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ELECTROMAGNETIC SCATTERING  
FROM CONDUCTORS IN A CONDUCTIVE  
HALF-SPACE NEAR A GROUNDED CABLE OF FINITE LENGTH

By

Jorge O. Parra

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A Thesis submitted to the Faculty and Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Geophysics.

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## ABSTRACT

The problem of determining the electromagnetic fields of a harmonically excited grounded cable of finite length in the presence of a conducting rectangular body is reduced to the solution of an integral equation. This equation is reduced to a matrix equation, and is solved for the electric field in the inhomogeneity.

Because the source is finite and the inhomogeneity is two-dimensional, the volume integral of electric field scattering is reduced to a surface integral by using a Fourier transform method. The electromagnetic fields in the conductor and at the surface of the earth are theoretically developed and calculated numerically in the wave number domain. The magnetic field response of a two-dimensional thin dike is computed and plotted in the space domain. Horizontal location of the conductor is best determined through measurements of vertical and horizontal magnetic field amplitude rather than phase of the magnetic field relative to current in the source.

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TABLE OF SYMBOLS

$\bar{E}$	electric field vector
$\bar{J}_s$	electric current density vector
I	electric current
$\bar{H}$	magnetic field vector
$\epsilon$	electrical permeativity
$\sigma_1$	electrical conductivity of half-space
$\sigma_2$	electrical conductivity of the conductor
$\mu$	magnetic permeability
$\omega$	frequency in radians/second
$k_y$	wave number in the Y-direction
$\gamma$	propagation constant
$\epsilon_0$	dielectric constant of free space
$\mu_0$	magnetic permeability of free space
q	$q = (x^2 + z^2)^{\frac{1}{2}}$
r	$r = (y^2 + x^2)^{\frac{1}{2}}$
R	$R = (x^2 + y^2 + z^2)^{\frac{1}{2}}$
$E_x, E_y, E_z$	orthogonal Cartesian components of $\bar{E}$
$H_x, H_y, H_z$	orthogonal Cartesian components of $\bar{H}$
$\bar{K}$	Green's tensor
$K_{xx}, K_{xy}, K_{xz},$ $K_{yx}, K_{yy}, K_{yz},$ $K_{zx}, K_{zy}, K_{zz}$	elements of the Green's tensor.

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$$u_1 \quad u_1 = (\lambda^2 + \gamma_1^2)^{\frac{1}{2}}$$

$$u_{1y} \quad u_{1y} = (\gamma_1^2 + k_y^2)^{\frac{1}{2}}$$

$$v_1 \quad v_1 = (\lambda^2 - k_y^2)^{\frac{1}{2}}$$

$$\delta \quad \delta = \left( \frac{2}{\omega \mu_0 \sigma_1} \right)^{\frac{1}{2}}$$

$$\delta_2 \quad \delta_2 = \left( \frac{2}{\omega \mu_0 \sigma_2} \right)^{\frac{1}{2}}$$

$$u \quad u = (2i + g^2)^{\frac{1}{2}}$$

$$v \quad v = (g^2 - g_y^2)^{\frac{1}{2}}$$

$$u_y \quad u_y = (g_y^2 + 2i)^{\frac{1}{2}}$$

$$g \quad g = \delta \lambda$$

$$g_y \quad g_y = \delta k_y$$

$$\rho \quad \rho = q/\delta$$

$$\xi \quad \xi = (\sigma_2 - \sigma_1)$$

l length of the line source

$$L \quad l/\delta$$

N, P Sommerfeld's integrals

$$X \quad X = x/\delta$$

$$Y \quad Y = y/\delta$$

$$Z \quad Z = z/\delta$$

$$X' \quad X' = x'/\delta$$

$$Y' \quad Y' = y'/\delta$$

$$Z' \quad Z' = z'/\delta$$



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## INTRODUCTION

One of the primary limitations in the use of electromagnetic methods in mineral exploration has been the lack of theoretical solutions for electromagnetic fields scattered by conductive inhomogeneities in the earth.

Since the advent of high-speed digital computers, however, previously intractable electromagnetic scattering problems have begun to yield to numerical solutions.

Electromagnetic prospecting methods are designed to detect conducting inhomogeneities when a primary source is applied. The primary source used in EM techniques may be natural uniform field, i.e., an electric or magnetic field caused by natural phenomena on or in the earth, or an artificial field associated with currents artificially maintained in the subsurface by a finite source.

The conventional electromagnetic systems used in geophysical exploration are conductive systems and inductive systems. In the conductive systems the primary field is caused by a current flow in the earth through a wire grounded at each end. Inductive systems have a source field created by means of a coil or loops of wire on the ground.

A long grounded wire or Turam method is extensively used in the search for metallic sulfide orebodies. Only two components of the electromagnetic field from a long wire can be observed at the surface of a uniform earth: the parallel component of the electric field and the vertical component of the magnetic field.

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By in large, with the notable exception of Hohmann (1973), the theory of a fixed source and a inhomogeneity has been restricted to two-dimensional time harmonic analysis. The electromagnetic scattering problem with a conductor buried in a half-space have been solved for a line-source excitation or plane-wave by Parry (1971), Hohmann (1971), Swift (1971), Coggon (1971), Dey and Morrison (1973), and others. These solutions have been developed using integral equations, network analysis, and finite-element techniques.

The general problem of electromagnetic scattering by three-dimensional conductive bodies buried in the earth, which are excited by an arbitrary source, has been examined by Hohmann (1973).

It is the purpose of this thesis to investigate the electromagnetic field scattering by a subsurface two-dimensional conductive inhomogeneity, which is excited by a finite electric current element located at the surface of the earth. A solution for the electromagnetic field in the form of an integral equation is developed by a point-matching technique (Richmond, 1965, 1966), (Harrington, 1968), Hohmann (1971, 1973), and Dey and Morrison (1973), and numerical evaluation is performed.

The finite grounded cable can be also used for locating massive sulfide ore deposits, and geothermal systems. With this source all six components of the electromagnetic field can be observed at the surface of the earth.

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GENERAL CONSIDERATIONS

The subsurface electromagnetic fields of an electric horizontal dipole have been evaluated by Wait (1961) and Baños (1966). Recently, Wait and Hill (1973) have extended the theory of conduction from current dipole to the case of a grounded cable of finite length.

With respect to Cartesian coordinate system, the half-space  $Z < 0$  is assumed to be a homogeneous earth with dielectric constant  $\epsilon_1$ , magnetic permeability  $\mu_1$ , and conductivity  $\sigma_1$ , except for a two-dimensional inhomogeneity which is described by electrical parameters  $\epsilon_2$ ,  $\mu_2$ ,  $\sigma_2$ . Air with a dielectric constant  $\epsilon_0$  and permeability  $\mu_0$  occupies the space  $Z > 0$ . The magnetic permeability in the lower half-space and the conductor is assumed to be the same as that of free-space. The exciting dipole source of length  $dl$  carrying an electric current  $I$  is assumed to have a time dependence  $e^{+j\omega t}$ .

THEORYFormulation of the Integral Equation

If  $\bar{E}^i$  is the incident electric field due to impressed electric currents  $\bar{J}_i$ , and  $\bar{E}^s$  is the scattered electric field due to polarization or scattering current  $\bar{J}_s$ , which exists only in the inhomogeneity, then the total electric field vector  $\bar{E}$  generated by a current dipole in the presence of the conductor in the half-space is given by the sum of the incident and scattered intensities,

$$\bar{E} = \bar{E}^i + \bar{E}^s$$

The scattered electric field  $\bar{E}^s$  is obtained multiplying the appropriated dyadic Green's function by  $\bar{J}_s$  and integrate over the volume  $V'$  of the inhomogeneity (Hohmann, 1973). Thus

$$\bar{E}^s = \int_{V'} \bar{K}(x, y, z; x', y', z') \bar{J}_s(x', y', z') dV'$$

where the scattering current density from the inhomogeneity must be

$$\bar{J}_s = [(\sigma_2 - \sigma_1) + j\omega(\epsilon_2 - \epsilon_1)] \bar{E}$$

The kernel  $\bar{K}$  is derived in Appendix A, and is the dyadic Green's function relating the electric field at a point  $(X, Y, Z)$  to a current element at a point  $(X', Y', Z')$ , as shown in Figure 1b.

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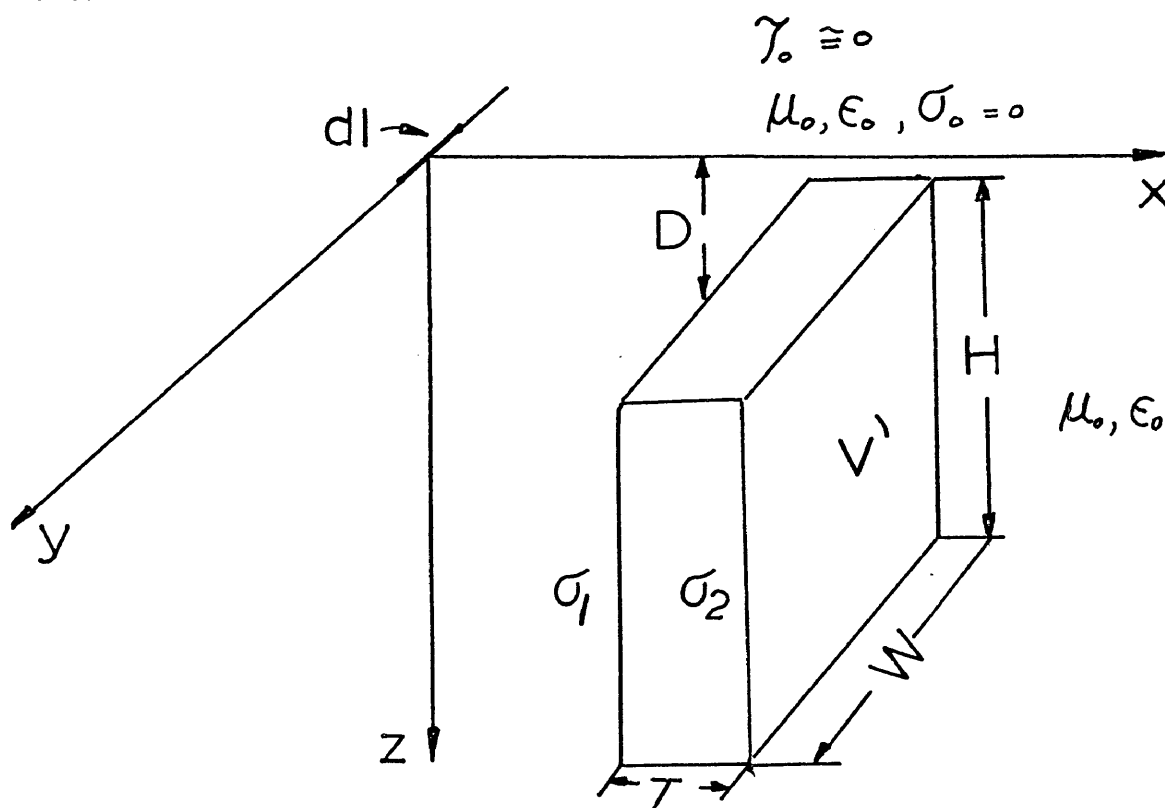


Figure 1a. Orientation of the electric dipole-source and the rectangular conductive inhomogeneity.

The conductivity  $\sigma_1$  of the earth surrounding the buried body is homogeneous; in addition, the conductivity  $\sigma_2$  of the body is considered to be constant throughout the body. Both conductivities are taken to be linear and isotropic.

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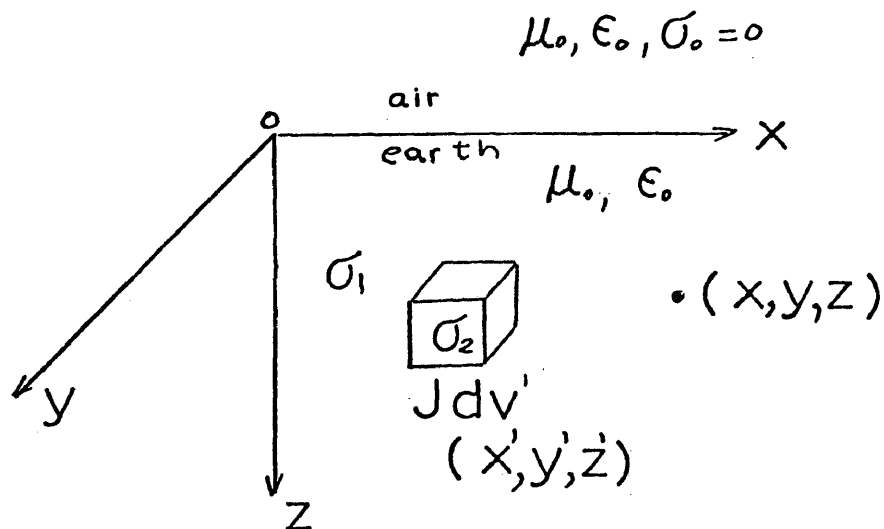


Figure 1b. Green's function geometry

The total unknown electric field vector anywhere can be expressed by the following integral equation,

$$\bar{E}(x, y, z) = \bar{E}^i(x, y, z) + \int_{V'} [(\sigma_2 - \sigma_1) + j\omega(\epsilon_2 - \epsilon_1)] \bar{K}(x, y, z; x', y', z') \cdot \bar{E}(x', y', z') dx' dy' dz' \quad (1)$$

where  $\bar{E}(x', y', z')$  is the excitation field at the surface of the inhomogeneity in the absence of that inhomogeneity. The electric constant in the conductor and in the half-space is assumed to be that of free-space ( $\epsilon_1 = \epsilon_2 = \epsilon_0$ ), so that the scattering current is related to the total electric field scattering by the constitutive relationship

$$J_s = (\sigma_2 - \sigma_1) \bar{E}(x', y', z')$$

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With  $\xi = (\sigma_2 - \sigma_1)$ , the integral equation is given by

$$\bar{E}(X, Y, Z) = \bar{E}^i(X, Y, Z) + \xi \int_{-w/2}^{w/2} \int_{X_0 - T/2}^{X_0 + T/2} \int_D^H \bar{K}(X, Y, Z; X', Y', Z') \bar{E}(X', Y', Z') dX' dY' dZ' \quad (2)$$

All distances have been normalized with respect to the plane wave skin depth, and will be denoted by capital letters, i.e.,

$$X = x/\delta, \quad Y = y/\delta, \quad Z = z/\delta$$

$$X' = x'/\delta, \quad Y' = y'/\delta, \quad Z' = z'/\delta, \quad \text{etc.}$$

$$\delta = \left( \frac{2}{\omega \mu_0 \sigma_1} \right)^{1/2}$$

Equation (2), where  $\bar{E}^i$  and  $\bar{K}$  are given functions and where  $\xi$  and the limits of integration are constant, is known as a Fredholm integral equation of the second kind.

The function  $\bar{E}$  is to be determined. The specified vector function  $\bar{K}$  which depends upon the spacial variable  $(X, Y, Z)$  as well as the auxiliary variable  $(X', Y', Z')$ , is the kernel of the integral equation.

A conducting rectangular body of infinite length along strike is considered. The source is finite, and the current is not uniform throughout the body. Under this condition the integral equation becomes



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$$E(X, Y, Z) = E^i(X, Y, Z) + \xi \int_{-\infty}^{\infty} \int_{X_a - T/2}^{X_a + T/2} \int_D^H \bar{K}(X, Y, Z; X', Y', Z') \bar{E}(X', Y', Z') dx' dy' dz' \quad (3)$$

if we define

$$\bar{F}(X, X'; Z, Z'; Y) = \int_{-\infty}^{\infty} \bar{K}(X, Y, Z; X', Y', Z') \bar{E}(X', Y', Z') dY \quad (4)$$

Then (3) becomes

$$\bar{E}(X, Y, Z) = \bar{E}^i(X, Y, Z) + \xi \int_{X_a - T/2}^{X_a + T/2} \int_D^H \bar{F}(X, X'; Z, Z'; Y) dx' dz' \quad (5)$$

The Fourier transform of the integral (4) is

$$F(X, X'; Z, Z'; g_y) = \bar{K}(X, X'; Z, Z'; g_y) \bar{E}(X', g_y, Z') \quad (6)$$

where  $g_y$  is the wave number in the y direction.

After Fourier transforming expression (5), and after substituting relation (6), the integral equation in the one-dimensional wave number domain becomes

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$$\bar{E}(X, g_y, Z) = \bar{E}^i(X, g_y, Z) + \xi \int_{X_a - T/2}^{X_a + T/2} \int_D^H \bar{K}(X, X'; Z, Z'; g_y) \bar{E}(X', g_y, Z') dX' dZ'$$

(7)

Consideration of the mutual coupling effect between the electric components yields the following integral equation in matrix form

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} E_x^i \\ E_y^i \\ E_z^i \end{pmatrix} + \xi \int_{X_a - T/2}^{X_a + T/2} \int_D^H \begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} dX' dZ'$$

(8)

The elements of the dyadic matrix may be given a physical interpretation, i.e.,  $\bar{K}_{ij}$  is the  $i$ th component of the electric field  $\bar{E}$  at  $(X, g_y, Z)$  due to a  $J$ -direct current element.

The dyadic elements  $\bar{K}_{ij}$  have been evaluated in the space domain by Hohmann (1973) and are defined in the one-dimensional wave number domain in Appendix B.

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Dipole in the Presence of a Conducting Half-Space

The current dipole of infinitesimal length  $dl$  and oriented in the  $Y$  direction, is located at the origin of a Cartesian coordinate system. The components of the incident electric field may be written (Wait, 1961):

$$dE_x = -\frac{I dl}{2\pi\sigma_1} \frac{\partial^3 N}{\partial x \partial y \partial z} \quad (9)$$

$$dE_y = -\frac{I dl}{2\pi\sigma_1} \left( -\frac{\partial^3 N}{\partial x^2 \partial z} + \frac{\partial^2 P}{\partial z^2} \right) \quad (10)$$

$$dE_z = -\frac{I dl}{2\pi\sigma_1} \frac{\partial^2 P}{\partial z \partial y} \quad (11)$$

where  $N$  and  $P$  are the Sommerfeld (1926) integrals given by

$$N = \int_0^{\infty} \frac{e^{-\sqrt{\lambda^2 + \gamma_1^2} z}}{\sqrt{\lambda^2 + \gamma_1^2}} J_0(\lambda r) d\lambda \quad (12)$$

$$P = \int_0^{\infty} \frac{e^{-\sqrt{\lambda^2 + \gamma_1^2} z}}{\sqrt{\lambda^2 + \gamma_1^2}} J_0(\lambda r) \lambda d\lambda \quad (13)$$

Introducing the attenuation factor

$$u_1 = \left( \lambda^2 + \gamma_1^2 \right)^{1/2} \xrightarrow{\lambda \rightarrow 0} \gamma_1$$

where the determination of the square root is such that  $\text{Re} \{ u_1 \} > 0$  over the entire path of integration,  $0 \leq \lambda < \infty$ .

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The integral  $N$  is equal to

$$I_0 \left[ \frac{\gamma_1}{2} (R+z) \right] K_0 \left[ \frac{\gamma_1}{2} (R-z) \right] \quad , \text{ Foster (1931)}$$

where  $I_0$  and  $K_0$  are modified Bessel functions of order zero.

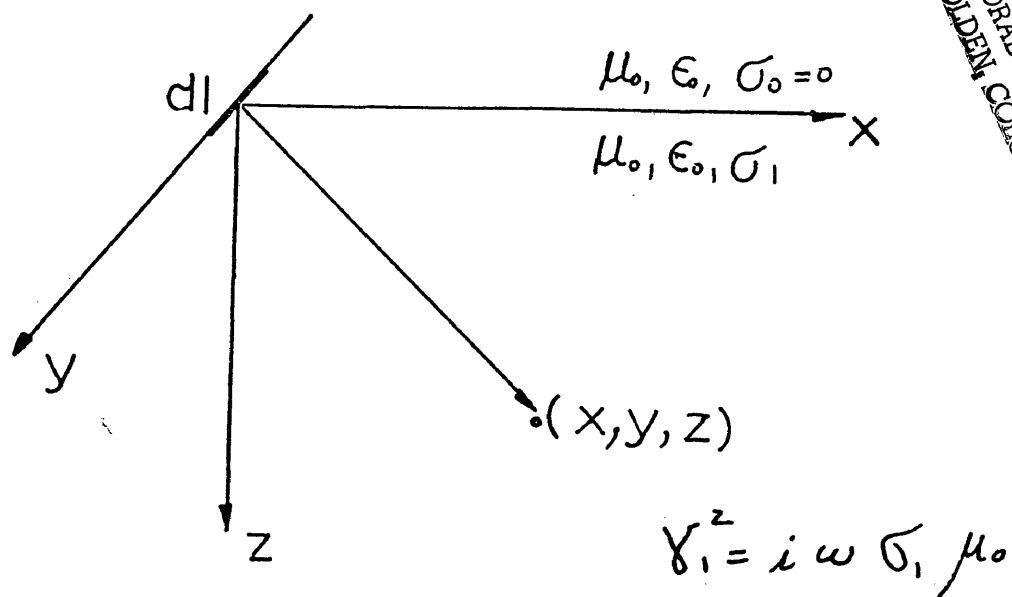


Figure 2. A dipole source on a homogeneous half-space

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The Fourier Integral of the Incident Electric Fields of Current Dipole

Consider first the Sommerfeld integrals N and P in one-dimensional wave number domain.

$$P_1(k_Y) = \int_{-\infty}^{\infty} \frac{e^{-\gamma_1 R}}{R} e^{-i k_Y y} dy \quad (14)$$

and

$$N_1(k_Y) = \int_{-\infty}^{\infty} \int_0^{\infty} \frac{e^{-u|z|}}{u} J_0(\lambda r) e^{-i k_Y y} d\lambda dy \quad (15)$$

where

$$k_Y = q_Y / \delta$$

$$r = (x^2 + y^2)^{1/2}$$

The kernel of the integral (14) is an even function of  $y$ . Thus

$$P_1(k_Y) = 2 \int_0^{\infty} \frac{e^{-\gamma_1 R}}{R} \cos(k_Y y) dy$$

This integral has been evaluated by Erdelyi (1954, Vo. I, P. 17),

$$P_1(k_Y) = 2 K_0(u_{1Y} q) \quad (16)$$

where

$$u_{1Y} = (\gamma_1^2 + k_Y^2)^{1/2}$$

$$q = (x^2 + z^2)^{1/2}$$

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The expression (15) can be rewritten as

$$N_1(k_Y) = \int_0^{\infty} \frac{e^{-u|z|}}{u|z|} \left[ \int_{-\infty}^{\infty} J_0(\lambda r) e^{-i k_Y y} dy \right] d\lambda$$

$$r = (x^2 + y^2)^{1/2}$$

Again, the kernel of this integral is an even function of  $y$ ; thus:

$$N_1(k_Y) = 2 \int_0^{\infty} \frac{e^{-u|z|}}{u|z|} \left[ \int_0^{\infty} J_0(\lambda r) \cos(k_Y y) dy \right] d\lambda$$

From Erdelyi (1954, Vol. I, P. 55) this integral becomes

$$N_1(k_Y) = 2 \int_{k_Y}^{\infty} \frac{e^{-u|z|}}{u|z|} \frac{\cos [x(\lambda^2 - k_Y^2)^{1/2}]}{(\lambda^2 - k_Y^2)^{1/2}} d\lambda,$$

(17)

Then expressions (9), (10), and (11) for  $dE_x$ ,  $dE_y$ , and  $dE_z$  in the one-dimensional wave number domain becomes

$$dE_x = -\frac{i g_Y I dL}{\pi \sigma_1} \int_{g_Y}^{\infty} e^{-uZ} \sin(XV) dg$$

(18)

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$$dE_Y = - \frac{I dL}{\pi \sigma_1} \left[ - \int_{g_y}^{\infty} e^{-uz} \nu \cos(x\nu) dg \right. \\ \left. + u_y^2 K_0(u_y \rho) \frac{z^2}{\rho^2} + \frac{u_y K_1(u_y \rho)}{\rho^3} (z^2 - x^2) \right] \quad (19)$$

$$dE_z = \frac{I dL}{\pi \sigma_1} g_y i u_y \frac{z}{\rho} K_1(u_y \rho) \quad (20)$$

where

$$g = \lambda \delta, \quad g_y = k_y \delta$$

$$u = (g^2 + 2i)^{1/2}$$

$$u_y = (g_y^2 + 2i)^{1/2}$$

$$\nu = (g^2 - g_y^2)^{1/2}$$

$$\rho = r/\delta$$

$$L = 1/\delta$$

and the distances are now measured in skin depths in the half-space.

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Subsurface Electromagnetic fields of a Grounded Cable of Finite Length

The geometry of the cable of length  $2l$  situated on the half-space carrying a current  $Ie^{j\omega t}$ , and an observer located at the point  $(X,Y,Z)$  are shown in Figure 3.

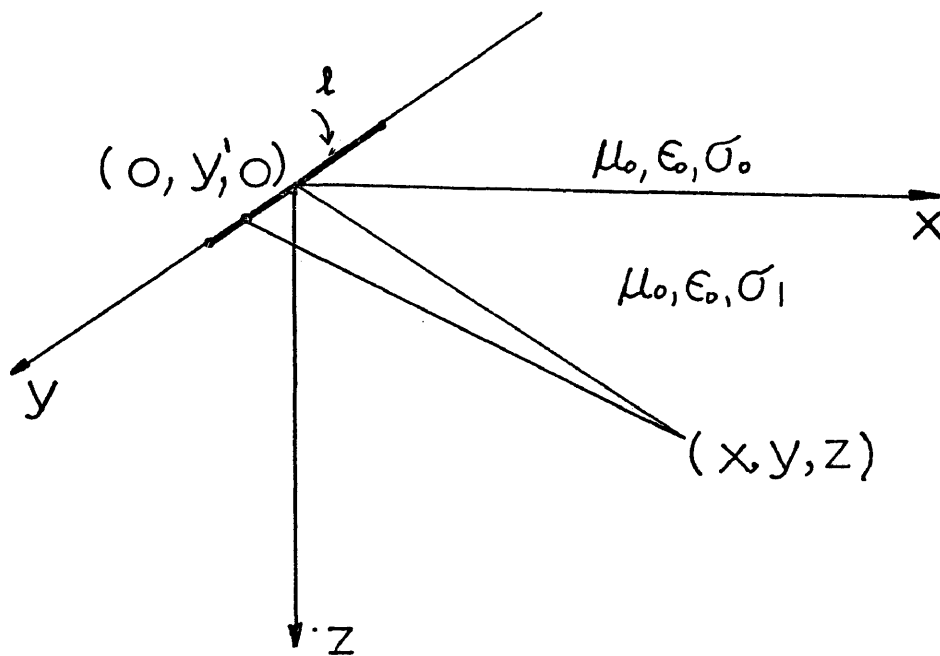


Figure 3. Finite line source on a homogeneous half-space.



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The electric field components (Wait, 1971) obtained by integrating  $dE_x$ ,  $dE_y$ , and  $dE_z$  from equations (18), (19) and (20) over the length  $2l$ :

$$E_x = -\frac{I}{2\pi\sigma_1} \int_{-l}^l \frac{\partial^3 N}{\partial x \partial y \partial z} dy' \quad (21)$$

$$E_y = -\frac{I}{2\pi\sigma_1} \int_{-l}^l \left( \frac{\partial^2 P}{\partial z^2} - \frac{\partial^3 N}{\partial x^2 \partial z} \right) dy' \quad (22)$$

$$E_z = -\frac{I}{2\pi\sigma_1} \int_{-l}^l \frac{\partial^2 P}{\partial y \partial z} dy' \quad (23)$$

The Fourier transformation of the Sommerfeld integrals  $N$  and  $P$  in this case are given by

$$P_2(k_y) = \int_{-\infty}^{\infty} \frac{e^{-\gamma_1 R}}{R} e^{-ik_y y} dy \quad (24)$$

and

$$N_2(k_y) = \int_{-\infty}^{\infty} \int_0^{\infty} \frac{e^{-u|z|}}{u} J_0(\lambda r) e^{-ik_y y} d\lambda dy \quad (25)$$

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In order to evaluate the integrals (24) and (25) it is convenient to effect a change of variables,

$$y = y' + a$$

By differentiation

$$dy = da$$

The integrals (24) and (25) become

$$P_2(k_Y) = e^{-i y' k_Y} \int_{-\infty}^{\infty} \frac{e^{-\gamma_1 R}}{R} e^{-i k_Y a} da \quad (26)$$

and

$$N_2(k_Y) = e^{-i y' k_Y} \int_0^{\infty} \int_{-\infty}^{\infty} \frac{e^{-u|z}}{u|} J_0(\lambda r) e^{-i k_Y a} da d\lambda \quad (27)$$

Since the integrand of the integrals (26) and (27) is an even function of  $a$ , the complex integral transform can be written as the cosine transform; thus

$$P_2(k_Y) = 2 \int_0^{\infty} \frac{e^{-\gamma_1 R}}{R} \cos(k_Y a) da$$

and

$$N_2(k_Y) = 2 \int_0^{\infty} \int_0^{\infty} \frac{e^{-u|z}}{u|} J_0(\lambda r) \cos(k_Y a) da d\lambda$$

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From equations (16) and (17), the Fourier transformation of the Sommerfeld integrals N and P in this case are given by

$$P_2(k_Y) = 2 e^{-i k_Y y'} K_0(u_Y \rho) \quad (28)$$

and

$$N_2(k_Y) = 2 e^{-i k_Y y'} \int_{k_Y}^{\infty} \frac{e^{-u z}}{u l} \frac{\cos(x v l)}{v l} d k_Y, \quad (29)$$

After normalization of the distances with respect to the plane-wave skin depth  $\delta$ , the integrals (28) and (29) become

$$P_2(q_Y) = 2 e^{-i q_Y Y'} K_0(u_Y \rho) \quad (30)$$

and

$$N_2(q_Y) = 2 e^{-i q_Y Y'} \int_{q_Y}^{\infty} \frac{e^{-u z}}{u} \frac{\cos(x v)}{v} d q, \quad (31)$$

Then the incident electric field components due to a grounded cable in the wave number domain are:

$$E_x = \frac{-2 i q_Y I}{\pi \sigma_i \delta} \frac{\sin q_Y L}{q_Y} \int_{q_Y}^{\infty} e^{-u z} \sin(x v) d q \quad (32)$$

$$E_Y = \frac{2 I}{\pi \sigma_i \delta} \frac{\sin(q_Y L)}{q_Y} \left[ - \int_{q_Y}^{\infty} \frac{e^{-u z}}{v} \cos(x v) d q + \frac{u_Y}{\rho^3} K_1(u_Y \rho) (Z^2 - X^2) \right] \quad (33)$$

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$$E_z = -\frac{2I}{\pi \sigma_1 \delta} \frac{\sin(q_y L)}{q_y} i q_y u_y \frac{z}{\rho} K_1(u_y \rho) \quad (34)$$

where the factor  $2 \frac{\sin q_y L}{q_y}$  is obtained for the evaluation of the integral

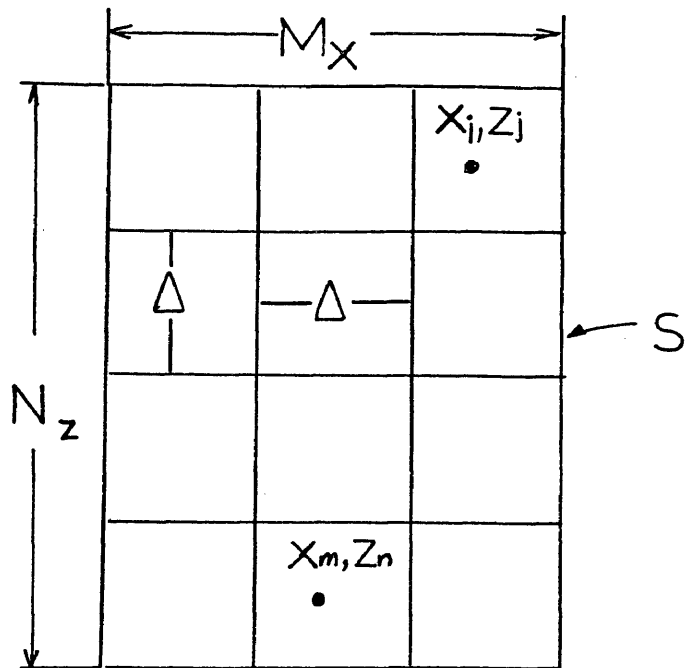
$$\int_{-L}^L e^{-i q_y y'} dy'.$$

### Numerical Solution of the Integral Equation

A point-matching method is used to solve the integral equation with pulse functions taken as subsectional bases. A function which exists over only one subinterval is the pulse function, (Harrington, 1968).

A simple way to obtain approximate solutions is to require that the integral equation be satisfied at discrete points in the cross-sectional area  $S'$ . The area is divided into subsections  $M_x$  by  $N_z$  squares cells of side dimension  $\Delta$ , as shown in Figure 4a. The cells are small enough so that the electric field intensity is nearly uniform in each cell. The electric field scattering in each cell is initially considered to be an unknown quantity and a system of linear equations is obtained by enforcing at the center  $(x_m, y, z_n)$  of each square the condition that the total field is equal to the sum of the incident and scattered fields. The solution of the linear system of equations is the scattered electric field in the conductor.

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Figure 4a. Division of body into  $M_x \times N_z$  cells for numerical solution.

The integral equation (7) by using the point-matching technique is approximated as

$$\begin{pmatrix} E_x(X_i, q_Y, Z_j) \\ E_Y(X_i, q_Y, Z_j) \\ E_z(X_i, q_Y, Z_j) \end{pmatrix} = \begin{pmatrix} E_x^{inc}(X_i, q_Y, Z_j) \\ E_Y^{inc}(X_i, q_Y, Z_j) \\ E_z^{inc}(X_i, q_Y, Z_j) \end{pmatrix} +$$

$$\sum_{m=1}^{M_x} \sum_{n=1}^{N_z} \bar{G}_{mn} \begin{pmatrix} E_x(X_m, q_Y, Z_n) \\ E_Y(X_m, q_Y, Z_n) \\ E_z(X_m, q_Y, Z_n) \end{pmatrix}$$

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where the dyadic matrix  $\bar{G}_{mn}$  is given by the sum of the non-singular dyadic function plus the singular dyadic function. The non-singular part is assumed to be constant over each cell, and the integration is carried out only for the singular part because the distances between cells are small.

The non-singular Green's function  $\bar{G}_{smn}$  is given by

$$\bar{G}_{smn} = \Delta^2 \begin{pmatrix} K_{sxx} & K_{sxy} & K_{sxz} \\ K_{syx} & K_{syy} & K_{syz} \\ K_{szx} & K_{szy} & K_{szz} \end{pmatrix} \quad (36)$$

and the singular Green's function  $\bar{G}_{pmn}$  is given by

$$\bar{G}_{pmn} = \int_{S'} \begin{pmatrix} K_{pxx} & K_{pxy} & K_{pxz} \\ K_{pyx} & K_{pyy} & K_{pyz} \\ K_{pzx} & K_{pzy} & K_{pzz} \end{pmatrix} dS' \quad (37)$$

Where

$$K_s = K_s (X_i - X_m ; Z_j + Z_n) ; \quad i \neq m, j \neq n$$

$$K_p = K_p (X_i - X' ; Z_j - Z') ; \quad \begin{matrix} i = m, \\ j = n \end{matrix}$$

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In order to reduce computation time each square cell is replaced by an equivalent circular cell of the same cross-sectional area (Richmond, 1965, 1966).

The expression (35) may be simplified in the following way:

$$\begin{pmatrix} {}^{ij}E_x \\ {}^{ij}E_y \\ {}^{ij}E_z \end{pmatrix} = \begin{pmatrix} E_x^{inc} \\ E_y^{inc} \\ E_z^{inc} \end{pmatrix} + \sum_{mn=1}^{MN} \begin{pmatrix} {}^{ij}K_{xx}^{mn} & {}^{ij}K_{xy}^{mn} & {}^{ij}K_{xz}^{mn} \\ {}^{ij}K_{yx}^{mn} & {}^{ij}K_{yy}^{mn} & {}^{ij}K_{yz}^{mn} \\ {}^{ij}K_{zx}^{mn} & {}^{ij}K_{zy}^{mn} & {}^{ij}K_{zz}^{mn} \end{pmatrix} \begin{pmatrix} E_x^{mn} \\ E_y^{mn} \\ E_z^{mn} \end{pmatrix}$$

(38)

$$ij = (i-1) N_z + j$$

where

$$mn = (m-1) N_z + n$$

$$MN = M_x \cdot N_z$$

and

$${}^{ij}K^{mn} = \Delta^2 K_s (X_i - X_m, q_r, Z_j + Z_n)$$

$$+ \int_{X_m - \Delta/2}^{X_m + \Delta/2} \int_{Z_n - \Delta/2}^{Z_n + \Delta/2} K_p (X_i - X', q_r, Z_j - Z') \cdot dx' dz'$$

$$ij = 1, 2, 3, \dots, MN$$

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Equation (38) can be rewritten as a system of linear equations.

$$\begin{pmatrix} \bar{G}_{xx} & \bar{G}_{xy} & \bar{G}_{xz} \\ \bar{G}_{yx} & \bar{G}_{yy} & \bar{G}_{yz} \\ \bar{G}_{zx} & \bar{G}_{zy} & \bar{G}_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = - \begin{pmatrix} E_x^{inc} \\ E_y^{inc} \\ E_z^{inc} \end{pmatrix}$$

(39)

where  $\bar{G}_{xx}$ ,  $\bar{G}_{xy}$ , etc., are submatrices with MN by MN elements, the vector  $(E_x, E_y, E_z)$  is the unknown electric field in the conductor, and  $(E_x^{inc}, E_y^{inc}, E_z^{inc})$  is the incident electric field vector.

The dyadic matrix  $\bar{G}$  is given by

$$\begin{pmatrix} ij G_{xx}^{mn} & ij G_{xy}^{mn} & ij G_{xz}^{mn} \\ ij G_{yx}^{mn} & ij G_{yy}^{mn} & ij G_{yz}^{mn} \\ ij G_{zx}^{mn} & ij G_{zy}^{mn} & ij G_{zz}^{mn} \end{pmatrix} = \text{(next page)}$$



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$$\begin{pmatrix} \sum_{ij}^{ij} K_{xx}^{mn} - \delta_{mn}^{ij} & \sum_{ij}^{ij} K_{xy}^{mn} - \delta_{mn}^{ij} & \sum_{ij}^{ij} K_{xz}^{mn} - \delta_{mn}^{ij} \\ \sum_{ij}^{ij} K_{yx}^{mn} - \delta_{mn}^{ij} & \sum_{ij}^{ij} K_{yy}^{mn} - \delta_{mn}^{ij} & \sum_{ij}^{ij} K_{yz}^{mn} - \delta_{mn}^{ij} \\ \sum_{ij}^{ij} K_{zx}^{mn} - \delta_{mn}^{ij} & \sum_{ij}^{ij} K_{zy}^{mn} - \delta_{mn}^{ij} & \sum_{ij}^{ij} K_{zz}^{mn} - \delta_{mn}^{ij} \end{pmatrix}$$

with  $\delta_{mn}^{ij} = 0$  if  $ij \neq mn$

and  $\delta_{mn}^{ij} = 1$  if  $ij = mn$

(40)

The electromagnetic fields observed at the surface of the earth are obtained by integrating the half-space kernel functions of the scattered electric field over each cross-sectional area of the rectangular body. The electric field vector at the point  $(X, y, Z)$  outside of the conductor is given by the following expression,

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} E_x^{inc} \\ E_y^{inc} \\ E_z^{inc} \end{pmatrix} + \Delta^2 \sum_S \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} \begin{pmatrix} \bar{K}_{xx} & \bar{K}_{xy} & \bar{K}_{xz} \\ \bar{K}_{yx} & \bar{K}_{yy} & \bar{K}_{yz} \\ \bar{K}_{zx} & \bar{K}_{zy} & \bar{K}_{zz} \end{pmatrix} \begin{pmatrix} \bar{E}_x^s \\ \bar{E}_y^s \\ \bar{E}_z^s \end{pmatrix}$$

(41)

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where  $\bar{E} = E(X, g_y, Z)$  is the electric field vector anywhere.

$\bar{E}^{inc} = E^{inc}(X, g_y, Z)$  is the incident electric field vector at a given point.

$\bar{E}^S = \bar{E}^S(X_m, g_y, Z_n)$  is the unknown scattering electric field in the inhomogeneity.

$\bar{K}_{ij}$  is the sum of a singular element plus a nonsingular element.

### Integration of the Primary Part of the Dyadic Matrix

The non-singular dyadic Green's function is assumed to be uniform in each cell. The numerical integration is carried out for the singular dyadic Green's function over each cell, because it is quite sensitive to the small distances between cells (Hohmann, 1971).

The integration over each singular cell is obtained analytically. It is found that the surface integral over the elements situated only on the diagonal of the dyadic matrix are non-zero (Appendix C).

The elements of the primary dyadic matrix in the wave number are defined in Appendix B.

Integration of the singular dyadic Green's function in the wave number domain over the surface of the body yields,

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$$\int_{S'} K_{PXX} dS' = \left( \frac{u_Y^2}{2} + 2i \right) Q$$

$$\int_{S'} K_{PYY} dS' = \left( g_Y^2 + 2i \right) Q$$

$$\int_{S'} K_{PZZ} dS' = \left( \frac{u_Y^2}{2} + 2i \right) Q$$

(42)

where for the singular cell the function (Hohmann, 1971),

$$Q = \frac{2\pi}{u_Y} \left\{ -a K_1(u_Y a) + \frac{1}{u_Y} \right\}$$

(42)

and  $u_Y = (g_Y^2 + 2i)^{1/2}$ ,  $K_1$  is modified Bessel function of order unity, and "a" is the radius of the equivalent cell, given by

$$a = \Delta / (\pi)^{1/2}$$

The value of Q over non-singular cells is given by (Hohmann, 1971).

$$Q = -\frac{2\pi}{u_Y} K_0(u_Y \rho_0) I_1(u_Y a)$$

(43)

Where  $I_1$  is the modified Bessel function of the first kind of order unity,  $K_0$  is the modified Bessel function of second kind of order zero, and

$$\rho_0 = \left[ (X_i - X_m)^2 + (Z_j - Z_n)^2 \right]^{1/2}$$

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The Magnetic Field Components at the Surface of the Earth as a Function of the Scattering Electric Field in a Three-dimensional Conductor.

