BETA-DECAY STUDIES OF NEUTRON-RICH

$^{11}$LI AND $^{32}$NA

by

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ABSTRACT

Two experiments were performed at the radioactive ion beam facility TRIUMF-ISAC in October 2003 and November 2004 with the goal of studying properties of short-lived, neutron-rich nuclei. Beta-particles and gamma-rays from the $\beta$-decays of $^{32}$Na and $^{11}$Li were detected using the $8\pi$ spectrometer, an advanced detector array consisting of 20 high-purity Germanium $\gamma$-ray detectors, along with 20 plastic scintillating detectors (‘SCEPTAR’) for detection of electrons, and a moving tape collector for removal of long-lived daughter products.

The $\beta$-decay of $^{32}$Na populated excited states in several light, deformed nuclei including $^{32}$Mg at the N=20 shell closure. Based on $\gamma$-$\gamma$ coincidences, several revisions to the level scheme of $^{32}$Mg have been proposed, including two previously unknown energy levels. Evidence concerning the location of a disputed 4$^+$ state is presented.

The $\beta$-decay of the halo nucleus $^{11}$Li was studied as part of an ongoing search for evidence of ‘halo survival’ through $\beta$-decay. Doppler broadened peaks appear in the $^{11}$Li decay spectrum, resulting from the $\beta$-delayed neutron emission to $^{10}$Be. These Doppler broadened peaks have been simulated using Monte Carlo techniques in order to extract information on the neutron-emitting states in $^{11}$Be. This work may help resolve disagreements between two recent works that also explored the $^{11}$Li decay, yet came to inconsistent conclusions.
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CHAPTER 1

INTRODUCTION

Great diversity is found among the nuclei observed in nature. Diversity is seen for example in nuclear lifetimes: radioactive nuclei may have lives as short as milliseconds or as long as billions of years, while other nuclei appear to be stable. Nuclei may also have a variety of shapes, from spherical to prolate or oblate, and are subject to many different radioactive decay processes. The goal of nuclear structure physics is to explore the properties and behavior of all these nuclei, and to help form a cohesive model describing the nucleus.

Nuclear structure has enjoyed some significant successes: sixty years ago, nuclear physicists were striving to explain the structure of stable and long-lived nuclei, yet today these stable nuclei are fairly well understood. The new frontiers of nuclear structure physics now lie among short-lived nuclei, where unusual and unexpected nuclear properties are observed. Examples of experiments in nuclear structure today include searches for super-heavy isotopes, studies of collective properties in heavy deformed nuclei, and experiments probing the structure of the lightest nuclei near the drip-lines.

Two experiments on light neutron-rich nuclei are discussed in this thesis. Nuclear $\beta$-decay is used in these experiments as a probe to explore the properties of the daughters of the short-lived neutron-rich nuclei $^{11}$Li and $^{32}$Na. Both of these nuclei display unexpected behavior: $^{32}$Na lies in the middle of a cluster of light, deformed nuclei, while $^{11}$Li has an unusually large matter distribution attributed to a two-
neutron ‘halo’. Through their $\beta$-decays, we seek to learn more about the origins of these unusual configurations.

Experimental difficulties are the norm when studying such short-lived nuclei: for example, the radioactive nuclei must be quickly produced, extracted and delivered to an experimental station before they decay away. Also, ‘background’ events from stray radiation or electronic noise must be reduced as much as possible, so that the ‘true’ events are statistically significant.

The two experiments discussed here were performed under very different conditions: in the first experiment, a low count rate of approximately 2-3 $^{32}$Na ions/second was delivered to the experimental facility. In this case, the true events would have been statistically insignificant except for the discriminating power provided by the detector array that permitted the removal of several important background contributions. In the second experiment, a much higher rate of approximately 12000 $^{11}$Li ions/second was collected. In this case, peaks in the $^{11}$Li decay spectrum were observed with high signal-to-noise ratio, and the main challenge arose instead from the extremely complex decay of $^{11}$Li, and the complicated features revealed by the high count rate.

These two experiments were performed in different regions of the table of isotopes, and had different goals and outcomes. However, a common thread links these experiments: in both cases, a precise understanding of the experimental facility and data acquisition along with the full complement of $\gamma$-spectroscopy tools were required to study nuclear structure through $\beta$-decay. This thesis presents two different approaches to the study of $\beta$-decay of neutron-rich nuclei. This illustrates the kind of work made possible in two different experimental conditions: one, in which beam intensity is severely limited, the second, where higher beam intensity is available
revealing complicated features requiring in-depth analysis.

The main body of this thesis consists of four chapters: in chapter 2, we discuss the basics of nuclear structure, $\beta$-decay and the nuclear shell model. In chapter 3, the details of the experimental facility are described. The experiments ($\beta$-decays of $^{32}\text{Na}$ and $^{11}\text{Li}$) are described in chapters 4 and 5. A conclusion is presented in chapter 6.
CHAPTER 2

THEORETICAL BACKGROUND

The atomic nucleus was discovered in 1911 by Ernest Rutherford (1871-1937), who observed a large cross section for α-particles scattering into backward angles from a thin gold foil. Rutherford attributed this strong back scattering to the presence of a dense, positively charged core inside the otherwise mostly empty atom. His hypothesis departed from earlier views such as the ‘plum-pudding’ model put forward by J.J. Thomson, in which the atom was thought to consist of negative point charges (electrons) vibrating inside an atom made mostly of diffuse, positively charged matter.

Our current model of the nucleus follows Rutherford’s hypothesis: the atom is made up of an electron cloud held in a Coulomb-potential well around a dense nucleus made up of protons and neutrons. Typical atomic radii are measured in units of $10^{-10}$ m, or angstroms (Å), whereas the nucleus is much smaller with a radius measured in units of $10^{-15}$ m, known as femtometers or fm (abbreviating ‘fermi’ in honor of physicist Enrico Fermi, 1901-1954). Despite the very small size of the nucleus it contains most of the mass of the atom: protons and neutrons have masses approximately 1836.2 and 1838.7 times greater respectively than that of the electron.

The nucleons are bound together by the strong nuclear force, that must be at least strong enough to overcome mutual Coulomb repulsion by the protons. Within the potential created by this strong force, regular features appear, suggesting that the nucleus may be described by a theoretical approach similar to the atomic shell model. For example, as in chemistry the noble gases are more stable than other elements due
to their atomic structure, some nuclei with certain ‘magic numbers’ of either protons or neutrons are also found to be more stable than their neighbors.

Nuclei are observed to undergo spontaneous transformations through radioactive decays. The first radioactive decay was observed in 1896 by A.H. Becquerel, who noticed that photographic plates were blackened after proximity to uranium salts despite being heavily wrapped in black paper. He attributed the discovery to penetrating radiation, able to pass through the protective black wrapping where visible light could not. Several forms of nuclear radiation were subsequently identified, including α, β and γ-rays.

From these basic insights discovered more than one century ago, physicists have built a complex view of nuclear properties, radiation, and structure. In this chapter, we will briefly discuss the makeup of the nucleus, fundamentals of nuclear β and γ-decay, and give an overview of the nuclear shell model.

2.1 Nuclear Properties

Physicists have so far observed more than 3500 unique nuclear isotopes, from the proton to a newly-discovered heavy, short-lived isotope of element 118. Each isotope can be characterized in its ground state by several key properties, including the binding energy, half-life, and the spin and parity. These measurable nuclear properties help characterize the isotope.

2.1.1 Binding Energy

When Z protons and N neutrons are assembled to form an atomic nucleus $^{A}_{Z}X_{N}$, the final ground-state rest energy is less than the sum of $Zm_{p}c^{2} + Nm_{n}c^{2}$. An adjustment must be made for the binding energy of the new composite nucleus. The
binding energy keeps the nucleus from breaking apart into its constituents. Binding energy is defined:

\[ B = Zm_p c^2 + Nm_n c^2 - [m(^A X) - Zm_e]c^2 \]  

(2.1)

where \( m(^A X) \) is the measured atomic mass of the nucleus of interest [1].

Several factors impact the nuclear binding energy. For example, Coulomb repulsion between the protons reduces the binding, and nuclei with large \( N/Z \) ratios are usually less bound. On the other hand, nuclei with even numbers of protons or neutrons have greater binding energy than their immediate neighbors. This increase in binding energy arises since two nucleons with opposite spins form a spin-zero ‘pair’ with lower energy.

From the definition of the binding energy, we may define the neutron ‘separation energy’ \( S_n \), the energy required to remove a single neutron from the nucleus. The single-particle separation energy is however not always useful, since the energies become ‘staggered’ due to the pairing effect. Instead, the two-particle separation energy \( S_{2n} \) is defined:

\[ S_{2n} = B(^A X_N) - B(^{A-2}_2 X_{N-2}) \]

\[ = [m(^{A-2}_2 X_{N-2}) - m(^A X_N) + 2m_n]c^2 \]  

(2.2)

In figure 2.1 the two-neutron separation energies for calcium and nickel isotopes are shown. The separation energy decreases suddenly above two of the so-called ‘magic numbers’, demonstrating the underlying shell structure of the nucleus: isotopes that fill a shell are more stable, with tightly bound nucleons, whereas nucleons lying just outside a closed shell are less tightly bound.
2.1.2 Half-life

While some nuclei on the chart of nuclides are stable, the overwhelming majority are subject to radioactive decay. If a uniform sample of $N_0$ radioactive nuclei is collected, the number of surviving nuclei after time $t$ is expressed:

$$N(t) = N_0 e^{-\frac{ln(2)t}{t_{1/2}}}$$  \hspace{1cm} (2.3)$$

where $t_{1/2}$ is the half-life for the radioactive decay.

Radioactive half-lives vary greatly across the chart of nuclides. Half-lives near the neutron and proton drip-lines are often of the order of milliseconds or shorter while closer to stability half-lives are typically measured in days, years, or longer.
2.1.3 Angular Momentum and Parity

Each nucleon in the nucleus has an orbital angular momentum \( \ell \) and spin \( s \), that may be combined to find its total angular momentum \( j \). From the total angular momenta of the nucleons, we may also find the total ‘nuclear spin’ \( J \) of the entire nucleus: \( J = \) the vector sum over all \( j \) for the constituent nucleons. Each nucleus, and each excited state in the nucleus, can therefore be identified by a characteristic \( J \)-value that can change only when angular momentum is added or carried away during a reaction or decay.

These states can also be identified by their parity. The parity operation \( P \) inverts all coordinates through the origin: \( r \rightarrow -r \). If, as a result, \( \psi(-r) = \psi(r) \) the function \( \psi \) has \( \pi = + \) or ‘even parity’, whereas if \( \psi(-r) = -\psi(r) \), the function has \( \pi = - \) or ‘odd parity’. For a single nucleon in a central potential, the parity \( \pi \) is related to the orbital angular momentum: \( \pi = (-1)^\ell \), and as the number of nucleons increases the parity can be determined as \( \pi_{\text{tot}} = \pi_1 \pi_2 \ldots \pi_n \).

If the wave function of the nucleus (including all the nucleons) were known completely, \( J \) and \( \pi \) could be determined analytically. In practice, the wavefunction is not generally known and so the spin and parity are treated as “overall” properties of the nucleus, to be measured through experimental techniques.

Two important generalizations can be made about the spin and parities of nuclear isotopes. First, the ground states of \textit{all} known even-N, even-Z nuclei are observed to have \( J^\pi = 0^+ \), a result of pairing between nucleons. Also, since both neutrons and protons are spin \( \frac{1}{2} \) particles, nuclei with an odd number of nucleons have half-integer angular momentum \( (\frac{1}{2}, \frac{3}{2}, \frac{5}{2} \text{ etc}) \) while nuclei with even \( A \) have integer \( J \).
Figure 2.2. The table of isotopes for $1 \leq Z \leq 13$. Neutron number $N$ increases along the x-axis, proton number $Z$ increases along the y-axis.

### 2.2 $\beta$-Decay

Most radioactive nuclei decay through the $\beta$-decay process, where in the nucleus a proton is converted to a neutron or neutron to proton, reducing the overall energy of the nucleus. A $\beta^-$ decay occurs in neutron-rich nuclei: a neutron is converted to a proton and an electron and antineutrino are emitted:

$$\frac{A}{Z}X_N \rightarrow \frac{A}{Z+1}Y_{N-1} + e^- + \bar{\nu}_e$$

The electron is required for conservation of charge, and the antineutrino $\bar{\nu}_e$ is required in order to conserve the number of leptons during the $\beta$-decay: leptons are particles including $\bar{\nu}_e$ and $e^-$ that are not subject to the strong nuclear force. During $\beta$-decay, one lepton and one anti-lepton are created, resulting in net conservation of lepton number.

A $\beta^+$ decay occurs in proton-rich nuclei: a proton is converted to a neutron while
an anti-electron (or ‘positron’) and neutrino are emitted:

\[ \frac{\gamma}{z}X_N \rightarrow_{z-1} ^{A}Y_{N+1} + e^+ + \nu_e \]

Electron capture is also possible in proton-rich nuclei, and competes with $\beta^+$ decay:

\[ \frac{\gamma}{z}X_N + e^- \rightarrow_{z-1} ^{A}Y_{N+1} + \nu_e \]

where the electron is captured from an inner atomic orbital.

The lightest nuclei of the table of isotopes are shown in figure 2.2. In this light region, $\beta$-decay is the most common radioactive process. Isotopes drawn in red and orange are proton-rich, unstable to $\beta^+$ decay. The rest are mostly neutron-rich, and include both nuclei unstable to $\beta^-$ decay (in blue) and nuclei unstable to $\beta$-delayed neutron emission (in green). More details about $\beta$-delayed neutron emitters, which include both $^{11}$Li and $^{32}$Na, will be presented shortly.

### 2.2.1 $\beta$-Decay Selection Rules

Not all energetically available states in the daughter nucleus can be populated through the $\beta$-decay process. Possible excited states populated by the decay are determined by the selection rules, that determine the allowed changes in angular momentum and parity through the decay.

In an ‘allowed decay’, the (anti) electron and (anti) neutrino are assumed to carry no orbital angular momentum away from the decay. Any change in the total angular momentum $J$ of the nucleus must therefore result from the sum of the spins of the two leptons. Since both leptons are spin 1/2 particles, their total spin may sum to either 0 or 1.
Two types of allowed decays are thus observed. In Fermi decays the lepton spins are anti-parallel, and the change in total nuclear angular momentum $\Delta J = 0$. In Gamow-Teller decays the spins are parallel, and $\Delta J = 0$ or 1. Parity is conserved in both cases: $\pi = (-1)^\ell$ and so if $\ell = 0$ under the allowed approximation, $\Delta \pi = 0$. The selection rules for allowed decays are therefore summarized:

Table 2.1. $\beta$-decay selection rules under the allowed approximation

\[
\begin{array}{c|c}
\Delta J & \Delta \pi \\
\hline
0,1 & 0
\end{array}
\]

In some cases, the electron and neutrino may be formed with non-zero orbital angular momentum. In this case, $\Delta J > 1$ and parity change become possible. The contribution of these ‘forbidden decays’ is normally strongly suppressed relative to allowed decays.

The selection rules play an important role in determining which levels in the daughter nucleus are populated during $\beta$-decay. For example, if the spin and parity of the ground state of the parent nucleus are very different from the spin and parity of the daughter’s ground state, excited states in the daughter may still be populated permitting exploration of the daughter through subsequent $\gamma$-ray emission.

### 2.2.2 Beta-Delayed Particle Emission

Very neutron-rich and proton-rich nuclei may emit extra particles following $\beta$-decay. To understand this process, we define the $Q$-value of the beta decay:

\[
Q_\beta = (m_{\text{parent}} - m_{\text{daughter}} - m_e - m_{\nu})c^2
\] (2.4)
Figure 2.3. Theoretical $\beta$-decay spectrum, showing the number of electrons emitted vs. electron kinetic energy. $Q_\beta=2.5$ MeV

For our purposes, the neutrino mass $m_\nu$ is negligible. As the Q-value grows large near the proton and neutron drip-lines, more high-lying excited states in the daughter nucleus can be populated.

As the $\beta$-decay Q-value grows very large, excited states above the particle emission thresholds may be populated. The excitation energy of these states exceed the separation energies for particle emission (neutron, proton, or other charged particles). These states therefore quickly emit particles, populating states in isotopes closer to stability. For example, in a very neutron-rich nucleus $\beta$-decay may be followed by emission of a neutron:

$$^{A-1}X_{N-2}^{(s)} \rightarrow ^{A-1}X_{N-1}^{(s)} + n$$
Figure 2.4. Two examples of $\beta$-decay in neutron-rich nuclei.

(a) $\beta$-decay with a moderate $Q$-value

(b) A decay with a large $Q$-value, populating states above the neutron separation energy (green).

In light nuclei, the final daughter nucleus will recoil significantly after emitting this neutron or proton, due to conservation of momentum. Figure 2.4 shows a schematic $\beta$-delayed neutron emission in a neutron-rich nucleus.

Half-lives of particle-emitting states are typically much shorter than half-lives for $\gamma$-decay, and these short half-lives have important consequences per the energy-time uncertainty relationship. The uncertainty relationship states that $\Delta E \Delta t \geq \hbar/2$, and may be written in the alternate form $\Gamma \tau \geq \hbar/2$ where $\Gamma$ is the energy width of a state and $\tau$ ($=t_{1/2}/\ln(2)$) is the mean lifetime of the state. As the lifetime of a state grows very short its width $\Gamma$ must grow larger. These excited states may be expressed as Lorentzian probability distributions about central energy $E_0$ with full-width at half-maximum $\Gamma$:

$$P(E) = \frac{1}{\pi} \frac{\Gamma/2}{(E - E_0)^2 + (\Gamma/2)^2} \quad (2.5)$$

This is also known as a ‘Breit-Wigner’ distribution.
2.3 Gamma-Ray Transitions

When a nucleus is left in an excited state (below any particle emission threshold) following $\beta$-decay, it usually de-excites via gamma-rays as shown in figures 2.4(a and b). The $\gamma$-rays constitute changes in the electromagnetic initial and final states of the nucleus, along with the emission of a high-energy photon (the $\gamma$-ray):

$$E_\gamma = E_i - E_f - \Delta_r$$  \hspace{1cm} (2.6)

where $\Delta_r$ is a small adjustment, for the recoil of the nucleus during $\gamma$-decay. The $\Delta_r$ term is necessary for conservation of linear momentum, but the term can usually be neglected except in precision measurements or in the case of a high energy $\gamma$-ray from a light nucleus.

Gamma rays are an extremely useful form of radiation for the study of nuclear structure. Since $E_\gamma$ is approximately equal to the difference between initial and final energy levels, we may use detected $\gamma$-rays to reconstruct the level scheme of a given nucleus. Gamma ray coincidences, consisting of two or more photons detected within a very short time span, are even more useful as they provide direct evidence for $\gamma$-ray cascades from subsequent excited states.

2.4 Elements of the Nuclear Shell Model

Early nuclear physicists discovered that some nuclei were more tightly bound than their neighbors. These nuclei appear at the magic numbers: 2,8,20,28,..., and served as some of the first evidence of an underlying shell structure in the nucleus.

Work on the nuclear shell model was inspired by success in atomic physics, where electrons are described as being bound within quantized orbitals with well-
Figure 2.5. As a central potential approximation for the nuclear strong force, the Woods-Saxon potential may be used as a basis for the shell model.

defined energy and angular momentum. The nucleus is, however, quite different from the atomic electrons: while electrons are bound by an external Coulomb potential, nucleons are bound by mutual strong force interactions.

Unlike the Coulomb force, the nuclear strong force has little long-range effect. At distances of 1-2 fm, however, it becomes attractive, and at even closer range it becomes strongly repulsive (due to Pauli exclusion). Many nucleons, all interacting through the strong force, create a nucleus of approximately constant density, with a radius approximately proportional to the number of nucleons: \( R \approx R_0 A^{1/3} \) with \( R_0 = 1.2 \) fm. Rather than cutting off sharply at \( R \), the nuclear matter distribution is observed to tail off over a relatively short distance. The mean-field of the nucleus therefore consists of a central region of approximately constant density, with a thin ‘skin’ where the density falls off. The ‘Wood-Saxon’ potential, shown in figure 2.5, captures this behavior:

\[
V(r) = \frac{-V_0}{1 + e^{(r-R)/a}}
\]  

(2.7)
When the Woods-Saxon potential is used in the time-independent Schrödinger equation, large gaps are found between the resulting energy eigenvalues. These gaps support the theory of nuclear shell closures since more energy would be required to cross the gap into the next shell. However, the locations of these shell gaps do not agree with the experimentally measured magic numbers beyond \( N,Z=20 \).

In 1949, Mayer and a collaboration of Haxel, Seuss and Jensen realized independently that a strong, attractive spin-orbit term \( V_{ts}(r) = -U(r) \ell \cdot S \) in the nuclear potential could resolve this disagreement [2, 3]. The \( \ell \cdot s \) coupling term splits each (degenerate) state with \( \ell > 0 \):

\[
\Delta E_{\ell} = \frac{(2\ell + 1)\hbar^2}{2} \int_0^\infty r^2 \, dr \, R_{\ell m}(r) U(r)
\]  

(2.8)

The spin-orbit splitting therefore increases approximately as \( (2\ell + 1) \) as shown in figure 2.6. The gaps in the resulting orbitals agreed with the magic numbers, laying the foundation for a successful shell-model description of the nucleus.

According to this original view of the shell model, nucleons fill each orbital in succession. Filled shells are assumed to be inert, so that the properties of the nucleus are usually determined only by valence nucleons or holes on top of the filled shells. Using this model, also known as the ‘independent particle model’, properties of many stable nuclei near the shell closures are fairly well understood: electric and magnetic multi-pole moments may be reproduced along with masses and radii. However, this model is not as well suited for studying large, deformed nuclei, or for moving away from shell closures. Also, significant deviations from the typical shell structure is observed when considering nuclei far from stability.
Figure 2.6. The shell model: A spin-orbit correction is added to the Woods-Saxon potential to produce energy gaps at the experimentally observed ‘magic numbers’ (2, 8, 20 etc)
2.4.1 Improved Shell Models

The independent particle model described in section 2.4 was designed to explain the shell closures observed in nuclei near the valley of stability, with certain magic numbers of protons or neutrons. This model has been very successful at describing these nuclei, and can be extended to also describe the properties of their neighbors (single-particle or single-hole states built on shell closures).

Away from the shell closures (and away from stability), however, calculations with the independent particle model become more difficult. Interactions between the valence particles, or between particles and holes, become non-negligible and must be taken into account, creating a large computational problem.

