COMPARATIVE GEOMECHANICAL INVESTIGATION OF EMPIRICAL, ANALYTICAL, AND NUMERICAL METHODS UTILIZED IN DESIGNING FLAT-ROOF EXCAVATIONS IN DISCONTINUOUS AND LAMINATED ROCKMASSES

by

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ABSTRACT

Simplification of complex geologic systems has been a necessary hallmark of geological engineering research and design to date. However, oversimplification and subsequent over-application of existing methods leaves room for significant improvement in our understanding of rockmass response to excavation. Although indisputable advancements have been made in increasing the safety of underground workings, falls of ground continue to injure or kill personnel and delay production. The overapplication of existing simplified methods is particularly problematic in discontinuous and laminated systems, where the response to excavation can be anisotropic and significantly impacted by the orientation, intensity, and condition of discontinuities. With the advancement of computational power and numerical modeling techniques, more of the mechanical complexities associated with discontinuous systems can be explicitly considered. Therefore, the goal of this research is to identify the geomechanical considerations for a wide range of discontinuous and laminated geologic conditions that should be incorporated into analytical and empirical methods to increase the safety and productivity of mining and civil works.

This thesis focuses on addressing and overcoming two of the most significant simplifications often employed in the design of flat-roof excavations: assuming that the overburden has no self-supporting capacity, and representing discontinuous systems as continua. To that end, this research utilizes the explicit discrete element method (DEM) to identify and account for the relevant geologic and mining conditions that control local and global stability. Model complexity and scale is increased incrementally, and model results are compared to existing, well-established analytical and empirical methods to validate, confirm, or frame the implications of the numerical results and their relationship with “reality”.

The first objective of this thesis is to evaluate roof self-stability and stress arching capacity through application and enhancement of the voussoir beam analog. Gaps in existing analytical calculations are identified and addressed through the methodical variation of geometry, material properties, and boundary conditions in explicit DEM voussoir beam numerical models. An adjusted voussoir beam analog is developed that can account for novel aspects of complexity such as post-peak material behavior, horizontal stress, and layered roofs that are passively bolted. The adjusted voussoir beam
analytical method is then applied to a case study of the Bondi Pumping Chamber excavation in Sydney, New South Wales, Australia.

The second objective of this thesis is to analyze roof self-supporting capacity and bolted stability through a parametric sensitivity analysis of 8,640 unique explicit DEM models of hypothetical coal-mine entries conducted with a particular focus on discontinuity properties. Additional considerations include in-situ stress magnitude and horizontal stress ratio, as well as material stiffness, strength, and anisotropy. Model inputs are utilized to assign a Coal Mine Roof Rating (CMRR) value to each model case, and the Analysis of Roof Bolt Systems (ARBS) is subsequently used to assess the reliability of the model results and focus future statistical analysis. Multivariate binary logistic regression is used to identify the statistically significant parameter inputs that determine the probability of a stable roof condition in unsupported and bolted models. Recommendations such as adjusting the cohesion-roughness rating and consideration of joint orientation in CMRR, as well as accounting for in-situ horizontal stress ratio in ARBS, are posited.

The last objective of this thesis is to identify how excavation roofs and pillars are mechanically linked. A calibrated, confinement-dependent coal pillar constitutive model is combined with the significant controls on roof stability identified through the course of this study to assess pillar-overburden interaction in single-entry and multi-entry models. Entry-scale models are used to identify the interaction between roof stress arching capacity and pillar confinement, and panel-scale models are subsequently developed to incorporate in-situ complexities such as panel width-to-height ratio, lithologic heterogeneity, and depillaring to assess overburden stress arching capacity and pillar response. Lastly, the panel-scale model results are compared to state-of-practice analytical and empirical methods such as tributary area theory (TAT), the Analysis of Retreat Mining Pillar Stability (ARMPS), the abutment angle concept, and the Mark-Bieniawski pillar strength equation. Results confirm that properties that increase stress arching in the overburden tend to decrease pillar loads and increase pillar strength.

The results of this study identify that increasing both the accuracy and applicability of existing analytical and empirical methods, as well as our holistic understanding of flat-roof excavation stability requires mechanically coupling the pillars to the roof and floor. Without this explicit consideration, state-of-practice and state-of-knowledge cannot advance towards both safer and more efficient excavations.
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DEDICATION

This thesis is dedicated to all those who have perished or been injured working underground and their friends and family, as well as to all the curious folks who came before me and who will come after me; those that are willing to start from “I don’t know” and engage with the world around them to bridge that gap.

Every machine has had the same history — a long record of sleepless nights and of poverty, of disillusions and of joys, of partial improvements discovered by several generations of nameless workers, who have added to the original invention these little nothings, without which the most fertile idea would remain fruitless. More than that: every new invention is a synthesis, the resultant of innumerable inventions which have preceded it in the vast field of mechanics and industry.

Science and industry, knowledge and application, discovery and practical realization leading to new discoveries, cunning of brain and of hand, toil of mind and muscle — all work together. Each discovery, each advance, each increase in the sum of human riches, owes its being to the physical and mental travail of the past and the present.

By what right then can any one whatever appropriate the least morsel of this immense whole and say — This is mine, not yours?

-Peter Kropotkin
CHAPTER 1
INTRODUCTION

1.1 Project Motivation & Overview

The existing state of academic research and practical design of underground excavations in both mining and civil applications relies heavily on simplifying assumptions regarding the mechanical behavior of rocks and rockmasses. Although some degree of simplification is required to study a given natural phenomenon, oversimplification and subsequent broad application can lead to decoupling of a given method from reality. Furthermore, designs of excavation layouts (i.e. pillar size and spacing) and installed supports (e.g. mesh, steel sets, shotcrete, bolts, anchors, etc.) are predominantly based on empirical approaches developed from limited datasets in specific geologic and mining conditions (e.g. Barton et al., 1974; Hudyma, 1986; Mark, 1987; Mark et al., 1997; Mark & Chase, 1997; Mark et al., 2001), analytical approaches that assume continuous, homogenous, isotropic, and linearly elastic (CHILE) material properties, or previous experience of individual operators and contractors (Galvin, 2016).

Simplification, conservativism, and component isolation are often substituted for mechanical understanding in the design of flat-roof excavations. For example, overburden dead-loading (i.e. no self-supporting capacity or stress arching, also known as tributary area theory (TAT)) reflects all three of these descriptors. It transforms a heterogeneous lithology with different material properties it to a single uniform block with no stress arching capacity, independent of any potential interaction with the pillars below. However unrealistic this is, it is accepted because it generally leads to a conservative final design.

Application of existing methods, particularly in discontinuous, anisotropic, inhomogeneous, non-elastic (DIANE) rockmasses that cannot always be approximated as a continuum, may lead to inadequate excavation design. Highly prevalent anisotropic discontinuity networks that influence rockmass behavior are most common in sedimentary and metamorphic rockmasses. Underground excavations in these rockmasses are prevalent throughout the mining and civil industries as sedimentary and metamorphic rock accounts for 83% (± 3.5%) of all rock types exposed at the Earth’s surface (Blatt & Jones, 1975) and act as host rocks for mined resources such as coal, stone, and metal ores.
Although the mechanics of flat-roofed underground excavations in horizontally laminated discontinuous rockmasses are relevant to a number of application areas, the analyses presented in this thesis are primarily (though not exclusively) presented in the context of underground coal mining. Coal is predominantly found in sub-horizontal, layered sedimentary lithology, has been mined world-wide for millennia, and has been researched globally for decades. Furthermore, the most dangerous subsurface work in the USA recently is coal mining (CDC, 2015).

Although a catastrophic global (i.e. total) underground failure has not occurred in the United States of America (USA) since the Crandall Canyon mine disaster in 2007, and mine fatalities have steadily decreased since 1990 thanks to the indisputable contribution of decades of existing research, falls of ground still kill and injure miners every year in the USA (Mark et al., 2011; MSHA, 2020). Analytical, empirical, and continuum numerical methods for roof and pillar stability evaluation (Molinda & Mark, 1993; Mark et al., 2005; Zhang & Heasley, 2013; Mark, 2015) have been developed and comprise the bulk of current industry-standard practices for ground-control operations. However, despite the large and diverse body of research regarding ground control, ground falls accounted for 112 fatalities in underground bituminous coal mines from 1995 to 2008 (Mark et al., 2011). During the same time period, smaller ground falls between installed roof support elements injured up to 400 miners annually (Mark et al., 2011). More recently, between 2008 and 2018, ground falls in underground coal mines were responsible for 39 fatalities and 2,565 injuries (MSHA, 2020). For comparison, in other underground mines (e.g. stone, metal, gemstone, and uranium), 14 fatalities and 205 injuries occurred between 2008 and 2018 due to falls of ground (MSHA, 2020). Although not as thoroughly documented and on a smaller scale, unforeseen ground behavior can cause major delays, cost-overruns, and even injuries and fatalities in civil construction (Sousa, 2007).

As mines move deeper underground and as urban underground civil works become more prevalent, the likelihood of more catastrophic failures (i.e. those that cause injuries and fatalities) is expected to increase (Kaiser et al., 1996; Diederichs, 2007). Improving our understanding of subsurface rock mechanics as they relate to flat-roof excavations will help reduce the negative impacts to health, human safety, and productivity that affect underground construction projects.

Ultimately, there should be no falls of ground or catastrophic pillar failure if the typically applied “conservative” design methods are appropriately accounting for all relevant aspects of rockmass
mechanical behavior. This is clearly not the case, and additional complexity must be accounted for in an attempt to more realistically represent local (entry-scale) and global (panel- or mine-scale) stability of underground excavations. In order to accomplish these goals, this thesis relies heavily on numerical modeling. However, model complexity is increased incrementally throughout the thesis and each set of models is accompanied by comparative analysis or validation through the use of well-vetted empirical or analytical methods. In relatively simple cases, the numerical, analytical, and empirical results converge to the same solution. Where the numerical results diverge from the analytical and empirical methods, this thesis addresses why, and subsequently develops and verifies new analytical methods, or provides mechanically informed criticism of existing methods such that future research may address their limitations and broaden their applicability.

In order to better understand pillar-roof interaction, this thesis first investigates discontinuous roof behavior through the lens of the voussoir beam analog (Diederichs & Kaiser, 1999) in multiple iterations of numerical models with increasing complexity. Then roof stability is analyzed in single-excavation models through the lens of the empirical Coal Mine Roof Rating (CMRR) (Molinda & Mark, 1994) and Analysis of Roof Bolt Systems (ARBS) (Mark et al., 2001) methods. Finally, the mechanical interaction between roof stress arching capacity and pillar confinement is considered in realistic small and large-scale (i.e. multiple excavation) models through the lenses of TAT, pressure arch theory (PAT), version 6 of Analysis of Retreat Mining Pillar Stability (ARMPS2010) (Mark, 2010), the abutment angle concept (Mark, 1987), and the Mark-Bieniawski Pillar strength equation (Mark & Chase, 1997).

Sections 1.2 through 1.3 provide a general introduction to discontinuous and laminated rockmasses and the methods by which they are studied that are relevant to this thesis. Sections 1.4 through 1.6 present the proposed objectives, limitations, conventions, and general outline of this thesis.

1.2 Discontinuous & Laminated Rockmasses

Discontinuous and laminated rockmasses are prevalent throughout the world and often host both mining and civil underground workings. Many empirical methods focused on such rockmasses in mind have been developed in the context of underground bituminous coal mining. Accordingly, to allow the use of such systems to be used as a point of comparison for the numerical models
developed in this thesis, much of the thesis is focused on coal-measure rock and its respective mining applications.

1.2.1 Coal-Measure Rock

The variability seen coal-measure rock begins at deposition. Coal starts as peat formed in a wide variety of peatlands (e.g. bogs, marshes, swaps, etc.), each with a specific associated depositional environment and sedimentary sequence (i.e. facies) (Thomas, 2013). Coal deposits are interbedded with other sedimentary rocks such as shale, mudstone, siltstone, sandstone, conglomerate, and limestone in varying proportions depending on the depositional environment (Horne et al., 1978); these associated lithologies are collectively referred to as coal-measure rock. Peatlands form as parts of larger ecosystems with specific depositional characteristics. Peatlands can form adjacent to alluvial, lacustrine, or marine erosional-depositional environments (Dai et al., 2020) and thus, the range of possible coal and coal-measure geology is large, even prior to considering diagenetic, tectonic, and sea-level effects (Figure 1.1).

![Figure 1.1: Schematic of possible peat forming environments shown in green (from Dai et al., 2020).](image)

Coal that formed in warm, tropical climates (e.g. Carboniferous Period) tends to overlay highly weathered, well-developed soil material (i.e. paleosols) featuring numerous root systems. If the
paleosols are very fine-grained, they are called underclay or fire clay; they are commonly slickensided and contain iron disulfide or carbonate nodules (Diessel, 1992).

Coals that formed in cooler climates tend to overlay claystone, siltstones, or sandstones with less developed root systems, which allows original bedding to be maintained. Some coal seams overlay volcanoclastic sedimentary rocks deposited by volcanic eruption. Others can exhibit a transition zone that contains laminated and interbedded coaly shale and shaly coal prior to contact with the bottom of the main coal seam. This floor lithology is indicative of a continuous transition from inorganic sediment deposition to peat deposition. Coarse-grained seat-earths (i.e. “floor”/“underburden” rocks) are less likely to be slickensided and are typically hard and brittle (Diessel, 1992).

Following peat deposition, a combination of depositional, tectonic (i.e. subsidence), and climatic (i.e. eustatic sea-level change) forces influence the nature of the overlying strata (Diessel, 1992). The lithologies that are deposited above the main seam are either siliciclastic, volcanoclastic or carbonate sedimentary rocks. Coarse-grained siliciclastic rocks, such as sandstones, conglomerates, and breccias are indicative of higher energy (i.e. fluvial, alluvial, or wave) deposition. Finer-grained siliciclastic rocks such as shales, mudstones, siltstones and carbonate sedimentary rocks indicate a lower-energy (i.e. floodplain, lacustrine, or shallow marine) depositional environment. Similar to the floor, volcanoclastic sedimentary rock such as tuff can also sometimes be found in the immediate roof and further into the overburden.

The transition from a coal seam (i.e. organic deposition) to the immediate roof (i.e. inorganic deposition) can be smooth and laterally continuous (i.e. concordant) or rough and laterally discontinuous (i.e. discordant) depending on the interaction of depositional, tectonic, and climatic forces (Diessel, 1992). Furthermore, transitions from peat deposition may oscillate, depositing additional layers of well-developed soil and peat (i.e. “coal riders”) (Figure 1.2a) or interbedded sandstones and shales (i.e. “stackrock”) that are deposited in fluvial levees, crevasse splays, or tidal flats and channels (Figure 1.2b) above the main coal seam.
Smooth, laterally continuous transitions between coal and roof rock can be associated with either gradual or sudden divergence from the peat depositional environment. For example, gradual divergence may be in the form of an advancing river meander on a back-barrier peat deposit where the approaching channel gradually deposits coarser sediments transitioning through laminates of shaly coal, coaly shale, shale, and ultimately sandstone (Diessel, 1992). Sudden changes, such as rapid sea-level rise or volcanic activity, that do not cause significant erosional scours tend to form sharp contacts and sudden transitions between a coal seam and the roof strata in the form of massive limestone or tuff deposits (Diessel, 1992).

Large-scale tectonism that induces compressional, extensile, or shear forces can create joint networks in the roof and floor rocks sympathetic to those forces. Furthermore, syn-depositional and post-depositional faulting and folding can occur in any depositional environment and is largely independent of the factors controlling the lithology of the floor, roof, and seam. However, faulting can impact alteration of coal lithologies by acting as a conduit for fluids, changing stress fields and subsequent joint development, or splitting and offsetting seam, roof, and floor units (Nelson, 1981). Additionally, folding can lead to slickensided bedding planes, decreasing the strength of already weak discontinuities in the roof (Nelson, 1981). These impacts are highlighted in Figure 1.3.
Coal and coal-measure rock also have varying degrees of primary and secondary permeability dependent on their lithology and structural features (Thomas, 2013). Furthermore, local climatic and hydrological characteristics play an important role in determining the availability of groundwater and surface water that can impact a given coal geology. Moisture sensitivity of many coal-measure rock, in particular claystones, shales, and underclays, is well documented and can result in disintegration or swelling of those materials (Molina & Mark, 1994).

It is clear that coal-measure rock (i.e. coal, shale, sandstone, limestone) encompasses a wide range of possible discontinuous and laminated system behavior due to the depositional environments that produce and preserve coal. Furthermore, sedimentary systems that do not produce or preserve coal are still subject to similar deposition cycles, diageneses, and large-scale tectonic forces. Accordingly, although this thesis primarily focuses on coal mining applications, the findings are relevant to excavations in non-coal sedimentary systems and other laminated and discontinuous rockmasses (e.g. trona, salt, carbonate replacement, etc.)

1.2.2 Research & Design Method Overview

Excavation failure in laminated and discontinuous rockmasses can occur via multiple mechanisms (i.e. gravity vs. stress-driven), in one or more system components (i.e. roof, rib, or floor), and at various scales (i.e. local or global). Preventing these failures often relies on either component isolation, or system oversimplification such that the interaction between mechanism, component, and scale cannot be explicitly accounted for. These failures are categorized into fall of ground (i.e. 

Figure 1.3: Schematic of potential mining problems due to post-depositional and syn-depositional faulting and folding (from Kentucky Geological Survey – University of Kentucky, 2021)
while this thesis is not specifically focused on studying a certain type of excavation failure, it is concerned with the interaction of these system components and broadly evaluates the overall mechanics governing deformation and failure in these systems. The mechanical interaction between roof, floor, pillars, and installed support is poorly understood due to isolation of these interdependent systems and implementation of simplifying assumptions in both research and design applications (Reed et al., 2016). Frith & Reed (2018) argued that simplifying the relationship between these systems is, at best, inaccurate. At worst, independent considerations of overburden, underburden, and pillar may contribute to instability, regardless of the degree of conservatism used.

The majority of the research in this thesis is framed through a fundamental concept of all solid mechanics; the ability for stresses to “arch” or “flow” around disturbances similar to how a river flows around a large boulder (Figure 1.4). The way discontinuous material responds to this stress redistribution at multiple scales is the central theme of this research.

Material properties in discontinuous and laminated rockmasses are typically referred to as DIANE. Note that CHILE material properties can be particularly ill-suited for studying a DIANE system, but that all the components of DIANE and CHILE are not mutually exclusive nor binary. They are end-member cases that can each be represented using continuum or discontinuum methods. For
example, one could simultaneously consider discontinuous and inelastic (i.e. DIANE) material properties such as explicit inelastic joints, but linearly elastic intact material (i.e. CHILE) in the same rockmass. However, the appropriate numerical representation of these materials in the course of a given study depends on both the observation scale (Figure 1.5) and the expected response of those material properties to the magnitude and orientation of induced stresses (McLamore & Gray, 1967). For example, at the decimeter scale a given material may behave according to intact rock CHILE properties, but have excavation scale behavior controlled by DIANE properties associated with the rockmass joint network. Most existing design methods and academic research pertaining to flat-roof excavations in discontinuous and laminated systems represent DIANE rockmass conditions as continua when they would be more accurately captured using a discontinuum representation.

DIANE material properties are common features of bedded, jointed, laminated, or foliated rockmasses and can be significant controls on the mechanical behavior following excavation.
Fracture networks in these rockmasses generally consist of a combination of horizontal bedding planes and vertical cross-joints; alternatively, fractures may develop in a strong, massive roof layer as a result of bending-stress-induced tensile cracking. Depending on depositional environment, tectonic history, and current mining-induced stresses, the network of bedding planes, laminations, and fracture sets violate many of the CHILE material assumptions on which the majority of existing research and design methods rely on. These violations make it increasingly difficult to capture rockmass behavior with uniform measures of rock quality (Kaiser et al., 2018). Analytical, empirical, and numerical methods broadly fall into two approaches when applied to DIANE materials in-situ: they either adjust CHILE material properties to approximate DIANE material behavior (i.e. continuum), explicitly represent the DIANE conditions observed in-situ (i.e. discontinuum), or some hybrid of the two.

1.2.2.1 Analytical Methods

Analytical methods employed in understanding discontinuous systems are based on fundamental mechanical principles; they are often used as a validation tool when developing or implementing empirical, experimental, and numerical methods (such as those described in Section 1.2.2.2 and 1.2.2.3). Certain analytical methods applied to discontinuous and laminated rockmasses adjust elastic material properties (e.g. Young’s Modulus, Shear Modulus, Poisson’s Ratio, etc.) and peak strength to account for anisotropy and discontinuity by considering the spacing, orientation, and stiffness of discontinuities while maintaining a continuum representation of the rockmass (Salamon, 1968; Amadei & Goodman, 1981; Gerrard, 1982; Yoshinaka & Yamabe, 1986; Huang et al., 1995; Amadei, 1996; Sridevi & Sitharam, 2000; Sitharam et al., 2001; Cai et al., 2004). In other words, approximating a DIANE material using continuum representation.

Other analytical methods apply largely to low-stress conditions and consider the orientation, intersection, and strength of specific discontinuities that may create unstable blocks surrounding an excavation (Carvalho et al., 1991). These methods are largely utilized in studying gravity-driven wedge failure along explicit discontinuities and do not consider the impact of intact material yield.

An analytical method utilized in studying flat-roof excavation mechanics is the voussoir beam analog, first theorized by Evans (1941) based on previous observations and experimentation by Fayol (1885), Jones & Llewellyn-Davies (1929), and Bucky & Taborelli (1938). The voussoir beam analog in Diederichs & Kaiser (1999) utilizes a hybrid approach by resolving joint spacing.
and stiffness into a uniform rockmass stiffness, but also allows for explicit sliding failure of to occur along joints at the beam abutments. The voussoir beam analog calculates maximum displacement and horizontal stress but has not been validated for more complex loading conditions or the presence of rockbolt shear resistance in its current formulation. However, it has been implemented in previous research in coal-measure (Shabanimashcool & Li, 2014) and non-coal (Diederichs & Kaiser, 1999; Alejano et al., 2008; Bakun-Mazor et al., 2009; Crockford, 2012) discontinuous and laminated systems

1.2.2.2 Experimental & Empirical Methods

Experimental laboratory studies of discontinuous rockmasses focus on identifying failure criteria and validating adjustments to elastic material constants based on discontinuity properties (Herget & Unrug, 1976; Tsoutrelis & Exadaktylos, 1993; Yasar, 1998; Arzúa et al., 2014), or on the strength and stiffness of the discontinuities themselves (ASTM, 2008; Muralha et al., 2014; Hencher & Richards, 2015) (Figure 1.6).

![Figure 1.6: Comparison of laboratory-determined apparent Young’s Modulus (GPa) (points) and analytically determined apparent Young’s Modulus (line) for various orientations of discontinuities to major principal stress loading direction (modified from Amadei, 1996).](image-url)

Experimental methods also include analysis of large-scale or in-situ rock samples to assess the ultimate strength of pillars in underground excavations by accounting for the competing scale effects of heterogeneity and confinement (e.g. Bieniawski & Van Heerden, 1975; M. N. Das, 1986). These findings from a given mining district or rock type are then sometimes applied to
cases that are possibly outside of the original experimental confines (e.g. local variations in stress regime, geology, etc.)

Empirical methods have historically focused on the development of in-situ observational rockmass rating systems that allow the user to characterize a given rockmass based on the strength of intact material (often from experimental methods), intensity and condition of discontinuities, and other relevant geomechanical properties or excavation properties that may control the rockmass behavior (e.g. moisture sensitivity, multiple discontinuity sets, excavation orientation, in-situ stress, etc.). These methods include, but are not limited to, Rock Mass Rating (RMR) (Bieniawski, 1989), Geologic Strength Index (GSI) (Marinos & Hoek, 2000), Q Tunneling Index (Q) (Barton et al., 1974), Coal Mine Roof Rating (CMRR) (Molinda & Mark, 1994), and Coal Measure Classification (CMC) (Whittles et al., 2007). Similar to experimental methods, the conditions that the empirical methods were developed in are sometimes incorrectly assumed to be ubiquitous or analogous to other cases and are applied accordingly. Furthermore, specific rating categories and their scales may simply be assumed to be relevant to the overall behavior of the rockmass, as opposed to rigorously verified to be so.

1.2.2.3 Numerical Methods

Overall, the limitations of analytical, experimental, and empirical methods are similar: the effects of DIANE material properties are accounted for either by identifying an approximate continuum representation, or by focusing on simplified, specific applications or scenarios. Numerical methods represent an alternative that can explicitly incorporate multiple complexities and apply to a wide range of conditions. As computational power increases, numerical methods are being utilized more frequently to better understand complex rock mechanics in underground excavations (Zipf, 2007; Esterhuizen et al., 2010; Walton et al., 2015; Tulu et al., 2017). Numerical studies of underground mining typically isolate individual systems and focus either on pillar behavior (Mortazavi et al., 2009; Shabanimashcool & Li, 2013; Li et al., 2015; Walton et al., 2016; Zhang et al., 2018; Sinha & Walton, 2019), roof stability (Diederichs & Kaiser, 1999; Nomikos et al., 2007; Talesnick et al., 2007; Coggan et al., 2012; Gao et al., 2014; Bai et al., 2016; Sinha, 2016; Bai & Tu, 2020), surface subsidence (Fischer & Hegemann, 2008; Esterhuizen et al., 2010; Tulu et al., 2017), or installed support and reinforcement elements (de Buhan et al., 2008; Bahrani & Hadjigeorgiou, 2017; Sinha & Walton, 2019b; Mohamed et al., 2020).
Numerical methods consist of continuum, discontinuum, and hybrid methods. The best-known continuum methods include the Finite Difference Method (FDM), the Finite Element Method (FEM), and the Boundary Element Method (BEM) (Jing, 1998; Jing, 2003). Commercially available software that implement continuum methods include Fast Lagrangian Analysis of Continua (FLAC) (FDM), RS2, RS3 and Abaqus (FEM), and EX3 and Map3D (BEM). All continuum methods can account for varying degrees of discontinuity and anisotropy by homogenization through directionally dependent elastic properties, or by using implicit (i.e. strain limited) or explicit (i.e. fully separable) joint elements (Jing, 1998). However, the relationship between increasing numbers of joint elements and model run-time is exponential (Jing, 1998). Furthermore, implicit joint elements cannot fully detach and only capture small-strain joint deformation (Jing, 1998).

Even when considering discontinuous and anisotropic systems, the majority of numerical modeling is conducted using continuum methods, which require approximation of the rockmass as a continuous body (Karabin & Evanto, 1999; Fischer & Hegemann, 2008; Heasley, 2009; Esterhuizen et al., 2010; Tsesarsky, 2012; Li et al., 2014; Tulu et al., 2017; Zhang et al., 2018; Mohamed et al., 2020; Tuncay et al., 2020a). Anisotropy in a continuous body is commonly replicated through the use of ubiquitous joint elements (Whittles, 2000; Sainsbury & Sainsbury, 2017; Tulu et al., 2017; Carvalho et al., 2019).

This thesis is uniquely focused on how flat-roof excavations in horizontally laminated and discontinuous rockmasses behave, and therefore requires computational methods that are capable of accurately representing the geometric and mechanical impacts of discontinuities at the excavation and mine-scale. Discontinuum numerical methods, which include the implicit and explicit Discrete Element Methods (DEM), allow for evaluation of such geometric and mechanical considerations and accurately capture the large-strain behavior of discontinuous rockmasses (Jing, 2003).

The DEM has two primary sub-classifications that depend on the manner in which the models are solved (i.e. implicitly or explicitly) (Jing, 2003). They are known as the implicit DEM, also known as Discontinuous Deformation Analysis (DDA), and the explicit DEM (commonly referred to as simply “DEM”). DDA models are solved implicitly where blocks are discretized with finite elements, and matrix equations for motion and deformation of each individual block are solved,
requiring larger computer memory (Jing, 2003; Jing, 2007). Furthermore, the implicit solution methodology is based on finding an equilibrium solution for the potential energy in the model, possibly reducing the accuracy of results under quasi-static conditions (Khan, 2010). Theoretically, DDA can implement larger timesteps because of its implicit solution method, but Khan (2010) found that in practice DDA was less efficient and required more computational power than the explicit DEM. Conversely, explicit DEM models are solved with an explicit time-marching solution where blocks are discretized with finite differences and no matrix equations are needed (Jing, 2007). This allows for the development of instability and mechanical damping to be explicitly modeled (Khan, 2010). Therefore, this thesis utilizes the explicit DEM to study the mechanical behavior of discontinuous and laminated rockmasses.

Different commercially available software packages differ in the way discrete bodies are represented (i.e. particles or blocks). The explicit DEM is commercially available as the Universal Distinct Element Code (UDEC) that can generate discrete fracture networks (DFNs), which mimic the distribution of joints and bedding planes found in the subsurface based on stochastic input parameters. Alternatively, Particle Flow Code (PFC) is a DEM software that represents discrete bodies using circular, elliptical, or polygonal blocks and is generally used to simulate the mechanical behavior of granular systems (Potyondy et al., 1996; Jing, 2003). PFC can be used to model jointed rockmasses but has drawbacks when compared to blocky DEM methods. Most notably, individual PFC particles are rigid (i.e. elastic), and the input parameters governing the behavior of these particles and the contacts between them require extensive calibration to the properties of a given rockmass (Mehranpour & Kulatilake, 2017).

Existing discontinuum numerical studies of discontinuous and laminated rockmasses focus on joint constitutive model and input parameters (Hsiung et al., 1993; Souley et al., 1997; Yeung & Leong, 1997; Solak & Schubert, 2004; Gao, Stead, & Coggan, 2014), roof stability in bedded systems (Sofianos & Kapenis, 1998; Diederichs & Kaiser, 1999; Nomikos et al., 2007; Coggan et al., 2012; Shabanimashcool & Li, 2015; Bai et al., 2016; Bai & Tu, 2020), and pillar behavior (Garza-Cruz & Pierce, 2014; Vardar et al., 2019; Sinha, 2020). However, these studies do not consider the impact that individual and combined explicit DEM input parameters have on roof stability, pillar loading, and pillar deformation; they do not address the roof, floor, pillars and installed support of an excavation or a series of excavations as interdependent systems contributing to local and global stability. Furthermore, the sensitivity of explicit DEM model results to model
inputs (e.g. multiple rock and joint analogs) and setup attributes (e.g. mechanical damping, soft contact overlap) remain largely undocumented.

1.3 The Explicit Discrete Element Method

The explicit DEM originated from engineering disciplines, including rock mechanics, that require models for the interaction of multiple discrete bodies (Jing, 2003). Individual blocks can be represented in two dimensions as rigid or deformable with different strengths and stiffnesses. Similarly, different discontinuities (i.e. faults, joint sets, bedding planes) can also be represented with different material properties in the same model. Figure 1.7 depicts a schematic approximation of how a jointed and faulted rockmass could be represented in an explicit DEM model.

![Figure 1.7: Figure showing a hypothetical explicit DEM representation of a jointed a faulted rockmass (top) with faults highlighted in red (bottom). Note that discontinuities cannot terminate in the center of a block (modified from Jing, 2003).](image)

Discontinuous rockmass and anisotropic behavior are typical features in coal-measure rock that contain various horizontal (bedding) and sub-vertical (joints) fractures depending on their depositional environment, diagenesis, and historic and current stress states (Galvin, 2016). The impacts of such features can be captured using the explicit DEM.

The most commonly applied explicit DEM in rock mechanics is UDEC by Itasca Consulting Group Inc. (Itasca), formulated by Cundall (1971) and then programmed by Cundall (1980) and Cundall & Hart (1985). UDEC further divides discrete blocks into a finite difference zones of triangular mesh elements featuring unique gridpoints and uses an explicit, dynamic time-marching solution based on block masses and contact stiffnesses to resolve equations based on Newton’s second law of motion between zone gridpoints and deformations of springs and slip surfaces at block contacts (Jing, 2003). The calculation cycle for a given explicit timestep (Δt) and associated
formulae for contacts, as well as zones and gridpoints for deformable blocks, are shown in Figure 1.8.

![Diagram showing calculations](image)

Figure 1.8: Calculation cycle used in the explicit DEM time stepping solution method where (a) contact forces and displacements are solved and then used in the (b) zone and gridpoint calculations. Time is then stepped forward and the cycle repeats until a predetermined solution ratio is reached based on remaining unbalanced forces in the model (modified from Itasca, 2014).

Recent studies utilizing the explicit DEM have focused on specific aspects of pillar mechanics such as calibration of microparameters to laboratory testing, impact of rib support on pillar stability (Sinha & Walton, 2018; Sinha & Walton, 2019b), and yield pillar design (Li et al., 2015). Some studies have also used the explicit DEM to predict surface subsidence from multi-seam longwall mining (te Kook et al., 2008).

Major limitations of the explicit DEM are the inability to rupture intact blocks after yield, and the propensity for contact overlap to occur, which is an impossible physical phenomenon. However, contact overlap is crucial in resolving stress transfer between adjacent blocks and equations of motion. It is evident based on the body of research using the explicit DEM that it is an effective method of studying discontinuous and laminated systems.
1.4 Project Objectives

Improving safety in flat-roof excavations requires holistic consideration of the interaction between overburden and pillar (Frith & Reed, 2019). Existing methods largely ignore this interaction and rely instead on conservatism in an attempt to maintain safe excavations. Despite these best efforts, falls of ground continue to injure and kill miners in the USA (MSHA, 2020). In order to understand pillar loading, roof and overburden stability and self-supporting capacity must be addressed. This requires the use of discontinuum numerical models to overcome analytical and empirical method limitations and to account for in-situ complexity that occurs in discontinuous and laminated systems. However, numerical model results require targeted confirmation and validation through comparison to existing analytical and empirical methods.

The relevant parametric interactions in DIANE material properties and their combined effect on roof and pillar stability have not been fully identified or explored in the literature. Therefore, this research aims to advance this understanding through methodical increase in the complexity of discontinuum numerical models with rigorous comparison to state-of-knowledge and state-of-practice analytical, and empirical methods (Figure 1.9).
In pursuit of advancing the applicability of the voussoir beam analog, identifying explicit DEM sensitivities, factors influencing roof-stability, pillar-roof-overburden interactions, and required model complexity in discontinuous and laminated rockmasses, the following research objectives are addressed:

1. Confirm that explicit DEM models can be used to accurately represent discontinuous roofs and their self-supporting capacity using traditional voussoir beam mechanics and capture the effects of increasing complexity (i.e. more realistic boundary conditions, geometry, and presence of installed support elements) by developing adjustments to the existing analytical method; validate the proposed adjustments in a case study of the Bondi Pumping Chamber in Sydney, New South Wales, Australia.

2. Identify critical inputs governing self-stability and supported stability of flat-roof excavations in more complex rockmass and loading conditions through a parametric sensitivity analysis of single-entry explicit DEM models considering a wide range of geologic and mining conditions; utilize the CMRR and ARBS empirical methods to assess the realism of the model results. Then utilize statistical analysis to determine the most significant explicit DEM inputs that control roof stability and how they relate to their counterparts in applied empirical methods.
3. Utilize critical inputs governing roof stability with a calibrated coal-pillar model from Sinha (2020) to assess pillar-roof mechanical interaction and local stability in single-entry explicit DEM models; identify combinations of roof properties, pillar properties, and mining conditions that impact roof stress arching capacity and pillar confinement.

4. Expand consideration of pillar-roof mechanical interaction to the panel scale to assess pillar-overburden interaction and global stability under a wide range of conditions, including consideration of material heterogeneity and pillar extraction. Consider results within the framework of existing pillar loading, strength, and design safety factors to identify where application of state-of-practice is limited and where future alterations to such methods should focus.

Investigation of roof mechanics and self-supporting capacity throughout this thesis provides insight into the impact that roof conditions such as intact material and discontinuity properties, constitutive models, in-situ stress ratios, mining depths, and DFN properties have on roof stability, as well as pillar loading and deformation. Understanding the sensitivity of model results to changes in material properties, particularly DFN orientation and discontinuity strength, will advance the understanding of how the explicit DEM approach should be applied for multiple geologic conditions and flat-roof excavation geometries. Additionally, parameters statistically significant to effective rockmass characterization and roof stability determination are identified.

The mechanisms by which local and global stability are maintained in a given flat-roof excavation, or at the mine scale, are thoroughly considered through the use of numerical models and results are compared to existing analytical and empirical methods. Concrete adjustments to state-of-practice methods, specifically voussoir beam analog calculations, are proposed and verified. Considerations of pillar-overburden interaction for future studies are documented and the implications pillar design methods are addressed.

Completion of the aforementioned objectives advances the academic application and understanding of numerical modeling and its relationship to analytical and empirical methods, as well as the practical application of numerical, empirical, and analytical methods.

1.5 Limitations & Conventions

Key scope limitations of this study include modeling three-dimensional environments in two-dimensions using plane-strain conditions, the inability to explicitly model creation of new fractures
due to intact rock yield, the implicit representation of finer-scale (i.e. sub-bedding) planes of weakness using the strain-softening ubiquitous joint constitutive model, neglecting lithologic heterogeneity within modeled rockmasses (Chapters 3 & 4), and neglecting the influence of groundwater on loading and long-term stability.

This thesis utilizes positive sign convention when reporting downward displacement, downward velocity, or compressive stresses of the analytical and numerical results produced during the course of this research. The results of previous research by others are reported with their published sign conventions, unless being compared to the results of the research herein.

1.6 General Outline

Due to the diverse material covered in this thesis, the main content has been divided into four main chapters, each featuring a chapter-specific literature review. Each subsection following the literature review generally includes methodology, results, and conclusions.

Chapter 2 consists of a methodical analysis of the voussoir beam analog using the explicit DEM. The complexity of beam geometry, loading conditions, and material properties are methodically increased to account for more realistic roof conditions. The effects of horizontal stress, inelastic material, as well as multiple, passively bolted layers are assessed. Adjustments to the existing Diederichs & Kaiser (1999) analytical solution are proposed and evaluated using a case study.

Chapter 3 presents a parametric sensitivity analysis of roof stability and stress arching in unsupported and bolted single-entry models that assesses the self-supporting capacity of the roof for a wide range of geologic and mining conditions. Pillar deformation is held constant by using effectively elastic parameters to initially isolate the influences of roof material parameters from the influences of pillar deformability. The CMRR and ARBS empirical systems are applied to the model inputs and model results to verify that a wide range of loading conditions have been captured. Binary Logistic Regression (BLR) is then utilized to identify the significant controls on roof stability probability and how those inputs relate to the future considerations of the CMRR and ARBS empirical methods. Through application of the empirical systems to model results, potential shortcomings in state-of-practice empirical methods are identified.

Chapter 4 implements the findings of Chapter 3 regarding the critical controls on roof stability and assesses pillar-roof interaction and local entry stability using a calibrated coal pillar model from
Sinha (2020). The interaction between roof stress arching, pillar confinement, in-situ stress magnitude, and in-situ stress ratio are identified as fundamental controls on local entry stability. The parameters controlling roof stress arching and pillar confinement in unsupported and bolted models are explicitly detailed and bolting mechanisms are identified through changes in pillar and roof yield. Lastly, select model results are preliminarily compared to analytical and empirical pillar loading and strength calculations.

Chapter 5 builds on Chapter 4 by once again increasing the model complexity to analyze the effects of material heterogeneity, depillaring, panel dimensions, and the properties identified in Chapters 3 and 4 that control pillar-roof interaction at the single-entry scale. Chapter 5 ultimately assesses the limitations of concepts applied in the empirical Analysis of Retreat Mining Pillar Stability (ARMPS2010) (Mark, 2010) and Analysis of Longwall Pillar Stability (ALPS) (Mark, 1990; Mark et al., 1994) systems, as well as a traditional factor-of-safety (FoS) design approach using TAT loading.

Chapter 6 summarizes the findings of the previous chapters and presents a synthesized discussion of the overall findings of this thesis. Additionally, the major contributions are listed, practical implications for industry practitioners are summarized, and considerations for future work are proposed. Finally, journal and conference publications associated with this thesis are provided.

Appendix A contains a more thorough analysis of uncertainty related to the support installation at the Bondi Pumping Chamber case study discussed in Section 2.9 and documentation of how to apply pretension to rockbolt elements in UDEC v6.0. Appendix B contains documentation of permissions to reproduce images from previous studies by others.
CHAPTER 2
EXPANDING APPLICABILITY OF THE VOUSSOIR BEAM ANALOG

Prior to modeling more complex (i.e. realistic) roof conditions found in underground excavations, consideration of a simplified and well-established analytical framework is appropriate. Understanding deformation mechanics and self-supporting capacity of flat-roof excavations provides a foundation from which more complex considerations of roof stress arching can be incorporated into pillar-overburden mechanical interaction. Flat-roof deformation mechanics have previously been studied predominantly via elastic and voussoir beam analogs (Galvin, 2016). Application of both elastic and voussoir beam theories to flat-roof excavations have unique limitations depending on the geologic and mining characteristics of a given excavation. Recall the continuous, homogeneous, isotropic, and linearly elastic (CHILE) as well as the discontinuous, inhomogeneous, anisotropic, non-elastic (DIANE) material properties and their appropriate continuum and discontinuum representation (see Figure 1.5). As this thesis focuses on the discontinuous deformation of laminated and cross-jointed systems, the voussoir beam analog is generally more representative of roof self-supporting capacity of those systems. However, some elastic beams were considered in preliminary models and the results frame the subtle limitations of comparing analytical and numerical methods. Confirmation that the explicit Discrete Element Method (DEM) can accurately capture voussoir beam mechanics as reported in the literature, and analysis of the voussoir beam mechanical response to more realistic boundary conditions and geometries allows for the development of an adjusted voussoir beam analog. Subsequent comparison of the adjusted and baseline methods to model results, as well as empirical verification of the adjusted method’s accuracy are also presented.

Even though the explicit DEM has been previously implemented in researching voussoir beam mechanics, studies often focus specifically on joint properties (Ran et al., 1994), abutment compliance (Sofianos & Kapenis, 1996), intact material properties and verification of analytical methods (Sofianos & Kapenis, 1998; Diederichs & Kaiser, 1999), or individual case studies (Hatzor & Benary, 1998; Tsesarsky, 2005; Alejano et al., 2008; Bakun-Mazor et al., 2009; Tsesarsky, 2012). More recent studies have analyzed the impact of locked-in horizontal stress (i.e. non-mobile abutments) (Oliveira & Pells, 2014; Shabanimashcool & Li, 2015), presence of active roof support (Oliveira & Paramaguru, 2016), and the transition from elastic beam behavior (i.e. massive, intact) to the development of tensile fractures and formation of a voussoir beam (Please
et al., 2013). Notably, voussoir mechanical response sensitivities to explicit DEM model-specific settings (i.e. mesh size, block rounding, etc.) have not been documented, and realistic loading conditions have not been thoroughly considered prior to application of the voussoir beam analog to more complex case studies (Diederichs & Kaiser, 1999; Alejano et al., 2008).

In order to expand the applicability of the voussoir beam analog to more complex loading conditions, preliminary baseline voussoir beam models were analyzed and a combined zone size - block rounding sensitivity analysis was conducted. Results were compared to previous modeling efforts and analytical solutions found in Diederichs & Kaiser (1999). Observations of mesh size and block rounding sensitivity were implemented in analyzing more complex voussoir beam models featuring inelastic intact material, applied horizontal stress (i.e. mobile abutments), locked-in horizontal stress (i.e. immobile abutments), passively bolted multi-layer voussoir models, and single-entry mine models with voussoir geometry discrete fracture networks (DFNs). Existing modifications to the baseline Diederichs & Kaiser (1999) voussoir beam analytical solution proposed by Oliveira & Paramaguru (2016) were considered, and unique adjustments identified and developed in the course of this study were proposed, tested, and verified as more accurate than the baseline Diederichs & Kaiser (1999) analytical solution through simulation of increasingly complex voussoir beams. A guide on the limitations and implementation of the proposed modifications is presented for future analytical design considerations. Finally, the adjusted voussoir beam analog is applied to a case study of the Bondi Pumping Chamber in Sydney, New South Wales, Australia. The numerical models, their analyses, and results considered in this chapter are briefly summarized in Table 2.1.
Table 2.1: Summary of different numerical models in support of expanding understanding of voussoir beam mechanics, their location in the chapter, analysis, results, and any adjustments to the existing analytical solution. CY = continuously yielding, N/A = not applicable, FoS\text{crushing} = analytically determined factor of safety against crushing failure, UDEC = Universal Distinct Element Code.

<table>
<thead>
<tr>
<th>Name</th>
<th>Section</th>
<th>Analysis</th>
<th>Results</th>
<th>Analytical Solution Adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Euler Beams</td>
<td>2.2</td>
<td>Simple comparison to frame future model considerations</td>
<td>Model results were sensitive to mechanical assumptions and zone size even in the simplest cases</td>
<td>N/A</td>
</tr>
<tr>
<td>Block Rounding &amp; Mesh Sensitivity</td>
<td>2.3.2</td>
<td>Parametric analysis of block rounding and zone size interaction</td>
<td>Optimal block rounding and zone size identified to minimize deviation from analytical solution</td>
<td>N/A</td>
</tr>
<tr>
<td>Baseline Voussoir Beam</td>
<td>2.3.3</td>
<td>Confirmation of voussoir beam results based on Diederichs &amp; Kaiser (1999) with CY joints</td>
<td>Agreed with previous modeling; use of CY joints does not impact results</td>
<td>N/A</td>
</tr>
<tr>
<td>Surcharge Loading</td>
<td>2.3.4</td>
<td>Confirmation that surcharge loading in models is accurately captured by analytical solution</td>
<td>Uniformly distributed surcharge loading can be accurately applied in UDEC and captured in the baseline Diederichs &amp; Kaiser (1999) analytical solution</td>
<td>N/A</td>
</tr>
<tr>
<td>Inelastic Voussoir Beams</td>
<td>2.4</td>
<td>Modeling two inelastic rock analog voussoir beams under increasing surcharge pressure until collapse</td>
<td>Identified post-peak material behavior as a significant control on inelastic beams as modeled in the explicit DEM.</td>
<td>Brittleness multiplier for FoS\text{crushing}</td>
</tr>
<tr>
<td>Horizontal Stress</td>
<td>2.5</td>
<td>Variation of applied horizontal stress with mobile and immobile abutment blocks</td>
<td>Magnitude of horizontal stress equal to voussoir determined stress plus applied stress.</td>
<td>Add in-situ horizontal stress to voussoir stress (“Total Stress Method”)</td>
</tr>
<tr>
<td>Orthogonal Joints &amp; Support Elements</td>
<td>2.6</td>
<td>Multiple beam geometries featuring horizontal and vertical joints passively supported with rockbolts</td>
<td>Identified statistical relationship between number of voussoir layers and adjusted rockmass modulus to accurately predict displacement; effective thickness adjustment to more accurately capture midspan maximum horizontal stress</td>
<td>Use layer-adjusted rockmass modulus formula and bolted thickness to calculate beam displacement; use effective thickness formula and existing rockmass modulus to calculate stress</td>
</tr>
<tr>
<td>Single Entry In-Situ</td>
<td>2.7</td>
<td>Considered impact of varying entry depth, surcharge load, and horizontal stress ratio</td>
<td>Applied adjusted methods, and considered effect of in-situ stresses</td>
<td>N/A</td>
</tr>
<tr>
<td>Bondi Pumping Chamber Case Study</td>
<td>2.9</td>
<td>Utilized models to populate DFN from multiple joint sets, and to determine end-member impacts of uncertainty in construction sequence</td>
<td>Successful application of the adjusted voussoir beam method developed herein</td>
<td>N/A</td>
</tr>
</tbody>
</table>
The following sections outline the existing research on voussoir beams and relevant Universal Distinct Element Code (UDEC) model setup attributes, as well as the methodologies and results of this thesis in verifying and advancing the potential applications of the voussoir beam analog under more realistic loading conditions (Figure 2.1).

2.1 Literature Review

2.1.1 Simple Elastic Beams

Flat-roof excavations in horizontally bedded sedimentary geologic units are commonly implemented in mining and civil-infrastructure projects. The simplest conceptual model of such an excavation may consider roof layers as simply supported or fixed-end beams with CHILE material properties; beam-ends are supported by pillars and adjacent unmined rock, as shown in Figure 2.2. This representation is generally accurate if the immediate roof is massive and competent, featuring no discontinuities other than horizontal bedding planes. The deformation, deflection, and stress distribution of simply supported and fixed-end beams under static applied loads are governed by the thoroughly validated assumptions and calculations provided by either Euler-Bernoulli or Timoshenko beam theory (Gere & Timoshenko, 1991; Hibbeler, 1997).
Euler-Bernoulli beam theory predicts the deflection and internal stresses of slender beams (i.e. span to thickness ratio \(s/t > 10\)) and assumes: (1) fully elastic material properties, (2) plane cross-sections of the beam remain planar, and (3) deformed beam angles and transverse shear strains are neglected. These assumptions allow for bending moment to be resolved into pure compression and pure tension around the neutral axis of the beam (Wang, 1995; Hibbeler, 1997) and the equations for determining shear force, moment (i.e. bending stresses), and deflection for pin-pin and fixed-fixed beams are given as:

\[
V_{\text{pin-pin}} = w\left(\frac{L}{2} - x\right) \tag{2.1}
\]

\[
M_{\text{pin-pin}} = \frac{w x}{2}(L - x) \tag{2.2}
\]

\[
\delta_{\text{pin-pin}} = \frac{w x}{24EI} \left(L^3 - 2Lx^2 + x^3\right) \tag{2.3}
\]

\[
V_{\text{fixed-fixed}} = w\left(\frac{L}{2} - x\right) \tag{2.4}
\]

\[
M_{\text{fixed-fixed}} = \frac{w}{12} \left(6Lx - L^2 - 6x^2\right) \tag{2.5}
\]

\[
\delta_{\text{fixed-fixed}} = \frac{w x^2}{24EI} (L - x^2) \tag{2.6}
\]

where \(V\) = shear force, \(M\) = moment, \(\delta\) = displacement, \(w\) = weight of beam, \(L\) = length of beam, \(x\) = distance along beam, \(E\) = Young’s Modulus, and \(I\) = second moment of inertia. However, CHILE material properties as well as the pin-pin and fixed-fixed end-constraints are rarely applicable in most mining scenarios where flat-roof excavations are utilized. Furthermore, Euler-Bernoulli equations do not incorporate axial loading due to horizontal stress or the effect of support.
elements (Galvin, 2016). Multiple methods that incorporate more representative end-constraints (i.e. Euler strut, column buckling, Johnson’s formula), the transition from elastic to voussoir beams (Please et al., 2013), and approximate the effect of support (i.e. elastic restraints) are available in the literature (Galvin, 2016). None of the aforementioned methods are applicable to the deformation of a discontinuous roof beam, but rather are applicable to rockmasses that behave as a continuum.

2.1.2 Voussoir Beams

CHILE assumptions do not account for many observed in-situ rock mass and material properties such as: (1) fracture influence, (2) variation in rock strength and stiffness, (3) anisotropy, and (4) temporal effects of cyclical loading (Hudson et al., 2002). Removing the “continuous” assumption and introducing vertical joints at the beam midspan and abutments brings the roof beam analog one step closer to capturing the in-situ mechanical behavior of excavation roof deformation in a discontinuous rockmass. This segmented beam geometry is known as a voussoir beam, first theorized by Evans (1941) based on previous observations and experimentation by Fayol (1885), Jones & Llewellyn-Davies (1929), and Bucky & Taborelli (1938). Since then, voussoir beam behavior has been studied via the analytical, laboratory, field, and numerical methods outlined herein.

Generally, voussoir beam stability is governed by the s/t of the beam, and the strengths and stiffnesses of intact material and joints. Unlike simply supported elastic beams, voussoir beams carry zero or negligible tensile forces. The symmetric deflection of the bilateral beam spans, through elastic shortening of the beam, generates support via a horizontal thrust reaction force at discontinuities, which transfers load to the abutments and supports the weight of the voussoir beam (Figure 2.3) (Sofianos, 1998).
The voussoir beam analog is traditionally applied in low-confinement scenarios where the immediate roof beam can be isolated from the complex loading conditions seen in the field. As depicted in Figure 2.4, four main failure modes can occur in an idealized (single, midspan joint) voussoir beam geometry: (a) snap-through/buckling or elastic instability where the self-weight of the beam overcomes the reaction moment and no intact rock damage occurs; (b) crushing failure induced where maximum compressive stresses overcome the intact strength of rock at beam midspan and abutments; (c) vertical abutment slip prior to development of a sufficiently strong compressive arch or increasing surcharge load; and (d) diagonal tensile cracking normal to compressive forces (Diederichs & Kaiser, 1999).
Despite the long history of voussoir beam research, analytical solutions for voussoir beam deflection, and maximum stresses are not as well-developed or constrained as simple elastic beams. This is reflected in the literature as variations in initial assumptions, boundary conditions, solution methods, and results. Methods such as iterative loop calculations, laboratory experiments, and numerical modeling have been employed in constraining expected and observed deformations and stresses in voussoir beams. However, accounting for more realistic behavior through variations in boundary conditions or beam geometries and analyzing the effect of installed support has not been considered as extensively for voussoir beams as in the case of simple elastic beams (i.e. Euler strut, column buckling, Johnson’s equation). Precisely due to that lack of variation, the development of the voussoir beam analog and its current state are considered herein.

Fayol (1885) noted that undermining a layered system of stacked timber beams caused delamination of immediately unsupported layers, while deflection decreased and approached zero in subsequently higher layers. Fayol proposed that the remaining layers above the collapsed portion the system could be treated as a sequence of simply supported beams. The simply supported beam assumption was the prevailing theory in practice until Bucky (1933) showed experimentally that cracked beams could remain stable with sufficiently stiff abutments.

Evans (1941) noted that typical excavation roof geometries in rock were highly discontinuous and could not carry tensile stresses. Furthermore, intact portions of the roof were incapable of carrying such large bending stresses in an elastic manner as predicted by simple beam theory. Evans (1941)
proposed the theory of the voussoir beam, named after the voussoir-arch used in masonry bridge building for millennia. He correctly noted that voussoir beam stability relies on development of internal compressive forces forming a complete arch of sufficient thickness to counteract the reaction moment generated by the beam deflecting under self-weight.

In order to solve for a static voussoir beam system, Evans (1941) assumed the following: (1) elastic compressional behavior, (2) null tensile stresses, (3) discontinuity shear strength supported overall beam stability, (4) initial vertical beam displacement in relationship to adjacent strata was negligible, (5) elastic strain at beam abutments was negligible, (6) deflection induced by bending moment of each half-span was negligible, (7) a hinged connection at the bottom of each abutment (i.e. no shear displacements at the abutments), and (8) a parabolic distribution of compression arch of constant thickness \( n \), leading to a relationship between initial moment arm thickness \( Z_o \) and beam thickness \( t \):

\[
n = 0.5t \tag{2.7}
\]
\[
Z_o = 0.66t \tag{2.8}
\]

As the two component beams deflect via rotation about the bottom corner of the abutments, elastic compressional strain reduces the length of each beam span, and reduces the compression arch from \( L_o \) to \( L_1 \), which reduces the effective length of the reaction moment arm. This deflects the beam downward and increases the reaction force, keeping the beam in static equilibrium. From a calculation perspective, this process is repeated as necessary while comparing the changes in moment arm \( (Z_o \text{ to } Z_1) \) and its linear relationship with stress to ensure that they do not exceed the intact strength of the rock beam. These steps are described mathematically through the following relationships:

\[
L_o = s + \frac{8Z_o^2}{3S} \tag{2.9}
\]
\[
L_1 = L_o - \left( L_o \times \frac{11}{24} \times \frac{f_m}{E} \right) \tag{2.10}
\]
\[
Z_1 = \sqrt{\frac{3S}{8} \times (L_1 - s)} \tag{2.11}
\]
\[
f_1 = \frac{Z_o}{Z_1} \times f_m \tag{2.12}
\]
where \( s \) = beam span, \( f_m \) = maximum stress, and \( E \) = Young’s Modulus. Plugging \( f_1 \) in to \( f_m \) in Eqn. 2.10 and repeating the process obtains the change in \( Z \) which can be linearly related to the change in stress and compared to the intact strength. This process can be repeated \( n \) times to a limiting amount of stress where critical deflection (\( \delta \)) is given as:

\[
\delta = Z_n - Z_o \tag{2.13}
\]

Evans (1941) also conducted laboratory experiments on physical models of multiple-joint voussoir beams composed of brick and low tensile strength mortar in various configurations and loading conditions. Multiple experiments were conducted to confirm that voussoir mechanics do occur in reality and to justify the theoretical assumptions used to predict deflection and stresses.

Wright & Mirza (1963) published results of tests on elastic physical models that featured midspan and abutment joints. Their results indicated that compressive stresses were higher at the abutments than at the midspan, and compression arch thickness was actually smaller than theorized by Evans (1941).

Wright (1972) discounted Evans’ (1941) assumption that compression arch thickness (i.e. contact length \( n \)) was equal to half the thickness of the beam using both physical and finite element method (FEM) models. The model results indicated that compression arch thickness varied significantly throughout the voussoir beam half-span and that its distribution at midspan and abutment was not perfectly triangular but could be assumed as such for practical engineering purposes. Additionally, Wright (1972) noted that abutment stresses were typically twice as high as midspan stresses (Figure 2.5).

![Figure 2.5: Single-joint voussoir beam half-span showing results of FEM analysis dimensionless horizontal stress contour distribution in the voussoir beam half-span. \( \xi \) = centerline (from Wright, 1972).](image)

Wright (1974) also developed an iterative calculation loop that found a best-fit relationship between normalized arch thickness \( (n_a) \) at the abutment and normalized moment arm \( (Z/t) \):
Sterling (1980) conducted physical tests on 63 intact rock beams made of homogeneous, isotropic sandstones and limestones under various loading conditions and geometries. Single and multi-layer cases were examined and qualitatively described as behaving similarly to Fayol’s (1885) timber stacks with intact rock failure occurring in the lowest beams. Sterling (1980) confirmed that shear and slip failure were the critical failure modes at lower s/t (thicker beams), while buckling failure was more likely in higher s/t (thinner beams). Mid-range s/t failure modes were influenced by the magnitude of lateral stresses generated in the beam and how much confinement was provided at the abutments. Higher confinement promoted crushing or diagonal cracking failure, while lower confinement promoted abutment slip (see Figure 2.4).

Beer & Meek (1982) extended existing analytical calculations to develop design charts for beams and plates (i.e. three-dimensional discontinuous layers) of various s/t, and created formulas incorporating bedding dip for non-horizontal excavations. Diverging from Evans (1941), the value of n was not assumed a priori, but calculated in an iterative fashion for the critical span of the beam using a fourth-order differential equation relating equilibrium arch thickness to a minimum value of horizontal stress:

\[ s^4 + C_1 s^2 - C_2 = 0 \]  
\[ C_1 = A^2 * \frac{0.1718f_m}{E} \]  
\[ C_2 = A^2 Z_0^2 \left(1 - \frac{11f_m}{24E}\right) \]  
\[ A = \frac{4nf_c}{wc\cos\alpha} \]

where \( \alpha = \) angle of dipping beam from horizontal. The value of n for a voussoir beam in equilibrium as determined by Beer & Meek (1982) and subsequently replicated by Brady & Brown (1993) was 0.75t for very small deflections that incur minimal horizontal stress development in the beam.

Stimpson & Ahmed (1991) ran laboratory tests on thick, unjointed rock beams consisting of limestone, granite, and potash. The beams were loaded until failure and their deflections and strains recorded. Results were then compared with FEM models that allowed for crack nucleation. Their
model showed that diagonal tensile cracking, rather than crushing or buckling, dominated deformation of thicker, initially intact beams (i.e. < 5 s/t).

Ran et al. (1994) analyzed the influence of shear slip along discontinuities in single and multi-jointed voussoir beams using FEM numerical models. Based on model results, a set of equations were proposed to estimate maximum acceptable transverse (i.e. surcharge) load in relationship to abutment slip failure if joint material properties could be well-constrained. The FEM models found compression arch thickness for a voussoir beam at equilibrium to range from 0.4-0.65t at midspan and 0.13-0.33t at the abutments, depending on beam stability.

Sofianos (1996) and Sofianos & Kapenis (1998) built upon the analytical solutions presented by previous authors. They compared analytical, laboratory, and explicit DEM numerical modeling results to develop design charts incorporating factor of safety (FoS) for normalized roof spans and their susceptibility to buckling, crushing, or snap-through failure of single-jointed voussoir beams. Closed-form solutions for displacement and horizontal stress in small and large deflection cases were developed.

Sofianos (1996) assumed (1) intact rock was CHILE, (2) beams were uniformly loaded with only three vertical joints, one at mid-span and one at each abutment, (3) abutments were rigid (i.e. non-deformable) and joints were very stiff, (4) Poisson’s Ratio of rock was zero, and (5) zero abutment confining stress existed prior to beam deflection. Closed-form solutions for displacement ($\delta$) and horizontal stress ($\sigma_x$) in small (Eqns. 2.19 & 2.20) and large (Eqns. 2.21 & 2.22) deflection cases were evaluated mathematically as:

$$\delta = t \times \left[ \left( \frac{\gamma s}{E (1-0.66n)} \right) \times \left( \frac{8}{3} + \left( \frac{s}{t(1-0.66n)} \right)^2 \right) \times t(1 - 0.66n) \right]$$

$$\sigma_x = 0.66 \times \gamma \frac{s}{t} \times \frac{s}{E}$$

$$\delta = \left( \frac{\gamma s}{E (1-0.66n)} \right) \times \left( \frac{8}{3} + \left( \frac{s}{t(1-0.66n)} \right)^2 \right) \times t(1 - 0.66n)$$

$$\sigma_x = E \times \left( \frac{\gamma s}{4n(1-\delta)} \right)$$

where $\gamma = \text{unit weight of beam material}$. Furthermore, design charts were provided for normalized beam loading and s/t conditions against buckling, crushing, and sliding failure modes. In
agreement with Ran et al. (1994), the normalized arch thickness for larger deflections was found to range from 0.12-0.30t and was related to span using analytical and numerical methods:

\[ n = \frac{1}{0.22s^2 + 2.7} \quad (2.23) \]

A graphical comparison of multiple compression arch thickness results is presented in Figure 2.6.

![Figure 2.6: Comparison of methods evaluating the relationship between normalized span (s_n) and compression arch thickness (n) (from Sofianos, 1996).](image)

Diederichs & Kaiser (1999) built upon Beer & Meek’s (1982) iterative solution loop and found that a stable voussoir beam in equilibrium will have a compression arch thickness of approximately 0.75t for small deflections and 0.3t at incipient collapse. They presented voussoir beam explicit DEM numerical models with regularly spaced vertical joints for purposes of comparison and developed similar factors of safety and design charts as shown in Sofianos (1996). Their findings related to compression arch thickness development initially appeared to contradict previous studies, most notably Wright (1982) and Sofianos (1996), but partially agreed with the findings in Ran et al. (1994).

Diederichs & Kaiser (1999) noted that the discrepancies in compression arch thickness results between their work and that of Sofianos (1996) were likely due to differences in block and joint
constitutive models. Diederichs & Kaiser (1999) were able to replicate Sofianos & Kapenis (1996) UDEC model behavior by using rigid or deformable intra-span blocks coupled with elastic joints and rigid abutments. Diederichs & Kaiser (1999) claimed that this lead to stress concentrations at the abutments, and subsequent over-prediction of moment arm length and inaccurate deflection predictions. Additionally, Diederichs & Kaiser (1999) modeled multiple-joint voussoir beams, where Sofianos (1996) modeled single-joint voussoir beams. Sofianos (1999), replying to Diederichs & Kaiser (1999), attributed the difference in compression arch thickness between the voussoir beam models as a result of multiple soft joints (i.e. jkn = 5 GPa) present in the Diederichs & Kaiser (1999) numerical models, in which joint normal and joint shear stiffnesses were 10 times lower than in Sofianos’ models, allowing more contact to occur between blocks as the voussoir beam deflected, which created a thicker compression arch. Yiouta-Mitra & Sofianos (2018) have updated the original Sofianos (1996) analytical solution to account for multi-jointed beams with stiff joints. While neither method is necessarily the “true” or “correct” method, this thesis is uniquely focused on multiple vertical discontinuities and a range of joint stiffnesses, rather than a single midspan crack or very stiff joints. Therefore, the analytical solution from Diederichs & Kaiser (1999) was used a baseline herein; their findings are discussed in further detail in Section 2.1.3.

Talesnick et al. (2007) conducted laboratory experiments utilizing small plaster blocks and a centrifuge to measure deflection and compression arch behavior in multi-jointed voussoir beams. Results matched previously published parabolic compression arch distributions, but the midspan arch thickness was measured as 0.5t as previously hypothesized by Evans (1941). As these laboratory models were not accelerated to collapse, the relationship between the measured compression arch thickness and the stability of the modeled voussoir beam was considered inconclusive. Tsesarsky (2012) calibrated FEM models to the Talesnick et al. (2007) laboratory results and found the equilibrium compression arch thickness at the abutments (n_{ab}) to range from 0.3-0.4t and the midspan (n_{m}) to range from 0.4-0.5t.

Alejano et al. (2008) performed a back-analysis of a failed mine roof in jointed sedimentary rock using voussoir beam analogs found in Diederichs & Kaiser (1999), Sofianos (1996), and Ran et al. (1999); numerical analysis was also conducted using the explicit DEM. Findings confirmed that the methods for determining roof stability developed by Diederichs & Kaiser (1999) and Sofianos (1996) (e.g. factor of safety against crushing (FoS_{crushing}), sliding (FoS_{sliding}), and
Buckling Limit (BL)) were reliable predictors of stability, but that the method of Diederichs & Kaiser (1999) was more reliable, particularly when analyzing BL.

Oliveira & Pells (2014) conducted numerical modeling of more complex voussoir beam behavior using the explicit DEM. The beam models largely mimicked those found in Diederichs & Kaiser (1999) but they utilized beam abutments that had the same stiffness as the intra-span blocks. They found that with increasing applied horizontal stress, voussoir beam deflection decreased from voussoir beam analytical predictions (i.e. Diederichs & Kaiser, 1999) to elastic beam (i.e. Euler-Bernoulli) predictions of displacement. They also briefly discussed the effects of zone size on beam displacement, as well as the influence of inelastic weak zones in the beam under variable horizontal stresses.

Most notably, they evaluated orthogonally jointed and bolted composite beam models that were 3 m thick, with 1.0 m individual voussoir layers. A single model geometry was analyzed with various bedding plane properties, bolt orientations, and locked-in horizontal stresses (Figure 2.7). Oliveira & Pells (2014) noted that passive rockbolts installed on 1.75 m spacing, perpendicular to bedding did not promote composite beam behavior for very stable beams when comparing displacement results to the baseline Diederichs & Kaiser (1999) analytical solution. However, their results showed an approximately 50% increase in bolted beam displacement with weak bedding partings, which significantly exceeded the predicted analytical displacement for a 1 m (i.e. single layer) voussoir beam.

![Figure 2.7: Model Geometry of orthogonally jointed and passively bolted composite voussoir beam (from Oliveira & Pells, 2014).](image)

Oliveira & Pells (2014) then focused on rockbolt performance, where rockbolt orientation (i.e. perfectly vertical versus sub-vertical) was determined to have a significant influence on complex voussoir beam behavior with sub-vertical rockbolts inclined towards either abutment decreasing...
the deflection and measured stress of multi-layer voussoir beam models relative to cases with vertical rockbolts.

Oliveira & Paramaguru (2016) compared the impacts of complex loading and geometric conditions on voussoir beam numerical models to the analytical predictions of voussoir beam mechanics from Diederichs & Kaiser (1999). These complexities primarily consisted of added horizontal stress, the presence of adverse geologic features, the presence of roof support in the form of tensioned bolts, and the effect of a slightly arched excavation shape. A confinement adjustment factor based on Ramamurthy (1993) was proposed to alter the rockmass modulus of the beam to account for the decrease in displacement and is given as:

\[ E_{rm,\text{modified}} = \frac{E_{rm}}{1 - e^{-\alpha \frac{UCS*}{\sigma_h}}} \leq 1.5\text{ to } 2.5\ E \tag{2.24} \]

where \( E_{rm} \) = rockmass modulus (see Eqn. 2.25), \( UCS^* \) = in-situ compressive strength, \( \sigma_h \) = initial horizontal stress, \( \alpha = 0.03 \text{ – 0.06} \) is an empirical confinement effect, and \( E \) = Young’s Modulus of the intact rock. Note that the modified \( E_{rm} \) is bound by 1.5 to 2.5 times \( E \). However, this method has two major drawbacks. First, it requires the use of an \( UCS^* \) value, which if considered infinite in the case of fully elastic beams, the value of the modified \( E_{rm} \) becomes equal to the unaltered \( E_{rm} \). Second, increasing stiffness will account for the decrease in displacement caused by horizontal stress, but it will also decrease stresses generated through voussoir arching. Additionally, increased in-situ horizontal stresses may initially decrease internal beam stresses, but that trend should reverse as in-situ horizontal stresses increase significantly and close the voussoir joint.

Oliveira & Paramaguru (2016) also presented an equation that provided a reasonable adjustment to the rockmass modulus used in the Diederichs & Kaiser (1999) analytical solution based on the spacing of bedding partings. However, they only presented verification results for a single model case, leaving it inapplicable to passively bolted systems and its broader applicability uncertain. Lastly, Oliveira & Paramaguru (2016) accounted for arched excavations by adding the weight of the arched material as a uniform distributed surcharge pressure at the top of the modeled beams.

Shabanimashcool & Li (2015) developed equations relating the effect of locked-in horizontal stresses to a single-jointed voussoir beam and verified them using numerical analysis and comparison to previous laboratory studies. They assumed that abutment blocks may be restricted from rotating and therefore could develop non-negligible bending moments, and that the
compression arch distribution was bilinear, not parabolic. They found that at lower stresses, increased horizontal in-situ stress decreased beam BL and FoS\textsubscript{crushing}. As stress magnitudes increased, stability against snap-thru and crushing decreased, particularly for longer span beams (i.e. s/t > 35). However, these analytical solutions did not account for the joint normal stiffness of beam discontinuities, which had a significant effect on voussoir beam stability.

Carvalho & Carter (2020) revised the voussoir beam analytical solution found in Diederichs & Kaiser (1999) to analyze crown pillar stability (i.e. low s/t voussoir beams) and compared these analytical solutions to Finite Element Analysis (FEA) numerical models that featured multiple horizontal joints or Voronoi block tessellations. They noted that clamping stresses could be added to voussoir generated stress to estimate total stress in the beam but did not provide any analysis of this claim.

2.1.3 Baseline Diederichs & Kaiser (1999) Analytical Solution

The voussoir analytical solution analyzed and modified in this study is depicted in the iterative loop calculation shown in Figure 2.8. The Diederichs & Kaiser (1999) iterative loop method was selected based on its ability to account for deformable abutments and the presence of multiple vertical joints and varying joint normal stiffness. Furthermore, the results of Alejano et al. (2008) indicated that it was more accurate at capturing in-situ conditions than the analytical solution of Sofianos (1996).
Figure 2.8: Iterative loop voussoir analytical solution method utilized in this study (from Diederichs & Kaiser, 1999).

N = compression arch thickness, Z = moment arm length, T = beam thickness, L = compression arch length, S = span, Fm = maximum compressive stress, ρ = specific weight, E = Young’s Modulus, α = bedding dip, p = surcharge pressure, σc = compressive strength, φ = joint friction angle.
The spacing and normal stiffness of vertical joints in Diederichs & Kaiser (1999) were incorporated into a rockmass modulus ($E_{\text{rmx}}$):

$$\frac{1}{E_{\text{rmx}}} = \frac{1}{E} + \frac{1}{(jkn)s_j} \quad (2.25)$$

where $jkn$ = joint normal stiffness, and $s_j$ = joint spacing. This adjustment is based on the equivalent stiffness of springs (Figure 2.9) connected in series and assumes that the system is being loaded axially (i.e. perpendicular to the joint orientation).

![Figure 2.9: Schematic and equation for the equivalent stiffness of springs connected in series, $k$ = spring stiffness (from Wikipedia, 2021).](image)

Equations relating uniformly distributed surcharge and support pressures to the change in effective specific weight ($\gamma$) of the voussoir beam were also presented:

$$\gamma^* = \gamma \pm \frac{q}{T} \quad (2.26)$$

where $q$ = uniformly distributed pressure, and $T$ = beam thickness. They suggested using a triangular distribution to account for passive rock bolt elements providing suspension support and were able to consider the effect of cablebolts in a case study by using a negative pressure and decreasing the effective specific weight of the beam. However, this does not account for the doweling effect (i.e. beam building, shear resistance) of fully grouted rockbolts.

The BL was calculated as the percentage of normalized compression arch thickness ($N$) where the equilibrium change in moment arm length ($Z_{\text{chk}}$) < 0 (i.e. moment arm shortening). As BL approached 35% (i.e. as deflection approached 0.1T), regardless of rockmass modulus or beam thickness, any increase in midspan deflection was found to become non-linear. Ultimate snap-thru (i.e. buckling) failure of an elastic beam occurred between a BL of 80-100% or 0.25T. A factor of safety against crushing and abutment slip (i.e. sliding) was also calculated from the
iterative loop calculation presented Figure 2.8. In order to calculate a FoS\textsubscript{crushing}, Diederichs & Kaiser (1999) suggested multiplying the lab-scale UCS by 0.3-0.5 to account for brittle material behavior and scale effects. Alejano et. al (2008) implemented this adjustment in their case study. Diederichs & Kaiser (1999) also proposed a method of estimating $E_{\text{rmx}}$ by using the Q tunneling index (Barton et al., 1974) and the level of confinement of a given excavation. Furthermore, they suggested that in thinly laminated ground, grouted rebar should have a length equivalent to the desired beam thickness and that such beams should be designed to a FoS of 1.5 to 2.0. However, neither of these methods were fully validated and did not account for the doweling effect (i.e. shear resistance) of passive rockbolts, only the effect of suspension (i.e. axial resistance). Although considered by Oliveira & Pells (2014), the mechanical impact of passive bolts installed normal to the excavation roof has not been thoroughly explored in the context of the voussoir beam analog.

It is evident that the body of research regarding roof stability in discontinuous sedimentary rock has confirmed that the voussoir beam analog can reasonably approximate roof deformation mechanics in flat-roof underground excavations. Voussoir beam mechanics firmly ground the more complex numerical models featured in this study to peer-reviewed scientific observation and applied mathematics based on first principles. However, a connection between the simplified analogs presented in this literature review and the more complex stress-arching conditions of laminated, discontinuous, and supported roofs has not yet been fully developed and verified.

2.1.4 UDEC Block Rounding & Gridpoint Force Calculations

Results presented in this Chapter related to comparison of numerical and analytical analysis of voussoir beams were affected by an explicit DEM feature known as block rounding. UDEC software rounds the edges of two-dimensional blocks during block creation. Block rounding removes a radial portion of block corners to help prevent overlap errors of soft contacts (i.e. explicit DEM contacts) and block locking in the model. The rounding length is measured from where the corner of two blocks intersects and is shown in Figure 2.10 (Itasca, 2014).
Figure 2.10: Depiction of block rounding performed in UDEC using a constant rounding distance, rather than rounding radius. \( d = \) distance to corner, \( r = \) radius of rounded corner (modified from Itasca, 2014).

The magnitude of block rounding affects the length of contacts between blocks, and sufficiently large rounding values can affect model accuracy and gridpoint force calculations. A block rounding length of approximately 1% of representative block edge length is recommended to maintain good model accuracy (Itasca, 2014).

Following block creation, incomplete cracks (i.e. discontinuities that do not terminate at a block boundary) are deleted, and each block is then discretized into finite difference zones using three-noded triangular mesh elements. Each node then becomes a gridpoint on either side of a block boundary, and nodes are then used to determine the number of individual contact elements that comprise a single discontinuity, and ultimately control overall joint behavior (Figure 2.11).

Figure 2.11: Schematic showing two blocks, internal finite difference zones, gridpoints, contacts, domains, and contact lengths (modified from Itasca, 2014).
The joint contact lengths shown in Figure 2.11 govern the calculations of normal and shear displacement change for each corner-edge contact and the associated length, as well as equations of motion and forces for gridpoints situated along block boundaries. Gridpoint forces calculations are given as follows:

\[ F_i = F_i^l + F_i^c + F_i^z \]  \hspace{1cm} (2.27)

where \( F_i^l \) = applied external loads (i.e. boundary conditions, support, damping), \( F_i^c \) = contact forces that are represented by equal and opposite forces at the two associated gridpoints, and \( F_i^z \) = the effect of internal stresses within neighboring zones (Itasca, 2014).