The integral equation (2) can be approximated by a finite summation.

$$\bar{E}(x, y, z) = \bar{E}^{inc}(x, y, z) + (\sigma_2 - \sigma_1) \sum_{n=1}^{N_z} \sum_{m=1}^{M_x} \int_{V'} \bar{E}(x', y', z') \bar{K}(x, y, z; x', y', z') dV' \quad (44)$$

where the dyadic Green's function

$$\bar{K}(x, y, z; x', y', z') = -\frac{\gamma_1^2}{\sigma_1} \bar{K}(x, y, z; x', y', z') - \frac{1}{\sigma_1} \nabla \nabla' \bar{K}(x, y, z; x', y', z')$$

is given by (Van Blandel, 1961) with

$$\bar{K}(x, y, z; x', y', z') = \frac{e^{-\gamma_1 R}}{4\pi R}$$

and  $R = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2}$ ,  $\gamma_1^2 = i\omega\mu_0\sigma_1$

The magnetic field vector of an arbitrary field point in terms of the electric field in the body can be calculated applying Faraday's law to equation (44). Thus

$$\bar{H}(x, y, z) = \bar{H}^{inc}(x, y, z) + (\sigma_2 - \sigma_1) \sum_{m=1}^{N_z} \sum_{n=1}^{M_x} E_{mn} \int_{V'} \nabla \times \bar{K}(x, y, z; x', y', z') dV' \quad (45)$$

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The dyadic Green's function to compute the magnetic field at any point in the space domain is given in Appendix B,

$$\begin{pmatrix} \mathcal{T}_2 + \frac{e^{-\gamma_1 R_0}}{4\pi R_0} & 0 & 0 \\ 0 & \mathcal{T}_2 + \frac{e^{-\gamma_1 R_0}}{4\pi R_0} & 0 \\ 0 & 0 & \frac{e^{-\gamma_1 R_0}}{4\pi R_0} - \frac{e^{-\gamma_1 R}}{4\pi R} \end{pmatrix} \quad (46)$$

The curl of the dyadic matrix (46) is

$$\nabla \times \bar{\bar{K}} = \begin{pmatrix} \frac{\partial (P_0 - P)}{\partial y} - \frac{\partial (\mathcal{T}_2 + P_0)}{\partial z} & 0 & 0 \\ 0 & -\frac{\partial (P_0 - P)}{\partial x} + \frac{\partial (\mathcal{T}_2 + P_0)}{\partial z} & 0 \\ 0 & 0 & \frac{\partial (\mathcal{T}_2 + P_0)}{\partial y} - \frac{\partial (\mathcal{T}_2 + P_0)}{\partial x} \end{pmatrix} \quad (47)$$

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Thus the magnetic field components anywhere in terms of the scattered electric field in the body are:

$$H_x = H_x^{inc} + (\sigma_2 - \sigma_1) \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} \left[ \int_{V'} \left\{ -\frac{\partial}{\partial z} (\mathcal{T}_z + P_0) \right\} dV' \right] \cdot E_{mnx} \quad (48)$$

$$H_y = H_y^{inc} + (\sigma_2 - \sigma_1) \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} \left[ \int_{V'} \left\{ \frac{\partial}{\partial z} (\mathcal{T}_z + P_0) \right\} dV' \right] \cdot E_{mny} \quad (49)$$

$$H_z = H_z^{inc} + (\sigma_2 - \sigma_1) \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} \left[ \int_{V'} \left\{ -\frac{\partial}{\partial y} (\mathcal{T}_z + P_0) + \frac{\partial}{\partial x} (\mathcal{T}_z + P_0) \right\} dV' \right] \cdot E_{mnz} \quad (50)$$

where  $(P_0 - P) = 0$  at the surface of the earth.

#### Development of the Magnetic Components in the Wave Number Domain

The integral equation (48) for a two-dimensional inhomogeneity of cross-section  $S'$  can be written as follows:

$$H_x = H_x^{inc} + \frac{(\sigma_2 - \sigma_1)}{\delta} \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} E_{mnx} \int_{S'} \int_{-\infty}^{\infty} \left( -\frac{\partial \mathcal{T}_z}{\partial z} - \frac{\partial P_0}{\partial z} \right) ds' dy'$$

or

$$H_x = H_x^{inc} + \frac{(\sigma_2 - \sigma_1)}{\delta} \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} E_{mnx} \int_{S'} F_x(x, y, z; x', y', z') ds'$$

(51)

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where 
$$F_x(x, y, z; x', y', z') = - \int_{-\infty}^{\infty} \left( \frac{\partial \mathcal{T}_z}{\partial z} + \frac{\partial \mathcal{P}_0}{\partial z} \right) dy'$$

and 
$$\mathcal{T}_z + \mathcal{P}_0 = \frac{1}{4\pi\delta} \left\{ \int_0^{\infty} \left( \frac{u-g}{u+g} \right) \frac{g}{u} e^{-u(z+z')} J_0\left(\frac{gr}{\delta}\right) dg + \frac{e^{-\gamma_1 R_0}}{R_0} \right\}$$

(52)

The Fourier transform of (52) becomes

$$\mathcal{T}_z + \mathcal{P}_0 \longleftrightarrow \frac{1}{2\pi} \left\{ K_0(u_r \rho_0) + \int_{g_y}^{\infty} \left( \frac{u-g}{u+g} \right) \frac{g}{u} e^{-\frac{u(z+z')}{\cos[\nu(x-x')]} \frac{1}{\nu}} dg \right\}$$

where 
$$\nu = (g^2 - g_y^2)^{1/2}, \quad 0 < g_y < g,$$

(53)

It is desirable to effect a change of variable in the second term of (53) in order to evaluate the integral numerically. Note that

$$g^2 = \nu^2 + g_y^2 \quad ; \quad (g > 0)$$

and by differentiation

$$dg = \frac{\nu d\nu}{(\nu^2 + g_y^2)^{1/2}}$$

$$\nu_0 = 0$$

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The transformation pair in (53) become

$$\mathcal{T}_2 + P_0 \longleftrightarrow \frac{1}{2\pi} \left\{ K_0(u_Y P_0) + \int_0^\infty \left( \frac{u-g}{u+g} \right) \frac{e^{-u(z+z')}}{u} \cos[(x-x')v] dv \right\} \quad (54)$$

where  $u = (v^2 + g^2 + z^2)^{1/2}$

The derivative of this function with respect to Z at the surface of the earth is

$$\left. \frac{\partial}{\partial Z} (\mathcal{T}_2 + P_0) \right|_{Z=0} \longrightarrow \frac{1}{2\pi} \left\{ u_Y \frac{Z_n}{P_0} K_1(u_Y P_0) - \int_0^\infty \left( \frac{u-g}{u+g} \right) e^{-uZ_n} \cos[(x-x')v] dv \right\} \quad (55)$$

The magnetic field  $H_x$  at the surface of the earth is obtained by substituting (55) into (51)

$$H_x = H_x^{inc} + \frac{\Delta^2 (\sigma_2 - \sigma_1)}{2\pi \delta} \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} E_{mnx} \left\{ - \frac{u_Y Z_n}{P_0} K_1(u_Y P_0) + \int_0^\infty \left( \frac{u-g}{u+g} \right) e^{-uZ_n} \cos[(x-x')v] dv \right\} \quad (56)$$



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The second term of the magnetic component  $H_x$  is equal to the second term of the magnetic component  $H_y$  with opposite sign, therefore, the magnetic field  $H_y$  at the surface of the earth is obtained directly from (49)

$$H_y = H_y^{inc} + \frac{\Delta^2 (\sigma_2 - \sigma_1)}{2\pi \delta} \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} E_{mny} \left\{ \frac{u_y z_n}{\rho_0} k_1(u_y \rho_0) - \int_0^\infty \left( \frac{u-g}{u+g} \right) e^{-uzn} \cos[(x-x')v] dv \right\} \quad (57)$$

The vertical magnetic field component  $H_z$  from equation (50) may be written as

$$H_z = H_z^{inc} + \frac{(\sigma_2 - \sigma_1)}{\delta} \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} E_{mnz} \left\{ \int_{S'} \int_{-\infty}^{\infty} \left[ -\frac{\partial (\mathcal{T}_z + \mathcal{P}_0)}{\partial y} + \frac{\partial (\mathcal{T}_z + \mathcal{P}_0)}{\partial x} \right] dy' dx' \right\} \quad (58)$$

or

$$H_z = H_z^{inc} + \frac{(\sigma_2 - \sigma_1)}{\delta} \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} E_{mnz} \int_{S'} F_z(x, y, z; x', y', z') ds'$$

where

$$F_z = \int_{-\infty}^{\infty} \left\{ -\frac{\partial (\mathcal{T}_z + \mathcal{P}_0)}{\partial y} + \frac{\partial (\mathcal{T}_z + \mathcal{P}_0)}{\partial x} \right\} dy'$$

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The first term of the integrand in (58) in the wave number domain is given by

$$\frac{\partial}{\partial Y} (\mathcal{T}_2 + \mathcal{P}_0) \longrightarrow \frac{g_y i}{2\pi} \left\{ K_0(u_Y \rho_0) + \int_{g_y}^{\infty} \left( \frac{u-g}{u+g} \right) \frac{g}{u} e^{-u(z+z')} \cdot \frac{\cos[(X-X')v]}{v} dg \right\} \quad (59)$$

The second term of the integrand in (58) is obtained by taking the derivative with respect to X in (58).

$$\frac{\partial}{\partial X} (\mathcal{T}_2 + \mathcal{P}_0) \longrightarrow \frac{1}{2\pi} \left\{ -u_Y \frac{(X-X')}{\rho_0} K_1(u_Y \rho_0) - \int_{g_y}^{\infty} \left( \frac{u-g}{u+g} \right) \frac{g}{u} \cdot e^{-u(z+z')} \frac{\sin[(X-X')v]}{v} dg \right\} \quad (60)$$

at the surface of the earth (59) and (60) yield.

$$\frac{\partial}{\partial Y} (\mathcal{T}_2 + \mathcal{P}_0) \Big|_{z=0} \longrightarrow -\frac{g_y i}{2\pi} \left\{ K_0(u_Y \rho_0) + \int_{g_y}^{\infty} \left( \frac{u-g}{u+g} \right) \frac{g}{u} \cdot e^{-uz'} \frac{\cos[(X-X')v]}{v} dg \right\} \quad (61)$$

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and

$$\left. \frac{\partial (\mathcal{J}_z + \rho_0)}{\partial x} \right|_{z=0} \longleftrightarrow \frac{1}{2\pi\delta} \left\{ -\frac{u_y (x-x')}{\rho_0} K_1(u_y \rho_0) - \int_{g_y}^{\infty} \left( \frac{u-g}{u+g} \right) \frac{g}{u} e^{-uz'} \sin[(x-x')v] dg \right\} \quad (62)$$

Finally the magnetic field  $H_z$  in the wave number domain as a function of the electric field in the conductor is given by

$$H_z = H_z^i - \frac{\Delta^2 (\sigma_2 - \sigma_1)}{2\pi\delta} \sum_{m=1}^{M_x} \sum_{n=1}^{N_z} E_{mnz} \left( g_y i K_0(u_y \rho_0) + g_y i \int_{g_y}^{\infty} \frac{u-g}{u+g} \frac{g}{u} e^{-uz_n} \frac{\cos[(x-x')v]}{v} dg + \frac{u_y (x-x_m)}{\rho} K_1(u_y \rho) + \int_{g_y}^{\infty} \frac{u-g}{u+g} \frac{g}{u} e^{-uz_n} \sin[(x-x_m)v] dg \right)$$

(63)

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The Incident Magnetic Field Component  $H_x^i$  at the Surface of the Earth

The magnetic field component  $H_x^i$  due to a line source of finite length oriented in the Y-direction in a half-space of propagation constant  $\gamma_1$  is given by (Hill and Wait 1973)

$$H_x^i = -\frac{I}{2\pi \gamma_1^2} \int_{-l}^l \left( \frac{\partial^3 P}{\partial z^3} + \frac{\partial^3 P}{\partial y^2 \partial z} - \frac{\partial^4 N}{\partial z^2 \partial x^2} \right) dy' \quad (64)$$

where  $\gamma_1^2 = u_1^2 - \lambda^2$

and  $\gamma_1^2 = i\omega \mu_0 \sigma_1$

At the surface of the earth  $\lim_{z \rightarrow 0} \frac{\partial^3 P}{\partial z^3} \rightarrow \frac{\partial^3 P}{\partial y^2 \partial z} \rightarrow 0$

so that (64) becomes

$$H_x^i = \frac{I}{2\pi \gamma_1^2} \int_{-l}^l \frac{\partial^4 N}{\partial z^2 \partial x^2} dy' \quad (65)$$

Substituting (12) in (65) we obtain

$$H_x^i = \frac{I}{2\pi \gamma_1^2} \int_{-l}^l \int_0^\infty \frac{\partial^2}{\partial x^2} [u_1 e^{-u_1 z} J_0(\lambda r) d\lambda] dy' \quad (66)$$

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adding  $\lambda$  and subtracting  $\lambda$  to  $u_1$ , we obtain

$$H_x^{inc} = \frac{I}{2\pi\gamma_1^2} \int_{-l}^l \int_0^\infty \frac{\partial^2}{\partial x^2} \left\{ (u_1 - \lambda) J_0(\lambda r) \right\} e^{-u_1 z} d\lambda dy' \\ + \frac{I}{2\pi\gamma_1^2} \int_{-l}^l \frac{\partial^2}{\partial x^2} \int_0^\infty \left[ e^{-u_1 z} \lambda J_0(\lambda r) d\lambda dy' \right] \quad (67)$$

The second integral of (67) is related to the Sommerfeld (1926) integral as follows

$$\frac{\partial P}{\partial z} = \int_0^\infty \lambda e^{-u_1 z} J_0(\lambda r) d\lambda = -\frac{z}{R^2} e^{-\gamma_1 R} (\gamma_1 + 1)$$

where the function  $\frac{\partial P}{\partial z}$  is continuous for all values of  $z > 0$ . Then (67) becomes

$$H_x^{inc} = \frac{I}{2\pi\gamma_1^2} \int_{-l}^l \frac{\partial^2}{\partial x^2} \left\{ (u_1 - \lambda) J_0(\lambda r) \right\} e^{-u_1 z} d\lambda dy' \\ + \frac{I}{2\pi\gamma_1^2} \frac{\partial^2}{\partial x^2} \int_{-l}^l \left[ -z \frac{e^{-\gamma_1 R}}{R^2} (\gamma_1 + 1) \right] dy' \quad (68)$$

In order to obtain an expression for the horizontal magnetic field  $H_x^{inc}$  at the surface of the earth we take the limit of (68) as  $z$  approaches to zero.

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$$\lim_{z \rightarrow 0} \int_{-\ell}^{\ell} \int_0^{\infty} \frac{\partial^2}{\partial x^2} \left[ (u_1 - \lambda) J_0(\lambda r) e^{-u_1 z} \right] d\lambda dy' =$$

$$\int_{-\ell}^{\ell} \int_0^{\infty} \frac{\partial^2}{\partial x^2} \left[ (u_1 - \lambda) J_0(\lambda r) \right] d\lambda dy'$$

and

$$\lim_{z \rightarrow 0} \int_{-\ell}^{\ell} \frac{\partial^2}{\partial x^2} \left[ -\frac{z e^{-\gamma_1 R}}{R^2} (\gamma_1 + 1) \right] dy' = 0$$

then (68) becomes

$$H_x^{inc} = \frac{I}{2\pi \gamma_1^2} \int_{-\ell}^{\ell} \int_0^{\infty} \frac{\partial^2}{\partial x^2} \left\{ (u_1 - \lambda) J_0(\lambda r) \right\} d\lambda dy'$$

(69)

by multiplying numerator of (69) by  $u_1 + \lambda$  and then replacing  $u_1^2 - \lambda^2$  by its equivalent  $\gamma_1^2$ , which is a constant we pull out of the integrand, we obtain

$$H_x^{inc} = \frac{I}{2\pi} \int_{-\ell}^{\ell} \int_0^{\infty} \frac{\partial^2}{\partial x^2} \left( \frac{J_0(\lambda r)}{u_1 + \lambda} \right) d\lambda dy'$$

(70)

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The Fourier transform (70) yields

$$H_x^{inc} = \frac{I}{2\pi} \frac{\partial^2}{\partial X^2} \int_{-l}^l \int_0^\infty \int_{-\infty}^\infty \frac{J_0(\lambda r)}{\omega l + \lambda} e^{-i k_y y} dy d\lambda dy' \quad (71)$$

Making the change of variable  $y=t+y'$  in (71), we obtain

$$H_x^{inc} = \frac{I}{2\pi} \frac{\partial^2}{\partial X^2} \int_{-l}^l \int_0^\infty \int_{-\infty}^\infty \frac{J_0(\lambda r)}{\omega l + \lambda} e^{-i k_y t} e^{-i k_y y'} dt d\lambda dy' \quad (72)$$

The integrand of (72) is an even function of  $t$ ; therefore, the bilateral Fourier transform reduces to a Fourier cosine transform

$$H_x^{inc} = \frac{I}{\pi} \frac{\partial^2}{\partial X^2} \int_{-l}^l \int_0^\infty \int_0^\infty \frac{J_0(\lambda r)}{\omega l + \lambda} \cos(k_y t) e^{-i k_y y'} dt d\lambda dy' \quad (73)$$

substituting

$$2 \frac{\sin k_y l}{k_y} \quad \text{for} \quad \int_{-l}^l e^{-i k_y y'} dy'$$

and

$$\frac{\cos [X(\lambda^2 - k_y^2)^{1/2}]}{(\lambda^2 - k_y^2)^{1/2}} \quad \text{for} \quad \int_0^\infty J_0(\lambda r) \cos(k_y t) dt$$

(73) becomes

$$H_x^{inc} = \frac{2I}{\pi} \frac{\partial^2}{\partial X^2} \int_{k_y}^\infty \left\{ \frac{\cos [X(\lambda^2 - k_y^2)^{1/2}]}{(\omega l + \lambda)(\lambda^2 - k_y^2)^{1/2}} \right\} d\lambda \frac{\sin k_y l}{k_y}$$

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and evaluating the derivatives with respect  $x$  gives the horizontal magnetic  $H_x^i$  at the surface of the earth

$$H_x^{inc} = - \frac{2I}{\pi} \frac{\sin k_y l}{k_y} \int_{k_y}^{\infty} \frac{v_1 \cos(v_1 x)}{u_1 + \lambda} d\lambda, \quad (75)$$

If the distances are normalized in skin depths

$$H_x = - \frac{2I}{\pi} \frac{\sin g_y L}{g_y} \int_{g_y}^{\infty} \frac{v \cos(xv)}{u + g} dg \quad (76)$$

### The Incident Magnetic Field Components $H_y^i$ and $H_z^i$ at the Surface of the Earth

The magnetic field components  $H_y^i$  and  $H_z^i$  due to a line source of finite length oriented in the  $y$ -direction are given by (Hill and Wait, 1973)

$$H_y^{inc} = \frac{I}{2\pi\sigma_1^2} \int_{-l}^l \left( -\frac{\partial^4 N}{\partial x \partial y \partial z^2} + \frac{\partial^3 P}{\partial x \partial y \partial z} \right) dy' \quad (77)$$

and

$$H_z^{inc} = \frac{I}{2\pi\sigma_1^2} \int_{-l}^l \left( -\frac{\partial^4 N}{\partial x \partial z^3} + \gamma_1^2 \frac{\partial^2 N}{\partial x \partial z} - \frac{\partial^3 P}{\partial x \partial z^2} \right) dy' \quad (78)$$



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At the surface of the earth in (77)  $\frac{\partial^3 \rho}{\partial x \partial y \partial z}$  is equal to zero, so that

$$H_Y^{inc} = -\frac{I}{2\pi \gamma_1^2} \int_{-\ell}^{\ell} \frac{\partial^4 N}{\partial x \partial y \partial z^2} dy' \quad (79)$$

By inserting (12) into (79) we obtain

$$H_Y^{inc} = -\frac{I}{2\pi \gamma_1^2} \int_{-\ell}^{\ell} \frac{\partial^2}{\partial x \partial y} \left\{ \int_0^{\infty} u_1 e^{-u_1 z} J_0(\lambda r) d\lambda \right\} dy' \quad (80)$$

adding  $\lambda$  and subtracting  $\lambda$  to  $u_1$ , we obtain

$$H_Y^{inc} = -\frac{I}{2\pi \gamma_1^2} \int_{-\ell}^{\ell} \frac{\partial^2}{\partial x \partial y} \left\{ \int_0^{\infty} (u_1 - \lambda) J_0(\lambda r) e^{-u_1 z} d\lambda dy' \right\} \\ - \frac{I}{2\pi \gamma_1^2} \int_{-\ell}^{\ell} \frac{\partial^2}{\partial x \partial y} \left\{ \int_0^{\infty} \lambda J_0(\lambda r) e^{-u_1 z} d\lambda dy' \right\} \quad (81)$$

or

$$H_Y^{inc} = -\frac{I}{2\pi \gamma_1^2} \int_{-\ell}^{\ell} \frac{\partial^2}{\partial x \partial y} \left\{ (u_1 - \lambda) J_0(\lambda r) e^{-u_1 z} d\lambda dy' \right\} \\ + \frac{I}{2\pi \gamma_1^2} \frac{\partial^2}{\partial x \partial y} \left[ \int_{-\ell}^{\ell} -\frac{z}{R^2} e^{-\gamma_1 R} (\gamma_1 + 1) \right] dy' \quad (82)$$

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At the surface of the earth (82) becomes

$$H_y^{inc} = -\frac{I}{2\pi\delta_1^2} \int_{-l}^l \frac{\partial^2}{\partial x \partial y} \left\{ \int_0^\infty (\omega_1 - \lambda) J_0(\lambda r) d\lambda dy' \right\} \quad (83)$$

we now multiply the numerator and denominator of (83) by  $\omega_1 + \lambda$ , employ the identity  $\omega_1^2 - \lambda^2 = \delta_1^2$

and obtain

$$H_y^{inc} = -\frac{I}{2\pi} \int_{-l}^l \int_0^\infty \frac{\partial^2}{\partial x \partial y} \left( \frac{J_0(\lambda r)}{\omega_1 + \lambda} \right) d\lambda dy' \quad (84)$$

Thus the magnetic field component  $H_y^i$  in the wave number domain at the surface of the earth with the distance measured in skin depths becomes

$$H_y^{inc} = \frac{2I}{\pi} g_{yi} \frac{\sin g_y L}{g_y} \int_0^\infty \frac{\sin(\nu x)}{\omega_1 + g} dg \quad (85)$$

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Substituting the Sommerfeld integrals N and P into (78) yields

$$H_z^{inc} = \frac{I}{2\pi\gamma_1^2} \frac{\partial}{\partial x} \left\{ \int_{-\lambda}^{\lambda} \int_0^{\infty} \left\{ u^2 e^{-u|z|} J_0(\lambda r) - (u^2 - \lambda^2) e^{-u|z|} J_0(\lambda r) - u\lambda e^{-u|z|} J_0(\lambda r) \right\} d\lambda dy' \right\} \quad (86)$$

or

$$H_z^{inc} = \frac{I}{2\pi\gamma_1^2} \frac{\partial}{\partial x} \left\{ \int_{-\lambda}^{\lambda} \int_0^{\infty} \lambda(\lambda - u) e^{-u|z|} J_0(\lambda r) d\lambda dy' \right\} \quad (87)$$

By multiplying numerator and denominator of (87) by  $\lambda + u$  and then replacing  $u^2 - \lambda^2$  by  $\gamma_1^2$ , which is a constant we pull out of the integrand, we obtain

$$H_z^{inc} = -\frac{I}{2\pi} \frac{\partial}{\partial x} \left\{ \int_{-\lambda}^{\lambda} \int_0^{\infty} \frac{\lambda e^{-u|z|}}{u + \lambda} J_0(\lambda r) d\lambda dy' \right\} \quad (88)$$

At the surface of the earth (88) becomes

$$H_z^{inc} = -\frac{I}{2\pi} \frac{\partial}{\partial x} \left\{ \int_{-\lambda}^{\lambda} \int_0^{\infty} \frac{\lambda J_0(\lambda r)}{u + \lambda} d\lambda dy' \right\} \quad (89)$$

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Thus the magnetic field component  $H_z^i$  in the wave number domain at the surface of the earth with the distance measured in skin depths is given by

$$H_z^{inc} = \frac{2I}{\pi} \left\{ \int_{g_y}^{\infty} \frac{g}{g+v} \sin(xv) dg \right\} \frac{\sin g_y L}{g_y} \quad (90)$$

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METHOD OF COMPUTATION

The first step of numerical evaluation involves the computation of the incident electric field vector ( $E_x^{inc}$ ,  $E_y^{inc}$ ,  $E_z^{inc}$ ) at the center of each cell in the body.

The three electric field components are given by (32), (33), and (34), where the  $E_x^{inc}$  and  $E_y^{inc}$  incident fields are functions of the complex integrals.

$$A = \int_{g_y}^{\infty} e^{-uz} \sin(xv) dg \quad (91)$$

and

$$B = \int_{g_y}^{\infty} e^{-uz} v \cos(xv) dg \quad (92)$$

The second step is to calculate the elements of the dyadic Green's matrix. The singular part can be calculated in closed form and some of the elements of the non-singular dyadic Green's matrix must be evaluated numerically.

The integrals (B-18), (B-20), (B-21) and (B-23) of the non-singular part matrix (Appendix B) are given by

$$C = \int_{g_y}^{\infty} \left(2 - \frac{g}{u}\right) e^{-u(z+z')} v \cos[(x-x')v] dg \quad (93)$$

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$$D = \int_{g_y}^{\infty} \left(2 - \frac{g}{u}\right) e^{-u(z+z')} \frac{\sin[(x-x')v]}{v} dg \quad (94)$$

$$M = \int_{g_y}^{\infty} \left(2 - \frac{g}{u}\right) e^{-u(z+z')} \frac{\cos[(x-x')v]}{v} dg \quad (95)$$

and

$$\mathcal{T}_2(g_y) = \frac{1}{2\pi} \int_{g_y}^{\infty} \left(\frac{u-g}{u+g}\right) \frac{g}{u} e^{-u(z+z')} \frac{\cos[(x-x')v]}{v} dg \quad (96)$$

The third step is to solve the system of equations (40) for the electric field in the body by Gauss-Jordan technique. Finally, the electric and magnetic fields outside the inhomogeneity can be found using the integral equations (44) and (45).

### Numerical Evaluation of the Integrals

With the change of variable  $g = (\nu^2 + g_y^2)^{\frac{1}{2}}$  the integrals (91) to (96) at the center of each cell can be transformed to

$$A = \int_0^{\infty} e^{-uz} \frac{\nu \sin(x\nu)}{\sqrt{\nu^2 + g_y^2}} d\nu \quad (97)$$

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$$B = \int_0^{\infty} e^{-uz} \frac{v^2 \cos(Xv)}{\sqrt{v^2 + g_Y^2}} dv \quad (98)$$

$$C = \int_0^{\infty} \left( 2 - \frac{(v^2 + g_Y^2)^{1/2}}{u} \right) \frac{v^2 e^{-u(z+z')}}{\sqrt{v^2 + g_Y^2} \cos[v(x-x')]} dv \quad (99)$$

$$D = \int_0^{\infty} \left( 2 - \frac{(v^2 + g_Y^2)^{1/2}}{u} \right) \frac{v e^{-u(z+z')}}{\sqrt{v^2 + g_Y^2} \sin[v(x-x')]} dv \quad (100)$$

$$M = \int_0^{\infty} \left( 2 - \frac{(v^2 + g_Y^2)^{1/2}}{u} \right) \frac{e^{-u(z+z')}}{\sqrt{v^2 + g_Y^2} \cos[v(x-x')]} dv \quad (101)$$

$$\mathcal{I}_2(g_Y) = \frac{1}{2\pi} \int_0^{\infty} \left\{ \frac{u - (v^2 + g_Y^2)^{1/2}}{u + (v^2 + g_Y^2)^{1/2}} \right\} \frac{e^{-u(z+z')}}{u} \cos[v(x-x')] dv \quad (102)$$

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In order to calculate the integrals, the integrands are computed numerically and separated into real and imaginary parts as follows:

$$\int_0^{\infty} F(u, z, z', \nu) d\nu = \int_0^{\infty} \Psi d\nu + i \int_0^{\infty} \Phi d\nu$$

where  $\Psi = \mathcal{R}[F(u, z, z', \nu)]$

and  $\Phi = \mathcal{I}[F(u, z, z', \nu)]$

are the real and imaginary parts of  $F(u, z, z', \nu)$ , respectively.

$\Psi$  and  $\Phi$  are continuous and smooth functions in each integral. The numerical procedure used to evaluate the integrals is the same that described by Hohmann (1971) and similar to that described by Meinardus (1966, 1971) and Frischknecht (1967), which consists of writing the integrals  $\Psi$  and  $\Phi$  as the sum of the alternating series

$$\Psi = \Psi_1 - \Psi_2 + \Psi_3 \dots \dots \dots - \Psi_{20}$$

and

$$\Phi = \Phi_1 - \Phi_2 + \Phi_3 \dots \dots \dots - \Phi_{20}$$

where

$$\mathcal{R}[F] + i \mathcal{I}[F] = \int_{\frac{\pi}{2}n + \pi(n-1)}^{\frac{\pi}{2} + n\pi} \Psi_{n+1} \cos(\nu) d\nu +$$

$$i \int_{\frac{\pi}{2}n + \pi(n-1)}^{\frac{\pi}{2} + n\pi} \Phi_{n+1} \cos(\nu) d\nu$$



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and

$$R[F'] + iI[F'] = \int_{\pi(n-1)}^{n\pi} \Psi_n \sin(v) dv + i \int_{\pi(n-1)}^{n\pi} \phi_n \sin(v) dv$$

$$n = 1, 2, 3, \dots, \infty$$

Each term of the series is evaluated between zeros of the cosine and sine function by Gaussian quadrature.

The summation of these series after the twentieth term may be made more rapidly convergent by applying Euler's transformation.

The term for  $n = 0$  for the cosine function is integrated by a Simpson's rule algorithm.

At the point  $X_t = X - X' = 0$  the integrals (99) to (102) can be evaluated using Simpson's rule and they are reduced to

$$C = \int_0^{\infty} \left\{ 2 - \frac{(v^2 + g_Y^2)^{1/2}}{u} \right\} \frac{v^2 e^{-u(z+z')}}{\sqrt{v^2 + g_Y^2}} dv \quad (103)$$

$$M = \int_0^{\infty} \left\{ 2 - \frac{(v^2 + g_Y^2)^{1/2}}{u} \right\} \frac{e^{-u(z+z')}}{\sqrt{v^2 + g_Y^2}} dv \quad (104)$$

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and

$$\mathcal{I}_2(q_Y) = \frac{1}{2\pi} \int_0^\infty \left\{ \frac{u - (v^2 + q_Y^2)^{1/2}}{u + (v^2 + q_Y^2)^{1/2}} \right\} \frac{e^{-u(z+z')}}{u} dv \quad (105)$$

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### NUMERICAL RESULTS

A thin vertical dike of infinite strike length and small depth extent relative to the wave length in the surrounding half-space is chosen as a model, (Figure 4b).

The source is a 100-meter grounded cable that carries 10 amperes of current located 250 meters away from the dike. In this example the depth to the top of the dike is 25 m, the thickness is 15 m, the depth extent is 150 m, the incident field frequency  $f=1000$  hz, the conductivity of the host rock  $\sigma$ , is  $10^{-3}$  mho/m, and the conductivity of the dike is 1 mho/m.

In the present analysis all distances are expressed in units of skin-depth  $\delta$ , in the lower half-space.

The magnetic responses have been studied because they are more indicative in the shape than the electric field response.

The electromagnetic field components are calculated in the wave number domain for several values of  $k_y$  so the inverse Fourier transform can be carried out.

The transformation of the electromagnetic field component from  $(x, k_y, z)$  domain to  $(x, y, z)$  space requires evaluation of the integrals:

$\nu_2 / \sigma_1 = 10^3$   
 $H = 150 \text{ m}$   
 $D = 25 \text{ m}$   
 $T = 15 \text{ m}$   
 $X_a = 250 \text{ m}$   
 $L = 50 \text{ m}$

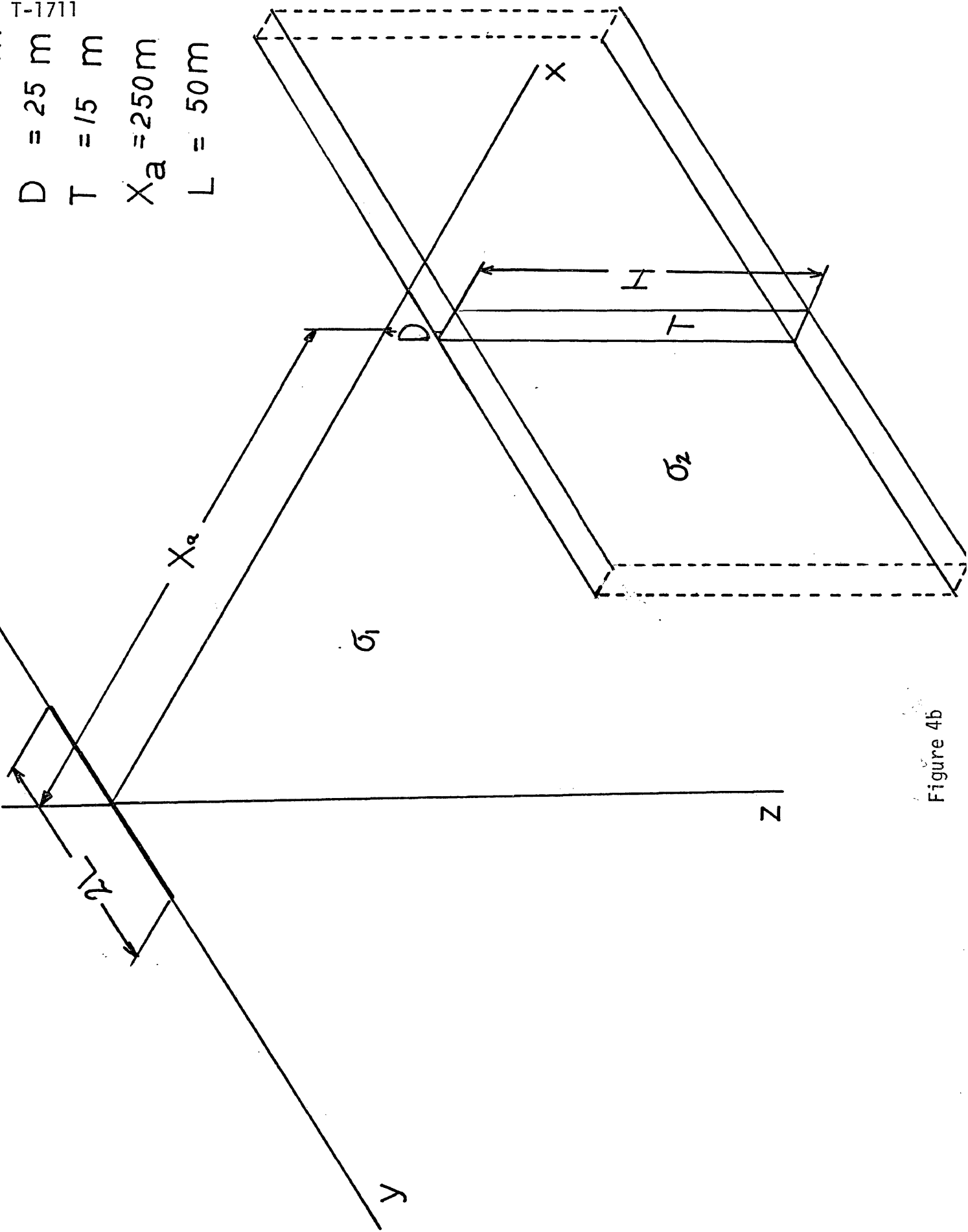


Figure 4b

$$H_x(x, y, z) = \frac{1}{\pi} \int_{-\infty}^{\infty} H_x(x, k_y, z) e^{j k_y y} dk_y$$

$$H_y(x, y, z) = \frac{1}{\pi} \int_{-\infty}^{\infty} H_y(x, k_y, z) e^{j k_y y} dk_y$$

$$H_z(x, y, z) = \frac{1}{\pi} \int_{-\infty}^{\infty} H_z(x, k_y, z) e^{j k_y y} dk_y$$

The electromagnetic components in the wave number domain are calculated for the same seven values of  $g_y$  as used by Coggon (1971); those chosen being .05, 0.3, 0.8, 1.7, 3.6, 7.2, 15.0. For each value of  $X$ , one electromagnetic component is calculated for seven values of  $g_y$ , and interpolated by a cubic spline function so as to provide 128 points. The new set of values in the wave number is transformed to the space domain by the Fast Fourier transform method.

Maps of the amplitude and phase of the magnetic components are shown in Figures (5) and (10). For comparison, maps of the amplitude and phase of the magnetic components for a uniform earth are shown in Figures (11) to (16). The magnetic components for a uniform earth are calculated by expressions developed in Appendix D.