Fortunately, the steady increase in computing power over the last few decades has encouraged the growth of large-scale shell-model calculations. Many different approaches have been taken, yet they share some common threads: the calculation requires a realistic interaction between the nucleons, inside a valence space large enough to take into account all possible interactions expected to contribute. For a much more in-depth overview of the shell model, and a comparison to other less computer-intensive views of nuclear structure, the reader could look at references [4] and [5].

2.5 Deformation

Deformation is a common phenomenon, particularly in relatively heavy nuclei. In deformed nuclei, the ground state configuration is not spherical. Nucleons instead form prolate or oblate shapes: ellipses rotated about their semi-major or semi-minor axes respectively (see figure 2.7 for familiar examples). The shape of the deformed
nucleus may be related to the spherical harmonic $Y_{20}$:

$$R(\theta, \phi) = R_{av}(1 + \beta Y_{20}(\theta, \phi))$$  \hfill (2.9)

where $\beta$ is the deformation parameter:

$$\beta = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R_{av}}$$  \hfill (2.10)

By convention, oblate deformations have $\beta < 0$ and prolate deformations have $\beta > 0$ [1].

Deformation gives rise to new rotational degrees of freedom for the nucleus: unlike a spherically symmetric nucleus, rotation of a deformed nucleus produces observable changes. Treating the nucleus (for now) as a rigid body rotating about its center of mass, the kinetic energy of rotation $E = \frac{1}{2}I\omega^2$ where $I$ is the moment of inertia and $\omega$ is the angular velocity. Then, since angular momentum $\ell = I\omega$, the energy may be re-written $E = L^2/(2I)$. The eigenvalues of the angular momentum operator squared is $L^2 = \hbar^2\ell(\ell+1)$ [6]. This applies to spin, orbital, and total angular

Figure 2.7. Familiar examples of oblate, spherical, and prolate shapes
<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E_\gamma$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24^+ \rightarrow 22^+$</td>
<td>508.3</td>
</tr>
<tr>
<td>$22^+ \rightarrow 20^+$</td>
<td>469.0</td>
</tr>
<tr>
<td>$20^+ \rightarrow 18^+$</td>
<td>428.6</td>
</tr>
<tr>
<td>$18^+ \rightarrow 16^+$</td>
<td>387.4</td>
</tr>
<tr>
<td>$16^+ \rightarrow 14^+$</td>
<td>345.5</td>
</tr>
</tbody>
</table>

Figure 2.8. Example of a rotational band, in $^{196}$Pb. The transitions and $\gamma$-ray energies are shown, along with the resulting spectrum. Data from [7]

momentum, so for a nucleus with total angular momentum $J$,

$$E_{J,\text{rotational}} = \frac{\hbar^2}{2I} J(J + 1)$$

(2.11)

While nuclei cannot be realistically treated as rigid bodies, equation 2.11 does give certain useful insights into the nature of deformed nuclei. In particular, it predicts a ‘rotational band’ of excited states that should be observed in the deformed rotators, with the energy depending on $J(J + 1)$. An example of a rotational band is observed in $^{196}$Pb, shown in figure 2.8.

### 2.5.1 Shell Model Treatment of Deformation

Several important effects combine to allow the description of deformation in the shell-model. The first important consideration is the fact that particle-hole excitations near shell closures always require less energy than expected using a single-particle Hamiltonian. The difference, arising from particle-hole correlations, may decrease the importance of the shell gap. This tendency can be enhanced near the drip-lines,
where the normal shell gaps have been observed to be weakened (see for example [8]).
These interactions may result in intruder states forming low-lying excited states, or
even displacing the normal ground-state configuration.

Particle-hole correlations help us understand low-lying intruder states, but to understand deformations in the context of the shell model, we must also consider p-n interactions. These interactions determine the mean field of the nucleus, including any possible deformation [5, 9].

2.6 Halos

We must now discuss one more unusual nuclear shape: the 'halo' configuration arising in some weakly-bound nuclei near the proton and neutron drip-lines. Halo systems appear both in nuclei and in atomic molecules, and stem from an inherently quantum effect. A halo occurs when the wave function of a particle bound loosely within a potential well tunnels out into the classically forbidden region outside the well. The simplest example is encountered in an introductory course in quantum mechanics: if a particle of mass $\mu$ is bound at energy $\epsilon$ within a finite, spherical square well potential $(V(r) = -V_0, r < R)$, the wave function may be expressed:

$$
\psi(r) \approx \begin{cases} 
\sin(kr)/r, & r < R \\
\sin(kR)e^{-\alpha(r-R)}/r, & r \geq R 
\end{cases}
$$

(2.12)

Here, the wave number $k = \sqrt{2\mu(V_0 - \epsilon)}$ and $\alpha = \sqrt{2\mu\epsilon}$. Thus lightly bound particles with small $\alpha$ are less damped for $r > R$ and so may have a significant fraction of the wave function outside the classical region [10].

Equation 2.12 is derived by assuming zero angular momentum ($\ell = 0$). This
Figure 2.9. Probability density for the potential well described by equation 2.12. The classical extent of the system is drawn in black.

...
Halo nuclei cannot be described using a 'mean-field' approach to the nucleus such as the Woods-Saxon potential described in section 2.4. Where the mean-field approach assumes that the nucleus has roughly constant density that dies off quickly at the nuclear radius, halo nuclei are instead characterized by two very different regions: the compact, high-density core and a diffuse, extended nuclear halo.
CHAPTER 3

EXPERIMENTAL FACILITY

Measuring the properties of very short-lived nuclei requires complex experimental facilities, since the nuclei must be produced on-site, transported to a detector, and measured within at most a few half-lives. In this chapter, we discuss the radioactive ion beam (or RIB) facility TRIUMF-ISAC, where beams of both $^{32}$Na and $^{11}$Li were produced. We also introduce the $8\pi$ $\beta$-decay spectrometer, where radiation from the $\beta$-decay of these nuclei was measured.

3.1 TRIUMF-ISAC

In order to study reactions or decays in nuclei far from stability with sufficient statistics, a beam of radioactive isotopes must be created. Among the various methods of producing these radioactive ion beams, the ‘Isotope Separation On-Line’ or ISOL technique is used at ISAC. Creating a RIB with ISOL requires a driver, usually high-energy charged particles from a cyclotron or linear accelerator, that strike a target, forming many radioactive nuclei through a combination of spallation, fission and fragmentation processes. These nuclei must then be ionized and accelerated to form a radioactive beam that can be steered to the experimental station.

Beams produced using ISOL techniques can potentially have very high flux, and relatively pure composition. The discriminating power of the mass separator depends inversely on beam velocity; since ISOL beams are extracted from the target with low energies contaminants can be effectively separated from the beam. Radioactive
ions may also be produced by fragmenting a high-energy beam on a thin foil, yet contaminants are more difficult to remove from a fragmentation beam using mass separation due to the higher beam energy.

ISOL techniques are used to produce radioactive ion beams at the TRIUMF-ISAC facility, in Vancouver, British Columbia. TRIUMF (originally the TRI University Meson Facility) is Canada’s national laboratory for nuclear and particle physics. The lab was built in 1972 for research in meson physics.

More recently, the proton beam has begun to be used as a driver for production of RIBs in the new experimental halls ISAC-I (completed in 1996-97) and ISAC-II (first beam in late 2006). These facilities make TRIUMF a leading facility for research in low-energy nuclear structure and astrophysics with radioactive beams.
3.1.1 The Cyclotron

The TRIUMF cyclotron is the largest in the world. It consists of six magnets in a pinwheel configuration, as seen in figure 3.1. The magnets create a large magnetic field of mean strength 4600 gauss in a vacuum chamber with an approximate radius of 28 feet. The six magnets form an ‘azimuthally varying field or AVF cyclotron’, in which ions encounter alternating regions of high and low magnetic field as they are accelerated. This AVF design of the cyclotron has the benefit of permitting higher beam energy and higher beam current simultaneously [12].

At TRIUMF unlike most other facilities the cyclotron is used to accelerate $H^-$ ions rather than $H^+$. Extracting the accelerated beam is therefore simpler for this cyclotron: the two electrons are ‘stripped’ with a 0.025 mm thick carbon foil, and the direction of the Lorentz force $\vec{f} = q(\vec{v} \times \vec{B})$ changes, steering the now-positive ions out of the cyclotron.

Several beamlines at varying energies may therefore be operated simultaneously using the TRIUMF cyclotron, a major advantage of this accelerator design. A proton beam of approximately 520 MeV may be achieved by inserting the stripping foil at the outer edge of the cyclotron, yet lower energy beamlines may also be operated by inserting foils closer to the center of the cyclotron.

One proton beam, operating at $\sim$70 MeV is dedicated to proton therapy. As a proton beam stops in matter, most energy is deposited near the end of the proton’s track, and this localized energy deposition may be used to destroy cancers of the eye. Proton therapy can be a very successful alternative to surgery, since the beam can be localized with collateral damage only in the immediate vicinity of the tumor.

Several other beam lines are currently operational and used for research. They include beam line BL1A, where protons are used to drive the meson experiment TWIST.
Figure 3.2. Proton beams extracted from the TRIUMF cyclotron may be used for proton therapy, for experiments in the ‘proton hall’, or sent to the ISAC radioactive ion beam facility (the TRIUMF Weak Interaction Symmetry Test). For our research, 500 MeV protons are extracted from the TRIUMF cyclotron in BL2A and sent to the radioactive ion beam facility ISAC.

3.1.2 The Isotope Separator and ACcelerator ISAC

The high-energy proton beam from the TRIUMF cyclotron serves as the driver for producing radioactive ions at the Isotope Separator and ACcelerator ISAC. ISAC uses ISOL techniques to produce radioactive beams: the 500 MeV beam of protons is impinged upon a solid high-Z target (usually of Si, Ta, or Zr; in both experiments discussed here a 22.4 g/cm² Tantalum target was used). The proton beam can be delivered to the ISAC production target with currents up to 100 µA. Radioactive ions are produced in the ensuing interactions (since fragments can be produced with extreme N/Z ratios, spallation is an efficient way to produce the radioactive nuclei of interest).
Figure 3.3. The high-power target is designed to receive greater proton flux from the cyclotron. The HPT was not available during the experiments discussed in this work, yet it promises higher beam yields for the future.

Fragments from spallation are extracted from the target by diffusion. To maximize diffusion (and RIB yield) without damaging the target, a temperature near 1200°C is sought. The target is heated partly by the proton beam, and also by supplemental electronic heating. Target damage is avoided by maintaining a wide focus of the proton beam, so that the proton flux/area is kept low, and additionally by capping the overall proton flux. Recently, a new, ‘high-power’ target (shown in figure 3.3) was developed with the goal of accepting a higher flux of protons, by dissipating heat more quickly [13]. The high power target was first used experimentally in June, 2006.

After radioactive ions are formed by spallation and fragmentation in the target, they must be ionized in order to be extracted, mass-separated and steered. Many possible ionizing processes are available, depending mostly on the chemical properties of the desired isotope. For example, surface ionization occurs when fragments strike metal with a high work function, removing the loosely bound valence electrons. Surface ionization is effective mainly for alkali atoms. Ions may also be formed via resonant laser excitation and ionization, in which atomic electrons are first excited to
Figure 3.4. Isotopes are selected using a magnetic mass separator. Ions with higher charge/mass ratios have smaller radii of curvature in the magnetic field.

bound orbits and then excited out of the atom with two or more tunable lasers. Laser ionization has recently been used at TRIUMF to study the decay of $^{62}$Ga [14]. An electron cyclotron resonance or ECR ion source is also under development at ISAC, with the goal of efficiently ionizing noble gases.

Once ionized, isotopes are extracted from the ion source and mass-separated with a magnetic dipole mass analyzer, in which the radius of curvature depends on the charge/mass ratio. By changing the strength of the B-field and the position and width of slits at the far end, the mass analyzer is tuned to select specific $q/m$ ratios. After the slits, a fairly pure beam of the desired isotope $\frac{A}{2}X$ in a single charge state remains. Some contaminants are still hard to remove, including isotopes with masses $n \times A$ in higher charge states, and isotopes on the same isobar with small mass differences from the desired isotope.

Development of new and more intense radioactive ion beams is an important ongoing endeavor at RIB facilities, as higher yield, more isotopically pure beams open up a new range of possible experiments. Different target materials are used in order to achieve the best yields in various regions of the table of isotopes. New targets
are also being developed for future use, including a Uranium Carbide target intended for creating extremely neutron rich isotopes in the medium mass range. Aside from target material, several factors are found to influence RIB yield, including proton flux (RIB yield is observed to increase non-linearly with proton current), heating of the production target, and age and wear of the target [15].

After a beam of ions is extracted from the target and separated in the mass analyzer, the ion beam envelope tends to diverge due to the Coulomb force. The beam must therefore be periodically refocused using electrostatic quadrupole lenses. Treating the z-direction as the beam axis, the nonuniform field inside an electric quadrupole focuses the ion beam in x, while defocusing in y. Two quadrupoles rotated by 90° will therefore result in a focused beam.
Figure 3.6. A three-dimensional view of the ISAC-I radioactive ion beam facility, including the 8π spectrometer

Once the radioactive nuclei are formed, extracted from the target, and ionized, the resulting beams can then be steered to experimental stations in two different experimental halls: ISAC-I and ISAC-II. ISAC-I is the original experimental hall, built in 1997. It features several experimental stations at low energy, plus a linear accelerator capable of delivering RIBs with energies up to 1.5 MeV/nucleon. A beamline has also recently been completed to the new ISAC-II experimental hall. Radioactive ions delivered to ISAC-II will be able to be accelerated to much higher energies: approximately 6.5 MeV/nucleon for heavy isotopes and up to \( \approx 15 \) MeV/nucleon for light masses. As these higher energy beams become available new detectors are being built to take advantage of them, such as the TRIUMF-ISAC Gamma-Ray Escape Suppressed Spectrometer TIGRESS, a gamma-ray spectrometer using segmented High-Purity Germanium or HPGe detectors [16].

In ISAC-I, radioactive ion beams may be steered to many different experimental facilities as shown in figure 3.5. A high energy beam line provides accelerated beams
to the TUDA (TRIUMF-UK Detector Array) and DRAGON (Detector of Recoils And Gammas Of Nuclear reactions) experiments, to study reaction of astrophysical interest. The low energy beam line also hosts several experiments. Among the main experimental setups, the $\beta$-NMR facility is used to study the structure of metals and semiconducting materials by measuring nuclear spin relaxation. At the TRIUMF Neutral Atom Trap TRINAT, lasers are used to trap alkali atoms and search, for example, for evidence of $\beta$-decay asymmetry. Last but not least, the $8\pi$ spectrometer is used for high-precision $\beta$-decay studies, and is now described in more detail.

3.2 The $8\pi$ Spectrometer

The $8\pi$ spectrometer [17] has served several purposes in different laboratories over its 20 year lifetime. It was first assembled at the Chalk River Laboratories in Ontario in 1985 and used to observe gamma-rays following fusion-evaporation reactions. In 1997, the spectrometer was moved to the 88-inch cyclotron at Lawrence Berkeley National Lab, and then to ISAC-I in March, 2000. At ISAC, the spectrometer has been reconfigured as a tightly packed, relatively high efficiency array meant to be used along with several auxiliary arrays for precision $\beta$-decay studies of nuclei far from stability. The array is shown in figure 3.7, and a schematic view including several important features to be discussed is shown in figure 3.8.

The $8\pi$ consists primarily of 20 high-purity germanium (HPGe) gamma-ray detectors arranged to fill the hexagonal gaps in a truncated icosahedra (soccer ball geometry) frame. Germanium is a semi-conducting material with a band gap at room temperature of 0.67 eV [18]. Gamma-rays are detected in germanium in the ‘depletion zone’, the region of the detector in which a free charge is swept out by an
Figure 3.7. Photo of the open $8\pi$ spectrometer array, including SCEPTAR and the tape transport system.

Figure 3.8. A simplified schematic view of the $8\pi$ spectrometer, showing the HPGe $\gamma$-ray detectors, Compton suppressors, SCEPTAR and the tape transport system.
electric field. The thickness of the depletion zone is given by:

\[ d = \left(\frac{2eV}{\epsilon N}\right)^{1/2} \]

where \( \epsilon \) is the dielectric constant, \( V \) is the applied reverse bias voltage, and \( N \) is the impurity concentration [19]. Since \( \gamma \)-rays are detected only in the depletion zone, the thickness \( d \) must be maximized by reducing the impurity concentration (creating high-purity germanium) and by increasing the reverse bias. The detectors are also cooled with LN\(_2\) to reduce random electronic noise.

Gamma rays can interact with the germanium crystal three ways, depending on the \( \gamma \)-ray energy. The photoelectric effect has the largest cross section below 1 MeV or so, Compton scattering from 1 to 4 or 5 MeV, and pair production at higher energies. These interactions are shown in figure 3.9. In each case, energy from the incoming \( \gamma \)-ray is converted to electrons in the conduction band in the Ge crystal, that are then
swept out by the electric field, causing a detectible signal. In some cases, however, the detector may miss the full energy of the \( \gamma \)-ray (such as when a photon Compton scatters out of the detector).

The \( \gamma \)-ray detection efficiency of of a HPGe detector depends on its size: as larger Ge crystals are grown higher detection efficiencies become possible. HPGe is frequently chosen not only for detection efficiency, however, but for its excellent resolution. A HPGe gamma-ray spectrum in the 0-8 MeV range displays clear photopeaks with full-width half maximum of 3-5 keV. However, these photopeaks sit upon a relatively large background from Compton-scattered gamma-rays that deposit only part of their energy in the germanium crystal. At the 8\( \pi \), this Compton background is reduced with the help of an external array of 20 bismuth germanate (BGO) crystals. BGO is a dense, high-Z material that provides high gamma-ray detection efficiency yet poor resolution. Coincidences between events detected in both BGO and HPGe crystals are vetoed, reducing the Compton background significantly and improving the signal/noise ratio for the 8\( \pi \).

A new auxiliary array for \( \beta \)-particle detection was also added to the inner volume of the 8\( \pi \) in 2003. The SCintillating Electron Positron Tagging ARray SCEPTAR \cite{20, 21} consists of 20 thin plastic scintillating detectors attached via plexiglass light-guides to photo-multiplier tubes located outside the array. Each scintillator is collinear to one HPGe detector such that the HPGe crystal views the center of the array through a single SCEPTAR element. With this setup, the 8\( \pi \) may be used to observe \( \beta \)-\( \gamma \) coincidences following \( \beta \)-decay. These \( \beta - \gamma \) coincidences are more likely than \( \gamma \) singles to be associated with the decay of interest. The \( \beta-\gamma \) coincidence spectrum from the 8\( \pi \) + SCEPTAR has overall fewer uncorrelated gammas and therefore improved signal/noise.
Figure 3.10. The correlation between γ-rays detected in HPGe detectors and β particles detected in SCEPTAR. The diagonal strip corresponds to higher activity in collinear detectors due to bremsstrahlung.

Figure 3.11. Energy loss of electrons in the plastic scintillating detectors. Most electrons from the β-decays of interest have energies in the ‘minimum ionizing’ region.
The background from bremsstrahlung radiation can also be reduced at the $8\pi$ with the help of the new SCEPTAR array. Bremsstrahlung, or ‘braking radiation’, is produced when charged particles such as electrons decelerate in matter. As these particles slow, they emit photons in the direction of travel to conserve linear momentum. If these photons are detected by the HPGe detectors, they create a potential source of background in our $\beta$-decay experiments. At the $8\pi$, the contribution of bremsstrahlung to the overall background may be reduced by vetoing $\beta$-$\gamma$ coincidences detected in collinear plastic and HPGe detectors. By vetoing all such coincidences we may reduce the background and improve signal/noise at the cost of also eliminating 1/20th of real events. The impact of bremsstrahlung is shown in figure 3.10.

The scintillating detectors in SCEPTAR are made of the extruded ‘fast plastic’ BC-404, typically used for fast counting experiments. The energy loss of electrons in this scintillator is shown in figure 3.11. For $\beta$-decays of nuclei far from stability, $e^{-}$ energies on the order of tens of MeV are expected; as we see in figure 3.11 these electrons are ‘minimum ionizing’ in the scintillator, and the full energy of the electrons cannot generally be recorded. For example, a 10 MeV $e^{-}$ deposits only about 0.099 MeV as it passes through the 1.5 mm thick SCEPTAR detector. The energy loss varies slowly with ion energy, yet SCEPTAR does permit discrimination between decays with different Q values. The plastic scintillating detectors are thin (1.5 mm) so as to cause little $\gamma$-ray attenuation.

Radioactive ions arriving at the $8\pi$ are implanted in a tape system running through the center of the spectrometer, inside the vacuum chamber. This tape transport system permits fast removal of long-lived daughters whose activity could otherwise contaminate the measurement of the decay of interest. The system was designed and built at Louisiana State University, and consists of an aluminized mylar tape fed
continuously from a reservoir in a cassette. A tensioning device is used to allow slack in the tape reservoir. The tape may be run on several different cycles, integrated with the ISAC beam controls and with the optional use of a beam on/off ‘kicker’, controlling when the beam is implanted in the tape.

The tape transport system is primarily meant to quickly remove isotopes from the center of the 8π array, so that the array mainly records γ-rays related to the β-decay of interest and not from decays of subsequent daughter products. However, since the tape cycle can be controlled directly along with data acquisition, it can sometimes play a larger role in the experiment. Analyzing data obtained in different tape cycles can help determine whether γ-rays are associated with the β-decay of the initial, short-lived, radioactive ion, or with the decays of longer-lived daughter products. Gamma rays associated with the parent should continue to arrive at a constant rate as long as the beam is on, but γ-rays associated with long-lived daughters will be detected more often if the tape is allowed to stop for a while in front of the detector. This analytical tool will be discussed further in chapter 4.

3.3 Data Acquisition

The reconfigured 8π consists of multiple detectors collecting data simultaneously in order to explore the β-decay process. During an experiment, any event recorded by a detector is sent to a raw data stream along with a time stamp from the computer recording when the event occurred. The raw data streams are kept separate, and are only later recombined by a ‘front-end’ computer running the data acquisition. This modular design ensures that a high count rate from one detector array does not impact the data from another array.

Data collection and transfer is performed using CAMAC-based modules from
the LeCroy corporation (CAMAC stands for “Computer Automation Measurement And Control”, and is a standard for data handling in nuclear and particle physics experiments). The system is capable of a high data-transfer rate, up to 10 MB/s. Data streams from the HPGe and SCEPTAR arrays respectively are read out into first-in first-out or FIFO buffers. These buffers are then periodically polled, and whenever any are found to contain data all are read out to a computer running the MIDAS data acquisition system [22] (MIDAS, the “Maximum Integrated Data Acquisition System”, developed jointly by TRIUMF and the Swiss Paul Scherrer Institute, for event based data acquisition). All the data read from the FIFO buffers in one pass make up one MIDAS event. A detailed description of the $8\pi$ data acquisition system is planned for publication in *Nuclear Instruments and Methods* [23].