### 2.1.5 Relevant Constitutive Models & Yield Calculation

The linearly elastic isotropic block constitutive model in UDEC is governed by Hooke’s Law under plane strain conditions in incremental form as:

\[
\begin{align*}
\Delta \sigma_{11} & = (K + \frac{4}{3}G) \Delta e_{11} + (K - \frac{2}{3}G) \Delta e_{22} \hspace{1cm} (2.28) \\
\Delta \sigma_{22} & = (K - \frac{2}{3}G) \Delta e_{11} + (K + \frac{4}{3}G) \Delta e_{22} \hspace{1cm} (2.29) \\
\Delta \sigma_{12} & = \Delta \sigma_{21} = 2G \Delta e_{12} \hspace{1cm} (2.30) \\
\Delta \sigma_{33} & = (K - \frac{2}{3}G) (\Delta e_{11} + \Delta e_{22}) \hspace{1cm} (2.31)
\end{align*}
\]

where \( \sigma = \) stress, \( K = \) bulk modulus, \( G = \) shear modulus, \( e = \) strain, \( 11 = \) major principal, \( 22 = \) minor principal, \( 12 = \) shear, and \( 33 = \) out-of-plane (Itasca, 2014). The incremental strain tensor \( (\Delta e_{ij}) \) for a given timestep \( (\Delta t) \) is calculated as:

\[ \Delta e_{ij} = \frac{1}{2} \left[ \frac{\delta \hat{u}_i}{\delta x_j} + \frac{\delta \hat{u}_j}{\delta x_i} \right] \Delta t \]  \hspace{1cm} (2.32)

where \( \hat{u}_i = \) displacement rate, and \( x_i = \) coordinate of block centroid (Itasca, 2014). This constitutive model cannot account for inelastic damage or anisotropy of block material in the numerical model. In order to account for damage of block material, the Mohr-Coulomb strain-softening constitutive model can be implemented in UDEC to simulate elastic-brittle-plastic material behavior. This constitutive model incorporates both shear and tensile yield functions and plastic flow rules to
simulate a peak strength and post-peak behavior (Itasca, 2014). The failure envelopes for shear ($f^s$) and tensile ($f^t$) failure are given as:

\[
f^s = \sigma_{11} - \sigma_{22} \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right) + 2c \sqrt{\frac{1 + \sin \varphi}{1 - \sin \varphi}}
\]

\[
f^t = \sigma^t - \sigma_{22}
\]

where $\varphi$ = friction angle, $c$ = cohesion, and $\sigma^t$ = tensile strength (Itasca, 2014). If a tensile strength is not specified, the strength is calculated as:

\[
\sigma^t = \frac{c}{\tan \varphi}
\]

Plastic flow rules and stress corrections are applied once a stress state above the failure limit has been obtained at a given timestep. Strain-weakening behavior is governed by a piece-wise linear function that defines the relationship between the plastic shear strain increment ($\Delta \varepsilon^{ps}$) and the instantaneous cohesion, friction, and dilation values. If the piecewise linear function is used to transition from peak to residual over a very small plastic shear strain increment, an elastic-brittle-plastic material behavior can be simulated.

The Mohr-Coulomb strain softening constitutive model has been validated as accurately replicating shear and tensile yield for a wide range of applications by Itasca (2014), and has been demonstrated to appropriate simulate yield of masonry brick (Nezami & Hashash, 2002; W. Chen et al., 2018) and inelastic voussoir beam behavior in previous studies (Alejano et al., 2008; Oliveira & Pells, 2014; Oliveira & Paramaguru, 2016).

### 2.2 Preliminary Simple Elastic Beam Models

Although simple elastic beams cannot explicitly capture the behavior of discontinuous mine roofs, Oliveira & Pells (2014) have shown that elastic beams provide a lower-bound estimate for in-situ roof beam deflection. Furthermore, the geometry and boundary condition of simple elastic beams are significantly less complex than voussoir beams, and their analytical solutions are well-constrained. In order to compare numerical and analytical results in the simplest cases, an initial modeling effort was undertaken to compare explicit DEM model results to analytical formulae for CHILE simple beams. The following sections outline the methods and results of simple elastic beam models developed in UDEC, including pin-pin and fixed-fixed beams. The results are compared to well-established Euler-Bernoulli analytical formulae.
2.2.1 Methodology & Model Inputs

Two UDEC models featuring a slender, elastic beam measuring 10 m wide and 0.75 m thick were developed; one was constrained with pin-pin supports and the other fixed-fixed by applying the appropriate zero-velocity boundary conditions at the ends of the beam (Figure 2.12).

Figure 2.12: Simple elastic beam models and their respective boundary conditions. Note the 500 times deformation magnification.

Beam material properties were assigned as 10 GPa bulk modulus (K), 8 GPa shear modulus (G), and 2,650 kg/m$^3$ density. Models were run to an equilibrium solution ratio of $1.0 \times 10^{-5}$ and maximum vertical displacement ($\delta_{yy}$), as well as flexural stresses ($\sigma_{xx}$) and shear stresses (V) at the midspan and abutment, were extracted from model results via FISH commands.

2.2.2 Elastic Beam Displacement and Stress Results

Model results are compared with analytical results based on Euler-Bernoulli beam equations in Table 2.2.

Table 2.2: Comparison of model results to Euler Bernoulli computational results. $\delta_{yy}$ = maximum vertical deflection (mm), $V$ = maximum shear stress, $\sigma_{xx}$ = maximum horizontal compressive stress, EB = Euler-Bernoulli analytical results.

<table>
<thead>
<tr>
<th></th>
<th>Pin-Pin</th>
<th></th>
<th>Fixed-Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Midspan</td>
<td>Beam-End</td>
<td>Midspan</td>
</tr>
<tr>
<td>Result</td>
<td>$\delta_{yy}$ (mm)</td>
<td>$V$ (MPa)</td>
<td>$\sigma_{xx}$ (MPa)</td>
</tr>
<tr>
<td>Model</td>
<td>3.7</td>
<td>0.05</td>
<td>2.56</td>
</tr>
<tr>
<td>EB</td>
<td>3.8</td>
<td>0.00</td>
<td>2.60</td>
</tr>
</tbody>
</table>

In both support-type cases, differences between model results and Euler-Bernoulli equations for midspan deflection and midspan maximum flexural stress were minimal. However, the difference
in maximum shear stresses at the midspan and beam-ends varied significantly between boundary condition cases. Euler-Bernoulli calculations assume shear stress only acts vertically, remains constant at a given cross section, and has no influence on flexural stress, while the numerical model solution explicitly accounts for variations in shear stress orientation, cross-sectional distribution, and the interaction shear and flexural stresses. These assumptions allowed the Euler-Bernoulli flexural stress distribution to be triangular and linear, with compression and tension acting perfectly parallel to the neutral axis of the beam. UDEC did not enforce these assumptions, and as such the values for maximum shear stress deviated from the analytical solution. Furthermore, given the nature of extracting model results from mesh elements adjacent to the midspan, which averaged stresses based on gridpoint calculations at the midspan and gridpoints on the other edge of the mesh element, the exact value at the model midspan or beam-end could not be determined. For example, as x deviated from 0 (i.e. left beam end), L (i.e. right beam end) or L/2 (i.e. midspan) in Eqns. (2.1-2.2, 2.4), analytically determined stresses deviated from 0; additionally, as mesh size in the numerical model increased, numerically determined shear stresses were averaged over a larger area, deviating from the exact value of the analytical solution.

In order to identify the main cause of these deviations (i.e. differences in assumptions vs. influence of mesh size) numerical and analytical results of the fixed-fixed beam model beam-end stresses were compared. Numerical and analytical results converged at the neutral axis of the beam where model shear and flexural stresses approached analytically predicted values (i.e. 0.10 MPa). This did not hold true for the top and bottom of the fixed-fixed beam, which exhibited exponential increases in shear and flexural stress towards the extreme fibers, due to deviations in shear stress from predicted values. Although the values converged, there remained some variation in the model results as indicated in Figure 2.13. This minute variation between the analytical and numerical results at the neutral axis (i.e. y = 0) was interpreted as the influence of mesh size, as this was where beam displacements were smallest and rotation at the fixed boundary was non-existent (i.e. approaching Euler-Bernoulli assumptions). Unsurprisingly, at the extreme fibers of the beam, where flexural stresses deviated from analytical solutions was also where the maximum shear stress and beam-end rotational displacements occurred in the fixed-fixed beam.
While this analysis was not explicitly applicable to laminated and discontinuous systems, it was important to note the potential discrepancies when comparing analytical and numerical methods even in the simplest cases. This preliminary effort confirmed that the UDEC elastic beam model was behaving as expected with respect to Euler-Bernoulli beam theory, but that transverse shear strains were not perfectly vertical and could not be replicated numerically, such that deviations from Euler-Bernoulli theory were encountered. Furthermore, the nature of extracting model results, particularly model zone stresses, did incur some deviation from the analytical predictions due to the averaging of gridpoint forces for a given zone stress state.

2.3 Baseline Elastic Voussoir Beam Models & Surcharge Loading

Previous explicit DEM modeling of voussoir beams by others (Diederichs & Kaiser, 1999; Oliveira & Pells, 2014) indicated that UDEC could accurately replicate the analytical solution presented in Diederichs & Kaiser (1999). The following sections outline the methods and results of elastic, multi-jointed voussoir beam models developed in this thesis based on those from Diederichs & Kaiser (1999). These models are referred to as “baseline” in this thesis because the boundary conditions, model geometries, and elastic material properties modeled did not deviate significantly from Diederichs & Kaiser (1999). However, the following analysis was required to
establish a reliable foundation from which to develop more complex models; as indicated in the previous section, even the simplest cases had discrepancies between analytical and numerical results. The results were compared to the analytical solutions and expected behavior discussed in the literature review.

2.3.1 Methodology & Model Inputs

Multi-jointed, elastic voussoir beam models were created in UDEC based on the range of material and discontinuity properties presented in Diederichs & Kaiser (1999). The presence of multiple vertical joints, rather than a single midspan joint, more closely resembled realistic roof conditions. The assumptions and boundary conditions utilized in this section deviated from the original theory developed by Evans (1941) but adhered closely to those in Diederichs & Kaiser (1999) with one exception: once voussoir arching was allowed to develop with effectively elastic joints (i.e. high cohesive strength, non-zero tensile strength), joint constitutive models were changed to continuously yielding in order to capture the effect of realistic inelastic joints on voussoir beam mechanics. The continuously yielding joint model has previously been shown to more accurately capture joint displacement under large deformation (Poock, 2016) and was therefore used for subsequent, more complex modeling efforts in this thesis.

1 m thick, vertically jointed voussoir beams of various length and rockmass modulus ($E_{rnx}$) were tested. All intact material was modeled as elastic, while discontinuities were effectively elastic in the first stage of the model and were gradually reduced to realistic strength frictional joints over three subsequent stages. Voussoir beam abutments were modeled as deformable blocks set to be functionally rigid (i.e. $K = 5.6(10)^{35}$ Pa). Recall that Diederichs & Kaiser (1999) noted that the use of rigid blocks in UDEC concentrated stress and led to inaccuracies. This is due to the fact that rigid blocks are not discretized and only require an assigned density, therefore impacting stress calculations. Furthermore, the most recent version of UDEC does not permit the use of fixed-velocity boundary conditions with rigid blocks, only deformable ones. The use of deformable blocks with a high stiffness allowed for the impact of abutment block properties to be minimized and approach the assumptions and boundary conditions used in both the baseline Diederichs & Kaiser (1999) analytical solution and the UDEC models featured therein. General model setup and magnified block deformation of the of an example baseline voussoir beam at equilibrium is shown in Figure 2.14. Note that the presence of horizontal joints in the abutments was critical in allowing
equal and opposite deformation conditions to exist between the beam and the abutment; this is discussed further in the results section in relation to the block rounding sensitivity analysis and is referred to as block rounding symmetry of the abutment-beam contact. Model parameter combinations were selected based on representative cases from existing literature (Diederichs & Kaiser, 1999). The relevant model inputs are given in Table 2.3.

![Figure 2.14: Baseline voussoir beam model boundary conditions shown at equilibrium state with a deformation magnification of 100 times. Beam is 10 m wide and 1 m thick.](image)

Table 2.3: Proposed material and geometric properties for simple elastic voussoir beam models. Note that highlighted values are varied concurrently and that the Rockmass Modulus is calculated from the joint spacing, stiffness, and intact Young’s Modulus using Eqn. (2.25).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Span (m)</td>
<td>5 10 15 20 25 30 35</td>
</tr>
<tr>
<td>Beam Thickness (m)</td>
<td>1 --- --- --- --- ---</td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>3000 --- --- --- --- ---</td>
</tr>
<tr>
<td>Rockmass Modulus (GPa)</td>
<td>3.3 33.3 --- --- --- ---</td>
</tr>
<tr>
<td>Joint Spacing (m)</td>
<td>1 0.5 --- --- --- ---</td>
</tr>
<tr>
<td>Joint Stiffness (jkn/jks) (GPa)</td>
<td>5 100 --- --- --- ---</td>
</tr>
<tr>
<td>Intact Young’s Modulus (GPa)</td>
<td>10 100 --- --- --- ---</td>
</tr>
</tbody>
</table>

The model solution method in Diederichs & Kaiser (1999) was replicated by running models in multiple stages. The first stage featured strong intra-span and abutment joints that allowed stable deflection to occur and horizontal stresses to develop in the voussoir beam. The second stage altered the intra-span joints to high friction, zero cohesion, and zero tensile strength. The third stage altered the intra-span joints from Mohr-Coulomb to continuously yielding in order to model the impact of more realistic discontinuities once voussoir arching had developed. The fourth stage changed the intra-span joint parameters to realistic values, but left abutment joints with high frictional strength to maintain the assumption of zero abutment slip as in the analytical solution. Every time joint parameters were altered (i.e. between each stage), the model was run to equilibrium. The joint properties and stage progression are depicted in Figure 2.15.
Preliminary model results matched expected deflections; however, model results for horizontal stress magnitude deviated significantly from analytical predictions. Based on this, further investigation was conducted on 10 m long, $E_{\text{rms}} = 3.3$ GPa beams. It was subsequently determined that the interaction between mesh size and block rounding had a significant effect on both deflection and stress magnitude results. Furthermore, initial models did not have the two horizontal joints in the abutment blocks, which promoted additional stress concentration at the bottom corners of the voussoir beam abutments. Following establishment of optimal mesh size, block rounding, and abutment block symmetry, the initial models were rerun, and their results confirmed UDEC’s ability to capture voussoir beam mechanics. Results were compared to voussoir analytical predictions of midspan deflection, maximum horizontal stress, and previous modeling efforts (Diederichs & Kaiser, 1999). Design criteria were also considered, such as the BL and $FoS_{\text{sliding}}$ given as:

$$BL(\%) = (1 - (n_{\text{max}} - n_{\text{min}})) \times 100 \quad (2.36)$$

$$FoS_{\text{sliding}} = \frac{\sigma_{\text{max}} n}{\gamma^* S \tan (\varphi)} \quad (2.37)$$

where $n$ = normalized compression arch thickness, $\sigma_{\text{max}}$ = maximum compressive stress, $\gamma^*$ = effective specific weight, $S$ = span, and $\varphi$ = joint friction angle. Select models were re-run with varying continuously yielding joint properties to analyze their impact on voussoir beam stability.
Once the results of the simple elastic voussoir beam models were verified, further analysis featuring the baseline (i.e. boundary conditions and general geometry shown in Figure 2.14) voussoir models subjected to surcharge loading were considered. Surcharge loading was applied using a vertical load boundary condition of 1 kN at each of the gridpoints at the top of the previously tested voussoir beam models; this resulted in a 7.9 kPa surcharge pressure for the 10 m long, $E_{\text{rmx}} = 3.3$ GPa baseline elastic voussoir beam (Figure 2.16). Eqn. (2.26) was used to calculate an effective specific weight of the beam to account for the uniformly applied surcharge pressure.

![Figure 2.16: Baseline voussoir beam geometry featuring uniformly applied surcharge pressure ($q$). No deformation magnification.](image)

2.3.2 Block Rounding & Zone Size Sensitivity Results

Initially, model results of simple elastic voussoir beams were able to match analytically predicted maximum vertical displacements and previous UDEC model results found in Diederichs & Kaiser (1999). However, model results of maximum horizontal stresses were consistently lower than their analytically predicted values and the reported values from Diederichs & Kaiser (1999). A parametric investigation was conducted on model inputs and settings that are not explicitly considered in the voussoir analytical solution, such as joint shear stiffness, abutment stiffness and block symmetry, block rounding, and zone size. Results indicated that stress development and distribution, as well as midspan deflection in the models were significantly influenced by the interaction between model zone size (i.e. mesh density) and the magnitude of block rounding, as well as the presence of horizontal joints in the abutment blocks imposing block rounding symmetry at the beam-abutment contact.

Block rounding symmetry of the abutment-beam contact is not an explicit consideration in the voussoir analytical solution, and preliminary models without horizontal joints in the abutment blocks coinciding with the top and bottom of the voussoir beam resulted in concentrations of
horizontal stress at the beam abutments. In particular, longer beam models had larger discrepancies between model results with and without abutment block symmetry. Figure 2.17 shows the impact that abutment symmetry had on concentrating maximum horizontal stresses due to the asymmetric interaction of the rounded block corner of the voussoir beam, and the non-rounded edge of the abutment block.

![Figure 2.17: Comparison of horizontal stress contours between 35 m long, simple elastic voussoir beam models showing approximately 3 MPa difference in maximum horizontal stress due to abutment symmetry and interaction of block corners and edges at the right beam-abutment contact. Top image shows generalized area of bottom image focus boxed in red and stress concentration due to asymmetric abutment encircled in black (bottom left).](image)

Itasca (2014) suggests that a block rounding value of 1% of the typical block edge length maintains accurate results. The baseline elastic voussoir beam models utilized in the block rounding sensitivity analysis have a minimum block edge length between 0.5 and 1.0 m, so rounding values ranging from 0.02 - 0.005 m and zone edge lengths of 0.5-0.05 m were analyzed for their impact on beam displacement and stress development (Figure 2.18). Changes in block rounding altered the contact area at block corners where maximum horizontal stresses occurred and where model results were extracted from. Conversely, changes in zone size (i.e. number of mesh elements, mesh element density) altered the number of individual contacts between blocks. Results clearly indicate that this interaction affects the measured deflection and stress magnitudes in the model, which deviate significantly from analytical predictions with decreasing zone size and constant block
rounding. Unlike purely continuum models, results did not necessarily converge to a consistent value with decreasing zone size.

Figure 2.18: 10 m, \( E_{\text{mm}} = 3.3 \) GPa, baseline elastic voussoir beam model results showing the impact of mesh element density and block rounding length on maximum vertical displacement, and maximum compressive stress at the midspan and abutments. Optimal mesh size and block rounding were selected based on this analysis and are circled in red.

Recall that stress results extracted from the equilibrium model state are calculated from equations of motion based on block geometries, zone properties, and gridpoint force calculations. As discussed in the literature review, gridpoint force calculations consider external applied loads, contact forces, and the internal forces of adjacent zones (Itasca, 2014).

Contrary to voussoir mechanics, model results showed decreased displacements associated with higher stress magnitudes across all mesh densities at identical rounding values. Theoretically, lower displacements should have resulted in lower stresses measured in the beam; furthermore, a larger contact length (i.e. compression arch) caused by smaller rounding should have also decreased the maximum horizontal stress as forces were distributed over a larger area. However, maximum contact length is strictly controlled by the block rounding in UDEC, even if the analytical voussoir beam solution assumes it ranges between 0 and 1 times the beam thickness.
with a stable value of 0.75t. The effect of block rounding on contact length and horizontal compression is shown in Figure 2.19. For larger zone sizes, smaller block rounding values increase the effective thickness of the voussoir beam by increasing the contact length at the edges of blocks. A larger section of joint normal stiffness is activated, which decreases displacement by restricting overlap at the edges of blocks. The increase in effective contact length simultaneously increases the contact forces in the gridpoint force calculation, resulting in increased stress magnitudes at the top of the midspan, and the bottom of the abutment joints.

Figure 2.19: The effect of block rounding on contact length and horizontal stress at the top midspan joint of a baseline elastic voussoir beam model (top, red box). Note the increase in contact length (white) and horizontal stress magnitude at the top of the beam in (b) the smaller rounding case. Zone elements are highlighted in green are 0.3 m maximum edge length. $\sigma_{xx}$ = horizontal compressive stress.

If the zone size was sufficiently small (i.e. high mesh element density), the previously established relationship of increased rounding resulting in larger displacements and lower stresses, reversed. This was due to the increase in the number of individual contacts, as dictated by the number of zone elements and gridpoints (Figure 2.20).
The displacement and stress magnitudes of the beam that featured more contacts (i.e. smaller zone size) were no longer controlled by the single contact at the top of the midspan joint or bottom of the abutment joint. Voussoir beam behavior was now controlled by the overall contact length, not the marginal increase in length of a single contact at the top or bottom of the beam. Smaller zones sizes coupled with larger rounding values now limited the total shear displacement that could occur between each block in the voussoir beam, which restricted overall beam displacement. Similarly, the stresses were no longer controlled by the marginal increase in effective contact length, but rather a concentration of internal and contact forces over a smaller contact, which increased the calculated stresses in the model. As mesh size decreased, a larger ratio of the upper zone at the midspan was no longer associated with a contact as the rounding value increased. As a result, the calculated gridpoint force decreased in accuracy, tending to overestimate the maximum horizontal stress in zones adjacent to the midspan and abutment joints.

Model sensitivity to zone size is a well-known phenomenon and mesh sensitivity analyses are routine. However, the identification of the critical interaction between zone size and block rounding and its impact on voussoir beam mechanics has not been studied previously in detail. This interaction also impacts the results of more complex blocky models and is considered in the
subsequent portions of this study. Generally, block rounding values should be approximately 1% of the representative block size, and no greater than 20% of the maximum zone size in the model to maintain high accuracy when considering stress distribution and displacement. Based on the results shown in Figure 2.18, the remaining voussoir beam models in this chapter utilize a block rounding of 0.015 m (i.e. 1.5-3.0% representative block length) and a zone maximum edge length of 0.2 m (i.e. block rounding < 20% maximum edge length).

2.3.3 Baseline Elastic Voussoir Beam Results

Following identification of optimum block rounding, zone size, and abutment-beam contact geometry, the baseline models were rerun and maximum midspan displacement, as well as maximum midspan and abutment horizontal stresses were compared to the model results and analytical solution from Diederichs & Kaiser (1999) (Figure 2.21). Note that Diederichs & Kaiser (1999) did not test $E_{rmx} = 33.3$ GPa beams in their stress analysis, which is why their model stress results are not presented in Figure 2.21.

Replacing fully elastic intra-span joints with average strength continuously yielding joints (i.e. 30° intrinsic friction angle) in the models resulted in nearly identical displacements and horizontal stresses to model results and their associated analytical solutions in Diederichs & Kaiser (1999). This indicated that the continuously yielding joint model did not interfere with voussoir mechanics.
if horizontal stresses were sufficient enough to support self-stability. Longer span beams featuring higher horizontal compressive stresses were analyzed with extremely weak joints (i.e. 15° intrinsic friction angle) and were able to maintain self-support through voussoir stress arching. However, shorter span beams with lower internal stresses could not overcome the influence of such weak joints and collapsed via abutment slip failure. If friction angles were decreased beyond approximately 20°, voussoir stability could not be maintained in shorter span beams (i.e. < 10 m). This corresponded with a decrease in analytically calculated FoS\textsubscript{sliding} to values below 1.0.

The agreement between the simple elastic beam models in this study with the model results and analytical solution presented in Diederichs & Kaiser (1999) provided a foundation to continue analyzing the influence of increasing complexity (i.e. realism) on voussoir beam behavior. If abutment joint intrinsic friction angle was decreased from 89° to match the intra-span joint strengths, displacement and stresses were slightly underpredicted by the analytical solution due to additional shear occurring along the abutment joints and concentrating stresses.

2.3.4 Surcharge Loading Results

Following verification of baseline voussoir beam results, the effect of surcharge loading on voussoir beam mechanics was considered. Diederichs & Kaiser (1999) presented Eqn. (2.26) but did not explicitly discuss the accuracy of this method when compared to UDEC model results. Surcharge and support pressures can be used to approximate the effects of a yielded back, groundwater, and suspensive roof support (Diederichs & Kaiser, 1999). Furthermore, inelastic models in Section 2.4 utilize surcharge pressure to test the accuracy of FoS\textsubscript{crushing}. Uniform surcharge loading was accounted for using Eqn. (2.26) to calculate an effective specific weight of the beam. A uniformly distributed surcharge load of 1 kN/gridpoint was applied to each of the previously tested voussoir beam models and the results are compared to analytical predictions in Figure 2.22. Note that the longest voussoir beams in both stiffness cases (i.e. 20 m for \(E_{\text{rmx}} = 3.3\) GPa, and 35 m for \(E_{\text{rmx}} = 33.3\) GPa) converged to a maximum stress and displacement for a given compression arch thickness in the iterative analytical loop, but the model results indicated that this was a borderline stability case that ultimately led to a buckling failure in both of the longer beams. The snap-thru failure of the beams occurred at an analytically determined BL of 79% and 71% for the softer and stiffer beam, respectively. Furthermore, the model midspan stresses for all beams...
were now equal to or even slightly exceeding the analytical calculation, whereas in the baseline models midspan stresses were more consistently overpredicted.

![Graph showing comparison of voussoir beam model results to analytical predictions](image)

**Figure 2.22:** Comparison of voussoir beam model results of (a) midspan deflection and (b) maximum horizontal stress to analytical predictions when subjected to a uniform surcharge load of 1 kN/m. DKC = Diederichs & Kaiser (1999) analytical solution, UDEC = model results from this study.

Clearly, the application of a uniformly distributed surcharge pressure was not solely impacting the specific weight of the beam. Once again, this highlighted the subtle limitations of even simple adjustments to the voussoir beam analytical solution, as well as the limitations of comparing analytical and numerical results.

Regardless, the results of surcharge load models further confirmed that UDEC can reasonably capture a wide range of voussoir beam behavior in accordance with governing equations found in the literature, regardless of the presence of inelastic joints coupled with surcharge pressure. This verification of surcharge loading allowed for simple inelastic voussoir beam models to be tested under changing loading conditions.

### 2.4 Inelastic Voussoir Beam Models

Although elastic material assumptions may approximate in-situ conditions in competent, massive rockmasses, discontinuous and layered systems frequently feature weak lithologies that deform inelastically. However, the consideration of inelastic materials in analytical calculations increases their complexity significantly. Therefore, the voussoir analytical solution utilizes elastic material assumptions in order to calculate $\text{FoS}_{\text{crushing}}$, even though crushing is an inelastic failure mode.
FoS\textsubscript{crushing} is traditionally determined by dividing the field-adjusted unconfined compressive strength (UCS) (i.e. in-situ compressive strength (UCS*)) of the rock by the analytically determined maximum horizontal compressive stress. The following sections investigate the accuracy of those assumptions that have not been considered in inelastic voussoir beam models (i.e. outside of case studies) previously. Furthermore, the difference in stress between beam midspan and abutment shown in Figure 2.21 and Figure 2.22 are explicitly explored in relationship to the accuracy of the baseline Diederichs & Kaiser (1999) analytical solution for FoS\textsubscript{crushing}. Three main causes contributing to differences between model results and the analytical solution were identified: midspan-abutment stress differential, analytical-model stress difference, and post-peak material behavior.

Note that only crushing failure was explicitly investigated in this thesis, while diagonal tensile cracking, another voussoir beam inelastic failure mode was not. This was due partly to the inability for the explicit DEM to explicitly model the rupture of intact material, and also because the inelastic beams modeled were not likely to incur diagonal tensile cracking failure due to their dimensions (i.e. s/t > 5) as indicated by physical models from Stimpson & Ahmed (1991). However, some model results indicated that a combination of crushing and diagonal tensile cracking failure was possible.

2.4.1 Methodology & Model Inputs

In order to quantify the impact of inelastic material yield on displacement and stress development in the voussoir beam, the previously stable 10 m span, 0.5 m joint spacing surcharge load beam was altered to run with inelastic Mohr-Coulomb intact material properties (Figure 2.23). Two rock-analogs representing a weak and a strong rock were selected (Table 2.4), and the appropriate material properties were assigned based on Tulu et. al (2017).
Figure 2.23: Simple inelastic voussoir beam model geometry and boundary conditions. All inelastic beams tested in this section are 10 m span, 1 m thick, and have a horizontal joint spacing of 0.5 m.

Table 2.4: Mohr-Coulomb parameters for intact material used in analyzing inelastic voussoir beam mechanical behavior. UCS* = in-situ compressive strength, E = Young’s Modulus, v = Poisson’s Ratio, Φi = initial friction angle, Φr = residual friction angle, ci = initial cohesion, cr = residual cohesion, ti = peak tensile strength, tr = residual tensile strength, ψ = dilation angle, εcr = critical plastic shear strain.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Strong</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS* (MPa)</td>
<td>47</td>
<td>5.7</td>
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<tr>
<td>Density (kg/m³)</td>
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<td>2500</td>
</tr>
<tr>
<td>E (GPa)</td>
<td>25</td>
<td>8</td>
</tr>
<tr>
<td>ν</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Φi</td>
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<td>20°</td>
</tr>
<tr>
<td>Φr</td>
<td>40°</td>
<td>20°</td>
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<tr>
<td>ci (MPa)</td>
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<td>2</td>
</tr>
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<td>ti (MPa)</td>
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<td>0.6</td>
</tr>
<tr>
<td>tr (MPa)</td>
<td>0.1ti</td>
<td>0.1ti</td>
</tr>
<tr>
<td>ψ</td>
<td>10°</td>
<td>5°</td>
</tr>
<tr>
<td>Strain-weakening εcr (strain)³</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Brittle εcr (strain)²</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Perfectly Plastic</td>
<td>Residual=Peak</td>
<td>Residual=Peak</td>
</tr>
</tbody>
</table>

FoS_{crushing} was calculated in accordance with the method from Diederichs & Kaiser (1999):

\[
FoS_{crushing} = \frac{UCS}{\sigma_{max}}
\]

(2.38)

where \(\sigma_{max}\) = maximum compressive stress determined by the voussoir analytical solution. The analytical solution assumes that when FoS_{crushing} < 1.0 (i.e. peak strength exceeded) the voussoir beam should immediately yield at the midspan and abutments, and collapse. Diederichs & Kaiser (1999) suggested multiplying the lab-scale UCS in Eqn. (2.38) by 0.3-0.5 to account for scale
effects. Alejano et. al (2008) implemented this adjustment in their case study as suggested by Diederichs & Kaiser (1999). However, as the properties listed in Table 2.4 are already field-scale values, and the heterogeneity of the voussoir beams tested was controlled (i.e. single layer, uniform block size and strength, uniform contact size and strength, Eqn. (2.38) was compared to model results with no additional adjustment.

Both rock analogs listed in Table 2.4 are stable under self-loading in the voussoir geometry tested, so models that used an increasing surcharge pressure were run. Applied surcharge pressure increased the effective unit weight of the voussoir beam, increasing the maximum horizontal compressive stress generated, and decreasing the $\text{FoS}_{\text{crushing}}$. Inelastic models were run by applying an 8 kPa surcharge pressure and solving through each of the four stages shown in Figure 2.15. Then the surcharge pressure was increased in increments of 2 kPa while the analytically predicted maximum stress was recalculated to identify the $\text{FoS}_{\text{crushing}}$ for each surcharge increment. After each increase in surcharge pressure, the model was solved to a standard equilibrium solution ratio of $1.0(10)^{-5}$ and stepped an additional 100,000 steps to ensure that the model had stopped moving. This process was repeated with a critical strain of $1.0(10)^{-6}$ and with perfectly plastic models to consider the impact of strain-weakening, brittle, and perfectly plastic failure on inelastic voussoir beam stability. Note that critical plastic strain is the amount of strain required to transition fully from peak to residual values of friction, cohesion, and tensile strength. Model histories of maximum horizontal stress at midspan and abutment, midspan deflection, and type of material yield were collected and compared to $\text{FoS}_{\text{crushing}}$ based on analytical predictions of displacement and horizontal stress.

**2.4.2 Initial Yield Results – Differential Stresses**

Regardless of post-peak strength, material yield initiated at the beam abutments as the analytically determined $\text{FoS}_{\text{crushing}}$ approached 1.8 (i.e. 22 kPa surcharge pressure) and 2.0 (i.e. 310 kPa surcharge pressure) for the weak and strong rock analogs, respectively. However, the distribution of yielded elements, and the nature of the yield incurred varied slightly with modeled post-peak behavior (Figure 2.24).
Recall that the voussoir beam analytical solution, which was used to calculate the FoS\textsubscript{crushing} shown in Figure 2.24, tended to underpredict stresses at the beam abutments. Therefore, the fact that yield initiated at approximately FoS\textsubscript{crushing} = 2.0 has no direct relationship with the FoS\textsubscript{crushing} adjustment factors proposed (0.3-0.5 UCS) by previous authors (Diederichs & Kaiser, 1999; Alejano et al., 2008) as those are based on material properties and scale, not differential stress concentrations. Furthermore, initial yield occurred when the abutment stresses approached the modeled UCS* as indicated by the model results (Table 2.5). Although yield initiated at an analytically determined FoS\textsubscript{crushing} indicating an optimistic prediction of stability by the analytical solution (i.e. FoS\textsubscript{crushing} > 1.0), the six beams tested remained stable immediately following yield, and analytically determined maximum displacements and stresses were generally consistent with model results.
Table 2.5: Comparison of strain-weakening (SW), brittle, and perfectly plastic (PP) model results and analytically determined values of displacement and maximum stress at the surcharge where yield initiated. UCS* - in-situ compressive strength, mid = midspan, abut = abutment.

<table>
<thead>
<tr>
<th></th>
<th>Strong Rock Analog</th>
<th>Weak Rock Analog</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>47 MPa UCS*</td>
<td>5.7 MPa UCS*</td>
</tr>
<tr>
<td></td>
<td>310 kPa Surcharge</td>
<td>22 kPa Surcharge</td>
</tr>
<tr>
<td>δv (cm)</td>
<td>σxx,mid (MPa)</td>
<td>σxx,abut (MPa)</td>
</tr>
<tr>
<td></td>
<td>2.4 24</td>
<td>0.80 3.1</td>
</tr>
<tr>
<td></td>
<td>2.5 28 45</td>
<td>0.67 3.3</td>
</tr>
<tr>
<td>Analytical Solution</td>
<td>-4.0 -14 -47</td>
<td>20 -6.1 -49</td>
</tr>
<tr>
<td>(Diederichs &amp; Kaiser, 1999)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PP &amp; SW</td>
<td>2.8 30 48</td>
<td>0.76 3.6 5.7</td>
</tr>
<tr>
<td>Analytical Error (%)</td>
<td>-14 -20 -50</td>
<td>5.3 -14 -46</td>
</tr>
<tr>
<td>Brittle</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

At the onset of zone element yield, the voussoir analytical solution underpredicted maximum stresses for all model cases, particularly those stresses measured at the beam abutments. This was due to a combination of the baseline analytical error (i.e. abutment-midspan discrepancy and surcharge error) and some yield-induced stress concentration. This discrepancy was generally consistent through all cases tested, but more pronounced in stronger and more brittle cases.

In summary, the divergence between model and analytical results at yield initiation was due to the midspan-abutment stress differential (assumed to be zero in the analytical solution) coupled with the difference in analytical and model results. However, regardless of this discrepancy, these results show that a beam capable of incurring inelastic deformation in intact material does not immediately collapse following yield due to stable post-yield deformation and the non-zero residual strength of the material modeled.

2.4.3 Incipient Beam Collapse Results – Post-Peak Material Behavior

Although the Diederichs & Kaiser (1999) voussoir analytical solution conservatively assumes that crushing failure of the beam occurs as soon as yield initiates, in reality, some stable beam deflection may occur between the onset of yield and the point of beam collapse. Accordingly, the analytically predicted FoS_{crushing} at failure will necessarily be less than or equal to the analytically predicted FoS_{crushing} at which yield initiates (observed to be FoS_{crushing} = 1.8-2.0 in Section 2.4.2). The degree to which FoS_{crushing} at failure deviates from FoS_{crushing} at yield depends on the post-peak behavior (i.e. combination of residual strength and critical plastic strain) of the intact rock material.

In the inelastic models considered in this section, as surcharge pressure was increased beyond the point of initial intact rock yield, additional zone elements began to yield in shear and tension at the beam abutments and midspan. Surcharge pressures were increased incrementally until the beams...
failed. Note that perfectly plastic voussoir beams remained stable at far greater surcharge pressure than strain-weakening inelastic voussoir beams, and strain-weakening beams at greater surcharge pressure than brittle models tested. All four brittle and strain-weakening voussoir beams tested failed at $\text{FoS}_{\text{crushing}} > 1.0$ as predicted by the Diederichs & Kaiser (1999) voussoir analytical solution and Eqn. (2.38), while the perfectly plastic beams failed at $\text{FoS}_{\text{crushing}} < 1.0$. The maximum stable surcharge pressures tested and their associated impact on the distribution of plastic shear strain and zone yield in the brittle, strain-weakening, and perfectly plastic cases are shown in Figure 2.25. Note that the surcharge pressure and analytical $\text{FoS}_{\text{crushing}}$ at which the inelastic voussoir beams collapsed was at the next surcharge increment after what is shown in Figure 2.25. However, this increase in surcharge pressure only marginally increased analytically determined maximum compressive stress and resulted in an identical $\text{FoS}_{\text{crushing}}$ when accounting for significant figures. Therefore, the penultimate surcharge pressure and associated $\text{FoS}_{\text{crushing}}$ can effectively be considered the surcharge load and $\text{FoS}_{\text{crushing}}$ at failure.

Analyzing the inelastic voussoir beam abutments under the maximum stable surcharge pressure allowed a clear description regarding the state of the beam immediately before collapse to be developed. The in-situ compressive strength of the intact rock had been exceeded, zone elements had yielded in shear and tension, and zone plastic strains had exceeded the critical plastic strain (i.e. zones were at residual strengths). Note that the maximum plastic strain at the beam abutment of the strain-weakening weak rock model depicted in Figure 2.25 was $3.0(10)^{-3}$, which was slightly below the assigned critical strain of $5.0(10)^{-3}$. This divergence (i.e. abutment zones in strain-weakening weak rock models below critical plastic strain) was partly due to the difference in strength between weak and strong rock material, and therefore maximum stable surcharge pressure. A weaker beam takes on less surcharge, leading to lower compressive stress and strain at the maximum stable surcharge. If the beam was highly brittle, the critical plastic strain was easily exceeded regardless of the maximum stable surcharge pressure. The critical strain was easily exceeded in the beam abutments of the brittle weak rock material because the critical strain was 1000 times lower than in the strain-weakening models. Conversely, the perfectly plastic weak rock model had no critical plastic strain (i.e. residual strength was equal to peak strength), and therefore it was able to incur the highest stable surcharge pressure of the cases tested.
Figure 2.25: Maximum stable surcharge pressure for simple inelastic voussoir beam model results showing yielded zone elements (right) and zones with plastic strains above the critical strain limit (left) at equilibrium for strong and weak beam models with brittle, strain-weakening, and perfectly plastic behavior. Surcharge pressure (q) and analytically determined FoS_{crushing} shown. SW = strain-weakening, PP = perfectly plastic.
In the strain-weakening beam models, midspan zones incurred significant shear and tensile yield, but the beams remained stable because the plastic strain at the midspan had not exceeded the critical strain and zone material properties had not reached residual values. Similarly, brittle beam models remained stable with insignificant or no yield at the midspan, while zones at the abutment had already reached residual strength values. Once the surcharge increment was increased to the next step (i.e. 2 kPa higher), the brittle and strain-weakening beam midspans exceeded the critical strain, reached residual strength values, and the beams failed. This confirmed that the midspan crushing is the critical control on crushing failure in brittle and strain-weakening material behavior. This finding was further highlighted by brittle and strain-weakening beam models remaining stable when maximum model abutment stresses approached nearly twice the UCS* of the beam material (Table 2.6). Interestingly, maximum stable surcharge model displacement and midspan stress results were generally consistent with analytical predictions and with analytical-model stress discrepancies in Section 2.4.2 (see Table 2.5). However, agreement diverged with increasing plastic strain and number of yielded elements. The plastic strain that the modeled beams incurred diverged from the voussoir analytical solution, where deflection is calculated by elastic shortening of the beam (i.e. no plastic strain was considered). As the critical control on beam stability was the state of the midspan, discrepancies in the abutment stresses were less important for the purposes of this study. At the maximum stable surcharge load, the voussoir analytical solution underpredicted maximum midspan stresses for strain-weakening and brittle beam models, underpredicted maximum midspan stresses in strong perfectly plastic beam models, and overpredicted maximum midspan stresses in weak perfectly plastic beam models. Increased brittleness and decreased strength resulted in better agreement with the analytical displacement predictions while marginally decreasing the agreement with midspan stress predictions.
Table 2.6: Comparison of strain-weakening (SW), brittle, and perfectly plastic (PP) model results and analytically determined values of displacement and maximum stress for the maximum stable surcharge for each material case tested. UCS* - in-situ compressive strength, mid = midspan, abut = abutment.

<table>
<thead>
<tr>
<th></th>
<th>Strong Rock Analog 47 MPa UCS*</th>
<th>Weak Rock Analog 5.7 MPa UCS*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>δv (cm) σxx,mid (MPa) σxx,abut (MPa)</td>
<td>δv (cm) σxx,mid (MPa) σxx,abut (MPa)</td>
</tr>
<tr>
<td>PP Surcharge (kPa)</td>
<td>740</td>
<td>79</td>
</tr>
<tr>
<td>Model Result</td>
<td>8.4 63 96 2.9 6.3 13</td>
<td></td>
</tr>
<tr>
<td>Analytical Solution</td>
<td>5.3 57 57 1.8 7.2 7.2</td>
<td></td>
</tr>
<tr>
<td>Analytical Error (%)</td>
<td>-37 -9.5 -41 -38 14 -45</td>
<td></td>
</tr>
<tr>
<td>SW Surcharge (kPa)</td>
<td>530</td>
<td>53</td>
</tr>
<tr>
<td>Model Result</td>
<td>5.1 48 90 1.4 5.8 10</td>
<td></td>
</tr>
<tr>
<td>Analytical Solution</td>
<td>3.9 40 40 1.3 5.3 5.3</td>
<td></td>
</tr>
<tr>
<td>Analytical Error (%)</td>
<td>-24 -17 -56 -7.1 -8.6 -47</td>
<td></td>
</tr>
<tr>
<td>Brittle Surcharge (kPa)</td>
<td>380</td>
<td>34</td>
</tr>
<tr>
<td>Model Result</td>
<td>3.4 36 59 0.99 4.7 7.5</td>
<td></td>
</tr>
<tr>
<td>Analytical Solution</td>
<td>2.8 28 28 0.99 4.0 4.0</td>
<td></td>
</tr>
<tr>
<td>Analytical Error (%)</td>
<td>-18 -22 -53 -0.0 -15 -47</td>
<td></td>
</tr>
</tbody>
</table>

Regardless of the accuracy of the analytically determined stress and displacement in comparison to inelastic beam model results, the analytically determined FoS\\textsubscript{crushing} had no method to account for post-peak material behavior and therefore was both inaccurate and inconsistently conservative relative to the more realistic UDEC model results. However, the impact of midspan-abutment and analytical-model stress differentials countered that inconsistency with consistently optimistic stability predictions (i.e. FoS\\textsubscript{crushing} > 1.0) at yield initiation and collapse in specific material cases (i.e. most brittle) that approach the analytical assumptions (i.e. beam collapse occurs at onset of material yield). The impact that beam material post-peak behavior had on model displacement and maximum horizontal stresses with increasing surcharge increment is summarized in Figure 2.26. Regardless of material strength, the post-peak material behavior largely dictated the ultimate stability of the beam. However, with decreasing strength, beam collapse of strain-weakening material approached the analytical solution. This indicates that the specific post-peak properties (i.e. friction, cohesion, tensile strength) also have an impact on the stability of inelastic beams and require further study outside the scope of this thesis. Furthermore, as surcharge pressure increased prior to beam collapse, the divergence between model midspan stresses, abutment stresses, and the baseline Diederichs & Kaiser (1999) analytical solution increased. This was previously observed in the Section 2.3.4 elastic models and cannot solely be explained by inelastic yield.
Figure 2.26: Model history results of abutment and midspan maximum horizontal stress (top) midspan vertical displacement (bottom) for strong (left) and weak (right) rock analogs. Vertical markers show the surcharge pressure at which (a) brittle beams collapse, (b) strain-weakening beams collapse, (c) analytical FoS\textsubscript{crushing} =1.0, (d) and perfectly plastic beams collapse. SW = strain-weakening, PP = perfectly plastic.

Note that the significantly higher abutment stresses were maintained throughout inelastic beam deformation partly due to the limitations of the explicit DEM. As brittle and strain-weakening zones approached residual strength, they could no longer transfer stresses effectively to the abutment (Figure 2.27). Recall that the explicit DEM cannot simulate the rupture of intact blocks, and although the abutment corner zone elements had yielded and were concentrating stresses towards the mid-height of the beam, those elements were no longer carrying significant horizontal stresses. However, they were still providing non-negligible confinement to the other zone elements and potentially preventing the full-crushing failure of the abutment corner. In reality, this area could rupture and detach, decreasing stability and potentially leading to collapse. This also explained why the deviation in perfectly plastic model displacement results was so significant,
while the deviation in stress results was not: perfectly plastic models maintained stress arching and compression arch integrity into the post-peak of the intact material behavior.

It is evident that although the voussoir analytical solution implements simplifying assumptions (i.e. does not consider post-peak) to calculate maximum beam displacement and horizontal stress, its accuracy in regard to midspan stress and displacement during inelastic deformation remains generally consistent until the point of collapse. Despite this consistency in determining displacement and stress, and consistent optimism when predicting crushing failure based on midspan-abutment and analytical-model stress differentials, FoS_{crushing} is still clearly dependent on post-peak material behavior as beams modeled with field-scale strength properties failed at three different values of analytically calculated FoS_{crushing}.

Although the strain-weakening and brittle beams collapsed at analytically determined FoS_{crushing} > 1.0 (i.e. too optimistic), consideration of realistic post-peak material properties can only increase the stability of a given beam as the analytical solution assumes that yield and collapse occur simultaneously. Therefore, the overly optimistic analytical solution in brittle and strain-weakening beams was based partially on the underprediction of maximum midspan stress at the maximum stable surcharge load (9-22%). If the analytical solution matched the modeled beam midspan...
stresses at ultimate load, the strain-weakening beams would fail at exactly \( \text{FoS}_{\text{crushing}} = 1.0 \), but the strong and weak brittle beams would fail at 1.3 and 1.2, respectively. To account for this remaining discrepancy, the collapse mechanics of the modeled beams must be considered, namely the influence of tensile yield.

2.4.4 Final Beam Collapse Results – Tensile Influence

Each of the post-peak behavior cases explored in Section 2.4.3 followed a distinct pattern of yield and collapse. As discussed previously, the brittle material cases both failed at \( \text{FoS}_{\text{crushing}} > 1.0 \) and neither incurred shear yield at the midspan prior to the surcharge increment that caused collapse (see Figure 2.25). Following application of the ultimate surcharge load, additional tensile yield occurred adjacent to zones that previously yielded in tension near the midspan and abutment. Shortly thereafter, the midspan yielded in shear and the voussoir compression arch immediately began to break down. Compression arch deterioration occurred in tandem with tensile yield propagation between the midspan and abutment, until total collapse of the beam occurred (Figure 2.28). This observed failure appeared to be tensile-yield induced stress concentration causing midspan crushing, followed by a concurrent diagonal tensile cracking due to the brittleness of the material, independent of its peak strength, as the weak rock brittle beam failed in a similar manner. However, abutment and midspan crushing clearly preceded the tensile yield propagation between the two, whereas diagonal tensile cracking failure occurs without crushing of the midspan or abutments.
When considering the strain-weakening beam collapse, a similar amount of shear yield and pattern of compression arch deterioration occurred, but the beam failed without tensile yield propagating fully from midspan to abutment and the shear failure of the midspan was not preceded by stress concentration due to tensile yield surrounding the midspan (Figure 2.29).
Conversely, perfectly plastic beams failed at analytically determined FoScrushing < 1.0 (i.e. conservative). The failure of the perfectly plastic beams can be conceptualized as an inelastic buckling-style failure where both elastic and plastic strains contributed to the deflection and eventual overcoming of a relatively constant maximum horizontal stress, rather than a sudden loss of strength and compression arch deterioration. This was clearly captured by the continuous and sustained horizontal compression arch as the moment arm decayed until ultimate collapse without associated tensile yield bridging between the abutment and midspan (Figure 2.30).
2.4.5 Inelastic Beam Discussion

While the analysis in this section does not cover every possible rock type, perfectly plastic material behavior corresponds to an upper-bound estimate of post-peak strength, and brittle material behavior represents the lower-bound. These results indicate that application of $F_{oS_{crushing}}$ to rocks and rockmasses with various post-peak material behavior can significantly overestimate (i.e. failure occurs at $F_{oS_{crushing}} > 1.0$) or underestimate (i.e. failure occurs at $F_{oS_{crushing}} < 1.0$) the safety of a given roof span. This is due to a combination of differences in analytically determined maximum stress and model stress results, post-peak behavior of the material modeled, and model tensile yield.

A correction factor of 0.3-0.5 applied to the lab-scale UCS of the rock was previously proposed by Diederichs & Kaiser (1999), based on the observations of (Martin, 1997), and implemented in the back analysis Alejano et al. (2008) to account for heterogeneities of the rockmass and scaling from laboratory strength to field strength. This value was based on the findings that damage could
initiate in massive, brittle rocks at the crack initiation value (i.e. ~0.4 UCS) at excavation boundaries. However, the peak strength of the beams tested herein was known and fixed across the brittle, strain-weakening, and perfectly plastic models. Furthermore, the UCS* implemented in the numerical models and utilized in Eqn. (2.38) was already selected to represent field-scale values. Therefore, it was clear that the baseline Diederichs & Kaiser (1999) analytical method, which already applies a correction factor of 0.3-0.5 to get from lab to field scale UCS, requires an additional adjustment based on the post-peak behavior of the rock. The results suggest that an additional 0.6- or 1.25-times adjustment be made to Eqn. (2.38) for brittle and perfectly plastic rock types, respectively, once the UCS has been adjusted to field scale values.

Results of inelastic voussoir beam models indicate that a yielded beam modeled using the explicit DEM can remain stable under voussoir arching even if yield occurs at the top of the midspan and bottom of the abutments. Under maximum stable surcharge pressure in brittle and strain-weakening cases, plastic strains did not exceed the critical plastic strain at the midspan, allowing the post-peak strength to maintain voussoir arching. The strain-weakening and brittle beams ultimately failed at an analytically calculated $\text{FoS}_{\text{crushing}}$ above 1.0. Conversely, perfectly plastic beams maintained peak cohesion, friction, and tensile strength in the post-peak and remained stable well below an analytically calculated $\text{FoS}_{\text{crushing}}$ of 1.0.

Inelastic voussoir beam results further confirmed the applicability of the voussoir beam analog to increasingly complex voussoir beams when strictly considering maximum stress and displacement. When considering design against crushing failure, post-peak behavior and field-scale adjustments should be acknowledged and considered in analysis. Furthermore, if horizontal stresses at model equilibrium were found to be significantly below the analytically determined stress, it was found to be reasonable to assume that the beam (i.e. roof) has lost all supporting capacity and has failed. This phenomenon will be considered when analyzing future models to verify stable or unstable roof conditions without explicit visual confirmation of thousands of roof stability models in Chapter 3.

### 2.5 Horizontal Stress Voussoir Beam Models

Prior to applying the voussoir beam analog to single-entry mine models with voussoir roofs or realistic DFNs and inelastic material, the potential in-situ stress impacts on elastic voussoir mechanics needed to be considered separately. The effects considered in this section included
horizontal confinement at beam abutments with either mobile or immobile abutment blocks. Model results were compared to the baseline Diederichs & Kaiser (1999) analytical solution and confinement adjustment methods used for stress determination were proposed.

2.5.1 Methodology & Model Inputs

The models in this section considered the impact of applied horizontal stresses on voussoir beam mechanical behavior. The previously modeled 10 m span, $E_{\text{rms}} = 3.3$ and 33.3 GPa simple elastic voussoir beams were analyzed with varying magnitudes of constant horizontal stress applied to the abutment blocks on both ends of the beam and solved to an equilibrium solution ratio (Figure 2.31). Abutment stresses ranging from 0.1 to 10 MPa were applied to both abutment blocks using a constant load boundary condition. This means that if 1.0 MPa was applied to each block, the total impact to the beam would be 2.0 MPa applied stress. Most notably, the abutment blocks were no longer fixed and could displace outwards if the applied stress did not overcome the internal stress of the beam. The voussoir beam was now allowed to lengthen, shorten, and deflect due to horizontal displacement of the abutment blocks, rather than purely due to elastic shortening of the beam. Furthermore, as this section was concerned with stress transfer between the beam and the abutments rather than the stresses generated internally by the beam, the abutment blocks were now the same stiffness as the beam material (i.e. realistic), rather than extremely stiff abutments as were used in Diederichs & Kaiser (1999) and the models in Sections 2.3 and 2.4.

Figure 2.31: General model setup of voussoir beam featuring an applied horizontal stress at the abutments and magnified deformed block geometry at equilibrium.

A second set of models was run with horizontal stresses “locked-in” by initializing horizontal stress, removing the load boundary condition, and replacing it with a zero-velocity boundary condition in a similar manner to Shabanimashcool & Li (2015).
Horizontal stress voussoir beam models were solved using the same four stage solution method per Section 2.3.1, and a wide range of horizontal stresses were considered. As these models were elastic, no crushing failure occurred due to increased levels of applied horizontal stress. Model results were compared to the voussoir analytical solution found in Diederichs and Kaiser (1999), and a linear relationship between applied horizontal stress and voussoir analytical maximum stress predictions was established.

2.5.2 Mobile Abutments & Continuously Applied Horizontal Stress

Continuously applied horizontal stress to both mobile abutments, when greater than 1.0 MPa, decreased midspan displacement of the $E_{\text{mix}} = 3.3$ and 33.3 GPa beams to limiting values of approximately 5.0 and 0.4 mm, respectively (Figure 2.32). Unsurprisingly, the 1.0 MPa applied to each mobile abutment block was equal to half the maximum horizontal stresses generated by both the $E_{\text{mix}} = 3.3$ and 33.3 GPa self-stabilizing beams (i.e. 2.0 MPa). This indicated that when forces are continuous, equal, and opposite, over comparable material stiffness and with fully mobile abutment blocks, the abutment joint had fully closed. Note that extremely high horizontal stresses were not considered, although they have been noted to induce beam buckling, particularly in longer span models ($> 30$ m) (Shabanimashcool & Li, 2015).

![Graph](image)

Figure 2.32: Effect of applied horizontal stress on maximum displacement of 10m span, $E_{\text{mix}} = 3.3$ and 33.3 GPa voussoir beams with mobile abutment blocks. Results compared to the baseline Diederichs & Kaiser (1999) analytical solution.

Maximum midspan horizontal stresses were first compared to variations in applied horizontal stress with mobile abutment blocks in Figure 2.33. Interestingly, at the same applied horizontal...
stress where displacement reached its limiting value (i.e. 1.0 MPa in mobile abutment models), the measured model stresses began to increase linearly. This further confirmed that abutment joint had fully closed, and the compression “arch” thickness was now equal to the thickness of the beam. Additionally, both beam models incurred nearly identical midspan stresses, as they did in the analytical solution.

![Figure 2.33: Effect of applied horizontal stress on maximum midspan stress of 10m span, $E_{mx}$ = 3.3 and 33.3 GPa voussoir beams with mobile abutment blocks. Results compared to the baseline Diederichs & Kaiser (1999) analytical solution.](image)

For mobile abutments, the simplest method was to consider a conservative assumption of stress and simply add the entire applied stress at all confinement levels tested to the stress determined by the baseline Diederichs & Kaiser (1999) analytical solution (Figure 2.34). Dubbed the “Total Stress Method”, this approach provided a consistently conservative stress estimate from the perspective of predicting midspan crushing failure. However, it could not account for the non-linear changes in stress or displacement incurred by mobile abutments and lower horizontal confining stress. The mobile abutment violated the boundary conditions of the baseline Diederichs & Kaiser (1999) voussoir beam analog and allowed for changes in moment arm length as well as compression arch distribution and thickness due to force reactions at the beam abutments, in addition to elastic shortening of the beam. An increase in applied stress increased the compression arch thickness (i.e. closes the abutment joint) and the resulting internal beam stresses simply increased by the same magnitude. Conversely, if the applied horizontal stress was not large enough, the voussoir beam becomes unstable and could fail via abutment slip or snap-through, depending on the s/t ratio of the voussoir beam and the strength of the voussoir beam joints.
Constraining this very specific mechanism over a small window of horizontal stress is outside the scope of this study.

![Plot depicting the accuracy of the “Total Stress Method” (TSM) in predicting the maximum model midspan stress of 10 m span, $E_{\text{rmx}} = 3.3$ and 33.3 GPa voussoir beams.](image)

**Figure 2.34**: Plot depicting the accuracy of the “Total Stress Method” (TSM) in predicting the maximum model midspan stress of 10 m span, $E_{\text{rmx}} = 3.3$ and 33.3 GPa voussoir beams.

### 2.5.3 Immobile Abutments & Locked-In Horizontal Stress

In order to constrain the impact of the mobile abutment blocks, similar models were run with locked-in applied horizontal stresses by using zero-velocity boundary conditions following stress initiation but prior to allowing beam deflection. A similar method was previously implemented by Shabanimashcool & Li (2015) on single-jointed voussoir beams and by Oliveira & Pells (2014) and Oliveira & Paramaguru (2016) on multi-jointed voussoir beams. This boundary condition represented the end-member case where no material yield occurred, and the abutment blocks and the beam abutment (i.e. roof-pillar boundary) remained competent and immobile.

If horizontal stresses are “locked-in” with a non-mobile abutment block, the baseline Diederichs & Kaiser (1999) analytical solution was naturally more accurate due to the limited range of displacements allowed by non-mobile abutment in comparison to the previous results (Figure 2.35). Interestingly, the softer and stiffer beam model results approached the analytical solution at opposite levels of confinement. This discrepancy can be explained by the vertical joint spacing. Although the $E_{\text{rmx}} = 33.3$ GPa beam was stiffer overall (i.e. stiffer material and stiffer joints), it had a 0.5 m joint spacing, increasing the degree relative softening of the beam response from abutment to midspan and allowing for deflection above what the analytical solution predicts. The $E_{\text{rmx}} = 3.3$ GPa beam was softer overall but had wider spaced joints (1.0 m) which decreased the
degree of relative softening from abutment to midspan. However, both cases approached limiting displacement values at a higher magnitude of confinement than their mobile abutment counterparts. This indicated that with immobile abutments, higher levels of confinement are required to restrict the opening of the abutment and midspan joints. Although Oliveira & Paramaguru (2016) presented a method of adjusting $E_{rnx}$ to account for the decrease in voussoir beam displacement due to in-situ horizontal stress, this method required a UCS and is not applicable to cases where only elastic intact rock is considered.

![Figure 2.35: Effect of locked-in horizontal stress on maximum displacement of 10m span, $E_{rnx} = 3.3$ and 33.3 GPa voussoir beams with immobile abutment blocks. Results compared to the baseline Diederichs & Kaiser (1999) analytical solution.](image)

The effect of “locked-in” horizontal stress on model maximum midspan and abutment stress was considered next (Figure 2.36). The results confirmed that the joint spacing was impacting the beam response. Note that the linear increase in stress in the beam with more joints (i.e. $E_{rnx} = 33.3$ GPa, 0.5 m spacing) initiated at a higher level of locked-in horizontal stress because the softening effect of joints compounds towards the beam midspan, requiring more confining stress to keep the midspan joint from opening. Furthermore, the relationship between abutment and midspan stress reversed in the stiffer beam case, with abutment stresses reducing due to larger compression arch thickness at the abutment joint, which decays in thickness towards the beam midspan (i.e. increasing stress) due to the softening effect of each joint.
2.5.4 Horizontal Stress - Discussion

Capturing all of the observed complexity prior to closure of the abutment and midspan joints in the baseline Diederichs & Kaiser (1999) analytical solution would require making certain assumptions regarding the boundary conditions of the beam abutment. A mobile abutment boundary condition represents an in-situ case where there is a significant decrease in strength and stiffness between beam midspan and abutments, coupled with low horizontal stress. In order to capture this with mechanical accuracy, one would need to account for the displacement of the abutment blocks’ effect on the moment arm and compression arch distribution. An immobile abutment boundary condition represents an in-situ case where excavation in adjacent drifts causes stress relaxation after initial voussoir stability is achieved (Alejano et al., 2008) and previously locked-in horizontal stresses deteriorate. It was clear that this could be accomplished by increasing the unit weight of the beam by varying amounts to account for the stress reaction occurring at the immobile abutment between the stress generated by the self-stabilizing voussoir beam and the decaying horizontal stress.

In most applications, the true behavior is likely somewhere between the mobile and immobile abutment end-members. This was explored further in Section 2.7 with in-situ single entry voussoir beam models. Due to the limited number of models encompassing a narrow range of possible
material and geometric properties, a verified accurate adjustment to effective unit weight was not possible within the scope of this study.

This relationship identified between applied horizontal stress and voussoir beam response has significant implications for future analysis of failure modes observed in more complex (i.e. “realistic”) models within this study. In-situ voussoir beam models discussed in Section 2.7 highlighted that the impact on stress and displacement prediction is negligible when using the “Total Stress Method” horizontal stress adjustment. However, the findings presented in this section were not focused on capturing the impact on displacement, but rather the impact on maximum horizontal stress in the voussoir beam, which is consistent between the two abutments modeled and consistently captured by the “Total Stress Method”.

The application of horizontal stress to mobile and immobile abutment blocks indicated that the internal beam horizontal stress under varying degrees of in-situ stress ratios could be reasonably predicted, but the impact on displacement could not be explicitly and accurately captured in the existing voussoir analytical solution from Diederichs & Kaiser (1999) with the available model results.

2.6 Horizontally Jointed & Bolted Beam Models

The models presented in this section featured geometries that deviated significantly from the voussoir beam models in the previous sections. The simultaneous effects of horizontal joint sets (i.e. bedding) and presence of roof support elements were considered to expand the application of existing voussoir theory to account for more realistic in-situ complexity.

2.6.1 Methodology & Model Inputs

In order to approach in-situ roof geometry of a discontinuous and layered rockmass, the previously analyzed baseline voussoir beam models were split into two 0.5 m thick voussoir beams and bolted using calibrated passive grouted rebar bolt parameters from the literature (Bahrani & Hadjigeorgiou, 2017). 1.0 m long bolts were installed through a faceplate on 0.5 m spacing using the beam and structural elements available in UDEC. An example bolted beam model geometry is shown in Figure 2.37. Note that the horizontal joints in the abutment blocks were modeled elastically with identical shear and normal stiffness values as the intra-span joints.
Models were run using the previously described method transitioning from elastic joints to continuously yielding. To ensure model stability, a velocity check of the beam bottom was conducted at model equilibrium to ensure that the average velocity of the beam did not exceed $1.0 \times 10^{-5}$ m/second. This threshold was chosen based on observations of beams that reached a standard equilibrium solution ratio, but when stepped forward became unstable. Note that model results discussed were from ultimately stable models at their equilibrium solution ratio to ensure a valid, direct comparison.

Initially, model displacement and horizontal stress results were compared to the Diederichs & Kaiser (1999) analytical solution, using the previously determined $E_{\text{rms}}$ of the jointed beams (i.e. 3.3 and 33 GPa, considering only vertical jointing), coupled with both the thickness of the entire beam (1 m) and the thickness of an individual layer (0.5 m), with poor agreement. Specifically, the presence of the orthogonal horizontal joint had a softening effect on the behavior of the overall voussoir beam when compared to a beam with identical dimensions, but no horizontal joint. Passive rockbolts tied the two beams together increasing the thickness and stable span but did not provide a clamping effect such that the voussoir beam would transfer stresses as a single, competent beam.

Preliminary model results identified that a trend existed in bolted beam behavior that could be reasonably captured by adjusting the analytical solution. However, the limited suite of preliminary models required additional parametric cases to establish a broader mechanical relationship. Therefore, an expanded suite of jointed and supported voussoir beam models was undertaken to analyze the inputs shown in Table 2.7.
This expanded suite included 810 unique numerical models. A method of resolving anisotropic stiffness into a single Young’s Modulus for use in the baseline Diederichs & Kaiser (1999) analytical solution could not be identified in the literature. Generalized application of anisotropic rockmass deformation moduli require knowing the unique states of stress (i.e. vertical, horizontal, and shear) and relating them to strains using a deformation modulus matrix (Salamon, 1968; Amadei & Goodman, 1981; Yoshinaka & Yamabe, 1986; Huang et al., 1995), rather than calculating an unknown maximum state of stress from elastic strains and a uniform stiffness.

Regardless, the baseline Diederichs & Kaiser (1999) analytical solution maintained accuracy for small-span and low-deflection beams when considering the individual layer thickness in the analytical solution. However, when considering the bolted thickness, as suggested in Diederichs & Kaiser (1999), accuracy of the analytical solution decreased significantly.

A statistical analysis of the stable model results was performed in an effort to identify potential modifications to the analytical solution. A method of resolving the anisotropic stiffness of a multi-layered bolted beam into a single effective Young’s Modulus based on the number of bolted layers was developed. This was done by using the fminsearch function in MATLAB to determine the rockmass modulus that minimized the difference between model displacement results and analytical predictions of maximum displacement for each case. As previously stated, the rockmass modulus calculated by consideration of only vertical joints in Eqn. (2.25) is referred to as $E_{rnx}$. New iterations of rockmass modulus ($E_{rm}$) are discussed in this section. $E_{rmy}$ is rockmass modulus calculated based on horizontal (i.e. bedding) joint normal stiffness and spacing. The rockmass modulus back calculated by using the fminsearch function is referred to as the effective or the
minimized $E_{rm}$. The rockmass modulus determined as a result of the statistical analysis of model inputs and the effective $E_{rm}$, is referred to as the layer-adjusted $E_{rm}$ ($E_{rmn}$).

Multiple three-dimensional linear regression surfaces were fit to the data set based on the calculated effective $E_{rm}$ (i.e. dependent variable), and its relationship with $E_{rmx}$, $E_{rmy}$, and the number of horizontal layers (i.e. independent variables). A trend was identified, and Eqn. (2.25) was adjusted to predict the displacement of multi-layered voussoir beams. This adjusted method was then verified as more accurate than the baseline Diederichs & Kaiser (1999) solution by comparison of predictions of both binary stability (i.e. stable or unstable) and maximum displacement of multi-layered bolted voussoir beam models in comparison to the baseline Diederichs & Kaiser (1999) method.

However, the adjusted method remained inaccurate for predicting horizontal stresses due to its significant reduction in rockmass modulus and increase in beam thickness. A similar statistical analysis was therefore utilized to back calculate an optimized effective thickness between the individual layer and bolted interval thickness for the purposes of estimating stress in multi-layered and bolted voussoir beams.

2.6.2 Horizontally Jointed and Bolted Beam Results

Multiple mathematical adjustments in the literature were reviewed, and no existing method was found that resolved anisotropic stiffnesses of multi-layered bolt voussoir beams into a single rockmass modulus value for use in the voussoir beam analytical solution. The Diederichs & Kaiser (1999) analytical solution significantly underpredicted displacements and maximum stress while over predicting the maximum stable span when considering the thickness to be the bolted interval. If one considered the beam thickness to be an individual layer, the analytical solution underpredicted the maximum stable span and overpredicted displacements and maximum horizontal stresses (Figure 2.38). Neither application of the Diederichs & Kaiser (1999) analytical solution could capture the displacement, stress, or maximum stable span of the preliminary horizontally jointed and bolted beam models. Note that the 20 m, $E_{rmx} = 3.3$ GPa and the 30 m, $E_{rmx} = 33$ GPa beams tested both collapsed via buckling failure and are therefore not shown in Figure 2.38.
Interestingly, model contours of horizontal stress highlighted two distinct stress arches in the layers of the stiffer bolted beams, and one stress arch in the softer bolted beams. Regardless of stiffness, stresses concentrated at the midspan of the top layer midspan and abutments of the bottom (Figure 2.39), further complicating comparison to the Diederichs & Kaiser (1999) analytical solution.

Figure 2.38: Comparison of (a) midspan displacement and (b) maximum horizontal stress model results featuring an orthogonal joint and rockbolt elements compared to the Diederichs & Kaiser (1999) analytical solution (D&K) considering the thickness to be a single layer (black), or the bolted interval (orange).

Figure 2.39: Horizontal stress distribution in bolted voussoir beam models for $E_{\text{rmx}} = 33$ GPa (top) and $E_{\text{rmx}} = 3.3$ GPa (bottom) highlighting the difference in compression arch distribution, bolts excluded for clarity.
This prompted comparison of a 1.0 m thick, single layer voussoir beam from Section 2.3 with equivalent thicknesses layered and bolted voussoir beams with varying bolt spacing, as well as a single layer 0.5 m thick beam, to clarify the transition in beam behavior (Figure 2.40). Results further confirmed that passive bolting of two 0.5 m beams changed overall beam behavior significantly. The behavior of a bolted beam existed somewhere between a single beam with the bolted thickness and a beam equal to the individual layer thickness. The behavior was further influenced by the spacing of the passive bolt elements. These preliminary results inspired a more rigorous parametric analysis featuring 2, 4, and 8-layer voussoir beams with varying geometric and properties listed in Table 2.7. Representative low-displacement models and their horizontal stress and displacement results are shown in Figure 2.41. Stable model results indicated that small displacements (i.e. mm to cm scale) in the presence of more realistic roof conditions (i.e. multiple layers and passive support elements) resulted in horizontal stresses transferred through each horizontal layer independently, while the bolted interval deflected uniformly. Similar to preliminary $E_{\text{mx}} = 3.3$ GPa models in Figure 2.40, the presence of realistic bolt spacing, and conservative bolt strength and stiffness allowed independent stress arches to form and for larger displacements to occur.
Figure 2.40: Comparison of horizontal stress (left) and vertical displacement (right) results of multiple beam geometries ranging from a single, unbolted, 1 m thick beam (top) to a single, unbolted, 0.5 m thick beam (bottom), and variations in between featuring different bolt spacings and properties. All beams have an $E_{max} = 3.3$ GPa.
Figure 2.41: Representative example stable model results showing 10 m span multi-layered and bolted voussoir beam models of varying thickness (T) and horizontal joint (i.e. bedding) spacing (s_y). The left half of each model is a contour of horizontal stress (σ_x) and the right half is a contour of vertical displacement (δ_y). Note that both contours are symmetric about the beam midspan.

Other models, particularly those with longer spans and either four or eight bolted layers, incurred significantly higher (i.e. cm to m scale) displacements but remained stable as support from passive, fully grouted rock bolt elements was activated. Additionally, the breakdown of the independent voussoir arches tending towards a single compression arch significantly increased beam abutment and midspan stresses at the bottom and top of the beam, respectively. The stability of these models is maintained by bolts approaching the limits of their load and deformation capacity, with bolt interface yield (i.e. spring and slider elements representing the bolt-grout-rock interface) occurring in bolts closer to the abutments, but not reaching the tensile failure strain of the “steel” portion of the fully grouted rockbolt element (Figure 2.42). Furthermore, bolt elements in the largest stable displacement models (i.e. > 60 cm) did not fully penetrate all midspan blocks, resulting in non-negligible, discontinuous deformation of the bottom of the beam. This clearly increased measured displacements, so select models were rerun with an additional midspan bolt. This additional bolt promoted continuous beam displacement and led to an approximately 10 cm decrease in maximum beam displacement (Figure 2.43).
Figure 2.42: Example large displacement stable model result showing horizontal stress contours (left) and vertical displacement contours (right). “X” represents bolt-grout-rock or faceplate-rock interface yield. Note that both contours are symmetric about the beam midspan.

Figure 2.43: Comparison of stable bolted beam displacement results with the addition of a single bolt (bottom) preventing discontinuous displacement and decreasing maximum vertical displacement by approximately 10 cm. “X” represents bolt-grout-rock or faceplate-rock interface yield.

The change in displacement shown in Figure 2.43 indicated that the discontinuous displacement of this magnitude could impact future statistical analyses. This was because an effective rockmass modulus was calculated based off of minimizing the difference in maximum model displacement and the analytically determined displacement.
As the beam displacement increased, the voussoir arching initially decayed at the bottom of the midspan, allowing blocks to slip if unsupported. Shorter span beams also experienced discontinuous displacement in the form of a localized, discontinuous shear displacement along intersecting joints, which increased with decreasing bolt spacing (i.e. more bolts) (Figure 2.44). At equilibrium in the 1.2 m spacing bolt model shown in Figure 2.44, the horizontal stress was not well distributed in or between each voussoir layer, but rather concentrated at the top of the bottom layer. The effective s/t ratio (i.e. span/bolted thickness) of the 1.2 m bolt spacing beam was such that the reduction in displacement lead to a reduction in horizontal stress arching, which lead to increased likelihood for shear displacement along vertical discontinuities (i.e. abutment slip failure). This discontinuity-driven displacement was then restricted as support from passive bolt elements was activated.

Figure 2.44: Model horizontal stress (left) and vertical displacement (right) results at equilibrium for identical spans, material properties, and discontinuity properties featuring variable bolt spacing at 2.4 m (top left) 1.8 m (top right), and 1.2 m (bottom). Note that both contours are symmetric about the beam midspan.

While the discontinuous deformation model results affected the accuracy of the statistical methods implemented in the following section, the overall goal of this thesis was to apply the voussoir beam analog to increasingly complex and realistic roof conditions. Seeing as discontinuous roof
displacements do occur in real excavation roofs, stable models were all considered in the statistical analyses, regardless of the continuity of the roof displacement.

Note that thicker beam model behavior was significantly impacted by the bolt node spacing. This impact was not discussed in this thesis, but the author cautions that consistent bolt node spacing be implemented in accordance with recommendations by Sinha (2020). The difference in results was significant and requires further study outside of the scope of this thesis. The bolt elements modeled in this chapter all utilized a bolt node spacing of 24/m.

2.6.3 Beam Stiffness-Displacement Analysis Results

The Diederichs & Kaiser (1999) analytical solution was tested against stable model displacement results considering both the entire beam thickness (as suggested by Diederichs & Kaiser (1999)), as well as the individual layer thickness with varying degrees of accuracy (Figure 2.45).

Figure 2.45: Comparison of the Diederichs & Kaiser (1999) analytical solution displacement and model results when considering the thickness input to be a single layer (left), or the entire bolted beam interval (right). Note that results that plot on the y-axis (i.e. \( x = 1.0(10)^5 \), circled in red) indicate that the analytical solution predicted an unstable result, but model results indicated stability had been maintained. A 1:1 trend line is shown in black, and the percent root mean square error for stable model results are provided; note the log-log axes.

The percent root mean squared error (RMSE) for each dataset was calculated as:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} z_i^2}{n}} \quad (2.39)
\]
where $\hat{Z}$ = percent error between the analytically predicted (i.e. 1:1 trend) and actual (i.e. model) results.

In stable model results, the accuracy of the baseline solution using the individual layer thickness was not significantly impacted, except for misclassification of 69 stable cases as unstable. If the misclassified models were not considered, this method maintained reasonable accuracy for predicting displacement with an RMSE of 39%. This result was intuitive; if the beams were more self-stable, the displacements were smaller, passive bolt support was not activated and the independent voussoir beams deflected more or less uniformly. As model displacements increased, the divergence of the Diederichs & Kaiser (1999) analytical solution with the individual layer thickness from the model results increased and eventually became completely unreliable for cm to m scale displacements. When using the bolted interval thickness, displacement is consistently and significantly underpredicted by the Diederichs & Kaiser (1999) analytical solution. However, distinct trends based on the number of passively bolted horizontal layers emerged from the dataset in log-log space.

Using the `fminsearch` function in MATLAB, an effective $E_{rm}$ was calculated by minimizing the difference between model and analytical displacement for $T = \text{bolted interval}$. The effective $E_{rm}$ was then compared to the $E_{rmx}$ and $E_{rmy}$, calculated using vertical or horizontal joint spacing and normal stiffness in Eqn. (2.25), respectively (Figure 2.46).
Three distinct groups of stable model results were noted and identified to be separated by the number of horizontal layers modeled; linear regression surfaces were fit to each data cluster. Interestingly, the impact of the parameters represented by $E_{rmx}$ had a small impact on the regression result, and the overall effective $E_m$ decreased with increasing number of horizontal layers independent of $E_{rmx}$ and $E_{rmy}$. The RMSE remained consistent between increasing number of horizontal layers, indicating that a viable effective $E_m$ estimate could be calculated from model inputs and reasonably account for the changes in complex voussoir beam geometry. Finally, a relationship between the $E_{rmx}$ and $E_{rmy}$ constants and number of horizontal layers is plotted in Figure 2.47.
Figure 2.47: Plot of coefficients of $E_{rmx}$ and $E_{rm}$ (Cx and Cy, respectively) from the surface regressions presented in Figure 2.46 versus the number of horizontal layers in the complex voussoir beam model.

Power functions fit to the data set suggest a limiting trend of decreasing stiffness, predominantly governed by $E_{rmx}$. This prompted a bivariate regression analysis that only considered $E_{rmx}$ and effective $E_m$. Figure 2.48 shows the results of the regression and associated Cx values.