The amplitude and phase maps of the magnetic components indicate the presence of the thin vertical conductor. All the maps of the electromagnetic field display symmetry about the  $X$ -axis. Figure 5 shows the horizontal magnetic field map  $H_x$  in amp/m. There are two peaks in the magnetic fields directly over the body, 0.25 amp/m each of them. The phase  $\psi_x$  is indicative for determining the location of a conductor. In Figure 6 the minimum value of the phase is situated over the center of

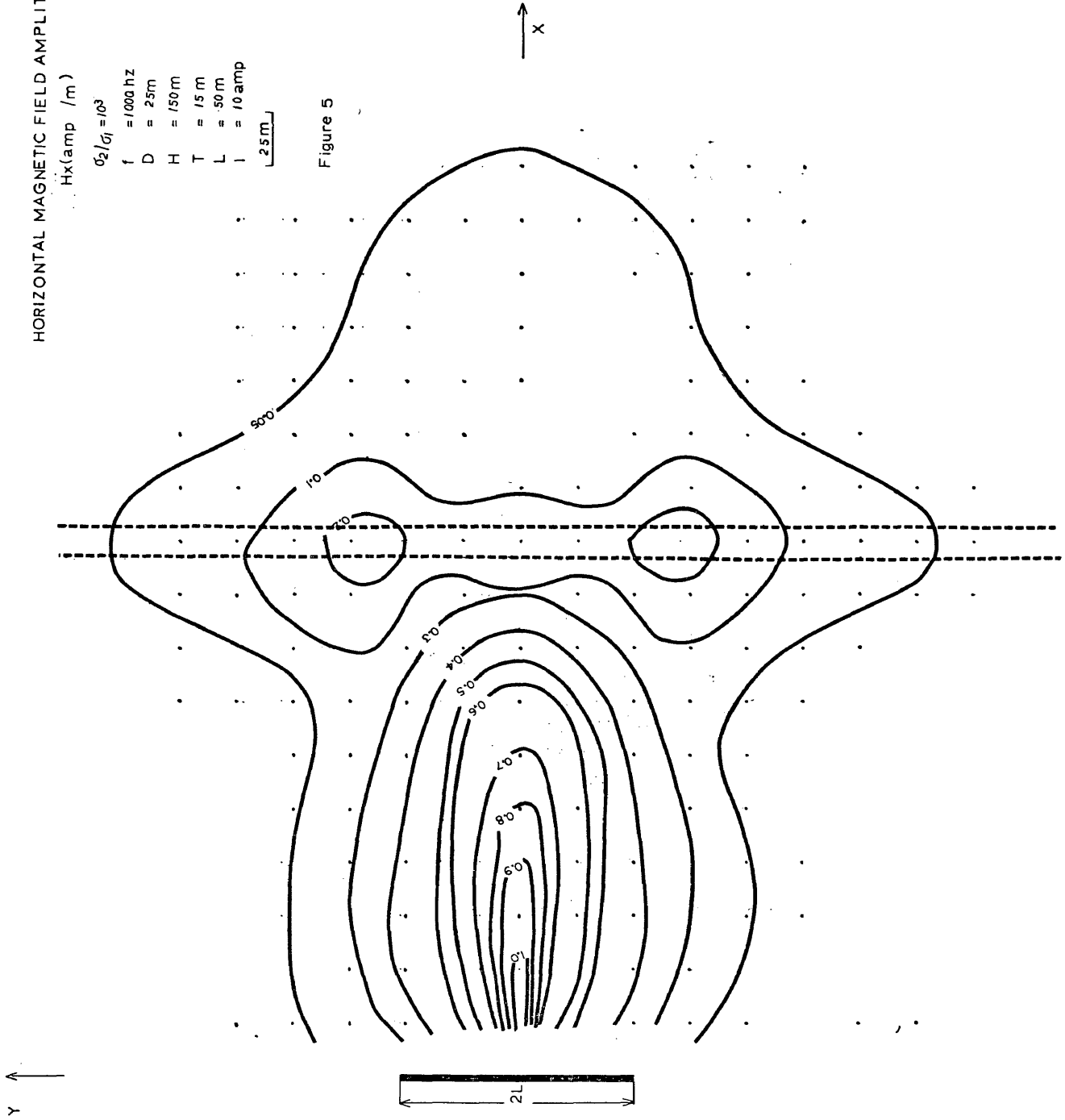
HORIZONTAL MAGNETIC FIELD AMPLITUDE

$H_x(\text{amp} / \text{m})$

- $\sigma_2/\sigma_1 = 10^3$
- $f = 1000 \text{ hz}$
- $D = 25 \text{ m}$
- $H = 150 \text{ m}$
- $T = 15 \text{ m}$
- $L = 50 \text{ m}$
- $I = 10 \text{ amp}$

25m

Figure 5

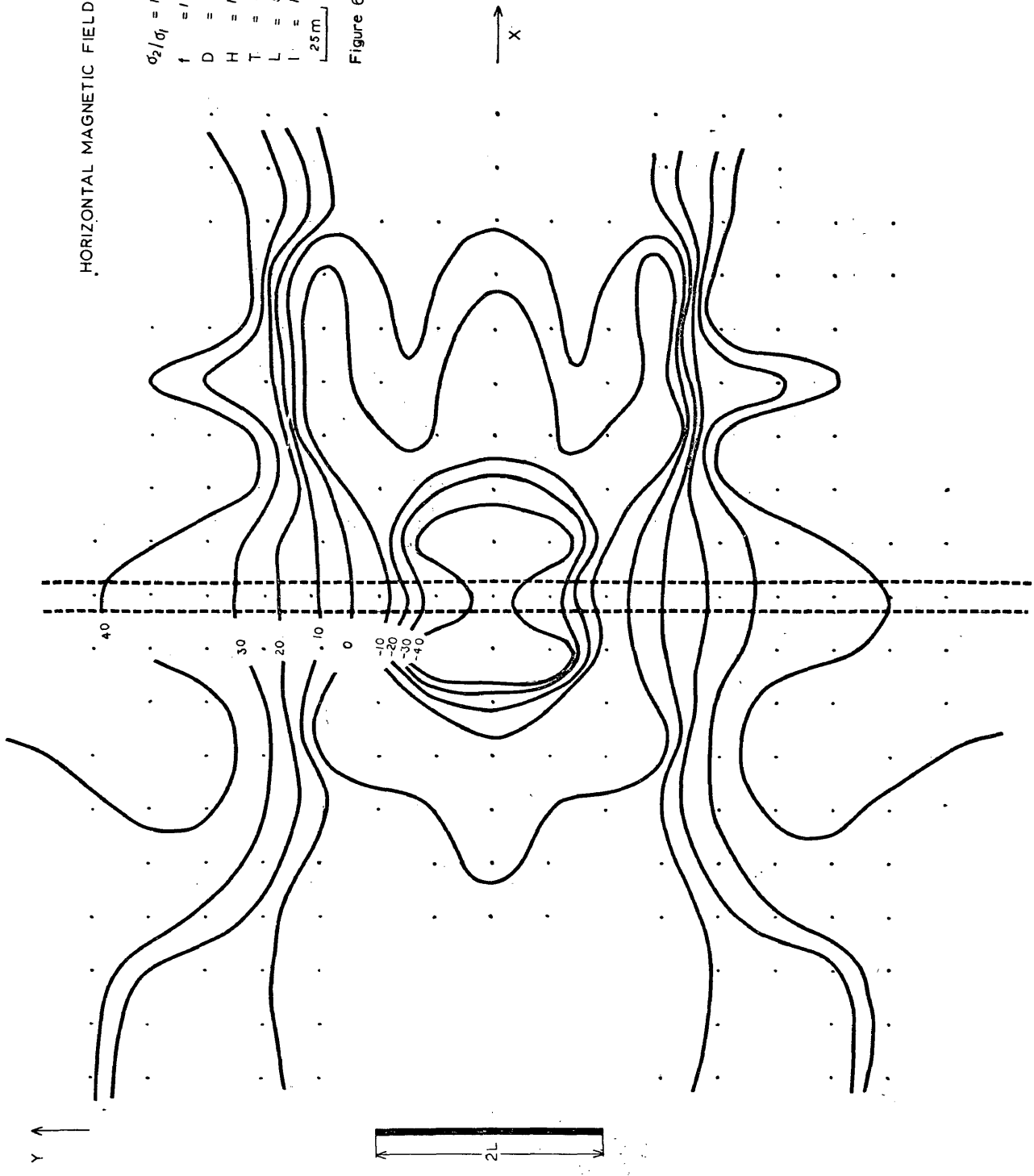


HORIZONTAL MAGNETIC FIELD PHASE,  $\Psi_x$ (deg.)

- $\sigma_2/\sigma_1 = 10^3$
- $f = 1000$  hz.
- $D = 25$  m
- $H = 150$  m
- $T = 15$  m
- $L = 50$  m
- $I = 10$  amp

25 m

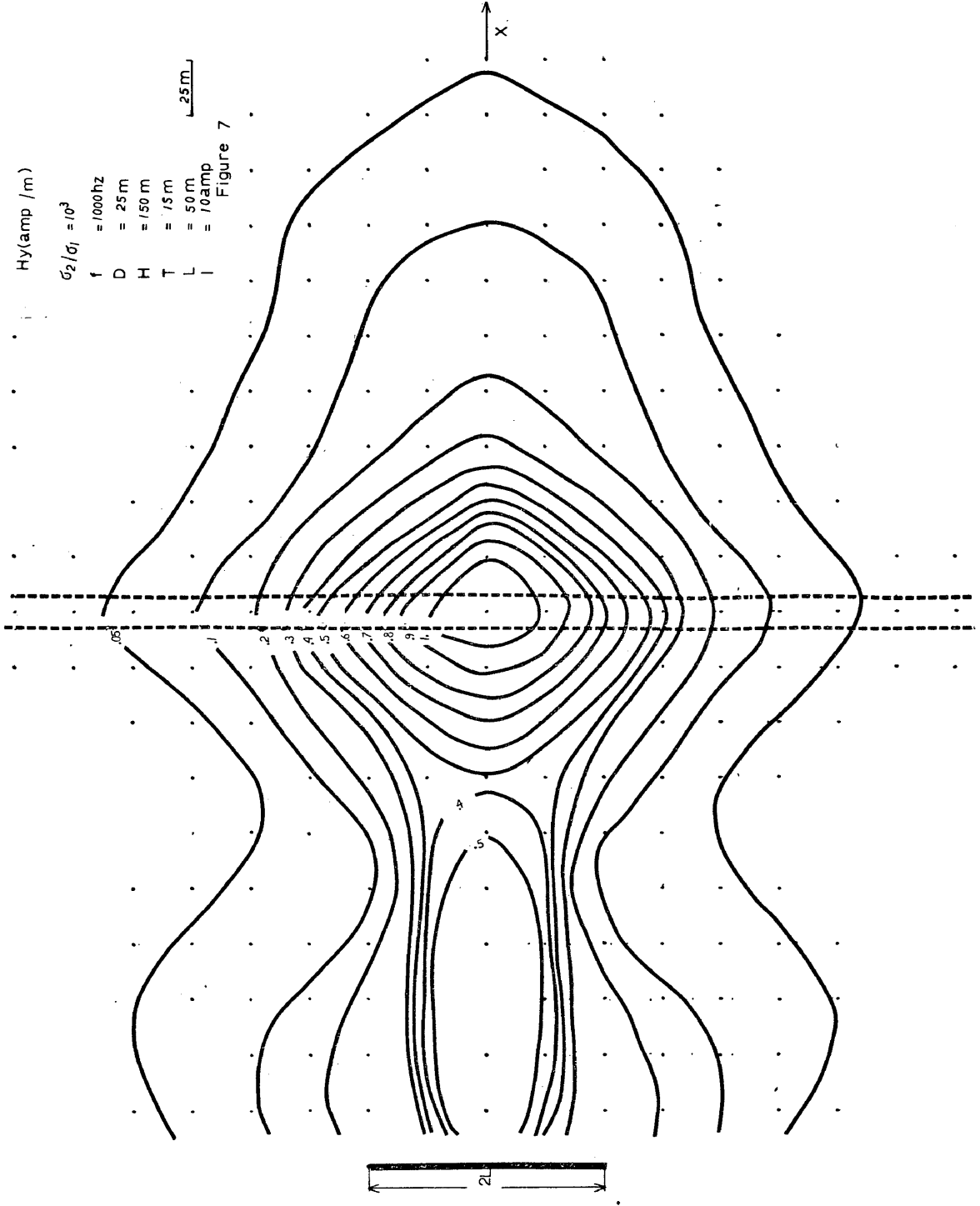
Figure 6



HORIZONTAL MAGNETIC FIELD AMPLITUDE

$\sigma_2/\sigma_1 = 10^3$   
 $H_y(\text{amp} / \text{m})$   
 $f = 1000\text{hz}$   
 $D = 25\text{ m}$   
 $H = 150\text{ m}$   
 $T = 15\text{ m}$   
 $L = 50\text{ m}$   
 $I = 10\text{ amp}$

Figure 7



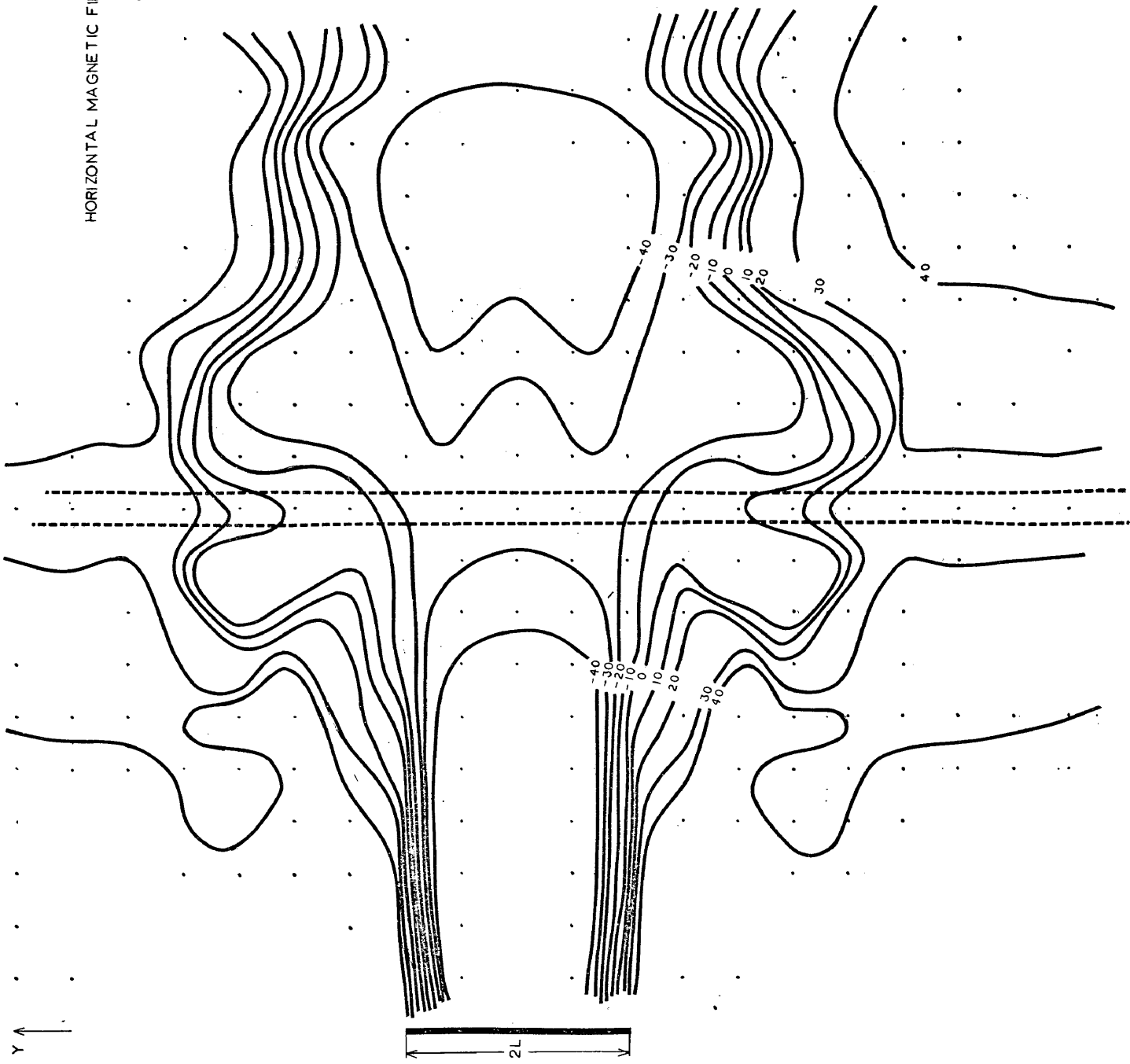


HORIZONTAL MAGNETIC FIELD PHASE,  $\psi_y$  (degrees)

$\phi_2(\phi_1) = 10^3$   
 $f = 10000$  hz  
 $D = 25$  m  
 $H = 150$  m  
 $T = 15$  m  
 $L = 50$  m  
 $I = 10$  amp

25 m

Figure 8



X

Y

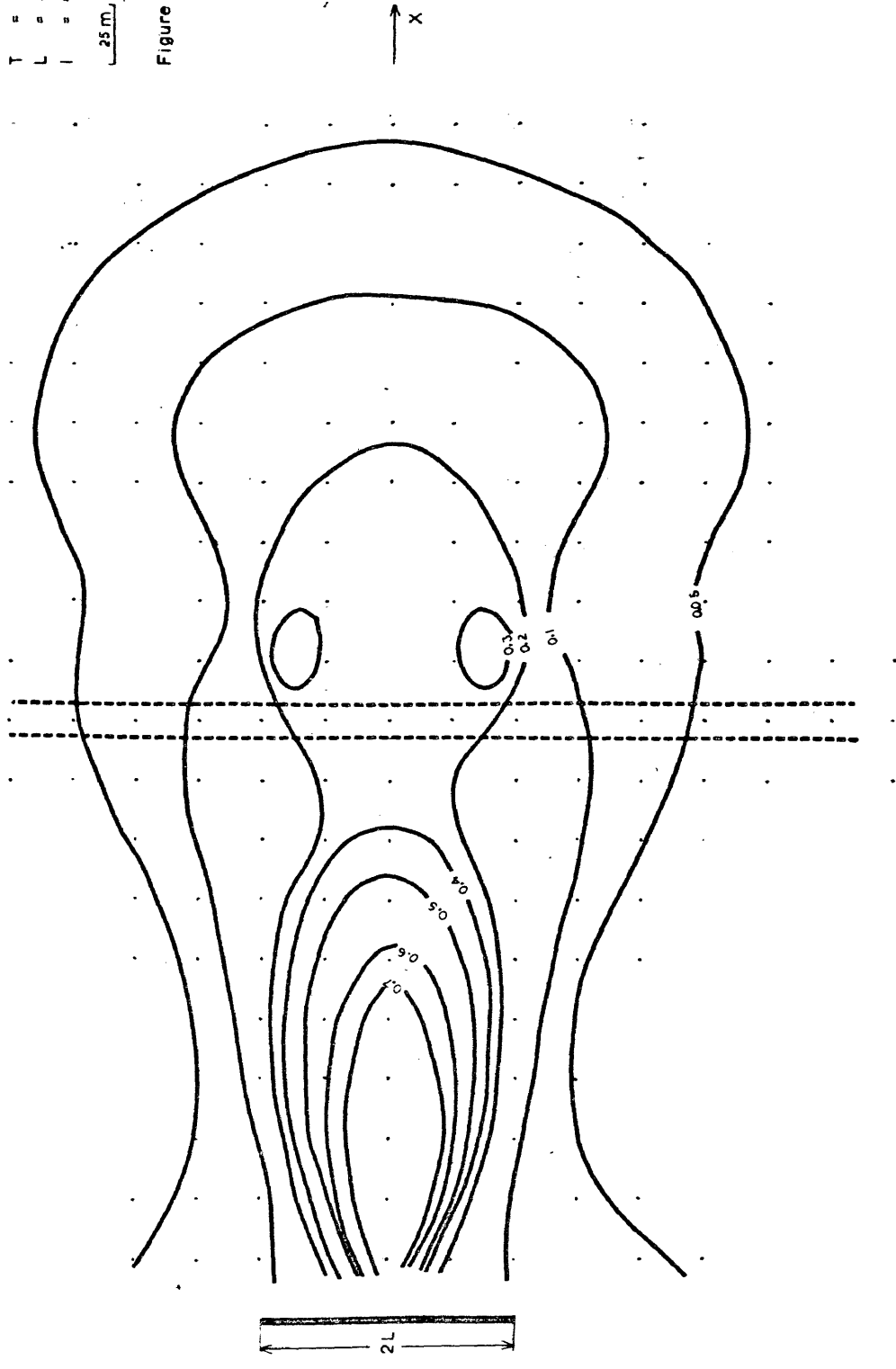
Y ↑

VERTICAL MAGNETIC FIELD AMPLITUDE, Hz (amp /m)

- $\sigma_2/\sigma_1 = 10^3$
- f = 1000 hz
- D = 25 m
- H = 150 m
- T = 15 m
- L = 50 m
- I = 10 amp

25 m

Figure 9



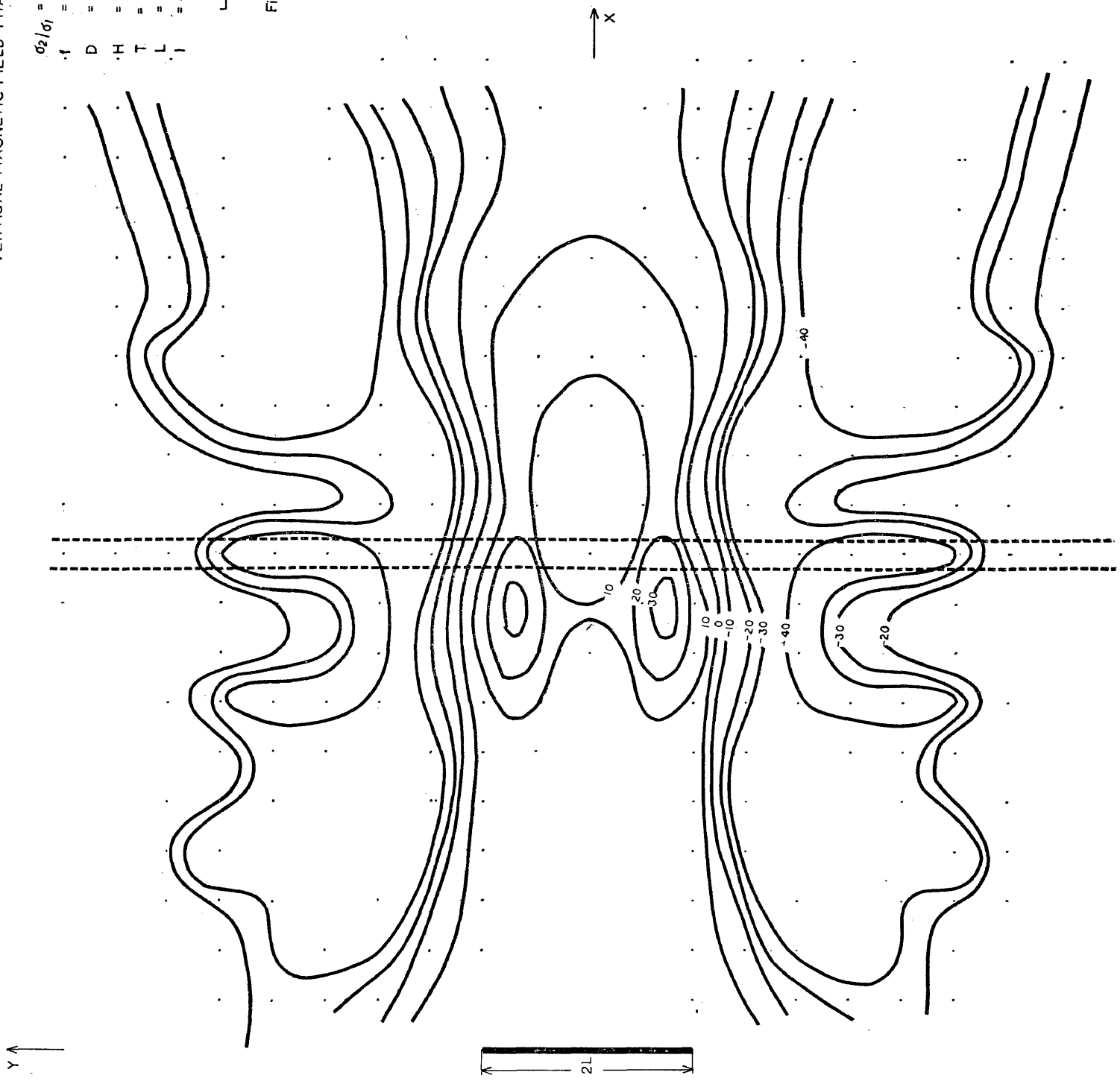
X →

VERTICAL MAGNETIC FIELD PHASE,  $\Psi_z$  (degrees)

- $\sigma_2/\sigma_1 = 10^3$
- $f = 1000$  hz
- D = 25 m
- H = 150 m
- T = 15 m
- L = 5.0 m
- I = 10 amp

25 m

Figure 10



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of the body.

The horizontal magnetic field map  $H_y$  is shown in Figure 7 in amp/m. The maximum amplitude is located directly over the center of the body and corresponds to 1.0 amp/m.

The amplitude of the magnetic field in the Y-direction decay exponentially away from the center of the dike.

The location of the subsurface conductors also can be determined by noting the position of maximum gradient for Hz measurements in Figure 9. In general, neither  $\psi_y$  nor  $\psi_z$  measurements are very useful for determining the location of the conductor.

HORIZONTAL MAGNETIC FIELD AMPLITUDE  
ON A CONDUCTING HALF-SPACE

$H_x$  (amp/m)  $10^2$

$\sigma = 0.001$  mho/m

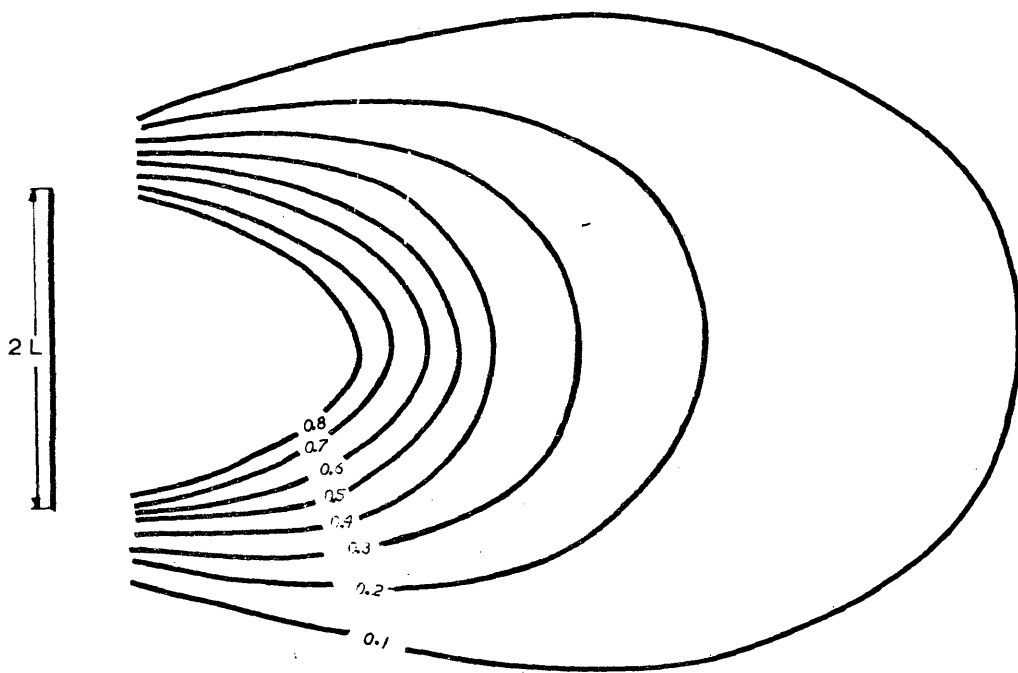
$f = 1000$  hz.

$L = 50$  m

$I = 10$  amp

25m.

Figure 11



HORIZONTAL MAGNETIC FIELD PHASE  
ON A CONDUCTING HALF-SPACE

$\Psi_x$  (degree)

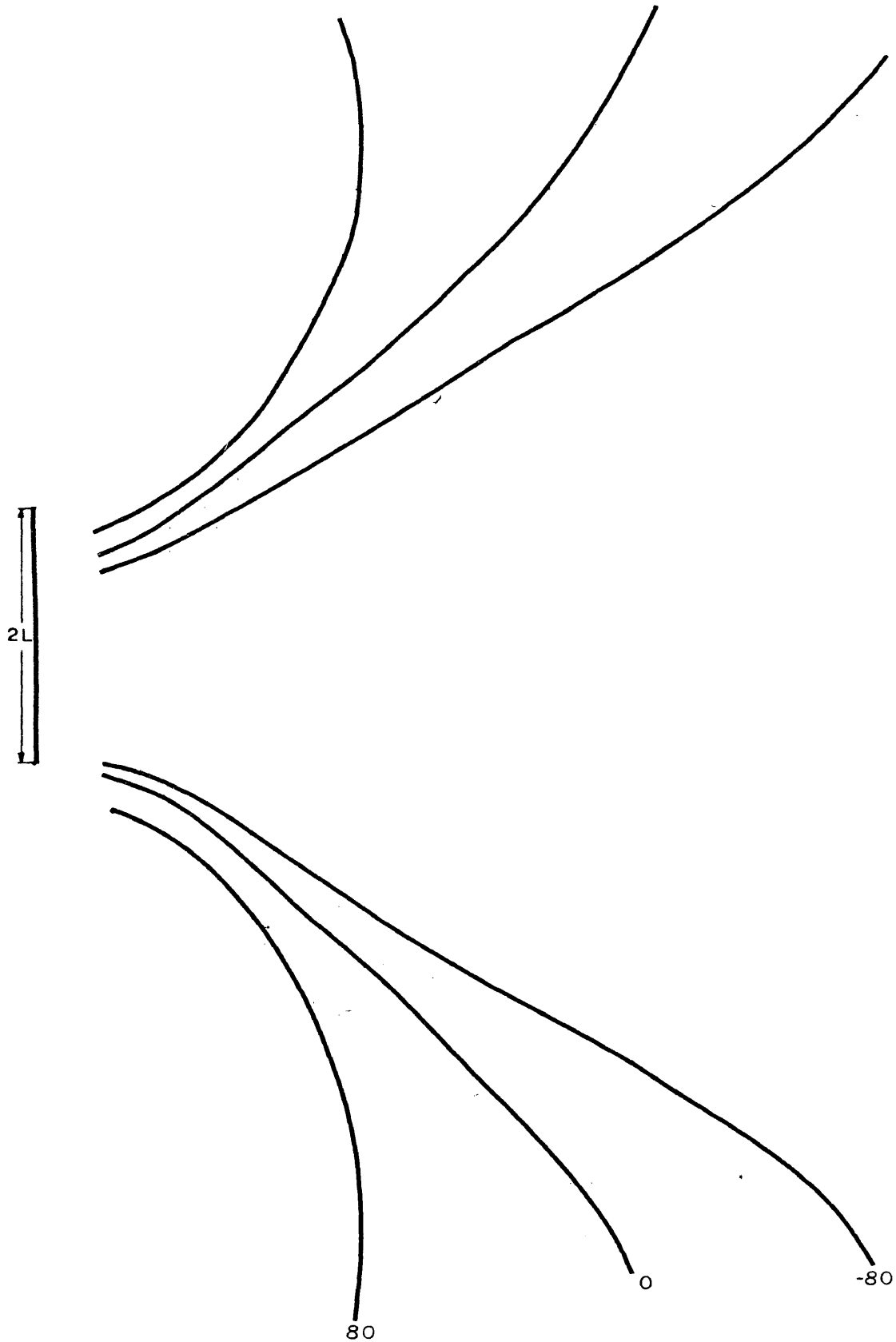
$\sigma = 0.001 \text{ mho/m}$

$f = 1000 \text{ Hz}$

$L = 50 \text{ m}$

$I = 1.0 \text{ amp}$  25m

Figure 12



HORIZONTAL MAGNETIC FIELD AMPLITUDE  
ON A CONDUCTING HALF-SPACE

$H_y$  (amp/m)  $10^2$

$\sigma = 0.001$  mho/m

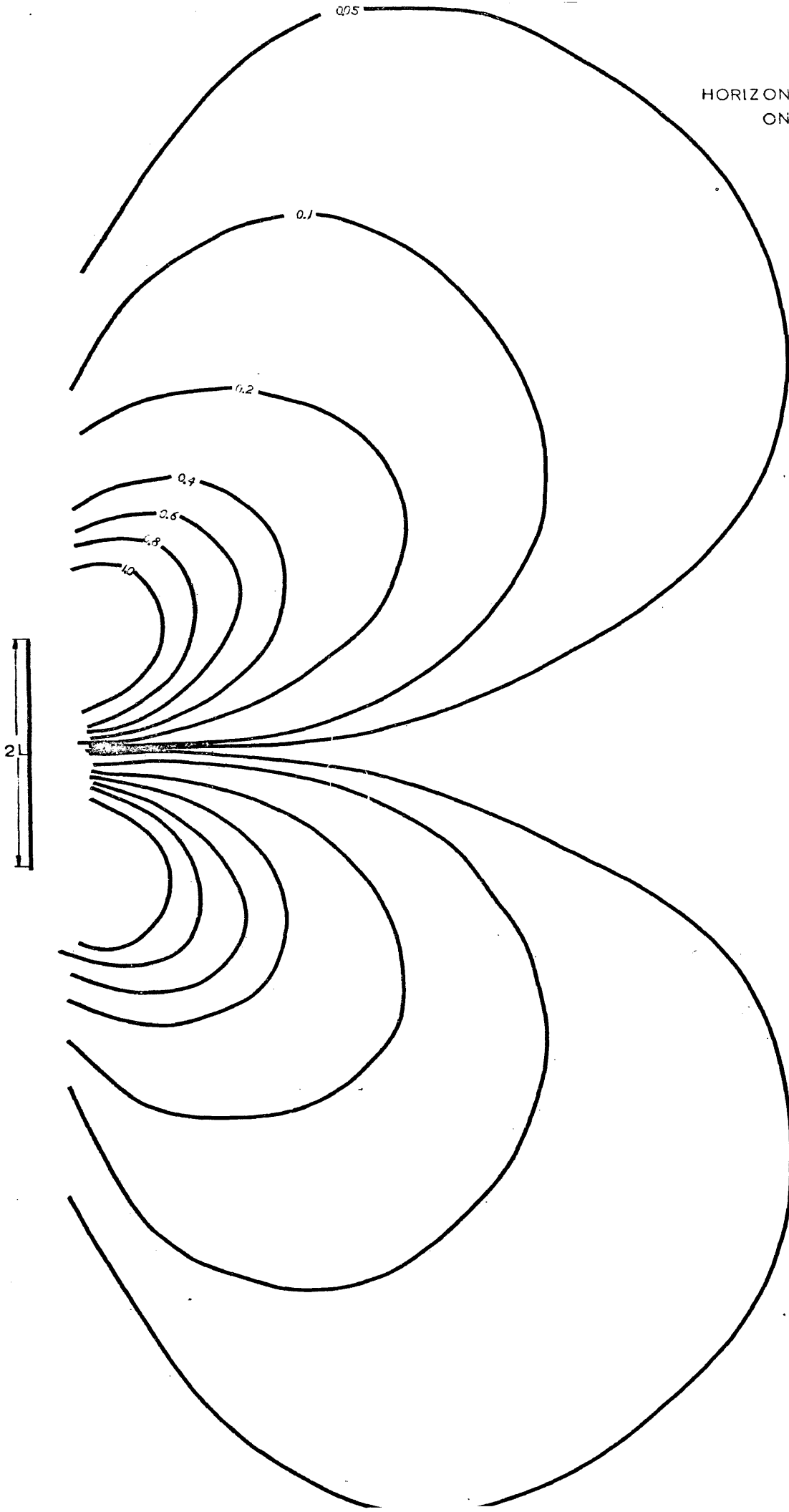
$f = 1000$  hz

$L = 50$  m

$I = 10$  amp

$\perp$  25m

Figure 13

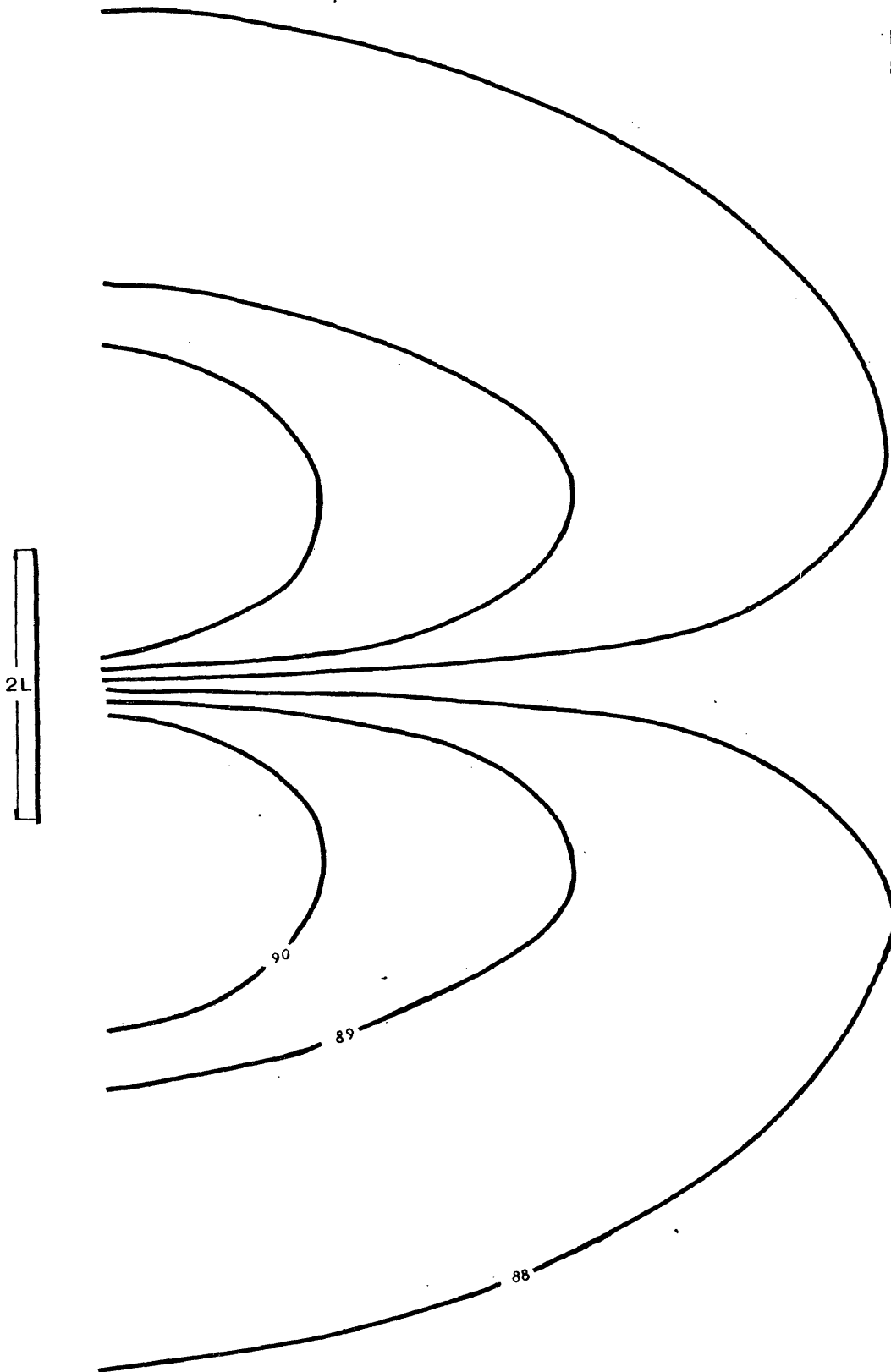


HORIZONTAL MAGNETIC FIELD PHASE  
ON A CONDUCTING HALF-SPACE

$\Psi_y$  (degrees)  
 $\sigma = 0.001$  mho/m  
 $f = 1000$  hz  
 $L = 50$  m  
 $I = 10$  amp

25m

Figure 14



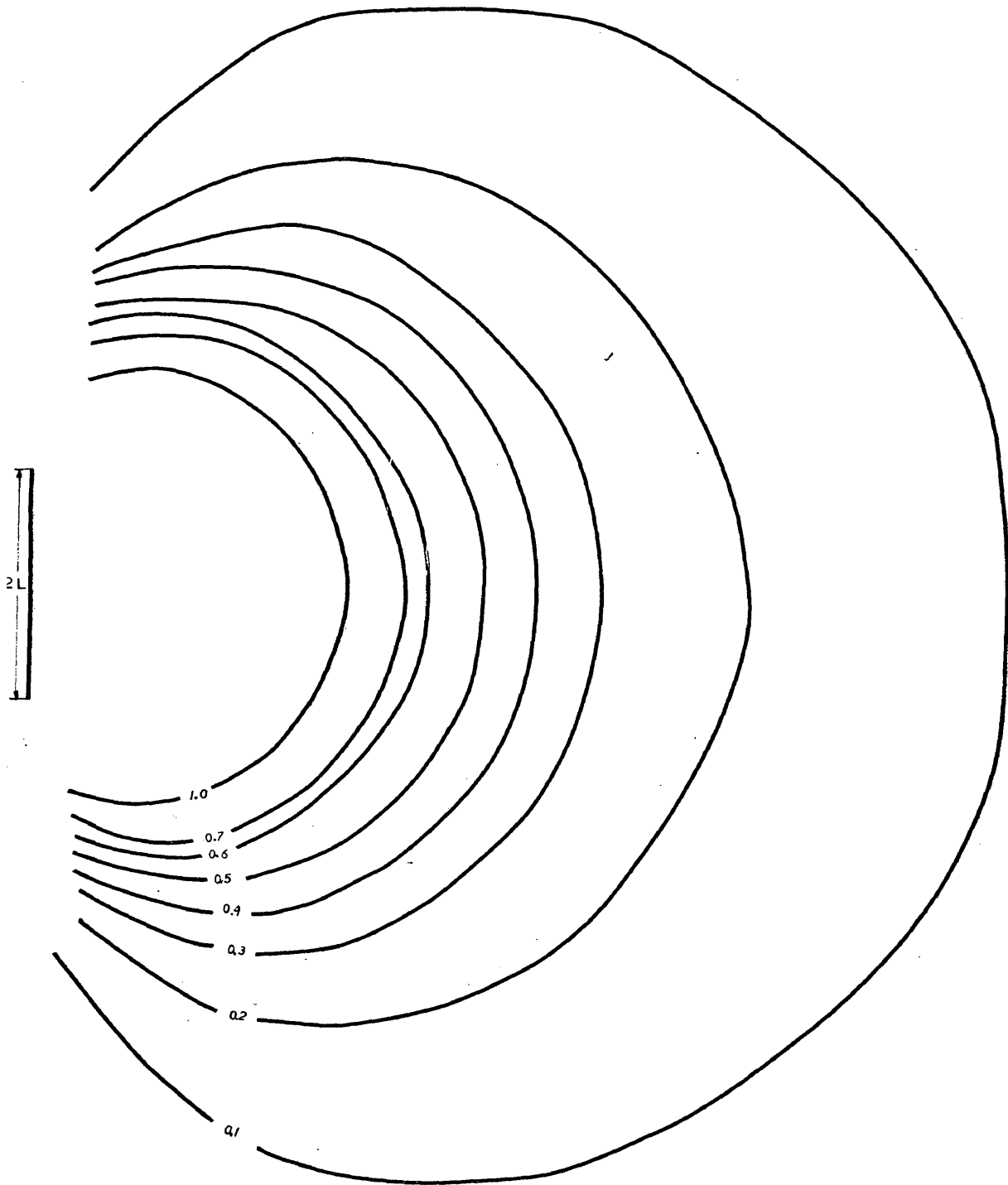


VERTICAL MAGNETIC FIELD AMPLITUDE  
ON A CONDUCTING HALF-SPACE  
Hz(amp/m)  $10^2$

$\sigma = 0.001$  mho/m  
 $f = 1000$  hz  
 $L = 50$  m  
 $I = 10$  amp

$\square 25$  m

Figure 15



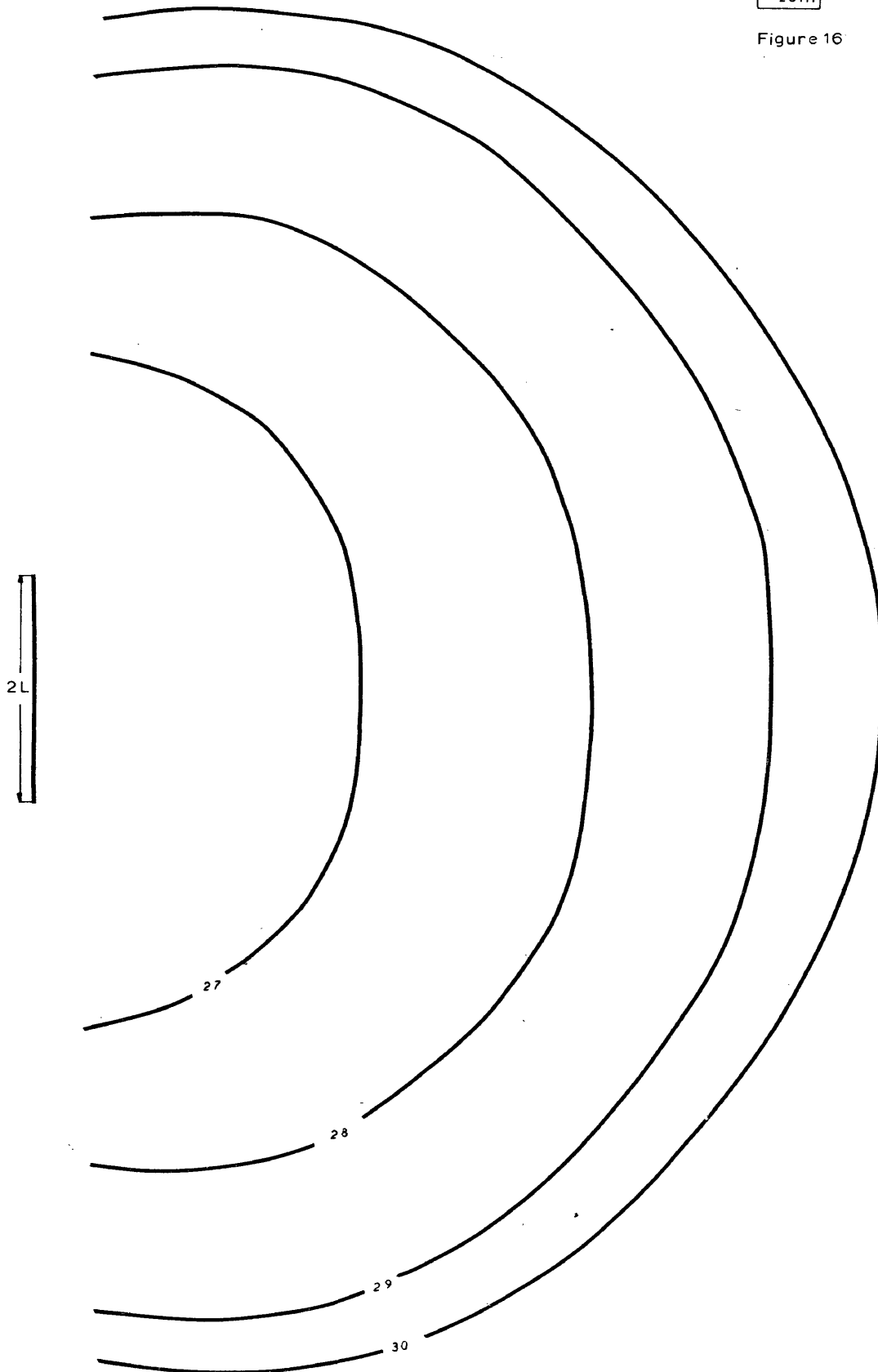
VERTICAL MAGNETIC FIELD PHASE  
ON A CONDUCTING HALF-SPACE

$\Psi_z$  (degrees)

$\sigma = 0.001$  mho/m  
 $f = 1000$  hz  
 $L = 50$  m  
 $I = 10$  amp

25m

Figure 16



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## CONCLUSIONS

The electromagnetic response of a two-dimensional conductor buried in the earth, for grounded cable source excitation, is theoretically formulated in the wave number domain using an integral equation solution and is calculated numerically by a point matching technique. The electromagnetic components are obtained in the space domain by using inverse Fourier transformation.