Since the data streams are separated, complex events (such as $\beta$-$\gamma$ coincidences) must be reconstructed later by inspecting the time stamp on each event and associating those arriving with close time stamps as coincidences. Thus the data acquisition system allows us both to rapidly collect data from different sources and to re-associate the data in post-analysis. However, in some cases it may not be possible to reconstruct events using the time stamp. For example, in chapter 5 we will discuss the impact of a power disruption to the clock in the CAMAC governing the SCEPTAR data stream. Due to the power failure, the time stamp was no longer reliable and other methods for reconstructing coincidence events had to be considered.

### 3.4 Performance of the $8\pi$

Before using the $8\pi$ spectrometer to study $\beta$-decays of exotic nuclei, we must quantify the performance of the array in order to understand the experimental results. The performance of the array includes efficiency of $\gamma$ and $\beta$-$\gamma$ coincidence detection,
and energy and time resolution.

HPGe detectors such as those used in the 8π have an energy-dependent efficiency for γ detection. The low-energy detection efficiency is reduced in p-type detectors as gamma-rays are absorbed by a thick diffused contact (n-type detectors feature a thinner contact and higher low-energy efficiencies). At higher energy, gammas grow more likely to penetrate through the detector without depositing all their energy. To quantify the efficiency of the 8π, several well-studied radioactive isotopes (152Eu, 133Ba, and 56Co) were placed in the center of the 8π + SCEPTAR array.

Table 3.1. Intensities of some γ-rays used to find the efficiency of the 8π detector array

<table>
<thead>
<tr>
<th>Parent</th>
<th>γ-ray (keV)</th>
<th>Intensity</th>
<th>Parent</th>
<th>γ-ray (keV)</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>152Eu</td>
<td>121.78</td>
<td>28.67(7)</td>
<td>60Co</td>
<td>1173.2</td>
<td>99.85(3)</td>
</tr>
<tr>
<td></td>
<td>964.08</td>
<td>14.65(2)</td>
<td></td>
<td>1332.49</td>
<td>99.98(3)</td>
</tr>
<tr>
<td></td>
<td>1408.0</td>
<td>21.07(2)</td>
<td>66Ga</td>
<td>1039.2</td>
<td>100</td>
</tr>
<tr>
<td>56Co</td>
<td>1238.3</td>
<td>66.9(6)</td>
<td></td>
<td>2189.6</td>
<td>15.12(15)</td>
</tr>
<tr>
<td></td>
<td>2598.5</td>
<td>17.3(3)</td>
<td></td>
<td>2751.9</td>
<td>63.1(5)</td>
</tr>
<tr>
<td></td>
<td>3253.4</td>
<td>8.1(2)</td>
<td></td>
<td>4295.2</td>
<td>10.6(1)</td>
</tr>
</tbody>
</table>

Selected γ-ray energies and intensities from these sources are found in table 3.1. The experimental intensities were compared to these tabulated values to find the relative efficiency response of the array from 200 keV to 3.5 MeV. This energy range has also been extended to higher energies by adding data from the electron capture decay of 66Ga (observed in December 2004 at the 8π with an identical setup). The intensities from the 66Ga decay are known to good precision [24], and their addition extends the relative efficiency calibration to 4.6 MeV. A simulation of the array efficiency was implemented at the University of Guelph using the detector geometry simulation libraries in GEANT4 [25], and agrees with observation extending the calibration to high energies. The array was also measured at TRIUMF using a calibrated 60Co
source to have an absolute efficiency for photon conversion of 0.94% at 1.332 MeV. The experimental and simulated results (scaled to agree with this measured absolute efficiency) are shown in figure 3.12. While the low-energy efficiencies (below 200-300 keV) vary greatly with electronic settings, the calibration above 4 MeV depends mainly on the size and geometry of the HPGe crystals, and should remain valid for future experiments.

Figure 3.12 depicts the efficiency calibration of the entire 8π detector array, averaging over the efficiencies of the 20 HPGe detectors. The 8π detectors are the same size and geometry and were very similar when first manufactured, but have now been in use for more than twenty years; neutron damage and annealing of the detectors have led to variations in individual detector efficiencies possibly by up to
These differences between individual detectors can usually be ignored in a $\beta$-decay experiment, as long as the radiation produced by the decay is isotropic. If so, the efficiency of the average of all detectors as shown in figure 3.12 is appropriate. In chapter 5, however, we will discuss a special case where a geometric asymmetry is present in the experiment. Some radioactive ions escape from the target toward the ‘upstream’ detectors following the $\beta$-decay of $^{11}$Li. These ions can emit $\gamma$-rays while
in motion, and in the resulting Doppler broadening the upstream detectors record more blue-shift. In this anisotropic case, the detectors are best treated by ‘ring’, where each ring of detectors is at constant $\theta$ relative to the beam axis as shown in figure 3.13. The efficiencies of the detector rings vary by about 15%.

In addition to the detector efficiency, the energy-dependent resolution for the $8\pi$ array was also extracted. Photopeaks recorded by HPGe detectors have a characteristic shape, consisting of a Gaussian, plus a ‘skewed Gaussian’ with a low-energy exponential tail representing incomplete charge collection, and a background:

$$F(x) = c_1 \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} + c_2 e^{(x-\mu)/\beta} \text{Erfc}(\frac{x-\mu}{\sqrt{2\sigma}} + \frac{\sigma}{\sqrt{2}\beta}) + c_3 \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$  \hspace{1cm} (3.2)

The parameter $\beta$ in the skew Gaussian determines the length of the exponential tail. The resolution of the Gaussian depends mostly on the number of charge carriers produced in the germanium crystal, thus $\sigma = \sqrt{F \epsilon \gamma}$ where $\epsilon$ is the energy necessary to create an electron-hole pair ($\approx 2.96$ eV for germanium) and $F (<1$ for germanium) is the ‘Fano factor’, effectively reducing the width of the peak [19].

To measure the resolution for as wide an energy range as possible, $\sigma$ was extracted from many $\gamma$-rays, from calibration sources and from $\gamma$-ray transitions in $^{11}$B following the decay of $^{11}$Be (gammas in $^{11}$B helped to extend the resolution measurement to nearly 8 MeV). The $\sigma/E$ relationship was obtained both for the collective $8\pi$ array, and for individual detector rings. The resolution $\sigma$ was expected to depend on the square root of the $\gamma$-ray energy, yet as seen in figure 3.14 a linear dependance was instead observed.

The performance of SCEPTAR is also important to the overall array. The plastic scintillating detectors are very efficient at detecting charged particles, but incomplete solid-angle coverage and losses in light transmitted to the pre-amplifiers combine to re-
Figure 3.14. Resolution of a single HPGe detector (blue) and of the full HPGe array (black). Resolution degrades slightly as data from the 20 detectors are summed.

Figure 3.15. Plot with $\gamma$ singles, $\beta-\gamma$ coincidences (in red), and bremsstrahlung-removed (in blue) spectra.
duce the $\beta$ conversion efficiency to approximately 80%. Thus, when a $\beta$-$\gamma$ coincidence requirement is applied, approximately 20% of real events are lost. Also, applying the $\beta$-$\gamma$ anti-coincidence to veto bremsstrahlung events also unavoidably vetoes some real events in the process. Both vetoes are worthwhile, however, since they provide an overall improvement in the signal/noise ratio. Figure 3.15 shows these improvements: the reductions in background from applying a $\beta$-coincidence requirement are approximately constant, while the improvement from vetoing bremsstrahlung is more important at low energy. This resolving power makes the combination of $8\pi$+SCEPTAR an exceptional tool for $\beta$-decay spectroscopy.

### 3.5 Concluding Remarks

The recent additions of SCEPTAR and the moving tape collector to the $8\pi$ are part of the necessary effort of increasing the overall sensitivity of the array to keep it competitive in the study of rare radioactive beams. The next chapter describes an experiment where the $\beta$-decay of neutron-rich $^{32}$Na is investigated with a beam of only 2-3 atoms/second.
CHAPTER 4

\textit{\textbf{\beta-decay of $^{32}$Na and the Island of Deformation}}

Nuclear deformation was introduced in chapter 2. Most deformed nuclei are massive, with $A \geq 150$. However, deformation also appears in some light neutron-rich nuclei. A well-known example is the ‘Island of Deformation’, a cluster of several deformed, neutron-rich nuclei lying near $^{31}$Na and $^{32}$Mg at the $N=20$ shell closure.

The Island of Deformation was first discovered by Thibault et al. through mass measurements of Na isotopes in 1975 [26]. Na isotopes were produced through the ISOL technique at CERN (the European Organization for Nuclear Research), and sent to a mass spectrometer for measurements. Both $^{31}$Na and $^{32}$Na exhibited smaller mass than expected, indicating higher binding energy than was predicted on the assumption these were spherical nuclei (an assumption based on the presence of the $N=20$ shell closure). Investigating further, Thibault et al. found that the two-neutron separation energy $S_{2n}$ increases for $^{31,32}$Na rather than decreasing as expected across a shell closure. They suggested the surprising results could be attributed to a sudden onset of deformation in the region.

In 1979, Detraz et al. continued the exploration of the deformed region at CERN, using Lithium-drifted Germanium or Ge(Li) detectors for $\gamma$-ray spectroscopy [27]. In this experiment, the $\beta$-decays of $^{27-32}$Na were observed for the first time. In the decay of $^{32}$Na, Detraz et al. discovered a strong $\gamma$-ray transition at 885.7 keV, and suggested that it must belong to $^{32}$Mg, as a $2^+_1 \rightarrow 0^+$ transition (data from this experiment is shown in table 4.1).
Figure 4.1. Shell closures in Ca isotopes, and the impact of deformation in neutron-rich Mg.

(a) Calcium isotopes near stability show clearly the effect of shell closures: here we observe the impact of the neutron shells on $E_{2^+}$, the energy of the first excited state.

(b) In the Magnesium isotopes, $^{32}\text{Mg}$ also lies at the $N=20$ shell closure yet $E_{2^+}$ drops due to the onset of deformation.

Table 4.1. $\gamma$-ray transitions following the $\beta$-decay of $^{32}\text{Na}$, observed by Detraz et al. with Lithium-drifted Germanium detectors [27]

<table>
<thead>
<tr>
<th>$E_\gamma$</th>
<th>$I_\gamma$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>885.7 $\pm$ 2.0</td>
<td>80 $\pm$ 8</td>
</tr>
<tr>
<td>1232 $\pm$ 4</td>
<td>6 $\pm$ 3</td>
</tr>
<tr>
<td>1440 $\pm$ 4</td>
<td>7 $\pm$ 3</td>
</tr>
<tr>
<td>1970 $\pm$ 5</td>
<td>11 $\pm$ 5</td>
</tr>
<tr>
<td>2153 $\pm$ 5</td>
<td>48 $\pm$ 10</td>
</tr>
<tr>
<td>2556 $\pm$ 5</td>
<td>12.5 $\pm$ 6.0</td>
</tr>
<tr>
<td>3920 $\pm$ 5</td>
<td>10.5 $\pm$ 5.0</td>
</tr>
</tbody>
</table>

The low energy of this first excited $2^+$ state provided further evidence of deformation in $^{32}\text{Mg}$. Since the nucleus lies at the $N=20$ shell closure, it was expected to have a large $E(2^+ \rightarrow 0^+)$, since according to the independent particle model discussed in chapter 2, collectively exciting a pair of particles from a closed shell generally requires more energy resulting in a higher first excited state. The first excited states in the doubly-magic nuclei $^{40}\text{Ca}$ and $^{48}\text{Ca}$ are indeed higher compared to those in other Ca isotopes as seen in figure 4.1(a). In $^{32}\text{Mg}$ however, the first excited state drops relative to other Mg isotopes as shown in figure 4.1(b), further suggesting the idea of
Figure 4.2. The table of isotopes, with part of the island of deformation highlighted in yellow (the boundaries of the island are not certain). The $\beta$-decay channels from $^{32}$Na are also shown, including 1-n and 2-n delayed emission branches.

decoration.

After the study by Detraz et al. the deformed region continued to draw attention. Two 'survey studies' were undertaken by Guillemaud-Mueller et al. in 1984 [28] and by Klotz et al. in 1993 [29]. Guillemaud-Mueller et al. observed the $\beta$-decays of $^{27-34}$Na and $^{30-33}$Mg at CERN, measuring the subsequent $\gamma$-rays with Li-drifted Germanium, or Ge(Li), detectors. Klotz et al. later measured the decays of $^{31,32}$Na and $^{31}$Mg using the on-line isotope mass separator ISOLDE at CERN, and detectors for $\beta$s, neutrons, and $\gamma$-rays. Since these two studies explored a wide range of neutron-rich nuclei near $N=20$, they provide some of our most useful information about deformed Na and Mg isotopes. They confirmed the 885 keV level proposed by Detraz et al. and assigned several more transitions to $^{32}$Mg.
Along with many other reaction studies, these experiments helped to map out an entire region of deformed nuclei near N=20: the ‘Island of Deformation’. The boundaries of this region are still being tested today, due mostly to the experimental challenges inherent in measurements near the neutron drip-line. As stated by Warburton et al. in 1990, however, $^{32}$Na and all the isotopes adjacent to it appear to belong to the deformed region, as seen in figure 4.2.

In light of the experimental evidence supporting a new deformed region, theorists also worked to explain the phenomenon. In 1979, Campi et al. suggested that the deformed shapes observed by [26, 27] could be handled in the context of the shell model by allowing intruders from higher orbitals. They proceeded to perform Hartree-Fock calculations, with an intruder from the neutron 1f$\frac{7}{2}$ orbital [30].

Subsequent calculations by Storm et al. and by Wildenthal and Chung proved quite successful in reducing the predicted mass (thus adjusting to the observed over-binding in the deformed region) [31, 32]. Like Campi et al., these groups allowed an intruder from the neutron 1f$\frac{7}{2}$ shell. Although these efforts were successful in reducing the predicted mass, they produced no deformation. Predictions for $E_x(2^+)$ in Mg from Wildenthal and Chung are shown in table 4.2.

<table>
<thead>
<tr>
<th>A</th>
<th>Theory</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1.5</td>
<td>1.37</td>
</tr>
<tr>
<td>26</td>
<td>2.0</td>
<td>1.81</td>
</tr>
<tr>
<td>28</td>
<td>1.8</td>
<td>1.47</td>
</tr>
<tr>
<td>30</td>
<td>1.9</td>
<td>1.48</td>
</tr>
<tr>
<td>32</td>
<td>2.1</td>
<td>0.89</td>
</tr>
</tbody>
</table>

In 1987, Poves and Retamosa successfully recreated the mass and the deformation
by including both 1f7/2 and 2p3/2 orbitals in the valence space [33]. Their success was reproduced by several other groups, including Warburton et al. in 1990 [34]. The authors also performed an improved calculation in 1994 with similar results [35].

These successful theories differ in some details, but share important elements: in each case, an intruding two-particle, two-hole (2p-2h) configuration competes with low-lying states in the deformed region. The energy required to promote two neutrons across the shell closure is offset both by an n-n interaction in the form of pairing, and by an n-p interaction. In some of the deformed nuclei including 32Mg, the intruder configuration displaces the lowest energy state to form a deformed ground state. For an overview of these theoretical models, see [5, 36].

Two other models have been published recently. In 2004, Yamamoto et al. calculated the B(E2) value of 32Mg using a Hartree-Fock-Bogoliubov approach [37]. Also, Kimura and Horiuchi calculated the level scheme of 32Mg in 2003 using antisymmetrized molecular dynamics or AMD [38]. Their prediction features several negative-parity states around 3-4 MeV, as shown in table 4.3.

Table 4.3. Excited states in 32Mg predicted by Kimura and Horiuchi

<table>
<thead>
<tr>
<th>J^*</th>
<th>E (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0^+</td>
<td>0.0</td>
</tr>
<tr>
<td>2^+</td>
<td>0.8</td>
</tr>
<tr>
<td>1^-</td>
<td>2.1</td>
</tr>
<tr>
<td>4^+</td>
<td>2.3</td>
</tr>
<tr>
<td>2^-</td>
<td>2.5</td>
</tr>
<tr>
<td>3^-</td>
<td>3.2</td>
</tr>
</tbody>
</table>
4.1 The Key Nucleus $^{32}\text{Mg}$

Great attention was paid to $^{32}\text{Mg}$ ever since its deformed properties were first discovered. $^{32}\text{Mg}$ lies at a neutron shell closure, and is an even-even nucleus ($N=20$, $Z=12$), so the deformation came as a great surprise, and nuclear structure physicists were eager to explore the properties of the nucleus further. Today, although we appear to have a sound explanation for the appearance of deformation at $N=20$ (as described in the previous section), several important questions remain for $^{32}\text{Mg}$.

For example, while the $2^+$ first excited state was identified by Detraz et al. in 1979, no other excited states have yet received firm $J^\pi$ assignments. We are particularly interested in the location of the $4^+$ state: the ratio $E(4^+)/E(2^+)$ should help us determine the source of the deformation in $^{32}\text{Mg}$, and determine whether it is a deformed rotator or has vibrational properties.

The level scheme established by Klotz et al. and Azaiez et al. [29, 39] is shown in figure 4.3. The state at 2322 keV has been suggested as the most likely candidate for the $4^+$ state by several groups, starting with Azaiez et al. [40]. Using in-beam $\gamma$-ray spectroscopy at GANIL, the group observed a 1430 keV transition in coincidence with the 885 keV $\gamma$-ray (agreeing with previous results from [28, 29]). The parent state of the 1430 keV transition at 2315(15) keV was identified as a probable $4^+$ candidate. This placement was based on two considerations: first, $2^+$ and $4^+$ states were expected to be more heavily populated based on selection rules. Also, the transitions were compared to Monte Carlo shell model calculations from Utsuno et al. [41]. The level at 2315 keV agreed closely with a predicted $4^+$ from the theoretical work.

A subsequent experiment by Bazin et al. in 2003 provided further evidence supporting the placement of the 2322 keV level as a $4^+$ [42]. The state was observed
Figure 4.3. Level scheme of $^{32}$Mg as established by previous $\beta$-decay and in-beam $\gamma$-spectroscopy experiments. All energies are in keV. The 2322 keV level (shown in blue) is considered a good $4^+$ candidate.
during the 2-proton knockout reaction of $^{34}\text{Si}$. Two-proton knockout has a higher cross section for removing correlated proton pairs, leaving the final nucleus in $2^+, 4^+$ etc. states.

However, two other studies cast doubt on the $4^+$ assignment. First, the 2321 keV state was observed by Pritychenko et al. in 1999 during an intermediate energy Coulomb excitation experiment. At the National Superconducting Cyclotron Laboratory (NSCL), $^{32}\text{Mg}$ was impinged upon a gold foil and excited to various states. From the experimental cross section for the 2321 keV yield, its possible $J^\pi$ values were suggested to be $(1^-, 1^+, 2^+)$ [43].

The 2321 keV level was also strongly populated during an inelastic nuclear scattering experiment performed by Mittig et al. in 2002 [44]. The authors suggest that since the inelastic scattering preferentially populated $3^-$ states in other nuclei in the study, (and since it should populate relatively low-spin states), the 2321 keV level could be a $3^-$ state.

The $\beta$-decay selection rules further complicate the placement of the $4^+$. The ground state of $^{32}\text{Na}$ is tentatively assigned as $(3^-, 4^-)$ [29], and $^{32}\text{Na}$ decays with a half-life of 12.9(7) ms [45]. With such a short half-life, there is no doubt that the bulk of the decay progresses through allowed channels to populate ‘high’ spin, negative-parity states in $^{32}\text{Mg}$: if the $^{32}\text{Na}$ ground state is a $3^-$, $(2^-, 3^-, 4^-)$ states are populated while if $^{32}\text{Na}$ has a $4^-$ ground state $(3^-, 4^-, 5^-)$ states are populated. Previous $\beta$-decay experiments uncovered no evidence that the 2321 keV level is fed by gamma-rays from higher-lying levels [28, 29], suggesting either that this level could have negative parity compatible with the $\beta$-decay selection rules, or that the state is fed by a previously unobserved $\gamma$-ray transition.
4.1.1 Need for a New $\beta$-decay Experiment

Due to these conflicting reports, and to the importance of the $4^+$ location in determining the source of the deformation in $^{32}\text{Mg}$, a new $\beta$-decay study of $^{32}\text{Na}$ was considered potentially profitable. The $^{32}\text{Na}$ $\beta$-decay is a good way to probe the $^{32}\text{Mg}$ level scheme, for several reasons: first, $^{32}\text{Na}$ decays with a very large Q-value of 20.02 MeV. The decay may therefore populate several excited states in $^{32}\text{Mg}$, as well as states in $^{31}\text{Mg}$ and $^{30}\text{Mg}$ through $\beta$-delayed neutron emission. Also, $^{32}\text{Na}$ is thought to have a ground state $J^\pi$ of $(3^-,4^-)$ as previously mentioned, whereas the ground and first excited states in $^{32}\text{Mg}$ have positive parity. It was anticipated that ‘high’ spin negative parity states would be populated during the decay, and that the subsequent $\gamma$-decay from these states would populate lower positive parity states, possibly including the $4^+$.

While several previous experiments had observed the $\beta$-decay of $^{32}\text{Na}$ ([27, 28, 29]), new detector arrays and advances in radioactive ion beam development, data extraction and analysis suggested that a new experiment could be useful. In particular, we wished to observe this decay with the new combination of the $8\pi$ $\beta$-decay station and SCEPTAR.

4.2 The Experiment

A new $^{32}\text{Na}$ $\beta$-decay experiment took place in October, 2003 at the $8\pi$ facility at ISAC. The $8\pi$ was configured as described in chapter 3, with HPGe crystals for $\gamma$-ray detection, Compton suppressors and the newly-added SCEPTAR array providing $\beta$ detection and removal of bremsstrahlung events.