Figure 2.48: (a) Linear regression fitting the Effective $E_m$ to $E_{rmx}$ only and (b) the resulting impact on the relationship between Cx and number of horizontal layers.
Based on the statistical analysis summarized in Figure 2.48, the following equation for the layer-adjusted $E_{rm}(E_{rmn})$ was developed:

$$E_{rmn} = (Cx)E_{rmx}$$  \hspace{1cm} (2.40)

Note that in a single layer case, $Cx$ is equal to 1.1 and Eqn. (2.40) is nearly equal to Eqn. (2.25). A 10% change in rockmass modulus results in an insignificant difference in analytically determined displacement and maximum stress using the Diederichs & Kaiser (1999) analytical solution for a single layer voussoir beam. The layer-adjusted $E_{rm}(E_{rmn})$ was then utilized in the Diederichs & Kaiser (1999) analytical solution and compared to model results of maximum displacement (Figure 2.49). Using the $E_{rmn}$ in the Diederichs & Kaiser (1999) analytical method allowed consideration of the bolted interval thickness, reduced RMSE to 36% when compared to the baseline method, and accurately classified the stability of 67 formerly misclassified stable models, leaving only two misclassified model results in total. These results verified that even with variations in bolt spacing and deformation, as well as discontinuous beam displacement, that the $E_{rmn}$ method more accurately captured orthogonally jointed and bolted beam displacement than the baseline Diederichs & Kaiser (1999) analytical solution.
Figure 2.49: Comparison of the layer-adjusted $E_{rm}$ analytical method displacement and stable bolted model results. Note that the layer-adjusted $E_{rm}$ is used in conjunction with the thickness of the bolted interval. Note that results that plot on the y-axis (i.e. $x = 1.0(10)^{-5}$, circled in red) indicate that the analytical solution predicted an unstable result, but model results indicated stability had been maintained. A 1:1 trend line is shown in black, and the percent root mean square error for stable model results are provided; note the log-log axes.

Some variance remains unaccounted for by this simplified approach. To that end, multiple linear regressions considering various combinations of other model inputs identified to be potentially important through visual analysis (i.e. bolted thickness and bolts/m) were undertaken (Table 2.8).
Table 2.8: Coefficients, p-values, and R²-Adjusted for multiple linear regressions considering the effect of additional model inputs on the effective rockmass modulus.

<table>
<thead>
<tr>
<th>N Layers</th>
<th>Variable</th>
<th>Coefficient</th>
<th>p-value</th>
<th>R² Adjusted</th>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>( E_{mx} )</td>
<td>0.30</td>
<td>&lt; 1.0(10)^{-6}</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Bolted Thickness</td>
<td>-0.27</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>4</td>
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<td>0.08</td>
<td>&lt; 1.0(10)^{-6}</td>
<td>0.97</td>
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<td></td>
</tr>
<tr>
<td>8</td>
<td>( E_{mx} )</td>
<td>0.02</td>
<td>&lt; 1.0(10)^{-6}</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>Bolted Thickness</td>
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<td>0.140</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Regression 2a-c</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( E_{mx} )</td>
<td>0.30</td>
<td>&lt; 1.0(10)^{-6}</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Bolted Thickness</td>
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<td>0.005</td>
<td></td>
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<tr>
<td>4</td>
<td>( E_{mx} )</td>
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<td>&lt; 1.0(10)^{-6}</td>
<td>0.97</td>
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<tr>
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<td>Bolted Thickness</td>
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<tr>
<td>8</td>
<td>( E_{mx} )</td>
<td>0.02</td>
<td>&lt; 1.0(10)^{-6}</td>
<td>0.95</td>
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<tr>
<td></td>
<td>Bolted Thickness</td>
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<td>0.140</td>
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<tr>
<td></td>
<td>Regression 3a-c</td>
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<td></td>
</tr>
<tr>
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<td>&lt; 1.0(10)^{-6}</td>
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<tr>
<td></td>
<td>Bolts/m</td>
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<td>0.0007</td>
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<tr>
<td>4</td>
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<td>0.08</td>
<td>&lt; 1.0(10)^{-6}</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Bolts/m</td>
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<td>0.0010</td>
<td></td>
</tr>
<tr>
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<td>&lt; 1.0(10)^{-6}</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>Bolts/m</td>
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<td>&lt; 1.0(10)^{-6}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Regression 4a-c</td>
<td></td>
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<td></td>
</tr>
<tr>
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<td>&lt; 1.0(10)^{-6}</td>
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</tr>
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<td>Bolts/m</td>
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<td>0.0006</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( E_{mx} )</td>
<td>0.08</td>
<td>&lt; 1.0(10)^{-6}</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Bolted Thickness</td>
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</tr>
<tr>
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<td>Bolts/m</td>
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<tr>
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<td>( E_{mx} )</td>
<td>0.02</td>
<td>&lt; 1.0(10)^{-6}</td>
<td>0.96</td>
</tr>
<tr>
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<td>Bolted Thickness</td>
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<td>1.0(10)^{-6}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bolts/m</td>
<td>0.23</td>
<td>&lt; 1.0(10)^{-6}</td>
<td></td>
</tr>
</tbody>
</table>

Although some of the individual and combined additional considerations had statistically significant (i.e. p-value < 0.05) coefficients, they had no meaningful impact on the R²-adjusted value of the original two-dimensional regression presented in Figure 2.48. Consideration of the bolted thickness coefficient had statistical significance in two-layer models, but this did not hold true for increasing layers. This analysis indicates that the regression presented in Figure 2.48 sufficiently accounts for the statistically relevant complexities of the model cases tested herein, particularly regarding multi-layered and orthogonally jointed voussoir beam displacement.

2.6.4 Beam Thickness-Stress Analysis Results

The baseline (i.e. no layer-adjusted Erm) Diederichs & Kaiser (1999) analytical solution was tested against stable model midspan and abutment stress results considering both the entire beam thickness (as suggested by Diederichs & Kaiser (1999)), as well as the individual layer thickness,
with varying degrees of accuracy (Figure 2.50). This comparison showed similar trends as the
displacement comparison in Figure 2.45. Most notably, the baseline Diederichs & Kaiser (1999)
analytical solution with $T =$ single voussoir layer remained too conservative, overpredicting
maximum stresses and instability. Consideration of $T =$ bolted interval revealed distinct trends
based on both span and bolted thickness, rather than number of horizontal layers. Note that the
same 69 stable models were classified as unstable by the analytical solution when considering $T =$
layer thickness. Interestingly, the misclassified model results had a wide range of maximum
midspan and abutment stresses, but their displacements were restricted to the highest results (see
Figure 2.45). This was due to the significant impact that beam thickness had on compression arch
thickness and associated stress magnitude. Interestingly, 4 of the misclassified cases, all of which
were modeled with softer beams that had 0.5 m thick layers and 2.0 m spaced vertical joints, had
model results where the midspan stresses exceeded the abutment stresses at equilibrium. This
behavior diverges significantly from the baseline voussoir beams discussed in Section 2.3 and is
due to complex interaction between bolt spacing and beam stability that is outside the scope of this
thesis. Additionally, all the remaining misclassified model cases featured 20 m spans and either 4
or 8 bolted layers.
Figure 2.50: Comparison of the Diederichs & Kaiser (1999) analytical stress solution and model midspan (top) and abutment (bottom) horizontal stress results when considering the thickness input to be a single layer (left), or the entire bolted beam interval (right). Note that results that plot on the y-axis (i.e. $x = 0$, circled in red) indicate that the analytical solution predicted an unstable result, but model results indicated stability had been maintained. A 1:1 trend line is shown in black and a percent RMSE has been calculated for results excluding $x = 0$.

If the misclassified models were not considered, the baseline method maintained reasonable accuracy for predicting model stresses, with RMSE values of 48% and 29% for midspan and abutment stresses, respectively. These results also agreed with the baseline voussoir beam results.
where midspan stresses were overpredicted by the analytical solution, however, the abutment stresses were not significantly or consistently underpredicted. This was likely due to the fact that using the individual layer thickness overpredicted stresses, causing the analytical solution to match elevated abutment stresses. When using the bolted interval thickness, both midspan and abutment stresses were consistently and significantly underpredicted by the baseline Diederichs & Kaiser (1999) analytical solution. However, distinct trends based on layer thickness and bolted thickness were apparent in the results. In considering both analyses (single layer vs. bolted thickness), it appeared that the maximum horizontal stress could be related to some interaction between individual layer thickness and bolted thickness as governed by the properties of the beam material, discontinuities, and bolt elements.

These observations prompted another optimization using \textit{fminsearch} in MATLAB to identify the effective beam thickness in the analytical solution that best predicted the maximum midspan stress in a multi-layered bolted beam. However, this was complicated by the difference in midspan and abutment stress (i.e. which should be considered?) and the consideration of rockmass modulus (i.e. should \( E_{rmn} \) or \( E_{rmx} \) be used in back calculating an effective beam thickness, and which should be used when implementing an adjusted thickness method?). Recall that smaller-displacement cases tended to contain voussoir arching within individual layers, and that midspan crushing controls inelastic beam deformation. Therefore, the analysis minimized the difference between the model midspan and analytical stresses using \( E_{rmx} \), as the majority of models had developed largely independent voussoir arches in each layer of the composite beam. Subsequent statistical analysis of the relationship between model inputs and back calculated effective thickness identified that the individual beam thickness, \( E_{rmx} \), and number of layers were critical controls on the effective thickness required to most accurately account for model maximum midspan stresses using the analytical solution. When calculating the effective thickness ratio for use in the baseline Diederichs & Kaiser (1999) analytical solution, \( E_{rmx} \) and number of layers were considered simultaneously through the previously developed layer-adjusted \( E_{rm} (E_{rmn}) \) (Figure 2.51).
Figure 2.51: Results of thickness optimization with respect to minimized model stresses between model maximum midspan horizontal stress and the voussoir analytical solution. Note that the effective thickness is normalized to the individual layer thickness and the span is normalized to the vertical joint spacing. The layer-adjusted $E_{mn}$ is calculated using Eqn. (2.40) based on model inputs.

The largest influence on effective thickness was clearly a combination of the individual layer thickness, the rockmass modulus of the individual layer, and the number of individual layers. These were captured by normalizing the effective thickness to the layer thickness (i.e. z-axis) and using the previously verified $E_{mn}$ (i.e. y-axis). However, the influence of bolted thickness and bolt spacing for otherwise identical model cases should also be noted. All else being equal, thicker bolted beams had a slightly lower effective thickness ratio, meaning that the effective thickness decreased towards the individual layer thickness, particularly in stiffer beams. Higher bolt spacing (i.e. fewer bolts) had a similar effect, allowing for the bolted beam to behave more like a series of individual beams. The effect of bolt spacing was magnified for softer beam models where effective
thickeneses for identical model cases with various bolt spacings could vary by more than 100%. Span and vertical joint spacing also had some influence, but no consistent trend could be identified. Prior to consideration of these various influencing factors, a simplified two-dimensional non-linear regression considering only the primary influence on effective thickness ratio (i.e. layer-adjusted $E_{rm}$) was performed (Figure 2.52). This resulted in a 12% RMSE, an $R^2$-adjusted of 52%, a $p$-value $< 0.05$, and the following best-fit equation:

$$\frac{T_e}{T_i} = 0.19 \times \frac{1}{\sqrt{E_{rmn}}} + 1.05 \tag{2.41}$$

where $T_e$ = effective thickness, $T_i$ = individual layer thickness, and $E_{rmn}$ = layer-adjusted rockmass modulus (Eqn. 2.40) As the number of layers in the bolted beams decreases, or as the material stiffness, joint stiffness, and joint spacing increase (i.e. layer-adjusted $E_{rm}$ increases), the effective thickness ratio approaches 1.05.

![Figure 2.52: Two-dimensional non-linear regression of effective thickness ratio results and layer-adjusted $E_{rm}$ calculated using Eqn. (2.40).](image)

The $T_e$ values derived from Eqn. 2.41 were then utilized in the Diederichs & Kaiser (1999) analytical solution (using the baseline $E_{rmx}$) to evaluate the accuracy of the effective thickness method in predicting model midspan stresses (Figure 2.53). This method provided much higher accuracy than either method that directly utilized the baseline Diederichs & Kaiser (1999) solution,
as shown in Figure 2.50. The percent RMSE decreased by half and only 16 of the 69 misclassified models were still misclassified. The remaining error was restricted to models featuring 2 or 4 m bolted thicknesses and thin individual layers (i.e. 0.5 m thick). This was related to the combined influence of bolt spacing and bolted thickness, previously discussed discontinuous beam deformation, and select cases with inverted stresses (i.e. abutment stresses lower than midspan).

![Comparison of the effective thickness method with stable bolted model midspan stress results. Note that the effective thickness is used in conjunction with the baseline (i.e. single-layer) E_em. Note that results that plot on the y-axis (i.e. x = 0, circled in red) indicate that the effective thickness analytical solution predicted an unstable result, but model results indicated stability had been maintained. A 1:1 trend line is shown in black, and the percent root mean square error for stable model results are provided.](image)

Potential additional factors influencing effective thickness were quantitatively investigated through additional non-linear regressions, with three distinct trends noted based on bolt spacing (Figure 2.54). This analysis shows that the accuracy of the effective thickness method increases significantly when considering models with more bolts and decreases with increasing bolt spacing (i.e. fewer bolts), as indicated by the variation in $R^2$-adjusted values. This error highlights the limitations of considering bolted beams, namely that bolt density must be sufficient enough to promote increasing the effective thickness of the beam. Utilizing this method of performing separate regressions for each bolt spacing case resulted in slightly better agreement between the analytical method and the model results than only using Eqn. (2.41), even with the decrease in
accuracy associated with the high bolt spacing case. However, incorrectly classified models were still represented by all bolt spacings tested, indicating that consideration of additional model inputs may be required. To that end, additional non-linear regressions considering individual model inputs and various combinations were undertaken (Table 2.9).

Figure 2.54: Two-dimensional non-linear regression of effective thickness ratio results and layer-adjusted $E_{\text{rm}}$ calculated using Eqn. (2.40) separated by model bolt spacing.
Table 2.9: Coefficients, p-values, and $R^2$-Adjusted for multiple linear regressions considering the effect of additional model inputs on the effective thickness. $E_{rm} = \text{Layer-adjusted } E_m$.

<table>
<thead>
<tr>
<th>Bolt Spacing</th>
<th>Variable</th>
<th>Coefficient</th>
<th>p-value</th>
<th>$R^2$ Adjusted</th>
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<tr>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Regression 1 - Figure 2.52</td>
<td>1/sqrt($E_{rm}$)</td>
<td>0.19</td>
<td>$&lt; 1.0(10)^{-6}$</td>
<td>0.52</td>
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<tr>
<td></td>
<td>Intercept</td>
<td>1.06</td>
<td>$&lt; 1.0(10)^{-6}$</td>
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<td></td>
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<tr>
<td>Regression 3</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>1/sqrt($E_{rm}$)</td>
<td>0.19</td>
<td>$&lt; 1.0(10)^{-6}$</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>Bolts/m</td>
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<tr>
<td></td>
<td>Intercept</td>
<td>0.87</td>
<td>$&lt; 1.0(10)^{-6}$</td>
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<tr>
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<td></td>
<td>Intercept</td>
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<td>$&lt; 1.0(10)^{-6}$</td>
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<tr>
<td>Regression 5a-c</td>
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<td>0.20</td>
<td>$&lt; 1.0(10)^{-6}$</td>
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<tr>
<td></td>
<td>Intercept</td>
<td>0.95</td>
<td>$&lt; 1.0(10)^{-6}$</td>
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<td>Regression 6</td>
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<td></td>
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<tr>
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<td>1/sqrt($E_{rm}$)</td>
<td>0.20</td>
<td>$&lt; 1.0(10)^{-6}$</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>Bolts/m</td>
<td>0.31</td>
<td>$&lt; 1.0(10)^{-6}$</td>
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</tr>
<tr>
<td></td>
<td>Vertical Joint Spacing</td>
<td>0.10</td>
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<tr>
<td></td>
<td>Intercept</td>
<td>0.74</td>
<td>$&lt; 1.0(10)^{-6}$</td>
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</tr>
</tbody>
</table>

Based on the results of the non-linear regressions, there are two methods that stand out as potential improvements to the two regressions presented in Figure 2.52 and Figure 2.54. Either Regression 5a-5c, which separates datasets based on modeled bolt spacing and accounts for vertical joint spacing, or Regression 6, which accounts for all bolt density with a single coefficient and incorporates vertical joint spacing as well are the two most likely to improve predictions over the original regression (i.e. Regression 1). Both methods were tested in the Diederichs & Kaiser
analytical solution to compare the applied accuracy of each regression (Figure 2.55). Surprisingly, even though Regression 5a-c and Regression 6 had significantly higher R^2-adjusted values, they were not substantially more accurate at predicting maximum midspan stresses than the original regression that only considered the layer-adjusted E_{rm} as an independent variable. Furthermore, similar numbers of models were misclassified: 14 and 16 for Regression 5a-c and Regression 6, respectively. Although the added complexity in both regressions presented in Figure 2.55 was more accurate in estimating the “correct” effective thickness, the stresses calculated by those effective thicknesses were not significantly more accurate. Therefore, the original regression given as Eqn. (2.41) remains the method suggested for practical use for bolt spacings analyzed (i.e. 1.2 – 2.4 m).

Figure 2.55: Comparison of two different effective thickness methods with and stable bolted model midspan stress results. The regression utilized to calculate effective thickness in (a) has a single coefficient to account for the bolt density and is referred to as “Regression 6” in Table 2.9, while the regressions utilized to calculate the effective thickness used in (b) are referred to as “Regression 5a-c”. Note that the effective thickness is used in conjunction with the baseline (i.e. single-layer) E_{rm}. Note that results that plot on the y-axis (i.e. x = 0, circled in red) indicate that the effective thickness analytical solution predicted an unstable result, but model results indicated stability had been maintained. A 1:1 trend line is shown in black, and the percent root mean square error for stable model results are provided.

2.6.5 Adjusted Voussoir Beam Analog Discussion

The displacement of multiple horizontal layers tied together with passive rockbolt elements can be predicted in the existing voussoir analytical solution through adjustment of the E_{rm} using a
constant based on the number of horizontal layers. However, horizontal layers tend to segregate horizontal stress transfer through the roof in stiffer and less densely bolted beams. Note that no models contained bolts that were installed partially through a given layer, so the effect of bolt length under less idealized circumstances remains unknown. Model midspan stresses can be most accurately predicted by using an effective thickness based on the previously verified layer-adjusted $E_{rm}(E_{rmn})$, the individual layer thickness, and the bolt spacing. Both methods showed an improved accuracy over the baseline Diederichs & Kaiser (1999) analytical solution, due to their ability to account for significantly more complex mechanical behavior. In the event that the effective thickness and layer-adjusted $E_{rm}$ methods have conflicting results regarding the prediction of stability or instability, the layer-adjusted $E_{rm}$ method should be used, as it was more accurate in predicting model results.

2.7 Voussoir Beams in Single-Entry Models

Following the study of voussoir beam behavior under individual variations to boundary conditions, model geometry, and material properties, the voussoir mechanical response was then tested in multiple single-entry models. The first set of single-entry models featured unsupported roofs and analyzed the parametric impact of varying in-situ stress ratio, depth, and surcharge loading. The second set of single-entry models considered the effect of passive roof support coupled with the previously mentioned in-situ effects. Model results were compared to the adjusted voussoir analytical solutions described in the previous sections (i.e. “Total Stress Method”, Layer-Adjusted $E_{rm}$, and Effective Thickness) and the baseline Diederichs & Kaiser (1999) analytical solution.

2.7.1 Unsupported Single-Entry Voussoir Beam Methodology & Model Inputs

In order to confirm that the voussoir beam analog could be applied in future modelling efforts and verify the previous findings regarding complex voussoir analytical behavior, the immediate roof and overburden of an underground excavation was modeled as either a single or a series of contiguous elastic voussoir beams. Single-entry elastic voussoir models were developed using previously analyzed voussoir geometries and rockmass moduli in the immediate roof and overburden and the effects of multiple depths, and in-situ stress ratios ($k_o$-ratios) were considered. Surcharge loading (i.e. height of the multi-layered voussoir beam) was not explicitly considered but increasing the number of contiguous voussoir layers in the roof inevitably lead to some degree of surcharge loading as each voussoir layer deflected more than a non-voussoir equivalent elastic.
material. Furthermore, the impact of excavation-induced stresses (i.e. stresses that are redistributed to the roof and floor due to the presence of an excavation) at in-situ stress ratios greater than or equal to 1.0 are implicitly modeled, but not explicitly accounted for in the analytical solutions adjustments considered herein.

The 10 m, $E_{\text{rmx}} = 3.3$ GPa elastic voussoir beam from Section 2.3 was placed into a single-entry mine model with equivalent continuum elastic moduli assigned to the non-voussoir areas of the roof and floor. Half coal pillars were modeled using elastic properties to isolate the impact of yielding pillars from the impact of explicit DFN representation, as well as stress magnitudes and ratios in the subsurface. The models were tested at entry depths of 30 and 100 m below grade, in-situ stress ratios ranging from 0.25-2.0, and explicit DFN heights (i.e. number of voussoir layers) of 1, 2, 5, and 30 m. The general model setup is depicted in Figure 2.56 showing end-member examples of explicit voussoir DFN heights.
Models were run in the same four-stage solution method transitioning from very strong Mohr-Coulomb joints to realistic continuously yielding joints. However, once the modeled joints were changed to the continuously yielding joint model (Stage 3), a 15° initial and intrinsic friction angle was utilized for the intra-span joints to ensure that abutment sliding failure would occur at low confining stress. Zero-velocity boundary conditions at each half-pillar boundary imposed a horizontal symmetry condition, effectively modeling an infinite array of pillars in either direction and equal and opposite deformation on either side of the boundary condition. Model results were first considered in relationship to the baseline Diederichs & Kaiser (1999) analytical solution.
without accounting for surcharge loading. Then the “Total Stress Method” developed in Section 2.5 was considered in relationship to model midspan stress results and broad considerations regarding surcharge loading and \( \text{FoS}_{\text{sliding}} \) are discussed to illustrate the complexity of utilizing the voussoir beam analog in-situ.

### 2.7.2 Unsupported Elastic Voussoir Single-Entry Results

Model displacement and maximum midspan stress results of the immediate voussoir layer in unsupported entries are compared to analytical predictions and results of previous modeling efforts of this chapter in Figure 2.57. Note that no effective unit weight was utilized for any of the analytical solutions in order to make more direct comparisons between various model results and the baseline Diederichs & Kaiser (1999) analytical solution. However, the difference in joint frictional strength did impact direct comparisons between the model results, particularly at low confining stresses.

![Figure 2.57: Unsupported single-entry voussoir model](image)

Regardless, when comparing model displacement results of complex stress (i.e. in-situ beam) versus simplified stress (i.e. mobile and immobile abutment horizontal stress) conditions, it was clear that for continuous displacement (i.e. no abutment slip failure), the in-situ models behaved...
more similarly to the locked-in stress models from Section 2.5.3 than the mobile abutment models from Section 2.5.2. Interestingly, the opposite was true for the results of maximum midspan stresses, in that the in-situ model results were more similar to the mobile abutment models. This was a result of excavation-induced stress redistribution increasing the magnitude of stress transfer and the resulting midspan stress, but not altering the mobility of the abutment, and therefore the displacement of the beam remained similar to the locked-in stress model results.

Most notably, the model results from Section 2.5 were not subject to any in-situ vertical stress, whereas the 1-layer models in this section were theoretically free from surcharge loading (i.e. deflecting back) but not the impacts of in-situ vertical stress. This allowed for consideration of mechanical behavior under in-situ stress conditions without the effect of surcharge pressure. With increasing vertical layers (i.e. 2, 5, and 30), the effects of surcharge loading and vertical stress could be considered simultaneously.

In the 30 m and 100 m entry cases, under high horizontal stress it was clear that increasing surcharge pressure (i.e. additional voussoir layers) had a consistent impact on beam displacement. At low confinement, the effect of surcharge loading became non-linear as abutment slip failure initiated and the roof began to deform discontinuously. Interestingly, the 1, 2 and 5-layer models for both 30 m and 100 m entry cases, converged at similar displacement values for an in-situ stress ratio of $k_o = 0.5$. This indicated that under no to low surcharge pressure, coupled with low horizontal stress, the voussoir beam displacement was not significantly impacted by changing depth (i.e. in-situ vertical stress). Note that due to the increased depth the in-situ horizontal stress magnitude (shown in Figure 2.57) for the 100 m deep entries was higher at the same in-situ stress ratio.

Explicit voussoir DFN height and depth to entry (i.e. surcharge loading, vertical stress) had consistent impacts on maximum model displacement. It was evident in the cases of stable (i.e. continuous) roof deflection, increased depth (i.e. higher vertical stress) and voussoir layers (i.e. higher surcharge loading) increased roof displacement, while higher horizontal stress decreased displacement to a limiting value. Similar behaviors were noted for the displacement of 2, 5, and 30-layer models. In the unsupported cases tested, the traditional voussoir mechanical response with respect to roof displacement was impeded by the presence of high confinement and exacerbated by vertical stress and surcharge loading.
Both the 30 m and 100 m deep, 30-layer voussoir models tested at in-situ stress ratios of 0.5 and 0.75 experienced initiation of abutment slip failure where horizontal stress arching in the roof began to be overcome by surcharge pressure incurred by the presence of contiguous voussoir beams, as shown with exaggerated displacement magnitude in Figure 2.58.

![Figure 2.58: 30 m deep, 0.5 k, single-entry voussoir DFN model at equilibrium shown with 10 times block magnification and sliding along discontinuities near the abutments.](image)

Recall that the abutment joints were modeled with a high (i.e. 89°) initial and intrinsic friction angle to maintain minimal shear displacement at the abutment-beam boundary contact. This was highlighted by the model results in Figure 2.58 where the sliding occurred at the contacts modeled with realistic initial and intrinsic friction angles. After adjusting the span from 10 m to 8 m to account for the high-friction abutment joints, the baseline Diederichs & Kaiser (1999) analytical solution calculated a $\text{FoS}_{\text{sliding}} = 1.7$ using Eqn. (2.29) for all the model cases tested, yet sliding initiated only in the 30-layer voussoir models with low vertical and horizontal stresses. This confirmed that in shallow, narrow span entries with sufficiently thick bedding planes and low k-ratios, abutment slip failure was more likely than buckling or crushing failure, and that likelihood decreased with increasing vertical stress or decreasing surcharge load. However, increasing vertical stresses appeared to lead to less impacts of surcharge due to the associated increase in horizontal stress. The onset of sliding and associated increase in measured horizontal stress was associated with a decrease in compression arch thickness, increasing the stresses measured at the
midspan as shown in Figure 2.57(b). This increase in measured stress indicated that the initiation of sliding failure did not necessarily mean that the roof would fail, stress concentrations had stopped the sliding and the models converged; this was reflected in the $\text{FoS}_{\text{sliding}} > 1.0$.

Due to the complex combined impact that vertical stress, surcharge loading, and horizontal stress had on the thickness of the compression arch, the limited number of parametric cases, and the limited impact on displacement results for the 10 m span cases tested, an optimized effective unit weight could not be verified in the same manner as the effective $E_m$ or effective thickness from Section 2.6.

Nevertheless, voussoir mechanical behavior in unsupported single-entry configurations matched both results from both mobile and immobile abutments, for maximum midspan stress and displacement, respectively. Vertical deflection approached a minimum value as horizontal stress increased, but midspan stresses increased linearly following closure of the abutment joint. However, the models featured in Section 2.5 did not account for any surcharge loading or other vertical stress influence, which had a significant effect on model displacement, but not model midspan horizontal stress, as indicated by the difference in 30 m and 100 m depth results with the same magnitude of in-situ horizontal stress. The “Total Stress Method” adequately accounted for the increase in measured midspan stress due to horizontal stress for stable (i.e. continuous) deformation in unsupported cases.

2.7.3 Bolted Single Entry Voussoir Beam Methodology

Following consideration of the effects of in-situ loading conditions on unsupported single-entry voussoir beam models, a subset of the unsupported configurations analyzed in Section 2.7.2 were analyzed in bolted conditions using the rockbolt element in UDEC with material properties from Bahrani & Hadjigeorgiou (2017). The same four-stage solution methodology was utilized, transitioning from Mohr-Coulomb joints to continuously yielding, but bolts were installed in Stage 1 after in-situ stresses were decreased to 70% along the excavation boundary and solved to equilibrium to mimic elastic stress relaxation prior to bolt installation following the findings of Vlachopoulos & Diederichs (2009). Following bolt installation, the stress boundary was removed, and the joint constitutive model and properties were changed in accordance with the previously described four-stage method. The supported model configurations for all model depth entries are shown in Figure 2.59.
Figure 2.59: Three different supported single entry elastic voussoir beam models configurations showing the number of voussoir layers, each layer thickness, and the length and spacing of the passive bolt elements and faceplates. These configurations were run at entry depths of 30 and 100m, simulated via initial stress condition, as well as under horizontal stress ratios ranging from 0.5-2.0. Model bottoms and boundary conditions excluded for clarity.

The supported voussoir roofs featured passive rockbolt elements that were 2.0 m long. Longer bolts would impact the 2-layer model results due to the suspension effect that would be caused by longer bolts extending into an unjointed (i.e. non-voussoir) portion of the overburden. Model results were compared to the baseline Diederichs & Kaiser (1999) analytical solution. These trends were then combined with the previously verified adjusted methods developed herein. Namely, model displacements were compared with the layer-adjusted $E_{rm}$ ($E_{rmn}$) method developed and verified in Section 2.6, while the model midspan stresses were compared to the effective thickness adjustment from Section 2.6 and the “Total Stress Method” developed in Section 2.5 and confirmed in unsupported single entry models in Section 2.7.2.

2.7.4 Bolted Single Entry Voussoir Beam Results

The results of bolted, single-entry elastic voussoir models were compared to the baseline Diederichs & Kaiser (1999) analytical solution as well as the layer-adjusted $E_{rm}$ ($E_{rmn}$) method developed in Section 2.6. As previously discussed in the unsupported cases, the explicit effects of vertical stress and contiguous voussoir layers were not accounted for in the adjusted analytical solution to focus on the comparison of baseline and adjusted methods (Figure 2.60).
Figure 2.60: Single-entry bolted model displacement results under various in-situ stress (i.e. 30 m and 100 m deep, $k_o=0.5 – 2.0$) and surcharge loading conditions (i.e. 2, 5, and 30 contiguous voussoir layers). Results for different entry depths and levels of horizontal stress compared to the baseline Diederichs & Kaiser (1999) analytical solution and the layer-adjusted $E_{rm}$ method for displacement from Section 2.6.

The combined effects of in-situ stress and surcharge loading on bolted single-entry models were similar to their unsupported counterparts. In some 30 m deep, 2-layer cases with moderate in-situ stress ratios (i.e. 1.0 - 1.5), the bolted models behaved more similarly to their 5-layer counterparts, indicating that at low vertical stresses the interplay between in-situ stresses and bolt elements is influencing the previously described trends from unsupported cases. As expected, at low levels of horizontal stress, the onset of abutment slip failure occurred, and roof displacement became discontinuous.

Due to the stable nature of the selected roof properties and voussoir geometries tested, the differences in bolted and unsupported model displacement results were small, although the layer-adjusted $E_{rm}$ method did increase displacement prediction accuracy for the cases where the voussoir beam analog is traditionally applied (i.e. shallow, low horizontal stress excavations).

Considering midspan stress results, the maximum horizontal stress was extracted from the bolted interval at the beam midspan and compared to baseline Diederichs & Kaiser (1999) analytical
solution, the “Total Stress Method” from Section 2.5, and the effective thickness method from Section 2.6.4 combined with the “Total Stress Method” (Figure 2.61).

![Graph showing stress comparison](image)

Figure 2.61: Single entry bolted model maximum midspan stress of the bolted interval for different entry depths (30 m and 100 m) and levels of horizontal stress ($k_o = 0.5-2.0$) compared to the baseline Diederichs & Kaiser (1999) analytical solution for $T =$ single layer (blue), the “Total Stress Method” from Section 2.5 (red), and the effective thickness method from Section 2.6, combined with the TSM (orange).

The effective thickness method combined with the TSM provided the most accurate predictions of measured midspan horizontal stress from model results. This is because the effective thickness method accounts for the decrease in stress due to multiple bolted layers, and the in-situ horizontal stress is added to that value.

The results of the analysis of various surcharge loading scenarios on bolted single-entry elastic voussoir beams showed that application of both proposed analytical adjustments to the voussoir analytical solution improve the accuracy of predictions of maximum displacement and maximum midspan horizontal stress. Bolted single-entry model results further confirmed that in-situ roof conditions and mechanical behavior can be generally represented by the voussoir beam analog.

### 2.8 Guidelines for Application of the Adjusted Analytical Method

A step-by-step guide is provided in this section to present how the adjusted analytical methods explored and developed in this chapter should be implemented in practice. Note that the limitations
of these adjustments are largely the same as those of the baseline Diederichs & Kaiser (1999) analytical method. The adjusted methods should only be applied to flat-roof excavations in homogeneous, well-jointed rockmasses (i.e. RMR > 50) under low-confinement conditions. Furthermore, the adjusted methods developed in this thesis have not been verified for dipping excavations, and the impact of bolt spacing has not been explicitly considered but the effective thickness method presented is valid for bolt spacings of 1.2-2.4 m.

2.8.1 Maximum Displacement Determination

In order to determine maximum midspan displacement, the required analytical inputs include beam span, bolted thickness, layer thickness, specific weight, Young’s Modulus, joint normal stiffness, and horizontal (i.e. bedding) joint spacing. Account for anticipated surcharge pressure (i.e. groundwater or weak back) or support pressure from suspension elements (i.e. cable bolts) by adjusting the beam specific weight using the appropriate formulae from Diederichs & Kaiser (1999). In fact, the only adjustments to the baseline Diederichs & Kaiser (1999) analytical solution required are to the beam thickness and the rockmass modulus.

The beam thickness should be set equal to the bolted interval and the rockmass modulus in the horizontal direction should be calculated as:

\[ \frac{1}{E_{rcx}} = \frac{1}{E} + \frac{1}{(jkn)s_j} \]  

(2.42)

The effective rockmass modulus of the beam should be calculated based on the number of layers (n) in the bolted interval as:

\[ E_{rms} = Cx * E_{rcx} \]  

(2.43)

where:

\[ Cx = \frac{1.1}{n^2} \]  

(2.44)

The iterative solution loop shall then be run as it is in the baseline Diederichs & Kaiser (1999) analytical solution. However, the only valid outputs from this analysis will be the displacement and the BL.
2.8.2 Maximum Stress Determination

In order to determine the maximum stress the analytical inputs required are identical to those in the previous section. The single-layer rockmass modulus (i.e. $E_{rmx}$) (Eqn. 2.42) shall be utilized as the beam stiffness and the effective thickness shall be calculated as:

$$T_e = (0.19 * \frac{1}{\sqrt{E_{rmn}}} + 1.05) * T_i$$

(2.45)

where $T_i$ = individual layer thickness and $E_{rmn}$ = layer-adjusted rockmass modulus in GPa.

The iterative solution loop shall then be run as it is in the baseline Diederichs & Kaiser (1999) analytical solution. The magnitude of in-situ horizontal stress ($\sigma_{xx}$) shall be calculated as:

$$\sigma_{xx} = g * \gamma * D * k_o$$

(2.46)

where $g$ = gravitational constant, $\gamma$ = specific weight, $D$ = depth to entry, $k_o$ = in-situ horizontal stress ratio, and added to the maximum horizontal stress calculated by the iterative solution loop to determine the total horizontal stress ($\sigma_{TSM}$) as:

$$\sigma_{TSM} = \sigma_{max} + \sigma_{xx}$$

(2.47)

This horizontal stress ($\sigma_{TSM}$) can then be utilized to calculate a FoS\textsubscript{crushing} as:

$$\text{FoS}_{\text{crushing}} = \frac{\text{UCS}^*}{\sigma_{TSM}} * B$$

(2.48)

where UCS\textsuperscript{*} = field scale unconfined compressive strength (e.g. in the absence of other information, adjusted from lab scale to field scale values with a 0.5 multiplier), and $B = 0.6–1.25$ multiplier depending on the post-peak behavior of the material (i.e. elastic-brittle-plastic = 0.6, perfectly plastic = 1.25). Lastly, FoS\textsubscript{sliding} can be calculated as:

$$\text{FoS}_{\text{sliding}} = \frac{\sigma_{TSMN}}{\gamma * S} \tan(\phi)$$

(2.49)

where $N$ = compression arch thickness and $\phi$ = friction angle of the discontinuity. Note that if $\sigma_{xx}$ is significantly higher than the beam $\sigma_{max}$, and adjacent excavations are unlikely to reduce horizontal stresses, voussoir arching is unlikely to occur.
2.9 Bondi Pumping Chamber Case Study

In order to verify the applicability of the adjusted voussoir beam analog developed in this chapter, a suitable case study was identified and selected for numerical and analytical analyses. This case study is known as the Bondi Pumping Chamber and has many features that make it a viable natural laboratory to test the adjusted voussoir beam analytical method. In particular, the adjustment for the presence of multiple horizontal layers and roof support. Specifically, the excavation was advanced through horizontally bedded and cross-jointed sedimentary rock with a flat-roof supported by 24 mm diameter, fully grouted roof bolts on 1.2 m spacing. Additionally, the shallow nature of the excavation and its close proximity (i.e. ~ 30 m) to an unconfined escarpment make the loading conditions as close to the suite of bolted voussoir beam models analyzed in this chapter as naturally possible. Recall that the models utilized to develop the adjusted voussoir beam analytical method in Section 2.6 utilized beam abutments with no stored or applied horizontal stresses. The development of the unconfined cliff face through uplift and erosion has dissipated any stored horizontal stress in the local rockmass (Pells, 1994).

Previously, engineers who developed the excavation design utilized FEM numerical models and the voussoir beam analog with a significantly reduced Young’s Modulus input and overestimated surcharge loading for a “conservative approach” (Pells, 1994). With the development of the adjusted voussoir analog in this thesis, a more mechanically accurate approach can be applied to the Bondi Pumping Chamber. Furthermore, explicit DEM numerical models of the site were utilized to approximate joint spacing based on DFN generation and observe the roof response to the range of possible rockmass conditions (i.e. thickness of bolted layers, discontinuity strength, etc.) noted in the literature.

The high strength of the rockmass relative to the in-situ stress magnitudes, coupled with the negligible horizontal stresses allows for independent evaluation of the in-situ impacts of discontinuities and support (i.e. the focus of the adjusted analytical method presented in Section 2.6). Successful application of the adjusted voussoir beam analog under low-confinement to the Hawkesbury Sandstone is the first step in determining how realistic in-situ vertical stress impacts bolted beam behavior, as well as how to apply the adjusted method to more complex stress regimes seen in underground mines and civil excavations.
2.9.1 Site Location & Geologic Setting

The Bondi Pumping Chamber is a 17 m deep, 70 m long, 19 m high, 12.5 m span flat-roof excavation completed in 1989 in Sydney, New South Wales, Australia (Pells & Best, 1991). It is primarily utilized in support of water treatment and ocean sewer outfall operations at the Bondi Sewage Treatment Plant (STP) (McQueen, 2004; Nye et al., 2005). The Bondi STP is one of three major ocean outfalls that pump treated sewage from the City of Sydney out into the Pacific Ocean (Figure 2.62). The Bondi STP is located between Military Road and the edge of the coastal cliffs along the Pacific Ocean, north of the Bondi Golf and Diggers Club. The general site plan of the Bondi STP and the proposed location of the pumping chamber prior to its construction are also shown in Figure 2.62. An isometric view of the Bondi STP from the northwest showing the location of the pumping chamber relative to other underground works is depicted in Figure 2.63.

![Figure 2.62: Location of the North Head, Bondi, and Malabar ocean outfall tunnels (left) (modified from Nye et al., 2005). Site plan of the Bondi STP indicating proposed pumping chamber location prior to construction (right) (modified from Clancy, 1984).](image)
A generalized cross section of the outfall tunnel at Bondi STP and associated pumping chamber is shown in Figure 2.64.
The pumping chamber was excavated in the Middle Triassic-age Hawkesbury Sandstone, which is a medium to coarse-grained quartz sandstone with minor shale and mudstone lenses (Pells, 2004). The formation is subdivided into three facies with the “sheet” and “massive” sandstone facies comprising 95% of the formation, and the mudstone only 5% (Pells, 2004). Cross-bedding in the sheet facies indicates that the Hawkesbury Sandstone was deposited in a fluvial delta system. This type of depositional environment is very common in coal-measure rock, and in fact, while the Hawkesbury Sandstone is not coal-bearing, it conformably overlies the Narrabeen Group coal-measures of the Sydney-Gunnedah Basin and represents continued deposition of a coal-bearing sedimentary sequence (Hutton, 2009). Furthermore, the presence of multiple intra-unit depositional sequences (i.e. bedding) and subsequent impact of diagenesis and tectonic forces make the Hawkesbury Sandstone an excellent example of a laminated and discontinuous rockmass.

An image of typical Hawkesbury Sandstone outcrop from the North Head outfall tunnel project is given in Figure 2.65, and a geologic map of the Sydney region is provided in Figure 2.66.
Figure 2.65: Images of typical Hawkesbury Sandstone massive facies from the North Head outfall project north of the Bondi Pumping Chamber (see Figure 2.62) (from P. Pells, personal communication 03/23/21).

Figure 2.66: Map depicting general site location and geologic formations of the Bondi STP (shapefile from data.gov.au, 2021).
2.9.2 Field Data

The material properties, roof displacement data, and construction sequence used to develop and assess both the numerical model and the adjusted analytical solution were based on the available site-specific (Henderson & Windsor, 1988; Pells, 1991; Pells & Best, 1991; Nye et al., 2005) and regional (Bertuzzi & Pells, 2002) literature, as well as personal communication with the engineer that designed the original excavation using the voussoir beam analog (P. Pells, personal communication 03/23/21, 03/28/21, 04/03/21). The generalized excavation shape, support design, and surrounding geology are shown in Figure 2.67.

Figure 2.67: Generalized dimensions, support, and local geology of the Bondi Pumping Chamber (from Pells, 1994). Figure 2.67, coupled with descriptions in Pells & Best (1991) allowed for a construction sequence, excavation geometries, and roof geometries for numerical modeling to be established. First, a 4 m
wide central heading was excavated by drill-and-blast method through the “Unit 1 Lower Laminate”, approximately 3 bolts were installed on 1.2 m spacing, and roof extensometers and survey monuments installed to begin displacement monitoring. Second, the right and left headings were excavated, with the roof being bolted on 1.2 m spacing during excavation advance. After excavation advance, the initial 50 mm of shotcrete was applied to the entire excavation, and angled bolts were installed through the initial shotcrete in the right excavation corner to support the weaker laminate layer. Last, a final liner of 75 – 100 mm thick shotcrete was applied, and the remainder of the chamber benched down via drill-and-blast to the final height of 19 m. This sequence was confirmed by engineers who helped design the excavation, however the exact timing of support installation (i.e. minutes, hours, or days) remains unknown (P. Pells, personal communication 04/03/21). Furthermore, conflicting evidence in the literature indicates that the bolts installed in the roof may have been passive (Pells & Best, 1991; Pells, 1994) or pretensioned to 60 kN (Henderson & Windsor, 1988; Pells et al., 2018). Contact with the design engineer was unable to resolve this discrepancy. Accordingly, numerical models were utilized to confirm that the use passive or pretensioned bolts would result in negligible differences in excavation response under such low-stress and high-strength rockmass conditions (Appendix A).

Deflection measurements were collected by roof extensometers and surface survey points to account for the total displacement due to the shallow nature of the excavation. Following full advancement of the central heading and installation of the roof extensometers, the surveys found that between 4 and 10 mm of roof sag occurred at the midspan as the excavation was widened from 4 m to 12.5 m and benched down to its final height of 19 m (Pells & Best, 1991) (Figure 2.68).
The variability in the measured roof sag is likely due to variation in the thickness of the main sandstone roof (i.e. “Unit 3 Sandstone” in Figure 2.67), possible variation in support installation timing, and local geologic heterogeneity. Numerical models were used to consider and constrain the effects of support installation timing on ultimate roof deflection.

UCS testing on the sandstone from the Bondi Pumping Chamber excavation resulted in a mean UCS of 30.7 MPa and dry tangent Young’s modulus of 13.8 GPa (Pells, 2004). Site-specific geotechnical testing data for the Upper and Lower Laminate were not available, but data from other sites indicate that laminated shales in the Hawkesbury Sandstone formation have UCS values ranging from 1 to 40 MPa, intact rock moduli between 5 and 15 GPa, and rockmass moduli between 0.5 and 2.5 GPa, depending on the class of shale (Bertuzzi & Pells, 2002). Both sandstone and laminate rock types are reported as having a unit weight of 24.0 kN/m³ (Pells, 2004). The relevant Hawkesbury formation shale (i.e. laminate) engineering properties are summarized in Table 2.10.
Table 2.10: Relevant intact (i.e. lab-scale) and rockmass engineering parameters for laminated shales in the regional laminated shale formations.

<table>
<thead>
<tr>
<th>Shale Class</th>
<th>Intact UCS (MPa)</th>
<th>Intact E (GPa)</th>
<th>Rockmass E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I/II</td>
<td>7-40</td>
<td>7-15</td>
<td>0.7-2.5</td>
</tr>
<tr>
<td>III</td>
<td>2-15</td>
<td>5-10</td>
<td>0.2-1.2</td>
</tr>
<tr>
<td>IV/V</td>
<td>1-2</td>
<td>---</td>
<td>0.05-0.5</td>
</tr>
</tbody>
</table>

Additionally, the thickness of the “Unit 5 Upper Laminate” is uncertain, but a 1983 site investigation report indicated that the shale occurs in beds up to 6 m thick (P. Pells, personal communication 03/23/21). The shale class of the laminate units in the Bondi Pumping Chamber are not explicitly identified in the available literature so a rockmass modulus of 1.0 GPa was assumed to reasonably approximate shale classes I-III.

Regarding discontinuity properties, Pells (1994) stated that there were two horizontal bedding planes in the sandstone roof. Furthermore, Pells & Best (1991) stated that the sandstone bedding plane spacing was 1 to 1.5 m. This correlates well with the upper and lower bounds of the total sandstone unit thickness (3.0 – 5.2 m thick) (see Figure 2.67). Regarding the strength and stiffness of those bedding planes, no site-specific data were available. However, Bertuzzi & Pells (2002) provided generalized Hawkesbury Sandstone rockmass parameter estimates for both bedding and joint strength and stiffness, which are shown in Table 2.11.

Table 2.11: Relevant discontinuity strength and stiffness parameters in the regional sandstone and laminated shale deposits.

<table>
<thead>
<tr>
<th>Description</th>
<th>Thickness (mm)</th>
<th>Friction Angle (°)</th>
<th>Normal Stiffness (GPa/m)</th>
<th>Shear Stiffness (GPa/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bedding Plane</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tight</td>
<td></td>
<td>35-45</td>
<td>4000</td>
<td>400</td>
</tr>
<tr>
<td>1-5</td>
<td></td>
<td>30-35</td>
<td>200</td>
<td>20</td>
</tr>
<tr>
<td>5-10</td>
<td></td>
<td>20-25</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Joint</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tight</td>
<td></td>
<td>35-40</td>
<td>4000</td>
<td>400</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>22-28</td>
<td>1500</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>18-22</td>
<td>500</td>
<td>50</td>
</tr>
</tbody>
</table>

Site-specific joint-set geometric data was provided from a 1983 site investigation report for the Bondi Pumping Chamber (P. Pells, personal communication 03/23/21), which identified three major joint sets (Table 2.12).
Table 2.12: Discontinuity orientation, persistence, spacing, and conditions noted during site investigation activities at the Bondi STP.

<table>
<thead>
<tr>
<th>Bondi Joint Set</th>
<th>Dip (°)</th>
<th>Strike (°)</th>
<th>Length (m)</th>
<th>Spacing (m)</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bedding</td>
<td>0-5 W</td>
<td>0</td>
<td>Continuous</td>
<td>0.3-5.0</td>
<td>Planar, clean, occasional clay seams</td>
</tr>
<tr>
<td>Set 2</td>
<td>90 ± 15</td>
<td>25 ± 10</td>
<td>1-5</td>
<td>1-3</td>
<td>Planar, rough, clean, fresh to slightly weathered wall rock</td>
</tr>
<tr>
<td>Set 3</td>
<td>90 ± 15</td>
<td>110 ± 10</td>
<td>1-5</td>
<td>&gt;3.0</td>
<td>Planar, rough, clean, fresh to slightly weathered wall rock</td>
</tr>
<tr>
<td>Set 4a</td>
<td>90 ± 10</td>
<td>150 ± 10</td>
<td>1-5</td>
<td>&gt;3.0</td>
<td>Planar, rough, clean, fresh to slightly weathered wall rock</td>
</tr>
<tr>
<td>Set 4b</td>
<td>90 ± 10</td>
<td>50 ± 10</td>
<td>1-5</td>
<td>&gt;3.0</td>
<td>Planar, rough, clean, fresh to slightly weathered wall rock</td>
</tr>
</tbody>
</table>

These joint set orientations relative to the alignment of the excavation and their apparent dip in relation to the tunnel cross-section are shown in Figure 2.69.

Finally, Enever (1999) notes that the regional in-situ horizontal stress ranges from 1 to 2 (Enever, 1999). However, the Bondi Pumping Chamber’s 30 m proximity to an unconfined cliff face is assumed to result in locally minimal horizontal stress, such that the voussoir beam analog applies without the need for stress adjustments. This assumption was evaluated as valid based on preliminary numerical model results that estimated horizontal stresses in the roof to be below 0.1 MPa.

2.9.3 Methodology & Model Inputs

An example UDEC model geometry is depicted in Figure 2.70, showing boundary conditions, as well as generalized model lithology and stochastically generated roof discrete fracture network.
(DFN) based on the projection of the four joint sets identified in the 1983 site investigation report into the plane of the model.

The models utilized a mesh size identical to that from the previous voussoir beam models in this chapter. A graded mesh was utilized for the remainder of the model since minimal deformation was expected away from the jointed and bolted immediate roof (Figure 2.71).
Figure 2.71: Graded mesh detail example for Bondi Pumping Chamber numerical models. Note that elastic vertical joints are used to create horizontal variation in mesh densities.

A DFN based on the set of cross-joint strikes identified in Table 2.12, was generated in the same region of the roof above the excavation and approximately 5 m past the horizontal bounds (Table 2.13).

Table 2.13: DFN geometry parameter mean values and standard deviations (sd) used in the Bondi Pumping Chamber model. Note that the minimum apparent dip shown in Figure 2.69 was subtracted from the mean dip (90°) and divided by 3 to get the standard deviation for the DFN theta input.

<table>
<thead>
<tr>
<th>Bondi Joint Set</th>
<th>Theta [sd] (°)</th>
<th>Spacing [sd] (m)</th>
<th>Persistence [sd] (m)</th>
<th>Gap [sd] (m)</th>
<th>Random Seed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 2</td>
<td>90 [6.0]</td>
<td>1.0 [0.0]</td>
<td>3.0 [0.5]</td>
<td>0.0 [0.0]</td>
<td>4001</td>
</tr>
<tr>
<td>Set 3</td>
<td>90 [19]</td>
<td>6.0 [1.0]</td>
<td>3.0 [0.5]</td>
<td>0.0 [0.0]</td>
<td>5001</td>
</tr>
<tr>
<td>Set 4a</td>
<td>90 [4.3]</td>
<td>4.0 [0.3]</td>
<td>3.0 [0.5]</td>
<td>0.0 [0.0]</td>
<td>600</td>
</tr>
<tr>
<td>Set 4b</td>
<td>90 [6.3]</td>
<td>4.0 [0.3]</td>
<td>3.0 [0.5]</td>
<td>0.0 [0.0]</td>
<td>700</td>
</tr>
</tbody>
</table>

Due to the extremely shallow and low-confinement nature of the excavation, the potential for intact block material yield was deemed to be insignificant, so elastic block models were utilized to study the bolted roof response. Note that the DFN was restricted to the sandstone areas directly above the excavation, and the remaining sandstone was modeled as an elastic equivalent continuum with
a rockmass modulus calculated using Eqn. (2.25) from intact sandstone elastic modulus, joint stiffness, and the mean spacing of the model generated DFNs. The mean model joint spacing was taken by calculating the average of joint spacing in each roof layer (i.e. span divided by number of cross-joints). Other model inputs associated with low uncertainty or low potential to impact the model results were not varied in the numerical modeling portion of the case study (Table 2.14).

Table 2.14: Input parameters that remained constant between Bondi Pumping Chamber models.

<table>
<thead>
<tr>
<th>Static Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry/Loading</td>
<td></td>
</tr>
<tr>
<td>Depth to Entry (m)</td>
<td>17</td>
</tr>
<tr>
<td>Excavation Bench Height (m)</td>
<td>2.5</td>
</tr>
<tr>
<td>Span (m)</td>
<td>12.5</td>
</tr>
<tr>
<td>In-Situ Stress Ratio</td>
<td>2.0</td>
</tr>
<tr>
<td>No. Sandstone Layers</td>
<td>3.0</td>
</tr>
<tr>
<td>Bolt Spacing (m)</td>
<td>1.2</td>
</tr>
<tr>
<td>Bolt Properties</td>
<td>See Table 2.7</td>
</tr>
<tr>
<td>Bolt Length (m)</td>
<td>3.9</td>
</tr>
<tr>
<td>Mean Joint Spacing</td>
<td>0.8 m</td>
</tr>
<tr>
<td>Block Material Properties</td>
<td></td>
</tr>
<tr>
<td>DFN Block Sandstone K (GPa)</td>
<td>7.7</td>
</tr>
<tr>
<td>DFN Block Sandstone G (GPa)</td>
<td>5.8</td>
</tr>
<tr>
<td>Laminate K (GPa)</td>
<td>0.56</td>
</tr>
<tr>
<td>Laminate G (GPa)</td>
<td>0.42</td>
</tr>
<tr>
<td>Discontinuity Material Properties</td>
<td></td>
</tr>
<tr>
<td>Bedding Roughness (mm)</td>
<td>1.0</td>
</tr>
<tr>
<td>Joint Roughness (mm)</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The mixture of site-specific and more generalized formation-level engineering data required that critical inputs were tested over a range of possible values. Specifically, multiple values were tested for the reported thickness of the sandstone roof, joint distribution, and joint strength and stiffness within the sandstone roof (Table 2.15).

Table 2.15: Input parameters that were varied between Bondi Pumping Chamber models; note that variables in rows highlighted with the same color were varied concurrently.

<table>
<thead>
<tr>
<th>Variable Parameters</th>
<th>Values Tested</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
</tr>
<tr>
<td>Geometry/Loading</td>
<td></td>
</tr>
<tr>
<td>Sandstone Roof Thickness (m)</td>
<td>3</td>
</tr>
<tr>
<td>Discontinuity Material Properties</td>
<td></td>
</tr>
<tr>
<td>Bedding jkn/jks (GPa/m)</td>
<td>200/20</td>
</tr>
<tr>
<td>Bedding Initial/Intrinsic Friction Angle (°)</td>
<td>35/30</td>
</tr>
<tr>
<td>Joint jkn/jks (GPa/m)</td>
<td>1500/150</td>
</tr>
<tr>
<td>Joint Initial/Intrinsic Friction Angle (°)</td>
<td>28/25</td>
</tr>
</tbody>
</table>
This resulted in four main Bondi models: two sandstone roof thickness values each being tested with both minimum and maximum discontinuity strengths and stiffnesses. Two additional cases were tested that considered the impact of hypothetical reduced bolt penetration in thin-roof (i.e. 3 m thick) models. These six models are presented in Section 2.9.4, however additional models that varied the type and timing of support installation were also considered to constrain the possible roof behavior in absence of robust site-specific data (Appendix A). These model cases were used as a point of comparison because the adjusted voussoir beam analog developed in Section 2.6 does not account for the effects of partial bolt penetration into another layer, pretensioned rockbolts, or shotcrete.

The modeled cliff face at the right boundary was left as a free surface and initial model stresses were set based on a $k_o = 2.0$; these stresses decayed significantly during stress initialization due to the unconfined right boundary. This simulated the release of regional elevated horizontal stress as the escarpment formed. A zero-velocity boundary condition was applied in the normal direction to the left side of the model, and a pinned (i.e. both x- and y-velocity restricted) boundary condition was used for the model bottom. The pinned boundary condition utilized at the model bottom was considered more realistic than implementing a roller boundary (i.e. only y-velocity restricted) due to the unconfined nature of the escarpment. The roller boundary effectively places a continuous and frictionless joint, unconstrained at the cliff face, between the explicitly modeled rock at the bottom of the model and the in-situ rock that is not represented in the model (i.e. the other side of the boundary). Preliminary model runs using a roller boundary condition at the bottom of the model resulted in unrealistic tensile stresses forming on the restricted side of the model (i.e. opposite the cliff face) after stress initialization and tensile stresses in the walls of the excavation. These tensile stresses promoted roof collapse in a similar manner as observed in the mobile abutment models in Section 2.5.2.

In order to simulate the roof displacement associated with an advancing excavation, the model was run in five stages. First, stresses were initialized and allowed to decay. Second, the 4 m central heading was excavated, and a 70% internal boundary stress was applied to the excavation interior and solved to an equilibrium solution ratio of $1.0(10)^{-5}$ to mimic relaxation prior to bolt installation in accordance with longitudinal displacement profiles from Vlachopoulos & Diederichs (2009). This assumed any deformation ahead of the excavation was elastic in nature (Walton & Diederichs, 2015). Third, the central heading bolts were installed, the relaxation stress boundary removed, and
the model was run to an equilibrium solution ratio. Bolt material properties were identical to those used in Section 2.6. Fourth, the right and left headings were excavated simultaneously, and the same stress relaxation and bolt installation procedure was repeated along the newly excavated boundary, and the model was solved to an equilibrium solution ratio. Fifth, the excavation was benched to its final height and solved a final time. Maximum roof midspan vertical displacement was tracked and extracted using a *history* command, and the final stress state of the immediate roof saved for comparison to the adjusted voussoir analytical method.

In Section 2.9.4, the reported displacement results from the adjusted analytical solution and numerical modeling efforts account for the timing of field displacement measurements in order to make direct comparisons. In the adjusted analytical solution, displacement is assumed to vary continuously as a function of increasing span, and this allows displacement of a 4 m span (i.e. central heading) beam to be subtracted from the displacement of a 12.5 m span beam for comparison to the field-measured displacements. In the numerical models, the recorded displacements are set to 0 after Stage 3 (i.e. completion of the 4 m central heading and support installation). In both cases the reported stresses are total stresses for the 12.5 m span with no adjustment, as there were no field measurements of stress to compare to. Note that the adjusted analytical method and numerical model results presented required no calibration and were intended to represent an approximation of what could have been produced prior to construction using available field data.

2.9.3.1 Support Installation Uncertainty

Due to the uncertainty regarding support installation timing and the conflicting literature on passive or pretensioned state of the installed roof bolts, additional numerical model cases were run to constrain the impacts of possible support configuration and timing. One case installed supplemental corner support and applied 50 mm of shotcrete at the same model stage where the 12.5 m span excavation roof was supported. This represents the largest possible difference between the model sequence considered in this case study (i.e. passive roof support only during beam deflection) and the earliest possible shotcrete support timing. This end-member case with simultaneous installation of all support featured a 3.0 m thick roof, as well as the higher discontinuity strength and stiffness, resulted in only a 0.4 mm (i.e. 9% ) decrease in roof sag. The
limited impact of the added support is hypothesized to be due to the low stress conditions, flat-roof excavation geometry, and low shotcrete thickness relative to the excavation span.

The other tested support case applied a pretension load of 60 kN to the roof bolts following their installation, which similarly resulted in a 0.4 mm (i.e. 9%) decrease in the model roof displacement. This agrees well with the findings of Boon et al. (2015), which show that pretensioned bolts in a shallow excavation in a well-jointed, competent rockmass had a minimal influence on excavation convergence. For further discussion on the method of applying bolt pretension and documentation of the supplemental support cases refer to Appendix A.

Note that the timing of support installation is not identical to the models used to develop the adjusted analytical solution in Section 2.6, however, the amount of pre-installation (i.e. 70% in-situ stress) displacement is insignificant in relation to the scale of measured deflection. The pre-support displacement for the 4.0 m central heading was between 0.01 and 0.02 mm. Following excavation expansion, the pre-support displacement was between 0.04 and 0.05 mm.

2.9.4 Adjusted Analytical Method Comparison

Based on the DFN populated in the numerical model using the inputs estimated using the site investigation report (see Table 2.13), the mean joint spacing was calculated to be 0.8 m. This value was used together with the intact Young’s Modulus, joint normal stiffness, total roof thickness, and number of bolted layers, from the site-specific and region-specific investigations. The adjusted voussoir beam analog was applied as outlined in Section 2.8, accounting for the unmeasured displacement of the central heading in the field, with good agreement to the range of roof displacement observed in the field (Table 2.16).
Table 2.16: Results of the adjusted voussoir analytical solution using applied to the known geometric and rockmass conditions of the Bondi Pumping Chamber. BT = bolted thickness, q = triangularly distributed surcharge pressure, \( \delta_{\text{max}} \) – maximum midspan displacement, BL = Buckling Limit, \( \sigma_{\text{max}} \) – maximum midspan stress, FoS\(_{\text{crushing}} \) – factor of safety against crushing, FoS\(_{\text{sliding}} \) – factor of safety against sliding.

<table>
<thead>
<tr>
<th>Case</th>
<th>BT (m)</th>
<th>Contact Properties (Table 2.15)</th>
<th>( \delta_{\text{max}} ) (mm)</th>
<th>BL</th>
<th>( \sigma_{\text{max}} ) (MPa)</th>
<th>FoS(_{\text{crushing}} )</th>
<th>FoS(_{\text{sliding}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0</td>
<td>Minimum</td>
<td>4.6</td>
<td>9.8</td>
<td>1 1</td>
<td>2.4</td>
<td>5.1</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
<td>Maximum</td>
<td>4.6</td>
<td>9.7</td>
<td>1 1</td>
<td>2.4</td>
<td>5.1</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
<td>Minimum</td>
<td>2.2</td>
<td>3.7</td>
<td>1 1</td>
<td>1.6</td>
<td>2.8</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
<td>Maximum</td>
<td>2.2</td>
<td>3.7</td>
<td>1 1</td>
<td>1.6</td>
<td>2.8</td>
</tr>
</tbody>
</table>

The adjusted analytical solution results accurately captured the range of reported roof displacement, particularly for the lower-bound 3 m thick sandstone roof. Altering the triangularly distributed surcharge load from 0 to 120 kPa (i.e. the conservative value used in Pells & Best, 1991) captured the whole range of reported roof displacement. Notably, the changes in contact material properties (i.e. jkn reduced from 4000 GPa/m to 1500 GPa/m and joint initial friction from 45° to 28°) had no noticeable effect on the voussoir analytical solution except to lower the factor of safety against sliding failure. The upper-bound roof bolted thickness (i.e. 4.5 m) underpredicted displacement as reported by Pells & Best (1991) but note that the FoS\(_{\text{sliding}} \) decreased significantly for the minimum contact strength case (i.e. Case 3).

2.9.5 Numerical Model Results

Model roof displacement for the 3.0 m thick roof cases were within the specified range predicted by the adjusted voussoir beam analog (Figure 2.72).
Figure 2.72: Bondi pumping chamber numerical model displacement results for the lower-bound 3 m thick roof featuring (a) weaker and softer discontinuities and (b) stronger and stiffer discontinuities.

While no field stress measurements were available, model results of horizontal stress were also compared to the value calculated to the adjusted analytical solution (Figure 2.73).

Figure 2.73: Bondi pumping chamber numerical model horizontal stress results for the lower-bound 3 m thick roof featuring (a) weaker and softer discontinuities and (b) stronger and stiffer discontinuities.

Maximum midspan stress was significantly overpredicted by the adjusted voussoir analytical solution in the stronger and stiffer discontinuity case (Figure 2.73b). Furthermore, the distribution of horizontal stresses in the weaker discontinuity case was highly non-uniform (Figure 2.73a). This was thought to be partially due to the bolt penetration into the upper laminate, a condition that was not tested in the development of the adjusted analytical solution. Therefore the 3.0 m thick...
A sandstone roof model was rerun with 2.9 m long bolts and both the displacement, and stress results matched the adjusted analytical solution (Figure 2.74). However, the model midspan stress was just approaching the lower-bound estimate of the adjusted analytical solution. The idealized voussoir conditions utilized to develop the adjusted method represented the maximum stress transfer possible. Therefore, the presence of non-vertical joints (as in this case) should strictly reduce the stress arching capacity of the supported roof.

Figure 2.74: Bondi pumping chamber numerical model (a) vertical displacement and (b) horizontal stress results for the lower-bound 3m thick roof featuring stronger and stiffer discontinuities with shorter bolts.

Further evidence in support of this explanation was found in the onset of sliding failure in thicker roof model results with weaker and softer cross-joints (Figure 2.75).
Figure 2.75: Bondi pumping chamber numerical model vertical displacement results for the upper-bound 4.5m thick roof featuring weaker and softer discontinuities.

Similar to the FoS_{crushing} assumptions noted in Section 2.4, FoS_{sliding} assumes that the roof fails as soon as the contact strength is exceeded. However, this is clearly not the case when the joints have non-zero residual strength, as is the case in reality. Therefore, the stress overprediction by the adjusted voussoir beam analog is due to the influence of sub-vertical cross-joints, which decrease the stress arching capacity of the beam. The model stresses diverge from the stress prediction based on perfectly vertical cross-joints, decreasing the applicability of the analytical FoS_{sliding} in Table 2.16. However, even if the analytical FoS_{sliding} is less than 1.0, the roof may initiate sliding and not completely detach due to residual frictional strength and stress generation along a yielding discontinuity.

In thicker roof models with stronger cross-joints (i.e. in the absence of abutment slip failure) the model results matched the analytical solution for displacement with a surcharge load between 0 and 120 kPa, while the midspan stress results matched the lower surcharge load adjusted analytical results. This further confirms that the sub-vertical joints are decreasing the bolted roof beam’s stress arching capacity relative to the maximum expected in an idealized voussoir scenario for a given displacement (Figure 2.76)
Analysis of model vertical stress results above the bolted roofs indicate that the traditional method of considering surcharge loading (i.e. as a dead-load) is not an accurate representation of surcharge. Pells & Best (1991) assumed a triangularly distributed surcharge pressure with a maximum value of 120 kPa for a conservative approach to approximate yielding of the “Unit 5 – Upper Laminate” material above the bolted roof. Model results indicate that the distribution of “surcharge” pressure in the absence of material yield is not triangular, but rather u-shaped (Figure 2.77) due to the stress arching capacity of the modeled material.

Figure 2.76: Bondi pumping chamber numerical model (a) vertical displacement and (b) horizontal stress results for the lower-bound 3m thick roof featuring stronger and stiffer discontinuities with shorter bolts.

Figure 2.77: Comparison of model “surcharge” pressure for the 3m thick, 2.9 m bolt, strong and stiff discontinuity model at equilibrium and assumed surcharge pressure from Pells & Best (1991).
This stress distribution was observed in all the model results to varying degrees with a mean range of 80 to 150 kPa, depending on the modeled roof stability. The u-shaped distribution is likely influencing the FoS_{sliding} in a way that is not accounted for in the existing or adjusted analytical solution; specifically, for a given total surcharge load (i.e. average surcharge pressure) the increased concentration of surcharge loads towards the beam abutments is hypothesized to result in a reduction in the stability of the roof beam regarding sliding failure, but an enhanced degree of stability against buckling failure.

2.9.6 Discussion

The results of the Bondi Pumping Chamber case study highlighted the strengths and limitations of the layer-adjusted E_{rm} method developed through the course of this chapter. Rockmass parameters and construction methodology information obtained from the available literature and through personal communication with the design engineer were incorporated to develop a DFN that accounted for the apparent dip of the joint sets relative to the alignment of the tunnel. A mean joint spacing was recorded from the generated DFN and applied to the adjusted voussoir analytical method with excellent agreement between the analytically predicted displacement, the in-situ measurements, and the 3 m thick roof model cases. However, the thicker roof displacement was underpredicted by the adjusted voussoir analytical solution in relationship to the measured roof sag. This difference can be explained by multiple mechanisms including but not limited to non-continuous roof deformation in the field (i.e. abutment slip failure), continuous roof deformation in the direction of excavation (i.e. sag governed by thinnest portion of the roof), location of extensometer installation (i.e. not in the thicker portions of the roof), and the influence of joint interactions in three dimensions (i.e. due to their intersection with other joint sets sub-parallel to the excavation axis).

Numerical model results featuring continuous roof displacement were typically closer to the lower-bound surcharge load used in the adjusted analytical solution, indicating that that the 120 kPa triangularly distributed surcharge dead-loading assumptions implemented by Pells & Best (1991) were likely inaccurate and that variations in roof thickness, blast damage, joint swarms, and non-continuous deformation caused the bulk of variation in recorded roof sag during construction of the Bondi Pumping Chamber. However, the roof stratigraphy was inferred based on the available site data, which did not include the full overburden stratigraphy to grade. The adjusted voussoir
beam analog applied to maximum midspan stress predictions remains an upper-bound estimate of model stress due to the presence of sub-vertical joints. Overall, however, the validity of the adjusted voussoir beam analog for practical application has been demonstrated through this case study. The method has provided a mechanical basis by which Young’s Modulus should be reduced that can be used in lieu of more arbitrary conservative assumptions.

2.10 Conclusions

Systematic analysis of voussoir beam mechanical behavior under a large range of loading conditions has verified that simple alterations to the existing voussoir analytical solution can capture the behavior of relatively complex voussoir beam scenarios as determined using numerical models. Accounting for applied horizontal stresses, horizontal joints in multi-layer systems, and the presence of passive rockbolts allows for practical use of the voussoir analog in more realistic scenarios. First, a parametric sensitivity analysis of simple elastic multi-jointed voussoir beam models identified a key interaction between block rounding and zone size that significantly impacted model results of maximum displacement and horizontal stress. Although recommendations are provided in Itasca (2014) regarding block rounding length and zone size independently, their interaction is not explicitly considered in the user manual. Generally, block rounding values should be approximately 1% of the representative block size, and no greater than 20% of the maximum zone size in the model to maintain high accuracy when considering stress distribution and displacement. Baseline voussoir beam models were largely based on those from Diederichs & Kaiser (1999), but considered joints represented with the continuously yielding constitutive model.

Following establishment of optimum block rounding and zone size for baseline voussoir beam models, the effect of surcharge loading was considered on elastic beams and found to match analytical predictions well. This prompted an investigation into the accuracy of FoS\textsubscript{crushing} and the mechanics of inelastic voussoir beam failure. This analysis demonstrated that the accuracy of FoS\textsubscript{crushing} relies heavily on the post-peak behavior of the modeled material. Furthermore, inelastic beams did not fail unless the midspan reached post-peak. The results suggest that an additional 0.6- or 1.25-times adjustment should be made to Eqn. (2.38) for brittle and perfectly plastic post-peak behavior end-members, respectively, once the UCS has been adjusted to field scale values (e.g. using a multiplier 0.3-0.5 as suggested by Diederichs & Kaiser, 1999). This adjustment also
accounts for the midspan-abutment stress discrepancy, and the propensity for brittle material to yield in tension.

The subsequent set of voussoir beam models were then used to analyze the impact of more complex boundary conditions and geometries, including the presence of support elements. First the influence of applied horizontal stress on mobile abutment blocks, as well as locked-in horizontal stresses were considered on baseline voussoir beam geometries. For the 10 m spans tested herein, in-situ horizontal stresses decreased displacement of elastic voussoir beams, except in cases where the abutments were mobile and very low applied stresses allowed for the beam self-weight to displace mobile abutments and collapse. Most importantly, adjustment of analytical stress prediction by simply adding the magnitude of in-situ horizontal stress as a simplified estimate was confirmed as adequate.

A parametric sensitivity analysis of 810 models analyzed the impacts of different beam sizes, joint spacings, layer thicknesses, material properties, joint properties, and bolt spacing on voussoir beam model displacement and stress distribution. Stable model results were analyzed statistically to determine an effective $E_{rm}$ that minimized the difference between the displacement results of numerical and analytical methods. This adjustment value was determined to be influenced by the number of horizontal layers in a given model. The layer-adjusted $E_{rm}$ ($E_{rmn}$) was then used to predict the deflection of the bolted interval with greater accuracy than the baseline Diederichs & Kaiser (1999) analytical solution. In order to predict model stresses, an analysis of effective beam thickness was conducted, and it was determined that for increasing material and joint stiffness or decreasing vertical joint spacing or number of horizontal layers, the optimal beam thickness ratio tended to 1.05. The effective thickness method for midspan stress prediction also proved more accurate than the baseline Diederichs & Kaiser (1999) analytical solution.

The voussoir beam analog was then tested in realistic single-entry models that analyzed the impacts of in-situ stresses, as well as the influence of surcharge loading on supported and unsupported elastic voussoir beam roofs. The results were compared against the traditional and adjusted analytical methods developed in this study and found that the adjusted analytical methods provided a better estimate of stress and displacement predictions in stable models. Implementation of the adjusted voussoir analytical solution to the Bondi Pumping Chamber case study further verified its applicability to supported flat-roof excavations. It also highlighted the impact that sub-
vertical joints can have on stress arching and the associated impact on the FoS_{sliding} estimated by the adjusted analytical method, even in supported roof conditions.

These findings also validate the use of the voussoir beam analog for model analysis and identification of jointed roof stability. Voussoir beam mechanics may not be applicable in all laminated and discontinuous systems (e.g. more massive, very weak, heavily jointed, extremely high stress), but represent a valuable reference case for the more complex geologic and mining conditions considered in the explicit DEM models in this study. A foundation for analyzing more complex roof behavior has been successfully developed through simple adjustments to existing analytical solutions. Accounting for applied horizontal stresses, orthogonal joints, and the presence of rockbolts are not only practically applicable to the use of the voussoir analog in field cases but are useful tools in the analysis of numerical model results going forward.
CHAPTER 3
PARAMETRIC SENSITIVITY ANALYSIS OF ROOF STABILITY IN SINGLE-ENTRY MODELS

Deformation of excavations in discontinuous and laminated rockmasses is governed by the mechanical interactions of multiple geologic (i.e. intact material properties, discontinuity orientations and properties) and mining (i.e. depth, entry span, and installed support) conditions. Furthermore, flat roofs are a common excavation shape in traditionally discontinuous rockmasses (i.e. layered sedimentary systems such as coal and associated lithologies). This excavation shape limits the amount of non-ore material removed from the mine and promotes the self-supporting, and reinforcement-supported capacity of the immediate roof to redistribute mining induced stresses. In order to understand how the pillar and roof interact following excavation advancement, the critical geologic and mining controls on roof stability must first be identified. The most applicable numerical tool to investigate layered and discontinuous systems, with a specific focus on the effects of discontinuities, is the explicit discrete element method (DEM).

Even though the explicit DEM has been previously implemented in researching these mechanical interactions at the excavation scale in flat roofs, studies often address isolated or specific impacts ranging from joint orientations and constitutive models (e.g. continuously yielding, Mohr-Coulomb, etc.) (Hsiung et al., 1993; Ran et al., 1994; Souley et al., 1997), excavation stability specifically related to local support interaction (Chen et al. 2001; Gale et al. 2004), horizontal stress influence (Garg, 2018), development of new explicit DEM algorithms (Boon, 2013), progressive damage due to moisture sensitivity (Bai et al., 2016), simulation of fracture propagation in intact roofs through the use of the bonded block modeling (BBM) technique (Gao et al., 2014), or back analysis of individual case studies (Hatzor & Benary, 1998; Hoelle, 2003; Alejano et al., 2008; Coggan et al., 2012; Tsesarsky, 2012).

In order to identify the combinations of critical inputs governing roof stability under a wide range of discontinuous rockmass behavior, a full parametric sensitivity analysis was conducted on hypothetical, single-entry coal mine models using the explicit DEM as implemented in Itasca’s Universal Distinct Element Code (UDEC). Model inputs were based on previous experimental (Bastola & Chugh, 2015) and numerical (Hsiung et al., 1993; Esterhuizen et al., 2010b; Tulu et al., 2017) studies, coupled with engineering judgement to simulate multiple geologic analogs (e.g.
laminated shale, shale, sandstone, limestone). Model results were verified as realistic by applying the empirical Coal Mine Roof Rating (CMRR) (Molinda & Mark, 1994) and associated Analysis of Roof Bolt Systems (ARBS) (Mark et al., 2001) in numerical models through consideration of model inputs and model results. Self-supporting capacity was investigated using unsupported models and their results considered through the lens of the voussoir beam analog. Binary logistic regression (BLR) was utilized to identify the combinations of critical inputs governing roof stability in unsupported and bolted explicit DEM models. The statistically significant controls on roof stability in model results were then considered in relationship to the weighting of parameters in the empirical CMRR and ARBS methods. Novel observations regarding the impacts of model setup attributes (e.g. damping, zone size, etc.) were documented.

The following section outlines the pertinent literature related to explicit DEM constitutive models, model setup attributes, existing explicit DEM research on flat-roof excavations, as well as the relevant empirical, analytical, and statistical methods used for verification in this chapter. Sections 3.2 through 3.5 discuss the methodologies, results, and conclusions of this thesis regarding roof stability in single-entries.

3.1 Literature Review

3.1.1 Relevant Constitutive Models

Constitutive models are components of numerical models that govern the mechanical response of model elements (i.e. block material, discontinuities, support) to changes in loading. They are typically governed by elastic relationships between stress and strain, plastic failure criteria, and post-peak behavior models (Jing, 2003). The explicit DEM can utilize multiple constitutive models to arrive at a solution; constitutive models relevant to this thesis are discussed in this section.