The transformation of the incident electromagnetic components and dyadic Green's matrix to wave number domain increases the number of integrals which must be evaluated numerically. The calculation of these integrals requires a large amount of computer time.

All test runs were executed on a Digital Equipment Corporation Model PDP-10 computer, a  $1\mu$  sec cycle-time machine. The computer run-time to calculate the three-magnetic components for one station was 8.6 seconds. The computer time in this case could be reduced if the integrals were computed once for given values of  $g_y$ ,  $X-X'$ , and  $Z + Z'$ , and then were supplied to the program as data.

Further investigation of this problem might involve the effect of conductivity variation, the effect of frequency variation, location of the dike relative to the source, and depth variation of the body can be studied using different models. Also, length of the grounded cable could be varied to ascertain its effect on the electromagnetic components at the surface of the earth.

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The results presented here are for a grounded cable as a source. However, the source can be arbitrary by changing the expressions for the incident electromagnetic fields, and then transforming to the wave number domain.

By changing the expressions for the incident fields and for the Green's dyadic matrix, the effect of a conductive overburden can be studied.

Some important factors which have not been considered in this thesis, multiple conductors, dip and arbitrary source can be analysed using the same formulation.

APPENDIX A

Derivation of the Green's Tensor (\*)

Consider the following geometry

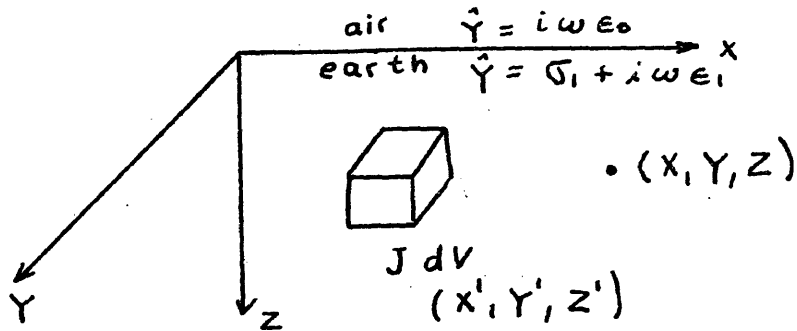


Figure A-1: Green's function geometry

We wish to find the dyadic Green's function relating the electric field at  $(X, Y, Z)$  to the elemental source current volume at  $(X', Y', Z')$ . That is, we want to find  $\bar{\bar{K}}$ , such that

$$\bar{E}(x, y, z) = \bar{\bar{K}}(x, y, z; x', y', z') \bar{J}(x', y', z') dV \quad (\text{A.1})$$

Then for a large volume of current, we can write

$$\bar{E} = \int_V \bar{\bar{K}} \cdot \bar{J} dV \quad (\text{A.2})$$

We begin with Maxwell's equations written in the form

(\*) (Hohmann, 1973)

$$-\nabla \times \bar{E} = \hat{z} \bar{H} \quad (\text{A.3})$$

$$\nabla \times \bar{H} = \hat{y} \bar{E} + \bar{J} \quad (\text{A.4})$$

Let the vector potential  $A$  be defined by the relation

$$\bar{H} = \nabla \times \bar{A} \quad (\text{A.5})$$

Then, from (A.3)

$$\nabla \times (\bar{E} + \hat{z} \bar{A}) = 0$$

which implies that

$$\bar{E} + \hat{z} \bar{A} = -\nabla \varphi,$$

or

$$\bar{E} = -\hat{z} \bar{A} - \nabla \varphi \quad (\text{A.6})$$

Taking the curl of (A.3) and substituting (A.4), we obtain

$$(\nabla^2 + k_1^2) \bar{E} = \nabla (\nabla \cdot \bar{E}) + \hat{z} \bar{J} \quad (\text{A.7})$$

where we have used the relation

$$\nabla \times \nabla \times \bar{F} = \nabla (\nabla \cdot \bar{F}) - \nabla^2 \bar{F} \quad (\text{A.8})$$

Substituting (A.6) into (A.7),

$$-\hat{z} (\nabla^2 + k_1^2) \bar{A} - (\nabla^2 + k_1^2) \nabla \varphi = -\nabla \left[ \hat{z} (\nabla \cdot \bar{A}) + \nabla^2 \varphi \right] + \hat{z} \bar{J}$$

Choosing 
$$\nabla \cdot \bar{A} = -\bar{y} \varphi, \quad (\text{A.9})$$

we obtain

$$(\nabla^2 + k_1^2) \bar{A} = -\bar{J} = -\bar{J} dv \delta(x') \delta(y') \delta(z') \quad (\text{A.10})$$

Substituting (A.9) into (A.6), we find

$$E = -\bar{z} \bar{A} + \frac{1}{\bar{y}} \nabla (\nabla \cdot \bar{A}) \quad (\text{A.11})$$

When (A.10) has been solved for A, the Green's tensor can be derived from A using (A.11).

Define a Fourier transform pair as follows:

$$F(\bar{K}) = \int_{-\infty}^{\infty} f(\bar{x}) e^{-i\bar{K} \cdot \bar{x}} d\bar{x}^3 \quad (\text{A.12})$$

$$f(\bar{x}) = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} F(\bar{K}) e^{i\bar{K} \cdot \bar{x}} d\bar{K}^3 \quad (\text{A.13})$$

Then, Fourier transforming (A.10) we obtain

$$\bar{A}(\bar{K}) = \frac{\bar{J} dv e^{-i\bar{K} \cdot \bar{x}}}{|\bar{K}|^2 - k_1^2} \quad (\text{A.14})$$

where

$$|\bar{K}|^2 = k_x^2 + k_y^2 + k_z^2$$

$$k_1^2 = \omega^2 \mu_1 \epsilon_1 - i \omega \mu_1 \sigma_1$$

Performing the inverse transform,

$$\bar{A}(k_x, k_y, z) = \frac{\bar{J} dV}{2\pi} \int_{-\infty}^{\infty} \frac{e^{i k_z (z-z')} e^{-i(k_x X + k_y Y)}}{k_z^2 + (k_x^2 + k_y^2 - k_i^2)} dk_z \quad (\text{A.15})$$

From Erdelyi (1954, v. 1, p. 8, #11),

$$\bar{A}(k_x, k_y, z) = -\bar{J} dV e^{-i(k_x X' + k_y Y')} \frac{e^{-\omega |z-z'|}}{\omega} \quad (\text{A.16})$$

where

$$\omega = (k_x^2 + k_y^2 - k_i^2)^{1/2}$$

Using the relation (Baños, 1966, p. 19),

$$\iint_{-\infty}^{\infty} f(k_x^2 + k_y^2) e^{i(k_x X + k_y Y)} dk_x dk_y = 2\pi \int_0^{\infty} f(\lambda) \bar{J}_0(\lambda r) \lambda d\lambda \quad (\text{A.17})$$

where

we can write

$$\lambda^2 = k_x^2 + k_y^2,$$

$$\bar{A}(x, y, z) = \frac{\bar{J} dV}{8\pi^2} \iint_{-\infty}^{\infty} \frac{e^{-\omega |z-z'|}}{\omega} e^{i[k_x(X-X') + k_y(Y-Y')]} dk_x dk_y$$

and

$$\bar{A}(r, z) = \frac{\bar{J} dV}{4\pi} \int_0^{\infty} \frac{e^{-\omega |z-z'|}}{\omega} \bar{J}_0(\lambda r) \lambda d\lambda \quad (\text{A.18})$$

where

$$r = [(x-x')^2 + (y-y')^2]^{1/2}$$



From Erdelyi (1954, v. 2, p. 9, #24) we find

$$\bar{A} = \frac{\bar{J} dV}{4\pi} \frac{e^{-ik_1 R}}{R} \quad (\text{A.19})$$

with

$$R = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2} \quad (\text{A.20})$$

The primary part of the Green's tensor can be derived from (A.19) using (A.11). To find the secondary part of the Green's tensor, we divide the field into TM and TE modes. The TM and TE vector potentials are defined by

$$\bar{A} = \varphi \bar{u}_z \quad (\text{TM}) \quad (\text{A.21})$$

$$\bar{F} = \theta \bar{u}_z \quad (\text{TE}) \quad (\text{A.22})$$

where  $\bar{u}_z$  is a unit vector in the Z direction, and the scalar potentials are solutions of

$$(\nabla^2 + k^2) \begin{Bmatrix} \varphi \\ \theta \end{Bmatrix} = 0 \quad (\text{A.23})$$

The solutions to (A.23) in transform space are

$$\varphi(k_x, k_y, z) = \varphi^+(k_x, k_y) e^{-uz} + \varphi^-(k_x, k_y) e^{+uz}$$

$$\theta(k_x, k_y, z) = \theta^+(k_x, k_y) e^{-uz} + \theta^-(k_x, k_y) e^{+uz}$$

While the electromagnetic fields are given by

$$\begin{aligned}
 E_x &= \frac{1}{\bar{y}} \frac{\partial^2 \varphi}{\partial x \partial z} - \frac{\partial \theta}{\partial y} \quad ; \quad H_x = \frac{\partial \varphi}{\partial y} + \frac{1}{z} \frac{\partial^2 \theta}{\partial x \partial z} \\
 E_y &= \frac{1}{\bar{y}} \frac{\partial^2 \varphi}{\partial y \partial z} + \frac{\partial \theta}{\partial x} \quad ; \quad H_y = -\frac{\partial \varphi}{\partial x} + \frac{1}{z} \frac{\partial^2 \theta}{\partial y \partial z} \quad (\text{A.26}) \\
 E_z &= \frac{1}{\bar{y}} \left( \frac{\partial^2}{\partial z^2} + k^2 \right) \varphi \quad ; \quad H_z = \frac{1}{z} \left( \frac{\partial^2}{\partial z^2} + k^2 \right) \theta
 \end{aligned}$$

For a homogeneous earth, we set up solutions in the different regions as shown below:

$$\begin{array}{c}
 (\varphi_0, \theta_0) e^{u_0 z} \\
 \hline
 (\varphi_1^+, \theta_1^+) e^{-u_1 z} + (\varphi_1^-, \theta_1^-) e^{u_1 z} \\
 \hline
 (\varphi_1^+, \theta_1^+) e^{-u_1 z} \\
 z = z'
 \end{array}
 \quad \begin{array}{l}
 z = 0 \\
 \\
 z = z'
 \end{array}$$

Figure A.2

where  $\varphi_1^- e^{u_1 z}$  and  $\theta_1^- e^{u_1 z}$  are the TM and TE components, respectively, of the primary field.

By analogy with the plane wave solution of Wait (1962, p. 11 ff) we can write the secondary potentials as

$$\varphi_1^+ = R_{TM} \varphi_1^- \quad (\text{A.27})$$

$$\theta_1^+ = R_{TE} \theta_1^-$$

where the reflection coefficients are given by

$$R_{TE} = \frac{N_1 - N_0}{N_1 + N_0}$$

and

(A.28)

$$R_{TM} = \frac{k_1 - k_0}{k_1 + k_0}$$

with

$$N = \frac{u}{z} \quad \text{and} \quad k = \frac{u}{y}$$

The next step is to find the primary TM and TE potentials

From (A.5) and (A.11), the primary vertical fields for an X-directed current element are

$$H_z = -i k_y A_x \quad (A.29)$$

$$E_z = i \frac{k_x}{\sigma_1} \frac{\partial A_x}{\partial z} \quad (A.30)$$

for a Z-directed current element, we have

$$\begin{aligned} E_z &= \left( \frac{\partial^2}{\partial z^2} + k_1^2 \right) A_z \quad (A.31) \\ &= \frac{(k_x^2 + k_y^2)}{\sigma_1} A_z \end{aligned}$$

Now, from (A.16), we have

$$\begin{Bmatrix} A_x \\ A_z \end{Bmatrix} = \begin{Bmatrix} J_x \\ J_z \end{Bmatrix} \frac{dv}{z} \frac{e^{\pm \omega(z-z')}}{\omega} e^{-i(k_x X' + k_y Y')} \quad (\text{A.32})$$

where (+) stands for (above) and (-) for (below) the current element.

Combining (A.26), (A.30) and (A.32), we find

$$\frac{k_x^2 + k_y^2}{\sigma_1} \psi_{ix}^- = \pm \frac{J_x dv}{2\sigma_1} i k_x e^{\mp \omega(z-z')} e^{-i(k_x X' + k_y Y')}$$

so that

$$(\text{TM}): \quad \psi_{ix}^- = \pm \frac{J_x dv}{2} \frac{i k_x}{k_x^2 + k_y^2} e^{\mp \omega(z-z')} e^{-i(k_x X' + k_y Y')} \quad (\text{A.33})$$

For the TE potential, we combined (A.26), (A.29), and (A.32)

$$\frac{k_x^2 + k_y^2}{z} \theta_{ix}^- = - \frac{J_x dv}{2\omega} i k_y e^{\mp \omega(z-z')} e^{-i(k_x X' + k_y Y')}$$

so that

$$(\text{TE}): \quad \theta_{ix}^- = - i\omega\mu_0 \frac{J_x dv}{2} \frac{i k_y}{\omega(k_x^2 + k_y^2)} e^{\mp \omega(z-z')} e^{-i(k_x X' + k_y Y')} \quad (\text{A.34})$$

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For a Z-directed current element, there is only a TM potential.

From (A.26), (A.31), and (A.32),

$$(TM): \varphi_{12}^- = \frac{J_z dV}{2} \frac{e^{\mp \omega_1 z'}}{\omega_1} e^{-i(k_x X' + k_y Y')} \quad (A.35)$$

Then, in the earth above the dipole,

$$\begin{Bmatrix} \varphi \\ \theta \end{Bmatrix} = \begin{Bmatrix} \varphi_1^- \\ \theta_1^- \end{Bmatrix} \left[ e^{+\omega_1 z} + \begin{Bmatrix} R_{TM} \\ R_{TE} \end{Bmatrix} e^{-\omega_1 z} \right] \quad (A.36)$$

While, below the dipole,

$$\begin{Bmatrix} \varphi \\ \theta \end{Bmatrix} = \begin{Bmatrix} \varphi_1^- \\ \theta_1^- \end{Bmatrix} \left[ e^{-\omega_1 z} + \begin{Bmatrix} R_{TM} \\ R_{TE} \end{Bmatrix} e^{-\omega_1(z+2z')} \right] \quad (A.37)$$

For a homogeneous earth at low frequencies with  $\mu_1 = \mu_0$ , the reflection coefficients are given by

$$R_{TE} = \frac{\omega_1 - \omega_0}{\omega_1 + \omega_0} \approx \frac{\omega_1 - \lambda}{\omega_1 + \lambda} \quad (A.38)$$

$$R_{TM} = \frac{\frac{\hat{y}_0}{\hat{y}_1} \omega_1 - \omega_0}{\frac{\hat{y}_0}{\hat{y}_1} \omega_1 + \omega_0} \approx -1 \quad (A.39)$$

$$\lambda = (k_x^2 + k_y^2)^{1/2} \quad (A.40)$$

Let us now derive the dyadic Green's function in Fourier transform space. Using (A.26), (A.33), (A.34), (A.36) and (A.37), we find

$$\begin{aligned} K_{xx} = \frac{1}{2\sigma_1} \left\{ \left[ e^{\pm \omega_1(z-z')} - R_{TM} e^{-\omega_1(z+z')} \right] \frac{k_x^2 \omega_1}{k_x^2 + k_y^2} \right. \\ \left. + \left[ e^{\pm \omega_1(z-z')} + R_{TE} e^{-\omega_1(z+z')} \right] \frac{k_y^2 k_1^2}{(k_x^2 + k_y^2) \omega_1} \right\} e^{-i(k_x x' + k_y y')} \end{aligned} \quad (A.41)$$

In order to separate the Green's function into current and charge parts, we rewrite (A.41) as

$$\begin{aligned} K_{xx} = \frac{1}{2\sigma_1} \left\{ -k_x^2 \left[ \frac{\omega_1 + \frac{k_1^2}{\omega_1}}{k_x^2 + k_y^2} \right] e^{\pm \omega_1(z-z')} + \frac{\left[ \left( \frac{\omega_1 - \lambda}{\omega_1 + \lambda} \right) \frac{k_1^2}{\omega_1} + \omega_1 \right] e^{-\omega_1(z+z')}}{k_x^2 + k_y^2} \right. \\ \left. + \frac{k_1^2}{\omega_1} \left[ e^{\pm \omega_1(z-z')} + \left( \frac{\omega_1 - \lambda}{\omega_1 + \lambda} \right) e^{-\omega_1(z+z')} \right] \right\} \cdot e^{-i(k_x x' + k_y y')} \end{aligned} \quad (A.42)$$

The primary part of the first term reduces us  $1/\omega_1$ , while the secondary part reduces to

$$\frac{\left( \frac{\omega_1 - \lambda}{\omega_1 + \lambda} \right) \frac{k_1^2}{\omega_1} + \omega_1}{\lambda^2} = \frac{-\omega_1^2 + 2\lambda\omega_1 - \lambda^2 + \omega_1^2}{\lambda^2 \omega_1} = \frac{2}{\lambda} - \frac{1}{\omega_1}$$

Thus, finally, in  $K_x, K_y$  space, we have

$$K_{XX} = \frac{1}{2\sigma_1 \omega_1} \left\{ -k_x^2 \left[ e^{\pm \omega_1(z-z')} + \left( \frac{2\omega_1}{\lambda} - 1 \right) e^{-\omega_1(z+z')} \right] \right. \\ \left. + k_x^2 \left[ e^{\pm \omega_1(z-z')} + \left( \frac{\omega_1 - \lambda}{\omega_1 + \lambda} \right) e^{-\omega_1(z+z')} \right] \right\} \cdot e^{-i(k_x x' + k_y y')} \quad (\text{A.43})$$

Similarly,

$$K_{YX} = \frac{1}{2\sigma_1} \left\{ \left[ e^{\pm \omega_1(z-z')} + e^{-\omega_1(z+z')} \right] \cdot \frac{-k_x k_y \omega_1}{k_x^2 + k_y^2} \right. \\ \left. + k_x^2 \left[ e^{\pm \omega_1(z-z')} + \frac{(\omega_1 - \lambda)}{(\omega_1 + \lambda)} e^{-\omega_1(z+z')} \right] \cdot \frac{-k_x k_y}{(k_x^2 + k_y^2) \omega_1} \right\} e^{-i(k_x x' + k_y y')} \quad (\text{A.44})$$

Combining terms,

$$K_{YX} = \frac{1}{2\sigma_1 \omega_1} \left\{ -k_x k_y \left[ e^{\pm \omega_1(z-z')} + \left( \frac{2\omega_1}{\lambda} - 1 \right) e^{-\omega_1(z+z')} \right] \right\} e^{-i(k_x x' + k_y y')} \quad (\text{A.45})$$

Also,

$$K_{ZX} = \frac{1}{2\sigma_1} \left\{ i k_x \left[ \pm e^{\pm \omega_1(z-z')} - e^{-\omega_1(z+z')} \right] \right\} e^{-i(k_x x' + k_y y')} \quad (\text{A.46})$$

From (A.26) and (A.35), we have

$$K_{xz} = \frac{i k_x}{2\sigma_1} \left[ \pm e^{\pm \omega_1(z-z')} + e^{-\omega_1(z+z')} \right] e^{-i(k_x x' + k_y y')} \quad (\text{A.47})$$

$$K_{zz} = \frac{\omega_1^2 + k_z^2}{2\sigma_1 \omega_1} \left[ e^{\pm \omega_1(z-z')} - e^{-\omega_1(z+z')} \right] e^{-i(k_x x' + k_y y')} \quad (\text{A.48})$$

or,

$$K_{zz} = \frac{1}{2\sigma_1 \omega_1} \left\{ \omega_1^2 \left[ e^{\pm \omega_1(z-z')} - e^{-\omega_1(z+z')} \right] + k_z^2 \left[ e^{\pm \omega_1(z-z')} - e^{-\omega_1(z+z')} \right] \right\} e^{-i(k_x x' + k_y y')} \quad (\text{A.49})$$

The Green's function in X, Y, Z space is given by inverse Fourier transforming according to (A.13). Alternatively, we can use (A.17) to find the solution in terms of a Hankel transform. Then, from (A.43), we have

$$K_{xx} = \frac{1}{4\pi\sigma_1} \int_0^\infty \left\{ \frac{\partial^2}{\partial x^2} \left[ \left( \frac{\lambda}{\omega_1} \right) e^{\pm \omega_1(z-z')} + \left( 2 - \frac{\lambda}{\omega_1} \right) e^{-\omega_1(z+z')} \right] + k_z^2 \left[ \left( \frac{\lambda}{\omega_1} \right) e^{\pm \omega_1(z-z')} + \left( \frac{\omega_1 - \lambda}{\omega_1 + \lambda} \right) \frac{\lambda}{\omega_1} e^{-\omega_1(z+z')} \right] \right\} J_0(\lambda r) d\lambda \quad (\text{A.50})$$



From (A.45),

$$K_{YX} = \frac{1}{4\pi\sigma_1} \left\{ \frac{\partial^2}{\partial x \partial y} \int_0^\infty \left[ \left( \frac{\lambda}{\omega_1} \right) e^{\pm \omega_1(z-z')} + \left( z - \frac{\lambda}{\omega_1} \right) e^{-\omega_1(z+z')} \right] J_0(\lambda r) d\lambda \right\} \quad (\text{A.51})$$

From (A.46),

$$K_{ZX} = \frac{1}{4\pi\sigma_1} \left\{ \frac{\partial^2}{\partial x \partial z} \int_0^\infty \left[ e^{\pm \omega_1(z-z')} + e^{-\omega_1(z+z')} \right] \frac{\lambda}{\omega_1} J_0(\lambda r) \right\} d\lambda \quad (\text{A.52})$$

From (A.47),

$$K_{XZ} = \frac{1}{4\pi\sigma_1} \frac{\partial^2}{\partial x \partial z} \int_0^\infty \left[ e^{\pm \omega_1(z-z')} - e^{-\omega_1(z+z')} \right] \frac{\lambda}{\omega_1} J_0(\lambda r) d\lambda \quad (\text{A.53})$$

From (A.49),

$$K_{ZZ} = \frac{1}{4\pi\sigma_1} \int_0^\infty \left[ \frac{\partial^2}{\partial z^2} \left( e^{\pm \omega_1(z-z')} - e^{-\omega_1(z+z')} \right) + k_1^2 \left[ e^{\pm \omega_1(z-z')} - e^{-\omega_1(z+z')} \right] \right] \frac{\lambda}{\omega_1} J_0(\lambda r) d\lambda \quad (\text{A.54})$$

Now, using (A.19), and letting

$$\mathcal{Y}_1 = \frac{1}{r} \int_0^\infty \left( z - \frac{\lambda}{\omega_1} \right) e^{-\omega_1(z+z')} \lambda J_1(\lambda r) =$$

$$\frac{1}{R^3} + \frac{1}{r} \int_0^\infty \left[ \left( z - \frac{\lambda}{\omega_1} \right) e^{-\omega_1(z+z')} - e^{-\lambda(z+z')} \right] \lambda J_1(\lambda r) d\lambda \quad (\text{A.55})$$

$$\tau_2 = \int_0^{\infty} \left( \frac{\omega_1 - \lambda}{\omega_1 + \lambda} \right) \frac{\lambda}{\omega_1} e^{-\omega_1(z+z')} J_0(\lambda r) d\lambda \quad (\text{A.56})$$

Other necessary relations are

$$\frac{\partial}{\partial x} [J_0(\lambda r)] = - \frac{(x-x')}{r} \lambda J_1(\lambda r) \quad (\text{A.57})$$

$$\int_0^{\infty} e^{-\lambda(z+z')} J_0(\lambda r) d\lambda = \frac{1}{R_s} \quad (\text{A.58})$$

$$\nabla \left( \frac{e^{-ik_1 R}}{R} \right) = - \left[ (x-x') \hat{x} + (y-y') \hat{y} \pm (z-z') \hat{z} \right] \tau_3 \quad (\text{A.59})$$

with 
$$\tau_3 = (ik_1 R + 1) \frac{e^{-ik_1 R}}{R^3} \quad (\text{A.60})$$

$$\nabla \left( \frac{e^{-ik_1 R_s}}{R_s} \right) = - \left[ (x-x') \hat{x} + (y-y') \hat{y} + (z+z') \hat{z} \right] \tau_4 \quad (\text{A.61})$$

with 
$$\tau_4 = (ik_1 R_s + 1) \frac{e^{-ik_1 R_s}}{R_s^3}$$

with 
$$R = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2} \quad (\text{A.62})$$

$$R = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2} \quad (\text{A.63})$$

$$R_s = [(x-x')^2 + (y-y')^2 + (z+z')^2]^{1/2} \quad (\text{A.64})$$

Thus, the dyadic Green's function is given by

$$\begin{aligned}
 K_{XX} &= \frac{1}{4\pi\sigma_1} \left\{ -\frac{\partial}{\partial X} \left[ (X-X') (\mathcal{T}_3 + \mathcal{T}_1) \right] + k_1^2 \left[ \frac{e^{-ik_1 R}}{R} + \mathcal{T}_2 \right] \right\} \\
 K_{YX} &= \frac{1}{4\pi\sigma_1} \left\{ -\frac{\partial}{\partial X} \left[ (Y-Y') (\mathcal{T}_3 + \mathcal{T}_1) \right] \right\} \\
 K_{ZX} &= \frac{1}{4\pi\sigma_1} \left\{ -\frac{\partial}{\partial X} \left[ (Z-Z') \mathcal{T}_3 + (Z+Z') \mathcal{T}_4 \right] \right\} \\
 K_{XY} &= \frac{1}{4\pi\sigma_1} \left\{ -\frac{\partial}{\partial Y} \left[ (X-X') (\mathcal{T}_3 + \mathcal{T}_1) \right] \right\} \\
 K_{YY} &= \frac{1}{4\pi\sigma_1} \left\{ -\frac{\partial}{\partial Y} \left[ (Y-Y') (\mathcal{T}_3 + \mathcal{T}_1) \right] + k_1^2 \left[ \frac{e^{-ik_1 R}}{R} + \mathcal{T}_2 \right] \right\} \\
 K_{ZY} &= \frac{1}{4\pi\sigma_1} \left\{ -\frac{\partial}{\partial Y} \left[ (Z-Z') \mathcal{T}_3 + (Z+Z') \mathcal{T}_4 \right] \right\} \quad (\text{A.65}) \\
 K_{XZ} &= \frac{1}{4\pi\sigma_1} \left\{ -\frac{\partial}{\partial Z} \left[ (X-X') (\mathcal{T}_3 - \mathcal{T}_4) \right] \right\} \\
 K_{YZ} &= \frac{1}{4\pi\sigma_1} \left\{ -\frac{\partial}{\partial Z} \left[ (Y-Y') (\mathcal{T}_3 - \mathcal{T}_4) \right] \right\} \\
 K_{ZZ} &= \frac{1}{4\pi\sigma_1} \left\{ -\frac{\partial}{\partial Z} \left[ (Z-Z') \mathcal{T}_3 - (Z+Z') \mathcal{T}_4 \right] + k_1^2 \left[ \frac{e^{-ik_1 R}}{R} - \frac{e^{-ik_1 R_s}}{R_s} \right] \right\}
 \end{aligned}$$

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APPENDIX B

The derivation of the Green's tensor for three-dimensional bodies buried in the earth is given by Hohmann (1973),

$$K_{xx} = \frac{1}{4\pi\sigma_1} \left\{ \frac{\partial}{\partial x'} [(x-x')(\tau_3 + \tau_1) - \gamma_1^2 \left[ \frac{e^{-\gamma_1 R_0}}{R_0} + \tau_2 \right]] \right\}$$

$$K_{yx} = \frac{1}{4\pi\sigma_1} \frac{\partial}{\partial x'} [(x-x')(\tau_3 + \tau_1)]$$

$$K_{zx} = \frac{1}{4\pi\sigma_1} \frac{\partial}{\partial x'} [(z-z')\tau_3 + (z+z')\tau_4]$$

$$K_{xy} = \frac{1}{4\pi\sigma_1} \frac{\partial}{\partial y'} [(x-x')(\tau_3 + \tau_1)]$$

$$K_{yy} = \frac{1}{4\pi\sigma_1} \left\{ \frac{\partial}{\partial y'} [(y-y')(\tau_3 + \tau_1)] - \gamma_1^2 \left[ \frac{e^{-\gamma_1 R}}{R_0} + \tau_2 \right] \right\}$$

$$K_{zy} = \frac{1}{4\pi\sigma_1} \frac{\partial}{\partial y'} [(z-z')\tau_3 + (z+z')\tau_4]$$

$$K_{xz} = \frac{1}{4\pi\sigma_1} \frac{\partial}{\partial z'} [(x-x')(\tau_3 + \tau_4)]$$

$$K_{yz} = \frac{1}{4\pi\sigma_1} \frac{\partial}{\partial z'} [(y-y')(\tau_3 + \tau_4)]$$

$$K_{zz} = \frac{1}{4\pi\sigma_1} \left\{ \frac{\partial}{\partial z'} [(z-z')\tau_3 + (z+z')\tau_4] - \gamma_1^2 \left[ \frac{e^{-\gamma_1 R_0}}{R_0} - \frac{e^{-\gamma_1 R}}{R} \right] \right\}$$

(B-1)

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where

$$\gamma_1^2 = i \omega \mu_0 \sigma_1$$

$$\mathcal{J}_1 = \frac{1}{4\pi\rho_0} \int_0^\infty \left(2 - \frac{\lambda}{\omega_1}\right) \frac{e^{-\omega_1(z+z')}}{\lambda J_1(\lambda\rho_0)} d\lambda \quad (\text{B-2})$$

$$\mathcal{J}_2 = \frac{1}{4\pi} \int_0^\infty \left(\frac{\omega_1 - \lambda}{\omega_1 + \lambda}\right) \frac{\lambda}{\omega_1} \frac{e^{-\omega_1(z+z')}}{J_0(\lambda\rho_0)} d\lambda \quad (\text{B-3})$$

$$\mathcal{J}_3 = (-\gamma_1 R_0 + 1) \frac{e^{-\gamma_1 R_0}}{4\pi R_0^3} \quad (\text{B-4})$$

$$\mathcal{J}_4 = (-\gamma_1 R + 1) \frac{e^{-\gamma_1 R}}{4\pi R} \quad (\text{B-5})$$

$$\omega_1 = (\lambda^2 + \gamma_1^2)^{1/2}$$

$$\rho_0 = [(x-x')^2 + (y-y')^2]^{1/2}$$

$$R_0 = [\rho_0^2 + (z-z')^2]^{1/2}$$

$$R = [\rho_0^2 + (z+z')^2]^{1/2}$$

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The Green's tensor can be divided into primary and secondary parts. The primary or singular part represents the effect of the whole space of conductivity  $\sigma_1$ , and the secondary or nonsingular part represents the effect air earth interface (Hohmann, 1973). Then

$$\overset{=}{K} = \overset{=}{K}_s + \overset{=}{K}_p$$

The primary part of the Green's tensor is

$$\overset{\parallel}{K}_p = \begin{pmatrix} \frac{\partial}{\partial x'} (x-x') \tau_3 + P_0 & \frac{\partial}{\partial y'} (x-x') \tau_3 & \frac{\partial}{\partial z'} (x-x') \tau_3 \\ \frac{\partial}{\partial x'} (y-y') \tau_3 & \frac{\partial}{\partial y'} (y-y') + P_0 & \frac{\partial}{\partial z'} (y-y') \tau_3 \\ \frac{\partial}{\partial x'} (z-z') \tau_3 & \frac{\partial}{\partial y'} (z-z') \tau_3 & \frac{\partial}{\partial z'} (z-z') \tau_3 + P_0 \end{pmatrix}$$

where

(B-6)

The secondary part of the Green's tensor is

$$\overset{\parallel}{K}_s = \begin{pmatrix} \frac{\partial}{\partial x'} (x-x') \tau_1 + \tau_2 & \frac{\partial}{\partial y'} (x-x') \tau_1 & \frac{\partial}{\partial z'} (x-x') \tau_4 \\ \frac{\partial}{\partial x'} (y-y') \tau_1 & \frac{\partial}{\partial y'} (y-y') \tau_1 + \tau_2 & \frac{\partial}{\partial z'} (y-y') \tau_4 \\ \frac{\partial}{\partial x'} (z+z') \tau_4 & \frac{\partial}{\partial y'} (z+z') \tau_4 & \frac{\partial}{\partial z'} (z+z') \tau_4 - P \end{pmatrix}$$

(B-7)

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Some of the elements of the singular matrix can be rewritten as follows:

$$\begin{aligned} (X-X') \mathcal{T}_3 &= \frac{1}{4\pi} \frac{\partial}{\partial X'} \left( \frac{e^{-\gamma_1 R_0}}{R_0} \right) \\ (Y-Y') \mathcal{T}_3 &= \frac{1}{4} \frac{\partial}{\partial Y'} \left( \frac{e^{-\gamma_1 R_0}}{R_0} \right) \\ (Z-Z') \mathcal{T}_3 &= \frac{1}{4\pi} \frac{\partial}{\partial Z'} \left( \frac{e^{-\gamma_1 R_0}}{R_0} \right) \end{aligned} \quad (\text{B-8})$$

Then, the singular Green's tensor is given by

$$\overline{\overline{\mathcal{K}}_p} = \begin{pmatrix} \frac{\partial^2 P_0}{\partial X'^2} + P_0 & \frac{\partial^2 P_0}{\partial Y' \partial X'} & \frac{\partial^2 P_0}{\partial Z' \partial X'} \\ \frac{\partial^2 P_0}{\partial X' \partial Y'} & \frac{\partial^2 P_0}{\partial Y'^2} + P_0 & \frac{\partial^2 P_0}{\partial Z' \partial Y'} \\ \frac{\partial^2 P_0}{\partial X' \partial Z'} & \frac{\partial^2 P_0}{\partial Y' \partial Z'} & \frac{\partial^2 P_0}{\partial Z'^2} + P_0 \end{pmatrix} \quad (\text{B-9})$$

where

$$P_0 = \frac{e^{-\gamma_1 R_0}}{R_0}$$

The nonsingular Green's tensor is given by

$$\overline{\overline{\mathcal{K}}_s} = \begin{pmatrix} \frac{\partial}{\partial X'} (X-X') \mathcal{T}_1 + \mathcal{T}_2 & \frac{\partial}{\partial Y'} (X-X') \mathcal{T}_1 & \frac{\partial^2 P}{\partial Z' \partial X'} \\ \frac{\partial}{\partial X'} (Y-Y') \mathcal{T}_1 & \frac{\partial}{\partial Y'} (Y-Y') \mathcal{T}_1 + \mathcal{T}_2 & \frac{\partial^2 P}{\partial Z' \partial Y'} \\ \frac{\partial^2 P}{\partial X' \partial Z'} & \frac{\partial^2 P}{\partial Y' \partial Z'} & \frac{\partial^2 P}{\partial Z'^2} - P \end{pmatrix} \quad (\text{B-10})$$

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If we multiply the integral (B-2) by  $(x-x')$ , we obtain

$$(x-x') \mathcal{T}_1 = \frac{1}{4\pi} \int_0^{\infty} \left(2 - \frac{\lambda}{u_1}\right) e^{-u_1 \frac{(z+z')}{\rho_0}} \frac{\lambda (x-x')}{\rho_0} J_1(\lambda \rho_0) d\lambda$$

and after substituting the relations

(B-11)

$$\frac{\partial}{\partial x'} (J_0(\lambda \rho_0)) = \lambda J_1(\lambda \rho_0) \frac{(x-x')}{\rho_0}$$

(B-12)

in (B-11), we obtain

$$(x-x') \mathcal{T}_1 = \frac{1}{4\pi} \int_0^{\infty} \left(2 - \frac{\lambda}{u_1}\right) e^{-u_1 \frac{(z+z')}{\rho_0}} \frac{\partial}{\partial x'} (J_0(\lambda \rho_0)) d\lambda$$

In the same way we can write

(B-13)

$$(y-y') \mathcal{T}_1 = \frac{1}{4\pi} \int_0^{\infty} \left(2 - \frac{\lambda}{u_1}\right) e^{-u_1 \frac{(z+z')}{\rho_0}} \frac{\partial}{\partial y'} (J_0(\lambda \rho_0)) d\lambda$$

(B-14)

### The Green's Tensor in the Wave Number Domain

The elements of the singular and nonsingular Green's tensor are calculated analytically in the wave number domain.