$^{32}\text{Na}_{21}$ is very neutron rich (for comparison, $^{23}\text{Na}_{12}$ is the only stable sodium isotope). Further from stability, and as the half-life decreases, radioactive ion beam
Table 4.4. Experimental yield of Na isotopes at ISAC, using Ta targets and approximately 40\(\mu\)A of p\(^+\) current

<table>
<thead>
<tr>
<th></th>
<th>Yield (/s)</th>
<th>Half-life</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{26})Na</td>
<td>1.4e+6</td>
<td>1.07(9) s</td>
</tr>
<tr>
<td>(^{27})Na</td>
<td>3.0e+5</td>
<td>301(6) ms</td>
</tr>
<tr>
<td>(^{28})Na</td>
<td>2.0e+3</td>
<td>30.5(4) ms</td>
</tr>
<tr>
<td>(^{29})Na</td>
<td>4.7e+2</td>
<td>44.9(12) ms</td>
</tr>
<tr>
<td>(^{30})Na</td>
<td>1.4e+2</td>
<td>48(2) ms</td>
</tr>
<tr>
<td>(^{32})Na(^a)</td>
<td>2.0e+0</td>
<td>12.9(7) ms</td>
</tr>
</tbody>
</table>

\(^a\)\(^{32}\)Na yield recorded using an earlier Ta target

yields generally grow smaller: extremely neutron rich nuclei like \(^{32}\)Na are less likely to be formed by spallation, and due to their short half lives they are less likely to survive long enough to be extracted from the target (for example, table 4.4 shows the yields obtained at ISAC for various Na isotopes [46]). After lengthy optimization, a beam of \(^{32}\)Na ions was created at ISAC despite the difficulties, but the yield was low with only 2-3 ions/second delivered to the Stt.

Due to the low count rate, the \(^{32}\)Na decay signal was overwhelmed by noise from unrelated background decays when all \(\gamma\)-rays were considered. Therefore, in order to isolate the \(^{32}\)Na decay, \(\beta-\gamma\) coincidences were used instead (see discussion in chapter 3). As seen in figure 4.4, even the strongest transition in \(^{32}\)Mg at 885 keV is obscured in the \(\gamma\)-singles spectrum yet clearly appears when considering \(\beta-\gamma\) coincidences.

The data shown in figure 4.4 were collected during a test run in August 2003. The test run served as a proof of principle, demonstrating that even with very low count rate the combination of collinear HPGe and plastic scintillating detectors yields a spectrum with good signal/noise. Also, after the test run several important decisions were made as to how best to change the experimental setup at the 8\(\pi\) to optimize
Figure 4.4. Data from a test run prior to the experiment. The top panel shows the full data set; in the bottom panel a $\beta$ coincidence requirement has been added. The 885 keV transition is clearly seen with the coincidence requirement in place.

data collection with a low count rate.

First, since the $\beta$ coincidence requirement was necessary to produce a meaningful $^{32}$Na decay spectrum, the requirement was hard-coded into the data acquisition system. Rather than triggering on all $\gamma$-singles events, the logic modules were programmed to only accept $\beta$-$\gamma$ coincidences, with the goal of reducing the quantity of irrelevant data and streamlining the data analysis. The trigger was relaxed later in the experiment to allow both $\beta$-$\gamma$ and $\gamma$-$\gamma$ coincidences, yet ultimately a $\beta$-$\gamma$ coincidence requirement was also put in place during our offline analysis.

The moving tape collector (MTC) was also used extensively during the experi-
Table 4.5. Approximate hours of data collection using each trigger and tape cycle

<table>
<thead>
<tr>
<th>Trigger:</th>
<th>( \beta - \gamma )</th>
<th>51</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta - \gamma + \gamma - \gamma )</td>
<td>71</td>
</tr>
<tr>
<td>Tape cycle</td>
<td>Continuous</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>6-second</td>
<td>24</td>
</tr>
</tbody>
</table>

The main purpose of the tape collector was to remove longer-lived daughter products that would otherwise have built up over time and resulted in higher background contaminating the decay spectrum. In this experiment, the tape was run in two different modes: for some runs, ions were continuously removed from the center of the array, and during other runs the tape was held motionless for six seconds before removal (‘continuous’ and ‘6-second’ cycles, respectively). In both cases, \(^{32}\text{Na}\) ions were continuously implanted, so that daughters had some chance to build up during the 6-second cycle. As we will see, by comparing the intensities collected using these two tape cycles we were able to discriminate between \(\gamma\)-rays arising from immediate \(^{32}\text{Na}\) daughters (\(^{32,31,30}\text{Mg}\)), and other daughters further down the decay chain.

While the tape collector efficiently removed \(^{32}\text{Na}\) daughters, another source of contamination was present within the vacuum chamber at the center of the \(8\pi\) throughout the experiment. Long-lived \(A=28\) isotopes were found in the chamber after a test beam of \(^{28}\text{Na}\) was briefly run at the beginning of the experiment. The \(^{28}\text{Na}\) beam focus was apparently too wide, so that some ions missed the tape and were implanted instead in SCEPTAR or in the walls of the vacuum chamber. Contaminants including \(^{28}\text{Mg}\) and \(^{28}\text{Al}\) (\(t_{1/2}=20.92\) hours and 2.25 minutes respectively [45]) therefore complicated the data analysis.

With some extra steps taken to reduce the impact of the contaminating \(^{28}\text{Na}\) daughters upon the \(^{32}\text{Na}\) decay scheme, the planned experiment then went forward.
$^{32}$Na ions were collected intermittently over the next two weeks, October 8$^{th}$-19$^{th}$ 2003, resulting in approximately 122 hours of total beam on target. In the resulting spectrum, approximately $2 \times 10^5$ $\gamma$-rays were recorded.

![$^{32}$Na $\beta$-decay spectrum](image)

Figure 4.5. Partial $^{32}$Na $\beta$-decay spectrum (0-2.5 MeV)

4.3 Analysis

Since the 8$\pi$ consists of 20 individual HPGe crystals, the first step in analyzing the data was to properly calibrate and gain match the entire array, to produce a final spectrum. Gain matching was performed using data from high-intensity sources (mainly $^{152}$Eu as well as $^{133}$Ba, $^{56}$Co and $^{88}$Y) placed in the 8$\pi$ at intervals throughout the experiment. Application of a first-order gain calibration, gain x channel + offset, was found insufficient, so second-order and eventually third-order calibrations were...
applied in order to correct for non-linearities in the ADCs and for drift in the electronics. After the calibration, the resulting histograms are binned by 1 keV/channel. The resulting spectrum is shown in figure 4.5.

Figure 4.5 includes many transitions almost certainly not related to our decay of interest. For example, the 1460 keV transition results from the EC decay of $^{40}$K, not from a $^{32}$Na daughter. Fortunately, $\beta$-detection from SCEPTAR helps us eliminate many of these transitions since they do not arrive in coincidence with a $\beta$-particle. Also, as described in chapter 3, SCEPTAR allows us to significantly reduce the background arising from bremsstrahlung radiation. The improved spectrum is also shown in figure 4.5 in blue.

Table 4.6. Recent studies near A=32, used to identify $\gamma$-rays in our $^{32}$Na decay

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{32}$Na $\rightarrow$ $^{32}$Mg</td>
<td>Klotz et al. [29]</td>
</tr>
<tr>
<td>$^{32}$Na $\rightarrow$ $^{31}$Mg</td>
<td>Klotz et al. Mach et al. [47]</td>
</tr>
<tr>
<td>$^{32}$Na $\rightarrow$ $^{30}$Mg</td>
<td>Klotz et al.</td>
</tr>
<tr>
<td>$^{32}$Mg $\rightarrow$ $^{32}$Al</td>
<td>Grévy et al. [48]</td>
</tr>
<tr>
<td>$^{32}$Al $\rightarrow$ $^{32}$Si</td>
<td>Fornal et al. [49]</td>
</tr>
<tr>
<td>$^{31}$Mg $\rightarrow$ $^{31}$Al</td>
<td>Maréchal et al. [50]</td>
</tr>
<tr>
<td>$^{31}$Al $\rightarrow$ $^{31}$Si</td>
<td>Détraz et al. [27]</td>
</tr>
<tr>
<td>$^{28}$Mg $\rightarrow$ $^{28}$Al</td>
<td>Endt et al. [51]</td>
</tr>
<tr>
<td>$^{28}$Al $\rightarrow$ $^{28}$Si</td>
<td>Endt et al.</td>
</tr>
</tbody>
</table>

Once these spectra were obtained we sought to identify as many $\gamma$-rays as possible, using previous $\beta$-decay studies of Na, Mg and Al isotopes as our guide. By identifying the known $\gamma$-rays we hoped to isolate previously unobserved transitions that warranted further study. Up-to-date information on the $\gamma$-rays expected following $\beta$-decays from these isotopes was therefore obtained from various sources, as shown in table 4.6.
With the help of recent data from these earlier studies, assignments were made to many $\gamma$-rays in our decay spectrum. A total of approximately 70 photopeaks were recorded with intensities greater than 1% relative to the strongest transition at 885 keV; of these 50 were identified from previous studies. The location and identification (if known) of these $\gamma$-rays is shown in table 4.7.

Table 4.7 contains several unknown transitions, including some at higher energies up to about 5.4 MeV. We therefore used two complementary methods for determining if these transitions were good candidates for belonging to the $^{32,31,30}$Mg level scheme. Through these selective filters we hoped to determine which $\gamma$-rays were likely associated with direct $^{32}$Na daughters, and which were more likely associated with the $^{28}$Na $\beta$-decay chain or with the $\beta$-decays of $^{32,31,30}$Mg. We first worked to identify $\gamma$-rays associated with N=28 isotopes, with the help of SCEPTAR.

As discussed in chapter 3, most electrons from the $^{32}$Na $\beta$-decay are in the minimum-ionizing energy range as they pass through the plastic scintillating detectors, and so only deposit a fraction of their energy in SCEPTAR. Figure 3.11 shows, however, that above the minimum ionization energy (at 1 MeV) the energy loss begins to increase with incident electron energy. In this case, $Q_\beta$ for the $^{32}$Na decay is about 20 MeV, while $Q_\beta$ for the $^{28}$Mg decay is 1.8 MeV, and electrons related to the $^{32}$Na $\beta$-decay deposit more energy on average in SCEPTAR. The presence of several decays with dramatically different $Q_\beta$ is obvious in the spectrum from SCEPTAR as shown in figure 4.6 (note the energy is in arbitrary units).

Gamma-rays coincident with higher energy $\beta$s are less likely to belong to A=28 isotopes, since the $Q_\beta$ values for $^{28}$Mg ($Q_\beta = 1.8$ MeV) and $^{28}$Al ($Q_\beta = 4.6$ MeV) are relatively low. We therefore created two spectra: $\gamma$-rays coincident with all $\beta$s, and $\gamma$-rays coincident only with $\beta$s above a certain threshold, shown in blue on figure 4.6.
Table 4.7. Identification of γ-rays observed in the $^{32}\text{Na}$ decay after β-coincidence and bremsstrahlung vetoes are applied: $E_\gamma$ (keV) and the associated decay (if known)

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>Decay Path</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$^{32}\text{Na} \rightarrow ^{31}\text{Mg}$</td>
</tr>
<tr>
<td>220.80(7)</td>
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</tr>
<tr>
<td>244.17(5)</td>
<td>$^{30}\text{Mg} \rightarrow ^{30}\text{Al}$</td>
</tr>
<tr>
<td>400.67(4)</td>
<td>$^{28}\text{Mg} \rightarrow ^{28}\text{Al}$</td>
</tr>
<tr>
<td>436.4(9)</td>
<td>?</td>
</tr>
<tr>
<td>443.75(7)</td>
<td>$^{30}\text{Mg} \rightarrow ^{30}\text{Al}$</td>
</tr>
<tr>
<td>486.2(2)</td>
<td>?</td>
</tr>
<tr>
<td>623.1(2)</td>
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</tr>
<tr>
<td>666.4(2)</td>
<td>$^{31}\text{Mg} \rightarrow ^{31}\text{Al}$</td>
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</tr>
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<td>697(2)</td>
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</tr>
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<td>735.06(7)</td>
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</tr>
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<td>4575(1)</td>
<td>?</td>
</tr>
</tbody>
</table>

*aAssigned to the $^{31}\text{Mg}$ level scheme by [47]. Possibly also part of the $^{32}\text{Mg}$ level scheme (see text).*
Figure 4.6. The $\beta$ spectrum associated with the $^{32}$Na decay. Decays with dramatically different Q-values are observed, assumed to correspond to $^{32}$Na and $^{28}$Mg respectively.

Figure 4.7. 'Cutting' low-energy $\beta$-particles reduces the contribution from lower-Q$_\beta$ decays such as $^{28}$Mg, yet makes only a small impact on $\gamma$-rays from the decay of $^{32}$Na.
We then extracted the intensities from each spectrum \( \text{I} = N/\epsilon(E) \) where \( N \) is the total number of counts recorded in the peak and \( \epsilon(E) \) is the energy-dependent efficiency of the 8\( \pi \) array), and normalized to the intensity of the 885 keV transition. We found that the intensities of \( \gamma \)-rays known to be associated with the A=28 decay were dramatically reduced by the \( \beta \) cut. The intensities of some unknown transitions were also reduced suggesting that they might also be related to decays with lower Q\( _{\beta} \). The effect of the \( \beta \) energy cut is seen on figure 4.7.

We also filtered out unknown transitions with the help of the moving tape collector. As mentioned before, data was collected with the tape running both in ‘continuous’ and in ‘6-second’ modes. Longer-lived daughter products had some chance to build up during the 6-second tape cycle, and so we expected \( \gamma \)-ray transitions related to these long-lived daughters to be more intense relative to the known \( ^{32}\text{Mg} \) transition at 885 keV when collected with the 6-second cycle.

Two more spectra were thus extracted, containing the data collected using 6-second and continuous tape cycles respectively. Intensities were extracted from each spectrum, and the ratio of intensities (normalized, once again, to unity for the 885 keV transition in \( ^{32}\text{Mg} \)) was considered. Since \( ^{32}\text{Na} \) has a very short half-life, its activity was not expected to change between the two spectra, whereas activities of longer-lived daughter products was expected to change drastically. A ratio \( I_{\text{6second}}/I_{\text{continuous}} \) near one was therefore considered a good indication that the \( \gamma \)-ray belongs to the level scheme of a \( ^{32}\text{Na} \) daughter: \( ^{32,31,30}\text{Mg} \), while \( \gamma \)-ray transitions with ratios far from unity were considered poor candidates for belonging to these level schemes.

All these results are compiled together on table 4.8, where we have displayed all of the unidentified \( \gamma \)-rays along with some of the stronger known transitions for reference. The intensities found with and without a \( \beta \) energy cut are displayed, along
Table 4.8. Change in intensity when a $\beta$ energy cut was applied, and when data from different tape cycles were compared. Intensities of several previously unknown $\gamma$-rays change very little with the tape cycle or the $\beta$ cut. These $\gamma$-rays are considered good candidates for belonging to the level scheme of a $^{32}$Na daughter, and are marked with a *. All intensities normalized to 100 for the 885 keV transition.

<table>
<thead>
<tr>
<th>E$_{\gamma}$</th>
<th>I(total)</th>
<th>I(after cut)</th>
<th>Ratio(6sec/cont)</th>
<th>Nucleus</th>
</tr>
</thead>
<tbody>
<tr>
<td>170.84(3)</td>
<td>9.9(6)</td>
<td>9.6(6)</td>
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<td>$^{30}$Al</td>
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<td>1.3(3)</td>
<td>1.3(6)</td>
<td>*</td>
</tr>
<tr>
<td>682.9(7)</td>
<td>0.6(4)</td>
<td>1.2(3)</td>
<td>1.7(8)</td>
<td>?</td>
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<tr>
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<td>1.1(2)</td>
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<td>2.4(4)</td>
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</tr>
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</tr>
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<td>0.9(4)</td>
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</table>

$^a$The 1779 keV peak is composed of transitions in both $^{28}$Si and $^{32}$Mg.
Figure 4.8. Gamma-ray spectrum from $^{32}$Na $\beta$-decay. Transitions in $^{32,31,30}$Mg are labeled with arrows, and transitions assigned in this work to the level scheme of $^{32}$Mg are identified with an *.

with the ratio of intensities $I_{\text{second}}/I_{\text{continuous}}$. As we can see, intensities of transitions known to belong to $^{32}$Na daughters tend to be unchanged by both the $\beta$ cut and by the tape cycle; these are shown in blue. Several unknown transitions are also unchanged by our filters and are also shown in blue.

Transitions known to belong to $^{32}$Na daughters are shown in figure 4.8 along with transitions considered good candidates for placement in the level scheme of a $^{32}$Na daughter. These unknown transitions are all weak, with small intensities. Using $\gamma$-$\gamma$ coincidences, however, we seek to place some of the transitions in the level scheme of a $^{32}$Na daughter.

4.3.1 (Beta)-Gamma-Gamma Coincidences

A small portion (approximately one percent) of events detected at the $8\pi$ in this experiment were coincidences: two or more $\gamma$-rays detected by different germanium detectors within a very short time gate. These coincidence events are extremely
Figure 4.9. Gamma-gamma coincidences before and after adding beta-particle coincidence requirement. From the very clean $\beta$-$\gamma$-$\gamma$ spectrum we gain confident coincidence analysis.

valuable as they allow us to analyze potential cascades of more than one gamma-ray, and to reconstruct a level scheme for the nucleus of interest. Part of the $\gamma$-$\gamma$ coincidence spectrum from this experiment is shown in grey in figure 4.9. Notice that this coincidence spectrum has very few counts compared to the $\gamma$-singles spectrum seen in figure 4.8.

The $\gamma$-$\gamma$ coincidence spectrum may be improved in our experiment with the help of SCEPTAR. While the $\gamma$-$\gamma$ coincidence spectrum contains some background due to occasional random coincidences, this background is greatly suppressed by adding a $\beta$ coincidence requirement: $\beta$-$\gamma$ (or $\beta$-$\gamma$-$\gamma$-... etc.) coincidences are more likely to be associated with the decay of interest than lone $\gamma$-rays or even $\gamma$-$\gamma$ coincidences. The $\beta$-$\gamma$-$\gamma$ coincidence spectrum is shown in black in figure 4.9, and has very little background. This spectrum was therefore used for the bulk of our coincidence analysis.

Exploration of the level scheme was carried out by setting energy gates on $\gamma$-rays of interest, and discovering which other $\gamma$-rays were observed in coincidence with them (coincidence techniques are also discussed in chapter 3). For example, the spectra of
Figure 4.10. Initial coincidence analysis, including coincidences with 885 keV (the $2^+ \rightarrow 0^+$ transition), and with the 486 keV, 1666 keV and 2268 keV newly identified transitions.

The $\gamma$-rays coincident with the 885 keV transition in figure 4.10 shows many counts at 1973 keV and 2152 keV, as expected from previous experiments (see figure 4.3). The coincidence spectrum from the 885 keV transition largely supports the level scheme of $^{32}$Mg as postulated earlier [29].

Some new additions appear to be in order, as well. Three of the unknown transitions are found to be in coincidence with known transitions and also match the energy difference of known energy levels in $^{32}$Mg. The 1666 keV transition is in coincidence with the 885 keV transition, and fits the energy difference between the known states at 2550 and 885 keV. The 486 keV $\gamma$-ray is also found in coincidence with 885 keV, and is placed as a transition between the known $^{32}$Mg states at 3037 keV and 2551 keV. Also, the 2269 keV line is observed in coincidence with the aforementioned
Figure 4.11. Coincidence spectra using both tape cycle data sets supporting the evidence of a new state at 3553 keV and possibly another at 4785 keV in $^{32}$Mg. The total number of events in each coincidence spectrum is indicated in the title together with the transition around which the gate was placed. Spectra labeled (a), (c) and (d) are obtained in $\beta$-\(\gamma\)-\(\gamma\) coincidences, while spectrum (b) is obtained using $\gamma$-$\gamma$ coincidences only.

1666 keV transition, and is assigned as a transition between the 4820 and 2551 keV states. These coincidences are also shown in figure 4.10.

The 693.4 keV $\gamma$-ray was seen but not placed by Guillemaud-Mueller et al. [28]. Recently, Mach et al. assigned this transition to $^{31}$Mg as an $11/2^- \rightarrow 7/2^-$ transition from a new level at 1154 keV to the previously seen 461 keV level [47]. $\beta$-$\gamma$-$\gamma$ coincidences observed in this work support the placement as made by Mach et al. (see figure 4.11(a)). However, the coincidence spectrum also suggests another slightly higher-energy line (about 695 keV) may belong in the $^{32}$Mg decay scheme, as the same coincidence gate produces coincidences with the 885 keV $\gamma$-ray (see figure 4.11(b)). As additional evidence, this transition is also seen in $\gamma$-$\gamma$ coincidences with the 1973 keV $\gamma$-ray, another known transition in $^{32}$Mg. Since these two $\gamma$-rays cannot be resolved,
the total intensity of the two is reported in table 4.8. To explain the coincidences observed between the 695 keV transition and the known $\gamma$-rays, we propose a new level in $^{32}$Mg at 3553 keV. This state decays to the known state at 2858 keV via the 695 keV $\gamma$-ray, and subsequently the nucleus de-excites via the 1973 keV and 885 keV $\gamma$-rays to the ground state.

Further evidence for the 3553 keV state exists. A gate was placed on the 1436 keV line, producing coincidences with 885 keV as expected but also unexpected coincidences with the 1232 keV $\gamma$-ray (see figure 4.11(c)). Coincidences between 1436 keV and 1232 keV $\gamma$-rays should not be possible, according to the level scheme established by Klotz et al., where both transitions correspond to de-excitations to the first excited state (at 885 keV). At present, the only published evidence of the 2117 keV state in $^{32}$Mg comes from Klotz et al., where the state is called for as the parent of the 1232 keV $\gamma$-ray. Due to our contradictory evidence, we suggest that the 2117 keV state postulated by Klotz et al. may not exist and that the 1232 keV transition results instead from the decay of the new 3553 keV state to the known 2321 keV state in $^{32}$Mg.

The 1232 keV transition is also found in coincidence with itself (figure 4.11(d)), suggesting the existence of an even higher excited state at 4785 keV in $^{32}$Mg. No coincidence is observed with the 695 keV $\gamma$-ray originating from the 3553 keV state, which could have constituted supporting evidence for the existence of such a state. A revised level scheme will be shown in the conclusion of this chapter.

4.3.2 Comparison with Previous Works

Many transitions found on table 4.8 have been previously observed in the earlier $\beta$-decay experiments by Guillemaud-Mueller et al. and by Klotz et al. [28, 29]. Both
previous experiments reported intensities for transitions in $^{32,31}$Mg, so we are able to compare intensities extracted in this work to their reported intensities. These intensities are collected on table 4.9, normalized in each case to the 885 keV transition in $^{32}$Mg.

