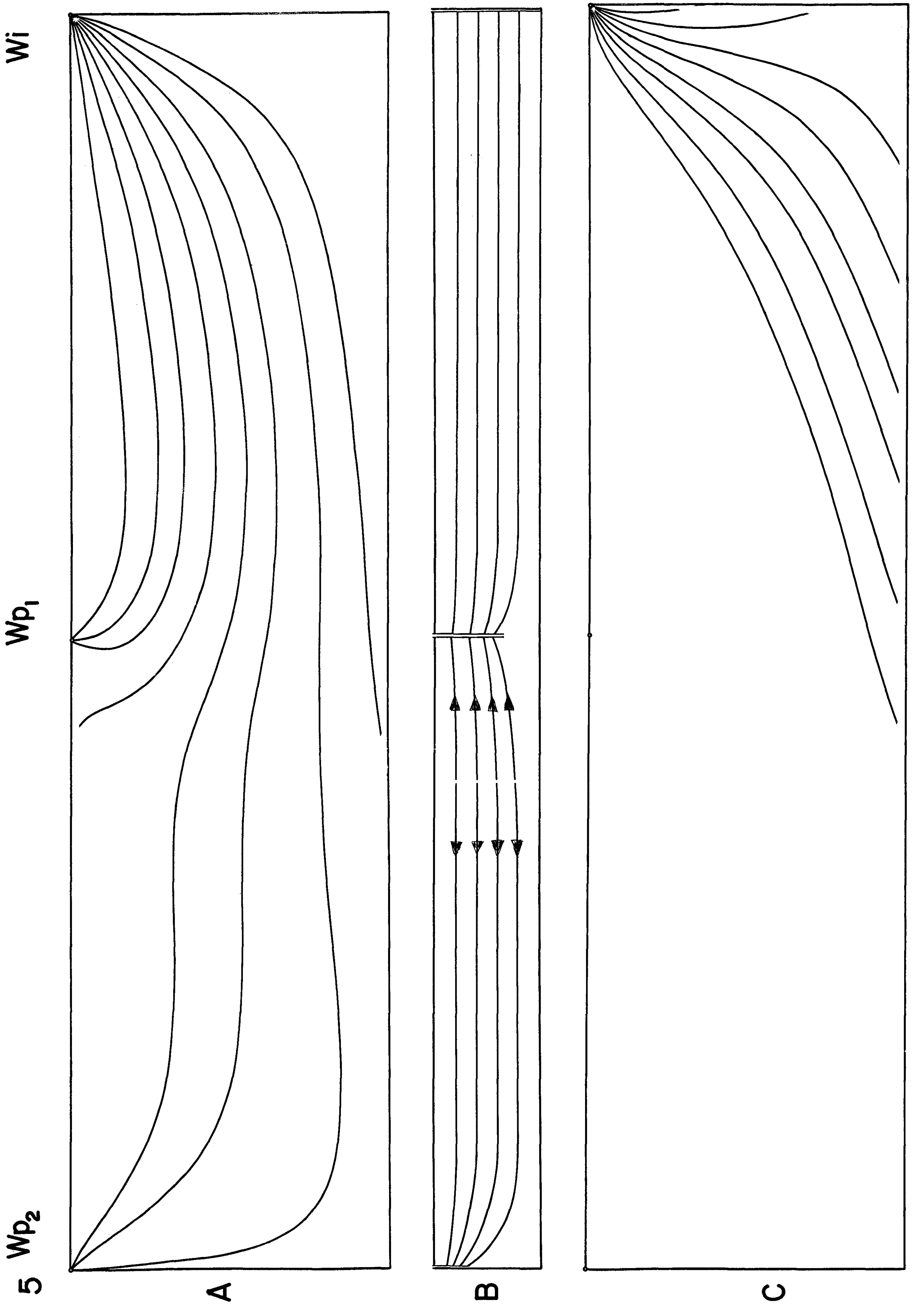
Wi
Wp$_1$
Wp$_2$
5
A
B
C

Figure 9

Case 6: Streamline maps

A - surface of model

B - vertical plane through the wells

C - bottom of model

	First producing well W_{p_1}	Second producing well W_{p_2}
Well penetration	1/3	Full
Relative output rate (percent of input rate)	50	50