The elements of the singular Green's tensor (B-9) are

$$\frac{\partial^2 \mathcal{P}_0}{\partial x'^2} \rightarrow -2 \left\{ u_1^2 \frac{(x-x')^2}{\rho_0^2} \kappa_0(u_1 \rho_0) + \frac{u_1}{\rho_0^3} \kappa_1(u_1 \rho_0) [(x-x')^2 - (z-z')^2] \right\}$$

$$\frac{\partial^2 \mathcal{P}_0}{\partial z'^2} \rightarrow -2 \left\{ u_1^2 \frac{(z-z')^2}{\rho_0^2} \kappa_0(u_1 \rho_0) + \frac{u_1}{\rho_0^3} \kappa_1(u_1 \rho_0) [(z-z')^2 - (x-x')^2] \right\}$$

$$\frac{\partial^2 \mathcal{P}_0}{\partial y'^2} \rightarrow -2 u_1^2 \kappa_0(u_1 \rho_0)$$

$$\mathcal{P}_0 \rightarrow 2 \kappa_0(u_1 \rho_0)$$



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$$\frac{\partial^2 P_0}{\partial x' \partial y'} \longrightarrow 2 i g_Y u_Y \frac{(x-x')}{\rho_0} \kappa_1(u_Y \rho_0)$$

$$\frac{\partial^2 P_0}{\partial x' \partial z'} \longrightarrow 2 u_Y \frac{(z-z')(x-x')}{\rho_0^2} \left\{ \kappa_0(u_Y \rho_0) u_Y + 2 \kappa_1(u_Y \rho_0) \right\}$$

$$\frac{\partial^2 P_0}{\partial y' \partial z'} \longrightarrow 2 i g_Y \frac{u_Y (z-z')}{\rho_0} \kappa_1(u_Y \rho_0) \quad (B-15)$$

The elements of the nonsingular Green's tensor in the wave number domain are given by

$$\frac{\partial}{\partial x'} (x-x') \mathcal{T}_i \longrightarrow -\frac{1}{2\pi} \int_{g_0}^{\infty} \left(2 - \frac{g}{u}\right) v e^{-u(z+z')} \frac{e^{-i(x-x')v}}{\cos[(x-x')v]} dg \quad (B-16)$$

$$\frac{\partial}{\partial x'} (y-y') \mathcal{T}_i \longrightarrow \frac{g_Y i}{2\pi} \int_{g_0}^{\infty} \left(2 - \frac{g}{u}\right) e^{-u(z+z')} \frac{e^{-i(x-x')v}}{\sin[(x-x')v]} dg \quad (B-17)$$

$$\frac{\partial}{\partial y'} (y-y') \mathcal{T}_i \longrightarrow -\frac{g_Y^2}{2\pi} \int_{g_0}^{\infty} \left(2 - \frac{g}{u}\right) e^{-u(z+z')} \frac{e^{-i(x-x')v}}{\cos[(x-x')v]} dg \quad (B-18)$$

$$\frac{\partial^2 P}{\partial x' \partial z'} \longrightarrow 2 u_Y (z+z')(x-x') \left\{ \kappa_0(u_Y \rho_0) u_Y + 2 \kappa_1\left(\frac{u_Y \rho_0}{\rho_0}\right) \right\} \quad (B-19)$$

$$\frac{\partial P}{\partial y' \partial z'} \longrightarrow -2 i g_Y u_Y (z+z') \kappa_1(u_Y \rho_0) \quad (B-20)$$

$$\frac{\partial^2 P}{\partial z'^2} \longrightarrow 2 \left\{ u_Y^2 \frac{(z+z')}{\rho^2} \kappa_0(u_Y \rho_0) + \frac{u_Y \kappa_1(u_Y \rho_0)}{\rho^3} [(z+z')^2 - (x-x')^2] \right\} \quad (B-21)$$

$$\frac{\partial (x-x') \mathcal{T}_i}{\partial y'} = \frac{\partial (y-y') \mathcal{T}_i}{\partial x'} \quad (B-22)$$

Finally we obtain

$$\mathcal{T}_i(g_Y) \longrightarrow \frac{1}{2\pi} \int_{g_0}^{\infty} \left(\frac{v-g}{u+g}\right) \frac{g}{u} e^{-u(z+z')} \frac{e^{-i(x-x')v}}{\cos[(x-x')v]} dg \quad (B-23)$$

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APPENDIX CIntegration of the Elements of the Singular Green's Tensor

In order to calculate the electric field at a point  $(X_j, y_j, Z_j)$  due to a square cylinder of current centered at a point  $(X_m, y_m, Z_m)$ , it is necessary to evaluate the following integrals:

$$Q_1 = \int_{S'} \frac{(x-x')}{\rho_0^2} K_0(u_r \rho_0) dS'$$

$$Q_2 = \int_{S'} \frac{[(x-x')^2 - (z-z')^2]}{\rho_0^3} K_1(u_r \rho_0) dS'$$

$$Q_3 = \int_{S'} K_0(u_r \rho_0) dS'$$

$$Q_4 = \int_{S'} \frac{(x-x')}{\rho_0} K_1(u_r \rho_0) dS'$$

$$Q_5 = \int_{S'} \frac{(z-z')(x-x')}{\rho_0^2} K_0(u_r \rho_0) dS'$$

$$Q_6 = \int_{S'} \frac{(z-z')(x-x')}{\rho_0^3} K_1(u_r \rho_0) dS'$$

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Consider first the case where  $i=m$ , and  $j=n$ . Consider the following change of variables:

$$\begin{aligned} \alpha &= X_i - X' \\ \beta &= Z_j - Z' \\ \rho_0 &= (\alpha^2 + \beta^2)^{1/2} \end{aligned}$$

so that

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Then the limits of integration in  $X'$  and  $Z'$  change from:

$$\begin{aligned} &(X_m + \Delta/2, X_m - \Delta/2) && \text{to} && \Delta/2, -\Delta/2 \\ \text{and} &(Z_n + \Delta/2, Z_n - \Delta/2) \end{aligned}$$

and changing to cylindrical coordinates and approximately the square cell by a circular cell of the same - cross-section area. (Richmond, 1965), we obtain:

$$Q_1 = \int_0^{2\pi} \int_0^a \cos^2 \theta K_0(u_r \rho_0) \rho_0 d\rho_0 d\theta$$

$$Q_2 = \int_0^{2\pi} \int_0^a \cos 2\theta K_1(u_r \rho_0) \rho_0 d\rho_0 d\theta$$

$$Q_3 = \int_0^{2\pi} \int_0^a K_0(u_r \rho_0) \rho_0 d\rho_0 d\theta$$

$$Q_4 = \int_0^{2\pi} \int_0^a \sin \theta K_1(u_r \rho_0) \rho_0 d\rho_0 d\theta$$

$$Q_5 = \int_0^{2\pi} \int_0^a \sin 2\theta K_0(u_r \rho_0) \rho_0 d\rho_0 d\theta$$

$$Q_6 = \int_0^{2\pi} \int_0^a \sin 2\theta K_1(u_r \rho_0) \rho_0 d\rho_0 d\theta$$

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$$Q_1 = \int_0^{2\pi} \int_0^a \left( \frac{1 - \cos 2\theta}{2} \right) K_0(u_r \rho_0) \rho_0 d\rho_0 d\theta$$

or

$$Q_1 = \pi \int_0^a K_0(u_r \rho_0) \rho_0 d\rho_0$$

It is readily seen that only  $Q_1$  and  $Q_3$  are non-zero.

$Q_1$  can be written as

$$Q_1 = \pi \int_0^a K_0(u_r \rho_0) \rho_0 d\rho_0$$

and

$$Q_3 = \int_0^a K_0(u_r \rho_0) \rho_0 d\rho_0$$

Integrals of this form have been evaluated analytically by Hohmann (1971).

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APPENDIX DMagnetic Field components at the surface of the Earth.

Equations (64), (77) and (78) at the surface of the earth become

$$H_x = \frac{I}{2\pi \gamma_1^2} \int_{-l}^l \frac{\partial^4 N}{\partial z^2 \partial x^2} dy' \quad (D-1)$$

$$H_y = \frac{I}{2\pi \gamma_1^2} \int_{-l}^l \frac{\partial^4 N}{\partial x \partial y \partial z^2} dy' \quad (D-2)$$

$$H_z = \frac{I}{2\pi \gamma_1^2} \int_{-l}^l \left( \frac{\partial^4 N}{\partial x \partial z^3} + \gamma_1^2 \frac{\partial^2 N}{\partial x \partial z} - \frac{\partial^3 P}{\partial x \partial z^2} \right) dy' \quad (D-3)$$

The derivatives of N can be simplified by replacing the derivatives of the modified Bessel functions by Bessel functions of zero and first order, (Hill and Wait, 1973). Thus,

$$\frac{\partial^2 N}{\partial x \partial z} = \frac{\gamma_1 x}{z} \left[ A_{00} I_0 K_0 + A_{11} I_1 K_1 + A_{01} I_0 K_1 + A_{10} I_1 K_0 \right]$$

where  $A_{00} = \frac{\gamma_1 z}{R^2}$ ,  $A_{11} = -\frac{\gamma_1 z}{R^2}$ ,  $A_{01} = -1 + \frac{z}{R}$ ,

$$A_{10} = -\frac{1}{R^2} \left( 1 + \frac{z}{R} \right)$$

(D-4)

At the surface of the earth (D-4) becomes

$$\lim_{z \rightarrow 0} \frac{\partial^2 N}{\partial x \partial z} = -\frac{\gamma_1 x}{2R^2} (I_0 K_1 + I_1 K_0)$$

(D-5)

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$$\frac{\partial^4 N}{\partial x^2 \partial z^2} = \frac{\gamma_1}{2} \left[ C_{00} I_0 K_0 + C_{11} I_1 K_1 + C_{01} I_0 K_1 + C_{10} I_1 K_0 \right]$$

where

$$C_{00} = \frac{\gamma_1 z^2}{R^4} \left[ -3 + \gamma_1^2 x^2 + \frac{15x^2}{R^2} \right]$$

$$C_{11} = \frac{\gamma_1^2}{R^2} \left[ -2 + z^2 \frac{(3 - \gamma_1^2 x^2)}{R^2} + \frac{6x^2}{R^2} - \frac{15x^2 z^2}{R^4} + \frac{2x^2(R^2 - 2z^2)}{R^2(x^2 + y^2)} \right]$$

$$C_{01} = \frac{1}{R^3} \left[ \frac{z(3 - \gamma_1^2 x^2)}{R} - \gamma_1^2 (x^2 + z^2) - \frac{3z^2(1 - 2\gamma_1^2 x^2)}{R^2} - \frac{15z^2 x^2}{R^3} + \frac{15x^2 z^2}{R^4} + \frac{\gamma_1^2 x^2 z (R + z)}{(x^2 + y^2)^{1/2}} \right]$$

$$C_{10} = \frac{1}{R^3} \left[ \frac{z(3 - \gamma_1^2 x^2)}{R} + \gamma_1^2 (x^2 + z^2) + \frac{3z^2(1 - 2\gamma_1^2 x^2)}{R^2} - \frac{15z^2 x^2}{R^3} - \frac{15x^2 z^2}{R^4} + \frac{\gamma_1^2 x^2 z (R - z)}{(x^2 + y^2)} \right]$$

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At the surface of the earth (D-6) is given by

$$\lim_{z \rightarrow 0} \frac{\partial^2 N}{\partial x^2 \partial z^2} = \frac{\gamma_1}{2} \left[ -C I_1 K_1 + C I_0 K_1 + C_0 I_1 K_0 \right]$$

(D-7)

$$\lim_{z \rightarrow 0} C_{01} = - \frac{\gamma_1^2 x^2}{R^3} = -C$$

$$\lim_{z \rightarrow 0} C_{10} = \frac{\gamma_1^2 x^2}{R^3} = C$$

$$\lim_{z \rightarrow 0} C_{11} = \frac{\gamma_1}{R^2} \left[ -2 + \frac{8x^2}{R^2} \right] = C_0$$

$$\frac{\partial^4 N}{\partial x^2 \partial z^3} = \frac{\gamma_1 x}{2} \left[ D_{00} I_0 K_0 + D_{11} I_1 K_1 + D_{01} I_0 K_1 + D_{10} I_1 K_0 \right]$$

(D-8)

where

$$D_{00} = \frac{\gamma_1 z}{R^4} \left( -3 + \frac{15z^2}{R^2} + \gamma_1^2 z^2 \right),$$

$$D_{11} = \frac{\gamma_1 z}{R^4} \left( 13 - \frac{15z^2}{R^2} - \gamma_1^2 z^2 \right)$$

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$$D_{01} = \frac{1}{R^2} \left[ \frac{3}{R^2} - \frac{3Z}{R^3} - \frac{15Z^2}{R^4} + \frac{15Z^3}{R^5} - \gamma_1^2 \right. \\ \left. - \frac{3\gamma_1^2 Z}{R} - \frac{\gamma_1^2 Z^2}{R^2} + \frac{6\gamma_1^2 Z^3}{R^3} \right]$$

$$D_{10} = \frac{1}{R^2} \left( \frac{3}{R^2} + \frac{3Z}{R^3} - \frac{15Z^2}{R^4} - \frac{15Z^3}{R^5} - \gamma_1^2 \right. \\ \left. + \frac{3\gamma_1^2 Z}{R} - \frac{\gamma_1^2 Z^2}{R^2} - \frac{6\gamma_1^2 Z^3}{R^3} \right)$$

At the surface of the earth (D-8) is given by

$$\lim_{z \rightarrow 0} \frac{\partial^4 N}{\partial x^2 \partial z^2} = \frac{\gamma_1 \times D}{2} (I_0 K_1 + I_1 K_0)$$

(D-9)

where

$$\lim_{z \rightarrow 0} D_{01} = \frac{1}{R^2} \left( \frac{3}{R^2} - \gamma_1^2 \right) = D$$

and

$$\lim_{z \rightarrow 0} D_{10} = \frac{1}{R^2} \left( \frac{3}{R^2} - \gamma_1^2 \right) = D$$



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From (Ward, 1967, p. 157)

$$\lim_{z \rightarrow 0} \frac{\partial^2 N}{\partial z^2} = \frac{\gamma_1^2}{2} [I_0 k_0 + I_1 k_1]$$

and

$$\lim_{z \rightarrow 0} \frac{\partial^3 N}{\partial x \partial z^2} = \frac{\gamma_1^2}{2} [I_0' k_0 + I_0 k_0' + I_1' k_1 + I_1 k_1']$$

or

$$\lim_{z \rightarrow 0} \frac{\partial^3 N}{\partial x \partial z^2} = -\gamma_1^2 \frac{I_1 k_1 x}{R^2}$$

The derivative of P required in (D-3) is given by

$$\begin{aligned} \frac{\partial^3 P}{\partial x \partial z^2} = & \frac{x}{R^3} e^{-\gamma_1 R} \left[ \frac{3}{R^2} \left( 1 - \frac{5z^2}{R^2} \right) + \frac{3\gamma_1}{R} \left( 1 - \frac{5z^2}{R^2} \right) \right. \\ & \left. + \gamma_1^2 \left( 1 - \frac{6z^2}{R^2} \right) - \gamma_1^3 \frac{z^2}{R} \right] \end{aligned}$$

then

$$\lim_{z \rightarrow 0} \frac{\partial^3 P}{\partial x \partial z^2} = \frac{x}{R^3} e^{-\gamma_1 R} \left[ \frac{3}{R^2} + \frac{3\gamma_1}{R} + \gamma_1^2 \right]$$

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## APPENDIX E

DATA OF ELECTROMAGNETIC FIELD  
COMPONENTS OF A CONDUCTOR IN A  
CONDUCTIVE HALF-SPACE NEAR A GROUNDED  
CABLE OF FINITE LENGTH

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Horizontal Magnetic Field Component  
Perpendicular to the Grounded Cable

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X (m)	Y (m)	H <sub>y</sub> (amp/m)	φ (degrees)
25.	17.	0.2341002E 01	0.1166849E 01
25.	34.	0.1042863E 00	0.2173062E 02
25.	51.	0.6524715E 00	0.2665483E 01
25.	68.	0.7143986E-01	0.1770728E 02
25.	85.	0.1418756E 00	0.7207269E 01
25.	102.	0.1883762E-01	0.3628287E 02
25.	119.	0.5430068E-01	0.1101728E 02
25.	136.	0.7920168E-02	0.4473889E 02
25.	153.	0.2715648E-01	0.1180509E 02
25.	170.	0.4174128E-02	0.4344495E 02
25.	187.	0.1605310E-01	0.9578839E 01
25.	204.	0.2560274E-02	0.3566505E 02
50.	17.	0.1694376E 01	0.1315221E 01
50.	34.	0.2041805E 00	0.1057053E 02
50.	51.	0.4327187E 00	0.3929985E 01
50.	68.	0.9176213E-01	0.1351455E 02
50.	85.	0.1246122E 00	0.8471458E 01
50.	102.	0.3174372E-01	0.2464813E 02
50.	119.	0.4760158E-01	0.1336390E 02
50.	136.	0.1157515E-01	0.3494710E 02
50.	153.	0.2233272E-01	0.1415484E 02
50.	170.	0.3763141E-02	0.4356291E 02
50.	187.	0.1077548E-01	0.1164954E 02
50.	204.	0.2101125E-02	0.3024208E 02
75.	17.	0.1230413E 01	0.1436995E 01
75.	34.	0.2479581E 00	0.6673319E 01
75.	51.	0.2847446E 00	0.5134826E 01
75.	68.	0.1086532E 00	0.1085333E 02
75.	85.	0.1080297E 00	0.8230814E 01
75.	102.	0.3647600E-01	0.1817702E 02
75.	119.	0.3842527E-01	0.1390124E 02
75.	136.	0.1428535E-01	0.2896255E 02
75.	153.	0.1956031E-01	0.1863290E 02
75.	170.	0.7423162E-02	0.3669350E 02
75.	187.	0.1063742E-01	0.2352684E 02
75.	204.	0.4289147E-02	0.4256834E 02
100.	17.	0.9018496E 00	0.8879659E 00
100.	34.	0.2615195E 00	0.5129119E 01
100.	51.	0.1809082E 00	0.5194946E 01
100.	68.	0.1119507E 00	0.4867461E 01
100.	85.	0.9089780E-01	0.9402422E 01
100.	102.	0.3716425E-01	0.2204367E 02
100.	119.	0.3090172E-01	0.2480153E 02
100.	136.	0.1590423E-01	0.3469461E 02
100.	153.	0.1648016E-01	0.2787807E 02
100.	170.	0.8580048E-02	0.3615915E 02
100.	187.	0.9093143E-02	0.2819188E 02
100.	204.	0.4659362E-02	0.3663855E 02

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X (m)	Y (m)	$H_x$ (amp/m)	$\psi_x$ (degrees)
125.	17.	0.6677254E 00	0.8174175E-01
125.	34.	0.2542994E 00	0.5088752E 01
125.	51.	0.1103181E 00	0.5297115E 01
125.	68.	0.1077173E 00	0.1047756E-01
125.	85.	0.7737184E-01	0.1246137E 02
125.	102.	0.3866112E-01	0.2795500E 02
125.	119.	0.2951796E-01	0.3536900E 02
125.	136.	0.1837358E-01	0.3945491E 02
125.	153.	0.1492431E-01	0.3683870E 02
125.	170.	0.8319199E-02	0.3916739E 02
125.	187.	0.6555445E-02	0.3497444E 02
125.	204.	0.3129807E-02	0.3573170E 02
150.	17.	0.4574160E 00	-0.9276484E 00
150.	34.	0.2640070E 00	0.4003930E 01
150.	51.	0.7669252E-01	0.3879741E 01
150.	68.	0.1423129E 00	-0.1915878E 01
150.	85.	0.4764394E-01	0.2012013E 02
150.	102.	0.2340409E-01	0.4484889E 02
150.	119.	0.3162133E-01	0.3855052E 02
150.	136.	0.2048215E-01	0.4343559E 02
150.	153.	0.1591549E-01	0.4427483E 02
150.	170.	0.1753367E-01	0.3168867E 02
150.	187.	0.1974579E-01	0.2323001E 02
150.	204.	0.2269443E-01	0.1419649E 02
175.	17.	0.3951921E 00	-0.5038747E 01
175.	34.	0.2185184E 00	-0.7040080E 00
175.	51.	0.2084460E-01	-0.2470813E 02
175.	68.	0.7185930E-01	-0.9059607E 01
175.	85.	0.5026852E-01	0.2199786E 02
175.	102.	0.3862557E-01	0.3749841E 02
175.	119.	0.3360814E-01	0.4307249E 02
175.	136.	0.2400544E-01	0.4395396E 02
175.	153.	0.1718343E-01	0.4468686E 02
175.	170.	0.1039821E-01	0.4499146E 02
175.	187.	0.7254098E-02	0.4359258E 02
175.	204.	0.4495863E-02	0.3698587E 02
200.	17.	0.2389416E 00	-0.1136056E 02
200.	34.	0.1567026E 00	-0.8969538E 01
200.	51.	0.5218739E-01	-0.1708945E 02
200.	68.	0.1156889E 00	-0.1604495E 01
200.	85.	0.6529540E-01	0.2083839E 02
200.	102.	0.4761250E-01	0.3618259E 02
200.	119.	0.3845443E-01	0.4319409E 02
200.	136.	0.3192837E-01	0.4280580E 02
200.	153.	0.2537611E-01	0.4319031E 02
200.	170.	0.2017082E-01	0.4186160E 02
200.	187.	0.1582667E-01	0.4227068E 02
200.	204.	0.1297981E-01	0.4016774E 02

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X (m)	Y (m)	$H_x$ (amp/m)	$\varphi_x$ (degrees)
225.	17.	0.9214866E-01	-0.4026384E 02
225.	34.	0.6152999E-01	-0.4181325E 02
225.	51.	0.1363598E 00	-0.5237270E 01
225.	68.	0.1938655E 00	0.5265221E 01
225.	85.	0.1207958E 00	0.1631204E 02
225.	102.	0.8632267E-01	0.2672815E 02
225.	119.	0.6120066E-01	0.3580727E 02
225.	136.	0.5318922E-01	0.3784357E 02
225.	153.	0.4373487E-01	0.4057581E 02
225.	170.	0.3828890E-01	0.4135593E 02
225.	187.	0.3232082E-01	0.4300926E 02
225.	204.	0.2869184E-01	0.4311571E 02
250.	17.	0.1440068E 00	-0.3550253E 02
250.	34.	0.9857017E-01	-0.2072171E 02
250.	51.	0.2161133E 00	-0.1948648E 01
250.	68.	0.2599523E 00	0.1750232E 01
250.	85.	0.1817638E 00	0.1660905E 02
250.	102.	0.1403956E 00	0.2612015E 02
250.	119.	0.1059633E 00	0.3406589E 02
250.	136.	0.8371937E-01	0.3593521E 02
250.	153.	0.6352353E-01	0.3915231E 02
250.	170.	0.4881928E-01	0.4059930E 02
250.	187.	0.3808367E-01	0.4308318E 02
250.	204.	0.3050775E-01	0.4374367E 02
275.	17.	0.8752280E-01	-0.4278627E 02
275.	34.	0.5895841E-01	-0.4455147E 02
275.	51.	0.1213789E 00	-0.7264377E 01
275.	68.	0.1715579E 00	0.5133183E 01
275.	85.	0.1067532E 00	0.1747751E 02
275.	102.	0.7923979E-01	0.2806270E 02
275.	119.	0.5767051E-01	0.3688181E 02
275.	136.	0.5095033E-01	0.3859126E 02
275.	153.	0.4228457E-01	0.4115663E 02
275.	170.	0.3735322E-01	0.4175063E 02
275.	187.	0.3176003E-01	0.4325729E 02
275.	204.	0.2830049E-01	0.4329453E 02
300.	17.	0.8237970E-01	-0.3324210E 02
300.	34.	0.8556885E-01	-0.1980510E 02
300.	51.	0.2707342E-01	-0.3910136E 02
300.	68.	0.7182270E-01	-0.6447036E 01
300.	85.	0.3823408E-01	0.2923471E 02
300.	102.	0.3686265E-01	0.4130611E 02
300.	119.	0.3466903E-01	0.4473851E 02
300.	136.	0.2901838E-01	0.4423726E 02
300.	153.	0.2330777E-01	0.4442128E 02
300.	170.	0.1848806E-01	0.4318068E 02
300.	187.	0.1466006E-01	0.4338988E 02
300.	204.	0.1205368E-01	0.4121506E 02

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X (m)	Y (m)	$H_x$ (amp/m)	$\varphi_x$ (degrees)
325.	17.	0.1240031E 00	-0.1953633E 02
325.	34.	0.1092046E 00	-0.6270284E 01
325.	51.	0.4447229E-01	-0.2264609E 02
325.	68.	0.1930558E-01	-0.4406584E 02
325.	85.	0.1532614E-01	0.4472125E 02
325.	102.	0.2620292E-01	0.4480893E 02
325.	119.	0.2883744E-01	0.4498091E 02
325.	136.	0.2123578E-01	0.4493922E 02
325.	153.	0.1596912E-01	0.4421716E 02
325.	170.	0.1010684E-01	0.4267781E 02
325.	187.	0.7913399E-02	0.3771402E 02
325.	204.	0.5351875E-02	0.2755925E 02
350.	17.	0.5097616E-01	-0.2140387E 02
350.	34.	0.1223195E 00	0.2668805E 01
350.	51.	0.1946082E-01	-0.1921251E 02
350.	68.	0.5701250E-01	-0.1401893E 02
350.	85.	0.1894359E-01	0.2949232E 02
350.	102.	0.3547511E-01	0.2760210E 02
350.	119.	0.4277481E-01	0.2697610E 02
350.	136.	0.2337226E-01	0.3574954E 02
350.	153.	0.1445011E-01	0.4325566E 02
350.	170.	0.1326178E-01	0.3535268E 02
350.	187.	0.1580064E-01	0.2520071E 02
350.	204.	0.2063451E-01	0.1351290E 02
375.	17.	0.8472079E-01	-0.1032954E 02
375.	34.	0.8801633E-01	0.3961671E 01
375.	51.	0.3733749E-01	-0.7274615E 01
375.	68.	0.1168421E-01	-0.4468121E 02
375.	85.	0.9022187E-02	0.4441864E 02
375.	102.	0.1472447E-01	0.4500031E 02
375.	119.	0.1686777E-01	0.4478221E 02
375.	136.	0.1197540E-01	0.4498764E 02
375.	153.	0.8867227E-02	0.4482854E 02
375.	170.	0.5035825E-02	0.4500067E 02
375.	187.	0.3337614E-02	0.4468523E 02
375.	204.	0.1295662E-02	0.4480795E 02
400.	17.	0.8233941E-01	-0.4766868E 01
400.	34.	0.8274794E-01	0.1919129E 01
400.	51.	0.4245668E-01	-0.2016026E 01
400.	68.	0.1023280E-01	-0.2112190E 02
400.	85.	0.8300778E-02	0.3304510E 02
400.	102.	0.8419555E-02	0.4449980E 02
400.	119.	0.9426802E-02	0.4468835E 02
400.	136.	0.7323034E-02	0.4494818E 02
400.	153.	0.5940169E-02	0.4495337E 02
400.	170.	0.4376668E-02	0.4396442E 02
400.	187.	0.3475509E-02	0.4398914E 02
400.	204.	0.2672631E-02	0.4103485E 02

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X (m)	Y (m)	$H_x$ (amp/m)	$\psi_x$ (degrees)
425.	17.	0.8495778E-01	0.3088096E 01
425.	34.	0.7689583E-01	0.2772563E 01
425.	51.	0.4833370E-01	0.5750572E 01
425.	68.	0.2292211E-01	0.1386997E 02
425.	85.	0.1030443E-01	0.2469350E 02
425.	102.	0.4441306E-02	0.4413081E 02
425.	119.	0.4076928E-02	0.4351143E 02
425.	136.	0.4149035E-02	0.4167729E 02
425.	153.	0.3681341E-02	0.4285645E 02
425.	170.	0.3397304E-02	0.4322453E 02
425.	187.	0.2984263E-02	0.4409036E 02
425.	204.	0.2697885E-02	0.4431734E 02
450.	17.	0.9048194E-01	0.3560102E 01
450.	34.	0.7481307E-01	0.5934169E 01
450.	51.	0.5575013E-01	0.6735083E 01
450.	68.	0.3608762E-01	0.8587133E 01
450.	85.	0.1555181E-01	0.2310127E 02
450.	102.	0.6843291E-02	0.4169527E 02
450.	119.	0.5199585E-02	0.4418427E 02
450.	136.	0.3518299E-02	0.4402057E 02
450.	153.	0.2103130E-02	0.4487343E 02
450.	170.	0.1201468E-02	0.3879199E 02
450.	187.	0.1107563E-02	0.1394408E 02
450.	204.	0.1521686E-02	-0.9016585E 01
475.	17.	0.8137292E-01	0.3430767E 01
475.	34.	0.7234734E-01	0.4599913E 01
475.	51.	0.5371730E-01	0.5749821E 01
475.	68.	0.3446419E-01	0.8114401E 01
475.	85.	0.1887972E-01	0.1544414E 02
475.	102.	0.9576339E-02	0.2593878E 02
475.	119.	0.5248059E-02	0.3594783E 02
475.	136.	0.3191998E-02	0.4018257E 02
475.	153.	0.1925360E-02	0.4278844E 02
475.	170.	0.1022723E-02	0.4488799E 02
475.	187.	0.6121101E-03	0.3993803E 02
475.	204.	0.5378583E-03	0.1506540E 02



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Horizontal Magnetic Field Component  
Parallel to the Grounded Cable

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X (m)	Y (m)	$H_y$ (amp/m)	$\varphi_y$ (degrees)
25.	17.	0.2317797E 01	-0.4499786E 02
25.	34.	0.8459514E-01	-0.3974681E 02
25.	51.	0.6781754E 00	0.4496773E 02
25.	68.	0.9601128E-01	0.4416910E 02
25.	85.	0.1712593E 00	0.4445134E 02
25.	102.	0.5204754E-01	0.3805402E 02
25.	119.	0.8637446E-01	0.4333572E 02
25.	136.	0.3518888E-01	0.3929224E 02
25.	153.	0.5522775E-01	0.4407648E 02
25.	170.	0.2552018E-01	0.4357970E 02
25.	187.	0.4014551E-01	0.4483595E 02
25.	204.	0.2050233E-01	0.4494183E 02
50.	17.	0.1670756E 01	-0.4499213E 02
50.	34.	0.1862159E 00	-0.4416289E 02
50.	51.	0.4571098E 00	0.4492308E 02
50.	68.	0.1203172E 00	0.4426373E 02
50.	85.	0.1521606E 00	0.4443050E 02
50.	102.	0.6106261E-01	0.4166452E 02
50.	119.	0.7651722E-01	0.4349162E 02
50.	136.	0.4013977E-01	0.4159538E 02
50.	153.	0.5094161E-01	0.4390047E 02
50.	170.	0.2987033E-01	0.4340765E 02
50.	187.	0.3706112E-01	0.4449699E 02
50.	204.	0.2311550E-01	0.4442754E 02
75.	17.	0.1233063E 01	-0.4500046E 02
75.	34.	0.2498280E 00	-0.4490128E 02
75.	51.	0.2853177E 00	0.4499379E 02
75.	68.	0.1138362E 00	0.4500053E 02
75.	85.	0.1195609E 00	0.4476959E 02
75.	102.	0.5473141E-01	0.4297675E 02
75.	119.	0.6182415E-01	0.4314349E 02
75.	136.	0.4022142E-01	0.4187973E 02
75.	153.	0.4728612E-01	0.4344987E 02
75.	170.	0.3449257E-01	0.4320920E 02
75.	187.	0.3821727E-01	0.4399704E 02
75.	204.	0.2905274E-01	0.4395453E 02
100.	17.	0.9371388E 00	-0.4498697E 02
100.	34.	0.2780719E 00	-0.4497398E 02
100.	51.	0.1600527E 00	0.4483200E 02
100.	68.	0.9780562E-01	0.4320236E 02
100.	85.	0.9301645E-01	0.4480838E 02
100.	102.	0.5434287E-01	0.4157875E 02
100.	119.	0.5865814E-01	0.4012556E 02
100.	136.	0.4221942E-01	0.3972864E 02
100.	153.	0.4313502E-01	0.4210809E 02
100.	170.	0.3246208E-01	0.4304691E 02
100.	187.	0.3288648E-01	0.4398750E 02
100.	204.	0.2582870E-01	0.4454494E 02

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X (m)	Y (m)	H <sub>y</sub> (amp/m)	$\varphi$ (degrees)
125.	17.	0.7273124E 00	-0.4493658E 02
125.	34.	0.2761061E 00	-0.4497412E 02
125.	51.	0.7949579E-01	0.4316058E 02
125.	68.	0.8511585E-01	0.3879129E 02
125.	85.	0.7692719E-01	0.4467392E 02
125.	102.	0.5933273E-01	0.4035864E 02
125.	119.	0.6310308E-01	0.3767174E 02
125.	136.	0.4653221E-01	0.3834531E 02
125.	153.	0.4170416E-01	0.4066652E 02
125.	170.	0.3022008E-01	0.4265211E 02
125.	187.	0.2766069E-01	0.4381856E 02
125.	204.	0.2135848E-01	0.4480780E 02
150.	17.	0.5857411E 00	-0.4463991E 02
150.	34.	0.2760016E 00	-0.4472406E 02
150.	51.	0.6918740E-01	0.8863387E 01
150.	68.	0.9277868E-01	0.2533578E 02
150.	85.	0.5876660E-01	0.4336705E 02
150.	102.	0.4451241E-01	0.4462956E 02
150.	119.	0.4891374E-01	0.4108714E 02
150.	136.	0.4152558E-01	0.4033246E 02
150.	153.	0.4000226E-01	0.3977173E 02
150.	170.	0.3310341E-01	0.3977216E 02
150.	187.	0.3093431E-01	0.3946797E 02
150.	204.	0.2559371E-01	0.3990550E 02
175.	17.	0.4953336E 00	-0.4292546E 02
175.	34.	0.3367981E 00	-0.4259232E 02
175.	51.	0.1355900E 00	-0.1318863E 02
175.	68.	0.1281013E 00	0.2491138E 02
175.	85.	0.5035486E-01	0.2804701E 02
175.	102.	0.1434455E-01	0.4122902E 02
175.	119.	0.1529201E-01	0.1968134E 02
175.	136.	0.2084649E-01	0.2954829E 02
175.	153.	0.2502913E-01	0.3120424E 02
175.	170.	0.2679301E-01	0.3558743E 02
175.	187.	0.2731010E-01	0.3573561E 02
175.	204.	0.2653048E-01	0.3807545E 02
200.	17.	0.5870831E 00	-0.3728207E 02
200.	34.	0.4617789E 00	-0.3840121E 02
200.	51.	0.2896437E 00	-0.1949757E 02
200.	68.	0.2309740E 00	0.6918481E 01
200.	85.	0.9195626E-01	0.2721781E 01
200.	102.	0.2539341E-01	0.4538651E 01
200.	119.	0.1184291E-01	-0.1939154E 02
200.	136.	0.1313156E-01	0.2000978E 02
200.	153.	0.1588406E-01	0.2430293E 02
200.	170.	0.1619539E-01	0.3777924E 02
200.	187.	0.1660924E-01	0.3757185E 02
200.	204.	0.1676558E-01	0.4220256E 02

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X (m)	Y (m)	H <sub>y</sub> (amp/m)	ψ (degrees)
225.	17.	0.8736998E 00	-0.3084204E 02
225.	34.	0.7688807E 00	-0.3182759E 02
225.	51.	0.5366609E 00	-0.1811217E 02
225.	68.	0.3801786E 00	0.4740536E 00
225.	85.	0.2091293E 00	-0.6510958E 01
225.	102.	0.1048968E 00	-0.9740965E 01
225.	119.	0.4555515E-01	-0.2497267E 02
225.	136.	0.1384481E-01	-0.1477277E 02
225.	153.	0.6775253E-02	0.2240045E 02
225.	170.	0.2131489E-01	0.3197034E 02
225.	187.	0.2899918E-01	0.2759059E 02
225.	204.	0.3272973E-01	0.3112915E 02
250.	17.	0.1160698E 01	-0.2710512E 02
250.	34.	0.9916070E 00	-0.2870183E 02
250.	51.	0.7376812E 00	-0.1726979E 02
250.	68.	0.5335313E 00	-0.1648742E 01
250.	85.	0.2759289E 00	-0.7413957E 00
250.	102.	0.1313555E 00	0.6278036E 01
250.	119.	0.4735458E-01	0.6547071E 01
250.	136.	0.2182463E-01	0.1955315E 02
250.	153.	0.8735517E-03	0.2097792E 02
250.	170.	0.7139798E-02	0.2336060E 01
250.	187.	0.1612940E-01	-0.1416661E 02
250.	204.	0.1598738E-01	-0.1002349E 02
275.	17.	0.8277334E 00	-0.2798720E 02
275.	34.	0.7460921E 00	-0.3050896E 02
275.	51.	0.5412242E 00	-0.1945517E 02
275.	68.	0.3797085E 00	-0.2902677E 01
275.	85.	0.2106288E 00	-0.1028665E 02
275.	102.	0.1060915E 00	-0.1361554E 02
275.	119.	0.4756756E-01	-0.2829749E 02
275.	136.	0.1477080E-01	-0.2397948E 02
275.	153.	0.6240487E-02	0.5677528E 01
275.	170.	0.2042297E-01	0.3012886E 02
275.	187.	0.2842128E-01	0.2644600E 02
275.	204.	0.3223163E-01	0.3058920E 02
300.	17.	0.4705607E 00	-0.3036749E 02
300.	34.	0.4052826E 00	-0.3573906E 02
300.	51.	0.3020632E 00	-0.2438597E 02
300.	68.	0.2280082E 00	-0.4188289E 01
300.	85.	0.9389961E-01	-0.1471906E 02
300.	102.	0.2781689E-01	-0.2529913E 02
300.	119.	0.1738098E-01	-0.3680379E 02
300.	136.	0.1250622E-01	-0.4146958E 01
300.	153.	0.1451642E-01	0.1096477E 02
300.	170.	0.1408845E-01	0.3463751E 02
300.	187.	0.1498439E-01	0.3557173E 02
300.	204.	0.1543899E-01	0.4182915E 02

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X (m)	Y (m)	H <sub>y</sub> (amp/m)	$\varphi$ (degrees)
325.	17.	0.2593840E 00	-0.3533580E 02
325.	34.	0.2401267E 00	-0.3985278E 02
325.	51.	0.1583633E 00	-0.2999748E 02
325.	68.	0.1109332E 00	-0.3737079E 01
325.	85.	0.4437569E-01	-0.2155098E 02
325.	102.	0.1118736E-01	-0.4064125E 02
325.	119.	0.1691126E-01	-0.2512535E 02
325.	136.	0.1790283E-01	0.1157297E 02
325.	153.	0.2176732E-01	0.2148198E 02
325.	170.	0.2402079E-01	0.3224660E 02
325.	187.	0.2497051E-01	0.3343124E 02
325.	204.	0.2483696E-01	0.3714436E 02
350.	17.	0.1932948E 00	-0.4118686E 02
350.	34.	0.1383535E 00	-0.4390933E 02
350.	51.	0.1068707E 00	-0.3846298E 02
350.	68.	0.8966631E-01	-0.2610663E 02
350.	85.	0.1763834E-01	-0.1610219E 02
350.	102.	0.1779289E-01	0.4115794E 02
350.	119.	0.3482071E-01	0.3505576E 02
350.	136.	0.3365065E-01	0.3674463E 02
350.	153.	0.3302072E-01	0.3642032E 02
350.	170.	0.2904299E-01	0.3790021E 02
350.	187.	0.2725633E-01	0.3773276E 02
350.	204.	0.2339463E-01	0.3907655E 02
375.	17.	0.1511484E 00	-0.4334230E 02
375.	34.	0.1123850E 00	-0.4479813E 02
375.	51.	0.7700473E-01	-0.4363651E 02
375.	68.	0.6030841E-01	-0.3257813E 02
375.	85.	0.1465153E-01	-0.7777869E 01
375.	102.	0.3726812E-01	0.2459518E 02
375.	119.	0.4827987E-01	0.2756094E 02
375.	136.	0.3853304E-01	0.3335477E 02
375.	153.	0.3246389E-01	0.3686705E 02
375.	170.	0.2519032E-01	0.4160690E 02
375.	187.	0.2204307E-01	0.4328775E 02
375.	204.	0.1870592E-01	0.4484128E 02
400.	17.	0.1223304E 00	-0.4408273E 02
400.	34.	0.1025860E 00	-0.4471515E 02
400.	51.	0.6579405E-01	-0.4452245E 02
400.	68.	0.4167953E-01	-0.3585495E 02
400.	85.	0.1491707E-01	-0.1696117E 02
400.	102.	0.2967581E-01	0.2155887E 02
400.	119.	0.3834828E-01	0.2724559E 02
400.	136.	0.3347024E-01	0.3475417E 02
400.	153.	0.3080278E-01	0.3856818E 02
400.	170.	0.2734834E-01	0.4230626E 02
400.	187.	0.2558948E-01	0.4349417E 02
400.	204.	0.2328926E-01	0.4459123E 02

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X (m)	Y (m)	H <sub>y</sub> (amp/m)	ψ (degrees)
425.	17.	0.8977520E-01	-0.4488718E 02
425.	34.	0.8498973E-01	-0.4376738E 02
425.	51.	0.4953332E-01	-0.4400278E 02
425.	68.	0.1666040E-01	-0.4390642E 02
425.	85.	0.1900781E-01	0.3863942E 01
425.	102.	0.2641102E-01	0.2827061E 02
425.	119.	0.3099553E-01	0.3384282E 02
425.	136.	0.3107141E-01	0.3883504E 02
425.	153.	0.3084246E-01	0.4101851E 02
425.	170.	0.3006475E-01	0.4276608E 02
425.	187.	0.2885742E-01	0.4343976E 02
425.	204.	0.2710390E-01	0.4405139E 02
450.	17.	0.8125877E-01	-0.4031264E 02
450.	34.	0.7497841E-01	-0.3697281E 02
450.	51.	0.5258855E-01	-0.3215067E 02
450.	68.	0.3390618E-01	-0.1554393E 02
450.	85.	0.3550773E-01	0.1634056E 02
450.	102.	0.3719972E-01	0.3039281E 02
450.	119.	0.3680515E-01	0.3585129E 02
450.	136.	0.3326720E-01	0.3944209E 02
450.	153.	0.3013130E-01	0.4166705E 02
450.	170.	0.2720656E-01	0.4328798E 02
450.	187.	0.2485347E-01	0.4411769E 02
450.	204.	0.2246692E-01	0.4462787E 02
475.	17.	0.7180494E-01	-0.4009708E 02
475.	34.	0.7304913E-01	-0.3420943E 02
475.	51.	0.5018953E-01	-0.3102011E 02
475.	68.	0.2978025E-01	-0.1788376E 02
475.	85.	0.3790214E-01	0.1293294E 02
475.	102.	0.3962347E-01	0.2619756E 02
475.	119.	0.3850578E-01	0.3240280E 02
475.	136.	0.3289694E-01	0.3794846E 02
475.	153.	0.2901141E-01	0.4142000E 02
475.	170.	0.2583143E-01	0.4381842E 02
475.	187.	0.2380941E-01	0.4469133E 02
475.	204.	0.2182712E-01	0.4499757E 02