The most well-known constitutive model governing the behavior of a discontinuity is the Mohr-Coulomb failure criterion. It consists of a set of linear relationships between major and minor principal stresses, or shear and normal stresses, and calculates the stress conditions at which sliding failure will initiate along a given discontinuity. The two-dimensional Mohr-Coulomb failure envelope in principal stress and shear-normal stress space are given as:

\[
\sigma_{11} = \sigma_{33} \frac{1+\sin\phi}{1-\sin\phi} - 2c \sqrt{\frac{1+\sin\phi}{1-\sin\phi}}
\]  (3.1)
\[ \tau_f = c + \sigma_n \tan \varphi \]  
(3.2)

where \( \sigma_{11} = \) major principal stress, \( \sigma_{33} = \) minor principal stress, \( c = \) cohesion, \( \varphi = \) friction angle, \( \tau_f = \) shear stress at failure, and \( \sigma_n = \) normal stress acting on the discontinuity.

The continuously yielding joint constitutive model is an alternative to traditional Mohr-Coulomb failure criterion that has been shown to more accurately model larger displacements (Poeck, 2016). It incorporates more “realistic” behavior by assuming zero tensile strength and continuously relates the shear strength of the joint to the decay of friction from an initial (i.e. peak) to an intrinsic (i.e. residual) value, as well as a decrease in effective dilatancy as a function of plastic shear strain and normal stress acting on the discontinuity.

The peak strength of a joint modeled as continuously yielding is initially controlled by the joint roughness parameter and the initial friction angle. After shearing onsets, the intrinsic friction angle governs deformation. This is depicted by the asperities featured in Figure 3.1.

![Figure 3.1: Schematic representation of continuously yielding joints and significant input parameters (modified from Poeck, 2016)](image)

A continuously yielding joint allows sudden displacements to occur due to the decreasing shear strength with increasing plastic shear strain. Normal and shear loading under the continuously yielding joint constitutive model are given as:

\[ \Delta \sigma_n = k_n * \Delta u_n \]  
(3.3)

\[ \Delta \tau = F k_s * \Delta u_s \]  
(3.4)

where \( \sigma_n = \) normal stress, \( k_n = \) joint normal stiffness, \( u_n = \) normal displacement, \( \tau = \) shear stress, \( F = \) tangent modulus factor, \( k_s = \) joint shear stiffness, \( u_s = \) shear displacement (Itasca, 2014). The tangent modulus factor (F) is controlled by the distance between the bounding shear strength (\( \tau_m \)) curve and actual shear strength curve (\( \tau \)) (see Figure 3.2) and is given by:
\[ F = \frac{1 - \tau / \tau_m}{1 - r} \]  

(3.5)

where \( r \) is initially 0 and upon load reversal (i.e. change in shearing direction) is set to \( \tau / \tau_m \), making \( F = 1 \). This immediately restores the shear stiffness in Eqn. (3.4) to its elastic state. \( \tau_m \) is given by:

\[ \tau_m = \sigma_n \tan \varphi_m \times \text{sgn}(\Delta u_s) \]  

(3.6)

where \( \varphi_m \) is the effective friction angle (Itasca, 2014). The relationship between shear loading and bounding shear strength is depicted in Figure 3.2.

\[ \Delta \varphi_m = -\frac{1}{R} \times (\varphi_m - \phi) \times \Delta u_s^p \]  

(3.7)

where \( R \) = joint roughness parameter, \( u_s^p \) = plastic displacement, and \( \phi \) = intrinsic friction angle (Itasca, 2014). The incremental change in effective friction angle is equal to:

\[ \varphi_m = \left( \varphi_m^{(i)} - \phi \right) \times \exp \left( -u_s^p / R \right) + \phi \]  

(3.8)

and effective dilatancy \( i \) is given as:
\[
i = \tan^{-1}(\frac{\tau}{\sigma_n}) - \varphi
\]  
(3.9)

(Itasca, 2014). Through Eqns. (3.1-3.7), the continuously yielding joint model simulates the progressive damage of joints under shear loading (Souley & Homand, 1996).

Intact (i.e. block) material in this study is simulated using three constitutive models to capture a large range of discontinuous rockmass behavior. Refer to Section 2.1.5 for a review of elastic isotropic and Mohr-Coulomb strain-softening constitutive models.

The anisotropy in discontinuous and laminated systems is dominated by the intensity and orientation of the laminations. The explicit DEM can account for these explicitly by using fully separable contacts, but the computational demand of an explicit DEM model increases linearly with the number of explicit discontinuities and blocks in the model (Jing, 2003). In order to approximate these rockmass conditions, previous studies have shown that the strain-softening ubiquitous joint (SUBI) constitutive model can be used to model fine-scale bedding, planes of weakness, or anisotropy (Hutchinson & Diederichs, 1995; Zipf, 2007; Sinha, 2016; Esterhuizen et al., 2017; Sainsbury & Sainsbury, 2017; Tulu et al., 2017; Abousleiman et al., 2019). SUBI implements the Mohr-Coulomb failure criterion and associated plastic flow and stress correction rules, but accounts for an implicit plane of weakness in reference to the global coordinate system of the model with a separate failure criterion (Figure 3.3). The orientation of the weak plane is adjusted as deformable blocks rotate due to large-strain deformation. If the stress state incurs failure along the weak plane, local plastic flow rules are used to calculate local stress corrections, which are resolved back to the global axis and applied to the stress state of the next timestep.
Figure 3.3: Illustration of a weak implicit plane in a Mohr-Coulomb solid oriented at angle (θ) between global (x,y) and local (x’,y’) coordinate axes (from Itasca, 2014).

Figure 3.4 features a schematic representation of how the SUBI constitutive model can be implemented within a three-dimensional zone element to represent implicit planes of weakness. This is not a direct comparison to the implementation of SUBI elements in this thesis, where zone elements are two-dimensional, SUBI planes are one-dimensional, and explicit discontinuities are also modeled. Rather, this three-dimensional continuum implementation of the SUBI constitutive model serves to clarify the component parts that govern its behavior.
The representation of sub-bedding planes of weakness is critical in approximating the overall behavior of particularly anisotropic or thinly laminated sedimentary rockmasses without significantly increasing model runtimes. However, SUBI material properties should be implemented with care, as the constitutive model does not consider joint spacing or stiffness and cannot match true deformation profiles, particularly at laboratory scales, while undergoing shear deformation (Sainsbury, 2016; Carvalho et al., 2019).

### 3.1.2 Rockbolt Representation

The rockbolt element (*struct rock* command) available in UDEC is a two-dimensional global reinforcement model that provides resistance to axial, shear, and bending moment loads throughout its entire length in a given modeled rockmass (Itasca, 2014). Rockbolt elements utilize spring and slider connections between structural nodes and adjacent finite difference zones to represent the bolt, bolt-grout, and grout-rock mechanical behavior (Figure 3.5).
Perfectly plastic inelastic behavior occurs normal to the rockbolt element when the limiting plastic moment is reached. Brittle inelastic behavior occurs when the tensile failure strain limit is reached and the element separates at the nodes (Itasca, 2014). Faceplates can also be modeled using the *struct beam* command and have been shown to replicate rockbolt behavior more realistically by tying the displacement of the bolt to the displacement of the excavation boundary (Hyett & Spearing, 2012).

### 3.1.3 Model Setup Attributes & Algorithms

Once appropriate constitutive models and support representation have been selected, models can be developed. However, multiple options involved in setting up and running numerical models are available. In this thesis, those options are collectively referred to as “model setup attributes”. They include parameters that are not unique to the explicit DEM, like zone element (i.e. mesh) size and mechanical damping, and others that are, like algorithms that control block rounding, block overlap tolerance, as well as block and contact detection logics. Refer to Sections 2.1.4 and 2.3.2 for discussion and analysis of block rounding and mesh size.

Although the development and verification of these algorithms are well-documented (Boon, 2013), the practical effects that model damping, overlap tolerance, and block detection logic broadly have
on the results of models of flat-roof stability are seldom reported in the literature. Understanding their individual and combined effects on roof stability are critical in confirming that the reliability of a given model matches the user’s assumptions.

Mechanical damping is used in UDEC to efficiently reach an equilibrium solution ratio in static and dynamic (i.e. seismic) applications. Mechanical damping is a real physical phenomenon, albeit one that is difficult to capture in discontinuous systems such as those modeled in UDEC. There are three major difficulties with selecting the degree, location, and behavior of damping within a given model. These difficulties include the propensity to restrict plastic flow (i.e. over-damping), the computational burden of finding the optimal damping value for individual nodes, and the inaccuracy of applying uniform damping to the entire model.

UDEC utilizes two main types of mechanical damping, “global” (i.e. “auto”) and “local”. “Global” damping applies a damping value based on a constant ratio related to the rate of change of nodal kinetic energy in the whole system. “Local” damping is applied in proportion to the unbalanced force of a given node (Itasca, 2014). Generally, it is apparent that “auto” damping tends to over-damp a given system and provide results that may incorrectly imply system stability, while “local” damping tends to under-damp in relation to “auto” damping, potentially allowing energy to propagate through the system unrealistically (Itasca, 2014). “Combined” damping is a form of “local” damping that does not rely on velocity sign changes to effectively damp a system. It is useful for models with unidirectional block movement (Itasca, 2014). Sinha et al. (2020) identified that “local” damping over-damps roof stability models when compared to “combined” damping for the same modeled conditions.

Conversely, block overlap is not a physical phenomenon, but tolerance some limited degree of overlap is critical in allowing the explicit DEM algorithm, which uses a soft-contact system, to run. If two block contacts propagate through each other too far, the resultant forces based on displacement and normal stiffness will create unacceptable errors and provide inaccurate results (Itasca, 2014). The default model overlap tolerance is set to 5 mm, such that any contact overlap greater than that will cause the model to stop running. No documentation of how changes to the overlap tolerance practically affects model results was found in the literature. This study presents model results that discuss the impact of increasing the overlap tolerance to 10 mm when investigating roof stability.
Cell mapping is an alternative contact detection logic to the default domain detection logic. It can be useful for blocky models because it maps all possible contacts within a given range prior to the contacts touching. Furthermore, contact searches are initiated for moving blocks only, and multiple blocks may be defined when using cell logic rather than making a single block and splitting it into sub-blocks with domain logic (Itasca, 2014). Similar to overlap tolerance, no documentation of how changes to the contact detection logic practically affects model results was found in the literature. This study presents model results that discuss the impact of cell mapping on model results in models featuring various degrees of block intensity. The terms “blocky” and “very blocky” are defined in Section 3.2.2.

3.1.4 Explicit DEM Modeling of Excavation Stability in Discontinuous Rockmasses

Explicit DEM models have been previously used to study roof stability in subsurface coal mining and other discontinuous and laminated rockmasses. Early implementation of the explicit DEM focused on the then-novel impact of explicit discontinuity properties and intensity (i.e. block size) on model behavior, particularly when compared to results of continuum modeling methods. Kripakov (1987) compared the results of FEM, BEM, and explicit DEM modeling techniques, when applied to typical mine design problems such as roof and floor stability (Figure 3.6), chain pillar stability, and multiple seam mining, to theoretical predictions and field measurements.

![Figure 3.6: Comparison of floor heave model results when implementing (a) the FEM and (b) the explicit DEM (modified from Kripakov, 1987).](image)

Fairhurst and Pei (1990) conducted a comparative analysis of FEM models representing joints as existing within an equivalent continuum to the results of explicitly jointed explicit DEM models.
Christianson (1989) compared the results of FDM models featuring ubiquitous joints and explicit DEM models with explicit joints, along with the impact of varying joint constitutive models (i.e. Barton-Bandis, Cundall-Lemos), and their respective inputs (i.e. friction angle, cohesion, dilation) on excavation stability related to thermal effects of underground nuclear waste storage. Similarly, Stephansson & Shen (1991) utilized the three-dimensional explicit DEM found in 3DEC to explore the impacts of joint conditions, thermal effects, glacial rebound, and dynamic loading (i.e. earthquakes) on jointed crystalline rocks for underground nuclear waste storage. These early applications of the explicit DEM validated the capacity to capture large-scale, discontinuity-driven deformation and excavation behavior in underground workings.

Barton et al. (1991) utilized the explicit DEM to investigate the impact of joint geometry on unsupported and supported roof stability of the 60 m span underground Olympic ice hockey rink in Lillehammer, Norway. Their findings indicated that for the three discrete fracture networks (DFNs) tested, unsupported deformation in the roof was minimal and matched the findings of empirical investigations conducted using the Q Tunneling Index. Del Greco et al. (1993) utilized the explicit DEM to conduct a case study of excavation stability focused on the unsupported walls of exploitation rooms in a talc block-caving mine. Garga & Wang (1993) and Indraratna & Singh (1994) utilized the hydro-mechanically coupled constitutive model to investigate the impact of groundwater loading and ingress on excavation stability in hypothetical and case-study based models.

Souley & Homand (1996) and Souley et al. (1997) also studied the impact of joint constitutive model on overall excavation stability and compared results to in-situ measurements. Kwon et al. (2000) compared joint deformation and stress concentration in UDEC and 3DEC models. Barla & Barla (2000) further identified that explicit DEM models were superior in capturing complex behavior and instability of discontinuous systems, when compared to finite difference method (FDM) equivalent continuum methods.

More recently, the explicit DEM has been utilized in specifically analyzing the stability of flat-roof excavations in laminated and discontinuous systems. Most applications of the explicit DEM in the context of flat-roof stability focus on expanding understanding of the voussoir beam analog in idealized flat-roof systems (Diederichs & Kaiser, 1999; Tsesarsky, 2005, 2012; Nomikos et al., 2007; Shabanimashcool & Li, 2015). Alejano et al. (2008) utilized the explicit DEM to conduct a
back analysis of roof collapse in a jointed and laminated carbonate deposit. They identified that stress redistribution (i.e. relaxation of horizontal stress) due to excavation of adjacent rooms, was the mechanism behind roof instability (Figure 3.7). However, all intact block material except for the immediate roof was modeled elastically, leaving the broader impacts of inelastic damage outside of the immediate roof unexplored.

Figure 3.7: Model roof collapse in previously stable center entry, following excavation of the left and right-hand side entries, confirming the destabilizing effect that decreasing compressive stress at the roof abutments decreases stability in this laminated carbonate environment (from Alejano et al., 2008).

Perino (2011) utilized the explicit DEM to focus on a unique case study of the ancient caverns at Tell es-Seba excavated in a bedded, jointed chalk formation. The study focuses on a comparative analysis of the development of the cavern, subsequent collapse, and self-stabilization of the remaining roof block materials under static conditions (i.e. non-seismic loading) using the two- and the three-dimensional explicit DEM. Intact block material was modeled using the elastic isotropic constitutive model and joints were modeled using the Mohr-Coulomb joint constitutive model. Results indicated that horizontal stress ratio \(k_o\) controlled roof deformation and that collapse occurred when \(k_o < 0.5\) in two-dimensional models (Figure 3.8).
Figure 3.8: Two dimensional models of progressive roof instability of the cavern roof (modified from Perino, 2011). This collapse was also captured in three-dimensional models, however it occurred at lower values of $k_o = 0.3$. This was attributed to the stabilizing effect of the intermediate horizontal stress on three-dimensional blocks coupled with the slightly larger block sizes used to decrease model runtime.

Coggan et al. (2012) validated that the progressive failure and fracture propagation in thinly bedded mudstones of an unsupported coal mine entry roof due to high horizontal stress could be captured using the explicit DEM, rather than a hybrid finite element method/discrete element method (FEM/DEM). Laminated mudstone was modeled by implementing Voronoi BBM in the explicit DEM to capture fracture propagation and explicit bedding planes to capture large-strain behavior (Figure 3.9a). Gao & Stead (2014) conducted a similar proof-of-concept investigation of the Trigon BBM approach in modeling fracture propagation of an unsupported massive shale roof (Figure 3.9b).

Figure 3.9: Roof failure model results using (a) Voronoi BBM and explicit bedding planes and (b) a Trigon BBM with no bedding planes (modified from Coggan et al., 2012; Gao & Stead, 2014).
Gao et al. (2014a) investigated the impact of implementing support elements in the Trigon BBM roof models from Gao & Stead (2014). Cable bolts and steel straps using the “Cable” and “Liner” elements available in UDEC were modeled and significantly reduced roof displacement and fracture growth.

Bai et al. (2016) utilized the Voronoi BBM in UDEC to conduct a case study of progressive failure of a wide entry, water-bearing roof in a Chinese coal mine. Model results generally matched observed deformations in the field, and the effects of supplemental supports (i.e. cribs and posts) and excavation sequence were accounted for. Strength degradation was modeled by reducing the strength of contacts to 0 as the model was stepped forward. This degradation was calibrated to strength degradation of saturated laboratory specimens of the modeled coal. Strength degradation had the largest impact on roof convergence in the models. Bai & Tu (2020) expanded on Bai et al. (2016) by analyzing the impact of explicit bedding planes and vertical joints, coupled with multiple support schemes, on roof stability of similar Voronoi BBM-DEM models. Explicit bedding and joint properties were calibrated to measured roof displacement from the Wujita coal mine. Following joint property calibration, the effectiveness of multiple support schemes was evaluated, and it was found that while grouted rockbolts suppressed macroscopic fracture growth and lamination separation, they failed to prevent skin failure and reduce loading of yielded superincumbent strata on to the immediate roof. Modeled wire mesh and grouted cable bolts were able to remedy those failure mechanisms effectively.

It is evident that use of the explicit DEM in researching excavation stability, particularly roof stability, has largely been focused on highlighting the differences in continuum and discontinuum methods, the influence of joint constitutive models and their respective inputs, and for application in back analysis of individual case studies. The contribution of these studies is indisputable, but their applications are typically quite broad (i.e. continuum vs. discontinuum) or quite specific (i.e. individual case studies). No study in the existing literature parametrically examines the generalized influence of a wide range of geologic and mining conditions, as well as model setup attributes, that control flat-roof excavation roof stability in explicit DEM models.

3.1.5 Coal Mine Roof Rating

The Coal Mine Roof Rating (CMRR) system is a field-implemented characterization method for quantifying the overall condition of the roof based on observationally determined geotechnical
qualities of intact rock, discontinuities, and groundwater considerations (Molinda & Mark, 1994). CMRR was initially developed based on previous rockmass rating systems and observation of 96 exposures at 75 US coal mines. Alternative rockmass rating systems such as the Q Tunneling Index (Q), Geologic Strength Index (GSI), and Rock Mass Rating (RMR) do not account for specific aspects of rockmass conditions that are prevalent in coal-measure rock. CMRR considers many geotechnical aspects specific to coal-measure rock and coal mine roofs, such as the presence of weak laminations (i.e. laminated shales) or other thinly bedded sedimentary layers, moisture sensitivity, and bolted intervals (Molinda & Mark, 1994).

Implementation of CMRR requires the user to rate the unconfined compressive strength (UCS), spacing, persistence, and shear strength of discontinuities, and moisture sensitivity of individual geologic units in and above the immediate roof via observation at surface outcrop, roof fall sites, or from borehole data. Additional adjustments for strong beds in the bolted interval, number of units in the bolted interval, groundwater conditions, strength of rocks above the bolted interval, and multiple discontinuity sets must also be considered (Molinda & Mark, 1994). Figure 3.10 depicts an example flow chart for CMRR calculation.

![Flow chart depicting typical process for determining CMRR using underground and surface data. RQD = rock quality designation (from Mark & Molinda, 2005)](image)

CMRR has been incorporated into multiple empirical design tools such as Primary Roof Support (PRSUP) (Molinda et al., 2000), Analysis of Roof Bolt Systems (ARBS) (Mark et al., 2001) and Analysis of Longwall Pillar Stability (ALPS) (Mark, 1989; Mark, 1990; Mark et al., 1994).
Furthermore, the calculation of CMRR can be modified to include core-based determination (Mark & Molinda, 1996), and a computer program has been developed to streamline implementation and interface with other programs such as AutoCAD.

The original database has also been expanded to 264 observations from more than 200 mines in the US (Mark & Molinda, 2005). CMRR has also been successfully implemented internationally in Australia to develop the Analysis of Longwall Tailgate Stability (ALTS) system (Colwell et al., 1999), South Africa (Butcher, 2001; Van Der Merwe et al., 2001), Canada (Forgerson et al., 2001), and China (Wang et al., 2018). However, Butcher (2001) noted that CMRR could be improved by accounting for joint orientation and $k_o$-ratio. Additional applications of CMRR include evaluation of potential highwall mining reserves (Hoelle, 2003), tailgate support guidelines in the Support Technology Optimization Program (STOP) (Barczak, 2000), derived inputs in Boundary Element Method (BEM) numerical models (Karabin, 1999), roof stability assessment in multi-seam mining (Luo et al., 1997), and viability assessment of extended cuts (Mark, 1999).

Calleja (2006) developed the “Rapid Rating” method for large amounts of drilling data which utilizes the digital lithology log, fracture log, geophysical log, core photos, and rock testing results to identify individual units and assign a unit rating. A modified CMRR (mCMRR) was developed by Taheri et al. (2017) to incorporate entry span and density of overburden rock into CMRR calculations and inform bolting requirements. Young (2018) noted that CMRR was most applicable in mines with higher incidence of discontinuities in the roof, but that in general, other variables such as topography, depth to entry, and intersection span should also be considered. Depth to entry and intersection span are both considered in the ARBS method, which is discussed in Section 3.1.6.

The evidence presented in the aforementioned literature indicates that CMRR is a versatile tool that can be applied to multiple coal mining applications in diverse geologic conditions. CMRR can also be adjusted from its original form and applied to new technologies or methods. Most importantly for this study, previous research suggests that CMRR may be used in conjunction with other methods, such as numerical models, to understand and link complex numerical parameter interaction to empirical methods and the field data that they are based on.
3.1.6 Analysis of Roof Bolt Systems

The analysis of Roof Bolt Systems (ARBS) method is built on empirical relationships between CMRR and depth of cover (H), intersection span (Is), and the bolt intensity value (ARBS_i). The ARBS_i of a given supported entry is calculated as:

\[ ARBS_i = \frac{L \times N \times C}{S \times W} \]  \hspace{1cm} (3.10)

where L = bolt length (ft), N = number of bolts per row, C = bolt capacity (kips), S = spacing between rows of bolts (ft), and W = entry width (ft). Observations of successful and failed cases were recorded along with their H, Is, and ARBS_i. The data was then analyzed as shown in Figure 3.11. Based on these empirical relationships established by measured CMRR, ARBS_i, and Is values, suggested ARBS and Is (ARBS_G and Is_G) values based on a given CMRR were developed using the best-fit linear delimiter between unstable and stable cases observed in the field. This resulted in the following equations:

\[ Is_G = 20 + 0.26CMRR \]  \hspace{1cm} (3.11)

\[ ARBS_G = (5.7 \log_{10} H) - 0.35CMRR + 6.5 \]  \hspace{1cm} (3.12)

Figure 3.11: Results of statistical analyses on coal mine entry case studies determining the relationship between (a) CMRR and Is, and (b) CMRR and Bolt Intensity (ARBS_i) (modified from Mark et. al., 2001).

These relationships were then used to determine Is_G and ARBS_G values for the multiple entries observed with varying roof bolt requirements, geologic conditions, and mine geometries. The differences between the actual and suggested values of intersection span and bolt intensity were then plotted against each other. A higher negative ARBS Difference (x-axis) value indicates that a given entry is more supported (i.e. higher bolt intensity) than what the empirical discriminant determined is needed for stability. A negative Span Difference (y-axis) indicates that the actual
span is larger than what is recommended based on empirical relationships. The difference between the suggested and actual values are plotted in Figure 3.12 and a linear trend was established between successful (stable) and failed cases, establishing the ARBS Discriminant. The ARBS Discriminant equation is given as:

\[ ARBS_{\text{Discriminant}} = ARBS_G - 0.3(I_sG - I_s) \]  

(3.13)

Figure 3.12: Plot showing the empirical relationship between ARBS Difference and Is Difference based on previously established relationships with CMRR. Note the 76% agreement with the discriminant line predicting correct classification of stable and unstable observed cases (from Mark et al., 2001).

Mark et al. (2001) note that there are cases where the roof was extremely weak and required additional support beyond roof bolts as shown by the failed cases with a very low ARBS difference (i.e. high degree of installed bolt intensity). Note that success and failures in Mark et. al. (2001) were classified as roof fall rates less than 0.4 falls per 10,000 ft drivage and greater than 1.5 falls per 10,000 ft of drivage, respectively. Intermediate cases between 0.4 and 1.5 falls per 10,000 ft of drivage were not considered in the calculation of the ARBS discriminant. Furthermore, while horizontal stress is not explicitly accounted for in the ARBS system, Mark et al. (2001) assume that based on the relationship between measured horizontal stress and depth of cover in Eastern US coal mines established by Mark & Mucho (1994), that depth of cover sufficiently captures the impact of stress level on roof stability for the case studies in Mark et al. (2001).
3.1.7 Binary Logistic Regression

Binary Logistic Regression (BLR) is a subset of Generalized Linear Models (GLM) and is a statistical tool utilized in multiple disciplines (McCulloch, 2001). BLR is utilized to estimate the probability of a given binary dependent variable (i.e. stable or unstable) occurring based on a dataset consisting of a single or multiple, categorical or continuous independent variables. Generalized models of linear regression and BLR for a single, continuous variable are shown in Figure 3.13.

![Figure 3.13: Graphic depicting generalized models of linear regression and BLR (from Dankers et al., 2018).](image)

BLR transforms the probability of dependent variable occurrence as determined by a standard general linear model:

\[ p(x) = a + bx \]  

(3.14)

where \( p = \) probability. To ensure that \( p(x) \) for all values of \( x \) is between 0 and 1, the logistic function is utilized to force the value to exist between 0 and 1:

\[ l(x) = \frac{1}{1+e^{-x}} \]  

(3.15)

When the logistic is combined with the probability function it forms in a special logistic function called the logit:
\[ \text{logit}(p) = \log \left( \frac{p}{1-p} \right) \]  

(3.16)

This is also known as the log(odds) and transforms the probability between 0 and 1 to a log(odds) between negative and positive infinity. This results in the generalized estimated regression equation:

\[ \hat{p} = \frac{e^{a+bx}}{1+e^{a+bx}} \]  

(3.17)

where \( \hat{p} \) = estimated probability, \( a \) = coefficient of the intercept, \( b \) = coefficient of the independent variable, and \( x \) = a unit of the continuous independent variable. Statistical outputs such as error, confidence intervals, goodness of fit parameters can all be calculated through the transformation between probability and log(odds).

3.2 Methodology

3.2.1 Model Setup

In order to advance the understanding of roof-stability in discontinuous and laminated systems, single-entry models were developed to analyze a wide range of geologic and mining conditions that are not simultaneously considered in the literature. In particular, the geomechanical properties and mining conditions that are captured in the CMRR (i.e. intact strength and discontinuity intensity and condition) and ARBS (i.e. mining depth, bolt intensity) empirical methods were varied. This would allow for more thorough application of empirical methods to model inputs and results in order to verify their realism.

As shown in Figure 3.14, the single-entry model geometry consists of two bounding half-pillars of \( w/h = 8.0 \) (\( w/h = 4.0 \) modeled explicitly), 6.0 m entry span, 2.5 m entry height, and features 30 m of explicitly represented, simplified, and homogeneous roof and overburden (i.e. no geologic heterogeneity besides the stochastic DFN). Modeling half of each bounding pillar with horizontal zero-velocity boundary conditions imposed horizontal symmetry, allowing the models to focus on local roof stability and limit the horizontal extent of the single-entry models. This assumed that deformation was mirrored to an infinite set of additional entries on either side of the model. Note that in this chapter, the term “roof” is utilized to discuss the area directly above the entry and pillars and below the height of installed support, while the term “overburden” generally refers to all the material above the entry and pillars (i.e. inclusive of the roof). Therefore, in this chapter, the terms
“roof properties” and “overburden properties” are synonymous. Note that this naming convention changes slightly in the following chapters.

Figure 3.14: Example single entry model depicting boundary conditions, model geometry, mesh density, and roof and overburden DFN.

The top of the model was a free surface for 30 m deep entries and a fixed velocity boundary for deeper cases. The remaining boundaries had zero-velocity conditions imposed in the direction of their normal. Once stresses were initialized in deeper model cases, zero-velocity boundary conditions were then applied to the top of the model, rather than a continually applied stress. Note that both boundary conditions (i.e. zero-velocity and continuously applied stress) represent possible approximations of in-situ behavior, and neither can be definitively established as more representative of “realistic” conditions. This is because at an unknown vertical height above a stable or unstable mine entry in the real world, deformation due to excavation approaches zero and stresses return to in-situ values. However, where this occurs is unknown a priori and depends heavily on the geologic and mining conditions of a given mine entry. If the conditions are such
that the entry is extremely stable, the stress and velocity boundary conditions approximate the same state at the model top. However, as deformation approaches the upper boundary of the model, the two possible boundary conditions begin to deviate in their effects. The zero-velocity boundary condition will stop additional deformation and represent full stress-arching above the explicitly modeled overburden, while the continuously applied stress will increase deformation and assume zero stress arching above the explicitly modeled overburden (Figure 3.15). As this portion of the study focused uniquely on immediate roof stability as a binary condition and was not concerned with stress transfer after the roof has failed, the simpler zero-velocity boundary condition was selected to prevent excess deformation and “contact overlap” error in the event that failure propagated to the upper boundary of the model in the deeper cases tested (i.e. 100 m and 200 m depth to entry).

![Figure 3.15: Comparison of the representations of different upper boundary conditions on the single-entry models if models are stable (left), or unstable and deformation approaches the upper model boundary (right).](image)

A graded mesh was applied to the model, focusing the finest mesh density on the immediate roof above the excavation. Graded mesh sensitivity results are discussed further in Section 3.3.1. Contacts between the pillar and the roof were modeled with the same strength and stiffness.
properties as the overburden DFN. The model floor was considered elastic with a stiffness calculated to approximate an equivalent continuum rockmass modulus of the fractured roof using Eqn. (2.25).

While some coal mine roofs do not exhibit cross-jointing as pronounced as is shown in Figure 3.14, such geometries were necessary to allow block separation to occur in the models. This is one of the stated limitations of explicit DEM modeling. However, jointed roofs in coal mines have been documented by Molinda & Mark (2010) and are described as contributing to roof failure. Coal pillars were modeled using an inelastic constitutive model from Sinha (2020) but were assigned parameters that led to negligible pillar yield and uniform deformation in the models to isolate the impact of pillar yield and deformation on roof stability. This was because the empirical methods used to verify the model results (i.e. CMRR and ARBS) as realistic do not explicitly consider the effect of pillar deformation and yield. Models discussed in later chapters utilize calibrated coal pillar parameters that lead to realistic pillar deformation and significant inelastic pillar yield.

A four-stage solution method was implemented for all unsupported model runs. Stage 1 initialized in-situ stresses based on block material densities, depth to entry, and in-situ stress ratio. Note that some models incurred “contact overlap” error during stress initialization. These models utilized a stepped joint stiffness from elevated (i.e. 20 times) joint normal and shear stiffnesses down to intended values in order to prevent “contact overlap” error during stress initialization. The effect of this method on model results is further discussed in Section 3.3.6.

Stage 2 applied an internal stress boundary equivalent to 70% in-situ stress following excavation. 70% was selected based on elastic material behavior in the longitudinal displacement profiles from Vlachopoulos & Diederichs (2009) and was meant to replicate realistic (i.e. three-dimensional) excavation mechanics and account for non-zero displacement prior to support installation. 70% in-situ stress effectively assumed that any roof deformation ahead of the excavation was elastic in nature (Walton and Diederichs, 2015). Note that in some cases the reduction in stresses from 100% to 70% to 0% may have influenced model stability relative to a more gradually staged relaxation process. However, this was only expected to be the case for models with borderline stability.

Stage 3 removed the internal stress boundary and solved the model to a standard equilibrium solution ratio of $1.0(10)^{-5}$ or until model error occurred, in both bolted and unsupported cases. An
additional velocity check stage (Stage 4) was implemented to ensure that the overall model equilibrium condition was indeed indicative of local roof stability. This solution stage sequence is depicted in Figure 3.16. All unsupported models were restored at the Stage 2 save state and standard size (19 mm diameter x 2.4 m long) and spaced (1.2 m) roof bolts were installed using the built-in rockbolt element in UDEC.

![Figure 3.16](image_url)  
Figure 3.16: Schematic depiction of stages for unsupported and bolted model cases.

### 3.2.2 Input Parameters

Model parameter combinations were selected based on existing numerical and laboratory studies (Hsiung et al., 1993, Esterhuizen et al., 2010; Bastola & Chugh, 2015; Tulu et al., 2017) to cover a range of realistic possible behaviors of coal-measure rock. Geologic parameters were selected to approximate the mechanical behavior of sedimentary rockmasses ranging from laminated shale to limestone (Table 3.1). Three zone constitutive models were used to achieve this. The SUBI constitutive model was selected to implicitly account for closely spaced bedding planes between explicitly modeled discontinuities. Stronger, massively bedded rocks were modeled using elastic-brittle-plastic (EBP) and isotropic elastic constitutive models. Regardless of the inelastic zone constitutive model (i.e. EBP or SUBI), all “intact” zone critical plastic strains were set to 1.0(10)\textsuperscript{−}.
to model brittle failure of intact material. Similarly, when the SUBI constitutive model was implemented, ubiquitous joint cohesion and tensile strength were modeled with elastic-brittle-plastic post-peak behavior replicate the brittle tensile failure and separation of laminations. However, ubiquitous joint friction angle was modeled as perfectly plastic (i.e. peak = residual) in accordance with Tulu et al. (2017) so that if a ubiquitous joint element failed in shear (i.e. slip) it would have some post-peak frictional strength. The continuously yielding joint constitutive model was implemented to gradually decay explicit discontinuity (i.e. DFN, bedding) strength from an initial (i.e. peak) to an intrinsic (i.e. residual) friction angle as a function of plastic shear displacement.
Table 3.1: Model parameters analyzed in this study for parametric sensitivity analyses; note that variables in rows highlighted with the same color were varied concurrently. w/h = width-to-height, $\phi_i$ = peak friction angle, $\psi$ = dilation angle, $C_r$ = residual cohesion, $\phi_r$ = residual friction, $T_r$ = residual tensile strength, $\text{SUBI}_{ji}$ = implicit joint peak cohesion, $\text{SUBI}_{jt}$ = ubiquitous joint tensile strength, $\text{SUBI}_{j\phi}$ = ubiquitous joint peak friction angle, $\text{SUBI}_{jdil}$ = ubiquitous joint dilation angle, $\text{SUBI}_{jcr}$ = implicit joint residual cohesion, $\text{SUBI}_{jtr}$ = ubiquitous joint residual tensile strength, $\text{SUBI}_{j\phi}$ = ubiquitous joint residual friction angle, $\varepsilon_{ps}$ = plastic strain, $jkn$ = joint normal stiffness, $jks$ = joint shear stiffness, $jen/jes$ = joint normal/shear exponent, $jr$ = joint roughness, $sd$ = standard deviation, BT = Bedding Thickness.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Depth to Entry (m)</th>
<th>30</th>
<th>100</th>
<th>200</th>
<th>---</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Span (m)</td>
<td>6</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Pillar w/h</td>
<td>8</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>In-Situ Stress Ratio</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Bedding Thickness (m)</td>
<td>0.5</td>
<td>1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Roof &amp; Overburden Block Material Properties (Field Scale)</th>
<th>Weak SUBI</th>
<th>Moderate SUBI</th>
<th>Strong SUBI</th>
<th>EBP</th>
<th>Elastic Soft</th>
<th>Elastic Stiff</th>
</tr>
</thead>
<tbody>
<tr>
<td>G (GPa)</td>
<td>0.42</td>
<td>3.33</td>
<td>3.33</td>
<td>12.5</td>
<td>0.42</td>
<td>12.5</td>
</tr>
<tr>
<td>K (GPa)</td>
<td>0.56</td>
<td>4.44</td>
<td>4.44</td>
<td>16.7</td>
<td>0.56</td>
<td>16.7</td>
</tr>
<tr>
<td>E (GPa)</td>
<td>1.0</td>
<td>8.0</td>
<td>8.0</td>
<td>30</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>Poisson’s Ratio (v)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
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<tr>
<td>Density (kg/m$^3$)</td>
<td>2350</td>
<td>2350</td>
<td>2350</td>
<td>2350</td>
<td>2350</td>
<td>2350</td>
</tr>
<tr>
<td>Cohesion (MPa)</td>
<td>2.5</td>
<td>5.0</td>
<td>7.5</td>
<td>15.0</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Tensile Strength (MPa)</td>
<td>0.5</td>
<td>0.75</td>
<td>1.0</td>
<td>2.0</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\phi_i$ (◦)</td>
<td>25</td>
<td>30</td>
<td>32</td>
<td>39</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\psi$ (◦)</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>12</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$C_r$ (Pa)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\phi_r$ (◦)</td>
<td>25</td>
<td>28</td>
<td>30</td>
<td>35</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$T_r$ (Pa)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

All combinations of the parameters, coupled with 2 random DFN seeds (100, 1234) resulted in 4,320 unique models. Two random seed numbers were utilized to determine the impact that
explicit joint location, rather than joint property distributions, had on model roof stability. Each case was run unsupported and with a standard bolt pattern (4 bolts across the roof span) resulting in 8,640 unique models.

Each random seed created eight unique DFNs based on the joint network geometry parameters listed in Table 3.1, their geologic characteristics are described in Table 3.2 and their geometries are depicted in Figure 3.17.

<table>
<thead>
<tr>
<th>DFN ID</th>
<th>Cross-Joints</th>
<th>Persistence</th>
<th>Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vertical</td>
<td>High</td>
<td>Close</td>
</tr>
<tr>
<td>2</td>
<td>Vertical</td>
<td>High</td>
<td>Wide</td>
</tr>
<tr>
<td>3</td>
<td>Vertical</td>
<td>Low</td>
<td>Wide</td>
</tr>
<tr>
<td>4</td>
<td>Vertical</td>
<td>Low</td>
<td>Close</td>
</tr>
<tr>
<td>5</td>
<td>Sub-Vertical</td>
<td>High</td>
<td>Close</td>
</tr>
<tr>
<td>6</td>
<td>Sub-Vertical</td>
<td>High</td>
<td>Wide</td>
</tr>
<tr>
<td>7</td>
<td>Sub-Vertical</td>
<td>Low</td>
<td>Wide</td>
</tr>
<tr>
<td>8</td>
<td>Sub-Vertical</td>
<td>Low</td>
<td>Close</td>
</tr>
</tbody>
</table>
Figure 3.17: Eight stochastic DFN representations in single-entry models from random seed 100.
Throughout this thesis the terms “blocky” and “very blocky” are used to describe the models analyzed. While no quantitative measure exists to determine if an explicit DEM model is blocky or very blocky, the descriptors in the geological strength index (GSI) for jointed rocks chart (Hoek & Marinos, 2000) qualify the models in this study as massive to very blocky and even seamy, depending on the DFN utilized and presence of the SUBI material. In the absence of SUBI material, DFNs 2, 3, 6, and 7 all fall under the “massive” to “blocky” regions based on their wide joint spacing. DFNs 1 and 4 were considered “blocky” because of their consistent rectangular block shapes, while DFNs 5 and 8 were considered “very blocky” due to their angular blocks.

Modeled bolts were 2.4 m long, 19 mm diameter, and installed on 1.2 m spacing with associated 10 cm long faceplates defined using UDEC’s liner elements. Bolt parameters were based on models calibrated by Bahrani and Hadjigeorgiou (2017) to laboratory axial and shear load tests on fully grouted rockbolts installed in concrete blocks (Table 3.3). Each bolt element was installed through and mechanically coupled with a faceplate modeled using the structural command in UDEC. Faceplates were modeled elastically using typical dimensions and appropriate properties of steel.

Table 3.3: Material properties for rockbolt and structural (i.e. faceplates) elements utilized in this study. E = Young’s Modulus, YS = Yield Strength, TFS = Tensile Failure Strain

<table>
<thead>
<tr>
<th>Area (mm²)</th>
<th>Density (kg/m³)</th>
<th>E (GPa)</th>
<th>Bolt YS (kN)</th>
<th>Bolt TFS (strain)</th>
<th>Plastic Moment (kN-m)</th>
<th>Shear Stiffness (MN/m²)</th>
<th>Normal Stiffness (MN/m²)</th>
<th>Shear Cohesion (MN/m)</th>
<th>Normal Cohesion (MN/m)</th>
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</thead>
<tbody>
<tr>
<td>Bolts</td>
<td>280</td>
<td>8050</td>
<td>200</td>
<td>176</td>
<td>0.15</td>
<td>2</td>
<td>50</td>
<td>100000</td>
<td>1.2</td>
</tr>
<tr>
<td>Plates</td>
<td>600</td>
<td>8050</td>
<td>200</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

3.2.3 Model CMRR & ARBS; Determination

In order to compare UDEC models with empirical observations, a CMRR value was estimated for each model using the tables provided in Molinda & Mark (1994). Block material cohesion and friction angle values were used to determine intact strength ratings by calculating a field-scale unconfined compressive strength (UCS*) via the following equation:

\[ UCS^* = \frac{2c\cdot \cos\varphi}{1-\sin\varphi} \]  

(3.18)

where \( c \) = cohesion, and \( \varphi \) = friction angle of the intact block material. Note that SUBI cohesion and friction angle were not considered in the determination of the CMRR strength rating. The reason for this is that CMRR has adjustment factors applied at the end of the process to account
for multiple discontinuity sets in laminated rockmass, which the SUBI constitutive model represents in the model. The UCS* values and their respective CMRR strength ratings are depicted in Table 3.4.

Table 3.4: Intact block material type, approximate UCS* from Eqn. (3.18), and assigned CMRR strength rating for roof stability models; note that elastic block materials effectively have infinite strength and were therefore assigned the highest possible intact strength rating.

<table>
<thead>
<tr>
<th>Block Material Type</th>
<th>UCS* (MPa)</th>
<th>CMRR Strength Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak SUBI</td>
<td>7.85</td>
<td>10</td>
</tr>
<tr>
<td>Moderate SUBI</td>
<td>17.3</td>
<td>13</td>
</tr>
<tr>
<td>Strong SUBI</td>
<td>24.0</td>
<td>15</td>
</tr>
<tr>
<td>EBP</td>
<td>62.9</td>
<td>22</td>
</tr>
<tr>
<td>Soft Elastic</td>
<td>N/A</td>
<td>30 (max)</td>
</tr>
<tr>
<td>Stiff Elastic</td>
<td>N/A</td>
<td>30 (max)</td>
</tr>
</tbody>
</table>

Determination of the cohesion-roughness, spacing-persistence, and multiple discontinuity adjustment relied on explicit joint (i.e. non-ubiquitous joints) initial friction angle, DFN spacing and persistence, bed thickness, and presence of the SUBI material model. First, a determination of the cohesion-roughness and spacing-persistence ratings due to explicit horizontal beds or presence of SUBI elements was made. If SUBI material properties were not used, the cohesion-roughness rating for both horizontal bedding planes (i.e. “bed rating”) and vertical or sub-vertical cross-joints (i.e. “joint rating”) were determined independently based on the explicit discontinuity strength (Table 3.5).

Table 3.5: Explicit discontinuity properties, joint analog description of cohesion and roughness, and assigned CMRR cohesion-roughness ratings for roof stability models.

<table>
<thead>
<tr>
<th>Initial Friction Angle</th>
<th>CMRR Description</th>
<th>CMRR Cohesion-Roughness Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>35°</td>
<td>Moderate Cohesion - Wavy</td>
<td>27</td>
</tr>
<tr>
<td>25°</td>
<td>Weak Cohesion – Planar</td>
<td>16</td>
</tr>
<tr>
<td>15°</td>
<td>Slickensided - Planar</td>
<td>10</td>
</tr>
</tbody>
</table>

Similarly, the spacing-persistence rating was determined for bedding planes based on their spacing, and for DFN cross-joints based on their average spacing and persistence (i.e. trace) (Table 3.6). Note that CMRR does not consider joint orientation when calculating spacing-persistence ratings.
Table 3.6: DFN ID, model inputs of discontinuity spacing and persistence, and assigned CMRR spacing-persistence rating.

<table>
<thead>
<tr>
<th>DFN ID</th>
<th>Spacing (m)</th>
<th>Persistence (m)</th>
<th>CMRR Spacing-Persistence Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bedding Plane</td>
<td>0.5</td>
<td>Continuous</td>
<td>20</td>
</tr>
<tr>
<td>Bedding Plane</td>
<td>1.0</td>
<td>Continuous</td>
<td>25</td>
</tr>
<tr>
<td>1 &amp; 5</td>
<td>0.25</td>
<td>4.0</td>
<td>20</td>
</tr>
<tr>
<td>2 &amp; 6</td>
<td>1.0</td>
<td>4.0</td>
<td>25</td>
</tr>
<tr>
<td>3 &amp; 7</td>
<td>1.0</td>
<td>2.0</td>
<td>27</td>
</tr>
<tr>
<td>4 &amp; 8</td>
<td>0.25</td>
<td>2.0</td>
<td>21</td>
</tr>
</tbody>
</table>

If the SUBI constitutive model was not used, then the smaller value between “bed rating” and “joint rating” was selected for the CMRR calculation. Then, the multiple discontinuity adjustment was applied by deducting 5, 4, or 2 points from the final rating if the two discontinuity ratings (i.e. “bed rating” and “joint rating”) were both lower than 30, 40, and 50, respectively. This adjustment, coupled with the intact strength rating, and the lower of the “bed rating” or “joint rating” were combined to determine the final CMRR of a given model roof. The groundwater sensitivity rating was not considered, as groundwater was not incorporated in the models.

The SUBI constitutive model was applied in three block material cases in this portion of the study. The SUBI reference plane of weakness was set to 0° from the global x-y plane on the model to mimic the anisotropic effect of sub-bedding planes of weakness (i.e. laminae). Therefore, the presence of the SUBI elements in a given model only adjusted the aforementioned “bed rating” by overriding the previously assigned cohesion-roughness and spacing-persistence “bed ratings” based on the maximum zone size of the model (0.125 m) (i.e. “spacing” of ubiquitous joints) and SUBI strength properties (i.e. “cohesion” of ubiquitous joints) (Table 3.7).

Table 3.7: Adjusted “bed rating” values for cohesion-roughness and spacing persistence ratings due to the presence of the SUBI elements.

<table>
<thead>
<tr>
<th>Block Material Type</th>
<th>Adjusted Cohesion-Roughness “Bed Rating”</th>
<th>Adjusted Spacing-Persistence “Bed Rating”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak SUBI</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>Moderate SUBI</td>
<td>24</td>
<td>11</td>
</tr>
<tr>
<td>Strong SUBI</td>
<td>27</td>
<td>13</td>
</tr>
</tbody>
</table>

The ARBS_i value (i.e. bolting intensity) is calculated using Eqn. (3.10) (Mark et al., 2001) based solely on bolt parameters (i.e. spacing, length, strength, etc.) and is therefore identical for each bolted model in this chapter. In unsupported models, validation of model results as an accurate approximation of reality through application of empirical systems required that the absence of bolts be represented using an ARBS_i value of 0. Note that both CMRR and ARBS were developed
based on empirical studies of supported mine entries. Therefore, while an ARBS; value of 0 is not necessarily outside of the empirical system, it is the unverified end-member case of that system. Finally, the suggested ARBS value \( \text{ARBS}_G \), Eqn. (3.12)) and the suggested intersection span \( \text{Is}_g \), Eqn. (3.11)) were calculated based on depth to entry and CMRR for each model.

3.2.4 Stability Determination

Models that did not converge to an equilibrium solution ratio, either due to “contact overlap” or “block rounding” error, were visually confirmed as unstable by plotting the deformed block geometries, displacements, and velocities at the timestep where error occurred. Most models that incurred these errors had significant roof collapse, while others had high block velocities (i.e. > 1.0 m/s) in the immediate roof at solution ratios far from equilibrium and were also classified as unstable.

Many unstable models and all stable models ultimately converged to a standard equilibrium solution ratio (i.e. \( 1.0(10)^{-5} \)). Prior to extracting equilibrium model results such as gridpoint displacements, velocities, and zone stresses, a FISH function was utilized to check the velocity of model roof and ensure that the roof had sufficiently stabilized. The use of the default solution ratio based on the total average unbalanced forces in the model can stop the model in a state of pseudo-equilibrium when looking strictly at roof stability. However, this was only the case for models at a borderline stability, so changing the solution ratio type, or decreasing the solution ratio magnitude below \( 1.0(10)^{-5} \) would unnecessarily be applied to extremely stable or extremely unstable models. Therefore a method based on observation of roof velocities was developed. If the average velocity of the roof was greater than 0.05 m/s, the model was stepped an additional 50,000 steps and visually checked to confirm that roof movement had either decelerated and stabilized or accelerated and collapsed. This threshold was selected based on visual inspection of approximately 100 preliminary cases where models with average immediate roof velocities higher than 0.2 m/s at the equilibrium solution ratio typically accelerated to collapse. A factor of 4 was applied to make sure that no cases were missed and that single falling blocks could be captured as well. Several cases that had reached the default equilibrium criterion in UDEC were ultimately unstable when this velocity check was implemented. Note that the velocities discussed in regard to explicit DEM models do not represent real velocities, because a given timestep in the model is not a unit of time in the real world, but part of the explicit DEM formulation.
Stability of model results could not realistically be determined by visual methods for all of the 8,640 roof-stability models. First, any model that incurred “contact overlap” or “block rounding” error was visually confirmed to be unstable at the time the error occurred. All the remaining models reached an equilibrium solution ratio and had vertical displacement and velocity of each gridpoint in the immediate roof, as well as zone stresses in the immediate roof extracted and saved for analysis. This allowed for an automatic stability classification scheme based on model results in MATLAB.

If any portion of the roof displaced greater than 0.5 m at model equilibrium (i.e. following velocity check), the model was classified as unstable. If the average velocity of the entire immediate roof (i.e. sum of the velocities of every roof gridpoint divided by the number of roof gridpoints) following the velocity check command was greater than 0.05 m/s, the model was classified as unstable. This velocity classification was redundant to ensure that all visually classified models were correctly recorded. Finally, if the maximum horizontal stress anywhere in the immediate roof beam was less than half of the predicted value based on the voussoir analog predicted stress plus the in-situ horizontal stress (i.e. “Total Stress Method”, see Section 2.5), then the model roof was classified as unstable. Note that the “Total Stress Method” had the propensity to overpredict single-entry stresses in Chapter 2 models by 1.5 times, requiring the 0.5 times adjustment. This was based on two observations. The first observation, from the results of Chapter 2 voussoir beam models, is that roof self-stability could not be maintained in elastic or inelastic block voussoir beams without maintaining horizontal stress arching. The second observation, from the model results presented in this chapter, was the propensity for some unsupported models to incur significant yield in the immediate roof and completely destress, but the model roof would not collapse due to the inability for the explicit DEM to model the rupture of intact blocks and the preferential orientation of some of the DFN joints. This would result in a stable classification based on displacement and velocity alone.

Under self-stable conditions, as the immediate roof deflected downwards, internal horizontal stresses developed in agreement with the voussoir beam analog described in Chapter 2. The baseline Diederichs & Kaiser (1999) analytical solution predicted that if the immediate roof layer in Figure 3.18c was a voussoir beam, it would generate approximately 0.7 MPa of horizontal compression. The in-situ horizontal stress was approximately 0.7 MPa, and therefore the “Total Stress Method” correctly indicated that 1.4 MPa would be the maximum midspan stress. If the
horizontal stress in the immediate roof at equilibrium was significantly lower than that value, it indicated that the immediate roof was not effectively transferring horizontal stress and was therefore likely unstable. If the model results indicated that all of the previously described conditions had not been met, then it was classified as stable.

Bolted models did not incorporate this voussoir stability check, as a yielded or destressed roof can be stabilized by bolt elements through skin support, suspension, or non-voussoir beam building. Bolted model stability classification only considered displacement and velocity as previously described.

### 3.3 Model Setup Attributes

Prior to the full sensitivity analysis considering all parametric combinations, an initial sensitivity analysis was conducted to evaluate the effects of the model setup attributes discussed in Section
3.1.3 on model results. Due to the large number of models, their wide range of geologic and mining conditions, and their blocky nature, issues related to model accuracy, roof stability, overburden behavior, “contact overlap” error, and model runtimes were anticipated. A model setup that would minimize “contact overlap” error, but still accurately capture roof stability was required. Additionally, from a practical standpoint, reducing model runtimes would be an auxiliary benefit, particularly for advancing the state-of-practice regarding explicit DEM modeling.

Eight baseline models spanning a range of material and loading conditions were selected to conduct a model setup attribute sensitivity analysis on; their material properties are listed in Table 3.8.

Table 3.8: Eight baseline models selected for model setup attribute sensitivity analysis based on a range of anticipated stability.

<table>
<thead>
<tr>
<th>Model ID</th>
<th>Depth (m)</th>
<th>In-Situ Stress Ratio</th>
<th>Block Material Type</th>
<th>DFN ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>30</td>
<td>1</td>
<td>EBP</td>
<td>1</td>
</tr>
<tr>
<td>P2</td>
<td>30</td>
<td>1</td>
<td>EBP</td>
<td>7</td>
</tr>
<tr>
<td>P3</td>
<td>30</td>
<td>1</td>
<td>Strong SUBI</td>
<td>7</td>
</tr>
<tr>
<td>P4</td>
<td>30</td>
<td>1</td>
<td>Weak SUBI</td>
<td>1</td>
</tr>
<tr>
<td>P5</td>
<td>200</td>
<td>1</td>
<td>EBP</td>
<td>1</td>
</tr>
<tr>
<td>P6</td>
<td>200</td>
<td>1</td>
<td>EBP</td>
<td>7</td>
</tr>
<tr>
<td>P7</td>
<td>200</td>
<td>1</td>
<td>Strong SUBI</td>
<td>7</td>
</tr>
<tr>
<td>P8</td>
<td>200</td>
<td>1</td>
<td>Weak SUBI</td>
<td>1</td>
</tr>
</tbody>
</table>

Model setup parameters analyzed in this section include the use of a FISH falling block deletion command, block and contact detection logic (i.e. `config cell` and `set cscan`), mechanical damping mode (i.e. “local” vs. “combined”), graded mesh density, and changes in overlap tolerance (i.e. `set ovtol`).

Each of the baseline models selected (P1-P8) were run with the default UDEC model setup parameters, then with each of the aforementioned model setup attributes implemented individually, as well as with all methods simultaneously. Binary classifications of roof stability were not impacted for stable (i.e. P1-P3 and P5-P7) or unstable (i.e. P4 and P8) models. Furthermore, impacts to other model results like height of caving, stable roof displacement, and maximum midspan stresses were minimal.

Note that the results of stepped joint stiffness on models that incurred “contact overlap” error during stress initialization are also considered in this section, but this analysis was conducted on 20 random stable and unstable models, not Model IDs P1-P8 listed in Table 3.8.
3.3.1 Graded Zone Size

Based on the findings in Section 2.3.2 and recommendations by Itasca (2014), an optimal block rounding should not be larger than approximately 20% of the zone edge length to match voussoir analytical predictions of stress and displacement. Additionally, the block rounding value should be approximately 1% the representative block edge length. Based on the results of block and mesh sensitivity analysis in Chapter 2, a sufficiently fine mesh was applied to the roof and overburden consisting of fine zones (0.125 m maximum edge length) above the entry and coarse zones (0.250 m maximum edge length) above the pillars, and coarser zones (0.5 maximum edge length) in the rest of the overburden (Figure 3.19a). The graded mesh had no impact on the binary stability outcome of the models tested and no discernable impact on stable models as indicated by the displacement contours in the roof and overburden (Figure 3.19b). Accordingly, the graded mesh was selected for use in the parametric sensitivity analysis.

![Image](https://example.com/image.png)

Figure 3.19: Comparison of uniform and graded (a) mesh densities and (b) their respective vertical displacement results of stable model P5.

3.3.2 Mechanical Damping

Model damping reduces the total kinetic energy of the model and is dependent on block velocities (Itasca, 2014). UDEC’s default damping method for static state solutions is “local”, set at a default
value of 0.8. “Auto” damping is typically used to absorb model vibrational energy as rapidly as possible (Itasca, 2014). Changing model damping from “local” to “auto” did not incur major changes to model results as vibrational energy in this model is zero. This is due to the lack of seismic waves, rockburst, or blasting being considered in the models.

Alternatively, “combined” damping is useful for models with constant, unidirectional block movement (Itasca, 2014). While initializing in-situ stresses, small changes in velocity sign may occur throughout blocky models due to compounding force reactions with neighboring blocks. This was confirmed by an approximately 3000% increase in in-situ stress initialization runtime with “combined” damping when compared to “local” damping, highlighting inappropriate application of the non-default damping. However, implementing “combined” damping after stress initialization had a negligible impact on changing stable model displacement and maximum compressive stress in the immediate roof, and runtime of unstable models decreased significantly. Furthermore, “combined” damping did increase the deformation above the roof, indicating that the “local” damping had overdamped the system due to the unidirectional downward block movement (Figure 3.20).

![Figure 3.20: Comparison of vertical displacement results of baseline default “local” damping (left) model P4 and model P4 implementing “combined” damping (right). Note the increased deformation (red line) in the overburden.](image)

180
Ultimately, “combined” damping has underdamped the system compared to “local” damping, allowing all the blocks that have yielded and displaced to continue displacing and detaching from stable portions of the overburden. Since “local” damping relies on velocity sign changes to efficiently damp a given system, each timestep where damping values are updated and the sign remains negative (i.e. downward displacement), “local” damping overdamps a given system in comparison to “combined”. While neither damping mode is inherently more realistic than the other, “combined” damping is more applicable to all the models in this study following stress initialization due to the generally unidirectional block movement during model runs, and the overall decrease in model runtimes. Furthermore, in a marginally stable case, “combined” damping would be a more conservative representation of roof stability, tending toward collapse rather than stability. Based on consideration of these results and those from other explicit DEM modeling cases (Sinha et al., 2020) it was concluded that “combined” damping mode is most appropriate for models where large deformations or block detachments are expected.

3.3.3 Contact Mapping & Detection

Mapping all possible contacts of a blocky model using the config cell command and updating the contact detection at every timestep via the set cscan 1 command are two model setup attributes, the effects of which are not well-documented in the literature. Since this chapter is particularly concerned with how the interaction between discontinuities and blocks affects roof stability, their impact had to be qualified prior to implementation in the full parametric sensitivity analysis. First, Itasca (2014) states that cell mapping is useful for blocky models as it creates and stores an address of every block and its possible contacts with every other surrounding block in a given model. Furthermore, updating the coordinates of block corners at every timestep using set cscan 1 (default is every 100 timesteps) will ensure that the highest degree of accuracy is maintained by reducing the potential for “contact overlap” error to occur at block contacts between the default 100 timesteps.

While the reported positive effects (e.g. increased accuracy, lower propensity for overlap tolerance) of these two model setup attributes were not captured in stable or unstable model results, the use of config cell and set cscan 1 did not significantly impact model results (Figure 3.21). This ensured that the stated benefit of their use was not negatively impacting the accuracy and consistency of the parametric sensitivity analysis.
3.3.4 Overlap Tolerance

Overlap tolerance is a required parameter in UDEC due to the soft contact algorithm utilized. When the contacts of two blocks overlap, the magnitude of overlap is utilized in resolving the equations of motion governing the model. If the overlap displacement exceeds the default tolerance value of 0.005 m, the displacements and stresses calculated by the model are subject to larger and larger error. However, due to the general blocky to very blocky nature of many of the models considered in this study, an overlap tolerance of 0.01 m was tested to reduce the propensity for “contact overlap” error to occur prior to the stabilization or collapse of the immediate roof. The effect of increased overlap on stress distribution in the stable 200 m deep “Strong SUBI” block material model is depicted in Figure 3.22.
Stable model results are indistinguishable from the default. Similarly, unstable model results were also identical with an increased overlap tolerance. Increased overlap tolerance in unstable models that incurred “contact overlap” error naturally led to higher deformation by virtue of the model stepping further than its overlapped default model counterpart. With all this in mind, an overlap tolerance increased from 0.005 (default) to 0.01 m was utilized in the parametric sensitivity analysis as it did not affect stable model results and allowed many unstable models to reach an equilibrium solution ratio by preventing “contact overlap” error.

3.3.5 Falling Block Deletion

In unstable models, deletion of fully separated and actively falling blocks (i.e. displacement greater than 0.8 m) via FISH function every 5000 model timesteps reduces the unbalanced forces and decreases the likelihood of a “contact overlap” error occurring between various falling blocks or falling blocks and the pillars or floor. This theoretically allows the model to finish running and reach an equilibrium solution ratio in a shorter amount of time without influencing roof stability results. Application of this method to the limited model suite was successful in both reducing the ratio of unbalanced forces and delaying “contact overlap” error in unstable models without significantly impacting the height of caving (Figure 3.23).
Figure 3.23: Comparison of vertical displacement results of default model P4 and model P4 implementing the automatic block deletion FISH function.

It is evident that in unstable cases where a caving style failure occurs, block deletion slightly increases deformation in the overburden because collapsed blocks are no longer providing support, and because the model was able to step further by delaying “contact overlap” error. Empirical observations show that the collapse height is typically 3 times the height of the mined seam (Adhikary & Guo, 2009; Majdi et al., 2012), indicating that the model results will only be significantly impacted by block deletion command in the event that caving reaches higher than 7.5 m above the entry. Additionally, block deletion is very effective at reducing model run-time when used in marginally unstable cases where only the immediate roof (i.e. one layer) or a single block fails and was therefore implemented in the full sensitivity analysis considering all parametric combinations. The block delete FISH function had no impact on stable model results, as no stable model blocks deflected enough to be selected for automatic deletion.

Note that the last recorded displacement and velocity are still stored when a block is deleted; therefore, models that reached an equilibrium solution ratio and had deleted blocks were able to be automatically classified via the previously described model extractions from the deleted roof block material.
3.3.6 Stepped Joint Stiffness Stress Initialization

Some DFNs analyzed in this study were susceptible to “contact overlap” issues during stress initialization due to the creation of thin, sliver-shaped blocks in the model roof and overburden. This affected approximately 30 models from DFN ID 5 for both random seeds tested. Models which incurred “contact overlap” error during stress initialization were re-initialized with higher joint stiffnesses (i.e. jkn = 5000 GPa/m, jks = 500 GPa/m). Following successful stress initialization with stiffer joints, the joint stiffness was decreased by an order of 10 and the model solved. This was repeated until the stresses were initialized with the originally planned joint normal and shear stiffness (jkn = 50 GPa/m, jks = 5 GPa/m). Following this modified stress initialization process, the models were run in accordance with the aforementioned four-stage solution method. For the purposes of comparison, this two-stage joint stiffness assignment process was utilized on a subset of 10 stable and 10 unstable models that did not originally encounter “contact overlap” error during stress-initialization, with no associated change in the model results. This indicates that results are comparable between the two methods, and that this is a viable solution to the “contact overlap” issue encountered during stress initialization.

3.4 Parametric Sensitivity Analysis Results

While visual evaluation of all 8,640 model results is not practical, selected results are valuable in visualizing the range of roof stability conditions simulated by the models. Model results were broadly classified into stable and unstable roofs; however, the types of instability noted during visual evaluation of unsupported model results were indicative that a wide range of geologic and mining conditions were captured (Figure 3.24).
Most cases were easily parsed into stable and unstable based on immediate roof displacement and velocity at model equilibrium. However, recall the horizontal stress stability determination step described in Section 3.2.4 and depicted in Figure 3.18c was required to confirm that roof self-stability was fully maintained through horizontal stress arching and avoid a false stable classification (i.e. pseudo-stability). Some models exhibited pseudo-stability following the velocity check in Stage 4 where velocities and displacements indicated a stable condition, but a large portion of the immediate roof had incurred yield and was transferring significantly lower levels of horizontal stress than the contiguous roof layers above the entry. The geometry of the DFN, coupled with the limitations of the explicit DEM (i.e. no intra-block rupture) may artificially stabilize this unsupported condition. As a result, all pseudo-stable models were classified as unstable because the immediate roof beam was not transferring horizontal stresses effectively in accordance with the aforementioned “Total Stress Method” (Figure 3.25).
Figure 3.25: Model results that detail the pseudo-stable condition from Figure 3.24 where (a) displacements and (b) velocity indicate that the roof is stable, but (c) horizontal stress transfer in the immediate roof, and material yield indicate that this model result would likely be unstable under in-situ conditions.

In many cases, the addition of roof support adequately stabilized previously unstable model roofs and decreased deflection of self-stable roofs. Similar to their unsupported counterparts, bolted model results exhibited a range of observed behaviors upon visual inspection (Figure 3.26).
Rockbolt elements could yield through failure of the bolt-grout-rock interface, where the stresses overcome the shear or normal strength of the interface, or through failure of the bolt itself when internal bolt forces lead to the tensile failure strain of the steel that the rockbolt element is modeling being exceeded. In the case of the “squeeze failure” in Figure 3.26, the bolt element (i.e. “steel” portion of the rockbolt) on the left side of the entry has failed and ruptured due to differential movement along the axis of the bolt element, simulating a fully yielded rockbolt. Conversely, the bolts in the relatively large stable displacement case in Figure 3.26 have all failed along their interfaces, while the bolt “steel” remains intact, indicating that more uniform displacement in that rockmass has separated the bolt from the interface, common for self-stabilizing roof conditions.

Many unsupported models were observed to transition from unstable to stable when bolts were added, and the mechanism by which that stability was enforced also encompassed the range of bolt
support mechanisms observed in-situ. Model bolts provided stability to the roof via simple skin support, suspension, and beam building, or some combination of the three, depending on the geologic and mining conditions of the model (Figure 3.27). Note that displacement contrasts between the bolted interval and the remaining overburden in bolted models was compared to the displacement and failure of unsupported models to broadly consider the bolt support mechanisms. For example, bolting that engages support in the roof via skin support and suspension does not significantly change the degree of deformation in the overburden, while the beam building mechanism decreases dilation between buckled layers in the overburden and decreases the height of major (i.e. > 10 mm) deformation by approximately 3 m.

![Figure 3.27: Vertical displacement contours of unstable unsupported models (top) and their stable bolted (bottom) counterparts at model equilibrium depicting the three main bolt support mechanisms encountered in this study. Bolt size exaggerated for clarity.](image)

Recall that the displacement and velocity of the model at equilibrium were sufficient to accurately classify bolted models as stable or unstable. If any portion of the immediate roof in bolted models
displaced greater than 0.5 m, or if the average velocity exceeded 0.05 m/s following implementation of the velocity check command, the bolted model result was classified as unstable. This is conservative, considering that many coal mine entries in the field remain serviceable even after large roof convergence and small block falls. However, due to the plane-strain nature of the two-dimensional models herein, any amount of failure that significantly deforms the roof is assumed to be occurring along the entirety of a theoretically infinite-length entry.

3.4.1 Verification of Model Results via Empirical Methods

While visual confirmation that a wide range of geologic and mining conditions is valuable, a more robust model verification method is required. Using the model-calculated CMRR along with model depth to entry and intersection span, model results for unsupported and bolted entries were analyzed in relation to the ARBS discriminant (Mark et al., 2001). The ARBS discriminant (Eqn. 3.13) is an empirically determined linear relationship that relates the difference between the actual (i.e. model) and recommended (i.e. CMRR and depth dependent) bolt intensity, with the difference between the actual and recommended Intersection Span (Is). The empirical discriminant correctly classifies 76% of coal mine entries observed in-situ as stable or unstable (Mark et al., 2001).

3.4.1.1 Unsupported Models

Unsupported models correspond to an actual bolt intensity value \( \text{ARBS}_i \) of 0, and an actual Intersection Span \( \text{Is} \) of 6 m. Suggested values were calculated based on model depth and the model CMRR estimates according to Eqns. (3.12) and (3.11). Results of the comparative analysis for all unsupported model results for each DFN random seed are presented in Figure 3.28. For a given point on the ARBS discriminant, a decrease in ARBS Difference (x-axis) value indicates that a given entry is more supported (i.e. higher bolt intensity) than what the empirical discriminant predicts is needed to maintain a stable entry. Conversely, a decrease in Span Difference (y-axis) indicates that the actual span is larger than what is recommended based on empirical relationships. Cases that plot above the ARBS discriminant are predicted to be stable, and cases that plot below are predicted to be unstable, while the actual model result is shown as green (stable) and red (unstable).
Figure 3.28: Results of unsupported models with random DFN seed (a) 100 and (b) 1234 and their predicted stability based on empirical formulae from Mark et al. (2001); models that plot above the ARBS discriminant are predicted to be stable by the empirical relationship; marker size indicates the number of models represented by a single data point.

The results indicated that empirical calculations and predictions based on model inputs accurately captured the relationship between CMRR, ARBS, entry depth, and intersection span with 90.8% correct classification for random seed 100 and 90.5% for random seed 1234. A confusion matrix of correctly and incorrectly predicted stable and unstable models is provided in Table 3.9. The 0.3% difference in unstable prediction accuracy between models with DFN random seed 100 and 1234 indicated that the specific stochastic distribution of joints in the immediate roof is not a critical control on roof stability for the DFN cases tested (i.e. where there is no distinct major discontinuity or fault that dominates roof behavior) and that generalized DFNs (i.e. non-unique) can be used to understand overall system behavior in unsupported models when considering a wide range of geologic and mining conditions.

Table 3.9: Confusion matrix showing percentage of stable and unstable unsupported model results predicted correctly (*italics*) and incorrectly by the ARBS discriminant based on model inputs for both random seeds.

<table>
<thead>
<tr>
<th>Model Results Seed 100</th>
<th>Stables</th>
<th>Unstables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable</td>
<td>63.4%</td>
<td>7.8%</td>
</tr>
<tr>
<td>Unstable</td>
<td>1.4%</td>
<td>27.4%</td>
</tr>
<tr>
<td>Model Results Seed 1234</td>
<td>Stables</td>
<td>Unstables</td>
</tr>
<tr>
<td>Stable</td>
<td>63.4%</td>
<td>8.1%</td>
</tr>
<tr>
<td>Unstable</td>
<td>1.4%</td>
<td>27.1%</td>
</tr>
</tbody>
</table>
In order to illustrate which conditions most impact the error between the ARBS method stability predictions and the model results, the percent correct classification for each model input and both random seed numbers are shown in Table 3.10.

Table 3.10: Changes in percent correct classification between ARBS method stability predictions for each unsupported parametric case. Percent correct classification for each input parameter below the total percent correct predicted depicted in Figure 3.28 are italicized. The absolute difference in error between seed numbers for every input tested, as well as the absolute difference in error range between the highest and lowest value in each parametric case for each random seed is reported.

<table>
<thead>
<tr>
<th>Parametric Case</th>
<th>Value</th>
<th>Percent Correct Classification</th>
<th>Seed Error [Difference]</th>
<th>Range [Difference]</th>
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<tbody>
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<td></td>
<td></td>
<td>Seed 100</td>
<td>Range</td>
<td>Seed 1234</td>
</tr>
<tr>
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<td>96.0%</td>
</tr>
<tr>
<td></td>
<td>100 m</td>
<td>89.5%</td>
<td>8.80%</td>
<td>88.8%</td>
</tr>
<tr>
<td></td>
<td>200 m</td>
<td>86.9%</td>
<td>8.70%</td>
<td>86.7%</td>
</tr>
<tr>
<td>In-Situ Stress Ratio</td>
<td>0.5</td>
<td>88.2%</td>
<td>4.20%</td>
<td>88.2%</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>91.8%</td>
<td>91.4%</td>
<td>91.8%</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>92.4%</td>
<td>91.8%</td>
<td>91.8%</td>
</tr>
<tr>
<td>Roof Block Material Model</td>
<td>Weak SUBI</td>
<td>100.0%</td>
<td>24.3%</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td>Moderate SUBI</td>
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<td>84.5%</td>
<td>84.5%</td>
</tr>
<tr>
<td></td>
<td>Strong SUBI</td>
<td>75.7%</td>
<td>75.9%</td>
<td>75.9%</td>
</tr>
<tr>
<td></td>
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<td>84.0%</td>
<td>84.3%</td>
<td>84.3%</td>
</tr>
<tr>
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<td>Soft Elastic</td>
<td>98.2%</td>
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<td>98.8%</td>
</tr>
<tr>
<td></td>
<td>Stiff Elastic</td>
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<td>99.3%</td>
<td>99.3%</td>
</tr>
<tr>
<td>Explicit Joint Strength</td>
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<td>78.9%</td>
<td>18.8%</td>
<td>79.5%</td>
</tr>
<tr>
<td></td>
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<td>95.7%</td>
<td>95.7%</td>
</tr>
<tr>
<td></td>
<td>Strong</td>
<td>97.7%</td>
<td>96.2%</td>
<td>96.2%</td>
</tr>
<tr>
<td>Bedding Thickness</td>
<td>0.5 m</td>
<td>89.7%</td>
<td>2.10%</td>
<td>90.6%</td>
</tr>
<tr>
<td></td>
<td>1.0 m</td>
<td>91.8%</td>
<td>90.6%</td>
<td>90.4%</td>
</tr>
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<td>DFN ID</td>
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</tr>
<tr>
<td></td>
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<td>94.8%</td>
<td>94.8%</td>
<td>94.8%</td>
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<tr>
<td></td>
<td>3</td>
<td>94.8%</td>
<td>94.8%</td>
<td>94.8%</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>7</td>
<td>93.5%</td>
<td>92.0%</td>
<td>92.0%</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>88.9%</td>
<td>87.0%</td>
<td>87.0%</td>
</tr>
</tbody>
</table>

The range in each random seed (i.e. columns 4 and 6 in Table 3.10) represents how variations in a given parameter (e.g. depth, block material model, etc.) impacted the accuracy of the ARBS method. The random seed error difference (i.e. column 7 in Table 3.10) represents how variations
in random seed (i.e. explicit joint location) influenced a tested value of each parameter (e.g. 30, 100, and 200 m deep entries). Lastly the difference in range error helps identify entire parameter sets that were sensitive to changes in DFN, rather than individual values.

As expected, all three joint-specific parameters (i.e. strength, DFN, and Bedding Thickness) were slightly more sensitive to the explicit changes in joint location and the sub-vertical joint DFNs (i.e. 5-8); all had higher variation in accuracy between the two seeds. Regardless, these results further confirm the previous findings that stochastic DFN random seed does not have a particularly critical role in impacting model results and thus the overall accuracy of the ARBS method applied to the numerical model results.

Interestingly, the accuracy of the ARBS method decreases with increasing depth but increases with increasing horizontal stress. Furthermore, prediction accuracy was much higher for end-member block material cases (i.e. “Weak SUBI” or “Stiff Elastic”), approaching 100% for both random seeds tested. Models with “Strong SUBI” roof blocks have the lowest accuracy of any tested value in any parametric case. This indicates that either the model CMRR values were not accurately calculated, the presence of SUBI joint elements cannot be accounted for in the CMRR empirical system through the bedding cohesion-roughness, persistence-spacing, and multiple discontinuity adjustment ratings, or other parameters are influencing the accuracy when applied to “Strong SUBI” roof blocks. Further investigation indicated that for the “Strong SUBI” block material model, the significant increase in error was for both stable and unstable predictions (Table 3.11). This increase in error tended towards overpredicting unstable conditions, which were in fact stable in the model result. These results initially indicated that the assigned CMRR value for the “Strong SUBI” roof block models may be overly conservative, but further isolation of parametric influence showed that when accounting for depth, the accuracy of the predictions decreased from 97.9% for 30 m deep entries to 52.8% for 200 m deep entries in models featuring “Strong SUBI” roof blocks. Similarly, decreases in explicit joint strength coupled with “Strong SUBI” roof blocks decreased accuracy from 88.9% to 54.2% with increasingly weaker explicit joints. This sensitivity to other parameters was because the average ARBS Difference for the “Strong SUBI” roof block model results were such that they all plotted very close to the ARBS discriminant, making changes in CMRR (i.e. explicit joint strength) or ARBS Difference (i.e. depth) have a significant effect on predicted stability.
Table 3.11: Confusion matrix showing percentage of stable and unstable unsupported model results predicted correctly (*italics*) and incorrectly by the ARBS discriminant for the Strong SUBI block material property and random seed 100.

<table>
<thead>
<tr>
<th>Model Results</th>
<th>Predicted Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seed 100</td>
<td>Stable</td>
</tr>
<tr>
<td>Strong SUBI</td>
<td>Stable</td>
</tr>
<tr>
<td></td>
<td>Unstable</td>
</tr>
</tbody>
</table>

Lastly, these results confirm that the ARBS method can be applied to unsupported cases with high accuracy by assuming an ARBS$_i$ value of 0, but that accuracy generally decreases for moderate to strong inelastic and anisotropic (i.e. SUBI) block material cases, particularly in tandem with increasing depth and decreasing joint strength.

3.4.1.2 Passively Bolted Models

Based on rockbolt element parameters and installation geometry utilized in the bolted models, an ARBS$_i$ value of 16.1 for the model support system was calculated from Eqn. (3.10) given by Mark et al. (2001). Based on the predictions of the ARBS method, all bolted models were expected to be stable (i.e. they all plotted above the ARBS discriminant) (Figure 3.29). A confusion matrix of correctly and incorrectly classified stable and unstable models is provided in Table 3.12.