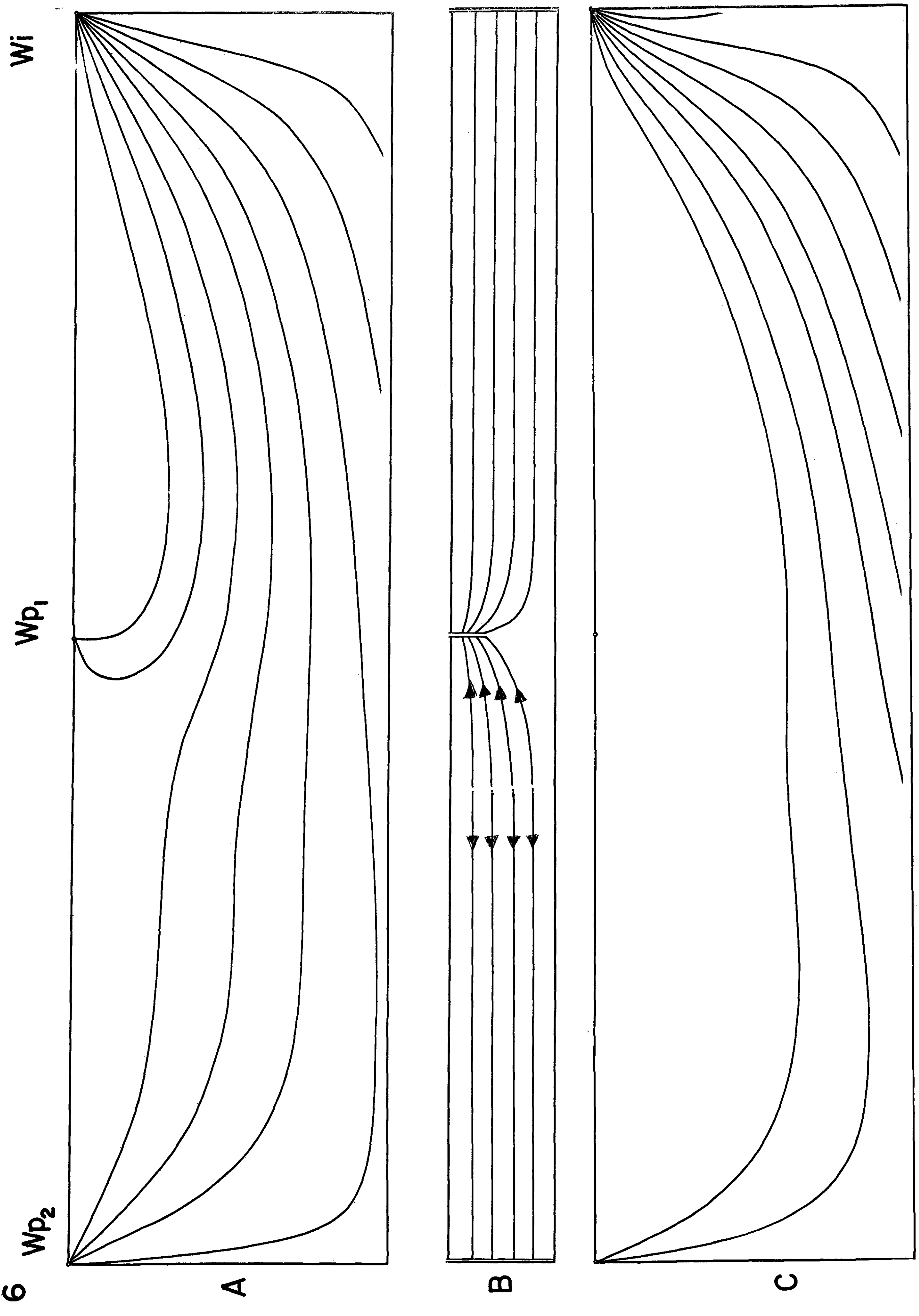

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Figure 10

Case 7: Streamline maps

A - surface of model

B - vertical plane through the wells

C - bottom of model

	First producing well W_{p_1}	Second producing well W_{p_2}
Well penetration	1/3	2/3
Relative output rate (percent of input rate)	50	50