T-1711

Vertical Magnetic Field Component

T-1711

X (m)	Y (m)	H <sub>z</sub> (amp/m)	ψ <sub>z</sub> (degrees)
25.	17.	0.7373427E 01	0.1023864E 00
25.	34.	0.2722682E 01	0.2125785E 00
25.	51.	0.1475310E 00	0.3792227E 01
25.	68.	0.1788038E 00	0.4336802E 00
25.	85.	0.8601773E-01	-0.7985085E 01
25.	102.	0.6098635E-01	-0.1510347E 02
25.	119.	0.3969911E-01	-0.2123151E 02
25.	136.	0.3033746E-01	-0.1963100E 02
25.	153.	0.2524640E-01	-0.1473147E 02
25.	170.	0.1481931E-01	-0.1181427E 02
25.	187.	0.1739160E-01	-0.2779425E 01
25.	204.	0.8499447E-02	0.5369274E 01
50.	17.	0.3240553E 01	0.2232989E 00
50.	34.	0.1396852E 01	0.4020245E 00
50.	51.	0.8521998E-01	0.5753868E 01
50.	68.	0.1008528E 00	-0.7151073E 00
50.	85.	0.2729754E-01	-0.2898630E 02
50.	102.	0.2746985E-01	-0.3521082E 02
50.	119.	0.2494949E-01	-0.3527217E 02
50.	136.	0.1387876E-01	-0.4141943E 02
50.	153.	0.1340771E-01	-0.2859264E 02
50.	170.	0.5528890E-02	-0.3029315E 02
50.	187.	0.6064858E-02	-0.5588556E 01
50.	204.	0.4288539E-02	0.1631326E 02
75.	17.	0.1858305E 01	0.5542610E 00
75.	34.	0.9228002E 00	0.1203179E 01
75.	51.	0.1452811E 00	0.4636459E 01
75.	68.	0.7124424E-01	-0.2560170E 01
75.	85.	0.1474445E-01	-0.4284575E 02
75.	102.	0.1775330E-01	-0.4434982E 02
75.	119.	0.2136208E-01	-0.3532475E 02
75.	136.	0.1163680E-01	-0.4257129E 02
75.	153.	0.1307479E-01	-0.2536592E 02
75.	170.	0.4518494E-02	-0.3200273E 02
75.	187.	0.7226102E-02	-0.3313984E 01
75.	204.	0.2290088E-02	0.2447087E 02
100.	17.	0.1170177E 01	0.5535944E 00
100.	34.	0.6712477E 00	0.2153512E 01
100.	51.	0.1507553E 00	0.2283821E 01
100.	68.	0.4620199E-01	-0.2043686E 02
100.	85.	0.1871378E-01	-0.4003172E 02
100.	102.	0.1959166E-01	-0.3812579E 02
100.	119.	0.1118894E-01	-0.4284671E 02
100.	136.	0.8500744E-02	-0.4053267E 02
100.	153.	0.3210258E-02	-0.4460580E 02
100.	170.	0.1090078E-02	-0.4273317E 02
100.	187.	0.2275579E-02	0.2716331E 02
100.	204.	0.2127213E-02	0.4113641E 02



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X (m)	Y (m)	H <sub>z</sub> (amp/m)	φ <sub>z</sub> (degrees)
125.	17.	0.7867802E 00	0.7164979E 00
125.	34.	0.5128583E 00	0.3400428E 01
125.	51.	0.1471072E 00	0.7079325E 00
125.	68.	0.4450610E-01	-0.3302391E 02
125.	85.	0.2955340E-01	-0.3617603E 02
125.	102.	0.2557280E-01	-0.3651648E 02
125.	119.	0.1733555E-01	-0.3321516E 02
125.	136.	0.1123624E-01	-0.3475159E 02
125.	153.	0.4608110E-02	-0.3095596E 02
125.	170.	0.1212931E-02	-0.2075771E 02
125.	187.	0.2321357E-02	0.4319839E 02
125.	204.	0.3330640E-02	0.3971095E 02
150.	17.	0.5448747E 00	0.1142879E 01
150.	34.	0.4088759E 00	0.7418149E 01
150.	51.	0.1358520E 00	0.1782167E 01
150.	68.	0.4916311E-01	-0.4133026E 02
150.	85.	0.3263123E-01	-0.3032173E 02
150.	102.	0.2473287E-01	-0.2760741E 02
150.	119.	0.1752359E-01	-0.1142420E 02
150.	136.	0.7110689E-02	-0.3294968E 02
150.	153.	0.2566263E-02	-0.3779222E 02
150.	170.	0.8873995E-02	-0.1801311E 02
150.	187.	0.1109821E-01	-0.6119763E 01
150.	204.	0.1369696E-01	-0.8184699E 01
175.	17.	0.4065822E 00	0.4512048E 01
175.	34.	0.2618890E 00	0.1229895E 02
175.	51.	0.1400083E 00	0.6052371E 01
175.	68.	0.1074168E 00	-0.1590825E 02
175.	85.	0.4825298E-01	-0.2771509E 02
175.	102.	0.2926822E-01	-0.4036563E 02
175.	119.	0.1680980E-01	-0.4492720E 02
175.	136.	0.1329062E-01	-0.4476366E 02
175.	153.	0.8255698E-02	-0.4124573E 02
175.	170.	0.5177565E-02	-0.4154820E 02
175.	187.	0.3646817E-02	-0.1873738E 02
175.	204.	0.2060917E-02	-0.9963552E 01
200.	17.	0.3095552E 00	0.1037095E 02
200.	34.	0.1713913E 00	0.2692804E 02
200.	51.	0.1349570E 00	0.1126909E 02
200.	68.	0.1488557E 00	-0.1444093E 02
200.	85.	0.5051315E-01	-0.3534409E 02
200.	102.	0.3707065E-01	-0.4457875E 02
200.	119.	0.4186414E-01	-0.3087245E 02
200.	136.	0.3357695E-01	-0.2881412E 02
200.	153.	0.2923894E-01	-0.1702672E 02
200.	170.	0.2110591E-01	-0.1124772E 02
200.	187.	0.1994772E-01	0.2803550E 01
200.	204.	0.1486464E-01	0.9638684E 01

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X (m)	Y (m)	H <sub>z</sub> (amp/m)	φ <sub>z</sub> (degrees)
225.	17.	0.2582056E 00	0.1018459E 02
225.	34.	0.1478565E 00	0.3849979E 02
225.	51.	0.1359650E 00	0.1666267E 02
225.	68.	0.1776215E 00	-0.1782967E 02
225.	85.	0.5538498E-01	-0.3917520E 02
225.	102.	0.4923110E-01	-0.4266391E 02
225.	119.	0.5929589E-01	-0.2571526E 02
225.	136.	0.4753147E-01	-0.2491586E 02
225.	153.	0.4104746E-01	-0.1373063E 02
225.	170.	0.2939287E-01	-0.1012597E 02
225.	187.	0.2694556E-01	0.3055520E 01
225.	204.	0.1950103E-01	0.7956002E 01
250.	17.	0.2257516E 00	0.8429499E 01
250.	34.	0.2409756E 00	0.2613069E 02
250.	51.	0.1569725E 00	0.5387477E 01
250.	68.	0.1301014E 00	-0.3481850E 02
250.	85.	0.6982553E-01	-0.3545345E 02
250.	102.	0.4439337E-01	-0.4174269E 02
250.	119.	0.1985984E-01	-0.4323557E 02
250.	136.	0.1695621E-01	-0.4498396E 02
250.	153.	0.8229647E-02	-0.4406230E 02
250.	170.	0.8050710E-02	-0.4070648E 02
250.	187.	0.5174413E-02	-0.1694002E 02
250.	204.	0.5637545E-02	-0.1912871E 02
275.	17.	0.2180870E 00	0.6874937E 01
275.	34.	0.3419011E 00	0.1465685E 02
275.	51.	0.1759745E 00	-0.3997923E 01
275.	68.	0.9884834E-01	-0.4438528E 02
275.	85.	0.8158094E-01	-0.3097186E 02
275.	102.	0.6864375E-01	-0.2638087E 02
275.	119.	0.6659436E-01	-0.1246453E 02
275.	136.	0.4585478E-01	-0.1797856E 02
275.	153.	0.3645089E-01	-0.1367672E 02
275.	170.	0.2278559E-01	-0.2222313E 02
275.	187.	0.1771443E-01	-0.1689517E 02
275.	204.	0.1071215E-01	-0.2950598E 02
300.	17.	0.2246734E 00	0.4600165E 01
300.	34.	0.2956502E 00	0.1349601E 02
300.	51.	0.1624558E 00	-0.8187270E 00
300.	68.	0.8512604E-01	-0.4112820E 02
300.	85.	0.7528359E-01	-0.3242845E 02
300.	102.	0.6127238E-01	-0.3325220E 02
300.	119.	0.5182983E-01	-0.2525587E 02
300.	136.	0.3633217E-01	-0.3017506E 02
300.	153.	0.2671519E-01	-0.2620052E 02
300.	170.	0.1685992E-01	-0.3248311E 02
300.	187.	0.1163393E-01	-0.2654945E 02
300.	204.	0.6834906E-02	-0.3606314E 02

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X (m)	Y (m)	H <sub>z</sub> (amp/m)	φ <sub>z</sub> (degrees)
325.	17.	0.2145247E 00	0.9679454E 01
325.	34.	0.2254802E 00	0.1463959E 02
325.	51.	0.1482477E 00	0.1831216E 01
325.	68.	0.8935827E-01	-0.2953484E 02
325.	85.	0.6672204E-01	-0.3558234E 02
325.	102.	0.5169527E-01	-0.4190314E 02
325.	119.	0.3652970E-01	-0.4316943E 02
325.	136.	0.2638016E-01	-0.4473772E 02
325.	153.	0.1580958E-01	-0.4499355E 02
325.	170.	0.1083305E-01	-0.4220483E 02
325.	187.	0.6997965E-02	-0.2922554E 02
325.	204.	0.7253628E-02	-0.1082334E 02
350.	17.	0.2121888E 00	0.1275209E 02
350.	34.	0.1352987E 00	0.1249681E 02
350.	51.	0.1467078E 00	0.4135988E 01
350.	68.	0.1428729E 00	-0.6273088E 01
350.	85.	0.6961364E-01	-0.3261649E 02
350.	102.	0.5411975E-01	-0.4384756E 02
350.	119.	0.4666845E-01	-0.4472963E 02
350.	136.	0.3296745E-01	-0.4487694E 02
350.	153.	0.2074152E-01	-0.4481233E 02
350.	170.	0.1080801E-01	-0.4433669E 02
350.	187.	0.4654702E-02	-0.3998732E 02
350.	204.	0.5399894E-02	0.7901516E 01
375.	17.	0.1820347E 00	0.1194150E 02
375.	34.	0.1250289E 00	0.1250843E 02
375.	51.	0.1285887E 00	0.4071217E 01
375.	68.	0.1208667E 00	-0.6568903E 01
375.	85.	0.6061118E-01	-0.2912929E 02
375.	102.	0.4315098E-01	-0.4257086E 02
375.	119.	0.3595948E-01	-0.4487662E 02
375.	136.	0.2752040E-01	-0.4452968E 02
375.	153.	0.2015141E-01	-0.4285471E 02
375.	170.	0.1336938E-01	-0.4156699E 02
375.	187.	0.9977382E-02	-0.3511276E 02
375.	204.	0.6413277E-02	-0.2915038E 02
400.	17.	0.1523859E 00	0.1012718E 02
400.	34.	0.1100162E 00	0.7396497E 01
400.	51.	0.1105796E 00	0.1156524E 01
400.	68.	0.1020088E 00	-0.7632277E 01
400.	85.	0.5796587E-01	-0.2923361E 02
400.	102.	0.4271671E-01	-0.4102420E 02
400.	119.	0.3373450E-01	-0.4483046E 02
400.	136.	0.2478419E-01	-0.4500095E 02
400.	153.	0.1722272E-01	-0.4433316E 02
400.	170.	0.1095453E-01	-0.4278723E 02
400.	187.	0.8025408E-02	-0.3507835E 02
400.	204.	0.5477205E-02	-0.2309999E 02

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X (m)	Y (m)	$H_z$ (amp/m)	$\varphi_z$ (degrees)
425.	17.	0.1060624E 00	0.6631080E 01
425.	34.	0.9422016E-01	0.4080403E 01
425.	51.	0.8336955E-01	-0.3480696E 01
425.	68.	0.7060117E-01	-0.1473029E 02
425.	85.	0.5397549E-01	-0.2750586E 02
425.	102.	0.4251175E-01	-0.3483830E 02
425.	119.	0.3161376E-01	-0.3867693E 02
425.	136.	0.2188157E-01	-0.4028609E 02
425.	153.	0.1350104E-01	-0.4214413E 02
425.	170.	0.7432412E-02	-0.4413364E 02
425.	187.	0.3801681E-02	-0.4392493E 02
425.	204.	0.2662022E-02	-0.2562917E 02
450.	17.	0.8063918E-01	0.8341716E 01
450.	34.	0.7350230E-01	0.5619040E 01
450.	51.	0.6280267E-01	-0.2202857E 01
450.	68.	0.5115040E-01	-0.1466519E 02
450.	85.	0.3996362E-01	-0.2797188E 02
450.	102.	0.3152631E-01	-0.3575227E 02
450.	119.	0.2337331E-01	-0.3997614E 02
450.	136.	0.1579938E-01	-0.4255424E 02
450.	153.	0.9787600E-02	-0.4476561E 02
450.	170.	0.6314427E-02	-0.4309088E 02
450.	187.	0.5529646E-02	-0.3013402E 02
450.	204.	0.5721048E-02	-0.1367545E 02
475.	17.	0.7403129E-01	0.6952264E 01
475.	34.	0.5885074E-01	0.1676526E 01
475.	51.	0.5570832E-01	-0.3875319E 01
475.	68.	0.4930566E-01	-0.1242457E 02
475.	85.	0.3358068E-01	-0.3092065E 02
475.	102.	0.2638302E-01	-0.3954843E 02
475.	119.	0.2036134E-01	-0.4330966E 02
475.	136.	0.1441039E-01	-0.4399234E 02
475.	153.	0.9293132E-02	-0.4480411E 02
475.	170.	0.5325507E-02	-0.4498535E 02
475.	187.	0.3098198E-02	-0.4145029E 02
475.	204.	0.1903513E-02	-0.2468948E 02

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Horizontal Incident Magnetic Field Component  
Perpendicular to the Grounded Cable

T-1711

X (m)	Y (m)	H <sub>x</sub> inc (amp/m)	$\varphi_x$ inc (degrees)
25.	17.	0.2337944E 01	0.1035887E 01
25.	34.	0.9785104E-01	0.2025333E 02
25.	51.	0.6563570E 00	0.2256533E 01
25.	68.	0.7402605E-01	0.1474386E 02
25.	85.	0.1467791E 00	0.6018428E 01
25.	102.	0.2201480E-01	0.2940384E 02
25.	119.	0.5924019E-01	0.9698047E 01
25.	136.	0.9861715E-02	0.4011983E 02
25.	153.	0.3084574E-01	0.1248572E 02
25.	170.	0.5798455E-02	0.4405109E 02
25.	187.	0.1871579E-01	0.1393168E 02
25.	204.	0.3890760E-02	0.4499808E 02
50.	17.	0.1689673E 01	0.1128252E 01
50.	34.	0.2028480E 00	0.8759265E 01
50.	51.	0.4359388E 00	0.3237400E 01
50.	68.	0.9703505E-01	0.1097736E 02
50.	85.	0.1265990E 00	0.6880289E 01
50.	102.	0.3234753E-01	0.2072675E 02
50.	119.	0.4887955E-01	0.1163144E 02
50.	136.	0.1374553E-01	0.3083804E 02
50.	153.	0.2588859E-01	0.1462393E 02
50.	170.	0.7562581E-02	0.3604179E 02
50.	187.	0.1533418E-01	0.1655342E 02
50.	204.	0.4507992E-02	0.4007373E 02
75.	17.	0.1239774E 01	0.1242702E 01
75.	34.	0.2529281E 00	0.6057511E 01
75.	51.	0.2784392E 00	0.4706365E 01
75.	68.	0.1023698E 00	0.1002032E 02
75.	85.	0.1071226E 00	0.7906799E 01
75.	102.	0.3793721E-01	0.1754753E 02
75.	119.	0.4140056E-01	0.1345615E 02
75.	136.	0.1692910E-01	0.2555995E 02
75.	153.	0.2246754E-01	0.1655658E 02
75.	170.	0.9454422E-02	0.2993925E 02
75.	187.	0.1306052E-01	0.1907474E 02
75.	204.	0.5527992E-02	0.3421559E 02
100.	17.	0.9242237E 00	0.1372400E 01
100.	34.	0.2697283E 00	0.4862151E 01
100.	51.	0.1675282E 00	0.7115066E 01
100.	68.	0.9664965E-01	0.1007647E 02
100.	85.	0.8920985E-01	0.9118617E 01
100.	102.	0.3983210E-01	0.1634349E 02
100.	119.	0.3590712E-01	0.1510083E 02
100.	136.	0.1893610E-01	0.2279010E 02
100.	153.	0.2003578E-01	0.1815985E 02
100.	170.	0.1074536E-01	0.2656462E 02
100.	187.	0.1149375E-01	0.2120197E 02
100.	204.	0.6270628E-02	0.3067159E 02

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X (m)	Y (m)	$H_x$ inc (amp/m)	$\varphi_x$ inc (degrees)
125.	17.	0.7007377E 00	0.1509505E 01
125.	34.	0.2674024E 00	0.4183213E 01
125.	51.	0.9062988E-01	0.1172811E 02
125.	68.	0.8478421E-01	0.1075879E 02
125.	85.	0.7319605E-01	0.1055590E 02
125.	102.	0.3920470E-01	0.1607144E 02
125.	119.	0.3172116E-01	0.1654164E 02
125.	136.	0.1990874E-01	0.2134792E 02
125.	153.	0.1822665E-01	0.1944780E 02
125.	170.	0.1144727E-01	0.2476566E 02
125.	187.	0.1038784E-01	0.2288956E 02
125.	204.	0.6691396E-02	0.2873624E 02
150.	17.	0.5409396E 00	0.1643134E 01
150.	34.	0.2545592E 00	0.3739681E 01
150.	51.	0.3901832E-01	0.2323123E 02
150.	68.	0.7010162E-01	0.1202643E 02
150.	85.	0.5914141E-01	0.1228157E 02
150.	102.	0.3697030E-01	0.1634875E 02
150.	119.	0.2836871E-01	0.1780333E 02
150.	136.	0.2007148E-01	0.2066792E 02
150.	153.	0.1679110E-01	0.2048802E 02
150.	170.	0.1168205E-01	0.2387242E 02
150.	187.	0.9572156E-02	0.2418805E 02
150.	204.	0.6855808E-02	0.2775879E 02
175.	17.	0.4255359E 00	0.1760202E 01
175.	34.	0.2364273E 00	0.3420270E 01
175.	51.	0.1481216E-01	0.4493216E 02
175.	68.	0.5475319E-01	0.1403689E 02
175.	85.	0.4697108E-01	0.1439102E 02
175.	102.	0.3379145E-01	0.1701527E 02
175.	119.	0.2553684E-01	0.1893596E 02
175.	136.	0.1963187E-01	0.2047073E 02
175.	153.	0.1556713E-01	0.2136317E 02
175.	170.	0.1156992E-01	0.2353958E 02
175.	187.	0.8931600E-02	0.2519080E 02
175.	204.	0.6834462E-02	0.2736670E 02
200.	17.	0.3412774E 00	0.1847052E 01
200.	34.	0.2161661E 00	0.3170819E 01
200.	51.	0.2832220E-01	0.2454449E 02
200.	68.	0.4007608E-01	0.1720828E 02
200.	85.	0.3654956E-01	0.1702972E 02
200.	102.	0.3012893E-01	0.1800581E 02
200.	119.	0.2302693E-01	0.1999747E 02
200.	136.	0.1876162E-01	0.2060930E 02
200.	153.	0.1445549E-01	0.2214757E 02
200.	170.	0.1121091E-01	0.2357915E 02
200.	187.	0.8392014E-02	0.2599289E 02
200.	204.	0.6685130E-02	0.2734653E 02

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X (m)	Y (m)	$H_x$ inc (amp/m)	$\psi_x$ inc (degrees)
225.	17.	0.2789996E. 00	0.1891398E 01
225.	34.	0.1956546E 00	0.2960567E 01
225.	51.	0.4239342E-01	0.1484022E 02
225.	68.	0.2690623E-01	0.2248306E 02
225.	85.	0.2772586E-01	0.2041969E 02
225.	102.	0.2629502E-01	0.1930565E 02
225.	119.	0.2071891E-01	0.2104404E 02
225.	136.	0.1759634E-01	0.2100369E 02
225.	153.	0.1339864E-01	0.2290105E 02
225.	170.	0.1068252E-01	0.2388414E 02
225.	187.	0.7907361E-02	0.2667331E 02
225.	204.	0.6451011E-02	0.2757127E 02
250.	17.	0.2323287E 00	0.1884046E 01
250.	34.	0.1759682E 00	0.2769709E 01
250.	51.	0.5154993E-01	0.1063083E 02
250.	68.	0.1595848E-01	0.3177632E 02
250.	85.	0.2036559E-01	0.2488737E 02
250.	102.	0.2249587E-01	0.2093503E 02
250.	119.	0.1854533E-01	0.2212700E 02
250.	136.	0.1624122E-01	0.2161081E 02
250.	153.	0.1236657E-01	0.2366907E 02
250.	170.	0.1004350E-01	0.2439073E 02
250.	187.	0.7450014E-02	0.2729204E 02
250.	204.	0.6163001E-02	0.2796211E 02
275.	17.	0.1968113E 00	0.1819893E 01
275.	34.	0.1576793E 00	0.2584089E 01
275.	51.	0.5687815E-01	0.8262398E 01
275.	68.	0.8985039E-02	0.4401498E 02
275.	85.	0.1438970E-01	0.3081886E 02
275.	102.	0.1886392E-01	0.2294579E 02
275.	119.	0.1647171E-01	0.2329477E 02
275.	136.	0.1477643E-01	0.2241084E 02
275.	153.	0.1134650E-01	0.2448636E 02
275.	170.	0.9337660E-02	0.2506013E 02
275.	187.	0.7004671E-02	0.2789124E 02
275.	204.	0.5842950E-02	0.2846779E 02
300.	17.	0.1693239E 00	0.1698029E 01
300.	34.	0.1410468E 00	0.2392742E 01
300.	51.	0.5949142E-01	0.6687314E 01
300.	68.	0.9743243E-02	0.3772302E 02
300.	85.	0.9853393E-02	0.3815134E 02
300.	102.	0.1548327E-01	0.2542270E 02
300.	119.	0.1448470E-01	0.2459564E 02
300.	136.	0.1326279E-01	0.2340013E 02
300.	153.	0.1033620E-01	0.2538055E 02
300.	170.	0.8597381E-02	0.2586902E 02
300.	187.	0.6563704E-02	0.2850024E 02
300.	204.	0.5505923E-02	0.2905469E 02



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X (m)	Y (m)	$H_x^{inc}$ (amp/m)	$\psi_x^{inc}$ (degrees)
325.	17.	0.1476685E 00	0.1521017E 01
325.	34.	0.1261382E 00	0.2186716E 01
325.	51.	0.6023199E-01	0.5513439E 01
325.	68.	0.1458110E-01	0.2389888E 02
325.	85.	0.7092632E-02	0.4430373E 02
325.	102.	0.1240848E-01	0.2848421E 02
325.	119.	0.1258319E-01	0.2608244E 02
325.	136.	0.1174605E-01	0.2458835E 02
325.	153.	0.9339198E-02	0.2637604E 02
325.	170.	0.7846370E-02	0.2680438E 02
325.	187.	0.6124314E-02	0.2913922E 02
325.	204.	0.5162153E-02	0.2970079E 02
350.	17.	0.1302914E 00	0.1293766E 01
350.	34.	0.1129078E 00	0.1958526E 01
350.	51.	0.5970651E-01	0.4563470E 01
350.	68.	0.1932156E-01	0.1577961E 02
350.	85.	0.6528247E-02	0.4322250E 02
350.	102.	0.9682387E-02	0.3226187E 02
350.	119.	0.1077341E-01	0.2781712E 02
350.	136.	0.1026047E-01	0.2599785E 02
350.	153.	0.8362185E-02	0.2749596E 02
350.	170.	0.7101972E-02	0.2786023E 02
350.	187.	0.5686615E-02	0.2982193E 02
350.	204.	0.4818652E-02	0.3039114E 02
375.	17.	0.1160901E 00	0.1022363E 01
375.	34.	0.1012470E 00	0.1701910E 01
375.	51.	0.5834485E-01	0.3746344E 01
375.	68.	0.2318920E-01	0.1106730E 02
375.	85.	0.7586237E-02	0.3535664E 02
375.	102.	0.7356603E-02	0.3679120E 02
375.	119.	0.9065926E-02	0.2987547E 02
375.	136.	0.8831747E-02	0.2766435E 02
375.	153.	0.7412978E-02	0.2876521E 02
375.	170.	0.6376650E-02	0.2903622E 02
375.	187.	0.5252309E-02	0.3055852E 02
375.	204.	0.4480083E-02	0.3111641E 02
400.	17.	0.1042780E 00	0.7130889E 00
400.	34.	0.9101635E-01	0.1411779E 01
400.	51.	0.5644942E-01	0.3010883E 01
400.	68.	0.2613264E-01	0.8033892E 01
400.	85.	0.9170331E-02	0.2697122E 02
400.	102.	0.5523570E-02	0.4161153E 02
400.	119.	0.7475261E-02	0.3234673E 02
400.	136.	0.7479738E-02	0.2963739E 02
400.	153.	0.6499637E-02	0.3021146E 02
400.	170.	0.5679447E-02	0.3033585E 02
400.	187.	0.4823901E-02	0.3135667E 02
400.	204.	0.4149560E-02	0.3187082E 02

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X (m)	Y (m)	$H_x$ inc (amp/m)	$\varphi_x$ inc (degrees)
425.	17.	0.9429127E-01	0.3717660E 00
425.	34.	0.8206546E-01	0.1084205E 01
425.	51.	0.5423146E-01	0.2326409E 01
425.	68.	0.2824213E-01	0.5895625E 01
425.	85.	0.1072861E-01	0.2039516E 02
425.	102.	0.4352298E-02	0.4484128E 02
425.	119.	0.6021526E-02	0.3531664E 02
425.	136.	0.6220791E-02	0.3197838E 02
425.	153.	0.5630054E-02	0.3186574E 02
425.	170.	0.5016830E-02	0.3176582E 02
425.	187.	0.4404094E-02	0.3222334E 02
425.	204.	0.3829095E-02	0.3265154E 02
450.	17.	0.8572191E-01	0.3420460E-02
450.	34.	0.7424533E-01	0.7164755E 00
450.	51.	0.5183770E-01	0.1673662E 01
450.	68.	0.2963734E-01	0.4277200E 01
450.	85.	0.1208775E-01	0.1548930E 02
450.	102.	0.4012648E-02	0.4322064E 02
450.	119.	0.4736163E-02	0.3878891E 02
450.	136.	0.5070671E-02	0.3474716E 02
450.	153.	0.4812002E-02	0.3376047E 02
450.	170.	0.4393540E-02	0.3333412E 02
450.	187.	0.3995590E-02	0.3316502E 02
450.	204.	0.3519975E-02	0.3345804E 02
475.	17.	0.7827294E-01	-0.3878545E 00
475.	34.	0.6741470E-01	0.3071017E 00
475.	51.	0.4936912E-01	0.1040122E 01
475.	68.	0.3043768E-01	0.2981641E 01
475.	85.	0.1321003E-01	0.1175741E 02
475.	102.	0.4393969E-02	0.3636876E 02
475.	119.	0.3674658E-02	0.4241946E 02
475.	136.	0.4048746E-02	0.3795117E 02
475.	153.	0.4053667E-02	0.3592122E 02
475.	170.	0.3813216E-02	0.3504732E 02
475.	187.	0.3600938E-02	0.3418808E 02
475.	204.	0.3223018E-02	0.3429131E 02

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Horizontal Incident Magnetic Field Component  
Parallel to the Grounded Cable

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X (m)	Y (m)	$H_y$ inc (amp/m)	$\varphi_y$ inc (degrees)
25.	17.	0.2325496E 01	-0.4500101E 02
25.	34.	0.7777786E-01	-0.4498712E 02
25.	51.	0.6679149E 00	0.4500108E 02
25.	68.	0.8146733E-01	0.4500107E 02
25.	85.	0.1553105E 00	0.4500108E 02
25.	102.	0.2669536E-01	0.4500107E 02
25.	119.	0.6634927E-01	0.4500108E 02
25.	136.	0.1268797E-01	0.4499992E 02
25.	153.	0.3698913E-01	0.4500064E 02
25.	170.	0.7870752E-02	0.4498953E 02
25.	187.	0.2411119E-01	0.4499936E 02
25.	204.	0.5606472E-02	0.4497115E 02
50.	17.	0.1676929E 01	-0.4500070E 02
50.	34.	0.1880846E 00	-0.4499376E 02
50.	51.	0.4470411E 00	0.4500108E 02
50.	68.	0.1063344E 00	0.4500104E 02
50.	85.	0.1362560E 00	0.4500096E 02
50.	102.	0.3973712E-01	0.4500023E 02
50.	119.	0.5676622E-01	0.4500053E 02
50.	136.	0.1906772E-01	0.4499736E 02
50.	153.	0.3222106E-01	0.4499956E 02
50.	170.	0.1169739E-01	0.4499193E 02
50.	187.	0.2054279E-01	0.4499802E 02
50.	204.	0.7821318E-02	0.4498389E 02
75.	17.	0.1227600E 01	-0.4500003E 02
75.	34.	0.2388278E 00	-0.4499261E 02
75.	51.	0.2890560E 00	0.4500090E 02
75.	68.	0.1111766E 00	0.4500108E 02
75.	85.	0.1166593E 00	0.4500078E 02
75.	102.	0.4600875E-01	0.4499844E 02
75.	119.	0.4951061E-01	0.4499811E 02
75.	136.	0.2319392E-01	0.4499071E 02
75.	153.	0.2892900E-01	0.4499503E 02
75.	170.	0.1440121E-01	0.4498329E 02
75.	187.	0.1829227E-01	0.4499142E 02
75.	204.	0.9516072E-02	0.4497418E 02
100.	17.	0.9119605E 00	-0.4499876E 02
100.	34.	0.2562540E 00	-0.4498955E 02
100.	51.	0.1778293E 00	0.4499907E 02
100.	68.	0.1056901E 00	0.4500092E 02
100.	85.	0.9852564E-01	0.4500090E 02
100.	102.	0.4793792E-01	0.4499866E 02
100.	119.	0.4372011E-01	0.4499675E 02
100.	136.	0.2542602E-01	0.4498857E 02
100.	153.	0.2637625E-01	0.4498970E 02
100.	170.	0.1600793E-01	0.4497437E 02
100.	187.	0.1668416E-01	0.4497876E 02
100.	204.	0.1058769E-01	0.4495442E 02

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X (m)	Y (m)	$H_y^{inc}$ (amp/m)	$\varphi_y^{inc}$ (degrees)
125.	17.	0.6883928E 00	-0.4499670E 02
125.	34.	0.2542935E 00	-0.4498535E 02
125.	51.	0.1003745E 00	0.4498753E 02
125.	68.	0.9398311E-01	0.4500020E 02
125.	85.	0.8235073E-01	0.4500108E 02
125.	102.	0.4730556E-01	0.4499939E 02
125.	119.	0.3931752E-01	0.4499643E 02
125.	136.	0.2649330E-01	0.4498816E 02
125.	153.	0.2445425E-01	0.4498482E 02
125.	170.	0.1684359E-01	0.4496695E 02
125.	187.	0.1550490E-01	0.4496298E 02
125.	204.	0.1114734E-01	0.4493391E 02
150.	17.	0.5286285E 00	-0.4499345E 02
150.	34.	0.2416685E 00	-0.4498010E 02
150.	51.	0.4710882E-01	0.4489845E 02
150.	68.	0.7928813E-01	0.4499783E 02
150.	85.	0.6816351E-01	0.4500096E 02
150.	102.	0.4509088E-01	0.4499998E 02
150.	119.	0.3586543E-01	0.4499623E 02
150.	136.	0.2672889E-01	0.4498750E 02
150.	153.	0.2294853E-01	0.4498001E 02
150.	170.	0.1713672E-01	0.4496013E 02
150.	187.	0.1461505E-01	0.4494637E 02
150.	204.	0.1135603E-01	0.4491498E 02
175.	17.	0.4133413E 00	-0.4498857E 02
175.	34.	0.2237160E 00	-0.4497369E 02
175.	51.	0.1166888E-01	0.4239349E 02
175.	68.	0.6380028E-01	0.4499081E 02
175.	85.	0.5583896E-01	0.4499985E 02
175.	102.	0.4191582E-01	0.4500049E 02
175.	119.	0.3298044E-01	0.4499606E 02
175.	136.	0.2633554E-01	0.4498653E 02
175.	153.	0.2167511E-01	0.4497511E 02
175.	170.	0.1704229E-01	0.4495311E 02
175.	187.	0.1390914E-01	0.4492947E 02
175.	204.	0.1133499E-01	0.4489629E 02
200.	17.	0.3292490E 00	-0.4498158E 02
200.	34.	0.2036407E 00	-0.4496579E 02
200.	51.	0.1462335E-01	-0.4287718E 02
200.	68.	0.4885958E-01	0.4497090E 02
200.	85.	0.4521144E-01	0.4499626E 02
200.	102.	0.3821921E-01	0.4500093E 02
200.	119.	0.3042230E-01	0.4499615E 02
200.	136.	0.2547844E-01	0.4498566E 02
200.	153.	0.2051753E-01	0.4497038E 02
200.	170.	0.1667361E-01	0.4494565E 02
200.	187.	0.1331211E-01	0.4491232E 02
200.	204.	0.1116421E-01	0.4487642E 02

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X (m)	Y (m)	$H_y$ inc (amp/m)	$\varphi_y$ inc (degrees)
225.	17.	0.2671710E 00	-0.4497203E 02
225.	34.	0.1833319E 00	-0.4495601E 02
225.	51.	0.2995329E-01	-0.4441728E 02
225.	68.	0.3522100E-01	0.4491254E 02
225.	85.	0.3610808E-01	0.4498657E 02
225.	102.	0.3431134E-01	0.4500107E 02
225.	119.	0.2805720E-01	0.4499678E 02
225.	136.	0.2429623E-01	0.4498521E 02
225.	153.	0.1941090E-01	0.4496617E 02
225.	170.	0.1611561E-01	0.4493782E 02
225.	187.	0.1277441E-01	0.4489494E 02
225.	204.	0.1089600E-01	0.4485468E 02
250.	17.	0.2207244E 00	-0.4495953E 02
250.	34.	0.1638653E 00	-0.4494394E 02
250.	51.	0.3974374E-01	-0.4462029E 02
250.	68.	0.2327326E-01	0.4471741E 02
250.	85.	0.2835886E-01	0.4496265E 02
250.	102.	0.3040211E-01	0.4500024E 02
250.	119.	0.2581493E-01	0.4499792E 02
250.	136.	0.2289987E-01	0.4498546E 02
250.	153.	0.1832284E-01	0.4496269E 02
250.	170.	0.1543193E-01	0.4492973E 02
250.	187.	0.1226542E-01	0.4487740E 02
250.	204.	0.1056557E-01	0.4483084E 02
275.	17.	0.1854500E 00	-0.4494395E 02
275.	34.	0.1458106E 00	-0.4492915E 02
275.	51.	0.4550579E-01	-0.4467818E 02
275.	68.	0.1324432E-01	0.4383022E 02
275.	85.	0.2180364E-01	0.4490518E 02
275.	102.	0.2662619E-01	0.4499706E 02
275.	119.	0.2366108E-01	0.4499937E 02
275.	136.	0.2137476E-01	0.4498645E 02
275.	153.	0.1723982E-01	0.4496013E 02
275.	170.	0.1466972E-01	0.4492154E 02
275.	187.	0.1176770E-01	0.4485976E 02
275.	204.	0.1019699E-01	0.4480511E 02
300.	17.	0.1582202E 00	-0.4492519E 02
300.	34.	0.1294253E 00	-0.4491113E 02
300.	51.	0.4847275E-01	-0.4469334E 02
300.	68.	0.5797494E-02	0.3517407E 02
300.	85.	0.1629885E-01	0.4476552E 02
300.	102.	0.2306478E-01	0.4498901E 02
300.	119.	0.2158172E-01	0.4500067E 02
300.	136.	0.1978485E-01	0.4498822E 02
300.	153.	0.1615920E-01	0.4495851E 02
300.	170.	0.1386349E-01	0.4491339E 02
300.	187.	0.1127220E-01	0.4484210E 02
300.	204.	0.9806789E-02	0.4477769E 02

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X (m)	Y (m)	$H_y$ inc (amp/m)	$\psi_y$ inc (degrees)
325.	17.	0.1368324E 00	-0.4490331E 02
325.	34.	0.1147754E 00	-0.4488936E 02
325.	51.	0.4952946E-01	-0.4468941E 02
325.	68.	0.5354460E-02	-0.2893672E 02
325.	85.	0.1172588E-01	0.4440622E 02
325.	102.	0.1976266E-01	0.4497159E 02
325.	119.	0.1957432E-01	0.4500105E 02
325.	136.	0.1817752E-01	0.4499071E 02
325.	153.	0.1508412E-01	0.4495793E 02
325.	170.	0.1303834E-01	0.4490544E 02
325.	187.	0.1077525E-01	0.4482449E 02
325.	204.	0.9406179E-02	0.4474886E 02
350.	17.	0.1197291E 00	-0.4487848E 02
350.	34.	0.1018138E 00	-0.4486324E 02
350.	51.	0.4930068E-01	-0.4467444E 02
350.	68.	0.9814329E-02	-0.4117929E 02
350.	85.	0.8008514E-02	0.4336923E 02
350.	102.	0.1674084E-01	0.4493657E 02
350.	119.	0.1764253E-01	0.4499925E 02
350.	136.	0.1658741E-01	0.4499373E 02
350.	153.	0.1402061E-01	0.4495848E 02
350.	170.	0.1221233E-01	0.4489783E 02
350.	187.	0.1027638E-01	0.4480701E 02
350.	204.	0.9002764E-02	0.4471886E 02
375.	17.	0.1058052E 00	-0.4485088E 02
375.	34.	0.9043038E-01	-0.4483220E 02
375.	51.	0.4822483E-01	-0.4465184E 02
375.	68.	0.1389542E-01	-0.4302036E 02
375.	85.	0.5166624E-02	0.3974355E 02
375.	102.	0.1400537E-01	0.4486859E 02
375.	119.	0.1579282E-01	0.4499326E 02
375.	136.	0.1503959E-01	0.4499693E 02
375.	153.	0.1297589E-01	0.4496022E 02
375.	170.	0.1139838E-01	0.4489078E 02
375.	187.	0.9776961E-02	0.4478970E 02
375.	204.	0.8601658E-02	0.4468797E 02
400.	17.	0.9427208E-01	-0.4482069E 02
400.	34.	0.8048415E-01	-0.4479568E 02
400.	51.	0.4660840E-01	-0.4462325E 02
400.	68.	0.1708571E-01	-0.4361838E 02
400.	85.	0.3471687E-02	0.2450972E 02
400.	102.	0.1155384E-01	0.4473814E 02
400.	119.	0.1403256E-01	0.4497974E 02
400.	136.	0.1355185E-01	0.4499974E 02
400.	153.	0.1195709E-01	0.4496320E 02
400.	170.	0.1060561E-01	0.4488445E 02
400.	187.	0.9279307E-02	0.4477267E 02
400.	204.	0.8206226E-02	0.4465645E 02

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X (m)	Y (m)	$H_y$ inc (amp/m)	$\varphi_y$ inc (degrees)
425.	17.	0.8456367E-01	-0.4478804E 02
425.	34.	0.7182318E-01	-0.4475307E 02
425.	51.	0.4466466E-01	-0.4458949E 02
425.	68.	0.1943663E-01	-0.4388411E 02
425.	85.	0.3351823E-02	-0.1108804E 02
425.	102.	0.9380240E-02	0.4448581E 02
425.	119.	0.1236863E-01	0.4495335E 02
425.	136.	0.1213643E-01	0.4500108E 02
425.	153.	0.1097068E-01	0.4496741E 02
425.	170.	0.9840332E-02	0.4487906E 02
425.	187.	0.8786082E-02	0.4475609E 02
425.	204.	0.7818751E-02	0.4462457E 02
450.	17.	0.7627088E-01	-0.4475304E 02
450.	34.	0.6429654E-01	-0.4470389E 02
450.	51.	0.4254123E-01	-0.4455110E 02
450.	68.	0.2106580E-01	-0.4402002E 02
450.	85.	0.4266515E-02	-0.3031046E 02
450.	102.	0.7479560E-02	0.4398511E 02
450.	119.	0.1080709E-01	0.4490503E 02
450.	136.	0.1080133E-01	0.4499922E 02
450.	153.	0.1002214E-01	0.4497279E 02
450.	170.	0.9106752E-02	0.4487486E 02
450.	187.	0.8299977E-02	0.4474007E 02
450.	204.	0.7440686E-02	0.4459256E 02
475.	17.	0.6909585E-01	-0.4471565E 02
475.	34.	0.5776153E-01	-0.4464775E 02
475.	51.	0.4033941E-01	-0.4450832E 02
475.	68.	0.2209419E-01	-0.4409277E 02
475.	85.	0.5383823E-02	-0.3654266E 02
475.	102.	0.5853191E-02	0.4294870E 02
475.	119.	0.9352915E-02	0.4481950E 02
475.	136.	0.9551402E-02	0.4499127E 02
475.	153.	0.9115852E-02	0.4497917E 02
475.	170.	0.8407488E-02	0.4487209E 02
475.	187.	0.7823430E-02	0.4472478E 02
475.	204.	0.7072940E-02	0.4456067E 02



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Vertical Incident Magnetic Field