![Figure 3.29: Results of bolted models with DFN random seed (a) 100 and (b) 1234 and their predicted stability based on empirical formulae from Mark et al. (2001); models that plot above the ARBS discriminant are predicted to be stable; marker size indicates the number of models represented by a single data point.](image-url)
Models with “Weak SUBI” block material were included in the bolted analysis even though they significantly decreased the overall accuracy of the empirical prediction. It was found that excessive bolting with an ARBS_i of 32.2 (i.e. 8 bolts) could not stabilize most of the “Weak SUBI” block material models, which comprised most of the incorrect predictions. Mark et al. (2001) noted that some of the case studies which make up the ARBS empirical data had extremely weak roofs such that they could not be supported by only roof bolts. Similarly, these “Weak SUBI” block material cases were outside of the empirical range within which ARBS is valid. Again, this relates to the inability of bolt systems to stabilize roof cases with extremely weak “intact” material (Mark et al., 2001) in both the real world and in the models. Further isolation of the parametric inputs provided insight into where the prediction error was most common (Table 3.13). Interestingly, many trends in accuracy reversed for bolted parametric cases when compared to their unbolted counterparts. Increases in in-situ stress ratio decreased prediction accuracy, and the impact of joint strength was reduced by the presence of roof support. Most notably, models featuring “Weak SUBI” roof blocks had very low accuracy, while the accuracy of the “Strong SUBI” roof block models increased substantially compared to the unsupported model results. Overall, the changes in absolute difference in error range remained consistent between bolted and unsupported models for depth, in-situ stress ratio, bedding thickness, and DFN. However, the accuracy between the random seeds for individual values in bolted model DFNs varied more than the unsupported models, indicating that the stochastic distribution of joints is more significant in determining stability in supported models. This is especially true in cases like vertical cross-joint DFNs 2 & 3, where wide joint spacing made the location of explicit discontinuities more impactful on prediction accuracy than in a more heavily jointed (i.e. close spacing) DFN or in an unsupported case.
Table 3.13: Changes in percent correct classification via model-applied CMRR/ARBS method for each bolted parametric case. Percent correct classification for each input parameter below the total percent correct predicted depicted in Figure 3.29 are italicized. The absolute difference in error between seed numbers for every input tested, as well as the absolute difference in error range between the highest and lowest value in each parametric case for each random seed is reported.

<table>
<thead>
<tr>
<th>Parametric Case</th>
<th>Value</th>
<th>Percent Correct Classification</th>
<th>Seed Error</th>
<th>Range</th>
<th>Difference</th>
<th>Range</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Seed 100</td>
<td>Range</td>
<td>Seed 1234</td>
<td>Range</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>30 m</td>
<td>84.6%</td>
<td>6.40%</td>
<td>83.5%</td>
<td>7.10%</td>
<td>1.10%</td>
<td>0.70%</td>
</tr>
<tr>
<td></td>
<td>100 m</td>
<td>83.2%</td>
<td>78.2%</td>
<td>76.4%</td>
<td>1.80%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>200 m</td>
<td>78.2%</td>
<td>82.5%</td>
<td>81.9%</td>
<td>4.70%</td>
<td>0.60%</td>
<td>2.00%</td>
</tr>
<tr>
<td>In-Situ Stress Ratio</td>
<td>0.5</td>
<td>83.9%</td>
<td>4.30%</td>
<td>82.3%</td>
<td>1.60%</td>
<td></td>
<td>0.40%</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>82.5%</td>
<td>79.6%</td>
<td>77.6%</td>
<td>2.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>79.6%</td>
<td>6.40%</td>
<td>83.5%</td>
<td>7.10%</td>
<td>1.10%</td>
<td>0.70%</td>
</tr>
<tr>
<td>Roof Block Material Model</td>
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<td>4.20%</td>
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<td>0.23%</td>
<td>3.97%</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>Moderate SUBI</td>
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<td>95.6%</td>
<td>95.6%</td>
<td>4.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strong SUBI</td>
<td>95.6%</td>
<td>95.6%</td>
<td>95.6%</td>
<td>4.00%</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>EBP</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>0.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Soft Elastic</td>
<td>99.5%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>0.30%</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>Stiff Elastic</td>
<td>99.8%</td>
<td>99.8%</td>
<td>99.8%</td>
<td>0.30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explicit Joint Strength</td>
<td>Weak</td>
<td>81.8%</td>
<td>0.50%</td>
<td>80.4%</td>
<td>0.50%</td>
<td>1.40%</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
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<td>80.9%</td>
<td>80.9%</td>
<td>1.40%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Strong</td>
<td>82.3%</td>
<td>82.3%</td>
<td>82.3%</td>
<td>1.40%</td>
<td></td>
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</tr>
<tr>
<td>Bedding Thickness</td>
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<td>0.0%</td>
<td>81.6%</td>
<td>2.00%</td>
<td>0.40%</td>
<td>2.00%</td>
</tr>
<tr>
<td></td>
<td>1.0 m</td>
<td>82.0%</td>
<td>0.0%</td>
<td>79.6%</td>
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</tr>
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<td>1.90%</td>
<td>1.00%</td>
</tr>
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<td>84.3%</td>
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<td>84.3%</td>
<td>81.2%</td>
<td>3.10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>81.2%</td>
<td>81.2%</td>
<td>82.1%</td>
<td>0.90%</td>
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<td></td>
</tr>
<tr>
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<td>78.7%</td>
<td>78.1%</td>
<td>0.60%</td>
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</tr>
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<td>80.6%</td>
<td>81.5%</td>
<td>0.90%</td>
<td></td>
<td></td>
</tr>
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<td>7</td>
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<td>88.0%</td>
<td>83.3%</td>
<td>4.70%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>82.4%</td>
<td>82.4%</td>
<td>75.0%</td>
<td>7.40%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Similarly, changes in random seed also increased the range of accuracy when applied to DFNs 7 & 8, which featured the low-persistence, sub-vertical cross-joints. The increase in error was compounded by a significant decrease in accuracy with decrease in joint spacing (i.e. DFN 8). These significant changes in accuracy between random seeds were not present in the high-persistence, sub-vertical joint cases (i.e. DFN 5 & 6) because the increase in joint length, which is accounted for in ARBS, overrides the effect of any single joint location.
These results highlight the local interaction of specific discontinuity location and other joint network geometric parameters on roof stability that are not accounted for by the CMRR system. Regardless, results of the ARBS empirical verification in the bolted models further confirmed that a wide range of geologic and mining conditions were accurately captured in the parametric sensitivity analysis of roof stability. Both bolted and unsupported model results were accurately predicted by the CMRR and ARBS empirical methods applied to the models. The accuracy of the actual ARBS method based on the original study was 76%, whereas its application to numerical models herein exceeded that accuracy in both unsupported and bolted model suites. This was likely due to two main effects: first, the geologic heterogeneity of a given entry in-situ is far greater than the homogeneous, uniform roof stability models in this study; second, the intersection span for all models remained constant (i.e. 6 m) for this portion of the study, while the intersections of the empirical data set did not. This can be observed in the narrow range of vertical axis values in Figure 3.28 and Figure 3.29 when compared to the wide range of horizontal axis values. Regardless, the results are consistent with the empirically derived ARBS discriminant and indicate that the models are behaving realistically and have captured a wide range of geologic and mining conditions.

3.4.2 Binary Roof Stability Sensitivities

3.4.2.1 Unsupported Roofs

With the roof stability models having been confirmed to approximate a wide range of realistic conditions, the ultimate objective of this section is to identify the most statistically significant inputs governing roof stability. Models resulting in stable or unstable conditions were tallied for different parametric cases (Figure 3.30). Since there was no meaningful difference in roof stability model results between random seed 100 and 1234, the random seed 100 and 1234 results are combined for the remainder of the analyses presented in this chapter.
Figure 3.30: Number of unsupported models that were classified as stable or unstable for every parametric category tested from both random seeds. $\phi_i$ = initial friction angle, Block Material Type 1 = “Stiff Elastic”, 2 = “EBP”, 3 = “Soft Elastic”, 4 = “Strong SUBI”, 5 = “Moderate SUBI”, 6 = “Weak SUBI”. Refer to Table 3.2 for DFN ID properties.

Entry depth increases in-situ stresses, thus inducing more yield along discontinuities and within intact material and increasing the number of unstable models. The “EBP” (i.e. Block Material Type 2 in Figure 3.30) and “Stiff Elastic” (i.e. Block Material Type 1 in Figure 3.30) cases showed similar results with respect to roof stability. DFN type and bedding thickness had minor effects on model stability, with sub-vertical cross-joints resulting in more unstable conditions than vertical cross-joints. Sub-vertical joints increase the likelihood of shear sliding under the action of horizontal stresses, diminishing compression arch formation and increasing instability. This phenomenon has been studied previously in simplified roof stability models by Ran et al. (1994) and observed in Section 2.9 of this thesis. However, some cases of sub-vertical joints with preferential orientation and spacing (i.e. DFN 6 and 7), coupled with the “Moderate SUBI” and “Strong SUBI” block material, increased the possibility of false stable classification, as discussed in Section 3.4.1.
Changes in joint strength did not significantly increase or decrease the stability of the immediate roof if the horizontal stress transfer was sufficient across discontinuities to mobilize friction and stabilize individual blocks, in accordance with voussoir beam mechanics governing abutment slip failure. This agrees well with the variable accuracy of the ARBS method applied to unsupported model results, where the decreased model joint strength resulted in a decreased CMRR and increased ARBS Difference. This resulted in a model that was stable, but that was predicted to be unstable by the ARBS empirical method. If explicit joint strength had a significant impact on unsupported model stability, this would be captured as less variation in prediction accuracy (Table 3.10). The mechanism behind this may be due to multiple factors, none of which are mutually exclusive. Are joint conditions too heavily weighted in CMRR, are the joint frictions modeled in UDEC not representative of a wide enough range of joint conditions, or are other parameters unaccounted for in CMRR affecting the influence of joint strength? Further analysis was needed to clarify why modeled joint strength in unsupported models was having an impact on prediction accuracy, and if other parameters were increasing or decreasing its overall effect.

A statistical analysis was required to better understand the interaction of explicit DEM inputs and their influence on model roof stability, rather than on a case-by-case basis. Specifically, BLR was utilized to identify the impact that categorical input variables (e.g. block material type, DFN ID) and continuous input variables (e.g. depth, in-situ stress ratio, etc.) have on a binary output (i.e. stable, unstable roof). While inputs like depth, stress ratio, and joint strength were modeled as categorical, they represent continuous variables in the real world. The goodness of fit was tested for parameters like depth, in-situ stress ratio, and joint strength as both categorical and continuous variables in preliminary BLRs. The difference was found to be negligible, so for consistency and ease of interpretation, all modeled parameters were treated as categorical variables. Note that BLR cannot account for “perfect delineators” where every unsupported model result was unstable, such as the “Weak SUBI” block material models. Therefore the results presented exclude the “Weak SUBI” roof block cases. First, parameters were individually considered for their impact on the probability of stability, standard error, and statistical significance (p-value) (Table 3.14). In the intercept cases, the stability probability is calculated from the intercept coefficient as:

\[ P_{\text{stability, intercept}} = \frac{e^{c_{\text{intercept}}}}{1+e^{c_{\text{intercept}}}} \]  

(3.19)
where \( C \) = coefficient listed in Table 3.14 and corresponds to the log(odds) that a model result was stable. As the independent variable is varied from the intercept, the coefficient is the change in the odds ratio from the intercept case, and therefore probability of stability is calculated as:

\[
p_{\text{stability}, n} = \frac{e^{C_{\text{intercept}} + C_n}}{1 + e^{C_{\text{intercept}} + C_n}}
\]  

(3.20)

Table 3.14: Results of BLR for both random seeds of each individual parameter against the binary outcome of stability. Mod = moderate, SE = standard error, t-Stat = t-statistic, NS CI = non-simultaneous confidence interval.

<table>
<thead>
<tr>
<th>Value</th>
<th>Coefficient</th>
<th>SE</th>
<th>t-Stat</th>
<th>p-Value</th>
<th>Stability Probability</th>
<th>95% NS CI</th>
<th>R²-Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Depth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(30 m)</td>
<td>3.72</td>
<td>0.17</td>
<td>21.4</td>
<td>&lt;1.00(10)⁶</td>
<td>0.98</td>
<td>0.97-0.98</td>
<td>0.07</td>
</tr>
<tr>
<td>100 m</td>
<td>-2.08</td>
<td>0.19</td>
<td>-11.1</td>
<td>&lt;1.00(10)⁶</td>
<td>0.84</td>
<td>0.82-0.86</td>
<td></td>
</tr>
<tr>
<td>200 m</td>
<td>-2.60</td>
<td>0.18</td>
<td>-14.1</td>
<td>&lt;1.00(10)⁶</td>
<td>0.75</td>
<td>0.73-0.78</td>
<td></td>
</tr>
<tr>
<td><strong>In-Situ Stress Ratio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.004</td>
</tr>
<tr>
<td>Intercept</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(( k_o = 0.5 ))</td>
<td>1.86</td>
<td>0.08</td>
<td>24.46</td>
<td>&lt;1.00(10)⁶</td>
<td>0.88</td>
<td>0.86-0.89</td>
<td></td>
</tr>
<tr>
<td>( k_o = 1.0 )</td>
<td>-0.08</td>
<td>0.11</td>
<td>-0.67</td>
<td>0.50</td>
<td>0.87</td>
<td>0.85-0.88</td>
<td></td>
</tr>
<tr>
<td>( k_o = 2.0 )</td>
<td>-0.42</td>
<td>0.11</td>
<td>-4.0</td>
<td>&lt;1.00(10)⁶</td>
<td>0.82</td>
<td>0.80-0.84</td>
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</tr>
<tr>
<td><strong>Roof Block Material Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.41</td>
</tr>
<tr>
<td>Intercept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Stiff Elastic)</td>
<td>4.96</td>
<td>0.41</td>
<td>12.1</td>
<td>&lt;1.00(10)⁶</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>EBP</td>
<td>0.00</td>
<td>0.58</td>
<td>0.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Soft Elastic</td>
<td>-0.78</td>
<td>0.50</td>
<td>-1.58</td>
<td>0.12</td>
<td>0.99</td>
<td>0.97-0.99</td>
<td></td>
</tr>
<tr>
<td>Strong SUBI</td>
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<td>0.42</td>
<td>-6.65</td>
<td>&lt;1.00(10)⁶</td>
<td>0.90</td>
<td>0.87-0.91</td>
<td></td>
</tr>
<tr>
<td>Mod SUBI</td>
<td>-5.31</td>
<td>0.41</td>
<td>-12.8</td>
<td>&lt;1.00(10)⁶</td>
<td>0.41</td>
<td>0.38-0.45</td>
<td></td>
</tr>
<tr>
<td><strong>Joint Initial Friction Angle</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0002</td>
</tr>
<tr>
<td>Intercept</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(35°)</td>
<td>1.84</td>
<td>0.08</td>
<td>24.02</td>
<td>&lt;1.00(10)⁶</td>
<td>0.86</td>
<td>0.84-0.88</td>
<td></td>
</tr>
<tr>
<td>25°</td>
<td>-0.02</td>
<td>0.11</td>
<td>-0.16</td>
<td>0.87</td>
<td>0.86</td>
<td>0.84-0.88</td>
<td></td>
</tr>
<tr>
<td>15°</td>
<td>-0.16</td>
<td>0.11</td>
<td>-1.48</td>
<td>0.14</td>
<td>0.84</td>
<td>0.82-0.86</td>
<td></td>
</tr>
<tr>
<td><strong>Bedding Thickness</strong></td>
<td></td>
<td></td>
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<td></td>
<td>0.002</td>
</tr>
<tr>
<td>Intercept</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(1.0 m)</td>
<td>1.91</td>
<td>0.06</td>
<td>29.8</td>
<td>&lt;1.00(10)⁶</td>
<td>0.87</td>
<td>0.86-0.88</td>
<td></td>
</tr>
<tr>
<td>0.5 m</td>
<td>-0.24</td>
<td>0.09</td>
<td>-2.77</td>
<td>0.005</td>
<td>0.84</td>
<td>0.83-0.86</td>
<td></td>
</tr>
<tr>
<td><strong>DFN ID</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>0.006</td>
</tr>
<tr>
<td>Intercept</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(DFN 1)</td>
<td>1.86</td>
<td>0.13</td>
<td>14.8</td>
<td>&lt;1.00(10)⁶</td>
<td>0.87</td>
<td>0.83-0.89</td>
<td></td>
</tr>
<tr>
<td>DFN 2</td>
<td>0.02</td>
<td>0.18</td>
<td>0.09</td>
<td>0.93</td>
<td>0.87</td>
<td>0.84-0.89</td>
<td></td>
</tr>
<tr>
<td>DFN 3</td>
<td>0.08</td>
<td>0.18</td>
<td>0.50</td>
<td>0.65</td>
<td>0.87</td>
<td>0.84-0.90</td>
<td></td>
</tr>
<tr>
<td>DFN 4</td>
<td>-0.08</td>
<td>0.18</td>
<td>-0.44</td>
<td>0.66</td>
<td>0.86</td>
<td>0.82-0.88</td>
<td></td>
</tr>
<tr>
<td>DFN 5</td>
<td>-0.37</td>
<td>0.17</td>
<td>-2.23</td>
<td>0.03</td>
<td>0.82</td>
<td>0.78-0.85</td>
<td></td>
</tr>
<tr>
<td>DFN 6</td>
<td>0.19</td>
<td>0.19</td>
<td>1.01</td>
<td>0.31</td>
<td>0.89</td>
<td>0.86-0.91</td>
<td></td>
</tr>
<tr>
<td>DFN 7</td>
<td>0.21</td>
<td>0.19</td>
<td>1.11</td>
<td>0.27</td>
<td>0.89</td>
<td>0.86-0.91</td>
<td></td>
</tr>
<tr>
<td>DFN 8</td>
<td>-0.47</td>
<td>0.17</td>
<td>-2.83</td>
<td>0.005</td>
<td>0.80</td>
<td>0.76-0.83</td>
<td></td>
</tr>
</tbody>
</table>
The preliminary, independent BLRs determined if changes in each parameter resulted in a significantly different stability outcome. Roof block material model was the most significant parametric input when analyzed independently, with the highest $R^2$-adjusted value and large changes in probability of stability for various values. Notably, the “Stiff Elastic”, “EBP”, and “Soft Elastic” block cases did not significantly impact model stability as indicated by the p-values greater than 0.05 in Table 3.14. Similarly, the intermediate in-situ stress ratio (i.e. 1.0) and joint strength (i.e. 25°) also were not significantly different from the intercept (i.e. most stable) values. Lastly, changes in vertical DFN parameters like spacing and persistence (i.e. DFNs 1-4) had no significant effect on roof stability, while the changes to sub-vertical joints with close spacing (i.e. DFNs 5 & 8) did.

As this chapter is specifically concerned with the combined impacts of the modeled parameters, all values were included in the multivariate BLR to determine if individually insignificant response parameters like “EBP” block material or joint persistence would increase in significance when considered in conjunction with other parameters. The results of the multivariate BLR are shown in Table 3.15. Note that the probability of stability reported in the table is calculated based on the intercept values, which are the model results from the most stable conditions (i.e. 30 m deep, $k_o=0.5$, “Stiff Elastic” block material, 35° initial friction angle, 1 m thick beds, and DFN ID 1), and the change of a given parameter to the value indicated in each row. Note that when all parameters are accounted for, the $R^2$-adjusted increases to 0.72 from the individual BLRs presented in Table 3.14, and joint initial friction angle becomes a statistically significant parameter. However, the probability of stability is not sensitive to changes in individual model parameters from the intercept condition (i.e. most stable case featuring 30 m deep, $k_o=0.5$, “Stiff Elastic” block material, 35° initial friction angle, 1 m thick beds, and DFN ID 1), as indicated by the near identical probabilities and confidence intervals. In order to calculate the probability of stability for any combination of inputs manually, Eqn. (3.20) is applied as:

$$p_{stability,ni} = \frac{e^{c_{intercept} + c_{n1} + c_{n2} \ldots + c_{ni}}}{1 + (e^{c_{intercept} + c_{n1} + c_{n2} \ldots + c_{ni}})}$$ (3.21)
Alternatively, the `plotSlice` command in MATLAB provides an interactive selector that can automatically calculate a probability of stability and 95% confidence intervals. In order to visualize the results presented in Table 3.15, the multivariate BLR `plotSlice` with the intercept values selected is depicted in Figure 3.31. For this combination of parameters, the minimum probability of stability by varying only one parameter at a time is 0.97 for the 5th tick mark from the left in the “Block Material” category in Figure 3.31, which corresponds with the models that feature “Moderate SUBI” block material and the BLR results in row 12 of Table 3.15.

Table 3.15: Results of BLR for both random seeds of each individual parameter against the binary outcome of stability. Mod = Moderate, SE = standard error, t-Stat = t-statistic, NS CI = non-simultaneous confidence interval.

<table>
<thead>
<tr>
<th>Value</th>
<th>Coefficient</th>
<th>SE</th>
<th>t-Stat</th>
<th>p-Value</th>
<th>Stability Probability</th>
<th>95% NS CI</th>
<th>R^2-Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>10.8</td>
<td>0.61</td>
<td>17.8</td>
<td>&lt;1.00(10)^6</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 m</td>
<td>-3.53</td>
<td>0.24</td>
<td>-14.5</td>
<td>&lt;1.00(10)^6</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>200 m</td>
<td>-5.11</td>
<td>0.28</td>
<td>-18.5</td>
<td>&lt;1.00(10)^6</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>In-Situ Stress Ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k_o = 1.0</td>
<td>-0.21</td>
<td>0.19</td>
<td>-1.13</td>
<td>0.26</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>k_o = 2.0</td>
<td>-1.22</td>
<td>0.19</td>
<td>-6.51</td>
<td>&lt;1.00(10)^6</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Roof Block Material Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EBP</td>
<td>0.00</td>
<td>0.58</td>
<td>0.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Soft Elastic</td>
<td>-0.81</td>
<td>0.51</td>
<td>-1.61</td>
<td>0.11</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Strong SUBI</td>
<td>-3.16</td>
<td>0.44</td>
<td>-7.24</td>
<td>&lt;1.00(10)^6</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Mod SUBI</td>
<td>-7.25</td>
<td>0.46</td>
<td>-15.8</td>
<td>&lt;1.00(10)^6</td>
<td>0.97</td>
<td>0.95-0.99</td>
<td></td>
</tr>
<tr>
<td>Joint Initial Friction Angle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25°</td>
<td>-0.05</td>
<td>0.18</td>
<td>-0.28</td>
<td>0.78</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>15°</td>
<td>-0.45</td>
<td>0.18</td>
<td>-2.50</td>
<td>0.01</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Bedding Thickness</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 m</td>
<td>-0.70</td>
<td>0.15</td>
<td>-4.63</td>
<td>3.70(10)^6</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN ID</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFN 2</td>
<td>0.05</td>
<td>0.31</td>
<td>0.15</td>
<td>0.89</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN 3</td>
<td>0.24</td>
<td>0.31</td>
<td>0.77</td>
<td>0.44</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN 4</td>
<td>-0.23</td>
<td>0.30</td>
<td>-0.75</td>
<td>0.45</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN 5</td>
<td>-1.11</td>
<td>0.30</td>
<td>-3.81</td>
<td>1.42(10)^4</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN 6</td>
<td>0.53</td>
<td>0.31</td>
<td>1.71</td>
<td>0.09</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN 7</td>
<td>0.59</td>
<td>0.31</td>
<td>1.87</td>
<td>0.06</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN 8</td>
<td>-1.38</td>
<td>0.29</td>
<td>-4.79</td>
<td>1.66(10)^6</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3.31: *plotSlice* of the multivariate BLR showing predicted stability and 95% non-simultaneous confidence interval (red) of a given model based on the parameter intercept input variables (vertical dashed lines). Note the y-axis ranges from 0.94 to 1.0. $k_o$ = in-situ horizontal stress ratio, $\phi_i$ = joint initial friction angle, BT = bedding thickness, DFN = Discrete Fracture Network.

If the categorical variable selectors were adjusted such that the lowest possible stability probability was obtained for the shallow entry models, the lowest stability probability was 0.45, primarily due to changes in the roof block material and horizontal stress ratio. However, every other parameter becomes statistically significant in impacting the stability probability by at least a magnitude of ±0.1, albeit with variable 95% confidence intervals (Figure 3.32). As entry depth increases or the CMRR decreases, the range of predicted stability in unsupported roofs varies from 1 (stable) to 0 (unstable). This is reflected in the empirically suggested ARBS value, $\text{ARBS}_G$ (Eqn. 3.12), which is based on entry depth and CMRR. Recall that CMRR is reflected in ratings assigned based on the categorical input variables. Note that while $k_o$-ratio is not explicitly accounted for in either the CMRR or ARBS methods, a relationship between entry depth and horizontal stress exists where observations were taken in Mark et al. (2001). However, this appears to be a major limitation to
application of the ARBS method to cases outside of the empirical study’s geographic and geologic bounds.

Figure 3.32: *plotSlice* of the multivariate BLR showing predicted stability (horizontal dotted line) and 95% non-simultaneous confidence interval (red dotted lines) of a given model condition based on the selected input variables (vertical dashed lines). Note the y-axis ranges from 0.0 to 1.0. \( k_o \) = in-situ horizontal stress ratio, Mod = Moderate, \( \phi_i \) = joint initial friction angle, BT = bedding thickness, DFN = Discrete Fracture Network.

The shallow entry results indicate that the ARBS method could improve by weighting of different factors in CMRR as a function of the roof intact strength rating. Furthermore, explicit consideration of in-situ stress ratio and the presence of closely spaced sub-vertical cross-joints could improve the accuracy of ARBS and CMRR, respectively. However, the same cannot be said about explicit joint strength. Based on unsupported model results, either too much emphasis is placed on explicit joint strength through the cohesion-roughness rating, or the frictional joints modeled have not captured a wide range of possible joint behavior. Recall that a rough, high cohesion joint has a maximum possible rating of 35 in the CMRR system, but the highest possible intact strength rating is 30.
As the model depth increases, and the roof block material strength remains low, the remaining parameters once again become less influential over modeled roof stability (Figure 3.33). These results indicate that at a minimum, the weighting between relevant rockmass properties in CMRR and its application in ARBS may need to be reconsidered depending on the in-situ stress to strength ratio of a given case. However, since both empirical methods were developed using supported roof cases, the bolted model results must be analyzed to determine how stability in more realistic roofs are dependent on parametric variations.

Figure 3.33: *plotSlice* of the multivariate BLR showing predicted stability (horizontal dotted line) and 95% non-simultaneous confidence interval (red dotted lines) of a given model condition based on the selected input variables (vertical dashed lines). Note the y-axis ranges from 0.0 to 1.0. $k_o =$ in-situ horizontal stress ratio, Mod = Moderate, $\phi_i =$ joint initial friction angle, BT = bedding thickness, DFN = Discrete Fracture Network.

### 3.4.2.2 Passively Bolted Roofs

A similar statistical analysis was undertaken on the results of bolted models for the combined results of random seeds 100 and 1234. As previously noted, the number of stable cases increased due to the presence of rockbolt elements (Figure 3.34).
Figure 3.34: Number of bolted models that were classified as stable or unstable for every parametric category tested from both random seeds. φ = initial friction angle, Block Material 1 = “Stiff Elastic”, 2 = “EBP”, 3 = “Soft Elastic”, 4 = “Strong SUBI”, 5 = “Moderate SUBI”, 6 = “Weak SUBI”. Refer to Table 3.2 for DFN ID properties.

The largest change in the number of stable model results between unsupported and bolted cases was incurred by the models featuring the “Moderate SUBI” block material properties. However, instability in other parametric cases decreased by similar magnitudes, indicating that the presence of bolts increased the self-supporting capacity of the roof by increasing the effective strength of the roof. In order to confirm this, a second multivariate BLR was conducted on bolted model results excluding the “Weak SUBI” block material property in order to make a direct comparison with the unsupported model results even though it was no longer a “perfect delineator” (Table 3.16).
Table 3.16: Results of multivariate BLR for both random seeds of each individual parameter against the binary outcome of stability in bolted models. Mod = Moderate, SE = standard error, t-Stat = t-statistic, NS CI = non-simultaneous confidence interval.

<table>
<thead>
<tr>
<th>Value</th>
<th>Coefficient</th>
<th>SE</th>
<th>t-Stat</th>
<th>p-Value</th>
<th>Stability Probability</th>
<th>95% NS CI</th>
<th>R²-Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>10.7</td>
<td>1.02</td>
<td>10.5</td>
<td>&lt;1.00(10)⁶</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.48</td>
</tr>
<tr>
<td>100 m</td>
<td>-1.23</td>
<td>0.54</td>
<td>-2.28</td>
<td>0.02</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>200 m</td>
<td>-4.12</td>
<td>0.51</td>
<td>-8.16</td>
<td>&lt;1.00(10)⁶</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>In-Situ Stress Ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_o = 1.0$</td>
<td>-0.88</td>
<td>0.38</td>
<td>-2.32</td>
<td>0.02</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>$k_o = 2.0$</td>
<td>-2.80</td>
<td>0.36</td>
<td>-7.72</td>
<td>&lt;1.00(10)⁶</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Roof Block Material Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EBP</td>
<td>-0.42</td>
<td>0.93</td>
<td>-0.45</td>
<td>0.65</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Soft Elastic</td>
<td>-0.42</td>
<td>0.93</td>
<td>-0.45</td>
<td>0.65</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>Strong SUBI</td>
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<td>0.79</td>
<td>-2.32</td>
<td>0.02</td>
<td>0.99</td>
<td>0.99-0.99</td>
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<tr>
<td>Mod SUBI</td>
<td>-5.09</td>
<td>0.75</td>
<td>-6.82</td>
<td>&lt;1.00(10)⁶</td>
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<td>0.99-0.99</td>
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</tr>
<tr>
<td>25°</td>
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<td>0.30</td>
<td>-0.59</td>
<td>0.56</td>
<td>0.99</td>
<td>0.99-0.99</td>
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</tr>
<tr>
<td>15°</td>
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<td>-1.29</td>
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<td></td>
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<tr>
<td>0.5 m</td>
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<td>0.24</td>
<td>2.55</td>
<td>0.01</td>
<td>0.99</td>
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</tr>
<tr>
<td>DFN ID</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFN 2</td>
<td>1.99</td>
<td>0.56</td>
<td>3.59</td>
<td>3.30(10)⁴</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN 3</td>
<td>1.79</td>
<td>0.53</td>
<td>3.37</td>
<td>7.60(10)⁴</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN 4</td>
<td>1.14</td>
<td>0.48</td>
<td>2.42</td>
<td>0.02</td>
<td>0.99</td>
<td>0.99-0.99</td>
<td></td>
</tr>
<tr>
<td>DFN 5</td>
<td>-0.69</td>
<td>0.38</td>
<td>-1.83</td>
<td>0.07</td>
<td>0.99</td>
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<td>DFN 6</td>
<td>0.66</td>
<td>0.44</td>
<td>1.50</td>
<td>0.14</td>
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<td>0.99-0.99</td>
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<td>DFN 7</td>
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<td>1.07</td>
<td>3.77</td>
<td>1.66(10)⁴</td>
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<td>0.99-0.99</td>
<td></td>
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<td>DFN 8</td>
<td>-0.57</td>
<td>0.38</td>
<td>-1.49</td>
<td>0.14</td>
<td>0.99</td>
<td>0.99-0.99</td>
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When compared to their unsupported counterparts, the $R^2$-adjusted decreased to 0.48 and the impact of increased depth and decreased block material strength on the intercept coefficient decreased. Similar to their unsupported counterparts, the bolted model probability of stability was not sensitive to changes in individual model parameters from the intercept conditions as indicated in the identical stability probability and 95% confidence intervals in Table 3.16. In the presence of passive roof bolts, the effect of joint initial friction angle became statistically insignificant while DFN ID became more significant overall. In particular, the high-persistence, closely spaced vertical cross-joints in DFN 1 now significantly impacted predicted stability along with the closely spaced sub-vertical cross-joints in DFNs 5 and 8. Recall that Eqn. (3.21) is used to calculate the probability of stability for any combination of parameter values and that plotSlice provides an interactive visual representation of those values. In 30 m deep entry models, the minimum possible
stability probability was increased from 0.45 in unsupported models to 0.85 in bolted models (Figure 3.35).

Overall, the multivariate BLR results for the bolted models exhibited similar parametric sensitivities to predicted stability as their unsupported counterparts. Increased depth and decreased block material strength still dominated the significant changes to probability of stability. However, the in-situ horizontal stress ratio and DFN ID become stronger controls in the presence of modeled passive roof bolts. Notably, high-persistence, closely spaced, vertical cross-joints in DFN ID 1 start to impact bolted stability even under low stress conditions. However, close spacing sub-vertical cross-joints in DFNs 5 and 8 still have more of a destabilizing effect due to the decreased stress arching associated with increased sub-vertical jointing. This indicates that bolted roofs under a wide range of realistic conditions also have varying degrees of stress arching capacity.

Figure 3.35: *plotSlice* of the multivariate BLR showing predicted stability (horizontal dotted line) and 95% non-simultaneous confidence interval (red dotted lines) of a given bolted model condition based on the selected input variables (vertical dashed lines). Note the y-axis ranges from 0.0 to 1.0. $k_o$ = in-situ horizontal stress ratio, Mod = Moderate, $\phi_i$ = joint initial friction angle, BT = bedding thickness, DFN = Discrete Fracture Network.
Recall that in unsupported models, when the in-situ stress to block material strength ratio was high, the remaining parameters had little effect on the probability of stability. This is not the case in the bolted model results; $k_o$-ratio and DFN significantly influence stability probability, even in the highest in-situ stress to roof block material strength ratio case (Figure 3.36).

Figure 3.36: *plotSlice* of the multivariate BLR showing predicted stability (horizontal dotted line) and 95% non-simultaneous confidence interval (red dotted lines) of a given bolted model condition based on the selected input variables (vertical dashed lines). Note the y-axis ranges from 0.0 to 1.0. $k_o =$ in-situ horizontal stress ratio, mod $= $ Moderate, $\phi_i =$ joint initial friction angle, BT $= $ bedding thickness, DFN $= $ Discrete Fracture Network.

Through analysis of bolted models, it has been confirmed that weighting between relevant rockmass properties in CMRR and its application in ARBS should be partially dependent on the in-situ stress to block material strength ratio of a given case and the material strength should be influenced by the presence of passive rockbolts in the roof. Furthermore, the impact of joint initial friction when considered with all other inputs is even less prominent in bolted model results, further confirming that CMRR may weigh joint strength too heavily. Additionally, based on the
minimal changes in stability probability between DFN ID 2 and 3, it appears that widely spaced, vertical joints are independent of the destabilizing effects of high persistence.

These results provide a statistically rigorous analysis of the interaction of multiple parameters on binary roof stability and indicate that the existing CMRR and ARBS methods explicitly account for the most significant impacts on roof stability (i.e. depth and overall rockmass strength). However, the impacts of geologic and mining conditions that remain explicitly unaccounted for (i.e. k_o-ratio, joint orientation), or possibly improperly accounted for (i.e. joint strength, spacing, and persistence), and their variable significance with changing stress and rock strength have been identified as potential considerations that could significantly increase the accuracy of existing empirical methods.

3.5 Conclusions

This chapter has presented a parametric sensitivity analysis of 8,640 unique numerical models capturing a wide range of geologic and mining factors that influence roof stability in laminated and discontinuous rockmasses using single-entry explicit DEM numerical models. Preliminary models were utilized to investigate the poorly documented influences of UDEC model setup attributes on model accuracy and model runtime.

Model results were confirmed as realistic via visual inspection of select models and verified as realistic through the unique application of empirical methods to model inputs and model results. Determination of CMRR based on model input parameters allowed for predictions regarding model stability to be assessed using the ARBS method. Visual inspection broadly categorized and identified behaviors unique to flat roof excavations in laminated and discontinuous rockmasses in unsupported and bolted cases. A robust automatic method was developed to determine model stability for 8,640 unique model cases, which incorporated roof displacement, velocity, and stress arching via the voussoir beam analog from Chapter 2 in its methodology. Application of the voussoir beam analog was critical in identifying unstable cases which were affected by explicit DEM limitations.

The ARBS method applied to the unsupported suite of models in this chapter was able to predict the model outcomes with a high degree (i.e. > 90 %) of accuracy based on model derived values of CMRR, bolt intensity, depth to entry, and intersection span. Roof self-supporting capacity was verified as realistic through the application of the ARBS method by assuming an end-member case
of bolt intensity equal to 0. The ARBS method applied to the bolted suite of models had decreased accuracy (i.e. > 80%) due to most of the “Weak SUBI” block material models requiring support in addition to, or instead of, fully grouted passive bolts. This issue was noted by Mark et al. (2001) in some of the ARBS case studies, further confirming the realistic behavior of the models. DFN random seed was determined to have a limited impact on roof stability, and subsequent statistical analyses combined the results of both random seed model runs.

Even though the influence of in-situ stress, joint orientation (i.e. vertical or sub-vertical), joint location (i.e. random seed number) and anisotropic strength (i.e. SUBI block material properties) cannot explicitly be accounted for in either CMRR or ARBS, their influence did not appear to significantly impact the overall accuracy of the empirical analysis and the verification of model results as realistic. However, subsequent univariate and multivariate BLR analyses revealed that while CMRR and ARBS broadly account for the most significant controls on roof stability (i.e. depth, rockmass rating), other relevant geologic and mining factors significantly control roof stability. Their significance level is highly dependent on the in-situ stress magnitude to roof block material strength ratio and the presence of roof support. Interestingly, the cohesion-roughness rating as applied to model inputs significantly reduced the accuracy of the ARBS method when applied to the model results, indicating that CMRR may weight joint strength too heavily or that the modeled joints were not capturing joint behavior in a realistic manner. This was further investigated through the use of BLR.

Similar to how the empirically derived ARBSG value is dependent on depth to entry and CMRR, model stability was most dependent on depth to entry followed by a combination of intact block material and discontinuity properties. In unsupported models, shallow (30 m) and deep (200 m) entry roof stability was primarily impacted by block material strength, while $k_o$-ratio, discontinuity strength, spacing, orientation, and persistence had either little or no significant impact. However, at intermediate depths (i.e. 100 m) or moderate strengths (i.e. “Strong SUBI”) the other parameters became more influential in controlling the roof self-supporting capacity. BLR verified that under most modeled conditions, the effect of joint initial friction angle on roof stability was statistically insignificant.

The bolted model results were also analyzed using multivariate BLR and the shallow model results largely resembled the stress-state to block-strength ratio-dependent parameter interaction
identified in the unsupported models. As stresses increased to 100 m, more parameters became increasingly significant. However, the decrease in parameter significance at 200 m depth, or with weaker block material strength was drastically reduced by the presence of passive bolts. In agreement with the findings of Butcher (2001), $k_o\text{-ratio}$ and joint persistence, spacing and orientation remained critical controls on bolted roof stability. This BLR further confirmed the effect of joint initial friction angle as increasingly insignificant in the presence of modeled roof bolts.

It remains unclear whether the cohesion-roughness rating has not been fully represented by the model inputs, has not been accurately applied to the model inputs, is overshadowed by the impact of ubiquitous joints, is weighed too heavily in CMRR, or some combination of the four. What is clear is that joint initial friction angle of continuously yielding joints has almost negligible impact on roof stability when combined with a wide range of geologic and mining conditions. The impact that it does have is dependent on depth, roof block material strength, the presence of roof support, and is consistently subject to low confidence intervals. Additional research is needed to completely rule out cohesion-roughness as a statistically meaningful parameter as it is currently applied in the CMRR-ARBS system. Alternatively, the shape of the flat-roof excavation can effectively transfer enough horizontal stresses through the roof and mute the effect of joint strength by reducing contact shear yield in the absence of intact material yield, making the effects of joint strength trivial.

The BLR has identified the critical combinations of inputs that most significantly affect roof stability when isolated from the effects of pillar yield. Those inputs will be tested with a calibrated pillar model in the following chapter to analyze the interaction between roof stability and pillar stability, which is not considered in the CMRR-ARBS empirical methods. Furthermore, successful implementation of empirical methods to numerical models has been proven to be a powerful tool in validating model results and has laid the groundwork for other studies to implement similar procedures. Incorporation of empirical methods through numerical models to capture end-member cases and test methods in previously untested conditions has the potential to advance the state-of-practice significantly.

Contrary to most pillar strength evaluation methods, the results of the parametric sensitivity analysis presented in this study confirm that many cases of bedded and jointed sedimentary roofs
do have self-supporting capacity as modeled herein. This self-supporting capacity is most-influenced by entry depth, in-situ stress ratio, fine-scale bedding (i.e. SUBI block material), as well as cross-joint orientation and spacing. However, at high stress to strength ratios, the influence of other parameters declines precipitously.

The findings of this research have provided groundwork information and methods about how to focus future empirical studies related to rockmass rating systems and mechanically relevant controls on roof stability, particularly in flat-roof excavations in laminated and discontinuous rockmasses. The use of numerical models to provide response data in untested and changing loading conditions, rather than static rockmass rating values based on unique cases and assumed mechanical relevance, has been shown to be both possible and beneficial.
CHAPTER 4
ANALYSIS OF PILLAR-ROOF MECHANICAL INTERACTION GOVERNING LOCAL STABILITY IN SINGLE-ENTRIES

4.1 Introduction

Following analysis of roof self-supporting capacity and verification of critical geologic and mining conditions that affect roof stability in single entries, identifying the critical mechanical interactions between the roof and pillars is the next logical step to expanding the holistic understanding of flat-roof excavation stability in laminated and discontinuous rockmasses. Of particular interest is the impact that critical controls on roof stability identified in Chapter 3 have on pillar loading and pillar strength, as well as how pillar properties such as size and stiffness impact roof stability. These interactions are either largely ignored in state-of-practice pillar and roof design, or are considered in the context of unique cases in state-of-knowledge pillar research (Reed et al., 2017; Frith & Reed, 2018, 2019).

In the context of the research presented in this chapter, the term “roof” refers to the area above the entry and below installed support (i.e. bolted interval), while “overburden” refers to the entire area between the seam and the upper boundary of the model (i.e. inclusive of the roof). As in Chapter 3, the roof, back, and overburden in this chapter are all modeled with the same block and discontinuity properties, therefore the terms “roof properties” and “overburden properties” are interchangeable. Note that this chapter largely focuses on the interaction between pillars and roof, although some general overburden considerations are also discussed. Additionally, the terms “yield”, “failure”, and “collapse” are not synonymous and are used to describe different phenomena observed in both the roof and pillar response to excavation.

The complex mechanical interactions between roof, pillar, and support, particularly in laminated and discontinuous systems, are poorly understood and conservatism is often used as a substitute for accurately capturing the mechanical behavior in analytical, empirical, and numerical design methods. Furthermore, application of empirical or site-specific numerical methods outside of the conditions in which those methods were developed is not wholly valid. While numerical methods may be more readily applicable to changing geologic and mining conditions, they rely on a significant amount of input data that may not be available, and furthermore, average users may lack understanding of the limitations of the various numerical methods (Galvin, 2016).
Nevertheless, numerical modeling remains a powerful tool in analyzing complex physical scenarios when properly applied and when results are interpreted within the limitations of a given numerical method. Galvin (2016) notes “numerical modelling currently provides the most reliable basis, albeit incomplete, for endeavoring to quantify the impacts on pillar system performance of low cohesion and friction interfaces, soft or weak foundations, and geological structure”. This chapter aims to help advance that endeavor via unique numerical models that investigate pillar-roof interaction. Existing research on pillar-roof interaction includes analytical studies that are limited by their assumptions (e.g. Salamon, 1970), empirical studies that are limited by their case study data and analytically determined pillar loading (e.g. Mark & Chase, 1997), and numerical studies that either simplify or isolate geologic and mining conditions (e.g. Shen & Duncan Fama, 2018).

The two-dimensional explicit discrete element method (DEM) is utilized in this chapter to consider a range of rockmass properties and mining conditions to understand changes in local stability in single-entry hypothetical coal mine models. Previous single-entry models presented in this thesis (Chapter 3) focused singularly on roof stability and its sensitivities to a wide range of geologic and mining conditions by utilizing effectively elastic pillar properties (i.e. peak cohesion artificially increased above the calibrated value) coupled with a single pillar w/h and stiffness. This simplified pillar deformability ensured that pillars responded to changing roof loads identically, allowing for only roof stability to be considered.

In this chapter, the most significant inputs controlling roof stability identified in Chapter 3 are considered with a calibrated coal pillar model from Sinha & Walton (2020) in supported and unsupported single-entry models. Similar to their Chapter 3 counterparts, the models in this chapter impose horizontal symmetry conditions to effectively model an infinite array of pillars and focus on the stability and pillar-roof interaction of a given entry. This chapter combines critical controls on roof stability (i.e. depth, block material) with variations in pillar material and geometric properties to explicitly (i.e. inelastic properties, pillar w/h) and implicitly (i.e. elastic properties) simulate various degrees of pillar deformation promote realistic pillar failure. The coal pillar model features inelastic material properties that were previously calibrated to the Mark-Bieniawski pillar strength equation (Sinha, 2020; Walton et al., 2020), coupled with realistic pillar-roof and pillar-floor interface strengths.
Results such as pillar stress development, roof and pillar yield, roof-pillar stress transfer, entry convergence, pillar peak average strength, and pillar confinement are considered and presented. Model results are compared to state-of-practice methods for prediction of pillar stresses and strengths such as tributary area theory (TAT) and the Mark-Bieniawski pillar strength equation. Although the general limitations of these methods are well-known, the specific impacts of various realistic geologic and mining conditions on their accuracy have not been fully considered.

Furthermore, the determination of which element fails first in local collapse, the roof or the pillars, is considered. The analyses considered in this chapter and their relation to existing research are outlined in Figure 4.1

![Image](image.png)

Figure 4.1: Graphical depiction of exiting explicit DEM research and the considerations developed, confirmed, and validated through the course of this research. BBM = bonded block method, MCSS = Mohr-Coulomb Strain-Softening, HBSS = Hoek-Brown Strain-Softening.

Section 4.2 outlines the existing research on pillars in discontinuous and laminated rockmasses, pillar design methods, and numerical modeling of pillar-roof interaction. The remaining sections present the methodologies and results of this chapter.

4.2 Literature Review

The following literature review focuses primarily on soft-rock (i.e. coal) pillars, but many concepts related to pillar loading and strength apply to both hard- and soft-rock pillars. Some concepts unique to hard-rock pillars are also discussed.
4.2.1 Pillars in Discontinuous Rockmass and Laminated Systems

As excavations in underground room and pillar mines are advanced to extract more ore in a given mine panel, the roadways parallel (i.e. headings) and perpendicular (i.e. cross-cuts) to the main mining direction leave unmined ore material in place. These unmined remnants are collectively referred to as pillars and they are the primary support preventing convergence of the roof and floor in underground workings (Galvin, 2016). Different pillars serve different functions depending on the spatial and temporal relationship of pillars with mining activity, as well as the pillar geometry. Broadly, the two pillar types considered in this thesis are panel pillars (Chapters 4 & 5) and barrier pillars (Chapter 5), each of which has a specific location, function, and relative size (Figure 4.2).

Panel pillars in underground coal mines can be between 15-50 m wide and 15-130 m long depending on cross-cut geometry, while barrier pillars can range from 30-100 m wide and generally run the length of a given panel (Galvin, 2016). Note that the consistency of spacing, size, shape and types of pillars utilized are highly dependent on geologic and mining conditions, as well as the design life of a given excavation and the excavation method utilized. These properties can vary significantly even within the same mine panel if lithology is highly laterally heterogenous.

While panel pillars provide short to long-term support to roof, rib, and floor in a localized area, barrier pillars limit stress field interactions between mine workings, and prevent migration of airflow (i.e. gas, inrush, combustion) in addition to providing more robust local and global support.
that limit mining-induced subsidence (Galvin, 2016). Panel pillars can be extracted partially or fully as the operations retreat out from the developed room-and-pillar panel in a process known as depillaring. As pillars are comprised of ore material, leaving them in place is often considered lost revenue for a mine. The effects of depillaring are explicitly considered in Chapter 5.

Regardless of the aforementioned geometric properties (i.e. size, shape, spacing), as dictated by geologic properties (i.e. ore seam thickness, strength, lateral extent) and mining conditions (i.e. in-situ stress, mining method), all pillars in laminated systems have common features. They are typically immediately bound by pillar-roof and pillar-floor interfaces that are governed by the change in lithology in which the ore resides. Furthermore, they are bound by roof and floor strata collectively referred to as the overburden and underburden, respectively (Galvin, 2016). Pillars in discontinuous systems may feature discontinuities within the pillars themselves. The destabilizing effect of these discontinuities can be mitigated through the use of support such as rockbolts and wire mesh. This thesis does not explicitly account for pillar discontinuities and assumes that pillars are free from large defects and throughgoing discontinuities such as major joints, faults, or inclusions and is therefore limited to consideration of mining scenarios where pillar yield and failure are primarily governed by damage to intact coal or rock material.

4.2.2 Pillar Yield, Failure, & Collapse

Pillar yield, failure, and collapse are complex processes that are inconsistently defined throughout the literature. Further complicating matters, a single pillar may simultaneously exist in multiple stages of yield, failure, or collapse depending on a number of conditions including, but not limited to, pillar rockmass and rock material properties, pillar width-to-height ratio (w/h), presence of support elements, and loading conditions (i.e. interface, overburden, and underburden properties) (Figure 4.3).

Figure 4.3: Schematic representation of various possible yield and failure states in a single pillar (adapted from Quinteiro et al., 1995 by Galvin, 2016).
Brady & Brown (2013) indicate that the onset of inelastic deformation (i.e. yielding) is not equivalent to failure, which is not equivalent to the point of peak strength (i.e. collapse). They suggest that a pillar has failed when it can no longer fulfill its engineering function. However, Galvin (2016) notes that the terminology is inconsistent throughout the literature and caution should be used when interpreting results that classify pillars as “failed”, “collapsed”, “stable”, “unfailed”, “serviceable”, and “functional”.

Pillar failure is broadly classified by Galvin (2016) into two main categories: conventional and dynamic confined core. Conventional failure refers to a group of pillars with lower w/h ratios (i.e. w/h < 6) that deform due to overburden loads in excess of pillar strengths. Conventional failure is further subdivided into controlled (i.e. squeeze) and uncontrolled failures, where signs of failure (i.e. rib spalling) are either apparent and continuous, or non-existent, respectively. Dynamic confined core failure is illustrated in Figure 4.3 and is associated with higher pillar w/h (i.e. > 6). With increasing pillar w/h, pillars can behave in a “perfectly plastic” or strain-hardening manner in the post-peak. However, these classifications do not account for the functional failure of a given pillar. For example, if a given entry is blocked with yielded slumped material, this would constitute a functional failure of the pillar without pillar collapse. Another form of failure, known as a “pressure burst” (i.e. coal bump, rock burst), refers to a sudden release of stored strain and can occur in conventional and dynamic confined core modes. It is known that the ultimate failure mode and the post-peak behavior of the pillar are largely controlled by the overburden response.

As the panel size increases, or as depillaring occurs, the effective stiffness and self-supporting capacity of the overburden is reduced. If that reduction is rapid and decreases below the stiffness of the yielding pillar, the strain increase incurred by the pillars is rapid and the failure is likely to be uncontrolled. Conversely, if the reduction in overburden stiffness and self-supporting capacity is uniform and continuous, the pillar failure is likely to be controlled and pillar collapse may never occur (Galvin, 2016).

Yield pillars are pillars intentionally designed to yield and redistribute stresses in a controlled manner that protects roadways against excessive roof deformation, floor heave, and coal bumps. Yield pillars are smaller than barrier pillars and increase the amount of coal that can be extracted between longwall panels and can also be used in room and pillar mining scenarios to control stress distribution. They are most commonly utilized to encourage stress arching in the roof between two
barrier pillars, while maintaining temporary support for the destressed roof directly above the yield pillar as shown in Figure 4.4 (Yavuz, 2001). While yield pillars have been shown to improve roof stability, particularly in longwall coal mining (Ozbay et al., 1995; Mark, 2010; Klemetti et al., 2019), little research has been conducted on how roof and overburden stability, and more broadly, ranges of realistic geomechanical and mining conditions can be accounted for holistically in determining pillar loading, pillar strength, and functional pillar failure for yield pillars and non-yield pillars alike.

![Figure 4.4: Schematic illustrating the difference in roof stress distribution when yield pillars are utilized in an entry system (modified from Yavuz, 2001)](image)

**4.2.3 Properties Influencing Pillar Behavior**

Pillars are most often studied, designed, and categorized according to their w/h, due to the significant impact w/h has on confinement development within the pillar, and consequently, peak strength and post-peak behavior (Figure 4.5). Note that pillar width and length are the only inherent pillar-specific parameters that can be engineered and designed, as opposed to other relevant properties such as in-situ rock strength and stiffness.
Figure 4.5: Generalized effect of increasing w/h on the stress and strain response of coal pillars, (a,b) approximate w/h < 3.0, (c) 3.0 < w/h < 6.0, (d) 6.0 < w/h < 10.0, (e) w/h > 10.0 (from Galvin, 2016).

The geometric effect on pillar behavior shown in Figure 4.5 is also seen in laboratory scale analyses of varying w/h small- and large-scale specimens (Das, 1986; Madden, 1990; Medhurst & Brown, 1996, 1998). Pillar strength and yielding variation with w/h was first researched by Salamon & Munro (1967) and has been the subject of numerous analytical, empirical, and numerical studies since (e.g. Wilson, 1983; Sheorey et al., 1987; Mark & Chase, 1997; Lunder & Pakalnis, 1997)

Another factor that impacts pillar confinement development but is not explicitly considered in state-of-practice pillar design is interface strength. The decrease in pillar confinement generation, peak strength, and post-peak strain-hardening with decreasing contact strength is well-documented in empirical studies on in-situ pillar strength (Mark & Bieniawski, 1986; Maleki, 1992; Hasenfus & Su, 1992; Parker, 1993), small and large-scale laboratory studies (Meikle, 1965; Khair, 1968; Brown & Gonano, 1974) (Figure 4.6), and results of numerical modeling (Iannacchione, 1990; Gale, 1996, 1998; Malan, 2012; Gu & Ozbay, 2014; Gale, 2017; Tuncay et al., 2020) (Figure 4.7).
Figure 4.6: Influence of rock-platen interface on stress-strain response of 51 mm diameter samples of varying height specimens of marble (adapted from Brown & Gonano, 1974 by Galvin, 2016).

Figure 4.7: Numerical model results showing the effect of strong (purple), moderate (light blue), and weak (green) floor-pillar-roof material and contact strengths and pillar w/h on average pillar strength compared to field data (adapted from Gale & Mills, 1994 by Galvin, 2016).
Although confinement generation increases the peak strength of a pillar, this allows additional strain energy to be stored, resulting in an increased likelihood of uncontrolled pillar bursting failure (Meikle, 1965; Khair, 1968; Newman, 2002; Gu & Ozbay, 2014). Lunder & Pakalnis (1997) developed a pillar strength formula for hard rocks from empirical observations that considers the combined confining effect of increasing pillar w/h and interface strength; this method is discussed further in Section 4.2.4.

In-situ horizontal stress ratio can affect the baseline confinement stored in the pillar and increases the peak-load carrying capacity of a given pillar (Coates, 1981; Galvin, 2016). This phenomenon has been documented in underground mines in both hard (i.e. non-coal) (Martin & Maybee, 2000; Maybee, 2000) and soft (i.e. coal) (Frith et al., 2020) rock, and has been reported in the results of previous numerical studies (Karabin & Evanto, 1999; Martin & Maybee, 2000; Maleki et al., 2009; Idris et al., 2015; Maleki, 2017; Heasley & Tulu, 2018). Similar to the effect of strong coal pillar interfaces, increased in-situ horizontal stress increases the peak pillar strength and therefore increases the likelihood of catastrophic failure in the event of rapid loss of confining stress due to roof failure, pillar “hourglassing”, or sudden failure of the pillar interface with the roof or floor (Gale, 2017; Frith et al., 2020) (Figure 4.8).

Figure 4.8: Depiction of possible mechanism of rapid reduction in pillar confinement (i.e. horizontal stress) and increase in vertical stress that can lead to catastrophic pillar failure (form Gale, 2017).
Sheorey et al. (1987) developed a pillar strength formula from empirical observations that considers the combined confining effects of increasing pillar w/h and in-situ horizontal stress; this method is discussed further in Section 4.2.5.

4.2.4 Analytical Pillar Load Formulae

TAT is a common analytical method for calculating average panel pillar stress in a room-and-pillar mine. TAT assumes that the overburden is continuous, has no self-supporting capacity (i.e. has no stiffness), deadloads equally onto regularly spaced pillars regardless of an individual pillar’s location in a given panel (i.e. ignores geologic heterogeneity and abutment load transfer to the barrier pillars and surrounding rockmass), and that only in-situ vertical stress impacts pillar loading (Van Der Merwe et al., 2003). The average pillar stress calculated via TAT ($\sigma_{TAT}$) is given as:

$$\sigma_{TAT} = \gamma H \ast \left( \frac{C_1 C_2}{w_1 w_2} \right)$$  \hspace{1cm} (4.1)

where $\gamma$ = unit weight of overburden material, $H$ = depth of cover, $C$ = center-to-center pillar spacing, $w$ = pillar width as shown in Figure 4.9. It is evident that TAT provides an overestimate of pillar loads, particularly when panel width to depth of cover ratio ($W/H$) is less than 1-1.5, or when overburden strata are very stiff (Galvin, 2016).

Figure 4.9: Schematic of geometric inputs utilized in calculating average pillar stress via TAT (from Van Der Merwe et al., 2003).
Coates (1981) developed an average pillar load ($\sigma_p$) calculation for deeper mines that considers the panel W/H, horizontal stress ratio ($k_o$), and elastic moduli ($E, \nu$) of the pillar ($p$) and properties of the overburden rockmass ($r$):

$$\sigma_p = \gamma H \times \left\{ \frac{2R - k_o H W}{1 - 2\nu_p} - \frac{\nu_p}{1 - \nu_p} \right\} \left\{ \frac{1}{k_o H W E_p} \right\}$$

$$\times \frac{H E_r}{W E_p} + \frac{2(1 - R)(1 + \frac{1}{N})}{W E_p} + 2 \frac{R B W}{W E_p}$$

(4.2)

where $N = \text{number of pillars across the panel width}$, $B = \text{entry span width}$, and $R = \text{extraction ratio}$, which is the ratio of extracted area to the original area and is calculated as:

$$R = \frac{C_1 C_2 - w_1 w_2}{C_1 C_2}$$

(4.3)

Refer to Figure 4.9 for $C$ and $w$ dimensions. However, this method has similar limitations as TAT in that it does not account for abutment load transfer to the barrier pillars, is only applicable in higher W/H panels, and only calculates a single average pillar stress.

In contrast with the aforementioned methods, version 6.0 of the Analysis of Retreat Mining Pillar Stability software (ARMPS2010) (Mark, 2010) utilizes pressure arch theory (PAT) to calculate panel pillar loads for panel geometries with $W/H < 1.0$ (Figure 4.10). A pressure arch factor ($F_{pa}$) is applied to tributary area load as follows:

$$F_{pa} = 1 - (0.28 \times \ln \left( \frac{H}{W} \right))$$

(4.4)

The remaining tributary area load is then applied to the barrier pillar load. This formula is based on the observations of load transfer distance (LTD) in laminated rockmasses presented by Abel (1988). Abel developed the LTD formula based on 55 case studies in laminated sedimentary deposits and Poulsen (2010) proposed assigning each pillar a zone of influence (ZI) equal to $LTD + w/2$ and a near-field extraction ratio as:

$$R_{nf} = \frac{E_A}{ZI}$$

(4.5)

where $E_A = \text{excavated area as shown in Figure 4.11}$. Note that the PAT does not account for local extraction ratios within a given ZI.
Figure 4.10: Schematic illustrating the consideration of pressure arching for panels with W/H < 1.0 utilizing the ARMP S v6.0 methodology (from Mark, 2010).

Figure 4.11: Schematic diagram of how the near-field extraction ratio is applied to PAT based on the ZI and EA of a given pillar (from Hauquin et al., 2016).
Mark et al. (2010) state that the strength of the ARMP52010 system is not the accuracy of its calculations, but rather the robustness of the empirical dataset that it is calibrated to. This statement remains valid for other available coupled empirical-analytical methods of pillar design such as the ALPS (Mark, 1990; Mark et al., 1994) and the Analysis of Longwall Tailgate Serviceability (ALTS) (Colwell et al., 1999). Rather than addressing the complex mechanical interactions governing the stability of a given excavation, conservativism and broadly applied case studies are substituted in order to develop easily applicable and reasonably safe excavations.

Note that the aforementioned methods for determining pillar loads calculate a conservative average stress for a given pillar or pillar array. Stress variation within a given pillar, at different stages of loading (i.e. peak, post-peak), adds an additional layer of complexity to the study of pillar loads (Figure 4.12). Note that even though stresses are concentrated towards the center of the pillar, the edges (i.e. ribs) of the pillar begin to yield at far lower levels of stress due to lack of confinement (i.e. lower peak strength) (Wagner, 1980). In order to conduct research on this topic, one requires well-instrumented empirical studies are required, or sophisticated numerical modeling methods (discussed in Section 4.2.7).

![Figure 4.12: Distribution of stress levels in a w/h = 2 pillar at different stages of loading (modified from Wagner, 1980)](image)

The enhanced confined core concept was developed by Salamon (1992) to account for the stress distribution within yield pillars. This method considers effects of overburden strata stiffness and...
seam thickness on pillar loading and pillar dimensions on load distribution, load distribution, pillar confinement, and contact strength on pillar edge yield, and finally the peak strength of the pillar that forms a confined core.

Thick, elastic beam deflection methods of determining pillar loads were also developed to account for the conservativism of TAT for deeper and lower W/H panels (Salamon, 1970; Sheorey & Singh, 1974; Jeremic, 1985). However, these rely heavily on simplifying material assumptions when applied to laminated and discontinuous environments in order to calculate roof displacement and subsequent pillar loads. These elastic beam deflection methods are not the preferred practical methods.

4.2.5 Empirical Pillar Strength Formulae

While there are some pillar strength formulae that rely on classic analytical methods (e.g. Salamon, 1995), the application of these methods is restricted by the simplification of complex mechanical interactions (Galvin, 2016). The most common, and well-documented methods of calculating the peak strength of both hard and soft rock pillars are based on empirically derived analytical equations that either adjust the intact strength of a standard laboratory-sized specimen to the dimensions of the pillar, are based on in-situ testing, back analyzed from performance of pillars in the field, or some combination of the aforementioned methods (Galvin, 2016). The back analyzed cases are then fit to a factor of safety (FoS) or stability factor (SF) based on analytical (Hedley & Grant, 1972; Mark & Chase, 1997) or numerical estimations (Von Kimmelmann et al., 1984; Krauland & Soder, 1987; Potvin et al., 1989) of the state of stress to determine the strength for various failed w/h pillars in a given empirical dataset.

This thesis is generally concerned with laminated and discontinuous rockmasses, and this chapter specifically models multiple geologic and mining conditions in single entries with a coal constitutive model calibrated to the Mark-Bieniawski pillar strength equation (Sinha & Walton, 2020). Therefore, the Mark-Bieniawski pillar strength equation, as well as those that consider more complex loading conditions (i.e. confinement due to in-situ stress and contact strength) (Sheorey et al., 1987; Lunder & Pakalnis, 1997) are discussed in this section.

The Mark-Bieniawski pillar strength equation is given as:

\[
S_p = S_i \ast (0.64 + 0.54 \frac{w}{h} - 0.18 \frac{w^2}{Lh})
\]  

(4.6)
where $S_p =$ average peak pillar strength, $S_1 =$ in-situ strength of coal (assumed to be 6.1 MPa) $w =$ pillar width, $h =$ pillar height, and $L =$ pillar length. This formula was altered from Bieniawski (1992) to integrate the stress profiles of Wagner (1974) and to account for the length of long barrier pillars. The Bieniawski pillar strength equation was developed based on 66 in-situ large-scale testing of square coal pillars in three South African coal mines (Bieniawski & Van Heerden, 1975). The in-situ testing of these pillars was conducted by removing the pillar tops using a universal coal cutter and replacing them with a loading frame that consisted of approximately 25 hydraulic jacks and cast in-place concrete of approximately 40 MPa strength which were reinforced with 12 mm rebar on 100 mm spacing formed and cured directly on the pillar specimen (Bieniawski & Van Heerden, 1975). Bieniawski and Van Heerden (1975) report that the end-constraints (i.e. the concrete-coal interface) and lateral constraints (i.e. restriction of lateral pillar movement at the top and bottom of the sample using a wood, steel, or concrete cap) were such that the loading condition of the pillar in-situ were “reasonably approximated” based on FEM models of sample stress distribution and the equivalent deformation of the upper and lower half of the large-scale sample.

The original ARMPS system consisted of empirical verification of 140 case studies where a SF was calculated based on assumed TAT loads and the Mark-Bieniawski pillar strength equation. This resulted in 93% of failed case studies with an ARMPS SF < 0.75 and 94% of successful case studies with an ARMPS SF > 1.5 (Mark & Chase, 1997). The empirical data set has since been expanded to include 692 cases as shown in Figure 4.13. Note that the only failed cases above an ARMPS SF of 1.5 are classified as pillar squeezes (i.e. controlled failure), and all catastrophic failures are below the 2010 guidelines. However, if the methods used to calculate SF were truly conservative, most of the failed cases should fall below a SF of 1.0. Additionally, statistical analysis of both roof strength and coal strength at each case study location were reported to not have statistically significant impacts on ARMPS SF (Mark, 2010). This finding runs counter to previous research presented in Section 4.2.3 and 4.2.4, the other pillar strength equations discussed in this section, and the numerical modeling research discussed in Sections 4.2.6 and 4.2.7.
There is clearly a large scatter in the dataset due to physical phenomena that remain unaccounted for in the simple, albeit relatively effective methods of state-of-practice pillar design. Other, less ubiquitous pillar strength formulae attempt to account for such physical phenomena, namely the extra-pillar properties that affect pillar confinement development.

Sheorey et al. (1987) developed an equation prior to testing its fit to an empirical dataset of 43 coal pillar case studies. Based on previously developed and case-studied pillar equations (Bieniawski, 1968; Sheorey et al., 1982; Salamon, 1983) and the requirements that a pillar strength equation should (1) fit the case study data, (2) be applicable to all w/h pillars, (3) account for end-constraints, w/h ratio, and size (i.e. account for coal seam discontinuities), (4) account for in-situ stresses, and (5) include a coal strength parameter that reduces in effect with increasing confinement, they proposed that the following equation form would account for the pillar w/h, in-situ stress, and end-constraints (i.e. pillar-roof and pillar-floor contacts):

$$ S_p = \sigma_c h^{-\gamma} + am \gamma \frac{d\sigma_x}{d\left(\frac{w}{h}\right)} \left(\frac{w}{h} - 1\right) $$  \hspace{1cm} (4.7)
where the first term accounts for the size effect \((h^{-\infty})\) on the laboratory sample strength \((\sigma_c)\) of a 2.5 cm cubic sample. This sample size was selected based on the historic data and sample sizes tested by the Central Mining Research Station in Dhanbad, India. The second term includes \(\frac{d\sigma_x}{d(h/w)} = \text{change in confinement with changes in pillar w/h}\). Note that when \(w/h = 1\), the second term goes to zero and the unconfined compressive strength of a cubic laboratory coal sample is obtained for \(h^{-\infty} = 1\). The equation was simplified with average values through trial-and-error fitting to the empirical dataset, and resulting in the following equation:

\[
S_p = 0.27\sigma_ch^{-0.36} + \frac{H}{160}(\frac{w}{h} - 1)
\]  

(4.8)

The \(H/160\) value was estimated by trial-and-error fitting to available case study data and can be adjusted to account for end-constraints by calibrating to other case studies or numerical analysis (Sheorey et al., 1987). However, a robust methodology for altering this \(H/160\) term is not presented or verified by Sheorey et al. 1987. They did note that the formula should only be applied to pillar w/h greater than 4 and for mining depths greater than 200 m.

Lunder & Pakalnis (1997) developed a formula for hard rock pillars based on a meta-analysis of previous studies and Boundary Element Method (BEM) numerical models. This formula is given as:

\[
S_p = (K * UCS)(C_1 + C_2 * \kappa)
\]  

(4.9)

where \(K = \text{rockmass strength size factor (0.44)}\), \(UCS = \text{unconfined compressive strength}\), \(C_1 \& C_2 = \text{constants (0.68 and 0.52, respectively)}\), and \(\kappa = \text{pillar friction term calculated as:}\)

\[
\kappa = \tan (\cos^{-1}(\frac{1-C_{pav}}{1+C_{pav}}))
\]  

(4.10)

where \(C_{pav} = \text{average pillar confinement is estimated as:}\)

\[
C_{pav} = 0.46(\log (\frac{w}{h} + 0.75))^{\frac{14h}{w}}
\]  

(4.11)

More recently, Prassetyo et al. (2019) developed a set of linear and power law pillar strength formulas to account for low and medium interface friction angles based on statistical analysis of the results of laboratory testing of small-scale coal pillar analogs.
It is evident based on the review of relevant literature that analytical methods for determining pillar strength employ many simplifying assumptions and that empirically derived analytical formulae may attempt to capture additional complexity, but those that do (Sheorey et al., 1987; Lunder & Pakalnis, 1997; Prassetyo et al., 2019) are based on limited datasets and unique constants that may not be broadly applicable. Additionally, the primary method of determining the strength of coal pillars in the USA (i.e. Mark-Bieniawski) suggests that such complexity has no statistically significant impact on pillar strength (Mark, 2010). However, the sources of uncertainty associated with the ARMP2010 method remain poorly studied. Furthermore, the numerous observations of the sensitivity of lab-scale specimen and in-situ pillar response to varied loading conditions indicate that there are other mechanical considerations that notably impact coal pillar strength other than pillar w/h.

4.2.6 Numerical Modeling of Pillar-Roof Interaction

Relative to empirical methods, numerical methods have allowed for more complex geologic and mining conditions to be considered when analyzing pillar loading in underground excavations. Numerical models can vary significantly in their scales, considerations, formulations, applications, constitutive models, and computational demand. Additionally, numerical models can be used to study either the isolated or combined impacts of geologic and mining conditions on pillar loading. Esterhuizen et al. (2010) evaluated the impact of overburden stiffness, panel W/H, and pillar w/h on pillar loading and mechanical response in three-dimensional finite difference method (FDM) models through the lens of the ground reaction curve concept. They found that overburden stiffness, coupled with panel W/H had a significant impact on pillar loading and deviation from TAT-calculated panel pillar loads. In particular, stiffer overburden combined with sub-critical panel widths increased stress transfer to barrier pillars and decreased loading of panel pillars.

Roberts et al. (2002) noted that the conservative nature of TAT, when applied to the determination of empirical pillar strength formulae, may significantly overestimate the strength of pillars. They utilized multiple numerical methods (i.e. Discontinuity Displacement – Boundary Element Method (DD-BEM) and Finite Element Method (FEM)) to consider the impacts of overburden stiffness, panel W/H, and extraction ratio. LaModel average pillar stress results were analyzed for their sensitivity to changes in number of laminations (i.e. lamination thickness), overburden stiffness, and seam stiffness, none of which measured greater than a 1% difference between TAT
calculations and model results. Roberts et al. (2002) found that the greatest deviations between TAT and model results of average pillar stress were due to extraction ratio (i.e. pillar w/h) and panel W/H, up to a 28% difference in the deepest (200 m) and highest extraction ratio (91%). Additional impacts on panel and barrier pillar loading due to pillar-roof-floor contact friction were considered in ELFEN and Phase2 models (Figure 4.14).

Figure 4.14: Results of (a) ELFEN and (b) Phase2 numerical models that show vertical stress results along the horizontal axis of the pillar for various pillar-roof and pillar-floor contact types as well as the number of laminations in the overburden (from Roberts et al., 2002).

These findings compliment those regarding the impact of contact strength on pillar strength and the inherent complexity of pillar loading, even with the use of simple numerical models. Roberts et al. (2002) concluded that using numerical models to estimate pillar loads, then calculating a factor of safety based on empirical strength equations results in unsafe pillars, particularly if the overburden yields significantly and imparts the full TAT load onto the panel pillars below. Inelastic material behavior, rockmass discontinuities, the impact of ground support, and horizontal stress were not accounted for in this particular study.

Esterhuizen & Mark (2009) compared the average pillar stress results in the active mining zone of three-dimensional FDM models with the ARMPS method applied to representative hypothetical case studies of eastern and western US coal mines. They found that in the western U.S model (i.e. deeper, stiffer and stronger overburden, larger pillars), ARMPS overpredicted average pillar stress between 9% and 28% at development and throughout depillaring of two panels. This is attributed
to the fact that the FDM model is accounting for pillar stiffness and load redistribution to unmined areas. However, in the eastern U.S. model (i.e. shallower, weaker and softer overburden, and smaller pillars), ARMPS underpredicted pillar stresses during depillaring in the second panel by 7.7%.

Hauquin et al. (2016) utilized two-dimensional elastic FDM models to compare TAT and PAT analytical methods and develop new analytical solutions for average pillar stress accounting for variation in pillar size as well as local and global extraction ratios through the relative extraction ratio (R$_{rel}$):

$$R_{rel} = \frac{R_L}{R_G}$$

(4.12)

Where $R_L$ = local extraction ratio, $R_G$ = global extraction ratio (i.e. average of all the $R_L$), and a regular mine layout has a $R_{rel} = 1$. Preliminary models considered the overburden, seam, and underburden as a continuous and homogenous material and only considered horizontal stresses due to the Poisson effect.

Yu et al. (2018) utilized two and three-dimensional inelastic explicit DEM models to determine that TAT should only be used for panel W/H > 3, far higher than estimates by previous studies. They statistically analyzed model results to develop an equation that accounts for overburden stiffness as well as panel and pillar geometry to determine pillar stress in lower W/H mine geometries. However, modeled pillar stresses were extracted for statistical analysis from the center of the pillars, rather than averaged over the whole pillar. This underrepresented the average state of stress in the stable pillar, as stresses tend to concentrate in the pillar corners and ribs in unyielded pillars. Furthermore, the only discontinuities modeled were the pillar-floor and pillar-roof interfaces and the equation developed was not compared against other existing methods or field data. The effect of inelastic damage to overburden or pillars was not explicitly considered therein other than to note the increase in central pillar stresses when pillar ribs yield.

While existing numerical research has identified important relationships between select geologic and mining conditions and their effect on pillar loading, most either employ numerical methods that often utilize extremely simplifying assumptions when considering laminated and discontinuous rockmass (i.e. DD-BEM, FEM), or only consider a single parameter such as coal-
roof interface strength. Furthermore, the complex interaction of inputs associated with roof DFN and their explicit impact on both pillar loading and failure is absent from the literature.

4.2.7 Numerical Modeling of Pillar Yield

Pillars in numerical models and the constitutive equations that govern their response to loading allow for more complete representation of the range of possible pillar behavior in complex geologic and mining conditions. As discussed in Chapter 1, the two primary approaches to numerical modeling are continuum and discontinuum methods. Although coal is highly brittle and may contain discontinuities such as cleats or other inclusions, coal pillars are commonly represented as continua in numerical models (e.g. Esterhuizen et al., 2010b; Sinha, 2020). Models investigating pillar yield require calibration to empirical pillar formulas, laboratory data, or field measurements from site-specific case studies.

Iannacchione (1990) modeled in-situ coal pillars using the FDM and the Mohr-Coulomb strain-softening (MCSS) constitutive model to test the influence of applied pressure and coal-roof interface on pillar yield. Overburden material was modeled as an elastic homogenous material and the coal-floor interface and underburden were not modeled. Results indicated that interface friction and cohesion had a significant impact on vertical stress distribution within pillars under stable loading (Figure 4.15a) and average pillar strength (Figure 4.15b).
Iannacchione (1990) noted that although this phenomenon has been observed in numerical modeling results and in the field, the linear MCSS failure criterion overpredicted the size of the yield zone at the pillar periphery and the magnitude of pillar core stresses when compared to expected behavior. Gale (1998) conducted a similar study, but modeled both the coal-roof and coal-floor interfaces, which resulted in even larger differences in average pillar strength for various interface strengths when compared to Iannacchione (1990). Badr (2004) modeled coal pillars using three-dimensional FDM with MCSS properties. They were able to calibrate material properties to agree with the expected peak strengths of the Bieniawski (1984) and Salamon (1967) empirical strength equations for w/h 1-8, when using a coal cubic strength value (K) of 9 MPa. However, when compared to in-situ pillar testing, average pillar stress calculated by the numerical model deviated with increasing sample w/h.
Esterhuizen et al. (2010) presented a calibrated coal pillar model fit to the Bieniawski pillar strength equation using the Hoek-Brown strain-softening (HBSS) failure criterion for pillar w/h 3-8. They were also able to capture coal rib stress profiles from case studies. Realistic coal roof interface properties were selected and then HBSS parameters adjusted to match the Bieniawski pillar strength equation. The calibrated coal pillar model was then implemented in multiple numerical models featuring heterogeneous MCSS roof lithologies based on case study data. Model results of chain and barrier pillar stress distributions were compared to empirical equations and in-situ stress measurements with good agreement.