Intensities reported by the three experiments were found to be mostly in agreement. Some discrepancies between the three works were found, however. Disagreements at low and high energy may be due to systematic errors in efficiency calibration in the other experiments. Another notable discrepancy at the 1972.9 keV transition may be explained by the observed presence in the experiment by Klotz et al. of the contaminant $^{128}$In that $\beta$-decays with a strong gamma-ray obscuring the 1972.9 keV gamma in $^{32}$Mg.

4.4 Concluding Remarks

The $^{32}$Mg level scheme established by this work is shown in figure 4.12. This level scheme bears some important differences from the previous scheme established by Klotz et al. In particular, according to the earlier level scheme, the state at 2321 keV was not found to be fed from any higher energy levels, and so was assumed to by directly fed during the $\beta$-decay. This led to a quandary (discussed in section 4.1) wherein some groups consider the 2321 keV state a good candidate for a $J^\pi = 4^+$ assignment, yet according to $\beta$-decay selection rules the state was restricted to negative parity.

The observation of a $\gamma$-ray feeding the 2321 keV state from a higher energy level is important as it re-opens the possibility that the 2321 may be assigned as $J^\pi = 4^+$. The low experimental count rate obtained in this experiment prevent us, however, from going further and attempting to measure $J^\pi$ for the state (with more
Table 4.9. Short comparison of intensities from this work to those from Guillemaud-Mueller et al. and Klotz et al. All intensities are normalized to the 885 keV transition in $^{32}$Mg.

<table>
<thead>
<tr>
<th>$E_\gamma$(keV)$^a$</th>
<th>$I_\gamma$</th>
<th>[29]</th>
<th>[28]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{32}$Mg</td>
<td>486.1$^b$</td>
<td>1.3(3)</td>
<td>-</td>
</tr>
<tr>
<td>$^{32}$Mg</td>
<td>~693.5$^{b,c}$</td>
<td>c</td>
<td>c</td>
</tr>
<tr>
<td>885.0</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>1231.7$^d$</td>
<td>3.8(5)</td>
<td>4.8(17)</td>
<td>4.9(14)</td>
</tr>
<tr>
<td>1436.1</td>
<td>9.8(6)</td>
<td>9.8(25)</td>
<td>10.2(20)</td>
</tr>
<tr>
<td>1665.6$^b$</td>
<td>2.4(4)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1783$^e$</td>
<td>-</td>
<td>-</td>
<td>8.3(20)</td>
</tr>
<tr>
<td>1972.9</td>
<td>11.6(8)</td>
<td>19.7(25)</td>
<td>14.3(25)</td>
</tr>
<tr>
<td>2151.7</td>
<td>47.0(17)</td>
<td>48.5(37)</td>
<td>52.6(60)</td>
</tr>
<tr>
<td>2268.5$^b$</td>
<td>2.5(3)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2550.7</td>
<td>6.4(6)</td>
<td>10.2(25)</td>
<td>9.1(20)</td>
</tr>
<tr>
<td>3034.5</td>
<td>12.0(9)</td>
<td>18.3(37)</td>
<td>13.3(30)</td>
</tr>
<tr>
<td>$^{31}$Mg</td>
<td>170.8</td>
<td>9.6(6)</td>
<td>21.9(35)</td>
</tr>
<tr>
<td>220.8</td>
<td>3.9(4)</td>
<td>8.9(18)</td>
<td>4.4(14)</td>
</tr>
<tr>
<td>239.9</td>
<td>9.5(6)</td>
<td>9.7(11)</td>
<td>27.6(32)</td>
</tr>
<tr>
<td>693.5$^c$</td>
<td>c</td>
<td>c</td>
<td>c</td>
</tr>
<tr>
<td>894.1</td>
<td>4.1(5)</td>
<td>5.1(26)</td>
<td>4.3(10)</td>
</tr>
<tr>
<td>$^{30}$Mg</td>
<td>1483.0</td>
<td>4.2(5)</td>
<td>4.9(22)</td>
</tr>
<tr>
<td>Unplaced</td>
<td>2869.2(8)</td>
<td>1.1(2)</td>
<td>-</td>
</tr>
<tr>
<td>2925.8(6)</td>
<td>3.3(4)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4575(3)</td>
<td>2.4(4)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5470(3)</td>
<td>3.3(6)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

$^a$Uncertainty is ± 0.5 keV unless otherwise specified
$^b$Assigned to $^{32}$Mg in this work
$^c$Doublet: centroid energy given; transitions in $^{31,32}$Mg - total intensity: $I_\gamma$: 6.9(6); [7]: - ; [15]: 3.8; Ratio: 1.1(2)
$^d$Possible doublet
$^e$Not extracted due to the presence of $^{28}$Mg (see text)
Figure 4.12. Level scheme of $^{32}$Mg, all energies in keV. Dashed lines are new in this work. Compare with figure 4.3
statistics, the $8\pi$ spectrometer would be well suited to making spin assignments based on angular distributions in $\gamma$-$\gamma$ coincidences).

Analysis of the $^{32}\text{Na}$ $\beta$-decay data was concluded with the publication in January, 2007 of a brief report in *Physical Review C* [52].
CHAPTER 5

β-DECAY OF $^{11}\text{Li}$

Nuclei display great variety, in lifetime, decay mode, shapes, and other properties. In chapter 4 we observed that deformation is able to arise in some very neutron-rich nuclei due to outside orbitals intruding in the normal valence space. We now encounter a few more light nuclei near the neutron drip-line with intruder states that create unexpected properties. These nuclei (including $^{11}\text{Be}$, $^{8}\text{He}$, $^{14}\text{Be}$, and the best-known example $^{11}\text{Li}$) are today known to have extended nuclear wavefunctions or ‘halos’.

5.1 Halo Nuclei

The theory of halo nuclei was introduced in chapter 2, and the deuteron ($^2\text{H}$) was introduced as the simplest halo system. Two other halo nuclei have recently received close attention: $^{11}\text{Li}$ and $^{11}\text{Be}$.

5.1.1 $^{11}\text{Be}$

The $\beta$-decay of the neutron-rich nucleus $^{11}\text{Be}$ was first observed by Wilkenson et al. in 1959, and seemed to show that the nucleus possessed a ground state with $J^\pi = \frac{1}{2}^+$ rather than $\frac{1}{2}^-$ as predicted by the independent particle model [53]. Several proposals were made to explain this surprising result: in 1961 Talmi and Unna first suggested that the energy of the $2s_{1/2}$ could be lowered relative to the $1p_{1/2}$ by interacting with holes in the proton $p_{3/2}$ shell [54]. Other more recent studies have
generally confirmed this idea, although some mixture of \(d_{5/2}\) neutrons have also been suggested (for a recent calculation performed in multi-\(\hbar\omega\) space, see for example [55]).

| \(^{10}\text{Be} + \text{n}\) | 10267.1123(4) |
| \(^{11}\text{Be}\) | 10266.608(6) |
| \(S_n\) | 0.504(6) |

Table 5.1. Binding Energy of \(^{11}\text{Be}\), masses from [56]

Talmi and Unna thus suggested that an intruder from a higher-lying \(s_{1/2}\) state forms the new ground state of \(^{11}\text{Be}\), while the ‘normal’ ground state ends up 320 keV higher. One other interesting property of the ground state becomes clear when we see the weak binding of the ground state as shown on table 5.1. Since the ground state is a weakly-bound s-wave, it is an excellent candidate for being a halo nucleus. Recent shell model treatments of \(^{11}\text{Be}\) have supported this view [5].

Although only the ground and first excited states of \(^{11}\text{Be}\) (\(\frac{1}{2}^+\) and \(\frac{1}{2}^-\) respectively) are bound with respect to particle emission, many ‘unbound resonances’ are observed. These excited states exist above the neutron separation threshold and are populated by the \(^{11}\text{Li}\) \(\beta\)-decay, then decay immediately by neutron emission.

### 5.1.2 \(^{11}\text{Li}\)

During the early exploration of light, neutron-rich nuclei, \(^{11}\text{Be}\) was assumed to lie at the neutron drip-line. However, in 1966, Poskanzer et al. used a silicon detector telescope (two \(\Delta E\) detectors and one \(E\) detector) to identify light ions, and observed \(^{11}\text{Li}\) for the first time [57]. Since \(^{11}\text{Li}\) had been predicted to be particle-unstable, observation of the nucleus was a surprise that led to immediate interest and the desire to carry on with the study of a new exotic, neutron rich nucleus.
Table 5.2. Binding Energy of $^{11}$Li, masses from [56]

<table>
<thead>
<tr>
<th>Mass (MeV/c²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{9}$Li+n+n</td>
</tr>
<tr>
<td>$^{11}$Li</td>
</tr>
<tr>
<td>$S_{2n}$</td>
</tr>
<tr>
<td>10287.531(2)</td>
</tr>
<tr>
<td>10287.232(19)</td>
</tr>
<tr>
<td>0.299(19)</td>
</tr>
</tbody>
</table>

Once $^{11}$Li was observed to be particle-stable, several follow-up studies were performed. First, the half-life was measured in 1969 to be 8.5(10) ms [58]. The same group (Klapisch et al.) also estimated that $^{11}$Li was bound by 2.6(5) MeV relative to $^{9}$Li+n+n. Shortly thereafter, Thibault et al. drastically revised this estimate downward when they measured the mass of $^{11}$Li [26]. More recent measurements suggest that Thibault et al. may have underestimated the binding energy of the halo neutrons [59]; the current accepted $S_{2n}$ value is seen on table 5.2.

Despite the weak binding energy of its ground state, the halo nature of $^{11}$Li was not revealed until two important experiments by Tanihata et al. in the 1980s. In 1985, Tanihata et al. demonstrated at the 88-inch cyclotron (Berkeley, CA) that $^{11}$Li has a dramatically larger matter radius than its isobaric neighbors. The discrepancy (seen in figure 5.1) was interpreted as evidence either of deformation, or of a ‘tail in the matter distribution’ forming in the extremely neutron-rich nucleus and dramatically extending the range of the wave function [60]. Tanihata et al. also performed a follow-up study in 1988, confirming the previous results [61].

Many subsequent studies of $^{11}$Li were undertaken in order to understand the unexpected results observed by Tanihata et al. In 1986, Arnold et al. measured the spin and magnetic moment of the $^{11}$Li ground state [62]. Both values ($J = \frac{3}{2}$ and $\mu_t=3.6673(25)$ n.m.) indicated that the protons are in a pure $1p_{3/2}$ state, with no deformation. The proposal of an extended matter distribution therefore gained
Figure 5.1. Matter radius measurements for light isotopes near A=11 from [60, 61]. The radius increases dramatically at $^{11}$Li

support.

Hansen and Jonson pointed out in 1987 that the matter radius of $^{11}$Li can be linked to the binding energy of the halo neutrons by treating the system as a ‘quasi-deuteron’, $^{9}$Li + 2n [63]. The wave-function of the two neutrons then behaves very much like equation 2.12, decaying exponentially away from the core.

The nature of the $^{11}$Li halo was explored by Borge et al. in 1997 [64]. In that experiment, the intensity of the 320 keV $1/2^- \rightarrow 1/2^+ \gamma$-ray transition in $^{11}$Be following $\beta$-decay from $^{11}$Li was measured. The $1/2^-$ is fed by a pure Gamow-Teller (GT) $\beta$-decay from the $3/2^-$ $^{11}$Li ground state, thus Borge et al. compared the GT transition probability to shell model predictions in order to measure the $^{11}$Li ground state composition. They found that the decay was best reproduced if $^{11}$Li was treated as a mixture of two configurations, the neutron closed shell and the $s^2_{1/2}p^{-2}_{1/2}$ (two-particle, two-hole) configuration. The admixture of distributions from these configurations forms the $^{11}$Li halo. The results were later confirmed by Simon et al. by dissociating $^{11}$Li on a carbon target and observing asymmetries in coincidences between ($^{9}$Be+n)
Figure 5.2. The ‘borromean’ nucleus $^{11}$Li is bound even though it consists of several unbound sub-systems and free neutron fragments following the breakup [65].

Despite these experimental successes, the nature of the $^{11}$Li ground state has not yet been fully explained in the context of the shell model, and continues to receive attention. Several groups have pointed out that, for an essentially three-body system such as $^{11}$Li ($^9$Li + 2n) the 2-body interactions must be taken into account [63, 66, 67]. In this case, neither 2-body system ($^9$Li+n) or (n+n) is bound. $^{11}$Li is therefore a ‘borromean’ system as seen on figure 5.2: the nucleus is particle-bound, yet removal of any element results in instability.

$^{11}$Li continues to attract attention and high-profile experiments. For example, another mass measurement at ISAC is planned for the near future, with the goal of measuring the mass to a precision of 1 keV or better [68]. Several groups have searched for evidence of excited resonances in $^{11}$Li, and one state at approximately 1 MeV is accepted [69] (no bound excited states are expected, since the ground state is bound by only about 0.3 MeV).
5.1.3 Excited Halo States

$^{11}$Be and $^{11}$Li both appear to possess halo ground states. In principle, excited halo states are expected to be present in some nuclei. Just like ground state halos, these excited halos arise due to weak binding energy and large s-wave components. Direct experimental evidence for excited halos is elusive, however: these states are generally short-lived (ps or fs), making probes of the mass or radius nearly impossible.

Although excited states generally cannot be experimented upon directly, indirect evidence may be collected to identify excited halos. For example, one experiment on the proton-rich nucleus $^{17}$F has claimed indirect evidence of an excited halo, in the form of the very large low-energy S-factor for direct capture of protons in a $(p,\gamma)$ reaction [70].

Jensen et al. suggested many possible 2-body and 3-body excited halo state candidates based on their low separation energies and the (theoretical) s-wave composition of each state [71]. Possible candidates include the $\frac{1}{2}^-$ first excited state in $^{11}$Be, and several excited states in $^{10}$Be, including the $2^-$ at 6263 keV. Interest in these potential excited halo states has since increased, prompting several new experiments searching for evidence of excited halos, and of ‘halo survival’.

5.2 $^{11}$Li and the Search for Halo Survival

The possibility of halo survival was raised by Timofeyuk and Descouvemont in 1996 [72], and subsequently by Jensen et al. [71]. They pointed out that the core of a halo nucleus appears to be more unstable to $\beta$-decay than the halo, and suggested that if a $\beta$-decay occurs within the core the halo configuration could be left more or less intact. The resulting states could then respond to the sudden change within the core, resulting in ‘halo-like’ configurations in the resulting excited states in the
daughter nuclei.

This suggestion has prompted nuclear physicists to search for evidence of halo survival during the $\beta$-decay of $^{11}$Li. Several recent experiments on the $^{11}$Li decay have been performed at TRIUMF-ISAC, with the aim of searching for evidence of halo-like states in $^{11}$Be and $^{10}$Be. If found, these halo-like states could serve as strong evidence for halo survival through the $\beta$-decay process.

### 5.2.1 The Complex $^{11}$Li $\beta$-decay Scheme

These experiments are made more difficult by the extremely complicated $\beta$-decay of $^{11}$Li. Since the Q-value for the decay is very large (20.61 MeV), many excited states in $^{11}$Be are populated, from the 0.320 MeV first excited state to a high-lying excited state in $^{11}$Be at approximately 18 MeV [73].

The highest excited states in $^{11}$Be are unstable to the emission of charged particles such as deuterons, tritons, and alphas. The charged particle emission may yield important information about halo survival: deuteron emission may for example result from a $\beta$-decay in the 2-neutron halo forming an excited p-n system. The charged-particle branches were studied at TRIUMF in spring 2005 ([74], to be published). In this experiment, a $^{11}$Li beam was implanted in a pixelated double-sided Silicon strip detector (DSSSD), and the signals from both beam implantation and re-emission of the charged particles were recorded.

These $\beta$-delayed charged-particle branches are very weak, making up only a small fraction of the overall decay. A somewhat larger $\beta$-decay branch proceeds about 10% of the time to the low-lying $\frac{1}{2}^-$ state in $^{11}$Be. As previously mentioned, this is the only bound excited state in $^{11}$Be; it decays to the ground state via a 320 keV $\gamma$-ray.

The largest $\beta$-decay modes proceed through intermediate energy excited states
in $^{11}$Be, via $\beta$-delayed one-neutron and two-neutron emission, to $^{10}$Be and $^9$Be. In particular, the $\beta$-delayed neutron emission to excited states in $^{10}$Be accounts for at least 70% of the overall $\beta$-decay. This decay mode is shown in figure 5.3. This ‘$\beta$-delayed neutron emission’ branch has been the subject of two recent experiments.

### 5.2.2 Previous $8\pi$ Experiment

In 2003, Sarazin et al. used the $8\pi$ spectrometer to explore the $\beta$-delayed one-neutron emission of $^{11}$Li to $^{10}$Be [75]. The arrangement of the $8\pi$ was nearly identical to the arrangement described in chapter 3, including HPGe detectors and Compton suppression, but the SCEPTAR array for $\beta$-detection had not yet been added to the $8\pi$.

Sarazin et al. focused on detecting Doppler broadened $\gamma$-rays in the $^{11}$Li decay spectrum. These Doppler broadened peaks are the result of $\beta$-delayed neutron emission populating excited states in $^{10}$Be. Since $^{10}$Be is a relatively light nucleus, the
neutron emission causes a significant recoil:

\[ E_{\text{recoil}} = E_{\text{neutron}} \times \frac{m_n}{M(^{10}\text{Be})} \]  

If \( \gamma \)-rays are emitted while the \(^{10}\text{Be} \) nucleus is still recoiling following neutron emission, the \( \gamma \)-rays may be Doppler shifted, and since the neutron emission is assumed to be isotropic with respect to the laboratory frame the total recorded \( \gamma \)-ray peak is Doppler broadened.

Sarazin et al. used Monte Carlo simulations to recreate the Doppler broadened \(^{10}\text{Be} \) transitions. By fitting the experimental results with Monte Carlo simulations, they explored the \(^{11}\text{Be} \rightarrow ^{10}\text{Be} \) neutron feeding branches, and the intensities and branching ratios of transitions in \(^{10}\text{Be} \). The resulting fits are seen in figure 5.4.

The experiment by Sarazin et al. may have been successful in finding evidence for halo survival. Al-Khalili and Arai [77] suggest that the weak \(^{10}\text{Be} \) \( 2^- \rightarrow 2_1^+ \) transition observed by Sarazin et al. [75], with a smaller-than-expected \( B(E1) \) value, may be explained if the \( 2^- \) state is in a halo configuration.

Some results from the experiment by Sarazin et al. have been disputed. First, another recent \( \beta \)-decay experiment was performed by Fynbo et al. at the ISOL facility ISOLDE at CERN. Fynbo et al. did not find evidence for halo survival during the decay [78]. Also, the neutron feeding branches suggested by Sarazin et al. have been disputed by another group following another recent experiment performed at TRIUMF-ISAC.

5.2.3 OSAKA Experiment

In 2003, Hirayama et al. also performed an experiment on the decay of \(^{11}\text{Li} \) at TRIUMF-ISAC [76]. The experiment was conceived by the Osaka University
Figure 5.4. Fits of Doppler broadened γ-rays by Sarazin et al. [75] Neutron feeding from the 8.81 MeV state seen in the 2590 keV and 5960 keV peaks is disputed by Hirayama et al. [76]

Figure 5.5. $^{11}\text{Li}$ decay scheme established by Sarazin et al.
Collaboration (OSAKA), and was similar to a previous experiment by Aoi et al. [79]. They made use of a polarized beam of $^{11}\text{Li}$, along with $\gamma$ and $\beta$ detectors, as well as time-of-flight neutron detectors. Hirayama et al. were therefore able to use $\beta - n - \gamma$ triple coincidences in their analysis, albeit with low statistics.

The OSAKA experiment has several important strengths: since a polarized $^{11}\text{Li}$ beam was used, some states in $^{11}\text{Be}$ were given firm $J^\pi$ assignments for the first time, based on the observed $\beta$-decay asymmetry:

$$W(\theta) \sim 1 + AP\cos(\theta)$$

(5.2)

where $A$ is the $\beta$-decay asymmetry from a given state, $P$ is the polarization, and $\theta$ the angle with respect to the polarization axis. Hirayama et al. were also able to directly measure neutrons in coincidence with some $\gamma$-rays in $^{10}\text{Be}$, aiding in their exploration of the neutron feeding.

The setup for the experiment by Hirayama et al. only included two HPGe $\gamma$-ray detectors, and as a result very few $\beta - n - \gamma$ triple coincidences were recorded. Thus, while the experiment is very sensitive to strong neutron branches they have little sensitivity to weaker transitions.

The analysis of Hirayama et al. has not reached complete agreement with the results from Sarazin et al. For example, neutrons observed by the OSAKA group in coincidence with the 2590 keV $\gamma$-ray transition do not support the suggestion by Sarazin et al. that a neutron branch from the $^{11}\text{Be}$ 8.81 MeV state is partially responsible for the shape of the 2590 and 5958 keV peaks (see figure 5.5). Instead, as seen in figure 5.6, the n-$\gamma$ coincidence spectrum suggests that only two neutron feeding branches (both from a $^{11}\text{Be}$ state near 8.03 MeV) are required to explain these peaks.
Figure 5.6. Neutron spectra observed by Hirayama et al. in coincidence with two prominent $^{10}\text{Be} \gamma$-rays [76]. A neutron branch from the $^{11}\text{Be}$ 8.82 MeV state was suggested by Sarazin et al. but does not appear in n-$\gamma$ coincidences (near 70 ns)
5.2.4 Need for a New Experiment

Since several discrepancies were found between the results reported by Sarazin et al. and Hirayama et al., a repeated study of the $^{11}\text{Li}$ $\beta$-decay at the $8\pi$ spectrometer was considered worthwhile. It was hoped that the new study would have several important advantages over both previous works: first, the $^{11}\text{Li}$ beam had seen continual development and improvement by the TRIUMF beam staff, resulting in improved fluxes of $^{11}\text{Li}$ ions. The $8\pi$ spectrometer had also recently been improved with the addition of the SCEPTAR array. A new experiment at the $8\pi$ would therefore have the advantage of detecting both $\gamma$-rays and $\beta$-particles, whereas Sarazin et al. detected only $\gamma$-rays.

5.3 Experimental Details

The new experiment was carried out during October/November, 2004. A beam of approximately 12000 $^{11}\text{Li}$ ions/second (the highest quality $^{11}\text{Li}$ beam in the world at that time, since surpassed by other experiments at ISAC) was delivered to the $8\pi$ over a 2-week period. Data collection was interrupted by a target vacuum leak, yet the beam was restored; eventually the experiment resulted in approximately 100 hours of data collection and about $9 \times 10^7$ recorded gamma events.