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X (m)	Y (m)	H <sub>z</sub> inc (amp/m)	$\varphi_z$ inc (degrees)
25.	17.	0.7360367E 01	-0.1347933E 00
25.	34.	0.2716605E 01	-0.2118430E 00
25.	51.	0.1584214E 00	-0.1235750E 01
25.	68.	0.1648539E 00	-0.7843280E 00
25.	85.	0.9095281E-01	-0.9407018E 00
25.	102.	0.5704409E-01	-0.1306586E 01
25.	119.	0.3497738E-01	-0.1532277E 01
25.	136.	0.3084284E-01	-0.1429070E 01
25.	153.	0.2085800E-01	-0.1450758E 01
25.	170.	0.1818706E-01	-0.1333079E 01
25.	187.	0.1336072E-01	-0.1214178E 01
25.	204.	0.1219693E-01	-0.1092314E 01
50.	17.	0.3224326E 01	-0.3926470E 00
50.	34.	0.1398605E 01	-0.5902832E 00
50.	51.	0.7391018E-01	-0.4692202E 01
50.	68.	0.8151561E-01	-0.2462542E 01
50.	85.	0.2438404E-01	-0.4882406E 01
50.	102.	0.2397260E-01	-0.3530740E 01
50.	119.	0.9759255E-02	-0.5781978E 01
50.	136.	0.1207243E-01	-0.3690834E 01
50.	153.	0.6892469E-02	-0.4586666E 01
50.	170.	0.6787147E-02	-0.3779044E 01
50.	187.	0.4374478E-02	-0.4199778E 01
50.	204.	0.4586637E-02	-0.3376141E 01
75.	17.	0.1857173E 01	-0.7495417E 00
75.	34.	0.9379008E 00	-0.1071496E 01
75.	51.	0.1462749E 00	-0.3457460E 01
75.	68.	0.6318176E-01	-0.4677421E 01
75.	85.	0.3832335E-02	-0.4028487E 02
75.	102.	0.1608461E-01	-0.7725799E 01
75.	119.	0.1568220E-02	-0.4343164E 02
75.	136.	0.6939787E-02	-0.8950229E 01
75.	153.	0.2011530E-02	-0.2116258E 02
75.	170.	0.3569622E-02	-0.9177710E 01
75.	187.	0.1332447E-02	-0.1694083E 02
75.	204.	0.2346815E-02	-0.7505242E 01
100.	17.	0.1202292E 01	-0.1199330E 01
100.	34.	0.6927574E 00	-0.1628998E 01
100.	51.	0.1701690E 00	-0.3762146E 01
100.	68.	0.5958023E-01	-0.6440745E 01
100.	85.	0.1678028E-01	-0.1506488E 02
100.	102.	0.1419954E-01	-0.1223497E 02
100.	119.	0.4516143E-02	-0.2593141E 02
100.	136.	0.5248949E-02	-0.1693990E 02
100.	153.	0.1305221E-02	-0.4183327E 02
100.	170.	0.2482051E-02	-0.1870943E 02
100.	187.	0.6296411E-03	-0.4382098E 02
100.	204.	0.1527433E-02	-0.1611105E 02

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X (m)	Y (m)	$H_z$ inc (amp/m)	$\varphi_z$ inc (degrees)
125.	17.	0.8346550E 00	-0.1738479E 01
125.	34.	0.5367967E 00	-0.2236420E 01
125.	51.	0.1734642E 00	-0.4283715E 01
125.	68.	0.6033698E-01	-0.7640788E 01
125.	85.	0.2515302E-01	-0.1257162E 02
125.	102.	0.1432142E-01	-0.1509841E 02
125.	119.	0.6929837E-02	-0.2209082E 02
125.	136.	0.4847169E-02	-0.2298781E 02
125.	153.	0.2436842E-02	-0.3209094E 02
125.	170.	0.2203329E-02	-0.2655573E 02
125.	187.	0.1211287E-02	-0.3366151E 02
125.	204.	0.1251291E-02	-0.2522858E 02
150.	17.	0.6087089E 00	-0.2362159E 01
150.	34.	0.4279808E 00	-0.2886867E 01
150.	51.	0.1672311E 00	-0.4892652E 01
150.	68.	0.6200277E-01	-0.8494090E 01
150.	85.	0.3032327E-01	-0.1212498E 02
150.	102.	0.1517608E-01	-0.1658272E 02
150.	119.	0.8449171E-02	-0.2141470E 02
150.	136.	0.5016692E-02	-0.2588911E 02
150.	153.	0.3263830E-02	-0.2916934E 02
150.	170.	0.2266753E-02	-0.3017834E 02
150.	187.	0.1622709E-02	-0.3093129E 02
150.	204.	0.1209675E-02	-0.3055205E 02
175.	17.	0.4611279E 00	-0.3064249E 01
175.	34.	0.3478752E 00	-0.3584530E 01
175.	51.	0.1566932E 00	-0.5563792E 01
175.	68.	0.6331176E-01	-0.9186600E 01
175.	85.	0.3345804E-01	-0.1228456E 02
175.	102.	0.1624181E-01	-0.1742825E 02
175.	119.	0.9511691E-02	-0.2153545E 02
175.	136.	0.5452115E-02	-0.2694194E 02
175.	153.	0.3881078E-02	-0.2815149E 02
175.	170.	0.2480079E-02	-0.3127403E 02
175.	187.	0.1902490E-02	-0.3029535E 02
175.	204.	0.1270448E-02	-0.3278127E 02
200.	17.	0.3602168E 00	-0.3836914E 01
200.	34.	0.2868541E 00	-0.4334943E 01
200.	51.	0.1444999E 00	-0.6289815E 01
200.	68.	0.6383866E-01	-0.9822305E 01
200.	85.	0.3521468E-01	-0.1273211E 02
200.	102.	0.1726861E-01	-0.1806557E 02
200.	119.	0.1031419E-01	-0.2199043E 02
200.	136.	0.5999357E-02	-0.2725401E 02
200.	153.	0.4380513E-02	-0.2786668E 02
200.	170.	0.2759239E-02	-0.3138895E 02
200.	187.	0.2116376E-02	-0.3036656E 02
200.	204.	0.1377013E-02	-0.3357967E 02

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X (m)	Y (m)	H <sub>z</sub> inc (amp/m)	ψ <sub>z</sub> inc (degrees)
225.	17.	0.2886059E 00	-0.4670323E 01
225.	34.	0.2392702E 00	-0.5141103E 01
225.	51.	0.1320310E 00	-0.7065156E 01
225.	68.	0.6351233E-01	-0.1044923E 02
225.	85.	0.3600102E-01	-0.1334407E 02
225.	102.	0.1813709E-01	-0.1866452E 02
225.	119.	0.1094310E-01	-0.2260237E 02
225.	136.	0.6568704E-02	-0.2735516E 02
225.	153.	0.4801314E-02	-0.2794360E 02
225.	170.	0.3057484E-02	-0.3125703E 02
225.	187.	0.2293542E-02	-0.3075647E 02
225.	204.	0.1501011E-02	-0.3388684E 02
250.	17.	0.2361544E 00	-0.5553370E 01
250.	34.	0.2015276E 00	-0.6003241E 01
250.	51.	0.1199974E 00	-0.7883730E 01
250.	68.	0.6241248E-01	-0.1108681E 02
250.	85.	0.3609335E-01	-0.1405538E 02
250.	102.	0.1880137E-01	-0.1927542E 02
250.	119.	0.1144077E-01	-0.2327419E 02
250.	136.	0.7107809E-02	-0.2744196E 02
250.	153.	0.5159803E-02	-0.2820529E 02
250.	170.	0.3346062E-02	-0.3114609E 02
250.	187.	0.2445610E-02	-0.3128801E 02
250.	204.	0.1625894E-02	-0.3408813E 02
275.	17.	0.1966593E 00	-0.6474907E 01
275.	34.	0.1711987E 00	-0.6919539E 01
275.	51.	0.1087460E 00	-0.8739403E 01
275.	68.	0.6067790E-01	-0.1174169E 02
275.	85.	0.3568939E-01	-0.1482774E 02
275.	102.	0.1925576E-01	-0.1990724E 02
275.	119.	0.1183091E-01	-0.2395360E 02
275.	136.	0.7588767E-02	-0.2757265E 02
275.	153.	0.5464148E-02	-0.2855508E 02
275.	170.	0.3608817E-02	-0.3112515E 02
275.	187.	0.2577634E-02	-0.3186040E 02
275.	204.	0.1742363E-02	-0.3430602E 02
300.	17.	0.1661848E 00	-0.7424945E 01
300.	34.	0.1465693E 00	-0.7886806E 01
300.	51.	0.9842014E-01	-0.9626526E 01
300.	68.	0.5845888E-01	-0.1241572E 02
300.	85.	0.3493419E-01	-0.1563819E 02
300.	102.	0.1951462E-01	-0.2055803E 02
300.	119.	0.1212875E-01	-0.2461682E 02
300.	136.	0.7999085E-02	-0.2775833E 02
300.	153.	0.5719654E-02	-0.2894293E 02
300.	170.	0.3838178E-02	-0.3119705E 02
300.	187.	0.2692372E-02	-0.3242151E 02
300.	204.	0.1846180E-02	-0.3456401E 02

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X (m)	Y (m)	$H_z$ inc (amp/m)	$\varphi_z$ inc (degrees)
325.	17.	0.1421594E 00	-0.8395156E 01
325.	34.	0.1263872E 00	-0.8900974E 01
325.	51.	0.8905184E-01	-0.1054004E 02
325.	68.	0.5589501E-01	-0.1310942E 02
325.	85.	0.3393548E-01	-0.1647272E 02
325.	102.	0.1960082E-01	-0.2122404E 02
325.	119.	0.1234533E-01	-0.2525575E 02
325.	136.	0.8335337E-02	-0.2799626E 02
325.	153.	0.5930442E-02	-0.2934592E 02
325.	170.	0.4031878E-02	-0.3134862E 02
325.	187.	0.2791601E-02	-0.3295139E 02
325.	204.	0.1935970E-02	-0.3486076E 02
350.	17.	0.1228458E 00	-0.9379834E 01
350.	34.	0.1097254E 00	-0.9956367E 01
350.	51.	0.8060664E-01	-0.1147611E 02
350.	68.	0.5310857E-01	-0.1382238E 02
350.	85.	0.3277292E-01	-0.1732266E 02
350.	102.	0.1953992E-01	-0.2190125E 02
350.	119.	0.1248870E-01	-0.2587100E 02
350.	136.	0.8599263E-02	-0.2827992E 02
350.	153.	0.6099839E-02	-0.2975581E 02
350.	170.	0.4190657E-02	-0.3156488E 02
350.	187.	0.2876573E-02	-0.3344769E 02
350.	204.	0.2012141E-02	-0.3518727E 02
375.	17.	0.1070645E 00	-0.1037394E 02
375.	34.	0.9586459E-01	-0.1104755E 02
375.	51.	0.7302737E-01	-0.1243038E 02
375.	68.	0.5019762E-01	-0.1455450E 02
375.	85.	0.3150636E-01	-0.1818196E 02
375.	102.	0.1935559E-01	-0.2258705E 02
375.	119.	0.1256577E-01	-0.2646552E 02
375.	136.	0.8794546E-02	-0.2860400E 02
375.	153.	0.6230906E-02	-0.3016974E 02
375.	170.	0.4316267E-02	-0.3183493E 02
375.	187.	0.2948256E-02	-0.3391408E 02
375.	204.	0.2075535E-02	-0.3553726E 02
400.	17.	0.9397763E-01	-0.1137453E 02
400.	34.	0.8425814E-01	-0.1216758E 02
400.	51.	0.6623775E-01	-0.1339947E 02
400.	68.	0.4724164E-01	-0.1530509E 02
400.	85.	0.3017982E-01	-0.1904622E 02
400.	102.	0.1906997E-01	-0.2327824E 02
400.	119.	0.1258229E-01	-0.2704312E 02
400.	136.	0.8926451E-02	-0.2896214E 02
400.	153.	0.6326173E-02	-0.3058809E 02
400.	170.	0.4411232E-02	-0.3214859E 02
400.	187.	0.3007242E-02	-0.3435770E 02
400.	204.	0.2127272E-02	-0.3590491E 02

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X (m)	Y (m)	$H_z^{inc}$ (amp/m)	$\psi_z^{inc}$ (degrees)
425.	17.	0.8298725E-01	-0.1237932E 02
425.	34.	0.7447666E-01	-0.1330923E 02
425.	51.	0.6016047E-01	-0.1437987E 02
425.	68.	0.4430133E-01	-0.1607321E 02
425.	85.	0.2882609E-01	-0.1991190E 02
425.	102.	0.1870257E-01	-0.2397198E 02
425.	119.	0.1254384E-01	-0.2760698E 02
425.	136.	0.9000745E-02	-0.2934856E 02
425.	153.	0.6388187E-02	-0.3101100E 02
425.	170.	0.4478183E-02	-0.3249744E 02
425.	187.	0.3054019E-02	-0.3478442E 02
425.	204.	0.2168371E-02	-0.3628496E 02
450.	17.	0.7365668E-01	-0.1338665E 02
450.	34.	0.6618178E-01	-0.1446501E 02
450.	51.	0.5471988E-01	-0.1536830E 02
450.	68.	0.4142208E-01	-0.1685777E 02
450.	85.	0.2746939E-01	-0.2077585E 02
450.	102.	0.1827027E-01	-0.2466557E 02
450.	119.	0.1245572E-01	-0.2815944E 02
450.	136.	0.9023327E-02	-0.2975777E 02
450.	153.	0.6419465E-02	-0.3143819E 02
450.	170.	0.4519813E-02	-0.3287363E 02
450.	187.	0.3089052E-02	-0.3519850E 02
450.	204.	0.2199797E-02	-0.3667255E 02
475.	17.	0.6566125E-01	-0.1439516E 02
475.	34.	0.5910474E-01	-0.1562749E 02
475.	51.	0.4984596E-01	-0.1636156E 02
475.	68.	0.3863694E-01	-0.1765736E 02
475.	85.	0.2612750E-01	-0.2163536E 02
475.	102.	0.1778753E-01	-0.2535655E 02
475.	119.	0.1232315E-01	-0.2870218E 02
475.	136.	0.9000048E-02	-0.3018488E 02
475.	153.	0.6422594E-02	-0.3186905E 02
475.	170.	0.4538782E-02	-0.3327042E 02
475.	187.	0.3112903E-02	-0.3560254E 02
475.	204.	0.2222473E-02	-0.3706290E 02

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APPENDIX F.

LISTING OF THE COMPUTER PROGRAMS

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COMPUTER PROGRAMMES

The subprogrammes required by each of the four main programmes are listed below.

Main Program.           EMXYZ

Sub-Progs.           GPS  
 GMNP  
 EXYZ  
 KPXYZ1  
 KPXYZ2  
 KSXYZ  
 SIMPON  
 CMATEQ  
 GSS  
 WABSS  
 GAUSS1  
 EULER  
 CBESNI  
 CBESNK  
 KEXYZ

Main Program.           INTGRL

Sub-Progs.           INTGLS  
 SUBINT  
 KK2  
 SIMPON

Main Program.           HXYZ

Sub-Progs.           WABSS  
 HXX  
 CBESNK  
 GSS  
 GGK  
 KK3  
 SUBINT  
 EULER  
 GAUSS1  
 KEXYZ  
 SIMPON  
 CBESNI



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Main Program.

HOMMO

Sub-Progs.

INTHX  
HX  
HY  
NT  
HZ  
BEKEP  
SIMPON

C\*\*\*\*\*PROGRAM EMXYZ\*\*\*\*\*

C  
C THIS PROGRAM CALCULATES THE ELECTRIC FIELD COMPONENTS  
C IN A CONDUCTOR BURIED IN THE CONDUCTIVE HALF-SPACE NEAR A  
C GROUNDED CABLE OF FINITE LENGTH.

C THE PROGRAM IS WRITTEN IN FORTRAN-4 LANGUAGE, AND WAS  
C DEVELOPED ON A DIGITAL COMPUTER CORPORATION MODEL PDP-10  
C COMPUTER.

C FOUR CONTROL CARDS ARE RECOGNIZED BY THE PROGRAM, THE  
C FORMAT OF STATEMENT NAME PUNCHED LEFT-JUSTIFIED IN COLS.  
C 1-4, FOLLOWED BY PARAMETERS PUNCHED IN COLS. 11-20, 21-30, 31-  
C 40, 41-50, 51-60, 61, 72, 71-80. THESE PARAMETERS ARE  
C READ WITH AN F10,0 FORMAT AND MUST THEREFORE ALL HAVE A  
C DECIMAL POINT, WHETHER INTEGER OR NOT.  
C CONTROL CARDS

C IDEN—THE CARD FOLLOWING THIS CONTROL CARD WILL BE READ  
C AND LISTED AS IDENTIFICATION, AN UNLIMITED NUMBER OF  
C "IDEN" CARDS MAY BE USED.

C DATA—THIS CONTROL CARD INITIATES READING OF THE FOLLOWING  
C PARAMETERS:

C F= FREQUENCY IN HZ.  
C SIGMA1 = CONDUCTIVITY OF THE HALF-SPACE. (MHO/M.)  
C SIGMA2 = CONDUCTIVITY OF THE CONDUCTOR. (MHO/M.)  
C L= HALF-LENGTH OF THE GROUNDED CABLE. (M.)  
C I= CURRENT IN AMPERES.

C PARA—THIS CONTROL CARD INITIATES READING OF THE  
C LOCATION AND GEOMETRY OF THE CONDUCTOR,  
C THE PARAMETERS ARE:

C WORD(1)= DISTANCE IN THE X-DIRECTION BETWEEN THE SOURCE  
C AND THE CENTER OF THE CONDUCTOR,  
C WORD(2)= DEPTH OF THE TOP OF THE CONDUCTOR. (M.)  
C WORD(3)=WORD(1)  
C WORD(4)=WORD(2)

C M= NUMBER OF CELLS IN THE X-DIRECTION.

C N= NUMBER OF CELLS IN THE Z-DIRECTION.

C EXEC—THIS CONTROL CARD INITIATES COMPUTATIONS AND  
C READING OF THE WAVE NUMBERS, AND THE INTEGRALS C, M, AND  
C TAU2 COMPUTED NUMERICALLY BY THE PROGRAM INTGR1.

C WAVE NUMBER

C EXIT—CALL EXIT.

C DIVICE SPECIFICATIONS:

C 1=IN INPUT CONTROL CARDS AND PARAMETERS  
C 11 INPUT OF THE INTEGRALS C, M, AND TAU2.  
C 0=IOUT - RESULTS TO DISK FILE.

C PROGRAM EMXYZ WAS DEVELOPED AT THE COLORADO SCHOOL OF  
C MINES AS PART OF A THESIS-RESEARCH STUDY FOR THE DEGREE OF  
C DOCTOR OF PHILOSOPHY IN GEOPHYSICS, FOR FURTHER INFORMATION SEE  
C CSM THESIS T-1711, "ELECTROMAGNETIC SCATTERING FROM CONDUCTORS  
C IN A CONDUCTIVE HALF-SPACE NEAR A GROUNDED CABLE OF FINITE LENGTH"  
C BY JORGE O. PARRA.

C JORGE O. PARRA DECEMBER 1974  
C COLORADO SCHOOL OF MINES.

C\*\*\*\*\*

REAL NO, LENGTH

INTEGER CONTRL, IDEN, PARA, DATA, EXEC

INTEGER EXEC

DATA EXEC/'EXEC'/

```

DATA IDEN,PARA,DATA,EXIT/'IDEN','PARA','DATA','EXIT'/
COMPLEX KPXX1,KPYY1,KPZZ1,KPXY1,KPXZ1,KPYX1,KPYZ1,KPZX1
* ,KPZY1
COMMON/SING1/KPXX1,KPYY1,KPZZ1,KPXY1,KPXZ1,KPYX1,KPYZ1
$ ,KPZX1,KPZY1
COMPLEX KPXX2,KPYY2,KPZZ2,KPXY2,KPXZ2,KPYX2,KPYZ2,
* KPZX2,KPZY2
COMMON/SING2/ KPXX2(60),KPYY2(60),KPZZ2(60),KPXY2(60),
* KPZX2(60),KPYX2(60),KPYZ2(60),KPZX2(60),KPZY2(60)
COMPLEX KSXX,KSXX,KSXX,KSXX,KSXX,KSXX,KSXX,KSXX,KSXX
COMMON/NONSIN/KSXX(121),KSXX(121),KSXX(121),KSXX(121),
* KSXX(121),KSXX(121),KSXX(121),KSXX(121),KSXX(121)
COMPLEX EX,EY,EZ,E
COMMON/ELXYZ/ EX(60),EY(60),EZ(60),E(180)
DIMENSION WORD(10),ID(20),ZD(121)
COMMON/VAR1/ Z(60),ZD(60),XM(60),X(60),DELTA,DELTA1
COMPLEX KMN(11,11),TKMN(31,31),I1,GAMMA2
COMMON/ABSWE/WIJ(160),RIJ(160),RS(160),WS(160)
COMPLEX IN1,IN2,CT,C,CGY,ARGA,FI(52)
COMMON/SSS/ IN1(121),IN2(121),GAMMA2(121)
COMPLEX UY,U
COMMON/DELTAP/DEL2,ALAMDA
CT=CMPLX(0.,0.)
CALL WABSS(WIJ,RIJ,WS,RS)

```

```

C
C
C
CONSTANTS AND PARAMETERS

```

```

PI=3.14159265
MO=4.*PI*1.E-07
PI2=SQRT(PI)

```

```

C
C
C
INPUT AND OUTPUT DEVICES

```

```

IN=1
IOUT1=4
IOUT2=4
IOUT=8

```

```

C
C
C
READ CONTROL CARDS

```

```

100 READ(IN,1) CONTRL,WORD
1  FORMAT(A4,6X,7F13.2)
IF(CONTRL.EQ.IDEN) GO TO 10
IF(CONTRL.EQ.DATA) GO TO 40
IF(CONTRL.EQ.PARA) GO TO 81
IF(CONTRL.EQ.EXEC) GO TO 30
IF(CONTRL.EQ.EXIT) GO TO 3
WRITE(IOUT2,4)
4  FORMAT(//10X,'ILLEGAL CONTROL-CARD STOP'/)
WRITE(IOUT,4) CONTRL,WORD
5  FORMAT(//10X,A4,6X,10F7.0)
3  STOP
10 INT1=WORD(1)
DO 141 I=1,INT1
READ(IN,16) ID
WRITE(IOUT2,17) ID
101 CONTINUE
16  FORMAT(20A4)
17  FORMAT(//10X,20A4)
GO TO 100

```

```

40      F=WORD(1)
        SIGMA1=WORD(2)
        SIGMA2=WORD(3)
        LENGTH=WORD(4)
        AI=WORD(5)
        W=2.*PI*F
        DELTA1=SQRT(2./(W*MO*SIGMA1))
        DELTA2=SQRT(2./(W*MO*SIGMA2))
        RK=SIGMA2/SIGMA1
        ALAMDA=RK-1.
        DECO=.9
        DELTA=DECO*DELTA2
        DEL2=(DELTA**2)/(DELTA1**2)
        A=DELTA/PI2
        A=A/DELTA1
        GO TO 100
81      X(1)=WORD(1)
        Z(1)=WORD(2)+ DELTA/2.
        XM(1)=WORD(3)
        ZN(1)=WORD(4)+DELTA/2.
        W=WORD(5)
        M=WORD(6)
        DO 44 K=1,N
        ZN(K+1)=ZN(K)+DELTA
44      Z(K+1)=Z(K)+DELTA
        DO 45 K=1,M
        XM(K+1)=XM(K)+DELTA
45      X(K+1)=X(K)+DELTA
        MN=M*N
C
C      PARAMETRIC RELATIONS
C
        GO TO 100
30      GY=WORD(1)
        PI1=2.*PI
        C=CMPLX(0.,1.)
        DO 56 I=1,60
        KSXX(I)=CT
        KSXY(I)=CT
        KSXZ(I)=CT
        KSYX(I)=CT
        KSYZ(I)=CT
        KSYI(I)=CT
        KSZX(I)=CT
        KSZY(I)=CT
56      KSZZ(I)=CT
        G=GY*GY
        U=CMPLX(G,2.)
        UY=CSQRT(U)
        SS=SIN(GY*LENGTH/DELTA1)/GY
        ARGA=UY*A
        CALL CBESNI(ARGA,2,FI)
        I1=FI(2)
        WRITE(IOUT2,79) I1
79      FORMAT(/ /10X,'I1=',2(E14.7,2X)/)
        NYT=21
C
C      CALCULATIONS OF THE SINGULAR PART
C
        CALL KPXYZ1(GY,C,UY,A,ARGA)

```

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```

WRITE(4,50)
50  FORMAT(/ /10X, 'HOLA' / /)
CALL KPXYZ2(GY,C,UY,A,N,I1)
DZA=2.*N
DZA=2.*Z(1)
DO 26 K=1,NZA
26  ZD(K)=(DZA+DELTA*(K-1))/DELTA1
WRITE(4,50)
READ(11,222) (IN2(I),GAMMA2(I),I=1,NZA)
222  FORMAT(4(2X,E14.7))
READ(11,25) (IN1(I),I=1,NZA)
CALL KSXYZ(N,ZD,IZA,GY,UY)
D2=DELTA1
CI=(2.*A1)/(PI*SIGMA1*D2)
JT=1
IT=1
DO 14 II=1,3
DO 15 JJ=1,3
KT=JJ+3*(II-1)
IF(KT.EQ.1) CALL GPS(KPXX1,KPXX2,KSXX,KMN,N)
IF(KT.EQ.2) CALL GPS(KPXY1,KPXY2,KSXY,KMN,N)
IF(KT.EQ.3) CALL GPS(KPXZ1,KPXZ2,KSXZ,KMN,N)
IF(KT.EQ.4) CALL GPS(KPYX1,KPYX2,KSXX,KMN,N)
IF(KT.EQ.5) CALL GPS(KPYY1,KPYY2,KSYY,KMN,N)
IF(KT.EQ.6) CALL GPS(KPYZ1,KPYZ2,KSZZ,KMN,N)
IF(KT.EQ.7) CALL GPS(KPZX1,KPZX2,KSZX,KMN,N)
IF(KT.EQ.8) CALL GPS(KPZY1,KPZY2,KSZY,KMN,N)
IF(KT.EQ.9) CALL GPS(KPZZ1,KPZZ2,KSZZ,KMN,N)
DO 21 I=1,N
DO 22 J=1,N
TKMN(IT,JT)=KMN(I,J)
22  JT=J+1*(JJ-1)
21  IT=I+1*(II-1)
15  CONTINUE
14  CONTINUE
WRITE(4,50)
CALL EXYZ(N,SS,CI,C,GY,UY)
WRITE(4,50)
NT=3*N
K=1
DO 18 I=1,N
18  E(K)=-EX(I)
K=K+1
DO 19 I=1,N
19  E(K)=-EY(I)
K=K+1
DO 20 I=1,N
20  E(K)=-EZ(I)
K=K+1
CALL CPATEQ(TKMN,E,NT,1,NT)
WRITE(IOUT,90) GY
90  FORMAT(F10.3)
WRITE(IOUT,25) (E(LL),LL=1,NT)
25  FORMAT(2(E14.7,2X))
GO TO 100
END

```

```

C*****SUBROUTINE GPS*****
C
C   GPS CONSTRUCTS THE SUBMATRICES OF THE DYADIC GREEN'S
C   TENSOR.
C
C           JORGE O. PARRA
C           COLORADO SCHOOL OF MINES
C

```

```

C*****
SUBROUTINE GPS(GP1,GA,GSX,KMN,N)
COMMON/DELTAP/DEL2,ALAMDA
COMPLEX GP1
COMPLEX GA(1),GB(11),GGP(11,11),KMN(11,11),GSX(1)
GA(1)=GP1
M=1
K=1
DO 102 JN=N,1,-1
102  GB(JN)=GA(K)
     K=K+1
     NZA=2*N
DO 103 I=1,N
     K=1
103  DO 103 J=I,N
     GGP(I,J)=GA(K)
     K=K+1
DO 104 I=1,N
     K=N-I+1
104  DO 104 J=1,I
     GGP(I,J)=GB(K)
     K=K+1
CALL GMNP(GSX,KMN,M,N,GGP)
RETURN
END

```

```

C*****SUBROUTINE GMNP*****
C
C   GMNP CONSTRUCTS THE DYADIC GREEN'S TENSOR.
C
C           JORGE O. PARRA
C           COLORADO SCHOOL OF MINES
C

```

```

C*****
SUBROUTINE GMNP(GSX,KMN,M,N,GGP)
COMPLEX GSX(1),KMN(11,11),GGP(11,11),GS,GT
COMMON/DELTAP/DEL2,ALAMDA
DO 5 I=1,M
DO 5 J=1,N
K=J
I1=(I-1)*N+J
DO 6 IK=1,M
DO 6 JN=1,N
J1=(JN-1)*N+JN
GS=GSX(K)
GT=DEL2*GS+GGP(I1,J1)
IF(I1.EQ.J1) DEL=0.0
IF(I1.EQ.J1) DEL=1.0
KMN(I1,J1)=ALAMDA*GT-CMPLX(DEL,0.)
6  K=K+1
5  CONTINUE
RETURN

```

END

```

C*****SUBROUTINE EXYZ*****
C
C   EXYZ CALCULATES THE INCIDENT ELECTRIC FIELD
C   COMPONENTS IN THE CONDUCTOR,
C
C           JORGE O. PARRA
C   COLORADO SCHOOL OF MINES
C
C*****
SUBROUTINE EXYZ(Y,SS,CI,C,GY,UY)
COMPLEX UY
COMPLEX EX,EY,EZ,E,FK(50)
COMMON/ABSWE/WIJ(160),RIJ(160),RS(160),WS(160)
COMMON/ELXYZ/EX(60),EY(60),EZ(60),E(180)
COMMON/VAR1/Z(60),ZN(60),XM(60),X(60),DELTA,DELTA1
COMPLEX ARG,K0,K1,B,C,QX,QY
X1=X(1)/DELTA1
X2=X1*X1
DO 1 I=1,N
Z1=Z(I)/DELTA1
Z2=Z1*Z1
R=SQRT(X2+Z2)
ARG=UY*R
CALL CBESNK(ARG,2,FK)
K0=FK(1)
K1=FK(2)
R1=R*R
R2=R1*R
B=UY*UY+K0*Z2/R1+UY*K1*(Z2-X2)/R2
CALL GSS(Z1,X1,GY,P,0,1)
QX=CMPLX(P,0)
EX(I)=CI*C*QX*SS*GY
CALL GSS(Z1,X1,GY,P,0,2)
QY=CMPLX(P,0)
EY(I)=-CI*(B+QY)*SS
EZ(I)=-CI*C*UY*Z1*K1*SS*GY/R
1 CONTINUE
RETURN
END

```

```

C*****SUBROUTINE KPXYZ1*****
C
C   KPXYZ1 COMPUTES THE SINGULAR PART OF THE GREEN'S
C   TENSOR FOR A SINGULAR CELL,
C
C           JORGE O. PARRA
C   COLORADO SCHOOL OF MINES
C
C*****
SUBROUTINE KPXYZ1(GY,C,UY,A,ARGA)
COMPLEX KPXX1,KPYY1,KPZZ1,KPXY1,KPXZ1,KPYX1,KPYZ1,KPZY1
* ,KPZX1,KPZY1,KPZY1
COMMON/SING1/ KPXX1,KPYY1,KPZZ1,KPXY1,KPXZ1,KPYX1,
* KPYZ1,KPZX1,KPZY1
COMPLEX C,CGY,UY,FK(50),ARGA,K1,CON,Q,CT
COMPLEX CP,UU
CP=CMPLX(Z,,2.)
CT=CMPLX(0,,0.)
PI=3.14159265
PI1=2.*PI

```

```

CALL CBESNK(ARG,2,FK)
UU=UY*UY
K1=FK(2)
COH=1./UY
Q=COH*(-A*K1+1./UY)
KPXX1=- (UU*.5+CP)*Q
KPHY1=CT
KPXZ1=CT
KPYX1=CT
KPHY1=- (GY*GY+CP)*Q
KPYZ1=CT
KPZX1=CT
KPHY1=CT
KPZZ1=KPXX1
RETURN
END

```

```

C*****SUBROUTINE KPXYZ2*****
C
C   KPXYZ2 COMPUTES THE SINGULAR PART OF THE GREEN'S TENSOR
C   FOR A NON-SINGULAR CELL,
C
C           JORGE O. PARRA
C   COLORADO SCHOOL OF MINES
C

```

```

C*****
SUBROUTINE KPXYZ2(GY,C,UY,A,N,I1)
COMPLEX KPXX2,KPHY2,KPZZ2,KPHY2,KPXZ2,KPHY2,KPHY2,
* KPZX2,KPHY2
COMMON/SING2/KPXX2(60),KPHY2(60),KPZZ2(60),KPHY2(60),
* KPXZ2(60),KPHY2(60),KPHY2(60),KPZX2(60),KPHY2(60)
COMPLEX C,CGY,UY,I1,CT,ARG,FK(50),KO,Q,QT
COMMON/VAR1/ Z(60),ZN(60),XM(60),X(60),DELTA,DELTA1
COMPLEX UU,CP
CP=CMPLX(0.,2.)
PI=3.14159265
PI1=2.*PI
CT=CMPLX(0.,1.)
XX=X(1)-XM(1)
UU=UY*UY
J=1
JN=2
DO 3 L=1,N-1
ZZ=(Z(J)-ZN(JN))/DELTA1
R=SQRT(XX*XX+ZZ*ZZ)
ARG=UY*R
CALL CBESNK(ARG,2,FK)
KO=FK(1)
Q=KO*PI1*A/UY
KPXX2(JN)=- (UU*.5+CP)*Q
KPHY2(JN)=CT
KPXZ2(JN)=CT
KPHY2(JN)=CT
KPHY2(JN)=- (GY*GY+CP)*Q
KPHY2(JN)=KPHY2(JN)
KPZX2(JN)=CT
KPHY2(JN)=CT
KPZZ2(JN)=KPXX2(JN)
JN=JN+1
CONTINUE
RETURN

```



END

```

C*****SUBROUTINE INTGLS*****
C
C   INTGLS CALCULATES THE NUMERICAL INTEGRATION OF THE
C   NON-SINGULAR ELEMENTS IN THE GREEN'S TENSOR.
C
C           JORGE O. PARRA
C           COLORADO SCHOOL OF MINES
C
C*****

```

```

SUBROUTINE INTGLS(IN1,IN2,GAMMA2,GY,ZD,ND)
  COMPLEX IN1(1),IN2(1),GAMMA2(1)
  DIMENSION ZD(1)
  DIMENSION XM(1)
  COMMON/ABSWE/WIJ(160),RIJ(160),RS(160),WS(160)
  COMPLEX C,K(600)
  DIMENSION R(602),P(602),Q(602)
  DIMENSION R1(602)
  PI=3.14159265
  PI1=2.*PI
  C=CMPLX(0.,1.)
  G=GY#GY
  NR=400
  DR=.25
  R(1)=0.0
  DO 1 I=1,NR
1  R(I+1)=R(I)+DR
  R1(1)=0.0
  DR1=.1
  NR1=600
  DO 7 II=1,NR1
7  R1(II+1)=R1(II)+DR1
  DO 6 I=1,ND
  Z1=ZD(I)
  CALL KK2(R,G,NR,Z1,K,1)
  CALL SUBINT(K,NR,P,Q,RY,QY,DR)
  GAMMA2(I)=CMPLX(RY,QY)/PI1
  CALL KK2(R,G,NR,Z1,K,2)
  CALL SUBINT(K,NR,P,Q,RY,QY,DR)
  IN2(I)=CMPLX(RY,QY)/PI1
  CALL KK2(R1,G,NR1,Z1,K,3)
  CALL SUBINT(K,NR1,P,Q,RX,QX,DR1)
  IN1(I)=CMPLX(RX,QX)/PI1
6  CONTINUE
  RETURN
  END

```

```

C*****SUBROUTINE SUBINT*****
C
C   SUBINT HANDLES THE NUMERICAL INTEGRATION OF THE REAL AND
C   IMAGINARY PART OF THE INTEGRALS.
C
C           JORGE O. PARRA
C           COLORADO SCHOOL OF MINES
C
C*****

```

```

SUBROUTINE SUBINT(K,NGX,P,Q,RX,QX,HG)
  COMPLEX K(1)
  DIMENSION P(1),Q(1),R(700)
  DO 1 I=1,NGX
  P(I)=REAL(K(I))

```

```

1      Q(I)=AIMAG(K(I))
      CALL SIMPON(P,R,NGX,NG)
      RX=R(NGX)
      CALL SIMPON(Q,R,NGX,NG)
      QX=R(NGX)
      RETURN
      END

```

```

C*****SUBROUTINE KK2*****

```

```

C
C      KK2 CALCULATES THE KERNEL OF THE INTEGRALS C,M
C      AND TAU2 AT X-X1=0.

```

```

C              JORGE O. PARRA
C              COLORADO SCHOOL OF MINES

```

```

C*****

```

```

      SUBROUTINE KK2(GX,G,NGX,Z,K,IK)
      COMPLEX K(1),U,UX,E
      DIMENSION GX(1)
      DO 1 I=1,NGX
      V=GX(I)
      V2=V*V
      A=V2+G
      U=CMPLX(A,2.)
      UX=CSQRT(U)
      E=CEXP(-UX*Z)
      S=SQRT(A)
      IF(IK.EQ.1) K(I)=(UX-S)*E/(UX*(UX+S))
      IF(IK.EQ.2) K(I)=(2.-S/UX)*E/S
      IF(IK.EQ.3) K(I)=(2.-S/UX)*V2*E/S
1      CONTINUE
      RETURN
      END

```

```

C*****SUBROUTINE KSXYZ*****

```

```

C
C      KSXYZ CALCULATES THE NON-SINGULAR ELEMENTS OF THE
C      DYADIC GREEN'S TENSOR,

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C              JORGE O. PARRA
C              COLORADO SCHOOL OF MINES

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C*****

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```

      SUBROUTINE KSXYZ(N,ZD,ND,GY,UY)
      COMPLEX IN1,IN2,GAMMA2,K0,K1,C,UA,FK(50),GC,UY
      COMMON/SSS/IN1(121),IN2(121),GAMMA2(121)
      DIMENSION ZD(1)
      COMMON/VAR1/ Z(60),ZN(60),XM(60),X(60),DELTA,DELTA1
      COMPLEX KSXX,KSXY,KSEX,KSXY,KSYY,KSZY,KSXZ,KSYZ,KSZZ
      COMMON/NONSIN/ KSXX(121),KSXY(121),KSZX(121),KSXY(121),
      * KSYY(121),KSZY(121),KSXZ(121),KSYZ(121),KSZZ(121)
      COMPLEX CT
      C=CMPLX(0.,1.)
      CT=CMPLX(0.,Z.)
      PI=3.14159265
      PI1=2.*PI
      XX=XN(1)-X(1)
      XX1=XX*XX
      GC=C*GY
      DO 26 K=1,ND
      Z1=ZD(K)

```

```
Z2=Z1*Z1
RS=SQRT(Z2+XX1)
UA=UY*RS
RS1=RS*RS
RS2=RS1*RS
CALL CBESHK(UA,2,FK)
K0=FK(1)/PI1
K1=FK(2)/PI1
KSXX(K)=GAMMA2(K)-IN1(K)
KSXY(K)=CT
KSXZ(K)=CT
KSYX(K)=CT
KSYZ(K)=GAMMA2(K)-GY*GY*IN2(K)
KSYZ(K)=-C*GY*UY*Z1*K1/RS
KSZX(K)=CT
KSZY(K)=-SYZ(K)
KSZZ(K)=UY*UY*Z2*K0/RS1 +UY*K1*Z2/RS2 -K0
26 CONTINUE
RETURN
END
```

```

C*****PROGRAM HXYZ*****
C
C   HXYZ PROGRAM CALCULATES THE MAGNETIC COMPONENTS AT THE
C SURFACE OF THE EARTH BY CONDUCTORS NEAR A GROUNDED CABLE OF
C FINITE LENGTH.
C   THREE CONTROL CARD ARE RECOGNIZED BY THE PROGRAM IN FORMAT
C (A4,6X,7F10.0),
C CONTROL CARDS
C   IDEN- THE CARD FOLLOWING THIS CONTROL CARD WILL BE READ
C AND LISTED AS IDENTIFICATION,AN UNLIMITED NUMBER OF "IDEN"
C CARDS MAY BE USED.
C   DATA- THIS CONTROL CARD INIATES READING OF THE FOLLOWING
C PARAMETERS:
C   F=FREQUENCY IN HZ,
C   SIGMA1=CONDUCTIVITY OF THE HALF-SPACE IN MHO/M.
C   SIGMA2=CONDUCTIVITY OF THE CONDUCTOR IN MHO/M.
C   I=CURRENT IN AMPERES.
C   PARA - THIS CONTROL CARD INIATES COMPUTATIONS AND READING
C OF THE WAVE NUMBERS,AND THE ELECTRIC FIELD COMPONENTS
C IN THE CONDUCTOR CALCULATED BY THE PROGRAM EMXYZ.
C   GY=WAVE NUMBER IN FORMAT(F10.3)
C   EXIT - CALL EXIT
C DEVICE SPECIFICATIONS:
C   INN=2 INPUT CONTROL CARDS AND PARAMETERS.
C   INPUT= 8 INPUT OF THE ELECTRIC FIELD COMPONENTS IN THE
C CONDUCTOR AND THE WAVE NUMBER.
C   IPUNCH=14 OUTPUT OF THE MAGNETIC FIELD COMPONENTS IN THE
C WAVE NUMBER DOMAIN.
C
C           JORGE O. PARRA
C           COLORADO SCHOOL OF MINES
C*****

```