A plethora of studies that follow a similar method as those previously discussed (i.e. FDM, MCSS or HBSS coal seam, explicit coal-roof interface, and various overburden and underburden representations that do not consider explicit discontinuities) can be found throughout the literature, where the numerical strength of coal pillars is calibrated to empirical pillar formulae (Mueller, 1990; Vervoort, 1992; Fama et al., 1995; Yavuz, 2001; Salamon et al., 2003; Lu et al., 2008; Oraee et al., 2009; Tesarik et al., 2013; Poulsen et al., 2014; A. J. Das et al., 2017). These calibrated coal pillar models are then typically verified through numerical analysis of instrumented case studies.

In the context of understanding pillar behavior, the discrete element method is largely implemented through the use of Bonded Block Method (BBM) models (i.e. Voronoi, Trigon) (Preston et al., 2013; Raffaldi, 2015; Sinha & Walton, 2018; Wu et al., 2019; Li & Bahrani, 2020), or when pillar discontinuities (e.g. rockmass fabric, throughgoing joint or fault) are controlling the deformation response (Esterhuizen et al., 2011; Esterhuizen et al., 2019). Other applications include studying strain energy release of rockburst due to deconfinement along interfaces about the pillar (Poecck et al., 2015, 2016; Zhang et al., 2015; Poecck, 2016; Khademian & Ozbay, 2018; Wang & Kaunda, 2019) or the influence of groundwater on overburden and pillar stability (Minkley et al., 2016).

In contrast to the aforementioned studies, a limited number of previous studies using the explicit DEM have represented coal pillars as continua, while analyzing the impact of discontinuous host rock on pillar yield as is done in this chapter. Gu & Ozbay (2015) utilized the explicit DEM to compare the difference in pillar-roof interface slip failure when using either the MC or continuously yielding (CY) joint constitutive model with fully elastic pillar, overburden, and underburden material properties. They found that MC joints could only capture stable slip failure
along the pillar-roof contact, while CY joints captured the sudden decrease in normal stress upon joint yield but did not analyze the impact on average pillar stresses.

Shen & Duncan Fama (2018) conducted a case study of pillar and overburden stability in a highwall coal mine failure using the explicit DEM. Pillars were modeled with the MCSS constitutive model, roof rocks were modeled as perfectly plastic MC materials and the underburden was modeled as fully elastic. Model results indicated that the pillars failed via axial splitting at a w/h of approximately 0.65, based on the tensile yielded elements in the model result (Figure 4.16).

![Figure 4.16: UDEC model results showing yielded elements of highwall mining pillars interpreted as failure via axial splitting (from Shen & Duncan Fama, 2018).](image)

Due to uncalibrated pillar behavior and post-peak uncertainty, yielded pillars were then deleted and roof stability was considered. Due to the increase in unsupported span, the model roof collapsed (Figure 4.17).
Considering these previous studies collectively, it was concluded that there exists a relative lack of research in the continuum representation of coal pillars using the explicit DEM to explicitly and accurately account for the impacts of non-unique, laminated, and discontinuous rockmasses on pillar yield.

4.2.8 The Progressive S-Shaped Yield Criterion

When utilizing a continuum material representation, modeled pillar behavior is heavily dependent on the constitutive model used (Lisjak & Grasselli, 2014). The most commonly applied constitutive models cannot accurately capture the full spectrum of pillar behavior and progressive damage under various levels of confinement and during various stages of load-deformation response (Sinha, 2020). To overcome this limitation, Sinha & Walton (2018) and Sinha (2020) developed a yield criterion based on the Ultimate S-shaped yield criterion (Diederichs, 2007; Kaiser & Kim, 2015). Known as the Progressive S-shaped yield criterion (Figure 4.18), it combines the low-confinement cohesion weakening friction strengthening (CWFS) criterion (Hajiabdolmajid et al., 2002) with classic shear yield criterion (i.e. MC, HB), and accounts for...
progressive damage and strength reduction due to plastic strain accumulation to capture the full range of pillar behavior without a near-exponential increase in strength with increasing confinement (i.e. pillar w/h) (Sinha, 2020).

![Progressive S-shaped yield criterion](image)

Figure 4.18: Conceptualization of the Progressive S-shaped yield criterion (from Sinha, 2020).

Progressive S-shaped yield criterion inputs for coal pillars were calibrated by Sinha (2020) to fit the Mark-Bieniawski pillar strength equation in FDM models for pillar w/h 2-8. No modeled coal-roof or coal-floor interface were utilized so that the lateral constraints used in the large-scale in-situ tests (Bieniawski & Van Heerden, 1975) were approximated (Figure 4.19).
The calibrated continuum representation was then validated in multiple case studies with comparisons to pillar loading, rib displacement, and stress distribution (Sinha, 2020). The Progressive S-shaped yield criterion was subsequently recalibrated to the Mark-Bieniawski pillar strength equation for implementation in a two-dimensional explicit DEM model (Figure 4.20).
Results captured both the pillar peak strengths and the transition from brittle to pseudo-ductile behavior with increasing w/h ratio. Additional cases were analyzed with a CY joint interface featuring moderate (i.e. 25° initial and 20° intrinsic friction angles) and weak (i.e. 15° initial and intrinsic friction angles) contacts, and a joint roughness parameter of 0.1 mm. This resulted in significant decreases in average peak strength for all w/h pillars tested, which is consistent with previous research on the impact of discontinuities/interfaces at pillar boundaries on pillar strength (Iannacchione, 1990; Gale, 1996; Gale, 2017).

4.2.9 Summary

Based on the review of relevant literature, it is evident that pillar-roof mechanical interaction is not explicitly considered in practical pillar design methods. Furthermore, considerations of pillar loading and pillar strength are often separated until calculation of an ostensibly “conservative” factor of safety to act as a substitute for mechanical accuracy. Existing numerical research has primarily utilized continuum or pseudo-discontinuum representations of the roof and overburden and the combined impact of a wide range of geologic and mining conditions has not been considered. In particular, the explicit DEM, which can account for these complex discontinuous and laminated conditions, is predominantly applied to specific case studies and utilizes either an overly simplified (i.e. elastic) or localized (i.e. BBM) approach to representing roof, overburden, and coal material. A broader application of the explicit DEM is required to understand specific
sensitivities of pillar-roof mechanical interaction and how they can be applied to a wide range of geologic and mining conditions. Lastly, the calibrated Progressive S-shaped yield criterion accurately captures the mechanically relevant controls on pillar confinement and subsequent peak strength and can account for the effects of interface (i.e. end-constraint) strength when combined with the continuously yielding joint constitutive model.

4.3 Methodology & Model Inputs

Based on the results of the binary logistic regression analysis in Chapter 3, key parameters influencing roof stability in the single-entry models were identified. In order to determine the impact that roof and overburden properties have on pillar loading and deformation, a subset of 27 models that represent various combinations of the explicit DEM inputs that most significantly impacted roof stability in supported and unsupported models were selected.

Models are classified and discussed in relationship to either their modeled roof or pillar properties. For clarity, each combination of roof properties, including baseline (i.e. Chapter 3) entry depth, unsupported stability result, bolted stability result, roof block material model, in-situ horizontal stress ratio, explicit joint strength, bedding thickness, and DFN, is assigned a roof property identification (RPID) number (Table 4.1). Note that in Chapter 3 models, pillar-interface strength was not independently considered from DFN joint strength in the roof, all inelastic contacts in the roof-stability models (i.e. bedding, cross-joints, pillar-floor and pillar-roof interfaces) were modeled with identical strength and stiffness for the weak, moderate, and strong joint cases tested in Chapter 3. Furthermore, these variations in contact strength identified little to no statistically significant impact on roof stability probability despite the magnitude of the cohesion-roughness rating in the Coal Mine Roof Rating (CMRR) empirical method. Although the condition of the pillar interface with the roof and floor has been identified by others (e.g. Roberts et al., 2002; Gale, 2017) as an important factor controlling peak pillar strength and loading, this chapter is focused on capturing the impact of a realistic range of roof stability behavior on pillar loading and the subsequent pillar-roof interaction. Therefore, the moderate strength contact representation was selected based on the interfaces modeled in Esterhuizen et al. (2010) and Roberts et al. (2002) to avoid promoting pillar stability or instability based on excessively strong or weak interfaces. Recall that the moderate strength contacts are represented using the continuously yielding joint constitutive model with an initial (i.e. peak) friction angle of 25° and an intrinsic (i.e. residual)
friction angle of 20°. The effects of stronger and weaker interfaces on pillar-overburden mechanical interaction are explicitly considered in Chapter 5 panel-scale models.

Table 4.1: DFN seed 100 models selected for analysis with calibrated Progressive S-shaped yield criterion modeled coal pillars, roof stability result from Chapter 3 indicated, refer to Chapter 3 for intact material strength and stiffness properties, RPID = roof property identification number, Mod = Moderate, SUBI = strain-softening ubiquitous joint constitutive model, BT = bedding thickness, DFN = discrete fracture network.

<table>
<thead>
<tr>
<th>RPID</th>
<th>Depth</th>
<th>Ch. 3 Unsupported Result</th>
<th>Ch. 3 Bolted Result</th>
<th>$k_o$ Ratio</th>
<th>Roof Block Material Type</th>
<th>Contact Strength</th>
<th>BT</th>
<th>DFN ID</th>
</tr>
</thead>
<tbody>
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<td>161</td>
<td>30 m</td>
<td>Stable</td>
<td>Stable</td>
<td>0.5</td>
<td>Strong SUBI</td>
<td>0.5 m</td>
<td></td>
<td>1</td>
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<td></td>
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<td>Stable</td>
<td>0.5</td>
<td>Strong SUBI</td>
<td>0.5 m</td>
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<tr>
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<td>Stable</td>
<td>0.5</td>
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<td>Stable</td>
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Recall that coal pillars in Chapter 3 single-entry models were modeled with higher peak cohesion such that the pillars behaved elastically and deformed uniformly under the loading conditions modeled therein. The models in this chapter were run with the coal pillar parameters developed through calibration of the Progressive S-shaped yield criterion (Walton et al., 2020), to the Mark-Bieniawiski pillar strength equation (Mark & Chase, 1997).

To determine the effect of pillar stability on roof stability, each of the RPID (i.e. set of roof properties) in Table 4.1 were run with varying model depth, pillar stiffness, and pillar w/h in both unsupported and bolted configurations for a total of 36 additional cases per model (Table 4.2).
This resulted in 972 additional models investigating the interaction between roof and pillar at the single-entry (i.e. local) scale.

Table 4.2: Pillar parameters and model geometries tested in Chapter 4 for each Chapter 3 model case listed in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values Tested</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth to Entry (m)</td>
<td>Baseline, 300, 450</td>
</tr>
<tr>
<td>Pillar w/h</td>
<td>8, 4</td>
</tr>
<tr>
<td>Pillar Stiffness (GPa)</td>
<td>3.0, 1.5, 4.5</td>
</tr>
<tr>
<td>Support</td>
<td>Unsupported, Bolted</td>
</tr>
</tbody>
</table>

Bolts were installed in an identical manner to Chapter 3, following 70% in-situ stress relaxation to mimic elastic displacement prior to bolt installation. Bolt geometric and material properties (Bahrani & Hadjigeorgiou, 2017) were also identical to those in Chapter 3 and are summarized in Table 4.3.

Table 4.3: Material properties for rockbolt and structural (i.e. faceplates) elements utilized in this study. E = Young’s Modulus, YS = Yield Strength, TFS = Tensile Failure Strain

<table>
<thead>
<tr>
<th></th>
<th>Area (mm²)</th>
<th>Density (kg/m³)</th>
<th>E (GPa)</th>
<th>Bolt YS (kN)</th>
<th>Bolt TFS (strain)</th>
<th>Plastic Moment (kN-m)</th>
<th>Shear Stiffness (MN/m²)</th>
<th>Normal Stiffness (MN/m²)</th>
<th>Shear Cohesion (MN/m)</th>
<th>Normal Cohesion (MN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolts</td>
<td>280</td>
<td>8050</td>
<td>200</td>
<td>176</td>
<td>0.15</td>
<td>2</td>
<td>50</td>
<td>100000</td>
<td>1.2</td>
<td>8</td>
</tr>
<tr>
<td>Plates</td>
<td>600</td>
<td>8050</td>
<td>200</td>
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</table>

Modeled depths were increased from their baseline value (i.e. either 30 or 200 m) to 300 and 450 m by increasing the initial in-situ stresses in order to push pillars to post-peak behavior. The boundary condition at the top of the model was also identical to Chapter 3 models, applying a vertical zero-velocity boundary condition for deeper model cases and a free surface for shallow (i.e. 30 m) models. Recall that this boundary condition assumes full stress arching at 30 m above the entry in the event that roof instability propagates to the upper boundary. This more similar to PAT than it is to TAT. However, the horizontal symmetry conditions imposed by the boundary conditions on either side of the model make comparisons to PAT impossible.

Variation in pillar stiffness was used to consider pillar deformation independent of any yield effects, particularly at depths where pillars were stable. The models were run using a uniformly small zone size of 0.125 m in the pillars and overburden. Due to the different pillar w/h values considered and associated change in model geometry, the DFNs for varying pillar w/h were not identical (Figure 4.21).
This variation was considered largely inconsequential based on the Chapter 3 results of roof stability models with different seeds (100, 1234) that provided nearly identical results, indicating that the DFN input mean and standard deviation values are the primary influences on DFN effects. Note that different depths featured identical DFNs, as the size of the model was kept constant by adjusting the initial stress condition of the model.

The history command in UDEC was utilized to track and extract the average pillar vertical stresses and strains, average pillar maximum and minimum principal stresses, roof midspan displacement, roof-pillar boundary displacement, as well as material yield in the roof and pillars every 100 steps. Pillar average vertical stress ($\sigma_{yy,av}$) was calculated as:

$$\sigma_{yy,av} = \frac{\sum_{i=1}^{n} \sigma_{yy,i} \times A_i}{A_{pillar}}$$ (4.13)

where $i =$ zone, $n =$ number of pillar mesh elements, $\sigma_{yy,zone} =$ zone vertical stress, $A_{zone} =$ zone area, and $A_{pillar} =$ pillar area. Average horizontal ($\sigma_{xx,av}$) and shear ($\sigma_{xy,av}$) stresses were calculated in identical manners from zone stresses. Pillar average major ($\sigma_{11,av}$) and minor ($\sigma_{33,av}$) principal stresses were calculated as:

$$\sigma_{11,av}, \sigma_{33,av} = \frac{\sigma_{xx,av} + \sigma_{yy,av}}{2} \pm \sqrt{\left(\frac{\sigma_{xx,av} - \sigma_{yy,av}}{2}\right)^2 + \sigma_{xy,av}^2}$$ (4.14)
This method allowed for calculation of the pillar average major and minor principal stresses, rather than the average of the individual zone principal stresses. Pillar average vertical strain was calculated from the vertical displacements between 10 equally spaced gridpoints at both the top and bottom of each pillar (Figure 4.22). Average pillar vertical strain ($\varepsilon_{yy,av}$) was calculated as:

$$\varepsilon_{yy,av} = \frac{\sum_{i=1}^{10} L_i}{10}$$

where $L_i$ and $L_o$ are the pillar dimensions defined in Figure 4.22.

Material yield in the roof and pillars was tracked using the FISH variable $z\_state$ (i.e. yielded or unyielded). First, the number of zones in the immediate roof beam and in each explicitly modeled pillar were counted via FISH function. The immediate roof beam was defined by the bounds of the excavation span (6 m wide) and the bedding thickness (0.5 or 1.0 m thick) at the top of the excavation of a given model. This allowed for the percent of yielded elements in a given area to be directly compared to one another. For example, a w/h = 4 and w/h = 8 pillar can be compared via the percentage of area that incurred yield, independent of their actual size.

The history files produced by the models allowed for a thorough analysis of pillar-roof interaction as yield propagated through the single-entry models. Additionally, at model equilibrium or overlap tolerance error, roof gridpoint displacements and velocities, roof zone stresses, pillar rib gridpoint displacement, and pillar-roof and pillar-floor contact shear displacement were extracted using the table command.
Consideration of model results is primarily separated into baseline depth cases (i.e. 30 and 200 m), and deeper entries to analyze the complex interaction more clearly between roof and pillar at multiple stress magnitudes. Further separation based on Chapter 3 roof stability results (i.e. stable vs. unstable) was also utilized to assess the explicit impact of roof stability on pillar-roof interaction. Sections 4.4.1 through 4.4.3 look at impacts and implications of specific subsets of model results on the mechanical interaction between roof and pillar. Namely, these sections are separated based on how overburden properties (i.e. roof stability) and variations in depth, pillar stiffness, and pillar w/h affected pillar loading, stress development, ultimate strength, and post-peak behavior at the entry scale. Finally, model results were considered more broadly via comparison to analytical predictions of pillar loading (i.e. TAT) and strength (i.e. Mark-Bieniawski pillar strength equation) in Section 4.4.4.

Models were run using the same model setup attributes outlined in Chapter 3 with the exception of the block delete command. The block delete command was removed to allow for analysis of the impact of material bulking on pillar stress development. Two main entry failure types were identified in unstable models: partial failure (i.e. immediate roof failure, yielding of pillar rib) and total failure (i.e. massive roof fall, total pillar collapse). In some cases, partial rib failure did not significantly intrude upon the entry are therefore considered stable in these cases. Partial and total failure were classified on a case-by-case basis through visual observation of pillar and roof displacement and velocity at model equilibrium or the point of model “contact overlap” error.

Many failed entries incurred “contact overlap” error prior to model equilibrium, making a direct comparison between all model results impossible. However, direct comparisons could be made between model results of average pre-peak pillar behavior, particularly at similar depths or between similar roof and pillar conditions. Furthermore, qualitative comparisons and results were considered for future panel-scale modeling efforts.

Single-entry models responded to changes in modeled depth and pillar properties (w/h and stiffness) with a wide range of behavior. Note that in some of the unstable model results presented, the roof and pillars are still moving at significant velocities, while others have reached a lower solution ratio prior to incurring “contact overlap” error. Regardless, examples of partial entry failure and total entry failure caused by instability in model roof, pillar, or both are discussed in this chapter and all non-equilibrium model results are identified as such.
4.4 Pillar-Roof Mechanical Interaction Results

4.4.1 Properties Promoting Roof Stability in Shallow and Moderate Depth Entries

540 of the 972 calibrated pillar single-entry models were developed from Chapter 3 roof stability models that were stable in both unsupported and bolted conditions with effectively elastic pillar properties (see Table 4.1). The effects of model inputs on stable roof conditions combined with calibrated pillar properties were first investigated through the shallow and moderate depth (i.e. 30 and 200 m) entry model results for pillar stress, strain, roof displacement, and yielded elements. Considering pillar-roof interaction in high stress loading conditions is complicated by unknowns such as the effects of excessive material yield, or when and where “contact overlap” error will occur. Identifying mechanical trends at lower stress levels, where roof stability is the baseline condition according to Chapter 3 model results, provides a foundation from which to analyze more complex results in higher stress models.

4.4.1.1 Average Pillar Loading & Yield in Shallow Unsupported Entries

In shallow entries (30 m deep), for the same pillar w/h and pillar stiffness, in-situ stress ratio (i.e. mining condition rather than geologic condition) was found to have the greatest impact on pillar loading and stress path (Figure 4.23). As expected, modeled geologic properties such as roof block material, bedding thickness, and DFN cross-joint orientation did not significantly impact average pillar stress under relatively low magnitude (i.e. stable) loading. Recall that low levels of stress coupled with changes in other inputs (e.g. material type, bedding thickness, DFN, etc.) had no statistically significant effect on binary roof stability in Chapter 3. However, the models with $k_o = 2.0$ (RPID 4577, 4578, and 4585) had a softer pillar response to loading in the complete absence of block material yield, incurring approximately twice as much strain vertical strain than $k_o = 0.5$ models without incurring any additional vertical stress.
Figure 4.23: Average (a) vertical pillar stress strain and (b) stress path plots for six stable roof models with various geologic and mining conditions (see Table 4.1), results depicted are from models with w/h = 8 pillars with 3.0 GPa Young’s Modulus. Note that pillar stress and strain measurements began after excavation, hence the baseline in-situ vertical stress of 0.7 MPa.

The mechanism behind this divergence in pillar response is due to the difference in the redistribution of excavation induced stresses for the two different $k_o$-ratios. When $k_o = 0.5$, the major principal stress is vertical and when $k_o = 2.0$ the major principal stress is horizontal (Figure 4.23b). When the excavation is advanced the elevated horizontal stresses in the pillar under $k_o = 2.0$ loading conditions must be transferred through the roof and floor due to the unconfined state of the rib. This results in elevated stress magnitudes with orientations sub-horizontal to the pillar interfaces, increased interface shear displacement, and increased horizontal pillar deformation. Since the pillar is a continuum and still behaving elastically at 30 m deep, the horizontal deformation (i.e. lateral strain) is accompanied by an increase in axial strain due to Poisson effects (Figure 4.24). This confirmed that the pillar behavior was responsive to the changes in other model parameters, even at low stress magnitudes. At equilibrium, the average major principal stress of the $k_o = 2.0$ pillars ultimately decayed to be equal to the average minor principal stress. In the $k_o = 0.5$ models, the average major principal (vertical) stress could not increase without an associated increase in average minor principal stress and was approximately twice as high as the minor principal stress at model equilibrium (see Figure 4.23).
Figure 4.24: Comparison of model contact shear displacement between (a) $k_o = 0.5$ and (b) $k_o = 2.0$, 30 m deep models at equilibrium. Generalized orientation of major principal stress depicted for both cases, not to scale.

Similar analyses for the other pillar w/h (i.e. w/h = 4) and pillar Young’s Moduli (i.e. 1.5 and 4.5 GPa) under the same roof and depth conditions, showed identical trends with smaller pillars incurring higher stress, and stiffer pillars incurring less strain, as expected.

4.4.1.2 Pillar-Roof Interaction & Bolt Effects in Shallow Entries

As the changes in modeled roof geologic properties (e.g. strength, stiffness, DFN) had little effect on pillar loading in shallow entries, and the changes in pillar properties (e.g. w/h, stiffness) had the expected effect on pillar loading, the effects of pillar properties on roof stability were expected to be similar. This section considers the subtle impacts of pillar-roof interaction in shallow entries through pillar behavior (i.e. average stress path), roof displacement and yield, and roof stability.

Overall, pillar stiffness and size had insignificant impacts on binary roof stability and zone material yield as seen in the midspan displacement and average percent immediate roof beam yield results for inelastic roof block models (Figure 4.25). Recall that percent immediate roof beam zone yield was calculated as the number of yielded zones divided by the total zones in a given area of the model. The immediate roof beam was considered to be the area bounded by the horizontal extents of the excavation and a single bedding plane into the roof, making the area one bed (i.e. 0.5 or 1 m) thick and as wide as the span of the excavation (6 m).
The most significant impacts were noted in the 30 m deep, $k_o = 2.0$, “Moderate SUBI” block material model (i.e. RPID 4578). The stiffest pillar version of this model incurred partial roof failure of the immediate roof following significant SUBI yield. This model featured the roof properties of RPID 4578 (see Table 4.1) combined with the stiffer (i.e. 4.5 GPa) and less squat (i.e. w/h = 4) calibrated pillar model. The change in DFN geometry due to change in model width could have some impact on roof stability, as the w/h = 4 pillar models happened to have fewer explicit joints in the immediate roof beam, increasing the local rockmass modulus and effective local roof stiffness. However, a distinct trend of increasing pillar stiffness causing greater roof damage at the same level of roof displacement was identified. Below 5 mm of roof displacement, the stiffer pillar model for RPID 4578 (see Figure 4.25, yellow dashed line, circular markers) has higher levels of roof yield. This reduces the stress arching capacity of the roof, promotes earlier roof collapse, which reduces the amount of overall yield possible in the immediate roof at model equilibrium (i.e. once the roof collapses it cannot transmit horizontal stress and therefore experiences less yield overall) (Figure 4.26). Counterintuitively, the stable roof has more inelastic zone yield at equilibrium because of the type of yield and the timing at which it occurred. Note the
SUBI slip yield (i.e. simulated bedding plane shear failure) at the roof corners in the stiffest pillar case (Figure 4.26, red circles) are largely absent in the 3.0 GPa pillar case. This resulted in the ultimate breakdown of voussoir arching, and collapse of the immediate roof.

Figure 4.26: Influence of pillar stiffness on roof deformation and zone yield in w/h = 4 pillar variations of RPID 4578, red circles highlight the major differences in roof yield between the two stiffer pillar cases.

The increased SUBI slip yield was not due to the differential displacements of the roof midspan versus abutment, which were comparable at approximately 1.1 mm for both pillar stiffnesses. This indicates the initial increased yield that led to roof failure in stiff pillar models was predominantly controlled by the concentration of decaying and rotating horizontal stress at the excavation roof corners, sub-horizontal to implicit ubiquitous joints, due to the stiffness of the pillar and roof, rather than increased differential roof displacement leading to increased roof yield.

Pillar stress path results in unsupported cases indicate that even though the pillars did not yield, the w/h = 4 pillars experienced a rotation of major principal stress from horizontal to vertical because the smaller pillars could not maintain sufficient levels of confining stress due to their
smaller size (Figure 4.27). However, if the w/h = 4 pillar is soft, it can undergo more deformation than the roof, reducing the amount of deformation and yield in the roof, allowing it to maintain voussoir stress arching, and preventing partial roof failure along explicit discontinuities.

Figure 4.27: Stress path comparison of multiple model results featuring roof conditions of RPID 4578 combined with different pillar w/h and pillar stiffnesses.

This sudden change in roof stability at low stress levels due to subtle changes in pillar properties illustrates how complex pillar-roof interaction can be even without considering the effects of pillar yield, failure, or collapse.

As expected, the addition of rockbolts to self-stable entries had a limited effect on pillar loading, particularly in w/h = 8 pillar models. Average peak stresses, strains, and stress paths remained approximately unchanged from their unsupported counterparts. However, results from w/h = 4 pillar models show some variation in pillar behavior under supported roof conditions. Most significantly, the roof fall induced by stiff w/h = 4 pillars as shown in Figure 4.26 was prevented by the installation of passive roof bolts (Figure 4.28).
Pillar stiffness influence on the roof mechanical behavior can still be seen in terms of increasing yield. The stiffest pillar w/h = 4 case tested still has the most roof yield, but only in the corners where bolt elements do not intersect the yielded zones. Both the passive nature and orientation of the bolts prevent them from completely reducing the yield above the pillar corners incurred by horizontal stress rotation and initial beam deflection (i.e. < 1.0 mm). However, once roof deflection increases, support from the passive bolts activates and prevents the SUBI tensile yield and loss of stress arching capacity seen in Figure 4.26.

4.4.1.3 Average Pillar Loading & Yield in Moderate Depth Unsupported Entries

In moderate depth (i.e. 200 m), unsupported models with previously stable roof properties, various degrees of pillar yield began to manifest under certain geologic and mining conditions with
interesting impacts on entry stability. Once again, $k_o = 2.0$ and “Soft Elastic” roof blocks had a softening effect on pillar mechanical response, but the roof properties with the most significant impacts were the “Soft Elastic” roof blocks coupled with the sub-vertical DFN and $k_o = 0.5$ (i.e. RPID 3337), and the “Stiff Elastic” roof blocks coupled with the vertical DFN and $k_o = 0.5$ (i.e. RPID 3233). Additionally, a difference in left and right pillar behavior emerged with the onset of pillar yield due to the stochastic nature of the DFN (Figure 4.29). In the “Stiff Elastic” roof block case (RPID 3233), stable pillar loads are significantly lower than for the other roof block properties tested. “Stiff Elastic” roof blocks allow for more excavation-induced stresses to be carried in the roof, reducing the excavation induced load imparted to the pillars. This was also the case in the other w/h and pillar stiffness tested with “Stiff Elastic” roof blocks.

![Figure 4.29: Comparison of left and right pillar behavior in 200 m deep models featuring roof properties resulting in self-stable roofs (see Table 4.1), pillar w/h = 8, and coal Young’s Modulus = 3.0 GPa.](image)

In the “Soft Elastic” roof block cases, explicit DFN properties had a greater impact on macroscopic pillar behavior. Sub-vertical DFN cross-joints (RPID 3337), induced partial failure and strain-weakening deformation of the right pillar only, while the left pillar yielded with minimal horizontal displacement of the pillar rib and incurred slightly higher average pillar stress (Figure 4.30).
However, the slight increase in left pillar average vertical stress shown in Figure 4.29 did not account for the entire amount of load that shed from the right pillar rib. The stresses in the right pillar dissipated through pillar yield and overburden loads were transferred towards the unyielded, confined center of the pillar, as indicated by model stress results. Even though the right pillar clearly incurred significant rib damage, the entry in Figure 4.30 was considered stable, since the model velocities at equilibrium indicated that the entry stabilized.

![Figure 4.30: Model RPID 3337 results at equilibrium of “Soft Elastic” roof block, $k_o = 0.5$, sub-vertical DFN, $w/h = 8$ pillars with a Young’s Modulus = 3.0 GPa showing the difference in pillar vertical stress distribution after significant rib failure of the right pillar.](image)

While the mechanism behind the differential pillar deformation is dependent on the interaction of material property and DFN geometry (i.e. the “Stiff Elastic” and “Strong SUBI” roof block models had no differential macroscopic pillar behavior), there are intricacies of this mechanism that are not immediately obvious. Did the physical presence and interconnectedness of multiple discontinuities in a given location (i.e. above the right pillar rib in Figure 4.30) increase the large-scale overburden loads imparted to the right pillar rib, did the presence of discontinuities cause
localized stress concentrations inducing additional pillar yield, or did the orientation and density of the sub-vertical discontinuities decrease stress arching capabilities and pillar confinement?

Looking at the vertical DFN counterpart to the entry shown in Figure 4.30 (i.e. RPID 3329) the differential pillar deformation was far less pronounced and the absolute difference in load between left and right pillar at equilibrium was negligible (see Figure 4.29). This indicates that the vertical DFN either promoted pillar stability by decreasing the overburden loads bearing on the pillar rib through the explicit location and interconnectedness of the cross-joints above the pillar, or the density of joints was such that localized stresses did not concentrate, or the orientation of joints more effectively transferred stresses and promoted pillar confinement, resulting in more equally loaded pillars.

Similar analysis for other pillar Young’s Moduli (i.e. 1.5 and 4.5 GPa) tested showed identical trends with smaller pillars incurring higher loads, and stiffer pillars incurring less strain, as expected. Softer w/h = 8 pillars had consistent, stable mechanical responses to loading between left and right pillars and prevented the significant pillar yield observed in the right pillar of the model roof conditions in Figure 4.30. The stiffer w/h = 8 pillars led to increased pillar yield and strain-weakening behavior in both the left and right pillars under the low horizontal stress, “Soft Elastic” roof block cases with both vertical (RPID 3329) and sub-vertical (RPID 3337) DFNs (Figure 4.31).
Stiffer pillars clearly exhibit more uniform pillar deformation in the post-peak when compared to their softer counterparts. However, the right pillar in both yielded cases (i.e. RPID 3329 and 3337) incurred more average axial strain and carried slightly less load than the left pillar. Additionally, the vertical DFN case (RPID 3329) had deformed the right pillar slightly more than the left, reversing the behavior seen in the previous model cases with moderate stiffness pillars. However, the differential pillar behavior under vertical DFN conditions (RPID 3329) was less pronounced than the corresponding case with a sub-vertical DFN, further confirming that vertical joints decreased average pillar stress differential. Although both pillars in the sub-vertical DFN case (RPID 3337) were in the post-peak due to rib damage, the entry remained stable as determined by final velocity of the roof and pillars at the model’s equilibrium solution ratio.

In w/h = 4 pillars, the overall behavior was remarkably similar, with “Stiff Elastic” roof blocks decreasing pillar average stress, and $k_0 = 2.0$ resulting in a softer pillar response. All w/h = 4 pillars were stable for the “Stiff Elastic” and “Strong SUBI” roof block cases (RPID 3041, 3042, 3049, and 3050) regardless of their pillar stiffness. Insignificant variations in differential loading were also noted in the high horizontal stress, “Soft Elastic” roof block models (RPID 4289 & 4297) for various pillar stiffnesses. RPID 3329 and 3337 (i.e. “Soft Elastic” roof blocks) models consistently
resulted in pillar strain-weakening behavior in one or both modeled pillars, across all three pillar stiffnesses for the w/h = 4 pillars (Figure 4.32).

In all vertical DFN (RPID 3329, Figure 4.32a) cases, the pillar results were not quantitatively comparable due to “contact overlap” error occurring in the 1.5 and 4.5 GPa pillar models. “Contact overlap” error did not occur in the 3.0 GPa pillar model, and the entry was stable despite significant pillar yield approaching the pillar center. Based on 3.0 GPa pillar equilibrium these results, it was assumed that the other pillars followed the same strain-weakening response. The pillar core maintained confining stresses, friction was fully mobilized around the core, and cohesion remained at peak values within the core, indicating that it was still functional at the equilibrium solution ratio (Figure 4.33).
Recall that the Progressive S-shaped yield criterion is based on the CWFS yield criterion for low confining stresses and the areas of the pillar that have undergone significant yield in tension have negligible or zero cohesion values, while their friction angles have mobilized. This behavior was also noted in the 1.5 and 4.5 GPa models, indicating they would likely equilibrate to a similar condition, however, the impact of these specific pillar properties on roof stability could not be determined with full certainty due to “contact overlap” error.

Conversely, all sub-vertical DFN (RPID 3337) cases converged to an equilibrium solution ratio without incurring “contact overlap” error and showed decreasing pillar stability and increasing pillar stress differential with decreasing pillar stiffness. Furthermore, in the soft pillar case where significant yield occurred in the left pillar, the right pillar maintained its stress and strain level,
further confirming that the sub-vertical DFN increased differences in behavior between the two pillars. All w/h = 4, k_o = 2.0 models incurred far less pillar yield than their k_o = 0.5 counterparts (regardless of DFN) due to the initial confined condition of the pillar reducing pillar rib tensile yield.

4.4.1.4 Pillar-Roof Interaction & Bolt Effects in Moderate Depth Entries

As the changes in modeled roof geologic properties began having an increased effect on pillar loading in moderate depth entries and the changes in pillar properties had the expected effect (i.e. smaller pillars take on more stress, softer pillars take on more strain), the effects of pillar properties on previously stable roofs were considered in a similar manner to those in Section 4.4.1.2.

Pillar stiffness and w/h had no significant impact on binary roof stability but had some influence on zone material yield as seen in the midspan displacement and average percent immediate roof beam yield results for inelastic roof block models in 200 m deep models (Figure 4.34). This is because the only inelastic roof block models with previously stable roof conditions tested at 200 m were the “Strong SUBI” roof blocks. However, overall trends were consistent with previous results discussed in Section 4.4.1.2, where stiffer or more slender pillars increase excavation-induced stress transfer to the roof and tend to increase immediate roof beam yield.

The one case where this did not hold true (RPID 3050) was due to the aforementioned limitation of explicit DFN changes with changing pillar w/h concentrating stresses and slightly increasing roof beam zone yield percentage in the squat pillar model.
4.4.1.5 Stable Roof Discussion

The analysis of the interaction of stable pillar loads and stable roof properties highlights some complexities in pillar-roof mechanical interaction, even in the lowest stress cases with the least variability in roof stability. Geologic and mining condition effects on pillar loading demonstrated in this section include the softening effect of high horizontal stress and differential pillar loading exacerbated by fracture location, interconnectivity, and orientation. Effects noted in the literature and captured in conjunction with the unique findings of this section include pillar load reduction under “Stiff Elastic” roof blocks, the load increase and propensity for pillar yield under “Soft Elastic” roof blocks, and the development of pillar confined cores providing support to the overburden and maintaining entry stability.

Most notably, stiff, low w/h pillars under high horizontal stresses imparted additional yield to inelastic roof blocks and resulted in unstable roof conditions, which were ultimately mitigated by the presence of passive bolt elements. However, the general effect of roof support on this set of models was negligible due to the self-stable nature of the roof properties tested in this section.
4.4.2 Properties Promoting Unstable Roofs in Moderate Depth Entries

432 of the 972 calibrated pillar single-entry models were developed from Chapter 3 roof stability models that were unstable under unsupported conditions with effectively elastic pillar properties (see Table 4.1). The effects of model inputs on unstable roof conditions combined with calibrated pillar properties were first investigated through the moderate depth (i.e. 200 m) entry model results for pillar stress, strain, roof displacement, and yielded elements.

4.4.2.1 Average Pillar Loading & Yield in Moderate Depth Unsupported Entries

When considering models featuring roof parameters that led to roof instability in unsupported Chapter 3 models (see Table 4.1), roof-pillar mechanical interaction was not particularly complicated, given that the roofs were modeled with properties that promoted roof failure. First, the unstable unsupported model results were considered in 200 m depth (i.e. baseline) entries, and a particular pillar response was noted throughout all the model cases. The pillar response to loading initially appears to be strain-weakening, but the unloading of the pillar following the “peak” load is largely elastic in nature and due to the collapse of the roof (Figure 4.35).
Figure 4.35: Average vertical left pillar stress-strain plots for 8 unstable roof models with various geologic and mining conditions (see Table 4.1), results depicted are from models with w/h = 8 pillars with 3.0 GPa Young’s Modulus and “Moderate SUBI” roof material properties. Note that pillar stress and strain measurements began after excavation, hence the baseline in-situ vertical stress of 4.6 MPa.

Although these pillars reached the same average peak stress as pillars under stable roof properties (see Figure 4.29), inelastic damage of the pillar is minimal at 200 m depth with unstable roof conditions. The models with larger roof deformation (i.e. RPID 51xx series, \(k_o = 2.0\)) resulted in significantly different average pillar stress and strain at model equilibrium than their \(k_o = 0.5\) counterparts (i.e. RPID 49xx series models). Note that some pillar stress-strain curves showed increased strain due to yield and deformation of pillar corners where one of ten strain measuring points was located (see Figure 4.22). As was the case in stable roof models in Section 4.4.1, the suite of unstable roof property model results showed high horizontal stress ratio as having a softening effect on pillar loading. Changes in DFN cross-jointing, bedding thickness, and the unstable block material types tested (i.e. only “Strong SUBI” and “Moderate SUBI”) had less significant impacts on pillar response in unstable roof models as their stable roof counterparts.

4.4.2.2 Pillar-Roof Interaction & Bolting Effects in Moderate Depth Entries

Pillar-roof mechanical interaction under weak or unstable roof properties was complicated by the presence of roof support elements, particularly when that support successfully prevented the failure
of the roof. As indicated by pillar and roof analysis methods discussed in Section 4.2 and in previous chapters, the impact of roof stabilization on pillar loading and the impact of pillar behavior on roof support efficacy are not considered in state-of-practice nor in published research. This interaction was first investigated by analyzing results from baseline (i.e. 200 m deep entry) models that featured geologic and mining properties that were unstable in unsupported configurations and stable in the presence of passive bolt elements (see Table 4.1). Average pillar mechanical responses to stabilized roofs in moderate depth (i.e. 200 m) entries were remarkably similar to those of their unsupported counterparts (Figure 4.36). Note that all pillar w/h and stiffnesses tested had identical unsupported versus bolted behavior.

Figure 4.36: Comparison of unsupported and bolted model cases where unstable unsupported model roofs were stabilized in the presence of the passive bolt elements and faceplates with (a) “Strong SUBI” roof blocks and $k_o = 2.0$ or (b) “Moderate SUBI” roof blocks and $k_o = 0.5$. Both (a) and (b) are models with pillar w/h = 8 and $E = 3.0$ GPa.

Under the conditions tested, roof stability had no impact on average pillar loading, although comparison of pillar and roof plastic yield results indicated that at this level of stress, the impacts were more subtle than the resolution of average pillar stress and strain could capture. Once again, the percent area of yielded elements was considered for a finer resolution examination of the observed impacts (Figure 4.37). Note that the degree of pillar yield at this depth was limited and can be conceptualized as pillar corner crushing; the effects of deeper entries and more thorough pillar yield are considered in Section 4.4.3.
Figure 4.37: Plots of percent zone yield for the immediate roof beam (y-axes) and the average percent zone yield between the left and right pillars (x-axes) in unsupported and bolted configurations at model equilibrium. (a) “Strong SUBI” roof blocks and $k_o = 2.0$, (b) “Moderate SUBI” roof blocks and $k_o = 0.5$. Both (a) and (b) are models with pillar \( w/h = 8 \) and \( E = 3.0 \) GPa.

The presence of roof bolts, in conjunction with high horizontal stress, “Strong SUBI” roof blocks, and vertical cross-joints (i.e. RPID 4001 & 4002), decreased the amount of pillar yield by varying degrees depending on the bedding thickness modeled. The presence of bolts in the 1.0 m bedding thickness case (i.e. RPID 4002) had a limited impact on roof and pillar yield (see red circle to red square in Figure 4.37a), while bolts in the 0.5 m bedding thickness case (i.e. RPID 4001) simultaneously increased roof yield and decreased pillar yield (see blue circle to blue square in Figure 4.37a). This decrease in pillar yield indicates that stresses and strains are more effectively carried by the roof and away from the pillars by virtue of the roofs bolted stability. The increase in roof yield is similar to the increase seen in Section 4.4.1.2, where a stable roof actually incurs more yielded elements whether it is self-stable or stabilized by bolts.

In models featuring sub-vertical cross-joints (RPID 4009 & 4010), the change in pillar yield was negligible compared to the significant decrease in roof yield associated with the presence of passive roof bolts. Furthermore, the small change in percent pillar yield was associated with the thick bed model (RPID 4010). Since stresses must either be transferred or dissipate via inelastic strain during yield, this indicates that if no change in pillar yield occurred, then the primary bolt mechanism was suspension and the decrease in roof yield was due to the decrease in roof...
displacement. If a change in pillar yield occurred, this indicates that some beam building was activated via increased stress transfer through the roof. This mechanism can easily be observed in model horizontal stress results between unsupported and bolted cases (Figure 4.38).

Figure 4.38: Depiction of the bolt support mechanism through model results of horizontal stress contours at equilibrium. Models feature unsupported (left) and bolted (right) “Strong SUBI” roof blocks with \( k_o = 2.0 \). All are models with pillar w/h = 8 and E = 3.0 GPa.
The models with no increase in horizontal-stress-bearing roof layers (i.e. RPID 4002 & 4009) also showed no change in percent pillar yield, while both models that increased horizontal stress transfer were associated with a decrease in percent pillar yield (see Figure 4.37). In RPID 4001, the immediate roof beam was nearly entirely destressed and held in place by the faceplates and bolts. The bolts also penetrated into previously deformed layers (see top panels of Figure 4.38) and were promoting horizontal stress transfer (i.e. beam building) in formerly unstable layers of the roof. This positive feedback effect was reinforced by the confinement provided by the immediate roof.

Figure 4.37b shows that the support mechanism in $k_o = 0.5$ models was more consistent, with roof bolts decreasing both roof and pillar yield in all cases. This can be explained in terms of the voussoir beam analog. Recall that roof bolts in voussoir beam models from Chapter 2 increased the effective thickness of a roof based on the material properties of the roof and the number of layers. While this is not an exact comparison, due to the inelastic nature of the models tested in this chapter, the increase in effective thickness decreased the maximum compressive stress. This decrease in roof stress concentration was accompanied by a simultaneous decrease in roof material yield and an increase in overall stress transfer through the roof due to its bolted stability, decreasing yield in the pillars.

The observations regarding pillar and roof yield were consistent throughout changing pillar stiffness for squat pillar models under inelastic roof blocks at 200 m modeled depth (Figure 4.39).
Figure 4.39: Plots of percent zone yield for the immediate roof beam (y-axes) and the average percent zone yield between the left and right pillars (x-axes) in unsupported (circle) and bolted (square) configurations at model equilibrium, (a) “Strong SUBI” roof blocks and $k_o = 2.0$, (b) “Moderate SUBI” roof blocks and $k_o = 0.5$. Both (a) and (b) are model results with pillar $w/h = 8$ and $E = 1.5$ GPa (hollow) and 4.5 GPa (filled).

Consistent with the shallow entry model results in Section 4.4.1.2, stiffer pillars modeled with $k_o = 2.0$ and the vertical DFN (i.e. RPID 4001 & 4002) resulted in significant increases in immediate roof beam zone yield. Overall, softer pillars incurred less pillar yield and were more compliant with the stiffness of the roof blocks, resulting in less variation in the influence of roof properties on pillar-roof mechanical interaction, particularly in $k_o = 0.5$ models. Even the stiffest pillars in the $k_o = 0.5$ models resulted in remarkably similar behavior (i.e. bolting decreased both roof and pillar yield), although with a higher level of equilibrium pillar yield.

Smaller $w/h$ pillars under the same geologic and mining conditions naturally incurred more pillar yield per area of the pillar, and pillar-roof mechanical interaction became less dependent on overburden properties overall (Figure 4.40). The exceptions to this more consistent behavior are the sub-vertical DFN cases coupled with thick beds in both horizontal stress conditions (i.e. RPID 4010 & 4970). These model results indicate that significant increases in both roof and pillar yield occur with increasing pillar stiffness, which are subsequently mitigated by the presence of bolt elements through a beam building mechanism.
When considering roof stability in a binary sense, the model results continue to be consistent with shallow entry models from Section 4.4.1.2 with larger, softer pillars promoting roof stability in unsupported cases. Recall that all of these models resulted in failed roofs when simulated with pillars that behaved elastically, further highlighting the importance of roof-pillar stiffness contrast, and documenting the importance of roof-pillar strength compliance and the influence of other properties on the potential stabilizing and destabilizing effects of horizontal stress (Figure 4.41).
Figure 4.41: Comparison of pillar size and stiffness influence on roof midspan displacement and immediate roof beam zone yield percentage for 200 m deep models with (a) “Strong SUBI” roof blocks and \( k_0 = 2.0 \), and (b) “Moderate SUBI” roof blocks and \( k_0 = 0.5 \).

All 24 \( k_0 = 0.5 \) models shown in Figure 4.41b, regardless of pillar w/h, pillar stiffness, DFN cross-joint orientation, or bedding thickness, resulted in roof failure. This shows no significant change in the macroscopic behavior (i.e. binary roof stability) between effectively elastic pillars and deformable pillars at low horizontal stress ratios. Conversely, many \( k_0 = 2.0 \) models shown in Figure 4.41a resulted in stabilized roof conditions, further confirming that pillar mechanical response under high-horizontal stresses is a critical control on roof self-supporting capacity. Similar to the results in 4.4.1.2, changes in pillar stiffness more significantly impacted roof self-stability in pillar w/h = 4 models. All unstable model conditions were stabilized when modeled with passive rockbolt elements.

In the \( k_0 = 2.0 \), “Moderate SUBI” roof blocks, 1.0 m bedding thickness, and sub-vertical cross-joint DFN models (RPID 5162), an interesting phenomenon occurred in the w/h = 4 pillar subset only. A near total loss of average vertical pillar stress occurred due to significant material yield of the overburden directly above the pillar (Figure 4.42). This behavior was not seen in similar models such as 5161 (i.e. 0.5 m bedding thickness), or 4970 (i.e. \( k_0 = 0.5 \)) due to changes in explicit DFN and stress state, respectively, hindering propagating yield in the overburden above the right pillar.
Note that the right pillar is totally destressed and has only begun to yield via low-confinement axial splitting.

The same model case was considered in its bolted configuration with significantly different results. The presence of support elements has prevented the massive roof collapse seen in Figure 4.42, reduced localized yield above the right pillar, and maintained greater stress transfer to the right pillar (Figure 4.43). Furthermore, there was more stress arching in the overburden. This is consistent with previous results regarding percent pillar and roof yield. However, there was no significant change in immediate roof yield, roof stress state, nor pillar zone yield. This observed change in pillar-overburden mechanical interaction illustrates clearly how roof and overburden properties, as well as roof stability are mechanically linked to pillar loading and pillar behavior, even in moderate stress regimes. The complex impacts of roof properties, pillar properties, and support considered in this section are more critically considered in deeper entry models (i.e. 300 and 450 m) in Section 4.4.3 where pillars are driven to total failure.
Figure 4.43: 200 m deep, bolted, “Moderate SUBI” roof blocks, sub-vertical DFN, \(k_o = 2.0\) model featuring \(w/h = 4\) and \(E = 3.0\) GPa pillars at equilibrium showing the major principal stress contours (left) and yielded elements (right).

### 4.4.2.3 Unstable & Stabilized Roof Discussion

The analysis of the interaction of stable pillar loads and geologic and mining conditions that promote roof instability (i.e. inelastic roof block material) confirmed the previously observed complexities in pillar-roof mechanical interaction and identified new considerations with increasing stress magnitudes. This section has further demonstrated the “softening” effect of high horizontal stress on pillar loading, differential pillar loading exacerbated by fracture location, interconnectivity, and orientation, and the effects of pillar stiffness and size on inelastic roof block yield.

Notably unique to this research is the identification of bolt mechanism response in roof and pillar yield, the stabilizing effect of pillar deformation under high horizontal stress, and the confinement provided by a stabilized roof to promote pillar loading. Furthermore, when modeling excavation stability, elastic pillars are likely to be a conservative assumption when modeling roof stability as they promote inelastic roof failure, particularly in high horizontal stress conditions.
4.4.3 Properties Promoting Self-Stable & Stabilized Roofs in Deep Entries

Following evaluation of geologic and mining conditions leading to stable and unstable roof conditions, and the resulting pillar-roof interaction in unsupported and supported cases, deeper entry model results were considered in order to analyze how more complete pillar failure affected roof-pillar interaction in single entries. Stability of model roofs with inelastic intact material (i.e. blocks) decreases with increasing depth, so a direct comparison of the effects noted in shallow and moderate depth model results (Sections 4.4.1 & 4.4.2) cannot be considered for inelastic roof blocks conditions. Therefore, this section first considers “Stiff Elastic” and “Soft Elastic” roof blocks in conjunction with the other variables listed in Tables 4.1 and 4.2 in order to more directly compare the effect of increasing stress magnitude with the combined impact of roof stiffness, DFN, bedding thickness, k_o-ratio, pillar w/h, and pillar stiffness. Then the effects of inelastic roof blocks in conjunction with the aforementioned roof and pillar properties, as well as passive roof support, are explored.

Note that the propensity for deeper models to incur “contact overlap” error was much higher, and many models did not reach an equilibrium solution ratio. However, peak pillar loads preceded contact overlap error in most of the models, making the macroscopic pillar behavior the focus of this section.

4.4.3.1 Pillar-Roof Interaction Under Elastic Roof Blocks in Deep Entries

First, elastic roof block conditions were considered in order to compare the combined effects of increasing depth and variable roof stiffness, stress ratio, DFN, bedding thickness, and pillar properties without the associated increase in intact roof block yield and associated load shedding. Note that the effect of roof support was considered but is not included due to the insignificant impact on macroscopic pillar behavior under elastic block roof conditions.

First, the influences of model depth, roof block stiffness, k_o-ratio, and DFN on average pillar stress and strain results for w/h = 8 and E = 3.0 GPa pillars were considered (Figure 4.44).
Figure 4.44: Average vertical right pillar stress strain plots for 5 elastic roof block models with various geologic and mining conditions (see Table 4.1), results depicted are from models with w/h = 8 pillars with 3.0 GPa Young’s Modulus. Note that pillar stress and strain measurements began after excavation, hence the variation in baseline in-situ vertical stress.

Under larger stress magnitudes, changes in explicit DFN from sub-vertical to vertical cross-joints had no discernible impact, and higher in-situ stress ratio still had a softening effect on the w/h = 8, E = 3.0 GPa pillar mechanical response. However, the stiffness of the elastic roof blocks and the magnitude of in-situ stresses clearly played a more significant role in deeper model pillar response. Deeper entries have higher stress magnitudes and therefore higher levels of confinement, increasing the peak strength of the pillar in accordance with previous observations by others (e.g. Meikle, 1965; Sheorey et al., 1987; Hasenfus & Su, 1992; Lunder & Pakalnis, 1997; Gale, 2017; Prassetyo et al., 2019). This is further explored at the panel-scale in Chapter 5.

Increasing pillar stiffness in w/h = 8 pillars had expected impacts (i.e. less axial strain) on macroscopic pillar behavior. However, decreased pillar stiffness, combined with “Soft Elastic” roof blocks and sub-vertical DFN cross-joints (i.e. RPID 3337 & 4297) became sensitive to changes in both stress magnitude and k_o-ratio. Furthermore, the k_o = 2.0 model that was not impacted (i.e. RPID 4289) by the changes in modeled depth had notably reduced peak average stresses when compared to the stiff pillar counterparts. This decrease was more pronounced in the deeper entry model results (Figure 4.45).
Figure 4.45: Average vertical right pillar stress strain plots for 5 elastic block roof models with various geologic and mining conditions (see Table 4.1), results depicted are from models with w/h = 8 pillars with 1.5 GPa Young’s Modulus. Note that pillar stress and strain measurements began after excavation, hence the variation in baseline in-situ vertical stress.

The 300 m deep, “Soft Elastic” roof block, sub-vertical DFN, $k_0 = 2.0$ (i.e. RPID 4297, solid green line in Figure 4.45) model, and the 450 m deep, “Soft Elastic” roof block, sub-vertical DFN, $k_0 = 0.5$ (i.e. RPID 3337, dotted yellow line in Figure 4.45) model both have macroscopic pillar behavior that significantly diverges from the other modeled pillars, as well as the stiffer pillar results shown in Figure 4.44. Both were unable to obtain the same peak average stress as the other models yet continued to incur vertical strain until post-peak average vertical stresses of all the pillars were similar at model equilibrium.

Recall in Section 4.4.1.2 that the presence of soft pillars in shallow entries had a stabilizing effect on select $k_0 = 2.0$ models featuring inelastic roof blocks. This was a result of roof-pillar compliance imparting less stress and strain to the roof, which decreased yield in the roof corners. In RPID 4297 (green line in Figure 4.45) the mechanism governing divergent pillar behavior is due to the combined effects of roof-pillar stiffness compliance, coupled with the decreased stress arching capability associated with sub-vertical joints. Average pillar stress path results indicated that average confinement for both the vertical and sub-vertical DFN cases initially decreased at a
similar rate and by a similar magnitude, but the sub-vertical DFN could not maintain nor take on additional vertical (i.e. minor principal) stresses (Figure 4.46). This pillar response is largely due to the yield development in the RPID 4297 pillar, again influenced by the decreased stress arching capacity of the sub-vertical DFN.

Figure 4.46: Plots of average stress path for pillar w/h = 8, E = 1.5 GPa in single-entry models featuring “Soft Elastic” block roof material, sub-vertical DFNs, and $k_o = 2.0$ at two modeled depths. Stress paths start at “o” and end at “x”. Note that the major principal stress is initially the horizontal stress.

The delay in pillar rib bulking due to horizontal stress decay is highlighted in Figure 4.47. Note that both models had equivalent stiffness elastic continuum floors, which maintained horizontal stress arching regardless of model roof DFN. This resulted in the propagation of rib yield from the floor to the roof in Figure 4.47(b)(iv) as the stress arching in the roof decayed in the presence of sub-vertical joints. This yield effect is non-existent in the deeper entry and stiffer pillar sub-vertical DFN cases because horizontal stress rotation is limited and stress magnitudes are maintained at the pillar rib by the external mechanism of higher in-situ stress state, or internal mechanism by virtue of the stiffness of the pillar.
Figure 4.47: Panel plot depicting (i,iii) major principal stress contours and (ii,iv) zone element yield at the onset of (a) pillar yield and (b) when pillar rib yield had traversed the height of the pillar for (i,ii) RPID 4289 and (iii,iv) 4297.
In the vertical joint case (i.e. RPID 4289), the DFN maintains stress arching through the roof and to the soft pillar rib, concentrating yield at the pillar periphery that leads to more rapid rib bulking and allows the pillar to take on the excess vertical stresses arching around the excavation while the roof and overburden maintain the excess horizontal stress arching. This behavior highlights the limitations of attempting to capture pillar behavior in a single stress-strain curve and designing to a so-called peak strength. All pillars modeled with soft roof blocks shown in Figure 4.45 attained very similar average stresses and strains at equilibrium, depending on their modeled depth. While the pillars in \( k_o = 2.0 \) models were able to support overburden loads, they ultimately all resulted in failed entries through a combination of rib failure and roof buckling collapse.

The increase in mining depth from 300 m to 450 m had the opposite effect on the \( k_o = 0.5 \) model coupled with sub-vertical joints (RPID 3337, dotted yellow line in Figure 4.45). The increased stress magnitude influenced the vertical stress more than the horizontal stress, reducing pillar’s ability to generate confinement under larger vertical loads. The confinement response can once again be visualized in terms of the average pillar stress paths (Figure 4.48).

![Figure 4.48: Plot of average stress path for pillars in single-entry models featuring “Soft Elastic” roof blocks, sub-vertical DFNs, and \( k_o = 0.5 \) at two modeled depths. Stress paths start at “o” and end at “x”. Note that the major principal stress is the vertical stress.](image-url)
Increases in major principal (i.e. vertical) stress must be accompanied by an increase in minor principal (i.e. horizontal, confining) stress for the pillar to maintain those vertical stress increases. The sub-vertical DFN under high in-situ stress magnitudes, and $k_o = 0.5$ cannot maintain the confinement that the pillar generates as it bulks due to its decreased arching capabilities. It is well-known that the internal confinement generation of a squat pillar is dependent on how the pillar periphery yields. Additionally, the mechanisms of pillar response to softer roof conditions are generally well-understood. However, documentation of the combined effects of explicit overburden joint orientation, $k_o$-ratio, and in-situ stress magnitude on preventing confinement loss or maintaining confinement generation represents a novel finding.

The response in more slender pillars was generally less variable, with macroscopic pillar behavior being largely independent of the interaction of pillar stiffness with changes in model depth, stress regime, and roof properties. This result is intuitive, as the smaller w/h pillar is less dependent on other properties to govern its confinement response. However, this is not the case under the “Stiff Elastic” roof blocks. Recall that the “Stiff Elastic” roof block models (i.e. RPID 353) had an assigned Young’s Modulus that was 30 times that of the “Soft Elastic” roof block model. This allowed all stiff roof elastic models to maintain functional stability of the model entry, regardless of the pillar w/h or pillar stiffness (within the ranges tested). The w/h = 4 pillars under “Stiff Elastic” roof blocks at 450 m depth incurred zone material yield in nearly 100% of associated zones and significant rib damage and horizontal displacement (approximately 20 cm) occurred, yet the average vertical stress in the pillars was reduced and the maximum stresses varied significantly with pillar stiffness (Figure 4.49). Unsurprisingly, in both 300 and 450 m deep entries, stiffer pillars incurred higher maximum average vertical stress. In the soft pillar cases, tensile yield reached the pillar core and fewer zones underwent high confinement shear failure, indicating that a softer w/h = 4 pillar could not internally generate as much confinement. This clearly resulted in a loss of significant load bearing capacity in the pillar, coupled with lower stress transfer in the roof beams as well as an elevated, yet uniform stress state in the floor. Nevertheless, the entry remained functional due to the arching capabilities of the very “Stiff Elastic” roof limiting the strain incurred by the pillars after the high levels of stress caused yield and were dissipated. This dissipation was exaggerated by the zero-velocity boundary condition at the top of the model, representing full stress arching 30 m above the entry. Following redistribution of excavation-
induced stresses via pillar yield and through the model roof and floor, there were no additional stresses to impart to the model.

Figure 4.49: Panel plot showing the pillar w/h = 4 models with “Stiff Elastic” roof block average pillar stress strain plots at 300 and 450 m depth entries. Softest and stiffest pillar model results of plastic yield and major principal stress at equilibrium shown.

The results of pillar response in models featuring “Soft Elastic” roof blocks were less clear, as many models incurred “contact overlap” error prior to reaching an equilibrium solution ratio. This complicated comparison and interpretation of model results with regards to pillar post-peak behavior. Regardless, the functional stability of the entry was not maintained in the majority of the “Soft Elastic” roof block models, even though a confined core was present at model equilibrium, or at the point of “contact overlap” error. The “Soft Elastic” roof block model could not limit the strain it imparted to the pillars in the post-peak like its stiff roof counterpart, which caused the pillar rib to either bulk in a stable manner or completely collapse into the excavation. Interestingly, the horizontal velocities of the pillar gridpoints at the time “contact overlap” error was incurred were up to 10 m/s, suggesting that a sudden and violent pillar failure (i.e. rockburst) has been simulated under soft roof conditions by the Progressive S-shaped yield criterion (Figure 4.50). This agrees with the findings of others (e.g. Salamon, 1970) that if the unloading curve of the roof is softer than the unloading curve of the pillar, then the potential for excess energy to be stored and released by the pillars increases. Note that due to the continuum nature of the modeled pillars, the
discontinuum aspects of the rockburst process (e.g. fracturing and ejection of rock) could not be explicitly simulated.

In many high $k_o$-ratio, soft roof cases, roof collapse accompanied pillar failure due to the increase in effective span as the pillar ribs yielded. While the presence of passive roof support had some impact on roof stability in the 300 m cases, pillar failure could not be mitigated by increasing the stability of the roof. In other cases, no change in the collapse behavior of the roof was noted, and supported entries still suffered massive roof collapse and were largely outside of the capacity of the modeled bolts to provide roof support.

4.4.3.2 Pillar-Roof Interaction Under Inelastic Roof Blocks in Deep Entries

Inelastic roof block models, much like their “Soft Elastic” block roof counterparts, were also affected by “contact overlap” errors. However, their peak average pillar stress results can still be compared, as all models were able to reach this point. This allows for comparison of the combined effects of increasing depth and variable roof strength, stress ratio, DFN, bedding thickness, and pillar properties on material yield and load shedding of the modeled roof and overburden. This section focuses on the subset of inelastic roof block models that were previously discussed in Section 4.4.2.2 (i.e. RPID 40xx and 49xx series). These model sets encompass realistic geologic
and mining conditions that were unstable in unsupported entries but were stabilized with the addition of standard rockbolts in Chapter 3 (see Table 4.1). They have been selected because they capture the largest potential effect of installed support and feature both “Strong SUBI” and “Moderate SUBI” inelastic block properties coupled with high and low $k_o$-ratios, respectively. Both are modeled with either sub-vertical or vertical cross-joints, as well as 0.5 m or 1.0 m bedding thickness.

First, the influences of model depth, roof strength, $k_o$-ratio, DFN, and bedding thickness in unsupported roofs on average pillar stress and strain results for $w/h = 8$ and $E = 3.0$ GPa pillars were considered. Note that all of these model results converged to an equilibrium solution ratio, allowing for post-peak pillar behavior to be explicitly compared. Similar to their elastic counterparts, $k_o = 2.0$ models had notably reduced peak average stresses that were also more pronounced in the deeper entry model results (Figure 4.51). Additionally, changes in explicit DFN from sub-vertical to vertical cross-joints had no discernible impact, and higher in-situ stress ratio still had a softening effect on the $w/h = 8$, $E = 3.0$ GPa pillar mechanical response.

![Figure 4.51: Average vertical right pillar stress strain plots for 8 inelastic roof models with various geologic and mining conditions (see Table 4.1); results depicted are from model subsets with $w/h = 8$ pillars with 3.0 GPa Young’s Modulus. Note that pillar stress and strain measurements began after excavation, hence the variation in baseline in-situ vertical stress.](image)
The overall pillar behavior appeared to be largely consistent between both depths modeled, and the strength and stiffness of the inelastic roof blocks and the magnitude of in-situ stresses played a proportional role in deeper model pillar response. Interestingly, the change in roof block material properties from elastic to inelastic did not significantly impact the peak average pillar load in any of the model results (see Figure 4.44). However, the post-peak behavior in both 300 and 450 m deep models was significantly influenced by the combined effects of $k_o$-ratio and roof block material strength. The RPID 40xx series models all had a $k_o = 2.0$ and “Strong SUBI” material properties, while the RPID 49xx series models all featured $k_o = 0.5$ and “Moderate SUBI” material properties. In order to isolate this impact, the RPID 30xx series model results were compared to the RPID 40xx series (i.e. same material strength, different $k_o$) (Figure 4.52).

Figure 4.52: Average vertical right pillar stress strain plots for 8 inelastic roof models with various geologic and mining conditions (see Table 4.1), results depicted are from models with w/h = 8 pillars with 3.0 GPa Young’s Modulus and “Strong SUBI” material roof properties with either $k_o = 5$ (solid) or $k_o = 2.0$ (dotted). Note that pillar stress and strain measurements began after excavation, hence the variation in baseline in-situ vertical stress.

Note that all the model results shown in Figure 4.52 reached an equilibrium solution ratio. However, the same was not true of all of the RPID 51xx models, preventing a direct comparison with model results for cases with “Moderate SUBI” roof block properties. Nevertheless, comparison of the RPID 30xx and 40xx series average pillar stress strain results depicted in Figure
4.52 confirmed that the stiffness of the pillar response and the peak strength were predominantly influenced by the modeled $k_o$, rather than the inelastic roof block material strength and stiffness. Additionally, the equilibrium strain levels indicate that the softening behavior continues into the post-peak as well.

Comparison of the post-peak behavior of the RPID 40xx and 49xx series in Figure 4.51 shows that inelastic roof block material strength and stiffness influence the post-peak load carrying capacity, with weaker, inelastic roof blocks inducing a more rapid strain-weakening response. This further confirms that once a given pillar begins to yield and cannot maintain or develop sufficient confinement due to internal (i.e. w/h) or external (i.e. DFN stress arching, in-situ stress) factors, its behavior is determined by the strength and stiffness of the loading system (i.e. roof and floor).

While the various inelastic roof block cases shown in this section tended to behave in a generally strain-weakening manner, significant differences were noted in the final average pillar stresses, particularly in the $k_o = 0.5$, “Moderate SUBI” roof block material cases (i.e. RPID 49xx series). Thinner beds and vertical cross-jointing resulted in the highest average pillar load at model equilibrium, while increases in bedding thickness and the presence of sub-vertical cross-joints significantly decreased the average pillar stress at equilibrium. The more instability or decrease in stress arching in the entry roof, the lower the pillar loads and the greater the pillar damage. This is highlighted by the comparison between unsupported and bolted model results for the $k_o = 0.5$, “Moderate SUBI” roof block material models (i.e. RPID 49xx series) (Figure 4.53).
Figure 4.53: Panel plot showing (a) the average stress-strain curves for $k_o = 0.5$, “Moderate SUBI” inelastic roof block model cases with pillar w/h = 8 and E = 3.0 GPa, in bolted and unsupported configurations. Select model results (b, c) of RPID 4962 (i.e. thick beds, vertical DFN) depicting (i,iii) plastic yielded zone elements and (ii,iv) vertical stress contours at model equilibrium in (i,ii) bolted and (iii,iv) unsupported cases show the impact of support on both roof and pillar stability.

Similar to the instability and decreased stress arching associated with sub-vertical joints in “Soft Elastic” roof block models in Section 4.4.3.1, analysis of model results in Figure 4.53 further confirmed that maintaining roof stability is key in maintaining pillar confinement and therefore post-peak load carrying capacity. Furthermore, the addition of support elements clearly promoted the support capacity of the modeled pillars. This trend was also noted in the $k_o = 2.0$, “Moderate SUBI” roof block material property models (i.e. RPID 51xx series), but due to “contact overlap” error, direct comparisons could not be verified. Direct analysis of other pillar w/h and stiffnesses were inhibited by the prevalence of “contact overlap” error in the model results, again complicating direct comparisons. However, smaller w/h pillars at these depths were generally less stable but incurred less damage and higher post-peak stress in the presence of a stabilized (i.e. bolted) roof.

4.4.3.3 Deep Entry Discussion

The previously observed complexities in pillar-roof mechanical interaction in shallow and moderate depth entries provided a framework for the analysis presented in this section. Analysis of the interaction of pillar geometric and material properties with a significant range of geologic
and mining conditions under increasing stress magnitudes, inelastic block material yield, and roof support efficacy has confirmed previous observations and identified unique mechanical interaction herein.

This section has verified that above all other considerations tested, horizontal stress ratio is key in controlling pillar-roof mechanical interaction. Previous studies by others generally considered horizontal stress as a purely positive influence on pillar strength, providing confinement and promoting stability in a strong roof. However, if the pillar-roof contact is sufficiently weak and the pillar is stiffer (i.e. stronger) than expected, the roof is weaker (or softer) than expected, or the roof has decreased stress arching capacity (e.g. sub-vertical DFN, weak block material, inclusions, etc.), horizontal stress can have a negative influence on confinement development and resultant peak pillar average stress, development of pillar rib bulking, and equilibrium post-peak stress and damage state. Furthermore, maintaining roof stability is a critical component of preventing the decay (i.e. high in-situ horizontal stress), or maintaining the generation (i.e. low in-situ horizontal stress) of, pillar confinement and load bearing capacity.

4.4.4 Analytical Method Comparison

In order to consider the combined mechanical impacts in a concise manner, model results were compared to the state-of-practice analytical and empirical methods that calculate pillar load (i.e. TAT) and strength (i.e. Mark-Bieniawski pillar strength equation). Regardless of the conditions modeled in this chapter, the traditional analytical methods discussed in the literature review were consistently and significantly inaccurate when attempting to calculate pillar loads or pillar strength based on model inputs (i.e. depth, pillar w/h) and compare them to model results. The model results considered in this section are all from bolted model cases, to approach the most realistic considerations modeled in this chapter.

4.4.4.1 Tributary Area Theory Comparison

The expected TAT pillar stresses calculated using Eqn. 4.1, for both w/h tested at each modeled depth in this chapter are compared to the pillar average stresses for at model equilibrium in Figure 4.54. Note that both the PAT and Coates (1981) methods require a panel width-to-height (W/H) ratio, which is effectively infinite under the conditions modeled in this chapter (i.e. single-entry horizontal symmetry), and therefore those methods cannot be applied as reported in the literature.
TAT error (%) was calculated by comparing the TAT error to the average pillar vertical stress at model equilibrium as follows:

\[
TAT \text{ Error } (%) = \frac{\sigma_{TAT} - \sigma_{yy,av}}{\sigma_{yy,av}} \times 100
\]  

(4.16)

Figure 4.54: Comparison of the impact of entry-scale model inputs on TAT-predicted loads. Pillar equilibrium average vertical stresses for both left and right pillars were averaged and compared as a percent change from the TAT-predicted loads. 1 = “Moderate SUBI”, 2 = “Strong SUBI”, 3 = “Soft Elastic”, 4 = “Stiff Elastic”.

As expected, TAT significantly overpredicted the equilibrium average pillar stresses in nearly every single-entry model result and TAT-error increased significantly with parameters that increased roof and pillar yield (i.e. increasing depth, decreasing roof strength, decreasing pillar size). These deviations due to inelastic yield are also exacerbated by the zero-velocity boundary condition at the top of the model. The magnitude of possible overburden stress arching was controlled by the zero-velocity boundary condition at the model top; as significant deformation or yield approached the model boundary, (see Figure 4.43) stresses dissipated. However, this destressed overburden and pillar response was not representative of the model results at their peak pillar loads. Therefore, a comparison between TAT and the maximum average pillar vertical stress for models that reached an equilibrium solution ratio was deemed more appropriate (Figure 4.55).
Figure 4.55: Comparison of the impact of entry-scale model inputs on TAT-predicted loads. Pillar maximum average vertical stresses for both left and right pillars were averaged and compared as a percent change from the TAT-predicted loads. 1 = “Moderate SUBI”, 2 = “Strong SUBI”, 3 = “Soft Elastic”, 4 = “Stiff Elastic”.

A deeper entry modeled in an infinite array of entries is theoretically independent from the known effects of panel width on TAT error (i.e. less accurate with increasing depth and constant width due to geometric controls on stress arching). As the stress magnitudes increased, the relative increase in excavation induced stress transfer through the roof and floor was greater than the relative increase of stresses transferred through the pillars, due to increasing pillar yield with depth. Both stress arching and pillar yield are violations of TAT assumptions and highlight the geologic controls on stress arching and TAT error, rather than the geometric controls discussed in Chapter 5. This is further evidenced by increasing TAT error with increasing k_o-ratio and roof block stiffness, and decreasing pillar w/h and stiffness, all of which have been demonstrated to increase roof stress arching in this chapter.

While the limitations of TAT are well-known, the method is typically assumed to be conservative due to the assumption that no stress arching of the overburden occurs. However, all 30 m entry
models resulted in TAT predictions that were overly optimistic (i.e. lower stresses, negative error) when compared to average pillar stress results. At first it would seem impossible that the TAT load could be exceeded, but since TAT assumes that the overburden bears uniformly on the pillar and does not account for pillar-overburden stiffness contrast, it cannot account for stress concentrations in the pillar corners. The increased deflection of the roof at the pillar edges concentrates stress and strain in the pillar corners, which in turn elevate average pillar vertical stresses above the TAT-predicted values. This only occurs in shallow entries because the magnitude of stress concentration relative to the level of in-situ vertical stress is higher than in deeper entries (Figure 4.56).

Figure 4.56: Vertical stress contours at model equilibrium for a 30 m deep, bolted, “Soft Elastic” roof block model with 0.5 m thick beds and vertical DFN (RPID 449) compared to the TAT calculated stress. Black dashed line indicates location of vertical displacement measuring points discussed in Figure 4.57.

Although most of the pillar in Figure 4.56 is equal to or less than 0.9 MPa, approximately 33% of the pillar is significantly above 0.9 MPa, particularly at the pillar corners. Note that this stress is concentrating without inducing pillar yield. This results in an average stress that ranges between 2-10 % higher than TAT-predicted loads. This is more thoroughly considered by comparing the average vertical displacement of the pillar top (dashed black line in Figure 4.56) to the associated TAT percent error for all 30 m deep bolted models (Figure 4.57). Since stiffer materials transfer stresses more effectively, less pillar top displacement is required for greater TAT underprediction. However, for any given roof block material model, smaller and softer pillars incur more vertical
displacement (i.e. strain), which translates to more horizontal strain, and results in higher stress concentration in the pillar corners.

Trends based on roof block material model, coupled with the pillar-top average displacement were identified. While softer roof blocks tend to reduce TAT error towards 0 (i.e. decrease pillar corner stress concentration) overall, changes in pillar-roof stiffness compliance that result in greater pillar displacement result in greater pillar corner stress concentration and subsequent TAT error. Practically, this would be a major concern for highly brittle or extremely weak pillars in shallow excavations.

All of the demonstrated TAT error results challenge not only the use of TAT to calculate design loads, but also the previously discussed empirical pillar strength equations that utilize TAT assumptions to back calculate the state of stress of a given pillar and determine the fit of a given strength equation.
4.4.4.2 Mark-Bieniawski Pillar Strength Equation Comparison

A similar analytical comparison was conducted between the peak average pillar stress of deep entry (i.e. 450 m) models and the Mark-Bieniawski pillar strength formula given in Eqn. (4.6). TAT error captured the impacts to pillar loading, and therefore the error of methods that utilize TAT loading to calibrate pillar safety or stability. However, additional consideration to the Mark-Bieniawski pillar strength equation must be given as it was developed independent of overburden loading through in-situ testing of large-scale samples.

Recall that the Progressive S-shaped yield criterion was originally calibrated to the Mark-Bieniawski pillar strength equation entirely separated from complex geologic and mining conditions modeled in this chapter. The interfaces were modeled elastically and more closely replicated the concrete-coal end-constraints and the lateral constraints (i.e. wood, steel, or concrete caps) utilized in the original in-situ testing (Bieniawski & Van Heerden, 1975). Walton et al. (2020) subsequently captured the decrease in pillar confinement and peak strength by combining the calibrated Progressive S-shaped yield criterion with realistic interfaces identical to those modeled in this chapter. The reduction in pillar strength closely resembled those reported by Gale and Mills (1994) (see Figure 4.7).

The focus of this section is to show the combined effects of realistic end-constraints (i.e. interfaces) and lateral constraints (i.e. none) with this chapter’s overall focus of pillar-roof interaction through the lens of parameters impacting roof stability and considered using an existing and widely applied pillar strength formula. The explicit effect of stronger and weaker interface strength on pillar strength is considered in Chapter 5.

Recall that Sections 4.4.1 through 4.4.3 demonstrated that pillar yield, failure, and collapse, are all different phenomena that are dependent on the material and geometric properties of the pillar coupled with the geologic and mining conditions of the roof and floor. Pillars can yield without failing or collapsing, particularly in deep entries with “Stiff Elastic” roof blocks. Pillars can also yield and incur significant rib failure without collapsing, however, in order to collapse, the pillar must yield and fail totally. This further complicates the comparison to analytical methods where technically the model result is stable and the pillar could be classified as a successful design, but in the event of depillaring or other stress changes, that same pillar would readily fail.
Therefore, pillar w/h = 4 models with “Soft Elastic” roof blocks were initially considered because of the total and complete pillar yield, failure, and collapse that occurred in all deep entry configurations. This allowed for their peak average vertical stress to be most closely compared to the ultimate strength captured by the Mark-Bieniawski pillar strength equation (Figure 4.58).