Aside from the vacuum pump failure, the experiment suffered another problem: one data acquisition crate had a faulty power supply, causing the time stamp to differ between $\gamma$-ray and $\beta$ events detected at the $8\pi$ and SCEPTAR. The problem was not discovered until after the experiment, when the two data streams were merged for $\beta$-$\gamma$ coincidence analysis. Fortunately, a solution was found and will be discussed in section 5.3.3.

For this experiment, the $8\pi$ was in nearly the same configuration as described
in chapter 3, with one major difference: the moving tape collector (or MTC) was 
not used at all during this experiment. Instead, $^{11}\text{Li}$ was implanted on a motionless 
strip of aluminized mylar in the center of the array. Although the decay daughters 
were therefore allowed to accumulate, only one daughter product makes a significant 
contribution to the decay spectrum: $^{11}\text{Be}$ decays to $^{11}\text{B}$ with a half-life of 13.8 seconds.

$\gamma$-rays emitted during this decay proved very useful in calibrating the $\gamma$-array.

### 5.3.1 Calibration

Calibrating the $8\pi$ for this experiment was quite similar to the calibration process 
described in chapter 4. In both cases, data from the decays of active $^{152}\text{Eu}$ and $^{56}\text{Co}$ 
sources formed the foundation of the calibration. In this case, however, $\gamma$-rays from 
$^{152}\text{Eu}$ and $^{56}\text{Co}$ are not sufficient: strong $^{56}\text{Co}$ $\gamma$-rays are only observed up to 3.2 MeV, 
whereas $\gamma$-rays in $^{10}\text{Be}$ are observed up to 5.8 MeV. Additional $\gamma$-rays in $^{11}\text{B}$ from 
the decay of $^{11}\text{Be}$ were therefore necessary for calibrating the array at high energy.
The energies and intensities of these $\gamma$-rays are well known from previous $\beta$-decay studies [80]. The $^{11}\text{B}$ $\gamma$-rays range in energy from 2.2 MeV to nearly 8 MeV, as seen on table 5.3. These transitions were also used to search for detector drift between calibration runs. The gamma-ray spectrum after calibration is shown in figure 5.7.

Table 5.3. $\gamma$-rays in $^{11}\text{B}$ from the $\beta$-decay of $^{11}\text{Be}$, used for calibration. From [80]:

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2124.47(3)</td>
<td>100</td>
</tr>
<tr>
<td>4665.9(4)</td>
<td>5.12(14)</td>
</tr>
<tr>
<td>5851.5(4)</td>
<td>6.01(24)</td>
</tr>
<tr>
<td>6789.8(5)</td>
<td>12.62(62)</td>
</tr>
<tr>
<td>7974.7</td>
<td>5.34(40)</td>
</tr>
</tbody>
</table>

5.3.2 First Comparison with Sarazin et al.

Once the calibration was complete, data from the current experiment was compared to data from the previous experiment performed by Sarazin et al. [75], described in section 5.2.2. The two experiments were both performed at the 8$\pi$ facility, yet differed significantly in several ways. For example, the more recent experiment enjoyed a much higher $^{11}\text{Li}$ yield, nearly 20 times the counts collected by Sarazin et al. Also, the SCEPTAR array for $\beta$-particle detection only became available after the experiment by Sarazin et al. Spectra from the two experiments are compared in figure 5.8; the $^{208}\text{Bi}$ contaminant is much more important relative to the $^{10}\text{Be}$ transition in the previous experiment since $^{11}\text{Li}$ counts were collected more slowly compared to background counts.
Figure 5.8. Comparison between current and previous $\pi$ experiments, with data scaled to match backgrounds. A contaminating $\gamma$-ray from $^{208}$Bi is seen here in the $\gamma$-singles, and is much more significant in data from the previous experiment

5.3.3 Treating $\beta$-$\gamma$ Coincidences

As mentioned before, we uncovered problems in correlating the data streams between $\gamma$-rays detected with HPGe crystals and $\beta$-particles detected with SCEPTAR. The problem was caused by a faulty power supply to one of the CAMAC crates, that led to large differences in the time stamps given to many recorded $\beta$ and $\gamma$ events. Normally, these events are correlated by the computer based on their time stamp (for an overview of the data acquisition system, see section 3.3). Since the time stamp was no longer valid, the decision was made to treat as $\beta - \gamma$ coincidences only physics events contained entirely inside single MIDAS events that consist of exactly one $\beta$ along with one or more $\gamma$-rays. This resulted in some coincidence counts being lost.

The impact of changing the treatment of $\beta - \gamma$ coincidences is seen in figure 5.9. For this sample run, almost all the events were lost when the time stamp was used, yet after changing the extraction process, we achieve approximately the performance expected from the $\pi$+SCEPTAR: only about 20% of $\gamma$-singles are lost as expected when applying a $\beta$ coincidence requirement.
Finally, once the $\beta - \gamma$ coincidence trigger was seen to work properly we vetoed bremsstrahlung events using anti-coincidences between Germanium and plastic scintillating detectors, as described in chapter 3. The results of applying both the $\beta$ coincidence requirement and the bremsstrahlung veto were shown on figure 3.15, as an example of the resolving power of SCEPTAR.

5.4 First Analysis

With $\beta - \gamma$ coincidences and bremsstrahlung veto working properly, the experimental spectrum was ready for analysis. The bulk of the analysis in this experiment (just like in the previous experiment by Sarazin et al.) was done by fitting Doppler broadened peaks in the experimental spectrum with Monte Carlo simulations. Some important information on $\gamma$-ray intensities and branching ratios was, however, extracted from the spectrum before resorting to Monte Carlo methods.
5.4.1 Extracting Intensities

The first step in the analysis was to extract intensities for all observed \( \gamma \)-ray transitions. The intensities were extracted (as described in chapter 4) by finding the number of counts in each peak as well as the approximate detector efficiency \( \epsilon(E) \):
\[ I = N / \epsilon. \]
In this case, however, since some \(^{10}\text{Be} \gamma\)-rays are Doppler broadened, they are not suitable for fitting with a simple gaussian or even with a more realistic shape such as a Gaussian with an exponential low-energy shoulder to simulate incomplete charge collection (as described in [81]). Instead, we generally fit these shapes with a sum of two or more Gaussians, plus a background consisting of a 2\(^{nd}\) order polynomial and a ‘rounded step function’:

\[
F_{\text{background}}(x) = a + bx + cx^2 + C \text{Erfc} \left( \frac{x - \mu}{\sqrt{2\sigma}} \right)
\]  

(5.3)

These fits are useful mainly as a way to estimate the behavior of the background beneath the Doppler broadened peak. Treating each peak with this background, we extracted the intensities for the \(^{10}\text{Be} \) transitions, shown on table 5.4.

Table 5.4. Intensities of (Doppler broadened) \(^{10}\text{Be} \) transitions in the \(^{11}\text{Li} \) \( \beta \)-decay spectrum. These results are in agreement with results from [75]

<table>
<thead>
<tr>
<th>( E_{\text{Level}}(\text{keV}) )</th>
<th>( E_{\gamma}(\text{keV}) )</th>
<th>Assignment</th>
<th>Intensity</th>
<th>BR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3368</td>
<td>3368</td>
<td>( 2^+_1 \rightarrow 0^+ )</td>
<td>100(5)</td>
<td></td>
</tr>
<tr>
<td>5958, 5960</td>
<td>2590</td>
<td>( (1^-, 2^+_2) \rightarrow 2^+_1 )</td>
<td>26.0(13)</td>
<td>( a )</td>
</tr>
<tr>
<td></td>
<td>5958</td>
<td>( (1^-, 2^+_2) \rightarrow 0^+ )</td>
<td>4.2(2)</td>
<td></td>
</tr>
<tr>
<td>6179</td>
<td>219</td>
<td>( 0^+_2 \rightarrow 1^- )</td>
<td>1.58(8)</td>
<td>34.9(16)</td>
</tr>
<tr>
<td></td>
<td>2811</td>
<td>( 0^+_2 \rightarrow 2^+_1 )</td>
<td>2.94(15)</td>
<td>65.1(16)</td>
</tr>
<tr>
<td>6265</td>
<td>2896</td>
<td>( 2^- \rightarrow 2^+_1 )</td>
<td>5.2(3)</td>
<td></td>
</tr>
</tbody>
</table>

\( a \) Intensities of \( \gamma \)-rays from the \( (1^-, 2^+_2) \) doublet cannot be resolved, so other methods must be used to extract the branching ratios: see text
5.4.2 Branching Ratios

Table 5.4 includes a measurement for the $\gamma$-ray branching ratio between the two $\gamma$-rays emitted by the $0^+_2$ state. This branching ratio, an expression of the relative probability of emission for each $\gamma$-ray, was determined directly from the intensities:

$$BR_{\gamma_1} = \frac{I_{\gamma_1}}{I_{\gamma_1} + I_{\gamma_2}}$$ (5.4)

The $1^-$ and $2^+_2$ states in $^{10}\text{Be}$ both also emit multiple $\gamma$-rays, but for these states the branching ratios could not be directly calculated from the $\gamma$-ray intensities, since the energies of these states are almost identical and subsequent Doppler broadened $\gamma$-rays are difficult to resolve.

In the case of the two $\gamma$-rays from the $1^-$ state, the branching ratio can still be extracted by taking advantage of $\gamma - \gamma$ coincidences. As seen on figure 5.10, the $1^-$ is fed from above by a 219 keV $\gamma$-ray from the $0^+_2$ state. A coincidence gate with this $0^+_2 \rightarrow 1^-$ transition therefore isolates the contribution from the $1^-$ to the 2590 keV and 5958 keV peaks.

The spectra obtained in coincidence with the 219 keV peak are shown in figure 5.11 in red. As expected, the $\gamma - \gamma$ coincidence isolates one portion of a larger Doppler broadened peak (the full peak is also shown for comparison, scaled down by at least an order of magnitude in each case). From these coincidences the branching ratios for the $1^-$ state are extracted:

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>BR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2592 $1^- \rightarrow 2^+_1$</td>
<td>34(4)</td>
</tr>
<tr>
<td>5960 $1^- \rightarrow 0^+$</td>
<td>66(4)</td>
</tr>
</tbody>
</table>

Table 5.5. Branching ratio from the $1^-$ state in $^{10}\text{Be}$
Figure 5.10. Energy levels and γ-ray transitions within \(^{10}\text{Be}\). γ-rays from the \(1^-\) and \(2^+_2\) are too close in energy to be resolved, but coincidence analysis using the \(0^+_2 \rightarrow 1^-\) may help resolve the problem.
Figure 5.11. $1^- \rightarrow 2^+$ and $1^- \rightarrow 0^+$ transitions in $^{10}$Be, observed using $\gamma - \gamma$ coincidences. The intensities permit extraction of a $\gamma$-ray branching ratio for the $1^-$ state. The full Doppler broadened peaks are also shown, scaled down to fit in the figure.

### 5.4.3 Need for Monte Carlo Methods

At this point, ‘typical’ $\gamma$-ray spectroscopy will take us no further. We have extracted intensities for each of the transitions in $^{10}$Be, and found branching ratios for two of these excited states. However, no further $\gamma - \gamma$ coincidence analysis can reveal the branching ratio from the third state (the $2^{+_2}$). More importantly, the $\gamma$-ray intensities can give no insight into the neutron-emitting states in $^{11}$Be, nor help untangle the neutron emission.

Instead, we must follow the example of Sarazin et al., and turn to the Doppler broadened $^{10}$Be $\gamma$-rays to fully unravel the $\beta$-delayed neutron emission. These Doppler broadened peaks contain much valuable information: as pointed out in equation 5.1, the energy of the $^{10}$Be recoil is proportional to the energy of the neutron, and the Doppler broadening is related to recoil energy, so the shape of broadened $\gamma$-rays can lead back to information on the neutron emission.
In all, five Doppler broadened peaks in $^{10}$Be must be considered. Four are shown in figure 5.12, at 2590, 2811, 2896, and 5958 keV respectively. The 3368 keV $2^+_1 \rightarrow 0^+_1$ transition is not shown, but will also be discussed.

In order to analyze and extract information from these complicated line shapes, Monte Carlo simulations were created to recreate the $\beta$-delayed one-neutron emission from $^{11}$Li as completely as possible, producing spectra that may be compared to the experimental result using the $\chi^2$ as a standard metric. Real physical quantities such as neutron energies, level half-lives etc. are entered as parameters into the simulations, so that the best values of these parameters may be measured by minimizing the $\chi^2$.

5.5 Ingredients of the Monte Carlo Simulations

The Monte Carlo simulations were implemented using C++ and libraries from ROOT (see [82] or visit http://root.cern.ch). The pseudo-random generator used in the Monte Carlo simulations is a version of the Mersenne twister implemented in the ROOT TRandom3 class: the Mersenne twister is a fast generator, yet also statistically
strong and has a long period of $2^{19937} - 1$, making it ideal for statistical studies [83].

The goal of the Monte Carlo code is to simulate the $\beta$-delayed neutron emission, taking as many physical effects into account as necessary to achieve good agreement with experimental results. The Monte Carlo simulates $\beta$-decay events one at a time, and for each event takes many effects into account including the energy loss of ions in the aluminum tape both during and after implantation, $\beta$-decay branching ratios, neutron emission, Doppler broadening of emitted $\gamma$-rays, resolution of the $8\pi$ detectors, and so on. These effects are explained in detail in the next few sections.

5.5.1 Implantation and $\beta$-decay

The simulation must first randomly assign the implantation depth, or distance into the aluminum tape that a $^{11}$Li ion implanted at 30.6 keV reaches before stopping. A long simulation was performed using SRIM, software for the Stopping and Range of Ions in Matter [84]. The resulting implantation profile is seen in figure 5.13; the Monte Carlo picks a random depth from this profile, with the most likely depth near 2200 Å.

After implantation, $^{11}$Li undergoes $\beta$-decay to excited states in $^{11}$Be. In the simulation, we may choose to populate all the known states based on their previously measured $\beta$-branching ratios. More frequently, though, only the state(s) with neutron branches that go on to fill $^{10}$Be levels important to the transition of interest are populated during the simulation, to save processing power and disk space.

The $\beta$-decay process was assumed to leave the resulting $^{11}$Be ion approximately at rest. As discussed in the conclusion to this chapter, this assumption turns out to be incorrect, and will be revised in future versions of the Monte Carlo simulations, causing potential significant changes in the analysis.
Figure 5.13. Probability distribution for the implantation depth of $^{11}$Li in aluminum, from SRIM

### 5.5.2 Neutron Emission

Most $^{11}$Be excited states populated by the $^{11}$Li decay are unstable to neutron emission, with extremely short lifetimes. These states exhibit intrinsic energy broadening due to the energy-time uncertainty relationship (as discussed in section 2.2.2). These excited states take the form of ‘Breit-Wigner’ distributions (described in equation 2.5), with full-width at half-maximum (FWHM) $\Gamma$ often in the hundreds of keV. Neutron emission from these states is assumed to be isotropic.

Many excited states have previously been observed in $^{11}$Be, and are the most likely candidates for neutron-emitting states in the simulations. Some of the states in the energy region of interest are shown on table 5.6.

In the simulation, the neutron energy is selected randomly; the energy equals the difference between a $^{11}$Be excited state energy selected from a Breit-Wigner dis-
Table 5.6. Previously measured excited states in \(^{11}\text{Be}\). Data from [85] if not otherwise specified. States above 7.03 MeV could be involved in the \(\beta\)-delayed neutron emission to \(^{10}\text{Be}\) states above the \(2^+_1\) state.

<table>
<thead>
<tr>
<th>(E_x) (MeV)</th>
<th>(J^\pi)</th>
<th>(\Gamma) (keV)</th>
<th>First observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(\frac{1}{2}^+)</td>
<td>13.81 s</td>
<td></td>
</tr>
<tr>
<td>0.320</td>
<td>(\frac{1}{2}^-)</td>
<td>115 fs</td>
<td></td>
</tr>
<tr>
<td>7.030(50)</td>
<td></td>
<td>300(100)</td>
<td></td>
</tr>
<tr>
<td>8.02(2)</td>
<td></td>
<td>230(55)</td>
<td>[79]</td>
</tr>
<tr>
<td>8.816(32)</td>
<td></td>
<td>200(50)</td>
<td></td>
</tr>
<tr>
<td>9.1</td>
<td></td>
<td>[86]</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td></td>
<td>[86]</td>
<td></td>
</tr>
<tr>
<td>10.590(50)</td>
<td></td>
<td>210(40)</td>
<td>[87]</td>
</tr>
<tr>
<td>(11.75)</td>
<td></td>
<td>[87]</td>
<td></td>
</tr>
<tr>
<td>14.0</td>
<td></td>
<td>[87]</td>
<td></td>
</tr>
<tr>
<td>18.5</td>
<td></td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>

The energy of the resulting excited state in \(^{10}\text{Be}\):

\[
E_n = (E(^{11}\text{Be}^*) - E(^{10}\text{Be}^*) - Q) \times M(^{10}\text{Be})/M(^{11}\text{Be})
\] (5.5)

where \(Q\) is the difference in ground-state energies of \(^{11}\text{Be}\) and \(^{10}\text{Be} + \text{n}\), shown on table 5.1. A schematic of the neutron emission is seen in figure 5.14.

Currently, recoil of \(^{11}\text{Be}\) following emission of \(\beta\)-particles is assumed to be negligible (this assumption will be discussed further in the conclusion). We therefore assume the \(^{11}\text{Be}\) ion is at rest in the lab frame before neutron emission, so that the center of mass of the \(^{10}\text{Be} - \text{n}\) system remains at rest after the emission. Then using equation 5.1 we calculate the energy of the \(^{10}\text{Be}\) recoil. Neutron emission from the 8.03 MeV \(^{11}\text{Be}\) state to the 6.2 MeV \(^{10}\text{Be}\) state is a typical example; in this case the mean neutron energy is about 1.35 MeV, and the mean energy of the \(^{10}\text{Be}\) recoils is about 135 keV. The mean recoil kinetic energy is much less than the rest energy of
Figure 5.14. Neutron emission from an excited state in $^{11}\text{Be}$. The state takes the form of a Breit-Wigner distribution, with centroid $E_0$ and width $\Gamma$.

$^{10}\text{Be}$ (approximately 9.3 GeV), so a non-relativistic treatment of the $^{10}\text{Be}$ particles is justified.

5.5.3 Energy Loss of the $^{10}\text{Be}$ Recoil

Immediately after neutron emission, the $^{10}\text{Be}$ ion begins to lose energy as it travels through the aluminum tape. We turn once again to SRIM for help in modeling the slowing of the Be ion.

SRIM’s predictions for the stopping power of a material with atomic number $Z$ are similar to the Bethe formula described in [88] and shown below (although the SRIM predictions are more complex):

$$\frac{dE}{dx} = 4\pi NZ \frac{z^2 e^4}{mc^2 \beta^2} \left[ \ln \left( \frac{2m_e c^2 \beta^2}{\hbar \langle \omega \rangle \cdot (1 - \beta^2)} \right) - \beta^2 \right]$$

(5.6)

Here, $\beta=v/c$, $z$ and $Z$ are the atomic numbers of the ion and target, and $\hbar \langle \omega \rangle$ is the ionization potential of the material. The Bethe formula provides the theoretical basis
for stopping of ions in matter.

The SRIM dE/dx database has been created by recognizing that energy loss is primarily a function of target material. The authors of SRIM therefore compile its tables by considering experiments of any ion striking the target of interest, and fitting the resulting energy loss curve (as seen in figure 5.15(a)) [84].

The resulting curve may disagree significantly with results from individual experiments. The disagreements are particularly important for low-energy particles (0-100 keV), where fewer experiments have been performed. For example, Mertens and Krist measured the stopping of beryllium ions on an aluminum target in 1979 [89]. In that experiment, an Al target was bombarded with Be ions ranging from 10 to 30 keV/amu, and the simplified stopping function \( S_e(10^{-15} \text{ eV cm}^2/\text{atom, } = 1.6582^{-1} \text{ eV/Å}) \) was measured, with a stated precision of 7%. The SRIM prediction for the stopping of Be in Al over-estimates the results from [89] as seen in figure 5.15(b), suggesting potentially larger uncertainties in this low-energy region.

Using the energy loss per distance dE/dx tabulated in SRIM, the Monte Carlo simulation steps the \(^{10}\text{Be}\) recoils through the target material. For every 1 fs time step, the initial energy of the recoil is re-calculated to determine both how far it will travel (\( v \text{ in Å/fs } = c \times \sqrt{\frac{2E_{\text{recoil}}}{M_{\text{recoil}}}} \)), and its energy loss in eV/Å. The recoil loses energy continuously until it stops (\( E_{\text{recoil}} = 0 \)), a \( \gamma \)-ray is emitted, or it emerges from the aluminum tape (a special case that will be discussed further in section 5.5.5).

5.5.4 Gamma Decay

Neutron emission often leaves the \(^{10}\text{Be}\) recoil in an excited state, so \( \gamma \)-rays are emitted while the recoil is slowing down. \( \gamma \)-emission and energy loss are closely linked, as longer-lived \(^{10}\text{Be}\) states have more time to lose energy and display less Doppler
Figure 5.15. Energy loss predictions from SRIM

(a) The SRIM data for aluminum is compiled from many stopping experiments using multiple ions. x-axis units in keV, Image from srim.org [84]

(b) Stopping of $^{10}\text{Be}$ ions in Al, as measured (in red) and extrapolated by SRIM (in black)
Figure 5.16. As the half-life of excited $^{10}$Be states grows longer, the ion slows more before $\gamma$-emission resulting in less Doppler broadening than short-lived states (figure 5.16 shows the relationship between half-life and energy loss for an ad hoc case). When a $\gamma$-ray is emitted, several factors determine the amount of Doppler broadening, including but not limited to the recoil energy and the direction of $\gamma$-emission.

The energy of the recoil at the moment of $\gamma$-ray emission depends on the initial recoil energy (from equation 5.1), and on the time the recoil spends slowing down in the aluminum tape. In the Monte Carlo, the time before $\gamma$-emission is chosen from an exponential distribution:

$$P(t) = e^{-t/\tau}$$

(5.7)

where $\tau ( = t_{1/2}/\ln(2) )$ is the 'mean lifetime' of the excited state.

The angle $\theta$ between recoil velocity and $\gamma$-ray emission also affects the Doppler broadening. The angle is usually selected from an isotropic distribution (although in some cases this isotropic assumption breaks down due to angular correlations between the neutron and $\gamma$-ray emission, discussed below). A small correction is then applied
to rotate the $\gamma$-ray back from the recoil frame to the lab frame, to discover whether it strikes a detector.