```

C
C   REAL MO, LG
C   INTEGER PARA
C   DATA PARA/'PARA'/
C   DIMENSION ID(20)
C   COMPLEX IN3,E(32),AUX(22),HX(22),HA(22),SUM1,HXT(22)
C   DIMENSION ZN(12),XM(12),X(22),WORD(12)
C   COMPLEX INGS3,ARG,CY,U,KT,SUM2
C   COMPLEX FK(50)
C   INTEGER IDEN,DATA,EXIT,CONTRL
C   DATA IDEN,DATA,EXIT/'IDEN','DATA','EXIT'/
C   COMPLEX HAZ(20),HZT(20),HZ(20),SY(20),ASY(20),AUX1(20)
C   COMPLEX SUM1,HAY(22),HYT(20),HY(20),IN4,INGS4,INGS5
C   COMPLEX KO,KT0,KT1,SUM3,SUM4,C22
C   COMMON/ARSWE/WIJ(160),RIJ(160),RSS(160),WSS(160)
C   CALL WABSS(WIJ,RIJ,WSS,RSS)
C
C   CONSTANTS
C
C   INN=2
C   INPUT=8
C   IDUT=4
C   IPUNCH=14
C   PI=3.14159265
C   PO=4, #PI+1, E-27
100 READ(INN,70) CONTRL,(WORD(I),I=1,7)
70  FORMAT(A4,7F10.0)
   IF(CONTRL.EQ.IDEN) GO TO 71

```

```

IF(CONTRL,EQ,DATA) GO TO 72
IF(CONTRL,EQ,PARA) GO TO 90
IF(CONTRL,EQ,EXIT) GO TO 73
WRITE(IOUT,74)
74  FORMAT(/ /10X,'ILLEGAL CONTROL CARD STOP!/')
73  STOP
71  CONTINUE
    ILP=WORD(1)
    DO 91 I=1,ILP
      READ(IGN,92) ID
      WRITE(IOUT,93) ID
91  CONTINUE
92  FORMAT(20A4)
93  FORMAT(/ /10X,20A4)
    GO TO 100
72  F=WORD(1)
    * = 2.*PI*F
    SIGMA1=WORD(2)
    SIGMA2=WORD(3)
    AI=WORD(4)
    DELTA1=SQRT(2./(M*MO*SIGMA1))
    DELTA2=SQRT(2./(M*MO*SIGMA2))
    DELTA=.9*DELTA2
    DEL=DELTA/DELTA1
    SIGD=SIGMA2-SIGMA1*
    PI1=2.*PI
    ALAMDA=SIGD/(PI1*DELTA1)
    ALAMDA=ALAMDA*DELTA*DELTA
    GO TO 120
90  CONTINUE
320 READ(INPUT,320) GY
    FORMAT(F12.3)
    N=10
    NX=19
    LG=170./DELTA1
    Z1=(25.+DELTA*.5)
    ZN(1)=Z1
    DO 1 I=1,N
1   ZN(I+1)=ZN(I)+DELTA
    X(1)=25.
    DO 20 I=1,NX
20  X(I+1)=X(I)+X(1)
    G=GY*GY
    G22=GY*CMPLX(G.,1.)
    B=CMPLX(G,2.)
    GY=CSQRT(G)
    XM(1)=25.
    NT=3.*
15  READ(INPUT,15) (E(LL),LL=1,NT)
    FORMAT(2(E14.7,2X))
    DO 27 I=1,NX
    X1=X(I)/DELTA1
    CALL HXX(HX(I),HY(I),HZ(I),X1,GY,AI,DELTA1,LG)
27  CONTINUE
    NXX=10
    * = 1
    LN=1
    IK=9
    DO 522 I=1,NXX
    XX=X(I)/DELTA1

```

```

SUM1=CMPLX(0,0,0,0)
SUM3=CMPLX(0,0,0,0)
SUM4=CMPLX(0,0,0,0)
SUM2=CMPLX(0,0,0,0)
JJ=1
DO 503 I=1,M
X1=XM(IM)/DELTA1
DO 503 IV=1,N
Z1=ZN(IN)/DELTA1
XL=XX-K1
RS=SQRT(XL*XL+Z1*Z1)
ARG=RS*UY
CALL CBESNK(ARG,3,FK)
K1=FK(2)
K0=FK(1)
KT=UY*K1*Z1/RS
KT1=UY*XL*K1/RS
KT0=C22*K0
IF(XL.EQ.0,0,0) GO TO 10
CALL GSS(Z1,XL,GY,P,0,IK)
INGS3=CMPLX(P,Q)
CALL GSS(Z1,XL,GY,P1,Q1,12)
INGS4=CMPLX(P1,Q1)
SUM3=SUM3+E(JJ+2*N)*(KT0+INGS4*C22)
CALL GSS(Z1,XL,GY,P2,Q2,14)
INGS5=CMPLX(P2,Q2)
SUM4=SUM4+E(JJ+2*N)*(KT1+INGS5)
SUM1=SUM1+E(JJ)*(INGS3-KT)
SUM2=SUM2+E(JJ+N)*(KT-INGS3)
GO TO 503
10 CALL G6K(Z1,GY,IN3,IN4)
SUM1=SUM1+E(JJ)*(IN3-KT)
SUM2=SUM2+E(JJ+N)*(KT-IN3)
SUM3=SUM3+E(JJ+2*N)*(KT0+IN4*C22)
503 JJ=JJ+1
AUX1(I)=SUM2
SY(I)=-SUM3
ASY(I)=-SUM4
502 AUX(I)=SUM1
IC=1
K=1
LN=1
DO 505 I=1,NX
IF(K.EQ.10) LN=-1
HA(I)=ALAMDA*AUX(K)
HAY(I)=ALAMDA*AUX1(K)
HAZ(I)=ALAMDA*(IC*ASY(K)+SY(K))
HXT(I)=HA(I)+HX(I)
HYT(I)=HAY(I)+HY(I)
HZT(I)=HAZ(I)+HZ(I)
IF(K.EQ.10) IC=-1
K=K+LN
505 CONTINUE
WRITE(IOUT,31)
WRITE(IOUT,300) GY
WRITE(IPUNCH,300) GY
WRITE(IOUT,31)
31 FORMAT(//10X,' ', '//')
WRITE(IOUT,14) (HA(I),HX(I),I=1,NX)
DO 240 I=1,NX

```

```

E1=REAL(HXT(I))
E2=AIMAG(HXT(I))
AE=SQRT(E1*E1+E2*E2)
PE=ATAN2(E2,E1)*180./PI
E3=REAL(HA(I))
E4=AIMAG(HA(I))
AE1=SQRT(E3*E3+E4*E4)
PE1=ATAN2(E4,E3)*180./PI
240 WRITE(IOUT,14) AE,PE,AE1,PE1
WRITE(IOUT,31)
14  FORMAT(4(2X,E14.7))
WRITE(IOUT,14) (HAY(I),HY(I),I=1,NX)
DO 250 I=1,NX
E1=REAL(HYT(I))
E2=AIMAG(HYT(I))
AE=SQRT(E1*E1+E2*E2)
PE=ATAN2(E2,E1)*180./PI
E3=REAL(HAY(I))
E4=AIMAG(HAY(I))
AE1=SQRT(E3*E3+E4*E4)
PE1=ATAN2(E4,E3)*180./PI
250 WRITE(IOUT,14) AE,PE,AE1,PE1
WRITE(IOUT,31)
WRITE(IOUT,14) (HAZ(I),HZ(I),I=1,NX)
DO 260 I=1,NX
E1=REAL(HZT(I))
E2=AIMAG(HZT(I))
AE=SQRT(E1*E1+E2*E2)
PE=ATAN2(E2,E1)*180./PI
E3=REAL(HAZ(I))
E4=AIMAG(HAZ(I))
AE1=SQRT(E3*E3+E4*E4)
PE1=ATAN2(E4,E3)*180./PI
260 WRITE(IOUT,14) AE,PE,AE1,PE1
WRITE(IPUNCH,14) (HXT(I),HX(I),I=1,NX)
WRITE(IPUNCH,14) (HYT(I),HY(I),I=1,NX)
WRITE(IPUNCH,14) (HZT(I),HZ(I),I=1,NX)
GO TO 130
END

```

```

C*****SUBROUTINE SIMPON*****
C
C   SIMPON CALCULATES THE VECTOR OF INTEGRAL VALUES FOR A
C   GIVEN EQUIDISTANT TABLE OF FUNCTION VALUES.
C
C*****
C   SUBROUTINE SIMPON(Y,Z,NMAXS,DR)
C
C   DIMENSION Y(1),Z(1)
C   HT=(1.0/3.0)*DR
C   NDIM=NMAXS
C   IF(NDIM-5)7,8,1
C
C   NDIM IS GT. THAN 5 ,PREPARATION OF INTEGRAL LOOP
C
C   1 SUM1=Y(2)+Y(2)
C     SUM1=SUM1+SUM1
C     SUM1=HT*(Y(1)+SUM1+Y(3))
C     AUX1=Y(4)+Y(4)
C     AUX1=AUX1+AUX1
C
C     AUX1=SUM1+HT*(Y(3)+AUX1+Y(5))
C     AUX2=HT*(Y(1)+3.375*(Y(2)+Y(5))+
1  2.625*(Y(3)+Y(4))+Y(6))
C     SUM2=Y(5)+Y(5)
C     SUM2=SUM2+SUM2
C     SUM2=AUX2-HT*(Y(4)+SUM2+Y(6))
C     Z(1)=2.
C     AUX=AUX+SUM1
C     AUX=AUX+AUX1
C     Z(2)=SUM2-HT*(Y(2)+AUX+Y(4))
C     Z(3)=SUM1
C     Z(4)=SUM2
C     IF(NDIM-6)5,5,2
C
C   INTEGRATION LOOP
C
C   2 DO 4 I=7,NDIM,2
C     SUM1=AUX1
C     SUM2=AUX2
C     AUX1=Y(I-1)+Y(I-1)
C     AUX1=AUX1+AUX1
C     AUX1=SUM1+HT*(Y(I-2)+AUX1+Y(I))
C     Z(I-2)=SUM1
C     IF(I-NDIM)3,6,6
C   3  AUX2=Y(I)+Y(I)
C     AUX2=AUX2+AUX2
C     AUX2=SUM2+HT*(Y(I-1)+AUX2+Y(I+1))
C   4  Z(I-1)=SUM2
C   5  Z(NDIM-1)=AUX1
C     Z(NDIM)=AUX2
C     RETURN
C   6  Z(NDIM-1)=SUM2
C     Z(NDIM)=AUX1
C     RETURN
C
C   END OF INTEGRATION LOOP
C
C   7 IF(NDIM-3)12,11,8
CNDIM IS EQUO

```



```

8      SUM2=1.125*HT*(Y(1)+3.0*Y(3)+3.0*Y(2)+Y(4))
      SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      Z(1)=S.
      AUX1=Y(3)+Y(3)
      AUX1=AUX1+AUX1
      Z(2)=SUM2-HT*(Y(2)+AUX1+Y(4))
      IF(NDIM-5)10,9,9
9      AUX1=Y(4)+Y(4)
      AUX1=AUX1+AUX1
      Z(5)=SUM1+HT*(Y(3)+AUX1+Y(5))
10     Z(3)=SUM1
      Z(4)=SUM2

```

```

C
C      NDIM IS EQUAL TO 3
C
11     SUM1=HT*(1.25*Y(1)+Y(2)+Y(2)-0.25*Y(3))
      SUM2=Y(2)+Y(2)
      Z(3)=HT*(Y(1)+SUM2+Y(3))
      Z(1)=S.
      Z(2)=SUM1
12     RETURN
      END

```

```

C*****SUBROUTINE BEKEP*****
C
C      BEKEP  CALCULATES THE MODIFIED BESSEL FUNCTIONS I0,I1,K0,
C      K1 FOR COMPLEX ARGUMENT X.  AMPLITUDE OF THE ARGUMENT
C      MUST BE LESS THAN 8.  PHASE IS 45.  DEGREES.

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C      JORGE O. PARRA.
C      COLORADO SCHOOL OF MINES.

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```

C*****
SUBROUTINE BEKEP(I0,I1,K0,K1,X,MX)
COMPLEX I0,I1,K0,K1,CC,IOA,I1A
COMPLEX CC1
S=1./SQRT(2.)
CC=-S*CMPLX(1.,1.)
CC1=S*CMPLX(1.,1.)
IF(MX.EQ.2) IOA=I0
IF(MX.EQ.2) I1A=I1
REAL KERX,KEIX,KERPX,KEIPX
DIMENSION A(8),B(7),C(8),D(7),E(7),F(7),G(7),H(7)
DATA A/1.,-64.,113.77777774,-32.36345652,2.64191397,-.0834966
* .00122552,-.00000901/
DATA B/16.,-113.77777774,72.81777742,-10.56765779,.52185615,
* -.01103667,.00011346/
DATA C/-.57721566,-59.05819744,171.36272133,-60.60977451,
* 5.65539121,-.19636347,.00009699,-.00002458/
DATA D/6.76454936,-142.91827687,124.23569650,-21.30060904,
* 1.17509064,-.02695875,.00029532/
DATA E/-4.,14.22222222,-6.06814810,.66047849,
* -.02609253,.00045957,-.00000394/
DATA F/.5,-12.66666666,11.37777772,-2.31167514,.14677204,
* -.00379386,.00004609/
DATA G/-3.69113734,21.42034017,-11.36433272,1.41304780,
* -.06136358,.00116137,-.00001075/
DATA H/.2139217,-13.39858846,19.41182758,-4.65950823,
* .33049424,-.00926707,.00011997/

```

```

PI=3.14159265358979
DATA U0,U,V,Z,X,Y,F,Q,U1,V1,Z1,W1,Y1,P1,Q1/0.,0.,0.,0.,
* 0.,0.,0.,0.,0.,0.,0.,0.,0.,0./
IF(X,LT.,00720001) GO TO 30
U0=X/8.
U=U0*U0
IF(X,LT.,00001) GO TO 30
V=U*U
IF(X,LT.,0001) GO TO 30
Z=V*U
IF(X,LT.,009) GO TO 30
W=Z*U
IF(X,LT.,04) GO TO 30
Y=W*U
IF(X,LT.,08) GO TO 30
P=Y*U
Q=P*U
IF(X,LT.,4) GO TO 30
U1=Q*U
V1=U1*U
IF(X,LT.,8) GO TO 30
Z1=V1*U
W1=Z1*U
Y1=W1*U
P1=Y1*U
Q1=P1*U

```

```

C
C
C

```

CALCULATION OF BER(X)

```

30 BERX=A(1)+A(2)*V+A(3)*W+A(4)*P+A(5)*U1+A(6)*Z1+A(7)*Y1+A(8)*Q

```

```

C
C
C

```

CALCULATION OF BEI(X)

```

BEIX=B(1)*U+B(2)*Z+B(3)*Y+B(4)*Q+B(5)*V1+B(6)*W1+B(7)*P1
IF(MX.EQ.1) GO TO 10
IF(MX.EQ.3) CONTINUE

```

```

C
C
C

```

CALCULATION OF KER(X)

```

AUXA=-ALOG(.5*X)*BERX+.25*PI*BEIX
AUXB=C(1)+C(2)*V+C(3)*W+C(4)*P+C(5)*U1+C(6)*Z1+C(7)*Y1+C(8)*Q
KERX=AUXA+AUXB

```

```

C
C
C

```

CALCULATION OF KEI(X)

```

AUXC=-ALOG(.5*X)*BEIX-.25*PI*BERX
AUXD=D(1)*U+D(2)*Z+D(3)*Y+D(4)*Q+D(5)*V1+D(6)*W1+D(7)*P1
KEIX=AUXC+AUXD

```

```

C
C
C

```

CALCULATION OF BER(X)'

```

10 CONTINUE
BERPX=X*(E(1)+U+E(2)*Z+E(3)*Y+E(4)*Q+E(5)*V1+E(6)*W1+E(7)*P1)

```

```

C
C
C

```

CALCULATION OF BEI(X)'

```

BEIPX=X*(F(1)+F(2)*V+F(3)*W+F(4)*P+F(5)*U1+F(6)*Z1+F(7)*Y1)
IF(MX.EQ.1) GO TO 20
IF(MX.EQ.3) CONTINUE

```

```

C

```

```

C      CALCULATION OF KER(X)'
C
AUXE=-ALOG(.5*X)*BERPX-BERX/X+.25*PI*BEIPX
AUXF=X*(G(1)*U+G(2)*Z+G(3)*Y+G(4)*Q+G(5)*V1+G(6)*W1+G(7)*P1)
KERPX=AUXE+AUXF
C
C      CALCULATION OF KEI(X)'
C
AUXG=-ALOG(.5*X)*BEIPX-BEIX/X-.25*PI*BERPX
AUXH=X*(H(1)+H(2)*V+H(3)*W+H(4)*P+H(5)*U1+H(6)*Z1+H(7)*Y1)
20  IO=CMPLX(BERPX,BEIX)
    I1=CC*CMPLX(-BEIPX,BERPX)
    IF(MX.EQ.1) GO TO 120
    K0=CMPLX(KERX,KEIX)
    KEIPX=AUXG+AUXH
    K1=CC1*CMPLX(-KEIPX,KERPX)
    IF(MX.EQ.3) GO TO 200
    IF(MX.EQ.2) IO=IOA
    IF(MX.EQ.2) I1=I1A
    GO TO 202
100  K0=CMPLX(0.,0.)
    K1=CMPLX(0.,0.)
200  U=0.0
    V=0.0
    W=0.0
    Z=0.0
    Y=0.0
    P=0.0
    Q=0.0
    U1=0.0
    V1=0.0
    Z1=0.0
    W1=0.0
    Y1=0.0
    P1=0.0
    Q1=0.0
    RETURN
    END
C*****SUBROUTINE CMATEQ*****
C
C      CMATEQ SOLVES A SET OF COMPLEX SIMULTANEOUS LINEAR
C      EQUATIONS BY GAUSS ELIMINATION COEFFICIENTS WITH
C      PARTIAL PIVOTING.
C*****
C      COMPLEX FUNCTION CMATEQ(A,B,III,JJJ,ID)
C      DIMENSION A(31,31),B(31,1)
C      COMPLEX A,B,S,D
C      COMPLEX F
C      F=CMPLX(0.,0.)
C      KK=III
C      NV=IABS(JJJ)
C      D=CMPLX(1.,0.)
C      IF(JJJ.LT.0) D=CMPLX(0.,0.)
C      KKM=KK-1
C      WRITE(4,111)
111  FORMAT(/'PAGE FOR 1')
      DO 90 I=1,KKM
        S=CMPLX(0.,0.)

```

```

DO 10 J=1, KK
R=CABS(A(J, I))
IF(R.LT.CABS(S)) GO TO 10
S=R
L=J
10 CONTINUE
IF(L.EQ.1) GO TO 50
DO 20 J=1, KK
S=A(I, J)
A(I, J)=A(L, J)
20 A(L, J)=S
IF(NV.LE.0) GO TO 40
DO 30 J=1, NV
S=S(I, J)
B(I, J)=B(L, J)
30 B(L, J)=S
40 D=-D
50 IF(CABS(A(I, I)).EQ.0.0) GO TO 90
IPO=I+1
DO 80 J=IPO, KK
IF(CABS(A(J, I)).EQ.0.0) GO TO 80
S=A(J, I)/A(I, I)
A(J, I)=CMPLX(Z., Z.)
DO 60 K=IPO, KK
60 A(J, K)=A(J, K)-A(I, K)*S
IF(NV.LE.0) GO TO 80
DO 70 K=1, NV
70 B(J, K)=B(J, K)-B(I, K)*S
80 CONTINUE
90 CONTINUE
WRITE(4, 122)
122 FORMAT(/'PASE POR 2')
DO 100 I=1, KK
100 D=D*A(I, I)
IF(NV.LE.0) GO TO 130
KMO=KK-1
DO 120 K=1, NV
B(KK, K)=B(KK, K)/A(KK, KK)
DO 120 I=1, KMO
N=KK-I
DO 110 J=N, KMO
110 B(N, K)=B(N, K)-A(N, J+1)*B(J+1, K)
120 B(N, K)=B(N, K)/A(N, N)
130 CMATEQ=D.
WRITE(4, 125) CMATEQ
125 FORMAT(/10X, 'ETER=', E14.7)
RETURN
END
C*****SUBROUTINE GSS*****
C
C GSS CALCULATES THE INTEGRALS OF THE NON-SINGULAR DYADIC
C GREEN'S FUNCTION AND THE INTEGRALS OF THE ELECTRIC
C FIELD BETWEEN THE ZEROS OF THE COSINE AND SINE FUNCTIONS
C
C JORGE O. PARRA
C COLORADO SCHOOL OF MINES
C*****
SUBROUTINE GSS(Z, Y, GX, P, U, IK)
COMMON/ABSWE/WIJ(160), RIJ(160), RS(160), WS(160)

```

```

DIMENSION PR(101),GR(101),R(101)
PI=3.14159265
N=100
H=PI/(2.*N)
IF(IK.EQ.1) GO TO 6
IF(IK.EQ.10) GO TO 6
IF(IK.EQ.14) GO TO 6
IF(IK.EQ.15) GO TO 6
R(1)=0,0
N2=N+1
DO 3 I=1,N2
G=H*(I-1)
Y=ABS(Y)
CALL KXYZ(G/Y,GX,Z,P,Q,IK)
PR(I)=P*COS(G)
GR(I)=Q*COS(G)
3 CONTINUE
CALL GAUSS1(Z,Y,YP,YQ,IK,GX)
H=H/Y
CALL SIMPON(PR,R,N2,H)
RY1=R(N2)
CALL SIMPON(GR,R,N2,H)
QY1=R(N2)
P=RY1+YP
Q=QY1+YQ
RETURN
6 CALL GAUSS1(Z,Y,P,Q,IK,GX)
RETURN
END
C*****SUBROUTINE WABSS*****
C
C   WABSS CALCULATES THE WEIGHT OF THE GAUSS QUADRATURE
C   METHOD OF INTEGRATION,
C
C           JORGE O. PARRA
C           COLORADO SCHOOL OF MINES,
C
C*****
SUBROUTINE WABSS(WIJ,RIJ,WS,RS)
DIMENSION WIJ(1),RIJ(1)
DIMENSION WS(1),RS(1)
DIMENSION X(8),W(8)
DATA X/-.96028986,-.79666648,-.52553241,-.18343464,
* .18343464,.52553241,.79666648,.96028986/
DATA W/.10122854,.22238103,.31370665,.36268378,.36268378,
* .31370665,.22238103,.10122854/
PI=3.14159265
P2=PI/2
K=1
DO 1 J=1,20
DO 2 I=1,8
X1=X(I)
W1=W(I)
RIJ(K)=P2*(X1+2.*J)
WIJ(K)=P2*W1
2 K=K+1
1 CONTINUE
X=1
DO 3 J=1,20
DO 4 I=1,8

```

```

      X1=X(I)
      W1=W(I)
      RS(K)=P2*(X1+2.*J-1.)
      WS(K)=P2*W1
4      K=K+1
3      CONTINUE
      RETURN
      END
C*****SUBROUTINE GAUSS1*****
C
C      GAUSS1 COMPUTES THE NUMERICAL INTEGRATION BETWEEN THE
C      ZEROS OF THE COSINE AND SINE FUNCTIONS.
C
C              JORGE O. PARRA
C      COLORADO SCHOOL OF MINES
C
C*****
      SUBROUTINE GAUSS1(Z,Y,YP,YQ,IK,GX)
      DIMENSION PR(20),QI(20)
      COMMON/ABSWE/WIJ(160),RIJ(160),RS(160),WS(160)
      IR=Z
      I=1
      TS1=0.0
      TS2=0.0
      R=.1E-05
      YP=0.0
      YQ=0.0
      DO 51 K=1,20
      PR(K)=0.0
      QI(K)=0.0
      DO 50 L=1,8
      Y=ABS(Y)
      IF(IK.EQ.1) GO TO 10
      IF(IK.EQ.10) GO TO 10
      IF(IK.EQ.14) GO TO 10
      IF(IK.EQ.15) GO TO 10
      CALL KEXYZ(RIJ(I)/Y,GX,Z,P,Q,IK)
      W=RIJ(I)*COS(RIJ(I))/Y
10     GO TO 20
      CONTINUE
      CALL KEXYZ(RS(I)/Y,GX,Z,P,Q,IK)
      W=RS(I)*SIN(RS(I))/Y
20     CONTINUE
      PR(K)=PR(K)+P*W
      QI(K)=QI(K)+Q*W
50     I=I+1
      YP=YP+PR(K)
      YQ=YQ+QI(K)
      FP=ABS(PR(K))
      FQ=ABS(QI(K))
      IF((FP.LT.R).AND.(FQ.LT.R)) GO TO 61
51     CONTINUE
      TS1=TS1+YP
      TS2=TS2+YQ
C
C      COMPUTE DIFFERENCES FOR PR(I)
C
      IF(FP.LT.R) GO TO 100
      CALL EULER(PR,Z3R,IR,YH1)
      YP=TS1+Z3R-YP

```

```

      GO TO 62
100    YP=TS1
      62    IF(FQ.LT.R) GO TO 200
          CALL EULER(G1,Z30,IR,VM2)
          YQ=TS2+Z30-YQ
          GO TO 63
200    YQ=TS2
      63    IF(IR.EQ.0) RETURN
          *WRITE(4,5) IR,Y
      5     FORMAT(/,1X,'IR=',I10,2X,E14,7/)
          *WRITE(4,59) VM1,VM2
      59    FORMAT(2(2X,E14,7))
          RETURN
      61    YP=TS1+YP
          YQ=TS2+YQ
          IR=0
          RETURN
          END

```

```

C*****SUBROUTINE EULER*****

```

```

C
C     EULER CALCULETES EULER'S TRANSFORMATION,
C           JORGE O. PARRA
C           COLORADO SCHOOL OF MINES,
C

```

```

C*****

```

```

      SUBROUTINE EULER(G,Z3,IR,VM)
      DIMENSION G(1)
      Z1=1.
      VM=Z1
      DO 57 K=1,15
      L=21-K
      Q=.5
      Z2=Q*ABS(G(L-1))
      DO 52 M=L,20
      Q=.5*Q
      G(M)=ABS(G(M-1))-ABS(G(M))
52     Z2=Z2+(Q*G(M))
      MS=L-2
      F=2.0
      DO 53 N=1,MS
53     F=F+G(M)
      Z2=SIGN(Z2,G(L-1))
      F=F+Z2
      IF(F) 55,54,55
54     V=0.2
      GO TO 56
55     V=ABS(Z1/F-1.)
      IF(V-VM) 56,56,57
56     VM=V
      Z3=F
57     Z1=F
      IF((VM-.1E-06).LT.0.0) GO TO 80
      IR=1
      RETURN
80     IR=0
      RETURN
      END

```

```

C*****SUBROUTINE BEKP1*****

```

```

C
C     BEKP1 CALCULATES THE MODIFIED BESSEL FUNCTIONS I0,I1,K0

```

C AND K1 FOR COMPLEX ARGUMENT X. AMPLITUDE OF X  
C MUST BE GREATER THAN 8. THE PHASE IS 45 DEGREES.

C JORGE O. PARRA  
C COLORADO SCHOOL OF MINES  
C

C\*\*\*\*\*

```

SUBROUTINE BEKP1(I0,I1,K0,K1,X,MX)
DIMENSION R(7),A(7),R1(7),A1(7),T(7),P(7)
DATA R/2.,.0110486,.0,-.0000906,-.0000252,-.0000034,.0000026/
DATA A/-.3926991,-.0110485,-.00009765,-.0000901,.0,.0000051,
* .0000019/
DATA R1/7071068,-.0625001,-.0013813,.0000005,.0000346,.00201
* .0000016/
DATA A1/7071068,-.0000001,.0013811,.0002452,.0000338,
* -.0000024,-.0000032/
COMPLEX T,P, TX,PX,K0,K1,I0,I1,TXX,GX,AUX,C4,C2,FX
COMPLEX PXX
DO 1 I=1,7
T(I)=CMPLX(R(I),A(I))
1 P(I)=CMPLX(R1(I),A1(I))
PI=3.1415965
U=8./X
V=-8./X
TX=T(1)+V*(T(2)+V*(T(3)+V*(T(4)+V*(T(5)+V*(T(6)+V*T(7))))))
PX=P(1)+V*(P(2)+V*(P(3)+V*(P(4)+V*(P(5)+V*(P(6)+V*P(7))))))
C1=SQRT(PI/(2.*X))
C2=(1./SQRT(2.))*CMPLX(1.,1.)
C4=(1./SQRT(2.))*CMPLX(1.,-1.)
FX=C1*CEXP(-C2*X+TX)
K0=FX
K1=C4*FX*PX
IF(MX.EQ.2) RETURN
TXX=T(1)+U*(T(2)+U*(T(3)+U*(T(4)+U*(T(5)+U*(T(6)+T(7)*U))))
PXX=P(1)+U*(P(2)+U*(P(3)+U*(P(4)+U*(P(5)+U*(P(6)+U*P(7))))))
C3=1./SQRT(2.*PI*X)
GX=C3*CEXP(C2*X+TXX)
I0=GX+CMPLX(2.,1.)*K0/PI
AUX=-CMPLX(2.,1.)*FX*PX/PI+GX*PXX
I1=C4*AUX
RETURN
END

```

C\*\*\*\*\*SUBROUTINE CBESNI\*\*\*\*\*

C  
C CBESNI CALCULATES THE MODIFIED BESSEL FUNCTIONS I<sub>ν</sub> FOR  
C ANY ORDER WHEN THE PHASE OF THE ARGUMENT IS LESS THAN  
C 45 DEGREES,  
C

C\*\*\*\*\*

```

SUBROUTINE CBESNI(Z,NMAX,FI)
COMPLEX FI(50),PI(200),Z,SUM,TZ,A
SUM=CMPLX(0.,0.)
RE=REAL(Z)
OE=AIMAG(Z)
R=SQRT(RE*RE+OE*OE)
I=R
JMAX=I+21
TZ=2./Z
9 FORMAT(4(2X,E14.7))
PI(JMAX+2)=CMPLX(0.,2.)

```



```

6      PI(JMAX+1)=CMPLX(1.E-38,0.0)
7      FORMAT(I10,I10)
      FORMAT(1X,8(1X,E14.7))
      DO 1 J=1,JMAX
      K=JMAX+2-J
      DK=K-1
      PI(K-1)=DK*TZ*PI(K)+PI(K+1)
1      SUM=SUM+PI(K)
      SUM=SUM+SUM
      A=CEXP(Z)/(PI(1)+SUM)
      DO 2 N=1,NMAX
2      FI(N)=A*PI(N)
      RETURN
      END

```

C\*\*\*\*\*SUBROUTINE CBESNK\*\*\*\*\*

C  
C       CBESNK CALCULATES THE MODIFIED BESSEL FUNCTION K, FOR  
C       ANY ORDER WHEN THE PHASE OF THE ARGUMENT IS LESS THAN  
C       45 DEGREES,  
C

C\*\*\*\*\*

```

C
      SUBROUTINE CBESNK(X,KMAX,FK)
      COMPLEX X,FI(50),FK(50),T,T2,T1
      XM=CABS(X)
      IF(XM-2.) 2,3,3
2      T=.5*X
      T=T*T
      FK(1)=((((((0.00000740*T+.00010750)*T+.00262693)*T+.0348859)*T+
      * .23069756)*T+.42278420)*T-.57721566
      FK(2)=(((((-.00004686*T-.00110404)*T-.01919402)*T-.18156897)*
      * .67278579)*T+.15443144)*T+1.
      CALL CBESNI(X,20,FI)
      T2=.5*LOG(T)
      FK(1)=FK(1)-T2*FI(1)
      FK(2)=FK(2)/X+T2*FI(2)
      T=2./X
      GO TO 1
3      T=2./X
      FK(1)=((((((0.00053208*T-.00251540)*T+.00587872)*T-.01062446)*T
      * .02189568)*T-.07832358)*T+1.25331414
      FK(2)=(((((-.00068245*T+.00325614)*T-.00760353)*T+.01504268)*
      * .03655620)*T+.23498619)*T+1.25331414
      T1=CEXP(-X)/CSQRT(X)
      FK(1)=FK(1)*T1
      FK(2)=FK(2)*T1
1      DO 4 N=3,KMAX
4      DK=N-2
      FK(N)=T*DK*FK(N-1)+FK(N-2)
      RETURN
      END

```

C\*\*\*\*\*SUBROUTINE KEXYZ\*\*\*\*\*

C  
C       KEXYZ CALCULATES COMPLEX KERNEL OF THE INTEGRALS,  
C

C               JORGE O. PARRA  
C               COLORADO SCHOOL OF MINES  
C

C\*\*\*\*\*

SUBROUTINE KEXYZ(GY,GX,Z,P,Q,IK)

```
COMPLEX U,UX,E,K,UY
G1=GY*GY
G2=GX*GX
A=G1+G2
U=CMPLX(A,2.)
UY=CSQRT(U)
S=SQRT(A)
E=CEXP(-UY*Z)
IF(IK.EQ.8) K=E*GY
IF(IK.EQ.3) K=UY*G1*E/S
IF(IK.EQ.4) K=UY*GY*E/S
IF(IK.EQ.5) K=E/S
IF(IK.EQ.6) K=UY*UY*E/S
IF(IK.EQ.1) K=E*GY/S
IF(IK.EQ.2) K=E*G1/S
IF(IK.EQ.7) K=G1/((UY+S)*S)
IF(IK.EQ.9) K=(UY-S)*E/(UY+S)
IF(IK.EQ.10) K=GY/((UY+S)*S)
IF(IK.EQ.12) K=E*(UY-S)/(UY*(UY+S))
IF(IK.EQ.14) K=GY*(UY-S)*E/(UY*(UY+S))
IF(IK.EQ.15) K=GY/(S+UY)
R=REAL(K)
G=AIMAG(K)
RETURN
END
```

```

C*****PROGRAM HOMMO*****
C
C   PROGRAM HOMMO CALCULATES THE MAGNETIC FIELD COMPONENTS
C AT THE SURFACE OF THE EARTH FOR THE CONDUCTIVE HALF-SPACE.
C
C CONTROL CARDS
C   IDEN- THE CARD FOLLOWING THIS CONTROL CARD WILL BE READ
C   AND LISTED AS IDENTIFICATION,
C   DATA- THIS CONTROL CARD INITIATES READING OF FOLLOWING
C   PARAMETERS:
C       F=FREQUENCY IN HZ.
C   SIGMA1= CONDUCTIVITY OF THE HALF-SPACE, (MHO/M.)
C       L=HALF-LENGTH OF THE GROUNDED CABLE(M.)
C       NX=NUMBER OF POINT IN THE X-DIRECTION
C       NY=NUMBER OF POINTS IN THE Y-DIRECTION,
C       X1=ABSCISSA OF THE FIRST STATION,
C       Y1=ORDINATE OF THE FIRST STATION.
C   EXIT- CALL EXIT
C DIVICE SPECIFICATIONS:
C   15=IN    INPUT CONTROL CARDS AND PARAMETERS,
C   16=IOUT  RESULTS TO DISKFILE,
C
C           JORGE O. PARRA
C           COLORADO SCHOOL OF MINES
C
C*****

```

```

REAL HO,L
INTEGER CONTRL,IDEN,DATA,EXIT
INTEGER CONTRL,IDEN,DATA,EXIT
DATA IDEN,DATA,EXIT/'IDEN','DATA','EXIT'/
DIMENSION X(30),Y(30),WORD(10),ID(20)
COMPLEX AX(30),CX,F
DIMENSION AX(30),PX(30)
PI=3.14159265
HO=4.*PI*1.E-07
WT=180./PI
IN=15
IOUT=16
100 READ(IN,1) CONTRL,WORD
1   FORMAT(A4,10F)
   IF(CONTRL.EQ.IDEN) GO TO 10
   IF(CONTRL.EQ.DATA) GO TO 40
   IF(CONTRL.EQ.EXIT) GO TO 2
   WRITE(IOUT,4)
4   FORMAT(/10X,'ILLEGAL CONTROL CARD STOP'/)
2   STOP
10  READ(IN,3) ID
3   FORMAT(20A4)
   WRITE(IOUT,5) ID
5   FORMAT(/10X,20A4)
   GO TO 100
40  F=WORD(1)
   SIGMA=WORD(2)
   L=WORD(3)
   AI=WORD(4)
   NX=WORD(5)
   NY=WORD(6)
   X1=WORD(7)
   Y1=WORD(8)
   K=2.*PI*F

```

```

DELTA=SQRT(2./(W*HC*SIGMA))
DY=Y1
DX=X1
X(1)=X1
Y(1)=Y1
C=AI/(4.*PI*DELTA)
CC=-CHPLX(C,2.)
50 DO 50 I=1,NX
X(I+1)=X(I)+DX
51 DO 51 I=1,NY
Y(I+1)=Y(I)+DY
WRITE(IOUT,72)
DH=.4
DO 64 I=1,NY
Y1=Y(I)/DELTA
60 DO 74 J=1,NX
X1=X(J)/DELTA
CALL INTHX(X1,Y1,H,L,DH,DELTA)
70 HX(J)=CC*H
DO 11 KK=1,NX
R1=REAL(HX(KK))
R2=AIMAG(HX(KK))
AX(KK)=SQRT(R1*R1+R2*R2)
PX(KK)=ATAN2(R2,R1)*PI
11 CONTINUE
WRITE(IOUT,71) (Y(I),X(K),AX(K),PX(K),K=1,NX)
60 CONTINUE
72 FORMAT(///13X,'.'/)
71 FORMAT(10X,F5.0,3X,F5.0,4X,E14.7,4X,E14.7)
GO TO 100
END
C*****SUBROUTINE INTHX*****
C
C   INTHX CALCULATES THE MAGNETIC FIELD COMPONENT HX OR HZ
C   AT THE SURFACE OF THE EARTH FOR DIFFERENT VALUES OF X AND Y
C
C           JORGE O. PARRA
C   COLORADO SCHOOL OF MINES
C
C*****
SUBROUTINE INTHX(X,Y,DHX,L,DY,DELTA)
COMPLEX DHX
REAL L
DIMENSION Y1(1000),U(1000),PR(1000),QR(1000)
NY=2*L/DY+1
Y1(1)=-L/DELTA
X2=X*X
DO 1 I=1,NY
YY=Y-Y1(I)
Y2=YY*YY
R=SQRT(X2+Y2)
CALL HZ(R,YY,Y,DHX,Y2,X2,X)
PR(I)=REAL(DHX)
QR(I)=AIMAG(DHX)
1 Y1(I+1)=Y1(I)+DY/DELTA
D=DY/DELTA
CALL SIMPON(PR,U,NY,D)
P=U(-Y)
CALL SIMPON(QR,U,NY,D)
Q=U(NY)

```

```

DHX=CMPLX(P,0)
RETURN
END

```

```

C*****SUBROUTINE HX*****

```

```

C
C HX CALCULATES THE MAGNETIC FIELD COMPONENT HX FOR THE
C CONDUCTIVE HALF-SPACE.

```

```

C
C JORGE O. PARRA
C COLORADO SCHOOL OF MINES
C

```

```

C*****

```

```

SUBROUTINE HX(R,YY,V,DHX,Y2,X2,X)
IMPLICIT COMPLEX (A-P)
GAMMA=CMPLX(1.,1.)
G=GAMMA*GAMMA
R2=R*R
R3=R2*R
R22=1./R2
XXX=X/SDRT(2.)
CALL BEKEP(I0,I1,K0,K1,XXX,3)
C11=GAMMA*R22*(-2.+8.*X2*R22)
C01=-G*X2/R3
C10=-C01
DHX=GAMMA*.5*(C11*I1*K1+C01*I0*K1+C10*I1*K0)
RETURN
END

```

```

C*****SUBROUTINE HY*****

```

```

C
C HY CALCULATES THE MAGNETIC FIELD COMPONENT HY FOR THE
C CONDUCTIVE HALF-SPACE.

```

```

C
C JORGE O. PARRA
C COLORADO SCHOOL OF MINES
C

```

```

C*****

```

```

SUBROUTINE HY(X,Y,L,HTY,DELTA)
COMPLEX HTY,N1,N2
REAL L
YL1=Y-L/DELTA
YL2=Y+L/DELTA
CALL NT(X,YL1,N1)
CALL NT(X,YL2,N2)
HTY=N1-N2
RETURN
END

```

```

C*****SUBROUTINE NT*****

```

```

C
C NT CALCULATES EACH TERM OF THE MAGNETIC FIELD
C COMPONENT HY,

```

```

C
C JORGE O. PARRA
C COLORADO SCHOOL OF MINES.
C

```

```

C*****

```

```

SUBROUTINE NT(X,Y,DHY)
COMPLEX I0,I1,K0,K1,GAMMA,DHY,B00,B11
COMPLEX B10,B01
GAMMA=CMPLX(1.,1.)
R=SDRT(X*X+Y*Y)
R1=R*R

```

```

ARG=1/SORT(2.)
CALL BEKEP(I0,I1,K0,K1,ARG,3)
DHZ=-GAMMA*GAMMA*I1*K1*X/R1
RETURN
END

```

```

C*****SUBROUTINE HZ*****

```

```

C
C
C
C
C
C
C
C
C

```

```

HZ CALCULATES THE MAGNETIC FIELD COMPONENT HZ FOR THE
CONDUCTIVE HALF-SPACE.

```

```

        JORGE O. PARRA
        COLORADO SCHOOL OF MINES

```

```

C*****

```

```

SUBROUTINE HZ(R,YY,Y,DHZ,Y2,X2,X)
IMPLICIT COMPLEX(A-P)
GAMMA=CMPLX(1.,1.)
G=GAMMA*GAMMA
R1=R*R
R2=R1*R
D01=(3./R1-G)/R1
D10=D01
XXX=1/SORT(2.)
CALL BEKEP(I0,I1,K0,K1,XXX,3)
DN1=GAMMA*X*.5*(D01*I0*K1+D10*I1*K0)
A01=CMPLX(-1./R1,0.0)
A10=A01
DN2=GAMMA*X*.5*(A01*I0*K1+A10*I1*K0)
F=3./R1+3.*GAMMA/R+G.
EX=F*CEXP(-GAMMA*R)
P=X*EX/R2
DHZ=DN1+2.*CMPLX(1.,0.0)*DN2-P
RETURN
END

```

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