Due in large part to the realistic pillar-roof and pillar-floor contacts, in accordance with the observations of numerous previous studies (e.g. Iannacchione, 1990; Gale, 2017; etc.), the Mark-Bieniawski pillar strength equation consistently overpredicted the strength of the w/h = 4 pillars under “Soft Elastic” roof blocks at 450 m deep. However, other parameters had a significant impact on the deviation between peak pillar strength and the Mark-Bieniawski pillar strength equation. For example, the softening effect of decaying horizontal stress allows for the same pillar to undergo more strain and decreases the peak strength of the pillar (refer to Figure 4.52). This appears counterintuitive to the known effects of confinement, however, the presence of realistic discontinuities at the pillar-roof and pillar-floor contact and realistic roof properties allow for that initial confinement to decay rapidly via horizontal displacement of the pillar and stress transfer through the roof. This also agrees with the results of decreasing modeled pillar stiffness, a softer pillar ultimately fails at a lower load and the Mark-Bieniawski pillar strength equation results in a higher error when compared to the model result.

Comparison between the Mark-Bieniawski pillar strength equation and model results with roof block material that is not “Soft Elastic” or with squatter pillars becomes less direct, as pillar behavior diverges from total collapse in every case modeled. This highlights the limitations of the
single-entry models and the boundary conditions utilized in this chapter. The pillars are not necessarily failing or collapsing due to the effect of geologic and mining conditions modeled, combined with the static nature of the state of stress by virtue of the boundary conditions utilized (i.e. zero-velocity). This highlights the need for more complex, panel scale models, that can model various states of stress and more thoroughly evaluate pillar-roof mechanical interaction, global stability, and the Mark-Bieniawski pillar strength equation. The findings in this chapter are further considered in the context of panel-scale models in Chapter 5.

4.5 Conclusions

The interaction between roof and pillar is more complex than accounted for by traditional analytical methods. Even those that attempt to account for the effects of confinement (e.g. Sheorey et al., 1987; Lunder & Pakalnis, 1997) on pillar strength fail to consider the negative impacts of increased horizontal stress when coupled with decreased roof stress arching and weak pillar-roof and pillar-floor contacts. Although the limitations of analytical methods are otherwise well-known, the specific combined impacts of multiple rockmass characteristics, their impact on roof stability, pillar loading, and the interaction between the two, have not been considered prior to this research. Furthermore, the consideration of rockmass characteristics when utilizing the calibrated Progressive S-shaped yield criterion and evaluating pillar-roof mechanical interaction in relationship to state-of-practice pillar design methods are also novel to this study. Distinct trends have been identified in single-entry models and shall be further analyzed at the panel scale in Chapter 5.

Functional failure of the entry can occur due to partial or total failure of the roof, pillars, or both. This depends heavily on the contrast in material properties between the roof and the pillar: a stronger, stiffer pillar is more likely to induce yield in the roof, whereas a stronger roof is more likely to induce yield in the pillar. In contrast, a softer roof is more likely to cause pillar failure while a stiffer roof will tend to limit pillar failure. The evidence suggests that entry collapse can initiate in either the roof or the pillars and neither element exercises absolute control on local entry stability. However, when the relative stiffness of the roof significantly exceeds that of the pillars, pillar yield may or may not be reduced, but pillar failure and collapse are certainly mitigated given a static stress state (i.e. zero-velocity boundary condition) and horizontal symmetry conditions (i.e. infinite pillar array) as modeled herein. Conversely, when pillars are exceedingly stiff or behave
elastically (i.e. Chapter 3), the roof is more likely to incur yield leading to roof instability. Notably, the Analysis of Roof Bolt Systems (ARBS) evaluated in Chapter 3 has no method of explicitly accounting for the effects of pillar properties on roof stability identified in this chapter.

These generalized mechanical responses of a given entry to excavation are further complicated by the combined effects of the aforementioned strength and stiffness contrasts, plus the in-situ state of stress (i.e. magnitude and orientation), the stress arching capacity of the roof and floor (i.e. DFN), and the efficacy and mechanism (i.e. beam building vs. suspension) of roof support. These effects can be broadly classified into internal and external controls on pillar loading, and internal and external controls on roof stress arching capability, none of which are mutually exclusive. Generally, an internal control on roof stress arching capacity (i.e. vertical vs. sub-vertical cross-joint orientation) is also an external control on pillar loading, and vice versa.

These controls can work in tandem or against each other to reduce the decay of existing pillar confinement and roof stress arching, generate pillar confinement and roof stress arching, or prevent imparting additional loads to yielded pillars as the entry components respond to excavation induced stresses. Note that increased pillar confinement and roof stress arching are not always going to increase the stability of an excavation. For example, a weak inelastic roof with increased stress arching capability will induce additional roof yield, potentially leading to collapse if not properly supported. Similarly, excess pillar confinement can allow pillars to take on significantly higher loads; in the event of a rapid loss of confinement, catastrophic pillar failure (i.e. rock burst) can occur.

4.5.1 External Controls on Pillar Response & Internal Controls on Roof Stress Arching

The external controls on pillar response and confinement explicitly considered in this study are the magnitude (i.e. depth) and orientation (i.e. k_o-ratio) of stresses, the strength of the pillar-roof and pillar-floor contacts (modeled with as a constant, moderate strength in this chapter), as well as roof block material stiffness and strength, DFN distribution and orientation, bedding thickness, and roof support (i.e. the internal controls on roof and floor stress arching capacity). First and foremost, the presence of the realistic pillar-roof and pillar-floor contacts was key in promoting the wide range of pillar responses to various loading conditions tested herein. In the presence of extremely strong or weak contacts, the pillar-roof mechanical interactions identified in this chapter would
not necessarily apply due to the existence of extreme pillar confinement or very little pillar confinement, respectively.

If stress magnitudes are low, other external controls on pillar response are largely insignificant. As model depth increases, the significance of the other external controls increases and overall pillar confinement and pillar peak strength increase (i.e. 300 vs. 450 m deep entry results).

The external control with the greatest impact on macroscopic pillar behavior under all stress magnitudes was the $k_0$-ratio, providing initial confinement and promoting a softer pillar response to overburden loading following excavation. As stress magnitude increased, the stiffness of the overburden became increasingly important in limiting the stress and strain imparted to the pillars by the overburden, promoting entry stability. Additionally, the presence of cross-joints, their orientation (i.e. sub-vertical or vertical), and their relationship with internal controls on pillar loading (i.e. pillar w/h) became increasingly significant in transferring and concentrating stresses differentially in both the roof and pillars under elastic and inelastic roof block models.

For higher stress (i.e. deeper) cases, the peak average stress and the stiffness of the pillar loading curve continued to be dependent on the roof block material stiffness and the $k_0$-ratio. However, the post-peak average pillar stress at model equilibrium became controlled by the roof block material strength while the post-peak equilibrium strain was controlled by the $k_0$-ratio. Higher overburden strengths resulted in higher post-peak pillar loads while higher $k_0$-ratio resulted in larger pillar strain at equilibrium. Furthermore, a wide range of pillar yield, failure, and collapse were noted in the model results. Roof stability was identified as crucial in allowing larger w/h pillars to maintain and generate confinement as their ribs yielded and bulked into the excavation. Bolt mechanisms (i.e. suspension and beam building) were identified and classified based on their impact on pillar and roof yield. Furthermore, supported roofs resulted in significantly higher post-peak pillar load carrying capacity, and reduced pillar yield towards the pillar core.

4.5.2 External Controls on Roof Stress Arching & Internal Controls on Pillar Response

The external controls on roof stress arching explicitly considered in this study are the magnitude (i.e. depth) and orientation (i.e. $k_0$-ratio) of stresses, presence and efficacy of roof support elements, as well as pillar w/h, stiffness, and strength (i.e. the internal controls on pillar confinement).
While the effects of depth, $k_o$-ratio, and roof support were thoroughly considered in Chapter 3, they were isolated from the impact of yielding pillars and pillar w/h considerations. Even at low stress magnitudes (i.e. 30 m deep), the external controls on roof stress arching affected roof stability. Increases in stiffness of w/h = 4 pillar under high horizontal stresses imparted additional yield to moderate strength inelastic roofs featuring vertical cross-joints and resulted in unstable unsupported roof conditions. No other combination of properties modeled at 30 m deep resulted in such divergent behavior, highlighting the combined negative impacts of multiple pillar (i.e. stiff, slender) and overburden (i.e. inelastic, high stress arching) properties. Increased roof stress arching by virtue of vertical cross-joints induced more roof yield leading to roof instability. However, the stresses imparted to the roof were due to the simultaneous inability of the pillar to prevent confinement loss (i.e. $k_o = 2.0$, pillar w/h = 4, weak contacts) and ability to more effectively transfer that stress to the roof due to its high stiffness (i.e. $E = 4.5$ GPa). This trend of stiffer pillars inducing additional roof yield carried through to deeper model cases but continued to be mitigated in the presence of roof bolts. Stiffer pillars also reduced the amount of pillar yield and were more likely to be able to maintain or generate confinement internally through more controlled rib bulking.

In moderate depth entries, the effects of roof support could not be identified in macroscopic average pillar stress and strain. Therefore, subtle changes in roof and pillar yield and displacement were utilized to analyze the mechanisms by which bolts stabilized the roof and increased stress arching capacity, which diverted stresses and yield away from the pillars. Furthermore, elastic pillar behavior has been demonstrated as a conservative assumption when modeling roof stability, as it promotes roof failure, particularly in high horizontal stress conditions.

4.5.3 Combined Impacts

Notably, the only properties that TAT accounts for are the stress magnitude and the pillar w/h. Arguments could be made that extraction ratio is an external control on pillar loading, however, it was not explicitly considered in single-entry models without associated changes to pillar w/h (an internal control). Regardless, TAT is absolutely accounting for the most significant external and internal controls on pillar loading. However, its accuracy decreases, and conservativism increases with increasing depth and decreasing pillar w/h. Single-entry model results also highlighted the controls that TAT is not accounting for still impact both the distribution and average level of pillar stresses. Namely, pillar-roof stiffness compliance. Other methods exist that account for external
pillar loading controls (Coates, 1981), but they cannot be directly applied without an explicit panel W/H.

The Mark-Bieniawski pillar strength equation only considers the internal controls on pillar stability through a constant intact specimen strength and a linear increase in ultimate pillar strength based on the pillar w/h. This was demonstrated as significantly optimistic under realistic pillar-interface strengths and “Soft Elastic” roof blocks. More importantly, even in the limited analysis presented in this chapter, the impacts of other external and internal controls on pillar loading are clear. Softer pillar response through decaying horizontal stress, or due to reduced stiffness of the pillar material itself reduce the ultimate strength of the pillar due to changes in loading conditions.

This chapter has also underscored the impact of horizontal stress ratio in tandem with other roof and pillar considerations such as realistic pillar interfaces, lower pillar deformability, soft or weak roof, as well as the roof strength to stress arching capacity ratio (i.e. increased stress arching capacity requires stronger roof to transfer and maintain those stresses). Maintaining roof stability is a key component of preventing the decay (i.e. high in-situ horizontal stress), or maintaining the generation (i.e. low in-situ horizontal stress) of, pillar confinement and load bearing capacity. Additionally, the idea that a single strength value adjusted solely based on pillar dimensions can be used and applied with certainty to accurately predict either the ultimate or functional stability of a given entry has been challenged. To more completely address the question of how pillars affect global overburden stability and how the overburden affects pillar loading, panel-scale models that incorporate panel W/H more realistic roof and overburden properties (i.e. heterogeneity), floor and underburden properties (i.e. inelastic, explicit discontinuities), and mining-induced loading (i.e. depillaring) are explored in Chapter 5.
CHAPTER 5
ANALYSIS OF PILLAR-OVERBURDEN MECHANICAL INTERACTION GOVERNING GLOBAL STABILITY IN PANEL-SCALE MODELS

5.1 Introduction

Following analysis of the critical mechanical interactions between roof and pillar stability at the local, single-entry scale, a more realistic consideration of mine geometry was required. In particular, the impact that combined overburden properties (e.g. intact material and discontinuity strength, stiffness, heterogeneity, etc.) and mining conditions (e.g. depth, mine geometry, in-situ stress, etc.) have on pillar loading and pillar strength was of interest. Panel width is a well-studied and significant control on pillar loading and must be accounted for in order to consider pillar-overburden mechanical interaction more completely at the mine scale. Additionally, consideration of block material heterogeneity in the form of more complex overburden and underburden lithology and realistic loading paths, such as those induced by depillaring, are crucial in advancing understanding of pillar and overburden mechanical behavior.

Calculations of panel and barrier pillar loads are generally performed by application of simplified analytical or semi-empirical methods such as tributary area theory (TAT), pressure arch theory (PAT), and abutment angle loading. Determination of pillar strength largely relies on equations developed from a combination of laboratory and field-based experimental studies that primarily consider the effect of pillar width-to-height ratio (w/h) on ultimate pillar strength. While geometric considerations regarding pillar and panel may account for the most significant aspects governing response to excavation, they are only valid for the geologic and mining conditions used to develop them.

Many numerical methods also employ similar simplifying assumptions and do not explicitly consider the individual or combined impacts of inelastic yield or presence of explicit discrete fracture networks (DFNs). Galvin (2016) notes that “parametric and sensitivity analyses of pillar design input parameters are important risk management measures”. This chapter continues the work of Chapter 4 in understanding those sensitivities under more realistic modeled conditions.

While others have considered simplified analogs for overburden behavior (e.g. Esterhuizen et al., 2010; Tulu et al., 2016) and pillar loading (Mark & Chase, 1997), this chapter presents unique numerical models that more fully investigate pillar-overburden interaction in laminated and
discontinuous systems. Furthermore, pillar mechanical response sensitivities to specific, previously calibrated continuum rockmass representations (Tulu et al. 2017) combined with empirically verified realistic discontinuum rockmass representations and other geometric and mining conditions have not been previously documented.

The two-dimensional explicit discrete element method (DEM) is utilized in this chapter to consider a range of rockmass properties and mining conditions to understand changes in global stability in multi-entry hypothetical coal mine models. First, preliminary panel-scale models were developed to confirm that a homogenous, anisotropic continuum representation of portions of the overburden outside of the caving zone via strain-softening ubiquitous joint (SUBI) constitutive model matched empirical observations of surface subsidence (Ditton & Frith, 2003). Additionally, the effects of model setup attributes (e.g. pillar extraction sequence, interim solution ratios, discontinuum gob bulking, etc.) and continuum overburden block material properties (i.e. SUBI) on panel and barrier pillar stress development, and entry convergence behavior were considered for implementation in a panel-scale sensitivity analysis.

With a reliable model setup established, a parametric sensitivity analysis that tested individual parameter changes was conducted on panel-scale, multi-entry systems to identify significant inputs governing the pillar-overburden mechanical interaction with a particular focus on panel geometry, pillar location, lithologic heterogeneity, and explicit discrete fracture network characteristics. Panel-scale models utilized horizontal symmetry to simulate an infinite array of multi-entry panels and were able to capture increasingly realistic mining scenarios such as depillaring operations, as well as realistic geologic conditions such as inelastic and discontinuous floor block material and lithologic heterogeneity.

Results such as pillar stress development, roof and pillar yield, overburden-pillar stress transfer, entry convergence, pillar peak strength, and pillar confinement are analyzed herein. Results are also compared to state-of-practice predictions of pillar stresses and strengths such as TAT, version 6.0 of the Analysis of Retreat Mining Pillar Stability (ARMPS2010) (Mark, 2010), the abutment angle concept (Mark, 1987), and the Mark-Bieniawski pillar strength equation (Mark & Chase, 1997). Finally, traditional factor of safety (FoS) and ARMPS2010 stability factor (SF) are calculated based on model inputs and compared to model results to assess the potential range of error when applying these simplified analogs to complex and discontinuous rockmass conditions.
Furthermore, the determination of which element fails first in global collapse, the overburden or the pillars, is explicitly considered in relation to the model results. The factors studied in this chapter and their relationship to existing research are outlined in Figure 5.1.

![Figure 5.1: Graphical depiction of exiting explicit DEM research and the explicit considerations developed, confirmed, and validated through the course of this research. BBM = bonded block method, MCSS = Mohr-Coulomb Strain-Softening, HBSS = Hoek-Brown Strain-Softening OB = Overburden.](image)

Section 5.2 outlines the existing research on barrier pillar loading, as well as numerical modeling and empirical studies regarding overburden response and surface subsidence. The remaining sections present the methodologies and results of this chapter in advancing the understanding of pillar-overburden mechanical interaction and existing pillar design limitations.

### 5.2 Literature Review

Much of this chapter is focused on room and pillar panel pillar behavior. For a robust discussion of panel pillar loading, strength, and numerical modeling, refer to the literature review in Chapter 4. However, panel-scale models do have unique considerations like barrier pillar loading, depillaring, and overburden mechanical response that are reviewed here.

#### 5.2.1 Barrier Pillar Loading

In order to determine barrier pillar loads from mined out panels in practice, the abutment angle concept is combined with the idea of sub-critical (i.e. $W/H < -0.8$) and super-critical (i.e. $W/H > -0.8$) panels (Figure 5.2). Specifically, average barrier pillar stress induced by side-abutment loads are calculated as follows:
where \( H = \) entry depth, \( W = \) panel width, \( w = \) barrier pillar width \( \gamma = \) overburden specific weight, and \( \beta = \) abutment angle. Mark & Chase (1997) recommended using an abutment angle of 21° for conservative design in US coal mines.

Figure 5.2: Illustration of how to consider barrier pillar loads using the abutment angle concept for (a) super-critical and (b) sub-critical panel widths (modified from Mark, 2010).

Tuncay et al. (2019) collected stress measurements from interpanel (i.e. barrier, abutment, chain) pillars in 28 U.S. and Australian coal mines with panel \( W/H \) ratios ranging from 0.29 to 2.2 and back calculated abutment angles (Figure 5.3).

Figure 5.3: Results of back calculated abutment angles from 28 case studies compared to (a) panel depth and (b) panel \( W/H \) ratio (b) (modified from Tuncay et al., 2019).
Tuncay et al. (2019) indicated that at depths above 200 m, a 21° abutment angle is applicable, but that at depths below 200 m, the following equation should be applied to determine abutment angle (β):

\[
\beta = 29.42 \times 0.68^{(H/W)}
\] (5.3)

Applying this adjustment to the ARMPS2010 database deep cover cases that utilize barrier pillars increased the accuracy of prediction of successful deep cover front- and side-abutment load cases from 34% to 47% with an associated increase in the required barrier pillar stability factor (SF) from 1.5 to 2.15 and no change in panel pillar SF (i.e. ARMPS2010 SF). However, accuracy for failed cases remained at 88% (Tuncay et al., 2019). Note that Tuncay et al. (2019) recommend using Eqn. (5.3) only for panel W/H cases ranging from 0.7 to 3.5.

5.2.2 Numerical Modeling of Barrier Pillar Post-Extraction Loading

Heasley (2000) utilized a variation of the displacement discontinuity method (DDM), also known as the indirect boundary element method (BEM), implemented in LaModel (Heasley, 1999) to investigate the accuracy of traditional pillar load and load distribution empirical-analytical formulae used in the ALPS method (Mark, 1990; Mark et al., 1994). Their findings indicated that at an overburden depth of 160 m and a panel width of 200 m (panel W/H = 1.25) the differences in peak abutment stress and abutment stress distribution varied significantly between the two methods (Figure 5.4a). Additional analysis of varying panel W/H indicated significant deviations between numerical and empirical methods (i.e. ALPS) in determining the abutment load percentage in the barrier pillar (Figure 5.4b).
Figure 5.4: Results of (a) side-abutment barrier pillar load distribution and (b) side-abutment load percentage when comparing the empirical ALPS method with two variations of LaModel numerical models. Note that the ratio shown in (b) is H/W, not W/H (modified from Heasley, 2000).

Note that at low panel W/H (i.e. sub-critical, right side of Figure 5.4b) the ALPS method overpredicts the proportion of the side-abutment load in the barrier pillar to the total load in comparison to the LaModel numerical results. Conversely, at higher W/H (i.e. super-critical, left side of Figure 5.4b) the ALPS method underpredicts that same ratio. This indicates that a constant abutment angle, particularly with increasing depth, is inaccurate.

Singh et al. (2011) analyzed the impact of various elastic and inelastic overburden and underburden material properties, as well as overburden heterogeneity on barrier pillar loading in FDM models featuring joint elements in the overburden. They identified geologic heterogeneity as a critical control through additional empirical observations in Indian coal mines.

Corkum & Board (2016) conducted a back analysis of a well-instrumented trona mine using three-dimensional FDM models. Longwall mining was utilized to excavate multiple panels approximately 460 - 490 m below grade and 150 - 165 m wide (i.e. subcritical). Chain pillars between each longwall panel were instrumented with stress cells and a single time domain reflectometry cable was installed through the overburden above the first panel to monitor subsidence and bedding separation. Corkum & Board (2016) noted “reasonable agreement” of change in vertical stresses in each mining stage between model results, stress cell data, and tributary area loading. However, reported values show differences up to 100% between numerical results and instrument data and instrument malfunction was postulated. Following extraction of all
four longwall panels, chain pillar stresses extracted from the numerical model approached tributary area loads.

5.2.3 Empirically Predicted & Numerically Modeled Overburden Mechanical Response

Overburden instability in response to excavation-induced deviatoric stresses can be planned (e.g. depillaring, longwall mining, block caving, etc.) or unplanned (i.e. massive roof fall in active workings). Regardless, this instability can propagate through the overburden depending on the overburden lithology, panel W/H, and seam thickness. This deformation is zoned according to the mechanical response of the overburden as shown in Figure 5.5. Both surface subsidence and caving (i.e. gob or goaf formation) are of particular research interest, as mining-induced subsidence can damage surface and subsurface infrastructure, and caving behavior impacts how stresses are transferred globally (i.e. across multiple entries and panels).

![Figure 5.5: Schematic representation of multiple zones of deformation above an unstable panel in a coal mine (from Galvin, 2016).](image-url)
Empirical studies of mining-induced subsidence focus on the impact of panel W/H and seam extraction height on the magnitude of surface subsidence (Figure 5.6).

Numerical methods analyzing panel-scale behavior often calibrate overburden material properties to empirical data such as those shown in Figure 5.6 (Esterhuizen et al., 2010a; Tulu et al., 2017). The caved zone, also known as the gob or goaf, is commonly modeled as a continuum using a constitutive model which replicates the strain-hardening behavior of caved material being compressed by superincumbent strata (Salamon, 1990). The remaining portions of the overburden (i.e. fractured and constrained zones) are also often represented with continuum methods (Wang et al., 2013; Bai et al., 2017; Lawson et al., 2017). The strain-softening ubiquitous joint (SUBI) constitutive model can be used to replicate the effect of anisotropic strength due to bedding and laminations found in most layered and discontinuous systems (Fahrman, 2016; Larson & Lavoie, 2016; Das et al., 2017; Esterhuizen et al., 2017; Tulu et al., 2017; Klemetti et al., 2018; Sears et al., 2019). For additional information on the SUBI constitutive model, refer to Chapter 3 of this thesis.
Overburden at the panel scale can also be modeled with discontinuum methods that implement varying degrees of complexity. However, discontinuum representation of overburden materials is often limited to specific case studies of individual excavations, where rockmass and rock material properties are well-constrained.

Poeck et al. (2008) compared the difference in caving behavior between Voronoi and traditional (i.e. rectangular) blocks using the explicit DEM and found that stratified, regular block geometries exhibit less bulking, and more surface subsidence.

Shabanimashcool et al. (2014) modeled longwall caving using two-dimensional explicit DEM models. Overburden and seam material were modeled as a linear-elastic material and discontinuities were modeled as frictional Mohr-Coulomb (MC) joints. A sensitivity analysis was performed on joint normal and shear stiffness, as well as joint spacing, to determine the distance to first caving and subsequent periodic caving of the gob (Figure 5.7). Results indicated that rock block size, in-situ horizontal stress, and mechanical properties of the overburden significantly impact the distance to first and periodic caving, as well as longwall shield loads (Shabanimashcool et al., 2014). However, the impact on pillar loading was not explicitly considered in this study.

Gao et al (2014b) used the Trigon BBM approach in a two-dimensional explicit DEM to model the effect of various geologic conditions on gob formation following longwall mining. Models were developed based on a heterogeneous roof stratigraphy from a German longwall coal mine.
The modeled geologic conditions included explicit bedding plane spacing, strength of the immediate roof, and effect of high horizontal stress. Findings indicated that all three were critical in controlling the behavior of caving roof following the longwall pass. Explicit bedding planes allowed for caving to occur, while thicker beds provided enhanced stability and allowed for larger unsupported spans. Similarly, a stronger and stiffer immediate roof delayed the onset of caving from 48 m outby to 60 m outby. Increased horizontal stress produced a “bearing beam” in the stronger layers of the overburden, but also increased the degree of shear failure occurring in the immediate roof (Figure 5.8).

Gao et al. (2014) utilized a similar method to model roadway damage in a Chinese coal mine following panel extraction and stress redistribution (Figure 5.9).
Wang et al. (2017) utilized explicit DEM models to analyze void ratio of fractures due to overburden subsidence and gob formation at a Chinese coal mine. Intact material was modeled with a perfectly plastic MC constitutive model and discontinuities were modeled using the MC joint constitutive model. Results were compared to subsidence measurements from field investigations for different mining stages with good agreement (Wang et al., 2017). Yao et al. (2017) utilized Taylor polygon BBM models to study the impact of seam dip on abutment loading and coal rib stability in a Chinese coal mine. Kang et al. (2017) performed large-scale physical modeling on massive and jointed overburden analogs and calibrated two-dimensional Trigon BBM models to match the failure mode of the physical models.

Le (2018) conducted a sensitivity analysis of individual parameters affecting longwall top coal caving operations modeled using the explicit DEM (Figure 5.10). These parameters were broadly categorized into overburden characteristics, coal seam characteristics, and pre-mining stresses. Le (2018) found that decreases in overburden strength, coal strength and stiffness, and discontinuity
spacing all facilitate top coal cavability. Increases in depth of cover also increase the caving of top coal. Interestingly, horizontal stress did impact top coal cavability directly, but the relationship was found to depend heavily on the spacing of discontinuities and properties of the overburden. Variations in joint orientation were not considered.

Figure 5.10: Longwall top coal caving model at 80 m of face advance showing principal stresses tensors and abutment vertical stress profiles in top coal and mining horizon (from Le, 2018).

5.3 Panel-Scale Continuum Overburden Representation

Prior to analyzing pillar-overburden interaction at the panel scale, limiting explicit DFN representation to the region immediately above the excavation level was deemed necessary to reduce computational demand in modeling of depillaring in room-and-pillar mining scenarios. From a physical perspective, deformation above the gob (i.e. caved zone) is not discontinuous (refer to Figure 5.5) and can be adequately represented using a continuum approach. Furthermore, the influence that discrete fractures have on pillar loading likely decays with increasing distance from the pillar-roof boundary and is therefore not mechanically relevant to pillar-overburden
interaction. However, the anisotropic behavior of the overburden must still be explicitly accounted for. Therefore, the SUBI constitutive model was considered as a viable representation method.

The SUBI constitutive model has been successfully implemented in continuum modeling (Fahrman, 2016; Larson & Lavoie, 2016; Das et al., 2017; Esterhuizen et al., 2017; Tulu et al., 2017; Klemetti et al., 2018; Sears et al., 2019) to capture overburden mechanical response in panel- and mine-scale models. This section outlines the verification of previously implemented SUBI constitutive model material properties (Tulu et al., 2017) in conjunction with the discontinuum overburden representation previously utilized in this thesis, through comparison to empirical subsidence values of longwall panel extraction (Holla, 1987; Ditton & Frith, 2003). Although the panel scale models presented in the remainder of this chapter are not simulating longwall mining, the nature of the two-dimensional model coupled with full pillar extraction is similar to longwall mining conditions, which have a plethora of available subsidence case study data. Furthermore, this verification effort is not intended to be a robust analysis on longwall extraction subsidence, but rather, a verification of a reasonably realistic continuum overburden mechanical response for use in future modeling efforts that capture pillar-overburden mechanical interaction more broadly (i.e. in any flat-roof excavation in laminated and discontinuous rockmass).

Diverging from Chapters 3 and 4, this chapter features roof, floor, overburden and underburden that are all modeled with various block material properties, therefore the terms “roof properties” and “overburden properties” are no longer interchangeable. The following terms and their definitions are utilized in the remainder of this chapter:

- “roof” – the portion of the model above the seam and below the height of installed support.
- “immediate roof” – the portion of the model within one bedding plane above the seam, extending from edge to edge.
- “floor” - the portion of the model up to 1 m below the seam.
- “overburden” – the entire area above the seam, inclusive of the roof.
- “underburden” - the entire area below the seam, inclusive of the floor.
- “continuum overburden/underburden block properties” – the material properties of blocks in the continuum areas of the overburden and underburden.
- “discontinuum overburden/underburden block properties” – the material properties of the blocks in the discontinuum areas of the overburden and underburden.
“gob” – the portion of the discontinuum overburden that collapses into the excavation following depillaring.

“yielded” – inelastic damage of intact material or along discontinuities has occurred.

“failed” – a natural (i.e. overburden, floor) or engineered (i.e. pillar, supported roof) excavation component is no longer behaving in its expected or designed manner and is no longer fulfilling its role in supporting the functional purpose and overall integrity of the excavation.

“collapsed” - a natural (i.e. overburden, floor) or engineered (i.e. pillar, supported roof) component cannot bear additional stress due to its residual state or physical separation from the other system components.

This chapter represents the gob explicitly using a discontinuum overburden, but comparisons are made to continuum gob simulation approaches in Section 5.3.2.

5.3.1 Methodology & Model Inputs

Three panel-scale geometries of W/H = 0.65 (140 m deep), 0.69 (70 m deep), and 1.3 (140 m deep) were developed with 8 m of explicit DFN in the roof (i.e. discontinuum overburden), and the remaining overburden was represented using no explicit discontinuities (i.e. continuum overburden); the 8 m DFN height was selected based on caving heights being known to approach 3 times the thickness of the extracted seam (2.5 m) (Adhikary & Guo, 2009; Majdi et al., 2012). All three models represented the overburden to surface (i.e. free surface boundary condition), and horizontal symmetry was imposed by modeling half of each barrier pillar (i.e. w/h = 16 modeled as w/h = 8) and zero-velocity boundary conditions normal to the edges and bottom of the model. All entries were 6 m wide with a mining height of 2.5 m and panel pillars were modeled as w/h = 6. 30 m of underburden was modeled as an elastic equivalent continuum using the equivalent rockmass modulus of the discontinuum overburden. Zone size and block rounding values were selected based on the results of previous modeling efforts and confirmed as adequate by conducting a panel-scale mesh sensitivity analysis. An example of final model setup, geometry, and mesh size is shown in Figure 5.11. Explicit bedding thicknesses of 1.0 m and 0.5 m were analyzed to identify the required number of explicit discontinuities required to model explicit gob formation and bulking.
Figure 5.11: SUBI continuum overburden calibration model setup for 70 m deep, W/H = 0.69 model showing boundary conditions. Explicit gob (i.e. discontinuum overburden) represented with a DFN featuring sub-vertical cross-joints and 1 m thick bedding planes (left). Magnified image (right) depicts zone element sizes throughout the model, roof bolt installation geometry, and elastic pillar slice joints that allowed for partial depillaring.

The block material, discontinuity material, and DFN properties tested are given in Table 5.1. The material properties of the blocks in the discontinuous roof were based on the “Moderate SUBI” material utilized in Chapter 3 and 4 (see Table 3.1), with a decreased zone material friction angle to promote roof failure and caving. The continuum overburden was modeled using the “Black Shale” material properties from Tulu et al. (2017). Similarly, DFN inputs and explicit discontinuity strength were also selected based on Chapter 3 roof stability model results to promote roof instability, gob formation, and bulking. Note that the roof and overburden blocks were not meant to model an identical rock type, but rather confirm that the combined use of a previously verified realistic discontinuum rockmass representation (i.e. “Moderate SUBI” from Chapter 3 and 4), with a previously calibrated and validated continuum overburden representation (i.e. “Black Shale” from Tulu et al., 2017) was a valid approach to modeling panel-scale pillar-overburden interaction.
Table 5.1: Intact material, discontinuity, and DFN properties used in the continuum overburden verification models. DO = discontinuum overburden, Mod = Moderate, CO = continuum overburden, φ = peak friction angle, ψ = dilation angle, Cr = residual cohesion, φr = residual friction, Tr = residual tensile strength, SUBI jci = implicit joint peak cohesion, SUBI jφ = ubiquitous joint tensile strength, SUBI jφi = ubiquitous joint peak friction angle, SUBI jdil = ubiquitous joint dilation angle, SUBI jcr = implicit joint residual cohesion, SUBI jtr = ubiquitous joint residual tensile strength, SUBI jφr = ubiquitous joint residual friction angle, jkn = joint normal stiffness, jks = joint shear stiffness, jen/jes = joint normal/shear exponent, jr = joint roughness, sd = standard deviation, BT = Bedding Thickness. DFN = discrete fracture network, CY = continuously yielding.

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<th>Mod</th>
<th>SUBI</th>
<th>CO</th>
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<td>SUBI Critical Plastic Strain</td>
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<tr>
<td>jen/jes</td>
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<tr>
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</tr>
<tr>
<td>Trace &lt;sd&gt; (m)</td>
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<td>---</td>
</tr>
<tr>
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</tr>
<tr>
<td>Bedding Thickness (m)</td>
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</tr>
</tbody>
</table>

As recommended by Tulu et al. (2017), the continuum overburden zone material was modeled with strain-weakening post-peak behavior and a critical plastic shear strain of 5.0(10)^-3, with zone cohesion and tensile strength degrading to 10% of their peak values and a constant friction angle. Furthermore, the continuum overburden ubiquitous joint elements were modeled with a perfectly plastic post-peak behavior to prevent unrealistic plastic flow of block material following SUBI yield.
“Intact” zones in the discontinuum overburden were modeled as they were in previous applications of the “Moderate SUBI” block material property in this thesis; they featured a critical plastic strain of $1.0 \times 10^{-6}$ (i.e. elastic-brittle-plastic) that degraded cohesion, tensile strength, and friction angle to their residual values in Table 5.1. Ubiquitous “joints” in the discontinuum overburden were modeled with a different post-peak behavior than their continuum overburden counterparts due to the presence of explicit joints in the DFN.

For the purposes of the subsidence evaluation described in this section, the coal seam was modeled with the Progressive S-shaped yield criterion using higher peak cohesion, making pillars effectively elastic to isolate the potential impacts of roof and overburden properties on subsidence from the complex pillar responses observed in Chapter 4. Pillars also featured vertical elastic joints to allow for sequential mining of each pillar. Similar to previous single-entry models discussed in this thesis, following stress initialization, the entries were excavated, and a 70% in-situ stress boundary was applied to the excavation boundary. This stress relaxation method allowed for bolts to be installed following elastic stress relaxation. 2.4 m long bolts were installed on 1.2 m spacing through 10 cm faceplates and were modeled using the calibrated bolt properties from Bahrani & Hadjigeorgiou (2017).

Following bolt installation, the models were solved to a standard equilibrium solution ratio of $1.0 \times 10^{-5}$ and in-plane depillaring was modeled by deleting pillar slices along vertical elastic joints from left to right. Following removal of each pillar slice, the model was run to a solution ratio of $5.0 \times 10^{-5}$. After the full removal of each pillar (i.e. every six slices), the model was run to a standard equilibrium solution ratio of $1.0 \times 10^{-5}$. This method was implemented in an attempt to capture the directionality of depillaring where pillars are commonly removed sequentially in a series of cuts.

Finally, model results of maximum surface subsidence were extracted and compared to the empirical observations in Ditton & Frith (2003) to demonstrate that the continuum SUBI representation of the portion of the overburden outside of explicit bulking and caving provides reasonable mechanical response to depillaring. Results were also compared to fully continuum (i.e. continuum gob, continuum overburden) two-dimensional finite difference method (FDM) model results from (Walton et al., 2020). Verification of the combined discontinuum and continuum overburden mechanical response to depillaring allowed for more complex analysis of pillar-overburden interaction to be considered in Sections 5.4 and 5.5.
5.3.2 Results & Empirical Verification

Surface subsidence results of the discontinuum-continuum overburden models featuring 1.0 m and 0.5 m thick beds are compared to the empirical dataset (Holla, 1987; Ditton & Frith, 2003) and implicit gob models from Walton et al. (2020) in Figure 5.12. All smaller panel W/H cases in this study fit the empirical data set accurately, particularly when considering the empirical data based on panel depth. Note that the empirical data from Holla (1987) shows that deeper mining depth for the same panel W/H generally results in higher $S_{\text{max}}/T$ values. Comparison of explicit gob model results from $W/H = 0.65$ (140 m deep) and $W/H = 0.69$ (i.e. 70 m deep) panels indicate that the model behavior is consistent with this trend.

![Figure 5.12](image)

Figure 5.12: Empirical confirmation of continuum SUBI overburden representation coupled with explicit DFN gob (i.e. discontinuum overburden) in this study and comparison to and implicit gob (FLAC) models from Walton et al. (2020). $S_{\text{max}} = \text{maximum surface subsidence, } T = \text{seam thickness, } H = \text{mining depth, } W = \text{panel width, } \text{DFN = discrete fracture network.}$

Larger panel W/H models, particularly with thicker discontinuum overburden beds, tended to underestimate subsidence when compared to the empirical data set. The divergence between the model results and the empirical dataset is a result of the sequential and directional (i.e. left to right) depillaring and subsequent solving to an equilibrium solution ratio between each pillar removal. This promoted additional non-uniform bulking of the discontinuum gob, particularly at the entry corners where the pillars provided support and led to the formation of two subsidence troughs in
both higher panel W/H models (Figure 5.13). Notably, the thick bed model increased bulking significantly and limited the subsidence of the continuum overburden. This is due to the increased stress-arching capability of thicker beds, delaying the onset of caving and inducing additional material yield, increasing the bulking modeled. This can readily be seen in the difference in collapsed block size when comparing blocks outside of the gob (i.e. above barrier pillars), to those within the gob for both cases in Figure 5.13; the increase in size of the blocks in the thick bed model gob is significantly greater than in the thin bed model. This indicates that the continuum representation is mechanically linked with the explicit gob bulking, and that the two phenomena are controlling the surface subsidence together. If explicit gob properties were not impacting subsidence, that would indicate that the discontinuum and continuum portions of the model were not mechanically linked, and therefore not realistic. This random variation in explicit gob bulking due to DFN and bedding thickness is absent in calibrated implicit gob models from Walton et al. (2020) where the entire seam was excavated in a single step and the gob material was emplaced with a strain-hardening constitutive model that modeled uniform bulking in response to symmetric overburden loads.
It is apparent that the continuum overburden, explicit gob (i.e. discontinuum overburden) approach more accurately fits the empirical dataset when compared to the implicit gob representation (Walton et al., 2020), particularly in lower panel W/H models. While maximum surface subsidence results between explicit and implicit gob model results were similar in many cases, the heterogeneous distribution of overburden displacement was not captured by the continuum implicit gob model results (Figure 5.14). These results verified that the calibrated continuum representation of the overburden in explicit gob models provides a reasonable approximation of the macroscopic overburden response to depillaring. Furthermore, the explicitly discontinuous roof was able to capture the heterogeneous and discontinuous nature of gob formation in a sequential excavation. Recall that the purpose of this study is not to analyze overburden subsidence, but pillar-overburden interaction. Combining these methods will allow for the impact of multiple complex geologic conditions on pillar-overburden interaction to be considered at the panel scale. Furthermore, panel-
scale models in the parametric sensitivity analysis in Section 5.5 feature maximum panel W/H of 0.6, well within the more accurate range of the continuum overburden response compared to the empirical data in Figure 5.12.

Figure 5.14: Vertical displacement contour plots of W/H = 0.65 models with (a) explicit DFN gob and (b) implicit gob (Walton et al., 2020) note that both panel-scale models matched expected displacement from Ditton & Frith (2003).

5.4 Panel-Scale Implicit Continuum Overburden Representation

Following the empirical verification of the discontinuum-continuum overburden representation for multiple panel dimensions, models were tested to identify the amount of explicitly modeled continuum overburden required to maintain consistent model stress, displacement, and yield results. As this chapter is focused on pushing pillars to their ultimate strength and assessing the pillar-overburden mechanical interaction at the panel-scale, deeper models were required. Single entry models from Chapter 4 indicated that 150 m to 450 m deep models would capture a wide range of pillar yield, failure, and collapse. However, explicitly representing 450 m of overburden to a free surface, irrespective of its continuum nature, is computationally inefficient and may be unnecessary in capturing the pillar-overburden interaction. A common numerical modeling technique is to replace explicitly modeled overburden with a stress boundary condition. However, since this chapter is concerned with pillar-overburden mechanical interaction, such a technique must be verified as generating consistent results.
Previous single-entry models in this thesis utilized a fixed-velocity boundary condition with appropriate initial stresses to simulate deeper model entries. While this boundary condition was suitable to investigate roof stability and pillar-roof interaction at the entry scale, recall that as deformation and yield approached the upper boundary, the fixed velocity boundary condition represented full stress arching above the boundary (i.e. pressure arch theory (PAT)). Based on the findings of Section 5.3 and empirical subsidence data, the assumption that at some point above the mine panel, in the event of full pillar extraction or pillar failure, the overburden displacement would be zero is far less realistic and far more optimistic than reality. Therefore a stress boundary condition at the top of the model that simulated deeper entries was assessed for its impact on pillar response.

5.4.1 Methodology & Model Inputs

A minimum depth of 150 m was chosen to capture the transition from elastic to inelastic pillar behavior as in-plane depillaring operations are simulated in Section 5.5. This was based on the results of Chapter 4 single-entry models that began to incur significant pillar yield and roof instability at approximately 200 m modeled depth, depending on the modeled rockmass conditions.

The general model setup considered in this section is similar to the models in Section 5.3. This section compares the pillar stress results between models that represented the continuum overburden to surface and those that implicitly represented the overburden above 150 m via stress boundary condition (Figure 5.15). Note that the block and discontinuity material properties utilized in this case are identical to those in Section 5.3 and are listed in Table 5.1.
Figure 5.15: Generalized preliminary model setups that either explicitly represent the entire overburden (left) or implicitly represent deeper cases with a stress boundary condition (right). Both have 8 m of explicit DFN in the roof and 4 m in the floor, pillar w/h = 3, span = 6.

Pillar and overburden stress distributions and yield between the two model setups were compared for a uniform, vertical cross-joint DFN (i.e. “Voussoir DFN”) (Figure 5.16).
While Section 5.3 was focused on verifying the continuum overburden mechanical response, this section focuses on mechanical interaction between coal pillars and surrounding materials. Therefore, the immediate floor and underburden material were no longer modeled as a homogenous equivalent elastic continuum in order to approach more realistic pillar loading conditions. The panel-scale models now featured 4 m of explicit DFN in the floor with block material properties identical to those in the roof; furthermore, the SUBI continuum representation used in the overburden was implemented in 26 m of explicitly modeled underburden. Whereas the zero-velocity boundary condition used in the single-entry models approximated PAT assumptions, the stress boundary condition approaches the assumptions of TAT, where no stress arching occurs above the boundary condition and the full weight of the implicitly modeled continuum overburden acts at the top of the model.

Models were run in a similar manner as those presented in Section 5.3. However, the coal seam was now modeled using the Progressive S-shaped yield criterion with calibrated material properties (Sinha, 2020), rather than with material properties that ensured uniform elastic pillar deformation. Calibration of the coal pillar model indicated that the presence of vertical construction joints that allowed for the multiple cut pillar extraction described in Section 5.3, despite their elastic properties, significantly impacted the behavior of the pillar (Walton et al.,
Therefore, in this section, no construction joints were included, meaning entire pillars were removed in a single step and the model solved to an equilibrium solution ratio of $1.0(10)^{-5}$ between each pillar removal. The history command in UDEC was utilized to track and extract the average pillar vertical stresses and strains, pillar maximum and minimum principal stresses, roof midspan displacement, roof-pillar boundary displacement, as well as material yield in the roof and pillars every 100 steps in an identical manner to that described in Chapter 4. The only difference in model result extraction is that there are more pillars from which to extract, and that the area of roof zone yield percentage now extends from model edge to edge within the first layer (i.e. 0.5 to 1.0 m) of the immediate roof.

Once model stresses were initialized, entries were excavated, stresses at the excavation boundaries were relaxed to 70% of in-situ values and the model was run to equilibrium. The stress boundary was then removed, bolts were installed with the same properties and geometry as those in Chapter 3, Chapter 4, and Section 5.3, and the model was run until “contact overlap” error occurred.

### 5.4.2 Stress Boundary Condition Effects on Pillar-Overburden Mechanical Interaction

Model results that either explicitly represented overburden to 300 m above the entry (i.e. “Explicit Depth”) or explicitly represented overburden to 150 m above the entry and represented the remaining continuum overburden via stress boundary condition (i.e. “Implicit Depth”) (see Figure 5.15) were compared. Comparison of panel pillar and barrier pillar stresses up to the point of “contact overlap” error, which occurred at similar points in the model, indicated that the stress boundary condition effectively captured the impact of increasing depth on pillar loading and failure without the need for explicitly modeled continuum overburden to surface (Figure 5.17).
The difference in pillar behavior, particularly for panel pillars was minimal and limited to the extreme post-peak (i.e. greater than 6.0% strain). Furthermore, models incurred “contact overlap” error at almost identical panel pillar strain and very similar pillar loads. Barrier pillar behavior in the different model configurations was less similar, with the “Explicit Depth” model reaching an average vertical stress approximately 10% higher than the “Implicit Depth” model. This is also reflected in the barrier pillar stress path, where confinement was not generated nor maintained as readily. However, this is not a direct comparison, as neither model reached an equilibrium solution ratio, and the barrier pillars did not yield like their panel pillar counterparts. Regardless, panel pillar loading is the focus of the deeper entry models where implicit overburden will be utilized, and therefore this observed trend of minor divergence in barrier pillar behavior is not considered to be practically significant.

Roof midspan displacement from three entries and percent immediate roof zone yield were also compared between the two models as an analog to evaluate possible changes to roof stress arching due to model geometry and boundary condition (Figure 5.18). Recall that the area of the immediate
roof was considered to be the first layer in the roof (i.e. 1 m thick) by the width of the entire model (i.e. 100 m).

![Graph](image)

Figure 5.18: Model results of roof midspan vertical displacement vs. immediate roof zone yield for the central entry, and the two leftmost entries modeled with 300 m of explicit overburden (solid line) or with 150 m of explicit overburden and a constant stress boundary condition (dotted line). See Figure 5.15 for entry locations.

These results further indicate that explicitly modeling 150 m of overburden is sufficient in capturing pillar-overburden mechanical interaction, and the remaining overburden can be implicitly modeled with a stress boundary condition with no significant impact to the model results of interest. However, consideration of barrier pillar behavior should be restricted to explicitly represented continuum overburden cases only.

5.5 Panel-Scale Parametric Sensitivity Analysis

Following the validation of implicit overburden representation with a stress boundary condition, pillar-overburden interaction could be considered at the panel scale in two-dimensional models featuring a wide range of geologic and mining conditions. The parameters considered were based on results of previous modeling efforts in this thesis, as well as results of previous numerical, empirical, and analytical studies by others (Hsiung et al., 1993; Esterhuizen et al., 2010; Bastola & Chugh, 2015; Tulu et al., 2017). A sensitivity analysis was conducted to identify critical inputs governing pillar-overburden interaction by varying individual parameters and noting the effects
on model results. This was followed by a limited expansion of select models to better capture the
divergence between the Mark-Bieniawski pillar strength equation and model behavior with respect
to pillar failure.

Although panel-scale models in this section simulate two-dimensional cross-sections of room-and-
pillar mines, the broader implications of the model results (i.e. influence of individual and
combined parameters) can be considered applicable to other mining methods implemented in
laminated and discontinuous rockmass (e.g. longwall, strip pillar, highwall, etc.). Model results
are compared to multiple analytical and empirical methods and are considered in relationship to
the previously categorized external and internal controls on pillar response and roof stress arching
in Chapter 4.

Previous single-entry models presented in Chapter 4 focused more on the subtle, local influences
of pillar, roof, and support properties on pillar-overburden mechanical interaction. However, these
results are broadly applicable because of the horizontal symmetry conditions imposed, effectively
modeling an infinite array of pillars. Section 5.5.2 builds on the single-entry model results and
focuses more on the sensitivity of macroscopic pillar behavior and local stability of entries in a
given panel to variations in both previously tested (e.g. DFN, k_r-ratio, etc.) and new (e.g.
heterogeneity, panel W/H, etc.) model parameters. Then pillar-overburden interaction is
considered in relation to unstable development loading (i.e. 300 and 450 m deep models).

Section 5.5.4 analyzes the model results through the lenses of multiple state-of-practice design
methods. Assessment of panel pillar loading through TAT and PAT methods, barrier pillar loading
through the abutment angle concept, and panel pillar strength through the Mark-Bieniawski pillar
strength equation.

5.5.1 Methodology & Model Inputs

For a description of model geometry, boundary conditions, and model setup attributes, refer to
Section 5.4.1. As this thesis is uniquely concerned with discontinuous and layered rockmasses, the
parametric sensitivity analysis was conducted from five baseline panel-scale models featuring
different DFNs. One DFN was designed to be fully uniform such that the presence of fractures
could be considered without local geometric effects influencing model results; the remaining
DFNs correspond to either vertical or sub-vertical cross-joints created with two different random
seed numbers each (Figure 5.19). DFN 1 is referred to as the “Voussoir DFN” due to its uniform
blocky geometry forming multiple voussoir layers in the roof and floor. DFN parameter mean and standard deviation are listed in Table 5.2. Note that changes in pillar w/h and number of entries altered the model geometry such that even DFNs with the same random seed were not identical, which is another justification for the use of the uniform “Voussoir DFN”.

![Diagram of DFN models](image)

Figure 5.19: Magnified view of the central entry in the baseline panel-scale models for each DFN tested showing zone size and bolt geometry. Note the change in random seed from 400 (i.e. DFN 2a & 3a) to 123456 (i.e. DFN 2b and 3b) results in the same DFN distribution but different explicit geometry.

Table 5.2: Stochastic DFN inputs for the DFNs tested in the baseline sensitivity analysis of panel-scale models. DFN 2 and 3 run with random seeds 400 and 123456 resulting in 5 different DFNs, sd = standard deviation.

<table>
<thead>
<tr>
<th>DFN Inputs</th>
<th>DFN 1</th>
<th>DFN 2</th>
<th>DFN 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle &lt;sd&gt; (°)</td>
<td>90 &lt;0&gt;</td>
<td>90 &lt;10&gt;</td>
<td>90 &lt;0&gt;</td>
</tr>
<tr>
<td>Gap &lt;sd&gt; (m)</td>
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<td>1 &lt;0.25&gt;</td>
<td>1 &lt;0.25&gt;</td>
</tr>
<tr>
<td>Trace &lt;sd&gt; (m)</td>
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<td>2.64 &lt;1&gt;</td>
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</tr>
<tr>
<td>Spacing &lt;sd&gt; (m)</td>
<td>0.5 &lt;0&gt;</td>
<td>0.3 &lt;0.1&gt;</td>
<td>0.3 &lt;0.1&gt;</td>
</tr>
</tbody>
</table>

The five baseline models utilized the properties listed in the first column of Table 5.3 with the exception of baseline continuum overburden properties, which are listed in column four. A sensitivity analysis was conducted by varying a single input parameter, or groups of input parameters (e.g. material properties, joint shear and normal stiffness) from the baseline condition.
For example, all DFNs were tested with pillar w/h = 1, 3 and 6 and the baseline pillar stiffness (i.e. 3 GPa), but all combinations of pillar w/h (i.e. 1, 3, or 6) and pillar stiffnesses (i.e. 1.5, 3.0, or 4.5 GPa) were not considered together to limit the number of model cases. To summarize, there are 5 baseline models that all have different DFNs but share the following properties that varied individually: 150 m entry depth, 5-entry panel, panel pillar w/h = 3, pillar E = 3 GPa, “DOU Moderate SUBI” block material properties in the discontinuum overburden and underburden, “COU Black Shale” block properties in the continuum overburden and underburden, homogeneous stratigraphy, joint friction angle = 15°, joint normal stiffness = 50 GPa/m, and joint shear stiffness = 5 GPa/m. All models also share the same barrier pillar w/h and installed roof support. This resulted in 125 unique panel-scale numerical models.
Table 5.3: Model parameters analyzed in this study for panel-scale sensitivity analyses; note that variables in rows highlighted with the same color are to be varied concurrently. The first column indicates the baseline model condition. Proposed lithologies and locations of strong beds are depicted in Figure 5.20 and Figure 5.21. COB = Continuum Overburden, Lith = lithology, DOU = discontinuum overburden/underburden, COU = continuum overburden/underburden, \( \varphi_i \) = peak friction angle, \( \psi \) = dilation angle, \( C_r \) = residual cohesion, \( T_r \) = residual tensile strength, \( \text{SUBI}_{j_\alpha} \) = implicit joint peak cohesion, \( \text{SUBI}_{j_t} \) = ubiquitous joint tensile strength, \( \text{SUBI}_{j_{\beta}} \) = ubiquitous joint peak friction angle, \( \text{SUBI}_{j_{\beta\beta}} \) = ubiquitous joint dilation angle, \( \text{SUBI}_{j_{\beta\beta\beta}} \) = implicit joint residual cohesion, \( \text{SUBI}_{j_r} \) = ubiquitous joint residual tensile strength, \( \text{SUBI}_{j_{\beta\beta\beta\beta}} \) = ubiquitous joint residual friction angle, \( j_{kn} \) = joint normal stiffness, \( j_{ks} \) = joint shear stiffness, \( j_{en}/j_{es} \) = joint normal/shear exponent, \( j_r \) = joint roughness, \( s_d \) = standard deviation, \( CY \) = continuously yielding.

<table>
<thead>
<tr>
<th>Panel Geometry/Pillar Properties/Heterogeneity</th>
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<tbody>
<tr>
<td>Depth to Entry (m)</td>
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<tr>
<td>Span (m)</td>
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<tr>
<td>Panel Pillar w/h</td>
</tr>
<tr>
<td>Barrier Pillar w/h</td>
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<tr>
<td>In-Situ Stress Ratio</td>
</tr>
<tr>
<td>No. of Entries</td>
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<tr>
<td>Strong Bed Location</td>
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<tr>
<td>Strong Bed Thickness</td>
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<tr>
<td>Hetero Models</td>
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<td>Pillar Stiffness (GPa)</td>
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<table>
<thead>
<tr>
<th>Block Material Properties (Field Scale)</th>
</tr>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>G (GPa)</td>
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<tr>
<td>K (GPa)</td>
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<tr>
<td>E (GPa)</td>
</tr>
<tr>
<td>Cohesion (MPa)</td>
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<tr>
<td>Tensile Strength (MPa)</td>
</tr>
<tr>
<td>( \varphi_i (\degree) )</td>
</tr>
<tr>
<td>( \psi (\degree) )</td>
</tr>
<tr>
<td>( C_r ) (MPa)</td>
</tr>
<tr>
<td>( \varphi_{j_t} (\degree) )</td>
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<tr>
<td>( T_r ) (MPa)</td>
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<tr>
<td>Critical Plastic Strain</td>
</tr>
<tr>
<td>( \text{SUBI}<em>{j</em>{\alpha}} ) (MPa)</td>
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<tr>
<td>( \text{SUBI}_{j_t} ) (MPa)</td>
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<tr>
<td>( \text{SUBI}<em>{j</em>{\beta}} ) (\degree)</td>
</tr>
<tr>
<td>( \text{SUBI}<em>{j</em>{\beta\beta}} ) (\degree)</td>
</tr>
<tr>
<td>( \text{SUBI}<em>{j</em>{\beta\beta\beta}} ) (MPa)</td>
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<tr>
<td>( \text{SUBI}<em>{j</em>{\beta\beta\beta\beta}} ) (MPa)</td>
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<tr>
<td>( \text{SUBI}<em>{j</em>{\beta\beta\beta\beta\beta}} ) (\degree)</td>
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<tr>
<td>( \text{SUBI}<em>{j</em>{\beta\beta\beta\beta\beta\beta}} ) Critical Plastic Strain</td>
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<table>
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<tr>
<th>CY Joint Material Properties</th>
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<tbody>
<tr>
<td>( j_{kn} ) (GPa/m)</td>
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<tr>
<td>( j_{ks} ) (GPa/m)</td>
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<tr>
<td>( j_{en}/j_{es} ) (\degree)</td>
</tr>
<tr>
<td>( \text{Initial (\varphi)} ) (\degree)</td>
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<tr>
<td>( \text{Intrinsic (\varphi)} ) (\degree)</td>
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<tr>
<td>( j_r ) (m)</td>
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</table>
Recall that moderate strength continuously yielding joints (i.e. 25° initial friction angle) were utilized in Chapter 4 for all contacts (i.e. bedding, joints, and pillar interfaces) based on previous findings in this thesis (i.e. Chapter 3) and the literature (Roberts et al., 2002; Esterhuizen et al., 2010). This chapter utilized a weaker discontinuity strength for the baseline case to promote roof failure following depillaring and intact roof block yield, and to assess the combined influence of other overburden and mining conditions with weaker interfaces (i.e. capture the lower-bound of Mark-Bieniawski pillar strength deviation).

Of the 125 unique panel-scale models, 85 featured homogenous block material properties in both the discontinuum and continuum overburden, respectively. In order to methodically approach more complex and heterogeneous (i.e. realistic) rockmass, the five baseline models (i.e. all 5 DFNs) were run with individual 2 m-thick strong beds modeled at various heights in the discontinuum (i.e. DFN) and continuum overburden areas of the model as listed in row seven of Table 5.3. This accounted for 30 “simple” heterogeneous stratigraphy models, six for each DFN tested. These strong bed locations are depicted in Figure 5.20, and if they are located in the DFN they have the “DOU Strong SUBI” properties, if they are located in the overburden they have “COU Limestone” properties.
In the 30 model cases with a single strong bed (Figure 5.20), the remaining material in the discontinuum and continuum overburden was modeled with the baseline material properties (i.e. “DOU Moderate SUBI” and “COU Black Shale” in Table 5.3), respectively.

In further pursuit of capturing more complex heterogeneous rockmass cases, two “complex” hypothetical heterogeneous lithologies roughly based on those used by Esterhuizen & Barczak (2006) were also modeled in the discrete sensitivity analysis by varying the baseline homogeneous models. This accounted for 10 of the 125 panel-scale models and brings the total heterogeneous model count to 40 (i.e. 30 “simple” single layer heterogeneous models, and 10 “complex” stratigraphies). Block material properties for both continuum and discontinuum overburden areas are listed in columns 2 through 6 and the rows highlighted in green in Table 5.3. These lithologies are depicted in Figure 5.20.
Figure 5.21: Figure depicting the two heterogeneous stratigraphy models to be tested for their impact on pillar-overburden mechanical interaction. Note that material properties for each lithologic unit are shown in Table 5.3 and that model tops (i.e. less than 70 m below grade) are excluded for clarity but the uppermost lithologic unit continues to the top of the model in the weak (top) and moderate strength (bottom) heterogeneous lithology model cases.
Similar to the models in Section 5.3, all block material in the continuum and discontinuum portions of the models in this section utilized the SUBI constitutive model. Recall that the continuum overburden and underburden block material properties are calibrated representations of various rock analogs from Tulu et al. (2017), while the block material properties represented in the DFN (i.e. discontinuum overburden and underburden) are not calibrated to a given rock type but developed and verified in Chapter 3 of this thesis as realistically approximating a wide range of geologic conditions. Furthermore, they were previously utilized in assessing pillar-roof interaction in Chapter 4. Rather than attempting to calibrate the rock analogs from Tulu et al. (2017) to account for explicit discontinuities, the hypothetical heterogeneous lithologies shown in Figure 5.21 simply address the impact of block material strength and stiffness heterogeneity in the overburden.

The term “development load” is utilized in this chapter to indicate the model equilibrium state, or timestep where “contact overlap” error occurred, after excavation of the entries, but prior to depillaring. In models with stable pillars under development loads (i.e. 150 m deep cases), two types of depillaring were modeled. First, in-plane depillaring was modeled by deleting each panel pillar from left to right and running the model to equilibrium after each pillar was removed, until all panel pillars were removed, and the model reached equilibrium (i.e. overburden stabilized) or “contact overlap” error occurred. Second, the model was restored to its development load stage (i.e. prior to in-plane depillaring) and was run with an increasing load applied to the top of the model to capture the effect of out-of-plane depillaring operations (i.e. front-abutment loading). This was done by assuming the out-of-plane length of the panel was twice that of the panel width and that the pillars in the model plane would take a maximum load equal to the total tributary load of that projected panel area. This maximum possible load was simply utilized as a benchmark to develop a standardized manner in which stresses were increased for a given panel W/H. The boundary condition at the top of the model was increased by 1% of that maximum load until the model incurred “contact overlap” error and history files of the pillar, roof, and overburden responses were extracted. While this method has limitations inherent to plane-strain conditions and significantly simplifies realistic front-abutment loads, determination of the effects of pillar geometry, pillar material properties, and overburden rockmass conditions on average peak strength and post-peak behavior requires pushing pillars to their ultimate strength.

Following comparison of out-of-plane depillaring, 300 m, and 450 m deep model results it became clear that DFN did not have a significant effect on pillar behavior under the conditions modeled.
Therefore, an additional set of 14 “Voussoir DFN” models that all simulated out-of-plane depillaring were run with variations in overburden and pillar properties to evaluate the non-DFN related impacts on pillar strength more completely; the results of these models are discussed in Section 5.5.4.3.

5.5.2 Parametric Sensitivities to DFN and Pillar Location

In order to build upon the critical findings of Chapter 4, the consideration of panel-scale model sensitivities follows a similar progression. First pillar-roof mechanical interaction is considered in conjunction with variations from the baseline condition under stable development loads. Then pillar-overburden interaction is considered under unstable development and depillaring loads for the geologic and mining conditions modeled.

5.5.2.1 Pillar-Roof Interaction Under Stable Development Loads

In 150 m deep models under development loading conditions, variations in individual parameters generally showed similar results to those obtained from the single-entry models in Chapter 4. However, new considerations at the panel scale (i.e. panel W/H, pillar location, lithologic heterogeneity, inelastic and discontinuous floor) and their interaction with variations in DFN properties were explored to identify critical inputs affecting pillar development loading.

First, the effects of changing panel geometry are presented in this section through changes in macroscopic panel pillar behavior based on pillar location and panel W/H. Then the significant internal and external controls on pillar behavior and roof stress arching identified in Chapter 4 are shown in terms of macroscopic pillar behavior, as well as immediate roof yield and displacement, and in some cases, pillar yield. Lastly, the effects of individual heterogeneous layers in the overburden and realistic heterogeneous stratigraphies are evaluated. Note that in cases where end-members bracket the baseline parameter value (i.e. pillar w/h = 1 and 6 with a baseline of 3), parameters with little to no effect are discussed in the text, but not explicitly plotted in the figures to simplify the presentation of results.

The average pillar stress-strain curves for the leftmost and center-left panel pillars at development loads in the baseline cases (i.e. all DFNs modeled with parameters from column two in Table 5.3) showed that explicit DFN and pillar location had very little impact on stable pillar loading in the 5-entry system (Figure 5.22a). Immediate roof zone yield and entry midspan displacement for the
center and left-most entries, indicated that at low, static stress levels, DFN also had minimal impact on roof behavior (Figure 5.22b). As expected, the “Voussoir DFN” is able to transfer stresses without incurring as much zone yield due to the relative lack of differential stress concentrations. Uniform, idealized stress arching in the “Voussoir DFN” led to less yield for similar entry displacement, and slightly elevated pillar average vertical stresses.

Increasing the panel W/H by adding two additional entries to form a 7-entry panel, changed the panel size such that it went from W/H = 0.40 to W/H = 0.58. This increased the differences between pillar and entry behavior for different DFN cases (Figure 5.23). The larger panel width slightly increased the difference between both the pillar and roof response in the center and edges of the panel. The slight increase in divergent behavior is also accompanied by a slight increase in both roof yield and displacement, while pillar stresses and strains are overall identical to the 5-entry model results. These changes are more pronounced in all of the non-uniform DFNs. This indicates that the decrease in stress arching through the overburden due to the increase in panel width is increasing stress overburden stress transfer to the local entries, which is manifesting as additional roof yield in the model.

Figure 5.22: (a) Average pillar vertical stress vs. axial strain at development loads of leftmost (solid line) and center left (dotted line) panel pillars and (b) entry midspan vertical displacement vs. immediate roof zone yield percentage of leftmost (solid line) and center (dotted line) entries at 150 m deep with various explicit DFNs in the immediate floor and roof.
Figure 5.23: (a) Average pillar vertical stress vs. axial strain at development loads of leftmost (solid line) and center left (dotted line) panel pillars and (b) entry midspan vertical displacement vs. immediate roof zone yield percentage of leftmost (solid line) and center (dotted line) entries in a 7-entry panel at 150 m deep with various explicit DFNs in the immediate floor and roof.

Increasing and decreasing pillar w/h while maintaining the 5-entry panel either decreased the panel W/H to 0.27 (i.e. pillar w/h = 1) or increased the panel W/H to 0.60 (i.e. pillar w/h = 6). Changing pillar w/h also changed explicit joint locations in non-voussoir DFNs, making a direct comparison between model results difficult. However, the “Voussoir DFN” coupled with the two random seeds for the non-uniform DFNs increases confidence in the comparison of the results based on their similar response of the pillar and the roof.

Models with squatter pillars (i.e. w/h = 6) had nearly identical pillar and roof behavior to the w/h = 3 models in Figure 5.22, with the exception of decreased average stresses due to the increased pillar area. Conversely, pillar w/h = 1 model results exhibited a slight increase in divergent response in relationship to pillar or entry location in the panel (i.e. left vs. center), DFN properties (i.e. different DFNs), and explicit joint location (i.e. different random seeds) (Figure 5.24). The smaller w/h, center-left panel pillars in non-voussoir DFN models had slightly elevated stresses and strains at development load equilibrium. This was observed to correspond to throughgoing shear yield in the center-left pillar (Figure 5.25). Overall, the interaction between pillar size and its subsequent effect on panel width is dominated by the former, but when pillar size remains
constant, panel width marginally affects pillar-roof mechanical interaction at development loads in the sub-critical panel cases tested in this thesis.

Figure 5.24: (a) Average pillar vertical stress vs. axial strain at development loads of leftmost (solid line) and center left (dotted line) panel pillars and (b) entry midspan vertical displacement vs. immediate roof zone yield percentage of leftmost (solid line) and center (dotted line) entries at 150 m deep with various explicit DFNs in the immediate floor and roof, as well as a pillar w/h = 1.
Figure 5.25: Model plastic yield results at development load equilibrium focused on the left barrier pillar to the center entry and highlighting the different states of yield between the leftmost pillar (LP) and the center-left pillar (CLP). Refer to Figure 5.24 for average vertical stress vs. average axial strain curves for the panel pillars depicted in this figure.

The effects of the most critical impact identified in Chapter 4 (i.e. $k_0 = 2.0$) were then considered on 150 m deep models at development loads (Figure 5.26).
Figure 5.26: (a) Average pillar vertical stress vs. axial strain at development loads of leftmost (solid line) and center
left (dotted line) panel pillars and (b) entry midspan vertical displacement vs. immediate roof zone yield percentage
of leftmost (solid line) and center (dotted line) entries at 150 m deep with various explicit DFNs in the immediate
floor and roof, as well as $k_0 = 2.0$.

Pillars responded to elevated in-situ horizontal stresses in a similar manner to their single-entry
counterparts. The same softer pillar response was noted in all DFNs tested and regardless of the
location of the pillar in the panel. When modeled with “Voussoir DFN” roof and floor, the left
most and center-left pillars with baseline pillar (i.e. w/h = 3, E = 3.0 GPa) and roof properties (e.g.
weak discontinuities, “DOU Moderate SUBI” roof and floor blocks and “COU Black Shale”
continuum overburden and underburden, etc.) coupled with $k_0 = 2.0$ had almost identical pillar
responses under development loads. This further confirmed the uniform and effective arching of
stresses by the “Voussoir DFN”. Conversely, the more realistic vertical cross-joint DFNs (i.e. DFN
2a and 2b) incurred slightly higher loads to the center-left pillar, indicating that stress arching is
effective, but less uniform than in the “Voussoir DFN”. Furthermore, the decreased roof stress
arching capacity associated with sub-vertical cross-joints (i.e. DFN 3a and 3b) can readily be seen
in the difference between different random seed number results of roof displacement and pillar
loading, particularly for DFN 3b. In both sub-vertical DFN cases, the entry at the center of the
panel displaces more, while the pillar towards the panel edge takes on higher stresses and strains.
This shows that even in generally stable conditions, as horizontal stress arching decays across each
entry, overburden loads are transferred away from the center of the panel and towards the barrier
pillars. Finally, the effect of explicit joint location is readily seen in the difference between DFN 3a and 3b’s respective roof and pillar responses to high horizontal stresses at development loads. However, these differences might be reduced if more appropriate supports (e.g. tensioned bolts, angled bolts, cable bolts, steel straps, etc.) were used to mitigate the localized destabilizing effects of horizontal stress. The aforementioned support methods could be used individually or in tandem to more effectively reduce the displacement and yield associated with high horizontal stresses through the roof (i.e. angled bolts loaded axially), increase the stress-arching capacity of the roof (i.e. tensioned bolts), provide skin support that allows the roof to deform more uniformly (i.e. steel straps), or anchor the deforming roof to a more stable portion of the overburden (i.e. cablebolts).

Recall that stiffer pillars led to more yield in inelastic roof block models in Chapter 4, especially in models with the vertical cross-joint DFNs. The panel-scale model results are similar, with stiffer pillars leading to slightly more roof yield, but slightly decreasing overall roof displacement under stable development loads, when compared to their softer counterparts (Figure 5.27). This is due to the presence of roof support mitigating the impact of more yield leading to more displacement, and the overall roof displacement becoming influenced by the vertical displacement of the pillar (i.e. stiffer pillar, less roof displacement).

![Graph](image)

Figure 5.27: Entry midspan vertical displacement vs. immediate roof zone yield percentage of leftmost (solid line) and center (dotted line) entries modeled with (a) soft (i.e. pillar E = 1.5 GPa) or (b) stiff (i.e. pillar E = 4.5 GPa) at 150 m deep with various explicit DFNs in the immediate floor and roof.
If the relative displacements of the roof midspans to the pillar tops are considered, the displacements are nearly identical between soft and stiff pillar model roofs. In stiff pillars, the pillar corners displaced approximately 3.0 mm less than their soft pillar counterparts, accounting for the difference seen in roof displacement. This further confirms the results in Chapter 4, specifically that increased excavation-induced stress transfer from the stiffer pillar to the roof, not differential roof displacement, is leading to increased yield.

Increases to roof block strength had no direct significant impact on pillar response in Chapter 4 single-entry models under stable loading conditions. As the baseline panel-scale roof conditions were stable, increasing the strength and stiffness had little impact on pillar response. However, binary roof stability was previously identified as a key external control on pillar loading. Therefore, analysis of weak roof block model results was performed. Notably, every entry in the weak roof models failed completely through significant roof collapse and floor heave under development loads. Every pillar, regardless of modeled DFN, pillar type, or location in the panel, incurred vertical bands of tensile yielded elements (Figure 5.28). This behavior is indicative of the onset of low-confinement axial splitting failure. Direct comparisons of model history results at development loads are impeded by the occurrence of “contact overlap” error in multiple weak roof models. However, those that did reach an equilibrium solution ratio at development loads (i.e. “Voussoir DFN” and DFN 3a) have similar average pillar stress strain and stress path curves (Figure 5.29)
Figure 5.28: Results of plastic yield at equilibrium under development loads for the “Voussoir DFN” (top) and DFN 3a (bottom) weak roof and floor block material models. LBP = Left Barrier Pillar, LP = Left Pillar, CLP = Center-Left Pillar.
First and foremost, the stress path results indicate that all pillars reach their peak average vertical stress as the pillar approaches and enters a tensile minor (i.e. horizontal) principal stress state. The almost complete lack of stress arching capability due to the weakness of the roof and floor allows for rapid deconfinement to go unmitigated and result in pillar behavior approaching axial splitting. However, the pillars have not failed or collapsed as indicated by the ability to take on load following full convergence in the entries in DFN 3a models. Note that a majority of the excessive strains are due to the crushing of pillar corners shown in Figure 5.28.

The “Weak SUBI” discontinuum overburden and underburden block material was unsurprisingly outside of the capacity for passive bolts to provide support (see Chapter 3 roof stability results). However, consideration of the weak roof results further confirm that roof and floor stress arching are critical external controls on pillar behavior. Furthermore, this is a clear end-member example of the overburden (and underburden) failing before the pillars at the panel scale.

The continuum overburden was not present in the single-entry models in Chapter 4, and therefore requires consideration in this section. Changing the continuum overburden from “Black Shale” to “Limestone” (see Table 5.3), but keeping the discontinuum overburden at the baseline, significantly increased the strength and stiffness of the overburden. As expected, this resulted in
slightly lower development loads and strains being imposed on the panel pillars, and less roof midspan displacement (Figure 5.30). Interestingly, the “Voussoir DFN” models coupled with the stronger and stiffer continuum overburden material reduced both immediate roof yield and displacement, while the non-uniform DFNs all increased yield, but, with the exception of the leftmost entry with DFN 3b, decreased displacement when compared to the results in Figure 5.22 (i.e. baseline models).

Figure 5.30: (a) Average pillar vertical stress vs. axial strain at development loads of leftmost (solid line) and center left (dotted line) panel pillars and (b) entry midspan vertical displacement vs. immediate roof zone yield percentage of leftmost (solid line) and center (dotted line) entries at 150 m deep with various explicit DFNs in the immediate floor and roof, as well as a stiffer and stronger continuum overburden block material.

Previous studies (e.g. Gale, 2017) have identified pillar-roof and pillar-floor contact strength as a critical external control on pillar confinement maintenance or generation. Furthermore, changes in discontinuity strength and stiffness were not considered in Chapter 4 and require investigation under otherwise stable loading conditions. Increasing the frictional strength of all the discontinuities in the baseline models from 15° to 35° promoted increased confinement generation and allowed for slightly higher development loads in the center-left pillar (Figure 5.31a), while increasing joint normal and shear stiffness by an order of magnitude had no impact when compared to baseline model results (Figure 5.31b). Note that based on previous model results in this section (see Figure 5.26), the pillar closer to the center of the panel was known to be more sensitive to
variations in roof and overburden properties. Therefore, for clarity, the leftmost pillar was excluded from the results in Figure 5.31a and b.

Figure 5.31: Center-left pillar stress path results for changing (a) joint strength and (b) stiffness at 150 m depth with various explicit DFNs in the immediate floor and roof.

Overall, the previously identified impacts in single-entry models were confirmed to apply at the panel scale under development loads. In particular, the combined negative impacts on roof stress arching capacity of sub-vertical DFNs, weak pillar-roof and pillar-floor contacts, and high in-situ stress ratio were confirmed at the panel scale. New considerations related to panel geometry, pillar location, extremely weak roofs, and joint strength and stiffness were analyzed in relationship to globally stable development loads.

5.5.2.2 Block Material Heterogeneity Under Stable Development Loads

Block material heterogeneity was analyzed by changing the baseline condition of homogeneous block strength and stiffness in the discontinuum and continuum overburden. Figure 5.32 shows the impact of a single, 2 m thick strong bed on roof displacement at the bottom and top of the discontinuum overburden for the various DFN cases. Minor variations in the pillar stress path are more clearly captured by significant differences in the displacement and roof zone yield results. Naturally, a stronger bed in the immediate roof reduces roof yield but does not significantly reduce roof displacement due to the identical stiffness and overall stability of both the “Moderate SUBI” and “Strong SUBI” block properties under 150 m development loads. The presence of a strong 2
A similar analysis was performed to evaluate the impact that a 2 m thick strong bed in the continuum overburden would have on pillar-overburden interaction under stable development loads (Figure 5.33).

Figure 5.32: Model results of (a) left pillar stress path and (b) left entry roof displacement and yield for baseline (dotted lines) model cases and for 2 m thick strong beds at the bottom (solid line) and top (dash-dot line) of the discontinuum overburden. Refer to Figure 5.20 for strong bed location depiction.

m thick bed at the top of the discontinuum overburden (i.e. “SBD Top”) resulted in responses identical to the baseline cases. Note that “SBD Mid” cases were also identical the baseline and “SBD Top” cases and were excluded from Figure 5.32 for clarity.
As expected, the effect of the strong bed in the continuum overburden on stable pillar-roof mechanical interaction is less than its strong bed counterpart in the discontinuum overburden. Interestingly, in “DFN 3a” the effect of the single strong bed 8 m into the continuum overburden, strongly resembled the roof response of the homogeneously strong continuum overburden shown in Figure 5.30. Inspection of the model with a single strong bed at 4 m into the overburden shows similar behavior to the 8 m strong bed and homogeneously strong continuum overburden. Notably, this behavior only occurs in the left-most entry roof and is due in part to explicit joint location based on the results of “DFN 3b” (i.e. same DFN inputs, different random seed). Regardless, this behavior is facilitated by the presence of a stronger and stiffer material in specific or entire areas of the continuum overburden. Interestingly, when the “Limestone” layer is directly above the discontinuum overburden, the additional roof yield and displacement does not occur. It only occurs when the 2 m thick “Limestone” layer is at 4 or 8 m above the discontinuum-continuum contact, or when the entire overburden is composed of “Limestone”. The additional yield and displacement incurred by the discontinuum overburden in those cases are due to the combined impacts of specific joint location and the change in stress distribution.