Once the recoil energy, and the angle $\theta$ between the recoil velocity and $\gamma$ emission are known we can calculate the Doppler shifted energy of the $\gamma$-ray:

$$E_{\gamma} = E_{\gamma 0} \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\frac{1}{1 - \frac{v}{c} \cos \theta}}$$

where $v/c = \sqrt{2E_{\text{recoil}}/M_{\text{recoil}}}$, and $E_{\gamma 0}$ is the energy of the $\gamma$ before Doppler correction, approximately equal to the difference between energy levels in $^{10}\text{Be}$.

**Angular Correlations** One more complication may arise in finding the angle between recoil velocity and emitted $\gamma$-ray: due to angular correlations between the neutron and $\gamma$-ray, $\gamma$-emission may not always be isotropic in the $^{10}\text{Be}$ frame. Angular correlations are discussed in a classic paper by Biedenharn and Rose [90]. $\alpha-\gamma$ correlations are addressed in that work, and the same formalism may be adjusted for $(n,p,d)-\gamma$ correlations. In this work we are interested in the correlation function between a neutron and the subsequent $\gamma$:

$$W_{n-\gamma}(\beta) = \sum_\nu A_\nu \frac{2\ell(\ell + 1)}{2\ell(\ell + 1) - \nu(\nu + 1)} P_\nu(\cos \beta)$$

where

$$A_\nu = F_\nu(L_1j_1j) F_\nu(L_2j_2j)$$

and the $F_\nu(Lj_1j)$ values have been tabulated by [90]. The first consequence is that, since $0^+$ states in the $\gamma$-emitter result in $\nu = 0$, no angular correlation is allowed for these states. For other neutron feeding branches we must consider the ‘channel spin’ ($^{3/2}_2^+ \rightarrow 2^-$ gives a channel spin of 1 or 2) and find a range of possible angular
As seen in figure 5.17, an n-γ correlation (positive or negative) has a very large potential impact on the shape of the resulting Doppler broadened peak. In this work we have mainly considered only very weak n-γ correlations, that have a smaller impact on the peak shape.

5.5.5 Geometric Asymmetry

Energy loss and gamma emission have very important links, as discussed in the previous section. The geometry of the target and detectors in the 8π is also responsible for a small but important anisotropy in the decay spectrum.

$^{11}$Li ions are typically implanted in the aluminum target from 0 to 3000 Å, yet the target is much thicker. Therefore, following neutron emission some $^{10}$Be ions may re-emerge from the aluminum only on the same side as they were implanted. If an ion escapes from the target into the vacuum chamber, it loses no more energy.

Since ions can escape only on one side of the aluminum tape, an interesting
Figure 5.18. A geometric asymmetry: $^{10}\text{Be}$ ions may escape from the aluminum tape only on the side where implantation occurred. If an ion escapes, it loses no more energy to Al.

Anisotropy is seen in the Doppler broadened $\gamma$-rays: ‘upstream’ detectors, closer to the beam implantation, record more blue-shifted $\gamma$-rays whereas ‘downstream’ detectors observe more red-shifted events. An example of an event with a $^{10}\text{Be}$ ion escaping from the tape is shown in figure 5.18.

This anisotropy becomes more important for $\gamma$-ray transitions from longer-lived $^{10}\text{Be}$ states. Since most $^{10}\text{Be}$ ions in these long-lived states have time to slow significantly or stop in the aluminum target before emitting any $\gamma$-rays, the resulting transitions display little Doppler broadening. $^{10}\text{Be}$ ions that escape from the aluminum add an important Doppler broadened contribution to these transitions (for example, see the 2811 keV transition in figure 5.19(a)). As we can see in figure 5.19, the geometric asymmetry is taken into account in the Monte Carlo simulations.
Figure 5.19. Geometry and resolution effects in the Monte Carlo simulation

(a) Simulated results from four detector ‘rings’, shown side by side to illustrate differences from the geometric asymmetry discussed in section 5.5.5

(b) The four detector rings drawn all together to illustrate differences in detector efficiency and resolution: one of the middle rings has a γ-ray detection efficiency almost 10% greater than the rest
5.5.6 Detection at the $8\pi$

After the $\gamma$-ray is emitted it must be recorded by one of the HPGe detectors. The resolution and efficiency of the detector array then need to be taken into account.

The intrinsic resolution of the HPGe detectors is described by equation 3.2: gaussian, with a low-energy shoulder for incomplete charge collection. In the Monte Carlo, a correction for the resolution is randomly selected from a realistic distribution and added to the Doppler-broadened $\gamma$-ray energy from equation 5.8. The parameters $\sigma$ and $\beta$ that determine the detector resolution were found by fitting peaks in $^{56}$Co, $^{66}$Ga, and $^{11}$B.

Due to the geometric asymmetry discussed in section 5.5.5, differences in efficiency between 'rings' of detectors are also important. The $8\pi$ array is arranged in four rings, each consisting of five detectors at the same angle $\theta$ with respect to the beam axis (as seen in figure 3.13). Since the 'upstream' rings of detectors records more blue-shifted $\gamma$-rays relative to the 'downstream' rings, differences in efficiency and resolution between the two may be important and must be considered in the simulation.

The differences in resolution and efficiency between detector rings are seen in figure 5.19(b). The relative efficiencies vary by about 10%.

5.5.7 Background Treatment

In order to properly compare the simulation to experimental data, both peak and background must be accounted for. The background is omitted from the Monte Carlo, partly to speed up the simulation and partly because the background is very complex, and treating it properly in the simulation would require adding extra degrees of freedom to account for Compton scattering, bremsstrahlung, single- and double-
escape, and neutron-induced peaks.

Instead of including the background in the Monte Carlo, we instead create a background model separately, and scale the background before adding to the simulated peak. The background model is usually simple, consisting of a $2^{nd}$-degree polynomial + a rounded step function (to account for an increase in Compton scattered $\gamma$-rays under the peak). In some cases the background becomes more complicated, such as the single-escape peak seen in figure 5.20; these will be discussed in greater detail in the next few sections.

Adding a random background impacts the uncertainties on the simulated peak. Typically, we seek to simulate at least 10 times as many events for each peak as are found in the experimental peak, up to 50 times if possible:

$$N_{\text{sim}} = 50 \times N_{\text{exp}}$$  \hspace{1cm} (5.11)
Even more counts are generally simulated in the artificial background, and then both are scaled down and added together before being compared to the experimental results:

\[ N = s(sb \cdot N_{bck} + N_{MC}) \]  \hspace{1cm} (5.12)

Here, the background scale factor ‘sb’ can be calculated from the integral \( I_s \) of the simulation, \( I_b \) of the background, and \( R \), the experimental peak/total ratio:

\[ sb = \frac{I_s}{I_b} \cdot \frac{1 - R}{R} \]  \hspace{1cm} (5.13)

The second scale factor, ‘s’, is required to scale the sum of simulated peak + background down to the experimental result. At the same time, the uncertainties for the simulation can be determined from the independent uncertainties in background and simulation by adding in quadrature:

\[ \sigma_{\text{sim}} = s\sqrt{s^2\sigma_{bck}^2 + \sigma_{MC}^2} \]  \hspace{1cm} (5.14)

Equation 5.14 is used to set the error per bin on each simulation.

### 5.6 Results

The Monte Carlo simulations described in the previous sections were designed to use real physical parameters as their input (parameters include half-lives, branching ratios, and so on). In our analysis we then sought the set of parameters that achieved the best agreement between simulated and experimental results, as measured by the \( \chi^2 \) test:

\[ \chi^2 = \sum_{i=1}^{N} \frac{(x_{\text{sim}}[i] - x_{\text{exp}}[i])^2}{\sigma_i^2} \]  \hspace{1cm} (5.15)
Due to the large number of parameters involved, and possible correlations between parameters, manually searching for the best fit would have been difficult and time-consuming. Instead, two algorithms were used to search for the best sets of parameters. The first made use of a grid-search technique, sequentially varying each important parameter by a pre-determined step in order to ‘map out’ the variations in $\chi^2$ with respect to each parameter.

The grid-search technique is simple to understand and easy to implement, but not very efficient especially when many parameters must vary simultaneously. Fortunately, a good alternative method exists, that actively searches for a minimum $\chi^2$ based on the derivatives with respect to each parameter. The Levenberg-Marquardt algorithm, described in more depth in appendix A, is particularly useful since the ‘Error Matrix’ $M$ is used during minimization; the matrix elements give the parameter errors and correlations between variables. In this work, we have used an implementation of Levenberg-Marquardt ported from the *fit.F* code described in reference [91].

By itself, the fitting algorithm under-reports the uncertainties on each parameter. In order to get an accurate idea of the uncertainties, we must consider both the statistical fluctuation in measured value, and systematic errors. Statistical uncertainties were obtained by measuring the same peak several times, varying only the random seed and possibly initial parameters for the Monte Carlo simulations. The measured value along with $\sigma_{\text{stat}}$ are then obtained from the resulting fitted parameters.

Straggling and energy loss were identified as the main potential source of systematic error in this experiment, since values from SRIM are typically trusted only to $\pm 10\%$. Thus, systematic errors $\sigma_{\text{sys}}$ were obtained by varying the energy loss by $\pm 10\%$ and finding the new best parameter values.
5.6.1 2811 keV

The first $\gamma$-ray modeled with the Monte Carlo simulations was the 2811 keV $0^{+}_2 \rightarrow 2^{+}_1$ transition. Modeling this peak was expected to be relatively simple, for a few reasons:

- Since the $\gamma$-ray is emitted from a $0^+$ state, no $n$-$\gamma$ angular correlations are allowed.

- Previous works [75, 76] suggest the $0^+_2$ state is fed by only one neutron branch, from the 8.03 MeV $\frac{3}{2}^-$ state.

The background for the 2811 keV peak is slightly more complicated. As seen in figure 5.20, the single-escape peak from the 3368 keV transition lies just above 2811 keV, and makes up a significant part of the background next to the high energy tail of the 2811 keV peak. Fortunately, the two peaks do not overlap to any significant degree, so we may approximate the background beneath the 2811 keV peak with a 2nd-order polynomial and rounded step function.

Monte Carlo simulations were performed using the $0^+_2$ half-life along with the energy and $\Gamma$ of the neutron-emitting state as free parameters. The best fit from these simulations is shown in figure 5.21, along with the fit residuals.

The results of our fit to the 2811 keV $\gamma$-ray are summarized in table 5.7, along with the previous results from [75, 76]. Our results support the previous conclusion that the $0^+_2$ state is fed by a single neutron from a state at approximately 8.02 MeV, with $\Gamma \approx 0.24$ MeV (the state has been given a $J^\pi$ assignment of $\frac{3}{2}^-$ [76]). Systematic uncertainties are much more significant in this work, due to the role of SRIM in our analysis.
Figure 5.21. Best fit for the 2811 keV transition, $\chi^2/\nu=1.88$.

Table 5.7. Measured parameters related to the 2811 keV $\gamma$-ray including statistical and systematic uncertainties, and comparison with previous works

<table>
<thead>
<tr>
<th></th>
<th>$0^+<em>2$ $t</em>{1/2}$, fs</th>
<th>$E_{(^{11}\text{Be})}$, MeV</th>
<th>$\Gamma_{(^{11}\text{Be})}$, keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>[75]</td>
<td>870 ±70 ±160</td>
<td>8.03(7)</td>
<td>-</td>
</tr>
<tr>
<td>[76]</td>
<td>-</td>
<td>8.02(2)</td>
<td>230(55)</td>
</tr>
<tr>
<td>This work</td>
<td>955 ±37 ±100</td>
<td>8.01 ±.02 +0.4/-0.7</td>
<td>240 ±70 +100/-10</td>
</tr>
</tbody>
</table>

The $0^+_2$ half-life found in this work is longer than that claimed by the previous work [75], yet the two results agree within a standard deviation. The longer half-life may be a result of fitting both $t_{1/2}$ and $\Gamma$ simultaneously, since the two parameters are correlated.

This long half-life plays an important role: $^{10}\text{Be}$ recoils are generally able to lose most of their energy in the aluminum target before emitting a $\gamma$-ray, so most of the Doppler broadening comes from recoils that escape from the tape (see figures 5.19(b) and 5.21). Since the 2811 keV and 219 keV $\gamma$-rays feed lower states in $^{10}\text{Be}$ (see figure 5.10), the long half-life will remain important in understanding other line-
The 2896 keV transition was next considered. Previous studies [75, 76] suggested that the $2^-$ like the $0_2^+$ is fed by only one neutron, in this case coming from a state in $^{11}\text{Be}$ lying at 8.82 MeV.

Analysis of the 2896 keV peak is made complicated by a convoluted background. The single escape from the $^{10}\text{Be} \ 2_1^+ \rightarrow 0_1^+$ transition at 3368 keV intrudes beneath the peak, and another transition is observed at 2895 keV, from the $\beta$-decay of $^{11}\text{Be}$ to $^{11}\text{B}$. These background contributions must therefore be taken into account. Fortunately, the behavior of the single-escape peak may be estimated based on the 3368 keV line-shape. Also, branching ratios for $^{11}\text{B}$ transitions following the $\beta$-decay of $^{11}\text{Be}$ have previously been measured [80], so the intensity of the $^{11}\text{B}$ peak can be estimated with the help of other $^{11}\text{B} \gamma$-rays.

The 2896 keV peak was first simulated with only one neutron branch feeding the $2^-$ state. The simulation is able to reproduce the central part of the peak, but the high and low-energy tails are not successfully reproduced as seen on figure 5.22. Thus, while the bulk of the peak can be explained with a neutron branch from the 8.82 MeV state in $^{11}\text{Be}$ as suggested by previous works [75, 76], strong evidence suggests a new branch from a higher energy $^{11}\text{Be}$ state is also necessary.

Under the assumption that the neutron-emitting state at 8.82 MeV was responsible for the bulk of the line shape, a small second neutron branch was introduced in the simulation to help explain the wide shoulders on the experimental peak. The results are seen in figure 5.23. First, the energy of the neutron emitting states was allowed to vary freely, resulting in the excellent fit seen in figure 5.23(a), but the energies of
these states do not overlap with known energy levels and are not in agreement with any previous experiment. Since these parameters had diverged, the neutron-emitting states were then ‘locked down’ to the accepted states at 8.82 MeV and 10.59 MeV (the two known states in the vicinity), and the peak was fit with only the $1^-$ half-life and the ratio of neutron feeding through the 10.59 MeV state allowed to vary. The $\chi^2$ for the resulting fit, seen in figure 5.23(b), increased.

Compared to the fairly successful simulation of the 2811 keV peak, the disagreement found between our best simulated results and the previously observed $^{11}$Be excited states is somewhat surprising, and deserves further discussion. The states

Table 5.8. Parameters measured for the 2896 keV peak for the fits shown in figure 5.23

<table>
<thead>
<tr>
<th>Neutron feeding, %</th>
<th>$2^- t_{1/2}$, fs</th>
<th>$E_1(^{11}$Be)</th>
<th>$E_2(^{11}$Be)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a: $11.4 \pm 1.0 \pm 2.0$</td>
<td>$50 \pm 3 \pm 10$</td>
<td>$8.55 \pm .02 + .04$</td>
<td>$11.2 \pm 0.2 - 0.3$</td>
</tr>
<tr>
<td>b: $10.8 \pm 0.7 \pm 0.2$</td>
<td>$90 \pm 5 \pm 12$</td>
<td>$8.82$(fixed)</td>
<td>$10.59$(fixed)</td>
</tr>
</tbody>
</table>
Figure 5.23. Fitting the 2896 keV peak with parameters shown in table 5.8

(a) The 2896 keV γ-ray fed by neutrons from (unknown) states at ~ 8.5 MeV and 11 MeV. $\chi^2/\nu = 1.3$

(b) Second fit, with the neutron-emitting states locked to 8.82 MeV and 10.6 MeV. $\chi^2/\nu = 3.3$

at 8.82 and 10.59 MeV are known from a compilation of several experiments to a precision of 32 keV and 50 keV respectively [73, 92]. They cannot be reconciled with the values (8.55 MeV ± 0.02 ± 0.04), (11.2 MeV ± 0.2 ± 0.3) corresponding to the neutron-feeding responsible for figure 5.23(a). Instead, either new, previously unobserved neutron-emitting states must be called for to explain the decay (which at this point is unlikely), or we must consider the possibility that, while neutron feeding does go through the known $^{11}\text{Be}$ states at 8.82 and 10.59 MeV, other effects are not properly taken into account in our model, increasing the $\chi^2$ as seen in figure 5.23(b).

The energies of these neutron-emitting states are also correlated with the half-life of the $^{10}\text{Be}$ $2^-$ state: the measurement of the $2^-$ half-life changes decreases dramatically when these energies vary.

One of the largest effects on the line shape comes from the slowing and stopping of the $^{10}\text{Be}$ recoil before it emits a γ-ray. One possibility, therefore, is that the stopping power $dE/dx$ is under- or over-estimated by SRIM. An effort has been made to improve the fit by allowing small deviations from the SRIM energy loss predictions,
with no success.

Attempts have also been made to change the model by including a possible third neutron branch from another state in $^{11}\text{Be}$, and also by including a strong angular correlation between neutron and $\gamma$-ray emissions (as mentioned in section 5.5.4). In both cases, improvements in the overall fit were deemed too small to justify keeping the effect.

The parameters measured for the two different fits to the 2896 keV peak are summarized on table 5.8. The disagreement between half-lives measured using the two methods is notable: these two fit results are not compatible, suggesting an underlying problem in the fit. We will discuss a possible solution to this discrepancy in the conclusion: accounting for the recoil from $\beta$-particle emission is likely to be sufficient to explain the Doppler broadened peak.

5.6.3 The $(1^-, 2^+_2)$ Doublet

Two more Doppler broadened peaks in the $^{11}\text{Li}$ decay spectrum (at 2590 keV and 5958 keV) each actually consist of two unresolved $\gamma$-ray transitions. These $\gamma$-ray transitions all arise from the $(1^-, 2^+_2)$ doublet in $^{10}\text{Be}$, a set of two states separated by only 1.5(6) keV. Since the $\gamma$-rays all originate from the same set of states, the shapes of the two Doppler broadened peaks are closely linked to one another, and must be treated together.

These peaks are more complicated to model than the peaks at 2811 and 2896 keV. First, at least two neutron branches are known to be involved in the decay: a direct branch feeding the $2^+_2$, and an indirect branch feeding the $1^-$ by way of the (long-lived) $0^+_2$ state. The decay scheme is seen in figure 5.24.

Assuming the level scheme and neutron feeding depicted in figure 5.24 is correct,
Figure 5.24. Neutron feeding for the $(1^-, 2^+_2)$ doublet

several branching ratios must be determined or used as free parameters in the Monte Carlo. They include the ratio of intensities between the two neutron feeding branches suggested by [76] and [79], and the γ-ray branching ratios from the $1^-$ and $2^+_2$ states. Fortunately, the ratio for the $1^-$ state was determined experimentally from $γ - γ$ coincidences (see section 5.4.2), removing one degree of freedom.

The 2590 keV and 5958 keV peaks were then simulated, with all neutron feeding proceeding through the 8.03 MeV state in $^{11}$Be. Parameters used in the simulation include the half-lives of the $1^-$ and $2^+_2$ states, the γ-ray branching ratio from the $2^+_2$, and the ratio between the intensities of the neutron feeding branches to the $0^+_2$ and $2^+_2$ states. The results are seen in figure 5.25. We find that the bulk of the decay, almost 85%, proceeds via a neutron branch directly feeding the $2^+_2$ state from the 8.03 MeV state in $^{11}$Be. Then, the $2^+_2$ decays about 90% of the time via a 2.59 MeV
Figure 5.25. Fitting the $(1^-, 2^+_2)$ doublet, neutron feeding from the 8.03 MeV state in $^{11}\text{Be}$

\[ \chi^2/\nu = 52.7 \]

\[ \chi^2/\nu = 5.66 \]

$\gamma$-ray, and only about 10% of the time via a 5.9 MeV $\gamma$-ray to the ground state. The half-lives of both $1^-$ and $2^+_2$ states appear to be short, less than 70 fs.

These fits also reveal that, while neutron feeding from the $^{11}\text{Be}$ 8.03 MeV state appears to be responsible for most of the shape of the peak, these two neutron branches are not enough to reproduce the width of the peak. A small second neutron branch from a higher $^{11}\text{Be}$ state appears to be necessary (particularly evident in of 5.25(b)).

Since neutron feeding from the 8.03 MeV state was found insufficient to explain the Doppler broadened line shape, additional feeding of the $^{10}\text{Be} \ 2^+_1$ state was considered (a direct feeding branch to the $1^-$ state is considered unlikely as the centroid of the new Doppler broadened contribution would be shifted by $\sim 1.5$ keV relative to the rest of the peak).

New fits were therefore performed, with additional free parameters for the energy of the unknown state in $^{11}\text{Be}$, and the $\beta$-decay branching ratio between the new $^{11}\text{Be}$ state and the 8.03 MeV state. The $^{11}\text{Be}$ states at 8.82, 9.1, 10.0, and 10.59 MeV were considered the most likely candidates for the new neutron-emitting state (see table 5.6), and the simulations suggest that the 9.1 MeV state agrees best with the
Figure 5.26. Fitting the doublet with a small additional neutron branch from the 9.1 MeV state

(a) 2590 keV, $\chi^2/\nu = 5.1$

(b) 5958 keV, $\chi^2/\nu = 2.3$

observed maximum Doppler broadening.

The results of the new simulation are shown in figure 5.26. With a fairly weak additional neutron branch, the fits for both peaks improve significantly. Parameters for the fit are shown on table 5.9.

The possibility of a new neutron branch has been addressed before in other works, and is the source of some controversy. As we discussed in sections 5.2.2 and 5.2.3, Sarazin et al. also found that the 2590 and 5958 keV peaks were wider than expected if neutron feeding came only from the $^{11}$Be 8.03 MeV state. They suggested [75] that a second neutron branch feeding the $2^+_2$ from the $^{11}$Be 8.82 MeV state could explain the extra width in these peaks. Furthermore, Sarazin et al. reported that the new neutron feeding branch was large, constituting about 30% of the total feeding of the $2^+_2$ state.

Hirayama et al. did not, however, see evidence of such a neutron branch in their time-of-flight neutron spectrum, or in $\beta$-n-$\gamma$ coincidences, seen in figures 5.6 and 5.27; a neutron branch from the 8.82 MeV state to the $2^+_2$ would be expected at about 2.4 MeV, corresponding to a time-of-flight of $\sim$70 ns, yet no such large neutron
Figure 5.27. Neutrons detected by Hirayama et al. in the $^{11}$Li decay [76]

feeding branch was observed.