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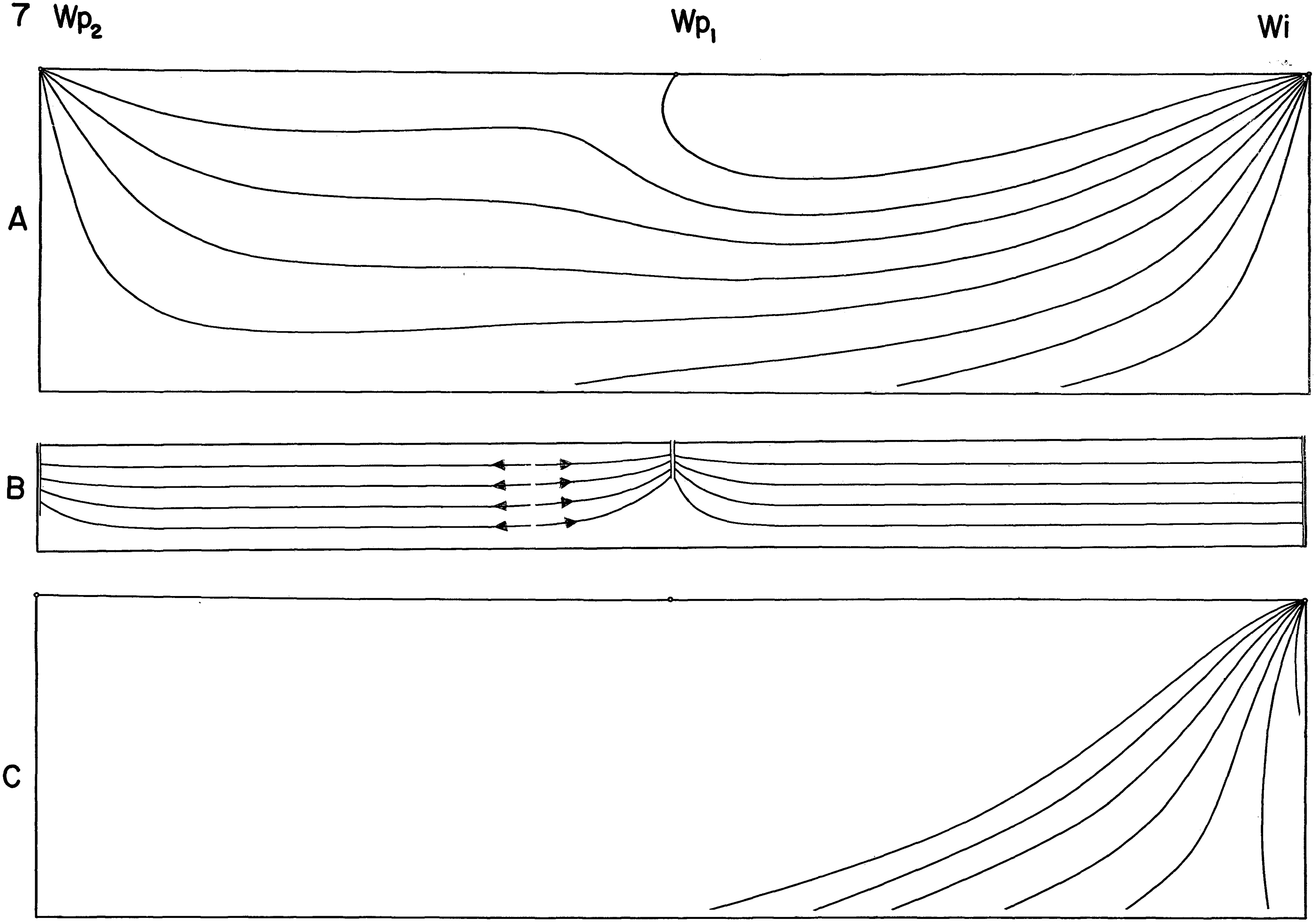