Moderate to strong heterogeneous lithology (i.e. “Lithology 2” in Figure 5.21) had similar impacts in that pillar loading was only slightly impacted at the macroscopic scale (i.e. stress path), but roof
response began to diverge from the baseline condition for all DFNs tested (Figure 5.34). Note that the increase in roof displacement and yield seen previously in “DFN 3a” and stronger continuum overburden lithologies, is now seen in all DFNs and in all entries. These subtle interactions under stable development loads are more completely explored under in-plane depillaring loads in Section 5.5.4.2.

![Figure 5.34: Model results of (a) left pillar stress path and (b) left entry roof displacement and yield for baseline (dotted lines) model cases and for moderate to strong heterogeneous lithology (solid line).](image)

The effects of weak heterogeneous lithology (see Figure 5.21) on pillar development loading were even more significant. In weak heterogeneous lithology models (i.e. “Lithology 1”), the only models that were able to reach an equilibrium solution ratio under development loads without “contact overlap” error were DFN 3a and 3b. The pillar responses of the leftmost and center left pillars of these models up to development load equilibrium are depicted in Figure 5.35. Notably, the difference in pillar response due to only changes in the DFN random seed, particularly in regard to average axial strain, appears significant. However, this is due to increased pillar corner crushing in DFN 3b due to changes in explicit joint location. The pillar response is similar to the homogenous weak roof models shown in Figure 5.28 and Figure 5.29, with the exception that confining stress never drops significantly below 0 (i.e. tensile stress conditions). This is due to the fact that the floor block material is not “Weak SUBI” and is able to maintain stress arching such that just enough pillar confinement is maintained to prevent excessive axial splitting. Accordingly,
the model results show signs of roof failure (Figure 5.36), but no floor heave as shown in Figure 5.28.

Figure 5.35: (a) Average pillar vertical stress vs. axial strain and (b) stress path at development load equilibrium of leftmost (solid line) and center left (dotted line) panel pillars, at 150 m deep with two explicit DFNs in the immediate floor and roof, as well as the weak heterogeneous lithology (i.e. “Lithology 1” from Figure 5.21).

Figure 5.36: Results of plastic yield at equilibrium under development loads for the “DFN 3a”, weak heterogeneous lithology (i.e. “Lithology 1”) model results. CLP = center-left pillar, CRP = center-right pillar.
The weak heterogenous lithology is another case where the overburden fails first at the panel scale; however, this overburden instability was mitigated by stronger heterogeneous layers in the overburden and underburden, resulting in less pillar yield.

5.5.2.3 Pillar-Overburden Interaction Under Unstable Development and Depillaring Loads

Increasing development loads were not tested with every combination of other model inputs. Three main sets of models were analyzed; each featured all 5 DFNs coupled with the remaining baseline model conditions under 300 and 450 m development loads, and the out-of-plane depillaring loads. This corresponded to 15 of the 125 unique panel-scale model cases.

This section is mainly concerned with the impacts of stress magnitude and explicit DFN on pillar-overburden interaction, both of which have been identified as significant external controls on pillar confinement and peak strength in entry-scale models in Chapter 4. Note that all of the models discussed in this section incurred “contact overlap” error, making direct comparisons between model results difficult. However, the peak average stresses, and the stress path of the pillar prior to “contact overlap” error can be compared to identify the impact of stress magnitude, stress modeling method (i.e. deep and static or shallow with increasing vertical load), and the orientation, location, and heterogeneity of explicit fractures in the roof and floor. Furthermore, the determination of pillar “yield”, “failure”, or “collapse” is described to ultimately allow for comparison of model results to Mark-Bieniawski pillar strength predictions. Refer to the definitions of these terms provided in Section 5.3.

First, the impacts of static deep development loads on pillar behavior were considered. 300 and 450 m model results for each DFN and for two pillar locations in the panel (i.e. leftmost and center-left) are compared in Figure 5.37a and b. The baseline model pillar responses to 300 m and 450 m development loads all followed the same general loading, unloading, and strain-hardening behavior. Pillar location and DFN introduced some variability, but no consistent trend could be identified. At the time of “contact overlap” error, all of the panel pillars had fully yielded, but were all still carrying significant overburden loads, meaning that they had not collapsed. Determining whether or not they have failed is slightly more complicated; the pillars have reached an initial peak, but some appear to be strain-hardening, particularly in the 450 m models (see Figure 5.37b).
Figure 5.37: Leftmost (solid line) and center-left (dotted line) average pillar stress strain results for model entry depth of (a) 300 m and (b) 450 m with various explicit DFNs in the immediate floor and roof.

The most significant impact on pillar behavior is the stress magnitude itself, increasing the initial pillar peak average stress by approximately 30% and the degree of strain-hardening behavior exhibited by the pillars. This behavior was previously seen in single entries in Chapter 4 and is due to the in-situ state of stress increasing pillar confinement in deeper entries (Figure 5.38).

Figure 5.38: Leftmost (solid line) and center-left (dotted line) average pillar stress path results for model entry depth of (a) 300 m and (b) 450 m with various explicit DFNs in the immediate floor and roof.
The impact of in-situ stress magnitude was further explored through the out-of-plane model results, where development loads were stable (i.e. 150 m deep) and out-of-plane depillaring was simulated by increasing the stress boundary condition at the top of the model. Stress-strain results more closely matched the 450 m deep model pillar response in Figure 5.37b, but the stress path results matched neither of the previously modeled development load conditions (Figure 5.39). Note that the extreme jumps in pillar average stresses in DFN 2a, 2b, and 3a are due to numerical error associated with significant amounts of “contact overlap”.

Figure 5.39: Average (a) pillar vertical stress vs. axial strain and (b) stress path at applied-load model “contact overlap” error of leftmost (solid line) and center left (dotted line) panel pillars, starting from a development load of 150 m with various explicit DFNs in the immediate floor and roof.

These results indicate that it is not only the magnitudes of the applied stresses that can affect pillar loading, but also the manner in which those stresses are applied. The relationships between pillar yield development and principal stresses, as well as pillar yield development and entry midspan convergence under the “Voussoir DFN” are shown in Figure 5.40.
Since the $k_o$-ratio in all of these models was 1.0, all pillars started with equal vertical and horizontal stress. Under the 300 m development loads, the major principal (i.e. vertical) stress, minor principal (horizontal) stress, and the development of pillar zone yield were such that the pillar lost most of its original confinement prior to the onset of pillar zone yield and achieved its peak strength at approximately 5% pillar zone yield. Under 450 m development loads, the initial stress magnitudes are higher and the pillar reaches its peak strength prior to onset of pillar zone yield, but initial confinement loss is slowed by 10% pillar zone yield and eventually reverses thanks to the floor heave and roof deflection providing additional external confinement to the yielding pillar. The decrease and reversal in the rate of confinement decay causes a more pronounced strain-hardening pillar behavior in the post-peak. In the out-of-plane depillaring case, pillar load is increased incrementally and solved to equilibrium, allowing the pillar, roof, and floor to yield more fully at each load step and maintain more confinement in the pillar.

Increasing roof and floor convergence shown in Figure 5.40 was observed to be coincident with the observed strain-hardening behavior (Figure 5.37). Further examination of the model results indicated that in every deep or out-of-plane depillaring model, failure of the floor and roof following the onset of pillar failure, provided significant confinement to pillars in the post-peak.
This increased the pillars’ load-bearing capacity and allowed the slender pillars to take on the sustained overburden loads from the applied stress boundary condition (Figure 5.41).

These model results resemble a “punch-through” or floor heave failure, which has previously been documented as promoting strain-hardening behavior in slender trona pillars (Board et al., 2007). This behavior was not seen in Chapter 4 model results because the floor was modeled as an elastic equivalent continuum and the zero-velocity boundary condition reduced overburden loads as yield approached the upper boundary. Recall that unstable conditions promoted shedding of pillar stresses in Chapter 4 single-entry models. Additionally, in weak roof and floor block models discussed in Section 5.5.2.1, the pillar response to excessively weak and yielding roof and floor was also to shed stresses. The results in Figure 5.41 represent the opposite behavior, where roof and floor material are unstable at 300 and 450 m depths, but yielded floor material is still strong enough to provide confinement to the failing pillar. Floor instability ultimately led to increased pillar confinement at the time of “contact overlap” error. Were the solution process to continue, it is unknown if this confinement would be maintained or if the pillar would ultimately collapse. Regardless, the functional failure of the entry has clearly occurred.
It should be noted that while these results arise in part due to a limitation of the explicit DEM where intact block rupture cannot be explicitly modeled, they also further confirm that the calibrated Progressive S-shaped yield criterion is accounting for the confinement-dependent nature and strength of pillar deformation, irrespective of the source of the confinement (i.e. end-constraints, lateral constraints, support, etc.) and that the panel-scale models are capturing a wide range of pillar responses when modeled with hypothetical but realistic overburden and underburden properties. The model result in Figure 5.41 shows a case where the roof and floor yield first, but the pillar fails first, inducing failure in the floor, which subsequently provides confinement to the failing pillar.

However, w/h = 3 pillars are generally not expected to strain-harden, as they cannot internally generate enough confinement to do so. Furthermore, the weak interfaces modeled in this Chapter are known to diminish confinement generation and maintenance. Finally, previous numerical investigations post-peak pillar behavior (Esterhuizen et al., 2010; Sinha, 2020) suggest that w/h = 3 pillars reach their peak strength well before 2% strain, while the pillar strain results shown in Figure 5.37 approach 6-8% strain at the time of “contact overlap” error. This suggests that the pillars can be considered failed prior to the failure of the floor, despite the fact that their ultimate strength (i.e. associated with complete pillar collapse and unloading) following strain-hardening may be higher than the initial apparent peak strength.

The Mark-Bieniawski pillar strength criterion was developed in such a manner that failure and collapse were coincident, which is clearly not the case in the model results shown herein. Although the Mark-Bieniawski strength is typically considered to represent the ultimate pillar strength (i.e. at collapse), it has been previously utilized in Mark (2010) to delineate “squeezed” and “burst” failed cases when developing the ARMPS2010 guidelines. The Mark-Bieniawski pillar strength for the w/h = 3 pillars presented in this section was calculated to be 14.0 MPa. Although this may (or may not) be an accurate representation of the ultimate peak strength associated with strain-hardening induced by confinement generated by roof and floor failure, it is clearly an optimistic estimate of the initial pillar stress peak that defines the onset of functional pillar failure for both the 300 and 450 m deep baseline cases.
5.5.3 Discussion of Parametric Sensitivities at the Panel-Scale

The results presented in this section further confirm the results of single-entry models and present new results regarding pillar-overburden mechanical interaction at the panel-scale. New considerations related to panel geometry, pillar location, inelastic and discontinuous floor and underburden, slender pillars, as well as development and depillaring loads have been evaluated.

Under 150 m development loads, changes in modeled rockmass conditions generally had the expected effects on pillar loading and roof stress arching. Explicit joint location, explored through two random DFN seeds, was found to locally impact pillar loading and roof stress arching, but overall the distribution and orientation of joints is more critical. The “Voussoir DFN” was confirmed as the most consistently optimistic representation of roof stress arching capacity. Excessively weak roof and floor material resulted in low confinement axial splitting of the w/h = 3 pillar models and w/h = 1 pillars with moderate strength roof and floor began developing cross-cutting planes of shear yield. Stronger pillar-roof and pillar-floor contacts were confirmed as controlling the level of confinement recorded in the pillars even under stable loading conditions. The effects of increasing development loads (i.e. 300 m and 450 m models) and depillaring loads showed that in-situ stress magnitude and development of pillar and host-rock yield control the measured average pillar stresses and their relationship to pillar yield and failure. Modeled depth and depillaring-induced stress changes are important external controls on initial pillar confinement and confinement maintenance throughout the loading of the pillar. Although the models incurred “contact overlap” error, and the ultimate state of the pillars in deeper model cases remains uncertain, based on previous research regarding the pillar strain at peak stress in w/h = 3 pillars (Board et al., 2007; Esterhuizen et al., 2010; Sinha, 2020), it can be said with confidence that the pillars considered in this section have failed. Based on comparison with the limited number of model cases tested in this section, the Mark-Bieniawski pillar strength equation appears to provide optimistic estimates of pillar strength, but this is further evaluated in Section 5.5.4.3.

5.5.4 Analytical & Empirical Method Comparisons

In order to consider the mechanical impacts of individual parameters in a concise and practical manner, panel-scale model results of pillar stresses and excavation stability were compared to the state-of-practice analytical and empirical methods that calculate pillar load (i.e. TAT, ARMPS2010 pressure arch factor (Fpa), abutment angle), strength (i.e. Mark-Bieniawski pillar strength equation)
strength equation), and overall pillar stability (i.e. traditional FoS and ARMPS2010 SF). For detailed discussion on these topics, refer to the Chapter 4 literature review (Sections 4.2.4 & 4.2.5). Regardless of the conditions modeled in this chapter, the traditional analytical and empirical methods discussed in the literature review were found to be mostly inaccurate when calculating pillar loads or pillar strength based on model inputs (i.e. depth, pillar w/h) and comparing them to model results. The intent of this was not to demonstrate that the existing methods are not viable, but rather to identify the areas where they fail to mechanically account for relevant rockmass conditions. These models are hypothetical, but realistic, and allow for conditions to be tested outside of what the empirical methods incorporated from field data and what the analytical methods assume and simplify regarding material behavior.

5.5.4.1 Tributary Area Theory & ARMPS2010 Fpa – Panel Pillar Development Loads

Previous model results in Section 5.5.2.1 indicated that the panel W/H and pillar location had little overall impact on the differential loading of the panel pillars across all development loads tested. In order to ensure consistent comparison between pillars in the various W/H panels, average vertical stress from the left-most pillar at the 150 m development load equilibrium stage was compared to the analytically predicted load. This comparison corresponds with the average stresses discussed in Sections 5.5.2.1 and 5.5.2.2 for the 150 m deep models. This method of comparing stresses of the panel pillar closest to the barrier pillar would typically be considered to increase the error due to increased stress arching near the barrier pillar. However, the results of Sections 5.4 and 5.5.2 indicate that the stress distribution under stable and unstable loads are generally consistent between pillars in a given panel for all panel W/H and pillar w/h tested. This diverges from continuum modeling efforts (e.g. Salamon, 1992; Yu et al., 2018; Dean-Pelikan & Walton, 2020) which show that pillar stresses decay significantly towards the barrier pillars in lower panel W/H configurations. This is once again due to the more realistic inelastic nature of the modeled overburden material and the stress distributing properties of the DFNs utilized.

The results in Figure 5.42 further highlight the inaccuracy of the various methods for modeled loading and material conditions. Note that deeper (i.e. 300 and 450 m deep models) and out-of-plane depillaring models all resulted in pillar failure prior to model equilibrium and are not considered in this comparison due to the prevalence of “contact overlap” errors and uncertainty regarding the final equilibrium load of the pillar. Additionally, recall that changes in panel W/H
and pillar w/h are linked with a constant number of entries and span width. The decrease in panel W/H from 0.58 to 0.40 represents the change from a seven-entry system to a five-entry system with identical pillar sizes, while the panel W/H = 0.20 and 0.60 represent the pillar w/h = 1 and 6 cases, respectively.

![Figure 5.42: Comparison of the impact of panel-scale model inputs on TAT-predicted loads. Leftmost panel pillar equilibrium average vertical stresses were compared as a percent change from the TAT predicted loads. BS = “Black Shale”, LS = “Limestone”, Mod = Moderate. Note that TAT Error above 50% is not shown for the baseline conditions (i.e. Panel W/H = 0.4, Pillar w/h = 3, Pillar E = 3.0 GPa, In-Situ Stress Ratio = 1.0, Continuum Block Strength = “Black Shale”, and Contact Strength = 15°), larger deviations are shown in Figure 5.43.]

As expected, TAT increasingly overpredicted panel pillar stresses with parameter changes that promoted overburden stress arching. These are primarily controlled by decreasing the panel W/H and pillar w/h, or increasing the strength and stiffness of the continuum overburden. Naturally, a narrower panel makes it easier for stresses to arch, increasing TAT overprediction. In the case of smaller w/h pillars, for the same in-situ vertical stress, they will have a higher state of pillar vertical stress than their squatter counterparts due to their smaller cross-sectional area, which TAT
accounts for through the excavation ratio. However, slender pillars also tend to transfer more stresses to the roof inducing additional yield and displacement, as shown in Section 4.4.1.2 (see Figure 4.27) and Section 5.5.2 (see Figure 5.24). This increased the deviation from TAT assumptions through inelastic yield and non-uniform displacement of the roof.

The increase in continuum overburden stiffness from 8.0 to 27 GPa, and the lab-scale UCS from 10 to 100 MPa resulted in an increase in the mean error from 10% to 17%. However, the presence of single strong layers in the discontinuum roof and continuum overburden, and changes in explicit discontinuity strength or stiffness had zero effect on the change in mean TAT error relative the baseline cases.

Increasing the deformability of the pillar by decreasing Young’s Modulus, also resulted in slightly higher TAT overprediction, consistent with the results in Section 4.4.4.1. Increases in in-situ stress ratio resulted in a slightly wider range of calculated TAT error. This was directly tied to the DFN orientation with DFNs that promoted stress arching (i.e. “Voussoir DFN”, and DFN 2a) increasing TAT error, while sub-vertical DFN 3a decreased error. Interestingly, TAT is more accurate than expected at the panel W/H = 0.6 tested in this chapter. This runs counter to the recommendations found in the literature (Van Der Merwe et al., 2003; Yu et al., 2018) that suggest TAT is only accurate for panel W/H > 1-3. This is likely due to the weak continuum overburden representation decreasing the stress arching capacity of the overburden and approaching TAT assumptions.

Changes in the strength of the discontinuum roof from “Moderate SUBI” to “Strong SUBI” had limited impact on TAT error, however, “Weak SUBI” immediate roof and weak heterogeneous stratigraphy (i.e. “Lithology 1”) increased TAT error significantly (Figure 5.30). In the case of the “Weak SUBI” discontinuum roof and weak heterogeneous lithology models, the collapse of each entry roof (see Figure 5.28 and Figure 5.36) dissipated stresses through material yield; this decreased the average panel pillar stress, increasing the TAT overprediction. However, this case is considered to represent an extreme end-member case, because it lacks sufficient roof support to prevent failure of an excessively weak roof.
Figure 5.43: Significant increase in TAT error associated with extremely unstable entry roof conditions. Mod = “Moderate SUBI”, Lith 1 = weak heterogenous lithology, Lith 2 = moderate to strong heterogeneous lithology, Homo = homogeneous baseline lithology.

The ARMPS2010 Fpa was developed through statistical analysis of pillar stability to account for overburden stress arching in panel dimensions of W/H < 1.0. Recall the logarithmic nature of the Fpa Eqn. (4.4); as panel W/H ratio decreases (i.e. as the modeled width decreases) the portion of load transferred to the barrier pillars initially increases significantly and then each subsequent increase decays as a function of the natural log. The AMPRS2010 Fpa development load calculation underpredicted panel pillar development loads (i.e. overestimated stress transfer to barrier pillars) significantly for all modeled rockmass conditions except where roof instability occurred, but was able to reduce the variability due to panel dimensions seen in the TAT comparison (Figure 5.44).
Figure 5.44: Comparison of the impact of geometric panel-scale model inputs on ARMPS2010 Fpa predicted development stress. Pillar equilibrium average vertical stresses were compared as a percent change from the ARMPS2010 Fpa predicted stress. Note that excessive error seen in “Weak SUBI” discontinuum overburden and weak heterogeneous lithology results from Figure 5.43 are excluded from these panel plots for clarity.

The ARMPS2010 Fpa is fit to demarcate “successful” and “failed” panel and barrier pillar cases at an ARMPS SF = 1.5, while using the Mark-Bieniawski pillar strength equation to estimate strength. As shown in Chapter 4 and Section 5.5.2.3, the Mark-Bieniawski pillar strength equation gives overly optimistic values particularly when applied to cases with typical or weak pillar interfaces. The Mark-Bieniawski strength of a barrier pillar is much higher than for a panel pillar. For example, a pillar w/h = 16 (i.e. barrier pillar) has a Mark-Bieniawski strength of 40.0 MPa, while a pillar w/h = 3 (i.e. panel pillar) has a strength of 14.0 MPa. If the barrier pillar experiences squeeze or burst failure that is more than likely limited to its ribs, the Fpa must overcompensate and transfer additional portions of the PAT stress away from the active mining zone and towards the barrier in order to yield the 40.0 MPa strength pillar. This results in the general under prediction of panel pillar stresses shown in Figure 5.44. Although the ARMPS2010 method reduced the overall range of error, it significantly underpredicted the development loads of the panel pillars, particularly at lower W/H panels. Furthermore, the explicit mechanical influences of overburden and roof properties on panel stress arching remain unaccounted for.

5.5.4.2 Abutment Angle Concept – Post-Extraction Barrier Pillar Loads

Barrier pillars provide global mine support, reduce surface subsidence, and protect critical underground mine infrastructure. Following the analysis of roof and overburden influence on panel pillar loading at 150 m development loads (i.e. prior to depillaring), the impact on barrier pillar loading was also investigated (i.e. after depillaring). Since 300 and 450 m models were unstable
under development loads, only 150 m deep models were considered. Of the 110 models that were 150 m deep, 77 converged to an equilibrium solution ratio of $1.0(10)^{-5}$ after full panel extraction.

Barrier pillar stress results at model equilibrium were extracted and compared to loads predicted by a recommended abutment angle of $21^\circ$, in accordance with suggested design parameters from ARMPs and ALPS (Mark, 1990; Mark et al., 1994; Tuncay et al., 2019). All models considered in this section were considered sub-critical, with panel W/H ranging from 0.13 (i.e. depth = 450 m, pillar w/h = 3, and a 5-entry panel) to 0.6 (i.e. depth = 150 m, pillar w/h = 6, and a 5-entry panel). Expected barrier pillar side-abutment loads following panel extraction for sub-critical widths were calculated using Eqn. 5.1 and compared to average vertical stress of the modeled barrier pillars as:

$$\text{Barrier Pillar Stress Error (\%)} = \frac{\sigma_{barrier, sub} - \sigma_{yy, av}}{\sigma_{yy, av}} \times 100 \tag{5.4}$$

This predicted side-abutment load was compared with average stresses of both left and right barrier pillars from shallow (150 m) models, where full pillar extraction was completed with no “contact overlap” error in explicit gob models (i.e. model reached an equilibrium solution ratio). As expected, a single abutment angle was highly inaccurate in predicting the side-abutment load transferred to the barrier pillar. Although adjustments to the $21^\circ$ abutment angle have been proposed (Tuncay et al., 2019) they are only recommended for the limits of the case studies used to develop them (i.e. $0.7 < \text{panel W/H} < 3.5$). Furthermore, the abutment angle recommendation for overburden depth less than 200 m remains $21^\circ$ (Tuncay et al., 2019). As the panel W/H cases tested in this chapter are all less than 0.7, and the overburden depths considered in this section are from 150 m deep models only, the original abutment angle approach from Mark (1987) was utilized.

In many model results, the barrier pillar loads for the left and right pillar were remarkably different due to the impact of pillar extraction direction (left to right), as well as the heterogeneity in overburden properties. The percent error between the model average barrier pillar vertical stress and that predicted by the recommended abutment angle calculations ranged from -34% (i.e. underpredicted) to 116% (i.e. overpredicted). Overall, the median results show that the abutment angle of $21^\circ$ is too large and overpredicts both left and right barrier pillar stresses for most model cases. Clearly, the recommended abutment angle is dependent on more than just panel geometry. First, the influence of explicit DFN on left and right barrier pillar stresses was independently
considered through comparison to the analytically calculated barrier pillar stress (Figure 5.45). The changes in barrier pillar stress error with DFN indicate that local entry stability and explicit gob bulking have some impact on overburden stress distribution, but are not the critical controls as they do not capture the whole range in the dataset. The uniform “Voussoir DFN” tends to load the left barrier pillar less, while the non-uniform vertical cross-joint DFNs (i.e. DFN 2a and 2b) result in more uniform loading, and the sub-vertical cross-joint DFNs tend to load the left barrier pillar more than the right.

Figure 5.45: Boxplots of 150 m deep, in-plane depillaring model left (L) and right (R) barrier pillar stress results for each DFN tested.

Further inspection of model results after removal of the first panel pillar show that this behavior is controlled by the caving and explicit bulking of the DFN gob (Figure 5.46). After material yield in the roof reduces the stress arching capacity of the discontinuum gob, the “Voussoir DFN” bulks less and begins caving precisely at the rib of the barrier pillar due to its uniform geometry. This promotes more overburden yield, reduction in the abutment angle, and a significant decrease in overburden stress arching. Conversely, both non-uniform DFNs promote more bulking and delay caving past the barrier pillar rib, maintaining overburden stress arching at this stage in the model. As subsequent pillars are extracted, the overburden stress arching deteriorated for all DFNs under the baseline conditions. However, the delayed decay of overburden stress arching in DFN 2
resulted in more evenly loaded barrier pillars, while the increase in explicit bulking in DFN 3 generally resulted in slightly higher left barrier pillar loads at full panel extraction.
Figure 5.46: Major principal stress contours of baseline 150 m deep panel-scale models at equilibrium following deletion of first panel pillar for three of the five DFN cases tested.
Next, panel dimensions and pillar w/h were considered simultaneously. Recall that the numbers of entries and span widths remained constant for pillar w/h = 1, 3, and 6 resulting in 40, 60, and 90 m wide panels, respectively. The 7-entry system was modeled using pillar w/h = 3 only, resulting in a panel width of 87 m (Figure 5.47).

![Boxplots of 150 m deep, in-plane depillaring model left (L) and right (R) barrier pillar stress results for each panel width tested.](image)

It is apparent that pillar w/h has some impact on barrier pillar stress differential, but that the trend overall is controlled by the panel width. As the panel width increases, the recommended 21° abutment angle increasingly overpredicts the barrier pillar stresses. Stress arching of the overburden is less effective over wider panels and more load is transferred onto the gob, decreasing the barrier pillar stresses and increasing the overprediction of the sub-critical abutment side load calculation. Initially, as panel pillars are removed, stresses increase in both barrier pillars evenly, but left barrier pillar stresses remain higher than the right. In wider panels, the left barrier pillar stresses increase at a faster rate and then drop below the right barrier pillar stresses upon deletion of the rightmost panel pillar (Figure 5.48). These results are the opposite of those from Heasley (2000), where the abutment angle method underpredicted the abutment load for increasing W/H panels. Note that Heasley (2000) utilized an elastic laminated overburden in LaModel, which assumes perfect stress arching unimpeded by inelastic damage. In the LaModel models, as the panel width increased, the stress transfer to the barrier pillars remained constant or increased,
depending on the thickness of the laminations used, because the overburden could not fail. This effectively kept super-critical panel dimensions behaving sub-critically.

Figure 5.48: Average pillar vertical stress results from DFN 2b in-plane depillaring models for (a) panel width = 40, (b) 60, and (c) 90. Note that when stress curve has as slope of 0, this indicates that the pillar has been extracted. LBP = left barrier pillar, LP = leftmost panel pillar, CLP = center-left panel pillar, CRP = center-right panel pillar, RP = rightmost panel pillar, RBP = right barrier pillar.

Once again, geometric changes in the model account for significant variation in the accuracy of the analytical methods which only consider geometric effects. Further analysis of the impact of variation in other geologic and mining properties indicate that their influence on barrier pillar stress should be explicitly accounted for in order to maintain accuracy in the application of empirical methods. The impact of increased and decreased horizontal stress was minimal, but followed a distinct trend (Figure 5.49). The results in Figure 5.49 are consistent with panel pillar loading results from Chapter 4 and Section 5.5.2.1, where changes in stable pillar peak average stress due to horizontal stress is minimal.
Deviation from the baseline pillar stiffness led to some variation in left and right barrier pillar loads (Figure 5.50). Stiffer pillars (i.e. panel and barrier) generally increased the error associated with the barrier pillar stress calculation for the left barrier pillars (i.e. left pillar took on less overburden loads than the analytical method predicted). However, these results are not fully representative, because neither sub-vertical DFN with stiff pillars were able to complete depillaring and arrive at an equilibrium solution ratio, allowing the voussoir DFN (see Figure 5.45) to skew the results of differential pillar loading when compared to the other pillar stiffnesses.
Explicit discontinuity strength had a minimal effect on barrier pillar loading (Figure 5.51). This agrees well with the results of previous models presented in this thesis. In Chapter 3, joint frictional strength had no statistically significant impact on binary roof stability, and under stable loading in Chapter 4 and Section 5.5.2.2, discontinuity strength had some influence on pillar confinement, but none on roof response.
The strength of discontinuum overburden blocks only had a significant impact in the weak roof and floor cases (Figure 5.52). As shown in Section 5.5.2.1, the weak roof and floor blocks had effectively zero roof stress arching capacity under development loads and they immediately failed. Only one weak discontinuum block model was able to model full panel extraction without “contact overlap” error. However, the differential loading in this model follows a similar trend: decreased roof and overburden stress arching capacity (i.e. wider panels, more uniform or less effective bulking) results in increased differential barrier pillar loading. Notably, increases in block material strength had minimal impacts.
Figure 5.52: Boxplots of 150 m deep, in-plane depillaring model left (L) and right (R) barrier pillar stress results for each discontinuum overburden block strength tested, Mod = Moderate.

Similarly, the minimal changes in barrier pillar loading due to the inclusion of strong beds in the discontinuum overburden are shown in Figure 5.53.

Figure 5.53: Boxplots of 150 m deep, in-plane depillaring model left (L) and right (R) barrier pillar stress results for each strong bed DFN location tested. Bot = Bottom, Mid = Middle.
Conversely, increases in the homogenous strength and stiffness of the continuum overburden block accounted for nearly all of the negative barrier pillar stress error (i.e. 21° abutment angle underpredicted barrier pillar side-abutment loads) (Figure 5.54). Although the 21° abutment angle is still unable to account for the direction of mining, the stronger and stiffer “Limestone” (see Table 5.1) in the continuum overburden results in much better agreement with the empirical abutment angle equation.

![Figure 5.54 Boxplots of 150 m deep, in-plane depillaring model left (L) and right (R) barrier pillar stress results for each continuum overburden block strength tested. BS = “Black Shale”, LS = “Limestone”.

The range in the differential pillar loading is highly influenced by the explicit DFN as highlighted by comparison of the major principal stress at model equilibrium for uniform, vertical, and sub-vertical cross-joint DFNs (Figure 5.55). Although all the models generally have higher left barrier pillar loads, the “Voussoir DFN” and DFN 2a both have more evenly loaded barrier pillars, while DFN 3a has heavily loaded the left barrier pillar with a significantly destressed right barrier pillar. Both pillars under DFN 3a discontinuum overburden have incurred significant yield towards their cores in comparison to the other DFNs modeled. Once again, the trend of increased left barrier pillar loads with increased stress arching capacity or explicit bulking is observed.
Figure 5.55: Major principal stress contours of “Limestone” continuum overburden 150 m deep panel-scale models at equilibrium following deletion of all four panel pillars for three of the five DFN cases tested.
Additional consideration of the “Limestone” continuum overburden DFN 3a model results at equilibrium before the final pillar was removed indicated that while this divergent barrier pillar behavior was developing throughout depillaring operations, it was the final pillar removal that significantly increased the stress differential in the barrier pillars (Figure 5.56).

Figure 5.56: Major principal stress contours of “Limestone” continuum overburden 150 m deep panel-scale models at equilibrium following deletion of three of four panel pillars in the DFN 3a discontinuum overburden case.

Barrier pillar stress path results reveal that the right barrier pillar failed to maintain confinement after removal of the right-most pillar (Figure 5.57). The right barrier pillar had higher levels of confinement up to the removal of the right-most panel pillar thanks to the local stability of the adjacent roof. However, after the last panel pillar was removed, the right barrier pillar rib lost confinement and began undergoing low-confinement tensile yield; overburden stresses were increasingly transferred to the left barrier pillar due to the strong and stiff “Limestone” overburden, which the left barrier pillar could in turn support due to confinement provided by the previously collapsed gob.
When the strong layer was limited to a “simple” heterogeneous stratigraphy featuring a 2 m thick, strong “Limestone” layer at various heights in the “Black Shale” continuum overburden, the behavior was more similar to the baseline cases than the uniformly strong continuum overburden. Notably, the variability in barrier pillar loading increased as the single strong bed was modeled further above the discontinuum-continuum boundary (Figure 5.58).
Figure 5.58: Boxplots of 150 m deep, in-plane depillaring model left (L) and right (R) barrier pillar stress results for each strong bed COB location tested.

The mechanism controlling the lower barrier pillar stress (i.e. higher barrier pillar stress error %) in the right barrier pillars with the presence of a single strong bed was related to the pillar losing its rib load as the discontinuum gob formation approached the barrier. Rather than losing its rib load and transferring stresses towards the other barrier pillar like in the homogeneous “Limestone” continuum overburden cases (Figure 5.55 and Figure 5.56), the stress arching of the 2 m “Limestone” bed transferred overburden loads (Figure 5.59) and yield (Figure 5.60) deeper into the right barrier pillar and discontinuum overburden above the barrier pillar, causing an increased pillar stress differential at model equilibrium. These results highlight the potential negative impacts of increased stress arching on barrier pillar stability. If the barrier pillar was perhaps replaced with chain pillars or bleeder pillars, the additional yield in the discontinuum portion of the roof could result in roof instability and leave the barrier pillar susceptible to damage from other abutment loads or even mine-induced seismicity because of its low-confinement state.
Figure 5.59: Major principal stress contours comparing the baseline homogeneous model (left) to the effect of a single “Limestone” bed at 4 m (center) and 8 m (right) into the continuum overburden. The average barrier pillar vertical stresses are highlighted to show the net effect on pillar stress distribution.

Figure 5.60: Comparison of zone and SUBI material yield around the right barrier pillar (RBP) between the baseline homogeneous model (left) to the effect of a single “Limestone” bed at 4 m (center) and 8 m (right) into the continuum overburden.

When the degree of heterogeneity is increased beyond a single layer in the continuum or discontinuum overburden, the degree of differential pillar loading remains elevated. However, comparison of results was complicated by the fact that only one weak lithology and two moderate strength lithology models reached an equilibrium solution ratio at full pillar extraction. The weak lithology converged for the DFN 3b case, while the moderate strength lithology converged for
DFN 2b and the “Voussoir DFN”. The equilibrium results are presented in Figure 5.61. As the degree of heterogeneity increased from single strong layers to hypothetical coal-measure stratigraphy, the differential barrier pillar loading increased, but the 21° abutment angle was consistently more accurate for the left barrier pillar loads. This is largely due to the presence of “Sandstone” and “Limestone” layers in the continuum overburden of both heterogeneous lithologies in different configurations (see Figure 5.21).

![Boxplots of 150 m deep, in-plane depillaring model left (L) and right (R) barrier pillar stress results for each heterogeneous lithology tested. Lith = Lithology, Homo = Homogeneous.](image)

In the weaker heterogeneous overburden lithology model (i.e. “Lithology 1”), a single thick limestone layer maintains stress arching across the panel, behaving similarly to the results in Figure 5.59 and Figure 5.60 that concentrate overburden stresses towards the core of the barrier pillar, inducing yield in the discontinuum overburden above the right barrier pillar (Figure 5.62a). In the stronger heterogeneous lithology model (i.e. “Lithology 2”), the majority of the overburden is either “Sandstone” or “Limestone” block material, and the pillar-overburden interaction is similar to the homogeneous strong continuum overburden model (Figure 5.62b). Most notably, although the abutment angle provides the same accurate value for barrier pillar loads in both lithologies, the distribution of barrier pillar stress and the overburden stress arching mechanisms diverge considerably.
Figure 5.62: Comparison of major principal stress contours at model equilibrium following full panel extraction for (a) weaker heterogeneous lithology (i.e. “Lithology 1) and (b) stronger heterogeneous lithology (i.e. Lithology 2).

Analysis of the in-plane depillaring model results indicates that the roof and overburden properties tested, most notably simple and complex heterogeneous lithologies, impact barrier pillar loading to a higher degree than the properties considered by the recommended abutment angle method: panel geometry and pillar properties. However, in sufficiently strong and stiff materials the recommended abutment angle is more accurate, indicating that it is likely capturing an important mechanism, but that it requires additional consideration of the geomechanical properties of the overburden in order to be made more broadly applicable.

5.5.4.3 Mark-Bieniawski Pillar Strength – Unstable Development and Extraction Loads

Following analysis of the effects of in-plane depillaring on barrier pillar loading, the impacts of unstable development loads and out-of-plane depillaring were considered through the lens of the Mark-Bieniawski pillar strength equation. An analysis comparing model results to the predicted Mark-Bieniawski pillar strength was conducted on all panel models with entry depths of 300 m and 450 m, as well as out-of-plane depillaring models, where panel pillars were loaded to peak strength. Recall that these 15 model results (i.e. 5 DFNs modeled with 3 loading conditions) did not vary any other rockmass characteristics of the roof and overburden. Recall that the pillars
began failing prior to yielding of the floor; the heaved floor provided additional artificial confinement.

Building on the stress results shown in Figure 5.40, analysis in this section indicated that the MB pillar strength equation overpredicted pillar strength under the out-of-plane depillaring conditions for 300 m entries by 35%, 450 m entries by 11%, and out-of-plane loading models by 5-15% (Figure 5.63). Explicit DFN geometry and random seed had a negligible impact on MB error when compared the influence of in-situ stress magnitude and out-of-plane loading. However, model results were most sensitive to the combined impacts of DFN and modeled stress condition.

![Graph](image)

Figure 5.63: Comparison of the impact of deep development load and out-of-plane depillaring on MB predicted pillar strength. Panel pillar peak average stresses were compared as a percent change from the MB predicted strength.

In order to better capture the effect of rockmass characteristics without the influence of random explicit DFNs, a second set of models that featured only the “Voussoir DFN” coupled with the applied-load out-of-plane depillaring method, as well as select previously discussed critical parameter variations (e.g. $k_o$-ratio, strong block material, strong discontinuities, etc.) were run until models incurred “contact overlap” error. Model pillar stress-strain and yield results indicated that similar to the model results in Section 5.5.2.3, the pillars were only strain-hardening due to collapse and failure of the roof and floor. Therefore, for similar reasons the peak average stress recorded by the pillar could be taken as the failure strength and compared to the Mark-Bieniawski strength.
This set contains 14 unique numerical models and their leftmost pillar peak average stress results are presented in relationship to Mark-Bieniawski pillar strength equation (Figure 5.64). Note that the model results featuring “Weak SUBI” roof block material are excluded as they were the only case where entries failed without associated pillar yield. However, this case is considered unrealistic, since a uniformly weak discontinuous roof extending well above and below the entry would likely be properly supported with cable bolts and steel straps, which are outside the scope of this study.

![Figure 5.64: Comparison of the impact of out-of-plane depillaring model inputs on Mark-Bieniawski (MB) predicted pillar strength.](image)

The Mark-Bieniawski pillar strength equation was quite sensitive to changes in rockmass characteristics and mine geometry modeled at the panel scale. As seen in the literature, increased strength of all the model discontinuities, including the contact between the pillar and the roof (i.e. “Strong Joints”) increased the strength of the pillar and approached the predicted Mark-Bieniawski strength. Furthermore, the amount of strength over prediction is consistent with the limited comparison presented in Chapter 4. Additionally, similar to Chapter 4 results, increased horizontal stress under similar loading conditions, and softer pillars decreased the peak strength of the pillar significantly when compared to the baseline condition. Heterogeneity also played a significant role...
as indicated by the model results from single strong beds at the base of the roof and overburden (i.e. other single strong bed locations excluded), as well as the “Lithology 2” (i.e. moderate strength bolted interval) models. Note that homogenously stronger overburden properties, both in the continuum and discontinuum portions of the overburden, deviated less from the baseline condition than the heterogeneous strong material models (i.e. “Strong Bed DFN”, “Strong Bed OB”, and “Lithology 2”). Interestingly, increasing panel width by modeling a 7-entry panel, rather than a 5-entry panel also decreased modeled pillar strength by changing the loading condition of the leftmost pillar. Recall that the out-of-plane depillaring is modeled by incrementally increasing the stress boundary condition at the top of the model. This increment is calculated based on the width of the panel; a wider panel increases the load in larger increments and does not allow for the same amount of pillar confinement to develop and the pillar fails at a lower load. Conversely, the Mark-Bieniawski strength was significantly conservative (i.e. underpredicted the strength) when applied to pillar w/h = 1 and k_o = 0.5. In the pillar w/h = 1 case, the pillar managed to maintain very low levels of confinement following an initial drop as it was loaded and approached its ultimate strength. In the k_o = 0.5 model results, the pillar managed to generate more confinement as it yielded than its k_o=1.0 counterpart, because lower horizontal stresses could not decay as readily along weak pillar interfaces.

5.5.4.4 Panel Pillar Factor of Safety and AMPRS2010 Stability Factor

In order to further tie the model results to practical methods and assess the state-of-practice, this section explores the concept of pillar engineering design from two approaches. First, a basic FoS where the expected strength is divided by the expected load and if that ratio is greater than 1.0 the design should be sufficient. Then, the same was done for the empirically derived ARMPS2010 SF which has the same Mark-Bieniawski strength as the FoS calculation, but accounts for pressure arching based on the panel dimensions.

Recall that all of the panel pillars in the 300 and 450 m deep models, irrespective of DFN, are considered to have reached their ultimate strength. ARMPS2010 considers “squeezes” and “bursts” as failed cases when applying the Mark-Bieniawski pillar strength equation, and the panel pillars modeled herein have met that threshold regardless of the potential for strain-hardening behavior. Therefore, it is necessary that the modeled FoS of the panel pillars were < 1.0. Conversely, all of the panel pillars in the 150 m deep development cases were successful at
Table 5.4: Comparison of unstable and stable panel pillars under various development loads to the state-of-practice pillar design methods. Note that ARMPS2010 SF should be > 1.5 for sufficient design (Mark, 2010). Incorrect design results italicized.

<table>
<thead>
<tr>
<th>Panel Pillar Model Result</th>
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<td></td>
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<td>450</td>
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<tr>
<td></td>
<td>150</td>
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<td>Pillar w/h</td>
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</tr>
<tr>
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<td>3</td>
<td>6</td>
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<tr>
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<td>7</td>
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<td></td>
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<tr>
<td></td>
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<tr>
<td>ARMPS2010 SF</td>
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<td>1.72</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>3.03</td>
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<td></td>
<td>2.66</td>
<td>5.80</td>
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Analytically determined FoS indicated that the modeled pillars under 300 m development loads should have been safe and the 450 m deep development load models agreed with FoS calculations. However, the ARMPS2010 SFs were both > 1.5 (i.e. the recommended design value to ensure stability). Conversely, the pillar w/h = 1, 150 m deep cases were predicted to be unstable by both FoS and ARMPS2010 SF calculations, and although some pillar yield occurred (see Figure 5.25), the entries remained stable and global support was maintained.

In the case of traditional FoS, the denominator (i.e. pillar loading) was increasingly overpredicted by TAT calculations with increasing depth and decreasing pillar w/h. This reduced the overall FoS such that it was accurate for the 450 m, pillar w/h = 3 case, but inaccurate for both the 300 m pillar w/h = 3, and 150 m pillar w/h = 1 cases. This was reinforced by the numerator (i.e. pillar strength) being more accurately predicted with increasing depth and lowering the error when compared to deeper modeled cases.

Even though the ARMPS2010 Fpa calculation (Eqn. 4.4) reduced the variability in pillar load error with changing panel w/h and pillar size, it more significantly underpredicted the pillar loads (i.e. denominator) than TAT, leading to a significant overprediction of stability in both deeper cases and a slight underprediction of stability for the 150 m deep pillar w/h = 1 cases.
5.5.4.5 Analytical and Empirical Method Discussion

Based on the model results, it is clear that state-of-practice methods account for the most significant controls on pillar behavior: vertical stress magnitude, pillar geometry, and panel geometry. However, when controlling for depth and pillar size, significant variation in both pillar loading and pillar failure strength have been documented for individual (i.e. discrete) parameter variations at the panel-scale.

TAT generally overestimated production loads (i.e. conservative), and MB generally overestimated pillar strength (i.e. optimistic) relative to the model results. These types of errors have the potential to result in a pillar factor of safety that is accurate due to the errors approximately offsetting one another, even though the mechanics of the system have been significantly simplified. A potential case where this could lead to improper design is highlighted by considering a simple example: a wide panel, where TAT loads are accurate to within 2% (see Figure 5.42), but the pillar strength has been over-estimated by 24% (see Figure 5.64), artificially inflating the factor of safety. This is even more problematic when looking at the ARMPS2010 Fpa load calculation, which significantly underpredicted pillar loads, albeit more consistently. This artificially inflated the SF such that the 300 m and 450 m development loads were both predicted to be safe.

Lastly, the simplified assumptions of the abutment angle concept were shown to be inaccurate, other than in the case of strong and stiff overburden lithologies, and could not account for mining direction of a panel pillar extraction sequence regardless of stiffness. When applied to uniformly weak lithologies, however uncommon they may be, the significant overestimation of barrier pillar loading proves that the mechanical complexity of the overburden is not being accounted for. Furthermore, the similar values of average left barrier pillar stress (see Figure 5.61) between the weaker and stronger heterogeneous lithologies, despite their significantly different overburden stress distributions and left barrier pillar load distribution (see Figure 5.62), indicate that there may be a non-unique component to the empirical data utilized to develop the abutment angle concept. The stress cells utilized to develop the abutment angle concept could only measure increases in stress magnitude of the barrier pillars. The measured increase was subsequently used to calculate a portion of the overburden deadload that arches to the barrier pillar in a single abutment angle value. Model overburden stress distributions clearly show that the two lithologies have
significantly different distributions of major principal stress, and that a single abutment angle of 21° is only coincidentally resulting in similar estimates of average barrier pillar vertical stresses.

5.6 Conclusions

The results of this chapter were used to quantify and summarize the combined influences of various geologic and mining conditions that have been historically neglected in pillar design and not previously considered in the application of the explicit DEM. The methodical increase in model complexity presented in this chapter has provided a reliable and repeatable set of results regarding pillar-overburden interaction. Future development of broadly applicable analytical-empirical methods must account for the mechanical influence of overburden properties. Namely, those that control local roof stability, and therefore panel pillar stability (see Chapter 4), and those that control overburden stress arching identified in this chapter.

First, it is clear that the strength of the pillar contact controls how the pillar interacts with the overburden and underburden, and therefore significantly influences pillar strength and post-peak behavior. This phenomenon was encountered throughout this chapter through changes in discontinuity strength. It is also apparent that the properties that affect global overburden stress arching and local roof stability, such as block material properties, cross-joint orientation, in-situ stress magnitude and ratio, all impact pillar loading and pillar strength. In particular, the stability of both the immediate roof and floor, the nature of gob bulking, and the stress arching capacity of the overburden are all critical system components. Although the contribution of analytical and empirical methods to the state-of-practice pillar design is significant, this chapter has shown that pillar design methods that utilize TAT, ARMPS2010 Fpa, abutment angle concept, and Mark-Bieniawski pillar strength equation with no consideration of roof and overburden properties have room for significant improvement to increase their applicability and the accuracy of their equations.

Stability of both the roof and pillar depends highly on the load transfer between the two systems in order to maintain local and global stability throughout a mine. Fortunately, the roof and overburden parameters that decrease the strength of the pillar also tend to promote stress transfer through the roof (i.e. lower pillar loads), and vice versa. However, the research presented herein has shown that the geologic and mining conditions considered result in overly optimistic pillar FoS and ARMPS2010 SF due to the significant overprediction of pillar strength at failure with
decreasing depth, and conservative design values through underprediction of pillar strength with decreasing pillar w/h.

Yield, failure, and collapse in the overburden, underburden, and pillar have been addressed in this chapter. Similar to the results of Chapter 4, at the panel-scale, yield can initiate in either the roof or the pillars depending on the state of stress and material properties. A stronger roof is likely to induce additional pillar yield. Global failure at the panel scale initiates in the pillar, but the peak strength and post-peak behavior are governed by the loading conditions as dictated by the roof and floor. This means that the question of which component is critical to system stability can be answered when the scale of the system and the material strength contrast between the components are well-defined. When the pillar, roof, and floor are of similar strength and stiffness, special consideration of the geologic and mining conditions that affect stress transfer across pillar interfaces must be undertaken.

A parametric analysis of panel-scale models was conducted, and it was determined that TAT consistently overestimated the stresses within the panel pillars. The analysis was conducted with multiple roof and overburden properties and input parameters such as panel width, roof lithology, explicit DFN geometry, pillar properties, in-situ stress ratio, and mining depth. Changes in depth and pillar w/h had the most significant impact on TAT error, with increasing depth and decreasing pillar w/h and panel width resulting in greater error. The roof and overburden properties that most significantly impacted TAT error were decreases in roof strength and increased horizontal stress, which increased TAT error. Assuming overburden dead-loads overestimated panel pillar loads by up to 50% across the range of conditions considered in this chapter.

Abutment side-loading of barrier pillars was thoroughly investigated from the same set of model results. Most notably, the fully excavated model results showed significant differences in side-abutment loads for the left and right barrier pillars due to the influence of mining direction, changes in block material strength, and heterogeneity in the explicit DFN gob (i.e. discontinuum overburden) and continuum overburden. Model inputs that increased or decreased the overburden’s capacity for stress arching significantly impacted the accuracy of the recommended 21° abutment angle. Furthermore, the DFN geometry had a non-negligible impact on the mean difference between left and right barrier pillar loads due to the variability in the degree of explicit
bulking. This was the opposite case under stable panel pillar loads, where DFN had almost zero impact and promoted more even panel pillar loading.

Pushing pillars to post-peak behavior by applying a stress boundary condition to the top of the model (simulating loading by out of plane depillaring) allowed for similar analysis of model geometry, as well as roof and overburden properties on Mark-Bieniawski pillar strength predictions. Panel width and pillar stiffness had the largest impact on Mark-Bieniawski error, but high horizontal stress and the strength and stiffness of the overburden and roof blocks also had significant impacts. Interestingly, a heterogeneous but generally strong overburden with a moderate strength immediate roof (“Lithology 2”) increased the Mark-Bieniawski error, while a homogeneously strong roof and overburden, or the presence of a single strong bed in the roof or overburden, led to pillar strength results closer to the Mark-Bieniawski pillar strength estimation. This indicates that accurately capturing heterogeneous stratigraphy is critical in estimating panel pillar strength. Ultimately, it was found that the Mark-Bieniawski pillar strength equation can overestimate panel pillar strength by up to 25%, which agreed well with the error found in less complex loading conditions analyzed in Chapter 4 (i.e. no floor yield, effectively infinite panel width).

Finally, the results were interpreted with respect to panel pillar FoS and ARMPS2010 SF and found that even in the single parameter variations tested at the panel scale, significant overprediction and underprediction of FoS and ARMPS2010 SF occurred in relationship to model pillar stress results. When calculating a traditional FoS using TAT, w/h = 1 pillars were incorrectly predicted to fail at 150 m development loads due to an overly conservative pillar strength and overpredicted pillar load. Similarly, ARMPS2010 SF had the same conservative strength, but underpredicted the pillar load, resulting in an overly conservative SF. For unstable panel pillars under 300 and 450 m development loads, the opposite was true, where both FoS and ARMPS2010 SF were optimistic due to an overprediction of pillar strength and loading by TAT, and underprediction of loading from ARMPS2010.
CHAPTER 6
CONCLUSIONS

This thesis methodically examined the impacts of explicitly represented discontinuities on roof and overburden self-supporting capacity in complex and realistic geologic and mining conditions at multiple scales. Roof and overburden mechanical interaction with barrier and panel pillars was also documented and state-of-practice methods for pillar and roof design were evaluated. The results of the previous chapters as well as the overarching themes, contributions, publications, and recommendations for future work resulting from completion of this thesis are summarized in the following sections.

6.1 Chapter-Specific Conclusions

6.1.1 Chapter 2: Expanding Applicability of the Voussoir Beam Analog

- Factor of safety against crushing failure \( \text{FoS}_{\text{crushing}} \) of inelastic voussoir beams is controlled by the post-peak behavior of the modeled material. In particular, the failure of the midspan controls beam stability in explicit DEM models. A 0.6- or 1.25-times adjustment should be made to Eqn. (2.38) for brittle and perfectly plastic post-peak behavior end-members, respectively, even when using field-scale unconfined compressive strength (UCS*).

- Statistical analysis of 810 unique numerical models that explored the behavior of horizontally layered and passively bolted voussoir beams were used to develop the layer-adjusted rockmass modulus \( E_{\text{nm}} \) and effective thickness methods to increase the accuracy of the baseline Diederichs & Kaiser (1999) analytical solution in predicting displacement and midspan stresses under these conditions.

- The adjusted analytical method was implemented in a case study of the Bondi Pumping Chamber located in Sydney, New South Wales, Australia. The adjusted analytical solution was able to accurately account for the range of reported roof deflection from extensometer measurements, and predictive (i.e. uncalibrated) numerical models were also able to capture the range of reported displacement. Notably, bolt pre-tensioning and shotcrete had minimal impacts on roof displacement and stress arching, indicating that the adjusted method may also be applied in some cases involving pretensioned fully grouted rockbolts. However, this requires further research to confirm.
6.1.2 Chapter 3: Parametric Sensitivity Analysis of Roof Stability in Single-Entry Models

- Roof self-supporting capacity in single entries was confirmed through unsupported model cases and a wide range of geologic and mining conditions were verified as realistic through the application of the Coal Mine Roof Rating (CMRR) (Molinda & Mark, 1994) and the Analysis of Roof Bolt Systems (ARBS) (Mark et al., 2001) empirical methods to model inputs and model results. The ARBS method was able to predict the unsupported model outcomes with a high degree (i.e. > 90%) of accuracy based on model-derived values of CMRR, bolt intensity, depth to entry, and intersection span. Discrete fracture network (DFN) random seed (i.e. positioning of specific joints within the relatively uniform networks tested) was determined to have a limited impact on roof stability.

- The ARBS method applied to bolted models had decreased accuracy (i.e. > 80%) due to the “Weak SUBI” block material models requiring additional (i.e. non-bolt) support. This weak roof behavior was noted by Mark et al. (2001) in some of the ARBS case studies and further confirmed the realism of the model results.

- Binary logistic regression (BLR) analyses confirmed that CMRR and ARBS broadly account for the most significant controls on roof stability (i.e. depth, rockmass rating). The significance level of other geologic and mining considerations is highly dependent on the in-situ stress magnitude to roof block material strength ratio and the presence of roof support. The most significant controls include modeled k_o-ratio and the combined impact of cross-joint orientation (i.e. vertical vs. sub-vertical) and cross-joint spacing and persistence.

- BLR verified that under most modeled conditions, the effect of joint initial friction angle on unsupported and bolted roof stability was statistically insignificant. This is preliminarily attributed to the excavation shape and effective roof stress arching muting the effects of weaker joints once sliding initiates. In roofs that do not effectively arch stresses (i.e. weaker block material), a stronger joint eventually shears due to loss of confining stress, and the impacts of joint strength are therefore limited.
6.1.3 Chapter 4: Analysis of Pillar-Roof Mechanical Interaction Governing Local Stability in Single Entries

- Roof stress arching (i.e. roof stability) and pillar confinement (i.e. pillar stability) were identified as mechanically linked through the pillar-roof interface; these can work in tandem or independently to govern the local stability of a given entry. All the tested parameters were broadly classified into internal and external controls on pillar loading, and internal and external controls on roof stress arching capability. All internal controls on roof stress arching capacity (i.e. vertical vs. sub-vertical cross-joint orientation) are also external controls on pillar loading, and vice versa.

- Across all modeled depths, the combined influence of block material model, k₀-ratio, and cross-joint orientation was prevalent. Namely, maintaining roof stability is a key component of preventing the decay (i.e. high in-situ horizontal stress), or maintaining the generation (i.e. low in-situ horizontal stress) of pillar confinement and load bearing capacity. The stiffness of the overburden became increasingly important with depth in limiting the vertical stress and strain imparted to the panel and promoting entry stability. Additionally, the presence of sub-vertical cross-joints resulted in differential stress transfer to pillars on either side of the excavation seen in all pillar conditions and roof block properties modeled, but was exacerbated by inelastic roof block material and high horizontal stresses. Peak average pillar stress and associated pillar strain were controlled by roof block stiffness and the modeled k₀-ratio. Post-peak behavior was heavily influenced by both the roof block material strength and stiffness. However, under the same roof block material, the equilibrium strain of the pillar was controlled by the k₀-ratio.

- Bolt mechanisms (i.e. suspension and beam building) were identified and classified based on their impact on pillar and roof yield. Furthermore, supported roofs were observed to result in significantly higher post-peak pillar load carrying capacity, and reduced unconfined pillar yield towards the pillar core.

- Conservativism of TAT increases significantly with decreasing pillar w/h, increasing depth, and inelastic material yield, particularly when full overburden stress arching is simulated with a model boundary condition. The Mark-Bieniawski pillar strength equation (Mark & Chase, 1997) was demonstrated to provide optimistic strength estimates as compared to cases modeled with realistic pillar interfaces and “Soft Elastic” roof blocks.
Increased pillar deformability through decaying horizontal stress, or reduced stiffness of the pillar material reduced the ultimate strength of the pillars in single-entry models.

6.1.4 Chapter 5: Analysis of Pillar-Overburden Mechanical Interaction Governing Global Stability in Panel-Scale Models

- A calibrated SUBI continuum overburden representation from Tulu et al. (2017) was modeled with various explicit gob properties and compared to empirical subsidence results for similar multiple panel width-to-height ratio (W/H) with reasonable agreement.
- A sensitivity analysis testing select combinations of geologic and mining conditions with particular interest in explicit gob DFN representation was conducted. The interaction between roof stress arching (i.e. roof stability), overburden stress arching, and both panel and barrier pillar loading and strength were further confirmed as mechanically linked through their explicit representation in 125 unique-panel scale models. Results confirmed that local roof stress arching and panel-wide overburden arching are mechanically coupled and significantly impact pillar loading and failure.
- Under stable development loads, multiple mechanisms observed in single-entry models carried through to the panel scale, including higher axial pillar strain under elevated horizontal stress, increased roof yield with less deformable pillars, and less effective roof stress arching in sub-vertical joint DFNs.
- Panel pillar stresses remained relatively consistent independent of DFN geometry and panel pillar location. However, with increasing in-situ stress magnitude, pillar peak average stresses increased due to increasing initial confining stress. Furthermore, observation of slender pillar strain-hardening behavior was observed following pillar “punch-through” failure and subsequent confinement from the heaved floor blocks. However, pillars were considered failed and compared to the Mark-Bieniawski pillar strength equation based on the significant levels of pillar strain (i.e. > 6%) and functional failure of each entry.
- As expected, with decreasing panel W/H and pillar w/h, TAT stress overprediction increased. However, changes in k_o-ratio, overburden lithology, and overburden strength and stiffness also had some impact. The TAT assumption of overburden dead-loads overestimated panel pillar loads by up to 50% across the range of conditions considered in this chapter. However, TAT was accurate (< 5% error) at panel W/H = 0.6, far lower than
previous continuum modeling (Salamon, 1992; Yu et al., 2018; Dean-Pelikan & Walton, 2020) suggests.

- ARMPS2010 pressure arch factor (Fpa) loading decreased the variability of stress error found in TAT predictions with pillar w/h and panel W/H, but still failed to account for the mechanical influences of other significant inputs. Furthermore, it consistently underpredicted panel pillar stresses due to the overall lack of consideration of overburden properties in the ARMPS2010 equations.

- In-plane depillaring model results indicated overburden stress arching and sequential pillar excavation had large impacts on barrier pillar loads that could not be captured by the recommended 21° abutment angle (Mark, 1989; Tuncay et al., 2019). Most notably, the fully excavated model results showed significant differences in side-abutment loads for the left and right barrier pillars due to the influence of mining direction, changes in block material strength, and heterogeneity in the explicit DFN gob (i.e. discontinuum overburden) and continuum overburden.

- Mark-Bieniawski pillar strength overpredicted pillar failure strength by an average of 15%, by underpredicting the strength of slender pillars and pillars in k_o = 0.5 models. Ultimately, it was found that the Mark-Bieniawski pillar strength equation can over-estimate panel pillar strength by up to 25%, which agreed well with the error estimates obtained for the less complex loading conditions analyzed in Chapter 4 (i.e. no floor yield, effectively infinite panel width).

- FoS and SF calculations indicated that both state-of-practice analytical and empirical pillar design methodologies overpredicted the stability of unstable pillars, and underpredicted the stability of stable pillars in the model cases presented herein.

6.2 Synthesis of Conclusions

6.2.1 Roof & Overburden Self-Stability, Supported Stability, & Stress Arching

TAT relies on simplifying assumptions that generally represent two extremes: the overburden simultaneously has no self-supporting capacity and is fully continuous with an infinite array of pillars below. Both assumptions ignore the stress arching that has been the focus of this thesis and a fundamental principal of solid mechanics.
PAT, and more specifically ARMPS2010 Fpa, as well as the abutment angle concept, attempt to correct for the errors inherent in TAT by adjusting the first assumption based on empirical evidence from various Eastern and Western US coal mines. Similar methods have been successfully implemented across the world and have undoubtedly increased the safety of underground coal mining globally. However, the mechanical relationship between overburden stress arching capacity and geomechanical properties of discontinuous and layered rockmasses is lost in the application of these methods to a wide range of geologic and mining conditions, and results in uniform design guidelines being used under conditions that are even more complex than the models presented in this study. This thesis has identified that stress arching and pillar confinement are mechanically linked and dependent on each other to maintain local and global stability. Critical combinations of overburden and pillar parameters identified in this study and others can help focus future efforts on refining analysis of flat-roof excavations in both mining and civil applications. Furthermore, use of the voussoir beam analog as shown in Diederichs & Kaiser (1999) in more complex conditions, without accounting for that complexity also relies on a conservative approach (i.e. design FoS = 1.5-2.0). This thesis has developed and verified adjustments to increase the applicability and accuracy of the voussoir beam analog.

Obviously, not all flat-roof excavations will have self-supporting or stress arching capacity in the immediate roof, but those that do are controlled by various parameters depending on the observation scale. In shallow flat-roof excavations for which the adjusted voussoir beam analog is applicable, span, thickness, intact material stiffness, post-peak material behavior, number of bolted layers, joint orientation, joint strength, abutment compliance, and horizontal stress have been identified as the critical considerations controlling local stability.

In shallow to moderate depth flat-roof excavations with variable geologic and mining conditions and negligible pillar deformation, local roof self-stability requires two conditions to be met. First, the roof must be able to transmit excavation-induced horizontal stress and second, it must be able to maintain that induced horizontal stress. This simply requires material and lamination strength to be higher than excavation induced loads, and predominantly vertical cross-jointing to transfer stresses effectively. Notably, joint strength plays a limited role in the presence of maintaining stable stress arching under these conditions. The addition of passive roof support decreases the role of joint strength and diminishes the effect of depth by increasing the ability for the roof to maintain and arch stresses. This is accomplished through three mechanisms: increasing the
effective thickness of each roof layer and reducing the stress concentrations in individual layers, taking on load in the bolts themselves, and reducing material yield by preventing stress-rotation-induced buckling and deconfinement. However, the addition of bolts can increase the impact of discontinuity persistence and spacing as unsupported blocks can fall from between roof support elements now that stress arching is transferred deeper into the roof and persistent joints can extend beyond the reach of standard installed support.

In shallow to deep flat-roof excavations with variable geologic and mining conditions and considerable pillar deformation, local roof stability requires two conditions to be met. It is clear that the pillar(s) supporting the roof must not collapse (i.e. global instability), but it must also deform in a manner that does not impart horizontal stress in excess of the roof’s supported stress bearing capacity, or induce buckling (spans > 30 m in Shabanimaschool & Li, 2015). This requires the pillar to maintain some in-situ confinement or to release it in a controlled manner, which in turn relies on the roof to transmit stresses effectively and the pillar interfaces to facilitate that stress transfer.

Overburden stress arching similarly depends on panel dimensions (i.e. span), the heterogeneity in strength and stiffness of the overburden material, the bulking behavior in the event of depillaring, and to a lesser extent, \( k_o \)-ratio. While single strong beds modeled in the overburden did facilitate more overburden stress arching, this did not result in purely positive effects and induced significant yield in the overburden above the barrier pillar core following pillar extraction. Depillaring directionality also impacts the overburden stress arching response and barrier pillar loads inby and outby the mining direction vary significantly depending on the material contrasts of a given lithology.

6.2.2 Pillar Confinement Controls

First, the influence that the pillar-floor and pillar-roof contact strength has on pillar strength and global stability cannot be overstated. It is well-documented in the literature and was explored in Chapter 5 of this thesis. The strength of the immediate roof contact had notable impacts on both stable pillar loading and pillar peak strength. It also controls pillar confinement, and therefore, pillar deformability, strength, and post-peak behavior. This phenomenon was encountered throughout this study at all model scales (i.e. entry to panel) when considering both inelastic and
elastic roof and floor block material. Future studies investigating pillar strength should always explicitly account for interface strength.

Roof and overburden self-supporting capacity and stability, as dictated by the strength and stiffness of intact materials, DFN properties, and in-situ stress magnitude and orientation, all impact pillar loading and pillar strength to varying degrees. To that end, the research presented herein has shown that the geologic and mining conditions considered resulted in overly optimistic or overly conservative pillar FoS and ARMPS2010 SF.

6.2.3 Global and Local Stability

Yield, failure, and collapse in excavation roofs and pillars have been addressed in this thesis. At both the entry (local) and the panel-scale (global), yield can initiate in either the roof or the pillars depending on the state of stress and material properties. A stronger roof is likely to induce additional pillar yield. Global failure at the panel scale initiates in the pillar, but the peak strength and post-peak behavior are governed by the loading conditions as dictated by the roof and floor. This means that the question of which component is critical to system stability can be answered when the scale of the system and the material strength contrast between the components are well-defined. When pillar, roof, and floor are of similar strength and stiffness, special consideration of the geologic and mining conditions that affect stress transfer across pillar interfaces should be conducted. While a universal mathematical relationship between these parameters and local or global stability cannot be developed from such a hypothetical, generalized study, the knowledge gained has the potential to guide future numerical, laboratory, and in-situ investigation towards the most critical aspects of mine stability that have been explored through the course of this thesis. It is clear that representing discontinuous systems explicitly has led to multiple academic and practical contributions regarding the study of flat-roof excavation stability.

6.3 Practical Implications

Although a “unified theory” of flat-roof excavation mechanics may never be fully realized and the voussoir beam analog and roof stress arching are not applicable to every flat-roof excavation, the findings documented herein have significant implications for practical implementation. This section briefly outlines the implications of this study for practicing mining and civil engineers.
In this thesis, the voussoir beam analog has been advanced beyond the critique that a “perfect knowledge of a perfect rockmass” (Oliveira & Pells, 2014) is required for its implementation. In particular, displacement and maximum horizontal stress in regularly bolted roofs in laminated and discontinuous rockmasses can now be calculated. First, based on rockmass and material observations and the in-situ stress state, one must determine if the roof has self-supporting capacity and will form a voussoir beam. This is largely based on engineering judgement coupled with the limitations described in Section 2.8 and Diederichs & Kaiser (1999). If the in-situ conditions indicate that low-confinement stress arching is a likely mechanical response, the method described in Section 2.8 can be applied to more accurately evaluate beam behavior than using the baseline (Diederichs & Kaiser, 1999) analytical method. However, note that the proposed method should be applied with caution until further independent validation can be conducted by others in future studies. Furthermore, note that the adjusted method was developed for implementation in single excavations; however, if the horizontal stress state between multi-entry systems is well-constrained, this method can be applied accurately.

Practitioners who implement the CMRR-ARBS empirical methods to determine required bolt intensity and intersection span with poor results, particularly when implemented in conditions with sub-vertical cross-joints, variable joint strength, and variable $k_o$-ratio, may consider how those conditions can impact the accuracy of CMRR and ARBS. Specifically, if the range of values of the cohesion-roughness and spacing-persistence ratings are decreased in a roof with purely vertical cross-joints, the range of CMRR values will be smaller and result in less variation in suggested bolt intensity and intersection span values. This is effectively accounting for the increased roof stress arching capacity and self-stability. If a negative adjustment is applied based on the mean and standard deviation of cross-joint dip, CMRR will decrease, increasing the suggested bolt intensity and decreasing the suggested intersection span. Finally, directly decreasing the suggested intersection span or increasing the suggested bolt intensity based on $k_o$-ratio can account for the increased yield in moderate to weak roof cases.

Those who implement other empirical rockmass rating and support requirement methods can use numerical models to methodically evaluate their accuracy when applied to conditions outside of the cases used to develop them. The methodology and findings in Chapter 3 can be used as a guideline for developing such a site-specific modeling program.
Those who implement TAT or PAT assumptions in designing panel pillars, ARMPS and ALPS in depillaring and longwall extraction design, or the Mark-Bieniawski pillar strength equation may consider adjusting their calculated FoS or SF to account for pillar interface strength, local roof stress arching capacity, and global overburden stress arching capacity. Particularly in extremely weak lithologies where loading conditions more closely resemble TAT than PAT and the Fpa in ARMPS may underpredict production pillar loads, extremely strong lithologies where TAT significantly overpredicts pillar loads, extremely weak pillar interfaces where pillar strength is significantly overpredicted, and extremely strong pillar interfaces where pillar strength is significantly underpredicted. However, these methods will largely result in a decrease in conservativism and should be applied with caution until independent verification by others can be conducted.

6.4 Future Work

Suggested future work based on the findings presented in this thesis includes the following:

- Expand the adjusted voussoir beam method to account for decreased stress arching due to sub-vertical joints and continue to verify its applicability through comparison to more case studies. Identify the interaction between in-situ vertical and horizontal stress, as well as excavation-induced stress on complex voussoir beam behavior.
- Re-run and analyze the suite of horizontally bedded and jointed voussoir beam models with inelastic blocks, sub-vertical joints, horizontal stresses, and tensioned rockbolts to evaluate the explicit impact of even more complex loading conditions on the adjusted analytical method.
- Apply the adjusted voussoir beam analog to supported model results from Chapters 3, 4 and 5. Classify each type of roof failure and identify if the adjusted voussoir beam analog can predict the buckling, crushing, or abutment slip failure of significantly more complex loading conditions.
- Analyze single-entry roof stability models with larger spans and various degrees of bolting and other support types to evaluate the CMRR and ARBS methods under an even wider range of conditions. Utilize model results to develop an ARBS method that can be applied for multiple types of roof support and accounts for roof self-supporting capacity.
Conduct a thorough re-evaluation of the CMRR based on the results of the BLR in Chapter 3. In particular, evaluate the importance of the cohesion-roughness rating in flat-roofs, utilize remote sensing techniques to determine joint set orientations and if sub-vertical joint orientation should be accounted for, and weight joint persistence based on its orientation and relationship with the bolted interval (i.e. do persistent joints exceed the height of installed support?).

Evaluate the ability of machine learning (e.g. BLR) to extrapolate stability prediction to untested cases by assuming additional model cases between tested, known outcomes. Analyze more model cases and develop logistic regressions for continuous variables such that stability probability equations can be developed for a wide range of geologic and mining conditions and shared broadly.

Investigate the moisture sensitivity CMRR rating’s impact on roof stability through use of the hydromechanical coupling in the explicit DEM on select Chapter 3 models.

Identify specific roof failure type with a more robust classification of model results, and test multiple ARBS discriminants to delineate between different types of roof failure.

Utilize the bonded block method to explicitly model calibrated pillars in Chapter 3, 4 and 5 models to capture the effects of pillar rib bulking and analyze state-of-practice rib support guidelines.

Rigorously evaluate the impacts of material stiffness, strength, post-peak behavior, and anisotropy on roof stress arching through use of SUBI, EBP, and elastic material properties by conducting a full parametric sensitivity analysis of various stiffnesses, inelastic material properties, and SUBI material in Chapter 3 and 4 models.

Apply the adjusted voussoir beam analog to determine if the span length to first caving of depillaring models can be accurately predicted. Utilize elastic blocks to evaluate buckling and sliding failure; utilize existing model cases to evaluate crushing failure.

6.5 Contributions

The most significant contributions of this thesis are summarized as:

1. Development and validation of the adjusted voussoir beam analog which incorporates inelastic post-peak behavior, horizontal stress, and passively bolted horizontal layers to
accurately account for displacement and stress transfer under more realistic roof conditions than the baseline Diederichs & Kaiser (1999) analytical solution.

2. Verification of five discontinuum rockmass analogs representing a wide range of realistic sedimentary rock behavior within the context of the CMRR and ARBS empirical systems.

3. Identification of the explicit DEM parameters that control self-supporting capacity and bolted stability, including a robust analysis of the impact of DFN geometry and discontinuity material properties.

4. Identification of the mechanical links between pillar and overburden, and the critical parametric combinations that impact local and global stability.

5. Confirmation that although the strength of empirical and analytical methods is not in the accuracy of their equations or assumptions, and while they are indeed accounting for the most significant impacts that govern pillar loading and strength, the path to improving their accuracy and applicability lies in the mechanical coupling of the pillars and overburden.

6.6 Thesis-Specific Publications

6.6.1 Journal Articles – Published


6.6.2 Journal Articles – Submitted or Under Preparation


6.6.3 Fully Refereed Conference Papers


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APPENDIX A: ALTERNATIVE SUPPORT IMPACT ON BONDI PUMPING CHAMBER NUMERICAL MODELS

Due to the uncertainty related to the effects of support type (i.e. passive vs. pretensioned) bolts and support timing installation on potential excavation behavior, additional model cases were considered to determine their impact. In particular, the 3 m thick, short bolt, strong discontinuity model described in Section 2.9.5 is presented in this appendix with two variations. The first model case progresses as follows:

1. 4 m central heading excavated
2. 70% stress relaxation along internal excavation boundary
3. Install 3, 2.9 m long, passive roof bolts on approximately 1.2 m spacing
4. Remove internal stress boundary and solve to an equilibrium solution ratio of $1.0(10)^{-5}$
5. Set model displacement to 0 and excavate full 12.5 m span
6. 70% stress relaxation along newly excavated internal boundary
7. Install remaining 8 passive roof bolts, 4 supplemental corner bolts, and apply 50 mm of shotcrete using parameters from Chryssanthakis et al. (1997)
8. Remove internal stress boundary and solve to an equilibrium solution ratio of $1.0(10)^{-5}$
9. Bench the excavation down to its final height of 19 m and solve to an equilibrium solution ratio of $1.0(10)^{-5}$

This support installation timing will provide a lower-bound prediction of roof displacement, since it assumes all supplemental support is installed at the same time as the initial passive bolt support for the full excavation span. Any delay in the installation of supplemental support relative to this case can only result in higher displacements, closer to those presented in Chapter 2.

The addition of the supplemental support decreased model displacement by 0.4 mm (i.e. 9%) and model maximum midspan stress by 0.1 MPa (Figure A.1). This indicated that the presence of supplemental corner support and 50 mm of shotcrete had little impact on the model roof behavior.
Figure A.1: Bondi pumping chamber numerical model (a) vertical displacement and (b) horizontal stress results for the lower-bound 3m thick roof featuring stronger and stiffer discontinuities with supplemental corner bolts and 50 mm shotcrete liner.

The second model variation used an identical excavation and support sequence to that presented in Chapter 2; however, bolts were assigned a pretension force of 60 kN in accordance with Henderson & Windsor (1988). Itasca’s universal distinct element code (UDEC) version 6.0 does not have built-in option to assign pretension to rockbolt structural elements as it does with cable structural elements. Therefore, a method was developed by which pretension could be assigned using FISH code.

Bolts and faceplates were installed as described in Section 2.9.3, but the common node that the bolt and faceplate shared was not attached using the connect command (Figure A.2). The 70% internal stress boundary was removed, and a zero-velocity internal boundary condition applied. Then the element (i.e. member between two nodes) between the faceplate and the second rockbolt node was deleted and a downward axial force of 60 kN was applied to the remaining rockbolt node. The model was solved to an equilibrium solution ratio of $1.0 \times 10^{-5}$ and the bolt axial force was checked to ensure it was approximately 60 kN. Then the axial force was removed from the node and the faceplate node and bolt node were reconnected with a new element; the faceplate was deleted and reinstalled using the “connect” command to attach the faceplate to the pretensioned bolt. The internal zero-velocity boundary condition was removed and the model solved as it was in Chapter 2.
Figure A.2: Depiction of bolt pretensioning processes implemented in UDEC.

This was repeated when the excavation was widened. Comparing bolt axial tension between the passive and pretensioned models at model equilibrium indicates that the pretensioning method developed was effective at maintaining bolt pretension as shown in Figure A.3, even though values decayed from 60 kN, which is a realistic behavior observed in pretensioned cable bolt elements (Itasca, 2014).

Figure A.3: Comparison of bolt axial load at final model equilibrium for (a) pretensioned bolt model and (b) passive bolts.
The use of pretensioned bolts decreased model displacement by 0.4 mm and model maximum midspan stress by 0.1 MPa (Figure A.4). This indicated that due to the competent nature of the rock and the low stress magnitudes, bolt pretensioning had a limited impact on roof behavior and the applicability of the adjusted analytical method.

Figure A.4: Bondi pumping chamber numerical model (a) vertical displacement and (b) horizontal stress results for the lower-bound 3m thick roof featuring stronger and stiffer discontinuities with 60 kN pretensioned bolts.
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