We note that, while the n-γ coincidences observed by Hirayama et al. do not appear to support more than two neutron branches involved in shaping the 2590 keV peak, very few coincidence counts were observed. Small additional branches from higher $^{11}$Be excited states may therefore have been missed. The neutron time-of-flight spectrum recorded by Hirayama et al. and reproduced here in figure 5.27 also

Table 5.9. Parameters related to the $1^-, 2^+_2$ doublet

<table>
<thead>
<tr>
<th>Neutron energy $E_n$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
</tr>
<tr>
<td>counts/ns</td>
</tr>
<tr>
<td>8000</td>
</tr>
<tr>
<td>6000</td>
</tr>
<tr>
<td>4000</td>
</tr>
<tr>
<td>2000</td>
</tr>
<tr>
<td>1000</td>
</tr>
<tr>
<td>500</td>
</tr>
</tbody>
</table>

- $5/2^- : A = 0.6$
- $3/2^- : A = -0.4$
- sum

TOF (ns)

10 | 20 | 40 | 60 | 80 | 100 | 120 | 140

Figure 5.27. Neutrons detected by Hirayama et al. in the $^{11}$Li decay [76]

feeding branch was observed.

We note that, while the n-γ coincidences observed by Hirayama et al. do not appear to support more than two neutron branches involved in shaping the 2590 keV peak, very few coincidence counts were observed. Small additional branches from higher $^{11}$Be excited states may therefore have been missed. The neutron time-of-flight spectrum recorded by Hirayama et al. and reproduced here in figure 5.27 also

Table 5.9. Parameters related to the $1^-, 2^+_2$ doublet

<table>
<thead>
<tr>
<th>$1^- t_{1/2}$</th>
<th>$2^+<em>2 t</em>{1/2}$</th>
<th>$\beta$ BR$^a$</th>
<th>Neutron BR$^b$</th>
<th>$2^+_2 \gamma$ BR$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.5(85) fs</td>
<td>57.1(2) fs</td>
<td>13.9(1)%</td>
<td>86.2(6)%</td>
<td>88.1(4)%</td>
</tr>
</tbody>
</table>

$^a$ Branching ratio between the 9.1 MeV and 8.03 MeV $^{11}$Be states during the $\beta$-decay

$^b$ BR between neutrons feeding the $0^+_2$ and $2^+_2$ (from the 8.03 MeV state)

$^c$ BR between 2590 and 5958 keV $\gamma$-rays
suggests that additional small neutron branches might be found in the 2-3 MeV region for example, and that the weak new branch from 9.1 MeV measured in this work is in agreement with the work by Hirayama et al.

Once again, however, we must point out that our fit results are not in excellent agreement with experimental data, suggesting that further effects must be taken into account in the Monte Carlo simulations. We expect that including recoil from $\beta$-emission may give much better agreement with experimental data.

5.6.4 3368 keV

The last Doppler broadened $^{10}\text{Be}$ peak comes from the 3368 keV $2^+_1 \rightarrow 0^+$ (first excited $\rightarrow$ ground state) transition. This peak has a very complicated structure: the $2^+_1$ state at 3368 keV is fed both indirectly by $\gamma$-ray transitions from higher-lying $^{10}\text{Be}$ states, and also directly by neutron feeding from several $^{11}\text{Be}$ excited states.

The 3368 keV transition was simulated, using the $\beta$-decay branches and neutron feeding ratios proposed by Hirayama et al. [76]. The results are shown in figure 5.28; the simulation clearly disagrees with experimental results, in particular overestimating the central portion of the peak.

The disagreement seen in figure 5.28 could simply result from incorrect values for the resolutions of the $8\pi$ detectors near 3368 keV. We investigated this possibility, but found that unrealistically poor resolutions were needed to achieve agreement with experiment (recall from section 5.5.6 that the detector resolutions have been measured at many different energies, from $\gamma$-rays in $^{56}\text{Co}$, $^{66}\text{Ga}$ and $^{11}\text{B}$).

Changing the detector resolution therefore cannot wholly explain the differences between simulation and experiment seen in figure 5.28. We instead consider the possibility that Hirayama et al. have either overstated the strength of low-energy
neutron branches, or understated the strength of high-energy branches, possibly due to a normalization problem between the high-energy and low-energy time-of-flight spectra recorded in that experiment.

Many parameters affect the shape of the 3368 keV peak. According to Hirayama et al., the $2^+$ state is fed (directly or indirectly) from seven different neutron-emitting states in $^{11}$Be ranging from 3.89 MeV to 10.6 MeV. Free parameters therefore include the $\beta$-branching ratios between these neutron-emitting states, the neutron branching ratio (if applicable), and the half-life of the $2^+$. Due to the large number of free parameters required, no organized search for the best fit to the 3368 keV peak has been performed. The shape of the 3368 depends on at least nine direct or indirect feeding branches (not including the two new branches discussed in sections 5.6.2 and 5.6.3). An attempt to reduce the strength of some low-energy neutron branches has been made, however, with some success. The results are seen on figure 5.29.
5.7 Review and Summary

The Monte Carlo simulations were developed with the purpose of recreating the $\beta$-delayed neutron emission from $^{11}$Li as carefully as possible, in order to fit the Doppler broadened $^{10}$Be $\gamma$-rays appearing in the $^{11}$Li $\beta$-decay spectrum. Each physical process that could significantly shape the Doppler broadened peak was addressed in the Monte Carlo (and discussed in section 5.5).

Unfortunately, our analysis suggests that the simulations have not fully achieved their purpose: the best simulated results still do not agree with experimental results, with $\chi^2/\nu$ ranging from 1.3 to 5 for various peaks. We must conclude therefore that the model is somehow incorrect or incomplete. We now discuss some of the key ingredients to the Monte Carlo simulations, and how they might be improved.
5.7.1 Energy Loss Corrections

The simulations depend heavily on accurate knowledge of energy loss and straggling of ions in matter. As discussed in section 5.5.3, this information is provided by SRIM along with generous uncertainties. Data from SRIM must be considered with extra caution for ions in the 0-200 keV region (including most of our $^{10}$Be recoils), since relatively little experimental data for this region exists.

Corrections to the energy loss for low-energy ions may therefore be necessary to achieve agreement with experiment. An effort has been made to improve the fit by ‘tweaking’ the SRIM predictions, with limited success. Directional straggling may also need to be taken into account: ions may change direction as they lose energy, possibly causing more ions to scatter out of the aluminum tape. An attempt has been made to implement directional straggling, with however only a small impact on the Monte Carlo.

5.7.2 Improved Backgrounds

Great care was taken to represent the background below each Doppler broadened peak as well as possible. The backgrounds of course could not be directly observed, however, and so the simulated backgrounds may not reflect reality. For example, a fairly strong contaminating $\gamma$-ray, the 2895.3 keV peak from the $\beta$-decay of $^{11}$Be, lies under the 2896 keV $^{10}$Be peak. This contaminant is known and has been included in the simulated background, yet we are concerned about other possible small contaminating $\gamma$-rays that may change the background behavior.

Such effects are likely to be small if present at all, however. All of the disagreements between simulation and experiment are observed to be roughly symmetric, suggesting that they arise from systematic effects in the simulation rather than $\gamma$-
rays missing from the background.

5.7.3 Better Knowledge of Detector Resolution

From 0-3 MeV, the resolution of the HPGe detectors can be determined with high-count peaks from the decays of $^{56}\text{Co}$ and $^{152}\text{Eu}$. Fitting these peaks, we find the resolution with high precision. Above 3 MeV, however, we must resort to lower-count $^{11}\text{B}$ peaks in the $^{11}\text{Li}$ decay spectrum to determine the energy-dependent resolution. The resolution determined from $^{11}\text{B}$ is less precise, and some large fluctuations are seen in the results. Unfortunately, no other $\gamma$-rays allow us to measure the resolution up to 6 MeV.

Instead of measuring it directly, we might allow the detector resolution to vary as another free parameter in the Monte Carlo simulation. This introduces new problems, however. The number of free parameters should be kept down for an optimal fit, and in this case the detector resolution is strongly anti-correlated with other free parameters (half-lives of $^{10}\text{Be}$ states). The resolution therefore should, if possible, be measured a priori.

5.7.4 Concluding Remarks

The $\beta$-decay of $^{11}\text{Li}$ was measured at TRIUMF-ISAC using the 8$\pi$ spectrometer. Doppler broadened peaks in the decay spectrum have been simulated with a Monte Carlo routine implemented in c++ and libraries from ROOT. With the Monte Carlo simulations, we have achieved good general agreement with previous experimental results on the half-lives of several $^{10}\text{Be}$ excited states, including the $0^+_2$, measured at about 0.9(1) ps, and the $2^-$, measured at about 85 fs. Our analysis suggests that some small neutron branches are required to fully explain the shape of the 2896 keV
transition, as well as the shape of $\gamma$-ray transitions from the $(1^-, 2^+_2)$ doublet.

The Monte Carlo simulations do not achieve excellent agreement with experimental results, however. Some revisions to our model of $\beta$-delayed neutron emission must therefore be considered.

In hopes of bringing about better agreement between simulated and experimental results, we are revisiting some of the simplifying assumptions made during the analysis. For example, as mentioned in section 5.5.2, recoil of the $^{11}$Be ion following $\beta$-emission has so far been assumed to be negligible. This assumption is now coming under scrutiny, however.

Momentum must be conserved during the $\beta$-decay so that if $^{11}$Li is initially at rest $\vec{p}^{^{11}\text{Be}} + \vec{p}_{e^-} + \vec{p}_\beta = 0$. In order to find an upper limit on the recoil of $^{11}$Be from the $\beta$-emission we assume for the moment that the electron carries all the momentum, and $\vec{p}^{^{11}\text{Be}} + \vec{p}_{e^-} = 0$. The electron momentum can be related to the $\beta$-decay $Q$-value:

$$E_{total}^2 = Q_{\beta}^2 = (m_e c^2)^2 + (p_e c)^2 \approx (p_e c)^2$$

(assuming the electron mass is small compared to the $Q$-value). Then,

$$|\vec{p}_e| = |\vec{p}^{^{11}\text{Be}}| = \frac{Q_{\beta}}{c}$$

and we may find an upper limit on the $^{11}$Be recoil energy following the $\beta$-emission:

$$E_{max} = \frac{p^2}{2 \cdot M(^{11}\text{Be})}$$

Thus, a $\beta$-decay from $^{11}$Li to the 8.03 MeV excited state in $^{11}$Be can potentially cause up to 7.7 keV of kinetic energy in the $^{11}$Be recoil.
For comparison, we consider the neutron emission from the 8.03 MeV state to the $2^-$ state in $^{10}\text{Be}$ (at 6.263 MeV). Following this neutron emission, $^{10}\text{Be}$ is left with a recoil energy of $\frac{M_n}{M(11\text{Be})}(E(11\text{Be}) - E(10\text{Be}) - 0.504) = 115.6 \text{ keV}$.

As we see, recoil following $\beta$-emission can in fact contribute a significant portion to the recoil energy, up to 6.3% in this example. This effect will therefore be included in the Monte Carlo simulation, and may help bring agreement between simulated and experimental results. Results presented in section 5.6 will be subject to significant change once the Monte Carlo simulations are updated to include the contribution of the $\beta$-decay to the recoil energy.
CHAPTER 6

CONCLUSION

The structure of the atomic nucleus is not yet fully understood. Both theoretical and experimental studies thus continue to be performed with the goal of probing short-lived nuclei far from stability. As we have discovered, these modern studies in nuclear structure are difficult both to carry out and to analyze: experimental challenges include the production and extraction of radioactive ions, measurement of resulting decays, and background suppression, while theoretical models of the nucleus are complicated by the large number of possible interactions. As these studies in nuclear structure continue to be carried out, our understanding of the nucleus further from stability is refined.

In this thesis, we have discussed the use of β-decay as a means of experimentally probing some exotic neutron-rich nuclei with unusual properties. The RIB facility TRIUMF-ISAC was discussed: at ISAC, radioactive ions were produced, and sent to the 8π β-decay spectrometer, an array of germanium γ-ray detectors and plastic scintillating β-particle detectors designed to study β-decay radiation even under difficult experimental circumstances (including low count rates, and strong background and neutron-induced contamination).

The 8π spectrometer was used to probe the β-decays of two radioactive isotopes. In the first experiment, the decay of 32Na allowed us to explore the region near the N=20 shell closure known as the ‘island of deformation’. Excited states in 32Mg were populated, permitting exploration of the level scheme of this deformed nucleus. De-
spite a very low count rate of 2-3 $^32\text{Na}$ ions/second, several new $\gamma$-ray transitions have been observed, and two new levels are suggested in this work [52]. An important $\gamma$-ray transition feeding the state at 2321 keV has been found, opening up the possibility of a $4^+$ spin assignment for that state.

In the future, a higher count rate of $^32\text{Na}$ ions may become available with the addition of an actinide target at the ISAC facility. A target of UC$_2$ is expected to produce high yields of mid-mass, neutron-rich nuclei through the spallation process. In this case, direct measurements of the spin and parity of excited states in the $^32\text{Na}$ daughters may become possible, by considering angular distributions between the $\gamma$-$\gamma$ coincidences (see figure 6.1 for example).

![Angular distributions](attachment:image.png)

Figure 6.1. Angular distributions in $\gamma$-$\gamma$ coincidences may permit $J^\pi$ assignments at the $8\pi$. Data from the decay of $^{150}\text{Eu}$, in the analysis of [93].

The spectrometer was also used to study the $\beta$-decay of the halo nucleus $^{11}\text{Li}$. Doppler broadened $\gamma$-rays appear in the resulting spectrum, due to the recoil of $^{10}\text{Be}$ following $\beta$-delayed neutron emission. These Doppler broadened peaks have been simulated using Monte Carlo methods, and the results suggest some small new neutron branches may be required to properly explain the peaks. The Monte Carlo simulation results are not yet in excellent agreement with experimental results, therefore
suggesting that our model of $\beta$-delayed neutron emission requires some revision.

As mentioned in the conclusion of chapter 5, the simulations will be revised to treat the $^{11}$Be recoil following $\beta$-decay. Some other improvements may be necessary, including for example changes in the energy loss predictions from SRIM, revisions of the background treatment, and changes to treatment of detector resolution. The background and detector resolution must be measured from the experimental data. Improving the energy loss predictions is more difficult: these experiments require for example precisely machined foils of uniform thickness (1500 Å or less), and a large analyzing magnet for identifying the energies of ions that exit the foils at various angles.

Eventually, however, a new experimental facility may offer the best chance for understand the $^{11}$Li $\beta$-delayed neutron emission. The TIGRESS detector array in ISAC-II is now partially complete. When finished, it will consist of 12 segmented HPGe crystals with a high efficiency for $\gamma$-ray detection [94]. In addition, TIGRESS may be operated with supplemental detectors such as DESCANT (the Deuterated Scintillor Array for Neutron Tagging). The combination would provide a very high-resolution $n-\gamma$ coincidence spectrum and permit a detailed view of some small neutron feeding branches that may have been missed so far. This is an example of a possible future combination of increasingly advanced detectors and refined analytical tools that give physicists a powerful way to advance the study of nuclear structure.
### LIST OF SYMBOLS

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<th>Symbol</th>
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<tr>
<td>$8\pi$:</td>
<td>$\beta$-decay detector array</td>
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<td>$\epsilon(E)$:</td>
<td>Energy-dependant detector efficiency</td>
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<td>BGO:</td>
<td>Bismuth Germanate ($\text{Bi}_4\text{Ge}<em>3\text{O}</em>{12}$)</td>
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<td>CAMAC:</td>
<td>Computer Automation Measurement and Control</td>
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<td>CERN:</td>
<td>European Organization for Nuclear Research (formerly the European Council for Nuclear Research)</td>
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<td>HPGe:</td>
<td>High-Purity Germanium</td>
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<td>ISAC:</td>
<td>Isotope Separator and ACcelerator</td>
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<td>MIDAS:</td>
<td>Maximum Integrated Data Acquisition System</td>
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<td>OSAKA:</td>
<td>Osaka University Collaboration</td>
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<td>RIB:</td>
<td>Radioactive Ion Beam</td>
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<td>SCEPTAR:</td>
<td>SCintillating Electron-Positron Tagging ARray</td>
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<tr>
<td>TRIUMF:</td>
<td>Houses the world’s largest cyclotron and the ISAC radioactive ion beam facility, previously known as the ‘Tri-University Meson Facility’</td>
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REFERENCES

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   http://www.triumf.ca/people/marik/homepage.html.
[68] J.Dilling and F.Sarazin, ground state mass determination of the halo nucleus $^{11}\text{Li}$.
Appendix

Statistical Methods

In chapter 5, we discussed how experimental data from the $^{11}$Li $\beta$-decay was collected using the $\delta$ spectrometer. Assuming the experimental spectrum has observed values $y_i$ in bins $i = 1 \ldots N$, the aim of our analysis was to find the model $f(x_i, \Theta)$ depending on parameters $\Theta = (\theta_1, \ldots, \theta_n)$ that produces the best agreement with the experimental values.

To this end, we define the likelihood function:

$$L(\Theta) = \prod_{i=1}^{N} f(x_i; \Theta)$$

(A.1)

$L(\Theta)$ gives the joint probability that all the experimental bins will be fit by a model $f$ with parameters $\Theta$. The best agreement between model and experiment is found by maximizing $L(\Theta)$. The problem can be simplified somewhat by taking the natural logarithm of both sides, reducing the product to a sum:

$$\ln L(\Theta) = \sum_{i=1}^{N} \ln f(x_i; \Theta)$$

Two methods have been used in searching for the maximum likelihood and are discussed here: a grid search, and a fitting routine based on the Levenberg-Marquardt algorithm. Also, for more information the reader could look at references [91, 95, 96].
A.1 Grid Search

The $\chi^2$ can be defined from the likelihood function, and determines the quality of fit between simulated and experimental data:

$$\chi^2 = -2 \ln L(\Theta) + \text{constant} = \sum_{i=1}^{N} \frac{(y_i - f(x_i; \Theta))^2}{\sigma_i^2}$$  \hspace{1cm} (A.2)

Note that in our analysis, the uncertainty $\sigma_i = \sqrt{\sigma_{\text{exp}}(i)^2 + \sigma_{\text{sim}}(i)^2}$. The sign on $L(\Theta)$ is inverted in equation A.2, so that by minimizing the $\chi^2$ we find the maximum likelihood. One method is to simply produce the model $f(x_i; \Theta)$ for many different parameters in a ‘grid’, and search for minima in the resulting $\chi^2$ values.

Figure A.1. Monte Carlo simulations were started with many different initial parameter values $\Theta_{\text{initial}}$. Each set of simulations converged on the same region, as the program actively sought to minimize the $\chi^2$. Only steps which reduced the $\chi^2$ are shown
This ‘grid search’ method was implemented with considerable success. The algorithm started with an initial set of parameters \( \Theta_{initial} \), and changed each parameter \((\theta_1, \theta_2, \ldots)\) by a set value, searching for a smaller \( \chi^2 \) (for efficiency, the simulations were performed on multiple computers using a client/server application). As shown in figure A.1, when several simulations were started with different initial values, they converged to the same minimum.

Using a grid search, we were able to map out the \( \chi^2 \) for many different sets of parameters. The grid search has a serious drawback, however: as the number of parameters \( n \) that determine the model increase, the number of points necessary for a grid also increases as \( 2^n + 1 \). This method therefore becomes inefficient for large numbers of parameters.

### A.2 Levenberg-Marquardt Algorithm

The Levenberg-Marquardt algorithm is another approach to finding the best set of parameters for the model \( f(x_i; \Theta) \). The algorithm is only briefly described here; for more detail the reader could see the chapter on data modeling in *Numerical Methods* [97]. Starting from equation A.1, we define the log-likelihood function \( \mathcal{L} = -\ln L(\Theta) \). We seek to minimize the log-likelihood function with respect to each parameter \( \theta_i \).

The gradient of \( \mathcal{L} \) with respect to the parameters \((\theta_1, \ldots, \theta_N)\) is:

\[
[b]_i \equiv -\frac{\delta \mathcal{L}}{\delta \theta_i} = 2 \sum_{k=1}^{N} \frac{\delta f(x_k; \Theta)}{\delta \theta_i} \frac{(y_k - f(x_k; \Theta))}{\sigma_k}
\]  

(A.3)

and is equal to zero where \( \mathcal{L} \) is minimized. Also, by taking another partial derivative
we arrive at the matrix $M^{-1}$, with matrix elements:

$$[M^{-1}]_{ij} \equiv \frac{\delta^2 L}{\delta \theta_i \delta \theta_j} = 2 \sum_{k=1}^{N} \frac{\delta^2 f(x_k; \Theta) \ 1}{\delta \theta_i \delta \theta_j \sigma_k^2}$$

(A.4)

Note that these expressions apply for gaussianly distributed random variables. Then, we have a set of linear equations that may be used to iteratively solve for the best values of the parameters:

$$\sum_{i=1}^{n} M^{-1} \delta \theta_i = b$$

(A.5)

The values $\delta \theta_i$ are used to constantly refine the model parameters $(\theta_1, \ldots, \theta_n)$ until a precision limit is reached.

If the initial model parameters $\Theta_{\text{initial}}$ are far from the minimum of $L$, equation A.5 is not very efficient at finding the minimum due to correlations between variables. The Levenberg-Marquardt algorithm therefore includes one more ingredient: the diagonal matrix elements of the matrix $M^{-1}$ are multiplied by $(1 + \lambda)$ to form the new matrix $(M^{-1})'$. When $\lambda$ is large, the matrix is dominated by the diagonal elements, and individual parameters are minimized without regard to correlations. As $\lambda$ becomes small, the correlations become more important.

The Levenberg-Marquardt algorithm has one more important benefit: as suggested by the name, the matrix $M^{-1}$ can be inverted to produce the error matrix $M$:

$$M = \begin{pmatrix}
\sigma_{\theta_1}^2 & \text{cov}(\theta_1, \theta_2) & \cdots \\
\text{cov}(\theta_1, \theta_2) & \sigma_{\theta_2}^2 & \\
\vdots & & \ddots
\end{pmatrix}$$

(A.6)

The diagonal elements of the error matrix correspond to the uncertainties in each parameter, and the off-diagonal elements correspond to correlations between measured
variables. For example, in chapter 5, Doppler broadened peaks were modeled with Monte Carlo simulations. The parameters $\Theta$ that determined the shape of the model included half-lives of $^{10}$Be excited states, branching ratios between $\gamma$-rays or neutrons, and energies of neutron-emitting states in $^{11}$Be. In some cases, two parameters had a similar effect on the shape of the simulation; these correlations appear in the matrix $M$ as off-diagonal elements.