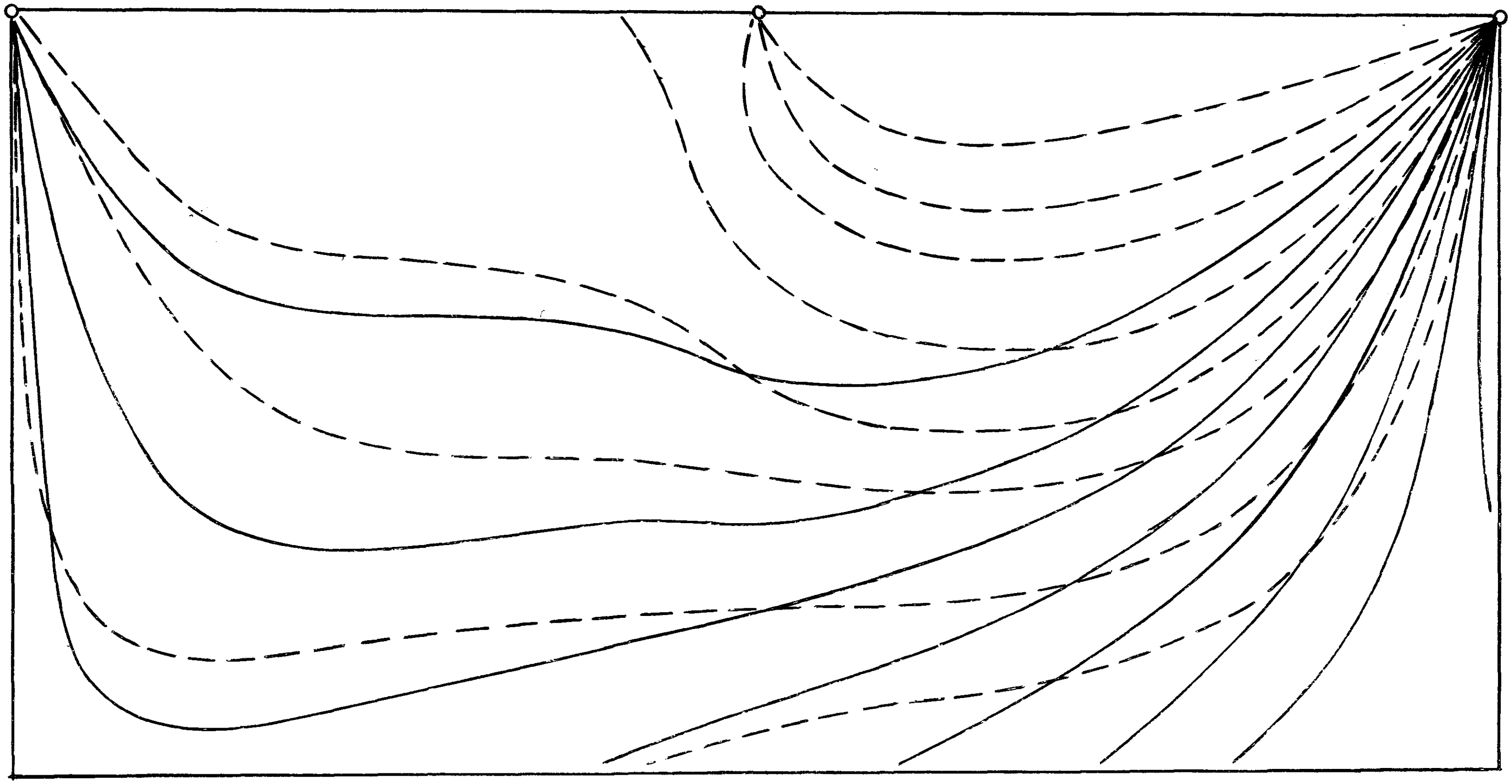

Figure 11

COMPARISON OF SURFACE STREAMLINES

Continuous lines: Case 1

Dashed lines: Case 5

Exaggeration in Y Direction: 2X

CONCLUSIONS

Electrolytic potentiometric models can conveniently be used to investigate the performance of programs of secondary recovery of oil. The accuracy with which predictions may depict actual reservoir performance is a function of both the accuracy with which the reservoir configuration is known and the accuracy with which reservoir and well characteristics are reproduced in the model.

The results obtained in this investigation prove that:

1. the potential distribution on any plane of the model depends on the length of the input and output sources;
2. error in performance predictions results from

two-dimensional potential mapping of only the surface of the reservoir model;

3. the error is larger with greater variation in the length of the output sources;

4. tri-dimensional analyses are possible by combining two-dimensional potential surfaces, and better results can be anticipated by this method for partial penetrating wells.

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